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16. Abstract

The primary objective of this study was to develop a mechanistic model to analyze the behavior of continuously reinforced concrete pavement (CRCP) for various material, structural, and environmental conditions. More specifically, we developed a two-dimensional model using the finite element methodology, which incorporates the temperature and moisture variations throughout the concrete depth, and a more realistic bond-slip relationship between concrete and longitudinal steel. Creep effect of concrete and crack spacing prediction are also included.

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# DEVELOPMENT OF A FINITE ELEMENT PROGRAM FOR CONTINUOUSLY REINFORCED CONCRETE PAVEMENTS

by

Seong-Min Kim
Moon C. Won
B. Frank McCullough

**Research Report 1758-S**

Research Project 0-1758
*Development of a Two-Dimensional Analysis Model for Continuously Reinforced Concrete Pavement*

Conducted for the

**Texas Department of Transportation**

in cooperation with the

**U.S. Department of Transportation**
**Federal Highway Administration**

by the

**CENTER FOR TRANSPORTATION RESEARCH**
Bureau of Engineering Research
THE UNIVERSITY OF TEXAS AT AUSTIN

November 1997

## IMPLEMENTATION RECOMMENDATIONS

Efforts have been made to develop a more realistic finite element model to analyze the behavior of continuously reinforced concrete pavement (CRCP) subjected to environmental loads. There are, however, some limitations in the developed finite element model that will need to be addressed. Thus, the implementation recommendations are the following:

1. The developed finite element model considers neither the warping effect nor the modeling of all underlying layers. This should be addressed in follow-up research.
2. More realistic modeling for creep effect also needs to be further studied.
3. Finally, to identify reasonable input values for the bond-slip relations and the creep effect, further investigation should also be undertaken.

## DISCLAIMERS



B. Frank McCullough, P.E. (Texas No. 19914)
*Research Supervisor*

## ACKNOWLEDGMENTS

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Research performed in cooperation with the Texas Department of Transportation and the U.S. Department of Transportation, Federal Highway Administration.

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# CHAPTER 1. INTRODUCTION

## 1.1 BACKGROUND AND OBJECTIVES

Continuously reinforced concrete pavement (CRCP), if designed and constructed properly, provides a durable pavement requiring little maintenance. Because of its durable nature, CRCP has been used primarily in urban areas where traffic is heavy and delays resulting from maintenance-related activities generate higher user costs. The Texas Department of Transportation (TxDOT) has built many kilometers of CRCP, especially in the urban areas of Houston and Dallas. While the performance of CRCP in Texas has been excellent, far exceeding its design life, localized distresses have nonetheless been observed.

The effectiveness of CRCP is ensured by, among other factors, tight crack widths and adequate transverse crack spacings, which are determined by design, materials, construction, and environmental variables. To ensure tight crack widths and adequate crack spacings, the effect of each variable and the interactions among these variables need to be investigated, so that optimum combinations of design, materials, and construction techniques can be determined. Mechanistic modeling provides a useful tool toward this end.

The first attempt to model CRCP behavior was made in the mid-seventies at the Center for Transportation Research of The University of Texas at Austin, under a study sponsored by the National Cooperative Highway Research Program (NCHRP). Since then, the model has been modified several times (Refs 1, 2, 3). The model has provided researchers with insights into how each of the design, materials, construction, and environmental variables affects CRCP behavior in terms of crack development. This insight, along with field observations, has resulted in improvements in TxDOT's CRCP design standards.

The main objective of this study is to develop a mechanistic model to analyze the behavior of CRCP for various material, structural, and environmental conditions. More specifically, we developed a two-dimensional model using a finite element methodology that incorporates the temperature and moisture variations throughout the concrete depth, as well as a more realistic bond-slip relationship between concrete and longitudinal steel. The creep effect of concrete and the crack spacing prediction are also included.

## 1.2 ORGANIZATION

This report consists of seven parts. The background and objectives are presented in Chapter 1. The finite element formulations and other mathematical materials are explained in Chapter 2. The finite element model developed in this study is also presented. In Chapter 3, preliminary studies are conducted with the developed model. The sensitivity of the input values is also explained. Parametric studies are performed in Chapter 4, where the results obtained from three different bond-slip models are compared. In Chapter 5, the main and subroutine programs are explained, with the input and output guides also presented. The summary and conclusions of this study and recommendations for further research are included in Chapter 6. Sample input and output, as well as the program list, are provided in the appendices.

# CHAPTER 2. FINITE ELEMENT MODELING OF CRCP

## 2.1 INTRODUCTION

This chapter describes the finite element formulations used to model CRCP. The concrete pavement layer is modeled using plane strain elements. The longitudinal steel is discretized using frame elements, so that both the bending resistance and the axial deformations can be considered. The bond-slip relation between concrete and longitudinal steel is modeled using various concepts of bond stress-slip relations. Complete modeling of underlying layers is not considered, on the assumption that there are vertical equivalent springs for the underlying layers. The horizontal stiffness of the underlying layers is assumed to be infinite. The frictional bond stress-slip relation between the concrete and base layers is modeled using linear elastic and perfect plastic springs. The geometric symmetry can be used because the pavement system and environmental loads can be assumed to be symmetric with respect to the center of the cracks. The horizontal degrees of freedom are restrained at the center of the cracks. At cracks, there is no restraint for concrete and no horizontal and rotational degrees of freedom for longitudinal steel. The environmental loads generated by the variations of temperature and drying shrinkage through the depth of the concrete layer are transformed to the equivalent nodal loads to calculate the displacements. The stresses are finally calculated using the mechanical and initial strains. The length of the pavement (crack spacing) considered first is selected to be the distance between the primary cracks. If the stresses of concrete at the center of the cracks are larger than the strength of concrete, there will be a new crack and the analysis is performed again with a reduced crack spacing (half of the former crack spacing). The creep effect is considered using the effective modulus method, whereby the modulus of concrete decreases as the time of consideration increases.

## 2.2 MODELING OF CONCRETE LAYER

The concrete pavement layer is discretized using plane strain elements. The shape functions for a four-noded bilinear rectangular element shown in Figure 2.1 can be written as

$$
\begin{aligned}
f_1 &= \frac{1}{4}(1-\xi)(1+\eta) \\
f_2 &= \frac{1}{4}(1-\xi)(1-\eta) \\
f_3 &= \frac{1}{4}(1+\xi)(1+\eta) \\
f_4 &= \frac{1}{4}(1+\xi)(1-\eta)
\end{aligned}
\tag{2.1}
$$

or

$$f_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \tag{2.2}$$

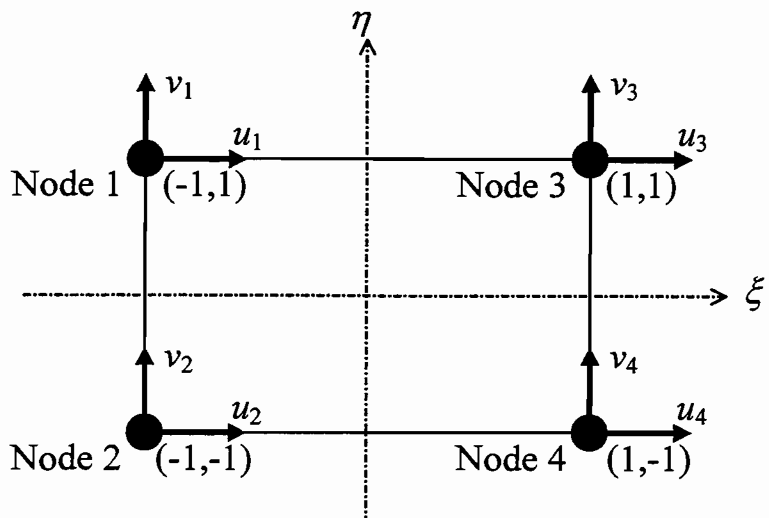



*Figure 2.1. Four-noded bilinear rectangular element*

where $\xi_i$ and $\eta_i$ are the coordinates at node *i*. The displacements *u* and *v* in the two principal directions can be written as

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} f_1 & 0 & f_2 & 0 & f_3 & 0 & f_4 & 0 \\ 0 & f_1 & 0 & f_2 & 0 & f_3 & 0 & f_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \mathbf{N}^{\mathrm{T}}\mathbf{U} \tag{2.3}$$

If the side lengths of a rectangular element in the $\xi$ and $\eta$ directions are *a* and *b*, respectively, strains can be determined by Eq. 2.4 (next page).

$$\begin{Bmatrix} \varepsilon_\xi \\ \varepsilon_\eta \\ \gamma_{\xi\eta} \end{Bmatrix} = \begin{bmatrix} \frac{2}{a}\frac{\partial f_1}{\partial \xi} & 0 & \frac{2}{a}\frac{\partial f_2}{\partial \xi} & 0 & \frac{2}{a}\frac{\partial f_3}{\partial \xi} & 0 & \frac{2}{a}\frac{\partial f_4}{\partial \xi} & 0 \\ 0 & \frac{2}{b}\frac{\partial f_1}{\partial \eta} & 0 & \frac{2}{b}\frac{\partial f_2}{\partial \eta} & 0 & \frac{2}{b}\frac{\partial f_3}{\partial \eta} & 0 & \frac{2}{b}\frac{\partial f_4}{\partial \eta} \\ \frac{2}{b}\frac{\partial f_1}{\partial \eta} & \frac{2}{a}\frac{\partial f_1}{\partial \xi} & \frac{2}{b}\frac{\partial f_2}{\partial \eta} & \frac{2}{a}\frac{\partial f_2}{\partial \xi} & \frac{2}{b}\frac{\partial f_3}{\partial \eta} & \frac{2}{a}\frac{\partial f_3}{\partial \xi} & \frac{2}{b}\frac{\partial f_4}{\partial \eta} & \frac{2}{a}\frac{\partial f_4}{\partial \xi} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \mathbf{BU} \qquad (2.4)$$

Finally, the element stiffness matrix can be obtained by

$$\mathbf{K} = \frac{ab}{4} t_w \int_{-1}^{1} \int_{-1}^{1} \mathbf{B}^{\mathrm{T}} \mathbf{D} \mathbf{B} \, d\xi d\eta \qquad (2.5)$$

where $\mathbf{D}$ is a material property matrix and $t_w$ is a thickness of the plane element. For CRCP, the thickness $t$ can be selected as the distance between longitudinal steel bars. For an isotropic material, if the problem is plane strain, the material property matrix $\mathbf{D}$ can be written as

$$\mathbf{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \qquad (2.6)$$

where $E$ is Young's modulus of elasticity and $\nu$ is Poisson's ratio.

## 2.3 MODELING OF LONGITUDINAL STEEL

The longitudinal steels are discretized using the frame elements shown in Figure 2.2.

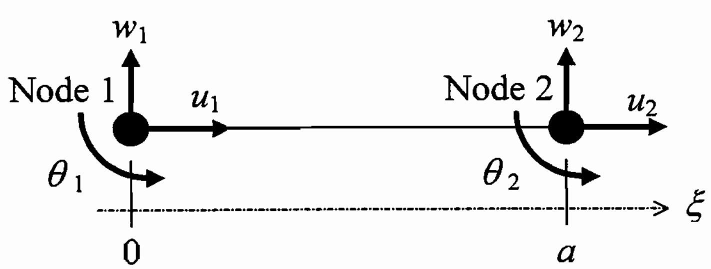



*Figure 2.2. Two-noded frame element*

If a length of an element is $a$, and a cross-sectional area ($A$), moment of inertia ($I$), and Young's modulus of elasticity ($E$) are constant in the element, the shape functions can be written as

$$
\begin{aligned}
f_1 &= 1 - \frac{\xi}{a} \\
f_2 &= 1 - \frac{3\xi^2}{a^2} + \frac{2\xi^3}{a^3} \\
f_3 &= \xi - \frac{2\xi^2}{a} + \frac{\xi^3}{a^2} \\
f_4 &= \frac{\xi}{a} \\
f_5 &= \frac{3\xi^2}{a^2} - \frac{2\xi^3}{a^3} \\
f_6 &= -\frac{\xi^2}{a} + \frac{\xi^3}{a^2}
\end{aligned}
\tag{2.7}
$$

The displacements can be written as

$$
\{u\} = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \end{bmatrix} \begin{Bmatrix} u_1 \\ w_1 \\ \theta_1 \\ u_2 \\ w_2 \\ \theta_2 \end{Bmatrix} = \mathbf{N}^{\mathbf{T}}\mathbf{U} \tag{2.8}
$$

and the strain in the $\xi$ direction and the curvature ($\kappa$) can be obtained by

$$
\begin{Bmatrix} \varepsilon_\xi \\ \kappa \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial \xi} & 0 & 0 & \dfrac{\partial f_4}{\partial \xi} & 0 & 0 \\ 0 & \dfrac{\partial^2 f_2}{\partial \xi^2} & \dfrac{\partial^2 f_3}{\partial \xi^2} & 0 & \dfrac{\partial^2 f_5}{\partial \xi^2} & \dfrac{\partial^2 f_6}{\partial \xi^2} \end{bmatrix} \begin{Bmatrix} u_1 \\ w_1 \\ \theta_1 \\ u_2 \\ w_2 \\ \theta_2 \end{Bmatrix} = \mathbf{BU} \tag{2.9}
$$

Finally, the element stiffness matrix can be obtained by (see top of next page)

$$\mathbf{K} = \int_0^a \mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\, d\xi = \frac{E}{a^3}\begin{bmatrix} Aa^2 & 0 & 0 & -Aa^2 & 0 & 0 \\ 0 & 12I & 6aI & 0 & -12I & 6aI \\ 0 & 6aI & 4a^2I & 0 & -6aI & 2a^2I \\ -Aa^2 & 0 & 0 & Aa^2 & 0 & 0 \\ 0 & -12I & -6aI & 0 & 12I & -6aI \\ 0 & 6aI & 2a^2I & 0 & -6aI & 4a^2I \end{bmatrix} \tag{2.10}$$

It should be noted that the vertical degrees of freedom on the longitudinal steel are the same vertical degrees of freedom on the nodes of concrete at contact faces.

An alternative to modeling for the longitudinal steel would be the use of only one-dimensional truss elements. The differences in the results when using the frame and the truss elements for the longitudinal steel are investigated in the next chapter.

## 2.4 MODELING OF BOND-SLIP BETWEEN CONCRETE AND LONGITUDINAL STEEL

If there is a relative displacement at the interface between concrete and longitudinal steel, the bond stress exists and the relative displacement is called *bond slip*. When there is no relative displacement at the interface, it is called *perfect bond*. To account for the effect of bond stress and bond slip, spring elements are used between the nodes at contact positions in the longitudinal direction, as shown in Figure 2.3.

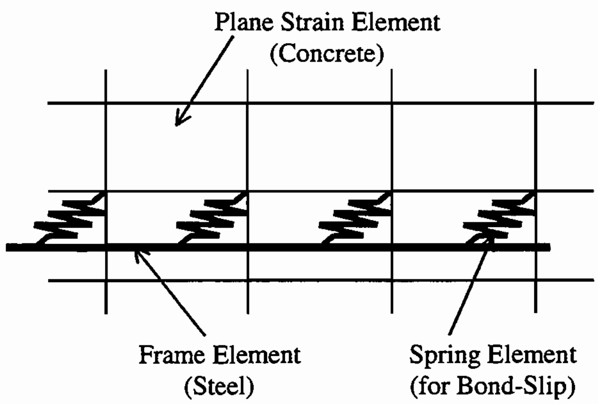



*Figure 2.3. Spring elements for effect of bond between concrete and steel*

The springs are horizontal springs and the stiffness of a spring can be obtained by multiplying the bond stress generated by the bond slip by the contact area. The contact area can be defined as

$$A_{contact} = \pi\ d\ (\frac{a_L}{2} + \frac{a_R}{2}) \tag{2.11}$$

where $d$ is a steel diameter, and $a_L$ and $a_R$ are the horizontal lengths of the elements on the left and right sides of a contact node, respectively. Thus, the stiffness of a spring can be obtained by

$$K_{spring} = k_{bond}\ A_{contact} \tag{2.12}$$

where $k_{bond}$ is the bond stiffness per unit area, which is a slope of the curve representing the relation between the bond stress and the bond slip. Various relations between bond stress and bond slip are considered, as shown in Figure 2.4. If perfect bond is considered, the two contact nodes linked by a spring element should be the same node, with the spring element not needed .Perfect bond can also be modeled using a large stiffness of the spring element in practice.

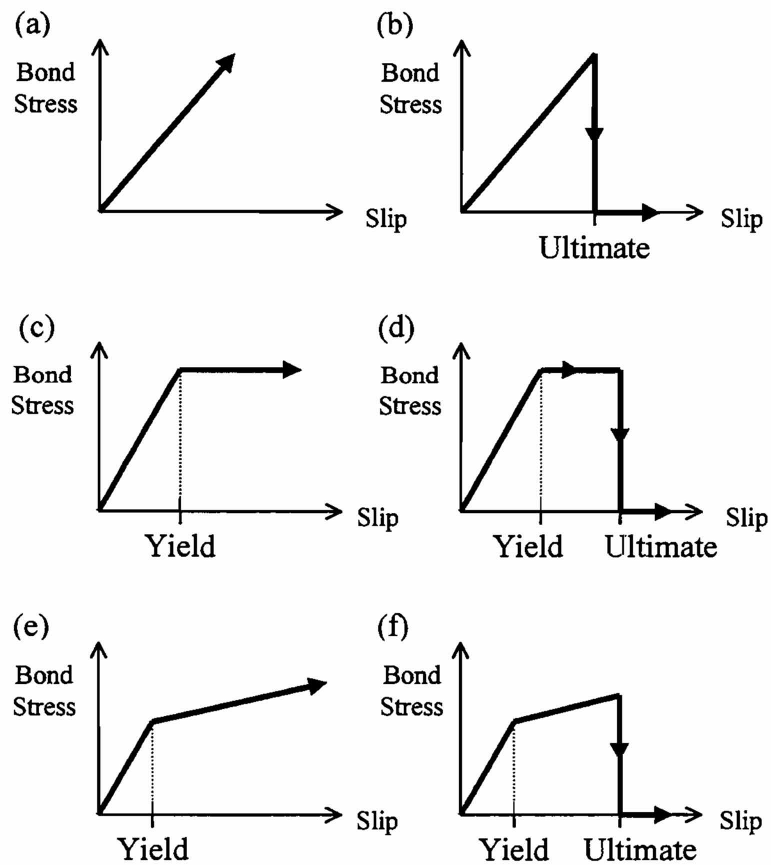



*Figure 2.4. Various models for bond stress-slip relation*

## 2.5 MODELING OF UNDERLYING LAYERS

Under the concrete layers there are various other layers, including the base, subbase, subgrade, and bedrock. Although all layers can be modeled using finite elements, computer memory and run time will be very large in such cases. An alternative would be to use equivalent springs for underlying layers. The stiffness of the equivalent spring would be a function of the depth and material properties of the layers as well as the pressure area if there is an external load. The vertical equivalent spring stiffness per unit area used in this study is 0.1085 MPa/mm (400 psi/in), a value that is a typical stiffness obtained from the field data. The horizontal stiffness of the base layer is assumed to be infinite. At the interface of the concrete layer and the base layer, there are frictional slips in the horizontal direction. The frictional bond-slip is modeled using spring elements, which is explained in the next section.

## 2.6 MODELING OF BOND-SLIP BETWEEN CONCRETE AND BASE LAYER

There are frictional stresses in the horizontal direction at the interface between the concrete layer and the base layer. The frictional bond-slip between the two layers can be modeled using spring elements, as was done for the bond-slip between concrete and steel. The relation between the bond stress and the slip is assumed to be linear up to a certain slip (yield slip) and constant after the slip, as shown in Figure 2.5. The spring stiffness can be obtained by Eq. 2.12, with the contact area as

$$A_{contact} = t_w\left(\frac{a_L}{2} + \frac{a_R}{2}\right) \tag{2.13}$$

where $a_L$ and $a_R$ are the horizontal lengths of the elements at the left and right sides of a contact node, and $t_w$ is the thickness of the plane element.

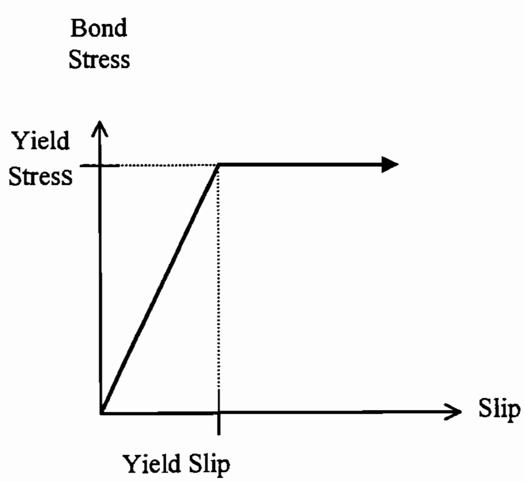



*Figure 2.5. Bond stress-slip relation between concrete and base layers*

## 2.7 BOUNDARY CONDITIONS

Geometric symmetry can be used because the pavement and the environmental loads are symmetric with respect to the centerline of the cracks. At cracks, there are no restraints for concrete and no degrees of freedom in the horizontal and rotational directions for longitudinal steel. At the center of the cracks, vertical degrees of freedom exist and the horizontal displacements and the rotational degree of freedom for steel are restrained. Figure 2.6 shows the boundary conditions of the finite element model.

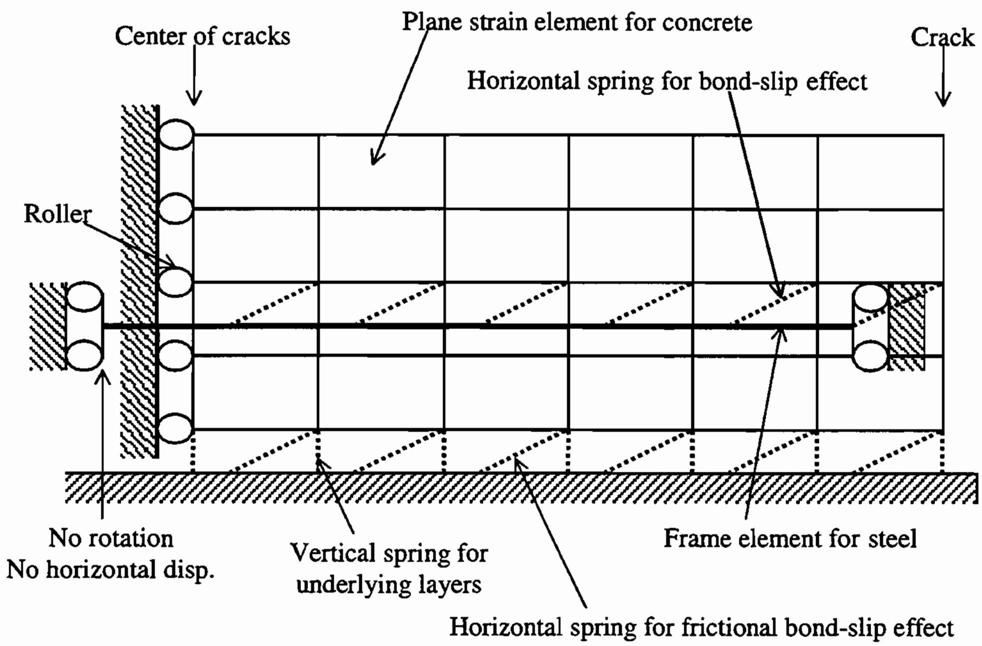



*Figure 2.6. Finite element model of CRCP*

## 2.8 EQUIVALENT LOADS FROM CHANGES IN TEMPERATURE AND DRYING SHRINKAGE

When there are initial strains, the equilibrium equation can be written as

$$\mathbf{F} = \int_V \mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\, dV\, \mathbf{U} - \int_V \mathbf{B}^{\mathrm{T}}\mathbf{D}\varepsilon_0 dV \tag{2.14}$$

where **F** is an external load vector and $\varepsilon_0$ is an initial strain vector. If there are no external loads, **F** is zero and Eq. 2.14 can be rewritten as

$$\int_V \mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\, dV\, \mathbf{U} = \int_V \mathbf{B}^{\mathrm{T}}\mathbf{D}\varepsilon_0 dV \tag{2.15}$$

The right-side term of Eq. 2.15 is the new load vector that makes the displacements. Total loads can be obtained by assembling all element loads. The initial strain vector resulting from temperature change for the plane strain problem can be written as

$$\begin{Bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{Bmatrix} = (1+\nu)\begin{Bmatrix} \alpha\ \Delta T \\ \alpha\ \Delta T \\ 0 \end{Bmatrix} \tag{2.16}$$

where $\nu$ is Poisson's ratio, $\alpha$ is the expansion coefficient, and $\Delta T$ is the change in temperature from the reference temperature.

If there are strains caused by drying shrinkage, the initial strain vector can be rewritten as

$$\begin{Bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{Bmatrix} = (1+\nu)\begin{Bmatrix} \alpha\ \Delta T + \varepsilon_{sh} \\ \alpha\ \Delta T + \varepsilon_{sh} \\ 0 \end{Bmatrix} \tag{2.17}$$

where $\varepsilon_{sh}$ is the drying shrinkage strain (negative sign for shrinkage).

When elements with low-order fields, such as linear or bilinear elements, are used, there can be errors in stresses when the interpolated temperatures from the nodes are used (Ref 4). An alternative is to use the uniform temperature for each element as the average temperature of the node temperatures. The differences in the results from the two methods will be given in the next chapter.

## 2.9 STRESS CALCULATION

If there are initial strains caused by temperature change and drying shrinkage, the stress vector can be obtained by

$$\sigma = \mathbf{D}(\varepsilon - \varepsilon_0) = \mathbf{D}(\mathbf{BU} - \varepsilon_0) \tag{2.18}$$

where $\varepsilon$ is the mechanical strain vector induced by the equivalent nodal loads and $\varepsilon_0$ is the initial strain vector due to temperature change and drying shrinkage. The stresses and strains are calculated at the integration points and the average values are used for each element in this study.

## 2.10 MODELING FOR CREEP EFFECT

The temperature drop does not occur at an instant but over some duration of time. Owing to slow changes in temperature and drying shrinkage, the creep of strain and the relaxation of stress may occur. There are many concepts used to analyze the creep effect for concrete (Ref 5). Some of them are the effective modulus method (Ref 6), the rate of creep method (Refs 7, 8), the rate of flow method (Ref 9), the improved Dischinger method (Ref 10), the principle of superposition of virgin creep curves (Refs 11, 12), and the Trost-Bazant method (Refs 13, 14). It is difficult to say which is the best method, since each method has its merits and drawbacks. In this study, the effective modulus method is used.

In the effective modulus method, the creep of concrete is accounted for by reducing Young's modulus of elasticity of concrete as

$$E_{eff}(t) = \frac{E(t_0)}{1+\phi(t,t_0)} \tag{2.19}$$

where $t$ is time of consideration, $t_0$ is the time at first application of load, $E(t_0)$ is modulus of elasticity at time $t_0$, and $\phi(t,t_0)$ is the creep coefficient at time $t$ for concrete loaded at time $t_0$. Since the creep at any given time depends on the current value of stress, and since the stress history is not considered, this method is used in the elastic analysis.

Assumptions are necessary when the creep effect is considered with the method. The creep strain is assumed to vary linearly with stress and to be independent of temperature. Actually, the creep coefficient $\phi(t,t_0)$ varies with temperature (Ref 5) and it will be significant for such viscoelastic materials as asphalt mixtures, though the creep coefficient $\phi(t,t_0)$ of concrete is assumed to be constant within the temperature changes in the field. The real modulus of concrete considered in the effective modulus method is the modulus at first application of load, $E(t_0)$. Therefore, it is assumed that the actual concrete modulus within the time of consideration is constant and the same as $E(t_0)$.

The creep coefficient used in this study is defined by

$$\phi(t,t_0) = \phi_{\max}[1-(1-\phi_x)^{\frac{t}{t_x}}] \tag{2.20}$$

where $\phi_{max}$ is the maximum creep ratio to the instantaneous elastic strain, $\phi_x$ is the ratio to $\phi_{max}$ to define a point on the creep curve, and $t_x$ is time corresponding to $\phi_x$. Figure 2.7 shows the curves of creep coefficient when $\phi_{max} = 1$ and $\phi_x = 0.9$.

## 2.11 DESCRIPTION OF TOTAL SYSTEM

CRCP is modeled using plane strain, frame, and spring elements. The concrete pavement layer is discretized using plane strain elements, with the total length being the distance between two cracks and with the thickness being the distance between longitudinal steels. The longitudinal steels are discretized using frame elements.

The spring elements are used to model the bond-slip effects between concrete and steel and between the concrete layer and the base. The stiffness of the underlying layers is also modeled using vertical springs. Figure 2.8 illustrates the concepts of modeling.

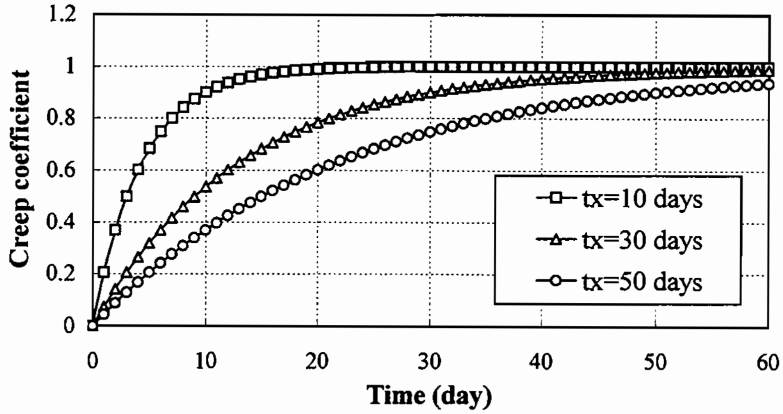



*Figure 2.7. Creep coefficient when $\phi_{max}$ =1 and $\phi_x$ =0.9*

In this study, the Gaussian elimination equation solver is used for a symmetric and nonsingular coefficient matrix stored in the band form (Ref 4). The element numbering is very important with this solver because the numbering governs the half-bandwidth that determines the computer memory. Figure 2.9 shows the numbering of elements and the degrees of freedom. With the numbering, the half-bandwidth can be obtained by

$$\text{Half-bandwidth} = (\text{NUMELY} + 3) \times 2 + 4 \tag{2.21}$$

where NUMELY is the number of four-node plane strain elements in the vertical direction.

The crack spacing prediction is included in the program. First, the primary crack spacing is given by a user. The program calculates the stresses through the depth at the center of the cracks. If any stress at the center of the cracks exceeds the tensile strength of concrete, it is assumed that there is another crack at the center of the two cracks and that the new crack spacing is half of the primary crack spacing. With the new crack spacing, the stresses are calculated again and those are compared with the tensile strength. This procedure continues until the stresses are less than the tensile strength. The tensile strength of concrete is determined in this study as the split-cylinder strength in psi by (Ref 15)

$$f'_{sp} = 7\sqrt{f'_c} = 7\frac{E}{33w^{\frac{3}{2}}} \tag{2.22}$$

where $f'_c$ is the compressive cylinder strength of concrete, $E$ is Young's modulus of concrete in psi, and $w$ is the unit weight of the hardened concrete in pcf.

The algorithm of the program is shown in Figure 2.10 and the descriptions of each subroutine and input guide are provided in Chapter 5.

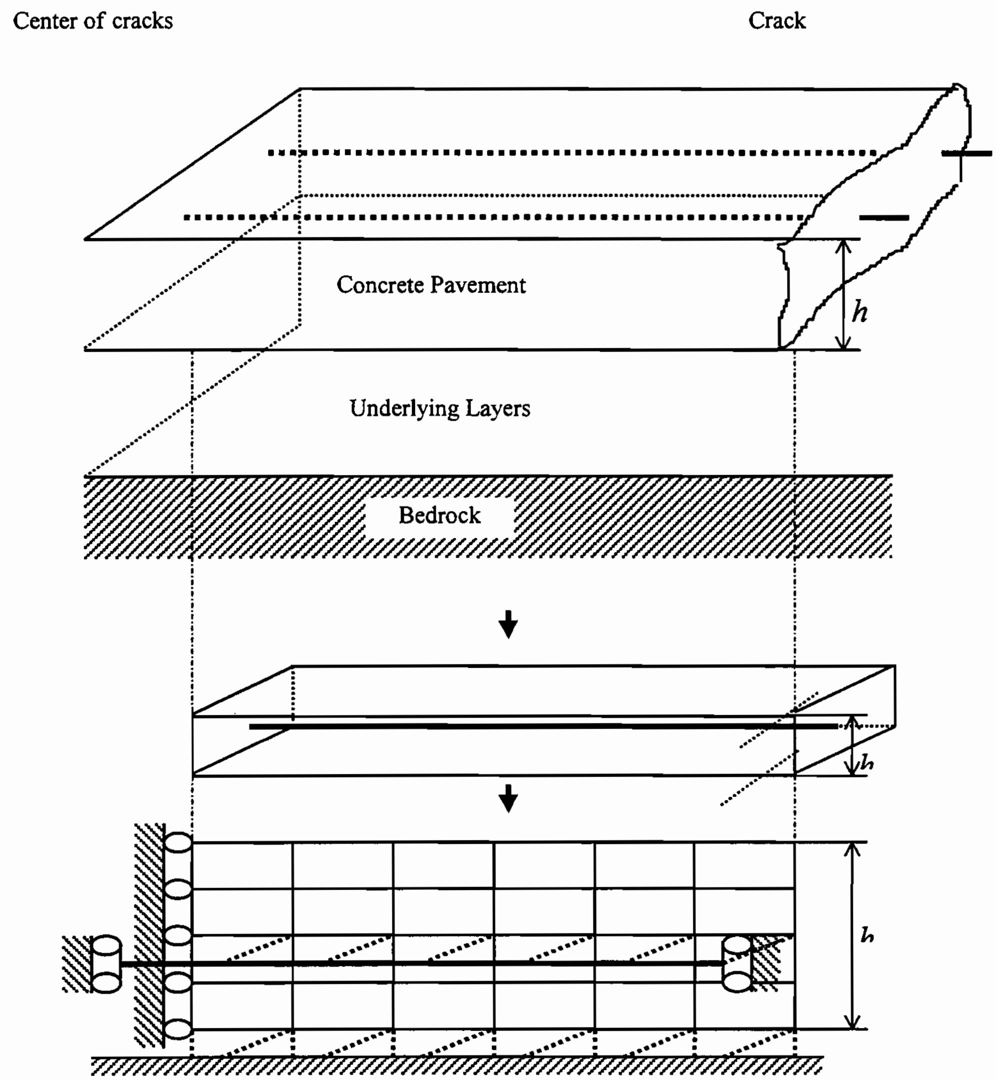



*Figure 2.8. Modeling of CRCP*

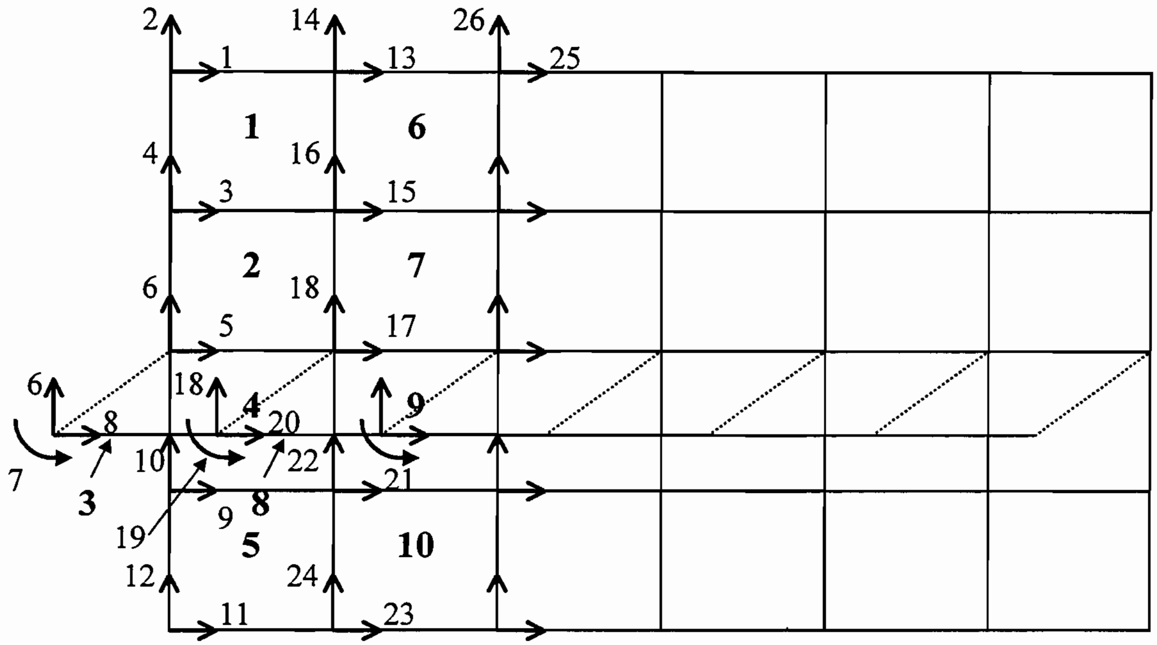



*Figure 2.9. Numbering of elements and degrees of freedom*

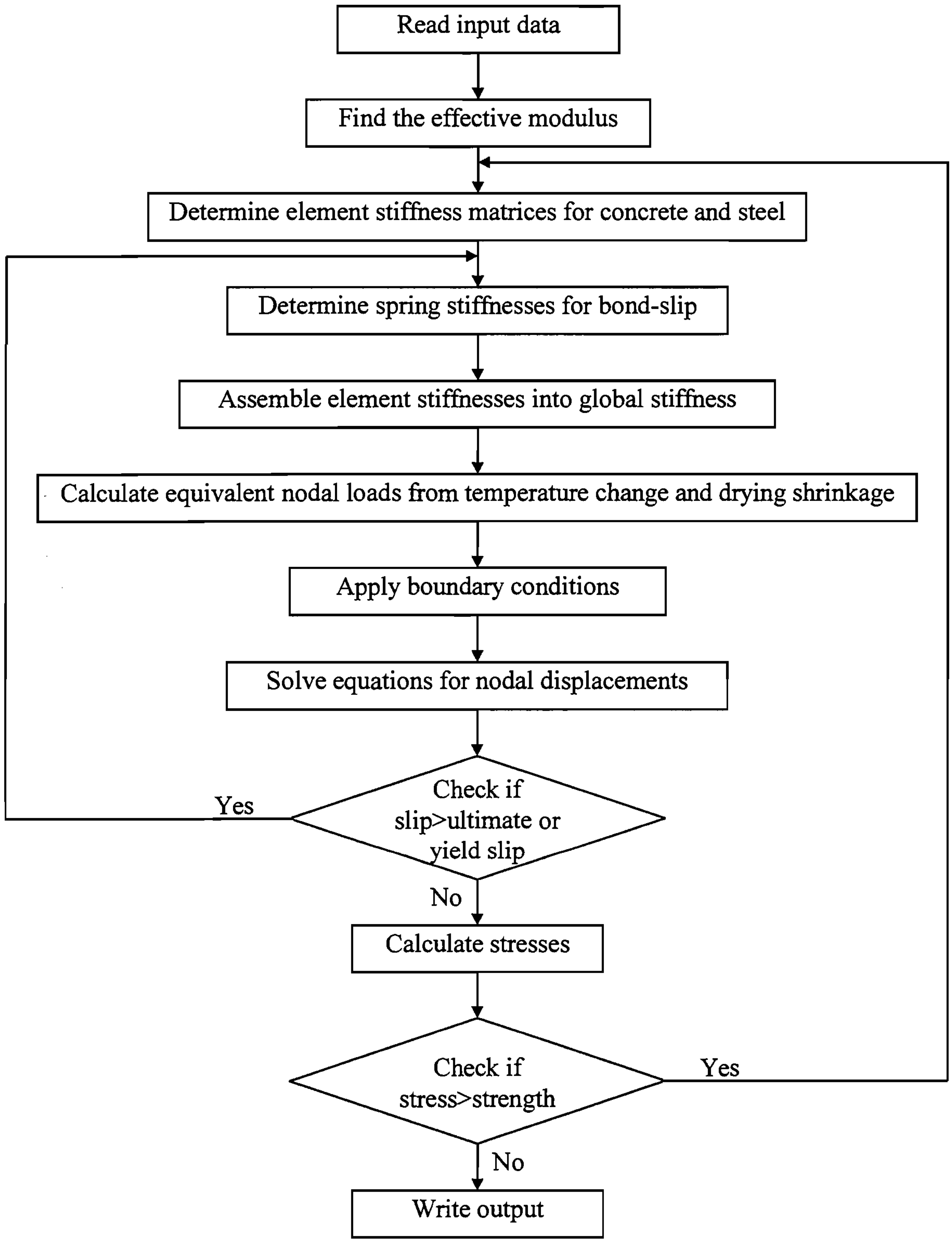



*Figure 2.10. Algorithm of the program*

# CHAPTER 3. PRELIMINARY STUDIES WITH A FINITE ELEMENT MODEL

## 3.1 INTRODUCTION

Preliminary studies were conducted using the developed finite element program. First, a convergence study was performed. The differences in the results when using different temperature definitions in the elements and when using different elements for the longitudinal steel were also compared. The effects of input values for various bond-slip relationships and those for the creep definitions were investigated next. The geometry and material properties used for this study are listed in Table 3.1. The temperatures in the concrete layer are assumed to vary linearly from the surface to the bottom of the slab. The drying shrinkage strains of concrete are also assumed to vary linearly throughout the slab depth.

## 3.2 CONVERGENCE STUDY

The convergence test was performed on the crack width with square plane strain elements assuming a linear bond-slip relationship between concrete and longitudinal steel. Figure 3.1 shows that crack widths converge with the number of elements more than 700, which corresponds to the element size of 25.4 mm by 25.4 mm (1.00 in. by 1.00 in.).

The equivalent nodal loads caused by changes in temperature can be determined two different ways. One is to use a uniform temperature for each element as an average of the node temperatures. The other is to use the node temperatures, and this is the linear temperature variation in an element in this study. It is noted that the use of the interpolated temperatures to find the strains and stresses with the elements with low-order fields can generate errors, and the use of a uniform temperature in each element can reduce such errors (Ref 4). As shown in Figure 3.2, the differences are very clear with a small number of elements, but those differences disappear as the number of elements increase.

*Table 3.1. Geometry and material properties of the CRCP model*

| | | | |
|---|---|---|---|
| Crack spacing | 3.048 m (10 ft) | Drying shrinkage strain | 0.00002 at surface<br>0 at bottom |
| Distance between longitudinal steels | 15.24 cm (6 in.) | Vertical stiffness of underlying layers | 0.1085 MPa/mm (400 psi/in.) |
| Concrete slab thickness | 30.48 cm (12 in.) | Bond-slip stiffness between conc. & steel | 190 MPa/mm (700000 psi/in.) |
| Steel location from surface | 15.24 cm (6 in.) | Second bond-slip stiffness | 19 MPa/mm (70000 psi/in.) |
| Concrete modulus | 13780 MPa (2000000 psi) | Yield slip between concrete and steel | 0.0254 mm (0.001 in.) |
| Poisson's ratio | 0.15 | Ultimate slip between concrete and steel | 0.1016 mm (0.004 in.) |

*Table 3.1.(continued) Geometry and material properties of the CRCP model*

| | | | |
|---|---|---|---|
| Steel diameter | 19.05 mm (0.75 in.) | Bond-slip stiffness between conc. & base | 0.0407 MPa/mm (150 psi/in.) |
| Expansion coefficient of concrete | 0.0000108/°C (0.000006/°F) | Yield slip between concrete and base | 0.508 mm (0.02 in.) |
| Expansion coefficient of steel | 0.000009/°C (0.000005/°F) | Maximum creep ratio | 2.0 |
| Surface temperature | 29.4 °C (85 °F) | Load duration | 12 hr |
| Bottom temperature | 37.8 °C (100 °F) | $\phi_r$ | 0.99 |
| Reference temperature | 48.9 °C (120 °F) | $t_s$ | 30 days |

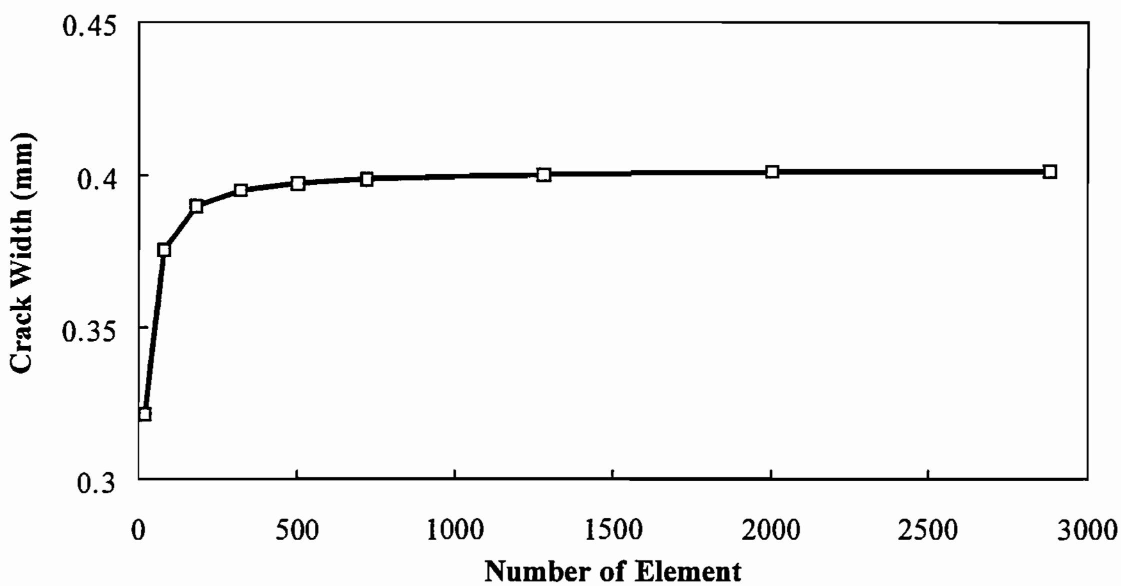



*Figure 3.1. Convergence test on crack width*

The longitudinal steel is modeled using the frame elements in this study. An alternative would be to use the truss elements for the longitudinal steel. When using the truss elements, the bending resistance of the steel is neglected and only the axial degrees of freedom are considered. As shown in Figure 3.3, there are minor differences in the convergence values. The crack widths with the frame elements are smaller than those with the truss elements. The difference in the convergence values in this case is about 1%.

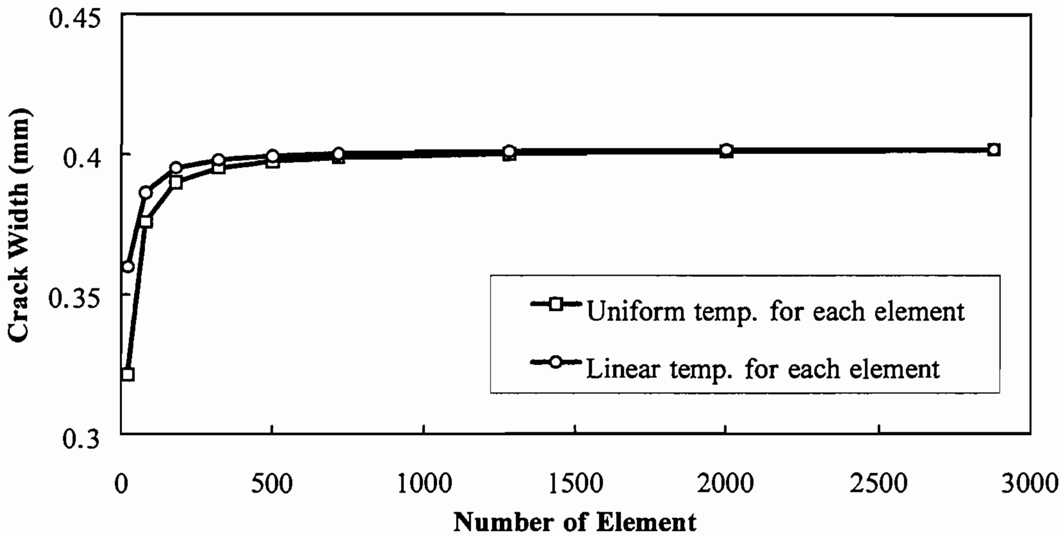



*Figure 3.2. Convergence test with different temperature definitions*

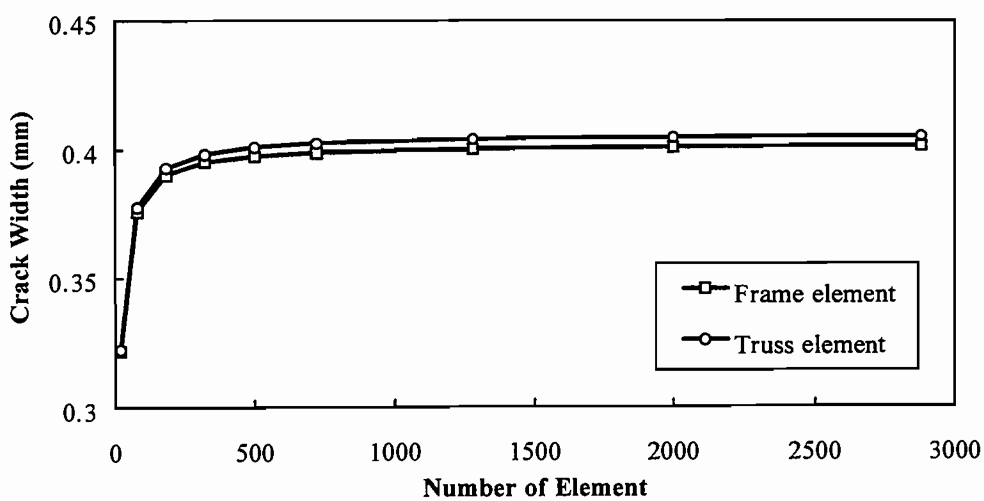



*Figure 3.3. Convergence test with different element types for longitudinal steel*

### 3.3 SENSITIVITY OF VARIABLES FOR BOND-SLIP

Four variables need to be determined to define the bond-slip relationship in this study. They are the primary and secondary bond-slip stiffnesses per unit area, yield slip, and ultimate slip. In order to evaluate the effect of the primary bond-slip stiffness on the crack width and stresses in concrete and steel, a sensitivity analysis was conducted assuming a linear relationship between bond slip and stress without yield slip. The results are shown in Figure 3.4. The three dependent variables — crack width, concrete stress at the surface of the mid-slab, and steel stress at crack — all vary widely within the range of the stiffness per unit area between 0.1 and 10000 MPa/mm. The figure also indicates that two convergence regions exist on the other ranges — one where bond stiffness is less than 0.1 MPa/mm, and a second where it is more than 10000 MPa/mm. Actual bond stiffness values are in the neighborhood of 190 MPa/mm, which indicates that the crack width and stresses in concrete and steel are fairly sensitive to the changes in bond stiffness, as illustrated in the laboratory experiment (Ref 16).

The effect of the yield slip was investigated using the bilinear bond-slip model shown in Figure 2.4*e*. As shown in Figure 3.5, the crack width decreases and the stresses in concrete and steel increase as the yield slip increases. Figure 3.6 shows the effect of the second bond-slip stiffness. The crack width and stresses are almost constant for the stiffnesses less than about 5 MPa/mm. For the stiffness values larger than about 5 MPa/mm, the crack width decreases and the stresses increase with increasing second bond-slip stiffness. The effect of the ultimate slip is shown in Figure 3.7. As the ultimate slip increases, the crack width decreases and the stresses in concrete and steel increase initially. The crack width and stresses are constant after a certain ultimate slip, about 0.15 mm in this case.

### 3.4 CREEP EFFECT

The effects of concrete creep were investigated for crack width and for stresses in concrete and steel. The creep effects were evaluated by varying the maximum creep ratio and the time duration of the changes in temperature and drying shrinkage. The creep coefficient was determined by using Eq. 2.20. Figure 3.8 shows the changes in effective modulus, crack width, and stresses in concrete and steel when the maximum creep ratio varies from 0 through 4. As the maximum creep ratio increases, the values of those four variables decrease. The stresses in concrete and steel are reduced by about 20%, while the decrease in crack width is about 10%. Figure 3.9 shows the effect of the load duration. As the load duration increases the four variables decrease. The rates of decrement of the stresses are larger than those of the crack width, as already shown in the previous case. This finding indicates the importance of including creep effect in the analysis of CRCP.

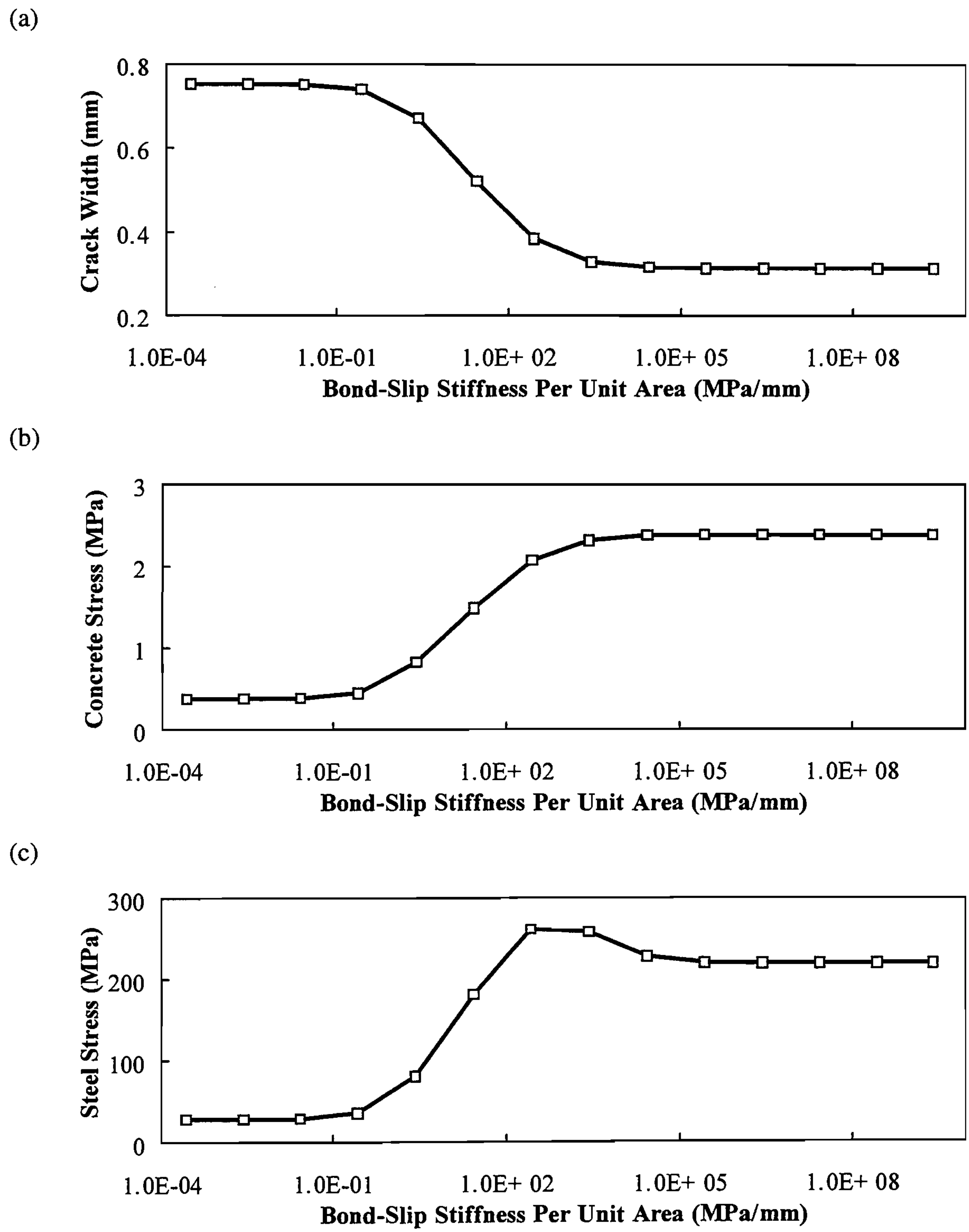



*Figure 3.4. Effect of bond-slip stiffness between concrete and steel: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

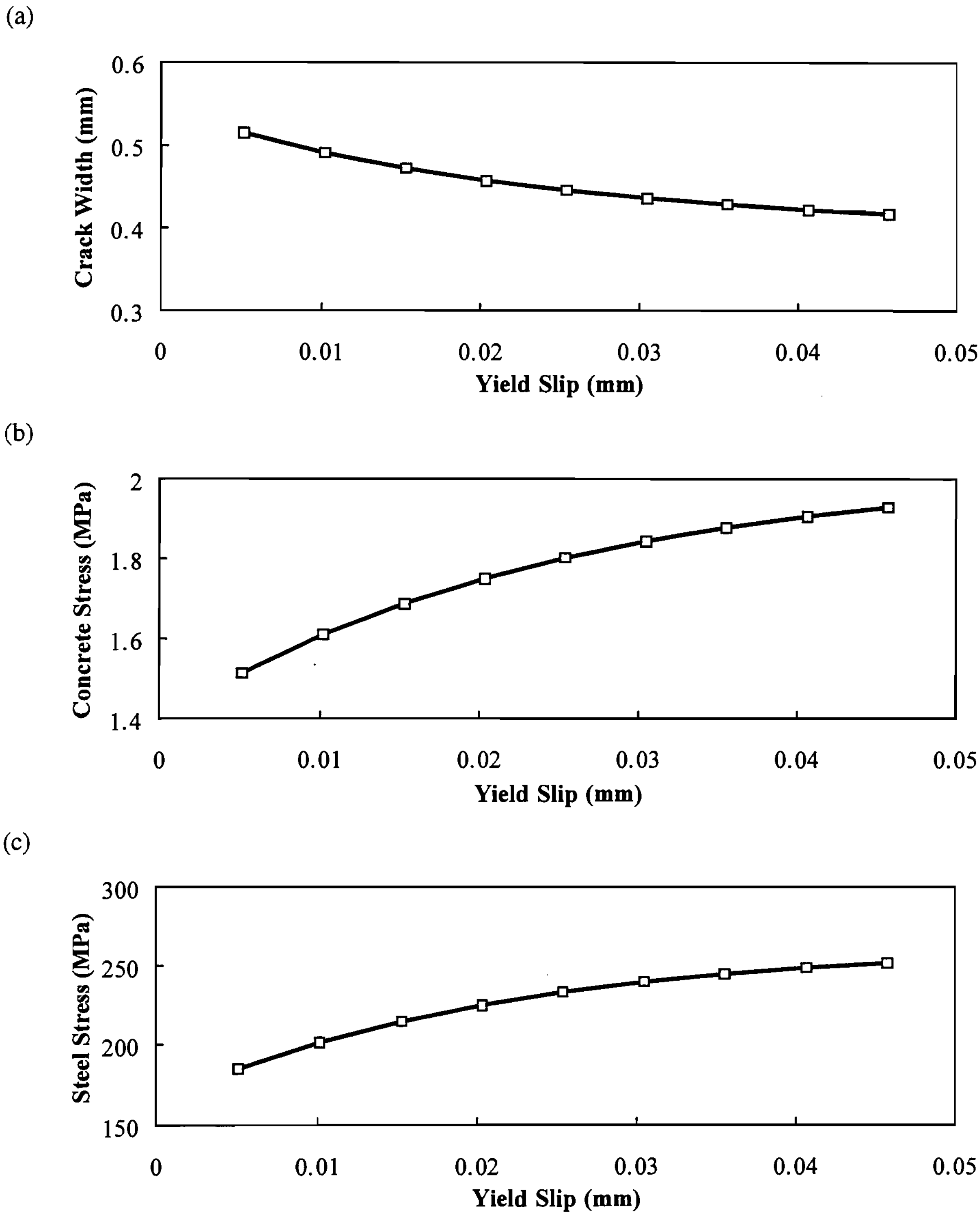



*Figure 3.5. Effect of yield slip between concrete and steel: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

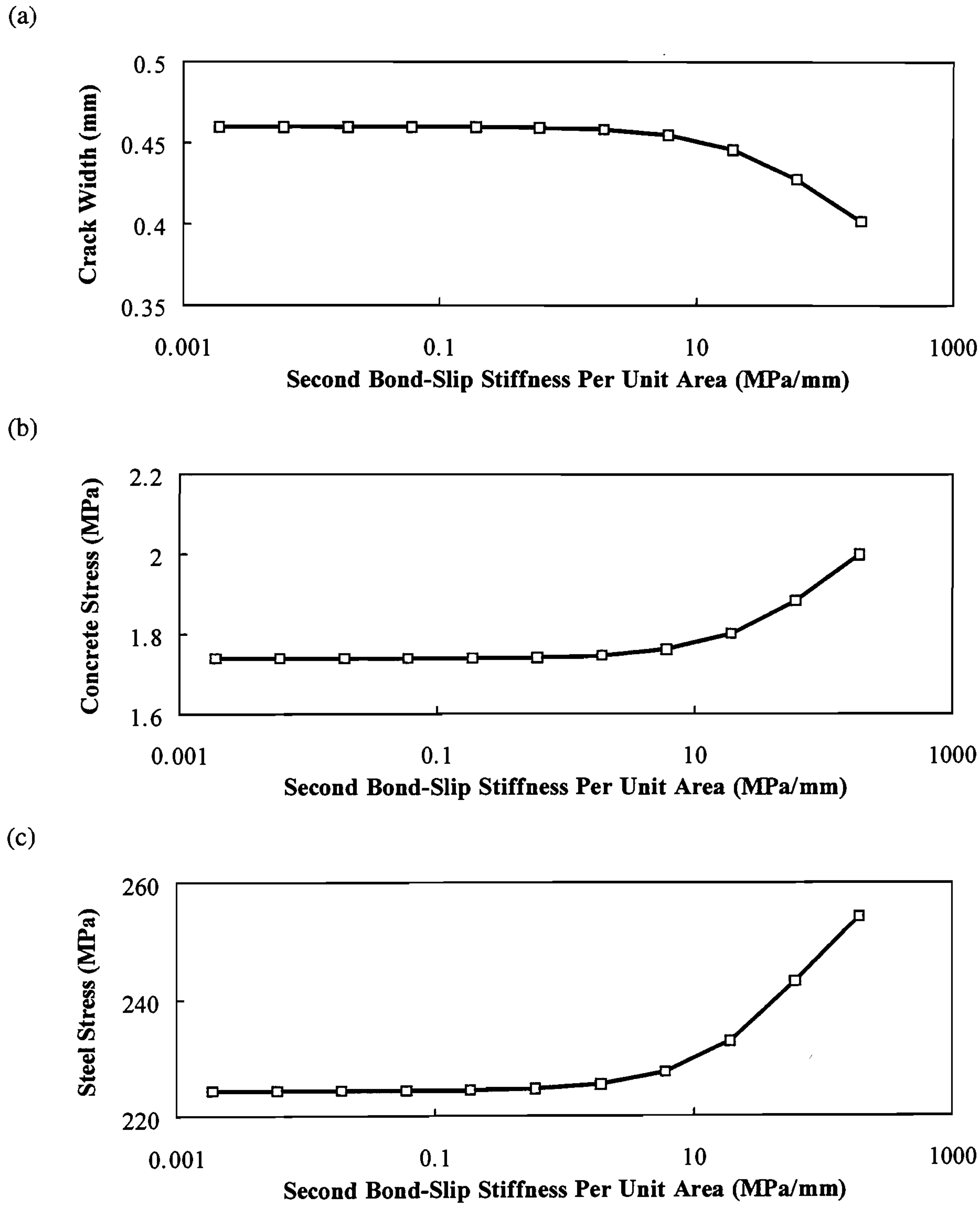
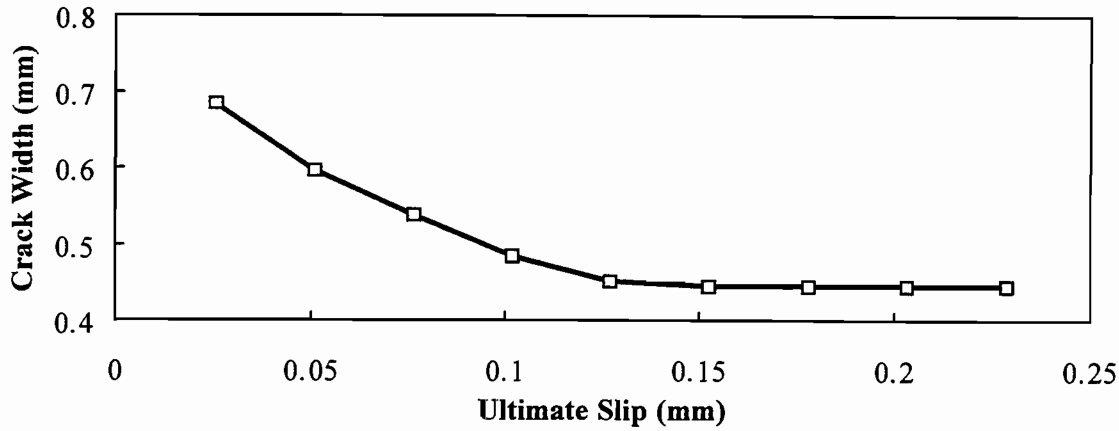
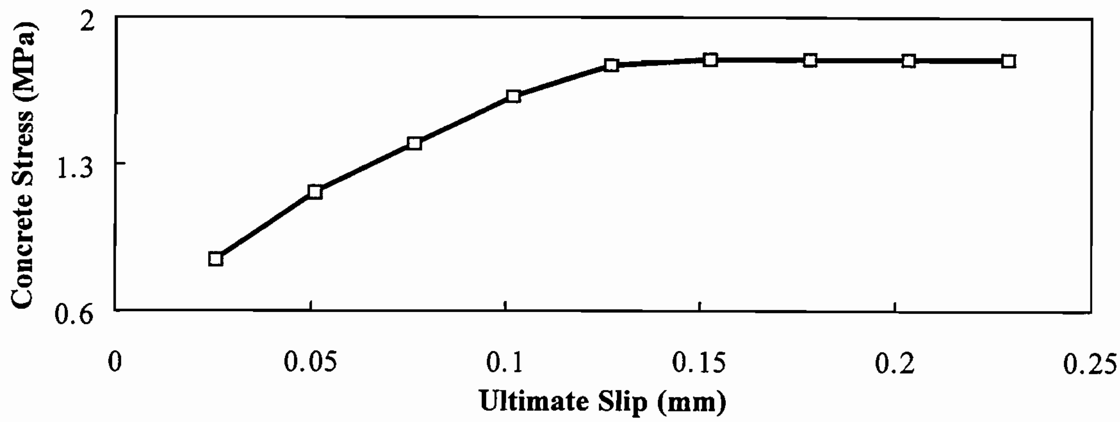



*Figure 3.6. Effect of second bond-slip stiffness between concrete and steel: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

(a)




(b)




(c)

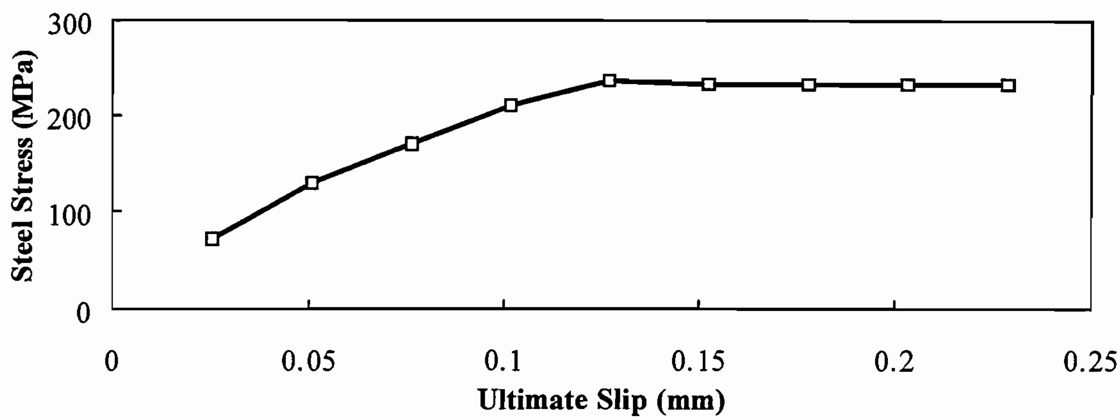



*Figure 3.7. Effect of ultimate slip between concrete and steel: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

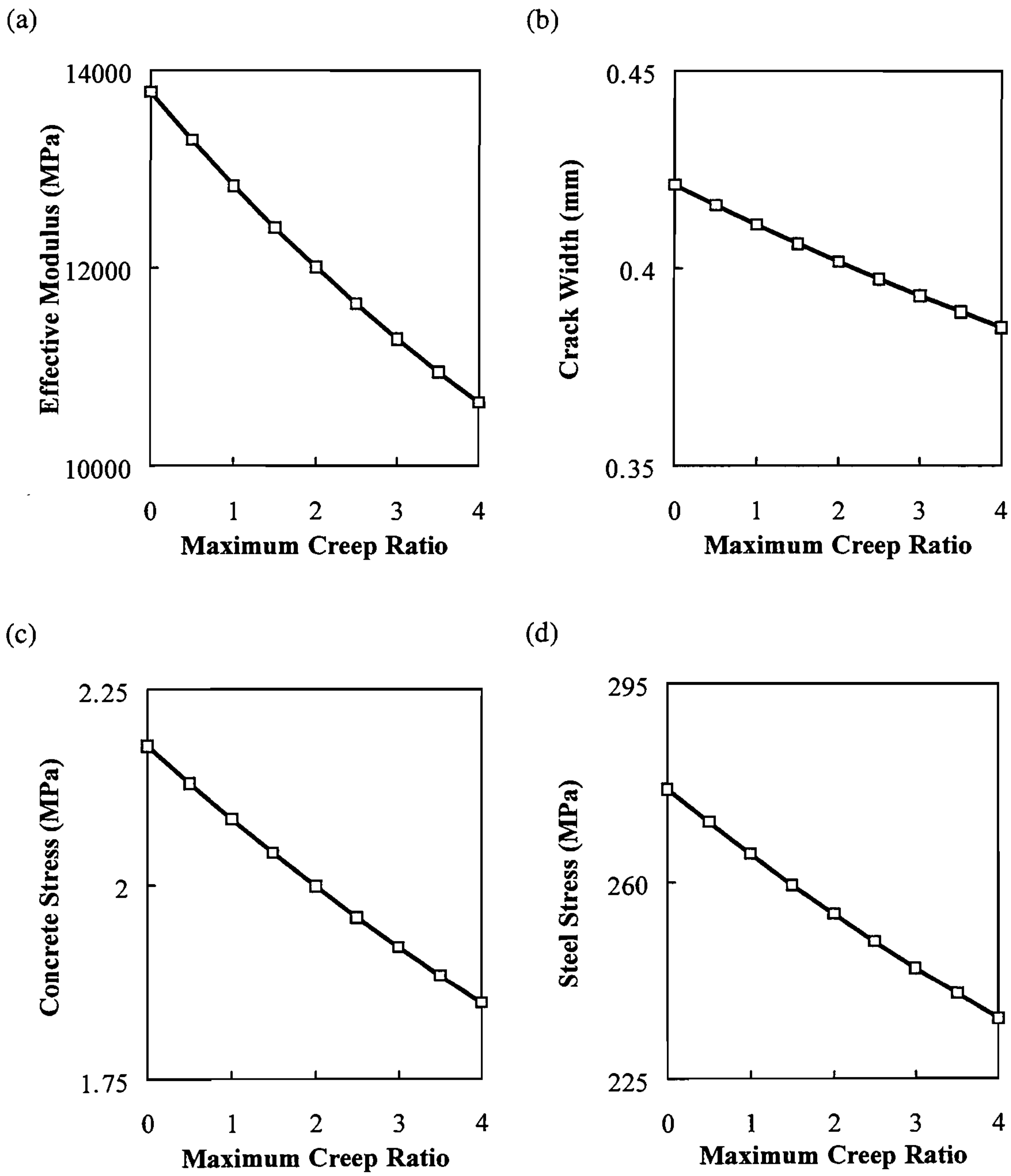



*Figure 3.8. Effect of maximum creep ratio: (a) effective modulus; (b) crack width; (c) concrete stress at top of slab center; (d) steel stress at crack*

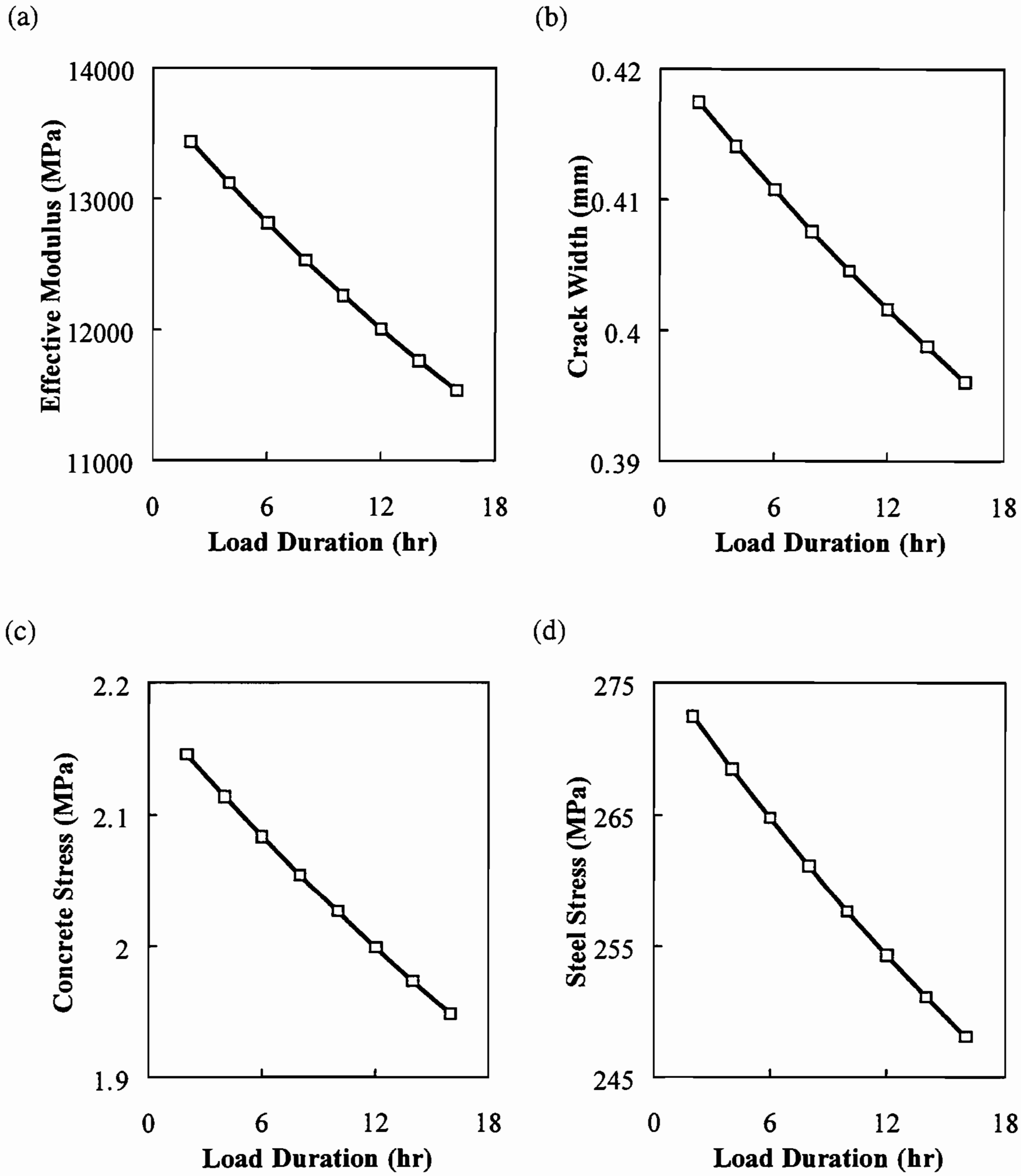



*Figure 3.9. Effect of load duration: (a) effective modulus; (b) crack width; (c) concrete stress at top of slab center; (d) steel stress at crack*

# CHAPTER 4. PARAMETRIC STUDIES

## 4.1 INTRODUCTION

Parametric studies were performed to evaluate the effects of design, materials, and construction variables on CRCP responses. Values in Table 3.1 were assigned to all the variables except for the variable under evaluation. Since preliminary studies show that CRCP responses are sensitive to the bond-slip relationship between concrete and steel, three bond-slip relationships were included throughout the parametric studies: they are linear (Fig. 2.4*a*), bilinear (Fig. 2.4*e*), and bilinear with an ultimate slip (Fig. 2.4*f*).

## 4.2 STRESS DISTRIBUTION

Figure 4.1 shows the distribution of stresses in concrete at the pavement surface and longitudinal steel along the slab. The maximum concrete stress occurs at the middle of the slab. The concrete stress decreases as it moves away from the middle of the slab — gradually initially, and then rather quickly as it approaches the crack. Different concrete stress distributions result from three bond-slip relationships. Linear bond-slip causes the largest concrete stress, while bilinear with ultimate slip causes the least. The maximum steel stress occurs at the crack, with the stress decreasing quickly as it moves away from the crack, as shown in Figure 4.1*b*. As in concrete, the linear bond-slip model provides the maximum steel stress at the crack, while bilinear with ultimate slip offers the least.

## 4.3 EFFECT OF CONCRETE PROPERTIES

The effects of Young's modulus of elasticity and the thermal expansion coefficient of concrete were also investigated. Figure 4.2 shows the variations of the crack width and stresses with increases in concrete elastic modulus. The three variables tend to increase as the modulus increases, with the exception of the stresses with the bilinear with ultimate bond-slip model. Figure 4.3 illustrates the effect of the concrete thermal coefficient. As the thermal coefficient increases, so do the crack width and stresses in concrete and steel, regardless of the bond-slip models. The only exception is the stresses in concrete and steel for the bilinear with ultimate bond-slip model. With this model, the concrete stress does not change, and the steel stress decreases as the thermal coefficient increases beyond 9 microstrains per degree Celsius in this case. This is believed to be due to the bond slip failure caused by large concrete volume changes.

Figure 4.3 explains the superior performance of CRCP with concrete containing coarse aggregates of a low thermal coefficient, such as certain limestone aggregates (Refs 17, 18). As the concrete thermal coefficient decreases, so do the concrete stress and the crack width. Larger transverse crack spacings result from lower concrete stresses. Tighter crack widths prevent the intrusion of water and foreign material from entering through the cracks.

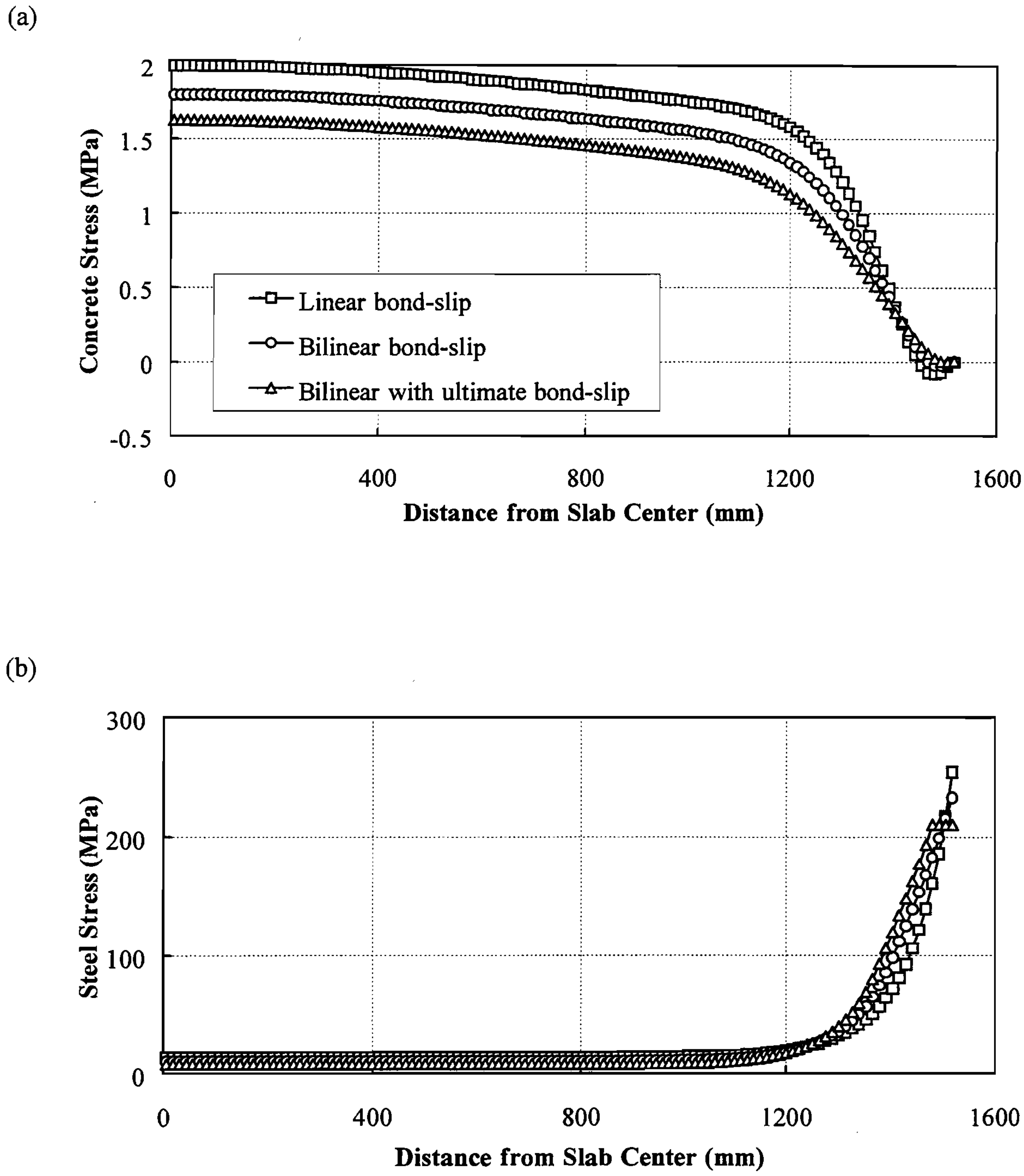



*Figure 4.1. Stress distribution in the longitudinal direction: (a) concrete stress at top of slab; (b) steel stress*

## 4.4 EFFECT OF LONGITUDINAL STEEL

The effects of the longitudinal steel ratio, steel diameter, and steel location were investigated next. Crack width decreases as the steel ratio increases (Fig. 4.4*a*). Concrete tensile stress at the top of the middle of the slab between transverse cracks increases with increasing steel ratio (Fig. 4.4*b*). Steel stress at the crack decreases as the steel ratio increases, except for the bilinear with the ultimate bond-slip model, which provides relatively constant steel stress for the range of steel ratios between 0.3% and 1.25% (Fig. 4.4*c*). The rates of change in crack width and concrete stress are largest for the bilinear with an ultimate slip model.

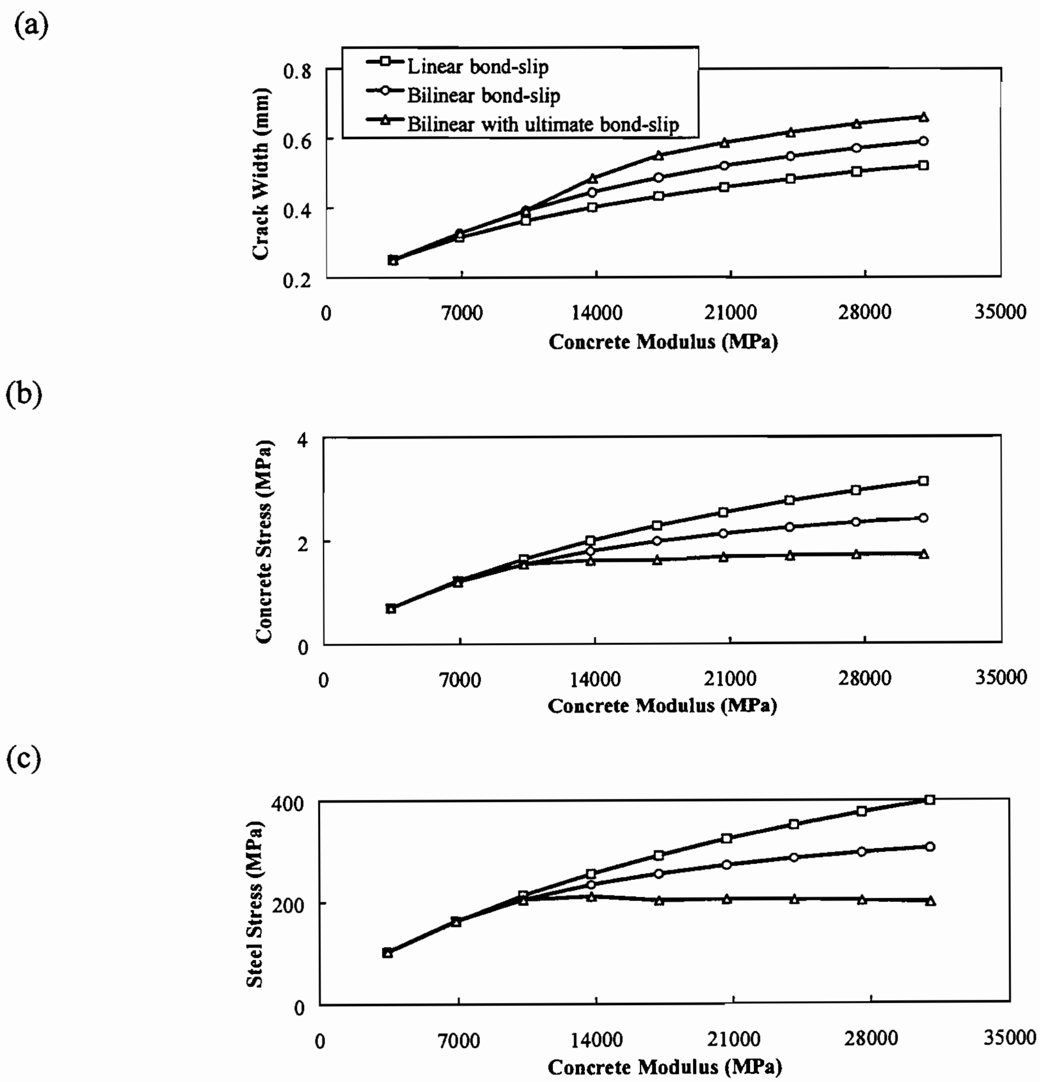



*Figure 4.2. Effect of concrete elastic modulus: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

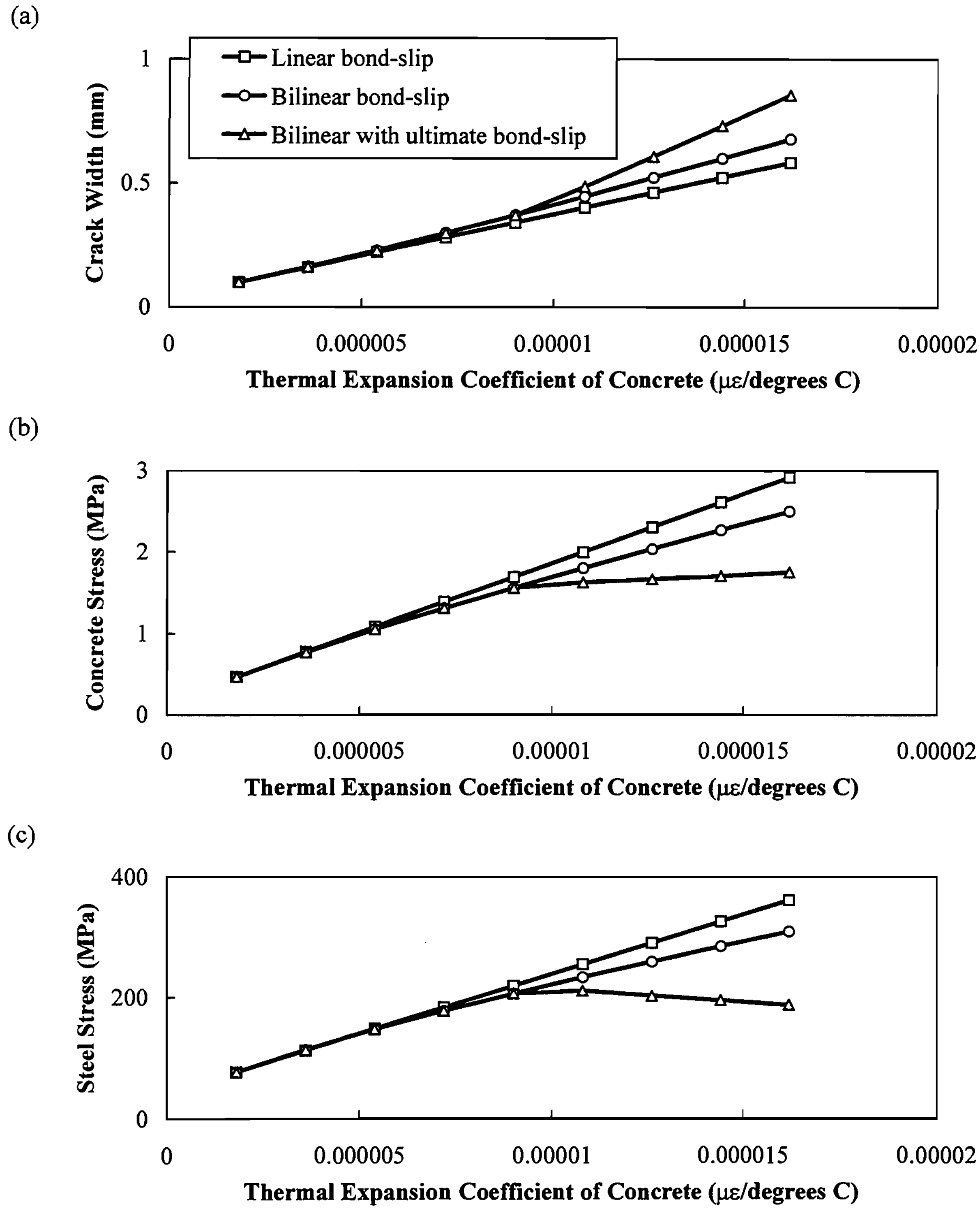



*Figure 4.3. Effect of thermal expansion coefficient of concrete: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

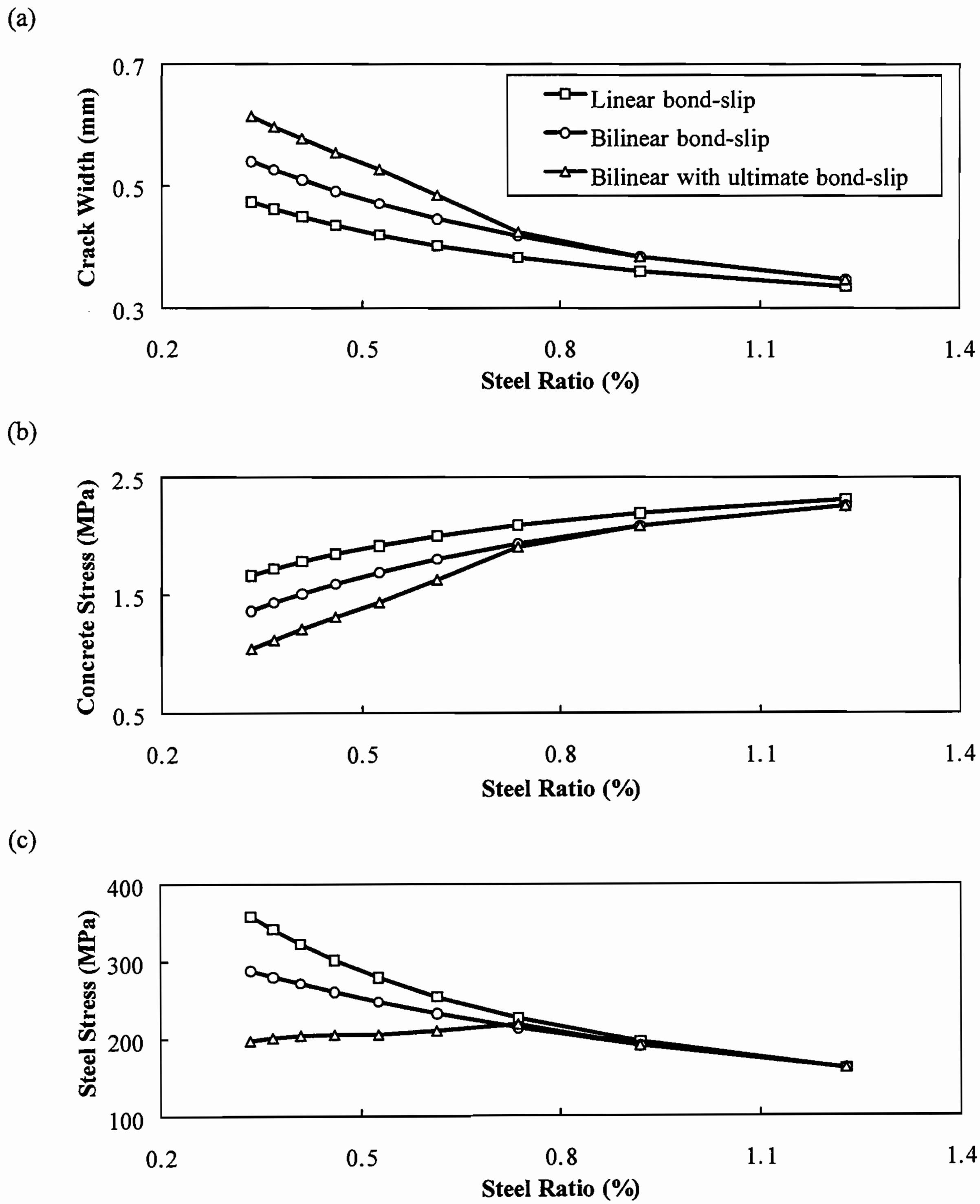



*Figure 4.4. Effect of steel ratio: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

Figure 4.5 shows the effect of the steel diameter when the steel ratio is fixed at 0.6%. Generally, the crack width increases while the stresses in concrete and steel decrease as the bar diameter increases, regardless of the bond-slip model. This is consistent with actual CRCP behavior (Ref 17). The effect of bar diameter is minimal with the linear bond-slip model, while the bilinear with ultimate slip model shows the maximum effect.

The effect of the steel location is shown in Figure 4.6. The crack width increases and the concrete stress decreases as the steel is placed further from the surface. This figure shows that the steel needs to be placed above the mid-depth to keep the cracks tight. It was found in Belgium and France that placing the steel at a depth of approximately one-third of the slab resulted in tighter crack widths and better performance (Ref 19). The effect of steel location on steel stress is shown in Figure 4.6*c*. Steel stress increases as the steel location moves from top to bottom, until it reaches mid-depth of the slab, and then decreases as the steel is placed further down.

## 4.5 RELATIONSHIP BETWEEN CRACK SPACING AND OTHER VARIABLES

The relationship between crack spacing and crack width, concrete stress, and steel stress is shown in Figure 4.7. The analysis reveals that the relationships generated by linear or bilinear bond-slip models are similar, while the bilinear with ultimate bond-slip model results in quite a different relationship. According to the linear and bilinear models, crack width becomes independent of crack spacing after the spacing exceeds 3 m (9.84 ft). However, the bilinear with ultimate bond-slip model produces a linear relationship between crack spacing and crack width up to 12 m (39.37 ft) (Fig 4.7*a*). With the linear and bilinear models, concrete and steel stresses increase rather quickly with crack spacings up to 6 m (19.69 ft), after which the rate of increase becomes insignificant (Fig 4.7*b* and *c*). For the bilinear with ultimate bond-slip model, concrete and steel stresses become independent of crack spacing after an initial increase.

Figure 4.8 shows the effect of steel ratio for a range of crack spacings up to 12 m (39.37 ft) using the bilinear bond-slip model. As the steel ratio increases, the crack width is reduced, as shown in Fig 4.8*a*. The reduction is the greatest when the steel ratio increases from 0.2% to 0.6%, compared with the minimal reduction from 0.6% to 1.0%. For a steel ratio of 0.2%, crack width continues to increase as the crack spacing becomes larger. On the other hand, for steel ratios over 0.6%, the crack width becomes independent of the crack spacing for the crack spacings over 3 m (9.84 ft). Figure 4.8*b* shows that for all steel ratios, there is a large rate of increase in concrete stress up to crack spacings of 6 m (19.7 ft), after which the rate becomes quite small. It also shows that more steel causes larger concrete stress, which will result in more cracks. The shape of the curves for steel stress is similar to that for crack width (Fig. 4.8*c*). Figure 4.8 indicates that an increasing steel ratio is beneficial as long as the crack spacing can be maintained above the minimum value.

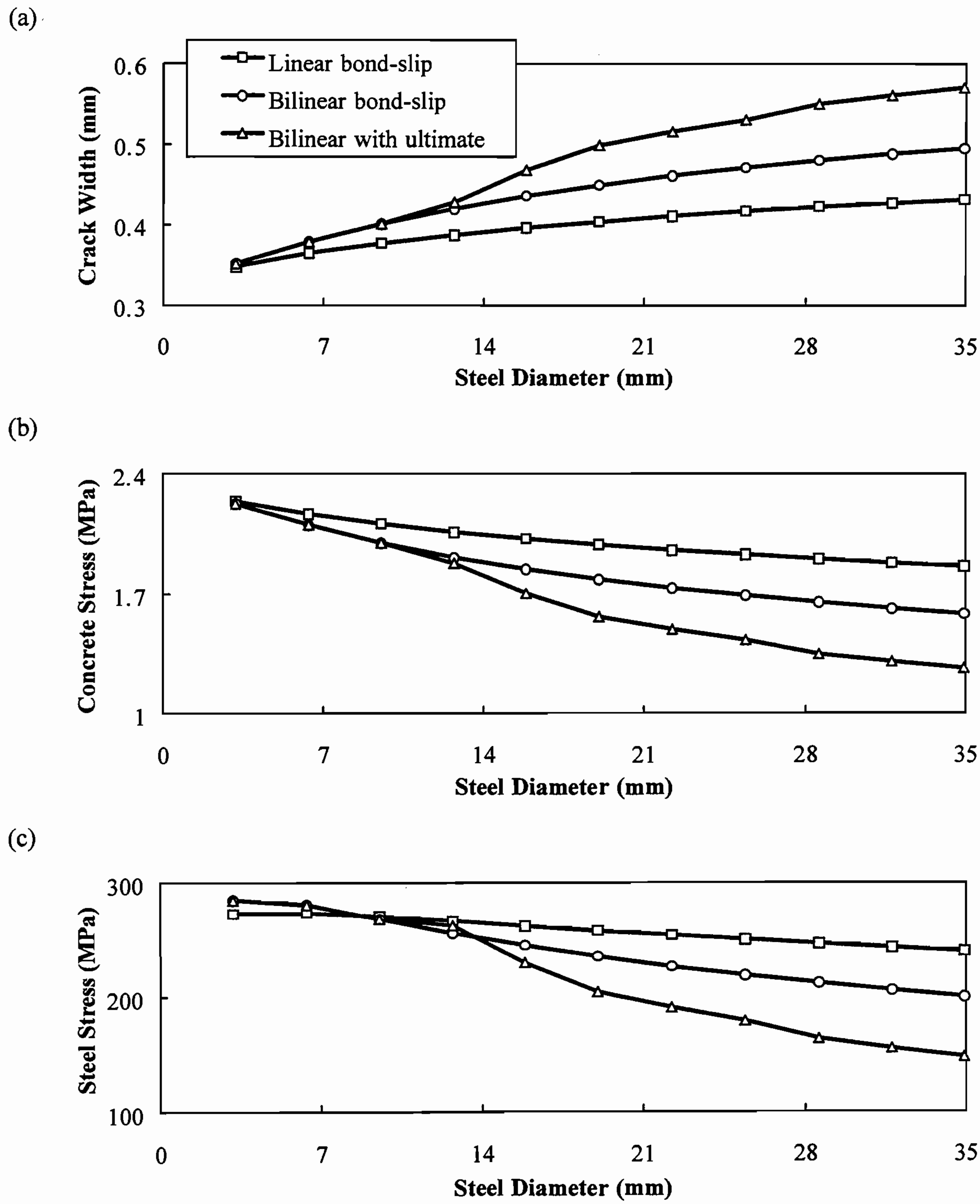



*Figure 4.5. Effect of steel diameter (when steel ratio=0.6%): (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

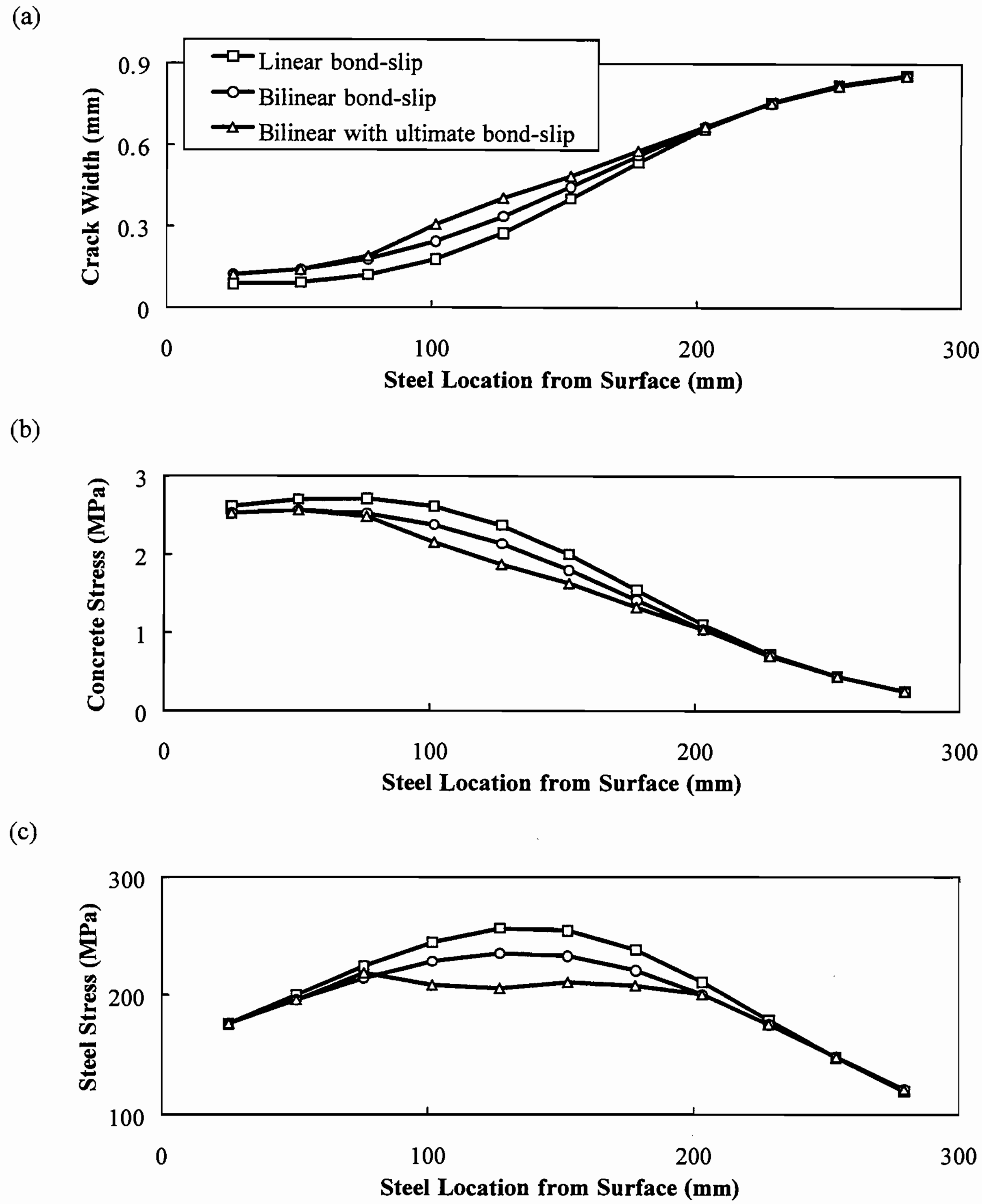



*Figure 4.6. Effect of steel location from surface: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

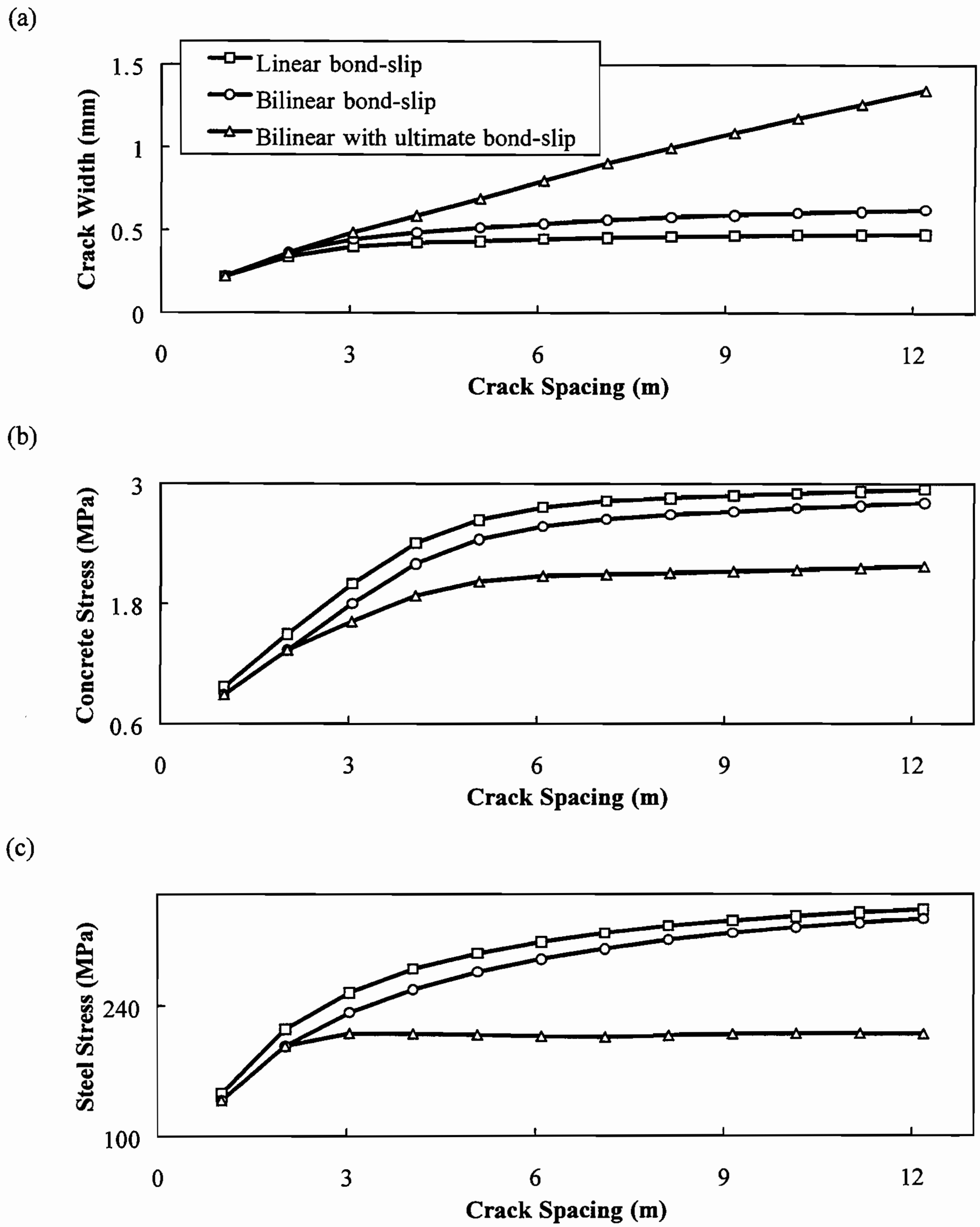



*Figure 4.7. Effect of crack spacing: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

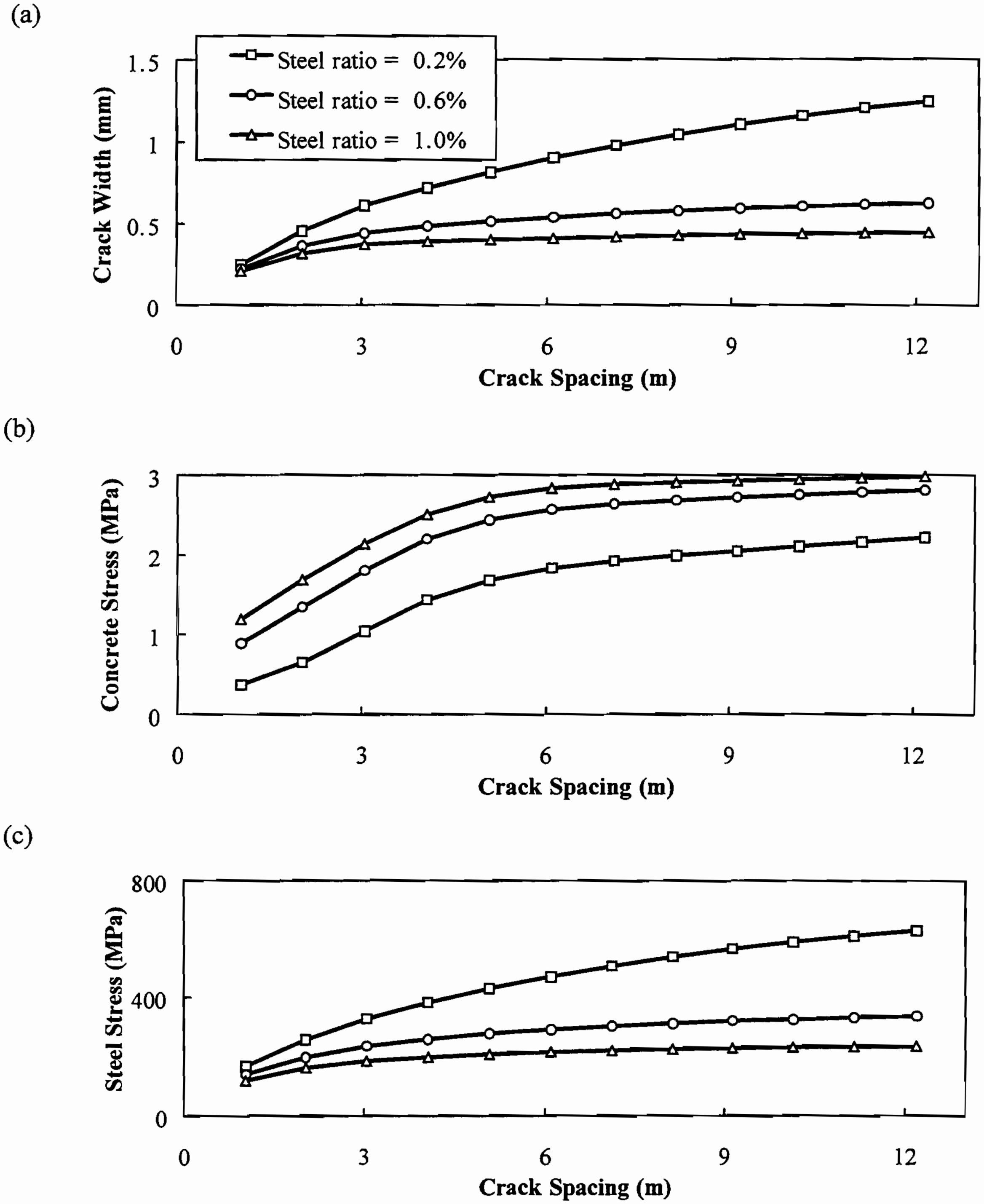



*Figure 4.8. Effect of crack spacing for various steel ratios: (a) crack width; (b) concrete stress at top of slab center; (c) steel stress at crack*

## CHAPTER 5. DESCRIPTION OF COMPUTER PROGRAM

### 5.1 MAIN PROGRAM

The main program can be divided into several distinct parts. First, the dimensions of needed arrays are determined and the input values are read. The basic variables are defined and needed subroutines are called. The slip lengths are checked to define the bond-slip stiffnesses. The tensile stresses of concrete are investigated next to determine if the tensile stresses are larger than the tensile strength of concrete. If the tensile stresses of concrete are larger than the tensile strength, it is assumed that there is another crack at the middle of the slab, and the analysis is performed again with a new crack spacing that is half of the former crack spacing. This procedure continues until the stresses are less than the strength. Finally, stresses of concrete and steel are calculated and printed out.

The dimensions of the arrays should be larger than the required dimensions. On the other hand, using very large dimensions can lead to hardware memory problems. The required dimensions of the arrays used in the program are as follows:

| | |
|---|---|
| ALOAD (ITNOD1) | DISP (ITNOD1) |
| SK (ITNOD1,MBAND) | BNSL (NUMELXX) |
| TEMLD (ITNOD1) | STRINI (NUMELY,4) |
| TLOAD (NUMELY,8) | INOD (INODDIM,8) |
| IMAX (NUMELXX) | SS (KS) |
| IMAXF (NUMELXX) | |

where

ITNOD1=2*(NUMELXX+1)*(NUMELY+2)
MBAND=(NUMELY+3)*2+4
KS=ITNOD1*MBAND
INODDIM=(NUMELY+1)*NUMELXX

and where NUMELXX and NUMELY are the numbers of the plane elements in the x and y directions, respectively.

### 5.2 SUBROUTINE PROGRAMS

The subroutine programs used in the code are as follows:

***CHMEMO***

This subroutine is used to check the required dimensions of the arrays. If a required dimension of an array is larger than the current dimension of the array, a notice that the program is checking the dimension of the array is given. The user must define the maximum dimensions of the array in the subroutine by changing the variables ITNOD1C, MBANDC, NUMELXC, NUMELYC, KSC, and INODC, which correspond to ITNOD1, MBAND, NUMELXX, NUMELY, KS, and INODDIM.

***CREEP***

This is used to calculate the reduced modulus of concrete using the effective modulus method.

***LINREC4***

This is a subroutine used to find the element stiffness matrix using four-noded bilinear isoparametric or rectangular elements. The material property matrix **D** is defined in this subroutine using the plane strain problem.

***MATPRO***

This is used to multiply matrices.

***FRAME***

The element stiffness matrix of the frame element, which includes horizontal, vertical, and rotational degrees of freedom, is defined here.

***ASSEM***

This subroutine assembles element stiffness matrices and nodal load vectors into the global system. The half-bandwidth is used for the global stiffness matrix.

***TEMPRT***

Calculations of initial stresses and equivalent nodal loads in elements resulting from changes in temperature and drying shrinkage are performed in this subroutine. Two options can be taken in the subroutine. One is to use an average temperature for an element, and the other is to use interpolated temperatures from the nodes at each integration point. The differences in the results are negligible when there is a sufficient number of elements, as explained in Chapter 3.

***SHRINK***

This is used to calculate equivalent temperature variations owing to drying shrinkage strains. The positive drying shrinkage strain represents the negative strain in the subroutine.

***BOUND***

The boundary conditions of the system are defined in this subroutine. The restraints are models using very large stiffnesses of the spring elements. The vertical stiffnesses of the base layer are also determined here.

***BNSLIP***

The bond-slip relation between concrete and longitudinal steel is defined here. The linear and bilinear bond-slip relations can be defined by controlling the input values.

***FRSLIP***

The bond-slip relation between the concrete layer and the base layer is defined here. The linear elastic and perfect plastic relation is used.

***SOLVE***

This subroutine is used to solve the equilibrium equations using the one-dimensional compacted array of banded stiffness matrix and Gaussian eliminations (Ref 4).

***CHSLIP***

This is used to determine if the slip length exceeds the limit values (yield slips).

***STRESS***

The stresses in the elements are calculated in this subroutine.

## 5.3 INPUT GUIDE

This section explains the program's required inputs. The input formats are freeform, such that only a space is needed between input values. Sample input is shown in Appendix A.

***LINE 1***

TOTLP: half of the primary crack spacing
TOTD: depth of the concrete layer
NUMELXX: number of elements of plane strain elements in the horizontal direction
NUMELY: number of elements of plane strain elements in the vertical direction

***LINE 2***

ECONC: Young's modulus of elasticity of concrete
POIS1: Poisson's ratio of concrete
THICK: thickness of the plane strain element (distance between longitudinal steels)
ALPC: coefficient of thermal expansion of concrete
WEIGHT: specific weight of concrete (pcf)

***LINE 3***

STLOC: steel location from surface
ESTL: Young's modulus of elasticity of steel
DIASTL: diameter of longitudinal steel
ALPS: coefficient of thermal expansion of steel
AKBUA: primary bond-slip stiffness per unit area between concrete and steel
AKBUA2: secondary bond-slip stiffness per unit area between concrete and steel
YLSLIP: yield slip between concrete and steel
ULSLIP: ultimate slip between concrete and steel

***LINE 4***

AKFRUA: bond-slip stiffness per unit area between concrete and base
YLFRSL: yield slip between concrete and base

*LINE 5*

TEMUPP: temperature at surface
TEMDNN: temperature at bottom of concrete
TEMF: reference temperature

*LINE 6*

SHRUP: drying shrinkage strain at surface
SHRDN: drying shrinkage strain at bottom of concrete

*LINE 7*

ICREEP: index for creep analysis (1=do creep analysis)
PHIMAX: maximum creep ratio to the instantaneous elastic strain
PERCEN: consideration percent to maximum creep
DAYPC: corresponding day to PERCEN
TIME: loading duration (hr)

*LINE 8*

ISTIND: index for stress calculation (1 = calculate stresses in the elements in the longitudinal direction, other = calculate stresses at center of cracks in the vertical direction)
IELST: first element number for stress calculation (must start with 1 if ISTIND is not 1)

## 5.4 OUTPUT GUIDE

Program outputs can be modified by users. This section explains the output of the current program, with sample output shown in Appendix B.

The output starts with CRACK=1, which means that the analysis is now performed with a primary crack spacing. CRACK=2 means that there is another crack at the center of the primary crack spacing, and that the analysis is performed with the crack spacing half of the primary crack spacing. Therefore, CRACK=*n* means that the analysis is performed with crack spacing $1/2^{n-1}$ of the primary crack spacing.

"Concrete-Steel Bond Slip" and "Concrete-Base Bond Slip" represent how much the yield slip develops. Concrete-Steel Bond Slip=0 means that no yield slip develops, and Concrete-Steel Bond Slip=1 means that the slip exceeds the yield slip at the first contact node at the crack. Therefore, Concrete-Steel Bond Slip=*n* means that the slips exceed the yield slip from the first contact node at the crack to the *n*th node from the crack. Concrete-Base Bond Slip represents the same relationship.

Crack spacing, crack width, and tensile strength of concrete are also written. The displacement at each degree of freedom is printed out, and the concrete stresses are written. (Numbering of the degrees of freedom and elements was explained in Chapter 2.) Finally, the initial thermal stress of the longitudinal steel and the final stresses in steel are also written.

# CHAPTER 6. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

## 6.1 SUMMARY

A finite element program to analyze continuously reinforced concrete pavement (CRCP) has been developed, and the behavior of CRCP resulting from environmental loads such as temperature change and drying shrinkage has been studied using the finite element model.

Concrete and longitudinal steel were discretized using the plane strain and the frame elements, respectively. Various bond-slip models between concrete and steel and between concrete and base layers were developed using spring elements. The creep effect and the crack spacing prediction were also included.

The sensitivity of the input values was investigated and parametric studies were conducted to evaluate the effects of design, materials, and construction variables on CRCP responses. Various bond-slip models provided differences in the results, but the overall trends were similar regardless of the bond-slip models.

Descriptions of the main and subroutine programs were explained and a user's guide for inputs was presented.

## 6.2 CONCLUSIONS

The preliminary and parametric studies provided the following conclusions:

1. The use of the interpolated temperatures from the nodes and the use of the average temperature in the element give the same results when the fine meshes are used.
2. Modeling of longitudinal steel using the frame and the truss elements gives very similar results, with the differences considered negligible.
3. Different concrete-steel bond-slip models produced varied results. An accurate bond-slip relationship needs to be investigated further.
4. Concrete creep reduces crack width and stresses in concrete and steel.
5. The thermal coefficient of concrete has a significant effect on crack width and stresses in concrete and steel. Using concrete having a low thermal coefficient will result in better CRCP performance.
6. Longitudinal steel variables — the amount of steel, bar diameter, and steel location — are important design variables that can influence CRCP behavior. For given environmental conditions, an optimum steel design can be developed using mechanistic models.

## 6.3 RECOMMENDATIONS

Efforts have been made to develop a more realistic finite element model to analyze the behavior of CRCP subjected to environmental loads. There are, however, some limitations in the developed finite element model, and those will need to be addressed in follow-up research. Some of the limitations include the omission of the warping effect and the modeling of all underlying layers. More realistic modeling for creep effect also needs to be further studied. Finally, further investigations should be undertaken to identify reasonable input values for the bond-slip relations and creep effect.

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## APPENDIX A

## SAMPLE INPUT

60. 12. 40 8

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6. 29000000. 0.75 0.000005 700000. 70000. 0.001 0.004

400. 150. 0.02

85. 100. 120.

0.00002 0.0

1 2. 99. 30. 2.

1 1

# APPENDIX B

# SAMPLE OUTPUT

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CRACK =            1
Concrete-Steel Bond-Slip =          0
Concrete-Steel Bond-Slip =          1
Concrete-Steel Bond-Slip =          2
Concrete-Steel Bond-Slip =          3
Concrete-Steel Bond-Slip =          4
CRACK =            2
Concrete-Steel Bond-Slip =          0
Concrete-Steel Bond-Slip =          1
Concrete-Steel Bond-Slip =          2
Concrete-Steel Bond-Slip =          3
CRACK SPACING (in) =   60.0000000000000
CRACK WIDTH (in) =  1.204811170658302E-002
TENSILE STRENGTH (psi) =   242.975280658800
DISPLACEMENTS AT DEGREES OF FREEDOM
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 136   0.906253E+02   -.110936E+01   -.142867E+02
 145   0.627576E+02   -.241341E-01   -.158452E+02
 154   0.341051E+02   0.247432E+01   -.137237E+02
 163   0.122837E+02   0.501976E+01   -.533824E+01
 172   0.446049E+01   -.394596E+00   0.346154E+01
STRESS IN STEEL BAR
INITIAL THERMAL STRESS=   3987.50000000000
ST. NO. FROM CENTER     STEEL STRESS
        1              0.527541E+03
        2              0.529004E+03
        3              0.532290E+03
        4              0.538195E+03
        5              0.548123E+03
        6              0.564373E+03
        7              0.590596E+03
        8              0.632502E+03
        9              0.698942E+03
       10              0.803569E+03
       11              0.967454E+03
       12              0.122330E+04
       13              0.162243E+04
       14              0.224685E+04
       15              0.323041E+04
       16              0.479728E+04
       17              0.733353E+04
       18              0.115303E+05
       19              0.172895E+05
       20              0.235438E+05
```

# APPENDIX C

# LIST OF PROGRAM CRCPFEM

# APPENDIX C

## LIST OF PROGRAM CRCPFEM

```
      PROGRAM CRCPFEM
C
C     Version 1.0
C     Program to find the response of the CRCP subjected to environmental
C     loads such as changes in temperature and drying shrinkage throughout
C     the depth of the concrete layer.
C     Concrete is discretized using plane strain elements, longitudinal
C     steel is modeled using frame elements, and the spring elements are
C     used to model the stiffness of the base layer and bond-slip relations.
C     The creep effect is included using the Effective modulus method.
C     Crack spacing prediction is included.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      CHARACTER*15 FILIN,FILOUT
      DIMENSION ALOAD(9664),DISP(9664),AKS(2,2),AKSB(4,4)
      DIMENSION SK(9664,70),AK(8,8),D(3,3),BNSL(150)
      DIMENSION TEMLD(9664),STRINI(30,4),TLOAD(30,8)
      DIMENSION INOD(4650,8),SS(676480),IMAX(150),IMAXF(150)
      COMMON NUMELX,NUMELY,A,B,E1,POIS1,THICK,ITNOD1,MBAND
C
C     minimum memory requirements
C
C       ALOAD(ITNOD1),DISP(ITNOD1),SK(ITNOD1,MBAND),BNSL(NUMELX)
C       TEMLD(ITNOD1),STRINI(NUMELY,4),TLOAD(NUMELY,8)
C       INOD(INODDIM,8),SS(KS),IMAX(NUMELX),IMAXF(NUMELX)
C
C     read names of input and output files
C
      WRITE (*,201)
  201 FORMAT (/'    ** ENTER INPUT FILE **   '/)
      READ (*,202) FILIN
      WRITE (*,203)
  203 FORMAT (//'   ** ENTER OUTPUT FILE **   '/)
      READ (*,202) FILOUT
  202 FORMAT (A)
C
C     open input and output files
C
      OPEN (5,FILE=FILIN,STATUS='OLD')
      OPEN (6,FILE=FILOUT,STATUS='UNKNOWN')
C
C     read input data
C
      READ(5,*) TOTLP,TOTD,NUMELXX,NUMELY
      READ(5,*) ECONC,POIS1,THICK,ALPC,WEIGHT
      READ(5,*) STLOC,ESTL,DIASTL,ALPS,AKBUA,AKBUA2,YLSLIP,ULSLIP
      READ(5,*) BOTKV,AKFRUA,YLFRSL
      READ(5,*) TEMUPP,TEMDNN,TEMF
      READ(5,*) SHRUP,SHRDN
      READ(5,*) ICREEP,PHIMAX,PERCEN,DAYPC,TIME
      READ(5,*) ISTIND,IELST
```

```
C
C     WHERE,
C       TOTLP,TOTD = half of primary crack spacing, and depth of concrete
C       NUMELXX,NUMELY = total number of plane elements in x and y directions
C       ECONC,POIS1,THICK = modulus,Poisson's ratio and thickness of concrete
C       WEIGHT = specific weight of concrete (pcf)
C       ALPC,ALPS = coefficient of thermal expansion for concrete and steel
C       STLOC = steel location from surface
C       ESTL,DIASTL = modulus and diameter of longitudinal steel
C       AKBUA,AKBUA2 = primary and secondary bond-slip stiffness per unit area
C       YLSLIP,ULSLIP = yield and ultimate slips
C       BOTKV = vertical stiffness per unit area of the base layer
C       AKFRUA = bond-slip stiffness per unit area between concrete and base
C       YLFRSL = yield slip between concrete and base
C       TEMUPP,TEMDNN,TEMF = surface, bottom and reference temperatures
C       SHRUP,SHRDN = surface and bottom drying shrinkage strains
C       ICREEP = index for creep analysis (1=creep analysis)
C       PHIMAX = maximum creep ratio to instantaneous strain
C       PERCEN = consideration percent to maximum creep
C       DAYPC = corresponding day to PERCEN
C       TIME = loading duration (hr)
C       ISTIND = index for stress output
C                (1: longitudinal, other: depth direction at center line)
C       IELST = first element number for stress calculation (must start
C                with 1 if ISTIND is not 1)
C
C     define basic variables
C
      ICRACK=1
  150 CONTINUE
      WRITE(*,*) 'CRACK =',ICRACK
C
      NUMELX=NUMELXX/(2**(ICRACK-1))
      TOTL=TOTLP/(2.**(ICRACK-1))
C
      A=TOTL/FLOAT(NUMELX)
      B=TOTD/FLOAT(NUMELY)
      ITNOD=(NUMELX+1)*(NUMELY+1)
      ITNOD1=ITNOD*2+(NUMELX+1)*2
      ASTL=(DIASTL/2.)*(DIASTL/2.)*3.141592654
      LSTLOC=STLOC/B+1
      AKSTL=ESTL*ASTL/A
      BNAREA=A*3.141592654*DIASTL
      AKBOND=AKBUA*BNAREA
      AKBOND2=AKBUA2*BNAREA
      BNFORCE=YLSLIP*BNAREA*(AKBUA-AKBUA2)
      DELT=TEMDNN+(TEMUPP-TEMDNN)*(1-STLOC/TOTD)-TEMF
      TEMUP=TEMUPP
      TEMDN=TEMDNN
      FRAREA=A*THICK
      AKFR=AKFRUA*FRAREA
      FRFORCE=YLFRSL*AKFR
      BOTK=BOTKV*A*THICK
      TSTRNG=7.*ECONC/(33.*WEIGHT*DSQRT(WEIGHT))
      MBAND=(NUMELY+3)*2+4
      KS=ITNOD1*MBAND
      INODDIM=(NUMELY+1)*NUMELX
C
      CALL CHMEMO(KS,INODDIM,ICHMEMO)
C     This program can take up to 150 by 30 elements
C
      IF(ICHMEMO.EQ.1) GO TO 1000
C
```

```
      IF(ICREEP.EQ.1) THEN
         CALL CREEP(TIME,ECONC,E1,PHIMAX,PERCEN,DAYPC)
      ELSE
         E1=ECONC
      ENDIF
C
      CALL LINREC4(AK,D)
C
      CALL FRAME(A,ESTL,DIASTL,AKSTL,AKS,AKSB)
C
      CALL SHRINK(SHRUP,SHRDN,TEMUP,TEMDN,ALPC)
C
      CALL TEMPRT(TEMUP,TEMDN,TEMF,ALPC,TLOAD,STRINI,D)
C
      KIM=1
      NA=1
      DO I=1,NUMELX
         IMAX(I)=0
         IMAXF(I)=0
      ENDDO
C
  100 CONTINUE
C
      DO I=1,ITNOD1
         ALOAD(I)=0.
      ENDDO
C
      CALL ASSEM(AK,SK,INOD,LSTLOC,AKS,TLOAD,TEMLD,AKSB)
C
      CALL BOUND(SK,LSTLOC,BOTK)
C
      CALL BNSLIP(SK,AKBOND,AKBOND2,LSTLOC,KIM,ALOAD,IMAX,BNFORCE,
     +            BNSLMAXS,BNSL,ULSLIP)
C
      CALL FRSLIP(SK,AKFR,NA,ALOAD,IMAXF,FRFORCE,FRSLMAXS)
C
      DO 101 I=1,ITNOD1
      DO 102 J=1,MBAND
         K=(J-1)*ITNOD1+I
         SS(K)=SK(I,J)
  102 CONTINUE
         ALOAD(I)=ALOAD(I)+TEMLD(I)
  101 CONTINUE
C
      CALL SOLVE(ITNOD1,MBAND,ALOAD,SS,KS)
C
      DO I=1,ITNOD1
         DISP(I)=ALOAD(I)
      ENDDO
C
      CALL CHSLIP(DISP,IMAX,KIM,BNSL,BNSLMAX,BNSLMAXS,LSTLOC,YLSLIP,
     +            YLFRSL,NA,FRSLMAX,FRSLMAXS,IMAXF)
C
      IF(BNSLMAX.GT.YLSLIP) GO TO 100
      IF(FRSLMAX.GT.YLFRSL) GO TO 100
C
C     Check if tensile stresses at center line of cracks are larger than
C     the tensile strength of concrete. If the stress on any point is
C     larger than the strength, a new crack is assumed to occur.
C
      INDST=0
      DO 400 JSTR=1,NUMELY+1
         IF(JSTR.EQ.LSTLOC) GO TO 400
```

```
         CALL STRESS(JSTR,STRINI,INOD,LSTLOC,ESTL,ALPS,DELT,DISP,D,
     +               CHSTRES,INDST)
         IF(CHSTRES.GT.TSTRNG) THEN
           ICRACK=ICRACK+1
           GO TO 150
         ENDIF
  400 CONTINUE
C
      TOTLF=TOTL*2.
      IWIDTH=ITNOD1-2*(NUMELY+2)+1
      DIWIDTH=DABS(2.*DISP(IWIDTH))
      WRITE(6,*) 'CRACK SPACING (in) =',TOTLF
      WRITE(6,*) 'CRACK WIDTH (in) =',DIWIDTH
      WRITE(6,*) 'TENSILE STRENGTH (psi) =',TSTRNG
C
      WRITE(6,*) 'DISPLACEMENTS AT DEGREES OF FREEDOM'
      DO I=1,ITNOD1
         WRITE(6,110) I,DISP(I)
  110 FORMAT(I10,5X,E15.6)
      ENDDO
C
      WRITE(6,*) 'CONCRETE STRESS'
      WRITE(6,*) 'EL NO    S XX           S YY           S XY'
      INDST=1
      IF(ISTIND.EQ.1) THEN
         DO I=1,NUMELX
            IELINP=IELST+(I-1)*(NUMELY+1)
            CALL STRESS(IELINP,STRINI,INOD,LSTLOC,ESTL,ALPS,DELT,DISP,
     +                  D,CHSTRES,INDST)
         ENDDO
      ELSE
         DO 402 I=1,NUMELY+1
            IELINP=IELST+I-1
            IF(IELINP.EQ.LSTLOC) GO TO 402
            CALL STRESS(IELINP,STRINI,INOD,LSTLOC,ESTL,ALPS,DELT,DISP,
     +                  D,CHSTRES,INDST)
  402 CONTINUE
      ENDIF
      INDST=2
      CALL STRESS(IELINP,STRINI,INOD,LSTLOC,ESTL,ALPS,DELT,DISP,D,
     +            CHSTRES,INDST)
C
 1000 CONTINUE
      STOP
      END
C*****************************************************************
      SUBROUTINE CHMEMO(KS,INODDIM,ICHMEMO)
C
C     to check required dimensions of the arrays
C
C     minimum memory requirements
C       ALOAD(ITNOD1),DISP(ITNOD1),SK(ITNOD1,MBAND),BNSL(NUMELX)
C       TEMLD(ITNOD1),STRINI(NUMELY,4),TLOAD(NUMELY,8)
C       INOD(INODDIM,8),SS(KS),IMAX(NUMELX),IMAXF(NUMELX)
C
C     where
C       ITNOD1=(NUMELY+1)*(NUMELX+1)*2+(NUMELX+1)*2
C       MBAND=(NUMELY+3)*2+4
C       KS=ITNOD1*MBAND
C       INODDIM=(NUMELY+1)*NUMELX
C
      IMPLICIT REAL*8 (A-H, O-Z)
      COMMON NUMELX,NUMELY,SKIP(5),ITNOD1,MBAND
```

```
C
C     the following values should be changed to fit current computer memory
C
      ITNOD1C=9664
      MBANDC=70
      NUMELXC=150
      NUMELYC=30
      KSC=676480
      INODC=4650
C
      ICHMEMO=0
      IF(ITNOD1.GT.ITNOD1C) THEN
        WRITE(*,*) 'memory fault: check dimensions of following arrays'
        WRITE(*,*) 'ALOAD, DISP, TEMLD, SK'
        ICHMEMO=1
      ENDIF
      IF(MBAND.GT.MBANDC) THEN
        WRITE(*,*) 'memory fault: check dimension of SK array'
        ICHMEMO=1
      ENDIF
      IF(NUMELX.GT.NUMELXC) THEN
        WRITE(*,*) 'memory fault: check dimensions of following arrays'
        WRITE(*,*) 'BNSL, IMAX, IMAXF'
        ICHMEMO=1
      ENDIF
      IF(NUMELY.GT.NUMELYC) THEN
        WRITE(*,*) 'memory fault: check dimensions of STRINI and TLOAD'
        ICHMEMO=1
      ENDIF
      IF(KS.GT.KSC) THEN
        WRITE(*,*) 'memory fault: check dimension of SS array'
        ICHMEMO=1
      ENDIF
      IF(INODDIM.GT.INODC) THEN
        WRITE(*,*) 'memory fault: check dimension of INOD array'
        ICHMEMO=1
      ENDIF
C
      RETURN
      END
C*****************************************************************
      SUBROUTINE CREEP(TIME,ECONC,E1,PHIMAX,PERCEN,DAYPC)
C
C     to find the reduced modulus using Effective Modulus Method
C
      IMPLICIT REAL*8 (A-H, O-Z)
C
      IF(PERCEN.EQ.100.) THEN
         WRITE(*,*) 'PUT LESS THAN 100 FOR PERCEN'
         WRITE(*,*) 'CREEP ANALYSIS WAS NOT PERFORMED'
         GO TO 806
      ENDIF
      CONST=(1./(1.-PERCEN/100.))**(TIME/(DAYPC*24.))
      PHI=PHIMAX*(1.-1./CONST)
      E1=ECONC/(1.+PHI)
C
  806 RETURN
      END
C*****************************************************************
      SUBROUTINE LINREC4(AK,D)
C
C     to find the element stiffness matrix using 4-node isoparametric
C     element (or 4-node bi-linear rectangular element)
```

```
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION AK(8,8),XI(4),ETA(4),XINT(2),YINT(2),WX(2),WY(2)
      DIMENSION D(3,3),BMAT(3,8),BT(8,3),F(4),FPXI(4),FPET(4),AUX(8,8)
      DIMENSION AJAC(2)
      COMMON NUMELX,NUMELY,A,B,E1,POIS1,THICK
      DATA XI/-1.,-1.,1.,1./,ETA/1.,-1.,1.,-1./
      DATA XINT/-0.57735027,0.57735027/,YINT/-0.57735027,0.57735027/
      DATA WX/1.,1./,WY/1.,1./
C
C     initialize matrices
C
      DO 1 I=1,3
      DO 3 J=1,3
    3 D(I,J)=0.
    1 CONTINUE
      DO 4 I=1,8
      DO 5 J=1,8
    5 AK(I,J)=0.
    4 CONTINUE
C
C     define D matrix (plane strain)
C
      COND=E1/((1.+POIS1)*(1.-2.*POIS1))
      D(1,1)=COND*(1.-POIS1)
      D(1,2)=COND*POIS1
      D(2,1)=D(1,2)
      D(2,2)=D(1,1)
      D(3,3)=COND*(1.-2.*POIS1)/2.
C
C     define D matrix (plane stress)
C
C      COND=E1/(1.-POIS1*POIS1)
C      D(1,1)=COND
C      D(1,2)=D(1,1)*POIS1
C      D(2,1)=D(1,2)
C      D(2,2)=D(1,1)
C      D(3,3)=E1/(2.+2.*POIS1)
C
C     number of integration points in x and y directions
C
      NPX=2
      NPY=2
C
      DO 100 IX=1,NPX
      DO 101 IY=1,NPY
C
      FACT=A*B*WX(IX)*WY(IY)*THICK/4.
      X=XINT(IX)
      Y=YINT(IY)
C
C     find derivatives of shape functions at integration points
C
      DO 10 I=1,4
         F(I)=(1.+X*XI(I))*(1.+Y*ETA(I))/4.
         FPXI(I)=XI(I)*(1.+Y*ETA(I))/4.
         FPET(I)=ETA(I)*(1.+X*XI(I))/4.
   10 CONTINUE
C
C     define B matrix and transpose of B
C
      DO I=1,3
      DO J=1,8
```

```
      BMAT(I,J)=0.
      ENDDO
      ENDDO
      AJAC(1)=A/2.
      AJAC(2)=B/2.
      DO 20 I=1,4
         I1=2*I-1
         I2=I1+1
         BMAT(1,I1)=FPXI(I)/AJAC(1)
         BMAT(2,I2)=FPET(I)/AJAC(2)
         BMAT(3,I1)=BMAT(2,I2)
         BMAT(3,I2)=BMAT(1,I1)
   20 CONTINUE
      DO 21 I=1,3
      DO 22 J=1,8
         BT(J,I)=BMAT(I,J)
   22 CONTINUE
   21 CONTINUE
C
C     define element stiffness matrix
C
      CALL MATPRO(D,BMAT,AUX,3,3,8,3,3,8)
C
      DO 23 I=1,3
      DO 24 J=1,8
         BMAT(I,J)=AUX(I,J)
   24 CONTINUE
   23 CONTINUE
C
      CALL MATPRO(BT,BMAT,AUX,8,3,8,8,3,8)
C
      DO 25 I=1,8
      DO 26 J=1,8
         AK(I,J)=AK(I,J)+FACT*AUX(I,J)
   26 CONTINUE
   25 CONTINUE
C
  101 CONTINUE
  100 CONTINUE
      RETURN
      END
C*****************************************************************
      SUBROUTINE MATPRO(AA,BB,CC,NA,NB,NC,M,N,L)
C
C     to multiply matrices
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION AA(NA,N),BB(NB,L),CC(NC,L)
      DO 56 I=1,M
      DO 57 J=1,L
      SUM=0.
      DO 58 K=1,N
      SUM=SUM+AA(I,K)*BB(K,J)
   58 CONTINUE
      CC(I,J)=SUM
   57 CONTINUE
   56 CONTINUE
      RETURN
      END
C*****************************************************************
      SUBROUTINE FRAME(A,ESTL,DIASTL,AKSTL,AKS,AKSB)
C
C     to define element stiffness matrix for frame (truss and beam) elements
```

```
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION AKS(2,2),AKSB(4,4)
C
C     element stiffness matrix for truss element
C
      AKS(1,1)=AKSTL
      AKS(1,2)=-AKSTL
      AKS(2,2)=AKS(1,1)
C
      AMINER=3.141592654*(DIASTL**4)/64.
      CONBE=ESTL*AMINER/(A**3)
C
C     element stiffness matrix for beam element
C
      AKSB(1,1)=12.*CONBE
      AKSB(1,2)=6.*A*CONBE
      AKSB(1,3)=-12.*CONBE
      AKSB(1,4)=6.*A*CONBE
      AKSB(2,2)=4.*A*A*CONBE
      AKSB(2,3)=-6.*A*CONBE
      AKSB(2,4)=2.*A*A*CONBE
      AKSB(3,3)=12.*CONBE
      AKSB(3,4)=-6.*A*CONBE
      AKSB(4,4)=4.*A*A*CONBE
C
      RETURN
      END
C*****************************************************************
      SUBROUTINE ASSEM(AK,SK,INOD,LSTLOC,AKS,TLOAD,TEMLD,AKSB)
C
C     to assemble element stiffness matrices and temperature load
C     vectors into global systems (using half-bandwidth for stiffness)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION INOD(4650,8),SK(9664,70),AK(8,8),AKS(2,2),AKSB(4,4)
      DIMENSION TLOAD(30,8),TEMLD(9664)
      COMMON NUMELX,NUMELY,SKIP(5),ITNOD1,MBAND
C
C     initialize banded stiffness matrix and temperature load vector
C
      DO 30 I=1,ITNOD1
      DO 31 J=1,MBAND
         SK(I,J)=0.
   31 CONTINUE
         TEMLD(I)=0.
   30 CONTINUE
C
C     element numbering and connectivity with degrees of freedom
C
      DO 32 KX=1,NUMELX
      DO 33 KY=1,NUMELY
         IF(KY.LT.LSTLOC) THEN
           IEL=(KX-1)*(NUMELY+1)+KY
           INOD(IEL,1)=IEL*2+2*KX-3
           INOD(IEL,2)=INOD(IEL,1)+1
           INOD(IEL,3)=INOD(IEL,1)+2
           INOD(IEL,4)=INOD(IEL,1)+3
           INOD(IEL,5)=INOD(IEL,1)+(NUMELY+2)*2
           INOD(IEL,6)=INOD(IEL,5)+1
           INOD(IEL,7)=INOD(IEL,5)+2
           INOD(IEL,8)=INOD(IEL,5)+3
         ELSE
```

```
          IEL=(KX-1)*NUMELY+KX+KY
          IF(KY.EQ.LSTLOC) THEN
            INOD(IEL,1)=IEL*2+2*KX-5
            INOD(IEL,2)=INOD(IEL,1)+1
            INOD(IEL,3)=INOD(IEL,1)+4
            INOD(IEL,4)=INOD(IEL,1)+5
            INOD(IEL,5)=INOD(IEL,1)+(NUMELY+2)*2
            INOD(IEL,6)=INOD(IEL,5)+1
            INOD(IEL,7)=INOD(IEL,5)+4
            INOD(IEL,8)=INOD(IEL,5)+5
          ELSE
            INOD(IEL,1)=IEL*2+2*KX-3
            INOD(IEL,2)=INOD(IEL,1)+1
            INOD(IEL,3)=INOD(IEL,1)+2
            INOD(IEL,4)=INOD(IEL,1)+3
            INOD(IEL,5)=INOD(IEL,1)+(NUMELY+2)*2
            INOD(IEL,6)=INOD(IEL,5)+1
            INOD(IEL,7)=INOD(IEL,5)+2
            INOD(IEL,8)=INOD(IEL,5)+3
          ENDIF
        ENDIF
C
C     assemble element stiffness matrix and load vector into global system
C
        DO 35 I=1,8
        DO 36 J=I,8
           SK(INOD(IEL,I),INOD(IEL,J)-INOD(IEL,I)+1)=
     +           SK(INOD(IEL,I),INOD(IEL,J)-INOD(IEL,I)+1)+AK(I,J)
   36   CONTINUE
           TEMLD(INOD(IEL,I))=TEMLD(INOD(IEL,I))+TLOAD(KY,I)
   35   CONTINUE
C
   33 CONTINUE
   32 CONTINUE
C
C     element and d.o.f. numbering for frame elements
C
      DO 37 KX=1,NUMELX
         IEL=LSTLOC+(KX-1)*(NUMELY+1)
         INOD(IEL,1)=IEL*2+2*KX-2
         INOD(IEL,2)=INOD(IEL,1)+1
         INOD(IEL,3)=INOD(IEL,1)+(NUMELY+2)*2
         INOD(IEL,4)=INOD(IEL,3)+1
         INOD(IEL,5)=IEL*2+2*KX
         INOD(IEL,6)=INOD(IEL,5)+(NUMELY+2)*2
C
C     assemble element stiffness matrix for beam elements
C
      DO 41 I=1,4
      DO 42 J=I,4
            SK(INOD(IEL,I),INOD(IEL,J)-INOD(IEL,I)+1)=
     +          SK(INOD(IEL,I),INOD(IEL,J)-INOD(IEL,I)+1)+AKSB(I,J)
   42 CONTINUE
   41 CONTINUE
C
C     assemble element stiffness matrix for truss elements
C
      DO 38 I=5,6
      DO 39 J=I,6
            SK(INOD(IEL,I),INOD(IEL,J)-INOD(IEL,I)+1)=
     +          SK(INOD(IEL,I),INOD(IEL,J)-INOD(IEL,I)+1)+AKS(I-4,J-4)
   39 CONTINUE
   38 CONTINUE
```

```
   37 CONTINUE
C
      RETURN
      END
C**************************************************************
      SUBROUTINE TEMPRT(TEMUP,TEMDN,TEMF,ALPC,TLOAD,STRINI,D)
C
C     to calculate initial stresses and applied loads due to
C     temperature variations.
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION D(3,3),XI(4),ETA(4),XINT(2),YINT(2),WX(2),WY(2)
      DIMENSION STRINI(30,4),AJAC(2),BMAT(3,8),BT(8,3)
      DIMENSION TLOAD(30,8),F(4),FPXI(4),FPET(4),TEML(4)
      COMMON NUMELX,NUMELY,A,B,E1,POIS1,THICK
      DATA XI/-1.,-1.,1.,1./,ETA/1.,-1.,1.,-1./
      DATA XINT/-0.57735027,0.57735027/,YINT/-0.57735027,0.57735027/
      DATA WX/1.,1./,WY/1.,1./
C
      DO I=1,NUMELY
      DO J=1,8
         TLOAD(I,J)=0.
      ENDDO
      ENDDO
C
C     number of integration points in x and y directions
C
      NPX=2
      NPY=2
C
      TEMDIF=TEMDN-TEMUP
      TEMINC=TEMDIF/FLOAT(NUMELY)
C
      DO 500 ITEMP=1,NUMELY
C
      TEMUPL=TEMUP+TEMINC*FLOAT(ITEMP-1)
      TEMDNL=TEMUPL+TEMINC
      TEMAV=(TEMUPL+TEMDNL)/2.
C
      INDEX=0
C
C     if INDEX=0 : use of an average temperature for an element
C     if INDEX=1 : use of interpolated temperature from the nodes
C                  at each integration point
C
      IF(INDEX.EQ.1) THEN
        TEML(1)=TEMUPL
        TEML(2)=TEMDNL
        TEML(3)=TEMUPL
        TEML(4)=TEMDNL
      ELSE
        DO I=1,4
           TEML(I)=TEMAV
        ENDDO
      ENDIF
C
      NUMIP=1
      DO 301 IX=1,NPX
      DO 302 IY=1,NPY
         FACT=A*B*WX(IX)*WY(IY)*THICK/4.
         X=XINT(IX)
         Y=YINT(IY)
         DO I=1,4
```

```
            F(I)=(1.+X*XI(I))*(1.+Y*ETA(I))/4.
            FPXI(I)=XI(I)*(1.+Y*ETA(I))/4.
            FPET(I)=ETA(I)*(1.+X*XI(I))/4.
         ENDDO
         CONTEM=0.
         DO I=1,4
            CONTEM=CONTEM+F(I)*TEML(I)
         ENDDO
C
         CONTEM=(CONTEM-TEMF)*ALPC*(1.+POIS1)
C
C        for plane stress:
C        CONTEM=(CONTEM-TEMF)*ALPC
C
         STRINI(ITEMP,NUMIP)=-1.*CONTEM*(D(1,1)+D(1,2))
C
C     define B matrix and transpose of B
C
      DO I=1,3
      DO J=1,8
      BMAT(I,J)=0.
      ENDDO
      ENDDO
      AJAC(1)=A/2.
      AJAC(2)=B/2.
      DO I=1,4
         I1=2*I-1
         I2=I1+1
         BMAT(1,I1)=FPXI(I)/AJAC(1)
         BMAT(2,I2)=FPET(I)/AJAC(2)
         BMAT(3,I1)=BMAT(2,I2)
         BMAT(3,I2)=BMAT(1,I1)
      ENDDO
      DO I=1,3
      DO J=1,8
         BT(J,I)=BMAT(I,J)
      ENDDO
      ENDDO
C
      DO I=1,8
         CONLO=-1.*(BT(I,1)+BT(I,2))*STRINI(ITEMP,NUMIP)*FACT
         TLOAD(ITEMP,I)=TLOAD(ITEMP,I)+CONLO
      ENDDO
      NUMIP=NUMIP+1
  302 CONTINUE
  301 CONTINUE
  500 CONTINUE
C
      RETURN
      END
C****************************************************************
      SUBROUTINE SHRINK(SHRUP,SHRDN,TEMUP,TEMDN,ALPC)
C
C     to calculate equivalent temperatures due to variations of drying
C     shrinkage strains
C     (positive shrinkage strain represents negative strain)
C
      IMPLICIT REAL*8 (A-H, O-Z)
C
      DTUP=SHRUP/ALPC
      DTDN=SHRDN/ALPC
      TEMUP=TEMUP-DTUP
      TEMDN=TEMDN-DTDN
```

```
C
      RETURN
      END
C*****************************************************************
      SUBROUTINE BOUND(SK,LSTLOC,BOTK)
C
C     to apply boundary conditions (using big spring method)
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION SK(9664,70)
      COMMON NUMELX,NUMELY
C
      BIGK=1.0E15
C
C     boundary condition for left side (no d.o.f. in x-direction)
C
      LSTLOC1=LSTLOC+1
      DO I=1,NUMELY+2
         IF(I.LE.LSTLOC) THEN
           J=I*2-1
         ELSE
           IF(I.EQ.LSTLOC1) THEN
             J=I*2
           ELSE
             J=I*2-1
           ENDIF
         ENDIF
         SK(J,1)=SK(J,1)+BIGK
      ENDDO
C
C     boundary condition for left side of steel (no rotational d.o.f.)
C
      J=LSTLOC1*2-1
      SK(J,1)=SK(J,1)+BIGK
C
C     bound. cond. for right side of steel(no dof in x- and rot. direc.)
C
      NSTLL=((NUMELY+2)*2)*NUMELX+LSTLOC*2+1
      SK(NSTLL,1)=SK(NSTLL,1)+BIGK
      SK(NSTLL+1,1)=SK(NSTLL+1,1)+BIGK
C
C     boundary condition for bottom
C
      NST1=(NUMELY+2)*2
      NST11=NST1+NST1*NUMELX
      SK(NST1,1)=SK(NST1,1)+BOTK/2.
      SK(NST11,1)=SK(NST11,1)+BOTK/2.
      DO I=2,NUMELX
         J1=NST1+NST1*(I-1)
         SK(J1,1)=SK(J1,1)+BOTK
      ENDDO
C
      RETURN
      END
C*****************************************************************
      SUBROUTINE BNSLIP(SK,AKBOND,AKBOND2,LSTLOC,KIM,ALOAD,IMAX,
     +                  BNFORCE,BNSLMAXS,BNSL,ULSLIP)
C
C     define bond-slip relation between concrete and steel bar
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION SK(9664,70),ALOAD(9664),IMAX(150),BNSL(150)
      COMMON NUMELX,NUMELY,SKIP(5),ITNOD1,MBAND
```

```
C
      NINC=(NUMELY+2)*2
      NST=LSTLOC*2-1+NINC
      DO I=1,NUMELX
         J1=NST+(I-1)*NINC
         J2=J1+3
         IF(I.EQ.NUMELX) THEN
           SK(J1,1)=SK(J1,1)+AKBOND/2.
           SK(J1,4)=SK(J1,4)-AKBOND/2.
           SK(J2,1)=SK(J2,1)+AKBOND/2.
         ELSE
           SK(J1,1)=SK(J1,1)+AKBOND
           SK(J1,4)=SK(J1,4)-AKBOND
           SK(J2,1)=SK(J2,1)+AKBOND
         ENDIF
      ENDDO
C
      IF(KIM.EQ.1) GO TO 889
C
      DO IKIM=1,KIM-1
         J1=NST+(IMAX(IKIM)-1)*NINC
         J2=J1+3
         IF(DABS(BNSL(IMAX(IKIM))).LT.ULSLIP) THEN
           IF(IMAX(IKIM).EQ.NUMELX) THEN
             AKB=AKBOND2/2.-AKBOND/2.
             BNFOR=BNFORCE/2.
           ELSE
             AKB=AKBOND2-AKBOND
             BNFOR=BNFORCE
           ENDIF
         ELSE
           BNFOR=0.
           IF(IMAX(IKIM).EQ.NUMELX) THEN
             AKB=-AKBOND/2.
           ELSE
             AKB=-AKBOND
           ENDIF
         ENDIF
         SK(J1,1)=SK(J1,1)+AKB
         SK(J1,4)=SK(J1,4)-AKB
         SK(J2,1)=SK(J2,1)+AKB
         IF(BNSLMAXS.LT.0.) THEN
           ALOAD(J1)=ALOAD(J1)+BNFOR
           ALOAD(J2)=ALOAD(J2)-BNFOR
         ELSE
           ALOAD(J1)=ALOAD(J1)-BNFOR
           ALOAD(J2)=ALOAD(J2)+BNFOR
         ENDIF
      ENDDO
C
  889 RETURN
      END
C*****************************************************************
      SUBROUTINE FRSLIP(SK,AKFR,NA,ALOAD,IMAXF,FRFORCE,FRSLMAXS)
C
C     define bond-slip relation between concrete and base
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION SK(9664,70),ALOAD(9664),IMAXF(150)
      COMMON NUMELX,NUMELY,SKIP(5),ITNOD1,MBAND
C
      NINC=(NUMELY+2)*2
      NSTFR=(NUMELY+2)*2-1+NINC
```

```
      DO I=1,NUMELX
         J1=NSTFR+(I-1)*NINC
         IF(I.EQ.NUMELX) THEN
           SK(J1,1)=SK(J1,1)+AKFR/2.
         ELSE
           SK(J1,1)=SK(J1,1)+AKFR
         ENDIF
      ENDDO
C
      IF(NA.EQ.1) GO TO 689
C
      DO INA=1,NA-1
         J1=NSTFR+(IMAXF(INA)-1)*NINC
         IF(IMAXF(INA).EQ.NUMELX) THEN
           AKF=-AKFR/2.
           FRFOR=FRFORCE/2.
         ELSE
           AKF=-AKFR
           FRFOR=FRFORCE
         ENDIF
         SK(J1,1)=SK(J1,1)+AKF
         IF(FRSLMAXS.LT.0.) THEN
           ALOAD(J1)=ALOAD(J1)+FRFOR
         ELSE
           ALOAD(J1)=ALOAD(J1)-FRFOR
         ENDIF
      ENDDO
C
  689 RETURN
      END
C*****************************************************************
      SUBROUTINE SOLVE(NEQ,MBAND,R,SS,KS)
C
C     to solve equilibrium equations using 1-D compacted array of
C     banded stiffness matrix and Gaussian eliminations
C
C     NEQ   = number of equations
C     MBAND = half-bandwidth
C     R     = constant vector (load vector)
C     SS    = 1-D array of coefficient matrix (banded stiffness matrix)
C     KS    = NEQ*MBAND
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION R(NEQ),SS(KS)
C
C     forward reduction of 1-D array of banded coefficient matrix
C
      DO 790 N=1,NEQ
      DO 780 L=2,MBAND
      KK1=(L-1)*NEQ+N
      IF(SS(KK1).EQ.0.) GO TO 780
      I=N+L-1
      C=SS(KK1)/SS(N)
      J=0
      DO 750 K=L,MBAND
      J=J+1
  750 SS((J-1)*NEQ+I)=SS((J-1)*NEQ+I)-C*SS((K-1)*NEQ+N)
      SS(KK1)=C
  780 CONTINUE
  790 CONTINUE
C
C     forward reduction of the vector of constants
C
```

```
      DO 830 N=1,NEQ
      DO 820 L=2,MBAND
      IF(SS((L-1)*NEQ+N).EQ.0.) GO TO 820
      I=N+L-1
      R(I)=R(I)-SS((L-1)*NEQ+N)*R(N)
  820 CONTINUE
  830 R(N)=R(N)/SS(N)
C
C     solve for unknowns by back-substitution
C
      DO 860 M=2,NEQ
      N=NEQ+1-M
      DO 850 L=2,MBAND
      IF(SS((L-1)*NEQ+N).EQ.0.) GO TO 850
      K=N+L-1
      R(N)=R(N)-SS((L-1)*NEQ+N)*R(K)
  850 CONTINUE
  860 CONTINUE
      RETURN
      END
C*****************************************************************
      SUBROUTINE CHSLIP(DISP,IMAX,KIM,BNSL,BNSLMAX,BNSLMAXS,LSTLOC,
     +                  YLSLIP,YLFRSL,NA,FRSLMAX,FRSLMAXS,IMAXF)
C
C     to check if the slip exceeds the limit value
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION BNSL(150),DISP(9664),IMAX(150),BNSLFR(150),IMAXF(150)
      COMMON NUMELX,NUMELY
C
      DO I=1,NUMELX
         BNSL(I)=0.
         BNSLFR(I)=0.
      ENDDO
      NINC=(NUMELY+2)*2
      NST=LSTLOC*2-1+NINC
      NSTFR=(NUMELY+2)*2-1+NINC
      DO I=1,NUMELX
         J1=NST+(I-1)*NINC
         J2=J1+3
         BNSL(I)=DISP(J1)-DISP(J2)
         J3=NSTFR+(I-1)*NINC
         BNSLFR(I)=DISP(J3)
      ENDDO
C
      BNSLMAX=0.
      DO 881 I=1,NUMELX
         IF(KIM.EQ.1) GO TO 883
         DO K=1,KIM-1
            IF(I.EQ.IMAX(K)) GO TO 888
         ENDDO
  883    ABNSL=DABS(BNSL(I))
         IF(ABNSL.GE.BNSLMAX) THEN
           BNSLMAX=ABNSL
           IMAX(KIM)=I
           BNSLMAXS=BNSL(I)
         ENDIF
  888 CONTINUE
  881 CONTINUE
C
      FRSLMAX=0.
      DO 891 I=1,NUMELX
         IF(NA.EQ.1) GO TO 893
```

```
         DO K=1,NA-1
            IF(I.EQ.IMAXF(K)) GO TO 898
         ENDDO
  893    ABNSLF=DABS(BNSLFR(I))
         IF(ABNSLF.GE.FRSLMAX) THEN
           FRSLMAX=ABNSLF
           IMAXF(NA)=I
           FRSLMAXS=BNSLFR(I)
         ENDIF
  898 CONTINUE
  891 CONTINUE
C
      BNSLRAT=BNSLMAX/YLSLIP
      FRSLRAT=FRSLMAX/YLFRSL
      IF(BNSLRAT.GE.FRSLRAT) THEN
        KIM=KIM+1
        KIMOUT=KIM-2
        write(*,*) 'Concrete-Steel Bond-Slip =',KIMOUT
      ELSE
        NA=NA+1
        NAOUT=NA-2
        write(*,*) 'Concrete-Base  Bond-Slip =',NAOUT
      ENDIF
C
      RETURN
      END
C*************************************************************
      SUBROUTINE STRESS(IELINP,STRINI,INOD,LSTLOC,
     +                  ESTL,ALPS,DELT,DISP,D,CHSTRES,INDST)
C
C     to find stresses for concrete and steel bar
C
      IMPLICIT REAL*8 (A-H, O-Z)
      DIMENSION STR(4,3),AJAC(2),STRINI(30,4),INOD(4650,8)
      DIMENSION XI(4),ETA(4),XINT(2),YINT(2),DISP(9664)
      DIMENSION D(3,3),BMAT(3,8),FPXI(4),FPET(4),AUX(8,8),UL(8)
      COMMON NUMELX,NUMELY,A,B,E1,POIS1,THICK,ITNOD1,MBAND
      DATA XI/-1.,-1.,1.,1./,ETA/1.,-1.,1.,-1./
      DATA XINT/-0.57735027,0.57735027/,YINT/-0.57735027,0.57735027/
C
C     number of integration points in x and y directions
C
      NPX=2
      NPY=2
C
      ITNUMEL=NUMELX*(NUMELY+1)
      MINUS=-1*(NUMELY+1)
      DO LOC=ITNUMEL,0,MINUS
         IF(LOC.LT.IELINP) GO TO 77
      ENDDO
   77 ILOC1=IELINP-LOC
      IF(ILOC1.LT.LSTLOC) THEN
        ILOC=ILOC1
      ELSE
        ILOC=ILOC1-1
      ENDIF
C
      DO I=1,8
         J=INOD(IELINP,I)
         UL(I)=DISP(J)
      ENDDO
C
      NUMIP=1
```

```
C
      DO 200 IX=1,NPX
      DO 201 IY=1,NPY
C
      X=XINT(IX)
      Y=YINT(IY)
C
C     find derivatives of shape functions at integration points
C
      DO 20 I=1,4
         FPXI(I)=XI(I)*(1.+Y*ETA(I))/4.
         FPET(I)=ETA(I)*(1.+X*XI(I))/4.
   20 CONTINUE
C
C     define B matrix and transpose of B
C
      DO I=1,3
      DO J=1,8
      BMAT(I,J)=0.
      ENDDO
      ENDDO
      AJAC(1)=A/2.
      AJAC(2)=B/2.
      DO 21 I=1,4
         I1=2*I-1
         I2=I1+1
         BMAT(1,I1)=FPXI(I)/AJAC(1)
         BMAT(2,I2)=FPET(I)/AJAC(2)
         BMAT(3,I1)=BMAT(2,I2)
         BMAT(3,I2)=BMAT(1,I1)
   21 CONTINUE
C
C     calculate element stresses at integration points
C
      CALL MATPRO(D,BMAT,AUX,3,3,8,3,3,8)
      DO 23 I=1,3
         STRTEMP=0.
         DO 24 J=1,8
            STRTEMP=STRTEMP+AUX(I,J)*UL(J)
   24 CONTINUE
         STR(NUMIP,I)=STRTEMP
   23 CONTINUE
      NUMIP=NUMIP+1
C
  201 CONTINUE
  200 CONTINUE
C
      DO I=1,4
      DO J=1,2
         STR(I,J)=STR(I,J)+STRINI(ILOC,I)
      ENDDO
      ENDDO
C
      CHSTRES=(STR(1,1)+STR(2,1)+STR(3,1)+STR(4,1))/4.
C
      IF(INDST.EQ.1) THEN
C        WRITE(6,*) 'CONCRETE STRESS'
        STROUT1=0.
        STROUT2=0.
        STROUT3=0.
        DO I=1,4
         STROUT1=STR(I,1)+STROUT1
         STROUT2=STR(I,2)+STROUT2
```

```
          STROUT3=STR(I,3)+STROUT3
         ENDDO
         STROUT1=STROUT1/4.
         STROUT2=STROUT2/4.
         STROUT3=STROUT3/4.
         WRITE(6,205) IELINP,STROUT1,STROUT2,STROUT3
C         WRITE(6,*) STRINI(ILOC,I)
  205    FORMAT(I5,3X,E12.6,3X,E12.6,3X,E12.6)
       ENDIF
       IF(INDST.EQ.2) THEN
         WRITE(6,*) 'STRESS IN STEEL BAR'
         THERMAL=-ESTL*ALPS*DELT
         WRITE(6,*) 'INITIAL THERMAL STRESS=',THERMAL
         WRITE(6,*) 'ST. NO. FROM CENTER     STEEL STRESS'
         DO I=1,NUMELX
            J=LSTLOC+(I-1)*(NUMELY+1)
            SIGST=ESTL*(DISP(INOD(J,6))-DISP(INOD(J,5)))/A+THERMAL
            WRITE(6,207) I,SIGST
  207       FORMAT(I10,10X,E15.6)
         ENDDO
       ENDIF
C
       RETURN
       END
```