CALCULATION OF THE ELASTIC MODULI
of a
TWO LAYER PAVEMENT SYSTEM
from
MEASURED SURFACE DEFLECTIONS
by
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Research Report Number 123-6
A System Analysis of Pavement Design
and Research Implementation
Research Study Number 1-8-69-123

conducted

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U. S. Department of Transportation
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Bureau of Public Roads

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Highway Design Division Research Section
Texas Highway Department

Texas Transportation Institute
Texas A&M University

Center for Highway Research
The University of Texas at Austin

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Preface

This is the sixth report issued under Research Study 1-8-69-123, A System Analysis of Pavement Design and Research Implementation. The study is being conducted jointly by principal investigators and their staffs in three agencies -- The Texas Highway Department, The Center for Highway Research at Austin and The Texas Transportation Institute -- as a part of the cooperative research program with the Department of Transportation, Federal Highway Administration.

Previous reports emanating from Study 123 are the following:


Report No. 123-4, "Developing A Pavement Feedback Data System," by R. C. G. Haas, describes the initial planning and development of a pavement feedback data system.


The authors are indebted to Messrs. Robert E. Long and James L. Brown, both of the Texas Highway Department, for furnishing the pavement deflection data used in the sample problems presented in Chapter 6.
The opinions, findings and conclusions expressed in this publication are those of the authors and not necessarily those of the Department of Transportation, Federal Highway Administration.
Abstract

This report gives the theoretical background and a description of a new computer program, ELASTIC MODULUS, capable of converting deflections measured by a Dynaflect on the surface of a highway pavement-subgrade (two-layer elastic) system, to the elastic moduli of the pavement and subgrade. Included with the report are instructions for the use of the program, a complete documentation of its operation, and the solutions of several example problems.
Summary

A sub-system of the flexible pavement design system described in the first report of Study 123 (see Preface), estimates the life of a trial design based solely on surface deflections computed from an empirical equation. In an attempt to improve the reliability of this sub-system (a primary objective of Study 123) it is intended, eventually, to base estimates of pavement life on stresses and strains computed from elasticity theory at critical points within the pavement structure. The use of elasticity theory, however, requires a knowledge of the in situ values of the elastic modulus, E, of each of the pavement materials in common use, as well as the subgrades, in the various Highway Department Districts.

According to elasticity theory, the moduli of a pavement and its subgrade can be estimated from surface deflections rather easily, provided the pavement structure above the subgrade is predominately a single material of known thickness, and the subgrade is reasonably uniform in stiffness to a considerable depth.

For determining the elastic moduli of the two materials composing such a pavement, a mathematical process has been developed, computerized, and is made available herewith to the Texas Highway Department. The method envisions the use of the Dynaflect for making the necessary measurements of surface deflections. The data collection and processing procedures, and the output format of the computer program, are exactly the same as those now employed in estimating the "stiffness coefficients"
used in the present version of the flexible pavement design system, with
the following exceptions:

1. The program described in this report prints elastic moduli in
lieu of stiffness coefficients.

2. The program prints a verbal description of both pavement and
subgrade, instead of the pavement alone.

The computer program has been given the name ELASTIC MODULUS. By a
slight modification, it can be used to predict Dynaflect deflections,
given the pavement thickness and the moduli of pavement and subgrade. In
this form the predictions of ELASTIC MODULUS were compared with those of
another program, BISTRO*. Agreement was excellent, except in the instance
of a pavement with a modulus much smaller than that of its subgrade, a
case not likely to arise often in practice.

To illustrate the results obtained when using ELASTIC MODULUS to
estimate pavement and subgrade moduli, Dynaflect data taken at several
points on seven short sections of flexible pavements near College Station,
Texas, were processed by the program. The ordering of the resulting
pavement moduli, as judged by the verbal descriptions of the materials
and local knowledge of their service performance, appeared reasonable.
In the case of the subgrade moduli, the range was too small to permit a
judgement of the validity of the results.

When using the results of the program to characterize materials in
a pavement design system based on elasticity theory, it is recommended
that the values of the computer moduli be halved before use. This recom-
mendation is based on extensive field correlation studies between deflections
produced by the Dynaflect and those produced by heavily loaded vehicles.

* Used by courtesy of Koninklijke/Shell-Laboratorium, Amsterdam.
Implementation Statement

The program ELASTIC MODULUS was written in the expectation that eventually the Texas Highway Department's Flexible Pavement Design System will, in the prediction of pavement life, use the stresses, strains and displacements computed throughout the structure from the theory of linear elastic layered systems, instead of solely the surface deflections calculated by the present empirical equation. When such a change occurs in the design system, in situ values of elastic moduli will be needed. This need probably can be met, at least to some degree, by the computer program described herein.

The published version of this report may be obtained by addressing your request as follows:

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11th and Brazos
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1. Introduction

Recently the Texas Highway Department began to implement, on a trial basis, a flexible pavement design system that characterizes each material in a proposed or existing pavement structure by a so-called "stiffness coefficient" (1, 2). The in situ coefficient for a material proposed for a new pavement is found from Dynaflect deflection data (3, 4) taken on existing highways that can be assumed to consist essentially of two layers -- a subgrade layer (regarded in theory to be infinitely thick), and a pavement layer composed predominately of a single material (for example, a base material with a surface treatment). The Dynaflect data are then used in an empirical equation that yields a composite stiffness coefficient for the pavement material or materials, and another (usually smaller) coefficient for the subgrade or foundation (1, 5). The coefficients, which vary numerically from about 0.15 for a weak, wet clay to about 1.00 for asphaltic concrete, are calculated by means of a Texas Highway Department computer program, STIFFNESS COEFFICIENT (6). The coefficients, along with other pertinent data, are used in the design process to predict a certain characteristic -- the "surface curvature index" -- of the deflection basin of a trial design composed of the tested materials, and from this characteristic, to predict the life of the design.

This report gives the theoretical background and a description of a new computer program, given the name ELASTIC MODULUS, that accepts and prints the same Dynaflect and other data (identification, location,
special comments, etc.) as the program STIFFNESS COEFFICIENT, but computes and prints out the in situ values of Young's modulus of pavement and subgrade instead of their stiffness coefficients. Linear elastic theory, with Poisson's ratio set to 1/2 for both layers, is used in the computations.

The program ELASTIC MODULUS was written in the expectation that eventually the Texas Highway Department's Flexible Pavement Design System will, in the prediction of pavement life, use the stresses, strains and displacements computed throughout the structure from the theory of linear elastic layered systems, instead of solely the surface deflections calculated by the present empirical equation (7). When such a change occurs in the design system, in situ values of elastic moduli will be needed. This need probably can be met, at least to some degree, by the computer program described herein.
2. Surface Deflection Equation for Two Layer Elastic System

This chapter describes the geometry of the Dynaflect loading and develops the applicable equation for surface deflections due to a point load acting perpendicular to the horizontal surface of a half-space consisting of two horizontal layers of infinite lateral extent.

2.1 The Loading Device (Dynaflect)

Through two steel wheels the trailer-mounted Dynaflect exerts two vertical loads, separated by 20 inches and varying sinusoidally in phase at 8 Hz, as indicated in Figure 1. The total load, exerted by rotating weights, varies from 500 pounds upward to 500 pounds downward. The upward thrust is overcome by the dead weight of the trailer so that the load wheels are always in contact with the pavement. The load-pavement contact areas are small and are considered to be points, rather than areas, in order to simplify the mathematics.

From the symmetry of Figure 1 it can be seen that one load of 1000 pounds can be substituted for the two loads shown, without affecting the vertical motion at points along the line of sensors. For this reason, in what follows only one point load, \( P \), of 1000 lbs., will be considered to be acting on the surface of the pavement.

2.2 List of Symbols

Following is a list of the mathematical symbols used in this report. A list of FORTRAN symbols used in ELASTIC MODULUS, together with their mathematical equivalents, will be found in Appendix 1.

\[ P = \text{vertical force acting at a point in the horizontal surface of a two-layer elastic half space.} \]
Figure 1: Relative position of Dynaflect loads and sensors. The sensors are usually placed in the outer wheel path, on a line paralleling the center line of the highway.
h = thickness of upper layer.

\( E_1 \) = Young's modulus of upper layer.

\( E_2 \) = Young's modulus of lower layer.

\( w \) = the vertical displacement of a point in the surface.

\( r, z \) = cylindrical coordinates. (The tangential coordinate, \( \theta \),

does not appear because only one load is used as explained

on page 3, and the resulting vertical deflections are

symmetrical about the z-axis.)

The load \( P \) acts downward at the point \( r = 0, z = 0 \). Positive \( z \) is

measured downward.

\( m \) = a parameter.

\( x = mr/h \).

\( \text{Jo}(x) \) = Bessel Function of the first kind and zero order with

argument \( x \).

\( V \) = a function of \( m \) and \( N \) (see Equations (1) and (2)).

\( N \) = a function of \( E_1 \) and \( E_2 \) (see Equation (2a)).

2.3 Development of the Equation

A vertical load, \( P \) (Figure 2), is applied at the point, \( 0 \), in the

horizontal, plane surface of a two-layer elastic system. The point of

load application is the origin of cylindrical coordinates, \( r \) and \( z \).

Positive values of \( z \) are measured vertically downward.

The thickness of the upper layer is \( h \) and its elastic modulus is \( E_1 \).

The thickness of the lower layer is infinite, and its elastic modulus

is \( E_2 \). Poisson's ratio for both layers is taken as \( 1/2 \).

It can be shown from Burmister's early work in elastic layered systems

(8) that the deflection, \( w \), of a surface point at the horizontal distance, \( r \),

from the point, \( 0 \), is related to the constants, \( h, E_1 \) and \( E_2 \), by the equation

\[
\frac{4\pi E_1}{3P} \int_{x=0}^{\infty} V \cdot \text{Jo}(x) dx = \int_{x=0}^{\infty} V \cdot \text{Jo}(x) dx
\]
FIGURE 2 - Two-layer elastic system loaded at a point on the surface.
where \( x = \frac{mr}{h} \),  
\[ m = \text{a parameter}, \]
\[ V = \frac{1 + 4Nme^{-2m} - N^2e^{-4m}}{1 - 2N(1 + 2m^2)e^{-2m} + N^2e^{-4m}}, \]
and  
\[ N = \frac{1 - E_2/E_1}{1 + E_2/E_1} = \frac{E_1 - E_2}{E_1 + E_2} \]

### 2.4 An Approximation of the Deflection Equation

The integration indicated in Equation 1 must be performed by numerical means. This task is made easier by taking advantage of the fact that (1) as \( x \) varies from zero to infinity in the integration process, \( m \) varies over the same range, while \( r \) and \( h \) are held constant, (2) as \( m \) varies from zero to infinity, the function \( V \) varies monotonically from \( E_1/E_2 \) to 1.0 and (3) for practical ranges of the ratio \( E_2/E_1 \), \( V \) approaches its limiting value of 1.0 at surprisingly low values of \( m \).

For example, it was found, as indicated in Table 1, that if \( m \) is set equal to 10, and \( E_2/E_1 \) is restricted to the range from zero to 1000, then \( V = 1.0 + .000001 \). Thus, we conclude that for practical purposes, when \( m \) is in the range from zero to 10, \( V \) is given by Equation 2, and when \( m \) is in the range from 10 to infinity, \( V = 1 \). This approximation can be expressed algebraically as follows:

\[
\int_0^{10r/h} V \cdot J_0(x) dx = \int_0^{10r/h} V \cdot J_0(x) dx + \int_0^{10r/h} J_0(x) dx 
\]

The second integral on the right side of Equation 3 is equivalent to the difference of two integrals, as indicated below:

\[
\int_{10r/h}^{x=10r/h} J_0(x) dx = \int_{0}^{10r/h} J_0(x) dx - \int_{0}^{x=0} J_0(x) dx = 1 - \int_{0}^{x=0} J_0(x) dx.
\]
Table 1: Values of the function, $V$, corresponding to selected values of the parameter $m$ and the modular ratio $E_2/E_1$.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\inf$</th>
<th>1000</th>
<th>100</th>
<th>10</th>
<th>1</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>Infinite</td>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
<td>100</td>
<td>1000</td>
</tr>
<tr>
<td>0.1</td>
<td>6012.</td>
<td>855.6</td>
<td>98.14</td>
<td>9.967</td>
<td>1</td>
<td>0.1006</td>
<td>0.01065</td>
<td>0.001655</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>50.49</td>
<td>47.94</td>
<td>32.98</td>
<td>8.056</td>
<td>1</td>
<td>0.1542</td>
<td>0.06727</td>
<td>0.05854</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>7.382</td>
<td>7.363</td>
<td>6.826</td>
<td>4.112</td>
<td>1</td>
<td>0.3250</td>
<td>0.2491</td>
<td>0.2414</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>1.137</td>
<td>1.137</td>
<td>1.134</td>
<td>1.110</td>
<td>1</td>
<td>0.9058</td>
<td>0.8888</td>
<td>0.8869</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>1.006</td>
<td>1.006</td>
<td>1.005</td>
<td>1.005</td>
<td>1</td>
<td>0.9955</td>
<td>0.9946</td>
<td>0.9945</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>1.000001</td>
<td>1.000001</td>
<td>1.000001</td>
<td>1.000001</td>
<td>1</td>
<td>0.999993</td>
<td>0.999991</td>
<td>0.999991</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inf.</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
<td>1</td>
<td>1</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
</tr>
</tbody>
</table>
By making the obvious substitution from Equation 4 in Equation 3 we have

\[ \int_0^\infty V \cdot J_0(x) \, dx \approx 10r/h \int_0^\infty V \cdot J_0(x) \, dx + 1 - \int_0^\infty J_0(x) \, dx, \]

or

\[ \int_0^\infty V \cdot J_0(x) \, dx = 1 + \int_0^\infty (V - 1) \cdot J_0(x) \, dx. \]

Comparing the last approximation, above, with Equation 1, we arrive at the approximation,

\[ \frac{4\pi E_1}{3P} \frac{w r}{h} = 1 + \int_0^{10r/h} (V - 1) \cdot J_0(x) \, dx \]  \hspace{1cm} (5)

where all symbols are as previously defined.

It is of interest to note from Equation 2 that \( V = 1 \) when \( E_2 = E_1 \) (that is, when the layered system of Figure 1 degenerates into a homogeneous elastic half-space), and that for this case Equation 5 reduces to

\[ \frac{4\pi E_1}{3P} \frac{w r}{h} = 1. \]

The correct equation for this case, according to Timoshenko (9), is

\[ \frac{4\pi E_1}{3P} \frac{w r}{h} = 1. \]

Thus, for the homogeneous case Equation 5 becomes exact.
3. Numerical Integration of Deflection Equation

To use Equation 5 it was necessary to employ some form of numerical integration process for evaluating the integral in that equation. The method known as Simpson's Rule was selected (11). This procedure required that a small but finite increment, \( \Delta x \), be chosen, and that the integral be calculated at \( x = 0, x = \Delta x, x = 2\Delta x, \) etc. over the specified range of integration. The smaller the value assigned to \( \Delta x \), the greater would be the accuracy of the result; on the other hand, the larger the value of \( \Delta x \), the less would be the required computer time. Thus a compromise between computer time and accuracy had to be made.

Noting that the integral of Equation 5 is the product of the factor, \( V - 1 \), which is a function of \( m \) and \( N \), and \( J_0(x) \), which is a function of \( x = mr/h \) (see Equation 1a), two safeguards against inaccurate results had to be incorporated into the program: (1) \( \Delta m \) had to be small enough to insure a sufficiently accurate numerical representation of the function \( V \), and (2) \( \Delta x \) had to be small enough to insure an accurate numerical representation of the function \( J_0(x) \).

After some study of the numerical values of \( V \) given in Table 1, and of the values of \( J_0(x) \) available from numerous sources (see, for example, Reference 10), the following rules were incorporated into the computer program for solving Equation 5:

(a) In the range \( m = 0 \) to \( m = 3 \), \( \Delta m \leq 0.01 \). (In FORTRAN, \( \text{DELML} .\leq \text{XK1} \).)

(b) In the range \( m = 3 \) to \( m = 10 \), \( \Delta m \leq 0.10 \). (In FORTRAN, \( \text{DELM2} .\leq \text{XK2} \).)
(c) In the entire range of x from 0 to 10r/h, not less than 61 values of Jo(x) are computed as x increases from any value x = c, to the value x = c + 3. This also insures that the number of values of Jo(x) computed between successive zeroes of that alternating function exceeds 61. (In FORTRAN, XNO = 61.)

Since Δx and Δm are interdependent according to Equation 1(a), that is,

$$Δx = Δm \cdot r/h,$$  \hspace{1cm} (1b)

the computer program had to insure that the rules (a), (b) and (c) given above were consistent with Equation 1(b). The details of how this was done may be found in the accompanying listing of the computer program and its flow diagram. Suffice it to say here that the accuracy of the solutions obtained (or the computer time used) can be changed by altering the values assigned to the FORTRAN variables XK1, XK2 and XNO mentioned in (a), (b) and (c) above and further defined in Appendix 1.

To explain briefly how Equation 5 is used in ELASTIC MODULUS to find pavement and subgrade moduli, consider the following:

Suppose that w1 has been measured on the surface of a pavement structure at the distance r1 from either Dynaflect load, and w2 at the distance r2. The thickness, h, of the pavement is known.

Now let F represent the function on the right side of Equation 5. We may then write two equations:

$$\frac{4\pi E_1}{3P} w_1 r_1 = F(E_2/E_1, r_1/h)$$ \hspace{1cm} (6a)

$$\frac{4\pi E_1}{3P} w_2 r_2 = F(E_2/E_1, r_2/h)$$ \hspace{1cm} (6b)
By dividing Equation 6a by 6b we obtain

\[
\frac{w_1 r_1}{w_2 r_2} = \frac{F(E_2/E_1, r_1/h)}{F(E_2/E_1, r_2/h)} \tag{7}
\]

where \(E_2/E_1\) is the only unknown.

By a convergent process of trial and error, a value of \(E_2/E_1\) usually can be found that satisfies Equation 7 to the desired degree of accuracy. After this has been done, \(E_1\) is calculated from Equation (6a), and finally \(E_2\) is found from the relation

\[
E_2 = E_1 \left(\frac{E_2}{E_1}\right).
\]
4. Accuracy Check

As mentioned earlier (Section 2.1) a point load was substituted in ELASTIC MODULUS for the area loads exerted by the Dynaflect. To check the effect of this assumption on accuracy, as well as the effect of the approximations described in Chapters 2 and 3, the following procedure was followed.

The contact area of each load wheel was measured approximately by inserting light sensitive paper between each wheel and the pavement, running the Dynaflect for a short time in strong sunlight, then removing the paper and measuring the unexposed areas.

From these measurements it was concluded that each 500 lb. load could be represented by a uniform pressure of 80 psi acting on a circular area with a radius of 1.41 inches. Furthermore, because of the symmetry of the load-geophone configuration, it was reasoned that the effect of both loads could be represented by a pressure of 160 psi acting on one circular area of the radius given above (1.41 inches).

The surface deflections $w_1$ and $w_2$ (see Figure 1) occurring at the distances $r = 10$ inches and $r = \sqrt{10^2 + 12^2} = 15.62$ inches from the center of the circle, could then be calculated from the program BISTRO, written by Koninklijke/Shell-Laboratorium, Amsterdam, and compared with deflections obtained by the program ELASTIC MODULUS modified slightly to receive as inputs $E_1$, $E_2$, $h$ and $r$ and to print out $w_1$ and $w_2$.

The two programs were compared as described above over a range of the ratio, $E_1/E_2$, from 0.1 to 1000, and a range of the thickness, $h$, from 5 to 40 inches. The results are recorded in Table 2 in the same manner that Dynaflect deflections are recorded -- that is, in milli-inches to two decimal places.
The table shows near perfect agreement in the range $1 \leq \frac{E_1}{E_2} \leq 1000$ for which the pavement is stiffer than the subgrade. On the other hand, with the subgrade much stiffer than the pavement ($\frac{E_1}{E_2} = 0.1$ in Table 2), the agreement was not as good. In addition, up-heavals occurred, as indicated by the negative signs of some of the deflections. In these cases the deflected surface is very irregular and Dynaflect data from such a pavement would be difficult to interpret since this device is not equipped to distinguish phase differences between load and geophone.

Since most pavements of the type illustrated in Figure 1 are obviously intended to be stiffer than their subgrades, and in view of the fact that irregular basin shapes are seldom encountered in practice, it is concluded from the data presented in Table 2 that ELASTIC MODULUS represents the theory of elasticity with sufficient accuracy to accomplish the purpose for which it was designed.
Table 2: Comparison of ELASTIC MODULUS with BISTRO

<table>
<thead>
<tr>
<th>E₁ (psi)</th>
<th>E₂ (psi)</th>
<th>E₁/E₂</th>
<th>h (in.)</th>
<th>( \omega_1 )</th>
<th>( \omega_2 )</th>
</tr>
</thead>
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<td>1,000</td>
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</tr>
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<td></td>
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<td></td>
<td>10</td>
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<td>-0.11</td>
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<td>40</td>
<td>14.9</td>
<td>14.9</td>
</tr>
</tbody>
</table>

Note: ELASTIC MODULUS: Point load of 1000 lbs.
BISTRO: Circular loaded area with radius of 1.41 in., pressure of 160 psi, load of 1000 lbs.
Both programs: Vertical deflection computed at the points \( r = 10'' \), \( z = 0 \) and \( r = 15.62'' \), \( z = 0 \).
5. Non-Unique Solutions

To investigate the possibility that the use of the program could lead to more than one solution -- that is, to more than one value of the ratio \( E_1/E_2 \) -- or perhaps to no solution at all in some cases, ELASTIC MODULUS was modified slightly to receive as inputs selected values of \( E_1/E_2 \) and the layer thickness \( h \), and to compute the corresponding ratio, \( w_1r_1/w_2r_2 \) (see Equation 7). The results of these computations were plotted as contours of the layer thickness, \( h \), in Figure 3. The range of input data was limited to the largest range that might be expected from field deflection tests made on real highways of the type illustrated in Figure 1.

To facilitate interpretation, Figure 3 has been divided into four quadrants as indicated on the graph. For example, by referring to quadrants I and II it can be seen that if the measured inputs to ELASTIC MODULUS satisfy the inequalities \( w_1r_1/w_2r_2 > 1 \) and \( h \geq 9.2" \) (see the dashed contour), a unique solution satisfying the inequality \( E_1/E_2 < 1 \) exists, and in this case the program finds and prints the two moduli. If, on the other hand, \( w_1r_1/w_2r_2 > 1 \) (as before) but \( h < 9.2" \), the possibility of two solutions exists -- or of no solution at all if the measured ratio \( w_1r_1/w_2r_2 \) is sufficiently great. In this case, i.e. \( w_1r_1/w_2r_2 > 1 \) and \( h < 9.2" \), the program abandons the search for a solution and prints the message "NO UNIQUE SOLUTION".

By examining quadrants III and IV, it can be concluded that if \( w_1r_1/w_2r_2 < 1 \) and \( h \geq 9.2" \), a unique solution satisfying the inequality \( E_1/E_2 > 1 \) exists. In this case the program finds the solution and prints the two moduli. On the other hand if \( w_1r_1/w_2r_2 < 1 \) as before, but \( h < 9.2" \) there are two possible solutions, one in quadrant III for \( E_1/E_2 > 1 \), and another
Figure 3: Contours of pavement thickness, h, plotted as a function of the ratios $E_1/E_2$ and $w_1r_1/w_2r_2$. 

-17-
in quadrant IV for $E_1/E_2 < 1$. Of these two solutions the one in quadrant III, representing a pavement whose elastic modulus is greater than that of the subgrade, is the more probable; therefore, the program seeks out the quadrant III solution, prints the corresponding moduli, and ignores the quadrant IV solution.

The information deduced above from Figure 3, and used in the control of the program ELASTIC MODULUS, is summarized in Table 3.
Table 3: Summary of Information from Figure 3 Used in the Control of the Program, ELASTIC MODULUS

<table>
<thead>
<tr>
<th>Measured Input Data</th>
<th>Unique Solution</th>
<th>Layer Having The Greater Modulus</th>
<th>Program Printout</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1 r_1/w_2 r_2$</td>
<td>Thickness, $h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greater than 1</td>
<td>Greater than 9.2</td>
<td>Yes</td>
<td>Subgrade</td>
</tr>
<tr>
<td>Greater than 1</td>
<td>Less than 9.2</td>
<td>No</td>
<td>&quot;NO UNIQUE SOLUTION&quot;*</td>
</tr>
<tr>
<td>Less than 1</td>
<td>Greater than 9.2</td>
<td>Yes</td>
<td>Pavement</td>
</tr>
<tr>
<td>Less than 1</td>
<td>Less than 9.2</td>
<td>No</td>
<td>Subgrade and pavement moduli</td>
</tr>
</tbody>
</table>

* When the experimental data $w_1 r_1/w_2 r_2$ exceeds unity, and $h$ is less than 9.2", some cases can arise for which no solution at all is possible.
6. Examples of Solutions Obtained by ELASTIC MODULUS

In May, 1968, Dynaflect deflections were measured at ten points in the outer wheel path on each of several 500-ft. sections of highways in the vicinity of College Station, Texas, originally for the purpose of gaining experience in the determination of the "stiffness coefficient" mentioned in the Introduction of this report (page 1). Some of these data, including thicknesses obtained by coring at five points in each section, were used as inputs to the computer program discussed herein for the purpose of illustrating its use in obtaining the elastic moduli of pavements and subgrades. The results are summarized in Tables 4 and 5, while the computer print-outs -- in the standard format of the program -- are shown in Tables 6a through 6g. In the latter group of tables the readings of each of the five geophones at each test station are given, although only the greatest deflections, \( w_1 \) and \( w_2 \), were actually used in estimating the moduli \( E_1 \) and \( E_2 \).

Tables 4 and 5 are arranged in descending order of the magnitude of the average modulus of pavement and subgrade, respectively. In comparing these two tables it is of interest to note that the variability of the pavement modulus, as indicated by the coefficient of variation in the last column, is generally greater than that of the subgrade. In addition it is apparent that the range of \( E_1 \) (13,900 psi to 283,200 psi) is much greater than the range of \( E_2 \) (11,700 psi to 20,000 psi). Finally, it should be pointed out that the pavement of Section 12, at the bottom of the list in Table 4, had an average modulus (13,900 psi) of approximately the same magnitude as that of its subgrade (14,400 psi).
The low pavement modulus found for Section 12 invites some discussion. The low value obtained may be due to the relatively poor quality of the major component of the pavement, a sandstone which, according to local engineers, has in some cases performed poorly. In any event the surfacing of this section had been overlayed -- because of map cracking -- shortly before it was tested in 1968, then again developed severe map cracking that required sealing in 1970. The seal coat failed to arrest the progress of surface deterioration, and at this writing (June, 1971) it is again being overlayed with one inch of hot-mix asphaltic concrete. In short, the contrast between the stiffness of the surfacing material and that of the base seems to be at the root of the trouble in this section.

Beyond these remarks concerning Section 12, and the additional fact that the ordering of the other materials appears reasonable, any other discussion of the ordering of the materials in Tables 3 and 4 is considered to be beyond the scope of this report.
Table 4: Average Pavement Modulus, $E_I$, for Seven 500-ft. Sections of Highways near College Station, Texas
(Deflection measurements made May 21, 1968)

<table>
<thead>
<tr>
<th>Test Section</th>
<th>Pavement Materials and Thicknesses</th>
<th>Pavement Thickness, $h$</th>
<th>Pavement Modulus, $E_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average Value (In.)</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>15</td>
<td>1.2&quot; Asph. Conc. 14.0&quot; Cement stabilized limestone</td>
<td>15.2</td>
<td>1.2</td>
</tr>
<tr>
<td>4</td>
<td>0.5&quot; Seal Coat 7.5&quot; Asphalt stabilized gravel</td>
<td>8.0</td>
<td>0.4</td>
</tr>
<tr>
<td>16</td>
<td>1.0&quot; Asph. Conc. 6.5&quot; Asph. emulsion stab. gravel</td>
<td>7.5</td>
<td>0.4</td>
</tr>
<tr>
<td>17</td>
<td>0.5&quot; Seal Coat 7.8&quot; Iron ore gravel</td>
<td>8.3</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>0.5&quot; Seal Coat 11.5&quot; Lime stabilized sandstone</td>
<td>12.0</td>
<td>2.8</td>
</tr>
<tr>
<td>3</td>
<td>0.5&quot; Seal Coat 12.0&quot; Red sandy gravel</td>
<td>12.5</td>
<td>1.0</td>
</tr>
<tr>
<td>12</td>
<td>3.7&quot; Asph. Conc. 16.2&quot; Sandstone</td>
<td>19.9</td>
<td>0.5</td>
</tr>
</tbody>
</table>

* Measurements were made at 10 locations in each section. Less than 10 solutions occur in cases where $\omega_1 r_1/\omega_2 r_2 > 1$ and $h < 9.2"$, as explained in Chapter 4.
Table 5: Average Subgrade Modulus, $E_2$, for Seven 500-ft. Sections of Highways near College Station, Texas
(Deflection measurements made May 21, 1968)

<table>
<thead>
<tr>
<th>Section</th>
<th>Thickness</th>
<th>Description</th>
<th>Formation</th>
<th>No.* Solutions</th>
<th>Average Value (PSI)</th>
<th>Standard Deviation</th>
<th>Coefficient of Variation (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>32&quot;</td>
<td>Red sandy clay, some gravel</td>
<td>Stone City</td>
<td>10</td>
<td>20,000</td>
<td>900</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>23&quot;</td>
<td>Sand over clay</td>
<td></td>
<td>10</td>
<td>19,000</td>
<td>1600</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>25&quot;</td>
<td>Grey sandy clay</td>
<td>Spiller Sandstone Member of Cook Mountain Formation</td>
<td>2</td>
<td>14,900</td>
<td>800</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>24&quot;</td>
<td>Tan sandy clay</td>
<td>Caddell</td>
<td>10</td>
<td>14,500</td>
<td>1400</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>22&quot;</td>
<td>Black stiff clay</td>
<td>Lagarto</td>
<td>10</td>
<td>14,400</td>
<td>900</td>
<td>6</td>
</tr>
<tr>
<td>17</td>
<td>21&quot;</td>
<td>Grey sandy clay</td>
<td>Spiller Sandstone Member of Cook Mountain Formation</td>
<td>8</td>
<td>12,700</td>
<td>1700</td>
<td>13</td>
</tr>
<tr>
<td>16</td>
<td>18&quot;</td>
<td>Brown clay</td>
<td>Alluvium deposit of Brazos River</td>
<td>10</td>
<td>11,700</td>
<td>700</td>
<td>6</td>
</tr>
</tbody>
</table>

* Measurements were made at 10 locations in each section. Less than 10 solutions occur in cases where $w_1r_1/w_2r_2 > 1$ and $h < 9.2"$, as explained in Chapter 4.
## Computer print-out for Section 3

<table>
<thead>
<tr>
<th>STATION</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
<th>W5</th>
<th>SCI</th>
<th>ES</th>
<th>EP</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - A</td>
<td>1.170</td>
<td>0.770</td>
<td>0.520</td>
<td>0.310</td>
<td>0.219</td>
<td>0.400</td>
<td></td>
<td></td>
<td>20200. 23500.</td>
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<tr>
<td>1 - B</td>
<td>1.140</td>
<td>0.770</td>
<td>0.510</td>
<td>0.310</td>
<td>0.213</td>
<td>0.370</td>
<td></td>
<td></td>
<td>20500. 28200.</td>
</tr>
<tr>
<td>2 - A</td>
<td>1.290</td>
<td>0.840</td>
<td>0.490</td>
<td>0.300</td>
<td>0.204</td>
<td>0.450</td>
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<td></td>
<td>18400. 20300.</td>
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<tr>
<td>2 - B</td>
<td>1.200</td>
<td>0.840</td>
<td>0.490</td>
<td>0.300</td>
<td>0.201</td>
<td>0.360</td>
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<td>19000. 33300.</td>
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<tr>
<td>3 - A</td>
<td>1.140</td>
<td>0.770</td>
<td>0.470</td>
<td>0.300</td>
<td>0.195</td>
<td>0.370</td>
<td></td>
<td></td>
<td>20500. 28200.</td>
</tr>
<tr>
<td>3 - B</td>
<td>1.110</td>
<td>0.770</td>
<td>0.460</td>
<td>0.300</td>
<td>0.201</td>
<td>0.340</td>
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<td>1.470</td>
<td>0.960</td>
<td>0.490</td>
<td>0.320</td>
<td>0.222</td>
<td>0.510</td>
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<tr>
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<td>1.380</td>
<td>0.900</td>
<td>0.470</td>
<td>0.310</td>
<td>0.213</td>
<td>0.480</td>
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<td></td>
<td>17200. 19000.</td>
</tr>
<tr>
<td>5 - A</td>
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<td>0.870</td>
<td>0.500</td>
<td>0.340</td>
<td>0.231</td>
<td>0.420</td>
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<td>18100. 24600.</td>
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<tr>
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<td>0.800</td>
<td>0.460</td>
<td>0.310</td>
<td>0.219</td>
<td>0.460</td>
<td></td>
<td></td>
<td>19000. 18100.</td>
</tr>
</tbody>
</table>

### Averages
- W1: 1.245
- W2: 0.829
- W3: 0.486
- W4: 0.310
- W5: 0.212
- SCI: 0.416
- ES: 1.8970
- EP: 24720

### Standard Deviation
- 0.057

### Number of Points in Average
- 10

### Remarks
- Table 6a: Computer print-out for Section 3.
```
STAT I

1 - A  1.650  1.200  0.870  0.660  0.500  0.450  14300.  84700.
1 - B  1.560  1.110  0.810  0.610  0.490  0.450  15500.  73100.
2 - A  2.310  1.470  0.930  0.710  0.530  0.840  NO UNIQUE SOLUTION
2 - B  2.310  1.410  0.900  0.670  0.510  0.900  NO UNIQUE SOLUTION
3 - A  2.430  1.550  0.930  0.670  0.490  0.930  NO UNIQUE SOLUTION
3 - B  2.490  1.530  0.930  0.670  0.500  0.960  NO UNIQUE SOLUTION
4 - A  2.490  1.470  0.900  0.640  0.480  1.020  NO UNIQUE SOLUTION
4 - B  2.430  1.410  0.840  0.610  0.470  1.020  NO UNIQUE SOLUTION
5 - A  2.340  1.440  0.870  0.620  0.450  0.900  NO UNIQUE SOLUTION
5 - B  2.430  1.470  0.930  0.650  0.470  0.960  NO UNIQUE SOLUTION

AVERAGES  2.244  1.401  0.891  0.651  0.489  0.843  14900.  78900.

STANDARD DEVIATION  0.214  849.  8202.

NUMBER OF POINTS IN AVERAGE = 10 2

W1  DEFLECTION AT GEOPHONE 1
W2  DEFLECTION AT GEOPHONE 2
W3  DEFLECTION AT GEOPHONE 3
W4  DEFLECTION AT GEOPHONE 4
W5  DEFLECTION AT GEOPHONE 5
SCI  SURFACE CURVATURE INDEX (W1 MINUS W2)
ES  ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
EP  ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2

Table 6b: Computer print-out for Section 4.
```
TEXAS HIGHWAY DEPARTMENT
DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

THIS PROGRAM WAS RUN - 06/21/71

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<th>COUNTY</th>
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<td>BURLESON</td>
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<table>
<thead>
<tr>
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<th>SECT.</th>
<th>JOB</th>
<th>HIGHWAY</th>
<th>DATE</th>
<th>DYNAFLECT</th>
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<td>1</td>
<td>FM 1361</td>
<td>5-21-68</td>
<td>1</td>
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</table>

PAV. THICK. = 12.00 INCHFS

SEAL COAT 0.50
LIME STAB. SANDSTONE 11.50
TAN SANDY CLAY SUBGR 0.0

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<tr>
<th>STATION</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
<th>W5</th>
<th>SCI</th>
<th>**</th>
<th>**</th>
<th>**</th>
<th>**</th>
<th>EP</th>
<th>**</th>
<th>**</th>
<th>**</th>
<th>**</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - A</td>
<td>1.500</td>
<td>1.110</td>
<td>0.710</td>
<td>0.470</td>
<td>0.330</td>
<td>0.39C</td>
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<td>41600.</td>
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<tr>
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<td>12700.</td>
<td>65800.</td>
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</tr>
<tr>
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<td>1.200</td>
<td>0.67C</td>
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<tr>
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<td>1.050</td>
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</tr>
<tr>
<td>3 - A</td>
<td>1.500</td>
<td>1.050</td>
<td>0.600</td>
<td>0.370</td>
<td>0.267</td>
<td>0.450</td>
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<td>27900.</td>
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<td>0.450</td>
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<td>25800.</td>
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AVERAGES 1.569 1.107 0.636 0.396 0.270 0.462 14480. 32340. |
STANDARD DEVIATION 0.112 1413. 15108. |
NUMBER OF POINTS IN AVERAGE = 10 10 10 |

W1 DEFLECTION AT GEOPHONE 1
W2 DEFLECTION AT GEOPHONE 2
W3 DEFLECTION AT GEOPHONE 3
W4 DEFLECTION AT GEOPHONE 4
W5 DEFLECTION AT GEOPHONE 5
SCI SURFACE CURVATURE INDEX (W1 MINUS W2)
ES ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
EP ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2

Table 6c: Computer print-out for Section 5.
Texas Highway Department

District 17 - Design Section

DynaFlect Deflections and Calculated Elastic Modulii

This program was run - 06/21/71

<table>
<thead>
<tr>
<th>Cont.</th>
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Pav. thick. = 19.90 inches

HOT MIX ASPH. CONC. 3.75 SANDSTONE 16.15

Black Clay Subgrade 0.0

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<tr>
<th>Station</th>
<th>W1</th>
<th>W2</th>
<th>W3</th>
<th>W4</th>
<th>W5</th>
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<th>** ES</th>
<th>** ** EP</th>
<th>** Remarks</th>
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Averages 1.692 1.065 0.660 0.477 0.365 0.627 14420. 13900.

Standard Deviation 0.091 861. 2661.

Number Of Points In Average = 10 10 10

W1  Deflection at Geophone 1
W2  Deflection at Geophone 2
W3  Deflection at Geophone 3
W4  Deflection at Geophone 4
W5  Deflection at Geophone 5
SCI Surface Curvature Index (W1 Minus W2)
FS  Elastic Modulus of the Subgrade from W1 and W2
EP  Elastic Modulus of the Pavement from W1 and W2

Table 6d: Computer print-out for Section 12.
## Table 6e: Computer print-out for Section 15.

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<th>STATION</th>
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<th>( W_3 )</th>
<th>( W_4 )</th>
<th>( W_5 )</th>
<th>( SCI )</th>
<th>( ES )</th>
<th>( ES )</th>
<th>( EP )</th>
<th>Remarks</th>
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<td>0.490</td>
<td>0.390</td>
<td>0.310</td>
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<td>230800.0</td>
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<td>0.600</td>
<td>0.490</td>
<td>0.390</td>
<td>0.310</td>
<td>0.080</td>
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<td>280500.0</td>
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<td>0.520</td>
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<td>0.100</td>
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<td>0.060</td>
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<tr>
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<td>21000.0</td>
<td>271700.0</td>
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**Averages:**
- \( W_1 \): 0.674
- \( W_2 \): 0.592
- \( W_3 \): 0.481
- \( W_4 \): 0.373
- \( W_5 \): 0.419
- \( ES \): 0.082

**Standard Deviation:**
- 0.016

**Number of Points in Average:**
- 10
**TEXAS HIGHWAY DEPARTMENT**

**DISTRICT 17 - DESIGN SECTION**

**DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII**

**THIS PROGRAM WAS RUN - 06/21/71**

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<tbody>
<tr>
<td>17</td>
<td>BRAZOS</td>
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<table>
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**PAV. THICK. = 7.50 INCHES**

**ASPHALT SURFACING** 1.00  **ASPH EMUL STAB GRAVL 6.50**

**BROWN CLAY SUBGRADE** 0.0

**Table 6f: Computer print-out for Section 16.**

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<th>W5</th>
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<th><strong>ES</strong></th>
<th><strong>EP</strong></th>
<th><strong>REMARKS</strong></th>
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<td>1.020</td>
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<td>0.490</td>
<td>0.630</td>
<td>10900.</td>
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**AVERAGES** 2.058 1.479 0.951 0.640 0.492 0.579 11740. 73910. 3843. 10 10 10

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<td>DEFLECTION AT GEOPHONE 4</td>
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<tr>
<td>W5</td>
<td>DEFLECTION AT GEOPHONE 5</td>
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<td>SCI</td>
<td>SURFACE CURVATURE INDEX (W1 MINUS W2)</td>
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<tr>
<td>ES</td>
<td>ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2</td>
</tr>
<tr>
<td>EP</td>
<td>ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2</td>
</tr>
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</table>

**STANDARD DEVIATION** 0.049 679. 13843.

**NUMBER OF POINTS IN AVERAGE** = 10 10 10.
TEXAS HIGHWAY DEPARTMENT

DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

THIS PROGRAM WAS RUN - 06/21/71

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PAV. THICK. = 8.30 INCHES

SEAL COAT = 0.50 IRON ORE GRAVEL 7.80

GREY SANDY CLAY SUBG = 0.0

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<th>W3</th>
<th>W4</th>
<th>W5</th>
<th>SCI</th>
<th>**</th>
<th>ES</th>
<th>**</th>
<th>**</th>
<th>EP</th>
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<td>0.630</td>
<td>0.480</td>
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<td>UNIQUE</td>
<td>SOLUTION</td>
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</tr>
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</tbody>
</table>

AVERAGES 2.037 1.362 0.917 0.671 0.528 0.675 12700. 36600.

| STANDARD DEVIATION | 0.146 | 1710. | 24675. |
| NUMBER OF POINTS IN AVERAGE | 10 | 8 | 8 |

| W1 | DEFLECTION AT GEOPHONE 1 |
| W2 | DEFLECTION AT GEOPHONE 2 |
| W3 | DEFLECTION AT GEOPHONE 3 |
| W4 | DEFLECTION AT GEOPHONE 4 |
| W5 | DEFLECTION AT GEOPHONE 5 |
| SCI | SURFACE CURVATURE INDEX (w1 MINUS w2) |
| ES | ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2 |
| EP | ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2 |

Table 6g: Computer print-out for Section 17.
7. Adjustment of Moduli for Practical Use in Pavement Design

As previously noted, the elastic moduli estimated by the computer program are based on deflections produced and measured by the Dynaflect system. Correlation studies of Dynaflect deflections with those produced by a 9000-lb. dual-tired wheel load and measured by means of the Benkelman Beam on highways in Illinois and Minnesota in 1967 (3) indicated that the 9000-lb. wheel load deflection could, with reasonable accuracy, be estimated from the Dynaflect deflection, \( w_1 \), by multiplying \( w_1 \) by 20.

But the peak-to-peak load of the Dynaflect is 1000-lbs.; thus, one would expect that the multiplying factor would be about 9, rather than 20 as found by actual field experience.

Various explanations could be advanced to explain this discrepancy. However, they would not alter the fact, brought out by the correlation study, that if one desires to use the values of \( E_1 \) and \( E_2 \) found from Dynaflect deflections to calculate the deflection of a linear elastic layered system acted on by a heavy vehicle, then he should approximately halve these moduli before using them in his calculations.
List of References


5. Scrivner, F. H. and W. M. Moore, "An Empirical Equation for Predicting Pavement Deflections," Research Report 32-12, Texas Transportation Institute, Texas A&M University, College Station, Texas.


Appendix 1

The variable names used in ELASTIC MODULUS are listed on the following pages. The variable names and their definitions are in alphabetical order in the following sequence:

- MAIN Program Variables
- Subroutine EMOD Variables
- Function BESJO Variables
- Function V Variables
MAIN Program Variables

A - Dummy array used with subroutine CORE to select the correct input format for each card read

AAP2 - Sum of pavement moduli

AAS2 - Sum of subgrade moduli

AAP2V - Average pavement modulus

AAS2V - Average subgrade modulus

AP2 - Elastic modulus of the pavement, rounded to nearest 100 (appears as EP on printout)

AS2 - Elastic modulus of the subgrade, rounded to nearest 100 (appears as ES on printout)

ASC1 - Sum of (W1 - W2). W1 - W2 = surface curvature index.

ASC1V - Average surface curvature index

AW1 - Sum of Geophone 1 deflections

AW2 - Sum of Geophone 2 deflections

AW3 - Sum of Geophone 3 deflections

AW4 - Sum of Geophone 4 deflections

AW5 - Sum of Geophone 5 deflections

AW1V - Average Geophone 1 deflection

AW2V - Average Geophone 2 deflection

AW3V - Average Geophone 3 deflection

AW4V - Average Geophone 4 deflection

AW5V - Average Geophone 5 deflection

COMM - Comments related to the project

CORE - Subroutine to re-read a card under format control

CO1, CO2, CO3, CO4 - County Name
D1 - Geophone 1 reading
D2 - Geophone 1 multiplier
D3 - Geophone 2 reading
D4 - Geophone 2 multiplier
D5 - Geophone 3 reading
D6 - Geophone 3 multiplier
D7 - Geophone 4 reading
D8 - Geophone 4 multiplier
D9 - Geophone 5 reading
D10 - Geophone 5 multiplier

DAP - Pavement elastic modulus (unrounded) as calculated in subroutine EMOD

DAS - Subgrade elastic modulus (unrounded) as calculated in subroutine EMOD

DATE - An IBM subroutine that returns the current month, day, & year

DP - Total pavement thickness

EMOD - Subroutine to calculate pavement & subgrade moduli

HWY1, HWY2 - Highway name & number

I - Pointer for data read into storage

ICK - Switch to indicate last data card

ICONT - Contract number for the highway

IDAY - Day the deflections were taken

IDIST - District number

IDYNA - Dynaflect number

IJOB - THD job number

ISECT - THD section number for the highway

IXDATE - Return arguments for subroutine DATE (month, day, year)

IYEAR - Year the deflections were taken
LA1 - Description of material in Layer 1
LA2 - Description of material in Layer 2
LA3 - Description of material in Layer 3
LA4 - Description of material in Layer 4
LA5 - Description of material in Layer 5
LA6 - Description of material in Layer 6
M - Month the deflections were taken
N - Counter for number of error free data cards read
NO - Counter for data cards omitted because of errors
NI - Counter to control printing of 30 lines per page
NCARD - Denotes card type
   100 = Project identification card
   200 = Existing pavement description card (layers 1, 2, & 3)
   300 = Existing pavement description card (layers 4, 5, & 6)
   400 = Data card (geophone readings and multipliers)
PN - Number of test points to be used in the analysis
REM - Any pertinent remarks related to any test point
ROUND - Statement function to round a given value of E1 or E2 to the nearest 100 psi
SCI - Surface curvature index, W1 - W2, in mils
SE1 - Standard deviation of surface curvature index
SE2 - Standard deviation of subgrade moduli
SE3 - Standard deviation of pavement moduli
SR1 - Variance of surface curvature index
SR2 - Variance of subgrade moduli
SR3 - Variance of pavement moduli
STA - Station number
T1 - Layer 1 thickness
T2 - Layer 2 thickness
T3 - Layer 3 thickness
T4 - Layer 4 thickness
T5 - Layer 5 thickness
T6 - Layer 6 thickness
W1 - Deflection at Geophone number 1
W2 - Deflection at Geophone number 2
W3 - Deflection at Geophone number 3
W4 - Deflection at Geophone number 4
W5 - Deflection at Geophone number 5
XLANE - Traffic lane & direction
Subroutine EMOD Variables

ACC - Test for convergence in iteration for finding E2/E1

AREA1 - Result of the integration from x = 0 to x = 3r/h

AREA2 - Result of the integration from X = 3r/h to X = 10r/h

BESJO - Function subroutine to return the Bessel Function Jo(x) for each x used

DELM1 - Increment of m used in interval from m = 0 to m = 3

DELM2 - Increment of m used in interval from m = 3 to m = 10

DELTA - Incremental value for E2/E1 (Subgrade modulus divided by Pavement modulus) used in iteration process

DELX1 - Increment of x in integration from x = 0 to x = 3r/h

DELX2 - Increment of x in integration from x = 3r/h to x = 10r/h

E1 - Pavement modulus (E1)

E2 - Subgrade modulus (E2)

ER - Input specifying the accuracy of the iteration in calculating the ratio E2E1

E2E1 - Ratio of subgrade modulus to pavement modulus (E2/E1)

ERROR - (F1F2 - RATIO), where F1F2 is calculated and RATIO is observed

FF - Function defined as \( \frac{4nE_1}{3P} \)\( w_1r_1 \) (See Eq. 5)

F1F2 - Ratio of FF with i = 1 to FF with i = 2

H - Pavement thickness, h

ISW - Switch used in iterating to find E2/E1, indicates first time through the iteration loop

MINUS - Switch used in iterating to find E2/E1, indicates a negative ERROR

N1 - Number of intervals used for integration from x = 0 to x = 3r/h
N2 - Number of intervals used for integration from \( x = 3r/h \) to \( x = 10r/h \)

P - Dynaflect load = 1000#

PART1 - Sum of interior ordinates of first integration

PART2 - Sum or end ordinates of first integration

PART3 - Sum of interior ordinates of second integration

PART4 - Sum of end ordinates of second integration

PLUS - Switch used in iterating to find \( E2/E1 \), indicates a positive error

RH - Radius (distance of geophone from load wheel) divided by pavement thickness, \((r/h)\)

R1 - Distance from load to Geophone 1

R2 - Distance from load to Geophone 2

RATIO - \((W1R1/W2R2)\)

SAVE - Contains the previous ERROR calculated that is closest to the convergence criterion in iterating to find \( E2/E1 \)

V - Function subroutine to return the value of \( V \) for each \( E2/E1 \) and \( XM1 \) values used

W1 - Geophone 1 deflection (mils)

W2 - Geophone 2 deflection (mils)

X1 - Value of any \( x \) in the interval \( x = 0 \) to \( x = 3r/h \)

X2 - Value of any \( x \) in the interval \( x = 3r/h \) to \( x = 10r/h \)

XK1 - Maximum value of \( \Delta m \) in the interval \( m = 0 \) to \( m = 3 \) (now set at 0.01)

XK2 - Maximum value of \( \Delta m \) in the interval \( m = 3 \) to \( m = 10 \) (now set at 0.10)

XM1 - Value of any \( m \) in the interval \( m = 0 \) to \( m = 3 \)

XM2 - Value of any \( m \) in the interval \( m = 3 \) to \( m = 10 \)

XN - \((E1 - E2)/(E1 + E2)\)
XNO - Minimum number of values of \( J_0(x) \) calculated in the interval from \( x \) to \( x + 3 \), in the calculation by Simpson's Rule of \( \text{AREA1} \) and \( \text{AREA2} \). XNO must be an odd number and is now set at XNO = 61.

Y - Array to store ordinates to be used in integration from \( x = 0 \) to \( x = 10r/h \).
Function BESJO Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Value x in the interval x = 0 to x = 10r/h</td>
</tr>
<tr>
<td>X3</td>
<td>X/3, or 3/X if X &gt; 3</td>
</tr>
<tr>
<td>X32</td>
<td>X3 Squared</td>
</tr>
<tr>
<td>X33</td>
<td>X3 Cubed</td>
</tr>
<tr>
<td>X34</td>
<td>X3 to the Fourth Power</td>
</tr>
<tr>
<td>X35</td>
<td>X3 to the Fifth Power</td>
</tr>
<tr>
<td>X36</td>
<td>X3 to the Sixth Power</td>
</tr>
</tbody>
</table>
Function V Variables

EXPM2M - Exponential, $e^{-2m}$

EXPM4M - Exponential, $e^{-4m}$

XM - Value of any m in the interval m = 0 to m = 10

XN - $(E_1 - E_2)/(E_1 + E_2)$
Appendix 2

A narrative of the procedure used by ELASTIC MODULUS to calculate pavement and subgrade elastic moduli is contained on the following pages.
Description of ELASTIC MODULUS Program

The ELASTIC MODULUS program consists of a main program, two subroutines, and two function subroutines. The main program reads the input data, performs certain data transformations, and outputs the results. Subroutine CORE is called by the main program and is used to allow the user to select the input format to be used to read a certain card. The elastic moduli of the pavement and subgrade for each test point are calculated in subroutine EMOD. The function subroutines BESJO and V are called by EMOD and are used in the numerical integration used in calculating the elastic moduli.

The main program reads each input data card into a storage area and uses subroutine CORE to select the read statement and data format to read each data card. Subroutine CORE allows a FORTRAN program to read under format control from a storage area which contains alphabetic character codes of a card image. Each data card has a code punched in the first three columns that designates the card type. If the code is 100 the card contains control information about the job, location, date, and total pavement thickness (see Appendix 4). Card code 200 indicates a card that contains word descriptions and thicknesses of the first three layers of the pavement (see Appendix 4). The word descriptions and thicknesses of layers 4, 5, and 6 (if present) are on cards with code 300 (see Appendix 4). If the card code is 400 or is blank, the card contains the station number and geophone readings and multipliers for each observation (see Appendix 4). Card code 100 also
indicates the beginning of data cards for each job (or set of observations) and all counters and sums are set to their initial values. The information on Data Cards 1 and 2 (and Data Card 3, if present) is read and printed in the heading of the output.

The deflections at each geophone are calculated from the geophone readings and multipliers on each Data Card 4. SCI = W1 - W2 is also calculated. If either W1 or W2 is zero, or if W1 is less than W2, an error message is printed, the observation is not included in the analysis, and the next card is read. If the quantity W1R1/W2R2 is greater than 1 and the total pavement thickness is less than 9.2" the observation is not included in the analysis, an error message is printed, and the next card is read. If W1 and W2 are valid observations they are converted to inches and are passed to Subroutine EMOD along with the total pavement thickness for the elastic moduli calculation.

The pavement and subgrade moduli returned from EMOD are rounded to the nearest 100, W1 and W2 are converted back to mils, the sums of the deflections, SCI, pavement modulus, and subgrade modulus are incremented by the individual observations of each of these variables. The counter N (the sum of the valid observations) is incremented and a line of output consisting of the station number, W1, W2, W3, W4, W5, SCI, subgrade modulus, pavement modulus, and remarks is printed. The program will then skip to a new page before going back to read the next card if 30 lines of output have been printed. If all the data cards have been read the program calculates and prints averages of all deflections, SCI, pavement modulus, and the subgrade modulus. The variances and standard deviations of SCI, pavement modulus and subgrade modulus are calculated and printed and the program reads Data Card 1 of the next set of observations or terminates normally if there is no more data.
Subroutine EMOD uses the W1, W2, and total pavement thickness from the main program in the integration process and iteration scheme used to calculate the pavement modulus, E1 and the subgrade modulus, E2. All calculations in EMOD and the function subroutines BESJO and V are done in double precision to preserve the accuracy of the numerical integration. The user has the option to change the following variables or leave them at their present values:

\[ P = 1000, \ ER = 0.001, \ XNO = 61.0, \ XK1 = 0.01, \ XK2 = 0.10 \]

All switches and counters are initialized, r/h ratios are calculated and the variable RATIO (used in determining the convergence criterion in iteration for finding E2/E1) is calculated.

DELM1 is calculated and tested against the maximum assigned value for this variable. DELM1 is then set to the maximum value or the calculated value (whichever is the smaller value) and is used in calculating DELX1. DELM2 is calculated and tested in the same manner. The starting values of DELTA and E2E1 are selected and XN for the first iteration is calculated. The iteration loop begins with the calculation of each XN value for each E2E1 value used.

The number of intervals for each integration, N1 and N2, are calculated. (Note -- N1 and N2 must be odd integers.) The ordinates for each x in each integration interval are calculated and stored in the vector Y. The area of each integration interval is calculated according to Simpson's Rule (See Reference 11) and the function FF(1) = AREA1 + AREA2 + 1 is calculated (See Equation 5). FF(2) is calculated as above except RH(2) is used in calculating DELM1 and DELM2.
ERROR is calculated and tested against the convergence criterion. If ERROR is < ACC, E1 and E2 are calculated and EMOD returns to the main program. Otherwise a new value of E2E1 is calculated and the iteration loop is repeated.

The iteration method consists of trying values of E2E1 until ERROR is within the convergence criterion. The ERROR for any trial value of E2E1 that is closest to the convergence criterion is saved so that E2E1 values for subsequent trials can be adjusted to "home in" on the convergence criterion in a minimum number of trials.

For the first trial value of E2E1, if ERROR is not within the convergence criterion, ERROR is stored in SAVE and ISW is set to 1. If ERROR > 0, PLUS is set to 1 and MINUS is set to 0. This indicates ERROR is positive since PLUS was set to 0 and minus was set to 1 initially. For each successive pass through the iteration loop the sign of ERROR determines the segment of code executed to adjust E2E1 for the next trial until the convergence criterion is met.

When ERROR is positive, E2E1 is adjusted for the next trial in the following manner. PLUS is set to 1 to indicate ERROR is positive. If ERROR from the previous trial was positive, E2E1 is decreased for the next trial and the iteration loop is repeated. If ERROR from the previous trial was negative, DELTA is decreased by 50%, and the test for SAVE < 0 is made. If SAVE is positive, ERROR is stored in SAVE, E2E1 is decreased for the next trial and the iteration loop is repeated. If SAVE is negative, the test for |SAVE| > ERROR is made. A true condition indicates the iteration method is approaching convergence on this trial from the positive direction so ERROR is stored in SAVE, E2E1 is decreased for the next trial and the iteration loop is repeated. A false condition indicates
ERROR is departing from convergence in the positive direction, so E2E1 is decreased for the next trial and the iteration loop is repeated.

E2E1 is adjusted for the next trial in the following manner when ERROR is negative. MINUS is set to 1 to indicate ERROR is negative. If ERROR from the previous trial was negative the iteration method is approaching the convergence criterion from the negative direction, so E2E1 is increased for the next trial and the iteration loop is repeated. If ERROR from the previous trial was positive, DELTA is decreased by 50% and the test for SAVE > 0 is made. A false condition indicates the previous ERROR was negative also so the test for |SAVE| > |ERROR| is made. A true condition indicates the iteration method is closer to convergence on this trial than on the previous trial so ERROR is stored in SAVE, and E2E1 is increased for the next trial and the iteration loop is repeated. A false condition to the above test indicates the previous ERROR was closer to convergence so increase E2E1 for the next trial and repeat the iteration loop. If the test for SAVE > 0 is true, then the test for |ERROR| > SAVE is made. A false condition indicates the iteration method is closer to convergence on this trial on the negative side than the previous trial was on the positive side, so ERROR is stored in SAVE, E2E1 is increased for the next trial and the iteration loop is repeated. For a true condition to the test for |ERROR| > SAVE the steps following the test for |SAVE| > |ERROR| are repeated.

The function subroutine BESJO calculates the Bessel Function Jo(x) using polynomial approximation (See Reference 12) for each value of X in the integration interval X = 0 to X = 10r/h.

The function subroutine V calculates the value of V (See Equation 2) for each XN and XM1 or XM2 value used.

B-6
Appendix 3

This appendix contains the ELASTIC MODULI program source deck set-up.
Deck Set-Up

The ELASTIC MODULUS program was written in FORTRAN IV, Version G. The user is advised to change the first read statement in the main program if the "END=" option of the FORTRAN read is not implemented at his installation. The user can also substitute any other "Re-read" routine for Subroutine CORE if this is desired. ELASTIC MODULUS requires approximately 100k of core storage and execution time is approximately 7 seconds per test point. The source deck set-up is shown on the following page.
Source Deck Set-Up for ELASTIC MODULUS Program

Cards for as Many Additional Problems as Desired

- Additional Problems
  - Data Card(s) 4
  - Data Card 3
  - Data Card 2
  - Data Card 1

- Control Cards

- Function V

- Function BESJO

- Subroutine EMOD

- ELASTIC MODULUS Main Program
  - Fortran Control Cards

- Subroutine CORE

- Assembler Control Cards

- Job Card and Hasp Control Cards
Appendix 4

The input format for each data card type used by ELASTIC MODULUS is included on the following pages. The fields of each card are delineated and examples of typical data entries for each field are shown.
<table>
<thead>
<tr>
<th>Field</th>
<th>Format</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMM (I), I = 1..7</td>
<td>7A4</td>
<td>Ex. 8 FT. LT. OF CENTERLINE</td>
</tr>
<tr>
<td>IDYNA</td>
<td>I2</td>
<td>Ex.</td>
</tr>
<tr>
<td>IDYEAR</td>
<td>I2</td>
<td>Ex. 70</td>
</tr>
<tr>
<td>IDAY</td>
<td>I2</td>
<td>Ex. 7</td>
</tr>
<tr>
<td>M</td>
<td>I2</td>
<td>Ex. 4</td>
</tr>
<tr>
<td>DP</td>
<td>F5.2</td>
<td>Ex. 15.0</td>
</tr>
<tr>
<td>XLANE</td>
<td>A3</td>
<td>Ex. SBL</td>
</tr>
<tr>
<td>HWY1, HWY2</td>
<td>A4, A3</td>
<td>Ex. US 290</td>
</tr>
<tr>
<td>IJOB</td>
<td>I2</td>
<td>Ex. 1</td>
</tr>
<tr>
<td>ISECT</td>
<td>I2</td>
<td>Ex. 8</td>
</tr>
<tr>
<td>ICONT</td>
<td>I4</td>
<td>Ex. 114</td>
</tr>
<tr>
<td>CO1, CO2, CO3, CO4</td>
<td>3A4, A2</td>
<td>Ex. WASHINGTON</td>
</tr>
<tr>
<td>IDIST</td>
<td>I2</td>
<td>Ex. 17</td>
</tr>
<tr>
<td>NCARD</td>
<td>I3</td>
<td>Ex. 100</td>
</tr>
<tr>
<td>CARD</td>
<td>FORMAT</td>
<td>Ex.</td>
</tr>
<tr>
<td>------</td>
<td>--------</td>
<td>-----</td>
</tr>
<tr>
<td>T3</td>
<td>F4.2</td>
<td>6.0</td>
</tr>
<tr>
<td>(LA3(I), I = 1,5)</td>
<td>5A4</td>
<td>LIME STAB. BASE</td>
</tr>
<tr>
<td>T2</td>
<td>F4.2</td>
<td>8.0</td>
</tr>
<tr>
<td>LA2(I), I = 1,5</td>
<td>5A4</td>
<td>IRON ORE BASE</td>
</tr>
<tr>
<td>T1</td>
<td>F4.2</td>
<td>1.5</td>
</tr>
<tr>
<td>(LA1(I), I = 1,5)</td>
<td>5A4</td>
<td>ASPHALTIC CONCRETE</td>
</tr>
<tr>
<td>NCARD</td>
<td>I3</td>
<td>200</td>
</tr>
<tr>
<td>T6</td>
<td>FORMAT F4.2</td>
<td>Ex. 4.0</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>---------</td>
</tr>
<tr>
<td>(LA6(I), I = 1,5)</td>
<td>FORMAT 5A4</td>
<td>Ex. SUBGRADE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T5</th>
<th>FORMAT F4.2</th>
<th>Ex. 4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(LA5(I), I = 1,5)</td>
<td>FORMAT 5A4</td>
<td>Ex. LIME STAB. SUBGRADE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T4</th>
<th>FORMAT F4.2</th>
<th>Ex. 6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(LA4(I), I = 1,5)</td>
<td>FORMAT 5A4</td>
<td>Ex. CEMENT STAB. BASE</td>
</tr>
</tbody>
</table>

NCARD | FORMAT I3 | Ex. 300 |
<table>
<thead>
<tr>
<th>Field</th>
<th>Format</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICK</td>
<td>Format I2</td>
<td>Ex. 99</td>
</tr>
<tr>
<td>(REM(I), I = 1,4)</td>
<td>Format 4A4</td>
<td>Ex. WEST BOUND LANE</td>
</tr>
<tr>
<td>D10</td>
<td>Format F3.2</td>
<td>Ex. 010</td>
</tr>
<tr>
<td>D9</td>
<td>Format F2.1</td>
<td>Ex. 37</td>
</tr>
<tr>
<td>D8</td>
<td>Format F3.2</td>
<td>Ex. 010</td>
</tr>
<tr>
<td>D7</td>
<td>Format F2.1</td>
<td>Ex. 45</td>
</tr>
<tr>
<td>D6</td>
<td>Format F3.2</td>
<td>Ex. 010</td>
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<tr>
<td>D5</td>
<td>Format F2.1</td>
<td>Ex. 60</td>
</tr>
<tr>
<td>D4</td>
<td>Format F3.2</td>
<td>Ex. 030</td>
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<td>D3</td>
<td>Format F2.1</td>
<td>Ex. 30</td>
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<td>D2</td>
<td>Format F3.2</td>
<td>Ex. 030</td>
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<td>D1</td>
<td>Format F2.1</td>
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<tr>
<td>STA</td>
<td>Format A7</td>
<td>Ex. 248+00</td>
</tr>
<tr>
<td>IYEAR</td>
<td>Format I2</td>
<td>Ex. 70</td>
</tr>
<tr>
<td>IDAY</td>
<td>Format I2</td>
<td>Ex. 7</td>
</tr>
<tr>
<td>M</td>
<td>Format I2</td>
<td>Ex. 4</td>
</tr>
<tr>
<td>ISECT</td>
<td>Format I2</td>
<td>Ex. 8</td>
</tr>
<tr>
<td>ICONT</td>
<td>Format I4</td>
<td>Ex. 118</td>
</tr>
<tr>
<td>NCARD</td>
<td>Format I3</td>
<td>Ex. 400</td>
</tr>
</tbody>
</table>
Appendix 5

This appendix contains the flowchart of the procedure used in ELASTIC MODULUS to calculate a pavement or subgrade modulus.
READ NCARD & remainder of card into the A array

CALL CORE

NCARD = 100?

YES

READ Card 1 - Control information, Highway, County, District, Date, & Comments

CALL DATE

WRITE Control information & date at top of page

Initialize all Counters, Sums & Variances to Zero

NO

NCARD ≠ 200?

YES

READ Card 2 - Layer 1, 2 & 3 Descriptions & Layer 1, 2 & 3 Thicknesses
WRITE
Layer 1, 2 & 3 Descriptions
& Layer 1, 2 & 3 Thicknesses

NCARD = 300
?

NO

READ
Card 3 - Layer 4, 5 & 6 Descriptions
& Layer 4, 5 & 6 Thicknesses

WRITE
Layer 4, 5 & 6 Descriptions
& Layer 4, 5 & 6 Thicknesses

Increment pointer I

READ
Card 4 - Control information,
Station, Geophone readings &
Multipliers

N GT 0
?

NO

YES

NO

NO GT 0
?

WRITE
Output column headings

Calculate W1, W2, W3, W4, W5 & SCI

W1 LT W2
?

YES

NO
YES

ADD to Sums

(W1*10)/(W2*246) LT 1
and
DP LE 9.1

NO

Convert W1 & W2 to inches

Call EMOD to calculate pavement modulus & subgrade modulus

Round pavement modulus to nearest 100

Add to sums
AAS2 = AAS2 + AS2
AAP2 = AAP2 + AP2

Increment N

WRITE Station, WI, W2, W3, W4, W5, SCI, sub-grade modulus, pavement modulus, remarks.

N1 GT 30

NO

WRITE Output column headings on next page

YES
Calculate Avg. deflections, Avg. SCI, Avg. pavement modulus, Avg. subgrade modulus

Calculate variance of SCI, Avg. pavement moduli & subgrade moduli

WRITE Avg. deflections, Avg. SCI, Avg. pavement modulus, Avg. subgrade modulus

Is PN = 1?

Calculate standard deviation of SCI, Pavement moduli & subgrade moduli

WRITE Standard deviation of SCI, Pavement moduli, subgrade moduli

WRITE No. of Test Points used in Analysis
SUBROUTINE EMOD

ENTRY

Set all switches, initialize all sums & constants

Calculate RH(1) & RH(2), RATIO, ACC

Do from 1 to 2

Calculate DELM1 = (1/RH) * (3/XNO - 1)

Is \( \frac{XX1}{DELMI} \) ?

YES

DELMI = XX1

NO

Calculate DELX1 = DELM1 * RH

Calculate DELM2 = (1/RH) * (3/XNO - 1)

Is \( \frac{XX2}{DELM2} \) ?

YES

DEL2 = XX2

NO

E-6
Calculate $\Delta X_2 = \Delta L_2 \times RH$

\[ \Delta = 0.5 \]
\[ E_2/E_1 = 0.001 \]

Calculate $X_N$, 
\[ X_N = (E_1 - E_2)/(E_1 + E_2) \]

Do from 1 to 2

Calculate $N_1$, No. of intervals for integration by Simpson's rule from $X = 0$ to $X = 3mr/h$

Calculate $N_2$, No. of intervals for integration by Simpson's rule from $X = 3mr/h$ to $X = 10mr/h$

Set
\[ X_M1 = 0.0 \]
\[ X1 = 0.0 \]
Do from 1 to N1

Calculate the ordinates for integration for interval $X = 0$ to $X = 3 \text{mr/h}$

$Y = [V(XN, XM) - 1] \ast \text{BESJO}(X1)$

Increment $XM1$ & $X1$, $XM1 = XM1 + DELM1$, $X1 = X1 + DELX1$

CONTINUE

$XM2 = XM1$, $X2 = X1$

Do from 1 to N2

Calculate the ordinates for integration for interval $X = 3 \text{mr/h}$ to $X = 10 \text{mr/h}$,

$Y = [V(XN, XM2) - 1] \ast \text{BESJO}(X2)$

Increment $XM2$ & $X2$, $XM2 = XM2 + DELM2$, $X2 = X2 + DELX2$

CONTINUE

$E-8$
PART1 = 0.0
PART2 = 0.0

Calculate N4, No. of interior ordinates under curve from X = 0 to X = 3mr/h, N4 = N1 - 3

Do from LL = 2 to N4, incrementing LL by 2

Sum interior ordinates,
PART1 = PART1 + 2 * Y(LL) + Y(LL + 1)

CONTINUE

Sum the end ordinates

PART2 = Y(1) + 4 * Y(N1 - 1) + Y(N1)

Calculate area of integration from X = 0 to X = 3mr/h
AREA1 = ((2 * DELX1)/3) * PART1 + (DELX1/3) * PART2

E-9
Calculate position of first interior ordinate for integration from $X = 3mr/h$ to $X = 10mr/h$, $N_5 = N_1 + 2$

Calculate position of last interior ordinate for integration from $X = 3mr/h$ to $X = 10mr/h$, $N_6 = N_2 - 3 + N_1$

Do from $L_M = N_5$ to $N_6$, incrementing $L_M$ by 2

Sum interior ordinates for interval $X = 3mr/h$ to $X = 10mr/h$

$\text{PART}_3 = \text{PART}_3 + 2 \times Y(L_M) + Y(L_M + 1)$

CONTINUE

Sum end ordinates of interval from $X = 3mr/h$ to $X = 10mr/h$

$\text{PART}_4 = Y(N_1 + 1) + 4 \times Y(N_1 + N_2 - 1) + Y(N_1 + N_2)$

Calculate area of integration from $X = 3mr/h$ to $X = 10mr/h$,

$\text{AREA}_2 = \left(\frac{2 \times \text{DELX}_2}{3}\right) \times \text{PART}_3 + \left(\frac{\text{DELX}_2}{3}\right) \times \text{PART}_4$

$E^{-10}$
Calculate function $FF$,
$FF = \text{AREA1} + \text{AREA2} + 1$

CONTINUE

Calculate $F1/F2$,
$F1F2 = FF(1)/FF(2)$

Calculate difference between calculated function & observed deflections,
$\text{ERROR} = F1F2 - \text{RATIO}$

Is $\text{ERROR} < \text{ACC}$?

NO

Is $\text{ISW} = 1$?

NO

First iteration save the error & set ISW
SAVE = ERROR
$\text{ISW} = 1$

Is $\text{ERROR} < 0$

NO

E-11
Error is positive, 
PLUS = 1
MINUS = 0

If ERROR

(+) SET
Positive error
Switch, PLUS = 1

Was
the previous
error negative,
MINUS = 1?

NO

Error is increasing, so decrease
E2E1 for next trial, calculate
E2E1 ratio for next iteration,
E2E1 = E2E1 - DELTA

Is
E2E1 < 0?

YES

NO

Error was negative before, but
positive this time, decrease size
of increment for E2E1,
DELTA = 0.5 * DELTA

E2E1 = 0.0001

E-12
Is $SAVE < 0$?

NO

Calculate $E2\text{E}1$ ratio for next iteration, $E2\text{E}1 = E2\text{E}1 - \Delta$  
$SAVE = ERROR$

Is $E2\text{E}1 \leq 0.0$?

NO

Is $|SAVE| > ERROR$?

NO

Error is increasing in positive direction, decrease $E2\text{E}1$ for next iteration,  
$E2\text{E}1 = E2\text{E}1 - \Delta$

Is $E2\text{E}1 < 0$?

YES $E2\text{E}1 = 0.0001$

NO

SET Negative error switch,  
MINUS = 1

E-13
Was the previous error positive? 

YES

Error is on Negative side. Increase E2E1 for next iteration. 
E2E1 = E2E1 + DELTA

NO

Is E2E1 > 1?

YES

H ≥ 9.2

NO

Delta too Large Iteration Process will be divergent, Decreases size of Delta, DELTA = 0.5 * DELTA

Error is on Negative side 
But the iteration process is Divergent, Decrease E2E1 for Next iteration E2E1 = E2E1 - DELTA

Error was positive before, but is negative this time, decrease size of Delta, DELTA = 0.5 * DELTA

E-14
Calculate E2EI ratio for next iteration,
E2EI = E2EI + DELTA

Is SAVE > 0 ?
YES

Is |SAVE| > |ERROR| ?
YES
SAVE = ERROR

NO

Calculate E2EI ratio for next iteration,
E2EI = E2EI + DELTA

NO

Is |ERROR| > SAVE ?
YES

Calculate E2EI ratio for next iteration,
E2EI = E2EI + DELTA
SAVE = ERROR

NO

Calculate E1 & E2,
E1 = (3 * P * FF(1))/(4 * P1 * W1 * R1)
E2 = E2EI * E1

CONTINUE

RETURN

END

E-15
FUNCTION BESJO

ENTRY

Calculate X3,
X3 = X/3

Is X > 3 ?
YES
X3 = 3/X

NO

Calculate Constants,
X32 = X3 * X3
X33 = X32 * X3
X34 = X32 * X32
X35 = X32 * X33
X36 = X33 * X33

Is |X| ≤ 3 ?
YES
Calculate BESJO(X) for X ≤ 3

NO

Calculate BESJO(X) for X > 3

RETURN

END

E-16
FUNCTION V

ENTRY

Set $V = 1$

Is $XM > 30$?

YES

Calculate Function $V$

RETURN

END

NO
Appendix 6

On the following pages is a listing of ELASTIC MODULUS with the output from a sample problem. The input data cards for the sample problem are listed after the program output.
ELASTIC MODULII -- MAIN PROGRAM

0001 DIMENSION STA(200),W1(200),W2(200),W3(200),W4(200),
* W5(200), AS2(200), A(20), SCI(200),
* IXDATE(3),COMM(7),REM(4)

0002 REAL * 8 STA, DAS, OAP, DBLE

NOTE -- THE PRINT & FORMAT STATEMENTS ARE FOR
OUTPUT ON 8 1/2 X 11 PAPER. FOR OUTPUT ON 11 X 14
PAPER USE THE PRINT & FORMAT STATEMENTS WITH 'C' IN
COLUMN 1.

STATEMENT FUNCTION TO ROUND 'X' TO NEAREST 'EVEN'
ROUND( X, EVEN ) = AINT( ( X + EVEN * .5 ) / EVEN )

10 CONTINUE

READ CARD CODE & REMAINDER OF CARD INTO A - ARRAY

READ(5,1,END=1000) NCARD, ( A(I), I = 1, 20 )

1 FORMAT( I3, 19A4, A1 )

CALL CORE ( A, 80 )

TEST FOR DATA CARD 1

IF(NCARD.EQ.100) GO TO 11

TEST FOR DATA CARD 2

IF(NCARD.EQ.200) GO TO 12

TEST FOR DATA CARD 3

IF(NCARD.EQ.300) GO TO 13

I IS A POINTER TO DATA IN STORAGE

14 I=N+1
READ DATA CARD 4

READ(5,6) ICONT,ISECT,M,IOAY,STA(I),D1,D2,D3,*D4,D5,D6,D7,D8,*D9,D10,(REM(I),J=1,4),ICK

6 FORMAT( 14,412,A7,3X, 5(F2.1,F3.2),8X,4A4,12)

IF(N.GT.0) GO TO 555
IF(NO.GT.0) GO TO 555

PRINT OUTPUT COLUMN HEADINGS

PRINT 61
61 FORMAT(/,1X,'STATION W1 W2 W3 W4 **','
W5 SCI ** ES ** EP ** REMARKS / )

CALCULATE DEFLECTIONS & SCI (DEFLECTIONS IN MILS)

555 W1(I)=D1*D2
0019 W2(I)=D3*D4
0020 W3(I)=D5*D6
0021 W4(I)=D7*D8
0022 W5(I)=D9*D10
0023 SCI(I)=W1(I)-W2(I)

TEST FOR W1 OR W2 = 0, AND W1 LESS THAN W2

IF(W1(I).EQ.0.0 OR W2(I).EQ.0.0) GO TO 64
IF(W1(I).LT.W2(I)) GO TO 66

AW1 =AW1 +W1(I)
0027 AW2 =AW2 +W2(I)
0028 AW3 =AW3 +W3(I)
0029 AW4 =AW4 +W4(I)
0030 AW5 =AW5 +W5(I)
0031 ASCII=ASCII+SCI(I)
0032 A52(I) = 0.0
0033 A53(I) = 0.0

IF( ( W1(I) * 10.0 ) / ( W2(I) * SQRT( 244.0 ) )
   = .GT. 1.0 .AND. DP .LE. 9.1 ) GO TO 60
CONVERT W1 & W2 TO INCHES

W1(I) = W1(I) / 1000.
W2(I) = W2(I) / 1000.

PASS W1, W2, & TOTAL PAVEMENT THICKNESS TO EMOD,
EMOD RETURNS UNROUNDED VALUES OF PAVEMENT & SUBGRADE
MODULE AS DAP & DAS

CALL EMOD ( DBLE(W1(I)), DBLE(W2(I)), DBLE(DP), DAP, DAS)

CONVERT W1 & W2 TO MILS

W1(I) = W1(I) * 1000.
W2(I) = W2(I) * 1000.

ROUND PAVEMENT & SUBGRADE MODULII TO NEAREST 100

DAS = ROUND(DAS, 100.)
DAP = ROUND(DAP, 100.)

PUT PAVEMENT & SUBGRADE MODULII IN STORAGE

AS2(I) = DAS
AP2(I) = DAP

ADD TO THE SUMS OF THE DEFLECTIONS, SCI, PAVEMENT,
AND SUBGRADE MODULII

AAS2 = AAS2 + AS2(I)
AAP2 = AAP2 + AP2(I)

ADD TO N, THE NUMBER OF VALID TEST POINTS

N = N + 1

PRINT A LINE OF OUTPUT

PRINT 63, STA(I), W1(I), W2(I), W3(I), W4(I), W5(I), SCI(I),
= AS2(I), AP2(I), (REM(J), J = 1, 4)

63 FORMAT(1X, A7, 3X, 5(F5.3, 2X), F5.3, 2F11.0, 2X, 2A4)

PRINT 63, STA(I), W1(I), W2(I), W3(I), W4(I), W5(I), SCI(I),
* AS2(I), AP2(I), (REM(J), J = 1, 2)

63 FORMAT(7X, A7, 1X, 6(F6.3), 2F10.0, 2X, 2A4)

N1 = N1 + 1
IF(N1.LT.30) GO TO 88
C SKIP TO NEXT PAGE & PRINT OUTPUT COLUMN HEADINGS IF
C THIRTY LINES HAVE BEEN PRINTED
C
84 CONTINUE
0052 PRINT 51
C PRINT 56
C PRINT 57, IDIST, C01, C02, C03, C04, ICONT, ISECT, IJOB, HWY1,
C * HWY2, XLANE, M, IDAY, IYEAR, IDYNA
C
0053 PRINT 56, IDIST, C01, C02, C03, C04
C
0054 56 FORMAT( T35, 'DISTRICT - COUNTY',/ T36, I2, 9X, 3A4)*A2 )
0055 PRINT 57, ICONT, ISECT, IJOB, HWY1, HWY2, M, IDAY, IYEAR, IDYNA
0056 57 FORMAT( T19, 'CONT. SECT. JOB HIGHWAY DATE',
* DYNAFLECT / T19, I4,2I7,4X;A4;A3;I4,2('-',I2),I9 )
0057 PRINT 61
0058 N1 = 0
0059 88 CONTINUE
C CHECK FOR LAST DATA CARD 4
C
0060 IF ( ICK.EQ.0) GO TO 10
0061 GO TO 80
C READ DATA CARD 1
C
0062 READ(5,2) IDIST, C01, C02, C03, C04, ICONT, ISECT, IJOB, HWY1,
* HWY2, XLANE, DP, M, IDAY, IYEAR, IDYNA, (COMM(I), I=1,7)
0063 2 FORMAT( 12,3A4, A2, I4, 2I2, A4, A3, A3, F5.2, 4I2, 7A4)
C PRINT HEADING
C
0064 PRINT 51
C 51 FORMAT( '1' )
C
0066 PRINT 52
C 52 FORMAT(33X,*TEXAS HIGHWAY DEPARTMENT',/)
0067 52 FORMAT(35X,*TEXAS HIGHWAY DEPARTMENT' )
C
0068 PRINT 53, IDIST
C 53 FORMAT(31X,*DISTRICT ',I2,' - DESIGN SECTION',/)
0069 53 FORMAT(33X,*DISTRICT ',I2,' - DESIGN SECTION' )
C
0070 PRINT 54
C 54 FORMAT(16X,*DYNAFLECT DEFLECTIONS AND CALCULATED ',
C * 'ELASTIC MODULII' / )
0071 54 FORMAT(21X,*DYNAFLECT DEFLECTIONS AND CALCULATED ',
* 'ELASTIC MODULII' )
C GET CURRENT DATE
C CALL DATE ( IXDATE(1), IXDATE(2), IXDATE(3) )

C PRINT 55, IXDATE
C 55 FORMAT(30X,'THIS PROGRAM WAS RUN - ', 2A3, A2 /
C 55 FORMAT(32X,'THIS PROGRAM WAS RUN - ', 2A3, A2 /)

C PRINT 56
C 56 FORMAT( 1X,'DIST. COUNTY CONT. SECT.*',
C * JOB HIGHWAY DATE DYNAFLECT*)

C PRINT 56, IDIST, CO1, CO2, CO3, CO4

C PRINT CONTROL INFORMATION FROM DATA CARD 1
C PRINT 57, IDIST, CO1, CO2, CO3, CO4, ICONT, ISECT, IJOB, HWY1,
C * HWY2, XLANE, M, IDAY, IYEAR, IDYNA
C 57 FORMAT( 2X, I2, 5X, 3A4, A2, 3X, I4, 4X, I2, 5X, I2, 2X, A4, A3,
C * A3, 2X, I2, ' - ', I2, ' - ', I2, 6X, I2 /)

C PRINT 57, ICONT, ISECT, IJOB, HWY1, HWY2, M, IDAY, IYEAR, IDYNA

C PRINT 58, (COMM(I), I=1,7), DP
C 58 FORMAT(10X,7A4, 2X, 'PAV. THICK. = ', F5.2, ' INCHES',/
C C INITIALIZE ALL SUMS & COUNTERS
C N=0
C N1 = 0
C NO = 0
C AW1 = 0.
C AW2=0.
C AW3=0.
C AW4=0.
C AW5=0.
C ASCI=0.
C AAS2=0.
C AAP2=0.
C SR1= 0.
C SR2= 0.
C SR3= 0.
C GO TO 10
C READ & PRINT INFORMATION ON DATA CARD 2
C 12 READ(5,3) (LA1(I), I=1,5), T1, (LA2(I), I=1,5), T2,
C * (LA3(I), I=1,5), T3
C 3 FORMAT( 5A4, F4.2, 5A4, F4.2, 5A4, F4.2)
FORTRAN IV G LEVEL 18 MAIN DATE = 71172

PRINT 59, (LA1(I), I=1,5), T1, (LA2(I), I=1,5), T2,
      * (LA3(I), I=1,5), T3
59 FORMAT (1X, 5A4, 1X, F5.2, 2X, 5A4, 1X, F5.2, 2X, 5A4, 1X, F5.2)
0096 PRINT 59, (LA1(I), I=1,5), T1, (LA2(I), I=1,5), T2
0097 PRINT 59, (LA3(I), I=1,5), T3
0098 59 FORMAT (16X, 5A4, 1X, F5.2, 5X, 5A4, 1X, F5.2)
0099 GO TO 10

READ & PRINT INFORMATION ON DATA CARD 3, IF PRESENT
13 READ(5,3) (LA4(I), I=1,5), T4, (LA5(I), I=1,5), T5,
      * (LA6(I), I=1,5), T6
13 PRINT 59, (LA4(I), I=1,5), T4, (LA5(I), I=1,5), T5,
      * (LA6(I), I=1,5), T6
0101 PRINT 59, (LA4(I), I=1,5), T4, (LA5(I), I=1,5), T5
0102 PRINT 59, (LA6(I), I=1,5), T6
0103 GO TO 10
0104 66 NO = NO + 1

PRINT NEGATIVE SCI MESSAGE
0105 PRINT 82, STA(I), W1(I), W2(I), (REM(J), J=1,4)
0106 82 FORMAT (1X, A7, 3X, F5.3, 2X, F5.3, 2X, 'NEGATIVE SCI OTHER',
      * 'CALCULATIONS OMITTED', 4X, 4A4)
0107 N1 = N1 + 1
0108 IF (N1 .LT. 30) GO TO 88
0109 GO TO 84
0110 64 NO = NO + 1

PRINT ERROR MESSAGE
0111 PRINT 81, STA(I), (REM(J), J=1,4)
0112 81 FORMAT (1X, A7, 3X, 'DATA ERROR ASSUMED A ZERO VALUE RE',
      * 'AD FOR W1 OR W2', 5X, 4A4)
0113 N1 = N1 + 1
0114 IF (N1 .LT. 30) GO TO 88
0115 GO TO 84

CONTINUE
0116 60 CONTINUE
0117 N = N + 1
0118 NO = NO + 1
0119 PRINT 85, STA(I), W1(I), W2(I), W3(I), W4(I), W5(I), SCI(I)
0120 85 FORMAT (7X, A7, 1X, 6F6.3, 2X, 'NO UNIQUE SOLUTION')
FORTRAN IV G LEVEL 18

0121  N1 = N1 + 1
0122  IF( N1 .LT. 30 ) GO TO 88
0123  GO TO 84

C ALL CARDS READ FOR AN ANALYSIS, CALCULATE AVERAGE DEFLECTIONS, AVERAGE SCI, AVERAGE PAVEMENT MODULUS, AND AVERAGE SUBGRADE MODULUS

0124  80  PN=N
0125  N1 = N - NO
0126  IF( N1 .LE. 0 ) N1 = 1
0127  AW1V= Aw1/PN
0128  AW2V= Aw2/PN
0129  AW3V= Aw3/PN
0130  AW4V= Aw4/PN
0131  AW5V= Aw5/PN
0132  ASCIV=ASI/PN
0133  AAS2V=AAS2/N1
0134  AAS2V=AAS2/N1

C CALCULATE VARIANCE OF SCI, SUBGRADE MODULUS & PAVEMENT MODULUS

0135  DO 62 I=1,N
0136  IF(W1(I).EQ.0.AND.W2(I).EQ.0) GO TO 62
0137  SR1 = SR1 + ((ASCIV-SCI(I))**2)
0138  IF( AS2(I) .EQ. 0.0) GO TO 62
0139  SR2= SR2+((AAS2V-AS2(I))**2)
0140  SR3= SR3+((AAP2V-AP2(I))**2)
0141  62 CONTINUE

C PRINT AVERAGES

0142  PRINT 65,AW1V,AW2V,AW3V,AW4V,AW5V,ASCIV,AAS2V,AAP2V
0143  65 FORMAT(/6X,'AVERAGES', 6(F5.3), 2F11.0 )

C CALCULATE STANDARD DEVIATION OF SCI, SUBGRADE MODULUS, AND PAVEMENT MODULUS

0144  IF( PN .EQ. 1 ) GO TO 90
0145  SE1 = SQRT(SR1/(PN-1))
0146  IF( N1 .LE. 1) GO TO 90
0147  SE2 = SQRT(SR2/(N1-1))
0148  SE3 = SQRT(SR3/(N1-1))

C PRINT STANDARD DEVIATIONS
C PRINT 71, SE1, SE2, SE3
C 71 FORMAT( 1X, 'STANDARD DEVIATION', 27X, F5.3, 2F11.0)
C
0149
C 90 CONTINUE
IF( N .EQ. 1 ) N1 = 1
PRINT 99, N, N1
C 99 FORMAT(1X, 'NUMBER OF POINTS IN AVERAGE = ',
C          114, 19, 110)
C
0153
C 99 FORMAT( 7X, 'NUMBER OF POINTS IN AVERAGE = ',
C          114, 19, 110)
C
0154
C PRINT 91
C 91 FORMAT( 7/5X, 'W1 DEFLECTION AT GEOPHONE 1' )
C
0159
C PRINT 92
C 92 FORMAT( 5X, 'W2 DEFLECTION AT GEOPHONE 2' )
C
0164
C PRINT 96
C 96 FORMAT( 5X, 'SCI SURFACE CURVATURE INDEX ( W1 MIN',
C          '*' US W2' )
C
0171
C 96 FORMAT( 10X, 'SCI SURFACE CURVATURE INDEX ( W1 MIN',
C                '*' US W2' )
C
0168
C PRINT 97
C 97 FORMAT( 5X, 'ES ELASTIC MODULUS OF THE SUBGRADE FROM',
C          '*' W1 AND W2' )
C
0173
C 97 FORMAT( 10X, 'ES ELASTIC MODULUS OF THE SUBGRADE FROM',
C                '*' W1 AND W2' )
C
0168
C PRINT 98
C 98 FORMAT( 5X, 'EP ELASTIC MODULUS OF THE PAVEMENT FROM',
C          '*' W1 AND W2' )
ELASTIC MODULUS OF THE PAVEMENT FROM:

98 FORMAT (10X,'EP
   * M W1 AND W2')

GO TO 10
1000 CONTINUE
0172 END
SUBROUTINE EMOD ( W1, W2, H, E1, E2 )

IMPLICIT REAL * 8 ( A-H, O-Z )
DIMENSION RHI(2), FF(2), Y(4000), DELM1(2), DELM2(2), DELX1(2), DELX2(2)

DATA P / 1000.000 , ER / .00100 /
DATA XNO / 61.000 , XK1 / 0.0100 , XK2 / 0.1000 /

P, XNO, XK1, XK2, R1 & R2 CAN BE CHANGED IF DESIRED

INTEGER PLUS

INITIALIZE SWITCHES & SAVE

R1 = 10.000
R2 = DSQRT( 244.000 )
MINUS = 1
PLUS = 0
ISW = 0
SAVE = 0.000

CALCULATE R/H, RATIO, & ACC ( ACC IS THE CONVERGENCE CRITERION )

RH(1) = R1 / H
RH(2) = R2 / H
RATIO = ( W1 * R1 ) / ( W2 * R2 )
ACC = ER * RATIO

DO 2 KL = 1, 2

CALCULATE AND TEST DELM1

DELM1(KL) = ( 1.000 / RH(KL)) * ( 3.000 /
* ( XNO - 1.000 ) )

IF( XK1 .LE. DELM1(KL) ) DELM1(KL) = XK1

CALCULATE DELX1

DELM1(KL) = DELM1(KL) * RH(KL)

CALCULATE AND TEST DELM2

DELM2(KL) = ( 1.000 / RH(KL)) * ( 3.000 /...
* ( XNO - 1.000 )

C 0022 IF( XK2 .LE. DELM2(KL) ) DELM2(KL) = XK2
C 0023 CALCULATE DELX2
C 0024 2 CONTINUE
C 0025 GET INITIAL VALUE OF E2/E1 AND DELTA
C 0026 DELTA = 0.500
E2E1 = 0.00100
C 0027 START ITERATION LOOP FOR EACH E2/E1 VALUE USED
C 0028 4 CONTINUE
XN = ( 1.000 - E2E1 ) / ( 1.000 + E2E1 )
C 0029 THE FUNCTIONS FF(1) AND FF(2) (SEE EQN. 5) ARE
C 0030 CALCULATED IN THE FOLLOWING DO LOOP.
C 0031 DO 29 KK = 1, 2
C 0032 CALCULATE NO. OF INTERVALS FOR SIMPSON'S RULE FOR
C 0033 EACH INTEGRATION. N1 & N2 MUST BE ODD INTEGERS.
C 0034 N1 = ( 3.000 * RH(KK) ) / DELX1(KK) + 1.000
0035 IF( (N1 / 2) * 2 .NE. N1 ) N1 = N1 + 1
C 0036 N2 = ( 7.000 * RH(KK) ) / DELX2(KK) + 1.000
0037 IF( (N2 / 2) * 2 .EQ. N2 ) N2 = N2 + 1
C 0038 CALCULATE ORDINATES FOR SIMPSON'S RULE FOR FIRST
C 0039 INTEGRATION
C 0040 XN1 = 0.000
0041 X1 = 0.000
0042 DO 28 JJ = 1, N1
0043 Y(JJ) = ( V( XN, XM1 ) - 1.000 ) * BESJ0( X1 )
0044 XM1 = XM1 + DELM1(KK)
0045 X1 = X1 + DELX1(KK)
0046 28 CONTINUE
C 0047 CALCULATE ORDINATES FOR SIMPSON'S RULE FOR SECOND
C 0048 INTEGRATION
C
0041  XM2 = XM1
0042  X2 = X1
0043  DO 27 KL = 1,N2
0044  Y(N1 + KL) = ( V(XN, XM2) - 1.000 ) * BESJ0(X2)
0045  XM2 = XM2 + DELM2(KK)
0046  X2 = X2 + DELX2(KK)
0047  27 CONTINUE
C
C SUM ORDINATES TO CALCULATE AREA UNDER THE CURVE OF FIRST
C INTEGRATION
C
0048  PART1 = 0.000
0049  PART3 = 0.000
C
C N4 IS NO. OF INTERIOR ORDINATES OF FIRST INTEGRATION
C
0050  N4 = N1 - 3
C
C SUM INTERIOR ORDINATES
C
0051  DO 26 LL = 2, N4, 2
0052  26 PART1 = PART1 + ( 2.000 * Y(LL) + Y(LL+1) )
C
C SUM END ORDINATES
C
0053  PART2 = Y(1) + 4.000 * Y(N1-1) + Y(N1)
C
C CALCULATE AREA OF FIRST INTEGRATION
C
0054  AREA1 = ((2.000 * DELX1(KK)) / 3.000) * PART1 + ( DELX1(KK) / 3.000 ) * PART2
C
C SUM ORDINATES TO CALCULATE AREA UNDER THE CURVE OF
C SECOND INTEGRATION
C
C THE LAST ORDINATE OF THE FIRST INTERVAL OF INTEGRAT-
CION IS ALSO THE FIRST ORDINATE OF THE SECOND INTERVAL
C
C N5 IS THE POSITION IN THE Y VECTOR OF THE FIRST
C INTERIOR ORDINATE OF THE SECOND INTEGRATION INTERVAL
C
0055  N5 = N1 + 2
C
C N6 IS THE POSITION IN THE Y VECTOR OF THE LAST
C INTERIOR ORDINATE OF THE SECOND INTEGRATION INTERVAL

F-13
C
N6 = N2 - 3 + N1

C
SUM INTERIOR ORDINATES

DO 25 LM = N5, N6, 2
25 PART3 = PART3 + ( 2.0D0 * Y(LM) + Y(LM+1) )

C
SUM END ORDINATES

PART4 = Y(N1+1) + 4.0D0 * Y(N1 + N2 - 1) + Y(N1 + N2)

C
CALCULATE AREA OF SECOND INTEGRATION.

AREA2 = ((2.0D0 * DELX2(KK)) / 3.0D0) * PART3 + ( DELX2(KK) / 3.0D0 ) * PART4

C
CALCULATE THE FUNCTION.

FF(KK) = AREA1 + AREA2 + 1.0D0

CONTINUE

C
CALCULATE F1/F2 AND CHECK FOR CONVERGENCE

F1F2 = FF(1) / FF(2)
ERROR = F1F2 - RATIO
IF(DABS( ERROR ) .LT. ACC ) GO TO 31

C
SET ISW AND SAVE ON FIRST TIME THROUGH ITERATION LOOP

IF( ISW .NE. 0 ) GO TO 6
ISW = 1
SAVE = ERROR
IF( ERROR .LT. 0.0D0 ) GO TO 6

C
SIGN OF FIRST ERROR IS **

PLUS = 1
MINUS = 0
CONTINUE

C
TEST FOR SIGN OF ERROR

IF( ERROR ) 30, 31, 32

C
SIGN OF ERROR IS **

32 PLUS = 1
IF (MINUS .NE. 0) GO TO 40
C ERROR IS POSITIVE, DECREASE E2E1 FOR NEXT TRIAL
C
E2E1 = E2E1 - DELTA
E2E1 = E2E1 .LE. 0.000
GO TO 4
C ERROR WAS NEGATIVE, NOW POSITIVE, CHANGE DELTA
C
DELTA = 0.500 * DELTA
IF (SAVE .LT. 0.000) GO TO 42
C SET SAVE = ERROR, DECREASE E2E1 FOR NEXT TRIAL
C
SAVE = ERROR
E2E1 = E2E1 - DELTA
E2E1 = E2E1 .LE. 0.000
GO TO 4
C ERROR IS INCREASING IN POSITIVE DIRECTION, DECREASE E2E1 FOR NEXT TRIAL
C
IF (DABS(SAVE) .GT. ERROR) GO TO 41
E2E1 = E2E1 - DELTA
E2E1 = E2E1 .LE. 0.000
GO TO 4
C SIGN OF ERROR IS '-'
C
MINUS = 1
IF (PLUS .NE. 0) GO TO 45
C ERROR IS NEGATIVE, INCREASE E2E1 FOR NEXT TRIAL
C
E2E1 = E2E1 + DELTA
E2E1 = E2E1 .GT. 1.000
GO TO 4
C CONTINUE
IF (H .GE. 9.200) GO TO 4
DELTA = 0.5 * DELTA
E2E1 = E2E1 - DELTA
GO TO 4
C ERROR IS NEGATIVE NOW, WAS POSITIVE BEFORE, CHANGE
```
C DELTA
0099   45 DELTA = 0.500 * DELTA
0100         IF( SAVE .GT. 0.000 ) GO TO 47
C TEST FOR ERROR LESS THAN SAVE
0101     46 IF(DABS ( SAVE ) .GT. DABS ( ERROR ) )SAVE = ERROR
C INCREASE E2E1 FOR NEXT TRIAL
0102     E2E1 = E2E1 + DELTA
0103     IF( E2E1 .GT. 1.000 ) GO TO 44
0104     GO TO 4
C TEST FOR ERROR GREATER THAN SAVE
0105    47 IF(DABS ( ERROR ) .GT. SAVE ) GO TO 46
C ERROR IS APPROACHING CONVERGENCE FROM NEGATIVE SIDE,
C SET SAVE = ERROR, INCREASE E2E1 FOR NEXT TRIAL
0106     SAVE = ERROR
0107     E2E1 = E2E1 + DELTA
0108     IF( E2E1 .GT. 1.000 ) GO TO 44
0109     GO TO 4
0110   31 CONTINUE
C CONVERGENCE CRITERION IS MET, CALCULATE E1 & E2
0111     E1 = (3.000 * P * FF(1))/ (4.000 * 3.1415926 * W1*R1)
0112     E2 = E2E1 * E1
C RETURN
0113
0114 END
```
REAL FUNCTION BESJC * 8 ( X )

A FUNCTION TO CALCULATE BESSEL FUNCTION J0(X) USING
POLYNOMIAL APPROXIMATION - REFERENCE HANDBOOK OF MATH.
FUNCTIONS, BUREAU OF STANDARDS, PAGES 369-370

DOUBLE PRECISION X3, X32, X33, X34, X35, X36, DCOS,
* DSQRT, DABS, X

CALCULATE X/3 OR 3/X

X3 = X/3.0

IF( X.GT. 3.0 ) X3 = 3.0/ X

CALCULATE POWERS OF X

X32 = X3*X3
X33 = X32*X3
X34 = X32*X32
X35 = X32*X33
X36 = X33*X33

2 IF ( DABS (X) .LE. 3.000 ) GO TO 3

CALCULATE BESJO(X) FOR VALUES OF X GREATER THAN 3

BESJO= 1.79788456 - .77E-6 * X3 - 0.5527400 -02 * X32 - .9512E-04 * X33 + .1372370 -02 * X34 -
* .72805E-03 * X35 + .14476E-03 * X36 ) / DSQRT(X) )
** DCOS( X - .78539816 - .04166397 * X3 - .3954E-04
** X32 + .262573D-02 * X33 - .541250-03 * X34 -
* .29333E-03 * X35 + .13558E-03 * X36 )

RETURN

CALCULATE BESJO(X) FOR VALUES OF X LESS THAN 3

BESJO= 1.0 - 2.2499997 * X32 + 1.2656208 * X34
* - .3163866 * X36 + .0444479 * ( X34 * X34 ) -
* .0039444 * ( X35 * X35 ) + .000210 * ( X36 * X36)

RETURN

END
REAL FUNCTION V * 8 ( XN, XM )

DOUBLE PRECISION XN, XM, EXPM2M, EXPM4M, DEXP

V - A FUNCTION OF 'E2E1', AND 'M'
'E2E1' IS THE E2/E1 RATIO, TESTED FROM .001 TO 1000.
'M' TESTED USING VALUES FROM 0.0 TO 150. WHICH IS
10 * (R/H)

V APPROACHES 1 FOR LARGE VALUES OF M

V = 1.0
IF( XM .GT. 30 ) RETURN

CALCULATE EXPONENTIALS

EXPM2M = DEXP ( -2.000 * XM )
EXPM4M = EXPM2M*EXPM2M

CALCULATE FUNCTION V FOR THE XN & XM1 OR XM2 VALUES

V = ( 1.000 + ( 4.000 * XN * XM * EXPM2M ) -
* ( XN * XN * EXPM4M ) ) / ( 1.000 - ( 2.000 * XN
* * ( 1.000 + 2.000 * XM * XM ) * EXPM2M ) +
* ( XN * XN * EXPM4M ) )

RETURN

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NOTE: Program printouts of sample problems will be found in the main body of the report, Tables 6a through 6g.
LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

1001741200S     1560  1  EM 168758L  12.5  52168  1

FAL COAT       0.5 RED SANDY GRAVEL       12.0 GREY & BRWN SAND SUB

1560  1  52168  1  -  A  39.3  77.1  52.1  31.1  73.03
1560  1  52168  1  -  B  38.3  77.1  51.1  31.1  73.03
1560  1  52168  2  -  A  41.3  28.3  49.1  30.1  68.03
1560  1  52168  2  -  B  40.3  29.3  49.1  30.1  67.03
1560  1  52168  3  -  A  39.3  77.1  47.1  30.1  65.03
1560  1  52168  3  -  B  37.3  77.1  46.1  30.1  67.03
1560  1  52168  4  -  A  49.3  32.3  49.1  32.1  74.03
1560  1  52168  4  -  B  46.3  30.3  47.1  31.1  71.03
1560  1  52168  5  -  A  43.3  29.3  50.1  34.1  77.03
1560  1  52168  5  -  B  42.3  80.1  46.1  31.1  73.03
LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

10017RRAZOS  2824  1FM  7776NBL  8.0  52168 1
200SEAL COAT  0.5 ASPHALT STAB GRAVEL 7.5GREY SANDY CLAY SUBG

| 2824 | 52168 | 1 - A | 55.3 | 40.3 | 29.3 | 66.1 | 50.1 |
| 2824 | 52168 | 1 - B | 52.3 | 37.3 | 81.1 | 61.1 | 49.1 |
| 2824 | 52168 | 2 - A | 77.3 | 49.3 | 31.3 | 71.1 | 59.1 |
| 2824 | 52168 | 2 - B | 77.3 | 47.3 | 30.3 | 67.1 | 57.1 |
| 2824 | 52168 | 3 - A | 81.3 | 50.3 | 31.3 | 67.1 | 49.1 |
| 2824 | 52168 | 3 - B | 83.3 | 51.3 | 31.3 | 67.1 | 50.1 |
| 2824 | 52168 | 4 - A | 83.3 | 49.3 | 30.3 | 64.1 | 48.1 |
| 2824 | 52168 | 4 - B | 81.3 | 47.3 | 28.3 | 61.1 | 47.1 |
| 2824 | 52168 | 5 - A | 78.3 | 48.3 | 29.3 | 62.1 | 45.1 |
| 2824 | 52168 | 5 - B | 81.3 | 49.3 | 31.3 | 65.1 | 47.1 |
### Listing of Data Cards for Sample Problems

**10017Burleson**  
1399 1 1FM 1361EBL 12.0 52168 1

**200 Seal Coat**  
0.5 Lime Stab. Sandstone 11.5 Ton Sandy Clay Subgr

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LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

10017 WASHINGTON 186 S 1SH 36 NRL 19.9 52168 1

200 HOT MIX ASPH. CONC. 75% SANDSTONE 1615 BLACK CLAY SUBGRADE

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 186 | 5 | 52168 | 1 | A | 56.3 | 34.3 | 61.1 | 42.1 | 30.1 |
| 186 | 5 | 52168 | 1 | B | 61.3 | 36.3 | 61.1 | 42.1 | 31.1 |
| 186 | 5 | 52168 | 2 | A | 58.3 | 36.3 | 67.1 | 47.1 | 36.1 |
| 186 | 5 | 52168 | 2 | B | 65.3 | 39.3 | 69.1 | 49.1 | 37.1 |
| 186 | 5 | 52168 | 3 | A | 56.3 | 36.3 | 68.1 | 50.1 | 38.1 |
| 186 | 5 | 52168 | 3 | B | 57.3 | 36.3 | 67.1 | 48.1 | 37.1 |
| 186 | 5 | 52168 | 4 | A | 56.3 | 37.3 | 75.1 | 57.1 | 46.1 |
| 186 | 5 | 52168 | 4 | B | 52.3 | 36.3 | 73.1 | 55.1 | 44.1 |
| 186 | 5 | 52168 | 5 | A | 50.3 | 32.3 | 59.1 | 44.1 | 33.1 |
| 186 | 5 | 52168 | 5 | B | 43.3 | 33.3 | 60.1 | 43.1 | 33.1 |

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LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

10017ROBERTSON

40 A 1US 190 NBL 15.2 52168 1

200HOT MIX ARCH. CONC. 1.25CEM. STAB. LIMESTONE1395RED SANDY CLAY SUBGR

| 40 | 52168 1 - A | 68.1 59.1 49.1 39.1 31.1 |
| 40 | 52168 1 - B | 69.1 60.1 49.1 39.1 31.1 |
| 40 | 52168 2 - A | 72.1 63.1 51.1 39.1 31.1 |
| 40 | 52168 2 - B | 70.1 62.1 49.1 39.1 31.1 |
| 40 | 52168 3 - A | 75.1 65.1 52.1 39.1 30.1 |
| 40 | 52168 3 - B | 76.1 66.1 51.1 39.1 30.1 |
| 40 | 52168 4 - A | 60.1 54.1 45.1 35.1 28.1 |
| 40 | 52168 4 - B | 58.1 52.1 43.1 33.1 28.1 |
| 40 | 52168 5 - A | 62.1 55.1 45.1 35.1 29.1 |
| 40 | 52168 5 - B | 65.1 57.1 47.1 36.1 28.1 |

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**LISTING OF DATA CARDS FOR SAMPLE PROBLEMS**

10017BRAZOS 1560 1 IFM 1687NRL 7.5 52168 1

200ASPHALT SURFACING 1.0 ASP F MUL STAB GRAVL 6.5BROWN CLAY SUBGRADE

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F-27
### Listing of Data Cards for Sample Problems

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| 540 3 52168 1 - A | 80.3 51.3 32.3 68.1 50.1 |
| 540 3 52168 1 - B | 75.3 48.3 30.3 63.1 48.1 |
| 540 3 52168 2 - A | 59.3 39.3 82.1 60.1 48.1 |
| 540 3 52168 2 - B | 60.3 40.3 82.1 62.1 49.1 |
| 540 3 52168 3 - A | 55.3 39.3 28.3 64.1 51.1 |
| 540 3 52168 3 - B | 53.3 39.3 28.3 61.1 51.1 |
| 540 3 52168 4 - A | 75.3 49.3 33.3 75.1 60.1 |
| 540 3 52168 4 - B | 78.3 53.3 35.3 79.1 63.1 |
| 540 3 52168 5 - A | 74.3 49.3 33.3 71.1 55.1 |
| 540 3 52168 5 - B | 70.3 47.3 32.3 68.1 53.1 |

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