A DISCRETE-ELEMENT ANALYSIS FOR ANISOTROPIC SKEW PLATES AND GRIDS

by

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Development of Methods for Computer Simulation of Beam-Columns and Grid-Beam and Slab Systems

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PREFACE

This report describes a numerical method for the analysis of anisotropic skew plates or slabs with grid-beams. Relations are developed which simplify the computation of anisotropic slab stiffnesses.

The method was programmed and coded for use on a digital computer. Although the program was written for the CDC 6600 computer, it is also compatible with IBM 360 systems. Copies of the program presented in this report may be obtained from File D-8 Research, Texas Highway Department, Austin, Texas, or from the Center for Highway Research at The University of Texas at Austin.

This work was sponsored by the Texas Highway Department in cooperation with the U. S. Department of Transportation Federal Highway Administration, under Research Project 3-5-63-56. The Computation Center of The University of Texas at Austin contributed the computer time required for this study. The authors are grateful to these organizations and the many individuals who have assisted them during this study.

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LIST OF REPORTS

Report No. 56-1, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns" by Hudson Matlock and T. Allan Haliburton, presents a finite-element solution for beam-columns that is a basic tool in subsequent reports.

Report No. 56-2, "A Computer Program to Analyze Bending of Bent Caps" by Hudson Matlock and Wayne B. Ingram, describes the application of the beam-column solution to the particular problem of bent caps.

Report No. 56-3, "A Finite-Element Method of Solution for Structural Frames" by Hudson Matlock and Berry Ray Grubbs, describes a solution for frames with no sway.

Report No. 56-4, "A Computer Program to Analyze Beam-Columns under Movable Loads" by Hudson Matlock and Thomas P. Taylor, describes the application of the beam-column solution to problems with any configuration of movable non-dynamic loads.

Report No. 56-5, "A Finite-Element Method for Bending Analysis of Layered Structural Systems" by Wayne B. Ingram and Hudson Matlock, describes an alternating-direction iteration method for solving two-dimensional systems of layered grids-over-beams and plates-over-beams.

Report No. 56-6, "Discontinuous Orthotropic Plates and Pavement Slabs" by W. Ronald Hudson and Hudson Matlock, describes an alternating-direction iteration method for solving complex two-dimensional plate and slab problems with emphasis on pavement slabs.

Report No. 56-7, "A Finite-Element Analysis of Structural Frames" by T. Allan Haliburton and Hudson Matlock, describes a method of analysis for rectangular plane frames with three degrees of freedom at each joint.

Report No. 56-8, "A Finite-Element Method for Transverse Vibrations of Beams and Plates" by Harold Salani and Hudson Matlock, describes an implicit procedure for determining the transient and steady-state vibrations of beams and plates, including pavement slabs.

Report No. 56-9, "A Direct Computer Solution for Plates and Pavement Slabs" by C. Fred Stelzer, Jr., and W. Ronald Hudson, describes a direct method for solving complex two-dimensional plate and slab problems.

Report No. 56-10, "A Finite-Element Method of Analysis for Composite Beams" by Thomas P. Taylor and Hudson Matlock, describes a method of analysis for composite beams with any degree of horizontal shear interaction.

Report No. 56-11, "A Discrete-Element Solution of Plates and Pavement Slabs Using a Variable-Increment-Length Model" by Charles M. Pearre, III, and W. Ronald Hudson, presents a method of solving for the deflected shape of freely discontinuous plates and pavement slabs subjected to a variety of loads.

Report No. 56-12, "A Discrete-Element Method of Analysis for Combined Bending and Shear Deformations of a Beam" by David F. Tankersley and William P. Dawkins, presents a method of analysis for the combined effects of bending and shear deformations.

Report No. 56-13, "A Discrete-Element Method of Multiple-Loading Analysis for Two-Way Bridge Floor Slabs" by John J. Panak and Hudson Matlock, includes a procedure for analysis of two-way bridge floor slabs continuous over many supports.

Report No. 56-14, "A Direct Computer Solution for Plane Frames" by William P. Dawkins and John R. Ruser, Jr., presents a direct method of solution for the computer analysis of plane frame structures.

Report No. 56-15, "Experimental Verification of Discrete-Element Solutions for Plates and Slabs" by Sohan L. Agarwal and W. Ronald Hudson, presents a comparison of discrete-element solutions with the small-dimension test results for plates and slabs, along with some cyclic data on the slab.

Report No. 56-16, "Experimental Evaluation of Subgrade Modulus and Its Application in Model Slab Studies" by Qaiser S. Siddiqi and W. Ronald Hudson, describes an experimental program developed in the laboratory for the evaluation of the coefficient of subgrade reaction for use in the solution of small dimension slabs on layered foundations based on the discrete-element method.

Report No. 56-17, "Dynamic Analysis of Discrete-Element Plates on Nonlinear Foundations" by Allen E. Kelly and Hudson Matlock, presents a numerical method for the dynamic analysis of plates on nonlinear foundations.

Report No. 56-18, "A Discrete-Element Analysis for Anisotropic Skew Plates and Grids" by Mahendrakumar R. Vora and Hudson Matlock, describes a tridirectional model and a computer program for the analysis of anisotropic skew plates or slabs with grid-beams.

Report No. 56-19, "An Algebraic Equation Solution Process Formulated in Anticipation of Banded Linear Equations" by Frank L. Endres and Hudson Matlock, describes a system of equation-solving routines that may be applied to a wide variety of problems by utilizing them within appropriate programs.

Report No. 56-20, "Finite-Element Method of Analysis for Plane Curved Girders" by William P. Dawkins, presents a method of analysis that may be applied to plane-curved highway bridge girders and other structural members composed of straight and curved sections.

Report No. 56-21, "Linearly Elastic Analysis of Plane Frames Subjected to Complex Loading Condition" by Clifford O. Hays and Hudson Matlock, presents a design-oriented computer solution of plane frame structures that has the capability to economically analyze skewed frames and trusses with variable cross-section members randomly loaded and supported for a large number of loading conditions.

ABSTRACT

A discrete-element method of analysis for anisotropic skew-plate and grid-beam systems is presented. The method can be used to solve a wide variety of problems. The principal features are

(1) formulation of six elastic stiffnesses and compliances in terms of three moduli of elasticity in any three directions and three Poisson's ratios related to these directions,

(2) representation of an anisotropic skew-plate and grid system by a discrete-element model consisting of a tridirectional arrangement of rigid bars and elastic joints,

(3) formulation of stress-strain and moment-curvature relations for the discrete-element model using concepts of a continuum composed of interconnected fibers,

(4) derivation of a stiffness matrix using equations of statics, and

(5) a recursion-inversion procedure to solve the stiffness equations.

The method allows free variation in stiffnesses, loads, and supports. Concentrated and distributed loads and supports and external couples in three directions, including grid-beams in three directions, are easily handled. A computer program has been written to check the formulation. The results compare well with the results from other approximate methods and with experimental data.

KEY WORDS: anisotropic elasticity, skew plate, skew grid, slab-grid system, skew bridge, discrete-element analysis, computers, bridges, plates.

SUMMARY

This study presents a method for the analysis of isotropic or anisotropic skew-plate and skew-grid-beam systems. The method can be used to solve a wide variety of problems and is particularly suited to analysis of skewed highway bridges and pavements.

A discrete-element analog is used to represent the actual structure, and formulation of the equations solved is based on this mechanical assembly. The assembly is such that the stiffnesses, geometric properties, loads, and restraints of the real system are represented in an accurate manner. The assembly is composed of an anisotropic plate with three tridirectional stiffnesses. Relations are developed in which the anisotropic plate stiffnesses are related to three moduli of elasticity in any three directions and three Poisson's ratios related to these directions. The six elastic constants can be determined by testing simple uniaxial specimens taken from the plate in any three directions. A grid beam assemblage may also be present and is oriented in the same three independent directions. These beams transfer only bending moment.

The included computer program, SLAB 44, is written in FORTRAN for the CDC 6600 computer and is easily made compatible with IBM 360, UNIVAC 1108, and other comparable computer systems.

A series of example problems is included to demonstrate and verify the method. No exact closed-form solution is available for even the simplest skew plate but the results compare favorably with several approximate methods. In addition, comparison is made with experimental results taken from a skewed, prestressed bridge modeled to a 5.5-to-1 ratio. The model represents a standard Texas Highway Department bridge structure. The computed results compare closely to the measured values within the elastic response range of the model.

A guide for data input is presented which allows routine application of the method of analysis with little necessary reference to the body of the main report. Any number of analyses may be run at the same time.

IMPLEMENTATION STATEMENT

The problems associated with analysis of skewed highway structures have long been difficult for the highway engineer to solve. The use of approximate distribution factors and strip methods has for years furnished the engineer with convenient design approximations, but it can be shown that extending these methods to heavily skewed structures may cause extreme complications associated with the inherent twisting which is not considered.

In this study, a computer program (SLAB 44) is developed for computer simulation and analysis of skewed slab and grid systems. The potential application of this work ranges from sensitivity studies of skewed bridge geometries to the day-to-day design of any skewed bridge structure. In addition, it may be used to study some other effects of skewed slabs such as skew angles, aspect ratios, and diaphragm placements. Furthermore, the coupling of research results of the skewed model test project with this program will make available to the highway engineer procedures which will permit better analysis of many types of structures.

Program SLAB 44 has recently been applied to the analysis of a skewed, post-stressed continuous slab structure. Very good correlations have been made between the analysis and a brief, full-scale load test of the structure located in Pasadena, Texas. The investigation was initiated by the non-load induced failure of a companion skewed structure.

Recommendations are made for further research in the area of nonlinear response, especially concerning concrete slab characteristics. It is possible to modify and extend the computer method presented in this work to include nonlinear effects.

It is further recommended that this program be put into test use by designers of the Texas Highway Department to further evaluate its uses, and to investigate needed extensions or modifications to make it more usable for the practicing design engineer.

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NOMENCLATURE

Symbol	Typical Units	Definition
a	--	Terms in stiffness matrix
$[a]$	--	Submatrix of stiffness matrix
$\{A\}$	--	Recursion coefficient vector
b	--	Terms in stiffness matrix
$[b]$	--	Submatrix of stiffness matrix
B	$lb\text{-}in^2/in$	Stiffness of plate model
$[B]$	--	Recursion coefficient matrix
c	lb/in^2	Elastic stiffness
c	--	Terms in stiffness matrix
$[c]$	--	Submatrix of stiffness matrix
$[C]$	--	Recursion coefficient matrix
d	--	Terms in stiffness matrix
$[d]$	--	Submatrix of stiffness matrix
D	$lb\text{-}in^2/in$	Plate stiffness
$[D]$	--	Recursion coefficient matrix
e	--	Terms in stiffness matrix
$[e]$	--	Submatrix of stiffness matrix

Symbol	Typical Units	Definition
E	lb/in^2	Modulus of elasticity
$[E]$	--	Recursion coefficient matrix
f	lb/in^2	Fiber stress
f	lb	Term in load vector
$\{f\}$	--	Load vector
F	$lb\text{-}in^2$	Beam stiffness
G	lb/in^2	Shear modulus
h	inch	Increment length
i	--	Numbering associated with a-direction
j	--	Numbering associated with c-direction
$[K]$	--	Stiffness matrix
ℓ	--	Cosine of an angle
m	--	Sine of an angle
M	in-lb/in	Moment per unit width in slab
M'	in-lb	Concentrated moment in slab model
$\overline{M}$	in-lb	Moment in beam
Q	lb	Load
s	in^2/lb	Elastic compliances
S	lb/in	Support spring
t	inch	Thickness of plate
T	in-lb	External couple
V	lb	Shear
w	inch	Deflection

Symbol	Typical Units	Definition
$\{w\}$	--	Deflection vector
γ	in/in	Shearing strain
ϵ	in/in	Normal strain
η	--	Coefficient of mutual influence
θ	degrees	Angle
μ	--	Directional Poisson effect
ν	--	Poisson's ratio
σ	lb/in^2	Normal stress
τ	lb/in^2	Shearing stress

CHAPTER 1. INTRODUCTION

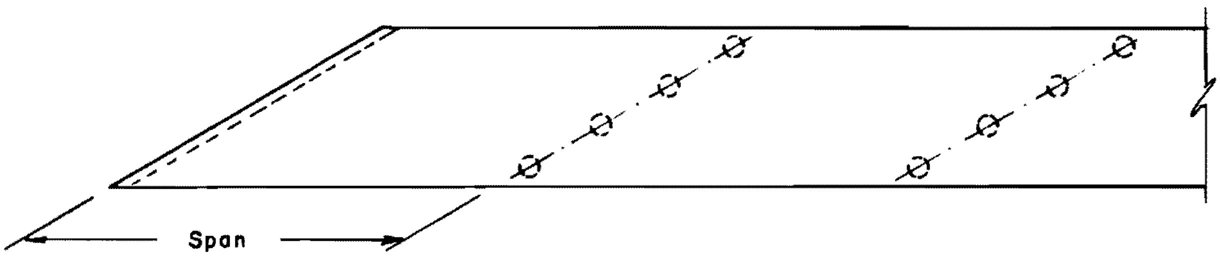
Skew slabs or plates with skew ribs occur frequently in modern structures such as airplane wings, highway bridges, and building floors, and their analysis is always difficult. There are no closed-form mathematical solutions available for even the simplest cases, except a simple triangular plate given by Timoshenko and Woinowsky-Krieger (Ref 40). The practicing engineer must use some approximate procedure for analysis. To analyze a continuous prestressed concrete skew slab bridge of two, three, or more spans, for example (Fig 1), he may choose a strip of slab in the span direction and consider it as a beam. This kind of approximation might be reasonable for a rectangular slab bridge but may be inappropriate in the case of a skew slab bridge. Generally, because of the presence of large twisting effects, the largest principal moments are not in the span direction.

The objective of this study is to develop relations for elastic compliances such that the computation of anisotropic plate stiffnesses is simplified, and to develop a discrete-element method of analysis for anisotropic skew-plate and grid systems in which the grid-beams may run in any three directions.

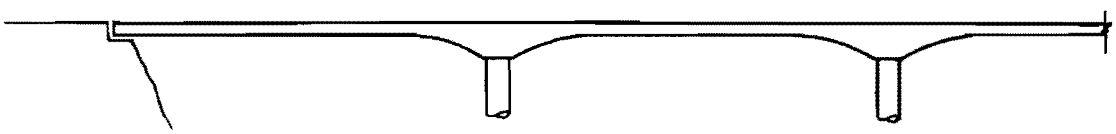
Previous Studies

Many investigators have attempted to analyze skew plate problems. Some of the methods are discussed here.

Finite Difference. In the finite difference approach, the partial differential equation and the boundary conditions are replaced by difference equations, which may be solved by any procedure. The parallelogram-shaped mesh could be used to fit the boundaries exactly. Using this type of mesh, Favre (Ref 9) solved simply supported skew plates, while Chen, Siess, and Newmark (Ref 6) solved a single-span, noncomposite skew bridge consisting of a concrete slab of uniform thickness supported by five identical steel beams. Morley (Ref 24) observed that when this type of mesh is used the convergence of solution, as indicated by advances to a finer mesh, deteriorates with increasing angle of skew in the case of simply supported uniformly loaded skew plates.



PLAN



LONGITUDINAL SECTION

Fig 1. Continuous prestressed concrete skew bridge.

Jensen et al (Refs 14 and 15) used an alternate system of finite difference equations. Robinson (Ref 35) and Naruoka (Ref 25) extended Jensen's finite difference procedure to compute influence coefficients for several skew plates while Naruoka et al (Refs 26, 27, 28, and 29) solved orthotropic parallelogram plates and gave several numerical and experimental results. All these studies are for a single span and for either isotropic or orthotropic plates.

Electrical Analog. In this method, the network of an electrical analog automatically solves the finite difference equations within the boundary while at the boundary the potentials are adjusted until the boundary conditions are satisfied. Rushton et al (Refs 11, 34, and 36) utilized this procedure to solve skew plates with various boundary conditions, including a four-span flat-slab 45-degree-skew bridge. He observed that for large angles of skew there was no apparent decrease in the accuracy of the deflections. Only isotropic plates were considered in his study.

Conformal Mapping. Aggarwala (Refs 1 and 2) used a conformal mapping procedure in which a parallelogram was mapped on the unit circle and obtained solutions for plates under transverse loadings. Only simply supported isotropic plates have been solved.

Finite-Element. In the finite-element method, the structure is idealized as an assemblage of deformable elements linked together at the nodal points, where the continuity and equilibrium are established. Using different types of elements, several investigators, including West (Ref 42), Mehrain (Ref 22), Cheung, King, and Zienkiewicz (Ref 7), Gustafson and Wright (Ref 10), and Sawko and Cope (Ref 37), have studied the problem. All of these studies were for either isotropic or orthotropic plates. Mehrain (Ref 22) has studied the skew problem extensively, making a comparative analysis of various forms of finite elements, and has observed that the accuracy of the finite-element solution drops rapidly when the angle of skew is increased in the case of simply supported uniformly loaded plates.

Series. In this method, with the fourth-order partial differential equation governing the deflection of the plate, a solution is obtained in which the deflection function is expressed in the form of a series. Quinlan (Ref 33), Kennedy and Huggins (Ref 16), and Morley (Refs 23 and 24) have presented solutions using different forms of series. These solutions are for single-span isotropic plates. Morley's results are the most extensive and several investigators have used these results as the basis for comparison with their methods.

Other Solutions. Akay (Ref 3) used a double-net model to solve for orthotropic skew plates with a boundary condition of either two opposite edges simply supported or all four edges simply supported. Several examples have been solved and the results compared with the solutions from other approaches. Suchar (Ref 39) dealt with anisotropic skew plates and obtained polynomial solutions to the governing differential equation using oblique coordinates. These polynomials were then used to calculate the influence surface for an orthotropic parallelogram plate with two opposite sides simply supported and the remaining edges free.

Present Study

It can be seen that except for Suchar (Ref 39) the studies were limited to either isotropic or orthotropic plates and also that most of the methods developed were for particular loading or boundary conditions.

In the present study a mechanical model consisting of a tridirectional system of rigid bars and elastic joints was used to simulate anisotropic skew plates plus slab-and-grid systems in which the grid-beams may run in any three directions. The model developed and the relations formulated are not limited to bending analysis but could also be adapted for plane stress analysis.

Discrete-Element Model

Chapter 2 describes a discrete-element model used to analyze anisotropic skew-plate and grid systems. Assumptions made for the solution of the model are also given.

Anisotropic Relations

Hearmon (Ref 12) and Lekhnitskii (Ref 17) have developed stress-strain relations in Cartesian coordinates for an anisotropic homogeneous body. For the problem of plane-stress in two dimensions, these relations require the computation of six elastic stiffnesses in terms of six independent elastic constants: moduli of elasticity in the x and y-directions, one Poisson's ratio, one shear modulus, and two coefficients of mutual influence of the first kind.

In Chapter 3, relations are developed in which the six elastic stiffnesses are related to three moduli of elasticity in any three directions and three Poisson's ratios related to these directions. This simplification is helpful in determining the six elastic constants by testing three simple uniaxial specimens taken from the plate at any three directions. Since the integration of stress-strain relations gives moment-curvature relations, the six anisotropic plate stiffnesses also may be computed in terms of three moduli of elasticity and three Poisson's ratios.

Using concepts of a continuum composed of interconnected fibers, stress-strain relations for the anisotropic discrete slab model are derived in Chapter 4. Moment-curvature relations for the slab and grid models are also derived.

Stiffness Matrix

In Chapter 5, equations of statics are used to derive a stiffness matrix for the discrete-element model. Chapter 6 describes the recursion-inversion solution procedure used to solve the stiffness equations.

Verification of Model

Chapter 7 describes a computer program written to verify the formulation. Several example problems are solved in Chapter 8, and results are compared with the closed-form solution for a triangular plate; with the solutions from other approximate methods, such as series, finite-element, conformal mapping, finite difference, and electrical analog; and with experimental results.

The appendices contain the guide for data input, general program flow chart, notations, program listing, listing of input data, and selected output.

CHAPTER 2. PROPOSED TRIDIRECTIONAL DISCRETE-ELEMENT MODEL

Introduction

In the discrete-element method of analysis a system (beam, plate, and plate and grid-beams) is replaced by an analogous physical model and then the analysis of the model is made. The mechanical assembly of this model should be such that it can represent the stiffnesses, geometric properties, loads, and restraints of the real system. This kind of approach has been used by several investigators including Matlock (Refs 18 and 19) for beam-column, Tucker (Ref 41) for rectangular grid-beam problems, and Newmark (Ref 30), Ang and Newmark (Ref 4), and Hudson (Ref 13) for rectangular plate problems.

A discrete-element model for a skew-plate and grid-beam system is proposed. In it the plate may be completely anisotropic and grid-beams may run in any three directions. In this chapter, the functions of different components of the model are explained and the assumptions required for the analysis of the model are listed.

Discrete-Element Model

A discrete-element model is to be worked out to be used to solve the following:

(1) an anisotropic skew plate or slab,

(2) a grid-beam system in which the beams may run in any three directions or less, and

(3) a combination problem, i.e., an anisotropic skew-plate and grid-beam system in which the beams may run in any three directions.

Figure 2 shows the proposed tridirectional model for plates and Fig 3 shows a typical grid-beam model.

Components of Model

The model of a plate (Fig 2) will consist of elastic joints connected by rigid bars running in directions a , b , and c .

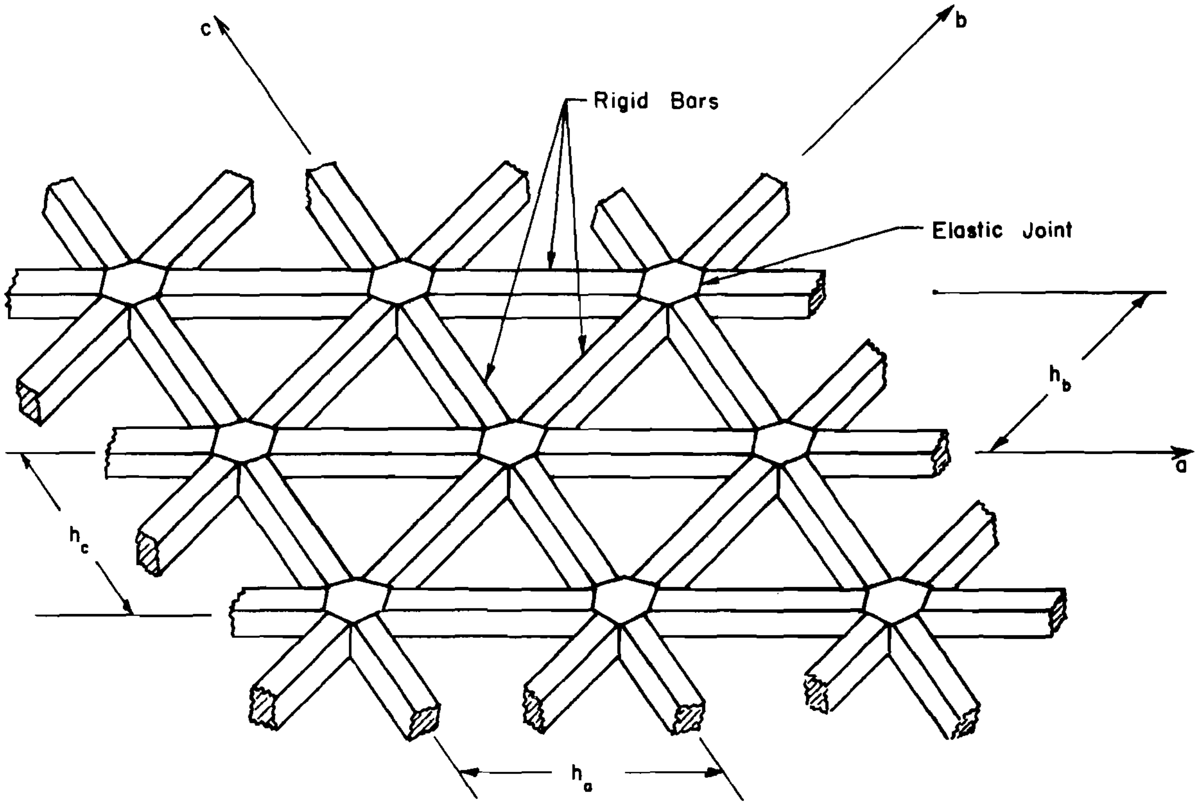


Fig 2. Discrete-element model for anisotropic skew plate showing all components.

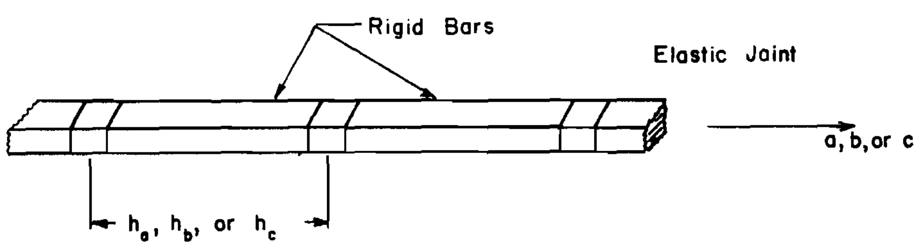


Fig 3. Discrete-element model for a typical grid-beam showing all components.

The model of a grid-beam (Fig 3) in a particular direction will consist of elastic joints connected by rigid bars running in that direction. The model of a grid-beam will be the same as the model of a beam-column worked out by Matlock (Refs 18 and 21).

Function of Each Component

The stiffnesses, loads, and restraints will be lumped at elastic joints, and hence all elastic action will take place at these joints. The only function of the rigid bars will be to transfer bending moments from one elastic joint to another without deforming.

Connection

The plate model and the three grid-beam models of the grid-beam system will be connected with one another at elastic joints. The rigid bars of different systems will have no connection with one another. Therefore at any particular elastic joint, the deflection of all the four systems should be the same.

Assumptions Related to Conventional Plates and Grid-Beams

The following assumptions are related to the conventional plates and grid-beams and are included in the subsequent discrete-element development. The first three are the same as shown by Timoshenko (Ref 40) for thin plates with small deflections.

(1) There is no axial deformation in the middle plane of the plate. This plane remains neutral during bending.

(2) Points of the plate lying initially on a normal-to-the-middle plane of the plate remain on a normal-to-the-middle plane of the plate after bending.

(3) The normal stresses in the direction perpendicular to the plate can be disregarded.

(4) All deformations are small with regard to the dimensions of the plate and grid system.

(5) The neutral axis of a plate with grid-beams is in the same level even though the cross sections of the plate and of each grid-beam may be nonuniform.

Assumptions Related to Discrete-Element Model for Plates and Grid-Beams

In addition to the above, the following assumptions are made for discrete-element model for plates and grid-beams.

(1) Each elastic joint is of infinitesimal size and composed of an elastic, but anisotropic, material. Curvature appears at the joint as concentrated angle change.

(2) The rigid bars of the models (Figs 2 and 3) are infinitely stiff and weightless. They transfer bending moments by means of equal and opposite shears. They are torsionally soft; i.e., they do not transfer twisting moment. They do not deform due to in-plane (axial) forces.

(3) The stiffnesses of plates and of grid-beams may vary from point to point.

(4) The spacing of elastic joints in the a and c-directions, designated h_a and h_c, respectively, need not be equal but must be constant. The spacing in the b-direction is equal to the length of the diagonal of the parallelogram having sides h_a and h_b (Fig 2).

Summary

The anisotropic plate and grid system is to be represented by a physical model having only one degree of freedom at each joint. The model will be helpful in visualization of the real problem. Discontinuous changes in stiffnesses, loads, and supports may be accommodated easily in the model. Where numerical word length is not a limitation, errors in the solution are due to approximating the real system with the model and not to the solution of model. Thus, accuracy of the solution will depend upon the number of increments used in the solution.

CHAPTER 3. ANISOTROPIC STRESS-STRAIN RELATIONS

Introduction

For plane stress problems, the anisotropic stress-strain relations require computation of six elastic compliances or six elastic stiffnesses. Hearmon (Ref 12) and Lekhnitskii (Ref 17) have shown that in Cartesian coordinates the compliances could be related to six independent elastic constants (moduli of elasticity in the x and y-directions, one shear modulus, one Poisson's ratio, and two coefficients of mutual influence of the first kind). Hearmon (Ref 12) has also described experiments required to determine the six compliances.

In this chapter, relations are worked out in which the six compliances and six stiffnesses are related to three moduli of elasticity with respect to any three directions and three Poisson's ratios related to these directions. This simplification is helpful in understanding and in computing the elastic compliances and stiffnesses.

Transformation relations have also been worked out whereby the modulus of elasticity and Poisson's ratio in any desired direction may be obtained from three moduli of elasticity and three Poisson's ratios related to any other three directions.

Hooke's Law

Hooke's law states that each stress component is directly proportional to each strain component. If σ represents stress and ϵ represents strain,

$$\sigma_{ij} = c_{ijk\ell}\epsilon_{k\ell} \tag{3.1}$$

and

$$\epsilon_{ij} = s_{ijk\ell}\sigma_{k\ell} \tag{3.2}$$

wherein i , j , k and ℓ take on all combinations of 1, 2 and 3.

The terms $c_{ijk\ell}$ are called elastic stiffnesses and $s_{ijk\ell}$ the elastic compliances. Equations 3.1 and 3.2 show that there are 81 stiffnesses and compliances. It can be shown (Ref 12) that $\sigma_{ij} = \sigma_{ji}$ and $\epsilon_{k\ell} = \epsilon_{\ell k}$. This results in $c_{ijk\ell} = c_{jik\ell} = c_{ij\ell k} = c_{ji\ell k}$ and $s_{ijk\ell} = s_{jik\ell} = s_{ij\ell k} = s_{ji\ell k}$ and reduces the number of stiffnesses and compliances to 36. It has been shown by Hearmon (Ref 12) that by thermodynamic argument $c_{ijk\ell} = c_{k\ell ij}$ and $s_{ijk\ell} = s_{k\ell ij}$ and these reciprocal relations further reduce stiffnesses and compliances to 21 in the most general case.

For plane stress problems, if σ_x and σ_y are the normal stresses in the x and y-directions, respectively, and τ_{xy} is the shearing stress, and if ϵ_x, ϵ_y, and γ_{xy} are the corresponding strains, as shown in Fig 4(a), then Hooke's law in Cartesian coordinates has the form

$$\sigma_x = c_{11}\epsilon_x + c_{12}\epsilon_y + c_{13}\gamma_{xy}$$

$$\sigma_y = c_{21}\epsilon_x + c_{22}\epsilon_y + c_{23}\gamma_{xy}$$

$$\tau_{xy} = c_{31}\epsilon_x + c_{32}\epsilon_y + c_{33}\gamma_{xy} \qquad (3.3)$$

or

$$\epsilon_x = s_{11}\sigma_x + s_{12}\sigma_y + s_{13}\tau_{xy}$$

$$\epsilon_y = s_{21}\sigma_x + s_{22}\sigma_y + s_{23}\tau_{xy}$$

$$\gamma_{xy} = s_{31}\sigma_x + s_{32}\sigma_y + s_{33}\tau_{xy} \qquad (3.4)$$

The stiffnesses and compliances in Eqs 3.3 and 3.4 satisfy the reciprocal relations which reduce the number of independent constants from nine to six. Hence, in general, for an anisotropic thin plate in a state of plane stress it is necessary to know the values of six different quantities to calculate elastic behavior. These reduce to two in the case of isotropic plates.

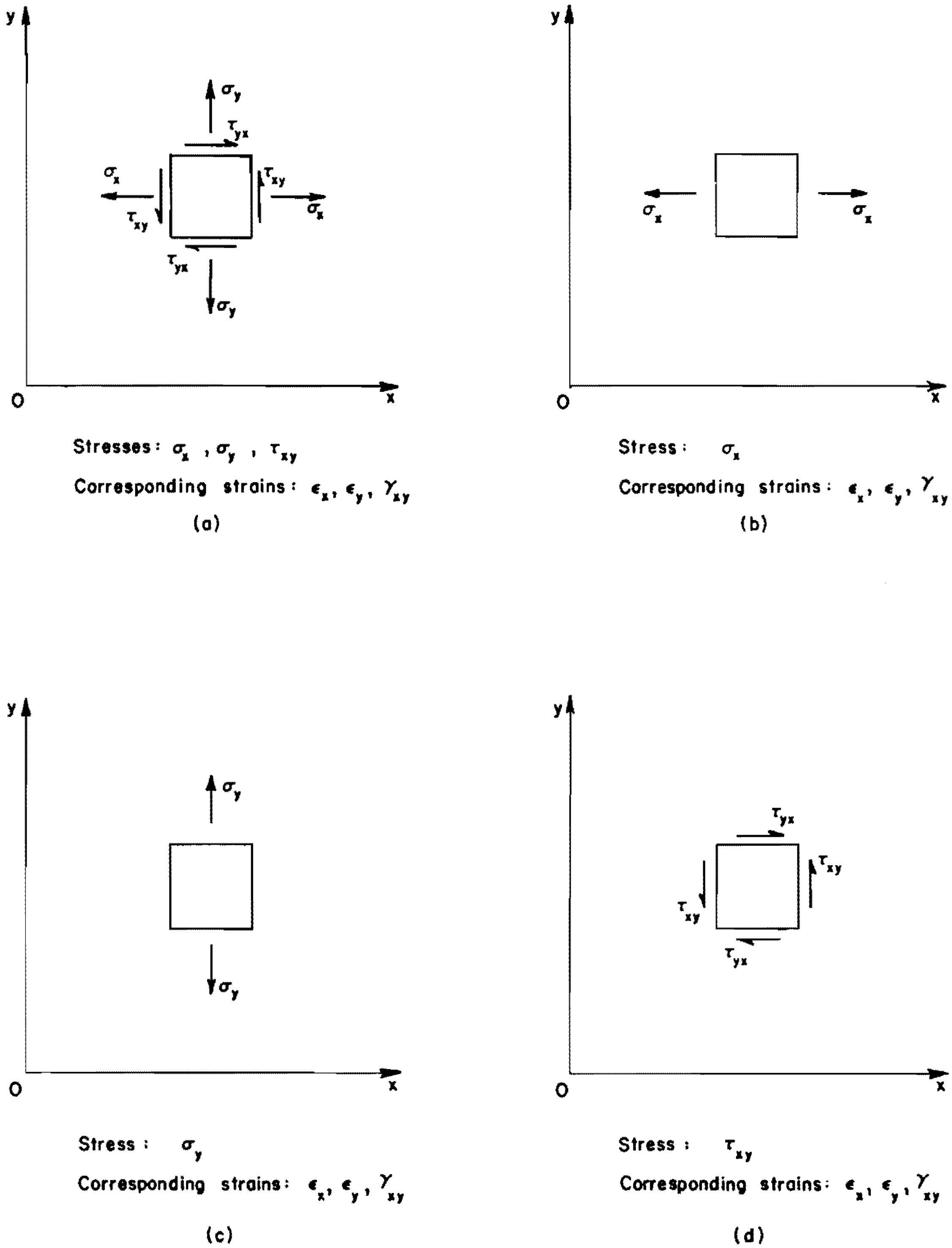


Fig 4. Stresses and corresponding strains for anisotropic plate element.

Elastic Compliances for Anisotropic Thin Plates

Consider a small rectangular element of a thin anisotropic plate: if the only stress acting on this element is σ_x , as shown in Fig 4(b), and the corresponding strains are ϵ_x , ϵ_y , and γ_{xy} , then from Eq 3.4

$$\epsilon_x = s_{11}\sigma_x \quad \underline{\text{or}} \quad s_{11} = \frac{\epsilon_x}{\sigma_x} = \frac{1}{E_x} \tag{3.5}$$

$$\epsilon_y = s_{21}\sigma_x \quad \underline{\text{or}} \quad s_{21} = \frac{\epsilon_y}{\sigma_x} = -\frac{\nu_{xy}}{E_x} \tag{3.6}$$

and

$$\gamma_{xy} = s_{31}\sigma_x \quad \underline{\text{or}} \quad s_{31} = \frac{\gamma_{xy}}{\sigma_x} = \frac{\eta_{xy,x}}{E_x} \tag{3.7}$$

If the only stress acting on the element is σ_y , as shown in Fig 4(c), and the corresponding strains are ϵ_x , ϵ_y , and γ_{xy} then

$$\epsilon_y = s_{22}\sigma_y \quad \underline{\text{or}} \quad s_{22} = \frac{\epsilon_y}{\sigma_y} = \frac{1}{E_y} \tag{3.8}$$

$$\epsilon_x = s_{12}\sigma_y \quad \underline{\text{or}} \quad s_{12} = \frac{\epsilon_x}{\sigma_y} = -\frac{\nu_{yx}}{E_y} \tag{3.9}$$

and

$$\gamma_{xy} = s_{32}\sigma_y \quad \underline{\text{or}} \quad s_{32} = \frac{\gamma_{xy}}{\sigma_y} = \frac{\eta_{xy,y}}{E_y} \tag{3.10}$$

Finally, if the only stress acting on the element is τ_{xy} , as shown in Fig 4(d), and the corresponding strains are ϵ_x , ϵ_y , and γ_{xy} then

$$\gamma_{xy} = s_{33}\tau_{xy} \quad \underline{or} \quad s_{33} = \frac{\gamma_{xy}}{\tau_{xy}} = \frac{1}{G_{xy}} \tag{3.11}$$

$$\epsilon_x = s_{13}\tau_{xy} \quad \underline{or} \quad s_{13} = \frac{\epsilon_x}{\tau_{xy}} = \frac{\eta_{x,xy}}{G_{xy}} \tag{3.12}$$

and

$$\epsilon_y = s_{23}\tau_{xy} \quad \underline{or} \quad s_{23} = \frac{\epsilon_y}{\tau_{xy}} = \frac{\eta_{y,xy}}{G_{xy}} \tag{3.13}$$

In Eqs 3.5 through 3.13, E_x and E_y are the Young's moduli (for tension-compression) with respect to the x and y-directions; G_{xy} is the shear modulus; ν_{xy} is the Poisson's ratio which characterizes the decrease in the y-direction for the tension in the x-direction; ν_{yx} is the Poisson's ratio which characterizes the decrease in the x-direction for the tension in the y-direction; $\eta_{xy,x}$ and $\eta_{xy,y}$ are the coefficients of mutual influence of the first kind (Ref 17) which involve the ratio of shearing strain to normal strain; and $\eta_{x,xy}$ and $\eta_{y,xy}$ are the coefficients of mutual influence of the second kind (Ref 17) which involve the ratio of normal strain to shearing strain.

Owing to the reciprocal relations,

$$s_{12} = s_{21} \quad \underline{or} \quad -\frac{\nu_{xy}}{E_x} = -\frac{\nu_{yx}}{E_y} \tag{3.14}$$

$$s_{13} = s_{31} \quad \underline{or} \quad \frac{\eta_{x,xy}}{G_{xy}} = \frac{\eta_{xy,x}}{E_x} \tag{3.15}$$

and

$$s_{23} = s_{32} \quad \underline{or} \quad \frac{\eta_{y,xy}}{G_{xy}} = \frac{\eta_{xy,y}}{E_y} \tag{3.16}$$

Hence for an anisotropic thin plate in a state of plane stress, six elastic compliances s_{11} , s_{12} , s_{13} , s_{22} , s_{23} , and s_{33} can be evaluated with known values of six independent constants E_x , E_y , G_{xy} , ν_{xy} (or ν_{yx}) , $\eta_{xy,x}$ (or $\eta_{x,xy}$) , and $\eta_{xy,y}$ (or $\eta_{y,xy}$) .

Combining the above results, the six compliances can be written as

$$s_{11} = \frac{1}{E_x}$$

$$s_{12} = -\frac{\nu_{xy}}{E_x}$$

$$s_{13} = \frac{\eta_{xy,x}}{E_x}$$

$$s_{22} = \frac{1}{E_y}$$

$$s_{23} = \frac{\eta_{xy,y}}{E_y}$$

and

$$s_{33} = \frac{1}{G_{xy}} \tag{3.17}$$

Elastic Compliances in Terms of Three Moduli of Elasticity and Three Poisson's Ratios

Another approach has been worked out to compute the six elastic compliances. In it the compliances are functions of three moduli of elasticity in any three directions a , b , and c , as shown in Fig 5(a), and the three Poisson's ratios related to these directions. Angle θ_1 between directions a and b , angle θ_2 between directions a and c , and angle θ_3 between directions b and c can have any value except 0 and 180 degrees. For convenience, directions x and a are taken as the same but, in general, the

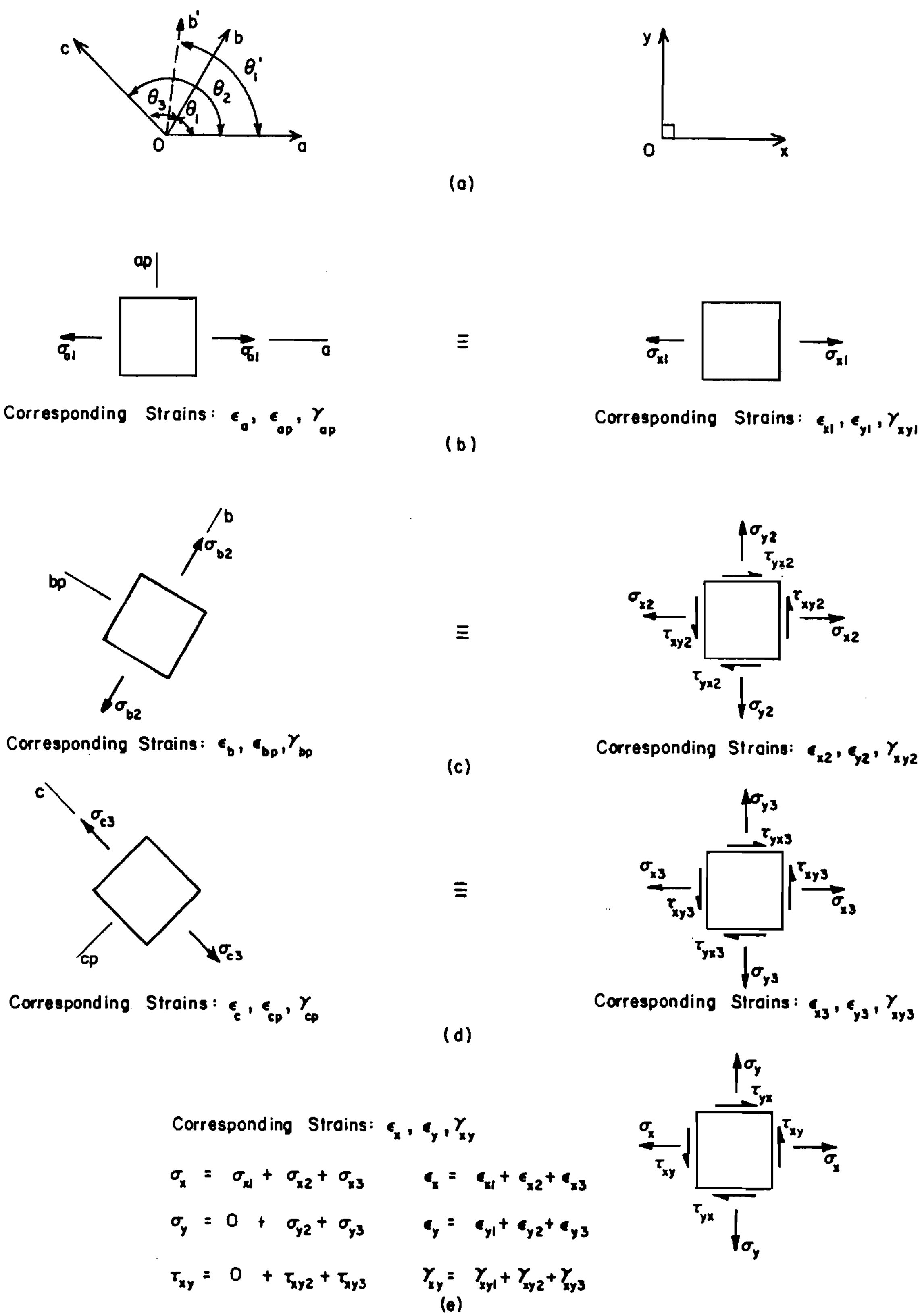


Fig 5. Stresses and corresponding strains for anisotropic plate elements in different directions.

angle between directions x and a need not be zero. This approach can be worked out as follows.

Consider a small rectangular element as shown in Fig 5(b). If the only stress acting on this element is σ_{a1} and the corresponding strains are ϵ_a, ϵ_{ap}, and γ_{ap}, where ϵ_a and ϵ_{ap} are strains in the a-direction and perpendicular to the a-direction (or ap-direction) and γ_{ap} is shearing strain, then

$$\epsilon_a = \frac{\sigma_{a1}}{E_a}$$

$$\epsilon_{ap} = -\frac{\nu_a \sigma_{a1}}{E_a}$$

$$\gamma_{ap} = \frac{\eta_a \sigma_{a1}}{E_a} \tag{3.18}$$

where E_a is the modulus of elasticity with respect to the a-direction; ν_a is the Poisson's ratio which characterizes the decrease perpendicular to the a-direction (or ap-direction) for tension in the a-direction; and η_a is the coefficient of mutual influence of the first kind related to the a-direction. Now for the same state of stress, if the element is oriented with respect to the x and y-directions, as shown in Fig 5(b) (the x and a-directions are the same in this case), then

$$\epsilon_{x1} = \frac{\sigma_{a1}}{E_a}$$

$$\epsilon_{y1} = -\frac{\nu_a \sigma_{a1}}{E_a}$$

$$\gamma_{xy1} = \frac{\eta_a \sigma_{a1}}{E_a} \tag{3.19}$$

and

$$\sigma_{x1} = \sigma_{a1}$$

$$\sigma_{y1} = 0$$

$$\tau_{xy1} = 0 \tag{3.20}$$

where σ_{x1} and σ_{y1} are the normal stresses in the x and y-directions, respectively; τ_{xy1} is the shearing stress; and ϵ_{x1}, ϵ_{y1}, and γ_{xy1} are the corresponding strains.

Now consider a small rectangular element as shown in Fig 5(c). If the only stress acting on this element is σ_{b2} and the corresponding strains are ϵ_b, ϵ_{bp}, and γ_{bp}, where ϵ_b and ϵ_{bp} are strains in the b-direction and perpendicular to the b-direction (or bp-direction) and γ_{bp} is the shearing strain, then

$$\epsilon_b = \frac{\sigma_{b2}}{E_b}$$

$$\epsilon_{bp} = -\frac{\nu_b \sigma_{b2}}{E_b}$$

$$\gamma_{bp} = \frac{\eta_b \sigma_{b2}}{E_b} \tag{3.21}$$

where E_b is the modulus of elasticity with respect to the b-direction; ν_b is the Poisson's ratio which characterizes the decrease perpendicular to the b-direction (or bp-direction) for tension in the b-direction; and η_b is the coefficient of mutual influence of the first kind related to the b-direction. Now for the same state of stress, if the element is oriented with respect to the x and y-directions, as shown in Fig 5(c), then by using Mohr's circle or transformation relations it can be shown that

$$\epsilon_{x2} = (\ell_1^2 - \nu_b m_1^2 + \eta_b \ell_1 m_1) \frac{\sigma_{b2}}{E_b}$$

$$\epsilon_{y2} = (m_1^2 - \nu_b \ell_1^2 - \eta_b \ell_1 m_1) \frac{\sigma_{b2}}{E_b}$$

$$\gamma_{xy2} = [-2\ell_1 m_1 - \nu_b 2\ell_1 m_1 + \eta_b (\ell_1^2 - m_1^2)] \frac{\sigma_{b2}}{E_b} \quad (3.22)$$

and

$$\sigma_{x2} = \ell_1^2 \sigma_{b2}$$

$$\sigma_{y2} = m_1^2 \sigma_{b2}$$

$$\tau_{xy2} = -\ell_1 m_1 \sigma_{b2} \quad (3.23)$$

where σ_{x2} and σ_{y2} are the stresses in the x and y-directions, respectively; τ_{xy2} is the shearing stress; ϵ_{x2}, ϵ_{y2}, and γ_{xy2} are the corresponding strains; ℓ_1 is cos θ_1; m_1 is sin θ_1; and θ_1 is the angle between the a and b-directions.

Finally, consider a small rectangular element as shown in Fig 5(d). If the only stress acting on this element is σ_{c3} and the corresponding strains are ϵ_c, ϵ_{cp}, and γ_{cp}, where ϵ_c and ϵ_{cp} are strains in the c-direction and perpendicular to the c-direction (or cp-direction) and γ_{cp} is the shearing strain, then

$$\epsilon_c = \frac{\sigma_{c3}}{E_c}$$

$$\epsilon_{cp} = -\frac{\nu_c \sigma_{c3}}{E_c}$$

$$\gamma_{cp} = \frac{\eta_c \sigma_{c3}}{E_c} \tag{3.24}$$

where E_c is the modulus of elasticity with respect to the c-direction; ν_c is the Poisson's ratio which characterizes the decrease perpendicular to the c-direction (or cp-direction) for tension in the c-direction; and η_c is the coefficient of mutual influence of the first kind related to the c-direction. Now for the same state of stress, if the element is oriented with respect to the x and y-directions, then by using Mohr's circle or transformation relations it can be shown that

$$\epsilon_{x3} = (\ell_2^2 - \nu_c m_2^2 + \eta_c \ell_2 m_2) \frac{\sigma_{c3}}{E_c}$$

$$\epsilon_{y3} = (m_2^2 - \nu_c \ell_2^2 - \eta_c \ell_2 m_2) \frac{\sigma_{c3}}{E_c}$$

$$\gamma_{xy3} = [-2\ell_2 m_2 - \nu_c 2\ell_2 m_2 + \eta_c (\ell_2^2 - m_2^2)] \frac{\sigma_{c3}}{E_c} \tag{3.25}$$

and

$$\sigma_{x3} = \ell_2^2 \sigma_{c3}$$

$$\sigma_{y3} = m_2^2 \sigma_{c3}$$

$$\tau_{xy3} = -\ell_2 m_2 \sigma_{c3} \tag{3.26}$$

where σ_{x3} and σ_{y3} are stresses in the x and y-directions, respectively; τ_{xy3} is the shearing stress; ϵ_{x3} , ϵ_{y3} , and γ_{xy3} are corresponding strains; ℓ_2 is $\cos \theta_2$; m_2 is $\sin \theta_2$; and θ_2 is the angle between the a and c-directions.

Now consider a case in which the above three sets of state of stresses act simultaneously. The method of superposition can be used in this case. Hence, as shown in Fig 5(e), if

$$\sigma_x = \sigma_{x1} + \sigma_{x2} + \sigma_{x3}$$

$$\sigma_y = \sigma_{y1} + \sigma_{y2} + \sigma_{y3}$$

$$\tau_{xy} = \tau_{xy1} + \tau_{xy2} + \tau_{xy3} \tag{3.27}$$

and

$$\epsilon_x = \epsilon_{x1} + \epsilon_{x2} + \epsilon_{x3}$$

$$\epsilon_y = \epsilon_{y1} + \epsilon_{y2} + \epsilon_{y3}$$

$$\gamma_{xy} = \gamma_{xy1} + \gamma_{xy2} + \gamma_{xy3} \tag{3.28}$$

then from Eqs 3.20, 3.23, and 3.26

$$\sigma_x = \sigma_{a1} + \sigma_{b2}\ell_1^2 + \sigma_{c3}\ell_2^2$$

$$\sigma_y = 0 + \sigma_{b2}m_1^2 + \sigma_{c3}m_2^2$$

$$\tau_{xy} = 0 - \sigma_{b2}\ell_1 m_1 - \sigma_{c3}\ell_2 m_2 \tag{3.29}$$

and from Eqs 3.19, 3.22, and 3.25

$$\epsilon_x = \frac{1}{E_a}\sigma_{a1} + (\ell_1^2 - \nu_b m_1^2 + \eta_b \ell_1 m_1)\frac{\sigma_{b2}}{E_b}$$

$$+ (\ell_2^2 - \nu_c m_2^2 + \eta_c \ell_2 m_2) \frac{\sigma_{c3}}{E_c}$$

$$\epsilon_y = -\frac{\nu_a}{E_a}\sigma_{a1} + (m_1^2 - \nu_b \ell_1^2 - \eta_b \ell_1 m_1) \frac{\sigma_{b2}}{E_b}$$

$$+ (m_2^2 - \nu_c \ell_2^2 - \eta_c \ell_2 m_2) \frac{\sigma_{c3}}{E_c}$$

$$\gamma_{xy} = \frac{\eta_a}{E_a}\sigma_{a1} + [-2\ell_1 m_1 - \nu_b 2\ell_1 m_1 + \eta_b(\ell_1^2 - m_1^2)] \frac{\sigma_{b2}}{E_b}$$

$$+ [-2\ell_2 m_2 - \nu_c 2\ell_2 m_2 + \eta_c(\ell_2^2 - m_2^2)] \frac{\sigma_{c3}}{E_c} \qquad (3.30)$$

Also ϵ_x , ϵ_y , and γ_{xy} can be related to σ_x , σ_y , and τ_{xy} using compliances from Eq 3.4 as follows:

$$\epsilon_x = s_{11}\sigma_x + s_{12}\sigma_y + s_{13}\tau_{xy}$$

$$\epsilon_y = s_{12}\sigma_x + s_{22}\sigma_y + s_{23}\tau_{xy}$$

$$\gamma_{xy} = s_{13}\sigma_x + s_{23}\sigma_y + s_{33}\tau_{xy} \qquad (3.31)$$

Substituting values of σ_x , σ_y , and τ_{xy} from Eq 3.29 into Eq 3.31

$$\epsilon_x = s_{11}(\sigma_{a1} + \sigma_{b2}\ell_1^2 + \sigma_{c3}\ell_2^2) + s_{12}(\sigma_{b2}m_1^2 + \sigma_{c3}m_2^2)$$

$$+ s_{13}(-\sigma_{b2}\ell_1 m_1 - \sigma_{c3}\ell_2 m_2)$$

$$\epsilon_y = s_{12}(\sigma_{a1} + \sigma_{b2}\ell_1^2 + \sigma_{c3}\ell_2^2) + s_{22}(\sigma_{b2}m_1^2 + \sigma_{c3}m_2^2)$$

$$+s_{23}(-\sigma_{b2}\ell_1 m_1 - \sigma_{c3}\ell_2 m_2)$$

$$\gamma_{xy} = s_{13}(\sigma_{a1} + \sigma_{b2}\ell_1^2 + \sigma_{c3}\ell_2^2) + s_{23}(\sigma_{b2}m_1^2 + \sigma_{c3}m_2^2)$$

$$+ s_{33}(-\sigma_{b2}\ell_1 m_1 - \sigma_{c3}\ell_2 m_2) \tag{3.32}$$

Since Eqs 3.30 and 3.32 should be the same, the coefficients of σ_{a1}, σ_{b2}, and σ_{c3} in both sets of equations should be equal. Comparison of the coefficients of σ_{a1}, σ_{b2}, and σ_{c3} results in the following nine relations:

$$s_{11} = \frac{1}{E_a} \tag{3.33}$$

$$s_{11}\ell_1^2 + s_{12}m_1^2 - s_{13}\ell_1 m_1 = \frac{1}{E_b}(\ell_1^2 - \nu_b m_1^2 + \eta_b \ell_1 m_1) \tag{3.34}$$

$$s_{11}\ell_2^2 + s_{12}m_2^2 - S_{13}\ell_2 m_2 = \frac{1}{E_c}(\ell_2^2 - \nu_c m_2^2 + \eta_c \ell_2 m_2) \tag{3.35}$$

$$s_{12} = -\frac{\nu_a}{E_a} \tag{3.36}$$

$$s_{12}\ell_1^2 + s_{22}m_1^2 - s_{23}\ell_1 m_1 = \frac{1}{E_b}(m_1^2 - \nu_b \ell_1^2 - \eta_b \ell_1 m_1) \tag{3.37}$$

$$s_{12}\ell_2^2 + s_{22}m_2^2 - s_{23}\ell_2 m_2 = \frac{1}{E_c}(m_2^2 - \nu_c \ell_2^2 - \eta_c \ell_2 m_2) \tag{3.38}$$

$$s_{13} = \frac{\eta_a}{E_a} \tag{3.39}$$

$$s_{13}\ell_1^2 + s_{23}m_1^2 - s_{33}\ell_1 m_1 = \frac{1}{E_b}[-2\ell_1 m_1 - \nu_b 2\ell_1 m_1$$

$$+ \eta_b(\ell_1^2 - m_1^2)] \tag{3.40}$$

$$s_{13}\ell_2^2 + s_{23}m_2^2 - s_{33}\ell_2 m_2 = \frac{1}{E_c}[-2\ell_2 m_2 - \nu_c 2\ell_2 m_2$$

$$+ \eta_c(\ell_2^2 - m_2^2)] \tag{3.41}$$

Solving the above nine relations for six elastic compliances s_{11} , s_{12} , s_{13} , s_{22} , s_{23} , and s_{33} and three coefficients of mutual influence of the first kind η_a , η_b , and η_c in terms of three moduli of elasticity E_a , E_b , and E_c and three Poisson's ratios ν_a , ν_b , and ν_c , the following results could be obtained:

$$\eta_a = \frac{(\ell_1 m_2 + m_1\ell_2)(\ell_1\ell_2 - \nu_a m_1 m_2)}{2\ell_1 m_1 \ell_2 m_2}$$

$$- \frac{(m_1\ell_3 + \ell_1 m_3)(\ell_1\ell_3 - \nu_b m_1 m_3)}{2\ell_1 m_1 \ell_3 m_3}\frac{E_a}{E_b}$$

$$- \frac{(\ell_2 m_3 - m_2\ell_3)(\ell_2\ell_3 + \nu_c m_2 m_3)}{2\ell_2 m_2 \ell_3 m_3}\frac{E_a}{E_c} \tag{3.42}$$

$$\eta_b = \frac{(\ell_1 m_2 - m_1\ell_2)(\ell_1\ell_2 + \nu_a m_1 m_2)}{2\ell_1 m_1 \ell_2 m_2}\frac{E_b}{E_a}$$

$$+ \frac{(m_1\ell_3 - \ell_1 m_3)(\ell_1\ell_3 + \nu_b m_1 m_3)}{2\ell_1 m_1 \ell_3 m_3}$$

$$+ \frac{(\ell_2 m_3 - m_2\ell_3)(\ell_2\ell_3 + \nu_c m_2 m_3)}{2\ell_2 m_2 \ell_3 m_3}\frac{E_b}{E_c} \tag{3.43}$$

$$\eta_c = -\frac{(\ell_1 m_2 - m_1\ell_2)(\ell_1\ell_2 + \nu_a m_1 m_2)}{2\ell_1 m_1 \ell_2 m_2}\frac{E_c}{E_a}$$

$$+\frac{(m_1\ell_3 + \ell_1 m_3)(\ell_1\ell_3 - \nu_b m_1 m_3)}{2\ell_1 m_1\ell_3 m_3}\frac{E_c}{E_b}$$

$$+\frac{(\ell_2 m_3 + m_2\ell_3)(-\ell_2\ell_3 + \nu_c m_2 m_3)}{2\ell_2 m_2\ell_3 m_3} \qquad (3.44)$$

$$s_{11} = \frac{1}{E_a} \qquad (3.45)$$

$$s_{12} = -\frac{\nu_a}{E_a} \qquad (3.46)$$

$$s_{13} = \frac{(\ell_1 m_2 + m_1\ell_2)}{2m_1 m_2}\frac{1}{E_a} - \frac{(\ell_1 m_2 + m_1\ell_2)}{2\ell_1\ell_2}\frac{\nu_a}{E_a} - \frac{m_2}{2m_1 m_3}\frac{1}{E_b}$$

$$+\frac{m_2}{2\ell_1\ell_3}\frac{\nu_b}{E_b} + \frac{m_1}{2m_2 m_3}\frac{1}{E_c} + \frac{m_1}{2\ell_2\ell_3}\frac{\nu_c}{E_c} \qquad (3.47)$$

$$s_{22} = \frac{\ell_1\ell_2}{m_1 m_2}\frac{1}{E_a} - \frac{(\ell_1\ell_2 - m_1 m_2)}{m_1 m_2}\frac{\nu_a}{E_a} - \frac{\ell_2}{m_1 m_3}\frac{1}{E_b} + \frac{\ell_2}{m_1 m_3}\frac{\nu_b}{E_b}$$

$$+\frac{\ell_1}{m_2 m_3}\frac{1}{E_c} - \frac{\ell_1}{m_2 m_3}\frac{\nu_c}{E_c} \qquad (3.48)$$

$$s_{23} = \frac{(\ell_1 m_2 + m_1\ell_2)}{2m_1 m_2}\frac{1}{E_a} + \frac{(\ell_1 m_2 + m_1\ell_2)(-2\ell_1\ell_2 + m_1 m_2)}{2\ell_1 m_1\ell_2 m_2}\frac{\nu_a}{E_a}$$

$$- \frac{m_2}{2m_1m_3}\frac{1}{E_b} + \frac{m_2(\ell_2 + \ell_1\ell_3)}{2\ell_1 m_1 \ell_3 m_3}\frac{\nu_b}{E_b} + \frac{m_1}{2m_2m_3}\frac{1}{E_c}$$

$$+ \frac{m_1(-\ell_1 - \ell_2\ell_3)}{2\ell_2 m_2 \ell_3 m_3}\frac{\nu_c}{E_c} \tag{3.49}$$

$$s_{33} = \frac{\ell_3}{m_1m_2}\frac{1}{E_a} + \frac{1}{\ell_1 m_1 \ell_2 m_2}[-m_1m_2(\ell_1\ell_2 - m_1m_2) - \ell_1^2 m_2^2 - m_1^2\ell_2^2]\frac{\nu_a}{E_a}$$

$$- \frac{\ell_2}{m_1m_3}\frac{1}{E_b} + \frac{1}{\ell_1 m_1 \ell_3 m_3}(m_1m_2\ell_3 + \ell_1^2 m_2^2 - m_1^2\ell_2^2)\frac{\nu_b}{E_b}$$

$$+ \frac{\ell_1}{m_2m_3}\frac{1}{E_c} + \frac{1}{\ell_2 m_2 \ell_3 m_3}(-m_1m_2\ell_3 + \ell_1^2 m_2^2 - m_1^2\ell_2^2)\frac{\nu_c}{E_c} \tag{3.50}$$

where ℓ_3 is $\cos\theta_3$; m_3 is $\sin\theta_3$; and θ_3 is the angle between the b and c-directions.

Equations 3.45 through 3.50 describe the relations in which the six elastic compliances are related to the three moduli of elasticity in any three directions a , b , and c and the three Poisson's ratios related to these directions. These relations are valid for any value of angles θ_1 , θ_2 , and θ_3 except 0 and 180 degrees. It might appear that the relations are not valid for 90 degrees but an additional relation between moduli of elasticity and Poisson's ratios exists at this angle. For example, if directions a and b are at 90 degrees to each other then

$$\frac{\nu_a}{E_a} = \frac{\nu_b}{E_b} \tag{3.51}$$

Using this additional relation, compliances can still be computed in terms of E_a , E_b , E_c , ν_a , ν_b , and ν_c

For the isotropic case $E_a = E_b = E_c = E$ and $\nu_a = \nu_b = \nu_c = \nu$; substituting these in Eqs 3.42 through 3.44 it can be seen that $\eta_a = \eta_b = \eta_c = 0$, which is as it should be.

Transformation Relation for Modulus of Elasticity

Knowing the values of six elastic compliances, the transformation relation for the modulus of elasticity can be worked out as follows.

Multiplying Eqs 3.34, 3.37, and 3.40 by ℓ_1^2 , m_1^2 , and $-\ell_1 m_1$, respectively, and adding the three resulting equations gives

$$\frac{1}{E_b} = \ell_1^4 s_{11} + 2\ell_1^2 m_1^2 s_{12} - 2\ell_1^3 m_1 s_{13} + m_1^4 s_{22}$$

$$- 2\ell_1 m_1^3 s_{23} + \ell_1^2 m_1^2 s_{33} \qquad (3.52)$$

where ℓ_1 is $\cos \theta_1$; m_1 is $\sin \theta_1$; θ_1 is the angle between the a and b-directions, as shown in Fig 5(a); and s_{11} , s_{12} , s_{13} , s_{22} , s_{23} , and s_{33} are elastic compliances (as shown in Fig 5(a), the x and a-directions are the same).

Now consider any direction b′ such that the angle between a (or x) and b′-directions is θ_1' . If E_b' is the modulus of elasticity with respect to the b′-direction then from Eq 3.52

$$\frac{1}{E_b'} = \ell_1'^4 s_{11} + 2\ell_1'^2 m_1'^2 s_{12} - 2\ell_1'^3 m_1' s_{13} + m_1'^4 s_{22} - 2\ell_1' m_1'^3 s_{23}$$

$$+ \ell_1'^2 m_1'^2 s_{33} \qquad (3.53)$$

where ℓ_1' is $\cos \theta_1'$ and m_1' is $\sin \theta_1'$.

Equation 3.53 describes the transformation relation for the modulus of elasticity in any direction. Similar relations have been worked out by Lekhnitskii (Ref 17) using a different approach.

Transformation Relation for Poisson's Ratio

Knowing the value of E_b' (or modulus of elasticity with respect to the b'-direction), the Poisson's ratio related to the b'-direction can be worked out. Consider Eq 3.34 as

$$s_{11}\ell_1^2 + s_{12}m_1^2 - s_{13}\ell_1 m_1 = \frac{1}{E_b}\left(\ell_1^2 - \nu_b m_1^2 + \eta_b \ell_1 m_1\right) \qquad (3.54)$$

Substituting the value of η_b from Eq 3.43 into Eq 3.54 and then following a procedure similar to that explained above for the transformation of the modulus of elasticity, the following relation could be written:

$$\nu_b' = -\frac{2\ell_1'^2\ell_3'}{m_1' m_2} E_b' s_{11} - \frac{2m_1'\ell_3'}{m_2} E_b' s_{12} + \frac{2\ell_1'\ell_3'}{m_2} E_b' s_{13}$$

$$+ \frac{\ell_1'\ell_3' m_3'}{m_1' m_2^2}\frac{E_b'}{E_a} + \frac{\ell_3' m_3'}{\ell_2 m_2}\frac{\nu_a E_b'}{E_a} - \frac{\ell_1' m_1' \ell_3'}{m_2^2 m_3'}\frac{E_b'}{E_c}$$

$$- \frac{\ell_1' m_1'}{\ell_2 m_2}\frac{\nu_c E_b'}{E_c} + \frac{\ell_1'\ell_3'}{m_1' m_3'} \qquad (3.55)$$

where ν_b' is the Poisson's ratio related to the b'-direction; ℓ_1' is $\cos\theta_1'$; m_1' is $\sin\theta_1'$; θ_1' is the angle between the a and b'-directions; ℓ_3' is $\cos\theta_3'$; m_3' is $\sin\theta_3'$; θ_3' is the angle between the b' and c-directions; ℓ_2 is $\cos\theta_2$; m_2 is $\sin\theta_2$; and θ_2 is the angle between the a and c-directions.

Elastic Stiffnesses for Anisotropic Thin Plate

Knowing elastic compliances, the elastic stiffnesses can be computed using the following procedure. Consider strain-stress relations

$$\epsilon_x = s_{11}\sigma_x + s_{12}\sigma_y + s_{13}\tau_{xy}$$

$$\epsilon_y = s_{12}\sigma_x + s_{22}\sigma_y + s_{23}\tau_{xy}$$

$$\gamma_{xy} = s_{13}\sigma_x + s_{23}\sigma_y + s_{33}\tau_{xy} \tag{3.56}$$

The values of elastic compliances can be computed by either of the above procedures or by some other means. Substituting these values of compliances in Eq 3.56 and solving for σ_x , σ_y , and τ_{xy} as

$$\sigma_x = c_{11}\epsilon_x + c_{12}\epsilon_y + c_{13}\gamma_{xy}$$

$$\sigma_y = c_{12}\epsilon_x + c_{22}\epsilon_y + c_{23}\gamma_{xy}$$

$$\tau_{xy} = c_{13}\epsilon_x + c_{23}\epsilon_y + c_{33}\gamma_{xy} \tag{3.57}$$

gives

$$c_{11} = \frac{1}{|Det|}(s_{22}s_{33} - s_{23}s_{23})$$

$$c_{12} = \frac{1}{|Det|}(s_{23}s_{13} - s_{12}s_{33})$$

$$c_{13} = \frac{1}{|Det|}(s_{12}s_{23} - s_{13}s_{22})$$

$$c_{22} = \frac{1}{|Det|}(s_{33}s_{11} - s_{13}s_{13})$$

$$c_{23} = \frac{1}{|Det|}(s_{13}s_{12} - s_{23}s_{11})$$

$$c_{33} = \frac{1}{|Det|}(s_{11}s_{22} - s_{12}s_{12}) \tag{3.58}$$

where

$$|Det| = \begin{vmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{vmatrix} \quad (3.59)$$

and where $|\ |$ is used for the determinant.

Equations 3.58 and 3.59 describe the relations between the elastic stiffnesses and the elastic compliances in which the compliances may be computed by using either Eq 3.17 or Eqs 3.45 through 3.50 or some other means.

Summary

The stiffnesses in Eq 3.58 could be related to three moduli of elasticity with respect to any three directions (a , b , and c) and three Poisson's ratios related to these directions through elastic compliances (Eqs 3.45 through 3.50). The six elastic constants could be experimentally determined by testing three specimens from the plate in unidirectional tension. These three specimens could be taken either from the three required directions (a , b , and c) or from any other three directions. The measured moduli of elasticity and Poisson's ratios may be transformed to the required directions using Eqs 3.53 and 3.55.

How elastic stiffnesses can be used in the moment-curvature relations for the discrete-element model of an anisotropic skew-plate and grid-beam system is shown in Chapter 4.

CHAPTER 4. STRESS-STRAIN RELATIONS FOR MODEL

Introduction

In this chapter, a small triangular element from an anisotropic plate is considered, and the conventional relations between the three normal stresses in any three directions and the corresponding three normal strains are worked out.

Since the rigid bars in the model of the anisotropic skew plate do not transfer any twisting moments, stress-strain relations for the plate model are derived from a small triangular element of the plate. The plate is assumed to be made up of three layers of interconnected fibers running in the three directions. The three fiber stresses are related to the three conventional normal strains for the stress-strain relations as derived for this element. This concept makes it clear what discretization choice is appropriate to develop the bar and spring model. Integration of these relations results in moment-curvature relations for the anisotropic skew-plate model.

Moment-curvature relations for grid-beam models are also derived.

Conventional Stress-Strain Relations for Triangular Elements

Consider a small rectangular differential element of a thin anisotropic plate. The stresses acting on this element are σ_x , σ_y , and τ_{xy} as shown in Fig 6 where σ_x and σ_y are the normal stresses in the x and y-directions, respectively, and τ_{xy} is the shearing stress. The corresponding strains are ϵ_x , ϵ_y , and γ_{xy} .

The anisotropic stress-strain relations for the rectangular element (Fig 6) may be written as

$$\sigma_x = c_{11}\epsilon_x + c_{12}\epsilon_y + c_{13}\gamma_{xy}$$

$$\sigma_y = c_{12}\epsilon_x + c_{22}\epsilon_y + c_{23}\gamma_{xy}$$

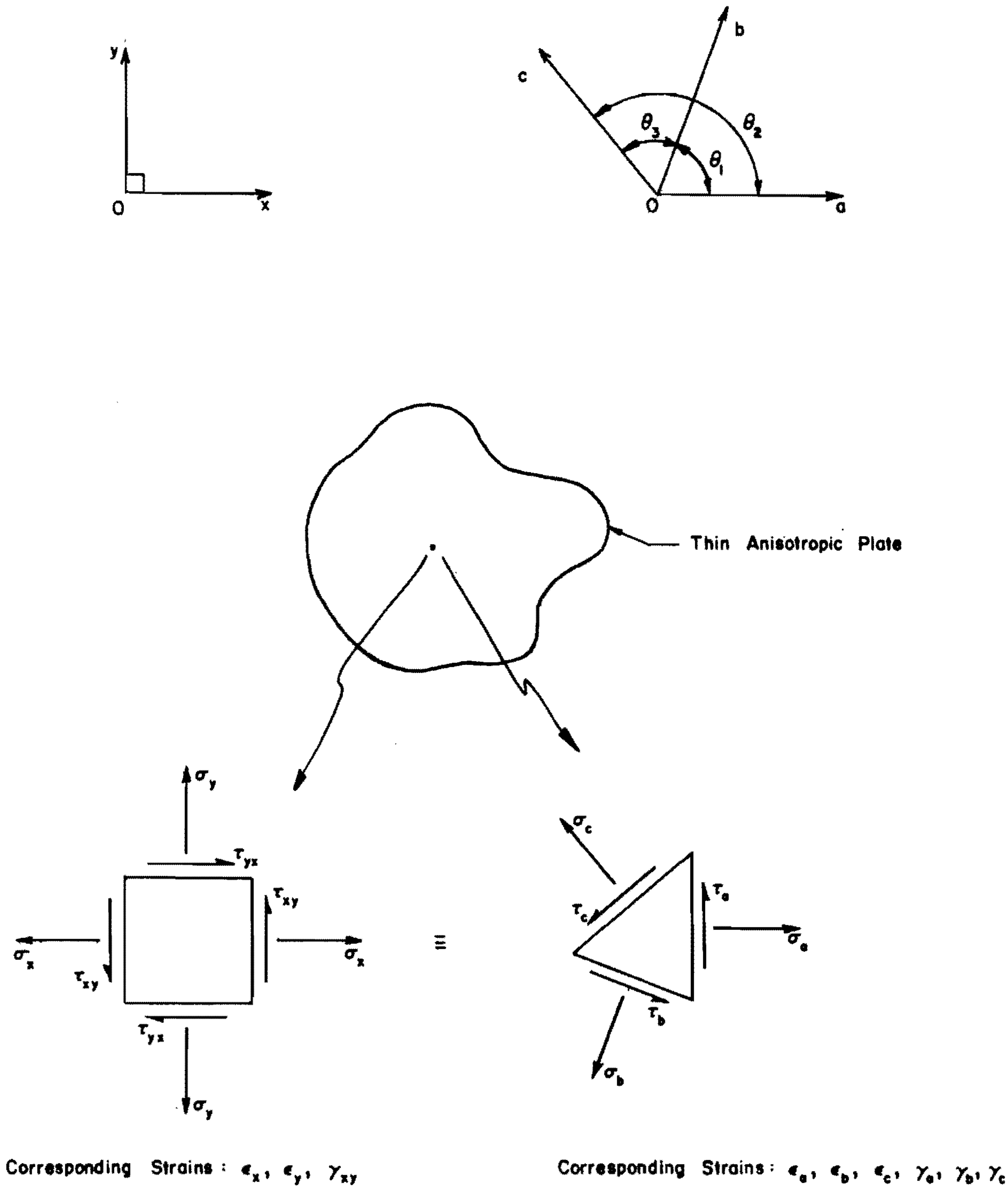


Fig 6. Stresses and corresponding strains for a rectangular element and equivalent triangular element for an anisotropic plate.

$$\tau_{xy} = c_{13}\varepsilon_x + c_{23}\varepsilon_y + c_{33}\gamma_{xy} \tag{4.1}$$

where c_{11} , c_{12} , c_{13} , c_{22} , c_{23} , and c_{33} are elastic stiffnesses.

Now consider a triangular differential element at the same location of the plate, as shown in Fig 6. The sides of this element are perpendicular to the a , b , and c-directions (a and x-directions are taken to be the same but, in general, are not necessarily the same). If the stresses acting on this triangular element are σ_a , σ_b , σ_c , τ_a , τ_b , and τ_c where σ_a , σ_b , and σ_c are the normal stresses in the a , b , and c-directions, respectively (Fig 6), and τ_a , τ_b , and τ_c are the shearing stresses related to these directions, and if ε_a , ε_b , ε_c , γ_a , γ_b , and γ_c are corresponding strains where ε_a , ε_b , and ε_c are normal strains and γ_a , γ_b , and γ_c are shearing strains, then by using Mohr's circle the following relations may be written:

$$\sigma_a = \sigma_x$$

$$\sigma_b = \sigma_x \cos^2 \theta_1 + \sigma_y \sin^2 \theta_1 - 2\tau_{xy} \sin \theta_1 \cos \theta_1$$

$$\sigma_c = \sigma_x \cos^2 \theta_2 + \sigma_y \sin^2 \theta_2 - 2\tau_{xy} \sin \theta_2 \cos \theta_2 \tag{4.2}$$

and

$$\varepsilon_a = \varepsilon_x$$

$$\varepsilon_b = \varepsilon_x \cos^2 \theta_1 + \varepsilon_y \sin^2 \theta_1 - \gamma_{xy} \sin \theta_1 \cos \theta_1$$

$$\varepsilon_c = \varepsilon_x \cos^2 \theta_2 + \varepsilon_y \sin^2 \theta_2 - \gamma_{xy} \sin \theta_2 \cos \theta_2 \tag{4.3}$$

Combining Eqs 4.1, 4.2, and 4.3, it is possible to develop the following anisotropic stress-strain relations in which the normal stresses in any three directions are related to the corresponding normal strains:

$$\sigma_a = \frac{1}{m_1 m_2 m_3}[P_{aa}\varepsilon_a + P_{ab}\varepsilon_b + P_{ac}\varepsilon_c]$$

$$\sigma_b = \frac{1}{m_1 m_2 m_3}[(P_{aa}\ell_1^2 + P_{ba}m_1^2 - 2P_{ca}\ell_1 m_1)\varepsilon_a$$

$$+ (P_{ab}\ell_1^2 + P_{bb}m_1^2 - 2P_{cb}\ell_1 m_1)\varepsilon_b + (P_{ac}\ell_1^2$$

$$+ P_{bc}m_1^2 - 2P_{cc}\ell_1 m_1)\varepsilon_c]$$

$$\sigma_c = \frac{1}{m_1 m_2 m_3}[(P_{aa}\ell_2^2 + P_{ba}m_2^2 - 2P_{ca}\ell_2 m_2)\varepsilon_a$$

$$+ (P_{ab}\ell_2^2 + P_{bb}m_2^2 - 2P_{cb}\ell_2 m_2)\varepsilon_b$$

$$+ (P_{ac}\ell_2^2 + P_{bc}m_2^2 - 2P_{cc}\ell_2 m_2)\varepsilon_c] \qquad (4.4)$$

where

$$P_{aa} = c_{11}m_1 m_2 m_3 + c_{12}\ell_1\ell_2 m_3 + c_{13}m_3(\ell_1 m_2 + m_1\ell_2)$$

$$P_{ab} = -c_{12}\ell_2 m_2 - c_{13}m_2^2$$

$$P_{ac} = c_{12}\ell_1 m_1 + c_{13}m_1^2$$

$$P_{ba} = c_{12}m_1 m_2 m_3 + c_{22}\ell_1\ell_2 m_3 + c_{23}m_3(\ell_1 m_2 + m_1\ell_2)$$

$$P_{bb} = -c_{22}\ell_2 m_2 - c_{23}m_2^2$$

$$P_{bc} = c_{22}\ell_1 m_1 + c_{23}m_1^2$$

$$P_{ca} = c_{13}m_1m_2m_3 + c_{23}\ell_1\ell_2m_3 + c_{33}m_3(\ell_1m_2 + m_1\ell_2)$$

$$P_{cb} = -c_{23}\ell_2m_2 - c_{33}m_2^2$$

$$P_{cc} = c_{23}\ell_1m_1 + c_{33}m_1^2 \qquad (4.5)$$

and

$$\ell_1 = \cos\theta_1$$

$$\ell_2 = \cos\theta_2$$

$$\ell_3 = \cos\theta_3$$

$$m_1 = \sin\theta_1$$

$$m_2 = \sin\theta_2$$

$$m_3 = \sin\theta_3 \qquad (4.6)$$

Stress-Strain Relations for Fiber Continuum

As explained in Chapter 2, the discrete-element model for the anisotropic skew plate consists of elastic joints connected by means of rigid bars running in any three directions (Fig 2). The rigid bars transfer bending moments from one elastic joint to the other elastic joint. Hence, to derive the stress-strain relations for the plate model, the following procedure is adopted.

It is assumed that the triangular element of Fig 6 is composed of three layers of infinitesimal fibers running in the a , b , and c-directions. These layers are so connected that the effect due to Poisson's ratio is transferred from one layer to the other two layers. This can be visualized by considering three layers of closely-spaced straps running in the a , b , and c-directions and pinned at the points of intersection as shown in Fig 7.

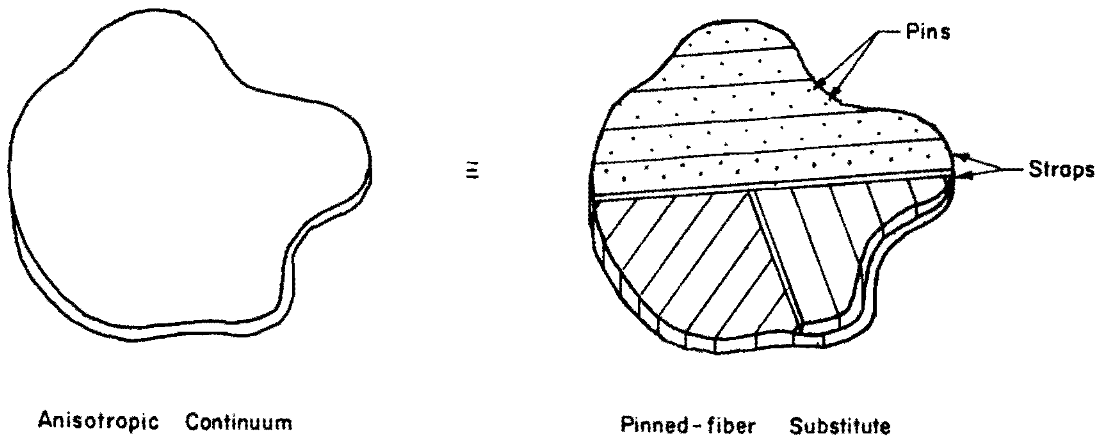


Fig 7. Simulation of anisotropic continuum with a fiber-element model.

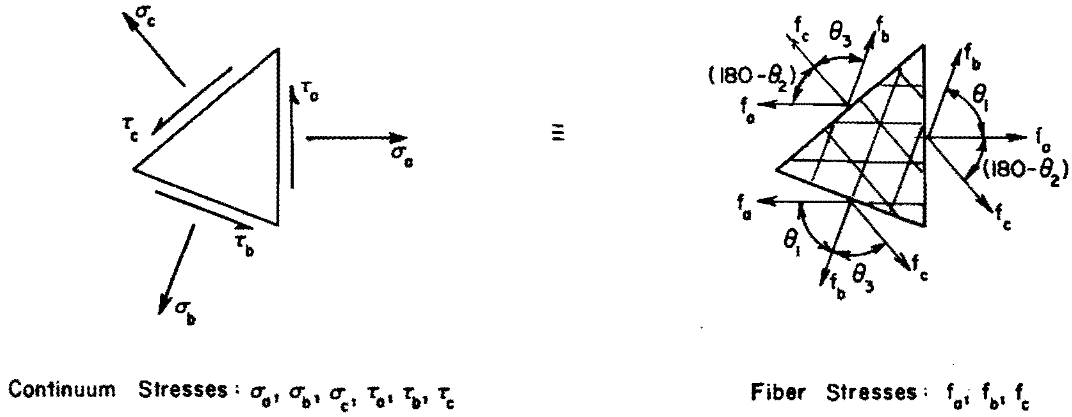


Fig 8. Stresses for a continuum element and equivalent fiber element.

Now if f_a , f_b , and f_c are stresses in the fibers running in the a , b , and c-directions, respectively, as shown in Fig 8, then, by statics, the following relations may be written

$$\sigma_a = f_a + f_b \cos^2 \theta_1 + f_c \cos^2 \theta_2$$

$$\sigma_b = f_a \cos^2 \theta_1 + f_b + f_c \cos^2 \theta_3$$

$$\sigma_c = f_a \cos^2 \theta_2 + f_b \cos^2 \theta_3 + f_c \tag{4.7}$$

and

$$\tau_a = f_b \cos \theta_1 \sin \theta_1 + f_c \cos \theta_2 \sin \theta_2$$

$$\tau_b = -f_a \cos \theta_1 \sin \theta_1 + f_c \cos \theta_3 \sin \theta_3$$

$$\tau_c = -f_a \cos \theta_2 \sin \theta_2 - f_b \cos \theta_3 \sin \theta_3 \tag{4.8}$$

Solving Eq 4.7 for f_a , f_b , and f_c ,

$$f_a = \frac{1}{D} [(1 - \ell_3^4)\sigma_a + (\ell_2^2\ell_3^2 - \ell_1^2)\sigma_b + (\ell_1^2\ell_3^2 - \ell_2^2)\sigma_c]$$

$$f_b = \frac{1}{D} [(\ell_2^2\ell_3^2 - \ell_1^2)\sigma_a + (1 - \ell_2^4)\sigma_b + (\ell_1^2\ell_2^2 - \ell_3^2)\sigma_c]$$

$$f_c = \frac{1}{D} [(\ell_1^2\ell_3^2 - \ell_2^2)\sigma_a + (\ell_1^2\ell_2^2 - \ell_3^2)\sigma_b + (1 - \ell_1^4)\sigma_c] \tag{4.9}$$

where $D = 1 - \ell_1^4 - \ell_2^4 - \ell_3^4 + 2\ell_1^2\ell_2^2\ell_3^2$ and ℓ_1 , ℓ_2 , and ℓ_3 are $\cos \theta_1$, $\cos \theta_2$, and $\cos \theta_3$, respectively (Eq 4.6).

Substituting the values of σ_a , σ_b , and σ_c from Eq 4.4 into Eq 4.9, the following stress-strain relations for the fiber continuum, which relate the fiber stresses to the conventional strains, may be obtained:

$$f_a = a_{11}\epsilon_a + a_{12}\epsilon_b + a_{13}\epsilon_c$$

$$f_b = a_{12}\epsilon_a + a_{22}\epsilon_b + a_{23}\epsilon_c$$

$$f_c = a_{13}\epsilon_a + a_{23}\epsilon_b + a_{33}\epsilon_c \qquad (4.10)$$

where

$$a_{11} = c_{11} + \frac{2\ell_1\ell_2}{m_1m_2}c_{12} + \frac{2(\ell_1m_2 + m_1\ell_2)}{m_1m_2}c_{13} + \frac{\ell_1^2\ell_2^2}{m_1^2m_2^2}c_{22}$$

$$+ \frac{2\ell_1\ell_2(\ell_1m_2 + m_1\ell_2)}{m_1^2m_2^2}c_{23} + \frac{(\ell_1m_2 + m_1\ell_2)^2}{m_1^2m_2^2}c_{33}$$

$$a_{12} = -\frac{\ell_2}{m_1m_3}c_{12} - \frac{m_2}{m_1m_3}c_{13} - \frac{\ell_1\ell_2^2}{m_1^2m_2m_3}c_{22}$$

$$- \frac{\ell_2(2\ell_1m_2 + m_1\ell_2)}{m_1^2m_2m_3}c_{23} - \frac{(\ell_1m_2 + m_1\ell_2)}{m_1^2m_3}c_{33}$$

$$a_{13} = \frac{\ell_1}{m_2m_3}c_{12} + \frac{m_1}{m_2m_3}c_{13} + \frac{\ell_1^2\ell_2}{m_1m_2^2m_3}c_{22}$$

$$+ \frac{\ell_1(\ell_1m_2 + 2m_1\ell_2)}{m_1m_2^2m_3}c_{23} + \frac{(\ell_1m_2 + m_1\ell_2)}{m_2^2m_3}c_{33}$$

$$a_{22} = \frac{\ell_2^2}{m_1^2 m_3^2} c_{22} + \frac{2\ell_2 m_2}{m_1^2 m_3^2} c_{23} + \frac{m_2^2}{m_1^2 m_3^2} c_{33}$$

$$a_{23} = -\frac{\ell_1 \ell_2}{m_1 m_2 m_3^2} c_{22} - \frac{(\ell_1 m_2 + m_1 \ell_2)}{m_1 m_2 m_3^2} c_{23} - \frac{1}{m_3^2} c_{33}$$

$$a_{33} = \frac{\ell_1^2}{m_2^2 m_3^2} c_{22} + \frac{2\ell_1 m_1}{m_2^2 m_3^2} c_{23} + \frac{m_1^2}{m_2^2 m_3^2} c_{33} \qquad (4.11)$$

and where c_{11} , c_{12} , c_{13} , c_{22} , c_{23} , and c_{33} are elastic stiffnesses, the values of which could be obtained as explained in Chapter 3.

The fiber continuum which is developed here could be made into a discrete-element tridirectional model like that in Fig 2 for plane stress instead of bending.

Moment-Curvature Relations for Fiber Continuum

Consider a differential triangular element, as shown in Fig 9, under the action of fiber stresses f_a , f_b , and f_c . If the three-layer element considered is at a distance z from the neutral surface then, based on assumptions in Chapter 2,

$$\epsilon_a = z \frac{\partial^2 w}{\partial a^2}$$

$$\epsilon_b = z \frac{\partial^2 w}{\partial b^2}$$

$$\epsilon_c = z \frac{\partial^2 w}{\partial c^2} \qquad (4.12)$$

where $\frac{\partial^2 w}{\partial a^2}$, $\frac{\partial^2 w}{\partial b^2}$, and $\frac{\partial^2 w}{\partial c^2}$ are curvatures in the a , b , and c-directions, respectively.

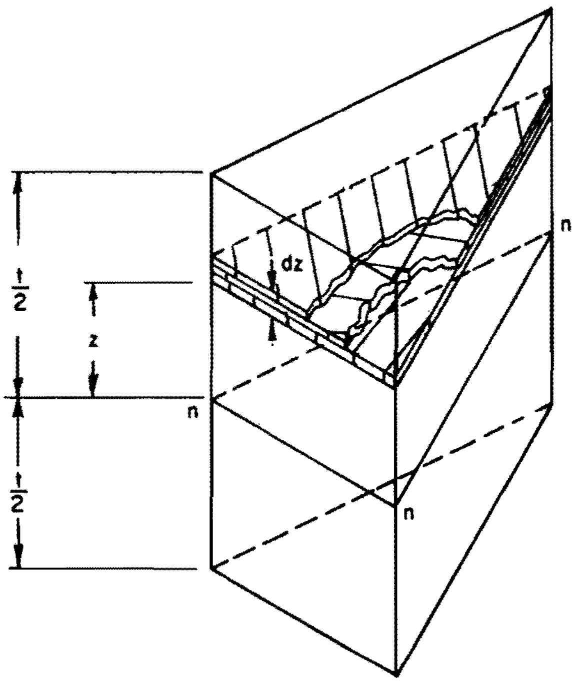


Fig 9. Differential element from plate.

If M_a, M_b, and M_c are bending moments per unit width in the plate continuum and t is the thickness of the plate, then, using Eqs 4.10 and 4.12,

$$M_a = \int_{-\frac{t}{2}}^{+\frac{t}{2}} z f_a dz = \int_{-\frac{t}{2}}^{+\frac{t}{2}} (a_{11}\epsilon_a + a_{12}\epsilon_b + a_{13}\epsilon_c) z dz$$

$$= \int_{-\frac{t}{2}}^{+\frac{t}{2}} \left(a_{11} \frac{\partial^2 w}{\partial a^2} + a_{12} \frac{\partial^2 w}{\partial b^2} + a_{13} \frac{\partial^2 w}{\partial c^2} \right) z^2 dz$$

$$= \left(a_{11} \frac{\partial^2 w}{\partial a^2} + a_{12} \frac{\partial^2 w}{\partial b^2} + a_{13} \frac{\partial^2 w}{\partial c^2} \right) \frac{t^3}{12} \qquad (4.13)$$

Deriving similar expressions for M_b and M_c and introducing the following relations

$$B_{11} = a_{11} \frac{t^3}{12}$$

$$B_{12} = a_{12} \frac{t^3}{12}$$

$$B_{13} = a_{13} \frac{t^3}{12}$$

$$B_{22} = a_{22} \frac{t^3}{12}$$

$$B_{23} = a_{23} \frac{t^3}{12}$$

$$B_{33} = a_{33} \frac{t^3}{12} \qquad (4.14)$$

and

$$D_{11} = c_{11} \frac{t^3}{12}$$

$$D_{12} = c_{12} \frac{t^3}{12}$$

$$D_{13} = c_{13} \frac{t^3}{12}$$

$$D_{22} = c_{22} \frac{t^3}{12}$$

$$D_{23} = c_{23} \frac{t^3}{12}$$

$$D_{33} = c_{33} \frac{t^3}{12} \tag{4.15}$$

it may be shown that

$$M_a = B_{11} \frac{\partial^2 w}{\partial a^2} + B_{12} \frac{\partial^2 w}{\partial b^2} + B_{13} \frac{\partial^2 w}{\partial c^2}$$

$$M_b = B_{12} \frac{\partial^2 w}{\partial a^2} + B_{22} \frac{\partial^2 w}{\partial b^2} + B_{23} \frac{\partial^2 w}{\partial c^2}$$

$$M_c = B_{13} \frac{\partial^2 w}{\partial a^2} + B_{23} \frac{\partial^2 w}{\partial b^2} + B_{33} \frac{\partial^2 w}{\partial c^2} \tag{4.16}$$

where

$$B_{11} = D_{11} + \frac{2\ell_1\ell_2}{m_1m_2} D_{12} + \frac{2(\ell_1m_2 + m_1\ell_2)}{m_1m_2} D_{13} + \frac{\ell_1^2\ell_2^2}{m_1^2m_2^2} D_{22}$$

$$+ \frac{2\ell_1\ell_2(\ell_1m_2 + m_1\ell_2)}{m_1^2m_2^2} D_{23} + \frac{(\ell_1m_2 + m_1\ell_2)^2}{m_1^2m_2^2} D_{33}$$

$$B_{12} = - \frac{\ell_2}{m_1m_3} D_{12} - \frac{m_2}{m_1m_3} D_{13} - \frac{\ell_1\ell_2^2}{m_1^2m_2m_3} D_{22}$$

$$- \frac{\ell_2(2\ell_1m_2 + m_1\ell_2)}{m_1^2m_2m_3} D_{23} - \frac{(\ell_1m_2 + m_1\ell_2)}{m_1^2m_3} D_{33}$$

$$B_{13} = \frac{\ell_1}{m_2m_3} D_{12} + \frac{m_1}{m_2m_3} D_{13} + \frac{\ell_1^2\ell_2}{m_1m_2^2m_3} D_{22} + \frac{\ell_1(\ell_1m_2 + 2m_1\ell_2)}{m_1m_2^2m_3} D_{23}$$

$$+ \frac{(\ell_1m_2 + m_1\ell_2)}{m_2^2m_3} D_{33}$$

$$B_{22} = \frac{\ell_2^2}{m_1^2m_3^2} D_{22} + \frac{2\ell_2m_2}{m_1^2m_3^2} D_{23} + \frac{m_2^2}{m_1^2m_3^2} D_{33}$$

$$B_{23} = - \frac{\ell_1\ell_2}{m_1m_2m_3^2} D_{22} - \frac{(\ell_1m_2 + m_1\ell_2)}{m_1m_2m_3^2} D_{23} - \frac{1}{m_3^2} D_{33}$$

$$B_{33} = \frac{\ell_1^2}{m_2^2m_3^2} D_{22} + \frac{2\ell_1m_1}{m_2^2m_3^2} D_{23} + \frac{m_1^2}{m_2^2m_3^2} D_{33} \qquad (4.17)$$

Equations 4.16 and 4.17 describe the moment-curvature relations for the anisotropic skew plate continuum in which each moment is related to the three curvatures in the a , b , and c-directions.

Moment-Curvature Relations for Grid-Beam Model

As discussed in Chapter 2, the discrete-element model for a grid-beam running in a particular direction consists of elastic joints connected by means of rigid bars running in that direction (Fig 3). Also each grid-beam model is considered as a beam with deflection compatibility at the elastic joints. The procedure used to derive moment-curvature relations for each grid-beam model is the same as shown by Matlock (Ref 18) for the beam-column model. Hence the final results may be written as

$$\bar{M}_a = F_a \frac{\partial^2 w}{\partial a^2}$$

$$\bar{M}_b = F_b \frac{\partial^2 w}{\partial b^2}$$

$$\bar{M}_c = F_c \frac{\partial^2 w}{\partial c^2} \qquad (4.18)$$

where $\bar{M}_a$, $\bar{M}_b$, and $\bar{M}_c$ are bending moments in grid-beams running in the a , b , and c-directions, respectively, and F_a , F_b , and F_c are flexural stiffnesses related to these directions.

It may be noted that if the plate stiffnesses D_{11} through D_{33} are computed in terms of three moduli of elasticity in any three directions and three Poisson's ratios related to these directions and if the three Poisson's ratios are set to zero, then the moment-curvature relations for the plate continuum (Eq 4.16) do not reduce to the moment-curvature relations for the grid-beam model (Eq 4.18).

Alternate Approach to Compute B_{11} Through B_{33}

An alternate approach is developed to compute B_{11} through B_{33} (Eq 4.16) in which they are related to three moduli of elasticity with respect to three directions and three directional Poisson effects. The directional Poisson effect characterizes the decrease in length in a particular direction for the tension in some other direction whereas the conventional Poisson's ratio characterizes the decrease in a particular direction for the tension in the direction perpendicular to it. The alternate approach is as follows.

Consider a small rectangular element, shown in Fig 10(b), with the only stress acting being σ_{a1} (the x and a-directions are the same). For the same state of stress, if a triangular element is considered, shown in Fig 10(b), and if σ_{a1}, σ_{b1}, and σ_{c1} are the normal stresses in the a, b, and c-directions, respectively, and ϵ_{a1}, ϵ_{b1}, and ϵ_{c1} are the corresponding normal strains, then

$$\epsilon_{a1} = \frac{\sigma_{a1}}{E_a}$$

$$\epsilon_{b1} = -\mu_{ab}\epsilon_{a1} = -\frac{\mu_{ab}\sigma_{a1}}{E_a}$$

$$\epsilon_{c1} = -\mu_{ac}\epsilon_{a1} = -\frac{\mu_{ac}\sigma_{a1}}{E_a} \tag{4.19}$$

and

$$\sigma_{a1} = \sigma_{a1}$$

$$\sigma_{b1} = \ell_1^2\sigma_{a1}$$

$$\sigma_{c1} = \ell_2^2\sigma_{a1} \tag{4.20}$$

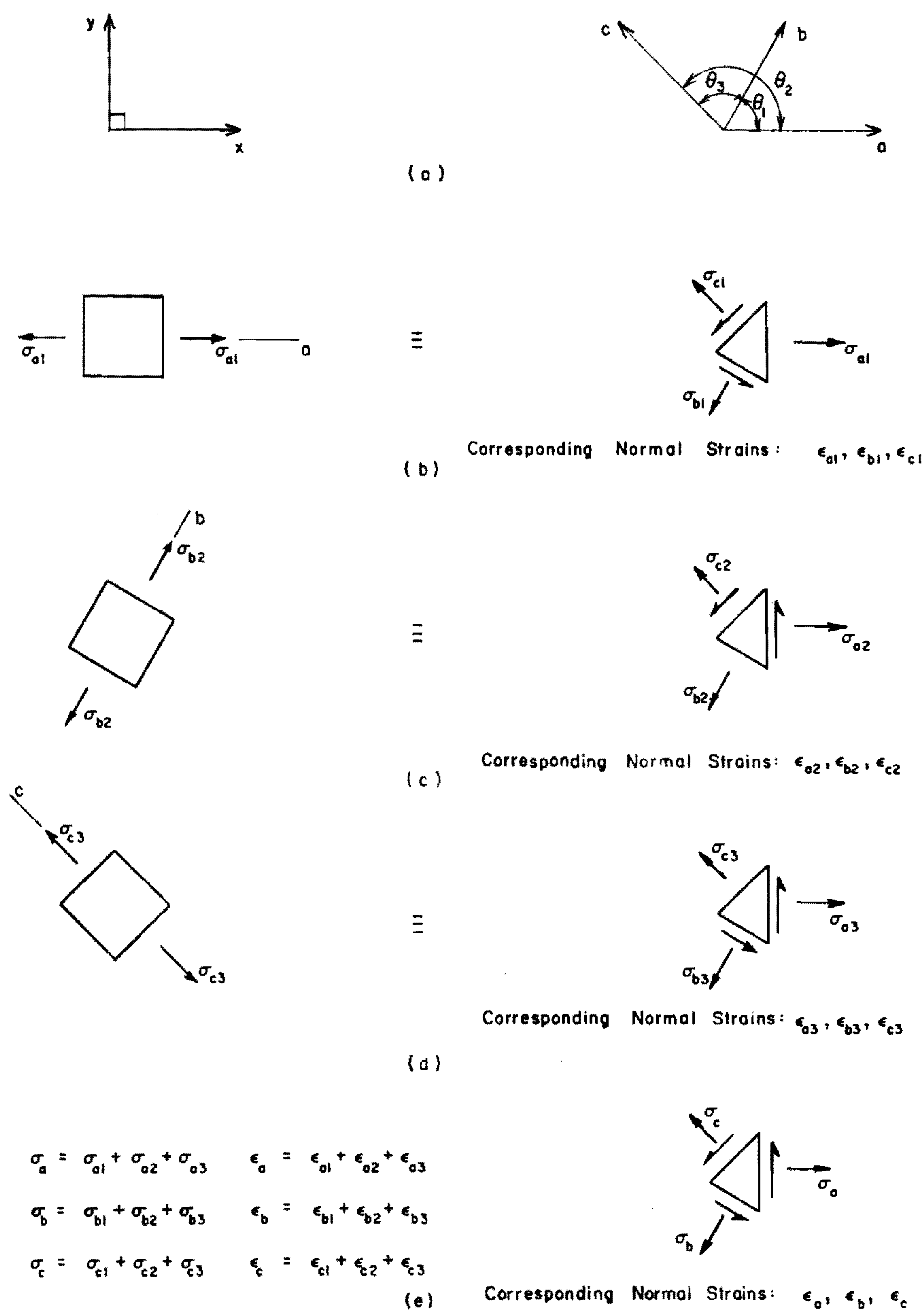


Fig 10. Stresses and corresponding strains for anisotropic rectangular and triangular plate elements in different directions.

where E_a is the modulus of elasticity with respect to the a-direction, μ_{ab} and μ_{ac} are the directional Poisson effects which characterize the decrease in the b and c-directions, respectively, for the tension in the a-direction, ℓ_1 is $\cos\theta_1$, and θ_1 is the angle between the a and b-directions.

Now consider a small rectangular element, shown in Fig 10(c), with the only stress acting being σ_{b2}. For the same state of stress, if a triangular element is considered, shown in Fig 10(c), and if σ_{a2}, σ_{b2}, and σ_{c2} are the normal stresses in the a, b, and c-directions, respectively, and ϵ_{a2}, ϵ_{b2}, and ϵ_{c2} are the corresponding normal strains, then

$$\epsilon_{b2} = \frac{\sigma_{b2}}{E_b}$$

$$\epsilon_{a2} = -\mu_{ba}\epsilon_{b2} = -\frac{\mu_{ba}\sigma_{b2}}{E_b}$$

$$\epsilon_{c2} = -\mu_{bc}\epsilon_{b2} = -\frac{\mu_{bc}\sigma_{b2}}{E_b} \qquad (4.21)$$

and

$$\sigma_{a2} = \ell_1^2\sigma_{b2}$$

$$\sigma_{b2} = \sigma_{b2}$$

$$\sigma_{c2} = \ell_3^2\sigma_{b2} \qquad (4.22)$$

where E_b is the modulus of elasticity with respect to the b-direction, μ_{ba} and μ_{bc} are the directional Poisson effects which characterize the decrease in the a and c-directions, respectively, for the tension in the b-direction, ℓ_3 is $\cos\theta_3$, and θ_3 is the angle between the b and c-directions.

Finally, consider a small rectangular element, shown in Fig 10(d), and with the only stress acting being σ_{c3}. For the same state of stress, if a triangular element is considered, shown in Fig 10(d), and if σ_{a3}, σ_{b3}, and σ_{c3} are the normal stresses and ϵ_{a3}, ϵ_{b3}, and ϵ_{c3} are the corresponding normal strains, then

$$\epsilon_{c3} = \frac{\sigma_{c3}}{E_c}$$

$$\epsilon_{a3} = -\mu_{ca}\epsilon_{c3} = -\frac{\mu_{ca}\sigma_{c3}}{E_c}$$

$$\epsilon_{b3} = -\mu_{cb}\epsilon_{c3} = -\frac{\mu_{cb}\sigma_{c3}}{E_c} \qquad (4.23)$$

and

$$\sigma_{a3} = \ell_2^2\sigma_{c3}$$

$$\sigma_{b3} = \ell_3^2\sigma_{c3}$$

$$\sigma_{c3} = \sigma_{c3} \qquad (4.24)$$

where E_c is the modulus of elasticity with respect to the c-direction, μ_{ca} and μ_{cb} are the directional Poisson effects which characterize the decrease in the a and b-directions, respectively, for the tension in the c-direction, ℓ_2 is $\cos\theta_2$, and θ_2 is the angle between the a and c-directions.

Now consider a triangular element in which the above three sets of state of stresses act simultaneously, as shown in Fig 10(e). The method of superposition can be used. Hence, if

$$\sigma_a = \sigma_{a1} + \sigma_{a2} + \sigma_{a3}$$

$$\sigma_b = \sigma_{b1} + \sigma_{b2} + \sigma_{b3}$$

$$\sigma_c = \sigma_{c1} + \sigma_{c2} + \sigma_{c3} \tag{4.25}$$

and

$$\epsilon_a = \epsilon_{a1} + \epsilon_{a2} + \epsilon_{a3}$$

$$\epsilon_b = \epsilon_{b1} + \epsilon_{b2} + \epsilon_{b3}$$

$$\epsilon_c = \epsilon_{c1} + \epsilon_{c2} + \epsilon_{c3} \tag{4.26}$$

then, using Eqs 4.19 through 4.24,

$$\sigma_a = \sigma_{a1} + \ell_1^2\sigma_{b2} + \ell_2^2\sigma_{c3}$$

$$\sigma_b = \ell_1^2\sigma_{a1} + \sigma_{b2} + \ell_3^2\sigma_{c3}$$

$$\sigma_c = \ell_2^2\sigma_{a1} + \ell_3^2\sigma_{b2} + \sigma_{c3} \tag{4.27}$$

and

$$\epsilon_a = \frac{1}{E_a}\sigma_{a1} - \frac{\mu_{ba}}{E_b}\sigma_{b2} - \frac{\mu_{ca}}{E_c}\sigma_{c3}$$

$$\epsilon_b = -\frac{\mu_{ab}}{E_a}\sigma_{a1} + \frac{1}{E_b}\sigma_{b2} - \frac{\mu_{cb}}{E_c}\sigma_{c3}$$

$$\epsilon_c = -\frac{\mu_{ac}}{E_a}\sigma_{a1} - \frac{\mu_{bc}}{E_b}\sigma_{b2} + \frac{1}{E_c}\sigma_{c3} \tag{4.28}$$

Now if Eqs 4.1, 4.2, and 4.3 are combined so that the three normal strains ϵ_a , ϵ_b , and ϵ_c are related to the three normal stresses σ_a ,

σ_b , and σ_c , which is the same as solving Eq 4.4 for three strains ϵ_a , ϵ_b , and ϵ_c , and if Eq 4.27 is substituted in these relations, then, by comparing coefficients of σ_{a1} , σ_{b2} , and σ_{c3} of the resulting equation and Eq 4.28, the following relations could be obtained:

$$\frac{\mu_{ba}}{E_b} = \frac{\mu_{ab}}{E_a}$$

$$\frac{\mu_{ca}}{E_c} = \frac{\mu_{ac}}{E_a}$$

$$\frac{\mu_{cb}}{E_c} = \frac{\mu_{bc}}{E_b} \tag{4.29}$$

Substituting Eq 4.29 into Eq 4.28 and solving for σ_{a1} , σ_{b2} , and σ_{c3} and then substituting the relations of σ_{a1} , σ_{b2} , and σ_{c3} into Eq 4.27, it is possible to develop the following anisotropic stress-strain relations, in which the normal stresses in any three directions are related to the corresponding normal strains.

$$\sigma_a = \frac{1}{|Det|}\Bigg[\left(-\frac{\mu_{cb}^2}{E_c^2} + \frac{\ell_1^2\mu_{ca}\mu_{cb}}{E_c^2} + \frac{1}{E_bE_c} + \frac{\ell_1^2\mu_{ba}}{E_bE_c}\right.$$

$$\left. + \frac{\ell_2^2\mu_{ba}\mu_{cb}}{E_bE_c} + \frac{\ell_2^2\mu_{ca}}{E_bE_c}\right)\epsilon_a + \left(\frac{\mu_{ca}\mu_{cb}}{E_c^2} - \frac{\ell_1^2\mu_{ca}^2}{E_c^2} + \frac{\mu_{ba}}{E_bE_c}\right.$$

$$\left. + \frac{\ell_2^2\mu_{ba}\mu_{ca}}{E_bE_c} + \frac{\ell_1^2}{E_aE_c} + \frac{\ell_2^2\mu_{cb}}{E_aE_c}\right)\epsilon_b + \left(\frac{\mu_{ba}\mu_{cb}}{E_bE_c} + \frac{\mu_{ca}}{E_bE_c}\right.$$

$$\left. + \frac{\ell_1^2\mu_{ba}\mu_{ca}}{E_bE_c} + \frac{\ell_1^2\mu_{cb}}{E_aE_c} + \frac{\ell_2^2}{E_aE_b} - \frac{\ell_2^2\mu_{ba}^2}{E_b^2}\right)\epsilon_c\Bigg]$$

$$\sigma_b = \frac{1}{|\mathrm{Det}|}\Bigg[\left(-\frac{\ell_1^2\mu_{cb}^2}{E_c^2}+\frac{\mu_{ca}\mu_{cb}}{E_c^2}+\frac{\ell_1^2}{E_bE_c}+\frac{\mu_{ba}}{E_bE_c}\right.$$

$$\left.+\frac{\ell_3^2\mu_{ba}\mu_{cb}}{E_bE_c}+\frac{\ell_3^2\mu_{ca}}{E_bE_c}\right)\epsilon_a+\left(\frac{\ell_1^2\mu_{ca}\mu_{cb}}{E_c^2}-\frac{\mu_{ca}^2}{E_c^2}\right.$$

$$\left.+\frac{\ell_1^2\mu_{ba}}{E_bE_c}+\frac{\ell_3^2\mu_{ba}\mu_{ca}}{E_bE_c}+\frac{1}{E_aE_c}+\frac{\ell_3^2\mu_{cb}}{E_aE_c}\right)\epsilon_b+\left(\frac{\ell_1^2\mu_{ba}\mu_{cb}}{E_bE_c}\right.$$

$$\left.+\frac{\ell_1^2\mu_{ca}}{E_bE_c}+\frac{\mu_{ba}\mu_{ca}}{E_bE_c}+\frac{\mu_{cb}}{E_aE_c}+\frac{\ell_3^2}{E_aE_b}-\frac{\ell_3^2\mu_{ba}^2}{E_b^2}\right)\epsilon_c\Bigg]$$

$$\sigma_c = \frac{1}{|\mathrm{Det}|}\Bigg[\left(-\frac{\ell_2^2\mu_{cb}^2}{E_c^2}+\frac{\ell_3^2\mu_{ca}\mu_{cb}}{E_c^2}+\frac{\ell_2^2}{E_bE_c}+\frac{\ell_3^2\mu_{ba}}{E_bE_c}+\frac{\mu_{ba}\mu_{cb}}{E_bE_c}\right.$$

$$\left.+\frac{\mu_{ca}}{E_bE_c}\right)\epsilon_a+\left(\frac{\ell_2^2\mu_{ca}\mu_{cb}}{E_c^2}-\frac{\ell_3^2\mu_{ca}^2}{E_c^2}+\frac{\ell_2^2\mu_{ba}}{E_bE_c}+\frac{\mu_{ba}\mu_{ca}}{E_bE_c}\right.$$

$$\left.+\frac{\ell_3^2}{E_aE_c}+\frac{\mu_{cb}}{E_aE_c}\right)\epsilon_b+\left(\frac{\ell_2^2\mu_{ba}\mu_{cb}}{E_bE_c}+\frac{\ell_2^2\mu_{ca}}{E_bE_c}+\frac{\ell_3^2\mu_{ba}\mu_{ca}}{E_bE_c}\right.$$

$$\left.+\frac{\ell_3^2\mu_{cb}}{E_aE_c}+\frac{1}{E_aE_b}-\frac{\mu_{ba}^2}{E_b^2}\right)\epsilon_c\Bigg] \tag{4.30}$$

where

$$|\text{Det}| = \begin{vmatrix} \frac{1}{E_a} & -\frac{\mu_{ba}}{E_b} & -\frac{\mu_{ca}}{E_c} \\ -\frac{\mu_{ba}}{E_b} & \frac{1}{E_b} & -\frac{\mu_{cb}}{E_c} \\ -\frac{\mu_{ca}}{E_c} & -\frac{\mu_{cb}}{E_c} & \frac{1}{E_c} \end{vmatrix} \tag{4.31}$$

and where $|\ |$ is used for the determinant.

Now substituting the values of σ_a, σ_b, and σ_c from Eq 4.30 into Eq 4.9, the following stress-strain relations for the fiber continuum may be obtained in which the fiber stresses are related to the conventional strains:

$$f_a = \frac{1}{|\text{Det}|}\left[\left(-\frac{\mu_{cb}^2}{E_c^2} + \frac{1}{E_b E_c}\right)\epsilon_a + \left(\frac{\mu_{ca}\mu_{cb}}{E_c^2} + \frac{\mu_{ba}}{E_b E_c}\right)\epsilon_b + \left(\frac{\mu_{ba}\mu_{cb}}{E_b E_c} + \frac{\mu_{ca}}{E_b E_c}\right)\epsilon_c\right]$$

$$f_b = \frac{1}{|\text{Det}|}\left[\left(\frac{\mu_{ca}\mu_{cb}}{E_c^2} + \frac{\mu_{ba}}{E_b E_c}\right)\epsilon_a + \left(-\frac{\mu_{ca}^2}{E_c^2} + \frac{1}{E_a E_c}\right)\epsilon_b + \left(\frac{\mu_{ba}\mu_{ca}}{E_b E_c} + \frac{\mu_{cb}}{E_a E_c}\right)\epsilon_c\right]$$

$$f_c = \frac{1}{|\text{Det}|}\left[\left(\frac{\mu_{ba}\mu_{cb}}{E_b E_c} + \frac{\mu_{ca}}{E_b E_c}\right)\epsilon_a + \left(\frac{\mu_{ba}\mu_{ca}}{E_b E_c} + \frac{\mu_{cb}}{E_a E_c}\right)\epsilon_b + \left(-\frac{\mu_{ba}^2}{E_b^2} + \frac{1}{E_a E_b}\right)\epsilon_c\right] \tag{4.32}$$

Solving Eq 4.32 for ϵ_a , ϵ_b , and ϵ_c simple relations are obtained between conventional strains and fiber stresses as follows:

$$\epsilon_a = \frac{1}{E_a} f_a - \frac{\mu_{ba}}{E_b} f_b - \frac{\mu_{ca}}{E_c} f_c$$

$$\epsilon_b = -\frac{\mu_{ba}}{E_b} f_a + \frac{1}{E_b} f_b - \frac{\mu_{cb}}{E_c} f_c$$

$$\epsilon_c = -\frac{\mu_{ca}}{E_c} f_a - \frac{\mu_{cb}}{E_c} f_b + \frac{1}{E_c} f_c \qquad (4.33)$$

It is interesting to note that the stress-strain relations of Eqs 4.32 and 4.33, which are developed above for the anisotropic fiber continuum, are analogous to the conventional stress-strain relations of Eqs 3.56 and 3.57 for a rectangular element of an anisotropic plate. For example, in the case of a rectangular element, if the only stress acting is σ_x , then $\sigma_y = \tau_{xy} = 0$. The same is true for a fiber continuum in which if the only fiber stress acting is f_a , then $f_b = f_c = 0$. Also, in the case of a rectangular element, the reciprocal relations exist for stiffnesses and compliances. Similar relations also exist for the fiber continuum in Eq 4.29.

Integration of stress-strain relations in Eq 4.32 results in moment-curvature relations similar to Eq 4.16 in which

$$B_{11} = \frac{1}{|Det|} \frac{t^3}{12} \left(-\frac{\mu_{cb}^2}{E_c^2} + \frac{1}{E_b E_c} \right)$$

$$B_{12} = \frac{1}{|Det|} \frac{t^3}{12} \left(\frac{\mu_{ca}\mu_{cb}}{E_c^2} + \frac{\mu_{ba}}{E_b E_c} \right)$$

$$B_{13} = \frac{1}{|Det|} \frac{t^3}{12} \left(\frac{\mu_{ba}\mu_{cb}}{E_b E_c} + \frac{\mu_{ca}}{E_b E_c} \right)$$

$$B_{22} = \frac{1}{|Det|} \frac{t^3}{12} \left(- \frac{\mu_{ca}^2}{E_c^2} + \frac{1}{E_a E_c} \right)$$

$$B_{23} = \frac{1}{|Det|} \frac{t^3}{12} \left(\frac{\mu_{ba}\mu_{ca}}{E_b E_c} + \frac{\mu_{cb}}{E_a E_c} \right)$$

$$B_{33} = \frac{1}{|Det|} \frac{t^3}{12} \left(- \frac{\mu_{ba}^2}{E_b^2} + \frac{1}{E_a E_b} \right) \tag{4.34}$$

where t is the thickness of the plate, and $|Det|$ is defined in Eq 4.31.

Equation 4.34 describes the six bending stiffnesses for an anisotropic fiber continuum. The stiffnesses are related to three moduli of elasticity E_a , E_b , and E_c with respect to the a , b , and c-directions, and three directional Poisson effects μ_{ba} (or μ_{ab}) , μ_{ca} (or μ_{ac}) , and μ_{cb} (or μ_{bc}) . These six constants could be experimentally determined by testing three specimens from the plate in unidirectional tension.

It may be observed that if in Eq 4.34 $\mu_{ba} = \mu_{ca} = \mu_{cb} = 0$, then the moment-curvature relations of Eq 4.16 for the fiber continuum reduce to the moment-curvature relations of Eq 4.18 for the grid-beam.

The relations between the directional Poisson effects and the conventional Poisson's ratios can also be established since

$$-\mu_{ba} = \ell_1^2 - \nu_b m_1^2 + \eta_b \ell_1 m_1$$

$$-\mu_{ca} = \ell_2^2 - \nu_c m_2^2 + \eta_c \ell_2 m_2$$

$$-\mu_{cb} = \ell_3^2 - \nu_c m_3^2 + \eta_c \ell_3 m_3 \tag{4.35}$$

Substitution of values of η_b and η_c from Eqs 3.43 and 3.44 into the above relations results in

$$-\mu_{ba} = \frac{m_3(\ell_1\ell_2 + \nu_a m_1 m_2)}{2\ell_2 m_2}\frac{E_b}{E_a} + \frac{m_2(\ell_1\ell_3 - \nu_b m_1 m_3)}{2\ell_3 m_3}$$

$$- \frac{m_1(\ell_2\ell_3 + \nu_c m_2 m_3)\ell_1 m_1}{2\ell_2 m_2 \ell_3 m_3}\frac{E_b}{E_c}$$

$$-\mu_{ca} = -\frac{m_3(\ell_1\ell_2 + \nu_a m_1 m_2)}{2\ell_1 m_1}\frac{E_c}{E_a} + \frac{m_2(\ell_1\ell_3 - \nu_b m_1 m_3)\ell_2 m_2}{2\ell_1 m_1 \ell_3 m_3}\frac{E_c}{E_b}$$

$$- \frac{m_1(\ell_2\ell_3 + \nu_c m_2 m_3)}{2\ell_3 m_3}$$

$$-\mu_{cb} = -\frac{m_3(\ell_1\ell_2 + \nu_a m_1 m_2)\ell_3 m_3}{2\ell_1 m_1 \ell_2 m_2}\frac{E_c}{E_a} + \frac{m_2(\ell_1\ell_3 - \nu_b m_1 m_3)}{2\ell_1 m_1}\frac{E_c}{E_b}$$

$$+ \frac{m_1(\ell_2\ell_3 + \nu_c m_2 m_3)}{2\ell_2 m_2} \qquad (4.36)$$

For the isotropic case, B_{11} through B_{33} reduce as follows:

$$B_{11} = \frac{Et^3}{12(1 - \nu^2)}\frac{(1 + \ell_3^2 - \nu m_3^2)}{2m_1^2 m_2^2}$$

$$B_{12} = \frac{Et^3}{12(1 - \nu^2)}\frac{(-\ell_1 - \ell_2\ell_3 + \nu m_2 m_3)}{2m_1^2 m_2 m_3}$$

$$B_{13} = \frac{Et^3}{12(1 - \nu^2)}\frac{(\ell_2 + \ell_1\ell_3 + \nu m_1 m_3)}{2m_1 m_2^2 m_3}$$

$$B_{22} = \frac{Et^3}{12(1 - \nu^2)} \frac{(1 + \ell_2^2 - \nu m_2^2)}{2m_1^2 m_3^2}$$

$$B_{23} = \frac{Et^3}{12(1 - \nu^2)} \frac{(-\ell_3 - \ell_1 \ell_2 + \nu m_1 m_2)}{2m_1 m_2 m_3^2}$$

$$B_{33} = \frac{Et^3}{12(1 - \nu^2)} \frac{(1 + \ell_1^2 - \nu m_1^2)}{2m_2^2 m_3^2} \qquad (4.37)$$

where E is the modulus of elasticity and ν is the conventional Poisson's ratio.

Summary

Equations 4.16 and 4.17 give moment-curvature relations for an anisotropic skew plate continuum in which moments are per unit width. To get the concentrated moments acting at a particular elastic joint in the corresponding discrete-element model, it is assumed that fibers running in a certain direction a , b , or c and having a certain width, as shown in Fig 11, are collected and lumped along each line of the model.

For a problem having only a grid, all the three grid-beams running in any three directions have deflection compatibility at the elastic joints. The effect of Poisson's ratio is not transferred from one grid-beam model to the other two grid-beam models. For the problem of an anisotropic plate plus grid-beams, the deflection compatibility is assumed at the elastic joints between the plate model and the three grid-beam models, and the effects due to Poisson's ratios are not transferred from plate model to any of the grid-beam models and vice versa.

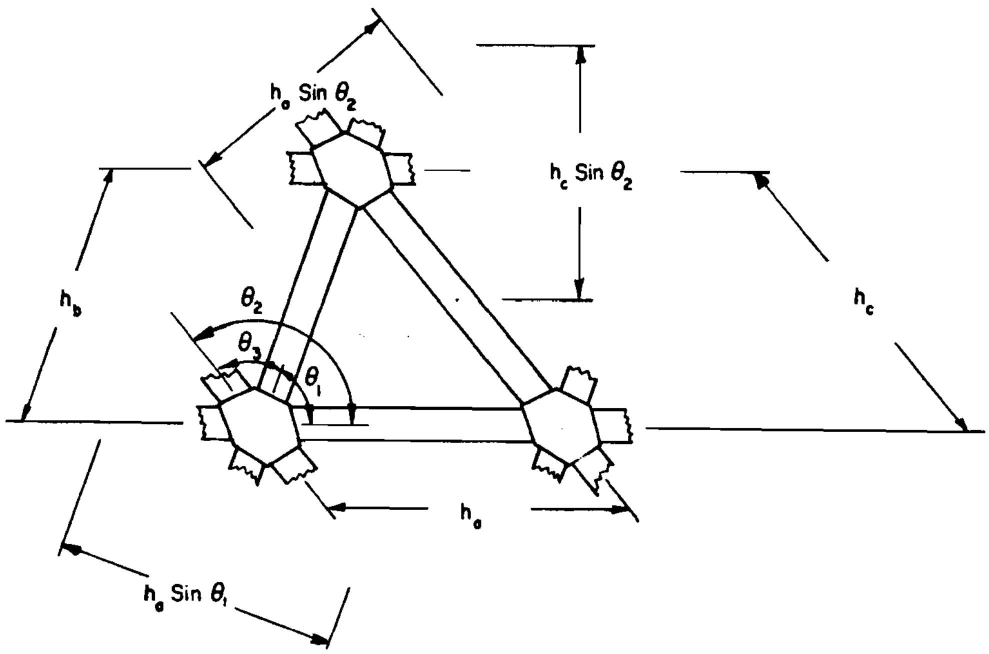


Fig 11. Plan of an anisotropic skew plate model showing the widths of fibers represented by each line of the model.

CHAPTER 5. FORMULATION OF STIFFNESS MATRIX

Introduction

In this chapter the equilibrium equation at Joint i,j is derived by considering the free-body of the joint and the rigid bars of the discrete-element slab and grid system. The operator resulting from the equilibrium equation is discussed. To form the appropriate stiffness matrix the operator is applied at each joint of the discrete-element model.

Free-Body Analysis

Figure 12 shows the free-body of Joint i,j of the slab and grid system with all appropriate internal and external forces acting on it. Any of the forces shown in Fig 12 may be zero but is considered to be acting for generality. The bars and joints are numbered as shown in Fig 13. For clarity, the following symbols are defined:

i = an integer used to index joints of the slab and grid system along the a-direction,

h_a = the increment length along the a-direction,

h_b = the increment length along the b-direction,

h_c = the increment length along the c-direction,

j = an integer used to index joints of the slab and grid system along the c-direction,

$M^{a'}_{i,j}$ = the concentrated bending moment in the a-direction at Joint i,j (equals $M_a h_c \sin \theta_2$) ,

$\overline{M}^a_{i,j}$ = the beam bending moment in the a-direction at Joint i,j,

$Q_{i,j}$ = the externally applied load at Joint i,j,

$S_{i,j}$ = support spring value at Joint i,j,

$T^a_{i-1,j}$ = external couple in the a-direction applied at Joint i-1,j,

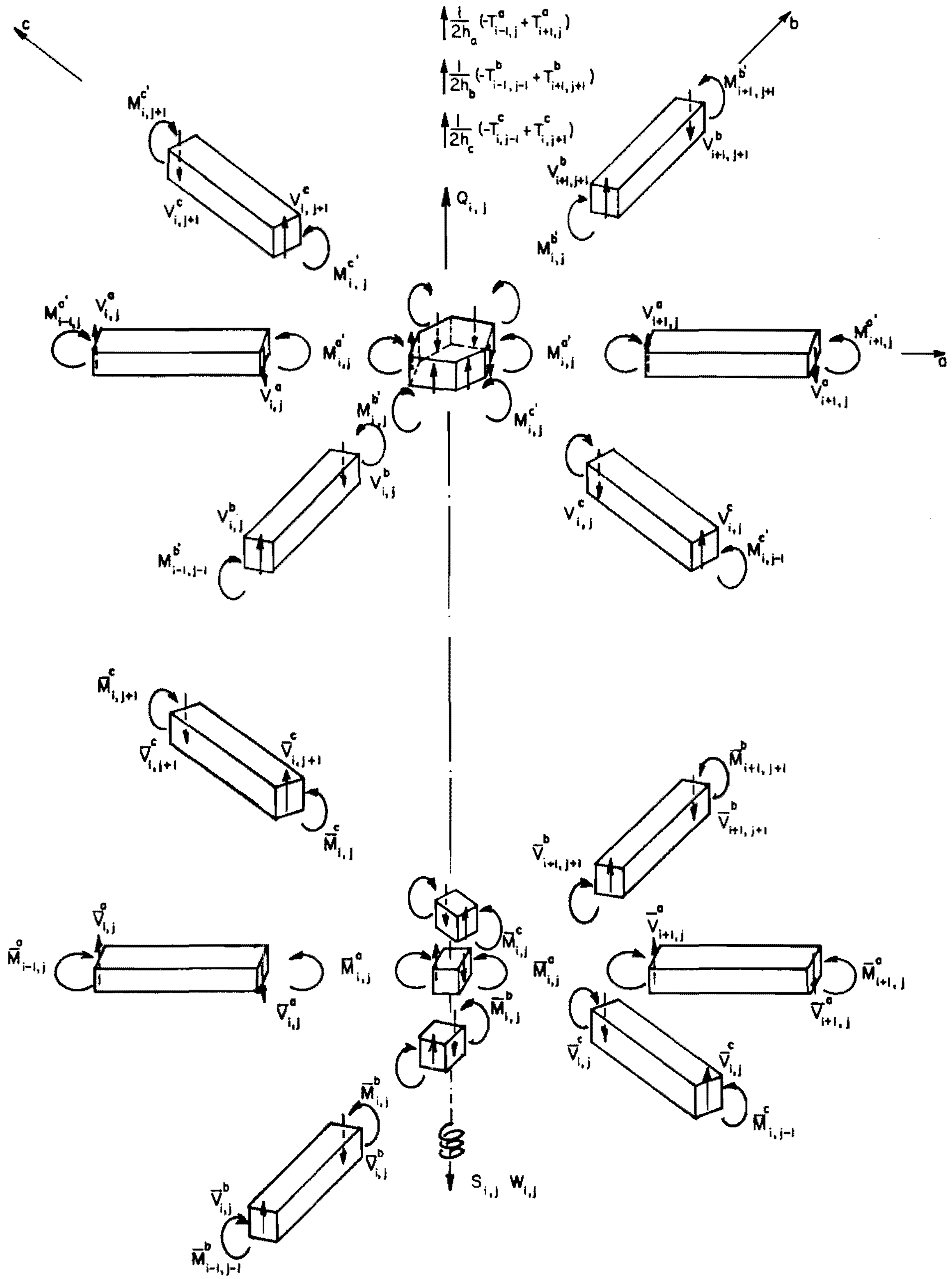


Fig 12. Free-body diagram of Joint i,j of an anisotropic skew slab and grid system model.

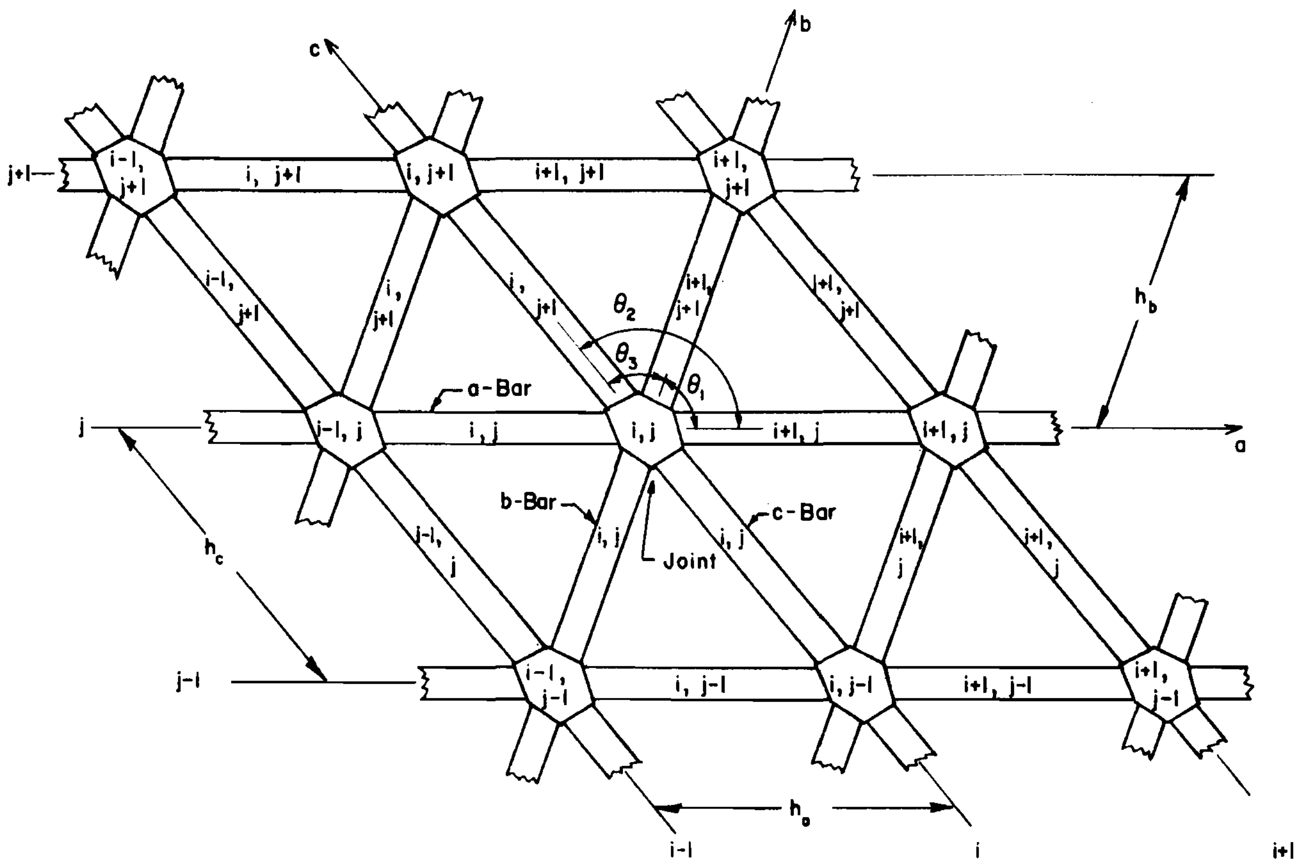


Fig 13. Plan view of skew slab and grid system model showing all parts with generalized numbering system.

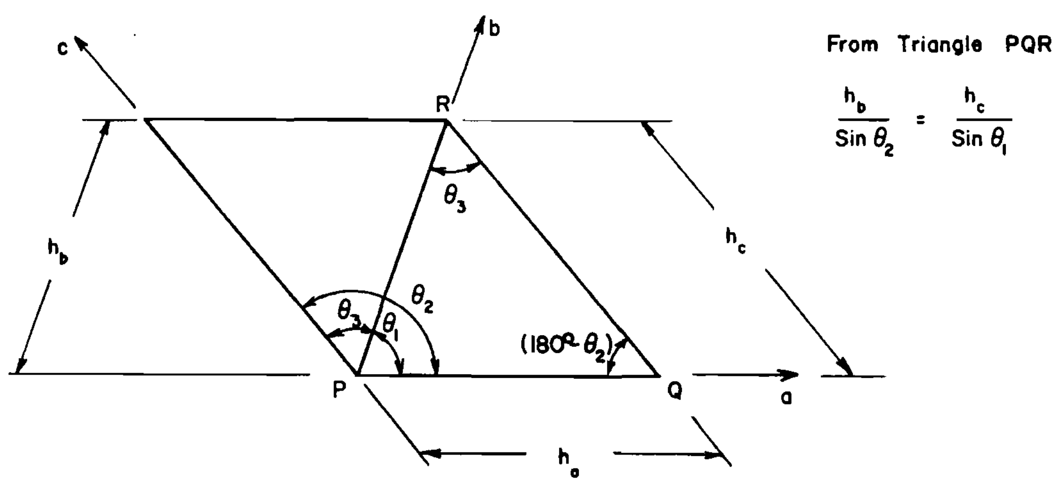


Fig 14. Geometric relations.

$V^a_{i,j}$ = shear in slab Bar i,j running in the a-direction,

$\overline{V}^a_{i,j}$ = shear in grid beam Bar i,j running in the a-direction,

$w_{i,j}$ = deflection at Joint i,j.

Summation of vertical forces at Joint i,j (Fig 12) with up taken as positive gives

$$V^a_{i,j} + V^b_{i,j} + V^c_{i,j} - V^a_{i+1,j} - V^b_{i+1,j+1} - V^c_{i,j+1} + \overline{V}^a_{i,j}$$

$$+ \overline{V}^b_{i,j} + \overline{V}^c_{i,j} - \overline{V}^a_{i+1,j} - \overline{V}^b_{i+1,j+1} - \overline{V}^c_{i,j+1} + Q_{i,j}$$

$$- S_{i,j}w_{i,j} + \frac{1}{2h_a}(-T^a_{i-1,j} + T^a_{i+1,j}) + \frac{1}{2h_b}(-T^b_{i-1,j-1}$$

$$+ T^b_{i+1,j+1}) + \frac{1}{2h_c}(-T^c_{i,j-1} + T^c_{i,j+1}) = 0 \qquad (5.1)$$

To eliminate shears from the above equation, the summation of moments about each individual bar is taken as follows:

$$-V^a_{i,j}h_a = M^{a'}_{i-1,j} - M^{a'}_{i,j}$$

$$-V^b_{i,j}h_b = M^{b'}_{i-1,j-1} - M^{b'}_{i,j}$$

$$-V^c_{i,j}h_c = M^{c'}_{i,j-1} - M^{c'}_{i,j}$$

$$V^a_{i+1,j}h_a = M^{a'}_{i+1,j} - M^{a'}_{i,j}$$

$$V^b_{i+1,j+1}h_b = M^{b'}_{i+1,j+1} - M^{b'}_{i,j}$$

$$V^{c}_{i,j+1}h_{c} = M^{c'}_{i,j+1} - M^{c'}_{i,j} \qquad (5.2)$$

and

$$-\overline{V}^{a}_{i,j}h_{a} = \overline{M}^{a}_{i-1,j} - \overline{M}^{a}_{i,j}$$

$$-\overline{V}^{b}_{i,j}h_{b} = \overline{M}^{b}_{i-1,j-1} - \overline{M}^{b}_{i,j}$$

$$-\overline{V}^{c}_{i,j}h_{c} = \overline{M}^{c}_{i,j-1} - \overline{M}^{c}_{i,j}$$

$$\overline{V}^{a}_{i+1,j}h_{a} = \overline{M}^{a}_{i+1,j} - \overline{M}^{a}_{i,j}$$

$$\overline{V}^{b}_{i+1,j+1}h_{b} = \overline{M}^{b}_{i+1,j+1} - \overline{M}^{b}_{i,j}$$

$$\overline{V}^{c}_{i,j+1}h_{c} = \overline{M}^{c}_{i,j+1} - \overline{M}^{c}_{i,j} \qquad (5.3)$$

where M'_a , M'_b , and M'_c are concentrated values of slab moments. Also

$$M'_a = M_a h_c \sin \theta_2$$

$$M'_b = M_b h_a \sin \theta_1$$

$$M'_c = M_c h_a \sin \theta_2 \qquad (5.4)$$

where M_a , M_b , and M_c are moments per unit width of slab.

Eliminating shears from Eq 5.1 by using Eqs 5.2 and 5.3 and rearranging gives the following expression:

$$\frac{1}{h_a}(M^{a'}_{i-1,j} - 2M^{a'}_{i,j} + M^{a'}_{i+1,j}) + \frac{1}{h_b}(M^{b'}_{i-1,j-1} - 2M^{b'}_{i,j}$$

$$+ M^{b'}_{i+1,j+1}) + \frac{1}{h_c}(M^{c'}_{i,j-1} - 2M^{c'}_{i,j} + M^{c'}_{i,j+1})$$

$$+ \frac{1}{h_a}(\overline{M}^a_{i-1,j} - 2\overline{M}^a_{i,j} + \overline{M}^a_{i+1,j}) + \frac{1}{h_b}(\overline{M}^b_{i-1,j-1}$$

$$- 2\overline{M}^b_{i,j} + \overline{M}^b_{i+1,j+1}) + \frac{1}{h_c}(\overline{M}^c_{i,j-1} - 2\overline{M}^c_{i,j} + \overline{M}^c_{i,j+1}) + S_{i,j}w_{i,j}$$

$$= Q_{i,j} + \frac{1}{2h_a}(-T^a_{i-1,j} + T^a_{i+1,j}) + \frac{1}{2h_b}(-T^b_{i-1,j-1}$$

$$+ T^b_{i+1,j+1}) + \frac{1}{2h_c}(-T^c_{i,j-1} + T^c_{i,j+1}) \qquad (5.5)$$

M'_a , M'_b , M'_c , $\overline{M}_a$, $\overline{M}_b$, and $\overline{M}_c$ expressions are found by introducing the finite-difference approximations for the second derivative of deflections into Eqs 4.16 and 4.18 and using Eq 5.4:

$$M^{a'}_{i-1,j} = B^{11}_{i-1,j}h_c \sin\theta_2 \frac{1}{h_a^2}(w_{i-2,j} - 2w_{i-1,j} + w_{i,j})$$

$$+ B^{12}_{i-1,j}h_c \sin\theta_2 \frac{1}{h_b^2}(w_{i-2,j-1} - 2w_{i-1,j} + w_{i,j+1})$$

$$+ B^{13}_{i-1,j}h_c \sin\theta_2 \frac{1}{h_c^2}(w_{i-1,j-1} - 2w_{i-1,j} + w_{i-1,j+1}) \qquad (5.6)$$

$$M^{a'}_{i,j} = B^{11}_{i,j} h_c \sin\theta_2 \frac{1}{h_a^2} (w_{i-1,j} - 2w_{i,j} + w_{i+1,j})$$

$$+ B^{12}_{i,j} h_c \sin\theta_2 \frac{1}{h_b^2} (w_{i-1,j-1} - 2w_{i,j} + w_{i+1,j+1})$$

$$+ B^{13}_{i,j} h_c \sin\theta_2 \frac{1}{h_c^2} (w_{i,j-1} - 2w_{i,j} + w_{i,j+1}) \quad (5.7)$$

$$M^{a'}_{i+1,j} = B^{11}_{i+1,j} h_c \sin\theta_2 \frac{1}{h_a^2} (w_{i,j} - 2w_{i+1,j} + w_{i+2,j})$$

$$+ B^{12}_{i+1,j} h_c \sin\theta_2 \frac{1}{h_b^2} (w_{i,j-1} - 2w_{i+1,j} + w_{i+2,j+1})$$

$$+ B^{13}_{i+1,j} h_c \sin\theta_2 \frac{1}{h_c^2} (w_{i+1,j-1} - 2w_{i+1,j} + w_{i+1,j+1}) \quad (5.8)$$

$$M^{b'}_{i-1,j-1} = B^{12}_{i-1,j-1} h_a \sin\theta_1 \frac{1}{h_a^2} (w_{i-2,j-1} - 2w_{i-1,j-1}$$

$$+ w_{i,j-1}) + B^{22}_{i-1,j-1} h_a \sin\theta_1 \frac{1}{h_b^2} (w_{i-2,j-2}$$

$$- 2w_{i-1,j-1} + w_{i,j}) + B^{23}_{i-1,j-1} h_a \sin\theta_1 \frac{1}{h_c^2} (w_{i-1,j-2}$$

$$- 2w_{i-1,j-1} + w_{i-1,j}) \quad (5.9)$$

$$M^{b'}_{i,j} = B^{12}_{i,j} h_a \sin\theta_1 \frac{1}{h_a^2} (w_{i-1,j} - 2w_{i,j} + w_{i+1,j})$$

$$+ B^{22}_{i,j} h_a \sin\theta_1 \frac{1}{h_b^2} (w_{i-1,j-1} - 2w_{i,j} + w_{i+1,j+1})$$

$$+ B^{23}_{i,j} h_a \sin\theta_1 \frac{1}{h_c^2} (w_{i,j-1} - 2w_{i,j} + w_{i,j+1}) \qquad (5.10)$$

$$M^{b'}_{i+1,j+1} = B^{12}_{i+1,j+1} h_a \sin\theta_1 \frac{1}{h_a^2} (w_{i,j+1} - 2w_{i+1,j+1}$$

$$+ w_{i+2,j+1}) + B^{22}_{i+1,j+1} h_a \sin\theta_1 \frac{1}{h_b^2} (w_{i,j} - 2w_{i+1,j+1}$$

$$+ w_{i+2,j+2}) + B^{23}_{i+1,j+1} h_a \sin\theta_1 \frac{1}{h_c^2} (w_{i+1,j}$$

$$- 2w_{i+1,j+1} + w_{i+1,j+2}) \qquad (5.11)$$

$$M^{c'}_{i,j-1} = B^{13}_{i,j-1} h_a \sin\theta_2 \frac{1}{h_a^2} (w_{i-1,j-1} - 2w_{i,j-1}$$

$$+ w_{i+1,j-1}) + B^{23}_{i,j-1} h_a \sin\theta_2 \frac{1}{h_b^2} (w_{i-1,j-2} - 2w_{i,j-1}$$

$$+ w_{i+1,j}) + B^{33}_{i,j-1} h_a \sin\theta_2 \frac{1}{h_c^2} (w_{i,j-2}$$

$$- 2w_{i,j-1} + w_{i,j}) \qquad (5.12)$$

$$M^{c'}_{i,j} = B^{13}_{i,j} h_a \sin\theta_2 \frac{1}{h_a^2} (w_{i-1,j} - 2w_{i,j} + w_{i+1,j})$$

$$+ B^{23}_{i,j} h_a \sin\theta_2 \frac{1}{h_b^2} (w_{i-1,j-1} - 2w_{i,j} + w_{i+1,j+1})$$

$$+ B^{33}_{i,j} h_a \sin\theta_2 \frac{1}{h_c^2} (w_{i,j-1} - 2w_{i,j} + w_{i,j+1}) \quad (5.13)$$

$$M^{c'}_{i,j+1} = B^{13}_{i,j+1} h_a \sin\theta_2 \frac{1}{h_a^2} (w_{i-1,j+1} - 2w_{i,j+1} + w_{i+1,j+1})$$

$$+ B^{23}_{i,j+1} h_a \sin\theta_2 \frac{1}{h_b^2} (w_{i-1,j} - 2w_{i,j+1} + w_{i+1,j+2})$$

$$+ B^{33}_{i,j+1} h_a \sin\theta_2 \frac{1}{h_c^2} (w_{i,j} - 2w_{i,j+1} + w_{i,j+2}) \quad (5.14)$$

$$\overline{M}^a_{i-1,j} = F^a_{i-1,j} \frac{1}{h_a^2} (w_{i-2,j} - 2w_{i-1,j} + w_{i,j}) \quad (5.15)$$

$$\overline{M}^a_{i,j} = F^a_{i,j} \frac{1}{h_a^2} (w_{i-1,j} - 2w_{i,j} + w_{i+1,j}) \quad (5.16)$$

$$\overline{M}^a_{i+1,j} = F^a_{i+1,j} \frac{1}{h_a^2} (w_{i,j} - 2w_{i+1,j} + w_{i+2,j}) \quad (5.17)$$

$$\overline{M}^b_{i-1,j-1} = F^b_{i-1,j-1} \frac{1}{h_b^2} (w_{i-2,j-2} - 2w_{i-1,j-1} + w_{i,j}) \quad (5.18)$$

$$\overline{M}^b_{i,j} = F^b_{i,j} \frac{1}{h_b^2} (w_{i-1,j-1} - 2w_{i,j} + w_{i+1,j+1}) \quad (5.19)$$

$$\overline{M}^b_{i+1,j+1} = F^b_{i+1,j+1} \frac{1}{h_b^2} (w_{i,j} - 2w_{i+1,j+1} + w_{i+2,j+2}) \tag{5.20}$$

$$\overline{M}^c_{i,j-1} = F^c_{i,j-1} \frac{1}{h_c^2} (w_{i,j-2} - 2w_{i,j-1} + w_{i,j}) \tag{5.21}$$

$$\overline{M}^c_{i,j} = F^c_{i,j} \frac{1}{h_c^2} (w_{i,j-1} - 2w_{i,j} + w_{i,j+1}) \tag{5.22}$$

$$\overline{M}^c_{i,j+1} = F^c_{i,j+1} \frac{1}{h_c^2} (w_{i,j} - 2w_{i,j+1} + w_{i,j+2}) \tag{5.23}$$

The terms defined by Eqs 5.6 through 5.23 are introduced in Eq 5.5. Collecting the terms, the final form of Eq 5.5 can be written as follows:

$$\begin{aligned} & a^1_{i,j}w_{i-2,j-2} + a^2_{i,j}w_{i-1,j-2} + a^3_{i,j}w_{i,j-2} + b^1_{i,j}w_{i-2,j-1} \\ & + b^2_{i,j}w_{i-1,j-1} + b^3_{i,j}w_{i,j-1} + b^4_{i,j}w_{i+1,j-1} + c^1_{i,j}w_{i-2,j} \\ & + c^2_{i,j}w_{i-1,j} + c^3_{i,j}w_{i,j} + c^4_{i,j}w_{i+1,j} + c^5_{i,j}w_{i+2,j} \\ & + d^2_{i,j}w_{i-1,j+1} + d^3_{i,j}w_{i,j+1} + d^4_{i,j}w_{i+1,j+1} + d^5_{i,j}w_{i+2,j+1} \\ & + e^3_{i,j}w_{i,j+2} + e^4_{i,j}w_{i+1,j+2} + e^5_{i,j}w_{i+2,j+2} = f_{i,j} \end{aligned} \tag{5.24}$$

where

$$a^1_{i,j} = B^{22}_{i-1,j-1} \frac{h_a \sin \theta_1}{h_b^3} + F^b_{i-1,j-1} \frac{1}{h_b^3}$$

$$a^2_{i,j} = B^{23}_{i-1,j-1} \frac{h_a \sin\theta_1}{h_b h_c^2} + B^{23}_{i,j-1} \frac{h_a \sin\theta_2}{h_b^2 h_c}$$

$$a^3_{i,j} = B^{33}_{i,j-1} \frac{h_a \sin\theta_2}{h_c^3} + F^c_{i,j-1} \frac{1}{h_c^3} \tag{5.25}$$

$$b^1_{i,j} = B^{12}_{i-1,j} \frac{h_c \sin\theta_2}{h_a h_b^2} + B^{12}_{i-1,j-1} \frac{\sin\theta_1}{h_a h_b}$$

$$b^2_{i,j} = B^{13}_{i-1,j} \frac{\sin\theta_2}{h_a h_c} - 2B^{12}_{i,j} \frac{h_c \sin\theta_2}{h_a h_b^2} - 2B^{12}_{i-1,j-1} \frac{\sin\theta_1}{h_a h_b}$$

$$- 2B^{22}_{i-1,j-1} \frac{h_a \sin\theta_1}{h_b^3} - 2B^{23}_{i-1,j-1} \frac{h_a \sin\theta_1}{h_b h_c^2}$$

$$- 2B^{22}_{i,j} \frac{h_a \sin\theta_1}{h_b^3} + B^{13}_{i,j-1} \frac{\sin\theta_2}{h_a h_c} - 2B^{23}_{i,j} \frac{h_a \sin\theta_2}{h_b^2 h_c}$$

$$- 2F^b_{i-1,j-1} \frac{1}{h_b^3} - 2F^b_{i,j} \frac{1}{h_b^3}$$

$$b^3_{i,j} = -2B^{13}_{i,j} \frac{\sin\theta_2}{h_a h_c} + B^{12}_{i+1,j} \frac{h_c \sin\theta_2}{h_a h_b^2} + B^{12}_{i-1,j-1} \frac{\sin\theta_1}{h_a h_b}$$

$$- 2B^{23}_{i,j} \frac{h_a \sin\theta_1}{h_b h_c^2} - 2B^{13}_{i,j-1} \frac{\sin\theta_2}{h_a h_c} - 2B^{23}_{i,j-1} \frac{h_a \sin\theta_2}{h_b^2 h_c}$$

$$- 2B^{33}_{i,j-1} \frac{h_a \sin\theta_2}{h_c^3} - 2B^{33}_{i,j} \frac{h_a \sin\theta_2}{h_c^3} - 2F^c_{i,j-1} \frac{1}{h_c^3} - 2F^c_{i,j} \frac{1}{h_c^3}$$

$$b^4_{i,j} = B^{13}_{i+1,j} \frac{\sin\theta_2}{h_a h_c} + B^{13}_{i,j-1} \frac{\sin\theta_2}{h_a h_c} \tag{5.26}$$

$$c^1_{i,j} = B^{11}_{i-1,j} \frac{h_c \sin\theta_2}{h_a^3} + F^a_{i-1,j} \frac{1}{h_a^3}$$

$$c^2_{i,j} = -2B^{11}_{i-1,j} \frac{h_c \sin\theta_2}{h_a^3} - 2B^{12}_{i-1,j} \frac{h_c \sin\theta_2}{h_a h_b^2}$$

$$- 2B^{13}_{i-1,j} \frac{\sin\theta_2}{h_a h_c} - 2B^{11}_{i,j} \frac{h_c \sin\theta_2}{h_a^3} + B^{23}_{i-1,j-1} \frac{h_a \sin\theta_1}{h_b h_c^2}$$

$$- 2B^{12}_{i,j} \frac{\sin\theta_1}{h_a h_b} - 2B^{13}_{i,j} \frac{\sin\theta_2}{h_a h_c} + B^{23}_{i,j+1} \frac{h_a \sin\theta_2}{h_b^2 h_c}$$

$$- 2F^a_{i-1,j} \frac{1}{h_a^3} - 2F^a_{i,j} \frac{1}{h_a^3}$$

$$c^3_{i,j} = B^{11}_{i-1,j} \frac{h_c \sin\theta_2}{h_a^3} + 4B^{11}_{i,j} \frac{h_c \sin\theta_2}{h_a^3} + 4B^{12}_{i,j} \frac{h_c \sin\theta_2}{h_a h_b^2}$$

$$+ 4B^{13}_{i,j} \frac{\sin\theta_2}{h_a h_c} + B^{11}_{i+1,j} \frac{h_c \sin\theta_2}{h_a^3} + B^{22}_{i-1,j-1} \frac{h_a \sin\theta_1}{h_b^3}$$

$$+ 4B^{12}_{i,j} \frac{\sin\theta_1}{h_a h_b} + 4B^{22}_{i,j} \frac{h_a \sin\theta_1}{h_b^3} + 4B^{23}_{i,j} \frac{h_a \sin\theta_1}{h_b h_c^2}$$

$$+ B^{22}_{i+1,j+1} \frac{h_a \sin\theta_1}{h_b^3} + B^{33}_{i,j-1} \frac{h_a \sin\theta_2}{h_c^3} + 4B^{13}_{i,j} \frac{\sin\theta_2}{h_a h_c}$$

$$+ 4B^{23}_{i,j} \frac{h_a \sin\theta_2}{h_b^2 h_c} + 4B^{33}_{i,j} \frac{h_a \sin\theta_2}{h_c^3} + B^{33}_{i,j+1} \frac{h_a \sin\theta_2}{h_c^3}$$

$$+ S_{i,j} + F^a_{i-1,j} \frac{1}{h_a^3} + 4F^a_{i,j} \frac{1}{h_a^3} + F^a_{i+1,j} \frac{1}{h_a^3}$$

$$+ F^b_{i-1,j-1} \frac{1}{h_b^3} + 4F^b_{i,j} \frac{1}{h_b^3} + F^b_{i+1,j+1} \frac{1}{h_b^3} + F^c_{i,j-1} \frac{1}{h_c^3}$$

$$+ 4F^c_{i,j} \frac{1}{h_c^3} + F^c_{i,j+1} \frac{1}{h_c^3}$$

$$c^4_{i,j} = -2B^{11}_{i,j} \frac{h_c \sin\theta_2}{h_a^3} - 2B^{11}_{i+1,j} \frac{h_c \sin\theta_2}{h_a^3}$$

$$- 2B^{12}_{i+1,j} \frac{h_c \sin\theta_2}{h_a h_b^2} - 2B^{13}_{i+1,j} \frac{\sin\theta_2}{h_a h_c} - 2B^{12}_{i,j} \frac{\sin\theta_1}{h_a h_b}$$

$$+ B^{23}_{i+1,j+1} \frac{h_a \sin\theta_1}{h_b h_c^2} + B^{23}_{i,j-1} \frac{h_a \sin\theta_2}{h_b^2 h_c} - 2B^{13}_{i,j} \frac{\sin\theta_2}{h_a h_c}$$

$$- 2F^a_{i,j} \frac{1}{h_a^3} - 2F^a_{i+1,j} \frac{1}{h_a^3}$$

$$c^5_{i,j} = B^{11}_{i+1,j} \frac{h_c \sin\theta_2}{h_a^3} + F^a_{i+1,j} \frac{1}{h_a^3} \qquad (5.27)$$

$$d^2_{i,j} = B^{13}_{i-1,j} \frac{\sin \theta_2}{h_a h_c} + B^{13}_{i,j+1} \frac{\sin \theta_2}{h_a h_c}$$

$$d^3_{i,j} = B^{12}_{i-1,j} \frac{h_c \sin \theta_2}{h_a h_b^2} - 2B^{13}_{i,j} \frac{\sin \theta_2}{h_a h_c} - 2B^{23}_{i,j} \frac{h_a \sin \theta_1}{h_b h_c^2}$$

$$+ B^{12}_{i+1,j+1} \frac{\sin \theta_1}{h_a h_b} - 2B^{33}_{i,j} \frac{h_a \sin \theta_2}{h_c^3} - 2B^{13}_{i,j+1} \frac{\sin \theta_2}{h_a h_c}$$

$$- 2B^{23}_{i,j+1} \frac{h_a \sin \theta_2}{h_b^2 h_c} - 2B^{33}_{i,j+1} \frac{h_a \sin \theta_2}{h_c^3}$$

$$- 2F^{c}_{i,j} \frac{1}{h_c^3} - 2F^{c}_{i,j+1} \frac{1}{h_c^3}$$

$$d^4_{i,j} = -2B^{12}_{i,j} \frac{h_c \sin \theta_2}{h_a h_b^2} + B^{13}_{i+1,j} \frac{\sin \theta_2}{h_a h_c} - 2B^{22}_{i,j} \frac{h_a \sin \theta_1}{h_b^3}$$

$$- 2B^{12}_{i+1,j+1} \frac{\sin \theta_1}{h_a h_b} - 2B^{22}_{i+1,j+1} \frac{h_a \sin \theta_1}{h_b^3}$$

$$- 2B^{23}_{i+1,j+1} \frac{h_a \sin \theta_1}{h_b h_c^2} - 2B^{23}_{i,j} \frac{h_a \sin \theta_2}{h_b^2 h_c}$$

$$+ B^{13}_{i,j+1} \frac{\sin \theta_2}{h_a h_c} - 2F^{b}_{i,j} \frac{1}{h_b^3} - 2F^{b}_{i+1,j+1} \frac{1}{h_b^3}$$

$$d^5_{i,j} = B^{12}_{i+1,j} \frac{h_c \sin \theta_2}{h_a h_b^2} + B^{12}_{i+1,j+1} \frac{\sin \theta_1}{h_a h_b} \qquad (5.28)$$

$$e^3_{i,j} = B^{33}_{i,j+1} \frac{h_a \sin \theta_2}{h^3_c} + F^c_{i,j+1} \frac{1}{h^3_c}$$

$$e^4_{i,j} = B^{23}_{i+1,j+1} \frac{h_a \sin \theta_1}{h_b h^2_c} + B^{23}_{i,j+1} \frac{h_a \sin \theta_2}{h^2_b h_c}$$

$$e^5_{i,j} = B^{22}_{i+1,j+1} \frac{h_a \sin \theta_1}{h^3_b} + F^b_{i+1,j+1} \frac{1}{h^3_b} \tag{5.29}$$

$$f_{i,j} = Q_{i,j} + \frac{1}{2h_a} (-T^a_{i-1,j} + T^a_{i+1,j}) + \frac{1}{2h_b} (-T^b_{i-1,j-1}$$

$$+ T^b_{i+1,j+1}) + \frac{1}{2h_c} (-T^c_{i,j-1} + T^c_{i,j+1}) \tag{5.30}$$

Also, from Fig 14,

$$\frac{h_b}{\sin \theta_2} = \frac{h_c}{\sin \theta_1} \tag{5.31}$$

Operator

The equilibrium equation (Eq 5.24) can be visualized as an operator. It has 19 points as shown in Figs 15(c) and 15(d), and is first applied to the bottom row of joints from left to right (Figs 15(a) and 15(b)), then to the second row, and so on, moving upward. It is interesting to note that Fig 15(b), which is a mirror image of Fig 15(a), does not form the same equilibrium equations as Fig 15(a). This is due to the lopsided operator. This means that the two problems (Figs 15(a) and 15(b)) would not give exactly the same results. It has been observed that the difference between these solutions is about 1 percent in maximum deflection for a 20-by-20 increment solution. This difference reduces as the number of increments is increased.

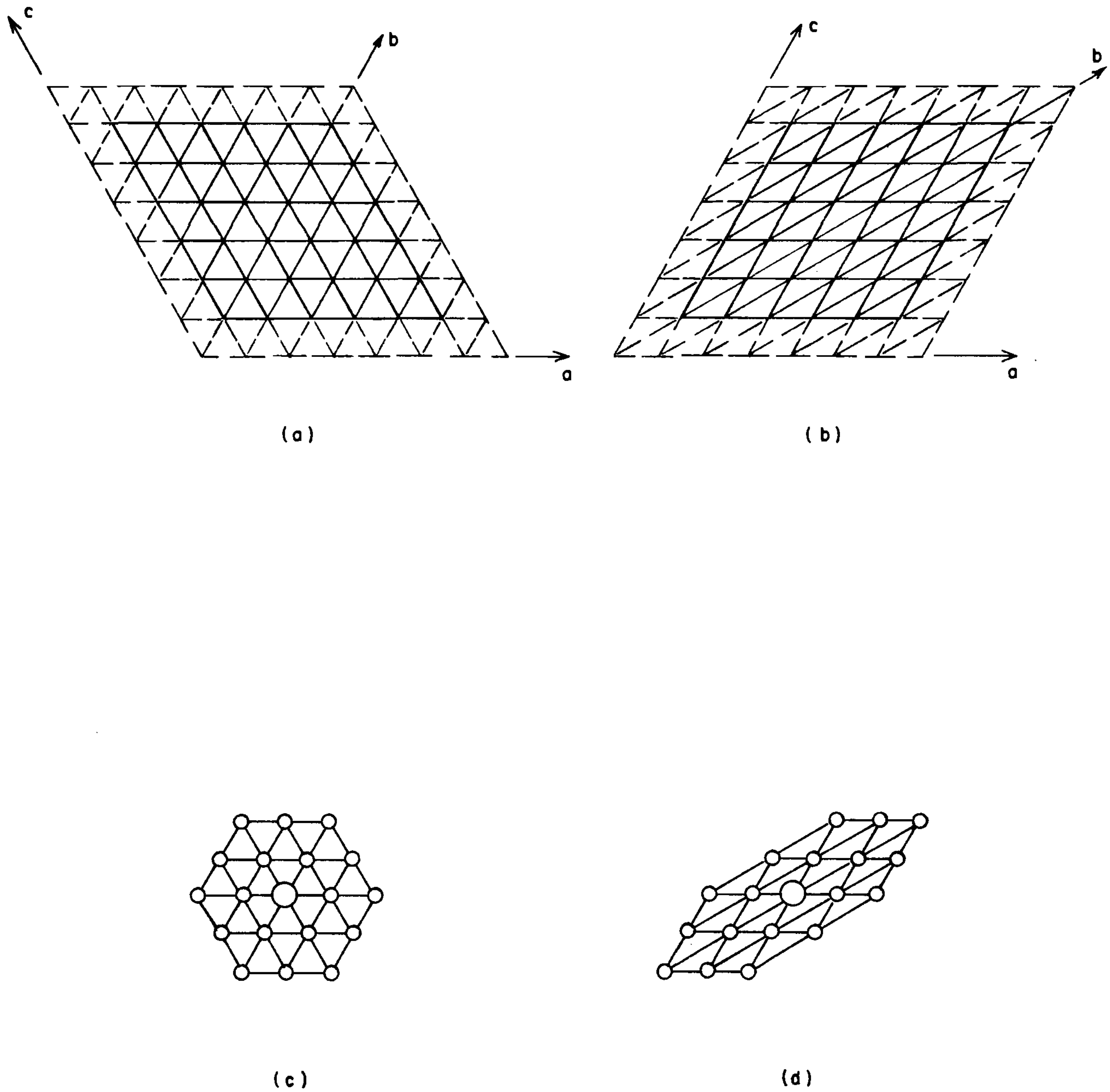


Fig 15. Discrete-element models and their operators.

Stiffness Matrix

Equations 5.24 through 5.30 describe the equilibrium equation at Joint i,j of the discrete-element model. Similar equations are written at each joint of the model, as explained above, to form the stiffness matrix.

In the matrix form, Eq 5.24 may be represented as

$$\left[K\right]\left\{w\right\} = \left\{f\right\} \tag{5.32}$$

The form of matrices $\left[K\right]$, $\left\{w\right\}$, and $\left\{f\right\}$ is shown in Fig 16. The stiffness matrix $\left[K\right]$ is symmetrical about its major diagonal and is also banded. The central band is five terms wide. The bands on either side of the central band are four terms wide and the two extreme bands are three terms wide. The stiffness matrix is partitioned as shown by the dashed line in Fig 16. If the skew slab and grid system to be solved is divided into m increments in the a-direction and n increments in the c-direction, then the stiffness matrix will have $n+3$ rows and $n+3$ columns of submatrices. Each submatrix will have $m+3$ rows and $m+3$ columns of terms. Because the recursion process is used to solve Eq 5.24, it is more efficient if m is smaller than n .

Summary

The stiffness matrix for the slab and grid system is obtained by writing equations of statics at each joint. The stiffness matrix is symmetric about its major diagonal. Advantage of this symmetry is taken in the solution of equations.

GENERAL SLAB EQUATION:

$$
\begin{aligned}
& a^1_{i,j} w_{i-2,j-2} + a^2_{i,j} w_{i-1,j-2} + a^3_{i,j} w_{i,j-2} \\
& + b^1_{i,j} w_{i-2,j-1} + b^2_{i,j} w_{i-1,j-1} + b^3_{i,j} w_{i,j-1} + b^4_{i,j} w_{i+1,j-1} \\
& + c^1_{i,j} w_{i-2,j} + c^2_{i,j} w_{i-1,j} + c^3_{i,j} w_{i,j} + c^4_{i,j} w_{i+1,j} + c^5_{i,j} w_{i+2,j} \\
& \qquad + d^2_{i,j} w_{i-1,j+1} + d^3_{i,j} w_{i,j+1} + d^4_{i,j} w_{i+1,j+1} + d^5_{i,j} w_{i+2,j+1} \\
& \qquad\qquad + e^3_{i,j} w_{i,j+2} + e^4_{i,j} w_{i+1,j+2} + e^5_{i,j} w_{i+2,j+2} = f_{i,j}
\end{aligned}
$$

OR IN MATRIX FORM:

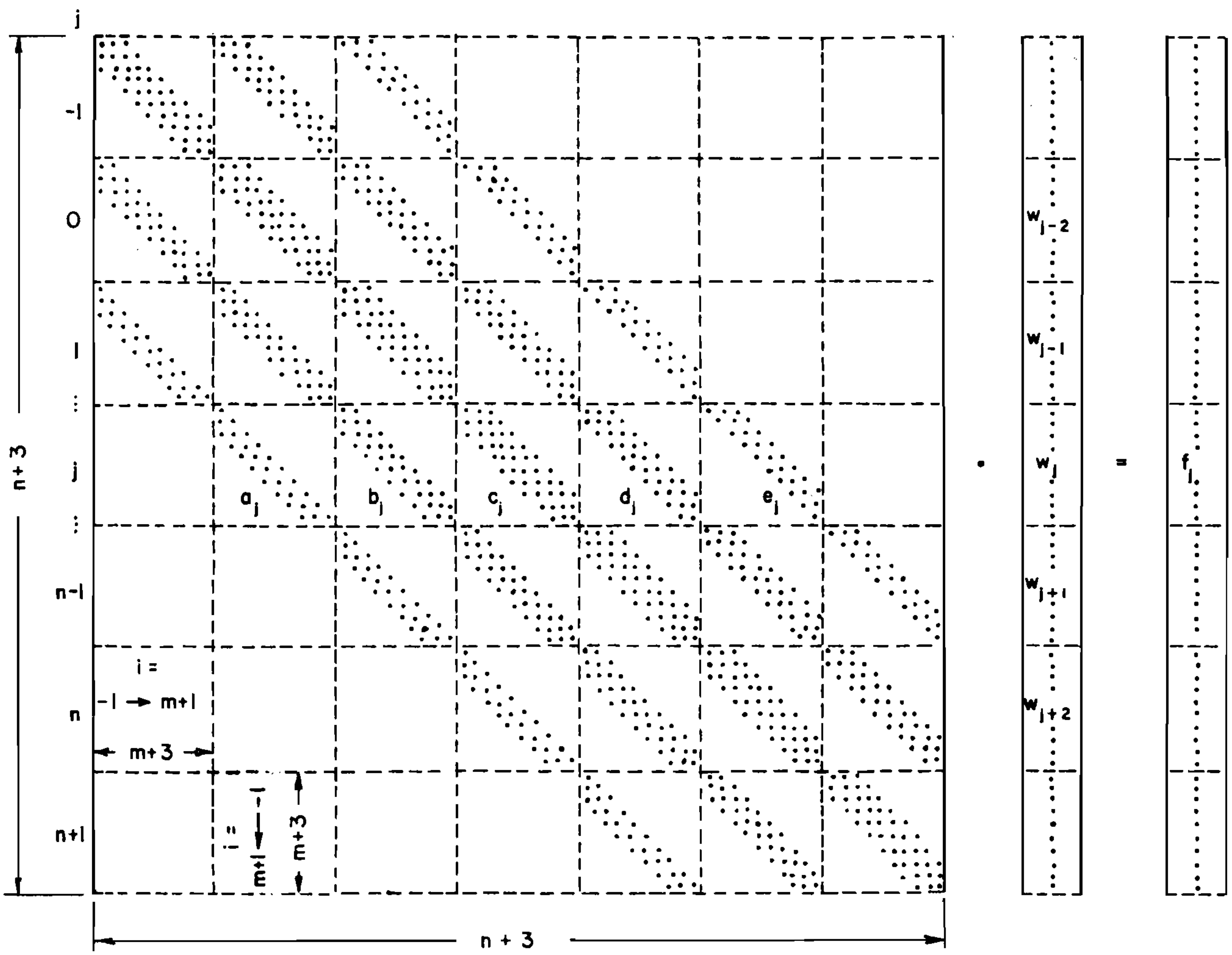


Fig 16. Form of the equations showing partitioned stiffness matrix.

CHAPTER 6. SOLUTION OF EQUATIONS

Introduction

The equilibrium equation is applied at each joint of the discrete-element model and resulting equations are arranged and partitioned. A recursion-inversion solution procedure is employed to solve the equations. A brief review of this procedure is made in this chapter. It has been shown that multiple load analysis can be handled efficiently also.

Arrangement of Equations

The problem to be solved, which may be an anisotropic skew plate, a skew grid, or a combination of both, should be divided into a skew grid work. If the number of increments is m in the a-direction and n in the c-direction then the number of joints becomes $m+1$ and $n+1$ in the a and c-directions, respectively. The equilibrium equation is written for each joint, including a fictitious joint, all around the actual problem, which makes the total number of equations to be solved $(m + 3) \times (n + 3)$.

The equations are arranged and partitioned as shown in Fig 16. Fig 17 shows the banding of different submatrices at a j^{th} horizontal partition.

Recursion-Inversion Solution Procedure

Matlock (Ref 18) described the recursion technique for the solution of equations for a beam-column. Stelzer (Ref 38) and Panak and Matlock (Ref 31) have used this technique to solve equations for the rectangular plate problems.

In the recursion procedure, a solution of the following form is assumed:

$$\left\{w_j\right\} = \left\{A_j\right\} + \left[B_j\right]\left\{w_{j+1}\right\} + \left[C_j\right]\left\{w_{j+2}\right\} \qquad (6.1)$$

At the j^{th} horizontal partition (Fig 16) the equation may be written in the form

$$a_j = \begin{bmatrix} a^3_{-1,j} & & & & & & \\ a^2_{0,j} & a^3_{0,j} & & & & & \\ a^1_{1,j} & a^2_{1,j} & a^3_{1,j} & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & a^1_{i,j} & a^2_{i,j} & a^3_{i,j} & & \\ & & & \ddots & \ddots & \ddots & \\ & & & a^1_{m-1,j} & a^2_{m-1,j} & a^3_{m-1,j} & \\ & & & & a^1_{m,j} & a^2_{m,j} & a^3_{m,j} \\ & & & & & a^1_{m+1,j} & a^2_{m+1,j} \; a^3_{m+1,j} \end{bmatrix} \qquad b_j = \begin{bmatrix} b^3_{-1,j} & b^4_{-1,j} & & & & & \\ b^2_{0,j} & b^3_{0,j} & b^4_{0,j} & & & & \\ b^1_{1,j} & b^2_{1,j} & b^3_{1,j} & b^4_{1,j} & & & \\ & \ddots & \ddots & \ddots & \ddots & & \\ & & b^1_{i,j} & b^2_{i,j} & b^3_{i,j} & b^4_{i,j} & \\ & & & \ddots & \ddots & \ddots & \ddots \\ & & & b^1_{m-1,j} & b^2_{m-1,j} & b^3_{m-1,j} & b^4_{m-1,j} \\ & & & & b^1_{m,j} & b^2_{m,j} & b^3_{m,j} \; b^4_{m,j} \\ & & & & & b^1_{m+1,j} & b^2_{m+1,j} \; b^3_{m+1,j} \end{bmatrix}$$

$$c_j = \begin{bmatrix} c^3_{-1,j} & c^4_{-1,j} & c^5_{-1,j} & & & & \\ c^2_{0,j} & c^3_{0,j} & c^4_{0,j} & c^5_{0,j} & & & \\ c^1_{1,j} & c^2_{1,j} & c^3_{1,j} & c^4_{1,j} & c^5_{1,j} & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & c^1_{i,j} & c^2_{i,j} & c^3_{i,j} & c^4_{i,j} & c^5_{i,j} & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & c^1_{m-1,j} & c^2_{m-1,j} & c^3_{m-1,j} & c^4_{m-1,j} & c^5_{m-1,j} \\ & & & c^1_{m,j} & c^2_{m,j} & c^3_{m,j} & c^4_{m,j} \\ & & & & c^1_{m+1,j} & c^2_{m+1,j} & c^3_{m+1,j} \end{bmatrix} \qquad d_j = \begin{bmatrix} d^3_{-1,j} & d^4_{-1,j} & d^5_{-1,j} & & & & \\ d^2_{0,j} & d^3_{0,j} & d^4_{0,j} & d^5_{0,j} & & & \\ d^1_{1,j} & d^2_{1,j} & d^3_{1,j} & d^4_{1,j} & d^5_{1,j} & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & d^1_{i,j} & d^2_{i,j} & d^3_{i,j} & d^4_{i,j} & d^5_{i,j} & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & d^1_{m-1,j} & d^2_{m-1,j} & d^3_{m-1,j} & d^4_{m-1,j} & d^5_{m-1,j} \\ & & & d^1_{m,j} & d^2_{m,j} & d^3_{m,j} & d^4_{m,j} \\ & & & & d^1_{m+1,j} & d^2_{m+1,j} & d^3_{m+1,j} \end{bmatrix}$$

$$e_j = \begin{bmatrix} e^3_{-1,j} & e^4_{-1,j} & e^5_{-1,j} & & & & \\ & e^3_{0,j} & e^4_{0,j} & e^5_{0,j} & & & \\ & & e^3_{1,j} & e^4_{1,j} & e^5_{1,j} & & \\ & & & \ddots & \ddots & \ddots & \\ & & & e^3_{i,j} & e^4_{i,j} & e^5_{i,j} & \\ & & & & \ddots & \ddots & \ddots \\ & & & & e^3_{m-1,j} & e^4_{m-1,j} & e^5_{m-1,j} \\ & & & & & e^3_{m,j} & e^4_{m,j} \\ & & & & & & e^3_{m+1,j} \end{bmatrix} \qquad w_j = \begin{bmatrix} w_{-1,j} \\ w_{0,j} \\ w_{1,j} \\ \vdots \\ w_{i,j} \\ \vdots \\ w_{m-1,j} \\ w_{m,j} \\ w_{m+1,j} \end{bmatrix} \qquad f_j = \begin{bmatrix} f_{-1,j} \\ f_{0,j} \\ f_{1,j} \\ \vdots \\ f_{i,j} \\ \vdots \\ f_{m-1,j} \\ f_{m,j} \\ f_{m+1,j} \end{bmatrix}$$

Fig 17. Banding of submatrices.

$$\left[a_j\right]\left\{w_{j-2}\right\} + \left[b_j\right]\left\{w_{j-1}\right\} + \left[c_j\right]\left\{w_j\right\} + \left[d_j\right]\left\{w_{j+1}\right\}$$

$$+ \left[e_j\right]\left\{w_{j+2}\right\} = \left\{f_j\right\} \qquad (6.2)$$

Substitution of equations similar to Eq 6.1 into Eq 6.2 to eliminate $\left\{w_{j-2}\right\}$ and $\left\{w_{j-1}\right\}$ gives

$$\left\{A_j\right\} = \left[D_j\right]\left[\left[E_j\right]\left\{A_{j-1}\right\} + \left[a_j\right]\left\{A_{j-2}\right\} - \left\{f_j\right\}\right]$$

$$\left[B_j\right] = \left[D_j\right]\left[\left[E_j\right]\left[C_{j-1}\right] + \left[d_j\right]\right]$$

$$\left[C_j\right] = \left[D_j\right]\left[e_j\right] \qquad (6.3)$$

where

$$\left[D_j\right] = -\left[\left[a_j\right]\left[C_{j-2}\right] + \left[E_j\right]\left[B_{j-1}\right] + \left[c_j\right]\right]^{-1}$$

$$\left[E_j\right] = \left[a_j\right]\left[B_{j-2}\right] + \left[b_j\right] \qquad (6.4)$$

Endres and Matlock (Ref 8) modified Eq 6.3 in order to make the solution procedure more efficient. The final form of equations for the solution of a symmetric stiffness matrix is

$$\left\{A_j\right\} = \left[D_j\right]\left[\left[E_j\right]\left\{A_{j-1}\right\} + \left[e_{j-2}\right]^t\left\{A_{j-2}\right\} - \left\{f_j\right\}\right]$$

$$\left[B_j\right] = \left[D_j\right]\left[E_{j+1}\right]^t$$

$$\left[C_j\right] = \left[D_j\right]\left[e_j\right] \qquad (6.5)$$

where

$$[D_j] = -\left[[e_{j-2}]^{-t} [C_{j-2}] + [E_j][B_{j-1}] + [c_j] \right]^{-1}$$

$$[E_{j+1}] = [e_{j-1}]^{t} [B_{j-1}] + [d_j]^{t} \qquad (6.6)$$

and t stands for transpose of matrix.

Solution of Deflections

In the forward pass of the recursion procedure, $\{A_j\}$, $\{B_j\}$, and $[C_j]$ are formed with the help of Eqs 6.5 and 6.6. On the reverse pass, deflections $\{w_j\}$ are computed using Eq 6.1.

Multiple-Loading Technique

This technique was discussed by Panak and Matlock (Ref 31).

For a multiple-loading solution, instead of re-solving the problem for each loading condition, the recursion coefficient vector $\{A_j\}$ in Eq 6.5 is modified for successive loadings and the other coefficients are retained on tape storage.

Summary

The recursion technique is advantageous in multiple-load analysis. The boundaries of the real problem are automatically taken care of due to the model.

CHAPTER 7. DESCRIPTION OF PROGRAM SLAB 44

Introduction

SLAB 44 is a computer program written to apply the discrete-element formulation of an anisotropic skew-plate and grid-beam system in which the grid-beams may run in any three directions. The number 44 simply means that this is the 44th significantly distinct program in the chronological sequence of development of various slab and grid programs. The program solves linear problems. In this chapter, Program SLAB 44 is discussed and the procedure for data input is explained. The error messages and other output information are also discussed.

The FORTRAN Program

The SLAB 44 computer program is written in FORTRAN and for the CDC 6600 computer. The program could be modified to make it compatible with IBM 360 computers, UNIVAC 1108 systems, or other computers.

A summary flow chart for the SLAB 44 program is given in Fig 18. A general flow diagram of the program is given in Appendix 2. A list of the variables used with their definitions is given in Appendix 3. A complete listing of the program is shown in Appendix 4.

Time and Storage Requirements

The compile time for the program is about 21 seconds on the CDC 6600 computer. The time required to run problems varies with the number of increments involved. On the CDC 6600, a ten-by-ten increment problem can be solved in about seven seconds, while a 20-by-20 increment problem can be solved in about 23 seconds, and a 40-by-40 increment problem can be solved in about 70 seconds.

The storage requirements are variable, depending upon the size of the problem to be run.

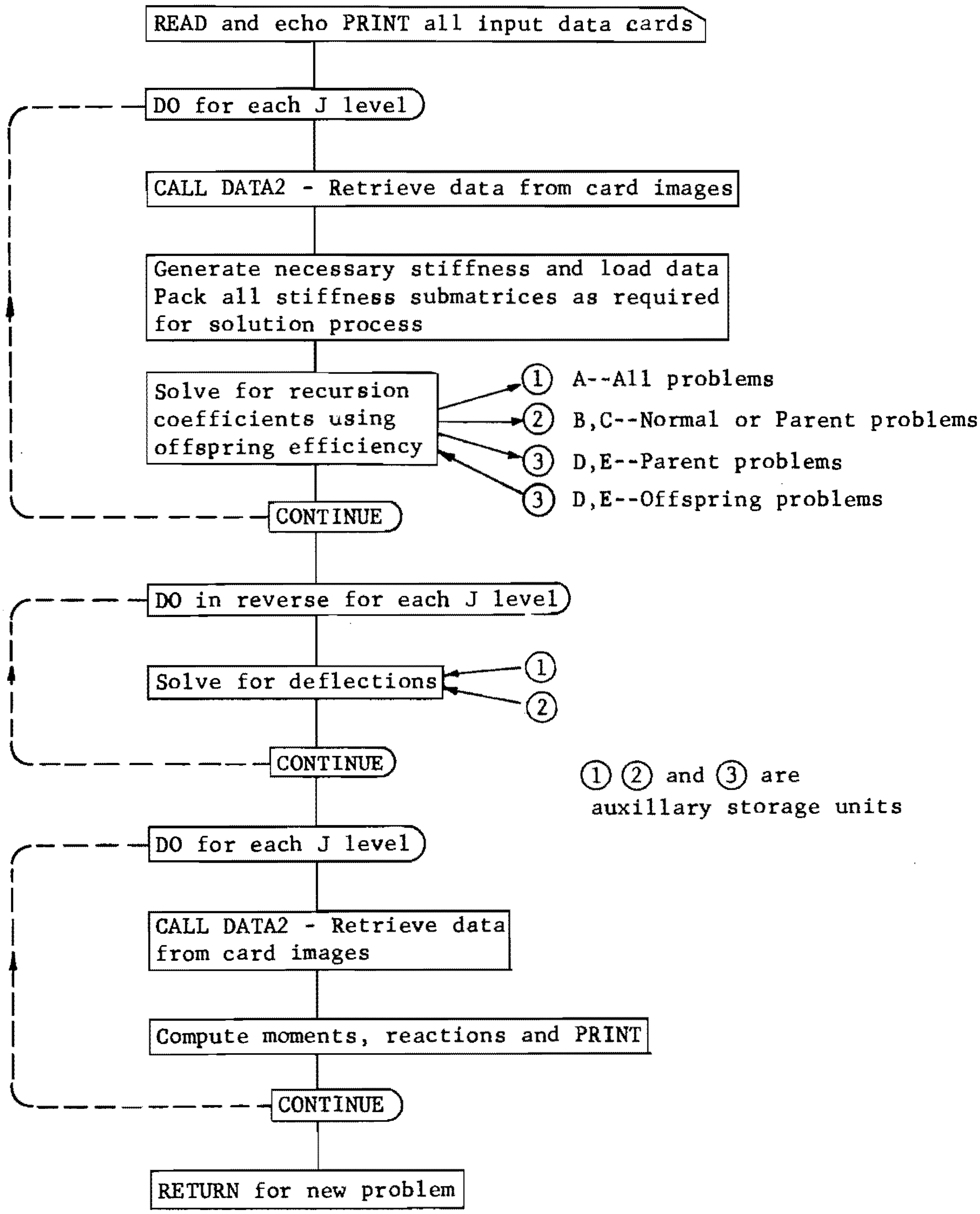


Fig 18. Summary flow chart for program SLAB 44.

Procedure for Data Input

A guide for data input in included in Appendix 1. The guide is designed so that additional copies may be made and used for routine reference. A parallel study of the guide will help to understand the following discussion.

Any number of problems may be run at the same time. A problem series is preceded by two cards which describe the run. The first card of each problem contains the program number and a brief description of the problem. The problem series terminates when a blank problem number is encountered.

Tables of Data Input

Table 1 is comprised of a single data card that includes options to hold data from a preceding problem, a count of cards added to each table in the current problem, multiple load option, print option, reaction output option, and stiffness input option.

The multiple load option in Table 1 is exercised for problem series in which only the load pattern and placement will vary and stiffness properties remain constant. The first problem in the loading series is the "parent" problem. Each successive loading is an "offspring" problem. The option is left blank for a normal problem.

The print option in Table 1 may be exercised to print bending moments in the a , b , and c-directions. The option is left blank to print bending moments in the x and y-directions and twisting moments (the x and a-directions are the same). In either case, the largest principal moments are computed and printed.

The reaction output option in Table 1 may be exercised to print a statics check for each joint, i.e., the summation of all shears, loads, and restraint reactions. The restraint reactions due to spring supports are printed if the option is left blank. In either case, the summation of support spring reactions and the maximum statics check error, with coordinates, are computed and printed at the end of the problem.

The stiffness input option in Table 1 may be exercised to input slab stiffnesses related to the three directions a , b , and c. The option is left blank to input slab stiffnesses related to the orthogonal directions x and y .

Table 2 is comprised of a single data card that includes number of increments in the a and c-directions, increment length in the a and c-directions, and the angle between the a and c-direction in degrees (Fig 13). For Table 2 a choice must be made between holding all the data from the preceding problem and entering entirely new data.

Table 3 is comprised of plate or slab bending stiffnesses. If the input value of stiffnesses is related to the orthogonal directions (D_{11} through D_{33}), then for isotropic and orthotropic plates and stiffnesses may be computed either as shown in the guide for data input in Appendix 1 or by some other procedure. For the anisotropic case the stiffnesses may be computed by using either three moduli of elasticity in any three directions and three Poisson's ratios related to the same three directions or another set of six constants, as explained in Chapters 3 and 4. Any other procedure may be used to find bending stiffnesses for an anisotropic plate. If the input value of stiffnesses is related to the directions a , b , and c (B_{11} through B_{33}), then it could be computed either in terms of stiffnesses related to the orthogonal directions (D_{11} through D_{33}) or in terms of three moduli of elasticity in any three directions and three directional Poisson effects, as explained in Chapter 4.

Table 4 is comprised of beam stiffnesses, loads, and support springs. Concentrated stiffness values for beams running in the a , b , and c-directions may be input. Load and support spring values for any joint are determined by multiplying the unit load or unit support values by the appropriate area of the real slab or plate assigned to that joint. Hinged supports are provided by using large spring values. Loads and stiffnesses that occur between joints may be fractionally proportioned to the adjacent joints.

Table 5 is comprised of external couples in the a , b , and c-directions. Concentrated values of couples may be input at joints.

In Tables 3, 4, and 5 all the data are described with a and c station numbers. To distribute data over an area, the lower left-hand and upper right-hand coordinates must be specified. To specify data at a single location, the same coordinates must be specified in both the "From" and "Through" coordinates. The "Through" coordinates must always be equal to or numerically greater than the "From" coordinates. All the data in Tables 3, 4, and 5 are algebraically accumulated and therefore values may be added or subtracted.

Error Messages

All input data are checked for possible errors. If any data errors are encountered, the problem is terminated and messages showing the number of data errors in each table and total data errors in the problem are printed. Typical input data errors are (1) misusing the multiple load option, such as an offspring problem following a normal problem; (2) having the number of increments in the a-direction greater than in the c-direction; (3) specifying a negative or zero increment length; (4) having the "Through" coordinates less than "From" coordinates; and (5) specifying data outside the limits of the slab and grid system.

In addition to the above, a general purpose error message "UNDESIGNATED ERROR STOP" is provided for a number of unlikely errors.

Computed Results

The output is arranged so that the input quantities of Tables 1 through 5 are printed with explanatory headings. The computed final results are printed in Table 6. The headings in Table 6 depend on the options used. This table is arranged to print the a and c-joint coordinates; the transverse deflection at each joint, with up positive; the bending moments in the x and y-directions and twisting moment, or bending moments in the a , b , and c-directions; the largest principal moment and its direction; and support reaction, or statics check at each joint. The summation of support reactions is computed and printed at the end of Table 6. Also, a statics check is made at each joint and the maximum statics check error is printed at the end of Table 6.

The interpretation of moments computed at the edge of a slab and grid-beam system needs some explanation. For example, for a simply supported, uniformly loaded, square plate, the moments at the edge, and in the direction perpendicular to the simple support, should be zero. With Program SLAB 44, these moments cannot be computed and printed as zeros, because in this program one fictitious joint all around the actual problem is considered and the moment at any boundary joint is computed in terms of three curvatures in the three directions. For the moment to be zero, either the three curvatures or all of the stiffnesses at the joint have to be zero. The moment could also be zero if its value in Eq 4.16 is zero. The moments at the fictitious joints would be computed as zero but at the actual boundary they might have some value.

Summary

SLAB 44 provides a solution for an anisotropic skew-plate and grid-beam system. The solution process used to solve the stiffness equations makes the program very efficient. To solve a particular problem, usually 15 to 20 increments in the a and c-directions are enough, although the number of increments depends upon the size of the real problem, accuracy required, and variation in parameters.

CHAPTER 8. EXAMPLE PROBLEMS AND COMPARISONS

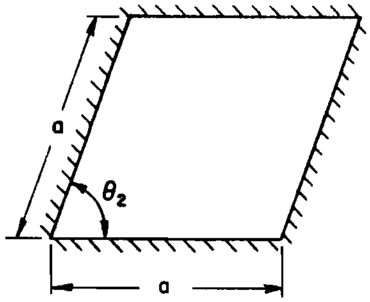
Introduction

To verify the formulation of an anisotropic skew-plate and grid system with the discrete-element approach, several example problems were solved using Program SLAB 44. Since there are no closed-form mathematical solutions for the skew plates, the results could be compared only with the results from other approximate methods. In this chapter, seven problem series are presented. In the first six series the results from the discrete-element solution are compared with series, finite-element, conformal mapping, finite difference, electrical analogue, and experimental results. The constants, e.g., stiffnesses and loads, used in each problem series are also given. In problem series 7, a brief sensitivity analysis is made for modeling of bending and torsional stiffnesses of a composite section of a single-span bridge.

A listing of input data and output for a selected problem is included in Appendices 5 and 6.

Problem Series 1. Simply Supported, Uniformly Loaded Rhombic Plates

A series of simply supported, uniformly loaded, isotropic, rhombic plates was solved using the SLAB 44 program with 20-by-20 increments. The results for the maximum deflections, maximum principal moments, and minimum principal moments are compared with the series solution by Morley (Ref 24), whose results are most extensive and have been used as a basis for comparison by several investigators, including Gustafson, and the finite-element solution by Gustafson (Ref 10) (Fig 19). For $\theta_2 = 90^\circ$, or 0° skew, the results are compared with the exact solution by Timoshenko (Ref 40). Figure 19 shows these comparisons. The overall differences between Morley's and SLAB 44 results are 4.1 percent in maximum deflection at $\theta_2 = 50^\circ$, or 40° skew; 9.4 percent in maximum principal moment at $\theta^2 = 30^\circ$, or 60° skew; and 5.8 percent in minimum principal moment at $\theta_2 = 50^\circ$, or 40° skew. At $\theta_2 = 90^\circ$, or 0° skew, the differences between the exact solution (Ref 40) and SLAB 44 results are



$a = 20$ in.

Stiffness : $D = \dfrac{Et^3}{12(1-\nu^2)} = 1.6 \times 10^6$ lb in.^{2}/in

Poisson's Ratio : $\nu = 0.3$

Load per Unit Area : $q_0 = 1 \times 10^4$ lb/in.^{2}

Simply supported, uniformly loaded, isotropic, rhombic plate

Angle	$w_{max} \times \dfrac{D}{q_0 a^4 10^{-3}}$				$M_{max} \times \dfrac{1}{q_0 a^2 10^{-2}}$				$M_{min} \times \dfrac{1}{q_0 a^2 10^{-2}}$			
θ_2	Ref 40 Exact	Ref 24	Ref 10 16 x 16	SLAB 44 20 x 20	Ref 40 Exact	Ref 24	Ref 10 16 x 16	SLAB 44 20 x 20	Ref 40 Exact	Ref 24	Ref 10 16 x 16	SLAB 44 20 x 20
90°	4.06	-	-	4.10	4.79	-	-	4.83	4.79	-	-	4.81
85°	-	4.01	-	4.06	-	4.86	-	4.90	-	4.66	-	4.70
80°	-	3.87	-	3.92	-	4.86	-	4.92	-	4.48	-	4.54
60°	-	2.56	2.59	2.65	-	4.25	4.26	4.41	-	3.33	3.37	3.46
50°	-	1.72	1.69	1.79	-	3.62	3.55	3.83	-	2.58	2.51	2.73
40°	-	0.958	-	0.996	-	2.81	-	3.04	-	1.80	-	1.90
30°	-	0.408	0.377	0.409	-	1.91	1.80	2.09	-	1.08	0.96	1.09

Ref 40 is exact solution by Timoshenko and Woinowsky-Krieger

Ref 24 is series solution by Morley

Ref 10 is finite-element solution by Gustafson and Wright

Fig 19. Comparison of results for simply supported, uniformly loaded, isotropic, rhombic plates.

1.0 percent in maximum deflection, 0.8 percent in maximum principal moment, and 0.4 percent in minimum principal moment.

Problem Series 2. Simply Supported Rhombic Plates with Concentrated Load

Aggarwala (Ref 1) has used conformal mapping to obtain the central deflection of the simply supported, centrally loaded, isotropic, rhombic plates. Twenty-by-twenty increment SLAB 44 solutions were made for different skew angles and the deflections at the centers of plates are compared with his results (Fig 20). The difference between Aggarwala's results and those from SLAB 44 at $\theta_2 = 50^\circ$, or 40° skew, is about 7 percent, which reduces to about 2.8 percent at $\theta_2 = 90^\circ$, or 0° skew.

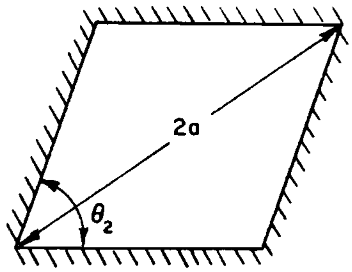
Problem Series 3. Simply Supported, Uniformly Loaded Triangular Plate

A closed-form solution for the deformation of a simply supported, uniformly loaded, isotropic, equilateral triangular plate has been given by Timoshenko (Ref 40). Using the SLAB 44 program, a 21-by-21 increment solution was made by inputting appropriate values of stiffnesses at each joint and using Poisson's ratio of 0.3. Figure 21 shows the comparison of deflection at point 0 of the triangular plate. The difference between the closed-form result and SLAB 44 result is about 0.78 percent.

Problem Series 4. Five-Beam, Noncomposite Skew Bridges

Chen, Siess, and Newmark (Ref 6) have considered a simple-span, noncomposite, skew bridge which consisted of a concrete slab of uniform thickness supported by five identical steel beams uniformly spaced and parallel to the direction of traffic. Using the finite difference approach, they have computed influence coefficients for moments and deflections for a number of skew bridges having an 8-by-8 mesh. Gustafson and Wright (Ref 10) have used the finite-element method with an 8-by-8 mesh to solve the same bridge problem for a few loading conditions and compared their results with the solutions of Chen, Siess, and Newmark.

The SLAB 44 program, with 8-by-8 increments, was used to solve the same bridge problem. Figure 22 shows a comparison of beam moments obtained from finite difference, finite-element, and discrete-element solutions for different skew angles. It is interesting to note that the results of finite-element



Simply supported, isotropic, rhombic plate with concentrated load at the center

Stiffness : $D = \frac{Et^3}{12(1-\nu^2)} = 4.0 \times 10^5$ lb in^2/in

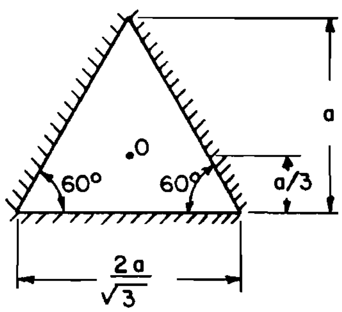
Poisson's Ratio : $\nu = 0.3$

Concentrated Load : $Q = 1.0 \times 10^3$ lb

Angle θ_2	Deflection at Center x $\frac{D}{Qa^2}$	
	Ref 1	SLAB 44 20 x 20
50°	0.00881	0.00918
60°	0.01200	0.01252
70°	0.01547	0.01604
80°	0.01920	0.01978
90°	0.02315	0.02380

Ref 1 is conformal mapping solution by Aggarwala

Fig 20. Comparison of results for simply supported, isotropic, rhombic plates with concentrated load at the center.



Simply supported, uniformly loaded, isotropic, triangular plate

Stiffness : $D = \frac{Et^3}{12(1-\nu^2)} = 1.0 \times 10^7$ lb-in^2/in

Poisson's Ratio : $\nu = 0.3$

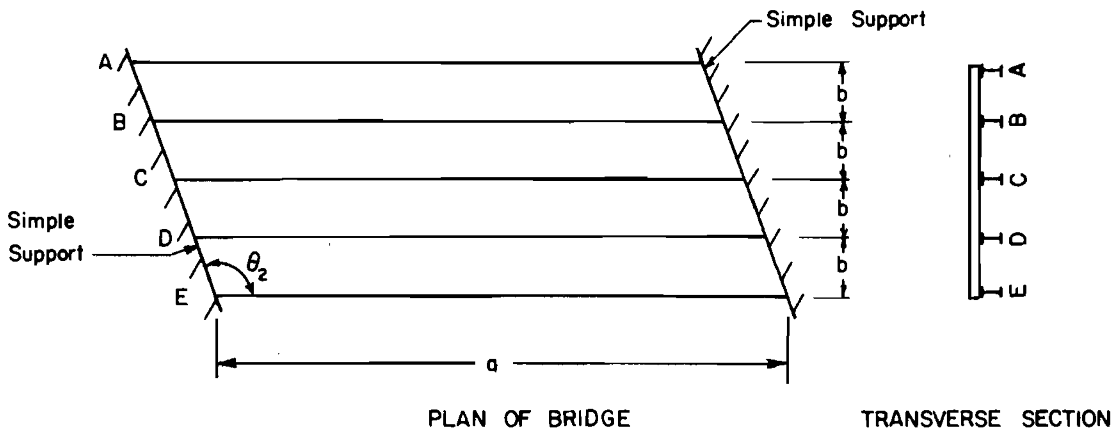
Load per Unit Area : $q = 1.0 \times 10^3$ lb/in^2

a = 10 inches

Deflection at O x $\frac{D}{qa^4 10^{-3}}$	
Ref 40	SLAB 44 21 x 21
1.029	1.037

Ref 40 is analytical solution by Timoshenko and Woinowsky-Krieger

Fig 21. Comparison of results for a simply supported, uniformly loaded, isotropic, triangular plate.



a = 20 in. b = 2 in.

Slab Stiffness: $D = \frac{Et^3}{12(1-\nu^2)} = 1 \times 10^5$ lb in^2/in.

Poisson's Ratio: $\nu = 0$

Beam Stiffness: $EI = 1 \times 10^7$ lb in^2

Concentrated Load: P = 5000 lb

$H = \frac{EI}{aD} = 5$

$b/a = 0.5$

} Non-dimensional parameters used in Ref 6 and 10

Angle θ_2	Midspan Moment in Beam	Midspan Position of Load P on Beam	Midspan Moment x $\frac{1}{Pa}$ Ref 6 8 x 8	Ref 10 8 x 8	Slab 44 8 x 8
150°	A	A	0.154	0.157	0.156
		C	0.015	0.021	0.022
		E	0.000	0.005	0.005
	B	A	0.049	0.050	0.049
		C	0.027	0.033	0.033
		E	0.004	0.012	0.011
	C	A	0.015	0.020	0.021
		C	0.070	0.085	0.085

Ref 6 is finite difference solutions by Chen, Siess, and Newmark.

Ref 10 is finite-element solution by Gustafson and Wright.

Fig 22(a). Comparison of results for five-beam, noncomposite skew bridges.

Angle θ_2	Midspan Moment in Beam	Midspan Position of Load P on Beam	Midspan Moment x $\frac{1}{Pa}$		
			Ref 6 8 x 8	Ref 10 8 x 8	Slab 44 8 x 8
135°	A	A	0.160	0.165	0.163
		C	0.015	0.022	0.023
		E	-0.009	-0.003	-0.003
	B	A	0.056	0.060	0.060
		C	0.033	0.043	0.044
		E	-0.001	0.006	0.006
	C	A	0.015	0.022	0.022
		C	0.078	0.096	0.095
120°	A	A	0.164	0.169	0.168
		C	0.016	0.022	0.022
		E	-0.013	-0.009	-0.009
	B	A	0.061	0.064	0.065
		C	0.038	0.048	0.048
		E	-0.003	0.002	0.002
	C	A	0.016	0.022	0.022
		C	0.083	0.099	0.098
90°	A	A	0.172		0.171
		C	0.022		0.022
		E	-0.017		-0.014
	B	A	0.067		0.068
		C	0.050		0.051
		E	0.000		0.000
	C	A	0.022		0.022
		C	0.101		0.099

Ref 6 is the finite difference solution by Chen, Siess, and Newmark.

Ref 10 is the finite-element solution by Gustafson and Wright.

Fig 22(b). Comparison of results for five-beam, noncomposite skew bridges.

and discrete-element (SLAB 44) solutions are almost identical, even though the finite-element method requires the solution of more than twice as many equations as SLAB 44 for the same mesh size when in-plane displacements are not considered. For $\theta_2 = 90^\circ$, or 0° skew, the three approaches give almost identical results.

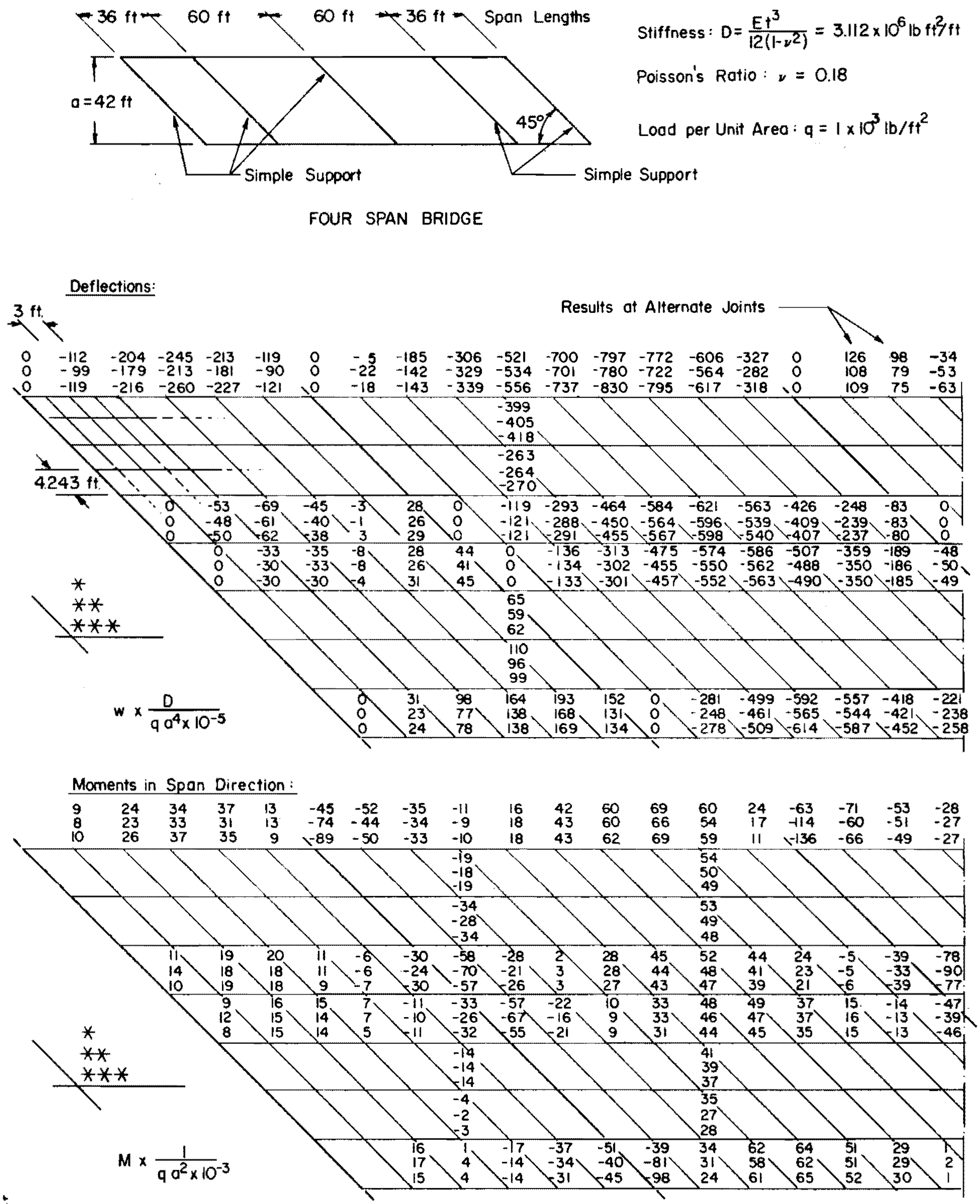
Problem Series 5. Four-Span Skew Bridge

Harnden and Rushton (Ref 11) have studied the deformation of a four-span 45° skew bridge using an electrical analogue computer. Sawko and Cope (Ref 37) have used the finite-element approach to solve the same bridge problem.

Using 14-by-64 increments, the SLAB 44 program was used to solve the same problem for a load uniformly distributed on the entire bridge. Figure 23 shows the deflections and moments in the span direction obtained from the three approaches. The results are superimposed on the grid used in Program SLAB 44. The difference in deflection between an electrical analogue and SLAB 44 solutions is about 4 percent at the location of maximum deflection (797, 780, and 830), and at other locations the difference is less than 5 percent with respect to the maximum deflection. The difference in deflection between finite-element and SLAB 44 results is about 6 percent at the location of maximum deflection and less than 6 percent at all other locations except one, where the difference is 8.8 percent with respect to the maximum deflection. In the case of bending moments, except for the locations over the supports, SLAB 44 results are very close to the other two approaches.

Problem Series 6. Verification with Experimental Results

Barboza (Ref 5) experimentally investigated the behavior of a skew, prestressed concrete bridge under various loading conditions. The bridge chosen was a Texas Highway Department standard, simply supported bridge with a skew angle of 30°. The bridge consisted of precast prestressed I-shaped girders with a cast-in-place deck slab. The slab was constructed to act compositely with the precast girders. The scale factor used for the model was 5.5. Figure 24 shows the dimensions of the model bridge. During the investigation, Barboza made a few auxiliary tests to determine experimentally the bending and torsional stiffnesses of a precast girder with the cast-in-place slab the width of which equaled the girder spacing in the model bridge (16.5 inches).

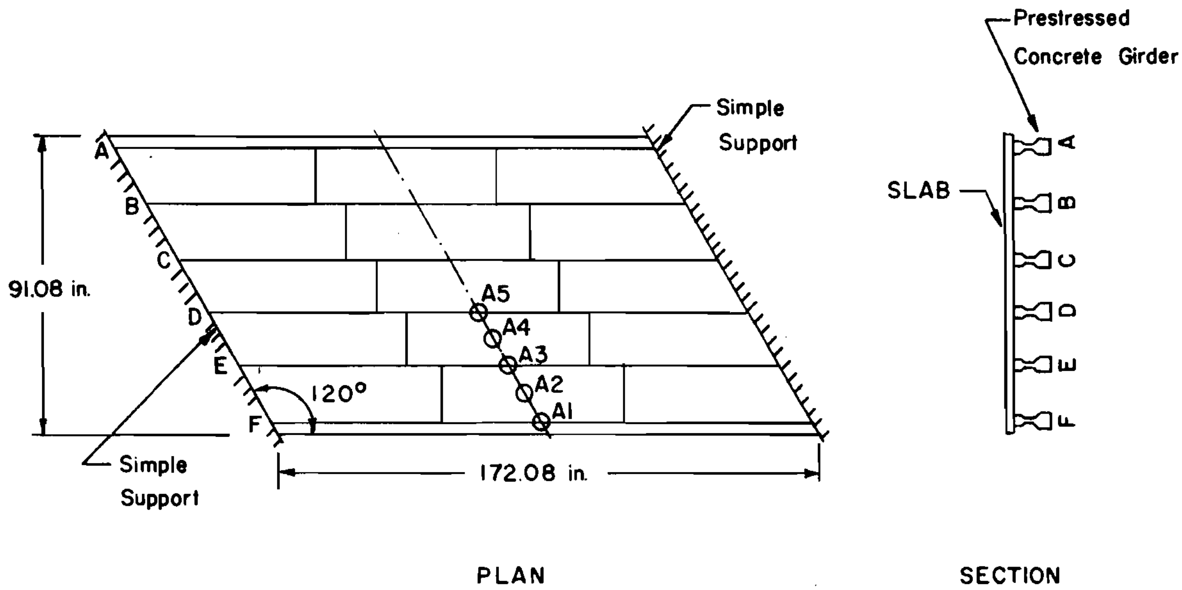


* is electrical analogue computer solution by Harnden and Rushton (Ref. 11)

** is finite-element solution by Sawko and Cope (Ref. 37)

*** SLAB 44

Fig 23. Comparison of results for a four-span skew bridge.



Number of Increments: 22 along skew by 72 along span

Slab Stiffness: $D_{11} = D_{22} = 7.46 \times 10^5$ lb in^2/in

Poisson's Ratio: $\nu = 0.167$

Girder Stiffness (Composite): 1.73×10^9 lb in^2

Diaphragm Stiffness: 2.34×10^7 lb in^2

Equivalent Girder Twisting Stiffness (distributed over 8.28 in. of slab width):
$D_{33} = 3.625 \times 10^6$ lb in^2/in

Fig 24. Experimental model tested by Barboza (Ref 5).

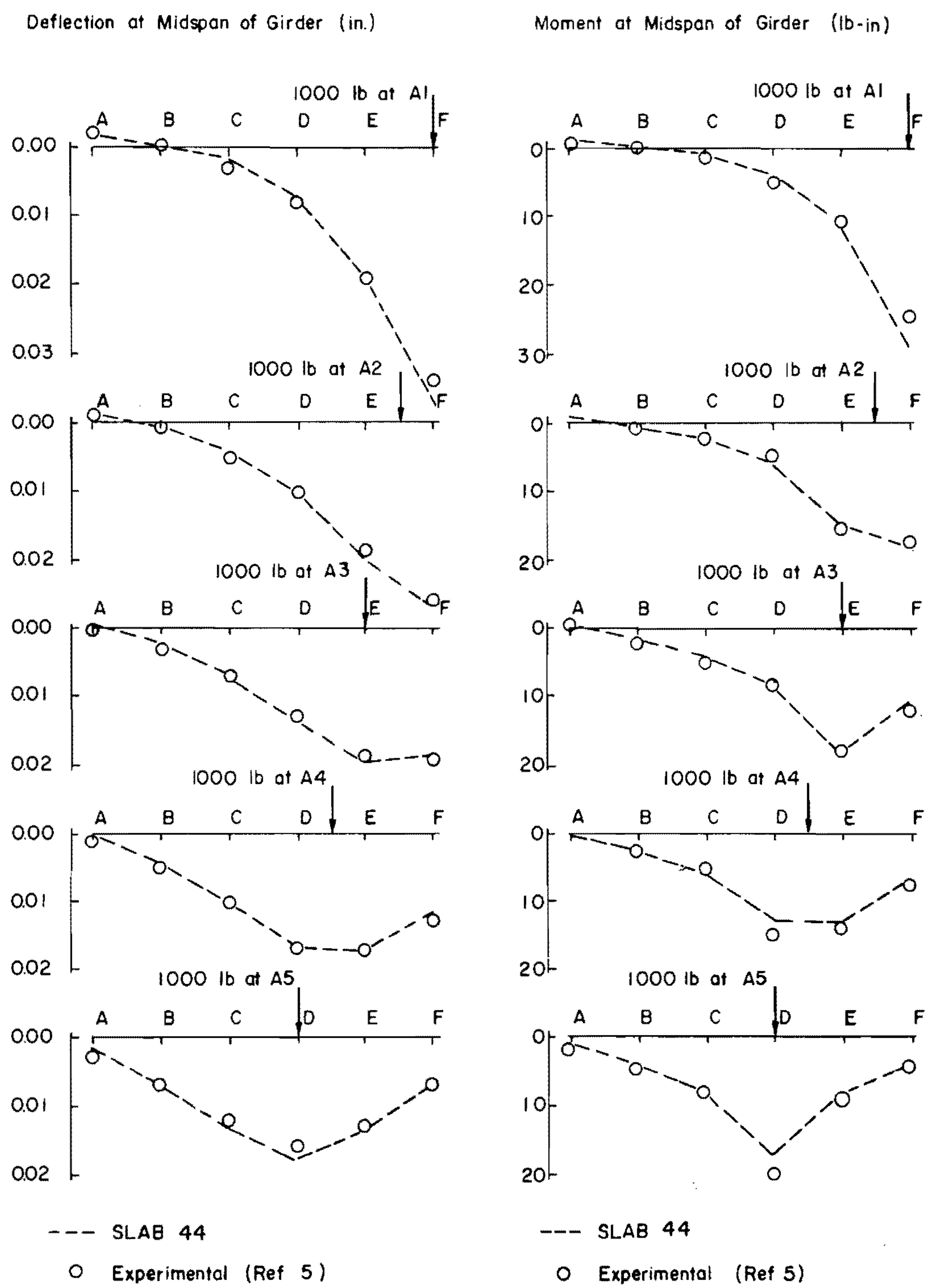


Fig 25. Comparisons with experimental results of Barboza (Ref 5).

The girders used in these auxiliary tests were fabricated in the same manner as those used in the model bridge structure.

Program SLAB 44 was used to analyze this bridge by inputting composite girder stiffnesses, which were obtained experimentally by Barboza, as beams. The torsional stiffness of the girders, also determined experimentally by Barboza, was input as additional twisting stiffness in a two-increment width of slab along the girders. The other parameters used were the same as given by Barboza and as shown in Fig 24.

The analysis was made for five different positions of concentrated load of 1,000 pounds, as tested by Barboza. At this load, the structure was still uncracked. The results of deflections and bending moments at the midspans of girders were compared with the experimental results. Figure 25 shows these comparisons. It is evident that there is a very close correlation between experimental (Ref 5) and SLAB 44 results.

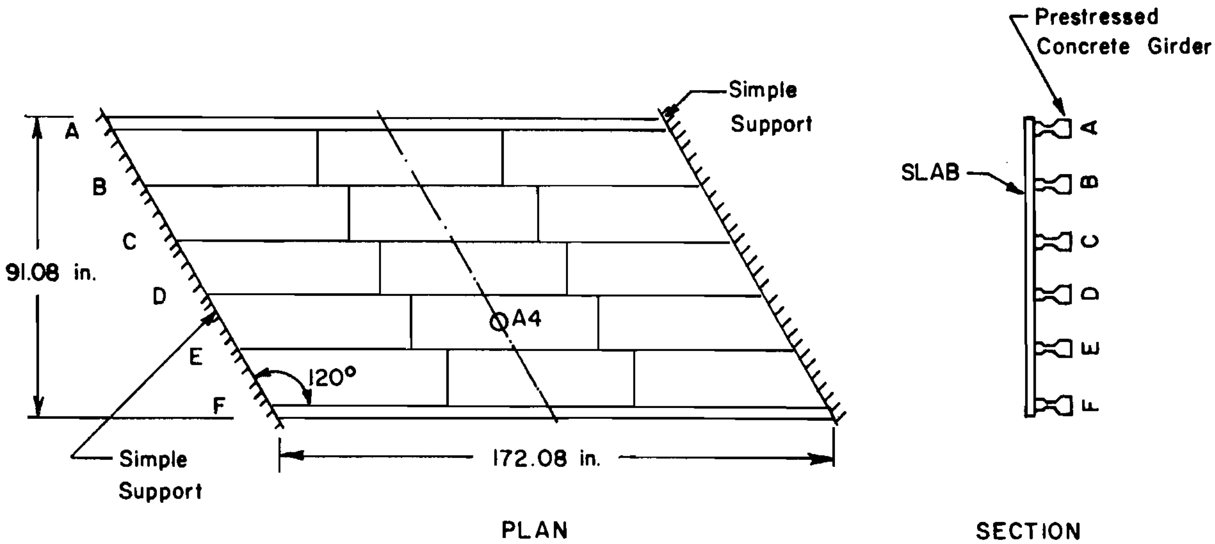
This problem series effectively demonstrates the modeling of composite action. It also shows that the diaphragms can be handled very simply even though they run in neither the span direction nor the skew direction.

Problem Series 7. Sensitivity of Modeling Bending and Torsional Stiffness of Composite Section

In this problem series, Program SLAB 44 was used to study the effects of variation of bending and torsional stiffnesses of a composite section of a single span bridge. This study is only analytical, even though the stiffnesses and constants of the bridge considered are the same as in problem series 6.

Figure 26 shows the dimensions and constants of the bridge. In the cases studied, the load of 1,000 pounds was considered to be acting at A4. The table in Fig 26 shows the variation in midspan deflections and midspan moments of girders D and E as the bending stiffness of the composite section was varied from 0.9 to 1.0 to 1.1 of the measured value (Ref 5), and the equivalent twisting stiffness was varied from 1.0 to 0.5 to 0.0 of the measured value (Ref 5).

It can be seen from the table in Fig 26 that the effect of variation of bending stiffness on deflection is more significant than the effect of variation of equivalent twisting stiffness. For example, consider the results of girder D. The deflection with a bending stiffness of 1.73×10^9 and a twisting stiffness of 3.625×10^6 is 0.01663. For the same bending stiffness, if the twisting stiffness is reduced by half then the deflection increases to



Number of Increments: 22 along skew by 72 along span

Slab Stiffness: $D_{11} = D_{22} = 7.46 \times 10^5$ lb in^2/in

Poisson's Ratio: $\nu = 0.167$

Diaphragm Stiffness: 2.34×10^7 lb in^2

Girder Number	Girder Bending Stiffness x 10^{-9}	Midspan Deflections Computed by SLAB 44 with Equivalent Girder Twisting Stiffness of			Midspan Moments Computed by SLAB 44 with Equivalent Girder Twisting Stiffness of		
		3.625×10^6 (Ref 5)	$0.5 \times 3.625 \times 10^6$	None	3.625×10^6 (Ref 5)	$0.5 \times 3.625 \times 10^6$	None
D	0.9 x 1.73 = 1.557	0.01805	0.01877	0.02043	12.55	12.90	13.72
	1.73 (Ref 5)	0.01663	0.01730	0.01883	12.79	13.15	13.98
	1.1 x 1.73 = 1.903	0.01544	0.01607	0.01749	13.01	13.38	14.22
E	0.9 x 1.73 = 1.557	0.01879	0.01940	0.02073	12.82	13.17	13.90
	1.73 (Ref 5)	0.01722	0.01779	0.01902	13.01	13.37	14.11
	1.1 x 1.73 = 1.903	0.01592	0.01645	0.01760	13.19	13.55	14.31

Note: All comparisons are for load at A4.
Ref 5 is experimental solution by Barboza.

Fig 26. Sensitivity of modeling bending and torsional stiffness of composite section.

0.01730, which is a 3.9 percent increase. If the twisting stiffness is kept the same (3.625×10^6) but the bending stiffness is reduced by only 10 percent, then the deflection increases to 0.01805, which is an 8.2 percent increase (over 0.01663). Compared to deflections, the bending moments are not appreciably affected. Even though only one load position was studied, this problem series demonstrates the necessity of computing composite girder bending stiffnesses with care.

Other Comparisons

In addition to use with the above problem series, SLAB 44 was used to solve several example problems for 0^o skew, or $\theta_2 = 90^o$, and the results were compared by solving the same problems using Program SLAB 36 (Ref 32). These problems were solved with different load and support conditions. The results of comparisons are not included here but it was observed that the difference in maximum deflection between the two solutions was about 1 percent, using ten-by-ten increments in both the cases.

Late in this study, there was an opportunity to apply the program to a real bridge. This study is reported elsewhere (Ref 20). Program SLAB 44 was used to study a failed structure, to study load placement on the test structure, to analyze the test structure, and to compare experimental results. The results indicated that for a severely skewed structure the strip method of analysis is not appropriate.

Summary

It has been observed by Mehrain (Ref 22) that in the case of simply supported uniformly loaded skew plates, the accuracy of finite difference and finite-element methods of solution drops rapidly as the angle of skew is increased. In the case of the finite-element method, this may be caused due to Kirchoff's hypothesis. This has not been observed (Problem Series 1) with the discrete-element approach presented here, even though the accuracy of the solution does depend upon the number of increments selected. In general, the results of the discrete-element model are in good agreement with the results of other approximate methods and with experimental data.

CHAPTER 9. USE OF THE METHOD

Summary

A discrete-element procedure of analysis of an anisotropic skew-plate and grid-beam system has been described. It has been observed from the literature studied that most of the work done on a skew plate is for either isotropic or orthotropic properties and for simple load and support conditions. The method presented here is not limited by these considerations. The stiffnesses, loads, and supports can be freely varied from point to point and in any direction. Concentrated and distributed loads and support springs, including external couples in three directions, can be easily handled. The principal features of this approach are summarized as follows:

(1) In the anisotropic stress-strain relations, the elastic compliances and stiffnesses are formulated in terms of three moduli of elasticity in any three directions and three Poisson's ratios related to these directions. This simplifies the computation of anisotropic plate stiffnesses in terms of six constants which could be experimentally determined by three tension tests.

(2) The anisotropic skew-plate and grid system is represented by a discrete-element model consisting of a tridirectional arrangement of rigid bars and elastic joints. The rigid bars are infinitely stiff and weightless and transfer bending moments. The elastic joints for the plate model are composed of elastic, but anisotropic, material The stiffnesses, loads, and restraints are lumped at elastic joints. All the elastic action takes place at these joints.

(3) To derive stress-strain relations for the plate model, it is assumed that an element of the plate is made up of three layers of interconnected fibers running in three directions. The fiber stresses are then related to conventional strains. The integration of these relations results in moment-curvature relations for the plate model. Each grid-beam is considered as a beam (Ref 18).

(4) Using equations of statics, the stiffness matrix is derived. The concentrated moments required in the equations of statics are obtained by assuming that the fibers running in a particular direction and having a certain width (Fig 11) are collected and lumped along each line of the model.

(5) A recursion-inversion procedure is used to solve the stiffness equations.

(6) A computer program, SLAB 44, is developed which is capable of determining deflections, bending and twisting moments, largest principal moment together with its direction, and reaction at each joint of the discrete-element model.

Comparisons with several different approaches such as series, finite-element, conformal mapping, finite difference, electrical analog, and experimental indicate that SLAB 44 produces excellent results.

Recommendations Pertaining to the Use of SLAB 44

This study and Program SLAB 44 are intended to provide a basic tool for use in design and to serve as a basis for future developments. The types of problems available in the literature are relatively simple, and SLAB 44 could be used to solve types of problems other than the example problems solved in Chapter 8 with SLAB 44.

Before coding a particular problem, the study of detailed rules and instructions would be helpful. Whenever it is necessary to make several solutions for the same structure in order to consider different load criteria or placements, the use of a multiple-load option switch is helpful in reducing the computer time.

Extensions of the Basic Method

The method developed could possibly be extended for several applications:

(1) The rotational restraint and axial thrust could be introduced in the formulation as is done in the beam-column and rectangular slab formulation by Matlock et al (Refs 18, 21, 13, and 31).

(2) The method could be extended to solve for nonlinear loads and supports in which the loads and supports are represented by load-deformation curves.

(3) The method could be utilized to study alternative designs for a particular problem. For example, in the bridge shown in Fig 24, the effect of different diaphragm configurations could be easily studied.

(4) All of the development of anisotropic stress-strain relations and discretization techniques developed here may be applied to plane stress problems.

(5) It may be possible to combine plane stress with bending analysis to solve for plates and pavement slabs in which in-plane forces are considered.

REFERENCES

1. Aggarwala, Bhagwan D., "Bending of Rhombic Plates," The Quarterly Journal of Mechanics and Applied Mathematics, Vol 19, London, 1966, pp 79-82.

2. Aggarwala, Bhagwan D., "Bending of Parallelogram Plates," Journal of the Engineering Mechanics Division, Vol 93, No. EM4, Proceedings of the American Society of Civil Engineers, August 1967.

3. Akay, H. U., "A Discrete-Element Solution of Skew Plates," Master's Thesis, The University of Texas at Austin, May 1969.

4. Ang, A. H. S., and N. M. Newmark, "A Numerical Procedure for the Analysis of Continuous Plates," Proceedings of the Second Conference on Electronic Computation, Structural Division, American Society of Civil Engineers, Pittsburgh, September 1960, p 379.

5. Barboza, N. J., "Load Distribution in a Skewed Prestressed Concrete Bridge," Master's Thesis, The University of Texas at Austin, August 1970.

6. Chen, T. Y., C. P. Siess, and N. M. Newmark, "Moments in Simply Supported Skew I-Beam Bridges," University of Illinois Engineering Experiment Station Bulletin 439, 1957.

7. Cheung, Y. K., I. P. King, and O. C. Zienkiewicz, "Slab Bridges with Arbitrary Shape and Support Conditions: A General Method of Analysis Based on Finite Elements," Proceedings, Vol 40, The Institution of Civil Engineers, May 1968.

8. Endres, F. E., and Hudson Matlock, "An Algebraic Solution Process Formulated in Anticipation of Banded Linear Equations," Research Report No. 56-19, Center for Highway Research, The University of Texas at Austin, January 1971.

9. Favre, H., "Le Calcul des Plaques Obliques par la Methode des Equations aux Differences," Publication of International Association for Bridge and Structural Engineering, Zurich, 1943.

10. Gustafson, W. C., and R. N. Wright, "Analysis of Skewed Composite Girder Bridges," Journal of the Structural Division, American Society of Civil Engineers, April 1968.

11. Harnden, C. T., and K. R. Rushton, "The Analysis of a Four-Span Bridge Using an Electrical Analogue Computer," Proceedings, Vol 36, The Institution of Civil Engineers, February 1967, pp 297-323.

12. Hearmon, R. F. S., An Introduction to Applied Anisotropic Elasticity, Oxford University Press, London, 1961.

13. Hudson, W. R., and Hudson Matlock, "Discontinuous Orthotropic Plates and Pavement Slabs," Research Report No. 56-6, Center for Highway Research, The University of Texas, Austin, May 1966.

14. Jensen, V. P., "Analysis of Skew Slabs," University of Illinois Engineering Experiment Station Bulletin 332, 1941.

15. Jensen, V. P., and J. W. Allen, "Studies of Highway Skew Slab Bridges with Curbs, Part 1, Results of Analyses," University of Illinois Engineering Experiment Station Bulletin 369, 1947.

16. Kennedy, J. B., and M. W. Huggins, "Series Solution of Skewed Stiffened Plates," Journal of the Engineering Mechanics Division, American Society of Civil Engineers, February 1964.

17. Lekhnitskii, S. G., Theory of Elasticity of an Anisotropic Elastic Body, Translated by P. Fern, Holden-Day, Inc., San Francisco, 1963.

18. Matlock, Hudson, and T. A. Haliburton, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns," Research Report No. 56-1, Center for Highway Research, The University of Texas, Austin, September 1966.

19. Matlock, Hudson, and W. B. Ingram, "Bending and Buckling of Soil-Supported Structural Elements," Proceedings, No. 32, Second Pan-American Conference on Soil Mechanics and Foundation Engineering, Brazil, July 1963.

20. Matlock, Hudson, J. J. Panak, M. R. Vora, and J. H. C. Chan, "Field Investigation of a Skewed, Post-Stressed Continuous Slab Structure," Interim Study Report to Texas Highway Department Bridge Division, Center for Highway Research, The University of Texas at Austin, May 25, 1970.

21. Matlock, Hudson, and T. P. Taylor, "A Computer Program to Analyze Beam-Column Under Movable Loads," Research Report No. 56-4, Center for Highway Research, The University of Texas at Austin, June 1968.

22. Mehrain, M., "Finite Element Analysis of Skew Composite Girder Bridges," Report No. 67-28, Structural Engineering Laboratory, University of California, Berkeley, California, November 1967.

23. Morley, L. S. D., "Bending of a Simply Supported Rhombic Plate Under Uniform Normal Loading," The Quarterly Journal of Mechanics and Applied Mathematics, Vol 15, Part 4, November 1962, pp 413-426.

24. Morley, L. S. D., Skew Plates and Structures, International Series of Monographs in Aeronautics and Astronautics, Vol 5, The Macmillan Company, New York, 1963.

25. Naruoka, M., "Untersuchung der schiefen Platte mit Benutzung des Rechenautomaten," Der Bauingenieur, 34, 1959, pp 401-407.

26. Naruoka, M., and H. Ohmura, "On the Analyses of a Skew Girder Bridge by the Theory of Orthotropic Parallelogram Plates," Publication International Association for Bridge and Structural Engineering, Zurich, 1959.

27. Naruoka, M., and H. Ohmura, "Uber die Berechnung der Einfluskoeffizienten fur Durchbiegung und Biegemoment der orthotropen Parallelogramm-Platte," Der Stahlbau, 28, 1959, pp 187-194.

28. Naruoka, M., H. Ohmura, and T. Yamamoto, "On the Model Tests of Skew Girder Bridges," Publication International Association for Bridge and Structural Engineering, Zurich, 1961.

29. Naruoka, M., and H. Yonezawa, "Uber die Anwendung der Biegungstheorie Orthotroper Platten auf die Berechnung schiefer Balkenbrucken," Der Bauingenieur, 32, 1957, pp 391-395.

30. Newmark, N. M., "Numerical Methods of Analysis of Bars, Plates, and Elastic Bodies," Numerical Methods of Analysis in Engineering, Edited by L. E. Grinter, Macmillan Company, New York, 1949, pp 139-168.

31. Panak, John J., and Hudson Matlock, "A Discrete-Element Method of Multiple-Loading Analysis for Two-Way Bridge Floor Slabs," Research Report No. 56-13, Center for Highway Research, The University of Texas at Austin, January 1970.

32. Panak, John J., and Hudson Matlock, "A Discrete-Element Method of Analysis for Orthogonal Slab and Grid Bridge-Floor System," Research Report, Center for Highway Research, The University of Texas at Austin, in progress.

33. Quinlan, P. M., "The λ-Method for Skew Plates," Fourth U. S. National Congress of Applied Mechanics, 1962, pp 733-750.

34. Redshaw, S. C., and K. R. Rushton, "Study of Various Boundary Conditions for Electrical Analogue Solutions of Extension and Flexure of Flat Plates," The Aeronautical Quarterly, Vol 11, Pt 3, London, August 1961, pp 275-282.

35. Robinson, K. E., "The Behaviour of Simply Supported Skew Bridge Slabs Under Concentrated Loads," Cement and Concrete Association, London, Research Report 8, 1959.

36. Rushton, K. R., "Electrical Analogue Solutions for the Deformation of Skew Plates," The Aeronautical Quarterly, Vol 15, Pt 2, London, May 1964, pp 169-180.

37. Sawko, F., and R. J. Cope, "The Analysis of Skew Bridge Decks - A New Finite Element Approach," The Structural Engineer, Vol 47, No. 6, June 1969, pp 215-224.

38. Stelzer, C. F., and W. R. Hudson, "A Direct Computer Solution for Plates and Pavement Slabs," Research Report No. 56-9, Center for Highway Research, The University of Texas, Austin, October 1967.

39. Suchar, M., "Obliczanie powierzchni wplywowych dla plyt rownoleglobocznych (Computation of the influence surfaces of plates in the form of parallelograms), Rosprawy Inz. 7, 1959, pp 237-260.

40. Timoshenko, S., and S. Woinowsky-Krieger, Theory of Plates and Shells, (Engineering Society Monographs), 2nd Edition, McGraw Hill, New York, 1959.

41. Tucker, Richard L., "A General Method for Solving Grid-Beam and Plate Problems," Ph.D. Dissertation, The University of Texas at Austin, 1963.

42. West, P. E., "A Finite Element Solution to Skew Slab Problems," Civil Engineering and Public Works Review, May 1966, p 619.

APPENDIX 1

GUIDE FOR DATA INPUT

GUIDE FOR DATA INPUT FOR SLAB 44

with supplementary notes

extract from

A DISCRETE-ELEMENT ANALYSIS FOR ANISOTROPIC SKEW PLATES AND GRIDS

by

Mahendrakumar R. Vora and Hudson Matlock

August 1970

SLAB 44 GUIDE FOR DATA INPUT - CARD FORMS

IDENTIFICATION OF RUN (two alphanumeric cards per run)

1	80

1	80

IDENTIFICATION OF PROBLEM (one alphanumeric card each problem: program stops if PROB NUM is left blank)

PROB NUM (1–5)		Description of problem (11–80)

TABLE 1. CONTROL DATA (one card for each problem)

	Enter "1" to Hold Prior					Number of Cards Added For							Reaction	Stiffness
	Table 2	Table 3	Table 4	Table 5		Table 2	Table 3	Table 4	Table 5		Multiple Load Option	Print Option	Output Option	Input Option
	10	15	20	25		35	40	45	50		60	70	75	80

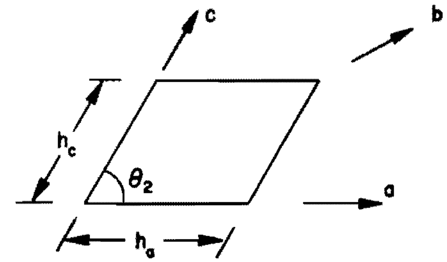
Multiple Load Option: Blank for Normal
+1 for Parent
-1 for Offspring

Print Option: Blank for M_x , M_y , and M_{xy}
1 for M_a , M_b , and M_c

Reaction Output Option: Blank for Support Reactions
1 for Statics Check

Stiffness Input Option: Blank for input value of slab stiffnesses in orthogonal directions (D_{11} through D_{33})
1 for input value of slab stiffnesses related to the a , b , and c-directions (B_{11} through B_{33})

TABLE 2. CONSTANTS (one card, or none if Table 2 or preceding problem is held)



Number of Increments a	Number of Increments c	Increment Length in a-Direction h_a	Increment Length in c-Direction h_c	Angle Between a and c-Directions θ_2
				(degrees)
5	10	20	30	40

TABLE 3. JOINT STIFFNESS DATA (number of cards according to Table 1)

From a	From c	Through a	Through c	D_{11} or B_{11}	D_{12} or B_{12}	D_{13} or B_{13}	D_{22} or B_{22}	D_{23} or B_{23}	D_{33} or B_{33}
5	10	15	20	30	40	50	60	70	80

D_{11} through D_{33} -	D_{11}	D_{12}	D_{13}	D_{22}	D_{23}	D_{33}
Isotropic:	$\left(\frac{E}{1 - \nu^2}\right)\frac{t^3}{12}$	$\left(\frac{\nu E}{1 - \nu^2}\right)\frac{t^3}{12}$	0	$\left(\frac{E}{1 - \nu^2}\right)\frac{t^3}{12}$	0	$\left(\frac{E}{2(1 + \nu)}\right)\frac{t^3}{12}$
Orthotropic:	$\left(\frac{E_x}{1 - \nu_x\nu_y}\right)\frac{t^3}{12}$	$\left(\frac{\nu_y E_x}{1 - \nu_x\nu_y}\right)\frac{t^3}{12}$	0	$\left(\frac{E_y}{1 - \nu_x\nu_y}\right)\frac{t^3}{12}$	0	$(G_{xy})\frac{t^3}{12}$

Anisotropic: May be computed by using Eqs 3.45 through 3.50, 3.58, and 4.15 in terms of three moduli of elasticity and three Poisson's ratios; or by using Eqs 3.17, 3.58, and 4.15 in terms of the other six constants; or by some other means.

B_{11} through B_{33} may be computed by using Eq 4.17 in terms of D_{11} through D_{33}; or by using Eq 4.34 in terms of three moduli of elasticity and three directional Poisson effects. In either case, the Stiffness Input Option in Table 1 must be exercised.

TABLE 4. BEAM STIFFNESS AND LOAD DATA (number of cards according to Table 1)

From		Through		Beam Bending Stiffnesses			Load	Spring
a	c	a	c	F_a	F_b	F_c	Q	S
5	10	15	20	30	40	50	60	70

TABLE 5. EXTERNAL COUPLE DATA (number of cards according to Table 1)

From		Through		External Couples		
a	c	a	c	T_a	T_b	T_c
5	10	15	20	30	40	50

STOP CARD (one blank card at end of run)
1 — 80

GENERAL PROGRAM NOTES

The data cards must be stacked in proper order for the program to run.

A consistent system of units must be used for all input data, for example, kips and feet.

All 2 to 5-space words are understood to be right-justified integers or whole decimal numbers . . . `+ 4 3 2 1`

All 10-space words are floating-point decimal numbers . . . . `- 4 . 3 2 1 E + 0 3`

TABLE 1. CONTROL DATA

For Table 2, the user must choose between holding all the data from the preceding problem or entering entirely new data. If the hold option is set equal to 1, the number of cards input for this table must be zero.

In Tables 3, 4, and 5, the data are accumulated by adding to previously stored data. The number of cards input is independent of the hold option, except that the cumulative total of cards in each of the tables cannot exceed the number allowed by program dimension statements.

Card counts in Table 1 should be rechecked after the coding of each problem is completed.

The multiple-load option is exercised for problem series in which only the load positions and magnitudes will vary. The first problem in a series is the Parent and is specified by entering +1, successive loadings are the Offspring and are specified by entering -1. If the option is left blank, the problem is complete within itself.

For Offspring problems, Tables 2, 3, and 5 are omitted.

The print option may be exercised for output moments. If specified 1, bending moments in a , b , and c-directions are printed. If left blank, bending moments in x and y-directions and twisting moments are printed (x and a-directions are the same). In either case the largest principal moments are computed and printed.

The reaction output option may be exercised by entering either 1 or leaving a blank. If 1 is entered, a statics check for each joint is printed (a statics check is a summation of all shears, loads, and restraint reactions). If a blank is left, restrain reactions due to spring supports are printed.

The stiffness input option may be exercised for input values of slab stiffnesses in Table 3. To input stiffnesses related to the a , b , and c-directions (B_{11} through B_{33}), 1 is entered. The option is left blank to input stiffnesses related to the orthogonal x and y-directions (D_{11} through D_{33}).

TABLE 2. CONSTANTS

The number of increments in the a-direction should be less than or equal to the number of increments in the c-direction.

The angle between the a and c-directions should be specified in degrees.

TABLES 3, 4, AND 5. STIFFNESS, LOAD, AND EXTERNAL COUPLE DATA

Variables:	D_{11} through D_{33}	B_{11} through B_{33}	F_a through F_c	Q	S	T_a through T_c
Typical Input Units:	$lb\text{-}in^2/in$	$lb\text{-}in^2/in$	$lb\text{-}in^2$	lb	lb/in	in-lb

All data are described with a coordinate system which is related to the a and c-station numbers (Fig 13). To distribute data over an area, it is necessary to specify the lower left-hand and the upper right-hand coordinates.

To specify data at a single location, it is necessary to specify the same coordinates in both the "From" and "Through" coordinates.

The "Through" coordinates must always be equal to or numerically greater than the "From" coordinates.

The user may input values on the edge of the slab and the corners to represent the proportionate area desired.

There are no restrictions on the order of cards in Tables 3, 4, and 5. Cumulative input is used, with full values at each coordinate.

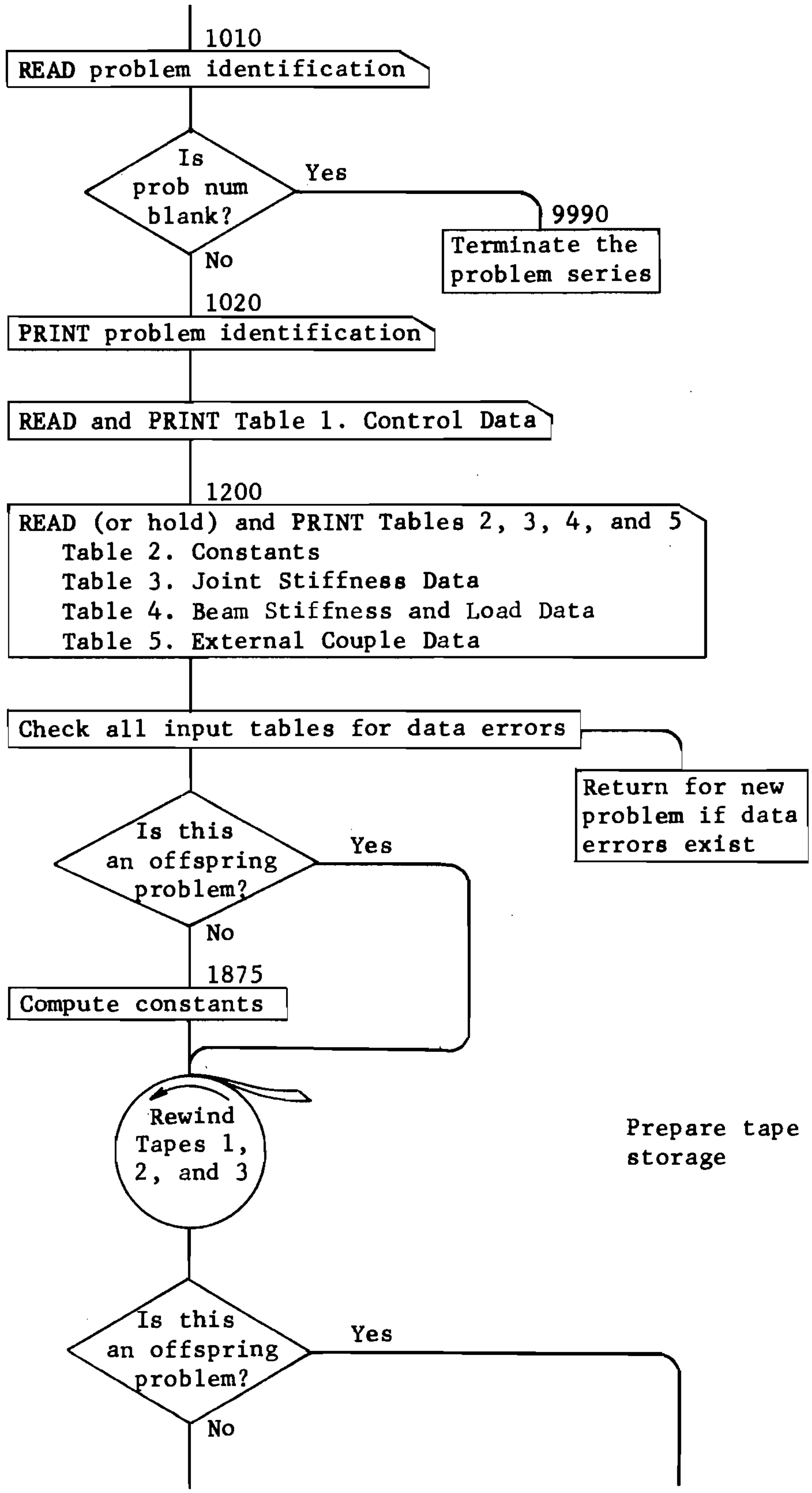
Unit stiffness values D_{11} through D_{33} for a slab or plate and concentrated stiffness values F_a through F_c for beams are input at full value joints. The values may be reduced proportionately for edges.

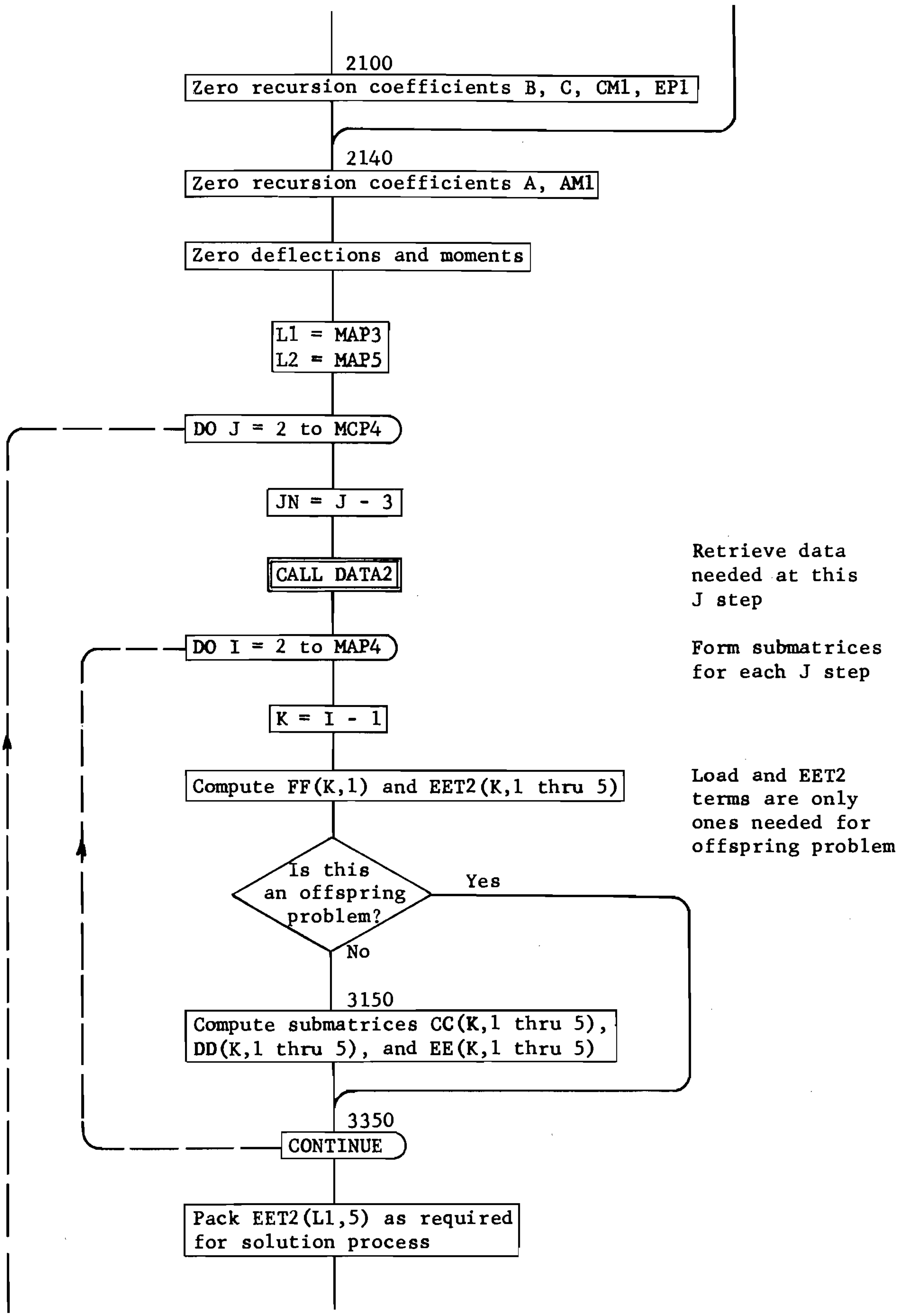
Load values Q and support springs S for any joint are determined by multiplying the unit load or unit support value by the appropriate area of the real slab or plate assigned to that joint. Hinged supports are provided by using large S values. Concentrated loads that occur between joints can be proportioned geometrically to adjacent joints.

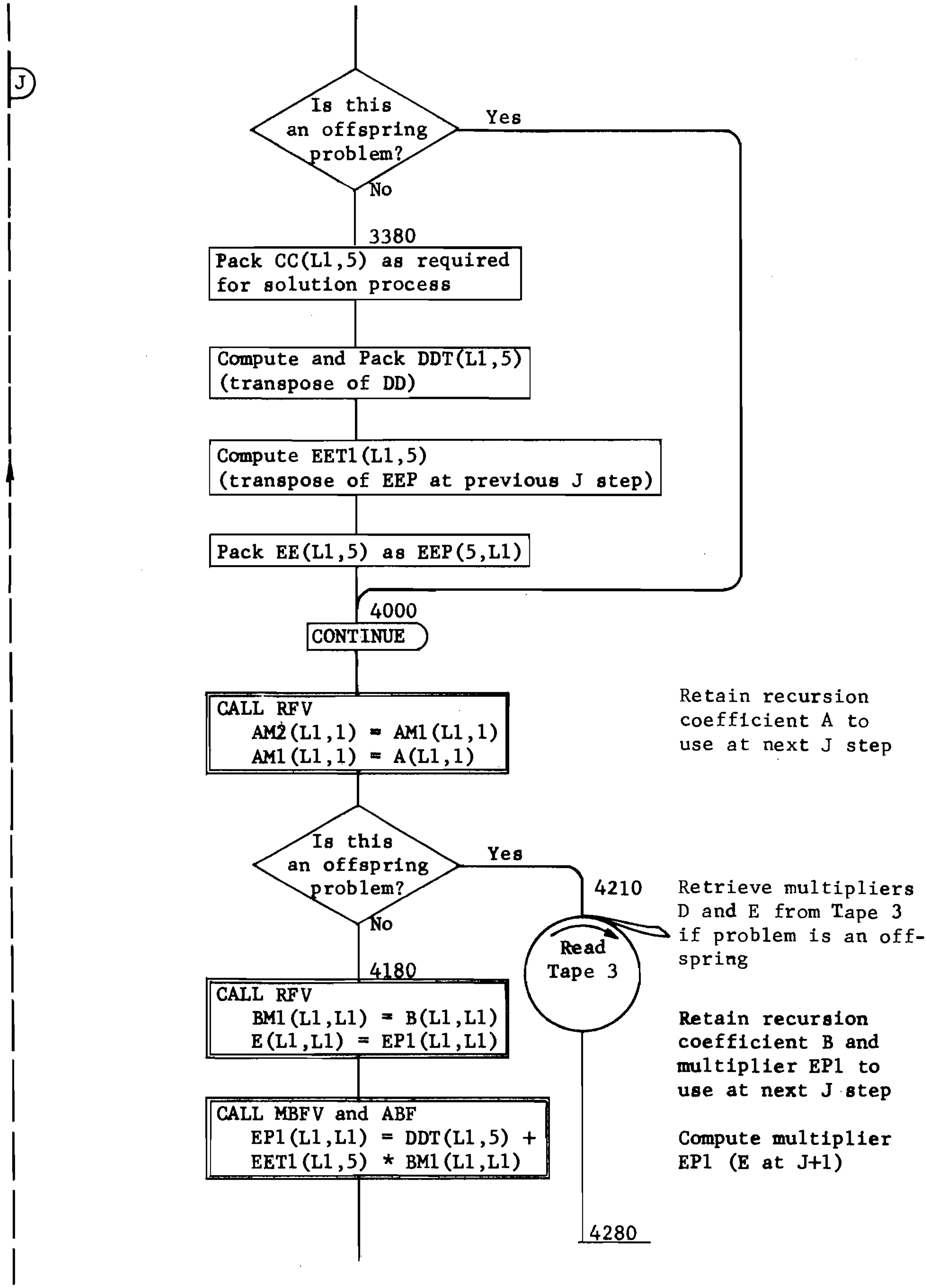
APPENDIX 2

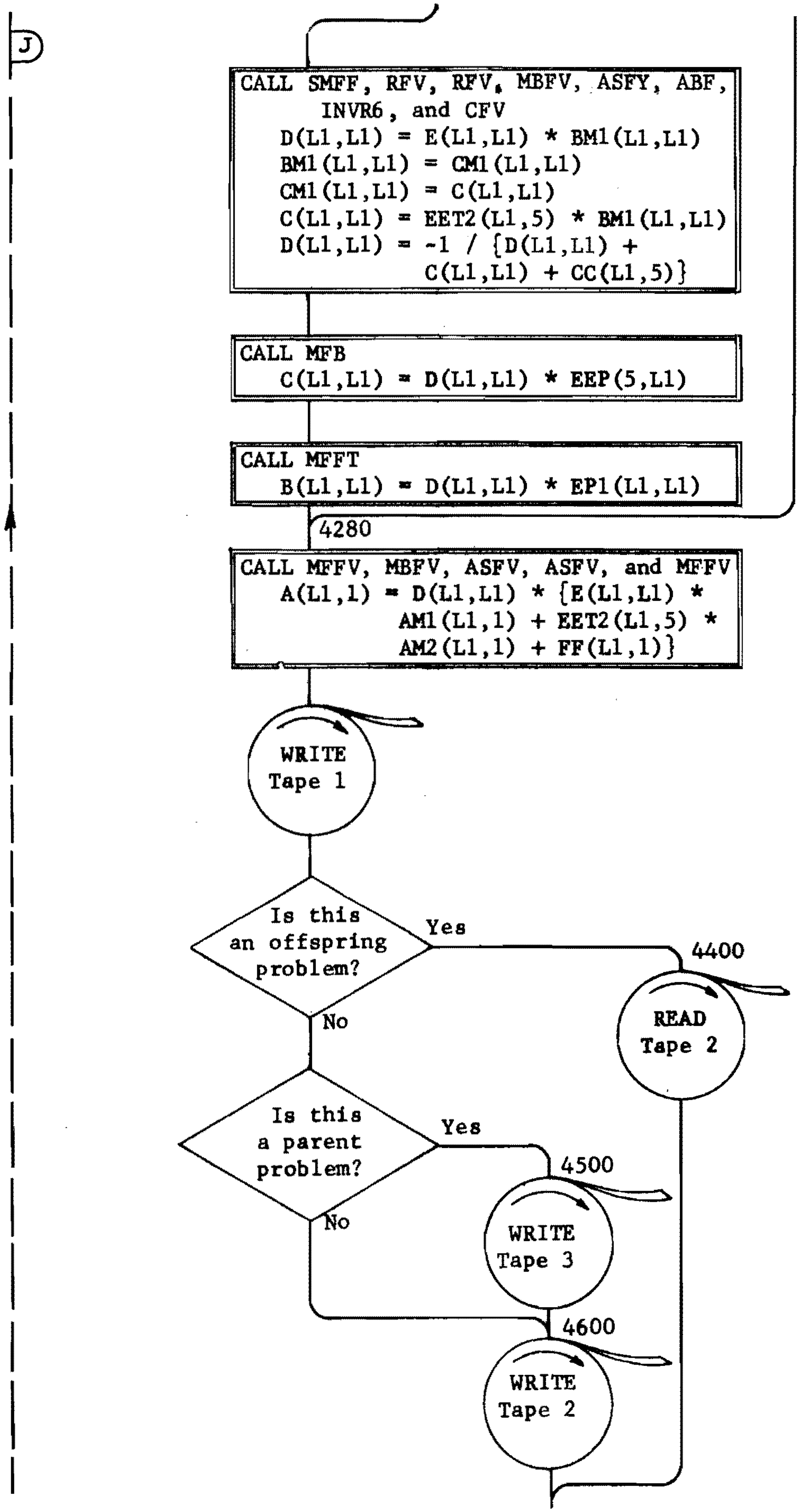
FLOW DIAGRAMS FOR PROGRAM SLAB 44

GENERAL FLOW DIAGRAM FOR SLAB 44




2100
Zero recursion coefficients B, C, CM1, EP1
2140
Zero recursion coefficients A, AM1
Zero deflections and moments
L1 = MAP3
L2 = MAP5
DO J = 2 to MCP4
JN = J - 3
CALL DATA2
Retrieve data needed at this J step
DO I = 2 to MAP4
Form submatrices for each J step
K = I - 1
Compute FF(K,1) and EET2(K,1 thru 5)
Load and EET2 terms are only ones needed for offspring problem
Is this an offspring problem?
Yes
No
3150
Compute submatrices CC(K,1 thru 5), DD(K,1 thru 5), and EE(K,1 thru 5)
3350
CONTINUE
Pack EET2(L1,5) as required for solution process


J
Is this an offspring problem?
Yes
No
3380
Pack CC(L1,5) as required for solution process
Compute and Pack DDT(L1,5) (transpose of DD)
Compute EET1(L1,5) (transpose of EEP at previous J step)
Pack EE(L1,5) as EEP(5,L1)
4000
CONTINUE
CALL RFV
AM2(L1,1) = AM1(L1,1)
AM1(L1,1) = A(L1,1)
Retain recursion coefficient A to use at next J step
Is this an offspring problem?
Yes
No
4210
Read Tape 3
Retrieve multipliers D and E from Tape 3 if problem is an off-spring
4180
CALL RFV
BM1(L1,L1) = B(L1,L1)
E(L1,L1) = EP1(L1,L1)
Retain recursion coefficient B and multiplier EP1 to use at next J step
CALL MBFV and ABF
EP1(L1,L1) = DDT(L1,5) +
EET1(L1,5) * BM1(L1,L1)
Compute multiplier EP1 (E at J+1)
4280



Compute multiplier D. Retain recursion coefficient C to use at next J step

Compute recursion coefficient C

Compute recursion coefficient B

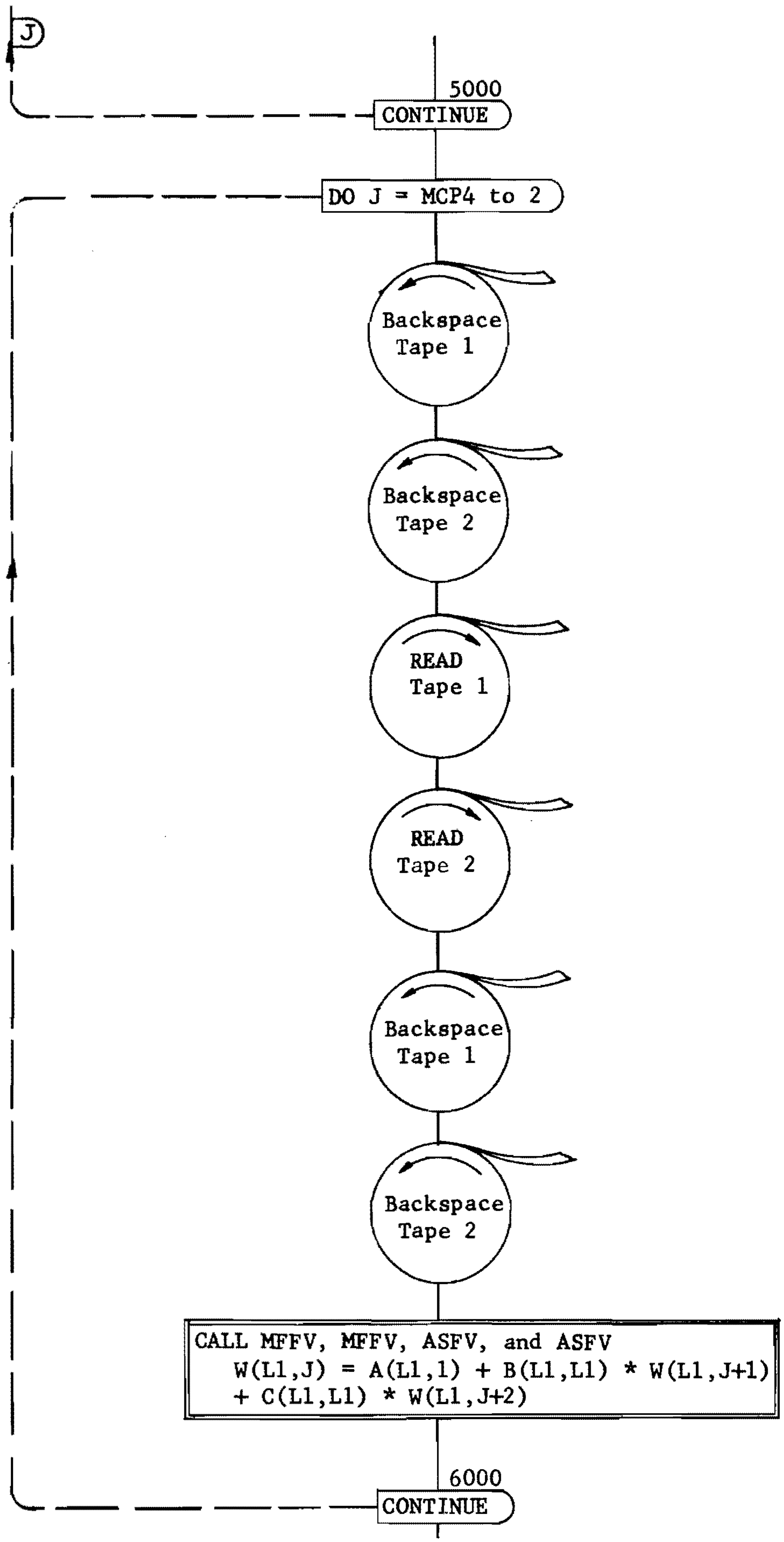
Compute recursion coefficient A

Retain recursion coefficient A on Tape 1

Move Tape 2 forward one J record

Retain multipliers D and E on Tape 3

Retain recursion coefficients B and C on Tape 2



Compute deflections starting at MCP4

Position Tape 1 for reading A

Position Tape 2 for reading B and C

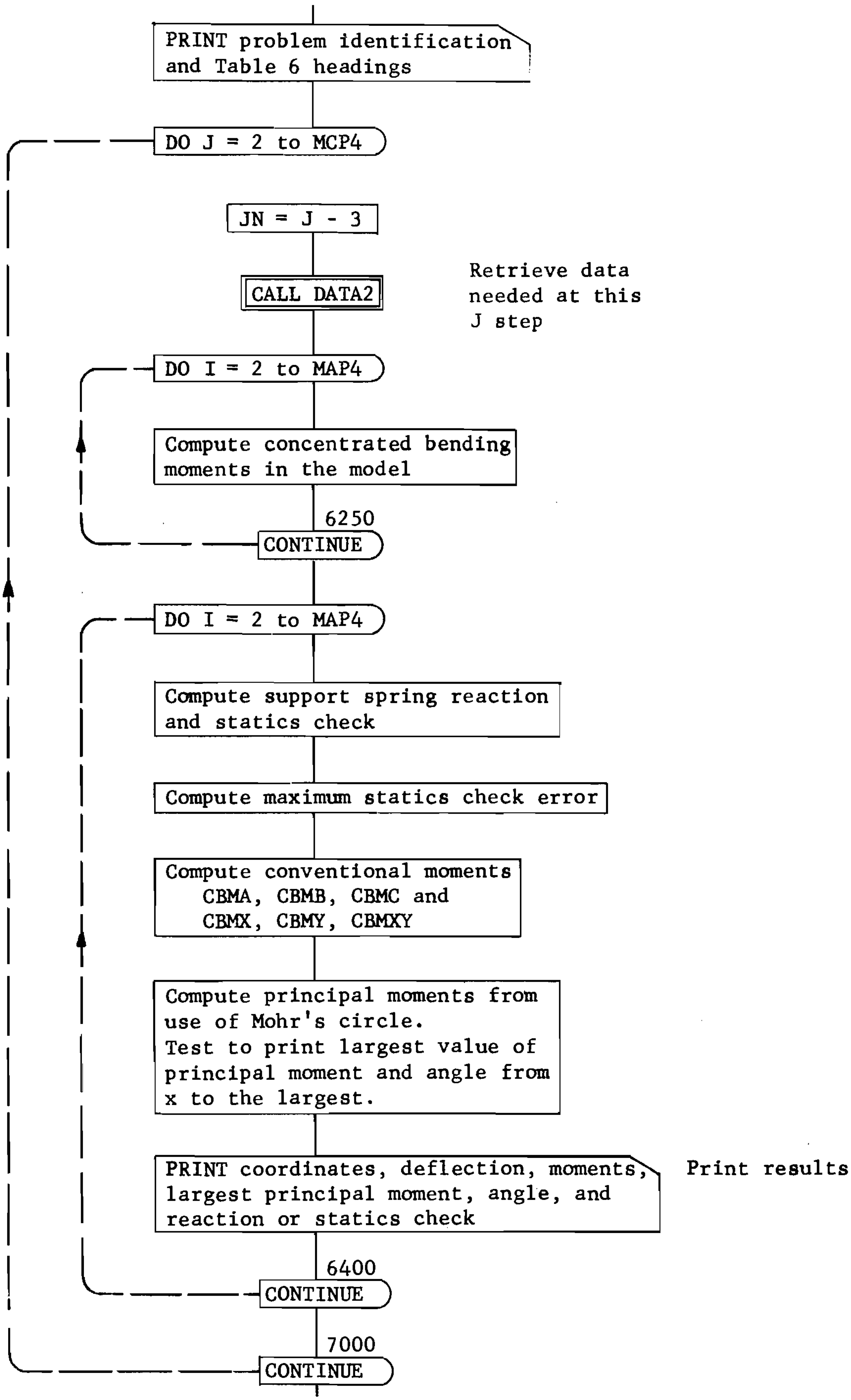
Retrieve the A recursion ceofficient to use at this J step

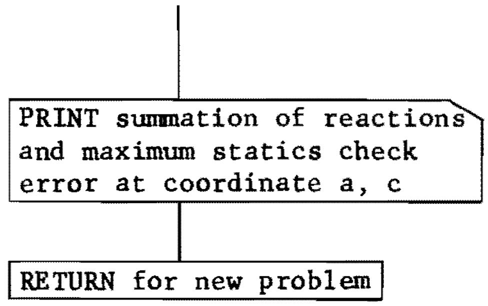
Retrieve the B and C recursion coefficients to use at this J step

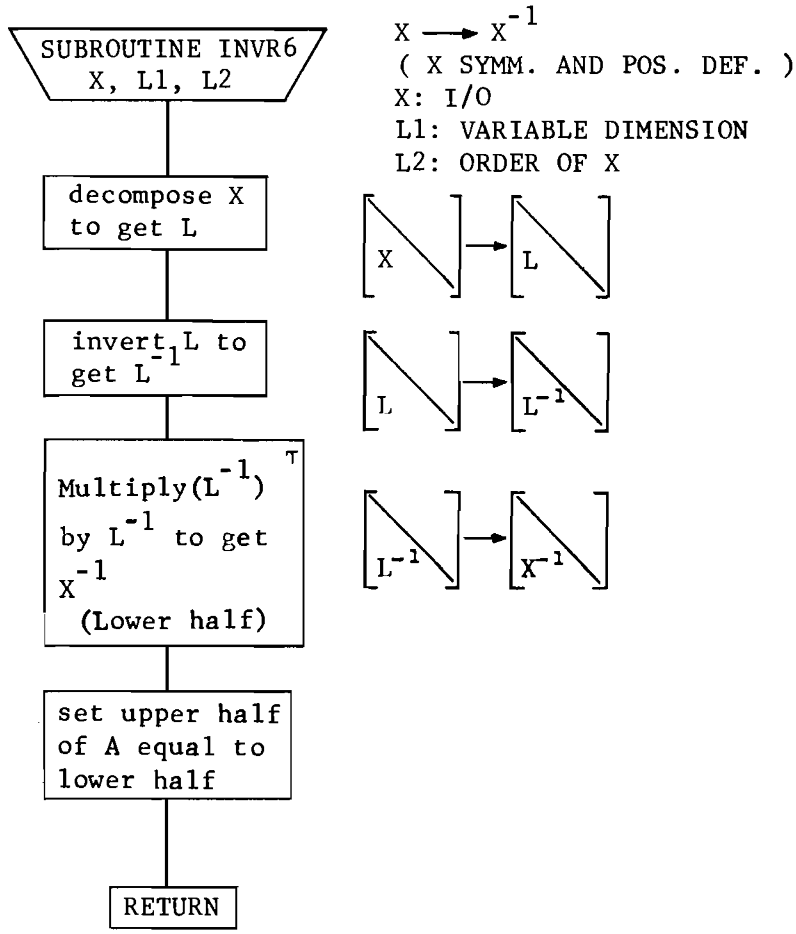
Reposition Tape 1

Reposition Tape 2

Compute deflections W at this J step


PRINT problem identification and Table 6 headings
DO J = 2 to MCP4
JN = J - 3
CALL DATA2
Retrieve data needed at this J step
DO I = 2 to MAP4
Compute concentrated bending moments in the model
6250
CONTINUE
DO I = 2 to MAP4
Compute support spring reaction and statics check
Compute maximum statics check error
Compute conventional moments CBMA, CBMB, CBMC and CBMX, CBMY, CBMXY
Compute principal moments from use of Mohr's circle.
Test to print largest value of principal moment and angle from x to the largest.
PRINT coordinates, deflection, moments, largest principal moment, angle, and reaction or statics check
Print results
6400
CONTINUE
7000
CONTINUE

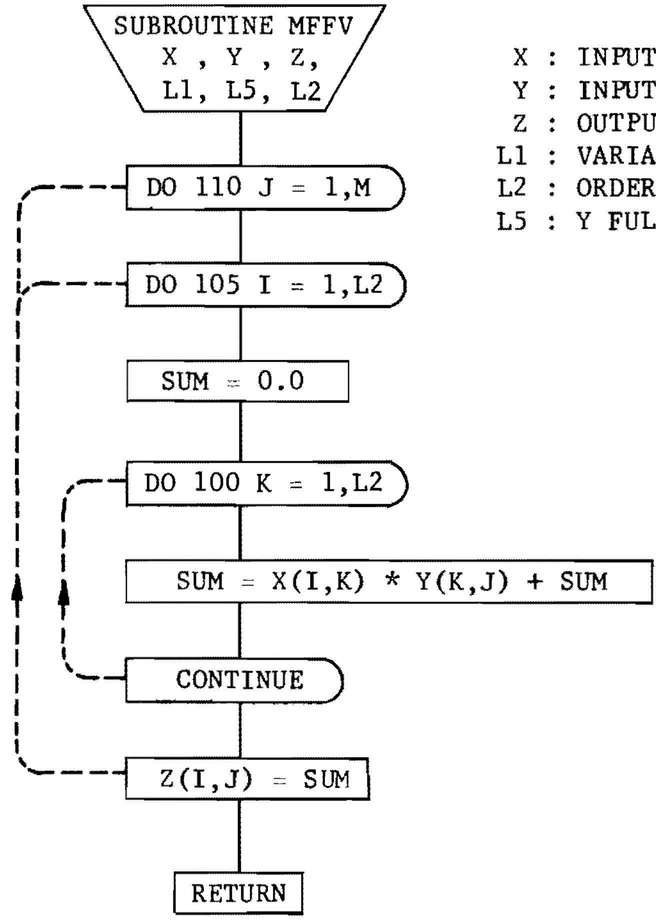

PRINT summation of reactions and maximum statics check error at coordinate a, c
RETURN for new problem



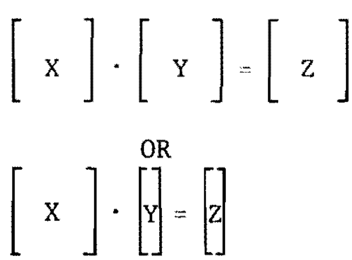
$$X = L \cdot L^T$$

$$X^{-1} = (L \cdot L^T)^{-1} = (L^T)^{-1} \cdot L^{-1} = (L^{-1})^T \cdot L^{-1}$$

This flow chart is extracted from Ref 8.



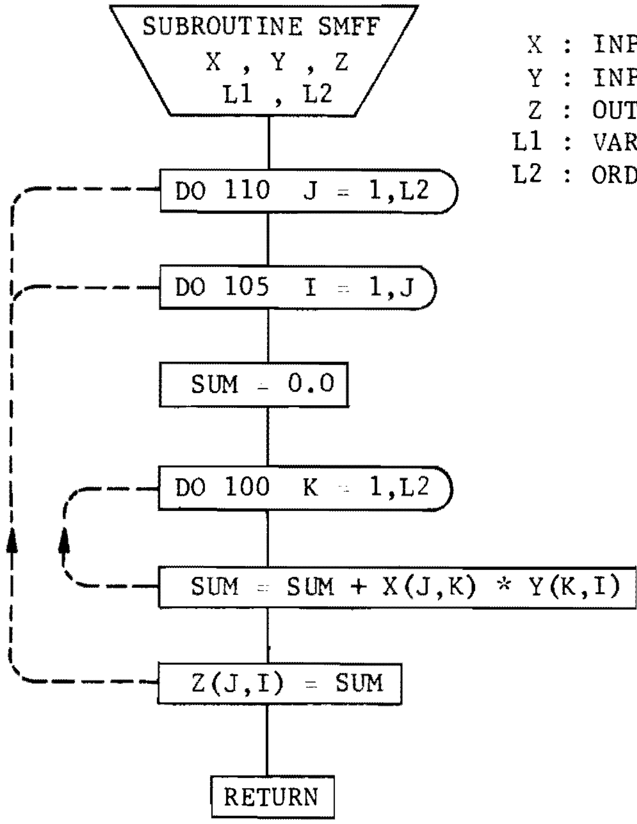
X : INPUT, FULL(Sq)
Y : INPUT, FULL OR VEC
Z : OUTPUT, FULL OR VEC
L1 : VARIABLE DIMENSION
L2 : ORDER OF X
L5 : Y FULL OR VECTOR ?



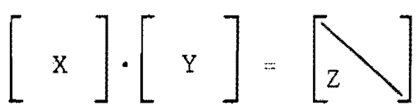
L5 = L1

L5 = 1

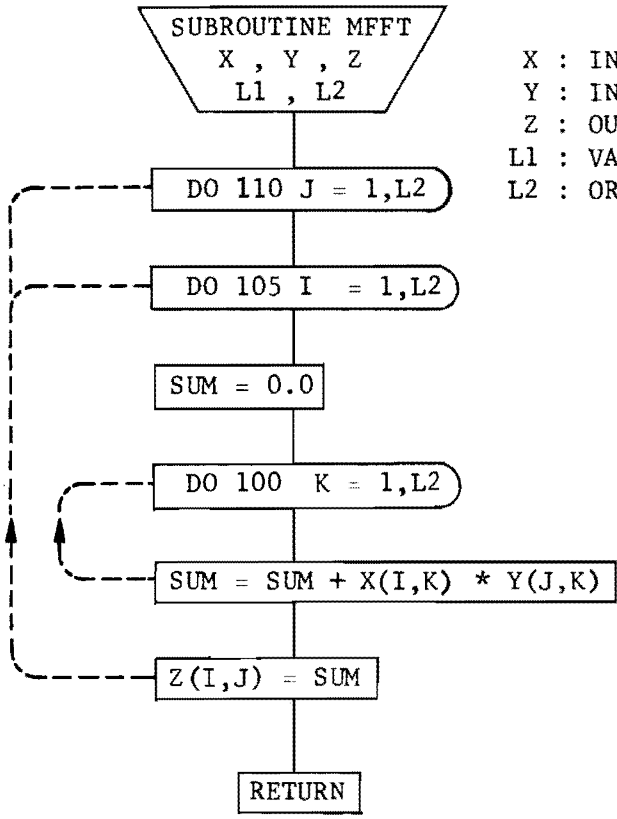
This flow chart is extracted from Ref 8.



X : INPUT, FULL(sq)
Y : INPUT,
Z : OUTPUT, LOWER HALF
L1 : VARIABLE DIMENSION
L2 : ORDER OF X , Y , Z



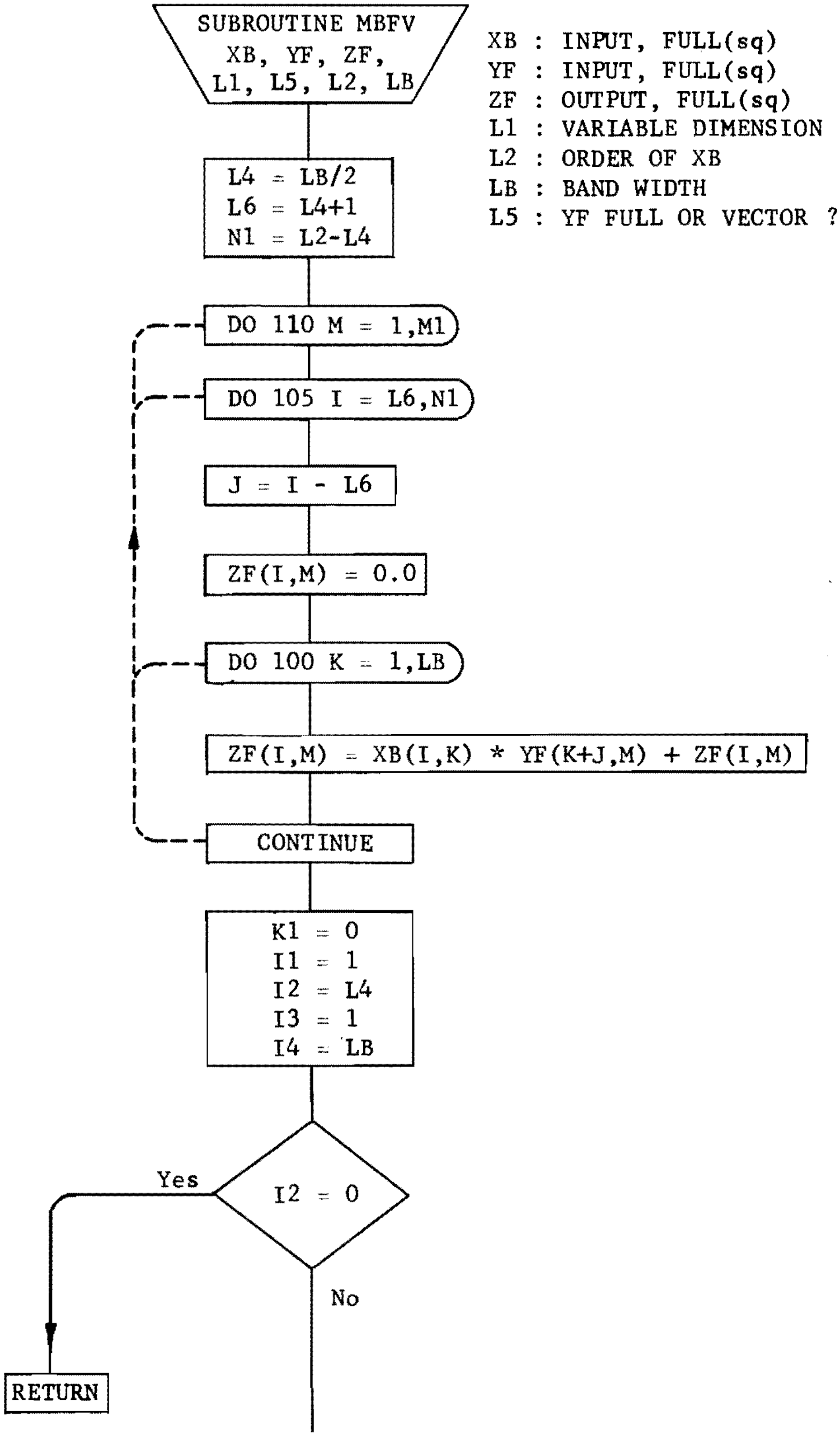
This flow chart is extracted from Ref 8.

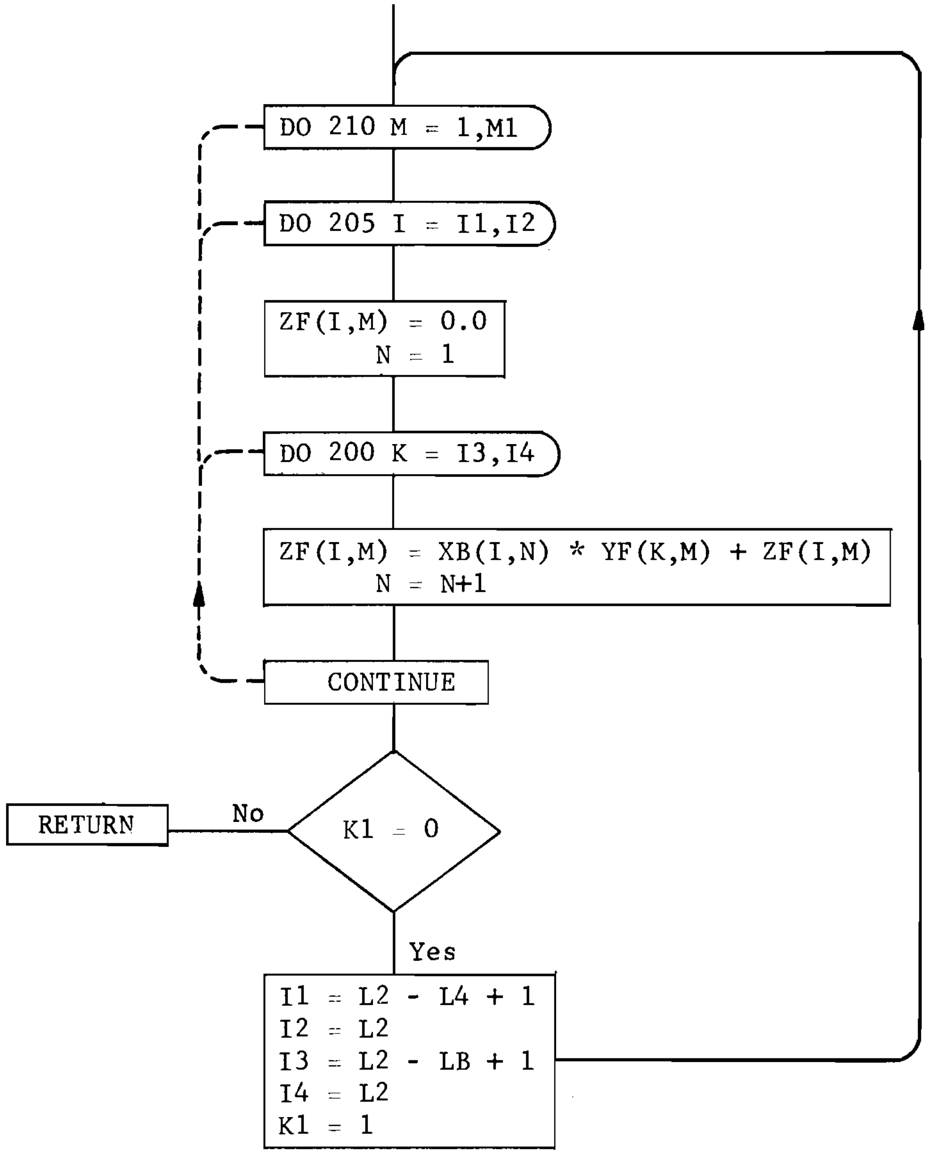


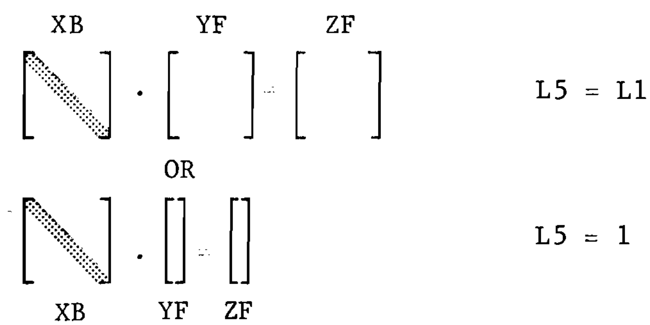
X : INPUT, FULL(sq)
Y : INPUT, FULL(sq)
Z : OUTPUT, FULL(sq)
L1 : VARIABLE DIMENSION
L2 : ORDER OF X , Y , Z

$$\begin{bmatrix} X \end{bmatrix} \cdot \begin{bmatrix} Y \end{bmatrix}^T = \begin{bmatrix} Z \end{bmatrix}$$

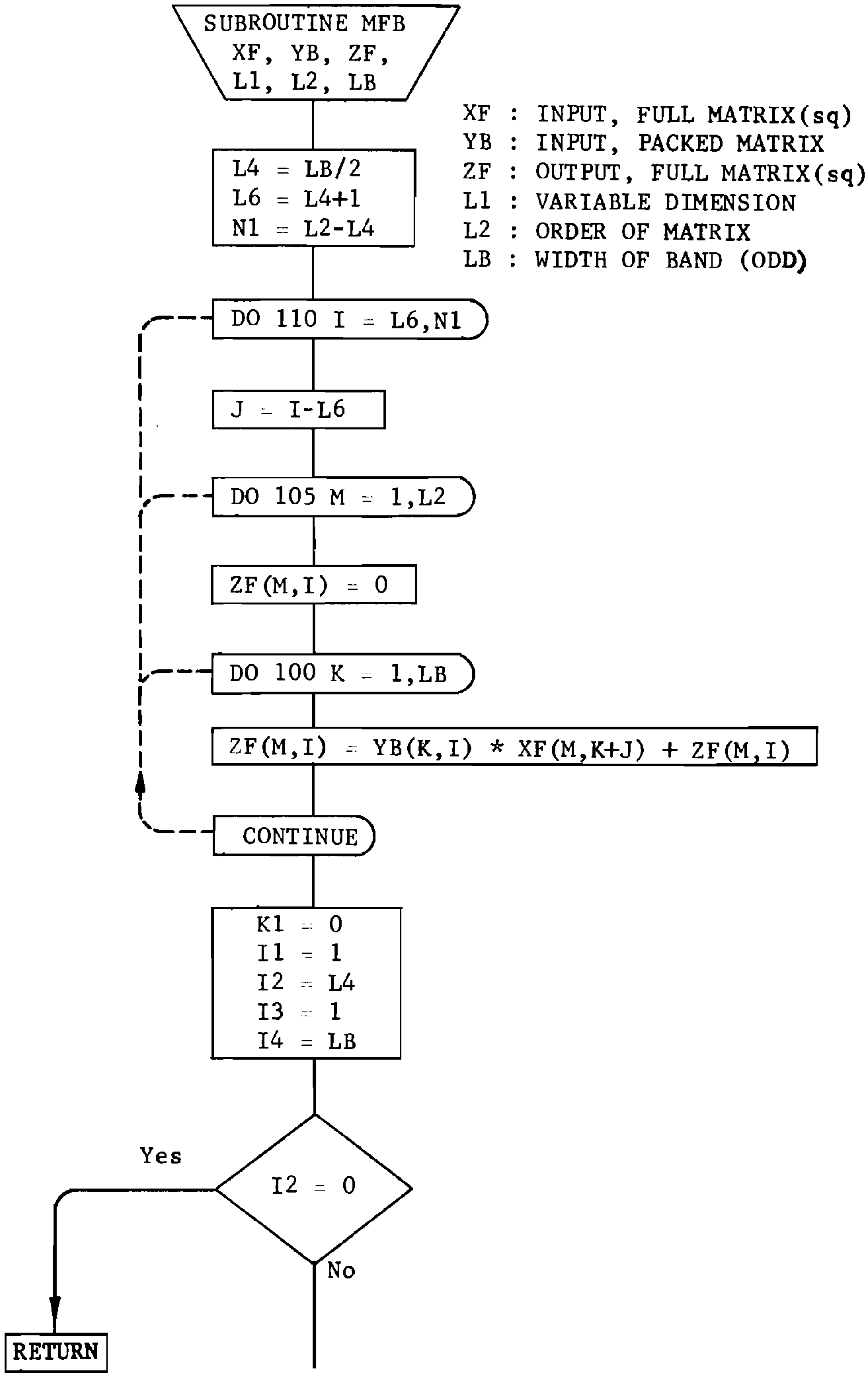
This flow chart is extracted from Ref 8.

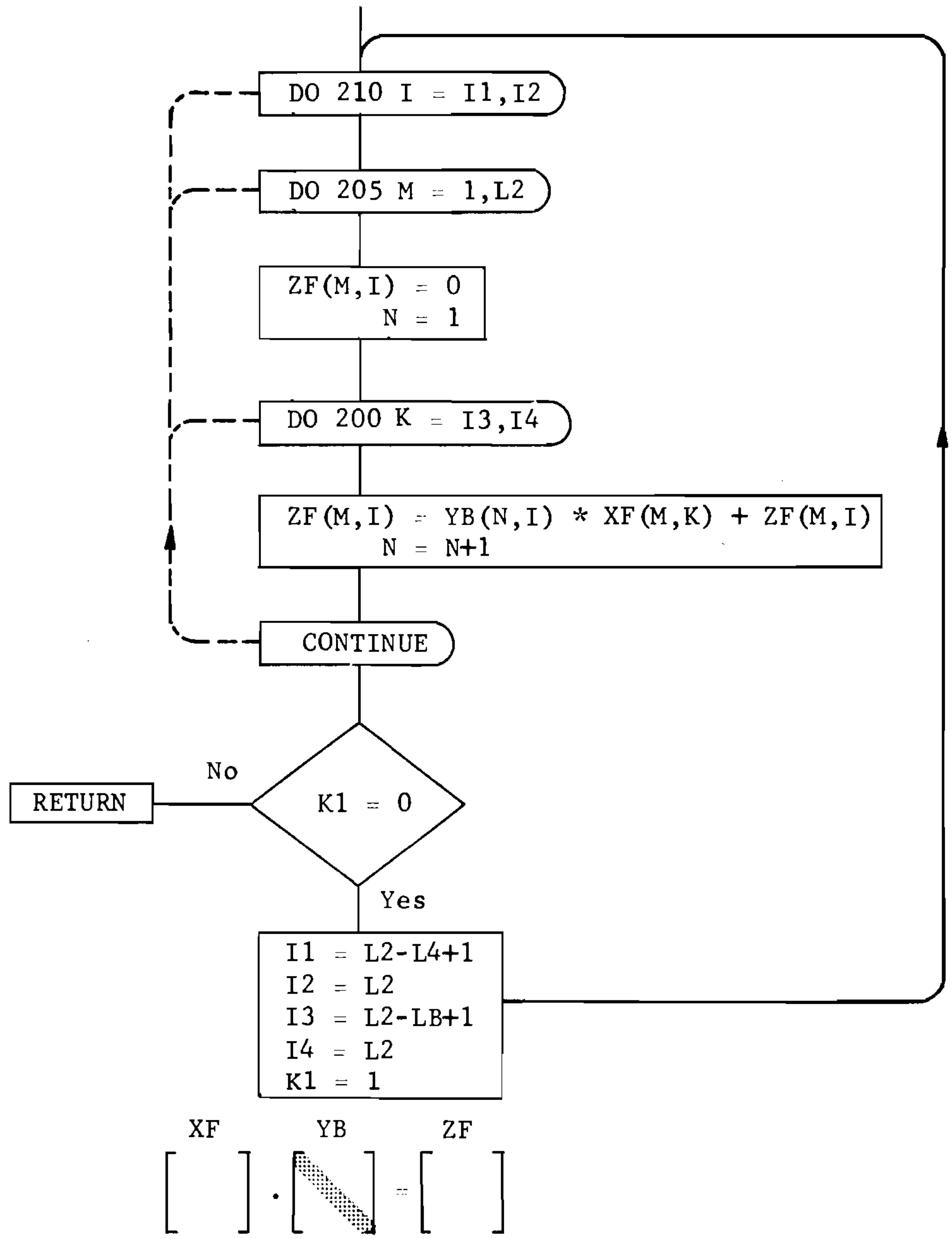

SUBROUTINE MBFV
XB, YF, ZF,
L1, L5, L2, LB
XB : INPUT, FULL(sq)
YF : INPUT, FULL(sq)
ZF : OUTPUT, FULL(sq)
L1 : VARIABLE DIMENSION
L2 : ORDER OF XB
LB : BAND WIDTH
L5 : YF FULL OR VECTOR ?
L4 = LB/2
L6 = L4+1
N1 = L2-L4
DO 110 M = 1,M1
DO 105 I = L6,N1
J = I - L6
ZF(I,M) = 0.0
DO 100 K = 1,LB
ZF(I,M) = XB(I,K) * YF(K+J,M) + ZF(I,M)
CONTINUE
K1 = 0
I1 = 1
I2 = L4
I3 = 1
I4 = LB
I2 = 0
Yes
No
RETURN



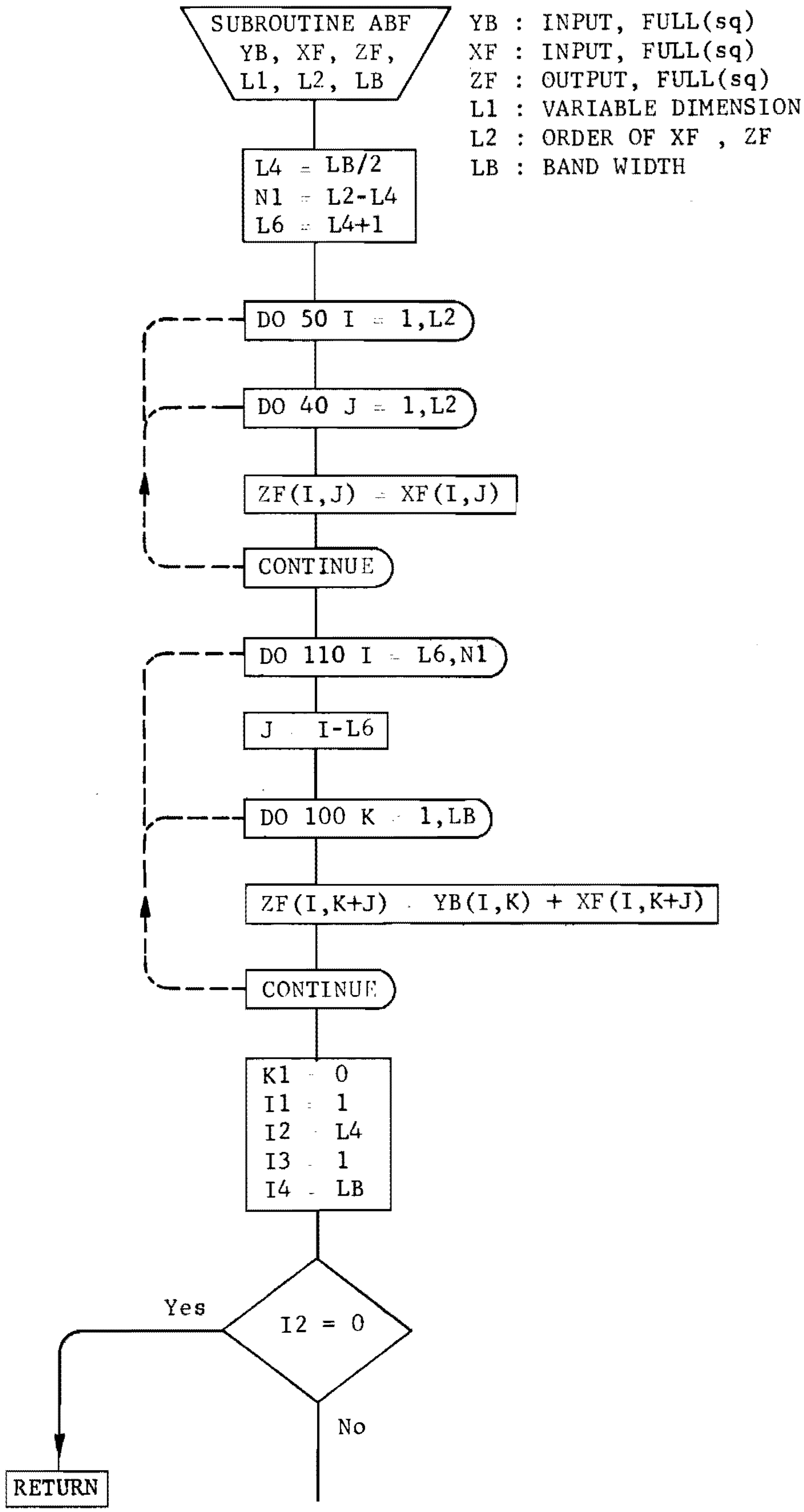


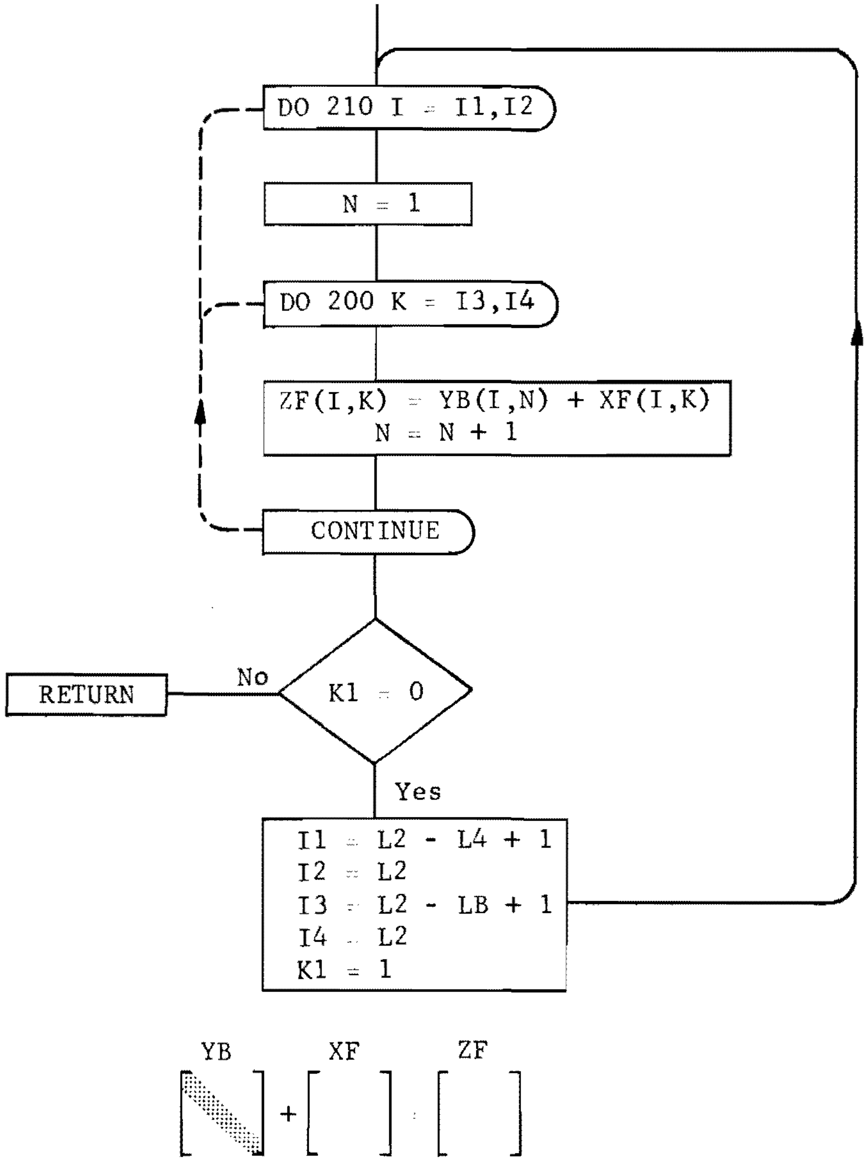
This flow chart is extracted from Ref 8.


SUBROUTINE MFB
XF, YB, ZF,
L1, L2, LB
XF : INPUT, FULL MATRIX(sq)
YB : INPUT, PACKED MATRIX
ZF : OUTPUT, FULL MATRIX(sq)
L1 : VARIABLE DIMENSION
L2 : ORDER OF MATRIX
LB : WIDTH OF BAND (ODD)
L4 = LB/2
L6 = L4+1
N1 = L2-L4
DO 110 I = L6,N1
J = I-L6
DO 105 M = 1,L2
ZF(M,I) = 0
DO 100 K = 1,LB
ZF(M,I) = YB(K,I) * XF(M,K+J) + ZF(M,I)
CONTINUE
K1 = 0
I1 = 1
I2 = L4
I3 = 1
I4 = LB
I2 = 0
Yes
No
RETURN

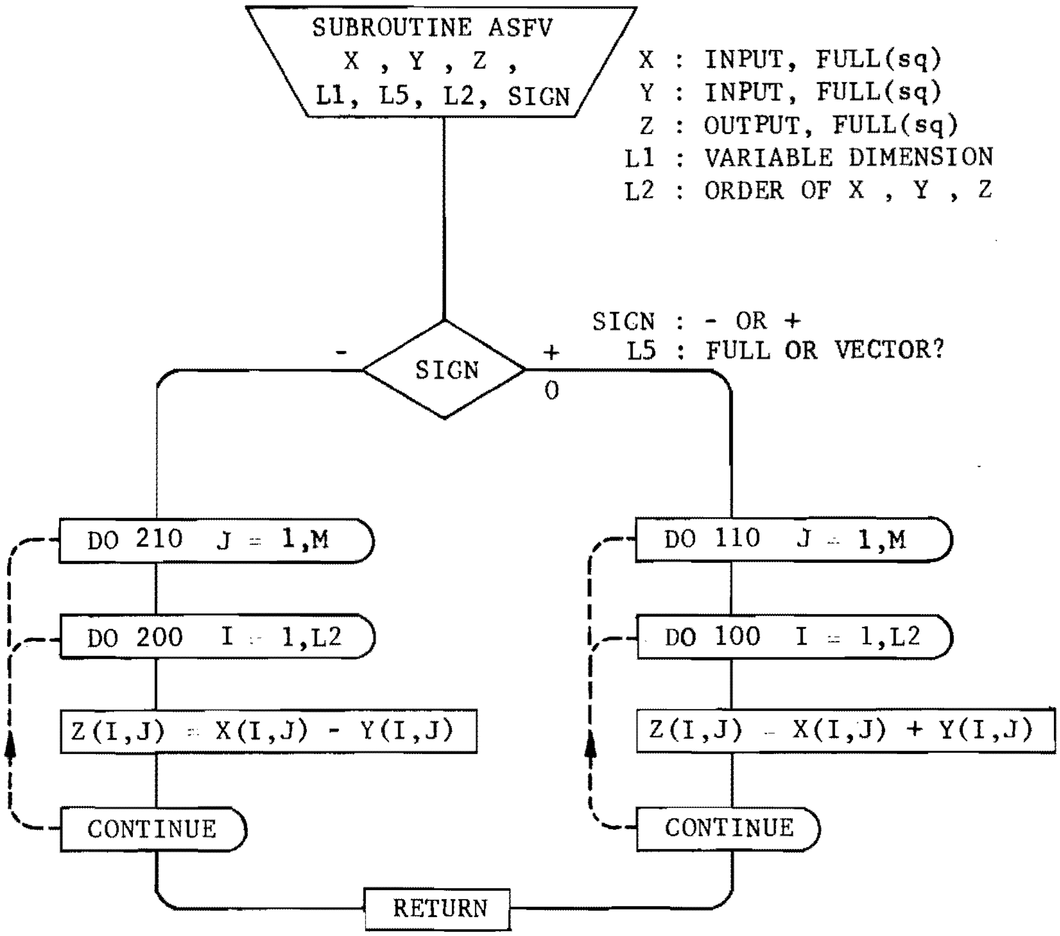


This flow chart is extracted from Ref 8.


SUBROUTINE ABF
YB, XF, ZF,
L1, L2, LB
YB : INPUT, FULL(sq)
XF : INPUT, FULL(sq)
ZF : OUTPUT, FULL(sq)
L1 : VARIABLE DIMENSION
L2 : ORDER OF XF , ZF
LB : BAND WIDTH
L4 = LB/2
N1 = L2-L4
L6 = L4+1
DO 50 I = 1,L2
DO 40 J = 1,L2
ZF(I,J) = XF(I,J)
CONTINUE
DO 110 I = L6,N1
J = I-L6
DO 100 K = 1,LB
ZF(I,K+J) = YB(I,K) + XF(I,K+J)
CONTINUE
K1 = 0
I1 = 1
I2 = L4
I3 = 1
I4 = LB
I2 = 0
Yes
No
RETURN

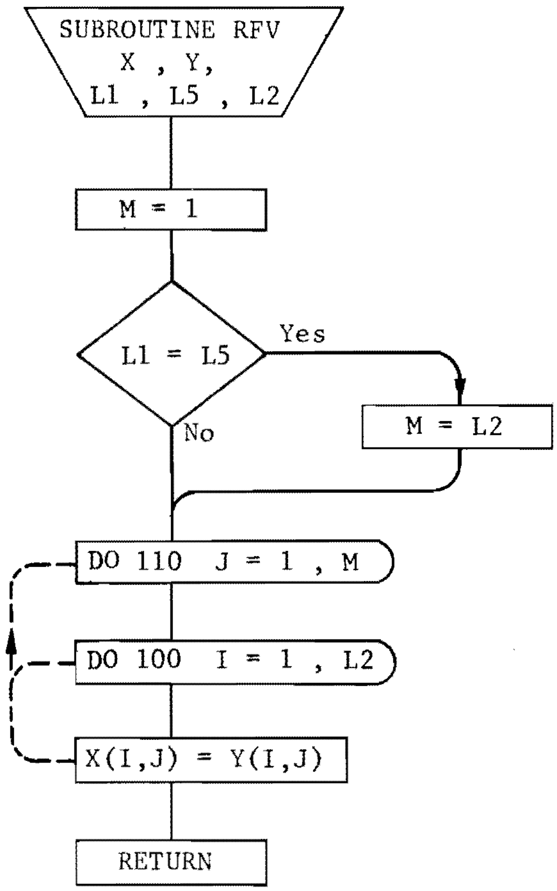


This flow chart is extracted from Ref 8.



$$[X] \pm [Y] = [Z]$$

This flow chart is extracted from Ref 8.

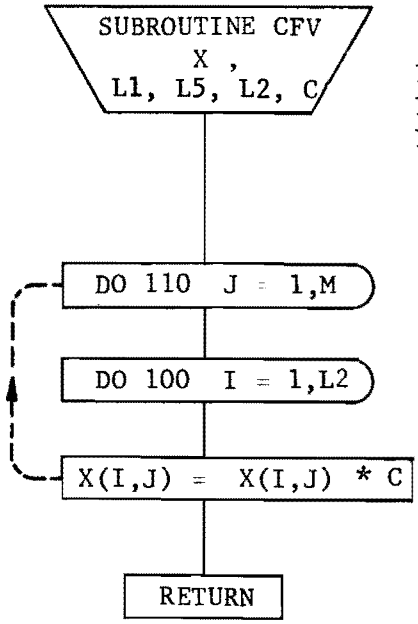


$$\begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} Y \end{bmatrix} \qquad L5 = L1$$

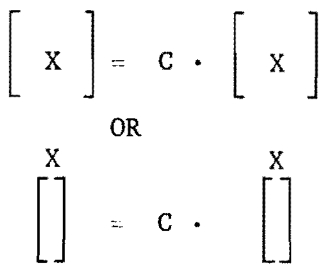
OR

$$\overset{X}{\begin{bmatrix} \; \end{bmatrix}} = \overset{Y}{\begin{bmatrix} \; \end{bmatrix}} \qquad L5 = 1$$

This flow chart is extracted from Ref 8.



X : I/O
L1 : VARIABLE DIMENSION
L2 : ORDER OR LENGTH OF X
L5 : FULL OR VECTOR
C : CONSTANT MULTIPLIER



This flow chart is extracted from Ref 8.

APPENDIX 3

GLOSSARY OF NOTATION FOR SLAB 44

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C---------NOTATION FOR SLAB44
C
C        A()                  RECURSION COEFFICIENT                      03MY0
C        ALF                  ANGLE ON MOHRS CIRCLE                      03MY0
C        AM1()                RECURSION COEFFICIENT A() AT J-1           03MY0
C        AM2()                RECURSION COEFFICIENT A() AT J-2           03MY0
C        AN1(),AN2()          IDENTIFICATION AND REMARKS (ALPHA-NUM)     03MY0
C        ATM()                TEMP STORAGE FOR A() RECURSION COEFF       03MY0
C        B()                  RECURSION COEFFICIENT                      03MY0
C        BETA                 ANGLE TO LARGEST PRINCIPAL MOMENT          03MY0
C        BETAT                TWICE BETA                                 03MY0
C        BMA(),BMB(),BMC()    CONCENTRATED MOMENTS IN SLAB MODEL IN      03MY0
C                             A B C DIRECTIONS                           03MY0
C        BMBA(),BMBB(),BMBC()CONCENTRATED MOMENTS IN BEAM MODEL IN       03MY0
C                             A B C DIRECTIONS                           03MY0
C        BMBBM1(),BMBCM1()    BMBB() AND BMBC() AT J-1                   03MY0
C        BMBBP1(),BMBCP1()    BMBB() AND BMBC() AT J+1                   03MY0
C        BMBM1(),BMCM1()      BMB() AND BMC() AT J-1                     03MY0
C        BMBP1(),BMCP1()      BMB() AND BMC() AT J+1                     03MY0
C        BM1()                RECURSION COEFFICIENT B() AT J-1           03MY0
C        B11VM,B11VP,B11VV    B11 AT (I-1,J), (I+1,J), (I,J)             03MY0
C        B12MV,B12PP,B12PV    B12 AT (I,J-1), (I+1,J+1), (I,J+1)         03MY0
C        B12VM,B12VP,B12VV    B12 AT (I-1,J), (I+1,J), (I,J)             03MY0
C        B13MV,B13PV,B13VM    B13 AT (I,J-1), (I,J+1), (I-1,J)           03MY0
C        B13VP,B13VV          B13 AT (I+1,J), (I,J)                      03MY0
C        B22MM,B22MV,B22PP    B22 AT (I-1,J-1), (I,J-1), (I+1,J+1)       03MY0
C        B22PV,B22VV          B22 AT (I,J+1), (I,J)                      03MY0
C        B23MM,B23MV,B23PP    B23 AT (I-1,J-1), (I,J-1), (I+1,J+1)       03MY0
C        B23PV,B23VV          B23 AT (I,J+1), (I,J)                      03MY0
C        B33MV,B33PV,B33VV    B33 AT (I,J-1), (I,J+1), (I,J)             03MY0
C        C()                  RECURSION COEFFICIENT                      03MY0
C        CBA                  ONE DIVIDED BY HA CUBED                    03MY0
C        CBB                  ONE DIVIDED BY HB CUBED                    03MY0
C        CBC                  ONE DIVIDED BY HC CUBED                    03MY0
C        CBMA,CBMB,CBMC       CONVENTIONAL BENDING MOMENTS PER UNIT      03MY0
C                             WIDTH OF SLAB IN A B C DIRECTIONS          03MY0
C        CBMO,CBMT            FIRST AND SECOND PRINCIPAL BENDING MOMENTS03MY0
C        CBMX,CBMY            CONVENTIONAL BENDING MOMENTS PER UNIT      03MY0
C                             WIDTH OF SLAB IN X AND Y DIRECTIONS        03MY0
C        CBMXY                CONVENTIONAL TWISTING MOMENT PER UNIT      03MY0
C                             WIDTH OF SLAB ABOUT X DIRECTION            03MY0
C        CC(,)                COEFFICIENTS IN STIFFNESS MATRIX           03MY0
C        CM1(,)               RECURSION COEFFICIENT C(,) AT J-1          03MY0
C        CS11 THRU CS33       MULT CONSTANTS FOR B11 THRU B33 IN STIFF   03MY0
C                             MATRIX                                     03MY0
C        C1,C2,C3             COSINE OF THETA1 THETA2 THETA3             03MY0
C        C1S,C2S,C3S          COSINE SQUARE OF THETA1 THETA2 THETA3      03MY0
C        D(,)                 RECURSION MULTIPLIER                       03MY0
C        DD(,)                COEFFICIENTS IN STIFFNESS MATRIX           03MY0
C        DDT(,)               TRANSPOSE OF DD(,)                         03MY0
C        D11() THRU D33()     BENDING STIFFNESS PER UNIT WIDTH OF SLAB   03MY0
C        D11N() THRU D33N()   INPUT VALUES OF D11() THRU D33()           03MY0
C        D12M1() THRU D33M1()D12() THRU D33() AT J-1                     03MY0
C        D12P1() THRU D33P1()D12() THRU D33() AT J+1                     03MY0
C        E(,)                 RECURSION MULTIPLIER                       03MY0
C        EE(,)                COEFFICIENTS IN STIFFNESS MATRIX           03MY0
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C         EEP(,)                  PACKED EE(,) AS REQUIRED FOR SOLUTION        03MYO
C         EET1(,)                 TRANSPOSE OF EEP(,) AT J-1                   03MYO
C         EET2(,)                 TRANSPOSE OF EE AT J-2                       03MYO
C         EP1(,)                  RECURSION MULTIPLIER                         03MYO
C         FA(),FB(),FC()          BEAM STIFFNESS IN A B C DIRECTIONS           03MYO
C         FAN(),FBN(),FCN()       INPUT VALUES OF FA() FB() FC()               03MYO
C         FBM1(),FCM1()           FB() FC() AT J-1                             03MYO
C         FBP1(),FCP1()           FB() FC() AT J+1                             03MYO
C         FF()                    COEFFICIENT IN LOAD VECTOR                   03MYO
C         HA,HB,HC                INCREMENT LENGTH IN A B C DIRECTIONS         03MYO
C         I                       STATION NUMBER IN A OR X DIRECTION           03MYO
C         IN13() THRU IN15()      INITIAL STATION IN A OR X DIRECTION          03MYO
C                                 USED IN TABLE 3 THRU 5                       03MYO
C         IN23() THRU IN25()      FINAL STATION IN A OR X DIRECTION            03MYO
C                                 USED IN TABLE 3 THRU 5                       03MYO
C         IPR                     PRINT OPTION SWITCH                          03MYO
C         ISTA                    EXTERNAL STATION IN A OR X DIRECTION         03MYO
C         ISTIFF                  STIFFNESS INPUT OPTION SWITCH                03MYO
C         ITEMP                   ISTA FOR MAXIMUM STATICS CHECK ERROR         03MYO
C         ITEST                   = 5 ALPHANUMERIC BLANKS USED TO              03MYO
C                                 TERMINATE PROGRAM                            03MYO
C         J                       STATION NUMBER IN C DIRECTION                03MYO
C         JN                      J-3                                          03MYO
C         JN13() THRU JN15()      INITIAL STATION IN C DIRECTION               03MYO
C                                 USED IN TABLE 3 THRU 5                       03MYO
C         JN23() THRU JN25()      FINAL STATION IN C DIRECTION                 03MYO
C                                 USED IN TABLE 3 THRU 5                       03MYO
C         JSTA                    EXTERNAL STATION IN C DIRECTION              03MYO
C         JTEMP                   JSTA FOR MAXIMUM STATICS CHECK ERROR         03MYO
C         K                       DO LOOP INDEX                                03MYO
C         KEEP2 THRU KEEP5        IF = 1, KEEP PRIOR DATA, TABLES 2 THRU 5     03MYO
C         KML                     KEEP ML FOR ERROR CHECKS                     03MYO
C         KPROB                   PROBLEM NUMBER FOR PARENT PROBLEM            03MYO
C         KROPT                   REACTION OUTPUT OPTION SWITCH                03MYO
C         L                       DO LOOP INDEX                                03MYO
C         L1,L2                   VARIABLE DIMENSION UNIT                      03MYO
C         MA                      NUMBER OF INCREMENTS IN A OR X DIRECTION     03MYO
C         MAP1 THRU MAP5          MA+1 THRU MA+5                               03MYO
C         MC                      NUMBER OF INCREMENTS IN C DIRECTION          03MYO
C         MCP1 THRU MCP5          MC+1 THRU MC+5                               03MYO
C         ML                      MULTIPLE LOADING SWITCH                      03MYO
C         N                       INDEX FOR READING CARDS                      03MYO
C         NCD2 THRU NCD5          NUMBER OF CARDS IN TABLES 2 THRU 5           03MYO
C                                 FOR THIS PROBLEM                             03MYO
C         NCT3 THRU NCT5          TOTAL NUMBER OF CARDS IN TABLES 3 THRU 5     03MYO
C         NC13 THRU NC15          INITIAL INDEX VALUE FOR THE INPUT TO         03MYO
C                                 TABLES 3 THRU 5                              03MYO
C         NDES                    NDE1 + NDE2 + NDE3  NDE4 + NDE5              03MYO
C         NDE1 THRU NDE5          NUMBER OF DATA ERRORS IN TABLES 1 THRU 5     03MYO
C         NF                      STARTING VALUE FOR DO LOOP                   03MYO
C         NK                      ORDER OF SUBMATRICES                         03MYO
C         NL                      MATRIX ORDER OF OVERALL COEFFICIENT MATRIX03MYO
C         NLM2                    NL-2                                         03MYO
C         NPROB                   PROBLEM NUMBER ( PROGRAM STOPS IF BLANK )    03MYO
C         N1,N2,N3                BAND WIDTH OF EE(,) DD(,) CC(,)              03MYO
C         PMMAX                   LARGEST PRINCIPAL MOMENT                     03MYO
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C          Q()                    TRANSVERSE LOAD PER JOINT                  03MYO
C          QN()                   INPUT VALUE OF Q()                         03MYO
C          REACT                  SUPPORT SPRING REACTION PER JOINT          03MYO
C          S()                    SPRING SUPPORT, VALUE PER JOINT            03MYO
C          SN()                   INPUT VALUE OF S()                         03MYO
C          STACH                  STATICS CHECK ERROR PER JOINT              03MYO
C          STEMP                  MAXIMUM STATICS CHECK ERROR                03MYO
C          SUMR                   SUMMATION OF REACTIONS                     03MYO
C          SWB,SWS                SWITCHES TO PRINT HEADINGS FOR OUTPUT      03MYO
C          S1,S2,S3               SINE OF THETA1 THETA2 THETA3               03MYO
C          S1S,S2S,S3S            SINE SQUARE OF THETA1 THETA2 THETA3        03MYO
C          TA()                   EXTERNAL COUPLE IN A OR X DIRECTION        03MYO
C          TAN(),TBN(),TCN()      INPUT VALUES FOR EXTERNAL COUPLE IN        03MYO
C                                 A B C DIRECTIONS                           03MYO
C          TBM1(),TCM1()          EXTERNAL COUPLES IN B AND C DIRECTIONS     03MYO
C                                 AT J-1                                     03MYO
C          TBP1(),TCP1()          EXTERNAL COUPLES IN B AND C DIRECTIONS     03MYO
C                                 AT J+1                                     03MYO
C          THETA                  ANGLE BETWEEN A AND C DIRECTIONS           03MYO
C                                 IN DEGREES                                 03MYO
C          THETA1                 ANGLE BETWEEN A AND B DIRECTIONS           03MYO
C                                 IN RADIANS                                 03MYO
C          THETA2                 THETA IN RADIANS                           03MYO
C          THETA3                 THETA2 - THETA1                            03MYO
C          W(,)                   DEFLECTION AT EACH JOINT                   03MYO
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APPENDIX 4

LISTING OF PROGRAM DECK OF SLAB 44

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      PROGRAM SLAB44 ( INPUT, OUTPUT, TAPE1, TAPE2, TAPE3 )
C
C-----THIS PROGRAM IS NOW DIMENSIONED TO SOLVE A 25 BY 75 GRID AND
C     FOR UPTO 70 CARDS IN EACH TABLF,  L1 = MA + 3, L2 = MA + 5
C
    1 FORMAT (    52H      PROGRAM   SLAB44    - MASTER DECK -     M. VORA
     1    ,                             28H  REVISION DATE 03 MAY 70     )REVISED
      DIMENSION AN1(32),  AN2(14)                                       120C9
C-----DIMENSION STATEMENT FOR NUMBER OF CARDS IN DIFFERENT TABLES
      DIMENSION IN13( 70), JN13( 70), IN23( 70), JN23( 70), D11N( 70),  120C9
     1          D12N( 70), D13N( 70), D22N( 70), D23N( 70), D33N( 70),  120C9
     2          IN14( 70), JN14( 70), IN24( 70), JN24( 70),  FAN( 70),  120C9
     3           FBN( 70),  FCN( 70),   QN( 70),   SN( 70),             120C9
     4          IN15( 70), JN15( 70), IN25( 70), JN25( 70),  TAN( 70),  120C9
     5           TBN( 70),  TCN( 70)                                    120C9
C-----DIMENSION STATEMENT WITH ( MA+5 )
      DIMENSION   D11(30),   D12(30),   D13(30),   D22(30),   D23(30),  REDIMEN
     1            D33(30), D12M1(30), D13M1(30), D22M1(30), D23M1(30),  REDIMEN
     2          D33M1(30), D12P1(30), D13P1(30), D22P1(30), D23P1(30),  REDIMEN
     3          D33P1(30),     FA(30),    FB(30),    FC(30),     Q(30), REDIMEN
     4              S(30),  FBM1(30),  FCM1(30),  FBP1(30),  FCP1(30),  REDIMEN
     5             TA(30),  TBM1(30),  TCM1(30),  TBP1(30),  TCP1(30)   REDIMEN
C-----DIMENSION STATEMENT WITH ( MA+3,   ) EXPECT FOR W WHICH
C     IS ( MA+5, MC+5 )
      DIMENSION   CC(28, 5),   DD(28, 5),   EE(28, 5),  DDT(28, 5),     REDIMEN
     1           EEP( 5,28), EET1(28, 5), EET2(28, 5),    FF(28, 1),    REDIMEN
     2             A(28    ),  AM1(28    ),  AM2(28    ),    B(28,28),   REDIMEN
     3           BM1(28,28),  EP1(28,28),  ATM(28    ),    C(28,28),    REDIMEN
     4           CM1(28,28),    D(28,28),    E(28,28),    W(30,80)      REDIMEN
C-----DIMENSION STATEMENT WITH ( MA+5 )
      DIMENSION    BMA(30),  BMBM1(30),    BMB(30),  BMBP1(30),         REDIMEN
     1           BMCM1(30),     BMC(30),  BMCP1(30),    BMBA(30),       REDIMEN
     2          BMBBM1(30),    BMBB(30), BMBBP1(30), BMBCM1(30),        REDIMEN
     3            BMBC(30), BMBCP1(30)                                  REDIMEN
      COMMON / DATA2 / IN13, JN13, IN23, JN23, IN14, JN14, IN24, JN24,  140C9
     1                 IN15, JN15, IN25, JN25,                          140C9
     2                 D11N, D12N, D13N, D22N, D23N, D33N,              140C9
     3                  FAN,  FBN,  FCN,   QN,   SN,  TAN,  TBN,  TCN,  140C9
     4                 NCT3, NCT4, NCT5, MAP5                           140C9
    6 FORMAT ( )                                                        120C9
   11 FORMAT (    5H1     , 80X, 10HI-----TRIM )                        120C9
   12 FORMAT (  16A5   )                                                120C9
   13 FORMAT (    5X, 16A5 )                                            120C9
   14 FORMAT ( A5, 5X, 14A5 )                                           120C9
   15 FORMAT (///10H      PROB  , /5X, A5, 5X, 14A5 )                   120C9
   16 FORMAT (///17H      PROB (CONTD), / 5X, A5, 5X, 14A5 )            230C9
   20 FORMAT ( 5X, 4I5, 5X, 4I5, 5X, I5, 5X, 3I5 )                      03MYO
   21 FORMAT ( 2I5, 3E10.3 )                                            120C9
   33 FORMAT ( 4( 2X, I3 ), 6E10.3 )                                    120C9
   43 FORMAT ( 4( 2X, I3 ), 5E10.3 )                                    120C9
   53 FORMAT ( 4( 2X, I3 ), 3E10.3 )                                    120C9
  100 FORMAT ( //27H      TABLE 1.  CONTROL DATA,                       120C9
     1  //  48X, 35H                         TABLE NUMBER,              120C9
     2   /  43X, 42H                             2    3    4    5,      120C9
     3  //   5X, 41H      HOLD FROM PRECEDING PROBLEM (1=HOLD),19X, 4I5, 120C9
     4   /   5X, 33H      NUM CARDS INPUT THIS PROBLEM, 27X, 4I5 ,      120C9
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    5  //    5X, 50H      MULTIPLE LOAD OPTION ( IF BLANK, PROBLEM IS S, 120C9
    6              16HINGLE LOADING --,                                120C9
    7   /   15X, 50HIF +1, PARENT FOR NEXT PROB -- IF -1, A OFFSPRING , 120C9
    8               5HPROB), 10X, I5,                                  120C9
    9   /    5X, 50H      PRINT OPTION (IF BLANK, MX MY MXY -- IF 1,    120C9
    A              25HMA MB MC PRINTED)          , I5,                 120C9
    B   /    5X, 50H      REACTION OUTPUT OPTION (IF BLANK, SUPPORT REA, 120C9
    C               9HCTION -- ,                                       120C9
    D   /   15X, 30HIF 1, STATICS CHECK PRINTED)  , 35X, I5,           03MY0
    E   /    5X, 51H      STIFFNESS INPUT OPTION (IF BLANK, D11 THRU D33,03MY0
    F   /   15X, 30HIF 1, B11 THRU B33 INPUT)      , 35X, I5 )         03MY0
  200 FORMAT ( //24H      TABLE 2.  CONSTANTS )                        120C9
  201 FORMAT ( / 50H           NUMBER OF INCREMENTS IN A DIRECTION  MA , 120C9
    1                32X, I3, /                                        120C9
    2       10X, 40HNUMBER OF INCREMENTS IN C DIRECTION  MC , 32X, I3, /120C9
    3       10X, 35HINCREMENT LENGTH IN A DIRECTION  HA, 30X, E10.3, /  120C9
    4       10X, 35HINCREMENT LENGTH IN C DIRECTION  HC, 30X, E10.3, /  120C9
    5       10X, 45HANGLE BETWEEN A AND C DIRECTION IN DEGREES    ,     120C9
    6       20X, E10.3 )                                               120C9
  300 FORMAT ( //35H      TABLE 3.  JOINT STIFFNESS DATA,              120C9
    1  //        50H        FROM    THRU       D11         D12        D13 , 120C9
    2            35H        D22         D23         D33     ,          120C9
    3   /        20H        JOINT   JOINT )                            120C9
  301 FORMAT ( //35H      TABLE 3.  JOINT STIFFNESS DATA,              03MY0
    1  //        50H        FROM    THRU       B11         B12        B13 , 03MY0
    2            35H        B22         B23         B33     ,          03MY0
    3   /        20H        JOINT   JOINT )                            120C9
  311 FORMAT ( 5X, 2(1X, I2, 1X, I3 ), 6E11.3 )                        120C9
  400 FORMAT ( //45H      TABLE 4.  BEAM STIFFNESS AND LOAD DATA  ,    120C9
    1  //        50H        FROM    THRU       FA          FB         FC  , 120C9
    2            35H         Q           S                  ,          120C9
    3   /        20H        JOINT   JOINT )                            120C9
  411 FORMAT ( 5X, 2(1X, I2, 1X, I3 ), 5E11.3 )                        120C9
  500 FORMAT ( //35H      TABLE 5.  EXTERNAL COUPLE DATA,              120C9
    1  //        50H        FROM    THRU       TA          TB         TC  , 120C9
    2   /        20H        JOINT   JOINT )                            120C9
  511 FORMAT ( 5X, 2(1X, I2, 1X, I3 ), 3E11.3 )                        120C9
  700 FORMAT ( //25H      TABLE 6. RESULTS      )                      230C9
  701 FORMAT ( //50H      TABLE 6. RESULTS -- USING STIFFNESS DATA FROM, 230C9
    1            18H PREVIOUS PROBLEM , A5 )                           230C9
  711 FORMAT ( / 49H                INPUT DATA IS SUCH THAT ONLY BEAM ,  230C9
    1            20HOUTPUT IS REQUIRED  ,                              230C9
    2    // 10X, 40H      BEAM MOMENTS ARE TOTAL PER BEAM     , / )     230C9
  712 FORMAT ( / 49H                INPUT DATA IS SUCH THAT ONLY SLAB ,  230C9
    1            20HOUTPUT IS REQUIRED  ,                              230C9
    2    // 10X, 40H      SLAB MOMENTS ARE PER UNIT WIDTH     ,        230C9
    3     / 10X, 47H      COUNTERCLOCKWISE BETA ANGLES ARE POSITIVE ,   230C9
    4   //  25X, 50H                                LARGEST    BETA   , 230C9
    5    /  25X, 50H                               PRINCIPAL    X TO   ) 230C9
  713 FORMAT ( / 50H                SLAB MOMENTS ARE PER UNIT WIDTH    , 230C9
    1    /  10X, 40H      BEAM MOMENTS ARE TOTAL PER BEAM     ,        230C9
    2    /  10X, 47H      COUNTERCLOCKWISE BETA ANGLES ARE POSITIVE ,//) 230C9
  721 FORMAT(25X,51HBEAM A      BEAM B      BEAM C                   S,03MY0
    1            6HUPPORT,                                             03MY0
    2   /        52H        A , C     DEFL       MOMENT      MOMENT      MOMEN230C9
    3            31HT                          REACTION )              03MY0
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722 FORMAT(25X,51HBEAM A      BEAM B      BEAM C                       S,03MY0
   1              6HTATICS,                                              03MY0
   2    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   3              31HT                         CHECK   )                 03MY0
731 FORMAT(25X,51HSLAB X      SLAB Y      SLAB XY     SLAB     LARGEST  S,03MY0
   1              6HUPPORT,                                              03MY0
   2    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   3              31HT    MOMENT    MOMENT   REACTION )                  03MY0
732 FORMAT(25X,51HSLAB A      SLAB B      SLAB C      SLAB     LARGEST  S,03MY0
   1              6HUPPORT,                                              03MY0
   2    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   3              31HT    MOMENT    MOMENT   REACTION   )                03MY0
733 FORMAT(25X,51HSLAB X      SLAB Y      SLAB XY     SLAB     LARGEST  S,03MY0
   1              6HTATICS,                                              03MY0
   2    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   3              31HT    MOMENT    MOMENT    CHECK   )                  03MY0
734 FORMAT(25X,51HSLAB A      SLAB B      SLAB C      SLAB     LARGEST  S,03MY0
   1              6HTATICS,                                              03MY0
   2    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   3              31HT    MOMENT    MOMENT    CHECK   )                  03MY0
741 FORMAT(25X,50HSLAB X      SLAB Y      SLAB XY   LARGEST     BETA    , 230C9
   1    /  25X, 50HMOMENT     MOMENT      MOMENT   PRINCIPAL    X TO    , 230C9
   2    /  25X, 51HBEAM A     BEAM B      BEAM C      SLAB     LARGEST  S,03MY0
   3              6HUPPORT,                                              03MY0
   4    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   5              31HT    MOMENT    MOMENT   REACTION )                  03MY0
742 FORMAT(25X,50HSLAB A      SLAB B      SLAB C    LARGEST     BETA    , 230C9
   1    /  25X, 50HMOMENT     MOMENT      MOMENT   PRINCIPAL    A TO    , 230C9
   2    /  25X, 51HBEAM A     BEAM B      BEAM C      SLAB     LARGEST  S,03MY0
   3              6HUPPORT,                                              03MY0
   4    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   5              31HT    MOMENT    MOMENT   REACTION )                  03MY0
743 FORMAT(25X,50HSLAB X      SLAB Y      SLAB XY   LARGEST     BETA    , 230C9
   1    /  25X, 50HMOMENT     MOMENT      MOMENT   PRINCIPAL    X TO    , 230C9
   2    /  25X, 51HBEAM A     BEAM B      BEAM C      SLAB     LARGEST  S,03MY0
   3              6HTATICS,                                              03MY0
   4    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   5              31HT    MOMENT    MOMENT    CHECK     )                03MY0
744 FORMAT(25X,50HSLAB A      SLAB B      SLAB C    LARGEST     BETA    , 230C9
   1    /  25X, 50HMOMENT     MOMENT      MOMENT   PRINCIPAL    A TO    , 230C9
   2    /  25X, 51HBEAM A     BEAM B      BEAM C      SLAB     LARGEST  S,03MY0
   3              6HTATICS,                                              03MY0
   4    /         52H       A , C     DEFL       MOMENT     MOMENT     MOMEN230C9
   5              31HT    MOMENT    MOMENT    CHECK     )                03MY0
751 FORMAT ( 5X, I2, 1X, I3, 4E11.3, 17X, E11.3 )                        230C9
752 FORMAT ( 5X, I2, 1X, I3, 5E11.3, F6.1, E11.3 )                       230C9
753 FORMAT ( 22X, 3E11.3 )                                               230C9
903 FORMAT ( / 25H                        NONE )                         120C9
905 FORMAT (   46H          USING DATA FROM THE PREVIOUS PROBLEM )       120C9
910 FORMAT (   43H          ADDITIONAL DATA FOR THIS PROBLEM   )         120C9
980 FORMAT (///40H      **** UNDESIGNATED ERROR STOP ****   )            120C9
991 FORMAT ( //10H      **** , I4,                                       120C9
   1              33H  DATA ERRORS IN THIS TABLE  ****   )               120C9
992 FORMAT (///30H      **** PROBLEM TERMINATED  , I4  ,                 120C9
   1              20H  DATA ERRORS ****   )                              120C9
993 FORMAT ( //50H      **** CAUTION. MULTIPLF LOAD OPTION MISUSED FO,  120C9
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     1              30HR THIS OR PRIOR PROBLEM ****  )                        120C9
  994 FORMAT ( //29X, 43HSUMMATION OF SUPPORT SPRING REACTION       =,        300C9
     1              E11.3 )                                                   300C9
  995 FORMAT ( / 29X, 35HMAXIMUM STATICS CHECK ERROR AT STA , I3, I3,         300C9
     1                    2H =, E11.3 )                                       300C9
C
C-----PROBLEM IDENTIFICATION
C
                ITEST = 5H                                                    120C9
                KML = 0                                                       120C9
      READ  12, ( AN1(N), N = 1, 32 )                                         120C9
      CALL TIC TOC (1)                                                        120C9
 1010 READ  14, NPROB, ( AN2(N), N = 1, 14 )                                  120C9
          IF ( NPROB - ITEST ) 1020, 9990, 1020                               120C9
 1020 PRINT 11                                                                120C9
      PRINT  1                                                                120C9
      PRINT 13, ( AN1(N), N = 1, 32 )                                         120C9
      PRINT 15, NPROB, ( AN2(N), N = 1, 14 )                                  120C9
C
C-----INPUT TABLE 1
C
      READ   20, KEEP2, KEEP3, KEEP4, KEEP5,                                  120C9
     1              NCD2, NCD3, NCD4, NCD5, ML, IPR, KROPT, ISTIFF            03MY0
      PRINT 100, KEEP2, KEEP3, KEEP4, KEEP5,                                  120C9
     1              NCD2, NCD3, NCD4, NCD5, ML, IPR, KROPT, ISTIFF            03MY0
                NDE1 = 0                                                      120C9
          IF ( KML ) 1115, 1110, 1112                                         120C9
 1110     IF ( ML ) 1111, 1115, 1115                                          120C9
 1111           NDE1 = NDE1 + 1                                               120C9
          GO TO 1115                                                          120C9
 1112     IF ( ML ) 1115, 1113, 1113                                          120C9
 1113 PRINT 993                                                               120C9
 1115           KML = ML                                                      120C9
          IF ( KEEP2 ) 9980, 1130, 1120                                       120C9
 1120     IF ( NCD2 ) 9980, 1130, 1125                                        120C9
 1125           NDE1 = NDE1 + 1                                               120C9
 1130     IF ( ML ) 1135, 1145, 1145                                          120C9
 1135     IF ( NCD2*NCD3*NCD5 ) 9980, 1145, 1140                              120C9
 1140           NDE1 = NDE1 + 1                                               120C9
 1145     IF ( NDE1 ) 9980, 1200, 1150                                        120C9
 1150 PRINT 991, NDE1                                                         120C9
C
C-----INPUT TABLE 2
C
 1200 PRINT 200                                                               120C9
          IF ( KEEP2 ) 9980, 1201, 1230                                       120C9
 1201           NDE2 = 0                                                      120C9
          IF ( NCD2 - 1 ) 1203, 1205, 1203                                    120C9
 1203           NDE2 = NDE2 + 1                                               120C9
 1205 READ   21, MA, MC, HA, HC, THETA                                        120C9
      PRINT 201, MA, MC, HA, HC, THETA                                        120C9
          IF ( MA - MC ) 1211, 1211, 1210                                     120C9
 1210           NDE2 = NDE2 + 1                                               120C9
 1211     IF ( HA * HC ) 1212, 1212, 1250                                     120C9
 1212           NDE2 = NDE2 + 1                                               120C9
          GO TO 1250                                                          120C9
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 1230      IF ( NCD2 ) 9980, 1240, 1235                                         120C9
 1235           NDE2 = NDE2 + 1                                                  120C9
 1240 PRINT 905                                                                  120C9
      PRINT 201, MA, MC, HA, HC, THETA                                           120C9
 1250      IF ( NDE2 ) 9980, 1300, 1270                                          120C9
 1270 PRINT 991, NDE2                                                            120C9
C
C-----INPUT TABLE 3
C
 1300      IF ( ISTIFF ) 9980, 1301, 1302                                        03MYO
 1301 PRINT 300                                                                  03MYO
           GO TO 1303                                                            03MYO
 1302 PRINT 301                                                                  03MYO
 1303      IF ( KEEP3 ) 9980, 1304, 1310                                         03MYO
 1304           NC13 = 1                                                         03MYO
                NCT3 = NCD3                                                      120C9
                NDE3 = 0                                                         120C9
                SWS  = 0.0                                                       120C9
           GO TO 1335                                                            120C9
 1310 PRINT 905                                                                  120C9
           DO 1325  N = 1, NCT3                                                  120C9
      PRINT 311, IN13(N), JN13(N), IN23(N), JN23(N), D11N(N),                    120C9
     1          D12N(N), D13N(N), D22N(N), D23N(N), D33N(N)                      120C9
 1325      CONTINUE                                                              120C9
      PRINT 910                                                                  120C9
                NC13 = NCT3 + 1                                                  120C9
                NCT3 = NCT3 + NCD3                                               120C9
 1335      IF ( NCD3 ) 9980, 1337, 1340                                          120C9
 1337 PRINT 903                                                                  120C9
           GO TO 1372                                                            120C9
 1340      DO 1370  N = NC13, NCT3                                               120C9
C-----IF STIFFNESS INPUT OPTION ISTIFF = 1, THEN B11 THROUGH B33 ARE
C     READ AND STORED AS D11 THROUGH D33
      READ   33, IN13(N), JN13(N), IN23(N), JN23(N), D11N(N),                    120C9
     1          D12N(N), D13N(N), D22N(N), D23N(N), D33N(N)                      120C9
      PRINT 311, IN13(N), JN13(N), IN23(N), JN23(N), D11N(N),                    120C9
     1          D12N(N), D13N(N), D22N(N), D23N(N), D33N(N)                      120C9
           IF ( IN13(N) - IN23(N) ) 1342, 1342, 1341                             120C9
 1341           NDE3 = NDE3 + 1                                                  120C9
 1342      IF ( JN13(N) - JN23(N) ) 1344, 1344, 1343                             120C9
 1343           NDE3 = NDE3 + 1                                                  120C9
 1344      IF ( IN23(N) - MA ) 1346, 1346, 1345                                  120C9
 1345           NDE3 = NDE3 + 1                                                  120C9
 1346      IF ( JN23(N) - MC ) 1350, 1350, 1347                                  120C9
 1347           NDE3 = NDE3 + 1                                                  120C9
 1350           SWS  = SWS + ABS ( D11N(N) + D12N(N) + D13N(N) + D22N(N)         120C9
     1                 + D23N(N) + D33N(N) )                                     120C9
 1370      CONTINUE                                                              120C9
 1372      IF ( NDE3 ) 9980, 1400, 1375                                          120C9
 1375 PRINT 991, NDE3                                                            120C9
C
C-----INPUT TABLE 4
C
 1400 PRINT 400                                                                  120C9
           IF ( KEEP4 ) 9980, 1401, 1410                                         120C9
 1401           NC14 = 1                                                         120C9
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                NCT4 = NCD4                                                    120C9
                NDE4 = 0                                                       120C9
                SWB  = 0.0                                                     120C9
            GO TO 1435                                                         120C9
 1410 PRINT 905                                                                120C9
            DO 1425  N = 1, NCT4                                               120C9
      PRINT 411, IN14(N), JN14(N), IN24(N), JN24(N), FAN(N),                   120C9
     1             FBN(N), FCN(N), QN(N), SN(N)                                120C9
 1425       CONTINUE                                                           120C9
      PRINT 910                                                                120C9
                NC14 = NCT4 + 1                                                120C9
                NCT4 = NCT4 + NCD4                                             120C9
 1435       IF ( NCD4 ) 9980, 1437, 1440                                       120C9
 1437 PRINT 903                                                                120C9
          GO TO 1472                                                           120C9
 1440     DO 1470  N = NC14, NCT4                                              120C9
      READ   43, IN14(N), JN14(N), IN24(N), JN24(N), FAN(N),                   120C9
     1             FBN(N), FCN(N), QN(N), SN(N)                                120C9
      PRINT 411, IN14(N), JN14(N), IN24(N), JN24(N), FAN(N),                   120C9
     1             FBN(N), FCN(N), QN(N), SN(N)                                120C9
          IF ( IN14(N) - IN24(N) ) 1442, 1442, 1441                            120C9
 1441           NDE4 = NDE4 + 1                                                120C9
 1442     IF ( JN14(N) - JN24(N) ) 1444, 1444, 1443                            120C9
 1443           NDE4 = NDE4 + 1                                                120C9
 1444     IF ( IN24(N) - MA ) 1446, 1446, 1445                                 120C9
 1445           NDE4 = NDE4 + 1                                                120C9
 1446     IF ( JN24(N) - MC ) 1450, 1450, 1447                                 120C9
 1447           NDE4 = NDE4 + 1                                                120C9
 1450           SWB  = SWB + ABS ( FAN(N) + FBN(N) + FCN(N) )                  120C9
          IF ( ML ) 1455, 1470, 1470                                           120C9
 1455     IF ( FAN(N)*FBN(N)*FCN(N)*SN(N) ) 1460, 1470, 1460                   120C9
 1460           NDE4 = NDE4 + 1                                                120C9
 1470       CONTINUE                                                           120C9
 1472     IF ( NDE4 ) 9980, 1500, 1475                                         120C9
 1475 PRINT 991, NDE4                                                          120C9
C
C-----INPUT TABLE 5
C
 1500 PRINT 500                                                                120C9
          IF ( KEEP5 ) 9980, 1501, 1510                                        120C9
 1501           NC15 = 1                                                       120C9
                NCT5 = NCD5                                                    120C9
                NDE5 = 0                                                       120C9
          GO TO 1535                                                           120C9
 1510 PRINT 905                                                                120C9
          DO 1525  N = 1, NCT5                                                 120C9
      PRINT 511, IN15(N), JN15(N), IN25(N), JN25(N),                           120C9
     1             TAN(N), TBN(N), TCN(N)                                      120C9
 1525     CONTINUE                                                             120C9
      PRINT 910                                                                120C9
                NC15 = NCT5 + 1                                                120C9
                NCT5 = NCT5 + NCD5                                             120C9
 1535     IF ( NCD5 ) 9980, 1537, 1540                                         120C9
 1537 PRINT 903                                                                120C9
          GO TO 1572                                                           120C9
 1540     DO 1570  N = NC15, NCT5                                              120C9
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      READ   53, IN15(N), JN15(N), IN25(N), JN25(N),                    120C9
     1           TAN(N), TBN(N), TCN(N)                                 120C9
      PRINT 511, IN15(N), JN15(N), IN25(N), JN25(N),                    120C9
     1           TAN(N), TBN(N), TCN(N)                                 120C9
          IF ( IN15(N) - IN25(N) ) 1542, 1542, 1541                     120C9
 1541          NDE5 = NDE5 + 1                                          120C9
 1542     IF ( JN15(N) - JN25(N) ) 1544, 1544, 1543                     120C9
 1543          NDE5 = NDE5 + 1                                          120C9
 1544     IF ( IN25(N) - MA ) 1546, 1546, 1545                          120C9
 1545          NDE5 = NDE5 + 1                                          120C9
 1546     IF ( JN25(N) - MC ) 1570, 1570, 1547                          120C9
 1547          NDE5 = NDE5 + 1                                          120C9
 1570     CONTINUE                                                      120C9
 1572     IF ( NDE5 ) 9980, 1600, 1575                                  120C9
 1575 PRINT 991, NDE5                                                   120C9
 1600          NDES = NDE1 + NDE2 + NDE3 + NDE4 + NDE5                  160C9
          IF ( NDES ) 9980, 1700, 1650                                  160C9
 1650 PRINT 992, NDES                                                   160C9
          GO TO 1010                                                    160C9
 1700     CONTINUE                                                      160C9
C
C-----COMPUTE FOR CONVENIENCE
C
          IF ( ML ) 1885, 1875, 1875                                    160C9
 1875          MAP1 = MA + 1                                            160C9
               MCP1 = MC + 1                                            160C9
               MAP2 = MA + 2                                            160C9
               MCP2 = MC + 2                                            160C9
               MAP3 = MA + 3                                            160C9
               MCP3 = MC + 3                                            160C9
               MAP4 = MA + 4                                            160C9
               MCP4 = MC + 4                                            160C9
               MAP5 = MA + 5                                            160C9
               MCP5 = MC + 5                                            160C9
               KPROB = NPROB                                            230C9
               THETA2 = THETA / 57.29578                                160C9
               HB     = SQRT ( HA*HA + HC*HC + 2.0*HA*HC*COS (THETA2) ) 160C9
               THETA1 = ASIN ( HC * SIN ( THETA2 ) / HB )               160C9
               THETA3 = THETA2 - THETA1                                 160C9
               CS11 = HC * SIN ( THETA2 ) / ( HA * HA * HA )            160C9
               CS12 = SIN ( THETA1 ) / ( HA * HB )                      160C9
               CS13 = SIN ( THETA2 ) / ( HA * HC )                      160C9
               CS22 = HA * SIN ( THETA1 ) / ( HB * HB * HB )            160C9
               CS23 = HA * SIN ( THETA2 ) / ( HB * HB * HC )            160C9
               CS33 = HA * SIN ( THETA2 ) / ( HC * HC * HC )            160C9
               CBA  = 1.0 / ( HA * HA * HA )                            160C9
               CBB  = 1.0 / ( HB * HB * HB )                            160C9
               CBC  = 1.0 / ( HC * HC * HC )                            160C9
               C1   = COS ( THETA1 )                                    160C9
               C2   = COS ( THETA2 )                                    160C9
               C3   = COS ( THETA3 )                                    160C9
               S1   = SIN ( THETA1 )                                    160C9
               S2   = SIN ( THETA2 )                                    160C9
               S3   = SIN ( THETA3 )                                    160C9
               C1S = C1 * C1                                            160C9
               C2S = C2 * C2                                            160C9
```

```
              C3S = C3 * C3                                                    160C9
              S1S = S1 * S1                                                    160C9
              S2S = S2 * S2                                                    160C9
              S3S = S3 * S3                                                    160C9
 1885     CONTINUE                                                             160C9
      REWIND 1                                                                  4JA8
      REWIND 2                                                                 04JA8
      REWIND 3                                                                 17JA8
          IF ( ML ) 2140, 2100, 2100                                           190C9
C-----SET INITIAL CONDITIONS
 2100     DO 2135  J = 1, MAP3                                                 190C9
          DO 2130  I = 1, MAP3                                                 190C9
              B(I,J)  = 0.0                                                    04JA8
              C(I,J)  = 0.0                                                    04JA8
              CM1(I,J) = 0.0                                                   04JA8
              EP1(I,J) = 0.0                                                   23MR8
              D(I,J)  = 0.0                                                    15JL9
 2130     CONTINUE                                                             190C9
 2135     CONTINUE                                                             190C9
 2140     DO 2150  I = 1, MAP3                                                 190C9
              A(I) = 0.0                                                       20MY8
              AM1(I) = 0.0                                                     20MY8
 2150     CONTINUE                                                             190C9
          DO 2350  J = 1, MCP5                                                 230C9
          DO 2300  I = 1, MAP5                                                 230C9
              W(I,J) = 0.0                                                     230C9
 2300     CONTINUE                                                             230C9
 2350     CONTINUE                                                             230C9
          DO 2400  I = 1, MAP5                                                 230C9
              BMA(I)    = 0.0                                                  230C9
              BMB(I)    = 0.0                                                  230C9
              BMC(I)    = 0.0                                                  230C9
              BMBM1(I)  = 0.0                                                  230C9
              BMBP1(I)  = 0.0                                                  230C9
              BMCM1(I)  = 0.0                                                  230C9
              BMCP1(I)  = 0.0                                                  230C9
              BMBA(I)   = 0.0                                                  230C9
              BMBB(I)   = 0.0                                                  230C9
              BMBC(I)   = 0.0                                                  230C9
              BMBBM1(I) = 0.0                                                  230C9
              BMBBP1(I) = 0.0                                                  230C9
              BMBCM1(I) = 0.0                                                  230C9
              BMBCP1(I) = 0.0                                                  230C9
 2400     CONTINUE                                                             230C9
C
C-----BEGIN FORWARD PASS -- SOLVE FOR RECURSION COEFFICIENTS
C
              NK = MAP3                                                        190C9
              NL = MCP4                                                        190C9
              NF = 2                                                           190C9
              L1 = 28                                                          REDIMEN
              L2 = 30                                                          REDIMEN
              N1 = 5                                                           190C9
              N2 = 5                                                           190C9
              N3 = 5                                                           190C9
          DO 5000  J = 2, MCP4                                                 190C9
```

```
            JN = J - 3
C-----RETRIEVE DATA NEEDED AT THIS J STEP
      CALL      DATA2 ( D11, D12, D13, D22, D23, D33, D12M1, D13M1,     130C9
     1          D22M1, D23M1, D33M1, D12P1, D13P1, D22P1, D23P1, D33P1,  130C9
     2          FA, FB, FC, Q, S, FBM1, FCM1, FBP1, FCP1, TA, TBM1, TCM1,130C9
     3          TBP1, TCP1, L2, JN, ML )                                 130C9
C
C-----FORM SUBMATRICES
C
         DO 3350  I = 2, MAP4                                            190C9
C-----COMPUTE TEMP CONSTANTS FOR SLAB STIFFNESS -- IF ISTIFF = 1, THE
C     D11 THROUGH D33 COEFFICIENTS ARE ACTUALLY B11 THROUGH B33(TABLE 3)
         IF ( ISTIFF ) 9980, 3100, 3110                                  03MYO
 3100       B22MM = ( C2S * D22M1(I-1) + 2.0 * C2 * S2 * D23M1(I-1)      03MYO
     1                + S2S * D33M1(I-1) ) / ( S1S * S3S )               160C9
            B23MM = ( - C1 * C2 * D22M1(I-1)                             160C9
     1                - ( C1 * S2 + S1 * C2 ) * D23M1(I-1)               160C9
     2                - S1 * S2 * D33M1(I-1) ) / ( S1 * S2 * S3S )       160C9
            B23MV = ( - C1 * C2 * D22M1(I)                               160C9
     1                - ( C1 * S2 + S1 * C2 ) * D23M1(I)                 160C9
     2                - S1 * S2 * D33M1(I) ) / ( S1 * S2 * S3S )         160C9
            B33MV = ( C1S * D22M1(I) + 2.0 * C1 * S1 * D23M1(I)          160C9
     1                + S1S * D33M1(I) ) / ( S2S * S3S )                 160C9
         GO TO 3120                                                      03MYO
 3110       B22MM = D22M1(I-1)                                           03MYO
            B23MM = D23M1(I-1)                                           03MYO
            B23MV = D23M1(I)                                             03MYO
            B33MV = D33M1(I)                                             03MYO
 3120    CONTINUE                                                        03MYO
C-----COMPUTE STIFFNESS VECTORS FF AND EET2
C-----K IS USED AS INDEX FOR FF AND EET2 SO THAT FF AND EET2 WILL BE
C     STORED FROM 1 TO MAP3 AS REQUIRED FOR SOLUTION PROCESS
            K = I - 1                                                    04NO9
            FF(K,1)   = Q(I) + 0.5 * ( - TA(I-1) + TA(I+1) ) / HA        190C9
     1                  + 0.5 * ( - TBM1(I-1) + TBP1(I+1) ) / HB         190C9
     2                  + 0.5 * ( - TCM1(I) + TCP1(I) ) / HC             190C9
            EET2(K,1) = CS22 * B22MM + CBB * FBM1(I-1)                   190C9
            EET2(K,2) = CS23 * ( B23MM + B23MV )                         190C9
            EET2(K,3) = CS33 * B33MV + CBC * FCM1(I)                     190C9
            EET2(K,4) = 0.0                                              190C9
            EET2(K,5) = 0.0                                              190C9
         IF ( ML ) 3350, 3150, 3150                                      190C9
C-----TEMP CONSTANTS B22MM, B23MM, B23MV AND B33MV ARE ALREADY COMPUTED
C     COMPUTE REMAINING REQUIRED CONSTANTS
 3150    IF ( ISTIFF ) 9980, 3160, 3170                                  03MYO
 3160       B11VM = ( S1S * S2S * D11(I-1)                               03MYO
     1                + 2.0 * C1 * S1 * C2 * S2 * D12(I-1)               160C9
     2                + 2.0 * S1 * S2 * ( C1 * S2 + S1 * C2 ) *          160C9
     3                D13(I-1) + C1S * C2S * D22(I-1)                    160C9
     4                + 2.0 * C1 * C2 * ( C1 * S2 + S1 * C2 ) *          160C9
     5                D23(I-1) + ( C1 * S2 + S1 * C2 ) ** 2 * D33(I-1)   160C9
     6                ) / ( S1S * S2S )                                  160C9
            B11VV = ( S1S * S2S * D11(I)                                 160C9
     1                + 2.0 * C1 * S1 * C2 * S2 * D12(I)                 160C9
     2                + 2.0 * S1 * S2 * ( C1 * S2 + S1 * C2 ) * D13(I)   160C9
     3                + C1S * C2S * D22(I)                               160C9
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     4          + 2.0 * C1 * C2 * ( C1 * S2 + S1 * C2 ) * D23(I) 160C9
     5          + ( C1 * S2 + S1 * C2 ) ** 2 * D33(I) )          160C9
     6          / ( S1S * S2S )                                  160C9
      B11VP = ( S1S * S2S * D11(I+1)                             160C9
     1          + 2.0 * C1 * S1 * C2 * S2 * D12(I+1)             160C9
     2          + 2.0 * S1 * S2 * ( C1 * S2 + S1 * C2 ) *        160C9
     3          D13(I+1) + C1S * C2S * D22(I+1)                  160C9
     4          + 2.0 * C1 * C2 * ( C1 * S2 + S1 * C2 ) *        160C9
     5          D23(I+1) + ( C1 * S2 + S1 * C2 ) ** 2 * D33(I+1) 160C9
     6          ) / ( S1S * S2S )                                160C9
      B12VM = ( - S1 * C2 * S2 * D12(I-1) - S1 * S2S * D13(I-1)160C9
     1          - C1 * C2S * D22(I-1)                            160C9
     2          - C2 * ( 2.0 * C1 * S2 + S1 * C2 ) * D23(I-1)    160C9
     3          - S2 * ( C1 * S2 + S1 * C2 ) * D33(I-1) )        160C9
     4          / ( S1S * S2 * S3 )                              160C9
      B12VV = ( - S1 * C2 * S2 * D12(I) - S1 * S2S * D13(I)      160C9
     1          - C1 * C2S * D22(I)                              160C9
     2          - C2 * ( 2.0 * C1 * S2 + S1 * C2 ) * D23(I)      160C9
     3          - S2 * ( C1 * S2 + S1 * C2 ) * D33(I) )          160C9
     4          / ( S1S * S2 * S3 )                              160C9
      B12VP = ( - S1 * C2 * S2 * D12(I+1) - S1 * S2S * D13(I+1)160C9
     1          - C1 * C2S * D22(I+1)                            160C9
     2          - C2 * ( 2.0 * C1 * S2 + S1 * C2 ) * D23(I+1)    160C9
     3          - S2 * ( C1 * S2 + S1 * C2 ) * D33(I+1) )        160C9
     4          / ( S1S * S2 * S3 )                              160C9
      B12PP = ( - S1 * C2 * S2 * D12P1(I+1)                      160C9
     1          - S1 * S2S * D13P1(I+1) - C1 * C2S * D22P1(I+1)  160C9
     2          - C2 * ( 2.0 * C1 * S2 + S1 * C2 ) * D23P1(I+1)  160C9
     3          - S2 * ( C1 * S2 + S1 * C2 ) * D33P1(I+1) )      160C9
     4          / ( S1S * S2 * S3 )                              160C9
      B13VM = ( C1 * S1 * S2 * D12(I-1) + S1S * S2 * D13(I-1)    160C9
     1          + C1S * C2 * D22(I-1)                            160C9
     2          + C1 * ( C1 * S2 + 2.0 * S1 * C2 ) * D23(I-1)    160C9
     3          + S1 * ( C1 * S2 + S1 * C2 ) * D33(I-1) )        160C9
     4          / ( S1 * S2S * S3 )                              160C9
      B13VV = ( C1 * S1 * S2 * D12(I) + S1S * S2 * D13(I)        160C9
     1          + C1S * C2 * D22(I)                              160C9
     2          + C1 * ( C1 * S2 + 2.0 * S1 * C2 ) * D23(I)      160C9
     3          + S1 * ( C1 * S2 + S1 * C2 ) * D33(I) )          160C9
     4          / ( S1 * S2S * S3 )                              160C9
      B13VP = ( C1 * S1 * S2 * D12(I+1) + S1S * S2 * D13(I+1)    160C9
     1          + C1S * C2 * D22(I+1)                            160C9
     2          + C1 * ( C1 * S2 + 2.0 * S1 * C2 ) * D23(I+1)    160C9
     3          + S1 * ( C1 * S2 + S1 * C2 ) * D33(I+1) )        160C9
     4          / ( S1 * S2S * S3 )                              160C9
      B13PV = ( C1 * S1 * S2 * D12P1(I) + S1S * S2 * D13P1(I)    160C9
     1          + C1S * C2 * D22P1(I)                            160C9
     2          + C1 * ( C1 * S2 + 2.0 * S1 * C2 ) * D23P1(I)    160C9
     3          + S1 * ( C1 * S2 + S1 * C2 ) * D33P1(I) )        160C9
     4          / ( S1 * S2S * S3 )                              160C9
      B22VV = ( C2S * D22(I) + 2.0 * C2 * S2 * D23(I)            160C9
     1          + S2S * D33(I) ) / ( S1S * S3S )                 160C9
      B22PP = ( C2S * D22P1(I+1) + 2.0 * C2 * S2 * D23P1(I+1)    160C9
     1          + S2S * D33P1(I+1) ) / ( S1S * S3S )             160C9
      B23VV = ( - C1 * C2 * D22(I)                               160C9
     1          - ( C1 * S2 + S1 * C2 ) * D23(I)                 160C9
```

```
     2                 - S1 * S2 * D33(I) ) / ( S1 * S2 * S3S )              160C9
             B23PV = ( - C1 * C2 * D22P1(I)                                  160C9
     1                 - ( C1 * S2 + S1 * C2 ) * D23P1(I)                    160C9
     2                 - S1 * S2 * D33P1(I) ) / ( S1 * S2 * S3S )            160C9
             B23PP = ( - C1 * C2 * D22P1(I+1)                                160C9
     1                 - ( C1 * S2 + S1 * C2 ) * D23P1(I+1)                  160C9
     2                 - S1 * S2 * D33P1(I+1) ) / ( S1 * S2 * S3S )          160C9
             B33VV = ( C1S * D22(I) + 2.0 * C1 * S1 * D23(I)                 160C9
     1                 + S1S * D33(I) ) / ( S2S * S3S )                      160C9
             B33PV = ( C1S * D22P1(I) + 2.0 * C1 * S1 * D23P1(I)             160C9
     1                 + S1S * D33P1(I) ) / ( S2S * S3S )                    160C9
          GO TO 3180                                                         03MY0
 3170        B11VM = D11(I-1)                                                03MY0
             B11VV = D11(I)                                                  03MY0
             B11VP = D11(I+1)                                                03MY0
             B12VM = D12(I-1)                                                03MY0
             B12VV = D12(I)                                                  03MY0
             B12VP = D12(I+1)                                                03MY0
             B12PP = D12P1(I+1)                                              03MY0
             B13VM = D13(I-1)                                                03MY0
             B13VV = D13(I)                                                  03MY0
             B13VP = D13(I+1)                                                03MY0
             B13PV = D13P1(I)                                                03MY0
             B22VV = D22(I)                                                  03MY0
             B22PP = D22P1(I+1)                                              03MY0
             B23VV = D23(I)                                                  03MY0
             B23PV = D23P1(I)                                                03MY0
             B23PP = D23P1(I+1)                                              03MY0
             B33VV = D33(I)                                                  03MY0
             B33PV = D33P1(I)                                                03MY0
 3180     CONTINUE                                                           03MY0
C-----COMPUTE STIFFNESSES CC, DD, AND EE
C-----K IS USED AS INDEX FOR CC, DD AND EE SO THAT CC, DD AND EE WILL BE
C     STORED FROM 1 TO MAP3 AS REQUIRED FOR SOLUTION PROCESS
             CC(K,1)   = CS11 * B11VM + CBA * FA(I-1)                        190C9
             CC(K,2)   = - 2.0 * CS11 * ( B11VM + B11VV )                    190C9
     1                    - 2.0 * CS12 * ( B12VM + B12VV )                   190C9
     2                    - 2.0 * CS13 * ( B13VM + B13VV )                   190C9
     3                    + CS23 * ( B23MM + B23PV )                         190C9
     4                    - 2.0 * CBA * ( FA(I-1) + FA(I) )                  190C9
             CC(K,3)   = CS11 * ( B11VM + 4.0 * B11VV + B11VP )              190C9
     1                    + CS22 * ( B22MM + 4.0 * B22VV + B22PP )           190C9
     2                    + CS33 * ( B33MV + 4.0 * B33VV + B33PV )           190C9
     3                    + 8.0 * ( CS12 * B12VV + CS13 * B13VV              190C9
     4                    + CS23 * B23VV ) + S(I)                            190C9
     5                    + CBA * ( FA(I-1) + 4.0 * FA(I) + FA(I+1) )        190C9
     6                    + CBB * ( FBM1(I-1) + 4.0 * FB(I) + FBP1(I+1)190C9
     7                    ) + CBC * ( FCM1(I) + 4.0 * FC(I) + FCP1(I) )190C9
          IF ( CC(K,3) ) 3320, 3310, 3320                                    190C9
 3310        CC(K,3)   = 1.0                                                 190C9
 3320        CC(K,4)   = - 2.0 * CS11 * ( B11VV + B11VP )                    190C9
     1                    - 2.0 * CS12 * ( B12VV + B12VP )                   190C9
     2                    - 2.0 * CS13 * ( B13VV + B13VP )                   190C9
     3                    + CS23 * ( B23MV + B23PP )                         190C9
     4                    - 2.0 * CBA * ( FA(I) + FA(I+1) )                  190C9
             CC(K,5)   = CS11 * B11VP + CBA * FA(I+1)                        190C9
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              DD(K,1)    = 0.0                                            190C9
              DD(K,2)    = CS13 * ( B13VM + B13PV )                       190C9
              DD(K,3)    = - 2.0 * CS13 * ( B13VV + B13PV )               190C9
    1                      - 2.0 * CS23 * ( B23VV + B23PV )               190C9
    2                      - 2.0 * CS33 * ( B33VV + B33PV )               190C9
    3                      + CS12 * ( B12VM + B12PP )                     190C9
    4                      - 2.0 * CBC * ( FC(I) + FCP1(I) )              190C9
              DD(K,4)    = - 2.0 * CS12 * ( B12VV + B12PP )               190C9
    1                      - 2.0 * CS22 * ( B22VV + B22PP )               190C9
    2                      - 2.0 * CS23 * ( B23VV + B23PP )               190C9
    3                      + CS13 * ( B13VP + B13PV )                     190C9
    4                      - 2.0 * CBB * ( FB(I) + FBP1(I+1) )            190C9
              DD(K,5)    = CS12 * ( B12VP + B12PP )                       190C9
              EE(K,1)    = 0.0                                            190C9
              EE(K,2)    = 0.0                                            190C9
              EE(K,3)    = CS33 * B33PV + CBC * FCP1(I)                   190C9
              EE(K,4)    = CS23 * ( B23PV + B23PP )                       190C9
              EE(K,5)    = CS22 * B22PP + CBB * FBP1(I+1)                 190C9
 3350     CONTINUE                                                        190C9
C-----PACK EET2 AS REQUIRED FOR SOLUTION PROCESS
              EET2(1,1) = EET2(1,3)                                       190C9
              EET2(1,2) = EET2(1,4)                                       190C9
              EET2(1,3) = EET2(1,5)                                       190C9
              EET2(1,4) = 0.0                                             190C9
              EET2(1,5) = 0.0                                             190C9
              EET2(2,1) = EET2(2,2)                                       190C9
              EET2(2,2) = EET2(2,3)                                       190C9
              EET2(2,3) = EET2(2,4)                                       190C9
              EET2(2,4) = EET2(2,5)                                       190C9
              EET2(2,5) = 0.0                                             190C9
              EET2(MAP2,5) = EET2(MAP2,4)                                 190C9
              EET2(MAP2,4) = EET2(MAP2,3)                                 190C9
              EET2(MAP2,3) = EET2(MAP2,2)                                 190C9
              EET2(MAP2,2) = EET2(MAP2,1)                                 190C9
              EET2(MAP2,1) = 0.0                                          190C9
              EET2(MAP3,5) = EET2(MAP3,3)                                 190C9
              EET2(MAP3,4) = EET2(MAP3,2)                                 190C9
              EET2(MAP3,3) = EET2(MAP3,1)                                 190C9
              EET2(MAP3,2) = 0.0                                          190C9
              EET2(MAP3,1) = 0.0                                          190C9
          IF ( ML ) 4000, 3380, 3380                                      190C9
C-----PACK CC AS REQUIRED FOR SOLUTION PROCESS
 3380         CC(1,1) = CC(1,3)                                           190C9
              CC(1,2) = CC(1,4)                                           190C9
              CC(1,3) = CC(1,5)                                           190C9
              CC(1,4) = 0.0                                               190C9
              CC(1,5) = 0.0                                               190C9
              CC(2,1) = CC(2,2)                                           190C9
              CC(2,2) = CC(2,3)                                           190C9
              CC(2,3) = CC(2,4)                                           190C9
              CC(2,4) = CC(2,5)                                           190C9
              CC(2,5) = 0.0                                               190C9
              CC(MAP2,5) = CC(MAP2,4)                                     190C9
              CC(MAP2,4) = CC(MAP2,3)                                     190C9
              CC(MAP2,3) = CC(MAP2,2)                                     190C9
              CC(MAP2,2) = CC(MAP2,1)                                     190C9
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            CC(MAP2,1) = 0.0                                            190C9
            CC(MAP3,5) = CC(MAP3,3)                                     190C9
            CC(MAP3,4) = CC(MAP3,2)                                     190C9
            CC(MAP3,3) = CC(MAP3,1)                                     190C9
            CC(MAP3,2) = 0.0                                            190C9
            CC(MAP3,1) = 0.0                                            190C9
C-----COMPUTE TRANSPOSE OF DD AS DDT
         DO 3450  I = 1, MAP3                                           190C9
            DDT(I,1)  = DD(I,5)                                         190C9
            DDT(I,2)  = DD(I,4)                                         190C9
            DDT(I,3)  = DD(I,3)                                         190C9
            DDT(I,4)  = DD(I,2)                                         190C9
            DDT(I,5)  = DD(I,1)                                         190C9
 3450    CONTINUE                                                       190C9
         DO 3505  L = 3, MAP3                                           190C9
            K = MAP3 - L + 3                                            190C9
            DDT(K,1)  = DDT(K-2,1)                                      190C9
 3505    CONTINUE                                                       190C9
            DDT(1,1)  = 0.0                                             190C9
            DDT(2,1)  = 0.0                                             190C9
         DO 3510  L = 2, MAP3                                           190C9
            K = MAP3 - L + 2                                            190C9
            DDT(K,2)  = DDT(K-1,2)                                      190C9
 3510    CONTINUE                                                       190C9
            DDT(1,2)  = 0.0                                             190C9
         DO 3515  K = 1, MAP2                                           190C9
            DDT(K,4)  = DDT(K+1,4)                                      190C9
 3515    CONTINUE                                                       190C9
            DDT(MAP3,4) = 0.0                                           190C9
         DO 3520  K = 1, MAP1                                           190C9
            DDT(K,5)  = DDT(K+2,5)                                      190C9
 3520    CONTINUE                                                       190C9
            DDT(MAP2,5) = 0.0                                           190C9
            DDT(MAP3,5) = 0.0                                           190C9
C-----PACK DDT AS REQUIRED FOR SOLUTION PROCESS
            DDT(1,1) = DDT(1,3)                                         190C9
            DDT(1,2) = DDT(1,4)                                         190C9
            DDT(1,3) = DDT(1,5)                                         190C9
            DDT(1,4) = 0.0                                              190C9
            DDT(1,5) = 0.0                                              190C9
            DDT(2,1) = DDT(2,2)                                         190C9
            DDT(2,2) = DDT(2,3)                                         190C9
            DDT(2,3) = DDT(2,4)                                         190C9
            DDT(2,4) = DDT(2,5)                                         190C9
            DDT(2,5) = 0.0                                              190C9
            DDT(MAP2,5) = DDT(MAP2,4)                                   190C9
            DDT(MAP2,4) = DDT(MAP2,3)                                   190C9
            DDT(MAP2,3) = DDT(MAP2,2)                                   190C9
            DDT(MAP2,2) = DDT(MAP2,1)                                   190C9
            DDT(MAP2,1) = 0.0                                           190C9
            DDT(MAP3,5) = DDT(MAP3,3)                                   190C9
            DDT(MAP3,4) = DDT(MAP3,2)                                   190C9
            DDT(MAP3,3) = DDT(MAP3,1)                                   190C9
            DDT(MAP3,2) = 0.0                                           190C9
            DDT(MAP3,1) = 0.0                                           190C9
C-----COMPUTE TRANSPOSE OF EEP AT PREVIOUS J STEP AS EET1 EXCEPT AT J = 2
```

```
          IF ( J - 2 ) 9980, 3550, 3580                                 190C9
 3550     DO 3575  K = 1, 5                                             190C9
          DO 3570  I = 1, MAP3                                          190C9
               EET1(I,K) = 0.0                                          190C9
 3570     CONTINUE                                                      190C9
 3575     CONTINUE                                                      190C9
          GO TO 3600                                                    190C9
 3580     DO 3590  K = 1, 5                                             190C9
          DO 3585  I = 1, MAP3                                          190C9
               EET1(I,K) = EEP(K,I)                                     190C9
 3585     CONTINUE                                                      190C9
 3590     CONTINUE                                                      190C9
 3600     CONTINUE                                                      190C9
C-----PACK EE AT THIS STEP AS EEP AND AS REQUIRED FOR SOLUTION PROCESS
          DO 3700  I = 1, MAP3                                          190C9
               EEP(1,I)  = EE(I,5)                                      190C9
               EEP(2,I)  = EE(I,4)                                      190C9
               EEP(3,I)  = EE(I,3)                                      190C9
               EEP(4,I)  = EE(I,2)                                      190C9
               EEP(5,I)  = EE(I,1)                                      190C9
 3700     CONTINUE                                                      190C9
          DO 3705  L = 3, MAP3                                          190C9
               K = MAP3 - L + 3                                         190C9
               EEP(1,K)  = EEP(1,K-2)                                   190C9
 3705     CONTINUE                                                      190C9
               EEP(1,1)  = 0.0                                          190C9
               EEP(1,2)  = 0.0                                          190C9
          DO 3710  L = 2, MAP3                                          190C9
               K = MAP3 - L + 2                                         190C9
               EEP(2,K)  = EEP(2,K-1)                                   190C9
 3710     CONTINUE                                                      190C9
               EEP(2,1)  = 0.0                                          190C9
          DO 3715  K = 1, MAP2                                          190C9
               EEP(4,K)  = EEP(4,K+1)                                   190C9
 3715     CONTINUE                                                      190C9
               EEP(4,MAP3) = 0.0                                        190C9
          DO 3720  K = 1, MAP1                                          190C9
               EEP(5,K)  = EEP(5,K+2)                                   190C9
 3720     CONTINUE                                                      190C9
               EEP(5,MAP2) = 0.0                                        190C9
               EEP(5,MAP3) = 0.0                                        190C9
C
               EEP(1,1) = EEP(3,1)                                      190C9
               EEP(2,1) = EEP(4,1)                                      190C9
               EEP(3,1) = EEP(5,1)                                      190C9
               EEP(4,1) = 0.0                                           190C9
               EEP(5,1) = 0.0                                           190C9
               EEP(1,2) = EEP(2,2)                                      190C9
               EEP(2,2) = EEP(3,2)                                      190C9
               EEP(3,2) = EEP(4,2)                                      190C9
               EEP(4,2) = EEP(5,2)                                      190C9
               EEP(5,2) = 0.0                                           190C9
               EEP(5,MAP2) = EEP(4,MAP2)                                190C9
               EEP(4,MAP2) = EEP(3,MAP2)                                190C9
               EEP(3,MAP2) = EEP(2,MAP2)                                190C9
               EEP(2,MAP2) = EEP(1,MAP2)                                190C9
```

```
              EEP(1,MAP2) = 0.0                                         190C9
              EEP(5,MAP3) = EEP(3,MAP3)                                 190C9
              EEP(4,MAP3) = EEP(2,MAP3)                                 190C9
              EEP(3,MAP3) = EEP(1,MAP3)                                 190C9
              EEP(2,MAP3) = 0.0                                         190C9
              EEP(1,MAP3) = 0.0                                         190C9
 4000     CONTINUE                                                      190C9
C-----INDICES NK, NL, NF, N1, N2, N3, L1, L2 FOR SOLUTION PROCESS
C     ARE DEFINED PRIOR TO DO 5000 LOOP
C-----REPLACE AM2, AM1, BM1 WITH PREVIOUS COEFFICIENTS
      CALL RFV   ( AM2, AM1, L1 , 1  , NK )                             190C9
      CALL RFV   ( AM1, A  , L1 , 1  , NK )                             190C9
         IF ( ML ) 4210, 4180, 4180                                     190C9
 4180 CALL RFV   ( BM1, B  , L1 , L1 , NK )                             190C9
         GO TO 4220                                                     190C9
C-----READ D AND E MULTIPLIERS FROM TAPE 3
 4210 READ (3)  (( D(I,K) , E(I,K) , I = 1,NK) , K = 1,NK )             190C9
         GO TO 4280                                                     190C9
C-----CALCULATE RECURSION MULTIPLIER E
 4220 CALL RFV   ( E   , EP1, L1 , L1 , NK )                            190C9
C-----CALCULATE RECURSION MULTIPLIER EP1
      CALL MBFV  (EET1, BM1, EP1, L1 , L1 , NK , N1 )                   190C9
      CALL ABF   (DDT , EP1, EP1, L1 , NK , N2 )                        190C9
C-----CALCULATE RECURSION MULTIPLIER D
      CALL SMFF  ( E   , BM1, D   , L1 , NK )                           190C9
      CALL RFV   ( BM1, CM1, L1 , L1 , NK )                             190C9
      CALL RFV   ( CM1, C   , L1 , L1 , NK )                            190C9
      CALL MBFV  (EET2, BM1, C   , L1 , L1 , NK , N1 )                  190C9
      CALL ASFV  ( D   , C   , D   , L1 , L1 , NK , +1 )                190C9
      CALL ABF   ( CC , D   , D   , L1 , NK , N3 )                      190C9
      CALL INVR6 ( D   , L1 , NK )                                      190C9
      CALL CFV   ( D   , L1 , L1 , NK , -1.)                            190C9
C-----CALCULATE RECURSION COEFFIECENT C
      CALL MFB   ( D   , EEP, C   , L1 , NK , N1 )                      190C9
C-----CALCULATE RECURSION COEFFIECENT B
      CALL MFFT  ( D   , EP1, B   , L1 , NK )                           190C9
C-----CALCULATE RECURSION COEFFIECENT A
 4280 CALL MFFV  ( E   , AM1, A   , L1 , 1   , NK )                     190C9
      CALL MBFV  (EET2, AM2, ATM, L1 , 1   , NK , N1 )                  190C9
      CALL ASFV  ( A   , ATM, AM2, L1 , 1   , NK , +1 )                 190C9
      CALL ASFV  ( AM2, FF , ATM, L1 , 1   , NK , -1 )                  190C9
      CALL MFFV  ( D   , ATM, A   , L1 , 1   , NK )                     190C9
C-----SAVE A COEFFICIENT ON TAPE 1
      WRITE (1)  ( A(I), I = 1,NK )                                     190C9
         IF ( ML ) 4400, 4600, 4500                                     190C9
 4400 READ (2)                                                          190C9
         GO TO 5000                                                     190C9
C-----SAVE D AND E MULTIPLIERS ON TAPE 3
 4500 WRITE (3)  (( D(I,K),E(I,K), I=1,NK), K=1,NK)                     190C9
C-----SAVE B AND C COEFFICIENTS ON TAPE 2
 4600 WRITE (2)  (( B(I,K),C(I,K), I=1,NK), K=1,NK)                     190C9
 5000     CONTINUE                                                      190C9
C
C-----BEGIN BACKWARD PASS -- COMPUTE RECURSION EQUATION
C
C-----BACKSUBSTITUTE AND COMPUTE DEFLECTIONS
```

```
      BACKSPACE 1                                                       20MY8
      BACKSPACE 2                                                       20MY8
      CALL RFV   ( W(NF,NL), A   , L1 , 1   , NK )                      18JL9
      BACKSPACE 1                                                       20MY8
      BACKSPACE 2                                                       20MY8
      READ (1)   ( A(I), I = 1,NK )                                     20MY8
      READ (2)   (( B(I,K) , C(I,K), I = 1,NK) , K = 1,NK)              20MY8
      BACKSPACE 1                                                       20MY8
      BACKSPACE 2                                                       20MY8
      CALL MFFV  ( B   , W(NF,NL), AM1, L1 , 1   , NK )                 18JL9
      CALL ASFV  ( A   , AM1, W(NF,NL-1)   , L1 , 1   , NK , +1 )       18JL9
                NLM2 = NL - 2                                           20MY8
C     NOTE THAT NLM2 = MCP2
          DO 6000  L = NF , NLM2                                        19OC9
                J = NLM2 + NF - L                                       19OC9
      BACKSPACE 1                                                       19OC9
      BACKSPACE 2                                                       19OC9
C-----READ A COEFFICIENT FROM TAPE 1
      READ  (1)   ( A(I), I = 1,NK )                                    19OC9
C-----READ B AND C COEFFICIENTS FROM TAPE 2
      READ (2)   (( B(I,K) , C(I,K), I = 1,NK) , K = 1,NK)              19OC9
      BACKSPACE 1                                                       19OC9
      BACKSPACE 2                                                       19OC9
      CALL MFFV  ( B   ,W(NF,J+1), AM1, L1 , 1   , NK )                 15JL9
      CALL MFFV  ( C   ,W(NF,J+2), AM2, L1 , 1   , NK )                 15JL9
      CALL ASFV  ( AM1, AM2, AM1, L1 , 1   , NK , +1 )                  19OC9
      CALL ASFV  ( A   , AM1,W(NF,J)   , L1 , 1   , NK , +1 )           15JL9
 6000     CONTINUE                                                      19OC9
C
C-----COMPUTE AND PRINT RESULTS
C
      PRINT 11                                                          23OC9
      PRINT 1                                                           23OC9
      PRINT 13, ( AN1(N), N = 1, 32 )                                   23OC9
      PRINT 16, NPROB, ( AN2(N), N = 1, 14 )                            23OC9
          IF ( ML ) 6115, 6110, 6110                                    23OC9
 6110 PRINT 700                                                         23OC9
          GO TO 6120                                                    23OC9
 6115 PRINT 701, KPROB                                                  23OC9
 6120     IF ( SWS ) 9980, 6125, 6140                                   30OC9
 6125 PRINT 711                                                         30OC9
          IF ( KROPT ) 9980, 6130, 6135                                 30OC9
 6130 PRINT 721                                                         30OC9
          GO TO 6215                                                    30OC9
 6135 PRINT 722                                                         30OC9
          GO TO 6215                                                    30OC9
 6140     IF ( SWB ) 9980, 6145, 6180                                   30OC9
 6145 PRINT 712                                                         30OC9
          IF ( KROPT ) 9980, 6150, 6165                                 30OC9
 6150     IF ( IPR ) 9980, 6155, 6160                                   30OC9
 6155 PRINT 731                                                         30OC9
          GO TO 6215                                                    30OC9
 6160 PRINT 732                                                         30OC9
          GO TO 6215                                                    30OC9
 6165     IF ( IPR ) 9980, 6170, 6175                                   30OC9
 6170 PRINT 733                                                         30OC9
```

```
          GO TO 6215                                                    300C9
 6175 PRINT 734                                                         300C9
          GO TO 6215                                                    300C9
 6180 PRINT 713                                                         300C9
          IF ( KROPT ) 9980, 6185, 6200                                 300C9
 6185     IF ( IPR ) 9980, 6190, 6195                                   300C9
 6190 PRINT 741                                                         300C9
          GO TO 6215                                                    300C9
 6195 PRINT 742                                                         300C9
          GO TO 6215                                                    300C9
 6200     IF ( IPR ) 9980, 6205, 6210                                   300C9
 6205 PRINT 743                                                         300C9
          GO TO 6215                                                    300C9
 6210 PRINT 744                                                         300C9
 6215     CONTINUE                                                      300C9
               SUMR = 0.0                                               230C9
               STEMP = 0.0                                              230C9
               ITEMP = - 2                                              230C9
               JTEMP = - 2                                              230C9
          DO 7000  J = 2, MCP4                                          230C9
               JN = J - 3                                               230C9
C------RETRIEVE DATA NEEDED AT THIS J STEP
      CALL        DATA2 ( D11, D12, D13, D22, D23, D33, D12M1, D13M1,   130C9
     1         D22M1, D23M1, D33M1, D12P1, D13P1, D22P1, D23P1, D33P1,  130C9
     2         FA, FB, FC, Q, S, FBM1, FCM1, FBP1, FCP1, TA, TBM1, TCM1,130C9
     3         TBP1, TCP1, L2, JN, ML )                                 130C9
          DO 6250  I = 2, MAP4                                          230C9
C------COMPUTE TEMP CONSTANTS FOR SLAB STIFFNESS -- IF ISTIFF = 1, THE
C     D11 THROUGH D33 COEFFICIENTS ARE ACTUALLY B11 THROUGH B33(TABLE 3)
          IF ( ISTIFF ) 9980, 6216, 6217                                03MY0
 6216          B11VV = ( S1S * S2S * D11(I)                             03MY0
     1                 + 2.0 * C1 * S1 * C2 * S2 * D12(I)               160C9
     2                 + 2.0 * S1 * S2 * ( C1 * S2 + S1 * C2 ) * D13(I) 160C9
     3                 + C1S * C2S * D22(I)                             160C9
     4                 + 2.0 * C1 * C2 * ( C1 * S2 + S1 * C2 ) * D23(I) 160C9
     5                 + ( C1 * S2 + S1 * C2 ) ** 2 * D33(I) )          160C9
     6                 / ( S1S * S2S )                                  160C9
               B12MV = ( - S1 * C2 * S2 * D12M1(I) - S1 * S2S * D13M1(I)160C9
     1                 - C1 * C2S * D22M1(I)                            160C9
     2                 - C2 * ( 2.0 * C1 * S2 + S1 * C2 ) * D23M1(I)    160C9
     3                 - S2 * ( C1 * S2 + S1 * C2 ) * D33M1(I) )        160C9
     4                 / ( S1S * S2 * S3 )                              160C9
               B12VV = ( - S1 * C2 * S2 * D12(I) - S1 * S2S * D13(I)    160C9
     1                 - C1 * C2S * D22(I)                              160C9
     2                 - C2 * ( 2.0 * C1 * S2 + S1 * C2 ) * D23(I)      160C9
     3                 - S2 * ( C1 * S2 + S1 * C2 ) * D33(I) )          160C9
     4                 / ( S1S * S2 * S3 )                              160C9
               B12PV = ( - S1 * C2 * S2 * D12P1(I) - S1 * S2S * D13P1(I)160C9
     1                 - C1 * C2S * D22P1(I)                            160C9
     2                 - C2 * ( 2.0 * C1 * S2 + S1 * C2 ) * D23P1(I)    160C9
     3                 - S2 * ( C1 * S2 + S1 * C2 ) * D33P1(I) )        160C9
     4                 / ( S1S * S2 * S3 )                              160C9
               B13MV = ( C1 * S1 * S2 * D12M1(I) + S1S * S2 * D13M1(I)  160C9
     1                 + C1S * C2 * D22M1(I)                            160C9
     2                 + C1 * ( C1 * S2 + 2.0 * S1 * C2 ) * D23M1(I)    160C9
     3                 + S1 * ( C1 * S2 + S1 * C2 ) * D33M1(I) )        160C9
```

```
     4             / ( S1 * S2S * S3 )                                 160C9
            B13VV = ( C1 * S1 * S2 * D12(I) + S1S * S2 * D13(I)        160C9
     1             + C1S * C2 * D22(I)                                 160C9
     2             + C1 * ( C1 * S2 + 2.0 * S1 * C2 ) * D23(I)         160C9
     3             + S1 * ( C1 * S2 + S1 * C2 ) * D33(I) )             160C9
     4             / ( S1 * S2S * S3 )                                 160C9
            B13PV = ( C1 * S1 * S2 * D12P1(I) + S1S * S2 * D13P1(I)    160C9
     1             + C1S * C2 * D22P1(I)                               160C9
     2             + C1 * ( C1 * S2 + 2.0 * S1 * C2 ) * D23P1(I)       160C9
     3             + S1 * ( C1 * S2 + S1 * C2 ) * D33P1(I) )           160C9
     4             / ( S1 * S2S * S3 )                                 160C9
            B22MV = ( C2S  * D22M1(I) + 2.0 * C2 * S2 * D23M1(I)       160C9
     1             + S2S * D33M1(I) ) / ( S1S * S3S )                  160C9
            B22VV = ( C2S * D22(I) + 2.0 * C2 * S2 * D23(I)            160C9
     1             + S2S * D33(I) ) / ( S1S * S3S )                    160C9
            B22PV = ( C2S  * D22P1(I) + 2.0 * C2 * S2 * D23P1(I)       160C9
     1             + S2S * D33P1(I) ) / ( S1S * S3S )                  160C9
            B23MV = ( - C1 * C2 * D22M1(I)                             160C9
     1             - ( C1 * S2 + S1 * C2 ) * D23M1(I)                  160C9
     2             - S1 * S2 * D33M1(I) ) / ( S1 * S2 * S3S )          160C9
            B23VV = ( - C1 * C2 * D22(I)                               160C9
     1             - ( C1 * S2 + S1 * C2 ) * D23(I)                    160C9
     2             - S1 * S2 * D33(I) ) / ( S1 * S2 * S3S )            160C9
            B23PV = ( - C1 * C2 * D22P1(I)                             160C9
     1             - ( C1 * S2 + S1 * C2 ) * D23P1(I)                  160C9
     2             - S1 * S2 * D33P1(I) ) / ( S1 * S2 * S3S )          160C9
            B33MV = ( C1S * D22M1(I) + 2.0 * C1 * S1 * D23M1(I)        160C9
     1             + S1S * D33M1(I) ) / ( S2S * S3S )                  160C9
            B33VV = ( C1S * D22(I) + 2.0 * C1 * S1 * D23(I)            160C9
     1             + S1S * D33(I) ) / ( S2S * S3S )                    160C9
            B33PV = ( C1S * D22P1(I) + 2.0 * C1 * S1 * D23P1(I)        160C9
     1             + S1S * D33P1(I) ) / ( S2S * S3S )                  160C9
        GO TO 6218                                                     03MY0
 6217       B11VV = D11(I)                                             03MY0
            B12MV = D12M1(I)                                           03MY0
            B12VV = D12(I)                                             03MY0
            B12PV = D12P1(I)                                           03MY0
            B13MV = D13M1(I)                                           03MY0
            B13VV = D13(I)                                             03MY0
            B13PV = D13P1(I)                                           03MY0
            B22MV = D22M1(I)                                           03MY0
            B22VV = D22(I)                                             03MY0
            B22PV = D22P1(I)                                           03MY0
            B23MV = D23M1(I)                                           03MY0
            B23VV = D23(I)                                             03MY0
            B23PV = D23P1(I)                                           03MY0
            B33MV = D33M1(I)                                           03MY0
            B33VV = D33(I)                                             03MY0
            B33PV = D33P1(I)                                           03MY0
 6218   CONTINUE                                                       03MY0
C-----COMPUTE CONCENTRATED BENDING MOMENTS IN MODEL
            BMA(I)   = B11VV * CS11 * HA                               230C9
     1               * ( W(I-1,J) - 2.0 * W(I,J) + W(I+1,J) )          230C9
     2               + B12VV * CS12 * HA                               230C9
     3               * ( W(I-1,J-1) - 2.0 * W(I,J) + W(I+1,J+1) )      230C9
     4               + B13VV * CS13 * HA                               230C9
```

```
     5                    * ( W(I,J-1) - 2.0 * W(I,J) + W(I,J+1) )            230C9
           BMB(I)       = B12VV * CS12 * HB                                   230C9
     1                    * ( W(I-1,J) - 2.0 * W(I,J) + W(I+1,J) )            230C9
     2                    + B22VV * CS22 * HB                                 230C9
     3                    * ( W(I-1,J-1) - 2.0 * W(I,J) + W(I+1,J+1) )        230C9
     4                    + B23VV * CS23 * HB                                 230C9
     5                    * ( W(I,J-1) - 2.0 * W(I,J) + W(I,J+1) )            230C9
           BMC(I)       = B13VV * CS13 * HC                                   230C9
     1                    * ( W(I-1,J) - 2.0 * W(I,J) + W(I+1,J) )            230C9
     2                    + B23VV * CS23 * HC                                 230C9
     3                    * ( W(I-1,J-1) - 2.0 * W(I,J) + W(I+1,J+1) )        230C9
     4                    + B33VV * CS33 * HC                                 230C9
     5                    * ( W(I,J-1) - 2.0 * W(I,J) + W(I,J+1) )            230C9
        IF ( J - 2 ) 9980, 6220, 6222                                         02FE0
 6220      BMBM1(I) = 0.0                                                     02FE0
           BMCM1(I) = 0.0                                                     02FE0
        GO TO 6224                                                            02FE0
 6222      BMBM1(I) = B12MV * CS12 * HB                                       02FE0
     1                    * ( W(I-1,J-1) - 2.0 * W(I,J-1) + W(I+1,J-1) )230C9
     2                    + B22MV * CS22 * HB                                 230C9
     3                    * ( W(I-1,J-2) - 2.0 * W(I,J-1) + W(I+1,J) )        230C9
     4                    + B23MV * CS23 * HB                                 230C9
     5                    * ( W(I,J-2) - 2.0 * W(I,J-1) + W(I,J) )            230C9
           BMCM1(I) = B13MV * CS13 * HC                                       230C9
     1                    * ( W(I-1,J-1) - 2.0 * W(I,J-1) + W(I+1,J-1) )230C9
     2                    + B23MV * CS23 * HC                                 230C9
     3                    * ( W(I-1,J-2) - 2.0 * W(I,J-1) + W(I+1,J) )        230C9
     4                    + B33MV * CS33 * HC                                 230C9
     5                    * ( W(I,J-2) - 2.0 * W(I,J-1) + W(I,J) )            230C9
 6224   IF ( MCP4 - J ) 9980, 6226, 6228                                      02FE0
 6226      BMBP1(I) = 0.0                                                     02FE0
           BMCP1(I) = 0.0                                                     02FE0
        GO TO 6230                                                            02FE0
 6228      BMBP1(I) = B12PV * CS12 * HB                                       02FE0
     1                    * ( W(I-1,J+1) - 2.0 * W(I,J+1) + W(I+1,J+1) )230C9
     2                    + B22PV * CS22 * HB                                 230C9
     3                    * ( W(I-1,J) - 2.0 * W(I,J+1) + W(I+1,J+2) )        230C9
     4                    + B23PV * CS23 * HB                                 230C9
     5                    * ( W(I,J) - 2.0 * W(I,J+1) + W(I,J+2) )            230C9
           BMCP1(I) = B13PV * CS13 * HC                                       230C9
     1                    * ( W(I-1,J+1) - 2.0 * W(I,J+1) + W(I+1,J+1) )230C9
     2                    + B23PV * CS23 * HC                                 230C9
     3                    * ( W(I-1,J) - 2.0 * W(I,J+1) + W(I+1,J+2) )        230C9
     4                    + B33PV * CS33 * HC                                 230C9
     5                    * ( W(I,J) - 2.0 * W(I,J+1) + W(I,J+2) )            230C9
 6230      BMBA(I)      = FA(I) / ( HA * HA )                                 02FE0
     1                    * ( W(I-1,J) - 2.0 * W(I,J) + W(I+1,J) )            230C9
           BMBB(I)      = FB(I) / ( HB * HB )                                 230C9
     1                    * ( W(I-1,J-1) - 2.0 * W(I,J) + W(I+1,J+1) )        230C9
           BMBC(I)      = FC(I) / ( HC * HC )                                 230C9
     1                    * ( W(I,J-1) - 2.0 * W(I,J) + W(I,J+1) )            230C9
        IF ( J - 2 ) 9980, 6232, 6234                                         02FE0
 6232      BMBBM1(I)= 0.0                                                     02FE0
           BMBCM1(I)= 0.0                                                     02FE0
        GO TO 6236                                                            02FE0
 6234      BMBBM1(I)= FBM1(I) / ( HB * HB )                                   02FE0
```

```
     1                  * ( W(I-1,J-2) - 2.0 * W(I,J-1) + W(I+1,J) )    230C9
               BMBCM1(I)= FCM1(I) / ( HC * HC )                          230C9
     1                  * ( W(I,J-2) - 2.0 * W(I,J-1) + W(I,J) )        230C9
 6236     IF ( MCP4 - J ) 9980, 6238, 6240                               02FE0
 6238          BMBBP1(I)= 0.0                                            02FE0
               BMBCP1(I)= 0.0                                            02FE0
          GO TO 6250                                                     02FE0
 6240          BMBBP1(I)= FBP1(I) / ( HB * HB )                          02FE0
     1                  * ( W(I-1,J) - 2.0 * W(I,J+1) + W(I+1,J+2) )    230C9
               BMBCP1(I)= FCP1(I) / ( HC * HC )                          230C9
     1                  * ( W(I,J) - 2.0 * W(I,J+1) + W(I,J+2) )        230C9
 6250     CONTINUE                                                       230C9
               JSTA = J - 3                                              230C9
      PRINT 6                                                            230C9
          DO 6400  I = 2, MAP4                                           230C9
               ISTA = I - 3                                              230C9
C-----COMPUTE REACTIONS, STATICS CHECK, SUMMATION OF REACTIONS AND
C     MAXIMUM STATICS CHECK ERROR
               REACT = - S(I) * W(I,J)                                   230C9
               SUMR  = SUMR + REACT                                      230C9
               STACH = ( BMA(I-1) - 2.0 * BMA(I) + BMA(I+1) ) / HA      230C9
     1               + ( BMBM1(I-1) - 2.0 * BMB(I) + BMBP1(I+1) ) / HB230C9
     2               + ( BMCM1(I) - 2.0 * BMC(I) + BMCP1(I) ) / HC      230C9
     3               + ( BMBA(I-1) - 2.0 * BMBA(I) + BMBA(I+1) ) / HA 230C9
     4               + ( BMBBM1(I-1) - 2.0 * BMBB(I) + BMBBP1(I+1) ) /230C9
     5               HB + ( BMBCM1(I) - 2.0 * BMBC(I) + BMBCP1(I) ) / 230C9
     6               HC - 0.5 * ( - TA(I-1) + TA(I+1) ) / HA            230C9
     7               - 0.5 * ( - TBM1(I-1) + TBP1(I+1) ) / HB           230C9
     8               - 0.5 * ( - TCM1(I) + TCP1(I) ) / HC - Q(I)        230C9
     9               + S(I) * W(I,J)                                    230C9
          IF ( ABS (STACH) - ABS (STEMP) ) 6254, 6254, 6252              230C9
 6252          STEMP = STACH                                             230C9
               ITEMP = ISTA                                              230C9
               JTEMP = JSTA                                              230C9
 6254     CONTINUE                                                       230C9
          IF ( SWS ) 9980, 6260, 6280                                    300C9
 6260     IF ( KROPT ) 9980, 6265, 6270                                  300C9
C-----PRINT OUTPUT IF ONLY BEAMS EXIST
 6265 PRINT 751, ISTA, JSTA, W(I,J), BMBA(I), BMBB(I), BMBC(I), REACT    230C9
          GO TO 6400                                                     300C9
 6270 PRINT 751, ISTA, JSTA, W(I,J), BMBA(I), BMBB(I), BMBC(I), STACH    230C9
          GO TO 6400                                                     300C9
C-----COMPUTE CONVENTIONAL BENDING MOMENTS PER UNIT WIDTH
 6280          CBMA  = BMA(I) / ( HC * S2 ) + C1S * BMB(I) / ( HA * S1 )300C9
     1               + C2S * BMC(I) / ( HA * S2 )                       230C9
               CBMB  = C1S * BMA(I) / ( HC * S2 ) + BMB(I) / ( HA * S1 )230C9
     1               + C3S * BMC(I) / ( HA * S2 )                       230C9
               CBMC  = C2S * BMA(I) / ( HC * S2 ) + C3S * BMB(I) /      230C9
     1               ( HA * S1 ) + BMC(I) / ( HA * S2 )                 230C9
               CBMX  = CBMA                                              230C9
               CBMY  = ( C1 * C2 * S3 * CBMA - C2 * S2 * CBMB           230C9
     1               + C1 * S1 * CBMC ) / ( S1 * S2 * S3 )              230C9
               CBMXY = ( S3 * ( C1 * S2 + S1 * C2 ) * CBMA - S2S * CBMB 230C9
     1               + S1S * CBMC ) / ( 2.0 * S1 * S2 * S3 )            230C9
C-----COMPUTE PRINCIPAL MOMENTS
               CBMO  = 0.5 * ( CBMX + CBMY )                             230C9
```

```
     1                 + SQRT ( 0.25 * ( CBMX - CBMY ) ** 2             230C9
     2                 + CBMXY * CBMXY )                                230C9
            CBMT   = 0.5 * ( CBMX + CBMY )                              230C9
     1                 - SQRT ( 0.25 * ( CBMX - CBMY ) ** 2             230C9
     2                 + CBMXY * CBMXY )                                230C9
C-----TEST TO PRINT ONLY MAXIMUM VALUE
         IF ( CBMX + CBMY ) 6316, 6318, 6318                            230C9
 6316          PMMAX = CBMT                                             230C9
         IF ( CBMX - CBMY ) 6340, 6330, 6320                            230C9
 6318          PMMAX = CBMO                                             230C9
         IF ( CBMX - CBMY ) 6320, 6330, 6340                            230C9
 6320          ALF = ATAN ( CBMXY / ( 0.5 * ( CBMX - CBMY ) ) )         230C9
     1                 * 57.29578                                       230C9
         IF ( ALF ) 6322, 6324, 6324                                    230C9
 6322          BETAT = - ALF - 180.0                                    230C9
         GO TO 6345                                                     230C9
 6324          BETAT = - ALF + 180.0                                    230C9
         GO TO 6345                                                     230C9
 6330    IF ( CBMXY ) 6332, 6334, 6336                                  230C9
 6332          BETAT = 90.0                                             230C9
         GO TO 6345                                                     230C9
 6334          BETAT = 0.0                                              230C9
         GO TO 6345                                                     230C9
 6336          BETAT = 90.0                                             230C9
         GO TO 6345                                                     230C9
 6340          ALF = ATAN ( CBMXY / ( 0.5 * ( CBMX - CBMY ) ) )         230C9
     1                 * 57.29578                                       230C9
               BETAT = - ALF                                            230C9
C-----CLOCKWISE ANGLES ARE NEGATIVE
 6345          BETA = 0.5 * BETAT                                       230C9
         IF ( KROPT ) 9980, 6350, 6365                                  300C9
 6350    IF ( IPR ) 9980, 6355, 6360                                    300C9
C-----PRINT SLAB OR COMBINED SLAB-BEAM OUTPUT
 6355 PRINT 752, ISTA, JSTA, W(I,J), CBMX, CBMY, CBMXY, PMMAX,          230C9
     1      BETA, REACT                                                 230C9
         GO TO 6380                                                     300C9
 6360 PRINT 752, ISTA, JSTA, W(I,J), CBMA, CBMB, CBMC , PMMAX,          230C9
     1      BETA, REACT                                                 230C9
         GO TO 6380                                                     300C9
 6365    IF ( IPR ) 9980, 6370, 6375                                    300C9
 6370 PRINT 752, ISTA, JSTA, W(I,J), CBMX, CBMY, CBMXY, PMMAX,          230C9
     1      BETA, STACH                                                 230C9
         GO TO 6380                                                     300C9
 6375 PRINT 752, ISTA, JSTA, W(I,J), CBMA, CBMB, CBMC , PMMAX,          300C9
     1      BETA, STACH                                                 230C9
 6380    IF ( SWB ) 9980, 6400, 6385                                    300C9
 6385 PRINT 753, BMBA(I), BMBB(I), BMBC(I)                              230C9
 6400    CONTINUE                                                       230C9
 7000    CONTINUE                                                       230C9
C-----PRINT SUMMATION OF REACTIONS AND MAX STATICS CHECK ERROR
      PRINT 994, SUMR                                                   300C9
      PRINT 995, ITEMP, JTEMP, STEMP                                    300C9
      CALL TIC TOC (4)                                                  120C9
         GO TO 1010                                                     120C9
 9980 PRINT 980                                                         120C9
 9990    CONTINUE                                                       120C9
```

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9999     CONTINUE                                                  120C9
      PRINT 11                                                     120C9
      PRINT 1                                                      120C9
      PRINT 13, ( AN1(N), N = 1, 32 )                              120C9
      CALL TIC TOC (2)                                             120C9
      END                                                          120C9
```

```
      SUBROUTINE DATA2 ( D11, D12, D13, D22, D23, D33, D12M1, D13M1,     130C9
     1          D22M1, D23M1, D33M1, D12P1, D13P1, D22P1, D23P1, D33P1,  130C9
     2          FA, FB, FC, Q, S, FBM1, FCM1, FBP1, FCP1, TA, TBM1, TCM1,130C9
     3          TBP1, TCP1, L2, JN, ML )                                 130C9
C
C-----THIS SUBROUTINE IS CALLED AT EACH J STEP IN THE STIFFNESS MATRIX   130C9
C         GENERATION AND AGAIN AT EACH J STEP WHEN COMPUTING RESULTS.    130C9
C
      DIMENSION   D11(L2),   D12(L2),   D13(L2),   D22(L2),   D23(L2),   130C9
     1              D33(L2), D12M1(L2), D13M1(L2), D22M1(L2), D23M1(L2), 130C9
     2            D33M1(L2), D12P1(L2), D13P1(L2), D22P1(L2), D23P1(L2), 130C9
     3            D33P1(L2),     FA(L2),    FB(L2),     FC(L2),     Q(L2),   130C9
     4                S(L2),  FBM1(L2),  FCM1(L2),  FBP1(L2),  FCP1(L2), 130C9
     5               TA(L2),  TBM1(L2),  TCM1(L2),  TBP1(L2),  TCP1(L2)  130C9
      DIMENSION IN13( 70), JN13( 70), IN23( 70), JN23( 70), D11N( 70),   120C9
     1          D12N( 70), D13N( 70), D22N( 70), D23N( 70), D33N( 70),   120C9
     2          IN14( 70), JN14( 70), IN24( 70), JN24( 70),  FAN( 70),   120C9
     3           FBN( 70),  FCN( 70),   QN( 70),   SN( 70),              120C9
     4          IN15( 70), JN15( 70), IN25( 70), JN25( 70),  TAN( 70),   120C9
     5           TBN( 70),  TCN( 70)                                     120C9
      COMMON / DATA2 / IN13, JN13, IN23, JN23, IN14, JN14, IN24, JN24,   140C9
     1                 IN15, JN15, IN25, JN25,                           140C9
     2                 D11N, D12N, D13N, D22N, D23N, D33N,               140C9
     3                  FAN,  FBN,  FCN,   QN,   SN,  TAN,  TBN,  TCN,   140C9
     4                 NCT3, NCT4, NCT5, MAP5                            140C9
   98 FORMAT ( //30H     UNDESIGNATED ERROR STOP    )                    130C9
C
C-----DISTRIBUTE DATA FROM TABLE 3
C
          DO 305  I = 1, MAP5                                            130C9
               D11(I)   = 0.0                                            130C9
               D12(I)   = 0.0                                            130C9
               D13(I)   = 0.0                                            130C9
               D22(I)   = 0.0                                            130C9
               D23(I)   = 0.0                                            130C9
               D33(I)   = 0.0                                            130C9
               D12M1(I) = 0.0                                            130C9
               D13M1(I) = 0.0                                            130C9
               D22M1(I) = 0.0                                            130C9
               D23M1(I) = 0.0                                            130C9
               D33M1(I) = 0.0                                            130C9
               D12P1(I) = 0.0                                            130C9
               D13P1(I) = 0.0                                            130C9
               D22P1(I) = 0.0                                            130C9
               D23P1(I) = 0.0                                            130C9
               D33P1(I) = 0.0                                            130C9
  305     CONTINUE                                                       130C9
          IF ( NCT3 ) 980, 400, 310                                      130C9
  310     DO 360  N = 1, NCT3                                            130C9
               I1 = IN13(N) + 3                                          130C9
               I2 = IN23(N) + 3                                          130C9
          IF ( JN - JN13(N) ) 345, 315, 315                              130C9
  315     IF ( JN23(N) - JN ) 330, 320, 320                              130C9
  320     DO 325  I = I1, I2                                             130C9
               D11(I) = D11(I) + D11N(N)                                 130C9
               D12(I) = D12(I) + D12N(N)                                 130C9
```

```
            D13(I) = D13(I) + D13N(N)                                      130C9
            D22(I) = D22(I) + D22N(N)                                      130C9
            D23(I) = D23(I) + D23N(N)                                      130C9
            D33(I) = D33(I) + D33N(N)                                      130C9
  325     CONTINUE                                                         130C9
  330     IF ( (JN-1) - JN13(N) ) 345, 333, 333                            130C9
  333     IF ( JN23(N) - (JN-1) ) 345, 335, 335                            130C9
  335     DO 340  I = I1, I2                                               130C9
            D12M1(I) = D12M1(I) + D12N(N)                                  130C9
            D13M1(I) = D13M1(I) + D13N(N)                                  130C9
            D22M1(I) = D22M1(I) + D22N(N)                                  130C9
            D23M1(I) = D23M1(I) + D23N(N)                                  130C9
            D33M1(I) = D33M1(I) + D33N(N)                                  130C9
  340     CONTINUE                                                         130C9
  345     IF ( (JN+1) - JN13(N) ) 360, 347, 347                            130C9
  347     IF ( JN23(N) - (JN+1) ) 360, 350, 350                            130C9
  350     DO 355  I = I1, I2                                               130C9
            D12P1(I) = D12P1(I) + D12N(N)                                  130C9
            D13P1(I) = D13P1(I) + D13N(N)                                  130C9
            D22P1(I) = D22P1(I) + D22N(N)                                  130C9
            D23P1(I) = D23P1(I) + D23N(N)                                  130C9
            D33P1(I) = D33P1(I) + D33N(N)                                  130C9
  355     CONTINUE                                                         130C9
  360     CONTINUE                                                         130C9
C
C-----DISTRIBUTE DATA FROM TABLE 4
C
  400     DO 405  I = 1, MAP5                                              130C9
            FA(I)    = 0.0                                                 130C9
            FB(I)    = 0.0                                                 130C9
            FC(I)    = 0.0                                                 130C9
            Q(I)     = 0.0                                                 130C9
            S(I)     = 0.0                                                 130C9
            FBM1(I)  = 0.0                                                 130C9
            FCM1(I)  = 0.0                                                 130C9
            FBP1(I)  = 0.0                                                 130C9
            FCP1(I)  = 0.0                                                 130C9
  405     CONTINUE                                                         130C9
          IF ( NCT4 ) 980, 500, 410                                        130C9
  410     DO 460  N = 1, NCT4                                              130C9
            I1 = IN14(N) + 3                                               130C9
            I2 = IN24(N) + 3                                               130C9
          IF ( JN - JN14(N) ) 445, 415, 415                                130C9
  415     IF ( JN24(N) - JN ) 430, 420, 420                                130C9
  420     DO 425  I = I1, I2                                               130C9
            FA(I) = FA(I) + FAN(N)                                         130C9
            FB(I) = FB(I) + FBN(N)                                         130C9
            FC(I) = FC(I) + FCN(N)                                         130C9
            Q(I)  =  Q(I) +  QN(N)                                         130C9
            S(I)  =  S(I) +  SN(N)                                         130C9
  425     CONTINUE                                                         130C9
  430     IF ( (JN-1) - JN14(N) ) 445, 433, 433                            130C9
  433     IF ( JN24(N) - (JN-1) ) 445, 435, 435                            130C9
  435     DO 440  I = I1, I2                                               130C9
            FBM1(I) = FBM1(I) + FBN(N)                                     130C9
            FCM1(I) = FCM1(I) + FCN(N)                                     130C9
```

```
  440      CONTINUE                                                    130C9
  445      IF ( (JN+1) - JN14(N) ) 460, 447, 447                       130C9
  447      IF ( JN24(N) - (JN+1) ) 460, 450, 450                       130C9
  450      DO 455  I = I1, I2                                          130C9
                FBP1(I) = FBP1(I) + FBN(N)                             130C9
                FCP1(I) = FCP1(I) + FCN(N)                             130C9
  455      CONTINUE                                                    130C9
  460      CONTINUE                                                    130C9
C
C-----DISTRIBUTE DATA FROM TABLE 5
C
  500      DO 505  I = 1, MAP5                                         130C9
                TA(I)   = 0.0                                          130C9
                TBM1(I) = 0.0                                          130C9
                TCM1(I) = 0.0                                          130C9
                TBP1(I) = 0.0                                          130C9
                TCP1(I) = 0.0                                          130C9
  505      CONTINUE                                                    130C9
           IF ( NCT5 ) 980, 600, 510                                   130C9
  510      DO 560  N = 1, NCT5                                         130C9
                I1 = IN15(N) + 3                                       130C9
                I2 = IN25(N) + 3                                       130C9
           IF ( JN - JN15(N) ) 545, 515, 515                           130C9
  515      IF ( JN25(N) - JN ) 530, 520, 520                           130C9
  520      DO 525  I = I1, I2                                          130C9
                TA(I) = TA(I) + TAN(N)                                 130C9
  525      CONTINUE                                                    130C9
  530      IF ( (JN-1) - JN15(N) ) 545, 533, 533                       130C9
  533      IF ( JN25(N) - (JN-1) ) 545, 535, 535                       130C9
  535      DO 540  I = I1, I2                                          130C9
                TBM1(I) = TBM1(I) + TBN(N)                             130C9
                TCM1(I) = TCM1(I) + TCN(N)                             130C9
  540      CONTINUE                                                    130C9
  545      IF ( (JN+1) - JN15(N) ) 560, 547, 547                       130C9
  547      IF ( JN25(N) - (JN+1) ) 560, 550, 550                       130C9
  550      DO 555  I = I1, I2                                          130C9
                TBP1(I) = TBP1(I) + TBN(N)                             130C9
                TCP1(I) = TCP1(I) + TCN(N)                             130C9
  555      CONTINUE                                                    130C9
  560      CONTINUE                                                    130C9
  600      CONTINUE                                                    130C9
       RETURN                                                          130C9
  980  PRINT 98                                                        130C9
       END                                                             130C9
```

```
      SUBROUTINE INVR6 ( X , L1 , L2 )                                  19FE8
C******* THIS ROUTINE TAKES THE INVERSE OF A SYMMETRIC POSITIVE - DEF   05MR8
C          MATRIX USING A COMPACTED CHOLESKI DECOMPOSITION PROCEDURE ,  05MR8
C          A FULL DIMENSIONED MATRIX IS REQUIRED BUT ONLY THE LOWER     05MR8
C          HALF IS USED BY THE 3 ROUTINES DRIVEN BY INVR6               05MR8
      DIMENSION X(L1,L1)                                                19FE8
      CALL DCOM1  ( X , L1 , L2 )                                       05MR8
      CALL INVLT1 ( X , L1 , L2 )                                       19FE8
      CALL MLTXL  ( X , L1 , L2 )                                       05MR8
          DO 100  I = 2 , L2                                            19FE8
               KC = I - 1                                               19FE8
          DO 50  J = 1 , KC                                             19FE8
               X(J,I) = X(I,J)                                          19FE8
   50     CONTINUE                                                      19FE8
  100     CONTINUE                                                      19FE8
      RETURN                                                            19FE8
      END                                                               19FE8
```

```
      SUBROUTINE DCOM1 ( X , L1 , L2 )                                    19FE8
C******* THIS SUBROUTINE PERFORMS DECOMPOSITION OF A GENERAL SYMMETRIC    03MY0
C         MATRIX TO A LOWER TRIANGULAR MATRIX BY CHOLESKI DECOMPOSITION   03MY0
      DIMENSION  X(L1,L1) , T(100)                                        12MR8
   10 FORMAT ( /85X,* NON-POSITIVE DEFINITE MATRIX ENCOUNTERED * )        12MR8
          DO 20  I = 1 , L2                                               12MR8
               T(I) = X(I,I)                                              12MR8
   20     CONTINUE                                                        12MR8
          IF ( X(1,1) .LE. 0.0 )  GO TO 4000                              05MR8
               X(1,1) = SQRT ( X(1,1) )                                   19FE8
               S1 = 1.0 / X(1,1)                                          19FE8
          DO 50  I = 2 , L2                                               19FE8
               X(I,1) = X(I,1) * S1                                       19FE8
   50     CONTINUE                                                        19FE8
               L2M1 = L2 - 1                                              19FE8
          DO 200  J = 2 , L2M1                                            19FE8
               S = 0.0                                                    19FE8
               JM1 = J - 1                                                19FE8
          DO 120  K = 1 , JM1                                             19FE8
               S = S + X(J,K) * X(J,K)                                    19FE8
  120     CONTINUE                                                        19FE8
          IF ( X(J,J) .LE. S )  GO TO 4000                                05MR8
               X(J,J) = SQRT ( X(J,J) - S )                               19FE8
              S1 = 1.0 / X(J,J)                                           19FE8
              JP1 = J + 1                                                 19FE8
          DO 190  I = JP1 , L2                                            19FE8
               S = 0.0                                                    19FE8
          DO 180  K = 1 , JM1                                             19FE8
               S = S + X(I,K) * X(J,K)                                    19FE8
  180     CONTINUE                                                        19FE8
               X(I,J) = ( X(I,J) - S ) * S1                               19FE8
  190     CONTINUE                                                        19FE8
  200     CONTINUE                                                        19FE8
               S = 0.0                                                    19FE8
          DO 250  K = 1, L2M1                                             19FE8
               S = S + X(L2,K) * X(L2,K)                                  19FE8
  250     CONTINUE                                                        19FE8
               S = X(L2,L2) - S                                           05MR8
          IF ( S .LE. 0.0 )  GO TO 4000                                   05MR8
               X(L2,L2) = SQRT( S )                                       05MR8
      RETURN                                                              19FE8
 4000 PRINT 10                                                            12MR8
               X(1,1) = T(1)                                              12MR8
          DO 400  I = 2 , L2                                              09AP8
               K = I - 1                                                  12MR8
               X(I,I) = T(I)                                              12MR8
          DO 350  J = 1 , K                                               12MR8
               X(I,J) = X(J,I)                                            12MR8
  350     CONTINUE                                                        12MR8
  400     CONTINUE                                                        12MR8
          DO 500  I = 1, L2                                               20MR9
  500 PRINT 15, ( X(I,J), J=1,L2 )                                        20MR9
   15 FORMAT ( /,5X,13E10.3 )                                             20MR9
      END                                                                 19FE8
```

```
      SUBROUTINE INVLT1 ( X , L1 , L2 )                                 19FE8
C******* THIS SUBROUTINE REPLACES A LOWER TRIANGULAR MATRIX BY          03MY0
C        ITS INVERSE                                                    03MY0
      DIMENSION X(L1,L1)                                                19FE8
          DO 50  I = 1 , L2                                             19FE8
               X(I,I) = 1.0 / X(I,I)                                    19FE8
   50     CONTINUE                                                      19FE8
               L2M1 = L2 - 1                                            19FE8
          DO 200  J = 1 , L2M1                                          19FE8
               JP1 = J+1                                                19FE8
          DO 150  I = JP1 , L2                                          19FE8
               IM1 = I-1                                                19FE8
               SUM = 0.0                                                19FE8
          DO 120  K = J , IM1                                           19FE8
               SUM = SUM - X(I,K) * X(K,J)                              19FE8
  120     CONTINUE                                                      19FE8
               X(I,J) = X(I,I) * SUM                                    19FE8
  150     CONTINUE                                                      19FE8
  200     CONTINUE                                                      19FE8
      RETURN                                                            19FE8
      END                                                               19FE8
```

```
      SUBROUTINE MLTXL ( X , L1 , L2 )                                  19FE8
C******* THIS SUBROUTINE MULTIPLIES A LOWER TRIANGULAR MATRIX BY ITS     03MY0
C        IMPLIED TRANSPOSE GENERATING ONLY THE LOWER HALF OF THE        03MY0
C        SYMMETRIC RESULTS IN THE STORAGE ALLOTED TO THE ORIGINAL       03MY0
C        LOWER TRIANGULAR MATRIX                                        03MY0
      DIMENSION X(L1,L1)                                                19FE8
          DO 200  I = 1 , L2                                            19FE8
          DO 150  J = 1 , I                                             19FE8
               SUM = 0.0                                                19FE8
          DO 100  K = I , L2                                            19FE8
               SUM = SUM + X(K,I) * X(K,J)                              19FE8
  100     CONTINUE                                                      19FE8
               X(I,J) = SUM                                             19FE8
  150     CONTINUE                                                      19FE8
  200     CONTINUE                                                      19FE8
      RETURN                                                            19FE8
      END                                                               19FE8
```

```
      SUBROUTINE MFFV ( X , Y , Z , L1 , L5 , L2 )                      03MR8
C******* THIS ROUTINE MULTIPLIES A FULL MATRIX                           13DE7
C         TIMES A FULL MATRIX OR A VECTOR                               13DE7
C         ( X * Y = Z )                                                 13DE7
      DIMENSION X(L1,L1) , Y(L1,L5) , Z(L1,L5)                          13DE7
              M = 1                                                     20MY8
          IF( L1 .EQ. L5 ) M = L2                                       20MY8
          DO 110  J = 1,M                                               13DE7
          DO 105  I = 1,L2                                              13DE7
              SUM = 0.0                                                 03MR8
          DO 100  K = 1,L2                                              13DE7
              SUM = SUM + X(I,K) * Y(K,J)                               03MR8
  100     CONTINUE                                                      13DE7
              Z(I,J) = SUM                                              03MR8
  105     CONTINUE                                                      13DE7
  110     CONTINUE                                                      13DE7
      RETURN                                                            13DE7
      END                                                               13DE7
```

```
      SUBROUTINE SMFF  ( X , Y , Z , L1, L2 )                           19FE8
C******* THIS ROUTINE MULTIPLIES TWO FULL MATRICES UNDER THE ASSUMPTION 05MR8
C        THAT THEIR PRODUCT WILL BE SYMMETRIC ( X,Y, AND Z ARE FULL     05MR8
C        DIMENSIONED BUT ONLY THE LOWER HALF OF EACH IS USED )          05MR8
      DIMENSION  X(L1,L1) , Y(L1,L1) , Z(L1,L1)                         19FE8
          DO 110  J = 1 , L2                                            19FE8
          DO 105  I = 1 , J                                             19FE8
              SUM = 0.0                                                 19FE8
          DO 100  K = 1 , L2                                            19FE8
              SUM = SUM + X(J,K) * Y(K,I)                               19FE8
  100     CONTINUE                                                      19FE8
              Z(J,I) = SUM                                              19FE8
  105     CONTINUE                                                      19FE8
  110     CONTINUE                                                      19FE8
      RETURN                                                            19FE8
      END                                                               19FE8
```

```
      SUBROUTINE MFFT ( X   , Y   , Z   , L1 , L2 )                     18MR8
C******* THIS ROUTINE MULTIPLIES A FULL MATRIX                          18MR8
C        TIMES THE TRANSPOSE OF A SECOND FULL MATRIX                    18MR8
C        ( X * YT = Z )                                                 18MR8
      DIMENSION  X(L1,L1) , Y(L1,L1) , Z(L1,L1)                         18MR8
          DO 110  J = 1 , L2                                            18MR8
          DO 105  I = 1 , L2                                            18MR8
               SUM = 0.0                                                18MR8
          DO 100  K = 1 , L2                                            18MR8
               SUM = SUM + X(I,K) * Y(J,K)                              18MR8
  100     CONTINUE                                                      18MR8
               Z(I,J) = SUM                                             18MR8
  105     CONTINUE                                                      18MR8
  110     CONTINUE                                                      18MR8
      RETURN                                                            18MR8
      END                                                               18MR8
```

```
      SUBROUTINE MBFV ( XB , YF , ZF , L1 , L5 , L2 , LB )              07DE7
C******* THIS ROUTINE MULTIPLIES A BANDED MATRIX                         07DE7
C        TIMES A FULL MATRIX OR A VECTOR                                 07DE7
C        ( XB * YF = ZF )                                                07DE7
      DIMENSION  XB( L1,LB ) , YF( L1,L5 ) , ZF( L1,L5 )                07DE7
                M1 = 1                                                   20MY8
         IF( L1 .EQ. L5 ) M1 = L2                                        20MY8
                L4 = LB/2                                                07DE7
                L6 = L4 + 1                                              07DE7
                N1 = L2 - L4                                             07DE7
         DO 110  M = 1,M1                                                13DE7
         DO 105  I = L6,N1                                               13DE7
                J = I - L6                                               07DE7
                SUM = 0.0                                                06MY8
         DO 100  K = 1,LB                                                07DE7
                SUM = XB(I,K) * YF(K+J,M) + SUM                          06MY8
  100    CONTINUE                                                        10NO7
                ZF(I,M) = SUM                                            06MY8
  105    CONTINUE                                                        10NO7
  110    CONTINUE                                                        10NO7
                K1 = 0                                                   10NO7
                I1 = 1                                                   10NO7
                I2 = L4                                                  07DE7
                I3 = 1                                                   07DE7
                I4 = LB                                                  07DE7
         IF( I2 ) 150, 900, 150                                          07DE7
  150    DO 210  M = 1,M1                                                13DE7
         DO 205  I = I1,I2                                               13DE7
                SUM = 0.0                                                06MY8
                N = 1                                                    07DE7
         DO  200 K = I3, I4                                              07DE7
                SUM = XB(I,N) * YF(K,M) + SUM                            06MY8
                N = N + 1                                                07DE7
  200    CONTINUE                                                        10NO7
                ZF(I,M) = SUM                                            06MY8
  205    CONTINUE                                                        10NO7
  210    CONTINUE                                                        10NO7
         IF( K1 ) 900,300,900                                            10NO7
  300           I1 = L2 - L4 + 1                                         07DE7
                I2 = L2                                                  07DE7
                I3 = L2 - LB + 1                                         07DE7
                I4 = L2                                                  07DE7
                K1 = 1                                                   10NO7
         GO TO 150                                                       10NO7
  900 RETURN                                                             10NO7
      END                                                                10NO7
```

```
      SUBROUTINE MFB ( XF , YB , ZF , L1 , L2 , LB )                  07DE7
C******* THIS ROUTINE MULTIPLIES A FULL MATRIX                        07DE7
C        TIMES A BANDED MATRIX                                        07DE7
C        ( XF * YB = ZF )                                             07DE7
      DIMENSION  XF( L1,L1 ) , YB( LB,L1 ) , ZF( L1,L1 )             07DE7
               L4 = LB/2                                              07DE7
               L6 = L4 + 1                                            07DE7
               N1 = L2 - L4                                           07DE7
          DO 110  I = L6,N1                                           07DE7
               J = I - L6                                             07DE7
          DO 105  M = 1,L2                                            07DE7
               SUM = 0.0                                              06MY8
          DO 100  K = 1,LB                                            07DE7
               SUM = YB(K,I) * XF(M,K+J) + SUM                        06MY8
  100     CONTINUE                                                    10NO7
               ZF(M,I) = SUM                                          06MY8
  105     CONTINUE                                                    10NO7
  110     CONTINUE                                                    10NO7
               K1 = 0                                                 10NO7
               I1 = 1                                                 10NO7
               I2 = L4                                                07DE7
               I3 = 1                                                 08DE7
               I4 = LB                                                07DE7
          IF( I2 ) 150, 900, 150                                      07DE7
  150     DO  210 I = I1, I2                                          10NO7
          DO 205  M =  1,L2                                           07DE7
               SUM = 0.0                                              06MY8
               N = 1                                                  08DE7
          DO  200 K = I3, I4                                          08DE7
               SUM = YB(N,I) * XF(M,K) + SUM                          06MY8
               N = N + 1                                              08DE7
  200     CONTINUE                                                    10NO7
               ZF(M,I) = SUM                                          06MY8
  205     CONTINUE                                                    10NO7
  210     CONTINUE                                                    10NO7
          IF( K1 ) 900,300,900                                        10NO7
  300          I1 = L2 - L4 + 1                                       07DE7
               I2 = L2                                                07DE7
               I3 = L2 - LB + 1                                       07DE7
               I4 = L2                                                07DE7
               K1 = 1                                                 10NO7
          GO TO 150                                                   10NO7
  900 RETURN                                                          10NO7
      END                                                             10NO7
```

```
      SUBROUTINE ABF ( YB , XF , ZF , L1 , L2 , LB )                     07DE7
C******* THIS ROUTINE ADDS A BANDED MATRIX                               07DE7
C        TO A FULL MATRIX                                                07DE7
C        ( YB + XF = ZF  OR  XF + YB = ZF )                              07DE7
      DIMENSION YB( L1,LB ) , XF( L1,L1 ) , ZF(L1,L1)                    07DE7
            L4 = LB/2                                                    07DE7
            N1 = L2 - L4                                                 07DE7
            L6 = L4 + 1                                                  07DE7
         DO 50  I = 1,L2                                                 07DE7
         DO 40  J = 1,L2                                                 07DE7
            ZF(I,J) = XF(I,J)                                            07DE7
   40    CONTINUE                                                        07DE7
   50    CONTINUE                                                        07DE7
         DO 110  I = L6,N1                                               07DE7
            J = I - L6                                                   07DE7
         DO 100  K = 1,LB                                                07DE7
            ZF(I,K+J) = YB(I,K) + XF(I,K+J)                              11DE7
  100    CONTINUE                                                        10NO7
  110    CONTINUE                                                        10NO7
            K1 = 0                                                       10NO7
            I1 = 1                                                       10NO7
            I2 = L4                                                      07DE7
            I3 = 1                                                       08DE7
            I4 = LB                                                      07DE7
         IF( I2 ) 150, 900, 150                                          07DE7
  150    DO 210 I = I1,I2                                                10NO7
            N = 1                                                        08DE7
         DO  200 K = I3, I4                                              08DE7
            ZF(I,K) = YB(I,N) + XF(I,K)                                  11DE7
            N = N + 1                                                    08DE7
  200    CONTINUE                                                        10NO7
  210    CONTINUE                                                        10NO7
         IF(K1) 900, 300, 900                                            10NO7
  300       I1 = L2 - L4 + 1                                             07DE7
            I2 = L2                                                      07DE7
            I3 = L2 - LB + 1                                             07DE7
            I4 = L2                                                      07DE7
            K1 = 1                                                       07DE7
         GO TO 150                                                       10NO7
  900    RETURN                                                          10NO7
         END                                                             10NO7
```

```
      SUBROUTINE ASFV ( X , Y , Z , L1 , L5 , L2 , SIGN )                20MY8
C******* THIS ROUTINE ADDS OR SUBTRACTS 2 FULL MATRICES OR 2 VECTORS     20MY8
C         ( X - Y = Z  OR  X + Y = Z )                                   13DE7
      DIMENSION X(L1,L5) , Y(L1,L5) , Z(L1,L5)                           13DE7
               M = 1                                                     20MY8
          IF( L1 .EQ. L5 ) M = L2                                        20MY8
          IF ( SIGN ) 190, 50, 50                                        13DE7
   50     DO 110  J = 1,M                                                13DE7
          DO 100  I = 1,L2                                               13DE7
               Z(I,J) = X(I,J) + Y(I,J)                                  13DE7
  100     CONTINUE                                                       13DE7
  110     CONTINUE                                                       13DE7
          GO TO 300                                                      13DE7
  190     DO 210  J = 1,M                                                13DE7
          DO 200  I = 1,L2                                               13DE7
               Z(I,J) = X(I,J) - Y(I,J)                                  13DE7
  200     CONTINUE                                                       13DE7
  210     CONTINUE                                                       13DE7
  300 RETURN                                                             13DE7
      END                                                                13DE7
```

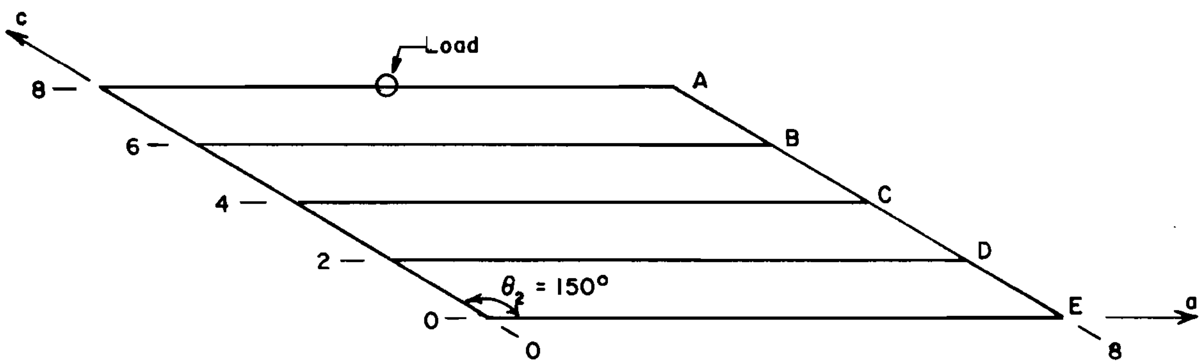
```
      SUBROUTINE RFV ( X , Y , L1 , L5 , L2 )                            23MR8
C******* THIS ROUTINE REPLACES A FULL MATRIX OR A VECTOR                 23MR8
C         ( X = Y )                                                      23MR8
      DIMENSION  X(L1,L5) , Y(L1,L5)                                     23MR8
               M = 1                                                     20MY8
          IF( L1 .EQ. L5 ) M = L2                                        20MY8
          DO 110  J = 1,M                                                23MR8
          DO 100  I = 1 , L2                                             23MR8
               X(I,J) = Y(I,J)                                           23MR8
  100     CONTINUE                                                       23MR8
  110     CONTINUE                                                       23MR8
      RETURN                                                             20MY8
      END                                                                23MR8
```

```
      SUBROUTINE CFV ( X , L1 , L5 , L2 , C )                          20MY8
C******* THIS ROUTINE MULTIPLIES A FULL MATRIX OR A VECTOR BY A CONSTANT13DE7
C          ( X = C*X )                                                 13DE7
      DIMENSION X(L1,L5)                                               13DE7
               M = 1                                                   20MY8
          IF( L1 .EQ. L5 ) M = L2                                      20MY8
          DO 110  J = 1,M                                              13DE7
          DO 100  I = 1,L2                                             13DE7
               X(I,J) = X(I,J) * C                                     13DE7
  100     CONTINUE                                                     13DE7
  110     CONTINUE                                                     13DE7
      RETURN                                                           13DE7
      END                                                              13DE7
```

```
      SUBROUTINE TIC TOC (J)                                           24OC6
C-----  TIC TOC (1)  =  COMPILE TIME                                   20DE7
C       TIC TOC (2)  =  ELAPSED TM TIME                                03MY0
C       TIC TOC (3)  =  TIME FOR THIS PROBLEM                          20DE7
C       TIC TOC (4)  =  TIME FOR THIS PROBLEM  AND  ELAPSED TM TIME    03MY0
   10 FORMAT(///30X19HELAPSED TM  TIME = I5,8H MINUTESF9.3,8H SECONDS ) 03MY0
   11 FORMAT(///30X15HCOMPILE TIME = ,I5,8H MINUTES,F9.3,8H SECONDS   ) 25SE6
   12 FORMAT(///30X24HTIME FOR THIS PROBLEM = ,I5,8H MINUTES,F9.3,     25SE6
     1       8H SECONDS  )                                             25SE6
               I = J - 2                                               21JY7
          IF ( I-1 ) 40, 30, 30                                        21JY7
   30          FI4 = F                                                 25SE6
   40 CALL SECOND (F)                                                  25SE6
               II1 = F                                                 25SE6
               I1 = II1 / 60                                           25SE6
               FI2 =  F - I1*60                                        25SE6
          IF ( I ) 50, 70, 60                                          24JL7
   50 PRINT 11, I1, FI2                                                21JY7
          GO TO 990                                                    25SE6
   60          FI3 = F - FI4                                           25SE6
               I2 = FI3 / 60                                           25SE6
               FI3 = FI3 - I2*60                                       25SE6
      PRINT 12, I2, FI3                                                25SE6
          IF ( I-1 ) 990, 990, 70                                      21JY7
   70 PRINT 10, I1, FI2                                                21JY7
  990     CONTINUE                                                     06SE7
      RETURN                                                           25SE6
      END                                                              25SE6
```

APPENDIX 5

LISTING OF INPUT DATA FOR SELECTED EXAMPLE PROBLEM



Problem 401 Five-beam Noncomposite Skew Bridge with Load at Midspan of Beam A

Fig A1. Geometry of the included problem.

```
CHG CEAQ0136   CODED AND RUN  21 MAY 70    INCH-LB UNITS
PROBLEM SERIES 4 - FIVE BEAM NONCOMPOSITE SKEW BRIDGE WITH 8 X 8 INCREMENTS
  401     5000 LB LOAD AT MIDSPAN OF BEAM A, ANGLE THETA2 = 150 DEG
                                       1    4    13
    8    8 2.500E+00 2.000E+00 1.500E+02
    0    0    8    8 2.500E+04 0.000E+00 0.000E+00 2.500E+04 0.000E+00 1.250E+04
    0    1    8    7 2.500E+04 0.000E+00 0.000E+00 2.500E+04 0.000E+00 1.250E+04
    1    0    7    8 2.500E+04 0.000E+00 0.000E+00 2.500E+04 0.000E+00 1.250E+04
    1    1    7    7 2.500E+04 0.000E+00 0.000E+00 2.500E+04 0.000E+00 1.250E+04
    0    0    8    0 5.000E+06
    1    0    7    0 5.000E+06
    0    2    8    2 5.000E+06
    1    2    7    2 5.000E+06
    0    4    8    4 5.000E+06
    1    4    7    4 5.000E+06
    0    6    8    6 5.000E+06
    1    6    7    6 5.000E+06
    0    8    8    8 5.000E+06
    1    8    7    8 5.000E+06
    0    0    0    8                                                  1.000E+20
    8    0    8    8                                                  1.000E+20
    4    8    4    8                                        -5.000E+03
```

APPENDIX 6

COMPUTED RESULTS FOR SELECTED EXAMPLE PROBLEM

PROGRAM SLAB44 - MASTER DECK - M. VORA REVISION DATE 03 MAY 70
CHG CEAQ0136 CODED AND RUN 21 MAY 70 INCH-LB UNITS
PROBLEM SERIES 4 - FIVE BEAM NONCOMPOSITE SKEW BRIDGE WITH 8 X 8 INCREMENTS

PROB
401 5000 LB LOAD AT MIDSPAN OF BEAM A. ANGLE THETA2 = 150 DEG

TABLE 1. CONTROL DATA

	TABLE NUMBER 2	3	4	5
HOLD FROM PRECEDING PROBLEM (1=HOLD)	-0	-0	-0	-0
NUM CARDS INPUT THIS PROBLEM	1	4	13	-0

MULTIPLE LOAD OPTION (IF BLANK, PROBLEM IS SINGLE LOADING -- IF +1. PARENT FOR NEXT PROB -- IF -1, A OFFSPRING PROB)	-0
PRINT OPTION (IF BLANK, MX MY MXY -- IF 1. MA MB MC PRINTED)	-0
REACTION OUTPUT OPTION (IF BLANK. SUPPORT REACTION -- IF 1. STATICS CHECK PRINTED)	-0
STIFFNESS INPUT OPTION (IF BLANK. D11 THRU D33 IF 1. B11 THRU B33 INPUT)	-0

TABLE 2. CONSTANTS

NUMBER OF INCREMENTS IN A DIRECTION MA	[illegible]
NUMBER OF INCREMENTS IN C DIRECTION MC	[illegible]
INCREMENT LENGTH IN A DIRECTION HA	2.500E+00
INCREMENT LENGTH IN C DIRECTION HC	2.000E+00
ANGLE BETWEEN A AND C DIRECTION IN DEGREES	1.500E+02

TABLE 3. JOINT STIFFNESS DATA

FROM JOINT		THRU JOINT		D11	D12	D13	D22	D23	D33
0	0	8	8	2.500E+04	0.	0.	2.500E+04	0.	1.250E+04
0	1	8	7	2.500E+04	0.	0.	2.500E+04	0.	1.250E+04
1	0	7	8	2.500E+04	0.	0.	2.500E+04	0.	1.250E+04
1	1	7	7	2.500E+04	0.	0.	2.500E+04	0.	1.250E+04

TABLE 4. BEAM STIFFNESS AND LOAD DATA

FROM JOINT		THRU JOINT		FA	FB	FC	Q	S
0	0	8	0	5.000E+06	-0.	-0.	-0.	-0.
1	0	7	0	5.000E+06	-0.	-0.	-0.	-0.
0	2	8	2	5.000E+06	-0.	-0.	-0.	-0.
1	2	7	2	5.000E+06	-0.	-0.	-0.	-0.
0	4	8	4	5.000E+06	-0.	-0.	-0.	-0.
1	4	7	4	5.000E+06	-0.	-0.	-0.	-0.
0	6	8	6	5.000E+06	-0.	-0.	-0.	-0.
1	6	7	6	5.000E+06	-0.	-0.	-0.	-0.
0	8	8	8	5.000E+06	-0.	-0.	-0.	-0.
1	8	7	8	5.000E+06	-0.	-0.	-0.	-0.

0	0	0	8	-0.	-0.	-0.	-0.	1.000E+20
8	0	8	8	-0.	-0.	-0.	-0.	1.000E+20
4	8	4	8	-0.	-0.	-0.	-5.000E+03	-0.

TABLE 5. EXTERNAL COUPLE DATA

FROM JOINT	THRU JOINT	TA	TB	TC
	NONE			

PROGRAM SLAB44 - MASTER DECK - M. VORA REVISION DATE 03 MAY 70
CHG CEAQ0136 CODED AND RUN 21 MAY 70 INCH-LB UNITS
PROBLEM SERIES 4 - FIVE BEAM NONCOMPOSITE SKEW BRIDGE WITH 8 X 8 INCREMENTS

PROB (CONTD)
401 5000 LB LOAD AT MIDSPAN OF BEAM A, ANGLE THETA2 = 150 DEG

TABLE 6. RESULTS

SLAB MOMENTS ARE PER UNIT WIDTH
BEAM MOMENTS ARE TOTAL PER BEAM
COUNTERCLOCKWISE BETA ANGLES ARE POSITIVE

A	C	DEFL	SLAB X MOMENT / BEAM A MOMENT	SLAB Y MOMENT / BEAM B MOMENT	SLAB XY MOMENT / BEAM C MOMENT	LARGEST PRINCIPAL SLAB MOMENT	BETA X TO LARGEST MOMENT	SUPPORT REACTION
-1	-1	1.402E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	-1	7.630E-04	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
1	-1	1.020E-04	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
2	-1	-6.563E-04	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
3	-1	-1.346E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
4	-1	-1.784E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
5	-1	-1.965E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
6	-1	-1.567E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
7	-1	-1.045E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
8	-1	-1.050E-04	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
9	-1	0.	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	0	1.080E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	0	2.034E-18	1.073E+00	2.265E+00	3.924E+00	5.638E+00	-49.3	-2.034E+02
			2.145E+02	0.	0.			
1	0	-8.118E-04	1.001E+00	-1.817E+00	2.516E+00	-3.292E+00	59.6	-0.
			2.002E+02	0.	0.			
2	0	-1.498E-03	1.555E+00	-2.276E+00	-2.823E+00	-3.772E+00	-62.1	-0.
			3.110E+02	0.	0.			
3	0	-1.991E-03	2.239E+00	-1.584E+00	-7.315E+00	7.888E+00	37.7	-0.
			4.477E+02	0.	0.			
4	0	-2.203E-03	2.738E+00	-4.071E-01	-9.236E+00	1.053E+01	40.2	-0.
			5.475E+02	0.	0.			
5	0	-2.073E-03	2.729E+00	-5.195E-01	-1.045E+01	1.168E+01	40.6	-0.
			5.459E+02	0.	0.			
6	0	-1.602E-03	2.068E+00	1.755E+00	-7.807E+00	9.721E+00	44.4	-0.

			4.137E+02	0.	0.			
7	0	-8.729E-04	1.147E+00	9.949E-01	-4.736E+00	5.808E+00	44.5	-0.
			2.294E+02	0.	0.			
8	0	-9.004E-19	4.662E-03	1.584E+00	-1.220E+00	2.251E+00	61.5	9.004E+01
			9.323E-01	0.	0.			
9	0	8.741E-04	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	1	2.753E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	1	-2.240E-19	9.800E+00	-4.553E+01	4.658E+00	-4.592E+01	85.2	2.240E+01
			0.	0.	0.			
1	1	-1.528E-03	9.069E+00	-3.786E+01	9.578E+00	-3.974E+01	78.9	-0.
			0.	0.	0.			
2	1	-2.488E-03	6.558E+00	-2.305E+01	-4.423E-01	-2.305E+01	-89.1	-0.
			0.	0.	0.			
3	1	-3.039E-03	7.754E+00	-8.411E+00	-1.185E+01	-1.467E+01	-62.1	-0.
			0.	0.	0.			
4	1	-3.106E-03	6.717E+00	4.111E+00	-1.937E+01	2.483E+01	43.1	-0.
			0.	0.	0.			
5	1	-2.752E-03	6.066E+00	1.851E+01	-2.247E+01	3.560E+01	52.7	-0.
			0.	0.	0.			
6	1	-2.020E-03	5.544E+00	2.350E+01	-2.020E+01	3.663E+01	57.0	-0.
			0.	0.	0.			
7	1	-9.405E-04	-2.219E+00	1.058E+01	-1.210E+01	1.787E+01	58.9	-0.
			0.	0.	0.			
8	1	4.566E-19	1.943E+00	1.283E+01	-5.386E+00	1.504E+01	67.7	-4.566E+01
			0.	0.	0.			
9	1	1.183E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	2	2.568E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	2	2.689E-18	4.395E+00	-1.492E+01	5.019E-01	-1.494E+01	88.5	-2.689E+02
			4.395E+02	0.	0.			
1	2	-2.018E-03	8.142E+00	-9.353E+01	5.346E+00	-9.381E+01	87.0	-0.
			8.142E+02	0.	0.			
2	2	-3.527E-03	1.099E+01	-7.294E+01	-1.141E+00	-7.295E+01	-89.2	-0.
			1.099E+03	0.	0.			
3	2	-4.350E-03	1.177E+01	-3.711E+01	-1.431E+01	-4.094E+01	-74.8	-0.
			1.177E+03	0.	0.			
4	2	-4.437E-03	1.089E+01	-6.404E+00	-2.276E+01	2.660E+01	34.6	-0.
			1.089E+03	0.	0.			
5	2	-3.843E-03	8.208E+00	1.620E+01	-2.770E+01	4.019E+01	49.1	-0.
			8.208E+02	0.	0.			
6	2	-2.735E-03	3.844E+00	4.434E+01	-3.034E+01	6.057E+01	61.9	-0.
			3.844E+02	0.	0.			
7	2	-1.388E-03	6.475E-01	5.303E+01	-1.446E+01	5.676E+01	75.6	-0.
			6.475E+01	0.	0.			
8	2	-6.249E-19	-5.602E-01	1.138E+01	-2.800E+00	1.201E+01	77.4	6.249E+01
			-5.602E+01	0.	0.			
9	2	1.318E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	3	6.953E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	3	7.043E-19	3.151E+01	-1.052E+02	3.084E+00	-1.053E+02	88.7	-7.043E+01
			0.	0.	0.			
1	3	-3.015E-03	1.638E+01	-1.320E+02	1.285E+01	-1.331E+02	85.1	-0.
			0.	0.	0.			

2	3	-5.006E-03	1.316E+01	-1.141E+02	-6.348E+00	-1.144E+02	-87.2	-0.
			0.	0.	0.			
3	3	-6.174E-03	1.679E+01	-6.485E+01	-2.355E+01	-7.116E+01	-75.0	-0.
			0.	0.	0.			
4	3	-6.293E-03	1.485E+01	-1.264E+01	-3.401E+01	3.779E+01	34.0	-0.
			0.	0.	0.			
5	3	-5.484E-03	1.171E+01	3.581E+01	-3.858E+01	6.418E+01	53.7	-0.
			0.	0.	0.			
6	3	-3.943E-03	9.860E+00	6.260E+01	-3.702E+01	8.168E+01	62.7	-0.
			0.	0.	0.			
7	3	-1.786E-03	-5.930E+00	4.727E+01	-2.603E+01	5.788E+01	67.8	-0.
			0.	0.	0.			
8	3	7.657E-19	-4.311E+00	4.599E+01	-9.543E+00	4.774E+01	79.6	-7.657E+01
			0.	0.	0.			
9	3	1.247E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	4	5.653E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	4	3.551E-18	1.007E+01	-3.766E+01	2.155E+00	-3.776E+01	87.4	-3.551E+02
			1.007E+03	0.	0.			
1	4	-4.395E-03	2.143E+01	-2.814E+02	8.429E+00	-2.816E+02	88.4	-0.
			2.143E+03	0.	0.			
2	4	-7.450E-03	2.474E+01	-2.091E+02	-1.048E+01	-2.096E+02	-87.4	-0.
			2.474E+03	0.	0.			
3	4	-9.959E-03	2.335E+01	-1.166E+02	-4.209E+01	-1.285E+02	-74.5	-0.
			2.335E+03	0.	0.			
4	4	-9.009E-03	2.089E+01	-4.035E+01	-5.182E+01	-6.592E+01	-60.3	-0.
			2.089E+03	0.	0.			
5	4	-7.753E-03	1.577E+01	2.308E+01	-5.802E+01	7.756E+01	46.8	-0.
			1.577E+03	0.	0.			
6	4	-5.512E-03	7.666E+00	6.914E+01	-5.886E+01	1.200E+02	62.3	-0.
			7.666E+02	0.	0.			
7	4	-2.792E-03	1.140E+00	1.151E+02	-3.139E+01	1.232E+02	75.6	-0.
			1.140E+02	0.	0.			
8	4	-2.403E-18	-1.933E+00	2.487E+01	-5.505E+00	2.596E+01	78.8	2.403E+02
			-1.933E+02	0.	0.			
9	4	2.550E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	5	1.352E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	5	-2.176E-18	4.694E+01	-2.309E+02	2.600E+01	-2.333E+02	84.7	2.176E+02
			0.	0.	0.			
1	5	-7.654E-03	5.922E+01	-2.547E+02	3.372E+01	-2.583E+02	83.9	-0.
			0.	0.	0.			
2	5	-1.161E-02	3.023E+01	-2.864E+02	-9.781E+00	-2.866E+02	-88.4	-0.
			0.	0.	0.			
3	5	-1.367E-02	3.899E+01	-2.008E+02	-6.205E+01	-2.159E+02	-76.3	-0.
			0.	0.	0.			
4	5	-1.330E-02	2.552E+01	-8.534E+01	-8.783E+01	-1.338E+02	-61.1	-0.
			0.	0.	0.			
5	5	-1.133E-02	2.179E+01	5.308E+01	-9.438E+01	1.331E+02	49.7	-0.
			0.	0.	0.			
6	5	-7.998E-03	1.550E+01	1.227E+02	-7.927E+01	1.648E+02	62.0	-0.
			0.	0.	0.			
7	5	-3.699E-03	-9.597E+00	1.096E+02	-5.511E+01	1.312E+02	68.6	-0.
			0.	0.	0.			
8	5	1.494E-18	-1.015E+01	9.775E+01	-1.943E+01	1.011E+02	80.1	-1.494E+02
			0.	0.	0.			

9	5	2.430E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	6	1.140E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	6	-1.482E-17	7.105E+00	-6.816E+01	1.352E+01	-7.052E+01	80.1	1.482E+03
			7.105E+02	0.	0.			
1	6	-1.052E-02	4.812E+01	-4.201E+02	2.653E+01	-4.216E+02	86.8	-0.
			4.812E+03	0.	0.			
2	6	-1.802E-02	6.661E+01	-3.927E+02	2.927E+01	-3.946E+02	86.4	-0.
			6.661E+03	0.	0.			
3	6	-2.137E-02	6.398E+01	-2.971E+02	-6.519E+01	-3.085E+02	-80.1	-0.
			6.398E+03	0.	0.			
4	6	-2.071E-02	4.917E+01	-1.816E+02	-1.227E+02	-2.346E+02	-66.6	-0.
			4.917E+03	0.	0.			
5	6	-1.699E-02	2.756E+01	-3.320E+01	-1.515E+02	-1.573E+02	-50.7	-0.
			2.756E+03	0.	0.			
6	6	-1.154E-02	6.107E+00	1.825E+02	-1.574E+02	2.747E+02	59.6	-0.
			6.107E+02	0.	0.			
7	6	-5.707E-03	-1.979E+00	2.469E+02	-6.248E+01	2.617E+02	76.7	-0.
			-1.979E+02	0.	0.			
8	6	-2.880E-18	-3.869E+00	7.397E+01	-1.800E+01	7.793E+01	77.6	2.880E+02
			-3.869E+02	0.	0.			
9	6	5.224E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	7	1.250E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	7	-6.370E-18	-2.159E+01	-1.451E+02	6.154E+01	-1.731E+02	67.9	6.370E+02
			0.	0.	0.			
1	7	-1.522E-02	9.120E+01	-7.349E+01	4.914E+01	9.549E+01	-16.2	-0.
			0.	0.	0.			
2	7	-2.536E-02	4.990E+01	-2.002E+02	5.057E+01	-2.101E+02	79.0	-0.
			0.	0.	0.			
3	7	-3.238E-02	1.068E+02	-1.866E+02	-1.828E+01	-1.977E+02	-86.4	-0.
			0.	0.	0.			
4	7	-3.272E-02	9.069E+01	-1.324E+02	-1.256E+02	-1.888E+02	-65.8	-0.
			0.	0.	0.			
5	7	-2.740E-02	5.691E+01	-6.726E+01	-1.778E+02	-1.935E+02	-54.6	-0.
			0.	0.	0.			
6	7	-1.852E-02	3.475E+01	3.612E+00	-2.098E+02	2.295E+02	42.9	-0.
			0.	0.	0.			
7	7	-7.471E-03	-5.729E+01	8.591E+01	-1.876E+02	2.151E+02	55.4	-0.
			0.	0.	0.			
8	7	8.802E-18	-1.428E+01	3.224E+02	-8.071E+01	3.408E+02	77.2	-8.802E+02
			0.	0.	0.			
9	7	5.686E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	8	1.650E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	8	-1.039E-17	-5.445E-02	-1.856E+01	1.425E+01	-2.630E+01	61.5	1.039E+03
			-1.089E+01	0.	0.			
1	8	-1.651E-02	1.266E+01	8.702E+00	3.971E+01	5.045E+01	-43.6	-0.
			2.532E+03	0.	0.			
2	8	-3.144E-02	2.952E+01	-3.860E+00	2.761E+01	4.509E+01	-29.4	-0.
			5.904E+03	0.	0.			
3	8	-4.268E-02	4.987E+01	1.892E+01	1.976E+01	5.950E+01	-26.0	-0.
			9.975E+03	0.	0.			
4	8	-4.769E-02	7.803E+01	2.345E+01	-3.101E+01	9.205E+01	24.3	-0.

			1.551E+04	0.	0.			
5	8	-4.294E-02	5.196E+01	1.007E+01	-7.936E+01	1.131E+02	37.6	-0.
			1.039E+04	0.	0.			
6	8	-3.169E-02	3.086E+01	5.418E+00	-1.010E+02	1.149E+02	41.4	-0.
			6.171E+03	0.	0.			
7	8	-1.659E-02	1.193E+01	-6.941E+00	-1.050E+02	1.079E+02	42.4	-0.
			2.386E+03	0.	0.			
8	8	-2.971E-17	-8.505E+00	-3.720E+01	-6.443E+01	-8.885E+01	-51.3	2.971E+03
			-1.791E+03	0.	0.			
9	8	1.447E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
-1	9	0.	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
0	9	1.226E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
1	9	-1.412E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
2	9	-3.392E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
3	9	-4.825E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
4	9	-5.965E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
5	9	-6.065E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
6	9	-4.990E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
7	9	-3.241E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
8	9	-1.144E-02	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			
9	9	9.740E-03	0.	0.	0.	0.	0.0	-0.
			0.	0.	0.			

SUMMATION OF SUPPORT SPRING REACTION = 5.000E+02

MAXIMUM STATICS CHECK ERROR AT STA 3 8 = 5.271E-09

TIME FOR THIS PROBLEM = 0 MINUTES 5.689 SECONDS

ELAPSED TM TIME = 0 MINUTES 30.116 SECONDS

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PROGRAM  SLAB44   - MASTER DECK -    M. VORA     REVISION DATE 03 MAY 70
CHG CEA00136   CODED AND RUN  21 MAY 70     INCH-LB UNITS
PROBLEM SERIES 4 - FIVE BEAM NONCOMPOSITE SKEW BRIDGE WITH 8 X 8 INCREMENTS


                    ELAPSED TM  TIME =     0 MINUTES   30.120 SECONDS
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THE AUTHORS

Mahendrakumar R. Vora was a Research Engineer Assistant with the Center for Highway Research, The University of Texas at Austin. His experience includes work with various private and government organizations in Bombay, India. His research interest is primarily in the area of discrete-element analysis.

Hudson Matlock is a Professor of Civil Engineering at The University of Texas at Austin. He has a broad base of experience, including research at the Center for Highway Research and as a consultant for other organizations such as the Alaska Department of Highways, Shell Development Company, Humble Oil and Refining Company, Penrod Drilling Company, and Esso Production Research Company. He is the author of over 50 technical papers and 25 research reports and received the 1967 ASCE J. James R. Croes Medal. His primary areas of interest in research include (1) soil-structure interaction, (2) experimental mechanics, and (3) development of computer methods for simulation of civil engineering problems.