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16. Abstract

This report presents the fundamental limit equilibrium slope stability equations required to compute the stability of earth slopes containing synthetic reinforcement ("geogrids"). The limit equilibrium equations were incorporated into a computer program, UTEXAS2, which was then used to perform a number of calculations for verification. Results of the calculations were compared with other published solutions and with the results of other limit equilibrium slope stability computations based on several simplified approximations.

An important consideration in the design of reinforced slopes is the magnitude of the force which can be developed in the reinforcement. A series of finite element calculations were performed to estimate potential elongation strain in reinforcement. The finite element computations simulated the expansion of soil in a compacted earth slope and were used to estimate probable strain in reinforcement.

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STABILITY COMPUTATION PROCEDURES FOR EARTH SLOPES CONTAINING INTERNAL REINFORCEMENT

by

Stephen G. Wright
Fernando Cuenca

Research Report Number 435-1

Repair of Slides in Earth Slopes
Research Project 3-8-85-435

conducted for

Texas State Department of Highways
and Public Transportation

in cooperation with the
U. S. Department of Transportation
Federal Highway Administration

by the

CENTER FOR TRANSPORTATION RESEARCH
BUREAU OF ENGINEERING RESEARCH
THE UNIVERSITY OF TEXAS AT AUSTIN

November 1986

PREFACE

The Texas State Department of Highways and Public Transportation has experienced a number of slope failures in recent years, which require some form of remedial action. Synthetic reinforcement materials, commonly referred to as "geogrids," provide at least one candidate for use in such repairs. Rational design of slopes using such soil reinforcement requires that stability computations be performed using procedures which model the effects of the internal reinforcement as close as practically possible. This report presents the results of a study conducted to develop limit equilibrium slope stability analysis procedures and a computer program for performing such stability calculations.

An important part of the design of reinforced earth slopes involves determination of the level of force which can be developed and relied upon in the reinforcement. A large number of the slope failures which have occurred in Texas and are of interest involve embankments constructed of compacted, highly plastic clay fills. Slides typically are confined to the compacted fill slope itself and occur a number of years after construction. The failures result from the gradual loss of strength of the soil due to expansion with time. Determination of the level of force which can be developed in reinforcement in such slopes requires taking into consideration the amount of strain which is developed in the reinforcement as a result of the gradual expansion of the soil with time. A series of finite element computations were performed as part of this study to investigate such strains and to estimate the potential magnitudes of the strains which might develop.

ABSTRACT

This report presents the fundamental limit equilibrium slope stability equations required to compute the stability of earth slopes containing synthetic reinforcement ("geogrids"). The limit equilibrium equations were incorporated into a computer program, UTEXAS2, which was then used to perform a number of calculations for verification. Results of the calculations were compared with other published solutions and with the results of other limit equilibrium slope stability computations based on several simplified approximations.

An important consideration in the design of reinforced slopes is the magnitude of the force which can be developed in the reinforcement. A series of finite element calculations were performed to estimate potential elongation strain in reinforcement. The finite element computations simulated the expansion of soil in a compacted earth slope and were used to estimate probable strain in reinforcement.

SUMMARY

Several approaches are examined for modeling internal soil reinforcement in stability computations for earth slopes. Two approaches based on using existing computer programs to approximate the effects of internal reinforcement were examined. Rigorous slope stability equations, which correctly model the reinforcement based on fundamental principles of limit equilibrium are also developed and presented. Spencer's limit equilibrium procedure was modified and incorporated into a computer program, UTEXAS2, for performing stability calculations using the rigorous, complete equilibrium slope stability equations. Several sets of parametric studies were conducted using the rigorous procedures and the computer program UTEXAS2.

All limit equilibrium slope stability procedures are based on the assumption that a certain level of force can be developed in the reinforcement. The development of such forces in slopes constructed of highly plastic soils, which swell and experience a reduction in shear strength with time, were of particular interest in the present study. Consequently, a series of finite element computations were performed to examine the levels of strain which might develop in a reinforced slope constructed of expansive soil. The elongation strain which would be anticipated in reinforcement was calculated based on an assumed tendency of the soil to swell. Based on the assumption of homogeneous linearly elastic conditions, it was found that the percent elongation which would be expected to develop in the reinforcement as a result of expansion would probably not exceed 5 percent for typical soils with a tendency for volumetric expansion not exceeding 25 percent. Such relatively small elongation strain (5 percent) in the reinforcement may require use of substantially less than ultimate strengths when establishing design forces in reinforcement in compacted fill of expansive soils.

IMPLEMENTATION STATEMENT

It is recommended that the rigorous limit equilibrium slope stability procedures developed in this report and implemented in the computer program, UTEXAS2, be used for stability computations for earth slopes. The program, UTEXAS2, contains a number of modifications and improvements to an earlier slope stability computer program, UTEXAS (Wright and Roecker, 1984), some of which were made in conjunction with the current research project and others of which were made under separate sponsorship by the U. S. Army Corps of Engineers. UTEXAS2, is recommended for implementation by the Texas State Department of Highways and Public Transportation to replace the earlier program (UTEXAS).

Further finite element computations are recommended to establish more clearly the magnitude of elongation strain which may be developed in reinforcement using synthetic materials ("geogrids"). It appears that only a relatively small fraction of the ultimate capacity of geogrids may be relied upon for design of embankments in expansive soils and further research is recommended to establish better the levels of force which can be used for design.

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CHAPTER ONE. INTRODUCTION

The Texas State Department of Highways and Public Transportation has experienced a number of slides in earth embankments, which require some form of remedial action. One candidate solution for such repairs is the use of synthetic reinforcement, now generally referred to as "geogrids." The SDHPT has used one such commercial geogrid (Tensar) for repair of a slide with considerable success. However, relatively little long-term experience exists with the use of such geogrids for slide repair and at least at the start of the current study presented in this report, rational procedures were not available for design of slopes employing geogrids for reinforcement. Accordingly, significant uncertainty existed at the time the remedial measure was installed and may still exist depending on the response of the soil in the slope with time.

A significant majority of the slope failures which have been experienced by the SDHPT in recent years have occurred in compacted earth fills many years after construction. A number of slope failures have occurred in slopes at least thirty years after initial construction. The foundation materials supporting the fills have generally been competent and did not contribute to the failure. Typically the soils involved in the slope failures have been highly plastic clays which swell gradually with time, accompanied by a gradual reduction in shear strength. Judging by the age of slopes at the time they failed, which in some cases is as much as 30 years after construction, the strength losses may require many years to develop. Much of the strength loss is directly attributable to swelling of the soil; however, additional losses in strength have recently been found to be due also to the effects of repeated wetting and drying, with the accompanying cracking leading to an apparent deterioration in strength.

In order to understand better the mechanics whereby reinforcement contributes to earth slope stability and to develop a rational procedure for computing the stability of reinforced slopes the studies described in this report were undertaken. In Chapter 2 the results of preliminary computations performed to obtain insight into the potential mechanics of reinforced earth slope stability are presented. These preliminary computations were also performed to explore the feasibility of using for design of reinforced slopes existing computer programs, which were not specifically designed to handle reinforcement, but could be used to approximate reinforcement. The fundamental equilibrium requirements for general limit equilibrium slope stability calculations with reinforcement are developed in Chapter 3, including modifications to a computer program, UTEXAS2, to incorporate reinforcement. In Chapter 4 the results of several series of computations to verify and illustrate the computer program are presented. To understand better the mechanism by which the forces in the soil reinforcement are developed in a compacted fill constructed of expansive soil a series of finite element computations was performed. The procedures and results of the finite element computations are presented in Chapter 5. Chapter 6 presents a summary and conclusions.

CHAPTER TWO. PRELIMINARY SLOPE STABILITY CALCULATIONS FOR EFFECTS OF SOIL REINFORCEMENT

INTRODUCTION

Several series of computations were performed at the outset of this study using existing computer codes. Although the computer codes which were used did not rigorously model the effects of soil reinforcement, they could be used to approximate the reinforcement in several ways. The purpose of these analyses was to examine if such approximations might lead to useful results and to identify potential difficulties which might arise in more rigorous modeling of the reinforcement. Two approaches were used to model the reinforcement in the preliminary calculations. In the first approach the reinforcement was modeled as thin layers of material ("soil"); each layer was assigned appropriate dimensions and shear strengths to simulate the reinforcement. In the second approach the reinforcement was represented by a series of external loads applied to the surface of the slope; the external loads were calculated to produce forces identical to those produced internally by the reinforcement.

MODELING AS EQUIVALENT "SOIL" LAYERS

Reinforcement was modeled as a series of thin layers of material by assigning an appropriate shear strength to each layer to represent the forces in the reinforcement. The thickness of the layers representing the reinforcement was arbitrarily selected as some small value; a thickness of 0.01 foot was used for the stability calculations performed in this study.

Theoretical Development

A typical reinforced slope and shear surface are shown in Fig. 2.1. The force in the reinforcement is designated as T and acts in the longitudinal direction of the reinforcement. The inclination of the shear surface measured from the horizontal is designated by the angle, α. The equivalent shear strength which is assigned to the layer representing the reinforcement is computed from the component of the reinforcement force which acts tangential

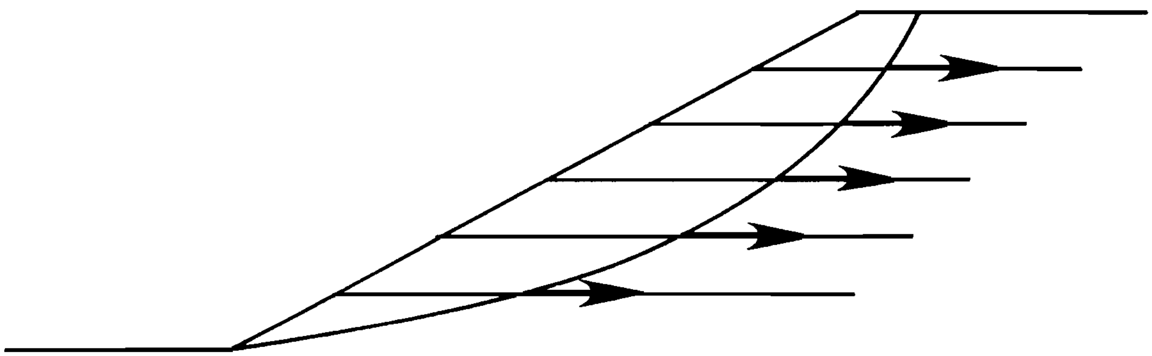


Fig. 2.1. Typical Reinforced Slope Illustrating Internal Forces on Shear Surface Due to Reinforcement.

to the shear surface. The component of the reinforcement force acting tangential to the shear surface, T_s, is given by,

$$T_S = T \cos \alpha \tag{2.1}$$

An equivalent shear strength to be assigned to a layer representing the reinforcement is obtained by dividing the tangential force (T_s) by an appropriate area. The area, designated as A_s, represents the area of the portion of the shear surface which intersects the reinforcement and is given by,

$$A_s = \frac{t\Delta z}{\sin \alpha} \tag{2.2}$$

where t is the thickness of the layer representing the reinforcement (Fig. 2.2); Δz is the length perpendicular to the two-dimensional plane of interest represented in Fig. 2.2. For a plane of unit thickness (Δz = 1) the area becomes,

$$A_s = \frac{t}{\sin \alpha} \tag{2.3}$$

The equivalent shear strength, $s_{eq.}$, is obtained from Eqs. 2.1 and 2.3 as,

$$s_{eq} = \frac{T_s}{A_s} = \frac{T \cos \alpha}{\frac{t}{\sin \alpha}} \tag{2.4}$$

The equivalent shear strength given by Eq. 2.4 varies with the orientation of the shear surface as shown in Fig. 2.3. The variation in shear strength with orientation of the shear plane is analogous to the variation in shear strength which might be found for a soil which is anisotropic; however, the specific pattern of variation in strength in an actual anisotropic soil would generally be different from the pattern shown in Fig. 2.3. Variations in shear strength with orientation of the shear plane (strength anisotropy) can be readily accommodated in stability computations with at least some computer programs, including UTEXAS (Wright and Roecker, 1984), which was used for the present computations. Thus, representation of reinforcement as a layer of

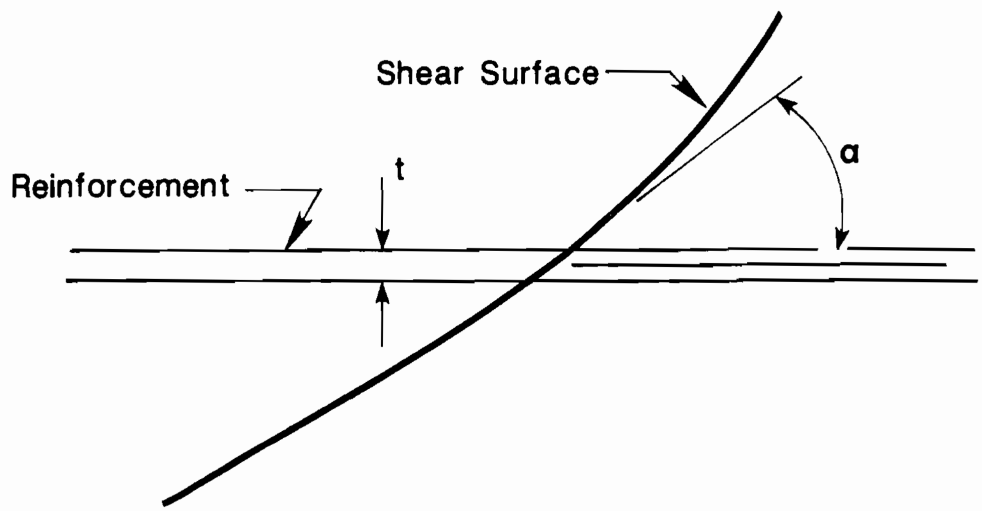


Fig. 2.2. Expanded View of Shear Surface and Reinforcement Showing Parameters Used to Compute Area, A_s.

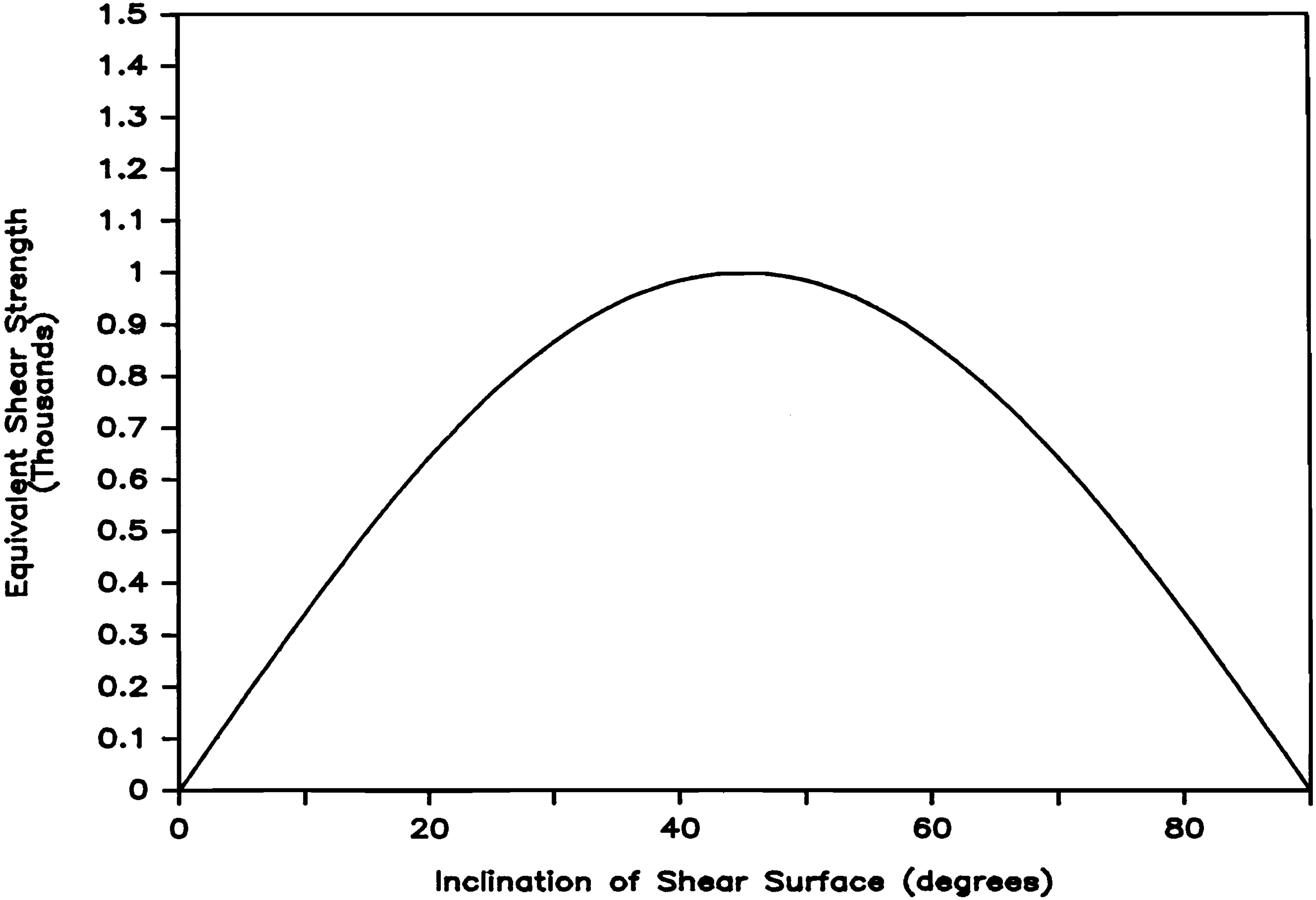


Fig. 2.3. Variation of Shear Strength with Shear Plane Orientation Used to Represent Reinforcement as an Equivalent Anisotropic Soil.

material with anisotropic shear strength properties is attractive from a practical point of view.

Computed Results

Several sets of slope stability computations were performed in which the reinforcement was modeled as a layer of material with anisotropic strength properties as described above. Computations were performed for several slopes using both circular and noncircular shear surfaces. A number of the calculations included attempts to locate a most critical noncircular (general-shaped) shear surface. During these computations it was discovered that if the shear surface was permitted to seek the path of least resistance with no constraints, the shear surface tended to cross each layer of material representing the reinforcement at a very steep angle as illustrated schematically in Fig. 2.4. Examination of Eq. 2.4 and Fig. 2.4 reveals that the shear strength approaches zero as the value of α approaches 90 degrees, i. e. as the shear surface becomes vertical. This was reflected in the results of the stability computations; the shear surface tended to become very steep where it crossed each layer of reinforcement. Although such abrupt changes in the slope of the shear surface were not permitted to occur when circular shear surfaces were used, it was clear that if one were to seek the most critical shear surface regardless of shape, the effects produced by characterizing the reinforcement as a material with anisotropic strength properties would be pronounced.

The effects of the inclination of the shear surface where it crosses reinforcement is illustrated by a series of computations for the slope shown in Fig. 2.5. The slope geometry and properties were taken from the publication "Guidelines for the Design and Construction of Embankments over Stable Foundations Using 'Tensar' Geogrids," which was prepared by Netlon, Ltd., England. For simplicity in these illustrative calculations the shear surface was first assumed to be a plane passing through the toe of the slope. The most critical plane yielding the minimum factor of safety was found by trial and error to be inclined at an angle of approximately 38 degrees from the horizontal plane. The corresponding minimum factor of safety was found to be 1.16. Next, the inclination of the shear surface was changed systematically to 50, 60, 70, 80 and 85 degrees locally where it crosses each layer of

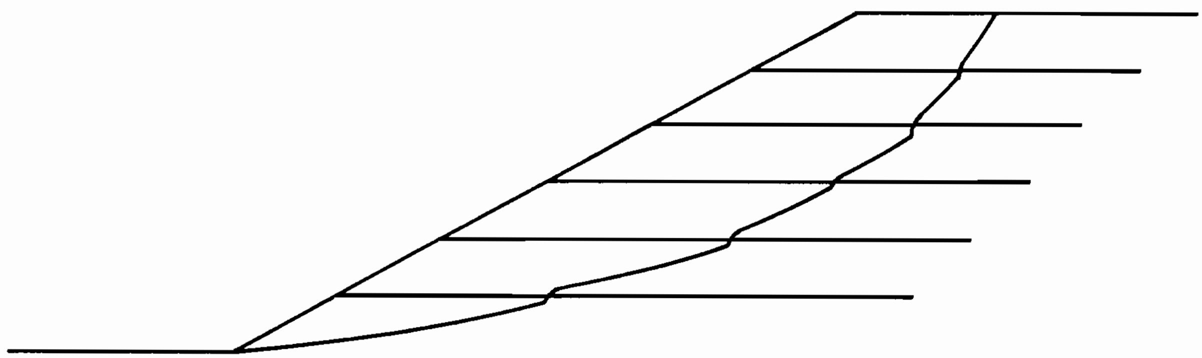
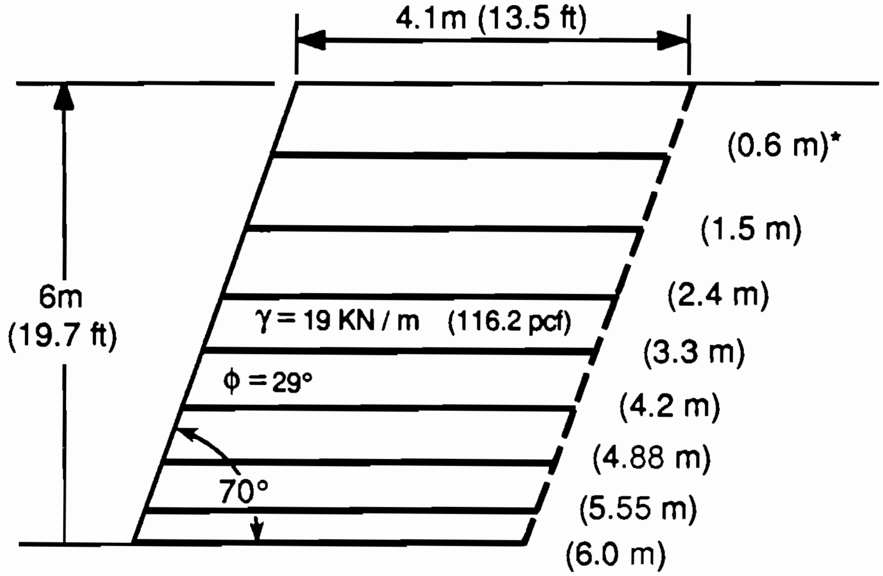


Fig. 2.4. Illustration of Local Distortion of Critical Shear Surface Where It Crosses Reinforcement When Reinforcement is Characterized as a Material with Anisotropic Strength Properties.



*Numbers in parentheses indicate depths to reinforcement layers.

Fig. 2.5. First Example Slope Used in Preliminary Calculations (After Netlon Limited, 1982).

reinforcement; the inclination of the shear surface in the soil above and below each layer of reinforcement was still retained at 38 degrees. The factor of safety was recomputed for each adjustment of the shear surface from local inclinations ranging from 50 to 85 degress. Results are summarized in Table 2.1. It can be seen that the factor of safety decreased from 1.16 to 0.79, or by more than 30 percent as the inclination of the shear surface was changed. Similar results were obtained with shear surfaces of other shapes; however, the sensitivity of the computed factor of safety to the shear surface inclination where it crossed the reinforcement depended on the contribution of the reinforcement to the overall stability of the slope. When the reinforcement had little effect, the shape of the shear surface where it crossed the reinforcement also had little effect.

Discussion

Several limitations were discovered using the approach where the soil reinforcement is represented as equivalent layers of material with anisotropic shear strength properties. Most importantly, the approach largely neglects the component of the reinforcement force which acts normal (perpendicular) to the shear surface. Although the solutions which were obtained for the factor of safety by this approach appear to neglect the normal component of the force, explicitly, the solutions satisfy static equilibrium and implicitly consider the component of the force normal to the shear surface. This is more easily understood by examining the fundamentals of the limit equilibrium slope analysis procedures which are used to compute the factor of safety. In these procedures, including Spencer's procedure which is used in UTEXAS, the stresses acting normal to the shear surface are considered to be unknown and are computed from the equations of static equilibrium. A complete equilibrium solution assures that the computed normal (and shear) stresses are in static equilibrium with the overlying soil mass. That is, the stresses, σ and τ, shown in Fig. 2.6 are in equilibrium with the weight of the soil mass bounded by points A, B and C in Fig. 2.6. Theoretically, the normal stresses which are computed would include any normal component of force in the reinforcement as well as the normal stresses in the soil. Thus, the procedures are at least still fundamentally correct on the basis of static equilibrium. However, the procedures of slices have been developed for cases where the normal stresses

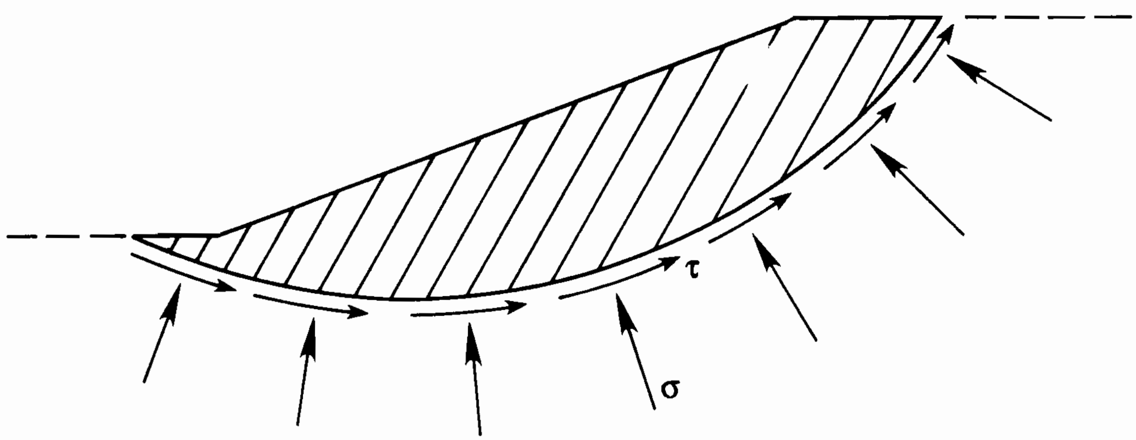


Fig. 2.6. Illustration of Stresses Along the Shear Surface and Corresponding Soil Mass for which Equilibrium is Satisfied in Limit Equilibrium Procedures.

TABLE 2.1. EFFECT OF "LOCAL" INCLINATION OF SHEAR SURFACE ON THE COMPUTED FACTOR OF SAFETY BY THE FIRST APPROACH.

Inclination of Shear Surface Where it Crosses Reinforcement (degrees)	Factor of Safety
38	1.16
50	1.10
60	1.02
70	0.95
80	0.85
85	0.79

vary gradually along the shear surface as suggested in Fig. 2.7a. The procedures are based on various assumptions about the internal distribution of stresses ("side forces"), which experience has shown produce reasonable distributions of stress for conventional, un-reinforced slopes. The procedures were not developed in a manner or with assumptions which would cause them to automatically reflect a transition from a moderate compressive stress in the soil to a suddenly large tensile stress in the reinforcement along the shear surface as suggested may occur for a slope with reinforcement (Fig. 2.7b). Consequently, the effect of the tensile stresses in the reinforcement do not appear to be reflected in the solutions which are obtained by conventional limit equilibrium procedures.

A second limitation which exists when the reinforcement is represented as an equivalent layer of material pertains to the manner in which the factor of safety is defined and applied. The factor of safety is applied equally to the strength of the soil and to the equivalent "strength" of the reinforcement. That is, when the reinforcement force is represented by the strength of an equivalent layer of soil, the reinforcement force is divided by the factor of safety when the equilibrium equations are solved. If the factor of safety is less than unity, the resulting solution, including the computed factor of safety will correspond to equilibrium of the slope with a reinforcing force greater than the actual force (T). Similarly if the factor of safety is greater than unity, the solution will correspond to equilibrium of the slope with a reinforcement force less than the actual value. In actual design practice, the reinforcement force should generally be determined independently of the stability calculations and might already include an appropriate factor of safety based on either tolerable deformations or allowable stresses. Accordingly, when the stability computations are performed, no additional factor of safety would normally be applied to the reinforcement.

MODELING BY EXTERNALLY APPLIED FORCES

The second approach for modeling reinforcement consists of representing the forces in the reinforcement as a series of externally applied forces. Although the forces actually exist as internal forces, many computer programs for slope stability analysis permit external loads (forces) to be represented,

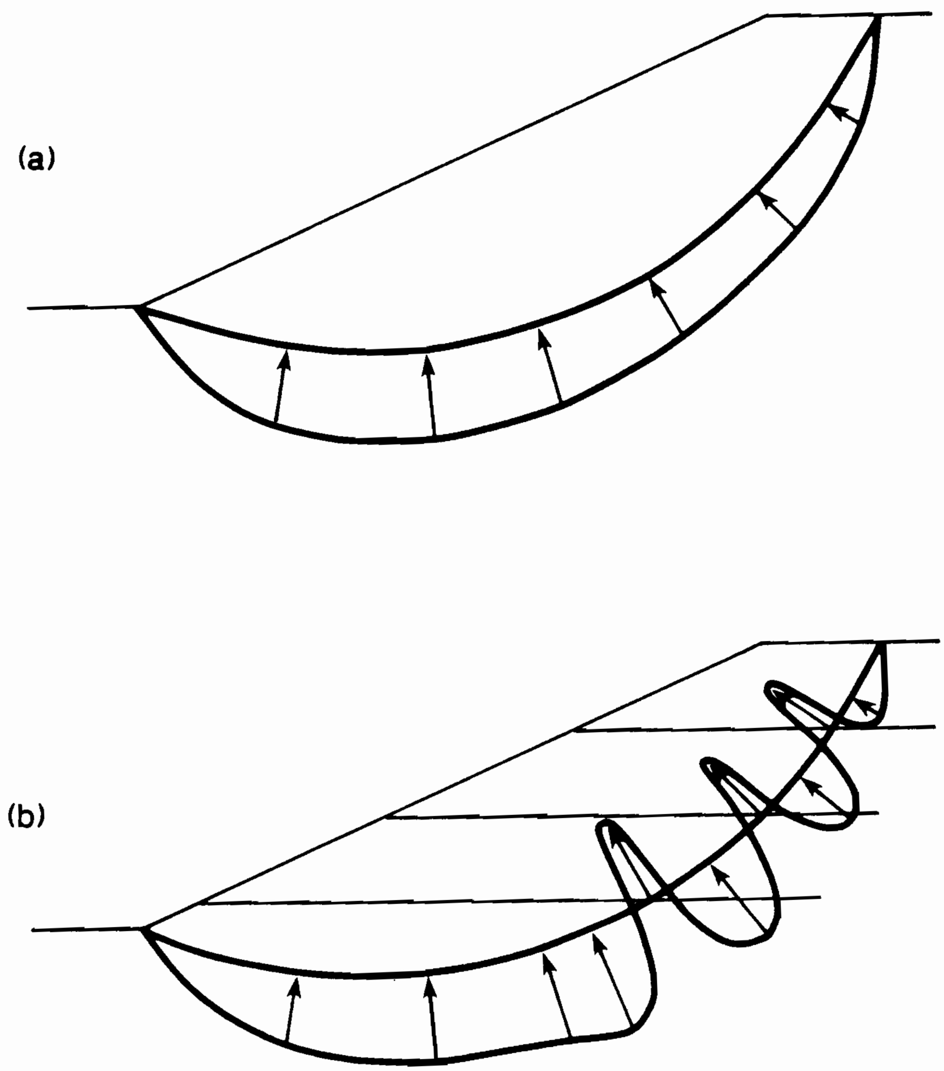


Fig. 2.7. Expected Distribution of Normal Stresses Along Shear Surfaces for (a) a Slope with no Internal Reinforcement, and (b) a Slope with Internal Reinforcement.

while internal forces are not represented. Thus, this approach is also attractive for use with existing computer programs.

Theoretical Basis

The internal force, T, in a layer of reinforcement can be replaced by an equivalent force, T', acting at the surface of the slope as shown in Fig. 2.8. The forces, T and T', are identical in magnitude and have the same line of action. Accordingly, both forces have the same contribution to the forces and moments acting on a soil mass such as the shaded mass $\overline{ABC}$ in Fig. 2.8. However, in a procedure of slices, where the soil mass is subdivided into a number of vertical slices as shown in Fig. 2.9, the forces (T and T') will actually be applied to different slices. The two forces will have somewhat different effects on the internal distribution of stresses between slices and along the shear surface. Consequently, they will not necessarily have the same effect on the computed factor of safety.

Computational Considerations

Many computer programs, including the one used in the current study do not permit concentrated forces to be applied at the surface of the slope. Instead they allow normal and shear stresses to be applied in directions perpendicular and parallel to the slope face. In order to represent the reinforcement forces as equivalent stresses the following equations are used:

$$\sigma = \frac{T' \sin \alpha}{\ell_s} \tag{2.5}$$

and

$$\tau = \frac{T' \cos \alpha}{\ell_s} \tag{2.6}$$

where ℓ_s is the distance along the slope face over which the stresses are applied. The distance is arbitrarily selected as some relatively small, convenient distance, e. g. 0.01 foot.

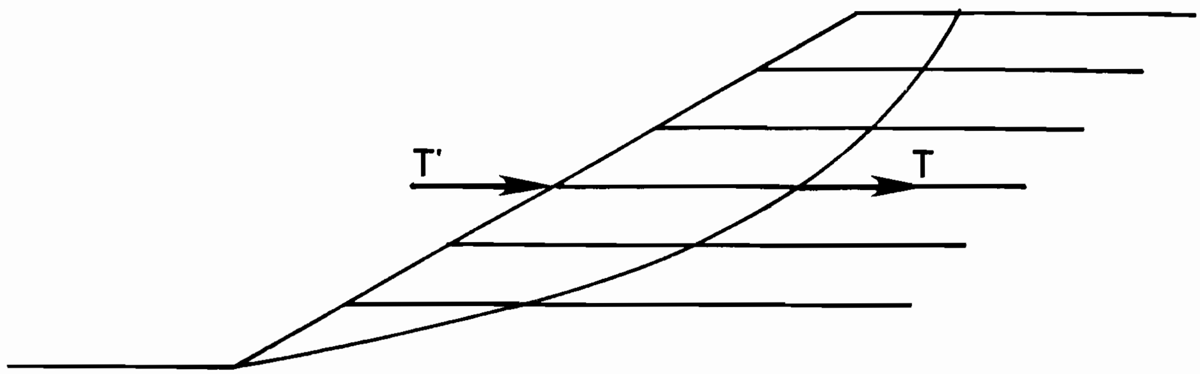


Fig. 2.8. Actual Reinforcement Forces (T) and Equivalent Surface Loads (T') Used to Represent Internal Reinforcement.

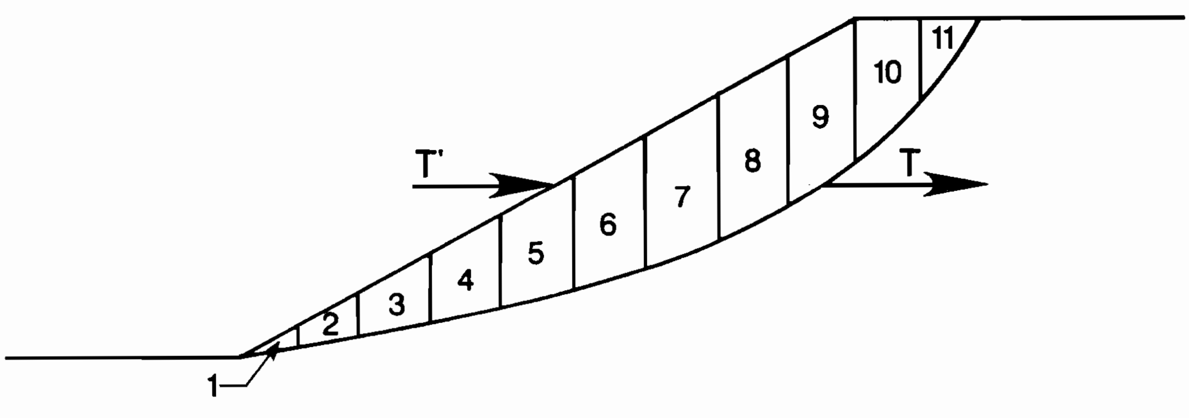


Fig. 2.9. Typical Slices Used in Procedures of Slices Showing Location of Actual (T) and Equivalent (T') Reinforcement Forces.

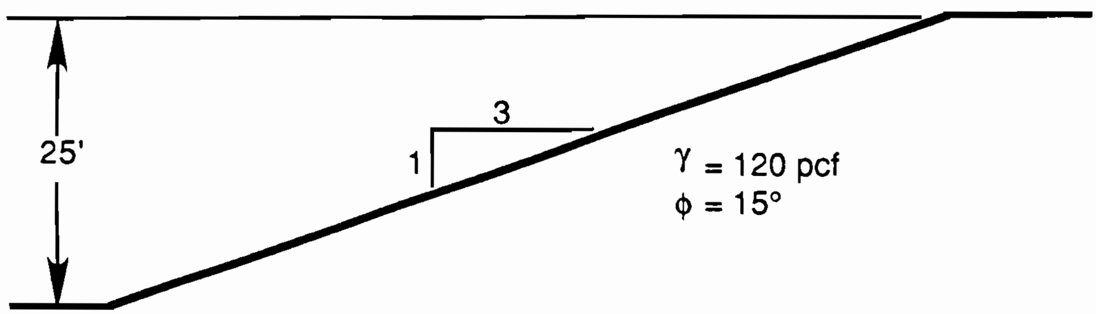


Fig. 2.10. Example 3:1 Slope Used for Comparative Calculations.

COMPARATIVE CALCULATIONS

Additional slope stability calculations were performed using both of the approaches described above for comparative purposes. Two slopes were selected for this purpose. The first slope is a relatively flat, 3:1 slope which was selected to be typical of a number of embankment slopes encountered by the Texas SDHPT, which are unstable or only marginally stable. The second slope is the same as the one previously considered in the calculations presented earlier in this chapter.

Example Slope No. 1

The first example slope is 25 feet high with a 3:1 side slope (Fig. 2.10). Soil properties are shown in Fig. 2.10. Calculations were performed for from one through five layers of reinforcement with the spacing as shown on Fig. 2.11. The reinforcement was assumed to be horizontal and to extend sufficiently far enough into the slope to be intersected by any potential shear plane. A force of 1527.9 pounds per lineal foot of slope was used for the reinforcement, which is the same force employed in the calculations presented previously in this chapter.

For simplicity all stability calculations were performed using a single planar shear surface inclined at one-half the slope angle from the horizontal plane. Calculations were performed by each of the two procedures described previously as well as by a modified form of the first procedure. For the modified form the force in the reinforcement was adjusted such that the force in the equilibrium solution was the actual force in the reinforcement, rather than the force divided by the factor of safety. The adjusted force (T) which was used to calculate the equivalent shear strength in the reinforcement using the modified form of the first approach was computed as,

$$T = 1527.9 \times F \tag{2.7}$$

where 1527.9 represents the actual reinforcement force and F is the factor of safety computed from the stability computations. A trial and error procedure was used to determine the factor of safety. That is, a factor of safety was assumed, the shear force was computed from Eq. 2.7, and stability computations were performed using the shear force from Eq. 2.7 to define an equivalent shear strength for the layers of material representing the reinforcement. The

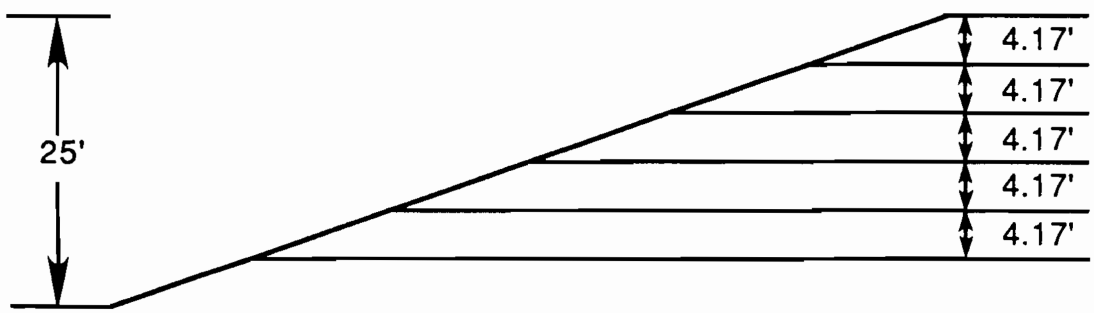


Fig. 2.11. Reinforcement Used for Example 3:1 Slope.

assumed and computed values of the factors of safety were then compared and a new value was assumed to repeat the process until the assumed and computed values were essentially equal. In this manner when the equivalent shear strength computed from Eq. 2.4 was factored in the stability computations by the factor of safety, the resulting stress and forces were the desired, actual values. The modified form of the first approach provides a solution which can be more easily compared with the second approach inasmuch as both approaches then correspond to the same force in the reinforcement under equilibrium conditions.

Computed factors of safety are summarized in Table 2.2 for each of the three approaches (Approach 1, Modified Approach 1 and Approach 2) and one through five layers of reinforcement. The factors of safety computed by the second approach are significantly higher than those computed by the first (unmodified) approach; however, when the first approach was modified so that the factor of safety was applied in the same way by the first and second approaches the differences became almost insignificant. When the factor of safety is applied in a consistent manner in both approaches the slightly higher values computed by the second approach are a result of the component of the reinforcement force acting normal to the shear surface, which is considered in the second approach and ignored in the first. In the second approach the normal component of force is multiplied by the tangent of the mobilized friction angle, tan ϕ_m (= tan ϕ/F), to produce a resisting force. The resisting force acts in addition to the component of the reinforcement force which is tangential to the shear surface. That is, the total resisting force due to the reinforcement, T_t, is

$$T_t = T\cos\alpha + T\sin\alpha\,\frac{\tan\phi}{F} \qquad (2.8)$$

The first term in this equation is identical to the component of the resisting force which is included in the modified form of the first approach; the second term represents the additional force included in the second approach. The differences between the factors of safety computed by the modified first approach and the second approach are small for this first example because the shear surface is relatively flat (α is small) and the mobilized frictional resistance (tanϕ/F) are relatively small. Accordingly, the second term in Eq.

TABLE 2.2. COMPARISON OF FACTOR OF SAFETY COMPUTED BY VARIOUS APPROACHES
EXAMPLE SLOPE NO. 1 - SHEAR PLANE AT $\beta/2$

APPROACH	1 Layer	2 Layers	3 Layers	4 Layers	5 Layers
Approach No. 1 Reinforcement represented as layers of material	1.73	1.81	1.89	1.97	2.04
Approach No. 1 - Modified Same as Approach No. 1, but factor of saftey not applied to reinforcement force	1.79	1.96	2.16	2.41	2.73
Approach No. 2 Reinforcement represented by external loads on the surface of the slope	1.80	1.97	2.18	2.44	2.77

2.8 is only a very small fraction of the first term. Much larger differences would exist if the second term were to become more dominant as illustrated by the next example.

Example Slope No. 2

The second example was illustrated previously in Fig. 2.5. Calculations were performed using the first approach, the modified form of the first approach discussed above, and the second approach. Calculations were performed for a single shear plane corresponding to the critical plane found earlier, inclined at 38 degrees from the horizontal. The factors of safety calculated by the three approaches are summarized in Table 2.3. In this example, the second approach where the reinforcement was modeled as applied forces, produced a factor of safety which was over twice as large as the values produced by either variation of the first approach, where the soil was modeled as a layer of equivalent soil. Even larger differences would be expected if the shear surface had been permitted to cross the reinforcement at a steep angle in the first approach.

Summary of Comparative Calculations

The magnitude of the differences between the factors of safety computed by the two approximate approaches described above depend on the relative role of the reinforcemnt on the satbility and the angle with which the reinforcement intersects the shear surface. However, the first approach appears to lead to unreasonable shapes for the most critical shear surfaces which in turn negate any beneficial effects of the reinforcement. Thus, it appears clear that the first approach, where the reinforcement is represented as a layer of equivalent soil, is much less reasonable than the second approach, where the reinforcement is represented as applied forces.

EFFECTS OF REINFORCEMENT ORIENTATION

In the theoretical developments and calculations presented above for the two approaches the reinforcement was always assumed to act horizontally and produce the internal horizontal force, T. However, if deformation occurs in a slope the reinforcement may be distorted from its original position. If the

TABLE 2.3. COMPARISON OF FACTOR OF SAFETY COMPUTED BY VARIOUS APPROACHES EXAMPLE SLOPE NO. 2 - CRITICAL SHEAR PLANE AT 38 DEGREES.

APPROACH	FACTOR OF SAFETY
Approach No. 1 Reinforcement represented as layers of material	1.16
Approach No. 1 - Modified Same as Approach No. 1, but factor of saftey not applied to reinforcement force	1.29
Approach No. 2 Reinforcement represented by external loads on the surface of the slope	2.73

reinforcement is initially horizontal, it may become reoriented to a direction approximately tangential to the shear surface, at least locally in the zone of highest stress and movement (Fig. 2.12). To examine the potential effects of such a reorientation of the reinforcement several additional sets of calculations were performed in which the reinforcement force was assumed to act tangential to the shear surface, rather than horizontally. Calculations were performed for the two example slopes and soil properties used in the previous section and illustrated in Figs. 2.5 and 2.10. For the first slope the number of layers of reinforcement was varied from a minimum of one layer to a maximum of five layers as shown previously in Fig. 2.10. All calculations were performed using the second approach, where the reinforcement is represented by a series of external loads.

Calculations were performed for the single planar shear surfaces used previously. Use of the planar shear surface considerably simplified the calculations when the reinforcement force was tangential to the shear surface: The reinforcement forces in all layers had the same, single line of action, which was coincident with the shear surface as illustrated in Fig. 2.13. Thus, only a single set of surface loads were required to represent all reinforcement forces.

Example Slope No. 1

Calculations for the first example slope (Fig. 2.10) were again performed for from one through five layers of reinforcement. Calculations were performed for two shear surfaces: One shear surface was inclined at one-half the slope angle (approximately 9.2 degrees); the other was inclined at 14 degrees. The factors of safety calculated with the reinforcement inclined tangential to the shear plane as well as horizontal are shown in Table 2.4. In the case of the first shear plane, inclined at one-half the slope angle, the factor of safety was always larger when the reinforcement force was assumed to be horizontal rather than tangent to the shear surface. For the shear plane inclined at 14 degrees the factor of safety was also higher when the reinforcement force acted horizontally for the case of from one to three layers of reinforcement. However, when the number of layers of reinforcement was increased to four and five the factor of safety was greatest when the reinforcement was tangential to the shear surface. Accordingly, the maximum factor of safety may occur

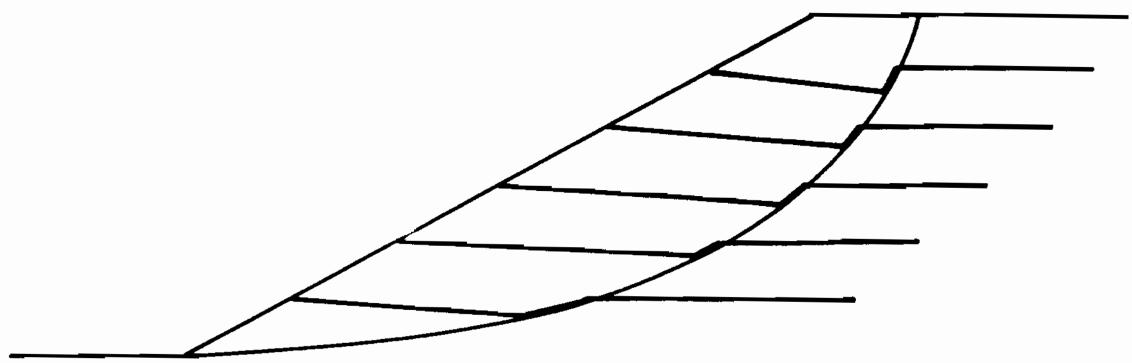


Fig. 2.12. Illustration of Potential Distortion of Reinforcement in a Slope where Large Shear Distortion has Occurred Along a Preferred Shear Plane.

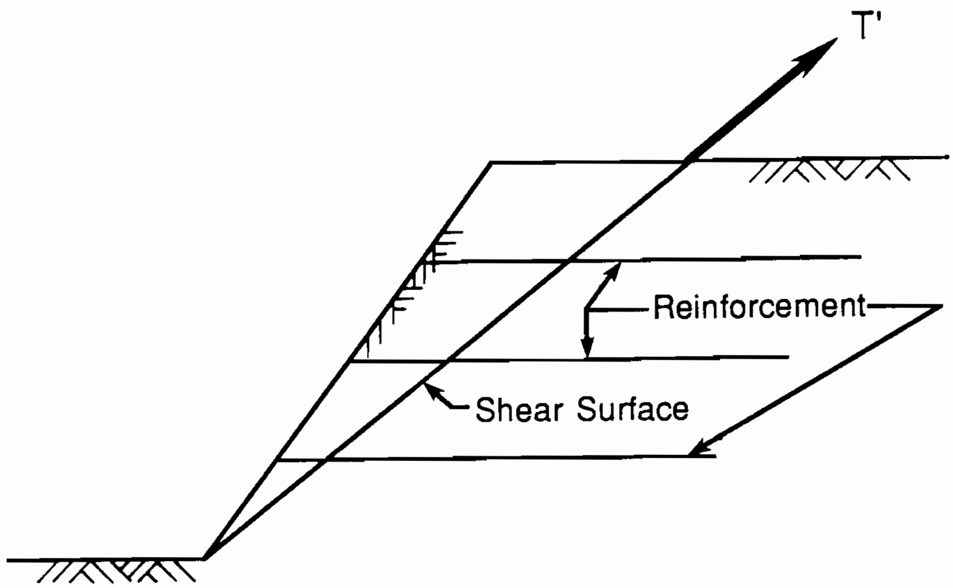


Fig. 2.13. Equivalent External Force for Planar Shear Surface When Reinforcement Force is Parallel to Shear Surface.

TABLE 2.4. EFFECT OF DIRECTION OF THE APPLIED REINFORCEMENT FORCE ON THE COMPUTED FACTOR OF SAFETY - EXAMPLE SLOPE NO. 1

	$\alpha = \beta/2$ (9.2*)		α = 14*	
Number of Layers	Horizontal Force	Tangential Force	Horizontal Force	Tangential Force
1	1.797	1.796	1.295	1.290
2	1.971	1.968	1.621	1.612
3	2.182	2.177	2.152	2.151
4	2.441	2.435	3.171	3.229
5	2.768	2.763	5.921	6.468

either when the reinforcement is horizontal or when it is tangent to the shear surface depending on the particular set of circumstances. This becomes apparent by comparing the net effect which the reinforcement produces when it acts horizontally with the effect of the reinforcement when it acts tangentially to the shear surface. The net effect of the reinforcement acting horizontally was shown previously to be,

$$T_t = T\cos\alpha + T\sin\alpha\,\frac{\tan\phi}{F} \qquad (2.9)$$

while when the reinforcement acts tangentially to the shear surface the net effect on the resistance is simply the force, T. Either the force T_t given by Eq. 2.+98 or the force T itself may be larger depending on the values of the inclination of the shear plane (α), the friction angle (ϕ), and the factor of safety. Accordingly, it is not possible to generalize on the relative magnitudes of the factors of safety calculated based on the two assumptions.

<u>Example Slope No. 2</u>

The factors of safety calculated with the reinforcement forces acting horizontally and tangentially to the shear surface for the second example slope are 2.73 and 3.74, respectively.

EFFECTS OF NUMBER OF LAYERS OF REINFORCEMENT

During the studies described above it was observed in some cases that after a certain number of layers of reinforcement were included in the slope the factor of safety became very large and the numerical solution for the factor of safety became unstable. This is illustrated, for example, by the computations for the 3:1 slope and a 14 degree shear plane which were summarized in Table 2.4. Examination of the computed factors of safety reveals a sharp increase as the number of layers of reinforcement is increased from four to five. Closer inspection of the solution in Table 2.4 and conditions which lead to instability in other examples revealed a potential problem in computing the stability where internal reinforcement is included as follows: Consider, first, the case where the reinforcement force is assumed to act tangentially to the shear surface. The total force due to the reinforcement acting tangential to the shear surface is,

$$T_{total} = T\ n \tag{2.10}$$

where n is the number of layers of reinforcement. For a plane shear surface the driving force, $F_{driving}$, due to the weight of the soil acting in a direction tangential to the shear surface is

$$F_{driving} = \frac{1}{2}\gamma H^2(\cot\alpha - \cot\beta)\ \sin\alpha \tag{2.11}$$

where γ is the unit weight of the soil, H is the height of the slope, and β is the slope angle. If the force in the reinforcement, T_{total}, is exactly equal to the driving force ($F_{driving}$) then no resisting force is required from the soil; theoretically, the factor of safety is infinite. If the force in the reinforcement is even larger and exceeds the driving force the resisting force in the soil must act in the same direction as the driving force to establish equilibrium. In this case the value of the factor of safety mathematically becomes negative. The reinforcement force and driving forces can be equated and solved for the number of layers of reinforcement, n_{eq}, where the two forces become equal. This gives,

$$n_{eq} = \frac{\gamma H^2(\cot\alpha - \cot\beta)\ \sin\alpha}{2T} \tag{2.12}$$

In the case of the 3:1 slope and 14 degree shear plane considered previously (γ = 120 pcf, H = 25 feet, and T = 1527.9 lbs/lin.ft.) the value of n computed from Eq. 2.12 is almost exactly 6.0. That is, if the number of layers of reinforcement is 6, or more the reinforcement force exceeds the driving force and the factor of safety becomes first infinite and, then, negative. The maximum number of layers of reinforcement was limited to 5 in the previous example to eliminate this potential problem.

In the case where the reinforcement force is horizontal or other than tangential to the shear surface (as was assumed in developing Eq. 2.12) or the shear surface is non-planar, similar, but more complex equations to Eq. 2.12

could conceivably be developed for the maximum number of layers of reinforcement which can exist before the factor of safety becomes infinite and, then, negative. In each case where the reinforcement is represented by a series of forces, there is a limiting amount of reinforcement beyond which a limiting equilibrium solution will become numerically unstable.

The problem of numerical instability does not exist in the case where the reinforcement is represented by an equivalent layer of material. In this case the reinforcement force is factored by the factor of safety and will never exceed the driving force. Although the factor of safety may become very large as more and more reinforcement is added, it will never approach infinity for any practical case.

SUMMARY

Two simplified approaches have been examined for representing the effects of internal soil reinforcement in slope stability computations using existing computer programs for slope stability computations. In the first approach the reinforcement is represented as layers of material with strength properties equivalent to those of an anisotropic material used to represent the component of the reinforcement force acting tangential to the shear surface; the normal component of the reinforcement force is explicitly ignored, although equilibrium is satisfied. In the second approach the reinforcement forces are represented as external boundary loads on the surface of the slope. The first approach appears to be inferior to the second approach: The first approach appears to lead to totally unrealistic results when attempts are made to locate the truly most critical shear surface.

It was found that the orientation assumed for the reinforcement will affect the computed stability and if a relatively large amount of reinforcement is included, the numerical solution can become unstable. Although numerical instabilities should only occur when the slope is very stable physically, such instabilities may cause difficulties when examining a number of alternatives to arrive at a suitable amount of reinforcement for design.

It appears that present computer programs for slope stability analysis may be used for analysis of reinforced slopes by representing the reinforcement as external loads. However, although such loads are statically equivalent to the internal loads, they must be applied at different locations on the soil mass than the actual internal loads being considered. This may lead to differences in the internal distribution of stresses and for complex cross-sections and more than one material could lead to errors. In addition when the reinforcement is represented by a series of external loads any variation in force along the reinforcement internally cannot be readily taken into account. In order to represent more correctly the effects of the reinforcement the more rigorous, fundamentally correct procedures described in the next chapter were examined.

CHAPTER THREE. DEVELOPMENT OF LIMIT EQUILIBRIUM SOLUTION PROCEDURE FOR REINFORCED EARTH SLOPES

Limit equilibrium analyses based on various procedures of slices have been widely used for many years to compute the stability of earth slopes. The computations presented in Chapter 2 were based on such limit equilibrium slope stability analysis procedures and showed that the procedures have promise for use in stability computations for reinforced earth slopes. However, the computations in Chapter 2 also indicated that the procedures and computer program (UTEXAS) employed probably need to be modified to model more properly the soil reinforcement and permit the added flexibility to account for variations in the force along the length of the reinforcement.

DEVELOPMENT OF LIMIT EQUILIBRIUM EQUATIONS

Limit equilibrium procedures of slices consider the mass of soil bounded by an assumed shear (sliding) surface to be subdivided into a finite number of vertical slices as shown in Fig. 3.1. The forces acting on a typical slice are illustrated in Fig. 3.2. The forces normally considered to act on the slice consist of the weight of the slice (W), a normal and shear force on the top of the slice (P and T, respectively), a normal and shear force on the base of the slice (N and S, respectively), a horizontal body force (KW), and forces on the left and right sides of the slice (Z_i and Z_{i+1}, respectively). The forces P and T on the top of the slice are used to represent any external forces on the surface of the slope, such as the force due to free surface water, stockpiles of material, or structural loads. In fact the forces P and T were used to represent the soil reinforcement in the second approximate method described in Chapter 2. The forces N and S on the base of the slice represent the respective normal and shear forces in the soil on the assumed shear surface. The horizontal force, KW, is used to approximate earthquake loadings when so-called "pseudo-static" analyses are performed. The force, KW, is zero in the case on any conventional static analysis. The forces, Z_i and Z_{i+1}, and

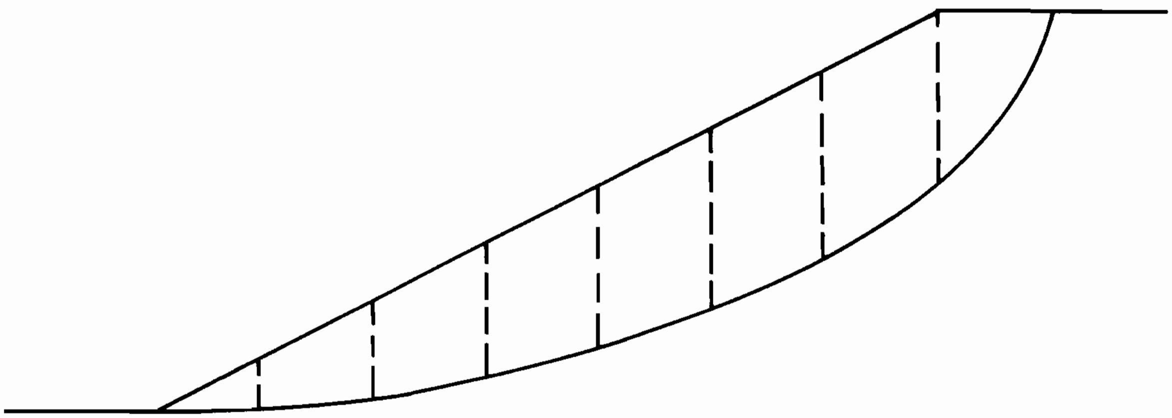


Fig. 3.1. Typical Shear Surface Subdivided into Slices.

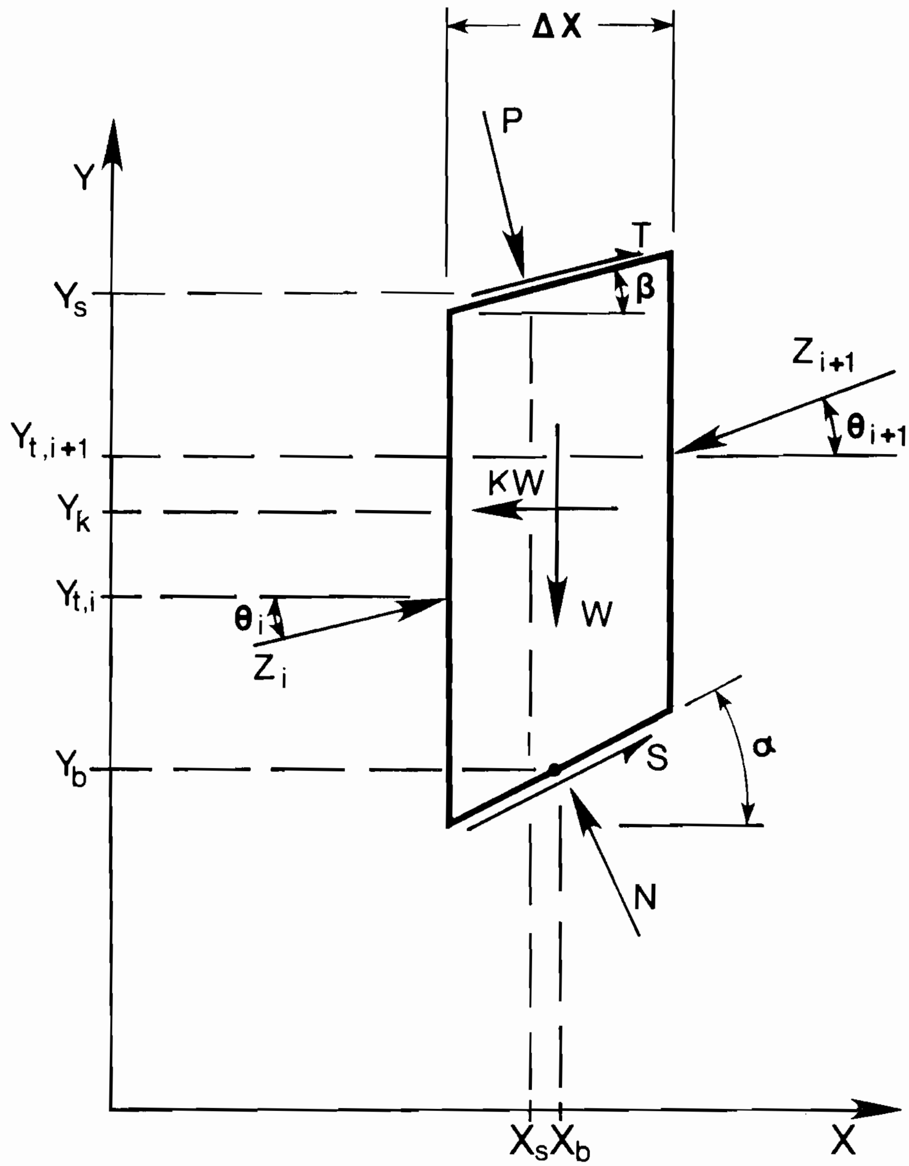


Fig. 3.2. Forces and Coordinates for a Typical Slice Used in Limit Equilibrium Slope Analysis Procedures.

their respective inclinations, θ_i and θ_{i+1}, represent the resultant total force on the left and right sides of the slice.

In the case of a reinforced slope an additional force can be introduced on the base of the slice as shown in Fig. 3.3. The force, R, represents the force developed in the reinforcement at the point where the reinforcement is intersected by the shear surface. The inclination of the force is designated by the angle, ψ, and the coordinates of the point where the reinforcement intersects the base of the slice are indicated by, x_r and y_r. The reinforcement force, R, is considered to act independently of the forces N and S shown previously in Fig. 3.2; the forces N and S represent forces transmitted through the soil, while the force R is transmitted through the reinforcement.

The forces P, T, R, W and KW must ordinarily be defined in order to compute the stability and, thus, it is convenient to combine these five forces and express them in terms of their resultant component in the horizontal and vertical directions and the resultant moment which they produce about a point on the center of the base of the slice. The resultant force in the horizontal direction, designated as F_h, is given by,

$$F_h = - KW + P\sin\beta + T\cos\beta + R\cos\psi \tag{3.1}$$

where forces acting to the right are considered to be positive. Similarly, the components of the known forces acting in the vertical direction are expressed by their resultant, F_v, given by,

$$F_v = - W - P\cos\beta + T\sin\beta + R\sin\psi \tag{3.2}$$

where forces acting in the upward direction are considered positive. The moment which the known forces produce about a point on the center of the base of the slice is expressed by,

$$\begin{aligned} M_o = &- P\sin\beta(y_s - y_b) - P\cos\beta(x_s - x_b) - T\cos\beta(y_s - y_b) \\ &+ T\sin\beta(x_s - x_b) + KW(y_k - y_b) - R\cos\psi(y_r - y_b) \\ &+ R\sin\psi(x_r - x_b) \end{aligned} \tag{3.3}$$

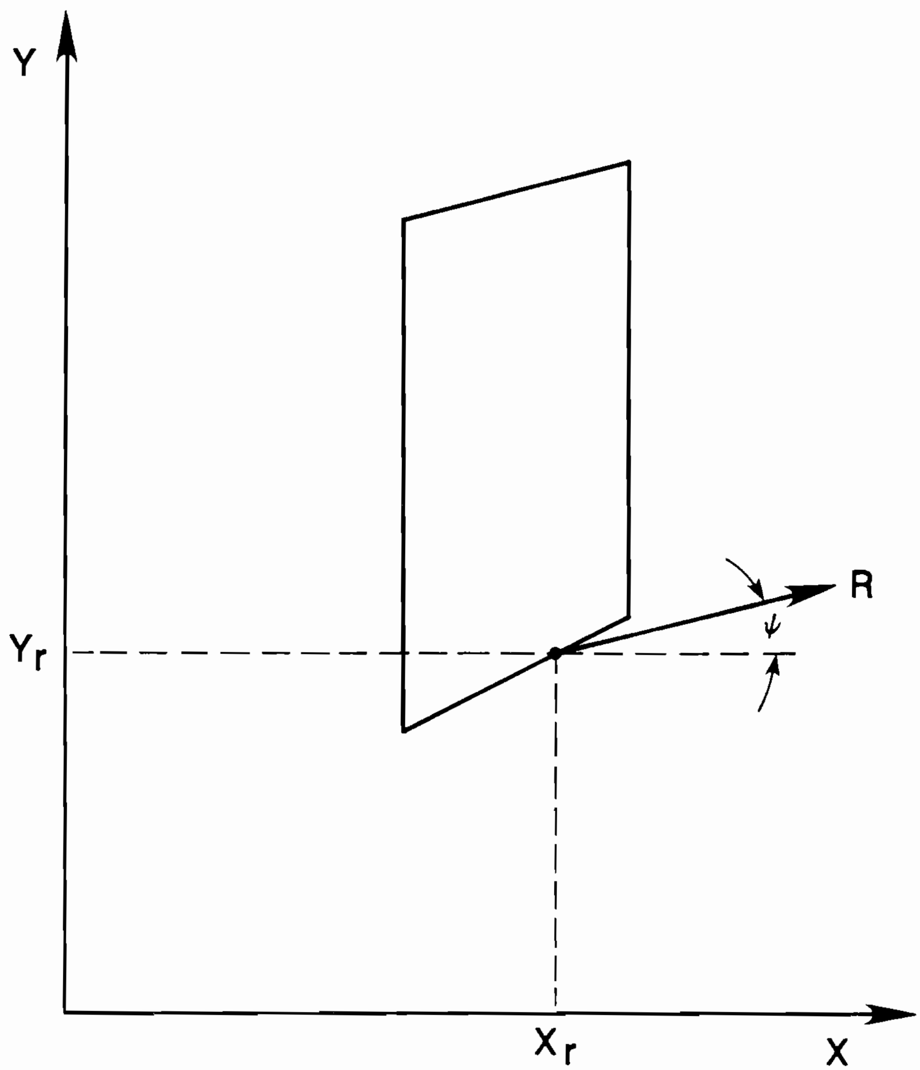


Fig. 3.3. Typical Slice with Additional Force and Coordinates Used to Represent Internal Soil Reinforcement.

The equations used to compute the stability by various limit equilibrium procedures are derived by considering the equilibrium of an individual slice. Summing forces in the vertical direction and noting that the summation must be equal to zero leads to the following equation for equilibrium of forces in the vertical direction:

$$F_v + Z_i \sin\theta_i - Z_{i+1}\sin\theta_{i+1} + N\cos\alpha + S\sin\alpha = 0 \tag{3.4}$$

Similarly, summation of forces in the vertical direction produces the following equation for equilibrium of forces in the horizontal direction:

$$F_h + Z_i \cos\theta_i - Z_{i+1}\cos\theta_{i+1} - N\sin\alpha + S\cos\alpha = 0 \tag{3.5}$$

Finally, summation of moments about a point on the center of the base of the slice produces the following equilibrium equation with respect to moment equilibrium:

$$\begin{aligned} M_o - Z_i \cos\theta_i (y_{t,i} - y_b) + Z_{i+1}\cos\theta_{i+1}(y_{t,i+1} - y_b) \\ - (Z_i \sin\theta_i + Z_{i+1}\sin\theta_{i+1})\frac{\Delta x}{2} = 0 \end{aligned} \tag{3.6}$$

Equations 3.4, 3.5 and 3.6 may be written for each slice to give a total of 3n equilibrium equations, where "n" is the number of slices used.

The shear force which appears in Eqs. 3.4 and 3.5 can be expressed in terms of the normal force on the base of the slice and the factor of safety. The factor of safety in all limit equilibrium procedures is defined in terms of shear strength and can be expressed by,

$$F = \frac{s}{\tau} \tag{3.7}$$

where τ is the shear stress on the base of the slice and s is the available shear strength. The shear stress, τ, is simply the shear force, S, on the base of the slice divided by the area of the base of the slice; in the case of a slice of unit thickness normal to the two-dimensional plane of interest the expression for the shear stress is,

$$\tau = \frac{S}{\Delta \ell} \tag{3.8}$$

where $\Delta \ell$ is the length of the base of the slice[1]. The shear strength can be expressed by the Mohr-Coulomb equation, which in the case of total stresses is written as

$$s = c + \sigma \tan\phi \tag{3.9}$$

where σ is the total normal stress on the base of the slice. The normal stress can be expressed in terms of the normal force, N, by,

$$\sigma = \frac{N}{\Delta \ell} \tag{3.10}$$

Finally, by combining Eqs. 3.8 through 3.10 with 3.7 and rearranging terms, the following equation is obtained for the shear force on the base of the slice:

$$S = \frac{1}{F}(c\Delta \ell + N\tan\phi) \tag{3.11}$$

The factor of safety which appears in Eq. 3.11 is assumed to be constant along the shear surface and is, thus, identical for each slice. Equation 3.11, which is derived using the Mohr-Coulomb strength equation and the definition of the factor of safety, but not using any of the equilibrium requirements, can be used to eliminate the shear force, S, from the equilibrium equations (3.4 and 3.5), replacing the shear force on each slice by a single unknown quantity, the factor of safety, F.

Once the shear force is eliminated form Eqs. 3.4 through 3.6, there are a number of unknowns which must still be considered. The unknowns are summarized in Table 3.1 for "n" slices. The total number of unknowns is 5n - 2, which clearly exceeds the number of equilibrium equations (3n) which are available

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[1]Although the shear force in the soil is actually only distributed over the portion of the base of the slice containing soil, excluding the portion containing reinforcement, the area of the base of a slice occupied by reinforcement is small and is neglected in Eq. 3.8 and subsequent equations.

TABLE 3.1. UNKNOWNS IN LIMIT EQUILIBRIUM PROCEDURES OF SLICES.

Description of Unknown	Quantity
Factor of safety (F)	1
Normal forces on base of slice (N)	n
Location of normal forces on base of slice	n
Side forces (Z)	n - 1
Inclination of side forces (θ)	n - 1
Location of side forces (y_t)	n - 1
TOTAL	5n - 2

for solution. One set of the unknowns, the locations of the normal forces on the base of the slice, has already been eliminated when the normal forces were assumed to act at a point on the center of the base of the slice. The location of the normal force on the base of the slice has been found to have little effect on a solution and, accordingly, this assumption appears to be entirely reasonable. However, there remain a total of n - 2 unknowns after this assumption has been made. A variety of assumptions can and have been made to reduce the number of unknowns and thereby obtain a statically determinant solution. One of the most widely used set of assumptions are those first suggested by Spencer. Spencer suggested that the side forces be assumed to act parallel and, thus, the unknown inclinations (θ) for the boundaries between slices are replaced by a single unknown value for θ. The unknowns that are considered in Spencer's procedures are summarized in Table 3.2 where it can be seen that the number of unknowns (3n) exactly balances the number of equilibrium equations available. Thus, a statically determinant solution can be obtained for the factor of safety and remaining unknowns. Spencer's procedure is employed in the computer program, UTEXAS, which was developed for the Texas SDHPT and was employed for all of the studies described in the present report. Detailed derivations of the equations and numerical procedures employed to solve the equations are presented by Wright (1986) for Spencer's procedure as well as the Simplified Bishop and Corps of Engineers' Modified Swedish procedures of slices. Implementation of the procedures into a computer program is described in the next section.

IMPLEMENTATION IN COMPUTER PROGRAM UTEXAS2

The limit equilibrium procedures described above for soil reinforcement have been incorporated into a computer program (UTEXAS2) for slope stability analyses (Wright, 1986)[2]. Each layer of soil reinforcement is represented in

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[2]The computer program, UTEXAS2, is an updated version of the computer program, UTEXAS, which was originally developed for the Texas State Department of Highways and Public Transportation as part of research project (Wright and Roecker, 1984). Substantial revisions, modifications and improvements were (Footnote Continued)

TABLE 3.2. UNKNOWNS IN SPENCER'S PROCEDURE.

Description of Unknown	Quantity
Factor of safety (F)	1
Normal forces on base of slice (N)	n
Side forces (Z)	n - 1
Inclination of side forces (θ)	1
Location of side forces (y_t)	n - 1
TOTAL	3n

the computer program by a piece-wise linear continuous line as illustrated in Fig. 3.4. The coordinates of points along the line of reinforcement are input as data to the program along with corresponding values of the force in the reinforcement at each point. The force is assumed to vary linearly between each pair of coordinates along the reinforcement line. Thus, depending on where a particular trial shear surface intersects the reinforcement the force is computed by linear interpolation between the appropriate pair of points. In this way any distribution of forces along the length of the reinforcement can be included as suggested, schematically in Fig. 3.5.

Three candidate models were considered for representing the way in which the reinforcement force was applied to the shear surface (base of slice) as shown in Fig. 3.6. The first model assumes that the reinforcement force acts in the direction of the reinforcement (Fig. 3.6a). Thus, depending on the orientation of the reinforcement, the force might be horizontal or inclined at some angle other than horizontal. The second model assumes that the reinforcement has been distorted to the point that the reinforcement force acts tangential to the shear surface (Fig. 3.6b). The third model assumes that the reinforcement force acts in the direction of the reinforcement, as is assumed in the first model, except only the component of the reinforcement force which is tangential to the shear surface is applied; the component acting normal to the shear surface is ignored. The latter model is conservative in terms of how the contribution of the reinforcement is included, but may not be realistic; the latter was selected primarily for use in comparison for the present study, rather than as a recommended procedure for design.

SUMMARY

The limit equilibrium procedures described herein based on Spencer's assumptions for the side forces between slices represent a rigorous set of

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(Footnote Continued)
made to UTEXAS for the U.S. Army Corps of Engineers (Wright, 1985); the revised computer program, including the provision for stability computations with reinforcement, is referred to as UTEXAS2 and user documentation is presented by Wright (1986).

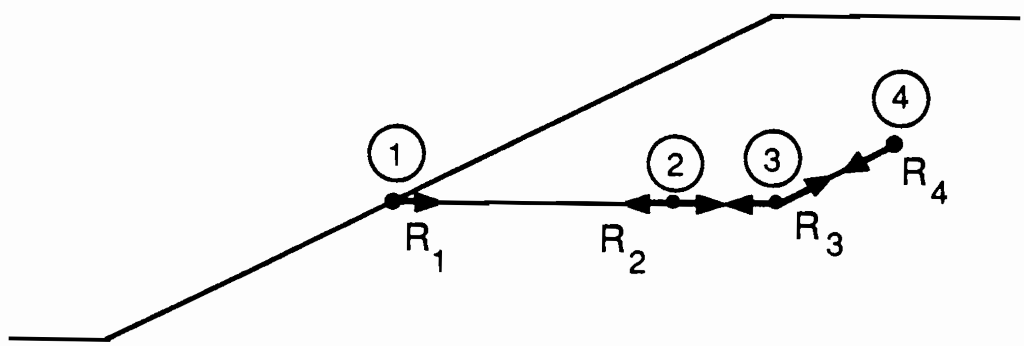

Fig. 3.4. Piece-wise Linear Representation of Internal Reinforcement and Forces Used in UTEXAS2.

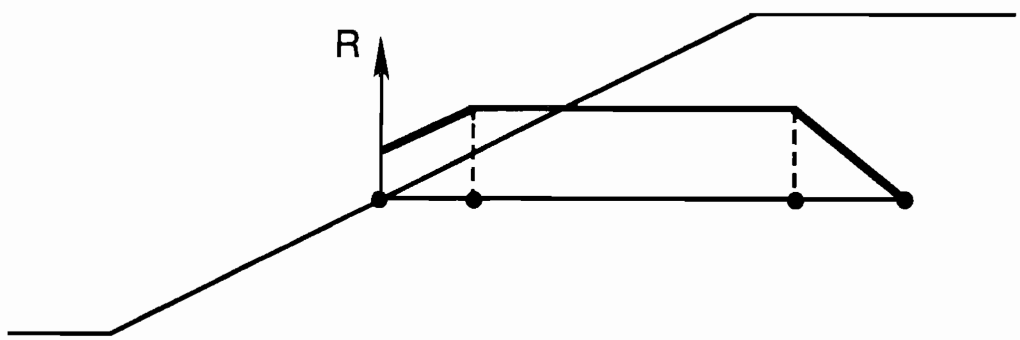

Fig. 3.5. Typical Pattern of Internal Forces in Reinforcement.

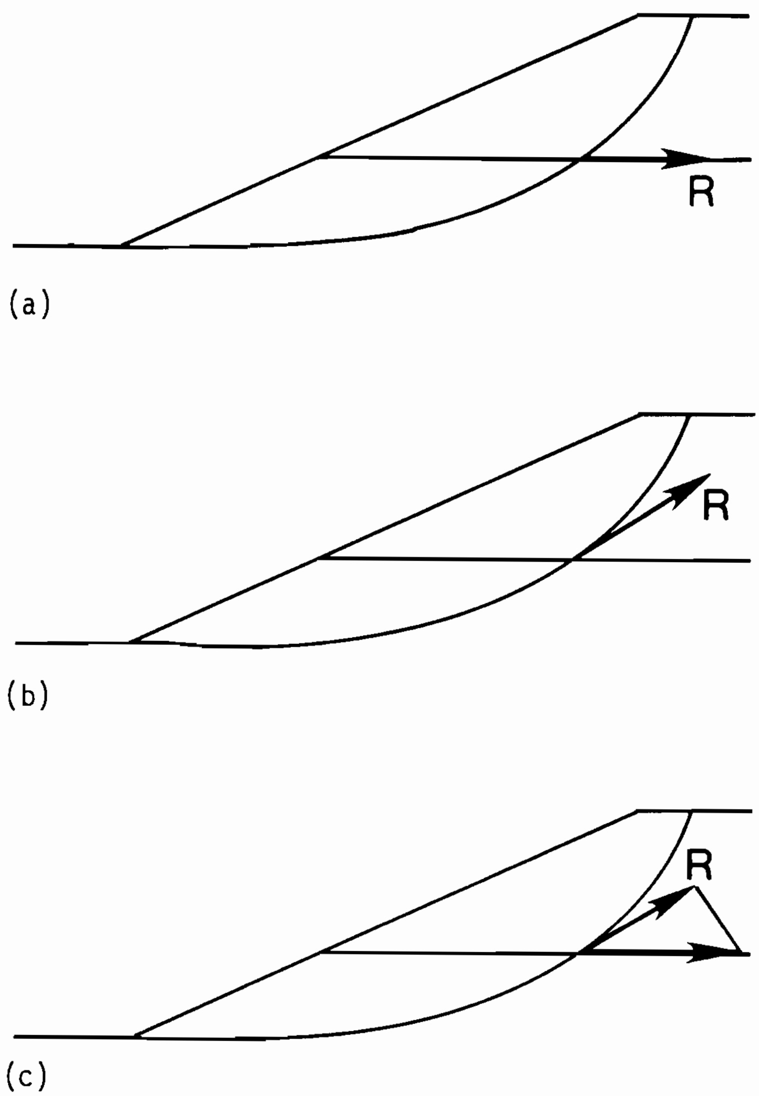

Fig. 3.6. Options Available for Representation of Internal Soil Reinforcement Forces in UTEXAS2: (a) Forces Parallel to Reinforcement, (b) Forces Parallel to Shear Surface, (c) Forces Parallel to Reinforcement, Only Component Tangential to Shear Surface Used.

equilibrium equations to account for the internal forces produced by soil reinforcement. The reinforcement forces are applied at the proper location on the soil mass and all requirements for static equilibrium are satisfied. Once the appropriate modifications were made to the computer program UTEXAS2 and verified for correctness a number of stability computations were performed to gain experience with the procedures and insight into their application. Results of the computations are presented in the next chapter.

CHAPTER FOUR. STABILITY COMPUTATIONS FOR TYPICAL REINFORCED SLOPE

Several series of stability calculations were performed using the procedures described in Chapter Three to demonstrate their application and to confirm their validity. An example problem developed and presented by the Tensar Corporation (1986) was selected as the basis for the example calculations. The example slope, illustrated in Fig. 4.1, is 38 feet high and inclined and 45 degrees (1:1). The soil is cohesionless with the properties shown in Fig. 4.1. Three "zones" (levels) of reinforcement are used as illustrated in Fig. 4.1.

COMPUTATIONS WITH CIRCULAR SHEAR SURFACES

The first series of calculations was performed using circular shear (potential sliding) surfaces. The reinforcement was assumed to have a working capacity of 1000 pounds per lineal foot. One set of calculations was performed assuming that the working capacity (1000 pounds/lineal foot) was developed along the entire length of the reinforcement with no reduction in force near the ends of the reinforcement. Two additional sets of calculations were performed assuming that the resistance in the reinforcement decreased to zero as the embedded end of the reinforcement was approached. In one of the additional sets of calculations it was assumed that the resistance was developed as frictional resistance along the top and bottom faces of the reinforcement. The resistance, dR, per incremental length, $d\ell$, along the reinforcement was calculated from:

$$dR = 2\,\gamma\,z\,\tan\phi\,d\ell \qquad (4.1)$$

where, γ is the unit weight of overlying soil, z is the vertical depth below the surface of the slope to the level of the reinforcement, and ϕ is the angle of internal friction of the soil (a cohesionless soil was assumed for the example slope). The resistance was then assumed to increase at the rate given by Eq. 4.1 ($dR/d\ell$) from zero at the embedded internal end of the reinforcement

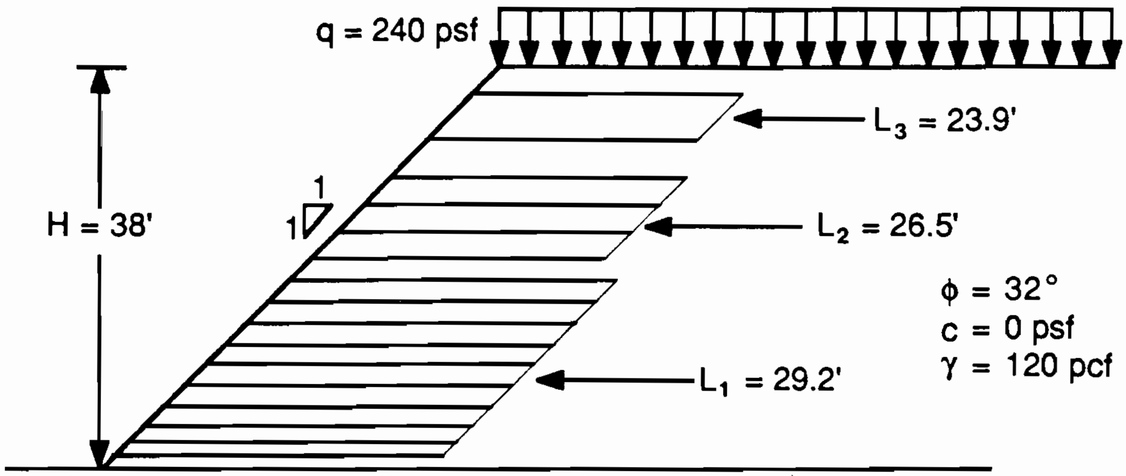

Fig. 4.1. Example Slope Used in Stability Computations (After Tensar Corporation. 1986).

up to a point where the resistance reached the "capacity" of 1000 pounds per lineal foot, after which the resistance was assumed to be constant. This is illustrated schematically in Fig. 4.2. In the next set of calculations it was assumed that the resistance was developed along only one side of the reinforcement. Thus, the resistance increased at a rate expressed by,

$$dR = \gamma\ z\ \tan\phi\ d\ell \tag{4.2}$$

Except for whether the resistance was developed on only one or on both sides of the reinforcement, the last two sets of calculations were identical.

The factors of safety calculated employing the three assumptions for the variation in the resistance along the reinforcement are summarized in Table 4.1. The corresponding critical circles are shown in Figs. 4.3 through 4.5. The factors of safety calculated for all three cases are very close, differing by less than 1 percent. The reason for the small difference is apparently because the critical shear surface only intersected one layer of reinforcement at a point where the resistance was less than the maximum assumed (1000 pounds/lineal foot). In the case considered, the assumption made regarding how the force in the reinforcement is developed is relatively unimportant. However, if the shear surface had intersected several layers of reinforcement at points where the resistance varied along the length of the reinforcement, the differences in the computed factors of safety would have been much greater. Additional examples would need to be studied to establish the importance of the variation in resistance along the reinforcement.

COMPUTATIONS WITH NONCIRCULAR SHEAR SURFACES

Two series of calculations were performed with noncircular shear surfaces. The first series employed a simple bilinear shear surface; the second series employed more general-shaped, noncircular shear surfaces.

Bilinear Shear Surfaces

The first series of computations with noncircular shear surfaces was performed using a bilinear surface like the one illustrated in Fig. 4.6. The elevation of the "hinge" point, designated as point B in Fig. 4.6, was

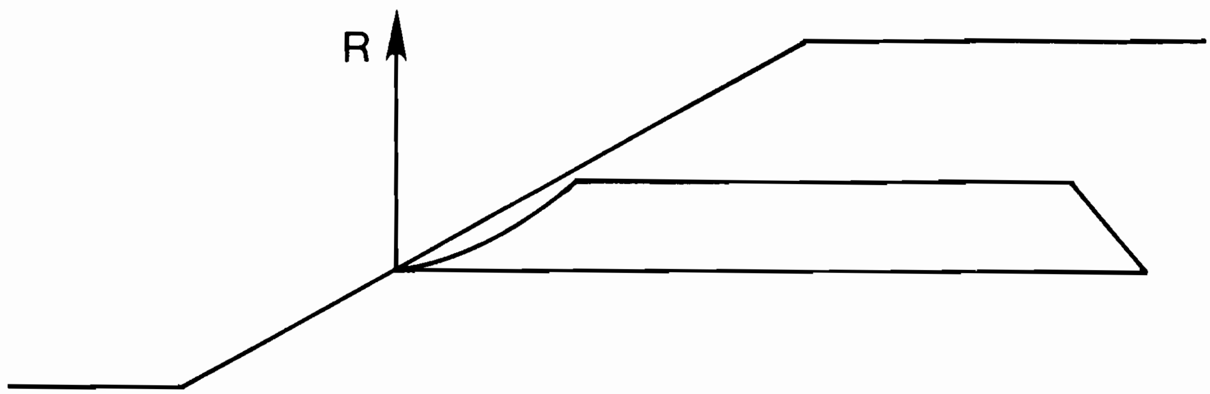

Fig. 4.2. Pattern of Force Developed Along the Length of Internal Reinforcement in a Typical Earth Slope.

TABLE 4.1. SUMMARY OF COMPUTED FACTORS OF SAFETY FOR CIRCULAR SHEAR SURFACES

Assumption	Factor of Safety
Full capacity along entire length of internal reinforcement.	1.410
Lowered capacity near embedded end of internal reinforcement - friction developed along both sides of internal reinforcement.	1.399
Lowered capacity near embedded end of internal reinforcement - friction developed along one side of internal reinforcement.	1.397

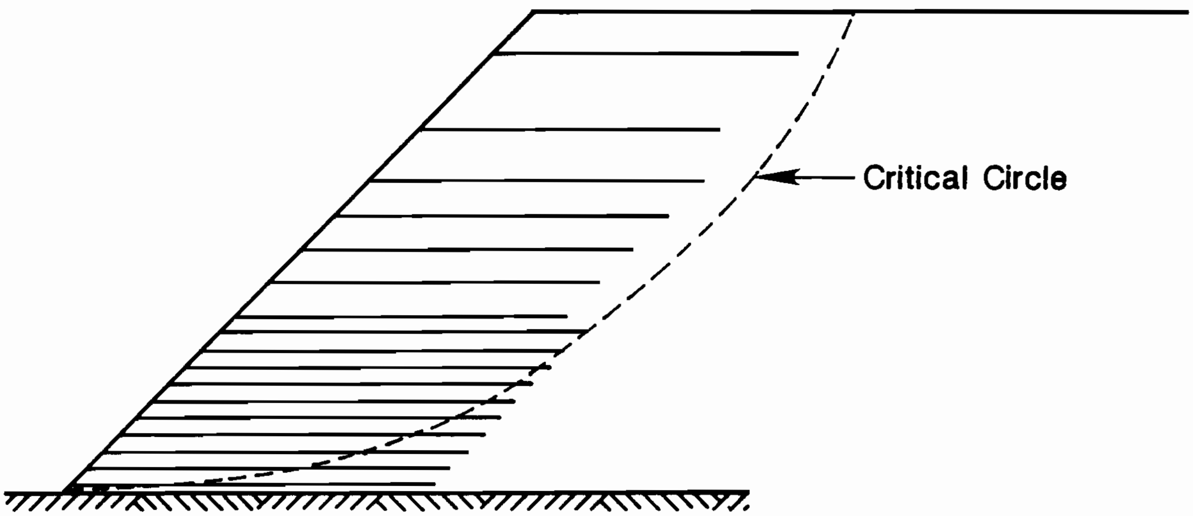

Fig. 4.3. Critical Circle for Example Slope: Full (Constant) Reinforcement Force Developed Along Entire Length of Reinforcement.

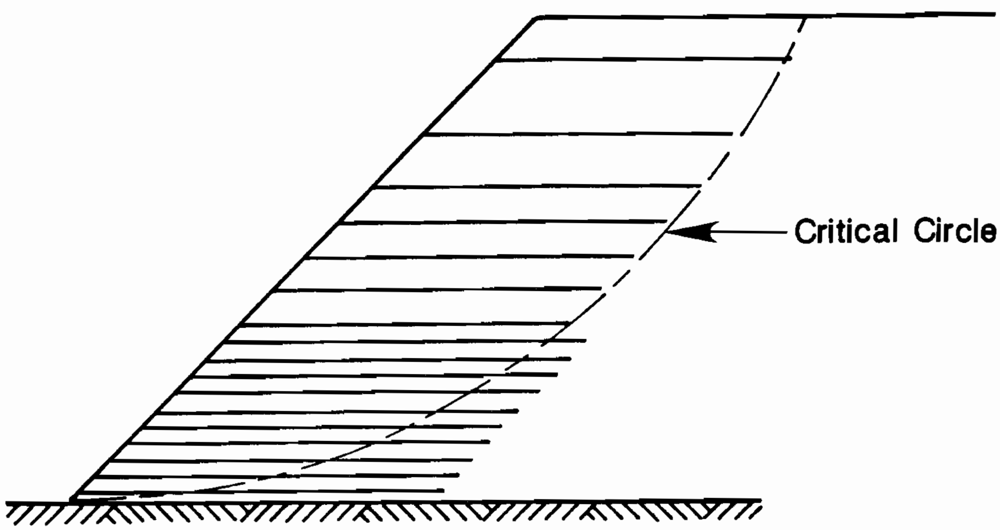

Fig. 4.4. Critical Circle for Example Slope: Reduced Reinforcement Force at Ends of Reinforcement Based on Friction Along Only One Side of Reinforcement.

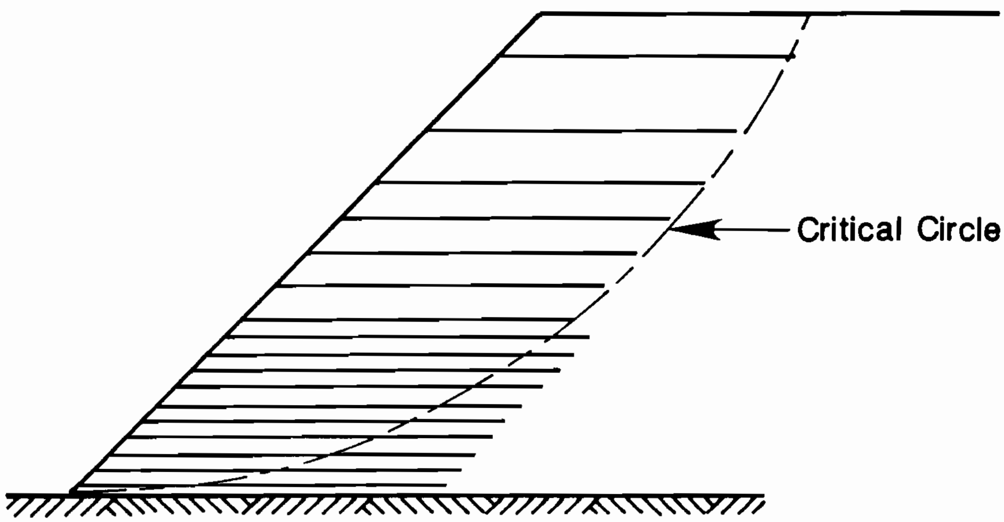

Fig. 4.5. Critical Circle for Example Slope: Reduced Reinforcement Force at Ends of Reinforcement Based on Friction Along Only One Side of Reinforcement.

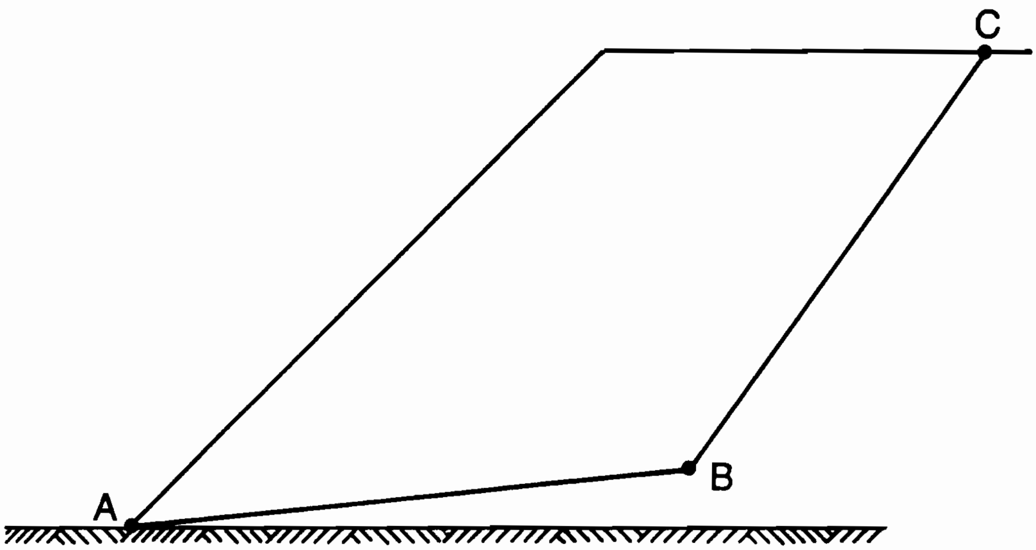

Fig. 4.6. Typical Bilinear Shear Surface Employed in Slope Stability Calculations.

selected and the computer program UTEXAS2 was used to automatically shift the hinge point horizontally and the point on the crest of the slope (Point C in Fig. 4.6) along the slope until a minimum factor of safety was found. Various elevations were assumed for the "hinge" point, and the most critical shear surface corresponding to each elevation was found using the computer program, UTEXAS2. The most critical shear surface located in this manner is illustrated in Fig. 4.7 and has a "hinge" point at an elevation of 2 feet above the toe of the slope. The factor of safety for the most critical surface is 1.431. This factor of safety (1.431) is slightly higher than the one computed for circular shear surfaces. Thus, circular shear surfaces are at least more critical than a simple bilinear shear surface.

General Noncircular Shear Surfaces

The next series of computations was performed to determine if a noncircular shear surface could be found which was more critical (produced a lower value for the factor of safety) than any of the shear surfaces considered previously. Two sets of computations were performed: The first set involved performing an automatic search using as an initial estimate for the position of the surface the most critical circle found previously; a total of sixteen points along the shear surface were defined and shifted to determine the most critical surface. The initial shear surface and points which were shifted are shown in Fig. 4.8. The second set of computations was identical to the first except that the search was initiated using the most critical bilinear shear surface located earlier and shown again in Fig. 4.9; a total of fifteen points, also shown in Fig. 4.9, were defined and shifted to locate the most critical shear surface. The computer program, UTEXAS2, readily permitted an automatic search to be conducted employing the various initial estimates for the shear surfaces. In these searches each point along the shear surface was systematically moved until a minimum factor of safety was computed. The direction of movement of each point was approximately perpendicular to the shear surface at that point.

The minimum factor of safety determined by starting with the circular shear surface as an initial estimate was 1.330. The minimum factor of safety based on the bilinear surface as an initial estimate was 1.366. These two values (1.330 and 1.366) are very similar and would be expected to become even

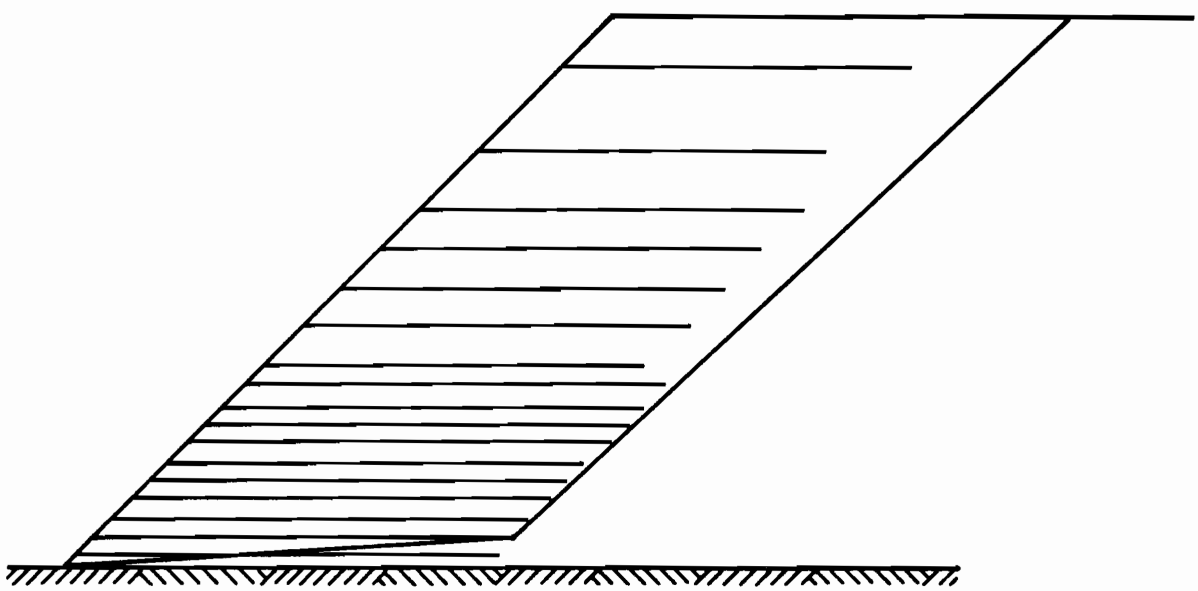

Fig. 4.7. Critical Bilinear Shear Surface for Example Slope.

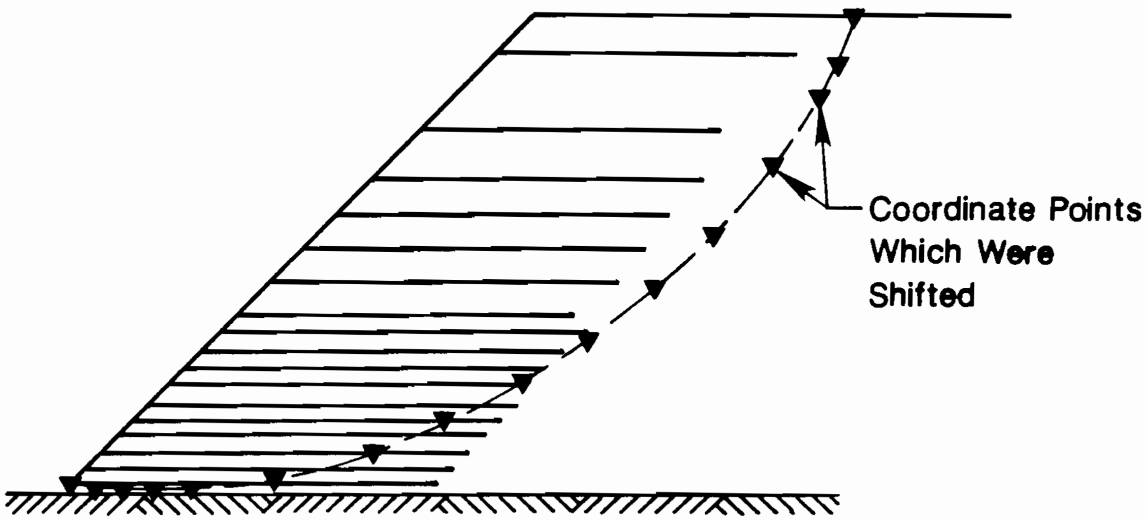

Fig. 4.8. Initial Trial Shear Surface Used in Search for Critical Noncircular Shear Surface: Based on Critical Circle.

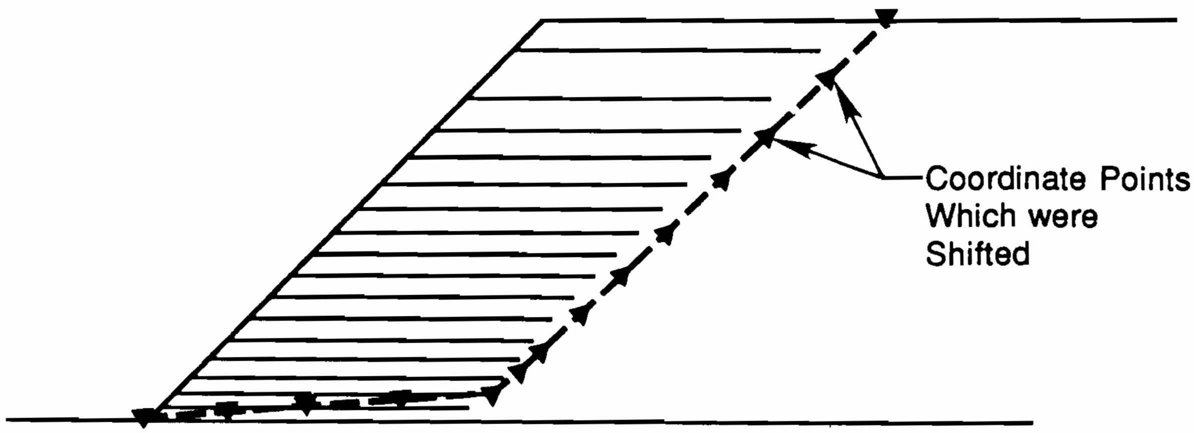

Fig. 4.9. Initial Trial Shear Surface Used in Search for Critical Non-Circular Shear Surface: Based on Critical Bilinear Shear Surface.

more similar as the number of points used to define the shear surface is increased; the number of points was found to be especially important along the lower portion of the shear surface The two minima values of the factor of safety (1.330 and 1.366) calculated with the general, noncircular shear surfaces are smaller than those obtained using any of the previous shear surfaces and, thus, the noncircular shear surfaces which were located are more critical and theoretically most correct. However, the differences between all of the factors of safety computed are very small and for practical purposes circular shear surfaces could have been used for the computations.

The critical shear surfaces found using the two initial estimates for the noncircular shear surfaces are shown together in Fig. 4.10. The two surfaces can be seen to be nearly the same, thus, being consistent with the similarities in the two factors of safety (1.330 and 1.366).

COMPARISON WITH OTHER SOLUTIONS

The example slope in Fig. 4.1 was presented by Tensar Corporation (1986) as part of a series of computations to illustrate their independent design procedure and a series of charts which they developed. Although all details of their procedure were not available at the time of this report, it was understood that the charts were based on limit equilibrium slope stability analysis procedures. The example which was developed, and is presented in Fig. 4.1, was developed with the criteria that the reinforcement in the slope should provide a factor of safety of at least 1.3. The calculations which were presented above confirm the stated factor of safety: The factor of safety is fully at least 1.3, but not excessively larger. Accordingly, the analysis procedures developed in Chapter 3 and their implementation in the computer program UTEXAS2, appear to be valid in that it is confirmed by the results of entirely independent calculations.

SUMMARY AND CONCLUSIONS

The analysis procedures developed in Chapter Three and incorporated into the computer program UTEXAS2 were found to be reliable and easy to use. The results of computations using these procedures showed for at least one case that the length over which the full resistance in the reinforcement is

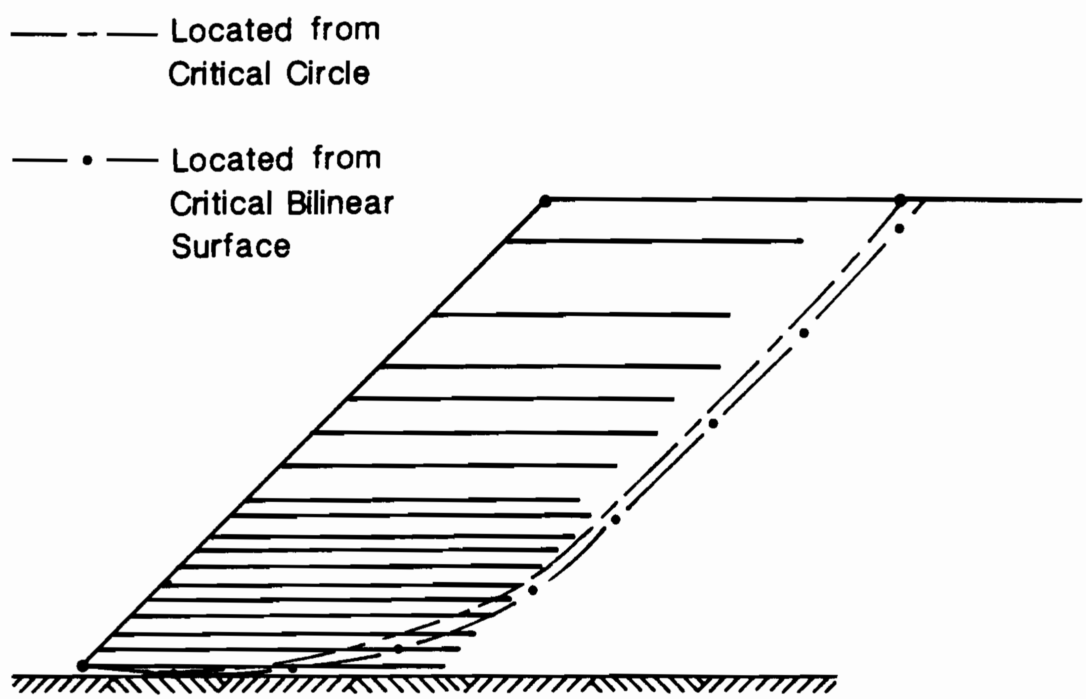

Fig. 4.10. Critical Noncircular Shear Surfaces Found Using Two Initial Estimates for Search.

developed is relatively unimportant due to the relatively sharp rate at which the resistance apparently built-up along the embedded length of the reinforcement. Whether this is true in most cases still needs to be studied; however, the results are promising and may suggest that the detail of development of force at the ends of the reinforcement ("end-effects") may not be of great concern. This would be at least true for "geo-grids" where high frictional resistance may be developed between the soil and reinforcement.

The computations for the case considered also revealed that circular shear surfaces were adequate for computing the minimum factor of safety. This also is encouraging because less effort is required to use circular shear surfaces than to use noncircular shear surfaces. Certainly noncircular shear surfaces may be important where relatively thin layers of soft material exist; however, for homogeneous embankments on firm foundations it appears that circular shear surfaces may generally be used.

CHAPTER FIVE. AN EXAMINATION OF STRAIN AND DEFORMATION LEVELS REQUIRED TO DEVELOP FORCES IN REINFORCEMENT

The theoretical procedures and analyses presented in the preceding chapters are all based on the assumption that a certain force is developed in the reinforcement. The development of this force requires that some deformation take place in the slope. The deformation may occur either during construction or with time after construction is completed. In the present study the major interest in soil reinforcement was for use in low embankments, typically less than 30 feet high, constructed of highly plastic clays, with liquid limits typically of at least 50, and resting on strong, relatively stiff foundations. In such cases relatively little deformation is expected to occur during construction. The soils and slopes of interest typically swell with time after construction is completed. The expansion and loss of strength with time is of particular interest and leads to the requirement for some type of reinforcement to provide adequate stability. In order for the reinforcement to become effective it must be capable of developing sufficient amounts of its resistance as the soil swells to at least partially compensate for the loss of strength. This requires that sufficient deformation develop in the soil and reinforcement, while at the same time the deformation cannot be so large that it causes the slope to become unserviceable.

In order to develop some insight into whether adequate amounts of deformation could occur to develop significant amounts of resistance in the reinforcement and at the same time not cause the slope to become unserviceable a series of finite element computations was performed. The finite element procedures and results are presented in this chapter.

MODELING CONCEPTS

As soil in a reinforced slope gradually expands it will produce deformations in the reinforcement. The deformation which occurs will depend on the inherent tendency of the soil to change volume as well as on the degree of restraint which the surrounding soil provides; the actual volume changes in the soil in the slope will be somewhat less than they would be if the soil were allowed to expand under a constant stress. The expansion of soil in a slope can be related to the manner in which a material in a body, e. g. a structure, will respond to temperature changes. If a given material expands a certain amount due to an increase in temperature at constant stress in an unrestrained condition, it will expand a lesser amount when subjected to the same increase in temperature with some additional restraint, and the stress will increase. The parallel between the strains associated with expansion of a soil and volume changes produced by temperature changes make it possible to model the volume changes due to expansion using existing finite element computer codes designed to simulate the effects of temperature change.

Finite element computations are routinely performed to model the effects of temperature change on stresses and deformations. In such computations the strain which would occur in an unrestrained condition is first specified in terms of a coefficient of thermal expansion, α, and a change in temperature, ΔT. For the three dimensions (x, y and z) the strains will be,

$$\varepsilon_x = \alpha\Delta T \tag{5.1}$$
$$\varepsilon_y = \alpha\Delta T \tag{5.2}$$
$$\varepsilon_z = \alpha\Delta T \tag{5.3}$$

The corresponding volumetric strain, ε_v, will be:

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z \tag{5.4}$$

or,

$$\varepsilon_v = 3\alpha\Delta T \tag{5.5}$$

An "equivalent" set of stresses which would be required to produce these strains is then computed and used to compute a corresponding set of loads (forces) which are applied in the finite element computations. In the case of a linear, elastic material the stresses are

$$\sigma_x = \frac{E\alpha\Delta T}{1 + \nu}\left(1 + \frac{3\nu}{1 - 2\nu}\right) \tag{5.6}$$

$$\sigma_y = \frac{E\alpha\Delta T}{1 + \nu}\left(1 + \frac{3\nu}{1 - 2\nu}\right) \tag{5.7}$$

$$\sigma_z = \frac{E\alpha\Delta T}{1 + \nu}\left(1 + \frac{3\nu}{1 - 2\nu}\right) \tag{5.8}$$

When these stresses are applied and the soil is unrestrained, the computed strains will be identical to those prescribed by the coefficient of thermal expansion and temperature change; if the soil is fully restrained such that no deformation can occur, there will, of course, be no strains, but the stresses will increase by an amount equal to the "equivalent" stresses. This same approach may be used to approximate the effects of volume changes associated with expansion of the soil as follows: The volumetric strains, $\varepsilon_{v\text{-swell}}$, which would occur in the soil at constant stress are first determined. These strains will depend on the soil type, access to moisture and ambient confining pressure. Once the strains associated with the unrestrained swell are determined, they are then related to an equivalent ("pseudo") coefficient of thermal expansion and temperature change such that,

$$\varepsilon_{v\text{-swell}} = 3\alpha\Delta T \tag{5.9}$$

Any pair of values of α and ΔT which produce the desired volume change may be selected. The values of α and ΔT may then be introduced into standard finite element codes designed to compute effects of temperature change on stresses and deformation.

Equations 5.6 through 5.8 are based on the assumption that the soil is free to expand equally in all directions. In the case of a slope the soil is more likely to be restricted to deform under conditions of plane strain with negligible strain along the longitudinal direction of the slope. In the case of plane strain in the z direction ($\varepsilon_z = 0$) the strains in the other two directions (x and y) for the case of constant stress in those directions are given by,

$$\varepsilon_x = -(\nu/E)\Delta\sigma_z + \alpha\Delta T \tag{5.10}$$

and,

$$\varepsilon_y = -(\nu/E)\Delta\sigma_z + \alpha\Delta T \tag{5.11}$$

where, ν is Poisson's ratio, E is Young's modulus, and $\Delta\sigma_z$ is the change in stress in the z (plane strain) direction produced by the temperature changes. The change in stress in the z direction is given by,

$$\Delta\sigma_z = - E\ \alpha\Delta T \tag{5.12}$$

where tension (decreases in stress) are considered to be positive to be consistent with the notation that expansive strains are positive. Substituting 5.12 into 5.10 and 5.11 then gives,

$$\varepsilon_x = \nu\alpha\Delta T + \alpha\Delta T \tag{5.13}$$

and,

$$\varepsilon_y = \nu\alpha\Delta T + \alpha\Delta T \tag{5.14}$$

The volumetric strain for plane strain is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y \tag{5.15}$$

which, upon introducing Eqs. 5.13 and 5.14 gives,

$$\varepsilon_v = 2(1 + \nu)\alpha\Delta T \tag{5.16}$$

Thus, for plane strain the relationship used to compute $\alpha\Delta T$ from the volumetric strain is,

$$\alpha\Delta T = \frac{\varepsilon_{v\text{-swell}}}{2(1 + \nu)} \tag{5.17}$$

Again, any combination of α and ΔT which satisfies Eq. 5.17 will produce the desired volumetric strain under conditions of plane strain in the z direction and constant stress in the x and y directions.

To illustrate how Eq. 5.17 can be applied, consider the following example: Suppose that it is desired to model the response of a soil in a slope where the soil would experience 10 percent volumetric strain under the present stresses if the stresses due to the surrounding soil do not change due to expansion. Also, suppose that a value of Poisson's ratio of 0.30 adequately describes the soil's response to changes in stress. From Eq. 5.17 we then have

$$\alpha \Delta T = \frac{0.01}{2(1 + 0.3)} = 0.0385 \qquad (5.18)$$

This ($\alpha\Delta T$ = 0.0385) could then be represented by α = 0.00385 and ΔT = 10, or α = .385 and ΔT = 0.1, or any other combination of values for α and ΔT which satisfies Eq. 5.18. The values for α and ΔT depend on the value selected for Poisson's ratio, but are independent of the value of Young's modulus.

FINITE ELEMENT COMPUTATIONS

A series of finite element computations was performed using the approach described above to simulate the volume changes associated with swell. Computations were performed for the slope illustrated in Fig. 5.1. The slope is 3(horizontal):1(vertical) and is 25 feet high. Although the height had an effect on the computed displacements, the height would not affect the computed strains. The slope was assumed to be homogeneous and no reinforcement was included. The purpose of the finite element computations was to assess an upper bound on the magnitude of the strain which might occur in the reinforcement; if the reinforcement had been included the computed strains would have been reduced.

The lateral boundary for the finite element mesh was assumed to be fixed against movement in the horizontal direction, while it was free to move in the vertical direction. The location of the lateral boundary was varied until it was observed to have no appreciable effect on the computed strains in the area immediately beneath the face of the slope, which was the area of interest. The

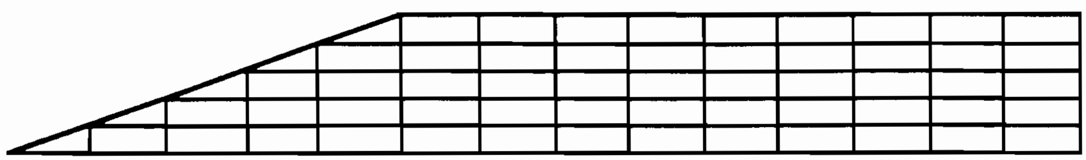

Fig. 5.1. Slope Used in Finite Element Computations to Determine Probable Strain Levels in Reinforcement Due to Soil Expansion.

location of the lateral boundary for the final calculations presented subsequently is shown in Fig. 5.1, a distance of 135 feet back from the crest of the slope.

Finite element computations were performed for two values of Poisson's ratio, 0.30 and 0.45, which are believed to be representative for many soils. Young's modulus was assumed to be constant, which appears to be a reasonable assumption for the depths of interest in slopes constructed of well-compacted, highly plastic clays.

COMPUTED STRAINS

Computed horizontal and vertical strains (ε_x and ε_y) are plotted in Figs. 5.2 and 5.3, respectively, for Poisson's ratio of 0.30, and in Figs. 5.4 and 5.5, respectively, for Poisson's ratio of 0.45. In general the larger the value of Poisson's ratio, the higher the strains, as would be expected.

The strains in the horizontal direction ,ε_x, are of particular interest. They (ε_x) represent the potential strain which would occur in horizontal reinforcement if the soil possessed the tendency to swell 10 percent in an unrestrained condition and the reinforcement provided only nominal resistance to prevent the strain. Referring to Figs. 5.2 (ν = 0.3) and 5.4 (ν = 0.45) it can be seen that the maximum strain does not exceed 2 percent.

DISCUSSION

For any given slope geometry and Poisson's ratio, the strains in the horizontal direction will be directly proportional to the amount of volumetric swell, $\varepsilon_{v\text{-}swell}$, assumed for the unrestrained condition. Thus, if 20 percent volumetric swell had been assumed for the calculations presented above, all of the computed strains (ε_x and ε_y) would have been twice as large. However, regardless of the volumetric strain assumed, within reasonable limits, it appears that the strains in the horizontal direction may be relatively small and deserve special attention when selecting force levels in the reinforcement for design. The small strains, not exceeding 2 percent, which were observed in the finite element computations are substantially below the strain of nearly 15 percent which may be required to fully mobilize the resistance in many synthetic reinforcement materials. Two percent strain is also several times

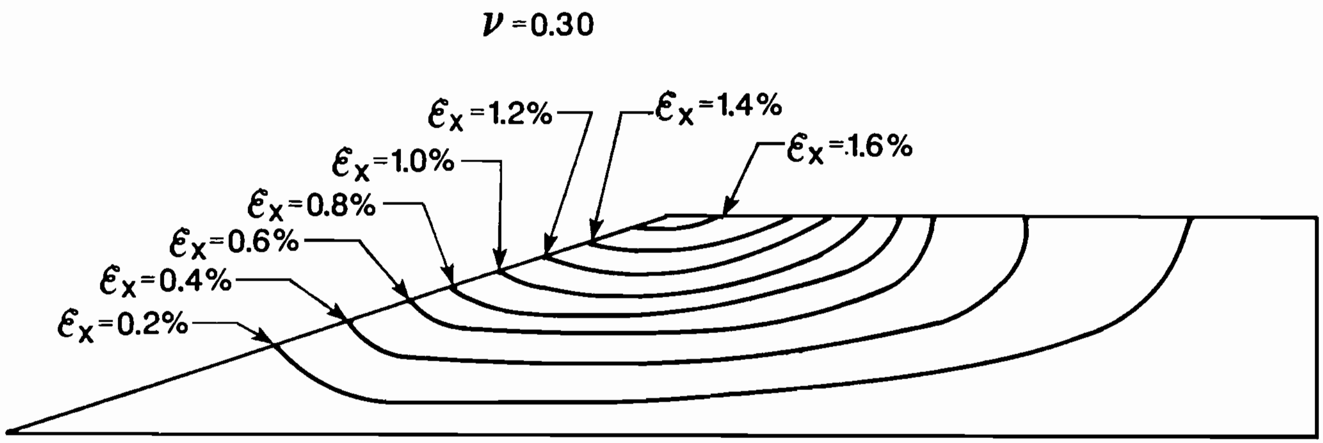

Fig. 5.2. Computed Horizontal Strains Produced by Soil with a Tendency for 10 Percent Volumetric Expansion in an Unrestrained Condition: Poisson's Ratio, ν = 0.3.

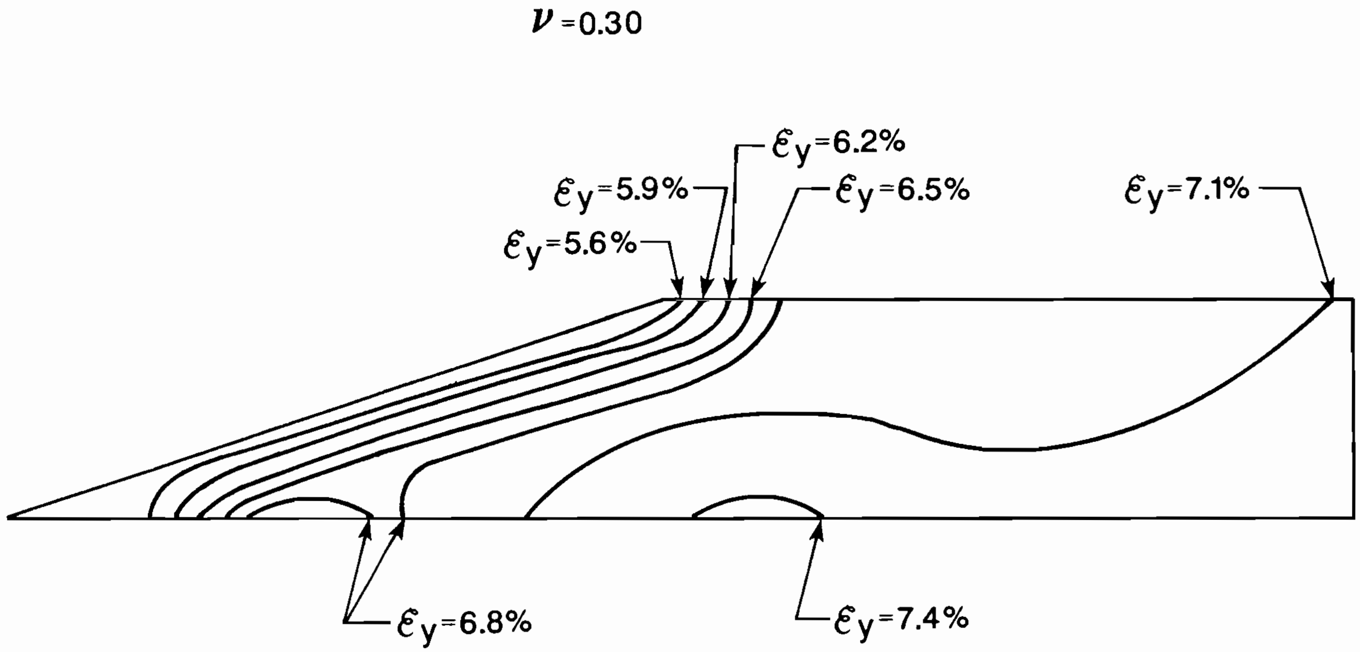

Fig. 5.3. Computed Vertical Strains Produced by Soil with a Tendency for 10 Percent Volumetric Expansion in an Unrestrained Condition: Poisson's Ratio, ν = 0.3.

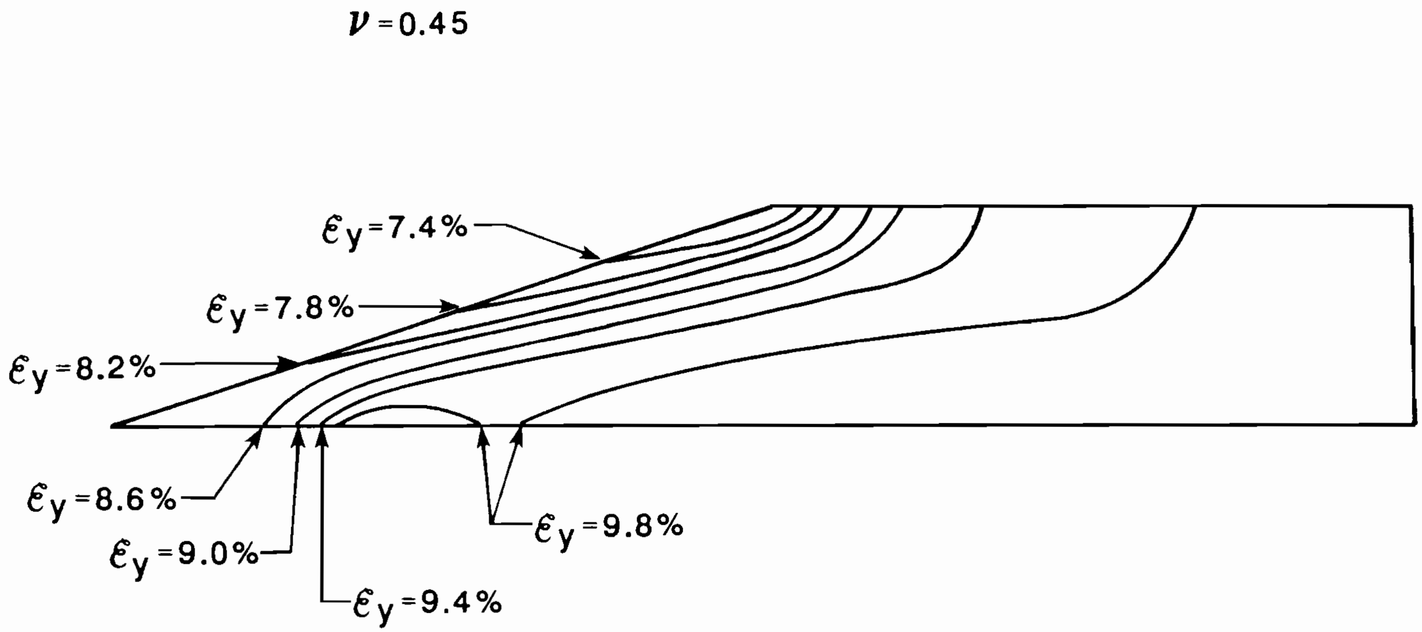

Fig. 5.4. Computed Horizontal Strains Produced by Soil with a Tendency for 10 Percent Volumetric Expansion in an Unrestrained Condition: Poisson's Ratio, ν = 0.45.

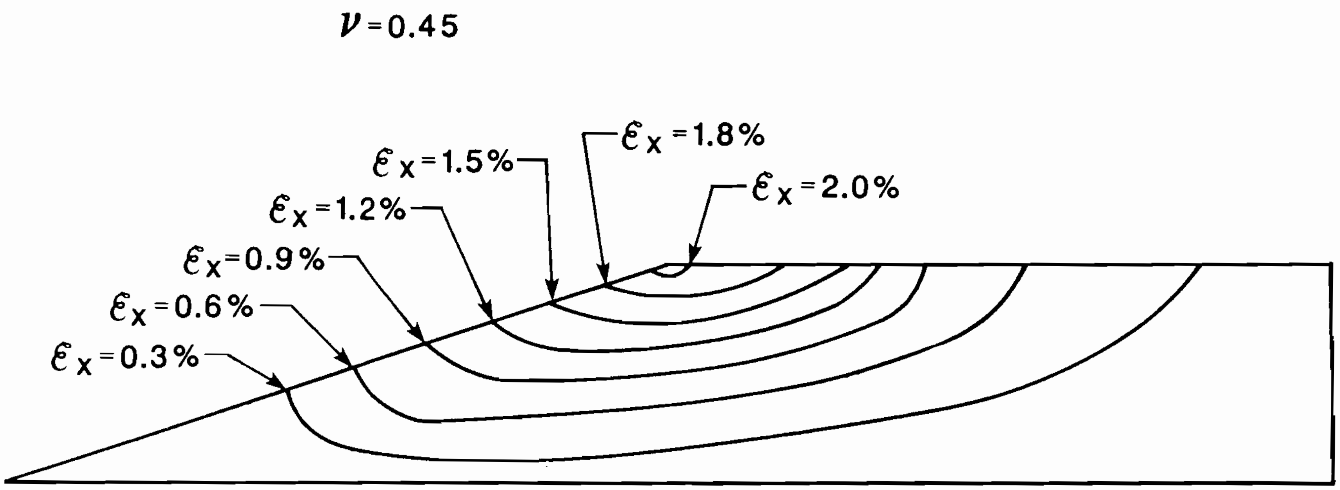

Fig. 5.5. Computed Vertical Strains Produced by Soil with a Tendency for 10 Percent Volumetric Expansion in an Unrestrained Condition: Poisson's Ratio, ν = 0.45.

less than the strain at lower levels of load, below ultimate, when the added effects of creep are included. The computed strains would have been even smaller if the reinforcement had been included in the finite element computations. Such small strains in reinforcement are particularly important in selecting an allowable working load for design of reinforced slopes. It is evident that for at least some types of reinforcement and some types slopes, only a small fraction of the total capacity of the reinforcement may be developed.

The purpose of the present finite computations was to gain some insight into potential strain levels which might develop in a slope as it expands. Additional finite element computations would be useful to examine the effects of other slope geometries on the computed strains and to determine if either significantly more or less favorable conditions may exist depending on slope geometry.

CHAPTER SIX. SUMMARY, CONCLUSION AND RECOMMENDATIONS

SUMMARY

Several approaches have been examined for modeling internal soil reinforcement in stability computations for earth slopes. In Chapter 2 two approaches based on using existing computer programs to approximate the effects of internal reinforcement were examined. One of these approaches involved treating the reinforcement as equivalent layers of material having anisotropic shear strength properties; the other approach involved representing the forces due to the reinforcement as externally applied loads on the surface of the slope. The approach in which the reinforcement was represented by external loads on the surface of the slope was judged to be the superior of the two approaches and produced good agreement with results obtained using the more rigorous procedures developed later in the study; the first approach was judged to be largely unacceptable. However, neither approach allowed variations in force along the length of the reinforcement to be considered, and both approaches represented at least some approximation. Rigorous limit equilibrium slope stability equations were developed in Chapter Three, which more correctly consider the effects of the internal reinforcement. The rigorous equations were incorporated into modifications of the slope stability computer program, UTEXAS2, and documented to permit their routine use for stability calculations of reinforced earth slopes. As actually implemented, the rigorous procedures are actually easier to use than the approximate procedures used earlier, which involved using features of most slope stability computer programs not actually intended for use in modeling internal reinforcement.

Several sets of parametric studies were conducted using the rigorous procedures and the computer program UTEXAS2. Results of these calculations were compared with similar calculations presented by Tensar corporation for reinforced slopes and good agreement was achieved. Also, good agreement was obtained between results using the rigorous procedures and results using the approximate procedures for those cases where the approximate procedures were judged to be reasonably correct based on fundamental equilibrium considerations. For example, in the case of a plane shear surface and a homogeneous slope it can be shown that the rigorous procedure and the approximate procedure, in which the reinforcement is represented by surface loads, should give identical values for the factor of safety. The parametric studies confirmed this observation.

Finite element computations were performed to examine the levels of strain which might develop in a reinforced slope constructed of expansive soil. The elongation strain which would be anticipated in reinforcement was calculated based on an assumed tendency of the soil to swell. For homogeneous linearly elastic conditions, a tendency for the soil to exoand in volume by 10 percent would produce no more than 2 percent elongation strain in the horizontal direction for a typical slope. Similarly a tendency for 20 percent swell would produce no more than 4 percent elongation. These strain levels (2-4 precent) are significantly below the strain at which the ultimate load is developed in at least some types of synthetic geogrids and are also much less than the strains at reduced working ("design") load levels when the additional effects of creep are added.

RECOMMENDATIONS

The rigorous limit equilibrium slope stability equations developed in Chapter 3 are consistent with the fundamental principles of limit equilibrium slope analysis, which have been widely used to compute factors of safety for earth slopes for many years. Accordingly, the procedures and the computer program, UTEXAS2, which incorporates the procedures are recommended for use in design of earth slopes employing soil reinforcement.

The stability computation procedures presented in this report are based on the assumption that a certain level of force will be developed in the soil reinforcement to contribute to the stability of the slope. The development of such forces will depend on the amount of strain in the reinforcement once it is first placed in the slope. Results of finite element computations presented in Chapter 5 suggest that the strains associated with expansion of typical soils may be only a few percent. These computations were based on the assumption that the <u>tendency</u> of the soil to swell was independent of initial confining stress and depth in the slope, and effects of nonlinearities and time were ignored. In addition, strains occurring during construction were ignored because of the strong foundations, stiff fill materials and low slope heights of interest. However, additional computations are needed to very that the strains during construction will be small. Further finite element computations are also recommended to account for effects of confining stress on swell, the effects of time, and the effects of nonlinearities in soil response. Until the results of such computations are available, it is recommended that the design forces used to compute stability be based on elongation strain levels in the reinforcement not exceeding 5 percent, including any effects of creep.

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