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DEPARTMENTAL RESEARCH

Report Number: 49 - 1

**CONSTRUCTION CONTROL
PROFILOGRAPH PRINCIPLES**

TEXAS HIGHWAY DEPARTMENT



CONSTRUCTION CONTROL PROFILOGRAPH PRINCIPLES

by

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Research Report 49-1

Development of a Construction Control Profilograph
Research Study 1-8-63-49



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In Cooperation With The
U. S. Department of Transportation
Federal Highway Administration
Bureau of Public Roads

June 1967

The opinions, findings, and conclusions expressed in this publication are those of the author and not necessarily those of the Bureau of Public Roads.

A C K N O W L E D G M E N T S

The research and development reported in this paper was performed by the Research Section of the Highway Design Division in cooperation with the U. S. Bureau of Public Roads. The work was under the supervision of Mr. R. L. Lewis, Research Engineer, and under the general direction of Mr. T. S. Huff, Chief Engineer of Highway Design.

Acknowledgment is given to Messrs. Don and Larry Hart of the Rainhart Corporation and particularly to Mr. Edward R. Hamilton for the design and construction of the model used in this study.

Appreciation is expressed to Mr. Harold Hans, Mr. Garlon Lawrence and Mr. Hugo Orellana for the collection and analysis of the data which made this report possible.

TABLE OF CONTENTS

	Page
LIST OF FIGURES	ii
ABSTRACTiii
BACKGROUND	1
OBJECT	4
DESCRIPTION OF EQUIPMENT	5
METHOD OF ANALYSIS	7
Model Studies	7
Number of Wheels	7
Wheel Placement	8
Output	9
Theoretical Studies	9
G.M. Equation	9
RESULTS OF ANALYSIS	18
Model Studies	18
Wheel Placement	18
Output	22
Theoretical Studies	22
G.M. Equation	22
Profilograph Length	25
SELECTION OF PROFILOGRAPH EQUIPMENT	25
CONCLUSIONS	32
BIBLIOGRAPHY	33
APPENDIX	34

LIST OF FIGURES

Figure		Page
1	Schematic of a Construction Control Profilograph	3
2	Harmonic Analyser	6
3	Oblique View	6
4	Model Study	10
5	Model Study	11
6	Model Study	12
7	Model Study	13
8	Component Study of a 28' Overall Length Profilograph	14
9	Component Study of a 28' Overall Length Profilograph	15
10	Component Study of a 42' Overall Length Profilograph	16
11	Study of Number of Averaging Wheels.	19
12	Study of Wheel Position	20
13	Study of Overall Length	21
14	Correlation of Harmonic Analyzer and G.M. Equation	24
15	Rainhart Profilograph In Road Position	27
16	Front View of the Rainhart Profilograph.	27
17	Outrigger Wheel Assembly	30
18	Profilograph Recording Mechanism	30

A B S T R A C T

In 1963, the Texas Highway Department in cooperation with the Bureau of Public Roads initiated a research project to develop a profilograph with which to control pavement surface roughness during construction operations.

To determine the desirable qualities of such an instrument, studies were made of the effect of the number of averaging wheels, the arrangement of the averaging wheels and framework, and the length of the instrument. Studies have indicated that the number of averaging wheels affects the accuracy or "readability" of the output results. Better results are produced if the averaging wheels are spaced evenly along the length of the instrument. The selection of the length of the equipment depends upon the magnitude of the length of the surface irregularities to be experienced. The studies were made using a mechanical system analyzer and confirmed using theoretical methods.

Based on the results of this study, a profilograph was leased from the Rainhart Company of Austin, Texas. This instrument incorporated the features that were found desirable in the system analyzer and theoretical studies.

REPORT ON
CONSTRUCTION CONTROL PROFILOGRAPH PRINCIPLES

Background

Almost every state uses some method of determining and correcting road roughness during construction, especially on concrete pavement. General Motors Proving Ground personnel⁽¹⁾ state that most agencies use a ten-foot straight edge with some permissible variation in the vertical direction. However, several agencies employ a "profilograph". There are profilographs capable of measuring almost any profile, from a profile of surface texture⁽²⁾ to a profile similar to an elevation profile found on construction plans⁽³⁾. A very interesting history of the development of devices to measure pavement roughness is recorded by Mr. F. N. Hveem⁽⁴⁾ which records instruments dated before 1900 through those of the present day.

At least two agencies have used a profilograph for construction control and one other has used a profilograph to measure the condition of existing roadways. These profilographs are of similar construction and since the profile interest lies in the roughness range which is perceived by vehicle operators, this equipment type is

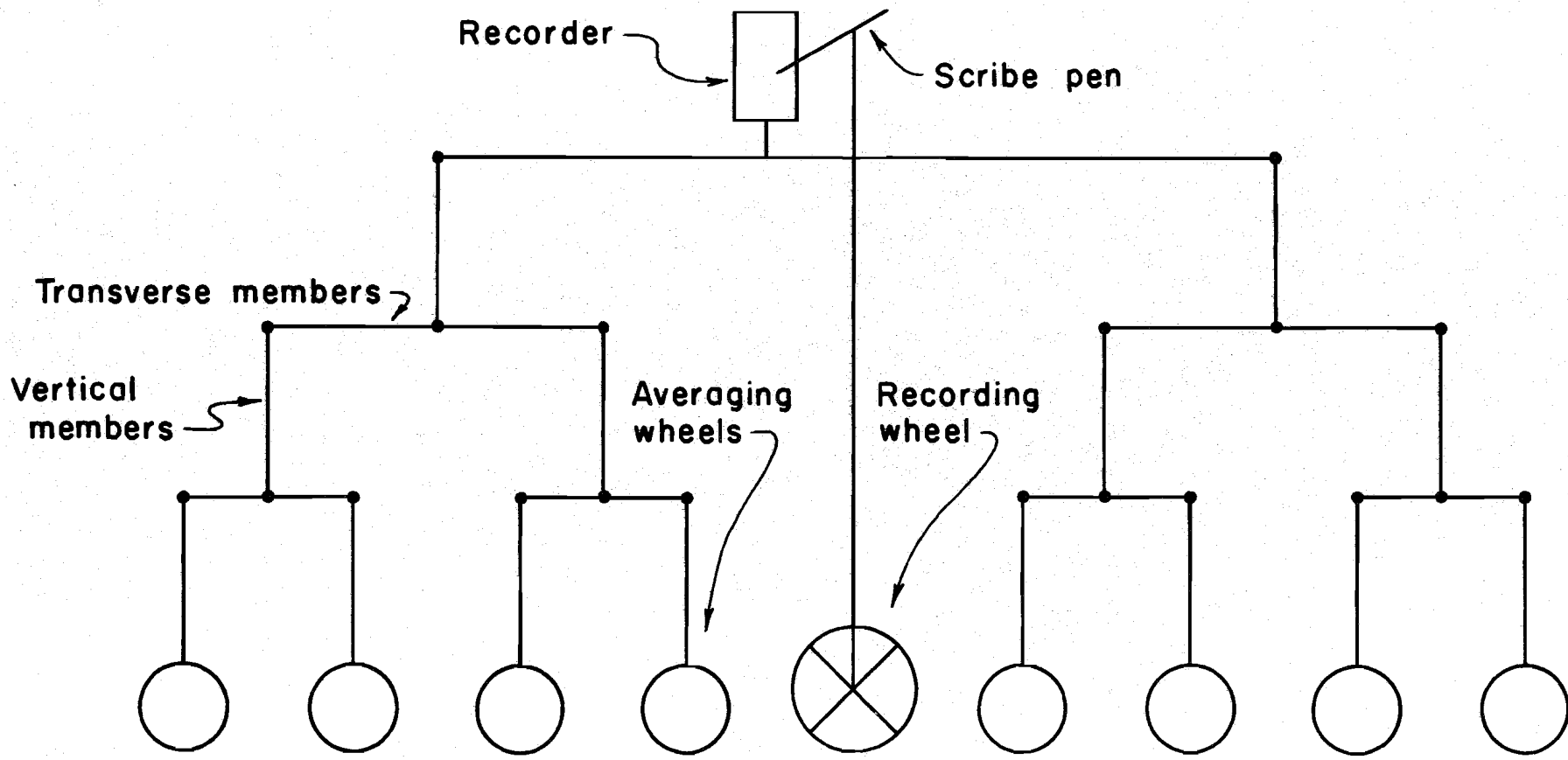
sensitive to a surface wave length range from near a pebble size to somewhere around 20 to 30 feet depending upon the length of equipment.

The specifications concerning roughness using the profilograph have been used in the following manner⁽⁴⁾:

1. A maximum allowable height of a single peak (bump or depression) in a certain prescribed distance.
2. A maximum allowable quantity of cumulative peaks per prescribed distance expressed in inches per mile.

The first specification is similar to the Texas specifications (360.8(3)), Surface Test and (340.4(11)), Straight Edges and Templates, which uses the straight edge on the pavement surface. The second deals with roughness as experienced by the continual vertical movement of the vehicle wheels.

Many of the roughness measuring profilographs are pushed manually (a few instruments are powerized but manual operation is generally used in the interest of economy) and consist of a framework generally in a whiffletree arrangement with several averaging wheels, a recording wheel, and a mechanical recorder (see Figure 1). The recording wheel and the averaging wheels rest on the pavement surface.



SCHEMATIC OF A CONSTRUCTION CONTROL PROFILOGRAPH

FIGURE 1

The averaging wheels provide a reference plane which supports the recorder and the recording wheel moves relative to the reference plane as it travels over the surface irregularities and gives a simulation of the profile. The whiffle-tree arrangement allows the averaging wheels to "average out" the same surface irregularities met by the recording wheel in such a manner that these vertical irregularities are sharply reduced at the recording instrument. This small vertical movement of the recording instrument allows the recorder wheel to feed to the recorder a relatively accurate road profile.

Theoretically, the recorder should be at some pre-selected distance above the pavement surface and be maintained at this distance, operating parallel to the theoretical pavement alignment. Herein lies the real problem with the construction control profilograph -- the recorder position depends entirely on the framework arrangement and averaging wheels.

Object

The object of this report is to study the effect of various design components on the profilograph output. This study was made by using both a mechanical analyzer and theoretical model analysis.

Description of Equipment

The equipment consisted of a "Harmonic Analyzer" developed by the Rainhart Corporation, Austin, Texas (see Figures 2 and 3). The Harmonic Analyzer was developed as a rapid method of studying the effect of pavement wave motion on a construction control profilograph. It consists of a 3/4 inch plywood member in which 3/8 inch deep grooves have been sawed parallel at equal intervals. The lower edge has been fixed with a member which supports a "pavement template". In Figure 2, eight thin wooden vertical members are inserted in the grooves and allowed to slide in a vertical direction. The contact of these and the top of the template represent the averaging wheels. Four lateral members are placed above the vertical members and allowed to slide along the plywood face to form a whiffletree arrangement. Four vertical members and two transverse members are then placed as illustrated. This procedure is repeated until a transverse member is placed above the remaining two vertical members. Figures 2 and 3 illustrate a symmetrical eight averaging wheel configuration. The road profile template has a frequency of slightly over half of the profilograph length. As the template is moved laterally, the pieces are kept in contact by applying a small force downward to the top center of the pyramid.

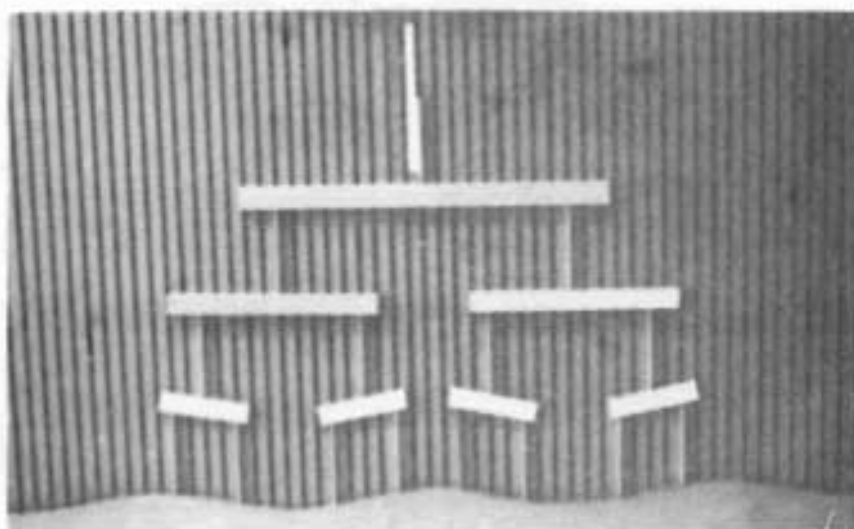


Figure 2
Harmonic Analyser

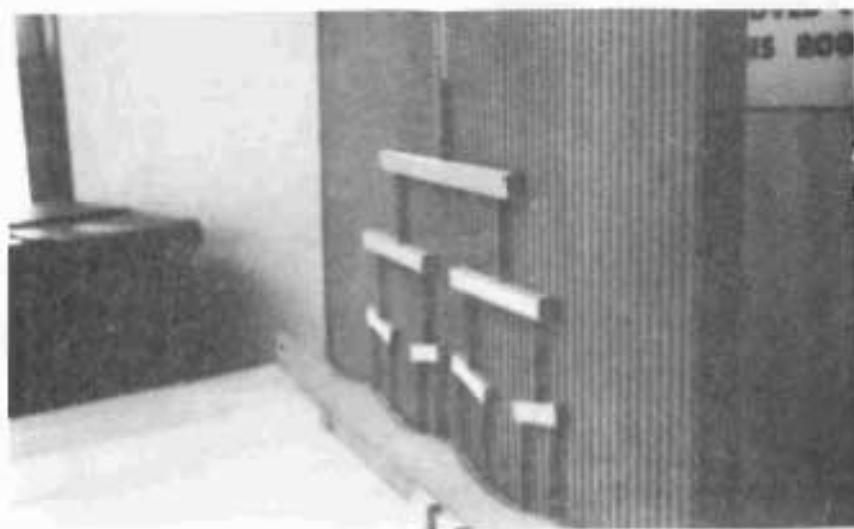


Figure 3
Oblique View

In practice, the top center (apex) is the location of the recorder chassis. The amount of travel of the apex (recorder) represents the basic error of the system.

If the model is completed with a simulated sensing wheel (tracer) and pen, an actual trace can be made on a mounted strip of paper. The paper is moved laterally in synchronism with the template and the pen is moved vertically by the apex. The template (input signal) can then be superimposed on the trace (output) for accurate direction comparison in evaluation of the accuracy of recording.

Method of Analysis

Model Studies:

Number of Wheels. A very quick analysis of the effect of the number of wheels is available if we number the lower vertical members in Figures 2 and 3, A1 through A8, from left to right, the second set B1 through B4, the third set C1 and C2 and the top vertical member D1. If it is assumed that all lower vertical members rest on a flat plane, the transverse members will all be horizontal. Then if A1 is raised one inch by a bump and A2 through A8 are not raised, B1 will be raised 1/2 inch, C1 will be raised

1/4 inch, and D1 will be raised 1/8 inch. This means that the recorder which is attached at D1 will record 1/8 inch movement, when theoretically the recorder should record a flat surface. Ideally the recorder should record a flat surface until the recording wheel reaches the one inch bump and then one inch vertical movement should be recorded. Since the undesirable record was a division of eight and eight wheels are used, it is evident that with more averaging wheels less undesirable vertical movement will occur at the recorder.

In the above response, due to the two dimensional construction of the analyzer, the assumption is made that the trace will have four peaks of 1/8 inch height, one of the full one inch height, and four more of 1/8 inch height. This occurs as the averaging wheels and the recording wheel pass progressively over the one inch bump. This action suggests that in actual construction operation, seven of the small peaks could possibly be eliminated if all averaging wheel paths were laterally separated.

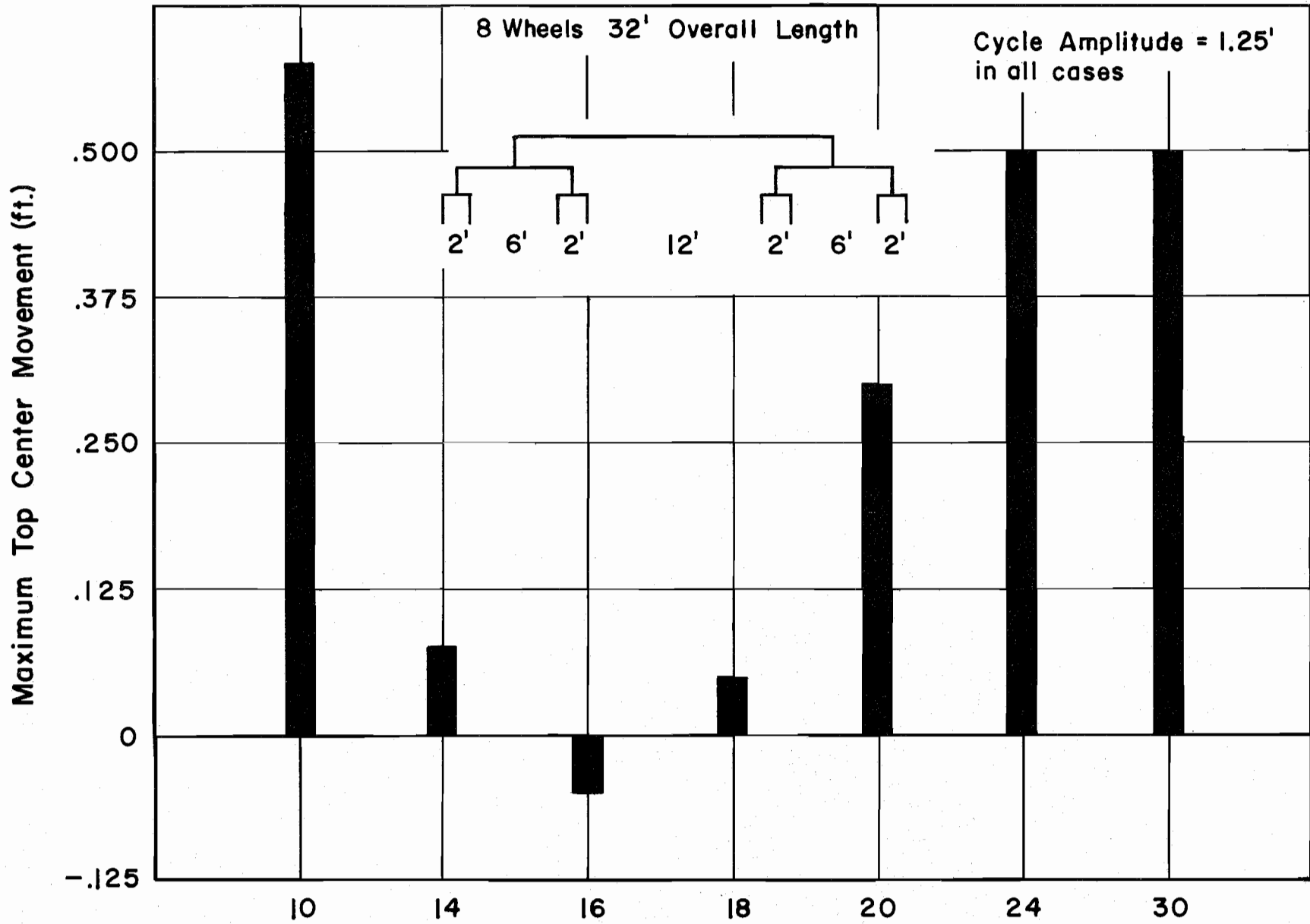
Wheel Placement. Various "pavement templates" were used in a wheel placement study in which the templates conformed to various cycle lengths. The maximum top center movement (or undesirable reference point movement) was

measured for each of the various cycle lengths. In all cases, the cycle amplitude was constant for each of the cycle lengths. Four profilograph mock-ups were used in which two odd wheel spacings and two even wheel spacings were studied. Assuming the distance between each of the equally spaced sawed grooves to be one foot, the information in Figures 4, 5, 6, and 7 was developed.

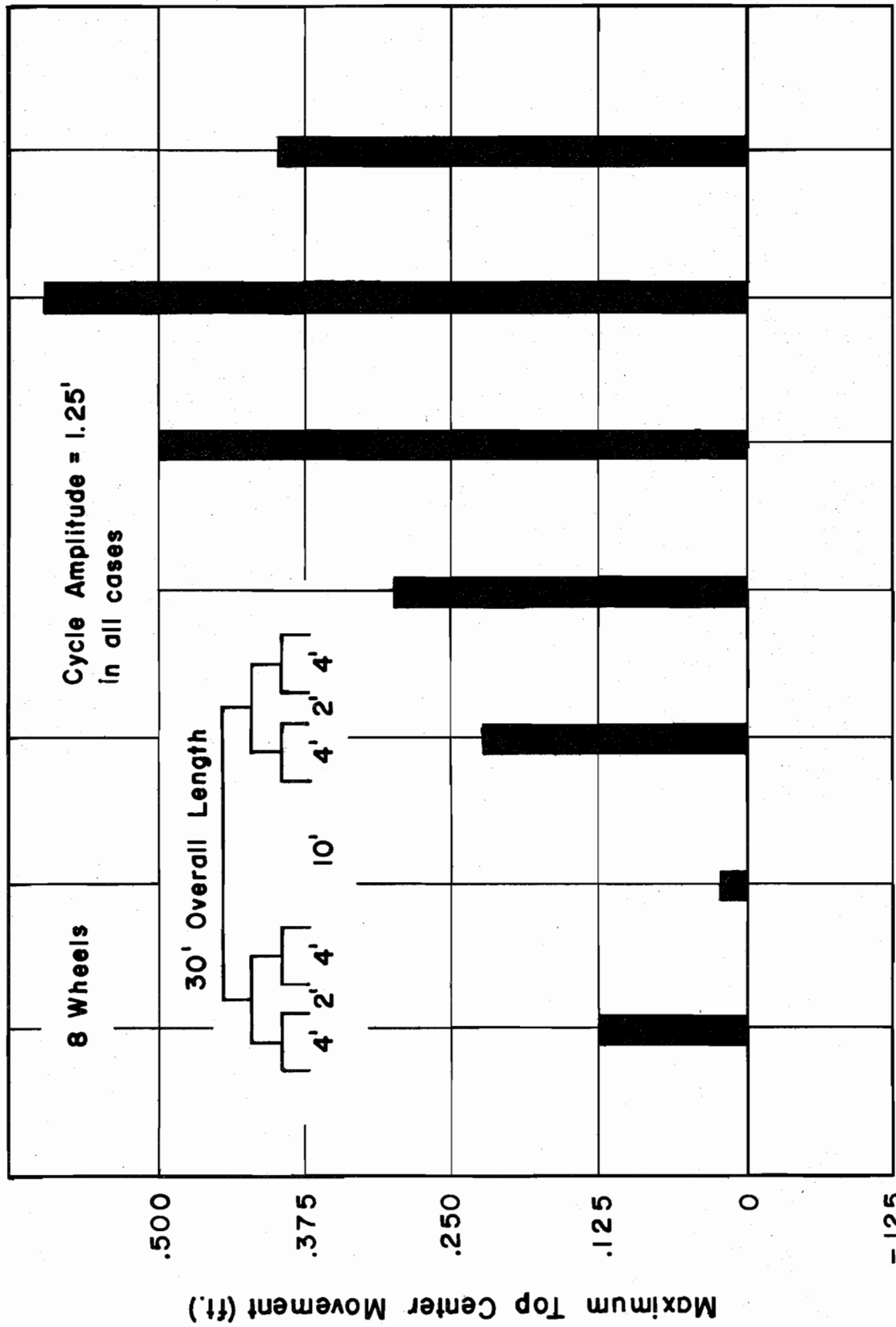
Output. Further study developed in which the actual profile of the "pavement template", the profile produced by the mock-up profilograph, and the maximum top center movement was recorded at selected intervals as the "pavement template" was pushed under the mock-up. Figures 8, 9, and 10 indicate the results produced for three different profilograph mock-ups.

Theoretical Studies

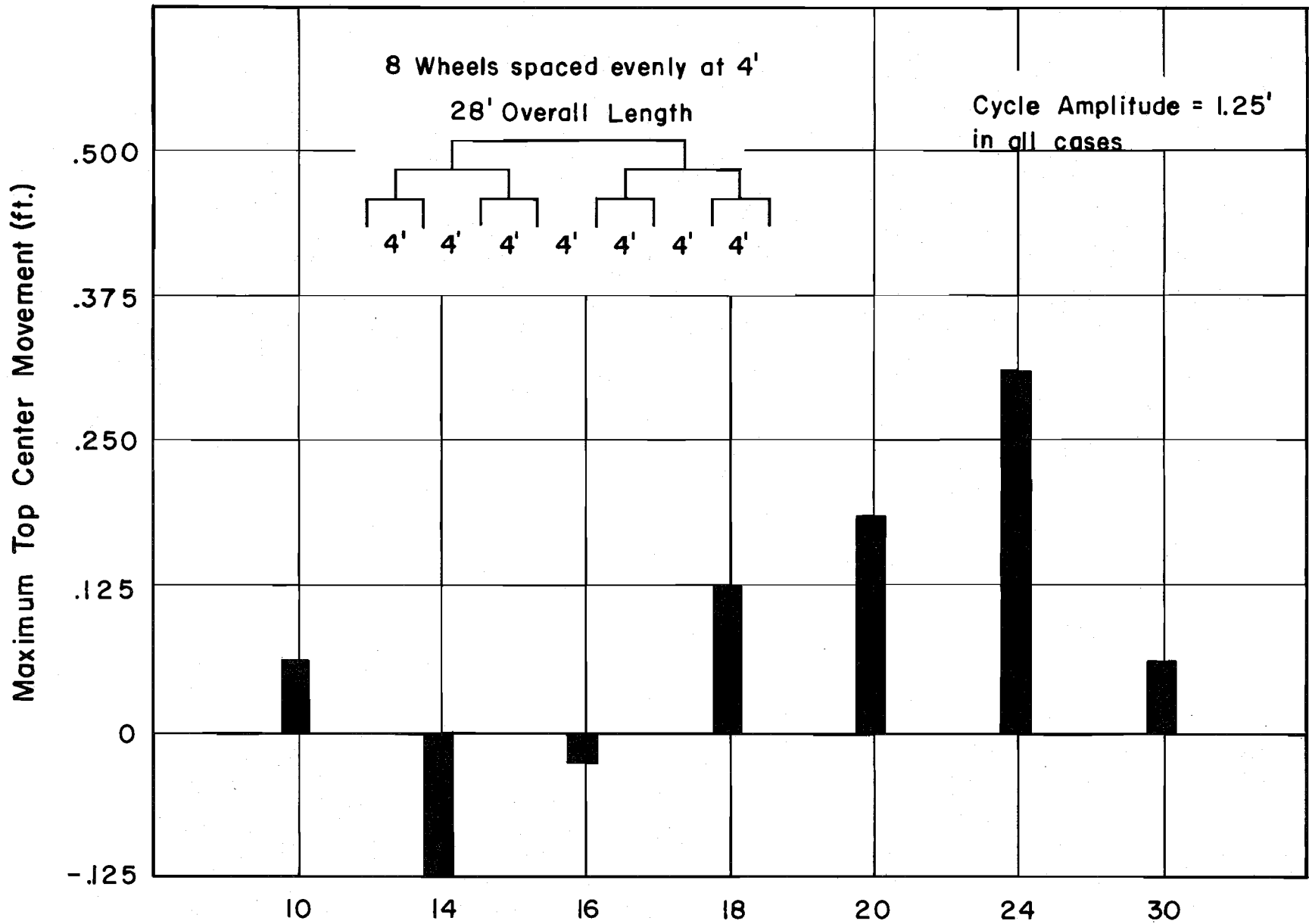
G.M. Equation. General Motors Proving Ground⁽¹⁾ made a complete study of the profilograph which they developed for construction control of a new circular test track. In one part of the report, two equations were developed which were studies of various theoretical wave lengths. The wave lengths were compared to a constant. This constant represented the amount the measured profile would deviate from the true



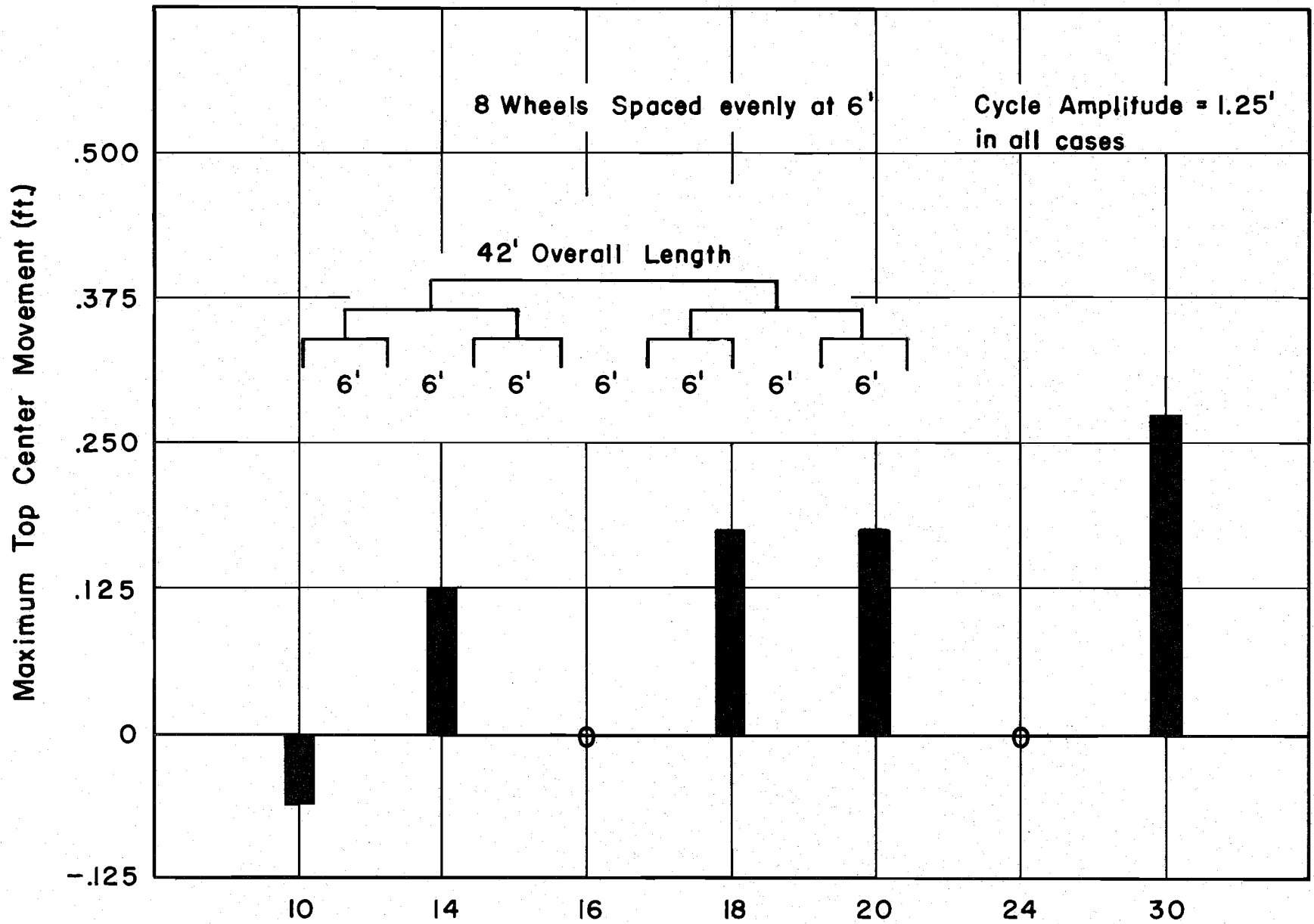
Cycle Length
 MODEL STUDY
 FIGURE 4



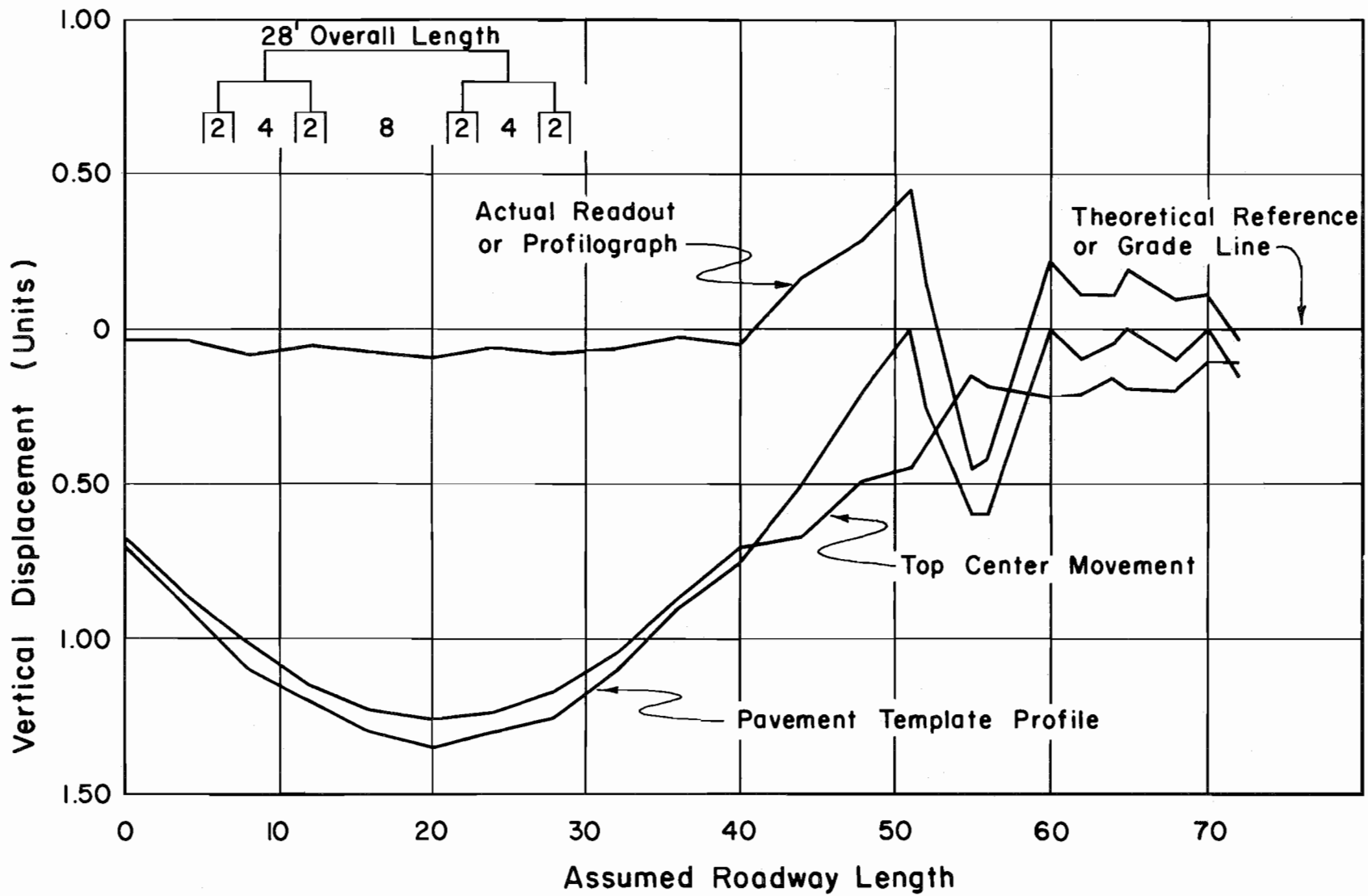
Cycle Length
MODEL STUDY
FIGURE 5



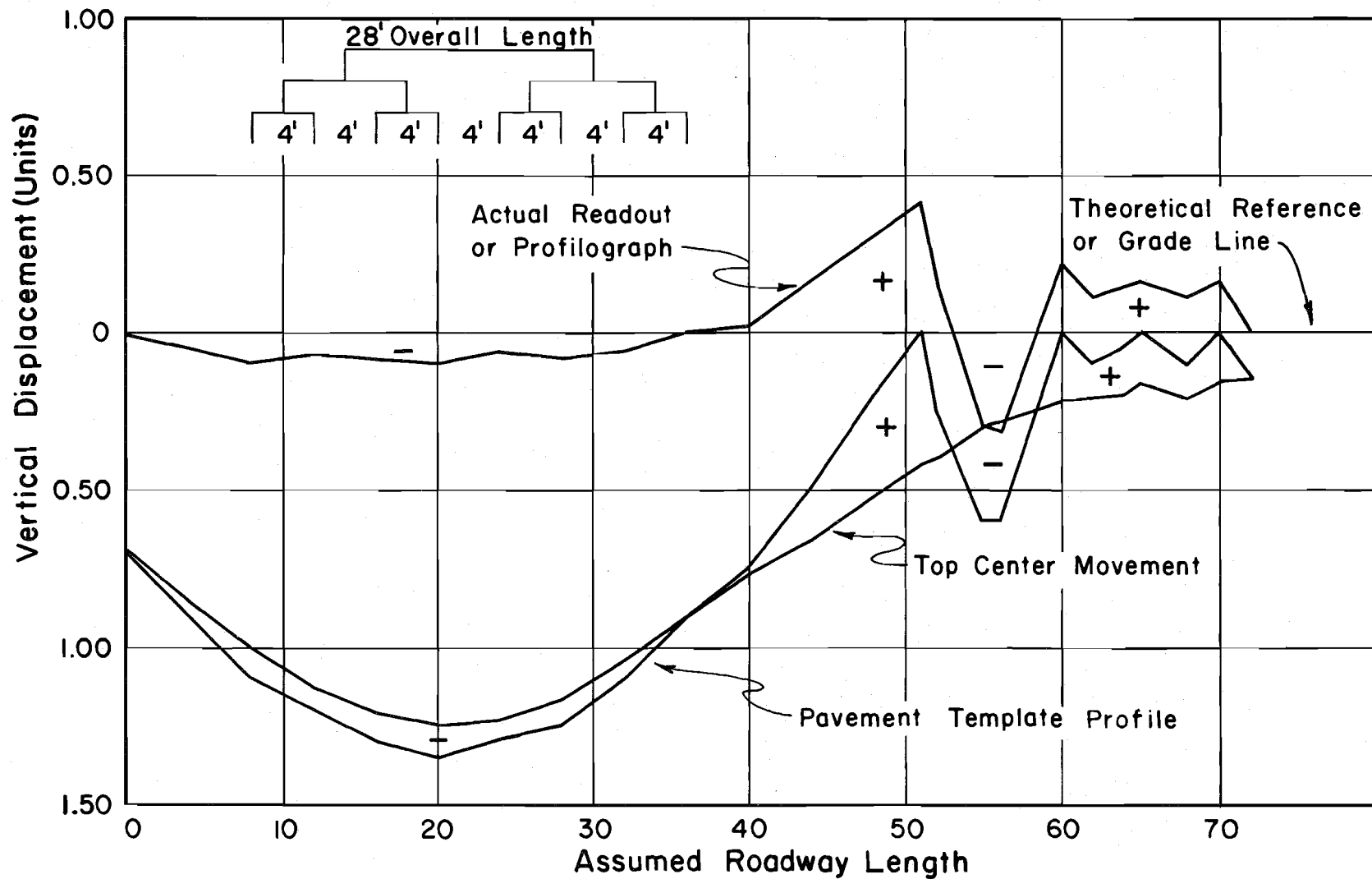
MODEL STUDY
FIGURE 6



MODEL STUDY
FIGURE 7

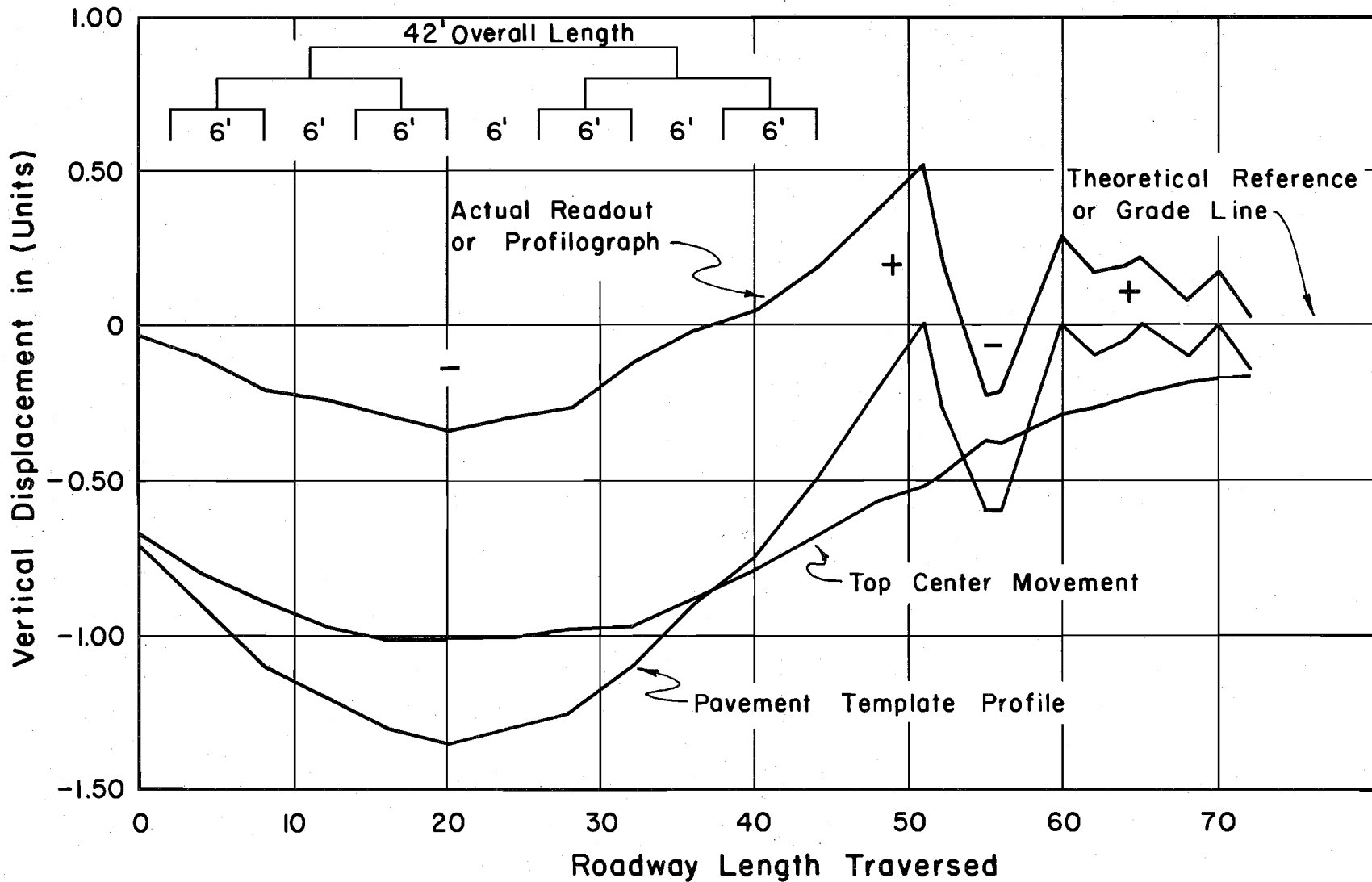


COMPONENT STUDY OF A 28' OVERALL LENGTH PROFILOGRAPH
 FIGURE 8



COMPONENT STUDY OF A 28' OVERALL LENGTH PROFILOGRAPH

FIGURE 9



COMPONENT STUDY OF A 42' OVERALL LENGTH PROFILOGRAPH
 FIGURE 10

profile given each surface as a constant wave length and constant amplitude. The equations are as follows:

$$1. A_1 = A(K)$$
$$2. K = 1 - \cos \frac{25\pi}{L} \left(\frac{\cos \frac{7.5\pi}{L} + \cos \frac{2.5\pi}{L} + \cos \frac{1.25\pi}{L}}{3} \right)$$

Where:

A_1 = measured amplitude of the wave

A = actual amplitude of the wave

L = wave length

K = constant

25 = the profilograph length and 7.5, 2.5 and 1.25 conform to averaging wheel spacings

It can be noted that as "K" approaches one, the closer the measured amplitude approaches the actual amplitude.

It was decided that several profilograph lengths would be studied using several wheel positions. Three wheel arrangements of 12 wheels odd spacing, 12 wheels even spacing, and eight wheels even spacing were studied. The various variables were substituted in the General Motor's equation and the results obtained with the use of a CDC-1604-A computer. The various odd wheel spacings used in this study are proportional to the 25-foot and 12 wheel odd spacing used in the General Motor's profilograph.

Figure 11 indicates a study of the number of averaging wheels and Figure 12 indicates the study of how the wheels should be spaced (odd or even). Figure 13 reveals the influence of differences in overall profilograph length.

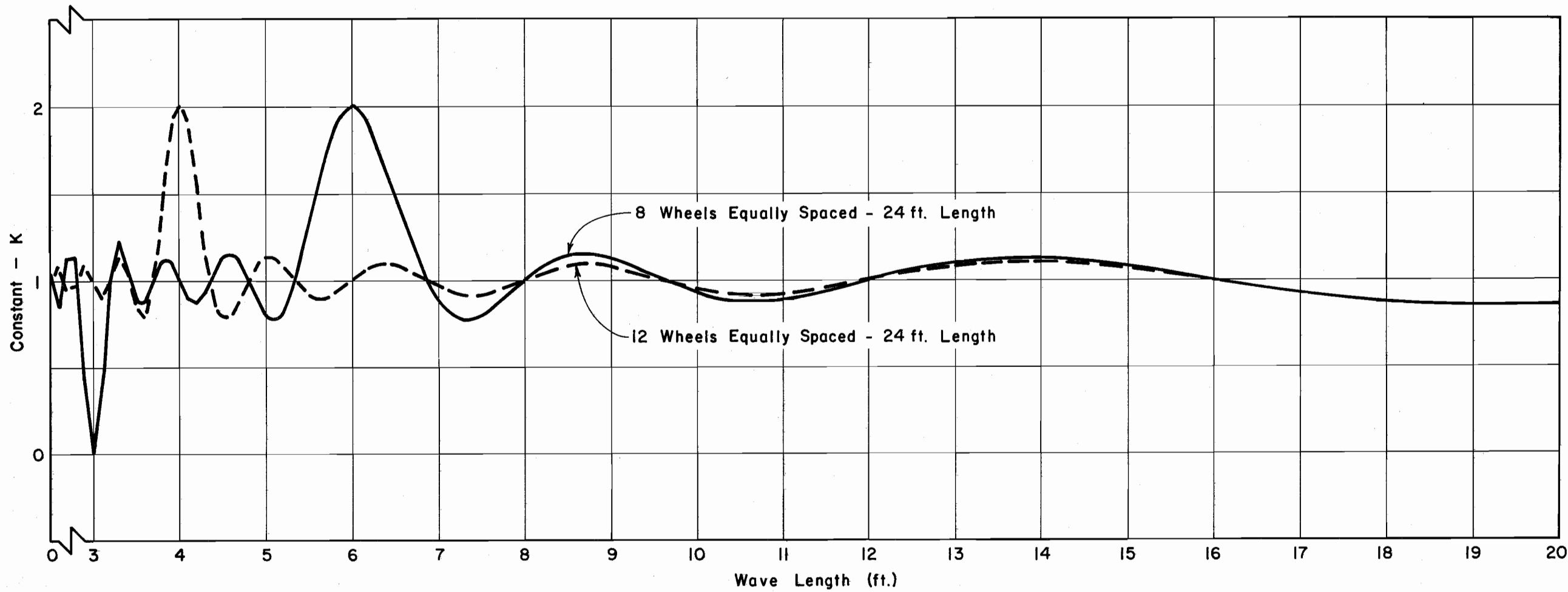
Results of Analysis

Model Studies:

Wheel Placement. Figure 4 presents the results of a 2-6-2-12-2-6-2 wheel spacing profilograph with an overall length of 32 feet. The maximum top center movements form a cyclic pattern when studied with various cycle lengths and the largest maximum top center movement occurred at a cycle length of 10 feet. (Movement of the top center represents movement of the reference plane, hence, undesirable movement.) The maximum reading was 0.575 foot assuming cycle length is in feet and the cycle amplitude equal to 1.25 feet.

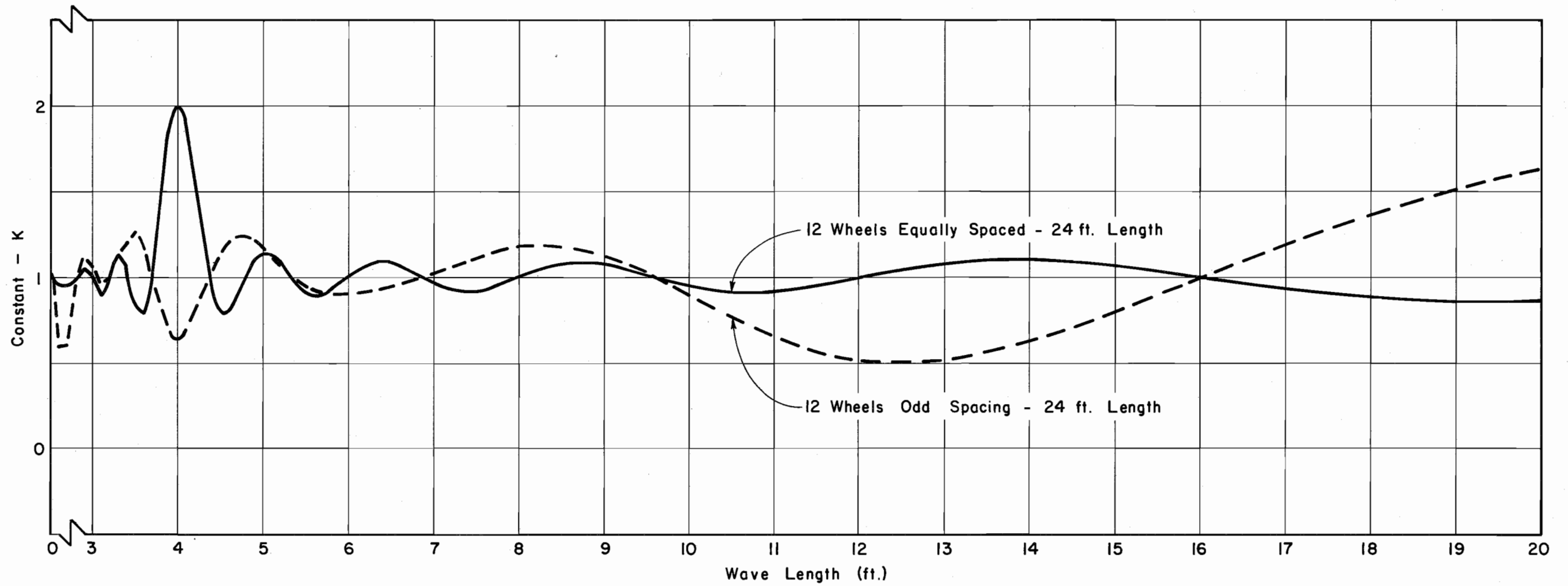
Figure 5 represents a staggered wheel spacing as shown with the greatest top center movement of 0.60 foot. The greatest movement occurred in the larger cycle lengths.

Figures 6 and 7 represent profilographs with equal wheel spacing of four feet and six feet respectively. The 28 foot overall length mock-up had a maximum magnitude of 0.30 foot and the 42 foot overall length mock-up obtained 0.275 foot. The pertinent point is the reduction in the



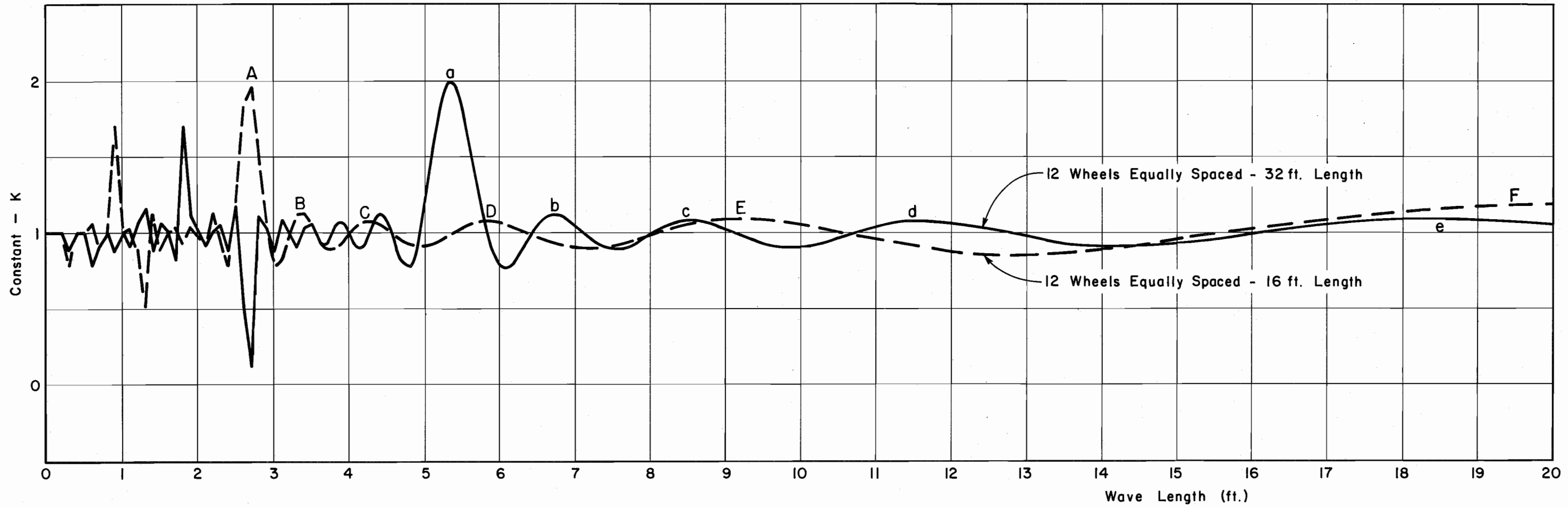
STUDY OF NUMBERS OF AVERAGING WHEELS

FIGURE II



STUDY OF WHEEL POSITION

FIGURE 12



STUDY OF OVERALL LENGTH

FIGURE 13

magnitude of the maximum top center movement between the odd spacing and the even spacing. With these facts known, it is immediately apparent that the equal wheel spacings produce less magnitude of movement at the top center. Some small difference in top center magnitude can be noted with the different lengths.

Output. Figures 8, 9, and 10 show the actual profilograph readout, the actual pavement template profile, and the maximum top center movement. The pavement template profile used in all three cases had high and low areas staggered with one long low dip near the zero end which was approximately 50 units in length. The right end was a series of small irregularities. It should be noted that the actual template profile less the maximum top center movement is actually the profilograph profile. The second apparent fact is that the vertical distance obtained with the profilograph is not equal to the actual template profile distances, but peaks both sag and crest occur at the same longitudinal distances in both cases.

Theoretical Studies:

G.M. Equation. The maximum top center movement of the 28 foot, eight wheel, even spacing profilograph was

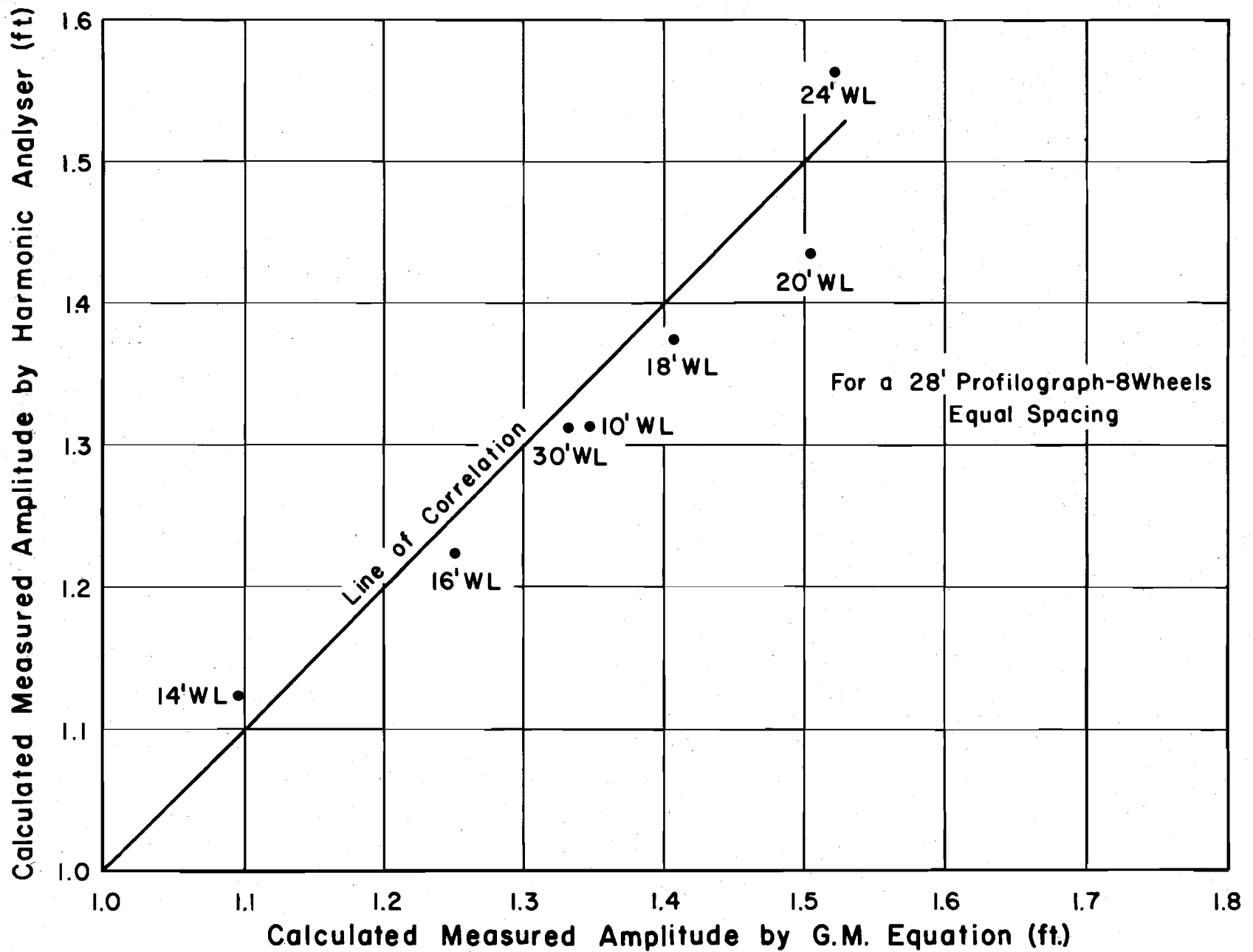
correlated to the constant, K , as obtained with the G.M. equation. A measured amplitude was calculated by both methods using an assumed constant amplitude of 1.25 feet. The calculations are given in Appendix A.

The maximum top center movement obtained from Figure 6 and the K values from the computer study of the G.M. equation can be compared to evaluate the relative results of the two methods. A graphical representation of these results is presented in Figure 14.

A close fit was obtained with the deviation from true correlation probably due to instrument and personnel error in gathering data with the Harmonic Analyzer.

The results of the study of the number of wheels (Figure 11) indicate 12 wheels are better than eight, up to a wave length of approximately 10 feet. At wave lengths over 10 feet, very little difference in K values was noted.

The study of the wheel position (Figure 12) indicates the value of K to be closer to 1.0 for the even wheel spacing with the exception of one large deviation which is indicated at various wave lengths depending upon the profilograph length. A large variation was found in the odd and even spacing in wave lengths over 10 feet which was



CORRELATION OF HARMONIC ANALYSER AND G.M. EQUATION

Figure 14

also indicated in the study with the Harmonic Analyzer.

Profilograph Length. It appears the height of the variation in K when comparing the same type profilograph, but with different lengths is always the same. The wave length at which they occur is dependent upon the profilograph length. Figure 13 indicates capital letters above the larger variations in K in the 16 foot length. Small letters are used over variations of the same quantity experienced in the 32 foot length. This means that a K value of the same magnitude can be expected at some wave length regardless of the length of the profilograph. It appears then that the profilograph length should be selected on the basis of the most likely wave length expected.

There is the question that the roadway will never have the characteristics of a true sine curve with constant amplitude and wave length. However, the roadway probably consists of irregularities which could conform to a combination of several curves and the constant sine curves were used as a method of comparative study.

Selection of Profilograph Equipment

As a result of this investigation, the Texas Highway Department has leased a profilograph from the Rainhart

Company of Austin, Texas. This instrument is approximately 25 feet in length and 52 inches wide (see Figure 15). The framework is made up of aluminum pipe and of aluminum juncture castings.

Any profilograph design must be a compromise. An approximate length of 25 to 30 feet was decided on because the studies showed that the roughness of interest would not be sensed by a shorter instrument. Cost and convenience of a longer machine could not be justified with the diminishing returns.

Twelve averaging wheels were used at 2'-3" centers longitudinally (see Figure 16). The averaging wheels track in individual paths four inches apart when pushed along the roadway. The longitudinal spacing of 2'-3" was selected because this spacing was not a multiple of standard construction joints and the twelve wheels were chosen because of the inherent virtues of a basic tripod design. The averaging wheels were arranged in four tripods. Each tripod contains three wheels attached in a transverse pattern for stability. Vertical movement of any one of the three wheels is transmitted through a "ball" joint or socket. The ball joint is located at the centroid of a triangle composed of angle points at each wheel when viewed in the plan dimension.



Rainhart Profilograph in Road Position

Figure 15



Front View of the Rainhart
Profilograph

Figure 16

From the "ball" joint, the vertical movement is transmitted through a whiffletree arrangement as described previously.

Actuation of a turning movement is accomplished by turning a cast aluminum steering wheel. This steering movement causes each tripod to be rotated with reference to the frame. All four tripods are rotated simultaneously by the steering wheel such that each is directed approximately tangent with a circular turning path. The profilograph is moved along the roadway by manually pushing the instrument through the steering wheel. Thus, the instrument is pushed and guided with the steering wheel.

Two outrigger wheels are attached to the frame so as to provide either additional averaging wheels or high speed trailer wheels. If the outrigger wheels are lowered by means of a scissor jack located near the recorder and the frame is locked in a rigid manner by two large pins, the system becomes a trailer which can be attached to a towing vehicle for high speed mobility. The twelve averaging wheels and the recording wheel are lifted from the pavement to eliminate wear or possible damage at high speeds. A shock absorber has been located in the outrigger wheel arrangement for decreasing impact loads when operating in

the roading mode (see horizontal mechanism near recorder - Figure 17). Maximum road speeds of 70 miles per hour were used until field crews were reminded by a member of the Texas Department of Public Safety that the speed limit of a passenger car with trailer is 60 miles per hour. However, both automobile and towed profilograph respond well at 60 miles per hour.

The recording wheel is composed of a sandwich of two cast aluminum disk halves bolted together and whose centers are mounted on the shaft of a double sealed ball bearing. These halves confine the V-belt tire which is backed up by a plywood disk. The tire, after mounting, is ground round on its own wheel pivot to exactly five feet in circumference.

Rotational movement of the recording wheel actuates both movement of a paper graph and distance counters. The distance counters record length and are located at the right on the recorder shown in Figure 18. The recorder utilizes a ball point pen as a stylus. Vertical movement of the recording wheel is noted not only on the paper graph but also on a counter system shown at the left in Figure 18. The top left counter sums vertical excursions above a theoretical grade line (see Figures 8, 9, & 10) and the



Outrigger Wheel Assembly

Figure 17



Profilograph Recording Mechanism

Figure 18

lower left counter sums vertical excursions below the theoretical grade line.

Road roughness measuring equipment of the type described herein should produce the following information

1. A numerical or index number for comparing various pavements and for establishing an acceptable roughness level.
2. Output to determine roughness location in order that correction can be imposed and for establishing an acceptable roughness level in short lengths.

The prime object of this apparatus was to discover, locate and evaluate roughness, primarily new construction with acceptance in mind. The vertical accumulation counters were conceived to tabulate index numbers to fulfill the requirement in No. 1 above and the graphical trace and distance counters fulfill the requirement in No. 2.

The vertical accumulation counter system allows various blanking tolerances to be used, and future work conducted with the profilograph will be concerned with selecting desirable blanking bands and with suggesting additional construction control specifications.

Conclusions

The following have been concluded from this study:

1. The profilograph length should depend upon the type of work for which the instrument is needed or the magnitude of wave lengths expected. This is not to indicate that a stable of profilographs should be maintained and used, rather a single, well compromised unit should be selected. Indications are that excellent measurements can be obtained especially since pavement surfaces do not conform to a uniform sine wave.
2. The even averaging wheel spacing has an advantage over the odd spacing usually.
3. The larger the number of averaging wheels the closer the measured amplitude conforms to the actual amplitude in general. The maximum number of averaging wheels used is dependent upon the geometric design of the profilograph and the economic factor.
4. Further research could reveal a method of obtaining the true amplitude from the measured amplitude through the use of a computer and given sufficient data.

B I B L I O G R A P H Y

1. Lundstrom, Louis C. and Skeels, Paul C., "Surface Control Methods Employed on General Motors Proving Ground Circular Test Track", UC-204, May 14, 1964.
2. Scrivner, Frank H., "A Modification of the AASHO Road Test Serviceability Index Formula", Research Report 32-1, Texas Transportation Institute, May 1963.
3. Hudson, W. Ronald, "High-Speed Road Profile Equipment Evaluation", Research Report, Center for Highway Research, The University of Texas, Project 3-8-63-73, July 15, 1965.
4. Hveem, F. N., "Devices for Recording and Evaluating Pavement Roughness", January 1960.

A P P E N D I X A

CORRELATION OF HARMONIC ANALYSER AND G.M. EQUATION

CORRELATION OF HARMONIC ANALYSER
AND G.M. EQUATION

28' Profilograph, 8 Wheels, Equally Spaced

10' Cycle

$$\begin{aligned} A &= 1.25 \\ (\text{HA}) A_1 &= 1.250 - (-0.036) = \underline{1.313} \\ K &= 1.077 \\ (\text{GM}) A_1 &= A(K) \\ (\text{GM}) A_1 &= 1.25 \times 1.077 = \underline{1.346} \\ \text{Diff.} &= (\text{GM})A_1 - (\text{HA})A_1 = 1.346 - 1.313 = \underline{\underline{0.033}} \end{aligned}$$

14' Cycle

$$\begin{aligned} A &= 1.25 \\ (\text{HA}) A_1 &= 1.250 - (0.125) = \underline{1.125} \\ K &= 0.875 \\ (\text{GM}) A_1 &= A(K) \\ (\text{GM}) A_1 &= 1.25 \times 0.875 = \underline{1.094} \\ \text{Diff.} &= (\text{GM})A_1 - (\text{HA})A_1 = 1.094 - 1.125 = \underline{\underline{-0.031}} \end{aligned}$$

16' Cycle

$$\begin{aligned} A &= 1.25 \\ (\text{HA}) A_1 &= 1.250 - (0.025) = \underline{1.225} \\ K &= 1.000 \\ (\text{GM}) A_1 &= A(K) \\ (\text{GM}) A_1 &= 1.25 \times 1.000 = \underline{1.250} \\ \text{Diff.} &= (\text{GM})A_1 - (\text{HA})A_1 = 1.250 - 1.225 = \underline{\underline{0.025}} \end{aligned}$$

18' Cycle

$$\begin{aligned} A &= 1.25 \\ (\text{HA}) A_1 &= 1.250 - (-.125) = \underline{1.375} \\ K &= 1.125 \\ (\text{GM}) A_1 &= A(K) \\ (\text{GM}) A_1 &= 1.25 \times 1.125 = \underline{1.406} \\ \text{Diff.} &= (\text{GM})A_1 - (\text{HA})A_1 = 1.406 - 1.375 = \underline{\underline{0.031}} \end{aligned}$$

20' Cycle

$$A = 1.25$$

$$(HA) A_1 = 1.250 - (-.188) = \underline{1.438}$$

$$K = 1.202$$

$$(GM) A_1 = A(K)$$

$$(GM) A_1 = 1.25 \times 1.202 = \underline{1.503}$$

$$Diff. = (GM)A_1 - (HA)A_1 = 1.503 - 1.438 = \underline{\underline{0.065}}$$

24' Cycle

$$A = 1.25$$

$$(HA) A_1 = 1.250 - (-.313) = \underline{1.563}$$

$$K = 1.217$$

$$(GM) A_1 = A(K)$$

$$(GM) A_1 = 1.25 \times 1.217 = \underline{1.521}$$

$$Diff. = (GM)A_1 - (HA)A_1 = 1.521 - 1.563 = \underline{\underline{-0.042}}$$

30' Cycle

$$A = 1.25$$

$$(HA) A_1 = 1.25 - (-0.063) = \underline{1.313}$$

$$K = 1.064$$

$$(GM) A_1 = A(K)$$

$$(GM) A_1 = 1.25 \times 1.064 = \underline{1.330}$$

$$Diff. = (GM)A_1 - (HA)A_1 = 1.330 - 1.313 = \underline{\underline{0.017}}$$