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# FLEXURAL FATIGUE DURABILITY OF SELECTED UNREINFORCED STRUCTURAL LIGHTWEIGHT CONCRETES

in cooperation with the Department of Transportation Federal Highway Administration Bureau of Public Roads

RESEARCH REPORT 81-4 STUDY 2-8-65-81 SYNTHETIC AGGREGATE RESEARCH

# FLEXURAL FATIGUE DURABILITY OF SELECTED UNREINFORCED STRUCTURAL LIGHTWEIGHT CONCRETES

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# PREFACE

This is the fourth report of a research study carried out in the Structural Research Department of the Texas Transportation Institute as part of the cooperative research program with the Texas Highway Department in cooperation with the U. S. Bureau of Public Roads. The first three reports are:

"Correlation Studies of Fundamental Aggregate Properties with Freeze-Thaw Durability of Structural Lightweight Concrete," by W. B. Ledbetter, Research Report 81-1, Texas Transportation Institute, August 1965.

"Effect of Degree of Synthetic Lightweight Aggregate Pre-Wetting on the Freeze-Thaw Durability of Lightweight Concrete," by C. N. Kanabar and W. B. Ledbetter, Research Report 81-2, Texas Transportation Institute, December 1966.

"Aggregate Absorption Factor as an Indicator of the Freeze-Thaw Durability of Structural Lightweight Concrete," by W. B. Ledbetter and Eugene Buth, Research Report 81-3, Texas Transportation Institute, February 1967.

The following staff personnel were actively engaged in the study: W. B. Ledbetter, Assistant Research Engineer and Principal Investigator; J. C. Chakrabarti, Research Assistant; Eugene Buth, Research Associate; Horace R. Blank, Research Geologist; and James T. Houston, Research Assistant. In addition, several undergraduate students were employed on various phases.

The authors are indebted to these people for their contribution to this research effort.

The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Bureau of Public Roads.

# ABSTRACT

In this research study, the fatigue durability of concrete was reviewed, a reproducible flexural fatigue test was developed, and the flexural fatigue durability of selected unreinforced concrete structural lightweight concretes was determined. Also, relationships between flexural fatigue durability and freeze-thaw durability of selected concretes were established.

Fatigue tests were conducted on unreinforced concrete prism specimens of three lightweight concretes and one regular weight concrete. Two of the lightweight concretes and the regular weight concrete were tested in a moist condition, and one lightweight concrete was tested in a dry condition. In the fatigue tests, all specimens were subjected to repeated sinusoidal stress cycles at the rate of 697 cpm. The fatigue stress levels were determined as a percentage of moduli of rupture (center point loading) of identical prism specimens. All concretes were cured for a minimum period of 28 days prior to testing. The following conclusions were suggested by this research:

1. The resistance of unreinforced structural concrete to flexural repeated load was dependent on the applied stress amplitude, the variation of log of fatigue life being inversely proportional to the applied stress up to 10 million repetitions of the load for all the concretes tested.

2. Type of coarse lightweight aggregate definitely affected the flexural fatigue behavior of unreinforced structural concrete.

3. Dry concrete exhibited longer fatigue life than wet concrete.

4. A relationship was found to exist between fatigue durability and freeze-thaw durability for the conditions of this study.

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### 1.1 General

The discovery of synthetic lightweight aggregate<sup>1</sup> was an important event in the history of structural concrete. It offered a suitable substitute for natural aggregate wherever natural aggregate was not available locally or within an economical hauling distance. Also, concrete made with lightweight aggregate, termed lightweight concrete, was considerably lighter than that made with natural aggregate and consequently, the dead weight of the component structural members was considerably reduced, causing appreciable saving in the total cost of the structure. Because of these and other advantages, lightweight concretes are being considered with ever increasing favor by the construction industry. Since about 1950, the potentialities of synthetic lightweight aggregates for the manufacture and use of lightweight concrete have become widely recognized and economically attainable. Today, this material is being extensively used for the construction of all kinds of concrete structures including highway pavements, bridges and buildings.

#### 1.2 Problems

Concrete, as a structural material, is almost always subjected to fatigue types of load (in addition to static or dead loads). The ability of concrete to withstand flexural fatigue, termed "fatigue durability" in this research study, has long been of importance to engineers. However, no standard measure of fatigue durability of concrete has been developed, and very little research has been conducted on the fatigue durability of lightweight concrete.

Just as the knowledge of fatigue durability of concrete is important, a knowledge of freeze-thaw durability of concrete is essential for predicting concrete's effective use in structures when subjected to the deteriorating influences of frost and weather. As with fatigue durability, researchers have long been interested in freezethaw durability and have devised test methods to evaluate the effects of freezing and thawing on concrete. But these methods are tedious and often produce conflicting results. Also, very little knowledge has been obtained on the freeze-thaw durability of lightweight concrete. Finally, to these authors' knowledge, no efforts have been made to compare fatigue durability with freezethaw durability of concrete.

#### 1.3 Objectives

The primary objectives of the present research were:

1. To review the literature concerned with the fatigue behavior of regular weight and lightweight concrete.

2. To develop a flexural fatigue durability test for unreinforced structural lightweight concretes.

3. To evaluate the effect of selected coarse light-

weight aggregate types on the fatigue durability of unreinforced concretes made with these aggregates.

4. To investigate the relationship between freezethaw durability and fatigue durability of the selected unreinforced structural lightweight concretes. For this purpose, freeze-thaw test data for similar concretes from prior research were used.

#### 1.4 Scope and Limitations

This research study represents one of the phases of a parent study undertaken in cooperation with the Texas Highway Department and U. S. Bureau of Public Roads.

Three formal reports have been prepared to date. Complete information concerning the overall purpose, scope, limitations, and background of the parent research study is given in these three reports (1, 2, 3).<sup>2</sup>

This study is primarily concerned with the evaluation of flexural fatigue response of unreinforced lightweight concrete. As the literature concerned with the freeze-thaw durability of lightweight concrete has been reported (2), only the literature involving the fatigue durability of concrete will be discussed.

In order to determine the fatigue durability of the concretes, a flexural fatigue durability test was developed and unreinforced concrete prism specimens, made with three selected aggregates, were subjected to repeated center point loading in simple flexure. Additional specimens were cast to determine ultimate static flexural capacity and compressive strength.

Concretes made with two lightweight aggregates and one regular weight coarse aggregate were selected for the study. For each type of concrete, three load levels were chosen for the fatigue durability tests. Depending on the scatter of the results, two to six specimens for each type of concrete were subjected to each fatigue load. Three prism specimens and three cylinders were subjected to static flexural and compressive strength tests, respectively, for each type of concrete.

In order to reduce the number of variables, as well as to isolate the effects of coarse aggregate type from other factors affecting fatigue durability of unreinforced concrete, the following variables were held as constant as possible throughout this study.

1. Cement Factor. Five sacks of cement per cubic yard of concrete were used.

2. Cement Type. Type I cement from the same manufacturer was used.

3. Fine Aggregate Type. All mixes used the same fine aggregate—a siliceous, river-run, regular weight sand.

4. Slump. All mixes were prepared with three to four in. slump.

5. Air. No air entraining admixtures were used.

6. Laboratary Procedures. See Section 7.2 for details.

7. Specimen Size. All prism specimens were cast in  $3 \ge 4 \ge 16$  in. steel molds.

<sup>&</sup>lt;sup>1</sup>For the purpose of this report, synthetic lightweight aggregates are defined as structural quality aggregates produced by fusing and bloating raw shales or clays in a rotary kiln under intense heat into predominantly amorphous silicates.

<sup>&</sup>lt;sup>2</sup>Numbers in parentheses correspond to the reference numbers contained in Section 7.3,

### 2.1 General Remarks

The fatigue behavior of concrete has been a matter of concern to the researchers of concrete since the turn of this century. Many studies had been made and conclusions drawn regarding the fatigue behavior of concrete. In order to have a better understanding of the problem of fatigue of concrete, the results of the investigations of the prior researchers are reviewed. This review spans the literature dealing primarily with the behavior of unreinforced concrete when subjected to both axial and bending repetitive loads. Most of these investigations, as noted below, were concerned with conventional sand and gravel concrete.

## 2.2 Conventional Sand and Gravel Concrete

2.2.1 Fatigue Behavior Under Axial Loads. The first known investigation related to the behavior of the material under axial tensile loads was conducted by De Joly in 1898. Mills and Dawson (4) state that De Joly's specimens were "cement mixtures in tension briquettes." This description does not give definite composition of the concrete. The specimens were two to 20 days old when tested. The loading frequencies ranged from 26 to 92 cpm. Results indicated a fatigue limit of approximately 50 percent of the static strengths of the briquettes. The number of cycles of load sustained prior to failure was inversely proportional to the frequency of application, and rest periods appeared to be beneficial. According to these authors there apparently were no further axial tension fatigue tests of concrete.

There were several investigations, however, of the fatigue behavior of unreinforced concrete in axial compression. In 1903, Van Ornum (5) conducted compression tests on two-inch cubes of neat cement, four weeks old. He also performed similar tests on seven-inch concrete cubes. In both cases his tests indicated a fatigue strength of approximately 55 percent of the static ultimate strength at about 7000 cycles of load. Van Ornum's work was important because it established the existence of the fatigue phenomenon for concrete and recorded the observation of progressive failure.

In 1907, Van Ornum conducted similar tests on  $5 \ge 5 \ge 12$  in. prisms of concrete aged both one month and one year (6). The repeated loads varied from near zero to a maximum value, and were applied at frequencies of two to four cpm. The data obtained from these tests indicated an increase in fatigue strength from that obtained in previous tests for a given number of repetitions of load. This increase may have been the result of the difference in age of the specimens, the use of better cement, or the difference in specimen dimensions.

Following Van Ornum's investigations, there were several studies which were practically a continuation of the work he started. Probst, Heim, Trieber, Yoshida, Ban, and Graf and Brenner, as cited by Nordby (7), carried out investigations which were mainly concerned with the progressive deformations exhibited by concrete under fatigue loading. These tests provided a firm basis for further work on the mechanism of fatigue failure.

Graf and Brenner observed that initial surface cracking frequently appeared in the specimens after only

a few cycles of load. Although this cracking was not necessarily an indication of impending failure, it served to illustrate the progressive nature of the failure. Speed of testing was, in general, an insignificant factor, although very low frequencies of application of load were generally associated with somewhat smaller fatigue strengths and greater permanent deformations. When expressed as a percentage of the ultimate strength, there was a slight (and perhaps insignificant) decrease in fatigue strength with corresponding increases in ultimate strength, water-cement ratio, or cement content.

After Graf and Brenner, more than 20 years passed without any significant new literature appearing concerning the compressive fatigue strength of unreinforced concrete. In 1959, Antrim and McLaughlin (8) published a study of the fatigue behavior of air-entrained and non-air-entrained concrete. This study compared the fatigue response of 3 in. by 6 in. air-entrained concrete cylinders, to non-air-entrained concrete cylinders of the same size subjected to repeated axial compressive loads applied at a frequency of 1000 cpm. Thirty-one of the former specimens and 34 of the latter were tested. No significant difference was found between the response of the two types of concrete, although the air-entrained concrete evidenced less data scatter. The data, when extrapolated, indicated a fatigue strength of 55 percent of the static ultimate strength at 10 million repetitions of load. No fatigue limit, a stress level which could be sustained indefinitely, was evidenced in any of the test series.

2.2.2 Fatigue Behavior Under Bending Loads. The first investigation into the problem of flexural fatigue was initiated by Clemmer (9). He used unreinforced concrete beams  $(6 \times 6 \times 36 \text{ in.})$  which were supported as cantilevers and subjected to repeated loads applied at 40 cpm. He mounted seven specimens like the spokes of a wheel from a central hub, and the loads were applied through a truck wheel and axle which traveled a circular track about a fixed vertical central shaft. This method of loading was selected because it was desired to duplicate, insofar as possible, the manner in which the service loads were applied to highway pavements. Clemmer tested about 100 specimens and incorporated as the principal variable the richness of the concrete mix. From these tests, he made the following conclusions:

1. Fatigue phenomenon did exist when concrete specimens were subjected to repeated flexural loads.

2. Concretes of richer mixes were somewhat less susceptible to failure than are those of leaner mixes.

3. The concretes tested in flexure exhibited a fatigue limit of between 51 and 54 percent of the static modulus of rupture.

4. The fatigue resistance increased by repeated applications at a stress level below the fatigue limit.

Clemmer's tests have been most valuable although his results were influenced by several factors. His test specimens were clamped in position and loaded as cantilever beams. Consequently, the complex stress patterns present at the point of support may have affected the

conclusions since this point was also the general point of failure. Also, because the free end of each specimen formed a segment of a discontinuous tract, there was some possibility that impact load was applied. Seven specimens were tested simultaneously, and the apparatus was stopped whenever any specimen failed. Thus an additional variable-intermittent periods of rest of varying duration-was introduced. Clemmer observed the effect of this variable and concluded that the rest periods introduced appeared to be beneficial in extending the fatigue life of a specimen. When a set of specimens being tested sustained in excess of one million repetitions of load without failure, Clemmer increased the intensity of the load and continued the tests until failures oc-This variation in the loading pattern imposed curred. still another variable, and Clemmer observed that specimens which had sustained a number of repetitions of load of a given intensity were better able to sustain repetitions of load of a greater intensity than were specimens without prior load history.

Almost concurrently with Clemmer, Hatt and Crepps conducted fatigue tests of cement-mortar beams at Purdue University and reported the results during 1923-25 (10, 11, 12). They subjected 4 x 4 x 36 in. specimens to complete reversal of stresses in pure bending at 10 cpm. The Purdue investigation provided the following information on the behavior of concrete subjected to complete reversal of stress.

1. A fatigue limit between 50 and 54 percent of the static breaking load was clearly discernible for specimens four months or more in age. No such definite fatigue limit was, however, evidenced for specimens 28 days old at the time of test.

2. The fatigue limit could be increased by fatiguing the material first at a stress below the fatigue limit.

3. There was progressive deformation in the extreme fibers of the specimens until failure took place. Rupture or failure of the bond occurred first on the extreme outer fibers where the deformation was maximum. This action was progressive toward the center of the beam until the complete failure was imminent. Stresses above the fatigue limit caused continual progressive deformation within certain limits.

4. The periods of rest appeared to have only a temporary effect on the fatigue resistance of concrete.

In 1953, Kesler reported the results of an investigation made to determine the effect of the speed of testing on the flexural fatigue response of normal concrete (13). In his studies, he used concretes of two different strengths—3600 psi and 4600 psi—and the frequency of the repeated loads varied from 70 to 230 to 440 cpm for each concrete. The data from the six series of tests indicated that there was no significant difference in fatigue response regardless of the speed of testing or the quality of the concrete when the applied loads were expressed in terms of the static ultimate flexural strength. No fatigue limit was found even though the tests were continued through 10 million cycles of loading. Kesler used the ultimate compressive strength of standard companion cylinders to estimate the flexural capacity of the flexure specimens: the average static flexural strength of broken "halves" of specimens was taken to be the ultimate strength of the specimen tested. It is highly doubtful that there exists a general relationship between ultimate compressive and flexural

strengths. The static strength of broken "halves" of flexural specimens is at best an approximation of the strength at the plane of failure—it cannot represent the true value of the static strength at that plane under repeated loads.

From 1954 until 1956, Kesler and Murdock conducted a series of fatigue tests, the main purpose of which was to determine the effect of range of stress on the flexural fatigue behavior of unreinforced concrete (14). The study included some results of other variables as well. Concretes of wet and dry mixes were tested, as were beams of increased moisture content. One set of specimens was tested with load periods interrupted by intermittent rest periods. All specimens in the investigation were subjected to repeated loads and no reversals of loading were employed. Range of stress, however, was not investigated. Instead, specimens were subjected to a loading pattern in which the lower load was a fixed but increasing (from series to series) percentage of the maximum. As a consequence, neither the mean nor the alternating components of applied stress were constant in any test series. Each diminished with diminishing peak stresses. There was, however, the single ratio of minimum to maximum nominal stress which did remain constant and hence did reflect the response of the specimen to variation of mean and alternating components of stress. The results of these tests appeared to be independent of whether the concrete mix was of a wet or dry consistency, and the average of the two sets was employed. The tests in which five-minute periods of rest were inserted between 10-minute periods of loading gave evidence that rest periods were beneficial in raising the fatigue strength determined at 10 million repetitions of load. No test series indicated a fatigue limit. Since these tests used more than 200 specimens, this evidence seems more convincing than that of Clemmer. Some evidence was found to verify the observation by Clemmer, Hatt, and Crepps that initial fatigue loading at loads too small to produce failure seems to be beneficial in raising the fatigue strength in subsequent tests. Repeated loading offered a limited opportunity to investigate the effect of variation in the range of stress in its correct sense. The data obtained in this investigation may be considered as only qualitative evidence.

In 1958, McCall published the results of an investigation of the fatigue of concrete in which he sought to establish a relationship among the applied stress level, the cycles to failure, and the probability of failure (15). The study was an initial step in adding a most important "third dimension" to the fatigue studies. He utilized small flexural specimens of air-entrained concrete subjected to complete reversals of stress at the rate of 1800 cpm. Twenty specimens were tested in obtaining the data. From the test, McCall concluded that concrete did not exhibit any fatigue limit. At 20 million cycles, he found a fatigue strength of about 50 percent of static ultimate strength, with a probability of failure somewhat less than one half. Fatigue strengths at 20 million cycles of reversed loading ranged from approximately 35 percent of static ultimate strength to about 58 percent, for probabilities of failure of 0.1 to 0.8, respectively. At 10 million reversals of stress, the fatigue strengths for all probabilities of failure were substantially unchanged from the values determined at 20 million cycles. McCall determined the applied stress level on the assumption that the static strength of each specimen was identical

to the mean strength of three specimens cast in the same batch.

In 1960, Hilsdorf and Kesler reported the results of an important investigation on the fatigue behavior of concrete subjected to varying repeated loads (16). This study sought to define the response of concrete to a pattern of loading which varied throughout the life of the specimens, and from this, to evaluate the applicability of the Miner Hypothesis of cumulative fatigue damage. Studies of the effects of rest periods on the fatigue strength of the material were also made in this program. Hilsdorf adopted a new technique for accurate determination of the static ultimate strength of the specimens. He performed a series of static tests on companion specimens and "halves" of broken fatigue specimens. From these, he obtained an empirical expression for the static ultimate strength which yielded estimates of substantially improved accuracy. Subsequent studies of the relationship between applied loads and maximum tensile strains allowed predictions of ultimate strengths which were superior to any previously obtained. However, these relations were applicable only to Hilsdorf's investi-From his tests, Hilsdorf concluded that the gation. Miner Theory, which assumes that fatigue damage is accumulated linearly, was not applicable in the case of fatigue of unreinforced flexural concrete specimens. The fatigue response was found to depend on parameters which reflected the difference in stress levels, and the number of cycles of load at each level which was applied in a given "block"; that is,  $n_1$  cycles at level  $S_1$  alter-nated with  $n_2$  cycles at level  $S_2$  form a "block" of cycles  $n_{\rm b}$  which is some fractional part of the total cycles to failure. As the number of cycles at the greater stress level became a greater percentage of the total within one "block," the fatigue strength was reduced. As the difference in stress level was increased, the total number of cycles required to produce failure was found to decrease for given values of the maximum stress level. Hilsdorf also found that rest period was beneficial, but the duration of the rest period was significant only for periods up to five minutes. Above five minutes, no significant further increase in the fatigue strength was observed.

### 2.3 Lightweight Concrete

2.3.1 Fatigue Behavior Under Axial Loads. In 1961, Gray, McLaughlin, and Antrim (17) reported the results of a compressive fatigue test of lightweight con-They attempted to establish the relationship becrete. tween the stress level in percent of static ultimate strength and number of cycles to failure of two lightweight concretes, and compared their results with those previously established for a normal weight concrete (8). They also included tests which afforded a comparison of the fatigue response at frequencies of loading of both 500 and 1000 cpm. The fine and coarse aggregates used in this study were expanded shale products manufactured in a rotary kiln. An air entraining agent was used in the concrete to produce around seven percent air content. The low strength concrete had an average ultimate compressive cylinder strength of about 3700 psi and the high strength, 6200 psi. Twenty-five specimens of the former and 28 of the latter were tested. The specimens were 3 x 6 in. cylinders. The stress levels used in fatigue tes ing were determined from an estimate of the average batch strength which, in turn, was determined

There was no significant difference between the fatigue behavior of the lightweight aggregate concrete specimens and those of the normal concrete of earlier studies (8). No fatigue limit was found within 10 million repetitions of loading. No effect of the rate of load application was found on the fatigue properties of lightweight concrete when the rate of load application was between 500 and 1000 cycles per minute. The fatigue strength, when extrapolated, was approximately 55 percent of the static ultimate strength at 10 million repetitions of load.

2.3.2 Fatigue Behavior Under Bending Loads. The only known fatigue test of lightweight concrete under bending loads was reported by Williams (18) in 1943. He reactivated flexural fatigue studies after a lull of about 18 years with a limited investigation of the fatigue properties of lightweight concretes. He chose to subject a set of Haydite<sup>3</sup> aggregate concrete specimens to complete reversals of stress applied at a frequency of 15 cpm. A second set of similar specimens was subjected to repeated stress cycles applied at the same frequency. Specimens of Gravelite<sup>4</sup> aggregate concrete were subjected to repeated stress cycles at a frequency of 115 cpm. In the latter tests, the apparatus was considered unsatisfactory, the impact being estimated at between 10 and 20 percent. All the specimens were 4 x 5- $\frac{1}{8}$  x 32- $\frac{1}{2}$  in. Williams found no true fatigue limit through one million reversals or repetitions of stress. For the Haydite specimens, approximate fatigue strengths of 40 and 50 percent were determined at one million cycles for reversed and repeated loadings, re-spectively. The fatigue strengths determined by Wil-liams were lower than "typical" values, in opposition to the basic identity of behavior of normal weight and lightweight concretes in axial compression. This suggested that flexural fatigue behavior could not be extrapolated to predict the fatigue response of axial tension, since the behavior might have been affected by aggregate qualities or the proximity of aggregate to a surface. On an examination of beam failures, it was found that the crack frequently occurred at a point where a weakened piece of aggregate was located close to the tension surface. Some pieces of aggregate were softer than others and some were weakened by cleavage planes. Such points of weakness were particularly significant in the case of alternating loads, since, as Williams pointed out, once a crack started the stress concentration at the root of the fissure was very high. There was rather wide scatter of values of modulus of rupture from static tests, and a similar scatter in fatigue data. Initiation of failures was thus associated with aggregate imperfections and crack propagation was assisted, if not caused,

<sup>&</sup>lt;sup>a</sup>Patented process of manufacture of lightweight aggregate.

<sup>&</sup>lt;sup>4</sup>Patented process of manufacture of lightweight aggregate.

by the "stress risers" at the top of the crack. He further noticed that rough corners and surface holes resulting from air and water becoming trapped in the fresh concrete also acted as "stress risers." In the more prolonged tests under the lighter loads, cracks developed at many of these irregularities and a few gradually extended laterally to the corners and then into the beam. He noted that all cracks did not always progress until failure occurred. Sometimes the crack which ultimately produced failure appeared on the surface quite late in the test and then developed rapidly until the specimen failed. He explained that, in such cases, a piece of aggregate near the surface failed and developed a more critical stress condition. When the beams were more heavily loaded, a crack usually started at a "stress riser" and progressed to failure since the stress at the root of the crack was high enough to break through any pieces of aggregate that might have obstructed its progress. Williams recorded strains throughout the test of the beam. A strain pattern was indicated which was, in general, cycle dependent in which the strains progressively increased. The peak strains rose sharply at failure and described a curve which possessed a vertical asymptote. The abruptness of the final rise was associated with the intensity of loading and was sharper at greater loads. The initial rate of change of strain with applied load cycles showed a lesser degree of dependence on the intensity of load.

# 3. Theoretical Considerations

# 3.1 Mechanism of Fatigue Failure

### of Concrete

3.1.1 General Remarks. Fatigue failure is essentially a tensile failure and generally results in a brittle fracture of the material which is characterized by rapid rate of crack propagation, with no gross deformation at the fracture. On a microscopic scale, the fracture surface is usually normal to the direction of the principal tensile stress. A failure in fatigue usually occurs at a point of stress concentration such as a sharp corner or a crack at a stress well within the ordinary elastic range (19).

Three basic factors are necessary to cause fatigue failure (19). These are (a) a maximum tensile stress of sufficiently high value, (b) a large enough variation or fluctuation in the applied stress, and (c) a sufficiently large number of cycles of the applied stress.

3.1.2 Stress Cycle. The general types of fluctuating stresses which can cause fatigue are illustrated in Fig. 3-1. Fig. 3-1a illustrates a completely reversed cycle of stress of a sinusoidal form. Fig. 3-1b illustrates a repeated stress cycle in which the maximum stress and the minimum stress are not equal. In this illustration they are both tension, but a repeated stress cycle could just as well contain maximum and minimum stresses of opposite signs or both in compression. Fig. 3-1c illustrates a complicated stress cycle which might be encountered in a structural component in service.

A fluctuating stress cycle can be considered to be made up of two components; a mean, or steady, stress  $\sigma_{\rm m}$  and an alternating, or variable, stress  $\sigma_{\rm a}$ . The range of stress is the algebraic difference between the maximum and the minimum stress in a cycle.

3.1.3 Microcracking of Concrete. Fatigue is strongly influenced by what may appear to be minor discontinuities in the structure, as for example, microcracks in concrete. It was established in a study by Hsu et al. (20) that microcracks existed in hardened unreinforced concrete even before it was subjected to any load. Microcracks in concrete can generally be divided into three types, viz., cracks at the interface between aggregate and mortar (bond cracks), cracks through the mortar (mortar cracks) and cracks through the aggregate. The bond cracks may exist even before the concrete is subjected to any load, while the mortar cracks remain negligible until a later loading stage. The reason for the existence of these microcracks even before loading has been attributed to the existence of large tensile stresses at the aggregate mortar interface when the clear distance between the aggregate is small (21). These interface tensile stresses, due chiefly to volume change during hydration, cause the microcracks to ap-



Fig. 3-1. Typical Fatigue Stress Cycles: (a) Reversed Stress; (b) Repeated Stress; and (c) Irregular or Random Stress Cycles.

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pear. If the cement expands during hydration, bond cracks should occur without any mortar cracks; if the cement shrinks during hydration, bond cracks should appear simultaneously with mortar cracks.

There is also considerable evidence to show that further microcracking in concrete occurs when loaded. Bond cracks increase in length, width, and number with increasing strain in both the ascending and descending branches of the stress-strain curve. Evans (22) used a microscope to detect microcracks on the tension face of beams in flexure and on the surface of concrete cylinders in uniaxial compression. These cracks were visible at loads considerably less than that required to cause fracture of the concrete. Blakey and Beresford (23) used electrical wire resistance strain gages to determine the onset of cracking of beams in flexure. Jones (24) and Jones and Kaplan (25) used an ultrasonic pulse velocity technique to indicate the development and orientation of microcracks in concrete cubes, uniaxially loaded in compression. L'Hermite reported, as cited by Kaplan (26), how the noise caused by cracking could be heard by means of a microphone.

These studies indicate the presence of cracks at a load very much less than that required to cause ultimate failure of the concrete. The length, width, and number of these cracks increase with the increasing load. However, the increase in bond crack is negligible at loads lower than 30 percent of the ultimate load. Mortar cracks begin to increase noticeably and form continuous crack patterns at about 70 to 90 percent of the ultimate load. These mortar cracks always bridge between nearby cracks, and usually where distances between coarse aggregates are relatively small.

3.1.4 Basic Studies of Fatigue Mechanism of Concrete. The mechanisms which produce fatigue failure in concrete are still not completely known. Basic research for studying the mechanisms of fatigue failure in concrete were not undertaken with this expressed purpose in mind until about 1959. There had, of course, been some observations and speculations, and remarks had been made very casually about the mechanisms. Van Ornum (6) observed as early as 1907 that the stressstrain curve of concrete in axial compression, originally convex upward, became linear after a few repetitions of load. When the maximum load was sufficiently large, he observed that the curve became progressively concave upward, and finally near failure it became S-shaped. At lesser loads the stress-strain curve simply became linear and remained so, although the modulus of elasticity was reduced to about 70 percent of its initial value. Van Ornum's observations might afford a basis for the explanation of the mechanism of fatigue failure since this change in the phenomenon of elasticity gives an indication of a progressive nature of the failure. Works by Probst, Heim, Trieber, Yoshida, Ban, and Graf and Brenner, as cited by Nordby (7), substantiated the results of Van Ornum's and suggested that the accumulated deformation, the instantaneous ratio of elastic to permanent deformation, or the mechanical hysteresis loss per cycle of applied load, may be a suitable measure of fatigue damage. Since those studies were made, one or more authors observed the progressive nature of fatigue fracture but the literature reveals nothing conclusive about this very important aspect of the mechanisms of fatigue failure. Williams (18) suggested that the initiation of failure might have been associated with aggregate imperfections and that crack propagation was assisted, if not caused, by the "stress-risers" at the top of the crack. He noted that all cracks did not propagate. Sometimes the cracks which ultimately produced failure appeared (on the surface) quite late in the test and then developed rapidly until the specimen failed. The origin of the significant late developing crack was only surmised.

Kesler and Murdock (14) speculated concerning the mechanism of failure, suggesting that aggregatepaste bond might be critical, but did not verify the theory. Hsu et.al. (20) indicated in a study that the strength of bond between mortar and aggregate is the weakest link in the strength of concrete.

It was only in 1959 that an investigation with the sole purpose of determining the mechanism of fatigue failure in concrete was initiated at the University of Illinois (27). As of this writing, the investigation was still in progress. This program sought data which would provide some understanding of the initiation and development of fatigue failures in concrete. It incorporated tests from which a correlation of some properties and subsequent fatigue response could be made. The degree of applicability of fracture mechanics was likewise studied. In the initial phase of the investigation, simplified flexural models were employed in which a single type of preshaped natural aggregate of different cross-sections was placed near the tension surface of the constant moment region of the specimen. The purpose of these models was to permit an evaluation of the contribution of bond failure to the fatigue failure. It was noticed that the aggregate exhibited the same initial modulus as the matrix under compressive loads, and it was assumed that this was true in tension as well. It was argued that with this assumption a model containing an aggregate inclusion ought to be identical in behavior to a specimen without any aggregate inclusion if the bond between the aggregate and matrix maintained continuity of the specimen. But, in fact, the static behavior differed while the fatigue response was unaltered. This strongly suggested that the bond might well be significant in precipitating failure. A hypothesis of failure was formulated from these tests which in essence attributed the initiation of failure to progressive deterioration of bond between coarse aggregate and the mortar matrix. The final failure was not determined, but was assumed to be similar in nature; that is, a bond failure between paste and fine aggregate. The degree to which this final failure is progressive was not known. In other phases of the program, models containing unbonded aggregate inclusions were investigated and the earlier findings were more or less corroborated.

It is evident from the above discussion that the state of knowledge about the mechanism of fatigue failure in concrete is not complete. The results obtained from the experiments do not afford an answer to the "why" of fatigue response of concrete. There still remains the question of where and how cracking was initiated. The current state of knowledge regarding the mechanism of fatigue failure of concrete can be summarized as follows:

1. No adequate description of the mechanism of fatigue failure exists. There is reasonable evidence to suspect that the progressive deterioration of bond between aggregate and binding matrix is a significant factor in the failure.

2. There appears to be a parameter dependent on a critical nominal stress and crack length, which can define failure.

3. The effects of several variables have not been isolated, and the relative contributions of each to the failure in fatigue are undefined. Among these variables are aggregate quality, size, gradation, and properties, together with mix design and service environment.

# 3.2 Mechanism of Freeze-Thaw Failure of Concrete

There are three known basic mechanisms which cause deterioration of concrete when exposed to alternate cycles of freezing and thawing (28). These are:

# 4. Laboratory Investigation

#### 4.1 General Remarks

The laboratory investigations were divided into two The first phase consisted of developing the phases. flexural fatigue durability test, and the second consisted of obtaining fatigue data of three unreinforced lightweight concretes made with aggregates R and E.

All the laboratory investigations were conducted in the Materials Testing Laboratory of the Civil Engineering Department and in the Structural Research Annex of the Texas A&M University.

# 4.2 Development of the Flexural Fatigue Test

In order to develop the flexural fatigue test, it was necessary to have (a) a fatigue machine and (b) proper instrumentation for checking the maximum fatigue load transmitted by the apparatus and its mode of variation with time.

The fatigue machine was developed by modifying an existing apparatus, termed the deflectometer (29). On modification, the fatigue machine was capable of exerting a constant alternating load of 248 lbs. and a variable dead load at a rate of 697 cpm. To avoid impact, the dead load was always greater than the alternating load. The apparatus developed, thus, subjected the fatigue specimens to repeated type of stress cycles only. It was equipped with a counter for recording the number of cycles of load applied at any time and a cut-off switch for automatically stopping further cycling of the load after the specimen broke.

In order to verify the stress-time relationship of the fatigue test, a load cell was designed and constructed. It had a calculated capacity of 7500 lbs. and a sensitivity of 5.44 lbs. per micro-inch per inch of indicated strain. The load cell was hooked up with a Sanborn recorder, model 127, and a strain gage amplifier model 140 for recording the stress-time relationship. Fig. 4-1 shows the entire instrumentation.

### 4.3 Fatigue Tests

4.3.1 General Remarks. Fatigue resistance of unreinforced structural concrete is believed to be influ-

1. Build up of hydraulic pressure in the gel structure of the cement paste from free-water freezing.

2. Growth of capillary ice during sustained cold periods when the paste is relatively dense.

3. Deterioration caused by concrete aggregates.

In the first two of these mechanisms, the primary effect of the freezing and thawing is the development of alternating, or fluctuating, internal stresses in the concrete. This fluctuating stress damage may be relatable to the fatigue damage discussed previously.

As the mechanisms of freeze-thaw deterioration were discussed fully in Research Report 81-2 of this study (2), they will not be repeated here.

enced more or less by the following factors:

- 1. Type of coarse aggregate
- 2. Stress level
- Pmin. 3. Range of stress, Pmax.
- 4. Type of stress cycles, repeated or reversed



Fig. 4-1. Testing Machine and Instrumentation For the Flexural Fatigue Test.

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- 5. Microcracking of concrete-stress concentration
- 6. Rest period
- 7. Air entrainment
- 8. Shrinkage stress
- 9. Specimen size

Within the limited scope of this investigation, the variables were restricted to the following:

- 1. Type of coarse aggregate
- 2. Stress level

4.3.2 Aggregates and their Physical Properties. Concrete made with two lightweight coarse aggregates designated aggregate R and aggregate E—were investigated. For comparison purposes, a regular weight concrete was also used (coarse aggregate H). All concretes contained regular weight, river-run, silicious sand. Coarse aggregates R and E were expanded shales commercially produced in Texas in a rotary kiln. Coarse aggregate H was a river-run, silicious, gravel. The physical properties of all the aggregates are shown in Tables 4-1 and 4-2. The aggregates chosen for study in this research program were also studied for the freeze-thaw behavior of the resulting concretes in prior research at Texas A&M (1).

4.3.3 Concrete Mix Design. In order to compare results, the same concrete mix designs as those of the freeze-thaw tests of prior research (1), with a nominally cement factor of five sacks per cubic yard, were used in this study. Mix design data are given in Section 7.1.

4.3.4 Fatigue Specimens. Unreinforced concrete prism specimens  $(3 \times 4 \times 16 \text{ in.})$  were used for the fatigue tests. The shape and size of the specimens were more or less determined by the maximum size of the aggregates, availability of steel molds, prior freeze-thaw research (1), the dimensions of the base of the testing machine which supported the specimens, and most important of all, the capacity of the testing machine.

4.3.5 Stress Level. The fatigue stress level of the concrete was evaluated as a certain percentage of the modulus of rupture of the prism specimens. In this study, it was the intention to subject the specimens to repeated stress cycles at four different stress levels equal to 50, 60, 70 and 80 percent of the concrete's modulus of rupture. But preliminary investigation revealed that

TABLE 4-1. AGGREGATE SIEVE ANALYSIS DATA

	C	umulative Pe	rcent Retain	ed
Sieve Size	Coarse Aggre- gate R	Coarse Aggre- gate E	Coarse Aggre- gate H	Fine Aggre- gate
34 in. 12 in. 38 in. #4 #16 #30 #50 #100	$\begin{array}{c} 2.4\\ 30.5\\ 54.3\\ 93.0\\ 99.4\\ 100.0 \end{array}$	$\begin{array}{c} 0.2 \\ 12.3 \\ 46.8 \\ 100.0 \\ 100.0 \\ 100.0 \end{array}$	12.139.465.395.898.2100.0	$0.4 \\ 9.5 \\ 23.4 \\ 43.7 \\ 83.5 \\ 96.0$

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Fig. 4-2. Arrangement of Concrete Prism Specimen For Fatigue Testing.

the concretes did not fail within  $10 \ge 10^6$  cycles when subjected to stresses below 70 percent of the modulus of rupture. Therefore, it was decided to evaluate the fatigue strength of all the concretes at stress levels of approximately 70, 80 and 90 percent of the modulus of rupture. In one particular instance, fatigue strength was arbitrarily evaluated at a stress level of 102 percent of the modulus of rupture.

4.3.6 Testing Procedures. The concretes made with the aggregates R and H were tested wet and those made with the aggregate E were tested both dry and wet. Special precautions were taken for those specimens which were tested wet to see that they did not lose moisture during the period of testing. This was necessary because it has been shown that concrete loses flexural strength rapidly as it starts losing moisture (31), which makes the determination of fatigue stress level impossible. (Ref. Section 7.2.2 for treatment of the wet concrete specimens.)

The specimens were placed in the testing machine and simply supported by two rollers at the two ends over a  $14\frac{1}{2}$  in span. The load was transmitted to the specimens through a steel ball, a steel disc, and a 3% in. square by 4 in. long steel bar. The complete arrangement is shown in Fig. 4-2. The steel bar was placed on the top surface of the specimen with its ends flush with the 16 in. sides of the latter and its width symmetrical with the loading plane of the specimen. Plaster of Paris paste was used in between the specimen and steel bar to make the top surface of the latter level and to insure uniform contact between the concrete and steel surfaces. The top surface of the steel disc and the bottom surface of the circular loading shaft of the machine had  $\frac{1}{2}$  in. diameter concentric grooves between which the steel ball was placed during loading. Care was taken to align the center of the beam between the supports so that the applied load would be vertical only and the beam would be in simple bending only. The applied load and support reactions were uniformly distributed across the width of the specimen. Tests were generally continued to failure of the specimens. In cases, however, where the specimens did not fail, tests were terminated anywhere between one and ten million cycles depending on the availability of time.

TABLE 4-2.	AGGREGATE	ENGINEERING	PROPERTIES

Coarse Aggregate	Absorption 3 Days <sup>a</sup> 14 Days <sup>a</sup>		Bulk Gra (S	Specific avity SD)	Dry Unit	Fineness Modulus <sup>c</sup>
Designation	%	%	3 Days <sup>b</sup>	14 Days <sup>b</sup>	Wt. (pcf)	(%)
R E H Fine	$\begin{array}{c} 4.7 \\ 6.5 \\ 1.2 \\ 0.8 \end{array}$	8.0 8.6 	$1.47 \\ 1.42 \\ 2:62 \\ 2.61$	1.50 $1.46$ $$	$\begin{array}{r} 46.8 \\ 43.8 \\ 101.0 \\ 99.0 \end{array}$	$7.0 \\ 6.5 \\ 7.1 \\ 2.57$

<sup>a</sup>Immersed in water for the period indicated. <sup>b</sup>As determined according to Bryant's method (30). <sup>c</sup>As defined in ASTM Designation C125-58.

# 4.4 Modulus of Rupture and Compressive Strength Tests

The modulus of rupture tests were conducted to obtain the maximum breaking loads of the prism specimens with a center point load applied parallel to the 4 in. axis over a  $14\frac{1}{2}$  in. simply supported span. The results were used to determine the fatigue stress levels as described earlier in Section 4.3.5. Except for the span, this test was conducted according to ASTM Method C293.

A total of 18 prism specimens were molded in steel molds from each batch of concrete. The specimens were then cured as described in Section 7.2.2. Three specimens were chosen at random from among the 18 for determination of the modulus of rupture. The values of the maximum flexural loads along with those of modulus of rupture are given in Table 4-3. The reported values of the breaking loads represent the average of three individual tests in each case.

In order to properly identify the concrete mixes, a total of three 6 in. diameter by 12 in. long cylinders from each batch of concrete were molded and cured in accordance with ASTM C192. Steel molds were used throughout. The cylinders were cured for a minimum of 28 days prior to testing. Compressive testing was done in accordance with ASTM C39.

The compressive strength values for each concrete are also reported in Table 4-3. Each of these values represents the average of three individual tests. Notice that in all cases structural quality concrete was achieved with 28-day compressive strengths ranging from 3570 psi to 4350 psi.

Coarse Aggregate Designation	Cement Factor (sks/cy)	Slump (in.)	28-Day Compressive Strength, (psi)	Static Ultimate Flexural Load (lb.)ª	Modulus of Rupture, (psi)ª	Remarks
R E	5.0 4.9	4¼ 3	3845 4050	$1080 \\ 950 \\ 1130$	650 570 680	Concrete was moist during
E	5.0	4	4350	1240	760	testing. Concrete was dry during
H (Regular Weight)	4.9	3	3570	1420	850	testing.

#### TABLE 4-3. CONCRETE PROPERTIES

<sup>a</sup>Strength determinations were made just prior to fatigue testing.

# 5. Analysis and Discussion of Results

# 5.1 General Remarks

This section contains the results of fatigue tests performed on the lightweight and regular weight concrete prism specimens. The results were analyzed for the relationship between stress level and the number of cycles of loading to concrete failure as a function of the aggregate type. Analyses were also made to establish relationship between (a) fatigue durability, (b) freezethaw durability, (c) aggregate absorption and (d) aggregate freeze-thaw loss.

#### 5.2 Fatigue Curves

Fatigue curves, in the form of number of cycles to failure (N) versus applied stress (S) were established for moist concretes made with coarse aggregates R, E



Number of Fatigue Cycles to Failure of Concrete, N(log)

Fig. 5-1. Relationship Between Aggregate Type and Fatigue Durability of Selected Lightweight Concretes.

and H as well as for a dry concrete made with aggregate E. All the fatigue curves were fitted, wherever possible, to the median of each group of values of N at the applied stress levels. These curves are estimates of the relationship between the applied stress and the number of cycles to failure that 50 percent of the population<sup>5</sup> would survive.

<sup>5</sup>Population is defined as the hypothetical collection of all possible test specimens that could be prepared in the specified way from the material under consideration (32).

The results of the fatigue tests have been summarized in Tables 5-1 through 5-4 for the various concretes studied. The S-N curves for the concretes are plotted in Fig. 5-1.

# 5.3 Effect of Aggregate Type on Fatigue Durability on Concrete

Fig. 5-1 reveals the effect of type of aggregate on the fatigue durability of unreinforced concretes made with the three different coarse aggregates. It can be

<b>TABLE 5-1.</b>	FATIGUE	TEST	RESULTS	$\mathbf{OF}$	AGGREGATE	$\mathbf{R}$	CONCRETE	(WET)	SERIES
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Illtimate Static	Fatigue Load		Total Fatima			
Flexural Load	Alternating Load (lb.)	Dead Load (lb.)	Load (lb.) (P <sub>f</sub> )	Stress Level % of P (S)	No. of Cycles to Failure (N)	Remarks
950	248	727	975	102	120	
$\begin{array}{c} 1080 \\ 1080 \end{array}$	248 248	$\begin{array}{c} 579\\615\end{array}$	827 863	87 80	$6.74 \times 10^4$ $2.29 \times 10^5$ $1.76 \times 10^6$ $1.08 \times 10^6$	
1080	248	505	753	70	$\begin{array}{c} 1.98 \times 10 \\ 11.04 \times 10^6 \\ 4.71 \times 10^5 \end{array}$	Test Terminated
					10.78 x 10 <sup>6</sup>	Test Terminated

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Illtimate Static	Fatigue Load		Total Fatigue				
Flexural Load (lb.) (P)	Alternating Load (lb.)	Dead Load (lb.)	Load (lb.) (P <sub>1</sub> )	Stress Level % of P (S)	No. of Cycles to Failure (N)	Remarks	
1240	248	859	1107	90	$5.89 \times 10^4$ $1.18 \times 10^4$ $7.80 \times 10^3$ $1.08 \times 10^4$		
1240	248	740	988	80	$\begin{array}{c} 1.08 \times 10^{5} \\ 8.86 \times 10^{5} \\ 6.58 \times 10^{5} \\ 3.77 \times 10^{5} \\ 6.62 \times 10^{5} \end{array}$		
1240	248	615	863	70	$\begin{array}{c} 6.02 \times 10 \\ 8.10 \times 10^6 \\ 10.00 \times 10^6 \end{array}$	Test Terminated Test Terminated	

TABLE 5-2. FATIGUE TEST RESULTS OF AGGREGATE E CONCRETE (DRY) SERIES

seen that the aggregate type definitely affects the fatigue behavior of these types of unreinforced concretes, which is a major finding in this phase of the study. When tested wet, all the three concretes made with the coarse aggregates R, E and H evidenced different fatigue lives at the same fatigue stress level. It was thus found that the fatigue durabilities of the concretes made with the two types of aggregates, aggregates R and E, were widely different. The fatigue resistance of the concrete made with the natural aggregate H was again different from the rest. Aggregate R concrete was found to be the most durable one with the longest fatigue life at all stress levels. Aggregate E concrete came next and, surprisingly enough, the natural aggregate H concrete evidenced the shortest fatigue life when the stress level was about 74 percent of the modulus of rupture or higher within the scope of this study. At a lower stress level than this, aggregate H concrete (wet) still evidenced a shorter fatigue life than the aggregate R concrete (wet) and the aggregate E concrete (dry); but how it compared with the fatigue life of the aggregate E concrete (wet), was not definitely known because the S-N curve of the latter was not established in this study between the stress levels of 70 and 80 percent of modulus of rupture.

The finding that fatigue durabilities of concretes vary depending on the type of lightweight coarse aggregates used is understandable and also agrees with prior findings (18). It is understood here that the typification of the aggregates is associated with the aggregate engineering properties. But when the engineering properties of two aggregates exhibit similarity, the difference in fatigue durability of the resulting concretes becomes hard to explain. That is what actually has happened in this study. An examination of the engineering properties of the aggregates R and E indicates a close resemblance between the two (see Table 4-2). Yet the fatigue durabilities of the two resulting concretes were found to differ widely. Why this is so, is not known, but this definitely brings out the fact that the similarity of the engineering properties of two or more lightweight aggregates may not be a criterion for estimating the fatigue durabilities of their resulting concretes. Basic studies of the structures and compositions of the synthetic lightweight aggregates may throw some light into this deviatoric behavior of the resulting concretes.

The figure also reveals another important aspect of fatigue life of concrete. Confining the attention to the two curves for the aggregate E concrete (wet) and the aggregate E concrete (dry), it is found that the dry concrete has a longer fatigue life than the wet concrete at all stress levels; and at lower stress levels, this difference becomes wider. This trend was also noticed by Hatt (12). The reason for this behavior is not known, but it may be hypothesized that internal tensile stresses resulting from hydrostatic pressure applied by the water in the concrete under saturated condition may be responsible for the shorter span of fatigue life for wet concrete. The different slopes of the two curves further indicate that the rate of damage done to the concretes by the repeated loads is not the same in both cases. The steeper slope of the aggregate E concrete (wet) suggests that its rate of damage is faster than that of the dry concrete

Illtimate Static	Fatigue Load		Total Fatigue				
Flexural Load (lb.) (P)	Alternating Load (lb.)	Dead Load (lb.)	Load (lb.) (P <sub>t</sub> )	Stress Level % of P (S)	No. of Cycles to Failure (N)	Remarks	
1130	248	789	1037	92	960 620 1160		
1130	248	671	919	81	$\begin{array}{c} 1.10 \\ 1.10 \\ \times 10^{6} \\ 1.46 \\ \times 10^{4} \\ 1.21 \\ \times 10^{4} \\ 1.77 \\ \times 10^{4} \\ 4.06 \\ \times 10^{4} \\ 9.84 \\ \times 10^{3} \end{array}$	Test	
1130	248	561	809	72	4.00 x 10 <sup>6</sup>	Test Terminated	

TABLE 5-3. FATIGUE TEST RESULTS OF AGGREGATE E CONCRETE (WET) SERIES

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under the conditions of the present study. The reason for this behavior is not known. Further research is needed to explore this aspect.

It was further noticed that the aggregate R concrete (wet) and the aggregate E concrete (dry) behave almost identically under repeated flexural loads. The slopes of the two curves are found to be almost the same indicating that the rate of damage of concrete is approximately the same in both cases. There is only slight horizontal shift between the two curves showing that the fatigue life of the concrete made with aggregate R (wet) is slightly longer than that of the concrete made with aggregate E (dry) at all stress levels above 70 percent. It would have been of interest to see how aggregate R concrete would behave when tested dry and how that would compare with the concrete made with aggregate E (dry).

The natural regular weight gravel concrete is seen to be the worst performer of the lot so far as the fatigue behavior is concerned. This is somewhat surprising because natural aggregate concretes are generally considered to exhibit better fatigue properties than lightweight aggregate concretes. The coarse aggregate type was the only variable in the concretes, and all the concretes were identically batched and cured. Under these conditions, it would seem reasonable to believe that the regular weight concrete would perform the best. This, however, was not the case, and in examining the concretes a possible explanation was found.

Fig. 5-2 shows typical fractured surfaces of all concretes. An eye examination reveals that in the cases of the lightweight aggregate concretes, whether wet or dry, failure of the specimens was always accompanied by the fracture of almost all the coarse aggregates along the fracture surface. This was not found in the case of the natural aggregate concrete. On the contrary, none of the coarse aggregates fractured; in all instances the failure surface took place between the mortar and the aggregates. It is not known where, when, or how the critical crack initiated in both the cases of lightweight and regular weight concretes. But once the crack initiated, it propagated through the coarse aggregates in its path in the case of lightweight concrete and around those in the case of the regular weight concrete of this study. As most fractures of sound concrete occur through a portion of the aggregate, it seems logical that in this case the aggregate H contained some type of surface coating which reduced the bond and caused the lower-than-expected fatigue strengths.



Fig. 5-2. Fractured Surface of Concrete Fatigue Specimens: (a) Aggregate E Concrete (wet); (b) Aggregate E Concrete (dry); (c) Aggregate R Concrete (wet); and (d) Aggregate H Concrete (wet).

Thus, no comparisons can be made between the lightweight and regular weight concretes used in this study.

Aggregate type does not seem to have much effect on the fatigue life at lower stress levels, i.e., stress levels below 70 percent. In the cases of the concretes made with aggregates R (wet), E (dry), and H (wet), the trend is such that there should be no failure of the concrete specimens at 70 percent stress level and below. More tests need to be conducted if it is desired to predict the fatigue behavior of concrete made with aggregate E (wet) in the neighborhood of 70 percent stress level, before any conclusions can be reached concerning this condition.

# 5.4 Relationship Between Fatigue Durability and Freeze-Thaw Durability

The results of the TTI<sup>6</sup> freeze-thaw tests along with the respective concrete properties are presented in Tables 5-5 and 5-6. The freeze-thaw results have been reported in Research Reports 81-1 and 81-2 from prior research at Texas A&M (1,2). An examination of Tables 5-5 and 5-6 shows that similar lightweight concretes were batched for both experimental programs. The results of

<sup> $^{\circ}$ </sup>Slow freezing in air and thawing in water. See Research Report 81-1 (1).

Illtimate Static	Fatigue Load		Total Fatigue			· · · · ·
Flexural Load (lb.) (P)	Alternating Load (lb.)	Dead Load (lb.)	Load (lb.) (P <sub>1</sub> )	Stress Level % of P (S)	No. of Cycles to Failure (N)	Remarks
1420	248	876	1124	80	$\begin{array}{c} 1.06 \ \ \mathrm{x} \ \ 10^3 \\ 1.28 \ \ \mathrm{x} \ \ 10^3 \\ 6.18 \ \ \mathrm{x} \ \ 10^3 \\ 1.42 \ \ \mathrm{x} \ \ 10^3 \\ 4.60 \ \ \mathrm{x} \ \ 10^3 \\ 2.28 \ \ \mathrm{x} \ \ 10^3 \end{array}$	
1420	248	740	988	70	$\begin{array}{c} 3.38 \times 10^{3} \\ 1.38 \times 10^{3} \\ 4.00 \times 10^{6} \\ 1.06 \times 10^{6} \end{array}$	Test Terminated Test Terminated

TABLE 5-4. FATIGUE TEST RESULTS OF AGGREGATE H CONCRETE (WET) SERIES

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Coarse Aggregate	Coarse Aggregate Degree of	Cement Factor	28-Day Compressive Strength	Number of Cycles to Failure at Stress Levels <sup>1</sup>		
Designation	Saturation	(sks/cy)	(ps1)	80%	85%	90%
R	14-Day	5.0	2945	1 5106	0.0105	9.6.104
E	14-Day Soaked	5.0 5.0	4350	$5.5 \times 10^{-5}$	$2.0 \times 10^{4}$ 7.0 × 10 <sup>4</sup>	$2.6 \times 10^{-1}$ 9.2 x 10 <sup>3</sup>
	(Concrete dried inside before tested)					
E	14-Day Soaked	4.9	4050	$2.3 \mathrm{x10^{4}}$	$6.0 \mathrm{x10^{3}}$	$1.5 \mathrm{x10^{3}}$

TABLE 5-5. FATIGUE TEST RESULTS AND THE CONCRETE PROPERTIES

<sup>1</sup>Results read out from curves of Figure 5-1.

the fatigue test at the stress levels shown in the tables mentioned above were read out from curves of Fig. 5-1. The relationship between the number of cycles to failure at 80, 85, and 90 percent stress levels in fatigue test and those in the Texas Transportation Institute freeze-thaw test is shown in Fig. 5-3. In this figure, the number of cycles to failure in freeze-thaw test, termed freezethaw durability, and those in fatigue test, termed fatigue durability, are plotted; the former as the ordinate in cartesian coordinates and the latter as the abscissa in logarithmic coordinates, for different concretes at different fatigue stress levels. Though the actions of fatigue and freeze-thaw were believed to be similar, it was not known before if there was any relationship between the fatigue durability and the freeze-thaw durability of concrete. For the concretes studied, Fig. 5-3 reveals a trend indicating that at a certain fatigue stress level, fatigue durability of a lightweight concrete bears a relationship with the freeze-thaw durability of the same concrete. It is further seen that the concrete which has a higher fatigue durability has also a higher freeze-thaw durability. Of course, insufficient data were obtained to clearly define this relationship, but it is believed that the data indicate such a relationship does exist.



Fig. 5-3. Relationship Between Freeze-Thaw Durability and Fatigue Durability of Selected Lightweight Concrete.

Coarse Aggregate Designation	Coarse Aggregate Degree of Saturation	Cement Factor (sks/cy)	28-Day Compressive Strength (psi)	Number of Freeze-Thaw Cycles to Failure
R	14-Day	5.0	4120	228
${f E}$	Oven Dry 14-Day Soaked	5.0 4.7	$\begin{array}{c} 5530\\ 4070 \end{array}$	200 <sup>b</sup> 14

TABLE 5-6. TTI FREEZE-THAW TEST RESULTS AND THE CONCRETE PROPERTIES<sup>a</sup>

aResults obtained from Research Reports 81-1 (1) and 81-2 (2). <sup>b</sup>Removed from freeze-thaw cycle after 200 cycles.

Fig. 5-3 further reveals that aggregate type has important influence on durability of concrete-be it freeze-thaw durability or fatigue durability. Aggregate R is found to produce the most durable concrete, regardless of the type of test, when the aggregate was presoaked for 14 days before batching. Compared to this, aggregate E, under the same conditions, produces a less durable concrete. At the same time, however, this study reveals that the aggregate E concrete evidences almost equal durability as the aggregate R concrete when either the aggregate was pre-soaked but the concrete was dried prior to testing or the concrete was wet when tested but the aggregate was oven dried prior to batching. Thus, if satisfactory durability can be defined as withstanding 200 or more cycles of freezing and thawing (TTI), performance of aggregate R concrete is found to be satisfactory even under high aggregate saturation conditions. Aggregate E concrete, however, showed unsatisfactory durability under high aggregate saturation conditions but proved satisfactory otherwise from the durability point of view.

### 5.5 Reproducibility of the Results

An examination of the results of the fatigue tests reveals some scatter of the values of N for all the concretes. This is not unusual in the case of concrete primarily because of the uncertainties inherent in its structure. Concrete is generally considered to be a viscoelastic, heterogeneous material which exhibits innumerable microcracks in the form of bond and mortar cracks. Also, the surface of the concrete specimens are rough and may contain shrinkage cracks or holes left by entrapped air bubbles which act as sources of stress concentration. Because of these facts, two specimens are never exactly identical. Even metals, which exhibit much more uniformity in structure as compared to concrete exhibit considerable scatter in their fatigue results. Dieter (19), in describing fatigue of metals, observes:

"It will generally be found that there is a considerable amount of scatter in the results, although a smooth curve can usually be drawn through the points without much difficulty. However, if several specimens are tested at the same stress, there is a great amount of scatter in the observed values of number of cycles to failure, frequently as much as one log cycle between the minimum and the maximum value."

In the light of this observation, the scatter of the results as evidenced in Fig. 5-1 is not beyond what is naturally expected to occur in fatigue testing of unreinforced concrete. As for the cyclic loading, the machine exerted regular sinusoidal types of loads of the desired amplitude. Thus, it can be concluded that the flexural fatigue test developed in section 4.2 is reliable and suitable for unreinforced concrete and gives reproducible results.

### 5.6 Evaluation of the Fatigue Test

The fatigue behavior of unreinforced concrete, in general, exhibits (a) no fatigue limit, and (b) a straight line variation of logarithm of fatigue life with fatigue load level (8, 9, 13, 17, 18). These characteristics agreed very well with the findings of the present study (Sec. 5.2 and 5.3). The finding of Williams (18) that different types of lightweight aggregate affect the fatigue life of the resulting concretes was also found true in this study (Sec. 5.3). According to Hatt (12), the same concrete when dry was more durable in fatigue than when wet. It was found in this study, too, that the aggregate E concrete when dry had a longer fatigue life than when wet (Sec. 5.3).

# 6. Conclusions and Recommendations

### 6.1 Conclusions

In an endeavor to evaluate the fatigue behavior of three unreinforced lightweight and one regular weight concrete, a flexural fatigue test was developed. Evaluation of the apparatus developed for testing unreinforced concrete prism specimens under repeated flexural stress cycles indicates that it yields reproducible results and that, within the scope of this study, it is sensitive to the factors which affect the fatigue behavior of unreinforced lightweight concrete.

Relationships between stress level and number of cycles to failure under repeated flexural stress cycles for three unreinforced lightweight concretes and one regular weight concrete were tentatively established. Although the data obtained were limited, the following conclusions can be drawn from this investigation. 1. The resistance of unreinforced structural concrete to flexural repeated load was dependent on the applied stress amplitude, the variation of log of fatigue life being inversely proportional to the applied stress up to 10 million repetitions of the load for all the concretes tested.

2. Type of coarse lightweight aggregate definitely affected the flexural fatigue behavior of unreinforced structural concrete.

3. It was found that the fatigue properties of the aggregate R concrete (wet) did not differ from those of the aggregate E concrete (dry), the S-N curves being of the same slope (Fig. 5-1). Fatigue properties of other concretes differed appreciably depending on the type of aggregate and the moisture condition of the concretes.

4. The S-N curves revealed a trend which indicated that there did not exist any fatigue limit for any of the concretes studied in this program up to 10 million repetitions of load.

5. Moisture condition of concrete affected its fatigue behavior. Preliminary findings indicated that dry concrete exhibited longer fatigue life than wet concrete made with the same lightweight coarse aggregate.

6. No comparisons were made between the fatigue durability of regular weight and lightweight concretes because of the uncharacteristic behavior exhibited by the regular weight concrete.

7. All concretes, except aggregate E (wet), exhibited a minimum fatigue life of 10 million cycles at 70 percent stress level.

8. A relationship was found to exist between fatigue durability and freeze-thaw durability (TTI test) of concretes made with the different lightweight coarse aggregates studied. It is revealed that a particular aggregate concrete which was durable in the freeze-thaw test was also durable in the fatigue test.

These conclusions are entirely dependent upon the conditions for which this research was conducted and should not be further generalized.

#### 6.2 Recommendations

In the light of the findings of this investigation, the following recommendations are offered:

1. The fatigue data obtained are limited for the aggregates H and E concretes (wet), particularly at the lower stress levels. More data should be collected to establish the S-N curves for these two concretes in this region.

2. The fatigue behavior of the concretes studied in this investigation should be investigated when they are air dried in the laboratory for the purpose of comparison.

3. Effect of cement content and air-entrainment on the flexural fatigue durability of lightweight concrete should be studied.

4. More types of aggregates should be included in the testing program to further establish the relationship between fatigue durability and freeze-thaw durability.

5. Effect of reinforcement should also be studied to completely evaluate the fatigue behavior of light-weight concrete.

6. Another regular weight concrete should be used which exhibits good fatigue strength to offer a comparison between lightweight and regular weight concrete.

#### 7.1 Concrete Mix Design Data

The concrete mix designs, in terms of percent absolute volume of the various constituents, are given in Table 7-1.

### 7.2 Laboratory Procedures

7.2.1 Concrete Mixing Procedure. Because of high rate and amount of absorption of water by the light-weight aggregates included in this program, a definite mixing procedure was followed in all cases where light-weight aggregates were involved. For details, refer to Research Reports 81-1 and 81-2 (1, 2).

7.2.2 Casting and Curing of Prism Specimens. The prism specimens were cast in steel molds, as described in Research Report 81-2 (1). A total of 18 prism specimens were cast from each batch of concrete. Four such batches were made using three different coarse aggregates, viz., one each with aggregates R and H, and two with aggregate E.

The specimens which were tested dry were cured for a minimum period of 27 days (after removal from the mold) in the moist room at approximately  $73^{\circ}$ F and 100 percent relative humidity. These were then taken out from the moist room and were allowed to dry in air inside the laboratory until tested.

The specimens which were tested wet were cured for a minimum of 27 days in the moist room after removal from the mold. Prior to testing, each specimen surface was coated with a membrane curing compound, Horncure 50D. Also, at frequent intervals during the test the specimen was recoated. Prior research at Texas A&M University (33) found this compound quite suitable for preventing loss of moisture from the concrete surface when sprayed thoroughly over the concrete.

#### 7.3 References

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Coarse Aggr. Designation	Cement Factor (sks/cy)	Percent Absolute Volume			Air	Slumn	Initial Unit Weight	
		Cement	Water	F.A.	C.A.	(%)	(in.)	(pcf)
R E (for dry conc)	$5.0 \\ 5.0$	$\begin{array}{c} 8.6\\ 8.2\end{array}$	$\begin{array}{c} 20.0\\ 21.0\end{array}$	$35.0 \\ 37.2$	$\begin{array}{c} 33.9\\31.5\end{array}$	$\begin{array}{c} 2.5\\ 2.1\end{array}$	4¼ 4	$\begin{array}{c} 121.6\\ 128.8\end{array}$
E (for wet	4.9	8.1	22.0	36.7	31.1	2.1	3	126.0
H	4.9	8.6	15.5	29.2	45.0	1.7	3	149.6

TABLE 7-1. CONCRETE MIX DESIGN DATA

<sup>a</sup>Determined in accordance with ASTM Test Designation C 231.

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