

CRITICAL MECHANICAL PROPERTIES OF STRUCTURAL LIGHT-  
WEIGHT CONCRETE AND THE EFFECTS OF THESE PROPERTIES  
ON THE DESIGN OF THE PAVEMENT STRUCTURE

by

William B. Ledbetter  
Ervin S. Perry  
James T. Houston  
J. Neils Thompson

Research Report Number 55-3F

Research Project Number 3-8-63-55

Determination of Critical Mechanical Properties of Structural Light-  
weight Concrete and the Effects of these Properties Upon the Design  
of the Pavement Structure

Conducted for

The Texas Highway Department  
Interagency Contract Nos. 4413-767 and 4413-828

In Cooperation with the  
U. S. Department of Commerce, Bureau of Public Roads

by

CENTER FOR HIGHWAY RESEARCH

THE UNIVERSITY OF TEXAS

AUSTIN, TEXAS

January 1965

Performed in the Laboratories  
of  
The Civil Engineering Department  
of  
The University of Texas  
Austin, Texas

## PREFACE

This is the third report in the series of three reports to be written covering the work of this project. The reports are:

Report No. 1 - "Relationship Between Critical Mechanical Properties and Age for Structural Lightweight Concrete" by W. B. Ledbetter and J. Neils Thompson; this report was concerned with the development of a technique to measure the tensile stress-strain characteristics of lightweight aggregate concrete and how this property is affected by restraint from volume change.

Report No. 2 - "Volume Changes in Unrestrained Structural Lightweight Concrete" by James T. Houston and J. Neils Thompson; this report is concerned with the development of a method to accurately determine the coefficient of linear thermal expansion as well as the unrestrained shrinkage characteristics of structural lightweight concrete.

Report No. 3 (Final Report) - "Critical Mechanical Properties of Structural Lightweight Concrete and the Effects of These Properties on the Design of the Pavement Structure" by W. B. Ledbetter, Ervin S. Perry, James T. Houston, and J. Neils Thompson; the current report summarizes the findings of the first two reports, and provides some interpretation of the effects of environment and restraint upon the design of the pavement structure.

The authors are grateful to the U. S. Bureau of Public Roads and the Texas Highway Department for their sponsorship in this program. Appreciation is also extended to the staff of these two organizations for their suggestions and comments during this undertaking.

On behalf of the authors, gratitude is expressed for the contributions of the following undergraduate students: Thomas B. Bell, John R. Tushek, and Daniel Schodek.

The responsible staff in this research were:

J. Neils Thompson, Professor of Civil Engineering (Faculty)

Ervin S. Perry, Assistant Professor of Civil Engineering  
(Faculty)

William B. Ledbetter, Assistant Professor of Civil Engineering,  
(Faculty), Texas A & M University, College Station, Texas

James T. Houston, Research Engineer Assistant I

J. Neils Thompson

January 11, 1965

## TABLE OF CONTENTS

	Page
PREFACE . . . . .	iii
LIST OF FIGURES . . . . .	viii
LIST OF TABLES . . . . .	xi
ABSTRACT . . . . .	xii
1. INTRODUCTION . . . . .	1
1.1 General . . . . .	1
1.2 Objectives and Limitations . . . . .	2
1.3 Background . . . . .	4
2. VOLUME CHANGES . . . . .	5
2.1 General . . . . .	5
2.2 Volume Changes - Thermal . . . . .	5
Effect of Air Content . . . . .	7
2.3 Volume Changes - Moisture . . . . .	7
Unrestrained . . . . .	7
Restrained . . . . .	8
3. COMPRESSIVE, FLEXURAL AND SPLIT-CYLINDER STRENGTH COMPARISONS . . . . .	12
3.1 General . . . . .	12
3.2 Flexural Compressive . . . . .	13
3.3 Flexural Split-Cylinder . . . . .	15
3.4 Effect of Curing Condition on Flexural Strength . .	15
4. TENSILE STRENGTH PROPERTIES . . . . .	19
4.1 General . . . . .	19

TABLE OF CONTENTS  
(Continued)

	Page
4.2 Effects of Environment on Tensile Stress-Strain Properties . . . . .	19
4.3 Effect of Curing Condition on Split-Cylinder Strength . . . . .	34
4.4 Relationship Between Direct Tensile and Compressive Strengths . . . . .	36
4.5 Relationship Between Direct Tensile and Split-Cylinder Strengths . . . . .	39
5. STATIC MODULUS OF ELASTICITY . . . . .	43
5.1 General . . . . .	43
Effect of Age and curing conditions . . . . .	43
5.2 Comparison of E in Tension and Compression . . . . .	45
5.3 Comparison of Measured and Calculated E . . . . .	45
6. DISCUSSION OF LIGHTWEIGHT CONCRETE DESIGN AND PAVEMENT PERFORMANCE . . . . .	50
6.1 General . . . . .	50
6.2 Concrete Pavement Design Formulations . . . . .	51
Concrete thickness . . . . .	51
Contraction joint spacing . . . . .	55
Distributed steel . . . . .	58
Continuous reinforcement . . . . .	59
6.3 Concrete Pavement Performance . . . . .	61
Pavement deflection under load . . . . .	61
Concrete warping stresses . . . . .	62
Volume changes - moisture . . . . .	65

TABLE OF CONTENTS  
(Continued)

	Page
7. CLOSURE . . . . .	66
7.1 Conclusions . . . . .	66
7.2 Recommendations . . . . .	70
7.3 Correlation of Results with Research Objectives .	72
8. APPENDIX . . . . .	73
8.1 Data . . . . .	73
8.2 List of Symbols . . . . .	79
8.3 Bibliography . . . . .	80

## LIST OF FIGURES

Figure		Page
2-1	Concrete Strain Temperature Properties of Normal Weight and Lightweight Concretes . . . . .	6
2-2	Restrained Concrete Volume Change Stresses Expressed as Percent Direct Tensile Strength of the Concrete for C. F. = 5 sk/cy . . . . .	10
2-3	Restrained Concrete Volume Change Stresses Expressed as Percent Direct Tensile Strength of the Concrete for C. F. = 4 sk/cy . . . . .	11
3-1	Relationship Trends Between Flexural and Compressive Strengths for Various Curing Conditions .	14
3-2	Relationship Trends Between Flexural and Split-Cylinder Strengths for Various Curing Conditions .	16
3-3	Effect of Curing on Flexural Strength . . . . .	17
4-1	Stress-Strain Curves For Three Curing Conditions for C. F. = 5 sk/cy, 6 % Air, and Age of 7 Days .	22
4-2	Stress-Strain Curves For Three Curing Conditions for C. F. = 5 sk/cy, 6 % Air, and Age of 28 Days .	23
4-3	Stress-Strain Curves for Three Curing Conditions for C. F. = 4 sk/cy, 6 % Air, and Age of 7 Days .	24
4-4	Stress-Strain Curves for Three Curing Conditions for C. F. = 4 sk/cy, 6 % Air, and Age of 28 Days .	25
4-5	Stress-Strain Curves for 7-Day Bag Cured Specimens with Different Cement Factors and Air Contents . . . . .	26
4-6	Stress-Strain Curves for 28-Day Bag Cured Specimens with Different Cement Factors and Air Contents . . . . .	27
4-7	Stress-Strain Curves for 7-Day Air Cured Specimens with Different Cement Factors and Air Contents . . . . .	28

LIST OF FIGURES  
(Continued)

Figure		Page
4-8	Stress-Strain Curves for 28-Day Air Cured Specimens with Different Cement Factors and Air Contents . . . . .	29
4-9	Stress-Strain Curves for 7-Day Oven Cured Specimens with Different Cement Factors and Air Contents . . . . .	30
4-10	Stress-Strain Curves for 28-Day Oven Cured Specimens with Different Cement Factors and Air Contents . . . . .	31
4-11	Effect of Curing Conditions on Split-Cylinder Strengths . . . . .	35
4-12	Relationship Between Direct Tensile Strength ( $f_t$ ) and Compressive Strength ( $f_c$ ) . . . . .	37
4-13	Percent of Total Available Tensile Strength for Restrained Structural Lightweight Concrete as a Function of Curing Conditions . . . . .	38
4-14	Relationship Between Direct Tensile and Split-Cylinder Strengths . . . . .	40
4-15	Relationships Between Useable Tensile Strength and Split-Cylinder Strengths . . . . .	41
5-1	Effect of Age and Curing Conditions on the Static Modulus of Elasticity . . . . .	44
5-2	Relationship Between Secant Modulus of Elasticity and Compressive Strength and Unit Weight of Concrete . . . . .	49
6-1	Modified AASHO Design Chart for Rigid Pavements . . . . .	53
6-2	Design Thickness for CPCR of Regular and Lightweight Concrete as a Function of Load Application and Subgrade Modulus . . . . .	56

LIST OF FIGURES  
(Continued)

Figure		Page
6-3	Design Thickness for CPJ of Regular and Lightweight Concrete as a Function of Load Application and Subgrade Modulus . . . . .	57
6-4	Difference in the Edge Deflection Parameters Between Regular Weight and Structural Lightweight Concrete Pavement . . . . .	63

LIST OF TABLES

Table		Page
4-1	Structural Lightweight Concrete Direct Tensile Test Data Tabulation (Average Values) . . . . .	20
5-1	Comparison of Compressive and Tensile Modulus of Elasticity . . . . .	46
6-1	Pavement Design Parameters . . . . .	54
8-1	Compressive, Flexural, and Indirect Tensile Strength Values . . . . .	74
8-2	Summary of Modulus of Elasticity Data . . . . .	77

## ABSTRACT

In this study, critical mechanical properties of structural lightweight concrete were determined and utilized in the evaluation of a design of concrete pavements. Also presented are the critical mechanical properties resulting from unrestrained and restrained volume changes. Particular attention is given to compressive, direct tensile, and indirect tensile (split cylinder) strength at various ages of the concrete.

The critical properties determined in this study indicate that concrete pavements can be designed with lightweight concrete and that expected performance in regard to the effects of warping stresses and pavement deflection will be better when lightweight concretes are used. However, the effects of restrained volume change of lightweight concrete on pavement performance can be detrimental if improper curing, or curing for too short a time, occurs. The need for further research into the effects of curing on lightweight concrete pavement performance is emphasized.

## 1. INTRODUCTION

### 1.1 General

Structural lightweight concrete has been used for many years in the construction of structural elements such as concrete buildings and bridges, generally with very excellent results. Occasionally failures have occurred and the causes have been credited to the failure of structural lightweight concrete, whether justifiable or unjustifiable. This perhaps is understandable because to many individuals structural lightweight concrete is still a new material, and as they may not have been previously exposed to its use, a quick judgement may lead to the conclusion that the material is unsound. Also, although lightweight concrete has been in use for some time, there remains much that is unknown concerning its behavior and performance in service. This is especially true for a concrete pavement structure. There are almost no concrete pavement structures constructed of lightweight concrete, and before this material can be used for this purpose, additional information must be known.

Therefore, in an attempt to find out more about this material and its use in concrete pavement structures, this study was undertaken. This is the third and final report in this study. The first report, Relationship Between Critical Mechanical Properties and Age For Structural Lightweight Concrete, was published in February 1964.<sup>1\*</sup> The scope of this first report

---

\*Numbers indicate references as listed in the Bibliography.

included the (1) development of a method whereby the direct tensile properties of the concrete could reliably and consistently be determined, (2) determination of the direct tensile as well as other critical mechanical properties of structural lightweight concrete which affect the design and performance of pavement structures, (3) analysis of concrete properties in terms of age as a test parameter, and (4) development of the relationships between these critical mechanical properties.

The second report,<sup>2</sup> was entitled Volume Change in Unrestrained Structural Lightweight Concrete. The objectives of this report were (1) to develop a method of accurately measuring small dimensional changes in concrete specimens, (2) to determine the coefficient of linear thermal expansion of lightweight and regular-weight concrete, and (3) to study unrestrained shrinkage characteristics of lightweight concrete.

## 1.2 Objectives and Limitations

This report has as its objectives: (1) to explore the properties of volume changes of a structural lightweight concrete due to temperature changes and moisture changes during curing, (2) to explore trends of relationships between compressive, split cylinder, and flexural strengths, (3) to report further information on the direct tensile properties, (4) to present relationships between direct tensile, compressive, and split cylinder strengths, and to show how these are affected by different curing conditions, (5) to explore properties of the static modulus of elasticity in both tension and compression, and (6) to determine the effects of all lightweight structural concrete properties investigated in this study on the design and performance of concrete pavement structures constructed with structural lightweight concrete.

The above objectives were determined for concrete made with one structural lightweight coarse aggregate and for concrete made with one regular-weight coarse aggregate. A river run sand was used as the fine aggregate in both concrete types. A complete description of the parameters involved in this investigation is given in the first report.<sup>3</sup> However, the variables will be summarized below. A structural lightweight, semi-coated, expanded shale with a nominal maximum size of 3/4 in. was used as a coarse aggregate in all tests of lightweight concrete. Cement factors of 4 sacks per cubic yard and 5 sacks per cubic yard were used with air contents of 2 per cent (no air-entrainment) and 6 per cent (using an air-entrainment additive). Three curing conditions were employed and were termed bag cured, oven cured, and air cured. The bag-cured specimens consisted of moist curing the specimens in sealed polyethylene plastic bags at 75F. The specimens in the bags were therefore cured under a relative humidity of approximately 100 per cent. The oven-cured specimens were cured in an oven at approximately 110F and low humidity. The temperature in the oven probably varied  $\pm 5$ F. The air-cured specimens were cured at approximately 50 per cent relative humidity and 75F generally prevailing in the laboratory. Concrete properties were determined at ages of 1/2 day, 2 days, 7 days, and 28 days in conjunction with the variables mentioned above.

In order to isolate the relationships between test parameters involved in this study, the following variables were held constant throughout this study:

1. Mixing time and sequence.
2. Cement type.

3. Batch size.
4. Air-entrainment type.
5. Consistency.
6. Test procedure (specimen size, rate of loading, etc).
7. Fine-aggregate type.

Discussions concerning materials, mixing techniques, and testing procedures can be found in the first two reports.<sup>4, 5</sup>

### 1.3 Background

Structural lightweight concrete was first produced in 1917. Stephen J. Hayde developed a process for expanding shale and clay into sound, hard, lightweight particles suitable for use as aggregate in structural concrete. This type of concrete has been used extensively since that time. For a complete development of the background of this material, including the research conducted recently at the Texas Transportation Institute, Texas A and M University, the reader is referred to the first report in this study.<sup>6</sup>

## 2. VOLUME CHANGES

### 2.1 General

All concrete changes dimensionally when subjected to changes in temperature and changes in moisture environment. Lightweight aggregate concrete in general is more susceptible to volume changes than regular-weight concrete because of its porous nature. This is particularly pronounced during the curing period.

The coefficient of linear thermal expansion, hereafter called coefficient of expansion ( $K_T$ ) is a measure of volume changes due to temperature variations, which is an important design consideration. The coefficient of expansion of a lightweight concrete and a regular-weight concrete are discussed and compared briefly in this chapter. A more complete description of testing techniques and results are presented by Houston.<sup>7</sup>

Shrinkage and expansion characteristics of one lightweight concrete are also presented in this chapter and compared to a regular-weight concrete.

### 2.2 Volume Changes-Thermal

Two lightweight and one regular-weight concrete specimens (6 x 12-in. cylinders) were tested for the determination of coefficient of expansion ( $K_T$ ). All specimens were sealed in polyethylene bags to eliminate any effects that humidity changes would have produced on the concrete during the testing period.

Values of  $K_T$  are reported as  $\times 10^{-6}$  in./in. per degree Fahrenheit. The results of the coefficient of expansion series shown in Fig. 2-1 indicate

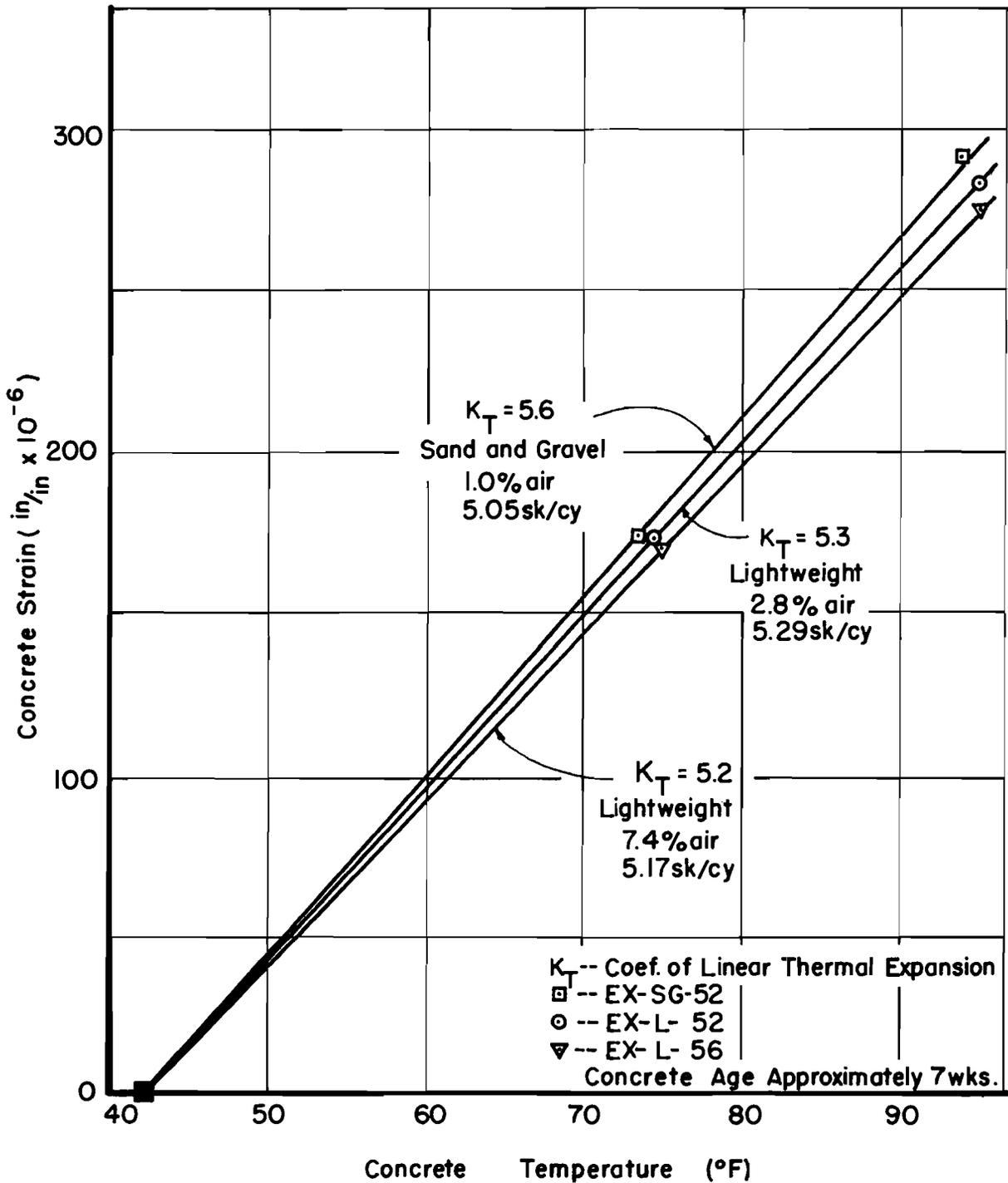


Fig. 2-1- Concrete Strain - Temperature Properties of Normal Weight and Lightweight Concretes.

that this property does not vary greatly between the regular-weight concrete and the lightweight concrete studied in this project. Note that the data points shown fall very near to a straight line in all three cases. In fact, values of  $K_T$  obtained by a straight line through the end points and those obtained by the use of at least-squares fit differ by a maximum of only one unit in the third decimal place. However, as a matter of practicality, results are reported only to the second place.

In comparison, it is seen that the values of  $K_T$  of both lightweight specimens are slightly lower than that of the regular-weight mix. This relationship between normal-weight and lightweight concretes has also been reported by Monfore and Lentz,<sup>8</sup> and Philleo.<sup>9</sup>

A value of 4.5 for  $K_T$  has been reported for concrete using expanded shale aggregate<sup>10</sup> for both the coarse and fine sizes. When it is considered that the lightweight concrete used in this report contained a natural sand as the fine aggregate, the higher values of 5.2 and 5.3 seem reasonable.

Effect of Air Content. In general, the addition of air entraining agents decreases the value of the coefficient of expansion of concrete. Peterson<sup>11</sup> reports a reduction in the value of  $K_T$  of 0.4 for expanded shale concrete containing 13.9 per cent air over that of the same concrete with no entrained air. The reduction in  $K_T$  of 0.1 reported here therefore seems logical when the magnitudes of the air contents are noted, (see Fig. 2-1).

### 2.3 Volume Changes-Moisture

Unrestrained. Four lightweight concrete specimens (6 x 12-in. cylinders) were used to determine the unrestrained shrinkage characteristics for two extreme curing conditions (bag at 75F and, oven at 110F).

Due to the nature of the measuring system which was used, it was impossible to obtain directly shrinkage strains occurring during the first day after mixing. However, embedment strain gages which were placed in the specimens at the time of pouring indicated sizeable expansions<sup>12</sup> during this period.

Specimens which were subjected to oven curing after about one day of age showed expansions of approximately  $200 \times 10^{-6}$  in./in. during the first day after they were placed in the oven. Thereafter, the concrete exhibited a relatively large shrinkage rate which continued for approximately 15 days. At this point the shrinkage rate decreased to a fairly constant value which continued through 100 days (last data points taken to this date). The total shrinkage after 100 days of oven curing was approximately  $290 \times 10^{-6}$  in./in. with a continuing strain rate of  $0.42 \times 10^{-6}$  in./in. per day. From these results and those reported by Shideler<sup>13</sup> on a wide variety of lightweight aggregates, this particular aggregate used in conjunction with a regular-weight sand showed low shrinkage characteristics.

Specimens which were placed in polyethelene bags after about one day of age showed expansions of approximately  $30 \times 10^{-6}$  in./in. during the first few days of bag curing. At this point, due to an inadvertent loss of moisture from the bags, the specimens began to shrink at different rates. If proper sealing had been accomplished, both specimens most probably would have shown a constant value of shrinkage strain of approximately  $30 \times 10^{-6}$  in./in. during the entire test period. This expansion phenomenon has previously been shown to be the case.<sup>14</sup>

Restrained. Volume-change stresses during curing due to the restraint in the concrete imposed by the steel bar in the direct tensile

test were determined for all of the parameters listed in the first report. The direct tensile test was described in detail in the first paper.

Figures 2-2 and 2-3 represent the restrained concrete volume change stresses expressed as percentages of the direct tensile strengths for concretes made with cement factors of 4 and 5 sacks per cubic yard. Actual values of the stresses may be seen in Table 4-1. The expansion of the concrete when bag cured was apparent. In one case the compressive stresses due to restrained expansion have been found to be as high as 35 or 40 percent of the direct tensile stresses. In certain cases, such as a reinforced concrete pavement where small deflections are expected, these compressive stresses can be advantageous in reducing the tensile stress from the application of load.

Specimens which were oven dried developed very high restrained tensile stresses. Notice in Fig. 2-3 that these stresses might approach the actual tensile strength, leaving very little useable tensile capacities.

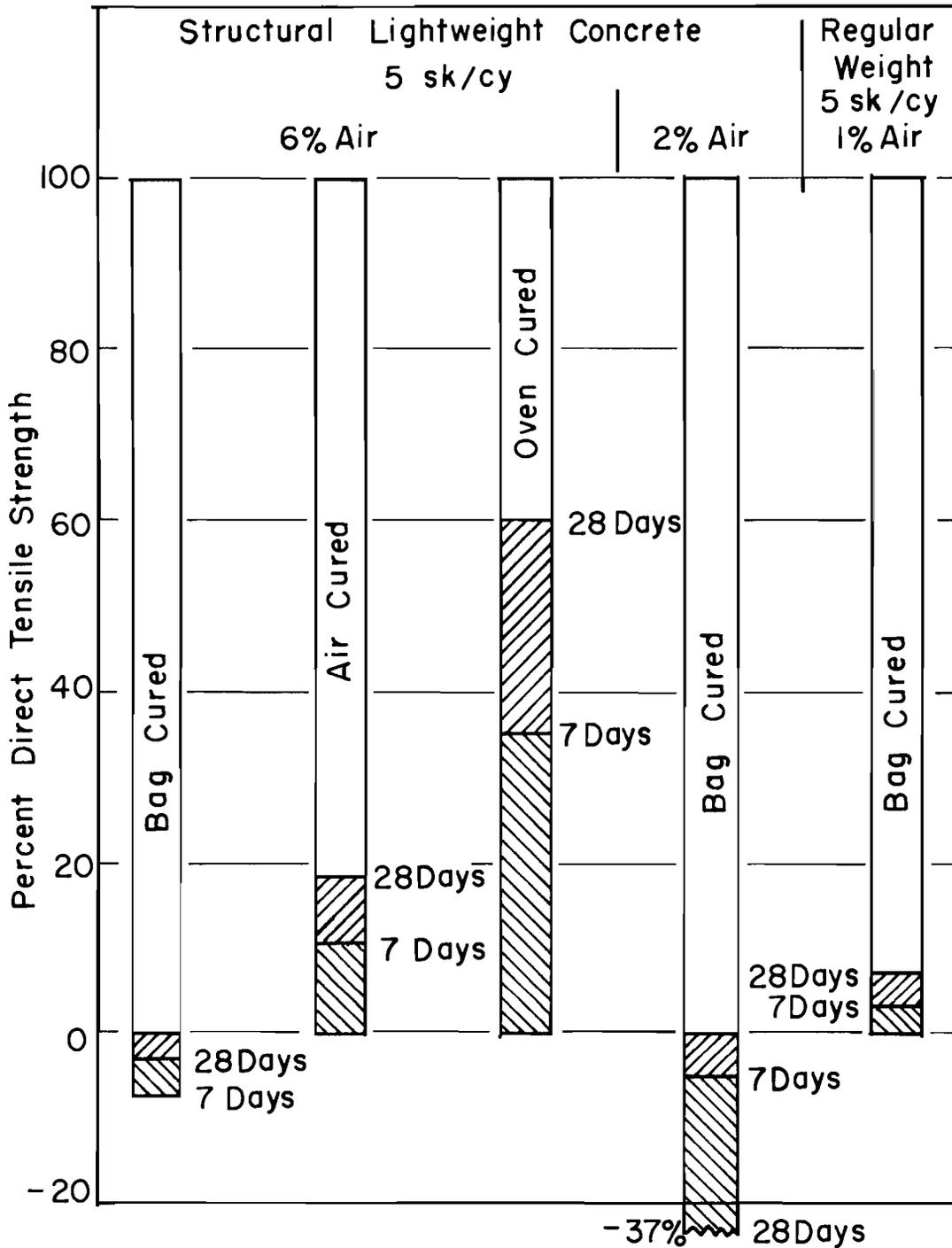
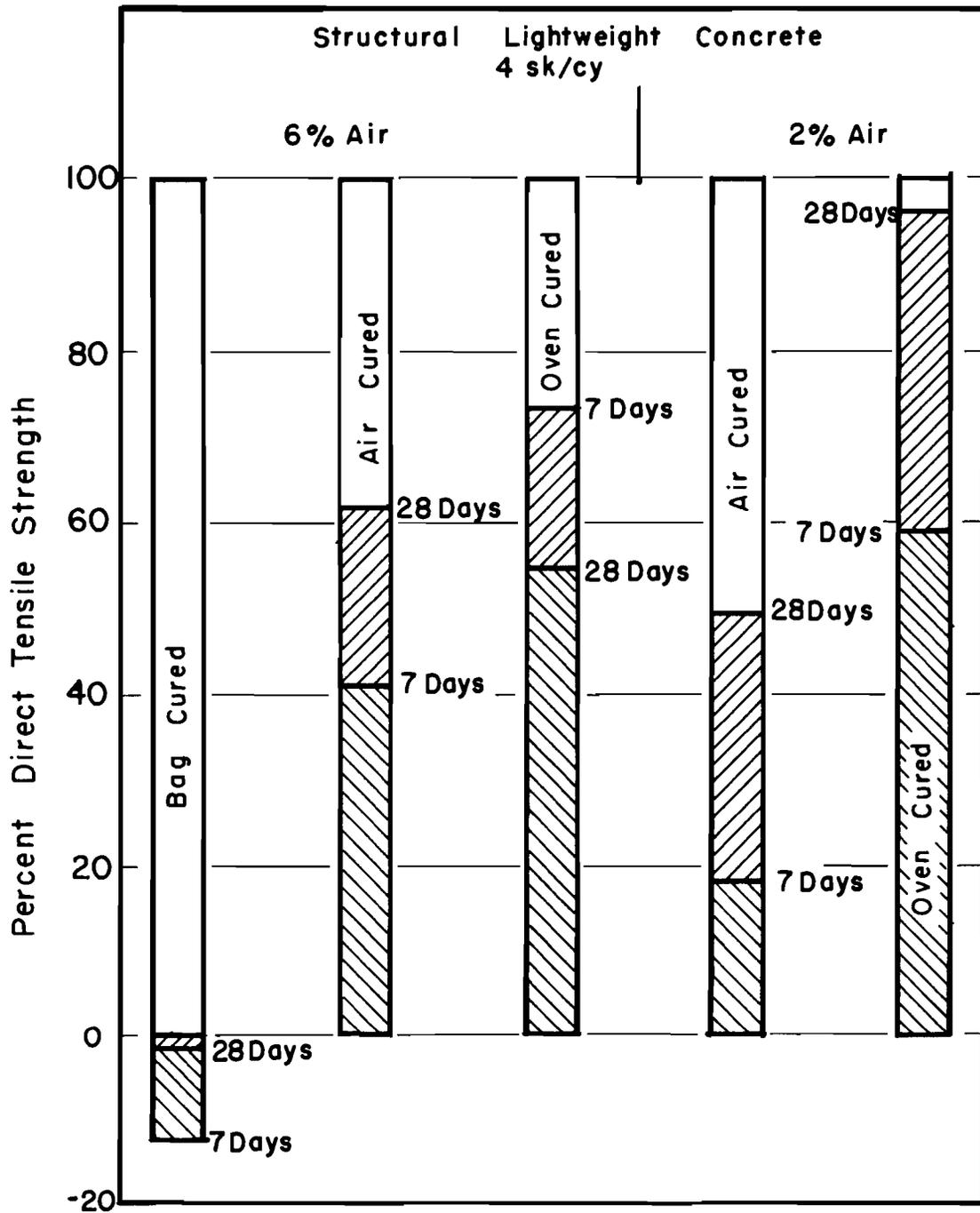


Fig. 2-2 Restrained Concrete Volume Change Stresses Expressed as Percent Direct Tensile Strength of the Concrete for CF = 5 sk/cy



**Fig. 2.3 - Restrained Concrete Volume Change Stresses Expressed as Percent Direct Tensile Strength of the Concrete for C.F.=4 sk/cy**

### 3. COMPRESSIVE, FLEXURAL AND SPLIT - CYLINDER STRENGTH COMPARISONS

#### 3.1 General

The Texas Highway Department uses the flexural strength as a measure of the concrete strength in the design of rigid pavements. For this reason, a brief discussion of some trends which were noticed during this investigation is presented in this chapter. Flexure tests were made with third-point loading eventhough the Texas Highway Department uses mid-point loading. A tabulation of all of the compressive, flexural, and split-cylinder strengths obtained for the various parameter studies is given in Table 8-1.

The indirect tension test, or split cylinder test, is described in detail in the first report.<sup>15</sup> In equation form, the split cylinder strength  $f_{sp}$  is equal to

$$f_{sp} = \frac{2P}{\pi DT}$$

where

P = Maximum applied load

D = diameter of specimen

T = length or thickness of specimen

It should be kept in mind, during the reading of this chapter, that flexural-strength determinations are quite dependent upon surface conditions existing at the outermost fibers of the concrete beam. Therefore considerable data scatter which usually occurs in flexural testing sometimes causes difficulty in presenting definite relationships. It is not the purpose of the

following discussion to present hard and fast relationships, but rather to present trends which were noticed in this data.

### 3.2 Flexural-Compressive

The relationship between flexural strength and compressive strength was statistically analyzed in the first report.<sup>16</sup> The ratio  $f_f/f_c$  ranged from 0.146 to 0.207 with a coefficient of variation of 43.4 per cent. This indicated that any suggested correlation between flexural and compressive strength would be questionable. However, a plot of flexural strength versus compressive strength for all of the parameters and curing conditions revealed the trends shown in Fig. 3-1. Even though a fairly large scatter in data is seen in Fig. 3-1, some correlation exists between the data points for a particular curing condition. Each curve shown was plotted by interpolating between a second and third-order fit of the data points. A computer solution was used to obtain the fits for the data points using a least squares technique.

It can be seen readily that for a particular compressive strength the bag-cured specimens tended to have higher flexural strengths than either the air-cured or oven-cured specimens. The oven-cured specimens resulted in the lowest flexural strengths. This was another indication that the bag-cured and oven-cured specimens represented two extreme curing conditions. The differences for the three curing conditions shown in Fig. 3-1 were undoubtedly caused by residual stresses or checking, crazing, or cracking in the surface fibers of flexural specimens. From the standpoint of flexural strength, it was apparent that any curing condition other than one that provides an environment which protects concrete from volume

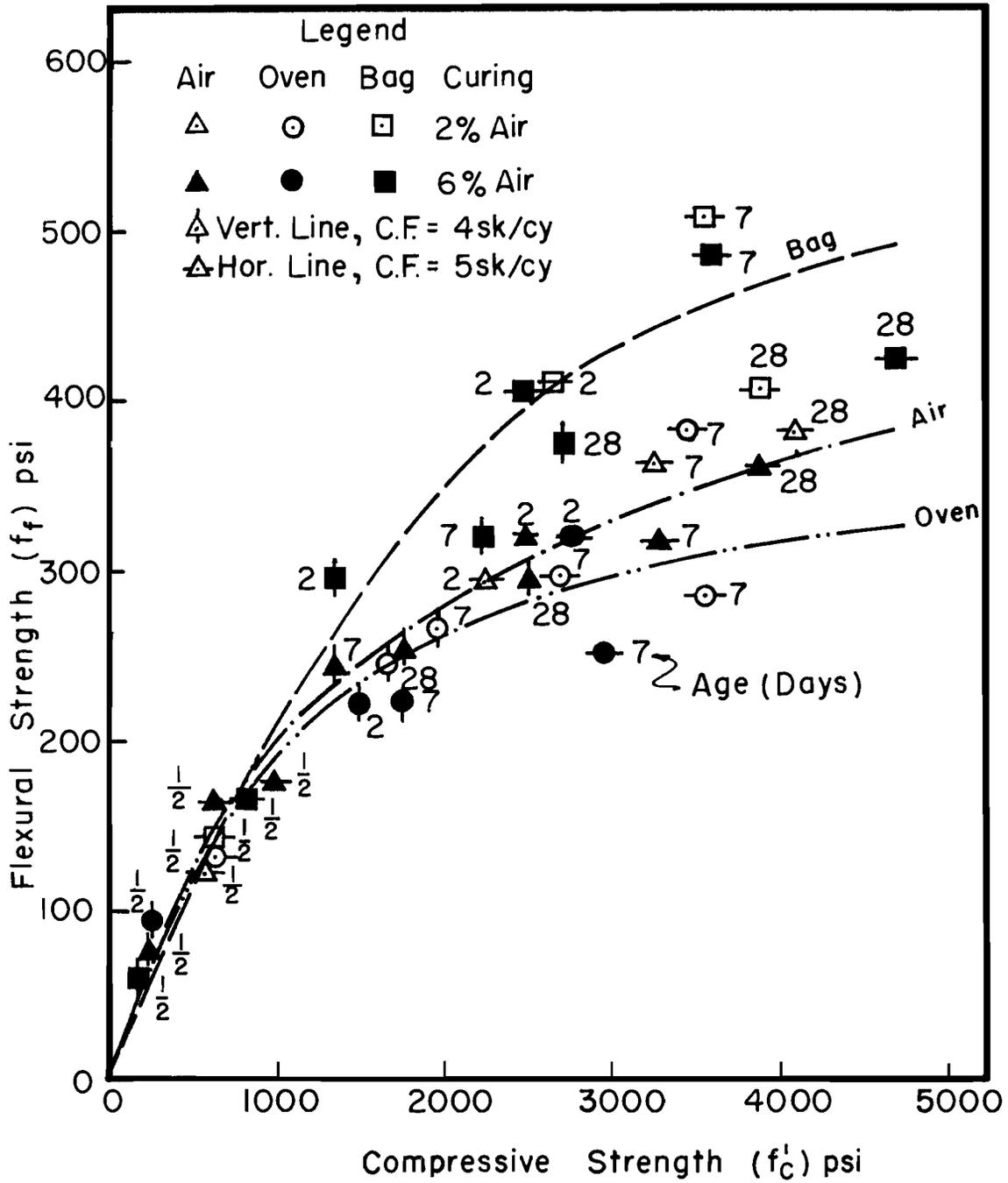


Fig 3.1-Relationship Trends Between Flexural and Compressive Strengths for Various Curing Conditions

change due to loss of moisture will result in loss of flexural strength because of residual tensile stresses.

### 3.3 Flexural-Split Cylinder

The relationship between split-cylinder strength and flexural strength was statistically analyzed in the first report<sup>16</sup> and little correlation was found. However, since split-cylinder strength and flexural strength have been frequently thought of as being a measure of the tensile strength of concrete, a plot of flexural strength versus split cylinder strength is shown in Fig. 3-2 for the various curing conditions studied. Notice that during the early ages for all curing conditions, the flexural tests developed strength values higher than the split cylinder tests. As the ages increased, the flexural strength tended to approach the split cylinder strength except for the bag-cured specimens which continued to develop higher strength values. This trend or relationship emphasizes the uncertainty of using the flexural-test to indicate the useable strength of lightweight concrete in situ.

### 3.4 Effect of Curing Condition on Flexural Strength

The effects of curing environment on the flexural strength are demonstrated in Fig. 3-3, in which air-cured and oven-cured strengths are plotted versus bag-cured strengths. Notice that for more than 2 days curing, the air-cured flexural strengths were approximately 78 per cent of the bag cured flexural strengths. The oven-cured strengths were approximately 72 per cent of the bag-cured strengths. These reductions in flexural strengths for air- and oven-cured environments were greater than reductions in split cylinder strengths caused by similar environments (see

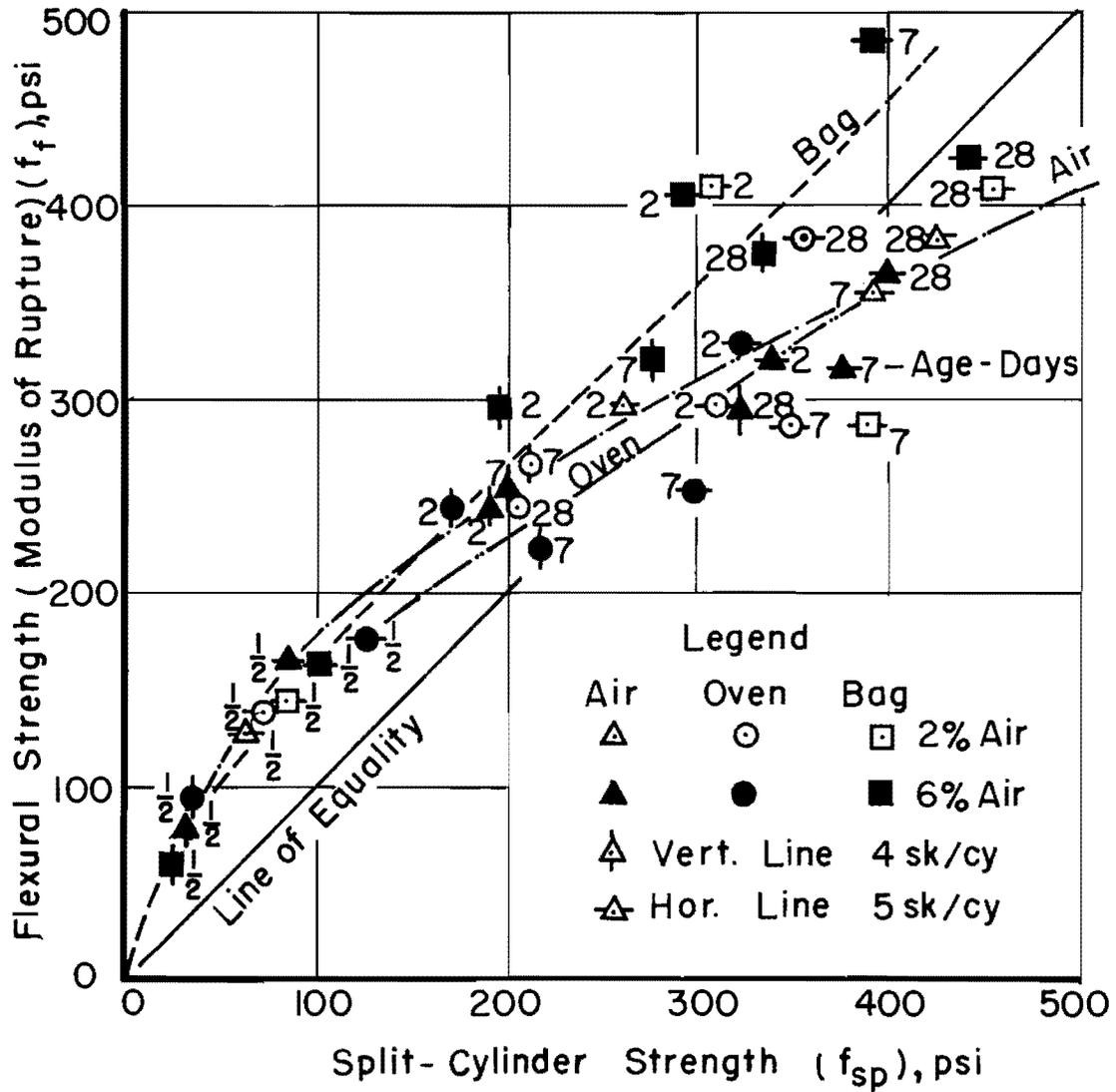


Fig. 3-2- Relationship Trends Between Flexural and Split-Cylinder Strengths for Various Curing Conditions.

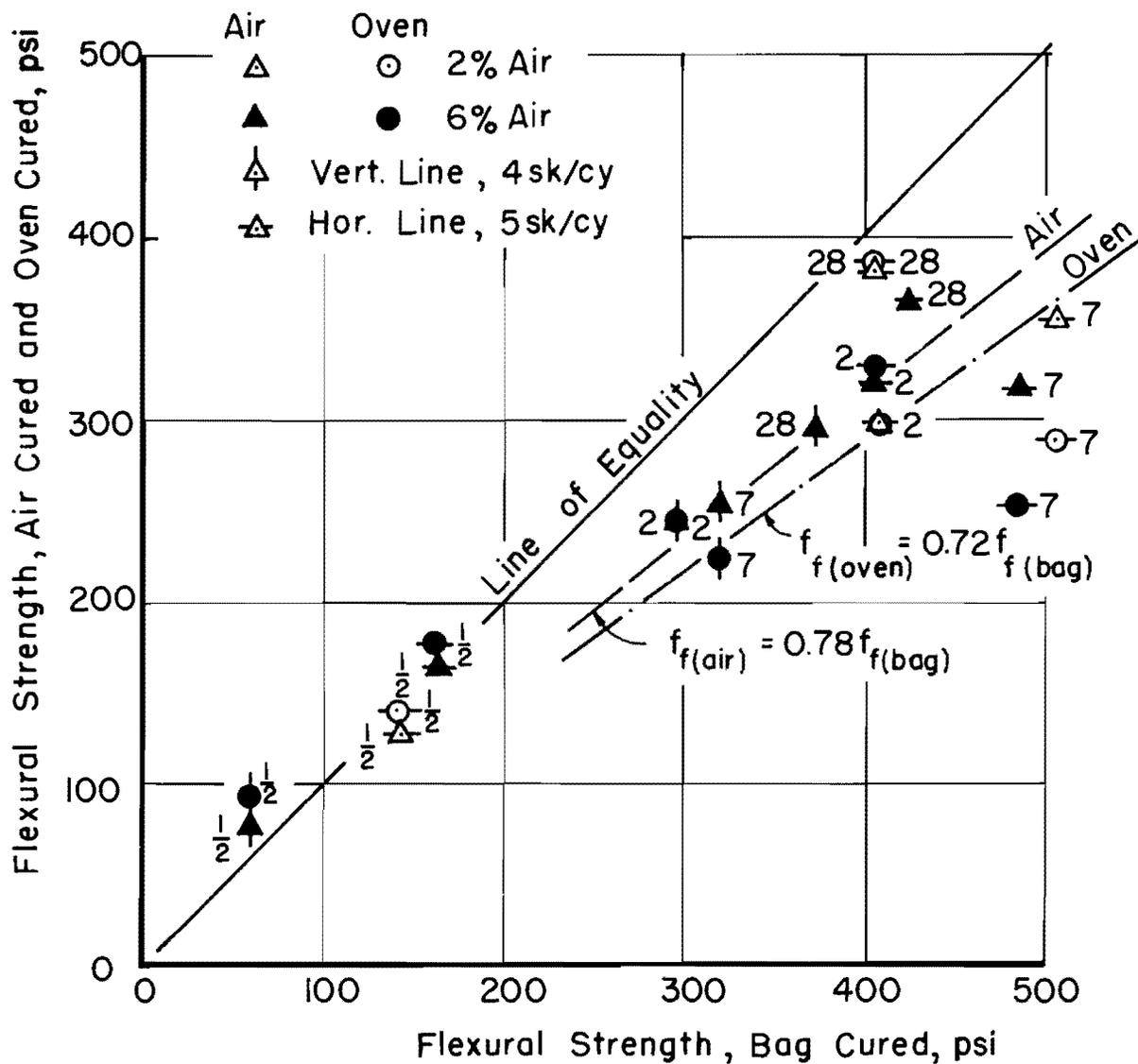


Fig. 3-3 Effect of Curing on Flexural Strength

Section 4.3). Apparently, at one-half day of age, surface cracking and crazing had not occurred sufficiently to alter the flexural-strength values.

## 4. TENSILE-STRENGTH PROPERTIES

### 4.1 General

Tensile stress-strain properties, the effects of curing conditions on tensile strengths, and the various relationships between direct tensile strengths, compressive strengths, and split-cylinder strengths will be presented in this chapter. Data used in plotting some of the curves in this chapter are shown in Table 4-1.

The flexural strength as an indicator of tensile strength of concrete is a property which is used by the Texas Highway Department in the design of continuously reinforced concrete pavements without transverse joints. Because the flexural strength and the tensile strength are used in the determination of the thickness of pavement and the amount of steel required, discussion in this chapter will be concerned with reductions in tensile strengths due to restraint under certain environmental conditions.

### 4.2 Effects of Environment on Tensile Stress-Strain Properties

All of the tensile stress-strain curves obtained in this investigation are presented in Figs. 4-1 through 4-10. Each curve represents a single test. The dashed portion of each of the curves, whose ordinate is labeled  $\sigma_{cz}$ , represents the amount of restrained concrete-volume-change stress present in the concrete prior to testing. The solid portion represents the stress-strain characteristics obtained from the direct tension test. The dashed portion of the curves do not represent the actual stress and strain behavior during the period prior to testing, but does represent the residual stress that existed in the concrete at the beginning of the tension

TABLE 4-1

STRUCTURAL LIGHTWEIGHT CONCRETE DIRECT TENSILE  
TEST DATA TABULATION (AVERAGE VALUES)

Cement Factor sks/cu yd	Air %	Age Days	Curing	$f'_c$ psi	$f_{sp}$ psi	$f_t$ psi	$\sigma_{cz}$ psi	$f_{tu}$ psi	$\epsilon_{ct} \times 10^6$	$\epsilon_{cz} \times 10^6$
5.0	6.0	7	Bag	3160	398	289	- 21	310	177	-10
			Air	2860	326	183	20	163	93	11
			Oven	2870	336	249	89	160	148	40
5.0	6.0	28	Bag	4920	505	375 <sup>1</sup>	- 24	399	--- <sup>1</sup>	- 6
			Air	3500	468	331	61	270	184	25
			Oven	3260	326	237	143	94	146	74
4.0	6.0	7	Bag	2170	296	180	- 22	202	99	- 9
			Air	1979	249	130	53	77	87	28
			Oven	1600	214	119	87	32	88	54
4.0	6.0	28	Bag	2620	280	178	- 3	182	84	- 1
			Air	2345	255	156	96	60	95	45
			Oven	1860	216	225	123	102	177	91

- Notes:
1. Bond, rather than tensile, failure occurred in this specimen
  2. This mix is regular-weight concrete (SG)
  3. For explanation of symbols used, see section 8-2

TABLE 4-1  
(Cont'd)

Cement Factor sks/cu yd	Air %	Age Days	Curing	$f'_c$ psi	$f_{sp}$ psi	$f_t$ psi	$\sigma_{cz}$ psi	$f_{tu}$ psi	$\epsilon_{ct} \times 10^6$	$\epsilon_{cz} \times 10^6$
5.0 <sup>2</sup>	1.0	7	Bag	4780	456	221	7	214	84	2
5.0 <sup>2</sup>	1.0	28	Bag	4750	496	372	26	346	92	5
5.0	2.0	7	Bag	3827	420	222	-11	233	102	-4
			Oven	3093	330	245	136	109	129	63
5.0	2.0	28	Bag	5475	455	223	-77	300	142	-24
4.0	2.0	7	Air	1944	285	166	30	136	105	12
			Oven	1807	195	158	94	64	95	48
4.0	2.0	28	Bag	2601	329	190	0	190	113	0
			Air	2210	368	207	102	105	78	20
			Oven	1740	210	142	136	6	54	41

- Notes:
1. Bond, rather than tensile, failure occurred in this specimen
  2. This mix is regular-weight concrete (SG)
  3. For explanation of symbols used, see section 8-2.

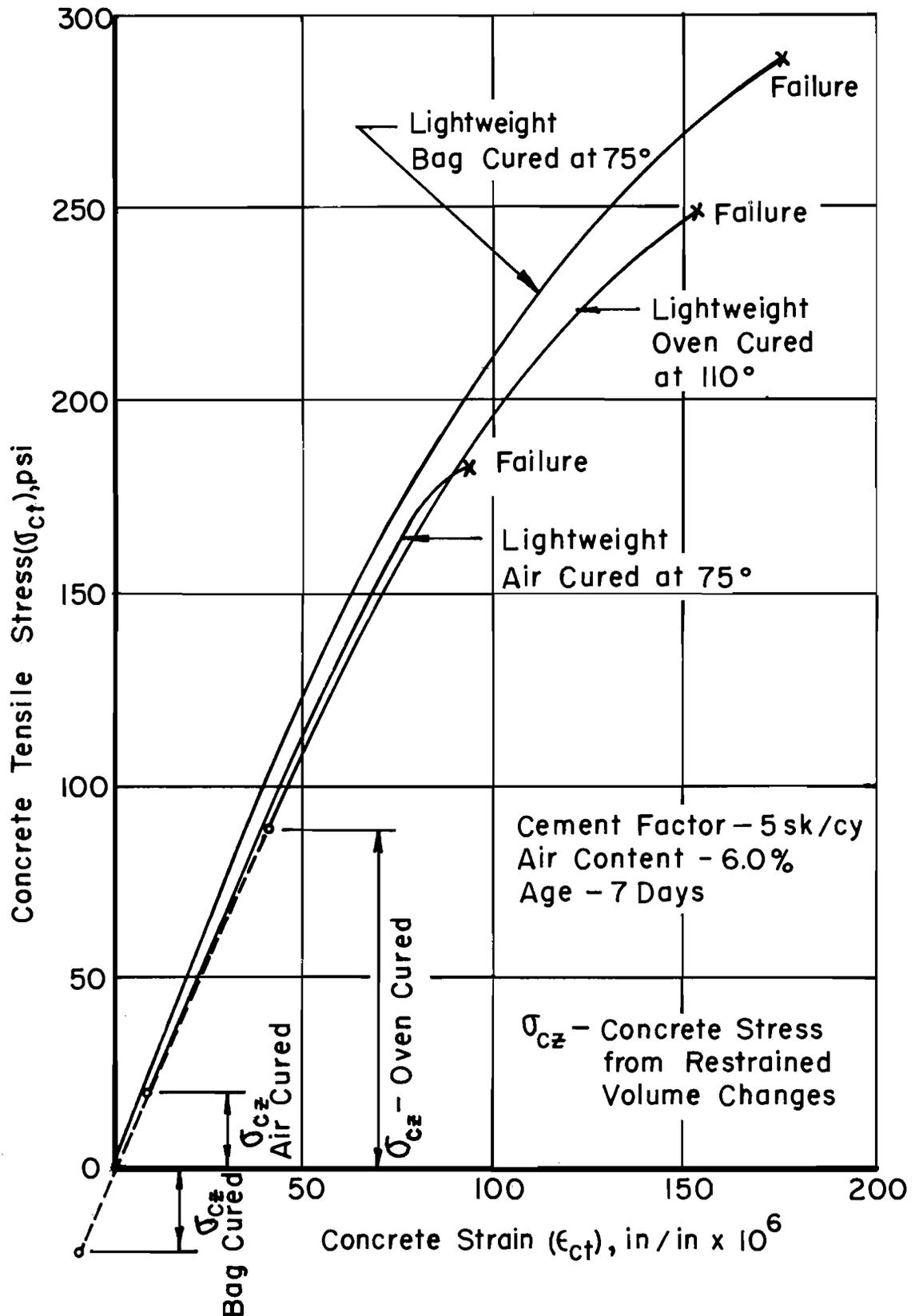


Fig. 4-1 - Stress - Strain Curves For Three Curing Conditions For C.F. = 5sk/cy, 6% Air, and Age of 7 Days

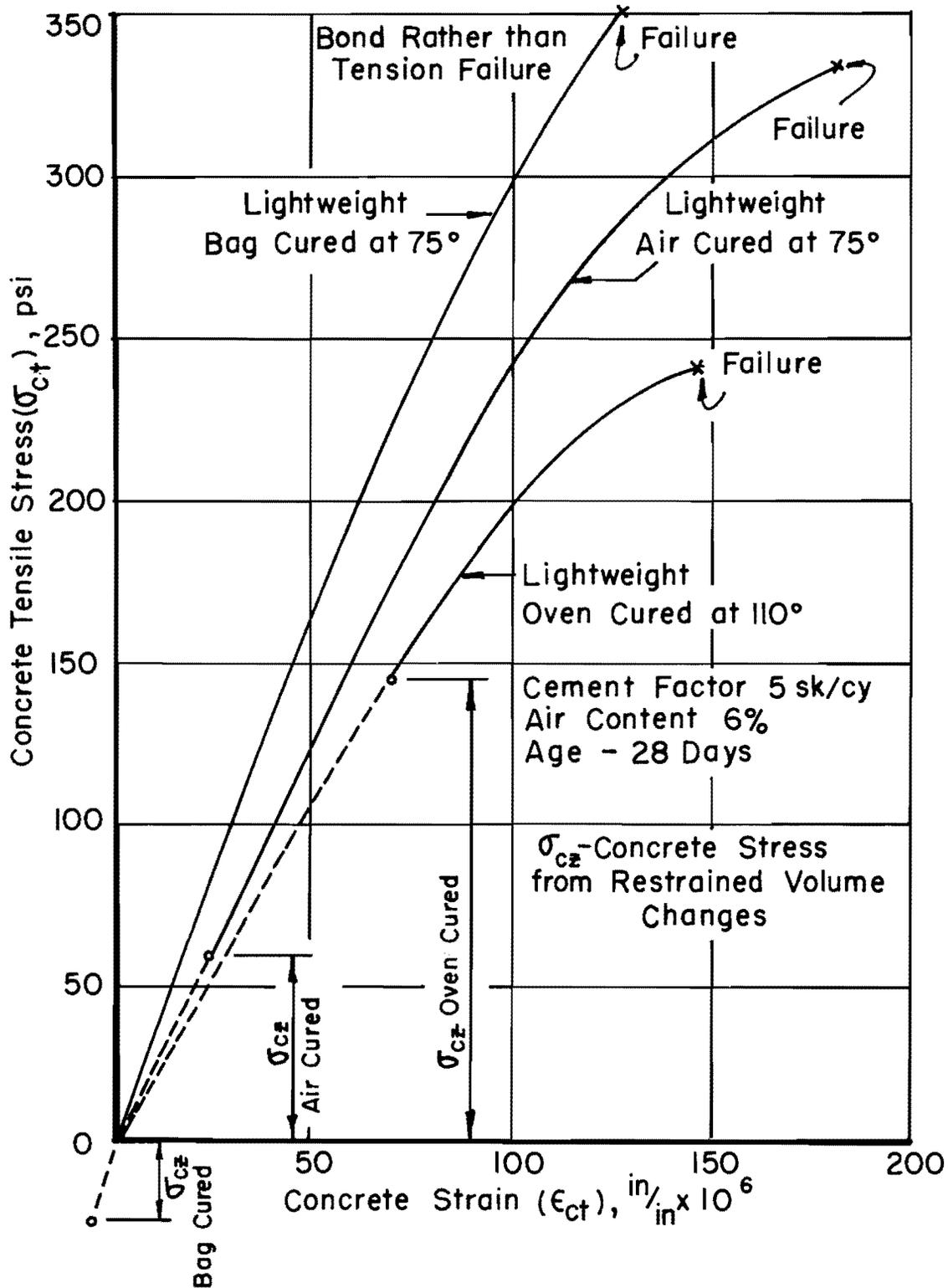


Fig. 4-2- Stress-Strain Curves for Three Curing Conditions for C.F. = 5sk/cy, 6% Air, and Age of 28 Days.

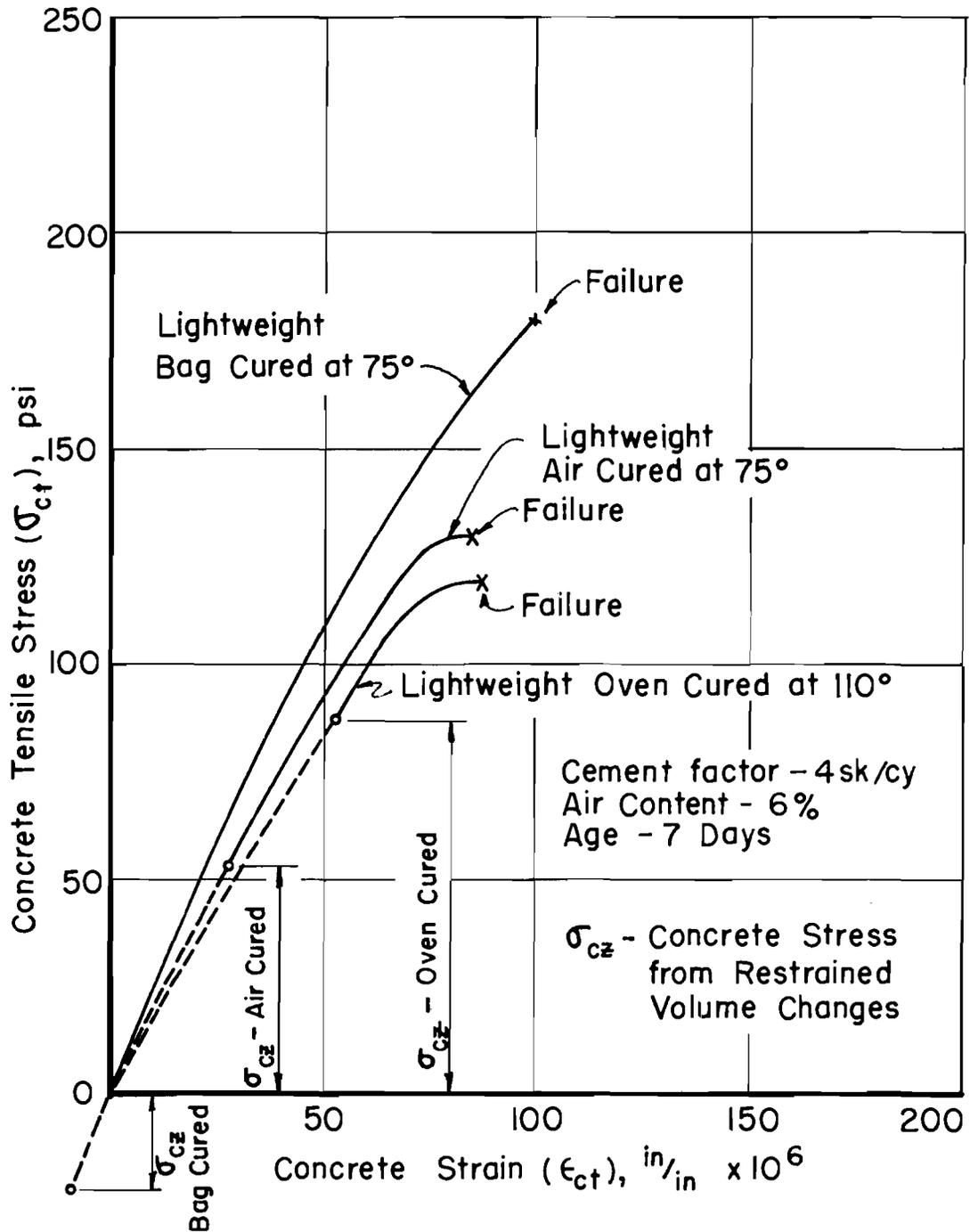


Fig 4-3 - Stress-Strain Curves for Three Curing Conditions for C.F = 4 sk/cy, 6% Air, and Age of 7 Days

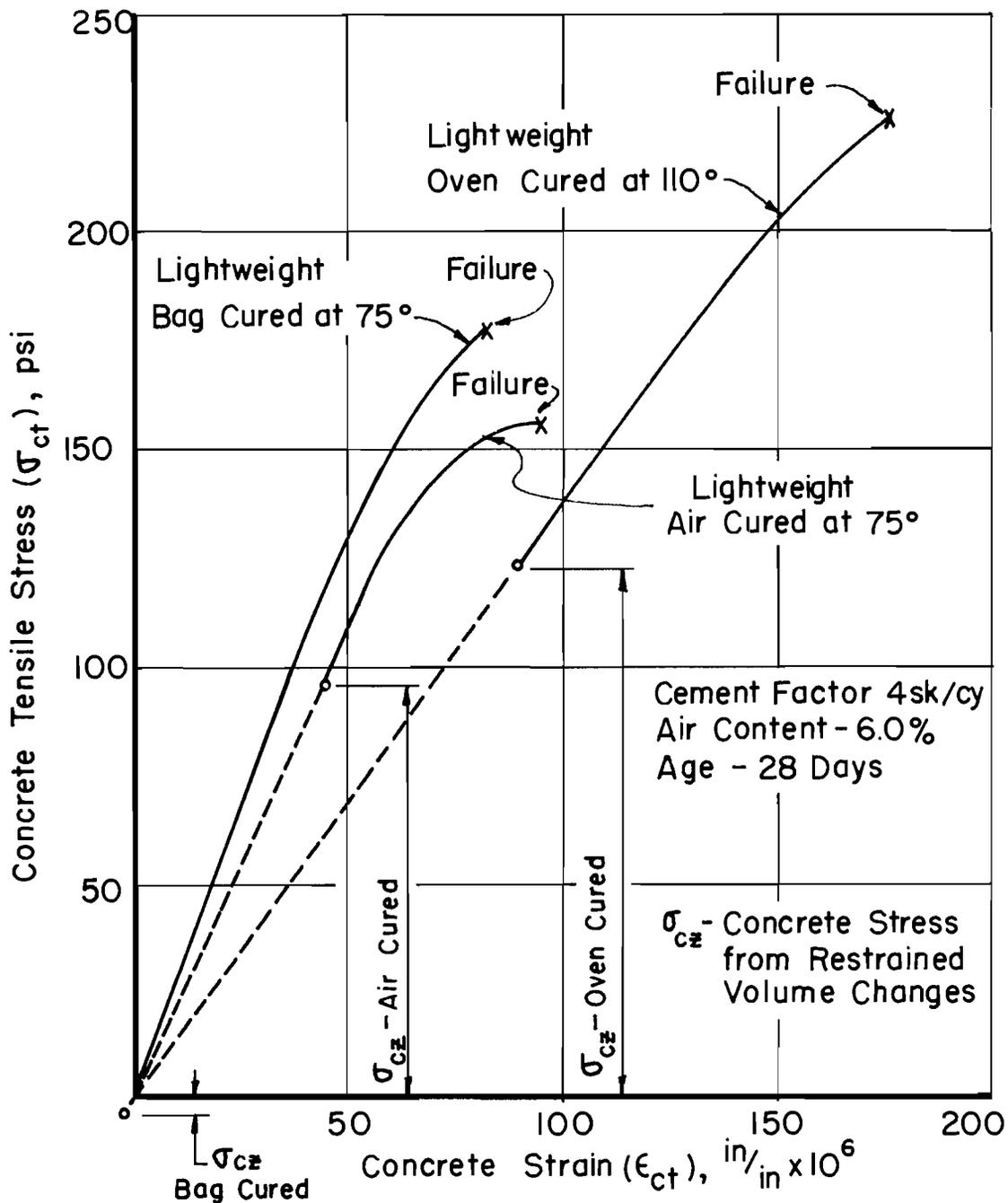


Fig. 4-4 - Stress-Strain Curves for Three Curing Conditions for C.F. = 4 sk cy, 6% Air, and Age of 28 Days

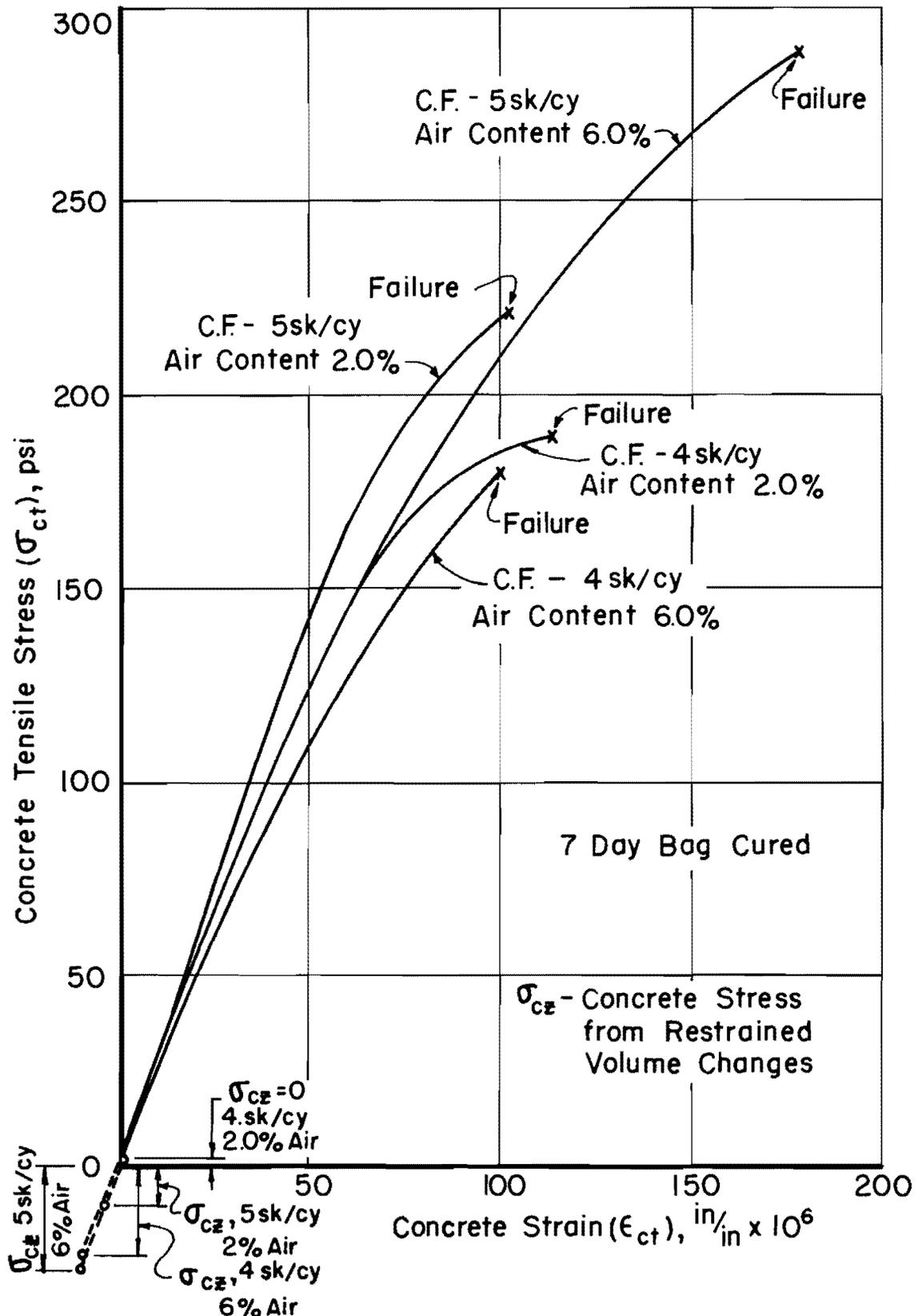


Fig.4-5 - Stress-Strain Curves for 7-Day Bag Cured Specimens with Different Cement Factors and Air Contents .

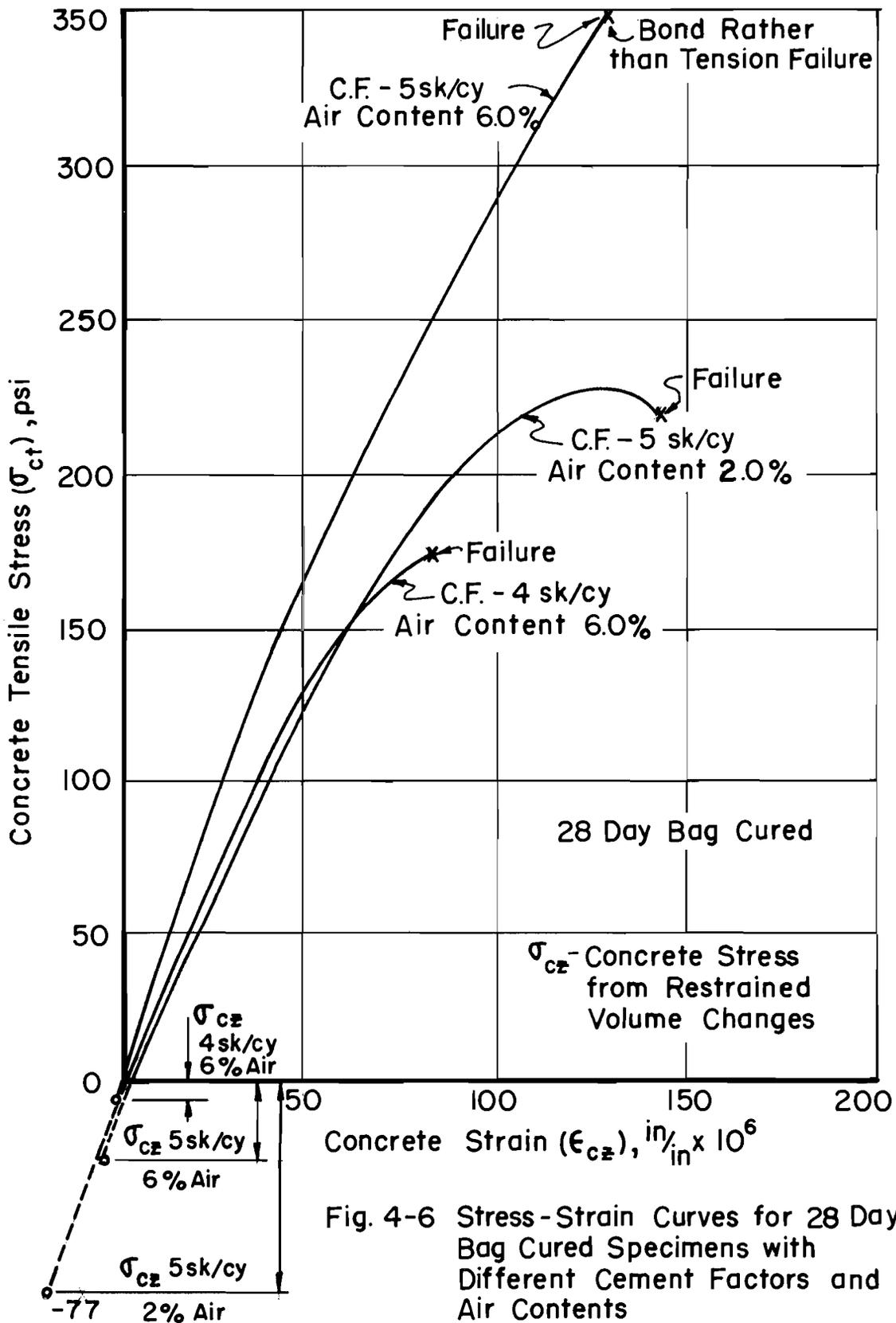


Fig. 4-6 Stress-Strain Curves for 28 Day Bag Cured Specimens with Different Cement Factors and Air Contents

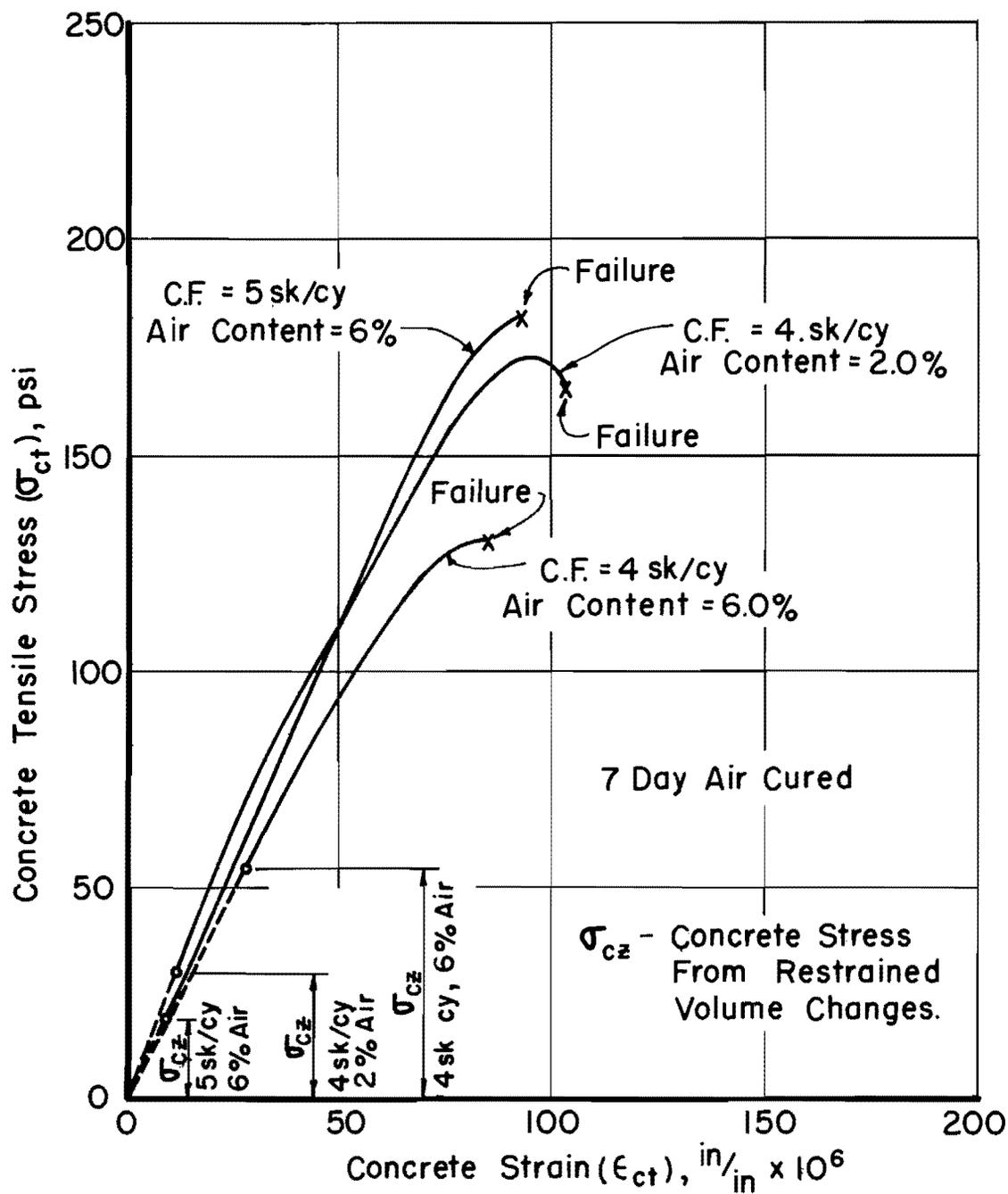


Fig 4-7 Stress Strain Curves For 7-Day Air Cured Specimen with Different Cement Factors and Air Contents.

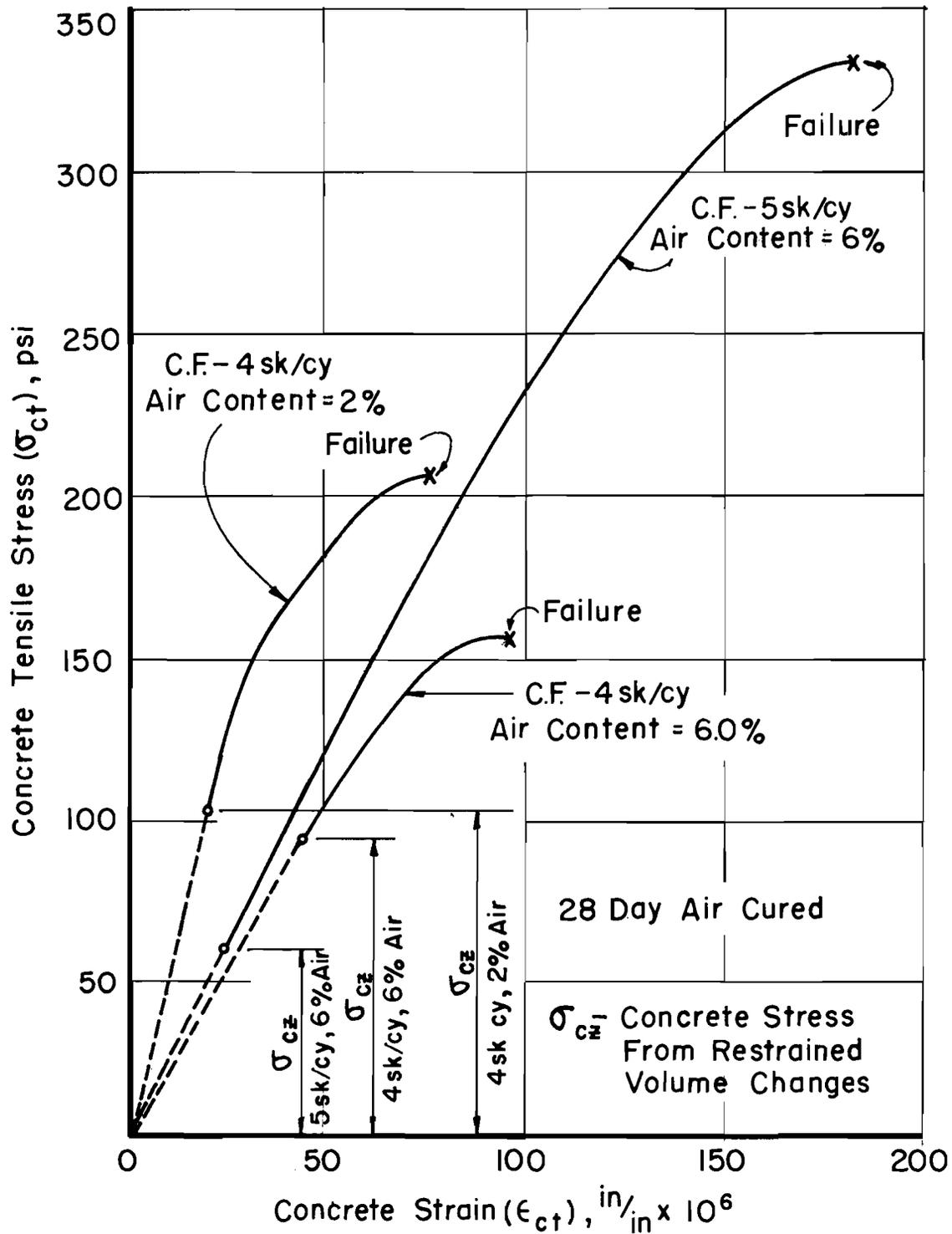


Fig. 4-8 - Stress Strain Curves for 28 Day Air Cured Specimens with Different Cement Factors and Air Contents.

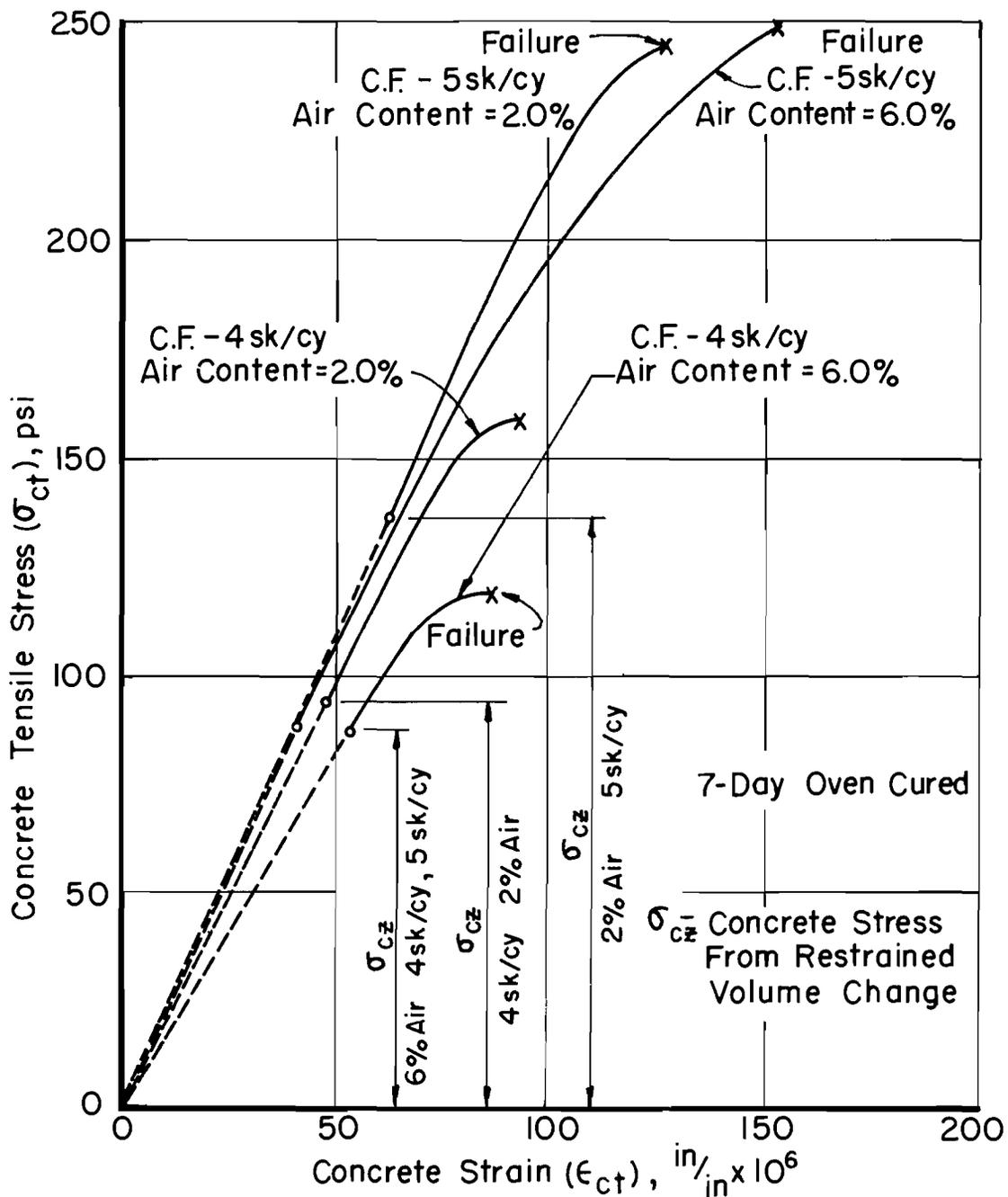


Fig. 4-9 - Stress Strain Curves for 7-Day Oven-Cured Specimens with Different Cement Factors and Air Contents.

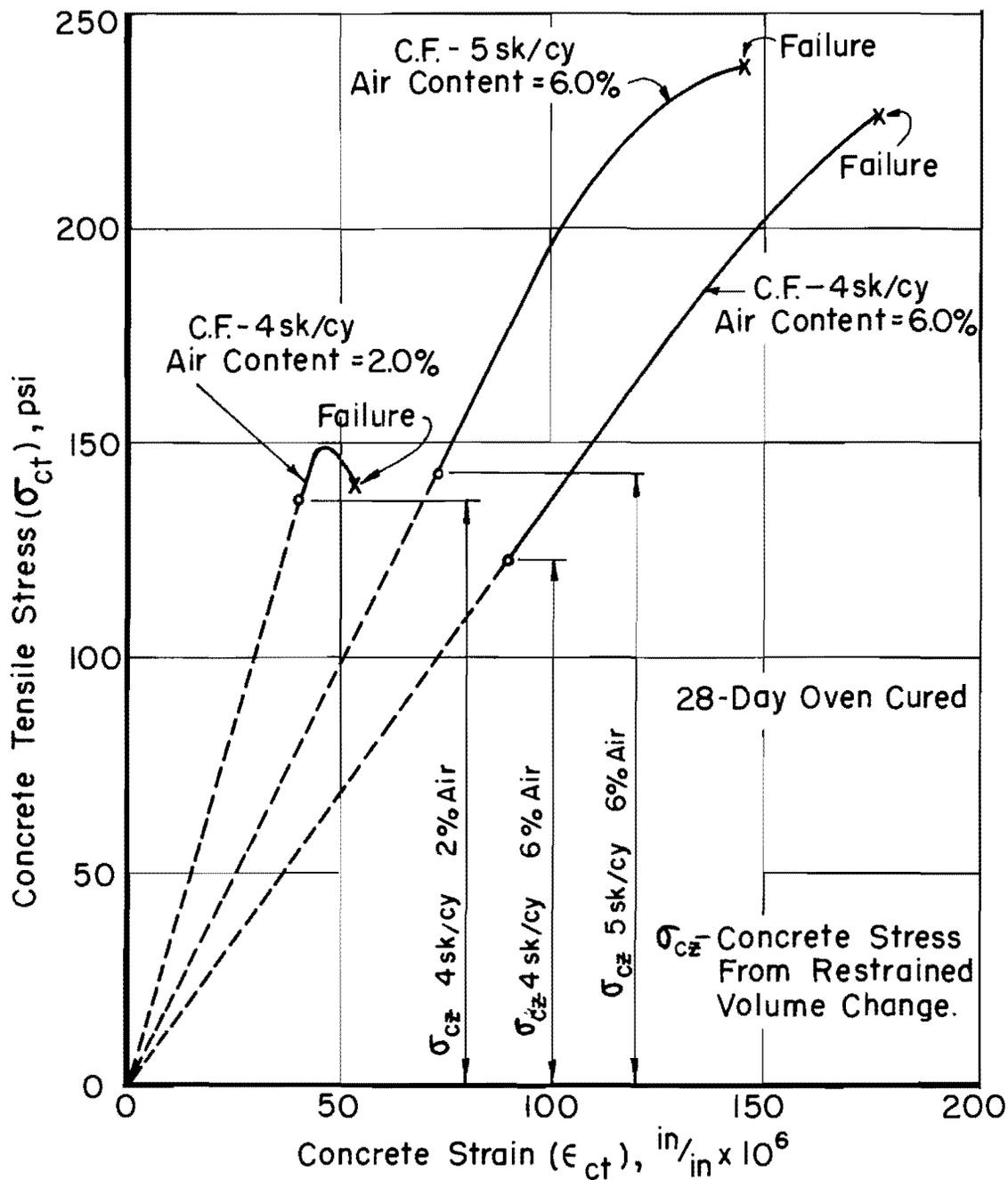


Fig. 4-10 - Stress Strain Curves for 28-Day Oven Cured Specimens With Different Cement Factors and Air Contents.

test. The corresponding residual strain was determined by the procedure given in the following paragraph.

The strain at the beginning of the solid portion of the stress-strain curves was determined by projecting a straight line with a slope of  $E$ , determined from the loading phase of the tension test, back to zero stress. This represents the strain that would relieve the residual stress using the modulus of elasticity existing at the time of test. The actual residual strain, which was affected by creep and changes in the modulus of elasticity with age, was obviously larger.

Since each curve represents only a single test, values of tensile strength, tensile strain, and modulus of elasticity as found from these curves may not be the generally expected value.

Observation of these stress-strain curves revealed several important factors which affect the tensile strength of concrete. In general, the bag-cured specimens exhibited the highest tensile strength and the highest modulus of elasticity. Air-cured specimens had the next highest tensile strength and oven-cured specimens had the lowest tensile strength.

By comparing Fig. 4-1 with Fig. 4-2, and Fig. 4-3 with Fig. 4-4, it was seen that the ultimate tensile strength at 28 days was generally larger than that at 7 days regardless of environment. Note also that by comparing Fig. 4-1 with Fig. 4-3, and Fig. 4-2 with Fig. 4-4, the slope's (modulus of elasticity) for the concretes with 5 sacks per cubic yard are steeper than for the concretes with 4 sacks per cubic yard. This will be discussed further in Chapter 5.

The results shown in Figs. 4-5 and 4-6 for bag-cured specimens were about as they were expected to be except that in regard to air-cured the

tensile strength of the 5-sack, 6-per cent air specimen was high, and the 5-sack, 2-per cent air specimen was low. The 4-sack tests resulted in lower values than the 5-sack tests, and the higher air contents resulted in lower values of modulus of elasticity. Note also in Figs. 4-5 and 4-6 that the residual stresses for bag cured specimens tended to be in compression for both 7 and 28-day tests, varying from 0 psi to 77 psi in compression.

Figures 4-7 and 4-8 show residual tensile stresses due to shrinkage of 20 to 53 psi at 7 days and 61 to 102 psi at 28 days. These curves tended to be in their anticipated positions except for the 4-sack, 2-per cent air, 28-day test which resulted in a curve that was too steep with respect to the generally accepted relationship.

The stress-strain curves for the oven-cured specimens shown in Figs. 4-9 and 4-10 show residual tensile stresses due to shrinkage of 87 psi to 136 psi at 7 days, and 123 to 143 psi at 28 days. In general these curves fell into proper position except for the 4-sack, 2-per cent air, 28-day test.

Figures 4-9 and 4-10 for oven curing show very high tensile stresses in the concrete due to restrained volume changes. On the other hand, Figs. 4-5 and 4-6 for bag curing showed expansion before testing, thus causing a compressive stress in the concrete. This aspect has been discussed in Section 2.3 of this report, but is mentioned again to emphasize once more that bag and oven curing represented two extreme curing conditions.

One other factor, which is shown in Figs. 4-2 and 4-6, was the failure of the bag-cured specimen in bond rather than tension. This probably suggests that bond strength was reduced as a result of an environment such as bag curing which reduces the tendency for shrinkage to occur. This aspect

merits further investigation.

The restraint which is given to the concrete volume changes by the bar in the direct tensile test is quite similar to that of a reinforcing bar in concrete pavements. The percentage of reinforcement is different, however. Deep concrete which has no volume change except near the surface where environmental conditions vary has this same type of restraint. The difference between the ultimate tensile strength ( $f_t$ ) and the restrained concrete stress ( $\sigma_{cz}$ ) represents the useable tensile strength ( $f_{tu}$ ). Note in Figs. 4-9 and 4-10 that the useable tensile strength was greatly affected by oven curing.

#### 4.3 Effect of Curing Condition on Split Cylinder Strength

From the results presented thus far, bag curing provided an environment that was more favorable for hydration resulting in higher tensile strengths. Fig. 4-11 is a plot of air-cured and oven-cured split cylinder strengths versus bag-cured split-cylinder strengths. Bag curing is used here as a base and the line of quality represents equal air-cured and oven-cured strengths with companion bag-cured strengths. A least squares fit of the first order to data points was made with a computer for the two curing conditions. Notice that the air-cured specimens exhibited only about 91 per cent of the split-cylinder strengths for the bag cured specimens. The oven-cured strengths were only about 77 per cent of the bag-cured strengths.

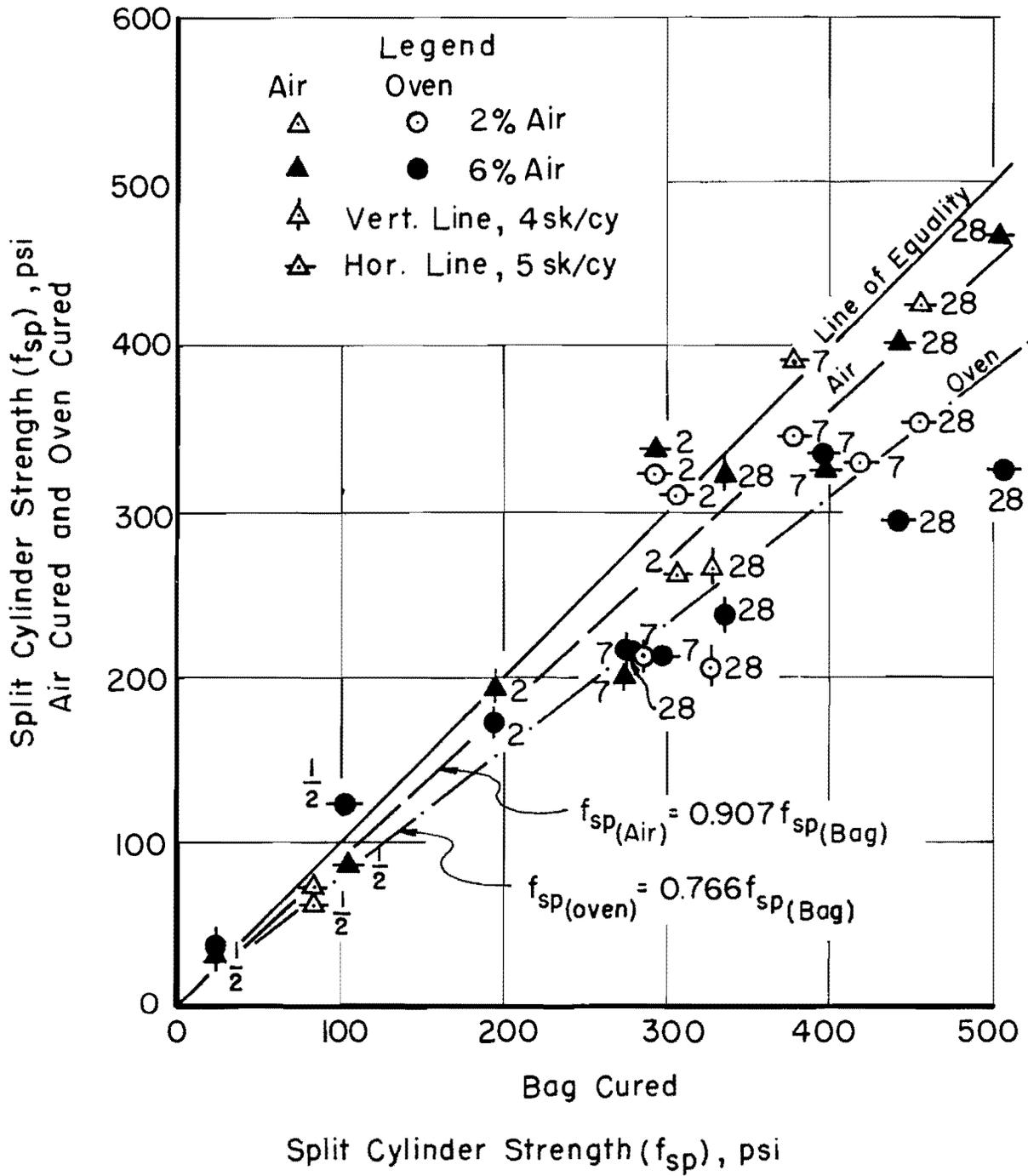


Fig.4-II- Effect of Curing Conditions on Split Cylinder Strengths.

#### 4.4 Relationship Between Direct Tensile and Compressive Strengths

Figure 4-12 shows a plot of all of the compressive-strength and companion direct-tensile-strength data which were collected in this investigation. Some of the plotted points were average values from two or more tests. Even though some scatter in the data exists, particularly for the bag cured, 2 per cent air-content specimens, a definite straight-line correlation was determined between the direct tensile strength and compressive strength. The tensile strength was determined to be approximately 7.0 per cent of the compressive strength for all curing conditions and strengths investigated in this study. There is a wide range of strengths represented in Fig. 4-12 which helps to validate the results.

As discussed earlier, the loss in tensile capacity of structural lightweight concrete that is restrained from volume changes by reinforcement has been found to be significant, if improper curing procedures are followed. To emphasize this, in Fig. 4-13 the per cent of the useable tensile strength has been plotted against type of curing from the results reported in Table 1. Note the significant loss in available or useable tensile strength between the bag cured and either the air or oven dried specimens. The following relationship between useable tensile strength ( $f_{tu}$ ) and compressive strength was determined to exist for data collected:

$$f_{tu} = 0.08 f'_c - 64 \quad - - - - - (4-1)$$

It should be emphasized that this is not a fixed relationship because of the variable environmental conditions.

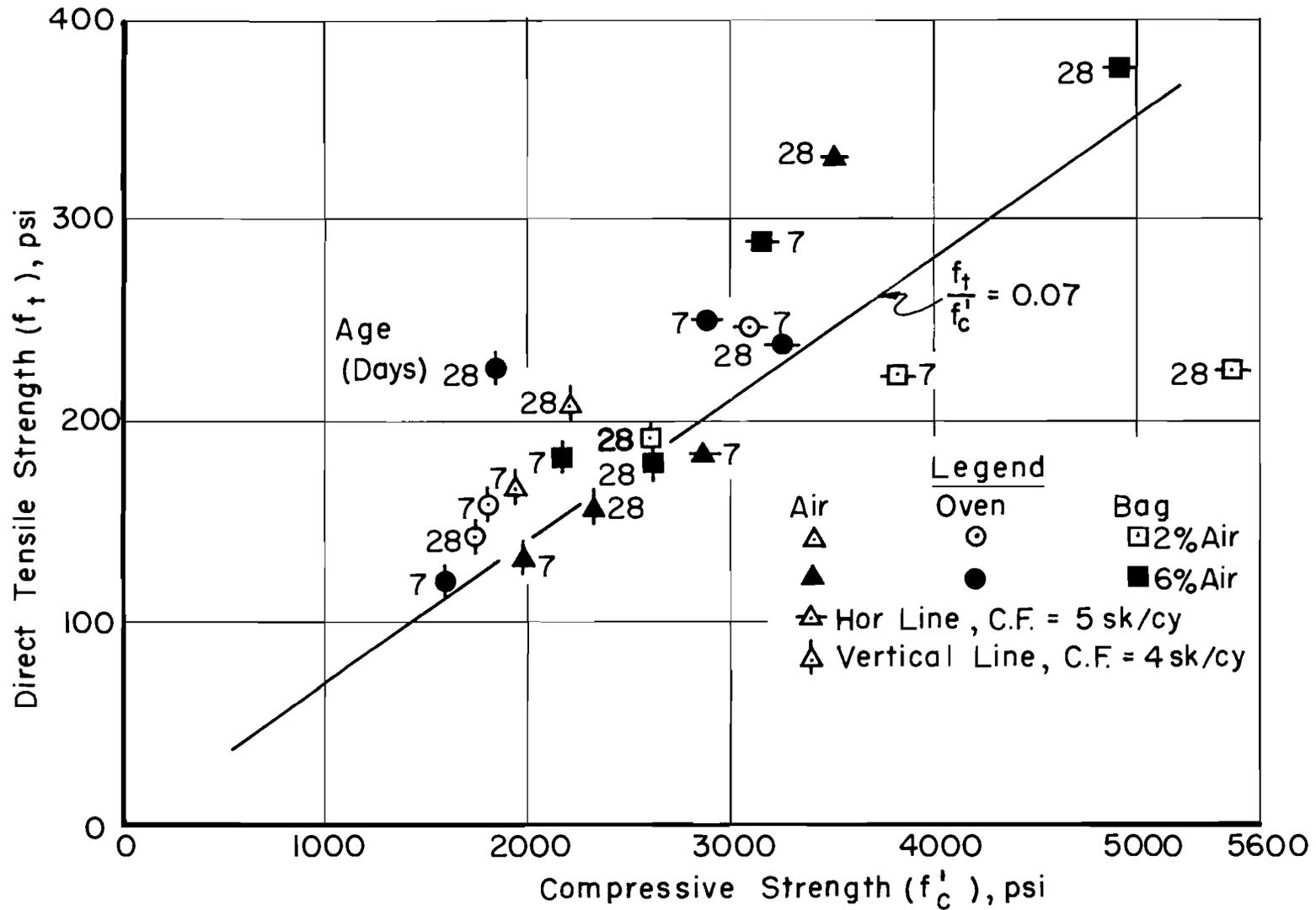
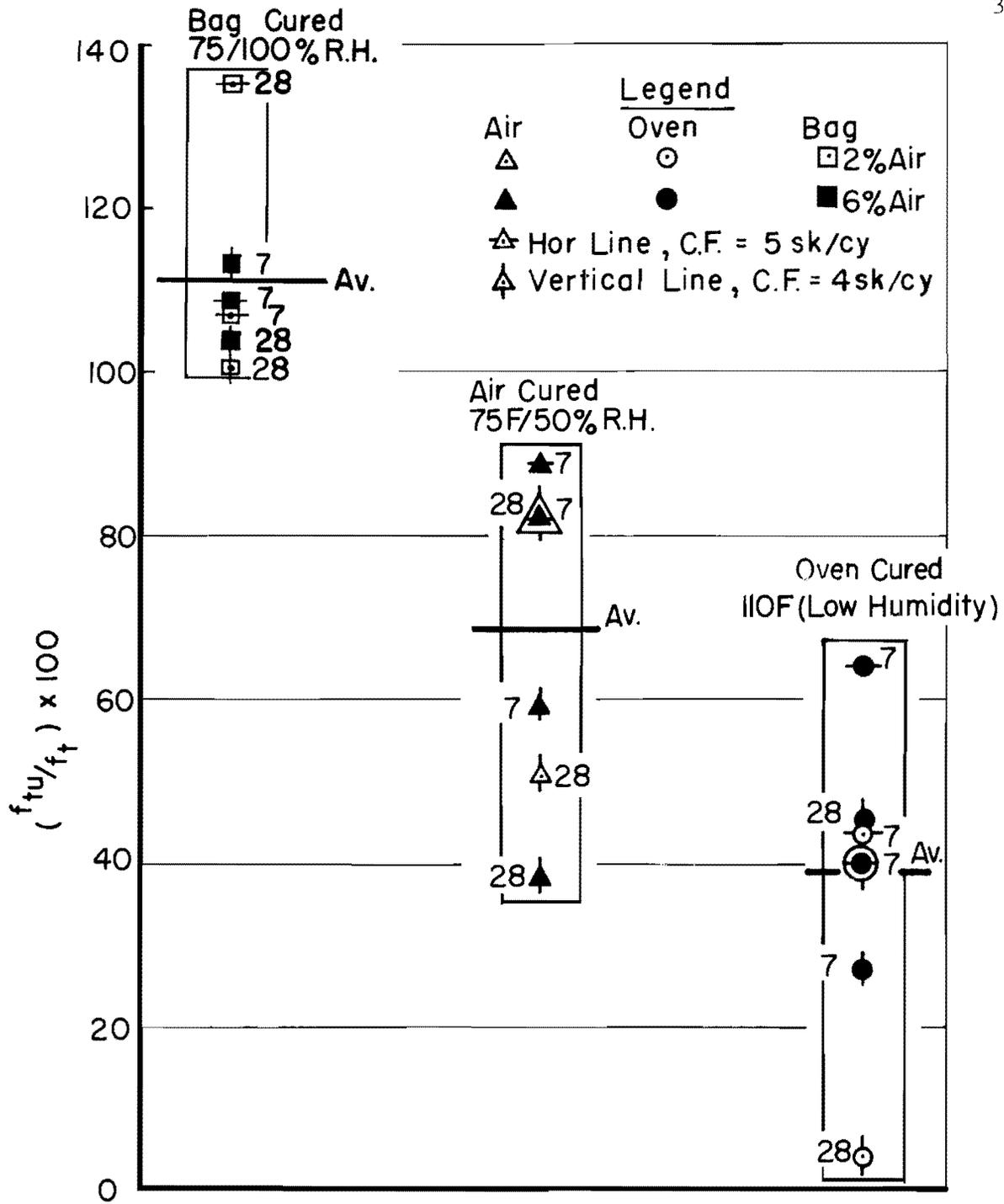


Fig. 4-12-Relationship Between Direct Tensile Strength ( $f_t$ ) and Compressive Strength ( $f'_c$ ).



Concrete Curing Conditions

Fig. 4-13- Percent of Total Available Tensile Strength for Restrained Structural Lightweight Concrete as a Function of Curing Conditions.

#### 4.5 Relationship Between Direct Tensile and Split-Cylinder Strengths

The relationship between direct tensile strength and split-cylinder strength is shown in Fig. 4-14. Here again, a very good strain-line correlation exists. The split-cylinder strength should be higher than the direct tensile strength mainly because of the difference in the stress conditions of the two tests. With the aid of Fig. 4-14, the direct tensile strength can be approximated from the following equation:

$$f_t = 0.07 f'_c = 0.66 f_{sp} \quad - - - - - (4-2)$$

In cases where the tensile strength of the concrete is used and where restraint to concrete volume change exists, the useable tensile strength should be a more accurate design value than tensile strength, flexural strength, or a percentage of the compressive strength. Fig. 4-15 is a plot of useable tensile strength versus the split cylinder strength for all three environmental conditions. The general trend as determined from a computer analysis for the three environmental conditions indicates that there is no fixed relationship between useable tensile strength and split-cylinder strength. For comparison purposes the relationship for all data, although not a firm relationship, was computed to be:

$$f_{tu} = 1.03 f_{sp} - 168 \quad - - - - - (4-3)$$

This emphasizes the fact that a high value of the useable tensile strength can not be assumed without proper curing and close control. The general trend for compressive strength (Eq. 4-1) and split cylinder strength (Eq. 4-2)

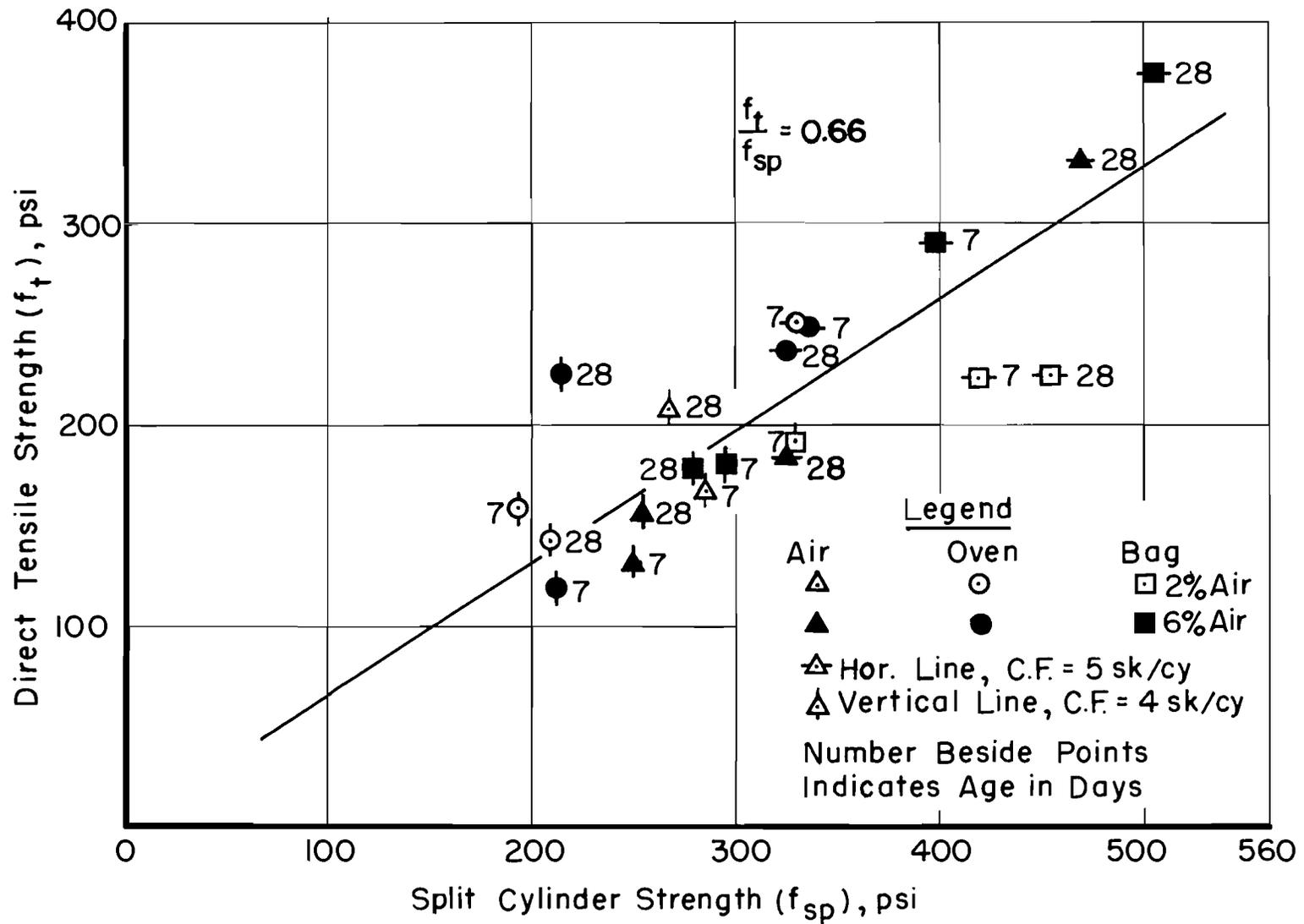


Fig. 4-14- Relationship Between Direct Tensile and Split Cylinder Strengths.

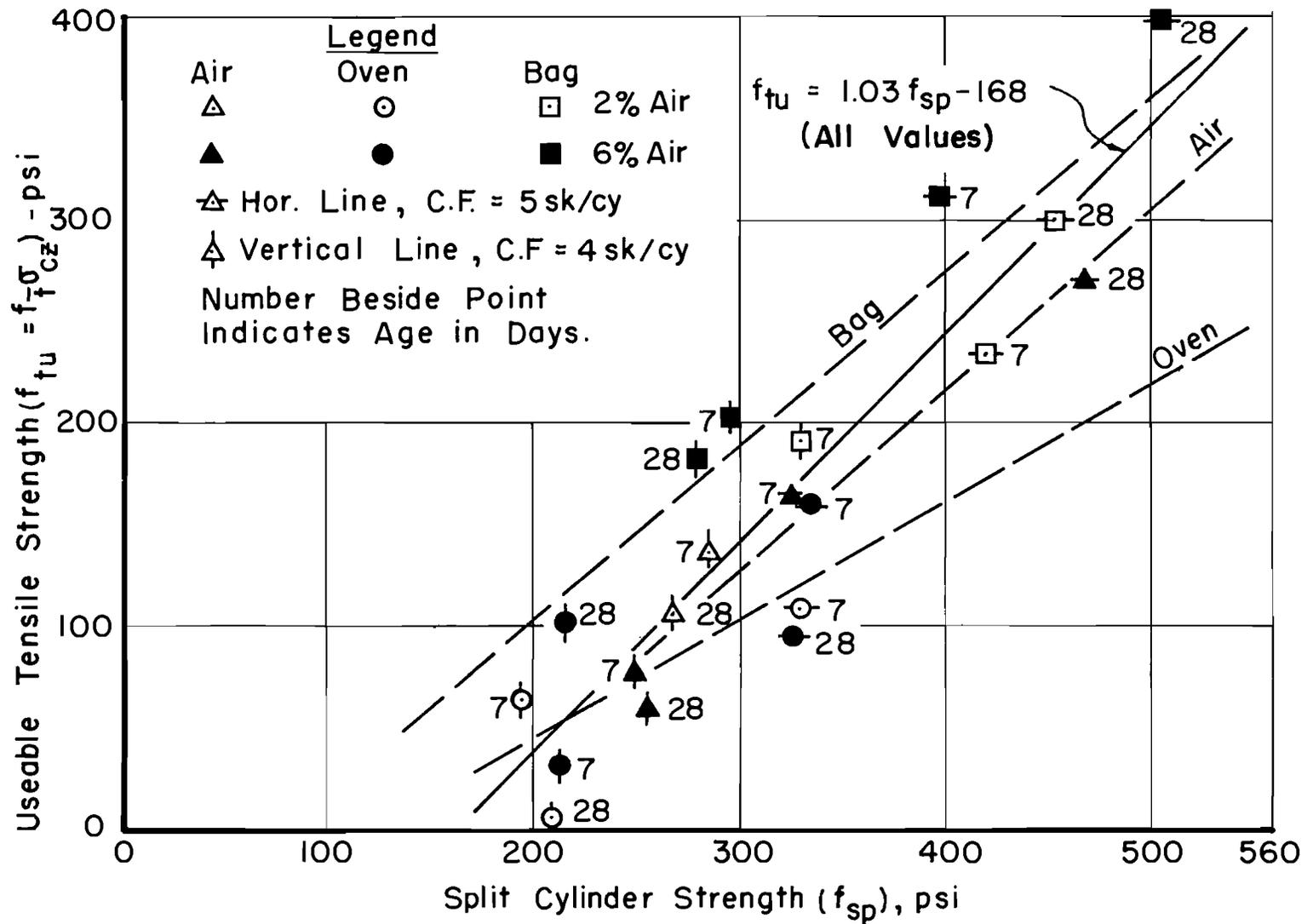


Fig. 4-15- Relationships Between Useable Tensile Strength and Split Cylinder Strengths

are shown in:

$$f_{tu} = 0.08 f'_c - 64 = 1.03 f_{sp} - 168 \quad \text{---(4-4)}$$

## 5. STATIC MODULUS OF ELASTICITY

### 5.1 General

The value of the modulus of elasticity of concrete is not a readily definable quantity when compared to some of the other physical properties of concrete. It might have a large range of values for any given set of conditions. Even though at least four different methods (ASTM E6) are recognized for defining and computing the modulus of elasticity of concrete, a wide range usually exists when any one method is used.

The modulus of elasticity of structural lightweight concrete is a very important property in determining deflections and strains of the concrete under load. The static modulus of elasticity was determined along with other test properties mentioned in this report. For purpose of comparison, the modulus of elasticity of a regular-weight concrete was determined.

A summary of all modulus of elasticity data is given in Table 8-2. All modulus-of-elasticity values given in Table 8-2 are secant moduli at 50 per cent of the ultimate strength in compression or 50 per cent of the ultimate tensile strength.

Effect of age and curing conditions. The effect of age upon the modulus of elasticity for three different curing conditions and the same mix design is represented in Fig. 5-1. In this figure it is seen that the modulus of elasticity approaches its maximum value at about 7 days with little or no increase thereafter. In fact, the value might decrease thereafter as shown for the air-cured curve. This decrease in value was quite common for

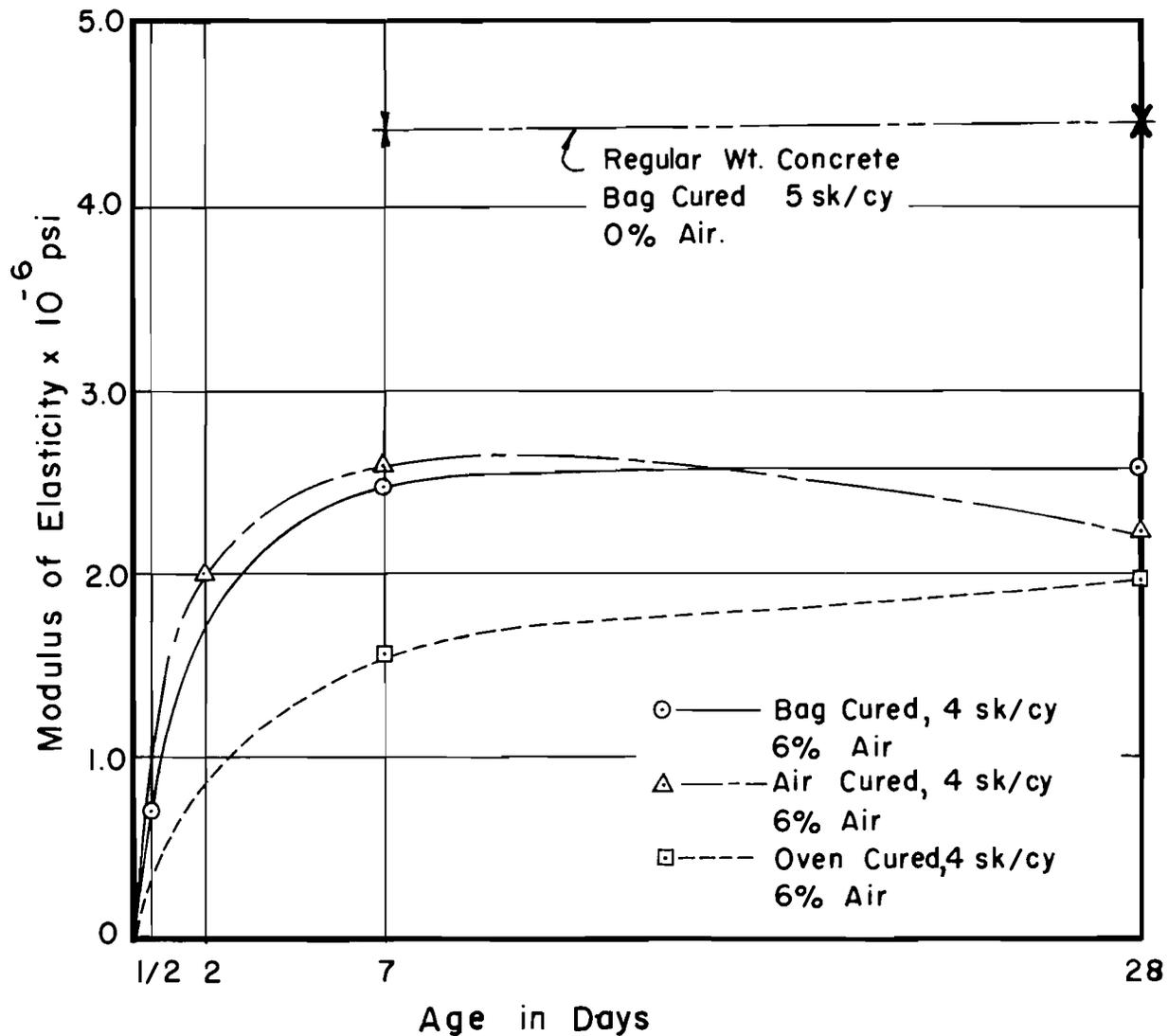


Fig. 5.1 - Effect of Age and Curing Conditions on the Static Modulus of Elasticity.

other mix designs as well (see Table 8-2) and is not uncommon even for regular-weight concrete.<sup>18</sup> This is a partial contradiction of the belief that the modulus of elasticity increases with age. Therefore, caution should be exercised in design when the 7-day modulus is used. On the other hand, it is conceivable that structural lightweight concrete will increase in modulus-of-elasticity values with time if it is loaded. This is common for regular-weight concrete.<sup>19</sup>

The curing condition of structural lightweight concrete seems to have some effect on the modulus of elasticity. In Fig. 5-1, the specimens which were oven cured have noticeably lower values than those which were air or bag cured. This was especially true for the mixes made with a cement factor of 4 sacks per cubic yard.

### 5.2 Comparison of E in Tension and Compression

A comparison of the values of modulus of elasticity obtained from tension and compression tests on companion specimens is given in Table 5-1. For all practical purposes the values in tension and compression may be considered the same since the average difference was less than 10 per cent. However, the tensile values of E tend to be slightly less than the compressive values.

### 5.3 Comparison of Measured and Calculated E

According to Pauw,<sup>20</sup> the modulus of elasticity of either regular-weight concrete or structural lightweight concrete may be approximated by

$$E = 33.6 w^{3/2} \sqrt{f'_c} \quad \text{--- (5-1)}$$

TABLE 5-1

COMPARISON OF COMPRESSIVE AND TENSILE MODULUS  
OF ELASTICITY

C. F. sks / cy	Per Cent	Age in Days	Curing Condition	Modulus of Elasticity		Percent Difference Based on Comp.
				$E \times 10^{-6}$ , psi Compressive	Tensile	
4	2	7	Air	2.33	2.42	+ 3.9
		28	Bag	2.81	2.74	- 2.5
		7	Oven	1.80	2.00	+11.1
4	6	7	Air	2.59	1.86	-28.2
		28	Air	2.21	2.12	- 4.1
		7	Bag	2.43	2.25	- 7.4
		28	Bag	2.60	2.70	+ 3.8
		7	Oven	1.55	1.62	+ 4.5
		28	Oven	1.83	1.33	-27.3
		7	Bag	2.64	2.92	+10.6
5	2	28	Bag	3.09	2.52	-18.4
		7	Oven	2.32	2.14	- 7.7
5	6	7	Air	2.62	2.38	- 9.2
		28	Air	2.46	2.45	- 0.4
		7	Bag	2.81	2.29	-18.5
		28	Bag	3.29	3.36	+ 2.1

TABLE 5-1

C. F. sks / cy	Per Cent Air	Age in Days	Curing Condition	Modulus of Elasticity		Percent Difference Based on Comp.
				$E \times 10^{-6}$ , Compressive	psi Tensile	
		7	Oven	2.64	2.15	-18.6
		28	Oven	2.55	1.93	-24.3
						- 7.3 % Avg.

where

$w$  = unit weight of the concrete in pounds per cubic foot at time of test

$f'_c$  = compressive strength of the concrete in psi

This formula was used to calculate the modulus of elasticity of various specimens for which the unit weights and compressive strengths were recorded in this investigation. The calculated values were compared with the measured values and were found to be about 14 per cent lower. The data was then plotted similar to Pauw<sup>21</sup> and fitted with a straight line through the origin using a least-squares technique. This straight-line fit is shown in Fig. 5-2.

From the data obtained in this investigation, the modulus of elasticity of structural lightweight concrete can be better approximated by Eq. (5-2).

$$E = 37.6 w^{3/2} \sqrt{f'_c} \quad \text{--- (5-2)}$$

It should be recognized that in determining the slope mathematically, the line was forced to go through zero.

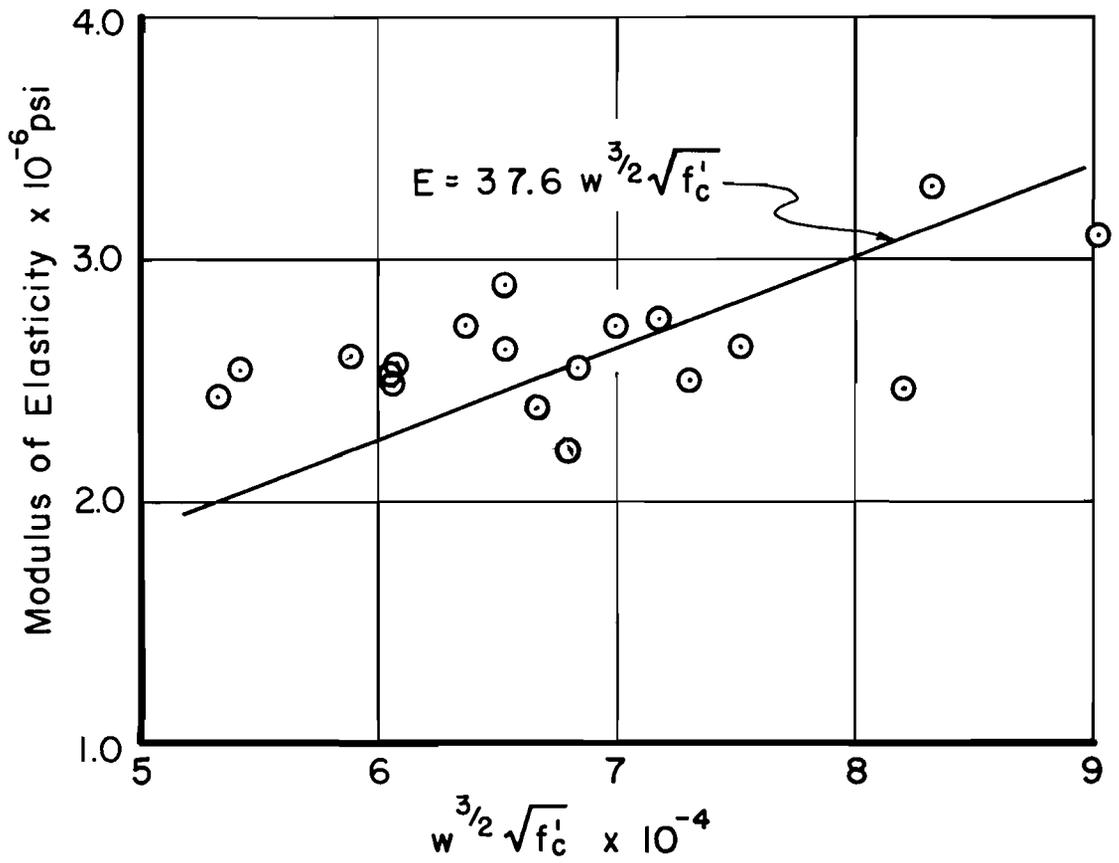


Fig.5-2- Relationship Between Secant Modulus of Elasticity and Compressive Strength and Unit Weight of Concrete.

## 6. DISCUSSION OF LIGHTWEIGHT CONCRETE DESIGN AND PAVEMENT PERFORMANCE

### 6.1 General

With the properties of structural lightweight concrete determined in this study, it is possible to analyze the design and performance of a concrete pavement structure built with this material. Of course, the results of this analysis are limited by the fact that only one lightweight aggregate was used, so generalizations involving all lightweight concretes cannot be made. However, test methods and procedures have been developed and analyzed to determine critical properties of structural concrete made with any material and their effects upon pavement structure performance.

As mentioned in the first report of this study,<sup>22</sup> design formulations include only a portion of those properties which are critical to the behavior and performance of a structure. In the case of concrete pavement structures, the concrete properties which are accounted for directly in the design formulations are the concrete's strength and modulus of elasticity. Additional properties of importance to the performance of a concrete pavement structure include:

1. Coefficient of thermal expansion
2. Restrained volume changes
3. Unrestrained volume changes
4. Properties at early ages
5. Durability
6. Correlation of fundamental aggregate properties with end product performance
7. Skid resistance.

Of these seven properties, properties 1 through 4 were investigated in this study for one structural lightweight aggregate concrete; property 5 is being investigated at Texas A & M University;<sup>23</sup> and properties 6 and 7 are proposed for future investigation at Texas A & M University.<sup>24</sup> In the following sections of this chapter, the various properties determined in this study will be examined in the light of concrete pavement design and performance. This examination will be in the form of a comparison of the effects of lightweight concrete properties with the effects of regular-weight concrete properties on pavement design and performance.

## 6.2 Concrete Pavement Design Formulations

The present-day design formulations include a design determination of (1) concrete thickness, (2) contraction joint spacing, (3) distributed steel requirements, and (4) continuous reinforcement.

Concrete thickness. The design formulation of the thickness of concrete pavement was one of the major results of the AASHO road test.<sup>25</sup> Coupling the results of the road test with prior research studies and experience, the American Association of State Highway Officials have published an Interim Guide for the design of rigid pavement structures.<sup>26</sup> This guide has been extended by the Texas Highway Department to cover the various types of concrete pavement constructed in Texas.<sup>27</sup>

From this development, a pavement-thickness-design equation has emerged and is given as follows:

$$\text{Log}\Sigma L = - 8.682 - 3.513 \log \left[ \frac{J}{S_c D^2} \left( 1 - \frac{2.64 a}{20.25 D^{0.75}} \right) \right] + 0.9155 \frac{G}{\beta} \quad \text{--- (6-1)}$$

where

- $\Sigma L$  = Number of accumulated equivalent 18-kip single axle loads
- $J$  = A coefficient dependent upon load transfer characteristics for slab continuity
- $S_c$  = Modulus of rupture (flexural strength) of concrete at 28 days (psi)
- $D$  = Nominal thickness of concrete pavement (in.)
- $Z$  =  $\frac{E_c}{k}$
- $E_c$  = Modulus of elasticity for concrete (psi)
- $k$  = Modulus of subgrade reaction (psi/in.)
- $a$  = Radius of equivalent loaded area = 7.15 in. for road test 18-kip axles
- $G$  =  $\frac{4.5 - P_t}{3}$
- $P_t$  = Servicability at end of time,  $t$
- $\beta$  =  $1 + \frac{(1.624)(10^7)}{(D + 1)^{8.46}}$

At first glance, Eq. (6-1) appears rather formidable and cumbersome to solve for the design pavement thickness ( $D$ ). However, it was a simple matter to program the equation on the computer and solve for ( $\Sigma L$ ) for all combinations of variables, and then take the results and present them in form of a design nomograph.<sup>28</sup> The nomograph is reproduced in Fig. 6-1 for ready reference. It is relatively easy to enter the nomograph with the design parameters and concrete properties involved and arrive at a design pavement thickness.

As stated in Section 6.1, this design formulation involves only two concrete properties directly--flexural strength and modulus of elasticity.

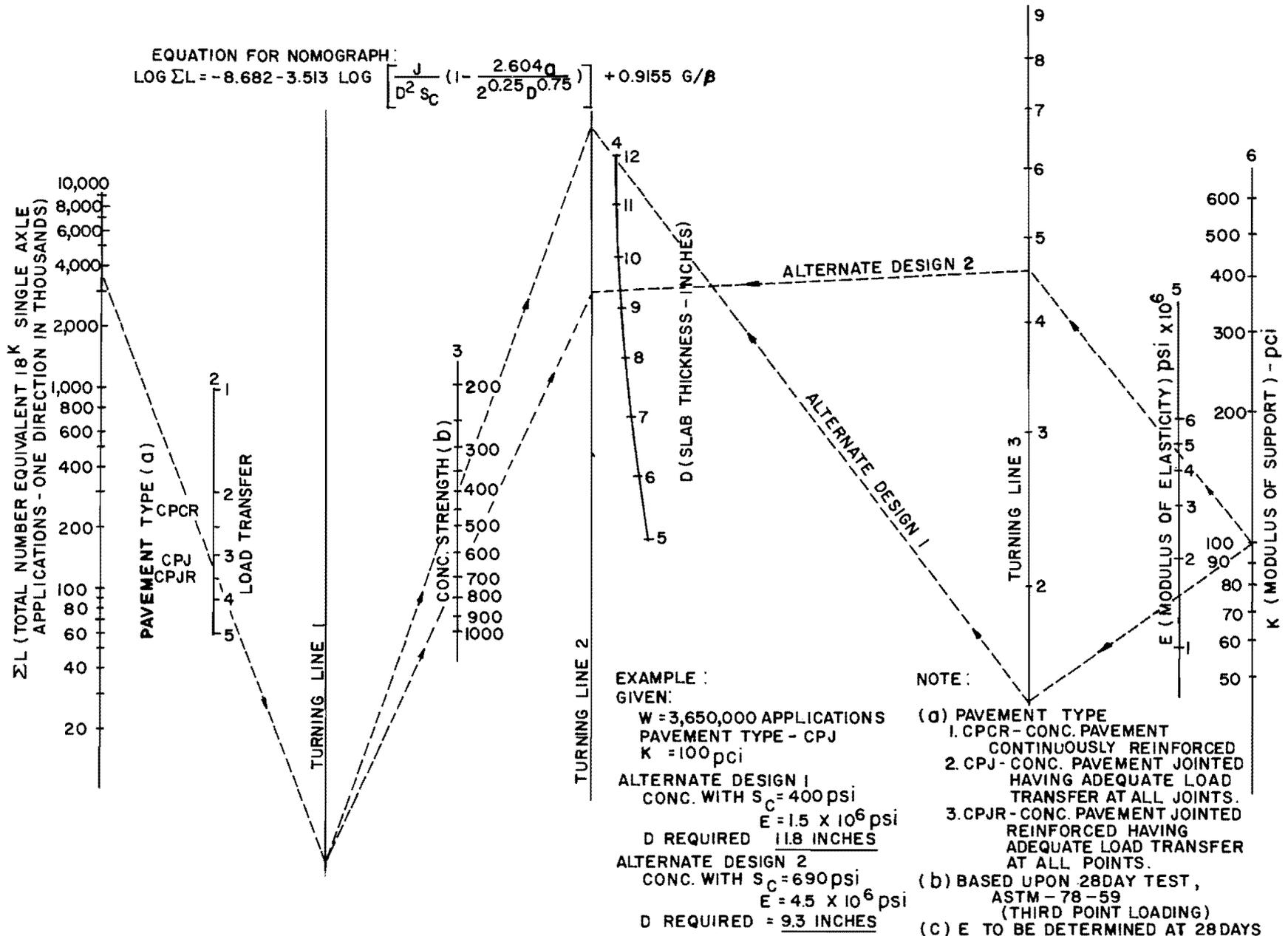


FIG. 6-1 - Modified AASHTO Design Chart-Rigid Pavements

The structural strength index used is the modulus of rupture, or flexural strength, of the concrete, which has been found to be a questionable index of concrete strength in that it is highly sensitive to the curing conditions imposed (see Sections 3.2 and 3.3). However, assuming values obtained from the flexural test were valid; comparisons were made of the design thickness of lightweight and regular-weight concrete. A complete list of assumed design parameters and example properties are given in Table 6-1. The comparisons of the equation for the various parameters are given in Fig. 6-2 for continuously reinforced concrete pavement (CPCR). Similar comparisons for jointed concrete pavement (CPJ) are given in Fig. 6-3. It is interesting to note that, in all cases, the required thickness of structural lightweight concrete was less than the required thickness for regular-weight concrete. As the example strengths were the same for the two types of concrete, the difference in design thickness must be due to the lower modulus of elasticity for structural lightweight concrete. This is in line with the theoretical development of slabs on elastic foundations by Westergaard.<sup>29</sup> The lower the modulus of elasticity, the lower the tensile stresses in the concrete, and hence the thinner the pavement can be to withstand the applied traffic load. Structural lightweight concrete, then, with its lower modulus of elasticity, offers an advantage over regular-weight concrete, in that less material is required to carry the load. Of course, this savings must be compared to the relative cost of lightweight and regular-weight concrete before determining which type offers the greatest over-all economy.

Contraction joint spacing. Jointed unreinforced concrete pavement requires transverse contraction joints spaced along the pavement length.

TABLE 6-1  
PAVEMENT DESIGN PARAMETERS

Example Design Quantities

$\Sigma L$	(Accumulated Equivalent 18 kip single Axle Loads)	4000;	6000;	8000
$k$	(Subgrade Modulus - psi/in.)	50;	100;	150; 200
$L_d$	(Applied Load - lb)	12,000;	16,000;	20,000
$F$	(Friction Factor)	2.0		
$E_s$	(Modulus of Elasticity of Steel - psi)	30 x 10 <sup>6</sup>	psi	
$f_s$	(Allowable working Stress in the steel - psi)	45000		

Concrete Properties

<u>Property</u>	<u>Regular Weight Concrete</u>	<u>Lightweight Concrete</u>
$S_c$ (concrete flexural strength-psi - 28-day third point)	500	500
$S'_c$ (concrete 28-day tensile strength - psi)	260	260
$\mu$ (Poisson's ratio)	0.15	0.15
$E_c$ (Modulus of Elasticity - psi)	4.5 x 10 <sup>6</sup>	2.25 x 10 <sup>6</sup>
$K_T$ (thermal coefficient in. /in. /°F)	5.55 x 10 <sup>-6</sup>	5.19 x 10 <sup>-6</sup>
$w$ (concrete unit weight - PCF)	150	115

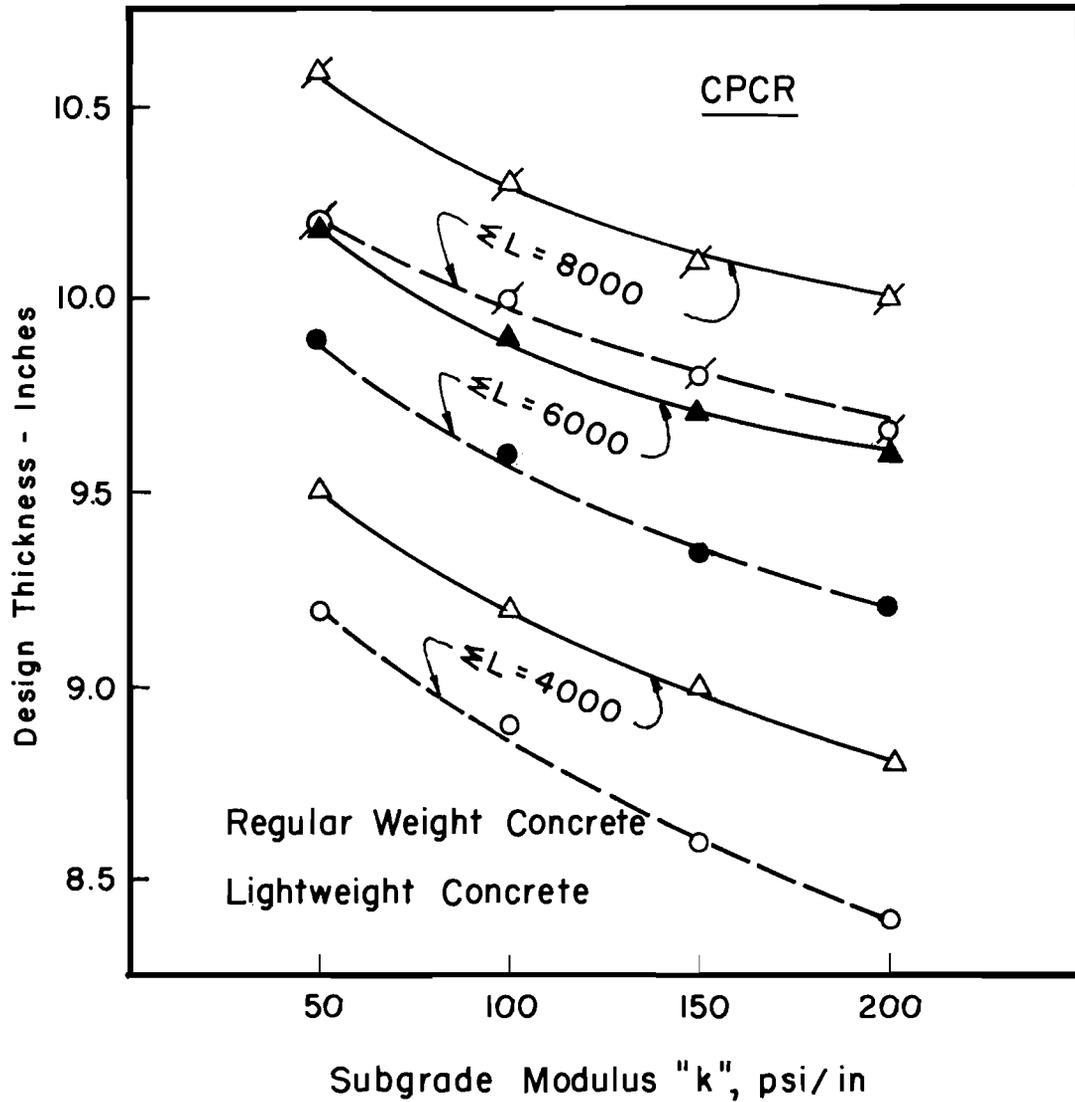


Fig 6-2 Design Thickness for CPCR of Regular and Lightweight Concrete as a Function of Load Application and Subgrade Modulus.

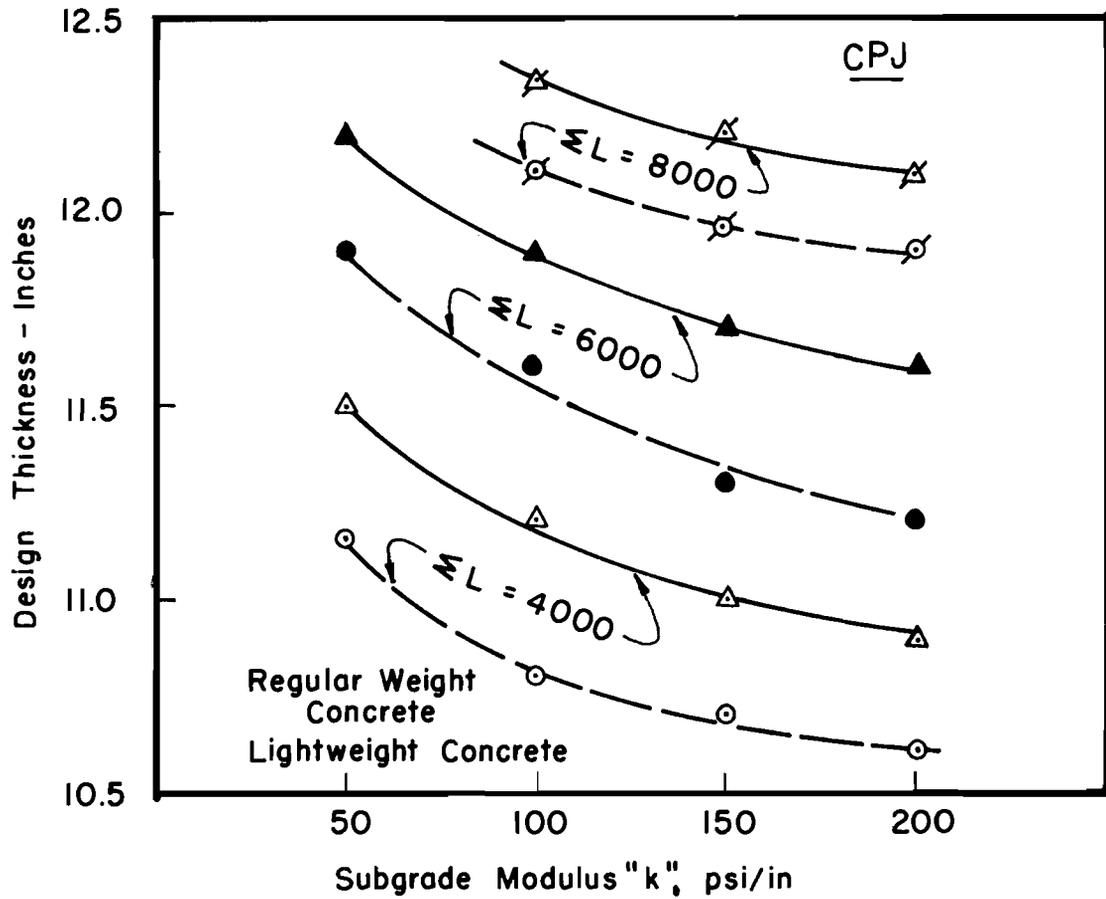


Fig. 6-3. Design Thickness for CPJ of Regular and Lightweight Concrete as a Function of Load Application and Subgrade Modulus.

Unfortunately, there are no design formulations to determine the required joint spacing,<sup>30</sup> and, therefore, for the most part, past experience has been relied upon to arrive at suitable designs.

The required joint spacing is strongly dependent upon the volume changes of the concrete, which is a critical property and is discussed in Section 6.3.

Distributed Steel. The amount of distributed steel in jointed, reinforced concrete pavement is determined from the following equation:

$$A_s = \frac{FLW}{2f_s} \quad \text{--- (6-2)}$$

where

$A_s$  = Area of steel required per foot width of slab (in. )

$L$  = Slab length (ft)

$W$  = Unit weight of concrete (pcf)

$F$  = Friction factor between the slab and the subbase

$f_s$  = Allowable working stress in the reinforcing steel (psi)

Note that the amount of steel is dependent upon the weight of the slab, all other factors being equal. Inherent in this formulation is the assumption that the concrete is sufficiently strong to support the load. It is reasonable to assume that the friction factor  $F$  between the pavement and the subgrade will be practically the same for each type of concrete. Since the concrete strengths for each type are the same for the concrete unit weights as shown in Table 6-1, and for a given slab length,  $L$ , the regular-weight concrete slab would require 30 per cent more distributed steel per foot width of slab than structural lightweight concrete. Therefore, as with the required

thickness; less material is required for structural lightweight concrete.

Of course, another important assumption is inherent in using this formula for lightweight concrete is that the lightweight concrete volume changes are no greater than regular-weight concrete volume changes. This will be discussed further in Section 6.3.

Continuous reinforcement. In the design of continuously reinforced concrete pavement without transverse joints, enough steel is placed in the slab to force the concrete to develop numerous transverse, hairline cracks. The steel does not prevent cracking; on the contrary, it induces cracking. However, it keeps the cracks tightly closed. This type of pavement has been used extensively in Texas with excellent results, as well as in many other states. The basic design equation for the steel percentage is given by:<sup>31</sup>

$$P_s = \frac{S'_c}{f_s - nS'_c} \quad \text{--- (3)}$$

$P_s$  = Required steel percentage (per cent)

$S'_c$  = Tensile strength of the concrete (psi)

$f_s$  = Allowable working stress in the steel (psi)

$n$  =  $E_s/E_c$

$E_s$  = Steel modulus of elasticity (psi)

$E_c$  = Concrete modulus of elasticity (psi)

This basic equation has been modified to include a term for the sub-base friction factor,  $F$ , for inclusion in the AASHO Interim Guide:<sup>32</sup>

$$P_s = (1.3 - 0.2F) \frac{S'_c}{f_s - nS'_c} \quad \text{--- (6-4)}$$

where

$F$  = Friction factor between the slab and the subbase

Using this equation to compare the relative amounts of steel required for regular-weight and lightweight concrete pavement, the only difference in the example design parameters (Table 6-1) between the two types of concrete is the modulus of elasticity, which affects  $n$ . For the selected values, solving for the steel percentage,  $P_s$ , yields:

For regular-weight concrete,  $P_s = 0.54\%$

For lightweight concrete  $P_s = 0.56\%$

In other words, slightly more steel is required for lightweight concrete than for regular-weight concrete, but this difference should not affect the cost of the resulting pavement any significant amount. One other point should be mentioned here. It has been shown that the "useable tensile strength" of lightweight concrete can be seriously reduced if unfavorable curing conditions occur. While this is detrimental to the capacity of the concrete for carrying external loads, an examination of equation 6-4 reveals that this reduction in tensile capacity would require less steel reinforcement for this type of pavement. While it would be unsafe to reduce the required amount of steel, it is an advantage to know that unfavorable curing will not adversely affect the purpose of steel in the concrete pavement for this type of pavement construction.

### 6.3 Concrete Pavement Performance

In the previous Section (6.2), comparisons were made between regular-weight and lightweight concrete pavement structures based upon existing pavement design formulations. As is the case with almost all design procedures, the capability of a given product to meet design requirements does not always insure the design product performance in service. There are many material properties which are considered as critical to the performance of the concrete pavement structure which are not considered directly in the design. Some of these critical properties will be discussed here.

Pavement deflection under load. As discussed, the lower modulus of elasticity of lightweight concrete means that it will deflect more under load and hence pickup additional subgrade support reducing concrete tensile stresses. The fundamental equation for the edge deflection of a slab under a concentrated load and on an elastic foundation has been developed by Westergaard,<sup>33</sup> and is:

$$Z_e = \frac{1}{\sqrt{6}} (1 + 0.4 \mu) \frac{L_d}{kl^2} \quad \text{--- (6-5)}$$

where

$Z_e$  = Edge deflection (in.)

$\mu$  = Poisson's ratio

$L_d$  = Applied Load (lb)

$$kl^2 = \left( \frac{E_c D^3 k}{12(1 - \mu^2)} \right)^{1/2}$$

$D$  = Slab thickness (in.)

$k$  = Modulus of subgrade reaction (psi/in.)

$E_c$  = Concrete modulus of elasticity (psi)

Notice that the  $k\ell$  term is in the denominator and contains the  $E$  term, hence, the lower the value of  $E$ , the greater the deflection for a given load  $L_d$ . Substituting  $\mu = 0.15$  and rewriting the equation 6-5 yields:

$$Z_e D^{3/2} = \frac{1.483 L_d}{(Ek)^{1/2}} \quad \text{--- (6-6)}$$

In this form, the equation solves for the parameter,  $Z_e h^{3/2}$  in terms of the applied load, modulus of elasticity, and subgrade modulus. Using example values given in Table 6-1, edge-deflection parameters for each concrete type were calculated for three different applied loads and compared by obtaining the numerical difference in the deflection parameter  $Z_e h^{3/2}$  between regular-weight and lightweight concrete. The results were plotted in Fig. 6-4. As expected, the greater the load and lower the value of subgrade modulus, the greater the difference in edge deflections between regular-weight and lightweight concrete pavement. The resulting deflection can be converted to stress and a required thickness can be established,<sup>34</sup> but the resulting thickness is based upon a single applied load. Therefore, the modified AASHO formula for design thickness (Eq. 6-1), which is based on a repetitive load, cannot be correlated directly to the deflection equation of Westergaard. However the two equations show similar results.

Concrete warping stresses. The effects of concrete volume changes

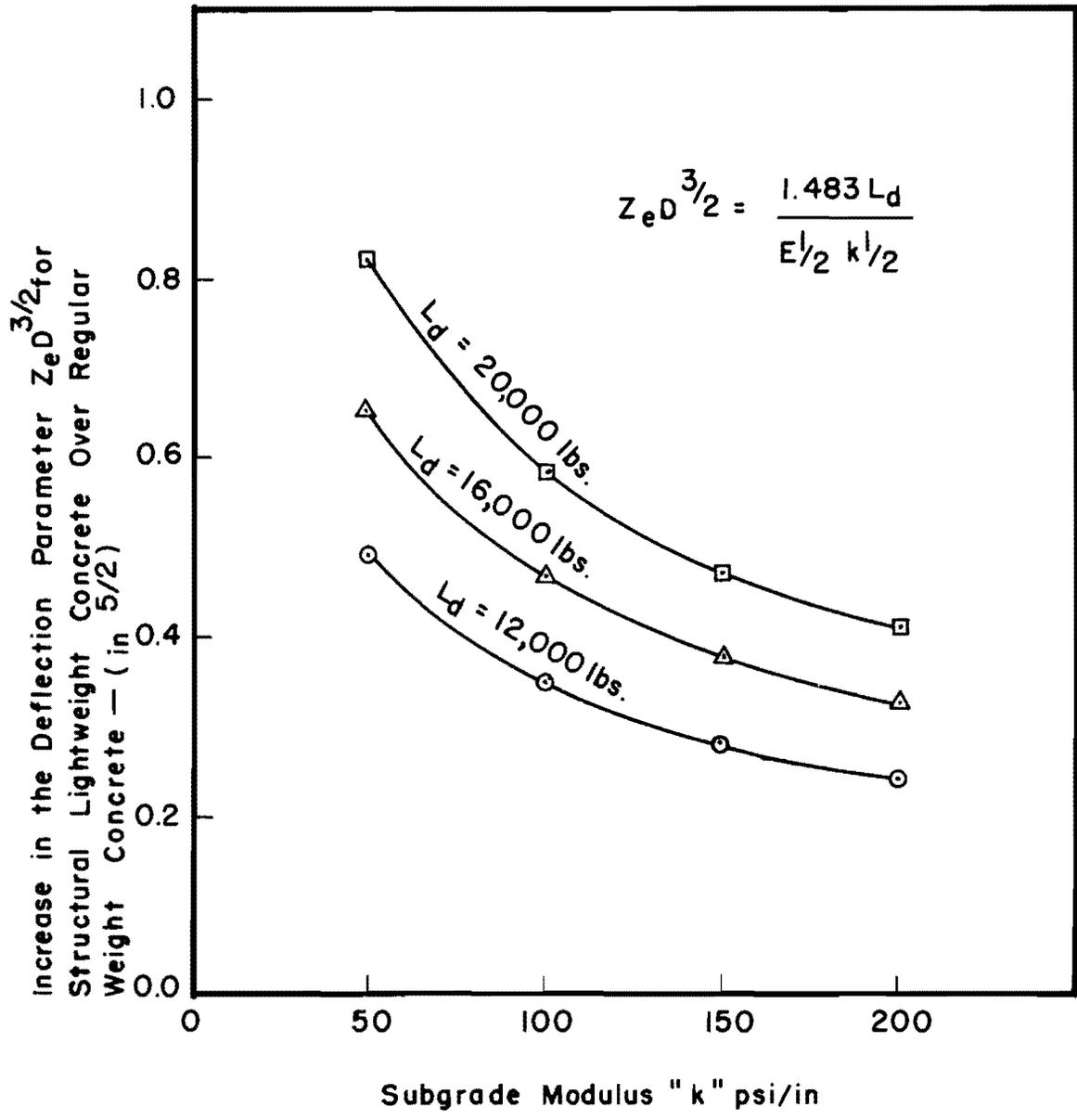


Fig.6-4. Difference in the Edge Deflection Parameters Between Regular Weight and Structural Lightweight Concrete Pavement.

with changes in temperature are an important consideration when evaluating a pavement material which will be subjected to rather severe temperature changes and differentials between the top and bottom of the slab. The theoretical warping stresses developed from temperature differentials has been formulated by Bradbury.<sup>35</sup>

$$\sigma_t = \frac{CE_c K_\tau T}{2} \text{-----}(6-7)$$

where

$\sigma_t$  = maximum stress (psi in the extreme fiber at the edge of the slab, in the direction of slab length.

C = Coefficient, directly proportional to slab length

$K_\tau$  = Coefficient of thermal expansion (in./in./°F).

T = Difference in temperature between the top and bottom of the slab.

The value for the warping stress is directly proportional to the product of coefficient of thermal expansion  $K_\tau$  and the modulus of elasticity  $E_c$ . Using the typical properties given in Table 6-1, for pavement slabs of the same length and thickness, regular-weight concrete would contain 114 per cent more stress due to warping than structural lightweight concrete. This means that the distance between transverse contraction joints, which is reflected in the coefficient C, can be increased on pavements constructed with structural lightweight concrete, and thereby effect a savings in construction costs. This also means that structural lightweight concrete undergoes less volume change from changes in temperature and therefore is more dimensionally stable over long periods of time. This points to an

expected increase in the performance capability of structural lightweight concrete in terms of warping stresses.

Volume changes-moisture. Volume changes from changes in moisture content of the concrete constitute another important property which seriously affects pavement structure performance. As discussed earlier, these volume changes, if the concrete is restrained, can produce severe tensile stresses in the concrete thereby drastically reducing the useable concrete tensile strength. This reduction in useable tensile strength could drastically reduce the concrete's capacity to carry external loads which could constitute a major disadvantage to the use of structural lightweight concrete if not properly accounted for in design and construction. By that, it is meant that proper curing procedures must be employed to prevent early excessive drying of the concrete causing high-tensile concrete stresses. Referring to Figs. 2-2 and 2-3, it can be seen that the restrained concrete volume change stresses can be almost eliminated during the critical early life of the concrete by proper curing. Of course, additional research is needed on lightweight concretes made with other aggregates and with curing conditions which more nearly approximate current field curing practices before this factor can be fully appraised. However, from the results obtained in this study, a clear warning has been sounded that restrained lightweight concrete volume changes could be very serious and should be investigated further. Pavement structures constructed with lightweight concrete should be watched closely for any performance effects which may show up as a result of restrained volume changes.

## 7. CLOSURE

### 7.1 Conclusions

The following conclusions appear to be valid for the parameters studied in this investigation.

1. The coefficient of thermal expansion for concrete produced with an expanded shale coarse aggregate and normal-weight fine aggregate was found to be  $5.3 \times 10^{-6}$  in./in. per  $^{\circ}\text{F}$  which was slightly lower than the  $5.6 \times 10^{-6}$  in./in. per  $^{\circ}\text{F}$  for concrete produced with sand and gravel. While the differences between these two coefficients may be less than anticipated, it can be explained by the fact that regular weight fines, along with a fully coated, low porosity, light-weight coarse aggregate, were used in this study. It should be emphasized that concrete made with other aggregates may exhibit markedly different thermal properties.

2. An increase in air content of 4.6 per cent caused a decrease in the coefficient of thermal expansion of  $0.1 \times 10^{-6}$  in/in. per  $^{\circ}\text{F}$  in the concrete using this particular expanded shale coarse aggregate.

3. This concrete with an expanded shale coarse aggregate in an unrestrained condition expanded initially as much as 300 micro in./in. and then in a bag environment remained in the expanded condition for the period observed (28 days). In the other extreme of an oven environment, the shrinkage at 65 days was as much as 289 micro in./in. which is low as compared with concretes produced with other lightweight aggregates. It is apparent that the volume change can be kept to a minimum under a favorable environment.

4. The correlation of flexural strengths with either compressive strengths or split cylinder strengths was poor which further indicated the difficulty of using flexure as an index of strength and more particularly tensile strength. When comparing the compressive strengths of the bag cured lightweight concrete with the bag cured regular weight concrete compressive strength, there was reasonable good correlation. This was also true for the direct tensile strengths and split cylinder strengths. However, the flexural strengths of the lightweight concrete were substantially lower and more irregular, preventing little correlation. Therefore, the flexural strength test appears to be a poor indicator of lightweight concrete strength and quality.

5. The split cylinder strengths for the three different environments resulted in the air environment specimens testing at approximately 91 per cent of the bag environment and the oven environment specimens testing at approximately 77 per cent of the bag environment. This test shows considerable promise as an indicator of concrete strength and quality.

6. It appears that from this investigation has come a procedure that not only accurately determines the tensile strength of concrete but also provides a measure of residual tensile stress developed by volume change. It can also be used to establish the influence of different aggregates, mix designs, admixtures, environments, etc. on tensile strength and the development of residual stress from volume changes.

7. When this lightweight aggregate concrete was restrained by a reinforcing bar, it developed either compressive or tensile residual stresses depending on the curing environment. Residual compressive stresses as high as 80 psi were developed with a bag environment whereas residual tensile stresses as high as 140 psi were developed in an oven

The intermediate environment in air resulted in residual tensile stresses as much as 100 psi.

8. For this lightweight aggregate concrete the (direct) tensile strength was found to be related to the split cylinder strength and compressive strength as follows:

$$f_t = 0.66 f_s = 0.07 f'_c$$

9. The useable tensile strength for this lightweight aggregate concrete was found to be significantly reduced under unfavorable environmental conditions during the curing period. Further it was determined that neither the split cylinder strength test nor the compression strength test indicated the useable tensile strength of the reinforced concrete.

10. Since the expanded shale used in this investigation is a low absorption aggregate, it is anticipated that aggregates with higher absorptions might result in even lower useable tensile strengths under restrained volume change and unfavorable environmental conditions.

11. For all practical purposes the modulus of elasticity for both tension and compression for concrete made with this particular expanded shale coarse aggregate was found to be the same except for several values which were affected by experimental inconsistencies in curing.

12. For the tests made in this investigation, the relationship of modulus of elasticity to unit weight and compressive strength was found

to be:

$$E = 37.6 w^{3/2} f'_c{}^{1/2}$$

This relationship is reasonably close to Pauw's<sup>19</sup> value of:

$$E = 33.6 w^{3/2} f'_c{}^{1/2}$$

13. Using the values obtained in this study for the properties investigated, the following design comparisons can be made for the various types of concrete pavements.

- a. The required pavement thickness for lightweight concrete is around 0.3 in. less than regular-weight concrete of the same strength. A higher Poisson's ratio for lightweight concrete would reduce this difference.
- b. For jointed reinforced concrete pavement, regular-weight concrete requires 30 per cent more distributed steel than lightweight concrete of the same joint spacing.
- c. For continuously reinforced concrete pavement, lightweight concrete requires slightly more steel (0.56 per cent) than regular-weight concrete (0.54 per cent).

14. In evaluating the expected pavement performance the following comparisons can be made.

- a. Lightweight concrete, with its lower modulus of elasticity, will deflect more under load than regular-weight concrete, thereby reducing extreme fiber tensile stresses. This is reflected in the required thickness, but, in addition, this greater deflection indicates a better compatibility and interaction between the pavement and its supporting subbase. This

interaction should result in better performance for lightweight concrete, from an external load standpoint.

- b. For given slab dimensions, concrete warping stresses due to temperature differentials between the top and bottom of the slab for regular-weight concrete will be 114 per cent greater than for lightweight concrete.
- c. Volume change of lightweight concrete, if unfavorably cured, can result in sizeable residual stresses in the concrete; and if restrained such as they would be in a concrete pavement, these volume changes could be extremely detrimental to the performance of the pavement structure.

## 7.2 Recommendations

1. Since the technique developed in this investigation provides a measure of both the tensile strength and useable tensile strength that might occur under a restrained volume change condition it is recommended that the effects of the following be investigated:

- a. properties of aggregates, particularly absorption and Poisson's ratio.
- b. mix design
- c. limiting and practicable curing environment
- d. use of molecular films to reduce evaporation
- e. percentages of steel.

2. Additional tests for the determination of the coefficient of expansion should be made with regard for the following parameters:

- a. age of concrete
- b. cement factor
- c. air content

d. aggregate type.

Also testing of several specimens of each mix investigated would minimize error due to experimental inconsistencies.

3. The effects of curing environment on bond strength should be investigated for at least two lightweight aggregate types: one with a relatively high absorption capacity, and one with a relatively low absorption capacity.

4. Dynamic tensile properties should be investigated for a structural lightweight concrete.

5. Test sections of structural lightweight concrete pavement should be constructed and evaluated over a period of time to verify the laboratory conclusions reached in this study.

### 7.3 Correlation of Results With Research Objectives

Inasmuch as this is the third and final report under this contract with the Texas Highway Department and Bureau of Public Roads, it is desirable to review the research objectives of the original contract and indicate wherein these three reports met the objectives.

The first objective was to establish the critical mechanical properties of structural lightweight concrete. This objective was covered in all three reports for one lightweight concrete. The critical properties of strength, curing environment, and age were investigated.

The second objective was to develop mathematical relationships between such factors as strength, modulus of elasticity, volume changes, and concrete age for various design parameters. Using the design parameters of two cement factors, three curing environments, and two air contents; relationships between volume changes and age were introduced in the first report and

developed further in the second report, and relationships between modulus of elasticity and age were developed in this the third report. Finally, in this third report additional information was presented and interrelationships between all properties and parameters were presented.

The third objective was an ultimate objective whereby enough information would be furnished to the highway designer to enable him to reliably predict the properties of structural lightweight concrete of importance in design. In this investigation a definite start has been made toward this ultimate objective. In the final report design comparisons were made between lightweight and regular weight concrete pavements. An overall plan of research has been formulated which will yield meaningful results if the research is continued.

## 8. APPENDIX

### 8.1 Data

This section contains a tabulation of some data used to prepare figures in this report.

TABLE 8-1  
 COMPRESSIVE, FLEXURAL, AND INDIRECT TENSILE  
 STRENGTH VALUES

C/F sks/cu yd	Air %	Cure	Age	Compressive		split cylinder		
				f' <sub>c</sub> psi	f <sub>f</sub> psi	f <sub>sp</sub> psi		
5.0	2	Bag	1/2	612	143	84		
			2	2645	410	308		
			7	3516	507	379		
			28	3863	406	453		
		Air	1/2	557	128	63		
			2	2223	296	261		
			7	3244	354	391		
			28	4070	381	425		
		Oven	1/2	607	130	71		
			2	2690	297	310		
			7	3539	287	348		
			28	3424	383	355		
		5.0	6	Bag	1/2	813	164	101
					2	2470	406	293
					7	3590	486	391
					28	4685	425	442
Air	1/2			630	164	86		
	2			2475	320	339		
	7			3285	317	376		
	28			3780	364	400		

TABLE 8-1  
(Cont'd)

C/F sks/cu yd	Air %	Cure	Age	Compressive		split cylinder
				f' <sub>c</sub> psi	f <sub>f</sub> psi	f <sub>sp</sub> psi
		Oven	1/2	970	176	126
			2	2765	326	323
			7	2945	252	299
			28	2365	455	297
4.0	6	Bag	1/2	182	60	23
			2	1342	296	196
			7	2220	320	276
			28	2770	374	337
		Air	1/2	236	78	33
			2	1325	243	191
			7	1775	253	203
			28	2480	295	322
		Oven	1/2	248	92	35
			2	1370	244	172
			7	1750	223	218
			28	1930	390	238
5.0 <sup>1</sup>	1	Bag	1/2	450	---	60.5
			2	2420	---	369
			7	3720	---	460
			28	4510	---	525
4.0	2	Bag	7	2260	---	289
			28	2785	---	329

TABLE 8-1  
(Cont'd)

C/F sks/cu yd	Air %	Cure	Age	Compressive		split cylinder
				$f'_c$ psi	$f_f$ psi	$f_{sp}$ psi
		Air	7	1590	---	213
			28	2257	---	268
		Oven	7	1963	266	213
			28	1664	245	206

Note: For explanation of symbols, see section 8-2  
1. This mix is regular-weight concrete (SG)

TABLE 8-2  
SUMMARY OF MODULUS OF ELASTICITY DATA

Cement Factor Sacks / cy	Air Content Per Cent	Age in Days	Curing Condition	Modulus* of Elasticity x 10 <sup>-6</sup> psi	
				Compression	Tension
4	2	7	Air	2.17	
		7	Bag	2.46	2.56
		7	Oven	1.87	
		28	Oven	1.77	
4	2	7	Air	2.33	2.42
		28	Air	2.02	
		28	Bag	2.81	2.74
		7	Oven	1.80	2.00
		28	Oven	1.77	3.33
4	6	2	Air	2.00	
		1/2	Bag	0.705	
		7	Bag	2.56	
		28	Bag	2.52	
		28	Oven	1.96	
4	6	7	Air	2.59	1.86
		28	Air	2.21	2.12
		7	Bag	2.43	2.25
		28	Bag	2.60	2.70
		7	Oven	1.55	1.62
		28	Oven	1.83	1.33
5	2	2	Air	2.19	
		28	Air	2.64	
		2	Bag	2.52	

TABLE 8-2  
(Cont'd)

Cement Factor Sacks / cy	Air Content Per Cent	Age in Days	Curing Condition	* Modulus of Elasticity x 10 <sup>-6</sup> psi	
				Compression	Tension
		7	Bag	2.72 <sup>+</sup>	
		2	Oven	2.35	
5	2	7	Bag	2.64	2.92
		28	Bag	3.09	2.52
		7	Oven	2.32	2.14
		28	Oven	2.02	
5	6	28	Air	2.50	
		28	Bag	2.46	
		7	Oven	2.38	
5	6	7	Air	2.62 <sup>+</sup>	2.38 <sup>+</sup>
		28	Air	2.46	2.45
		7	Bag	2.81 <sup>+</sup>	2.29 <sup>+</sup>
		28	Bag	3.29	3.36
		7	Oven	2.64 <sup>+</sup>	2.15
		28	Oven	2.55	1.93
5	1	7	Bag	4.43 <sup>+</sup>	4.30
		Regular-Weight	28	Bag	4.45 <sup>+</sup>

\* Secant modulus of elasticity taken as slope to 0.5 f'<sub>c</sub> or 0.5 f<sub>t</sub>.

<sup>+</sup> These values are averages of 2 or more tests.

## 8.2 List of Symbols

C.F.	Cement factor - sacks per cubic yard
cy	Cubic yard
$\epsilon_{ct}$	Concrete tensile strain (Total)
$\epsilon_{cz}$	Concrete strain from restrained volume changes
E	Concrete static modulus of elasticity, psi
$^{\circ}F$	Degree Fahrenheit
F	Friction Factor
$f'_c$	Concrete compressive strength, psi
$f_{sp}$	Concrete split cylinder strength, psi
$f_t$	Concrete direct tensile strength, psi
$f_{tu}$	Concrete useable tensile strength, psi
$K_{\tau}$	Coefficient of linear thermal expansion, in./in. per $^{\circ}F$
sk	Sack
$\sigma_{ct}$	Concrete tensile stress
w	Unit weight of concrete, pounds per cubic foot

Other symbols used in this report which are not defined here are defined where they are used.

### 8.3 Bibliography

1. Ledbetter, W. B. and J. Neils Thompson, Relationship Between Critical Mechanical Properties and Age For Structural Lightweight Concrete, The University of Texas, Center for Highway Research, February, 1964.
2. Houston, J. T. and J. Neils Thompson, Volume Changes in Unrestrained Structural Lightweight Concrete, The University of Texas, Center for Highway Research, August, 1964.
3. Ledbetter, W. B., Op. Cit. pp. 2-4
4. Ledbetter, W. B., Op. Cit. pp. 20-42, pp. 90-96
5. Houston, J. T., Op. Cit. pp. 37-40
6. Ledbetter, W. B., Op. Cit. pp. 10-19
7. Houston, J. T., Op. Cit. pp. 10-19
8. Monfore, G. E. and A. E. Lentz, "Physical Properties of Concrete at Very Low Temperature," P. C. A. Research and Development Laboratories, Vol. 4, May 1962, p. 37.
9. Philleo, Robert, "Some Physical Properties of Concrete at High Temperatures," Journal of the American Concrete Institute, April 1958, Vol. 29, pp. 860-1.
10. Monfore, G. E. and A. E. Lentz, Op. Cit. p. 37.
11. Petersen, P. H. "Properties of Some Lightweight Aggregate Concretes With and Without Air-Entraining Admixture," Journal of the American Concrete Institute, Oct. 1958, Vol. 20, p. 167, 173.
12. Houston, J. T., Op. Cit. pp. 31-2
13. Shideler, J. J., "Lightweight Aggregate Concrete For Structural Use," Journal of the American Concrete Institute, Oct. 1957, Vol. 29, pp. 303, 324-5.
14. Ledbetter, W. B., Op. Cit. p. 52
15. Ledbetter, W. B., Op. Cit. pp. 101-107
16. Ledbetter, W. B., Op. Cit. p. 76
17. Ledbetter, W. B., Op. Cit. p. 78

18. Freudenthal, A.M., and Frederic Roll, "Creep and Creep Recovery of Concrete Under High Compressive Stress," American Concrete Institute Journal, June 1958, Vol. 54, p. 1118.
19. Washa, George W., and Paul G. Fluck, "Extent of Sustained Loading on Compressive Strength and Modulus of Elasticity of Concrete," American Concrete Institute, May 1950.
20. Pauw, Adrain, "Static Modulus of Elasticity of Concrete as Effected By Density," Proceedings, Journal of the American Concrete Institute, December 1960, Vol. 57, No. 6, pp. 679-687.
21. Ibid.
22. Ledbetter, W.B., and J. Neils Thompson, Relationship Between Critical Mechanical Properties and Age for Structural Lightweight Concrete, The University of Texas, Center for Highway Research, June 1964, p. 1.
23. "Durability of Lightweight Aggregate Concrete," Texas A & M University Cooperative Highway Research Project 2-5-62-35 with the Texas Highway Department.
24. "Synthetic Aggregate Research," Proposed Texas A & M University Cooperative Research Project with the Texas Highway Department.
25. "The AASHO Road Test Report 5, Pavement Research," Highway Research Board Special Report 61E 1962, p. 352.
26. AASHO Interim Guide for the Design of Rigid Pavement Structures, AASHO AASHO Committee on Design, April 1962.
27. Hudson, W.R., and B.F. McCullough, An Extension of Rigid Pavement Design Methods, Texas Highway Department Departmental Research Report, No. 64-1, 1964, p. 50.
28. Ibid, p. 18-19
29. Westergaard, H. M., "Stresses in Concrete Pavements Computed by Theoretical Analysis," Public Roads, Vol. 27, No. 2, April 1926.
30. Yoder, E. J., Principles of Pavement Design, John Wiley & Sons, New York, 1959, p. 473
31. McCullough, B.F. and W.B. Ledbetter, "LTS Design of Continuously Reinforced Concrete Pavement," Transactions of the ASCE. Vol. 127, 1962, Part IV, p. 365.
32. AASHO Interim Guide for the Design of Rigid Pavement Structures, AASHO Committee on Design, April 1962.

33. Westergaard, H. M., "Stresses in Concrete Pavement Computed by Theoretical Analysis," Public Roads, Vol. 27, No. 2, April 1926.
34. Ibid.
35. Bradbury, Royall D., Reinforced Concrete Pavements, The Wire Reinforcement Institute, Washington, D. C., 1938.