The exact nature of dynamic traffic assignment (DTA) equilibrium is not fully known in simulation-based models. Universal solutions for general networks may not exist and multiple equilibria are possible. This is problematic for transportation practitioners since projects are evaluated at a unique equilibrium state. By formulating traffic assignment as a large-scale game, techniques and literature from game theory can be applied to address these equilibrium issues. Two network examples are presented: one displays a scenario where no DTA equilibrium exists and the other showcases a scenario with multiple equilibria. The first network is shown to have a mixed strategy Nash equilibrium. In the second network, the amount of multiple equilibria is reduced by applying the trembling hand refinement from game theory.

A game is characterized by three elements: (1) set \( I \) consisting of all entities/players, (2) a set of actions/strategies \( A_i \) for every \( i \in I \), and (3) the utility or satisfaction \( u_i \), player \( i \) will expect from the given set of strategies, \( u_i : A \rightarrow \mathbb{R} \). The most widely used and recognized notion of an equilibrium state is the concept of Nash equilibrium. Nash equilibrium can be categorized into two basic types of equilibrium: pure and mixed strategy. Pure strategy Nash equilibrium can be defined as the stable state where no player can improve his/her utility by changing strategies. It is expressed formally below, where \( a_i \) indicates the actions of all players except player \( i \) and \( q_i \) represents all other strategies available to player \( i \) besides \( a'_i \).

\[ u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i}) \quad \forall i \in I \]

A game can have multiple pure strategy Nash equilibria or none at all. However, any game with a finite set of players and a finite set of actions is guaranteed to have a mixed strategy Nash equilibrium. In a mixed strategy solution, players are allowed to randomize among their various actions - to choose a probability distribution over their strategy set as to maximize their expected utility. Consider the network below. Vehicle 1 travels from A to B. Vehicle 2 travels from C to D.

There are three user equilibrium solutions associated with the network shown below: (1) Vehicle 1 chooses the left path and Vehicle 2 chooses the top path, (2) Vehicle 1 chooses the left path and Vehicle 2 chooses the bottom path, and (3) Vehicle 1 chooses the rightmost path and Vehicle 2 chooses the bottom path. These solutions correspond to three pure strategy Nash equilibrium points.

By applying the concept of trembling hand perfect equilibrium to the network, weakly dominated strategies can be eliminated. This includes the user equilibrium associated with Vehicle 1 using the rightmost path. Vehicle 1 should only choose the rightmost path if Vehicle 2 chooses the top path 100% of the time; the travel time on the left path will always be less than or equal to the travel time on the right path, and so forth ad infinitum. There is no user equilibrium solution. However, there is a mixed strategy Nash equilibrium: Vehicle 1 will choose the left path 50% of the time, and Vehicle 2 will choose the top path 50% of the time. This added information from game theory can aid practitioners; this fixed ratio of departure can be used to approximate traffic flows, leading to other indices of interest (e.g., crash rates).