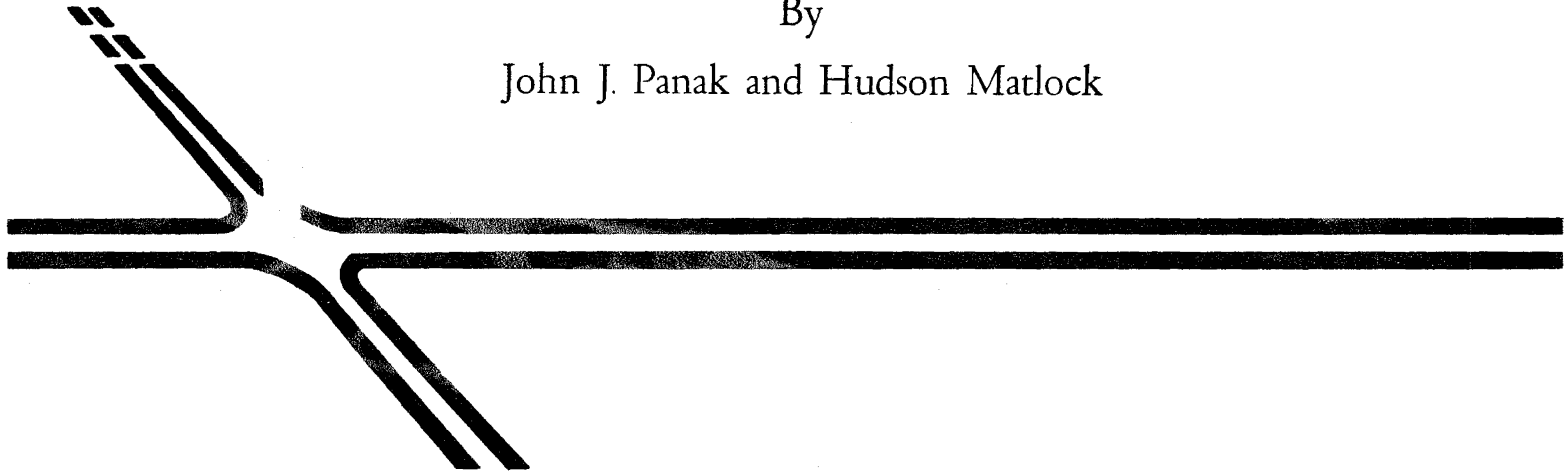


# A DISCRETE-ELEMENT METHOD OF ANALYSIS FOR ORTHOGONAL SLAB AND GRID BRIDGE FLOOR SYSTEMS

By

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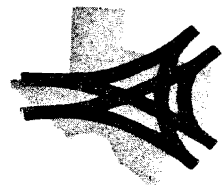
SUMMARY REPORT 56-25 (S)

SUMMARY OF  
RESEARCH REPORT 56-25

PROJECT 3-5-63-56

COOPERATIVE HIGHWAY RESEARCH PROGRAM  
WITH TEXAS HIGHWAY DEPARTMENT  
AND  
U. S. DEPARTMENT OF TRANSPORTATION  
FEDERAL HIGHWAY ADMINISTRATION

CENTER FOR HIGHWAY RESEARCH  
THE UNIVERSITY OF TEXAS AT AUSTIN  
MAY 1972



# SUMMARY REPORT 56-25 (S)

## Foreword

Research Report 56-25 describes a numerical method of analysis for orthogonal slabs and plates with an integral grid-beam system. The method is particularly suited for application to highway bridge structures composed of slabs with supporting beams and diaphragm systems. Other structures such as highway and airfield pavements on elastic foundations, two-way mat foundations, and grid type sub-assemblages of more complex structures can also be investigated. The report is the twenty-fifth in a series of reports that describes work in Research Project 3-5-63-56, "Development of Methods for Computer Simulation of Beam-Columns and Grid-Beam and Slab Systems."

## Introduction

Analysis of highway bridge structures has become a much more complex problem in recent years due to better materials, more complex structural configurations, and the need for greater economy. Presently, most structures are one of three distinct types: (1) concrete slab spans, (2) slab and girder arrangements, or (3) a special case of the second which has two-way reinforced decks over widely spaced girders and floorbeams. All of these structures have one characteristic in common, the stiffening effect of the basic structure by outstanding members. These outstanding members are the girders of slab-and-girder-type structures, or the sidewalks and parapets of slab-type structures. This stiffening contributes significantly to the overall structural action and must be included for realistic analyses. The method described in this report can effectively solve these general structure types for the majority of highway bridges and is therefore most useful to the bridge design engineer.

The computer program which is described in the report is the result of several years of development by this research project. It provides an excellent general solution for most highway bridge structures. Other structures such as highway or airfield pavements on elastic foundations, flat or haunched building slabs, prestressed slabs and grids, one-way or two-way mat foundations, stiffened plates, and any plate or grid-type substructure of more complex configurations can also be investigated.

## Method of Analysis

The basic solution procedure for line or one-dimensional members was originally presented in this project for several beam-column applications (Refs

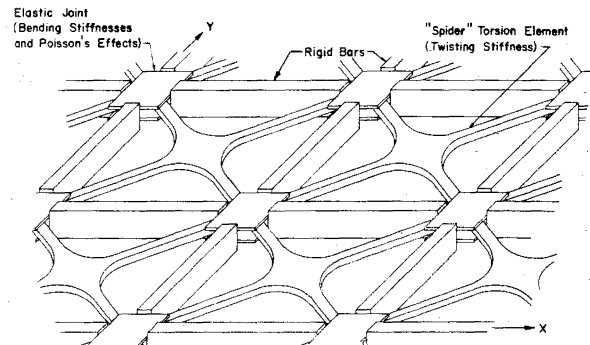


Fig 1. Discrete-element model of slab or plate.

6, 7, 8, and 12). The procedure was extended in various forms and applied to a variety of problems including static, dynamic, and nonlinear solutions of beams and plates (Refs 3, 4, 5, 9, 10, and 11). All of these developments laid the ground work for the discrete-element solution procedure utilized within this report. The mathematical analysis process used handles most general banded equation systems and is efficient for computer operations (Ref 2).

During the development of this program, a parallel procedure for skewed anisotropic plate structures was developed (Ref 13) which also includes the capability to handle added beam stiffnesses. When applied to square or orthogonal structures, however, the skewed plate program is significantly more inefficient. The orthogonal slab and grid program has several additional output features such as the ability to plot selected profile lines of deflection and moment and the ability to plot a three-dimensional view of the exaggerated deflected shape of the structure.

The basic discrete-element model used in the program is essentially the same as that used in previous developments on this project (Refs 3, 4, 5, 9, 10, and 11). The slab twisting stiffness is represented as a twisting element connected directly to the joints, as shown in Fig 1. Figure 2 shows the discrete-element model of the grid system which acts in conjunction with the slab or plate model shown in Fig 1. Both

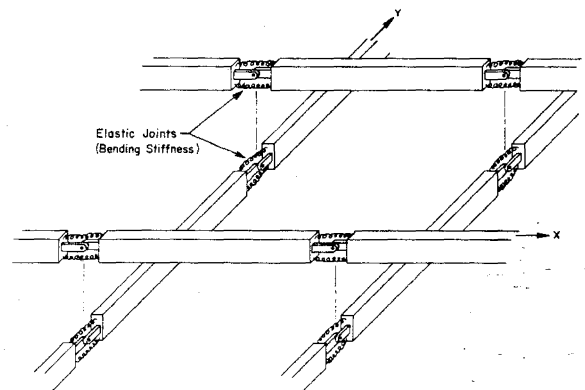


Fig 2. Discrete-element model of grid system.

models are connected at the joints by what may be thought of as ball and socket connections which ensure that the deflection is the same at the common joint locations. This composite mechanical model is used to simulate any slab and grid system. The model allows for the free linear elastic variation of stiffnesses and support characteristics; loads can be applied at each joint to represent any degree of concentrated or uniform loading.

A complete set of simultaneous equations is written based on the discrete-element model and solved by a direct solution process. The ordered system of equations forms a diagonally banded, positive definite, and symmetrical stiffness matrix in which the coefficient matrix is partitioned into smaller banded submatrices. The efficient solution procedures of Ref 2 are used to recursively solve the resulting sparsely banded set of equations. The solution procedure includes a multiple-loading technique presented in Ref 9. This technique allows multiple loading solutions to be made of the same structure. Loads are placed at various locations to study their effects without the necessity of completely re-solving the problem. A fortunate property of the technique is that as the problem gets larger, the percentage of solution time for each multiple load solution becomes smaller. A time as low as 8 percent of the initial problem time has been observed for the largest problems.

The orthogonal slab and grid program is called SLAB 49, which simply indicates that it is the forty-ninth in a sequence of developments for plate and grid-type structures on this project. The program is written in FORTRAN and follows the basic guidelines given for ASA FORTRAN (Ref 1). The program is written for IBM 360 and CDC 6600 computers and is easily made compatible with other similar systems. The program storage requirements are variable depending on the size of the problem to be run. Less than ten statements need be changed at the beginning of the program to change the dimension size for various size problems.

A significant feature of the program is the technique through which all data input are retained for each problem in the series. Each data card is stored as a data card image which is searched at each level in the recursive solution process; only the necessary terms are generated at each level, thus saving a significant amount of storage. This technique of storing data card images is also the most convenient method of transferring information from a data-generation routine written for certain specific problem types. The basic general slab and grid program is then used for solution of the specific problems.

A summary flow diagram of program SLAB-49 is shown in Fig 3. The program checks all data for

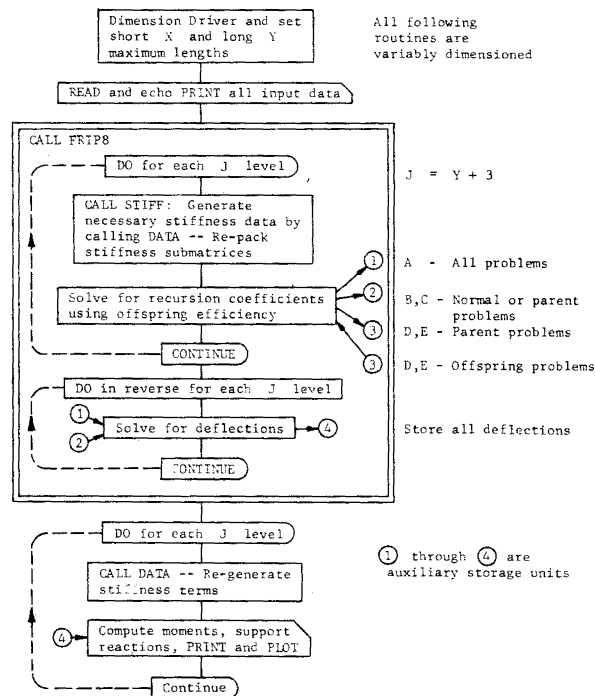


Fig 3. Summary flow diagram for SLAB 49.

general compatibility with the geometry of the specific problem and consistency of coordinate input. A count is made of the number of common data errors usually made. The program output consists of deflections, slab bending moments, slab twisting moments, maximum principal moments and their directions, and support reactions. The bending moments in the supporting grid system are also computed. Selected areas of deflection of moment output may be plotted by means of an optional printer plot routine which is included. These printer plots have been found to be especially valuable because the user obtains them with the rest of his printed output. Time is not spent unnecessarily waiting for line plot output or in hand plotting. Three-dimensional deflection plots are also obtainable with the program as shown in Fig 4.

### Example Problems

Application of the SLAB 49 program is demonstrated by a series of problems which present the analysis of a typical Texas Highway Department bridge structure, briefly shown in Fig 5. The structure consists of a concrete deck resting on wide flange longitudinal main beams and is continuous over three spans. Stiffnesses of the individual members were computed as if complete composite action is achieved in the positive moment regions. One included solution is illustrated in Fig 6, which shows the structure loaded with two HS20 trucks. The trucks are positioned to produce a possible maximum negative support moment for the first interior beam. The re-

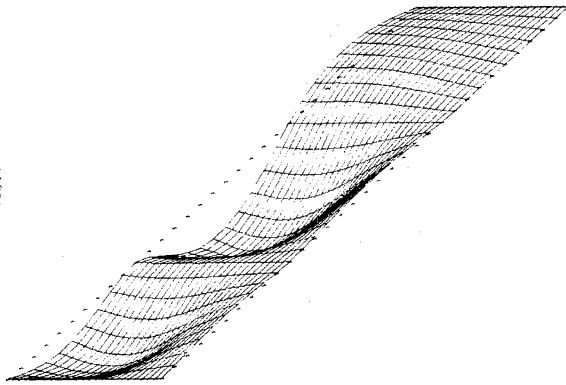


Fig 4. Three-dimensional plot of deflections.

sulting computed moment variations are also shown in Fig 6.

## Implementation

Research Report 56-25 presents a rational method of analysis for orthogonal bridge floor systems. This program could be considered the culmination of work on this research project in this area. The method is not limited to bridge floor systems however; pavement slabs, building slabs, stiffened and anchored bulkheads, and other two-dimensional plate or grid systems can also be analyzed.

The computer program is written and organized in a manner which will facilitate its extension for specific applications. Of primary need is a series of user-oriented, data-generation routines which will use this program as the basic solver. Among these would be data-generation routines for constant thickness slab structures, pavements, and beam and slab systems such as presented as the example problem series.

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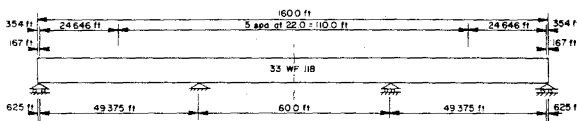


Fig 5. Three-span highway bridge.

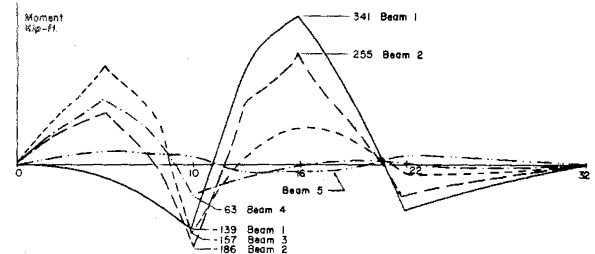
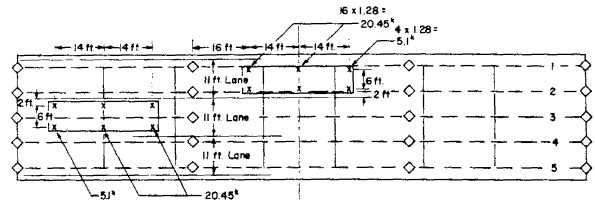


Fig 6. Plan of example structure with results.

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KEY WORDS: bridge decks, orthotropic plates, discrete-element analysis, grid systems, computers.

The full text of Research Report 56-25 can be obtained from R. L. Lewis, Chairman, Research and Development Committee, Texas Highway Department, File D-8 Research, 11th and Brazos Streets, Austin, Texas 78701 (512/475-2971).