AN ALGEBRAIC EQUATION SOLUTION PROCESS FORMULATED IN ANTICIPATION OF BANDED LINEAR EQUATIONS

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Foreword

Research Report 56-19 describes a numerical tool which is particularly useful for the solving of a system of linear algebraic equations resulting from the analysis of problems by the discrete-element method. This is the nineteenth in a series of reports that describe the work in Research Project 3-5-63-56, entitled "Development of Methods for Computer Simulation of Beam-Columns and Grid-Beam and Slab Systems."

Introduction

All physical problems, when subjected to some type of discrete analysis, lead to a system of simultaneous equations which possess certain favorable properties. This is especially true of most structural analysis techniques. It is the purpose of Research Report 56-19 to present an algorithm that is general enough to handle all types of physical problems and at the same time be efficient. The algorithm takes advantage of all the known properties of the resulting stiffness matrix, incorporates the ability to handle problems that have the same stiffness properties but differ in their load conditions, minimizes the core storage, and packages all this into an easy-to-use group of subroutines.

Computer Subroutines

The routines take maximum advantage of two major properties of the system of equations. First, the coefficient matrix is symmetric and positive definite. Second, the routines handle the more general problem of

\[ AW = F \]

where

\[ W = (w_1, w_2, \ldots, w_L) \]
\[ F = (f_1, f_2, \ldots, f_L) \]

For structural problems this represents a series of problems that, instead of being treated as independent solutions, can be handled as a family which possesses the same stiffness matrix \( A \) but differs in the load vector \( f_j \).

The only limitation imposed on the subroutines is that the operator width is limited to five, which means that the number of sub-bands is limited to five, although the band width for any particular submatrix is arbitrary up to the maximum of a full matrix.

Conclusions

A solution process is presented which efficiently handles coefficient matrices that are partitioned into submatrices that possess the property of banding. While this is especially useful for problems analyzed by finite differences, it can also be applied to problems analyzed by sub-structuring.

A more general routine which removes the restriction of an operator width of five and is completely general for all widths is in the developmental stage.

The full text of Research Report 56-19 can be obtained from R. L. Lewis, Chairman, Research and Development Committee, Texas Highway Department, File D-8 Research, 11th and Brazos Streets, Austin, Texas 78701 (512/475-2971).

KEY WORDS: structural analysis, numerical analysis, computers, mathematics, banded equations, finite differences.