# CALCULATION OF THE ELASTIC MODULI
# of a
# TWO LAYER PAVEMENT SYSTEM
# from
# MEASURED SURFACE DEFLECTIONS

by

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Research Report Number 123-6

A System Analysis of Pavement Design
and Research Implementation
Research Study Number 1-8-69-123

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U. S. Department of Transportation
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# Preface

This is the sixth report issued under Research Study 1-8-69-123, A System Analysis of Pavement Design and Research Implementation. The study is being conducted jointly by principal investigators and their staffs in three agencies -- The Texas Highway Department, The Center for Highway Research at Austin and The Texas Transportation Institute -- as a part of the cooperative research program with the Department of Transportation, Federal Highway Administration.

Previous reports emanating from Study 123 are the following:

Report No. 123-1, "A Systems Approach Applied to Pavement Design and Research," by W. Ronald Hudson, B. Frank McCullough, Frank H. Scrivner, and James L. Brown, describes a long-range comprehensive research program to develop a pavement systems analysis and presents a working systems model for the design of flexible pavements.

Report No. 123-2, "A Recommended Texas Highway Department Pavement Design System Users Manual," by James L. Brown, Larry J. Buttler, and Hugo E. Orellana, is a manual of instructions to Texas Highway Department personnel for obtaining and processing data for flexible pavement design system.

Report No. 123-3, "Characterization of the Swelling Clay Parameter Used in the Pavement Design System," by Arthur W. Witt, III, and B. Frank McCullough, describes the results of a study of the swelling clay parameter used in pavement design system.

Report No. 123-4, "Developing A Pavement Feedback Data System," by R. C. G. Haas, describes the initial planning and development of a pavement feedback data system.

Report No. 123-5, "A Systems Analysis of Rigid Pavement Design," by Ramesh K. Kher, W. R. Hudson, and B. F. McCullough, describes the development of a working systems model for the design of rigid pavements.

The authors are indebted to Messrs. Robert E. Long and James L. Brown, both of the Texas Highway Department, for furnishing the pavement deflection data used in the sample problems presented in Chapter 6.

## Abstract

This report gives the theoretical background and a description of a new computer program, ELASTIC MODULUS, capable of converting deflections measured by a Dynaflect on the surface of a highway pavement-subgrade (two-layer elastic) system, to the elastic moduli of the pavement and subgrade. Included with the report are instructions for the use of the program, a complete documentation of its operation, and the solutions of several example problems.

# Summary

A sub-system of the flexible pavement design system described in the first report of Study 123 (see Preface), estimates the life of a trial design based solely on surface deflections computed from an empirical equation. In an attempt to improve the reliability of this sub-system (a primary objective of Study 123) it is intended, eventually, to base estimates of pavement life on stresses and strains computed from elasticity theory at critical points within the pavement structure. The use of elasticity theory, however, requires a knowledge of the in situ values of the elastic modulus, E, of each of the pavement materials in common use, as well as the subgrades, in the various Highway Department Districts.

According to elasticity theory, the moduli of a pavement and its subgrade can be estimated from surface deflections rather easily, provided the pavement structure above the subgrade is predominately a single material of known thickness, and the subgrade is reasonably uniform in stiffness to a considerable depth.

For determining the elastic moduli of the two materials composing such a pavement, a mathematical process has been developed, computerized, and is made available herewith to the Texas Highway Department. The method envisions the use of the Dynaflect for making the necessary measurements of surface deflections. The data collection and processing procedures, and the output format of the computer program, are exactly the same as those now employed in estimating the "stiffness coefficients"

used in the present version of the flexible pavement design system, with the following exceptions:

1. The program described in this report prints elastic moduli in lieu of stiffness coefficients.
2. The program prints a verbal description of both pavement and subgrade, instead of the pavement alone.

The computer program has been given the name ELASTIC MODULUS. By a slight modification, it can be used to predict Dynaflect deflections, given the pavement thickness and the moduli of pavement and subgrade. In this form the predictions of ELASTIC MODULUS were compared with those of another program, BISTRO*. Agreement was excellent, except in the instance of a pavement with a modulus much smaller than that of its subgrade, a case not likely to arise often in practice.

To illustrate the results obtained when using ELASTIC MODULUS to estimate pavement and subgrade moduli, Dynaflect data taken at several points on seven short sections of flexible pavements near College Station, Texas, were processed by the program. The ordering of the resulting pavement moduli, as judged by the verbal descriptions of the materials and local knowledge of their service performance, appeared reasonable. In the case of the subgrade moduli, the range was too small to permit a judgement of the validity of the results.

When using the results of the program to characterize materials in a pavement design system based on elasticity theory, it is recommended that the values of the computer moduli be halved before use. This recommendation is based on extensive field correlation studies between deflections produced by the Dynaflect and those produced by heavily loaded vehicles.

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* Used by courtesy of Koninklijke/Shell-Laboratorium, Amsterdam.

## Implementation Statement

The program ELASTIC MODULUS was written in the expectation that eventually the Texas Highway Department's Flexible Pavement Design System will, in the prediction of pavement life, use the stresses, strains and displacements computed throughout the structure from the theory of linear elastic layered systems, instead of solely the surface deflections calculated by the present empirical equation. When such a change occurs in the design system, in situ values of elastic moduli will be needed. This need probably can be met, at least to some degree, by the computer program described herein.

The published version of this report may be obtained by addressing your request as follows:

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## Table of Contents

## List of Figures

# List of Tables

# 1. Introduction

Recently the Texas Highway Department began to implement, on a trial basis, a flexible pavement design system that characterizes each material in a proposed or existing pavement structure by a so-called "stiffness coefficient" (1, 2). The in situ coefficient for a material proposed for a new pavement is found from Dynaflect deflection data (3, 4) taken on existing highways that can be assumed to consist essentially of two layers -- a subgrade layer (regarded in theory to be infinitely thick), and a pavement layer composed predominately of a single material (for example, a base material with a surface treatment). The Dynaflect data are then used in an empirical equation that yields a composite stiffness coefficient for the pavement material or materials, and another (usually smaller) coefficient for the subgrade or foundation (1, 5). The coefficients, which vary numerically from about 0.15 for a weak, wet clay to about 1.00 for asphaltic concrete, are calculated by means of a Texas Highway Department computer program, STIFFNESS COEFFICIENT (6). The coefficients, along with other pertinent data, are used in the design process to predict a certain characteristic -- the "surface curvature index" -- of the deflection basin of a trial design composed of the tested materials, and from this characteristic, to predict the life of the design.

This report gives the theoretical background and a description of a new computer program, given the name ELASTIC MODULUS, that accepts and prints the same Dynaflect and other data (identification, location, special comments, etc.) as the program STIFFNESS COEFFICIENT, but computes and prints out the in situ values of Young's modulus of pavement and subgrade instead of their stiffness coefficients. Linear elastic theory, with Poisson's ratio set to 1/2 for both layers, is used in the computations.

The program ELASTIC MODULUS was written in the expectation that eventually the Texas Highway Department's Flexible Pavement Design System will, in the prediction of pavement life, use the stresses, strains and displacements computed throughout the structure from the theory of linear elastic layered systems, instead of solely the surface deflections calculated by the present empirical equation (7). When such a change occurs in the design system, in situ values of elastic moduli will be needed. This need probably can be met, at least to some degree, by the computer program described herein.

## 2. Surface Deflection Equation for Two Layer Elastic System

This chapter describes the geometry of the Dynaflect loading and develops the applicable equation for surface deflections due to a point load acting perpendicular to the horizontal surface of a half-space consisting of two horizontal layers of infinite lateral extent.

### 2.1 The Loading Device (Dynaflect)

Through two steel wheels the trailer-mounted Dynaflect exerts two vertical loads, separated by 20 inches and varying sinusoidally in phase at 8 Hz, as indicated in Figure 1. The total load, exerted by rotating weights, varies from 500 pounds upward to 500 pounds downward. The upward thrust is overcome by the dead weight of the trailer so that the load wheels are always in contact with the pavement. The load-pavement contact areas are small and are considered to be points, rather than areas, in order to simplify the mathematics.

From the symmetry of Figure 1 it can be seen that one load of 1000 pounds can be substituted for the two loads shown, without affecting the vertical motion at points along the line of sensors. For this reason, in what follows only one point load, P, of 1000 lbs., will be considered to be acting on the surface of the pavement.

### 2.2 List of Symbols

Following is a list of the mathematical symbols used in this report. A list of FORTRAN symbols used in ELASTIC MODULUS, together with their mathematical equivalents, will be found in Appendix 1.

$P$ = vertical force acting at a point in the horizontal surface of a two-layer elastic half space.

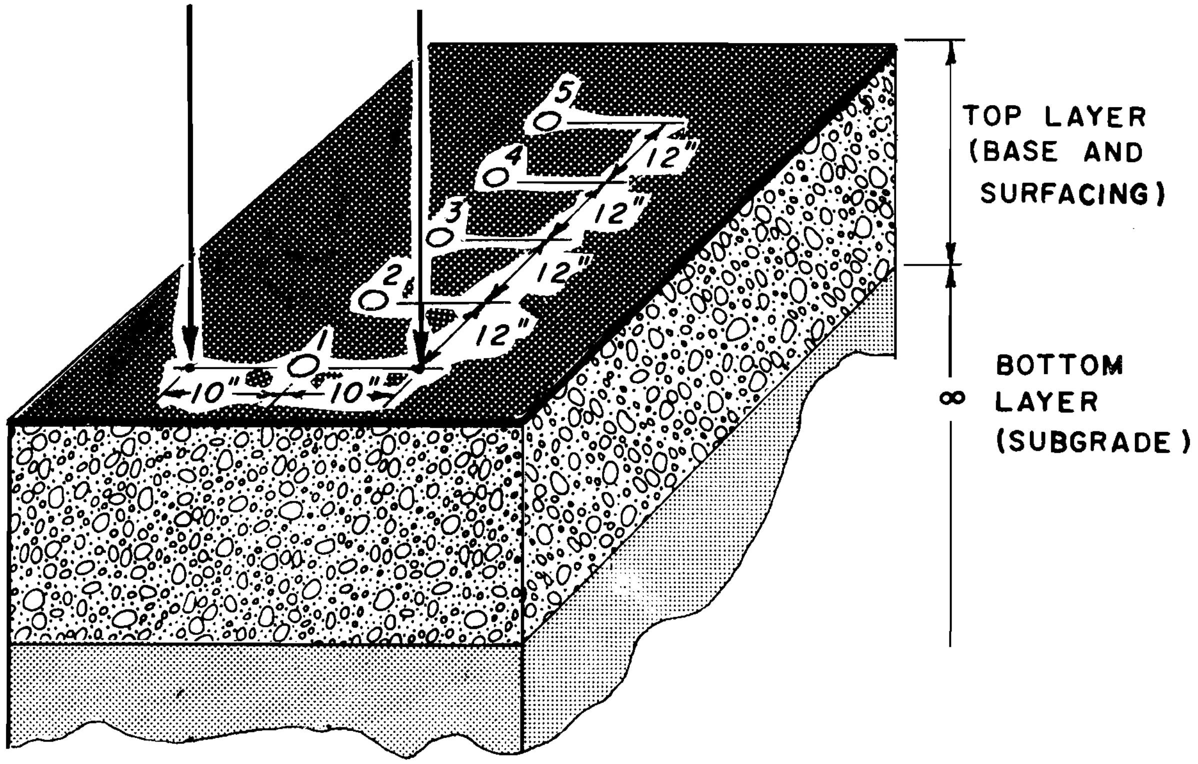

Figure 1: Relative position of Dynaflect loads and sensors. The sensors are usually placed in the outer wheel path, on a line paralleling the center line of the highway.

$h$ = thickness of upper layer.

$E_1$ = Young's modulus of upper layer.

$E_2$ = Young's modulus of lower layer.

$w$ = the vertical displacement of a point in the surface.

$r$, $z$ = cylindrical coordinates. (The tangential coordinate, $\Theta$, does not appear because only one load is used as explained on page 3, and the resulting vertical deflections are symmetrical about the z-axis.)

The load P acts downward at the point $r = 0$, $z = 0$. Positive z is measured downward.

$m$ = a parameter.

$x = mr/h$.

$Jo(x)$ = Bessel Function of the first kind and zero order with argument x.

$V$ = a function of m and N (see Equations (1) and (2)).

$N$ = a function of $E_1$ and $E_2$ (see Equation (2a)).

## 2.3 Development of the Equation

A vertical load, P (Figure 2), is applied at the point, 0, in the horizontal, plane surface of a two-layer elastic system. The point of load application is the origin of cylindrical coordinates, r and z. Positive values of z are measured vertically downward.

The thickness of the upper layer is h and its elastic modulus is $E_1$. The thickness of the lower layer is infinite, and its elastic modulus is $E_2$. Poisson's ratio for both layers is taken as 1/2.

It can be shown from Burmister's early work in elastic layered systems (8) that the deflection, w, of a surface point at the horizontal distance, r, from the point, 0, is related to the constants, h, $E_1$ and $E_2$ by the equation

$$\frac{4\pi E_1}{3P} wr = \int_{X=0}^{\infty} V \cdot Jo(x)dx, \qquad (1)$$

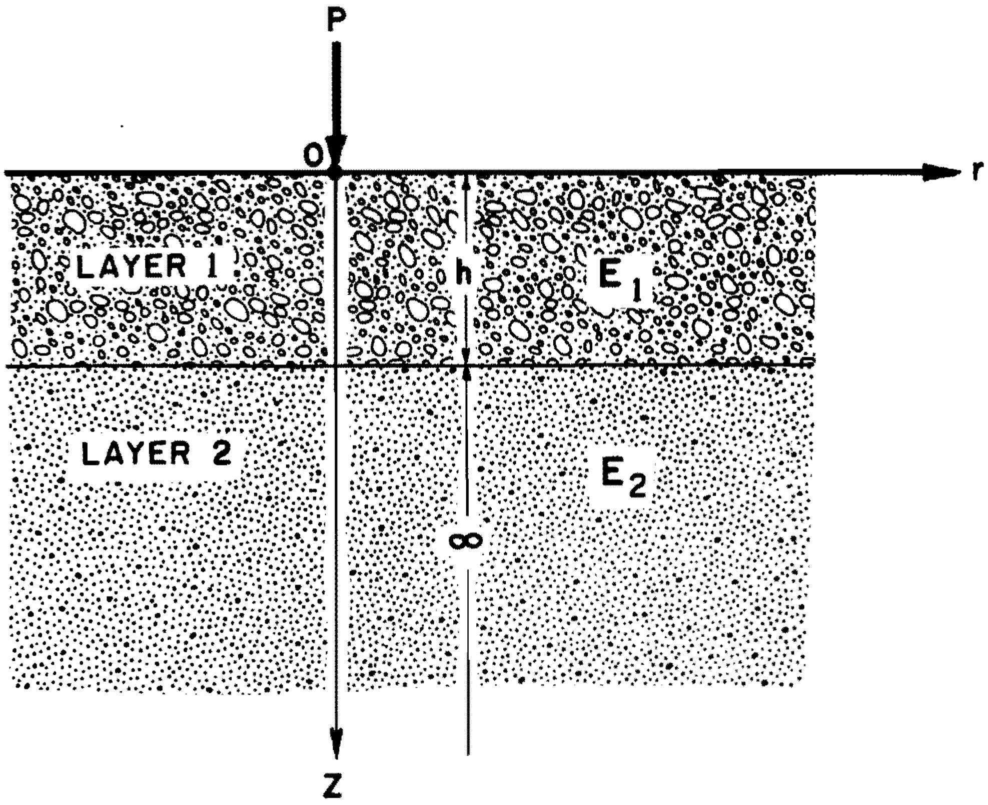

FIGURE 2 - Two-layer elastic system loaded at a point on the surface.

where $x = mr/h$, (1a)

$m$ = a parameter,

$$V = \frac{1 + 4Nme^{-2m} - N^2e^{-4m}}{1 - 2N(1 + 2m^2)e^{-2m} + N^2e^{-4m}}, \tag{2}$$

and $$N = \frac{1 - E_2/E_1}{1 + E_2/E_1} = \frac{E_1 - E_2}{E_1 + E_2} \tag{2a}$$

## 2.4 An Approximation of the Deflection Equation

The integration indicated in Equation 1 must be performed by numerical means. This task is made easier by taking advantage of the fact that (1) as x varies from zero to infinity in the integration process, m varies over the same range, while r and h are held constant, (2) as m varies from zero to infinity, the function V varies monotonically from $E_1/E_2$ to 1.0 and (3) for practical ranges of the ratio $E_2/E_1$, V approaches its limiting value of 1.0 at surprisingly low values of m. For example, it was found, as indicated in Table 1, that if m is set equal to 10, and $E_2/E_1$ is restricted to the range from zero to 1000, then $V = 1.0 \pm .000001$. Thus, we conclude that for practical purposes, when m is in the range from zero to 10, V is given by Equation 2, and when m is in the range from 10 to infinity, V = 1. This approximation can be expressed algebraically as follows:

$$\int_{x=0}^{\infty} V \cdot Jo(x)dx \approx \int_{x=0}^{10r/h} V \cdot Jo(x)dx + \int_{x=10r/h}^{\infty} Jo(x)dx \tag{3}$$

The second integral on the right side of Equation 3 is equivalent to the difference of two integrals, as indicated below:

$$\int_{x=10r/h}^{\infty} Jo(x)dx = \int_{x=0}^{\infty} Jo(x)dx - \int_{0}^{10r/h} Jo(x)dx = 1 - \int_{x=0}^{10r/h} Jo(x)dx. \tag{4}$$

Table 1: Values of the function, V, corresponding to selected values of the parameter m and the modular ratio $E_2/E_1$.

| m | $E_2/E_1$<br>0 | .001 | .01 | .1 | 1 | 10 | 100 | 1000 |
|---|---|---|---|---|---|---|---|---|
| 0.0 | Infinite | 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |
| 0.1 | 6012. | 855.6 | 98.14 | 9.967 | 1 | 0.1006 | 0.01065 | 0.001655 |
| 0.5 | 50.49 | 47.94 | 32.98 | 8.056 | 1 | 0.1542 | 0.06727 | 0.05854 |
| 1.0 | 7.382 | 7.363 | 6.826 | 4.112 | 1 | 0.3250 | 0.2491 | 0.2414 |
| 3.0 | 1.137 | 1.137 | 1.134 | 1.110 | 1 | 0.9058 | 0.8888 | 0.8869 |
| 5.0 | 1.006 | 1.006 | 1.005 | 1.005 | 1 | 0.9955 | 0.9946 | 0.9945 |
| 10.0 | 1.000001 | 1.000001 | 1.000001 | 1.000001 | 1 | 0.9999993 | 0.9999991 | 0.9999991 |
| Inf. | 1. | 1. | 1. | 1. | 1 | 1. | 1. | 1. |

By making the obvious substitution from Equation 4 in Equation 3 we have

$$\int_{x=0}^{\infty} V \cdot Jo(x)dx \approx \int_{x=0}^{10r/h} V \cdot Jo(x)dx + 1 - \int_{x=0}^{10r/h} Jo(x)dx, \text{ or}$$

$$\int_{x=0}^{\infty} V \cdot Jo(x)dx \approx 1 + \int_{x=0}^{10r/h} (V - 1)\ Jo(x)dx.$$

Comparing the last approximation, above, with Equation 1, we arrive at the approximation,

$$\frac{4\pi E_1}{3P} wr \approx 1 + \int_{x=0}^{x=10r/h} (V - 1)\ Jo(x)dx \qquad (5)$$

where all symbols are as previously defined.

It is of interest to note from Equation 2 that $V = 1$ when $E_2 = E_1$ (that is, when the layered system of Figure 1 degenerates into a homogeneous elastic half-space), and that for this case Equation 5 reduces to

$$\frac{4\pi E_1}{3P} wr \approx 1.$$

The correct equation for this case, according to Timoshenko (9), is

$$\frac{4\pi E_1}{3P} wr = 1.$$

Thus, for the homogeneous case Equation 5 becomes exact.

### 3. Numerical Integration of Deflection Equation

To use Equation 5 it was necessary to employ some form of numerical integration process for evaluating the integral in that equation. The method known as Simpson's Rule was selected (11). This procedure required that a small but finite increment, $\Delta x$, be chosen, and that the integral be calculated at $x = 0$, $x = \Delta x$, $x = 2\Delta x$, etc. over the specified range of integration. The smaller the value assigned to $\Delta x$, the greater would be the accuracy of the result; on the other hand, the larger the value of $\Delta x$, the less would be the required computer time. Thus a compromise between computer time and accuracy had to be made.

Noting that the integral of Equation 5 is the product of the factor, $V - 1$, which is a function of m and N, and Jo(x), which is a function of $x = mr/h$ (see Equation 1a), two safeguards against inaccurate results had to be incorporated into the program: (1) $\Delta m$ had to be small enough to insure a sufficiently accurate numerical representation of the function V, and (2) $\Delta x$ had to be small enough to insure an accurate numerical representation of the function Jo(x).

After some study of the numerical values of V given in Table 1, and of the values of Jo(x) available from numerous sources (see, for example, Reference 10), the following rules were incorporated into the computer program for solving Equation 5:

(a) In the range $m = 0$ to $m = 3$, $\Delta m \leq 0.01$. (In FORTRAN, DELM1 .LE. XK1.)

(b) In the range $m = 3$ to $m = 10$, $\Delta m \leq 0.10$. (In FORTRAN, DELM2 .LE. XK2.)

(c) In the entire range of x from 0 to 10r/h, not less than 61 values of Jo(x) are computed as x increases from any value x = c, to the value x = c + 3. This also insures that the number of values of Jo(x) computed between successive zeroes of that alternating function exceeds 61. (In FORTRAN, XNO = 61.)

Since $\Delta x$ and $\Delta m$ are interdependent according to Equation 1(a), that is,

$$\Delta x = \Delta m \cdot r/h, \tag{1b}$$

the computer program had to insure that the rules (a), (b) and (c) given above were consistent with Equation 1(b). The details of how this was done may be found in the accompanying listing of the computer program and its flow diagram. Suffice it to say here that the accuracy of the solutions obtained (or the computer time used) can be changed by altering the values assigned to the FORTRAN variables XK1, XK2 and XNO mentioned in (a), (b) and (c) above and further defined in Appendix 1.

To explain briefly how Equation 5 is used in ELASTIC MODULUS to find pavement and subgrade moduli, consider the following:

Suppose that $w_1$ has been measured on the surface of a pavement structure at the distance $r_1$ from either Dynaflect load, and $w_2$ at the distance $r_2$. The thickness, h, of the pavement is known.

Now let F represent the function on the right side of Equation 5. We may then write two equations:

$$\frac{4\pi E_1}{3P} w_1 r_1 \approx F(E_2/E_1,\ r_1/h) \tag{6a}$$

$$\frac{4\pi E_1}{3P} w_2 r_2 \approx F(E_2/E_1,\ r_2/h) \tag{6b}$$

By dividing Equation 6a by 6b we obtain

$$\frac{w_1 r_1}{w_2 r_2} = \frac{F(E_2/E_1,\ r_1/h)}{F(E_2/E_1,\ r_2/h)}\ , \qquad (7)$$

where $E_2/E_1$ is the only unknown.

By a convergent process of trial and error, a value of $E_2/E_1$ usually can be found that satisfies Equation 7 to the desired degree of accuracy. After this has been done, $E_1$ is calculated from Equation (6a), and finally $E_2$ is found from the relation

$$E_2 = E_1 (\frac{E_2}{E_1}) .$$

## 4. Accuracy Check

As mentioned earlier (Section 2.1) a point load was substituted in ELASTIC MODULUS for the area loads exerted by the Dynaflect. To check the effect of this assumption on accuracy, as well as the effect of the approximations described in Chapters 2 and 3, the following procedure was followed.

The contact area of each load wheel was measured approximately by inserting light sensitive paper between each wheel and the pavement, running the Dynaflect for a short time in strong sunlight, then removing the paper and measuring the unexposed areas.

From these measurements it was concluded that each 500 lb. load could be represented by a uniform pressure of 80 psi acting on a circular area with a radius of 1.41 inches. Furthermore, because of the symmetry of the load-geophone configuration, it was reasoned that the effect of both loads could be represented by a pressure of 160 psi acting on one circular area of the radius given above (1.41 inches).

The surface deflections $w_1$ and $w_2$ (see Figure 1) occurring at the distances $r = 10$ inches and $r = \sqrt{10^2 + 12^2} = 15.62$ inches from the center of the circle, could then be calculated from the program BISTRO, written by Koninklijke/Shell-Laboratorium, Amsterdam, and compared with deflections obtained by the program ELASTIC MODULUS modified slightly to receive as inputs $E_1$, $E_2$, h and r and to print out $w_1$ and $w_2$.

The two programs were compared as described above over a range of the ratio, $E_1/E_2$, from 0.1 to 1000, and a range of the thickness, h, from 5 to 40 inches. The results are recorded in Table 2 in the same manner that Dynaflect deflections are recorded -- that is, in milli-inches to two decimal places.

The table shows near perfect agreement in the range $1 \leq E_1/E_2 \leq 1000$ for which the pavement is stiffer than the subgrade. On the other hand, with the subgrade much stiffer than the pavement ($E_1/E_2 = 0.1$ in Table 2), the agreement was not as good. In addition, up-heavals occurred, as indicated by the negative signs of some of the deflections. In these cases the deflected surface is very irregular and Dynaflect data from such a pavement would be difficult to interpret since this device is not equipped to distinguish phase differences between load and geophone.

Since most pavements of the type illustrated in Figure 1 are obviously intended to be stiffer than their subgrades, and in view of the fact that irregular basin shapes are seldom encountered in practice, it is concluded from the data presented in Table 2 that ELASTIC MODULUS represents the theory of elasticity with sufficient accuracy to accomplish the purpose for which it was designed.

Table 2: Comparison of ELASTIC MODULUS with BISTRO

| | | | | Computed Deflections (mils) | | | |
|---|---|---|---|---|---|---|---|
| | | | | $w_1$ | | $w_2$ | |
| $E_1$ (psi) | $E_2$ (psi) | $E_1/E_2$ | h (in.) | ELASTIC MODULUS | BISTRO | ELASTIC MODULUS | BISTRO |
| 10,000,000 | 10,000 | 1,000 | 5 | 0.99 | 0.99 | 0.93 | 0.93 |
| | | | 10 | 0.52 | 0.52 | 0.51 | 0.51 |
| | | | 20 | 0.26 | 0.26 | 0.26 | 0.26 |
| | | | 40 | 0.13 | 0.13 | 0.13 | 0.13 |
| 1,000,000 | 10,000 | 100 | 5 | 1.86 | 1.85 | 1.55 | 1.55 |
| | | | 10 | 1.07 | 1.07 | 0.99 | 0.99 |
| | | | 20 | 0.57 | 0.57 | 0.55 | 0.55 |
| | | | 40 | 0.30 | 0.30 | 0.30 | 0.30 |
| 100,000 | 10,000 | 10 | 5 | 2.65 | 2.65 | 1.77 | 1.77 |
| | | | 10 | 1.94 | 1.93 | 1.56 | 1.56 |
| | | | 20 | 1.20 | 1.20 | 1.06 | 1.06 |
| | | | 40 | 0.74 | 0.74 | 0.64 | 0.64 |
| 10,000 | 10,000 | 1 | 5 | 2.39 | 2.39 | 1.53 | 1.53 |
| | | | 10 | 2.39 | 2.39 | 1.53 | 1.53 |
| | | | 20 | 2.39 | 2.39 | 1.53 | 1.53 |
| | | | 40 | 2.39 | 2.39 | 1.53 | 1.53 |
| 1,000 | 10,000 | 0.1 | 5 | -0.11 | -0.40 | 0.85 | 0.86 |
| | | | 10 | -0.15 | -0.06 | -0.58 | -0.57 |
| | | | 20 | 7.45 | 7.52 | 1.30 | 1.32 |
| | | | 40 | 14.9 | 14.9 | 6.68 | 6.69 |

Note: ELASTIC MODULUS: Point load of 1000 lbs.
BISTRO: Circular loaded area with radius of 1.41 in., pressure of 160 psi, load of 1000 lbs.
Both programs: Vertical deflection computed at the points r = 10", z = 0 and r = 15.62", z = 0.

## 5. Non-Unique Solutions

To investigate the possibility that the use of the program could lead to more than one solution -- that is, to more than one value of the ratio $E_1/E_2$ -- or perhaps to no solution at all in some cases, ELASTIC MODULUS was modified slightly to receive as inputs selected values of $E_1/E_2$ and the layer thickness h, and to compute the corresponding ratio, $w_1r_1/w_2r_2$ (see Equation 7). The results of these computations were plotted as contours of the layer thickness, h, in Figure 3. The range of input data was limited to the largest range that might be expected from field deflection tests made on real highways of the type illustrated in Figure 1.

To facilitate interpretation, Figure 3 has been divided into four quadrants as indicated on the graph. For example, by referring to quadrants I and II it can be seen that if the measured inputs to ELASTIC MODULUS satisfy the inequalities $w_1r_1/w_2r_2 > 1$ and $h \geq 9.2''$ (see the dashed contour), a unique solution satisfying the inequality $E_1/E_2 < 1$ exists, and in this case the program finds and prints the two moduli. If, on the other hand, $w_1r_1/w_2r_2 > 1$ (as before) but $h < 9.2''$, the possibility of two solutions exists -- or of no solution at all if the measured ratio $w_1r_1/w_2r_2$ is sufficiently great. In this case, i.e. $w_1r_1/w_2r_2 > 1$ and $h < 9.2''$, the program abandons the search for a solution and prints the message "NO UNIQUE SOLUTION".

By examining quadrants III and IV, it can be concluded that if $w_1r_1/w_2r_2 < 1$ and $h \geq 9.2''$, a unique solution satisfying the inequality $E_1/E_2 > 1$ exists. In this case the program finds the solution and prints the two moduli. On the other hand if $w_1r_1/w_2r_2 < 1$ as before, but $h < 9.2''$ there are two possible solutions, one in qudarant III for $E_1/E_2 > 1$, and another

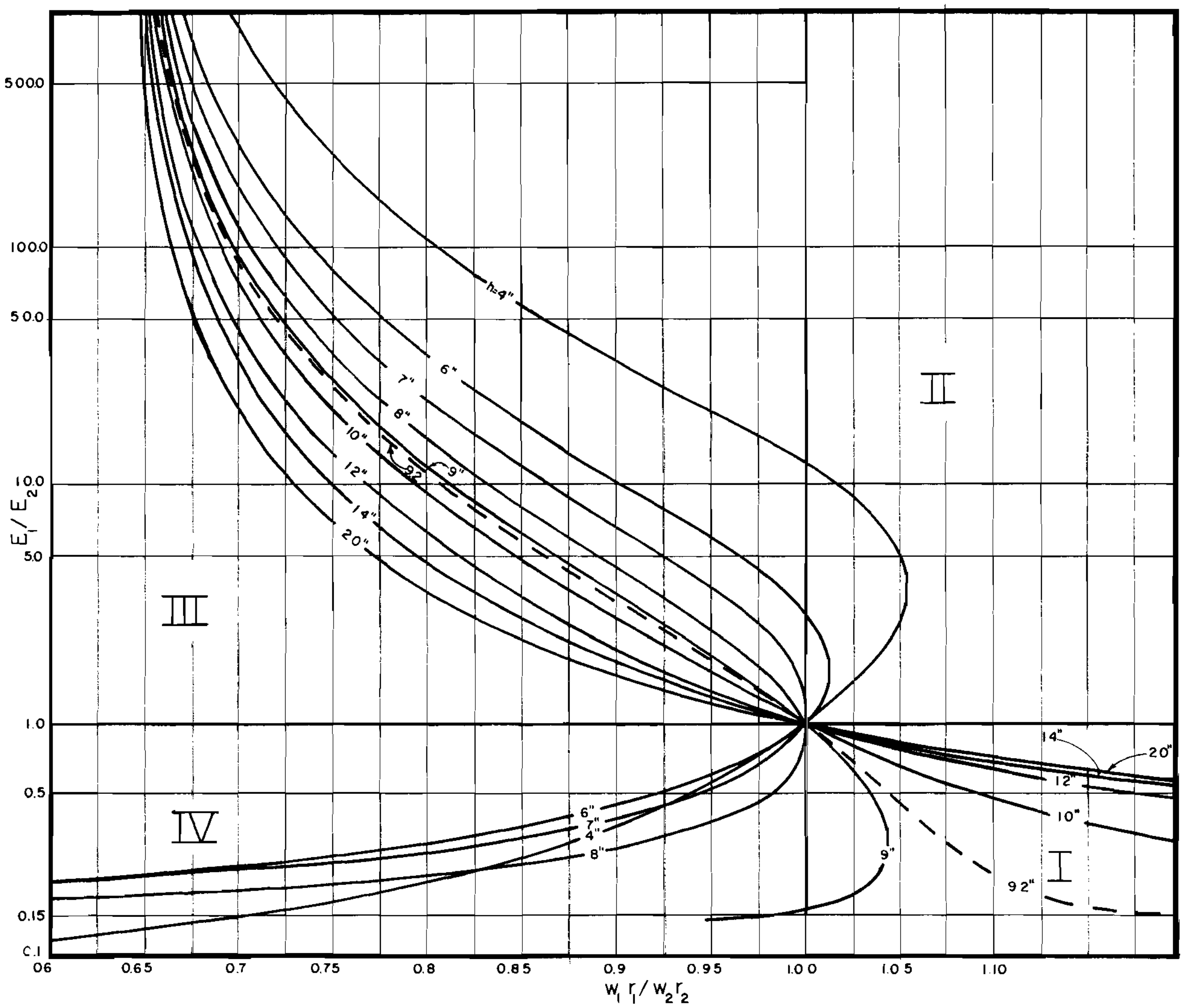

Figure 3: Contours of pavement thickness, h, plotted as a function of the ratios $E_1/E_2$ and $w_1r_1/w_2r_2$.

in quadrant IV for $E_1/E_2 < 1$. Of these two solutions the one in quadrant III, representing a pavement whose elastic modulus is greater than that of the subgrade, is the more probable; therefore, the program seeks out the quadrant III solution, prints the corresponding moduli, and ignores the quadrant IV solution.

The information deduced above from Figure 3, and used in the control of the program ELASTIC MODULUS, is summarized in Table 3.

Table 3: Summary of Information from Figure 3 Used in the Control of the Program, ELASTIC MODULUS

| Measured Input Data | | | | |
|---|---|---|---|---|
| $w_1r_1/w_2r_2$ | Thickness, h (in.) | Unique Solution | Layer Having The Greater Modulus | Program Printout |
| Greater than 1 | Greater than 9.2 | Yes | Subgrade | Subgrade and Pavement moduli |
| Greater than 1 | Less than 9.2 | No | May be either | "NO UNIQUE SOLUTION"* |
| Less than 1 | Greater than 9.2 | Yes | Pavement | Subgrade and pavement moduli |
| Less than 1 | Less than 9.2 | No | May be either, but the more probable of two possible solutions is selected | Subgrade and pavement moduli for solution having $E_1/E_2 > 1$ |

* When the experimental data $w_1r_1/w_2r_2$ exceeds unity, and h is less than 9.2", some cases can arise for which no solution at all is possible.

## 6. Examples of Solutions Obtained by ELASTIC MODULUS

In May, 1968, Dynaflect deflections were measured at ten points in the outer wheel path on each of several 500-ft. sections of highways in the vicinity of College Station, Texas, originally for the purpose of gaining experience in the determination of the "stiffness coefficient" mentioned in the Introduction of this report (page 1). Some of these data, including thicknesses obtained by coring at five points in each section, were used as inputs to the computer program discussed herein for the purpose of illustrating its use in obtaining the elastic moduli of pavements and subgrades. The results are summarized in Tables 4 and 5, while the computer print-outs -- in the standard format of the program -- are shown in Tables 6a through 6g. In the latter group of tables the readings of each of the five geophones at each test station are given, although only the greatest deflections, $w_1$ and $w_2$, were actually used in estimating the moduli $E_1$ and $E_2$.

Tables 4 and 5 are arranged in descending order of the magnitude of the average modulus of pavement and subgrade, respectively. In comparing these two tables it is of interest to note that the variability of the pavement modulus, as indicated by the coefficient of variation in the last column, is generally greater than that of the subgrade. In addition it is apparent that the range of $E_1$ (13,900 psi to 283,200 psi) is much greater than the range of $E_2$ (11,700 psi to 20,000 psi). Finally, it should be pointed out that the pavement of Section 12, at the bottom of the list in Table 4, had an average modulus (13,900 psi) of approximately the same magnitude as that of its subgrade (14,400 psi).

The low pavement modulus found for Section 12 invites some discussion. The low value obtained may be due to the relatively poor quality of the major component of the pavement, a sandstone which, according to local engineers, has in some cases performed poorly. In any event the surfacing of this section had been overlayed -- because of map cracking -- shortly before it was tested in 1968, then again developed severe map cracking that required sealing in 1970. The seal coat failed to arrest the progress of surface deterioration, and at this writing (June, 1971) it is again being overlayed with one inch of hot-mix asphaltic concrete. In short, the contrast between the stiffness of the surfacing material and that of the base seems to be at the root of the trouble in this section.

Beyond these remarks concerning Section 12, and the additional fact that the ordering of the other materials appears reasonable, any other discussion of the ordering of the materials in Tables 3 and 4 is considered to be beyond the scope of this report.

Table 4: Average Pavement Modulus, $E_1$, for Seven 500-ft. Sections of Highways near College Station, Texas

(Deflection measurements made May 21, 1968)

| | Pavement Materials and Thicknesses | | Pavement Thickness, h | | Pavement Modulus, $E_1$ | | | |
|---|---|---|---|---|---|---|---|---|
| Test Section | Surfacing | Base | Average Value (In.) | Standard Deviation | No.* Solutions | Average Value (PSI) | Standard Deviation | Coefficient of Variation (percent) |
| 15 | 1.2" Asph. Conc. | 14.0" Cement stabilized limestone | 15.2 | 1.2 | 10 | 283,200 | 76,100 | 27 |
| 4 | 0.5" Seal Coat | 7.5" Asphalt stabilized gravel | 8.0 | 0.4 | 2 | 78,900 | 8,200 | 10 |
| 16 | 1.0" Asph. Conc. | 6.5" Asph. emulsion stab. gravel | 7.5 | 0.4 | 10 | 73,900 | 13,800 | 19 |
| 17 | 0.5" Seal Coat | 7.8" Iron ore gravel | 8.3 | 0.7 | 8 | 36,600 | 24,700 | 67 |
| 5 | 0.5" Seal Coat | 11.5" Lime stabilized sandstone | 12.0 | 2.8 | 10 | 32,300 | 15,100 | 47 |
| 3 | 0.5" Seal Coat | 12.0" Red sandy gravel | 12.5 | 1.0 | 10 | 24,700 | 6,000 | 24 |
| 12 | 3.7" Asph. Conc. | 16.2" Sandstone | 19.9 | 0.5 | 10 | 13,900 | 2,700 | 19 |

* Measurements were made at 10 locations in each section. Less than 10 solutions occur in cases where $w_1r_1/w_2r_2 > 1$ and $h < 9.2"$, as explained in Chapter 4.

Table 5: Average Subgrade Modulus, $E_2$, for Seven 500-ft. Sections of Highways near College Station, Texas

(Deflection measurements made May 21, 1968)

| | | Subgrade Material | | Subgrade Modulus, $E_2$ | | | |
|---|---|---|---|---|---|---|---|
| Section | Thickness Investigated | Description | Formation | No.* Solutions | Average Value (PSI) | Standard Deviation | Coefficient of Variation (percent) |
| 15 | 32" | Red sandy clay, some gravel | Stone City | 10 | 20,000 | 900 | 5 |
| 3 | 23" | Sand over clay | Spiller Sandstone Member of Cook Mountain Formation | 10 | 19,000 | 1600 | 8 |
| 4 | 25" | Grey sandy clay | Spiller Sandstone Member of Cook Mountain Formation | 2 | 14,900 | 800 | 5 |
| 5 | 24" | Tan sandy clay | Caddell | 10 | 14,500 | 1400 | 10 |
| 12 | 22" | Black stiff clay | Lagarto | 10 | 14,400 | 900 | 6 |
| 17 | 21" | Grey sandy clay | Spiller Sandstone Member of Cook Mountain Formation | 8 | 12,700 | 1700 | 13 |
| 16 | 18" | Brown clay | Alluvium deposit of Brazos River | 10 | 11,700 | 700 | 6 |

* Measurements were made at 10 locations in each section. Less than 10 solutions occur in cases where $w_1r_1/w_2r_2 > 1$ and $h < 9.2"$, as explained in Chapter 4.

```
                            TEXAS HIGHWAY DEPARTMENT

                         DISTRICT 17 - DESIGN SECTION

               DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

                        THIS PROGRAM WAS RUN - 06/21/71

                             DIST.        COUNTY
                              17          BRAZOS

          CONT.    SECT.    JOB    HIGHWAY    DATE       DYNAFLECT
          1560      1        1     FM 1687   5-21-68         1

                                   PAV. THICK. = 12.50 INCHES

        SEAL COAT                 0.50     RED SANDY GRAVEL     12.00

        GREY & BRWN SAND SUB  0.0

STATION   W1     W2     W3     W4     W5     SCI   ** ES   ** ** EP   **  REMARKS

 1 - A   1.170 0.770 0.520 0.310 0.219 0.400     20200.     23500.
 1 - B   1.140 0.770 0.510 0.310 0.213 0.370     20500.     28200.
 2 - A   1.290 0.840 0.490 0.300 0.204 0.450     18400.     20300.
 2 - B   1.200 0.840 0.490 0.300 0.201 0.360     19000.     33300.
 3 - A   1.140 0.770 0.470 0.300 0.195 0.370     20500.     28200.
 3 - B   1.110 0.770 0.460 0.300 0.201 0.340     20700.     33900.
 4 - A   1.470 0.960 0.490 0.320 0.222 0.510     16100.     18100.
 4 - B   1.380 0.900 0.470 0.310 0.213 0.480     17200.     19000.
 5 - A   1.290 0.870 0.500 0.340 0.231 0.420     18100.     24600.
 5 - B   1.260 0.800 0.460 0.310 0.219 0.460     19000.     18100.

AVERAGES 1.245 0.829 0.486 0.310 0.212 0.416     18970.     24720.
STANDARD DEVIATION                     0.057      1551.      5996.
NUMBER OF POINTS IN AVERAGE =             10        10         10

  W1    DEFLECTION AT GEOPHONE 1
  W2    DEFLECTION AT GEOPHONE 2
  W3    DEFLECTION AT GEOPHONE 3
  W4    DEFLECTION AT GEOPHONE 4
  W5    DEFLECTION AT GEOPHONE 5
  SCI   SURFACE CURVATURE INDEX ( W1 MINUS W2)
  ES    ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
  EP    ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2
```

Table 6a: Computer print-out for Section 3.

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                         TEXAS HIGHWAY DEPARTMENT

                       DISTRICT 17 - DESIGN SECTION

             DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

                     THIS PROGRAM WAS RUN - 06/21/71

                           DIST.        COUNTY
                            17          BRAZOS

           CONT.    SECT.    JOB    HIGHWAY    DATE       DYNAFLECT
           2824       2       1     FM 2776   5-21-68         1

                                    PAV. THICK. =  8.00 INCHES

         SEAL COAT                0.50      ASPHALT STAB. GRAVEL  7.50

         GREY SANDY CLAY SUBG  0.0

STATION    W1     W2     W3     W4     W5     SCI   ** ES   ** ** EP   **   REMARKS

  1 - A  1.650 1.200 0.870 0.660 0.500 0.450     14300.     84700.
  1 - B  1.560 1.110 0.810 0.610 0.490 0.450     15500.     73100.
  2 - A  2.310 1.470 0.930 0.710 0.530 0.840   NO UNIQUE SOLUTION
  2 - B  2.310 1.410 0.900 0.670 0.510 0.900   NO UNIQUE SOLUTION
  3 - A  2.430 1.500 0.930 0.670 0.490 0.930   NO UNIQUE SOLUTION
  3 - B  2.490 1.530 0.930 0.670 0.500 0.960   NO UNIQUE SOLUTION
  4 - A  2.490 1.470 0.900 0.640 0.480 1.020   NO UNIQUE SOLUTION
  4 - B  2.430 1.410 0.840 0.610 0.470 1.020   NO UNIQUE SOLUTION
  5 - A  2.340 1.440 0.870 0.620 0.450 0.900   NO UNIQUE SOLUTION
  5 - B  2.430 1.470 0.930 0.650 0.470 0.960   NO UNIQUE SOLUTION

AVERAGES 2.244 1.401 0.891 0.651 0.489 0.843     14900.     78900.
STANDARD DEVIATION                     0.214       849.      8202.
NUMBER OF POINTS IN AVERAGE =             10         2          2

   W1     DEFLECTION AT GEOPHONE 1
   W2     DEFLECTION AT GEOPHONE 2
   W3     DEFLECTION AT GEOPHONE 3
   W4     DEFLECTION AT GEOPHONE 4
   W5     DEFLECTION AT GEOPHONE 5
   SCI    SURFACE CURVATURE INDEX ( W1 MINUS W2)
   ES     ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
   EP     ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2
```

Table 6b: Computer print-out for Section 4.

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                         TEXAS HIGHWAY DEPARTMENT

                       DISTRICT 17 - DESIGN SECTION

           DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

                      THIS PROGRAM WAS RUN - 06/21/71

                            DIST.       COUNTY
                             17         BURLESON

          CONT.   SECT.   JOB   HIGHWAY    DATE       DYNAFLECT
          1399      1      1    FM 1361   5-21-68         1

                                PAV. THICK. = 12.00 INCHES

        SEAL COAT                 0.50     LIME STAB. SANDSTONE 11.50

        TAN SANDY CLAY SUBGR  0.0

STATION   W1     W2     W3     W4     W5     SCI   ** ES   ** ** EP   **   REMARKS

 1 - A  1.500 1.110 0.710 0.470 0.330 0.390     14500.      41600.
 1 - B  1.560 1.230 0.780 0.480 0.330 0.330     12700.      65800.
 2 - A  1.650 1.200 0.670 0.400 0.243 0.450     13500.      33100.
 2 - B  1.440 1.050 0.640 0.380 0.246 0.390     15400.      38500.
 3 - A  1.500 1.050 0.600 0.370 0.267 0.450     15300.      27900.
 3 - B  1.440 0.990 0.580 0.370 0.261 0.450     16100.      25800.
 4 - A  1.500 1.050 0.560 0.340 0.216 0.450     15300.      27900.
 4 - B  1.380 0.990 0.540 0.330 0.213 0.390     16300.      35900.
 5 - A  1.920 1.260 0.650 0.400 0.280 0.660     12400.      14400.
 5 - B  1.800 1.140 0.630 0.420 0.310 0.660     13300.      12500.

AVERAGES 1.569 1.107 0.636 0.396 0.270 0.462    14480.      32340.
STANDARD DEVIATION                        0.112     1413.      15108.
NUMBER OF POINTS IN AVERAGE =                10        10          10

   W1     DEFLECTION AT GEOPHONE 1
   W2     DEFLECTION AT GEOPHONE 2
   W3     DEFLECTION AT GEOPHONE 3
   W4     DEFLECTION AT GEOPHONE 4
   W5     DEFLECTION AT GEOPHONE 5
   SCI    SURFACE CURVATURE INDEX ( W1 MINUS W2)
   ES     ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
   EP     ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2
```

Table 6c: Computer print-out for Section 5.

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                          TEXAS HIGHWAY DEPARTMENT

                        DISTRICT 17 - DESIGN SECTION

               DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

                       THIS PROGRAM WAS RUN - 06/21/71

                         DIST.        COUNTY
                          17          WASHINGTON

           CONT.    SECT.    JOB    HIGHWAY     DATE        DYNAFLECT
            186       5       1     SH 36      5-21-68          1

                                    PAV. THICK. = 19.90 INCHES

          HOT MIX ASPH. CONC.   3.75      SANDSTONE            16.15

          BLACK CLAY SUBGRADE   0.0

STATION    W1     W2     W3     W4     W5     SCI   ** ES   ** ** EP   **  REMARKS

 1 - A   1.680 1.020 0.610 0.420 0.300 0.660     15200.     12400.
 1 - B   1.830 1.080 0.610 0.420 0.310 0.750     14400.     10700.
 2 - A   1.740 1.080 0.670 0.470 0.360 0.660     14300.     12700.
 2 - B   1.950 1.170 0.690 0.490 0.370 0.780     13200.     10400.
 3 - A   1.680 1.080 0.680 0.500 0.380 0.600     14100.     14400.
 3 -     1.710 1.080 0.670 0.480 0.370 0.630     14200.     13500.
 4 - A   1.680 1.110 0.750 0.570 0.460 0.570     13600.     15600.
 4 - B   1.560 1.080 0.730 0.550 0.440 0.480     13800.     19400.
 5 - A   1.500 0.960 0.590 0.440 0.330 0.540     15900.     15900.
 5 - B   1.590 0.990 0.600 0.430 0.330 0.600     15500.     14000.

AVERAGES 1.692 1.065 0.660 0.477 0.365 0.627     14420.     13900.
STANDARD DEVIATION                     0.091       861.      2661.
NUMBER OF POINTS IN AVERAGE =             10         10         10

   W1    DEFLECTION AT GEOPHONE 1
   W2    DEFLECTION AT GEOPHONE 2
   W3    DEFLECTION AT GEOPHONE 3
   W4    DEFLECTION AT GEOPHONE 4
   W5    DEFLECTION AT GEOPHONE 5
   SCI   SURFACE CURVATURE INDEX ( W1 MINUS W2)
   ES    ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
   EP    ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2
```

Table 6d: Computer print-out for Section 12.

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                              TEXAS HIGHWAY DEPARTMENT

                             DISTRICT 17 - DESIGN SECTION

                  DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

                            THIS PROGRAM WAS RUN - 06/21/71

                               DIST.        COUNTY
                                17          ROBERTSON

              CONT.    SECT.    JOB    HIGHWAY    DATE      DYNAFLECT
               49        8       1     US 190    5-21-68        1

                                    PAV. THICK. = 15.20 INCHES

            HOT MIX ASPH. CONC.    1.25     CEM. STAB. LIMESTONE 13.95

            RED SANDY CLAY SUBGR  0.0

STATION    W1     W2     W3     W4     W5     SCI   ** ES    ** ** EP    **   REMARKS

  1 - A   0.680 0.590 0.490 0.390 0.310 0.090     21000.    230800.
  1 - B   0.680 0.600 0.490 0.390 0.310 0.080     19500.    280500.
  2 - A   0.720 0.630 0.510 0.390 0.310 0.090     19100.    240900.
  2 - B   0.700 0.620 0.490 0.390 0.310 0.080     18600.    284100.
  3 - A   0.750 0.650 0.520 0.390 0.300 0.100     19000.    209200.
  3 - B   0.760 0.650 0.510 0.390 0.300 0.110     19800.    179000.
  4 - A   0.600 0.540 0.450 0.350 0.280 0.060     20100.    402600.
  4 - B   0.580 0.520 0.430 0.330 0.880 0.060     21000.    405200.
  5 - A   0.620 0.550 0.450 0.350 0.910 0.070     20800.    327800.
  5 - B   0.650 0.570 0.470 0.360 0.280 0.080     21000.    271700.

AVERAGES 0.674 0.592 0.481 0.373 0.419 0.082     19990.    283180.
STANDARD DEVIATION                         0.016       926.     76113.
NUMBER OF POINTS IN AVERAGE =                 10        10         10

   W1     DEFLECTION AT GEOPHONE 1
   W2     DEFLECTION AT GEOPHONE 2
   W3     DEFLECTION AT GEOPHONE 3
   W4     DEFLECTION AT GEOPHONE 4
   W5     DEFLECTION AT GEOPHONE 5
   SCI    SURFACE CURVATURE INDEX ( W1 MINUS W2)
   ES     ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
   EP     ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2
```

Table 6e: Computer print-out for Section 15.

```
                          TEXAS HIGHWAY DEPARTMENT

                        DISTRICT 17 - DESIGN SECTION

               DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

                        THIS PROGRAM WAS RUN - 06/21/71

                           DIST.       COUNTY
                            17         BRAZOS

            CONT.    SECT.    JOB    HIGHWAY    DATE       DYNAFLECT
            1560       1       1     FM 1687   5-21-68         1

                                     PAV. THICK. =   7.50 INCHES

          ASPHALT SURFACING      1.00      ASPH EMUL STAB GRAVL  6.50

          BROWN CLAY SUBGRADE    0.0

STATION   W1     W2     W3     W4     W5     SCI   ** ES    ** ** EP   **   REMARKS

 1 - A   2.160 1.500 0.960 0.660 0.520 0.660     11500.      49700.
 1 - B   2.130 1.530 0.960 0.650 0.510 0.600     11300.      70200.
 2 - A   1.920 1.410 0.930 0.640 0.490 0.510     12300.      94300.
 2 - B   1.860 1.350 0.900 0.630 0.500 0.510     12800.      88100.
 3 - A   2.040 1.470 0.930 0.630 0.490 0.570     11800.      75000.
 3 - B   2.070 1.500 0.960 0.650 0.500 0.570     11600.      77300.
 4 - A   2.220 1.620 1.020 0.670 0.490 0.600     10700.      77500.
 4 - B   2.220 1.590 1.020 0.650 0.490 0.630     10900.      65300.
 5 - A   1.980 1.380 0.900 0.610 0.470 0.600     12500.      56900.
 5 - B   1.980 1.440 0.930 0.610 0.460 0.540     12000.      84800.

AVERAGES 2.058 1.479 0.951 0.640 0.492 0.579     11740.      73910.
STANDARD DEVIATION                      0.049      679.      13843.
NUMBER OF POINTS IN AVERAGE =              10        10          10

   W1     DEFLECTION AT GEOPHONE 1
   W2     DEFLECTION AT GEOPHONE 2
   W3     DEFLECTION AT GEOPHONE 3
   W4     DEFLECTION AT GEOPHONE 4
   W5     DEFLECTION AT GEOPHONE 5
   SCI    SURFACE CURVATURE INDEX ( W1 MINUS W2)
   ES     ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
   EP     ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2
```

Table 6f: Computer print-out for Section 16.

```
                         TEXAS HIGHWAY DEPARTMENT

                        DISTRICT 17 - DESIGN SECTION

             DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULII

                     THIS PROGRAM WAS RUN - 06/21/71

                          DIST.       COUNTY
                           17         BRAZOS

          CONT.    SECT.    JOB   HIGHWAY    DATE        DYNAFLECT
           540       3       1    FM 974    5-21-68          1

                                  PAV. THICK. =  8.30 INCHES

        SEAL COAT               0.50     IRON ORE GRAVEL       7.80

        GREY SANDY CLAY SUBG  0.0

STATION    W1     W2     W3     W4     W5     SCI    ** ES   ** ** EP   **   REMARKS

 1 - A   2.400 1.530 0.960 0.680 0.500 0.870   NO UNIQUE SOLUTION
 1 - B   2.250 1.440 0.900 0.630 0.480 0.810   NO UNIQUE SOLUTION
 2 - A   1.770 1.170 0.820 0.600 0.480 0.600     14000.      25600.
 2 - B   1.800 1.200 0.820 0.620 0.490 0.600     13800.      28000.
 3 - A   1.650 1.170 0.840 0.640 0.510 0.480     14600.      60100.
 3 - B   1.590 1.170 0.840 0.610 0.510 0.420     14600.      88300.
 4 - A   2.250 1.470 0.990 0.750 0.600 0.780     11000.      16700.
 4 - B   2.340 1.590 1.050 0.790 0.630 0.750     10600.      27000.
 5 - A   2.220 1.470 0.990 0.710 0.550 0.750     11200.      21000.
 5 - B   2.100 1.410 0.960 0.680 0.530 0.690     11800.      26100.

AVERAGES 2.037 1.362 0.917 0.671 0.528 0.675     12700.      36600.
STANDARD DEVIATION                     0.146      1710.       24675.
NUMBER OF POINTS IN AVERAGE =            10          8           8

   W1    DEFLECTION AT GEOPHONE 1
   W2    DEFLECTION AT GEOPHONE 2
   W3    DEFLECTION AT GEOPHONE 3
   W4    DEFLECTION AT GEOPHONE 4
   W5    DEFLECTION AT GEOPHONE 5
   SCI   SURFACE CURVATURE INDEX ( W1 MINUS W2)
   ES    ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W2
   EP    ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W2
```

Table 6g: Computer print-out for Section 17.

## 7. Adjustment of Moduli for Practical Use in Pavement Design

As previously noted, the elastic moduli estimated by the computer program are based on deflections produced and measured by the Dynaflect system. Correlation studies of Dynaflect deflections with those produced by a 9000-lb. dual-tired wheel load and measured by means of the Benkelman Beam on highways in Illinois and Minnesota in 1967 (3) indicated that the 9000-lb. wheel load deflection could, with reasonable accuracy, be estimated from the Dynaflect deflection, $w_1$, by multiplying $w_1$ by 20.

But the peak-to-peak load of the Dynaflect is 1000-lbs.; thus, one would expect that the multiplying factor would be about 9, rather than 20 as found by actual field experience.

Various explanations could be advanced to explain this discrepancy. However, they would not alter the fact, brought out by the correlation sutdy, that if one desires to use the values of $E_1$ and $E_2$ found from Dynaflect deflections to calculate the deflection of a linear elastic layered system acted on by a heavy vehicle, then he should approximately halve these moduli before using them in his calculations.

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11. Eshback, Ovid W., "Handbook of Engineering Fundamentals," Second Edition, John Wiley & Sons, New York, New York, 1953, page 2-114.

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# Appendix 1

The variable names used in ELASTIC MODULUS are listed on the following pages. The variable names and their definitions are in alphabetical order in the following sequence:

MAIN Program Variables

Subroutine EMOD Variables

Function BESJO Variables

Function V Variables

## MAIN Program Variables

A - Dummy array used with subroutine CORE to select the correct input format for each card read

AAP2 - Sum of pavement moduli

AAS2 - Sum of subgrade moduli

AAP2V - Average pavement modulus

AAS2V - Average subgrade modulus

AP2 - Elastic modulus of the pavement, rounded to nearest 100 (appears as EP on printout)

AS2 - Elastic modulus of the subgrade, rounded to nearest 100 (appears as ES on printout)

ASC1 - Sum of (W1 - W2). W1 - W2 = surface curvature index.

ASC1V - Average surface curvature index

AW1 - Sum of Geophone 1 deflections

AW2 - Sum of Geophone 2 deflections

AW3 - Sum of Geophone 3 deflections

AW4 - Sum of Geophone 4 deflections

AW5 - Sum of Geophone 5 deflections

AW1V - Average Geophone 1 deflection

AW2V - Average Geophone 2 deflection

AW3V - Average Geophone 3 deflection

AW4V - Average Geophone 4 deflection

AW5V - Average Geophone 5 deflection

COMM - Comments related to the project

CORE - Subroutine to re-read a card under format control

CO1, CO2, CO3, CO4 - County Name

D1 - Geophone 1 reading

D2 - Geophone 1 multiplier

D3 - Geophone 2 reading

D4 - Geophone 2 multiplier

D5 - Geophone 3 reading

D6 - Geophone 3 multiplier

D7 - Geophone 4 reading

D8 - Geophone 4 multiplier

D9 - Geophone 5 reading

D10 - Geophone 5 multiplier

DAP - Pavement elastic modulus (unrounded) as calculated in subroutine EMOD

DAS - Subgrade elastic modulus (unrounded) as calculated in subroutine EMOD

DATE - An IBM subroutine that returns the current month, day, & year

DP - Total pavement thickness

EMOD - Subroutine to calculate pavement & subgrade moduli

HWY1, HWY2 - Highway name & number

I - Pointer for data read into storage

ICK - Switch to indicate last data card

ICONT - Contract number for the highway

IDAY - Day the deflections were taken

IDIST - District number

IDYNA - Dynaflect number

IJOB - THD job number

ISECT - THD section number for the highway

IXDATE - Return arguments for subroutine DATE (month, day, year)

IYEAR - Year the deflections were taken

LA1 - Description of material in Layer 1

LA2 - Description of material in Layer 2

LA3 - Description of material in Layer 3

LA4 - Description of material in Layer 4

LA5 - Description of material in Layer 5

LA6 - Description of material in Layer 6

M - Month the deflections were taken

N - Counter for number of error free data cards read

NO - Counter for data cards omitted because of errors

NI - Counter to control printing of 30 lines per page

NCARD - Denotes card type

100 = Project identification card

200 = Existing pavement description card (layers 1, 2, & 3)

300 = Existing pavement description card (layers 4, 5, & 6)

400 = Data card (geophone readings and multipliers)

PN - Number of test points to be used in the analysis

REM - Any pertinent remarks related to any test point

ROUND - Statement function to round a given value of E1 or E2 to the nearest 100 psi

SCI - Surface curvature index, W1 - W2, in mils

SE1 - Standard deviation of surface curvature index

SE2 - Standard deviation of subgrade moduli

SE3 - Standard deviation of pavement moduli

SR1 - Variance of surface curvature index

SR2 - Variance of subgrade moduli

SR3 - Variance of pavement moduli

STA - Station number

T1 - Layer 1 thickness

T2 - Layer 2 thickness

T3 - Layer 3 thickness

T4 - Layer 4 thickness

T5 - Layer 5 thickness

T6 - Layer 6 thickness

W1 - Deflection at Geophone number 1

W2 - Deflection at Geophone number 2

W3 - Deflection at Geophone number 3

W4 - Deflection at Geophone number 4

W5 - Deflection at Geophone number 5

XLANE - Traffic lane & direction

## Subroutine EMOD Variables

ACC - Test for convergence in iteration for finding E2/E1

AREA1 - Result of the integration from x = 0 to x = 3r/h

AREA2 - Result of the integration from X = 3r/h to X = 10r/h

BESJO - Function subroutine to return the Bessel Function Jo(x) for each x used

DELM1 - Increment of m used in interval from m = 0 to m = 3

DELM2 - Increment of m used in interval from m = 3 to m = 10

DELTA - Incremental value for E2/E1 (Subgrade modulus divided by Pavement modulus) used in iteration process

DELX1 - Increment of x in integration from x = 0 to x = 3r/h

DELX2 - Increment of x in integration from x = 3r/h to x = 10r/h

E1 - Pavement modulus(E1)

E2 - Subgrade modulus (E2)

ER - Input specifying the accuracy of the iteration in calculating the ratio E2E1

E2E1 - Ratio of subgrade modulus to pavement modulus (E2/E1)

ERROR - (F1F2 - RATIO), where F1F2 is calculated and RATIO is observed

FF - Function defined as $\frac{4\pi E_1}{3P} w_i r_i$ (See Eq. 5)

F1F2 - Ratio of FF with i = 1 to FF with i = 2

H - Pavement thickness, h

ISW - Switch used in iterating to find E2/E1, indicates first time through the iteration loop

MINUS - Switch used in iterating to find E2/E1, indicates a negative ERROR

N1 - Number of intervals used for integration from x = 0 to x = 3r/h

N2 - Number of intervals used for integration from x = 3r/h to x = 10r/h

P - Dynaflect load = 1000#

PART1 - Sum of interior ordinates of first integration

PART2 - Sum or end ordinates of first integration

PART3 - Sum of interior ordinates of second integration

PART4 - Sum of end ordinates of second integration

PLUS - Switch used in iterating to find E2/E1, indicates a positive ERROR

RH - Radius (distance of geophone from load wheel) divided by pavement thickness, (r/h)

R1 - Distance from load to Geophone 1

R2 - Distance from load to Geophone 2

RATIO - (W1R1/W2R2)

SAVE - Contains the previous ERROR calculated that is closest to the convergence criterion in iterating to find E2/E1

V - Function subroutine to return the value of V for each E2/E1 and XM1 values used

W1 - Geophone 1 deflection (mils)

W2 - Geophone 2 deflection (mils)

X1 - Value of any x in the interval x = 0 to x = 3r/h

X2 - Value of any x in the interval x = 3r/h to x = 10r/h

XK1 - Maximum value of $\Delta m$ in the interval m = 0 to m = 3 (now set at 0.01)

XK2 - Maximum value of $\Delta m$ in the interval m = 3 to m = 10 (now set at 0.10)

XM1 - Value of any m in the interval m = 0 to m = 3

XM2 - Value of any m in the interval m = 3 to m = 10

XN - (E1 - E2)/(E1 + E2)

XNO - Minimum number of values of Jo(x) calculated in the interval from x to x + 3, in the calculation by Simpson's Rule of AREA1 and AREA2. XNO must be an odd number and is now set at XNO = 61

Y - Array to store ordinates to be used in integration from x = 0 to x = 10r/h

## Function BESJO Variables

X - Value x in the interval $x = 0$ to $x = 10r/h$

X3 - X/3, or 3/X if $X > 3$

X32 - X3 Squared

X33 - X3 Cubed

X34 - X3 to the Fourth Power

X35 - X3 to the Fifth Power

X36 - X3 to the Sixth Power

## Function V Variables

EXPM2M - Exponential, $e^{-2m}$

EXPM4M - Exponential, $e^{-4m}$

XM - Value of any m in the interval m = 0 to m = 10

XN - (E1 - E2)/(E1 + E2)

# Appendix 2

A narrative of the procedure used by ELASTIC MODULUS to calculate pavement and subgrade elastic moduli is contained on the following pages.

## Description of ELASTIC MODULUS Program

The ELASTIC MODULUS program consists of a main program, two subroutines, and two function subroutines. The main program reads the input data, performs certain data transformations, and outputs the results. Subroutine CORE is called by the main program and is used to allow the user to select the input format to be used to read a certain card. The elastic moduli of the pavement and subgrade for each test point are calculated in **subroutine** EMOD. The function subroutines BESJ0 and V are called by EMOD and are used in the numerical integration used in calculating the elastic moduli.

The main program reads each input data card into a storage area and uses subroutine CORE to select the read statement and data format to read each data card. Subroutine CORE allows a FORTRAN program to read under format control from a storage area which contains alphabetic character codes of a card image. Each data card has a code punched in the first three columns that designates the card type. If the code is 100 the card contains control information about the job, location, date, and total pavement thickness (see Appendix 4). Card code 200 indicates a card that contains word descriptions and thicknesses of the first three layers of the pavement (see Appendix 4). The word descriptions and thicknesses of layers 4, 5, and 6 (if present) are on cards with code 300 (see Appendix 4). If the card code is 400 or is blank, the card contains the station number and geophone readings and multipliers for each observation (see Appendix 4). Card code 100 also indicates the beginning of data cards for each job (or set of observations) and all counters and sums are set to their initial values. The information on Data Cards 1 and 2 (and Data Card 3, if present) is read and printed in the heading of the output.

The deflections at each geophone are calculated from the geophone readings and multipliers on each Data Card 4. SCI = W1 - W2 is also calculated. If either W1 or W2 is zero, or if W1 is less than W2, an error message is printed, the observation is not included in the analysis, and the next card is read. If the quantity W1R1/W2R2 is greater than 1 and the total pavement thickness is less than 9.2" the observation is not included in the analysis, an error message is printed, and the next card is read. If W1 and W2 are valid observations they are converted to inches and are passed to Subroutine EMOD along with the total pavement thickness for the elastic moduli calculation.

The pavement and subgrade moduli returned from EMOD are rounded to the nearest 100, W1 and W2 are converted back to mils, the sums of the deflections, SCI, pavement modulus, and subgrade modulus are incremented by the individual observations of each of these variables. The counter N (the sum of the valid observations) is incremented and a line of output consisting of the station number, W1, W2, W3, W4, W5, SCI, subgrade modulus, pavement modulus, and remarks is printed. The program will then skip to a new page before going back to read the next card if 30 lines of output have been printed. If all the data cards have been read the program calculates and prints averages of all deflections, SCI, pavement modulus, and the subgrade modulus. The variances and standard deviations of SCI, pavement modulus and subgrade modulus are calculated and printed and the program reads Data Card 1 of the next set of observations or terminates normally if there is no more data.

Subroutine EMOD uses the W1, W2, and total pavement thickness from the main program in the integration process and iteration scheme used to calculate the pavement modulus, E1 and the subgrade modulus, E2. All calculations in EMOD and the function subroutines BESJO and V are done in double precision to preserve the accuracy of the numerical integration. The user has the option to change the following variables or leave them at their present values:

P = 1000, ER = 0.001, XNO = 61.0, XK1 = 0.01, XK2 = 0.10

All switches and counters are initialized, r/h ratios are calculated and the variable RATIO (used in determining the convergence criterion in iteration for finding E2/E1) is calculated.

DELM1 is calculated and tested against the maximum assigned value for this variable. DELM1 is then set to the maximum value or the calculated value (whichever is the smaller value) and is used in calculating DELX1. DELM2 is calculated and tested in the same manner. The starting values of DELTA and E2E1 are selected and XN for the first iteration is calculated. The iteration loop begins with the calculation of each XN value for each E2E1 value used.

The number of intervals for each integration, N1 and N2, are calculated. (Note -- N1 and N2 must be odd integers.) The ordinates for each x in each integration interval are calculated and stored in the vector Y. The area of each integration interval is calculated according to Simpson's Rule (See Reference 11) and the function FF(1) = AREA1 + AREA2 + 1 is calculated (See Equation 5). FF(2) is calculated as above except RH(2) is used in calculating DELM1 and DELM2.

ERROR is calculated and tested against the convergence criterion. If ERROR is < ACC, E1 and E2 are calculated and EMOD returns to the main program. Otherwise a new value of E2E1 is calculated and the iteration loop is repeated.

The iteration method consists of trying values of E2E1 until ERROR is within the convergence criterion. The ERROR for any trial value of E2E1 that is closest to the convergence criterion is saved so that E2E1 values for subsequent trials can be adjusted to "home in" on the convergence criterion in a minimum number of trials.

For the first trial value of E2E1, if ERROR is not within the convergence criterion, ERROR is stored in SAVE and ISW is set to 1. If ERROR > 0, PLUS is set to 1 and MINUS is set to 0. This indicates ERROR is positive since PLUS was set to 0 and minus was set to 1 initially. For each successive pass through the iteration loop the sign of ERROR determines the segment of code executed to adjust E2E1 for the next trial until the convergence criterion is met.

When ERROR is positive, E2E1 is adjusted for the next trial in the following manner. PLUS is set to 1 to indicate ERROR is positive. If ERROR from the previous trial was positive, E2E1 is decreased for the next trial and the iteration loop is repeated. If ERROR from the previous trial was negative, DELTA is decreased by 50%, and the test for SAVE < 0 is made. If SAVE is positive, ERROR is stored in SAVE, E2E1 is decreased for the next trial and the iteration loop is repeated. If SAVE is negative, the test for |SAVE| > ERROR is made. A true condition indicates the iteration method is approaching convergence on this trial from the positive direction so ERROR is stored in SAVE, E2E1 is decreased for the next trial and the iteration loop is repeated. A false condition indicates

ERROR is departing from convergence in the positive direction, so E2E1 is decreased for the next trial and the iteration loop is repeated.

E2E1 is adjusted for the next trial in the following manner when ERROR is negative. MINUS is set to 1 to indicate ERROR is negative. If ERROR from the previous trial was negative the iteration method is approaching the convergence criterion from the negative direction, so E2E1 is increased for the next trial and the iteration loop is repeated. If ERROR from the previous trial was positive, DELTA is decreased by 50% and the test for SAVE > 0 is made. A false condition indicates the previous ERROR was negative also so the test for |SAVE| > |ERROR| is made. A true condition indicates the iteration method is closer to convergence on this trial than on the previous trial so ERROR is stored in SAVE, and E2E1 is increased for the next trial and the iteration loop is repeated. A false condition to the above test indicates the previous ERROR was closer to convergence so increase E2E1 for the next trial and repeat the iteration loop. If the test for SAVE > 0 is true, then the test for |ERROR| > SAVE is made. A false condition indicates the iteration method is closer to convergence on this trial on the negative side than the previous trial was on the positive side, so ERROR is stored in SAVE, E2E1 is increased for the next trial and the iteration loop is repeated. For a true condition to the test for |ERROR| > SAVE the steps following the test for |SAVE| > |ERROR| are repeated.

The function subroutine BESJO calculates the Bessel Function Jo(x) using polynomial approximation (See Reference 12) for each value of X in the integration interval X = 0 to X = 10r/h.

The function subroutine V calculates the value of V (See Equation 2) for each XN and XM1 or XM2 value used.

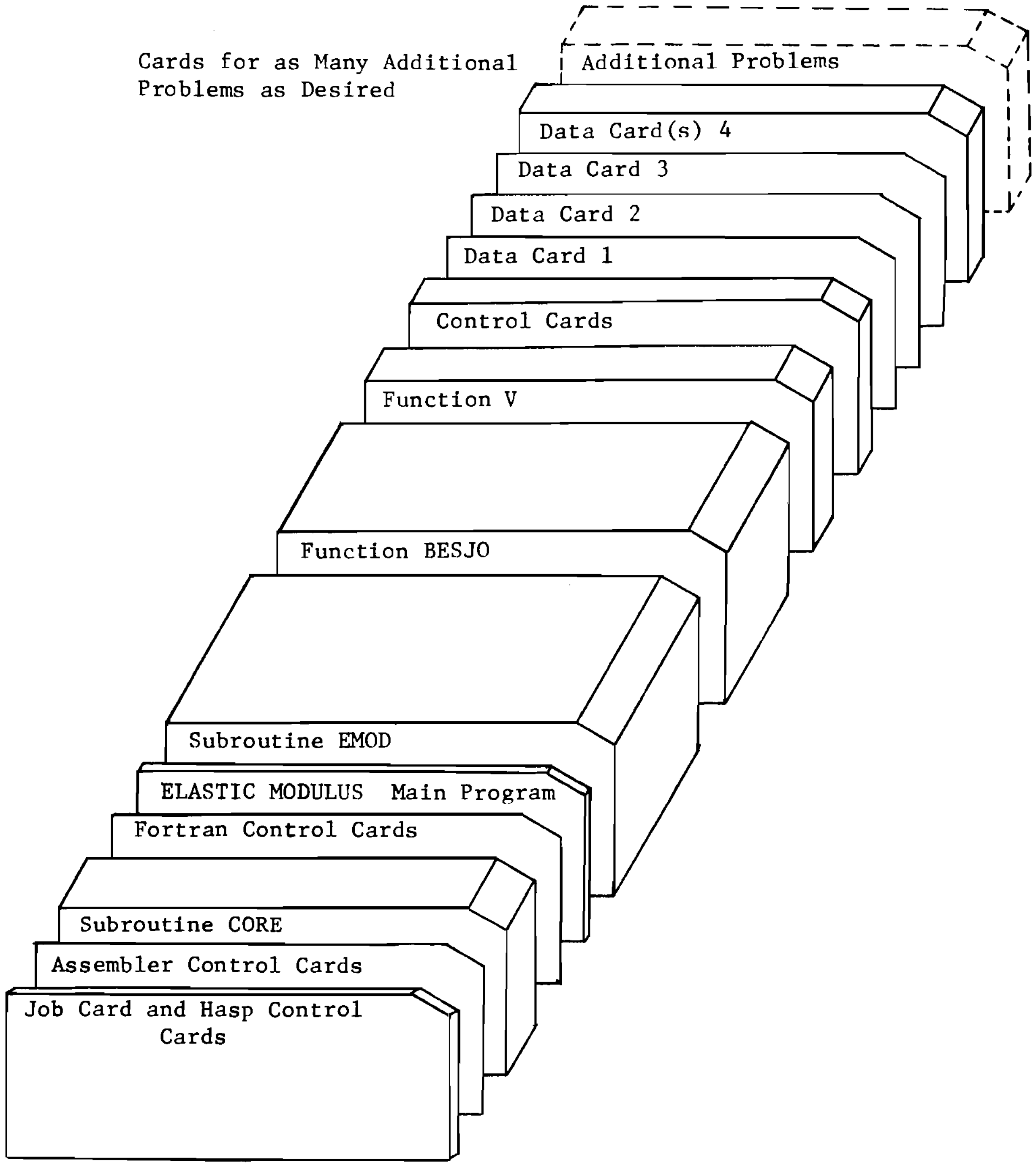
# Appendix 3

This appendix contains the ELASTIC MODULI program source deck set-up.

## Deck Set-Up

The ELASTIC MODULUS program was written in FORTRAN IV, Version G. The user is advised to change the first read statement in the main program if the "END=" option of the FORTRAN read is not implemented at his installation. The user can also substitute any other "Re-read" routine for Subroutine CORE if this is desired. ELASTIC MODULUS requires approximately 100k of core storage and execution time is approximately 7 seconds per test point. The source deck set-up is shown on the following page.

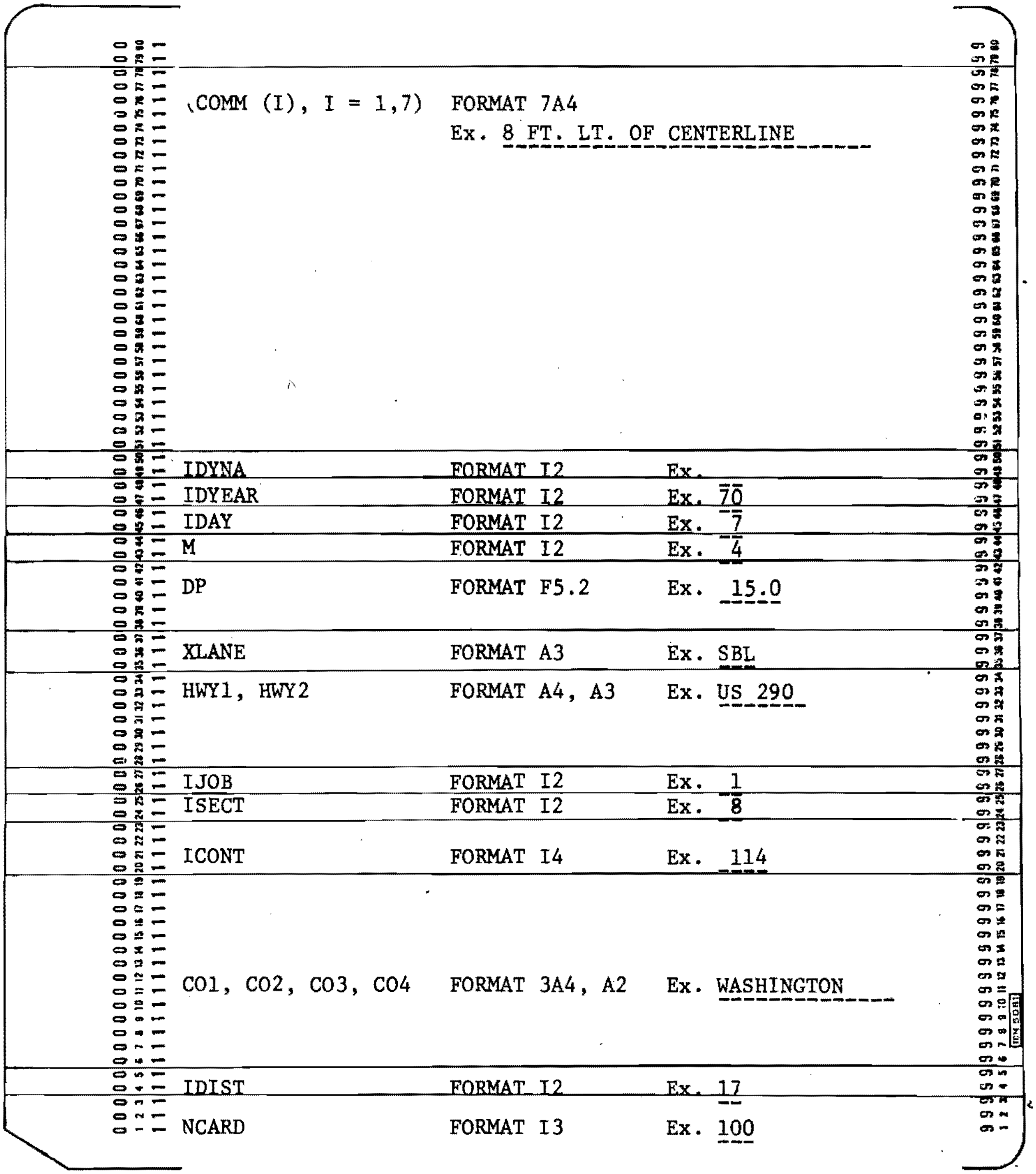
# Source Deck Set-Up for ELASTIC MODULUS Program



# Appendix 4

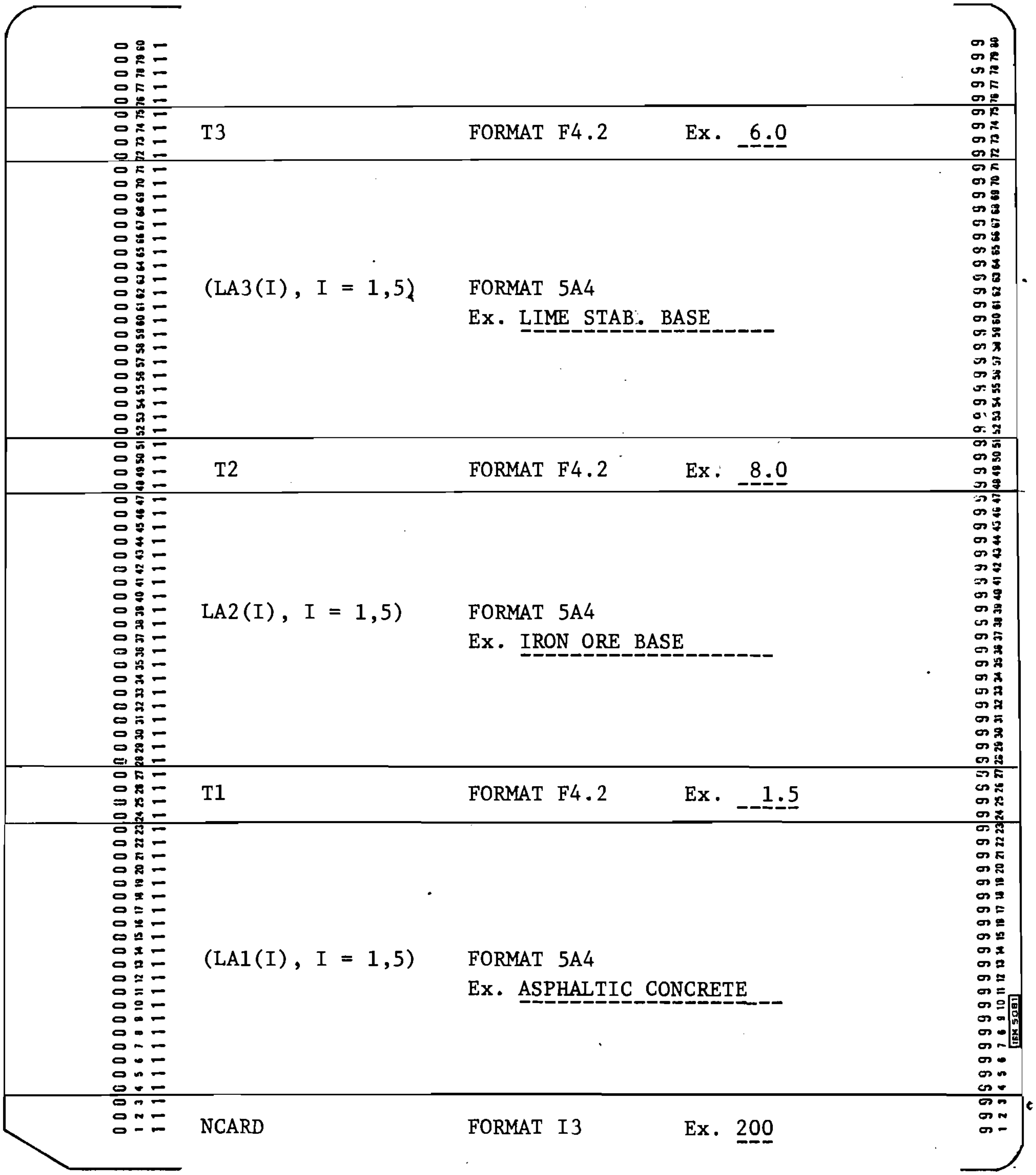
The input format for each data card type used by ELASTIC MODULUS is included on the following pages. The fields of each card are delineated and examples of typical data entries for each field are shown.

CARD 1



| | | |
|---|---|---|
| COMM (I), I = 1,7) | FORMAT 7A4 | Ex. 8 FT. LT. OF CENTERLINE |
| IDYNA | FORMAT I2 | Ex. |
| IDYEAR | FORMAT I2 | Ex. 70 |
| IDAY | FORMAT I2 | Ex. 7 |
| M | FORMAT I2 | Ex. 4 |
| DP | FORMAT F5.2 | Ex. 15.0 |
| XLANE | FORMAT A3 | Ex. SBL |
| HWY1, HWY2 | FORMAT A4, A3 | Ex. US 290 |
| IJOB | FORMAT I2 | Ex. 1 |
| ISECT | FORMAT I2 | Ex. 8 |
| ICONT | FORMAT I4 | Ex. 114 |
| CO1, CO2, CO3, CO4 | FORMAT 3A4, A2 | Ex. WASHINGTON |
| IDIST | FORMAT I2 | Ex. 17 |
| NCARD | FORMAT I3 | Ex. 100 |

CARD 2

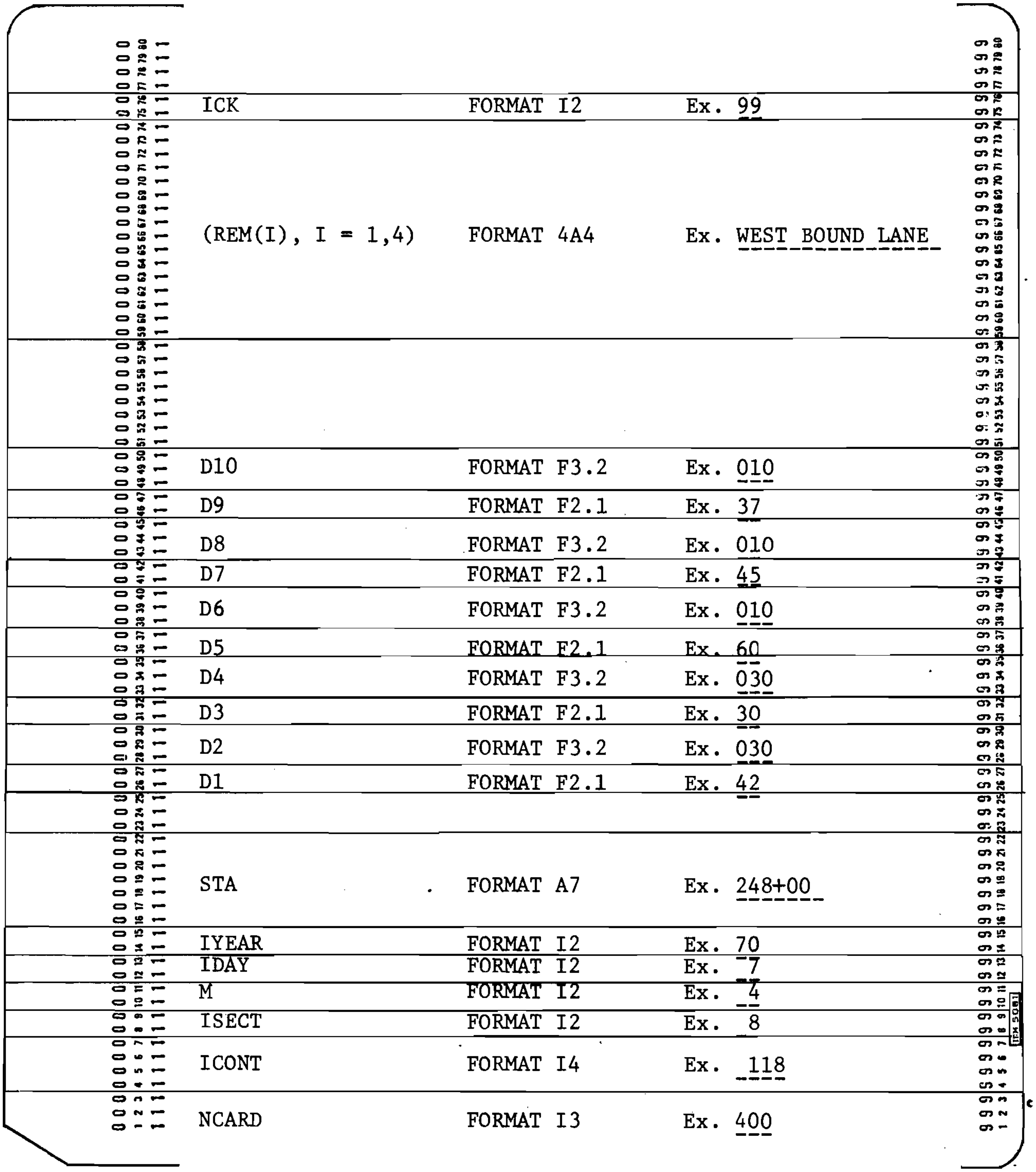


| | | |
|---|---|---|
| T3 | FORMAT F4.2 | Ex. 6.0 |
| (LA3(I), I = 1,5) | FORMAT 5A4<br>Ex. LIME STAB. BASE | |
| T2 | FORMAT F4.2 | Ex. 8.0 |
| LA2(I), I = 1,5) | FORMAT 5A4<br>Ex. IRON ORE BASE | |
| T1 | FORMAT F4.2 | Ex. 1.5 |
| (LA1(I), I = 1,5) | FORMAT 5A4<br>Ex. ASPHALTIC CONCRETE | |
| NCARD | FORMAT I3 | Ex. 200 |

CARD 3

| | | |
|---|---|---|
| T6 | FORMAT F4.2 | Ex. ____ |
| (LA6(I), I = 1,5) | FORMAT 5A4<br>Ex. SUBGRADE | |
| T5 | FORMAT F4.2 | Ex. 4.0 |
| (LA5(I), I = 1,5) | FORMAT 5A4<br>Ex. LIME STAB. SUBGRADE | |
| T4 | FORMAT F4.2 | Ex. 6.0 |
| (LA4(I), I = 1,5) | FORMAT 5A4<br>Ex. CEMENT STAB. BASE | |
| NCARD | FORMAT I3 | Ex. 300 |

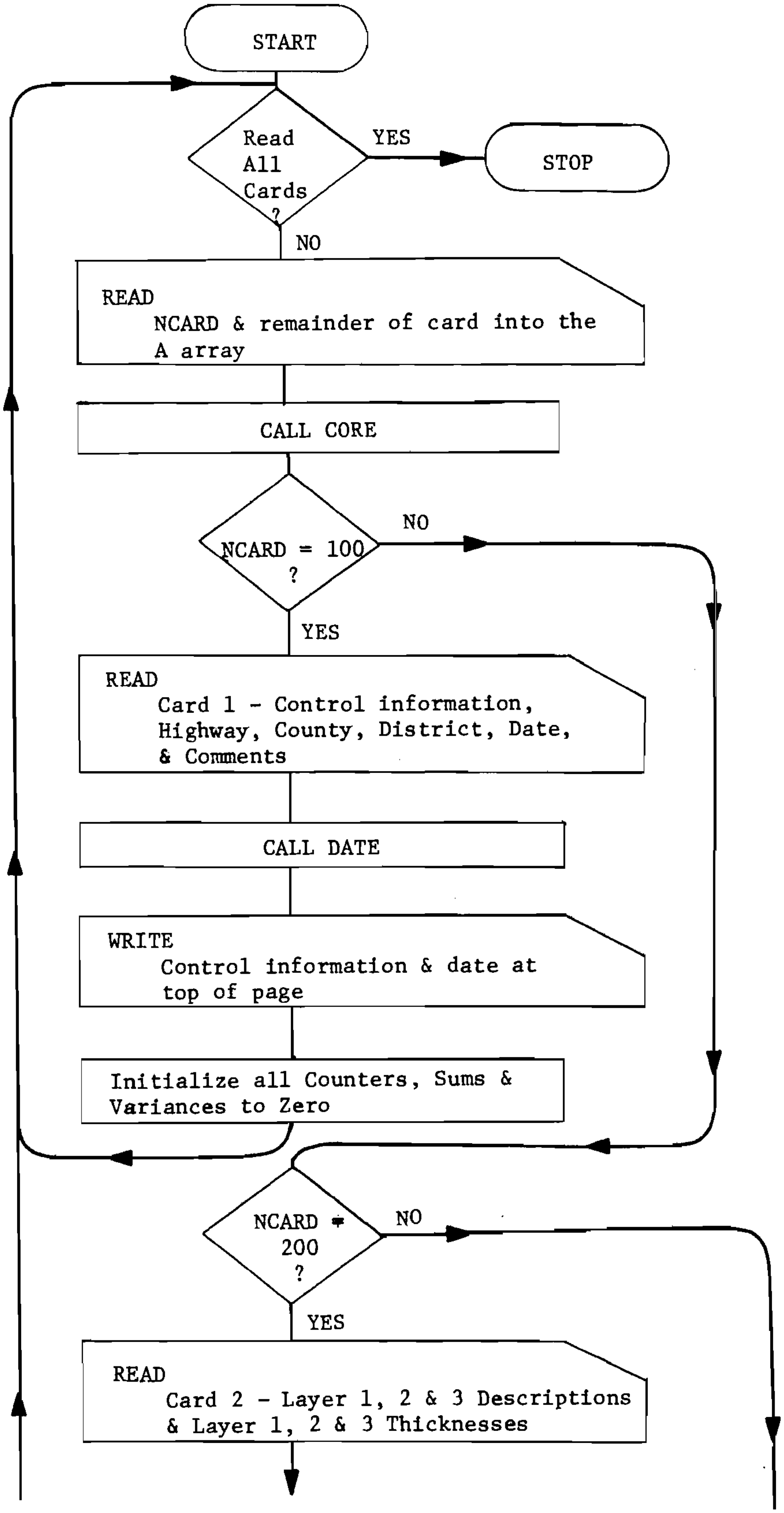
CARD 4

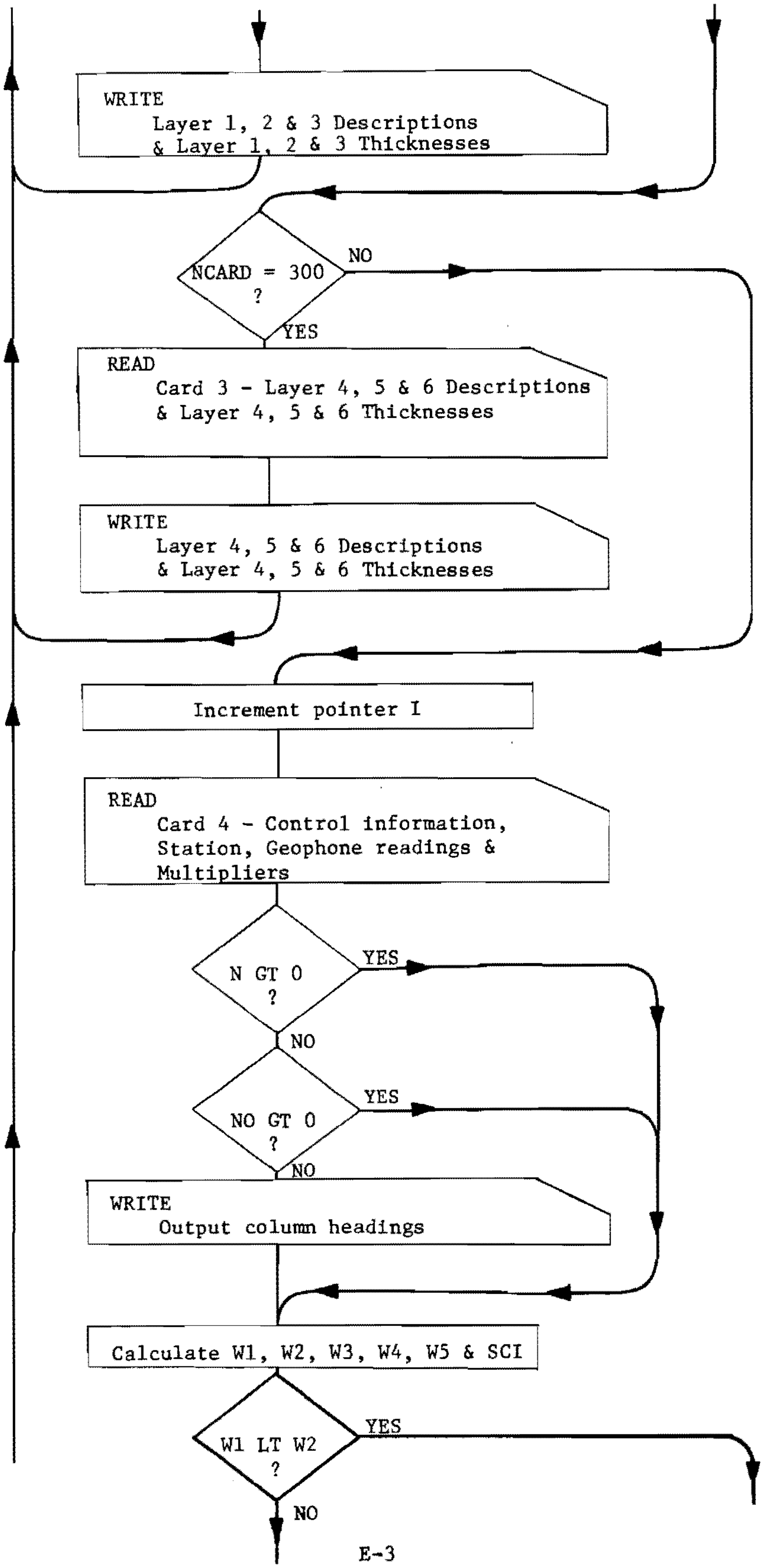


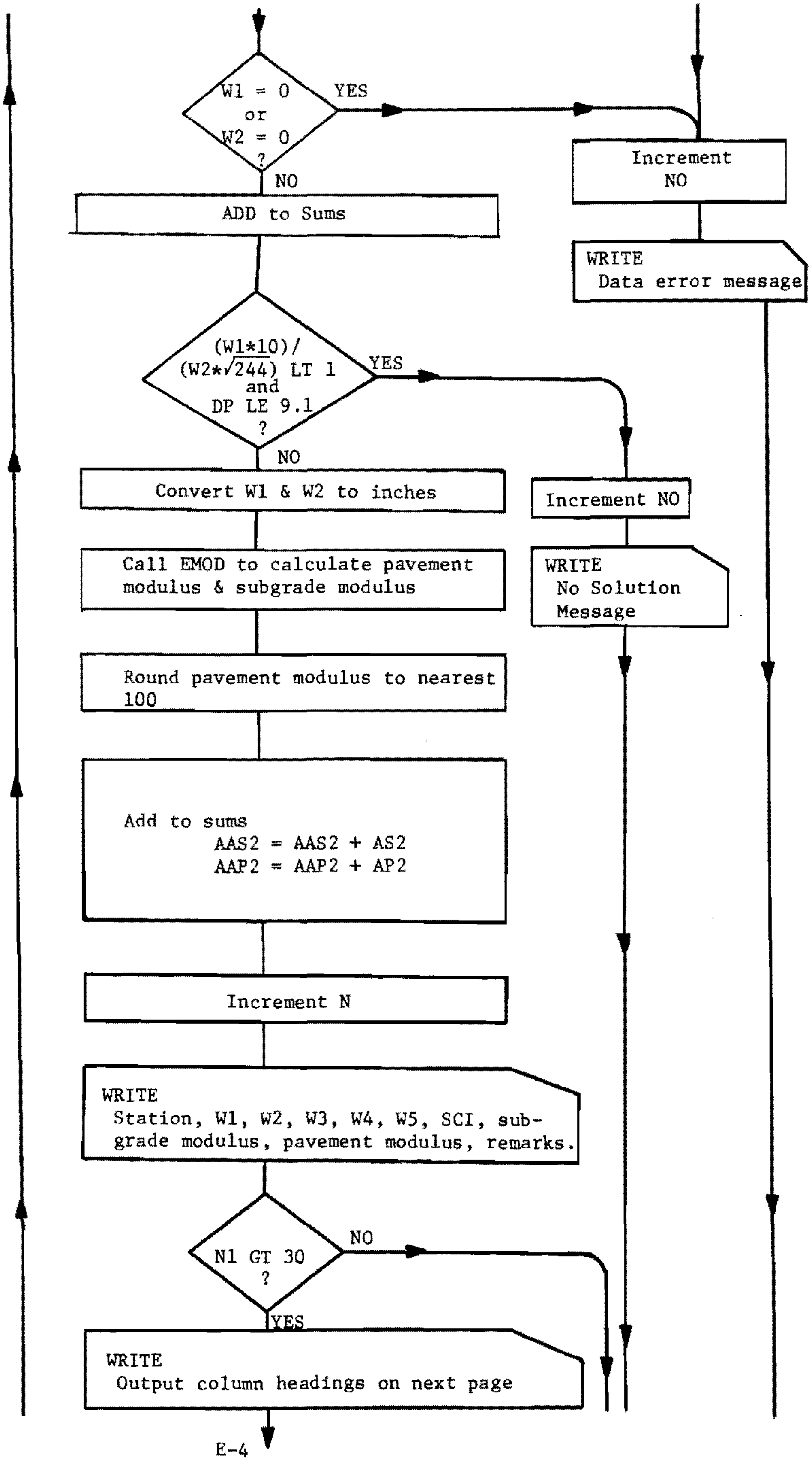
| | | |
|---|---|---|
| ICK | FORMAT I2 | Ex. 99 |
| (REM(I), I = 1,4) | FORMAT 4A4 | Ex. WEST BOUND LANE |
| D10 | FORMAT F3.2 | Ex. 010 |
| D9 | FORMAT F2.1 | Ex. 37 |
| D8 | FORMAT F3.2 | Ex. 010 |
| D7 | FORMAT F2.1 | Ex. 45 |
| D6 | FORMAT F3.2 | Ex. 010 |
| D5 | FORMAT F2.1 | Ex. 60 |
| D4 | FORMAT F3.2 | Ex. 030 |
| D3 | FORMAT F2.1 | Ex. 30 |
| D2 | FORMAT F3.2 | Ex. 030 |
| D1 | FORMAT F2.1 | Ex. 42 |
| STA | FORMAT A7 | Ex. 248+00 |
| IYEAR | FORMAT I2 | Ex. 70 |
| IDAY | FORMAT I2 | Ex. 7 |
| M | FORMAT I2 | Ex. 4 |
| ISECT | FORMAT I2 | Ex. 8 |
| ICONT | FORMAT I4 | Ex. 118 |
| NCARD | FORMAT I3 | Ex. 400 |

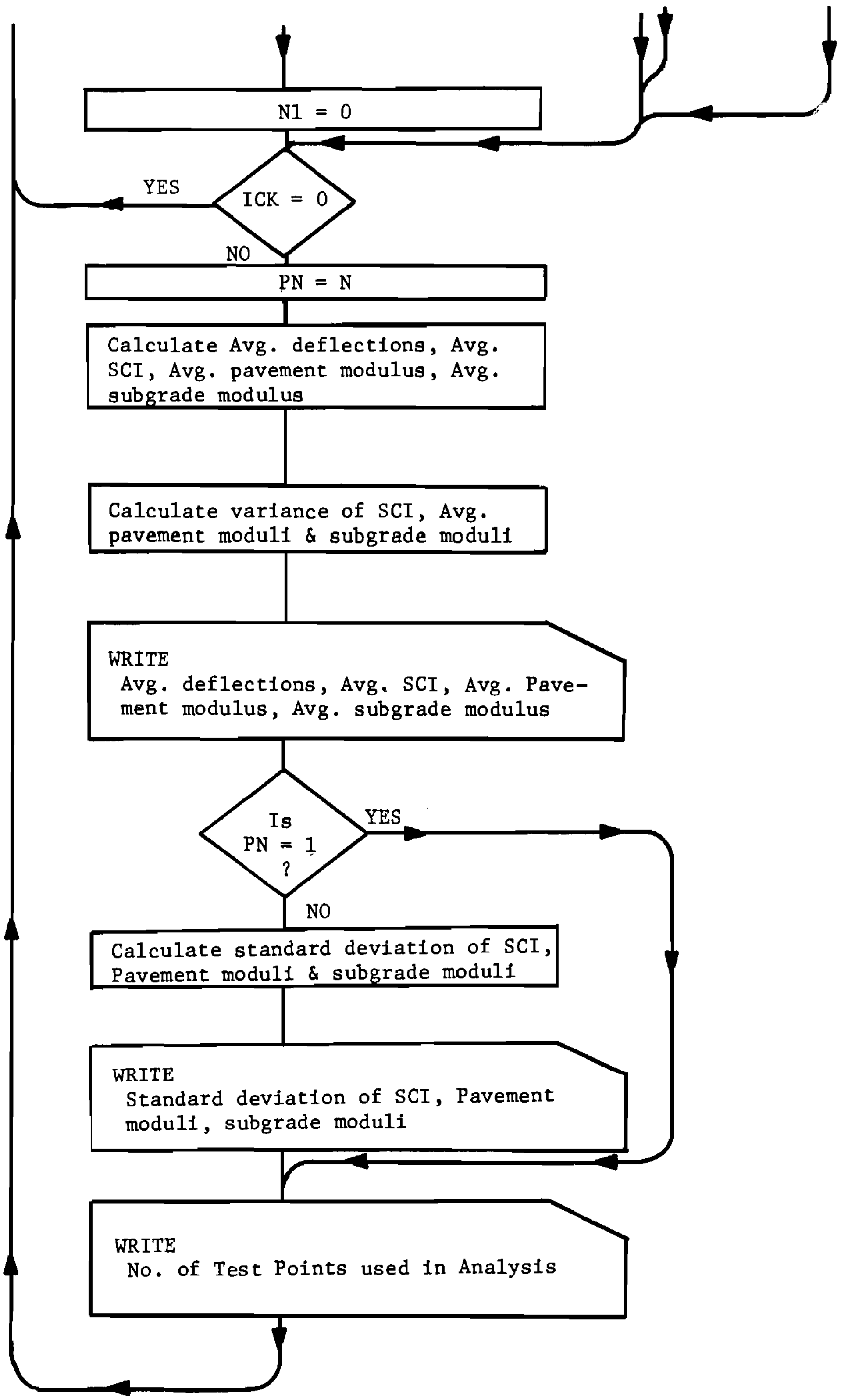
# Appendix 5

This appendix contains the flowchart of the procedure used in ELASTIC MODULUS to calculate a pavement or subgrade modulus.

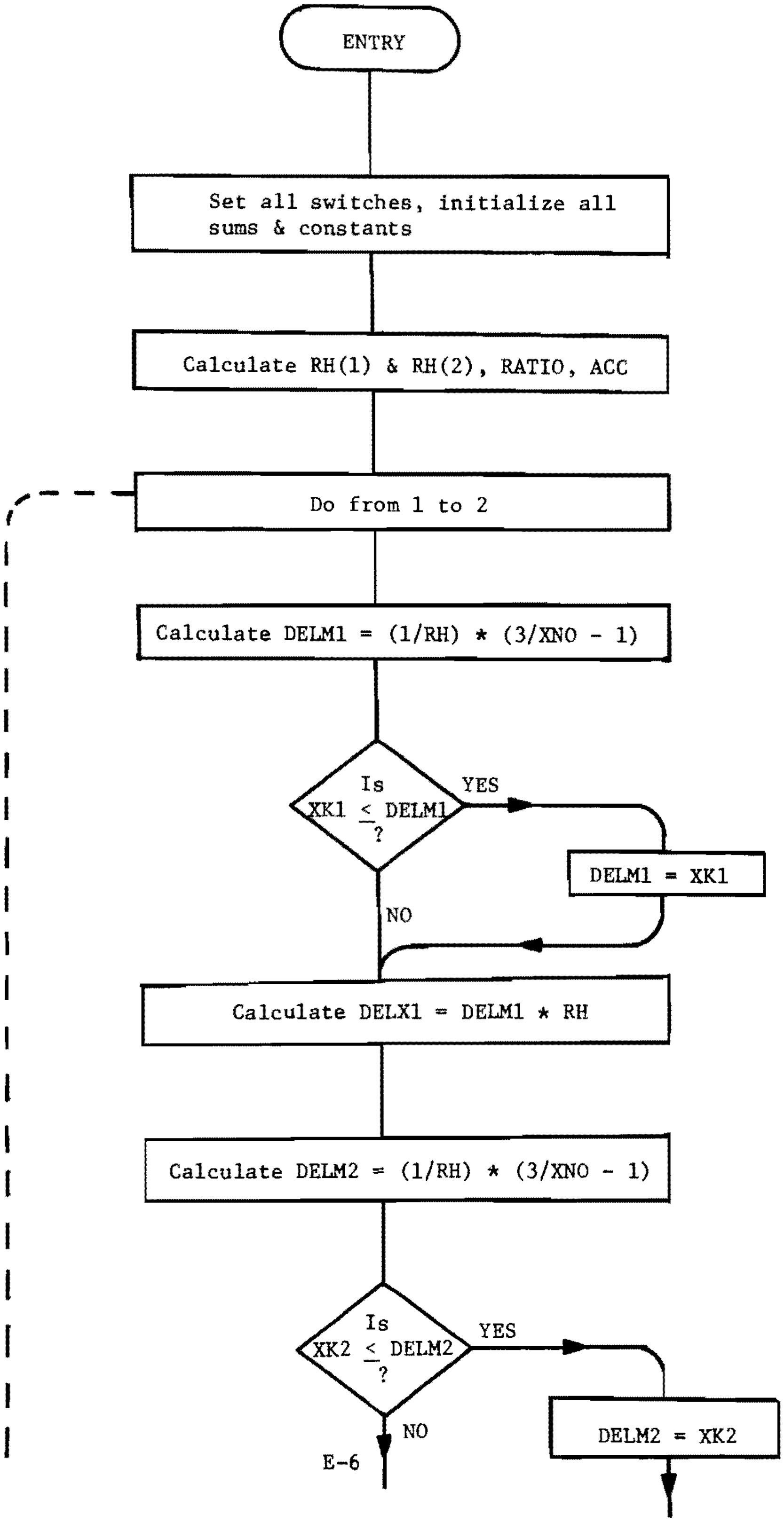

START
Read All Cards ?
YES
STOP
NO
READ
NCARD & remainder of card into the A array
CALL CORE
NCARD = 100 ?
NO
YES
READ
Card 1 - Control information, Highway, County, District, Date, & Comments
CALL DATE
WRITE
Control information & date at top of page
Initialize all Counters, Sums & Variances to Zero
NCARD = 200 ?
NO
YES
READ
Card 2 - Layer 1, 2 & 3 Descriptions & Layer 1, 2 & 3 Thicknesses

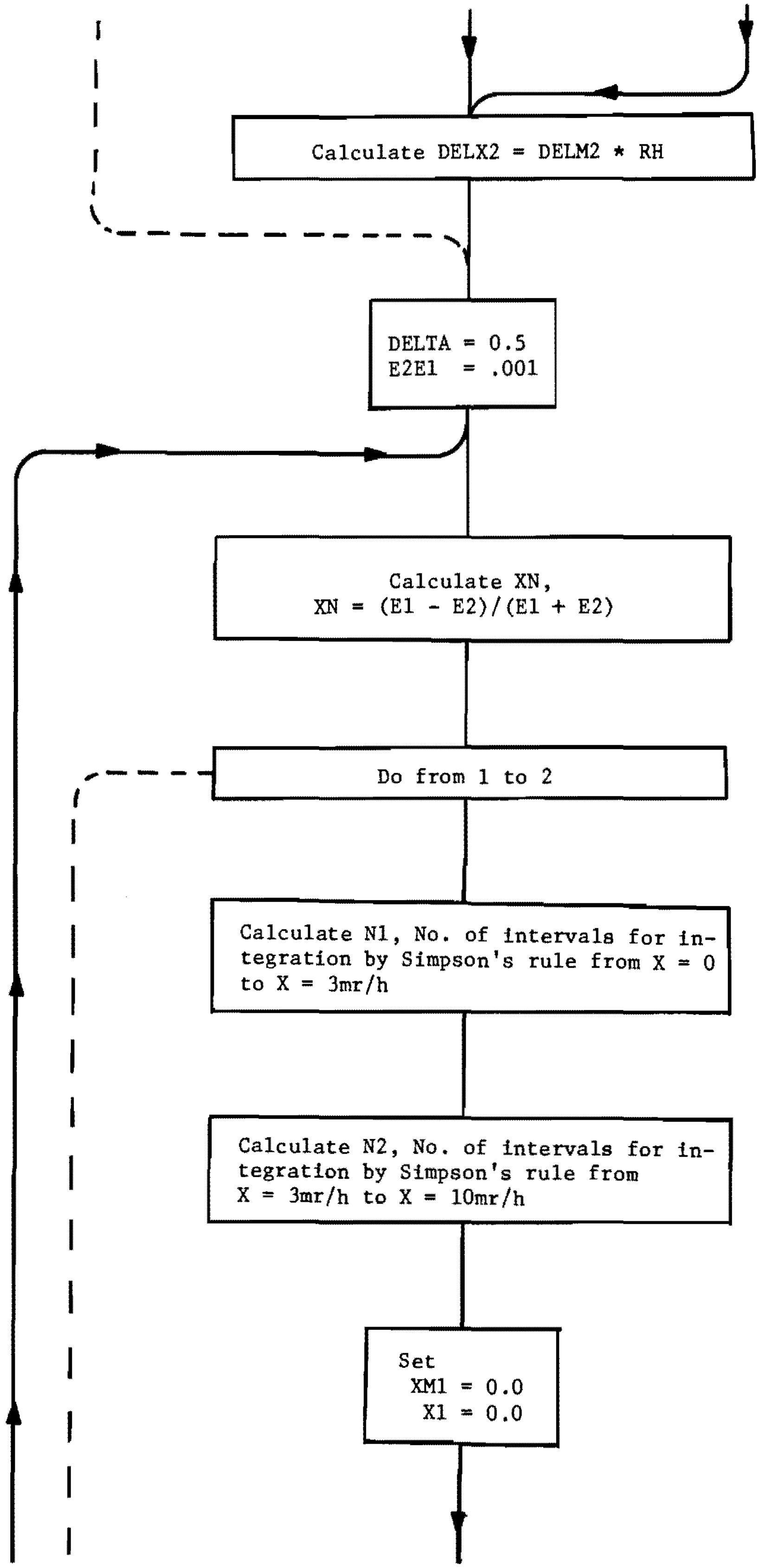

WRITE
Layer 1, 2 & 3 Descriptions
& Layer 1, 2 & 3 Thicknesses
NCARD = 300
?
NO
YES
READ
Card 3 - Layer 4, 5 & 6 Descriptions
& Layer 4, 5 & 6 Thicknesses
WRITE
Layer 4, 5 & 6 Descriptions
& Layer 4, 5 & 6 Thicknesses
Increment pointer I
READ
Card 4 - Control information,
Station, Geophone readings &
Multipliers
N GT 0
?
YES
NO
NO GT 0
?
YES
NO
WRITE
Output column headings
Calculate W1, W2, W3, W4, W5 & SCI
W1 LT W2
?
YES
NO

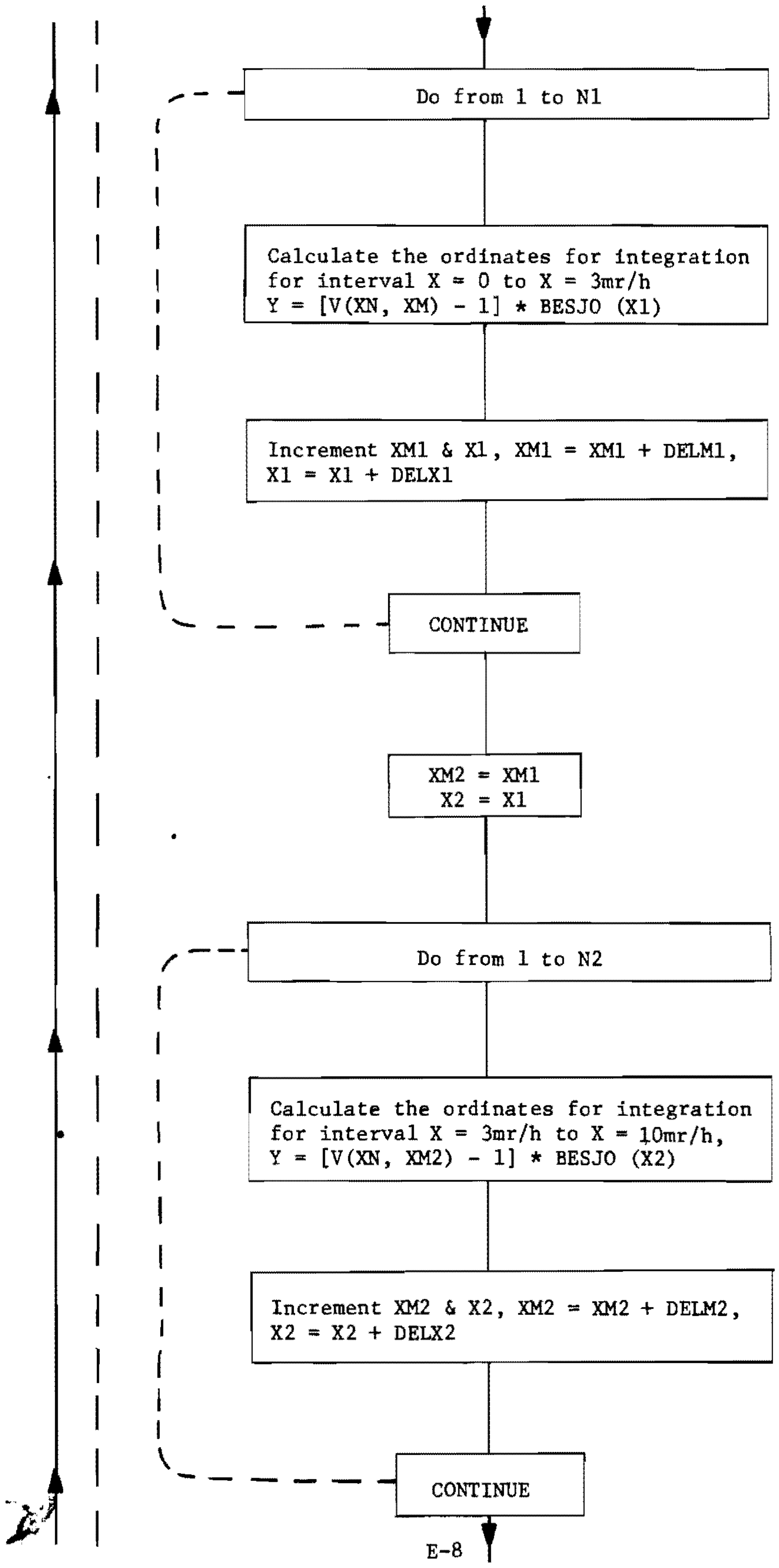

W1 = 0
or
W2 = 0
?
YES
NO
Increment
NO
ADD to Sums
WRITE
Data error message
(W1*10)/
(W2*√244) LT 1
and
DP LE 9.1
?
YES
NO
Convert W1 & W2 to inches
Increment NO
Call EMOD to calculate pavement
modulus & subgrade modulus
WRITE
No Solution
Message
Round pavement modulus to nearest
100
Add to sums
AAS2 = AAS2 + AS2
AAP2 = AAP2 + AP2
Increment N
WRITE
Station, W1, W2, W3, W4, W5, SCI, sub-
grade modulus, pavement modulus, remarks.
N1 GT 30
?
NO
YES
WRITE
Output column headings on next page

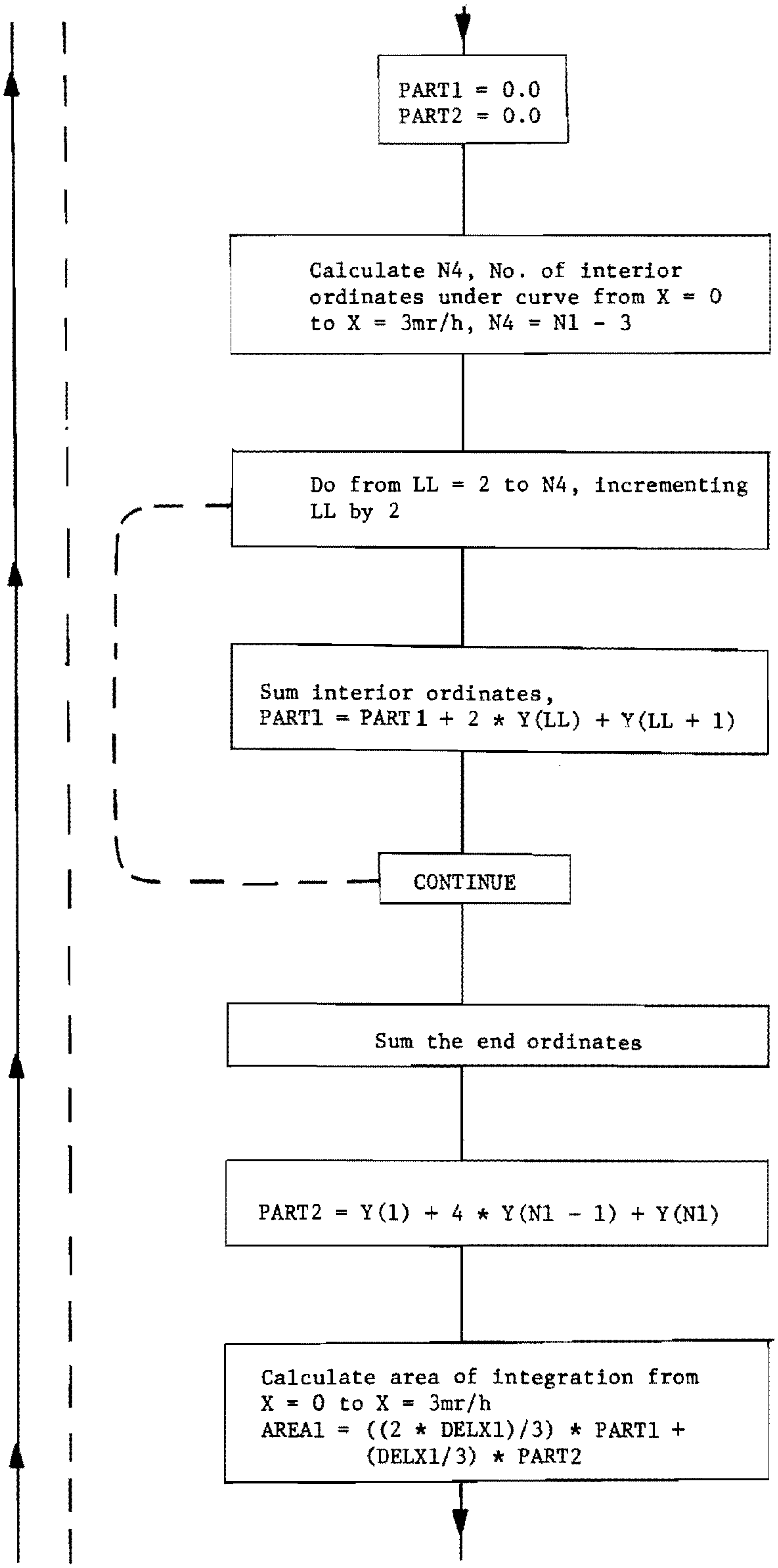

N1 = 0
YES
ICK = 0
NO
PN = N
Calculate Avg. deflections, Avg. SCI, Avg. pavement modulus, Avg. subgrade modulus
Calculate variance of SCI, Avg. pavement moduli & subgrade moduli
WRITE
Avg. deflections, Avg. SCI, Avg. Pavement modulus, Avg. subgrade modulus
Is PN = 1 ?
YES
NO
Calculate standard deviation of SCI, Pavement moduli & subgrade moduli
WRITE
Standard deviation of SCI, Pavement moduli, subgrade moduli
WRITE
No. of Test Points used in Analysis

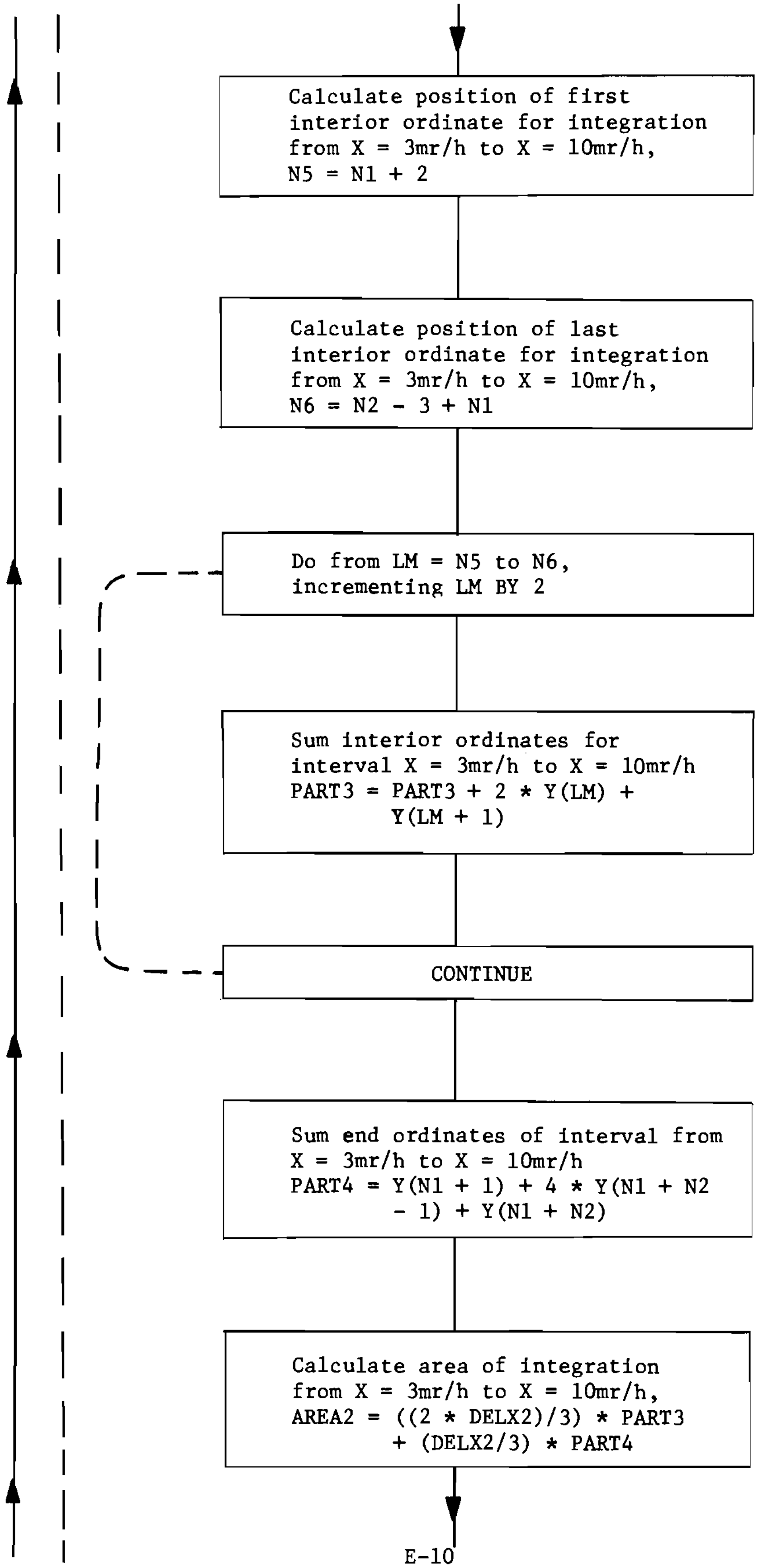
SUBROUTINE EMOD




Calculate DELX2 = DELM2 * RH
DELTA = 0.5
E2E1 = .001
Calculate XN,
XN = (E1 - E2)/(E1 + E2)
Do from 1 to 2
Calculate N1, No. of intervals for integration by Simpson's rule from X = 0 to X = 3mr/h
Calculate N2, No. of intervals for integration by Simpson's rule from X = 3mr/h to X = 10mr/h
Set
XM1 = 0.0
X1 = 0.0


Do from 1 to N1
Calculate the ordinates for integration for interval X = 0 to X = 3mr/h Y = [V(XN, XM) - 1] * BESJO (X1)
Increment XM1 & X1, XM1 = XM1 + DELM1, X1 = X1 + DELX1
CONTINUE
XM2 = XM1
X2 = X1
Do from 1 to N2
Calculate the ordinates for integration for interval X = 3mr/h to X = 10mr/h, Y = [V(XN, XM2) - 1] * BESJO (X2)
Increment XM2 & X2, XM2 = XM2 + DELM2, X2 = X2 + DELX2
CONTINUE


PART1 = 0.0
PART2 = 0.0
Calculate N4, No. of interior ordinates under curve from X = 0 to X = 3mr/h, N4 = N1 - 3
Do from LL = 2 to N4, incrementing LL by 2
Sum interior ordinates, PART1 = PART1 + 2 * Y(LL) + Y(LL + 1)
CONTINUE
Sum the end ordinates
PART2 = Y(1) + 4 * Y(N1 - 1) + Y(N1)
Calculate area of integration from X = 0 to X = 3mr/h
AREA1 = ((2 * DELX1)/3) * PART1 + (DELX1/3) * PART2



Calculate position of first interior ordinate for integration from X = 3mr/h to X = 10mr/h,
N5 = N1 + 2

Calculate position of last interior ordinate for integration from X = 3mr/h to X = 10mr/h,
N6 = N2 - 3 + N1

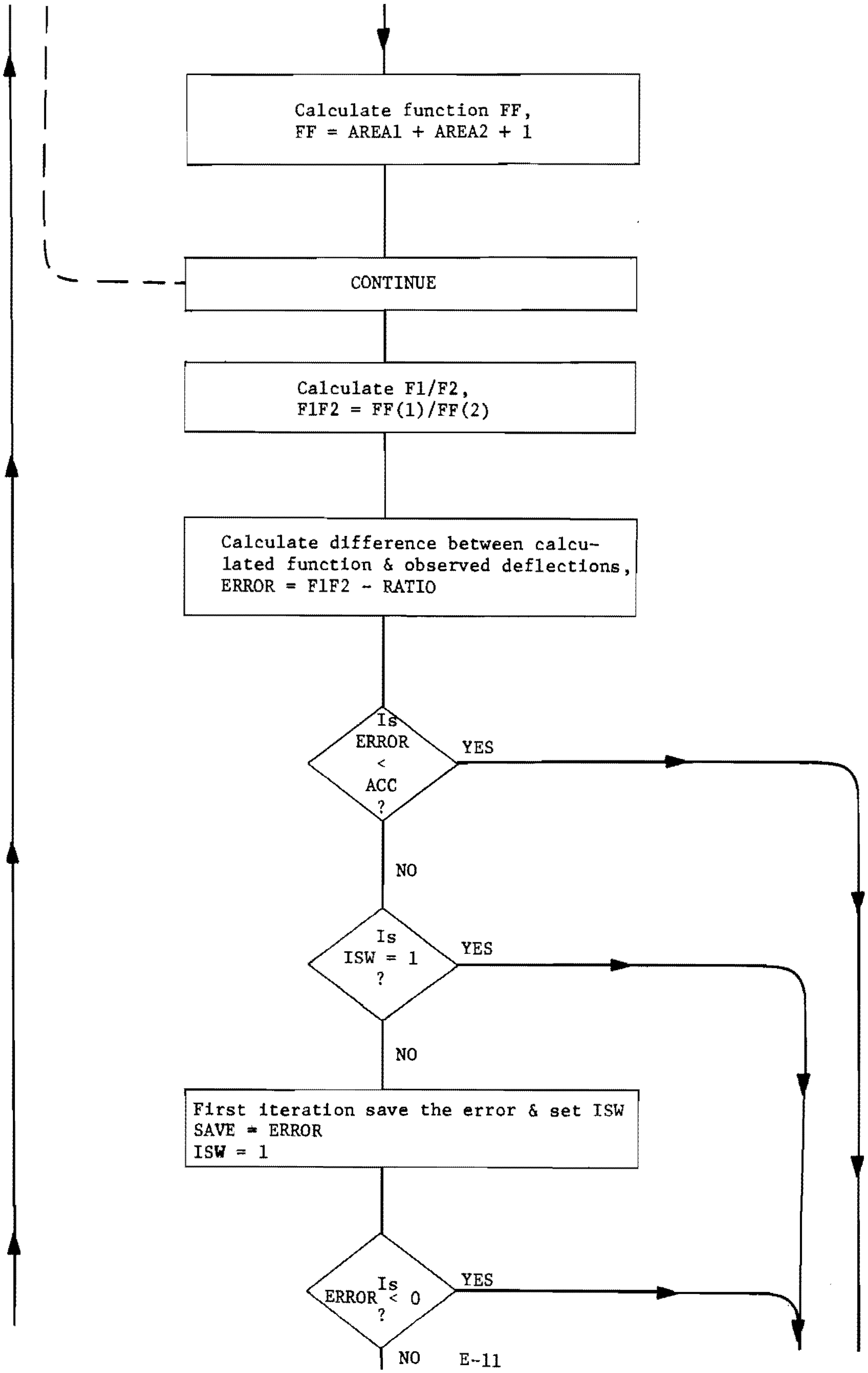
Do from LM = N5 to N6, incrementing LM BY 2

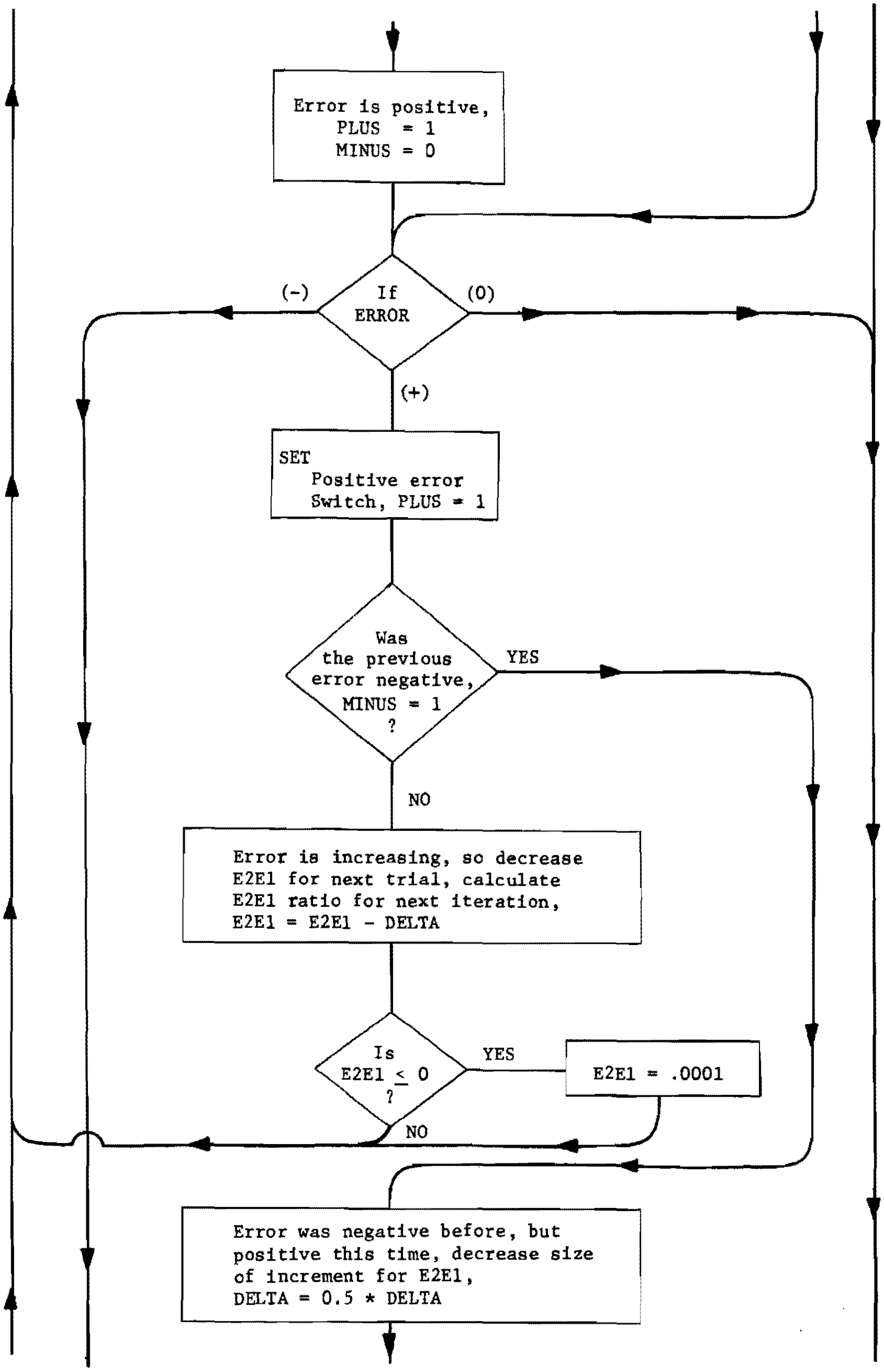
Sum interior ordinates for interval X = 3mr/h to X = 10mr/h
PART3 = PART3 + 2 * Y(LM) + Y(LM + 1)

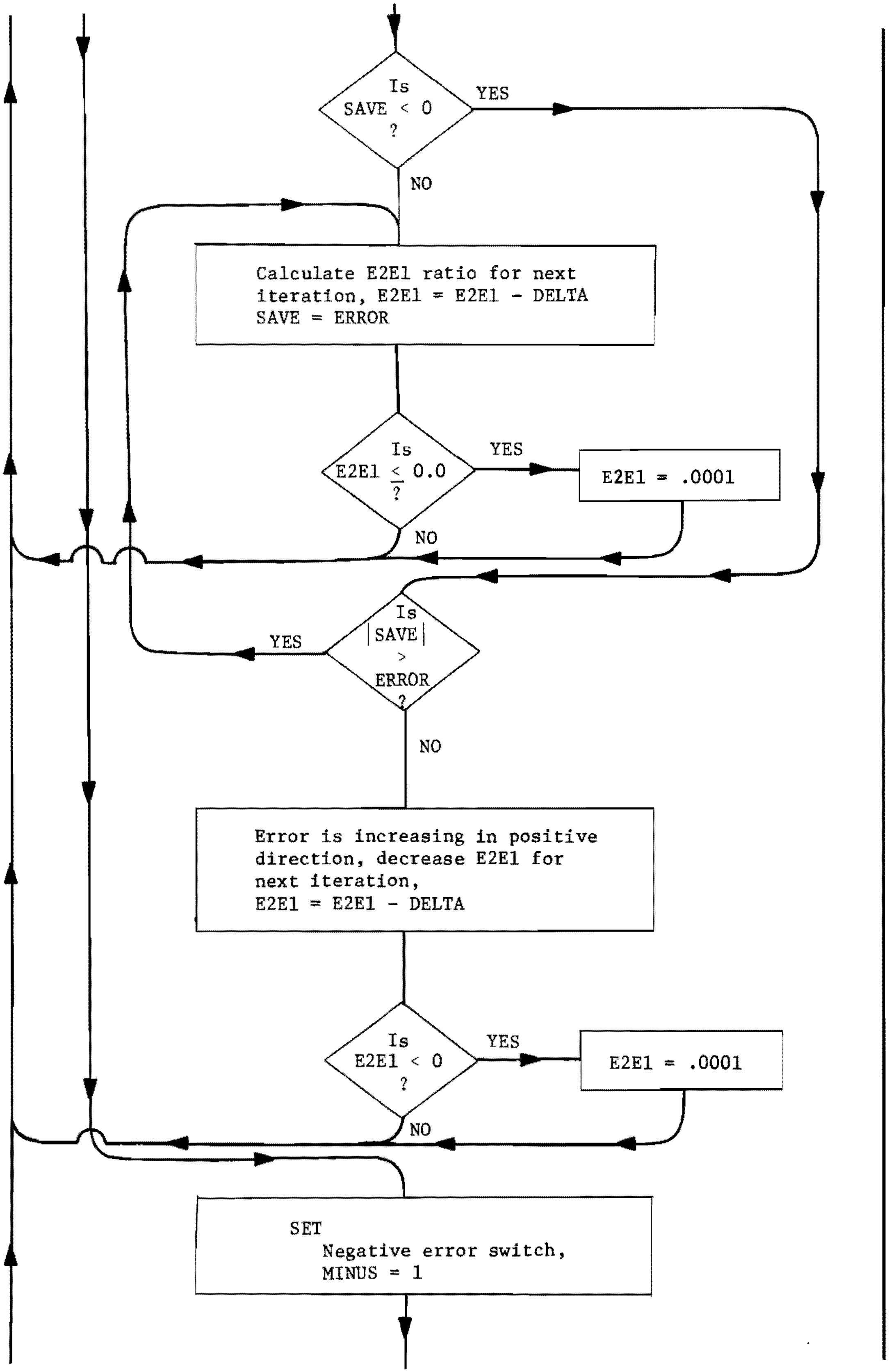
CONTINUE

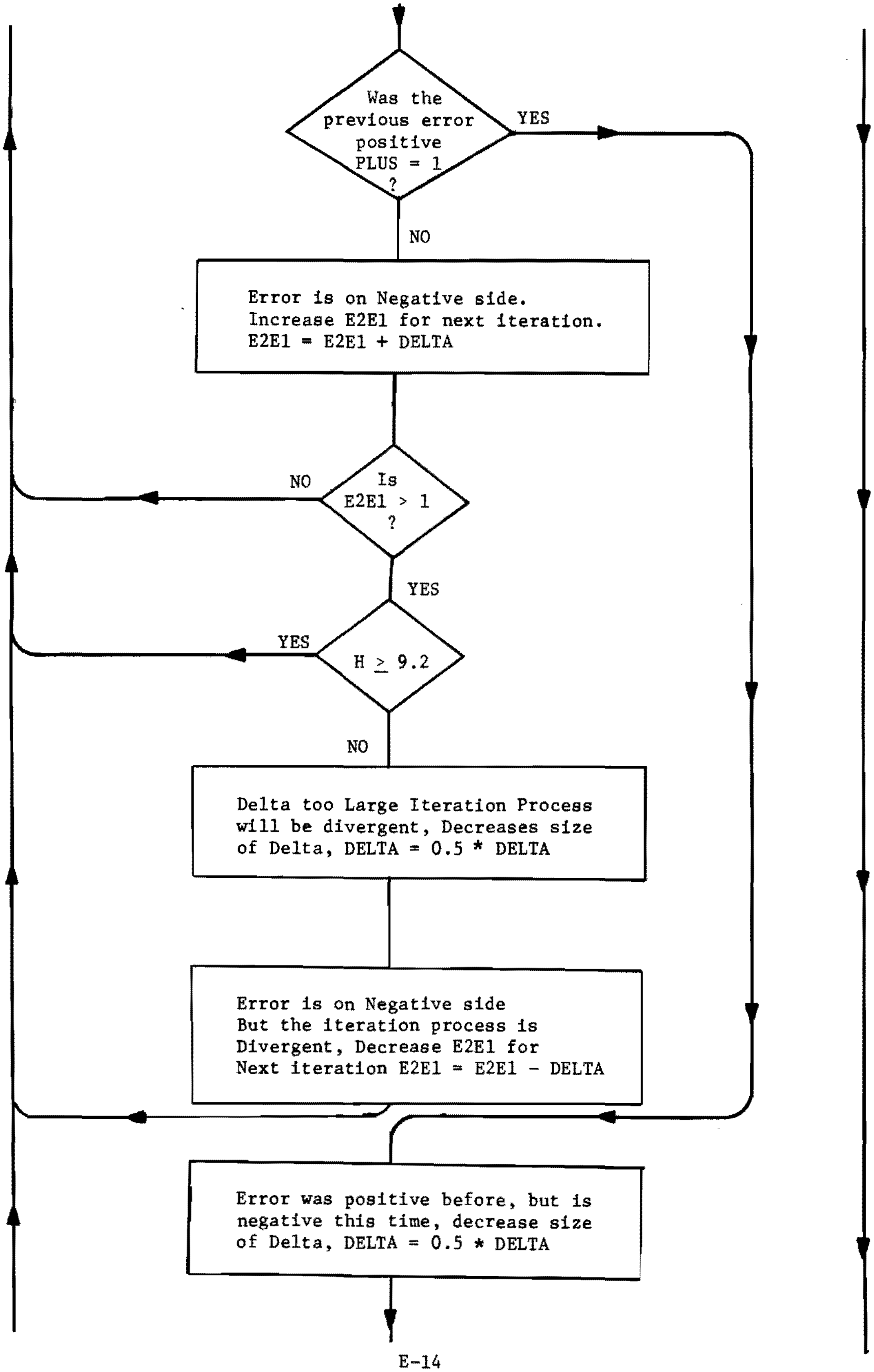
Sum end ordinates of interval from X = 3mr/h to X = 10mr/h
PART4 = Y(N1 + 1) + 4 * Y(N1 + N2 - 1) + Y(N1 + N2)

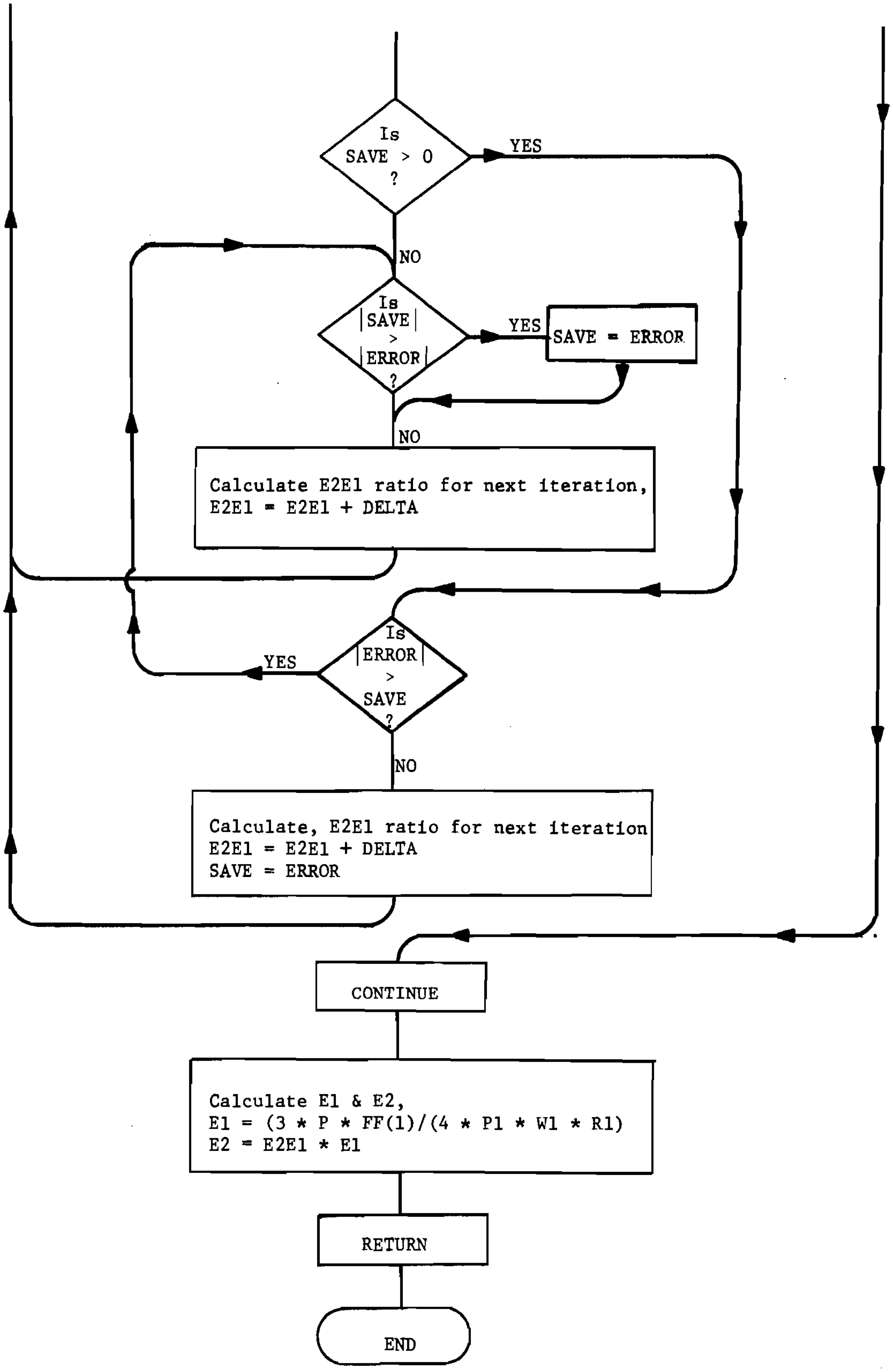
Calculate area of integration from X = 3mr/h to X = 10mr/h,
AREA2 = ((2 * DELX2)/3) * PART3 + (DELX2/3) * PART4


Calculate function FF,
FF = AREA1 + AREA2 + 1
CONTINUE
Calculate F1/F2,
F1F2 = FF(1)/FF(2)
Calculate difference between calculated function & observed deflections,
ERROR = F1F2 - RATIO
Is ERROR < ACC ?
YES
NO
Is ISW = 1 ?
YES
NO
First iteration save the error & set ISW
SAVE = ERROR
ISW = 1
Is ERROR < 0 ?
YES
NO

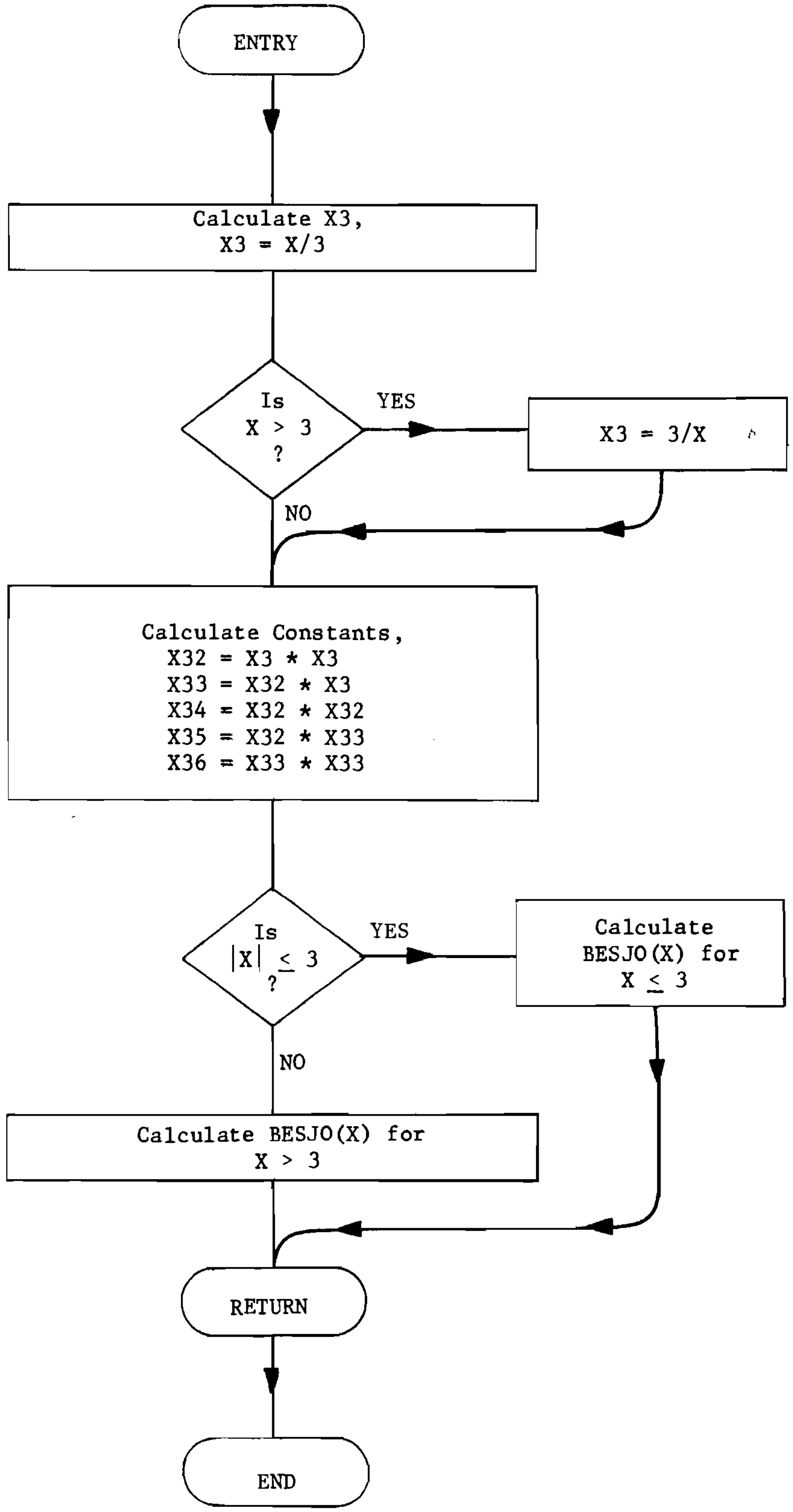

Error is positive,
PLUS = 1
MINUS = 0
(-)
If ERROR
(0)
(+)
SET Positive error Switch, PLUS = 1
Was the previous error negative, MINUS = 1 ?
YES
NO
Error is increasing, so decrease E2E1 for next trial, calculate E2E1 ratio for next iteration, E2E1 = E2E1 - DELTA
Is E2E1 ≤ 0 ?
YES
E2E1 = .0001
NO
Error was negative before, but positive this time, decrease size of increment for E2E1, DELTA = 0.5 * DELTA


Is
SAVE < 0
?
YES
NO
Calculate E2E1 ratio for next iteration, E2E1 = E2E1 - DELTA
SAVE = ERROR
Is
E2E1 ≤ 0.0
?
YES
E2E1 = .0001
NO
Is
|SAVE|
>
ERROR
?
YES
NO
Error is increasing in positive direction, decrease E2E1 for next iteration,
E2E1 = E2E1 - DELTA
Is
E2E1 < 0
?
YES
E2E1 = .0001
NO
SET
Negative error switch,
MINUS = 1

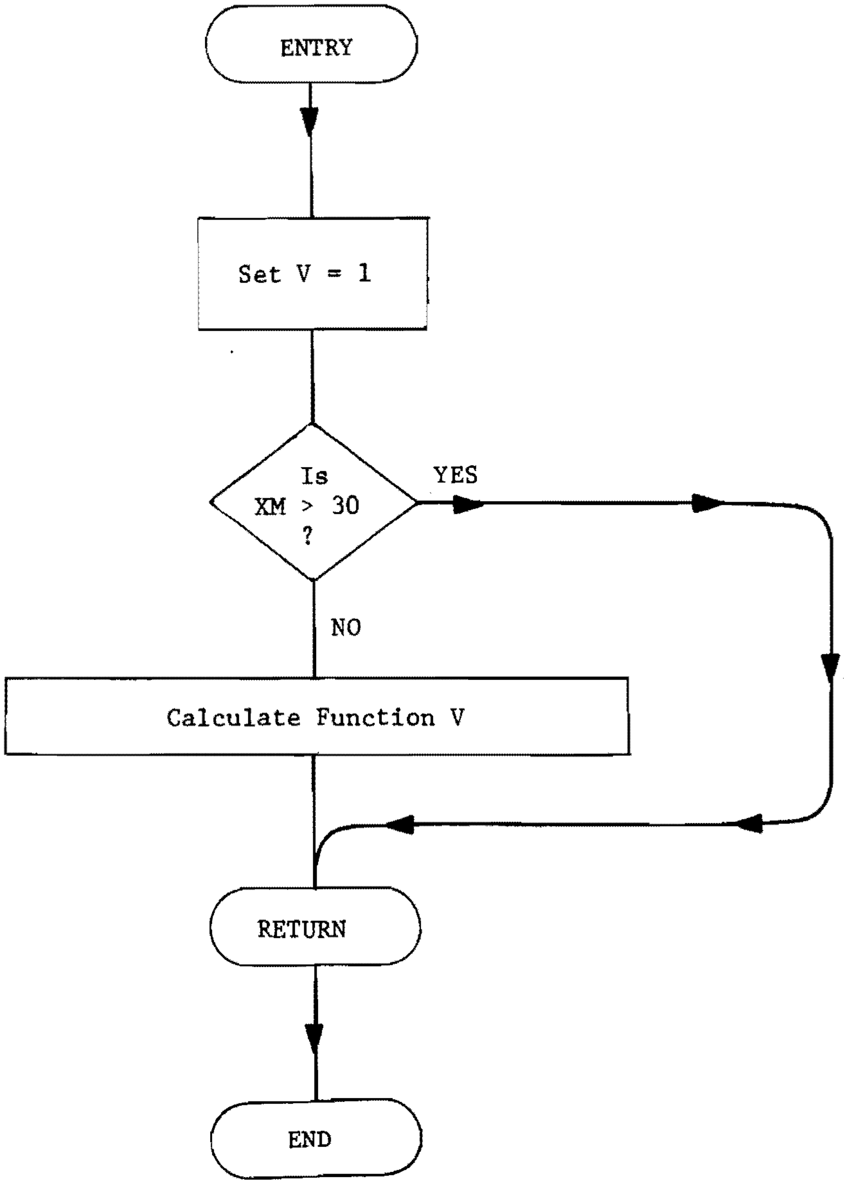

Was the previous error positive PLUS = 1 ?
YES
NO
Error is on Negative side. Increase E2E1 for next iteration. E2E1 = E2E1 + DELTA
NO
Is E2E1 > 1 ?
YES
YES
H ≥ 9.2
NO
Delta too Large Iteration Process will be divergent, Decreases size of Delta, DELTA = 0.5 * DELTA
Error is on Negative side But the iteration process is Divergent, Decrease E2E1 for Next iteration E2E1 = E2E1 - DELTA
Error was positive before, but is negative this time, decrease size of Delta, DELTA = 0.5 * DELTA


Is
SAVE > 0
?
YES
NO
Is
|SAVE|
>
|ERROR|
?
YES
SAVE = ERROR
NO
Calculate E2E1 ratio for next iteration,
E2E1 = E2E1 + DELTA
Is
|ERROR|
>
SAVE
?
YES
NO
Calculate, E2E1 ratio for next iteration
E2E1 = E2E1 + DELTA
SAVE = ERROR
CONTINUE
Calculate E1 & E2,
E1 = (3 * P * FF(1)/(4 * P1 * W1 * R1)
E2 = E2E1 * E1
RETURN
END

FUNCTION BESJO



FUNCTION V



# Appendix 6

On the following pages is a listing of ELASTIC MODULUS with the output from a sample problem. The input data cards for the sample problem are listed after the program output.

```
            C
            C
            C     ELASTIC MODULII  --  MAIN PROGRAM
            C
            C
 0001             DIMENSION STA(200),W1(200),W2(200),W3(200),W4(200),
                 *  AP2(200),LA1(5),LA2(5),LA3(5),LA4(5),LA5(5),LA6(5),
                 * W5(200), AS2(200), A(20), SCI(200),
                 *  IXDATE(3),COMM(7),REM(4)
 0002             REAL * 8 STA, DAS , DAP, DBLE
            C
            C
            C     NOTE -- THE PRINT & FORMAT STATEMENTS ARE FOR
            C     OUTPUT ON 8 1/2 X 11 PAPER.  FOR OUTPUT ON 11 X 14
            C     PAPER USE THE PRINT & FORMAT STATEMENTS WITH 'C' IN
            C     COLUMN 1.
            C
            C
            C
            CCC   STATEMENT FUNCTION TO ROUND 'X' TO NEAREST 'EVEN'
            C
 0003             ROUND( X, EVEN )  =  AINT( ( X + EVEN * .5 ) / EVEN  )
                 * *  EVEN
            C
 0004          10 CONTINUE
            C
            C     READ CARD CODE & REMAINDER OF CARD INTO A - ARRAY
            C
 0005             READ(5,1,END=1000)  NCARD, ( A(I), I = 1 , 20 )
            C
 0006           1 FORMAT( I3, 19A4, A1 )
 0007             CALL  CORE  ( A,  80 )
            C
            C     TEST FOR DATA CARD 1
            C
 0008             IF(NCARD.EQ.100) GO TO 11
            C
            C     TEST FOR DATA CARD 2
            C
 0009             IF(NCARD.EQ.200) GO TO 12
            C
            C     TEST FOR DATA CARD 3
            C
 0010             IF(NCARD.EQ.300) GO TO 13
            C
            C     I IS A POINTER TO DATA IN STORAGE
            C
 0011          14 I=N+1
```

```
               C
               C      READ DATA CARD 4
               C
 0012                 READ(5,6) ICONT,ISECT,M,IDAY,IYEAR,STA(I),D1,D2,D3,
                     * D4, D5, D6, D7, D8,
                     *D9,D10,(REM(J),J=1,4),ICK
               C
 0013               6 FORMAT(    I4,4I2,A7,3X, 5(F2.1,F3.2),8X,4A4,I2)
               C
 0014                 IF( N.GT. 0 ) GO TO 555
 0015                 IF(NO.GT.0) GO TO 555
               C
               C      PRINT OUTPUT COLUMN HEADINGS
               C
 0016                 PRINT 61
               C   61 FORMAT(/,1X,'STATION    W1     W2     W3     W4     ',
               C     = 'W5     SCI    ** ES   **  ** EP   **     REMARKS' / )
 0017              61 FORMAT(/ 7X,'STATION    W1    W2     W3     W4     W5',
                     = '    SCI   ** ES   ** ** EP   **  REMARKS'/)
               C
               C      CALCULATE DEFLECTIONS & SCI ( DEFLECTIONS IN MILS )
               C
 0018             555 W1(I)=D1*D2
 0019                 W2(I)=D3*D4
 0020                 W3(I)=D5*D6
 0021                 W4(I)=D7*D8
 0022                 W5(I)=D9*D10
 0023                 SCI(I)=W1(I)-W2(I)
               C
               C      TEST FOR W1 OR W2 = 0, AND W1 LESS THAN W2
               C
 0024                 IF (W1(I).EQ.0.OR.W2(I).EQ.0) GO TO 64
 0025                 IF(W1(I).LT.W2(I)) GO TO 66
               C
 0026                 AW1 =AW1 +W1 (I)
 0027                 AW2 =AW2 +W2 (I)
 0028                 AW3 =AW3 +W3(I)
 0029                 AW4 =AW4 +W4(I)
 0030                 AW5 =AW5 +W5(I)
 0031                 ASCI=ASCI+SCI(I)
 0032                 AS2(I)  =  0.0
 0033                 AP2(I)  =  0.0
               C
               C
 0034                 IF( ( W1(I) * 10.0 ) / ( W2(I) * SQRT( 244.0 ) )
                     =  .GT.  1.0  .AND.  DP  .LE.  9.1 )  GO  TO  60
               C
               C
```

```
               C      CONVERT W1 & W2 TO INCHES
               C
 0035                 W1(I)  =  W1(I) / 1000.
 0036                 W2(I)  =  W2(I) / 1000.
               C
               C      PASS W1, W2, & TOTAL PAVEMENT THICKNESS TO EMOD,
               C      EMOD RETURNS UNROUNDED VALUES OF PAVEMENT & SUBGRADE
               C      MODULII AS DAP & DAS
               C
 0037                 CALL EMOD ( DBLE(W1(I)), DBLE(W2(I)), DBLE(DP),DAP, DAS)
               C
               C      CONVERT W1 & W2 TO MILS
               C
 0038                 W1(I) = W1(I) * 1000.
 0039                 W2(I) = W2(I) * 1000.
               C
               C
               C
               C
               C      ROUND PAVEMENT & SUBGRADE MODULII TO NEAREST 100
               C
 0040                 DAS  =  ROUND( DAS, 100. )
 0041                 DAP  =  ROUND( DAP, 100. )
               C
               C      PUT PAVEMENT & SUBGRADE MODULII IN STORAGE
               C
 0042                 AS2(I)  =  DAS
 0043                 AP2(I)  =  DAP
               C
               C      ADD TO THE SUMS OF THE DEFLECTIONS, SCI, PAVEMENT,
               C      AND SUBGRADE MODULII
               C
 0044                 AAS2=AAS2+AS2(I)
 0045                 AAP2=AAP2+AP2(I)
               C
               C      ADD TO N, THE NUMBER OF VALID TEST POINTS
               C
 0046                 N=N+1
               C
               C      PRINT A LINE OF OUTPUT
               C
               C      PRINT 63,STA(I),W1(I),W2(I),W3(I),W4(I),W5(I),SCI(I),
               C     = AS2(I), AP2(I), ( REM(J),J=1,4 )
               C   63 FORMAT(1X, A7, 3X, 5(F5.3,2X ), F5.3,2F11.0,5X,4A4 )
 0047                 PRINT 63,STA(I),W1(I),W2(I),W3(I),W4(I),W5(I),SCI(I),
                     * AS2(I), AP2(I), (REM(J),J=1,2)
 0048              63 FORMAT( 7X,A7,1X,6(F6.3), 2F10.0,2X, 2A4  )
 0049                 N1 = N1 + 1
```

```
0050                  IF(N1.LT.30) GO TO 88
                C
                C     SKIP TO NEXT PAGE & PRINT OUTPUT COLUMN HEADINGS IF
                C     THIRTY LINES HAVE BEEN PRINTED
                C
0051               84 CONTINUE
0052                  PRINT 51
                C     PRINT 56
                C     PRINT 57,IDIST,CO1,CO2,CO3,CO4,ICONT,ISECT,IJOB,HWY1,
                C    * HWY2, XLANE, M, IDAY, IYEAR, IDYNA
0053                  PRINT 56, IDIST, CO1, CO2, CO3, CO4
0054               56 FORMAT( T35,'DIST.       COUNTY'/ T36, I2,9X, 3A4,A2 /)
0055                  PRINT 57, ICONT, ISECT,IJOB,HWY1,HWY2,M,IDAY,IYEAR,IDYNA
0056               57 FORMAT( T19, 'CONT.   SECT.   JOB   HIGHWAY    DATE',
                     '      DYNAFLECT' / T19,I4,2I7,4X,A4,A3,I4,2('-',I2),I9 /)
0057                  PRINT 61
0058                  N1 = 0
0059               88 CONTINUE
                C
                C     CHECK FOR LAST DATA CARD 4
                C
0060                  IF ( ICK.EQ.0) GO TO 10
0061                  GO TO 80
                C
                C     READ DATA CARD 1
                C
0062               11 READ(5,2) IDIST,CO1,CO2,CO3,CO4,ICONT,ISECT,IJOB,HWY1,
                     * HWY2, XLANE, DP,M,IDAY,IYEAR,IDYNA,(COMM(I),I=1,7)
0063                2 FORMAT(    I2,3A4,A2,I4,2I2,A4,A3,A3,F5.2,4I2,7A4)
                C
                C     PRINT HEADING
                C
0064                  PRINT 51
0065               51 FORMAT( '1' )
                C
0066                  PRINT 52
                C  52 FORMAT(33X,'TEXAS HIGHWAY DEPARTMENT',/)
0067               52 FORMAT(35X,'TEXAS HIGHWAY DEPARTMENT' /)
                C
0068                  PRINT 53,IDIST
                C  53 FORMAT(31X,'DISTRICT ',I2,' - DESIGN SECTION',/)
0069               53 FORMAT(33X,'DISTRICT ',I2,' - DESIGN SECTION' /)
                C
0070                  PRINT 54
                C  54 FORMAT(16X,'DYNAFLECT DEFLECTIONS AND CALCULATED ',
                C    = 'ELASTIC MODULII'  /  )
0071               54 FORMAT(21X,'DYNAFLECT DEFLECTIONS AND CALCULATED ',
                     * 'ELASTIC MODULII' / )
```

```
C
C     GET CURRENT DATE
C
      CALL DATE ( IXDATE(1), IXDATE(2), IXDATE(3)   )
C
      PRINT 55,IXDATE
C  55 FORMAT(30X,'THIS PROGRAM WAS RUN - ', 2A3,A2  /    )
   55 FORMAT(32X,'THIS PROGRAM WAS RUN - ', 2A3,A2 /  )
C
C     PRINT 56
C  56 FORMAT( 1X,'DIST.       COUNTY       CONT.    SECT.',
C    *'   JOB   HIGHWAY    DATE      DYNAFLECT')
      PRINT 56, IDIST, C01, C02, C03, C04
C
C     PRINT CONTROL INFORMATION FROM DATA CARD 1
C
C     PRINT 57,IDIST,C01,C02,C03,C04,ICONT,ISECT,IJOB,HWY1,
C    * HWY2, XLANE, M, IDAY, IYEAR, IDYNA
C  57 FORMAT( 2X,I2,5X,3A4,A2,3X,I4,4X,I2,5X,I2,2X,A4,A3,
C    * A3, 2X, I2, '-', I2, '-', I2, 6X, I2 / )
      PRINT 57, ICONT, ISECT,IJOB,HWY1,HWY2,M,IDAY,IYEAR,IDYNA
C
      PRINT 58,(COMM(I),I=1,7),DP
   58 FORMAT(10X,7A4,2X,'PAV. THICK. = ',F5.2,' INCHES',/)
C
C     INITIALIZE ALL SUMS & COUNTERS
C
       N=0
      N1 = 0
      NO = 0
      AW1= 0.
      AW2=0.
      AW3=0.
      AW4=0.
      AW5=0.
      ASCI=0.
      AAS2=0.
      AAP2=0.
      SR1= 0.
      SR2= 0.
      SR3= 0.
C
      GO TO 10
C     READ & PRINT INFORMATION ON DATA CARD 2
C
   12 READ(5,3) (LA1(I),I=1,5),T1,(LA2(I),I=1,5),T2,
     * (LA3(I),I=1,5), T3
    3 FORMAT(     5A4,F4.2,5A4,F4.2,5A4,F4.2)
```

```
C         PRINT 59,(LA1(I),I=1,5),T1,(LA2(I),I=1,5),T2,
C       * (LA3(I),I=1,5), T3
C      59 FORMAT(  1X,5A4,1X,F5.2,2X,5A4,1X,F5.2,2X,5A4,1X,F5.2)
          PRINT 59,(LA1(I),I=1,5),T1,(LA2(I),I=1,5),T2
          PRINT 59, ( LA3(I), I=1,5), T3
       59 FORMAT(16X, 5A4, 1X, F5.2, 5X, 5A4, 1X, F5.2/)
          GO TO 10
C
C         READ & PRINT INFORMATION ON DATA CARD 3, IF PRESENT
C
       13 READ(5,3)  (LA4(I),I=1,5),T4,(LA5(I),I=1,5),T5,
        * (LA6(I),I=1,5), T6
C         PRINT 59,(LA4(I),I=1,5),T4,(LA5(I),I=1,5),T5,
C       * (LA6(I),I=1,5), T6
          PRINT 59,(LA4(I),I=1,5),T4,(LA5(I),I=1,5),T5
          PRINT 59, ( LA6(I), I=1,5), T6
          GO TO 10
       66 NO = NO+1
C
C         PRINT NEGATIVE SCI MESSAGE
C
          PRINT 82,STA(I),W1(I),W2(I),(REM(J),J=1,4)
       82 FORMAT(1X,A7,3X,F5.3,2X,F5.3,2X,'NEGATIVE SCI OTHER ',
        * 'CALCULATIONS OMITTED', 4X, 4A4)
C
          N1=N1+1
          IF( N1  .LT.  30 )  GO  TO  88
          GO  TO  84
       64 NO = NO + 1
C
C         PRINT ERROR MESSAGE
C
          PRINT 81,STA(I),(REM(J),J=1,4)
       81 FORMAT( 1X,A7,3X,'DATA ERROR ASSUMED A ZERO VALUE RE',
        * 'AD FOR W1 OR W2', 5X, 4A4 )
C
          N1 = N1+ 1
          IF( N1  .LT.  30 )  GO  TO  88
          GO  TO  84
C
C
       60 CONTINUE
          N  =  N  +  1
          NO  =  NO  +  1
          PRINT 85,  STA(I), W1(I), W2(I), W3(I), W4(I),
            W5(I),  SCI(I)
       85 FORMAT( 7X, A7, 1X, 6F6.3, 2X,
            'NO UNIQUE SOLUTION'   )
```

```
 0121                N1  =  N1  +  1
 0122                IF( N1  .LT.  30 )  GO  TO  88
 0123                GO  TO  84
             C
             C
             C
             C       ALL CARDS READ FOR AN ANALYSIS, CALCULATE AVERAGE
             C       DEFLECTIONS, AVERAGE SCI, AVERAGE PAVEMENT MODULUS,
             C       AND AVERAGE SUBGRADE MODULUS
             C
 0124             80 PN=N
 0125                N1  =  N  -  NO
 0126                IF( N1  .LE.  0 )  N1  =  1
 0127                AW1V= AW1/PN
 0128                AW2V= AW2/PN
 0129                AW3V= AW3/PN
 0130                AW4V= AW4/PN
 0131                AW5V= AW5/PN
 0132                ASCIV=ASCI/PN
 0133                AAS2V=AAS2/N1
 0134                AAP2V=AAP2/N1
             C
             C       CALCULATE VARIANCE OF SCI, SUBGRADE MODULUS & PAVE-
             C       MENT MODULUS
             C
 0135                DO 62 I=1,N
 0136                IF(W1(I).EQ.0.OR.W2(I).EQ.0) GO TO 62
 0137                SR1 = SR1 + ((ASCIV- SCI(I))**2)
 0138                IF( AS2(I)  .EQ.  0.0)  GO TO 62
 0139                SR2= SR2+((AAS2V-AS2(I))**2)
 0140                SR3= SR3+((AAP2V- AP2(I))**2)
 0141             62 CONTINUE
             C
             C       PRINT AVERAGES
             C
 0142                PRINT 65,AW1V,AW2V,AW3V,AW4V,AW5V,ASCIV,AAS2V,AAP2V
             C    65 FORMAT(/1X,'AVERAGES', 6(2X, F5.3 ), 2F11.0    )
 0143             65 FORMAT(/ 7X, 'AVERAGES', 6(F6.3), 2F10.0  )
             C
             C       CALCULATE STANDARD DEVIATION OF SCI, SUBGRADE
             C       MODULUS, AND PAVEMENT MODULUS
             C
 0144                IF( PN  .EQ.  1 )  GO  TO  90
 0145                SE1  =   SQRT(SR1/(PN-1))
 0146                IF( N1 .LE. 1)  GO TO 90
 0147                SE2 = SQRT(SR2/(N1-1))
 0148                SE3 = SQRT(SR3/(N1-1))
             C
             C       PRINT STANDARD DEVIATIONS
```

```
               C
 0148                PRINT 71,SE1,SE2,SE3
               C  71 FORMAT( 1X,'STANDARD DEVIATION',27X,F5.3, 2F11.0    )
 0149             71 FORMAT(  7X,'STANDARD DEVIATION', 20X,F6.3,2F10.0)
               C
 0150             90 CONTINUE
 0151                IF( N  .EQ.  1 )  N1  =  1
 0152                PRINT 99,N, N1,  N1
               C  99 FORMAT(1X,'NUMBER OF POINTS IN AVERAGE = ',
               C    =  I14, I9, I10 )
 0153             99 FORMAT(  7X,'NUMBER OF POINTS IN AVERAGE = ',
                    =  I14, I9, I10 )
               C
 0154                PRINT 91
               C  91 FORMAT( /,5X,'W1   DEFLECTION AT GEOPHONE 1')
 0155             91 FORMAT(/10X,'W1    DEFLECTION AT GEOPHONE 1')
               C
 0156                PRINT 92
               C  92 FORMAT(   5X,'W2   DEFLECTION AT GEOPHONE 2')
 0157             92 FORMAT( 10X,'W2    DEFLECTION AT GEOPHONE 2')
               C
 0158                PRINT 93
               C  93 FORMAT(    5X,'W3   DEFLECTION AT GEOPHONE 3')
 0159             93 FORMAT( 10X,'W3    DEFLECTION AT GEOPHONE 3')
               C
 0160                PRINT 94
               C  94 FORMAT(    5X,'W4   DEFLECTION AT GEOPHONE 4')
 0161             94 FORMAT( 10X,'W4    DEFLECTION AT GEOPHONE 4')
               C
 0162                PRINT 95
               C  95 FORMAT(    5X,'W5   DEFLECTION AT GEOPHONE 5')
 0163             95 FORMAT( 10X,'W5    DEFLECTION AT GEOPHONE 5')
               C
 0164                PRINT 96
               C  96 FORMAT(   5X,'SCI   SURFACE CURVATURE INDEX ( W1 MIN',
               C    = 'US W2)' )
 0165             96 FORMAT( 10X,'SCI   SURFACE CURVATURE INDEX ( W1 MIN',
                    * 'US W2)'    )
               C
 0166                PRINT 97
               C  97 FORMAT( 5X,'ES    ELASTIC MODULUS OF THE SUBGRADE FRO',
               C    = 'M W1 AND W2'  )
 0167             97 FORMAT( 10X,'ES    ELASTIC MODULUS OF THE SUBGRADE FRO',
                    * 'M W1 AND W2' )
               C
 0168                PRINT 98
               C  98 FORMAT( 5X,'EP    ELASTIC MODULUS OF THE PAVEMENT FROM',
               C    = ' W1 AND W2'    )
```

```
 0169            98 FORMAT( 10X,'EP    ELASTIC MODULUS OF THE PAVEMENT FRO',
                   * 'M W1 AND W2' )
               C
               C
 0170               GO TO 10
 0171          1000 CONTINUE
 0172               END
```

```
 0001                 SUBROUTINE  EMOD (  W1, W2, H, E1, E2 )
                C
                C
 0002                 IMPLICIT REAL * 8  ( A-H, O-Z )
 0003                 DIMENSION  RH( 2), FF( 2), Y(4000), DELM1(2),
                        DELM2(2), DELX1(2), DELX2(2)
                C
 0004                 DATA P  /   1000.0D0 / , ER / .001D0/
 0005                 DATA XNO / 61.0D0 /, XK1 / 0.01D0 /, XK2 / 0.10D0 /
                C
                C     P, XNO, XK1, XK2, R1 & R2 CAN BE CHANGED IF DESIRED
                C
 0006                 INTEGER  PLUS
                C
                C     INITIALIZE SWITCHES & SAVE
                C
 0007                 R1   =  10.0D0
 0008                 R2   =  DSQRT( 244.0D0 )
 0009                 MINUS  =  1
 0010                 PLUS  =  0
 0011                 ISW  =  0
 0012                 SAVE  =  0.0D0
                C
                C     CALCULATE R/H, RATIO, & ACC ( ACC IS THE CONVERGENCE
                C     CRITERION )
                C
 0013                 RH(1)  =  R1 / H
 0014                 RH(2)  =  R2 / H
 0015                 RATIO  =  ( W1 * R1 ) / ( W2 * R2 )
 0016                 ACC  =  ER * RATIO
                C
                C
 0017                 DO   2  KL  =  1 ,  2
                C
                C     CALCULATE AND TEST DELM1
                C
 0018                 DELM1(KL) = ( 1.0D0 / RH(KL)) * ( 3.0D0 /
                     *  ( XNO - 1.0D0 ) )
                C
 0019                 IF( XK1  .LE.  DELM1(KL) )  DELM1(KL)  =  XK1
                C
                C     CALCULATE DELX1
                C
 0020                 DELX1(KL)  =  DELM1(KL)  *  RH(KL)
                C
                C     CALCULATE AND TEST DELM2
                C
 0021                 DELM2(KL) = ( 1.0D0 / RH(KL)) * ( 3.0D0 /
```

```
                     *  ( XNO - 1.0D0 ) )
              C
 0022                IF( XK2  .LE.  DELM2(KL) )  DELM2(KL)  =  XK2
              C
              C      CALCULATE DELX2
              C
 0023                DELX2(KL)  =  DELM2(KL)  *  RH(KL)
 0024              2 CONTINUE
              C
              C
              C      GET INITIAL VALUE OF E2/E1 AND DELTA
              C
 0025                DELTA  =  0.5D0
 0026                E2E1  =  0.001D0
              C
              C
              C      START ITERATION LOOP FOR EACH E2/E1 VALUE USED
              C
 0027              4 CONTINUE
 0028                XN  =  ( 1.0D0  -  E2E1 )  /  ( 1.0D0  +  E2E1 )
              C
              C      THE FUNCTIONS FF(1) AND FF(2) (SEE EQN. 5) ARE
              C      CALCULATED IN THE FOLLOWING DO LOOP.
              C
 0029                DO  29  KK  =  1  ,   2
              C
              C       CALCULATE NO. OF INTERVALS FOR SIMPSON'S RULE FOR
              C      EACH INTEGRATION.  N1 & N2 MUST BE ODD INTEGERS.
              C
 0030                N1  =  ( 3.0D0 * RH(KK) ) / DELX1(KK) + 1.0D0
 0031                 IF((N1 / 2) *  2  .EQ. N1)  N1 = N1 + 1
              C
 0032                N2  =  ( 7.0D0 * RH(KK) ) / DELX2(KK) + 1.0D0
 0033                  IF((N2 /2) * 2 .EQ. N2)  N2 = N2 + 1
              C
              C       CALCULATE ORDINATES FOR SIMPSON'S RULE FOR FIRST
              C      INTEGRATION
              C
 0034                XM1  =  0.0D0
 0035                X1  =  0.0D0
 0036                  DO  28   JJ  =  1, N1
 0037                Y(JJ)  =  ( V( XN, XM1 )  -  1.0D0 )  *  BESJ0( X1 )
 0038                   XM1 =  XM1 + DELM1(KK)
 0039                   X1  =  X1  +  DELX1(KK)
 0040             28   CONTINUE
              C
              C       CALCULATE ORDINATES FOR SIMPSON'S RULE FOR SECOND
              C      INTEGRATION
```

```
              C
 0041                   XM2 =  XM1
 0042                   X2  =  X1
 0043                   DO  27   KL =  1,N2
 0044                Y(N1 + KL)  =  ( V( XN, XM2 )  -  1.0D0 )  *  BESJO( X2 )
 0045                   XM2 = XM2 + DELM2(KK)
 0046                   X2  =  X2  +  DELX2(KK)
 0047             27    CONTINUE
              C
              C      SUM ORDINATES TO CALCULATE AREA UNDER THE CURVE OF FIRST
              C      INTEGRATION
              C
 0048                PART1  =  0.0D0
 0049                PART3  =  0.0D0
              C
              C      N4 IS NO. OF INTERIOR ORDINATES OF FIRST INTEGRATION
              C
 0050                  N4 =  N1 - 3
              C
              C      SUM INTERIOR ORDINATES
              C
 0051                  DO  26  LL = 2 , N4, 2
 0052             26 PART1  =  PART1  +  ( 2.0D0 * Y(LL)  +  Y(LL+1) )
              C
              C      SUM END ORDINATES
              C
 0053                PART2  =  Y(1)  +  4.0D0  *  Y(N1-1)  +  Y(N1)
              C
              C       CALCULATE AREA OF FIRST INTEGRATION.
              C
 0054                AREA1 = ((2.0D0 * DELX1(KK)) / 3.0D0) *
                       PART1 + ( DELX1(KK) / 3.0D0) * PART2
              C
              C       SUM ORDINATES TO CALCULATE AREA UNDER THE CURVE OF
              C       SECOND INTEGRATION
              C
              C
              C      THE LAST ORDINATE OF THE FIRST INTERVAL OF INTEGRAT-
              C      ION IS ALSO THE FIRST ORDINATE OF THE SECOND INTERVAL
              C
              C
              C      N5 IS THE POSITION IN THE Y VECTOR OF THE FIRST
              C      INTERIOR ORDINATE OF THE SECOND INTEGRATION INTERVAL
              C
 0055                  N5 = N1 + 2
              C
              C      N6 IS THE POSITION IN THE Y VECTOR OF THE LAST
              C      INTERIOR ORDINATE OF THE SECOND INTEGRATION INTERVAL
```

```
              C
 0056                 N6 =  N2 - 3  + N1
              C
              C     SUM INTERIOR ORDINATES
              C
 0057                 DO  25   LM =  N5 , N6 , 2
 0058            25 PART3  =  PART3  +  ( 2.0D0 * Y(LM)  +  Y(LM+1) )
              C
              C     SUM END ORDINATES
              C
 0059               PART4  =  Y(N1+1) + 4.0D0 * Y(N1 + N2 - 1) + Y(N1 + N2)
              C
              C      CALCULATE AREA OF SECOND INTEGRATION.
              C
 0060               AREA2 = ((2.0D0 * DELX2(KK)) / 3.0D0) *
                      PART3 + ( DELX2(KK) / 3.0D0) * PART4
              C
              C      CALCULATE THE FUNCTION.
              C
 0061               FF(KK)  =  AREA1  +  AREA2  +  1.0D0
 0062            29  CONTINUE
              C
              C     CALCULATE F1/F2 AND CHECK FOR CONVERGENCE
              C
 0063               F1F2  =  FF(1)  /  FF(2)
 0064               ERROR  =  F1F2  -  RATIO
 0065               IF(DABS( ERROR ) .LT. ACC )  GO  TO  31
              C
              C     SET ISW AND SAVE ON FIRST TIME THROUGH ITERATION LOOP
              C
 0066               IF( ISW .NE. 0 )  GO  TO   6
 0067               ISW  =  1
 0068               SAVE  =  ERROR
 0069               IF( ERROR  .LT.  0.0D0 )  GO  TO  6
              C
              C     SIGN OF FIRST ERROR IS '+'
              C
 0070               PLUS  =  1
 0071               MINUS  =  0
 0072             6 CONTINUE
              C
              C     TEST FOR SIGN OF ERROR
              C
 0073               IF( ERROR )  30, 31, 32
              C
              C     SIGN OF ERROR IS '+'
              C
 0074            32 PLUS  =  1
```

```
0075                 IF( MINUS .NE. 0 )  GO  TO  40
               C
               C     ERROR IS POSITIVE, DECREASE E2E1 FOR NEXT TRIAL
               C
0076                 E2E1  =  E2E1  -  DELTA
0077                 IF( E2E1  .LE.  0.0D0 )  E2E1  =  0.0001D0
0078                 GO  TO   4
               C
               C
               C     ERROR WAS NEGATIVE, NOW POSITIVE, CHANGE DELTA
               C
0079              40 DELTA  =  0.5D0  *  DELTA
0080                 IF( SAVE  .LT.  0.0D0 )  GO  TO  42
               C
               C     SET SAVE = ERROR, DECREASE E2E1 FOR NEXT TRIAL
               C
0081              41 SAVE  =  ERROR
0082                 E2E1  =  E2E1  - DELTA
0083                 IF( E2E1  .LE.  C.0D0 )  E2E1  =  0.0001D0
0084                 GO  TO  4
               C
               C     ERROR IS INCREASING IN POSITIVE DIRECTION, DECREASE
               C     E2E1 FOR NEXT TRIAL
               C
0085              42 IF(DABS( SAVE ) .GT. ERROR )  GO  TO  41
0086                 E2E1  =  E2E1  - DELTA
0087                 IF( E2E1  .LE.  0.0D0 )  E2E1  =  0.0001D0
0088                 GO  TO  4
               C
               C     SIGN OF ERROR IS '-'
               C
0089              30 MINUS  =  1
0090                 IF( PLUS .NE. 0 )   GO  TO  45
               C
               C     ERROR IS NEGATIVE, INCREASE E2E1 FOR NEXT TRIAL
               C
0091                 E2E1  =  E2E1  +  DELTA
0092                 IF( E2E1  .GT.  1.0D0 )  GO  TO  44
0093                 GO  TO  4
               C
               C
0094              44 CONTINUE
0095                 IF( H  .GE.  9.2D0 )  GO  TO  4
0096                 DELTA  =  0.5  *  DELTA
0097                 E2E1  =  E2E1  -  DELTA
0098                 GO  TO  4
               C
               C     ERROR IS NEGATIVE NOW, WAS POSITIVE BEFORE, CHANGE
```

```
                C      DELTA
                C
 0099              45 DELTA  =  0.5D0  *  DELTA
 0100                 IF( SAVE  .GT.  0.0D0 )  GO  TO  47
                C
                C     TEST FOR ERROR LESS THAN SAVE
                C
 0101              46 IF(DABS ( SAVE )  .GT.DABS ( ERROR ) )SAVE = ERROR
                C
                C     INCREASE E2E1 FOR NEXT TRIAL
                C
 0102                 E2E1  =  E2E1  +  DELTA
 0103                 IF( E2E1  .GT.  1.0D0 )  GO  TO  44
 0104                 GO  TO  4
                C
                C     TEST FOR ERROR GREATER THAN SAVE
                C
 0105              47 IF(DABS ( ERROR )  .GT. SAVE )  GO  TO  46
                C
                C     ERROR IS APPROACHING CONVERGENCE FROM NEGATIVE SIDE,
                C     SET SAVE = ERROR, INCREASE E2E1 FOR NEXT TRIAL
                C
 0106                 SAVE  =  ERROR
 0107                 E2E1  =  E2E1  +  DELTA
 0108                 IF( E2E1  .GT.  1.0D0 )  GO  TO  44
 0109                 GO  TO  4
 0110              31 CONTINUE
                C
                C     CONVERGENCE CRITERION IS MET, CALCULATE E1 & E2
                C
 0111                 E1 = (3.0D0 * P * FF(1))/ (4.0D0 * 3.14159D0 *W1*R1)
 0112                 E2  =  E2E1  *  E1
                C
 0113                 RETURN
 0114                 END
```

```
0001                  REAL FUNCTION BESJO * 8 ( X )
                C
                C
                C     A FUNCTION TO CALCULATE BESSEL FUNCTION JO(X) USING
                C     POLYNOMIAL APPROXIMATION - REFERENCE HANDBOOK OF MATH.
                C     FUNCTIONS, BUREAU OF STANDARDS, PAGES 369-370
                C
0002                  DOUBLE PRECISION X3, X32, X33, X34, X35, X36,DCOS,
                     *  DSQRT, DABS, X
                C
                C     CALCULATE X/3 OR 3/X
                C
0003                  X3 = X/3.0
0004                  IF( X.GT. 3.0) X3 = 3.0/ X
                C
                C     CALCULATE POWERS OF X
                C
0005                  X32= X3*X3
0006                  X33=X32*X3
0007                  X34=X32*X32
0008                  X35=X32*X33
0009                  X36=X33*X33
                C
0010                2 IF ( DABS (X)  .LE. 3.0D0 )  GO  TO  3
                C
                C     CALCULATE BESJO(X) FOR VALUES OF X GREATER THAN 3
                C
0011                  BESJO=((.79788456-.77E-6   * X3  -  0.552740D-02 *
                     * X32 - .9512E-04 * X33 + .137237D-02 * X34 -
                     * .72805E-03 * X35 + .14476E-03 * X36 ) / DSQRT(X) )
                     * * DCOS( X - .78539816 - .04166397 * X3 - .3954E-04
                     * * X32 + .262573D-02 * X33 - .54125D-03 * X34 -
                     * .29333E-03 * X35 + .13558E-03 * X36 )
                C
0012                  RETURN
                C
                C     CALCULATE BESJO(X) FOR VALUES OF X LESS THAN 3
                C
0013             3    BES JO= 1.0  -  2.2499997 * X32  +  1.2656208 * X34
                     * - .3163866 * X36 + .0444479 * ( X34 * X34 ) -
                     * .0039444 * ( X35 * X35 ) + .000210 * ( X36 * X36)
                C
0014                  RETURN
0015                  END
```

```
0001                REAL FUNCTION V * 8 ( XN , XM )
            C
            C
0002                DOUBLE PRECISION  XN, XM, EXPM2M, EXPM4M, DEXP
            C
            C       V - A FUNCTION OF 'E2E1', AND 'M'
            C       'E2E1' IS THE E2/E1 RATIO, TESTED FROM .001 TO 1000.
            C       'M' TESTED USING VALUES FROM 0.0 TO 150. WHICH IS
            C       10 * (R/H)
            C
            C
            C       V APPROACHES 1 FOR LARGE VALUES OF M
            C
0003                       V  =  1.0
0004                       IF( XM .GT.  30 )    RETURN
            C
            C
            C       CALCULATE EXPONENTIALS
            C
0005                EXPM2M  =  DEXP ( -2.0D0 * XM )
0006                EXPM4M= EXPM2M*EXPM2M
            C
            C       CALCULATE FUNCTION V FOR THE XN & XM1 OR XM2 VALUES
            C
0007                V  =  ( 1.0D0 + ( 4.0D0 * XN * XM * EXPM2M ) -
                   * ( XN * XN * EXPM4M ) )  /  ( 1.000 - ( 2.0D0 * XN
                   * * ( 1.0D0 + 2.0D0 * XM * XM )  * EXPM2M )  +
                   *  ( XN * XN * EXPM4M )  )
            C
0008                RETURN
0009                END
```

```
  LOC  OBJECT CODE    ADDR1 ADDR2  STMT   SOURCE STATEMENT

000000                               1 CORE#    START 0
                                     2          ENTRY CORE
                                     3          EXTRN IBCOM#
                                     4          EXTRN FIOCS#
000000                               5          USING *,15
000000 47F0 F00C            0000C    6 CORE     BC    15,*+12
000004 07000000                      7          DC    XL4'07000000'
000008 C3D6D9C5                      8          DC    CL4'CORE'
00000C 90E4 D00C            0000C    9          STM   14,4,12(13)
000010 9823 1000            00000   10          LM    2,3,0(1)
000014 5833 0000            00000   11          L     3,0(3)
000018 9023 F090            00090   12          STM   2,3,BUFADR
00001C 4110 F02E            0002E   13          LA    1,CORE2
000020 4130 F074            00074   14          LA    3,CLOAD
000024 0523                         15          BALR  2,3
000026 98E4 D00C            0000C   16          LM    14,4,12(13)
00002A 1BFF                         17          SR    15,15
00002C 07FE                         18          BCR   15,14
                                    19          DROP  15
00002E                              20          USING *,1
00002E 5040 106E            0009C   21 CORE2    ST    4,SAVE4
000032 1841                         22          LR    4,1
00002E                              23          USING CORE2,4
                                    24          DROP  1
000034 1810                         25          LR    1,0
000036 910F 1001      00001         26          TM    1(1),X'0F'
00003A 4710 4022            00050   27          BC    1,OUTPUT
00003E 5810 4076            000A4   28          L     1,VFIOCS
000042 4130 4046            00074   29          LA    3,CLOAD
000046 0523                         30          BALR  2,3
000048 9823 4062            00090   31          LM    2,3,BUFADR
00004C 47F0 4036            00064   32          BC    15,RETURN
000050 9823 4062            00090   33 OUTPUT   LM    2,3,BUFADR
000054 9240 2000      00000         34          MVI   0(2),X'40'
000058 0630                         35          BCTR  3,0
00005A 0630                         36          BCTR  3,0
00005C 4430 4040            0006E   37          EX    3,DMOVE
000060 4133 0002            00002   38          LA    3,2(3)
000064 5840 406E            0009C   39 RETURN   L     4,SAVE4
000068 1810                         40          LR    1,0
                                    41          DROP  4
00006A 47F1 0006            00006   42          BC    15,6(1)
00006E D200 2001 2000 00001 00000   43 DMOVE    MVC   1(0,2),0(2)
000074                              44          USING *,3
000074 50F0 3024            00098   45 CLOAD    ST    15,SAVE
000078 58F0 302C            000A0   46          L     15,VIBCOM
00007C 9250 F04A      0004A         47          MVI   74(15),X'50'
000080 440F 004A            0004A   48          EX    0,74(15)
000084 9258 F04A      0004A         49          MVI    74(15),X'58'
000088 58F0 3024            00098   50          L     15,SAVE
00008C 07F2                         51          BCR   15,2
000090                              52 BUFADR   DS    2F
000098                              53 SAVE     DS    F
00009C                              54 SAVE4    DS    F
0000A0 00000000                     55 VIBCOM   DC    A(IBCOM#)
```

```
  LOC   OBJECT CODE     ADDR1 ADDR2  STMT   SOURCE STATEMENT

0000A4 00000000                        56 VFIOCS    DC     A(FIOCS#)
                                       57           END
```

NOTE: Program printouts of sample problems will be found in the main body of the report, Tables 6a through 6g.

# LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

```
10017BRAZOS         1560 1 1FM 1687SBL 12.5 52168 1

200SEAL COAT            0.5 RED SANDY GRAVEL    12.0GREY & BRWN SAND SUB

   1560 1 52168 1 - A    39.3 77.1 52.1 31.1 73.03
   1560 1 52168 1 - B    38.3 77.1 51.1 31.1 71.03
   1560 1 52168 2 - A    43.3 28.3 49.1 30.1 68.03
   1560 1 52168 2 - B    40.3 28.3 49.1 30.1 67.03
   1560 1 52168 3 - A    38.3 77.1 47.1 30.1 65.03
   1560 1 52168 3 - B    37.3 77.1 46.1 30.1 67.03
   1560 1 52168 4 - A    49.3 32.3 49.1 32.1 74.03
   1560 1 52168 4 - B    46.3 30.3 47.1 31.1 71.03
   1560 1 52168 5 - A    43.3 29.3 50.1 34.1 77.03
   1560 1 52168 5 - B    42.3 80.1 46.1 31.1 73.03
```

# LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

```
10017BRAZOS          2824 2 1FM 2776NBL  8.0 52168 1

200SEAL COAT              0.5 ASPHALT STAB. GRAVEL 7.5GREY SANDY CLAY SUBG

   2824 2 52168 1 - A    55.3 40.3 29.3 66.1 50.1
   2824 2 52168 1 - B    52.3 37.3 81.1 61.1 49.1
   2824 2 52168 2 - A    77.3 49.3 31.3 71.1 53.1
   2824 2 52168 2 - B    77.3 47.3 30.3 67.1 51.1
   2824 2 52168 3 - A    81.3 50.3 31.3 67.1 49.1
   2824 2 52168 3 - B    83.3 51.3 31.3 67.1 50.1
   2824 2 52168 4 - A    83.3 49.3 30.3 64.1 48.1
   2824 2 52168 4 - B    81.3 47.3 28.3 61.1 47.1
   2824 2 52168 5 - A    78.3 48.3 29.3 62.1 45.1
   2824 2 52168 5 - B    81.3 49.3 31.3 65.1 47.1
```

# LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

```
10017BURLESON       1399 1 1FM 1361EBL 12.0 52168 1

200SEAL COAT            0.5 LIME STAB. SANDSTONE11.5TAN SANDY CLAY SUBGR

   1399 1 52168 1 - A    50.3 37.3 71.1 47.1 33.1
   1399 1 52168 1 - B    52.3 41.3 78.1 48.1 33.1
   1399 1 52168 2 - A    55.3 40.3 67.1 40.1 81.03
   1399 1 52168 2 - B    48.3 35.3 64.1 38.1 82.03
   1399 1 52168 3 - A    50.3 35.3 60.1 37.1 89.03
   1399 1 52168 3 - B    48.3 33.3 58.1 37.1 87.03
   1399 1 52168 4 - A    50.3 35.3 56.1 34.1 72.03
   1399 1 52168 4 - B    46.3 33.3 54.1 33.1 71.03
   1399 1 52168 5 - A    64.3 42.3 65.1 40.1 28.1
   1399 1 52168 5 - B    60.3 38.3 63.1 42.1 31.1
```

# LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

```
10017WASHINGTON     186 5 1SH 36  NBL 19.9 52168 1

200HOT MIX ASPH. CONC. 3.75SANDSTONE          1615BLACK CLAY SUBGRADE

    186 5 52168 1 - A    56.3 34.3 61.1 42.1 30.1
    186 5 52168 1 - B    61.3 36.3 61.1 42.1 31.1
    186 5 52168 2 - A    58.3 36.3 67.1 47.1 36.1
    186 5 52168 2 - B    65.3 39.3 69.1 49.1 37.1
    186 5 52168 3 - A    56.3 36.3 68.1 50.1 38.1
    186 5 52168 3 -      57.3 36.3 67.1 48.1 37.1
    186 5 52168 4 - A    56.3 37.3 75.1 57.1 46.1
    186 5 52168 4 - B    52.3 36.3 73.1 55.1 44.1
    186 5 52168 5 - A    50.3 32.3 59.1 44.1 33.1
    186 5 52168 5 - B    53.3 33.3 60.1 43.1 33.1
```

## LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

```
10017ROBERTSON        49 8 1US 190 NBL 15.2 52168 1

200HOT MIX ASPH. CONC. 1.25CEM. STAB. LIMESTONE1395RED SANDY CLAY SUBGR

     49 8 52168 1 - A    68.1 59.1 49.1 39.1 31.1
     49 8 52168 1 - B    68.1 60.1 49.1 39.1 31.1
     49 8 52168 2 - A    72.1 63.1 51.1 39.1 31.1
     49 8 52168 2 - B    70.1 62.1 49.1 39.1 31.1
     49 8 52168 3 - A    75.1 65.1 52.1 39.1 30.1
     49 8 52168 3 - B    76.1 65.1 51.1 39.1 30.1
     49 8 52168 4 - A    60.1 54.1 45.1 35.1 28.1
     49 8 52168 4 - B    58.1 52.1 43.1 33.1 88.1
     49 8 52168 5 - A    62.1 55.1 45.1 35.1 91.1
     49 8 52168 5 - B    65.1 57.1 47.1 36.1 28.1
```

LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

```
10017BRAZOS         1560 1 1FM 1687NBL   7.5 52168 1

200ASPHALT SURFACING   1.0 ASPH EMUL STAB GRAVL 6.5BROWN CLAY SUBGRADE

   1560 1 52168 1 - A     72.3 50.3 32.3 66.1 52.1
   1560 1 52168 1 - B     71.3 51.3 32.3 65.1 51.1
   1560 1 52168 2 - A     64.3 47.3 31.3 64.1 49.1
   1560 1 52168 2 - B     62.3 45.3 30.3 63.1 50.1
   1560 1 52168 3 - A     68.3 49.3 31.3 63.1 49.1
   1560 1 52168 3 - B     69.3 50.3 32.3 65.1 50.1
   1560 1 52168 4 - A     74.3 54.3 34.3 67.1 49.1
   1560 1 52168 4 - B     74.3 53.3 34.3 65.1 49.1
   1560 1 52168 5 - A     66.3 46.3 30.3 61.1 47.1
   1560 1 52168 5 - B     66.3 48.3 31.3 61.1 46.1
```

# LISTING OF DATA CARDS FOR SAMPLE PROBLEMS

```
10017BRAZOS          540 3 1FM 974 FBL  8.3 52168 1

200SEAL COAT             0.5 IRON ORE GRAVEL      7.8GREY SANDY CLAY SUBG

    540 3 52168 1 - A    80.3 51.3 32.3 68.1 50.1
    540 3 52168 1 - B    75.3 48.3 30.3 63.1 48.1
    540 3 52168 2 - A    59.3 39.3 82.1 60.1 48.1
    540 3 52168 2 - B    60.3 40.3 82.1 62.1 49.1
    540 3 52168 3 - A    55.3 39.3 28.3 64.1 51.1
    540 3 52168 3 - B    53.3 39.3 28.3 61.1 51.1
    540 3 52168 4 - A    75.3 49.3 33.3 75.1 60.1
    540 3 52168 4 - B    78.3 53.3 35.3 79.1 63.1
    540 3 52168 5 - A    74.3 49.3 33.3 71.1 55.1
    540 3 52168 5 - B    70.3 47.3 32.3 68.1 53.1
```