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noise and vibration control • sound quality • test facility design • instruction

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INTRODUCTION

Nelson Acoustics performed acoustical measurements at CTR's request along I-30 in Dallas TX in order to demonstrate a method for assessing the *in situ* reflectivity of the retaining wall opposite the Kessler Park neighborhood, prior to a proposed installation of sound absorbing material. An Executive Summary of the project and results is provided in Report 1052-01. This report gives a more detailed explanation of the method and results.

1. OVERVIEW

Nelson Acoustics performed acoustical measurements along I-30 in Dallas TX on November 30, December 1, 21, and 30, 2010. All measurement locations on the freeway proper were within 100 yards east and west of the Edgefield Ave. bridge. A receptor location in the Kessler Park neighborhood was used for one test (see Figure 1 below). Tests were designated by letter codes as follows. Test "K" was designated the "keeper" that best represented the reflection properties of the retaining wall.

Measurements at receptor locations were performed using a ½" Bruel and Kjaer microphone, a National Instruments 9234 data acquisition card, Nelson Acoustics Trident Multichannel Real-Time Analyzer Software. Post-processing analysis was performed using Nelson Acoustics' proprietary LabVIEW analysis routines.

Generation at the source location was driven by Nelson Acoustics' proprietary LabVIEW-based *MLS* generation routines and a National Instruments 4431 analog output board.

Test A/B and E: November 30 and December 1, 2010. Two loudspeakers were located 5m from the retaining wall directly on the pavement in the northernmost westbound traffic lane near *"1052 A Approximate"*. A microphone was located directly above the highway barrier at altitudes of 12 and 16 feet near *"1052 B"* at the Kessler Ct. gated community.

Results indicated reflection coefficients smaller than expected. We inferred from this that the reflected sound was attenuated by traveling an additional 10m over sound-absorbing PFC pavement.

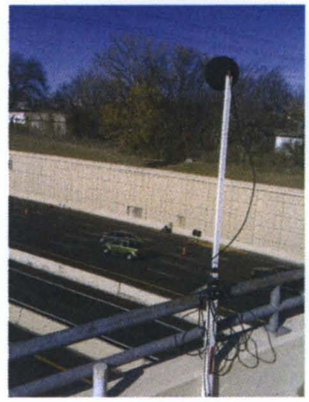
Test C/D: December 1, 2010. Two loudspeakers were located 5m from the retaining wall directly on the pavement in the northernmost westbound traffic lane near *"1052 C Approximate"*. A microphone was located at a height of 16 feet AGL in the Kessler Park neighborhood near *"1052 D"*.

Results indicated reflection coefficients smaller than expected. We inferred from this that the reflected sound was attenuated by traveling an additional 10m over sound-absorbing PFC pavement.

Closer inspection revealed a strong and un-attenuated reflection from the face of the Edgefield Ave. bridge. This reinforced the idea that the sound-absorbing PFC was a complicating factor and suggested changes to the procedure that were adopted for subsequent measurements.

Test F: December 20, 2010. Two loudspeakers were located on poles 4m from the retaining wall approximately 4 ft. above ground level in the northernmost westbound traffic lane near *"1052 F Source"*. The area between the loudspeakers and the retaining wall was covered with 7/16" OSB plywood laid on the pavement in an effort to eliminate the "extra" attenuation along the reflected sound path. A microphone was located directly above the highway barrier at an altitude of 8 feet above the bridge near *"1052 F Receptor"*.

Test G: December 20, 2010. Two loudspeakers were located on poles 4m from the retaining wall approximately 4 ft. above ground level in the northernmost westbound traffic lane near *"1052 G Source"*. The area between the loudspeakers and the retaining wall was covered with 7/16" OSB plywood laid on the pavement in an effort to eliminate the "extra" attenuation along the reflected sound path. A microphone was located directly above the highway barrier at an altitude of 8 feet above the bridge near *"1052 G,K Receptor"*.



Test H: December 20, 2010. Two loudspeakers were located on poles 5m from the retaining wall approximately 4 ft. above ground level in the northernmost westbound traffic lane near *"1052 A Approximate"*. The area between the loudspeakers and the retaining wall was covered with 7/16" OSB plywood laid on the pavement in an effort to eliminate the "extra" attenuation along the reflected sound path. A microphone was located directly above the highway barrier at an altitude of 12 feet near *"1052 B"*.

Results for F, G and H indicated reflection coefficients larger than expected. We inferred from this that the reflected sound was augmented by a ground-reflected image source that was not present along the direct sound path.

Arrangement "F" was abandoned because of the low angle of incidence to the wall.

An attempt to post-process the data while still on site was thwarted by a software error initiated by instability within Microsoft Windows XP, followed by a later full-scale shredding of the XP disk partition.

Test I: December 30, 2010. Two loudspeakers were located 5m from the retaining wall, standing vertically on a plywood reflecting plane in the northernmost westbound traffic lane near *"1052 A Approximate"*. The area between the loudspeakers and the retaining wall was covered with 7/16" OSB plywood laid on the pavement in an effort to eliminate the "extra" attenuation along the reflected sound path. Another area of plywood was added on the south side of the loudspeakers to augment ground reflections along the direct sound path, although the area was smaller because of proximity to traffic. A microphone was located directly above the highway barrier at an altitude of 12 feet near *"1052 B"*.

Test J: December 30, 2010. As Test I but with the entire reflecting plane removed. This is a repeat of test E.

Test K: December 30, 2010. Two loudspeakers were located 4m from the retaining wall in the northernmost westbound traffic lane near *"1052 G Source"* laying on their sides to provide better vertical coverage by the high frequency horns. The area between the loudspeakers and the retaining wall was covered with 7/16" OSB plywood laid on the pavement in an effort to eliminate the "extra" attenuation along the reflected sound path. Another area of plywood was added on the south side of the loudspeakers to augment ground reflections along the direct sound path, although the area was smaller because of proximity to traffic. A microphone was located directly above the highway barrier at an altitude of 12 feet above the bridge near *"1052 G,K Receptor"*.

Test L: December 30, 2010. As Test K but with the south "half" of the reflecting plane removed. This is a repeat of test G, to check our understanding of the role of the reflecting plane.

Test M: December 30, 2010. As Test K but with the entire reflecting plane removed, to check our understanding of the role of the reflecting plane.

Results for tests *I-M* corroborated our understanding of the effect of the reflecting plane on the calculated reflection coefficients: removing the south half of the reflecting plane caused computed results to be larger, and removing the entire reflecting plane caused computed results to be smaller. The influence of HOV lane jersey barriers also raised questions that brought the results of Tests *I* and *J* into question.

After allowing for the phase cancellation caused by ground-reflected image sources, results for arrangement "*K*" at 500, 1000, and 2000 Hz octave bands converged very well to the expected values near 1.0. The computed results for 250 Hz and below continued to trend anomalously high, probably because of the reduced size of the reflecting plane under the direct path.

Figure 1: Measurement Locations

2. TEST METHOD

The simplest and most natural way of determining the strength of the reflection would be to use a large impulse source such as a small yachting cannon. This approach was deemed undesirable because of the likelihood of startling drivers and alarming nearby residents.

Nelson Acoustics proposed that a "maximum length sequence" method be used to assess the reflection. The sequence belongs to a special class of binary signals that correlate only with themselves. Copies of the sequence can be cleanly recovered from a noisy environment into which it is broadcast. A simple cross-correlation between the original signal and the received signal more or less eliminates the traffic noise and permits detection of the direct and reflected sound. See Figure 2 below for a simplified schematic. A rather rigorous derivation of the test method mathematics follows.

No test method can ever be "perfect". A well-crafted test method minimizes the number of uncontrolled factors, accounts for their presence, and leads to a robust and repeatable procedure. The test method described below is believed to well-crafted in this sense.

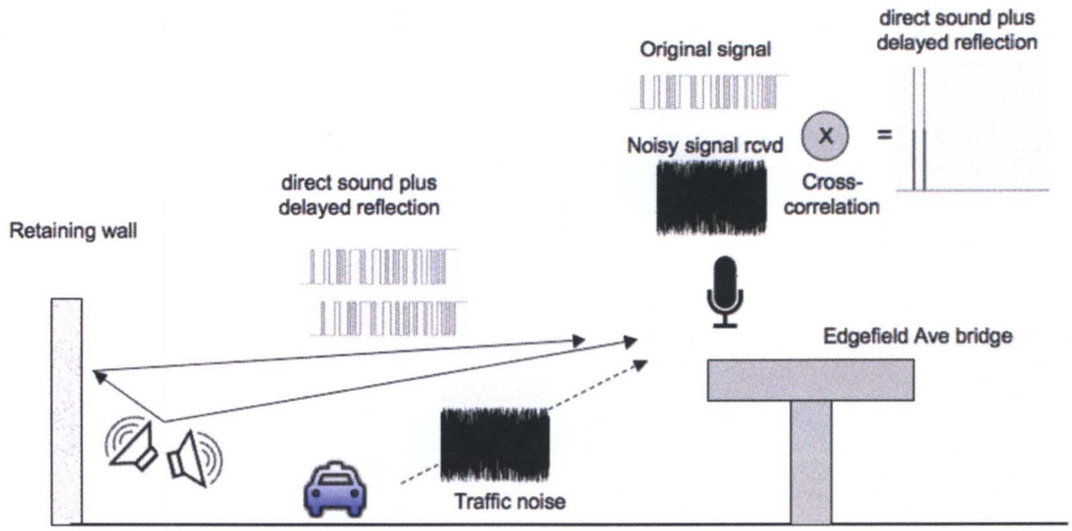


Figure 2: Schematic Diagram of Measurement Method

2.1 Test Signal

The *MLS* signal selected was of 15-th order and consisted of 32,767 samples. It was broadcast at a sampling rate of 25,600 Hz, giving it a nominal bandwidth of 0 – 12,800 Hz. The duration of one *MLS* "frame" was roughly 1.2s and the overall measurement time was 400s, so that the signal was repeated upwards of 300 times.

The spectrum of the signal is "white", that is, equal energy per Hertz. To obtain more low-frequency output, to better match the vehicle noise spectrum, and to protect high-frequency drivers, the *MLS* signal was subjected to a "pinking" filter that produced roughly equal energy per octave.

The filtered (i.e., convolved) MLS signal traveling through the atmosphere at sound speed *c* can be written as:

$$MLS(t-\frac{r}{c})\otimes F_P(t).$$

The impulse response of the pinking filter is very brief compared to the gap between reflected and direct sound and does not influence the results.

2.2 Loudspeakers

Two Eon15 G2 400W two-way powered speakers were used. The 1500 Hz crossover directed low-frequency sound to the 300W 15" woofer and high-frequency sound to the 100W 60° x 90° constant–directivity horn. Four speakers mounted in a back-to-back cluster would approximate very well an omni-directional source in the plane of the array. However, only two were needed for the test to cover the direct and reflected directions.

The speakers have a variable gain setting which was immobilized after setting the speakers as close as possible to the same power output. The sound pressure of speakers *1* and *2* at a distance r_0 of 1m are expressed as p_1 and p_2.

Loudspeakers were pointed at the microphone (for the direct sound) and at the point of the corresponding reflection on the retaining wall. The directivity of each speaker is assumed to be θ_F in the forward direction and θ_B in the backward direction, so that the sound pressure at some distance *r* along the direct path, without a ground reflection, can be expressed as:

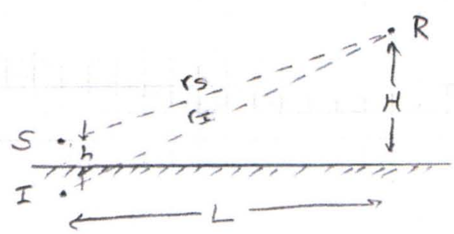
$$p_D(t) = \frac{r_0}{r}\left(p_1 D(\theta_F) + p_2 D(\theta_B)\right) MLS(t - \frac{r}{c}) \otimes F_P(t)$$

2.3 Ground Reflection

For each physical source above a reflecting plane there exists an image source below the reflecting plane. Although the sources are perfectly synchronized, slight differences in distance to a receptor cause the signals from the image to arrive slightly out of phase. When integrated across an octave band of pink noise, shading factors χ_D and χ_R can be computed for the direct and reflected paths.

The geometry is defined as

$$r_S = \sqrt{L^2 + (H-h)^2}$$
$$r_I = \sqrt{L^2 + (H+h)^2}$$
$$\Delta r = r_I - r_S$$



The frequencies in an octave band are defined as

$$f_1 = f_0 / \sqrt{2}$$
$$f_2 = f_0 \cdot \sqrt{2}$$
$$f_2 - f_1 = \Delta f = f_1$$
$$\frac{f_2}{f_1} = \sqrt{2}$$

The relative strength of the reflection coefficient from the reflecting plane is

$$D\rho e^{j\phi}$$

where D is the decimal directivity ratio between the angle from the Source to the Receptor and the Image to the Receptor (recalling that the Image source is "upside down"), ρ is the reflection strength and ϕ is the complex phase angle of the reflection.

The transfer function of the direct-plus-reflected signal relative to the direct signal only is, at a given frequency ω, is:

$$H(\omega) = \frac{p_0 e^{j\omega t}\left(\dfrac{e^{jkr_S}}{r_S} + D\rho e^{j\phi}\dfrac{e^{jkr_I}}{r_I}\right)}{p_0 e^{j\omega t}\left(\dfrac{e^{jkr_S}}{r_S}\right)} = 1 + D\frac{r_S}{r_I}\rho e^{j\phi}e^{jk\Delta r}$$

The real-valued squared magnitude of this expression is

$$|H(f)|^2 = \left(1 + D\frac{r_S}{r_I}\rho e^{j\phi}e^{jk\Delta r}\right)\left(1 + D\frac{r_S}{r_I}\rho e^{-j\phi}e^{-jk\Delta r}\right)$$

$$|H(f)|^2 = 1 + \left(D\frac{r_S}{r_I}\rho\right)^2 + D\frac{r_S}{r_I}\rho\left(e^{j(k\Delta r+\phi)} + e^{-j(k\Delta r+\phi)}\right)$$

$$|H(f)|^2 = 1 + \beta^2 + 2\beta\cos(k\Delta r + \phi)$$

where $k = 2\pi f/c$ and constants are combined to define β.

Because pink noise has constant energy per octave, the energy per Hertz drops as $1/f$. The energy in an octave band can be expressed as

$$\left\langle |H(f)|^2 \right\rangle = \frac{f_0}{\Delta f}\int_{f_1}^{f_2} |H(f)|^2\, df$$

$$\left\langle |H(f)|^2 \right\rangle = \frac{f_0}{\Delta f}\int_{f_1}^{f_2} \frac{1+\beta^2}{f}\, df + \frac{f_0}{\Delta f}2\beta\int_{f_1}^{f_2} \frac{\cos(k\Delta r + \phi)}{f}\, df$$

$$\left\langle |H(f)|^2 \right\rangle = \frac{f_0}{\Delta f}\left(1+\beta^2\right)\ln\left(\frac{f_2}{f_1}\right) + \frac{f_0}{\Delta f}2\beta\int_{k_1}^{k_2} \frac{\cos(k\Delta r + \phi)}{k\Delta r}\, d(k\Delta r)$$

$$\left\langle |H(f)|^2 \right\rangle = \frac{f_0}{\Delta f}(1+\beta^2)\ln\left(\frac{f_2}{f_1}\right) + \frac{f_0}{\Delta f}2\beta\left(Ci\left({k_0\Delta r}/{\sqrt{2}}\right) - Ci\left(k_0\Delta r\cdot\sqrt{2}\right)\right)$$

$$\left\langle |H(f)|^2 \right\rangle = \sqrt{2}(1+\beta^2)\ln(2) + 2\sqrt{2}\beta\left(Ci\left({k_0\Delta r}/{\sqrt{2}}\right) - Ci\left(k_0\Delta r\cdot\sqrt{2}\right)\right)$$

$$\left\langle |H(f)|^2 \right\rangle \approx 2\sqrt{2}\left[\ln(2) + \left(Ci\left({k_0\Delta r}/{\sqrt{2}}\right) - Ci\left(k_0\Delta r\cdot\sqrt{2}\right)\right)\right]$$

Ci represents the Cosine Integral, a special function. At the fourth line we make the assumption that $\phi = 0$ and in the last line that $\beta = 1$.

We then define χ_D and χ_R for the direct and reflected paths (note that the Δr are different for each) as

$$\chi_D(f) = \sqrt{\left\langle |H(f)|^2 \right\rangle_D}$$
$$\chi_R(f) = \sqrt{\left\langle |H(f)|^2 \right\rangle_R}$$

Applying the ground-reflection shading coefficient we expect that the sound pressure at some distance r_1 along the direct path can be expressed as:

$$p_D(t,f) = \frac{r_0}{r_1}\chi_D(f)\left(p_1 D(\theta_F) + p_2 D(\theta_B)\right)MLS(t-\frac{r}{c}) \otimes F_P(t)$$

2.4 Ground- and Air-Attenuation

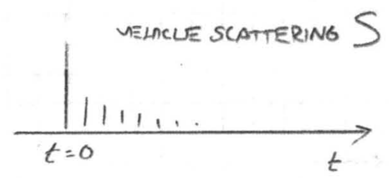
Sound waves are subject to excess attenuation due to ground attenuation and air attenuation. These factors are grouped into one value *A(r)*. With the reflecting plane in place the values $A(r_1)$ along the direct path and $A(r_2)$ along the reflected are expected to be similar. The direct pressure signal is now described as:

$$p_D(t,f) = \frac{r_0}{r}\chi_D(f)A(r,f)\left(p_1 D(\theta_F) + p_2 D(\theta_B)\right)MLS(t-\frac{r}{c}) \otimes F_P(t) \otimes S_D(t-\frac{r}{c})$$

2.5 Scattering from Vehicles

The method is designed to "work" in the presence of live, moving traffic. Vehicles block the signal for a portion or all of a given frame. At other times sound scatters off of vehicles near the line of flight in such a way that some of this energy also arrives at the receptor point. We define a time function S(t) that represents this scattering.

The value at $t = 0$ is less than 1.0 to the extent that the sound path is completely blocked and/or scattered. Additional returns are expected with decreasing strength as t increases because later returns imply longer distance traveled and hence additional attenuation with distance.



It must be assumed that there are two functions, S_D for the direct path and S_R for the reflected path because vehicles are slightly more likely to intercept the direct path than the reflected path. In addition each function can be expected to have a different form for each frame i:

$$p_{D,i}(t,f) = \frac{r_0}{r}\chi_D(f)A(r,f)\left(p_1D(\theta_F) + p_2D(\theta_B)\right)MLS(t-\frac{r}{c}) \otimes F_P(t) \otimes S_{D,i}(t-\frac{r}{c})$$

2.6 <u>Noise from traffic</u>

Traffic noise adds to the pressure measured but is uncorrelated with the MLS signal.

$$P_{D,i}(t,f) = \frac{r_0}{r}\chi_D(f)A(r,f)\left(p_1D(\theta_F) + p_2D(\theta_B)\right)\left[MLS(t-\frac{r}{c}) \otimes F_P(t) \otimes S_{Di}(t-\frac{r}{c})\right] + N_i(t)$$

2.7 <u>Reflection from Retaining Wall</u>

We assume that the retaining wall pressure reflection coefficient is a real number R. It can be shown later that the phase angle of the reflection is unimportant because the direct and reflected wave packets are separated by enough time so that there is no phase cancellation.

The distance traveled for the direct wave is expressed as r_1 and that for the reflected wave as r_2. Hence the two wave fields for the i-th frame, in the orientation with speaker 1 serving the direct field and speaker 2 the reflected field, are expressed as:

$$P_{D1,i}(t,f) = \frac{r_0}{r_1}\chi_D(f)A(r_1,f)\left(p_1D(\theta_F) + p_2D(\theta_B)\right)\left[MLS(t-\frac{r_1}{c}) \otimes F_P(t) \otimes S_{Di}(t-\frac{r_1}{c})\right] + N_i(t)$$

$$P_{R2,i}(t,f) = R_{12}(f)\frac{r_0}{r_2}\chi_R(f)A(r_2,f)\left(p_2D(\theta_F) + p_1D(\theta_B)\right)\left[MLS(t-\frac{r_2}{c}) \otimes F_P(t) \otimes S_{Ri}(t-\frac{r_2}{c})\right] + N_i(t)$$

Given that there are small differences in p_1 and p_2, the measured reflection coefficient is expressed as R_{12} to designate the speaker arrangement.

2.8 Cross-correlation with *MLS*

Cross-correlation with the original *MLS* signal eliminates the traffic noise and replaces the MLS signal with a delta-function, producing an impulse response for the *i*-th frame.

$$P_{D1,i}(t,f) = \frac{r_0}{r_1}\chi_D(f)A(r_1,f)\left(p_1D(\theta_F) + p_2D(\theta_B)\right)\left[\partial(t-\frac{r_1}{c})\otimes F_P(t)\otimes S_{Di}(t-\frac{r_1}{c})\right]$$

$$P_{R2,i}(t,f) = R_{12}\frac{r_0}{r_2}\chi_R(f)A(r_2,f)\left(p_2D(\theta_F) + p_1D(\theta_B)\right)\left[\partial(t-\frac{r_2}{c})\otimes F_P(t)\otimes S_{Ri}(t-\frac{r_2}{c})\right]$$

2.9 Average over many frames and octave-band filter

We then average over many frames in attempt to make the scattering of the cars "vanish" much as a long-time exposure of a freeway seems to make the vehicles vanish. In their place we expect a reduced "brightness" at $t = 0$ related to the percentage of time that the sound path is totally blocked.

At this point the average impulse reponse is octave-band filtered so that the ground reflection can be applied. The impulse response of the octave band filters is long at lower frequencies, the geometry of the test was selected to provide adequate spacing between the direct and reflected packets.

$$P_{D1}(t,f) = \frac{r_0}{r_1}\chi_D(f)A(r_1,f)\left(p_1D(\theta_F) + p_2D(\theta_B)\right)\left[\partial(t-\frac{r_1}{c})\otimes F_P(t)\otimes\langle S_D(t)\rangle\otimes F_{OB}(t)\right]$$

$$P_{R2}(t,f) = R_{12}\frac{r_0}{r_2}\chi_R(f)A(r_2,f)\left(p_2D(\theta_F) + p_1D(\theta_B)\right)\left[\partial(t-\frac{r_2}{c})\otimes F(t)\otimes\langle S_R(t)\rangle\otimes F_{OB}(t)\right]$$

2.10 Integrate energy in the direct and reflected packets

The impulse response at the receptor consists of the combined direct and reflected wave packets. For each octave band we integrate the squared sound pressure level comprising each packet:

$$\int P_{D1}(t,f)^2\,dt = \left(\frac{r_0}{r_1}\chi_D(f)A(r_1,f)\left(p_1D(\theta_F) + p_2D(\theta_B)\right)\right)^2 \int\left(F_P(t)\otimes\langle S_D(t)\rangle\otimes F_{OB}(t)\right)^2 dt$$

$$\int P_{R2}(t,f)^2\,dt = \left(R_{12}\frac{r_0}{r_2}\chi_R(f)A(r_2,f)\left(p_2D(\theta_F) + p_1D(\theta_B)\right)\right)^2 \int\left(F(t)\otimes\langle S_R(t)\rangle\otimes F_{OB}(t)\right)^2 dt$$

2.11 Solve for R

We first take the ratio of the reflected energy to the direct energy:

$$\frac{\int P_{R2}(t,f)^2\,dt}{\int P_{D1}(t,f)^2\,dt}=\frac{\left(R_{12}\frac{r_0}{r_2}\chi_R(f)A(r_2,f)\left(p_2D(\theta_F)+p_1D(\theta_B)\right)\right)^2}{\left(\frac{r_0}{r_1}\chi_D(f)A(r_1,f)\left(p_1D(\theta_F)+p_2D(\theta_B)\right)\right)^2}\frac{\int\left(F_P(t)\otimes\langle S_R(t)\rangle\otimes F_{OB}(t)\right)^2dt}{\int\left(F_P(t)\otimes\langle S_D(t)\rangle\otimes F_{OB}(t)\right)^2dt}$$

then solve for R_{12} as

$$R_{12}(f)=\sqrt{\frac{\int P_{R2}(t,f)^2\,dt}{\int P_{D1}(t,f)^2\,dt}\frac{\left(\frac{r_0}{r_1}\chi_D(f)A(r_1,f)\left(p_1D(\theta_F)+p_2D(\theta_B)\right)\right)^2}{\left(\frac{r_0}{r_2}\chi_R(f)A(r_2,f)\left(p_2D(\theta_F)+p_1D(\theta_B)\right)\right)^2}\frac{\int\left(F_P(t)\otimes\langle S_D(t)\rangle\otimes F_{OB}(t)\right)^2dt}{\int\left(F_P(t)\otimes\langle S_R(t)\rangle\otimes F_{OB}(t)\right)^2dt}}$$

and assume that, through well-selected geometry, the scattering terms are sufficiently similar and therefore the right hand integral quotient approaches unity:

$$R_{12}(f)\approx\sqrt{\frac{\int P_{R2}(t,f)^2\,dt}{\int P_{D1}(t,f)^2\,dt}\frac{\left(r_2\chi_DA(r_1,f)\left(p_1D(\theta_F)+p_2D(\theta_B)\right)\right)^2}{\left(r_1\chi_RA(r_2,f)\left(p_2D(\theta_F)+p_1D(\theta_B)\right)\right)^2}}.$$

We switch from integration to summation to prepare for the digital implementation:

$$R_{12}(f)\approx\frac{r_2}{r_1}\frac{\chi_D(f)}{\chi_R(f)}\frac{A(r_1,f)}{A(r_2,f)}\sqrt{\frac{\sum P_{R2}(t,f)^2}{\sum P_{D1}(t,f)^2}\frac{\left(p_1D(\theta_F)+p_2D(\theta_B)\right)^2}{\left(p_2D(\theta_F)+p_1D(\theta_B)\right)^2}}$$

Several of the terms on the right hand side are inconvenient and can be eliminated by measuring again with the speakers reversed (that is, move speaker 1 from the direct position to the reflecting position). Multiply the results together.

$$R_{12}(f)R_{21}(f)\approx\left(\frac{r_2}{r_1}\frac{\chi_D(f)}{\chi_R(f)}\frac{A(r_1,f)}{A(r_2,f)}\right)^2\sqrt{\frac{\sum P_{R2}(t,f)^2}{\sum P_{D1}(t,f)^2}}\sqrt{\frac{\sum P_{R1}(t,f)^2}{\sum P_{D2}(t,f)^2}}\frac{\left(p_1D(\theta_F)+p_2D(\theta_B)\right)}{\left(p_2D(\theta_F)+p_1D(\theta_B)\right)}\frac{\left(p_2D(\theta_F)+p_1D(\theta_B)\right)}{\left(p_1D(\theta_F)+p_2D(\theta_B)\right)}$$

The gain and directivity terms on the right side cancel, leading to a simplified equation:

$$R_{12}(f)R_{21}(f)\approx\left(\frac{r_2}{r_1}\frac{\chi_D(f)}{\chi_R(f)}\frac{A(r_1,f)}{A(r_2,f)}\right)^2\sqrt{\frac{\sum P_{R2}(t,f)^2}{\sum P_{D1}(t,f)^2}}\sqrt{\frac{\sum P_{R1}(t,f)^2}{\sum P_{D2}(t,f)^2}}$$

Finally we solve for the barrier reflection coefficient *R(f)* as the square-root of the product of *R12* and R_{21}:

$$R(f) \approx \frac{r_2}{r_1}\frac{\chi_D(f)}{\chi_R(f)}\frac{A(r_1,f)}{A(r_2,f)}\sqrt{\sqrt{\frac{\sum P_{R2}(t,f)^2}{\sum P_{D1}(t,f)^2}}\sqrt{\frac{\sum P_{R1}(t,f)^2}{\sum P_{D2}(t,f)^2}}}$$

We now turn our attention to the attenuation terms *A(r,f)* which consist of factors for ground- and air-attenuation. The reflecting plane is intended to create identical ground reflections for the direct and reflected paths, which appears to be a good assumption for 500 Hz and above.

Air attenuation is not expected to be a factor because the distances traveled in this test geometry are short. For the conditions tested (roughly 68F, 70% RH) air attenuation in the 2 kHz band is 3.3 dB/1000 ft. The difference in air attenuation for the direct and reflected sound is about 0.08 dB (3.3 dB/1000 ft x 25 ft path length difference). The inferred pressure attenuation would be on the order of only 1%.

Thus we conclude that the *A(r,f)* terms can be safely eliminated and arrive at a final form for the octave-band reflection coefficient:

$$R(f) \approx \frac{r_2}{r_1}\frac{\chi_D(f)}{\chi_R(f)}\sqrt{\sqrt{\frac{\sum P_{R2}(t,f)^2}{\sum P_{D1}(t,f)^2}}\sqrt{\frac{\sum P_{R1}(t,f)^2}{\sum P_{D2}(t,f)^2}}}$$

2.12 Absorption coefficient

The absorption coefficient of the retaining wall is:

$$\alpha_i = 1 - R_i^2$$

where *i* refers to each of the four principal traffic noise octave bands: 250 Hz, 500 Hz, 1000 Hz, and 2000 Hz.

2.13 Added energy due to reflection

The added energy due to the presence of the reflection, for distant receptors, is

$$\Delta dB_i = 10 \cdot \log_{10}(1 + R_i^2) = 10 \cdot \log_{10}(2 - \alpha_i)$$

where *i* refers to each of the four principal traffic noise octave bands: 250 Hz, 500 Hz, 1000 Hz, and 2000 Hz.

3. RESULTS FOR TEST ARRANGEMENT "K"

Computed results are tabulated in Table 1 below.

The 500 Hz and 1000 Hz results indicate a near-perfect reflector. The 2000 Hz result suggests scattering from surface irregularities on the concrete barrier. Because this scattering causes the sound to not arrive at the receptor, it "looks like" sound absorption to the method.

The 250 Hz result exceeds 1.0, most likely due to the absence of an adequate ground reflection along the direct path because of the limited extent of the reflecting plane. The 125 Hz result is even larger, which supports this supposition. The complete loss of an in-phase pressure reflection would reduce the direct signal by 6 dB (pressure-doubling), a factor of 4 in pressure. If we assume that the 125 Hz wave suffered a complete loss of the ground reflection and the 250 Hz a 50% loss, we would apply *ad-hoc* divisors of *4* and *2* in the 125 Hz and 250 Hz bands respectively to obtain the adjusted results. A small adjustment is made at 1000 Hz as well to bring the result below 1.00.

The ground-reflection shading is not applied at 2000 Hz because the sound does not come in contact with the ground because of horn directionality, hence the factor is forced to 0.99.

Table I: Arrangement "K" results

	125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	A
Energy Ratio (Refl/Direct)	2.03	1.11	0.62	0.77	0.65	---
Distance Ratio r_2/r_1	1.18	1.18	1.18	1.18	1.18	---
Ground Reflection χ_D/χ_R	1.40	1.38	1.33	1.14	N/A	---
Reflection Coefficient [1]	**3.35**	**1.81**	**0.97**	**1.04**	**0.77**	---
Ad-hoc Divisor	4.00	2.00	1.00	1.07	1.00	---
Reflection Coefficient [1], adjusted	**0.84**	**0.91**	**0.97**	**0.99**	**0.77**	---
Absorption Coefficient [1]	0.30	0.18	0.05	0.03	0.41	---
Added Energy [dB]	2.3	2.6	2.9	3.0	2.0	---
Benchmark Spectrum [dB]	62.1	67.8	75.7	79.2	68.3	81.2
"Anechoic" Spectrum [dB]	59.8	65.2	72.8	76.2	66.3	78.4

The benchmark spectrum consists of A-weighted octave-band sound pressure levels measured atop the Edgefield Avenue bridge.

The final row shows the "anechoic" spectrum determined by subtracting the added energy *ΔdB* from the initial SPL spectrum. *Note: this spectrum is not likely to be an actual SPL result because the effect of the sound absorption is dependent on the distance of the sound source to the barrier and to the receptor location.*

By comparing the A-weighted results in the right hand column it appears that the retaining-wall reflection adds *81.2 – 78.4 = 2.8 dBA* compared to a completely anechoic (non-reflective) wall. The theoretical limit (for a perfect reflector) is 3.0 dBA.

3.1 Energy-Time Curves

Energy-time curves are presented in Figures 2-6 for the octave bands 125 Hz, 250 Hz, 500 Hz, 1000 Hz, and 2000 Hz. The differing arrival times is caused by propagation time through the octave-band filters. The direct and reflected wave packets arrive approximately 22 ms apart, indicating a path-length difference of about 25 ft. This is consistent with the test geometry (24 ft. inferred from physical measurements). The blue cursors mark the boundaries around the direct and reflected wave packets.

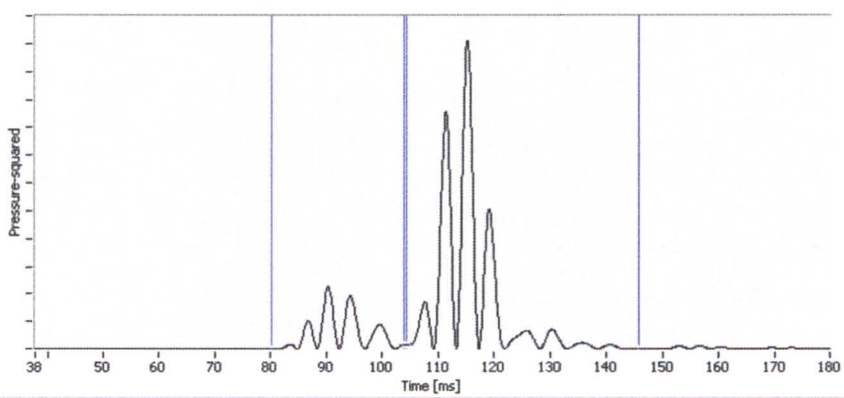


Figure 2: 125 Hz energy-time curve

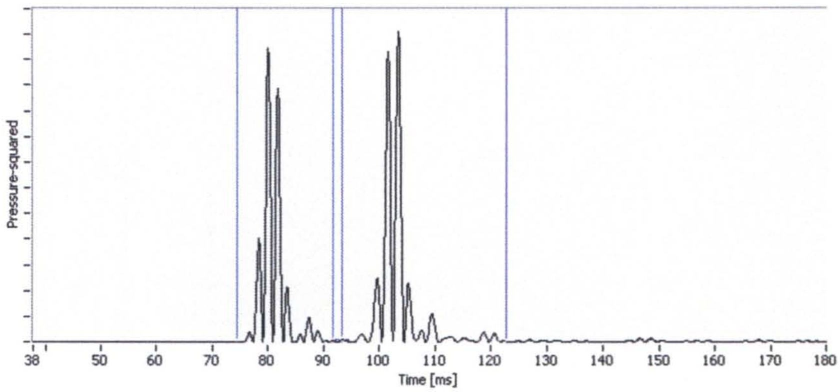


Figure 3: 250 Hz energy-time curve

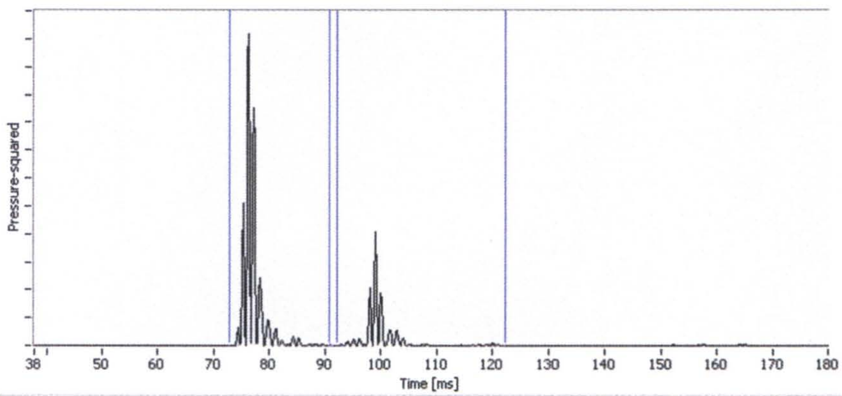


Figure 4: 500 Hz energy-time curve

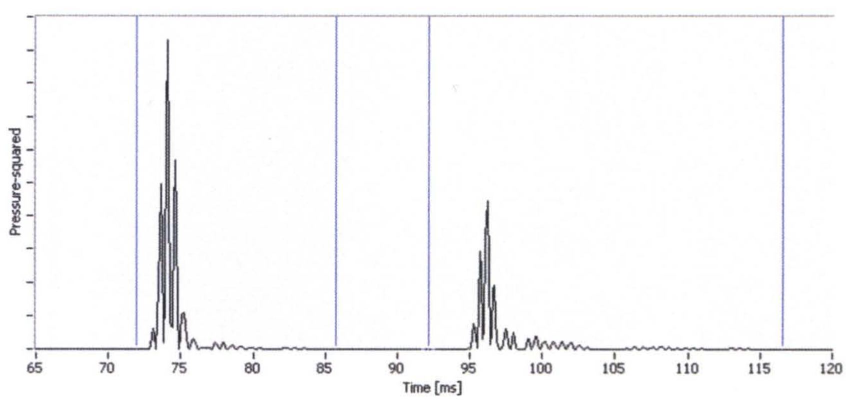


Figure 2: 1000 Hz energy-time curve

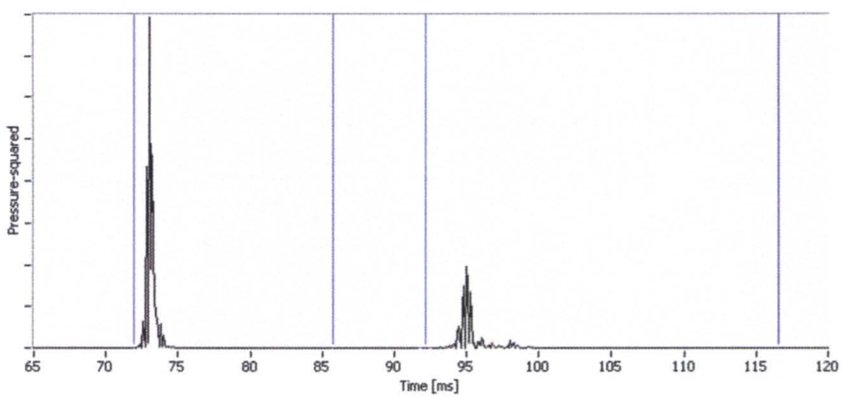


Figure 6: 2000 Hz energy-time curve

4. PERFORMANCE ASSESSMENT AFTER INSTALLATION

Nelson Acoustics proposes that the test be repeated in Arrangement "K" after the installation and again some time later to assess the long-term performance of the sound-absorbing treatment.

Reflection coefficient results at 125 Hz should be ignored because of the relatively weak contribution to the A-weighted sound level. The same *ad-hoc* divisors should be applied for consistency with the benchmark case.

The *ΔdB* for the treated state should be computed from the adjusted reflection coefficients, and then applied to the "anechoic" sound pressure level spectrum. The aggregate A-weighted sound level is then calculated from the "boosted" spectrum and compared to the 81.24 dBA result for the wall in its current state. The A-weighted difference *ΔdBA* then represents the benefit provided by the retaining wall sound-absorbing treatment:

$$\Delta dB_i = 10\log(1 + R_i^{\,2})$$

$$L_{P,i} = 65.2 \quad 72.8 \quad 76.2 \quad 66.3$$

$$\Delta dBA = 81.24 - \sum_i 10^{0.1\left(L_{P,i} + \Delta dB_i\right)}$$

where once again *i* refers to each of the four principal traffic noise octave bands: 250 Hz, 500 Hz, 1000 Hz, and 2000 Hz.

5. CONCLUSION

This report presents a rigorous derivation of the test method, reports the results for test arrangement "K", and proposes a method for assessing the "dB reduction" afforded by the sound-absorbing treatment.

The method is very mathematically and physically robust except for low-frequency effects that must be adjusted for at 250 Hz and below. In any event these are of minor importance because of the relatively minor contribution of low-frequency sound to the overall A-weighted sound level.

Please feel free to contact me with questions about the test method, the results, or the assessment method.

Sincerely,

NELSON ACOUSTICS (TX F-3001)

Electronically Reproduced Signature

David A. Nelson, INCE Bd. Cert., PE (OR 17635, TX 81329)

Principal Consultant