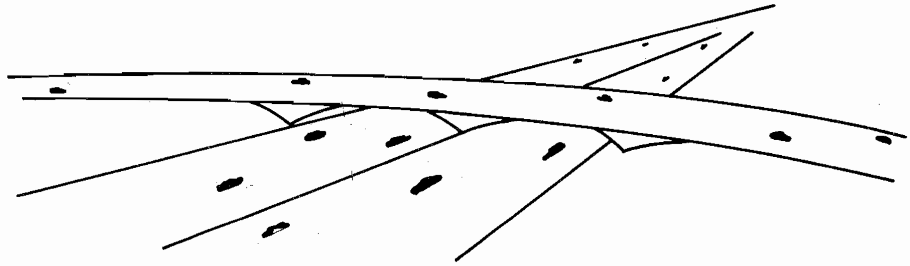

DEPARTMENTAL RESEARCH

Report Number SS 13.1

SENSITIVITY ANALYSIS OF THE EXTENDED AASHO RIGID PAVEMENT DESIGN EQUATION

TEXAS HIGHWAY DEPARTMENT

SENSITIVITY ANALYSIS OF THE EXTENDED AASHO RIGID PAVEMENT DESIGN EQUATION

by

Harvey J. Treybig
Associate Design Engineer

Special Study 13.1

Research Section-Highway Design Division
TEXAS HIGHWAY DEPARTMENT

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The systems approach to pavement design has, since 1965, received the attention of many researchers and design engineers. The use of systems engineering has not developed any startling new inputs for the solution of pavement design problems, but rather organizes the various aspects of the total problem into a manageable form. The systems concept emphasizes those factors and ideas which are common to the successful operation of relatively independent parts of the whole pavement problem.

The output response of the pavement system (Ref. 1) is the serviceability or performance that the pavement experiences. This serviceability performance concept was first developed by Carey and Irick (Ref. 2) at the AASHO Road Test. The AASHO Interim Guide for the Design of Rigid Pavement Structures (Ref. 3) was the first design method which incorporated the performance criteria.

The pavement design variables are of a stochastic nature. Variations occur in the parameters which must be recognized by designers. The effects of these stochastic variations are not always known. The design variables usually considered in rigid pavement design are flexural strength, modulus of elasticity, thickness, type of rigid pavement, environmental conditions, expected traffic, modulus of subgrade reaction, Poissons ratio, and subbase. All of these variables affect the problem seriously.

Objective

The objectives of this research are 1) to determine the importance of rigid pavement design variables, 2) interactions among the variables, and 3) to evaluate the effects of stochastic variations for each variable using an extension of the AASHO Interim Guide (Ref 4).

Scope

The rigid pavement design variables considered are the modulus of elasticity, flexural strength, slab thickness, slab continuity, modulus of subgrade reaction, initial serviceability index and terminal serviceability index. Each variable was evaluated for two levels except the modulus of subgrade reaction at three levels and the initial and terminal serviceability indexes at one level each.

CHAPTER II

THE PROBLEM AND APPROACH

The evaluation of rigid pavement design variables requires the use of some type of mathematical model relating the variables. If a systems approach such as that of Hudson, et. al. (Ref 1) is to ultimately be applicable to the model, then a system output response such as performance must be included as a variable in the model.

Selection of Model

Pavement performance criteria was developed and used at the AASHO Road Test, hence any design models developed using these performance data could have been used in this study. Two mathematical models for the rigid pavement design variables including the performance variable were considered. The first model is that from the AASHO Interim Guide for the Design of Rigid Pavement Structures (Ref 3). Realizing the shortcomings of the Interim Guide (Ref 3), Hudson and McCullough (Ref 4) extended the Interim Guide (Ref 3) to include modulus of elasticity, modulus of subgrade reaction, and a term for pavement continuity to include continuously reinforced concrete pavement.

Hudson and McCullough's extension of the Interim Guide was selected for use in the study described here, because it was the best available mathematical model based on performance which contained the desired design variables. The model is as follows:

$$\text{Log } \Sigma W = -9.483 - 3.837 \log \left[\frac{J}{S_x D^2} \left(1 - \frac{2.61a}{Z^{1/4} D^{3/4}} \right) \right] + \frac{G}{\beta} \text{ -----(1)}$$

ΣW = number of accumulated equivalent 18 kip single axle loads

J = a coefficient dependent upon load transfer characteristics or slab continuity

S_x = modulus of rupture of concrete at 28 days (psi)

D = nominal thickness of concrete pavement (inches)

Z = E/k

E = modulus of elasticity for concrete (psi)

k = modulus of subgrade reaction (psi/inch)

a = radius of equivalent loaded area = 7.15 for Road Test

$$G = \log\left(\frac{Po - Pt}{Po - 1.5}\right)$$

P_o = serviceability index immediately after construction

P_t = terminal serviceability index assumed as failure

$$\beta = 1 + \frac{1.624 \times 10^7}{(D + 1)^{8.46}}$$

Approach

A sensitivity analysis is a procedure to determine the change in a dependent variable due to a unit change in an independent variable. A sensitivity analysis can be used to evaluate a whole system of variables and interactions between the variables that compose the system. The rigid pavement design variables were evaluated in this research by means of a sensitivity analysis which determined the change in pavement life due to changes in the variables. However, the unit of study for each variable was chosen to represent statistical variations which have been observed in actual engineering practice. The analysis involved the levels of the variables shown in Table 1.

In a previous sensitivity study by Buick, (Ref 5) the theoretical importance of design variables was evaluated by the use of an instantaneous rate of change which was quantified by first order partial derivatives. Buick also evaluated the practical importance with a

TABLE 1

VARIABLES AND LEVELS FOR

THE EXPERIMENT

VARIABLE	LEVEL		
	LOW	MEDIUM	HIGH
Modulus of Elasticity (E)	$2.0(10)^6$		$5.5(10)^6$
Flexural Strength (S_x)	400		800
Thickness (D)	6		12
Modulus of Subgrade Reaction (k)	25	200	1000
Continuity (J)	3.20		2.33
Initial Serviceability Index (P_o)		4.2	
Terminal Serviceability Index (P_t)		3.0	

study of the parameter variations tolerated by selected thickness change constraints and a least squares fitting of appropriate equations to parameter-thickness data.

In another sensitivity analysis McCullough, Van Til, Vallerga, and Hicks (Ref 6) formulated the necessary partial derivatives but used numerical techniques to evaluate the "error" in total traffic in terms of percent change in each variable. McCullough, et. al. evaluated only the AASHO Interim Guide (Ref 3) while Buick evaluated the AASHO Interim Guide Design Method, Corps of Engineers Rigid Pavement Design Method for Streets and Roads, and the Portland Cement Association Design Method for Rigid Pavements.

For this research a full factorial of the variables listed in Table 1 was evaluated. In each factorial cell, each variable was evaluated for the effect of its perturbations around the mean. In order to compare meaningful variations of a variable, the standard deviation was chosen rather than the standard unit change in the variable.

Every material property, modulus of elasticity, flexural strength, etc., has a statistical distribution with a mean and a variation or a dispersion about this mean. Therefore, standard deviations have been developed for all the design variables to characterize their variation. The development of these standard deviations is covered in Chapter III.

The changes in expected pavement life have been computed for variations in each variable in each block of the factorial (Figure 1). Each block of the factorial involves a fixed level of the seven variables.

Continuity	Mod. of Elas.	Subg. Mod.	Flex. Stren. 400, Thickness 6	Flex. Stren. 400, Thickness 12	Flex. Stren. 800, Thickness 6	Flex. Stren. 800, Thickness 12
3.20	$2(10)^6$	25	1	13	25	37
		200	2	14	26	38
		1000	3	15	27	39
	$5.5(10)^6$	25	4	16	28	40
		200	5	17	29	41
		1000	6	18	30	42
2.33	$2(10)^6$	25	7	19	31	43
		200	8	20	32	44
		1000	9	21	33	45
	$5.5(10)^6$	25	10	22	34	46
		200	11	23	35	47
		1000	12	24	36	48

FACTORIAL FOR ANALYSIS OF CHANGE

IN PAVEMENT LIFE

Figure 1

One solution for each block was made for the expected pavement life for the mean values in the factorial. Solutions were then run with each of the seven variables allowed to vary, once positive and once negative, thus totaling fourteen additional solutions for the expected pavement life.

The changes in expected pavement life due to these perturbations were computed in terms of percent increase or decrease in expected pavement life. The change was computed with the following formula.

$$L_i = \frac{W(j, i \pm \Delta) - W_j}{W_j} \times 100 \quad \text{----------------------------------} (2)$$

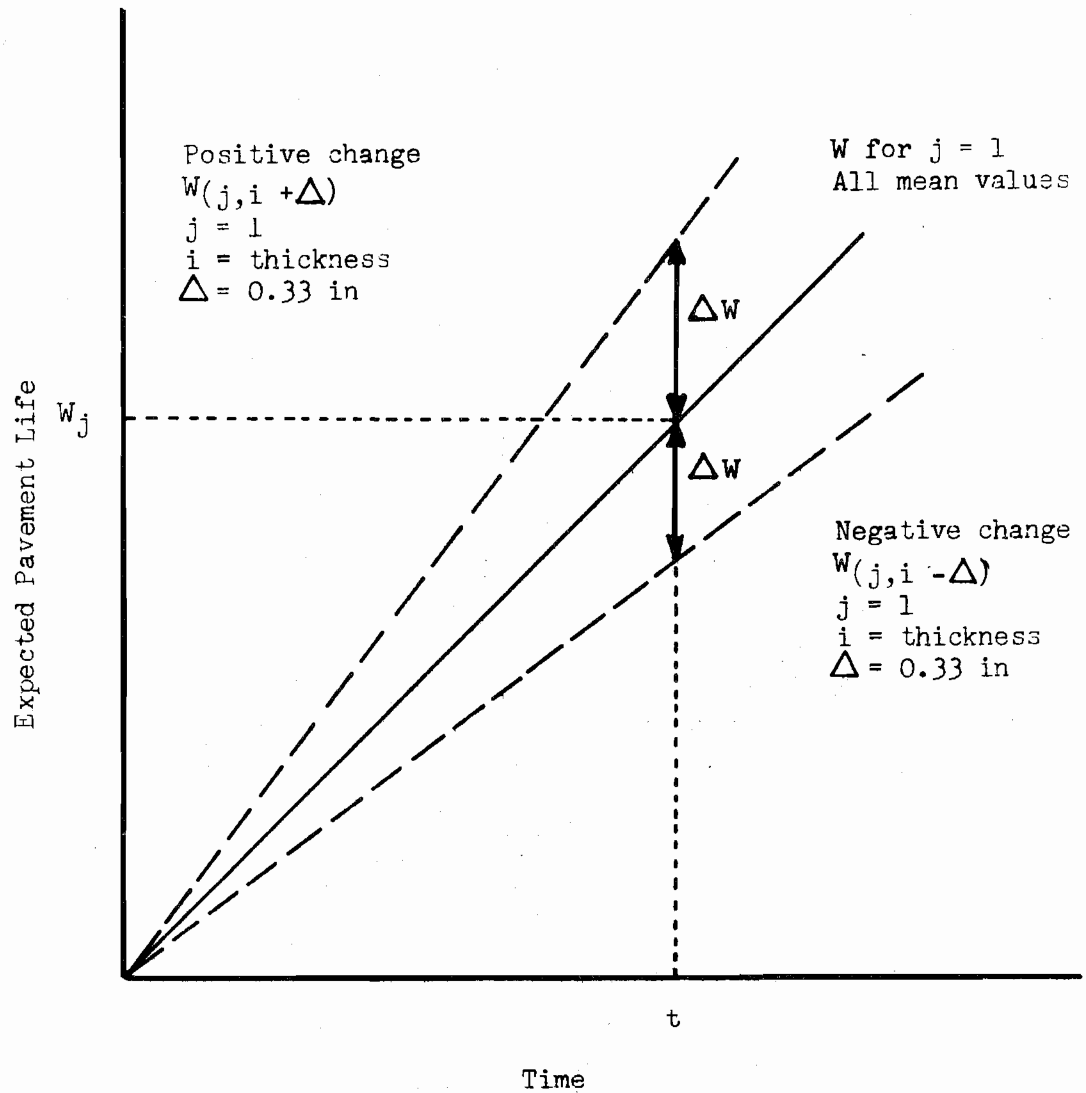
Where

L_i = Percent change in expected pavement life due to a variation in design variable (i)

W_j = expected pavement life for mean values in factorial. j indicates block of factorial by number listed in Figure 1.

$W(j, i \pm \Delta)$ = expected pavement life for factorial block j with variable i incremented plus or minus some change.

A positive sign on L_i indicates increased pavement life while a negative sign indicates a reduction in pavement life. The meaning of the change in expected life is illustrated in Figure 2. Three hypothetical lines are shown. The center line represents the expected pavement life for one of the factorial blocks, j, example j = 1 (Figure 1). The upper line represents the expected life of a pavement with all variables fixed at that level, except one, for example thickness, which is increased two standard deviations. The lower line represents the expected pavement life for the same factorial block with a negative variation in thickness. The effects of some variations will of course



GRAPHICAL EXPLANATION OF CHANGE IN

EXPECTED PAVEMENT LIFE

Figure 2

be reversed, that is an increase in the variable gives a decreased life.

CHAPTER III

FORMULATION OF DESIGN VARIABLE VARIATIONS

Highway contractors in general do not use statistical quality control in construction. Thus there is little exact data on expected variations in the design variables. The two largest sources of data for the development of the standard deviations are the permanent construction files of the Texas Highway Department and the reports on the AASHO Road Test. The author's experience also aided in the development of some of the standard deviations. A search of the literature yielded little more than what was available through the above mentioned sources.

Development of Standard Deviations

Using available data together with some assumptions, standard deviations were developed for each level of each variable selected for this investigation (Table 1).

Concrete Properties. Two properties of concrete are of interest in this study : 1) the modulus of elasticity (E) and 2) the flexural strength (S_x). The two levels of modulus of elasticity were 2.0×10^6 psi and 5.5×10^6psi. Test data for modulus of elasticity were obtained from Shafer of the Texas Highway Department (Ref 7) and from Johnson's work (Ref 8) at the Iowa Engineering Experiment Station. Analysis of these data yielded standard deviations of 0.2×10^6 psi and 0.6×10^6 psi respectively, for the low and high levels of modulus of elasticity (Appendix 1).

Two levels of flexural strength are involved, 400 psi and 800 psi. Data related to variation of flexural strength were obtained from several sources. The Texas Highway Department maintains permanent files of flexural strength on all concrete paving projects. From these records (Ref 9) much data was used. Hudson has extended some of the AASHO Road Test Work (Ref 10) wherein flexural strength data were also available. Schleider reviewed large amounts of flexural strength data from the Texas Highway Departments construction files (Ref 11). Data for the high level, 800 psi came from all three sources and the average standard deviation was 60 psi. The data for the low level of flexural strength came only from Schleider (Ref 11). The standard deviation for the low level was 45 psi.

Slab Thickness (D). Slab thickness was involved at two levels, 6 inches and 12 inches. Sample data were obtained for several other thicknesses from the Texas Highway Department (Ref 9), but were not available for these thicknesses. From these data (See Appendix 1), the average coefficient of variation was obtained. The coefficient of variation was assumed to be constant. The standard deviations for the two thicknesses in this experiment were computed using the obtained coefficient of variation. They are 0.165 inch and 0.330 inch for the six and twelve inch slab thicknesses, respectively.

Modulus of Subgrade Reaction (k). Although k-values are used in design, records show little evidence that agencies designing and building pavements actively measure modulus of subgrade reactions by use of plate load tests (Ref 6). However, data were available from the AASHO Road Test (Ref 10). These data were for k-values averaging about 100 pci. The standard deviation for this data was 16 pci. In this investigation, k-values of 25, 200, and 1000 were used. Thus, the assumption was again made that the coefficient of variation would be a constant over a range of values of the parameter. The three necessary standard deviations were computed using the coefficient of variation, 15 percent, (See Appendix 1) from the AASHO data. The standard deviations calculated for the three levels were 4, 30, and 150 pci, respectively (Appendix 1).

Slab Continuity (J). The slab continuity term was used by Hudson and McCullough to characterize the particular type of rigid pavement being considered. Hudson and McCullough found J = 2.20 for continuously reinforced concrete pavement. This value was based on Texas Highway Department design procedures and pavement performance. Since that time McCullough and Treybig (Ref 12) have conducted extensive studies on deflection of jointed and continuously reinforced concrete pavement. This research has yielded a new J-value for continuous pavement, J = 2.33 (Appendix 2). Since this term is not a measureable material property but a pavement characteristic, its dispersion could not be determined in the normal way. Based on variability of deflection measurements and experience, standard deviations were estimated as 0.19 for continuous pavement and 0.13 for jointed pavement.

Performance Variables (P_o, P_t). In this design method, performance is a function of the serviceability indexes. The initial serviceability index at time of construction has never been evaluated for rigid pavements other than those at the AASHO Road Test. Scrivner (Ref 13) indicated that the average initial serviceability index on flexible pavement in Texas is 4.2. Having no better estimate for

Texas, this same value was assumed for rigid pavement for this study. Performance studies by Treybig (Ref 14) indicate that the initial serviceability index of in-service pavements is well below the 4.5 measured at the AASHO Road Test (Ref 15). Based on a knowledge of construction variability and numerous observations of pavements under construction together with comments by Hudson and McCullough (Ref 16), the standard deviation of the initial serviceability index was estimated to be 0.5.

For the standard deviation of the terminal serviceability index, actual measurements of serviceability index were used. Measurements at the AASHO Road Test (Ref 15) as well as performance studies in Texas indicate a standard deviation of 0.3.

Discussion of Variables

The standard deviations developed for each of the design variables cannot all be used with the same confidence. Those that are true standard deviation developed from real data are thought to be good estimates of variation in service. These include the modulus of elasticity, flexural strength, and the terminal serviceability index.

For both the modulus of subgrade reaction and thickness the assumption was made that the coefficient of variation for each of the two respective variables would be constant for all levels of the variables in the ranges under study. Since the thickness data that were used (Ref 9) gave reasonable standard deviations and also a reasonably constant coefficient of variation over the range, the standard deviations for the thicknesses in this experiment are believed to be reasonable. The coefficient of variation on the k-value resulted from data recorded at the AASHO Road Test (Ref 10) which was conducted under somewhat more careful control than might be expected in the field. Furthermore, there is little data to substantiate the assumption of constant coefficient of variation, although wide variations would not be expected. The values used herein should be verified in future studies.

The standard deviations of the remaining two variables are associated with a somewhat lower level of confidence. The standard deviations for both the initial serviceability index and the J-value are estimated and not computed from numerical data.

Variations for Sensitivity Study

Each of the design variables has some distribution. For the analysis of changes in expected pavement life the total variation for each variable was either one or two standard deviations. Table 2 indicates the levels of each variable as well as the respective standard deviations.

The variation in modulus of elasticity was selected as plus or minus one standard deviation.

The variation selected for thickness, flexural strength, and modulus of subgrade reaction was plus or minus two standard deviations, since 95 percent of the total variation (assuming a normal distribution for each variable) would be included within the bounds.

The level of variation for slab continuity was selected as plus or minus one standard deviation since the variation was estimated rather than based on real data.

TABLE 2

STANDARD DEVIATIONS FOR VARIABLES

VARIABLE	LEVEL	STANDARD DEVIATION
Modulus of Elasticity (E)	$2.0(10)^6$	$0.2(10)^6$
	$5.5(10)^6$	$0.6(10)^6$
Flexural Strength (S_x)	400	45
	800	60
Thickness (D)	6	0.165
	12	0.330
Modulus of Subgrade Reaction (k)	25	4
	200	30
	1000	150
Continuity (J)	3.20	0.13
	2.33	0.19
Initial Serviceability Index (P_o)	4.2	0.5
Terminal Serviceability Index (P_t)	3.0	0.3

The serviceability parameters were both varied plus and minus one standard deviation. The initial serviceability index has an upper limit of 5.0; thus, with a mean of 4.2 and two standard deviations, unreasonable answers would result. For the terminal serviceability index, plus and minus one standard deviation was also selected.

The deviation levels of all variables are listed in Table 3.

TABLE 3

DEVIATION LEVELS OF INDEPENDENT VARIABLES

Variable	Level	Deviation Level
Modulus of Elasticity (E)	$2(10)^6$	$1.8(10)^6$ $2.2(10)^6$
	$5.5(10)^6$	$4.9(10)^6$ $6.1(10)^6$
Flexural Strength (S_x)	400	305 495
	800	680 920
Thickness (D)	6	5.67 6.33
	12	11.34 12.66
Modulus of Subgrade Reaction (k)	25	17 33
	200	140 260
	1000	700 1300
Continuity (J)	3.20	3.07 3.22
	2.33	2.14 2.52
Initial Serviceability Index (P_o)	4.2	3.7 4.7
Terminal Serviceability Index (P_t)	3.0	2.7 3.3

CHAPTER IV

RESULTS

The total expected pavement life as predicted by equation (1) has been determined for the factorial. Figure 3 shows the predicted pavement life for each case in total 18 kip single axle applications. The effects of positive and negative variations in each of the variables in each factorial block have been evaluated in terms of percent change in life. Table 4 contains the percent change in life for variations in each variable in each respective factorial block.

Analysis

An analysis of variance was made on the mean values of the expected pavement life shown in Figure 3. The analysis of variance considered only five of the seven factors since the initial and terminal serviceability indexes were studied only at one level.

Those factors and interactions found to be significant at selected alpha levels are shown in Table 5. All other possible interactions were found insignificant.

A graphic presentation was chosen to relate the effects of variations in each variable to pavement life. Pavement life change is characterized by the percent change in the summation of traffic loads for the performance period. The performance period is the time required for the pavement serviceability index to decrease from 4.2 to 3.0. Table 6 contains the factors analyzed in their order of importance with respect to change in life due to factor variations.

Continuity	Mod. of Elas.	Subg. Mod. \ Flex. Stren.	400		800	
		Thickness	6	12	6	12
3.20	$2(10)^6$	25	87,000	6,432,000	1,240,000	91,918,000
		200	310,000	11,542,000	4,302,000	164,946,000
		1000	3,384,000	27,319,000	48,356,000	390,407,000
	$5.5(10)^6$	25	62,000	5,406,000	889,000	77,259,000
		200	144,000	8,271,000	2,064,000	118,198,000
		1000	567,000	14,965,000	8,106,000	213,860,000
2.33	$2(10)^6$	25	293,000	21,731,000	4,190,000	310,543,000
		200	1,017,000	38,994,000	14,533,000	557,239,000
		1000	11,432,000	92,296,000	163,370,000	1,318,957,000
	$5.5(10)^6$	25	210,000	18,265,000	3,003,000	261,013,000
		200	488,000	27,943,000	6,971,000	399,324,000
		1000	1,916,000	50,560.000	27,385,000	722,519,000

PREDICTED PAVEMENT LIFE IN

18 KIP EQUIVALENCIES

Figure 3

TABLE 4
PERCENT CHANGE IN PREDICTED LIFE

Factorial Number	Positive Variation in Factors						
	E	Sx	D	K	J	Po	Pt
1	-3	127	31	12	-14	9	-13
2	-8	127	21	29	-14	9	-13
3	-21	127	-5	114	-14	9	-13
4	-3	127	34	8	-14	9	-13
5	-6	127	28	18	-14	9	-13
6	-12	127	15	42	-14	9	-13
7	-3	127	31	12	-26	9	-13
8	-8	127	21	29	-26	9	-13
9	-21	127	-5	114	-26	9	-13
10	-3	127	33	9	-26	9	-13
11	-6	127	27	17	-26	9	-13
12	-12	127	15	42	-26	9	-13
13	-2	127	46	6	-14	19	-25
14	-4	127	42	11	-14	19	-25
15	-7	127	35	23	-14	19	-25
16	-2	127	47	4	-14	19	-25
17	-3	127	44	8	-14	19	-25
18	-5	127	40	14	-14	19	-25
19	-2	127	46	6	-26	19	-25
20	-4	127	42	11	-26	19	-25
21	-7	127	35	23	-26	19	-25
22	-1	127	47	4	-26	19	-25
23	-3	127	44	8	-26	19	-25
24	-5	127	40	14	-26	19	-25
25	-4	71	31	12	-14	9	-13
26	-8	71	21	29	-14	9	-13
27	-21	71	-5	114	-14	9	-13
28	-3	71	33	8	-14	9	-13
29	-6	71	27	17	-14	9	-13
30	-12	71	15	42	-14	9	-13
31	-4	71	31	12	-26	9	-13
32	-8	71	21	29	-26	9	-13
33	-21	71	-5	114	-26	9	-13
34	-3	71	33	8	-26	9	-13
35	-6	71	27	17	-26	9	-13
36	-12	71	15	42	-26	9	-13
37	-2	71	46	6	-14	19	-25
38	-4	71	42	11	-14	19	-25
39	-7	71	34	23	-14	19	-25
40	-1	71	47	4	-14	19	-25
41	-3	71	44	8	-14	19	-25
42	-5	71	40	14	-14	19	-25
43	-2	71	46	6	-26	19	-25
44	-4	71	42	11	-26	19	-25
45	-7	71	34	23	-26	19	-25
46	-1	71	47	4	-26	19	-25
47	-3	71	44	8	-26	19	-25
48	-5	71	40	14	-26	19	-25

TABLE 4 Cont.

Factorial	Negative Variation in Factors						
Number	E	Sx	D	K	J	Po	Pt
1	5	-65	-25	-14	17	-14	11
2	10	-65	-18	-26	17	-14	11
3	33	-65	+12	-54	17	-14	11
4	3	-65	-26	-10	17	-14	11
5	8	-65	-22	-17	17	-14	11
6	16	-65	-12	-33	17	-14	11
7	4	-65	-25	-13	39	-14	11
8	10	-65	-18	-26	39	-14	11
9	33	-65	+12	-54	39	-14	11
10	3	-65	-26	-54	39	-14	11
11	7	-65	-22	-18	39	-14	11
12	16	-65	-12	-33	39	-14	11
13	2	-65	-33	-7	17	-28	25
14	5	-65	-30	-12	17	-28	25
15	8	-65	-26	-21	17	-28	25
16	2	-65	-33	-5	17	-28	25
17	3	-65	-32	-9	17	-28	25
18	6	-65	-29	-15	39	-28	25
19	2	-65	-33	-7	39	-28	25
20	4	-65	-30	-12	39	-28	25
21	8	-65	-26	-21	39	-28	25
22	2	-65	-33	-5	39	-28	25
23	3	-65	-32	-9	39	-28	25
24	6	-65	-29	-15	39	-28	25
25	4	-46	-25	-13	17	-14	11
26	10	-46	-18	-26	17	-14	11
27	33	-46	+12	-54	17	-14	11
28	3	-46	-26	-10	17	-14	11
29	7	-46	-22	-18	17	-14	11
30	16	-46	-12	-33	17	-14	11
31	4	-46	-25	-13	39	-14	11
32	10	-46	-18	-26	39	-14	11
33	33	-46	+12	-54	39	-14	11
34	3	-46	-26	-10	39	-14	11
35	7	-46	-22	-18	39	-14	11
36	16	-46	-13	-33	39	-14	11
37	2	-46	-33	-7	17	-28	25
38	4	-46	-30	-12	17	-28	25
39	8	-46	-26	-21	17	-28	25
40	2	-46	-33	-5	17	-28	25
41	3	-46	-32	-9	17	-28	25
42	6	-46	-29	-15	39	-28	25
43	2	-46	-33	-7	39	-28	25
44	4	-46	-30	-12	39	-28	25
45	8	-46	-26	-21	39	-28	25
46	2	-46	-33	-5	39	-28	25
47	3	-46	-32	-9	39	-28	25
48	6	-46	-29	-15	39	-28	25

TABLE 5

ANALYSIS OF VARIANCE FOR EXPECTED PAVEMENT LIFE

Combination of Factors	Degrees of Freedom	Mean Squares	F Value	Significance Level, %
5	1	44,961,433	161.1	1
4	1	43,452,302	155.7	1
4x5	1	33,969,002	121.7	1
1	1	16,971,287	60.8	1
1x5	1	13,267,448	47.5	1
1x4	1	12,822,076	45.9	1
1x4x5	1	10,023,472	35.9	1
3	2	8,946,193	32.0	1
3x4	2	6,758,959	24.2	1
3x5	2	5,269,082	18.9	1
3x4x5	2	3,980,810	14.3	1
2	1	3,605,810	12.9	1
1x3	2	2,639,904	9.5	1
1x3x4	2	1,994,450	7.2	5
2x5	1	1,740,530	6.2	5
2x3	2	1,601,046	5.7	5
1x3x5	2	1,554,808	5.6	5
2x3x4	2	1,209,627	4.3	5

Legend of Factors

1 - Continuity
2 - Modulus of Elasticity
3 - Modulus of Subgrade Reaction
4 - Flexural Strength
5 - Thickness

TABLE 6

ORDER OF IMPORTANCE OF VARIABLES WITH RESPECT TO CHANGE IN EXPECTED PAVEMENT LIFE

FLEXURAL STRENGTH

THICKNESS

CONTINUITY

TERMINAL SERVICEABILITY INDEX

INITIAL SERVICEABILITY INDEX

MODULUS OF SUBGRADE REACTION

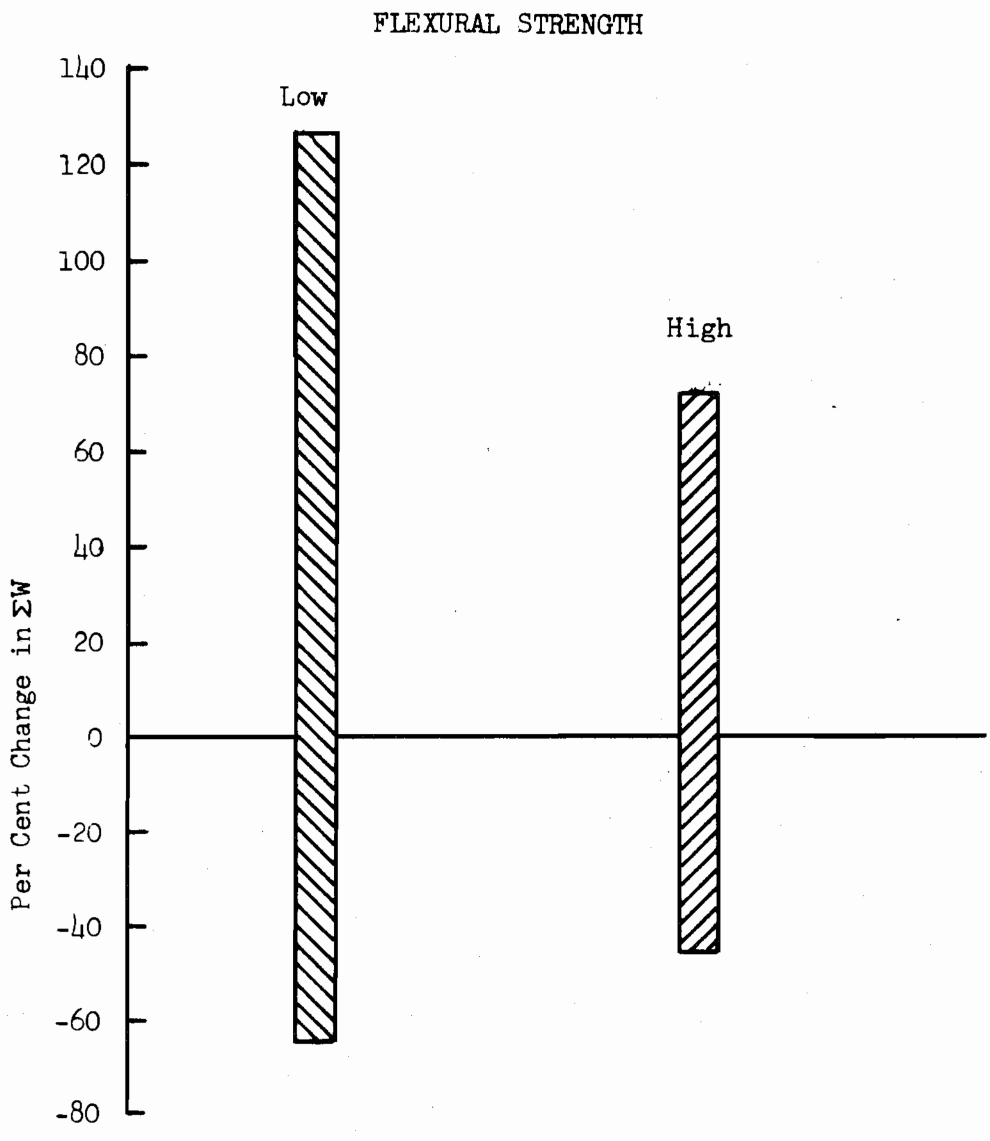
MODULUS OF ELASTICITY

Flexural Strength. The change in predicted accumulated traffic or pavement life is more severe for the lower level of strength or lightweight concrete (Figure 4). The change in life due to variation in flexural strength is independent of the concrete modulus of elasticity, thickness, modulus of subgrade reaction and slab continuity.

Thickness. The changes in pavement life due to variations in thickness are shown in Figure 5. The positive variations in thickness resulted in longer life while thinning the slab shortened the predicted life. The change in life which results from thickness variations is independent of the continuity and flexural strength but dependent on the concrete modulus of elasticity, modulus of subgrade reaction and thickness. The changes in life due to positive variations in thickness are greater per standard deviation than that due to negative changes (Figure 5).

Slab Continuity. Variations in slab continuity show that the change in expected pavement life is greater for continuously reinforced concrete pavement than for jointed concrete. The change in expected pavement life due to variations in continuity of the slab is independent of all factors that were evaluated at more than one level. In actual highway engineering practice this independence may be conjectured, but for the mathematical model used herein the independence does exist. Figure 6 shows the effects of variations in continuity on the change in expected pavement life for both jointed and continuously reinforced concrete pavement.

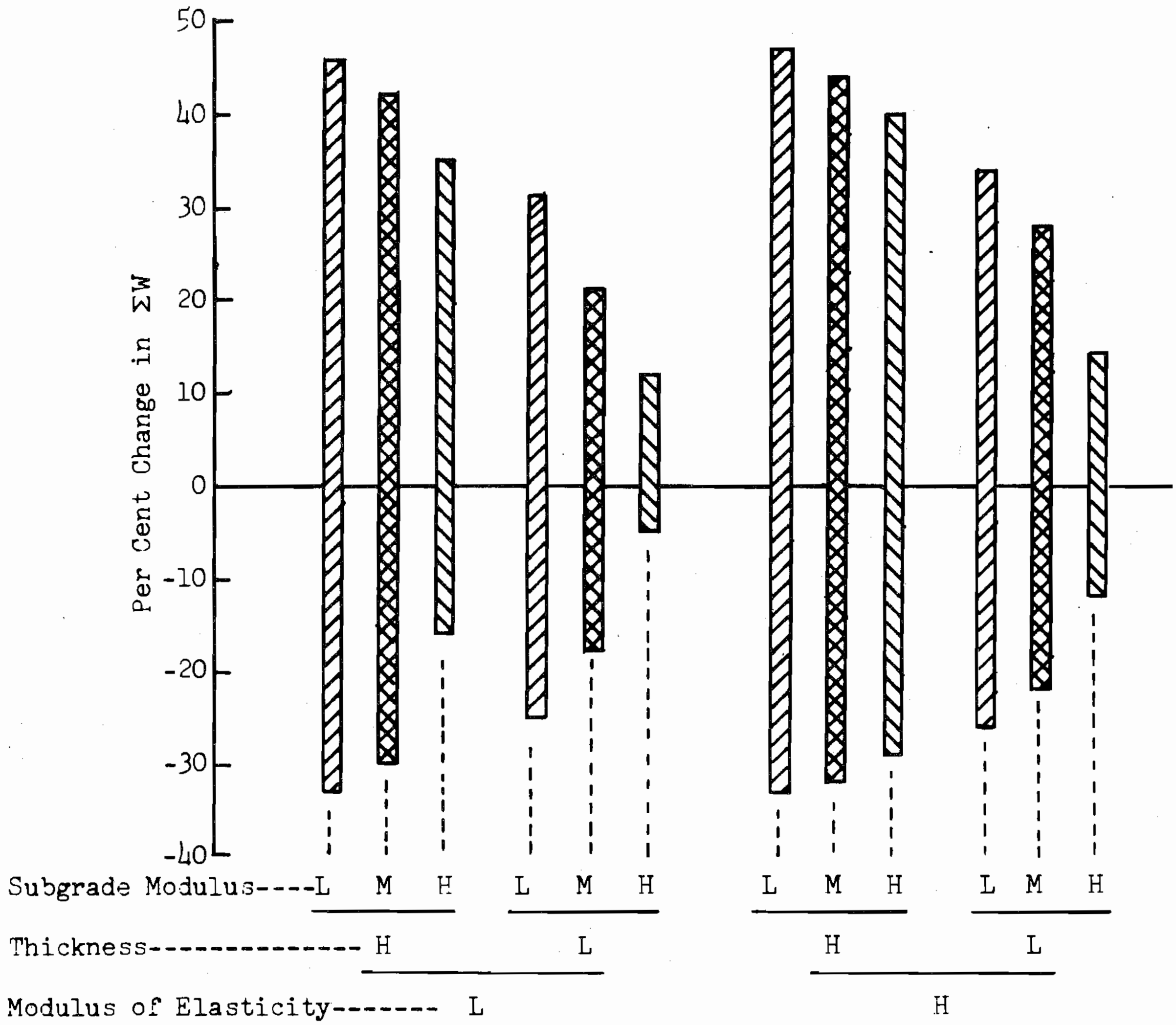
Terminal Serviceability Index. One level of terminal serviceability index, 3.0, was evaluated. The change in expected pavement life is dependent only on the level of thickness and is more severe at



EFFECT OF FLEXURAL STRENGTH VARIATIONS

ON PREDICTED LIFE

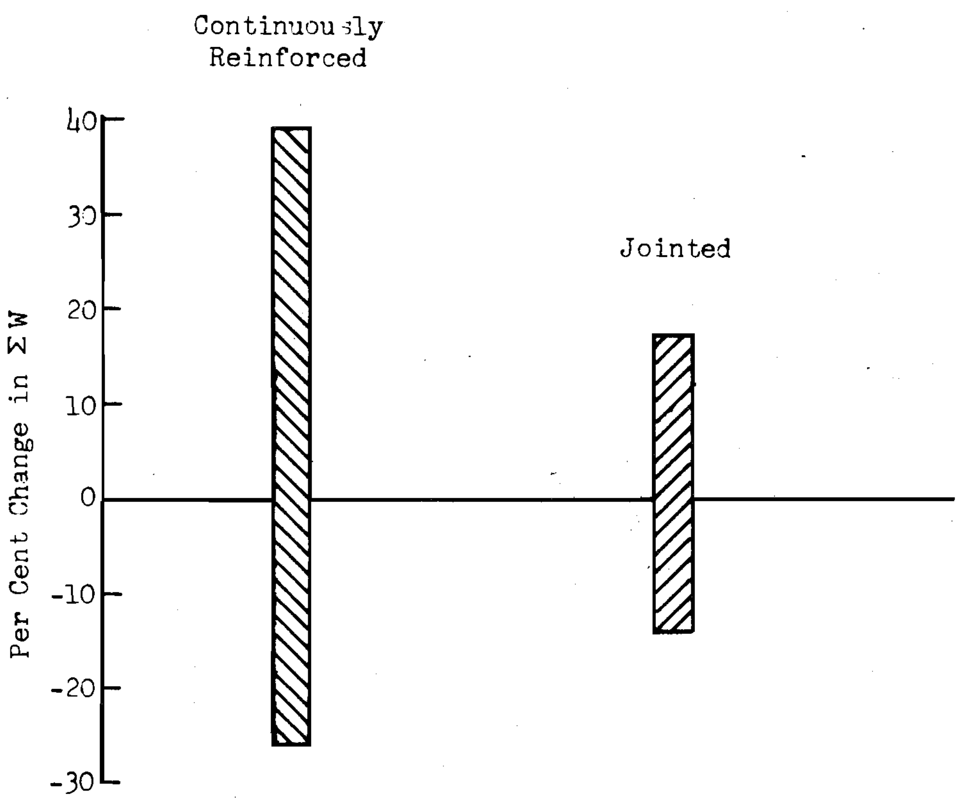
Figure 4



EFFECT OF THICKNESS VARIATIONS

ON PREDICTED LIFE

Figure 5



EFFECT OF SLAB CONTINUITY VARIATIONS

ON PREDICTED LIFE

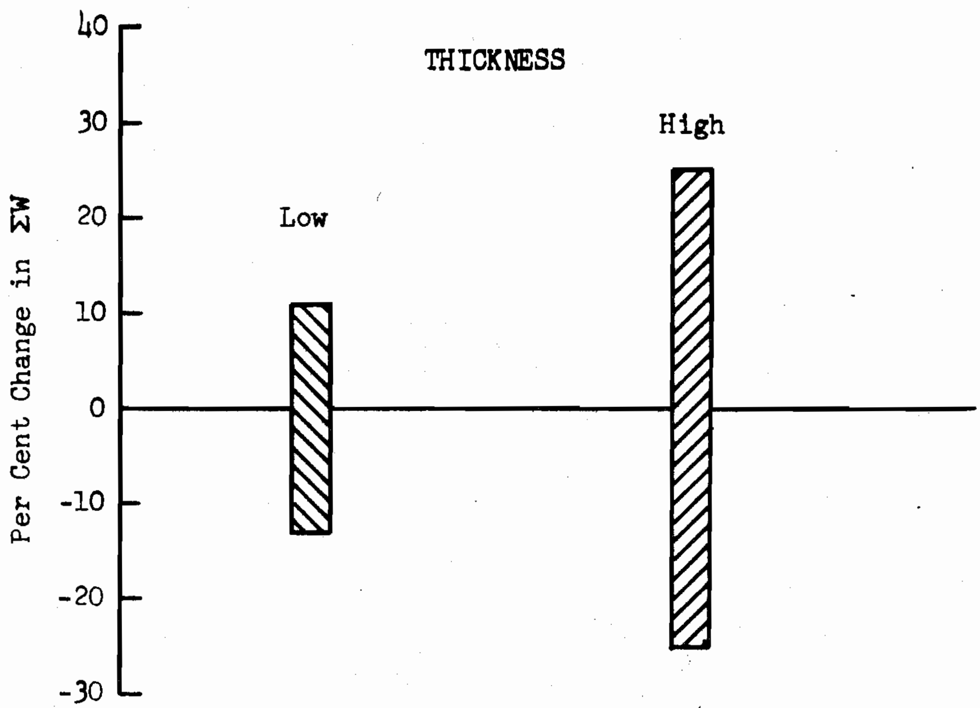
Figure 6

the higher level of thickness as related in Figure 7.

Initial Serviceability Index. All work in this experiment was for one level of the initial serviceability index, 4.2. It was found that change in expected pavement life due to variations in the initial serviceability index depends only on the level of thickness. The change in expected life is independent of all other factors in the inference as shown in Figure 8 where the change in life for variations in the initial serviceability index is greater for the high level of thickness.

Modulus of Subgrade Reaction. The changes in pavement life due to variations in k-value are dependent on the level of the variable, modulus of elasticity and thickness as shown in Figure 9. It is independent of the level of flexural strength or the slab continuity. For the two lower levels of the subgrade modulus, the positive and negative changes in expected life are approximately equal, while for the high level of the subgrade modulus they are not. The changes in expected pavement life due to variations in the modulus of subgrade reaction are much higher for the lower level of thickness than for the higher level of thickness.

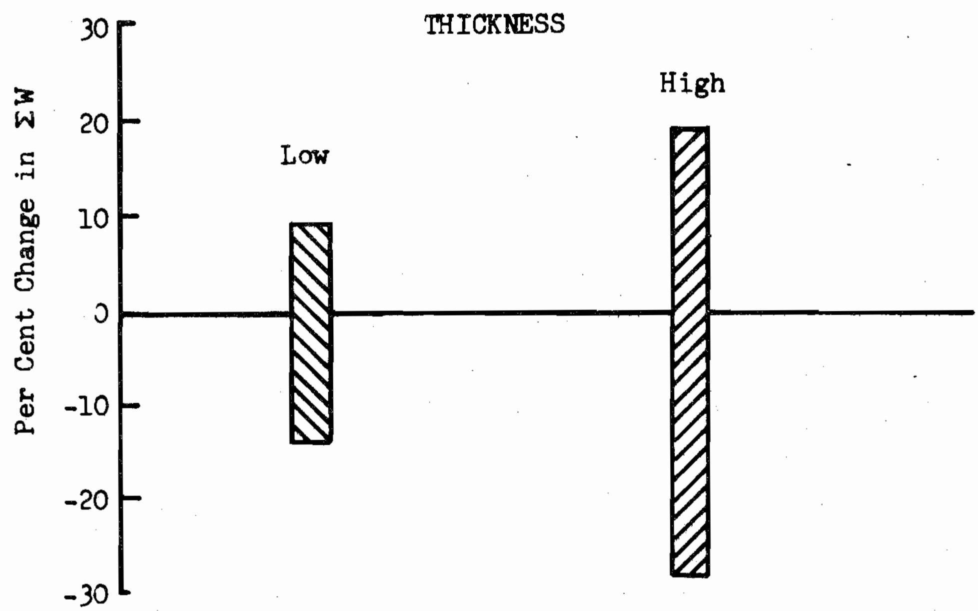
Modulus of Elasticity. The change in expected pavement life is positive for a decrease and negative for an increase in the modulus of elasticity (Figure 10). The magnitude of change in life is dependent on the level of the concrete modulus, the pavement thickness, and the modulus of subgrade reaction, but is independent of flexural strength and slab continuity.



EFFECT OF TERMINAL SERVICEABILITY INDEX

VARIATIONS ON PREDICTED LIFE

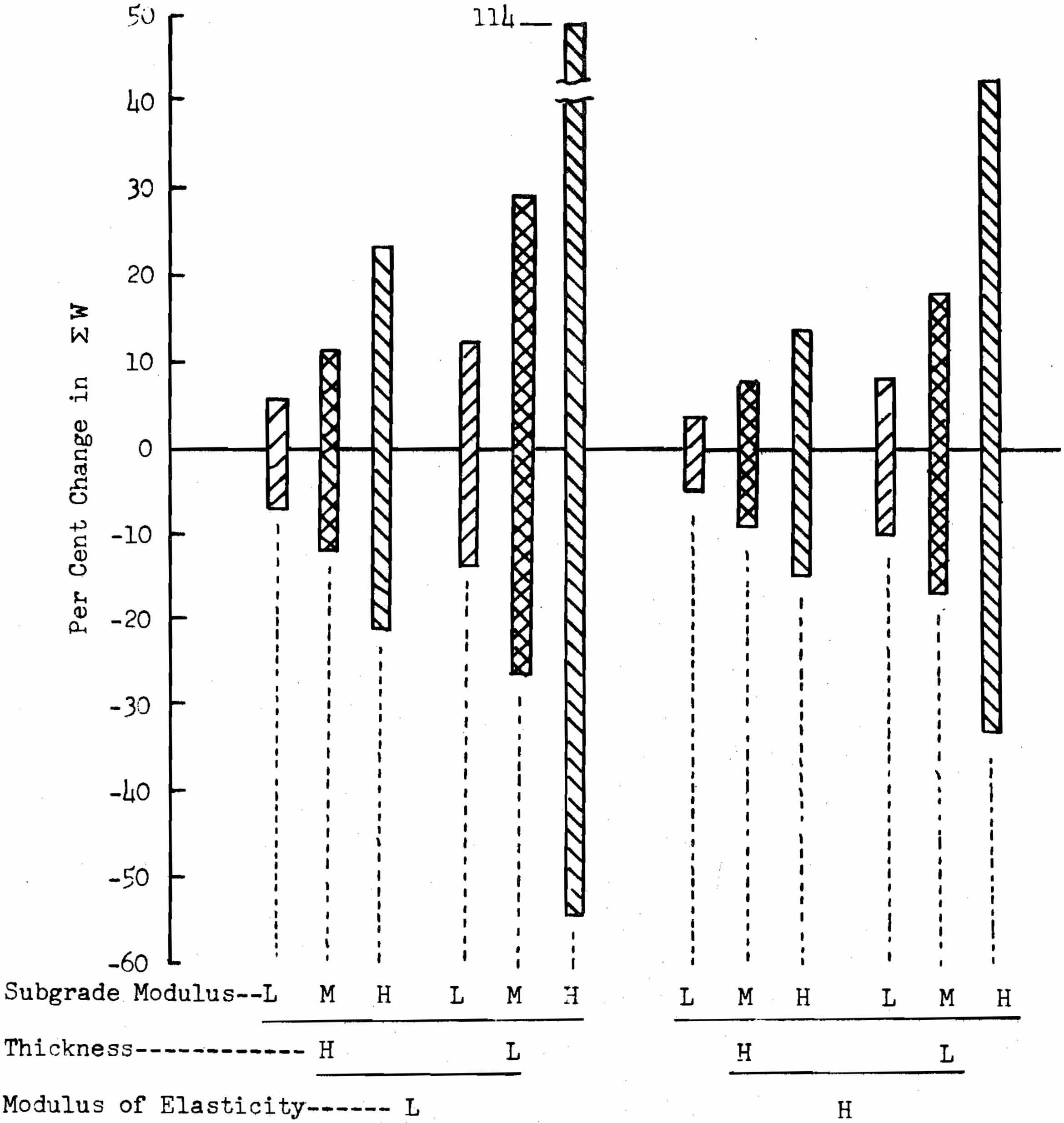
Figure 7



EFFECT OF INITIAL SERVICEABILITY INDEX

VARIATIONS ON PREDICTED LIFE

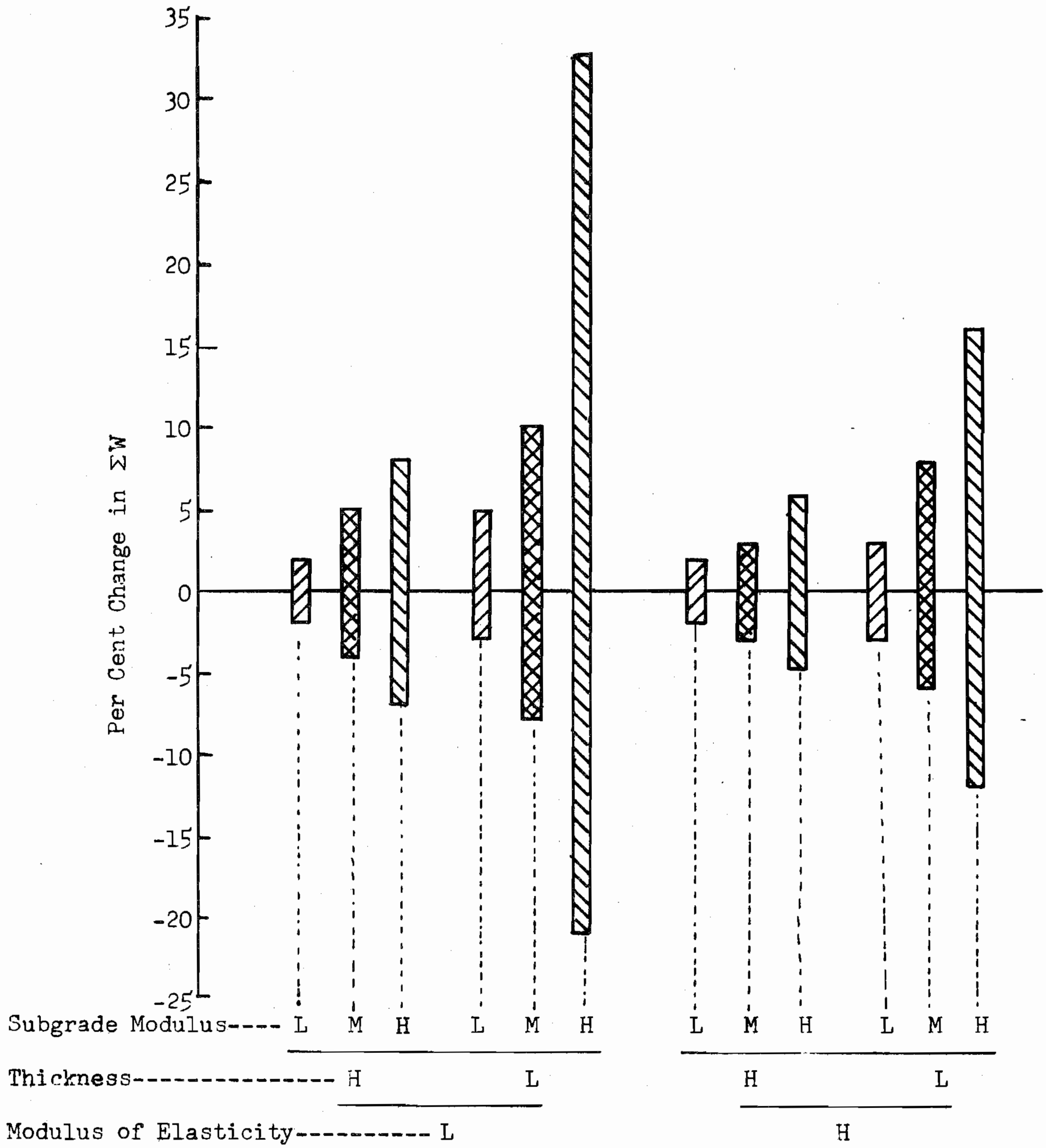
Figure 8



EFFECT OF MODULUS OF SUBGRADE REACTION

VARIATIONS ON PREDICTED LIFE

Figure 9



EFFECT OF MODULUS OF ELASTICITY

VARIATIONS ON PREDICTED LIFE

Figure 10

Discussion of Results

Examination of the data in Figure 3 indicates that the design equation (Eq. 1) predicts extremely high estimates of the total pavement life or accumulated traffic when combinations of variables occur at the high level. These results alone are not meaningful. The percent change in pavement life due to variations in the factors over this entire range is useful in evaluating the importance of factors. The use of a digital computer made it a relatively simple matter to evaluate the full factorial experiment.

The analysis of variance for these data has indicated eight two-factor interactions and five three-factor interactions to be significant. Thus variations in expected pavement life other than that due to the main factors was not random, but was due to a relationship between the factors. The analysis of variance thus indicates for the extended AASHO design equation for rigid pavement that the design variables are truly significant as well as the combinations or interactions cited in Table 5. The analysis of variance yielded essentially the same ordering on the design variables as did the analysis of changes in expected pavement life for factor variations.

According to AASHO Road Test results (Ref 15), the average absolute residual for the total expected traffic is about ten percent; thus, computed changes in the expected pavement life or total traffic of ten percent or less have little or no meaning.

As expected and as indicated by this study the two most significant factors in the extended AASHO rigid pavement design equation (Table 5 and Figure 4 and 5) are the flexural strength and the pavement thickness. The analysis of variance for the mean values of expected pavement life also showed the thickness and flexural strength to be very important (Table 5). These two are closely followed by the slab continuity term. Buick, in his "Analysis and Synthesis of Highway Pavement Design" (Ref 5) found that for the AASHO rigid pavement design method, the flexural strength was one of the two most important parameters. McCullough, et. al. (Ref 6) also found the effects of flexural strength highly significant. Thus it appears logical that variations in the flexural strength should also be highly significant in the extension of the AASHO rigid pavement design method. Under actual construction conditions the flexural strength of the concrete has a high probability of variation (Ref. 6). The magnitude of change in expected pavement life is not equal for equal magnitude positive and negative variations in the flexural strength (Figure 4).

The expected change in pavement service life due to overestimated or deficiences in thickness was over 20 percent in most cases (Figure 5). The study by McCullough, (Ref 6) also indicates the thickness to be a very significant factor. The chance for variation in thickness is always present during construction where a section might be built thicker or thinner than the desired plan dimensions. The magnitude of change in expected pavement life is not equal for equal magnitude positive and negative variations in thickness.

The slab continuity factor in this design equation does nothing more than designate the type of rigid pavement. In actual construction the continuity of continuously reinforced pavement would certainly depend on the percent longitudinal steel, concrete strength and probably several other parameters. The continuity of a jointed pavement would certainly depend on joint spacings and mechanical load transfer supplied. Thus the continuity or load transfer term might be somewhat misleading. The variations in the load transfer term show what might be expected in practice due to construction variations as well as variation in material properties.

Variations in the serviceability factors showed significant changes in the traffic service life. They were not as great as the flexural strength or the thickness, but greater than 10 percent, the average absolute residual on total traffic (Ref 15) required for significance. This design method indicates the change in life due to variations in the serviceability parameters to be dependent on thickness. Buick, (Ref 5) in his evaluation of relative practical importance which parallels this experiment, found the serviceability parameters to be less important than flexural strength which agrees with the findings herein.

The numerical evaluation of changes in expected pavement life in terms of percent change in total traffic shows that of the variables in the design equation, variations in the modulus of elasticity of the concrete have the least effect on the pavement service life within reasonable variations. Except for the high levels of modulus of subgrade reaction on the low level of thickness, all changes in expected life (Figure 10), positive or negative, are less than ten percent. This confirms McCullough's finding (Ref 6).

Another factor whose variations showed insignificant effects was the modulus of subgrade reaction. For the factorial of this experiment only the variations at the high level of the subgrade modulus created significant changes (10%) in the expected traffic service life. This corroborates the McCullough work.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This investigation was conducted to determine the sensitivity of pavement life as predicted by practical variations in the design variables. The conclusions are limited to the range of variables studied in this experiment and are of course only as good as the model being investigated. These findings however can provide reasonable information for use in design in selecting those variables which require the most intensive study effort, those which promise to yield the best improvements. This study illustrates a method which can be used to set priorities to upgrade any design model.

Based on the premise that the more important factors produce a greater change in the expected pavement life, the following conclusions are warranted.

1. The flexural strength and thickness are the most important factors whose negative variations may critically shorten pavement life.
2. The continuity factor was found to be the third most important factor.
3. The terminal and initial serviceability indexes were found to be important factors whose variations produce significant changes in the expected pavement life.
4. The modulus of subgrade reaction and the modulus of elasticity are the least important design factors whose variations do not significantly affect the expected pave-

ment life.

5. Significant interactions between the design factors do occur indicating that the variations in the pavement life are not completely defined by the design variables alone.

Furthermore this study shows those variables where tighter quality control restrictions are needed in order to improve life estimates.

Recommendations

Based on this investigation, it is recommended that:

1. Practicing highway engineers recognize that variations in the pavement variables are real and have a significant effect on pavement life and that better quality control of concrete flexural strength and thickness be incorporated to improve pavement performance.
2. Roughness specifications of new pavements should be updated to obtain smoother pavement and higher initial serviceability indices.
3. In future model development attention should be given to these variables in the priority listing presented.

APPENDIX 1

DATA FOR DEVELOPMENT OF STANDARD DEVIATIONS

FLEXURAL STRENGTH

High Level

	Standard Deviations
From Texas Highway Dept. (Ref. 9)	60
	46
	70
Hudson (Ref. 10)	45
From Schleider (Ref. 11)	82
	56

Average Standard Deviation = 60 psi.

Low Level

	Standard Deviations
From Schleider (Ref. 11)	67
	20

Average Standard Deviation = 45 psi.

MODULUS OF ELASTICITY

Low Level

Shafer* (Ref. 7)	Johnson (Ref. 8)
2.11×10^6	2.62×10^6
2.31×10^6	2.65×10^6
1.96×10^6	2.99×10^6
1.88×10^6	2.90×10^6
2.33×10^6	3.12×10^6
2.43×10^6	
2.26×10^6	
Mean = 2.183×10^6 psi	Mean = 2.856×10^6 psi
Standard Deviation = 0.194×10^6 psi	Standard Deviation = 0.217×10^6 psi

Average Standard Deviation for low level = 0.2×10^6 psi

High Level

Shafer* (Ref. 7)

4.19×10^6

5.81×10^6

5.21×10^6

5.84×10^6

5.68×10^6

5.99×10^6

Mean = 5.453×10^6 psi
Standard Deviation = 0.633×10^6 psi

Standard Deviation selected for high level = 0.6×10^6 psi

* Average of three values

THICKNESS

From Texas Highway Department (Ref. 9)

Thickness (in)	Standard Deviation	Coefficient of Variation
8 - CRCP	0.229	2.7
8 - CRCP	0.227	2.7
8 - CRCP	0.236	2.8
8 - JCP	0.215	2.6
9 - JCP	0.336	3.5
10 - JCP	0.298	2.9

Mean coefficient of variation = 2.75

Assume coefficient of variation constant for all thicknesses, thus the standard deviations for the low and high levels are:

D = 6 in σ = 0.165

D = 12 in σ = 0.330

MODULUS OF SUBGRADE REACTION

From AASHO Road Test (Ref. 10)

Outer Wheel Path	Inner Wheel Path
94	101
107	103
85	85
124	117
113	135
78	97
107	123
105	113
122	
Mean = 104	Mean = 109
Standard Deviation = 16.8	Standard Deviation = 15.9
Coefficient of Variation = 16.1%	Coefficient of Variation = 14.6%

Average coefficient of variation = 15%

Assume coefficient of variation constant for all values of modulus of subgrade reaction.

Thus for this experiment the standard deviations for each level of k-value are:

level	k-value	Standard Deviation
low	25	4
medium	200	30
high	1000	150

APPENDIX 2

DEVELOPMENT OF A NEW CONTINUITY COEFFICIENT FOR CONTINUOUSLY REINFORCED CONCRETE PAVEMENT

DEVELOPMENT OF A NEW CONTINUITY COEFFICIENT FOR CONTINUOUSLY REINFORCED CONCRETE PAVEMENT

The "J" term which is a coefficient dependent on load transfer characteristics or slab continuty came about in Hudson and McCullough's extension of the AASHO Rigid Guide. It was part of Spanglers stress equation which was used to modify the original AASHO equation (Ref. 17). The value of J for jointed pavement was 3.2. For continuously reinforced concrete pavement, Hudson and McCullough made J = 2.2 based on comparisons of previous design procedures and performance studies.

In 1968 McCullough and Treybig completed a comprehensive rigid pavement deflection study (Ref. 12) which included both continuously reinforced and jointed concrete pavements. This research provided data for revision of the value of J for continuously reinforced pavement. The new value of J is based on the relative differences in deflections of eight inch pavements.

For continuously reinforced concrete pavements, the difference in edge deflection was noted for measurements made at and midway between shrinkage cracks. The same type data were developed for jointed pavements with edge deflections measured at joints and midway between. The percent difference in each of the two respective measurements was used in a ratio to compute J for continuous pavement as

$$J_c = J_j \frac{A}{B}$$

Where:

J_c = Slab continuity coefficient for continuously reinforced pavement.

J_j = Slab continuity coefficient for jointed pavement = 3.2.

A = Percent difference in deflection for continuously reinforced pavement.

B = Percent difference in deflection for jointed pavement.

$$J_c = 3.20 \times \frac{12.87}{17.66}$$

$$J_c = 2.33$$

APPENDIX 3

COMPUTER PROGRAM LISTING

```
      DIMENSION  SIGMAL(400), ISX(400),  D(400), ME(400), KSUB(400),
     1PO(400),  TP(400),  TL(400),  SIGLG(400), CHECK(400), SICK(400)
C
C     SIGMAL = COMPUTED VALUE OF ACCUMULATED TRAFFIC
C     SIGLG  = LOG OF SIGMAL
C     ME     = MODULUS OF ELASTICITY OF CONCRETE
C     ISX    = FLEXURAL STRENGTH,(28 DAY, THIRD POINT LOADING)
C     KSUB   = MODULUS OF SUBGRADE REACTION
C     D      = SLAB THICKNESS
C     TL     = LOAD TRANSFER
C     PT     = TERMINAL SERVICEABILITY INDEX
C     PO     = INITIAL SERVICEABILITY INDEX
C
      NN = 0
  200 I = NN + 1
      READ 100, ISX(I), D(I),ME(I), KSUB(I), PO(I), TP(I), TL(I)
  100 FORMAT( I3, F4.2, I7, I4, F3.2, F3.2, F3.2 )
      IF(ISX(I).EQ.0 )   GO TO  211
      NN = NN + 1
      GO TO 200
  211 PRINT 111
  111 FORMAT(1H1)
      PRINT 103
  103 FORMAT( 6X, 'SX        D          E        K        J         PT        PO
     1      LOG L                   L' // )
C
C     COMPUTE SIGLG AND SIGMAL
C
      DO 310   I = 1, NN
C
C         CHECK FOR LOG OF NEGATIVE NUMBER
C
  400     CHECK(I)    =                         ((TL(I) /(ISX(I) * D(I) ** 2
     1                )) * (1 - ((2.604 * 7.15) /((ME(I) / KSUB(I)) **0.25
     2                * D(I) ** 0.75))))
      IF (CHECK(I).LT.0.0 )      GO TO 213
  410     SICK(I) = ((PO(I) - TP(I)) / (PO(I) - 1.5 ))
      IF( SICK(I).LT.0.0 )       GO TO 214
      SIGLG(I)   =    - 9.483 - (3.837* ALOG10((TL(I) /(ISX(I) * D(I) ** 2
     1                )) * (1 - ((2.604 * 7.15) /((ME(I) / KSUB(I)) **0.25
     2                * D(I) ** 0.75)))))) + (ALOG10((PO(I) - TP(I)) / (PO(
     3                I) - 1.5 ))) / (1 + (1.624 * 10**7 / ((D(I) + 1) **
     4                8.46)))
      SIGMAL(I)   =   10**SIGLG(I)
C
C          PRINT DATA SET
C
  212 PRINT 104,ISX(I), D(I),ME(I), KSUB(I), TL(I), TP(I), PO(I),
```

```
      1SIGLG(I), SIGMAL(I)
104 FORMAT( 5X, I3, 4X, F5.2, 4X, I7, 3X,I4, 4X, F4.2, 4X, F4.2, 4X,
      1F4.2, 4X, F8.6, 4X,F12.0/ )
       GO TO 310
213 PRINT 105,ISX(I), D(I),ME(I), KSUB(I), TL(I), TP(I), PO(I),
      1CHECK(I)
105 FORMAT(//5X,I3, 4X, F5.2, 4X, I7, 3X,I4, 4X, F4.2, 4X, F4.2, 4X,
      1F4.2,4X, 'CHECK(I) =   ' F10.6 //)
       GO TO  410
214 PRINT 106,ISX(I), D(I),ME(I), KSUB(I), TL(I), TP(I), PO(I),
      1SICK(I)
106 FORMAT(//5X,I3, 4X, F5.2, 4X, I7, 3X,I4, 4X, F4.2, 4X, F4.2, 4X,
      1F4.2, 4X, 'SICK(I) =   ' F10.6 //)
310 CONTINUE
       END
```

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