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# Precast, Prestressed Concrete Bent Caps: Volume 2 Design Recommendations and Design Examples

Technical Report 0-6863-R1-Vol2

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16. Abstract


Recommendations for design of pretensioned bent caps are developed based on the findings of full-scale experimental tests of bent cap subassemblages. Companion examples are provided to demonstrate implementation of the design recommendations. First, recommendations are made for converting existing reinforced concrete bent cap designs to pretensioned designs, providing contractors the option to choose pretensioned caps at any point during the construction process. Next, design recommendations are made for the design of pretensioned caps as part of the original substructure design. Flexural design is based on the concept of zero tension under dead load to ensure that any cracks that form close upon removal of live load. Shear design recommendations modify the American Association of State Highway and Transportation Officials sectional design method to ensure that crack angles used are physically admissible. New design recommendations are first demonstrated through examples for standard bridge designs. Additional examples demonstrate use of internal voids to reduce weight and how pretensioned bent caps can be used to eliminate column lines to save construction time and cost. Finally, recommendations and examples are included for end region detailing and for incorporation of a pocket connection that minimizes the effects of prestressing and provides considerable tolerance for construction.


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# PRECAST, PRESTRESSED CONCRETE BENT CAPS: VOLUME 2 – DESIGN RECOMMENDATIONS AND DESIGN EXAMPLES

by

Anna C. Birely, Ph.D.
Assistant Professor
Zachry Department of Civil Engineering
Assistant Research Scientist
Texas A&M Transportation Institute

John B. Mander, Ph.D.
Zachry Professor of Design and Construction Integration
Zachry Department of Civil Engineering
Research Scientist
Texas A&M Transportation Institute

and

Codi D. McKee and Ju Dong Lee
Graduate Assistant Researchers
Texas A&M Transportation Institute

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# 1 INTRODUCTION

## 1.1 OVERVIEW AND BACKGROUND

With hundreds of new and replacement bridges constructed every year in Texas, improving the efficiency of construction using accelerated bridge construction can have a significant impact on the traveling public. For accelerated construction of substructures, the Texas Department of Transportation (TxDOT) first began using precast reinforced concrete (RC) bent caps in the early 1990s. Since then, connections for precast RC bent caps have become a standard detail that offer contractors flexibility in construction in the method. To provide further options for contractors, bent caps may be prestressed. Prestressed caps can be built by precast fabricators, eliminating the need for on-site construction.

Beyond constructability, the use of pretensioned bent caps can allow for improved performance of standard bridge designs or can permit design of optimized substructures. Prestressing enables designs of caps that have limited or no cracking under design loads. In instances where cracks form, widths are considerably smaller in size and number than in RC designs. The reduced cracking in pretensioned bent caps can allow for longer spans, enabling the design of bridges with reduced number of column lines.

To avoid costly mistakes and to fully realize the performance benefits of pretensioned bent caps over RC bent caps, it is necessary to establish a thorough understanding of the behavior of pretensioned bent caps for multicolumn substructures, to address any detailing concerns, and to establish appropriate design procedures. From a performance standpoint, validation of improved resistance to cracking is needed, along with stress limits for use in design. For shear, American Association of State Highway and Transportation Officials (AASHTO) LRFD sectional design procedures were developed for thin-web girders, and while they may lead to reasonable shear reinforcement design for RC bent caps, their appropriateness for pretensioned bent caps must be assessed. The introduction of prestressing can lead to the potential for end-region cracking, for which adequate detailing must be validated. Finally, cap-to-column connection options must be assessed to determine the best option for pretensioned bent caps. To address these concerns, TxDOT initiated project 0-6863. This document and companion document (Birely et al. 2018) summarizes the findings of the project.

## 1.2 OBJECTIVES AND WORK PLAN

The work in this report was conducted under TxDOT Project 0-6863. The overall objective of the project is to investigate the use of precast, pretensioned bent caps to enable implementation in multicolumn bridges. Specific technical objectives are to:

- Evaluate the overall behavior and serviceability of precast, pretensioned bent caps through large-scale experimental testing.
- Evaluate precast cap connections and develop connections ideal for pretensioned bent caps.
- Evaluate the use of interior voids to reduce substructure weight and enhance constructability.
- Develop details and design recommendations to enable implementation in multicolumn substructures.

The investigation of the precast, pretensioned bent caps was conducted via four tasks aimed at meeting the technical objectives. Task 1 provided a comprehensive review of literature and the state-of-practice for precast bent caps, including connections. Task 2 identified design objectives and provided preliminary designs for pretensioned bent caps. In Task 3, a full-scale experimental test program investigated behavior of pretensioned bent caps and the influence of key design variables. Volume 1 documents the finding of Tasks 1 to 3. In Task 4, design recommendations and design procedures were developed; the results of Task 4 are contained in this document.

## 1.3 SUMMARY OF VOLUME 1

Volume 1 presented 1) a detailed literature review including a review of column-to-cap connections to guide the development of a new precast connection suitable for pretensioned bent caps, 2) design considerations and preliminary design recommendations for pretensioned bent caps, including flexure, shear, and end region detailing, and 3) an experimental test program to investigate performance of bent caps under demands representative of the indeterminate demands seen in multicolumn bridge substructures. Six full-scale sub-assemblages were tested, including a RC control test, a baseline pretensioned design, and four variations of the baseline pretensioned design that investigated the impact of shear reinforcement spacing, the amount of prestressing, use of internal voids, and void detailing.

A summary of preliminary design recommendations and application to a suite of standard bridges, included the following:

- The amount of prestressing can be determined by design for zero-tension under dead load and selecting concrete strength to ensure that the desired service and or ultimate limit states are met. For the suite of bridges found in TxDOT standards, enforcement of a minimum concrete strength of 6 ksi led to many designs expected to remain uncracked even under ultimate demands.
- Application of AASHTO LRFD Sectional Design procedures to pretensioned bent caps leads to many designs that are controlled by the minimum area of steel and/or spacing requirements. The crack angles used in design are often times physically inadmissible, highlighting potential problems in implementing sectional design methods for pretensioned bent caps.
- A pocket connection with dowel bars was proposed to enable large construction tolerances and use of concrete in place of grout. The pocket is formed by a corrugated metal pipe that also provides resistance to prestressing forces and confinement/shear reinforcement during loading of the bridge. The size of the pocket and dowel configuration can be optimized to enable the desired tolerances while providing adequate wall thickness for prestressing. The use of a side configuration of prestressing strands ensures that the bottom of the pocket is clear for dowels to fit easily within the pocket.

Key findings of the experimental test program included:

- Flexure and shear cracking appeared sooner than expected in pretensioned caps, although the observed cracks were significantly lower in quantity, length, and width than the companion RC bent cap. Upon removal of loads, the cracks in the pretensioned bent caps closed, validating the proposed flexural design philosophy.
- Reducing the amount of shear reinforcement in solid bent caps had a minimal impact on the observed performance. This was primarily on account of the prestressing increasing the cracking shear strength significantly such that shear cracks only formed in deep beam regions where the shear resistance is provided mainly by a strut mechanism. The shear reinforcement primarily affected the flexural cracks, with cracks forming at the locations

of the shear reinforcement. When greater spacing was used, fewer cracks formed, and the crack width was typically larger than in the bent cap with smaller spacing, although average crack widths were consistent.

- The use of interior voids had minimal effect on flexural cracking under design loads; however, shear cracks were observed along the void. Peak load carrying capacity of the solid and voided bent caps was similar, but the failure was more brittle in the voided specimen. Increasing the amount of prestressing was found to delay flexural cracking significantly, but had only a minor influence on the cracking shear. Modification of the void length had a slight impact on the shear crack details at service level loads but the impact was negligible at higher loads. In deep beam regions, chamfered corners appeared beneficial, but conclusive recommendations cannot be made on account of different loading conditions.

## 1.4 OVERVIEW OF VOLUME 2

In Volume 2, the results of the experimental test program are used to inform the development of revised design recommendations. For flexure, the preliminary design procedure of zero tension under dead load is retained, with modifications to account for the early onset of initial flexural cracking. For shear, the AASHTO sectional design procedure is modified to ensure crack angles used in design are physically admissible. Design recommendations for both shear and flexure include alternatives that allow for conversion of a RC design to a pretensioned alternative. Recommendations for design and detailing of end regions and connections are made. To demonstrate implementation of the proposed design recommendations, four design scenarios are considered.

In the first design scenario, the conversion of a RC cap to a pretensioned cap is demonstrated. Such a scenario is the origin of the use of pretensioned bent caps in TxDOT multicolumn bridges and will arise when pretensioned caps are desired not as a means to improve performance of caps, but to give contractors an additional option for fabrication. While precast RC bent caps have been used in TxDOT since the early 1990s and have been a standard option since early 2000s, pretensioned bent caps can allow fabrication to occur off-site, with prestressing allowing for avoidance of tensile stresses during shipping and placement.

In the second design scenario, bent caps are designed to illustrate use of the proposed design procedure for new designs. This may be done 1) in parallel with RC design to offer contractors options, 2) for the development of pretensioned standards, or 3) as the solo option for design, providing a means to deliver enhanced performance of bent caps that eliminate cracking under service level loads, thus extending the expected service life of the substructure. Two examples are provided. The first, Example #2, is a new design of the bridge in design scenario #1 to demonstrate application of the design procedures for the development of standards and to provide context for the conversion design. The second, Example #3, is a single cap for a four-lane divided highway.

In the first two design scenarios, only solid sections are considered for bent caps. In the case of longer bent caps, it may become prohibitive to ship and/or place the bent caps on account of the large weights. Thus, the use of internal voids is explored. The addition of a void can lighten the bent cap to allow it to be more manageable, but additional concerns arise due to cracking, particularly shear. Example #4 demonstrates implementation of a void in the same substructure designed in Example #3.

In the final design scenario (Example #5), the use of pretensioned bent caps is explored as a means to reduce construction time and costs by eliminating column lines from multicolumn bent caps. To illustrate this, the substructure in Example #3 is economized by reducing the number of columns from six to four columns; longer overhangs used are used in Example #5.

Detailing of the end region and design of the pocket connection are included in Example #2.

## 1.5 ORGANIZATION OF VOLUME 2

Chapter 2 presents the proposed design procedure for transforming a RC design to a pretensioned design. For new designs, the flexure and shear procedures are described in Chapters 3 and 4, respectively. Detailing is discussed in Chapter 5, with draft design specifications and drawings provided in Appendices A and B. Chapter 6 presents an overview of the five design examples, along with a discussion of the results. Finally, Chapter 7 provides a summary of findings and recommendations for future work.

# 2 DESIGN BY TRANSFORMATION OF RC DESIGNS

## 2.1 OVERVIEW AND BASIS FOR DESIGN TRANSFORMATION

When a RC bent cap design is put out for bid, the general contractor may wish to submit a bid based on construction that uses an alternative precast prestressed concrete solution. It may be highly likely that such a solution is very competitive. The economies of such a solution may be negated if a costly redesign is conducted by consultants retained by the general contractor.

To enable the use of precast pretensioned alternatives, the contractor must provide a functionally equivalent design. As an alternative to a full redesign requiring analysis of demands, a formal transformation of the reinforced solution (provided in the bid documents) to a pretensioned solution is desired.

This chapter presents a procedure for transformation of a reinforced solution to a prestressed solution. Section 2.2 presents the transformation for flexural design. Section 2.3 presents the transformation for shear design..

## 2.2 FLEXURAL DESIGN TRANSFORMATION

The concept of the transformation from reinforced to prestressed bent caps is to ensure the flexural strength is not reduced. The required number of strands ($n_{req'd}$) can be simply calculated by the equation below:

$$n_{req'd} A_{ps} (0.75 f_{pu}) \geq (A_{s\_top} + A_{s\_bot}) f_y$$

$$n_{req'd} \geq \frac{(A_{s\_top} + A_{s\_bot}) f_y}{0.75 A_{ps} f_{pu}} \tag{2-1}$$

where $A_{ps}$ = area of a single prestressing strand (in.$^2$); $f_{pu}$ = ultimate strength of prestressing steel (ksi); $A_{s\_top}$ = area of longitudinal reinforcement in top half of the section (in.$^2$); $A_{s\_bot}$ = area of longitudinal reinforcement in bottom half of the section (in.$^2$); and $f_y$ = specified yield strength of reinforcing bars.

The number of strands is rounded up or adjusted to be placed evenly along the sides of the section. For a concentric layout, $n_{req'd}$ needs to be a multiple of 4, with a symmetric layout about x and y axes.

After the selection of the strand layout, a cracking moment ($M_{cr}$) and nominal flexural resistance ($M_n$) of the section shall be calculated. It shall be checked if a factored flexural resistance ($M_r$) is greater than or equal to $M_{cr}$ to prevent a brittle failure.

## 2.3 SHEAR DESIGN TRANSFORMATION

In transforming a RC bent cap to a pretensioned bent cap, the shear strength of the bent cap will be influenced by the prestressing selected for flexure design. Prestressing will have the effect of decreasing the crack angle, resulting in more transverse reinforcement providing shear resistance. Additionally, prestressing will have the effect of increasing the shear resistance of the concrete. With increased steel and concrete contributions, the nominal shear strength ($V_n$) will increase. It would therefore be permissible to preserve the shear reinforcement provided in the RC design. However, the benefits of prestressing may be used to reduce the shear reinforcement if so desired. In the event a redesign is desired, a transformation procedure is required.

Using AASHTO LRFD design provisions requires the use of the shear and moment demands to calculate the nominal shear strength ($V_n$). If this information is not provided on bid documents and the contractor wishes to avoid a full redesign, it is necessary to provide an alternative design approach that is independent of the demands. To enable this, the capacity design philosophy is used.

In general, ductile failure modes are preferable to brittle failure modes. In other words, undesirable failure modes may be intentionally prevented by increasing their strength compared to those of the preferable failure modes; this is the basic philosophy of the capacity design (Mander et al. 1998).

For concrete structures, it is desirable that the shear strength exceeds the flexural strength to prevent a brittle premature shear failure, accompanied by a rapid deterioration of strength and stiffness. Thus, in the shear transformation it is ensured that a factored shear capacity ($\phi V_n$) is always greater than or equal to the shear demand ($V_u$) imposed by a plastic flexural mechanism of the structure. This inhibits a brittle shear failure prior to flexural failure. Such a transformation may be achieved without analyzing actual moment and shear demands. Instead, a RC design may be transformed using the bent cap geometry and material properties.

Required steps for capacity design of shear are detailed below, with a flowchart of the procedure provided in Figure 2.1. In-depth explanation will be presented in the following paragraphs:

- Step 1: Calculate assumed angle of shear cracks.
- Step 2: Calculate shear demand $V_u$ associated with the plastic collapse mechanism.
- Step 3: Calculate transverse reinforcement required.
- Step 4: Check minimum transverse reinforcement.

Figure 2.2 illustrates variables required for the shear transformation, where $A_{s\_top}$ = area of longitudinal reinforcement in the top half of the section (in.$^2$); $A_{s\_bot}$ = area of longitudinal reinforcement in the top half of the section (in.$^2$); $A_{st}$ = area of both tension and compression reinforcement (in.$^2$); ($A_{st}$=$A_{s\_top}$+$A_{s\_bot}$) ; $A_v$ = area of a transverse reinforcement (in.$^2$); $f_y$ = specified yield strength of longitudinal reinforcement (ksi); $f_{yh}$ = specified yield strength of transverse reinforcement (ksi); $f'_c$ = specified compressive strength of concrete (ksi); $b_v$ = width of cross section at contact surface being investigated for horizontal shear (in.); $d$ = distance from extreme compressive fiber to centroid of longitudinal tension reinforcement (in.); $d'$ = distance from extreme compression fiber to centroid of longitudinal compression reinforcement (in.); $d_v$ = effective shear depth (in.); ($d$ – $d'$) can be used for simplicity; $a$ = distance between column face and bearing pad face (in.), referred as to shear span; and $L$ = distance from the center of a column to the center of a girder (in.).

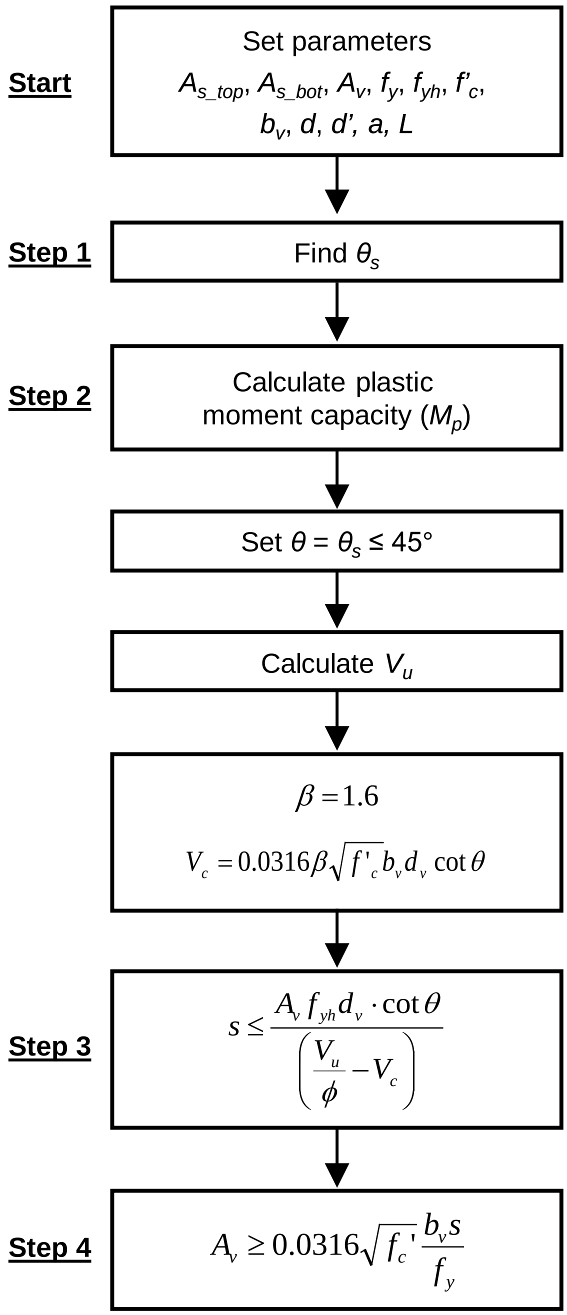



**Figure 2.1. Flowchart for the Proposed Shear Conversion.**

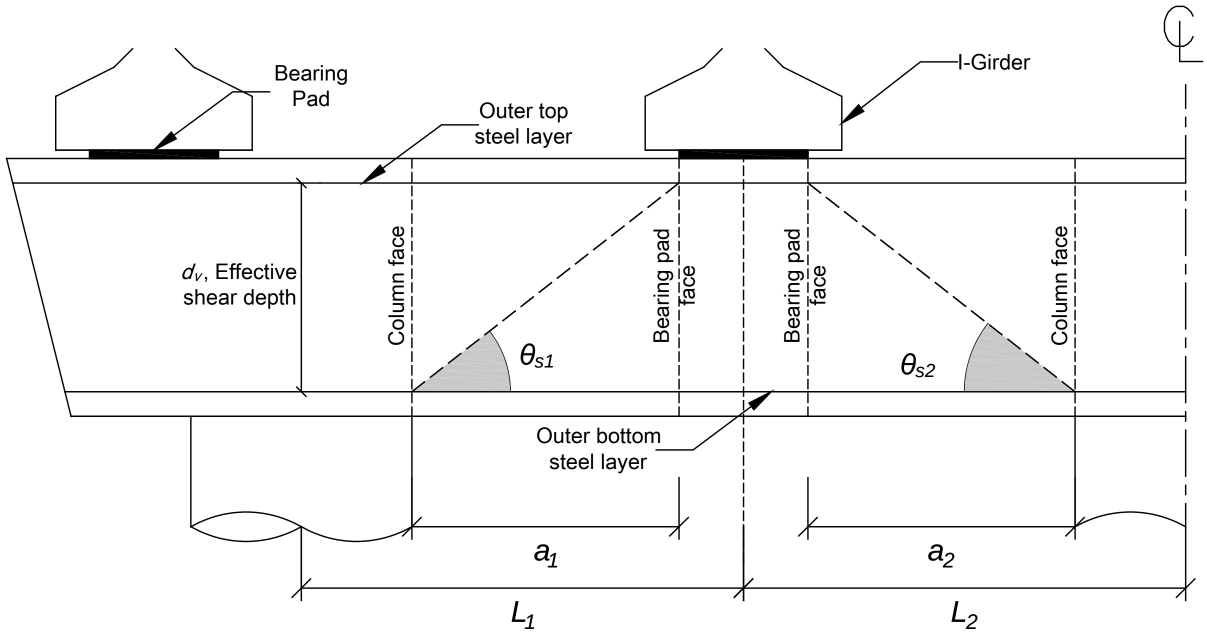



**Figure 2.2. Variables Required in the Shear Conversion.**

### 2.3.1 Step 1: Calculate Assumed Angle of Shear Cracks

A feasible crack angle, referred to as the compressive strut angle, $\theta_s$, shall be found from the bent cap configuration. The compressive strut angle indicates an angle between the column face and the bearing pad face within the shear depth, as shown in Figure 2.2. The crack angle $\theta$ used for calculation of the shear strength shall be taken as $\theta_s$, but no larger than 45°.

### 2.3.2 Step 2: Calculate Shear Demand $V_u$ Associated with the Plastic Collapse Mechanism

The total plastic moment is calculated as:

$$M_p^- + M_p^+ = A_{st} f_y d_v \tag{2-2}$$

where $M_p^+$ = positive plastic moment capacity; $M_p^+ = A_{s\_bot} f_y d_v$ (k-ft); and $M_p^-$ = negative plastic moment capacity, $M_p^- = A_{s\_top} f_y d_v$ (k-ft).

Using the plastic moment capacities, the associated shear demand ($V_u$) can be determined based on the length span between the moments. The factored nominal shear capacity ($V_r$) shall be greater than this demand as specified in AASHTO LRFD 1.3.2.1-1.

$$V_u = \frac{M_p^{\ +} + M_p^{\ -}}{L} \tag{2-3}$$

$$V_u \leq V_r = \phi V_n$$

where $L$ = distance from the center of a column to the center of a girder (in.).

### 2.3.3 Step 3: Calculate Transverse Reinforcement Required

The nominal shear capacity is the sum of the shear resistances provided by concrete and transverse reinforcement as given in AASHTO LRFD 5.8.3.3-1. AASHTO LRFD determines the concrete contribution as a function of strain in the longitudinal reinforcement. The strain in turn is calculated as a function of the geometry, longitudinal reinforcement, and the ultimate demands. As the transformed design are done without knowing the demands, an alternative method is needed. The shear resistance can be modified from the equation in AASHTO LRFD 5.8.3.3-3 was modified to consider the actual shear crack angle and the geometric characteristics of bent cap members as:

$$V_c = 0.0316\beta\sqrt{f'_c}\, b_v d_v \cot\theta \tag{2-4}$$

where $\beta$ = 1.6, and is derived from lower limits of the values, with consideration for the appropriate angle $\theta$.

Shear resistance provided by transverse reinforcement is obtained by:

$$V_s = \frac{A_v f_{yh} d_v \cot\theta}{s} \tag{2-5}$$

The required transverse reinforcement spacing, $s$, can be obtained as:

$$s \leq \frac{A_v f_{yh} d_v \cdot \cot\theta}{\left(\frac{V_u}{\phi} - V_c\right)} \tag{2-6}$$

By substituting Eqs (2-2) through (2-5) into (2-6), it can be shown:

$$s \le \frac{A_v f_{yh} d_v}{\left( \frac{A_{st} f_y d_v \tan\theta}{\phi \cdot L} - 0.0316\beta\sqrt{f'_c} b_v d_v \right)} \tag{2-7}$$

where $\beta$ = 1.6.

### 2.3.4 Step 4: Check Minimum Transverse Reinforcement Requirement

Finally, check that the spacing from Step 3 satisfies the minimum transverse reinforcement requirement in accordance with AASHTO LRFD 5.8.2.5.

$$A_{v\_min} = 0.0316\sqrt{f'_c}\frac{b_v s}{f_y} \qquad \text{(AASHTO LRFD Eq. 5.8.2.5-1)}$$

$$A_{v\_prov'd} > A_{v\_min}$$

# 3 DESIGN FOR FLEXURE

## 3.1 OVERVIEW

The previous chapter detailed design of pretensioned bent caps based on RC bent cap design drawings. To provide design of pretensioned caps as an original design, a design procedure based on the calculated demands is required. Design of pretensioned caps may be motivated by the desire for improved performance (resistance to cracking), use of interior voids to reduce weight, or to reduce the number of column lines. This chapter presents the proposed design procedure for flexure design. Chapter 4 documents shear design. Chapter 5 documents detailing and connections. Chapter 6 presents application of the design recommendations.

The recommended procedure for the design for flexure largely follows the preliminary flexure design procedure documented in the companion report to this document (Birely et al. 2018). Modifications to the design procedure are made based on the findings of full-scale bent cap tests documented in the companion report. These modifications account for the observed flexural cracking, the impact of the pocket connection on the flexural response, and the use of eccentric strand layouts. Section 3.2 discusses the cracking moment and implications for design. Section 3.3 details the modified flexure design procedure. Section 3.4 provides a discussion of application of flexure design for sections with eccentric strand layouts.

## 3.2 CRACKING MOMENT

The full-scale pretensioned bent caps cracked at moment demands lower than those predicted by pre-testing calculations. Using measured concrete compressive strengths on test day, the tensile stress associated with first observed cracking was found to range from $0.131\text{-}0.145\sqrt{f'_c}$ ($4.1\text{-}4.6\sqrt{f'_c}$ in psi) for negative bending, and $0.119\text{-}0.152\sqrt{f'_c}$ ($3.8\text{-}4.8\sqrt{f'_c}$ in psi) for positive bending. For positive moment calculations, the appropriate solid or voided cross-section was used to back calculate the tensile stress. For negative moment calculations, the net section at the center of the pocket was used. Based on these findings, the preliminary design procedure is modified to use a tensile stress limit of $0.126\sqrt{f'_c}$ ($4\sqrt{f'_c}$ in psi) to determine the concrete compressive strength that will ensure that cracking is avoided at service demands. This stress limit is in addition to stress limits in AASHTO LRFD.

The impact of these findings does not have an effect on the design of bent caps to provide zero tension under dead load, but does influence the expected performance. In the preliminary design studies by Birely et al. (2018) and Barooah (2016), standard TxDOT bridges were designed using the preliminary flexural design procedure. For all design, no cracking was expected at Service 1 demands. For most design, no cracking was expected at Strength 1 demands. The implication of the experimental findings is that the no cracking design objective may not be met; however, the preliminary flexural design procedure step used the service demand to establish a concrete strength that would ensure no cracking. In the preliminary design studies, this value was less than the minimum specified and the minimum was used in design.

## 3.3 PROPOSED FLEXURE DESIGN APPROACH

This section presents the recommended flexure procedure for the design for flexure of prestressed concrete bent caps. The proposed procedure is based on selecting strands to achieving zero tension under dead loads. This is done to allow any cracks formed under overload condition (up to ultimate strength) to close under the full removal of live loads. Once the number of strands is selected, remaining steps are to verify the design satisfies AASHTO LRFD requirements, and if desired, to minimize the possibility of cracking under all other loading conditions, in particular to avoid cracking under normal service loads. The steps for flexural design are summarized below, with elaboration in the subsequent paragraphs:

- Step 1: Determine number of strands.
- Step 2: Determine required minimum concrete compressive strength.
- Step 3: Check that the minimum number of strands is satisfied.
- Step 4: Check ultimate strength capacity.
- Step 5: Check deflections.

### 3.3.1 Step 1: Determine Number of Strands

Figure 3.1 presents the governing design stress conditions that generally determine the number of strands for a specific bent cap design. The aim is to achieve zero tension under dead load. Therefore, flexural stresses under dead load should remain compressive at the extreme tension fiber:

$$-\frac{F}{A}+\frac{M_{DL}}{S_x}< f_t =0 \qquad (3\text{-}1)$$

and within the normal service limits at the extreme compression fiber:

$$-\frac{F}{A}-\frac{M_{DL}}{S_x}> f_c =-0.45 f_c^{'} \qquad (3\text{-}2)$$

in which $F$ = prestress force after losses; $M_{DL}$= dead load moment; $f'_c$ = specified compression strength of the concrete; $A$ = cross-sectional area of the bent cap at the section of interest; $S_x$ = section modulus of the bent cap at the section of interest. When determining the bent cap sectional properties, it is important to consider the cap-to-column connection type and the use of interior voids. Due to potential variation in the cross-sections, both the positive and negative moment regions must be considered in the design. For the use of a pocket column connection, use the untransformed, hollow section properties at the location of the pocket connection for considering the negative moment region.

The number of strands to achieve the required prestressing force to achieve zero tension under dead load is calculated as:

$$n_t =\frac{F_t}{T_{strand}} \qquad (3\text{-}3)$$

The number of strands to achieve the required prestressing force to reach the compressive stress limit under dead load is calculated as:

$$n_c =\frac{F_c}{T_{strand}} \qquad (3\text{-}4)$$

where $T_{strand}$ = prestressing force per strand and is calculated as:

$$T_{strand} = f_{pbt} A_{PS} (1-\Delta f_{pT}) \qquad (3\text{-}5)$$

in which $f_{pbt}$ = $0.75f_{pu}$ = stress limit in low relaxation strand immediately prior to transfer; $f_{pu}$ = specified tensile strength of prestressing strand = 270 ksi (AASHTO LRFD Table 5.4.4.1-1); $A_{ps}$ = area of each strand = 0.217 in.$^2$ for 0.6-in. diameter strand; and $\Delta f_{pT}$ = prestress loss, in lieu of a more precise analysis 20 percent time dependent losses shall suffice.

The number of strands, $n$, is selected from Equation (3-3) and is rounded up to the nearest multiple of 2 or 4 for a symmetric arrangement of strands in the bent cap. To satisfy compressive stress limits, $n$ must not exceed $n_c$.

The provided prestressing force, $F$, is determined from the selected number of strands, $n$, multiplied by the prestressing force per strand, $T_{strand}$.

### 3.3.2 Step 2: Determine Required Minimum Concrete Compressive Strength

To ensure that the bent cap does not crack at service loads, a minimum concrete compressive strength should be provided such that the service stresses are less than or equal to the service stress limits specified in AASHTO LRFD or the recommended limit specified by the research team.

The tensile and compressive stresses are calculated from the service load moments (see Figure 3.1). The tensile and compressive stress should be computed in both the positive and negative moment regions due to the variation in cross-section.

$$-\frac{F}{A}+\frac{M_{SL}}{S_x} \leq f_t = k\sqrt{f'_c} \tag{3-6}$$

$$-\frac{F}{A}-\frac{M_{SL}}{S_x} \geq f_c = -0.45 f'_c \tag{3-7}$$

in which $M_{SL}$ = service load moment due to dead load and live load with impact; $f_t$ = tension stress; and $f_c$ = compression stress.

The design concrete compressive strength must be selected such that all service stress limits are met. For tension, the AASHTO tension limits (Table 5.9.4.2.2-1) must be satisfied at a minimum. Alternatively, the enhanced tension limits proposed by researchers may be used to minimize the possibility of cracking at service loads. For compression, the AASHTO compressive (Table 5.9.4.2.1-1) service stress limits are met. Stress limits are presented as a function of the concrete compressive strength, in the format:

$$f_t \le k\sqrt{f_c^{'}} \tag{3-8}$$

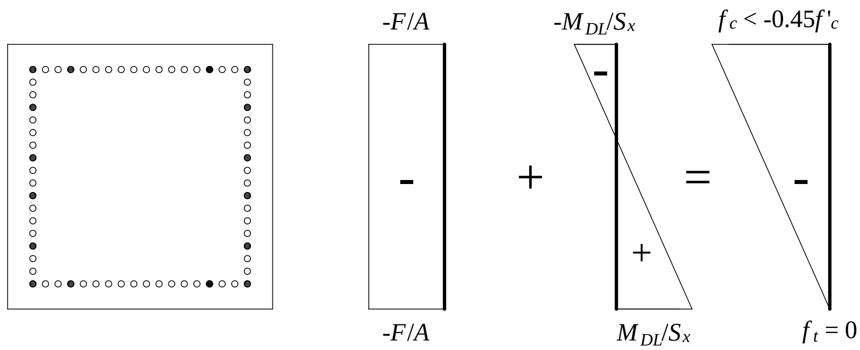
$$f_c \ge -0.45 f_c^{'} \tag{3-9}$$

where $k$ is the multiplier found from AASHTO Table 5.9.4.2.2-1, or from the enhanced tensile limit proposed by researchers. Values of $k$ are summarized as follows:

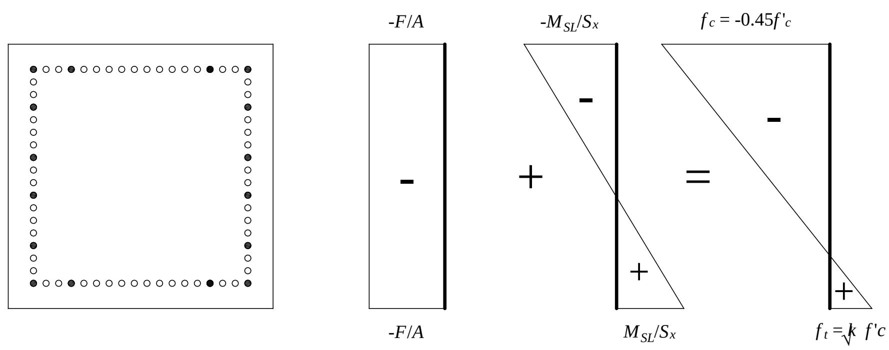
| | | |
|---|---|---|
| AASHTO Class I Exposure: | Moderate corrosion environment | $k = 0.19$ |
| Enhanced Class I Exposure: | Moderate corrosion environment | $k = 0.126$ |
| AASHTO Class II Exposure: | Severe corrosion environment | $k = 0.0948$ |

Depending on the exposure class, and the objective to limit cracking under service load demands, the appropriate tensile stress limit multiplier, $k$, is chosen and used in Equation (3-8) to determine the limiting tensile stress.

Solving Equations (3-8) and (3-9) for $f'_c$, one obtains the required minimum concrete compressive strength to meet both tensile and compressive stress limits. The minimum concrete stress $f'_{c\_min}$ is determined from the greater of the calculated minimum $f'c$ for both tensile and compressive stress limits. If the calculated value of $f'_{c\_min}$ is less than 5 ksi, a minimum design concrete compressive strength of 5 ksi is recommended. If the required minimum concrete compressive strength exceeds the maximum allowed by the TxDOT BDM-LRFD (8.5 ksi), adjust the number of strands and recompute Step 2.




(a) *Stresses under Dead Load: No tension*




(b) *Stresses under Service Load: Establish Minimum Concrete Strength*

**Figure 3.1. Stress Diagrams for Prestressed Concrete Bent Caps with Side Strand Configuration.**

### 3.3.3 Step 3: Check That the Minimum Number of Strands is Satisfied

To preclude a brittle failure of the bent cap it is necessary to check that the flexural resistance is greater than the cracking moment.

AASHTO LRFD Section 5.7.3.3.2 specifies that the amount of prestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, $M_r$, which is at least equal to the lesser of a) 1.33 times the ultimate moment, $M_u$, and b) cracking moment, $M_{cr}$. The cracking moment is given by AASHTO for a non-composite section as:

$$M_{cr} = \gamma_3(\gamma_1 f_r + \gamma_2 f_{cpe})S_x \tag{3-10}$$

in which $M_{cr}$ = cracking moment; $f_r = 0.24\sqrt{f'_c}$ (AASHTO LRFD 5.4.2.6); $f_{cpe}$ = compressive stress due to the prestressing; and $S_x$ = section modulus. For precast, pretensioned components with bonded flexural reinforcement: $\gamma_1 = 1.6$; $\gamma_2 = 1.1$; $\gamma_3 = 1.0$. The cracking moment, $M_{cr}$, should be calculated using the calculated minimum concrete compressive strength determined in Step 2.

By providing the minimum number of strands, $n_{min}$, the following requirements are satisfied:

$$\phi M_n = M_r \geq M_{cr} \tag{3-11}$$

$$M_r \geq 1.33 M_u \tag{3-12}$$

where $M_n$ = the approximate moment capacity of the bent cap at the section of interest; $\phi = 1.0$; and $M_u$ = the applied ultimate moment demand at the section of interest.

$M_n$ can be approximated by:

$$M_n \geq n A_{strand} f_y jd \tag{3-13}$$

where $n$ = number of prestressing strands; $A_{strand}$ = area of prestressing strand (0.217 in.$^2$); $f_y$ = yield strength of prestressing strand; and $jd$ = approximate internal moment arm of the section (0.45$D$).

By rearranging Equations (3-10) and (3-13) and solving for $n$, the number of strands to satisfy Equation (3-11) is determined from:

$$n \geq \frac{\gamma_1 f_r S_x}{A_{strand} f_y jd - \frac{\gamma_2 T_{strand} S_x}{A}} \tag{3-14}$$

By rearranging Equations (3-10), (3-11), and (3-12) and solving for $n$, the number of strands to satisfy Equation (3-12) is determined from:

$$n \geq \frac{1.33 M_u}{A_{strand} f_y jd} \tag{3-15}$$

The minimum number of strands, $n_{min}$, to prevent a brittle failure is determined from the maximum of the number of strands, $n$, determined from Equations (3-14)and (3-15). If the $n$ selected during Step 1 is less than the $n_{min}$, increase the number of strands and recheck the service stress limits in Step 2.

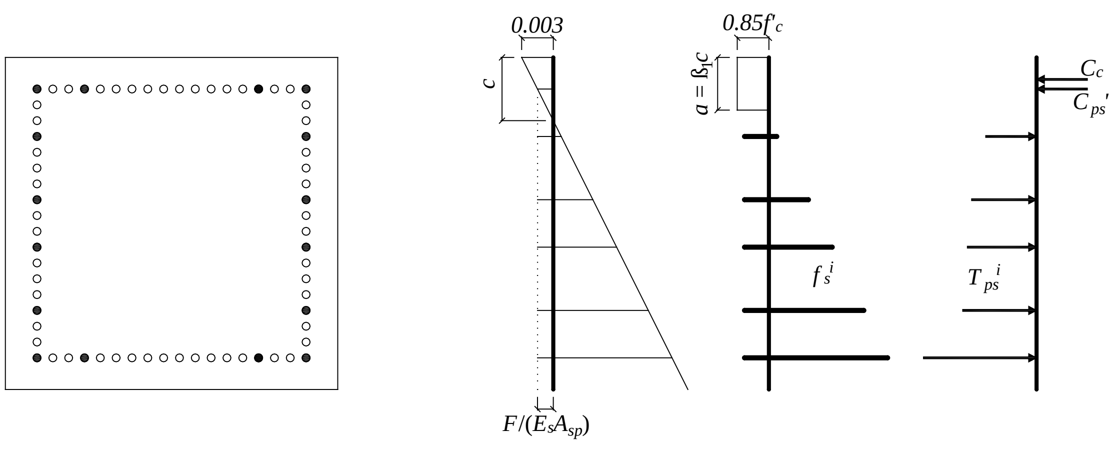
### 3.3.4 Step 4: Check Ultimate Strength Capacity

The bent cap should have at least the nominal strength capacity such that it does not fail under ultimate loads. The ultimate flexural moment capacity ($M_n$) is calculated per AASHTO LRFD 5.7.3.2 (see Figure 3.2) and evaluated against the demands:

$$\phi M_n \geq M_u \tag{3-16}$$

in which $M_u$ = flexural demand under ultimate loads; $\phi$ = 1.0 for tension-controlled prestressed concrete sections (AASHTO LRFD 5.5.4.2.1). If $\phi M_n < M_u$, the prestressing force should be increased such that Equation (3-16) is satisfied. For a side configuration prestressing strand layout, $M_n$ should be calculated using the strain compatibility method.

Once the final strand layout, sectional properties, and material properties are chosen and the flexural resistance, $M_r$, of the bent cap is determined, AASHTO LRFD 5.7.3.3.2 requirements should be rechecked to ensure that the factored flexural resistance is adequate to prevent brittle failure.




*Note: $A_{sp}$= area of prestressing strand; $F/E_sA_{sp}$ = prestrain, after losses; F = prestressing force, after losses*

**Figure 3.2. Ultimate Strength Capacity of Prestressed Concrete Bent Caps with Side Strand Configuration.**

### 3.3.5 Step 5: Check Deflections

To ensure that the deflection of the bent cap does not affect serviceability, the deflection should be checked to be within the specified limit.

The deflection, $\Delta$, under vehicular loading should be less than the limit specified in AASHTO LRFD 2.5.2.6.2, specifically:

$$\Delta < Span / 800 \tag{3-17}$$

## 3.4 DESIGN WITH ECCENTRIC STRAND LAYOUT

For many bent cap designs, the use of concentric strand layouts is sufficient to provide the necessary strength for both positive and negative bending; however, some bridges, particularly those with larger column spacing, will have considerably different demands for positive and negative bending. For these bridges, designs with a concentric layout may result in an excessive number of strands or large deflections. To alleviate this, an eccentric strand layout may be used. For eccentric design, stress limits, including tensile stresses at dead load and the enhanced service tensile stress limits, can be used to establish a Magnel diagram to optimize the number of strands and eccentricity. An alternative approach that may be more palatable within the proposed design procedure is to modify the selection of the strands to design for the average moment demand and assign eccentricity to account for the difference. To do so, the average moment is calculated and used to calculate the number of strands that will produce zero tension under dead load at the section with the larger magnitude demand. The difference is then applied to calculate the necessary eccentricity and from that the strand layout. Once this is established, the remaining steps are unchanged except for accounting for the eccentricity in stress checks.

# 4 DESIGN FOR SHEAR

## 4.1 OVERVIEW

In this chapter, a shear design procedure for prestressed concrete bent caps is proposed. This shear design approach is devised to ensure that present AASHTO LRFD shear design provisions are not misapplied in an inadmissible fashion as described in the companion document (Birely et al. 2018). Before establishing the shear design procedure, several key points that are necessary in accurately establishing shear resistance, which is primarily related to crack angles, are discussed in Sections 4.2 and 4.3. Section 4.4 gives the consideration of cracking shear in the design. Section 4.5 presents an overview of the proposed design philosophy and design procedure. Finally, the detailed design procedure for is presented in Section 4.6.

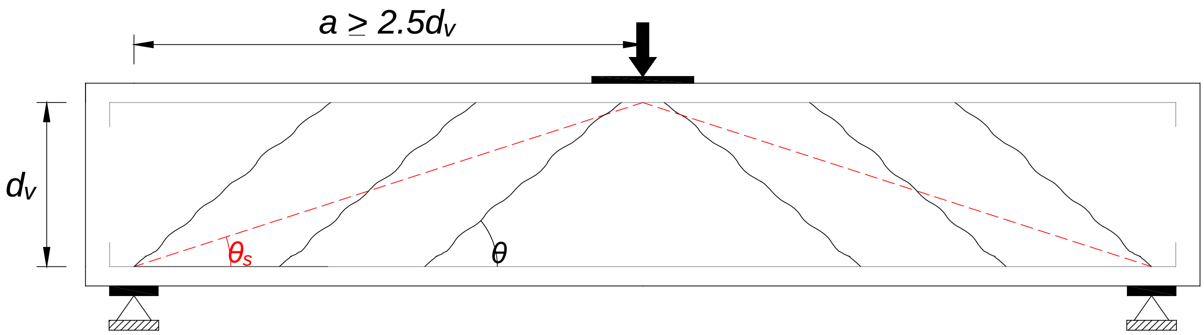
## 4.2 SHEAR CRACK ANGLE

The diagonal shear crack angle is an important factor in concrete structures since it influences the post-cracking behavior of concrete members (Marti 1985; Rogowsky and MacGregor 1986; Vecchio and Collin 1986; Aoyama 1993; Collins et al. 1996; Kim and Mander, 1999; 2007). Such studies have shown that the shear span-depth ratio is the major variable affecting the shear strength and the diagonal crack behavior of concrete members. The shear span-depth ratio may be defined as:

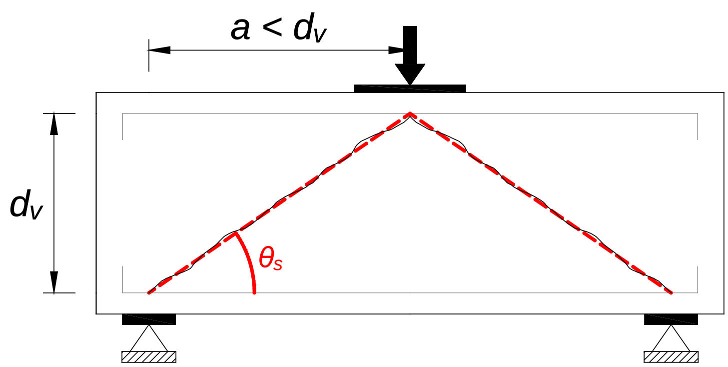
$$a / d = \cot\theta_S \tag{4-1}$$

in which $a$ = shear span; $d$ = effective member depth; and $\theta_s$ is the associated corner-to-corner angle for the shear span under consideration.

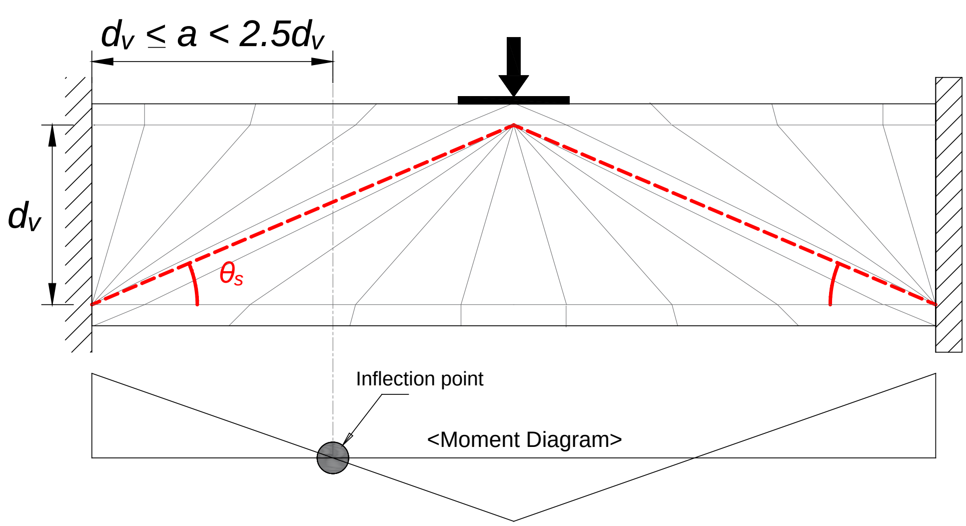
Figure 4.1 presents commonly accepted classifications of beams based on the shear span-depth ratio. In general, a member is considered slender if $a/d \geq 2.5$ ($\theta_s \leq 22°$) or deep if $a/d < 1.0$ ($\theta_s >$ 45), and are governed generally by flexure and shear, respectively. Between these limits, intermediate beams may be governed by either flexure or shear (Kani 1964; Zararis and Papadakis 2001; Brown et al. 2006; Brown and Bayrak 2008a; Brown and Bayrak 2008b; Choi et al. 2007a; Choi et al. 2007b; and Choi et al. 2016).




(a) *Slender beam; Cracks tend to be parallel and steeper than θs*
*Behavior is governed by truss action (θ > θs).*




(b) *Squat beam; A large diagonal crack may form.*
*Behavior is governed primarily by arch action (θ > 45°).*




(c) *Intermediate beam; Cracks tend to form a fan shape, although limit behavior is governed by the principal diagonal crack ($\theta < \theta_s < 45°$)*

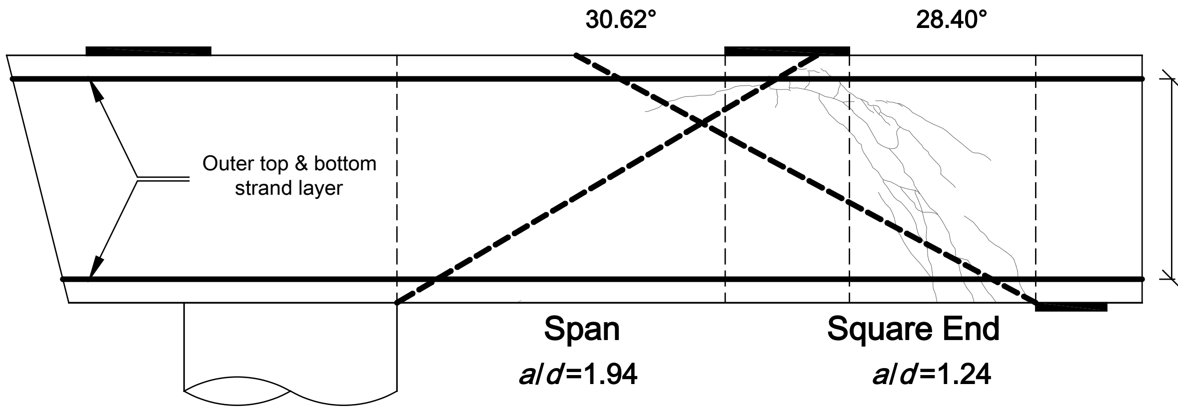
**Figure 4.1. Determination of Slender, Intermediate, and Deep Beams.**

Generally, bent caps have a relatively short shear span with a large shear depth, categorizing them as either intermediate or deep beams. It is well recognized that shear force is transferred to an adjacent support through two principal mechanisms in deep beams depending on the angle, $\theta_s$. For intermediate beams, the combined truss and direct strut actions govern. For very deep beams, the direct strut action (arch action) governs (Matamoros and Wong 2003; He et al. 2012; Mander et al. 2012; Tuchscherer et al. 2014).

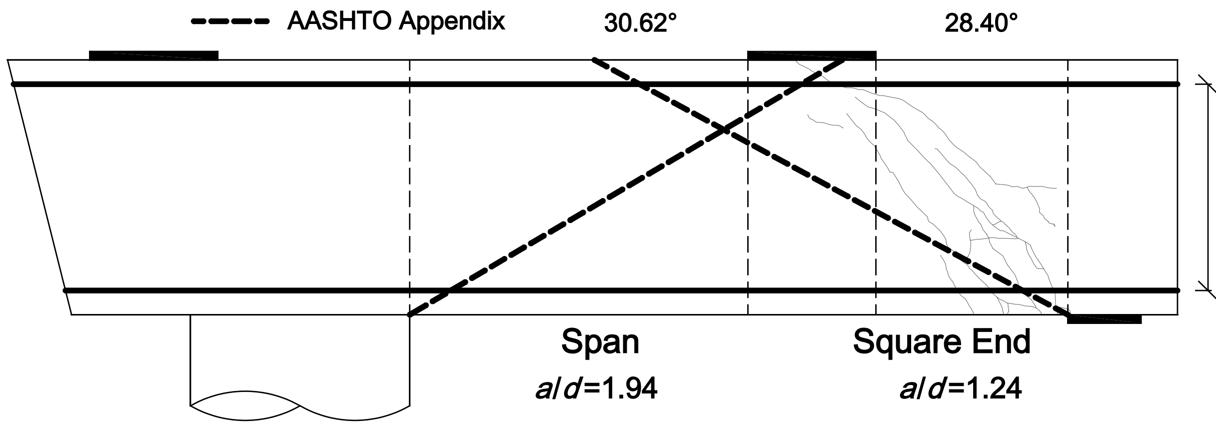
The AASHTO LRFD shear design approach was developed with roots in the Modified Compression Field Theory. The Modified Compression Field Theory was based on tests on shear panel in which truss action mainly governed the shear transfer, and does not take the direct strut action into account in the theory. Therefore, the use of the diagonal crack angle in accordance with AASHTO provisions in calculating the shear resistance, while appropriate for slender beams, may be inadmissible for intermediate and certainly for squat beams where direct strut action governs the shear resistance. To evaluate the appropriateness of the AASHTO shear provisions, the experimental tests by Birely et al. (2018) are considered. Figure 4.2 presents the key observed crack patterns at failure for the experimental tests on the pretensioned concrete cap beams.

Figure 4.3 shows two alternative methods for determining the crack angle that is admissible for use in the shear design of cap beams that will not lead to unconservative design outcomes. Strictly, one should use the azimuth trajectory of the compression strut, as shown by the dashed blue lines. Without developing a full truss solution, the commencement and termination of these lines are ambiguous. Therefore, it is proposed that the corner-to-corner diagonal angles, which are slightly deeper (and, thus more conservative) be used as depicted by the thin red lines in Figure 4.3.

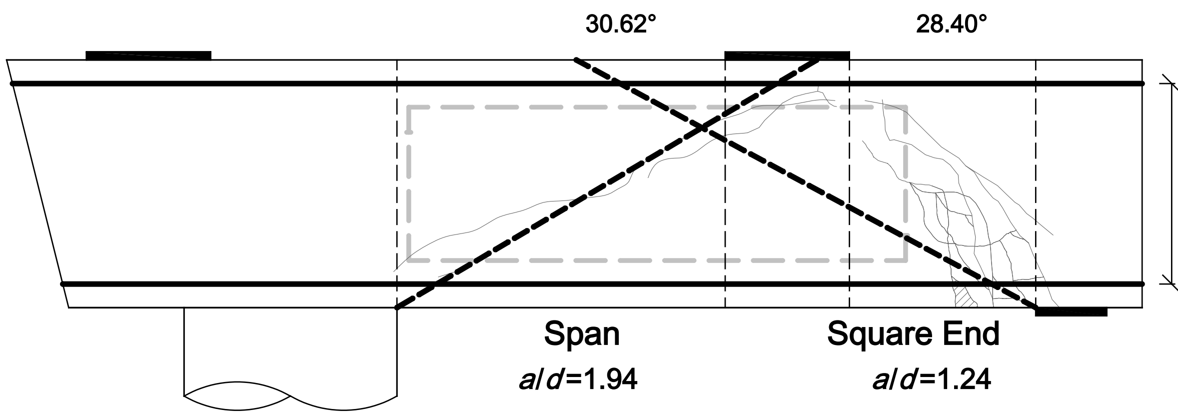
The crack angles observed generally comply with the corner to corner crack angles, $\theta_s$, determined in Figure 4.1. By contrast, the crack angles $\theta$ calculated in accordance with AASHTO provisions are generally smaller than $\theta_s$ ($\theta < \theta_s$). As the calculated shear resistance depends on $\cot\theta$, the use of AASHTO provisions may lead to unconservative resistance due to the physically inadmissible angles used.




(a) *PSS-16-12*

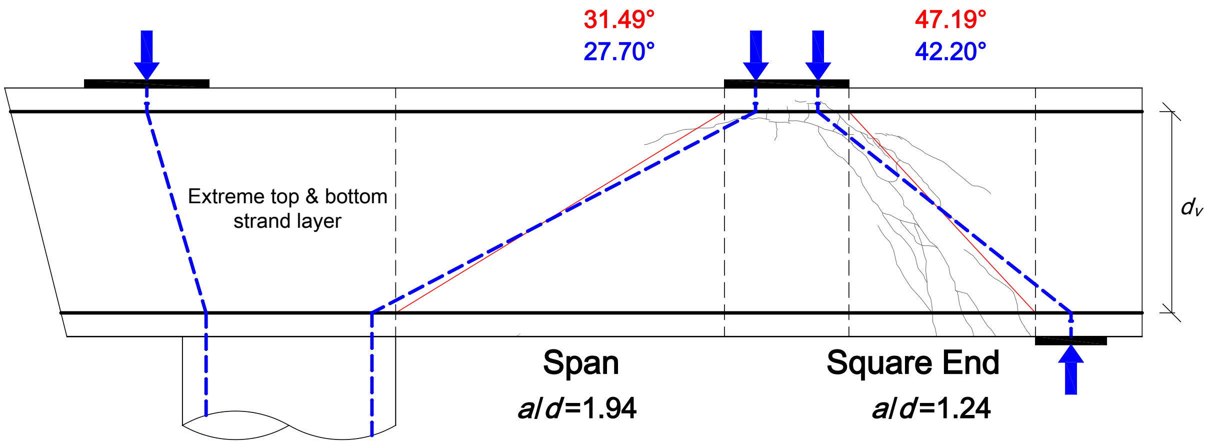



(b) *PSS-16-24*

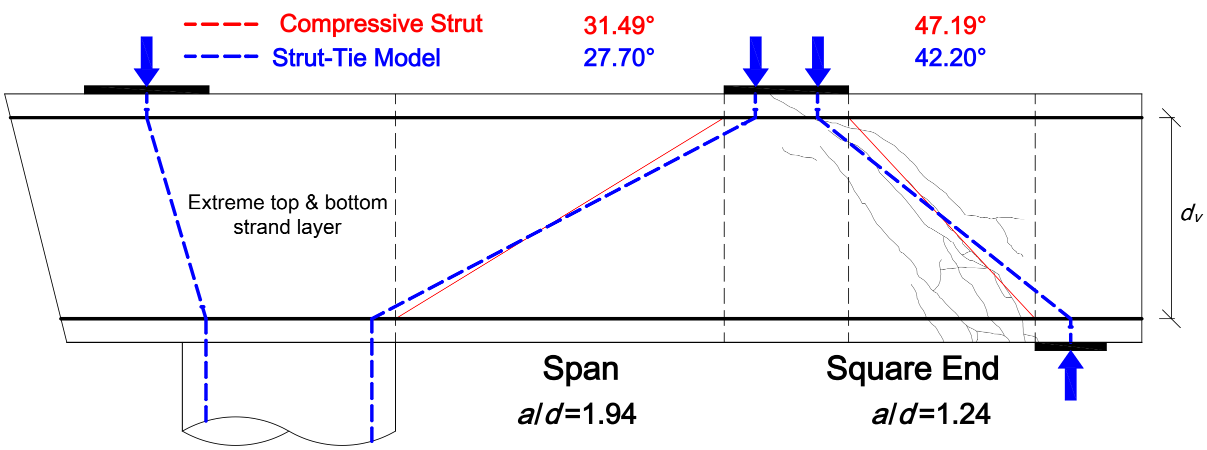



(c) *PSV-16-12*

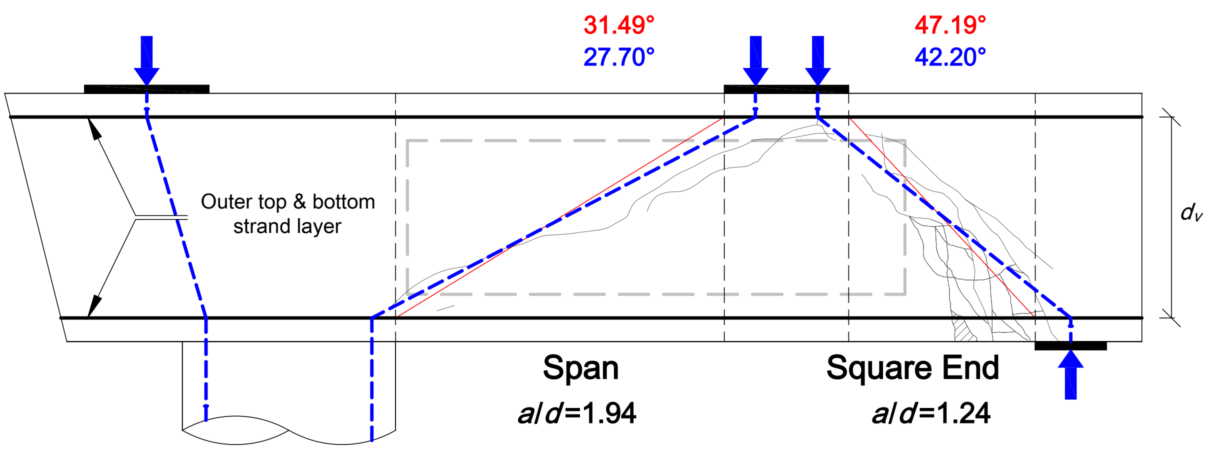
**Figure 4.2. Comparisons of Crack Angle Calculated by AASHTO and Observed Shear Cracks.**




(a) *PSS-16-12*




(b) *PSS-16-24*




(c) *PSV-16-12*

**Figure 4.3. Comparisons of the Proposed Conservative Strut Model Angles (Red) with Truss Model Angles (Blue).**

## 4.3 CONCRETE CONTRIBUTION ON SHEAR STRENGTH

AASHTO provisions specifies the nominal shear resistance provided by concrete, $V_c$, as $0.0316\beta\sqrt{f'_c}b_vd_v$ (AASHTO LRFD 5.8.3.3-3) based on the work by Collins and Mitchell (1991; 1996). Collins and Mitchell calculate $\beta$ as:

$$\beta = \frac{4\cot\theta}{1+\sqrt{500\varepsilon_1}} \tag{4-2}$$

where $\beta$ = tensile stress factor indicating ability of cracked concrete to transmit shear; $\theta$ = angle of inclination of principal compressive stress in cracked concrete with respect to longitudinal axis of member; and $\varepsilon_1$ = principal tensile strain in cracked concrete.

The value of $\beta$ varies as a function of the ratio of the shear stress to nominal concrete compressive strength ratio ($v_u/f'_c$) and the longitudinal strain ($\varepsilon_x$). Discrete values were tabulated by Collins and Mitchell, with slight changes made in adaption for AASHTO shear design provisions with slight changes. By setting $\beta$ = 1.6, the shear resistance for concrete may be simplified to give:

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v \cot\theta \tag{4-3}$$

where $\beta$ = 1.6.

For this to hold in Eq. 5-2,$\varepsilon_1$ = 0.0045, which is a limiting value given by Collins and Mitchell. Moreover, for the preliminary design studies by Birely et al. (2018), $\beta\cot\theta$ = 1.6 on average.

## 4.4 CRACKING SHEAR

The experimental tests by Birely et al. (2018) showed that the formation of shear cracks can occur in hollow sections under service loads while solid sections did not display any shear cracks under 140 percent of factored design loads. From this observation, researchers recommend checking cracking shear strength when the concrete sections have an interior void or thin web, or when shear cracking needs to be severely restricted. The cracking shear can be calculated by analyzing the principle planes and stresses using Mohr's Circle. Shear strength corresponding to the occurrence of a principal tensile stress equal to the tensile strength of concrete is defined as the cracking shear. Researchers recommend using $0.0632\sqrt{f'_c}$ (ksi) for the tensile strength of concrete based on observed cracking in the experimental tests. The calculated cracking shear shall be greater than or equal to the shear demand applied to the section at service load.

## 4.5 OVERVIEW OF PROPOSED SHEAR DESIGN PROCEDURE

The design philosophy for shear retains the basic philosophy in AASHTO LRFD, in which the factored resistance must be greater than the demand, with the shear resistance calculated as the sum of the concrete and steel contributions:

$$V_u \leq V_r = \phi_v V_n \qquad \text{(AASHTO LRFD Eq. 1.3.2.1-1 and 5.8.2.1-2)}$$

$V_n$ is the lesser of

$$V_n = V_c + V_s + V_p \qquad \text{(AASHTO LRFD Eq. 5.8.3.3-1)}$$

$$V_n = 0.25 f'_c b_v d_v + V_p \qquad \text{(AASHTO LRFD Eq. 5.8.3.3-2)}$$

where: $V_u$ is the factored shear force at section (kip); $V_r$ is the factored shear resistance (kip); $V_c$ is the nominal shear resistance provided by tensile stresses in the concrete (kip); $V_s$ is the shear resistance provided by shear reinforcement (kip); $V_p$ is the component in the direction of the applied shear of the effective prestressing force (kip); and $\phi_v$ is the shear resistance factor (= 0.9 for normal weight concrete as specified in AASHTO LRFD 5.5.4.2.1).

The steel contribution is established from the amount of steel crossing the crack angle $\theta$. The concrete contribution is a function of both the crack angle and the factor $\beta$. AASHTO LRFD design provisions provide two methods to calculate $\theta$ and $\beta$: 1) iterative method using tables in AASHTO LRFD Appendix B5; and 2) simplified method using equations in AASHTO LRFD 5.8.3.4.2. Each method has advantages and disadvantages. While the method from Appendix B5 may be more accurate, it requires an iterative solution, making it cumbersome for hand calculations. The method from AASHTO LRFD 5.8.3.4.2 is simple for hand calculations, but it is less accurate and may be excessively conservative in some cases (Hawkins et al. 2005; TxDOT 2010). However, both methods do not consider physical admissibility of cracks for deep beams with relatively small shear span-depth ratios. This results in a need for a shear design procedure that overcomes the deficiencies of the AASHTO provisions, both in accuracy and admissibility when used for prestressed concrete bent cap designs.

In the proposed shear design, AASHTO 5.8.3.4.2 may be used to calculate $\theta$ and $\beta$ in lieu of AASHTO Appendix B5 to simplify the design procedure by removing the need for iteration. To account for physical admissibility of cracks and to avoid excessively conservative designs, additional design steps are introduced. In the additional steps, $\theta$ calculated in accordance with AASHTO 5.8.3.4.2 is compared to the compressive strut angles, $\theta_s$, obtained from the geometry of the bridge configuration. The larger of the two angles is used as the shear crack angle for the shear design. The shear crack angle shall be limited to 45° to avoid excessively conservative design. Depending on which angle controls, the shear resistance by concrete and shear reinforcement will be calculated differently.

If $\theta_s \leq \theta \leq 45°$, shear resistances of concrete and transverse reinforcement are calculated as given by the AASHTO LRFD shear design provisions in Section 5.8.3.3.

If $\theta \leq \theta_s \leq 45°$, set $\theta = \theta_s$ and calculate $V_c$ and $V_s$ using:

$$V_c = 0.0316\beta\sqrt{f'_c}\,b_v d_v \cot\theta \quad (4\text{-}4)$$

$$V_s = \frac{A_v f_{yt} d_v \cot\theta}{s} \quad \text{(AASHTO LRFD Eq. 5.8.3.3-4)}$$

where $\beta = 1.6$; and $\theta$ shall not be taken neither less than $\theta_s$, nor greater than 45°.

If the larger of $\theta$ and $\theta_s$ is greater than 45°, an arch mechanism is the main shear resistance mechanism. In this mechanism, a large portion of the total shear force from the girder load is transferred directly to the adjacent column following the compression strut (Matamoros and Wong 2003). Considering that the concrete is capable of providing a significant compressive load capacity, additional transverse reinforcement may be not required. Therefore, in this case it is recommended to provide a spacing based on the minimum area of steel in AASHTO LRFD 5.8.2.5.

## 4.6 PROPOSED SHEAR DESIGN APPROACH

The steps in the proposed design procedure are:

Step 0: Check cracking shear.

Step 1: Determine $d_v$.

Step 2: Determine limiting compressive strut angle, $\theta_s$.

Step 3: Calculate $\varepsilon_s$ and $\theta$ using AASHTO LRFD.

Step 4: Determine $\theta$ for use in shear design.

If $\theta_s < \theta \leq 45°$, use $\theta$ for the further calculation.

If $\theta < \theta_s \leq 45°$, set $\theta = \theta_s$.

If the larger of $\theta$ and $\theta_s > 45°$, skip to Step 6.

Step 5: Determine required spacing of transverse reinforcement.

Step 6: Check minimum transverse reinforcement.

Step 7: Check maximum spacing of transverse reinforcement.

Figure 4.4 gives a flowchart. The procedure shall be repeated at those regions where critical shear demand may be expected, between column and girder locations. The governing spacing shall be provided throughout the length of the bent cap.

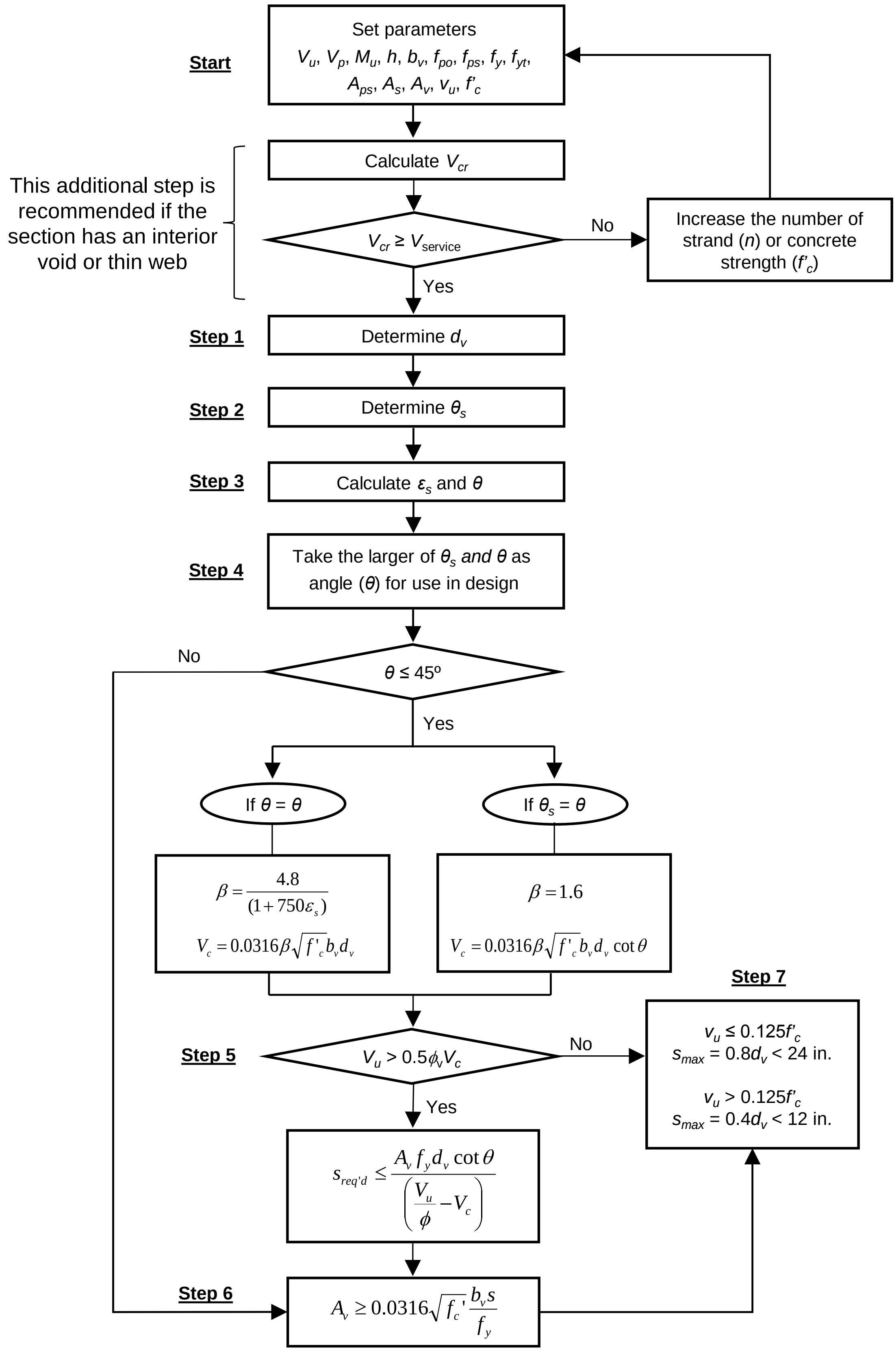



**Figure 4.4. Flowchart for the Proposed Shear Design Procedure.**

### 4.6.1 Step 0: Check Cracking Shear

This step is recommended if shear cracking needs to be checked. This is particularly important for voided cap beams or thin web members. Shear strength corresponding to the occurrence of a principal tensile stress equal to the tensile strength of concrete is defined as the cracking shear. A tensile strength of $0.0632\sqrt{f'_c}$ is recommended based on the observations in the experimental program (Birely et al. 2018). A compressive force provided by prestressing needs to be taken into consideration in a prestressed concrete section. The cracking shear is computed by:

$$V_{cr} = \frac{I_g b_v}{Q}\sqrt{f_t^{\,2} + f_t\left(\frac{nT}{A}\right)} \tag{4-5}$$

where $I_g$ = moment of inertia of the gross concrete section about the centroidal axis, neglecting the reinforcement (in.$^4$); $b_v$ = width of web adjusted for the presence of ducts (in.) or width of the interface (in.); $Q$ = first moment of area (in.$^3$); $f_t$ = tensile strength of concrete (ksi) and $0.0632\sqrt{f'_c}$ is recommended; $n$ = number of strands provided from the flexural design; $T$ = prestressing force per strand after loss (kip); and $A$ = area of cross section (in.$^2$).

The calculated cracking shear strength shall be greater than or equal to the shear demand at service load. If the occurrence of shear cracks may be expected under service loads (DL+LL+IM), consider increasing $f'_c$, providing additional concentric axial prestress, or increasing the shear width.

### 4.6.2 Step 1: Determine $d_v$

AASHTO provisions provide an equation to calculate $d_v$ (AASHTO Eq. C5.8.2.9-1). However, this equation was developed for flexural members and it may not be appropriate when steel reinforcement is placed along the full depth of the wide bent cap sections. Therefore, the effective shear depth is taken as the distance between the outer steel layers. This shear depth, $d_v$, shall not be less than the greater of $0.9d_e$ or $0.72h$. Figure 4.5 shows shear parameters and assumed force free body diagram in a prestressed concrete section with side configuration.

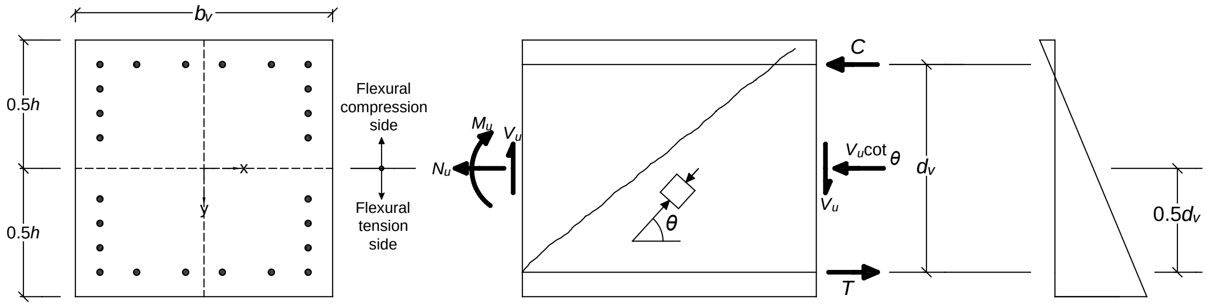



**Figure 4.5. Illustration of Shear Parameters for Prestressed Concrete Section (Adapted from AASHTO 5.8.3.4.2-1).**

Figure 4.5 shows $A_{ps}$ = area of prestressing steel on the flexural tension side of the member (in.$^2$). Consider the strands in the top half or bottom half depending on the direction of moment; $A_s$ = area of nonprestressed steel on the flexural tension side of the member at the section under consideration (in.$^2$); $d_e$ = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in.), $d_e = (A_{ps}f_{ps}d_p+A_sf_yd_s)/(A_{ps}f_{ps}+A_sf_y)$; $d_p$ = distance from extreme compression fiber to the centroid of the prestressing strands (in.); $d_s$ = distance from extreme compression fiber to the centroid of the nonprestressed tensile reinforcement (in.); $f_{ps}$ = average stress in prestressing steel at the time of which the nominal resistance of member is required (ksi); $f_y$ = specified yield strength of reinforcing bar (ksi); and $h$ = overall thickness or depth of the member (in.).

### 4.6.3 Step 2: Determine Limiting Compressive Strut Angle, $\theta_s$

The second step in the shear design process is to find an angle between the column face and bearing pad face under the closest girder within the shear depth ($d_v$) from the bridge geometries. Figure 4.6 shows a configuration of a 38-ft TxDOT standard bent cap with Tx62 girders as an example. Only half of this bent cap is shown as the configuration is symmetrical. All angles at shear critical sections shall be considered and compared with angles from AASHTO 5.8.3.4.2 in Step 3.

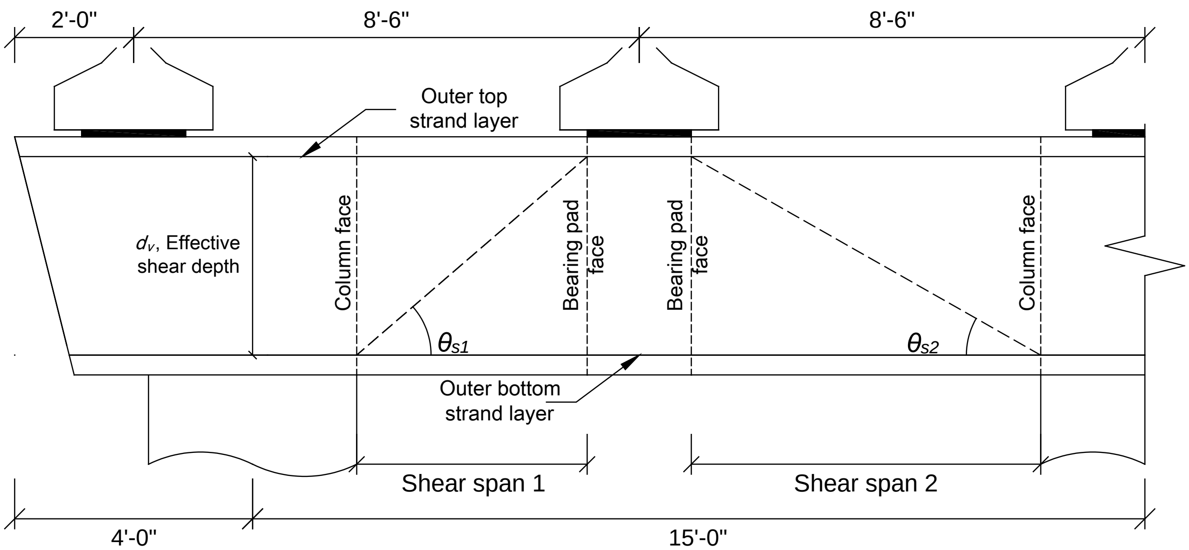



**Figure 4.6. Determination of the Compressive Strut Angle ($\theta_s$).**

### 4.6.4 Step 3: Calculate $\varepsilon_s$ and θ Using AASHTO LRFD

The net longitudinal tension strain in the section at the centroid of the tension reinforcement, $\varepsilon_s$, is calculated in accordance with AASHTO Eq. 5.8.3.4.2-4. The calculated $\varepsilon_s$ shall be taken as not greater than $6.0\times10^{-3}$ or less than $-0.40\times10^{-3}$.

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} f_{po}\right)}{E_s A_s + E_P A_{ps}} \qquad \text{(AASHTO LRFD Eq. 5.8.3.4.2-4)}$$

If $\varepsilon_s < 0$ use

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} f_{po}\right)}{E_s A_s + E_p A_{ps} + E_c A_c}$$

where $A_c$ = area of concrete on the flexural tension side of the member as shown in Figure 4.5 (in.$^2$); $A_{ps}$ = area of prestressing steel on the flexural tension side of the member, as shown in Figure 4.5 (in.$^2$); For simplicity, consider the strands in the top or bottom depending on the direction of moment; $A_s$ = area of nonprestressed steel on the flexural tension side of the member at the section under consideration, as shown in Figure 4.5 (in.$^2$); $d_v$ = effective shear depth (in.); $E_c$ = modulus of elasticity of concrete (ksi); $E_s$ = modulus of elasticity of reinforcing bars (ksi); $E_p$ = modulus of elasticity of prestressing strands (ksi); $f_{po}$ = parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi). For the usual levels of prestressing, a value of $0.7f_{pu}$ will be appropriate for both pretensioned and post-tensioned members; $N_u$ = factored axial force, taken as positive if tensile and negative if compressive (kip); $|M_u|$ = absolute value of the factored moment, not to be taken less than $|V_u - V_p|\ d_v$ (kip-in.); and $V_u$ = factored shear force (kip).

The calculated value of $\varepsilon_s$ is used to calculate $\theta$ by:

$$\theta = 29 + 3500\varepsilon_s \qquad \text{(AASHTO LRFD Eq. 5.8.3.4.2-3)}$$

### 4.6.5 Step 4: Determine θ for Use in Shear Design

The calculated $\theta$ and compressive strut angle, $\theta_s$, shall be compared in this step. The larger angle will be considered as a governing shear crack angle and shall be used for further calculations. If the larger angle is greater than 45°, then move to Step 6.

When $\theta_s < \theta \leq 45°$,
$V_c$ shall be calculated as follows:

$$\beta = \frac{4.8}{(1+750\varepsilon_s)} \qquad \text{(AASHTO LRFD Eq. 5.8.3.4.2-1)}$$

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v \qquad \text{(AASHTO LRFD Eq. 5.8.3.3-3)}$$

When $\theta < \theta_s \leq 45°$, set $\theta = \theta_s$ with $\beta = 1.6$, $V_c$ can be obtained by:

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v \cot\theta \qquad (4\text{-}6)$$

When the larger of $\theta$ and $\theta_s > 45°$, skip to Step 6.

### 4.6.6 Step 5: Determine Required Spacing of Transverse Reinforcement

By comparing the shear demand and the nominal concrete shear resistance calculated in Step 4, a required spacing of transverse reinforcement for the section will be determined. The two cases possible are:

<u>Case 1:</u> $V_u < 0.5\phi_v$, provide minimum spacing of transverse reinforcement in Step 7.

<u>Case 2:</u> $V_u \geq 0.5\phi_v$, provide additional hoop steel reinforcement so that:

$$V_s \geq \frac{V_u}{\phi_v} - V_c \tag{4-7}$$

where:

$$V_s = \frac{A_v f_{yt} d_v \cot\theta}{s}$$ (AASHTO LRFD Eq. 5.8.3.3-4)

Provide transverse reinforcement spacing satisfying:

$$s \leq \frac{A_v f_{yt} d_v \cot\theta}{\left(\frac{V_u}{\phi_v} - V_c\right)} \tag{4-8}$$

Calculated transverse reinforcement spacing shall satisfy the minimum area of transverse and maximum spacing requirements presented in Step 6 and 7.

### 4.6.7 Step 6: Check Minimum Transverse Reinforcement

The minimum transverse reinforcement is calculated as:

$$A_{v\_min} = 0.0316\sqrt{f'_c}\,\frac{b_v s}{f_y}$$ (AASHTO LRFD Eq. 5.8.2.5-1)

$$A_{v\_prov'd} > A_{v\_min}$$

### 4.6.8 Step 7: Check Maximum Spacing of Transverse Reinforcement

The minimum spacing required is determined depending on the shear stress under shear demand as following:

$$v_u = \frac{|V_u - \phi_v V_p|}{\phi_v b_v d_v}$$ (AASHTO LRFD Eq. 5.8.2.9-1)

$v_u < 0.125f'_c$; $s_{max}$ is the minimum of $0.8d_v$ and 24 in. (AASHTO LRFD Eq. 5.8.2.7-1)

$v_u \geq 0.125f'_c$; $s_{max}$ is the minimum of $0.4d_v$ and 12 in. (AASHTO LRFD Eq. 5.8.2.7-2)

# 5 DETAILING AND DESIGN SPECIFICATIONS

## 5.1 OVERVIEW

The previous chapters provided procedures for flexure and shear design for transformation of reinforced designs and for new designs. For both design approaches, additional design specifications and detailing are needed. Appendix A provides a draft of TxDOT Bridge Design Manual design (TxDOT 2015) specifications for pretensioned bent caps. Appendix B provides drawings for recommended standard details. This chapter discusses detailing for the end region (Section 5.2), pocket connection (Section 5.3), and cross section detailing (Section 5.4)

## 5.2 END REGION DETAILING

Prestressing introduces the possibility of cracks forming at the ends due to bursting and spalling stress. To account for this, specific end region detailing is required. First, spalling reinforcement in accordance with AASHTO LRFD code provisions. Second, bursting reinforcement in accordance with the recommendations of recent TxDOT research (O'Callaghan and Bayrak 2008). Additional transverse reinforcement consisted of six vertical and horizontal C-bars was placed at the end region for several specimens as recommended by the precaster (Bexar Concrete Works). The test results demonstrated that provisions of spalling and bursting reinforcement were effective in controlling cracks during the transfer of prestressing forces at release. However, some cracks formed within the transfer length; these were not adequately prevented in the young concrete after the release of strands.

Both spalling and bursting reinforcement should be provided in end regions as described below and illustrated in Figure 5.1:

1. Provide spalling reinforcement in accordance with AASHTO LRFD 5.10.10.1 to handle spalling stresses in the region *D*/4 from the member end.
2. Provide bursting reinforcement immediately after spalling reinforcement, from *D*/4 to the transfer length as suggested by O'Callaghan and Bayrak (2008). The transfer length shall be taken as 60 strand diameters.

A required area of steel ($A_s$) can be obtained by the following equation for both spalling and bursting reinforcement. The calculated area of steel shall be provided in the region $D/4$ from the member end and from $D/4$ to the transfer length, respectively.

$$A_s \geq 0.04\frac{P_i}{f_s}, \tag{5-1}$$

where $A_s$ = required area of reinforcement for spalling and bursting (in.$^2$); $D$ = overall dimension of precast member (in.); $P_i$ = total prestressing force at transfer (kips); and $f_s$ = stress in steel not to exceed 20 ksi (ksi).

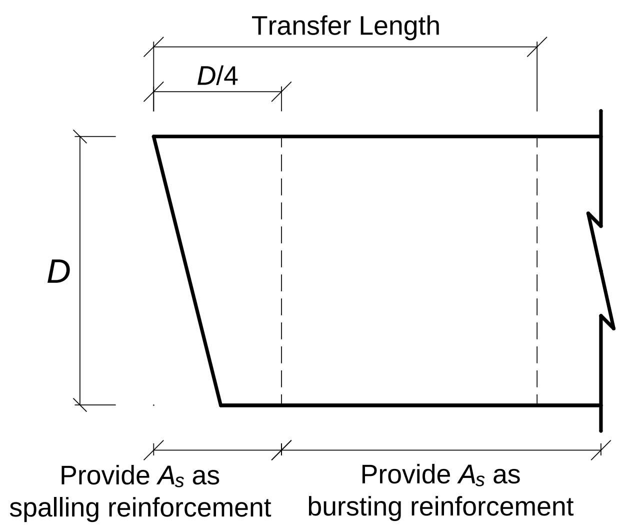



**Figure 5.1. End Region Design Recommendation.**

Although advantages of the use of the additional vertical and horizontal C-bars at the end of the member were not obvious in the test results, it may be helpful to mitigate the end splitting effect and prevent expansion of cracks in both horizontal and vertical directions by bridging cracks. For this reason, providing additional transverse hoop steel at the ends of the member is recommended. The spacing between each C-bar can be determined by $B/6 - 2$ for a horizontal spacing and $D/6 - 2$ for a vertical spacing, respectively, where $D$ is the depth and $B$ is the width of the bent cap.

## 5.3 POCKET CONNECTION DETAILING

The use of the large single pocket connection had benefits such as 1) the easy and rapid installation of the bent cap onto the column by providing large misalignment tolerances; 2) the increased constructability of the connection by filling the pocket with normal weight concrete instead of a specialty grout.

The assembly of the bent cap beam on the column proceeded in a straightforward fashion as follows:

1. Place column.
2. Lift bent cap and locate onto the column.
3. Fit over the dowel bars into the corrugated pipe.
4. Fill the pocket with a normal weight concrete.

The entire procedure to place the bent cap on in the right position took approximately 12 minutes, and this may reduce the working time significantly on the construction sites.

The connection performed as intended in the experimental program with no yielding of dowel bars and no crack formations at the bedding layer and the column under normal bridge demands. The test demonstrated the appropriateness of the design presented in Birely et al. (2018).

Under joint opening and closing demands, minor cracks were observed in the column and bedding layer, followed by yielding of dowel bars near the bedding layer; evidence that the pocket connection was capable of load transfer from the cap to the column.

Minor cracks formed at the top of the pocket in prestressed bent caps. This is primarily attributed to drying shrinkage and large compression stresses under load. Providing hoops at the top of the pocket and transverse reinforcement adjacent to the pocket may reduce these cracks. The depth of the corrugated steel pipe did not influence the connection performance. It may be somewhat useful to have some cover concrete at the top of the pocket to prevent the corrosion of the pipe.

Based on the observations in the experimental program, the following recommendations are made:

1. Determine a pipe thickness to serve as shear reinforcement to resist the shear demand, and also to reduce stress concentration arising from prestressing force. The detailed procedure of the determination of the pipe thickness is provided by Birely et al. (2018).
2. Place #5 hoops at the top of the pocket and #5 J-Bars at 6-in. maximum spacing in the joint area to provide crack control.

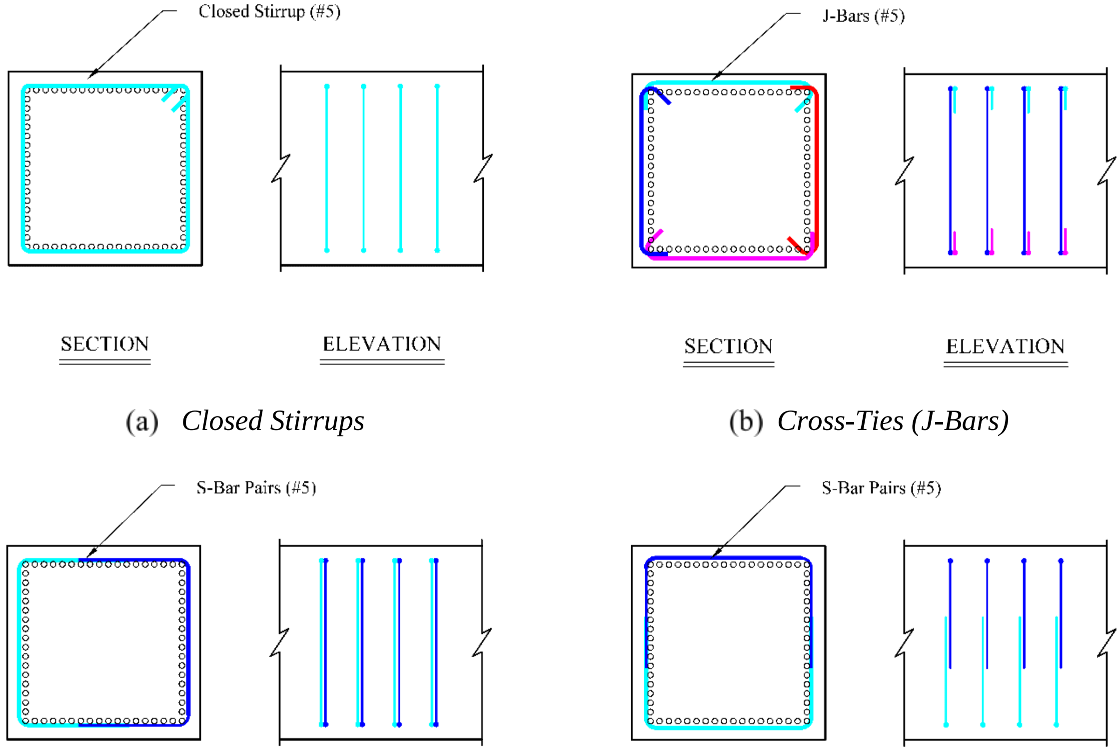
3. Detail a 3-in. concrete cover on the top of the galvanized steel pocket to provide room for placing the hoop and reinforcement and also provide a measure of corrosion protection.

## 5.4 CROSS-SECTION DETAILING

Strands should be placed primarily along the sides of the bent caps to accommodate the pocket pipe, with a few strands provided in the top and bottom layers to act as skin reinforcement. Strands adjacent to the pocket can provide additional stability during construction. In designs with eccentric strand configurations, strands may pass through the top of the pocket; however, the bottom should be left clear to avoid potential conflict when placing the pocket around the column dowel bars.

Four options exist for detailing of transverse reinforcement. The first is to provide closed stirrups as is standard practice for RC construction; this is typically not a favorable option for the standard practice of many precast plants. One alternative is to use four single cross-ties as shown in Figure 5.2(b). More common methods are the use of spliced U-shaped bars. The detail in Figure 5.2(c) is preferable, as the splice is located away from the direction of action of the shear forces. The detail in Figure 5.2(d) may be used if the design is for non-seismic loads, sufficient splice length is provided, and the splice is securely tied.

If voids are used, strand layouts should consider constructability of transverse reinforcement, as tightly spaced strands may create challenges to tying spliced bars. Wall thickness should be a minimum of 8-in. The walls may need to be thicker to accommodate larger numbers of strands or to provide sufficient cracking strength. If the shear demand is well below the cracking shear, voids with square corners are sufficient. If the possibility of shear cracks exist, chamfered corners are recommended; experimental tests were inconclusive but suggest the chamfer may help reduce the extent of shear cracks after they form.




(a) *Closed Stirrups*

(b) *Cross-Ties (J-Bars)*

(c) *Top Splice*

(d) *Side Splice*

**Figure 5.2. Transverse Reinforcement Detailing Options.**

# 6 DESIGN EXAMPLES

## 6.1 OVERVIEW

To demonstrate implementation of the design procedures put forth in Chapter 2 through Chapter 5, four design scenarios are considered:

1. Transformation based design, discussed in Section 6.2 and Example #1 (Appendix C).
2. Demand based design for typical bridge designs, discussed in Section 6.3 and Examples #2 and #3 (Appendices D and E).
3. Design for reduced weight cross-sections, discussed in Section 6.5 and Example #4 (Appendix F).
4. Design for reduced number of column lines, discussed in Section 6.6 and Example #5 (Appendix G).

In the sections that follow, each design scenario is summarized and a discussion of the example, including bridge characteristics and resulting design, is presented. Example #2 includes calculations for end region detailing and for pocket connection design.

## 6.2 DESIGN SCENARIO #1 – TRANSFORMED DESIGN

The use of the transformation design procedure presented in Sections 2.2 and 2.3 are demonstrated for the design a bent cap in a standard TxDOT I-girder bridge. The design selected is the BIG-38-Tx62 bent cap design for a 38-ft wide bridge with Tx62 girders. The bent consists of 42-in. diameter columns spaced at 15-ft, supporting five girders spaced at 8.5-ft. The RC bent cap is 4-ft square, with 7-#11 bars and 12-#11 bars for top and bottom flexural reinforcing steel, respectively; skin reinforcement is not considered to contribute to the strength. Transverse reinforcement is #5 double legged stirrups at 8-in. spacing. Design material properties are $f'_c$ = 3.6 ksi and $f_y$ = 60 ksi.

The conversion of the bent cap from RC to pretensioned concrete is done in Example #1, provided in Appendix C. Here a summary of the design is provided for brevity. The pretensioned cap is designed using a concrete strength of 5 ksi and 0.6-in. diameter prestressing strand with a strength of $f_{pu}$ = 270 ksi. The strands are assumed to have 20 percent losses.

Using the flexural transformation procedure of Section 2.2, 44 strands are needed to provide an equivalent area of flexural reinforcement. The strands are uniformly distributed along the sides of the bent cap to accommodate the pocket connection, shown in Figure 6.2. The nominal strength was verified to exceed the cracking moment in both positive and negative bending; for negative bending, the cross-section was reduced to the net section to conservatively exclude any contributions of the fill concrete in the pocket connection. An evaluation of the factored resistance for the RC and pretensioned designs indicates that the converted design provides a factored flexural resistance greater than or equal to the RC factored resistance in both the positive and negative moment regions.

As discussed in Section 2.3, the shear design can be held constant from the RC design as shear strength contributions of the concrete and steel are expected to increase with the addition of prestressing. To evaluate if the shear reinforcement could be reduced, the proposed shear transformation procedure is applied for the first interior span of the bent (shown in Figure 6.1). Strut angles were 40.7 and 29.6°. Shear demands were calculated using the plastic moments. The spacing required to provide sufficient strength was 2.27-in. and 6.61-in.

For the region to the left of the girder, the strut angle is not sufficient to be considered a deep region in which minimum reinforcement could be provided. The short distance leads to high shear demands and the need for tight 2-in. spacing of reinforcement. For the region to the right of the girder, the strut angle is shallower and leads to a more reasonable 6-in. spacing. Considering either region, the capacity based design does not allow for a reduction in transverse reinforcement.

The results of the design example raise the question as to whether the shear design transformation procedure is appropriate. It is important to note that without demands, the design of transverse reinforcement to satisfy AASHTO requirements becomes difficult. The proposed transformation is intentionally conservative to ensure a brittle shear failure is avoided. Further, some factors unique to the design example may contribute to the tight spacing of the transverse reinforcement. First, the standard bridges are valid for spans from 40-ft to 130-ft, leading to more reinforcement that is needed for shorter span lengths. In the case of individual designs of bridges, the plastic moments generated by flexural reinforcement may be relatively smaller than that used in Example #1, leading to smaller shear demands and thus the need for less reinforcement. Second, the girder is in close proximity to the column, leading to larger shear demands and steeper angles. The shear design transformation procedure was applied to other standard bent cap designs to evaluate other bridge configurations. For the BIG-32 bent for Tx54 girders or smaller (42-in. square), the capacity design led to a reduction of the transverse reinforcement from 6-in. spacing in the RC design, to 11-in. spacing in the pretensioned design, with the AASHTO LRFD minimum amount of reinforcement controlling.

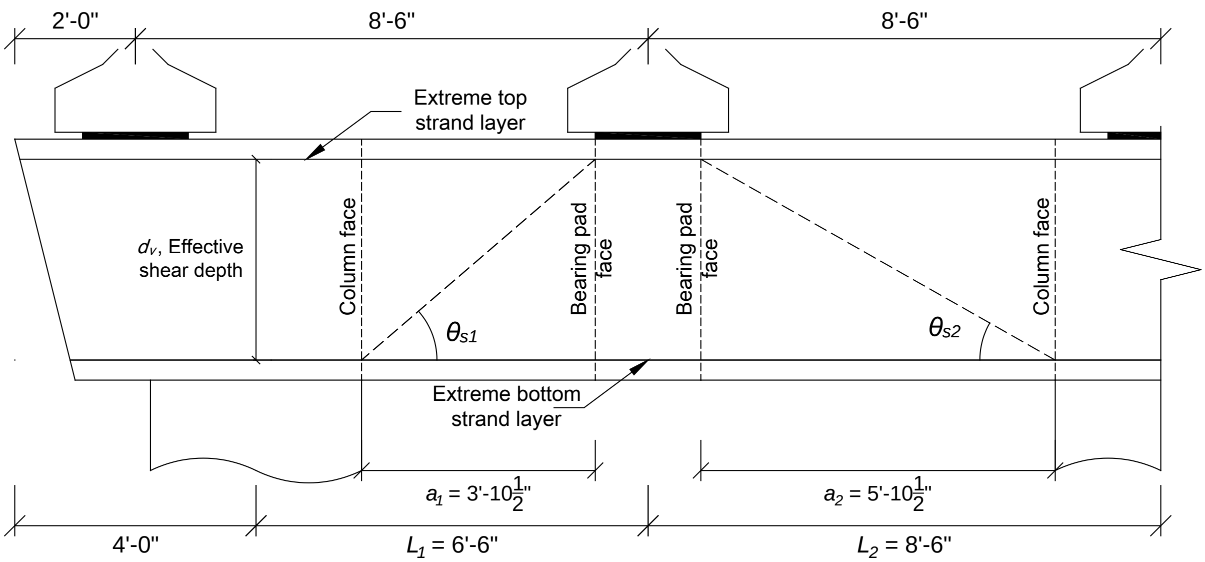



**Figure 6.1. Determination of the Compressive Strut Angle ($\theta_s$) for 6 Column Bent from Design Example #1.**

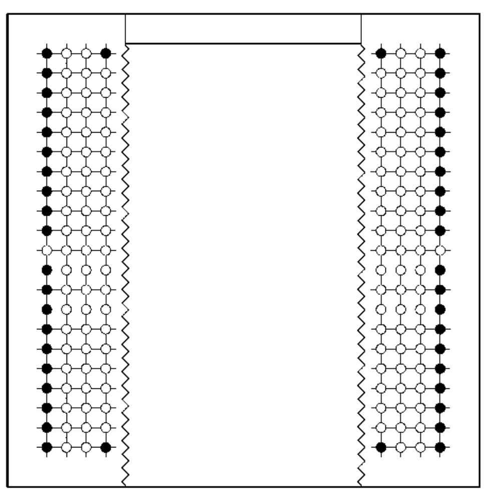


**Figure 6.2. Side Strand Configuration from Design Example #1.**

## 6.3 DESIGN SCENARIO #2 – DEMAND BASED DESIGN

The use of the proposed procedures in Chapters 3 and 4 are demonstrated for the design of solid bent caps in two bridges. The first bridge is the same used to demonstrate transformed design in Section 6.2, with the design based on calculated demands rather than a RC design.

The second bridge is a 4-lane divided highway, Tx62 girders and a 4-ft square bent cap are used to provide consistency with the previous example. The bent cap length is 80-ft and is supported by six 3.5-ft diameter columns. Ten girders are spaced at 8.5-ft.

Appendices D and E illustrate the designs in detail for Examples #2 and #3. A summary of the two designs is provided here for brevity. A concentric layout of 0.6-in. diameter strands is used with a side configuration to accommodate a pocket connection.

For both bent caps, the moment demands at dead load are similar, resulting in the same number of strands (28) to achieve zero tension under dead load. For the BIG-38 bent cap, the enhanced tensile stress limits at service were used to provide a design that would not crack at service. The calculated concrete strength was found to be 6.0 ksi, higher than the minimum concrete strength. For the 4-lane divided highway, the AASHTO tensile stress limits at service were used. The concrete compressive strength to satisfy the limits was well below the minimum, so the minimum (5 ksi) was used for design. Both designs satisfied all remaining checks for the flexural design procedure.

For design of transverse reinforcement for both bent caps, the shear angle provided by the AASHTO design provisions is shallower than the compressive strut angle and therefore physically inadmissible. If design were based on this angle, the possibility of insufficient shear reinforcement arises. Instead, the proposed design procedure that uses the compressive strut angle is used. The shear demand and crack angle is similar for the two bent caps, resulting in a 10-in. spacing used in both.

## 6.4 COMPARISON OF TRANSFORMED AND DEMAND BASED DESIGNS

The BIG-38 bent cap design in Example #2 is the same bent cap used in Example #1 to demonstrate transformation of a RC design to a pretensioned design. A comparison of the design provides insight into the adequacy of the transformation procedure. The transformed design resulted in a greater number of strands (44) than the demand based design (28 strands), highlighting the conservative design resulting from the transformation. For shear design, the demand based design required a transverse reinforcement spacing not greater than 10-in. By comparison, the proposed transformation procedure of retaining the spacing from the RC design (8-in.) or using the capacity design philosophy (6-in.) provides a more conservative design.

## 6.5 DESIGN SCENARIO #3 – REDUCED WEIGHT

To demonstrate the use of pretensioned bent caps for the design of precast caps that have reduced weight, the design of the 4-lane divided highway in Example #3 is revised to include an interior void. No other characteristics of the bridge or design properties are modified. The detailed design is documented in Example #4 in Appendix F. A summary of the voided design in comparison to the solid design is discussed here.

The bent cap was left solid in the overhang and at the columns to provide adequate section for the connection. The void provided a 24-in. × 24-in. opening in the cross-section between the columns starting 2-in. from the face of the columns. The use of the void resulted in a reduction of the weight from 180 kips to 146 kips.

The flexure design was unaffected by the addition of the void as negative moment demand controlled the design. The minimum recommended concrete strength satisfied service stress limits. The number of strands remained the same. The addition of the void had a greater impact on the shear design as the shear width was reduced to one-half of that used in the solid section. Consequently, the required shear spacing was reduced from 10-in. in the solid design to 8.5-in. in the voided design. From the experimental tests of voided bent caps, a concern arises in the formation of diagonal cracks at relatively low demands. The cracking shear was calculated for the section and found to be greater than the service demands. If the shear demand had exceeded the expected cracking strength, the design should be adjusted to have a higher concrete strength and/or thicker walls to increase the cracking shear.

## 6.6 DESIGN SCENARIO #4 – ELIMINATE COLUMN LINES

In the final design scenario, the use of pretensioning to enable reduction of column lines is explored. In bridges with multiple interior bents and a large width that results in the need for many columns, a major cost of the construction is driven by the foundation costs. Consequently, considerable savings may be achieved by reducing the number of column lines. This may not be practical with the use of RC bent caps as the larger column spacing and the associated increase in demands will result in designs that are significantly cracked under service loads, with the possibility of cracks forming from just the dead load alone. The enhanced crack resistance of pretensioned caps can make a reduction in column lines practical, although it is likely that the design will be unable to accommodate the enhanced service stress limits that would eliminate cracking at dead load.

To illustrate the design of a bent cap with reduced column lines, the 4-lane divided highway with six columns used in Examples #3 and #4 is redesigned with only four columns. In reducing the number of columns, the overhangs are lengthened. The overhang length is the same used in Phase 2 experimental tests documented by Birely et al. (2018), and shown to provide the desired performance under design loads.

Design for flexure used an eccentric strand layout and followed the recommendations of Section 3.4. The initial calculation of the number of strands for zero tension under dead load required a concrete strength that exceeded the maximum of 8.5 ksi to satisfy AASHTO stress limits at service. The concrete strength was set to 8.5-ksi and the number of strands increased to 56 with an eccentricity of −1.19-in., to satisfy AASHTO stress limits at service. With the higher required concrete strength, it is evident that the use of enhanced limits to reduce the amount of cracking is not practical in this design scenario. For shear design, 11.0-in. spacing of transverse reinforcement is required to satisfy shear strength requirements, and 8.4-in. spacing is required to satisfy the minimum transverse reinforcement requirement.

# 7 SUMMARY AND CONCLUSIONS

Recommendations for pretensioned bent caps were developed based on the findings of full-scale experimental tests of bent caps. First, recommendations were made for converting existing RC bent cap designs to pretensioned designs, thereby providing contractors the option to select pretensioned caps at any point during the construction process. Next, demand based design recommendations were made to enable design of pretensioned caps as part of the original substructure design. Design for flexure was based on the philosophy of zero tension under dead load, ensuring that any cracks that form close upon removal of live loads. Design for shear was based on AASHTO shear design provisions, with modifications made to ensure that crack angles used in design were physically admissible. Recommendations were made for detailing of cross-sections, end regions, and connections. Companion examples were provided to demonstrate implementation of the design procedures.

The transformation design procedure was applied to the TxDOT BIG-38-62 RC bent cap standard. The following observations were made:

1. The capacity design based shear transformation did not allow for a reduction of the transverse reinforcement from that of the original RC design due to the close proximity of the girder to the column. When the procedure is applied to girders with a greater shear span length, reduction of the reinforcement may be permitted.
2. By comparing to demand based design of the same bridge, the transformation procedure was shown to provide conservative results for both longitudinal and transverse reinforcement.

The demand based design procedure was first applied to bent caps for bridges with solid bent caps and column spacing meeting typical designs (BIG-38-62 and a 4-lane divided highway with six columns). The following observations were made:

1. Design using the zero tension under dead load philosophy resulted in designs providing sufficient strength.

2. The use of enhanced service stress limits to avoid cracking controlled the concrete strength in one example. In the other example, the AASHTO stress limits were used, resulting in the minimum concrete strength controlling the design.
3. The AASHTO shear design provisions resulted in physically inadmissible crack angles. The design was completed using the proposed modification to crack angle and concrete shear strength contribution.

A solid bent cap was also designed for a bridge with a reduced number of column lines (4-lane divided highway with four columns). The following observations were made:

1. Design using the zero tension under dead load philosophy was insufficient to satisfy service stress limits. The design was modified to use the maximum recommended concrete compressive strength and the strands adjusted to satisfy service limits.
2. The use of a higher concrete strength (8.5-ksi) than in previous examples led to a larger minimum transverse reinforcement. The minimum required transverse reinforcement controlled shear design.

To demonstrate the potential use of interior voids to reduce the weight of the bent caps, the 4-lane divided highway example with six columns was modified. The following observations were made:

1. Negative moment demands controlled the design of the number of strands and the concrete strength; thus design for flexure was unaffected by the presence of the void.
2. The void reduced the shear width, leading to the need for an increase in the transverse reinforcement.
3. The shear cracking strength exceeded the service demands, so the thickness of the walls was considered acceptable and chamfered corners are not needed.

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# APPENDIX A: DRAFT TXDOT BRIDGE DESIGN MANUAL DESIGN SPECIFICATIONS FOR PRETENSIONED BENT CAPS

# Chapter 4 — Substructure Design

## Section 9 — Pretensioned Concrete Bent Caps

### Materials

Use Class H Concrete with a minimum $f'_{ci}$ = 4.0 ksi and $f'_c$ = 5.0 ksi and maximum $f'_c$ = 8.5 ksi.

Use pretensioning strand with a specified tensile strength, $f_{pu}$ of 270 ksi.

Use Grade 60 mild reinforcing steel.

### Geometric Constraints

See Chapter 4 Section 4.

### Structural Analysis

See Chapter 4 Section 4.

### Design Criteria

Check limit states using Strength I, Service I, and Dead Only load combinations. For Dead Only load combination, limit flexural tensile stress at extreme tension fiber to $f_t$ = 0 ksi.

Check that Article 5.9.4.2.2 Tensile Stress Limits are satisfied. For Class I exposure, limit tensile stress to $0.19\sqrt{f'c}$ (ksi). For Class II exposure, limit tensile stress to $0.0948\sqrt{f'c}$ (ksi). To further limit cracking under Service and Ultimate Load conditions, it is recommended to limit tensile stress to $0.126\sqrt{f'c}$ (ksi) for Class I exposure conditions.

Check Article 5.7.3.3.2 for minimum reinforcement.

For multi-column bent caps, take design negative moments at the center line of the column.
For multi-column bent caps with columns 4 ft. wide or wider, take design negative moments at the effective face of the column.

For shear, follow the requirements of Article 5.8.3.4.2.

If $\theta$ is found to be smaller than $\theta_s$ as defined in Figure 1, calculate the nominal shear resistance provided by concrete with

$$V_c = 0.0316\beta\sqrt{f'_c}\,b_v d_v \cot\theta$$

where $\theta = \theta_s$ and $\beta = 1.6$.

If $\theta > 45^{\circ}$, transverse reinforcement need only satisfy the provisions of Articles 5.8.2.5 and 5.8.2.7.

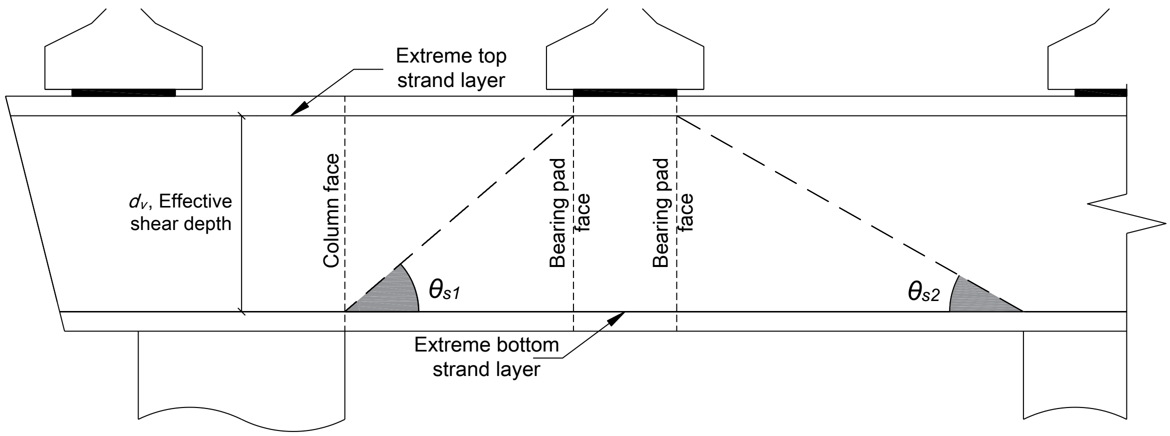



**Figure 1. Determination of the Compressive Strut Angle ($\theta_s$)**

## Detailing

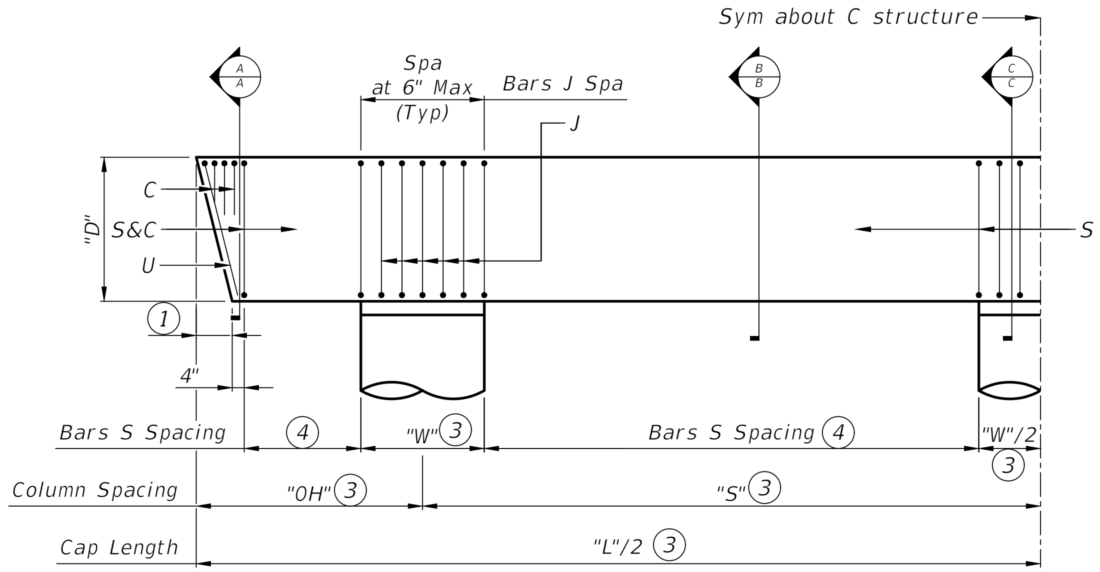
For flexural reinforcement, use 0.6-in diameter 7-wire prestressing strands.

Use #5 stirrups, except as noted, with a 4-in. minimum and a 12-in. maximum spacing. Do not use stirrups larger than #6. Use double stirrups if required spacing is less than 4-in. If torsional resistance is explicitly addressed in the design, ensure that the stirrup detailing is consistent with AASHTO requirements.

If voids are needed to reduce the weight for shipping and placement, provide walls with a minimum wall thickness of 8-in. If shear cracks are expected to form under design demands, provide chamfered corners on the voids.

End region detailing should be calculated to provide spalling resistance according to Article 5.10.10.1 for a length D/4 from the member end. Spalling reinforcement should be extended from D/4 to the transfer length to provide bursting resistance with the transfer length calculated as 60 times the diameter of the strand.

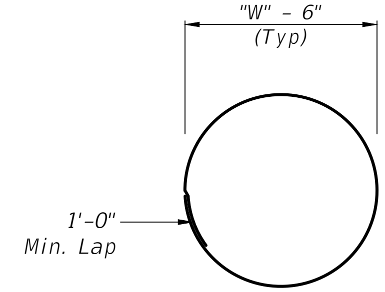
# APPENDIX B: RECOMMENDED STANDARD DETAILS



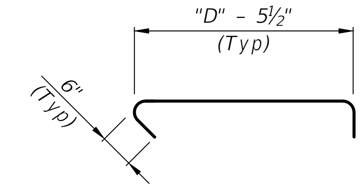

3 COLUMN BENT

INTERIOR BENT HALF ELEVATION

Strands, B Bars, and Column Reinforcement not shown for clarity

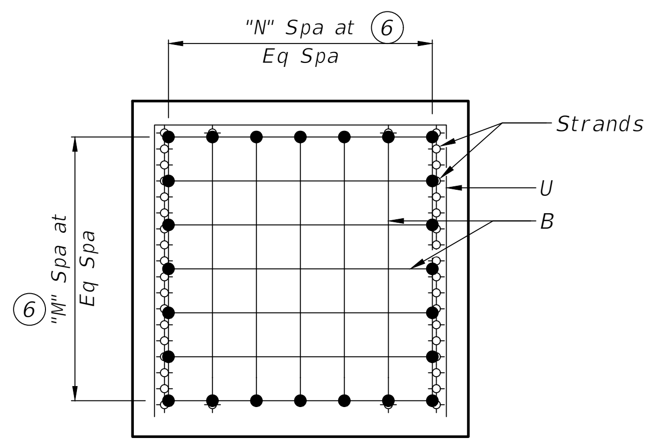



Bar H




Bar J

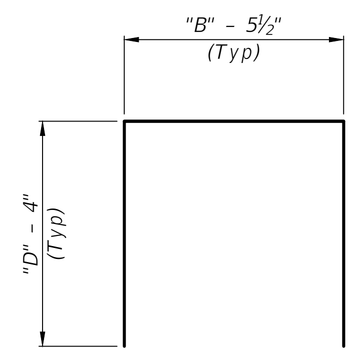
1. Variable. See Interior Bents sheet for dimension. Measured parallel to top of cap cross-slope.
2. Dimensioned to center of strand.
3. See Interior Bents Sheet.
4. Reinforcement spacing within transfer length specified by design, with max spa of 6". From transfer length to face of column use spacing for shear, with max spa of 6". Shear reinforcement in span according to design, with 12" maximum.
5. It is recommended to place strands in available strand locations nearest the corrugated steel pipe to prevent movement during concrete placement
6. "N" = "B"/6 - 2
   "M" = "D"/6 - 2



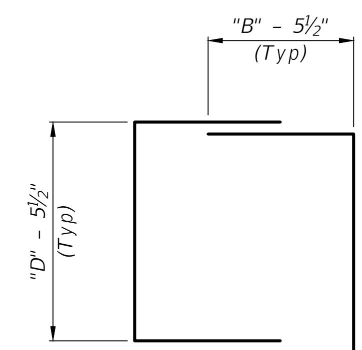

End Region

SECTION A-A

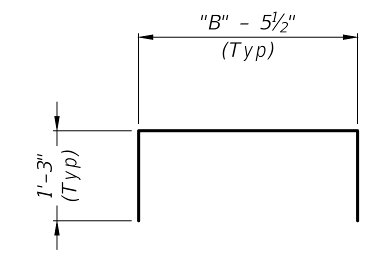
Bars C not shown for clarity



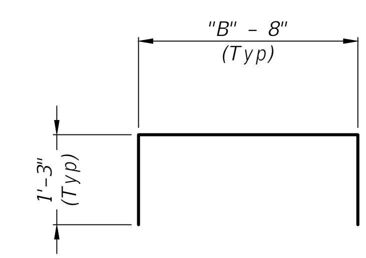

Bar U



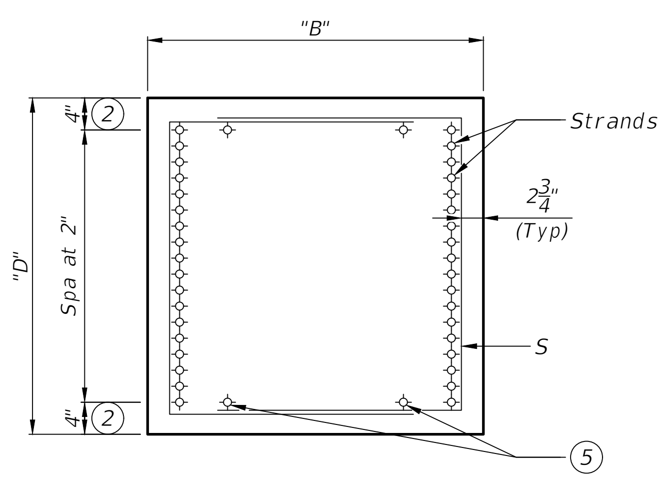

Bar S




Bar C

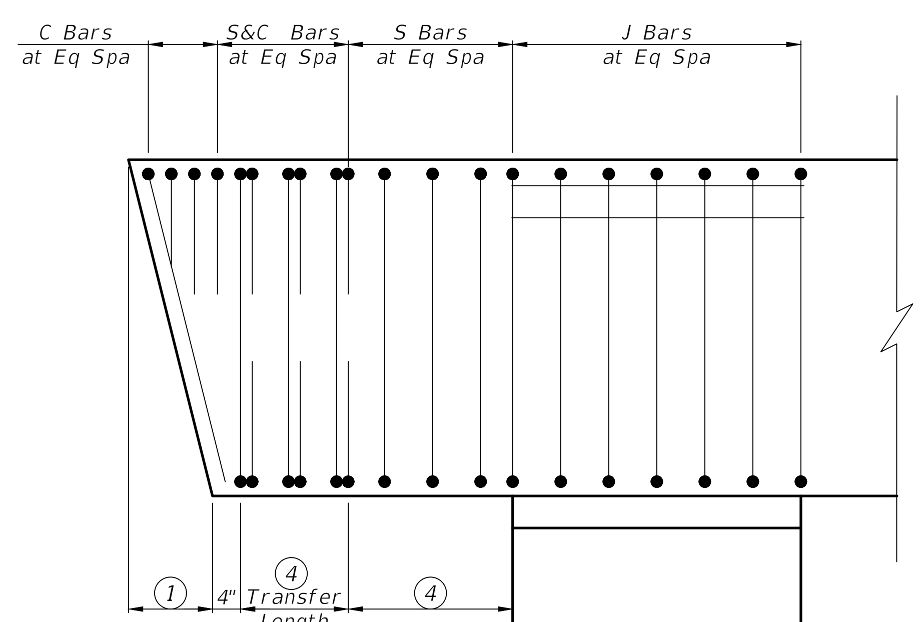



Bar B




Typical Cross-Section

SECTION B-B




End Region

ELEVATION A

Strands and Column Reinforcement not shown for clarity

TxDOT 0-6863
Precast Pretensioned
Bent Caps

Recommended Details
SHEET 1 of 2

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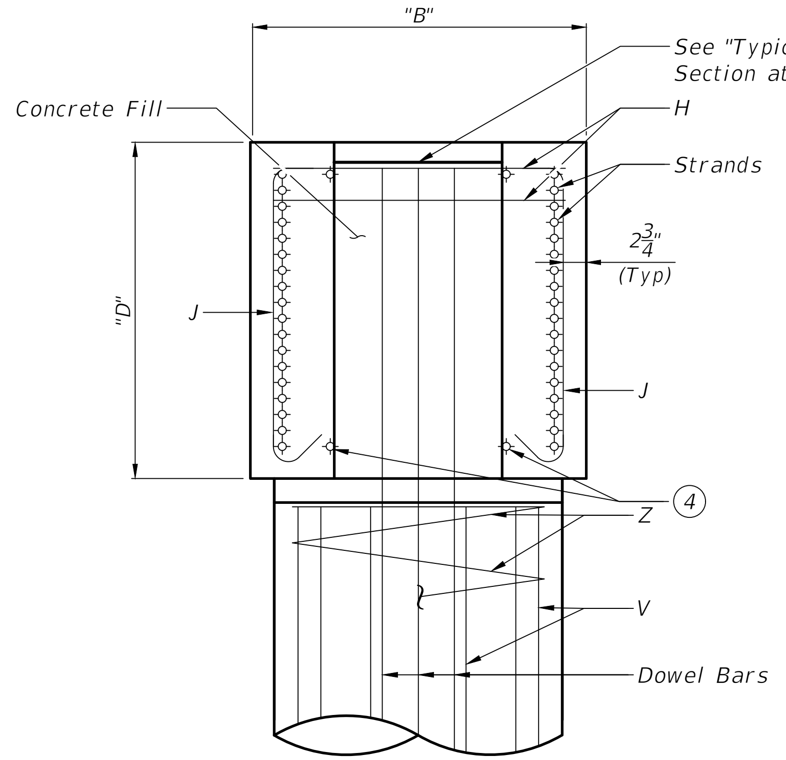
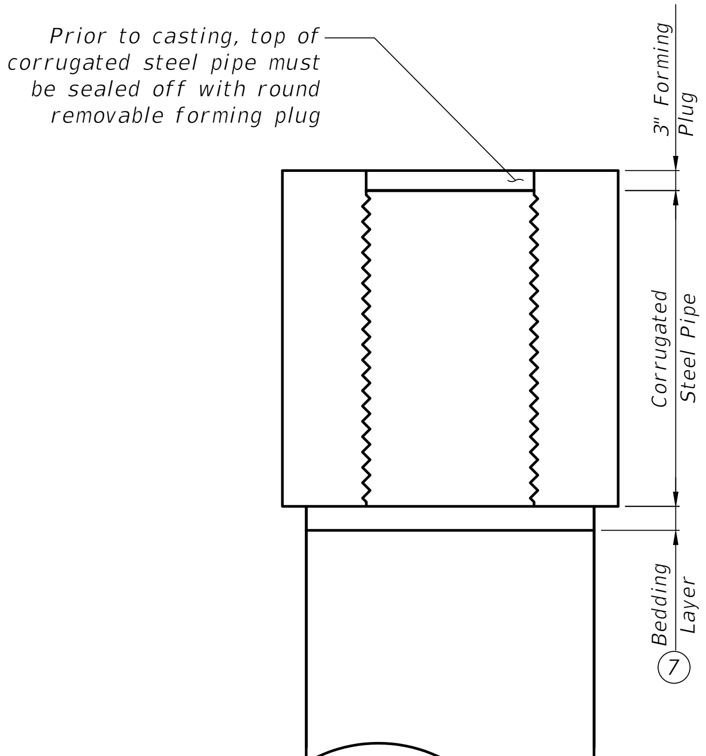



SECTION C-C




TYPICAL CAP SECTION AT COLUMN

Cap and Column Reinforcement not shown for clarity

(7) Bedding layer Min. Thickness of 2x Nominal Agg. size of concrete fill. Maximum depth 4".

## CONSTRUCTION NOTES:

Cap Fabrication:

Fabricate in accordance with Item 425, "Precast Prestressed Concrete Structural Members".

Secure corrugated metal pipes to prevent their movement during concrete placement. Location tolerance of pipes is ¼" from plan location, transversely and longitudinally. Seal pipes to prevent intrusion of concrete.

Chamfer or round all exposed corners ¾".

Repair cracks exceeding 0.005 in. in width as directed. The Fabricator must take approved corrective actions if cracks greater than 0.005 in. form. All work, material, and engineering related to the cracks will be at the Contractor's expense.

Caps can be set level or at grade. If required or needed, build bearing seats/pedestals to achieve final grade. Bearing seats/pedestals may be precast with the initial cast. Bearing seats/pedestals that conflict with column locations may not be precast with cap. Do not locate lift points at bearing seats/pedestals if bearing seats/pedestals are precast. If bearing seats/pedestals are not precast, cast in accordance with Item 420.4.9, "Treatment and Finishing of Horizontal Surfaces". Do not slope the top of caps between bearing areas from the center slightly towards the edge. If pedestals are not precast, drill and epoxy anchor bars EB1 and EB2 into top of cap in accordance with Item 420.7.10, "Installation of Dowels and Anchor Bolts".

If earwalls are required, see Interior Bent sheets for details.

If shear keys are required elsewhere in plans, submit details. Shear keys may not be precast. Drill and epoxy shear key anchor reinforcement into top of cap in accordance with Item 420.4.7.10, "Installation of Dowels and Anchor Bolts".

Limit flexural stress in cap to 250 psi during handling and storage. Store and handle caps in accordance with Item 425, "Precast Prestressed Concrete Structural Members". Do not stack caps.

Prior to releasing strands ensure that bent cap concrete reaches the minimum release strength of 4.0 ksi, unless otherwise noted.

Cap-to-Column Connection:

Construct a mock-up of the cap-to-column connection that must demonstrate the ability of the Contractor to provide a connection free of voids. In the presence of the Engineer use trial batch of concrete fill using the same materials, equipment, and personnel to be used for actual concreting operations and fill the mock-up at least one week before concreting. Field test the trial batch of concrete fill to the same levels required for the actual concreting.

Caps may be placed on columns/drilled shafts after column/drilled shaft concrete has achieved a flexural strength of 355 psi (or 2,500 psi compressive strength). Use plastic shims or friction collars to support the cap at the proper elevation prior to concreting. Total area of plastic shims used on top of each column may not exceed 6 percent of the column area. Column/drilled shaft curing may be interrupted a maximum of 2 hours for placement of plastic shims or friction collars and cap placement.

Provide mortar tight forms. Ensure the top of the column is in a saturated surface dry (SSD) condition just before placing concrete fill. Deposit concrete such that all voids in the bedding layer and bent cap are completely filled. Deposit concrete through the top opening of the cap pocket in a manner that deposits concrete from the bedding layer on the bottom of the connection upward. Vibrate concrete in the pocket in accordance with Item 420.4.7.9, "Consolidation". Trowel finish top surface of cap pockets flush with top of cap. Wet mat cure these locations for at least 48 hours. When lifting loops are removed, recess loops 3/8" minimum and fill void with Type VII epoxy mortar in accordance with Chapter 2, Section 7 of the Concrete Repair Manual. Subsequent loading can occur when the concrete fill reaches its required 28 day compressive strength.

## MATERIAL NOTES:

Provide 12 gage, Type I, lock-seam, helical corrugated pipe conforming to Item 460, "Corrugated Metal Pipe".

Provide Grade 60 reinforcement. Do not epoxy coat reinforcement even if column reinforcement is epoxy coated.

Provide Class "H" (HPC) Concrete for Cap Concrete.

Provide Class "C" or "S" Concrete for Cap-to-Column Connection concrete fill with nominal aggregate size no larger than 1/2 times the specified bedding layer thickness.

Use low relaxation strands, each pretensioned to 75% of fpu.

## GENERAL NOTES:

Designed in accordance with AASHTO LRFD Bridge Design Specifications.

Prestress loss calculated according to Research Report FHWA/TX-12/0-6374-2 Table 6.6 using a relative humidity of 60 percent.

The Contractor has the option to provide prestressed, precast bent caps in accordance with the details shown. No additional payment will be made if the Contractor uses prestressed, precast bent caps.

Submit shop drawings of prestressed, precast bent caps for approval prior to construction.

Indicate lifting attachments and locations on shop drawings.

Corrugated Pipe and Concrete Fill are subsidiary to Item 425, "Precast Prestressed Concrete Structural Members".

See standard Interior Bents sheets for details and notes not shown.

TxDOT 0-6863
Precast Pretensioned Bent Caps

Recommended Details
SHEET 2 of 2

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# APPENDIX C: DESIGN EXAMPLE #1

Date: February, 2018

Standard Rectangular Reinforced Concrete Bent Cap Conversion to Prestressed Concrete

This design example is in accordance with the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012) and TxDOT Bridge Design Manual - LRFD (October 2015)

# Design Parameters

Material Properties

$f'_c := 5 \cdot ksi$ *Concrete Compressive Strength*

$f_r := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi$ $\quad f_r = 0.54\ ksi$ *Modulus of Rupture (AASHTO LRFD Eq. 5.4.2.4-1)*

$f_{pu} := 270 \cdot ksi$ *Ultimate Strength of Prestressing Steel*

$f_y := 60\ ksi$ *Yield Strength of Mild Steel*

$f_{yp} := 0.9 \cdot f_{pu}$ $\quad f_{yp} = 243\ ksi$ *Yield Strength of Prestressing Steel (AASHTO LRFD Table 5.4.4.1-1)*

$w_{cE} := 145 \cdot pcf$ *Unit Weight of Concrete for $E_c$ Calc*

$E_c := 33000 \cdot \left(\frac{w_{cE}}{1000 \cdot pcf}\right)^{1.5} \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi = 4074\ ksi$ *Modulus of Elasticity of Concrete, (AASHTO LRFD Eq. 5.4.2.4-1)*

$E_s := 29000 \cdot ksi$ *Modulus of Elasticity of Mild Steel*

$E_p := 28500 \cdot ksi$ *Modulus of Elasticity of Prestressing Steel*

Section Properties

$B := 48 \cdot in$ *Width of Cap*

$D := 48 \cdot in$ *Depth of Cap*

$Dia_{pipe} := 24\ in$ *Diameter of pocket connection*

$A_{strand} := 0.217 \cdot in^2$ *Area of Prestressing Strand*

$A_{s\_bot}$ = 12 #11 bars $\quad A_{s\_bot} := 12 \cdot 1.56\ in^2 = 18.72\ in^2$ *Area of "bottom" RC flexural steel*

$A_{s\_top}$ = 7 #11 bars $\quad A_{s\_top} := 7 \cdot 1.56\ in^2 = 10.92\ in^2$ *Area of "top" RC flexural steel*

$A_s := A_{s\_bot} + A_{s\_top} = 29.64\ in^2$ *Total area of RC flexural steel*

$\Delta_{f.pt} := 0.2$ *Assumed prestress loss in pretensioned members*

$T_{strand} := 0.75 \cdot f_{pu} \cdot A_{strand} \cdot \left(1 - \Delta_{f.pt}\right)$ $\quad T_{strand} = 35.15\ kip$

## Design Parameters (Con't)

**Section Properties (Con't)**

$I_{pos} := \frac{B \cdot D^3}{12}$ $I_{pos} = 442368\ in^4$ *Moment of Inertia of the solid section at the positive moment region*

$I_{neg} := \frac{B \cdot D^3}{12} - \frac{Dia_{pipe} \cdot D^3}{12}$ $I_{neg} = 221184\ in^4$ *Moment of Inertia of the hollow section at the negative moment region*

$S_{pos} := \frac{I_{pos}}{\left(\frac{D}{2}\right)}$ $S_{pos} = 18432\ in^3$ *Section Modulus of solid Rectangular Section at the positive moment region*

$S_{neg} := \frac{I_{neg}}{\left(\frac{D}{2}\right)}$ $S_{neg} = 9216\ in^3$ *Section Modulus of hollow connection section at the negative moment region*

$A_{pos} := B \cdot D$ $A_{pos} = 2304\ in^2$

$A_{neg} := \left(B - Dia_{pipe}\right) \cdot D$ $A_{neg} = 1152\ in^2$

# Flexure Conversion

## Determine Number of Strands

$A_{ps} := \frac{A_s \cdot f_y}{0.75 \cdot f_{pu}} = 8.78\ in^2$ *Equivalent area of prestrssing steel*

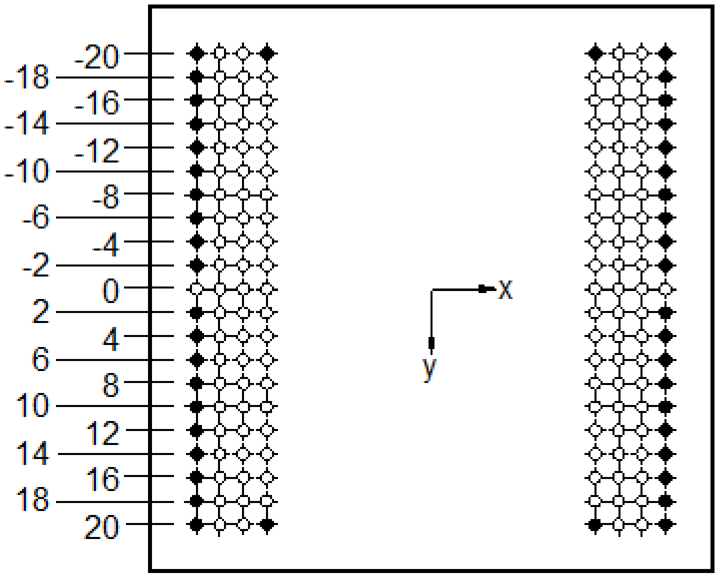
$n := \mathrm{Ceil}\left(\frac{A_{ps}}{A_{strand}}, 4\right)$ $n = 44$ *Number of strands to have concentric strand layout with the equivalent area of prestressing steel.*

$F_{provided} := T_{strand} \cdot n$ $F_{provided} = 1546.78\ kip$ *Prestressing force provided from the selected number of strands*

## Flexural Conversion (Con't)

## Check Ultimate Strength Capacity

Determine strand configuration




The strand layout is limited based on the configuration of the cap-to-column connection. For this example, the cap-to-column connection is assumed to be formed by a 24-inch nominal diameter pocket connection.

**Define Variables**

$\Delta_{\varepsilon p} := \dfrac{T_{strand}}{E_p \cdot A_{ps}}$ $\qquad \Delta_{\varepsilon p} = 0.0001$

*Pre-strain, after losses*

$\beta := \max\left(\left(0.85 - \left(\dfrac{f'_c}{ksi} - 4\right) \cdot 0.05\right), 0.65\right)$ $\qquad \beta = 0.8$

*(AASHTO LRFD 5.7.2.2)*

$\varepsilon_{cu} := 0.003$

*Maximum strain at extreme compression fiber (AASHTO LRFD 5.7.2.1)*

$\phi := 1.0$

*Strength Reduction Factor (AASHTO LRFD 5.5.4.2.1)*

$Q := 0.03$

$R := 6$

*Q and R are constants in the Menegotto-Pinto"equation used to determine the stress at $i^{th}$ layer of prestressing steel. Since a side configuration layout of prestressing steel was used instead of the conventional top & bottom layout, the stresses in the prestressing steel must be determined at each individual layer.*

$c_i := -15 \cdot in$

*Initial location of neutral axis used in the iterative solution of determining the moment capacity.*

## Flexural Conversion (Con't)

### Calculate Strain and Stress in Each Steel Layer

$$\varepsilon_{ti} = \varepsilon_{cu} \cdot \left( \frac{d_i - c}{\frac{D}{2} + c} \right)$$

*Tension strain at the $i^{th}$ layer. $d_i$ is the depth of the prestressing layer, as shown in the strand layout (note the convention and origin of distance measurements)*

$$\varepsilon_{si} = \varepsilon_{ti} + \Delta_{\varepsilon p}$$

*Total strain on each layer, considering the pre-strain*

$$f_{psi} = E_p \cdot \varepsilon_{si} \cdot \left( Q + \frac{1 - Q}{\left( 1 + \left( \left| \frac{\varepsilon_{si} \cdot E_s}{f_y} \right| \right)^R \right)^{\frac{1}{R}}} \right)$$

*Menegotto-Pinto equation to determine the stress in the $i^{th}$ layer*

$$T_i = f_{psi} \cdot A_{psi}$$

*Tension force in the $i^{th}$ layer of steel. $A_{psi}$ is the area of prestressing steel in that layer.*

$$jd_i = \frac{D}{2} - d_i - \frac{a}{2}$$

*Moment arm between compressive stress block and the $i^{th}$ layer of prestressing steel*

$$a = \frac{-D}{2} + \left( \frac{-\beta}{2} \cdot \left( \frac{-D}{2} - c \right) \right)$$

*Depth of the equivalent compression block, with respect to the center of the bent cap*

$$M_i = T_i \cdot j \cdot d_i$$

*Moment in the $i^{th}$ layer*

$$C_c = -0.85 \cdot f'_c \cdot \beta \cdot \left( \frac{D}{2} - c \right) \cdot B$$

*Compressive force from the equivalent compressive stress block*

*The previous equations are calculated using the $c_i$ value, and iterated with changing values of $c$ until the sum of the tensile forces equals the magnitude of the compressive force:*

$$\Sigma T_t = \left| C_c \right|$$

*This process is completed in Microsoft Excel, and the results are presented in the following table*

## Flexural Conversion (Con't)

**Moment Capacity**

| di (in) | ni | εti | Δεp | εsi | fpsi (ksi) | Ti (kips) | jdi (in) | Mi (k-in) |
|---|---|---|---|---|---|---|---|---|
| -20 | 4 | -0.00197 | 0.00568 | 0.00371 | 105.69 | 91.74 | -0.67 | 61.3 |
| -18 | 2 | -0.00146 | 0.00568 | 0.00423 | 120.17 | 52.15 | 1.33 | 69.4 |
| -16 | 2 | -0.00094 | 0.00568 | 0.00474 | 134.47 | 58.36 | 3.33 | 194.4 |
| -14 | 2 | -0.00043 | 0.00568 | 0.00525 | 148.47 | 64.44 | 5.33 | 343.5 |
| -12 | 2 | 0.00008 | 0.00568 | 0.00577 | 161.99 | 70.30 | 7.33 | 515.4 |
| -10 | 2 | 0.00060 | 0.00568 | 0.00628 | 174.81 | 75.87 | 9.33 | 708.0 |
| -8 | 2 | 0.00111 | 0.00568 | 0.00680 | 186.69 | 81.02 | 11.33 | 918.1 |
| -6 | 2 | 0.00163 | 0.00568 | 0.00731 | 197.40 | 85.67 | 13.33 | 1142.1 |
| -4 | 2 | 0.00214 | 0.00568 | 0.00782 | 206.77 | 89.74 | 15.33 | 1375.8 |
| -2 | 2 | 0.00265 | 0.00568 | 0.00834 | 214.73 | 93.19 | 17.33 | 1615.1 |
| 0 |  | 0.00317 | 0.00568 | 0.00885 | 221.32 | 0.00 | 19.33 | 0.0 |
| 2 | 2 | 0.00368 | 0.00568 | 0.00937 | 226.68 | 98.38 | 21.33 | 2098.5 |
| 4 | 2 | 0.00420 | 0.00568 | 0.00988 | 230.97 | 100.24 | 23.33 | 2338.7 |
| 6 | 2 | 0.00471 | 0.00568 | 0.01040 | 234.38 | 101.72 | 25.33 | 2576.8 |
| 8 | 2 | 0.00523 | 0.00568 | 0.01091 | 237.11 | 102.90 | 27.33 | 2812.5 |
| 10 | 2 | 0.00574 | 0.00568 | 0.01142 | 239.29 | 103.85 | 29.33 | 3046.2 |
| 12 | 2 | 0.00625 | 0.00568 | 0.01194 | 241.07 | 104.62 | 31.33 | 3278.0 |
| 14 | 2 | 0.00677 | 0.00568 | 0.01245 | 242.53 | 105.26 | 33.33 | 3508.4 |
| 16 | 2 | 0.00728 | 0.00568 | 0.01297 | 243.76 | 105.79 | 35.33 | 3737.8 |
| 18 | 2 | 0.00780 | 0.00568 | 0.01348 | 244.81 | 106.25 | 37.33 | 3966.4 |
| 20 | 4 | 0.00831 | 0.00568 | 0.01399 | 245.72 | 213.29 | 39.33 | 8388.9 |
| 0.00 | 44 | Cc = | -1904.8 | kips | ∑Ti = | 1904.8 | ∑Mi = | 3557.96 |

$c := -14.06 \cdot in$ *Final location of N/A, from iterations*

$\Sigma M_i = 3557.96\ kip \cdot ft$ *Sum of $M_i$ from the iterations*

$M_n := \Sigma M_i$ $M_n = 3558\ kip \cdot ft$

$M_r := \phi \cdot M_n$ $M_r = 3558\ kip \cdot ft$ *Factored Flexural Resistance (AASHTO LRFD 5.7.3.2.1)*

## Flexural Conversion (Con't)

## Check Minimum Capacity

*Calculate the $M_{cr}$ and check if the $M_r$ meets AASHTO LRFD 5.7.3.3.2*

$\gamma_1 := 1.6$ $\gamma_2 := 1.1$ $\gamma_3 := 1.0$

Negative Moment Region

$f_{cpe} := \frac{F_{provided}}{A_{neg}}$ $f_{cpe} = 1.34\ ksi$

$M_{cr\_neg} := \gamma_3 \cdot \left( \left( \gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe} \right) \cdot S_{neg} \right)$ $M_{cr\_neg} = 1794\ kip \cdot ft$

$M_{rCheck} := \mathbf{if} \left( M_r \geq M_{cr\_neg}, \text{“GOOD”}, \text{“NOT GOOD”} \right)$

$M_{rCheck} = \text{“GOOD”}$

Positive Moment Region

$f_{cpe} := \frac{F_{provided}}{A_{pos}}$ $f_{cpe} = 0.67\ ksi$

$M_{cr\_pos} := \gamma_3 \cdot \left( \left( \gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe} \right) \cdot S_{pos} \right)$ $M_{cr\_pos} = 2453\ kip \cdot ft$

$M_{rCheck} := \mathbf{if} \left( M_r \geq M_{cr\_pos}, \text{“GOOD”}, \text{“NOT GOOD”} \right)$

$M_{rCheck} = \text{“GOOD”}$

## Design Summary

Concrete Strength

$f'_c = 5\ ksi$

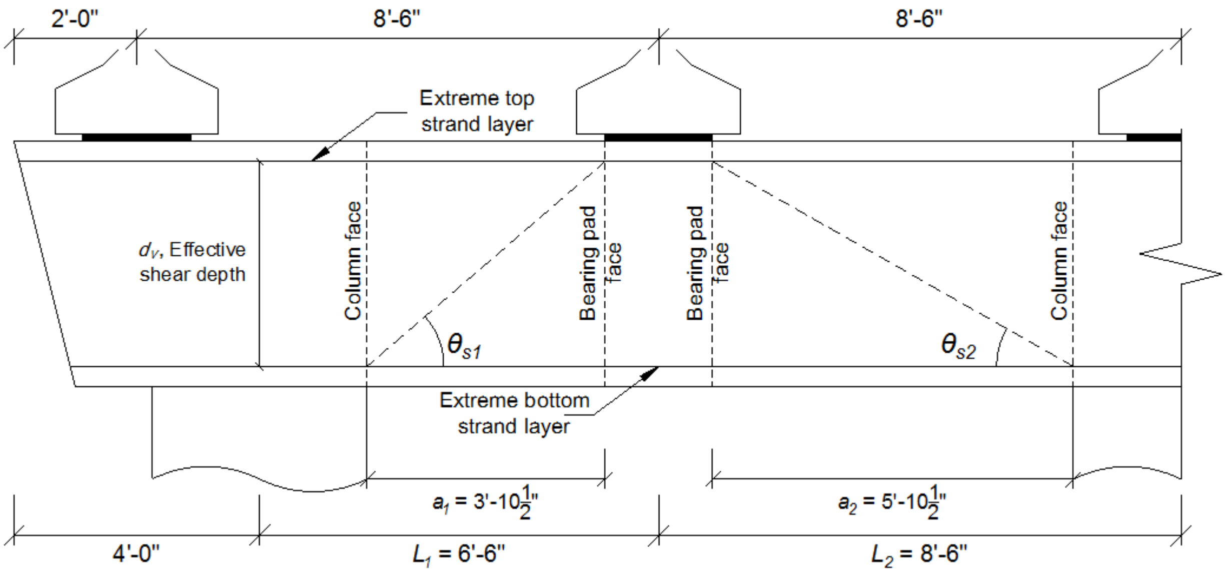
Prestressing

$n = 44$

$F_{provided} = 1546.8\ kip$

# Shear Conversion

## Bridge Configuration




### Section Properties

| | |
|---|---|
| $A_{s\_bot}$ = 12 #11 bars $\quad A_{s\_bot} := 12 \cdot 1.56\ \textbf{in}^2 = 18.72\ \textbf{in}^2$ | *Area of bottom RC flexural steel* |
| $A_{s\_top}$ = 7 #11 bars $\quad A_{s\_top} := 7 \cdot 1.56\ \textbf{in}^2 = 10.92\ \textbf{in}^2$ | *Area of top flexural steel* |
| $A_s := A_{s\_bot} + A_{s\_top} = 29.64\ \textbf{in}^2$ | *Total area of RC flexural steel* |
| $A_v := 0.62\ \textbf{in}^2$ | *Area of transverse steel* |
| $f_y = 60\ \textbf{ksi}$ | *Yield strength of flexural steel* |
| $f_{yh} := 60\ \textbf{ksi}$ | *Yield strength of transverse steel* |
| $b_v := B = 48\ \textbf{in}$ | *Width of cross section* |
| $d := 44\ \textbf{in}$ | *Effective depth of bottom flexural steel* |
| $cover := 4 \cdot \textbf{in}$ | *Measured from Center of flexural steel* |
| $d_v := d - cover = 40\ \textbf{in}$ | *Effective shear depth* |
| $L_1 := 78\ \textbf{in}$ & $L_2 := 102\ \textbf{in}$ | *Distance between the center of the column and the center of the girder* |
| $a_1 := 46.5\ \textbf{in}$ & $a_2 := 70.5\ \textbf{in}$ | *Shear span* |
| $\phi_v := 0.9$ | |

## Shear Conversion (Con't)

### Find $\theta_s$

$$\theta_{s1} := \text{atan}\left(\frac{d_v}{a_1}\right) \quad \& \quad \theta_{s2} := \text{atan}\left(\frac{d_v}{a_2}\right)$$

*Compressive strut angle, $\theta_s$, is the angle between column face and bearing pad face within the shear depth, $d_v$*

$$\theta_{s1} = 40.7\ \textbf{deg} \quad \& \quad \theta_{s2} = 29.6\ \textbf{deg}$$

### Calculate shear demand causing plastic failure mechanism

$M_{p\_pos} := A_{s\_bot} \cdot f_y \cdot d_v$ $\quad M_{p\_pos} = 3744\ \textbf{kip} \cdot \textbf{ft}$ *Positive plastic moment capacity*

$M_{p\_neg} := A_{s\_top} \cdot f_y \cdot d_v$ $\quad M_{p\_neg} = 2184\ \textbf{kip} \cdot \textbf{ft}$ *Negative plastic moment capacity*

$M_{p\_total} := M_{p\_pos} + M_{p\_neg}$ $\quad M_{p\_total} = 5928\ \textbf{kip} \cdot \textbf{ft}$ *Total plastic moment capacity*

$V_{u1} := \dfrac{M_{p\_total}}{L_1}$ $\quad V_{u1} = 912\ \textbf{kip}$ *Shear demand based on the plastic moment capacity at shear span, $a_1$*

$V_{u2} := \dfrac{M_{p\_total}}{L_2}$ $\quad V_{u2} = 697.4\ \textbf{kip}$ *Shear demand based on the plastic moment capacity at shear span, $a_2$*

### Check a required transverse reinforcement

$\beta := 1.6$ *Factor relating concrete shear capacity of concrete*

$$V_{c1} := 0.0316\ \beta \cdot \sqrt{\frac{f'_c}{\textbf{ksi}}} \cdot \textbf{ksi} \cdot b_v \cdot d_v \cdot \cot\left(\theta_{s1}\right) = 252.3\ \textbf{kip}$$

*Concrete shear resistance in $a_1$*

$$s_1 := \frac{A_v \cdot f_{yh} \cdot d_v \cdot \cot\left(\theta_{s1}\right)}{\left(\dfrac{V_{u1}}{\phi_v} - V_{c1}\right)} = 2.27\ \textbf{in}$$

*Required transverse reinforcement spacing in $a_1$*

$$V_{c2} := 0.05 \cdot \sqrt{\frac{f'_c}{\textbf{ksi}}} \cdot \textbf{ksi} \cdot b_v \cdot d_v \cdot \cot\left(\theta_{s2}\right) = 378.34\ \textbf{kip}$$

*Concrete shear resistance in $a_2$*

$$s_2 := \frac{A_v \cdot f_{yh} \cdot d_v \cdot \cot\left(\theta_{s2}\right)}{\left(\dfrac{V_{u2}}{\phi_v} - V_{c2}\right)} = 6.61\ \textbf{in}$$

*Required transverse reinforcement spacing in $a_2$*

*Required shear reinforcement exceeds that for the reinforced concrete design*
*$\therefore$ use reinforced concrete spacing or 6 in. maximum spacing to provide conservative design*

$s := 6\ \textbf{in}$

## Shear Conversion (Con't)

### Check Minimum Transverse Reinforcement

$$A_{v\_min} := 0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot \frac{b_v \cdot s}{f_y} \qquad A_{v\_min} = 0.34\ in^2 \qquad \textit{(AASHTO LRFD Eq. 5.8.2.5-1)}$$

$$A_v = 0.62\ in^2 \quad > \quad A_{v\_min}$$

$$MinimumSteelCheck := \mathbf{if}\left(A_v > A_{v\_min}, \text{“Okay”}, \text{“Not okay”}\right)$$

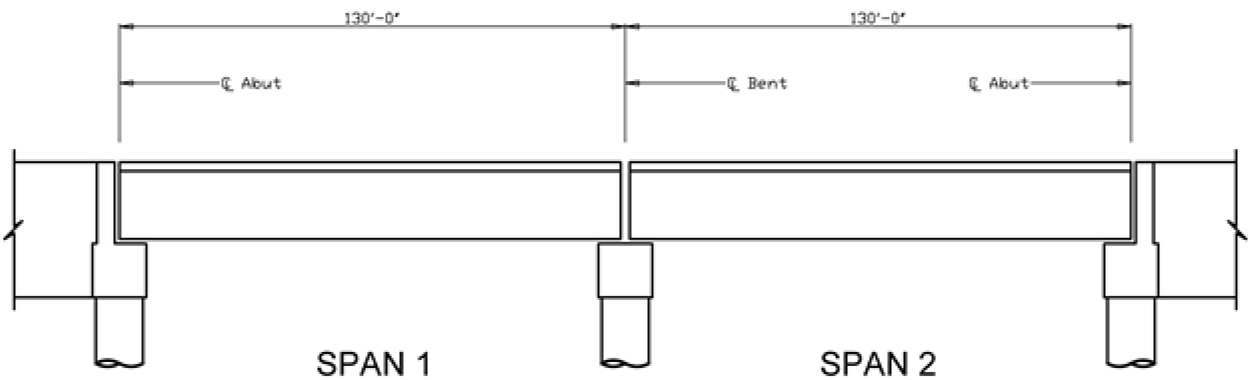
$$MinimumSteelCheck = \text{“Okay”}$$

# APPENDIX D: DESIGN EXAMPLE #2

Date: February, 2018

Rectangular Pretensioned Bent Cap Design Example, with a Symmetric, Concentric Strand Layout for BIG-62-38 Equivalent
This design example is in accordance with the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012) and TxDOT Bridge Design Manual - LRFD (October 2015)

# Design Parameters




## Span 1

130' Type Tx62 Girders ($0.948 \frac{k}{ft}$)
5 Girders Spaced @ 8.50 with 3' deck overhangs

*"AASHTO LRFD" refers to the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012)*

## Span 2

*"BDM-LRFD" refers to the TxDOT Bridge Design Manual - LRFD (October 2015)*

130' Type Tx54 Girders ($0.948 \frac{k}{ft}$)
5 Girders Spaced @ 8.50 with 3' overhangs

*"TxSP" refers to TxDOT guidance, recommendations, and standard practice*

## All Spans

Deck is 40' wide
Type T551 Rail ($0.382 \frac{k}{ft}$)
8.5" Thick Slab (0.100 ksf)
Assume 2" Overlay @ 140 pcf (0.023 ksf)

## Assume

*(TxSP)*

4'-0" X 4'-0" Cap
3~42" Columns Spaced @ 15'-0"
Cap will be modeled as a continuous beam with simple supports using TxDOT's CAP18 program.

*TxDOT does not consider frame action for typical multi-column Rectangular Reinforced Concrete Bents. The same methodology is applied to the structural analysis of multi-column Rectangular Pretensioned Concrete Bents. (BDM-LRDSFD, Ch. 4, Sect. 4, Structural Analysis)*

## Design Parameters (Con't)

### Define Variable

Back Span | Forward Span

| Back Span | Forward Span | |
|---|---|---|
| $Span1 := 130 \cdot ft$ | $Span2 := 130 \cdot ft$ | *Span Length* |
| $GdrSpa1 := 8.5 \cdot ft$ | $GdrSpa2 := 8.5 \cdot ft$ | *Girder Spacing* |
| $GdrNo1 := 5$ | $GdrNo2 := 5$ | *Number of Girders in Span* |
| $GdrWt1 := 0.948 \cdot \frac{kip}{ft}$ | $GdrWt2 := 0.948 \cdot \frac{kip}{ft}$ | *Weight of Girder* |

Bridge

$RailWt := 0.382 \cdot \frac{kip}{ft}$ *Weight of Rail*

$SlabThk := 8.5 \cdot in$ *Thickness of Bridge Slab*

$OverlayThk := 2 \cdot in$ *Thickness of Overlay*

$w_c := 150 \cdot pcf$ *Unit Weight of Concrete for Load Calcs*

$w_{olay} := 140 \cdot pcf$ *Unit Weight of Overlay*

Other Variables:

$station := 0.5 \cdot ft$ *Station Increment for CAP18 Analysis*

$IM := 33\%$ *Dynamic Load Allowance, (AASHTO LRFD Table 3.6.2.1-1)*

Cap Dimensions:

$CapWidth := 48 \cdot in$

$CapDepth := 48 \cdot in$

$cover := 4 \cdot in$ *Measured from Center of Prestressing Strand*

Material Properties:

$f'_c := 5 \cdot ksi$ *Concrete Compressive Strength*

$f_{pu} := 270 \cdot ksi$ *Ultimate Strength of Prestressing Steel*

$A_{strand} := 0.217 \cdot in^2$ *Area of Prestressing Strand*

$w_{cE} := 145 \cdot pcf$ *Unit Weight of Concrete for $E_c$ Calc*

## Design Parameters (Con't)

### Define Variable (con't)

$$E_c := 33000 \cdot \left(\frac{w_{cE}}{1000 \cdot pcf}\right)^{1.5} \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi = 4074\ ksi$$

*Modulus of Elasticity of Concrete, (AASHTO LRFD Eq. 5.4.2.4-1)*

$$E_s := 29000 \cdot ksi$$

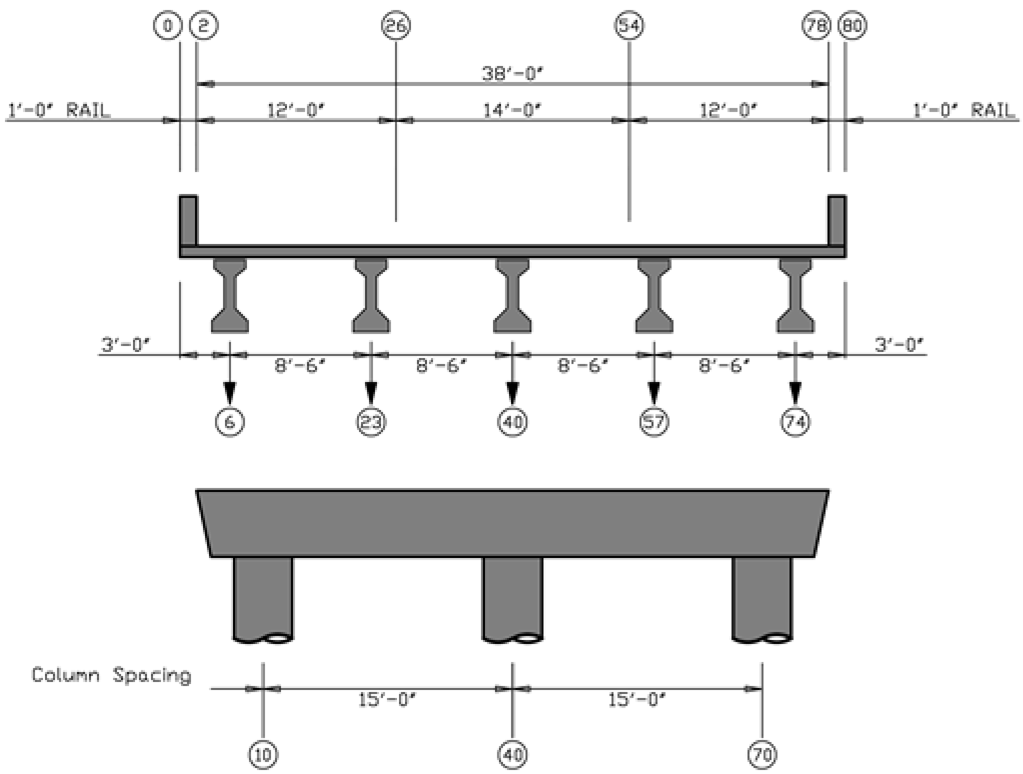
*Modulus of Elasticity of Mild Steel*

$$E_p := 28500 \cdot ksi$$

*Modulus of Elasticity of Prestressing Steel*

# Cap Analysis

### Cap Model




*The circled numbers are the stations that are used for the CAP18 Input file. One station is 0.5ft in the direction perpendicular to the pgl.*

## Cap Analysis (Con't)

**Dead Load**

Span 1

$Rail1 := \frac{2 \cdot RailWt \cdot \frac{Span1}{2}}{min(GdrNo1, 6)}$ $Rail1 = 9.93\ kip$ *Rail weight is distributed evenly among stringers, up to 3 stringers per rail. (TxSP)*

$Slab1 := w_c \cdot GdrSpa1 \cdot SlabThk \cdot \frac{Span1}{2} \cdot 1.1$ $Slab1 = 64.57\ kip$ *Slab DL is increased by 10% to account for haunch and thickened slab ends.*

$Girder1 := GdrWt1 \cdot \frac{Span1}{2}$ $Girder1 = 61.62\ kip$ *Weight of girder acting on bent*

$DLRxn1 := Rail1 + Slab1 + Girder1$ $DLRxn1 = 136.13\ kip$ *Dead load reaction per girder, not considering overlay. (Overlay is calculated separately due to possibility of applying a different load factor)*

$Overlay1 := w_{olay} \cdot GdrSpa1 \cdot OverlayThk \cdot \frac{Span1}{2}$ $Overlay1 = 12.89\ kip$ *Design for future overlay, per girder*

Span 2

$Rail2 := \frac{2 \cdot RailWt \cdot \frac{Span2}{2}}{min(GdrNo2, 6)}$ $Rail2 = 9.93\ kip$

$Slab2 := w_c \cdot GdrSpa2 \cdot SlabThk \cdot \frac{Span2}{2} \cdot 1.1$ $Slab2 = 64.57\ kip$

$Girder2 := GdrWt2 \cdot \frac{Span2}{2}$ $Girder2 = 61.62\ kip$

$DLRxn2 := Rail2 + Slab2 + Girder2$ $DLRxn2 = 136.13\ kip$

$Overlay2 := w_{olay} \cdot GdrSpa2 \cdot OverlayThk \cdot \frac{Span2}{2}$ $Overlay2 = 12.89\ kip$

Cap

$A_g := CapWidth \cdot CapDepth$ $A_g = 2304\ in^2$ *Gross Area of Cap*

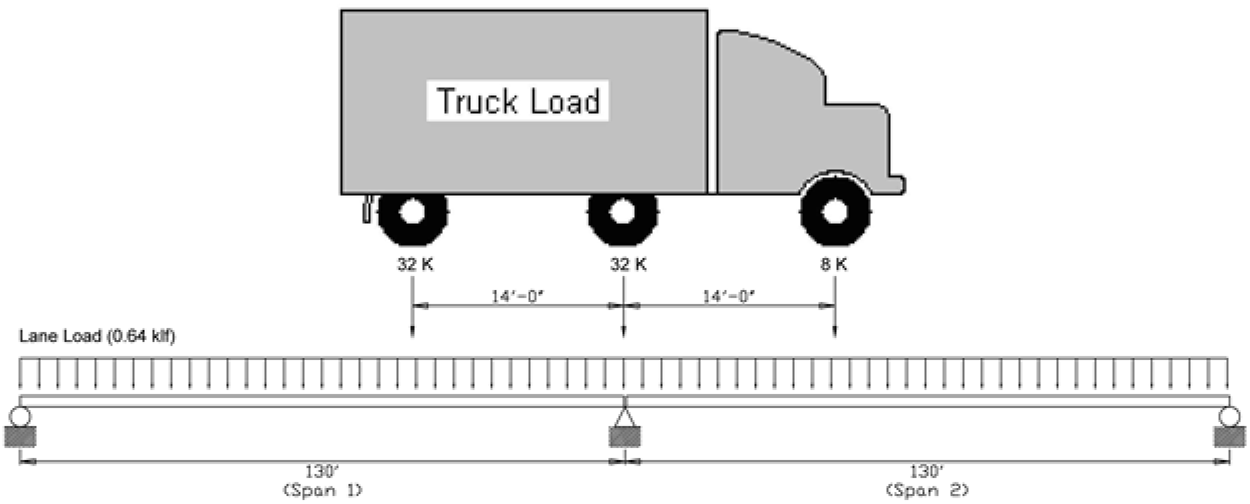
$Cap := w_c \cdot A_g \cdot station$ $Cap = 1.2\ kip$ *Dead Load of Cap, per station*

$I_g := \frac{1}{12} \cdot CapWidth \cdot CapDepth^3$ $I_g = (4.42 \cdot 10^5)\ in^4$ *Gross Moment of Inertia of Cap*

$E_c = 4074\ ksi$ $E_c \cdot I_g = (1.25 \cdot 10^7)\ kip \cdot ft^2$ *Bending Stiffness of Cap*

## Cap Analysis (Con't)

**Live Load** *(AASHTO LRFD 3.6.1.2.2 and 3.6.1.2.4)*




$\text{LongSpan} := \max(\text{Span1}, \text{Span2})$

$\text{ShortSpan} := min(\text{Span1}, \text{Span2})$

$\text{LongSpan} = 130\ ft$

$\text{ShortSpan} = 130\ ft$

$\text{IM} = 0.33$

Live Load Model

*Use HL-93 Live Load. Maximum reaction at an interior bent, the Design Truck" will govern over Design Tandem." With the Long Span less than twice as long as the Short Span, the maximum reaction occurs when the middle axle (32 kip) is placed over the support, the front axle (8 kips) is placed on the Short Span, and the rear axle (32 kips) is placed on the Long Span.*

$$\text{Lane} := 0.64 \cdot klf \cdot \left(\frac{\text{LongSpan} + \text{ShortSpan}}{2}\right)$$

$\text{Lane} = 83.2\ kip$

$$\text{Truck} := 32 \cdot kip + 32 \cdot kip \cdot \left(\frac{\text{LongSpan} - 14 \cdot ft}{\text{LongSpan}}\right) + 8 \cdot kip \cdot \left(\frac{\text{ShortSpan} - 14 \cdot ft}{\text{ShortSpan}}\right)$$

$\text{Truck} = 67.69\ kip$

$\text{LLRxn} := \text{Lane} + \text{Truck} \cdot (1 + \text{IM})$

$\text{LLRxn} = 173.23\ kip$

*Combine Design Truck" and Design Lane" loadings.*
*(AASHTO LRFD 3.6.1.3)*

$\text{P} := 16.0 \cdot kip \cdot (1 + \text{IM})$

$\text{P} = 21.28\ kip$

*Dynamic Load Allowance (IM) does not apply to the Design Lane"*
*(AASHTO LRFD 3.6.1.2.4)*

$$\text{W} := \frac{\text{LLRxn} - (2 \cdot \text{P})}{10 \cdot ft} \cdot \text{station}$$

*Live Load is applied to the deck slab by two 16 kip wheel loads increased by IM, with the remainder of the live load distributed over a 10ft design lane width*
*(AASHTO LRFD 3.6.1.2.1) (TxSP)*

$\text{W} = 6.53\ kip$

*Live Load applied to the slab is distributed to the beams assuming the slab is hinged at each beam, except the outside beam.*
*(BDM-LRFD, Ch. 4, Sect. 4, Structural Analysis)*

# Cap Analysis (Con't)

### Cap18 Input

Multiple Presence Factors, m *(AASHTO LRFD Table 3.6.1.1.2-1)*

| No. of Lanes | Factor "m" |
|---|---|
| 1 | 1.20 |
| 2 | 1.00 |
| 3 | 0.85 |
| >3 | 0.65 |

Limit States *(AASHTO LRFD 3.4.1)*

*The cap design only needs to consider Strength I, Service I, and Service I (Dead Load Only)*

Strength I

| | |
|---|---|
| Live Load and Dynamic Load Allowance | LL + IM = 1.75 |
| Dead Load Components | DC = 1.25 |
| Dead Load Wearing Surface (Overlay) | DW = 1.50 |

*TxDOT allows Overlay Factor to be reduced to 1.25 (TxSP).*

Service I

| | |
|---|---|
| Live Load and Dynamic Load Allowance | LL + IM = 1.00 |
| Dead Load and Wearing Surface | DC & DW = 1.00 |

*CAP18 Input is included in an Appendix to this example*

### Cap18 Output

| | Max +M | Max -M |
|---|---|---|
| Dead Load | $M_{DLpos} := 578.6 \cdot kip \cdot ft$ | $M_{DLneg} := 615.3 \cdot kip \cdot ft$ |
| Service Load | $M_{SLpos} := 1000.9 \cdot kip \cdot ft$ | $M_{SLneg} := 884.2 \cdot kip \cdot ft$ |
| Ultimate Load | $M_{ULpos} := 1462.2 \cdot kip \cdot ft$ | $M_{ULneg} := 1239.7 \cdot kip \cdot ft$ |

*Maximum loads from the CAP18 Output file. The output is included in an Appendix to this design example.*

$M_{DLmax} := \max\left(M_{DLpos}, M_{DLneg}\right)$ $\qquad M_{DLmax} = 615.3\ kip \cdot ft$

$M_{SLmax} := \max\left(M_{SLpos}, M_{SLneg}\right)$ $\qquad M_{SLmax} = \left(1 \cdot 10^{3}\right)\ kip \cdot ft$

$M_{ULmax} := \max\left(M_{ULpos}, M_{ULneg}\right)$ $\qquad M_{ULmax} = 1462.2\ kip \cdot ft$

# Flexural Design

*The flexural design of the bent cap is based on the philosophy of "Zero Tension Under Dead Load." The design follows the following steps:*

- *Design for Zero Flexural Tension under Dead Load*
- *Determine Minimum Concrete Compressive Strength and Check Stresses at Service Loads*
- *Check the Minimum Number of Strands*
- *Check the Ultimate Strength Capacity*
- *Check that Minimum Capacity is satisfied*

## Define Constants and Variables

$B := CapWidth$ $\quad D := CapDepth$ $\quad Dia_{pipe} := 24\ in$

$f_r := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi$ $\quad f_r = 0.54\ ksi$ *Modulus of Rupture (AASHTO LRFD Eq. 5.4.2.4-1)*

$jd := 0.45 \cdot CapDepth$ $\quad jd = 21.6\ in$ *Approximate Moment Arm between Tension and Compression section to estimate nominal moment capacity for determination of Minimum Number of Strands*

$f_y := 0.9 \cdot f_{pu}$ $\quad f_y = 243\ ksi$ *Yield Strength of Prestressing Steel (AASHTO LRFD Table 5.4.4.1-1)*

$\Delta_{f.pt} := 0.2$ *Assumed prestress loss in pretensioned members*

$T_{strand} := 0.75 \cdot f_{pu} \cdot A_{strand} \cdot (1 - \Delta_{f.pt})$ $\quad T_{strand} = 35.15\ kip$

$I_{pos} := \frac{B \cdot D^3}{12}$ $\quad I_{pos} = 442368\ in^4$ *Moment of Inertia of the solid section at the positive moment region*

$I_{neg} := \frac{B \cdot D^3}{12} - \frac{Dia_{pipe} \cdot D^3}{12}$ $\quad I_{neg} = 221184\ in^4$ *Moment of Inertia of the hollow section at the negative moment region*

$S_{pos} := \frac{I_{pos}}{\left(\frac{D}{2}\right)}$ $\quad S_{pos} = 18432\ in^3$ *Section Modulus of solid Rectangular Section at the positive moment region*

$S_{neg} := \frac{I_{neg}}{\left(\frac{D}{2}\right)}$ $\quad S_{neg} = 9216\ in^3$ *Section Modulus of hollow connection section at the negative moment region*

$A_{pos} := B \cdot D$ $\quad A_{pos} = 2304\ in^2$

$A_{neg} := (B - Dia_{pipe}) \cdot D$ $\quad A_{neg} = 1152\ in^2$

## Flexural Design (Con't)

## Design for Zero Flexural Tension Under Dead Load

Tension Limit:

$$\frac{-F_t}{A}+\frac{M_{DL}}{S_x}=f_t=0$$

*Set the stress at the extreme tension fiber to zero*

$$F_{t\_neg}:=\frac{M_{DLneg}\cdot A_{neg}}{S_{neg}}$$ $F_{t\_neg}=923\ kip$

*Determine the prestressing force required to achieve zero tension stress in the negative moment region*

$$F_{t\_pos}:=\frac{M_{DLpos}\cdot A_{pos}}{S_{pos}}$$ $F_{t\_pos}=867.9\ kip$

*Determine the prestressing force required to achieve zero tension stress in the positive moment region*

$$F_t:=\max\left(F_{t\_neg},F_{t\_pos}\right)$$ $F_t=923\ kip$

*Use the calculated $F_t$ to determine the corresponding number of strands required*

$$n_{flex\_t}:=\mathrm{Ceil}\left(\left(\frac{F_t}{T_{strand}}\right),4\right)$$ $n_{flex\_t}=28$

*For symmetric, concentric layouts- the number of strands must be a multiple of 4*

Compression Limit:

$$\frac{-F_c}{A}-\frac{M_{DL}}{S_x}\geq f_c=-0.45\cdot f'_c$$

*Set the stress at the extreme compression fiber to the compressive stress limit (AASHTO LRFD Table 5.9.4.2.1-1*

$$F_{c\_neg}:=\left(0.45\cdot f'_c-\frac{M_{DLneg}}{S_{neg}}\right)\cdot A_{neg}$$ $F_{c\_neg}=1669.1\ kip$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the negative moment region*

$$F_{c\_pos}:=\left(0.45\cdot f'_c-\frac{M_{DLpos}}{S_{pos}}\right)\cdot A_{pos}$$ $F_{c\_pos}=4316.1\ kip$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the positive moment region*

$$F_c:=min\left(F_{c\_neg},F_{c\_pos}\right)$$ $F_c=1669.1\ kip$

$$n_{flex\_c}:=\mathrm{Floor}\left(\left(\frac{F_c}{T_{strand}}\right),4\right)$$ $n_{flex\_c}=44$

*Use the calculated $F_c$ to determine the corresponding number of strands required to reach the limiting compressive stress*

$$n:=min\left(n_{flex\_t},n_{flex\_c}\right)$$ $n=28$

*The number of strands selected for the should be between $n_{flec\_t}$ and $n_{flex\_c}$*

$$F_{provided}:=n\cdot T_{strand}$$ $F_{provided}=984.3\ kip$

*Determine the provided prestressing force from the selected number of strands*

## Flexural Design (Con't)

## Determine Minimum Concrete Compressive Strength from Service Stress

Tension Stress:

$$\frac{-F}{A}+\frac{M_{SL}}{S_x}\geq f_t = k\cdot\sqrt{f'_c}$$

*The stress at the extreme tension fiber must not exceed the service stress limit, which is* $k\cdot\sqrt{f'_c}$ .
*(AASHTO LRFD Table 5.9.4.2.2-1)*

*Values of* $k$ *are different for various corrosion conditions. For AASHTO LRFD 5.9.4.2.2,* $k$ *values are:*

- *Moderate Exposure (Class I)* $k := 0.19$
- *Severe Exposure (Class II)* $k := 0.0948$

*Recommended value of* $k$ *to limit cracking under service loads was shown to be smaller than the value imposed by AASHTO. This value is* $k := 0.126$

*The tensile stresses under service conditions should not exceed values specified by AASHTO LRFD 5.9.4.2.2. However, to further reduce cracking under service conditions in Class I conditions, the tensile stresses should not exceed the recommended value concluded from experimental testing.*

*For the purpose of this design example, the recommended value of* $k$ *will be used to check the service level stresses and compute the minimum concrete compressive strength.*

*The assumed minimum* $f'_c$ *value was lower than the required minimum concrete compressive strength determined by the service stress limits. Therefore, the* $f'_c$ *was increased to meet this requirement.*

$$f'_c := 6\ ksi$$

Negative Moment Region:

$k := 0.126$ *Multiplier for* $k\cdot\sqrt{f'_c}$ *tensile stress limit*

$$f_{t\_sl\_neg} := \frac{-F_{provided}}{A_{neg}}+\frac{M_{SLneg}}{S_{neg}}$$ $f_{t\_sl\_neg} = 0.297\ ksi$ *Tensile Stress at extreme tension fiber under Service Loads*

$$f_{t\_slim\_neg} := k\cdot\sqrt{\frac{f'_c}{ksi}}\cdot ksi$$ $f_{t\_slim\_neg} = 0.309\ ksi$ *Limiting tensile stress at the extreme tension fiber*

$$TenLimit := \textbf{if}\left(f_{t\_sl\_neg}\leq f_{t\_slim\_neg}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$$ $TenLimit = \text{“GOOD”}$

Positive Moment Region:

$$f_{t\_sl\_pos} := \frac{-F_{provided}}{A_{pos}}+\frac{M_{SLpos}}{S_{pos}}$$ $f_{t\_sl\_pos} = 0.224\ ksi$ *Tensile Stress at extreme tension fiber under Service Loads*

$$f_{t\_slim\_pos} := k\cdot\sqrt{\frac{f'_c}{ksi}}\cdot ksi$$ $f_{t\_slim\_pos} = 0.309\ ksi$ *Limiting tensile stress at the extreme tension fiber*

$$TenLimit := \textbf{if}\left(f_{t\_sl\_pos}\leq f_{t\_slim\_pos}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$$ $TenLimit = \text{“GOOD”}$

## Flexural Design (Con't)

### Check Minimum Concrete Compressive Strength with Service Stress (Con't)

Compression Stress:

$$\frac{-F}{A} - \frac{M_{SL}}{S_x} > f_c = -0.45 \cdot f'_c$$

*The stress at the extreme compression fiber must not exceed the service stress limit. Considering the connection section as solid in compression (AASHTO LRFD Table 5.9.4.2.1-1)*

$$f_{c\_sl} := \left(\frac{-F_{provided}}{A_{pos}}\right) - \left(\frac{M_{SLmax}}{S_{pos}}\right) \qquad f_{c\_sl} = -1.079\ ksi$$

*Compressive Stress at extreme compression fiber under Service Loads*

$$f_{c\_slim} := -0.45 \cdot f'_c \qquad f_{c\_slim} = -2.7\ ksi$$

*Limiting compressive stress at the extreme compression fiber*

$$CompLimit := \textbf{if}\left(f_{c\_sl} \geq f_{c\_slim}, \text{“GOOD”}, \text{“NOT GOOD”}\right) \qquad CompLimit = \text{“GOOD”}$$

Minimum Concrete Strength:

$$f'_{c\_tmin} := \left(\left(\frac{\max\left(f_{t\_sl\_neg}, f_{t\_sl\_pos}\right)}{k \cdot ksi}\right)^2\right) \cdot ksi \qquad f'_{c\_tmin} = 5.55\ ksi$$

*Determine minimum concrete compressive strength to achieve the limiting tensile stress*

$$f'_{c\_cmin} := \frac{f_{c\_sl}}{-0.45} \qquad f'_{c\_cmin} = 2.4\ ksi$$

*Determine minimum concrete compressive strength to achieve the limiting compressive stress*

$$f'_{c\_min} := \max\left(f'_{c\_cmin}, f'_{c\_tmin}, 5 \cdot ksi\right) \qquad f'_{c\_min} = 5.55\ ksi$$

*Minimum Concrete Compressive Strength. If calculated values are significantly low, use a practical minimum achievable concrete strength (consider $5 \cdot ksi$ for precast, prestressed concrete)*

*If the required Minimum Concrete Compressive strength is larger than the maximum allowed design strength from Chapter 4 of BDM-LRFD, adjust the number of strands and/or strand layout to reduce the service stresses.*

*If the calculated Minimum Concrete Compressive strength is larger than the assumed strength, recheck stress limits with the "new" compressive strength.*

## Flexural Design (Con't)

## Determine Minimum Number of Strands

*AASHTO LRFD 5.7.3.3.2 specifies that the factored flexural resistance* $M_r$ *should be at least greater than the lesser of* $M_{cr}$ *or* $1.33 \cdot M_u$ *. The derivation of the formula to determine* $n_{min}$ *is shown:*

$$-\left(\frac{F}{A_g}\right)+\left(\frac{M_{cr}}{S_x}\right) = f_r$$

*Stress at extreme tension fiber is equal to the modulus of rupture*

$$M_{cr} = \gamma_3\left(\gamma_1 \cdot f_r + \gamma_2\left(\frac{F}{A}\right)\right)\cdot S_x \qquad \gamma_1 := 1.6 \quad \gamma_2 := 1.1 \quad \gamma_3 := 1.0$$

*Where* $F$ *is the prestressing force*

$$\phi M_n = M_r \geq M_{cr} \text{ , where } \phi := 1.0$$

*(AASHTO LRFD 5.5.4.2.1)*

*Approximate nominal moment capacity*

$$M_n \geq n \cdot A_{ps} \cdot f_y \cdot jd$$

Substituting, simplifying, and equating $M_{cr}$ and $M_n$ :

$$\gamma_1 f_r \cdot S_x + \gamma_2 \cdot \left(n \cdot T_{strand}\right)\frac{S_x}{A} = n \cdot A_{ps} \cdot f_y \cdot jd$$

Thus:

$$n = \frac{\gamma_1 \cdot f_r \cdot S_x}{A_{ps} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_x}{A}}$$

Also $M_r \geq 1.33 \cdot M_{UL}$

$$n = \frac{1.33 \cdot M_{UL}}{A_{ps} \cdot f_y \cdot jd}$$

$$n_{min\_pos} := \mathrm{ceil}\left(\max\left(\frac{\gamma_1 \cdot f_r \cdot S_{pos}}{A_{strand} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_{pos}}{A_{pos}}}, \frac{1.33 \cdot M_{ULpos}}{A_{strand} \cdot f_y \cdot jd}\right)\right)$$

*n must be checked for both positive and negative bending regions, as the cross-sections are not the same.*

$$n_{min\_pos} = 21$$

$$n_{min\_neg} := \mathrm{ceil}\left(\max\left(\frac{\gamma_1 \cdot f_r \cdot S_{neg}}{A_{strand} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_{neg}}{A_{neg}}}, \frac{1.33 \cdot M_{ULneg}}{A_{strand} \cdot f_y \cdot jd}\right)\right)$$

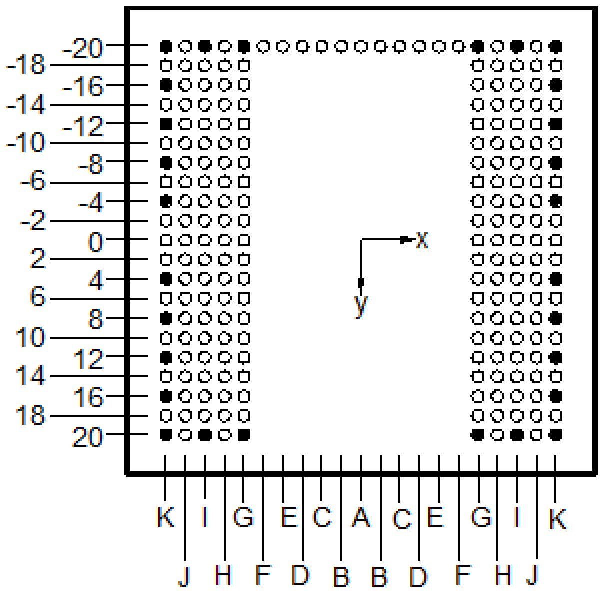
$$n_{min\_neg} = 18$$

$$n_{min} := \max\left(n_{min\_pos}, n_{min\_neg}\right) \qquad n_{min} = 21$$

$$n_{min\_check} := \mathbf{if}\left(n \geq n_{min}, \text{“GOOD”}, \text{“NOT GOOD”}\right) \qquad n_{min\_check} = \text{“GOOD”}$$

## Check Ultimate Strength Capacity

Determine strand configuration




*The strand layout is limited based on the configuration of the cap-to-column connection. For this example, the cap-to-column connection is assumed to be formed by a 24-inch nominal diameter pocket connection.*

**Define Variables**

$$\Delta_{\varepsilon p} := \frac{T_{strand}}{E_p \cdot A_{strand}}$$ $\Delta_{\varepsilon p} = 0.0057$

*Pre-strain, after losses*

$$\beta := \max\left(\left(0.85 - \left(\frac{f'_c}{ksi} - 4\right) \cdot 0.05\right), 0.65\right)$$ $\beta = 0.75$

*(AASHTO LRFD 5.7.2.2)*

$\varepsilon_{cu} := 0.003$

*Maximum strain at extreme compression fiber (AASHTO LRFD 5.7.2.1)*

$\phi := 1.0$

*Strength Reduction Factor (AASHTO LRFD 5.5.4.2.1)*

$Q := 0.03$

$R := 6$

*Q and R are constants in the "Menegotto-Pinto" equation used to determine the stress at i$^{th}$ layer of prestressing steel. Since a side configuration layout of prestressing steel was used instead of the conventional top & bottom layout, the stresses in the prestressing steel must be determined at each individual layer.*

$c_i := -15 \cdot in$

*Initial location of neutral axis used in the iterative solution of determining the moment capacity.*

## Flexural Design (Con't)

**Calculate Strain and Stress in Each Steel Layer**

$$\varepsilon_{ti} = \varepsilon_{cu} \cdot \left( \frac{d_i - c}{\frac{D}{2} + c} \right)$$

*Tension strain at the $i^{th}$ layer. $d_i$ is the depth of the prestressing layer, as shown in the strand layout (note the convention and origin of distance measurements)*

$$\varepsilon_{si} = \varepsilon_{ti} + \Delta_{\varepsilon p}$$

*Total strain on each layer, considering the pre-strain*

$$f_{psi} = E_p \cdot \varepsilon_{si} \cdot \left( Q + \frac{1 - Q}{\left( 1 + \left( \left| \frac{\varepsilon_{si} \cdot E_s}{f_y} \right| \right)^R \right)^{\frac{1}{R}}} \right)$$

*Menegotto-Pinto equation to determine the stress in the $i^{th}$ layer*

$$T_i = f_{psi} \cdot A_{psi}$$

*Tension force in the $i^{th}$ layer of steel. $A_{psi}$ is the area of prestressing steel in that layer.*

$$jd_i = \frac{D}{2} - d_i - \frac{a}{2}$$

*Moment arm between compressive stress block and the $i^{th}$ layer of prestressing steel* ,

$$a = \frac{-D}{2} + \left( \frac{-\beta}{2} \cdot \left( \frac{-D}{2} - c \right) \right)$$

*Depth of the equivalent compression block, with respect to the center of the bent cap*

$$M_i = T_i \cdot j \cdot d_i$$

*Moment in the $i^{th}$ layer*

$$C_c = -0.85 \cdot f'_c \cdot \beta \cdot \left( \frac{D}{2} - c \right) \cdot B$$

*Compressive force from the equivalent compressive stress block*

*The previous equations are calculated using the $c_i$ value, and iterated with changing values of $c$ until the sum of the tensile forces equals the magnitude of the compressive force:*

$$\Sigma T_t = \left| C_c \right|$$

*This process is completed in Microsoft Excel, and the results are presented in the following table*

## Flexural Design (Con't)

**Moment Capacity:**

| $d_i$ (in) | $n_i$ | $\varepsilon_{ti}$ | $\Delta\varepsilon_p$ | $\varepsilon_{si}$ | $f_{psi}$ (ksi) | $T_i$ (kips) | $jd_i$ (in) | $M_i$ (k-in) |
|---|---|---|---|---|---|---|---|---|
| -20 | 6 | -0.00146 | 0.00568 | 0.00423 | 120.19 | 156.48 | 0.89 | 139.1 |
| -16 | 2 | 0.00009 | 0.00568 | 0.00577 | 162.03 | 70.32 | 4.89 | 343.8 |
| -12 | 2 | 0.00163 | 0.00568 | 0.00731 | 197.44 | 85.69 | 8.89 | 761.7 |
| -8 | 2 | 0.00317 | 0.00568 | 0.00886 | 221.36 | 96.07 | 12.89 | 1238.2 |
| -4 | 2 | 0.00471 | 0.00568 | 0.01040 | 234.41 | 101.73 | 16.89 | 1718.2 |
| 4 | 2 | 0.00780 | 0.00568 | 0.01348 | 244.82 | 106.25 | 24.89 | 2644.5 |
| 8 | 2 | 0.00934 | 0.00568 | 0.01503 | 247.27 | 107.32 | 28.89 | 3100.3 |
| 12 | 2 | 0.01089 | 0.00568 | 0.01657 | 249.16 | 108.13 | 32.89 | 3556.4 |
| 16 | 2 | 0.01243 | 0.00568 | 0.01811 | 250.77 | 108.84 | 36.89 | 4014.8 |
| 20 | 6 | 0.01397 | 0.00568 | 0.01966 | 252.26 | 328.44 | 40.89 | 13429.5 |
| | | $C_c$ = | -1269.3 | kips | $\sum T_i$ = | 1269.3 | $\sum M_i$ = | 30946.5 |

$c := -16.97 \cdot in$ *Final location of N/A, from iterations*

$\Sigma M_i := 30946.5 \cdot kip \cdot in$ *Sum of $M_i$ from the iterations*

$M_n := \Sigma M_i$ $M_n = 2578.9\ kip \cdot ft$

$M_r := \phi \cdot M_n$ $M_r = 2579\ kip \cdot ft$ *Factored Flexural Resistance (AASHTO LRFD 5.7.3.2.1)*

$\text{CapacityCheck} := \textbf{if}\left(M_r \geq M_{ULmax}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$\text{CapacityCheck} = \text{“GOOD”}$

## Flexural Design (Con't)

## Check Minimum Capacity:

*Calculate the* $M_{cr}$ *and check if the* $M_r$ *meets AASHTO LRFD 5.7.3.3.2*

$\gamma_1 := 1.6$ $\gamma_2 := 1.1$ $\gamma_3 := 1.0$

Negative Moment Region:

$f_{cpe} := \dfrac{F_{provided}}{A_{neg}}$ $f_{cpe} = 0.85\ \boldsymbol{ksi}$

$M_{cr\_neg} := \gamma_3 \cdot \left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{neg}\right)$ $M_{cr\_neg} = 1381\ kip \cdot ft$

$M_{rCheck} := \textbf{if}\left(M_r \geq min\left(M_{cr\_neg}, 1.33 \cdot M_{ULneg}\right), \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$M_{rCheck} = \text{“GOOD”}$

Positive Moment Region:

$f_{cpe} := \dfrac{F_{provided}}{A_{pos}}$ $f_{cpe} = 0.43\ \boldsymbol{ksi}$

$M_{cr\_pos} := \gamma_3 \cdot \left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{pos}\right)$ $M_{cr\_pos} = 2041\ kip \cdot ft$

$M_{rCheck} := \textbf{if}\left(M_r \geq min\left(M_{cr\_pos}, 1.33 \cdot M_{ULpos}\right), \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$M_{rCheck} = \text{“GOOD”}$

## Design Summary:

Concrete Strength:

$f'_c = 6\ ksi$

Prestressing:

$n = 28$

$F_{provided} = 984.3\ kip$

# Shear Design

## Design Philosophy:

$V_u$ *(Ultimate Shear) must be less than* $V_r$ (Shear Resistance)

$V_u \le V_r$ *(AASHTO LRFD Eq. 1.3.2.1-1)*

$V_r = \phi_v \cdot V_n$ *(AASHTO LRFD Eq. 5.8.2.1-2)*

$\phi_v := 0.9$ *Reduction factor (AASHTO LRFD 5.5.4.2.1)*

$V_n$ *is the lesser of* $V_{n1}$ and $V_{n2}$

*where*

$V_{n1} = 0.25 f_c\, b_v\, d_v + V_p$ *(AASHTO LRFD Eq. 5.8.3.3-2)*

$V_{n2} = V_c + V_s + V_p$ *(AASHTO LRFD Eq. 5.8.3.3-1)*

$V_c = 0.0316\, \beta \cdot \sqrt{f'_c} \cdot b_v \cdot d_v$ *Shear Resistance of the Concrete (AASHTO LRFD Eq.5.8.3.3-3)*

$V_s = \dfrac{A_v \cdot f_y \cdot d_v \cdot cot(\theta)}{s}$ *Shear Resistance of the Transverse Steel (AASHTO LRFD Eq. C5.8.3.3-1)*

## Define Demands

*Since shear is dependent on location, let's look at STA 13:* *At the interior face of the exterior column*

$V_{service} := 269.1 \cdot kip$ *Shear demand at service*

$V_u := 377.6 \cdot kip$ *Ultimate shear demand*

$M_u := 565.7 \cdot kip \cdot ft$ *Ultimate moment demand*

$N_u := 0 \cdot kip$ *Ultimate axial force*

$V_p := 0\ kip$ *Vertical component of the prestress force There is no vertical component of the prestressing force since straight strands are used*

## Shear Design (Con't)

### Define Variables

| | | |
|---|---|---|
| $f'_c = 6\ ksi$ | | *Depth of the bent cap* |
| $f_y := 60 \cdot ksi$ | | *Yield strength of mild steel* |
| $f_{pu} = 270\ ksi$ | | *Tensile strength of prestressing steel* |
| $f_{po} := 0.7 \cdot f_{pu}$ | $f_{po} = 189\ ksi$ | *Parameter taken as modulus of elasticity of prestressing tendon* |
| $n = 28$ | | *Number of strand provided* |
| $h := CapDepth$ | $h = 48\ in$ | *Depth of the bent cap* |
| $b_v := CapWidth - D_{void}$ | $b_v = 48\ in$ | *Width of the bent cap* |
| $c := c + \frac{h}{2}$ | $c = 7.03\ in$ | *Neutral axis from the top extreme concrete* |
| $A_{ps} := \frac{n}{2} \cdot A_{strand}$ | $A_{ps} = 3.04\ in^2$ | *Area of strands in tension side* |
| $A_s := 0 \cdot in^2$ | | *Area of mild steel reinforcement* |
| $d_s := 0 \cdot in$ | | *Effective depth of mild steel reinforcement* |
| $d_p := \frac{D}{2} + e_o$ | $d_p = 24\ in$ | *Distance from extreme compression fiber to the centroid of the prestressing strands*<br>*Note:* $e_o := 0$ *for concentric strand layout* |
| $k := 0.28$ | | *For low relaxation strand (AASHTO LRFD C5.7.3.1.1)* |
| $A_{ct} := \frac{h \cdot b_v}{2}$ | | *Area of concrete on the flexural tension side of the cap, from the extreme tension fiber to on half the cap depth.* |
| $f_{ps} := f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right)$ | $f_{ps} = 248\ ksi$ | *Average stress in prestressing steel (AASHTO LRFD Eq.5.7.3.1.1-1)* |
| $d_e := \frac{A_{ps} \cdot f_{ps} \cdot d_p + A_s \cdot f_y \cdot d_s}{A_{ps} \cdot f_{ps} + A_s \cdot f_y}$ | $d_e = 24\ in$ | *Effective depth from extreme compression fiber to the centroid of the tensile force (AASHTO LRFD Eq. 5.8.2.9-2)* |

## Shear Design (Con't)

### Check Cracking Shear

This step is recommended for the section with an interior void or thin web

*Shear demand at service load shall be less than* $V_{cr}$

$$V_{service} \le V_{cr}$$

Look at STA 13 where the interior void is located with large shear demand

$V_{service} = 269.1\ kip$ — *Shear demand at the interior face of the exterior column under service load*

$A_{pos} = 2304\ in^2$ — *Area of the hollow section at the positive moment region*

$I_{pos} = 442368\ in^4$ — *Moment of inertia of the hollow section at the positive moment region*

$b_w := D - D_{void}$ $\quad b_w = 48\ in$ — *Width of the hollow section*

$Q_{solid} := \dfrac{B \cdot D^2}{8}$ $\quad Q_{solid} = 13824\ in^3$ — *First moment of area of the solid section*

$Q_{void} := \dfrac{D_{void}{}^3}{8}$ $\quad Q_{void} = 0\ in^3$ — *First moment of area of the void*

$Q_{pos} := Q_{solid} - Q_{void}$ $\quad Q_{pos} = 13824\ in^3$ — *First moment of area of the voided section*

$f_t := 0.0632 \cdot \sqrt{\dfrac{f'_c}{ksi}} \cdot ksi$ $\quad f_t = 0.15\ ksi$ — *Tensile strength of concrete for shear*

$T_{strand} = 35.15\ kip$ — *Tension force by single strand*

$n = 28$ — *Number of strand provided*

$V_{cr} := \dfrac{I_{pos} \cdot b_w}{Q_{pos}} \cdot \sqrt{\left(f_t\right)^2 + \left(\dfrac{f_t \cdot n \cdot T_{strand}}{A_{pos}}\right)}$ — *Cracking shear*

$V_{cr} = 461.06\ kip$

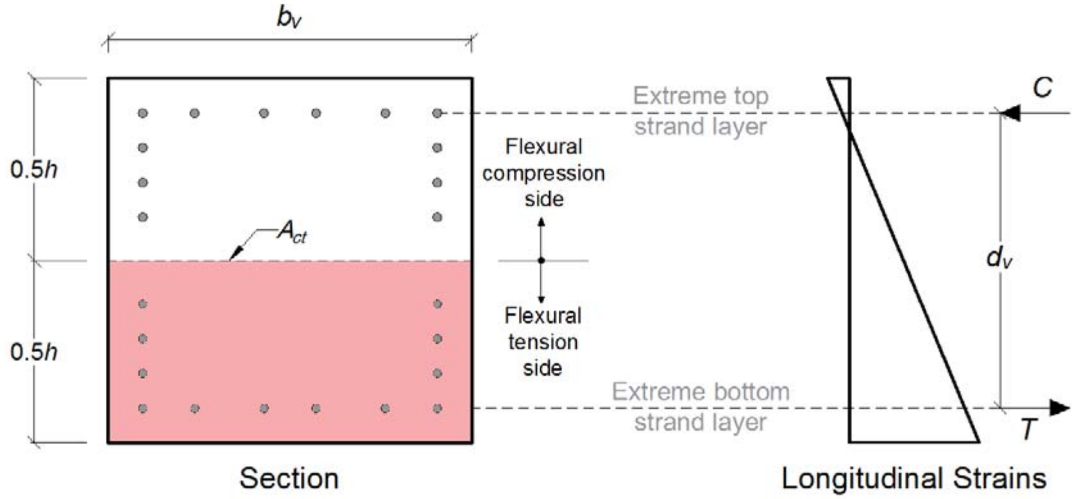
ShearCrackCheck := **if** $\left(V_{service} \le V_{cr}, \text{"Okay"}, \text{"Not Okay"}\right)$

ShearCrackCheck = "Okay"

## Shear Design (Con't)

### Find Effective Shear Depth

$d_v := CapDepth - 2 \cdot cover$ $\quad d_v = 40\ in$ *Effective shear depth; taken as the distance between outer steel layers*




$d_v$ need not be less than the greater of $0.9\,d_e$ and $0.72\,h$ *(AASHTO LRFD 5.8.2.9)*

$d_v$ = maximum of: $\quad d_v := h - 2 \cdot cover = 40\ in$

$0.9 \cdot d_e = 21.6\ in$

$0.72 \cdot h = 34.56\ in$

$d_v := \max\left(d_v, 0.9 \cdot d_e, 0.72 \cdot h\right)$ $\quad d_v = 40\ in$

*Since* $V_n$ must be lesser of $V_{n1}$ and $V_{n2}$ (as per AASHTO LRFD 5.8.3.3), then $V_u$ must be less than both $\phi V_{n1}$ and $\phi V_{n2}$. $V_{n1}$ is dependent on the section properties and the flexural reinforcement. $V_{n2}$ is dependent on the section properties, the flexural reinforcement, and the shear reinforcement. $V_{n1}$ is independent of the shear steel, therefore if $V_u$ is greater than $\phi V_{n1}$ the cap fails in shear regardless of transverse steel.

### Check AASHTO 5.8.3.3-2

$V_{n1} := 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p$ $\quad V_{n1} = \left(2.88 \cdot 10^3\right)\ kip$ *(AASHTO LRFD Eq.5.8.3.3-2)*

$V_{r1}$ must be greater than $V_u$ $\quad V_{r1} := \phi_v \cdot V_{n1}$

$V_{r1} = 2592\ kip > V_u = 377.6\ kip$ *(AASHTO LRFD Eq.5.8.2.1-2)*

$V_{r1}Check := \textbf{if}\left(V_{r1} > V_u, \text{“Okay”}, \text{“Not Okay”}\right)$

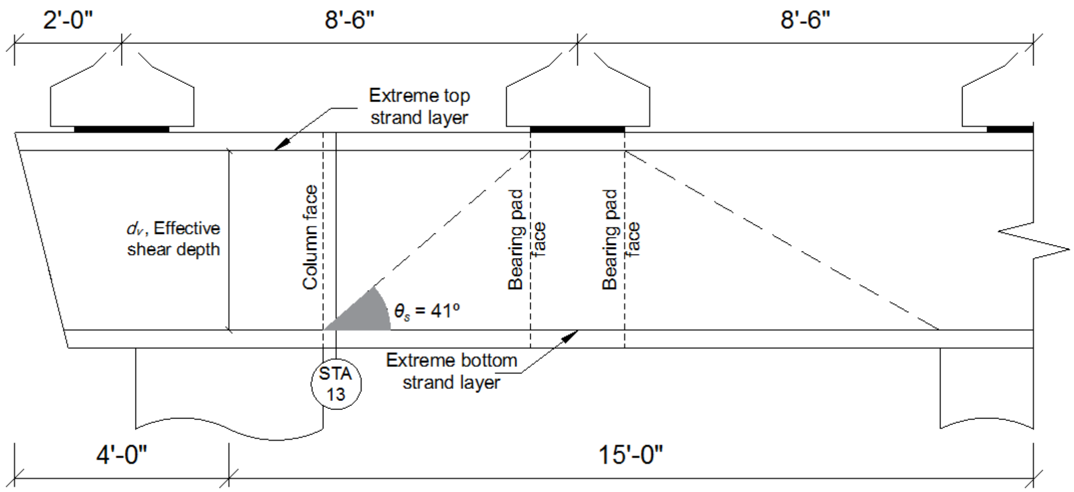
*If $V_{r1}$ is greater than $V_{u1}$, then use a larger cap depth in order to satisfy shear requirements.*

$V_{r1}Check = \text{“Okay”}$

## Shear Design (Con't)

### Determine the Compressive Strut Angle

*Find $\theta_s$ from the bent cap geometry* $\theta_s := 40.7$ deg. *Angle between the column face and the bearing pad face*




### Calculate Determine $\varepsilon_s$ and $\theta$

*The method for calculating $\varepsilon_s$ and $\theta$ used in this design example is from AASHTO LRFD 5.8.3.4.2.*

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$

*The net longitudinal tensile strain in the section at the centroid of the tension reinforcement (AASHTO 5.8.3.4.2-1). If $\varepsilon_s$<0, then assume $\varepsilon_s$=0 or recalculate with the denominator of the equation replaced by ($E_sA_s + E_pA_{ps} + E_cA_c$); however $\varepsilon_s$ should not be taken as less than $-0.40 \cdot 10^{-3}$ or greater than $6.0 \cdot 10^{-3}$ (AASHTO LRFD. 5.8.3.4.2)*

If $\varepsilon_s < 0$, then use $\varepsilon_s$=0 or an equation below

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{\left(E_s \cdot A_s + E_p \cdot A_{ps} + E_c \cdot A_{ct}\right)}$$

where, $|M_u| = 565.7$ ***kip·ft*** must be greater than $|V_u - V_p| \cdot d_v = 1259$ ***kip·ft***

$M_u := \max\left(|M_u|, |V_u - V_p| \cdot d_v\right)$ $M_u = 1259$ ***kip·ft***

$A_{ps} = 3.04$ ***in***$^2$

$$\varepsilon_s := \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$ $\varepsilon_s = 2.09 \cdot 10^{-3}$

$\theta := 29 + 3500\ \varepsilon_s$ $\theta = 36.3$ deg. *(AASHTO LRFD Eq.5.8.3.4.2-3)*

## Shear Design (Con't)

### Determine $\theta$ for Use in the Design and Calculate $V_c$

The controlling angle is the larger of $\theta$ and $\theta_s$

If $\theta$ is larger

$$\beta_1 := \frac{4.8}{(1 + 750 \cdot \varepsilon_s)} \qquad \beta_1 = 1.87$$

*(AASHTO LRFD Eq. 5.8.3.4.2-1)*
*This equation is for section contaning at least the minimum amount of transverse reinforcement.*
*AASHTO LRFD Eq. 5.8.3.4.2-2 provides $\beta$ calculation for section without the minimum amount of shear reinforcement*

$$V_{c1} := 0.0316\ \beta_1 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot b_v \cdot d_v \qquad V_{c1} = 277.78\ kip$$

*(AASHTO LRFD Eq.5.8.3.3-3)*

If $\theta_s$ is larger

$$\beta_2 := 1.6$$

$$V_{c2} := 0.0316 \cdot \beta_2 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot b_v \cdot d_v \cdot \cot\left(\frac{\theta_s \cdot \pi}{180}\right)$$

$$V_{c2} = 276.45\ kip$$

$$V_c := \textbf{if}\left(\theta_s > \theta, V_{c2}, V_{c1}\right) \qquad V_c = 276.45\ kip$$

$$\theta := \max\left(\theta, \theta_s\right) \qquad \theta = 40.7 \text{ deg.}$$

### Check if Shear Reinforcement is Required

ShearRequired := **if** ($V_u > 0.5 \cdot \phi_v \cdot V_c$, “Required”, “"Not Required”)

ShearRequired = “Required”

∴ Shear reinforcement is required

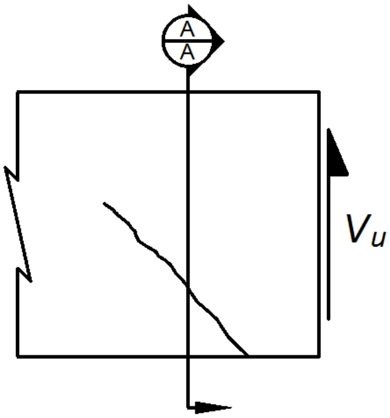
## Shear Design (Con't)

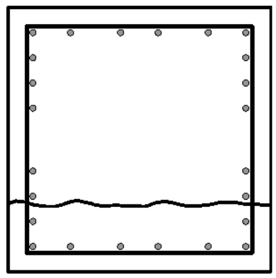
### Provide Shear Reinforcement

$A_v := 2 \cdot (0.31)\ in^2$ $\qquad A_v = 0.62\ in^2$

Assuming #5 stirrups at $s := 10\ in$ spacing

*TxDOT limits transverse reinforcement spacing to a maximum of 12"and a minimum of 4"*
*(BDM-LRFD, Ch. 4, Sec. 4, Detailing)*
*Trial and error is used to determine the stirrup spacing required for the section*



Section A-A

*The transverse reinforcement, "$A_v$" is a closed stirrup. The failure surface intersects two legs of the stirrup, therefore the area of the shear steel is two times the stirrup bar's area (0.31 in² for #5 bar). See the sketch of the failure plan to the left*

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot\left(\frac{\theta \cdot \pi}{180}\right)}{s} \qquad V_s = 173\ kip$$

*(AASHTO LRFD Eq. C5.8.3.3-1)*

$V_n$ = minimum of:

$V_c + V_s + V_p = 449.45\ kip$ *(AASHTO LRFD Eq. 5.8.3.3-1)*

$0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p = \left(2.88 \cdot 10^3\right)\ kip$ *(AASHTO LRFD Eq. 5.8.3.3-2)*

$V_n := min\left(V_c + V_s + V_p, 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p\right)$ *Nominal Shear Resistance*

$V_n = 449.45\ kip$

$V_r := \phi_v \cdot V_n$ $\qquad V_r = 404.5\ kip$ *Factored Shear Resistance*

$V_u = 377.6\ kip$ *Factored Shear Force*

ShearResistance := **if** ($V_u \le V_r$, "Okay", "Not Okay")

ShearResistance = "Okay"

## Shear Design (Con't)

### Check Minimum Transverse Reinforcement

$$A_{v\_min} := 0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot \frac{b_v \cdot s}{f_y}$$ $A_{v\_min} = 0.62\ in^2$ *(AASHTO LRFD Eq. 5.8.2.5-1)*

$A_v = 0.62\ in^2 \quad > \quad A_{v\_min}$

MinimumSteelCheck := **if** $(A_v > A_{v\_min}$, "Okay", "Not okay")

MinimumSteelCheck = "Okay"

### Check Maximum Spacing of Transverse Reinforcement

*Shear Stress*

$$v_u := \frac{V_u - (\phi_v \cdot V_p)}{\phi_v \cdot b_v \cdot d_v}$$ $v_u = 0.219\ ksi$

*Average factored shear stress on the concrete*
*(AASHTO LRFD Eq.5.8.2.9-1)*

$0.125 \cdot f'_c = 0.75\ ksi$

*if* $v_u < 0.125 \cdot f'_c$, $s_{max}$ *= maximum of:* *(AASHTO LRFD Eq.5.8.2.7-1)*

$0.8 \cdot d_v = 32\ in$ *& 24in.*

*if* $v_u \geq 0.125 \cdot f'_c$, $s_{max}$ *= maximum of:* *(AASHTO LRFD Eq.5.8.2.7-2)*

$0.4 \cdot d_v = 16\ in$ *& 12in.*

*Since* $v_u < 0.125 \cdot f'_c$, $s_{max}$ *=24.0 in.*

*TxDOT limits the maximum transverse reinforcement spacing to 12,"therefore:* *(BDM-LRFD, Ch.4, Sec.4, Detailling)*

$s_{max} := 12.00\ in$

$s = 10\ in \quad < \quad s_{max}$

**SpacingCheck** := **if** $(s < s_{max}$, "Okay"", "Not okay")

**SpacingCheck** = "Okay""

*Shear capacity and checks should be repeated at ALL points of critical shear. Note: in the overhangs, the stirrups need to be spaced @ 5in because shear is higher. Similarly the stirrups need to be spaced @ 5in near the center column. When the spacing needed is less than 4in, use double stirrups. (BDM-LRFD, Ch. 4, Sec. 4, Detailing) When using double stirrups,* $A_v$ *is four times the stirrups bar's area.*

# End Region Detailing

## Spalling and Bursting Resistance Design

*For splitting resistance, reinforcement should be provided in the end region within a distance of h/4 (h=overall width of the member) from the member end.* *(AASHTO LRFD 5.10.10.1)*

*Set Variables*

$n = 28$ — *Number of strand*

$A_{strand} = 0.22\ in^2$ — *Sectional area of single strand*

$f_{pu} = 270\ ksi$ — *Ultimate stress of strand*

$f_s := 20\ ksi$ — *Stress in steel (not to exceed 20ksi)*

$h = 48\ in$ — *Depth of the section*

Prestressing force at transfer (before loss of prestress) is calculated by

$P_i := n \cdot 0.75\ f_{pu} \cdot A_{strand}$ $\quad P_i = 1230.4\ kip$ — *Initial prestressing force in release stage (assume 75% $f_{pu}$ is released)*

The required area of steel for splitting resistance in the end region

$A_{s\_req'd} := 0.04\ \dfrac{P_i}{f_s}$ $\quad A_{s\_req'd} = 2.46\ in^2$ — *(AASHTO LRFD 5.10.10.1) The resistance shall not be less than 4% of the total prestressing force*

$A_{s\_req'd}$ is distributed from the member end to h/4

$L_{spall} := \dfrac{h}{4}$ $\quad L_{spall} = 12\ in$ — *Required length for spalling reinforcedment*

$\dfrac{A_{s\_end}}{s}$ shall be greater than $\dfrac{A_{s\_req'd}}{L_{spall}}$ $\quad \dfrac{A_{s\_req'd}}{L_{spall}} = 2.46\ \dfrac{in^2}{ft}$

Try #5 hoop ( $A_{s\_end} := 0.62\ in^2$ ) at 3 in. spacing ( $s := 3\ in$ )

$$\frac{A_{s\_end}}{s} = 2.48\ \frac{in^2}{ft} \quad > \quad \frac{A_{s\_req'd}}{L_{spall}} = 2.46\ \frac{in^2}{ft}$$

$$SpallingCheck := \textbf{if}\left(\frac{A_{s\_end}}{s} > \frac{A_{s\_req'd}}{L_{spall}}, \text{"Okay"}, \text{"Not okay"}\right)$$

$SpallingCheck = \text{"Okay"}$

$\therefore$ Provide #5 hoop at 3 in. spacing within h/4 from the member end.

## End Region Detailing (con't)

### Spalling and Bursting Resistance Design (Con't)

*Additional reinforcement for busting stress is requred immediately after spalling reinforcement from h/4 to the transfer length for bursting* — *(IAC-88-5DD1A003-1)*

*Transfer length is taken as 60 strand diamerers* — *(AASHTO LRFD 5.11.4.1)*

$StrandDia := 0.6\ in$ — *(Diameter of the single strand)*

$TransferLength := 60 \cdot StrandDia$ — *(AASHTO LRFD 5.11.4.1)*

$TransferLength := \max(60 \cdot StrandDia, 36\ in)$ — *(IAC-88-5DD1A003-1)*

$TransferLength = 36\ in$

*The required area of steel for busting resistance is the same as spalling resistance* — *(IAC-88-5DD1A003-1)*

$A_{s\_req'd} := 0.04 \frac{P_i}{f_s}$ $\qquad A_{s\_req'd} = 2.46\ in^2$

$A_{s\_req''d}$ is distributed within the region bounded by the distance h/4 from the end of the member to the transfer length

$L_{burst} := TransferLength - L_{spall}$ $\qquad L_{burst} = 24\ in$ — *Required length for bursting reinforcedment*

$\frac{A_{s\_end}}{s}$ *shall be greater than* $\frac{A_{s\_req'd}}{L_{burst}}$ $\qquad \frac{A_{s\_req'd}}{L_{burst}} = 1.23\ \frac{in^2}{ft}$

*Try #5 hoop (* $A_{s\_end} := 0.62\ in^2$ *) at 6 in. spacing (* $s := 6\ in$ *)*

$\frac{A_{s\_end}}{s} = 1.24\ \frac{in^2}{ft} \quad > \quad \frac{A_{s\_req'd}}{L_{burst}} = 1.23\ \frac{in^2}{ft}$

$BurstingCheck := \mathbf{if}\left(\frac{A_{s\_end}}{s} > \frac{A_{s\_req'd}}{L_{burst}}, \text{“Okay”}, \text{“Not okay”}\right)$

$BurstingCheck = \text{“Okay”}$

Provide #5 hoop at 6 in spacing within $L_{burst}$

# Pocket Connection Detailing

## Joint Shear Capacity Design

*$V_{uj}$ (Ultimate joint shear) must be less $V_{rj}$ (Join shear resistance)*

$$V_{uj} \le V_{rj}$$

$$V_{rj} = \phi_v \cdot V_{nj}$$ $\phi_v := 0.9$

$$V_{nj} = V_{cj} + V_{sj}$$

*where*

$$V_{cj} = 0.0632 \sqrt{f'_{c\_pocket}}\ A_{vj}$$ *Joint shear strength provided by concrete*

$$V_{sj} = \frac{1}{2} \rho_t \cdot \left(\frac{\pi}{4}\ d_{pocket}\right)^2$$ *Joint shear strength provided by steel corrugated pipe*

Define parameters required for pocekt connection design

$t_{pocket} := 0.109\ in$ (for 12-gage pipe) — *Thickness of the corrugated pipe*

$d_{pocket} := Dia_{pipe}$ — $d_{pocket} = 24\ in$ — *Diameter of the pocket*

$d_{col} := 42\ in$ — *Diameter of the column*

$A_{gc} := \frac{\pi \cdot d_{col}^2}{4}$ — $A_{gc} = 1385.4\ in^2$ — *Gross area of the column,*

$A_{vj} := 0.8 \cdot A_{gc}$ — $A_{vj} = 1108.4\ in^2$ — *Joint shear area ( $A_{vj} := 0.8\ A_{gc}$)*

$A_b := 0.20\ in^2$ (for #4 spiral) — *Sectional area of the spirals in the column*

$s := 6\ in$ — *Spiral reinforcement spacing*

$f_{yp} := 33\ ksi$ — *Nominal yield stress of the steel corrugated pipe*

$f'_{c\_pocket} := 3.6\ ksi$ — *Compressive strength of pocket concrete*

*Check equivalent shear strength to the spiral reinforcement in the column*

$t_{req'd\_j} := \frac{A_b}{s}$ — $t_{req'd\_j} = 0.03\ in$

$t_{pocket} = 0.11\ in\ >\ t_{req'd\_j}$ — *If $t_{req'd}$ is greater than $t_{pocket}$, change pocket thickness greater than $t_{req'd}$*

## Pocket Connection Detailing (con't)

### Joint Shear Capacity Design (Con't)

$$\rho_t := \frac{4 \cdot t_{pocket}}{d_{pocket}} \qquad \rho_t = 0.02$$

*Reinforcement ratio of the steel corrugated Pipe*

*Calculate $V_{sj}$ , $V_{cj}$, and $V_{nj}$*

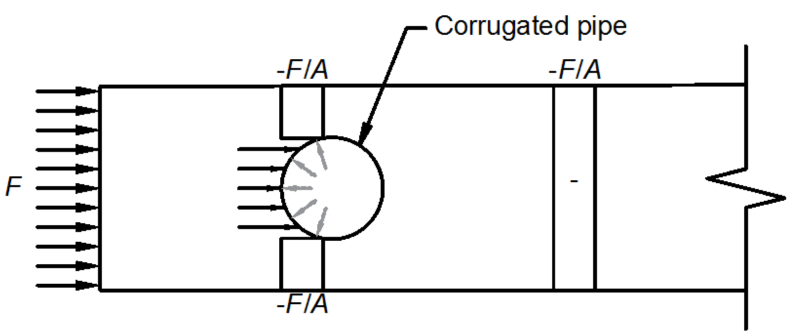
$$V_{cj} := 0.0632 \cdot A_{vj} \cdot \sqrt{\frac{f'_{c\_pocket}}{ksi}} \cdot ksi \qquad V_{cj} = 132.91\ kip$$

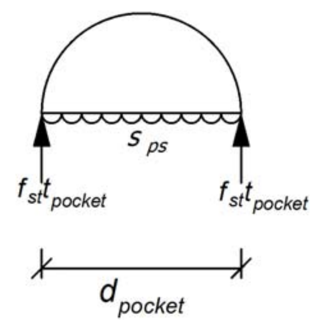
$$V_{sj} := \frac{1}{2} \cdot \rho_t \cdot f_{yp} \cdot \left(\frac{\pi}{4}\ d_{pocket}\right)^2 \qquad V_{sj} = 106.5\ kip$$

$$V_{nj} := V_{cj} + V_{sj} \qquad V_{nj} = 239.41\ kip$$

## Prestress Impact on Voided Pocket Area

*The thickness of the corrugated pipe should be thick enougth to prevent stress concentration*




$f_{st} := 33\ ksi$ — *Allowable stress of the corrugate pipe*

$B = 48\ in$ — *Width of the bent cap*

$D = 48\ in$ — *Depth of the bent cap*

$P_i = 1230\ kip$ — *Initial prestressing*

Required pocket thickness causing uniform stress in the bent cap is

$$t_{req'd\_2} := \frac{P_i \cdot d_{pocket}}{2\ f_{st} \cdot B \cdot D} \qquad t_{req'd\_2} = 0.19\ in$$

*Required pocket thickness for prestress impact*

$t_{pocket} = 0.11\ in\ <\ t_{req'd\_2}$

The corrugated pipe with required thickness may not be commercially available.
Select the largest gage pipe available

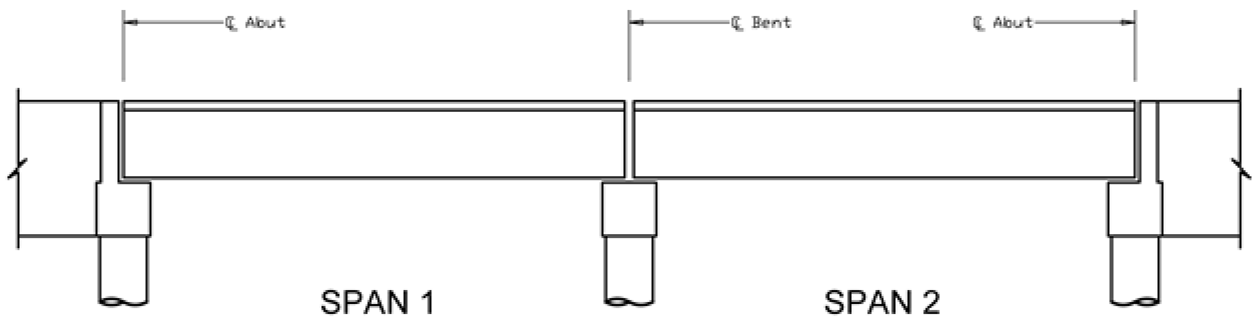
# APPENDIX E: DESIGN EXAMPLE #3

Date: February, 2018

Rectangular Pretensioned Bent Cap Design Example, with a Cocentric Strand Layout

This design example is in accordance with the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012) and TxDOT Bridge Design Manual - LRFD (October 2015)

# Design Parameters




## Span 1

100' Type Tx62 Girders ($0.948 \frac{k}{ft}$)

10 Girders Spaced @ 8.50' with 3' deck overhangs, with middle two girders spaced @ 8'

*"AASHTO LRFD" refers to the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012)*

*"BDM-LRFD" refers to the TxDOT Bridge Design Manual - LRFD (October 2015)*

## Span 2

100' Type Tx54 Girders ($0.948 \frac{k}{ft}$)

10 Girders Spaced @ 8.50' with 3' deck overhangs, with middle two girders spaced @ 8'

*"TxSP" refers to TxDOT guidance, recommendations, and standard practice*

## All Spans

Deck is 40' wide

Type T551 Rail ($0.382 \frac{k}{ft}$)

Type SSCB(1) Median Barrier ( $0.717 \cdot \frac{k}{ft}$)

8.5" Thick Slab (0.100 ksf) *(TxSP)*

Assume 2" Overlay @ 140 pcf (0.023 ksf)

## Assume

4'-0" X 4'-0" Cap
4~42" Columns Spaced @ 22'-0"
Cap will be modeled as a continuous beam with simple supports using TxDOT's CAP18 program.

*TxDOT does not consider frame action for typical multi-column Rectangular Reinforced Concrete Bents. The same methodology is applied to the structural analysis of multi-column Rectangular Pretensioned Concrete Bents. (BDM-LRDSFD, Ch. 4, Sect. 4, Structural Analysis)*

## Design Parameters (Con't)

**Define Variable**

| Back Span | Forward Span | |
|---|---|---|
| $Span1 := 100 \cdot ft$ | $Span2 := 100 \cdot ft$ | *Span Length* |
| $GdrSpa1 := 8.5 \cdot ft$ | $GdrSpa2 := 8.5 \cdot ft$ | *Girder Spacing* |
| $GdrNo1 := 10$ | $GdrNo2 := 10$ | *Number of Girders in Span* |
| $GdrWt1 := 0.948 \cdot \frac{kip}{ft}$ | $GdrWt2 := 0.948 \cdot \frac{kip}{ft}$ | *Weight of Girder* |

Bridge

$RailWt := 0.382 \cdot \frac{kip}{ft}$ — *Weight of Rail*

$MedianWt := 0.717 \cdot \frac{kip}{ft}$ — *Weight of Median Barrier*

$SlabThk := 8.5 \cdot in$ — *Thickness of Bridge Slab*

$OverlayThk := 2 \cdot in$ — *Thickness of Overlay*

$w_c := 150 \cdot pcf$ — *Unit Weight of Concrete for Load Calcs*

$w_{olay} := 140 \cdot pcf$ — *Unit Weight of Overlay*

Other Variables:

$station := 0.5 \cdot ft$ — *Station Increment for CAP18 Analysis*

$IM := 33\%$ — *Dynamic Load Allowance, (AASHTO LRFD Table 3.6.2.1-1)*

Cap Dimensions:

$CapWidth := 48 \cdot in$

$CapDepth := 48 \cdot in$

$cover := 4 \cdot in$ — *Measured from Center of Prestressing Strand*

Material Properties:

$f'_c := 5 \cdot ksi$ — *Assumed Concrete Compressive Strength*

$f_{pu} := 270 \cdot ksi$ — *Ultimate Strength of Prestressing Steel*

## Design Parameters (Con't)

**Define Variable** **(Con't)**

$A_{strand} := 0.217 \cdot in^2$ *Area of Prestressing Strand*

$w_{cE} := 145 \cdot pcf$ *Unit Weight of Concrete for $E_c$ Calc*

$$E_c := 33000 \cdot \left(\frac{w_{cE}}{1000 \cdot pcf}\right)^{1.5} \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi = 4074\ ksi$$

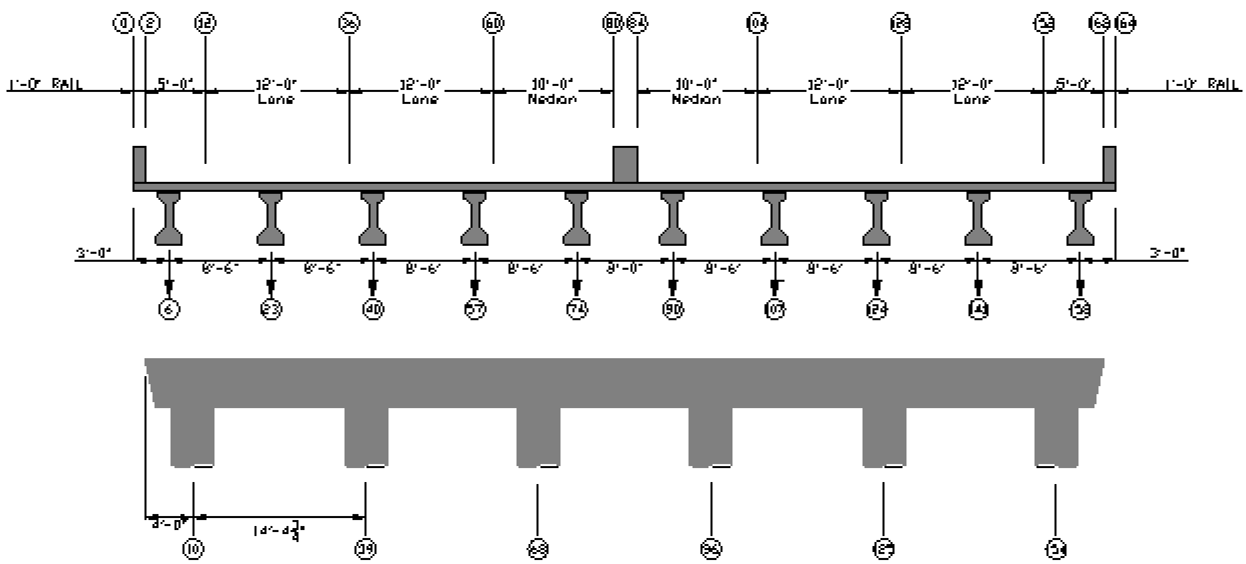
*Modulus of Elasticity of Concrete, (AASHTO LRFD Eq. 5.4.2.4-1)*

$E_s := 29000 \cdot ksi$ *Modulus of Elasticity of Mild Steel*

$E_p := 28500 \cdot ksi$ *Modulus of Elasticity of Prestressing Steel*

# Cap Analysis

**Cap Model**




*The circled numbers are the stations that are used for the CAP18 Input file. One station is 0.5ft in the direction perpendicular to the pgl.*

## Cap Analysis (Con't)

### Dead Load

Span 1

$$\text{Rail1} := \frac{(2 \cdot \text{RailWt} + \text{MedianWt}) \cdot \frac{\text{Span1}}{2}}{min(\text{GdrNo1}, 9)}$$ $\text{Rail1} = 8.23\ kip$ *Rail weight is distributed evenly among stringers, up to 3 stringers per rail. (TxSP)*

$$\text{Slab1} := w_c \cdot \text{GdrSpa1} \cdot \text{SlabThk} \cdot \frac{\text{Span1}}{2} \cdot 1.1$$ $\text{Slab1} = 49.67\ kip$ *Slab DL is increased by 10% to account for haunch and thickened slab ends.*

$$\text{Girder1} := \text{GdrWt1} \cdot \frac{\text{Span1}}{2}$$ $\text{Girder1} = 47.4\ kip$ *Weight of girder acting on bent*

$$\text{DLRxn1} := \text{Rail1} + \text{Slab1} + \text{Girder1}$$ $\text{DLRxn1} = 105.3\ kip$ *Dead load reaction per girder, not considering overlay. (Overlay is calculated separately due to possibility of applying a different load factor)*

$$\text{Overlay1} := w_{olay} \cdot \text{GdrSpa1} \cdot \text{OverlayThk} \cdot \frac{\text{Span1}}{2}$$ $\text{Overlay1} = 9.92\ kip$ *Design for future overlay, per girder*

Span 2

$$\text{Rail2} := \frac{(2 \cdot \text{RailWt} + \text{MedianWt}) \cdot \frac{\text{Span2}}{2}}{min(\text{GdrNo2}, 9)}$$ $\text{Rail2} = 8.23\ kip$

$$\text{Slab2} := w_c \cdot \text{GdrSpa2} \cdot \text{SlabThk} \cdot \frac{\text{Span2}}{2} \cdot 1.1$$ $\text{Slab2} = 49.67\ kip$

$$\text{Girder2} := \text{GdrWt2} \cdot \frac{\text{Span2}}{2}$$ $\text{Girder2} = 47.4\ kip$

$$\text{DLRxn2} := \text{Rail2} + \text{Slab2} + \text{Girder2}$$ $\text{DLRxn2} = 105.3\ kip$

$$\text{Overlay2} := w_{olay} \cdot \text{GdrSpa2} \cdot \text{OverlayThk} \cdot \frac{\text{Span2}}{2}$$ $\text{Overlay2} = 9.92\ kip$

Cap

$$A_g := \text{CapWidth} \cdot \text{CapDepth}$$ $A_g = 2304\ in^2$ *Gross Area of Cap*

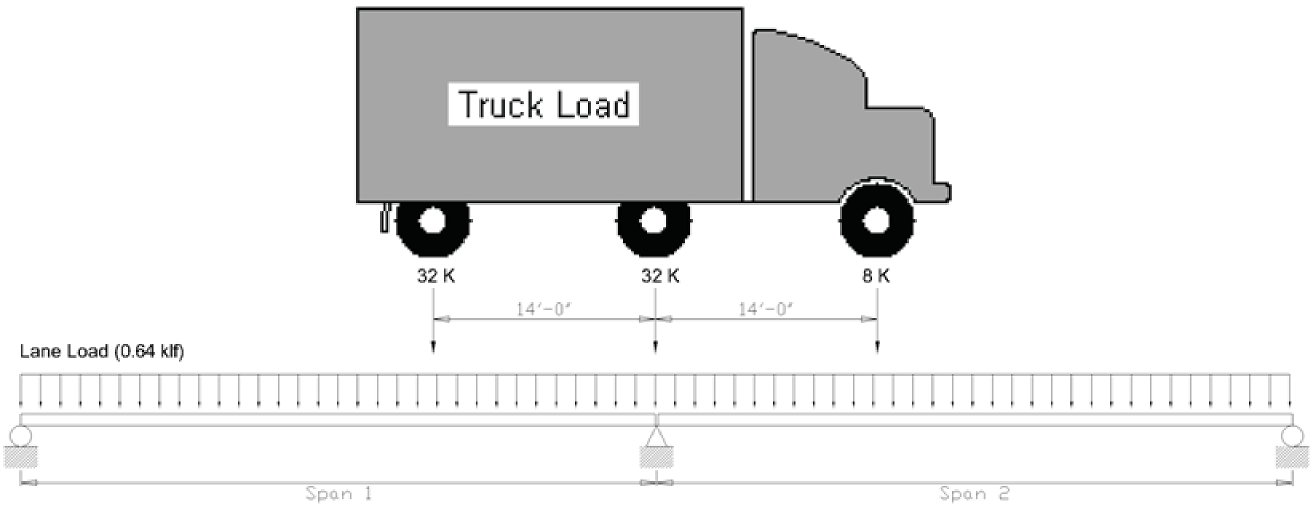
$$\text{Cap} := w_c \cdot A_g \cdot \text{station}$$ $\text{Cap} = 1.2\ kip$ *Dead Load of Cap, per station*

$$I_g := \frac{1}{12} \cdot \text{CapWidth} \cdot \text{CapDepth}^3$$ $I_g = (4.42 \cdot 10^5)\ in^4$ *Gross Moment of Inertia of Cap*

$E_c = 4074\ ksi$ $E_c \cdot I_g = (1.25 \cdot 10^7)\ kip \cdot ft^2$ *Bending Stiffness of Cap*

## Cap Analysis (Con't)

**Live Load** *(AASHTO LRFD 3.6.1.2.2 and 3.6.1.2.4)*




Live Load Model

$\text{LongSpan} := \max(\text{Span1}, \text{Span2})$

$\text{ShortSpan} := \min(\text{Span1}, \text{Span2})$

$\text{LongSpan} = 100\ ft$

$\text{ShortSpan} = 100\ ft$

$\text{IM} = 0.33$

$$\text{Lane} := 0.64 \cdot klf \cdot \left(\frac{\text{LongSpan} + \text{ShortSpan}}{2}\right)$$

$\text{Lane} = 64\ kip$

$$\text{Truck} := 32 \cdot kip + 32 \cdot kip \cdot \left(\frac{\text{LongSpan} - 14 \cdot ft}{\text{LongSpan}}\right) + 8 \cdot kip \cdot \left(\frac{\text{ShortSpan} - 14 \cdot ft}{\text{ShortSpan}}\right)$$

$\text{Truck} = 66.4\ kip$

$\text{LLRxn} := \text{Lane} + \text{Truck} \cdot (1 + \text{IM})$

$\text{LLRxn} = 152.31\ kip$

$\text{P} := 16.0 \cdot kip \cdot (1 + \text{IM})$

$\text{P} = 21.28\ kip$

$$\text{W} := \frac{\text{LLRxn} - (2 \cdot \text{P})}{10 \cdot ft} \cdot \text{station}$$

$\text{W} = 5.49\ kip$

*Use HL-93 Live Load. Maximum reaction at an interior bent, the "Design Truck" will govern over "Design Tandem." With the Long Span less than twice as long as the Short Span, the maximum reaction occurs when the middle axle (32 kip) is placed over the support, the front axle (8 kips) is placed on the Short Span, and the rear axle (32 kips) is placed on the Long Span.*

*Combine "Design Truck" and "Design Lane" loadings.*
*(AASHTO LRFD 3.6.1.3)*

*Dynamic Load Allowance (IM) does not apply to the "Design Lane"*
*(AASHTO LRFD 3.6.1.2.4)*

*Live Load is applied to the deck slab by two 16 kip wheel loads increased by IM, with the remainder of the live load distributed over a 10ft design lane width*
*(AASHTO LRFD 3.6.1.2.1) (TxSP)*

*Live Load applied to the slab is distributed to the beams assuming the slab is hinged at each beam, except the outside beam. (BDM-LRFD, Ch. 4, Sect. 4, Structural Analysis)*

# Cap Analysis (Con't)

**Cap18 Input**

Multiple Presence Factors, m *(AASHTO LRFD Table 3.6.1.1.2-1)*

| No. of Lanes | Factor "m" |
|---|---|
| 1 | 1.20 |
| 2 | 1.00 |
| 3 | 0.85 |
| >3 | 0.65 |

Limit States *(AASHTO LRFD 3.4.1)*

*The cap design only needs to consider Strength I, Service I, and Service I (Dead Load Only)*

Strength I

| | |
|---|---|
| Live Load and Dynamic Load Allowance | LL + IM = 1.75 |
| Dead Load Components | DC = 1.25 |
| Dead Load Wearing Surface (Overlay) | DW = 1.50 |

*TxDOT allows Overlay Factor to be reduced to 1.25 (TxSP).*

Service I

| | |
|---|---|
| Live Load and Dynamic Load Allowance | LL + IM = 1.00 |
| Dead Load and Wearing Surface | DC & DW = 1.00 |

*CAP18 Input is included in an Appendix to this example*

**Cap18 Output**

| | Max +M | Max -M |
|---|---|---|
| Dead Load | $M_{DLpos} := 432.8 \cdot kip \cdot ft$ | $M_{DLneg} := 566.9 \cdot kip \cdot ft$ |
| Service Load | $M_{SLpos} := 781.8 \cdot kip \cdot ft$ | $M_{SLneg} := 775.4 \cdot kip \cdot ft$ |
| Ultimate Load | $M_{ULpos} := 1151.7 \cdot kip \cdot ft$ | $M_{ULneg} := 1089.4 \cdot kip \cdot ft$ |

*Maximum loads from the CAP18 Output file. The output is included in an Appendix to this design example.*

$M_{DLmax} := \max\left(M_{DLpos}, M_{DLneg}\right)$ $\quad M_{DLmax} = 566.9\ kip \cdot ft$

$M_{SLmax} := \max\left(M_{SLpos}, M_{SLneg}\right)$ $\quad M_{SLmax} = 781.8\ kip \cdot ft$

$M_{ULmax} := \max\left(M_{ULpos}, M_{ULneg}\right)$ $\quad M_{ULmax} = 1151.7\ kip \cdot ft$

# Flexural Design

*The flexural design of the bent cap is based on the philosophy of "Zero Tension Under Dead Load." The design follows the following steps:*

- *Design for Zero Flexural Tension under Dead Load*
- *Determine Minimum Concrete Compressive Strength and Check Stresses at Service Loads*
- *Check the Minimum Number of Strands*
- *Check the Ultimate Strength Capacity*
- *Check that Minimum Capacity is satisfied*

## Define Constants and Variables

$B := CapWidth$ $\quad$ $D := CapDepth$ $\quad$ $Dia_{pipe} := 24\ in$

$f_r := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi$ $\quad$ $f_r = 0.54\ ksi$ $\quad$ *Modulus of Rupture (AASHTO LRFD Eq. 5.4.2.4-1)*

$jd := 0.45 \cdot CapDepth$ $\quad$ $jd = 21.6\ in$ $\quad$ *Approximate Moment Arm between Tension and Compression section to estimate nominal moment capacity for determination of Minimum Number of Strands*

$f_y := 0.9 \cdot f_{pu}$ $\quad$ $f_y = 243\ ksi$ $\quad$ *Yield Strength of Prestressing Steel (AASHTO LRFD Table 5.4.4.1-1)*

$\Delta_{f.pt} := 0.2$ $\quad$ *Assumed prestress loss in pretensioned members*

$T_{strand} := 0.75 \cdot f_{pu} \cdot A_{strand} \cdot \left(1 - \Delta_{f.pt}\right)$ $\quad$ $T_{strand} = 35.15\ kip$

$I_{pos} := \frac{B \cdot D^3}{12}$ $\quad$ $I_{pos} = 442368\ in^4$ $\quad$ *Moment of Inertia of the solid section at the positive moment region*

$I_{neg} := \frac{B \cdot D^3}{12} - \frac{Dia_{pipe} \cdot D^3}{12}$ $\quad$ $I_{neg} = 221184\ in^4$ $\quad$ *Moment of Inertia of the hollow section at the negative moment region*

$S_{pos} := \frac{I_{pos}}{\left(\frac{D}{2}\right)}$ $\quad$ $S_{pos} = 18432\ in^3$ $\quad$ *Section Modulus of solid Rectangular Section at the positive moment region*

$S_{neg} := \frac{I_{neg}}{\left(\frac{D}{2}\right)}$ $\quad$ $S_{neg} = 9216\ in^3$ $\quad$ *Section Modulus of hollow connection section at the negative moment region*

$A_{pos} := B \cdot D$ $\quad$ $A_{pos} = 2304\ in^2$

$A_{neg} := \left(B - Dia_{pipe}\right) \cdot D$ $\quad$ $A_{neg} = 1152\ in^2$

## Flexural Design (Con't)

### Design for Zero Flexural Tension Under Dead Load

Tension Limit:

$$\frac{-F_t}{A}+\frac{M_{DL}}{S_x} = f_t = 0$$

*Set the stress at the extreme tension fiber to zero*

$F_{t\_neg} := \frac{M_{DLneg} \cdot A_{neg}}{S_{neg}}$ $F_{t\_neg} = 850.4\ kip$

*Determine the prestressing force required to achieve zero tension stress in the negative moment region*

$F_{t\_pos} := \frac{M_{DLpos} \cdot A_{pos}}{S_{pos}}$ $F_{t\_pos} = 649.2\ kip$

*Determine the prestressing force required to achieve zero tension stress in the positive moment region*

$F_t := \max\left(F_{t\_neg}, F_{t\_pos}\right)$ $F_t = 850350\ lbf$

$n_{flex\_t} := \mathrm{Ceil}\left(\left(\frac{F_t}{T_{strand}}\right), 4\right)$ $n_{flex\_t} = 28$

*Use the calculated $F_t$ to determine the corresponding number of strands required*

$e_o := 0\ in$

*For symmetric, concentric layouts- the number of strands must be a multiple of 4 with no eccentricity*

Compression Limit:

$$\frac{-F_c}{A}-\frac{M_{DL}}{S_x} \geq f_c = -0.45 \cdot f'_c$$

*Set the stress at the extreme compression fiber to the compressive stress limit (AASHTO LRFD Table 5.9.4.2.1-1*

$F_{c\_neg} := \left(0.45 \cdot f'_c - \frac{M_{DLneg}}{S_{neg}}\right) \cdot A_{neg}$ $F_{c\_neg} = 1741.7\ kip$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the negative moment region*

$F_{c\_pos} := \left(0.45 \cdot f'_c - \frac{M_{DLpos}}{S_{pos}}\right) \cdot A_{pos}$ $F_{c\_pos} = 4534.8\ kip$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the positive moment region*

$F_c := \min\left(F_{c\_neg}, F_{c\_pos}\right)$ $F_c = 1741.7\ kip$

$n_{flex\_c} := \mathrm{Floor}\left(\left(\frac{F_c}{T_{strand}}\right), 4\right)$ $n_{flex\_c} = 48$

*Use the calculated $F_c$ to determine the corresponding number of strands required to reach the compressive stress limit*

$n := \min\left(n_{flex\_t}, n_{flex\_c}\right)$ $n = 28$

*The number of strands selected for the should be between $n_{flec\_t}$ and $n_{flex\_c}$*

$F_{provided} := n \cdot T_{strand}$ $F_{provided} = 984.3\ kip$

*Determine the provided prestressing force from the selected number of strands*

# Flexural Design (Con't)

**Design for Zero Flexural Tension under Dead Load** **(Con't)**

Compression Limit:

$$\frac{-F_c}{A} - \frac{M_{DL}}{S_x} \geq f_c = -0.45 \cdot f'_c$$

*Set the stress at the extreme compression fiber to the compressive stress limit (AASHTO LRFD Table 5.9.4.2.1-1*

$$F_{c\_neg} := \left(0.45 \cdot f'_c - \frac{M_{DLneg}}{S_{neg}}\right) \cdot A_{neg}$$ $F_{c\_neg} = 1741.7\ kip$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the negative moment region*

$$F_{c\_pos} := \left(0.45 \cdot f'_c - \frac{M_{DLpos}}{S_{pos}}\right) \cdot A_{pos}$$ $F_{c\_pos} = 4534.8\ kip$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the positive moment region*

$$F_c := min\left(F_{c\_neg}, F_{c\_pos}\right)$$ $F_c = 1741.7\ kip$

$$n_{flex\_c} := \text{Floor}\left(\left(\frac{F_c}{T_{strand}}\right), 4\right)$$ $n_{flex\_c} = 48$

*Use the calculated $F_c$ to determine the corresponding number of strands required*

$$n := min\left(n_{flex\_t}, n_{flex\_c}\right)$$ $n = 28$

*The number of strands selected for the should be between $n_{flex\_t}$ and $n_{flex\_c}$*

$$F_{provided} := n \cdot T_{strand}$$ $F_{provided} = 984.3\ kip$

*Determine the provided prestressing force from the selected number of strands*

## Flexural Design (Con't)

## Check Minimum Concrete Compressive Strength with Service Stress

Tension Stress:

$$\frac{-F}{A} + \frac{M_{SL}}{S_x} \geq f_t = k \cdot \sqrt{f'_c}$$

*The stress at the extreme tension fiber must not exceed the service stress limit, which is* $k \cdot \sqrt{f'_c}$ .
*(AASHTO LRFD Table 5.9.4.2.2-1)*

*Values of* $k$ *are different for various corrosion conditions. For AASHTO LRFD 5.9.4.2.2,* $k$ *values are:*

- *Moderate Exposure (Class I)* $k := 0.19$
- *Severe Exposure (Class II)* $k := 0.0948$

*Recommended value of* $k$ *to limit cracking under service loads was shown to be smaller than the value imposed by AASHTO. This value is* $k := 0.126$

*The tensile stresses under service conditions should not exceed values specified by AASHTO LRFD 5.9.4.2.2. However, to further reduce cracking under service conditions in Class I conditions, the tensile stresses should not exceed the recommended value concluded from experimental testing.*

*For this design example, the AASHTO value of* $k$ *for Class I expoure will be used to check the service level stresses and compute the minimum concrete compressive strength.*

Negative Moment Region:

$k := 0.19$ *Multiplier for* $k \cdot \sqrt{f'_c}$ *tensile stress limit*

$$f_{t\_sl\_neg} := \frac{-F_{provided}}{A_{neg}} + \frac{M_{SLneg}}{S_{neg}} \qquad f_{t\_sl\_neg} = 0.155\ ksi$$

*Tensile Stress at extreme tension fiber under Service Loads*

$$f_{t\_slim\_neg} := k \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \qquad f_{t\_slim\_neg} = 0.425\ ksi$$

*Limiting tensile stress at the extreme tension fiber*

$\text{TenLimit} := \textbf{if}\left(f_{t\_sl\_neg} \leq f_{t\_slim\_neg}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$ $\text{TenLimit} = \text{“GOOD”}$

Positive Moment Region:

$$f_{t\_sl\_pos} := \frac{-F_{provided}}{A_{pos}} + \frac{M_{SLpos}}{S_{pos}} \qquad f_{t\_sl\_pos} = 0.082\ ksi$$

*Tensile Stress at extreme tension fiber under Service Loads*

$$f_{t\_slim\_pos} := k \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \qquad f_{t\_slim\_pos} = 0.425\ ksi$$

*Limiting tensile stress at the extreme tension fiber*

$\text{TenLimit} := \textbf{if}\left(f_{t\_sl\_pos} \leq f_{t\_slim\_pos}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$ $\text{TenLimit} = \text{“GOOD”}$

# Flexural Design (Con't)

## Check Minimum Concrete Compressive Strength with Service Stress (Con't)

Compression Stress:

$$\frac{-F}{A} - \frac{M_{SL}}{S_x} > f_c = -0.45 \cdot f'_c$$

*The stress at the extreme compression fiber must not exceed the service stress limit. Considering the connection section as solid in compression (AASHTO LRFD Table 5.9.4.2.1-1)*

$$f_{c\_sl} := \left(\frac{-F_{provided}}{A_{pos}}\right) - \left(\frac{M_{SLmax}}{S_{pos}}\right) \qquad f_{c\_sl} = -0.936\ ksi$$

*Compressive Stress at extreme compression fiber under Service Loads*

$$f_{c\_slim} := -0.45 \cdot f'_c \qquad f_{c\_slim} = -2.25\ ksi$$

*Limiting compressive stress at the extreme compression fiber*

CompLimit := **if** ($f_{c\_sl} \geq f_{c\_slim}$, "GOOD", "NOT GOOD")     CompLimit = "GOOD"

Minimum Concrete Strength:

$$f'_{c\_tmin} := \left(\left(\frac{\max\left(f_{t\_sl\_neg}, f_{t\_sl\_pos}\right)}{k \cdot ksi}\right)^2\right) \cdot ksi \qquad f'_{c\_tmin} = 0.67\ ksi$$

*Determine minimum concrete compressive strength to achieve the limiting tensile stress*

$$f'_{c\_cmin} := \frac{f_{c\_sl}}{-0.45} \qquad f'_{c\_cmin} = 2.08\ ksi$$

*Determine minimum concrete compressive strength to achieve the limiting compressive stress*

$$f'_{c\_min} := \max\left(f'_{c\_cmin}, f'_{c\_tmin}, 5 \cdot ksi\right) \qquad f'_{c\_min} = 5\ ksi$$

*Minimum Concrete Compressive Strength. If calculated values are significantly low, use a practical minimum achievable concrete strength (consider $5 \cdot ksi$ for precast, prestressed concrete)*

*If the required Minimum Concrete Compressive strength is larger than the maximum allowed design strength from Chapter 4 of BDM-LRFD, adjust the number of strands and/or strand layout to reduce the service stresses.*

*If the calculated Minimum Concrete Compressive strength is larger than the assumed strength, recheck stress limits with the "new" compressive strength.*

## Flexural Design (Con't)

## Determine Minimum Number of Strands

*AASHTO LRFD 5.7.3.3.2 specifies that the factored flexural resistance* $M_r$ *should be at least greater than the lesser of* $M_{cr}$ *or* $1.33 \cdot M_u$ *. The derivation of the formula to determine* $n_{min}$ *is shown:*

$$-\left(\frac{F}{A_g}\right)+\left(\frac{M_{cr}}{S_x}\right) = f_r$$

*Stress at extreme tension fiber is equal to the modulus of rupture*

$$M_{cr} = \gamma_3\left(\gamma_1 \cdot f_r + \gamma_2\left(\frac{F}{A}\right)\right)\cdot S_x \qquad \gamma_1 := 1.6 \quad \gamma_2 := 1.1 \quad \gamma_3 := 1.0$$

*Where* $F$ *is the prestressing force*

$$\phi M_n = M_r \geq M_{cr} \text{ , where } \phi := 1.0$$

*(AASHTO LRFD 5.5.4.2.1)*

$$M_n \geq n \cdot A_{ps} \cdot f_y \cdot jd$$

*Approximate nominal moment capacity*

Substituting, simplifying, and equating $M_{cr}$ and $M_n$ :

$$\gamma_1 f_r \cdot S_x + \gamma_2 \cdot \left(n \cdot T_{strand}\right)\frac{S_x}{A} = n \cdot A_{ps} \cdot f_y \cdot jd$$

Thus:

$$n = \frac{\gamma_1 \cdot f_r \cdot S_x}{A_{ps} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_x}{A}}$$

Also $M_r \geq 1.33 \cdot M_{UL}$

$$n = \frac{1.33 \cdot M_{UL}}{A_{ps} \cdot f_y \cdot jd}$$

$$n_{min\_pos} := \text{ceil}\left(\max\left(\frac{\gamma_1 \cdot f_r \cdot S_{pos}}{A_{strand} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_{pos}}{A_{pos}}}, \frac{1.33 \cdot M_{ULpos}}{A_{strand} \cdot f_y \cdot jd}\right)\right)$$

*$n$ must be checked for both positive and negative bending regions, as the cross-sections are not the same.*

$$n_{min\_pos} = 20$$

$$n_{min\_neg} := \text{ceil}\left(\max\left(\frac{\gamma_1 \cdot f_r \cdot S_{neg}}{A_{strand} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_{neg}}{A_{neg}}}, \frac{1.33 \cdot M_{ULneg}}{A_{strand} \cdot f_y \cdot jd}\right)\right)$$

$$n_{min\_neg} = 16$$

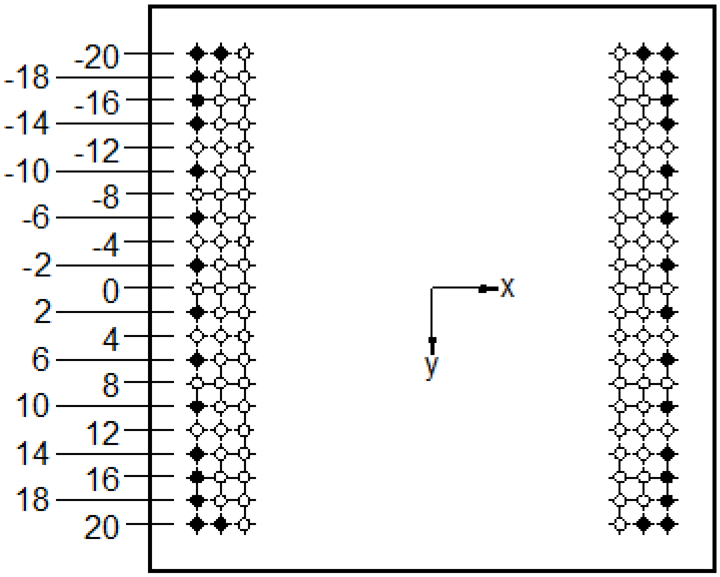
$$n_{min} := \max\left(n_{min\_pos}, n_{min\_neg}\right) \qquad n_{min} = 20$$

$$n_{min\_check} := \textbf{if}\left(n \geq n_{min}, \text{“GOOD”}, \text{“NOT GOOD”}\right) \qquad n_{min\_check} = \text{“GOOD”}$$

## Flexural Design (Con't)

## Check Ultimate Strength Capacity

Determine strand configuration




*The strand layout is limited based on the configuration of the cap-to-column connection. For this example, the cap-to-column connection is assumed to be formed by a 24-inch nominal diameter pocket connection.*

**Define Variables**

$\Delta_{\varepsilon p} := \dfrac{T_{strand}}{E_p \cdot A_{strand}}$ $\quad \Delta_{\varepsilon p} = 0.0057$

*Pre-strain, after losses*

$\beta := \max\left(\left(0.85 - \left(\dfrac{f'_c}{ksi} - 4\right) \cdot 0.05\right), 0.65\right)$ $\quad \beta = 0.8$

*(AASHTO LRFD 5.7.2.2)*

$\varepsilon_{cu} := 0.003$

*Maximum strain at extreme compression fiber*
*(AASHTO LRFD 5.7.2.1)*

$\phi := 1.0$

*Strength Reduction Factor*
*(AASHTO LRFD 5.5.4.2.1)*

$Q := 0.03$

$R := 6$

*Q and R are constants in the "Menegotto-Pinto" equation used to determine the stress at $i^{th}$ layer of prestressing steel. Since a side configuration layout of prestressing steel was used instead of the conventional top & bottom layout, the stresses in the prestressing steel must be determined at each individual layer.*

$c_i := -15 \cdot in$

*Initial location of neutral axis used in the iterative solution of determining the moment capacity.*

## Flexural Design (Con't)

**Calculate Strain and Stress in Each Steel Layer**

$$\varepsilon_{ti} = \varepsilon_{cu} \cdot \left( \frac{d_i - c}{\frac{D}{2} + c} \right)$$

*Tension strain at the $i^{th}$ layer. $d_i$ is the depth of the prestressing layer, as shown in the strand layout (note the convention and origin of distance measurements)*

$$\varepsilon_{si} = \varepsilon_{ti} + \Delta_{\varepsilon p}$$

*Total strain on each layer, considering the pre-strain*

$$f_{psi} = E_p \cdot \varepsilon_{si} \cdot \left( Q + \frac{1 - Q}{\left(1 + \left( \left| \frac{\varepsilon_{si} \cdot E_s}{f_y} \right| \right)^R \right)^{\frac{1}{R}}} \right)$$

*Menegotto-Pinto equation to determine the stress in the $i^{th}$ layer*

$$T_i = f_{psi} \cdot A_{psi}$$

*Tension force in the $i^{th}$ layer of steel. $A_{psi}$ is the area of prestressing steel in that layer.*

$$jd_i = \frac{D}{2} - d_i - \frac{a}{2}$$

*Moment arm between compressive stress block and the $i^{th}$ layer of prestressing steel*

$$a = \frac{-D}{2} + \left( \frac{-\beta}{2} \cdot \left( \frac{-D}{2} - c \right) \right)$$

*Depth of the equivalent compression block, with respect to the center of the bent cap*

$$M_i = T_i \cdot j \cdot d_i$$

*Moment in the $i^{th}$ layer*

$$C_c = -0.85 \cdot f'_c \cdot \beta \cdot \left( \frac{D}{2} - c \right) \cdot B$$

*Compressive force from the equivalent compressive stress block*

*The previous equations are calculated using the $c_i$ value, and iterated with changing values of $c$ until the sum of the tensile forces equals the magnitude of the compressive force:*

$$\Sigma T_t = \left| C_c \right|$$

*This process is completed for both Positive and Negative Bending in Microsoft Excel, and the results are presented in the following tables*

## Flexural Design (Con't)

**Moment Capacity:**

| $d_i$ (in) | $n_i$ | $\varepsilon_{ti}$ | $\Delta\varepsilon_p$ | $\varepsilon_{si}$ | $f_{psi}$ (ksi) | $T_i$ (kips) | $jd_i$ (in) | $M_i$ (k-in) |
|---|---|---|---|---|---|---|---|---|
| -20 | 4 | -0.00149 | 0.00568 | 0.00419 | 119.21 | 103.48 | 0.82 | 84.6 |
| -16 | 2 | 0.00002 | 0.00568 | 0.00570 | 160.24 | 69.54 | 4.82 | 335.0 |
| -14 | 2 | 0.00077 | 0.00568 | 0.00645 | 178.91 | 77.65 | 6.82 | 529.4 |
| -10 | 2 | 0.00228 | 0.00568 | 0.00796 | 209.04 | 90.73 | 10.82 | 981.4 |
| -6 | 2 | 0.00379 | 0.00568 | 0.00947 | 227.63 | 98.79 | 14.82 | 1463.8 |
| -2 | 2 | 0.00530 | 0.00568 | 0.01098 | 237.44 | 103.05 | 18.82 | 1939.1 |
| 2 | 2 | 0.00680 | 0.00568 | 0.01249 | 242.63 | 105.30 | 22.82 | 2402.7 |
| 6 | 2 | 0.00831 | 0.00568 | 0.01400 | 245.73 | 106.65 | 26.82 | 2860.0 |
| 10 | 2 | 0.00982 | 0.00568 | 0.01550 | 247.90 | 107.59 | 30.82 | 3315.6 |
| 14 | 2 | 0.01133 | 0.00568 | 0.01701 | 249.64 | 108.34 | 34.82 | 3772.3 |
| 16 | 2 | 0.01208 | 0.00568 | 0.01777 | 250.42 | 108.68 | 36.82 | 4001.5 |
| 20 | 4 | 0.01359 | 0.00568 | 0.01928 | 251.90 | 218.65 | 40.82 | 8924.6 |
| 0.00 | 28 | $C_c$ = | -1298.4 | kips | $\sum T_i$ = | 1298.4 | $\sum M_i$ = | 30609.9 |

$c := -16.04 \cdot in$ — *Final location of N/A, from iterations*

$\Sigma M_i = 30609.94\ kip \cdot in$ — *Sum of $M_i$ from the iterations*

$M_n := \Sigma M_i$ $\qquad M_n = 2550.8\ kip \cdot ft$

$M_r := \phi \cdot M_n$ $\qquad M_r = 2550.8\ kip \cdot ft$ — *Factored Flexural Resistance (AASHTO LRFD 5.7.3.2.1)*

CapacityCheck := **if** $(M_r \geq M_{ULmax}$, “GOOD”, “NOT GOOD”$)$

CapacityCheck = “GOOD”

## Flexural Design (Con't)

## Check Minimum Capacity:

*Calculate the $M_{cr}$ and check if the $M_r$ meets AASHTO LRFD 5.7.3.3.2*

$\gamma_1 := 1.6$ $\gamma_2 := 1.1$ $\gamma_3 := 1.0$

Negative Moment Region:

$f_{cpe} := \frac{F_{provided}}{A_{neg}}$ $f_{cpe} = 0.85\ ksi$

$M_{cr\_neg} := \gamma_3 \cdot \left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{neg}\right)$ $M_{cr\_neg} = 1381\ kip \cdot ft$

$M_{rCheck} := \textbf{if}\left(M_r \geq min\left(M_{cr\_neg}, 1.33 \cdot M_{ULneg}\right), \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$M_{rCheck} = \text{“GOOD”}$

Positive Moment Region:

$f_{cpe} := \frac{F_{provided}}{A_{pos}}$ $f_{cpe} = 0.43\ ksi$

$M_{cr\_pos} := \gamma_3 \cdot \left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{pos}\right)$ $M_{cr\_pos} = 2041\ kip \cdot ft$

$M_{rCheck} := \textbf{if}\left(M_r \geq min\left(M_{cr\_pos}, 1.33 \cdot M_{ULpos}\right), \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$M_{rCheck} = \text{“GOOD”}$

**Design Summary:**

Concrete Strength:

$f'_c = 5\ ksi$

Prestsessing:

$n = 28$

$F_{provided} = 984.3\ kip$

# Shear Design

## Design Philosophy:

$V_u$ *(Ultimate Shear) must be less than* $V_r$ (Shear Resistance)

$V_u \leq V_r$ *(AASHTO LRFD Eq. 1.3.2.1-1)*

$V_r = \phi_v \cdot V_n$ *(AASHTO LRFD Eq. 5.8.2.1-2)*

$\phi_v := 0.9$ *Reduction Factor (AASHTO LRFD 5.5.4.2.1)*

$V_n$ *is the lesser of* $V_{n1}$ and $V_{n2}$

*where*

$V_{n1} = 0.25 f_c\, b_v\, d_v + V_p$ *(AASHTO LRFD Eq. 5.8.3.3-2)*

$V_{n2} = V_c + V_s + V_p$ *(AASHTO LRFD Eq. 5.8.3.3-1)*

$V_c = 0.0316\, \beta \cdot \sqrt{f'_c} \cdot b_v \cdot d_v$ *Shear Resistance of the Concrete (AASHTO LRFD Eq.5.8.3.3-3)*

$V_s = \dfrac{A_v \cdot f_y \cdot d_v \cdot cot(\theta)}{s}$ *Shear Resistance of the Transverse Steel (AASHTO LRFD Eq. C5.8.3.3-1)*

## Define Demands

Since shear is dependent on location, let's look at STA 13: *At the interior face of the exterior column*

$V_{service} := 207.2 \cdot kip$ *Shear demand at service*

$V_u := 290.8 \cdot kip$ *Ultimate shear demand*

$M_u := -474.2 \cdot kip \cdot ft$ *Ultimate moment demand*

$N_u := 0 \cdot kip$ *Ultimate axial force*

$V_p := 0\ kip$ *Vertical component of the prestress force There is no vertical component of the prestressing force since straight strands are used*

## Shear Design (Con't)

### Define Variables

| | | |
|---|---|---|
| $f'_c = 5\ ksi$ | | *Depth of the bent cap* |
| $f_y := 60 \cdot ksi$ | | *Yield strength of mild steel* |
| $f_{pu} = 270\ ksi$ | | *Tensile strength of prestressing steel* |
| $f_{po} := 0.7 \cdot f_{pu}$ | $f_{po} = 189\ ksi$ | *Parameter taken as modulus of elasticity of prestressing tendon* |
| $n = 28$ | | *Number of strand provided* |
| $h := CapDepth$ | $h = 48\ in$ | *Depth of the bent cap* |
| $b_v := CapWidth - D_{void}$ | $b_v = 48\ in$ | *Width of the bent cap* |
| $c := c + \frac{h}{2}$ | $c = 7.96\ in$ | *Neutral axis from the top extreme concrete* |
| $A_{ps} := \frac{n}{2} \cdot A_{strand}$ | $A_{ps} = 3.04\ in^2$ | *Area of strands in tension side* |
| $A_s := 0 \cdot in^2$ | | *Area of mild steel reinforcement* |
| $d_s := 0 \cdot in$ | | *Effective depth of mild steel reinforcement* |
| $d_p := \frac{D}{2} + e_o$ | $d_p = 24\ in$ | *Distance from extreme compression fiber to the centroid of the prestressing strands*<br>*Note:* $e_o := 0$ *for concentric strand layout* |
| $k := 0.28$ | | *For low relaxation strand (AASHTO LRFD C5.7.3.1.1)* |
| $A_{ct} := \frac{h \cdot b_v}{2}$ | | *Area of concrete on the flexural tension side of the cap, from the extreme tension fiber to on half the cap depth.* |
| $f_{ps} := f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right)$ | $f_{ps} = 245\ ksi$ | *Average stress in prestressing steel (AASHTO LRFD Eq.5.7.3.1.1-1)* |
| $d_e := \frac{A_{ps} \cdot f_{ps} \cdot d_p + A_s \cdot f_y \cdot d_s}{A_{ps} \cdot f_{ps} + A_s \cdot f_y}$ | $d_e = 24\ in$ | *Effective depth from extreme compression fiber to the centroid of the tensile force (AASHTO LRFD Eq. 5.8.2.9-2)* |

## Shear Design (Con't)

### Check Cracking Shear

This step is recommended for the section with an interior void or thin web

*Shear demand at service load shall be less than* $V_{cr}$

$$V_{service} \le V_{cr}$$

Look at STA 13 where the interior void is located with large shear demand

$V_{service} = 207.2\ kip$ — *Shear demand at the interior face of the exterior column under service load*

$A_{pos} = 2304\ in^2$ — *Area of the hollow section at the positive moment region*

$I_{pos} = 442368\ in^4$ — *Moment of inertia of the hollow section at the positive moment region*

$b_w := D - D_{void}$ $\quad b_w = 48\ in$ — *Width of the hollow section*

$Q_{solid} := \dfrac{B \cdot D^2}{8}$ $\quad Q_{solid} = 13824\ in^3$ — *First moment of area of the solid section*

$Q_{void} := \dfrac{{D_{void}}^3}{8}$ $\quad Q_{void} = 0\ in^3$ — *First moment of area of the void*

$Q_{pos} := Q_{solid} - Q_{void}$ $\quad Q_{pos} = 13824\ in^3$ — *First moment of area of the voided section*

$f_t := 0.0632 \cdot \sqrt{\dfrac{f'_c}{ksi}} \cdot ksi$ $\quad f_t = 0.14\ ksi$ — *Tensile strength of concrete for shear*

$T_{strand} = 35.15\ kip$ — *Tension force by single strand*

$n = 28$ — *Number of strand provided*

$$V_{cr} := \frac{I_{pos} \cdot b_w}{Q_{pos}} \cdot \sqrt{\left(f_t\right)^2 + \left(\frac{f_t \cdot n \cdot T_{strand}}{A_{pos}}\right)}$$

*Cracking shear*

$V_{cr} = 435.38\ kip$

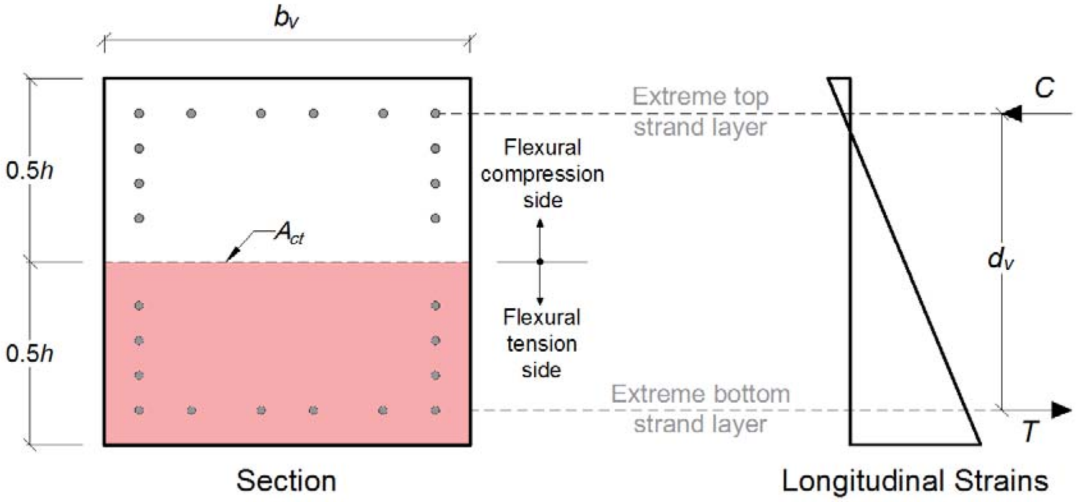
$ShearCrackCheck := \mathbf{if}\left(V_{service} \le V_{cr}, \text{“Okay”}, \text{“Not Okay”}\right)$

$ShearCrackCheck = \text{“Okay”}$

## Shear Design (Con't)

### Find Effective Shear Depth

$d_v := CapDepth - 2 \cdot cover$ $\qquad d_v = 40\ in$ $\qquad$ *Effective Shear Depth; taken as the distance between outer steel layers*




$d_v$ need <u>not</u> be <u>less</u> than the <u>greater</u> of $0.9\,d_e$ and $0.72\,h$ $\qquad$ *(AASHTO LRFD 5.8.2.9)*

$d_v$ = maximum of: $\qquad d_v := h - 2 \cdot cover = 40\ in$

$0.9 \cdot d_e = 21.6\ in$

$0.72 \cdot h = 34.56\ in$

$d_v := \max\left(d_v, 0.9 \cdot d_e, 0.72 \cdot h\right)$ $\qquad d_v = 40\ in$

*Since* $V_n$ must be lesser of $V_{n1}$ and $V_{n2}$ (as per AASHTO LRFD 5.8.3.3), then $V_u$ must be less than both $\phi V_{n1}$ and $\phi V_{n2}$. $V_{n1}$ is dependent on the section properties and the flexural reinforcement. $V_{n2}$ is dependent on the section properties, the flexural reinforcement, and the shear reinforcement. $V_{n1}$ is independent of the shear steel, therefore if $V_u$ is greater than $\phi V_{n1}$ the cap fails in shear regardless of transverse steel.

### Check AASHTO 5.8.3.3-2

$V_{n1} := 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p$ $\qquad V_{n1} = \left(2.4 \cdot 10^3\right)\ kip$ $\qquad$ *(AASHTO LRFD Eq.5.8.3.3-2)*

$V_{r1}$ must be greater than $V_u$ $\qquad V_{r1} := \phi_v \cdot V_{n1}$

$V_{r1} = 2160\ kip > V_u = 290.8\ kip$ $\qquad$ *(AASHTO LRFD Eq.5.8.2.1-2)*

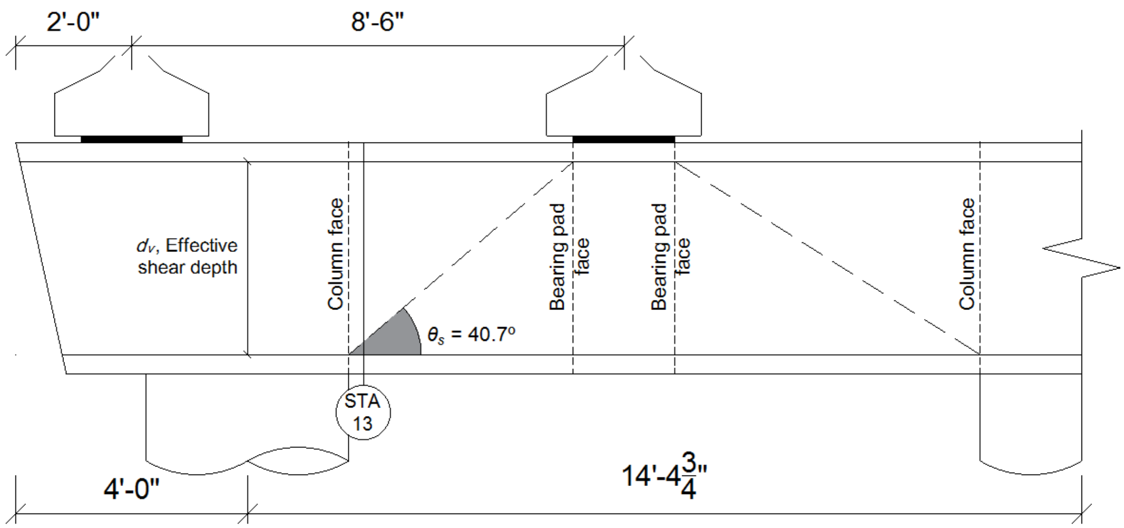
$V_{r1}Check := \mathbf{if}\left(V_{r1} > V_u, \text{“Okay”}, \text{“Not Okay”}\right)$ $\qquad$ *If $V_{r1}$ is greater than $V_{u1}$, then use a larger cap depth in order to satisfy shear requirements.*

$V_{r1}Check = \text{“Okay”}$

## Shear Design (Con't)

### Determine the Compressive Strut Angle

Find $\theta_s$ from the bent cap geometry $\theta_s := 40.7$ deg. *Angle between the column face and the bearing pad face*




### Calculate Determine $\varepsilon_s$ and $\theta$

The method for calculating $\varepsilon_s$ and $\theta$ used in this design example is from AASHTO LRFD 5.8.3.4.2.

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$

*The net longitudinal tensile strain in the section at the centroid of the tension reinforcement (AASHTO 5.8.3.4.2-1). If $\varepsilon_s$<0, then assume $\varepsilon_s$=0 or recalculate with the denominator of the equation replaced by $(E_sA_s + E_pA_{ps} + E_cA_c)$; however $\varepsilon_s$ should not be taken as less than $-0.40 \cdot 10^{-3}$ or greater than $6.0 \cdot 10^{-3}$ (AASHTO LRFD. 5.8.3.4.2)*

If $\varepsilon_s < 0$, then use $\varepsilon_s$=0 or an equation below

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{(E_s \cdot A_s + E_p \cdot A_{ps} + E_c \cdot A_{ct})}$$

where, $|M_u| = 474.2$ ***kip·ft*** must be greater than $|V_u - V_p| \cdot d_v = 969$ ***kip·ft***

$M_u := \max(|M_u|, |V_u - V_p| \cdot d_v)$ $\qquad M_u = 969$ ***kip·ft***

$$\varepsilon_s := \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}} \qquad \varepsilon_s = 8.57 \cdot 10^{-5}$$

$\theta := 29 + 3500\ \varepsilon_s$ $\qquad \theta = 29.3$ deg. *(AASHTO LRFD Eq.5.8.3.4.2-3)*

## Shear Design (Con't)

### Determine $\theta$ for Use in the Design and Calculate $V_c$

The controlling angle is the larger of $\theta$ and $\theta_s$

If $\theta$ is larger

$$\beta_1 := \frac{4.8}{(1 + 750 \cdot \varepsilon_s)} \qquad \beta_1 = 4.51$$

*(AASHTO LRFD Eq. 5.8.3.4.2-1) This equation is for section contaning at least the minimum amount of transverse reinforcement. AASHTO LRFD Eq. 5.8.3.4.2-2 provides $\beta$ calculation for section without the minimum amount of shear reinforcement*

$$V_{c1} := 0.0316\ \beta_1 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot b_v \cdot d_v \qquad V_{c1} = 611.88\ kip$$

*(AASHTO LRFD Eq.5.8.3.3-3)*

If $\theta_s$ is larger

$$\beta_2 := 1.6$$

$$V_{c2} := 0.0316 \cdot \beta_2 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot b_v \cdot d_v \cdot \cot\left(\frac{\theta_s \cdot \pi}{180}\right)$$

$$V_{c2} = 252.36\ kip$$

$$V_c := \mathbf{if}\left(\theta_s > \theta, V_{c2}, V_{c1}\right) \qquad V_c = 252.36\ kip$$

$$\theta := \max\left(\theta, \theta_s\right) \qquad \theta = 40.7 \text{ deg.}$$

### Check if Shear Reinforcement is Required

ShearRequired := **if** $\left(V_u > 0.5 \cdot \phi_v \cdot V_c\right.$, “Required” , “"Not Required”)

ShearRequired = “Required”

$\therefore$ Shear reinforcement is required

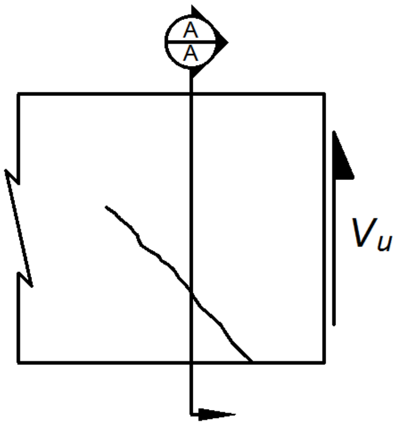
## Shear Design (Con't)

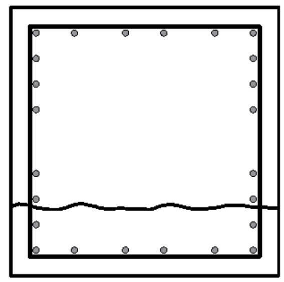
### Provide Shear Reinforcement

$A_v := 2 \cdot (0.31)\ in^2$ $\qquad A_v = 0.62\ in^2$

*TxDOT limits transverse reinforcement spacing to a maximum of 12"and a minimum of 4"*
*(BDM-LRFD, Ch. 4, Sec. 4, Detailing)*
*Trial and error is used to determine the stirrup spacing required for the section*

Assuming #5 stirrups at $s := 10\ in$ spacing



Section A-A

*The transverse reinforcement, "$A_v$" is a closed stirrup. The failure surface intersects two legs of the stirrup, therefore the area of the shear steel is two times the stirrup bar's area (0.31 $in^2$ for #5 bar). See the sketch of the failure plan to the left*

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot\left(\frac{\theta \cdot \pi}{180}\right)}{s} \qquad V_s = 173\ kip$$

*(AASHTO LRFD Eq. C5.8.3.3-1)*

$V_n$ = minimum of:

$V_c + V_s + V_p = 425.36\ kip$ *(AASHTO LRFD Eq. 5.8.3.3-1)*

$0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p = \left(2.4 \cdot 10^3\right)\ kip$ *(AASHTO LRFD Eq. 5.8.3.3-2)*

$V_n := min\left(V_c + V_s + V_p, 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p\right)$ *Nominal shear resistance*

$V_n = 425.36\ kip$

$V_r := \phi_v \cdot V_n$ $\qquad V_r = 382.82\ kip$ *Factored shear resistance*

$V_u = 290.8\ kip$ *Factored shear force*

ShearResistance := **if** ($V_u \le V_r$, "Okay", "Not Okay")

ShearResistance = "Okay"

## Shear Design (Con't)

### Check Minimum Transverse Reinforcement

$A_{v\_min} := 0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot \frac{b_v \cdot s}{f_y}$ $\qquad A_{v\_min} = 0.57\ in^2$ $\qquad$ *(AASHTO LRFD Eq. 5.8.2.5-1)*

$A_v = 0.62\ in^2 \quad > \quad A_{v\_min}$

$MinimumSteelCheck := \mathbf{if}\left(A_v > A_{v\_min}, \text{"Okay"}, \text{"Not okay"}\right)$

$MinimumSteelCheck = \text{"Okay"}$

### Check Maximum Spacing of Transverse Reinforcement

*Shear Stress*

$v_u := \frac{V_u - \left(\phi_v \cdot V_p\right)}{\phi_v \cdot b_v \cdot d_v}$ $\qquad v_u = 0.168\ ksi$ $\qquad$ *Average factored shear stress on the concrete (AASHTO LRFD Eq.5.8.2.9-1)*

$0.125 \cdot f'_c = 0.63\ ksi$

if $v_u < 0.125 \cdot f'_c$, $s_{max}$ = maximum of: $\qquad$ *(AASHTO LRFD Eq.5.8.2.7-1)*

$0.8 \cdot d_v = 32\ in$ & 24in.

if $v_u \geq 0.125 \cdot f'_c$, $s_{max}$ = maximum of: $\qquad$ *(AASHTO LRFD Eq.5.8.2.7-2)*

$0.4 \cdot d_v = 16\ in$ & 12in.

Since $v_u < 0.125 \cdot f'_c$, $s_{max}$=24.0 in.

*TxDOT limits the maximum transverse reinforcement spacing to 12," therefore:* $\qquad$ *(BDM-LRFD, Ch.4, Sec.4, Detailling)*

$s_{max} := 12.00\ in$

$s = 10\ in \quad < \quad s_{max}$

$\mathbf{SpacingCheck} := \mathbf{if}\left(s < s_{max}, \text{"Okay""}, \text{"Not okay"}\right)$

$\mathbf{SpacingCheck} = \text{"Okay""}$

*Shear capacity and checks should be repeated at ALL points of critical shear. Note: in the overhangs, the stirrups need to be spaced @ 5in because shear is higher. Similarly the stirrups need to be spaced @ 5in near the center column. When the spacing needed is less than 4in, use double stirrups. (BDM-LRFD, Ch. 4, Sec. 4, Detailing) When using double stirrups,* $A_v$ *is four times the stirrups bar's area.*

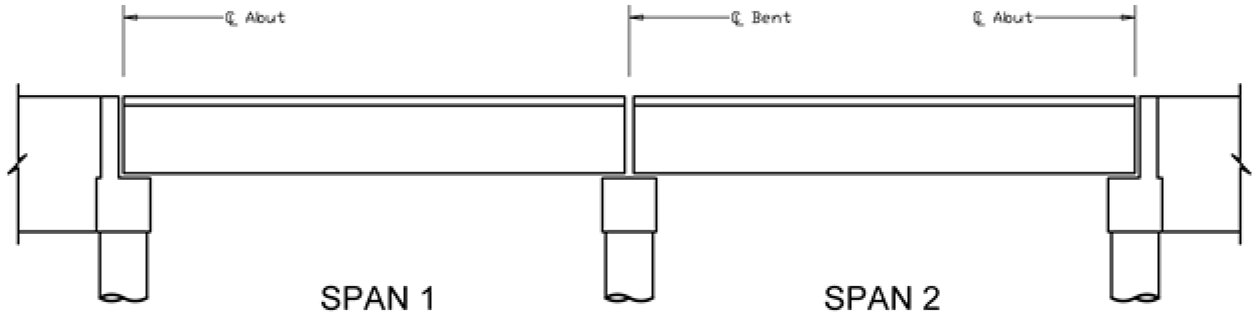
# APPENDIX F: DESIGN EXAMPLE #4

Date: February, 2018

Rectangular Pretensioned Bent Cap Design Example, with a Cocentric Strand Layout

This design example is in accordance with the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012) and TxDOT Bridge Design Manual - LRFD (October 2015)

# Design Parameters




## Span 1

100' Type Tx62 Girders ($0.948 \frac{k}{ft}$)

10 Girders Spaced @ 8.50' with 3' deck overhangs, with middle two girders spaced @ 8'

*"AASHTO LRFD" refers to the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012)*

*"BDM-LRFD" refers to the TxDOT Bridge Design Manual - LRFD (October 2015)*

## Span 2

100' Type Tx54 Girders ($0.948 \frac{k}{ft}$)

10 Girders Spaced @ 8.50' with 3' deck overhangs, with middle two girders spaced @ 8'

*"TxSP" refers to TxDOT guidance, recommendations, and standard practice*

## All Spans

Deck is 40' wide

Type T551 Rail ($0.382 \frac{k}{ft}$)

Type SSCB(1) Median Barrier ( $0.717 \cdot \frac{k}{ft}$)

8.5" Thick Slab (0.100 ksf)

Assume 2" Overlay @ 140 pcf (0.023 ksf) *(TxSP)*

## Assume

4'-0" X 4'-0" Cap
4~42" Columns Spaced @ 22'-0"
Cap will be modeled as a continuous beam with simple supports using TxDOT's CAP18 program.

*TxDOT does not consider frame action for typical multi-column Rectangular Reinforced Concrete Bents. The same methodology is applied to the structural analysis of multi-column Rectangular Pretensioned Concrete Bents. (BDM-LRDSFD, Ch. 4, Sect. 4, Structural Analysis)*

## Design Parameters (Con't)

**Define Variable**

| Back Span | Forward Span | |
|---|---|---|
| $Span1 := 100 \cdot ft$ | $Span2 := 100 \cdot ft$ | *Span Length* |
| $GdrSpa1 := 8.5 \cdot ft$ | $GdrSpa2 := 8.5 \cdot ft$ | *Girder Spacing* |
| $GdrNo1 := 10$ | $GdrNo2 := 10$ | *Number of Girders in Span* |
| $GdrWt1 := 0.948 \cdot \frac{kip}{ft}$ | $GdrWt2 := 0.948 \cdot \frac{kip}{ft}$ | *Weight of Girder* |

Bridge

$RailWt := 0.382 \cdot \frac{kip}{ft}$ — *Weight of Rail*

$MedianWt := 0.717 \cdot \frac{kip}{ft}$ — *Weight of Median Barrier*

$SlabThk := 8.5 \cdot in$ — *Thickness of Bridge Slab*

$OverlayThk := 2 \cdot in$ — *Thickness of Overlay*

$w_c := 150 \cdot pcf$ — *Unit Weight of Concrete for Load Calcs*

$w_{olay} := 140 \cdot pcf$ — *Unit Weight of Overlay*

Other Variables:

$station := 0.5 \cdot ft$ — *Station Increment for CAP18 Analysis*

$IM := 33\%$ — *Dynamic Load Allowance, (AASHTO LRFD Table 3.6.2.1-1)*

Cap Dimensions:

$CapWidth := 48 \cdot in$

$CapDepth := 48 \cdot in$

$cover := 4 \cdot in$ — *Measured from Center of Prestressing Strand*

Material Properties:

$f'_c := 5 \cdot ksi$ — *Assumed Concrete Compressive Strength*

$f_{pu} := 270 \cdot ksi$ — *Ultimate Strength of Prestressing Steel*

## Design Parameters (Con't)

### Define Variable (Con't)

$A_{strand} := 0.217 \cdot in^2$ — *Area of Prestressing Strand*

$w_{cE} := 145 \cdot pcf$ — *Unit Weight of Concrete for $E_c$ Calc*

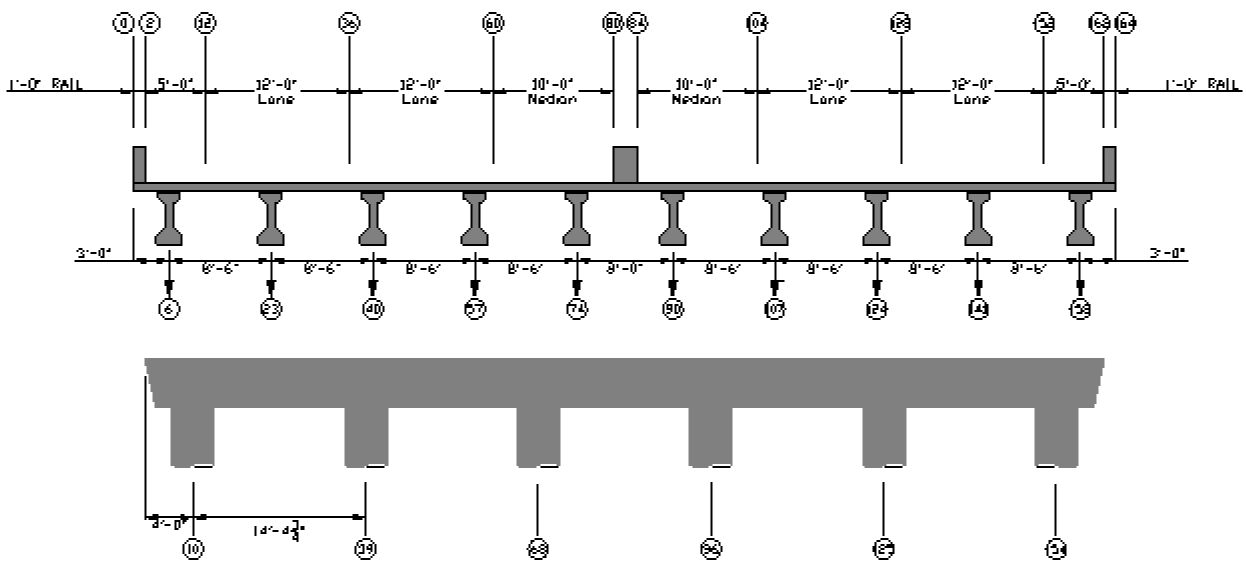
$E_c := 33000 \cdot \left(\frac{w_{cE}}{1000 \cdot pcf}\right)^{1.5} \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi = 4074\ ksi$ — *Modulus of Elasticity of Concrete, (AASHTO LRFD Eq. 5.4.2.4-1)*

$E_s := 29000 \cdot ksi$ — *Modulus of Elasticity of Mild Steel*

$E_p := 28500 \cdot ksi$ — *Modulus of Elasticity of Prestressing Steel*

# Cap Analysis

### Cap Model




*The circled numbers are the stations that are used for the CAP18 Input file. One station is 0.5ft in the direction perpendicular to the pgl.*

# Cap Analysis (Con't)

## Dead Load

Span 1

$$Rail1 := \frac{(2 \cdot RailWt + MedianWt) \cdot \frac{Span1}{2}}{min(GdrNo1, 9)}$$ $Rail1 = 8.23\ kip$ *Rail weight is distributed evenly among stringers, up to 3 stringers per rail. (TxSP)*

$$Slab1 := w_c \cdot GdrSpa1 \cdot SlabThk \cdot \frac{Span1}{2} \cdot 1.1$$ $Slab1 = 49.67\ kip$ *Slab DL is increased by 10% to account for haunch and thickened slab ends.*

$$Girder1 := GdrWt1 \cdot \frac{Span1}{2}$$ $Girder1 = 47.4\ kip$ *Weight of girder acting on bent*

$$DLRxn1 := Rail1 + Slab1 + Girder1$$ $DLRxn1 = 105.3\ kip$ *Dead load reaction per girder, not considering overlay. (Overlay is calculated separately due to possibility of applying a different load factor)*

$$Overlay1 := w_{olay} \cdot GdrSpa1 \cdot OverlayThk \cdot \frac{Span1}{2}$$ $Overlay1 = 9.92\ kip$ *Design for future overlay, per girder*

Span 2

$$Rail2 := \frac{(2 \cdot RailWt + MedianWt) \cdot \frac{Span2}{2}}{min(GdrNo2, 9)}$$ $Rail2 = 8.23\ kip$

$$Slab2 := w_c \cdot GdrSpa2 \cdot SlabThk \cdot \frac{Span2}{2} \cdot 1.1$$ $Slab2 = 49.67\ kip$

$$Girder2 := GdrWt2 \cdot \frac{Span2}{2}$$ $Girder2 = 47.4\ kip$

$$DLRxn2 := Rail2 + Slab2 + Girder2$$ $DLRxn2 = 105.3\ kip$

$$Overlay2 := w_{olay} \cdot GdrSpa2 \cdot OverlayThk \cdot \frac{Span2}{2}$$ $Overlay2 = 9.92\ kip$

Cap

$$A_g := CapWidth \cdot CapDepth$$ $A_g = 2304\ in^2$ *Gross Area of Cap*

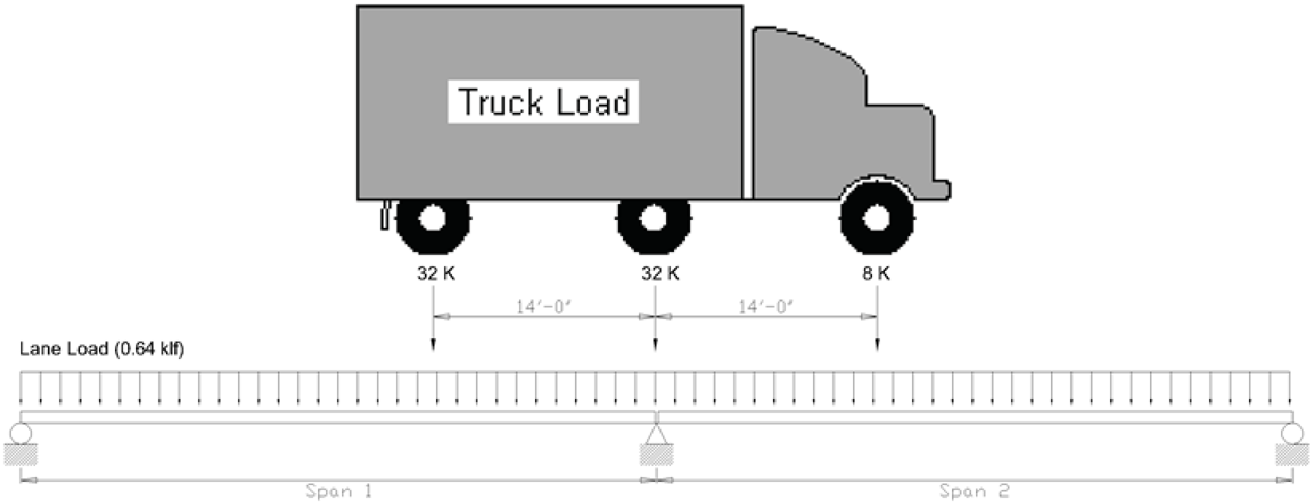
$$Cap := w_c \cdot A_g \cdot station$$ $Cap = 1.2\ kip$ *Dead Load of Cap, per station*

$$I_g := \frac{1}{12} \cdot CapWidth \cdot CapDepth^3$$ $I_g = (4.42 \cdot 10^5)\ in^4$ *Gross Moment of Inertia of Cap*

$E_c = 4074\ ksi$ $E_c \cdot I_g = (1.25 \cdot 10^7)\ kip \cdot ft^2$ *Bending Stiffness of Cap*

## Cap Analysis (Con't)

**Live Load** *(AASHTO LRFD 3.6.1.2.2 and 3.6.1.2.4)*




Live Load Model

$\text{LongSpan} := \max(\text{Span1}, \text{Span2})$

$\text{ShortSpan} := \min(\text{Span1}, \text{Span2})$

$\text{LongSpan} = 100\ ft$

$\text{ShortSpan} = 100\ ft$

$\text{IM} = 0.33$

$$\text{Lane} := 0.64 \cdot klf \cdot \left(\frac{\text{LongSpan} + \text{ShortSpan}}{2}\right)$$

$\text{Lane} = 64\ kip$

$$\text{Truck} := 32 \cdot kip + 32 \cdot kip \cdot \left(\frac{\text{LongSpan} - 14 \cdot ft}{\text{LongSpan}}\right) + 8 \cdot kip \cdot \left(\frac{\text{ShortSpan} - 14 \cdot ft}{\text{ShortSpan}}\right)$$

$\text{Truck} = 66.4\ kip$

$\text{LLRxn} := \text{Lane} + \text{Truck} \cdot (1 + \text{IM})$

$\text{LLRxn} = 152.31\ kip$

$\text{P} := 16.0 \cdot kip \cdot (1 + \text{IM})$

$\text{P} = 21.28\ kip$

$$\text{W} := \frac{\text{LLRxn} - (2 \cdot \text{P})}{10 \cdot ft} \cdot \text{station}$$

$\text{W} = 5.49\ kip$

*Use HL-93 Live Load. Maximum reaction at an interior bent, the "Design Truck" will govern over "Design Tandem." With the Long Span less than twice as long as the Short Span, the maximum reaction occurs when the middle axle (32 kip) is placed over the support, the front axle (8 kips) is placed on the Short Span, and the rear axle (32 kips) is placed on the Long Span.*

*Combine "Design Truck" and "Design Lane" loadings.*
*(AASHTO LRFD 3.6.1.3)*

*Dynamic Load Allowance (IM) does not apply to the "Design Lane"*
*(AASHTO LRFD 3.6.1.2.4)*

*Live Load is applied to the deck slab by two 16 kip wheel loads increased by IM, with the remainder of the live load distributed over a 10ft design lane width*
*(AASHTO LRFD 3.6.1.2.1) (TxSP)*

*Live Load applied to the slab is distributed to the beams assuming the slab is hinged at each beam, except the outside beam.*
*(BDM-LRFD, Ch. 4, Sect. 4, Structural Analysis)*

## Cap Analysis (Con't)

### Cap18 Input

Multiple Presence Factors, m *(AASHTO LRFD Table 3.6.1.1.2-1)*

| No. of Lanes | Factor "m" |
|---|---|
| 1 | 1.20 |
| 2 | 1.00 |
| 3 | 0.85 |
| >3 | 0.65 |

Limit States *(AASHTO LRFD 3.4.1)*

*The cap design only needs to consider Strength I, Service I, and Service I (Dead Load Only)*

Strength I

| | |
|---|---|
| Live Load and Dynamic Load Allowance | LL + IM = 1.75 |
| Dead Load Components | DC = 1.25 |
| Dead Load Wearing Surface (Overlay) | DW = 1.50 |

*TxDOT allows Overlay Factor to be reduced to 1.25 (TxSP).*

Service I

| | |
|---|---|
| Live Load and Dynamic Load Allowance | LL + IM = 1.00 |
| Dead Load and Wearing Surface | DC & DW = 1.00 |

*CAP18 Input is included in an Appendix to this example*

### Cap18 Output

| | Max +M | Max -M |
|---|---|---|
| Dead Load | $M_{DLpos} := 432.8 \cdot kip \cdot ft$ | $M_{DLneg} := 566.9 \cdot kip \cdot ft$ |
| Service Load | $M_{SLpos} := 781.8 \cdot kip \cdot ft$ | $M_{SLneg} := 775.4 \cdot kip \cdot ft$ |
| Ultimate Load | $M_{ULpos} := 1151.7 \cdot kip \cdot ft$ | $M_{ULneg} := 1089.4 \cdot kip \cdot ft$ |

*Maximum loads from the CAP18 Output file. The output is included in an Appendix to this design example.*

$M_{DLmax} := \max\left(M_{DLpos}, M_{DLneg}\right)$ $\qquad M_{DLmax} = 566.9\ kip \cdot ft$

$M_{SLmax} := \max\left(M_{SLpos}, M_{SLneg}\right)$ $\qquad M_{SLmax} = 781.8\ kip \cdot ft$

$M_{ULmax} := \max\left(M_{ULpos}, M_{ULneg}\right)$ $\qquad M_{ULmax} = 1151.7\ kip \cdot ft$

# Flexural Design

*The flexural design of the bent cap is based on the philosophy of "Zero Tension Under Dead Load." The design follows the following steps:*

- *Design for Zero Flexural Tension under Dead Load*
- *Determine Minimum Concrete Compressive Strength and Check Stresses at Service Loads*
- *Check the Minimum Number of Strands*
- *Check the Ultimate Strength Capacity*
- *Check that Minimum Capacity is satisfied*

## Define Constants and Variables

$B := CapWidth$ $\quad D := CapDepth$ $\quad Dia_{pipe} := 24\ in$ $\quad D_{void} := 24\ in$

$f_r := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi$ $\quad f_r = 0.54\ ksi$ $\quad$ *Modulus of Rupture (AASHTO LRFD Eq. 5.4.2.4-1)*

$jd := 0.45 \cdot CapDepth$ $\quad jd = 21.6\ in$ $\quad$ *Approximate Moment Arm between Tension and Compression section to estimate nominal moment capacity for determination of Minimum Number of Strands*

$f_y := 0.9 \cdot f_{pu}$ $\quad f_y = 243\ ksi$ $\quad$ *Yield Strength of Prestressing Steel (AASHTO LRFD Table 5.4.4.1-1)*

$\Delta_{f.pt} := 0.2$ $\quad$ *Assumed prestress loss in pretensioned members*

$T_{strand} := 0.75 \cdot f_{pu} \cdot A_{strand} \cdot (1 - \Delta_{f.pt})$ $\quad T_{strand} = 35.15\ kip$

$I_{pos} := \frac{B \cdot D^3}{12} - \frac{D_{void}^{\ 4}}{12}$ $\quad I_{pos} = 414720\ in^4$ $\quad$ *Moment of Inertia of the solid section at the positive moment region*

$I_{neg} := \frac{B \cdot D^3}{12} - \frac{Dia_{pipe} \cdot D^3}{12}$ $\quad I_{neg} = 221184\ in^4$ $\quad$ *Moment of Inertia of the hollow section at the negative moment region*

$S_{pos} := \frac{I_{pos}}{\left(\frac{D}{2}\right)}$ $\quad S_{pos} = 17280\ in^3$ $\quad$ *Section Modulus of solid Rectangular Section at the positive moment region*

$S_{neg} := \frac{I_{neg}}{\left(\frac{D}{2}\right)}$ $\quad S_{neg} = 9216\ in^3$ $\quad$ *Section Modulus of hollow connection section at the negative moment region*

$A_{pos} := (B \cdot D) - D_{void}^{\ 2}$ $\quad A_{pos} = 1728\ in^2$

$A_{neg} := (B - Dia_{pipe}) \cdot D$ $\quad A_{neg} = 1152\ in^2$

## Flexural Design (Con't)

### Design for Zero Flexural Tension Under Dead Load

Tension Limit:

$$\frac{-F_t}{A}+\frac{M_{DL}}{S_x}=f_t=0$$

*Set the stress at the extreme tension fiber to zero*

$F_{t\_neg} := \frac{M_{DLneg} \cdot A_{neg}}{S_{neg}}$ $F_{t\_neg} = 850.4\ kip$

*Determine the prestressing force required to achieve zero tension stress in the negative moment region*

$F_{t\_pos} := \frac{M_{DLpos} \cdot A_{pos}}{S_{pos}}$ $F_{t\_pos} = 519.36\ kip$

*Determine the prestressing force required to achieve zero tension stress in the positive moment region*

$F_t := \max\left(F_{t\_neg}, F_{t\_pos}\right)$ $F_t = 850350\ lbf$

$n_{flex\_t} := \mathrm{Ceil}\left(\left(\frac{F_t}{T_{strand}}\right), 4\right)$ $n_{flex\_t} = 28$

*Use the calculated $F_t$ to determine the corresponding number of strands required*

$e_o := 0\ in$

*For symmetric, concentric layouts- the number of strands must be a multiple of 4 with no eccentricity*

Compression Limit:

$$\frac{-F_c}{A}-\frac{M_{DL}}{S_x}\geq f_c=-0.45 \cdot f'_c$$

*Set the stress at the extreme compression fiber to the compressive stress limit (AASHTO LRFD Table 5.9.4.2.1-1*

$F_{c\_neg} := \left(0.45 \cdot f'_c - \frac{M_{DLneg}}{S_{neg}}\right) \cdot A_{neg}$ $F_{c\_neg} = 1741.7\ kip$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the negative moment region*

$F_{c\_pos} := \left(0.45 \cdot f'_c - \frac{M_{DLpos}}{S_{pos}}\right) \cdot A_{pos}$ $F_{c\_pos} = 3368.6\ kip$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the positive moment region*

$F_c := \min\left(F_{c\_neg}, F_{c\_pos}\right)$ $F_c = 1741.7\ kip$

$n_{flex\_c} := \mathrm{Floor}\left(\left(\frac{F_c}{T_{strand}}\right), 4\right)$ $n_{flex\_c} = 48$

*Use the calculated $F_c$ to determine the corresponding number of strands required to reach the compressive stress limit*

$n := \min\left(n_{flex\_t}, n_{flex\_c}\right)$ $n = 28$

*The number of strands selected for the should be between $n_{flec\_t}$ and $n_{flex\_c}$*

$F_{provided} := n \cdot T_{strand}$ $F_{provided} = 984.3\ kip$

*Determine the provided prestressing force from the selected number of strands*

## Flexural Design (Con't)

### Design for Zero Flexural Tension under Dead Load (Con't)

Compression Limit:

$$\frac{-F_c}{A} - \frac{M_{DL}}{S_x} \geq f_c = -0.45 \cdot f'_c$$

*Set the stress at the extreme compression fiber to the compressive stress limit (AASHTO LRFD Table 5.9.4.2.1-1*

$$F_{c\_neg} := \left(0.45 \cdot f'_c - \frac{M_{DLneg}}{S_{neg}}\right) \cdot A_{neg}$$

$F_{c\_neg} = 1741.7$ ***kip***

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the negative moment region*

$$F_{c\_pos} := \left(0.45 \cdot f'_c - \frac{M_{DLpos}}{S_{pos}}\right) \cdot A_{pos}$$

$F_{c\_pos} = 3368.6$ *kip*

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load at the positive moment region*

$$F_c := min\left(F_{c\_neg}, F_{c\_pos}\right)$$

$F_c = 1741.7$ ***kip***

$$n_{flex\_c} := \mathrm{Floor}\left(\left(\frac{F_c}{T_{strand}}\right), 4\right)$$

$n_{flex\_c} = 48$

*Use the calculated* $F_c$ *to determine the corresponding number of strands required*

$$n := min\left(n_{flex\_t}, n_{flex\_c}\right)$$

$n = 28$

*The number of strands selected for the should be between* $n_{flex\_t}$ *and* $n_{flex\_c}$

$$F_{provided} := n \cdot T_{strand}$$

$F_{provided} = 984.3$ ***kip***

*Determine the provided prestressing force from the selected number of strands*

## Flexural Design (Con't)

## Check Minimum Concrete Compressive Strength with Service Stress

Tension Stress:

$$\frac{-F}{A}+\frac{M_{SL}}{S_x}\geq f_t = k\cdot\sqrt{f'_c}$$

*The stress at the extreme tension fiber must not exceed the service stress limit, which is* $k\cdot\sqrt{f'_c}$ .
*(AASHTO LRFD Table 5.9.4.2.2-1)*

*Values of* $k$ *are different for various corrosion conditions. For AASHTO LRFD 5.9.4.2.2,* $k$ *values are:*

- *Moderate Exposure (Class I)* $k := 0.19$
- *Severe Exposure (Class II)* $k := 0.0948$

*Recommended value of* $k$ *to limit cracking under service loads was shown to be smaller than the value imposed by AASHTO. This value is* $k := 0.126$

*The tensile stresses under service conditions should not exceed values specified by AASHTO LRFD 5.9.4.2.2. However, to further reduce cracking under service conditions in Class I conditions, the tensile stresses should not exceed the recommended value concluded from experimental testing.*

*For this design example, the AASHTO value of* $k$ *for Class I expoure will be used to check the service level stresses and compute the minimum concrete compressive strength.*

Negative Moment Region:

$k := 0.19$ *Multiplier for* $k\cdot\sqrt{f'_c}$ *tensile stress limit*

$$f_{t\_sl\_neg} := \frac{-F_{provided}}{A_{neg}}+\frac{M_{SLneg}}{S_{neg}}$$ $f_{t\_sl\_neg} = 0.155\ ksi$ *Tensile Stress at extreme tension fiber under Service Loads*

$$f_{t\_slim\_neg} := k\cdot\sqrt{\frac{f'_c}{ksi}}\cdot ksi$$ $f_{t\_slim\_neg} = 0.425\ ksi$ *Limiting tensile stress at the extreme tension fiber*

$\text{TenLimit} := \textbf{if}\left(f_{t\_sl\_neg}\leq f_{t\_slim\_neg}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$ $\text{TenLimit} = \text{“GOOD”}$

Positive Moment Region:

$$f_{t\_sl\_pos} := \frac{-F_{provided}}{A_{pos}}+\frac{M_{SLpos}}{S_{pos}}$$ $f_{t\_sl\_pos} = -0.027\ ksi$ *Tensile Stress at extreme tension fiber under Service Loads*

$$f_{t\_slim\_pos} := k\cdot\sqrt{\frac{f'_c}{ksi}}\cdot ksi$$ $f_{t\_slim\_pos} = 0.425\ ksi$ *Limiting tensile stress at the extreme tension fiber*

$\text{TenLimit} := \textbf{if}\left(f_{t\_sl\_pos}\leq f_{t\_slim\_pos}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$ $\text{TenLimit} = \text{“GOOD”}$

# Flexural Design (Con't)

## Check Minimum Concrete Compressive Strength with Service Stress (Con't)

Compression Stress:

$$\frac{-F}{A} - \frac{M_{SL}}{S_x} > f_c = -0.45 \cdot f'_c$$

*The stress at the extreme compression fiber must not exceed the service stress limit. Considering the connection section as solid in compression (AASHTO LRFD Table 5.9.4.2.1-1)*

$$f_{c\_sl} := \left(\frac{-F_{provided}}{A_{pos}}\right) - \left(\frac{M_{SLmax}}{S_{pos}}\right) \qquad f_{c\_sl} = -1.113\ ksi$$

*Compressive Stress at extreme compression fiber under Service Loads*

$$f_{c\_slim} := -0.45 \cdot f'_c \qquad f_{c\_slim} = -2.25\ ksi$$

*Limiting compressive stress at the extreme compression fiber*

$$CompLimit := \textbf{if}\left(f_{c\_sl} \geq f_{c\_slim}, \text{“GOOD”}, \text{“NOT GOOD”}\right) \qquad CompLimit = \text{“GOOD”}$$

Minimum Concrete Strength:

$$f'_{c\_tmin} := \left(\left(\frac{\max\left(f_{t\_sl\_neg}, f_{t\_sl\_pos}\right)}{k \cdot ksi}\right)^2\right) \cdot ksi \qquad f'_{c\_tmin} = 0.67\ ksi$$

*Determine minimum concrete compressive strength to achieve the limiting tensile stress*

$$f'_{c\_cmin} := \frac{f_{c\_sl}}{-0.45} \qquad f'_{c\_cmin} = 2.47\ ksi$$

*Determine minimum concrete compressive strength to achieve the limiting compressive stress*

$$f'_{c\_min} := \max\left(f'_{c\_cmin}, f'_{c\_tmin}, 5 \cdot ksi\right) \qquad f'_{c\_min} = 5\ ksi$$

*Minimum Concrete Compressive Strength. If calculated values are significantly low, use a practical minimum achievable concrete strength (consider $5 \cdot ksi$ for precast, prestressed concrete)*

*If the required Minimum Concrete Compressive strength is larger than the maximum allowed design strength from Chapter 4 of BDM-LRFD, adjust the number of strands and/or strand layout to reduce the service stresses.*

*If the calculated Minimum Concrete Compressive strength is larger than the assumed strength, recheck stress limits with the "new" compressive strength.*

## Flexural Design (Con't)

## Determine Minimum Number of Strands

*AASHTO LRFD 5.7.3.3.2 specifies that the factored flexural resistance $M_r$ should be at least greater than the lesser of $M_{cr}$ or $1.33 \cdot M_u$ . The derivation of the formula to determine $n_{min}$ is shown:*

$$-\left(\frac{F}{A_g}\right)+\left(\frac{M_{cr}}{S_x}\right) = f_r$$

*Stress at extreme tension fiber is equal to the modulus of rupture*

$$M_{cr} = \gamma_3\left(\gamma_1 \cdot f_r + \gamma_2\left(\frac{F}{A}\right)\right) \cdot S_x \qquad \gamma_1 := 1.6 \quad \gamma_2 := 1.1 \quad \gamma_3 := 1.0$$

*Where $F$ is the prestressing force*

$$\phi M_n = M_r \geq M_{cr} \text{ , where } \phi := 1.0$$

*(AASHTO LRFD 5.5.4.2.1)*

$$M_n \geq n \cdot A_{ps} \cdot f_y \cdot jd$$

*Approximate nominal moment capacity*

Substituting, simplifying, and equating $M_{cr}$ and $M_n$ :

$$\gamma_1 f_r \cdot S_x + \gamma_2 \cdot \left(n \cdot T_{strand}\right) \frac{S_x}{A} = n \cdot A_{ps} \cdot f_y \cdot jd$$

Thus:

$$n = \frac{\gamma_1 \cdot f_r \cdot S_x}{A_{ps} \cdot f_y \cdot jd - \frac{\gamma_2 \cdot T_{strand} \cdot S_x}{A}}$$

Also $M_r \geq 1.33 \cdot M_{UL}$

$$n = \frac{1.33 \cdot M_{UL}}{A_{ps} \cdot f_y \cdot jd}$$

$$n_{min\_pos} := \mathrm{ceil}\left(\max\left(\frac{\gamma_1 \cdot f_r \cdot S_{pos}}{A_{strand} \cdot f_y \cdot jd - \frac{\gamma_2 \cdot T_{strand} \cdot S_{pos}}{A_{pos}}}, \frac{1.33 \cdot M_{ULpos}}{A_{strand} \cdot f_y \cdot jd}\right)\right)$$

*$n$ must be checked for both positive and negative bending regions, as the cross-sections are not the same.*

$$n_{min\_pos} = 20$$

$$n_{min\_neg} := \mathrm{ceil}\left(\max\left(\frac{\gamma_1 \cdot f_r \cdot S_{neg}}{A_{strand} \cdot f_y \cdot jd - \frac{\gamma_2 \cdot T_{strand} \cdot S_{neg}}{A_{neg}}}, \frac{1.33 \cdot M_{ULneg}}{A_{strand} \cdot f_y \cdot jd}\right)\right)$$

$$n_{min\_neg} = 16$$

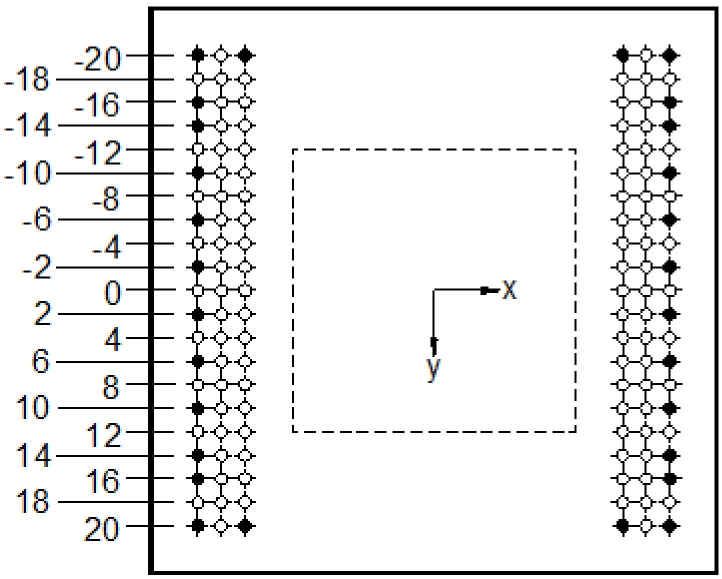
$$n_{min} := \max\left(n_{min\_pos}, n_{min\_neg}\right) \qquad n_{min} = 20$$

$$n_{min\_check} := \textbf{if}\left(n \geq n_{min}, \text{“GOOD”}, \text{“NOT GOOD”}\right) \qquad n_{min\_check} = \text{“GOOD”}$$

## Flexural Design (Con't)

## Check Ultimate Strength Capacity

Determine strand configuration




*The strand layout is limited based on the configuration of the cap-to-column connection. For this example, the cap-to-column connection is assumed to be formed by a 24-inch nominal diameter pocket connection.*

**Define Variables**

$\Delta_{\varepsilon p} := \dfrac{T_{strand}}{E_p \cdot A_{strand}}$ $\qquad \Delta_{\varepsilon p} = 0.0057$

*Pre-strain, after losses*

$\beta := \max\left(\left(0.85 - \left(\dfrac{f'_c}{ksi} - 4\right) \cdot 0.05\right), 0.65\right)$ $\qquad \beta = 0.8$

*(AASHTO LRFD 5.7.2.2)*

$\varepsilon_{cu} := 0.003$

*Maximum strain at extreme compression fiber*
*(AASHTO LRFD 5.7.2.1)*

$\phi := 1.0$

*Strength Reduction Factor*
*(AASHTO LRFD 5.5.4.2.1)*

$Q := 0.03$

$R := 6$

*Q and R are constants in the "Menegotto-Pinto" equation used to determine the stress at $i^{th}$ layer of prestressing steel. Since a side configuration layout of prestressing steel was used instead of the conventional top & bottom layout, the stresses in the prestressing steel must be determined at each individual layer.*

$c_i := -15 \cdot in$

*Initial location of neutral axis used in the iterative solution of determining the moment capacity.*

## Flexural Design (Con't)

### Calculate Strain and Stress in Each Steel Layer

$$\varepsilon_{ti} = \varepsilon_{cu} \cdot \left( \frac{d_i - c}{\frac{D}{2} + c} \right)$$

*Tension strain at the $i^{th}$ layer. $d_i$ is the depth of the prestressing layer, as shown in the strand layout (note the convention and origin of distance measurements)*

$$\varepsilon_{si} = \varepsilon_{ti} + \Delta_{\varepsilon p}$$

*Total strain on each layer, considering the pre-strain*

$$f_{psi} = E_p \cdot \varepsilon_{si} \cdot \left( Q + \frac{1 - Q}{\left( 1 + \left( \left| \frac{\varepsilon_{si} \cdot E_s}{f_y} \right| \right)^{R} \right)^{\frac{1}{R}}} \right)$$

*Menegotto-Pinto equation to determine the stress in the $i^{th}$ layer*

$$T_i = f_{psi} \cdot A_{psi}$$

*Tension force in the $i^{th}$ layer of steel. $A_{psi}$ is the area of prestressing steel in that layer.*

$$jd_i = \frac{D}{2} - d_i - \frac{a}{2}$$

*Moment arm between compressive stress block and the $i^{th}$ layer of prestressing steel*

$$a = \frac{-D}{2} + \left( \frac{-\beta}{2} \cdot \left( \frac{-D}{2} - c \right) \right)$$

*Depth of the equivalent compression block, with respect to the center of the bent cap*

$$M_i = T_i \cdot j \cdot d_i$$

*Moment in the $i^{th}$ layer*

$$C_c = -0.85 \cdot f'_c \cdot \beta \cdot \left( \frac{D}{2} - c \right) \cdot B$$

*Compressive force from the equivalent compressive stress block*

*The previous equations are calculated using the $c_i$ value, and iterated with changing values of $c$ until the sum of the tensile forces equals the magnitude of the compressive force:*

$$\Sigma T_t = \left| C_c \right|$$

*This process is completed for both Positive and Negative Bending in Microsoft Excel, and the results are presented in the following tables*

## Flexural Design (Con't)

**Moment Capacity:**

| $d_l$ (in) | $n_l$ | $\varepsilon_{tl}$ | $\Delta\varepsilon_p$ | $\varepsilon_{sl}$ | $f_{psl}$ (ksi) | $T_l$ (kips) | $jd_l$ (in) | $M_l$ (k-in) |
|---|---|---|---|---|---|---|---|---|
| -20 | 4 | -0.00149 | 0.00568 | 0.00419 | 119.21 | 103.48 | 0.82 | 84.6 |
| -16 | 2 | 0.00002 | 0.00568 | 0.00570 | 160.24 | 69.54 | 4.82 | 335.0 |
| -14 | 2 | 0.00077 | 0.00568 | 0.00645 | 178.91 | 77.65 | 6.82 | 529.4 |
| -10 | 2 | 0.00228 | 0.00568 | 0.00796 | 209.04 | 90.73 | 10.82 | 981.4 |
| -6 | 2 | 0.00379 | 0.00568 | 0.00947 | 227.63 | 98.79 | 14.82 | 1463.8 |
| -2 | 2 | 0.00530 | 0.00568 | 0.01098 | 237.44 | 103.05 | 18.82 | 1939.1 |
| 2 | 2 | 0.00680 | 0.00568 | 0.01249 | 242.63 | 105.30 | 22.82 | 2402.7 |
| 6 | 2 | 0.00831 | 0.00568 | 0.01400 | 245.73 | 106.65 | 26.82 | 2860.0 |
| 10 | 2 | 0.00982 | 0.00568 | 0.01550 | 247.90 | 107.59 | 30.82 | 3315.6 |
| 14 | 2 | 0.01133 | 0.00568 | 0.01701 | 249.64 | 108.34 | 34.82 | 3772.3 |
| 16 | 2 | 0.01208 | 0.00568 | 0.01777 | 250.42 | 108.68 | 36.82 | 4001.5 |
| 20 | 4 | 0.01359 | 0.00568 | 0.01928 | 251.90 | 218.65 | 40.82 | 8924.6 |
| 0.00 | 28 | $C_c$ = | -1298.4 | kips | $\sum T_l$ = | 1298.4 | $\sum M_l$ = | 30609.9 |

$c := -16.04 \cdot in$ — *Final location of N/A, from iterations*

$\Sigma M_i = 30609.94\ kip \cdot in$ — *Sum of $M_i$ from the iterations*

$M_n := \Sigma M_i$ $\qquad M_n = 2550.8\ kip \cdot ft$

$M_r := \phi \cdot M_n$ $\qquad M_r = 2550.8\ kip \cdot ft$ — *Factored Flexural Resistance (AASHTO LRFD 5.7.3.2.1)*

CapacityCheck := **if** $(M_r \geq M_{ULmax}$, "GOOD", "NOT GOOD")

CapacityCheck = "GOOD"

## Flexural Design (Con't)

## Check Minimum Capacity:

*Calculate the $M_{cr}$ and check if the $M_r$ meets AASHTO LRFD 5.7.3.3.2*

$\gamma_1 := 1.6$ $\gamma_2 := 1.1$ $\gamma_3 := 1.0$

Negative Moment Region:

$$f_{cpe} := \frac{F_{provided}}{A_{neg}}$$ $f_{cpe} = 0.85$ ***ksi***

$M_{cr\_neg} := \gamma_3 \cdot \left( \left( \gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe} \right) \cdot S_{neg} \right)$ $M_{cr\_neg} = 1381$ ***kip·ft***

$M_{rCheck} := \textbf{if} \left( M_r \geq min \left( M_{cr\_neg}, 1.33 \cdot M_{ULneg} \right), \text{"GOOD"}, \text{"NOT GOOD"} \right)$

$M_{rCheck} = \text{"GOOD"}$

Positive Moment Region:

$$f_{cpe} := \frac{F_{provided}}{A_{pos}}$$ $f_{cpe} = 0.57$ ***ksi***

$M_{cr\_pos} := \gamma_3 \cdot \left( \left( \gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe} \right) \cdot S_{pos} \right)$ $M_{cr\_pos} = 2139$ ***kip·ft***

$M_{rCheck} := \textbf{if} \left( M_r \geq min \left( M_{cr\_pos}, 1.33 \cdot M_{ULpos} \right), \text{"GOOD"}, \text{"NOT GOOD"} \right)$

$M_{rCheck} = \text{"GOOD"}$

**Design Summary:**

Concrete Strength:

$f'_c = 5$ *ksi*

Prestsessing:

$n = 28$

$F_{provided} = 984.3$ ***kip***

# Shear Design

## Design Philosophy:

$V_u$ *(Ultimate Shear) must be less than* $V_r$ (Shear Resistance)

$V_u \le V_r$ *(AASHTO LRFD Eq. 1.3.2.1-1)*

$V_r = \phi_v \cdot V_n$ *(AASHTO LRFD Eq. 5.8.2.1-2)*

$\phi_v := 0.9$

*Reduction factor*
*(AASHTO LRFD 5.5.4.2.1)*

$V_n$ *is the lesser of* $V_{n1}$ and $V_{n2}$

*where*

$V_{n1} = 0.25 f_c\, b_v\, d_v + V_p$ *(AASHTO LRFD Eq. 5.8.3.3-2)*

$V_{n2} = V_c + V_s + V_p$ *(AASHTO LRFD Eq. 5.8.3.3-1)*

$V_c = 0.0316\, \beta \cdot \sqrt{f'_c} \cdot b_v \cdot d_v$

*Shear resistance of the concrete*
*(AASHTO LRFD Eq.5.8.3.3-3)*

$$V_s = \frac{A_v \cdot f_y \cdot d_v \cdot cot(\theta)}{s}$$

Shear resistance of the transverse steel
*(AASHTO LRFD Eq. C5.8.3.3-1)*

## Define Demands

*Since shear is dependent on location, let's look at STA 13:*

*At the interior face of the exterior column*

$V_{service} := 207.2 \cdot kip$ *Shear demand at service*

$V_u := 290.8 \cdot kip$ *Ultimate shear demand*

$M_u := -474.2 \cdot kip \cdot ft$ *Ultimate moment demand*

$N_u := 0 \cdot kip$ *Ultimate axial force*

$V_p := 0\ kip$

*Vertical component of the prestress force*
*There is no vertical component of the prestressing force since straight strands are used*

## Shear Design (Con't)

### Define Variables

| | | |
|---|---|---|
| $f'_c = 5\ ksi$ | | *Depth of the bent cap* |
| $f_y := 60 \cdot ksi$ | | *Yield strength of mild steel* |
| $f_{pu} = 270\ ksi$ | | *Tensile strength of prestressing steel* |
| $f_{po} := 0.7 \cdot f_{pu}$ | $f_{po} = 189\ ksi$ | *Parameter taken as modulus of elasticity of prestressing tendon* |
| $n = 28$ | | *Number of strand provided* |
| $h := CapDepth$ | $h = 48\ in$ | *Depth of the bent cap* |
| $b_v := CapWidth - D_{void}$ | $b_v = 24\ in$ | *Width of the bent cap* |
| $c := c + \frac{h}{2}$ | $c = 7.96\ in$ | *Neutral axis from the top extreme concrete* |
| $A_{ps} := \frac{n}{2} \cdot A_{strand}$ | $A_{ps} = 3.04\ in^2$ | *Area of strands in tension side* |
| $A_s := 0 \cdot in^2$ | | *Area of mild steel reinforcement* |
| $d_s := 0 \cdot in$ | | *Effective depth of mild steel reinforcement* |
| $d_p := \frac{D}{2} + e_o$ | $d_p = 24\ in$ | *Distance from extreme compression fiber to the centroid of the prestressing strands*<br>*Note:* $e_o := 0$ *for concentric strand layout* |
| $k := 0.28$ | | *For low relaxation strand (AASHTO LRFD C5.7.3.1.1)* |
| $A_{ct} := \frac{h \cdot b_v}{2}$ | | $A_{ct}$ *is the area of concrete on the flexural tension side of the cap, from the extreme tension fiber to on half the cap depth.* |
| $f_{ps} := f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right)$ | $f_{ps} = 245\ ksi$ | *Average stress in prestressing steel (AASHTO LRFD Eq.5.7.3.1.1-1)* |
| $d_e := \frac{A_{ps} \cdot f_{ps} \cdot d_p + A_s \cdot f_y \cdot d_s}{A_{ps} \cdot f_{ps} + A_s \cdot f_y}$ | $d_e = 24\ in$ | *Effective depth from extreme compression fiber to the centroid of the tensile force (AASHTO LRFD Eq. 5.8.2.9-2)* |

## Shear Design (Con't)

### Check Cracking Shear

*This step is recommended for the section with an interior void or thin web*

Shear demand at service load shall be less than $V_{cr}$

$$V_{service} \le V_{cr}$$

Look at STA 20 where the interior void is located with large shear demand

$V_{service} = 207.2\ kip$ — *Shear demand at the interior face of the exterior column under service load*

$A_{pos} = 1728\ in^2$ — *Area of the hollow section at the positive moment region*

$I_{pos} = 414720\ in^4$ — *Moment of inertia of the hollow section at the positive moment region*

$b_w := D - D_{void}$ $\quad b_w = 24\ in$ — *Width of the hollow section*

$Q_{solid} := \dfrac{B \cdot D^2}{8}$ $\quad Q_{solid} = 13824\ in^3$ — *First moment of area of the solid section*

$Q_{void} := \dfrac{D_{void}{}^3}{8}$ $\quad Q_{void} = 1728\ in^3$ — *First moment of area of the void*

$Q_{pos} := Q_{solid} - Q_{void}$ $\quad Q_{pos} = 12096\ in^3$ — *First moment of area of the voided section*

$f_t := 0.0632 \cdot \sqrt{\dfrac{f'_c}{ksi}} \cdot ksi$ $\quad f_t = 0.14\ ksi$ — *Tensile strength of concrete for shear*

$T_{strand} = 35.15\ kip$ — *Tension force by single strand*

$n = 28$ — *Number of strand provided*

$$V_{cr} := \frac{I_{pos} \cdot b_w}{Q_{pos}} \cdot \sqrt{\left(f_t\right)^2 + \left(\frac{f_t \cdot n \cdot T_{strand}}{A_{pos}}\right)}$$ — *Cracking shear*

$V_{cr} = 260.82\ kip$

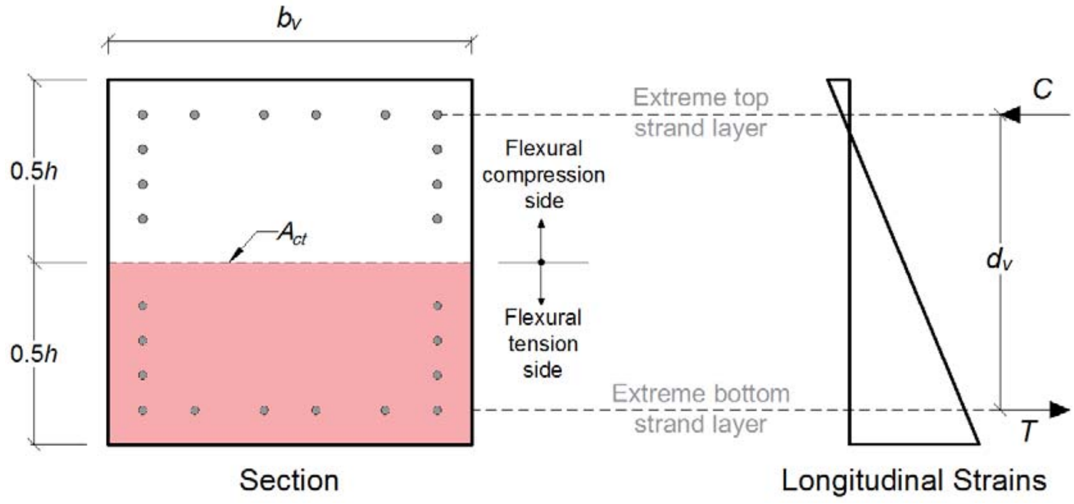
$ShearCrackCheck := \mathbf{if}\left(V_{service} \le V_{cr}, \text{“Okay”}, \text{“Not Okay”}\right)$

$ShearCrackCheck = \text{“Okay”}$

## Shear Design (Con't)

### Find Effective Shear Depth

$d_v := CapDepth - 2 \cdot cover$ $\quad$ $d_v = 40\ in$ $\quad$ *Effective Shear Depth; taken as the distance between outer steel layers*




$d_v$ need not be less than the greater of $0.9\, d_e$ and $0.72\, h$ *(AASHTO LRFD 5.8.2.9)*

$d_v$ = maximum of: $\quad$ $d_v := h - 2 \cdot cover = 40\ in$

$0.9 \cdot d_e = 21.6\ in$

$0.72 \cdot h = 34.56\ in$

$d_v := \max\left(d_v, 0.9 \cdot d_e, 0.72 \cdot h\right)$ $\quad$ $d_v = 40\ in$

*Since* $V_n$ must be lesser of $V_{n1}$ and $V_{n2}$ (as per AASHTO LRFD 5.8.3.3), then $V_u$ must be less than both $\phi V_{n1}$ and $\phi V_{n2}$. $V_{n1}$ is dependent on the section properties and the flexural reinforcement. $V_{n2}$ is dependent on the section properties, the flexural reinforcement, and the shear reinforcement. $V_{n1}$ is independent of the shear steel, therefore if $V_u$ is greater than $\phi V_{n1}$ the cap fails in shear regardless of transverse steel.

### Check AASHTO 5.8.3.3-2

$V_{n1} := 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p$ $\quad$ $V_{n1} = 1200\ kip$ $\quad$ *(AASHTO LRFD Eq.5.8.3.3-2)*

$V_{r1}$ must be greater than $V_u$ $\quad$ $V_{r1} := \phi_v \cdot V_{n1}$

$V_{r1} = 1080\ kip > V_u = 290.8\ kip$ $\quad$ *(AASHTO LRFD Eq.5.8.2.1-2)*

$V_{r1}Check := \textbf{if}\left(V_{r1} > V_u, \text{“Okay”}, \text{“Not Okay”}\right)$

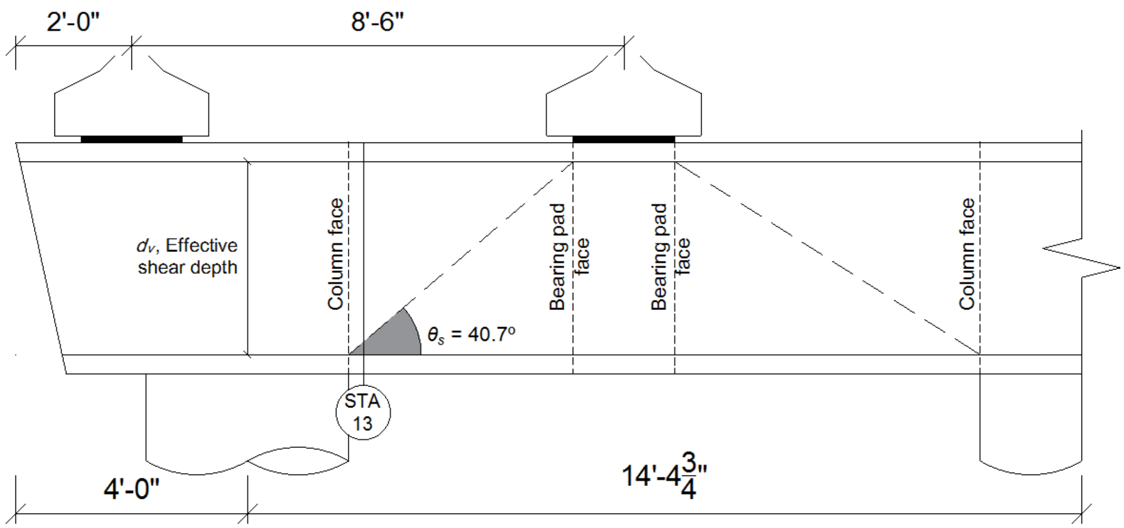
If $V_{r1}$ is greater than $V_{u1}$, then use a larger cap depth in order to satisfy shear requirements.

$V_{r1}Check = \text{“Okay”}$

## Shear Design (Con't)

### Determine the Compressive Strut Angle

Find $\theta_s$ from the bent cap geometry $\theta_s := 40.7$ deg. Angle between the column face and the bearing pad face




### Calculate Determine $\varepsilon_s$ and $\theta$

The method for calculating $\varepsilon_s$ and $\theta$ used in this design example is from AASHTO LRFD 5.8.3.4.2.

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$

*The net longitudinal tensile strain in the section at the centroid of the tension reinforcement (AASHTO 5.8.3.4.2-1). If $\varepsilon_s$<0, then assume $\varepsilon_s$=0 or recalculate with the denominator of the equation replaced by $(E_sA_s + E_pA_{ps} + E_cA_c)$; however $\varepsilon_s$ should not be taken as less than $-0.40 \cdot 10^{-3}$ or greater than $6.0 \cdot 10^{-3}$ (AASHTO LRFD. 5.8.3.4.2)*

If $\varepsilon_s < 0$, then use $\varepsilon_s$=0 or an equation below

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{\left(E_s \cdot A_s + E_p \cdot A_{ps} + E_c \cdot A_{ct}\right)}$$

where, $|M_u| = 474.2$ ***kip·ft*** must be greater than $|V_u - V_p| \cdot d_v = 969$ ***kip·ft***

$M_u := \max\left(|M_u|, |V_u - V_p| \cdot d_v\right)$ $M_u = 969$ ***kip·ft***

$$\varepsilon_s := \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}} \qquad \varepsilon_s = 8.57 \cdot 10^{-5}$$

$\theta := 29 + 3500\ \varepsilon_s$ $\theta = 29.3$ deg. *(AASHTO LRFD Eq.5.8.3.4.2-3)*

## Shear Design (Con't)

### Determine $\theta$ for Use in the Design and Calculate $V_c$

The controlling angle is the larger of $\theta$ and $\theta_s$

If $\theta$ is larger

$$\beta_1 := \frac{4.8}{(1 + 750 \cdot \varepsilon_s)} \qquad \beta_1 = 4.51$$

*(AASHTO LRFD Eq. 5.8.3.4.2-1)*
*This equation is for section contaning at least the minimum amount of transverse reinforcement.*
*AASHTO LRFD Eq. 5.8.3.4.2-2 provides $\beta$ calculation for section without the minimum amount of shear reinforcement*

$$V_{c1} := 0.0316\ \beta_1 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot b_v \cdot d_v \qquad V_{c1} = 305.94\ kip$$

*(AASHTO LRFD Eq.5.8.3.3-3)*

If $\theta_s$ is larger

$$\beta_2 := 1.6$$

$$V_{c2} := 0.0316 \cdot \beta_2 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot b_v \cdot d_v \cdot \cot\left(\frac{\theta_s \cdot \pi}{180}\right)$$

$$V_{c2} = 126.18\ kip$$

$$V_c := \textbf{if}\left(\theta_s > \theta, V_{c2}, V_{c1}\right) \qquad V_c = 126.18\ kip$$

$$\theta := \max\left(\theta, \theta_s\right) \qquad \theta = 40.7 \text{ deg.}$$

### Check if Shear Reinforcement is Required

ShearRequired := **if** $\left(V_u > 0.5 \cdot \phi_v \cdot V_c\right.$, “Required”, “"Not Required”)

ShearRequired = “Required”

∴ Shear reinforcement is required

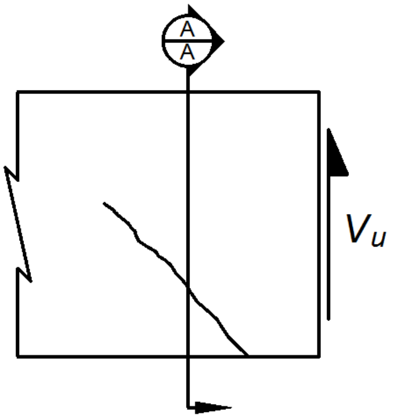
## Shear Design (Con't)

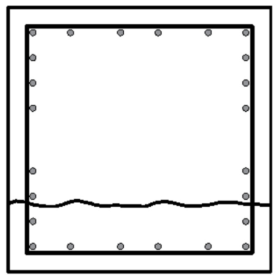
### Provide Shear Reinforcement

$A_v := 2 \cdot (0.31)\ in^2$ $\qquad A_v = 0.62\ in^2$

*TxDOT limits transverse reinforcement spacing to a maximum of 12"and a minimum of 4"*
*(BDM-LRFD, Ch. 4, Sec. 4, Detailing)*
*Trial and error is used to determine the stirrup spacing required for the section*

Assuming #5 stirrups at $s := 8.5\ in$ spacing



Section A-A

*The transverse reinforcement, "$A_v$" is a closed stirrup. The failure surface intersects two legs of the stirrup, therefore the area of the shear steel is two times the stirrup bar's area (0.31 in$^2$ for #5 bar). See the sketch of the failure plan to the left*

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot\left(\frac{\theta \cdot \pi}{180}\right)}{s} \qquad V_s = 203.52\ kip$$

*(AASHTO LRFD Eq. C5.8.3.3-1)*

$V_n$ = minimum of:

$V_c + V_s + V_p = 329.71\ kip$ *(AASHTO LRFD Eq. 5.8.3.3-1)*

$0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p = \left(1.2 \cdot 10^3\right)\ kip$ *(AASHTO LRFD Eq. 5.8.3.3-2)*

$V_n := min\left(V_c + V_s + V_p, 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p\right)$ *Nominal Shear Resistance*

$V_n = 329.71\ kip$

$V_r := \phi_v \cdot V_n \qquad V_r = 296.74\ kip$ *Factored Shear Resistance*

$V_u = 290.8\ kip$ *Factored Shear Force*

ShearResistance := **if** ($V_u \le V_r$, “Okay”, “Not Okay”)

ShearResistance = “Okay”

## Shear Design (Con't)

### Check Minimum Transverse Reinforcement

$$A_{v\_min} := 0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot \frac{b_v \cdot s}{f_y} \qquad A_{v\_min} = 0.24\ in^2$$ *(AASHTO LRFD Eq. 5.8.2.5-1)*

$A_v = 0.62\ in^2 \quad > \quad A_{v\_min}$

MinimumSteelCheck := **if** $(A_v > A_{v\_min}$, “Okay”, “Not okay”)

MinimumSteelCheck = “Okay”

### Check Maximum Spacing of Transverse Reinforcement

*Shear Stress*

$$v_u := \frac{V_u - (\phi_v \cdot V_p)}{\phi_v \cdot b_v \cdot d_v} \qquad v_u = 0.337\ ksi$$

*Average factored shear stress on the concrete*
*(AASHTO LRFD Eq.5.8.2.9-1)*

$0.125 \cdot f'_c = 0.63\ ksi$

if $v_u < 0.125 \cdot f'_c$, $s_{max}$ = maximum of: *(AASHTO LRFD Eq.5.8.2.7-1)*

$0.8 \cdot d_v = 32\ in$ & 24in.

if $v_u \geq 0.125 \cdot f'_c$, $s_{max}$ = maximum of: *(AASHTO LRFD Eq.5.8.2.7-2)*

$0.4 \cdot d_v = 16\ in$ & 12in.

Since $v_u < 0.125 \cdot f'_c$, $s_{max}$=24.0 in.

*TxDOT limits the maximum transverse reinforcement spacing to 12,"therefore:* *(BDM-LRFD, Ch.4, Sec.4, Detailling)*

$s_{max} := 12.00\ in$

$s = 8.5\ in \quad < \quad s_{max}$

**SpacingCheck** := **if** $(s < s_{max}$, “Okay"”, “Not okay”)

**SpacingCheck** = “Okay"”

*Shear capacity and checks should be repeated at ALL points of critical shear. Note: in the overhangs, the stirrups need to be spaced @ 5in because shear is higher. Similarly the stirrups need to be spaced @ 5in near the center column. When the spacing needed is less than 4in, use double stirrups. (BDM-LRFD, Ch. 4, Sec. 4, Detailing) When using double stirrups,* $A_v$ *is four times the stirrups bar's area.*

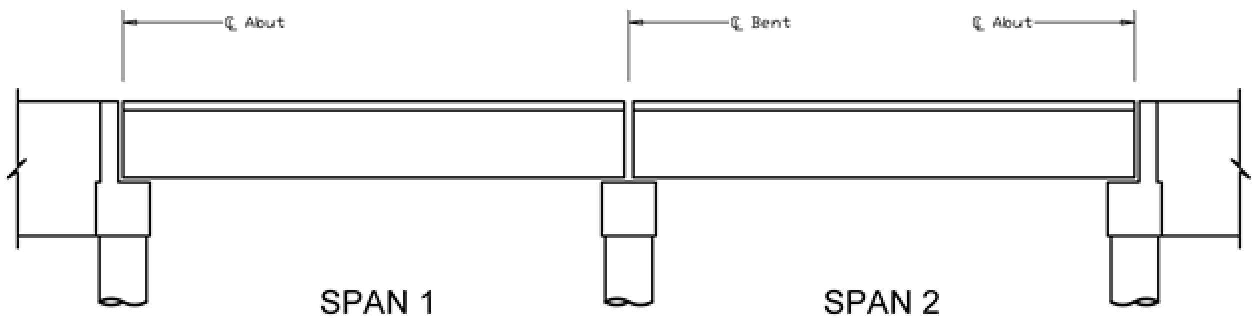
# APPENDIX G: DESIGN EXAMPLE #5

Date: February, 2018

Rectangular Pretensioned Bent Cap Design Example, with an Eccentric Strand Layout and Interior Void

This design example is in accordance with the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012) and TxDOT Bridge Design Manual - LRFD (October 2015)

# Design Parameters




## Span 1

*"AASHTO LRFD" refers to the AASHTO LRFD Bridge Design Specifications, 6th Ed. (2012)*

*"BDM-LRFD" refers to the TxDOT Bridge Design Manual - LRFD (October 2015)*

*"TxSP" refers to TxDOT guidance, recommendations, and standard practice*

100' Type Tx62 Girders ($0.948\frac{k}{ft}$)

10 Girders Spaced @ 8.50' with 3' deck overhangs, with middle two girders spaced @ 8'

## Span 2

100' Type Tx54 Girders ($0.948\frac{k}{ft}$)

10 Girders Spaced @ 8.50' with 3' deck overhangs, with middle two girders spaced @ 8'

## All Spans

Deck is 40' wide

Type T551 Rail ($0.382\frac{k}{ft}$)

Type SSCB(1) Median Barrier ( $0.717\ \frac{k}{ft}$)

8.5" Thick Slab (0.100 ksf) *(TxSP)*

Assume 2" Overlay @ 140 pcf (0.023 ksf)

## Assume

4'-0" X 4'-0" Cap

4~42" Columns Spaced @ 22'-0"

Cap will be modeled as a continuous beam with simple supports using TxDOT's CAP18 program.

*TxDOT does not consider frame action for typical multi-column Rectangular Reinforced Concrete Bents. The same methodology is applied to the structural analysis of multi-column Rectangular Pretensioned Concrete Bents. (BDM-LRDSFD, Ch. 4, Sect. 4, Structural Analysis)*

## Design Parameters (Con't)

### Define Variables

| Back Span | Forward Span | |
|---|---|---|
| $Span1 := 100 \cdot ft$ | $Span2 := 100 \cdot ft$ | *Span Length* |
| $GdrSpa1 := 8.5 \cdot ft$ | $GdrSpa2 := 8.5 \cdot ft$ | *Girder Spacing* |
| $GdrNo1 := 10$ | $GdrNo2 := 10$ | *Number of Girders in Span* |
| $GdrWt1 := 0.948 \cdot \frac{kip}{ft}$ | $GdrWt2 := 0.948 \cdot \frac{kip}{ft}$ | *Weight of Girder* |

Bridge

$RailWt := 0.382 \cdot \frac{kip}{ft}$ — *Weight of Rail*

$MedianWt := 0.717 \cdot \frac{kip}{ft}$ — *Weight of Median Barrier*

$SlabThk := 8.5 \cdot in$ — *Thickness of Bridge Slab*

$OverlayThk := 2 \cdot in$ — *Thickness of Overlay*

$w_c := 150 \cdot pcf$ — *Unit Weight of Concrete for Load Calcs*

$w_{olay} := 140 \cdot pcf$ — *Unit Weight of Overlay*

Other Variables:

$station := 0.5 \cdot ft$ — *Station Increment for CAP18 Analysis*

$IM := 33\%$ — *Dynamic Load Allowance, (AASHTO LRFD Table 3.6.2.1-1)*

Cap Dimensions:

$CapWidth := 48 \cdot in$

$CapDepth := 48 \cdot in$

$cover := 4 \cdot in$ — *Measured from Center of Prestressing Strand*

Material Properties:

$f'_c := 5 \cdot ksi$ — *Assumed Concrete Compressive Strength*

$f_{pu} := 270 \cdot ksi$ — *Ultimate Strength of Prestressing Steel*

## Design Parameters (Con't)

### Define Variable (Con't)

$A_{strand} := 0.217 \cdot in^2$ — *Area of Prestressing Strand*

$w_{cE} := 145 \cdot pcf$ — *Unit Weight of Concrete for $E_c$ Calc*

$$E_c := 33000 \cdot \left(\frac{w_{cE}}{1000 \cdot pcf}\right)^{1.5} \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi = 4074\ ksi$$

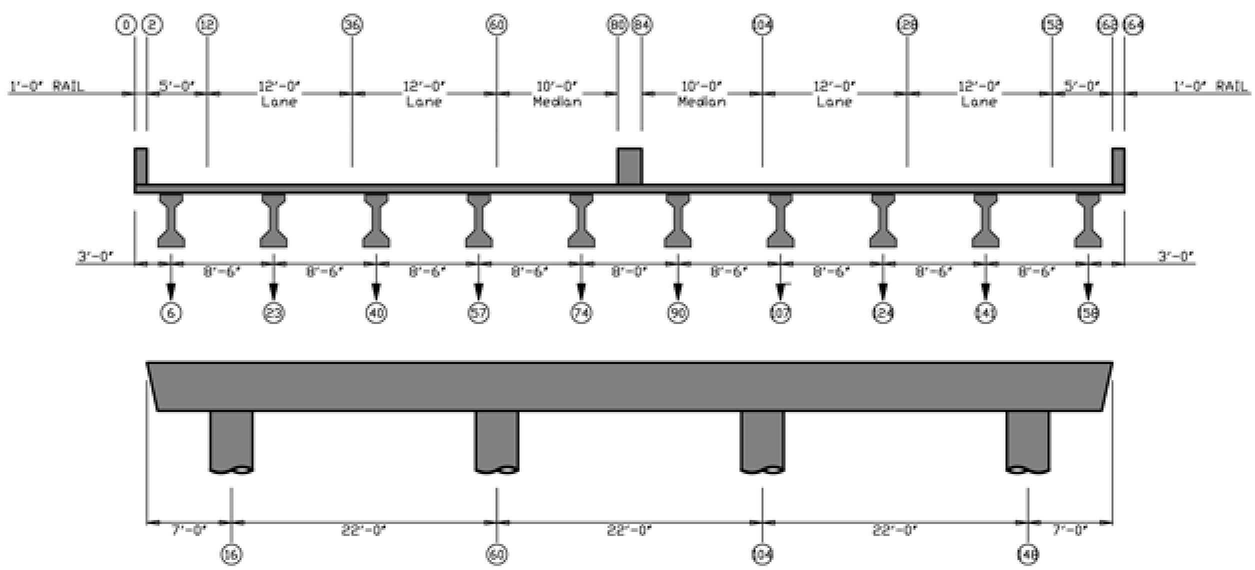
*Modulus of Elasticity of Concrete, (AASHTO LRFD Eq. 5.4.2.4-1)*

$E_s := 29000 \cdot ksi$ — *Modulus of Elasticity of Mild Steel*

$E_p := 28500 \cdot ksi$ — *Modulus of Elasticity of Prestressing Steel*

# Cap Analysis

### Cap Model




*The circled numbers are the stations that are used for the CAP18 Input file. One station is 0.5ft in the direction perpendicular to the pgl.*

## Cap Analysis (Con't)

**Dead Load**

Span 1

$Rail1 := \frac{(2 \cdot RailWt + MedianWt) \cdot \frac{Span1}{2}}{min(GdrNo1, 9)}$ $Rail1 = 8.23\ kip$ *Rail weight is distributed evenly among stringers, up to 3 stringers per rail. (TxSP)*

$Slab1 := w_c \cdot GdrSpa1 \cdot SlabThk \cdot \frac{Span1}{2} \cdot 1.1$ $Slab1 = 49.67\ kip$ *Slab DL is increased by 10% to account for haunch and thickened slab ends.*

$Girder1 := GdrWt1 \cdot \frac{Span1}{2}$ $Girder1 = 47.4\ kip$ *Weight of girder acting on bent*

$DLRxn1 := Rail1 + Slab1 + Girder1$ $DLRxn1 = 105.3\ kip$ *Dead load reaction per girder, not considering overlay. (Overlay is calculated separately due to possibility of applying a different load factor)*

$Overlay1 := w_{olay} \cdot GdrSpa1 \cdot OverlayThk \cdot \frac{Span1}{2}$ $Overlay1 = 9.92\ kip$ *Design for future overlay, per girder*

Span 2

$Rail2 := \frac{(2 \cdot RailWt + MedianWt) \cdot \frac{Span2}{2}}{min(GdrNo2, 9)}$ $Rail2 = 8.23\ kip$

$Slab2 := w_c \cdot GdrSpa2 \cdot SlabThk \cdot \frac{Span2}{2} \cdot 1.1$ $Slab2 = 49.67\ kip$

$Girder2 := GdrWt2 \cdot \frac{Span2}{2}$ $Girder2 = 47.4\ kip$

$DLRxn2 := Rail2 + Slab2 + Girder2$ $DLRxn2 = 105.3\ kip$

$Overlay2 := w_{olay} \cdot GdrSpa2 \cdot OverlayThk \cdot \frac{Span2}{2}$ $Overlay2 = 9.92\ kip$

Cap

$A_g := CapWidth \cdot CapDepth$ $A_g = 2304\ in^2$ *Gross Area of Cap*

$Cap := w_c \cdot A_g \cdot station$ $Cap = 1.2\ kip$ *Dead Load of Cap, per station*

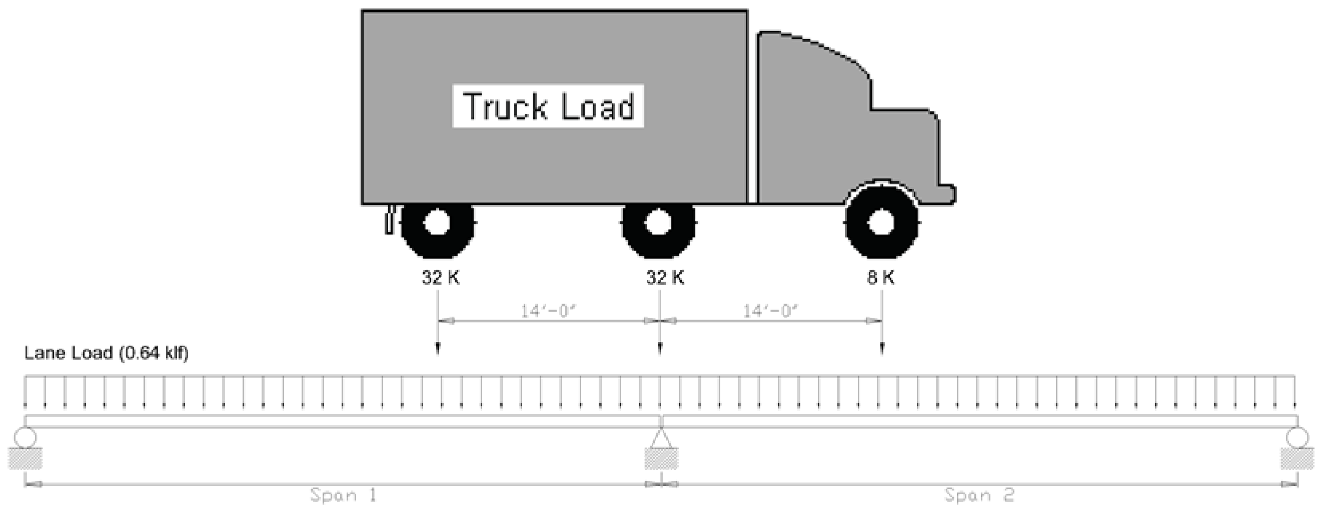
$I_g := \frac{1}{12} \cdot CapWidth \cdot CapDepth^3$ $I_g = (4.42 \cdot 10^5)\ in^4$ *Gross Moment of Inertia of Cap*

$E_c = 4074\ ksi$ $E_c \cdot I_g = (1.25 \cdot 10^7)\ kip \cdot ft^2$ *Bending Stiffness of Cap*

$S_x := \frac{I_g}{\frac{CapDepth}{2}}$ $S_x = (1.84 \cdot 10^4)\ in^3$

## Cap Analysis (Con't)

**Live Load** *(AASHTO LRFD 3.6.1.2.2 and 3.6.1.2.4)*




Live Load Model

$\text{LongSpan} := \max(\text{Span1}, \text{Span2})$

$\text{ShortSpan} := \min(\text{Span1}, \text{Span2})$

$\text{LongSpan} = 100\ ft$

$\text{ShortSpan} = 100\ ft$

$\text{IM} = 0.33$

*Use HL-93 Live Load. Maximum reaction at an interior bent, the Design Truck" will govern over Design Tandem." With the Long Span less than twice as long as the Short Span, the maximum reaction occurs when the middle axle (32 kip) is placed over the support, the front axle (8 kips) is placed on the Short Span, and the rear axle (32 kips) is placed on the Long Span.*

$$\text{Lane} := 0.64 \cdot klf \cdot \left(\frac{\text{LongSpan} + \text{ShortSpan}}{2}\right)$$

$\text{Lane} = 64\ kip$

$$\text{Truck} := 32 \cdot kip + 32 \cdot kip \cdot \left(\frac{\text{LongSpan} - 14 \cdot ft}{\text{LongSpan}}\right) + 8 \cdot kip \cdot \left(\frac{\text{ShortSpan} - 14 \cdot ft}{\text{ShortSpan}}\right)$$

$\text{Truck} = 66.4\ kip$

*Combine Design Truck" and Design Lane" loadings.*
*(AASHTO LRFD 3.6.1.3)*

$\text{LLRxn} := \text{Lane} + \text{Truck} \cdot (1 + \text{IM})$

$\text{LLRxn} = 152.31\ kip$

*Dynamic Load Allowance (IM) does not apply to the Design Lane"*
*(AASHTO LRFD 3.6.1.2.4)*

$P := 16.0 \cdot kip \cdot (1 + \text{IM})$

$P = 21.28\ kip$

*Live Load is applied to the deck slab by two 16 kip wheel loads increased by IM, with the remainder of the live load distributed over a 10ft design lane width*
*(AASHTO LRFD 3.6.1.2.1) (TxSP)*

$$W := \frac{\text{LLRxn} - (2 \cdot P)}{10 \cdot ft} \cdot \text{station}$$

$W = 5.49\ kip$

*Live Load applied to the slab is distributed to the beams assuming the slab is hinged at each beam, except the outside beam.*
*(BDM-LRFD, Ch. 4, Sect. 4, Structural Analysis)*

# Cap Analysis (Con't)

## Cap18 Input

Multiple Presence Factors, m *(AASHTO LRFD Table 3.6.1.1.2-1)*

| No. of Lanes | Factor "m" |
|---|---|
| 1 | 1.20 |
| 2 | 1.00 |
| 3 | 0.85 |
| >3 | 0.65 |

Limit States *(AASHTO LRFD 3.4.1)*

*The cap design only needs to consider Strength I, Service I, and Service I (Dead Load Only)*

Strength I

| | |
|---|---|
| Live Load and Dynamic Load Allowance | LL + IM = 1.75 |
| Dead Load Components | DC = 1.25 |
| Dead Load Wearing Surface (Overlay) | DW = 1.50 |

*TxDOT allows Overlay Factor to be reduced to 1.25 (TxSP).*

Service I

| | |
|---|---|
| Live Load and Dynamic Load Allowance | LL + IM = 1.00 |
| Dead Load and Wearing Surface | DC & DW = 1.00 |

## Cap18 Output

| | Max +M | Max -M |
|---|---|---|
| Dead Load | $M_{DLpos} := 748.9 \cdot kip \cdot ft$ | $M_{DLneg} := 1211.0 \cdot kip \cdot ft$ |
| Service Load | $M_{SLpos} := 1427.6 \cdot kip \cdot ft$ | $M_{SLneg} := 1910.0 \cdot kip \cdot ft$ |
| Ultimate Load | $M_{ULpos} := 2123.8 \cdot kip \cdot ft$ | $M_{ULneg} := 2737.0 \cdot kip \cdot ft$ |

*Maximum loads from the CAP18 Output file.*

$M_{DLavg} := \text{mean}(M_{DLpos}, M_{DLneg})$ $\quad M_{DLavg} = 980\ kip \cdot ft$ $\quad \Delta M_{DL} := M_{DLavg} - M_{DLneg}$

$M_{SLmax} := \max(M_{SLpos}, M_{SLneg})$ $\quad M_{SLmax} = 1910\ kip \cdot ft$ $\quad \Delta M_{DL} = -231.05\ kip \cdot ft$

$M_{ULmax} := \max(M_{ULpos}, M_{ULneg})$ $\quad M_{ULmax} = 2737\ kip \cdot ft$

# Flexural Design

*The flexural design of the bent cap is based on the philosophy of "Zero Tension Under Dead Load." The design follows the following steps:*

- *Design for Zero Flexural Tension under Dead Load*
- *Determine Minimum Concrete Compressive Strength and Check Stresses at Service Loads*
- *Check the Minimum Number of Strands*
- *Check the Ultimate Strength Capacity*
- *Check that Minimum Capacity is satisfied*

## Define Constants and Variables

$B := CapWidth$ $D := CapDepth$ $Dia_{pipe} := 24\ in$

$f_r := 0.24 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi$ $f_r = 0.54\ ksi$ *Modulus of Rupture (AASHTO LRFD Eq. 5.4.2.4-1)*

$jd := 0.45 \cdot CapDepth$ $jd = 21.6\ in$ *Approximate Moment Arm between Tension and Compression section to estimate nominal moment capacity for determination of Minimum Number of Strands*

$f_y := 0.9 \cdot f_{pu}$ $f_y = 243\ ksi$ *Yield Strength of Prestressing Steel (AASHTO LRFD Table 5.4.4.1-1)*

$\Delta_{f.pt} := 0.2$ *Assumed prestress loss in pretensioned members*

$T_{strand} := 0.75 \cdot f_{pu} \cdot A_{strand} \cdot (1 - \Delta_{f.pt})$ $T_{strand} = 35.15\ kip$

$I_{pos} := \frac{B \cdot D^3}{12}$ $I_{pos} = 442368\ in^4$ *Moment of Inertia of the section at the positive moment region*

$I_{neg} := \frac{B \cdot D^3}{12} - \frac{Dia_{pipe} \cdot D^3}{12}$ $I_{neg} = 221184\ in^4$ *Moment of Inertia of the hollow section at the negative moment region*

$S_{pos} := \frac{I_{pos}}{\left(\frac{D}{2}\right)}$ $S_{pos} = 18432\ in^3$ *Section Modulus of Rectangular Section at the positive moment region*

$S_{neg} := \frac{I_{neg}}{\left(\frac{D}{2}\right)}$ $S_{neg} = 9216\ in^3$ *Section Modulus of hollow connection section at the negative moment region*

$A_{pos} := B \cdot D$ $A_{pos} = 2304\ in^2$

$A_{neg} := (B - Dia_{pipe}) \cdot D$ $A_{neg} = 1152\ in^2$

## Flexural Design (Con't)

## Design for Zero Flexural Tension Under Dead Load

*For this eccentric design, the average absolute maximum moments will be used to determine a number of strands required to satisfy the Zero Tension Under Dead Load concept. A required eccentricity to balance the remaining negative moment will be determined.*

Tension Limit:

$$\frac{-F_t}{A} + \frac{M_{DL}}{S_x} = f_t = 0$$

*Set the stress at the extreme tension fiber to zero*

$$F_{t\_neg} := \frac{M_{DLavg} \cdot A_{neg}}{S_{neg}} \qquad F_{t\_neg} = \left(1.5 \cdot 10^3\right)\ kip$$

*Determine the prestressing force required to achieve zero tension stress in the negative moment region*

$$F_{t\_pos} := \frac{M_{DLavg} \cdot A_{pos}}{S_{pos}} \qquad F_{t\_pos} = \left(1.47 \cdot 10^3\right)\ kip$$

*Determine the prestressing force required to achieve zero tension stress in the positive moment region*

$$F_t := \max\left(F_{t\_neg}, F_{t\_pos}\right) \qquad F_t = 1469925\ lbf$$

$$n_{flex\_t} := \mathrm{Ceil}\left(\left(\frac{F_t}{T_{strand}}\right), 4\right) \qquad n_{flex\_t} = 44$$

$$e_o := \frac{\Delta M_{DL}}{n_{flex\_t} \cdot T_{strand}} \qquad e_o = -1.79\ in$$

*Eccentricity required to balance the remaing negative moment*

Compression Limit:

$$\frac{-F}{A} + \left(\frac{F_c \cdot e_o}{S_x}\right) - \frac{M_{DL}}{S_x} \geq f_c = -0.45 \cdot f'_c$$

*Set the stress at the extreme compression fiber to the compressive stress limit (AASHTO LRFD Table 5.9.4.2.1-1*

$$F_{c\_neg} := \frac{A_g \cdot \left(-0.45 \cdot f'_c + \frac{M_{DLneg}}{S_x}\right)}{\left(\frac{e_o \cdot A_g}{S_x}\right) - 1} \qquad F_{c\_neg} = 2751.1\ kip$$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load, with the previously determined eccentricityat the negative moment region*

$$F_{c\_pos} := \frac{A_{pos} \cdot \left(-0.45 \cdot f'_c + \frac{M_{DLpos}}{S_{pos}}\right)}{\left(\frac{e_o \cdot A_{pos}}{S_{pos}}\right) - 1} \qquad F_{c\_pos} = 3317.4\ kip$$

*Determine the prestressing force required to achieve the compressive stress limit under Dead Load, with the previously determined eccentricity*

$$F_c := min\left(F_{c\_neg}, F_{c\_pos}\right) \qquad F_c = 2751.1\ kip$$

## Flexural Design (Con't)

$n_{flex\_c} := \text{Floor}\left(\left(\frac{F_c}{T_{strand}}\right), 2\right)$ $n_{flex\_c} = 78$

*Use the calculated $F_c$ to determine the corresponding number of strands required to reach the limiting compressive stress*

$n := min\left(n_{flex\_t}, n_{flex\_c}\right)$ $n = 44$

*The number of strands selected for the should be between $n_{flec\_t}$ and $n_{flex\_c}$*

$F_{provided} := n \cdot T_{strand}$ $F_{provided} = 1546.8\ \textit{kip}$

*Determine the provided prestressing force from the selected number of strands*

## Flexural Design (Con't)

## Check Minimum Concrete Compressive Strength with Service Stress

Tension Stress:

$$\frac{-F}{A}+\frac{F\cdot e_o}{S_x}+\frac{M_{SL}}{S_x}\geq f_t = k\cdot\sqrt{f'_c}$$

*The stress at the extreme tension fiber must not exceed the service stress limit, which is* $k\cdot\sqrt{f'_c}$ . *(AASHTO LRFD Table 5.9.4.2.2-1)*

*Values of* $k$ *are different for various corrosion conditions. For AASHTO LRFD 5.9.4.2.2,* $k$ *values are:*

- *Moderate Exposure (Class I)* $k := 0.19$
- *Severe Exposure (Class II)* $k := 0.0948$

*Recommended value of* $k$ *to limit cracking under service loads was shown to be smaller than the value imposed by AASHTO. This value is* $k := 0.126$

*The tensile stresses under service conditions should not exceed values specified by AASHTO LRFD 5.9.4.2.2. However, to further reduce cracking under service conditions in Class I conditions, the tensile stresses should not exceed the recommended value concluded from experimental testing.*

*For this design example, the AASHTO value of* $k$ *for Class I exposure will be used to check the service level stresses and compute the minimum concrete compressive strength.*

*The assumed minimum* $f'_c$ *value was lower than the required minimum concrete compressive strength determined from the service stress limits. Therefore, the* $f'_c$ *was increased to the maximum allowed by BDM LRFD Chapter 4. Even with the increased concrete compressive strength, the initial values for* $n$ *and* $e_o$ *did not pass service stress checks. Therefore, the following geometry was chosen to satisify service stress checks:*

$n := 56$ $\qquad e_o := -1.19\cdot in$ $\qquad F_{provided} := n\cdot T_{strand} = 1968.6\ kip$ $\qquad f'_c := 8.5\cdot ksi$

Negative Moment Region:

Tension:

$k := 0.19$

*Multiplier for* $k\cdot\sqrt{f'_c}$ *tensile stress limit*

$$f_{t\_sl\_neg} := -\left(\frac{F_{provided}}{A_{neg}}\right)+\left(\frac{F_{provided}\cdot e_o}{S_{neg}}\right)+\frac{M_{SLneg}}{S_{neg}}$$

*Tensile Stress at extreme tension fiber under Service Loads*

$f_{t\_sl\_neg} = 0.524\ ksi$

$$f_{t\_slim\_neg} := k\cdot\sqrt{\frac{f'_c}{ksi}}\cdot ksi$$

$f_{t\_slim\_neg} = 0.554\ ksi$

*Limiting tensile stress at the extreme tension fiber*

$TenLimit := \mathbf{if}\left(f_{t\_sl\_neg} \leq f_{t\_slim\_neg}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$TenLimit = \text{“GOOD”}$

## Flexural Design (Con't)

### Check Minimum Concrete Compressive Strength with Service Stress (Con't)

Compression:

*The stress at the extreme compression fiber must not exceed the service stress limit. Considering the connection section as solid in compression (AASHTO LRFD Table 5.9.4.2.1-1)*

$$\frac{-F}{A} - \frac{M_{SL}}{S_x} > f_c = -0.45 \cdot f'_c$$

$$f_{c\_sl\_neg} := \left(\frac{-F_{provided}}{A_g}\right) - \left(\frac{F_{provided} \cdot e_o}{S_x}\right) - \left(\frac{M_{SLneg}}{S_x}\right)$$

*Compressive Stress at extreme compression fiber under Service Loads*

$$f_{c\_sl\_neg} = -1.971 \ ksi$$

$$f_{c\_slim\_neg} := -0.45 \cdot f'_c \qquad f_{c\_slim\_neg} = -3.825 \ ksi$$

*Limiting compressive stress at the extreme compression fiber*

CompLimit := **if** $(f_{c\_sl\_neg} \ge f_{c\_slim\_neg}$, "GOOD", "NOT GOOD")

CompLimit = "GOOD"

Positive Moment Region:

Tension:

$$f_{t\_sl\_pos} := -\left(\frac{F_{provided}}{A_{pos}}\right) - \left(\frac{F_{provided} \cdot e_o}{S_{pos}}\right) + \frac{M_{SLpos}}{S_{pos}}$$

*Tensile Stress at extreme tension fiber under Service Loads*

$$f_{t\_sl\_pos} = 0.202 \ ksi$$

$$f_{t\_slim\_pos} := k \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \qquad f_{t\_slim\_pos} = 0.554 \ ksi$$

*Limiting tensile stress at the extreme tension fiber*

TenLimit := **if** $(f_{t\_sl\_pos} \le f_{t\_slim\_pos}$, "GOOD", "NOT GOOD")

TenLimit = "GOOD"

Compression:

*The stress at the extreme compression fiber must not exceed the service stress limit. (AASHTO LRFD Table 5.9.4.2.1-1)*

$$\frac{-F}{A} - \frac{M_{SL}}{S_x} > f_c = -0.45 \cdot f'_c$$

$$f_{c\_sl\_pos} := \left(\frac{-F_{provided}}{A_{pos}}\right) + \left(\frac{F_{provided} \cdot e_o}{S_{pos}}\right) - \left(\frac{M_{SLpos}}{S_{pos}}\right)$$

*Compressive Stress at extreme compression fiber under Service Loads*

$$f_{c\_sl\_pos} = -1.911 \ ksi$$

$$f_{c\_slim\_pos} := -0.45 \cdot f'_c \qquad f_{c\_slim\_pos} = -3.825 \ ksi$$

*Limiting compressive stress at the extreme compression fiber*

CompLimit := **if** $(f_{c\_sl\_pos} \ge f_{c\_slim\_pos}$, "GOOD", "NOT GOOD")

CompLimit = "GOOD"

## Flexural Design (Con't)

### Check Minimum Concrete Compressive Strength with Service Stress (Con't)

Minimum Concrete Strength:

$$f'_{c\_tmin} := \max\left(\left(\frac{f_{t\_sl\_neg}}{k \cdot ksi}\right)^2, \left(\frac{f_{t\_sl\_pos}}{k \cdot ksi}\right)^2\right) \cdot ksi$$

$f'_{c\_tmin} = 7.6\ ksi$

*Determine minimum concrete compressive strength to achieve the limiting tensile stress*

$$f'_{c\_cmin} := \frac{min\left(f_{c\_sl\_neg}, f_{c\_sl\_pos}\right)}{-0.45}$$

$f'_{c\_cmin} = 4.38\ ksi$

*Determine minimum concrete compressive strength to achieve the limiting compressive stress*

$$f'_{c\_min} := \max\left(f'_{c\_cmin}, f'_{c\_tmin}, 5 \cdot ksi\right)$$

$f'_{c\_min} = 7.6\ ksi$

*Minimum Concrete Compressive Strength. If calculated values are significantly low, use a practical minimum achievable concrete strength (consider $5 \cdot ksi$ for precast, prestressed concrete)*

*If the required Minimum Concrete Compressive strength is larger than the maximum allowed design strength from Chapter 4 of BDM-LRFD, adjust the number of strands and/or strand layout to reduce the service stresses.*

*If the calculated Minimum Concrete Compressive strength is larger than the assumed strength, recheck stress limits with the "new" compressive strength.*

## Flexural Design (Con't)

## Determine Minimum Number of Strands

*AASHTO LRFD 5.7.3.3.2 specifies that the factored flexural resistance* $M_r$ *should be at least greater than the lesser of* $M_{cr}$ *or* $1.33 \cdot M_u$ *. The derivation of the formula to determine* $n_{min}$ *is shown:*

$$-\left(\frac{F}{A_g}\right)+\left(\frac{M_{cr}}{S_x}\right) = f_r$$

*Stress at extreme tension fiber is equal to the modulus of rupture*

$$M_{cr} = \gamma_3\left(\gamma_1 \cdot f_r + \gamma_2\left(\frac{F}{A}\right)\right) \cdot S_x \qquad \gamma_1 := 1.6 \quad \gamma_2 := 1.1 \quad \gamma_3 := 1.0$$

*Where* $F$ *is the prestressing force*

$$\phi M_n = M_r \geq M_{cr} \text{ , where } \phi := 1.0$$

*(AASHTO LRFD 5.5.4.2.1)*

*Approximate nominal moment capacity*

$$M_n \geq n \cdot A_{ps} \cdot f_y \cdot jd$$

Substituting, simplifying, and equating $M_{cr}$ and $M_n$ :

$$\gamma_1 f_r \cdot S_x + \gamma_2 \cdot \left(n \cdot T_{strand}\right) \frac{S_x}{A} = n \cdot A_{ps} \cdot f_y \cdot jd$$

Thus:

$$n = \frac{\gamma_1 \cdot f_r \cdot S_x}{A_{ps} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_x}{A}}$$

Also $M_r \geq 1.33 \cdot M_{UL}$

$$n = \frac{1.33 \cdot M_{UL}}{A_{ps} \cdot f_y \cdot jd}$$

$$n_{min\_pos} := \text{ceil}\left(\max\left(\frac{\gamma_1 \cdot f_r \cdot S_{pos}}{A_{strand} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_{pos}}{A_{pos}}}, \frac{1.33 \cdot M_{ULpos}}{A_{strand} \cdot f_y \cdot jd}\right)\right)$$

*$n$ must be checked for both positive and negative bending regions, as the cross-sections are not the same.*

$$n_{min\_pos} = 30$$

$$n_{min\_neg} := \text{ceil}\left(\max\left(\frac{\gamma_1 \cdot f_r \cdot S_{neg}}{A_{strand} \cdot f_y \cdot jd - \dfrac{\gamma_2 \cdot T_{strand} \cdot S_{neg}}{A_{neg}}}, \frac{1.33 \cdot M_{ULneg}}{A_{strand} \cdot f_y \cdot jd}\right)\right)$$

$$n_{min\_neg} = 39$$

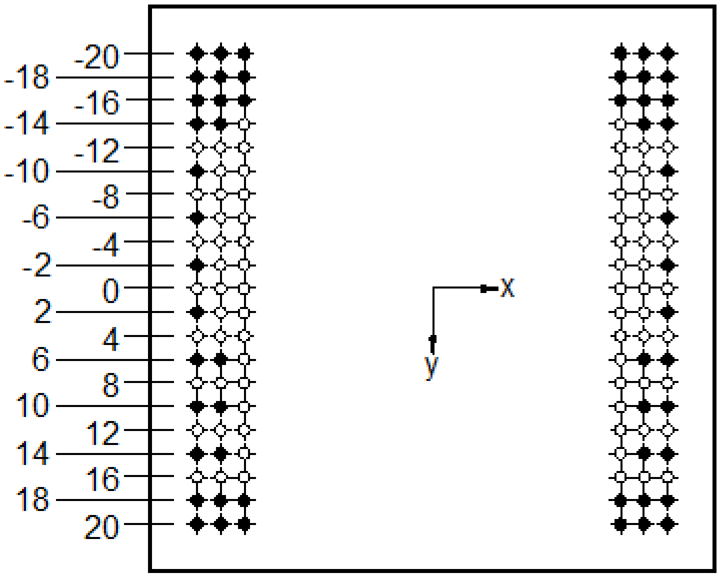
$$n_{min} := \max\left(n_{min\_pos}, n_{min\_neg}\right) \qquad n_{min} = 39$$

$$n_{min\_check} := \textbf{if}\left(n \geq n_{min}, \text{“GOOD”}, \text{“NOT GOOD”}\right) \qquad n_{min\_check} = \text{“GOOD”}$$

## Flexural Design (Con't)

## Check Ultimate Strength Capacity

Determine strand configuration




*The strand layout is limited based on the configuration of the cap-to-column connection and the use of an interior void. For this example, the cap-to-column connection is assumed to be formed by a 24-inch nominal diameter pocket connection.*

**Define Variables**

$$\Delta_{\varepsilon p} := \frac{T_{strand}}{E_p \cdot A_{strand}} \qquad \Delta_{\varepsilon p} = 0.0057$$

*Pre-strain, after losses*

$$\beta := \max\left(\left(0.85 - \left(\frac{f'_c}{ksi} - 4\right) \cdot 0.05\right), 0.65\right) \qquad \beta = 0.65$$

*(AASHTO LRFD 5.7.2.2)*

$$\varepsilon_{cu} := 0.003$$

*Maximum strain at extreme compression fiber (AASHTO LRFD 5.7.2.1)*

$$\phi := 1.0$$

*Strength Reduction Factor (AASHTO LRFD 5.5.4.2.1)*

$$Q := 0.03$$

$$R := 6$$

*Q and R are constants in the "Menegotto-Pinto" equation used to determine the stress at $i^{th}$ layer of prestressing steel. Since a side configuration layout of prestressing steel was used instead of the conventional top & bottom layout, the stresses in the prestressing steel must be determined at each individual layer.*

$$c_i := -15 \cdot in$$

*Initial location of neutral axis used in the iterative solution of determining the moment capacity.*

## Flexural Design (Con't)

**Calculate Strain and Stress in Each Steel Layer**

$$\varepsilon_{ti} = \varepsilon_{cu} \cdot \left( \frac{d_i - c}{\frac{D}{2} + c} \right)$$

*Tension strain at the $i^{th}$ layer. $d_i$ is the depth of the prestressing layer, as shown in the strand layout (note the convention and origin of distance measurements)*

$$\varepsilon_{si} = \varepsilon_{ti} + \Delta_{\varepsilon p}$$

*Total strain on each layer, considering the pre-strain*

$$f_{psi} = E_p \cdot \varepsilon_{si} \cdot \left( Q + \frac{1 - Q}{\left( 1 + \left( \left| \frac{\varepsilon_{si} \cdot E_s}{f_y} \right| \right)^R \right)^{\frac{1}{R}}} \right)$$

*Menegotto-Pinto equation to determine the stress in the $i^{th}$ layer*

$$T_i = f_{psi} \cdot A_{psi}$$

*Tension force in the $i^{th}$ layer of steel. $A_{psi}$ is the area of prestressing steel in that layer.*

$$jd_i = \frac{D}{2} - d_i - \frac{a}{2}$$

*Moment arm between compressive stress block and the $i^{th}$ layer of prestressing steel*

'

$$a = \frac{-D}{2} + \left( \frac{-\beta}{2} \cdot \left( \frac{-D}{2} - c \right) \right)$$

*Depth of the equivalent compression block, with respect to the center of the bent cap*

$$M_i = T_i \cdot j \cdot d_i$$

*Moment in the $i^{th}$ layer*

$$C_c = -0.85 \cdot f'_c \cdot \beta \cdot \left( \frac{D}{2} - c \right) \cdot B$$

*Compressive force from the equivalent compressive stress block*

*The previous equations are calculated using the $c_i$ value, and iterated with changing values of $c$ until the sum of the tensile forces equals the magnitude of the compressive force:*

$$\Sigma T_t = \left| C_c \right|$$

*This process is completed for both Positive and Negative Bending in Microsoft Excel, and the results are presented in the following tables*

## Flexural Design (Con't)

**Positive Moment Capacity:**

| $d_i$ (in) | $n_i$ | $\varepsilon_{ti}$ | $\Delta\varepsilon_p$ | $\varepsilon_{si}$ | $f_{psi}$ (ksi) | $T_i$ (kips) | $jd_i$ (in) | $M_i$ (k-in) |
|---|---|---|---|---|---|---|---|---|
| -20 | 6 | -0.00181 | 0.00568 | 0.00387 | 110.12 | 143.38 | 0.71 | 101.7 |
| -18 | 6 | -0.00122 | 0.00568 | 0.00446 | 126.75 | 165.03 | 2.71 | 447.2 |
| -16 | 6 | -0.00063 | 0.00568 | 0.00505 | 143.07 | 186.28 | 4.71 | 877.3 |
| -14 | 4 | -0.00004 | 0.00568 | 0.00565 | 158.85 | 137.89 | 6.71 | 925.1 |
| -10 | 2 | 0.00115 | 0.00568 | 0.00683 | 187.48 | 81.36 | 10.71 | 871.4 |
| -6 | 2 | 0.00233 | 0.00568 | 0.00802 | 209.92 | 91.10 | 14.71 | 1340.1 |
| -2 | 2 | 0.00352 | 0.00568 | 0.00920 | 225.09 | 97.69 | 18.71 | 1827.7 |
| 2 | 2 | 0.00470 | 0.00568 | 0.01039 | 234.34 | 101.70 | 22.71 | 2309.7 |
| 6 | 4 | 0.00589 | 0.00568 | 0.01157 | 239.85 | 208.19 | 26.71 | 5560.7 |
| 10 | 4 | 0.00707 | 0.00568 | 0.01276 | 243.29 | 211.18 | 30.71 | 6485.1 |
| 14 | 4 | 0.00826 | 0.00568 | 0.01394 | 245.64 | 213.21 | 34.71 | 7400.6 |
| 18 | 6 | 0.00945 | 0.00568 | 0.01513 | 247.41 | 322.13 | 38.71 | 12469.4 |
| 20 | 6 | 0.01004 | 0.00568 | 0.01572 | 248.17 | 323.11 | 40.71 | 13153.8 |
| -1.19 | 54 | $C_c$ = | -2282.3 | kips | $\sum T_i$ = | 2282.3 | $\sum M_i$ = | 53769.8 |

$c := -13.88 \cdot in$ *Final location of N/A, from iterations*

$\Sigma M_i = 53769.82 \; kip \cdot in$ *Sum of $M_i$ from the iterations*

$M_n := \Sigma M_i$ $M_n = 4480.8 \; kip \cdot ft$

$M_{r\_pos} := \phi \cdot M_n$ $M_{r\_pos} = 4480.8 \; kip \cdot ft$ *Factored Flexural Resistance (AASHTO LRFD 5.7.3.2.1)*

$CapacityCheck := \textbf{if}\left(M_{r\_pos} \geq M_{ULpos}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$CapacityCheck = \text{“GOOD”}$

*Calculate the $M_{cr}$ and check if the $M_r$ meets AASHTO LRFD 5.7.3.3.2*

$\gamma_1 := 1.6$ $\gamma_2 := 1.1$ $\gamma_3 := 1.0$

$$f_{cpe} := \frac{F_{provided}}{A_{pos}} + \frac{F_{provided} \cdot e_o}{S_{pos}}$$ $f_{cpe} = 0.73 \; ksi$

$M_{cr\_pos} := \gamma_3 \cdot \left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{pos}\right)$ $M_{cr\_pos} = 2547.8 \; kip \cdot ft$

$M_{rCheck} := \textbf{if}\left(M_{r\_pos} \geq min\left(M_{cr\_pos}, 1.33 \cdot M_{ULpos}\right), \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$M_{rCheck} = \text{“GOOD”}$

## Flexural Design (Con't)

**Negative Moment Capacity:**

| di (in) | ni | εti | Δεp | εsi | fpsi (ksi) | Ti (kips) | jdi (in) | Mi (k-in) |
|---|---|---|---|---|---|---|---|---|
| -20 | 6 | -0.00186 | 0.00568 | 0.00383 | 108.98 | 141.89 | 0.59 | 84.2 |
| -18 | 6 | -0.00128 | 0.00568 | 0.00440 | 125.06 | 162.83 | 2.59 | 422.2 |
| -14 | 4 | -0.00014 | 0.00568 | 0.00555 | 156.21 | 135.59 | 6.59 | 894.0 |
| -10 | 4 | 0.00101 | 0.00568 | 0.00669 | 184.33 | 160.00 | 10.59 | 1694.9 |
| -6 | 4 | 0.00215 | 0.00568 | 0.00784 | 206.94 | 179.63 | 14.59 | 2621.3 |
| -2 | 2 | 0.00330 | 0.00568 | 0.00898 | 222.76 | 96.68 | 18.59 | 1797.5 |
| 2 | 2 | 0.00444 | 0.00568 | 0.01013 | 232.69 | 100.99 | 22.59 | 2281.6 |
| 6 | 2 | 0.00559 | 0.00568 | 0.01127 | 238.69 | 103.59 | 26.59 | 2754.8 |
| 10 | 2 | 0.00673 | 0.00568 | 0.01241 | 242.44 | 105.22 | 30.59 | 3218.9 |
| 14 | 4 | 0.00788 | 0.00568 | 0.01356 | 244.96 | 212.62 | 34.59 | 7355.3 |
| 16 | 6 | 0.00845 | 0.00568 | 0.01413 | 245.95 | 320.23 | 36.59 | 11718.1 |
| 18 | 6 | 0.00902 | 0.00568 | 0.01470 | 246.82 | 321.36 | 38.59 | 12402.3 |
| 20 | 6 | 0.00959 | 0.00568 | 0.01528 | 247.60 | 322.38 | 40.59 | 13086.4 |

$c := -13.52 \cdot in$ *Final location of N/A, from iterations*

$\Sigma M_i = 60331.6\ kip \cdot in$ *Sum of $M_i$ from the iterations*

$M_n := \Sigma M_i$ $M_n = 5027.6\ kip \cdot ft$

$M_{r\_neg} := \phi \cdot M_n$ $M_{r\_neg} = 5027.6\ kip \cdot ft$ *Factored Flexural Resistance (AASHTO LRFD 5.7.3.2.1)*

$CapacityCheck := \mathbf{if}\left(M_{r\_neg} \geq M_{ULneg}, \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$CapacityCheck = \text{“GOOD”}$

*Calculate the $M_{cr}$ and check if the $M_r$ meets AASHTO LRFD 5.7.3.3.2*

$\gamma_1 := 1.6$ $\gamma_2 := 1.1$ $\gamma_3 := 1.0$

$$f_{cpe} := \frac{F_{provided}}{A_{neg}} - \frac{F_{provided} \cdot e_o}{S_{neg}}$$ $f_{cpe} = 1.96\ ksi$

$M_{cr\_neg} := \gamma_3 \cdot \left(\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{neg}\right)$ $M_{cr\_neg} = 2317.8\ kip \cdot ft$

$M_{rCheck} := \mathbf{if}\left(M_{r\_neg} \geq min\left(M_{cr\_neg}, 1.33 \cdot M_{ULneg}\right), \text{“GOOD”}, \text{“NOT GOOD”}\right)$

$M_{rCheck} = \text{“GOOD”}$

# Design Summary

Concrete Strength:

$f'_c = 8.5\ ksi$

Prestressing:

$n = 56$

$F_{provided} = 1968.6\ \boldsymbol{kip}$

$e_o = -1.19\ \boldsymbol{in}$

# Shear Design

## Design Philosophy:

$V_u$ *(Ultimate Shear) must be less than* $V_r$ (Shear Resistance)

$V_u \le V_r$ — *(AASHTO LRFD Eq. 1.3.2.1-1)*

$V_r = \phi_v \cdot V_n$ — *(AASHTO LRFD Eq. 5.8.2.1-2)*

$\phi_v := 0.9$ — *Reduction factor (AASHTO LRFD 5.5.4.2.1)*

$V_n$ *is the lesser of* $V_{n1}$ and $V_{n2}$

*where*

$V_{n1} = 0.25 f_c\, b_v\, d_v + V_p$ — *(AASHTO LRFD Eq. 5.8.3.3-2)*

$V_{n2} = V_c + V_s + V_p$ — *(AASHTO LRFD Eq. 5.8.3.3-1)*

$V_c = 0.0316\, \beta \cdot \sqrt{f'_c} \cdot b_v \cdot d_v$ — *Shear resistance of the concrete (AASHTO LRFD Eq.5.8.3.3-3)*

$V_s = \dfrac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s}$ — *Shear resistance of the transverse steel (AASHTO LRFD Eq. C5.8.3.3-1)*

## Define Demands

*Since shear is dependent on location, let's look at STA 63:* — *At the interior face of the exterior column*

$V_{service} := 351.7 \cdot kip$ — *Shear demand at service*

$V_u := 489.4 \cdot kip$ — *Ultimate shear demand*

$M_u := -1515.4 \cdot kip \cdot ft$ — *Ultimate moment demand*

$N_u := 0 \cdot kip$ — *Ultimate axial force*

$V_p := 0\ kip$ — *Vertical component of the prestress force There is no vertical component of the prestressing force since straight strands are used*

## Shear Design (Con't)

### Define Variables

| | | |
|---|---|---|
| $f'_c = 8.5\ ksi$ | | *Depth of the bent cap* |
| $f_y := 60 \cdot ksi$ | | *Yield strength of mild Steel* |
| $f_{pu} = 270\ ksi$ | | *Tensile strength of prestressing steel* |
| $f_{po} := 0.7 \cdot f_{pu}$ | $f_{po} = 189\ ksi$ | *Parameter taken as modulus of elasticity of prestressing tendon* |
| $n = 56$ | | *Number of strand provided* |
| $h := CapDepth$ | $h = 48\ in$ | *Depth of the bent cap* |
| $b_v := CapWidth - D_{void}$ | $b_v = 48\ in$ | *Width of the bent cap* |
| $c := c + \frac{h}{2}$ | $c = 10.48\ in$ | *Neutral axis from the top extreme concrete* |
| $A_{ps} := \frac{n}{2} \cdot A_{strand}$ | $A_{ps} = 6.08\ in^2$ | *Area of strands in tension side* |
| $A_s := 0 \cdot in^2$ | | *Area of mild steel reinforcement* |
| $d_s := 0 \cdot in$ | | *Effective depth of mild steel reinforcement* |
| $d_p := \frac{D}{2} - e_o$ | $d_p = 25.19\ in$ | *Distance from extreme compression fiber to the centroid of the prestressing strands* |
| $k := 0.28$ | | *For low relaxation strand (AASHTO LRFD C5.7.3.1.1)* |
| $A_{ct} := \frac{h \cdot b_v}{2}$ | | *Area of concrete on the flexural tension side of the cap, from the extreme tension fiber to on half the cap depth.* |
| $f_{ps} := f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right)$ | $f_{ps} = 239\ ksi$ | *Average stress in prestressing steel (AASHTO LRFD Eq.5.7.3.1.1-1)* |
| $d_e := \frac{A_{ps} \cdot f_{ps} \cdot d_p + A_s \cdot f_y \cdot d_s}{A_{ps} \cdot f_{ps} + A_s \cdot f_y}$ | $d_e = 25.19\ in$ | *Effective depth from extreme compression fiber to the centroid of the tensile force (AASHTO LRFD Eq. 5.8.2.9-2)* |

## Shear Design (Con't)

### Check Cracking Shear

This step is recommended for the section with an interior void or thin web

*Shear demand at service load shall be less than* $V_{cr}$

$$V_{service} \le V_{cr}$$

Look at STA 63 where the interior void is located with large shear demand

$V_{service} = 351.7\ kip$ — *Shear demand at the interior face of the exterior column under service load*

$A_{pos} = 2304\ in^2$ — *Area of the hollow section at the positive Moment Region*

$I_{pos} = 442368\ in^4$ — *moment of inertia of the hollow section at the positive moment region*

$b_w := D - D_{void}$    $b_w = 48\ in$ — *Width of the hollow section*

$Q_{solid} := \dfrac{B \cdot D^2}{8}$    $Q_{solid} = 13824\ in^3$ — *First moment of area of the solid section*

$Q_{void} := \dfrac{D_{void}^{\ 3}}{8}$    $Q_{void} = 0\ in^3$ — *First moment of area of the void*

$Q_{pos} := Q_{solid} - Q_{void}$    $Q_{pos} = 13824\ in^3$ — *First moment of area of the voided section*

$f_t := 0.0632 \cdot \sqrt{\dfrac{f'_c}{ksi}} \cdot ksi$    $f_t = 0.18\ ksi$ — *Tensile strength of concrete for shear*

$T_{strand} = 35.15\ kip$ — *Tension force by single strand*

$n = 56$ — *Number of strand provided*

$V_{cr} := \dfrac{I_{pos} \cdot b_w}{Q_{pos}} \cdot \sqrt{\left(f_t\right)^2 + \left(\dfrac{f_t \cdot n \cdot T_{strand}}{A_{pos}}\right)}$ — *Cracking shear*

$V_{cr} = 671.97\ kip$

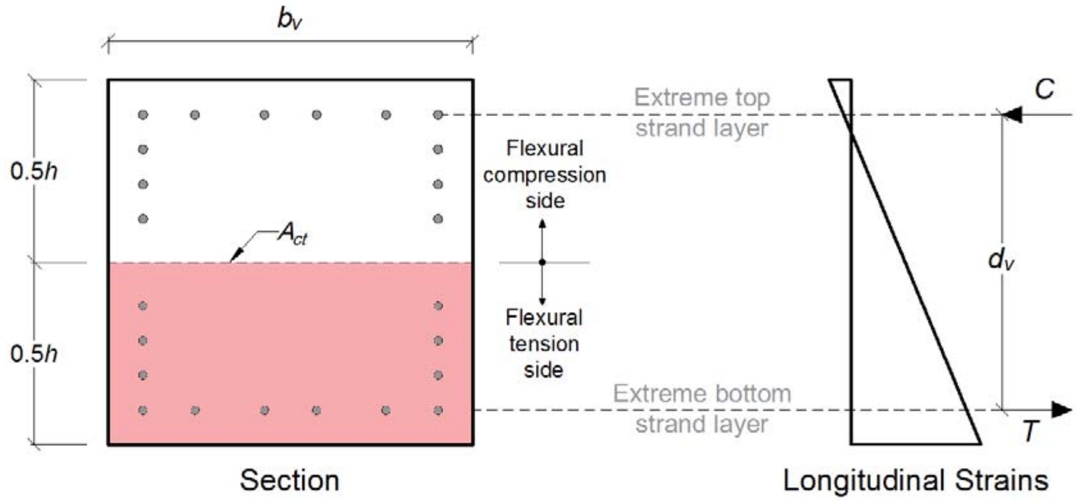
ShearCrackCheck := **if** ($V_{service} \le V_{cr}$, "Okay", "Not Okay")

ShearCrackCheck = "Okay"

## Shear Design (Con't)

### Find Effective Shear Depth

$d_v := CapDepth - 2 \cdot cover$ $\qquad d_v = 40\ in$ $\qquad$ *Effective shear depth; taken as the distance between outer steel layers*




$d_v$ need not be less than the greater of $0.9\,d_e$ and $0.72\,h$ $\qquad$ *(AASHTO LRFD 5.8.2.9)*

$d_v$ = maximum of: $\qquad d_v := h - 2 \cdot cover = 40\ in$

$0.9 \cdot d_e = 22.67\ in$

$0.72 \cdot h = 34.56\ in$

$d_v := \max\left(d_v, 0.9 \cdot d_e, 0.72 \cdot h\right)$ $\qquad d_v = 40\ in$

*Since* $V_n$ must be lesser of $V_{n1}$ and $V_{n2}$ (as per AASHTO LRFD 5.8.3.3), then $V_u$ must be less than both $\phi V_{n1}$ and $\phi V_{n2}$. $V_{n1}$ is dependent on the section properties and the flexural reinforcement. $V_{n2}$ is dependent on the section properties, the flexural reinforcement, and the shear reinforcement. $V_{n1}$ is independent of the shear steel, therefore if $V_u$ is greater than $\phi V_{n1}$ the cap fails in shear regardless of transverse steel.

### Check AASHTO 5.8.3.3-2

$V_{n1} := 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p$ $\qquad V_{n1} = \left(4.08 \cdot 10^3\right)\ kip$ $\qquad$ *(AASHTO LRFD Eq.5.8.3.3-2)*

$V_{r1}$ must be greater than $V_u$ $\qquad V_{r1} := \phi_v \cdot V_{n1}$

$V_{r1} = 3672\ kip > V_u = 489.4\ kip$ $\qquad$ *(AASHTO LRFD Eq.5.8.2.1-2)*

$V_{r1}Check := \mathbf{if}\left(V_{r1} > V_u, \text{“Okay”}, \text{“Not Okay”}\right)$

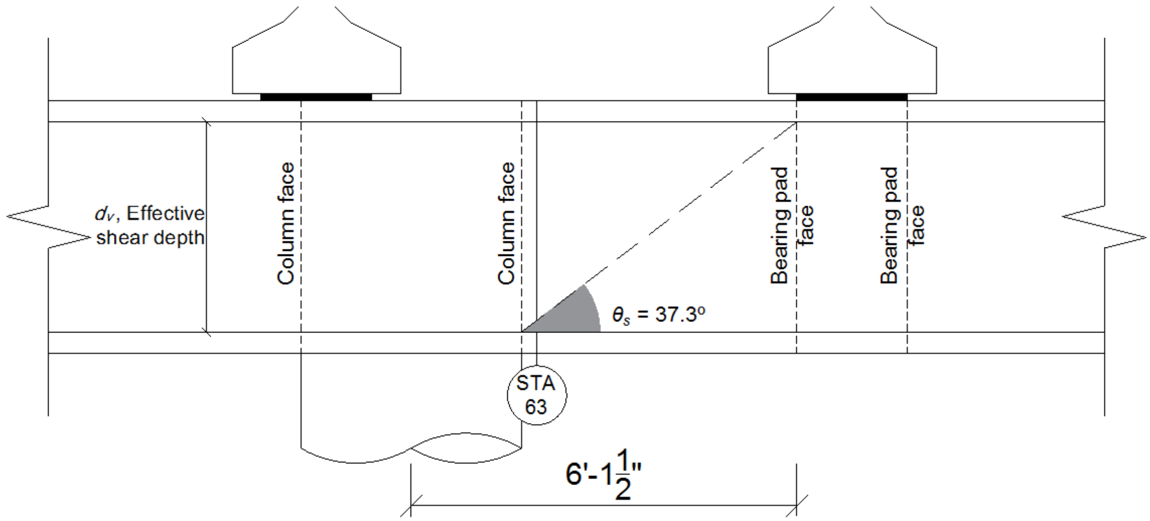
*If $V_{r1}$ is greater than $V_{u1}$, then use a larger cap depth in order to satisfy shear requirements.*

$V_{r1}Check = \text{“Okay”}$

## Shear Design (Con't)

### Determine the Compressive Srut Angle

*Find $\theta_s$ from the bent cap geometry* $\theta_s := 37.3 \text{ deg.}$ *Angle between the column face and the bearing pad face*




### Calculate Determine $\varepsilon_s$ and $\theta$

*The method for calculating $\varepsilon_s$ and $\theta$ used in this design example is from AASHTO LRFD 5.8.3.4.2.*

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$

*The net longitudinal tensile strain in the section at the centroid of the tension reinforcement (AASHTO 5.8.3.4.2-1). If $\varepsilon_s$<0, then assume $\varepsilon_s$=0 or recalculate with the denominator of the equation replaced by $(E_sA_s + E_pA_{ps} + E_cA_c)$; however $\varepsilon_s$ should not be taken as less than $-0.40 \cdot 10^{-3}$ or greater than $6.0 \cdot 10^{-3}$ (AASHTO LRFD. 5.8.3.4.2)*

If $\varepsilon_s < 0$, then use $\varepsilon_s = 0$ or an equation below

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{\left(E_s \cdot A_s + E_p \cdot A_{ps} + E_c \cdot A_{ct}\right)}$$

where, $|M_u| = 1515.4 \; kip \cdot ft$ must be greater than $|V_u - V_p| \cdot d_v = 1631 \; kip \cdot ft$

$M_u := \max\left(|M_u|, |V_u - V_p| \cdot d_v\right)$ $M_u = 1631 \; kip \cdot ft$

$$\varepsilon_s := \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}} \qquad \varepsilon_s = -9.79 \cdot 10^{-4}$$

*Use* $\varepsilon_s := 0$

$\theta := 29 + 3500 \, \varepsilon_s$ $\theta = 29 \text{ deg.}$ *(AASHTO LRFD Eq.5.8.3.4.2-3)*

## Shear Design (Con't)

### Determine $\theta$ for Use in the Design and Calculate $V_c$

The controlling angle is the larger of $\theta$ and $\theta_s$

If $\theta$ is larger

$\beta_1 := \dfrac{4.8}{(1 + 750 \cdot \varepsilon_s)}$ $\qquad \beta_1 = 4.8$

*(AASHTO LRFD Eq. 5.8.3.4.2-1)*
*This equation is for section contaning at least the minimum amount of transverse reinforcement.*
*AASHTO LRFD Eq. 5.8.3.4.2-2 provides $\beta$ calculation for section without the minimum amount of shear reinforcement*

$V_{c1} := 0.0316\ \beta_1 \cdot \sqrt{\dfrac{f'_c}{ksi}} \cdot ksi \cdot b_v \cdot d_v$ $\qquad V_{c1} = 849.06\ kip$ $\qquad$ *(AASHTO LRFD Eq.5.8.3.3-3)*

If $\theta_s$ is larger

$\beta_2 := 1.6$

$V_{c2} := 0.0316 \cdot \beta_2 \cdot \sqrt{\dfrac{f'_c}{ksi}} \cdot ksi \cdot b_v \cdot d_v \cdot \cot\left(\dfrac{\theta_s \cdot \pi}{180}\right)$

$V_{c2} = 371.52\ kip$

$V_c := \mathbf{if}\left(\theta_s > \theta, V_{c2}, V_{c1}\right)$ $\qquad V_c = 371.52\ kip$

$\theta := \max\left(\theta, \theta_s\right)$ $\qquad \theta = 37.3$ deg.

### Check if Shear Reinforcement is Required

ShearRequired := **if** ($V_u > 0.5 \cdot \phi_v \cdot V_c$, "Required", ""Not Required")

ShearRequired = "Required"

$\therefore$ Shear reinforcement is required

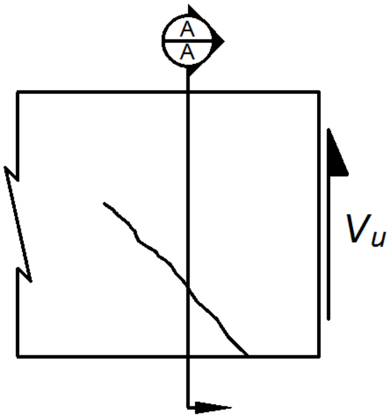
## Shear Design (Con't)

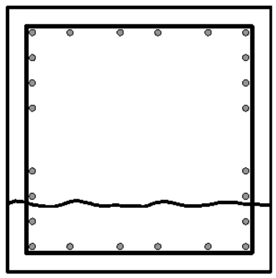
### Provide Shear Reinforcement

$A_v := 2 \cdot (0.31)\ in^2$ $\qquad A_v = 0.62\ in^2$

*TxDOT limits transverse reinforcement spacing to a maximum of 12"and a minimum of 4"*
*(BDM-LRFD, Ch. 4, Sec. 4, Detailing)*
*Trial and error is used to determine the stirrup spacing required for the section*

Assuming #5 stirrups at $s := 8.4\ in$ spacing



Section A-A

*The transverse reinforcement, "$A_v$" is a closed stirrup. The failure surface intersects two legs of the stirrup, therefore the area of the shear steel is two times the stirrup bar's area (0.31 $in^2$ for #5 bar). See the sketch of the failure plan to the left*

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot\left(\frac{\theta \cdot \pi}{180}\right)}{s}$$

$V_s = 232.53\ kip$

*(AASHTO LRFD Eq. C5.8.3.3-1)*

$V_n$ = minimum of:

$V_c + V_s + V_p = 604.05\ kip$ *(AASHTO LRFD Eq. 5.8.3.3-1)*

$0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p = \left(4.08 \cdot 10^3\right)\ kip$ *(AASHTO LRFD Eq. 5.8.3.3-2)*

$V_n := min\left(V_c + V_s + V_p, 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p\right)$ *Nominal shear resistance*

$V_n = 604.05\ kip$

$V_r := \phi_v \cdot V_n$ $\qquad V_r = 543.65\ kip$ *Factored shear resistance*

$V_u = 489.4\ kip$ *Factored shear force*

ShearResistance := **if** $(V_u \le V_r$, "Okay", "Not Okay")

ShearResistance = "Okay"

## Shear Design (Con't)

### Check Minimum Transverse Reinforcement

$A_{v\_min} := 0.0316 \cdot \sqrt{\frac{f'_c}{ksi}} \cdot ksi \cdot \frac{b_v \cdot s}{f_y}$  $A_{v\_min} = 0.62\ in^2$  *(AASHTO LRFD Eq. 5.8.2.5-1)*

$A_v = 0.62\ in^2 \quad > \quad A_{v\_min}$

$\text{MinimumSteelCheck} := \textbf{if}\left(A_v > A_{v\_min}, \text{“Okay”}, \text{“Not okay”}\right)$

$\text{MinimumSteelCheck} = \text{“Okay”}$

### Check Maximum Spacing of Transverse Reinforcement

*Shear Stress*

$v_u := \frac{V_u - (\phi_v \cdot V_p)}{\phi_v \cdot b_v \cdot d_v}$  $v_u = 0.283\ ksi$  *Average factored shear stress on the concrete (AASHTO LRFD Eq.5.8.2.9-1)*

$0.125 \cdot f'_c = 1.06\ ksi$

if $v_u < 0.125 \cdot f'_c$, $s_{max}$ = maximum of:  *(AASHTO LRFD Eq.5.8.2.7-1)*

$0.8 \cdot d_v = 32\ in$ & 24in.

if $v_u \geq 0.125 \cdot f'_c$, $s_{max}$ = maximum of:  *(AASHTO LRFD Eq.5.8.2.7-2)*

$0.4 \cdot d_v = 16\ in$ & 12in.

Since $v_u < 0.125 \cdot f'_c$, $s_{max}$=24.0 in.

*TxDOT limits the maximum transverse reinforcement spacing to 12," therefore:*  *(BDM-LRFD, Ch.4, Sec.4, Detailling)*

$s_{max} := 12.00\ in$

$s = 8.4\ in \quad < \quad s_{max}$

$\textbf{SpacingCheck} := \textbf{if}\left(s < s_{max}, \text{“Okay""”}, \text{“Not okay”}\right)$

$\textbf{SpacingCheck} = \text{“Okay""”}$

*Shear capacity and checks should be repeated at ALL points of critical shear. Note: in the overhangs, the stirrups need to be spaced @ 5in because shear is higher. Similarly the stirrups need to be spaced @ 5in near the center column. When the spacing needed is less than 4in, use double stirrups. (BDM-LRFD, Ch. 4, Sec. 4, Detailing) When using double stirrups, $A_v$ is four times the stirrups bar's area.*