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TWO-DIMENSIONAL ANALYSIS OF BACKWATER AT BRIDGES
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This report represents the results of a study to develop, modify and apply a microcomputer version of the FESWMS-TX model for two-dimensional analysis of backwater at bridges.

The authors wish to thank the Texas State Department of Highways and Public Transportation for their sponsorship of the work and express appreciation to the contact persons, Mr . Eric Friedrich and Mr. Peter Smith. The manuscript was typed at the center for Research in Water Resources, and appreciation is extended to their staff.

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#### Abstract

A finite element modeling system for analyzing twodimensional hydrodynamics in the horizontal plane has been modified to run on an IBM personal computer. Some theoretical aspects about the method of finite elements in relation to the governing equations are discussed. An extensive guideline is given for data preparation and model implementation. A review of previous work using the same finite element program done within and outside the United States has been given to show wide acceptance and applicability of this particular computer code. Two example applications on the mainframe version of the same computer code are reproduced to demonstrate some of the capabilities of the PC version. A natural stream with a bridge crossing was analyzed to show that early convergence of the finite element solution to the two-dimensional free-surface flow equations is possible by the specification of an initial solution obtained from one-dimensional backwater calculations. A fully interactive and graphics-oriented computer modeling system is suggested as the primary goal for future research.


The objective of this project has been to develop a PC version of the FESWMS-TX model for the two-dimensional analysis of backwater at highway bridges. Modifications and additions to the type and amount of input data to the FiniteElement Surface-Water Modeling System (FESWMS-TX) were incorporated so that: a) cross sectional information in a format directly compatible with existing one-dimensional backwater analysis programs (HY-7 and HEC-2) can be extracted from the finite element network; b) input data in the new as well as the old (prior to FESWMS-TX) format can be read-in and processed accordingly; c) an additional method of subdividing total flow along an inlet section of the finite element network can be used as an option; d) initial conditions supplied by the user (e.g., from one-dimensional backwater calculations) can be incorporated; and e) all data manipulation, calculations, and plotting can be performed using an IBM personal computer.

Input requirements were discussed in more detail than in previous applications. A user's manual was written for FESWMS-TXPC, the microcomputer version of the Einite-Element Surface-Water Modeling System (FESWMS-TX). The possibility of providing a good initial "guess" (and thus, guarantee early convergence) to the iterative procedure in solving the governing two-dimensional equations for free-surface flow was explored. This is accomplished by using results obtained from a one-dimensional backwater analysis program (HY-7) which was especially written to handle flow computations in the vicinity of bridges. From the results obtained, a onedimensional code can indeed be a valuable support program. Several demonstration runs were made to illustrate the capabilities of the PC version.

## IMPLEMENTATION STATEMENT

The FESWMS-TX microcomputer model can be applied by the Texas State Department of Highways and Public Transportation. The user's manual should make the application of the model self-explanatory once the model is put on the Texas state Department of Highways and Public Transportation computer facilities.
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CHAPTER 1. INTRODUCTION

### 1.1 Background

Many problems in hydraulic engineering require the determination of water depths and velocities in the twodimensional horizontal domain, such as bays, estuaries, harbors and rivers with wide flat flood plains. Of particular interest in this report is the determination of water depths and velocities in the vicinity of bridges located on highway crossings.

In investigating backwater effects at bridges several approaches may be used - analytical solutions, numerical models, and physical models (McAnally, et al., 1983). Analytical solutions are those in which depth and velocities are obtained directly from mathematical expressions. Unfortunately, no known closed-form solution (a solution obtained by substituting values of the independent variables into the equations) exists for the complex two-dimensional free-surface flow equations. As such, the mathematical modeler is left with no choice but to substantially simplify the governing equations. For example, the one-dimensional Manning equation incorporates all effects of energy dissipation into a single parameter $n$ and is thus an easy equation to evaluate. In general, the usefulness of analytical solutions decline with increasing complexity of geometry or increasing detail of results desired.

Physical models, as the name implies, physically scale down and reproduce a given hydraulic phenomenon in the laboratory. In addition to the fact that conflicts in similitude requirements usually arise with this kind of modeling, the apparent advantage of physical models is overshadowed by the high costs.

Numerical modeling, on the other hand, employs special computational methods, such as iteration and approximation, to solve mathematical expressions that do not have closed-form solutions. A numerical model or modeling system, as used in this report, is the combination of a) computer programs (codes), b) a geometric mesh or grid (network), and c) a set of input parameters representing a physical location or site (study area). With economy as a main consideration the logical way to go about analyzing backwater at bridges would be through numerical modeling.

Earlier numerical models employed the finite difference technique in arriving at a solution to the flow equations at bridges (Lee and Froehlich, 1985). Basically, the finite difference method (FDM) approximates derivatives by taking
differences in the values of the variables at finite intervals of space and time. A shortcoming of this method is that it only allows discretization of space into regular grids producing "rough" edges at the boundaries of the study area. Complex topography and irregular boundaries prompted engineers to resort to more sophisticated numerical methods such as finite element methods (FEM), and more recently, boundary element methods (BEM).

The finite element method approximates a solution to flow equations by subdividing the area of interest into smaller subareas, referred to as finite elements. The dependent variables (e.g., water surface elevations and velocities) are approximated over each element by continuous functions which are interpolated in terms of unknown point (nodal) values of the variables. An error, defined as the deviation of the approximated solution from the correct solution, is minimized. Then, when the boundary conditions are imposed, a set of solvable simultaneous equations in terms of nodal values of the dependent variables are formed. The solution is smooth and continuous over each individual finite element.

As new and more sophisticated computer methods in modeling hydraulic problems are devised, unprecedented progress in computer hardware development is also being made. In particular, personal computers (or microcomputers), have had tremendous improvements, in terms of speed and amount of data that can be manipulated, over the past few years. Graphic displays on a microcomputer's screen provide visual opportunities that not only extend one's modeling capabilities, but also encourage the enthusiastic involvement of the modeler in the real physical behavior of a hydraulic system such as bridge approaches.

### 1.2 The Finite-Element Surface-Water Modeling System

In 1971-1972, the U.S. Army Corps of Engineers commissioned Resource Management Associates, Lafayette, CA to develop twodimensional finite element models for fluid flow. One of these hydrodynamic models came to be known as RMA-2. Although developed to be as general as possible, this model has gone through several modifications with a number of individuals or groups of individuals attempting to verify the model at different locations within the United States. The model allows steady state and time dependent solutions. Preprocessor and postprocessor have been developed to simplify network preparation and allow automatic plotting of networks, velocity vectors, and contours of water depth. The United States Geological Survey (USGS) used RMA-2 as the core program to develop the Finite-Element Surface-Water Modeling System
(FESWMS) (Lee et al., 1983). FESWMS combined RMA-2 with preprocessing and postprocessing programs to make the system more accessible to the user. Mays and Taur (1983) of the University of Texas made it further user-oriented by modifying some input data requirements more applicable to field data available. The latter modification is known as FESWMS-TX. It consists of four interacting programs that can be run in a semiinteractive mode as explained in the flowchart shown in Fig. 1.1. The PC version developed in this project is called FESWMSTXPC.

## 1.3 objectives

This report focuses on two-dimensional flow analysis at bridges related to, but not limited to the use of microcomputers. Specifically, the following are the objectives:

1. Evaluate the performance of a finite element code (RMA2) in a microcomputer (IBM personal computer or IBM PC) environment applied to a steady-state analysis of highway crossings.
2. Describe the performance of RMA-2 with specified initial starting conditions obtained from one-dimensional backwater computations.

### 1.4 Scope of Work

In order to meet the above objectives, the following scope of work has been completed.

1. Revise one subroutine in RMA-2 based on the need to more accurately define the upstream boundary condition in freesurface flow. The original version of RMA-2 required the specification of unit discharge (flow per unit width) for all nodal points defining an inflow section. From a user's point of view, this requirement is quite cumbersome to satisfy. Mays and Taur (1983) modified the code such that the user need only to input a total discharge for each inlet section; the program, in turn, subdivides total flow in proportion to the cross sectional area tributary to each node at the inlet section in question. Subroutine USSET was specifically written for this purpose. Henderson (1966) suggests a different method to subdivide flow within a cross section based on velocity and energy loss considerations. This work incorporates this alternative method (refer to Chapter 3) and subroutine USSET was modified accordingly.


Fig. 1.1. - Application Procedure for FESWMS-TXPC


Fig. 1.1. - Application Procedure for FESWMS-TXPC
(continued)
2. Develop a microcomputer version of RMA-2 and all its support programs for the IBM personal computer. Microsoft's Fortran Compiler version 3.3 (1985) and PLOTWORKS' Plot88 Software Library (1986) were used to perform the conversion from the revised mainframe version to the microcomputer version.
3. Improved model documentation and computer-aided graphical display capabilities are both essential areas to make two-dimensional modeling methods more practical for engineering applications in general. To meet this end, the user's manual for computer program RMA-2 and its support programs (RMA-1, RMA1PLT, and RMA2PLT) were rewritten, giving particular attention to correct card sequencing and explanation of some options (discussed in Chapter 3) added by the author and incorporated in RMA-1 and RMA-2. Likewise, the plotting programs RMA1PLT and RMA2PLT were rewritten to enhance user visualization of the problem in a microcomputer environment.
4. Explore the possibility of using a one-dimensional backwater code, such as HY-7 in conjunction with the twodimensional backwater code.
a.) Unlike one-dimensional codes where specification of flows and/or water surface elevations is required at each end of a river reach, the two-dimensional code RMA-2 requires a specification of initial conditions for all nodal points in the finite element grid. A good estimate of the downstream water surface elevation can be made by adding more cross sections beyond the downstream extent of the flood plain and doing a step backwater analysis up to and including the downstream section of the original network (Lee and Bennett, 1981). This preliminary task can be performed using any one-dimensional backwater code that can handle subcritical and supercritical flows.
b.) Convergence of the Newton-Raphson algorithm requires that the first "guess" at the solution be fairly close to the final solution (Gee, 1983). RMA-2, by default, makes an initial guess of zero velocity for all nodal locations and a horizontal water surface (specified by the user). Since the main concern of this report is the determination of flows and water surface elevations at the vicinity of bridges, a better initial guess could possibly be obtained from one-dimensional water surface profile computations.

It was shown (Shearman, et al., 1979) that HY-7 has the capability of determining water surface profiles through bridges based on relatively recent developments in bridge backwater analysis. More of HY-7's capabilities and advantages over other existing step-backwater analysis models are discussed in Chapter 3.

### 2.1 Depth-Averaged Two-Dimensional Flow Equations

Computer code RMA-2 was written to solve the Navier-Stokes equations (or simply flow equations) reduced to two-dimensional form. Assuming hydrostatic pressure distribution and momentum correction factors of unity, the depth-averaged two-dimensional flow can be described by three nonlinear partial differential equations (PDE's), one for the conservation of mass and two for the conservation of momentum (Pritchard, 1971):

Continuity:

$$
\begin{equation*}
\frac{\partial h}{\partial t}+\frac{\partial(u h)}{\partial x}+\frac{\partial(v h)}{\partial y}=0 \tag{2.1}
\end{equation*}
$$

Momentum:

$$
\begin{align*}
& \frac{\partial u}{\partial t}+u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}+g\left(\frac{\partial h}{\partial x}+\frac{\partial z_{o}}{\partial x}\right)-\frac{1}{\rho h}\left[\frac{\partial}{\partial x}\left(\varepsilon_{x x} h \frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon_{x y} h \frac{\partial u}{\partial y}\right)\right] \\
& -2 \omega v \sin \phi+\frac{g u}{c^{2} h}\left(u^{2}+v^{2}\right)^{1 / 2}-\frac{\zeta}{h} v_{a}^{2} \cos \psi=0  \tag{2.2}\\
& \frac{\partial v}{\partial t}+u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}+g\left(\frac{\partial h}{\partial y}+\frac{\partial z_{0}}{\partial y}\right)-\frac{1}{\rho h}\left[\frac{\partial}{\partial x}\left(\varepsilon_{y x} h \frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon_{y y} h \frac{\partial v}{\partial y}\right)\right] \\
& -2 \omega u \sin \phi+\frac{g v}{c^{2} h}\left(u^{2}+v^{2}\right)^{1 / 2}-\frac{\zeta}{h} v_{a}^{2} \sin \psi=0 \tag{2.3}
\end{align*}
$$

where

$$
\begin{aligned}
\mathrm{x}, \mathrm{y}= & \text { Cartesian coordinates in the positive } \\
& \text { east and north directions, respectively } \\
& \text { (feet) } \\
\mathrm{t}= & \text { time (seconds) } \\
\mathrm{u}, \mathrm{v}= & \text { depth-averaged velocity components in } \\
& \text { the x- and y-directions, respectively } \\
& \text { (feet per second), } \\
\mathrm{h}= & \text { depth (feet) } \\
\mathrm{z}_{0} \quad= & \text { bed elevation (feet) } \\
\mathrm{\rho} \quad= & \text { density of water (slugs per second) }
\end{aligned}
$$

```
        \omega = rate of the earth's angular rotation (per second)
        \phi = latitude (degrees)
        g = acceleration due to earth's gravity
        (feet per square second)
    C = Chezy coefficient (feet to the one-half
        power per second)
\varepsilon}\mp@subsup{\varepsilon}{xx}{},\mp@subsup{\varepsilon}{\textrm{xy}}{},\mp@subsup{\varepsilon}{yx}{},\mp@subsup{\varepsilon}{yy}{}= eddy viscosities (pound second per
        square foot)
    \zeta = water surface resistance coefficient
        (nondimensional)
    Va = local wind velocity (feet per second), and
    \psi = angle between the wind direction and the
    x-axis (degrees)
```

These equations were derived based on a unit control volume. The first three terms in the momentum (or motion) equations refer to the contribution of inertial forces while the remaining five terms pertain to forces associated with pressure, Coriolis effects, bottom friction and wind resistance, respectively. It should be noted that the two-dimensional freesurface equations account for energy losses through two mechanisms: bottom friction and turbulent stresses. In light of this, the Chezy equation for bottom friction in open-channel and the Boussinesq's eddy viscosity concept (turbulent stresses are proportional to velocity gradients) are incorporated in the flow equations.

Boundary conditions are set by specifying flow components (or water surface elevations) at open boundaries and zero normal (or tangential flow) at lateral boundaries. Likewise, initial conditions must be specified for time-dependent problems. The set of three PDE's in addition to the boundary and initial conditions present a well-posed initial-boundary-value problem.

A variational formulation of the above equations can be obtained by introducing the following substitutions:

$$
\begin{aligned}
& r=u h, \\
& s=v h
\end{aligned}
$$

where

$$
\begin{array}{rl}
u, ~ & v=\text { velocity in the } x \text { and } y \text { directions, respectively } \\
& \text { (primary variables) } \\
\mathrm{r}, \mathrm{~s}=\text { unit flow in the } \mathrm{x} \text { and } \mathrm{y} \text { directions, respectively } \\
& \text { (transformed variables) }
\end{array}
$$

The primary variables and their derivatives with respect to $x$ can be expressed in terms of their corresponding transformed variables as:

$$
\begin{align*}
& u=r h^{-1}  \tag{2.4}\\
& v=s h^{-1}  \tag{2.5}\\
& \frac{\partial u}{\partial x}=h^{-1} \frac{\partial r}{\partial x}-r^{-2} \frac{\partial h}{\partial x}  \tag{2.6}\\
& \frac{\partial v}{\partial x}=h^{-1} \frac{\partial s}{\partial x}-\operatorname{sh}^{-2} \frac{\partial h}{\partial x}  \tag{2.7}\\
& \frac{\partial^{2} u}{\partial x^{2}}=h^{-1} \frac{\partial^{2} r}{\partial x^{2}}-r h^{-2} \frac{\partial^{2} h}{\partial x^{2}}-2 h^{-2} \frac{\partial h}{\partial x} \frac{\partial r}{\partial x}+2 r h^{-3}\left(\frac{\partial h}{\partial x}\right)^{2}  \tag{2.8}\\
& \frac{\partial^{2} v}{\partial x^{2}}=h^{-1} \frac{\partial^{2} s}{\partial x^{2}}-s^{-2} \frac{\partial^{2} h}{\partial x^{2}}-2 h^{-2} \frac{\partial h}{\partial x} \frac{\partial s}{\partial x}+2 s h^{-3}\left(\frac{\partial h}{\partial x}\right)^{2} \tag{2.9}
\end{align*}
$$

Substituting these values, along with derivatives with respect to $y$, into Eqs. 2.1-2.3 yield the following final form of the governing flow equations.

Continuity:

$$
\begin{equation*}
\frac{\partial h}{\partial t}+\frac{\partial r}{\partial x}+\frac{\partial s}{\partial y}=0 \tag{2.10}
\end{equation*}
$$

Motion Equations:

$$
\begin{aligned}
& \left(h^{-1} \frac{\partial r}{\partial t}-h^{-2} r \frac{\partial h}{\partial t}+r h^{-2} \frac{\partial r}{\partial x}-r^{2} h^{-3} \frac{\partial h}{\partial x}+\operatorname{sh}^{-2} \frac{\partial r}{\partial y}-\operatorname{srh}^{-3} \frac{\partial h}{\partial y}\right) \\
& +g\left(\frac{\partial h}{\partial x}+\frac{\partial a_{0}}{\partial x}\right)+\frac{-\varepsilon_{x x}}{\rho}\left[h^{-1} \frac{\partial^{2} r}{\partial x^{2}}-r h^{-2} \frac{\partial^{2} h}{\partial x^{2}}-2 h^{-2} \frac{\partial h}{\partial x} \frac{\partial r}{\partial x}+2 r h^{-3}\left(\frac{\partial h}{\partial x}\right)^{2}\right]
\end{aligned}
$$

$$
\begin{align*}
& +\frac{-\varepsilon_{x y}}{\rho}\left[h^{-1} \frac{\partial^{2} r}{\partial y^{2}}-r h^{-2} \frac{\partial^{2} h}{\partial y^{2}}-h^{-2} \frac{\partial h}{\partial y} \frac{\partial r}{\partial y}+r h^{-3}\left(\frac{\partial h}{\partial y}\right)^{2}\right]-2 \omega s h^{-1} \sin \phi \\
& +\frac{g h^{-3}}{c^{2}} r\left(r^{2}+s^{2}\right)^{1 / 2}-\frac{\zeta v_{a}^{2}}{h} \cos \psi=0 \\
& \left(h^{-1} \frac{\partial s}{\partial t}-h^{-2} s \frac{\partial h}{\partial t}+\operatorname{sh}^{-2} \frac{\partial s}{\partial y}-s^{2} h^{-3} \frac{\partial h}{\partial y}+r h^{-2} \frac{\partial s}{\partial x}-r \operatorname{sh}^{-3} \frac{\partial h}{\partial x}\right) \\
& +g\left(\frac{\partial h}{\partial y}+\frac{\partial a_{0}}{\partial y}\right)+\frac{-\varepsilon_{y x}}{\rho}\left[h^{-1} \frac{\partial^{2} s}{\partial x^{2}}-s h^{-2} \frac{\partial^{2} h}{\partial x^{2}}-2 h^{-2} \frac{\partial h}{\partial x} \frac{\partial s}{\partial x}+2 s^{-3}\left(\frac{\partial h}{\partial x}\right)^{2}\right. \\
& +\frac{-\varepsilon_{y y}}{\rho}\left[h^{-1} \frac{\partial^{2} s}{\partial y^{2}}-\operatorname{sh}^{-2} \frac{\partial^{2} h}{\partial y^{2}}-h^{-2} \frac{\partial h}{\partial y} \frac{\partial r}{\partial y}+\operatorname{sh}^{-3}\left(\frac{\partial h}{\partial y}\right)^{2}\right]-2 \omega r h^{-1} \sin \phi \\
& +\frac{g h^{-3}}{c^{2}} s\left(r^{2}+s^{2}\right)^{1 / 2}-\frac{\zeta v_{a}^{2}}{h} \sin ^{2} \psi=0 \tag{2.12}
\end{align*}
$$

### 2.2 The Method of Weighted Residuals and the Finite Element Method

Method of weighted residuals is the name given to a variety of approximate methods for the solution of partial differential equations. All of these methods are characterized by a common process described briefly as follows.

The unknown function (the solution to the differential equation) is expressed in terms of a series of basis functions multiplied by unknown coefficients. This series is substituted in the differential equation. Generally, the resulting differential equation is not satisfied since a residual ( $\mathrm{Re}_{\mathrm{e}}$ ) appears and the unknown coefficients are computed through a residual minimization process (Zienkiewicz, 1977).

A brief mathematical illustration is given. Assume a differential operator $£$ generating the equation:

$$
\begin{equation*}
£(\varphi)=f \tag{2.13}
\end{equation*}
$$

where $\varphi=\varphi(x, y)$ is defined in a domain $D$ bounded by a surface $\Gamma$. The equation is completed by appropriate boundary conditions on $\Gamma$.

The method of weighted residuals is analyzed in the following steps:
(1) Assume a functional form of the field variable (the unknown function)

$$
\begin{equation*}
\hat{\varphi}=\sum_{i=1}^{n} N_{i} \varphi_{i} \tag{2.14}
\end{equation*}
$$

where $N_{i}(x, y, z)$ are the assumed basis functions satisfying (approximately) the differential equation and the boundary conditions; $\varphi_{i}$ are the undetermined coefficients which can be either constants or functions of time (in the case of time dependent problems); and $n$ is the total number of discrete locations (nodes) in the whole solution domain.
(2) As the approximate form of $\varphi$ does not satisfy the differential equation the substitution of $\hat{\varphi}$ in Eq. 2.13 generates a residual

$$
\begin{equation*}
\mathrm{R}_{\mathrm{e}}=£(\hat{\varphi})-£ \tag{2.15}
\end{equation*}
$$

The optimization in the approximation of $j$ to exact solution $j$ is achieved through the minimization of the residual $R_{e}$. According to the method of weighted residuals minimization requires 'orthogonality' of $R_{e}$ to a set of weighting functions expressed mathematically as,

$$
\begin{equation*}
\int_{D}\left(£(\hat{\varphi}) W_{i} d D=0, i=1 \ldots n\right. \tag{2.16}
\end{equation*}
$$

(3) Integration leads to $n$ algebraic equations, usually coupled to a system of simultaneous equations with respect to
the $n$ unknown coefficients $\varphi_{i}$. The solution of the system gives the function in open form.

It can be proved mathematically that $\hat{\varphi}$ converges to $\varphi$ with the proper selection of basis function $N_{i}$, weighting function $W_{i}$, and increasing $n$. If $W_{i}$ are functions of a complete set and $n \rightarrow \infty$, then the only function orthogonal to all the elements of a complete set is zero.

Special names are assigned to the method according to the form of the $W_{i}$ functions. For $W_{i}=N_{i}$, the method is known as the Galerkin method. Since the $N_{i}$ functions are defined and extended over the solution domain they all contribute to the computation of the values of the field variable. The finite element method (FEM) provides a systematic way of forming basis functions. The idea of using basis functions defined over only part of the solution domain is the opening step in finite element analysis.

First, consider domain discretization. From finite difference methods, a mesh comes in the form of a grid over which the solution is established. In finite element methods, the meshes may be line segments, triangles or quadrilaterals, cubes or orthogonal parallelepipeds for $1-D, 2-D$ and $3-D$ problems, respectively. For the 2-D problem, rectilinear triangular finite elements will be described here in more detail, although many other elements have been proposed and described by Chung (1978), Heubner (1975), and Zienkiewicz (1977).

Second, let us define the basis functions of the field variable (unknown function) locally, extending over some elements only. Several functions may be used: linear, quadratic, cubic and higher-degree polynomials. With these in mind, it is appropriate to call these local basis functions shape functions. The variation of the field variable over an element is expressed in terms of the shape functions and the values of the field variable, a process known as piecewise approximation. An example for the 2-D case will clarify these ideas.

Consider a rectilinear triangular element (e) defined by the coordinates ( $x_{i}, Y_{i}, x_{j}, Y_{j}, x_{k}, y_{k}$ ). (note: The subscript i used in this example is used in terms of the local system defined by element (e) while the subscript $i$ mentioned earlier refers to the global system defined by domain D.) A piecewise approximation of the field variable $\varphi$ can be achieved through the nodal values and linear shape functions $N_{i}, N_{j}, N_{k}$ having the form of pyramids. Fig. 2.1 shows a typical rectilinear
triangular element (on the $x-y$ plane) with the linear shape function $N_{i}$ over it.


Fig. 2.1. - Finite Element 'Linear' Triangle (e)

It can be observed that the shape function $N_{i}$ takes on the value 1 at node $i$ and zero for the other nodes defining element (e). This characteristic is typically referred to in finite elements as the basis function having "local support". $N_{i}$ can be expressed analytically by the equations:

$$
\begin{equation*}
N_{i}^{(e)}=\frac{a_{i}^{(e)}+b_{i}^{(e)}+c_{i}^{(e)} Y}{2 \Delta} \tag{2.17}
\end{equation*}
$$

where

$$
\begin{equation*}
a_{i}^{(e)}=x_{j} y_{k}-x_{k} y_{j} ; \quad b_{i}^{(e)}=y_{j}-y_{k} ; \quad c_{i}^{(e)}=x_{k}-x_{j} \tag{2.18}
\end{equation*}
$$

and

$$
2 \Delta=\left|\begin{array}{ccc}
1 & x_{i} & y_{i}  \tag{2.19}\\
1 & x_{j} & y_{j} \\
1 & x_{k} & y_{k}
\end{array}\right|
$$

Similarly, the $N_{j}, N_{k}$ shape functions can be expressed through cyclic permutation of the indices $i \rightarrow j \rightarrow k \rightarrow i . E q .2 .19$ tells us that the value of the determinant happens to be twice the
area of the triangular element we considered in the $x-y$ plane. The requirement of the coordinates of the vertices of the triangle being specified in the counterclockwise direction (see element identity card as defined in Appendix B) is explained by the fact that areas are always positive and that the value of the determinant, which is one-half of this area, depends on the order by which the coordinates of the vertices of the triangle are specified.

The $\hat{\varphi}^{(\mathrm{e})}$ expression is:

$$
\begin{equation*}
\hat{\varphi}^{(\mathrm{e})}=\mathrm{N}_{i}^{(\mathrm{e})} \varphi_{i}+\mathrm{N}_{\mathrm{j}}^{(\mathrm{e})} \varphi_{\mathrm{j}}+\mathrm{N}_{\mathrm{k}}^{(\mathrm{e})} \varphi_{\mathrm{k}}=[\mathrm{N}]^{(\mathrm{e})}\{\varphi\}^{(\mathrm{e})} \tag{2.20}
\end{equation*}
$$

where the term on the left-hand side of the equation refers to the field variable and the term on the right-hand side of the equation refers to the product of the row vector of shape functions [ $\mathrm{N}^{(e)}$ and column vector of nodal values of the field variable $\{\varphi\}^{(e)}$, respectively.

Along the whole solution domain, divided into $M$ elements, the function $\varphi$ is expressed approximately by the sum:

$$
\begin{equation*}
\hat{\varphi}=\sum_{e=1}^{M} \hat{\varphi}^{(e)}=\sum_{e=1}^{M}[N]^{(e)}\{\varphi\}^{(e)} \tag{2.21}
\end{equation*}
$$

For higher order shape functions (and thus, non-linear variation of the field variable over the element) more than three nodal points are required, usually spread over the sides of the triangular elements. Elements of other than triangular form and higher order shape functions are described in Chung (1978). Quadratic and cubic elements are the names given to elements used with 2nd and 3rd order shape functions.

For practical purposes we are more interested in the nodal values of the field variable $\{\varphi\}(e)$ rather than the field variable itself (the unknown function). The application of the approximate forms $\varphi^{(e)}$ to the differential equation and the use of the Galerkin method lead to equations for the element (e) containing as unknowns the nodal values $\varphi_{i}, \varphi_{j}, \varphi_{k}$ for the twodimensional case. Substituting the last equation into Eq. 2.16 leads to :

$$
\begin{equation*}
\int_{D}[£(\hat{\phi})-f] N_{i} d D=\int_{D} R_{e} N_{i} d D=0, \quad i=1,2, \ldots n \tag{2.22}
\end{equation*}
$$

Eq. 2.22 represents a set of $n$ equations which may be solved for all nodal values of the field variable $\left\{\varphi_{i}\right\}$ where subscript i is now in terms of the global system. The last step is known as the assemblage stage wherein a set of simultaneous equations is formed for the entire domain. Each element is accounted for in the global system by direct addition of its influence on the field variable at appropriate nodal location.

The nonlinearity of a problem, as the case is for the set of partial differential equations governing two-dimensional free-surface flow, implies nonlinearity in the system of equations formed by the assemblage stage. By using the NewtonRaphson method, a derivative approach based upon the error of the previous solution can be used to obtain a solution to the nonlinear system of equations. a brief description follows.

The objective of this technique is to determine $\mathrm{x}_{1}$, . . . , $\mathrm{x}_{\mathrm{n}}$ such that the functionals $\mathrm{f}_{1}$, : . . , $\mathrm{f}_{\mathrm{n}}$ are approximately zero. These functionals which form the system of nonlinear equations are evaluated in terms of $\mathrm{x}_{1}$, . . . , $\mathrm{x}_{\mathrm{n}}$ :

$$
\begin{gather*}
f_{1}\left(x_{1}, x_{2}, \cdot \cdot \cdot, x_{n}\right)=f_{1}\left(x_{i}\right) \\
f_{2}\left(x_{1}, x_{2}, \cdot \cdot \cdot, x_{n}\right)=f_{2}\left(x_{i}\right) \\
\cdot  \tag{2.23}\\
\cdot \\
f_{n}\left(x_{1}, x_{2}, . . .\right.
\end{gather*}
$$

Using Taylor series expansion, one can evaluate these functionals at $x_{1}+h_{1}$, . . , $x_{n}+h_{n}$ by expanding about the initial values $x_{1}, \ldots, x_{n}$. Thus

$$
\begin{gathered}
f_{1}\left(x_{1}+h_{1}, x_{2}+h_{2}, \ldots, x_{n}+h_{n}\right)=f_{1}\left(x_{i}\right) \\
f_{2}\left(x_{1}+h_{1}, x_{2}+h_{2}, \ldots, x_{n}+h_{n}\right)=f_{2}\left(x_{i}\right) \\
\dot{!} \\
f_{n}\left(x_{1}+h_{1}, x_{2}+h_{2}, \ldots, x_{n}+h_{n}\right)=f_{n}\left(x_{i}\right)
\end{gathered}
$$

$$
\begin{gather*}
f_{1}\left(\bar{x}_{i}\right)=f_{1}\left(x_{i}\right)+\sum_{j=1}^{n} h_{j} \frac{\partial f_{1}\left(x_{i}\right)}{\partial x_{j}}  \tag{2.24}\\
f_{2}\left(\bar{x}_{i}\right)=f_{2}\left(x_{i}\right)+\sum_{j=1}^{n} h_{j} \frac{\partial f_{2}\left(x_{i}\right)}{\partial x_{j}} \\
\cdot \\
\cdot \\
f_{n}\left(\bar{x}_{i}\right)=f_{n}\left(x_{i}\right)+\sum_{j=1}^{n} h_{j} \frac{\partial f_{n}\left(x_{i}\right)}{\partial x_{j}}
\end{gather*}
$$

Eq. 2.24 represents a truncated Taylor series in which only the linear terms are retained, which is why this method is sometimes referred to as the "linear method." Therefore, forcing the functions $f_{1}\left(x_{i}\right), ., \quad, f_{n}\left(x_{i}\right)$ to be zeroes permits the derivation of an iterative formula for determining roots of nonlinear algebraic equations. That is,

$$
\begin{gather*}
0=f_{1}\left(x_{i}\right)+\sum_{j=1}^{n} h_{j} \frac{\partial f_{1}\left(x_{i}\right)}{\partial x_{j}} \\
0=f_{2}\left(x_{i}\right)+\sum_{j=1}^{n} h_{j} \frac{\partial f_{2}\left(x_{i}\right)}{\partial x_{j}}  \tag{2.25}\\
\cdot \\
0 \\
0=f_{n}\left(x_{i}\right)+\sum_{j=1}^{n} h_{j} \frac{\partial f_{n}\left(x_{i}\right)}{\partial x_{j}}
\end{gather*}
$$

Expanding, then using the matrix form yields
or more simply

$$
\begin{equation*}
[J]\{h\}=\{f\} \tag{2.27}
\end{equation*}
$$

Here $\{h\}$ is the increments vector and $\{f\}$ is the function matrix; the square matrix of partial derivatives is denoted by [J] and is called the Jacobian. Therefore, solving for the increments vector gives

$$
\begin{equation*}
\{h\}=-[J]^{-1}\{f\} \tag{2.28}
\end{equation*}
$$

However, since $h_{1}=\bar{x}_{1}-\mathrm{x}_{1}, \mathrm{~h}_{2}=\overline{\mathrm{x}}_{2}-\mathrm{x}_{2}, \ldots, \mathrm{~h}_{\mathrm{n}}=\overline{\mathrm{x}}_{\mathrm{n}}-\mathrm{x}_{\mathrm{n}}$, substituting into Eq. 2.27 yields

$$
\begin{equation*}
\{\bar{x}\}=\{x\}-[J]^{-1}\{f\} \tag{2.29}
\end{equation*}
$$

Using $k$ to designate the number of iterations performed, we can express Eq. 2.29 in the following general form:

$$
\begin{equation*}
\{x\}_{k+1}=\{k\}_{k}-[J]_{k}^{-1}\{f\}_{k}, \quad k=1,2, \ldots \tag{2.30}
\end{equation*}
$$

This solution procedure consists of making initial approximations for each of the $n$ variables, then evaluating the Jacobian and the $n$ equations so that

$$
\begin{equation*}
\operatorname{det}[J] \neq 0 \tag{2.31}
\end{equation*}
$$

after which a test for errors is made. This is accomplished by using the maximum $h$ value as a basis for accuracy. Thus

$$
\begin{equation*}
T \geq \max \left|h_{i}\right|, \quad i=1, \ldots, n \tag{2.32}
\end{equation*}
$$

where $T$ is an appropriate tolerance value.

Since some problems may not converge, a limit on the maximum number of iterations should be included in any computer program pertaining to roots of nonlinear algebraic equations. Unfortunately, there are no general means by which an initial solution can be obtained. A knowledge of the physical problem being solved is one way of obtaining an initial estimate. For the solution of the two-dimensional flow equations, the use of a one-dimensional backwater analysis program is one way to obtain an initial solution. This possibility will be discussed in Chapter 5. The lack of a closed form solution to the modified Navier-Stokes equations prompts us to use numerical modeling to get an approximate solution to the equations. Norton (1980) went through the rigors of deriving the entries in the Jacobian matrix required in the Newton-Raphson solution of Eqs. 2.10 to 2. 12 .

### 3.1 The Preprocessor (RMA-1)

The primary purpose of computer program RMA-1 is to generate two-dimensional finite element grids or networks for use by the computer program RMA-2. As a data preprocessor, the specific capabilities of RMA-1 are the following:

1. To read, edit and print all geometric data required for the computer program RMA-2;
2. To refine, update, modify and change a network which has been generated by a previous run of RMA-1;
3. To calculate the coordinates of mid-side node of an element side so as to exactly achieve specified slopes at corner node points. This capability is required for specification of zero normal (tangential) flow along network boundaries;
4. To develop an internal element order which will result in most efficient run times for RMA-2; and
5. To write a data file which can be used as input to RMA-2 and the plotting programs RMA1PLT and RMA2PLT.

### 3.1.1 Basic Operation

RMA-1 is the preprocessing program. Its main function is to transform the input data into a form appropriate as direct input to the processor RMA-2, and to the two postprocessors which are essentially plotting programs.

RMA-1 consists of one short master routine, RMA1, and several subroutines as shown in Fig. 3.1. Curved-sided elements are plotted by RMA1 using the same mathematical functions as are used by the finite element functions.

The main routine, RMA1, is used as a starting point and subsequent calls to the different subroutines are made. Program execution is initiated in RMA1, after which input data is read from file "TAPE5". If the program is instructed to calculate mid-side nodal coordinates, appropriate parameters are input and the calculations made. Finally, control is passed back to RMA1 and the nodal coordinates and element identities are printed for the final network.


Fig. 3.1. - Program Structure of RMA-1 in FESWMS-TXPC
As a user option, the program will calculate an element order which will minimize the computational effort required by RMA-2 (Collins, 1973). This is performed by a call to subroutine REORD which in turn calls subroutines ORDER and ADJPT to complete its calculations. This option should be invoked only when the finite element network is guaranteed to be geometrically satisfactory.

RMA1 will then write its results to files "TAPE8" and "TAPE3" for subsequent use by RMA1PLT and RMA-2, respectively. Finally, a whole or subsection plot of the network will be done by a series of calls to subroutine FIT. Program execution is then terminated.

### 3.1.2 Modifications Made

Minor changes were made on RMA-1. The reader is advised to refer to the user's guide when going through the following points.

1. Options to dump plot to screen or any one of four IBM PC-compatible pen plotters were added.
2. RMA-1 can extract geometric information from the input data deck and create GR cards (cross section geometry cards) for both one-dimensional step-backwater codes HY-7 and HEC-2. To generate GR cards for HY-7, place a negative sign before the variable NA (card no. 10 in input data deck for RMA-1). This will calculate horizontal station numbers and elevations for each corner node between NA and NB, inclusive. Also, the order by which the nodes are inputted should be from left to right (looking downstream) in order to conform with the input requirements of $\mathrm{HY}-7$. The same procedure applies for the case when the user wishes to obtain GR cards for HEC-2 from the finite element network; a negative sign is placed before the value of $N B$. Files "HY7GR.DAT" and "HEC2GR.DAT" will be generated by invoking these options.

The conversion of RMA-1 to the PC version was done using Microsoft Fortran Compiler (version 3.3) and PLOT88 Software Library. Initially, the computer program was broken down to separate subroutines. These subroutines or source codes were compiled individually producing a set of object codes, one for each subroutine. In turn, these object codes were combined (linked) with a set of libraries (that was part of the compiler software package) and PLOT88 Software Library to form a single executable code "RMA1.EXE". This preprocessor can handle as many as 600 elements or 1,100 nodes (whichever is greater for a given finite element network) and takes a minimum of 426,968 bytes to load (i.e., minimum random access memory requirement). It runs on an IBM XT, AT or compatible (as all other computer codes in FESWMS-TXPC) .

### 3.1.3 Data Preparation

When preparing data for RMA-1, several key points should be remembered:

1. In order to determine the proper coordinates of a midside node, specification of slopes at both end nodes is a must. The position of the mid-side node is then calculated by a quadratic formula to exactly satisfy both slopes.
2. File "TAPE3" containing nodal information (coordinates, slopes and bottom elevations) and element information (type and node connectivity list) is written.
3. Coordinate scale factors (XFACT and YFACT) for scaling to prototype dimensions are included in RMA-1. However, the
author finds no immediate use for such factors and as such, are not functional in the IBM PC version of the RMA-1.
4. The plotting scale factors (XSCALE and YSCALE) are applied to the input coordinates independent of the coordinate scale factors.
5. When rotation is desired prior to plotting, the network is rotated AR degrees (clockwise from the X-axis) using the origin of the global coordinate system as a pivot and not the specified origin for plotting.

### 3.2 The Processor (RMA-2)

The generalized computer program RMA-2 solves the depthintegrated equations of fluid mass and momentum conservation in two horizontal directions. These equations are solved by the finite element method using the Galerkin method of weighted residuals, a Newton-Raphson iteration scheme, and a numerical integration using seven-point Gaussian quadrature (Zienkiewicz, 1971). The elements may either be triangles or quadrilaterals and may have curved (parabolic) sides. Quadratic shape functions are used for approximating velocity components while linear shape functions are utilized for water depths (Hood and Taylor, 1974). Model topography is defined by assigning a ground surface elevation to each element vertex and requiring the ground surface to vary linearly within the element. Time derivatives are handled by an implicit finite-difference scheme.

The overall objective of the solution technique (the FE method) is to satisfy (within certain limits of accuracy) all three PDE's simultaneously. Errors in the conservation of mass equation may become significant such that deviations from physical reality (continuity of flow) may become troublesome. For a well-constructed finite element network, however, errors in mass conservation are small. As a check, the model has the capability of integrating the flow across a line following element sides and beginning and ending at element vertices.

### 3.2.1 Basic Operations

The program RMA-2 is composed of a main executive routine, RMA2, a block data subroutine called BLOCKD, and ten other subroutines. The general structure and linkages between the various routines is indicated in Fig. 3.2. A brief description of the intended purpose and functional operation of each subroutine is provided in Appendix $F$.


Fig. 3.2. - Program Structure of RMA-2 in FESWMS-TXPC

### 3.2.2 Modifications Made

1. Subroutine USSET was modified to account for an alternative way to distribute total flow among the nodes in an inflow section. A brief theoretical discussion follows.

The problem of subdividing flow among distinct regions or subsections in a channel cross section is indeed a difficult problem with which to deal (Henderson, 1966). Consider the case of overbank flow. Figure 3.3 shows a situation where water
surface elevation is assumed to be substantially constant over the whole section of flow, a condition that must exist if hydrostatic pressure must remain constant along any horizontal line drawn across the section. The different subsections of flow (1, 2 and 3 ) will almost certainly have different velocities yielding an energy grade line varying across the section.

The problem reduces to defining a total head $H(=p / \gamma+$ $\mathrm{V} 2 / 2 \mathrm{~g}+\mathrm{z}_{0}$ ) applicable to the entire cross section. Streeter and Wylie (1979); and Henderson (1966) showed that the solution is to use a constant mean velocity head $\alpha V_{m}{ }^{2} / 2 \mathrm{~g}\left(\mathrm{~V}_{\mathrm{m}}=Q_{T} / A_{T}\right)$ that sits on top of the free surface (Fig. 3.4).

The scenario leads us back to the idea that each subsection must have its unique flow characteristics - distinct head loss, different flow depths, etc. This situation leads to Fig. 3.5.

Mays and Taur (1983), in an effort to make flow specification in RMA-2 as convenient as possible to the user, subdivided the total flow across a section in proportion to the subareas tributary to each nodal point (Fig. 3.6). The fraction of the total discharge in each of the subsections is:

$$
\begin{equation*}
\frac{Q_{i}}{Q_{T}}=\frac{A_{i}}{A_{T}} . \tag{3.1}
\end{equation*}
$$

An alternative method is proposed in this report.
The total discharge $Q_{T}$ is equal to the sum of the individual discharges from all subsections.

$$
\begin{equation*}
Q_{T}=Q_{1}+Q_{2}+\ldots+Q_{i}+\ldots+Q_{n-1}+Q_{n} \tag{3.2}
\end{equation*}
$$

where: $i=$ nodal point representing a subsection.

Assuming uniform total head $H$, and any head deduced from it, across each section, the same value of friction slope $S_{f}$ applies to each subsection. From Manning's equation, we have

$$
\begin{equation*}
Q_{1}=K_{i} s_{f}^{1 / 2} \quad, \quad i=1 ; n \tag{3.3}
\end{equation*}
$$



Fig. 3.3. - Divided Flow: Horizontal Water Line; Different Energy Lines


Fig. 3.4. - Divided Flow: Horizontal Water Line; Horizontal Energy Line


Fig. 3.5. - Divided Flow: Different Water Lines; Horizontal Energy Line


Fig. 3.6. - Divided Flow: Subareas Tributary to Each Point with Horizontal Water Line
where: $\quad K_{i}=$ conveyance of subsection $i$

$$
\begin{aligned}
& =\frac{1.49}{n_{i}} A_{i} R_{i}^{2 / 3} \\
A_{i} & =\text { area of subsection } i \\
R_{i} & =\text { hydraulic radius of subsection } i \\
& =\text { ratio of area to wetted } \\
& \text { perimeter of subsection } i \\
& =A_{i} / P_{i}
\end{aligned}
$$

Summing up all subsection flows

$$
\begin{align*}
Q_{T} & =\sum_{i=1}^{n} K_{i} S_{f}^{1 / 2} ; \text { and } \\
S_{f}^{1 / 2} & =\frac{Q_{T}}{\sum_{i=1}^{n} K_{i}} \tag{3.4}
\end{align*}
$$

Substituting back to Eq. 3.3 and solving for $Q_{i} / Q_{T}$, we have

$$
\begin{equation*}
\frac{Q_{i}}{Q_{T}}=\frac{K_{i}}{\sum_{i=1}^{n} K_{i}} \tag{3.5}
\end{equation*}
$$

Therefore, the fraction of the total discharge in each of the subsections is given by Eq. 3.5. This equation gives results that are accurate enough for practical purposes and is normally adopted in practice (Henderson, 1966). Thus, given $Q_{T}$ and the geometric descriptions of an inlet section (nodal coordinates and elevations), subroutine USSET will (1) compute for the area and hydraulic radius of each subsection (tributary to each node), (2) calculate the conveyance of each subsection (Eq. 3.3), and (3) solve for each subsection flow using Eq. 3.5.

The series of flow subdivision computations involving Eq. 3.1 is referred to as area ratio method and Eq. 3.5 is used in the conveyance ratio method; $\mathrm{NPROP}=0$ and $\mathrm{NPROP}=1$, respectively in the user's guide for RMA-2 (Appendix B).
2. RMA-2 can be run without using geometry output file "TAPE3" obtained from a previous run of preprocessor RMA-1. This can be done by combining geometry input cards with hydraulic
parameter cards in one input file "TAPE5". Parameter IFILE (card no. 2 in input data deck for RMA-2) should be set to zero to invoke this option.
3. The old (prior to FESWMS-TX) input format for boundary conditions is an option in FESWMS-TXPC. This enhancement makes it possible for a.) data used in the older version of RMA-2 to be rerun in the IBM PC version; and b.) the user to override the procedure used by subroutine USSET in distributing total discharge through an inlet section if he feels that flow can be more accurately described using the old format - specification of individual unit flows for each inflow node. This option is activated by preceding the value of NBX with a negative sign (card no. 2 in input data deck for RMA-2).
4. Initial (or restart) conditions (e.g. solution obtained from one-dimensional step-backwater computations) for the first simulation run of RMA-2 can be made part of the input deck by specifying a value of -1 for the variable NB.
5. Once the geometry of a finite element network is defined (i.e., after running RMA-1) vector plotting routine RMA2PLT can be run without running RMA-2 by supplying the former with a formatted solution data (file "TAPE9") and if also desired, a formatted continuity line data (file "TAPE10"). One of the example problems shows this capability.

The conversion of RMA-2 into the IBM-PC version was done using the same procedure as the one used for converting RMA-1. An 8087 or 80287 math coprocessor would speed-up the numbercrunching performed by RMA-2 but is not necessary for the program to function properly. The executable code "RMA2.EXE" handles the same size of network as "RMA1.EXE" but needs at least 505,546 bytes of random access memory.

### 3.2.3 Data Preparation

Some special considerations in preparing the input data set for RMA-2 are presented as follows.

1. All files created from one FESWMS-TXPC program as inputs to another are unformatted. This was determined to be the most efficient way of transferring information from one program to the other written for the IBM PC.
2. Variable ELEV (card no. 2 in input data deck for RMA2PLT) is used only in the first simulation run of RMA-2. As pointed out in Chapter 1, RMA-2 assumes a horizontal water surface elevation at the very first simulation run. An initial
value for depth of flow (one of the unknowns) is then calculated by subtracting bed elevation at each nodal location from ELEV.
3. Nodes comprising an inlet section should be specified from right to left bank (looking downstream) as required by variable NNXUS (card no. 12 in input data deck for RMA2).
4. RMA-2 can handle time-dependent problems, the discussion of which is beyond the scope of this report. The reader is referred to Norton (1980) for input requirements and an example application.

### 3.3 The Postprocessors (RMA1PLT and RMA2RLT)

The postprocessors for FESWMS-TXPC are used to aid the user in visualizing the hydraulics involved in the analysis. These programs will be able to produce plots using one of the four different plotters - IBM6180, IBM7372, HP7470, HP7475 and the screen.

The postprocessor RMA2PLT was completely rewritten to accommodate vector plotting features unique to the IBM personal computer.

RMA1PLT works in a similar fashion to RMA-1. In fact, it can be referred to as a short version of the latter. It was written to take in data from RMA-1; do some preliminary calculations and plot the finite element grid to one of the four plotters or the computer screen. Therefore, if the user intends to take an initial look at the network, it would be the best (in terms of speed) for him to choose the screen dump option when running RMA1 and later on make a separate run using RMA1PLT for a hard copy of the same network. The executable code for RMA1PLT is "RMA1PLT.EXE".

RMA2PLT, on the other hand, has the following features. Vector representation can be done in terms of nodal velocity or unit discharge. To add a sense of direction to the plotted network, a north arrow can be placed anywhere within the plotting area. Continuity can be checked along specified continuity lines by specifying a maximum of 41 nodal coordinate pairs. The program then makes use of numerical flow integration based on the line integration technique (Borse, 1986). The executable code for RMA2PLT is "RMA2PLT.EXE".

Figure 3.7 shows data flow and program linkage between the four computer codes in FESWMS-TXPC.


Fig. 3.7. - Data Flow and Program Linkage in FESWMS-TXPC

### 3.4 Model Implementation

### 3.4.1 Definition of Physical Boundaries

A prerequisite in running RMA-2, like all other finite element simulation programs, is the definition of the physical boundaries of a study area.

RMA-2 will accept any combination of triangular or quadrilateral elements with curved sides. The use of curvedsided elements to define channel bends (1) facilitates a more realistic representation of prototype conditions, and (2) improves the accuracy of the solution as documented by Gee and MacArthur (1978), King and Norton (1978), and Walters and Cheng (1978, 1980) for mixed-interpolation formulation of the freesurface flow equations. Each side of each element is defined in terms of the spatial coordinates of the three nodal points which lie along the element side (Fig. 3.8). In all cases there exists a nodal point at each end of an element side with an internal, or mid-side node at an arbitrary location between the end points. For straight element sides the position of the mid-side node is internally calculated from the position of the end nodes, and lies exactly midway between these ends.

The need for curved elements arises from the desire to allow flow to move parallel to the network's outer boundaries. To do this, the boundary must have a continuous and unique slope at all points or flow will "leak" in and out of the system (King and Norton, 1978; and Norton, 1980). Again, construction of a smooth continuous network with curved sides provides this capability with the added benefit that irregular physical geometries may be reasonably approximated.

The only practical way for a user to calculate and examine the precise shape of a curved sided element is to plot the internally generated function using RMA-1 and RMA1PLT (discussed in Sec. 3.1 and 3.3).

### 3.4.2 Discretization of Finite Elements

Although discretization into finite elements by RMA-2 accommodates both triangular and quadrilateral elements emphasis will be given to partitioning of the study area into an equivalent network of triangular elements only. Subdivision lines between elements are laid down where abrupt changes in vegetative cover of topography occur. Each element is designed to represent an area of nearly homogeneous vegetative cover. Additional network detail should be used in (1) large prototype


Curved Element

Fig: 3.8. - Finite Element Descriptions
(from Mays and Taur, 1983)
ground surface gradients, such as those between overbanks and main channel bottoms; and (b) areas where velocity and water surface gradients are expected to be relatively large, such as near bridge openings.

Another significant point to stress at this stage is element aspect ratio. It is defined as the ratio of the longest element side to the shortest element side. This ratio should be kept to a minimum (about 10) in order to comply with the "smoothness" requirements of the finite element method (Lee, et al., 1983). If the direction of local velocities and magnitudes of depth gradients can be estimated beforehand, it would be better to align the shortest element side with the largest velocity and/or depth change and the longest side with the smallest change. For example, the longest element side should be aligned parallel to the longitudinal channel axis, along which velocity and depth changes are typically small.

Also, it might be advisable to break the study area into two or more subsections in order to reduce the cost of design and preliminary calibration (Lee, et al., 1982). Estimated boundary conditions can be used at the upstream and downstream boundaries of each subsection on a preliminary basis. Then, all subsections can be combined prior to the final calibration and subsequent analysis.

Ideally, the true solution will be better approximated as the finite element mesh (or network) becomes finer (i.e., increase in element density); only to be limited by the amount of computer memory available and at the expense of increased computer time. Thus, good judgement should be made by the modeler in developing the finite element network.

### 3.4.3 Determination of Nodal and Element Identities

First, number the outer nodes (the vertices) and mid-side nodes of the finite element network. There is no rule as to the correct order by which numbers are assigned to the nodes within an element but there is a requirement that these node numbers be specified in a counterclockwise direction to define the element bounded by these nodes (see Sect. 2.2) In order to efficiently utilize the available storage of the computer the difference in nodal numbers within each element should be minimized to the optimum. A utility program SUBSTI was written to satisfy this requirement, was applied to the Embarras River study (Wheelock, 1985), and is presented in Chapter 4. Second, assign numbers to the triangular elements in sequence. The other input variables for this stage of the model construction can be found in Appendix B.

### 3.4.4 Boundary Conditions

There are four types of boundary conditions to consider for each nodal point.

1. "No-Boundary" Condition

When a "no-boundary" condition is assigned to a node for a particular variable, then this variable simply enters the calculation as one of the unknowns. Normally, this condition is used at the internal points of the network.
2. Exact Boundary Condition

An exact boundary condition is one where specified flow components (discharge per unit width) are to be assigned to a particular nodal point. This is the type of specification that is used to set flows at inflow sections. Two points have to be brought out. First, under this classification, a user may also specify zero flow (stagnation boundary condition). Stagnation boundary condition was applied at extremities (lateral boundaries) of the finite element network by Gee and MacArthur (1981) and Resource Management Associates (1977). In both cases, the modelers expressed preference of parallel flow boundary condition (described next) over stagnation boundary condition. Second, as mentioned earlier, the user also has the option to input a total flow for each inflow section, and RMA-2, in turn, will subdivide that flow among the nodal points in the section in question by the methods explained in Sec. 3.2.2.

## 3. Parallel Flow Boundary Condition

A feature has been incorporated into the computer code which allows flow to move parallel to a fixed boundary (tangential or zero normal flow). This feature is useful a) in reducing the required level of element detail when straightsided systems are being simulated, and b) if the tangential flow along curved boundaries is desired. When using the parallel flow option it is important to insure that the fixed boundary along which flow is allowed to move is continuous in slope (Fig. 3.9). The corresponding input data requirement is given in Sec. 3.1.3.

## 4. Exit Boundary Condition

The specification of exit boundary conditions for head (target or control water surface elevation) is handled in a slightly different fashion from the exact boundary condition (Norton, 1980). From a user's point of view, this means that the

$\square-m i d-s i d e$ node (e.g. mn-1) whose coordinates will be computed based on the slopes of tangent 11 nes (e.g. $t-1$ and $t-2$ ) drawn at adjacent corner nodes (e.g. cn-l and cn-2)

Fig. 3.9. - Description of Curved Boundaries
head at an exit boundary will have to be identified as such, and that in most cases the model will not reproduce these head specifications with the same numerical precision as an exact boundary condition.

### 3.4.5 Specification of Coefficients

The selection of proper Manning's $n$ and Chezy's C are well documented in Chow (1959). It poses no serious problem in running RMA-2.

Specification of proper eddy viscosities (turbulent exchange coefficients multiplied by the mass density of water) can have an influence on the simulation results and should be selected with proper care. The optimum values to be used for these coefficients are not well known at this time (Lee, et al., 1983), and the most practical values for initial applications are those successfully applied in previous simulations. The general statement that can be made at this time is that the larger the element area is, the larger the numerical value that should be assigned to the element. Nonzero eddy viscosities are necessary for convergence of the model since they account for suppressing the nonlinear instabilities inherent in the partial differential equations to be solved (Lee, 1983). Whereas energy dissipation through the use of eddy viscosity generally has small effect upon the dynamics of large scale motions, it provides a necessary and controllable approximation to the physical energy dissipation process in free-surface flow (Walters, 1981).

Once convergence is achieved, by assigning high enough values of eddy viscosities, the solution becomes less sensitive to changes in their values than to changes in the values of Chezy coefficients or to changes in network detail (Lee, et al., 1983). Mays and Taur (1983) prepared a list of "typical" values for eddy viscosity based on some of the applications cited in Chapter 4.

### 3.4.6 Running Computer Code RMA-2

As mentioned earlier, the solution is obtained by using the Newton-Raphson method - an iterative procedure. Taking this into consideration, a systematic way to run RMA-2 is suggested (Lee and Bennett, 1981).

The downstream water surface elevation is initially set as high as necessary to avoid negative depths (an occurrence that may cause the numerical scheme to blow up) in the upstream part
of the study area (By default, RMA-2 will assume a zero velocity, horizontal water surface elevation.). It is then decreased in small steps (about 0.5-2.0 ft) per run until the specified value (the target or control water surface elevation) is reached. The number of iteration cycles per run can be increased until the change in the solution between two successive iterations (convergence parameters) is less than an acceptable value (say 1\%).

### 3.4.7 Selection of a One-Dimensional Flow Analysis Program

The choice for a one-dimensional flow analysis code to work as a support program for the two-dimensional backwater code RMA2 must satisfy the following requirements - (1) It must be compatible with conventional step-backwater analyses; (2) It should incorporate bridge backwater computational methods; and (3) It must provide a procedure for multiple bridge opening analyses. The theory behind one-dimensional water surface profile computation is beyond the scope of this report but basically it involves the satisfaction of energy balance between sections upstream and downstream from the bridge (Shearman, et al., 1985). Three existing codes satisfy all of the above requirements.

The most widely used computer code for water surface profile computations is HEC-2 from the Corps of Engineers (1981). USGS's E431 (Shearman, 1976) is a computer model which uses a modification of the Federal Highway Administration (FHWA) methodology (Bradley, 1970) for bridge backwater computations. A relatively new model called $\mathrm{HY}-7$ is the seventh member in a series of FHWA computer programs for hydraulic analyses. A limited comparison between these three one-dimensional codes based on how well they reproduce some observed water surface profiles was done by Shearman, et al. (1985).

From a series of Hydrologic Investigation Atlases published by the USGS, data from actual flood events at bridges in Alabama, Louisiana, and Mississippi were documented (Shearman, et al., 1985). The available data includes: (1) flood discharge; (2) water-surface profile based on recovered high-water marks; (3) geometry of the bridge and the valley for a significant distance both upstream and downstream of the bridge; and (4) roughness coefficients determined by water surface profile computations. In general, HY-7 tends to give better results and
appears to more completely define the water surface profile in the vicinity of a bridge. $H Y-7$ was therefore chosen as the support program for RMA-2.

## CHAPTER 4. A HISTORICAL REVIEW

### 4.1 General

RMA-2 has been satisfactorily applied in several locations within and outside the United States. In order to show the wide applicability of RMA-2, a short description of some of the past applications of this computer code is presented next.

### 4.2 Previous Applications

### 4.2.1 Johnsonville Steam Plant in TVA System near Johnsonville, Tennessee (Norton, 1977)

A series of simulations of a reservoir (Figs. 4.1-4.2) with a large power plant discharge was done by Norton (1977). The study was performed to evaluate the temperature behavior of the reservoir and the impact of thermal discharge to the whole study area. The model was calibrated based on observed conditions corresponding to May 29, 1969. Runs were made with and without flows from the Johnson Steam Plant in operation (Figs. 4.3a-b). It was concluded that there was insignificant influence of the local discharge of the cooling water from the steam plant into the general flow pattern in Kentucky Lake.

### 4.2.2 Tallahalla Creek in Mississippi River <br> (Resource Management Associates, 1977)

It was suggested that the apparent violation of mass conservation which can be observed in the results of the finite element program be resolved and quantified. The problem of analyzing a convergent-divergent flow regimen in a natural flood plain constricted by a bridge crossing was first investigated by Franques and Yannitell (1974) with a non-linear potential flow type finite element model. Tseng (1975) worked on the same problem and came up with a finite element network as shown in Fig. 4.4a. King and Norton (1977) prepared a series of runs with different levels of smoothness and network refinement for the Tallahalla application. Their work was one of the first applications to use curved element boundaries to improve numerical stability of the solution process. In addition to the original network (level 0) six other alternative networks were constructed, the last being that as shown in Fig. 4.4b (level 6). A total of seven element types corresponding to seven different turbulent exchange coefficients and Chezy's C values were used. Some of the significant results of this study were:


Fig. 4.2. - Finfte Element Network - Kentucky Reservoir in the Vicinity of Johnsonville Steam Plant
(from Norton, 1977)


> Fig. 4.3a - Velocity Field (No Cooling Water Discharge) Johnsonville Steam Plant Application (from Norton, 1977)


Fig. 4.3b - Velocity Field (Plant Operating) Johnsonville Steam Plant Application (from Norton, 1977)


Fig. 4.4a - Level 0 Network for Tallahalla Creek
Application
(from Resource Management Associates, 1977)


Fig. 4.4b - Level 6 Network for Tallahalla Creek Application
(from Resource Management Associates, 1977)

1. The errors in flow continuity tend to be local effects and do not significantly influence the results in distant (with respect to the finite element network) locations.
2. Velocities increase (through the bridge opening in particular) when the network is refined.
3. Energy grade line is less sensitive to changes of network than water surface elevation.

### 4.2.3 Rio Grande de Loiza Floodplain Simulation (Gee and MacArthur, 1978)

A 6 x 6 mi floodplain in Puerto Rico was modeled using computer code RMA-2. Using a Chezy C ranging from 10 to 40 fti/2/sec to account for a variable bottom topography and a turbulent exchange coefficient ranging from 260 to $500 \mathrm{ft} 2 / \mathrm{sec}$ varying with element size, a single-inlet, two-outlet network was constructed (Figs. 4.5a-c). Although no calibration of coefficients were done, the criteria for solution acceptability was set such that flow at all continuity lines deviate from the total inflow by less than $\pm 5 \%$. This criteria was met by varying network detail. Table 4.1 shows the continuity performance of the initial and final network configurations as well as execution times on a CDC 7600. From this application, it appears that velocity is somewhat more sensitive to errors in continuity than depth. Thus, if one is interested in flow velocities only, a more stringent continuity criterion can be imposed than if water surface elevations are of interest.

### 4.2.4 McNary Dam Second Powerhouse Study (Gee and MacArthur, 1981)

The objective of this study is the selection of a second powerhouse site for the McNary lock and dam on the Columbia River (Fig. 4.6). The roughness coefficient (Chezy C) was calibrated for this application to reproduce an observed flow condition.

An automatic reordering algorithm (Collins, 1973) was incorporated in the finite element program and was first used in this application. This numbering scheme is utilized for input/output and the system of equations are internally reordered to reduce storage. Also, at that stage of the computer program development it allowed two valid types of boundary conditions at boundaries where no flow enters or leaves the system. One is the slip boundary condition where the velocity on the boundary is tangential to the boundary; the other is the


Fig. 4.5. Rio Grande de Loiza Application (from Gee and MacArthur, 1981)

| Network | 3.1 | 3.3 | 3.5 |
| :---: | ---: | :---: | ---: |
| No. of Nodes | 310 | 375 | 432 |
| No. of Elements | 131 | 162 | 189 |
| CDC 7600 Execution Time (sec) | 22 | 31 | 45 |
| Check |  | Percent of Inflow |  |
| Line | 100.0 | 100.0 | 100.0 |
| 1 (inflow) | 89.2 | 90.8 | 96.2 |
| $2+3$ | 114.9 | 106.8 | 104.9 |
| 4 | 87.5 | 92.0 | 96.4 |
| $5+6$ | 79.3 | 90.1 | 98.2 |
| $7+8$ | 99.8 | 99.4 | 98.7 |
| $9+10$ | 100.0 | 100.0 | 100.0 |
| $11+12$ |  |  |  |
| (outflow) |  |  |  |

Table 4.1. - Continuity Performance of the Three Networks
Used in the Rio Grande de Loiza Application
(from Gee and MacArchur, 1981)


Fig. 4.6. - Study Area Showing Possible Second Powerhouse Locations for McNary Dam
stagnation point where both components of velocity are zero. Both boundary conditions were used but no significant difference in continuity performance was observed. (Problems conducted by Resource Management Associates (1977) indicated stagnation condition as inferior.) Also, velocity distributions were significantly different. The authors expressed preference of slip boundary condition using curved boundaries over the other method.

As in the preceding application, this study indicated that errors in continuity tend to be reflected more strongly in the velocity than the depth.

### 4.2.5 Proposed I-326 Crossing on Flood Stages of the Congaree River near Columbia, South Carolina (Lee and Bennett, 1981)

The hydraulic impact of the proposed Interstate Route 326 crossing of the Congaree River near Columbia, South Carolina (Fig. 4.7) was studied by the U.S. Geological Survey in 1981. By this time, the U.S. Geological Service had a fully operational modeling system (FESWMS) with RMA-2 as the core computer program. A major accomplishment of this application was the assessment of the finite element program as an operational tool for analyzing complex highway crossings and other modifications of river flood plains.

The model was calibrated using the flood of October 1976 with a peak discharge of $155,000 \mathrm{cfs}$ and a peak elevation of 142.8 ft NGVD. The maximum flood of record (peak discharge $=$ 364,000 cfs; peak water surface elevation $=152.8 \mathrm{ft}$ NGVD) which occurred in August 1908 was modeled in three cases: with proposed dike (Otarre) on the right bank (Fig 4.8a), with dike (Manning) on the left bank (Fig. 4.8b), and with dikes on both banks (Fig. 4.8c). Simulations were performed both with and without the highway embankments in place. Changes in flow distribution resulting from the embankments as well as local velocities in the bridge openings (Fig. 4.9) were obtained.

Water surface elevations at the downstream outflow boundary were estimated using profiles obtained from one-dimensional step-backwater analysis. A horizontal downstream water surface was assumed in all runs. note: Water surface elevations at the downstream boundary can also be based on historically recorded high-water marks (Lee, et al., 1983).

The South Carolina Department of Highways and Public Transportation changed the design of several bridge openings based on the results of this study (Table 4.2). Both velocities


Fig. 4.7. - Congaree River Flood Plain near Columbia, S.C. Showing Proposed Bridges


Fig. 4.8a - Finite Element Network for Congaree Application; Otarre Dike in Place


Fig. 4.8b - Finite Element Network for Congaree Application; Manning Dike in Place


Fig. 4.8c - Finite Element Network for Congaree Application; Otarre and Manning Dikes in Place


Fig. 4.9. - Section of the Finite Element Network Showing One Bridge Opening in the Congaree Application
(from Lee and Bennett, 1981)

| BriAge number | BriAge description | Original destan |  |  | Revised deaign |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { Aeqinning } \\ & \text { atation } \end{aligned}$ | $\begin{aligned} & \text { Ending } \\ & \text { station } \end{aligned}$ | Length (ft) | ```Boginnlng station``` | Ending etation | Length (ft) |
| 1 | Twin overflow bridges 1 | 448 + on | 455 + 90 | 780 | Bridqes not | revised |  |
| 2 | Twin overpases over Old state Poan (Poad S-66) | $502+60$ | $511+n 0$ | 840 | $495+10$ | $510+10$ | 1,4no |
| 3 | Twin briAqua over Congaree Creek | $516+25$ | $530+65$ | 1.440 | Aridges nnt | revisen |  |
| 4 | win hriAqes: over Congaree River | $549+35$ | $562+55$ | 1.320 | Brinqes not | revimen |  |
| 5 | Trin hridaes over Metro Lane | 57A + 25 | 579+75 | 150 | Arifaee not | revisen |  |
| 6 | Twin overflov bridges 2 | $6 n 4+0 n$ | $610+6 n$ | 660 | 601 + 00 | 614 + 20 | 1,320 |
| 7 | min overtlow bridges 3 | $634+75$ | $641+35$ | 667 | $635+65$ | $640+45$ | 480 |
| culvert | Three A-foot-by-A-foot reinforcenconcrete hox culverts | $656+25$ | - | 24 | Culverts not | revisen |  |

Table 4.2. - Bridges Proposed for the Interstate Route 326 Crossing of the Flood Plain of the Congaree River (from Lee and Bennett, 1981)
in the bridge openings and backwater were reduced as indicated by the results from a simulation incorporating the new design for the case with dikes on the left bank of the river.

### 4.2.6 I-10 Crossing on Pearl River near Slidell, Louisiana (Wiche, et al., 1982)

Similar to the Congaree application, the I-10 crossing on Pearl River located in the lower part of the basin on the Mississippi-Louisiana border (Fig. 4.10) provides an ideal site for applying two-dimensional backwater analysis.

Two alternative modifications (structural and nonstructural) for improving hydraulic characteristics of the highway crossing were studied. Both alternatives reduce backwater and average velocities on the overbanks and in the channels, and both eliminate roadway overtopping.

The first alternative involved improving hydraulic characteristics of the three existing bridge openings (Fig. 4.11a). This nonstructural alternative included the removal of spoil left after construction and the clearing of brush and trees. The second alternative, on the other hand, involved placing a new 2,000-ft opening in the I-10 embankment between West Pearl and Middle Rivers (Fig. 4.11b). The apparent improvements brought about by these two alternatives are graphically illustrated by comparing Fig. 4.12 with Figs. 4.13ab.

### 4.2.7 Model Verification By Simulating_Known Flow Patterns (Driscoll, 1981)

Laboratory experiments through an expanding section where flow in the supercritical state was allowed to pass were conducted by Blaisdell (1951). In a similar laboratory set-up, Ippen (1951) modeled flow in the subcritical state to pass through a contracting channel section. Both of these experiments were reproduced using the two-dimensional computer program and close agreement was established in both cases.

For the expanding section, an exit boundary condition was provided as a water surface elevation. Although not necessarily met by the program, it was required by the solution scheme. For the contracting section, no difficulties were encountered by the program in simulating both subcritical (upstream boundary) and supercritical (downstream boundary) flows coexisting in the same network. In both sections, the depth patterns of the observed laboratory data were adequately approximated.


Fig. 4.10. - Lower Pearl River Basin: MississippiLouisiana Border


Fig. 4.1la - Finite Element Network Showing Close-up View of Existing 3-Bridge Opening Along I-10 (from Wiche, et al., 1982)


Fig. 4.llb - Finite Element Network Showing Close-up View of Proposed $4-$ Bridge Opening Along I-10 (from Wiche, et al., 1982)


Fig. 4.12. - Contour Lines of Equal Backwater and Drawdown at the Study Area of the Lower Pearl River Basin in the Vicinity of $\mathrm{I}-10$ Crossing (Lee, et al., 1983)

(a)


Fig. 4.13-Close-up View of I-10 Crossing Showing
Contour Lines of Equal Backwater and Drawdown Due to a) Nonstructural and b) Structural Modifications (from Wiche, et al., i982)

### 4.2.8 Walnut Creek at Austin, Texas (Mays and Taur, 1983)

After modifying FESWMS to become FESWMS-TX, Mays and Taur (1983) applied the revised modeling system to a portion of Walnut Creek watershed in Austin, Texas. The study area is shown in Fig. 4.14.

The primary objective of this study was to simulate the Memorial Day flood ( $Q=14,300$ cfs) that occurred in May 1981. The need for two-dimensional flood flow analysis arose from the fact that previous floodings within the study area with any discharge of $10,000 \mathrm{cfs}$ or greater caused significant overbank flow. In fact, computed discharges for this storm on Walnut Creek at the Webberville Road (Martin Luther King Blvd.) USGS stream gage were not in agreement with data obtained from a field reconnaissance. Indeed, the storm of May 1981 warranted a more sophisticated way to study the hydrometeorological conditions associated with it.

To study the accuracy of the model, six different networks were used, level 1 being the coarsest and level 6 the finest. Fig. 4.15 shows one of the networks with the continuity lines (in bold) which were used consistently for all six levels. Due to uncertainties regarding the "true" values of the model coefficients unique to this particular application, a set of different Manning's roughness coefficients and turbulent exchange coefficients (eddy viscosities) were used and run separately for network level 6. A sensitivity analysis of such nature is necessary if the modeler wants to study changes in computed water surface elevations brought about by varying the two coefficients corresponding to the two mechanisms of energy dissipation accounted for by RMA-2. Fig. 4.16 is the vector plot obtained from the Walnut Creek study for network level 6.

### 4.2.9 Embarras River at Lake Charleston in Central Illinois (Wheelock, 1986)

An application done at The University of Texas involved the development of a preprocessor code written in LISP (LISt Processor) and embedded in a CAD (Computer-Aided Drafting) system to aid in a fast, error-free construction of the required geometry input data.

In 1980, a dike was constructed on the western portion of Lake Charleston along the Embarras River (Fig. 4.17). The dam on the southern tip of the study area (treated as a broad-crested


Fig. 4.14. - Walnut Creek Example Study Area


weir with length equal to 430 ft ) forms a downstream control with a water surface elevation of 589.6 ft above msl at $\mathrm{Q}=$ 34,000 cfs. Three questions were posed for this application: 1. what is the velocity distribution at the river in the bend?; 2. what are the velocities in the flood plain at the base of the dike?; and 3. what are the water surface elevations in the flood plain?

This application was rerun using FESWMS-TXPC and pertinent results are presented in Chapter 5.

### 4.2.10 Hybrid Models

RMA-2 has been jointly used with other solution methods (field observations, analytical solution, physical models) to solve problems in hydraulics. This type of hybrid modeling has been successfully implemented by the U.S. Army Engineer Waterways Experiment Station in Vicksburg, Mississippi in developing solutions to problems of constructing and maintaining a navigational channel through the Columbia River estuary. For a more thorough account of the entire hybrid model, the reader is encouraged to refer to McAnally, et al. (1983).

### 4.3 Microcomputer Implementation

To date, the author is unaware of any two-dimensional backwater analysis computer code written for a personal computer. Most existing finite element codes (one- or multidimensional) written for a microcomputer address problems in stress, thermal and vibration analyses (Engineering Microsoftware Review). This report would then be a pioneering work on two-dimensional free-surface flow analysis using the finite element method on a microcomputer.

## CHAPTER 5. EXAMPLE APPLICATIONS

The first two example applications were taken from previous work (Driscoll, 1981, and Wheelock, 1986) done at the University of Texas. They are presented here to illustrate some of the graphic capabilities of FESWMS-TXPC. Some comments on the printed output are also given. The third example application illustrates the use of the same set of computer programs applied to a natural stream with a bridge crossing.

### 5.1 Application to a Natural Open Channel

The first example application was taken from work done by Driscoll (1981). In addition to verifications for two laboratory data on expanding and contracting channel sections obtained by Blaisdell (1951) and Ippen (1951), respectively, he did a simulation run on a stream bend ( 185 nodes; 167 elements) under flood conditions (Fig. 5.1). Due to significant differences in the finite element formulation between the version of RMA-2 that Driscoll used from that version used by the author, the latter was not able to reproduce Driscoll's simulations. However, in Appendix $F$ of his report, he laid down a criteria for control elevation adjustments in the solution of his application problem. From Table 5.1, we note that seven, four and five reductions for the three control elevations were used. For each of these reductions, an unspecified number of iterations cycles was done (internal to the program). The version of RMA-2 in FESWMS-TXPC gives full control to the user as to the number of iterations per reduction that can be done. From a practical point of view, it is not always possible to predict the number of iteration cycles required to satisfactorily solve a particular problem, and some experimentation (a form of art rather than science) can be beneficial. Driscoll's work precludes the acceptance of the costly (in terms of computer time and user effort) trial and error approach which was required by users of RMA-2. His work served as a motivation to the author for looking at ways to obtain the solution to twodimensional free-surface flow problems in a more systematic way. In particular, the problem of determining two-dimensional depth and flow distribution in the vicinity of a bridge section using RMA-2 can possibly be enhanced by the incorporation of a onedimensional backwater analysis program, the details of which are given in the third example application.

The finite element grid and velocity vector plot for Driscoll's example application (Figs. 5.2 and Fig. 5.3) were generated using RMA-1 and RMA2PLT in FESWMS-TXPC. The corresponding input data cards are given in Appendix $C$.


Fig. S.1. - Topographic Representation of the Study Area Modeled by Driscoll (1981)

| Starting Water Surface Control Elevation | Ending Water Surface Control Elevation | 2STEP (FT) | ECHEC |
| :---: | :---: | :---: | :---: |
| 743.5 | 740.5 | 0.50 | 0.10 |
| 740.5 | 739.5 | 0.25 | 0.10 |
| 739.5 | 739.0 | 0.10 | 0.05 |
| * ZSTEP = Increment by which the control elevation is to be reduced when satisfactory convergence has been achieved. |  |  |  |
| HECK = The maximum chan fore a drop 1 | In depth allowed bet control elevation wi | uccessiv cur. | tion |

Table S.1. - Criteria for Control Elevation Adjustments
(from Driscoll, 1981)


Fig. 5.2 LAKE CREEK BEND AT FLOOD STAGE - RMAI RUN heriz. CRID SCRELE 1.3500 . VERT. GAID SCRE 1,3000.


Fig. 5.3. - LAKE CREEK BEND AT FLOOD STAGE
RMAZPLT RUN ON THE IBM-PC

GRID SCPLE 1 13000.
velocitt vector scalei 1 in - 15.95 fT/SEC

### 5.2 Application to an $0 x-$ Bow Bend Confined by a Levee

Another practical application of the two-dimensional hydrodynamics model was shown by Wheelock (1986). Fig. 5.4 shows the finite element network (408 nodes; 184 elements) originally used in this study. As mentioned in the previous chapters, smoothness requirements for finite element analysis dictate the amount of refinement necessary to produce acceptable results. The vector plot produced from this network after running RMA-2 is given in Fig. 5.5. Three possible areas where smoothness requirements might not have been met are: 1) at the bottom and 2) top portions of the bottleneck region, and 3) at the right side of the exit section right before the dam. For these areas, there appear to be flow reversals with no apparent reason (e.g., lateral inflow). The author refined the network such that: 1) more curve-sided elements are defined, and 2) the difference between node numbers within each element is minimized. The first criterion was met by defining more curvesided elements in the vicinity of the bottleneck as well as in the dam area. The second criterion was met with the help of a small program (SUBSTI) which was written specifically to renumber a node or a set of nodes to one value or a range of values specified by the user. The modified network is shown in Fig. 5.6 and the corresponding vector plot can be found in Fig. 5.7. All simulation runs were done in the same order (downstream elevation, number of iteration cycles and amount of drop in elevation of control elevation) that Wheelock (1986) did for his network. As a consequence of the "smoothening out" process, a more uniform flow distribution and zero backflow condition occurred in the bottleneck area. However, some irregularities in flow patterns still exist at the right-hand side of the outlet structure. Although convergence rates for both networks did not vary that much, the modified network showed improvements in the mass balance (continuity) check especially in continuity section 7 where the channel experienced a significant amount of contraction (Table 5.2).

### 5.3 Application to a Single-Opening Bridge Approach

The study area for this example application is in Round Rock, Texas. Fig. 5.8 shows a topographic map of a portion of Brushy Creek cut across by a bridge whose bottom chord extends down to 706.5 ft above msl.


Figure 5.5
EMBARRAS RIVER AT LAKE CHARLESTON





Fig. 5.8. - Topographic Map: Brushy Creek Study Area with Bridge Crossing

# Table 5.2. -- Comparison of Mass Conservation Between Original and Modified Networks 

| Line | (Run 1) | (Run 2) | (Run 3) | (Run 4) |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| 1 | $100.0(100.0)$ | $100.0(100.0)$ | $100.0(100.0)$ | $100.0(100.0)$ |
| 2 | $100.6(100.8)$ | $101.0(101.2)$ | $101.6(101.9)$ | $101.8(102.4)$ |
| 3 | $93.3(93.7)$ | $93.2(93.7)$ | $93.5(94.1)$ | $93.6(95.5)$ |
| 4 | $100.2(96.8)$ | $99.0(96.7)$ | $97.6(96.4)$ | $98.4(93.9)$ |
| 5 | $86.8(83.4)$ | $85.8(83.3)$ | $84.8(83.4)$ | $84.3(85.2)$ |
| 6 | N/A | N/A | N/A | $89.5(89.2)$ |
| 7 | N/A | N/A | N/A | $88.4(99.6)$ |
| 8 | N/A | N/A | N/A | $100.4(98.8)$ |

note: Numbers in parentheses represent results obtained from the modified network.

Initially, the finite element network was developed based on several guidelines pointed out in previous chapters (e.g. limits on aspect ratio, location of curved boundaries, etc.). The network was constructed such that necessary bottom elevation information can easily be extracted and translated to a portion of the input data deck of the one-dimensional backwater code HY7. Figs. 5.9-5.10 show the same finite element network (169 nodes; 72 elements) along with the numbering (nodes and elements) schemes used. Fig.5.11 shows bed elevations at all corner nodes within the network.

Before any simulation can be made, the most downstream section of the network (cross section bounded by nodes 157 and 169, inclusive) has to have an exit boundary condition (see Sec. 3.4.4). This information can be furnished if a rating curve exists at the section in question or if a simple step backwater profile computation can be performed. Taking the latter course, HY-7 was run using eight cross sections (Fig. 5.12) in addition to the network's most downstream cross section, EXIT. SEC8 in Fig. 5.12 refers to a cross section just upstream of an uncontrolled spillway. Assuming flow to be that over a broadcrested weir, at $Q=10,000$ cfs, the water surface elevation at SEC8 is 701.88 ft above msl (elevation of top of spillway $=$ 696.12 ft above msl; length of spillway $=234.38 \mathrm{ft}$ ). Doing a step backwater profile computation along the channel (Appendices $G$ and $H$ ) using HY-7, the target elevation for cross section EXIT becomes 702.61 ft above msl.

Out of the seven cross sections defined in the finite element network, only four are required in the one-dimensional code HY-7 (APPR, BRIDGE, FULLV, and EXIT). Because of this limitation some variables (i.e., velocities and water surface elevations at nodes located along a section not considered in



Fig. S. 10 - BRUSHY CREEK AT ROUND ROCK, TEXAS: RMAIPLT RUN


Fig. 5.11 BRUSHY CREEK AT ROUND ROCK, TEXAS: RMAIPLT RUN


HY-7) have to be interpolated for the other three cross sections. From Appendix $H$, the computed velocities are broken down to their $x$ and $y$ components (based on the direction of the main channel) and water surface elevations are assumed to be horizontal along the cross sections they represent.

Finally, two sets of simulation runs of RMA-2 are performed, first, using an initial zero-flow, horizontal water surface elevation condition (the default condition) for all nodes, and second, with the computed flows and elevations from HY-7 as initial values.

A total of five runs (control elevations $=705.00$, 704.00, $703.00,702.70,702.61)$ were made for each set. In all runs, the set with specified initial water surface elevations from HY-7 provided solutions to the governing flow equations that (1) converged faster (Table 5.3), and (2) violated the continuity equation to a lesser degree (Table 5.4). Figs. 5.13a-b and Figs. $5.14 \mathrm{a}-\mathrm{b}$ are the vector plots for the normal simulation and the simulation with given initial conditions from HY-7, respectively. Tables 5.3 and 5.4 give a comparison of the two sets of simulations in terms of convergence parameters and mass conservation. They represent values obtained after each computer run (five iteration cycles and one target or control water surface elevation per run).

Table 5.3. -- Comparison of Convergence Parameters (Average Change of Values of Variables Between Iterations) in Simulation Runs With and Without Specified Initial Conditions from HY-7

| WITHOUT HY-7 INPUT |  |  |  | WITH HY-7 INPUT |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RUN | X-FLOW | Y-FLOW | DEPTH | X-FLOW | Y-FLOW | DEPTH |
| NO. |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 1 | .1036 | .1663 | .2227 | .0284 | .0432 | .0642 |
| 2 | .0252 | .0593 | .0993 | .0078 | .0219 | .0416 |
| 3 | .0242 | .0425 | .0667 | .0087 | .0214 | .0405 |
| 4 | .0070 | .0206 | .0429 | .0039 | .0137 | .0294 |
| 5 | .0027 | .0131 | .0322 | .0026 | .0099 | .0232 |

The results indicate that the model performed better (i.e., converged faster) given an approximate solution from less sophisticated methods. The intention of the author is not to combine HY-7 with RMA-2 to make the latter more robust but to show how one would benefit from the other. From a research viewpoint, more work should be done on flow analysis (and backwater effects) at bridges with more complex geometry (i.e., multiple openings, skewed approach channel, etc.).

Table 5.4. -- Comparison of Mass Conservation Between Simulation Runs With and Without Specified Initial Conditions from HY-7

| Line | (Run 1) | (Run 2) | (Run 3) | (Run 4) | (Run 5) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| 1 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
|  | $(100.0)$ | $(100.0)$ | $(100.0)$ | $(100.0)$ | $(100.0)$ |
| 2 | 102.2 | 102.1 | 101.6 | 101.2 | 100.9 |
|  | $(102.1)$ | $(102.1)$ | $(101.6)$ | $(101.2)$ | $(100.9)$ |
| 3 | 97.2 | 97.6 | 97.9 | 97.9 | 97.9 |
|  | $(97.1)$ | $(97.6)$ | $(97.8)$ | $(97.9)$ | $(97.9)$ |
| 4 | 93.8 | 94.2 | 94.4 | 94.2 | 94.1 |
|  | $(93.6)$ | $(94.2)$ | $(94.4)$ | $(94.2)$ | $(94.1)$ |
| 5 | 96.7 | 96.8 | 97.0 | 97.0 | 97.0 |
|  | $(97.2)$ | $(97.8)$ | $(97.8)$ | $(97.8)$ | $(97.7)$ |
| 6 | 96.8 | 96.5 | 95.7 | 95.8 | 95.8 |
|  | $(96.5)$ | $(98.0)$ | $(96.9)$ | $(96.6)$ | $(96.5)$ |
| 7 | 49.6 | 74.7 | 82.7 | 89.7 | 93.6 |
|  | $(89.7)$ | $(92.0)$ | $(91.3)$ | $(94.1)$ | $(96.2)$ |

note: Numbers in parentheses represent results obtained with specified initial conditions from HY-7.


FRBJSHYY ${ }^{513 \mathrm{a}}$ CREEK AT ROUND ROCK, TEXAS WITH ZERO VELOCITY INITIAL CONDITION

GRIO SCALE 1:1200.
UNIT DISCHARGE VECTOR SCALE: I IH $9: 14 \mathrm{CFS} / \mathrm{FT}$


Fi8. 5.14 a
BRUSHY CREEK AT ROUND ROCK, TEXAS WITH INITIAL CONDITION FROM HY-7

GRID SCALE I:1700.
UNIT DISCHAREC VECTOR SCALE: I IN - $92.31 \mathrm{CFS/ft}$


### 6.1 Summary and Conclusions

The study has shown that a computer code written for an IBM PC employing a fairly sophisticated numerical method (finite elements) applied to a set of equations governing free-surface flow can adequately model a natural stream in the vicinity of a bridge crossing. The major accomplishments in this thesis are:

1. Some theoretical aspects of the two-dimensional backwater finite element analysis program RMA-2 were described.
2. Modifications and additions to the type and amount of input data to the Finite-Element Surface-Water Modeling System (FESWMS-TX) were incorporated so that:
a) cross-sectional information in a format directly compatible with existing one-dimensional backwater analysis programs (HY-7 and HEC-2) can be extracted from the finite element network;
b) input data in the new as well as the old (prior to FESWMS-TX) format can be read-in and processed accordingly;
c) an additional method of subdividing total flow along an inlet section of the finite element network can be used as an option;
d) initial conditions supplied by the user (e.g., from onedimensional backwater calculations) can be incorporated; and
e) all data manipulation, calculations, and plotting can be performed using an IBM personal computer.
3. Input requirements were discussed in more detail than in previous applications. A user's manual was written for FESWMSTXPC, the microcomputer version of the Einite-Element SurfaceWater Modeling System (FESWMS) originally created by the U.S. Geological Service (Lee and Bennett, 1981) and modified by Mays and Taur (1983) to become FESWMS-TX.
4. The possibility of providing a good initial "guess" (and thus, guarantee early convergence) to the iterative procedure in solving the governing two-dimensional equations for free-surface flow was explored. This is accomplished by using results obtained from a one-dimensional backwater analysis program (HY7) which was especially written to handle flow computations in
the vicinity of bridges. From the results obtained, a onedimensional code can indeed be a valuable support program for RMA-2.
5. Several demonstration runs were made to show the capabilities of the IBM PC version.

### 6.2 Recommendations

The limitations and therefore grounds for further research in FESWMS-TXPC are based not only on the limited amount of data that a microcomputer can handle nor the speed with which it processes these data (all simulation runs took less than 30 minutes on an IBM AT) but are also due to the following:

1. The present input procedure requires manually describing the topography of the floodplain to be modeled. As such, it is tedious, time-consuming, and expensive. A more sophisticated input procedure, e.g., the use of a computer-aided design (CAD) hardware attached to the personal computer would greatly minimize this inconvenience.
2. The use of HY-7 as a "support" program for RMA-2 was quite cumbersome. All runs made for HY-7 had to be done on the mainframe and as such results have to be downloaded to the IBM PC before FESWMS-TXPC can perform its functions. HY-7 has approximately 7,000 lines with 100 subroutines. On the other hand, RMA-2 is half as long with only 11 subroutines. If HY-7 can be cut down to a set of subroutines pertinent to bridge computations only then it can be converted to a PC version and probably be a part of RMA-2 as a subroutine.
3. HY-7 was found to be useful in two ways. First, it was able to set the exit boundary conditions (see Sect 3.4.4) necessary before any computations can be made. Second, it provided a good initial estimate for the solution of the twodimensional free-surface flow equations. A third benefit that can be derived from HY-7 would be its capability to delineate the extent of a floodplain for two-dimensional flow analysis. The current version of RMA-2 cannot handle substantial "drying" of the nodes describing the finite element network. (In a computational viewpoint, lack of detail in one part of a network that causes some nodes to dry up has relatively small influence at remote locations within the network.) Therefore, the modeler must make sure that the limits of the floodplain be well defined for all design discharges he is considering. Essentially, this task requires the incorporation of the one-dimensional code with an algorithm which will automatically generate the finite element network.

## A P P E N D I X A

PROGRAM LISTING OF REVISED SUBROUTINE USSET


```
C
    subroutine usset
C
```



```
C THE Program IS dEVElOPEd for SETTING FLOW rates for u/S nodes
            COMMON /BLKB/ CORD(825,2),NBC(825,3),VEL(3,825),SPEC(825,3).
        * ALFA(825),AO(825),SIGMA(825,2),VOLD(3,825),VDOT(3,825),NOP(371,
        * 8),IMAT(371),ORT(50,5),NFIXH(825),NFIX(825),NLOC(825),VDOTO(3;
        - 825).TH(371)
            COMMON /CNTRL/ LE,LP,NBUS,NNXUS(83)
c
            DIMENSION PERIM(83),AREA(83),CONVY(83),QSEC(83),PROP(83),DXY(83)
c
    NBUS : OF U/S NODES (MID-POINTS INCLUDED)
    NPROP: WEIGHTING TECHNIQUE OPTION:
        1 r conveyance ratio method
            else g area ratio method
        the order of nodes shown on the u/s inlet must obey the
        the rule of initIal node being on the right hand SIde when
        facing the inlet along the direction of flow.
        an UPSTREM SECTION MUST START AND END WITH CORNER NODES
```




```
C
        READ (5,10) mProp,NbuS,QDISCH,WSElEV
        10 FORMAT (2I5,2F10.2)
        READ (5.20) (NNXUS(N),N-1,NBUS)
    20 FORMAT (16I5)
c
C Calculate coordinates for mid-points
    DO 30 ILL - 1,NBUS-2,2
                    IKK - NNXOS(ILL)
                    IKK1 - NNXUS(ILL+1)
                IKK2 - NNXUS(ILL+2)
                CORD(IKK1,1) = 0.5*(CORD(IKK,1) +CORD(IKKK2,1))
                    CORD(IKK1,2) = 0.5*(CORD(IKK,2) +CORD(IKK2,2))
                AO(IKK1)=- 0.5*(AO(IKKK)+AO(IKK2))
    30 CONTINUE
C
    J = 1
        K - j+1
    40 N = NnXUS(J)
    Ns - nNXUS(K)
```

```
C
C ******** FOR FIRST AND LAST NODES
C
C COMPUTE FOR PERIMETER
C ON HORIZONTAL (XY) PLANE
C
    DX2 = ABS(CORD(N,1)-CORD(NS,1))
    DY2 - ABS(CORD(N,2)-CORD(NS,2))
    DXY(J)=SQRT(DX2*DX2*DY2*DY2)/2.
C
C ON VERTICAL (Z) PLANE
DZ-ABS(AO(N)-AO(NS))/2.
C
C ON XYZ SPACE
C
    PERIM(J) = SQRT(DXY(J)*DXY(J)+DZ*DZ)+(HSELEV-AO(N))
C
C COMPUTE FOR TRIBUTARY AREA
    x1 - 0.
    Y1-AO(N)
    IF (J.EQ.1) THEN
        x2 - 0:
        Y2 - WSELEV
        X3 - - DXY(J)
        Y3 - WSELEV
        X4-X3
        Y4-0.5*(AO(NS)+AO(N))
        ELSE
        X2 - DXY(J)
        Y2-0.5*(AO(NS)+AO(N))
        x3-x2
        Y3 - WSELEV
        X4=0.
        Y4 = WSELEV
        ENDIF
C
C USE DMD METKOD TO COMPUTE FOR THE AREA OF A POLYGON HITH GIVEN COORDIN
```



```
        - XI))
C
C COMPUTE FOR CONVEYANCE
C CONVY(J) =(AREA(J)**(5./3.))/(PERIM(J)**(2.13.))
        IF (J.NE.1) GO TO 50
        J - NBUS
        K - NBUSnl
        GO TO 40
C
C ******** FOR MIDOLE NODES
C
    5O CONTINUE
        DO 60 I - 2,NBUS-1
            NLEFT = NNXUS(I+1)
            NMID - NNXUS(I)
            NRIGHT - NNXUS(I+1)
```

```
C
C COHPUTE FOR PERIMETER
C
C ON THE HORIZONTAL (XY) PLANE
C
    DX2L - ABS(CORD(NMID,1)-CORD(NLEFT,1))
    DX2R - ABS(CORD(NRICHT,1)&CORD(NMID;1))
    DY2L - ABS(CORD(NMID,2) & CORD(NLEFT,2))
    DY2R - ABS(CORD(NRIGHT,2)пCORD(NMID,2))
C
    DXYL - SQRT(DX2L*DX2L+DY2L+DY2L)/2.
    DXYR = SQRT(DX2R*DX2R+DY2R*DY2R)/2.
    DXY(I) = DXYL+DXYR
C
C ON THE VERTICAL (Z) PLANE
C
    DZL - ABS(AO(NMID)-AO(NLEFI))/2.
    D2R - ABS(AO(NRIGHT)\capAO(NMID))/2.
C
C ON THE XYZ SPACE
C
    DL - SQRT(DXYL*DXYL+DZL*DZL)
    DR - SQRT(DXYR*DXYR+DZR"DZR)
C
    PERIM(I) - DL+DR
C
C COMPUTE FOR TRIBUTARY AREA
C DENOTE NMID AS THE ORICIN (XI,Y1) WITH COORDINATES (O.,AO(NMID))
    XI=0.
    Y1 - AO(NMID)
    X2 - DXYR
    Y2 - 0. 5*(AO(NRIGHT) +AO(NMID))
    X3-X2
    Y3 - WSELEV
    X4-0.
    Y4 = WSELEV
    X5 - - DXYL
    Y5 - WSECEV
    X6 = X5
    Y6 - 0.5*(AO(NLEFT) + AO(NMID))
C
C USE DMD METHOD TO COMPUTE FOR THE AREA OF A POLYGON WITK GIVEN COORDIN
    AREA(I) = 0.5*((X)*Y2+X2*Y 3+X * Y4+X4*Y5 X XY6*X6*Y1)-(Y1*X2+Y2*
        * X X Y 3*X4+Y4*X5+Y5X6+Y6*X1))
C
C COMPUTE FOR CONVEYANCE
C
    CONVY(I) = (AREA(I)**(5.13.))/(PERIM(I)**(2./3.))
    6 0 ~ C O N T I N U E ~
```

```
C
C COMPUTE TOTAL PERIMETER, TOTAL AREA AND TOTAL CONVEYANCE
C
        PERIMT - O.
        AREAT - 0.
        CONVYT - O.
        DO 70 I - 1,NBUS
            PERIMT - PERIMT+PERIM(I)
            AREAT - AREAT +AREA(I)
            CONVYT - CONVYT+CONVY(I)
    TO CONTINUE
        DO 8O NNSEC - 1,NBUS
c
c calculate flow rate for each section (node)
        IF (MPROP.EQ.1) THEM
                PROP(NNSEC) - CONVY(NNSEC)/CONVYT
            ELSE
                prop(nNSEC) - area(nNSEC)/areat
            ENDIF
            QSEC(NNSEC) - PROP(NNSEC)*QdISCH/DXY(NNSEC)
C
    80 CONTINUE
    compute angle
        JRR - NNXUS(1)
        JlL - nNXUS(nbus)
        DX - CORD(JRR,1)-CORD(JLL.1)
        DY - CORD(JRR,2)OCORD(JLL,2)
        IP (DX.EQ.O.0) XSIN - 1.0
        IF (DX.EQ:O:O) XCOS - 0.0
        IF (DY:EQ.0:0) XSIN - 0.0
        IF (DY.EQ.0.0) XCOS - 1.0
        IP (DX:EQ:O:O.OR.DY.EQ:O.0) GO TO 90
c
        AN - ABS(DY)/ABS(DX)
        ANGLE - ATAM(AN)
        XSIN - SIN(ANGLE)
        xcos - COS(AMGLE)
c
    90 CONTINUE
c
        DO 100 nnsec - 1.nbus
            JM - nNXUS(NNSEC)
            SPEC(JM.1) - XSIN*QSEC(NNSEC)
            SPEC(JM,2) - XCOS*QSEC(NNSEC)
    CONSIDER EIGHT DIfferent conditions
c first
c
    IF (DX.GT.O.O.AND.DY.LT.O.0) SPEC(JM,1) = SPEC(JM,1)
    IF (DX.GT.O.O.AND.DY.LT.O.O) SPEC(JM,2) - SPEC(JM,2)
C
C SECOND
IF (DX.LT.O.0.AND.DY.GT.0.0) SPEC(JM,1) = - SPEC(JM,1)
    IF (DX.LT.O.O.AND.DY.GT.0.0) SPEC(JM,1) = -SPEC(JM,1)
C
c THIRD
    IF (DX.GT.O.O.AND.DY.GT.0.0) SPEC(JM,1) = -SPEC(JM.1)
    IF (DX.GT.0.0.AND.DY.GT.0.0) SPEC(JM,2) = SPEC(JM,2)
```

```
C
C FOURTH
C
    IF (DX.LT.O.O.AND.DY.LT.O.0) SPEC(JM,1) - SPEC(JM,1)
    IF (DX.LT.O.O.AND.DY.LT.0.0) SPEC(JM,2) = -SPEC(JM;2)
C
C FIFTH
C
C
C SIXTH
C
    IF (DX.EQ.O.O.AND.DY.LT.O.O) SPEC(JM,1) = SPEC(JM,1)
C
C SEVENTH
C
EIGHTH
C
C
    100 cONTINUE
C
C COMPUTE FOR QTOT USING SIMPSON'S RULE
C
        KITER = O
        MAXIM - NBUSe2
    110 CONTINUE
        SUMX = 0.0
        SUMY = 0:0
        DO 120 K = 1,MAXIM.2
            N1-NNXOS(K)
            N2 - NNXUS(K+1)
            N3 = NNXUS (K+2)
            DX = (CORD(N3,1)-CORD(N1,1))/6.
            DY = (CORD(N3,2)-CORD(N1,2))/6.
            DT - HSELEV-AO(NT)
            D3 - HSELEVaAO(N3)
            D2 - (D1+D3)/2.
            SUMX = SUMX*DY*(SPEC(N1,1)+4.0*SPEC(N2,1)+SPEC(N3,1))
            SUMY = SUMY - DX* (SPEC(N1,2)+4:O*SPEC(N2,2)+SPEC(N3,2))
    120 CONTINUE
        QTOT - SQRT (SUMX SUMX + SUHY *SUMY)
        DIFFER - QDISCH-QTOT
        EPSIL - ABS(DIFFER/QDISCH)
        IF (EPSIL.LE.O.005) CO TO 140
        DO 130 L - 1,NBUS
            MNO - NNXUS(L)
            FACIUR = QDISCH/OTOT
            SPEC(MNO,1) - SPEC(MNO,1)*FACTOR
            SPEC(MNO,2) - SPEC(MNO,2)*FACTOR
    130 CONTINUE
        KITER - KITER+1
        IF (KITER.LT.3) GO TO 110
C
    140 RETURN
    END
```


# A P P EN D I X B 

USER'S GUIDE

Card Input
No.
(1) TITLE - 20A4

TITLE Any 80 column comment; this comment will appear on both the printed and plotted output.
(2) ISLP, IPNN, IPEN, IPO, IRO, IPP, NXZL, IPLT - 8 I5

ISLP Parameter to input corner node slope:
0 : no input required, or
1 : corner node slope input required in card no. 6.

IPNN Parameter to plot node numbers: 0 : off,
1 : all node numbers to be plotted, or
2 : ground-surface elevations at corner nodes to be plotted.

IPEN Parameter to plot element numbers: 0 : off,
1 : element numbers to be plotted, or
2 : element types to be plotted.
IPO Parameter to draw network plot:
0 : off,
1 : network plot to be drawn, or
2 : additional plots on the same data written in unit 8 (filename $=$ "TAPE8").

IRO Parameter to arrange element order:
0 : off; or
1 : program is to internally arrange the element order for a more efficient hydrodynamic solution, input required in card no. 11.

IPP

NXZL Number of network line segments for which the program will internally calculate exact coordinates to insure a straight line of equal slope. Additional input required in card no. 10.

IPLT User's option to dump plot to a plotter or the screen:
0 : Plot is dumped on the screen, 1 : Plot is to be made on a HP7470 (or IBM6180) plotter, or 2 : Plot is to be made on a HP7475 (or IBM7372) plotter.
(3) HORIZ, VERT, XSCALE, YSCALE, AR - 5F10.0

HORIZ Maximum horizontal size of plot, inches. If scale plot factors are used, set to zero.

VERT Maximum vertical size of plot, inches. If scale plot factors are used, set to zero.


NXPMAX Node number of maximum $X$ location in a partial network plot.

NYPMIN Node number of minimum $Y$ location in a partial network plot.

NYPMAX Node number of maximum $Y$ location in a partial network plot.
note: - Use card no. 5-a only if IPP $=1$. - Use card no. 5-b only if $I P P=2$.
(7) $\quad \operatorname{MSN}(\mathrm{N}) \quad-\quad \mathrm{I} 10$

MSN(N) Midside node on element with curved side/s for which coordinates are to be calculated using specified slopes of adjacent corner nodes.
note: - Use this card only if ISLP $=1$.

- For each midside node specified (one per card) its corresponding adjacent corner nodes and their respective slopes should have been given in card no. 6.
- Terminate with a card containing 9999 in columns 7-10.
(8) J, $\operatorname{NOP}(J, 1), \operatorname{NOP}(J, 2), \operatorname{NOP}(J, 3), \operatorname{NOP}(J, 4), \operatorname{NOP}(J, 5)$, NOP (J, 6), bf, IMAT(J) - 7I5, 10X, I5
$J \quad$ Element number.
$\operatorname{NOP}(J, 1)$
:
:

```
    NOP(J,6) Element node numbers starting at any
    corner node and moving counter-
    clockwise around the element.
    bf Leave columns 36-45 blank.
    IMAT(J) Element type number.
    note: - Use this card for each element in the
        network and terminate with a card containing
        9999 in columns 2-5.
(9) J, CORD(1), CORD(2), WD(J), ALPHA(J) - I10, 4E10.0
    J Node number.
    CORD(1) X-coordinate of node J.
    CORD(2) Y-coordinate of node J.
    WD(J) Bottom elevation of node J above
    m.s.l.
    ALPHA(J) Slope to be assigned to node J.
    note: - ALPHA(J) on this card, if different from
        zero, overrides input on card no. 6.
        - Use this card for each node in the network
                and terminate with a card containing 9999 in
        columns 7-10.
(10) NA, NB, NIP(J) - 16I5
    NA A corner node at one end of a
        straight line segment.
    NB The corner node at the other end of
    the straight line segment.
```

NIP (J) Corner nodes between nodes NA and NB for which the $Y$-coordinate is to be interpolated using the input values of the X -coordinate ( thus replacing the inputted values for the $Y$ coordinate, CORD(2) in card no. 9, if any); J goes from 1 to a maximum of 14.
note: - Use this card only if NXZL (card no. 2) > 0 .

- Use this card NXZL times.
- Terminate with a card containing 9999 in columns 2-5 if IRO $=0$ else, proceed below.
(11) NLIST (N) - I5

NLIST(N) A list of node points (from 1 to 160) from which the program will reorder the internal sequence of elements to obtain the most efficient operation of the simulation programs. As a general rule at least two starting locations should be tried, one at each end of the network. Enter all above cards and terminate with a card containing 9999 in columns 2-5. (This terminal card is also called a sentinel card.)
note: - Use this card only if $\operatorname{IRO}=1$.

- If used:
a) Enter 16 node numbers per line, i.e., $16 I 5$ per card.
b) The end of the list is a blank card field. If the last value occupies the last field (columns 76 to 80 ) in the card, terminate with a blank card and a sentinel card; otherwise, just a sentinel card is required.

INPUT DATA STRUCTURE FOR RMA1PLT-PC

```
Card Input
    No.
    Description
(1) TITLE - 11A4
    TITLE Any 44 column comment; this comment
        will appear on both the printed and
        plotted output.
    (2) IPNN, IPEN, IPP, IPLT - 4I5
    IPNN Parameter to plot node numbers :
        0 : off,
        1 : all node numbers to be plotted,
        Or
        2 : ground-surface elevations at
        corner nodes to be plotted.
    IPEN Parameter to plot element numbers :
        0 : off,
        1 : element numbers to be plotted, or
        2 : element types to be plotted.
        IPP Parameter to produce partial network
        plot :
        0 : off, no partial plot is produced,
        no input of coordinates of the
        lower left-hand corner of the
        plot is needed in card no. 4,
        minimum X and minimum Y are to be
        used as lower left-hand corner of
        plot; or
    1 : partial plot to be produced,
        Additional input for lower left-
        hand corner (origin) of the plot
        required in card no. 4.
```

IPLT User's option to dump plot to a plotter or the screen :
0 : Plot is dumped on the screen, 1 : Plot is to be made on a HP7470 (or IBM6180) plotter, or
2 : Plot is to be made on a HP7475 (or IBM7372) plotter.
(3) XSCALE, YSCALE - 2F10.0

XSCALE Plotting scale factor for X (horizontal) inputs.

YSCALE Plotting scale factor for Y (vertical) inputs.
(4) $\mathrm{XO}, \mathrm{YO}-2 \mathrm{~F} 10.0$

X0 X -coordinate of point that will be lower left-hand corner of network plot.

Y0 Y-coordinate of point that will be lower left-hand corner of network plot.
note: - Use this card only if $\operatorname{IPP}=1$.

## INPUT DATA STRUCTURE FOR RMA2-PC

## Card Input

No.
Description
(1) TITLE - 20A4

TITLE Any 80 column comment; this comment will appear on both the printed and plotted outputs.
(2) NE, NMAT, NPX, NBX, IPRT, NCL, IWIND, NB, NLL, IFILE, NOPT - 11 I5

NE The number of element cards to be read. Set to zero if geometry file (unit IFILE) from RMA1 is to be used.

NMAT The number of different sets of turbulent exchange coefficients (eddy viscosities) and Manning's "n".

NPX The number of node coordinate cards to be read. Set to zero if geometry file (unit IFILE) from RMAl is to be used.

NBX The number of nodes with parallel and/or exit boundary conditions specified.

IPRT Control for output printing :
0 : node and element data output suppressed,
1 : all input data printed, or
2 : print only the initial conditions for velocity and depth.

NCL The number of line segments along which flow continuity is to be checked.

IWIND Control for wind field input : 0 : no wind field input, or
1 : wind field input.

| NB | Logical unit for file from which to read initial conditions (enter 0 if no file is to be read; -1 if inital conditions will be read-in from card $7-\mathrm{C}$ ) . |
| :---: | :---: |
| NLL | Logical unit for file upon which to write restart conditions (enter 0 if no file is to be written). |
| IFILE | Logical unit for file containing geometric data created by program RMA1 (enter 0 if geometry file is to be read-in from TAPE5; card nos. 7-a and $7-\mathrm{b}$; otherwise, use logical unit $3)$. |
| NOPT* | Logical unit for file upon which to write a complete time history of simulation (enter 0 if no file is to be written; otherwise, use logical unit 4). |
| note: - If for wil | egative sign preceeds NBX, the old for inputting boundary conditions followed. See card no. 8. |
| OMEGA, ELE | MP, XSCALE, ZSCALE - 5E10.0 |
| OMEGA | Average local latitude in degrees. |
| ELEV | Maximum initial water surface elevation (feet) with respect to the m.s.l. This value will be used to determine the initial water depths for all nodes in the network if variable NB (card no. 2) is zero. |
| TEMP | Average initial water temperature ( ${ }^{\circ} \mathrm{C}$ ). |
| XSCALE | Scale factor to convert X -coordinate inputs (from RMA1) to feet. |
| ZSCALE | Scale factor to convert $Y$-coordinate inputs (from RMA1) to feet. |

(4) NITI, NITN, NCBC, NSTART, DELT, TMAX - 4I5, 2E10.0

NITI Number of iterations for initial solution.

NITN* Number of iterations per dynamic solution step.

NCBC* Number of time steps between updates of boundary conditions.

NSTART* Starting time step.
DELT* Length of time step (hours).
TMAX* Total simulation time (hours).
note: - Variables with asterisks (*) are required for dynamic simulation.
(5) J, $\operatorname{ORT}(J, 1), \operatorname{ORT}(J, 2), \operatorname{ORT}(J, 3), \operatorname{ORT}(J, 4), \operatorname{ORT}(J, 5)$ - I10, 5E10.0
$J$ Identification number for a set of eddy viscosities. This variable corresponds to the element type number input for card no. 8 in program RMA1.

ORT (J,1) Turbulent exchange coefficient associated with the X direction in the X plane (lb- sec/ft2) for elements of type J.

ORT(J,2) Turbulent exchange coefficient associated with the $Y$ direction in the X plane (lb-sec/ft2) for elements of type J.

ORT(J,3) Turbulent exchange coefficient associated with the X direction in the $Y$ plane (lb-sec/ft2) for elements of type J.

```
        ORT(J,4) Turbulent exchange coefficient
        associated with the Y direction in
        the Y plane (lb-sec/ft2) for elements
        of type J.
    ORT(J,5) Manning's "n" for element of type J.
    note: - Use this card NMAT times.
(6) LINE (J,K) - 14I5
    LINE(J,K) List of corner node numbers which
        define line segment J across which
        total flow is to computed for
        continuity. K goes from 1 to a
        maximum of 42, i.e., }14\mathrm{ node numbers
        per line. The last node number in
        continuity line J must be negative
        unless it is in a field position less
        than the 14th on a card.
    note: - J goes from 1 to NCL.
(7-a) J, NOP(J,1), NOP(J,2), NOP(J,3), NOP(J,4),
NOP (J,5),
    NOP(J,6), bf, IMAT(J) - 7I5, 10X, I5
        J Element number.
        NOP (J,1)
        :
        :
            NOP(J,6) Element node numbers starting at any
                                    corner node and moving counter-
                                    clockwise around the element.
            bf Leave columns 36-45 blank.
            IMAT(J) Element type number.
    note: - Use this card only if IFILE = 0.
            - Use this card NE (card no. 2) times.
```

```
(7-b) J, CORD (J,1), CORD (J,2), AO(J) - I10, 3E10.0
    J Node number.
    CORD (J,1) X-coordinate of node J.
    CORD (J,2) Y-coordinate of node J.
    AO(J) Slope to be assigned to node J.
    note: - Use this card only if IFILE = 0.
        - Use this card NPX (card no. 2) times.
(7-c) M, TV(3) - I5, 3E10.0
    J Node number.
    TV(1) Initial velocity in the x-direction
                                at node M.
    TV(2) Initial velocity in the Y-direction
        at node M.
        TV(3) Initial water surface elevation at
        node M.
    note: - Use this card only if NB = -1.
(8) J, bf1, NFIX1, NFIX2, NFIX3, bf2, SPEC(J, 1),
    SPEC (J,2), SPEC (J,3) - I10, 5X, 3I1, 2X, 3E10.0
        J Node number.
        bf Leave columns 11-15 blank.
        NFIX1 Enter 1 if the X-direction flow is to
        have an exact boundary condition;
        otherwise leave blank.
        NFIX2 Enter 2 if the Y-direction flow is to
        have an exact boundary condition;
        otherwise leave blank.
    NFIX3 Enter 2 if the head is to have an
        exit boundary condition; otherwise
        leave blank.
```

```
    bf Leave columns 19-20 blank.
    SPEC(J,1) The specified velocity in the x-
    direction at node J.
    SPEC(J,2) The specified velocity in the y-
        direction at node J.
    SPEC(J,3) The specified water surface elevation
        at node J.
note: - Use this card only if the old format for 
(9) N, NFIX(N), bf, SPEC(N,3) - 2I10, 20X, E10.0
    N Node number.
    NFIX(N) Integer sum of the following values
        which sets computational parameters
        within the program :
    200 : If node N is a D/S boundary
        node
        1000 : If node N is a parallel flow
        boundary node
    bf Leave columns 21-40 blank.
    SPEC(N,3) The specified water surface elevation
        at node N.
note: - If NFIX(N) = 1200, then flow will be
        allowed to move parallel to the boundary at
        D/S node N. This card should be used only
        for parallel flow and/or downstream boundary
        nodes. Nodes at the upstream section are
        handled by input card nos. 10 and 11.
    - Use this card NBX times.
```

(10) NINLET ..... I5
NINLET Number of inlet sections.
(11) NPROP, NBUS, QDISCH,WSELEV - 2I5, 2F10.2
NPROP Parameter to specify the techniqueused in subdividing the totaldischarge in an inlet section :0 : area ratio method, or
1 : conveyance ratio method.
NBUS Number of nodes (midside nodesincluded) in the inlet section.
QDISCH Discharge at the inlet section.
WSELEV Average water surface elevation(above msl) at the inlet section.
note: - WSELEV should not be changed from oneiteration to the next. Otherwise, thecomputed discharge per unit width for eachnode at the inlet section will vary.
(12) NNXUS (N) - $16 I 5$NNXUS (N) Node numbers (corner and midsidenodes combined) at the inlet section.N goes from 1 to NBUS, 16 values to aline.
note: - The order by which the nodes should be specified should be from right to left (looking downstream).

- Card sequence 11 and 12 is used NINLET times.

Card Input
No.
Description
(1) TITLE - 11A4

TITLE Any 44 column comment; this comment will appear on both the printed and plotted output.
(2) STITLE - 11A4

STITLE Any 44 column comment; this comment will appear on the printed output and may or may not appear on the plotted output.
note: - TITLE and STITLE should be centered within the 44 columns allotted to them. See note after card no. 5.
(3) NOPTO, IPLT - $2 I 5$

NOPTO Origin option : 0 : Use XMIN, YMIN as origin, or 1 : Input origin required.

IPLT User's option to dump plot to a plotter or the screen :
0 : Plot is dumped on the screen, 1 : Plot is to be made on a HP7470 (or IBM6180) plotter, or
2 : Plot is to be made on a HP7475 (or IBM7372) plotter.
(4) HORIZ, VERT, XSCALE, YSCALE, AR, X0, YO, NOPT6 7E10.0, I5

HORIZ Horizontal size (in.) of plot.
VERT Vertical size (in.) of plot.

```
    XSCALE Plotting scale factor (in./ft.) in
    the x-direction.
    YSCALE Plotting scale factor (in./ft.) in
    the y-direction.
    AR Plot rotation (clockwise) in degrees
    from the X-axis.
    X0 X-coordinate of point that will be
        origin of region to be plotted.
    Y0 Y-coordinate of point that will be
    origin of region to be plotted.
    NOPT6 Parameter to plot boundary :
    0 : Not to plot boundary, or
    1 : Plot boundary.
note: - Use X0 and Y0 only if NOPT0 = 1.
    - X0 and Y0 need not correspond to the
        coordinates of a node (or nodes) in the
        network.
(5) IFLAG, AN, XON, YON, XOS, YOS - I5, 5F10.2
    IFLAG Legend option :
    0 : No titles and north arrow,
    1 : Titles and north arrow using
        values read-in,
    2 : Titles and north arrow using
        default values, or
    3 : Vertical title only.
    AN Rotation (clockwise) of north arrow
    in degrees from vertical.
    XOS X-coordinate (in.) for TITLE.
    YOS Y-coordinate (in.) for TITLE.
    X0N X-coordinate (in.) of base of north
    arrow.
    Y0N Y-coordinate (in.) of base of north
    arrow.
```

```
note: - X0S and Y0S marks the lower left-hand corner
    of the first character (including leading
    blank characters) to be drawn.
(6) NOPT3, IPNN, NOPT1, NOPT2, NDEC1, NDEC2 - 6I5
NOPT3 Node Velocity/Node Unit Dischrarge Option :
                                    0 : To plot velocity vectors at the
                nodes, or
1 : To plot unit discharge vectors at
                the nodes.
    IPNN Number (0, 1 or 2) of variables whose
        values are to be plotted beside a
        node.
    NOPT1 TYpe of variable whose value is to be
        plotted beside a node (Variable
        referred to by IPNN = 1 or first
        variable referred to by IPNN = 2) :
        0 : Node number,
        1 : Velocity or Unit discharge,
        2 : Water Surface Elevation, or
        3 : Depths.
        Similar to NOPT1 (second variable
        referred to by IPNN = 2).
        NDEC1 Number of digits to the right of the
        decimal point of the value of the
        variable denoted by NOPT1.
    Number of digits to the right of the
        decimal point of the value of the
        variable denoted by NOPT2.
note: - Use NOPT2 and NDEC2 only if IPNN = 2.
    - If NDEC1 (or NDEC2) = 0, the numbers are
        rounded off to the nearest integer.
    - If NDEC1 (or NDEC2) = -1, only integer parts
        of numbers are plotted, i.e. truncation is
        done.
```

```
NOPT4, VLNGTH, VSCALE - I5, 2F10.0
    NOPT4 Option for scaling :
        0 : To scale velocity (or unit
                                discharge) by max. abs. value
                equals VLNGTH, or
    1 : To scale velocity (or unit
                                discharge) by 1 in. equals
        VSCALE.
    VLNGTH Length (in.) of max. abs. velocity
        (or unit discharge) vector.
    VSCALE Number of ft/sec (or cfs/ft)
        corresponding to. 1 in. length of
                            velocity (or unit disharge) vector.
note: - If NOPT4 = 0, VLNGTH should be greater than
        zero and VSCALE (if supplied by the user)
        will be overwritten with an internally
        computted value.
    - If NOPT4 = 1, VSCALE should be greater than
        zero and VLNGTH (if supplied by the user)
        will be overwritten with an internally
        computted value.
```

NOPT8, NOPT7, NOPT10, NOPT11, NOPT5 - 5 I5
NOPT8 Node identity and boundary option :
0 : Plot velocity (or unit discharge)
vector (using the symbol "--->")
at the nodes,
1 : Plot outline of boundary only, or
2 : Plot symbol " + " at the nodes.
NOPT7 Solution data file option :
0 : To read unformatted plot file
from "TAPE9", or
1 : To read formatted plot file from
"TAPE9".
NOPT10 Continuity plot option :
0 : Do not plot continuity lines and
discharges, or
1 : Plot continuity lines and
discharges.

NOPT11 Continuity line data file option : 0 : To read unformatted plot file from "TAPE10", or
1 : To read formatted plot file from "TAPE10".

NOPT5 Contouring input data file option : 0 : Do not write a file ("TAPE8") for contouring program input, or 1 : Write a file ("TAPE8") for contouring program input.
note: - TAPE8 in this program is different from the I/O files used in programs RMA1 and RMA1PLT that also come under the same filename.

DEFINITION OF CARDS IN FORMATTED SOLUTION FILE (TAPE9)
(1) NNODE - I5

NNODE Node number.
(2) XVEL, YVEL, WSELEV - $3 F 10.0$

XVEL Velocity (ft/sec) in the $x$-direction.
YVEL Velocity (ft/sec) in the $y$-direction.
WSELEV Water surface elevation for node NNODE.
note: - Use formatted TAPE9 only if NOPT7 $=1$.

```
DEFINITION OF CARDS IN FORMATTED CONTINUITY FILE (TAPE10)
(1) NCLINES - I5
NCLINES Number of continuity lines where mass
                    balance will be checked.
(2) LIMIT - I5
LIMIT Number of nodes (corner and midside)
                    define in continuity line LINE(K)
                    below.
(3) LINE (K) - 16I5
LINE (K) Node numbers in a continuity line; \(K\) goes from 1 to LIMIT.
note: - Card sequence \(2-3\) is used NCLINES times. - Use formatted TAPE10 only if NOPT10 = 1 and NOPT11 = 1 .
```

A P P ENDIXC

INPUT DATA FOR EXAMPLE 1

210
198
175
165
153
141
127
112 -99 85 71
59 46 35 336 353 279 305 315 321 323 3
22 62 91
119
145
184
247
258 9999

| 1 | 15 | 16 | 17 | 2 | 8 | 3 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 2 | 8 | 2 | 17 | 1 | 10 | 9 |
| 3 | 10 | 1 | 17 | 4 | 12 | 11 |
| 4 | 17 | 18 | 19 | 5 | 12 | 4 |
| 5 | 12 | 5 | 19 | 6 | 13 | 245 |
| 6 | 41 | 29 | 30 | 23 | 15 | 22 |
| 7 | 15 | 23 | 30 | 24 | 17 | 16 |
| 8 | 17 | 24 | 30 | 31 | 32 | 25 |
| 9 | 17 | 25 | 32 | 26 | 19 | 18 |
| 10 | 19 | 26 | 32 | 33 | 34 | 27 |
| 11 | 19 | 27 | 34 | 20 | 13 | 6 |
| 12 | 13 | 20 | 34 | 28 | 21 | 244 |
| 13 | 75 | 63 | 53 | 52 | 41 | 62 |
| 14 | 41 | 52 | 53 | 42 | 30 | 29 |
| 15 | 30 | 42 | 53 | 54 | 47 | 43 |
| 16 | 30 | 43 | 47 | 36 | 32 | 31 |
| 17 | 47 | 48 | 44 | 37 | 32 | 36 |
| 18 | 32 | 37 | 44 | 38 | 34 | 33 |
| 19 | 44 | 45 | 40 | 39 | 34 | 38 |
| 20 | 34 | 39 | 40 | 35 | 21 | 28 |
| 21 | 75 | 78 | 79 | 64 | 53 | 63 |
| 22 | 53 | 64 | 79 | 66 | 67 | 65 |
| 23 | 53 | 65 | 67 | 55 | 47 | 54 |
| 24 | 67 | 68 | 57 | 56 | 47 | 55 |
| 25 | 47 | 56 | 57 | 49 | 44 | 48 |
| 26 | 44 | 49 | 57 | 58 | 51 | 50 |
| 27 | 44 | 50 | 51 | 46 | 40 | 45 |
| 28 | 105 | 106 | 107 | 92 | 75 | 91 |
| 29 | 75 | 92 | 107 | 93 | 79 | 78 |
| 30 | 79 | 80 | 81 | 72 | 67 | 66 |
| 31 | 67 | 73 | 74 | 69 | 57 | 68 |
| 32 | 74 | 70 | 61 | 60 | 57 | 69 |
| 33 | 57 | 60 | 61 | 59 | 51 | 58 |



| 99 | 229 | 239 | 212 | 238 | 219 | 243 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | 225 | 232 | 233 | 227 | 219 | 226 | 1 |
| 101 | 233 | 231 | 229 | 243 | 219 | 227 | 1 |
| 102 | 218 | 236 | 233 | 232 | 225 | 213 | 1 |
| 103 | 218 | 240 | 241 | 237 | 233 | 236 | 1 |
| 104 | 241 | 234 | 229 | 231 | 233 | 237 | 1 |
| 105 | 81 | 82 | 74 | 73 | 67 | 72 | 1 |
| 106 | 95 | 96 | 88 | 87 | 81 | 86 | 1 |
| 107 | 13 | 244 | 21 | 7 | 14 | 246 | 2 |
| 108 | 255 | 250 | 218 | 214 | 221 | 249 | 2 |
| 109 | 257 | 251 | 218 | 250 | 255 | 256 | 2 |
| 110 | 257 | 252 | 241 | 240 | 218 | 251 | 2 |
| 111 | 268 | 259 | 253 | 258 | 270 | 269 | 3 |
| 112 | 266 | 260 | 253 | 259 | 268 | 267 | 3 |
| 113 | 266 | 261 | 255 | 254 | 253 | 260 | 3 |
| 114 | 264 | 263 | 257 | 256 | 255 | 262 | 2 |
| 115 | 264 | 262 | 255 | 261 | 266 | 265 | 2 |
| 116 | 268 | 269 | 270 | 278 | 298 | 277 | 2 |
| 117 | 282 | 273 | 266 | 267 | 268 | 274 | 2 |
| 118 | 280 | 271 | 264 | 265 | 266 | 272 | 2 |
| 119 | 280 | 272 | 266 | 273 | 282 | 281 | 2 |
| 120 | 288 | 284 | 280 | 281 | 282 | 285 | 2 |
| 121 | 288 | 285 | 282 | 286 | 290 | 289 | 2 |
| 122 | 290 | 286 | 282 | 287 | 292 | 291 | 2 |
| 123 | 292 | 287 | 282 | 283 | 294 | 293 | 2 |
| 124 | 282 | 274 | 268 | 275 | 294 | 283 | 2 |
| 125 | 294 | 275 | 268 | 276 | 296 | 295 | 2 |
| 126 | 296 | 297 | 298 | 302 | 306 | 301 | 2 |
| 127 | 298 | 303 | 308 | 307 | 306 | 302 | 2 |
| 128 | 298 | 278 | 270 | 279 | 300 | 299 | 2 |
| 129 | 298 | 299 | 300 | 304 | 308 | 303 | 2 |
| 130 | 300 | 305 | 310 | 309 | 308 | 304 | 2 |
| 131 | 306 | 307 | 308 | 312 | 316 | 311 | 2 |
| 132 | 316 | 312 | 308 | 313 | 318 | 317 | 2 |
| 133 | 308 | 309 | 310 | 314 | 318 | 313 | 2 |
| 134 | 318 | 314 | 310 | 315 | 320 | 319 | 2 |
| 135 | 316 | 317 | 318 | 325 | 337 | 324 | 2 |
| 136 | 318 | 319 | 320 | 327 | 339 | 326 | 2 |
| 137 | 337 | 325 | 318 | 326 | 339 | 338 | 2 |
| 138 | 339 | 327 | 320 | 321 | 322 | 328 | 2 |
| 139 | 337 | 338 | 339 | 347 | 354 | 346 | 2 |
| 140 | 339 | 355 | 372 | 362 | 354. | 347 | 2 |
| 141 | 339 | 328 | 322 | 330 | 10 | 329 | 2 |
| 142 | 339 | 329 | 10 | 340 | 356 | 348 | 2 |
| 143 | 10 | 331 | 341 | 349 | 356 | 340 | 1 |
| 144 | 322 | 323 | 8 | 9 | 10 | 330 | 3 |
| 145 | 356 | 349 | 341 | 350 | 358 | 357 | 2 |
| 146 | 385 | 365 | 356 | 357 | 358 | 366 | 2 |
| 147 | 385 | 366 | 358 | 367 | 382 | 381 | 2 |
| 148 | 354 | 362 | 372 | 371 | 370 | 361 | 2 |
| 149 | 339 | 348 | 356 | 363 | 372 | 355 | 2 |
| 150 | 370 | 371 | 372 | 374 | 375 | 373 | 2 |
| 151 | 375 | 374 | 372 | 376 | 378 | 377 | 1 |
| 152 | 372 | 363 | 356 | 364 | 378 | 376 | 1 |
| 153 | 378 | 364 | 356 | 365 | 385 | 379 | 1 |
| 154 | 378 | 379 | 385 | 381 | 382 | 380 | 1 |
| 155 | 382 | 367 | 358 | 368 | 384 | 383 | 2 |
| 156 | 358 | 352 | 345 | 353 | 360 | 359 | 3 |
| 157 | 358 | 351 | 343 | 344 | 345 | 352 | 3 |
| 158 | 341 | 331 | 10 | 11 | 12 | 332 | 2 |
| 159 | 253 | 248 | 221 | 217 | 189 | 247 | 3 |
| 160 | 255 | 249 | 221 | 248 | 253 | 254 | 3 |
| 161 | 341 | 342 | 343 | 351 | 358 | 350 | 2 |
| 162 | 341 | 333 | 13 | 334 | 343 | 342 | 2 |
| 163 | 343 | 334 | 13 | 246 | 14 | 335 | 2 |


| 164 | 343 | 33514 | 336 345 | 344 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 165 | 358 | 359360 | 369384 | 368 | 3 |
| 166 | 341 | 33212 | 24513 | 333 | 1 |
| 167 | 296 | 276268 | 277 298 | 297 | 2 |
| 9999 |  |  |  |  |  |
|  | 8 | 1003.0 | 446.0 | 737:08 |  |
|  | 10 | 1106.0 | 442.0 | 735:00 |  |
|  | 12 | 960:0 | 262.0 | 727:80 |  |
|  | 13 | 988.0 | 168.0 | 734:00 |  |
|  | 14 | 1051:0 | -90:0 | 737:07 |  |
|  | 15 | 861.0 | 351.0 | 737.12 |  |
|  | 17 | 922:0 | 291.0 | 733:41 |  |
|  | 19 | 876.0 | 195.0 | 726.20 |  |
|  | 21 | 892.0 | 106:0 | 737:25 |  |
|  | 30 | 733.0 | 210.0 | 733.85 |  |
|  | 32 | 785:0 | 162:0 | 725:20 |  |
|  | 34 | 834.0 | 122.0 | 734.26 |  |
|  | 40 | 758:0 | 109:0 | 737:33 |  |
|  | 41 | 737.0 | 308.0 | 737.10 |  |
|  | 44 | 719:0 | 119:0 | 734:42 |  |
|  | 47 | 700.0 | 147.0 | 724.20 |  |
|  | 51 | 664:0 | 113:0 | 737:38 |  |
|  | 53 | 624.0 | 205.0 | 734.08 |  |
|  | 57 | 640:0 | 123:0 | 734:54 |  |
|  | 61 | 589.0 | 118:0 | 737.73 |  |
|  | 67 | 598:0 | 153:0 | 724:20 |  |
|  | 74 | 563.0 | 130.0 | 734.66 |  |
|  | 75 | 596.0 | 278:0 | 737:59 |  |
|  | 77 | 523.0 | 126.0 | 737.87 |  |
|  | 79 | 504:0 | 218:0 | 734:34 |  |
|  | 81 | 518.0 | 166:0 | 725.50 |  |
|  | 88 | 472:0 | 147:0 | 734:80 |  |
|  | 90 | 447.0 | 145.0 | 738:10 |  |
|  | 95 | 441:0 | 188:0 | 726:90 |  |
|  | 102 | 411.0 | 166.0 | 734:89 |  |
|  | 104 | 379:0 | 167:0 | 738:33 |  |
|  | 105 | 435.0 | 340:0 | 738.16 |  |
|  | 107 | 445:0 | 256:0 | 734:50 |  |
|  | 114 | 343.0 | 236.0 | 728:20 |  |
|  | 116 | 333:0 | 200:0 | 735:02 |  |
|  | 118 | 305.0 | 200.0 | 738.42 |  |
|  | 128 | 268:0 | 242:0 | 734.99 |  |
|  | 129 | 365.0 | 455.0 | 738.47 |  |
|  | 131 | 297:0 | 393:0 | 734.91 |  |
|  | 133 | 249.0 | 301.0 | 728.50 |  |
|  | 137 | 224:0 | 261:0 | 738.70 |  |
|  | 142 | 196.0 | 307.0 | 735.13 |  |
|  | 144 | 165:0 | 335:0 | 738:75 |  |
|  | 149 | 192.0 | 402.0 | 728.90 |  |
|  | 154 | 163:0 | 384.0 | 735.26 |  |
|  | 156 | 137.0 | 423.0 | 738.80 |  |
|  | 157 | 375:0 | 603:0 | 739.01 |  |
|  | 159 | 251.0 | 510.0 | 735.18 |  |
|  | 166 | 145:0 | 471:0 | 735:39 |  |
|  | 169 | 164.0 | 538.0 | 729.50 |  |
|  | 173 | 123:0 | 517:0 | 738:93 |  |
|  | 177 | 195.0 | 606.0 | 735.41 |  |
|  | 181 | 136:0 | 573:0 | 735:54 |  |
|  | 185 | 121.0 | 617.0 | 739.14 |  |
|  | 189 | 430.0 | 746.0 | 739:74 |  |
|  | 191 | 240.0 | 783.0 | 735:81 |  |
|  | 194 | 157.0 | 642.0 | 729.80 |  |
|  | 199 | 138:0 | 686:0 | 735:71 |  |
|  | 201 | 122.0 | 712.0 | 739:62 |  |
|  | 202 | 159:0 | 719:0 | 730:50 |  |


|  | 212 | 126.0 | 790:0 | 739.90 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 215 | 157:0 | 800.0 | 731:90 |  |  |  |
|  | 218 | 232.0 | 920.0 | 734.00 |  |  |  |
|  | 219 | 151:0 | 830:0 | 735:92 |  |  |  |
|  | 221 | 302.0 | 852.0 | 736.00 |  |  |  |
|  | 225 | 180:0 | 874:0 | 733:80 |  |  |  |
|  | 228 | 140.0 | 749.0 | 735.80 |  |  |  |
|  | 229 | 136:0 | 872:0 | 740:00 |  |  |  |
|  | 233 | 170.0 | 883.0 | 736:00 |  |  |  |
|  | 241 | 202:0 | 931:0 | 740:10 |  |  |  |
|  | 253 | 492:0 | 822.0 | 740.73 |  |  |  |
|  | 255 | 344:0 | 988:0 | 734:00 |  |  |  |
|  | 257 | 286:0 | 990:0 | 740.14 |  |  |  |
|  | 264 | 364:0 | 1076:0 | 740:43 |  |  |  |
|  | 266 | 423.0 | 1082:0 | $734: 00$ |  |  |  |
|  | 268 | 537:0 | 1002:0 | 738:00 |  |  |  |
|  | 270 | 550.0 | . 819.0 | 740.50 |  |  |  |
|  | 280 | 406:0 | 1180.0 | 740:91 |  |  |  |
|  | 282 | 483.0 | 1212.0 | 732.80 |  |  |  |
|  | 288 | 424:0 | 1270:0 | 741:21 |  |  |  |
|  | 290 | 483.0 | 1270:0 | 732.00 |  |  |  |
|  | 292 | 601:0 | 1270:0 | 741:55 |  |  |  |
|  | 294 | 630.0 | 1122.0 | 741.05 |  |  |  |
|  | 296 | 750:0 | 1031:0 | 740:46 |  |  |  |
|  | 298 | 680.0 | 898.0 | 738.00 |  |  |  |
|  | 300 | 619:0 | 789:0 | 740:05 |  |  |  |
|  | 306 | 838.0 | 900.0 | 739.56 |  |  |  |
|  | 308 | 817:0 | 763:0 | 738:00 |  |  |  |
|  | 310 | 719.0 | 700:0 | 739.14 |  |  |  |
|  | 316 | 945:0 | 778:0 | $738: 14$ |  |  |  |
|  | 318 | 973.0 | 648.0 | 735.00 |  |  |  |
|  | 320 | 887:0 | 601:0 | 738.20 |  |  |  |
|  | 322 | 1020.0 | 544.0 | 736.88 |  |  |  |
|  | 337 | 1076:0 | 710:0 | 737:32 |  |  |  |
|  | 339 | 1116.0 | 573.0 | 735.00 |  |  |  |
|  | 341 | 1119:0 | 353:0 | 730:00 |  |  |  |
|  | 343 | 1138.0 | 245.0 | 734.00 |  |  |  |
|  | 345 | 1195:0 | 48:0 | 736.89 |  |  |  |
|  | 354 | 1242.0 | 675.0 | 736.96 |  |  |  |
|  | 356 | 1220:0 | 420:0 | 730.00 |  |  |  |
|  | 358 | 1286:0 | 344.0 | 732.50 |  |  |  |
|  | 360 | 1422:0 | 2:0 | $736: 72$ |  |  |  |
|  | 370 | 1422.0 | 583.0 | 736.80 |  |  |  |
|  | 372 | 1343:0 | 475 :0 | $731: 00$ |  |  |  |
|  | 375 | 1422.0 | 472:0 | 730.00 |  |  |  |
|  | 378 | 1422:0 | $453: 0$ | 726:50 |  |  |  |
|  | 382 | 1422.0 | 422.0 | 730.10 |  |  |  |
|  | 384 | 1422.0 | 293.0 | 732:00 |  |  |  |
|  | 385 | 1342:0 | 420:0 | 730:40 |  |  |  |
| 9999 |  |  |  |  |  |  |  |
| $\begin{array}{r} 370 \\ 9999 \end{array}$ | 373 | $375 \quad 377$ | 378380 | 382383 | 384 | 369 | 360 |

```
lake creek bend at flood stage
RMAR2PLT RUN ON THE IBM&PC
    O 1
    2 0.0
        0.0040
        0.0040
        0.
        O
        0.5
        O
    O
```


## A P P E N D I X D

INPUT DATA FOR EXAMPLE 2



| 80 | 176 | 181 | 198 | 197 | 196 | 182 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 81 | 174 | 175 | 176 | 182 | 196 | 183 | 1 |
| 82 | 172 | 173 | 174 | 183 | 196 | 184 | 1 |
| 83 | 172 | 184 | 196 | 195 | 194 | 185 | 1 |
| 84 | 172 | 185 | 194 | 193 | 192 | 186 | 1 |
| 85 | 170 | 171 | 172 | 186 | 192 | 187 | 1 |
| 86 | 143 | 169 | 170 | 187 | 192 | 188 | 1 |
| 67 | 143 | 188 | 192 | 191 | 190 | 189 | 1 |
| 88 | 190 | 210 | 214 | 213 | 212 | 211 | 1 |
| 89 | 190 | 191 | 192 | 209 | 214 | 210 | 1 |
| 90 | 192 | 193 | 194 | 208 | 214 | 209 | 1 |
| 91 | 194 | 207 | 216 | 215 | 214 | 208 | 1 |
| 92 | 194 | 206 | 218 | 217 | 216 | 207 | 1 |
| 93 | 194 | 195 | 196 | 205 | 218 | 206 | 1 |
| 94 | 196 | 204 | 220 | 219 | 218 | 205 | 1 |
| 95 | 196 | 197 | 198 | 203 | 220 | 204 |  |
| 96 | 198 | 199 | 200 | 202 | 220 | 203 | 1 |
| 97 | 220 | 202 | 200 | 221 | 242 | 222 | 1 |
| 98 | 220 | 222 | 242 | 241 | 240 | 223 | 1 |
| 99 | 116 | 142 | 143 | 189 | 190 | 201 | 1 |
| 100 | 220 | 223 | 240 | 239 | 238 | 224 | 1 |
| 101 | 218 | 219 | 220 | 224 | 238 | 225 | 1 |
| 102 | 216 | 217 | 218 | 225 | 238 | 226 | 1 |
| 103 | 216 | 226 | 238 | 237 | 236 | 227 | 1 |
| 104 | 216 | 227 | 236 | 235 | 234 | 228 | 1 |
| 105 | 214 | 215 | 216 | 228 | 234 | 229 | 1 |
| 106 | 212 | 213 | 214 | 229 | 234 | 230 | 1 |
| 107 | 212 | 230 | 234 | 233 | 232 | 231 | 1 |
| 108 | 232 | 251 | 255 | 254 | 253 | 252 |  |
| 109 | 232 | 233 | 234 | 250 | 255 | 251 | 1 |
| 110 | 234 | 235 | 236 | 249 | 255 | 250 | 1 |
| 111 | 236 | 248 | 257 | 256 | 255 | 249 | 1 |
| 112 | 236 | 247 | 259 | 258 | 257 | 248 | 1 |
| 113 | 236 | 237 | 238 | 246 | 259 | 247 | 1 |
| 114 | 238 | 239 | 240 | 245 | 259 | 246 | 1 |
| 115 | 240 | 241 | 242 | 244 | 259 | 245 | 1 |
| 116 | 242 | 243 | 260 | 261 | 259 | 244 | 1 |
| 117 | 260 | 243 | 242 | 262 | 280 | 264 | 1 |
| 118 | 260 | 264 | 280 | 279 | 278 | 265 | 1 |
| 119 | 259 | 261 | 260 | 265 | 278 | 263 | 1 |
| 120 | 257 | 258 | 259 | 263 | 278 | 266 | 1 |
| 121 | 257 | 266 | 278 | 277 | 276 | 267 | 1 |
| 122 | 257 | 267 | 276 | 275 | 274 | 268 | 1 |
| 123 | 255 | 256 | 257 | 268 | 274 | 269 | 1 |
| 124 | 255 | 269 | 274 | 270 | 253 | 254 | 1 |
| 125 | 253 | 270 | 274 | 273 | 272 | 271 | 1 |
| 126 | 291 | 290 | 272 | 289 | 293 | 292 | 1 |
| 127 | 272 | 273 | 274 | 288 | 293 | 289 | 1 |
| 128 | 293 | 288 | 274 | 275 | 276 | 287 | 1 |
| 129 | 293 | 287 | 276 | 286 | 295 | 294 | 1 |
| 130 | 295 | 286 | 276 | 285 | 297 | 296 | 1 |
| 131 | 297 | 285 | 276 | 277 | 278 | 284 | 1 |
| 132 | 297 | 284 | 278 | 283 | 299 | 298 | 1 |
| 133 | 299 | 283 | 278 | 279 | 280 | 282 | 1 |
| 134 | 299 | 282 | 280 | 281 | 301 | 300 | 1 |
| 135 | 321 | 303 | 301 | 302 | 323 | 322 | 1 |
| 136 | 321 | 304 | 299 | 300 | 301 | 303 | 1 |
| 137 | 319 | 305 | 299 | 304 | 321 | 320 | 1 |
| 138 | 319 | 306 | 297 | 298 | 299 | 305 | 1 |
| 139 | 319 | 307 | 295 | 296 | 297 | 306 | 1 |
| 140 | 317 | 308 | 295 | 307 | 319 | 318 | 1 |
| 141 | 315 | 309 | 295 | 308 | 317 | 316 | 1 |
| 142 | 315 | 310 | 293 | 294 | 295 | 309 |  |
| 143 | 291 | 292 | 293 | 310 | 315 | 311 |  |


| 144 | 291 | 311 | 315 | 314 | 313 | 312 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 145 | 313 | 314 | 315 | 330 | 333 | 331 | 1 |
| 146 | 333 | 343 | 344 | 332 | 313 | 331 | 1 |
| 147 | 333 | 330 | 315 | 316 | 317 | 329 | 1 |
| 148 | 333 | 329 | 317 | 328 | 335 | 334 | 1 |
| 149 | 335 | 328 | 317 | 327 | 337 | 336 | 1 |
| 150 | 317 | 318 | 319 | 326 | 337 | 327 | 1 |
| 151 | 337 | 326 | 319 | 320 | 321 | 325 | 1 |
| 152 | 337 | 325 | 321 | 322 | 323 | 324 | 1 |
| 153 | 350 | 338 | 337 | 324 | 323 | 351 | 1 |
| 154 | 350 | 339 | 335 | 336 | 337 | 338 | 1 |
| 155 | 348 | 340 | 335 | 339 | 350 | 349 | 1 |
| 156 | 346 | 341 | 335 | 340 | 348 | 347 | 1 |
| 157 | 346 | 342 | 333 | 334 | 335 | 341 | 1 |
| 158 | 344 | 343 | 333 | 342 | 346 | 345 | 1 |
| 159 | 344 | 359 | 363 | 362 | 361 | 360 | 1 |
| 160 | 344 | 345 | 346 | 358 | 363 | 359 | 1 |
| 161 | 346 | 347 | 348 | 357 | 363 | 358 | 1 |
| 162 | 348 | 356 | 365 | 364 | 363 | 357 | 1 |
| 163 | 348 | 355 | 367 | 366 | 365 | 356 | 1 |
| 164 | 348 | 349 | 350 | 354 | 367 | 355 | 1 |
| 165 | 350 | 351 | 323 | 153 | 367 | 354 | 1 |
| 166 | 367 | 353 | 323 | 352 | 369 | 368 | 1 |
| 167 | 387 | 371 | 369 | 388 | 408 | 391 | 1 |
| 168 | 367 | 368 | 369 | 371 | 387 | 372 | 1 |
| 169 | 367 | 372 | 387 | 386 | 385 | 373 | 1 |
| 170 | 365 | 366 | 367 | 373 | 385 | 374 | 1 |
| 171 | 365 | 374 | 385 | 384 | 383 | 409 | 1 |
| 172 | 365 | 409 | 383 | 382 | 381 | 375 | 1 |
| 173 | 363 . | 364 | 365 | 375 | 381 | 376 | 1 |
| 174 | 361 | 362 | 363 | 376 | 381 | 377 | 1 |
| 175 | 361 | 377 | 381 | 380 | 379 | 378 | 1 |
| 176 | 379 | 398 | 402 | 401 | 400 | 399 | , |
| 177 | 379 | 380 | 381 | 397 | 402 | 398 | 1 |
| 178 | 381 | 382 | 383 | 396 | 402 | 397 | , |
| 179 | 402 | 396 | 383 | 395 | 404 | 403 | 1 |
| 180 | 383 | 384 | 385 | 394 | 404 | 395 | 1 |
| 181 | 404 | 394 | 385 | 393 | 406 | 405 | 1 |
| 182 | 385 | 386 | 387 | 392 | 406 | 393 | 1 |
| 183 | 406 | 392 | 387 | 391 | 408 | 407 | 1 |
| 184 | 369 | 370 | 389 | 390 | 408 | 388 | 1 |
|  | 1 | 649 | 1.00 | 3456 | 6.00 | 590.00 |  |
|  | 3 | 679 | 9.00 | 363 | 4.00 | 580.00 |  |
|  | 5 | 689 | 4:00 | 368 | 6:00 | 566.00 |  |
|  | 7 | 699 | 6.00 | 374 | 8.00 | 580.00 |  |
|  | 9 | 707 | 6:00 | 379 | 7:00 | 582.00 |  |
|  | 11 | 773 | 3.00 | 416 | 9.00 | 585.00 |  |
|  | 13 | 78 | 5:00 | 426 | 0:00 | 590.00 |  |
|  | 28 | 661 | 2.00 | 414 | 4.00 | 580.00 |  |
|  | 30 | 682 | 4.00 | 419 | 5.00 | 580.00 |  |
|  | 37 | 603 | 5.00 | 435 | 7.00 | 590.00 |  |
|  | 39 | 600 | 5:00 | 450 | 0:00 | 580:00 |  |
|  | 42 | 61 | 1.00 | 452 | 1.00 | 580.00 |  |
|  | 44 | 65 | 7.00 | 456 | 8.00 | 580:00 |  |
|  | 46 | 682 | 8.00 | 458 | 8.00 | 580.00 |  |
|  | 48 | 695 | 9.00 | 460 | 6:00 | 580:00 |  |
|  | 50 | 742 | 2.00 | 465 | 4.00 | 585.00 |  |
|  | 52 | 785 | 0:00 | 469 | $9: 00$ | 590:00 |  |
|  | 70 | 597 | 7.00 | 466 | 5.00 | 590.00 |  |
|  | 71 | 59 | 2:00 | 482 | 6.00 | 590:00 |  |
|  | 73 | 612 | 1.00 | 480 | 0.00 | 580.00 |  |
|  | 75 | 665 | 3:00 | 506 | 4:00 | 580.00 |  |
|  | 77 | 703 | 4.00 | 491 | 1.00 | 580.00 |  |


| 79 | 7435:00 | 5002:00 | 580.00 |
| :---: | :---: | :---: | :---: |
| 89 | 6237.00 | 4996.00 | 590.00 |
| 90 | 6481.00 | 5166:00 | 590:00 |
| 93 | 6672:00 | 5341.00 | 590.00 |
| 95 | 6791:00 | 5309:00 | 575:00 |
| 97 | 6978.00 | 5260.00 | 570.00 |
| 99 | 7160:00 | 5212:00 | 577:00 |
| 101 | 7442.00 | 5137.00 | 580.00 |
| 103 | 7886.00 | 5019:00 | 590:00 |
| 116 | 6756.00 | 5761.00 | 590.00 |
| 118 | 6971.00 | 5765.00 | 577.00 |
| 120 | 7105.00 | 5769.00 | 570.00 |
| 122 | 7226.00 | 5769:00 | 575:00 |
| 124 | 7367.00 | 5771.00 | 577.00 |
| 126 | 7676:00 | 5776:00 | 580:00 |
| 128 | 8023.00 | 5782.00 | 590.00 |
| 143 | 6807:00 | 5918:00 | 590:00 |
| 145 | 6891.00 | 5950.00 | 577.00 |
| 147 | 7018:00 | 6001:00 | 567:00 |
| 149 | 7121.00 | 6039.00 | 575.00 |
| 151 | 7523:00 | 6195:00 | 577.00 |
| 153 | 7783.00 | 6295.00 | 580.00 |
| 155 | 8075:00 | 6408:00 | 582.00 |
| 157 | 8250.00 | 6476.00 | 590.00 |
| 170 | 6873:00 | 5970.00 | 575:00 |
| 172 | 6974.00 | 6052.00 | 566.00 |
| 174 | 7069:00 | 6128:00 | 575:00 |
| 176 | 7569.00 | 6532.00 | 582.00 |
| 178 | 7649:00 | 6596:00 | 590:00 |
| 190 | 6729.00 | 5985.00 | 590.00 |
| 192 | 6788:00 | 6063:00 | 575:00 |
| 194 | 6867.00 | 6167.00 | 565.00 |
| 196 | 6952.00 | 6279:00 | 575:00 |
| 198 | 7202.00 | 6607.00 | 582.00 |
| 200 | 7227:00 | 6639:00 | 590:00 |
| 212 | 6478.00 | 6201.00 | 590.00 |
| 214 | 6529:00 | 6290:00 | 580:00 |
| 216 | 6589.00 | 6396.00 | 564.00 |
| 218 | 6647:00 | 6500:00 | 570.00 |
| 220 | 6768.00 | 6709:00 | 582.00 |
| 232 | 6162:00 | 6360:00 | 590:00 |
| 234 | 6190.00 | 6457:00 | 575:00 |
| 236 | 6228:00 | 6592:00 | 563:00 |
| 238 | 6259.00 | 6705.00 | 567.00 |
| 240 | 6290:00 | 6814.00 | 582:00 |
| 242 | 6301.00 | 6863.00 | 590.00 |
| 253 | 5622:00 | 6582:00 | 590:00 |
| 255 | 5638.00 | 6684.00 | 570.00 |
| 257 | 5656:00 | 6810:00 | 562:00 |
| 259 | 5675.00 | 6911.00 | 577.00 |
| 260 | 5682:00 | 6955.00 | 582.00 |
| 272 | 5124.00 | 6608.00 | 590.00 |
| 274 | 5127:00 | 6706:00 | 575:00 |
| 276 | 5131.00 | 6838.00 | 562.00 |
| 278 | 5133.00 | 6933:00 | 582:00 |
| 280 | 5133.00 | 7009.00 | 590.00 |
| 291 | 4871.00 | 6462:00 | 590:00 |
| 293 | 4792.00 | 6648.00 | 570.00 |
| 295 | 4737.00 | 6779:00 | 561.00 |
| 297 | 4705.00 | 6854.00 | 572.00 |
| 299 | 4666.00 | 6946:00 | 582:00 |
| 301 | 4633.00 | 7025.00 | 590.00 |
| 313 | 4574:00 | 6290:00 | 590:00 |


|  | 315 | 4414.00 | 6525.00 | 577.00 |
| :---: | :---: | :---: | :---: | :---: |
|  | 317 | 4319.00 | 6664.00 | 561.00 |
|  | 319 | 4254:00 | 6759:00 | 570:00 |
|  | 321 | 4123.00 | 6956.00 | 582.00 |
|  | 323 | 4058:00 | 7047.00 | 590:00 |
|  | 333 | 4090.00 | 6444.00 | 577.00 |
|  | 335 | 4079:00 | 6642.00 | 561:00 |
|  | 337 | 4074.00 | 6737.00 | 567.00 |
|  | 344 | 3552:00 | 6344 :00 | 590:00 |
|  | 346 | 3705.00 | 6557.00 | 577.00 |
|  | 348 | 3787.00 | 6671.00 | 561:00 |
|  | 350 | 3847.00 | 6754.00 | 567.00 |
|  | 361 | 3248:00 | 6526:00 | 590:00 |
|  | 363 | 3420.00 | 6701.00 | 575.00 |
|  | 365 | 3550:00 | 6834:00 | 562:00 |
|  | 65 | 5978.00 | 4676.00 | 590.00 |
|  | 367 | 3697:00 | 6985:00 | 575:00 |
|  | 369 | 3846.00 | 7137.00 | 590.00 |
|  | 379 | 2892.00 | 6917:00 | 590:00 |
|  | 381 | 3148.00 | 6993.00 | 577.00 |
|  | 383 | 3299.00 | 7039:00 | 565:00 |
|  | 385 | 3419.00 | 7074.00 | 562.00 |
|  | 387 | 3661:00 | 7146:00 | 570:00 |
|  | 389 | 3808.00 | 7188.00 | 590.00 |
|  | 400 | 2892:00 | 6991:00 | 590:00 |
|  | 402 | 3126.00 | 7054.00 | 590.00 |
|  | 404 | 3280:00 | 7100:00 | 575.00 |
|  | 406 | 3407.00 | 7128.00 | 575.00 |
|  | 408 | 3698:00 | 7209.00 | 575:00 |
|  | 9999 |  |  |  |
| 212 | 253 | 232 |  |  |
| 157 | 200 | 178 |  |  |
| 280 | 323 | 301 |  |  |
| 9999 |  |  |  |  |



## A P P E N D I X E

INPUT DATA FOR EXAMPLE 3



```
    109 12201. 11960. 690.
    107 12224: 11902: 686:
    105 12241. 11862. 685.1
    103 12258. 11820: 688:
    101 12276. 11775. 700.9
    142 12225: 12205. 700.75
    12248.5 12145.3 700.3
    12272. 12085. 696:
    12291. 12037. 692.
    12309: 11990. 688.
    12349. 11914. 685.
    12357. 11872: 686.
    12367. 11844. 700.75
    12320. 12301.74 700.5
    12355. 12218. 700.
    12390: 12130: 695:
    2426. 12047. 687.
    2452: 11987: 685:
    12477. 11925. 685.5
9 9 9 9
    1
    27
    53
    7
    101
    128
    157}1169 159 161 163 165 167 
9 9 9 9
```



| 52 | 2.029 | 1:109 | 705:176 |
| :---: | :---: | :---: | :---: |
| 53 | 2.037 | 1.113 | 705.142 |
| 54 | 2.037 | 1:113 | 705:142 |
| 55 | 2.037 | $1: 113$ | 705.142 |
| 56 | 2.037 | 1:113 | 705:142 |
| 57 | 2.037 | 1.113 | 705.142 |
| 58 | 2:037 | 1:113 | 705:142 |
| 59 | 2.037 | 1.113 | 705.142 |
| 60 | 2:037 | 1:113 | 705:142 |
| 61 | 2.037 | 1.113 | 705.142 |
| 62 | 2:037 | 1:113 | 705:142 |
| 63 | 2.037 | 1.113 | 705.142 |
| 64 | 2:037 | 1.113 | 705:142 |
| 65 | 2.037 | 1.113 | 705.142 |
| 66 | 2:041 | 1:115 | 705:126 |
| 67 | 2:041 | 1.115 | 705.126 |
| 68 | 2.041 | 1:115 | 705:126 |
| 69 | 2.041 | 1.115 | 705.126 |
| 70 | 2:041 | 1:115 | 705:126 |
| 71 | 2:041 | 1.115 | 705.126 |
| 72 | 2.041 | $1: 115$ | 705:126 |
| 73 | 2.041 | 1.115 | 705.126 |
| 74 | 2.041 | 1.115 | 705:126 |
| 75 | 2.041 | 1.115 | 705.126 |
| 76 | 2.041 | 1:115 | 705:126 |
| 77 | 2.041 | 1.115 | 705.126 |
| 78 | 2.045 | 1:117 | 705:110 |
| 79 | 2.045 | 1.117 | 705.110 |
| 80 | 2:045 | 1:117 | 705:110 |
| 81 | 2.045 | 1.117 | 705.110 |
| 82 | 2.045 | 1:117 | 705:110 |
| 83 | 2.045 | 1.117 | 705.110 |
| 84 | 2.045 | 1:117 | 705:110 |
| 85 | 2.045 | 1.117 | 705.110 |
| 86 | 2.045 | 1:117 | 705:110 |
| 87 | 2.045 | 1.117 | 705.110 |
| 88 | 2.045 | 1:117 | 705:110 |
| 89 | 2.013 | 1.100 | 705.101 |
| 90 | 2:013 | 1:100 | 705:101 |
| 91 | 2.013 | 1.100 | 705.101 |
| 92 | 2:013 | 1:100 | 705:101 |
| 93 | 2.013 | 1.100 | 705.101 |
| 94 | 2:013 | 1.100 | 705.101 |
| 95 | 2:013 | 1:100 | 705:101 |
| 96 | 2.013 | 1.100 | 705.101 |
| 97 | 2:013 | 1:100 | 705:101 |
| 98 | 2.013 | 1.100 | 705:101 |
| 99 | 2.013 | 1:100 | 705:101 |
| 100 | 2.013 | 1.100 | 705.101 |
| 101 | 1.981 | 1:082 | 705:093 |
| 102 | 1.981 | 1.082 | 705.093 |
| 103 | 1.981 | 1.082 | 705:093 |
| 104 | 1.981 | 1.082 | 705.093 |
| 105 | 1.981 | 1.082 | 705:093 |
| 106 | 1.981 | 1.082 | 705.093 |
| 107 | 1.981 | 1:082 | 705:093 |
| 108 | 1.981 | 1.082 | 705.093 |
| 109 | 1.981 | 1.082 | 705:093 |
| 110 | 1.981 | 1.082 | 705.093 |
| 111 | 1.981 | 1.082 | 705:093 |
| 112 | 1.981 | 1.082 | 705.093 |
| 113 | 1.981 | 1:082 | 705:093 |
| 114 | 1.892 | 1.034 | 705.068 |



|  | 164 | 2 |  |  |  |  | 702.61 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 165 | 2 |  |  |  |  | 702.61 |  |  |  |  |
|  | 166 | 2 |  |  |  |  | 702.61 |  |  |  |  |
|  | 167 | 2 |  |  |  |  | 702.61 |  |  |  |  |
|  | 168 | 2 |  |  |  |  | 102.61 |  |  |  |  |
|  | 169 | 12 |  |  |  |  | 702.61 |  |  |  |  |
|  | 14 | 1 |  |  |  |  |  |  |  |  |  |
|  | 39 | 1 |  |  |  |  |  |  |  |  |  |
|  | 40 | 1 |  |  |  |  |  |  |  |  |  |
|  | 65 | 1 |  |  |  |  |  |  |  |  |  |
|  | 66 | 1 |  |  |  |  |  |  |  |  |  |
|  | 88 | 1 |  |  |  |  |  |  |  |  |  |
|  | 89 | 1 |  |  |  |  |  |  |  |  |  |
|  | 113 | 1 |  |  |  |  |  |  |  |  |  |
|  | 114 | 1 |  |  |  |  |  |  |  |  |  |
|  | 142 | 1 |  |  |  |  |  |  |  |  |  |
|  | 143 | 1 |  |  |  |  |  |  |  |  |  |
|  | 26 | 1 |  |  |  |  |  |  |  |  |  |
|  | 27 | 1 |  |  |  |  |  |  |  |  |  |
|  | 52 | 1 |  |  |  |  |  |  |  |  |  |
|  | 53 | 1 |  |  |  |  |  |  |  |  |  |
|  | 77 | 1 |  |  |  |  |  |  |  |  |  |
|  | 78 | 1 |  |  |  |  |  |  |  |  |  |
|  | 100 | 1 |  |  |  |  |  |  |  |  |  |
|  | 101 | 1 |  |  |  |  |  |  |  |  |  |
|  | 127 | 1 |  |  |  |  |  |  |  |  |  |
|  | 128 | 1 |  |  |  |  |  |  |  |  |  |
|  | 156 | 1 |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |  |  |  |  |  |
| 00 | 13 | 10000. |  | 705. |  |  |  |  |  |  |  |
| 1 | 2 | 34 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |

## A P P E N D I X F

SUBROUTINES USED IN RMA-2
(mostly from Norton, 1980)

The program RMA-2 is composed of a main executive routine, RMA2, ten subroutines, and a block data subroutine called BLOCKD. The general structure and linkages between the various routines is indicated in Fig. 3.2 in the main text. A description of the intended purpose and functional operation of each routine is provided with emphasis placed on purpose rather than technical detail. Each of the subroutines is discussed in the following paragraphs; they are presented in alphabetical order except for RMA2 which is presented first.

## MAIN SUBROUTINE RMA2

RMA2 is the executive routine for the hydrodynamic program. Its primary functions are to direct the flow of calls to supporting subroutines and to perform the various cycle counts on iterations and time loops. Program initiation and normal program termination are both found in RMA2.

Operationally, RMA2 begins by calling subroutine INPUT to read all geometric and run control data, and then subroutine LOAD to setup the equation numbers and check the problem size.

If the problem size is within the limits of the program, RMA2 either branches to the solution of time dependent or iterates the steady state solution; program execution may either continue or stop after completion of the steady state solution. If a time dependent problem has been specified, the program continues to cycle through time steps up to the total time limit. Subroutine CHECK is called at the end of each dynamic iteration cycle and a final results file is written if specified by the user. At the conclusion of the time loop program execution is terminated.

## SUBROUTINE CHECK

Subroutine CHECK has the function of computing and printing the total flow which crosses a user specified line which is defined in terms of a list of node numbers. The output from this routine is helpful in that a user can define lines along both internal and external sections of the network and the program will automatically determine the net flow and thus provide information on overall patterns of fluid movement. CHECK is called from subroutine OUTPUT for each print cycle, and makes no further subroutine calls itself.

To compute the flux across a line the program integrates the components of flow in each orthogonal direction and sums the components for the total. The program does this by entering a loop on the user specified list of nodal connections. Each set of two nodes in a line is assumed to define an element side, and is integrated by computing the flux at each node point and
weighting the end points by the factor $1 / 6$ and the mid-point by 2/3. This process is repeated until a zero node number, or the maximum node number has been reached. CHECK prints the flow rate across each line and makes a percentage comparison for each line compared to the first line computed. When the flow crossing all lines has been computed and printed control is returned to the calling routine.

SUBROUTINE COEFS
Subroutine COEFS is used to develop the element influence for each system variable for each nodal point equation. COEFS is called from FRONT for each active element at each iteration step (either steady state or dynamic) and subroutine COEFS makes no further subroutine calls; control is returned to FRONT.

Operationally, COEFS begins by setting the proper values into the eddy viscosity parameters, zeroing out the element coefficient matrices, copying the proper numerical integration values into a temporary array (depending on the order of numerical integration and if the element is triangular or quadrilateral) and computing the local coordinates. COEFS then enters the numerical integration loop. It first computes the transformed coordinate system and then develops the various functions for the linear and quadratic interpolating functions (all in the transformed coordinate system). Next, the right hand side (R.H.S.) vector is computed from the values at the previous iteration, as well as the various derivatives to be used in the Newton solution scheme. All of these operations are completed for the $X$ and $Y$ flow equations and continuity equations, respectively, at each integration point and summed into the element coefficient array ESTIFM, and R.H.S. vector, R1.

At the conclusion of the numerical integration loop, COEFS computes and adds to the R.H.S. vector the influence of the pressure related surface integrals at all external network boundaries. The calculation is keyed on element sides which have boundary condition specifications, and the integration is done with a four-point numerical integration scheme. For straightsided elements these calculations are redundant, but for curved sides component forces are produced for both the normal and tangential flow equations.

After computing the boundary forces, appropriate coordinate transformations are made for parallel flow to both the element coefficient matrix and the R.H.S. vector. The final operation is the elimination of the exact boundary conditions from the element matrices, after which control is sent back to FRONT.

FRONT is a subroutine which is used to form and solve a set of non-symmetric simultaneous equations using a wave front technique. This is accomplished by calling subroutine COEFS for each system element in the order specified by the array NFIXH (generated by RMA-1). A forward elimination is carried out on the equations as they are fully summed, with intermediate results written on the disk (via subroutine XWRT) as the local buffers are filled. Once all the contributions have been calculated, the back substitution is made by retrieving the stored information via succession calls to subroutine XRED. After all the equations have been solved, control is given back to the calling routine, RMA2.

SUBROUTINE INPUT
Subroutine INPUT is the program routine that inputs all card data to the program RMA-2. INPUT is divided into two sections. The first and largest section inputs the majority of problem data from either the card reader or specified files. The second section is used only in dynamic problems and only for updating boundary conditions. The first section of INPUT will be called once in any program execution and the second section will be called each time dynamic boundary conditions are to be updated. INPUT may call SIZE and/or OUTPUT depending on what information is to be printed.

Operationally, INPUT begins by initializing program arrays to either zero or a null value to be used later to determine if certain program inputs have been made; run control, element characteristics and continuity line data are then read and printed. If an RMA-1 generated geometry file has been specified (usually unit 3) it is read; otherwise the element cards are the next input. The program then determines the maximum node and element numbers and prints a message. The coordinate cards come next, followed by the data for coordinate interpolation, element reordering and nodal slopes. If coordinate data is from cards the program will calculate the coordinates at unspecified midside nodes and the slopes along the outer edges of the network. If the PRINT OPTION is greater than zero, subroutine SIZE is called and the element and node data printed.

Next, depending on user specifications, the initial conditions are either: a) internally set (zero flows, average depths), b) read from a restart file or c) read from the same set of cards where hydraulic parameters (Manning $n$, eddy viscosity, etc.) can be found. For the last case, a set of cards defining the initial $x$ and $y$ velocities and water surface elevations for each node is required. If the PRINT OPTION is greater than one (1) the initial conditions are printed by a
call to subroutine OUTPUT. Boundary conditions and wind data are then input (with the option to skip through the input file) and printed if the PRINT OPTION is greater than zero. Next, the boundary conditions are converted to incremental values for internal use and the side slopes zeroed for nonparallel flow specification. Lastly, the number of inlet sections has to be specified by the user. For each inlet section a call to USSET is made so that flow specification and subdivision can be made. Control is returned to RMA2.

If a dynamic problem is being run. INPUT is called at specified intervals to update non-zero boundary specifications. This is done by a call to INPUT with special parameters in its calling sequence which indicates skipping of the normal functions and triggers input of dynamic boundary conditions. After each call for input of dynamic boundary conditions, control is returned to the calling routine, RMA2.

SUBROUTINE LOAD
Subroutine LOAD has one major purpose: to compute the equation number for each nodal variable in the system, and to check the size of the resulting problem against the dimension limits of the program. This function is performed at least once in each program execution and thereafter each time the boundary conditions updated.

Operationally, the routine begins by finding those node points for which equations are to be formed. All nodes not mentioned in the definition of an element, or nodal variables for which exact boundary conditions have been specified are deleted from consideration. Next, the order of the equations is developed, and the program halts; otherwise, control is returned to the calling routine. In either case, a message is printed indicating the results of the storage allocation procedure.

SUBROUTINE OUTPUT
Subroutine OUTPUT is called from RMA2 and/or INPUT and performs up to four separate tasks depending on the status of the solution and the value of the parameter in its calling sequence. If there has been an update of the boundary conditions, OUTPUT adjusts the state variable array, VEL, to its specified values. Next, if a solution iteration has been performed, the solution corrections are added to the most recent estimate of state variables (array VEL) and the average and maximum solution corrections determined. The solution corrections are then printed along with the flow line calculations determined from a call to CHECK.

If printed output is requested the program computes the values of velocity and water surface position from the basic program variables, and prints the final results. Control is then returned to the calling program, either INPUT in the case of the initial conditions or RMA2 in the case of active simulation.

SUBROUTINE SIZE
Subroutine SIZE has the single purpose of calculating the area for each element in the network. This is needed as the calculation of the area of an element with curved sides is not straightforward.

To compute element areas, SIZE numerically integrates a constant function over the element with the result that the value of the integration is the element area. The technique is similar to the numerical integration used in the solution to the hydrodynamic equations, and relies on many of the same factors. SIZE is called from subroutine INPUT only if the element areas are to be printed, and makes no further subroutine calls.

Logically, the routine consists of one large Do loop for each network element. Within this loop the program finds the type of element (triangular or quadrilateral) and copies the proper numerical integration factors into working storage locations. A loop is then executed for each numerical integration point ( 7 for triangular and 9 for quadrilateral) for the particular element, and the result summed into XAREA. At the conclusion of these two loops, control returns to INPUT.

SUBROUTINE USSET
As defined in the main text, USSET is an option in FESWMS$T X$ where total flow specification is required of the user. Based on two methods of flow subdivision (see Chapter 3), flow for each inlet section is subdivided among inlet nodes (also specified by the user) in relation to the subsection area tributary to each node and roughness characteristic of the inlet section. Control is passed back to the calling routine INPUT after the calculations are completed.

SUBROUTINE XRED
This subroutine reads from scratch disk file one buffer block of information as written by subroutine XWRT. Both subroutines XRED and XWRT should make use of the most efficient block data transfers available on the users computer system, and will probably need to be written for a computer system.

## SUBROUTINE XWRT

This subroutine writes to the scratch disk file one buffer block of information. A buffer block is 2001 words long and is comprised of the variables LQ and the arrays LHS and QS in COMMON/BUFFR/. This routine and its associate read routine, should make use of the most efficient block data transfer available on the users computer system.

A P P EN D I X G

INPUT DATA FOR HY-7 RUNS

```
    BRUSHY CREEK AT ROUND ROCK, TEXAS
    DOWNSTREAM EXTENSION OF PINITE ELEMENT GRID
    POR INPUT AS INITIAL CONDITION FOR 2D MODEL
    10000.
    701.88
SEC8-1685.
    1000..708..1035..706..1075..686..1188..684..1284..686..
    1325.,690..1409.,694.,1450.,696.,1550.,700.,1663.,704.
    1075.00.1284.00
    .09. .09. .09
SEC7 -1332.
    1000.,710.,1025.,700.,1040..686.,1138.00,684.,1256.,686.,
    1260.,688.,1 345.,692..1450.,696.,1515.,700.,1675..704.
    1040.00. 1256.00
    .09. .09. .09
SEC6 -1169.5
    1000.,710.,1100.,688.,1175.,686.,1275.00,684.1,1400.,686.,
    1406.,688.,1444.,690.,1538.,694.,1600.,698..1700.,704.
    1175.,1400.
    .09. .09, .09
SEC5 -1038.
    1000.,710.,1188.,698.,1225.,692.,1315.,690.,1325.,686.,
    1420.,684.2;1520.,686.,1525..688.,1565.,690.;1738.,704.
    1325..1520.
    .09. .09, .09
SEC4-688.
    994.,706.,1000.,702.,1025.,696.,1138.,694.,1213.,688.,1225.,686.,
    1320.,684.4,1420.,686.,1430.,690.,1465.,708:,1525.,710.
    1225:,1420.
    .09. .09. .09
SEC 3-581.5
    994:,706.,1000..702.,1038.,696.,1163.,694.,1213.,690.,1338.,688.,
    1350.,686.,1394..684.5,1438..686.,1450.,690:.1488.,712.
    1350..1438.
    .09. .09. .09
SEC2 - 369.
    994.,706.,1000.,702.,1100.,698.,1250..692.,1275.,690.,1400.,688..
    1413..683.;1450..684.7;1500..686..1513..690.,1550.,708.
    1413:. 1500.
    .09, .09, .09
SECY - 300.
    994.,706.,1000.,702.,1125.,700.,1138.,696_,1350.,690.,1388.,686_.
    1438.,684.8,1494.,688:;1513.,690.,1575.,702.,1613.,706.
    1388..1494.
    .09. .09, .09
EXIT O.
    994..706.. 1000.00.700.50. 1028.57.685.50. 1093.51.685.00.
    1161.04.687.00, 1254.55.695.00, 1345.46,700.00,
    1436:38.700.50.1440..706.
    1028.57. 1345.46
    .09. .09. .09
```

```
    BRUSHY CREEK AT ROUND ROCK, TEXAS
    WATERWAY WITH SIMGLE GRIDGE OPENING
    FOR INPUT AS INITIAL CONDITION FOR 2D MODEL
    10000.
    702.61
EXIT O
    994..706.. 1000.00.700.50. 1028.57.685.50, 1093.51.685.00,
    1161.04.687.00, 1254.55.695.00. 1345.46.700.00,
    1436.38,700.50. 1440..706.
    1028.57, 1345.46
    .09. .09. .09
FULLV298.27
    965..710.. 970.00, 701.00, 1000.00.701.00, 1023.85.688.00,
    1087.44,685.20, 1153.03.688.00, 1259.67.695:00.
    1286:16.701.00. 1316:16.701:00. 1321.16,710:00
    1023.85,1259.67
    .09. .09. .09
BRDGE298.27
    4.0, 710.50
    286.17, 1000.00, 1286.16
    1. 28.00, 0.0, 000.00, 0.00, 0.00, 0.00
    .09
APPR 556.50
    996..710.. 1000.00,701.50. 1019.15,686.20, 1057.46.685.50,
    1117.65.688.20. 1221.63.694.20. 1259.94.698.20.
    1350.23.701.50, 1365.00,710.00
    1019.15,1259.94
    .09. .09. .09
```


## A P P E N D I X H

OUTPUT DATA FOR HY-7 RUNS


```
HY-7 FEDERAL HIGHWAY ADHINISTRATION P U. S. GEOLOGICAL SURVEY
```



```
    BRUSHY CREEK AT ROUND ROCK, TEXAS
    WATERWAY WITH SINGLE BRIDCE OPENING
    FOR INPUT'AS INITIAL CONDITION FOR 2D MODEL
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline XSID：CODE & SLEN & LEW & AREA & \(V{ }^{\text {HD }}\) & HF． & EGL & CRWS & 0 & WSEL \\
\hline SRD & FLEN & REW & K & ALPH & HO & ERR & FR & VEL & \\
\hline EXIT ：XS 0. & \[
\begin{array}{r}
-9999 . \\
r 9999 .
\end{array}
\] & \[
\begin{array}{r}
998 . \\
1438 .
\end{array}
\] & \[
\begin{array}{r}
4292 . \\
353458 .
\end{array}
\] & \[
\begin{array}{r}
.09 \\
1.06
\end{array}
\] & \[
\begin{array}{r}
09.99 \\
-9.99
\end{array}
\] & \[
\begin{array}{r}
702.70 \\
-999: 99
\end{array}
\] & \[
\begin{array}{r}
691.04 \\
.14
\end{array}
\] & \[
\begin{array}{r}
10000 \\
2.33
\end{array}
\] & 702.61 \\
\hline FULLV：FV \({ }^{\text {298 }}\) & 298.
298. & 969.
1317. & \[
\begin{array}{r}
3748 . \\
336016 .
\end{array}
\] & .12
1.10 & ． 25 & 702.97
.00 & -999.99
.15 & \[
\begin{array}{r}
10000 \\
2.67
\end{array}
\] & 702.85 \\
\hline & くくくTHE & ABOVE & SULTS & ECT & n NORM & UN & STRICT & FLO & \\
\hline
\end{tabular}
APPR :AS 258. 999. 3611. .13 .25 703.22-999.99 10000. 703.09
    557. 258. 1353. 308411. 1.09 .00 .00 .16 2.77
        <<<<<THE ABOVE RESULTS'REFLQCT "NORMAL" (UNCONSTRICTED) FLOW>>>>>
                <<<<<<RESULTS REFLECTING THE CONSTRICTED FLOW FOLLOW\>>>>
```



```
        M(G) M(K) KQ XLKO XRKQ OTEL
        .191 .020 304089. 996. 1282. 702.89
                        <<<<<<END OF BRIDGE COMPUTATIONS>>>>>
ER
NORMAL END OF RUN BBWFH
```

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