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DEPARTMENT

COOPERATIVE
RESEARCH

AN ANALYSIS OF DYNAMIC DISPLACEMENTS MEASURED WITHIN PAVEMENT STRUCTURES

in cooperation with the
Department of Transportation
Federal Highway Administration

RESEARCH REPORT 136-6F
STUDY 2-8-69-136
FLEXIBLE PAVEMENTS

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16. Abstract

A mathematical model was developed for representing the measured displacement vector fields in thirty test sections at the Research Annex at Texas A&M University. While the model predicted the displacements fairly well in each test section, it was obvious that improvements were required in the model before attempting its use in the design of flexible pavements.

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An Analysis of Dynamic Displacements Measured Within Pavement Structures

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Research Report No. 136-6F
Design and Evaluation of Flexible Pavements
Research Study 2-8-69-136

Sponsored by

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Preface

This is the sixth and final report issued under Research Study 2-8-69-136, Design and Evaluation of Flexible Pavements, being conducted at the Texas Transportation Institute as part of the cooperative research program with the Texas Highway Department and the Department of Transportation, Federal Highway Administration. Previous reports were:

(1) "Seasonal Variations of Pavement Deflections in Texas", by Rudell Poehl and Frank H. Scrivner, Research Report 136-1, Texas Transportation Institute, January, 1971.

(2) "A Technique for Measuring the Displacement Vector throughout the Body of a Pavement Structure Subjected to Cyclic Loading", by William M. Moore and Gilbert Swift, Research Report 136-2, Texas Transportation Institute, August, 1971.

(3) "A Graphical Technique for Determining the Elastic Moduli of a Two-Layered Structure from Measured Surface Deflections", by Gilbert Swift, Research Report 136-3, Texas Transportation Institute, November, 1972.

(4) "An Empirical Equation for Calculating Deflections on the Surface of a Two-Layered Elastic System", by Gilbert Swift, Research Report 136-4, Texas Transportation Institute, November, 1972.

(5) "Elastic Moduli Determination for Simple Two Layer Pavement Structure Based on Surface Deflections", by William M. Moore, Research Report 136-5, Texas Transportation Institute, September, 1973.

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Disclaimer

The contents of this report reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

Abstract

A mathematical model was developed for representing the measured displacement vector fields in thirty test sections at the Research Annex at Texas A&M University. While the model predicted the displacements fairly well in each test section, it was obvious that improvements were required in the model before attempting its use in the design of flexible pavements.

Key Words: Pavements, Flexible Pavements, Pavement Deflections, Dynamic Deflections.

Summary

A basic prupose of Study 136 was to provide new experimental eivdence of the manner in which strains induced by dynamic surface loads are distributed throughout flexible pavement structures. Since strains in heterogeneous materials cannot be measured accurately, while displacements can (at least under cyclic loading), it was decided to measure the latter in the expectation that a mathematical model for the displacements could be found, and the model could be easily converted to strains by well known procedures. Accordingly, measurements were made on 30 specially designed test sections at the Texas A&M University Research Annex. Cyclic loading (8 cps, 1000 lbs. peak-to-peak) was supplied by a Dynaflect.

Measured Data: Nearly 7300 measurements of vertical and horizontal displacements, half of which were replicate measurements to be used in defining experimental error, were made and have been compiled in Appendix B, a separate volume (see page 3). With regard to these data, the following can be said:

a. Vertical motions at all points were downward as the load was increased, and had an average replication (or experimental) error of 15% of the mean displacement. This error is considered small and tends to support the reliability of the vertical displacement data.

b. At shallow depths in the pavement structure (about 3 inches for the thinnest pavements, up to approximately 20 inches for some thick pavements) points tended to move horizontally toward the load; at greater depths they moved horizontally away from the load. The average replication error was 32% of the mean value of the amplitude. Thus the

horizontal (or radial) displacement measurements appear to be less reliable than the vertical measurements.

c. Contour maps of a number of the sections suggested the existence of a horizontal line (or "neutral axis") in the vicinity of which radial displacements were near zero, and vertical displacements were near maximum, at least up to about 100 inches measured horizontally from the load.

d. Some of the data suggested that the sum of the three normal strains (the dilatation, or bulk strain) was negligible, compared to the largest of the three strains.

Application of Neutral Axis Concept: A commonly used formula for locating the depth to the neutral axis of a composite beam was employed as a regression model for an analysis of the appropriate data (layer thicknesses, material types, apparent depths to the neutral axis) from the 30 test sections. This analysis yielded ratios of the elastic moduli of the six types of materials used in bases, subbases and embankments, to the modulus of the asphaltic concrete material. The modular ratios appeared to be ordered reasonably, but their values should be checked by independent means.

Model for Displacements: The model adopted for displacements (which was so structured that the bulk strain was zero, and the depth to the neutral axis appeared in the model) contained four constants to be determined by regression on the data. When fitted to the vertical displacements section-by-section good fits were obtained, the value of the squared correlation coefficient, R^2, ranging from .94 to .99, but when fitted to the vertical displacement data from all 30 sections simultaneously, large prediction errors were encountered. When fitted to the radial (horizontal) displacement data section-by-section lower values of R^2 resulted, and the values of the four constants obtained were different for each section from the values previously found from the vertical displacement data for the same section.

Conclusions and Recommendation: Since the displacement vector model could not be extrapolated from one test section to the next, it obviously cannot be extrapolated to real highway sections, and therefore is not suitable, in its present form, for use as a practical design tool. The basic data, however, are considered unique in the field of pavement research, and deserve further study. It is recommended that such a study be pursued to a satisfactory conclusion.

Implementation Statement

The analysis reported herein resulted in a displacement vector model that only partially met the requirements of a practical pavement design tool. The basic data, available in Appendix B, should be analyzed further and a model developed that is suitable for use in the Texas Flexible Pavement Design system computer program to supplement or replace the presently used structural subsystem. Implementation must await the results of this further study.

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1. Introduction

More experimental evidence of the manner in which stresses and/or strains induced by heavy wheel loads are distributed through flexible pavement structures in essential to the continued development of basic pavement design theory. Various mathematical theories - e.g., linear elastic, non-linear elastic, visco-elastic, etc. - have in recent years been proposed for use in predicting these stresses or strains. A choice among these theories can be based, according to some researchers, on the behavior of the pavement materials when subjected to laboratory testing of small samples of the construction materials. Others have proposed to validate one theory or another through field or model testing, in some cases involving the measurement of stresses and/or strains by means of instruments imbedded in the structure.

In the present instance an oscillating load (8 cps, 1000 lbs. peak-to-peak) was applied by a Dynaflect to the surface of 30 test sections involving seven types of construction materials founded on a bed of plastic clay. The horizontal and vertical motions, or displacements, caused by the oscillating load were measured by portable instruments lowered to various depths into an open, small-diameter drilled hole.

By lowering the measuring instrument to a selected depth, and stationing the dynaflect at selected distances from the hole, horizontal displacements were measured in a vertical plane in each section at 117 points on a rectangular grid 9 points deep by 13 points long. Grid points were located at horizontal distances from the load ranging from approximately 12 inches to 216 inches, and at depths from 0 to 65 inches.

Vertical displacements were measured at the same points, and at 9 additional points in a vertical line located at a horizontal distance of 10 inches from the load. More details are given in Research Report 136-2(1)

An analysis of these displacements is the subject of this report.

2. Basic Data

The lay-out and cross-sections of the test sections used in this study are fully described in Research Report 32-8(2). It will suffice to say here that the data treated were gathered on 27 statistically designed test sections, each consisting of a surfacing material (Asphaltic concrete) a base material, a subbase material and an embankment. Materials and thicknesses were varied among the test sections in such a way as to make possible objective statistical analyses of the response of the sections to surface loads, and to isolate the response of each material. Three additional sections, not conforming to the original experiment design but lending additional strength to it were also included so that a total of 30 sections were tested.

The measured displacements and other basic data are compiled in Appendix b, published as a separate volume of this report. To illustrate the information available in Appendix B, a typical set of eight pages for one section (Section 5) is presented in Figures 2.1 and 2.2 and Tables 2.1 through 2.6. Average data of the type shown in Tables 2.5 and 2.6 were used in the regression analyses discussed in a later chapter. The replication error is treated in Chapter 3. The equipment and procedures used in making the measurements are discussed fully in a previous report (1).

Appendix B can be obtained on a loan basis from:

Engineer-Director, File D-10
Planning and Research Division
Texas Highway Department
P.O. Box 5051
Austin, Texas 78763

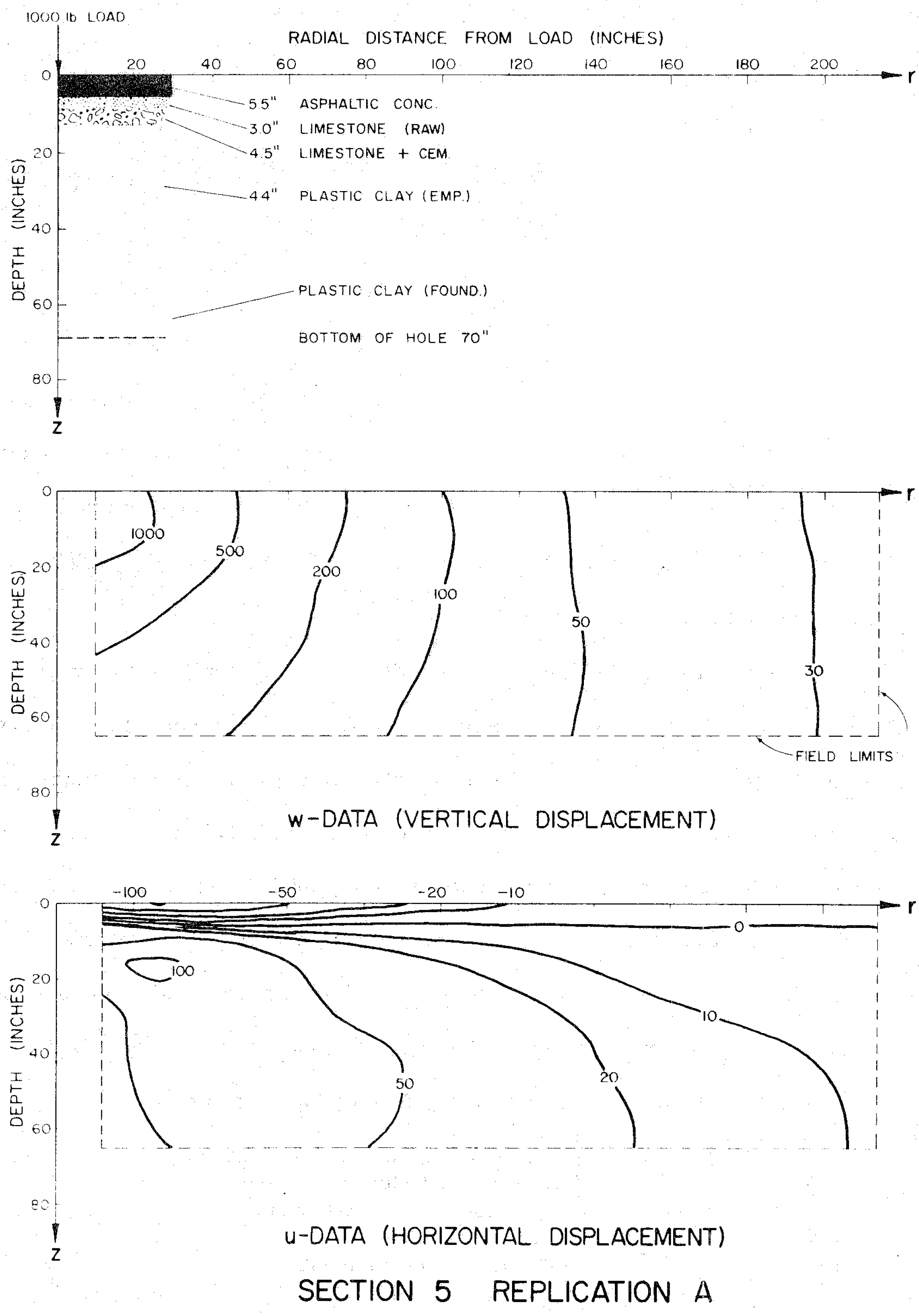
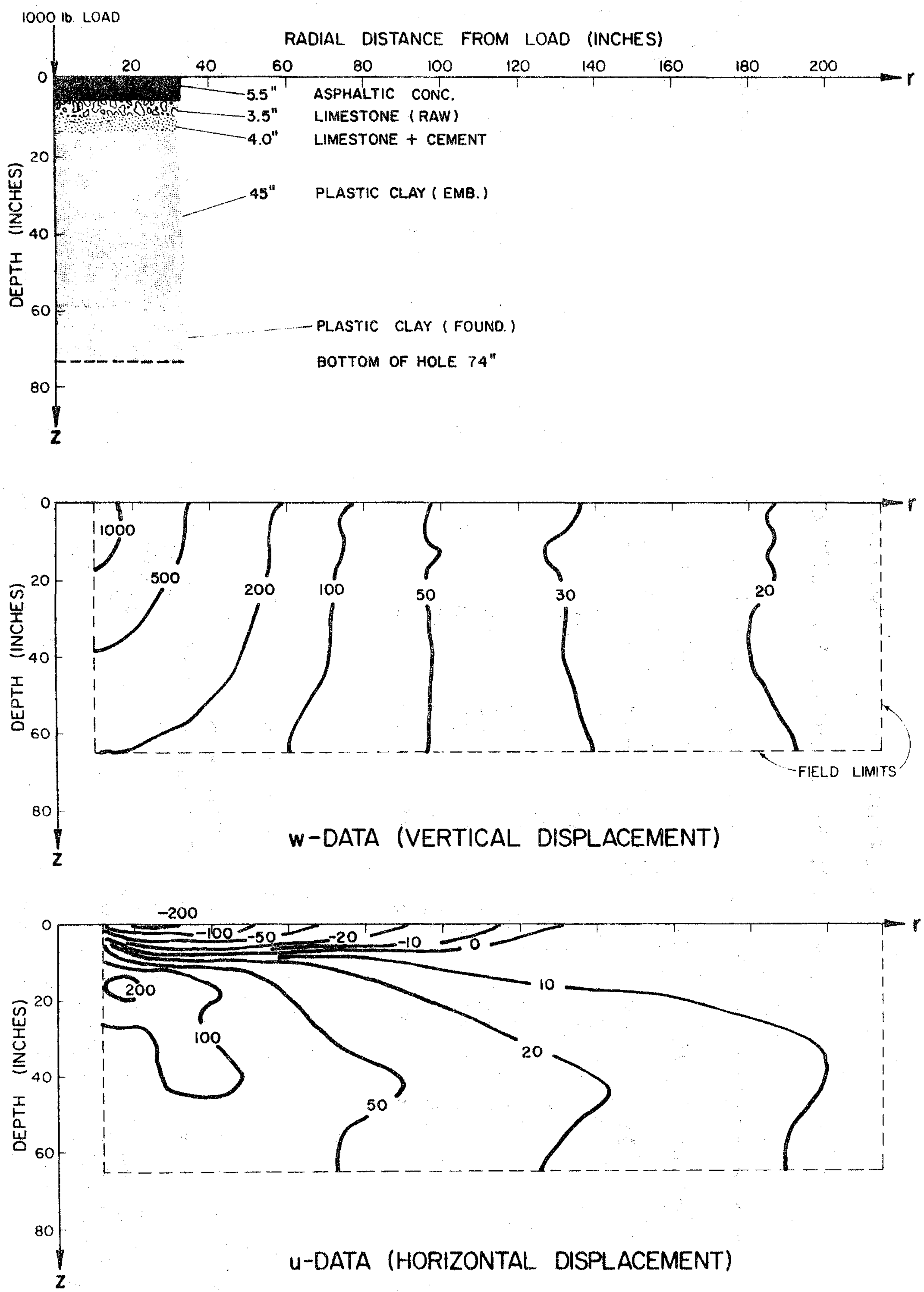

1000 lb LOAD
RADIAL DISTANCE FROM LOAD (INCHES)
0
20
40
60
80
100
120
140
160
180
200
r
DEPTH (INCHES)
5.5" ASPHALTIC CONC.
3.0" LIMESTONE (RAW)
4.5" LIMESTONE + CEM.
44" PLASTIC CLAY (EMP.)
PLASTIC CLAY (FOUND.)
BOTTOM OF HOLE 70"
z
1000
500
200
100
50
30
FIELD LIMITS
w-DATA (VERTICAL DISPLACEMENT)
-100
-50
-20
-10
0
100
10
50
20
u-DATA (HORIZONTAL DISPLACEMENT)
SECTION 5 REPLICATION A

Figure 2.1



SECTION 5 REPLICATION B

Figure 2.2

Table 2.1

SECTION 5 REPLICATION A

U - DATA (MICRO-INCHES) FOR SINGLE 1000 LB. LOAD

DEPTH Z (IN.)	10.0	11.7	15.6	20.6	26.0	37.4	49.0	60.8	72.7	96.5	120.4	144.3	180.3	216.2
	*************** R A D I A L					D I S T A N C E			R	(I N .)		***************		
0		-100	-93	-97	-102	-87	-72	-51	-33	-20	-9	-6	-2	-2
5.5		43	29	21	12	6	-4	-4	-1	0	0	-1	0	0
8.5		34	29	27	27	22	21	13	11	8	5	5	3	3
12.0		53	70	77	83	67	53	36	28	17	10	5	4	4
17.0		66	91	106	113	93	71	53	39	24	14	8	5	4
29.0		40	46	56	64	66	64	56	47	37	24	16	9	6
41.0		27	42	55	68	80	80	71	61	46	30	19	12	8
56.0		25	33	45	53	66	69	66	57	47	33	22	13	9
65.0		25	30	37	44	56	61	60	53	45	31	22	13	9

D R I L L I N G L O G D A T A

LAYER NO.	DEPTH Z (IN.) FROM	TO	LAYER THICKNESS MEASURED	DESIGN	**** MATERIAL ****	** BOND **	MOISTURE CONTENT
1	0.0	5.5	5.5	5.0	ASPHALTIC CONCRETE		
2	5.5	8.5	3.0	4.0	LIMESTONE (RAW)	BONDED	DRY
3	8.5	12.0	3.5	4.0	LIMESTONE + CEMENT	BONDED	DRY
4	12.0	55.0	43.0	40.0	PLASTIC CLAY (EMB)	BONDED	WET
5	55.0	70.0	15.0		PLASTIC CLAY (FOUND)	BONDED	MOIST

Table 2.2

SECTION 5 REPLICATION B

U - DATA (MICRO-INCHES) FOR SINGLE 1000 LB. LOAD

DEPTH Z (IN.)	10.0	11.7	15.6	20.6	26.0	37.4	49.0	60.8	72.7	96.5	120.4	144.3	180.3	216.2
	***************			R A D I A L		D I S T A N C E			R	(I N .)		***************		
00.0		-80	-180	-204	-208	-165	-109	-67	-41	-14	-9	10	9	9
05.5		64	38	-26	-38	-41	-31	-19	-14	-12	7	10	8	7
09.0		69	41	29	22	15	11	11	7	6	6	7	6	6
13.0		199	188	161	130	83	51	33	22	10	7	7	5	5
17.0		199	238	216	186	121	76	48	31	16	11	10	7	4
29.0		70	75	79	82	79	67	54	42	27	19	15	11	8
41.0		55	79	94	107	112	99	81	70	43	27	19	11	9
58.0		37	43	51	60	70	69	60	50	34	23	16	11	7
65.0		22	31	42	49	59	62	56	50	34	21	16	11	7

D R I L L I N G L O G D A T A

LAYER NO.	DEPTH Z (IN.) FROM	TO	LAYER THICKNESS MEASURED	DESIGN	**** MATERIAL ****	** BOND **	MOISTURE CONTENT
1	0.0	5.5	5.5	5.0	ASPHALTIC CONCRETE		
2	5.5	9.0	3.5	4.0	LIMESTONE (RAW)	BONDED	DRY
3	9.0	13.0	4.0	4.0	LIMESTONE + CEMENT	BONDED	DRY
4	13.0	58.0	45.0	40.0	PLASTIC CLAY (EMB)	BONDED	MOIST
5	58.0	74.0	16.0		PLASTIC CLAY (FOUND)	BONDED	MOIST

Table 2.3

SECTION 5 REPLICATION A

W - DATA (MICRO-INCHES) FOR SINGLE 1000 LB. LOAD

DEPTH Z (IN.)	RADIAL DISTANCE R (IN.) 10.0	11.7	15.6	20.6	26.0	37.4	49.0	60.8	72.7	96.5	120.4	144.3	180.3	216.2
0	1391	1344	1234	1047	953	669	462	325	209	111	62	40	32	27
5.5	1453	1328	1234	1094	969	681	456	319	216	112	65	42	32	27
8.5	1453	1359	1219	1094	984	669	478	319	209	117	65	42	32	27
12.0	1406	1297	1188	1078	925	662	453	313	203	116	66	42	34	27
17.0	1094	1047	984	906	806	603	425	297	192	114	65	43	33	26
29.0	662	647	612	587	544	434	331	242	166	106	64	43	33	26
41.0	525	500	491	469	434	369	287	220	150	105	64	45	33	26
56.0	325	316	311	308	284	253	211	169	127	90	60	45	32	25
65.0	269	269	263	257	248	225	183	153	114	90	59	45	32	25

DRILLING LOG DATA

LAYER NO.	DEPTH Z (IN.) FROM	TO	LAYER THICKNESS MEASURED	DESIGN	MATERIAL	BOND	MOISTURE CONTENT
1	0.0	5.5	5.5	5.0	ASPHALTIC CONCRETE		
2	5.5	8.5	3.0	4.0	LIMESTONE (RAW)	BONDED	DRY
3	8.5	12.0	3.5	4.0	LIMESTONE + CEMENT	BONDED	DRY
4	12.0	55.0	43.0	40.0	PLASTIC CLAY (EMB)	BONDED	WET
5	55.0	70.0	15.0		PLASTIC CLAY (FOUND)	BONDED	MOIST

Table 2.4

SECTION 5 REPLICATION B

W - DATA (MICRO-INCHES) FOR SINGLE 1000 LB. LOAD

DEPTH Z (IN.)	**************			R A D I A L		D I S T A N C E		R	(I N .)		**************			
	10.0	11.7	15.6	20.6	26.0	37.4	49.0	60.8	72.7	96.5	120.4	144.3	180.3	216.2
00.0	1210	1218	1000	823	677	440	276	184	112	53	35	28	21	17
05.5	1234	1153	1016	806	645	427	256	161	103	48	34	27	20	19
09.0	1298	1282	1048	847	661	416	263	159	104	48	32	27	21	18
13.0	1266	1129	1008	823	645	400	245	157	105	55	31	25	20	17
17.0	1032	976	919	750	581	389	239	161	100	49	33	26	21	17
29.0	629	726	597	540	463	324	215	143	97	50	34	26	20	17
41.0	471	461	460	398	348	265	187	130	92	51	34	26	20	17
58.0	244	245	244	231	213	187	142	102	81	50	35	28	21	17
65.0	216	197	200	195	187	161	133	100	78	50	37	28	21	18

D R I L L I N G L O G D A T A

LAYER NO.	DEPTH Z (IN.) FROM	TO	LAYER THICKNESS MEASURED	DESIGN	**** MATERIAL ****	** BOND **	MOISTURE CONTENT
1	0.0	5.5	5.5	5.0	ASPHALTIC CONCRETE		
2	5.5	9.0	3.5	4.0	LIMESTONE (RAW)	BONDED	DRY
3	9.0	13.0	4.0	4.0	LIMESTONE + CEMENT	BONDED	DRY
4	13.0	58.0	45.0	40.0	PLASTIC CLAY (EMB)	BONDED	MOIST
5	58.0	74.0	16.0		PLASTIC CLAY (FOUND)	BONDED	MOIST

Table 2.5

SECTION 5 AVERAGE OF REPLICATIONS A AND B

U - DATA (MICRO-INCHES) FOR SINGLE 1000 LB. LOAD

DEPTH Z (IN.)	10.0	11.7	15.6	20.6	26.0	37.4	49.0	60.8	72.7	96.5	120.4	144.3	180.3	216.2
	******************			R A D I A L		D I S T A N C E			R	(I N .)		******************		
0.0		-89	-136	-150	-154	-125	-90	-58	-36	-16	-8	2	3	3
5.50		53	33	-2	-13	-17	-17	-11	-7	-5	3	4	3	3
8.75		51	34	27	24	18	15	11	8	6	5	5	4	4
12.50		125	128	118	106	74	51	34	24	13	8	5	4	4
17.00		132	164	160	149	106	73	50	34	19	12	8	5	3
29.00		54	60	67	72	72	65	54	44	31	21	15	9	6
41.00		40	60	74	87	95	89	75	65	44	28	18	11	8
57.00		30	37	47	56	67	68	62	53	40	27	18	11	7
65.00		23	30	39	46	57	61	57	51	39	25	18	11	7

SECTION 5 REP ERROR IN U

I	MEAN DEPTH (IN)	REP. ERROR	ABS MEAN	REP. ERROR (PCT OF MEAN)
1	0.0	27	68	40.5
2	5.5	13	14	96.0
3	8.8	6	17	32.4
4	12.5	29	54	54.2
5	17.0	33	71	46.8
6	29.0	8	45	16.9
7	41.0	11	54	21.1
8	57.0	4	41	9.2
9	65.0	3	36	7.2
SECTION VALUES		18	44	42.0

Table 2.6

SECTION 5 AVERAGE OF REPLICATIONS A AND B

W - DATA (MICRO-INCHES) FOR SINGLE 1000 LB. LOAD

DEPTH Z (IN.)	**************** R A D I A L				D I S T A N C E				R	(I N .)				****************
	10.0	11.7	15.6	20.6	26.0	37.4	49.0	60.8	72.7	96.5	120.4	144.3	180.3	216.2
0.0	1300	1280	1116	934	814	554	368	254	160	81	48	33	26	21
5.50	1343	1240	1124	949	806	553	355	239	159	79	49	34	25	22
8.75	1375	1320	1133	970	822	542	370	238	156	82	48	34	26	22
12.50	1335	1212	1097	950	784	530	348	234	153	85	48	33	26	21
17.00	1062	1011	951	827	693	495	331	228	145	81	48	34	26	21
29.00	645	686	604	563	503	378	272	192	131	77	48	34	26	21
41.00	497	480	475	433	390	316	236	174	120	77	48	35	26	21
57.00	284	280	277	269	248	219	176	135	103	69	47	36	26	20
65.00	242	232	231	225	217	192	157	126	95	69	47	36	26	21

SECTION 5 REP ERROR IN W

I	MEAN DEPTH (IN)	REP. ERROR	MEAN	REP. ERROR (PCT OF MEAN)
1	0.0	79	500	15.8
2	5.5	91	499	18.2
3	8.8	82	510	16.1
4	12.5	82	491	16.7
5	17.0	60	426	14.1
6	29.0	33	299	11.0
7	41.0	31	238	13.1
8	57.0	28	157	18.2
9	65.0	24	138	17.6
SECTION VALUES		62	362	17.2

3. Replication Error

An estimate of displacement variability not amenable to explanation by the design constants of a test section is the replication error for the section. Pairs of replicate measurements were made in each of the thirty sections in accordance with the procedures described in a previous report of this study (1). As stated in that report (p. 7), "replication errors observed on a test section reflect not only the variability of the measuring process but also include the effects of variations in the structural properties of the section. The combined variability will define the limiting prediction accuracy for the displacement model being sought."

Although replicate measurements were made with the Dynaflect at identical horizontal distances from each of two instrumented bore holes in a section, the vertical distances from load to transducer were sometimes slightly different, mainly because of differences (usually small) in the depth to an interface between layers. When a measurement had previously been made at an interface in an instrumented hole, it was believed desirable to make the corresponding replicate measurement at the actual interface in the second hole rather than at the depth of the first measurement, if the two depths differed. In this way, the variability in layer thickness would be accounted for in the replication error.

Replication errors were calculated as follows.

Let d (i) = the algebraic difference between the $i^{\underline{th}}$ pair of n replicate measurements of a horizontal (or vertical) displacement. Then the section replication error for horizontal (or vertical) displacements was computed from

$$\text{Section Rep. Error} = \sqrt{\sum_{1}^{n} [d(i)/2]^2 /n} \qquad (3.1)$$

The horizontal displacement of a point was considered positive in sign if it moved away from the oscillating load while the load was increasing (or toward the load while it was decreasing); otherwise it was given a negative sign. A similar rule applied to vertical displacements, but since all points had a downward component of motion during periods of increasing load, all vertical displacements were positive. On the contrary, many of the horizontal displacements were negative. Thus, to express a replication error (Equation 3.1) as a percentage of the average displacement for a section, it was decided to use as a base the average of the absolute values of the observed displacements. In Table 3.1 are the replication errors, averaged over all 30 sections, expressed both in mils (thousandths of an inch) and as a percentage of the average of the absolute values of the observed displacements.

Replication errors for individual sections are given in Appendix A, Tables A1 and A2, and in more detail in Appendix B. It will be noted in the summary appearing in Table 3.1, that the mean replication error for horizontal displacements is small when measured in mils, but large when expressed as a percentage of the average absolute value of the displacement, as compared with similar statistics for the vertical displacements. The trend toward large percentage errors in replicate measurements of horizontal displacements may be due in part to the fact that the transducer for measuring them was more sensitive to small, unavoidable angular installation errors, than was the case for the vertical displacement transducer. Even if this source of error is discounted,

Table 3.1: Mean Values of Section Replication Errors for Thirty Sections

	Horizontal Displacement	Vertical Displacement
Mean Abs. Value, All Displacements (in Mils)	0.028	0.226
Replication Error		
In Mils	0.009	0.033
As a percentage of average absolute displacement measured in each section	32%	15%

the fact remains that the absolute values of the horizontal displacements were, on the average, only 12% of the vertical displacements , (.028 mils compared to .226 mils), so that the horizontal transducer was frequently operated near its limit of sensitivity even under the most favorable circumstances.

The role played by the replication errors in the development of a model for the displacement vector field will be discussed in the next chapter.

4. Displacement Vector Model

This chapter specifies the minimum requirements that should be met by a displacement vector model intended for eventual use as a part of the structural subsystem of the Texas Flexible Pavement Design System (FPS)(3). It also describes in general terms the type of model developed, and the degree of success achieved in satisfying the specified requirements.

4.1. Definitions: Most of the minimum requirements of the model can be more concisely expressed in symbols than in words. A list of the necessary symbols and their definitions follow.

The symbols r, z, θ represent cylindrical coordinates of a point located within a pavement structure. The origin of coordinates is in the pavement surface, which is considered to be a horizontal plane of infinite extent. Positive values of z are measured downward. The coordinate r, always positive, is measured horizontally. A load, acting vertically downward, is applied at the origin of coordinates. Each layer of material, bounded by horizontal planes of infinite extent, is assumed to be statistically homogeneous; thus, the vector field is assumed to be sensibly symmetrical about the z-axis so that the value of the angular coordinate, θ, is of no significance.

$u(r,z)$ = the radial component of the displacement of a point with coordinates r and z. $\hat{u}(r,z)$ is an estimate, computed from a model, of $u(r,z)$.

$w(r,z)$ = the vertical component of the displacement. $\hat{w}(r,z)$ is an estimate, computed from a model, of $w(r,z)$.

$\varepsilon_r(r,z)$, $\varepsilon_\theta(r,z)$, $\varepsilon_z(r,z)$ = the radial, tangential and vertical normal strains respectively. The corresponding estimates are $\hat{\varepsilon}_r(r,z)$, $\hat{\varepsilon}_\theta(r,z)$, $\hat{\varepsilon}_z(r,z)$.

As the load is increased, u is positive if the point moves away from the load, w is positive if it moves downward, and a normal strain ε is positive if the material in a small region surrounding the point is elongated in the direction indicated by the subscript on ε.

Normal strains are related to the displacements as follows:

$$\varepsilon_r = \frac{\partial u}{\partial r} \tag{4.1}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{4.2}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \tag{4.3}$$

h_1, h_2, h_3, h_4 = the thickness of the surfacing, base, subbase and embankment, respectively, of a test section. Values of these section constants are given in Table 4.1.

E_1, E_2, E_3, E_4, E_5 = a measure of the stiffness of the material (independent of its thickness or position in the structure) composing the surfacing, base, subbase, embankment and foundation soil, respectively, of the test section. The types of materials used are indicated in Table 4.1 and are described in more detail in Table 4.2.

f_i $(x_1, x_2, \ldots)$ represents an algebraic function of the quantities $x_1, x_2, \ldots$ The subscript, i, is used on f to distinguish between different functions of the same variables.

4.2. <u>Requirements of Model</u>: The minimum requirements to be placed on any model were deemed to be the following.

Table 4.1: Test Section Designs

	Thickness (In.)				Material Type (see also Table 2.2)			
Sec.	Surf.	Base	Subb.	Emb.	Surf.	Base	Subb.	Emb.
1	5	4	4	40	HMAC	LS+C	LS	Clay
2	1	12	4	36	HMAC	LS+C	LS	Clay
3	1	4	12	36	HMAC	LS+C	LS	Clay
4	5	12	12	24	HMAC	LS+C	LS	Clay
5	5	4	4	40	HMAC	LS	LS+C	Clay
6	1	12	4	36	HMAC	LS	LS+C	Clay
7	1	4	12	36	HMAC	LS	LS+C	Clay
8	5	12	12	24	HMAC	LS	LS+C	Clay
9	5	4	4	40	HMAC	LS	LS	Gr.
10	1	12	4	36	HMAC	LS	LS	Gr.
11	1	4	12	36	HMAC	LS	LS	Gr.
12	5	12	12	24	HMAC	LS	LS	Gr.
13	5	4	4	40	HMAC	LS+C	LS+C	Gr.
14	1	12	4	36	HMAC	LS+C	LS+C	Gr.
15	1	4	12	36	HMAC	LS+C	LS+C	Gr.
16	5	12	12	24	HMAC	LS+C	LS+C	Gr.
17	3	8	8	34	HMAC	LS+L	LS+L	SC
18	1	8	8	36	HMAC	LS+L	LS+L	SC
19	5	8	8	32	HMAC	LS+L	LS+L	SC
20	3	4	8	38	HMAC	LS+L	LS+L	SC
21	3	12	8	30	HMAC	LS+L	LS+L	SC
24	3	8	8	34	HMAC	LS	LS+L	SC
25	3	8	8	34	HMAC	LS+C	LS+L	SC
26	3	8	8	34	HMAC	LS+L	LS	SC
27	3	8	8	34	HMAC	LS+L	LS+C	SC
28	3	8	8	34	HMAC	LS+L	LS+L	Clay
29	3	8	8	34	HMAC	LS+L	LS+L	Gr.
31	0.5	6	0	0	ST	LS+C	--	--
32	0.5	6	0	0	ST	LS	--	--
33	0.5	6	0	0	ST	LS+L	--	--

Table 4.2: Materials Used in Embankment, Subbase, Base and Surfacing of Test Sections

Description	Abbreviation Used In Table 2.2	AASHO Class	Unified Soil Class	Texas Triaxial Class	Compressive Strength (psi)*
Compacted Plastic Clay	Clay	A-7-6(20)	CH	5.0	22
Sandy Clay	SC	A-2-6(1)	SC	4.0	40
Sandy Gravel	Gr.	A-1-6	SW	3.6	43
Crushed Limestone	LS	A-1-a	GS-GM	1.7	165
Crushed Limestone + 2% Lime	LS+L	A-1-a	GW-GM	1.0	430
Crushed Limestone + 4% Cement	LS+C	A-1-a	GW-GM	1.0	2270
Asphaltic Concrete	HMAC				
Surface Treatment	ST				

* By Texas triaxial procedure, at a lateral pressure of 5 psi

NOTE: The natural material below the embankments was a deep deposit of plastic clay similar to that described above.

A. The model should consist of a set of two equations,

$$\hat{u}(r,z) = f_1(h_1, \ldots, h_4; E_1, \ldots E_5; C_1, C_2, \ldots; r,z) \quad (4.4)$$

$$\hat{w}(r,z) = f_2(h_1, \ldots, h_4; E_1, \ldots E_5; C_1, C_2, \ldots; r,z) \quad (4.5)$$

B. Equations 4.4 and 4.5 should predict the measured values of u and w on the test sections with an overall error of approximately the same magnitude as the measured overall replication error.

C. Equations 4.4 and 4.5 should meet the following simple conditions at certain points outside the boundaries of the measured vector fields:

(1) $\hat{u}(o,z) = 0$ for all values of z.

(2) $\hat{w}(o,z)$ should be the maximum value of $\hat{w}(r,z)$ on the horizontal plane z = a constant of finite value.

(3) Both $\hat{u}(r,z)$ and $\hat{w}(r,z)$ should be zero at points infinitely distant from the load (r and/or z infinite).

D. The model should meet the following test for consistency: if two adjacent layers have the same value of E, then the model must yield the same result if the two layers are combined into a single layer with that modulus.

E. Expressions for normal strains found by operating on Equations 4.4 and 4.5 as indicated by Equations 4.1 and 4.3, should yield values that compare favorably with strains found by numerical differentiation of the basic data.

F. The model should be tractable to the extent that displacements and strains can be calculated with a minimum of computer time, perhaps less than one percent of the time required by available computer programs for linear elastic layered systems.

The requirements specified in paragraphs A through F appear to need no justification. However, it may be noted that a required state of equilibrium of the stresses associated with the displacements was not mentioned. This was omitted because it was taken for granted that if the selected model proved to predict the displacements within the replication error, then the stresses corresponding to the computed displacements could be assumed to be in equilibrium to within a tolerable error. It was also felt that any special conditions that might exist at the interface between dissimilar materials would automatically be duplicated by a model that predicted the displacements with the required accuracy.

4.3: Neutral Axis Concept: An examination of contour plots of the u-data collected on the 30 sections turned up many cases in which the contour line for $u = 0$ was approximately horizontal, at least within the range $11.7'' \leq r \leq 100''$. For example, see Figures 2.1 and 2.2. If such a contour line were approximated by a straight horizontal line it seemed clear that both u and the radial strain, $\varepsilon_r = \partial u/\partial r$, would be very small at points along that line, at least within the range, $11.7'' \leq r \leq 100''$, on many of the sections.

A study of the w-data showed that the maximum value of w (with r being fixed) nearly always occurred not at the surface, as might be expected, but at varying depths below the surface. The vertical strain $\varepsilon_z = \partial w/\partial z$ was necessarily zero at the point where w was maximum. The estimated points at which $\partial w/\partial z = 0$ in numerous sections fell near a straight horizontal line drawn through the field, at least within the range $10'' \leq r \leq 100''$.

The approximately horizontal line on which ε_r was near zero and that on which ε_z was near zero did not always coincide; in fact, there was poor correlation between the depths at which these two lines appeared to occur.

Nevertheless there seemed to be enough evidence in the data to warrant the adoption of the concept of a neutral axis, along which the normal strains ε_r and ε_z were zero. This assumption vastly simplified the introduction into the model of the constants E_1, E_2, E_3, and E_4. The simplification was brought about by treating a unit width of the pavement structure as a composite beam of unit width and a depth equal to the combined thickness of surface, base, subbase and compacted embankment - a total of 53 inches for the 27 main test sections, and 6.5 inches for the three special turn around sections. A formula in common use in structural engineering for determining the position of the neutral axis of a composite beam was used. It appears below.

$$Z = [h_1(h_1/2) + h_2(h_1 + h_2/2)\ (E_2/E_1)$$
$$+ h_3(h_1 + h_2 + h_3/2)\ (E_3/E_1)$$
$$+ h_4(h_1 + h_2 + h_3 + h_4/2)\ (E_4/E_1)]$$
$$/[h_1 + h_2(E_2/E_1) + h_3(E_3/E_1) + h_4(E_4/E_1)] \qquad (4.6)$$

where Z is the depth to the neutral axis, and the other symbols are as previously defined.

An estimate of Z was made from the contour map of u-data for each section. The layer thicknesses were known (Table 4.1). Thus, in Equation 4.6, only the modular ratios E_2/E_1, E_3/E_1 and E_4/E_1 were unknown for each section.

By a suitable transformation of Equation 4.6, it was possible to write a linear regression equation with six coefficients, each representing the ratio of the modulus of one of the six construction materials to the modulus of the surfacing material. A regression analysis using the appropriate data from all thirty sections (depth to neutral axis, Z; layer thicknesses, h_1, h_2, h_3 and h_4; and the type of material composing each layer), yielded the modular ratios given in Table 4.3.

The moduli appearing in Table 4.3 appear to be ordered logically, with the possible exception of the cement-stabilized limestone. However, because of difficulties encountered in selecting from the u-data a representative horizontal line corresponding to u=0, the application of the neutral axis concept to this study is admittedly open to question. The choice was made in the interest of simplicity and utility.

Table 4.3: Ratio of Modulii of Six Construction Materials to Modulus of Asphaltic Concrete

Material	Ratio
Asphaltic Concrete	1.00
Cr. Limestone + 4% Cement	0.60
Cr. Limestone + 2% Lime	0.34
Cr. Limestone	0.25
Sandy Gravel	0.0093
Sandy Clay	0.0066
Plastic Clay	0.00089

4.4: Bulk Strain Assumption: When data collection had been completed on the first three of the 30 test sections, the normal strains were estimated from the basic data in accordance with Equations 4.1, 4.2 and 4.3. The sum of the resulting trio of normal strains at most points tended to be very small compared to the largest of the three strains. It was therefore decided to make the simplifying assumption that the sum, $\varepsilon_r + \varepsilon_\theta + \varepsilon_z$ (or bulk strain), was zero at every point in the pavement structure. By application of Equation 4.1, 4.2 and 4.3, this assumption led to the following relationship between the vertical and horizontal displacements:

$$u = -\frac{1}{r} \int \frac{\partial w}{\partial z} \, r dr \tag{4.7}$$

Equation 4.7 made it possible to adopt a model for $w(r,z)$, determine the best fitting values of the constants C_1, C_2, . . ., by regression analysis using $w(r,z)$ as the dependent variable, and employ Equation 4.7 to find the corresponding model for $u(r,z)$. If the resulting model were found to fit the u-data with acceptable accuracy, the assumption that the bulk strain is negligible would be proved, and the main problem of this phase of the study - to produce a satisfactory model - would be solved.

It should be noted here that the geometry of a continuous medium leads to the conclusion that if the bulk strain is zero, and if the normal strains are sufficiently small, then the medium deforms with negligible change in volume. In applications of the theory of elasticity, such as medium would be said to have a Poisson's ratio of 0.5.

4.5. Type of Model and Results Achieved: Equations were developed for $\hat{u}(r,z)$ and $\hat{w}(r,z)$. These equations and details of their derivation are given in Appendix A. Here only their general form and the degrees of success achieved in meeting the requirements specified in Article 4.2 will be treated.

The letter preceding each of the following paragraphs indicates that the subject matter discussed is related to the paragraph designated by the same letter in the list of requirements, Article 4.2.

A. The original model developed consisted of two equations,

$$\hat{u}(r,z) = f3(Z, C, A1, A2, A3, r, z) \tag{4.8}$$

$$\hat{w}(r,z) = f4(Z, C, A1, A2, A3, r, z) \tag{4.9}$$

where C, A_1, A_2, A_3 were regarded as regression constants assumed to have the same value for all sections. Z was computed for each section from Equation 4.6, by substituting in that equation the appropriate modular ratios from Table 4.3, and the layer thicnkesses, h_1, h_2, h_3 and h_4 from Table 4.1. Thus, unlike Equations 4.4 and 4.5, representing a more desirable model, Equations 4.8 and 4.9 did not explicitly contain E_1 and E_5. It should be pointed out that neither quantity was a variable in the experiment design, and would therefore, from a statistical point of view, be difficult to quantify. E_1 perhaps could be estimated from laboratory tests: it would then be possible to quantify E_2, E_3 and E_4 from the ratios recorded in Table 4.3. Several schemes for including E_1 and E_5 in Equations 4.8 and 4.9 were considered, but none were satisfactory in terms of predicting the measured displacements.

Equations 4.8 and 4.9 satisfied the assumed condition that the bulk strain was zero; i.e., they satisfied Equation 4.7.

When Equation 4.9 was fitted to the w-data, section-by-section, it was found that the regression constants A_1, A_2, A_3 and C varied from section to section in such a way that they could not be related, with sufficient accuracy, to the section design variables. Thus, instead of one set of the regression constants, C, A_1, A_2, and A_3, thirty sets were determined from the w-data, one set per section. These, along with related statistical data, are given in Table A1 in Appendix A.

The equation for $\hat{u}(r,z)$ (Equation 4.8) was found from Equation 4.9 by application of Equation 4.7. Thus, the constants C, A_1, A_2, and A_3, already determined section-by-section from analysis of the w-data, carried over to the equation for $\hat{u}(r,z)$, section-by-section. When the resulting set of 30 equations were used to predict the u-data, large errors were found: the assumption that the bulk strain was zero at all points in the pavement structure was thus apparently invalidated.

The general form of Equation 4.8, however, produced fair results when a new set of the constants, C, A_1, A_2 and A_3 were determined directly from the u-data, again section-by-section. Thus another set of 30 constants were determined: these appear in Table A2.

Thus it is clear that the model only partially satisfied the requirements specified in paragraph 4.2A.

B. When the constants C, A_1, A_2 and A_3 recorded in Tables A1 and A2 were used in Equations 4.8 and 4.9, respectively, the prediction errors for each section were, on the whole, comparable with the replication errors for the section. Thus, requirement 4.2B was satisfied section-by-section, but not across sections.

C. The boundary conditions specified in 4.2C were met.

D. The consistency requirement specified in 4.2D was met.

E. The model failed to meet the requirement specified in 4.2E regarding strains, even though the displacement vector field was predicted within an error usually considered adequate. An examination of plots of $\hat{w}$ versus z (r fixed), and $\hat{u}$ versus r (z fixed), showed that the slopes of these curves (equivalent to ε_r and ε_z, respectively) did not agree closely enough with corresponding slopes of curves of the measured data to warrant the claim that predicted strains were sufficiently accurate for use in design. For example, there was a distinct tendency for the predicted compressive vertical strain to grow larger with increases in depth below the neutral axis, while the experimental data showed the opposite trend.

F. The specification regarding computer time (4.2F) was met.

5. Summary of Findings and Recommendations

The findings and recommendations that follow are based mainly on the material presented in the two chapters preceding this one.

5.1. Measured Data: With regard to the measured data, the following can be said:

a. The measured values of vertical displacements in the thirty test sections were all positive in sign (all downward in direction), averaged 0.226 mils, and had an average replication error of 0.033 mils or 15% of the mean displacement. This error is considered small and tends to support the reliability of the measured vertical displacement data.

b. The horizontal displacements observed in the same sections tended at shallow depths to be negative (points moved toward the load), and at greater depths to be positive (points moved away from the load). The mean of the absolute values of these displacements was 0.028 mils, and the average replication error was 0.009 mils or 32% of the mean of the absolute values observed. Thus it appears from their larger percentage replication error that the observed horizontal displacements were less consistent than the vertical displacements. One reason may have been that the horizontal transducer frequently operated near its sensitivity limit.

c. Contour maps of a number of the test sections suggested the existence of a horizontal line (or "neutral axis") in the vicinity of which radial displacements were near zero and vertical displacements were near maximum, at least in the approximate range $12'' \leq r \leq 100''$.

d. Some of the early displacement data, when converted to normal strains by an approximate numerical procedure, suggested that the sum of the normal strains (i.e. the bulk strain) was negligible when compared to the largest of the normal strains.

5.2. Application of Neutral Axis Concept: A commonly used formula for locating the depth to the neutral axis of a composite beam was employed as a regression model for an analysis of the appropriate data from the 30 test sections. This analysis yielded ratios of the moduli of the six types of materials used in bases, subbases and embankments, to the modulus of the asphaltic concrete. The moduli appeared to be ordered reasonably, but their values should be checked by some independent means.

5.3. Model for Vertical Displacements: With regard to the model selected for the vertical displacement data, the principal findings were as follows.

a. The model, which contained four constants to be evaluated from the data, could not be fitted to the data from all 30 test sections simultaneously without excessive prediction errors.

b. When fitted to the data section-by-section, the model prediction errors compared favorably with the section replication errors, and the squared correlation coefficients were high, ranging from 0.94 to 0.99.

c. From finding b. above, it is clear that the model for vertical displacements has some merit and that the data has a certain consistency, but from finding a. it is equally clear that in its present form the model can not be extrapolated to real highway sections, since it can not be extrapolated from one test section to another. Therefore, it is concluded that the model for vertical displacements is not suitable for use in design, and further analysis work is necessary.

5.4. Model for Radial Displacements:

a. A model for radial displacements in each test section was obtained from the corresponding equation for vertical displacements, (for which the values of all constants were already known) based on the assumption that the bulk strain was zero. The results were not acceptable because of large prediction errors. It was then concluded that the bulk strain was not sufficiently small to warrant use of the simplifying assumption that it was negligible.

b. The same general form of the equation for radial displacements described in a. above was then used, except that new constants were obtained by regression analysis on the radial displacement data, section-by-section. The prediction errors were greatly reduced, but were still, on the average, twice the size of the replication errors. Squared correlation coefficients ranged from .51 to .93 and averaged .73. Thus, both on the score of a rather poor fit to the data, and the fact that the equations developed section-by-section could not be extrapolated to real highway sections, it was concluded that the equation for radial displacements, like that for vertical displacements, is not suitable for use in design, and further analysis work is necessary.

5.5 Predicted Strains: After all regression constants appearing in the displacement models had been evaluated from the data, the models were differentiated to obtain expressions for the normal strains, ε_r and ε_z. The strains computed from these expressions did not agree well with strains indicated from plots of the basic data. The disparity was too large to permit use of these equations in design, even though at least one of the displacement equations fit the displacement data very well according to normal statistical standards.

5.6. Recommendation: In view of the fact that only partial success was achieved in this first attempt to model the observed vector fields measured in the 30 test sections used in this study, it is recommended that another trial or trials be undertaken and continued until a model meeting all the specifications stated in Article 4.2 is produced. The basic data, believed to be unique in the field of pavement research, is available in full in Appendix B for this purpose.

6. List of References

1. "A Technique For Measuring the Displacement Vector Throughout the Body of a Pavement Structure Subjected to Cyclic Loading," William M. Moore and Gilbert Swift, Research Report 136-2, Texas Transportation Institute, August, 1971.

2. "Evaluation Of the Stiffness of Individual Layers in a Specially Designed Pavement Facility From Surface Deflections," F. H. Scrivner and William M. Moore, Research Report 32-8, Texas Transportation Institute, June, 1966.

3. "A Systems Approach Applied to Pavement Design and Research," W. Ronald Hudson, B. Frank McCullough, Frank H. Scrivner, and James L. Brown, Texas Highway Department, Austin, Texas, 1970.

Appendix A

Equations Used In The Analysis

Two basic assumptions were made:

(1) A neutral axis exists.

(2) The dilatation (bulk strain) is zero.

Let Z = z at the neutral axis. Then Z is given by Equation 4.6 in the main body of the report.

If the bulk strain is zero, it follows that

$$\frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \tag{A1}$$

It can be shown that Equation A1 is satisfied if

$$u = \frac{1}{r}(\alpha_1 + \alpha_2 + \alpha_3)\frac{d\beta}{dz} \tag{A2}$$

$$w = -\frac{1}{r}\left(\frac{d\alpha_1}{dr} + \frac{d\alpha_2}{dr} + \frac{d\alpha_3}{dr}\right)\beta \tag{A3}$$

where α_1, α_2, α_3 are arbitrary functions of r only, and β is an arbitrary function of z only.

The form of α_i (i = 1, 2, 3) that was chosen is

$$\alpha_i = -\frac{Ai}{2b_i}(1 - e^{-b_i r^2}) \tag{A4}$$

where $b_1 = .005$, $b_2 = .0005$, $b_3 = .00005$ (A5)

and $\beta = ce^{-(\frac{z}{Z} - 1)^2 \ln c}$ (A6)

The values assigned to b_1, b_2 and b_3 were found by a trial-and-error method of regression, using Equation (A8) below, with z held constant. Once selected, these values of the b_i fitted w-data, with z fixed, very closely: R^2 was in the order of 0.99 for nearly all sections at all depths z.

From (A2), (A4) and (A6) the full expression for u was found to be

$$u = \left(\frac{c \ln c}{Z}\right)\left(\frac{z}{Z} - 1\right) e^{-\left(\frac{z}{Z} - 1\right)^2 \ln c}$$

$$\times \frac{1}{r}\left[\frac{A_1}{b_1}\left(1-e^{-b_1 r^2}\right) + \frac{A_2}{b_2}\left(1-e^{-b_2 r^2}\right) + \frac{A_3}{b_3}\left(1-e^{-b_3 r^2}\right)\right] \quad \text{(A7)}$$

From (A3), (A4) and (A6) the full expression for w was found to be

$$w = c\left(A_1 e^{-b_1 r^2} + A_2 e^{-b_2 r^2} + A_3 e^{-b_3 r^2}\right) e^{-\left(\frac{z}{Z} - 1\right)^2 \ln c} \quad \text{(A8)}$$

It can be shown from (A8) that along any vertical line (r = a constant), w(r,Z) is the maximum value of w(r, z) along that line. It can also be shown from A(8) that

$$c = \frac{w(r,Z)}{w(r,o)} \quad \text{(A9)}$$

where r is fixed in value. Thus, along any vertical line, r = a constant, c is the ratio of the maximum value of w to the value of w at the surface.

It can be shown with little effort that Equations (A7) and (A8) satisfy the simple boundary conditions specified in Article 4.2C in the main body of the report. It can also be shown from Equation 4.6 that Z meets the consistency test specified in Article 4.2D, and from this it follows that Equations A7 and A8 also meet that test.

When (A7) and (A8) were tested against the data by non-linear regression analysis section-by-section, it was found that the constants C, A_1, A_2, A_3 had different values in the two equations, and furthermore, that these values did not carry across from one section to the next, although high values of R^2 and small errors resulted from analyses of the w-data, section-by-section, and a fair fit was obtained using the u-data. A listing of the values of C, A_1, A_2, A_3 for each section, together with a comparison of prediction with replication errors, will be found in Tables A1 and A2.

By differentiating (A7) and (A8) the following expressions for strain were found.

Table A1: Results of Analyses of Horizontal Displacements, u (For model see equation A7)

Section	Z (in.)	C	A_1	A_2	A_3	R^2	Pred. Error (mils)	Rep. Error (mils)
1	4.9	1.102	.8090	.0242	-.0144	.77	.034	.017
2	7.3	1.029	.0497	.1998	-.0049	.83	.009	.009
3	6.5	1.067	.8854	.0820	-.0160	.74	.027	.011
4	10.6	1.065	-.0748	.1536	-.0024	.93	.004	.005
5	5.6	1.072	.7466	.0752	-.0173	.68	.031	.019
6	9.1	1.080	.3043	.2176	-.0259	.86	.014	.015
7	9.1	1.080	.0889	.1545	-.0108	.89	.008	.006
8	14.0	1.118	.0307	.1061	-.0025	.60	.013	.004
9	5.8	1.083	.6657	-.0273	-.0037	.62	.023	.009
10	9.0	1.080	.0577	.0630	-.0080	.72	.019	.006
11	9.0	1.159	.4543	-.0085	-.0004	.51	.025	.006
12	11.0	1.050	.4311	.1081	-.0004	.55	.017	.007
13	6.7	1.025	.2489	.1040	-.0047	.62	.013	.008
14	9.0	1.046	.1095	.0962	-.0009	.68	.009	.005
15	9.0	1.050	.0976	.0741	-.0020	.66	.009	.006
16	13.6	1.050	-.0061	.1233	-.0104	.92	.004	.002
17	8.4	1.065	.1375	.1332	-.0114	.79	.011	.007
18	8.7	1.064	.2374	.1331	-.0068	.68	.016	.011
19	8.6	1.050	.1163	.1434	-.0092	.76	.011	.011
20	6.8	1.027	.7987	.1796	-.0091	.78	.012	.007
21	10.0	1.054	-.1177	.1351	-.0026	.87	.006	.008
24	8.5	1.083	.4159	.0881	-.0111	.72	.018	.008
25	8.1	1.033	.0606	.1297	-.0022	.83	.006	.005
26	7.8	1.069	.4517	.0925	-.0102	.73	.016	.012
27	9.7	1.065	.1074	.1560	-.0080	.86	.008	.009
28	7.7	1.042	.0153	.1666	-.0097	.86	.007	.002
29	8.6	1.031	.0309	.1244	-.0001	.78	.007	.004
31	3.1	1.012	.4023	.1443	-.0330	.77	.032	.017
32	2.7	1.016	.4488	-.3429	-.0164	.66	.091	.022
33	2.9	1.009	.9650	.2393	-.0463	.77	.030	.015
				Average		.73	.018	.009

Table A2: Results of Analyses of Vertical Displacements, w (for Model see Equation A8)

Section	Z (in).	C	A1	A2	A3	R^2	Pred. Error (mils)	Rep. Error (mils)
1	4.9	1.014	.3756	.8089	.0914	.97	.062	.056
2	7.3	1.017	-.0258	.2570	.1867	.98	.193	.023
3	6.5	1.020	.3810	.5666	.1165	.98	.037	.068
4	10.6	1.026	-.0154	.1120	.1446	.98	.118	.050
5	5.6	1.019	.4295	.9163	.1215	.98	.062	.062
6	9.1	1.040	.1665	.6293	.1332	.98	.032	.058
7	9.1	1.030	-.0246	.2960	.1732	.98	.020	.028
8	14.0	1.057	.6711	.1469	.1642	.98	.017	.023
9	5.8	1.010	.3627	.3028	.1517	.96	.037	.010
10	9.0	1.029	.3503	.2743	.1385	.97	.032	.055
11	9.0	1.030	.3518	.2618	.1341	.96	.037	.032
12	11.0	1.032	.3298	.1900	.1484	.96	.030	.032
13	6.7	1.011	.0487	.2857	.1509	.98	.021	.028
14	9.0	1.019	.0059	.1641	.1735	.98	.016	.013
15	9.0	1.033	.0043	.1870	.2001	.94	.030	.020
16	13.6	1.026	.0087	.0688	.1745	.98	.010	.009
17	8.4	1.024	.0184	.2752	.1925	.98	.019	.017
18	8.7	1.030	.0447	.3667	.1979	.98	.028	.035
19	8.6	1.025	-.0245	.2847	.1841	.98	.019	.028
20	6.8	1.016	.2912	.3039	.2044	.98	.024	.030
21	10.0	1.030	-.0078	.1793	.1978	.99	.014	.013
24	8.5	1.035	.2763	.4331	.1575	.98	.030	.016
25	8.1	1.017	.0056	.1585	.1872	.99	.012	.012
26	7.8	1.024	.0671	.3767	.1765	.99	.021	.031
27	9.7	1.036	-.0176	.2751	.2009	.98	.021	.009
28	7.7	1.019	-.0020	.2825	.1882	.98	.019	.013
29	8.6	1.014	.0089	.1174	.1363	.98	.011	.019
31	3.1	1.005	.6868	.0152	.0942	.94	.104	.020
32	2.7	1.005	.7380	.6004	.0894	.94	.137	.062
33	2.9	1.005	.5449	.8895	.1044	.94	.092	.122
				Average		.97	.043	.033

$$\varepsilon_r = \left(\frac{c \ln c}{Z}\right) \left(\frac{z}{Z} - 1\right) e^{-\left(\frac{z}{Z} - 1\right)^2 \ln c}$$

$$\times \frac{1}{r^2} \left\{ \frac{A_1}{b_1} \left[(2b_1 r^2 + 1) e^{-b_1 r^2} - 1 \right] \right.$$

$$+ \frac{A_2}{b_2} \left[(2b_2 r^2 + 1) e^{-b_2 r^2} - 1 \right]$$

$$\left. + \frac{A_3}{b_3} \left[(2b_3 r^2 + 1) e^{-b_3 r^2} - 1 \right] \right\} \qquad \text{(A10)}$$

$$\varepsilon_z = \left(\frac{-2c \ln c}{Z}\right) \left(\frac{z}{Z} - 1\right)$$

$$\times (A_1 e^{-b_1 r^2} + A_2 e^{-b_2 r^2} + A_3 e^{-b_3 r^2}) e^{-\left(\frac{z}{Z} - 1\right)^2 \ln c} \qquad \text{(A11)}$$

where C, A_1, A_2, A_3 are to be taken from Table A1 for use in Equation A10, and from Table A2 for use in Equation A11.

At $r = 0$, for points directly beneath the load, (A10) and (A11) reduce to the following.

$$\varepsilon_r(o,z) = \left(\frac{c \ln c}{Z}\right) \left(\frac{z}{Z} - 1\right) e^{-\left(\frac{z}{Z} - 1\right)^2 \ln c}$$

$$\times (A_1 + A_2 + A_3) \qquad \text{(A12)}$$

$$\varepsilon_z(o,z) = \left(\frac{-2c \ln c}{Z}\right) \left(\frac{z}{Z} - 1\right) e^{-\left(\frac{z}{Z} - 1\right) \ln c}$$

$$\times (A_1 + A_2 + A_3) \qquad \text{(A13)}$$

where the values of C, A_1, A_2, A_3 are to be taken from Table A1 for use in Equation (A12) and from Table A2 for use in Equation (A13).

The **strains** predicted from (A12) and (A13) appeared unreasonable when compared with strains estimated directly from plots of the measured data at nearby points, as previously mentioned in the main body of the report.

The analysis leading to estimates of modular ratios and values of the depth, Z, of the neutral axis, was independent of the w-data, and required from the u-data only the apparent distance from the surface to the zero contour of u (see, for example, Figure 2.1). Because of the vagaries of this contour line in most sections at distances from the load exceeding about 100 inches ($r > 100''$), it was decided to select, with some subjective judgement, the value of Z from that position of the contour lying between $r = 11.7''$ and $r = 100''$. Even within this range, the depth to the contour line varied over a considerable range for many sections and these ranges were different for the two sets of replicate measurements. These ranges are shown graphically in Figure A1. Also shown in that figure is the computed value of Z for each section. It is clear that although the computed values lie within the observed overall range of values observed (when both replicates are considered), the concept of a horizontal neutral axis should be considered tentative. It was adopted because it seemed to yield a proper ordering of the six modular ratios from a very simple mathematical model (Equation 4.6).

To make clear what is meant by "computed" values of Z, the following explanation is offered.

Equation 4.6 was first transformed into a model in which "observed" values of Z were used, leaving only the modular ratios as unknown constants. These constants, or modular ratios, were found by linear regression analysis, and then used in Equation 4.6 (in the form in which it appears in the text) to compute Z for each test section. As indicated in Figure A1, the computed values of Z generally lie within the range observed in one or both replicates.

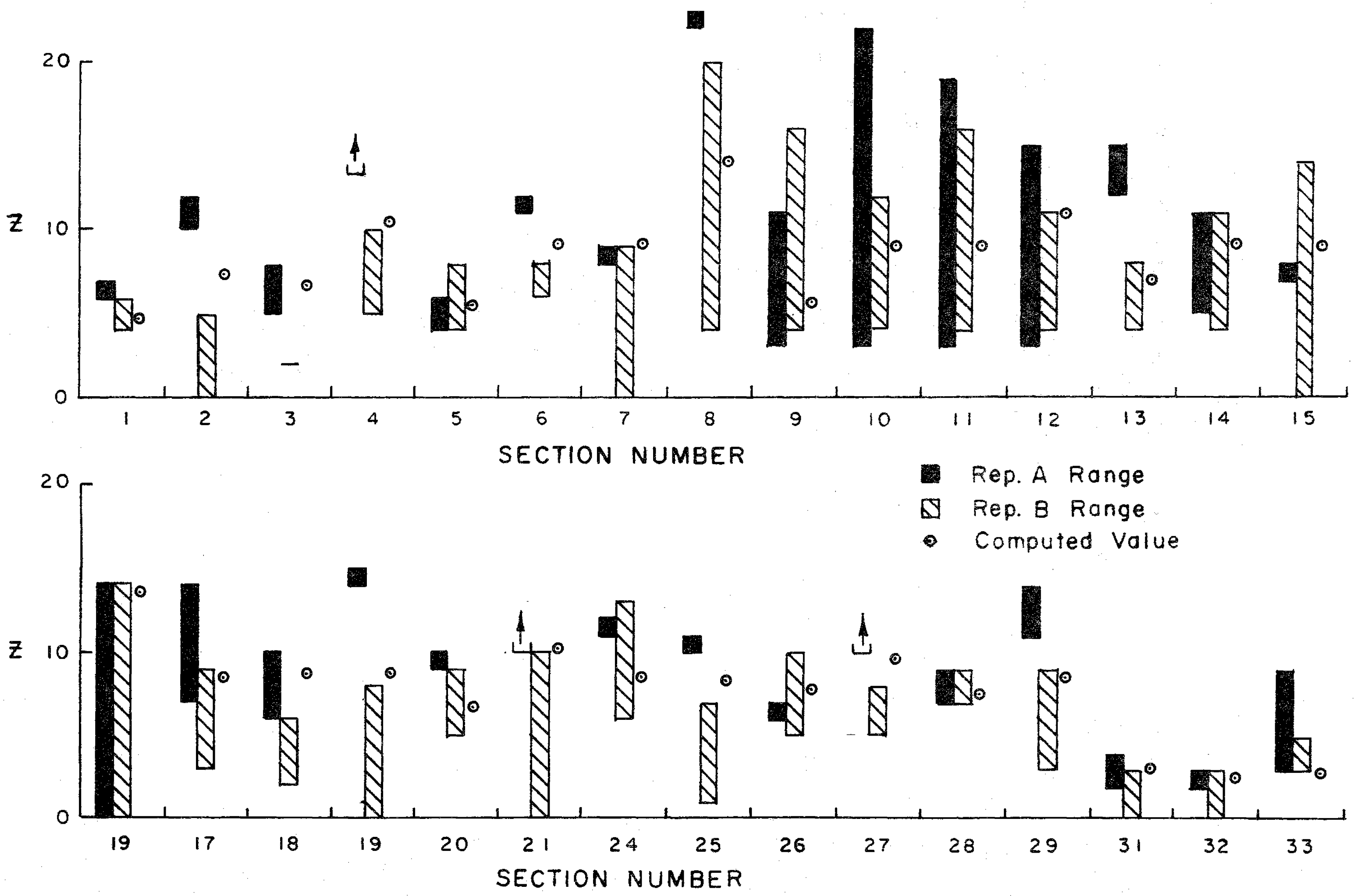


Figure A.1: Range of observed values of depth to contour, u = 0, compared with computed values of depth to the assumed neutral axis.

PREVIOUS REPORTS OF STUDY

Research Report 136-1, "Seasonal Variations of Pavement Deflections in Texas," by Rudell Poehl and Frank H. Scrivner.

Research Report 136-2, "A Technique for Measuring the Displacement Vector Throughout the Body of a Pavement Structure Subjected to Cyclic Loading," by William M. Moore and Gilbert Swift.

Research Report 136-3, "A Graphical Technique for Determining the Elastic Moduli of a Two-Layered Structure from Measured Surface Deflections," by Gilbert Swift.

Research Report 136-4, "An Empirical Equation for Calculating Deflections on the Surface of a Two-Layer Elastic System," by Gilbert Swift.

Research Report 136-5, "Elastic Moduli Determination for Simple Two Layer Pavement Structures Based on Surface Deflections," by William M. Moore.