PHYSICAL FORCES
AFFECTING AN AUTOMOBILE
IN MOTION

RESOURCE OUTLINE FOR
DRIVER EDUCATION INSTRUCTORS
IN SECONDARY SCHOOLS AND COLLEGES

TEXAS DEPARTMENT OF PUBLIC SAFETY
AUSTIN, TEXAS
The basic mission of the Texas Department of Public Safety is to develop and carry out, in cooperation with other governmental agencies, positive programs of police and regulatory services, within existing regulations, that will maintain order in our society, so that the people within the state may be secure in person and property and may enjoy the rights and privileges naturally theirs or secured to them by constitution or statute.

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DUTIES AND POWERS OF THE COMMISSION. Art. 4413(4) Vernon's Civil Statutes of the state of Texas

(1) The Commission shall formulate plans and policies...for the education of the citizens of the state in the promotion of public safety and law observance.

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Driver Education Instructor Resource Outline

PHYSICAL FORCES AFFECTING AN AUTOMOBILE IN MOTION

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PREFACE

The Recapitulation at the end is a concise review of the more important physical force concepts treated in the outline. This preface is only a preview of the key concepts by name and relation. Its purpose is to direct the teacher’s attention to those subjects which should receive emphasis.

The “square principle” in the distance formula, $s = ut \pm \frac{1}{2}at^2$, in the centrifugal force formula, $CF = \frac{mv^2}{r}$, and in the kinetic energy formula, $KE = \frac{1}{2}mv^2$, is a driver problem in preventing collisions. The lb-sec in the momentum formula, $mom = mv$, is a driver problem in reducing the severity of injury to car occupants resulting from impact forces during a collision.

The distance formula shows how acceleration and deceleration distances vary by the square of the time. It is related to kinetic energy which varies by the square of the velocity and gives the relation between force and distance. The distance formula shows how unequal parts of a given distance will be covered in equal parts of the time involved, as a car’s velocity increases or decreases.

Centrifugal force is a problem of steering control. It overturns cars or skids them off curved roads. It releases the kinetic energy of a car from a driver’s control. Its hazard increases as the kinetic energy increases since both CF and KE increase by the square of the velocity.

Kinetic energy continually threatens a driver with loss of control (collisions) on a straight road, due to the way braking distances vary by the square of the speed. KE involves a linear force and distance at all times and involves a side force (centrifugal) while a car is in a curve or in any change of direction from straight ahead.

Kinetic energy involves force and distance factors which cause collisions and determine the damage that can be done at a given speed. If a car has enough open distance in which to decelerate at normal braking rates, its energy will be harmless. When this distance is shortened a property of KE called momentum becomes a hazard. Momentum involves the force in KE and the time it takes a car to stop. Momentum is measured in lb-sec, a product of the force and the time. As the time (sec) is shortened at a given speed, the force (lb) increases.

When a car in collision stops in a short distance an occupant hurled forward stops in a short time against the car inside. The momentum of the occupant’s body is changed quickly. The time (sec) in the lb-sec becomes small and the force (lb) in the lb-sec becomes great. When the time is a small fraction of a second the force can be many times the weight of the occupant.

Simply, the outline is a study of an automobile in motion, which involves space and time—or velocity, which is space per unit of time, as ft/sec. Car control problems which grow out of the apparently harmless elements of mass, space, and time are as complex as the motion formulas may appear at first sight. The formulae however are the simplest ways to state the complex relations of the elements. That is the reason they are formulae. The most direct way then to grasp the concepts involved is to understand the formulas.

For example, the formula $s = ut \pm \frac{1}{2}at^2$, says the distance (s) traveled by a car in a time (t) while accelerating at a rate (a) is equal to the initial speed (u) times the time (t) plus (or minus) one-half the rate (a) times the time (t) squared. After reading that, one rushes back to the formula for simplicity and clarity. Now an analysis:

If a car had held a constant speed during the time (t), the distance (s) covered would be the average velocity ($\bar{v}$) times the time (t). That is the ($ut$) part. (We start with (u) so (u) is the average of itself.) The $\frac{1}{2}at^2$ part gives us the distance due only to that part of the speed each second which was in excess of (or less than) the initial speed (u).

Let us assume the speed is increasing. The car accelerates from the initial speed (u) to a final speed (v). The speed added to (u) is the difference between (u) and (v) and is equal to (at), which is the rate of increase of speed each second times the number of seconds. The average speed in accelerating from (u) to (v) is equal to $at$. The distance (s) covered during acceleration = average speed $\times$ time.
Therefore $s_a = (at)t = at^2$. 

We have $s_c = ut$ for the constant speed part and $s_a = \frac{at^2}{2}$ for the acceleration part. Together the two parts give $s = ut + \frac{at^2}{2}$.

Other formulas in the outline are even less difficult than this one once the reader understands the parts involved.
# TABLE OF CONTENTS

Preface ............................................................... i
Table of Contents ................................................................ iii
Introduction ........................................................................ v
Gravitational Force ........................................................... vii

## PART ONE

Speed ................................................................. 1
Acceleration ............................................................... 1
Acceleration, Mass and Force ............................................. 2
Friction Force ............................................................. 2
Coefficient of Friction .................................................... 3
Kinetic Energy ............................................................... 4
Kinetic Energy, Work and Braking Distance ....................... 6
Kinetic Energy, Work and Horsepower ................................. 8
Kinetic Energy and Momentum ........................................... 10
Inertia and Centrifugal Force ............................................. 14
Additional Forces Affecting a Car on a Curve ..................... 16
Concepts ..................................................................... 17
Study Questions for Teachers ............................................. 18
Formulae .................................................................... 20

## PART TWO

(Visual Aids)

Friction .................................................................. 26
Force Concepts .......................................................... 26
Inertia and Centrifugal Force ............................................ 29
Kinetic Energy and Braking Distance ................................. 30
Acceleration ............................................................... 31
Diagrams, Charts, Tables, etc. .......................................... 32

## PART THREE

Braking Effort and Coefficient of Friction

- Braking Effort or Braking Efficiency ............................... 77
- Coefficient of Friction between the Tires and Road Surface 77
- How Braking Effort and Coefficient of Friction are Related 78
- Miscellaneous Items on Braking .................................. 79

Inertia and Centrifugal Force

- Inertia .................................................................. 80
- Centrifugal Force .................................................... 80
- How Centrifugal Force Can be Reduced .......................... 81
- Factors which a Driver Should Analyze ........................... 82

Kinetic Energy and Braking Distance

- Energy .................................................................. 82
- Kinetic Energy must be Changed into Heat Energy .......... 83
- The Danger of Kinetic Energy ...................................... 83
- Conservation of Energy ............................................. 84
- Braking Distances in Tests ........................................... 84
- Impact Force in a Collision ......................................... 85
INTRODUCTION

Two aspects of the auto-accident problem cause considerable dismay. One is that while riders and pedestrians generally must know that in collision impacts between flesh and steel, flesh will be crushed, they often act as though they think the reverse is true. Some analysts attribute this defect to smooth running, comfortable cars which prevent occupants from sensing the danger of their motion. But riders when they are on foot can sense the motion of cars coming toward them yet they often display no more concern for a two-ton car brushing them than if the car were a walking pedestrian. We have come to consider the occasional occupant or pedestrian who is realist enough to see the danger for what it is as afflicted with a sort of phobia. This first aspect of the problem probably is related in some way to the second, which actually is appalling.

Everybody is ready to talk and often does at length about accidents—who, where, when, how, and why. We have elaborate control programs involving thousands of trained personnel who devote endless hours to minute details of needed legislation, equipment, administration, and operations. Literally reams of literature on how to dodge the traffic ogre pass annually to drivers. Public media admonish, advise, cajole, and plead for more intelligent driver conduct. More of all of these activities are sorely needed. But our second problem is that within this milieu of worthy effort little attention is given to the “What” which actually causes the damages, injuries, and deaths in traffic accidents. The “What” is difficult. It is abstract and uncommon to the senses. And we hear that it is too hard for drivers to digest. But subjects that are easy for drivers to digest cannot be substituted for the crux of the problem. Regrettably, we have come to treat the “What” much as we treat “The Thing.” We vaguely think of it as being real, but we know that it is intangible and invisible. The accident “What” is “Kinetic Energy”—which is just as abstract as “The Thing,” and its real meaning seems to concern drivers in about the same way. What appears to be driver disrespect for other users of the highway often is a lack of understanding of physical forces and a consequent failure to cope with kinetic energy and inertia.

It is time to stop ignoring the basic factor in traffic accidents just because it is difficult and to start popularizing interest in this killer. Our sophistry in disregarding physical forces does not become a space age nation; we waste more money in auto accidents each year than it should cost to go to the moon.

Energy and matter are two fundamental concepts of physics and of the world about us. The concept of a car’s mass is easy for a driver to grasp, but not concepts of energy and force. Defining energy is as difficult as defining electricity or gravity. Understanding thoroughly the phenomenon called energy is as difficult as defining it, but there are many things about energy a student can learn which will help him become an intelligent driver. For acceptable results in driver education a teacher should see that his students do understand (1) how destructive kinetic energy is, (2) how it develops with speed, (3) how much kinetic energy a car possesses when traveling at various speeds, (4) how it limits steering control, and (5) how it affects acceleration and deceleration distances. How energy limits a driver’s control of a car is perhaps the most important one bit of knowledge needed by all drivers. This knowledge is basic to improvement in driver judgment.

Basically the task of driving a car in modern traffic is a job of blending a mass-force (vehicle) into flows of tremendous energy (mass-forces in motion) along traffic lanes of relatively narrow limits. Getting a mass-force into and out of an energy stream safely involves acceleration rates. Avoiding an overlapping of energies within the stream involves both the frequency and the speeds of the mass-forces. The task involves problems of both lateral control (judgment in steering) and linear control (selecting and maintaining position). The problem of adjusting a mass in motion in space and time to the physical and mental limitations of drivers and pedestrians is too complex to be dismissed in driver education with an admonition to use “common sense.” The “common sense” approach to driving is costing the nation over 100 billion dollars annually while killing 50,000 + people and causing over 2,000,000 disabling injuries.

A driver who understands how kinetic energy influences acceleration and deceleration distances and steering control will possess knowledge upon which much better judgment can be based and without which he may not be motivated to discipline himself to avoid dangerous speeds or positions or to act early at any speed to escape a trap. Besides the vehicle itself and the ground, there is only one factor common to all motor vehicle accidents, and that is the velocity of a vehicle. Not even a driver is an essential factor, but there can be no accident without speed. The damage or injury in every accident is a mark of what was kinetic energy, present only when a vehicle is in motion.

Motion is the essence of all that is desirable in an automobile. A driver’s admiration of a car ceases abruptly when it won’t run. Therefore, in the study of an automobile in motion a teacher is at once analyzing the basic factor in damage
or injury and associating driver responsibility directly with motion, an approach which possesses high motivation value.

As a speedomoter needle moves up or down the driver should be able to visualize how the car's invisible energy is changing. Increasing speed from 60 mph to 61 mph seems so insignificant that it is hardly worth mentioning. Actually, however, this increase of 1 mph adds to the car's destructive power the same amount of energy as the car has at 11 mph. And with 50% braking it adds 8 feet to the car's braking distance. Other facts about kinetic energy treated in this outline are just as unbelievable.

Rules of the Road in traffic laws are based primarily on the hazards of kinetic energy. Yet some teachers devote more class time to the automobile at rest than to the automobile in motion. Such scheduling may be due to the fact that resource materials on the automobile at rest are more plentiful and better illustrated than are materials on physical forces. It may be due also to the fact that physical forces are more difficult to teach. But physical forces do the damage in traffic, not automobiles at rest. Driver education therefore does not attack the problem head-on unless it imparts an understanding of the automobile in motion. The purpose of this outline and its illustrations is to aid teachers in this job by providing them with subject matter not readily available in concise form from other sources. The more limited the time in which a teacher must teach concepts of a complex subject, the more important it is that the teacher understand the subject in detail. This is the justification for the technical treatment of some of the subjects. It is designed for a teacher who has not studied physics.

The outline consists of three parts. PART ONE contains the minimum information a teacher should have in preparing a unit on physical laws in a minimum classroom course of 30 hours in driver education. PART TWO contains visual aid materials including suggestions for preparing props, and a number of diagrams, charts, and tables illustrating special treatments of subjects selected to guide a teacher in preparing visual aids. PART THREE contains additional materials for a unit in a semester course and for a unit in college teacher preparation courses. While PART THREE is quite technical in parts, it contains several discussions related to driving procedures which one will find useful in any course on driving.

The knowledge obtained in studying this unit is basic to the development of sound judgment, which is a major objective in teaching driving procedures. It is logical therefore that a unit on physical laws as well as the one on highway traffic laws be taught prior to the unit on driving procedures. It is not intended that students learn how to work problems. It is more important that a student understand concepts and memorize facts that will mature his judgment and motivate restraint; he should gain an understanding of energy sufficient to improve his ability to recognize traffic hazards and to increase his respect for its power enough to cause him to associate self-discipline with self-survival.

We can train students or we can educate them. Here is an example of the difference:

A filling station owner trains an attendant with rules, one of which is to wash car engines with kerosene—never gasoline. A customer leaves a car for a quick, complete wash job. The engine is very hot. The attendant drives the car onto the wash rack and throws a gallon of diesel over the engine. The car and filling station burn down. The attendant is carried to the hospital.

Educating the attendant would have involved knowledge of flash points of gasoline which is $-45^\circ F$ and up and of diesel which is $122^\circ F$ and up; of requisites of a fire: fuel, oxygen, and heat; and that a hot auto engine can easily possess heat in excess of $122^\circ F$.

With this knowledge the attendant would be able to reason that if the temperature of this engine is higher than $122^\circ F$ this kerosene is just as dangerous as gasoline. Moreover, he would be able to apply the knowledge to prevent hazards in many situations too numerous to be covered with specific rules. In short he would be able to use knowledge with imagination and produce sound judgments. Rules of conduct should augment understanding of concepts, not replace them. (See teaching concepts page 17.)
GRAVITATIONAL FORCE

(Fact and Fiction)

Next to the astronomical amounts of energy existing in matter and the mysterious forces which hold matter together, the most awesome feature of nature is the gravitational force which keeps things, animals, and people firmly attached to the outside of a whirling sphere. This force creates what we call weight. It presses car wheels to the ground and thus makes it possible for a car to move when the engine turns the rear wheels.

We mention later that if gravity did not exist a car would hang in midair or stay on the ground where you place it. If it were in midair, you could not pull it down unless you caught hold of some object fixed to the earth with one hand and pulled with the other. To push a car on the ground you would have to get between the car and a fixed object such as a tree and push against the tree and the car at once. You would find it difficult to stand on the ground and move the car because your feet could not get much traction unless the push were at a lifting angle. You could lift the car from the ground by standing by a bumper and pulling up. Any time you moved the car, it would keep drifting slowly until it hit something.

If you should hold to the bumper after accelerating it upward you could not get back to earth by simply jumping off. If after you and the car ascended a few feet you decided to jump off, you wouldn't fall after you jumped. You would go off in the direction your feet pushed you and the car would be accelerated slightly in the opposite direction. You would accelerate to a higher speed than the car because the car's mass is much more dense per unit of volume than is your body. The car has more inertia. Its mass \( \times \) its speed would be equal to your mass \( \times \) your speed. Since your mass is much less than the car's mass your speed would be higher. If you kicked back with a 100 lb. force when you "jumped," the force would act against the car and the car would react with a 100 lb. force against your body.

If instead of jumping off the bumper you crawled underneath the car (this would be just as easy as crawling on top), aimed your head toward the ground and kicked against the bottom of the car, the action and reaction would increase the car's speed upward a little and accelerate your body back toward the ground. If the one impulse from your kick accelerated your body to say 10 ft/sec, that would be your constant speed and the speed at which you would hit the ground. You might break an arm if you hit something hard. If you hit something springy and couldn't grab hold of a fixed object you would bounce back off the earth, never to return unless you met an object drifting toward the earth and caught hold of it. The object's mass \( \times \) speed would have to be greater than your body's mass \( \times \) speed, else you would stop its motion toward the earth. The object's momentum would have to absorb your momentum and still have some speed left in the direction of the earth.

In order to arrive at the earth at a low speed you would need to kick off from the car with a very gentle push. You would take longer getting back but you would arrive safely. Knowing about physical laws would help you avert panic which might cause you to kick hard to get back fast.

Gravity makes us tired but life would be unpleasant without it. You couldn't run and romp, and if you jumped up out of your chair to oppose an argument you would literally hit the ceiling. Ceilings could be covered with foam rubber. Yell leaders would wear crash helmets and perform under a shed or perhaps inside plastic cages, anchored a few feet off the ground. That would be a show.

You would probably have to wear a tiny jet engine when outside so you could get back to earth in case you got pushed off by a jealous suitor or in case you stepped on something springy which would bounce you off. People would wear shoes with magnetic soles and walk on pavements impregnated with metal particles. If you bounced off you would need only enough power to stop you and start you back. Inertia would bring you on in.

Instead of having highway patrols to assist stranded motorists we would probably have air patrols wearing bigger jets that could overtake absentminded people who forgot to don their jet packs when they left home. And of course there would be some who went out without any gas in their tanks radioing back for service stations to send up some fuel. If one also forgot his radio helmet and an air patrol failed to see him waving his shirt, that would be all, unless he hitchhiked back on some drifting mass.

One does not have to understand the gravitational force in order to appreciate and respect it. The same reasoning holds for other forces we shall study during this course.
SPEED

Speed (or velocity) is a rate of covering distance per unit of time.

\[
\text{Speed} = \frac{\text{distance}}{\text{time}}, \text{ or } \text{Distance} = \text{speed} \times \text{time}, \text{ or } \text{Time} = \frac{\text{distance}}{\text{speed}}
\]

If distance is miles and time is hours, speed = miles/mph

hours

If distance is feet and time is seconds, speed = feet/ft/sec

seconds

\[
\text{mph} \times 1.467 = \text{ft/sec}. \quad \text{ft/sec} \times 0.682 = \text{mph}
\]

In discussing movement of vehicles to prevent collisions we find ft/sec more appropriate than mph.

ACCELERATION

Acceleration (or deceleration) is a rate of changing speed per unit of time.

\[
\text{Acceleration} = \frac{\text{change in speed}}{\text{time}} = \frac{\text{mph or ft/sec}}{\text{second}} \quad \text{sec}
\]

ft/sec = ft/sec/sec, or ft/sec^2 (read feet per second per second)

Acceleration is uniform if speed changes the same amount each second.

If a car accelerates from 30 mph to 60 mph in 10 seconds

\[
a = \frac{v - u}{t} = \frac{60 \text{ mph} - 30 \text{ mph}}{10 \text{ seconds}} = \frac{30 \text{ mph}}{10 \text{ sec}} = 3 \text{ mph/sec}
\]

v = final speed. \quad u = initial speed. \quad t = seconds

If we convert mph to ft/sec in above example

\[
a = \frac{88 \text{ ft/sec} - 44 \text{ ft/sec}}{10 \text{ sec}} = \frac{44 \text{ ft/sec}}{10 \text{ sec}} = 4.4 \text{ ft/sec/sec}
\]

If the car decelerates from 60 mph to 30 mph in 10 seconds

\[
a = \frac{v - u}{t} = \frac{30 \text{ mph} - 60 \text{ mph}}{10 \text{ sec}} = \frac{-30 \text{ mph}}{10 \text{ sec}} = -3 \text{ mph/sec}
\]

If the mph is converted to ft/sec, a = -4.4 ft/sec/sec.

The acceleration was negative and is called deceleration.
ACCELERATION, MASS, AND FORCE

Mass (matter) = weight = \( \frac{\text{lbs}}{\text{gravity} \ 32 \text{ ft/sec/sec}} \)

A car's mass has weight because gravity acts on it constantly. Weight then is a force pushing the car against the ground. Actually the earth is pulling the car to it.

Gravity will accelerate a car in free fall at a rate of 32 ft/sec/sec. When a car is at rest the weight force due to gravity is equalized by a ground force pushing up.

Force is that which puts a car in motion or changes its speed, or tends to do so. (If a parking brake is not set on a grade, the weight force due to gravity will put a car in motion.) Gravity tends to put a car at rest in motion. The ground force prevents the motion but the gravity force is still present.

In studying a car in motion we are interested in forces such as are created by friction, by the engine, or by one vehicle colliding with another vehicle, a fixed object, a pedestrian, or an animal. Weight is a special force because it is always present, and it is an important factor. Other forces are measured in lbs. just as is the gravity force called weight. The term "retarding force" is used herein frequently to identify a force acting in a direction opposite to the direction a car is moving. The force itself in effect is no different from any other force.

Newton’s 2nd law of motion states that (1) acceleration is proportional to the force; that is, the greater the force acting on a mass the faster the speed of the mass will increase and (2) acceleration is inversely proportional to the mass; that is, the less a mass weighs the faster a given force can change its speed. Check the acceleration formula below to see how obvious the two rules are. These are "common sense" concepts and much easier to understand than kinetic energy to which this study leads.

\[
\text{Acceleration} = \frac{\text{Force}}{\text{Mass}} = \frac{\text{mass} \times \text{acceleration}}{\text{gravity}} \quad \text{(Mass = weight)}
\]

\[
\text{Force (lbs)} = \frac{\text{weight (lbs)}}{\text{gravity (ft/sec/sec)}} = \frac{\text{force (lbs) \times \text{gravity (ft/sec/sec)}}}{\text{weight (lbs)}} = \text{ft/sec/sec}
\]

Note that the lbs. cancel out and we get ft/sec/sec as the unit of measure for acceleration. It means that a speed in ft/sec changes some given amount each sec: ft/sec/sec, or ft/sec, just as mph is mph/sec (mph could be m/h but usually is written mph).

Note also that when you have a rate of acceleration the weight has been involved in obtaining it.

If a 100 lb. force is applied to a 3200 lb. car for 1 sec.

\[
a\ =\ 100 \text{ lbs.} \times \frac{32}{\text{3200 lbs}} = 1 \text{ ft/sec/sec} \quad \text{(A gain in speed of one foot per second for every second the force acts.)}
\]

If the 100 lb. force is applied constantly for several seconds the car’s speed will increase 1 ft/sec during each second and the car’s acceleration is said to be 1 ft/sec/sec.

If we know the car’s weight is 3200 lbs. and the acceleration is 1 ft/sec/sec we can determine the force.

\[
F = ma = \frac{3200 \text{ lb.} \times 1 \text{ ft/sec/sec}}{32 \text{ ft/sec/sec}} = 100 \text{ lbs.}
\]

FRICION FORCE

When two objects are rubbed together friction transforms mechanical energy into heat energy. Rubbing your hands together will demonstrate this fact.
Suppose you have a 10 lb. rubber-covered object 2" x 4" x 8" long (similar to a brick) with a steel hook in one end. You hook a kitchen scale to the brick and pull it at a constant rate across an asphalt covered surface. Suppose the scale registers a 5 lb. pull which is 50% of the brick’s weight. If you place a 10 lb. weight on top of the brick, the scale would then register a 10 lb. pull which is still 50% of the weight of the brick and its load. The lb. pull force horizontal to the asphalt surface has doubled but so has the weight force pressing the object against the asphalt, and the ratio of the pull force to the weight force remains constant.

Suppose now you turn the brick onto its 2" side and pull it again. The lb. pull will be 5 lbs. as at first. The contact area between the brick and the asphalt is only half as many square inches, but the weight of the brick is the same and the weight pounds per square inch is twice as great as when the brick was pulled on its 4" side.

Theoretically the weight of the object and the area of the contact surface do not alter the ratio of the horizontal pull force to the vertical weight force. The pull force, however, must be exactly parallel to the plane between the contact surfaces of the brick and the asphalt, and in line with the center of mass of the object.

The weight force presses a car’s drive wheels against the ground; friction between the tires and the ground tends to keep the wheels from spinning as the engine turns the rear axle; and the car moves.

Friction between the brake shoes and brake drums decelerates a moving car, or if the brakes are locked friction between the tires and the ground causes the car to skid to a stop.

To keep a car in motion the engine must overcome rolling friction between the tires and the ground and wind friction (or air resistance), both of which increase rapidly as a car’s speed increases.

It is apparent that friction forces are important factors in moving and stopping a car.

**COEFFICIENT OF FRICTION BETWEEN TIRES AND PAVEMENTS**

When a car’s wheels are locked, as in an emergency stop, a retarding force acts in the direction opposite to the direction the car is moving. The interlocking roughnesses between the rubber and the pavement determine how much retarding force is exerted. The force formula, \( F = ma \), gives the relation between a force (retarding force in this case) and the rate of deceleration. Obviously, the greater the retarding force the higher the rate of deceleration and vice versa.

If the rate of deceleration were 32 ft/sec/sec the retarding force would be as great as the weight force which is caused by gravity acceleration of 32 ft/sec/sec. The retarding force would be 100% of the car’s weight. The braking effort would be 100%.

The coefficient of friction (f) is a ratio between the retarding force the pavement can create, and the weight of the car. We shall call it the friction value or simply the f value of the pavement. (It is also called the drag factor.)

\[
f = \frac{\text{Force}}{\text{weight}}, \quad \text{or} \quad f \times \text{weight} = \text{Force (retarding)}
\]

If a car weighs 3200 lbs, and if when skidded with all four wheels locked (a rigid body) the pavement is rough enough to decelerate the car at a rate equal to gravity (32 ft/sec/sec), the retarding force will be equal to the car’s weight and

\[
f = \frac{\text{Force}}{\text{weight}} = \frac{3200 \text{ lbs}}{3200 \text{ lbs}} = 1.0
\]

(f values of dry used pavements usually range from 0.5 to 0.8 and are never as high as 1.0)

If the pavement can decelerate the locked-wheel car at only 16 ft/sec/sec the rate is only half of gravity, the retarding force is only half the car’s weight and

\[
f = \frac{\text{Force}}{\text{weight}} = \frac{1600 \text{ lbs}}{3200 \text{ lbs}} = 0.5 \text{ (read point 5)}
\]
If a towing truck carrying a suitable spring scale should tow a car (with all wheels skidding) at a constant speed the scale would register the retarding force in lbs. and we could use the above formula to find the $f$ value of the pavement.

$$f = \frac{\text{Force}}{\text{weight}}$$

A more practical way to determine the $f$ value is to drive a car at a steady speed, lock the wheels with the brakes, measure the skid distance and place the data in the following formula:

$$f = \frac{(\text{mph})^2}{30 \times \text{skid distance}}$$

(The "30" is a constant which results when the formula is derived from the KE and Work formulas)

If the speed was 30 mph and the skid distance was 50 ft. then

$$f = \frac{30^2}{30 \times 50} = \frac{900}{1500} = 0.6$$

Note that the weight is not a factor in this formula. (For an explanation see FORMULAE.) So long as the car is a rigid body (all wheels skidding) it does not matter how much the car weighs. In the example the retarding force would be 60% of the weight and the car would decelerate at 0.6 of 32 (gravity) or 19.2 ft/sec/sec whether the car weighed 3200 lbs. or 4000 lbs.

If a car's brakes were not good enough to lock all wheels we could still use the same formula to determine the braking effort exerted by the brakes. Suppose we applied the brakes hard at 30 mph but the wheels would not lock and the braking distance was 70 ft. Then the BE (braking effort) would be

$$\text{BE} = \frac{30^2}{30 \times 70} = \frac{900}{2100} = 0.429 \text{ or 42.9%}$$

(a retarding force in lbs. equal to 42.9% of the car's weight was exerted.)

This gives the efficiency of the car's brakes but it does not give the $f$ value of the pavement because the car was not a rigid mass sliding on the pavement.

We did learn however that the car's brakes were illegal because the minimum braking effort required by law is 52.8%. The law requires a stop in 25 ft. from 20 mph.

If we had conducted the test on a pavement with an $f$ value less than 0.429 the brakes could have locked the wheels and the answer would have given us the $f$ value of the pavement.

It is apparent that the braking distance depends on both the efficiency of brakes and the $f$ value of a pavement. The lower of the values of these two factors determines how long the braking distance will be regardless of how high the value of the other factor is.

**KINETIC ENERGY**

We come now to the most important subject in our study of physical forces because it involves the forces that destroy cars and kill people. It is a concept very difficult for drivers to understand.

Mass and energy are the two basic phenomena of the universe. Energy exists in many forms: mechanical, heat, chemical, electrical, light, and atomic energy. Energy may be transformed from one form to another but the total amount in the universe is constant.

Two forms of mechanical energy are potential energy and kinetic energy.

Potential energy is the capacity of a mass or body to do work due to the body's position. Water behind a dam, a bullet in a gun, a compressed spring, and a car at rest on a hill are forms of potential energy.
If a car at rest on a hill is set in motion downhill by gravity, the potential energy begins to change to kinetic energy. As the car rolls on a level rolling friction and air resistance slowly change the kinetic energy into heat energy, and when all of the kinetic energy is transformed into heat energy the car will stop.

If the car’s brakes are applied, friction between the brake shoes and brake drums will change the kinetic to heat energy more quickly.

If the wheels are locked, friction between the tires and the pavement will change the kinetic energy into heat energy very quickly.

If the car should collide with a fixed object the impact will change the kinetic energy into heat energy instantly. In any case all of the kinetic energy must be dissipated before the car will come to rest.

If at the bottom of the hill the car had started rolling up another hill, its kinetic energy due to motion would have changed back into potential energy due to position of height. If there had been no rolling friction and air resistance the car would have rolled as high on the second hill as it was on the hill from which gravity set it in motion. Actually, of course, rolling friction and air resistance were changing some of the kinetic energy into heat energy all the time the car was rolling downhill. Consequently the car could never regain its original height.

Neither can a rubber ball dropped in free fall. Its PE and KE keep alternating, losing out to heat energy each time it hits the floor and to air friction while in motion, until it stair-steps down to zero PE and zero KE.

If the top of the original hill was 100 ft. above the bottom of the hill and the car’s weight was 3200 lbs., the car when at rest at the top of the hill possessed 320,000 ft. lbs. of potential energy.

Potential energy = weight \times height 
= 3200 \text{ lbs} \times 100 \text{ ft} = 320,000 \text{ ft lbs}

If we disregard the friction losses due to air and rolling, the car’s kinetic energy at the bottom of the hill was 320,000 ft. lbs., because Kinetic Energy = Potential Energy less such losses. If, therefore, we know the KE of a car in motion we can divide the KE by the car’s weight and find the height to which the KE would raise the car if the car were suddenly directed upward vertically. The textbook formula for KE is

\begin{equation}
\text{Kinetic Energy} = \frac{1}{2}mv^2 = \frac{1}{2} \text{ mass} \times (\text{ft/sec})^2
\end{equation}

If 32.2 is used as the value of gravity in the above formula it will give approximately the same answer as another KE formula in which the velocity in mph is used and which is simpler for illustrations:

\begin{equation}
\text{KE} = \frac{\text{Weight} \times (\text{mph})^2}{30} \quad (\text{The "30" is a constant})
\end{equation}

the KE of a 3000 lb. car traveling 60 mph is

\begin{equation}
\text{KE} = \frac{3000 \times 60^2}{30} = \frac{3000 \times 3600}{30} = 360,000 \text{ ft lbs}
\end{equation}

As stated before, \( \text{PE} = \text{weight} \times \text{height} \). Then

\begin{equation}
\text{height} = \frac{\text{PE}}{\text{weight}} \quad \text{Since KE = PE,}
\end{equation}

\begin{equation}
\text{height} = \frac{360,000 \text{ ft lbs}}{3000 \text{ lbs}} = 120 \text{ ft.}
\end{equation}

This is the height to which the car would be projected by its KE if it were suddenly directed upward vertically.

As the car left the ground moving 60 mph (88 ft/sec), gravity would start decelerating its speed at a rate of 32 ft/sec/sec and the car’s upward velocity would be zero after 2.75 sec.
If instead of being directed upward, the car had been skidded to a stop on a pavement with an \( f \) value of 1.0, the pavement, instead of gravity, would have decelerated the car at a rate of 32 ft/sec/sec and the car would have come to a stop in 120 ft. in 2.75 sec. (Or we might say that \( a = -32 \text{ ft/sec/sec} \))

The pavement with an \( f \) value of 1.0 retarded the car with a force equal to the weight of the car just the same as gravity retarded the car's upward motion with a force equal to the weight of the car.

The rate of acceleration or deceleration is the controlling factor in controlling a car. While engine horsepower is also a factor in positive acceleration, the \( f \) value of a pavement is a key factor both in positive acceleration and in negative acceleration or deceleration and, therefore, a key factor in the braking distance.

A formula which gives the distance required to change velocity when the rate of acceleration or deceleration and the time are known shows the relation of the rate of deceleration to braking distance. Keep in mind that acceleration is a rate of changing velocity per unit of time, and once we know the change in velocity and the rate of acceleration or deceleration we can easily find the time by dividing the amount of change by the rate. (See time formula above)

The distance formula is as follows:

\[
S = ut + \frac{at^2}{2}
\]

- \( S \) = distance in ft
- \( u \) = initial vel. in ft/sec
- \( t \) = time in sec
- \( a \) = acceleration in ft/sec/sec

\( a = \) acceleration in ft/sec/sec

(\(+a\) if speed is increasing
and \(-a\) if decreasing)

The initial velocity of the car when projected upward or when skidded on the pavement with an \( f \) value of 1.0 was 88 ft/sec. In each case acceleration was negative or \(-32 \text{ ft/sec/sec}\). We therefore can get the time, 2.75 sec. Substituting the data in the preceding formula we get

\[
S = (88 \times 2.75) - \frac{32 \times 2.75^2}{2} = 242 - 32 \times 7.55 = 121 \text{ ft}
\]

The difference of 1 ft. in the answer here and the 120 ft. in the PE formula is due to a higher value of gravity used in the KE formula. (See discussion of at bottom of pp 24 and 30.)

KINETIC ENERGY, WORK, AND BRAKING DISTANCE

Kinetic Energy = Work

Work = Force \times \text{distance}

To have a proper respect for the limitations which kinetic energy places on a driver in controlling a car a driver must understand

(1) What is meant by the statement that KE increases in proportion to the square of the speed.

(2) Why braking distances theoretically will increase in proportion to the KE.

(3) That if, due to variable factors such as slight changes in the \( f \) value during skid stops at different speed, the braking distance in demonstrations is not always proportional to the KE, still unquestionably the KE increases by the speed squared whether or not the measured braking distances reveal the fact exactly. Once the speed is there the KE is there. Therefore, the speed per se represents explicitly the destructive power of a moving car.

(4) That at speeds above 50 mph the formula braking distances are impractically short because to attain them a driver must lock his brakes and consequently lose control of his car. At high speed a skidding car will likely leave the traffic lane and might skid into opposing traffic or into a fixed object.
A thorough understanding of KE is the key to driver self-discipline. There would be few arguments about speed if all drivers really understood the implications of speed^2 in relation to the destructive power of the KE which speed^2 develops.

**Work** must be done to change the speed of a car, usually by the engine to increase the speed and by the brakes to decrease the speed. The difference between the kinetic energies at two different speeds, therefore, represents the work done. If one of the two speeds is 0 mph then all of the car’s energy would be the work done.

Suppose a car’s speed is changed from 30 mph to 60 mph.

KE of a 3000 lb car at 60 mph = 360,000 ft lbs

KE of a 3000 lb car at 30 mph = 90,000 ft lbs

KE of the change of 30 mph = 270,000 ft lbs

Note that the KE of the change of 30 mph here is 3 times the KE of 30 mph. This figures, since the KE at 60 mph is 4 times the KE at 30 mph.

Work done = 270,000 ft lbs = difference between the two KE’s.

Work = Force \times distance, or distance = \frac{Work}{Force}

If the car accelerated from 44 ft/sec (30 mph) to 88 ft/sec (60 mph) in 11 seconds its rate would be 4 ft/sec/sec, and the force required for this rate would be

\[ \text{Force} = ma = \frac{3000 \text{ lbs}}{32 \text{ ft/sec/sec}} \times 4 \text{ ft/sec/sec} = 375 \text{ lbs} \]

The distance required to accelerate from 30 mph to 60 mph would be

\[ \text{distance} = \frac{\text{Work}}{\text{Force}} = \frac{270,000 \text{ ft lbs}}{375 \text{ lbs}} = 720 \text{ ft} \]

If the car were decelerated from 88 ft/sec to 44 ft/sec at a rate of 16 ft/sec/sec the retarding force would be

\[ \text{Force} = ma = \frac{3000 \text{ lbs}}{32 \text{ ft/sec/sec}} \times 16 \text{ ft/sec/sec} = 1500 \text{ lbs} \]

The distance required would be \[ \frac{270,000 \text{ ft lbs}}{1500 \text{ lbs}} = 180 \text{ ft} \]

To get the distance required to skid the car to a stop from a speed of 88 ft/sec (60 mph) on a pavement with an f value of 0.5 by using the work formula we first get the retarding force by using the formula, Force = f \times weight and find Force = 0.5 \times 3000 \text{ lbs} = 1500 \text{ lbs}. Then

\[ \text{distance} = \frac{\text{Work}}{\text{Force}} = \frac{360,000 \text{ ft lbs}}{1500 \text{ lbs}} = 240 \text{ ft} \]

Note that 360,000 ft lbs., the KE at 60 mph, is the KE difference between 60 mph and 0 mph.

If the f value had been 1.0, the force would have been 3000 lbs. and the distance 120 ft.

By using the formula, Work = Force \times distance, we again verify the distances obtained in other ways and confirm the relations between Speed, Energy, Work, Braking Distance, f value, Force, and Acceleration.
KINETIC ENERGY, WORK, AND HORSEPOWER

To accelerate a 3200 lb. car at a rate of 4 ft/sec/sec requires the application of a constant force which by the force equation gives

\[
\text{Force} = ma = \text{weight} \times \text{acceleration} = 3200 \text{ lbs} \times \frac{32 \text{ ft/sec/sec}}{32 \text{ ft/sec/sec}} = 400 \text{ lbs}
\]

If the 400 lb. force is applied for 22 seconds the car will attain a speed of 88 ft/sec (60 mph).

\[
v = at = 4 \times 22 = 88
\]

The KE of a 3200 lb. car moving 60 mph = 384,000 ft lbs.

The “Work = Force \times distance” formula gives the distance required for the car to reach 60 mph; that is, to develop 384,000 ft lbs. of KE.

\[
distance = \frac{\text{Work}}{\text{Force}} = \frac{384,000 \text{ ft lbs}}{400 \text{ lbs}} = 960 \text{ ft}
\]

If the force were 800 lbs. instead of 400 lbs. the accelerating rate would be 8 ft/sec/sec and the distance would = 480 ft. When the force is doubled, the acceleration rate is doubled and the distance is halved.

But few stock cars can accelerate as fast as 8 ft/sec/sec. Our example of 4 ft/sec/sec is a fair rate for the average car.

Low horsepower cars can attain fairly high speeds but they require more time. Fast acceleration requires high horsepower. The shorter the time in which a given amount of work is done, the more horsepower required.

Power is the rate at which work is done per unit of time. KE tells us how the weight force of a car is “expanded” due to its speed. The KE difference between two speeds is a measure of the work done. Work tells us how far the KE present can project the weight force. Power determines how fast the work can be done.

\[
\text{Power} = \frac{\text{Work}}{\text{Time}} \quad \text{(Time may be minutes or seconds)}
\]

In the preceding example the work done was 384,000 ft lbs. and it took 22 seconds. The power required was

\[
\text{Power} = \frac{384,000 \text{ ft lbs}}{22 \text{ seconds}} = 17,455 \text{ ft lbs/sec}
\]

Horsepower (hp) is a conventional term for expressing power, somewhat like saying a horse is 15 hands high instead of saying 60 inches or 5 ft.

\[
\text{hp} = \frac{33,000 \text{ ft lbs}}{60 \text{ in}} = \frac{550 \text{ ft lbs}}{1 \text{ hand}}
\]

To do this work in half the time would require 64 hp.

Let us check the increase in hp required when the 3200 lb. car is loaded to 4000 lbs:

The work done in accelerating a 4000 lb. car from rest to 60 mph = 480,000 ft lbs.

If the car were accelerated at 8 ft/sec/sec the time required would be 11 sec (88 ft/sec divided by 8 ft/sec/sec = 11 sec), and the power required would be
Power = $480,000 \text{ ft lbs} = 43,636 \text{ ft lbs/sec}$. The horsepower would be
\[
\frac{43,636 \text{ ft lbs/sec}}{550 \text{ ft lbs/sec/hp}} = 79.3 \text{ horsepower}
\]

Accelerating the 3200 lb. car to 60 mph in 11 sec. takes 64 hp and accelerating a 4000 lb. car takes 70 hp, an additional 15 hp for the extra 800 lbs. This is the weight of a carload of adults added to the 3200 lb. car. About 80% of the power is used in accelerating the car’s dead weight.

The dead weight cost of fast acceleration is extreme when a driver is alone. The cost is justified when a driver uses horsepower to prevent position conflicts in traffic which cause collisions or congestion.

There may be a lot of difference between the rated horsepower of an engine and the horsepower available for changing positions quickly in traffic at normal driving speeds. One manufacturer cites the following example:

An auto engine rated 200 gross horsepower at 100 mph might have only 145 gross horsepower at 60 mph. It might produce only 100 net horsepower at the rear wheels due to power requirements of auxiliary equipment and incidental power losses caused by muffler, generator, fan, etc.

From the net horsepower at the rear wheels must be deducted the road-load horsepower used in normal operation of a car on smooth level roads to offset rolling friction and air resistance. This road-load need might range from 15 hp at 40 mph to 45 hp at 70 mph.

After road-load horsepower is deducted from the net horsepower available at the rear wheels, the power left is called reserve horsepower. This is the power available for accelerating, for climbing, and for driving on rough ground.

Brake horsepower relates to a method of rating an engine and has nothing to do with a car’s brakes, but braking requires power. Consider the example where a 4000 lb. car was accelerated at 8 ft/sec/sec to a speed of 88 ft/sec. If the 4000 lb. car is braked to a stop from 88 ft/sec with a braking effort of 50% the car will decelerate at a rate of 16 ft/sec/sec and require 5.5 sec to stop.

\[
\text{Power required} = \frac{480,000 \text{ ft lbs}}{5.5 \text{ sec}} = 87,272 \text{ ft lbs/sec}
\]

\[
\text{hp required} = \frac{87,272 \text{ ft lbs/sec}}{550 \text{ ft lbs/sec}} = 158.6
\]

If the braking is done with the wheels turning, the brakes must deliver this horsepower in the form of a retarding force created by friction between the brake shoes and the drums. The brakes must produce twice as much power in decelerating the car at 16 ft/sec/sec as the engine produced in accelerating the car at 8 ft/sec/sec. In accelerating to 60 mph the energy was developed in 11 seconds and in braking it is dissipated in 5.5 sec. However, the engine can repeat its performance immediately again and again. The brakes cannot. After a few consecutive braking operations from 60 mph the brakes would be so hot they would fade. Heat created in the engine is scientifically controlled. Brakes simply radiate heat into the surrounding air.

If the braking is done with the wheels locked, the tires and pavement must deliver this horsepower in the form of a retarding force created by friction at the tire-pavement contact points, the areas of which are much smaller than the contact areas between the shoes and the drums. Moreover, the rubber tires cannot compete with the tough pavement. Consequently many travel miles of rubber is peeled off the tires during skid stops. In a skid stop from high speed rubber loss may throw the wheels out of balance.

In a test with a new car weighing 4405 lbs., 5 hard-braking, rolling-wheel stops from 60 mph were made on a new, level pavement. On the 6th stop the driver tried to lock the brakes. The braking distance was 347 ft. This amounted to a braking effort of 34.6%. The car’s brakes when cold probably could have exerted an effort of 100%. This test would have required a pavement with an f value of 1.0. The braking distance would have been 120 ft.
KINETIC ENERGY AND MOMENTUM

Kinetic energy involves a force acting through a distance.

Momentum involves a force existing through a time.

Often persons relate momentum to braking distance hazards. If braking distances and destructive forces were proportional to momentum for a given car, they would be proportional to the speed, and speed would not be the problem it is in accident prevention program. For example, if a braking distance at 2 mph were 2 ft. the braking distance at 60 mph would be 60 ft. instead of 200 ft. as it is (with a braking effort of 60%). To show how the two concepts are related it is better to use the formula for KE which requires velocity in ft/sec instead of mph: \[ KE = \frac{1}{2} \text{mass} \times (\text{ft/sec})^2 \]

The unit of measure for KE is ft-lb and for Momentum it is lb-sec:

\[ KE = \frac{1}{2} \text{weight} \times \text{vel}^2 = \frac{\text{lb}}{\text{ft/sec/sec}} \times (\text{ft/sec})^2 = \text{lb} \times \text{ft}^2 \times \text{sec}^2 = \text{ft-lb} \]

\[ \text{Mom} = \text{mass} \times \text{vel} = \frac{\text{weight} \times \text{vel}}{\text{ft/sec/sec}} = \text{lb} \times \text{ft/sec} = \text{lb-sec} \]

*While lb-ft would seem as natural here as lb-sec for momentum, the term lb-ft is used in mechanics as a measure of the moment of force around an axis. The term ft-lb is used for KE in order to avoid confusion. In both cases lbs. are multiplied by ft.

A 3200 lb. car moving 88 ft/sec possesses (1) KE and (2) Momentum as follows:

1. \[ KE = \frac{3200 \times 88^2}{64} = 50 \times 7744 = 387,200 \text{ ft lbs} \]
2. \[ \text{Mom} = \frac{3200 \times 88}{32} = 100 \times 88 = 8800 \text{ lb-sec} \]

Since KE = Work, the KE represents a weight-force of 3200 lbs. exerted through a distance of 121 ft.

\[ \text{distance} = \frac{\text{Work}}{\text{Force}} = \frac{387,200 \text{ ft lbs}}{3200 \text{ lbs}} = 121 \text{ ft} \]

The Mom represents the impulse a weight-force can create against a resistance by virtue of the mass's speed.

\[ \text{Impulse} = \text{Force} \times \text{time} \]

\[ 8800 \text{ lb-sec} = \frac{3200 \text{ lb} \times \text{time}}{3200 \text{ lb}} = 2.75 \text{ sec} \]

That is the time required to decelerate the 3200 lb. car from a speed of 88 ft/sec to a stop, through a distance of 121 ft., against a retarding force equal to the weight of the car.

During this deceleration the average velocity (\( \bar{v} \)) of the car was 44 ft/sec, while the weight force was constant. The average Momentum then was \( \text{mass} \times \bar{v} = \frac{3200 \times 44}{32} = 4400 \text{ lb-sec} \). This is \( \frac{1}{2} \) the Momentum the car had at a speed of 88 ft/sec.

When the car accelerated from a stop to 88 ft/sec it had a different momentum for each change of 1 ft/sec in its speed.

\[ \text{At 1 ft/sec its mom was} \frac{3200 \times 1}{32} = 100 \text{ lb-sec} \]

\[ \text{Average Momentum During Each Second} \]

\[ \frac{50 \text{ lb-sec}}{50 \text{ lb-sec}} \]
At 2 ft/sec its mom was $\frac{3200 \times 2}{32} = 200$ lb-sec

At 3 ft/sec its mom was $\frac{3200 \times 3}{32} = 300$ lb-sec

... ... ...

At 44 ft/sec its mom was $\frac{3200 \times 44}{32} = 4400$ lb-sec

... ... ...

At 86 ft/sec its mom was $\frac{3200 \times 86}{32} = 8600$ lb-sec

At 87 ft/sec its mom was $\frac{3200 \times 87}{32} = 8700$ lb-sec

At 88 ft/sec its mom was $\frac{3200 \times 88}{32} = 8800$ lb-sec

Sum of averages of momenta from 1 ft/sec to 88 ft/sec = 387,200

If we sum the averages of each momentum the car had during each 1 ft/sec of speed it passed through, we get 387,200, the number of ft.lbs. of kinetic energy the car possessed while traveling 88 ft/sec. This is the way in which the impulse forces accumulate to make a mph added to a car’s speed of 60 mph so much more destructive than a mph added to some lower speed. For example, the 2nd ft/sec added 150 ft.lbs. to the energy of the 1st ft/sec. The 88th ft/sec added 8750 ft.lbs. to the energy of the 87th ft/sec. Note that the 8750 ft.lbs. of energy is an average of the momentum as the car’s speed increased from the 87th sec thru the 88th sec. The other figures in the right hand column are averages for given seconds.

At any given speed the KE will equal the sum of all the momenta a car had in arriving at the given speed.

The average of all the momenta is $\frac{3}{4}$ the momentum of the car at the speed it is moving. Note that if we multiply the average of all these momenta by the speed the car is moving we get the KE of the car at the speed it is moving.

\[
\text{KE} = \frac{3}{4} \times \text{momentum} \times \text{velocity} \quad \text{(Mom = mass} \times \text{velocity)}
\]

\[
\text{KE} = \frac{3}{4} \times (\text{mass} \times \text{velocity}) \times \text{velocity} = \frac{3}{4} \text{mass} \times \text{vel}^2 = \frac{3}{4} \text{mv}^2
\]

\[
\text{KE} = \frac{3}{4} \times \left(\frac{3200 \times 88}{32}\right) \times 88
\]

\[
= \frac{3}{4} \times (100 \times 88) \times 88
\]

\[
= \frac{3}{4} \times (8800) \times 88
\]

\[
\text{KE} = 4400 \text{ lb-sec} \times 88 \text{ ft} = 387,200 \text{ ft-lbs}
\]

Let us explore further the meaning of the ft-lbs of KE in terms of force since the concept force involves the weight of a car which is much more tangible than the term energy. One can try to lift a car and can imagine the injury resulting if only one wheel fell off a jack onto his foot.

If gravity were not present the absolute mass of a car would simply hang in air or stay on the ground, where you put it, without exerting any weight force.
Its mass, however, would still be just as real and if it hit you at a speed of 60 mph it would do the same damage as it does with gravity present. When we multiply the absolute mass of a car by the accelerating force of gravity we get weight, because gravity is what makes a mass on scale register a force in lbs. Weight (lb) = mass × gravity (ft/sec/sec). From this we find

\[
\text{mass} = \frac{w \text{ lb}}{g \text{ ft/sec}^2} = \frac{w \text{ lb}}{g \text{ ft}} \times \sec^2 = \frac{w \text{ lb-sec}^2}{g \text{ ft}}
\]

This unit of mass is called a slug, a term used to measure absolute mass. A slug is that mass which a 1 lb. force will accelerate at a rate of 1 ft/sec/sec. We find that any object weighing 32 lbs. is the required mass.

\[
\text{Force} = \text{mass} \times \text{acceleration}
\]

\[
1 \text{ lb} = \frac{32 \text{ lb-sec}^2}{32 \text{ ft}} \times a
\]

\[
1 \text{ lb} = \frac{1 \text{ lb-sec}^2}{\text{ft} \cdot \sec^2} \times a. \text{ Making } a \text{ the subject we get}
\]

\[
a = \frac{1 \text{ lb ft}}{1 \text{ ft} \cdot \text{sec}^2} \text{ or } 1 \text{ ft/sec/sec}
\]

If the 1 lb. force acts only for 1 sec. the

\[
\text{Impulse} = \text{Force} \times \text{time} = 1 \text{ lb} \times 1 \text{ sec} = 1 \text{ lb-sec}
\]

The sum of impuses = change in momentum

The velocity of the slug is changed from 0 ft/sec to 1 ft/sec. Its velocity is 1 ft/sec.

\[
\text{Momentum} = \text{mass} \times \text{vel (or slugs} \times \text{ft/sec)}
\]

\[
= \frac{w \text{ lb-sec}^2}{\text{g ft}} \times \text{vel}
\]

\[
\text{mom} = \frac{32 \text{ lb-sec}^2}{32 \text{ ft}} \times 1 \text{ ft/sec} = \frac{1 \text{ lb-sec}^2}{\text{ft} \cdot \sec^2} \times 1 \text{ ft} = 1 \text{ lb-sec}
\]

An impulse of 1 lb. acting 1 sec. on 1 slug (32 lbs.) accelerates the slug to a velocity of 1 ft/sec. thereby changing its momentum or in this case giving it a momentum of 1 lb-sec.

If the force had been 100 lbs. acting on a mass of 100 slugs (3200 lb. car) the acceleration would still be 1 ft/sec/sec

\[
F = ma
\]

\[
100 \text{ lb} = \frac{3200 \text{ lb-sec}^2}{32 \text{ ft}} \times a = 100 \text{ lb-sec}^2 \times a \quad \frac{a = 100 \text{ lb ft}}{100 \text{ lb-sec}^2} = 1 \text{ ft/sec/sec}
\]

If the 100 lb. force acted for only 1 sec. the car's velocity would be 1 ft/sec and its momentum would = mass × vel = 100 slugs × 1 ft/sec = 100 lb-sec.

(See p. 50 or p. 101 for detailed information on the impact force of the lb-sec.)
1. If you try to lift a car you get an idea of how its mass \( \times \) gravity = weight.

2. If you try to hold back a slowly rolling car you get an idea of how its momentum = mass \( \times \) velocity = weight \( \times \) gravity. But neither one of these concepts will give you an idea of kinetic energy.

3. If a 3200 lb. car is accelerated from a stop to 88 ft/sec, its momentum at the speed of 88 ft/sec is 8800 lb-sec.

\[
\text{Momentum} = \frac{\text{weight} \times \text{vel}}{\text{gravity}} = \frac{3200 \text{ lbs} \times 88 \text{ ft/sec}}{32 \text{ ft/sec/sec}} = 8800 \text{ lb-sec}
\]

But the car had a momentum of 8700 lb-sec at 87 ft/sec and a momentum of 8600 lb-sec at 86 ft/sec, and so on.

If you should figure the average momentum the car developed in accelerating to each ft/sec it passed through, and total the results, you would get 387,200 lb-sec. (The average from 0 ft/sec to 1 ft/sec would be 50, etc. The average from 87 ft/sec to 88 ft/sec would be 8750.)

4. The number (387,200) in the above total is the same as the number of the ft.lbs. of kinetic energy of a 3200 lb. car moving 88 ft/sec.

Keep in mind that the momentum (mass \( \times \) vel) of the 3200 lb. car going 88 ft/sec = 8800 lb-sec.

If we multiply the mom (8800 lb-sec) by \( \frac{1}{2} \) the vel (44 ft/sec) the time factor (sec) cancels out and we get 387,000 ft.lbs. of kinetic energy.

Or we could multiply \( \frac{1}{2} \) the mom (4400 lb-sec) by the vel (88 ft/sec) and get the same product.

5. The kinetic energy formula is written \( \frac{1}{2} \text{mv}^2 \) or \( \frac{1}{2} \text{mass} \times \text{vel}^2 \). This is just another way of stating what we have said above:

\[
\text{kinetic energy} = \frac{1}{2} \text{mom} \times \text{vel}. \text{Substituting mass} \times \text{vel} \text{for momentum, we get KE} = \frac{1}{2} (\text{mass} \times \text{vel}) \times \text{vel} = \frac{1}{2} \text{mass} \times \text{vel}^2 = \frac{1}{2} \text{mv}^2
\]

6. Momentum vs Time and Kinetic Energy vs Distance

a. The momentum can tell us how long it will take a resisting force to stop the mass (weight-force) of the moving car.

\[
\text{time required} = \frac{\text{momentum}}{\text{weight}} = \frac{8800 \text{ lb-sec}}{3200 \text{ lb}} = 2.75 \text{ sec.}
\]

(2) If the resisting force is 50% of the car's weight (a 50% braking effort) the time is \( \frac{8800 \text{ lb-sec}}{1600 \text{ lb}} = 5.5 \text{ sec.} \)

b. The kinetic energy can tell us how far the mass (weight-force) of the moving car will operate against a resisting force (such as is created by a pavement against locked wheels).

\[
\text{distance required} = \frac{\text{KE}}{\text{weight}} = \frac{387,200 \text{ ft lb}}{3200 \text{ lb}} = 121 \text{ ft.}
\]

(2) If the resisting force is 50% of the car's weight (a 50% braking effort) the distance is \( \frac{387,200 \text{ ft lb}}{1600 \text{ lb}} = 242 \text{ ft.} \)

c. In the above examples the denominator is obtained by multiplying the friction value (f) of the pavement by the weight of the car.
\[ f = F = \text{retarding Force, or transposing, } F = \frac{fw}{w} \]

If \( f = 0.5 \) the retarding Force = \( 0.5 \times 3200 \text{ lbs} = 1600 \text{ lbs} \)

d. It is evident that the KE of a given car moving at a given speed is related to the weight of the car and to every mph the car passed through as it was accelerated to the given speed (for which the KE is being calculated). KE increases in such a complex way that it is extremely difficult for a driver to understand its danger. Note in table below how momentum is related to time, and KE is related to distance. Note also that \( \frac{1}{2} \) the KE will develop during the upper 29% of the speed, between 21.2 mph and 30 mph.

**HOW TIME, DISTANCE, VELOCITY, MOMENTUM, AND KINETIC ENERGY VARY AS 3200 LB. CAR ACCELERATES AT 4 FT/SEC/SEC FROM STOP TO 30 MPH (Figures in 21.2 mph column are close approximations)**

<table>
<thead>
<tr>
<th></th>
<th>15 mph</th>
<th>21.2 mph</th>
<th>30 MPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>T = O</td>
<td>5½</td>
<td>7.8</td>
<td>11 sec</td>
</tr>
<tr>
<td>S = O</td>
<td>60½</td>
<td>121</td>
<td>242 ft</td>
</tr>
<tr>
<td>V = O</td>
<td>22</td>
<td>31.1</td>
<td>44 ft/sec</td>
</tr>
<tr>
<td>Mom =</td>
<td>2200 lb-sec</td>
<td>3100 lb-sec</td>
<td>4400 lb-sec</td>
</tr>
<tr>
<td>KE =</td>
<td>24200 ft lb</td>
<td>48400 ft lb</td>
<td>96800 ft lbs</td>
</tr>
<tr>
<td>( \frac{1}{2} ) the mom at ( \frac{1}{2} ) the time</td>
<td>71% of mom at 71% of time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{4} ) the KE at ( \frac{1}{4} ) the distance</td>
<td>( \frac{1}{2} ) the KE at ( \frac{1}{2} ) the distance</td>
<td></td>
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<tr>
<td>( \frac{1}{4} ) the KE at ( \frac{1}{2} ) the speed</td>
<td>( \frac{1}{2} ) the KE at 71% of speed</td>
<td></td>
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</tr>
</tbody>
</table>

**INERTIA AND CENTRIFUGAL FORCE**

Newton's 1st law of motion says in effect that a body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant speed and in a straight line.

It is apparent that a body at rest will remain in that state until some external force moves it or until the forces acting on it are unbalanced. The gravitational force of course acts constantly on a standing car, but the ground reacts with a force equal to the gravitational force and keeps the car at rest, provided it is on level ground. Letting the air out of the tires will unbalance the forces and gravity will move the car a few inches until the ground balances the forces again.

When a wheel is raised on a rickety jack to remove a flat tire it might seem that the car tries to fall off the jack. This appearance points up how subtle all natural forces are, including those affecting the control of an automobile in motion. You want the car to stay on the jack. The car wants to stay on the jack. If the car does not stay on the jack it will be due to your lack of knowledge of how forces work or to your lack of planning. The same causes hold generally for drivers who allow cars to collide or overturn.

When the forces acting on a car are unbalanced the car starts accelerating. It must accelerate some in order to move. If, after the car accelerates (to any speed) the forces are balanced again, the car tends to remain in motion at a constant speed in a straight line, indefinitely. However, rolling friction and air resistance are working constantly, and if all the engine power is cut off these two retarding forces soon bring the car to rest again.

The natural law we have been describing is called inertia. Inertia is inherent in the mass itself. The heavier the mass the greater the inertial force it exerts against change from a state of rest or change from a velocity. Actually velocity includes both a rate of speed and a direction. Once a car's direction is changed inertia tends to keep it on the new heading. It has no preference for a heading. It just opposes change. A car wants to go straight at a curve. It wants to keep going downhill or uphill and it wants to stay in the left lane when you are overtaking and passing. Inertia does not like turns because a turn is a change in direction.

The faster a car is going the more inertia it has to resist a change in direction. In fact its resistance to change increases in proportion to the speed squared. This is thorn No. 2 in a driver's halo. We examined thorn No. 1 in analyzing kinetic
energy, which also increases by the speed squared. These two thorns have extinguished over two million lives and maimed millions of bodies since the advent of the automobile some 80 years ago.

The two thorns are quite different. KE is a force in lbs. capable of being projected through a distance. It is "explosive" in effect and it persists so long as a car is in motion, regardless of whether the car is turning or going straight. It increases by the weight of a car and by the speed².

Inertia is a force in lbs. which increases by the weight of a car, by its speed² and inversely with length of the radius. Inertia acts instantly when the front wheels are turned off a straight line and it disappears instantly when the car is headed straight again.

KE is deadly enough by itself but when it teams up with inertia, the speed² which increases the two at the same time makes the two a vicious pair.

KE not only does the damage in a collision but also causes a collision when a driver over drives his braking distance, or locks his brakes to correct his error and loses directional control of his car, thus losing any opportunity to steer around an obstacle.

Inertia is an instant side force which snatches directional control of a car from its driver and causes the car either to overturn in the roadway or to skid straight ahead off a turn or curve. Inertia grabs control, returns the car to a straight path either in a roll or in a skid, and disappears. KE then operates alone skidding the car sideways on its wheels, or on its side, or top, or both into ditches, fences, bridges, trees, poles, or opposing traffic.

The higher the car's center of mass and/or the higher the f value of the pavement the easier it is for inertia to turn the car over. The lower the center of mass and the lower the value of road friction, the more likely the car is to skid. The higher the speed and the lower the f value, the smaller the turning angle needed to start a skid.

When a driver steers the front wheels into a turn the tires create side forces which oppose inertia. The force direction is toward the center of a circle along the radius of the circle. This force is called centripetal. The inertial force which opposes the centripetal force is called centrifugal. Centrifugal force is an inertial force. The force is generated by the motion of the car mass, not at the center of a circle.

The circle or centrifugal concept is appropriate because the radius of the curved path the car follows is a convenient factor in measuring inertial force.

But a student behind the wheel cannot cope with circles whose radii may extend 100 to 500 ft. out into a field or woods. The student should be taught to estimate the rate of increase of the turning angle between a car's horizontal axis and the car's original straight path. On a curve with a 100 ft. radius the turning angle will increase to 90° in half the time it takes on a curve with a 200 ft. radius. If a car's speed is the same on both curves the CF will be twice as great on the smaller circle. The faster a car's front end turns away from the original path the higher the CF, for a given weight and speed. The angle increases very rapidly in a right turn at an intersection. This is the reason the speed must be very low in order to make a legal turn.

A driver can look ahead to the deadpoint in a curve and estimate how rapidly his car will have to change direction if he stays in the roadway. The bend in the road is fixed, but he can reduce the rate at which the angle increases by reducing the rate at which his car is moving over the ground.

We said above that if a car's speed is the same on both curves (100 ft. and 200 ft.) the CF will be twice as great on the smaller circle as on the larger circle. A look at the formula for centrifugal force will show why this is true.

\[
\text{CF} = \frac{\text{mass} \times (\text{ft/sec})^2}{\text{radius}} = \frac{\text{weight} \times (\text{ft/sec})^2}{\text{gravity} \times \text{radius}}
\]

The weight and speed which will be the same on both curves are in the numerator of the equation. Gravity is a constant. The radius then is the factor which determines the CF. If the radius is 100 ft. the quotient will be twice as large as when the radius is 200 ft.
Now back to our statement that the only way a driver can reduce the rate at which his car's heading will change, on a curve ahead, is to reduce the rate at which his car is moving over the ground, that is, his speed. While one of the thorns in safe driving is the fact that CF increases by the speed^2, the fact that it does so makes it unnecessary to reduce the speed by half in order to have on a 100 ft. curve the same CF as on the 200 ft. curve.

To check this fact suppose a 3200 lb. car moves around a 200 ft. curve at 30 mph (44 ft/sec).

\[
CF = \frac{3200 \times 44^2}{32 \times 200} = \frac{3200 \times 1936}{6400} = 968 \text{ lbs}
\]

How much will the speed have to be reduced to have the same CF of 968 lbs. on a 100 ft. curve?

\[
968 = \frac{3200 \times v^2}{32 \times 100} = v^2 \quad (v = \text{ft/sec})
\]

\[
v = \sqrt{968} = 31.1 \text{ ft/sec or } 21.2 \text{ mph}
\]

A speed of 30 mph on a 200 ft. curve created a CF of 968 lbs.

A speed of 30 mph on a 100 ft. curve would create a CF of 1936 lbs.

A speed of 21.2 mph on a 100 ft. curve would reduce the 1936 lbs. to 968 lbs. Reducing the speed 29.3% reduces the CF 50%.

The CF acts outward against the car's center of mass while the centripetal pavement friction force acts inward at the tire-pavement contact points. The center of mass (C/M) is also called the center of gravity (C/G). It is the point around which the weights of all parts of the car are in balance. The point shifts slightly when a person enters the car, again when another person enters and so on. When the seats are filled and the trunk is loaded the C/M shifts considerably toward the rear. If it shifts very much beyond the rear of the mid point between the car's axles the car becomes less sensitive to steering pressures. The tail begins to wag the dog. Steering control becomes unstable.

In turning the car with two occupants into a curve the driver might experience a slight resistance to the turn but would automatically add a little pressure to keep the car in the curve. With the C/M shifted toward the rear due to extra passengers and a loaded trunk the driver might find upon steering into a curve that the front wheels tend to go toward the inside of the curve instead of resisting the turn. This surprises the driver and he quickly corrects the oversteer. He overcorrects and sets up a dangerous swinging action to the right and left of the center of the lane.

When rear wheels are overloaded it is very important that tire pressure be increased in order to prevent abnormal tire deflection which induces oversteer. Station wagons with full loads of passengers or cargo are especially subject to oversteer instability on curves.

**ADDITIONAL FORCES AFFECTING A CAR ON A CURVE**

The unstable steering condition mentioned above is related to cornering forces (side thrusts) and slip angles (the angle between the direction a turned wheel is headed and the direction it actually travels) which involve wheel loads, weight shifts, wheelbase, camber, toe-in, tire inflation, speed, etc., variables too involved for this unit but very important to a driver if his car is subject to oversteer, which at very high speed might cause the car to go out of control toward the inside of a curve in half a second.

Gyroscopic forces created by the turning wheels exert some pressures for or against driver control in a curve.

Wind force can be an important safety factor on a curve. Both the velocity and the angle at which wind strikes a car as it changes direction on a curve might tilt the scale either for or against a driver who is about to lose control of his car. Of two cars which have the same surface areas the lighter car is at a disadvantage when the wind force is already against the driver. Small cars usually are scaled down more in weight than in surface area and therefore are more likely to be less stable on curves in wind.
CONCEPTS

Concepts a teacher should understand are listed to aid him in preparing a teaching outline. The extent to which he can cover the concepts in a minimum course unit is limited, but he should be able to treat them properly in a unit of ten to twelve hours on physical forces in a semester course.

1. Acceleration and the meaning of ft/sec/sec
2. Relation of force to acceleration in changing the velocity of a mass
3. Inverse relation between acceleration and deceleration distances
4. Weight as a force and its relation to the acceleration of gravity
5. Friction force and its relation to acceleration and deceleration
6. Friction coefficient and its relation to gravity acceleration and weight
7. Relation of potential energy to weight and gravity acceleration
8. Kinetic energy and its relation to potential energy
9. Kinetic energy and its relation to work
10. Kinetic energy and its relation to velocity
11. Kinetic energy and its relation to braking distance
12. Braking distance and its relation to pavement friction
13. Brake efficiency and its relation to braking effort (and f value)
14. Kinetic energy and its relation to power
15. Relation of power to acceleration and brake performance
16. Kinetic energy and its relation to momentum
17. How the lb-sec of momentum becomes a destructive impact force.
18. Relative speed and relative energy
19. Inertia and centrifugal force
20. Relation between centripetal force and pavement friction
21. Relation of center of mass and tire deflection to directional control

In a minimum course unit a teacher should expect his students to understand at least

1. How ground distances covered per second are opposite in length for one car accelerating and one car decelerating at the same rate.

2. How ground distance covered in the first second when decelerating from high speed is related to the total braking distance percentagewise.

3. How speed changes kinetic energy and how kinetic energy changes braking distances, with emphasis on change between 30 mph and 42.4 mph.

4. How kinetic energy of top 5 or 10 mph of a given speed is related to the total kinetic energy at the given speed.

5. How friction value of pavement changes skid distances in an emergency stop and limits centripetal force in a curve.

6. How speed and turning radius change centrifugal force.

7. How location of center of mass (% of gross load on rear tires) and rear tire deflection (air pressure) affect steering control on curves.

8. How ground distance covered during perception time and reaction time compares with braking distance at a given speed.

9. How the lb-sec of momentum becomes a destructive impact force.

A student should acquire enough knowledge of these concepts to make recommended defensive driving procedures meaningful and acceptable. He does not have to be able to define terms precisely or discuss concepts with exactness in order to profit by a brief study, provided the study is well organized—that is, the concepts are logically related.
Before a flight instructor teaches students the basic defensive procedure of landing into the wind, he explains lift forces just enough to make the students respect the rule. The instructor is teaching flying, not physics, and he devotes only such time to the forces as will enable the students to make sound decisions. But he knows that flying is basically a skill in controlling forces. And he knows from his own experience that a rule without a reason may be sound but soon ignored. So he does not dare omit an explanation of the forces. One student may be able to analyze the forces involved and explain the principle while another might not be able to discuss them intelligently, yet both respect them and both practice the rule religiously. The instructor himself may not be adept in explaining the force concepts yet is able to prepare students for years of accident free flying by teaching procedures firmed up by a respect for physical forces, which he described before he started teaching procedures.

The task of teaching students to drive safely is probably more difficult than teaching them to fly safely. The exposure to hazards certainly is more immediate. And the hazards are physical forces.

**STUDY QUESTIONS FOR TEACHER**

1. It is 50 miles from A to B. A car averages 40 mph going from A to B and without stopping returns from B to A averaging 60 mph. What was the time for the round trip? Answer 2 hrs. 5 min.

   \[ \bar{v} = \bar{s}, \text{ or } \frac{t}{v} = \bar{s}. \text{ Caution: you cannot average averages.} \]

2. A car accelerates from 10 mph to 40 mph in 11 sec. What is its rate of acceleration in ft/sec/sec? Answer 4 ft/sec/sec.

   Use formula \( a = \frac{v - u}{t} \). Convert mph to ft/sec.

3. A car accelerates from 10 mph for 20 sec at a rate of 3 ft/sec/sec. What is the car's speed at the end of 20 sec? Answer 50.9 mph.

   Use formula \( v = u + at \). Convert mph to ft/sec and convert answer back to mph.

4. A 3200 lb. car is traveling 20 mph. An accelerating force of 200 lbs. is applied to the car for 20 sec. What is the car's speed in mph at the end of 20 sec? Answer 47.3 mph.

   Use formula, \( F = \text{mass} \times \text{acceleration} \) to get \( a \). Then, \( v = at \) to get speed from a stop. Convert ft/sec to mph.

5. What is the force required to accelerate a 3200 lb. car 7 ft/sec/sec? Answer 700 lbs.

   Use formula \( F = ma \)

6. What is the retarding force exerted against a 3200 lb. car skidding on a pavement with an \( f \) value of 0.7? Answer 2240 lbs.

   Use formula, \( f = \frac{\text{Force}}{\text{weight}} \)

7. What is the rate of deceleration in ft/sec/sec of the car in problem 6? Answer 22.4 ft/sec/sec.

   \[ a = \frac{F}{m} = \frac{F}{w} = \frac{g \times g}{g} = g \times g \]

8. If a truck requires 147 ft. to stop from 42 mph what is the % braking effort exerted by the truck's brakes? Answer 40%.

   Use formula, \( \text{Braking Effort} = \frac{V^2}{30 \times s} \)
9. The driver of a 6000 lb. pickup traveling 50 mph sees a car approaching his traffic lane from a stop sign. He locks his brakes and skids to a stop a few inches from the car as its rear end clears his path (The f value of the pavement is 0.6). If he had been moving 60 mph when he locked his brakes, at what speed would he have hit the car? Answer 33.2 mph.

You can use formula, Braking Distance = \( \frac{v^2}{30 \times f} \), and disregard the weight, since the car was skidding.

10. From what height would a 3200 lb. car have to drop in free fall in order to develop the kinetic energy the car possesses at a speed of 70 mph? Answer 163.3 ft.

Use formulas, KE = \( wV^2 \) and PE = \( \text{weight} \times \text{height} \). KE = PE.

11. The speed the car in problem 10 would have had after a free fall of 163.3 ft. would be the speed from which the car would skid 163.3 ft on a pavement with an f value of 1.0—that is, a pavement which would decelerate the car at the rate of gravity, 32 ft/sec/sec. What was the car's speed when it hit the ground? Answer 70 mph.

You can use formula, \( v^2 = u^2 + 2as \), or formula \( f = \frac{v^2}{30} \), but note that speeds v and u are ft/sec while V is mph.

12. A car loaded weighs 4000 lbs. and is traveling 60 mph. To what mph would the driver have to increase his speed in order to increase the car's skid distance (at 60 mph) an amount equal to the car's skid distance at 40 mph? The f value of the pavement is 0.6. Answer 72 + mph.

Suggestion: Use either braking distance formula or KE formula and check your answer with the other. This is a good example for showing students that the KE and BD of 12.8 mph when added to 60 mph is equivalent to the KE and BD of a speed of 40 mph.

13. In problem 12 if the rates of acceleration are the same show that it takes the driver traveling 60 mph less than \( \frac{5}{3} \) as long to add KE of 40 mph as it takes to accelerate from a stop to 40 mph. What is the exact time ratio? Answer 1:3.33 This can be done by dividing 40 mph by (72 mph - 60 mph).

14. KE and braking distances double at the following speeds: 5.3 mph - 7.5 mph - 10.6 mph - 15 mph - 21.2 mph - 30 mph - 42.4 mph - 60 mph - 84.8 mph. Use the KE formula to verify these values.

15. Use the braking distance formula with an f value of 0.5 to confirm the fact that the distances also double between any two of the speeds listed in problem 14.

16. Select from problem 15 one of the speeds with the distance found for an f value of 0.5 and check the speed and distance with the formula, \( v^2 = u^2 \pm 2as \), where \( v \) = final speed in ft/sec, \( u \) = initial speed in ft/sec, \( a \) = rate of acceleration and \( s \) = distance. You can assume the car accelerated at 16 ft/sec/sec, then \( u \) would be zero and \( a \) would be plus:

\[ v^2 = 2as = 2 \times 16 \times s. \]

Or you can assume the car is decelerated from the initial speed \( u \).

You then use \( -a \) and set \( v = 0: \)

\[ 0^2 = u^2 - 2as, \text{ or } -u^2 = -2as, \text{ or } u^2 = 2as = 2 \times 16 \times s. \]

Moreover, you can make a the subject of the formula and find the rate of acceleration or deceleration when you know the distance covered between two known speeds. In a deceleration problem the value of \( a \) can be converted to the braking effort exerted. In the above problem you will find \( a = 16 \text{ or } -16 \text{ and } 16 = 50\% \text{ braking effort. And more still. If the car skidded in decelerating from } u \text{ speed to } v \text{ speed, the braking effort becomes the } f \text{ value of the pavement or 0.5.} \]

17. Compute the maximum speed in mph at which a 3200 lb. car can stay in a flat curve of 200 ft. radius without skidding if the f value of the pavement is 0.6. Answer 42.3 mph.
First, determine the amount of CF the pavement can withstand:

\[ f = \text{Force}, \text{ or } \text{Force} = f \times w = 0.6 \times 3200 = 1920 \text{ lbs} \]

Then set this value for CF in the formula, \( \text{CF} = \frac{\text{weight} \times v^2}{32 \times \text{radius}} \), and solve for \( v \) (ft/sec).

Finally convert the speed in ft/sec to mph.

18. Compute the maximum speed in mph at which a 4000 lb. car can stay in the same curve as in problem 17 without skidding, to determine whether the weight of a car is a factor in averting a slide in a curve.

**FORMULAE**

- \( m = \frac{w}{g} \)
- \( h = \frac{PE}{w} \)
- \( F = ma \)
- \( F = wa \frac{g}{a} \)
- \( KE = PE = wh \)
- \( a = \frac{v - u}{t} \)
- \( v = u \pm at \)
- \( v = \frac{u + v}{2} \)
- \( v^2 = u^2 + 2as \)
- \( a = \frac{v^2 - u^2}{2s} \)
- \( s = ut \pm \frac{at^2}{2} \)
- \( \overline{v} = \frac{D}{T} \)
- \( D = VT \)
- \( T = \frac{D}{V} \)
- \( W = Fs \)
- \( F = \frac{W}{s} \)
- \( s = \frac{W}{F} \)

- \( a = \text{acceleration in ft/sec/sec} \)
- \( BD(s) = \text{braking distance} \)
- \( BE = \text{braking effort in } \% \text{ of car’s weight} \)
- \( D = \text{distance in miles} \)
- \( F = \text{force in lbs} \)
- \( f = \text{coefficient of friction (a ratio)} \)
- \( g = \text{gravity} \)
- \( h = \text{height in ft} \)
- \( hp = \text{horsepower} \)
- \( KE = \text{kinetic energy in ft-lbs} \)
- \( m = \text{mass in 32 lb slugs} \)
- \( \text{mom} = \text{momentum in lb-sec} \)
- \( P = \text{power in ft lbs/sec} \)
- \( PE = \text{potential energy in ft-lbs} \)
- \( s = \text{distance (space) in ft} \)
- \( t = \text{time in hrs} \)
- \( t = \text{time in sec} \)
- \( u = \text{initial speed in ft/sec} \)
- \( v = \text{final speed in ft/sec} \)
- \( \overline{v} = \text{average velocity in ft/sec} \)
- \( V = \text{speed in mph} \)
- \( \overline{V} = \text{average speed in mph} \)
- \( w = \text{weight in lbs} \)
- \( W = \text{Work in ft-lbs} \)
A driver's big problem consists of controlling a car while coping with kinetic energy and inertia. All the forces discussed in this outline are related to that problem. The forces act at the four tire-pavement contact points and at or around the point called the center of mass. The forces constantly threaten to trigger kinetic energy in a direction the driver does not want to go, as the driver alters speed and direction to follow a road or to steer around obstacles.

How formulas \( BD(s) = \frac{V^2}{30f} \) and \( KE = \frac{wV^2}{30} \) are related:

An explanation of why the braking distance formula does not contain a weight factor while the kinetic energy formula does may clarify the concept that changing the load of a skidding car does not alter the braking distance at a given speed. The \( f \) value factor in the \( BD \) formula is a ratio between the Force and weight factors in the \( KE \) formula:

\[
\begin{align*}
    s &= \frac{V^2}{30f} \\
    f &= \frac{F}{w} = \text{Force} / \text{weight} \\
    s &= \frac{V^2}{30} \times \frac{F}{w} \\
    s &= \frac{V^2}{30} \times \frac{w}{F} \\
    Fs &= V^2w, \quad Fs = \text{Work} = KE \quad \frac{30}{30} \\
    KE &= \frac{V^2w}{30} \\
\end{align*}
\]

When a car skids, 100% of its weight force is exerted vertically against the pavement. The \( f \) value tells us what percent the horizontal retarding force is of the weight force. The heavier a car is the greater the vertical force; and the greater the retarding force, which is a percent of the weight force.

How the textbook formula \( KE = \frac{1}{2}mv^2 \) is changed to \( KE = \frac{wV^2}{30} \) in which the speed is mph instead of ft/sec:

\[
\begin{align*}
    KE &= \frac{1}{2}mv^2 = \frac{1}{2} \times w \times v^2 = \frac{1}{2} \times \frac{w}{g} \times \frac{v^2}{64.4} = \frac{w}{g} \times \frac{v^2}{64.4} \\
    v &= 1.467 \times V \quad \text{V = mph} \quad (\text{ft/sec} = 1.467 \times \text{mph}) \\
    v^2 &= (1.467)^2 \times V^2 = 2.15 \times V^2 \\
    KE &= \frac{wV^2}{30} = \frac{w}{g} \times \frac{2.15}{64.4} \times V^2 = \frac{wV^2}{64.4} \times \frac{2.15}{30} \quad \text{Substitute} \quad (1.467^2 \times V^2) \quad \text{for} \quad v^2 \\
\end{align*}
\]

How the formula \( BD(s) = \frac{V^2}{30f} \) is changed to the velocity formula for determining the minimum speed from skid marks:

\[
\begin{align*}
    s &= \frac{V^2}{30f} \quad V = \text{mph} \quad s = \text{average length of four skid marks} \\
\end{align*}
\]
How the foot pound (ft-lb) and the pound second (lb-sec) become units of measure of KE and momentum.

It is necessary that proper units be used in the equations, as lb for force and ft/sec/sec for acceleration. While ft/sec may be converted to mph, a change in unit requires we add a proportionality constant or a conversion factor to give unity to the equation. This was done when KE = \( \frac{1}{2} mv^2 \) was made KE = \( wV^2 \). 

If proper units are stated properly, units may be cancelled. This fact makes equations of units valuable. While equations are often stated without writing in the units, it should be clear that the units determine the validity of an equation. One cannot appreciate the relations involved unless he can see how units are cancelled to give the measures. Keep in mind that *per* means "divided by."

1. Momentum = mass \( \times \) velocity
   
   \[ \text{Mom} = \frac{\text{mass} \times \text{velocity}}{\text{g ft/sec/sec}} \]

   First, let us see how ft/sec/sec becomes ft/ sec^2

   \[ \text{ft/sec/sec} = \text{sec} = \text{ft} \times \frac{1}{\text{sec} \times \text{sec}} = \text{ft} \times \frac{1}{\text{sec} \times \text{sec}} \]

   Here the main division is ft/sec divided by sec, the change in v, ft/sec, per unit of time, sec.

   \[ \frac{1}{\text{sec} \times \text{sec}} \]

   Then \( w \) becomes \( \frac{1}{\text{lb} \times \frac{\text{sec}^2}{\text{g ft sec}}} \)

   Here the main division is weight \( \text{lb} \) divided by \( \frac{\text{g ft sec}}{\text{sec}^2} \)

   \[ \frac{1}{\text{sec}^2} \]

   Now the whole equation:

   \[ \text{Mom} = \frac{\text{mass} \times \text{velocity}}{\text{g ft/sec/sec}} \]

   After we cancel units we have lb and sec in the numerator, \( \text{lb} \times \text{sec} = \text{lb-sec} \)

2. KE = \( \frac{1}{2} \) mass \( \times \) velocity^2

   \[ \text{KE} = \frac{\frac{1}{2} \text{lb} \times \text{sec}^2 \times (\text{ft})^2}{1 \text{ ft sec}} \]

   \[ = \frac{\frac{1}{2} \text{lb} \times \text{sec}^2 \times \text{ft}^2}{1 \text{ ft sec}^2} = \frac{\text{lb} \times \text{ft}}{2} \text{ ft-lb} \]

   The two sec^2 cancel and one ft cancels leaving a ft in the numerator. The \( \frac{1}{2} \) represents an average of the momentum involved as velocity is changed. It is a constant in this particular equation. (We cannot cancel 2 ft and ft^2.)
3. Work = \( Fs = ma \times s = \frac{w}{a} \times s \)

\[
= \frac{1}{g} \times \frac{m}{lb} \times \frac{a}{x \times ft \times ft} \times \frac{ft}{sec^2} \times \frac{sec^2}{sec^2} \\
= \frac{lb \times sec^2 \times ft \times ft}{1} \times \frac{lb \times ft}{ft \times sec^2} = \text{ft} \cdot \text{lb} \\
\]

The mechanical quantities in the formulae are simply combinations of the fundamental quantities mass, space (distance), and time. Here are examples:

**Velocity = distance per unit of time**

\[ v = \frac{S}{t} \]  (In these examples a captial S is used in v and a to distinguish its distance from s in Work = Fs.)

**Acceleration = change of velocity per unit of time**

\[ a = \frac{v}{t} \times \frac{1}{t} = \frac{S}{t} \times \frac{1}{t} = \frac{S}{t^2} \]

(Note that \( a = \frac{v - u}{t} \) but if \( u = 0 \) then \( a = \frac{v}{t} \))

**Force = mass times acceleration**

\[ F = ma = \frac{mS}{t^2} \]

**Energy = Work = force times distance**

**Work = Fs**

\[ Work = \frac{mS}{t^2} \times s \]  (while the S and s each represent distance they are different quantities)

**Power = Work per unit of time**

\[ = \frac{Work}{t} = \frac{Work}{t} \times \frac{1}{t} \]

\[ = \frac{mS}{t^2} \times s \times \frac{1}{t} = \frac{mS}{t^3} \times s \]

Let us check the value of \( a = \frac{S}{t^2} \) in the Force = \( \frac{mS}{t^2} \) equation above and then check the Work = Fs equation against the equation KE = \( \frac{1}{2}mv^2 \).

Assume a 3200 lb. car traveling 88 ft/sec decelerates at a rate of 32 ft/sec/sec. We may select any rate we like but we use 32 ft/sec/sec and a weight of 3200 lbs. because with the vaule of gravity it makes the arithmetic simpler.

\[
\text{Time to stop} = \frac{v}{a} = \frac{88 \text{ ft/sec}}{32 \text{ ft/sec}^2} = 2.75 \text{ sec} \\
\]

\[
a = \frac{S}{t^2}, \text{ Then } S = at^2 = \frac{32 \text{ ft}}{sec^2} \times (2.75 \text{ sec})^2 = \frac{32 \text{ ft}}{sec^2} \times 7.56 \text{ sec}^2 = 242 \text{ ft} \\
\]
\[ a = \frac{S}{t^2} = \frac{242 \, \text{ft}}{(2.75 \, \text{sec})^2} = \frac{32 \, \text{ft}}{7.56 \, \text{sec}^2} \]

(This checks with \( a \) in \( v \))

\[ F = ma = \frac{w \times S}{g} = \frac{3200 \, \text{lb} \times 242 \, \text{ft}}{32 \, \text{ft} \times 7.56 \, \text{sec}^2} = 3200 \, \text{lb} \]

(Retarding force when \( a = 32 \, \text{ft/sec}^2 \))

Work = \( Fs \). We can find the value of this \( s \) with \( s = ut - \frac{at^2}{2} \)

\[ s = \frac{88 \, \text{ft} \times 2.75 \, \text{sec} - 32 \, \text{ft} \times (2.75 \, \text{sec})^2}{2 \, \text{sec}^2} \]

\[ = 242 \, \text{ft} - \frac{32 \, \text{ft} \times 7.56 \, \text{sec}^2}{2 \, \text{sec}^2} \]

\[ = 242 \, \text{ft} - \frac{32 \, \text{ft} \times 7.56}{2} \]

\[ s = 242 \, \text{ft} - 242 \, \text{ft} = 121 \, \text{ft} \]

Work (ft-lb) = \( F \) lb \times \( s \) ft = 3200 lb \times 121 ft = 387,200 ft-lbs

\[ KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 3200 \times 88^2 = 3200 \times 7744 = 387,200 \, \text{ft-lbs} \]

(Note that work done on a mass involves motion of the mass. You might "work" hard trying to lift or push a mass but you do no work on it unless it moves.)

Factors used to Convert Measures of Velocity:

1. To convert miles per hour to feet per second:
   If a car moves 1 mile in 1 hour, it moves 5,280 feet in 3600 seconds, or 1.467 feet in 1 second
   \[ \frac{5280 \, \text{ft}}{3600 \, \text{sec}} = 1.467 \, \text{ft/sec}. \]
   Example: 60 mph \times 1.467 = 88 ft/sec.

2. To convert feet per second to miles per hour:
   If a car moves 1 foot in 1 second, it moves 3600 feet in 3600 seconds, or 0.682 mile (3600 ft) in 1 hour (3600 sec.)
   \[ \frac{5280 \, \text{ft}}{3600 \, \text{sec} \times 0.682} = \text{mph}. \]
   Example: 88 ft/sec \times 0.682 = 60 mph

Precaution in Working a Problem involving Velocity.

1. In Section IV, B, 3, the equation \( v = at \) will give the velocity from a stop, when the rate of acceleration and the time are known.

   If a car starts at 0 mph and accelerates 4 ft/sec/sec for 11 seconds its velocity will be \( a \times t \), or \( 4 \times 11 = 44 \, \text{ft/sec} \).

2. If a car starts at 44 ft/sec and accelerates 4 ft/sec/sec for 11 seconds, the final velocity would be the initial velocity \( (u) \) plus \( a \times t \), or \( v = u + at = 44 + (4 \times 11) = 88 \, \text{ft/sec} \).
3. If a car travels 10 sec at a velocity of 88 ft/sec the distance \( s = \bar{v} \times t = 88 \times 10 = 880 \) feet. The velocity here is assumed to be the average velocity which may be denoted by \( \bar{v} \). A constant speed becomes an average velocity.

4. If a car moving 44 ft/sec accelerates to 88 ft/sec in 11 sec, the distance covered during acceleration is determined by multiplying the average velocity by the time. The average velocity \( \bar{v} \) is \( \frac{u + v}{2} \) the sum of the initial velocity \( u \) and the final velocity \( v \).

\[
\bar{v} = \frac{u + v}{2} = \frac{44 + 88}{2} = 66 \text{ ft/sec.}
\]

\[
The \ distance(s) = \bar{v} \times t = 66 \times 11 = 736 \text{ ft.}
\]

Values of the Rate of Acceleration called Gravity.

While the acceleration rate due to the gravitational attraction of the earth is considered a constant, actually it varies slightly over the surface of the earth, ranging approximately from 32 ft/sec/sec at the equator to 32.2 ft/sec per sec at the poles.
PART TWO

VISUAL AIDS

PLEASE READ PREFACE AND INTRODUCTION

Each of the following pages in PART TWO contains a special treatment of related subjects. The purpose of this approach is to provide the instructor with condensed reviews of certain text materials in PART ONE and PART THREE together with illustrations which might be used as visual aids.

While the illustrations might be reproduced on large poster boards or drawn on the blackboard, a more economical and effective way to use them is in overhead projections on a wall or screen. Plastic sheets with which one can make transparencies on a duplicating machine within a few seconds are now available. In some instances an instructor might wish to revise the sketches and reduce the typed material before making transparencies.

After one or two trial runs an instructor should be able to improve his transparencies and make them so inclusive that he can cover most of the unit on physical forces by using physical demonstrations and screen projections. Concise statements defining concepts or describing complex relations projected along with drawings are as conducive to learning as are the drawings themselves.

I. Friction

A. Procure a toy vehicle weighing a few pounds or load a toy truck. Weigh vehicle and load. Lock all wheels so that the tires will skid. Attach a light hand scale to the truck and read the pounds registered by the scale when the truck is pulled steadily across a table. This number of pounds pull divided by the gross weight will give the coefficient of friction between the tires and the surface of the table. By using surfaces of different degrees of smoothness, different coefficients of friction can be shown.

B. Place a toy car with locked wheels on a smooth board about \( \frac{1}{4} \)" thick by 6" wide by 20" long. Raise one end of the board slowly beside an upright ruler until the car slips down the slope. Then lower the board slightly and measure the height of the raised end (in inches) at a position just below where the car will slip. The height divided by the horizontal distance between the ruler and the low end of the board gives the per cent grade: for example, \( \frac{10}{17} \) in = 0.59 = 59% grade. It also gives the static coefficient of friction between the tires and the surface:

\[
f = \frac{10}{17} = 0.59.
\]

C. Wet the surfaces used in A or B with soap foam or other suitable lubricant and repeat the demonstrations. Note the differences in the f values.

D. In demonstration A, pull the scale very slowly at the start and note that the pulling force required to move the car from rest is slightly higher than the force required to keep it moving at a constant rate after it is placed in motion. This demonstrates that the static coefficient of friction is higher than the kinetic coefficient of friction. After the car starts moving and at a very slow rate, it may start a jerking action which will decrease as the speed is increased. This action is called "stick and slip." Stick and slip makes a door with dry hinges squeak. It will squeak more when opened slowly than when opened fast. This stick and slip effect may cause the f value of a pavement at very low speed to be slightly higher than the f value at normal driving speeds.

II. Force Concepts

A. To visualize the "force" concept consider a 3600 lb. car standing on the ground. If you try to lift the car your effort is opposed by a force which is created by gravity acting on the car's mass. We say the car's weight is 3600 lbs. We might just as well say the car's force is 3600 lbs.

The force equation is: Force = mass × acceleration.
The mass equation is: mass = weight. Cross multiplying, we have weight = mass \times \text{gravity (acceleration)}. 

If an engine could accelerate our car as fast as gravity can the constant force the engine would have to exert would be

\[
\text{Force} = ma = \frac{w}{g} \times 32 = 3600 \text{ lbs}
\]

Attach secure handles to a 100 lb. cube and place it on a bathroom scale. Direct two students to pull up on the cube until the scale registers zero lbs. Each student will get the feel of a 50 lb. force. They will be balancing the gravitational pull on the cube mass.

B. To visualize the “force through distance” concept assume we get 36 men to lift the 3600 lb. car one foot off the ground. The men will expend at least 3600 ft. lbs. of energy. If they raise the car two feet, they double the work done on the car, which then possesses 7200 ft. lbs. of potential energy.

Work = \text{Force} \times \text{distance} = 3600 \text{ lbs} \times 2 \text{ ft} = 7200 \text{ ft lbs (potential energy stored up)}

If the men now push the car with its wheels locked a distance of one foot over a surface with an f value of 1.0, they will do the same work as in lifting the car one foot.

However, in raising the car the men’s work transformed the motion energy into potential energy, because the car could then fall and do the same amount of work. In skidding the car on the road surface, the men transformed the motion energy into heat energy in the form of friction between the tires and the surface.

Recall that for an f value of 1.0 the retarding force must equal the weight of a car, \( f = \frac{w}{g} = 3600 = 1.0 \). A moving car skidding on a surface with an f value of 1.0 decelerates at a rate of 32 ft/sec/sec, which is the value of gravity.

The men’s one-foot pushing job required the same amount of work as lifting the car one foot against the car’s force (weight) created by gravity. If the f value of the pavement were 0.5 instead of 1.0, the same work they did to push the car one foot would move it two feet, because the retarding force created by the pavement would be one-half as great. For the same reason a car going 60 mph will skid 240 feet on a pavement with an f value of 0.5 where it would skid only 120 feet if the f value were 1.0.

In raising the car one foot, each man produced 100 ft. lbs. of potential energy. The 100 lb. cube is too heavy for one student to lift, but have two students, after they feel the 50 lb. force, to raise the 100 lb. cube from the scale platform to the top of a stool which stands exactly one foot higher than the scale platform.

Have the students visualize first the 100 lb. cube at rest on a person’s hand or foot or head and second, the cube falling one foot onto a person’s hand or foot or head.

The difference in the severity of injury resulting is caused by the invisible kinetic energy created by the cube’s (force) movement through a distance under the acceleration of gravity. The rate of acceleration is of key importance so far as the amount of energy is concerned but only because it determines the velocity of the mass at the point of impact with the person’s hand or foot or head.

C. To visualize the relation between “weight \times \text{distance}” and “weight \times \text{velocity}”:

Procure a 10 lb. cube and a 5 lb. cube, preferably with bases the same dimensions as the 100 lb. cube. Mark a point on the classroom wall 10 feet above the floor.

First explain that the 10 lb. cube dropped from the 10-foot height can develop the same kinetic energy or the same destructive force as the 100 lb. cube dropped from a one-foot height.
The difference in weights is made up by the speed energy which one cannot see. The 10 lb. cube hits the floor going 3.162 times as fast as the 100 lb. cube. The square of 3.162 is 10, and $10 \times 10$ lbs. = 100 lbs.

Velocity of 100 lb. cube after dropping 1 ft:

\[
\begin{align*}
v^2 &= u^2 + 2a \hspace{1cm} v = \text{final velocity in ft/sec} \hspace{1cm} u = \text{initial velocity in ft/sec} \hspace{1cm} a = \text{rate of acceleration in ft/sec/sec} \hspace{1cm} s = \text{distance in feet} \\
v^2 &= 0 + 2 \times 32 \times 1 \\
v^2 &= 64 \\
v &= \sqrt{64} = 8 \text{ ft/sec}
\end{align*}
\]

Velocity of 10 lb. cube after dropping 10 ft:

\[
\begin{align*}
v^2 &= 0 + 2 \times 32 \times 10 \\
v^2 &= 640 \\
v &= \sqrt{640} = 25.298 \text{ ft/sec}
\end{align*}
\]

Kinetic Energy of 100 lb. cube after dropping 1 foot:

\[
\begin{align*}
KE &= \frac{1}{2} \text{mass} \times \text{velocity}^2 = \frac{1}{2} \text{weight} \times \frac{\text{velocity}^2}{\text{gravity}} \\
KE &= 100 \times 8^2 = 100 \times 64 = 100 \text{ ft lbs}
\end{align*}
\]

Kinetic Energy of 10 lb. cube after dropping 10 feet:

\[
\begin{align*}
KE &= 10 \times 25.298^2 = 10 \times 640 = 100 \text{ ft lbs}
\end{align*}
\]

After two students raise the 100 lb. cube one foot onto the stool, have them handle the 5 lb. cube and then explain that the 5 lb. cube dropping 10 feet will possess the same amount of energy each student expended in lifting 50 lbs. of the 100 lb. cube to a height of one foot. The difference in the feel is the kinetic energy created by the speed of the 5 lb. cube after it drops 10 feet.

The advantages of having a 100 lb. cube instead of a 50 lb. cube are (1) there is less danger of lift injuries with two students lifting a 100 lb. cube than with one student lifting a 50 lb. cube, because it is easier for students to keep their backs straight, holding to one handle on one side of a cube; (2) it is desirable that all students get the feel of a 100 lb. weight by trying to push or rock it even though you do not want them to try to lift it. In discussing the 432,000 ft. lbs. of energy of a 3600 lb. car going 60 mph you want all students to have some idea of how difficult it is to move an even 100 lb. force.

The areas of the bases of all cubes should be the same so that there will be no question about variations in damage a falling cube might do due to area distribution of the forces, when comparing the drops of the 100 ft. cube and the smaller cubes. For a 100 lb. cube you might place a heavy wooden box of compact dimensions on the scale and fill it with dry sand until you would have 100 lbs. with a top nailed on. With a trial weighing you can trim the box walls down to near the dimensions of the sand. You might drill two holes in opposite sides of the box for inserting heavy rope handles but heavy metal handles which will fit two hands would be better. For a more compact cube you might bury some scrap iron in the sand. A cement cube with a base at least 1 ft. square would be desirable. Since these demonstrations are extremely important and driver education is here to stay, it will be a good investment to procure or construct durable props.

D. To visualize the "force through time" and "impact force" concepts consider that it probably took the men
several seconds to push the car one foot. An occupant sitting in the car asleep might never know the car was moved. Suppose now a heavy truck bumps the car and skids it one foot in a split second. The same amount of work is done, the same amount of energy is expended but in a much shorter time, creating a more powerful blow due to a quick change in the momentum of the car. The sleeping occupant would certainly know the car had been moved. He might go to the hospital with a popped neck.

III. Inertia and Centrifugal Force

A. Place a toy car on a board so that it will be crosswise to the slope when one end of the board is raised. Raise one end of the board slowly. Measure the height of the raised end when the car skids. Then tape a small weight to the top of the car to raise the center of mass. Raise one end of the board slowly toward the position from which the car skidded in the first demonstration. If the center of mass was raised appreciably the car will tilt before the board is raised high enough for the car to skid. This will show how centrifugal force acting against a high center of mass can tilt a vehicle on a curve or sharp turn before the centrifugal force is great enough to overcome the friction between the tires and pavement and make the car skid off the curve.

B. Draw a curved roadway on a piece of plywood about 2 ft. x 3 ft. Lubricate the board with soap foam. Raise one end slightly. Head a toy car down the slope with front wheels turned to follow the curve to show that directional control is maintained when the front wheels are free to roll. The car will roll around the curve to the side of the board. Then lock all wheels with the front wheels turned to follow the curve as in the first demonstration. Note that the car skids straight ahead off the curve, demonstrating how locking the front wheels causes loss of directional control. When the wheels stop turning centripetal force is reduced and inertia makes the car go straight.

A lubricated surface is not essential, but it will help insure a skid instead of a tilt should the board have to be raised too high to overcome the surface friction.

When the wheels are locked, inertia tends to move the center of mass of the car in a straight line regardless of the f value of the surface. The front of the car is simply resting on so many square inches of rubber on the surface, and ordinarily on a smooth hard surface the angle the tires' contact points make with the ground does not matter. (It would matter on a soft surface where the sides of the front tires could dig in.)

If the friction value of the surface is low enough, the car's front wheels may be turned to follow the curve and even left free to roll. If the end of the board is raised a little faster to simulate a car going into a slick curve too fast, the car will still slide off the curve. This demonstration shows inertia working as in the preceding demonstration (because the centripetal force is reduced) but the centripetal force here is reduced in spite of the turning wheels because the surface friction is too weak to overcome inertia and make the car go in the direction the turning wheels are leading.

The fact that when rolling wheels are turned they change the direction of a car shows that they "channel" the drive of the rear wheels to one side. But they can do this only because the friction of the surface is strong enough to overcome inertia. If the speed is low the centrifugal force is low and a low f value will be sufficient to make the car turn. If the speed is too high for the f value, the turned rolling wheels still point the direction but the surface is too weak to set up enough side force to overcome inertia. In turns, therefore, the f value limits the speed at which directional control can be maintained, even with all wheels rolling.

C. Procure an electric multi-speed turntable which can be accelerated manually. If the equipment has a turntable smaller than 12 inches in diameter, a thin circular plywood or corrugated cardboard top might be placed over the regular table to provide a longer radius. At some place on the table fasten a wedge about 1 1/2" wide x 3 in. long to simulate a banked curve. One side of the wedge should be very thin and the other side about 1/8" thick. Draw sections of curved roadways on the table, one with a radius exactly twice as long as the other. Measure each radius to a point over which the center of a car will be located. Mark the point or spots where the car wheels will rest. If the radius of the spots on outside roadway is exactly twice as long as the radius of the spots on inside road, the speed of the outside car will be twice as high as that of the inside car. Procure small toy cars just large enough to be seen easily by a class, with at least two exactly alike.
The following demonstrations can be made:

1. Place a car on each roadway and select a turntable speed which will throw the outside car off while letting the inside car stay on. (On each trial move the turntable to the same rpm.) Then move the outside car closer toward the inside car until its reduced speed will permit it to stay on.

2. Take two cars exactly alike and place a weight near the center of mass of one car (or midway the wheel base on a level with the axles.) Place the cars on the same roadway to show that if the center of mass is similarly located on two cars that are alike except for weight, the weight will not cause one car to leave the roadway ahead of the other or to stay on the roadway longer. The f value, you will recall, is a ratio of the holding force to the weight of the car.

3. Take two cars alike except fasten a small weight to the top of one and place them on the same roadway to show that the car with the high center of mass will tilt over while the other car remains on the road.

4. Place two like cars the same distance from the center of the turntable with one on the banked section to show that the bank increases the effective f value of the pavement.

5. If you have a cardboard top, lubricate a small area with soap foam and place two cars the same distance from the center with one on the dry surface and one on the wet to show that the car on the low friction surface will slide off first, when the rpm of the turntable is increased slowly.

Accelerate the table slowly in tryout tests and mark the rpm control plate for speeds at which various demonstrations will be made. This preparation will prevent a waste of time in class.

In demonstration #1 you lower the centrifugal force on the outside car as you move it inward due to reductions in the car's speeds, but the shortening of the car's radius of turn increases the centrifugal force. This is a built-in flaw due to using a turntable to demonstrate centrifugal force. The speed of the outside car is lowered by shortening the radius of the car's path, while the rpm of the table remains constant. A look at the centrifugal force formula will show that the force will reduce rapidly (due to the square) as the speed is lowered but also that shortening the radius will increase the force at a given speed, due to the radius being in the denominator of the formula.

\[ CF = \frac{\text{mass} \times \text{speed}^2}{\text{radius}} \]

To avoid this error you would need two turntables with the two cars (one on each) on equal radii. You could then produce different speeds by changing the rpm of one turntable. In spite of this minor flaw, the one-turntable demonstration probably is more effective.

IV. Kinetic Energy and Braking Distances (Refer also to Section II Force Concepts.)

A. A 3600 lb. car falling 120 feet hits the ground going 60 mph and possesses 432,000 ft. lbs. (3600 lb. \( \times \) 120 ft.) of kinetic energy just before the impact. An object weighing 432,000 lbs. falling 1 foot would hit the ground with the same energy. 432,000 lbs \( \times \) 1 ft = 432,000 ft lbs.

We have translated the energy created by the speed, into weight in order to give the student a physical picture of the invisible energy possessed by a mass due to its speed. We have shown that the energy of a 3600 lb. car going 60 mph is comparable to 432,000 lbs. of mass or 216 tons (dropped 1 ft.).

If you will cut 120 small blocks of wood about the size of a toy car and preferably about the same weight, and stack the 120 blocks into a cube structure beside the toy car, the bulk of the structure will fairly well show figuratively how the energy of the car at 60 mph increases the size and weight of the car (3600 \( \times \) 120 = 432,000). Then arrange two cubes of 30 blocks each. One of these cubes will represent 30 mph. Both cubes will represent 42½ mph and will show how the energy doubles by adding 12½ mph to 30 mph.
B. Procure a semi-electric 22 caliber detonator with which reaction time distance and braking distance can be marked on the pavement and measured. After students have studied kinetic energy, conduct a field demonstration for the class.

Be sure that the speedometer is exactly correct for speeds at which stopping tests are to be run, the tires are properly inflated and the front brakes are equalized. For more accurate results see that the speedometer needle is steady on the mark when brakes are locked. A rising or falling needle may introduce an error. (Car’s speed may fall ½ mph during reaction time.)

Conduct two tests, one at 15 mph and one at 30 mph to show (1) how reaction time distance increases with speed and (2) how braking distance increases by the square of the speed.

Then conduct a test at 21 mph to show that the braking distance doubles when the speed is increased 6 mph (from 15 mph to 21 mph) and doubles again when the speed is increased 9 mph (from 21 mph to 30 mph). See section III C page 84.

V. Acceleration

A. In the acceleration section of the outline there is a table showing the ground distance covered each second during deceleration from 60 mph to a stop, with a braking effort of 50%. Procure a toy car about 3 inches long and let its length represent 18 feet, the length of an average car. Then cut to the car’s scale, slats of wood (about ½ in. sq.) to represent the distances covered each second of braking. Paint the slats different colors. On one side of each slat print the length in feet it represents. On the same side at the left end (assuming the car is traveling from left to right) print the speed in mph the car is going at the start of the second which the slat represents. The figures will aid you in discussing how the distances vary during each second of braking. Lay the slats end to end in front of the car according to the seconds they represent.

B. In the acceleration section there is a table showing ground distances covered each second by a car accelerating from a stop at a rate of 4 ft/sec/sec. Cut slats of wood to scale and label with distances and speeds similar to the procedure described in subsec. A above. Lay the slats end to end in front of the car according to the seconds they represent.

C. The props in A and B above can be used jointly to show where an overtaking car will be after each second of braking when a car starting from a stop enters the traffic lane ahead of the overtaking car. For this illustration it might help to paint slats representing the same second the same color.

Suggestion:

Reading the recapitulation (p. 100) at this juncture will help the teacher relate key concepts to impact injuries to vehicle occupants.

Precaution:

In working vehicle motion problems involving ft. lbs. of energy which might range from tens of thousands of ft. lbs. to over a million, do not be concerned about differences of a few hundred ft. lbs. which can show up due to variations in decimal points used, including the value of gravity. For example the constant “30” in the KE formula using mph is rounded off from 29.95, which itself is obtained from using a gravity value of 32.2 in the KE formula using ft/sec. If gravity 32 were used here the constant “30” would actually be 29.77.
GRAVITY ACCELERATION, DISTANCE, AND VELOCITY

**ANALYSIS**

Average velocity ($v$) during 1st sec:

\[
v = \frac{0 \text{ ft/sec} + 32 \text{ ft/sec}}{2} = 16 \text{ ft/sec}
\]

\[
s = vt = 16 \text{ ft/sec} \times 1 \text{ sec} = 16 \text{ ft}
\]

Average velocity during 1st & 2nd sec:

\[
v = \frac{0 \text{ ft/sec} + 64 \text{ ft/sec}}{2} = 32 \text{ ft/sec}
\]

\[
s = vt = 32 \text{ ft/sec} \times 2 \text{ sec} = 64 \text{ ft}
\]

Average velocity during 3 seconds:

\[
v = \frac{0 \text{ ft/sec} + 96 \text{ ft/sec}}{2} = 48 \text{ ft/sec}
\]

\[
s = vt = 48 \text{ ft/sec} \times 3 \text{ sec} = 144 \text{ ft}
\]

\[
a = 32 \text{ ft/sec/sec}
\]

For object starting from rest:

\[
v = at = 32 \text{ ft/sec/sec} \times 1 \text{ sec} = 32 \text{ ft/sec}
\]

\[
v = at = 32 \text{ ft/sec/sec} \times 2 \text{ sec} = 64 \text{ ft/sec}
\]

\[
v = at = 32 \text{ ft/sec/sec} \times 3 \text{ sec} = 96 \text{ ft/sec}
\]

For gravity acceleration from rest:

\[
s = ut \pm at^2
\]

\[
v^2 = u^2 \pm 2as
\]

becomes

\[
s = 16 t^2
\]

\[
v = \sqrt{64 s}
\]

<table>
<thead>
<tr>
<th>Time</th>
<th>Distance(s)</th>
<th>Velocity (v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 sec</td>
<td>16 ft</td>
<td>32 ft/sec</td>
</tr>
<tr>
<td>2 sec</td>
<td>64 ft</td>
<td>64 ft/sec</td>
</tr>
<tr>
<td>3 sec</td>
<td>144 ft</td>
<td>96 ft/sec</td>
</tr>
<tr>
<td>4 sec</td>
<td>256 ft</td>
<td>128 ft/sec</td>
</tr>
</tbody>
</table>

Ball and car would accelerate at same rate if there were no air resistance.

At the end of each second the object is moving 32 ft/sec faster than at the start of the sec, because the gravitational force is constant.

If the force had lasted only 1 sec the object would have continued at a constant speed of 32 ft/sec.

Weight is a measure of the constant gravitational force which just happens to accelerate objects at a rate of 32 ft/sec/sec. (The rate would be different on other planets and on the moon.)

\[
\text{Force} = \text{mass} \times (\text{any}) \text{ acceleration}
\]

\[
\text{Weight (Force)} = \text{mass} \times (\text{gravity}) \text{ acceleration}, \text{ or mass} = \frac{\text{weight}}{\text{gravity}}
\]

\[
\text{Weight (Force)} = \frac{\text{weight}}{32 \text{ ft/sec}^2} \times 32 \text{ ft/sec}^2 = \text{weight}
\]

Ball Force = \[1 \text{ lb} \times 32 \text{ ft/sec}^2 = \text{lb} \]

Car Force = \[3200 \text{ lb} \times 32 \text{ ft/sec}^2 = 3200 \text{ lbs}\]
A constant force of 1 lb will accelerate a 1 lb object at a rate of 32 ft/sec/sec. A constant force of 3200 lbs will accelerate a 3200 lb object at a rate of 32 ft/sec/sec.

A car engine would have to produce a constant force equal to a car's weight in order to accelerate the car at a rate of 32 ft/sec/sec. Actually, the force would have to be a little greater than the car's weight in order to overcome rolling friction and air resistance.
FORCE AND ACCELERATION

In physics a force is that which tends (1) to produce motion in a body, or (2) to produce a change of motion in a body.

The gravitational force tends to put your body in a downward motion. The earth keeps your body from moving but the gravity force is still present, luckily. If it were not there would be no friction between your shoes and the ground and you could not get traction to walk. You would weigh 0 lbs.

Distances covered during acceleration and deceleration are deceptive. A constant force of 1 lb. will accelerate a 32 lb. ball at a rate of 1 ft/sec/sec. The velocity increases 1 ft/sec, in direct proportion to the time, but the distance covered increases by the square of the time divided by 2.

The chart below shows the relationship between the distance and the time and speed for the ball accelerating at a rate of 1 ft/sec/sec from a stop to a speed of 20 ft/sec.

The next chart shows the ball decelerating at a rate of 1 ft/sec/sec from 20 ft/sec to a stop.

The charts show why drivers who pull into a traffic lane at low speed in front of a moving vehicle cannot escape being hit.

Compare the distances covered in 5 sec. and 10 sec. during deceleration with the distances covered in like times during the ball's acceleration from rest. In 5 sec. the accelerating ball covers 12½ ft. while the decelerating ball covers 87½ ft. In 10 sec. the accelerating ball covers 50 ft. while the decelerating ball covers 150 ft.

Distances in the charts will be proportional to those of a car accelerating at a normal rate of 3 ft/sec/sec (= 2 mph/sec) and a car decelerating due to engine's braking when a driver releases the gas pedal (also about 2 mph/sec).

While the rate of acceleration at 5 mph is just as high as it is at 20 mph, the ground distance covered each second is very much shorter. A high rate of acceleration means you are changing the speed a lot each second. Whether or not you cover a lot of ground each second depends on your speed when the acceleration starts. You can accelerate from a stop at a high rate in low gear for a few seconds, but you will not cover much ground. However, you will cover a short distance quickly. This is important when getting out of the path of another vehicle or getting farther ahead of it will prevent a collision.
WEIGHT AND ACCELERATION

Newton discovered that force = mass x acceleration. Your body is a mass. Your weight is a special force which is constantly acting on your body's mass with a special acceleration called gravity. In a free fall the speed of your body would increase 32 ft/sec every second.

Suppose you stand on a scale in a still elevator and the scale registers 150 lbs. If the elevator should start accelerating downward at a rate of 32 ft/sec/sec, the scale would register 0 lbs. Any time the elevator stopped accelerating and moved at a constant speed (any speed) the scale would again register 150 lbs. Your weight would double if the elevator accelerated upward at 32 ft/sec/sec.

When you sit in a car acceleration of gravity exerts a vertical force of 150 lbs. on your body against the seat. If the car is accelerated forward at a rate of 32 ft/sec/sec the acceleration will exert a 150 lb force on your body against the back of the seat. You then "weigh" 150 lbs. vertically and 150 lbs. horizontally.

If the car's brakes are locked (and the f value is 1.0) the car will decelerate at 32 ft/sec/sec and the 150 lb force which pushed your body backward will now push it forward toward the panel. (Actually a car can neither accelerate nor decelerate at such a high rate.)

Do not confuse acceleration with velocity. Acceleration = change in velocity / time

A car moving 32 ft/sec is traveling 21.8 mph. If the car stops in 1 second it decelerates at a rate of 32 ft/sec/sec. Its velocity is changed from 21.8 mph to 0 mph in 1 sec. To say a car decelerates 32 ft/sec/sec is the same as saying it decreases its speed 21.8 mph per sec.
ACCELERATION, BRAKING EFFORT, AND COEFFICIENT OF FRICTION

Acceleration is the rate of change of velocity, usually expressed as "feet per second per second" which may be abbreviated to "ft/sec/sec" or "ft/sec". If a car should increase its speed 1 mph each second, its rate of acceleration would be 1 mph/sec, or 1.467 ft/sec/sec. (1 mph = 1.467 ft/sec)

Acceleration is considered positive if the speed is increasing and negative if the speed is decreasing. Negative acceleration is also called deceleration.

Acceleration is constant if speed changes the same amount each second. When a car accelerates from rest, the speed ordinarily does not increase at a constant rate. Cars in traffic starting from rest normally will accelerate initially at rates ranging from 2 ft/sec/sec to 5 ft/sec/sec, although some cars can do better. As a car's speed increases the car's maximum rate of acceleration usually decreases until at the car's top speed the acceleration rate is zero, and the car's speed is constant.

A car specially designed and with an engine of 700 hp has been able to average an acceleration rate of 29.33 ft/sec/sec from rest over a quarter-mile course. The coefficient of friction (f) of the pavement must have been at least 0.917 (29.33 + 32) because the friction value of a pavement limits a car's rate of acceleration, regardless of how powerful the car's engine is. The same car would accelerate very slowly on ice.

To accelerate a car at the rate of gravity, 32 ft/sec/sec, the pavement's f value would have to be 1.0 (32 + 32).

To decelerate a car at the rate of gravity, 32 ft/sec/sec, the pavement's f value would have to be 1.0 and the car's brakes would have to exert a braking effort of 100%, that is, create a retarding force equivalent to 100% of the car's weight. The brakes could do this if they could lock the wheels on a pavement with an f value of 1.0.

If a car were projected vertically upwards at a velocity of 88 ft/sec (60 mph), gravity would exert a retarding force equivalent to 100% of the car's weight and would decelerate the car at a rate of 32 ft/sec/sec. After one second the car would be traveling 56 ft/sec (88 - 32). After two seconds its speed would be 24 ft/sec (56 - 32 or 88 - 64). The car would come to a "stop" in 2.75 sec (88 + 32).

The height to which the car would rise is 121 ft.

\[ s = ut - \frac{at^2}{2} \]

\[ s = (88 \times 2.75) - 32 \times (2.75)^2 \]

\[ s = 242 - 121 = 121 \text{ ft} \]

If the car going 88 ft/sec could lock its brakes on a pavement with an f value of 1.0, it would decelerate at the rate of gravity, 32 ft/sec/sec, and would come to a stop in 2.75 sec after skidding 121 ft.

If the f value were 0.5 instead of 1.0 the car could decelerate at a rate only \( \frac{1}{2} \) of 32 (gravity). The rate would be 16 ft/sec/sec and the distance would be twice as long, 242 ft.
THE DECELERATION—ACCELERATION TRAP

The distance covered each second by a car decelerating or accelerating varies in proportion to the square of the time. The square principle accounts for collision traps here just as it does in kinetic energy and centrifugal force. One second can easily be the difference between a safe margin and a collision.

\[ s = ut \pm at^2 / 2 \]

\( s \) = distance in ft \( t \) = time in sec \( u \) = initial speed in ft/sec \( a \) = rate of acceleration in ft/sec/sec

use \((+a)\) if speed is increasing

use \((-a)\) if speed is decreasing

Car A decelerates from a speed of 90 ft/sec at a rate of 18 ft/sec/sec (firm braking short of skid). The BE = 18 ft = 56.25% of 32

Distance covered during 1st sec is 9 times distance during 5th sec.

Car B enters traffic lane from a stop and accelerates at a rate of 3 ft/sec/sec (a fair rate for most starts).

Distance covered during 1st sec is 1/9 distance during 5th sec.

The trap occurs when car B enters traffic within the stopping distance of Car A. If entry is near the end of this distance Car A driver is prone to delay braking. Escape depends on Car A driver braking earlier or harder than he thinks is necessary and Car B driver accelerating faster than he thinks is necessary. Had Car A driver applied brakes 1 sec earlier he could have stopped at the 144 ft. marker, 24 ft. from point of rear bumper of Car B before it started.

Distances covered by decelerating Car A overtaking accelerating Car B show how hazardous the early seconds are:

<table>
<thead>
<tr>
<th></th>
<th>1st sec</th>
<th>2nd sec</th>
<th>3rd sec</th>
<th>4th sec</th>
<th>5th sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car A go:</td>
<td>81 ft</td>
<td>63 ft</td>
<td>45 ft</td>
<td>27 ft</td>
<td>9 ft</td>
</tr>
<tr>
<td>Car B go:</td>
<td>1½ ft</td>
<td>4½ ft</td>
<td>7½ ft</td>
<td>10½ ft</td>
<td>13½ ft</td>
</tr>
<tr>
<td>Differences:</td>
<td>79½ ft</td>
<td>58½ ft</td>
<td>37½ ft</td>
<td>16½ ft</td>
<td>-4½ ft</td>
</tr>
</tbody>
</table>

Distance Car A gains on Car B each second

To get time (t) and distances (s) for Car A to reach Car B:

Car A starts braking 168 ft from rear of Car B (car B is 19½ ft long). 225 - (19½ + 37½) = 168 ft

Distance front of Car A goes = Distance rear of Car B goes + 168 ft.

\[ s_A = 90t - 18t^2, \quad s_B = 0t + 3t^2. \]

Then set \( s_A = s_B + 168 \)

\[ 90t - 18t^2 = 3t^2 + 168 \]

\[ 21t^2 - 90t + 168 = 0 \]

Solve preceding equation for (t) and get \( t = 2.75 \) sec. Then \( s = 90t - 18 \times (2.75)^2 = 179 \) ft. Use \( v^2 = u^2 + 2as \) to get

speeds of cars at collision point.
TAILGATING

TAILGATING IS A TRUCKER'S TERM FOR "FOLLOWING TOO CLOSE"

"Following too close" is one of the ways in which a person can be "driving too fast for conditions."

The error results from a driver's inability to judge following distances accurately and/or a driver's lack of understanding of how time lag in braking affects the ground distances covered by two vehicles preceding a rear-end collision.

In the illustration two cars involved in a rear-end collision are pictured side by side to simplify the analysis.

DRIVER "A" GOING 60 MPH LOCKS BRAKES FOR AN EMERGENCY STOP.

DRIVER "B" GOING 60 MPH, 100 FT BEHIND "A", TAKES 2 SECONDS TO DETECT THE HAZARD AND REACT.

Assume an f value of 0.625 which gives deceleration rates of 20 ft/sec/sec. \( f \times \text{gravity} = 0.625 \times 32 = 20 \)

"A" locks brakes at 60 mph
"A" skids 136 ft. during "B"'s 2 sec. P—R time
"A"'s speed 32.7 mph when "B" locks brakes
"B" travels 176 ft. in 2 seconds Perception—Reaction time
"B" locks brakes at 60 mph
distance to close-60 ft. time to close-1.5 sec.
"B"'s speed 39.6 mph when it hits "A"

"B's" speed 60 mph

TO PREVENT THE COLLISION "B" WOULD HAVE TO LOCK BRAKES AT OR BEFORE POINT "A" LOCKED BRAKES.

To do this "B" would have to perceive the hazard and react in 1.14 sec. (time to travel 100 ft at 88 ft/sec).

Or he would have to be following at 176 ft. The extra 75 ft. required is the distance covered in the extra P—R time of 0.86 sec. \( 0.86 \text{ sec} \times 88 = 76 \).

MINIMUM SAFE FOLLOWING DISTANCE AT 60 MPH IS 120 FT. \((2 \times 60)\).

This distance gives a driver only 1.36 sec to perceive and react to a locked-wheel stop made by a vehicle he is following. The driver must have his eyes on traffic continuously. If his reaction time is 0.75 sec., he will have only 0.61 sec. in which to detect the hazard.

A MUCH SAFER FOLLOWING DISTANCE AT 60 MPH IS 180 FT. \((3 \times 60)\).

This distance gives a driver only 2.05 sec. to perceive and react to a locked-wheel stop. If a driver requires 0.75 sec. in reacting, he would have only 1.3 sec. perception time, in which to detect the hazard, but this is more than double the time he would have when following at 120 ft.
BRAKING EFFORT AND COEFFICIENT OF FRICTION

Braking effort is a measure of the retarding force which brakes can exert, expressed in percent of a car's weight. Brakes capable of exerting a braking effort of 100% can decelerate a car at a rate equal to gravity, 32 ft/sec/sec. (The braking efficiency rating of a car's brakes is the same as the highest braking effort the brakes can exert.) However, an effective braking effort of 100% could not actually be exerted unless the value of a pavement's coefficient of friction were equal to 1.0.

1. Rolling Wheel Braking
   a. The retarding force (braking effort) exerted by the brake shoes and the brake drums varies with the brake pedal pressure and the condition of the shoes and drums.
   b. The lower the braking effort for a given speed the longer the braking distance.
   \[ \text{Braking Distance} = \frac{30 \times \text{Braking Effort (expressed as a decimal)}}{(mph)^2} \]
   c. Texas law requires a braking efficiency of 44.4% or a braking distance of 30 ft. at 20 mph.
   \[ \text{Braking Distance} = \frac{20^2}{30 \times .444} = \frac{400}{13.32} = 30 \text{ ft} \]
   d. If the percent braking efficiency of a car's brakes is less than the friction value (coefficient of friction) of the pavement, the brakes cannot lock the wheels. The braking distance then could never be as short as it would be with locked wheels.
   e. A car with a low braking efficiency on a pavement with a high friction value is a hazard because the driver cannot stop as quickly as the car ahead of him.

2. Locked Wheel Braking
   a. A car's brakes cannot lock the wheels unless the car's braking efficiency is equal to, or greater than, the friction value of the pavement.
   b. Regardless of what a car's braking efficiency is, once the wheels are locked the braking distance depends exclusively upon the friction value of the pavement.
   \[ \text{Braking Distance} = \frac{30 \times (f \text{ value})}{(mph)^2} \]
   c. The lower the f value is, the longer the braking distance will be.
   d. While locked-wheel braking insures the shortest braking distance, a car with a high braking efficiency on a pavement with a low friction value can be dangerous, because it is easy for the driver to lock his brakes. This may happen when it is as important for a driver to change directions as it is to brake. With brakes locked a driver loses directional control because he cannot steer his car unless the front wheels are rolling.

The coefficient of friction (or friction value) is a ratio of the retarding force which a pavement can exert to a car's weight (when a car's wheels are locked). In effect then this ratio, expressed as a percent, becomes the best braking effort brakes can exert. Since the friction values of most dry pavements range from 0.5 to 0.8, the effective braking efforts which cars' brakes can exert therefore range from 50% to 80%. The friction value of a dry clean pavement usually is reduced appreciably by gravel, sand, mud, water, etc.
BRAKING FORCES AND ROLLING FRICTION

Turning-Wheel Braking

Opposing force of pavement is effected at axle and reduces axle's horizontal motion relative to the pavement.

Pavement grips tire and exerts a force which opposes the shoe-rim force and thus prevents wheel from sliding as the shoes slow wheel's rotation.

If the shoes lock the wheel, the chassis and wheel become a rigid body and the effective retarding force is exerted by the pavement, and the heat created in dissipating the car's kinetic energy is channeled to the tire and the pavement.

In turning-wheel braking, the shoes and drum share the heat with the tire and pavement.

Pavement is slightly deformed by vehicle's load. A rolling wheel is continually being pulled out of this depression over point X. (The figure is exaggerated.)

In effect rolling friction is a "braking" force which increases with car's speed. As speed increases it consumes more horsepower because in a given time the wheel must be pulled out of more depressions.
FRICITION, RETARDING FORCE, AND WEIGHT

If a car were to drop in a free fall (air resistance disregarded) from the top of a 12 story building 121 feet tall it would accelerate at a rate of 32 ft/sec/sec, and would reach the ground in 2.75 seconds traveling 88 ft/sec.

\[ v = \frac{2as}{2} = \frac{2 \times 32 \times 121}{2} = 7744 \text{ ft/sec} \quad t = \frac{v}{a} = \frac{88}{32} = 2.75 \text{ sec.} \quad s = \frac{at^2}{2} = \frac{32 \times (2.75)^2}{2} = 121 \text{ ft} \]

If the car on reaching the ground could start skidding on a (horizontal) pavement with a friction value of 1.0, the car would decelerate at a rate of 32 ft/sec/sec and would come to a stop in 121 feet, the length of the 12 story building, in 2.75 sec. (Actually pavements cannot decelerate a car this fast; most f values range from 0.5 to 0.8.)

A change in the car's weight would not alter the car's acceleration rate in the free fall or its deceleration rate in the skid, provided the changed weights on the four wheels remain proportional to the original weights.

The rate of deceleration is determined by the retarding force the pavement can exert. The retarding force a given pavement can exert determines its friction or f value. The force in pounds is some percent of the car's weight in pounds.

\[ f = \frac{\text{Force (retarding)}}{\text{weight}} \]

By transposing we get, \[ \text{Force} = f \times \text{weight} \]

To find the f value: \[ f = \frac{(\text{mph})^2}{30 \times \text{skid distance}} \]

(Note that the weight is not a factor in this formula. The f factor involves the weight and the f value means a pavement can exert a retarding force equal to a certain percent of a weight regardless of what the weight is.)

If the f value is determined with a car weighing 3000 lbs. and is found to be 0.6, the retarding force in a skid will be,

\[ \text{Force} = 0.6 \times 3000 \text{ lbs} = 1800 \text{ lbs.} \]

Skidding Car

\[
\begin{array}{c|c|c}
\text{Previous Weight} & \text{f value} & \text{Add a load of} \\
1800 \text{ lbs} & 0.6 & 1000 \text{ lbs} \\
3000 \text{ lbs} & & 4000 \text{ lbs}
\end{array}
\]

When the weight varies the retarding force varies. Due to this principle the skidding distance remains the same for a given speed even though the weight is changed. The f value is simply a ratio of the retarding force to the weight force acting against the pavement surface when all four wheels are skidding.

The principle would apply in rolling-wheel braking if the same percent braking effort were exerted after the load is increased as was exerted before. In the preceding examples in which the car's wheels are locked, the pavement exerts a braking effort of 60% of the car's weight. The car's wheels would not lock on a pavement with an f value of 0.6 unless the car's brakes could exert a braking effort of at least 60%, either before or after increasing the weight. As a vehicle's load is increased more braking power is required in order to maintain the same braking effort. Weight becomes an important factor in the braking distance when a vehicle's load is increased so much that the brakes cannot exert the same braking effort as they could prior to overloading.

In locked-wheel braking we know the braking effort is constant and we can determine its value by finding the f value of the pavement. Knowing the braking effort (through finding the pavement's f value) and the skid distance, we can estimate the speed.

To find the braking effort exerted in rolling-wheel braking we would have to measure the braking distance in some way other than with skid marks.
**KINETIC ENERGY**

Energy and mass (matter) are the two basic phenomena of the universe. The most famous formula of physics equates energy and mass: \( E = mc^2 \). It says that the atomic energy (in ergs) in mass is equal to the mass in grams \( \times \) the square of the velocity of light in cm/sec.

Energy is ability to do Work. Work = force \( \times \) distance (thru which force acts). Energy exists in many forms: mechanical energy, heat energy, chemical energy, electrical energy, light energy, and atomic energy.

Energy changes form but the total amount of energy in the universe remains constant.

Mechanical energy exists in two forms (mechanical pertains to laws of matter and motion):

1. **Potential energy** is the ability of a mass to do Work due to its position.

2. **Kinetic Energy** is the ability of a mass to do Work due to its motion.

---

**POTENTIAL ENERGY**

*(due to position)*

- water behind a dam
- bullet in a gun barrel
- wound clock spring

**KINETIC ENERGY**

*(due to motion)*

- water in motion
- bullet in motion
- clock works in motion
- car at rest on a hill
- car in motion

\[ \text{PE} = \text{weight} \times \text{height} = \text{ft lbs} \]

\[ \text{KE} = \text{Work} = \text{force} \times \text{distance} = \text{ft lbs} \]

Car rolling downhill due to its weight force (gravity) is changing its potential energy into kinetic energy. As it loses one it gains the other. When it reaches bottom of hill all of its PE will have been changed into KE. Its KE is then changed into heat energy thru rolling friction and air resistance and the car comes to a stop.
FORCE AND ENERGY

A car weighing 3600 lbs. traveling 60 mph possesses 432,000 ft. lbs. of kinetic energy. To get some idea of the damage which the car's weight force of 3600 lbs. can do as a result of such motion, construct and handle the weight props described in the illustration.

Consider the 100 lb. weight (a 100 lb. force) resting on your foot, as against the weight being dropped 1 ft. onto your foot (100 ft. lbs. of energy).

Now consider the 10 lb. weight (a 10 lb. force) resting on your foot. Compare this 10 lb. force with the 100 lb. force resting on your foot.

Then try to visualize that the 10 lb. weight after falling 10 ft. could do the same damage to your foot as the 100 lb. weight after falling 1 ft. This comparison should give you some idea of the part which motion plays in causing damages and injuries in auto collisions. A speed of 60 mph turns a 3600 lb. weight force into 432,000 ft. lbs. of energy.

The 10 lb. weight after falling 10 ft. is moving 25.3 ft/sec or 17.3 mph. The 10 lb. weight moving 17.3 mph = 100 ft. lbs. of energy. Consider first being hit by the 10 lb. weight moving 17.3 mph and then consider the potential injury to a pedestrian who might be struck by a 3600 lb. weight moving 17.3 mph. The 3600 lb. car will possess 360 times as much energy as the 10 lb. weight. While other factors influence the severity of injury, a pedestrian should understand that the energy of a car traveling 15 to 20 mph is sufficient to kill him instantly, without the car running over him.

1. Pull up on the weight until the scales register zero lbs. and you will experience the feel of a 100 lb. force.

2. Lift the weight onto the top of a stool 1 ft. above the top of the scale platform and you will get the feel of work producing potential energy, which is equal to weight \times height. The work done = force \times distance = 100 lbs \times 1 \text{ ft} = 100 \text{ ft lbs of potential energy}.

3. The 10 lb. weight raised 10 ft. above the floor will possess 100 ft. lbs. of potential energy and upon falling will develop the same kinetic energy as will the 100 lb. weight falling 1 ft. Let the weight fall upon a small box which can easily support the weight at rest, and you will see the energy of motion in action.
KINETIC ENERGY CURVE FOR 3600 LB. CAR
SHOWING THAT BOTH K.E. AND BRAKING DISTANCE INCREASE BY
THE SQUARE OF THE SPEED. (Distances based on braking effort of 50%)

\[
\text{K. E.} = \text{Weight} \times \text{MPH}^2
\]

\[
\text{B. D.} = \frac{\text{MPH}^2}{30} \times \text{K.E.}
\]

85-mph 864,000 ft. lbs. (twice the energy of 60 mph).

42.4 mph 107 ft.

192,000

CAR'S ENERGY AT 80 MPH
IS LIKE CAR FALLING 213 FT.
FROM THE TOP OF A 21 STORY
SKYSCRAPER

\[
\frac{768,000 \text{ ft. lbs.}}{3600 \text{ lbs.}} = 213 \text{ ft.}
\]
KINETIC ENERGY AND HEAT ENERGY

Inertia will keep a car traveling at a constant speed, say 60 mph, unless some force acts on the car's mass to change its speed.

Since air friction and rolling friction are always present these frictions act constantly as a retarding force, which the car's engine must balance in order to maintain a constant speed.

If the engine force is removed, these two friction forces retard the car's mass until all of its kinetic energy is dissipated. At 60 mph a car on level pavement might coast for two minutes and cover a distance of one mile before stopping.

Stopping or slowing a car is basically a problem of decreasing kinetic energy. The kinetic energy must be changed into some other form of energy. The car's brakes do this job. They change the kinetic energy into heat energy quickly. The brakes must be carefully designed to suit the weight and the speed of a vehicle (kinetic energy normally developed). Otherwise the brake shoes and brake drums cannot radiate the heat energy fast enough. When this state occurs the brakes are overloaded. They will fade until the excess heat is radiated. Since braking is normally intermittent, brakes have time to cool between applications. Fading is not a problem unless a vehicle's weight and/or speed create a heat load higher than that for which the brakes were designed; or unless the brakes are used too frequently or too long at one time.

Kinetic energy has a measuring unit called the foot pound (ft-lb).

Heat energy has a measuring unit called the British thermal unit (Btu).

1 Btu is the quantity of heat required to raise the temperature of 1 pound of water through 1°F.

1 Btu will raise the temperature of 1 pound of steel 10°F.

1 Btu is equivalent to 778 ft-lbs of kinetic energy.

A 3600 lb. car traveling 60 mph possesses 432,000 ft-lbs of kinetic energy. This is equivalent to 555 Btu \(432,000 \div 778\). This is sufficient heat to increase the temperature of 40 lbs. of brake drums by 138°F.

The brake shoes are attached to the car proper which possesses the kinetic energy to be dissipated. When the shoes are pressed against the drums attached to the turning wheels the invisible kinetic energy of motion is changed into friction heat which raises the temperature of the shoes and drums. The drums begin radiating the excess heat into the cooler air. The design of the drums and the area of their braking surface determine how rapidly the drums can radiate the heat.

If the brakes are used frequently, especially at high speeds, or constantly over a prolonged period such as down a mountain, heat will accumulate much faster than the drums can radiate it. The drums will expand causing them to be farther from the shoes. The brake pedal will become "soft." The brake linings on the shoes will become glazed much as a fabric will on pressing it hard and long with a hot iron. In this state the linings are ineffective in gripping the drums. The pedal will be hard.
HOW SPEED INCREASES KINETIC ENERGY

Kinetic Energy Increases by the Square of the Speed just as the Opening of a Pipe Increases by the Square of the Pipe's Radius.

The distance around any circle is 3.1416 times as long as the diameter. This ratio, 3.1416, is a constant called \( \pi \) (pi).
The area of a circle = \( \pi \times \text{radius}^2 \), or \( \pi r^2 \)

Area of opening of pipe with 3" diameter:
Area = \( \pi r^2 \)

\[ = 3.1416 \times \left( \frac{3}{2} \right)^2 \]
\[ = 3.1416 \times \frac{9}{4} \]
Area = 7.07 sq. in.

The diameter of a 6" pipe is only 2 times as long as that of a 3" pipe but the pipe's opening is 4 times as great.

Area = \( \pi r^2 \)

\[ = 3.1416 \times 3^2 \]
\[ = 3.1416 \times 9 \]
Area = 28.27 sq. in.

Area = \( \pi r^2 \)

\[ = 3.1416 \times 6^2 \]
\[ = 3.1416 \times 36 \]
Area = 113.1 sq. in.

Note: The areas of these circles give pictures of how the energy of a car is quadrupled when its speed is doubled.

The diameter of a 12" pipe is only 4 times as long as that of a 3" pipe but the pipe's opening is 16 times as great.
ENERGY VS. CAR MASS

Kinetic Energy of a 3200 lb. car moving at various speeds where the size of the car is increased to represent the weight force of the car plus the ft-lbs of energy created by the speed.

Figures over dotted lines give the time and distance it takes to reduce the speed and energy between the illustrations based on a braking effort of 50%.

This car represents 3200 ft lbs of KE

$w = 3200 \text{ lb} \quad V = 5.47 \text{ mph}$

How we determine from the weight-force, distance, and speed the basic unit in ft lbs to which the sizes of the car are scaled:

When a car requires 1 ft to stop with 100% braking effort its speed is 5.47 mph:

$BE = \frac{V^2}{30 \times s}$

$(100\%) \ 1.0 = \frac{V^2}{30 \times 1}$

$V^2 = 30$

$V = \sqrt{30} = 5.47 \text{ mph}$

If the car weighs 3200 lbs its KE at 5.47 mph is 3200 ft lbs

$KE = \frac{wV^2}{30}$

$= \frac{3200 \times (5.47)^2}{30}$

$= \frac{3200 \times 30}{30}$

$KE = 3200 \text{ ft lbs}$

The car at rest is a weight-force of 3200 lbs. Moving 5.47 mph the mass weighing 3200 lbs possesses 3200 ft lbs of KE.

We can check this with $PE = weight \times height = KE$. If the car is raised 1 ft it has 3200 ft lbs of PE. And in falling 1 ft its speed will be 5.47 mph.

$v^2 = u^2 - 2as = 0 - 2 \times 32.2 \times 1 = 64.4$

$v = \sqrt{64.4} = 8.02 \text{ ft/sec} = 5.47 \text{ mph}$

(a = gravity = 32.2 ft/sec/sec)
FT/SEC, ENERGY, AND BRAKING DISTANCE

1. HOW BRAKING DISTANCES DOUBLE
   (BASED ON BRAKING EFFORT OF 50%-OPTIMUM FOR CONTROLLED STOPS FROM HIGH SPEEDS)

2. MPH ON SPEEDOMETER SHOULD TELL A DRIVER:
   a. SPEED IN FT/SECOND
   b. HOW KINETIC ENERGY INCREASES BY THE SQUARE OF THE SPEED

NOTE HOW GASOLINE CONSUMPTION INCREASES AS SPEED INCREASES ABOVE 40 MPH.

3600 lb car
HOW KINETIC ENERGY OF TOP 5 MPH INCREASES AS SPEED INCREASES

Each Bar Represents the Kinetic Energy of the Top 5 mph of Speeds from 5 to 75 Miles per Hour.

<table>
<thead>
<tr>
<th>FT. LBS.</th>
<th>BRAKING DIST.</th>
</tr>
</thead>
<tbody>
<tr>
<td>OF KE</td>
<td>OF THE 5 MPH</td>
</tr>
<tr>
<td>(3600 lb Car)</td>
<td>(+0.6)</td>
</tr>
</tbody>
</table>

| 3,000 | --------------- | THIS BAR REPRESENTS THE KINETIC ENERGY OF 5 MPH AT A SPEED OF 5 MPH | 1.4 FT. |
| 9,000 | --------------- | THIS BAR REPRESENTS THE KINETIC ENERGY OF THE 5 MPH BETWEEN 5 AND 10 MPH | 4.2 FT. |
| 15,000 | --------------- | 10-15 MPH | 7.0 FT. |
| 21,000 | --------------- | 15-20 MPH | 9.8 FT. |
| 27,000 | --------------- | 20-25 MPH | 12.6 FT. |
| 33,000 | --------------- | 25-30 MPH | 15.4 FT. |
| 39,000 | --------------- | 30-35 MPH | 18.2 FT. |
| 45,000 | --------------- | 35-40 MPH | 21.0 FT. |
| 51,000 | --------------- | 40-45 MPH | 23.8 FT. |
| 57,000 | --------------- | 45-50 MPH | 26.6 FT. |
| 63,000 | --------------- | 50-55 MPH | 29.4 FT. |
| 69,000 | --------------- | 55-60 MPH | 32.2 FT. |
| 75,000 | --------------- | 60-65 MPH | 35.0 FT. |
| 81,000 | --------------- | 65-70 MPH | 37.8 FT. |
| 87,000 | --------------- | 70-75 MPH | 40.6 FT. |

THE TOP 5 MPH OF 35 MPH DEVELOPS KE THE CAR WOULD DEVELOP AT 18 MPH.
THE TOP 5 MPH OF 75 MPH DEVELOPS KE THE CAR WOULD DEVELOP AT 27 MPH.
THE KINETIC ENERGY AT 5 MPH IS 3000 FT LBS BUT THE KE OF 5 MPH ADDED TO 70 MPH IS 87000 FT LBS — 29 TIMES GREATER.
IMPULSE AND IMPACT FORCES

How stopping time causes injuries and how the time is related to deceleration distance which increases by the square of the speed:

**IMPULSE Force**

Suppose an engine accelerates a 3200 lb. car at a rate of 4 ft/sec/sec for 22 seconds. The force required to change the car's speed at this rate is 400 lbs. applied constantly.

\[
\text{Force} = \text{mass} \times \text{acceleration} = \frac{\text{weight} \times a}{\text{gravity}} = \frac{3200 \times 4}{32} = 400 \text{ lbs}
\]

(Note that \(a = \frac{1}{8}\) of gravity and the force is \(\frac{1}{8}\) of the weight.)

\[
\begin{align*}
\text{v} &= 0 \text{ ft/sec} \\
\text{when Force starts acting} \\
\text{400 lb} &\rightarrow \text{3200 lb} \\
\text{after speed is changed 4 ft/sec/sec for 22 sec} \\
\text{momentum} &= \frac{\text{mass} \times \text{velocity}}{\text{gravity}} = \frac{\text{weight} \times v}{32} \\
\text{mom} &= \frac{3200 \times 88}{32} = 8800 \text{ lb-sec}
\end{align*}
\]

Each second there was an impulse of 400 lb-sec. The sum of the Impulses in 22 sec = 8800 lb-sec.

The total Impulses = Force \(\times\) time = (mass \(\times\) acceleration) \(\times\) time = \(\frac{\text{weight} \times a \times t}{\text{gravity}}\) = \(\frac{3200 \times 4 \times 22}{32}\) = lb-sec

The total Impulse then = change in the momentum of the mass as its speed changed from 0 ft/sec to 88 ft/sec.

It does not matter what the combination of force and time was in changing the car's speed, once the 3200 lb. car is moving 88 ft/sec, the car's momentum is 8800 lb-sec. In decelerating the car the Impulses work in a similar manner each second to reduce the car's momentum.

**IMPACT Forces** which do damage and injury are impulses exerted in a very short time. When time in the formula, Impulse = Force \(\times\) time, is short, the lb in the lb-sec becomes very great. This is the killer in collisions.

If the car could lock its brakes on a pavement with an f value of 1.0, it would decelerate at a rate of 32 ft/sec/sec and would stop in 2.75 sec (88 ft/sec + 32 ft/sec/sec).

The retarding Force exerted = \(\frac{8800 \text{ lb-sec}}{2.75 \text{ sec}}\) = 3200 lb (weight of the mass, a 1 G force).

If the car in a collision stops in 1 sec, the Force = \(\frac{8800 \text{ lb-sec}}{1 \text{ sec}}\) = 8800 lb which is 2.75 times the weight of the car (2.75 G's).

Suppose the car strikes a fixed object and the bumper stops in 1/44 sec:

The retarding Force exerted = \(\frac{8800 \text{ lb-sec}}{1/44 \text{ sec}}\) = \(8800 \text{ lb-sec} \times 44\) = 387,200 lbs (121 G's).

\[v = 88 \text{ ft/sec}\]

\[\text{The retarding Force at the bumper} = 387,200 \text{ lb}\]

8800 lb-sec becomes a tremendous explosion when the time is short.
The stopping time is related to the deceleration distance as follows:

The average velocity of the car during the impact is 44 ft/sec (88/2). At a speed of 44 ft/sec the car would go 1 ft in 1/44 sec. A force of 387,200 lbs. exerted through a distance of 1 ft = 387,200 ft lbs. which is the kinetic energy of the car traveling 88 ft/sec and is the work done (Force × distance) in stopping the car.

One impact force occurs between the bumper and the object struck. Another impact force occurs between an occupant and the instrument panel or other part.

Suppose a 160 lb. occupant in the front seat is thrown forward at 88 ft/sec when the bumper of the car above strikes the fixed object. The occupant strikes the instrument panel in say 1/10 sec. The panel is still moving forward as the front end of the car collapses, although the bumper stopped in 1/44 sec.

Assume the velocity of the panel has been reduced from 88 ft/sec to 22 ft/sec when the occupant strikes the panel. The relative speed between the occupant and the panel is 66 ft/sec. The momentum of the occupant's body before impact = \( mv = \frac{160 \times 88}{32} = 440 \text{ lb-sec} \).

The momentum after impact = \( \frac{160 \times 22}{32} = 110 \text{ lb-sec} \). The change in momentum in reducing the occupant's speed from 88 ft/sec to 22 ft/sec = 440 lb-sec - 110 lb-sec = 330 lb-sec. Recall that Impulse = change in mom = 330 lb-sec. Impulse = Force × time.

Force = Impulse or change in mom = 330 lb-sec = 3300 lb

This is a force of 20.62 G's since it is 20.62 times the weight of the occupant.

Car's velocity before collision = 88 ft/sec

The instant the bumper stops against the fixed object, the occupant continues forward at 88 ft/sec. While the occupant travels from the seat to the panel, the panel (and seat) are decelerating rapidly but have not come to rest.

Front of car while collapsing absorbs considerable energy and increases slightly the distance and time for the panel to stop. This reduces the relative speed between occupant and panel and consequently reduces the G force at the panel as compared with the G force at the bumper.

The "G" is a conventional term used to indicate a force in terms of gravity or of weight which is a gravity force. When \( a = \frac{32 \text{ ft/sec/sec}}{} \) (the same as gravity) the Force = the weight of the mass.

\[ \text{Force} = ma = \frac{w \times a}{32} = \frac{w}{32} \text{ weight}. \]
The 160 lb occupant decelerated from 88 ft/sec to 22 ft/sec in 0.1 sec.

\[
a = \frac{88 - 22}{0.1} = 660 \text{ ft/sec/sec} \quad \text{(This is 20.62 times 32, the rate of gravity)}
\]

\[
F = \frac{w \times a}{g} = \frac{160 \times 660}{32} = 3300 \text{ lb} \quad \text{(This is 20.6 times the weight)}
\]

The number of G's in a force (or the G force) then is \( \frac{a}{g} \times 32 \).

**CONSERVATION OF MOMENTUM**

Momentum = mass \times \text{velocity} = \frac{\text{weight} \times \text{velocity}}{32}

When two masses collide, the velocity of each mass after the collision is different from what it was before the collision. How much the velocity of each mass will change depends on the difference in the weights of the two masses.

The principle of conservation of momentum says that the sum of the momenta of the two masses after the collision will equal the sum of the momenta of the two masses before the collision. If two masses have different weights the changes in their velocities must vary according to their weights in order for the momenta before and after a collision to balance.

\[
(mass_1 \times u_1) + (mass_2 \times u_2) = (mass_1 \times v_1) + (mass_2 \times v_2)
\]

Before the collision \hspace{1cm} (After the collision)

\( u = \text{initial speed in ft/sec} \hspace{1cm} v = \text{final speed in ft/sec} \)

Illustration:

Two cars weigh 3200 lbs. each. Car 1 traveling 88 ft/sec (60 mph) collides rear-end with car 2 traveling 44 ft/sec (30 mph).

\[
\begin{align*}
\text{Mom car}_1 &= \frac{3200 \times 88}{32} = 100 \times 88 \\
\text{Mom car}_2 &= \frac{3200 \times 44}{32} = 100 \times 44
\end{align*}
\]

Sum of momenta of both cars before collision \( = (100 \times 88) + (100 \times 44) \)

Problem: If speed of car 1, after collision is decreased to 66 ft/sec, what will be the speed of car 2?

\[
(100 \times 88) + (100 \times 44) = (100 \times 66) + (100 \times v_2)
\]

\[
8800 + 4400 = 6600 + 100v_2
\]

\[
100v_2 = 8800 + 4400 - 6600 = 6600
\]

\[
v_2 = \frac{6600}{100} = 66 \text{ ft/sec (45 mph)}
\]

Since the two cars weigh the same, the 15 mph lost by car 1 is gained by car 2. Had car 1 weighed more than car 2, car 2 would have gained more speed and vice versa.

In the illustration it is assumed that both masses are perfectly elastic—that is, they have a high degree of resilience and will not become deformed on impact. (Steel is very resilient whereas lead is not.)

![Illustration of a car collision](image-url)
If in the truck-car collision the truck's speed is reduced from 45 mph (66 ft/sec) to 40 mph (58.68 ft/sec) the car's speed will be increased from 30 mph (44 ft/sec) to what speed?

\[
\begin{align*}
(3200 \times 66) + (3200 \times 44) &= (3200 \times 58.68) + (3200 \times v_f) \\
&= 66000 + 4400 = 58680 + 100v_f \\
100v_f &= 66000 + 4400 - 58680 \\
v_f &= \frac{11720}{100} = 117.20 \text{ ft/sec (80 mph)}
\end{align*}
\]

Actually the car's speed would not be increased this much because the impact force would be so great that the mass of the car would probably be deformed. Moreover, if the car were in gear the rear wheels would skid under such rapid acceleration. However, the illustration points up the difference in a rear-end collision hazard due to weight of the overtaking vehicle. Note that if the two vehicles had weighed the same, the car would have gained the 5 mph lost by the truck. But the truck is 10 times as heavy as the car. Consequently the speed gained by the car is 10 times as much as the speed lost by the truck.
HOW ENERGY RELATES CAR'S WEIGHT FORCE TO DISTANCE

A 3200 lb car traveling 60 mph possesses 384,000 ft lbs of KE, enough to skid the car on a normal pavement \( (f = 0.6) \) a distance of 200 ft in 4.6 sec. \( KE = \frac{w \times (\text{MPH})^2}{30} \)

The KE extends the weight force of the car down the roadway as if the bumper were elongated 200 ft.

The KE is like a huge steel cube sliding down the roadway for 200 ft. After sliding 100 ft the speed of the cube is still 42.4 mph. After sliding 150 ft the speed of the cube is still 30 mph.

The KE is like a cannon firing 32 100 lb steel shells at a target 200 feet away.
FORCE THROUGH DISTANCE

(Work = Force × distance)

When the 200 lb. ball is rolled up the beam a distance 3 times as far from the fulcrum as the 600 lb. ball is, it balances the big ball, raising it 5 ft. (Weight of beam is disregarded.)

In balancing the big ball the 200 lb. ball in effect loses 3000 ft. lbs. of potential energy and the big ball gains 3000 ft. lbs. of potential energy.

In the balanced state the 200 lb. ball possesses 1000 ft. lbs. of PE while the 600 lb. ball possesses 3000 ft. lbs. of PE. Although the 200 lb. ball weights only 1/3 as much as the big ball, it is exerting a moment of force on the left side of the fulcrum equivalent to that exerted by the big ball on the right side of the fulcrum.

In this balance of “work” the small ball exerts a vertical weight force of 200 lbs. while the big ball exerts a vertical weight force of 600 lbs. The difference in the weight forces is balanced by the difference in the distances (to the fulcrum) through which the weight forces are exerted.

The “distance” in this illustration is not exactly analogous to the “distance” through which a car’s weight force can be exerted as a result of the car’s kinetic energy, but the illustration is useful in that it presents a “distance” factor in work, being done in a way that a person can see it. The “distance” factor in the work which a moving car can do as a result of its kinetic energy cannot be seen in operation. It becomes visible in skid marks only after a car skids to a stop.

The small ball at rest possesses 1000 ft. lbs. of potential energy and in falling 5 ft. can produce work equivalent to 1000 ft. lbs. of kinetic energy. This work would result from the weight force (200 lbs.) acting through a distance of 5 ft.

But the small ball at rest in the balancing position also is exerting its weight force of 200 lbs. through its distance from the fulcrum. The work being done is equivalent to the 3000 ft. lbs. of potential energy, which is the amount of work the big ball can do in falling 5 ft.

The balls will balance so long as the ratio of their distances from the fulcrum is inverse to the ratio of their weights. The 200 lb. ball is 30 ft. from the fulcrum. It is exerting a moment of force counter-clockwise around the fulcrum axis equal to 6000 lb-ft (200 × 30). The 600 lb. ball is 10 ft. from the fulcrum. It is exerting a moment of force clockwise around the fulcrum axis equal to 6000 lb-ft (600 × 10).

The term lb-ft is used to measure a moment of force around an axis. The fulcrum axis is perpendicular to the page. A turn axis might point in any direction. The fulcrum concept permits us to picture ft-lb and lb-ft in one diagram. In problems of force moments the distance a weight force is from the ground is immaterial.
INCREASING SPEED FROM 30 MPH TO 42 1/2 MPH DOUBLES KINETIC ENERGY
(ILLUSTRATIONS ARE LOCKED WHEEL STOPS ON AVERAGE PAVEMENT)

\[ f = .6 \]

30 mph

\[
\begin{align*}
\text{BD of Car Going 5 mph} & = 18' \\
\text{KE} & = 108,000 \text{ FT. LBS.} \\
& (3600 \text{ LB. CAR})
\end{align*}
\]

42 1/2 mph

\[
\begin{align*}
\text{BD of Car Going 5 mph} & = 100' \\
\text{KE} & = 216,000 \text{ FT. LBS.} \\
& (3600 \text{ LB. CAR})
\end{align*}
\]

INCREASING SPEED FROM 30 MPH TO 35 MPH ADDS 18' TO BRAKING DISTANCE

\[
\begin{align*}
\text{BD of Car Going 5 mph} & = 1.4' \\
\text{BD of Car Going 5 mph} & = 50'
\end{align*}
\]

BRAKING DISTANCE OF 5 MPH IS 1.4 FT. BUT WHEN 5 MPH IS ADDED TO 30 MPH THE BRAKING DISTANCE OF 5 MPH IS 18 FT.

ENERGY OF 5 MPH WHEN ADDED TO 30 MPH IS THE SAME AS THE ENERGY OF THE CAR GOING 18 MPH.

ENERGY OF 5 MPH WHEN ADDED TO 60 MPH IS THE SAME AS THE ENERGY OF THE CAR GOING 25 MPH, AND IT ADDS 35 FT. TO THE BRAKING DISTANCE.

FIXED OBJECT COLLISION AT 30 mph IS LIKE CAR FALLING FROM 3 STORIES

\[
\begin{align*}
42 1/2 \text{ mph} & \quad 6 \text{ STORIES} \\
60 \text{ mph} & \quad 12 \text{ STORIES}
\end{align*}
\]
RELATIVE SPEEDS AND COLLISION ENERGY

1. 3600 lb car going 60 mph hits rear end of 3600 lb car going 30 mph
   Collision Energy here is 108,000 ft lbs
   Relative Speed is 30 mph

2. 3600 lb car going 60 mph hits a bridge head
   Collision Energy here is 432,000 ft lbs
   (4 times No. 1)
   Relative Speed is 60 mph

3. 3600 lb car going 60 mph hits head-on a 3600 lb car going 60 mph
   Collision Energy here is 1,728,000 ft lbs
   (4 times No. 2 and 16 times No. 1)
   Relative Speed is 120 mph
A driver who does not understand how increases in energy involve the weights of vehicles and the squares of their speeds can be deceived by "relative speed" in two ways:

1. If the relative speed between two vehicles is say 15 mph the driver may think only of the energy involved in an actual speed of 15 mph with respect to the ground.

2. Or if he does not think about the relative speed at all the sense of motion he gets from the two vehicles alone will be the same regardless of the actual speeds of the two vehicles with respect to the ground. Stationary objects nearby can serve as cues to the actual speeds. If the roadway is not smooth, the sway of his car may help. These cues will obviously inform an untrained driver that the danger is greater than at 15 mph if he collides with one of the stationary objects.

But the lesson here does not involve collision with the stationary objects, at least not directly. It concerns only the differences in the energies of the two vehicles should they collide with one another. The hazard of the stationary objects is present also because the difference in the energies of the two vehicles can vary so much with a relative speed of 15 mph that in addition to an increased crushing of car bodies, one of the vehicles might carom or catapult the other into a fixed object.

The following examples will show that a relative speed can be very deceptive as a guide to the impact energy present in collisions involving cars with similar weights at different speeds and involving trucks of different weights and different speeds.

Each passenger car weighs 3000 lbs. Truck weights are given. Energy = weight x (mph)^2

<table>
<thead>
<tr>
<th>Speeds and Energies of Colliding Vehicles</th>
<th>Relative Energy Involved</th>
<th>Relative Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>7½ mph</td>
<td>7½ mph</td>
<td>15 mph</td>
</tr>
<tr>
<td>5625 ft lbs</td>
<td>5625 ft lbs</td>
<td>11,250 ft lbs</td>
</tr>
<tr>
<td>15 mph</td>
<td>at rest</td>
<td>15 mph</td>
</tr>
<tr>
<td>22500</td>
<td>0</td>
<td>22,500 ft lbs</td>
</tr>
<tr>
<td>30 mph</td>
<td>15 mph</td>
<td>15 mph</td>
</tr>
<tr>
<td>90,000</td>
<td>22,500</td>
<td>67,500 ft lbs</td>
</tr>
<tr>
<td>45 mph</td>
<td>30 mph</td>
<td>15 mph</td>
</tr>
<tr>
<td>202,500</td>
<td>90,000</td>
<td>112,500 ft lbs</td>
</tr>
<tr>
<td>60 mph</td>
<td>45 mph</td>
<td>15 mph</td>
</tr>
<tr>
<td>360,000</td>
<td>202,500</td>
<td>157,500 ft lbs</td>
</tr>
<tr>
<td>45 mph</td>
<td>30 mph</td>
<td>15 mph</td>
</tr>
<tr>
<td>7333 lbs</td>
<td>90,000</td>
<td>405,000 ft lbs</td>
</tr>
<tr>
<td>495,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35 mph</td>
<td>30 mph</td>
<td>5 mph</td>
</tr>
<tr>
<td>22041 lbs</td>
<td>90,000</td>
<td>810,000 ft lbs</td>
</tr>
</tbody>
</table>
WEIGHT AND ENERGY

So far as passenger cars are concerned it is most important that students understand the concept of how energy increases by the square of the speed, because the weights of most of the passenger cars vary less than a thousand pounds.

Weight, however, becomes very important when a passenger car is compared with a loaded truck. A knowledge of the tremendous energy developed by a heavily loaded truck is necessary in order for a student to develop a proper respect for trucks at intersections, when turning left in front of them, when applying brakes while one is following close, when meeting one head-on and when overtaking and passing one.

1. Kinetic energy of a loaded truck weighing 60,000 lbs. traveling 45 mph:
   \[ E = \text{weight} \times (\text{mph})^2 = \frac{60,000 \times 45^2}{30 \times 30} = \frac{60,000 \times 2025}{900} = 4,050,000 \text{ ft lbs} \]
   (The “30” in the equation is a constant)
   This is 20 times the energy of a 3000 lb car moving 45 mph.

2. At what speed must a 3000 lb car travel to develop that much energy?
   \[ \frac{4,050,000}{3000} = \frac{3000 \times (\text{mph})^2}{30 
   \text{(mph)}^2 = \frac{30 \times 4,050,000}{30 \times 30} = 40500 \text{ mph} \]
   \[ \text{mph} = \sqrt{40500} = 201.2 \text{ mph} \]

3. A 3000 lb car at 60 mph develops 360,000 ft lbs of energy. At what speed would a 60,000 lb truck develop the same amount of energy?
   \[ \frac{360,000}{3000} = \frac{60,000 \times (\text{mph})^2}{30 \times 30} \]
   \[ \text{(mph)}^2 = \frac{30 \times 360,000}{60,000} = 180 \text{ mph} = \sqrt{180} = 13.41 \text{ mph} \]

The answers in two of the illustrations, 201.2 mph and 13.4 mph, seem so preposterous that many adults cannot take them seriously. The energy concept is basically the crux of the whole driver accident problem. The instructor must move slowly and painstakingly with it or he may lose his best opportunity to turn out disciplined drivers.

The maximum gross weight for trucks in Texas is 72,000 lbs. After the students think about the examples given above, work similar examples using the maximum weight. Illustrations of both weights follow:

Energy of \[\text{60,000 lbs}\] at 45 mph = Energy of \[\text{3000 lbs}\] at 201.2 mph

Energy of \[\text{60,000 lbs}\] at 13.4 mph = Energy of \[\text{3000 lbs}\] at 60 mph

Energy of \[\text{72,000 lbs}\] at 45 mph = Energy of \[\text{3000 lbs}\] at 220.5 mph

Energy of \[\text{72,000 lbs}\] at 12.2 mph = Energy of \[\text{3000 lbs}\] at 60 mph
HOW TIME$^2$ AFFECTS DECELERATION DISTANCE

LOCKED WHEEL DISTANCE BY SECONDS FROM 75 MPH
(IF DRIVER COULD DO IT WITHOUT TURNING OVER OR
SKIDDING INTO OPPOSING TRAFFIC.)

IN FIRST SECOND NEARLY $\frac{1}{3}$ OF TOTAL BRAKING DISTANCE
100 FT. IS B.D. OF 13 MPH WHEN ADDED TO 62 MPH.
100 FT. ALSO IS B.D. AT 42.5 MPH

\[ f = 0.6 \]
\[ a = f \times \text{gravity} = 0.6 \times 32 = 19.2 \text{ ft./sec./sec.} \]

\[ v^2 = u^2 - 2as \]
\[ v = \text{final speed in ft/sec} \]
\[ u = \text{initial speed in ft/sec} \]
\[ a = \text{deceleration in ft/sec/sec} \]
\[ s = \text{distance in ft} \]
\[ t = \text{time in sec} \]
HOW BRAKING DISTANCE VARIATES WITH BRAKING TIME
AND WITH SPEED

SPEED 90 ft./sec. (61.4 mph)

SPEED 45 ft./sec. (30.7 mph)

STOP

202\(\frac{1}{2}\) ft.
3 sec.

67\(\frac{1}{2}\) ft.
3 sec.

270 ft.
6 sec.

1. DURING TOP HALF OF BRAKING TIME CAR GOES 3 TIMES AS FAR AS DURING BOTTOM HALF.

2. TOP HALF OF SPEED ACCOUNTS FOR \(\frac{3}{4}\) OF BRAKING DISTANCE.

3. BOTTOM HALF OF SPEED ACCOUNTS FOR \(\frac{1}{4}\) OF BRAKING DISTANCE.

RATE OF DECELERATION = 15 ft./sec./sec. (Firm braking. Will slide packages off seat.)

\[
\text{TIME} = \frac{\text{SPEED}}{\text{rate of dec.}} = \frac{90}{15} = 6 \text{ sec.}
\]

\[
s = ut - \frac{at^2}{2}
\]

where

- \(s\) = distance
- \(u\) = initial speed in ft/sec.
- \(a\) = rate of deceleration
- \(t\) = time
TIME AND DISTANCE REQUIRED TO OVERTAKE AND PASS A VEHICLE

Problem: Car A traveling 60 mph overtakes and passes Car B traveling 45 mph. We have A's speed but to get the passing distance we need the time; that is, how long any part of Car A will be left of the center line. To get the time we need the Distance of Overtake and the Rate of Overtake.

Solution: Time of Overtake = Distance of Overtake / Rate of Overtake

Do not confuse the Distance of Overtake with the passing distance which we are trying to find. Time of overtake = seconds passing car is blocking left lane.

Step 1. Draw a static diagram showing the Distance of Overtake, the combined length of "obstructions"

Step 2. Calculate Distance of Overtake: sum of lengths of all obstructions: 18 + 14 + 50 + 18 + 50 + 14 + 18 = 182 ft.

Step 3. Calculate Rate of Overtake: average speed Car A minus average speed car B = 88 - 66 = 22 ft/sec


Step 5. Calculate passing distance of Car A: Distance = speed x time = 88 ft/sec x 8.3 sec = 730.4 ft

Problem: What will passing distance be if Car A goes around two-50-ft trucks going 45 mph, 75 ft apart?

Instead of 18 ft for Car B we have 175 feet added to 164 ft (182 - 18) for a D of O of 339 ft.

T of O = 339 ft / 22 ft/sec = 15.4 sec. Passing distance = 88 ft/sec x 15.4 sec = 1355 ft

Problem: When Car A starts to pass how far away must a meeting car be if each car holds a speed of 60 mph?

1. In the Car B problem: 2 x 730.4 ft = 1461 ft. Plus 100 ft clearance required by law = 1561 ft.

2. In the truck problem: 2 x 1355 ft = 2710 ft. Plus 100 ft clearance = 2810 ft, over half a mile.

Note:
The 14 ft for changing lanes is the 11 ft the center of Car A would move in changing lanes sideways plus the extra 3 ft the car diagonal would extend if car were moved sideways at an angle.

Car A's length diagramed at the start is a simplified representation of the extra distance some part of the car is inside the passing lane at the start and at the end due to crossing the center line outside the 50 ft clearance points at an angle.
WHY DRIVERS MAKE POOR DECISIONS IN PASSING

PASSING DISTANCES AND DISTANCES MEETING CAR MUST BE FROM PASSING CAR

Based on field tests that averaged 12 sec. for Accelerated passes and 9 sec. for Constant Speed passes with vehicle speeds of 45 mph & 60 mph as illustrated

1. ACCELERATED PASS: Car A starts pass at 45 mph, same speed Car B is traveling, and accelerates to 60 mph

<table>
<thead>
<tr>
<th>45 mph</th>
<th>45 mph</th>
<th>45 mph</th>
<th>60 mph</th>
<th>60 mph</th>
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<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Averages 52.5 mph during pass</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 924 ft (12 sec at 52.5 mph)
- Legal Clearance 100 ft
- 1,056 ft (12 sec at 60 mph)
- Distance C must be from A when A starts pass (About 0.4 mile)

2. CONSTANT SPEED PASS: Car A starts pass at 60 mph around Car B traveling 45 mph

<table>
<thead>
<tr>
<th>60 mph</th>
<th>45 mph</th>
<th>45 mph</th>
<th>60 mph</th>
<th>60 mph</th>
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<tbody>
<tr>
<td>A</td>
<td>B</td>
<td></td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

- 792 ft (9 sec at 60 mph)
- 1,684 ft (9 sec at 60 mph)
- Distance C must be from A when A starts pass

- 396 ft

The distance that Car A is on the left side in the Constant Speed pass is 132 feet shorter than in the Accelerated pass.

The 132 feet is the difference covered by Car A traveling at an average of 52.5 mph for 12 sec. in the Accelerated pass and at an average of 60 mph in the Constant Speed pass.

But during the 3 sec. less time, Car C covers 264 fewer feet (3 \times 88). The total difference is 396 feet (132 + 264).

All drivers at times make both types of passes, but some do not realize that the meeting car must be 400 feet farther away in the accelerated pass to have the same margin of safety as in the constant speed pass. If meeting drivers did not slow down quickly to compensate for these drivers' errors, thousands more would be killed or injured.

Law requires only 100 feet clearance between meeting cars but 100 feet is unsafe because 2 cars meeting at 60 mph cover 100 feet in 0.57 sec.
Perception time distance and stopping distance are distinctly different driver problems.

**Perception Time Distance**
- PTD begins at the point at which a driver can physically see a hazard and ends at the point at which the driver starts reacting by changing speed, changing direction, signaling, etc.

**Stopping Distance**
- Reaction Time Distance
- Braking Distance

This distance can be short if a driver:
1. sees a hazardous situation
2. recognizes it as a hazard
3. starts reacting immediately

This distance will be long if a driver:
1. does not have his eyes on the roadway
2. sees but does not recognize a hazard
3. does not start reacting early

Perception Time can range from a split second to eternity.

The farther it extends into the Stopping Distance the more severe the collision.

If it extends to the point of collision (as in nodding at the wheel) and the driver is killed, then it extends to eternity because the driver will never start reacting.

Example:
- At 60 mph with \( \frac{3}{4} \) sec RT, RTD = 66 ft
- A given driver has a minimum reaction time distance at any given speed.

Example:
- At 60 mph with 50% Braking Effort BD = 240 ft
- On a given surface a given vehicle has a minimum braking distance at any given speed.

A Stopping Distance is a built-in hazard and is complex because:

**Reaction Time Distance** increases by the speed

**Braking Distance** increases by the square of the speed

<table>
<thead>
<tr>
<th></th>
<th>30 mph</th>
<th>60 mph</th>
<th>% Increase</th>
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</thead>
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<tr>
<td>RTD (( \frac{3}{4} ) sec)</td>
<td>33 ft</td>
<td>66 ft</td>
<td>100%</td>
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<tr>
<td>BD (50% BE)</td>
<td>60 ft</td>
<td>240 ft</td>
<td>300%</td>
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<tr>
<td>Stopping Distance</td>
<td>93 ft</td>
<td>306 ft</td>
<td>229%</td>
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</table>
PERCEPTION-TIME DISTANCE AND SPEED

At 50 mph Driver Has 1.5 Sec. to Recognize Hazard.
At 55 mph Driver Has 0.9 Sec. to Recognize Hazard.
At 60 mph Driver Has 0.4 Sec. to Recognize Hazard.
At 70 mph Driver Can Neither See Nor React in Time.
**INERTIA AND CENTRIFUGAL FORCE**

1. Inertia tends to keep a moving car going in the direction it is headed, and in a straight line.
   
a. Tire-ground friction force makes a car go right or left, off a straight line.

   b. Resistance of ground (upgrade) makes a car go up, off a straight line.

   c. Gravity makes a car go down, off a straight line.

![Centrifugal Force](image)

2. Inertia also tends to keep a moving car going at a constant speed.
   
a. A car moving at a constant speed does not want to speed up or slow down.

   (1) Engine (and/or gravity) force makes a car speed up.

   (2) Air resistance and rolling friction make a car slow down.

   On an up grade gravity slows a car.

3. When air and rolling friction forces are balanced by engine, Inertia keeps a car (on level road) at a constant speed.

   ![Diagram](image)

   - Car at rest: all external forces are balanced.
     (Inertia resists change of this state)

   - Car moving 50 mph: all external forces are balanced.
     (Inertia resists change of this state)

   Engine must balance resistance of air and rolling frictions. Otherwise, on a straight level road driver would not need engine.

   - When engine force is reduced
     Car B slows down

   - When engine force is increased
     Car B speeds up

   During acceleration the engine force must increase more than resistance forces increase.

   If air and rolling frictions were not present, some other retarding force would be needed to reduce the car's speed, just as a force is needed to increase the car's speed.
CENTRIFUGAL FORCE — 3,636 LB. CAR

\[ \text{C.F.} = \frac{\text{WEIGHT} \times \text{SPEED}^2}{\text{GRAVITY} \times \text{RADIUS}} \]

(Speed in ft./sec.)

(GRAVITY = 32)

In ② the radius of ① is cut in half and the C.F. is doubled.

In ③ the speed of ① is doubled and the C.F. is quadrupled.

If the friction value of the pavement is 0.6, the tires could exert a holding force of 2182 LBS. (60% of the car's weight). If C.F. exceeds this figure, the car will skid or overturn along a tangent to the curve.

In figures 2 and 3 the cars would skid.
CENTRIFUGAL FORCE

If Centrifugal Force Exceeds Friction Force Exerted by Pavement, Car Will Leave the Curve Along a Path Tangent to the Curved Path of the Car.

A tangent to a car's path in a left-turn 2-lane curve leads directly into a borrow ditch and/or fence, poles, trees, etc.

If the Center of Mass is High Enough, Centrifugal Force can Tilt a Car Over before the Tires Lose their grip on the Pavement.

Centrifugal force opposes Centripetal force but acts through the center of mass.

The longer the "arm" between the ground and the center of mass the less centrifugal force required to tilt a car over.

Pavement friction exerts a centripetal force at tire-pavement contact points and toward center of curve.

Curve center around which vehicle is turning in a left-turn curve.
HOW CENTRIFUGAL FORCE INCREASES

Centrifugal Force = weight × velocity² = lb × (ft/sec)² = lb × ft²/sec² = lb

1. If a car travels the same speed on a curve of 100 ft. radius as it went on a curve of 200 ft. radius, the CF will be **2 times as great**.

2. If a car travels twice as fast on a given curve, the CF will be **4 times as great**.

3. Now note that if a car travels twice as fast on a curve of 100 ft. radius as it went on a curve of 200 ft. radius, the CF will be **8 times as great**.

4. Incidentally, can Car A increase its speed any at all and stay in the short curve?

   a. If f = 0.6, the pavement can withstand a CF of 1920 lbs (0.6 × 3200)

      \[1920 = \frac{3200 \times v^2}{32 \times 100}\]

      \[v = \sqrt{1920} = 43.8 \text{ ft/sec} = 29.9 \text{ mph (top speed)}\]

   b. If f = 0.8, car can hold against CF of 2560 lbs.

      \[2560 = \frac{3200 \times v^2}{32 \times 100}\]

      \[v = \sqrt{2560} = 50.6 \text{ ft/sec} = 34.5 \text{ mph (top speed)}\]

Answer: It depends on the f value of the pavement
An illustration of how a driver who is unable to recognize road hazards or who fails to use his speedometer in adjusting his speed to road conditions can drive into a deadly trap. Car weighs 3200 lbs. Curve radius is 200 ft.

In this section of the road the driver enters rain. But the rain has washed off the traffic film and the driver feels his tires are keeping a good grip on the pavement, so he eases back up to his original speed. Although it is nighttime the pavement is light colored and he seems to have fair visibility in spite of the rain.

The driver enters a black top here. He cannot see as well but the rain is lighter. This compensates a little for losing the light pavement and after he gets accustomed to the black top he eases back up to his original speed. At the end of this section he notices a curve sign with a speed information warning of 45 mph. While he usually is able to make such curves at 50 mph in dry weather he decides to slow down to 45 mph to be safe. He does not check his speedometer however and actually enters the curve at 47 mph.

Just before the driver approaches this curve a light shower has fallen. The pavement in the curve is pock marked with tire rubber laid down by drivers who entered the curve in dry weather at 50 mph. There is just enough water on these smudges to cover the surface with oily globules. This condition has lowered the effective $f$ value of the curve’s surface (and bank) from 0.8 to 0.4. The smudges blend with the black top and the driver cannot see them. Nor does he know that only a light shower has fallen on the curve. The top speed of the curve at the time is 34 1/2 mph and the driver is going 47 mph. When the rear wheels spin a little he hits the brakes and goes out of control.

Centrifugal force acting against the car moving 47 mph (70 ft/sec) is 2450 lbs:

$$\text{CF} = \text{weight} \times \frac{(\text{ft/sec})^2}{\text{gravity} \times \text{radius}} = 3200 \times \frac{70^2}{32 \times 200} = 2450 \text{ lbs}$$

Centrifugal force which an $f$ value of 0.8 could withstand = $0.8 \times 3200 = 2560$ lbs. The car could barely have stayed in curve at 47 mph on dry pavement.

Centrifugal force which an $f$ value of 0.4 could withstand = $0.4 \times 3200 = 1280$ lbs. At 47 mph the CF was 1170 lbs too high (2450 – 1280).

The top speed at which the car could stay in the curve with the $f$ value of 0.4:

$$3200 \times \frac{v^2}{32 \times 200} = 1280 \text{ lbs}$$

$$(v = \text{ft/sec}) \quad v^2 = 2560, \text{ and } v = 50.59 \text{ ft/sec} = 34.5 \text{ mph}$$

The car was traveling 12.5 mph too fast to stay in the curve.

What does this 12.5 mph mean in terms of Kinetic Energy?

The car’s energy at 34.5 mph = $\frac{\text{weight} \times (\text{mph})^2}{30} = 3200 \times \frac{(34.5)^2}{30} = 3200 \times 1190.25 = 3808800 = 126,960 \text{ ft lbs}$

The car’s energy at 47 mph = $\frac{3200 \times 47^2}{30} = 3200 \times 2209 = 235,627 \text{ ft lbs}$

The car possessed 108,667 ft lbs. excess energy. This is the energy the car would develop at an actual speed of 31.9 mph.
CENTRIFUGAL FORCE VS KINETIC ENERGY

1. Car weight = 3600 lbs. Speed = 60 mph (88 ft/sec)
2. Kinetic Energy = Weight x MPH^2 = 3600 x (60)^2 = 432,000 ft lbs
3. KE of car at 27.6 mph = 92,741 ft lbs

DRIVER PROBLEMS

1. The driver must reduce the KE 78.5% in order to prevent car from sliding at the deadpoint.
2. The hazard at the deadpoint is one of a side force in pounds (in % of car's weight) which if higher than the f value of pavement will throw the car and its kinetic energy out of control. The problem is one of curve radius vs vehicle speed.
3. The hazard at point of entry is one of energy in ft lbs which if too high will prevent driver from reducing speed to 40 ft/sec at the deadpoint. The problem is one of deceleration distance.
4. The driver in illustration is 176 ft. from the deadpoint. How fast must he decelerate to reduce his speed from 88 ft/sec to 40 ft/sec at the deadpoint?

If his reaction time is average (¾ sec) he will travel 66 ft (0.75 x 88) while getting on his brakes. This leaves him 110 ft. for braking.

\[ v^2 = u^2 - 2as \]
\[ 40^2 = 88^2 - (2 \times 110)a \]
\[ 1600 = 7744 - 220a \]
\[ 220a = 7744 - 1600 = 6144 \]
\[ a = \frac{6144}{220} = 27.9 \text{ ft/sec/sec} \]

The driver cannot decelerate this fast because the f value of the pavement is only 0.63. Even in a skid he could decelerate only 20.2 ft/sec/sec.

The f value x gravity = maximum rate of deceleration: 0.63 x 32 = 20.2 ft/sec/sec

To decelerate at 27.9 ft/sec/sec the f value would have to be 0.87 (27.9 + 32). The f values of most pavements are much lower than 0.87.

5. To show how important ¾ sec is at 60 mph, if driver started applying brake when 176 ft. from the deadpoint, he could decelerate at 17.5 ft/sec/sec (a firm but safe rate) and be traveling 40 ft/sec at the deadpoint.
KINETIC ENERGY AND INERTIA

INERTIA TRIGGERS KE FORCE ON A CURVE

The 200 ft. dotted line in front of the car represents the distance the car's KE at 60 mph can project the car skidding on a pavement which exerts a retarding force equal to 60% of the car's weight.

As the car is turned from a straight path another force acts instantly against the side of the car. This force is created by the Inertia of the moving mass and is called centrifugal force. It is independent of the KE which represents a force acting through a distance. CF is simply a side push trying to overcome the steering force and to make the car move along in a straight path. In effect its job is to reduce the angle at which the front wheels have turned from the horizontal axis of the car.

Yet both the CF and the KE increase by the speed². The speed of 60 mph which was safe on the straight road makes the CF on the curved road too high for the holding force of the tires on the pavement.

The CF therefore returns the car to a straight path which leads off the curved road. When the car is moving straight again the CF disappears. Once the CF breaks the tires' grip on the pavement, there is no way to prevent the car from skidding 200 ft. provided it is not slowed down by a borrow ditch, fence, guard rail, tree or another vehicle.

In the illustration the CF is 3872 lbs. and the pavement's holding force is only 1920 lbs. But a CF of 2000 lbs. would be just as dangerous. Either one would trigger the KE to carry the car off the curve. Therein lies the hazard of CF. It is like drowning in just enough water to cover you. Being in the ocean wouldn't drown you any "deader."

Dotted lines represent skid distance of 200 ft.

- weight = 3200 lbs
- velocity = 60 mph (88 ft/sec)
- KE = \( \frac{1}{2} \times 3200 \times 88^2 = 387,200 \text{ ft lbs} \)
- \( f \) value = 0.6
- Braking Distance = \( \frac{60^2}{30 \times 0.6} = 200 \text{ ft} \)
- radius = 200 ft
- CF = \( \frac{3200 \times 88^2}{32 \times 200} = 3872 \text{ lbs} \)

Force pavement can exert to hold car = 
0.6 \times 3200 = 1920 \text{ lbs}
TURNING ANGLE AND CENTRIFUGAL FORCE

Estimating CF by judging time it will take to alter the direction of a car through the angle the curve exit makes with the approach road.

Both Cars A and B (each 3200 lbs.) have turned through an angle of 45° with the original path. In arriving at this 45° angle Car A traveled twice as far as Car B traveled. A's speed then was twice B's speed. But Car B's angle with OY increased at the same rate as Car A's angle with OY, since both cars reached the 45° angle in the same period of time.

Car A reached the 45° angle on a 200 ft. curve at 44 ft/sec and developed 968 lbs. of CF.

Car B reached the 45° angle on a 100 ft. curve at 22 ft/sec and developed 484 lbs. of CF.

But if Car B's speed had been the same as Car A's, Car B would have reached the 45° angle in half the time it took for Car A. The rate of increase of Car B's angle with OY would have been twice as great as Car A's and Car B would have turned through a 90° angle and arrived at position B' while Car A was turning through a 45° angle. Car B's CF would have been 1936 lbs.

Of course the CF of 1936 lbs acting on Car B as it approached position B' was present from the instant Car B got on the turning radius of 100 ft. But a driver approaching point O can look at the curve ahead and tell whether his car is going to complete 90° turn in 3 to 4 seconds as on B's path or 7 to 8 seconds as on A's path. This check will help him estimate the rate at which his turning angle will increase. He must also estimate the friction value of the pavement. These are the two more important checks he should make on the approach to a curve.

Car B could barely stay in the 100 ft. curve at 30 mph if the f value were 0.61.

\[ f = \text{Force} = 1936 \div 3200 = 0.61 \]

The pavement's grip on the tires could withstand a CF equal to 61% of the car's weight. If the f value were lower the car would start a spin.
CORNERING FORCE

When a car's wheels are turned into a curve the tires develop a side thrust by running at an angle of inclination between the plane of rotation and the direction of travel of the wheel. This angle between the direction a wheel is headed and the direction it actually travels is called the slip angle. The tire sorts of grabs more than it can hold onto. The side thrust developed is called a cornering force which opposes centrifugal force and enables the car to turn a corner so to speak.

When a driver has to add steering wheel pressure to keep a car in a curve, the car is said to understeer. The driver must increase the side thrust to make the car stay in a curved path. This condition is desirable from a safety standpoint and this understeer characteristic is considered stable. Understeering exists when the center of mass is nearer the front axle than the rear axle, that is, over 50% of the car's gross weight is on the front wheels.

When over 50% of a car's weight is on the rear wheels, the center of mass against which centrifugal force acts is nearer the rear wheel contact points where part of the inward thrust opposing centrifugal force is created. When a driver under this condition increases the slip angle at the front wheels to stay in a curve he may create more inward thrust at the front wheel contact points than is needed, and the front wheels being farther from the center of mass than the rear wheels will produce a greater moment of force around the center of mass than the rear wheels do and cause the front end of the car to tend to swing off the curve toward the inside. This characteristic of a car is called oversteer and it can be very dangerous. If the speed is high and the slip angle at the front wheels is increased rapidly the driver may not have time to make a correction before the front end swings off the curve inside.

Instability is aggravated also by weight distribution right and left. When the speed is high the centrifugal force moment acting at the center of mass (nearer the rear axle) causes a transfer of considerable weight from the inside rear wheel to the outside rear wheel. The cornering force resulting from the increased load on an outside wheel is not sufficient to offset the loss due to the decreased load on an inside wheel. That is, the heavier loaded tire undergoes a greater loss in cornering ability for the same degree of weight transfer.

Thus when the rear wheels are more heavily loaded than the front wheels steering stability is reduced, and when the moment of roll caused by centrifugal force redistributes the load on the rear wheels unequally steering stability is reduced further.

In the diagram the car at position A is in a gentle curve with constant radius at high speed. The load on the rear wheels is well over 50% of the car's gross weight. The deflection of the rear tires is abnormal because air pressure was not increased to compensate for the abnormal load.

At position B the turning radius shortens abruptly and the driver increases the steering angle quickly to stay in the lane. This act increases the slip angle of the front wheels and therefore the side thrust toward the center of the curve. It also increases the centrifugal force at the center of mass which shifts more of the rear axle load to the outside wheel and thus reduces the total cornering force of the two rear wheels. Less inward thrust at the rear wheels in effect aids centrifugal force which is acting at the center of mass near the rear wheels. At the same time the inward thrust at the front wheels increases. The car spins and rolls.

For some idea of the hazard of oversteer consider how quickly a castor can buckle when you push a piece of furniture across the floor too fast.
GYROSCOPIC FORCES

FLY WHEEL EFFECT

When a car is turned to the left, more weight is impressed on the front wheels than on the rear wheels. This increase of weight on the front tires tends somewhat to offset the adverse effects of too much rear wheel weight, which induces oversteer. (See cornering force.)

When a car is turned to the right, more weight is impressed on the rear wheels than on the front wheels. This increase of weight on the rear tires tends somewhat to aggravate the adverse effects of too much rear wheel weight, which induces oversteer.

CAR WHEELS EFFECT

Some curves are banked toward the outside. This superelevation enables the pavement to hold a car against more centrifugal force than it can in a flat curve. If the slope of the roadway at the approach or exit of a bank is not gradual, a gyroscopic effect is created as the outside wheels suddenly climb or descend the grade.

When a car enters an abruptly banked section of a curve, the rear end of the car tends to slew outward and the front end tends to slew inward. This action aggravates oversteer in either a right turn or a left turn curve, since oversteer itself tends to make the front of a car go toward the inside of a curve.

When a car leaves an abruptly banked section of a curve, the rear end of the car tends to slew inward and the front end outward. This action tends to offset the effects of oversteer in either a right turn or a left turn curve. However the help comes too late because the oversteer hazard is greatest at the deadpoint of a curve just after a car enters the banked section.

While these forces are real their hazard potential lies mainly in their influence on directional control while a driver is coping with hazards of centrifugal force and/or cornering force.

When a car enters a normal left turn curve the top of the car tends to tilt outward. The same effect occurs in a right turn curve, but in the right turn curve the flywheel effect tends to lessen the weight on the front tires. If the center of mass is nearer the rear axle due to an overload in the trunk, we have a condition where both centrifugal force and the car wheels effect tend to tilt the top of the car outward at a point nearer the rear axle. The flywheel effect tends to lessen the weight on the front wheels more still, and the cornering force at the front wheels induces oversteer toward the center of the curve. We have acting a down-force at the trunk, a roll-out force near the back seat, an up-force at the engine, and a pull-right force at the front wheels. While the gyro effects are minor compared with the centrifugal and cornering forces, they are real and can help to trigger kinetic energy in a direction a driver does not want to go. A driver has only four small tire-pavement contact friction points through which he must adjust the attitude and motion of his car to avoid loss of control.

Gyro effects can be demonstrated with a small electric motor (that fits the hand) equipped with a 3 to 4 inch rubber wheel. A better prop for the flywheel effect is a plastic model car about 4" x 10" (shell-type with nothing inside) equipped with a mounted 2" gyro wheel located where a flywheel would be. A motor with a rubber wheel will be needed to rev up the gyro (in the direction a flywheel turns). Obviously the effects these props produce will be exaggerated greatly due to differences in weights of wheels and car as compared with those of an actual car. Precaution: The electric motor should be grounded if operator is standing on the ground or on a floor that is not insulated from the ground.
**Braking Distance Chart in Feet**  
(Reaction Time Not Included)

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<th>MPH</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
<th>40%</th>
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Braking Distance equals Distance Traveled by Vehicle from the Time that Brake Pressure is Applied until Vehicle Comes to a Full Stop.

Braking Distance equals \( \frac{(\text{M.P.H.})^2}{30 \times \% \text{ of Braking Effort}} \)

% of Braking Effort equals \( \frac{(\text{M.P.H.})^2}{30 \times \text{Braking Distance}} \)

If all car wheels skid the % BE equals the f value of pavement. A 60% BE would be an f value of 0.6.
PART THREE

PHYSICAL FORCES AFFECTING VEHICLES IN MOTION

Please Read PREFACE and INTRODUCTION

I. BRAKING EFFORT and COEFFICIENT of FRICTION (Friction Forces Applied to Decelerate a Car)

A. Braking Effort or Braking Efficiency

1. When the brake shoes (which are attached to the chassis) are pushed against the brake drum (which is attached to the rolling wheel) the wheel’s rate of turning is reduced. The friction force exerted between the brake shoe and the brake drum therefore exerts on the car a force which is opposite to the direction in which the car is moving.

2. This retarding force is measured in percent of the car’s weight. The force may be spoken of as the braking effort which a car’s brakes can exert to slow down the car.

\[
\text{BE} = \frac{F}{w} = \text{braking effort or braking efficiency} \\
F = \text{retarding force in lbs} \\
w = \text{weight of car in lbs}
\]

a. If a car weighing 3600 lbs. has brakes which when applied hard can exert a constant retarding force of only 1800 lbs. the efficiency of the brakes is 50%.

\[
\text{BE} = \frac{1800}{3600} = 0.5 \text{ (or 50%)}
\]

b. If the car’s brakes can exert a retarding force of 3600 lbs. the braking effort exerted is 100% of the car’s weight.

\[
\text{BE} = \frac{3600}{3600} = 1.0 \text{ (or 100% braking efficiency)}
\]

c. The efficiency of a car’s brakes depends mainly upon the material of which the brake shoe linings are made: the area (square inches) of the surface between shoe linings and drums, and the metal of which the brake drums are made, its resistance to expansion when it is heated.

d. Whatever the initial efficiency of a car’s brakes may be, the efficiency will be reduced when the brakes are overheated, as when applied constantly for several minutes on down grades. Some brakes will fade out faster than others but all brakes will fade out if drivers allow them to get hot enough.

3. How is Braking Effort determined?

a. Garages have testing equipment with floor-level plates on which a car is stopped at very low speed. This and other devices which can be carried in a car measure the rate of deceleration or the retarding force when the brakes are applied.

b. Some Vehicle Inspection stations merely make hard stops at 20 mph on a dry pavement to determine whether the brakes meet the minimum legal braking effort of 45%. Since practically all clean, dry pavements have a friction value as high or higher than 0.45, it follows that brakes which can lock the wheels on a dry pavement have a braking efficiency of at least 45%.

B. Coefficient of Friction between the Tires and the Road Surface

1. When an object slides over a surface, a friction force retards the movement of the object. The amount of this retarding force created by friction between the object and the surface depends upon the materials of which the object and the surface are made.
2. In the case of a car decelerating with the wheels locked, we have rubber sliding over a concrete or asphalt surface. While the rubber of different types of tires may vary in texture, and the surfaces of pavements of different roads may vary in their ability to "grip" rubber, it may be said that for a given tire rubber and a given road surface, there is a constant ratio of the retarding force (which the pavement can exert) to the car's weight. This ratio is called the "coefficient of friction." It is an index of how efficient the pavement is in retarding the forward motion of a car when the car's wheels are locked—that is, when the car skids over the surface of the pavement.

3. To avoid repeating the long term, "coefficient of friction" we shall use the letter "f" instead and say "the friction value" or simply the "f value" of the pavement. The f value may also be thought of as the "drag factor," or the ability of a pavement to hold back a skidding car.

\[ f = \frac{F}{w} \]

where:
- \( f \) = coefficient of friction
- \( F \) = retarding force in lbs which a pavement can exert
- \( w \) = weight of car in lbs

If a pavement can exert a retarding force of 1800 lbs. on a 3600 lb. car which is sliding to a stop, the f value of the pavement is 0.5: \( f = \frac{1800}{3600} = 0.5 \). Likewise, if the brake shoes when pushed against the brake drums on turning wheels can exert a retarding force of 1800 lbs, the Braking Effort = \( \frac{1800}{3600} = 0.5 \), or 50%.

a. The f value of dry pavements range from 0.5 to 0.9 approximately. The f value decreases as a pavement becomes traffic polished. The f value is lower after sand or silt blows onto a pavement and is lower usually when a pavement is wet. A light shower on a pavement covered with traffic film (oily) will cause the f value to become dangerously low.

b. Suppose an f value (level) is 0.7; the effective f value on an upgrade of 3% (0.03) would be 0.73 (0.70 + 0.03); skidding downgrade it would be 0.67 (0.70 - 0.03).

4. How is a pavement's f value determined?

a. One way is to tow a car sliding on locked wheels with the tow bar attached to a heavy spring scale carried by the towing vehicle. The scale registers the pounds pull required to keep the car sliding at a constant speed.

b. Another way (and one which will obtain a more accurate f value for use in calculating braking distances) is to make a locked-wheel test stop with a car on level pavement. The speed of the car when the brakes were locked and the length of the skid distance are inserted in the following formula:

\[ f = \frac{\frac{V^2}{30 \times s}}{\text{coefficient of friction}} \]

\[ V = \text{speed in mph} \]
\[ s = \text{skid distance in test stop} \]

The "30" is a constant

If the test car's speed is 30 mph and the skid distance is 50 feet, \( f = \frac{\frac{30^2}{30 \times 50}}{900} = 0.6 \)

C. How the Braking Effort and the Coefficient of Friction are Related

1. We have reviewed two types of friction forces which are applied
a. between the brake shoes and brake drums

(1) expressed in the percent the retarding force is of the car's weight.

b. between the tires and the pavement

(1) expressed as a ratio of the retarding force to the car's weight.

2. So far as decelerating a car is concerned we may think of the f value of the pavement also in terms of a retarding force expressed in percent of the car's weight. In both cases a coefficient of friction is the key factor. Actually there is a coefficient of friction between the brake shoe linings and brake drums just as there is between the tires and a pavement.

a. If a car's brakes can lock the wheels on any pavement, then the f value of the pavement determines how much retarding force actually can be exerted. A car's brakes may be capable of a 100% braking effort, but if the car makes a locked-wheel stop on a pavement with an f value of 0.6, then the effective braking effort could not exceed 60%.

b. If a car's brakes were only good enough to meet the minimum braking efficiency required by law (about 45%), then no matter how high the f value of a pavement might be, the maximum braking effort the car's brakes could exert would be 45%. The car's brakes could not lock the wheels on a pavement with an f value higher than 0.45.

c. The weaker of the two friction forces applied (1) at the brakes and (2) on the pavement, determines how long a braking distance will be.

D. Miscellaneous Items on Braking

1. If a vehicle's load does not exceed the capacity for which the vehicle (including its brakes) was designed, the vehicle's brakes, if in good condition, should be able to lock the wheels on the best pavement (high f value). Some vehicles, both cars and trucks, have their brakes so overloaded at times that the brakes' efficiency is reduced appreciably. In such cases the car or truck's brakes cannot lock all wheels on dry pavements. It is more dangerous to drive immediately in front of such a vehicle than it is to follow it closely.

2. If a car's brakes can lock all wheels, the skid distance at a given speed will be the same length irrespective of the load (except for minor variations which will occur in different tests with the same load). The f value, \( f = \frac{F}{w} \)

is a ratio of the retarding force of the pavement to the car's weight. The retarding force, \( F = Fw \), will be a fixed percent of the vehicle's weight, whatever the weight may be. Visualize the weight as a force acting against a pavement. If the wheels are locked and the f value is constant, then the retarding force will be a fixed percent of the weight regardless of what the weight is.

a. A possible exception is an instance wherein the loads vary so much that the heavier load might actually lower the f value due to extreme heating of certain surfaces such as tar. This would not occur on a “clean” pavement such as brushed concrete, unless the weight were so great that it would melt the tire rubber abnormally. Such overweights do not occur in passenger car.

b. Another exception which might occur even though the f value is not lowered is one wherein the heavier load is so distributed that the center of mass is shifted to a location higher and/or forward, thus causing a greater percentage of the total weight to be shifted to the front wheels when the brakes are locked. The rear tires would then be doing a smaller percentage of the work in retarding the heavier load.

3. With a given load in a locked-wheel stop, a car's front tires exert a retarding force about twice that exerted by the rear tires. In DPS tests at 30 mph, four skidding tires stopped a car in 50 feet; this is a braking effort of 60%. The front tires alone stopped the car in 60 feet; this is a braking effort of 50%. The rear tires alone stopped the car in 120 feet; this is a braking effort of 25%. The importance of maintaining the front brakes with the best brake linings and keeping the front brakes equalized cannot be overemphasized. What's out front really counts in brakes.
4. Skid distances at a given speed may vary between two vehicles which have relatively short and long wheel bases, which, together with different locations of the centers of mass, might shift disproportionate percents of the vehicles' weights to the front wheels when the brakes are locked.

5. Variations of skid distances among vehicles moving at the same speed are due primarily to different percentages of the vehicles' weights being shifted to tires located ahead of the centers of mass. Tractor-trailer combinations present special cases because the trailer's center of mass (through which the weight of the trailer acts) forces the rear tires of the tractor to do a greater percentage of the total work done than they would do if the tractor were skidding alone. And the location of the center of mass of a loaded trailer influences the percent of the trailer's weight carried by the rear tires of the trailer when the brakes are locked. The farther the center of mass is to the rear of the mid-point of the trailer's wheel base, the less extra weight it will throw onto the rear wheels of the tractor, and the more weight the trailer wheels will carry during a skid.

II. INERTIA AND CENTRIFUGAL FORCE

A. Inertia is the tendency of an object at rest to remain at rest and of an object in motion to remain in motion, at a constant speed and in a straight line. Forces are balanced on an object moving at a constant speed just as they are when the object is at rest and the moving object will not change its speed or its direction until a force is applied to create a change.

1. Centrifugal force acting on a car in a curve therefore is actually an inertial force trying to make the car return to a straight path. Nature wants the car to go straight. When a driver turns the front wheel from straight ahead he creates a centripetal force which acts inward along the radius of the turn. Centrifugal force in effect acts outward along the turn's radius to make the car go straight, along a line tangent to the curve. But this force does not originate at the center of a circle. It is created at the car by inertia of the car when the driver turns from straight ahead. The centrifugal force concept which involves a circle and its radius is a man-made device created to simplify measurements of inertia.

2. Once the engine gets a car going at any constant speed on level ground the engine has to produce only enough force to overcome air resistance, chassis friction (gears, bearings etc.), and rolling friction (tires on pavement) in order to keep the car going at a constant speed. This fact is due to the law of inertia. Once these retarding forces are balanced by the engine, inertia keeps the car at a constant speed.

a. Streamlined body design, moderate speeds, good lubrication, an overdrive, and proper tire inflation reduce gasoline consumption because they reduce the amount of power needed to overcome the above mentioned frictions. A constant speed requires less gas than widely varying speeds, which average the constant speed because the above frictions will help reduce speed but will not help accelerate a car. Slight variations in cruising speeds, however, are desirable both for the engine and the driver.

B. The centrifugal force (CF) formula shows how different car weights and different speeds on curves of different radii determine the amount of CF against a car on a curve.

\[
\text{CF} = \frac{w \times v^2}{32 \times r}\]

\(w = \text{weight in pounds}\)

\(v = \text{speed in ft/sec}\)

\(r = \text{radius of curve in feet}\)

\(32 = \text{constant acceleration rate of gravity}\)

Note: The value of gravity varies over the earth from 32.0 to 32.2 approximately. To change mph to ft/sec multiply the mph by 1.467. To change ft/sec to mph multiply the ft/sec by 0.682.

A 3600 lb. car going 60 mph (88 ft/sec) on a curve with 300 ft. radius creates a CF of 2904 lbs.

\[
\text{CF} = \frac{3600 \times 88^2}{32 \times 300} = \frac{3600 \times 7744}{9600} = \frac{27878400}{9600} = 2904 \text{ lbs}
\]

1. The coefficient of friction (between the pavement and the tires) times a car's weight gives the maximum number of pounds CF which a car can withstand and stay in a curve. If the f value of the pavement is 0.6 and a
car weighs 3600 pounds, it can withstand only 2160 pounds of CF (0.6 × 3600). The car going 60 mph on a 300 ft. radius with f = 0.6 could not stay in the curve. The f value would have to be 0.81 or better.

\[ f = \frac{F}{w} = \frac{2904}{3600} = 0.807. \]

a. The percent of pavement elevation or bank in a curve increases the effective friction value of the pavement. If the f value is 0.6 and the bank is 3% (.03) the effective friction value is 0.63 (.60 plus .03) and the 3600 lb. car can withstand 2268 pounds of CF (.63 × 3600). A banked curve has the effect of increasing the f value of the pavement against CF.

2. When the tire grip on the pavement is broken, the car leaves a curve in a slide or skid along a tangent to the curved path of the car, and not along the radius of the curve. (A tangent to a curve is a straight line, along which inertia wants the car to go.)

a. If the tire-pavement grip does not break, the centrifugal force acting against the center of mass of the car can turn the car over. This can happen in any sharp turning movement on a straight road, if the speed is too high for the radius of the turn. A sharp cut-in at high speed after overtaking and passing on a wet road can start a skid and cause loss of directional control. On a dry road with a high f value it might tilt the car over. This result frequently occurs when a driver oversteers following a blowout.

   (1) The center of mass is a point within the bulk of the car around which the weight of all parts (top, bottom, front, rear) of the car is in balance. CF acts against the center of mass. Loading of a car may shift the center of mass forward, rearward, or upward.

   (2) The higher the center of mass of a vehicle is (such as a car with luggage on top) the easier it will be for the same amount of centrifugal force to turn the car over. The distance between the center of mass (against which the centrifugal force would be acting outward) and the pavement (which would be holding the wheels on a curve) serves as a sort of lever. The longer the lever, the less force required to move the center of mass outward. Centripetal force acts inward at points where tires and pavement meet and centrifugal force acts outward at the point of the center of mass.

3. A driver can feel an increase of centrifugal force against his body and can relate this feeling to the amount of centrifugal force which will turn his car over, but he cannot relate it equally well to the amount of centrifugal force which will break the tire grip on the pavement and cause the car to skid or slide along a path tangent to the curve.

   a. A tangent to the path of a car in a left turn curve leads into a borrow ditch, fence or trees, and a tangent to the path of a car in a right turn curve leads into opposing traffic.

C. Centrifugal force can be reduced appreciably by reducing the speed and/or straightening out the front wheels a little.

1. When CF is too high on a left turn curve where an open shoulder is available, the driver might, in the emergency, straighten the front wheels a little for an instant, and fan the brakes quickly at the same time (even if he has to go onto the shoulder) and reduce the CF enough for him to continue in the curve. If the front wheels are not almost straight the instant brakes are applied, the CF remaining may start a skid, especially if loose material is on the shoulder. However, while the wheels are straight there is no CF. What the driver must do in a second is steer straight to reduce CF, slap the brakes quickly, and steer back into the curve. This recovery procedure requires skill. If a driver does not have it, he can compensate by employing better judgment to enter curves at safe speeds. A defensive driver will rely first on judgment in any case.

2. On a right turn curve the recovery procedure in 1. above is more hazardous because it might carry the car across the center line. If opposing traffic is near, the driver should at all costs keep his car on the right side. This urgency places a lot more responsibility on the driver to know that he is entering a right turn curve at a safe speed. The left turn curve is a hazard to his life. The right turn hazard involves other people's lives.
D. Factors which a driver should habitually and quickly analyze in determining a safe speed for each curve he approaches:

1. Whether it is a right turn or a left turn curve, and width of traffic lanes. How difficult it will be to keep car away from center line.

2. Whether opposing traffic is close, especially if the curve is a right turn curve.

3. The curve’s radius, or radii (the radius might become shorter at some point in the curve).

4. The grade. If the road is not level, is it uphill or downhill and how steep? A downhill curve is more dangerous because gravity works with CF against a driver’s control of his car.

5. Super elevation (amount of bank) of the curve. Is there a good bank at the dead-point of the curve, where the curve is sharpest?

6. Friction value of pavement: texture of surface, such as rough or polished, and whether smooth or bumpy. A polished or bumpy pavement reduces pavement grip on tires.

7. Foreign matter on pavement: water, snow, sleet, sand, gravel, wet silt, oil slick, melting tar, etc.

8. Location of center of mass in his car: higher and/or farther back then usual, due to loading.

9. Condition of tires: inflation pressure, strength of sidewalls—are tires old or are walls cracked?

III. KINETIC ENERGY AND BRAKING DISTANCE

A. Energy is a measure of the capacity or power to do work. Energy exists in many forms such as chemical, electrical, heat, light, atomic, and mechanical. Kinetic energy is a form of mechanical energy possessed by a moving body due to its motion.

1. Kinetic energy is a deceptive killer in traffic because it increases by the square of the speed and an untrained driver cannot predict this fact. Actually the fact may seem illogical and a violation of “common sense.” A speed of 60 mph is simply twice a speed of 30 mph, just as 60 pounds is twice a weight of 30 pounds. But the energy of a car going 60 mph is 4 times its energy at 30 mph. It is dangerous therefore for a driver to think in terms of speed alone. Unless he can think in terms of energy he will never have a proper respect for the destructive power of a moving car and will be unable to judge what a “speed safe for conditions” really means. Students should understand the “square” concept and how it affects control of a car.

a. The textbook formula for calculating kinetic energy is

\[ KE = \frac{1}{2} \text{mass} \times v^2 = \frac{1}{2} \frac{w}{g} \times v^2 = \frac{w}{64.4} \times v^2 = \text{ft lbs} \]

\[ g = \text{gravity} = 32.2 \]

\[ w = \text{weight in lbs} \]

\[ v = \text{vel. in ft/sec} \]

b. Another formula for KE in which the speed in mph can be used is simpler for class illustrations an gives practically the same results:

\[ KE = \frac{w}{30} \times \frac{V^2}{30} = \text{ft lbs} \]

30 is a constant

\[ w = \text{weight in pounds} \]

\[ V = \text{speed in mph} \]

The energy of a 3600 lb. car going 60 mph is

\[ KE = \frac{3600}{30} \times \frac{60^2}{30} = \frac{3600}{30} \times \frac{3600}{30} = \frac{12960000}{30} = 432000 \text{ ft lbs} \]

You will find that the same car at 30 mph possesses 108,000 ft lbs of KE.
2. KE must be changed into heat energy when a car comes to a stop.

a. If a car free rolls to a stop, the energy is changed gradually through rolling friction between tires and pavement. This method is not practical because it takes too long to stop.

b. When the brakes are applied (with the wheels still turning) the brake shoes exert a drag force against the brake drums attached to the turning wheels, thus changing the KE to heat energy more rapidly. This “braking with turning wheels” method is more practical than the free roll.

c. When the wheels are locked, the KE is changed to heat energy through friction between the tires and the road surface. This method stops a car faster than the “braking with turning wheels” can, but otherwise it is not as practical, because a driver loses directional control of his car when the front wheels are locked. And the car usually will skid in the direction the center of mass was moving. (Ideal brakes would automatically slow the turning wheels as rapidly as possible without letting the wheels lock. Drivers cannot control present brakes this well, because they cannot control the pressure exerted on the brake pedal.)

d. When the wheels are locked, the skid distance represents the KE of the car, and that is why the braking distance increases by the square of the speed. But for any given braking effort exerted by the brakes, the braking distance (whether or not the wheels are locked) represents the KE and therefore will increase by the square of the speed.

(1) A chart or other device showing braking distances at various speeds usually is based on a single braking effort value. When two charts give different distances for the same speed, the charts are based on different braking efforts and each can be correct for the braking effort used as a base, provided a car’s brakes and the friction value of the pavement both are as good as the braking effort used in the chart.

The braking distance chart (table) in PART TWO of this paper contains several columns of braking distances headed by %’s of braking effort. A chart could be based upon the data in any one of these columns, but it would be reliable only for the % braking effort (or comparable f value) selected.

B. The danger of kinetic energy (KE) may be better understood by relating it to potential energy, which is another form of mechanical energy.

1. Potential energy is the ability a mass has to do work due to its position such as a compressed spring or water behind a dam. When a mass with potential energy is set in motion its potential energy is changed to kinetic energy.

a. Calculate the KE a car has at some speed. Then divide the foot pounds of KE by the weight of the car to find the height in feet from which the car would have to fall to develop the KE due to the car’s motion at the selected speed. (See 2. Conservation of Energy.)

Example: 3600 lb. car going 60 mph

\[
KE = \frac{3600 \times V^2}{30} = \frac{3600 \times 3600}{30} = \frac{1296000}{30} = 432000 \text{ ft lbs}
\]

Potential Energy = wh

\[
PE = wh, \text{ or } h = \frac{PE}{w}
\]

height = \frac{432000 \text{ ft lbs}}{3600 \text{ lbs}} = 120 \text{ feet}

The 3600 pound car raised 120 feet possesses a capacity (due to its position) to do the same amount of work it can do (by virtue of its motion) at 60 mph.
Example: Suppose a 3600 lb. car going 60 mph skids to stop on a pavement with an f value of 1.0 (actually pavements do not have f values this high):

An f value of 1.0 creates a braking effort of 100%. A braking effort of 100% is equivalent to a rate of deceleration of 32 ft/sec/sec (100% of 32, gravity)

Braking distance = \frac{V^2}{30 \times f} = \frac{3600}{30 \times 1} = 120 ft

In the first example the car starts from rest at a position 120 ft. high and accelerates at a rate of 32 ft/sec/sec (air resistance is disregarded). When it hits the ground it is going 60 mph. During the fall the car's potential energy is changed to kinetic energy and when it strikes the ground its kinetic energy is instantly changed into heat energy.

In the second example the car going 60 mph starts decelerating its speed at the same rate it increased its speed during the fall, 32/sec/sec; and in the same distance, 120 ft. it dissipates its kinetic energy and comes to rest. If the power exerted by the tires and pavement in this example were harnessed to hoist the car it could raise the car 120 ft. high in the same time it took for the car to skid to a stop.

2. Conservation of Energy. The total energy in the universe remains constant. Energy is never created or destroyed. Energy lost by one body exactly equals that gained by a second. One kind of energy may be transformed into an equivalent amount of one or more of the same or other kinds of energy, but the total energy is always conserved.

KE and PE are forms of mechanical energy. A car parked on a hill has potential energy. When the brake is released and it starts rolling its potential energy is being changed to kinetic energy.

When a car skids, its KE is being transformed into heat energy. When a car strikes a fixed object, all of its KE is transformed instantly into forces which bend, twist and heat the object, the car and the car's occupants. In a rolling-wheel stop the brakes change KE into heat energy which heats the brakes, which radiate heat into the atmosphere, raising the air temperature. If the air is set in motion we have kinetic energy again. A similar transformation takes place in a locked-wheel stop, in which the tires and pavement are heated.

a. As stated, when the car starts falling from a height of 120 feet, its potential energy (due to position) is changed into kinetic energy (due to motion) and its speed when it hits the ground will be 60 mph (88 ft/sec). Use gravity acceleration of 32.2 ft/sec/sec in the following formula to verify the speed:

\[ v = \sqrt{u^2 + 2as} \]

\[ v = \sqrt{0 + (2 \times 32.2 \times 120)} \]

\[ v = \sqrt{7728} = 88 \text{ ft/sec} \]

b. When a moving car strikes an object, a part of the car's KE is transformed into KE of the object (if the object is put into motion) and/or into heat energy and forces which deform the car and the object. If the object hit is fixed, such as a bridge-head or a tree, or if the object is moving in the opposite direction as in a head-on collision, and "explosion" of energy occurs, because the KE of the car(s) is dissipated in about one second.

c. Incidentally, one should not confuse KE which is measured in foot pounds with Centrifugal force (CF) which is measured in pounds. CF is a force in pounds "pushing" against the side of a car. KE is the measure of a force in pounds which a moving car is able to exert through a distance in feet.

C. While KE always increases by the square of the speed, braking distances in tests do not always exactly reflect the "square of the speed" phenomenon expressed in the formula, due to
1. slight changes in the amount of retarding friction between tires and pavement during skids at different speeds, when the same car is used in tests.

2. different effects of locations of centers of mass in vehicles (especially those with long and short wheel bases) when brakes are locked, causing different percents of car’s weights to be shifted to the front wheels.

3. different textures of synthetic rubber in tires, when different cars with different brands of tires are used in tests.

4. slight errors in speedometers at different speeds and in reading the speedometer when the brakes are locked in tests. At 30 mph, an error of 1 mph produces an error of over 3 feet.

D. Impact force in a collision

1. Knowing how kinetic energy increases with increases of speed, how it is related to ground distances covered in acceleration and deceleration, and how destructive it can be, is of first importance in improving driver judgment to avoid collisions. However, a driver’s problem not only is one of keeping his car’s energy within reasonable limits but also is one of keeping damage and injury to a minimum once a collision is imminent. Since the term “impact force” has some meaning to anyone who ever bumped his head, an analysis of the second part of the driver problem will be made here around the term “impact force.”

This analysis offers an opportunity also to clarify in part the commonly used term “momentum” (which students are bound to introduce into any discussion of moving objects) and to show the relation between momentum and kinetic energy.

2. An impulse is a force acting through a time. The shorter the time in which a force acts the more powerful the blow it produces.

\[ \text{Momentum} = \text{mass} \times \text{velocity} = \frac{\text{weight}}{32} \times \text{velocity.} \]

An impulse is measured by the change in momentum it produces divided by the time it takes to produce the change. If Car A hits and accelerates Car B, Car B’s momentum is changed. The change is in its speed since its weight is constant. If Car B’s momentum before the acceleration is labeled mom, and its momentum after the acceleration is labeled mom₂ then the

\[ \text{Impulse} = \frac{\text{mom}_2 - \text{mom}_1}{t \text{ (sec)}} \]

(in seconds) through which the impulse force acted. The change in momentum is equal to the force which produced the acceleration of the mass. The acceleration produced is the increase of speed divided by the time, or

\[ a = \frac{v_2 - v_1}{t}, v_2 = \text{speed after the pushing} \]
\[ v_1 = \text{speed before the pushing started} \]

The force which produced the acceleration is \( F = ma \) (mass \( \times \) acceleration), or

\[ F = m \frac{v_2 - v_1}{t}, \text{ or } F = \frac{mv_2}{t} - \frac{mv_1}{t}, \text{ or } F = \frac{\text{mom}_2}{t} - \frac{\text{mom}_1}{t} \]

Note by studying the preceding formulas that for a given time the change in the velocity is proportional to the force; the greater the force, the greater the change in velocity. Now note also the following:

a. If Car A going 45 mph (66 ft/sec) runs into another vehicle and Car A decelerates to 30 mph (44 ft/sec) in 4 seconds the
Impulse Force = \( \frac{mv_i - mv_f}{4} = \frac{m66 - m44}{4} = \frac{m22}{4} = \text{mass} \times 5.5 \)

If Car A going 60 mph (88 ft/sec) runs into another vehicle and Car A decelerates to 45 mph (66 ft/sec) in 4 seconds the

\[
\text{Impulse Force} = \frac{m88 - m66}{4} = \frac{m22}{4} = \text{mass} \times 5.5
\]

The impulse forces in the two examples are equivalent because the changes in momentum in the two cases was the same amount, mass \( \times 22 \).

b. If in either of the above examples the time was 1 second, the force would have to be 4 times as great. When the time factor in a collision is too small to measure accurately the time is disregarded and the impulse is called an impact force.

3. A quick change in a large momentum produces a powerful blow. A large kinetic energy produces a destructive blow. Kinetic energy indicates how far a body will move against a given resistance before it stops. Momentum indicates how long a body will move against a given resistance before it stops. A car's KE includes momentum. KE does work which is force through a distance. If a car's momentum is changed suddenly, the car's KE is "compressed" into a powerful blow. It does a given amount of work in a shorter time. This compression of energy destroys cars and people. (See p. 50 or p. 101 for detailed information on the impact force of the lb-sec.)

a. In the two preceding examples of changes in momentum the accelerating forces were equal, although the speeds involved were 66 ft/sec to 44 ft/sec in one example and 88 ft/sec to 66 ft/sec in the other.

It takes the same accelerating force to accelerate a given car to a given increase in ft/sec, no matter what the initial speed of the car may be, because once a car is going at a given speed inertia tends to keep it at that speed. The force required to accelerate a given car at a given rate is the same starting at 66 ft/sec as it is starting at 44 ft/sec, or starting at any other speed for that matter, so long as the car is moving. (Increases in air and rolling resistances are disregarded in order to simplify the concept.)

b. The changes in Kinetic energy, however, were not the same. Let us illustrate this point with a 4000 lb. car, using the textbook formula:

(1) Energy of 4000 lb car going 44 ft/sec:

\[
\text{KE} = \frac{\text{weight} \times \text{velocity}^2}{2 \times \text{gravity}} = \frac{wv^2}{64} \quad \text{(We use 32 as the value of gravity for this illustration)}
\]

\[
\text{KE} = \frac{4000 \times 44^2}{64} = \frac{4000 \times 1936}{64} = 121000 \text{ ft lbs}
\]

(2) Energy of 4000 lb car going 66 ft/sec:

\[
\text{KE} = \frac{4000 \times 66^2}{64} = \frac{4000 \times 4356}{64} = 272250 \text{ ft lbs}
\]

(3) Energy of 4000 lb car going 88 ft/sec:

\[
\text{KE} = \frac{4000 \times 88^2}{64} = \frac{4000 \times 7744}{64} = 484000 \text{ ft lbs}
\]

The energy increase from 44 ft/sec to 66 ft/sec was 125%.

The energy increase from 66 ft/sec to 88 ft/sec was 78%.
Yet the actual energy added in going from 66 ft/sec to 88 ft/sec was 1.4 times the energy added in going from 44 ft/sec to 66 ft/sec.

In trying to visualize how kinetic energy increases we must remember that the energy of a given mph depends upon the number of mph below it, or to which it is being added. For example, a chart in PART TWO will show that the energy of 5 mph when added to 30 mph is 15 times as large as the energy of the car going only 5 mph. And 5 mph added to 60 mph is 25 times as large; and 5 mph added to 70 mph is 29 times as large. And of course the number of feet added to the braking distance increases in a like manner.

Be sure the students understand that we are comparing 5 mph with 5 mph, one 5 mph as the actual speed of a car and the other 5 mph added to some other speed.

For example, when we compare 5 mph added to 70 mph with an actual speed of 5 mph, we are not comparing 5 mph + 70 mph with 5 mph. The energy of 5 mph + 70 mph (or 75 mph) is 225 times the energy at 5 mph:

\[
\frac{75^2}{5^2} = \frac{5625}{25} = 225
\]

4. Here are types of collisions comparing impact forces for a vehicle going at a given speed:

a. Most severe is a head-on collision in which the other vehicle also is moving.

b. The next less severe collision is one with a fixed object, such as a bridgehead or tree.

c. Third in line is a right angle collision with a vehicle either moving or parked, which has room to move sideways without being stopped by a curb or other fixed object.

d. Next is an end collision with a standing vehicle which is free to roll during the impact.

e. Next is a rear-end collision with a moving vehicle. The more slowly the struck vehicle is going the more dangerous this type is, of course.

f. Then comes a sideswipe collision—first, with a fixed object and, second, with a moving vehicle, provided the angles of collision are very small.

g. Least severe perhaps is a collision with stationary objects which will yield, bend, uproot, or break during impact, such as small shrubbery, a barbed wire fence, or brush which does not decelerate the car too fast. (Along this line, research is being conducted with the multiflora rose, a thickly matted bush which might be planted near curves, at T-intersections, along median-strips, and in other selected areas to catch cars which go out of control and decelerate them gradually.)

5. When a driver is in a trap and it is apparent that he cannot escape a collision of some sort he should look for an out which will involve the least impact force. Frequently drivers not conditioned to this sort of planning have stayed on the pavement and become involved in high impact force collisions when they might have gone to the borrow ditch or even into a field or pasture and escaped with relatively minor damage to their vehicles.

Such drivers have locked their brakes on a wet road to avoid hitting a car ahead rear-end and skidded head-on into opposing traffic. Others have wheeled their cars into opposing traffic to avoid a rear-end collision with a vehicle they are following too closely. In many such instances the drivers could have steered to the right and escaped with a minor fixed object collision, because they would have had more space in which to decelerate before hitting the fixed object. Every mph reduction in speed right up to the point of contact reduces the energy of a car rapidly.

Drivers in panic (unable to control their muscles for a few seconds immediately following sudden fright) have held their brakes locked and plowed into fixed objects, when, had they been conditioned to good planning,
could have braked just short of locked wheels, maintained steering control of their cars, and escaped with a sideswipe collision.

6. Either weak brakes or a low $f$ value makes a braking distance so long that more collisions could result, even though a low deceleration rate such as 10 ft/sec/sec is very desirable from the standpoint of passenger comfort. A deceleration rate of 10 ft/sec/sec is equivalent to a braking effort of 31% and to an $f$ value of 0.31 (deceleration rate divided by 32, gravity, gives the braking effort). A deceleration rate of 22 ft/sec/sec is considered a practical maximum, because at a higher rate passengers must brace themselves to stay in their seats. A rate of 22 ft/sec/sec is equivalent to a braking effort of 69% or an $f$ value of 0.69. This braking effort or $f$ value permits a braking distance of 174 ft. at 60 mph, but many car brakes are not that good and many pavements do not have an $f$ value that high. If all brakes and all pavements (dry or wet) were that good, the number of collisions could be reduced appreciably.

7. The present general condition of brakes and pavements presents the following alternatives to drivers. Either they must shorten their braking distances by adjusting their speeds to the limitations of present brakes and pavements or they will continue to shorten their braking distances through collisions. All other factors even, this is the crux of the collision problem and the challenge of driver education instructors. All the past and current ballyhoo and propaganda to the contrary notwithstanding, increases in speeds in a given environment will increase collisions and the severity of injuries. Present speeds are compromises with safety. The penalty of higher speeds, with other factors constant, is more collisions and more injuries.

E. Relation of Momentum to Kinetic Energy

1. Momentum should not be confused with kinetic energy, and the instructor should understand how the two are related.

The following equations show (1) the relation of Force to Momentum and Impulse, and (2) the relation of Force $\times$ Time (Momentum) to Force $\times$ distance (Kinetic Energy):

\[
\text{Force} = \text{mass} \times \text{acceleration} = \frac{\text{weight} \times a}{\text{gravity}} = \frac{\text{lb}}{32 \text{ ft/sec}^2} = \text{lb}
\]

\[
\text{Acceleration} = \frac{v_2 - v_1}{t} = \text{change in velocity} = \text{ft/sec} = \frac{\text{ft/sec}}{\text{sec}} \text{ (or ft/sec)$^2$)
\]

\[
\text{Momentum} = \text{mass} \times \text{velocity} = \frac{\text{weight} \times v}{\text{gravity}} = \frac{\text{lb}}{32 \text{ ft/sec}^2} = \text{lb-sec}
\]

\[
\text{Impulse} = \text{change in momentum} = \text{mom}_2 - \text{mom}_1 = \text{mass} \times v_2 - \text{mass} \times v_1
\]

\[
= \frac{\text{weight}}{\text{gravity}} \times v_2 - \text{gravity} \times v_1 = \frac{\text{lb} \times \text{ft/sec}}{\text{sec}} - \frac{\text{lb} \times \text{ft/sec}}{\text{sec}} = \text{lb-sec}
\]

\[
\text{Force} = \frac{\text{change in momentum}}{\text{time} \text{ required for change}} = \frac{\text{mom}_2 - \text{mom}_1}{\text{time}} = \text{mass} \times \frac{v_2 - v_1}{\text{time}} = \text{mass} \times \frac{v_2 - v_1}{\text{time}} \times \frac{\text{grav}_{y}}{\text{ft/}_{\text{sec}}^2} = \text{lb-sec}
\]

\[
\text{Ft} = \text{mass} \times v_2 - \text{mass} \times v_1 = \text{mass} (v_2 - v_1) = m (v_2 - v_1).
\]

\[
\text{s (distance)} = \text{average velocity} \times \text{time} = \frac{1}{2}(v_2 + v_1) \times t, \text{ or cross multiplying,}
\]

\[
\text{s} = \frac{1}{2}(v_2 + v_1). \text{ Multiplying corresponding sides of this equation and the Ft equation:}
\]

\[
\text{Ft} \times \frac{s}{t} = m (v_2 - v_1) \times \frac{1}{2}(v_2 + v_1) \text{ and we get}
\]

88
\[ Fs = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \text{change in Kinetic Energy} = \text{Work done} \]

Note: "lb-sec" is a compound word which relates force to time just as "ft-lb" relates force to distance. (See p. 50 or p. 101 for detailed information on the Impact force of the lb-sec.)

2. The force which accelerates a car is a "push" created and applied continually by the engine. Any time the "push" is discontinued or at least reduced until it just balances rolling friction and air resistance, the car continues at a constant speed. If these resistances were not present, inertia would keep the car moving at the speed it had attained when the accelerating force was discontinued and work would have to be done on the car in order to change its speed, either up or down, by the engine if the speed were increased and by the brakes if the speed were decreased.

a. A given force will change the velocity of a given mass a certain amount in a given time. It makes no difference to the force how fast the mass was moving when the force started acting on the mass. When the velocity of a given mass is changed a given amount, the momentum of the mass is changed in proportion to the change in speed.

The kinetic energy, however, is much concerned with how fast the mass was moving before its speed was changed, because KE changes in proportion to a change in the square of the speed and the amount of KE in a mph depends upon the speed before a change.

3. A car going at a constant speed is a mass which possesses a quantity of motion (mass \times velocity) and this quantity is called momentum. It is a convenient term for identifying a factor which is made up of two variables, weight and velocity, without having to be concerned with whether the factor results from a small weight \times a high velocity or a large weight \times a low velocity. Since momentum of a given mass depends upon its velocity in feet per second, the quantity of motion changes with time. The unit of momentum is lb-sec, a measure of force and time.

4. A term used to identify the relation between a change of momentum and the time during which the change takes place is called impulse. It is a force whose size is dependent on time. Since it represents a change in momentum, its unit also is lb-sec.

5. Look at the equation, Force = ma. The mass is the same as in the equation,

\[
\text{Impulse} = \frac{mv_f - mv_i}{t}, \text{ and the acceleration, } a = \frac{v_f - v_i}{t}, \text{ is the same as in the Impulse equation. It is clear then that Force also equals the change in momentum } = \frac{mv_f - mv_i}{t} \text{ and also that } \]

\[
Ft = mv_f - mv_i = m(v_f - v_i) \]

6. The use of the next equation, \( s = \frac{1}{2}(v_f + v_i) \times t \), and its transposed form, \( s = \frac{1}{2}(v_f + v_i) \), is the transition step in which the Force \times Time concept in Momentum evolves into the Force \times Distance concept in Kinetic Energy.

The equation, \( Fs = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \), states that the work (Fs) done by a force while it acts on a mass through a distance(s) is equal to the excess of \( \frac{1}{2} mv_f^2 \) over \( \frac{1}{2} mv_i^2 \). The term \( \frac{1}{2} mv^2 \) is called the kinetic energy, or the capacity of a mass to do work by virtue of its motion.

7. Finally, note that \( \frac{1}{2} mv^2 \) is equivalent to \( \frac{1}{2} \) of the momentum times the velocity:

\[
\text{Momentum} = \text{mass} \times \text{velocity} = mv \]

\[
\text{Kinetic Energy} = \frac{1}{2} \text{momentum} \times \text{velocity} = \frac{1}{2} \text{(mass} \times \text{velocity}) \times \text{velocity} = \frac{1}{2} \text{mass} \times \text{velocity} \times \text{velocity} 
\]

\[
\text{KE} = \frac{1}{2} mv^2
\]
F. Energy (speed), Collisions, Injuries, and Deaths

1. More collisions occur on city streets than on rural highways because
   a. Vehicles are continually stopping in traffic lanes to load or park and usually are backing against traffic either on entering or leaving a parking zone.
   b. Vehicles cross or enter each other's paths every 300 feet or so at intersections.
   c. Vehicles are continually entering traffic from private driveways within every block.
   d. Vehicles turn left in front of other vehicles at nearly every intersection and frequently between intersections entering private driveways.
   e. Pedestrians are continually crossing or entering the paths of vehicles at intersections and between intersections.
   f. Bicycles frequently are moving contrary to the normal traffic patterns.

2. Very few of the above mentioned position conflicts occur on rural highways.
   a. This is one reason, and if the same volume of traffic is present, the principal reason vehicles can move at higher speeds on rural roads with fewer collisions than occur on city streets.

3. In city traffic there are 155 property damage only accidents and 55 non-fatal injury accidents to 1 fatal accident, while on rural highways there are 25 property damage only accidents and 14 non-fatal injury accidents to 1 fatal accident. (Accepted ratios in 1979.)

<table>
<thead>
<tr>
<th>Type of Traffic</th>
<th>Ratios of Types of Accidents</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Property Damage Only Accidents</td>
<td>Non-Fatal Injury Accidents</td>
</tr>
<tr>
<td>Town &amp; City</td>
<td>155</td>
<td>55</td>
</tr>
<tr>
<td>Rural Roads</td>
<td>25</td>
<td>14</td>
</tr>
</tbody>
</table>

4. Vehicle miles driven in Texas are fairly evenly distributed between the streets in towns and cities and the roads outside of towns and cities. What, then accounts for
   a. the ratios of the numbers of a given type of accident in city and rural traffic such as the ratio 155 for property damage only accidents and the ratio 55 for non-fatal injury accidents? 25
   b. the great difference between the ratios of the three types of accidents in city traffic (155:55:1) and in rural traffic (25:14:1)?

5. There are two factors which account for the pattern in the ratios in the number of collisions and in the severity of injuries:
   a. One is the great difference in the number of points of conflict present in city and rural traffic.
   b. The second is the great difference in the amount of kinetic energy possessed by vehicles moving in city and in rural traffic.

6. Multiplying the ratio figures for types of accidents by the number of total fatal accidents in 1979 we get the following:
7. Analysis:

a. The principal way in which the larger number of property damage only accidents in town and city traffic can be reduced is by reducing the number of points of conflict, because the much smaller number of rural property damage only accidents shows that without the points of conflict traffic can move at higher speeds with fewer property damage only accidents. Of course, many of the major sources of conflicts such as private driveways and intersections cannot be eliminated. The only alternative to eliminating the physical defects is to reduce the energy of the moving vehicles which collide at the points of conflict.

b. Note in the numbers of non-fatal injury accidents that the rural advantage of fewer collisions due to fewer points of conflict becomes a disadvantage due to higher speeds.

1. 26% of all town and city accidents were non-fatal injury accidents.
2. 35% of all rural road accidents were non-fatal injury accidents.

c. A comparison of the ratios will reveal that an accident rate (number of accidents per 100,000 vehicle miles traveled) for town and city traffic will be much higher than an accident rate for the rural roads. Obviously, this difference is due to the many points of conflict in town and city traffic, but (and this is important) many persons uninformed about energy use these differences in accident rates to "prove" that speed does not cause accidents, because, they argue, in cities where speeds are lower the accident rate is higher. They conclude that 60 mph is safer than 30 mph, when actually 60 mph is four times as dangerous as 30 mph, for a given physical environment, no matter what it might be. They should conclude that the way to reduce the high accident rate in cities where drivers cannot escape many points of conflict is to reduce the speeds.

d. The ratios in the table of item 6 indicate the relative change in the safety factor of speed. The ratio of town and city non-fatal injury accidents to the rural road non-fatal injury accidents is less than two-thirds the ratio of property damage only accidents. This change points up how much more unsafe the higher rural road speeds are. In the case of fatal accidents, the ratio is completely reversed and we find more fatal accidents for the same amount of vehicle miles traveled.

e. Cornell University Auto Crash Research reports indicate that the exposure of car occupants to critical injury or death is nearly 3 times as great at speeds above 60 mph as it is at speeds under 60 mph. To be exact, the ratio reported is \frac{16}{6}.

f. If one understands energy he would expect such a finding. Suppose one drops 100 eggs from a height of 1 foot, another 100 from 2 feet and so on up to 100 feet. No matter how the eggs are packaged, so long as they are packed alike, and no matter what the surface is the eggs are dropped on, foam rubber or cement, the pattern of breakage will be as follows: The higher the drop the higher the percentage of cracked eggs.

g. In the thought experiment, all of the drops are considered to be "accidents" in which collisions occur. The experiment relates the energy involved to the severity of damage or injury, once the accidental contacts are made. The experiment does not relate energy to the frequency of points of conflict; that is, it does not show how avoiding collisions at points of conflict is made more difficult as speeds increase. However, it should be obvious that in town and city traffic where points of conflicts are numerous, there are two ways to reduce the number of collisions which occur at the points of conflict and which cause the accident rate in town and city traffic to be much higher than in rural road traffic.
One way is to steer around the point of conflict when a collision is impending. The other way is to stop short of the point of conflict when a collision is impending. Too much energy for conditions makes a driver helpless in following the second procedure and often either makes it difficult for him to steer around or makes him lose control of his car if he attempts to steer around. Energy, then, is a driver's major obstacle to steering and stopping safely, no matter where he drives or at what speed he drives. And the higher the speed in a given environment the more helpless a driver becomes. A student cannot become an intelligent driver unless he accepts and drives by these precepts.

G. How Energy, Braking Effort, Braking Distance and Work Formulas are Related

1. Formulas to be compared:

\[ KE = \frac{\text{weight} \times (\text{mph})^2}{30} \]

\[ \text{Braking Distance} = \frac{(\text{mph})^2}{30f} \]

\[ PE = \text{weight} \times \text{height} \]

\[ \text{Braking Effort or } f = \frac{\text{Force}}{\text{weight}} \]

\[ \text{KE} = \text{Potential Energy} \]

\[ \text{Braking Effort or } f = \frac{(\text{mph})^2}{30 \times \text{braking distance}} \]

\[ \text{Work} = \text{Force} \times \text{distance} \]

\[ \text{Energy} = \text{Work} \]

Since \( f = \frac{F}{w} \), \( \text{Force} = \text{weight} \times f \)

2. Example: 3600 lb car going 60 mph

a. \( KE = \frac{3600 \times 60^2}{30} = 432,000 \text{ ft lbs} = \text{Work} \)

b. \( \text{Work} = \text{Force} \times \text{distance} = (\text{weight} \times f) \times \text{distance} \)

(1) If \( f = 1.0 \), \( \text{Work} = (\text{weight} \times 1.0) \times \text{distance} \)

\[ 432,000 = 3600 \times \text{distance}, \text{or transposing, distance} = \frac{432,000}{3600} = 120 \text{ feet} \]

(2) If \( f = 0.5 \), \( \text{Work} = (\text{weight} \times 0.5) \times \text{distance} \).

\[ 432,000 = (3600 \times 0.5) \times \text{distance}, \text{or transposing, distance} = \frac{432,000}{1800} = 240 \text{ feet} \]

c. \( \text{Braking Distance} = \frac{(\text{mph})^2}{30f} \)

(1) If \( f = 1.0 \), \( \text{Braking Distance} = \frac{60^2}{30 \times 1.0} = 3600 = 120 \text{ feet} \)

(2) If \( f = 0.5 \), \( \text{Braking Distance} = \frac{60^2}{30 \times 0.5} = 3600 = 240 \text{ feet} \)

d. \( \text{Potential Energy} = \text{weight} \times \text{height} \) (or \( \text{force} \times \text{distance} \))

(1) If a 3600 lb car is raised 120 feet,

\[ \text{PE} = 3600 \times 120 = 432,000 \text{ ft lbs} = \text{Work done on the car} \]

(2) If a 3600 lb car falls 120 feet,
KE = PE = 432,000 ft lbs = Work car's energy does on the ground and on the car

3. Incidentally, if we arrange the braking distance formula to solve for mph, we get the formula used to calculate the least speed necessary to lay down a given set of skidmarks:

\[
BD = \frac{(\text{MPH})^2}{30 \times \%\text{BE}} \quad \text{(f value of pavement is the BE when all tires skid)}
\]

\[30 \times f \times BD = (\text{MPH})^2 \quad \text{(cross multiply)}\]

\[
\text{MPH} = \sqrt{30 \times f \times BD} \quad \text{(transpose and take the square root of both sides of the equation)}
\]

\[
\text{MPH} = 5.5 \sqrt{s} \quad \text{(s = BD, measured by taking the average length of all 4 skid marks)}
\]

IV. HORSEPOWER AND ACCELERATION

A. Horsepower is simply a convenient term used for measuring power.

1. Power = Work done per unit of time

   a. Power = Kinetic Energy in ft. lbs = \(\frac{\text{ft lbs}}{\text{sec}}\)

   b. 1 horsepower = 550 ft lbs of work done in 1 second or it will raise 550 lbs one foot in one second

   \[
   \text{Number of horsepower} = \frac{\text{Power in ft. lbs/sec}}{550 \text{ ft lbs/sec/hp}} = \text{number of horsepower (hp = horsepower)}
   \]

   The more horsepower an engine can exert the more work it can do in a given time.

   c. An engine in accelerating a 3600 lb. car to 88 ft/sec (60 mph) does work at least equivalent to 432,000 ft. lbs. of kinetic energy because that is the energy the car possesses at 60 mph. If this work is done in 11 seconds (which would require an acceleration rate of 8 ft/sec/sec) the power, or rate of doing the work, may be determined as follows:

   \[
   \text{Power} = \frac{\text{Work (ft lbs)}}{\text{time (sec)}}
   \]

   \[
   \text{Power} = \frac{432,000 \text{ ft lbs}}{11 \text{ sec}} = 39,273 \text{ ft lbs/sec}
   \]

   \[
   \text{Horsepower} = \frac{39,273 \text{ ft lbs/sec}}{550 \text{ ft lbs/sec/hp}} = 71.4 \text{ hp (required to accelerate 3600 lb car to 60 mph in 11 sec)}
   \]

   Most cars do not have the reserve horsepower to accelerate up to 60 mph this fast. Most cars can accelerate faster than this in low gear for a short distance, provided the f value of the pavement is high. The f value and the position of the center of mass limit acceleration, regardless of the power of the engine.

   If the car going 60 mph is braked to a stop in 5\(\frac{1}{2}\) seconds (which would be a deceleration rate of 16 ft/sec/sec or a braking effort of 50%) the work done by the brakes and/or the tires and pavement is 432,000 ft. lbs.

   The power is:

   \[
   \text{Power} = \frac{432,000 \text{ ft lbs}}{5.5 \text{ sec}} = 78,545 \text{ ft lbs/sec}
   \]
Horsepower = \frac{78454}{550} = 142.8 \text{ hp}

The brakes exert twice as much hp as the engine in doing the same amount of work in half the time.

d. Brake horsepower has nothing to do with the brakes of an automobile. The “brake” relates to the Prony brake, a braking device which “clamps” onto a wheel on the crankshaft of an engine mounted on a block in a laboratory. It registers in lb-ft the moment of force or the torque the engine can exert at a given rpm. This and similar devices are used to rate an engine’s gross horsepower.

e. A novel example for comparing the power required to lift a car with the power required to propel it is the air car which is being developed. The model of one manufacturer weighs 450 lbs. It uses 15 hp of push to hold it off the ground and 1\frac{1}{2} \text{ hp} to propel it at 15 mph.

2. An automobile engine rated 200 gross horsepower at 100 mph might have only 145 gross horsepower at 60 mph, according to one manufacturer.

Furthermore, according to the manufacturer, at 60 mph the engine might produce only 100 net horsepower available at the rear wheels. The difference (45 hp) between the gross horsepower and the net horsepower available to propel the car would be absorbed by atmospheric conditions (altitude), exhaust heat and spark, air cleaner, muffler, fan and generator, power steering, air conditioning, and transmission, and rear axle.

Keeping the car moving at a given speed on a smooth, level, paved road requires varying amounts of horsepower. This is called “road-load horsepower” which is needed to overcome rolling resistance, air resistance and chassis friction. The road-load horsepower required increases as the speed increases: at 40 mph it is 15 and at 70 mph it is 45. Air resistance accounts for much of the increase. Modern low pressure tires or underinflated tires increase rolling resistance. Rolling resistance may require up to 35 hp at very high speeds.

The difference between the net horsepower available at the rear wheels and the road-load horsepower is called reserve horsepower. This is the power available for accelerating and climbing and driving on rough, level terrain.

In the example engine under discussion (and described by a manufacturer) the reserve horsepower decreases as the speed increases or decreases, from 60 mph. On a smooth, level road the reserve horsepower varies as follows:

<table>
<thead>
<tr>
<th>Speed of Car</th>
<th>Reserve Horsepower</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 mph</td>
<td>30 hp</td>
</tr>
<tr>
<td>40 mph</td>
<td>58 hp</td>
</tr>
<tr>
<td>60 mph</td>
<td>70 hp</td>
</tr>
<tr>
<td>80 mph</td>
<td>60 hp</td>
</tr>
</tbody>
</table>

The car has maximum reserve horsepower at a speed of about 60 mph.

3. A low horsepower engine might be able to do the same amount of work as a high horsepower engine but it cannot do the work in as short a time as the high horsepower engine. A modern engine might accelerate a car from rest to 60 mph in half the time required by an engine 30 years ago.

4. Once a car reaches a given speed it takes much less work or power to keep it moving at a constant speed. This is due to nature’s law of inertia. Where it takes 70 horsepower to accelerate a car to 60 mph in 22 seconds it might take only 30 horsepower to keep it moving at 60 mph on a level road. The actual pushing force required to balance the retarding forces of air, tires, and chassis, at the constant speed, is 187.5 lbs. and may be computed as follows:

\[30 \text{ hp} = 16,500 \text{ ft lbs/sec} (30 \times 550)\]
Power = Force (lbs) \times \text{velocity (ft/sec)}

\[ 16,500 = \text{Force} \times 88 \]

\[ \text{Force} = \frac{16,500}{88} = 187.5 \, \text{lbs} \]

To accelerate a 3600 lb car at a rate of 4 ft/sec/sec requires a constant force of 450 lbs.

\[ \text{Force} = \text{mass} \times a = \text{weight} \times a \]

\[ F = \frac{3600 \times 4}{32} = 450 \, \text{lbs} \]

5. Gasoline consumption will be lower if a driver keeps his car lubricated and greased properly, keeps the front wheels properly aligned, keeps the tires properly inflated, and avoids fast acceleration. All of these precautions except fast acceleration reduce the road-load horsepower required to overcome chassis friction and rolling friction while cruising.

In accelerating, the engine must work against inertia which tends to keep the car at a constant speed. The car's weight therefore is the main factor. The faster the car accelerates, the more fuel energy it takes. (Chemical energy of the gasoline is changed into kinetic energy.)

Cruising at a high speed requires a higher rate of gas consumption because rolling friction and air resistance increase faster than the speed increases. A tire manufacturer reports that 25% of the fuel used by an economy car is consumed by the drag of its tires.

B. Acceleration is a term used to express the rate at which a car changes its speed. Acceleration can be positive or negative. Negative acceleration is also called deceleration.

1. Acceleration = \frac{\text{change in speed}}{\text{time}} = \text{rate of change of speed per unit of time required to change the speed}

a. If a car going 30 mph (44 ft/sec) increases its speed to 60 mph (88 ft/sec) in 11 seconds, it increases its speed 4 ft/sec during every second of acceleration: \[ \frac{88 - 44}{11} = 4 \, \text{ft/sec} \]

The car is going 44 ft/sec. At the end of the 1st second it will be going \[ 44 + 4 = 48 \, \text{ft/sec} \]; at the end of the 2nd second it will be going \[ 48 + 4 = 52 \, \text{ft/sec} \], etc. It is, therefore, increasing its speed 4 feet per second every second and its rate of acceleration is said to be 4 feet per second per second (written 4 ft/sec/sec or \( \frac{4 \, \text{ft/sec}}{\text{sec}^2} \) or 4 ft/sec or 4 ft).

b. If a car starts from a stop and reaches a speed of 30 mph (44 ft/sec) in 11 seconds, its rate of acceleration is \[ \frac{44 - 0}{11} = 4 \, \text{ft/sec/sec}. \]

c. If a car going 60 mph (88 ft/sec) brakes to a stop in 8 seconds its rate of deceleration is \[ \frac{0 - 88}{8} = -11 \, \text{ft/sec/sec} \] (neg. acc.).

2. The following formula was used in the preceding examples:

\[ a = \frac{v - u}{t} \]

\( v = \) final speed in ft/sec

\( u = \) initial speed in ft/sec

\( t = \) time in sec

\( a = \) rate of acc. in ft/sec/sec
3. When u is 0, \( a = \frac{v}{t} \) (Transposing, one can also find \( v = at \), and \( t = \frac{v}{a} \))

To convert mph to ft/sec multiply the mph by 1.467. To convert ft/sec to mph multiply the ft/sec by .682.

4. The most important lesson involving acceleration for a driver to learn is how the ground distance covered each second varies when a car is accelerating or decelerating.

\[
s = ut \pm at^2
\]

\( s = \) distance in feet

\( u = \) initial velocity in ft/sec

\( a = \) acceleration in feet per second per second

\( t = \) time in seconds

If the car is accelerating use the plus sign. If the car is decelerating use the minus sign.

5. If the car starts from rest the initial velocity (u) is zero and the formula is, \( s = \frac{at^2}{2} \)

6. Problem showing differences in ground distances covered by a car decelerating from 70 mph when a 50% braking effort is exerted; this example will point up how dangerous a few extra mph can be when a driver must decelerate quickly to avoid a hazard:

a. At 70 mph a car is going 103 ft per sec. \((70 \times 1.467)\). With a 50% braking effort a car will decelerate at a rate of 16 feet per second per second. The \( \text{BE} 50\% \times \text{gravity} 32 \text{ ft/sec/sec} = 16 \text{ ft/sec/sec} \). We now have the value of acceleration to be used in the distance formula to obtain the ground distance covered in 1 second, 2 seconds, etc:

\[
s = ut - at^2
\]

(The minus sign is used because the car is decelerating)

\[
s = (103 \times 1) - 16 \times 1^2\]

\( s = 103 - 8 = 95 \text{ feet (the ground distance covered the 1st second).} \)

What is the speed at the end of 1 second after brakes are applied:

\[
a = \frac{v - u}{t}, \text{ or transposed, } v = u - at.
\]

\( v = 103 - (16 \times 1) = 87 \text{ ft/sec or } 59 \text{ mph (87 \times 0.682)} \)

The car in 1 second decelerated from 70 mph to 59 mph and covered a ground distance of 95 feet.

b. Now determine the ground distance covered in 2 seconds after brakes are applied:

\[
s = (103 \times 2) - 16 \times 2^2\]

\( s = 206 - 32 = 174 \text{ feet, the ground distance covered during the 1st and 2nd seconds. The ground distance covered during the 2nd second } = 174 - 95 = 79 \text{ feet.} \)

What is the speed at the end of 2 seconds:

\( v = u - at \)

\( v = 103 - (16 \times 2) = 71 \text{ ft/sec, or } 48 \text{ mph (71 \times 0.682)} \)
c. Table of the above data:

<table>
<thead>
<tr>
<th>Brakes applied</th>
<th>Ground distance covered during 1st second</th>
<th>Speed at end of 1st second</th>
<th>Ground distance covered during 2nd second</th>
<th>Speed at end of 2nd second</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 mph</td>
<td>95 ft.</td>
<td>59 mph</td>
<td>79 ft.</td>
<td>48 mph</td>
</tr>
</tbody>
</table>

In the 1st second of braking the car covers 95 feet on the ground (about one-third of a city block), and during this one second the car’s speed is reduced from 70 mph to 59 mph. This example shows how hazardous only 11 mph can be when an emergency arises. The 11 mph adds 95 feet to the braking distance. If at 59 mph a driver could barely have stopped before hitting an object, at 70 mph he would have enough energy to go 95 feet more after reaching the object. An initial speed of 11 mph with 50% braking requires a distance of only 7.4 feet to stop. But when the 11 mph is added to 59 mph, the 11 mph requires a ground distance of 95 feet. Every student should understand these facts thoroughly.

d. Assume the driver is traveling a legal speed of 59 mph. Note how much he can reduce his braking distance by slowing down early when he sees that a hazard ahead might develop into an emergency. If the driver, upon seeing the hazard ahead, slacks off on the accelerator and reduces his speed 11 mph, to a speed of 48 mph, (it takes about 5 sec. and 400 feet for an engine in conventional gear to do this) he will have eliminated 79 feet from his braking distance. If the hazard should develop into an emergency, his defensive act of easing up on the accelerator early might enable him to stop short of a collision, whereas had he held his speed at 59 mph he would have collided at a speed of 34.5 mph. (34.5 mph requires a braking distance of 79 feet at 50% braking.)

While drivers of most cars on most pavements could exert more than 50% braking at speeds under 40 mph without too much danger, harder braking at a high speed of 60 mph might be extremely dangerous. Many drivers would lose control of their cars. The smart thing to do is to ease off on the gas pedal at the very first sign of danger.

7. Deceleration table showing how speed each second and ground distance covered each second decreases.

| Braking from 60 mph with Braking Effort of 50% (a = 16 ft/sec/sec) |
|-------------------------|-------------------------|-------------------------|-------------------------|
| Seconds of Braking From 60 mph | Distance Covered | Accumulated Distance Covered | Speed at End of Sec. Ft. per second | Miles per hour |
| 1st second              | 80 ft.                  | 80                      | 72 ft/sec              | 49 mph        |
| 2nd second              | 64 ft.                  | 144                     | 56 ft/sec              | 38 mph        |
| 3rd second              | 48 ft.                  | 192                     | 40 ft/sec              | 27 mph        |
| 4th second              | 32 ft.                  | 224                     | 24 ft/sec              | 16 mph        |
| 5th second              | 16 ft.                  | 240                     | 8 ft/sec               | 5 mph         |
| ½ of 6th sec            | 2 ft.                   | 242                     | 0 ft/sec               | 0 mph         |

5½ sec. total 242 ft., total braking distance

8. Acceleration from a stop presents a particularly difficult problem for drivers because the ground distance covered is very deceptive.

a. In braking, the long distances covered each second occur during the first half of the deceleration time, while in accelerating from a stop the short distances covered each second occur in the first half of the acceleration time.

b. In braking, high deceleration rates can be attained, up to 20 ft/sec/sec normally, while only relatively low acceleration rates are possible. An average rate of 4 ft/sec/sec from a stop to 30 mph is more than most drivers normally attain.
c. It is probably more difficult for a driver to visualize how short the ground distance he covers during the first half of an acceleration maneuver than it is to visualize how long the ground distance he covers during the first half of a deceleration maneuver. Not only are the proportional distances per second reversed in accelerating, but also the acceleration rates are much lower than the deceleration rates.

9. Acceleration table showing how ground distance covered each second increases from a stop to 30 mph with an acceleration rate of 4 ft/sec/sec:

<table>
<thead>
<tr>
<th>Seconds of Acceleration from a Stop</th>
<th>Distance Covered each sec.</th>
<th>Accumulated Distance Covered</th>
<th>Speed at End of Sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st second</td>
<td>2 ft</td>
<td>2 ft</td>
<td>4 ft/sec 2.7 mph</td>
</tr>
<tr>
<td>2nd second</td>
<td>6 ft</td>
<td>8 ft</td>
<td>8 ft/sec 5.5 mph</td>
</tr>
<tr>
<td>3rd second</td>
<td>10 ft</td>
<td>18 ft</td>
<td>12 ft/sec 8.2 mph</td>
</tr>
<tr>
<td>4th second</td>
<td>14 ft</td>
<td>32 ft</td>
<td>16 ft/sec 10.9 mph</td>
</tr>
<tr>
<td>5th second</td>
<td>18 ft</td>
<td>50 ft</td>
<td>20 ft/sec 13.6 mph</td>
</tr>
<tr>
<td>6th second</td>
<td>22 ft</td>
<td>72 ft</td>
<td>24 ft/sec 16.4 mph</td>
</tr>
<tr>
<td>7th second</td>
<td>26 ft</td>
<td>98 ft</td>
<td>26 ft/sec 19.2 mph</td>
</tr>
<tr>
<td>8th second</td>
<td>30 ft</td>
<td>128 ft</td>
<td>32 ft/sec 21.8 mph</td>
</tr>
<tr>
<td>9th second</td>
<td>34 ft</td>
<td>162 ft</td>
<td>36 ft/sec 24.6 mph</td>
</tr>
<tr>
<td>10th second</td>
<td>38 ft</td>
<td>200 ft</td>
<td>40 ft/sec 27.3 mph</td>
</tr>
<tr>
<td>11th second</td>
<td>42 ft</td>
<td>242 ft</td>
<td>44 ft/sec 30.0 mph</td>
</tr>
</tbody>
</table>

Note that in 3 seconds a car goes only 18 feet, one car length. A car approaching from the rear at 30 mph (44 ft/sec) would cover 132 feet during the 3 seconds. In 5 seconds the accelerating car goes only 50 ft, while the car approaching from the rear covers 220 ft, over \( \frac{3}{4} \) of a city block.

A car approaching at 60 mph (88 ft/sec) covers 264 ft. By the time the starting car covers 18 ft. By the time the starting car goes 50 ft. the car approaching at 60 mph covers 440 ft, nearly the length of 1\( \frac{1}{2} \) football fields.

C. Advantages of fast acceleration rates permitted by high reserve horsepower:

1. Shortens time and distance on left side of a two-lane, two-way roadway in an overtake and pass maneuver.

2. Shortens time to cross a lane of heavy traffic at an intersection.

3. Enables a car to blend with traffic after making turns at an intersection, on entering traffic from a shoulder or curb, and on entering a freeway from an entrance ramp.

4. Aids uniform flow of traffic going up hills and mountains.

D. Hazards of high reserve horsepower:

1. A dangerous increase in kinetic energy can be effortless and rapid.
   a. Increasing speed from 30 mph to 42\( \frac{1}{2} \) mph doubles the kinetic energy and the braking distance.
   b. If a driver's braking distance at 30 mph is 50 feet, it is 100 feet at 42\( \frac{1}{2} \) mph. Where the driver could barely skid to a stop from 30 mph and avoid hitting a vehicle, he would at 42\( \frac{1}{2} \) mph hit the vehicle with a speed of 30 mph. Every student should memorize this example along with many others in this outline.

2. A driver who does not habitually drive by his speedometer (check it frequently) cannot know how much he is increasing the kinetic energy of his car.
   a. Furthermore, his speedometer must be accurate or he must know what its error is. Otherwise, even though he checks it regularly, he may be going 5 mph faster than he thinks he is. If a braking distance at 30 mph is
50 feet (f = .6), a driver actually going 35 mph when he thinks he is going 30 mph unknowingly adds 18 feet to his braking distance. The energy of the 5 mph (when added to 30 mph) is equivalent to the energy developed at 18 mph. If at 30 mph he could lock the brakes and skid to a stop just short of another vehicle or a pedestrian, he would at 35 mph still be going 18 mph when he hit the vehicle or pedestrian.

3. A modern car may have its maximum reserve horsepower at 60 mph. Unless the driver checks his speedometer often he can, with a total unawareness, ease his speed up to 70 or 80 mph. It is easier still to slip into a dangerous speed if the car is in overdrive. Easing off on the gas pedal in overdrive does not permit the engine to retard his speed as it would in conventional drive. If he happens to be on a slight downgrade or if he has a tail wind, his car’s speed may decrease very slowly, even if he takes his foot off the gas pedal. If this situation occurs on close approach to an intersection, a curve, or a no-passing zone hazard, the driver may delay using his brakes until he is beyond the point of safe return. There is no substitute for checking and driving by the speedometer in maintaining control of an automobile. The destructive energy hidden by the quiet, smooth, floating motion of modern cars is a killer in sheep’s clothing. Instructors must get through to students with this fact or we shall never develop into a nation of intelligent drivers.

4. When accelerating from one speed to another, a driver should in checking the mph on his speedometer, think in terms of how the energy is increasing by the square of the speed:

\[
\begin{align*}
30^2 &= 900 = 2\frac{1}{2} \text{ times as much Energy at 30 mph as at 20 mph} \\
40^2 &= 1600 = 4 \text{ times as much Energy at 40 mph as at 20 mph} \\
42.5^2 &= 1806.25 = 2 \text{ times as much Energy at } 42.5 \text{ mph as at 30 mph} \\
60^2 &= 3600 = 4 \text{ times as much Energy at 60 mph as at 30 mph} \\
80^2 &= 6400 = 16 \text{ times as much Energy at 80 mph as at 20 mph}
\end{align*}
\]

5. Costs of using reserve horsepower can be reduced by reducing the rate of acceleration on leaving signal lights or stop signs when there is no need to accelerate fast; and by climbing grades at a moderate speed. The shorter the time to increase speed or elevation the more it costs. In other words, the faster you cover a given distance either horizontally or vertically the more you pay. You pay once for getting there and you pay again for getting there quickly.

E. Analysis of Accidents:

If one will measure the distance from the point of collision or overturn in an accident to the point(s) from which the driver(s) could first see the point of collision or overturn, one can illustrate the importance of a driver’s understanding the relationship between vehicle speeds in feet per second and the ground distances covered in feet. One can show that, in most accidents, either the driver(s) was already committed to the collision at the point from which he could first see the point of collision, because he could not decelerate fast enough, or the driver(s) could have avoided the accident after he could first see the point of collision had he not delayed 2 or 3 seconds before decelerating.
RECAPITULATION

A car mass is very dense matter evidenced by the fact that it is very heavy for its size.

The force that a car mass at rest exerts is called weight and is caused by gravity which tends to accelerate the car at a rate of 32 ft/sec/sec.

A horizontal force equal to gravity could accelerate the car mass at a rate of 32 ft/sec/sec. If under this force the car were immobilized against a stone wall, the car would "weigh" the same against the wall as it does against the ground. A force in lbs. would be exerted in each case but no work would be done by the force until the car moved. If the car moved either horizontally or vertically, it would possess ft-lbs of energy which would in turn enable the car itself to exert a force.

The car engine accelerates a car horizontally but never at a rate as high as gravity. A rate of 8 ft/sec/sec is very high for a stock car engine and 3 to 4 ft/sec/sec is a fair rate for an average car.

The rate of acceleration is proportional to the force exerted on the car by the engine. If the rate were the same as gravity, then the propelling force would have to be equal to the car's weight, because \( F = ma \) and \( a = \frac{w}{g} = \text{weight (of car)} \).

The force that accelerates a car or decelerates it determines the rate at which the car's speed will be changed. The propelling or retarding force then is related percentage-wise to the car's weight and the rate of acceleration or deceleration is related percentage-wise to gravity's rate of acceleration or deceleration. (Gravity decelerates an object projected upward at the same rate it accelerates a falling object.)

When a car accelerates from a stop the distance covered each second increases by the square of the time: \( s = at^2 \). If the rate is 4 ft/sec/sec, the distance by seconds will be \( s = \frac{4t^2}{2} = 2t^2 \). The distance covered in 1 sec will be 2 ft; in 2 sec, 8 ft; in 3 sec, 18 ft; in 4 sec, 32 ft, etc. This fact is very deceptive and traps drivers who turn from stops into traffic lanes ahead of cruising vehicles. Many drivers accelerate at a rate less than 4 ft/sec/sec and some are hit before they cover 50 ft.

Distance covered each sec by a decelerating car decreases by the square of the time; \( s = ut - \frac{at^2}{2} \). The car covers the longest distance the 1st sec, etc. When this car is overtaking the accelerating car mentioned above and the driver delays braking we have a double-action trap, often sprung by drivers who are unaware of the square principle which has carried hundreds of thousands of innocent victims to an untimely death. The square principle is the crux of driver error also when problems involve energy and centrifugal force.

A powerful engine can aid the accelerating driver and good brakes can aid the decelerating driver in escaping the double-action trap, provided the drivers use the aids early. Understanding the relation between the time\(^2\) and the distances covered is the motivation needed.

The distance factor in changing a car's speed is related the work done (Fs) which is equal to the car's weight force times the distance through which it acts. Since \( F = ma \), the larger the retarding force (F) is, the higher the rate of deceleration will be, and the shorter the distance. If the retarding force is small the rate of deceleration will be low and the distance long. But the work done in stopping a given car from a given speed will be the same in either case. A car mass at a given speed has exactly so much energy and can do exactly so much work. The rate at which it does this work is called power.

Since the difference in kinetic energy a car has at two different speeds is the work done in changing the speed and since this is true when one of the speeds is 0 mph, the kinetic energy at a given speed is equal to the work done to accelerate the car to the given speed, or to decelerate it from the given speed to a stop. Therefore, the weight force (lbs.) of the car mass times the distance (ft.) it takes to stop the car is equal to the ft-lbs of kinetic energy which is \( \frac{1}{2}mv^2 \). Accelerating and decelerating distances are proportional to the time\(^2\). KE is proportional to the speed\(^2\).

The momentum of a car is equal to its mass times its velocity but the car's energy is equal to \( \frac{1}{2} \) its momentum times its velocity or \( \frac{1}{2} (mass \times v) \times v \).

Impulse = force \times time = change in momentum = change in velocity for a given car, since the mass is constant. Therefore, the momentum added or subtracted due to an impulse is the mass \( \times (v - u) \) or the mass times the change in velocity. After impulse changes a car's speed the car's momentum = mass \( \times v \).

Impulse is the difference between two momenta of a car just as work is the difference between two KE's of a car. Impulse (and mom) involved force \times time while work (and KE) involve force \times distance. When time becomes a factor in work we have power which is work (ft-lb) per unit of time.

The KE of a car mass determines how far (distance) the car's weight force can be projected against a retarding force such as brake friction, or tire-pavement friction when the wheels are locked (there is always some retarding force in the form of air resistance and rolling friction; otherwise inertia would keep a car moving at a constant speed indefinitely).
The momentum of a car mass determines how long (time) it takes a retarding force to overcome the moving car's weight force. If the time is very short as when the car strikes another car, a post, or a bridgehead, the lbs-sec of momentum produce a powerful blow; a KE of ft-lbs is made destructive by a fast rate of onset and we have what is called impact force, an impulse (Ft) delivered in a very short period of time. Impact forces are destroyers of cars and occupants.

An impact force at low speed can cause critical injuries especially if an occupant's body strikes a part of the car which concentrates the force in a small area of the body. High speed is more deadly because the increase of KE is proportional to the increase of the speed$^2$. When a car is in collision with an object the impact force damages the car. The body of an occupant is hurled forward at the speed the car was going just before contact. The body then strikes the interior of the car which is decelerating at an extremely high rate. This second impact force injures the occupant. The force with which a body collides with the interior of a car is determined by the distance in which the car stops and the velocity of the occupant's body whose energy, of course, is proportional to the square of the body's speed.

A moving mass has a property called inertia which opposes any change in its velocity; that is, a constant speed in a straight line. When forces acting on the mass along the path of the mass, are unbalanced, the mass will be accelerated or retarded; that is, its linear speed will change. When forces acting on the mass at angles to the path of the mass are unbalanced, the mass will change direction.

The engine and brakes normally change the linear speed of a car. Friction between the tires and pavement when the front wheels are turned changes its direction. This force is called centripetal. It accelerates the car toward the center of a curved path. The inertial force that opposes centripetal force is called centrifugal.

When centripetal force exceeds centrifugal force the car moves on a curve with a decreasing radius. When the two factors are balanced the car moves on a curve with a constant radius. When centrifugal force exceeds centripetal force the car returns to a straight path.

Centripetal forces act at the tire-pavement contact points, while centrifugal force acts at the car's center of mass. When centrifugal force exceeds the centripetal force the car will roll over and/or slide along a straight line tangent to the path of the car in the curve. Centrifugal force simply returns the car's motion to a straight path. The hazard lies in the fact that the straight path leads off the roadway a distance determined by the car's KE, unless within this distance the car crashes into objects, which will shorten the distance but deform the car and occupants.

While centrifugal force increases as the radius of a curve decreases (note that the radius is in the denominator of the formula, CF = \( \frac{mv^2}{r} \)) this fact creates the hazard it does because the force increases also by the speed$^2$ which is a factor in the numerator of the formula. The speed$^2$ and the radius work together to make a car leave a curve and go straight. We have the principle of the square operating in three driver problems which confront every driver while his car is in motion:

\[
\frac{s}{2} = ut \pm at^2; \quad KE = \frac{1}{2}mv^2; \quad \text{and} \quad CF = \frac{mv^2}{r}
\]

Finally we relate the rate of deceleration (\(-a\)) and the stopping distance in a collision to a car's impact force, using a 3200 lb. car moving 60 mph (88 ft/sec):

\[
KE = \frac{1}{2}mv^2 = 387,200 \text{ ft lbs} = \text{Work} = \text{Force} \times \text{distance(s)}, \quad \text{Force} = \text{mass} \times \text{acceleration}.
\]

Note that if \(a = 32\), the deceleration would = gravity and the retarding force (F) would = 100% of the weight, or 3200 lbs. Then distance(s) = 121 ft.

The average velocity during the stop is 44 ft/sec. The time to stop is \(121 \frac{\text{ft}}{44 \text{ ft/sec}} = 2.75 \text{ sec.}\)

The time factor which determines the impact force is in the rate of (neg) acceleration. When \(a = 32 \text{ ft/sec/sec}\) (the value of gravity) we have a 1G force which is equal the weight of the mass; if \(a = 64\) we have a 2G force, etc. (Any mass accelerated at the rate of gravity produces a force of 1G.)

In the equation \(\text{Work} = Fs = (ma)\) the value of distance(s) will be large when \(a\) is small and vice versa. The rate of deceleration is high when the time is short and a short distance makes the time short.

Assume the car strikes a fixed object and its center of mass stops in 2 ft. The time to stop in 2 ft at 44 ft/sec is 1 sec. Then \(a\) will be 1836 (ft/sec/sec) or 60\(\frac{1}{2}\) G's.

If \(s\) were 1 ft, \(a\) would be 3872 ft/sec/sec and the G force would be 121. Recall that when \(s\) was 121 ft, we had a 1G force. Impulse = Force \times time. The mass in \(F = ma\) is constant for a given car. Then when \(t\) is short \(a\) is high and the impact force is great. For a given mass accelerated to a given speed the total impulse remains the same. However, the force (F) in the impulse equation varies as the \(a\) in \((ma)\) varies and \(a\) varies with the time required to change the velocity. The 3200 lb. car moving 88 ft/sec has a momentum of 8800 lb-sec. An accelerating force (F) of 400 lbs. would accelerate the car at 4 ft/sec/sec and develop a momentum of 8800 lb-sec. in 22 sec.
But in collisions where s is short, the t is small and the 8800 lb-sec of the 3200 lb. car moving 88 ft/sec produces a powerful force (F). The force (F) is the lb. and the time (t) is the sec. If t = 2.75 sec, F = 3200 lb. If t = 1 sec, F = 8800 lb. If t = 1 sec, F = 193,600 lb. The shorter the time, the greater the blow the 3200 lb. car can produce. If t = 1 sec, the force (F) = 8800 lb/sec × \( \frac{44}{22} \) = 8800 lb-sec = 8800 lb-sec × \( \frac{44}{1} \) sec = 387,200 lb (121 G's).

The average velocity of the car is 44 ft/sec while its speed is being reduced from 88 ft/sec to 0 ft/sec. At a speed of 44 ft/sec the car will go 1 ft in 1 sec. If the force (F) of 387,200 lb. is exerted through a distance of 1 ft we have 387,200 ft lbs. which is the KE of the car moving 88 ft/sec.

When the distance(s) to stop is 121 ft, the time (t) is 2.75 sec and the force (F) of the Impulse = 8800 lb-sec = 3200 lb, the weight of the car (1 G). Changing the stopping time from 2.75 sec to 1 sec increases the force (F) from 3200 lbs to 387,200 lbs. The G force is 121 times the weight of the car and 121 times the weight of the occupant if he stops in 1 sec. An occupant's body after impact with steel under such a force probably could be identified by fingerprints.

When a car collides with a fixed object the bumper and other parts collapse, thus increasing a few feet the distance through which the instrument panel moves before it comes to rest. As a result the relative velocity between an occupant's body and the instrument panel is reduced. Consequently the G force on an occupant is appreciably lower than the G force at the bumper.

Suppose a 160 lb. occupant is thrown forward at 88 ft/sec. He hits the panel in 0.1 sec, during which the speed of the panel has been reduced to 22 ft/sec. The momentum of the occupant's body is reduced during impact from 440 lb-sec to 110 lb-sec. Mom at 88 ft/sec = 160 × 88 = 440 lb-sec. Mom at 22 ft/sec = 160 × 22 = 3520 lb-sec. The force (F) = \( \frac{\text{change in mom}}{\text{time}} \) = \( \frac{3200 \text{ lb-sec}}{0.1 \text{ sec}} \) G's). An occupant's body after impact with steel under this force probably could be identified by relatives.

Note the following relations of mass, space and time, illustrated with a 3200 lb. car with a rate of motion (v) = 88 ft/sec and a rate of changing the rate of motion (a) = 32 ft/sec/sec.

\[ v = at \]
\[ \bar{v} = \frac{1}{2} \times \frac{v}{2} \]
\[ s = \frac{at^2}{2} \times \frac{(2.75 \text{ sec})^2}{2} = \frac{242}{2} = 121 \text{ ft} \]

Work = Fs = (3200 lb x 32 ft/sec²) x (3200 lb x 121 ft) = KE

Impulse = Ft = (3200 lb x 32 ft/sec²) x 8800 lb sec = 8800 lb sec = Mom

Mom = Fs = 3200 lb x 121 ft = 3200 lb x 121 ft x 2.75 sec = 387,200 ft lb = 44 ft

Impulse = Ft = 3200 lb x 2.75 sec = 44 ft = \( \bar{v} = \frac{1}{2} v = \frac{\text{KE}}{2} = \frac{387,200 \text{ ft lb}}{2} = 44 \text{ ft} \)
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**SQUARE ROOT TABLE**

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**SQUARE ROOT TABLE**—(Continued)
INDEX

—A—
Acceleration 1, 31, 93, 95
Charts 32, 34, 35, 36, 37, 38
Tables 97, 98
Accidents, Ratios, Types 90
Accidents, Analysis of 99

—B—
Braking 79
Braking Distance 6, 30, 82, 92
Charts 44, 48, 60, 61
Table 76
Braking Effort 77, 92
Charts 36, 39
Braking Forces 40

—C—
Centrifugal Force 14, 29, 66, 67, 68, 69, 70, 71, 80
and Center of Mass 68
and Coef. of Friction 70
and Kinetic Energy 71
and Turning Angle 73
Centripetal Force 15, 68, 80
Coefficient of Friction 3, 36, 39, 70, 77
and Turning Angle 73
on Grades 76
on Banked Curve 82
Concepts 17, 26
Cornering Force 74
Curve, Forces 16
Chart, Increasing Speed 56

—D—
Decisions in Passing 63
Deceleration, table 97
Distance 32, 61, 97
Force through 55
Required to Overtake and Pass 62, 63
Perception Time 64, 65
Deaths, Energy, Collisions 90

—E—
Energy 42, 43, 82
Collision and Relative Speeds 57
Conservation of 84
Heat 45
Kinetic 4, 6, 8, 10, 30, 45, 51, 71, 72
and Braking Distance 6, 29, 30, 82, 86
and Momentum 10, 50, 58
Charts 42, 43, 44, 46, 47, 48, 49, 54, 56
Impact and Relative Speeds 58
and Weight 59
Speed, Collisions, Injuries 90

—F—
Force 1, 41, 43, 55, 74
Gravitational vii
Braking 40
Centrifugal (see Centrif. Force)
Centripetal 68, 15, 80
Cornering 74
Friction 2
Gyroscopic 16, 75
Impact, in Collision 85, 51
Retarding 41
Through Distance 55
Wind 16, 66
Formule 20, 92
Friction 26
Coefficient of 3, 36, 39, 70, 77
Force 2
Retarding 41
Rolling 40, 56

—G—
G Force 51
Gravity 24, 31, 32, 34, 35, 80
Center of (see center of mass)
Gyroscopic Force 16, 75

—H—
Horsepower, Energy and Work 8
Horsepower and Acceleration 93

—I—
Impact Energy and Rel. Speeds 58
Impact Force in Collision 85, 101
Inertia (see centrifugal force)
and Cent. Force 14, 29, 66, 80
and Kinetic Energy 72
Impulse 85, 101, 50
Injuries, Energy, Collisions 90

—K—
Kinetic Energy (see Energy)

—L—
Lb—Sec (see Impulse & Mom.) 50, 101

—M—
Mass 1, 47
Center of 81, 16
Slug of 12
Momentum
Conservation of 52
and Kinetic Energy 10
Relation to Kinetic Energy 88

—P—
Passing
Time and Distance 62, 63
Driver Decisions 63
Pavement-Tire Friction 3
Perception vs Stopping Distance 64, 65

—Q—
Questions, Study 18

—R—
Reaction Time and Distance 64, 65
Recapitulation 100

—S—
Speed 1, 46, 48, 49, 56
Braking Distance and Time 61
Relative, & Collision Energy 57
Relative, & Impact Energy 58
Square Root Table 103

—T—
Tailgating 38
Teaching Concepts 16
Time 61, 64, 65
Reaction, table 76
and Distance 60

—V—
Velocity 1, 32
Visual Aids 26

—W—
Weight 4, 6, 8, 35, 41, 54, 59
Work (see Energy, K.) 6, 8, 27, 55

104