FREQUENCIES OF VARIOUS LEVELS OF STRESS IN HIGHWAY BRIDGES

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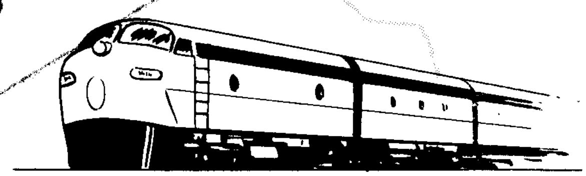

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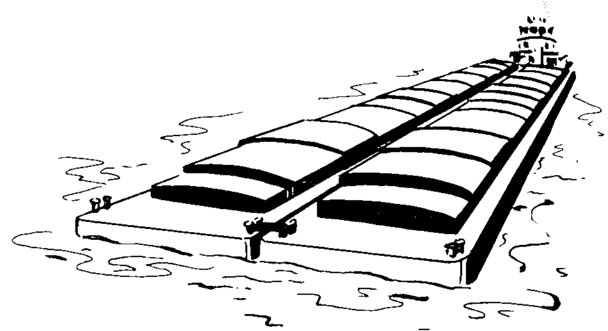

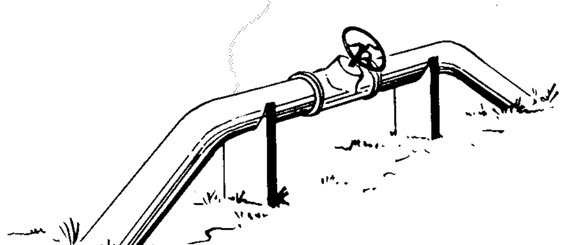

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Frequencies of Various Levels of Stress in Highway Bridges

HENSON K. STEPHENSON,
Research Engineer, Texas Transportation Institute, and
Professor of Civil Engineering,
Agricultural and Mechanical College of Texas

The paper presents a method for predicting the frequencies of various levels of stress to which highway bridges may be subjected as a result of the heavy vehicle loads encountered in various compositions and volumes of traffic for any given period of time or throughout the life of a given structure. In the literature on heavy vehicle loads and their stress-producing effects on highway bridges, certain facets of the problem have been treated by several authors from time to time. The procedures embodied in these past works are brought together for the first time and presented as a complete method for predicting the number of repetitions of various intensities of stress to which any particular member or part of a given structure may be subjected as a result of given traffic conditions. For the investigation of varying numbers of stress repetitions of various intensities of stress and how they may be related to present design criteria for fatigue, it is highly desirable that a reliable method be available for predicting the frequencies of such stresses. It is believed that the method presented accomplishes this objective by providing the means for predicting the frequencies of various levels of stress produced by heavy vehicle loads in any particular part or member of a given bridge corresponding with given traffic conditions.

• THIS PAPER presents a method for predicting the frequencies of various levels of stress produced by heavy vehicle loads in highway bridges. The method allows for variations in the sizes and weights of heavy vehicles and their stress-producing effects on spans of various lengths, types of construction, and design designation. It also allows for the effects of variations in the compositions and volumes of traffic to which a given bridge may be subjected during a specified period of time or throughout its expected life. In the literature on heavy vehicle loads and their stress-producing effects on highway bridges, a number of the various facets of this problem have been treated by several authors from time to time. The ideas and procedures embodied in these past works are brought together here for the first time and presented as a complete method for estimating the number of repetitions of various intensities of stress to which any member or part of a particular structure may be subjected as a result of given traffic conditions.

For the investigation of stress repetitions and how they may be related to present design criteria for fatigue, it is highly desirable that a reliable method be available for predicting the frequencies of such stresses. It is believed that the method presented here accomplishes this objective by providing the means for predicting the frequencies of various levels of stress produced by heavy vehicle loads in any member or part of a particular bridge corresponding with specified traffic conditions.

To make the presentation of this method as simple and as specific as possible, the entire discussion and all the illustrative examples are confined to bending moments and bending stresses in simple-span bridges of one construction type and one design designation. It might be well to mention, however, that the principles and procedures outlined here for predicting bending stress repetitions may be as readily applied to other types of stress or stress functions, such as direct tension, compression, or shear.

The bridges selected for illustrating the method here are of H 15 design and consist of a concrete deck of minimum thickness supported by unencased steel beams. For further simplicity it is also assumed that the steel beams in these bridges are so spaced that the maximum live load bending stress produced in an interior stringer by a single vehicle, in one lane only, will amount to $C = 75$ percent of that produced by identical vehicles in each lane simultaneously. This value of the coefficient $C=75$ percent will be very close to the actual values for most bridges of this type. Another way of saying this would be that if a given bridge were loaded with vehicles having identical H-equivalencies, one in each lane, the maximum live load stress produced in a typical interior stringer would be 4/3, or 133 percent, of that produced by only one of these vehicles in one lane only.

The reason for selecting this comparatively light type of construction is that the ratio of dead load stresses to total design stresses is smaller than would obtain for any of the heavier types of construction, such as reinforced concrete deck girder spans. Consequently, any conclusions arrived at concerning the stress-producing effect of a given vehicle or vehicles on any particular bridge are on the conservative rather than the unsafe side.

For the purpose of presentation here, it is convenient to break the method down into three separate but interrelated parts. Remembering now that the discussion and illustrative examples are confined to bending moments and bending stresses in simple-span steel beam bridges of H 15 design, these parts are as follows:

1. Design stress ratios.
2. Stress-producing effects of equivalent H truck loadings.
3. Frequencies of various levels of stress in highway bridges.

The nomenclature and definitions used herein are assembled in Appendix A for convenience of reference.

DESIGN STRESS RATIOS

Design stress ratios, Q, as the term is used here, refer to the ratios of total actual stresses to total design stresses in any particular member or part of a given highway bridge. For example, consider a 50-ft simple span steel stringer bridge with concrete deck of H 15 design. If the design calculations for this bridge show that the dead load produces a maximum stress of 8.28 ksi (kips per square inch), and the design live load plus impact produces a maximum stress of 9.72 ksi in one of the interior stringers, it will be seen that the total design stress for this stringer is $8.28+9.72=18.00$ ksi. A basic design stress of 18.00 ksi would be satisfactory for such a steel stringer inasmuch as this value corresponds with the maximum bending stress permitted by the 1957 AASHO bridge design specifications.

Now if further calculations indicate that a particular heavy vehicle load would produce a maximum live load plus impact stress of $K f_H=14.56$ ksi (see Appendix A for nomenclature) it will be seen that the maximum total actual stress in this stringer would be $8.28+14.56=22.84$ ksi. So, in accordance with the foregoing definition, the ratio of total actual stress to total design stress for this situation results in a design stress ratio of $Q=22.84/18.00=1.27$. This means that the particular heavy vehicle under consideration would result in total actual stresses 1.27 times as much as the total basic design stress of 18.00 ksi for which this stringer was designed. Another way of saying this would be that the vehicle under consideration on this bridge would result in an overstress of 27 percent; that is, total actual stresses 27 percent in excess of the basic design stress permitted by the AASHO design specifications. But, before proceeding further with the discussion of design stress ratios, a few comments concerning the stress-producing characteristics of heavy vehicle loads, measured in terms of equivalent H truck loadings or other standardized loadings, are in order.

A procedure for measuring the stress-producing characteristics of heavy motor vehicles was developed and presented in Bulletins 127, 131, 132, and 135 of the Texas Engineering Experiment Station (*1, 2, 3, 4*). These bulletins provide a part of the background material on which the method for predicting stress repetitions presented herein is based. In these bulletins the observation is made that each of the many heavy vehicle types and loadings has one thing in common — the capacity to induce a stress (bending, shear, or direct) of definite and calculable magnitude at any particular point in a given bridge. Consequently, a bridge of given type and span can be made to serve as a sort of weighing device by which the maximum stress (bending, shear, or direct stress) produced by any given heavy vehicle can be directly compared with that produced by any other vehicle or arbitrarily standardized loading. However, rather than directly comparing the actual stresses produced by a given heavy vehicle with those produced by others, it is more convenient to appraise the stress-producing effects of a given vehicle if they are expressed in terms of some arbitrary or standardized loading on a simple span of given length.

For this purpose a standard H truck, H-S truck, or any other arbitrary loading, could be used. In this paper, however, the standard H truck loading is used as a basis for measuring the stress-producing characteristics of all other vehicles because the load-carrying capacities of most existing highway bridges are rated in terms of the H loading design. And, as previously mentioned, bending moment is the stress function used to illustrate the method presented herein for measuring overstress because it is the bending stresses that ordinarily determine the load-carrying capacity of most highway bridges.

It should be mentioned here also that the overstress resulting from any other equivalent loading, such as an equivalent concentrated load or equivalent H-S truck loading, can be determined by converting these equivalent loadings into equivalent H truck loadings by use of the conversion coefficients in Table 10, Appendix B (see *3*, p. 73), which gives the conversion coefficients based on maximum moments for equivalent loadings on simple spans of various lengths. A brief explanation of these coefficients and several example problems also are included in Appendix B.

On a 50-ft simple span, for example, if it was determined that a given heavy vehicle produced a maximum live load moment of 445.6 kip-ft, with no allowance for impact, it would be found to be the same as the maximum live load moment produced by an H 20 truck on the same span. Based on its capacity to produce bending stresses in a simple span having a length of 50 ft the given heavy vehicle would be converted into or rated as an equivalent H truck load weighing 20 tons, or simply an equivalent H 20 truck loading. In a similar manner, if a given heavy vehicle produced as much direct stress in a particular member of a given through truss bridge as an H 21.6 truck, it would be rated as an equivalent H 21.6 truck loading insofar as its capacity to produce direct stress in that particular member is concerned. The logic would be similar for any type of stress or stress function at any point that might be of interest in any type of simple span or continuous bridge. The manner in which these equivalent design loads can be used for determining the degree of overstress, or design stress ratio, produced by any given vehicle at some particular point in a given bridge are explained presently in some detail.

Development of Equation

At this point it might be well to reexamine the stress relationships in the 50-ft simple span steel stringer bridge of H 15 design referred to at the beginning of this discussion of design stress ratios. A study of the stresses in this bridge, and how they are related to each other, shows how such relationships provide a basic and necessary tool for the further investigation of repeated stresses in highway bridges (also see 3). For this 50-ft bridge the design calculations show that the dead load produces a maximum stress of 8.28 ksi and the design live load plus impact produces a maximum stress of 9.72 ksi, or a maximum total design stress of 8.28+9.72=18.00 ksi, in one of the typical interior stringers. In accord with the nomenclature given in Appendix A, it will be seen from these data that the dead load ratio, R_D, which is defined as the ratio that the maximum dead load stress, f_D (moment, M_D; shear, V_D; or other stress function), bears to the maximum total design stress, f_T (moment, M_T; shear, V_T; or other stress function) would be

$$B_D = \frac{f_D}{f_T} = \frac{M_D}{M_T} = \frac{8.28}{18.00} = 0.460 \qquad (1)$$

Similarly, it will be seen from these data that the live load ratio, R_L, which is defined as the ratio that the maximum live load plus impact stress, Kf_L (moment, KM_L; shear, KV_L; or other stress function), bears to the maximum total design stress, f_T (moment, M_T; shear, V_T; or other stress function), would be

$$R_L = \frac{K\,f_L}{f_T} = \frac{K\,M_L}{M_T} = \frac{9.72}{18.00} = 0.540 \qquad (2)$$

But because the sum of the design dead load, live load and impact stresses (moments, shears, or other stress function) for a given member is equal to the total design stress, the sum of the dead load and live load ratios must equal 1.00, and it follows that

$$R_D + R_L = \frac{f_D + K\,f_L}{f_T} = \frac{8.28 + 9.72}{18.00}$$
$$= 0.460 + 0.540 = 1.00 \qquad (3)$$

Similarly, if these ratios were defined in terms of moments for an interior stringer or in terms of moments for a full lane, which would be proportional to those in the stringer, their values would remain the same and their sum equal 1.00. Thus,

$$R_D + R_L = \frac{M_D + K\,M_L}{M_T}$$
$$= 0.460 + 0.540 = 1.00 \qquad (4)$$

In Eq. 3 the maximum stress, f_L, in one of the interior stringers is produced by the standard design live loading (without impact), which for this 50-ft span consists of one standard H 15 truck in each lane, simultaneously: K is the coefficient by which the live load stress, f_L, is increased to include the specified allowance for impact. That is,

$$K = 1.00 + I \qquad (5)$$

in which I is the impact fraction as determined by the AASHO formula

$$I = 50/(S + 125) \qquad (6)$$

and S is the length in feet of that portion of the span which is loaded to produce maximum stress in the member under consideration.

For the 50-ft simple span under consideration this means that the impact would amount to $I = 50/(50+125) = 0.286$, which in turn would result in a coefficient $K = 1.000 + 0.286 = 1.286$. As previously stated, the design live load plus impact produces a maximum stress in an interior stringer of $K\,f_L = 9.72$ ksi. Inasmuch as this value includes an allowance of 28.6 percent for impact, it will be seen that the design live load stress without impact would be $f_L = 9.72/1.286 = 7.56$ ksi.

It was also stated previously that further calculations indicated that a particular heavy vehicle would produce a maximum live load plus impact stress of $K\,f_H = 14.56$ ksi in the most highly stressed interior stringer of that 50-ft simple span steel stringer bridge. The next question would be: What is the H-equivalence of this particular vehicle on a 50-ft span? In other words, a standard H truck of what weight would be required to produce a live load plus impact stress of 14.56 ksi in the most highly stressed interior stringer? This question can be answered by referring to previous calculations, which show that the design live load plus impact produces in this same interior stringer a maximum stress of $K\,f_L = 9.72$ ksi. The design live load in this case consists of one H 15 truck in each lane simultaneously. For this bridge, too, it was assumed that a single vehicle in one lane only would produce 75 percent as much stress in an interior stringer as that produced by identical vehicles, one in each lane simultaneously. On this basis, therefore, a single H 15 truck on this bridge in one lane only would produce in the same interior stringer a live load plus impact stress of $C\,K\,f_L = 0.75 \times 1.286 \times 7.56 = 7.28$ ksi.

Now if a single H 15 truck, in one lane only, produces a maximum live load plus impact stress of this magnitude, by direct proportion one can find the equivalent H truck required to produce a corresponding stress of $K\,f_H = 14.56$ ksi, or $K\,f_H / C\,K\,f_L = 14.56/7.28 = 2.00$ times as much live load plus impact stress as a single H 15 truck. Therefore, this given heavy vehicle would be rated as an equivalent H 30 truck on a 50-ft span. Symbolically, the equivalent H truck rating (EHT) for this particular vehicle would be $\text{EHT} = 15(K\,f_H / C\,K\,f_L) = 15(14.56/7.28 =$ Equiv. H 30 truck.

Based on the foregoing discussion of dead load, design live load, impact and actual live load plus impact stresses, and how they may be related for determining the design stress ratios which result from actual vehicle loadings, it is now possible to write a general expression for determining the design stress ratio (*3*) produced by a vehicle of given H-equivalency on a span of given length. In terms of stress produced by vehicles of given H-equivalency, the design stress ratio would be

$$Q = R_D + R_L \frac{K' f_H C}{K f_L} \qquad (7)$$

Similarly, if the stress function were in terms of maximum bending moments produced by vehicles of given H-equivalency, the design stress ratio would be

$$Q = R_D + R_L \frac{K' M_H C}{K\,M_L} \qquad (8)$$

in which

$$K' = 1.00 + I' \qquad (9)$$

is the coefficient by which the actual live load stress (moment or other stress function) is multiplied to obtain the live load plus impact stress (moment or other stress function) produced on a given span by a given vehicle under consideration; and I' is the impact fraction assumed in connection with the stress-producing effects of any given vehicle under consideration. Depending on the speed of the vehicle under consideration and other traffic conditions, the impact fraction, I', could be assumed at any reasonable value between zero and the full impact allowance, I, as defined by the AASHO design specifications.

In Eq. 7, if f_H represents the maximum live load stress in an interior stringer resulting from identical vehicles of given H-equivalency, one in each lane simultaneously, the coefficient $C = 1.00$ (or 100 percent) of the potential stress that would result from identical vehicles of given H-equivalency, one in each lane simultaneously. But if only one of these vehicles were placed in one lane only, C would be less than 1.00, and in the foregoing examples it has been assumed that $C = 0.75$ for the case of one vehicle in one lane only. Here, it will be remembered that C is a function of the stringer spacing and, for all lanes loaded, $C = 1.00$. Similarly, in Eq. 8, if M_H represents the maximum live load moment in an interior stringer resulting from identical vehicles of given H-equivalency, one in each lane simultaneously, $C = 1.00$. But if only one of these vehicles were placed in one lane only, this coefficient would be less than one, say $C = 0.75$, as has been assumed previously.

Likewise, if M_H represents the moment for one lane produced in a given span by a vehicle of given H-equivalency and M_L represents the live load moment for one lane produced by the design live load, the ratio M_H/M_L would be the same as would obtain if M_H were defined as the moment in an interior stringer resulting from vehicles of given H-equivalency, one in each lane simultaneously and M_L the moment in the same stringer resulting from the design live load in each lane simultaneously. Therefore, Eq. 8 provides a general expression for determining design stress ratios resulting from heavy vehicle loadings.

Use of Eq. 7 or Eq. 8

To illustrate the use of Eq. 7 or Eq. 8 for determining design stress ratios, suppose it is desired to determine the design stress ratio resulting from the live load plus impact stress of 14.56 ksi produced in an interior stringer by the equivalent H 30 truck in one lane only on the 50-ft span referred to earlier. Now if $K'=K=1.286$, and this stress of 14.56 ksi is 75 percent of what it would be if each lane were loaded, then if vehicles with identical H-equivalencies were placed one in each lane simultaneously the maximum actual live load stress in an interior stringer would be $K' f_H = 14.56/0.75 = 19.44$ ksi.

With this information it is now possible by use of Eq. 7 to determine the design stress ratio in an interior stringer of this 50-ft span. Therefore, by Eq. 7 it will be found that the design stress ratio for this situation is

$$Q = 0.460 + 0.540\left(\frac{19.44\times 0.75}{9.72}\right) = 0.460 + 0.810 = 1.270.$$

This shows that the given vehicle, which turned out to be an equivalent H 30 truck, would result in a design stress ratio of 1.27 or an overstress of 27 percent in an interior stringer if this vehicle were the only one on the span at one time.

What would the design stress ratio be if vehicles of identical H-equivalencies (equivalent H 30 trucks) were placed one in each lane simultaneously? An equivalent H 30 truck in each lane simultaneously on this 50-ft span would, by Eq. 7, result in a design stress ratio of

$$Q = 0.460 + 0.540\left(\frac{19.44\times 1.00}{9.72}\right) = 1.54$$

In other words, on this 50-ft span of H 15 design, an equivalent H 30 truck in each lane simultaneously would produce a maximum stress in an interior stringer 54 percent in excess of the basic design stress, or a maximum actual stress of $1.54\times 18.00 = 27.70$ ksi.

Evaluating H-Equivalencies

For any given span, if M_H is the moment for one lane produced by a single equivalent H truck weighing H tons and $M_{H(1)}$ the moment for one lane produced by a standard H truck weighing 1.0 ton, then the H rating or H-equivalency in tons for any particular vehicle on a given span would be

$$H = M_H/M_{H(1)} \qquad (10a)$$

or

$$M_H = H\, M_{H(1)} \qquad (10b)$$

Substitution of Eq. 10*b* in Eq. 8 gives

$$H = \frac{K\, M_L\, (Q - R_D)}{C\, K'\, M_{H(1)}\, R_L} \qquad (11)$$

an equation for determining the equivalent H truck loading that would be required on a given span to produce a design stress ratio, Q, of specified value.

Ratios Resulting from Equivalent H Truck Loadings

By rearranging Eq. 11 or by substituting the value of M_H, as given by Eq. 10*b*, in Eq. 8, it will be seen that the design stress ratios resulting from various weights of equivalent H trucks and other loading conditions would be

$$Q = R_D + R_L \frac{H\, C\, K'\, M_{H(1)}}{K\, M_L} \qquad (12)$$

This shows that the design stress ratio, Q, is a linear equation. Therefore, for any given member of a bridge of given span, Q varies directly with the values of H, C and K' in Eq. 12. Thus, Eq. 12 provides a simple and effective means for estimating the stress-producing effects of heavy vehicle loads on highway bridges of various spans, types of construction, and design designation.

The usefulness and variety of information to be obtained from Eq. 12 are discussed and illustrated in the following section.

STRESS-PRODUCING EFFECTS OF EQUIVALENT H TRUCK LOADINGS

Simple Span Steel Stringer Bridges of H 15 Design

The bridges selected for illustrating the degree of overstress or understress (design stress ratio) produced by equivalent H truck loadings on simple spans of H 15 design consist of a concrete deck of minimum thickness supported by unencased steel stringers or girders. The tables and charts which follow provide the means for quickly determining the overstress or understress (design stress ratio) in simple span bridges of this type and design designation which result from any of the heavy vehicle loads encountered in ordinary highway traffic. The design stress ratios given by these tables and charts are correct for the estimated percent of total design stresses represented by dead load and live load plus impact stresses given by Figure 1 for typical interior stringers of simple span steel stringer bridges of H 15 design. It might be added here that the stress relationships indicated are fairly representative of simple span bridges of this construction type and design designation. Moreover, because the dead load ratios shown are based on the lightest type of construction commonly used for simple span bridges of H 15 design, any estimate of overstress obtained from the tables or charts will be on the conservative side for the heavier types of bridge construction.

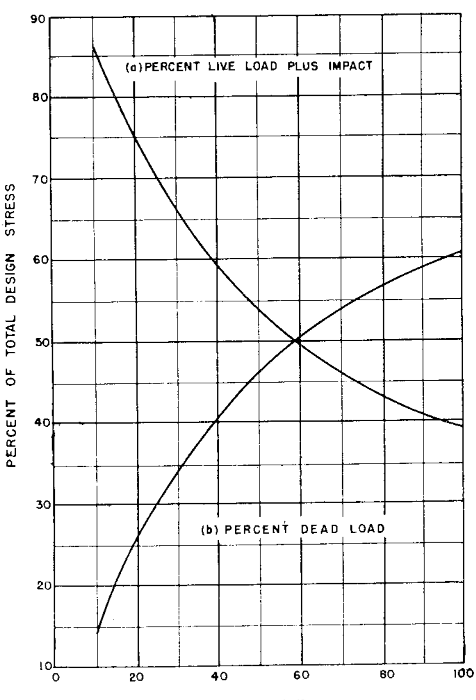

Figure 1. Estimated percentage of total design stresses represented by live load plus impact and dead load stresses for simple span deck girder bridges of H 15 design.

Stresses Produced by Equivalent H Trucks

Because Figure 1 gives the ratio of dead load stress to total design stress, R_D, and the ratio of live load plus impact stress to total design stress, R_L, it will be seen that the equivalent H truck loading corresponding with any degree of overstress or understress (design stress ratio, Q) and loading conditions may be determined by Eq. 11. Tables 1 through 4 give the equivalent H truck loadings, in tons, required to produce maximum bending stresses in an interior stringer, corresponding to a given design stress ratio, for four different conditions of loading. The four conditions of loading are as follows:

1. One vehicle in each lane with full allowance for impact.
2. One vehicle in each lane with no allowance for impact.
3. One vehicle in one lane with full allowance for impact.
4. One vehicle in one lane with no allowance for impact.

Referring to Eq. 12, it will be seen that the design stress ratio, Q, is a linear equation. For any given member of a particular bridge it will also be seen that Q varies directly with the values of H, C, and K' in Eq. 12. This is illustrated in Figures 2 to 13. Figures 2 to 7 give the design stress ratios produced by equivalent H trucks on simple span steel stringer bridges of H 15 design with one vehicle in each lane and varying allowance for impact. Figures 8 to 13, give the design stress ratios produced by equivalent H trucks on simple span steel stringer bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

On a 50-ft span, for example, Figure 4 shows that one equivalent H 30 truck in each lane simultaneously ($C = 1.00$) with full allowance for impact would result in a maximum design stress ratio, $Q = 1.54$.

TABLE 1

EQUIVALENT H TRUCK LOADING IN EACH LANE WITH FULL ALLOWANCE FOR IMPACT REQUIRED TO PRODUCE MAXIMUM STEEL STRESS CORRESPONDING TO GIVEN DESIGN STRESS RATIO

$I' = I$ $K' = 1.00 + I = K$ $C = 1.00$

Span	10	20	30	40	50	60	70	80	90	100
R_L	0.862	0.745	0.660	0.595	0.540	0.495	0.462	0.435	0.410	0.394
K	1.30	1.30	1.30	1.30	1.286	1.27	1.256	1.244	1.232	1.222
M_D	12.5	53.4	123.9	229.7	366.1	542.2	775.7	1056.7	1400.2	1762.0
$K\,M_L$	78.0	156.0	240.5	337.4	429.8	531.5	666.1	813.6	973.0	1145.6
M_T	90.5	209.4	364.4	567.1	795.9	1073.7	1441.8	1870.3	2373.2	2907.6
Design Stress Ratio, Q	Equivalent H Truck Loading									
1.50	23.7	25.1	26.4	27.6	28.9	30.9	34.2	37.7	41.5	45.0
1.40	22.0	23.1	24.1	25.1	26.1	27.8	30.7	33.7	36.9	40.0
1.30	20.2	21.0	21.8	22.6	23.3	24.7	27.1	29.7	32.4	35.0
1.20	18.5	19.0	19.5	20.1	20.6	21.6	23.6	25.6	27.8	29.9
1.10	16.7	17.0	17.3	17.5	17.8	18.5	20.0	21.6	23.3	24.9
1.00	15.0	15.0	15.0	15.0	15.0	15.4	16.4	17.6	18.7	19.9
0.90	13.3	13.0	12.7	12.5	12.2	12.2	12.9	13.5	14.1	14.8
0.80	11.5	11.0	10.5	10.0	9.4	9.1	9.3	9.5	9.6	9.8
0.70	9.8	9.0	8.2	7.4	6.7	6.0	5.8	5.4	5.0	4.7
0.60	8.0	6.9	5.9	4.9	3.9	2.9	2.2	1.4	0.5	—
0.50	6.3	4.9	3.6	2.4	1.1	—	—	—	—	—

This means that the maximum stress produced in one of the interior steel stringers by such a loading would be 154 percent of the basic allowable design stress, or an overstress of 54 percent. However, if the speed of these equivalent H 30 trucks were reduced to say 5 mph, which would result in little or no impact, it will be seen that the maximum amount of overstress in an interior stringer would be reduced to about 28 percent.

Similarly, on a 50-ft span Figure 10 shows that one equivalent H 30 truck in one lane only ($C = 0.75$) with full allowance for impact would result in a maximum design stress ratio, $Q = 1.27$, or an overstress of about 27 percent. However if the speed of this equivalent H 30 truck were reduced so as to result in little or no impact, the maximum amount of overstress in an interior stringer would only amount to about 8 percent. These illus-

TABLE 2

EQUIVALENT H TRUCK LOADING IN EACH LANE WITH NO ALLOWANCE FOR IMPACT REQUIRED TO PRODUCE MAXIMUM STEEL STRESS CORRESPONDING TO GIVEN DESIGN STRESS RATIO

$I' = 0.00$ $K' = 1.00 + I = K$ $C = 1.00$

Span	10	20	30	40	50	60	70	80	90	100
R_L	0.862	0.745	0.660	0.595	0.540	0.495	0.462	0.435	0.410	0.394
K	1.30	1.30	1.30	1.30	1.286	1.27	1.256	1.244	1.232	1.222
M_D	12.5	53.4	123.9	229.7	366.1	542.2	775.7	1056.7	1400.2	1762.0
$K\,M_L$	78.0	156.0	240.5	337.4	429.8	531.5	666.1	813.6	973.0	1145.6
M_T	90.5	209.4	364.4	567.1	795.9	1073.7	1441.8	1870.3	2373.2	2907.6
Design Stress Ratio, Q	Equivalent H Truck Loading									
1.50	30.8	32.6	34.3	35.9	37.2	39.2	43.0	47.0	51.1	55.0
1.40	28.6	30.0	31.3	32.6	33.6	35.3	38.5	41.9	45.5	48.9
1.30	26.3	27.4	28.4	29.3	30.0	31.3	34.0	36.9	39.9	42.7
1.20	24.0	24.7	25.4	26.1	26.4	27.4	29.6	31.9	34.3	36.6
1.10	21.8	22.1	22.4	22.8	22.9	23.4	25.1	26.9	28.6	30.4
1.00	19.5	19.5	19.5	19.5	19.3	19.5	20.6	21.8	23.0	24.3
0.90	17.2	16.9	16.5	16.2	15.7	15.6	16.2	16.8	17.4	18.1
0.80	15.0	14.3	13.6	12.9	12.1	11.6	11.7	11.8	11.8	11.9
0.70	12.7	11.6	10.6	9.7	8.6	7.7	7.2	6.8	6.2	5.8
0.60	10.4	9.0	7.7	6.4	5.0	3.7	2.8	1.8	0.6	—
0.50	8.2	6.4	4.7	3.1	1.4	—	—	—	—	—

TABLE 3

EQUIVALENT H TRUCK LOADING IN ONE LANE WITH FULL ALLOWANCE FOR IMPACT REQUIRED TO PRODUCE MAXIMUM STEEL STRESS CORRESPONDING TO GIVEN DESIGN STRESS RATIO

$I' = I$ $K' = 1.00 + I = K$ $C = 0.75$

Span	10	20	30	40	50	60	70	80	90	100
R_L	0.862	0.745	0.660	0.595	0.540	0.495	0.462	0.435	0.410	0.394
K	1.30	1.30	1.30	1.30	1.286	1.27	1.256	1.244	1.232	1.222
M_D	12.5	53.4	123.9	229.7	366.1	542.2	775.7	1056.7	1400.2	1762.0
$K\,M_L$	78.0	156.0	240.5	337.4	429.8	531.5	666.1	813.6	973.0	1145.6
M_T	90.5	209.4	364.4	567.1	795.9	1073.7	1441.8	1870.3	2373.2	2907.6
Design Stress Ratio, Q	Equivalent H Truck Loading									
1.50	31.6	33.4	35.1	36.8	38.5	41.1	45.6	50.3	55.3	60.1
1.40	29.3	30.7	32.1	33.5	34.8	37.0	40.9	45.0	49.2	53.3
1.30	27.0	28.1	29.1	30.1	31.1	32.9	36.2	39.6	43.2	46.6
1.20	24.6	25.4	26.1	26.7	27.4	28.7	31.4	34.2	37.1	39.9
1.10	22.3	22.7	23.0	23.4	23.7	24.6	26.7	28.8	31.0	33.2
1.00	20.0	20.0	20.0	20.0	20.0	20.5	21.9	23.4	24.9	26.5
0.90	17.7	17.3	17.0	16.7	16.3	16.3	17.2	18.0	18.8	19.8
0.80	15.4	14.6	13.9	13.3	12.6	12.2	12.4	12.7	12.8	13.0
0.70	13.0	11.9	10.0	9.9	8.9	8.1	7.7	7.3	6.7	6.3
0.60	10.7	9.3	7.9	6.6	5.2	3.9	2.9	1.9	0.6	—
0.50	8.4	6.6	4.9	3.2	1.5	—	—	—	—	—

trations should suffice to show the value and utility of the stress data to be obtained from Figures 2 through 13.

FREQUENCIES OF STRESS LEVELS

Mathematical Basis for Study

The study of stress repetitions, as well as that of arriving at the proper design live load for highway bridges, is not only a function of the sizes, weights and frequencies of individual heavy vehicles found on the highways, but also of the frequencies of various intensities of loading that might be expected to occur on a given part or length of bridge, as a result of the chance grouping of two or more of these heavy vehicles in traffic. Fortunately, it is only necessary to make a few simplifying assumptions concerning the behavior of highway traffic in order to apply the theory of probability (*5*) to

TABLE 4

EQUIVALENT H TRUCK LOADING IN ONE LANE WITH NO ALLOWANCE FOR IMPACT REQUIRED TO PRODUCE MAXIMUM STEEL STRESS CORRESPONDING TO GIVEN DESIGN STRESS RATIO

$I' = 0.00$ $K' = 1.00 + I' = 1.00$ $C = 0.75$

Span	10	20	30	40	50	60	70	80	90	100
R_L	0.862	0.745	0.660	0.595	0.540	0.495	0.462	0.435	0.410	0.394
K	1.30	1.30	1.30	1.30	1.286	1.27	1.256	1.244	1.232	1.222
M_D	12.5	53.4	123.9	229.7	366.1	542.2	775.7	1056.7	1400.2	1762.0
$K\,M_L$	78.0	156.0	240.5	337.4	429.8	531.5	666.1	813.6	973.0	1145.6
M_T	90.5	209.4	364.4	567.1	795.9	1073.7	1441.8	1870.3	2373.2	2907.6
Design Stress Ratio, Q	Equivalent H Truck Loading									
1.50	41.1	43.5	45.7	47.9	49.5	52.3	57.3	62.6	68.2	73.4
1.40	38.1	40.0	41.7	43.5	44.8	47.0	51.4	55.9	60.7	65.2
1.30	35.1	36.5	37.8	39.1	40.0	41.8	45.4	49.2	53.2	57.0
1.20	32.0	33.0	33.9	34.7	35.2	36.5	39.4	42.5	45.7	48.8
1.10	29.0	29.5	29.9	30.4	30.5	31.3	33.5	35.8	38.2	40.5
1.00	26.0	26.0	26.0	26.0	25.7	26.0	27.5	29.1	30.7	32.3
0.90	23.0	22.5	22.0	21.6	21.0	20.7	21.6	22.4	23.2	24.1
0.80	20.0	19.0	18.1	17.3	16.2	15.5	15.6	15.7	15.7	15.9
0.70	16.9	15.5	14.2	12.9	11.4	10.2	9.6	9.0	8.2	7.7
0.60	13.9	12.0	10.2	8.5	6.7	5.0	3.7	2.3	0.7	—
0.50	10.9	8.6	6.3	4.1	1.9	—	—	—	—	—

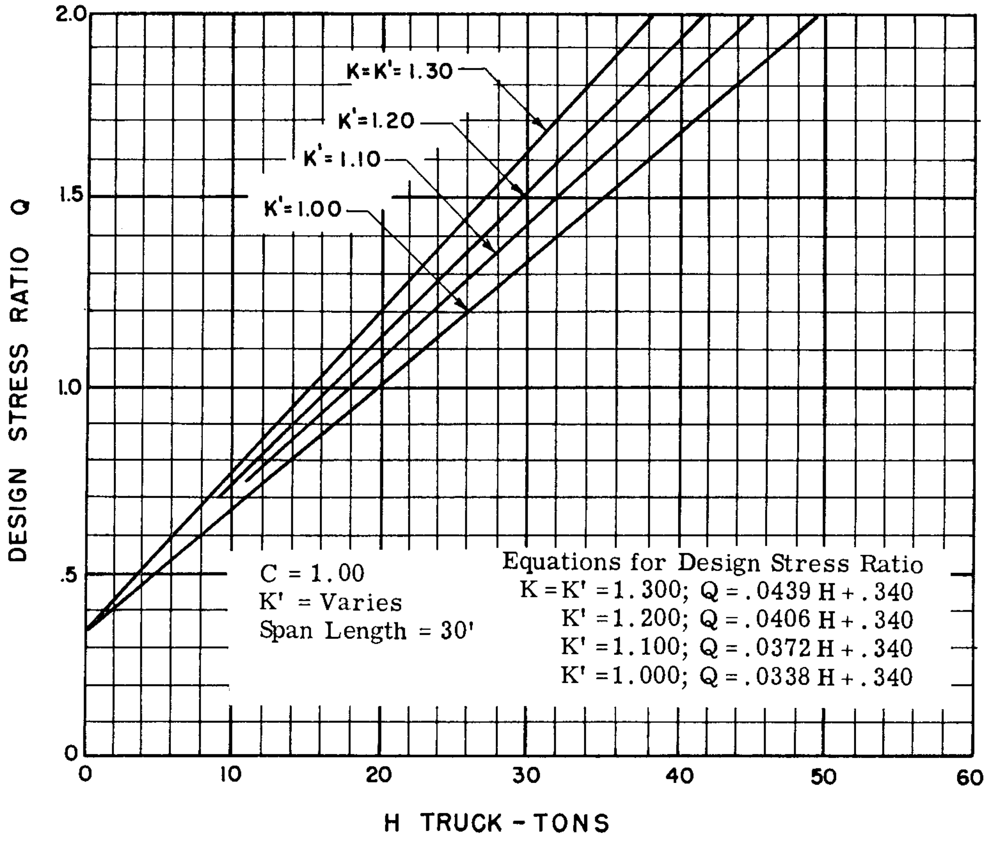

Figure 2. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

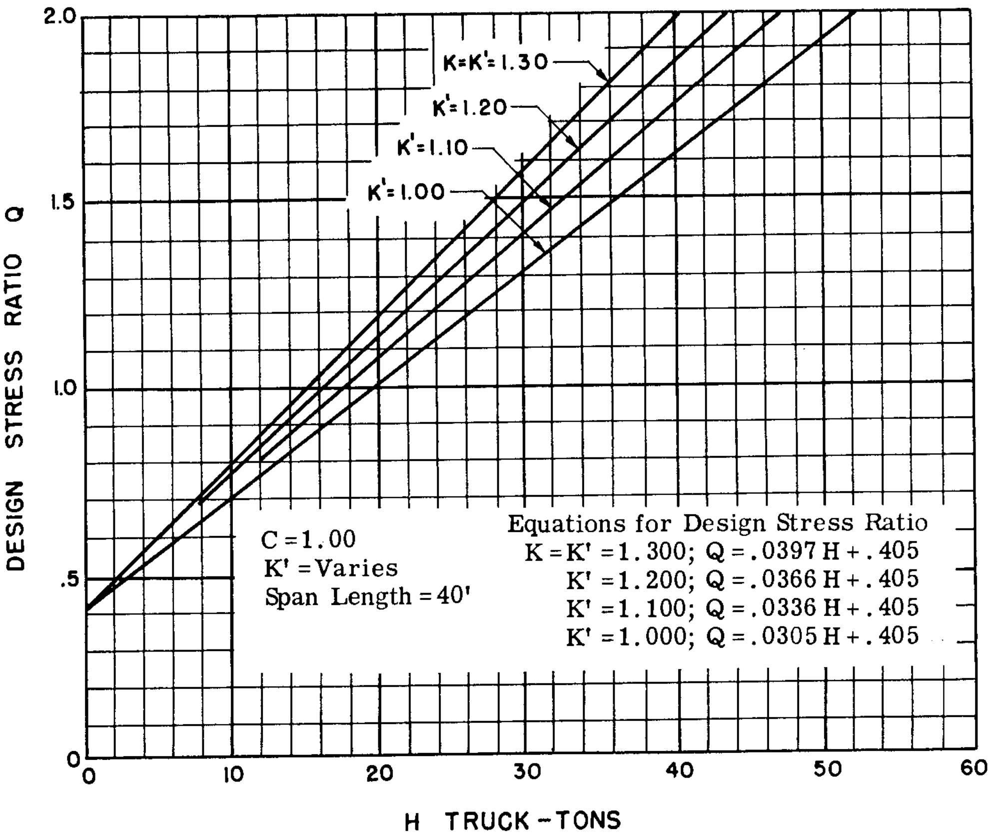

Figure 3. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

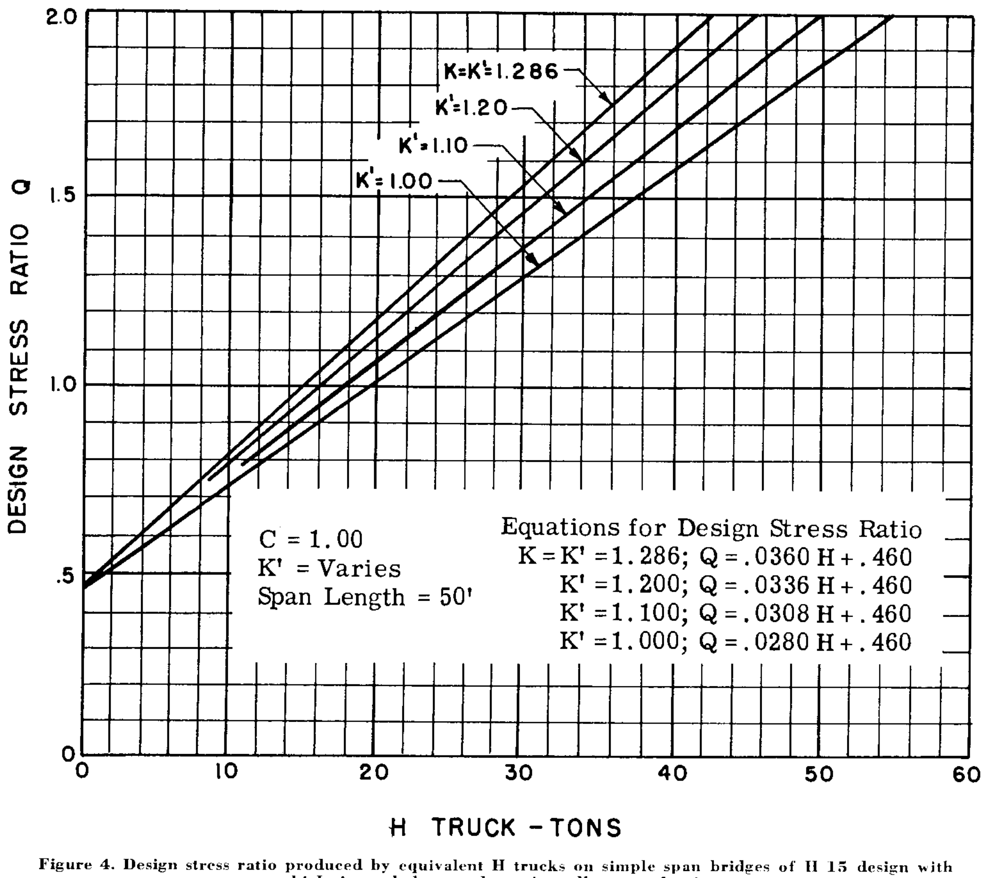

Figure 4. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

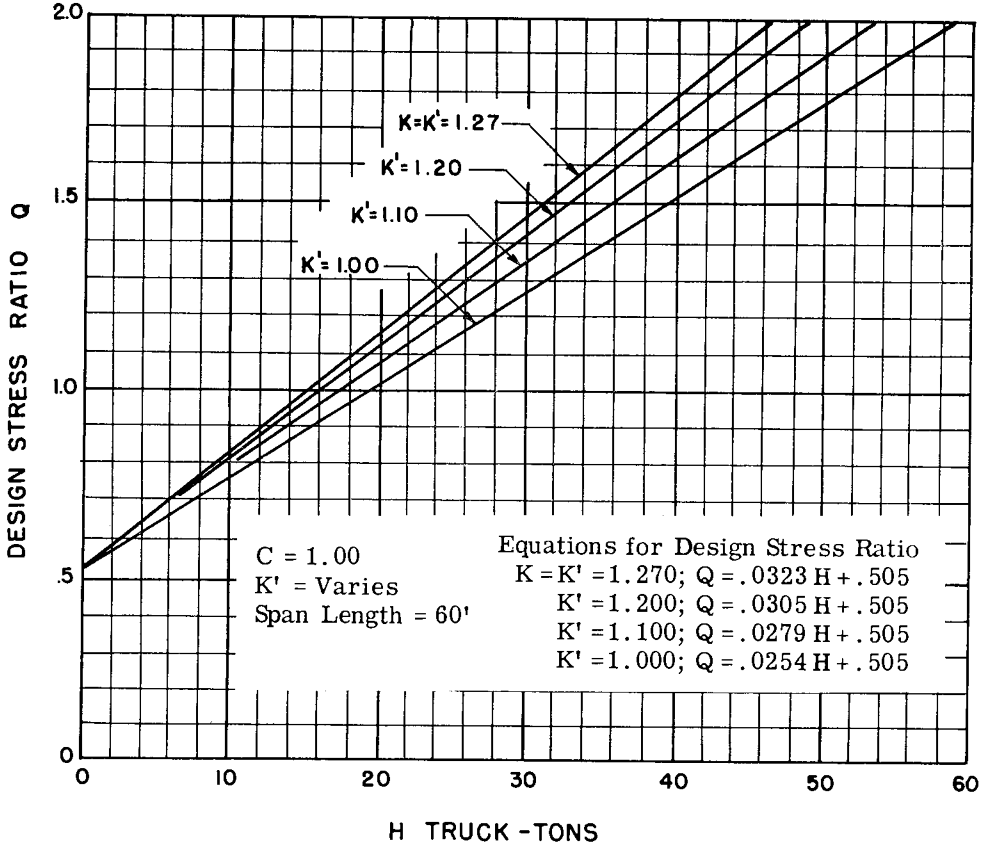

Figure 5. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

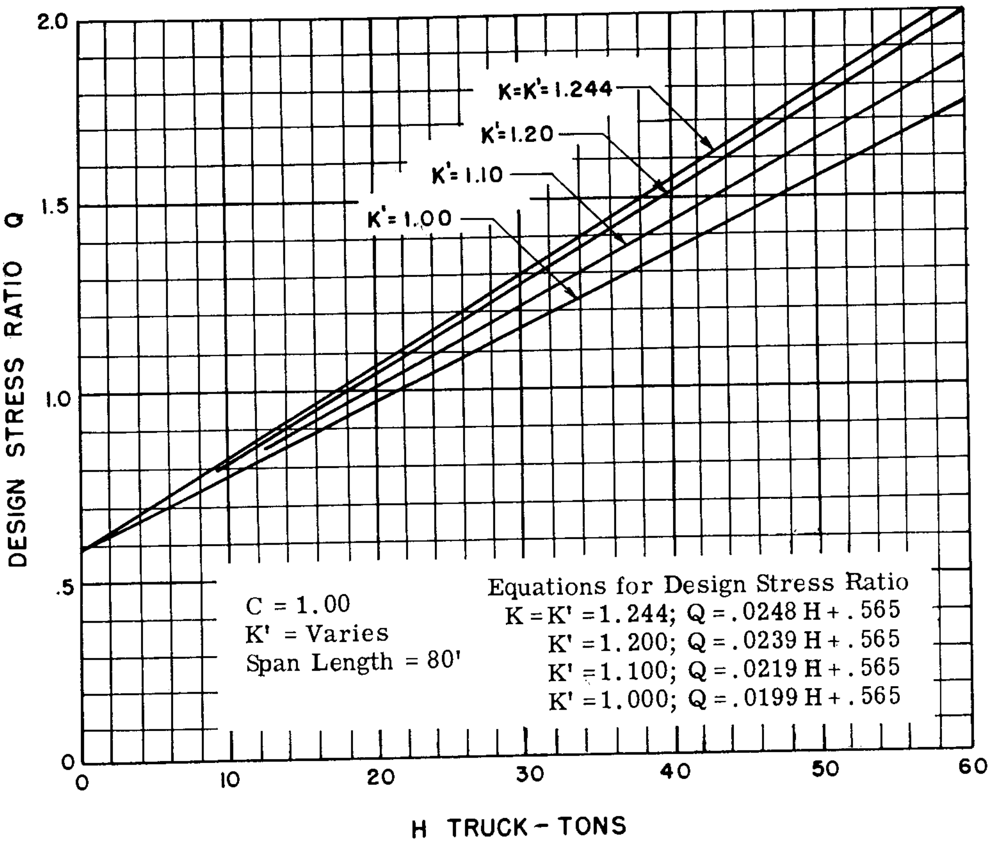

Figure 6. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

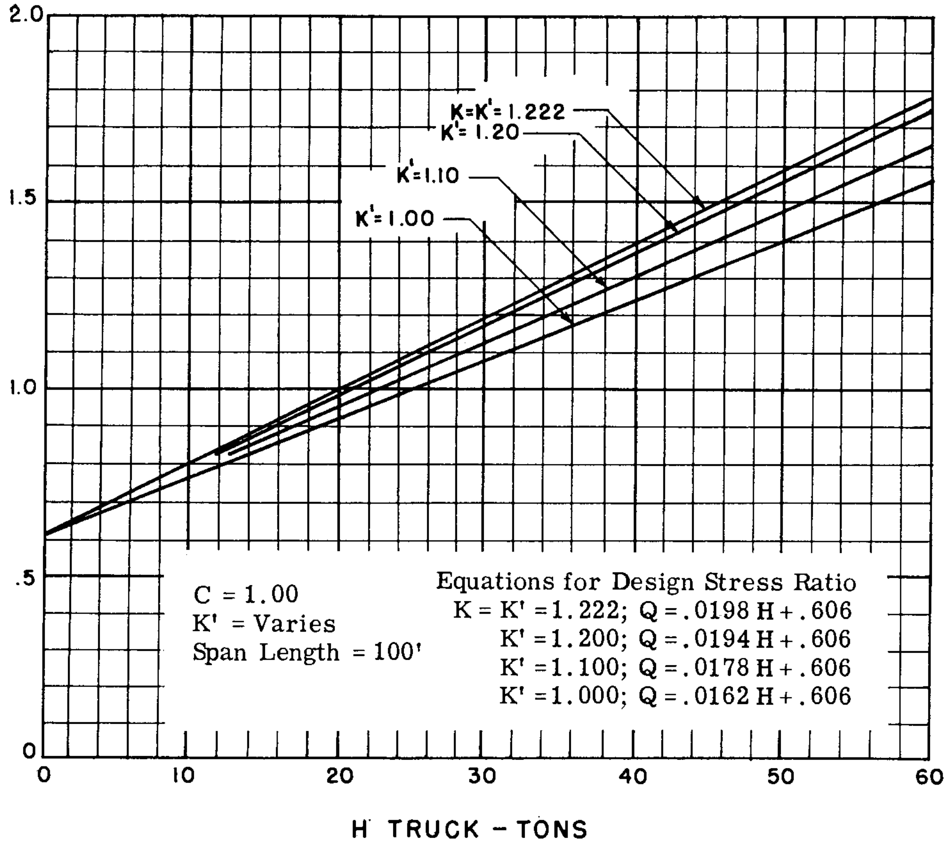

Figure 7. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

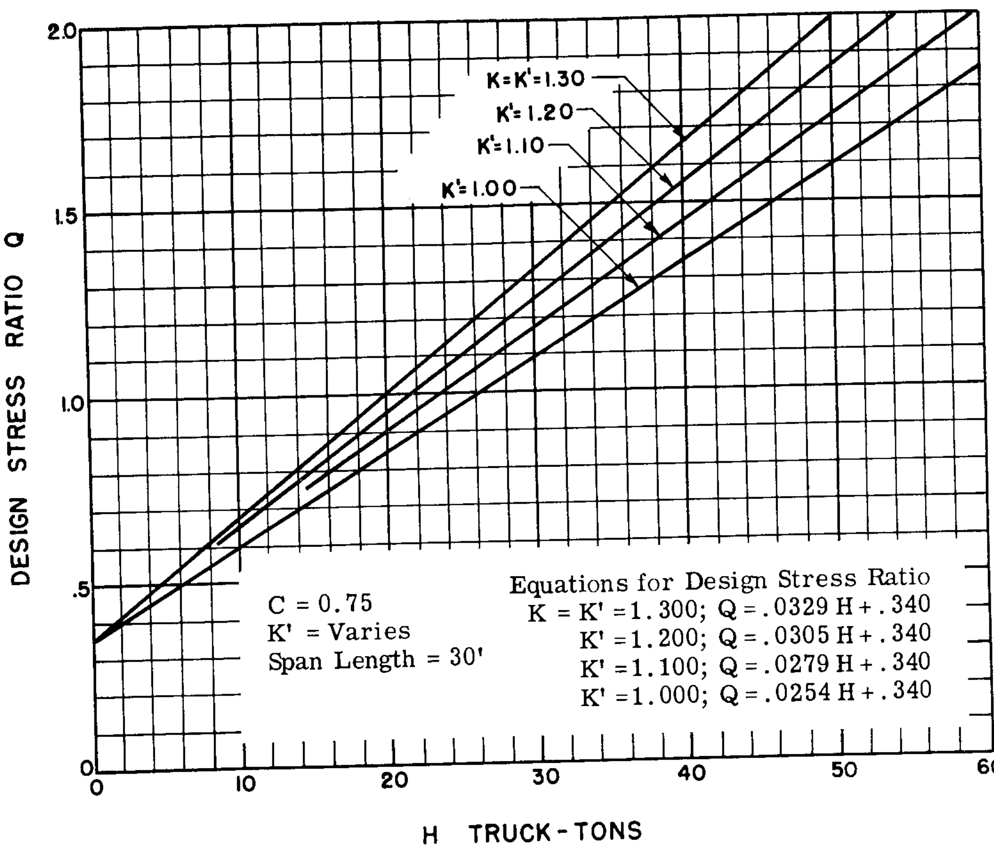

Figure 8. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

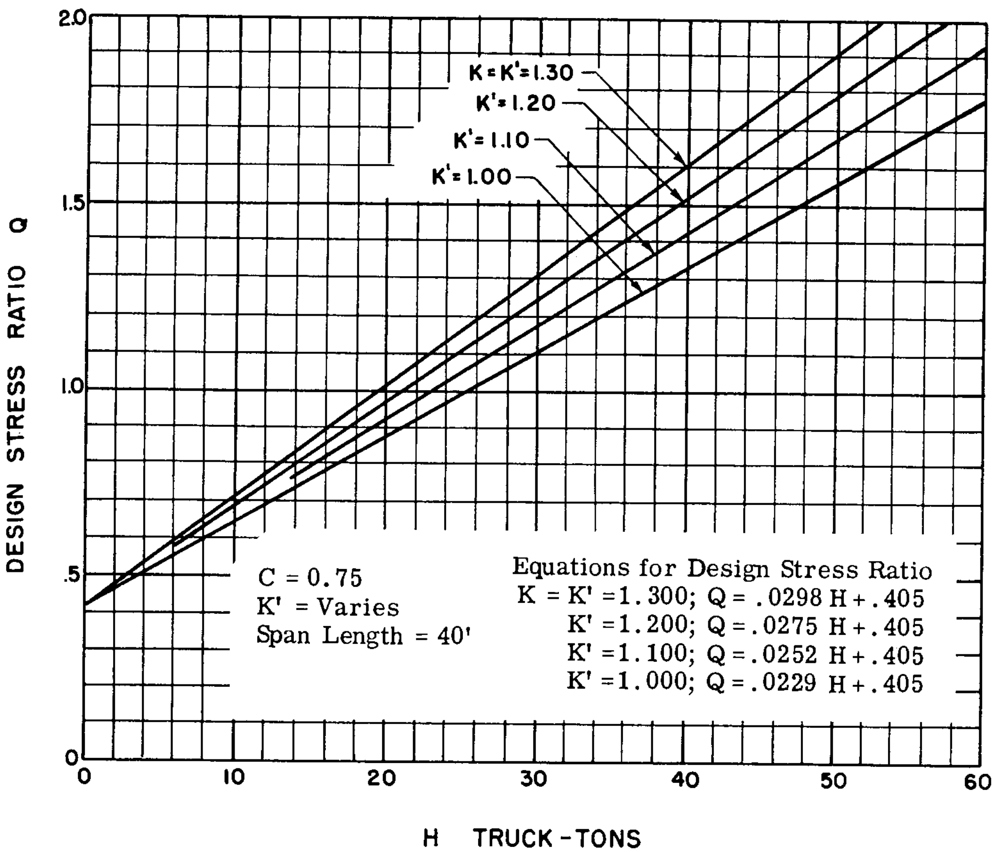

Figure 9. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

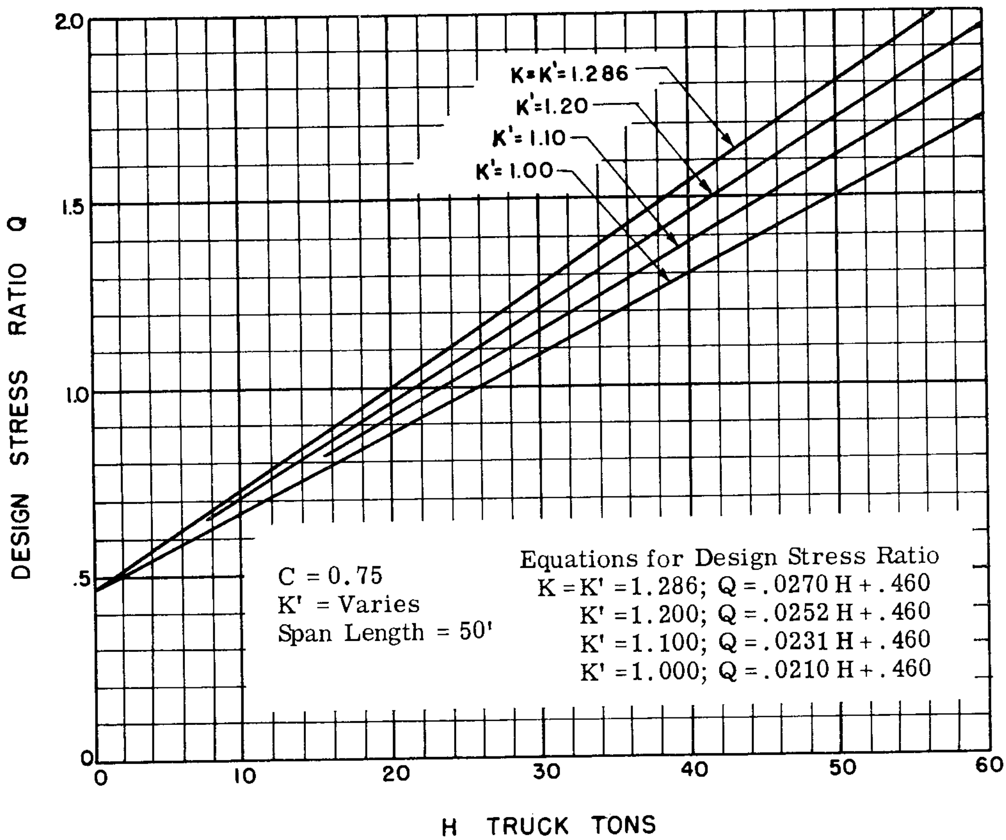

Figure 10. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

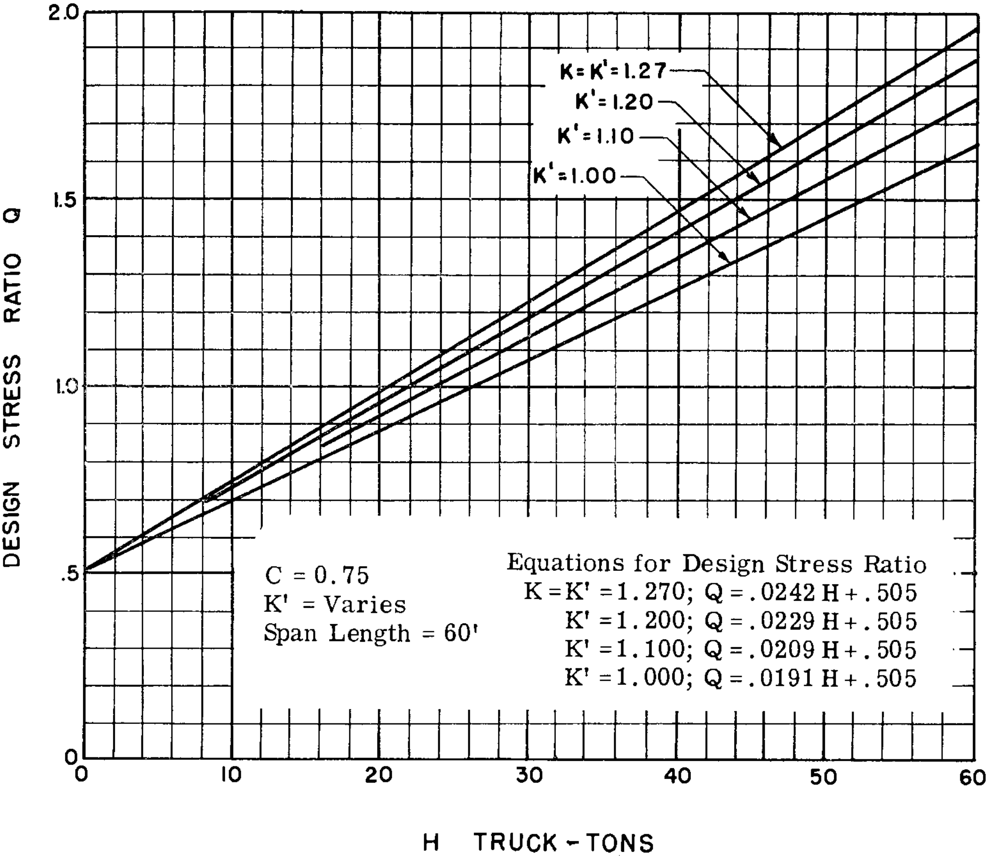

Figure 11. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

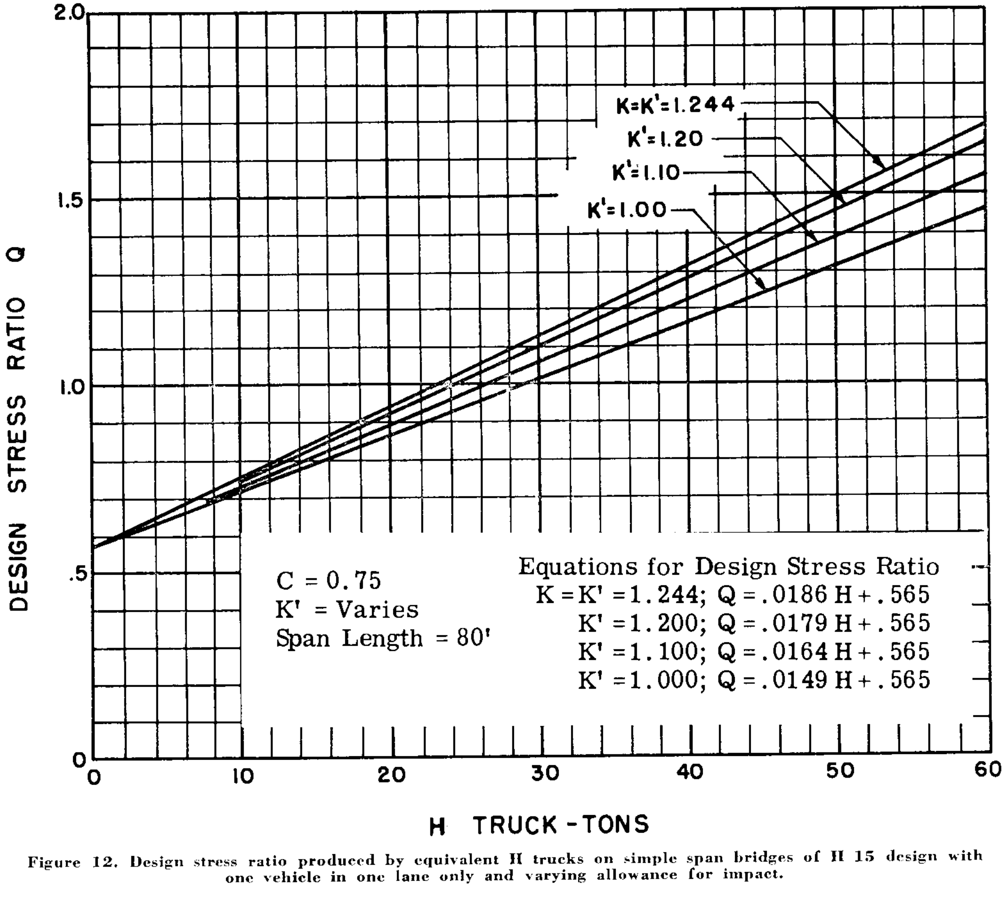

Figure 12. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

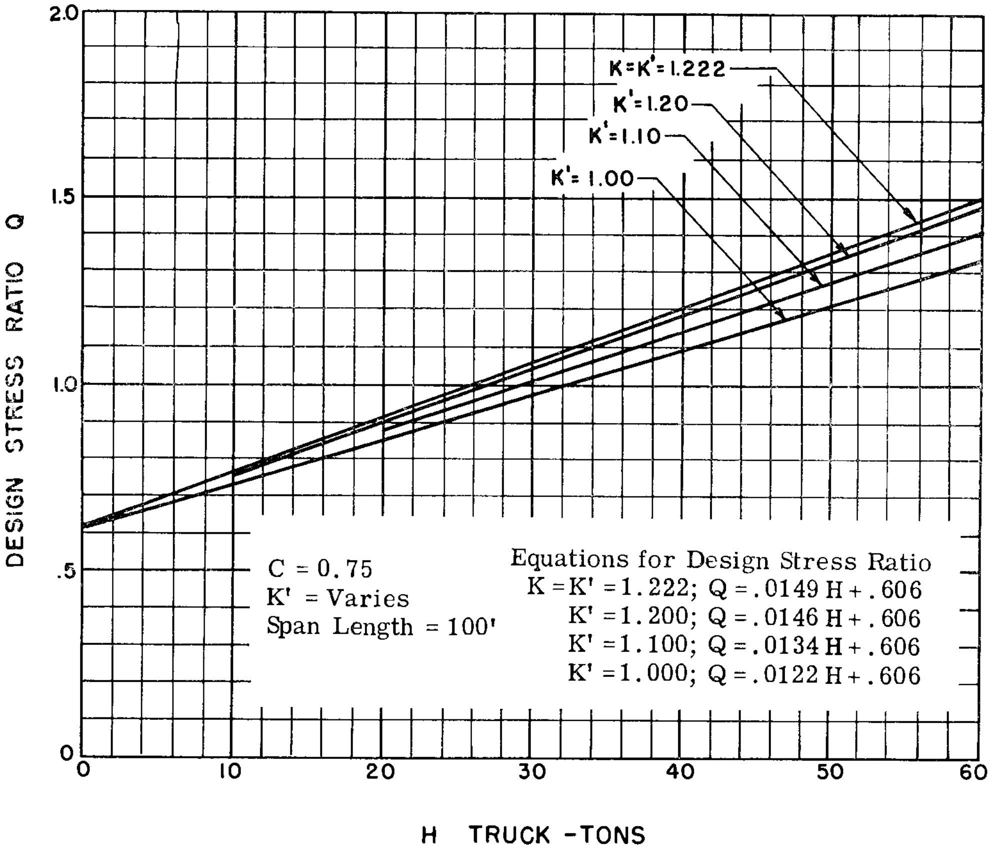

Figure 13. Design stress ratio produced by equivalent H trucks on simple span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

the chance grouping of vehicles and the frequency of specified vehicle groups. These assumptions may be stated as follows:

1. Vehicles, both individually and by types, are distributed at random in ordinary highway traffic.
2. The average composition, volume and speed of traffic remain constant during the time period under consideration.

The first assumption means that the time and distance spacings of vehicles occur entirely by chance and not as a result of artificial control. Similarly, it means that the various vehicle types (such as automobiles, buses, and trucks) occur entirely by chance throughout the traffic stream. The second assumption merely means that the time period under consideration must be of short enough duration to insure that the average composition, volume, and speed of the traffic remain constant during that time. At certain times this time period could be several hours; but at others, when the characteristics of the traffic are changing rapidly, the time period may be only 30, or even 15, minutes.

Numerous studies by the author and others (*6, 7, 8*) have demonstrated that the foregoing assumptions approximate the actual behavior of ordinary highway traffic sufficiently close for solving many types of traffic problems. Moreover, these studies have shown that the time and distance spacings of vehicles, both individually and by groups, in ordinary traffic agree rather closely with the distributions given by the Poisson frequency distribution formula. This means, therefore, that the probability of vehicle groups of unspecified types occurring within specified lengths of time or distance can be estimated mathematically by use of Poisson's law (*9*). Once this probability has been determined, the probability that the group consists of certain specified vehicles, or that they are arranged in some particular order, may be found by use of the basic theorems for calculating simple and compound probabilities (*1, 5, 9*). It should be mentioned also that Poisson's law has also been found to provide a good estimate of the frequency distribution of various intensities of heavy vehicle loads measured in terms of their H truck loading equivalencies on a given span (*1*, pp. 427-438). This last statement will be discussed presently in more detail.

For any given traffic conditions it has been shown elsewhere (*9*) that the Poisson frequency distribution formula provided a fairly simple mathematical means for predicting how often two or more heavy vehicles might be expected to occur in a specified manner within a distance of X feet on a bridge or along the highway. By way of illustration, Figures 14 and 15 show the time interval for typical unspecified vehicle groups occurring within specified lengths, based on 6,000 and 12,000 vehicles per day, respectively, traveling at an average speed of 39.5 mph. Also, Figures 16 and 17 show the time interval for typical specified vehicle groups occurring within specified lengths, based on 6,000 and 12,000 vehicles per day, respectively, traveling at an average speed of 39.5 mph.

For example, based on 6,000 vehicles per day at an average speed of 39.5 mph, it will be seen for $T(2{,}50;2)$ in Figure 14 that 2 vehicles, unspecified as to type, will occur in each of the 2 directions of travel within a particular 50-ft length of bridge or along the roadway about once every 0.1 years, or about 10 times a year. Similarly, for $T(4{,}50;a/2)$ in Figure 14, it will be seen that 4 vehicles will occur in some manner in either one or the other or both directions of travel within a particular 50-ft length about once every 0.05 years, or about 20 times a year.

If one were concerned with the time intervals for typical heavy vehicle groups occurring within specified lengths, based on 12,000 vehicles per day containing 5 percent heavy vehicles at an average speed of 39.5 mph, they will be found in Figure 17. For example, it will be found, for $T(2\text{H},50;2)$ in Figure 17, that 2 heavy vehicles will occur in each of the 2 directions of travel within a particular 50-ft length of bridge or along the roadway about once every 1,500 years. Similarly, for $T(4\text{H},50;a/2)$ in Figure 17, it will be seen that 4 heavy vehicles will occur in some manner in either one or the other or both directions of travel within a particular 50-ft length about once every 500 years.

The development of the Poisson frequency distribution and its use for predicting the time intervals for typical unspecified or specified vehicle groups occurring within specified lengths, as illustrated in Figures 14, 15, 16 and 17, are given in some detail elsewhere *(9)*, therefore are not repeated here. It is believed also that the foregoing illustrations will suffice to indicate the use of Figures 14-17, inclusive.

It should be added here, also, that one of the principal advantages of using the Poisson distribution is the comparative ease with which the successive terms of the Poisson series may be evaluated. Actually, there is rarely any occasion for making such calculations, as tables are available *(10; or 1,* pp. 380-384) that cover a wide range of values for Z.

Heavy Vehicles on Span One at a Time

As previously mentioned, it has been found that Poisson's law provides a very good estimate for the frequency distribution of the various intensities of heavy vehicle loads measured in terms of either their gross weights or their H truck loading equivalencies on a given span (*1*, pp. 427-483). For this purpose the Poisson equation is written:

$$P(n) = \frac{Z^n e^{-Z}}{n!} \qquad (13)$$

Each vehicle constitutes a sample whose gross weight or H-equivalency is measured in tons. For example if the H-equivalency of a given vehicle on a 60-ft span fell between 17.50 and 18.49 tons, it would be classified as an equivalent H 18 truck loading. Perhaps the simplest way to explain Eq. 13 is to discuss it with reference to Table 5.

Table 5 shows the calculated frequencies of equivalent H truck loadings, on various span lengths, for the 4,531 heavy trucks reported by the 1942 national loadometer survey, based on Eq. 13. With respect to the 50-ft span, for example, it will be noted that the vehicles producing the smallest moment were equivalent H 8 trucks; the largest were equivalent H 26 trucks and the average were equivalent H 15.2 trucks. For this distribution it will be noted that the Poisson coefficient, Z, is equal to the difference between the average and the least H-equivalencies. In other words, if the equivalent H 8 cell is thought of as the zero cell, the average H-equivalency would be 7.2 cells higher or greater than the zero cell.

Eq. 13, therefore, would be read thus: The probability that the H-equivalency of a given vehicle will be n cells (or in this case n is in tons) larger than the smallest or zero cell, when the average is Z cells greater than the zero cell, is equal to $Z^n\ e^{-Z}/n!$. The distribution shown in Table 5 for the 50-ft span indicates that 0.1 percent of the heavy vehicles reported were equivalent H 8 trucks, 0.5 percent were equivalent H 9 trucks, etc. The H-equivalencies for the other spans are interpreted similarly. Tables are available (*1, 10*) which give the Poisson distributions for a wide range of Z values, so it is seldom necessary to calculate frequencies from Eq. 13.

Calculated rather than actual frequencies of H-equivalencies are given in Table 5 merely to illustrate the use of Poisson's frequency law in situations where actual loadometer data may not be available. However, for evaluating the design stress ratios, Q, resulting from heavy vehicles on a given span, one at a time, it might be preferable to calculate the Q-values from observed loadometer data.

With the preceding information, suppose it is now desired to determine the frequencies of various levels of stress in simple span steel stringer bridges 50 ft and 100 ft in length. Suppose, further, that these bridges are subjected to a traffic volume of 12,000 vehicles per day containing 5 percent heavy trucks whose H-equivalencies are distributed for these spans as shown in Table 5. If the useful life of these bridges is assumed as 50 years, it would mean that a total of $12{,}000 \times 0.05 \times 365 \times 50 = 11{,}000{,}000$

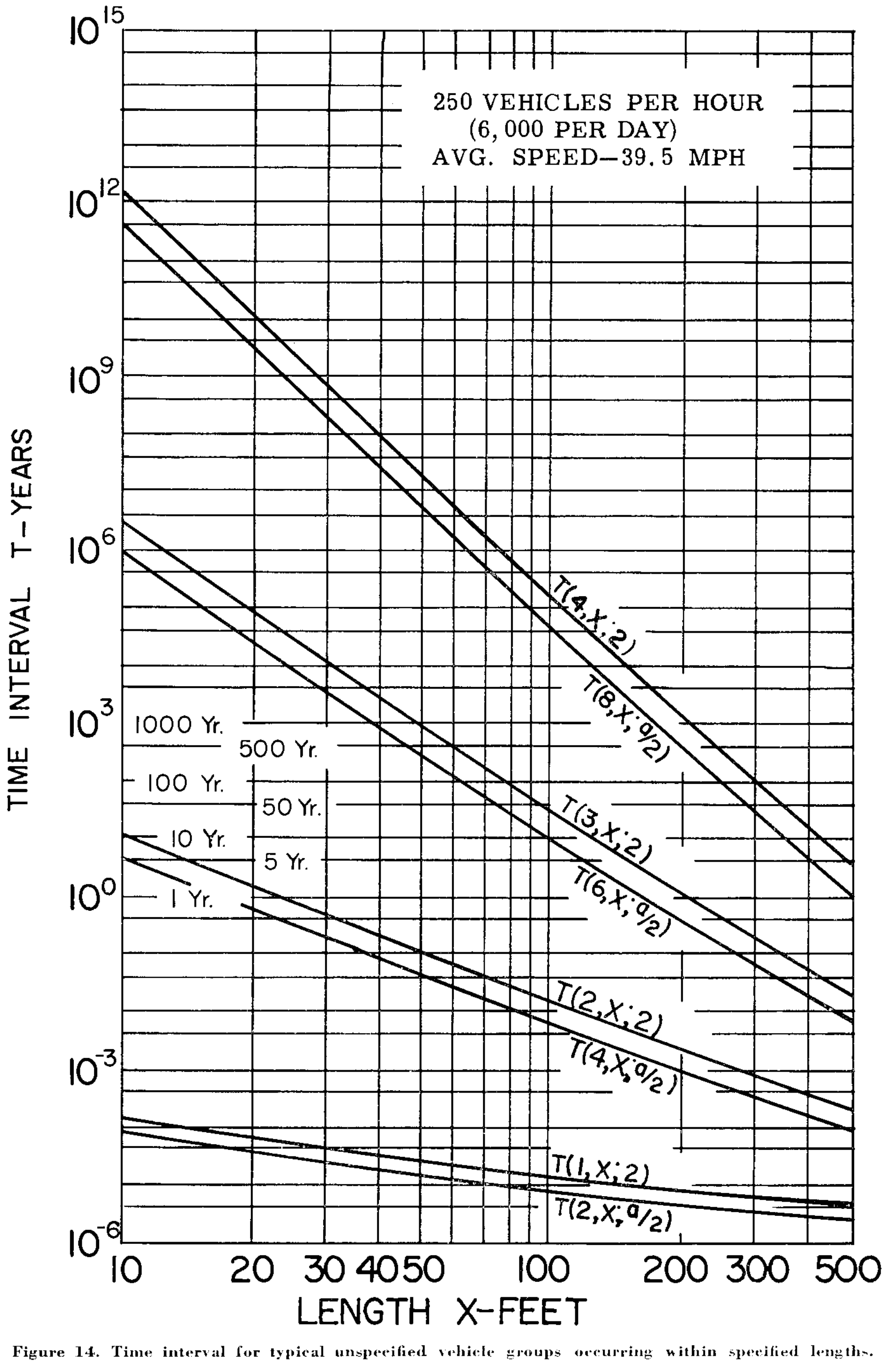

Figure 14. Time interval for typical unspecified vehicle groups occurring within specified lengths.

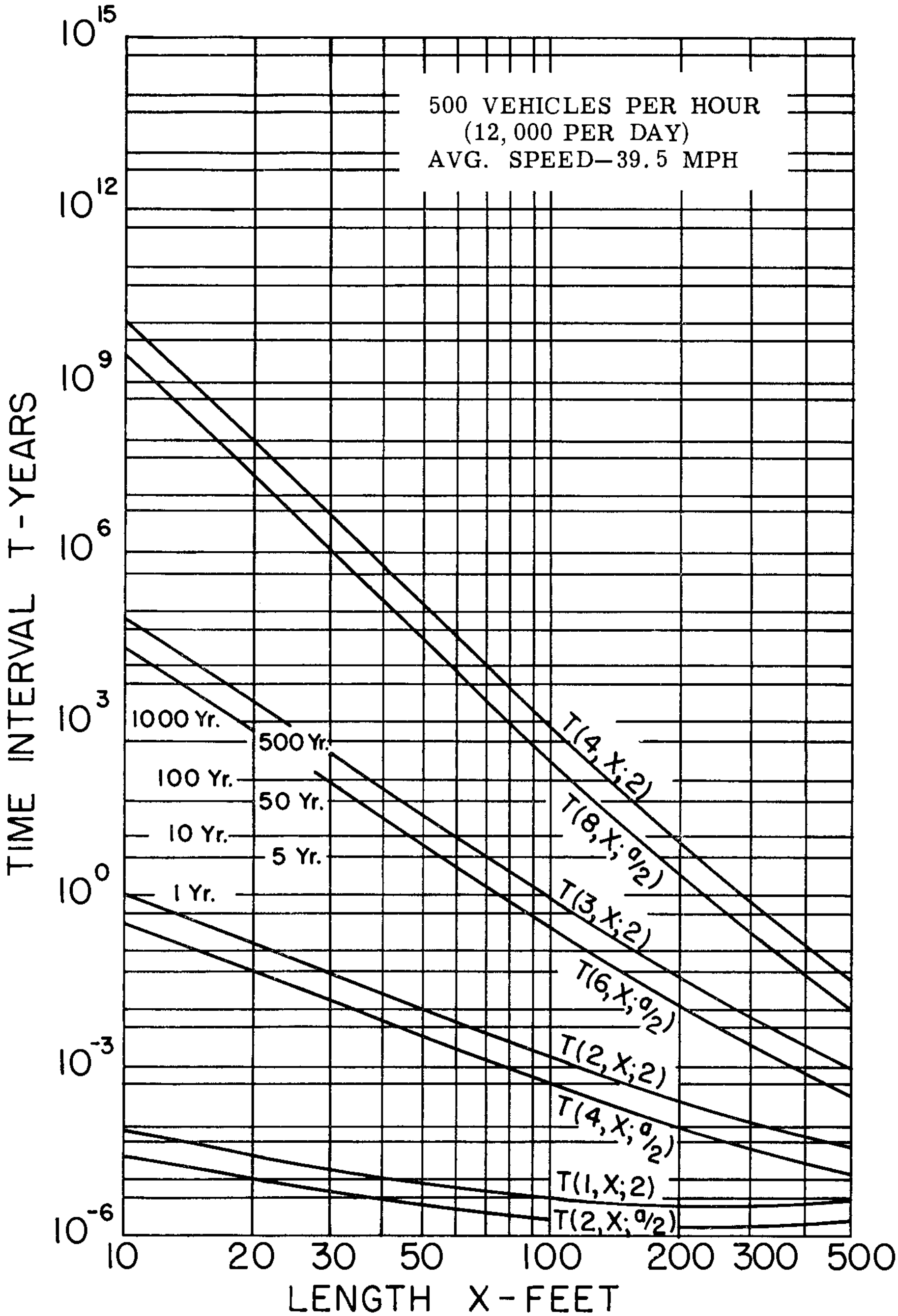

Figure 15. Time interval for typical unspecified vehicle groups occurring within specified lengths.

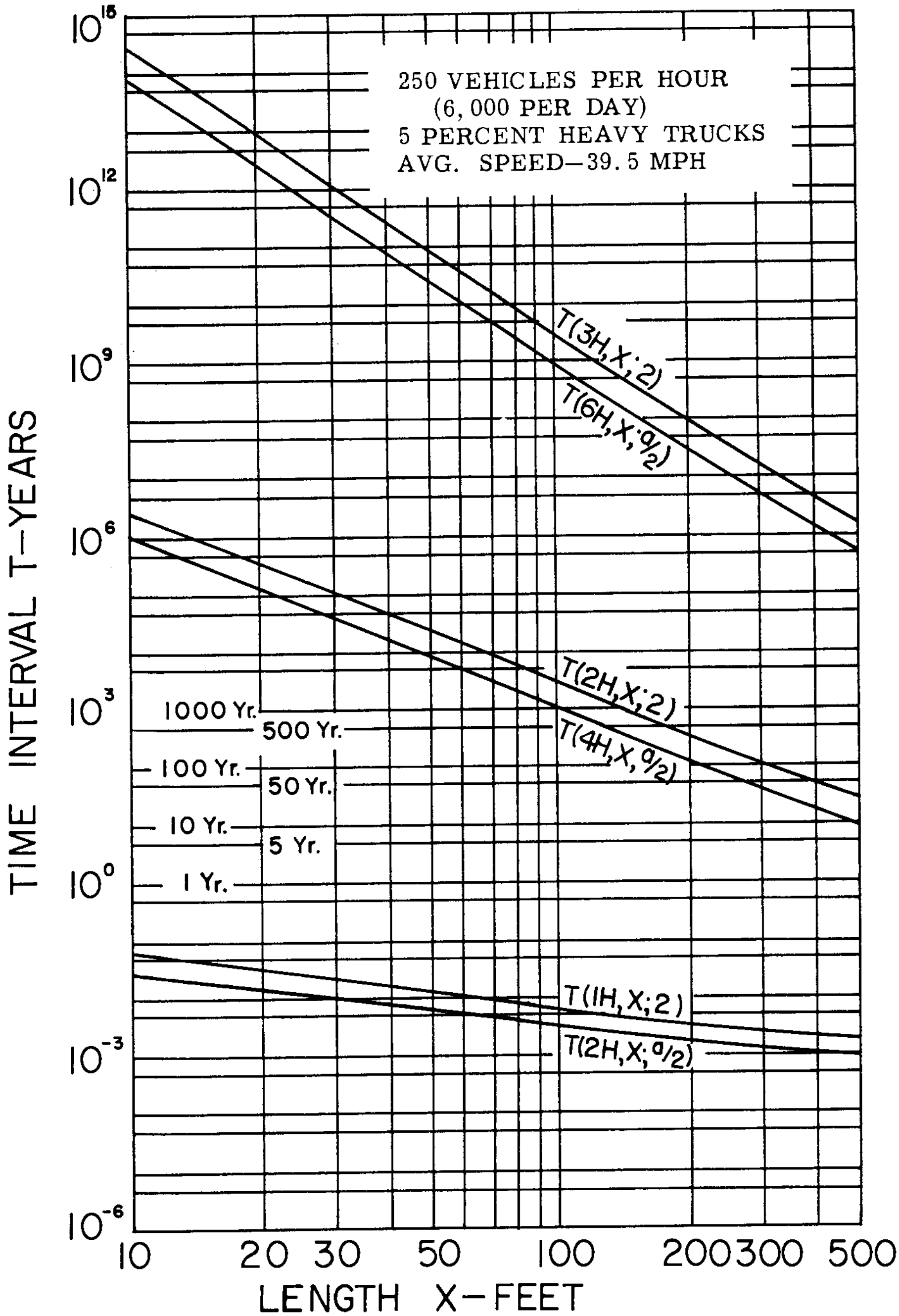

Figure 16. Time interval for typical specified vehicle groups occurring within specified lengths.

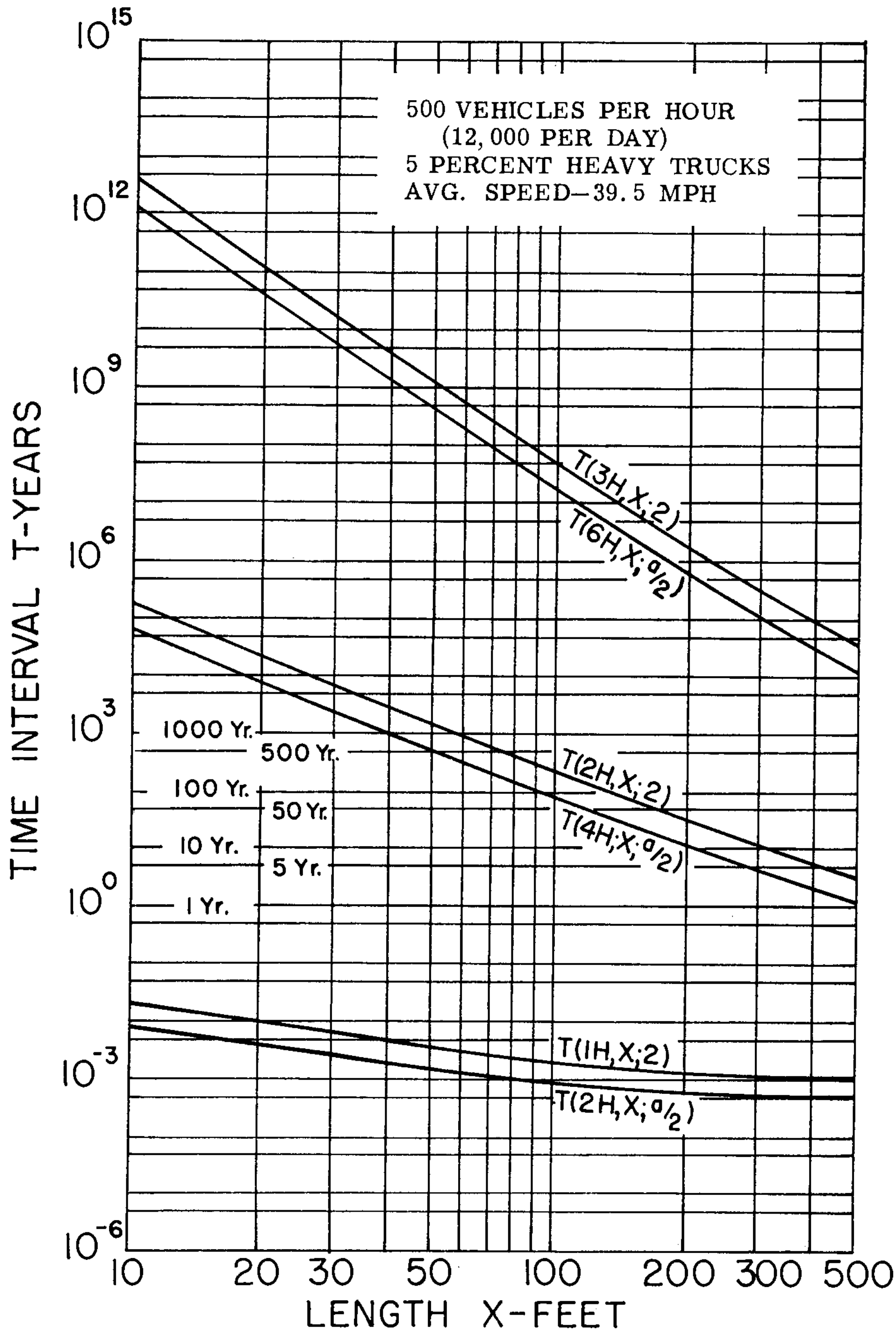

Figure 17. Time interval for typical specified vehicle groups occurring within specified lengths.

TABLE 5

CALCULATED FREQUENCIES[1] OF EQUIVALENT H TRUCK LOADINGS[2] FOR 4,531 HEAVY TRUCKS OF ALL TYPES REPORTED BY THE 1942 LOADOMETER SURVEY, BASED ON POISSON'S FREQUENCY DISTRIBUTION LAW

Equivalent H Truck Loadings	Span (ft)								
	10	20	30	40	50	60	80	100	Infinite G.V.W.
5	0.2	—	—	—	—	—	—	—	—
6	1.1	0.2	0.1	—	—	—	—	—	—
7	3.4	1.3	0.7	0.1	—	—	—	—	—
8	7.3	3.9	2.2	0.6	0.1	—	—	—	—
9	11.6	8.1	5.2	2.1	0.5	0.1	—	—	—
10	14.7	12.5	9.1	4.9	1.9	0.6	—	—	—
11	15.9	15.4	12.8	8.7	4.6	2.1	0.2	0.1	—
12	14.5	16.0	14.9	12.4	8.4	4.9	1.1	0.5	—
13	11.6	14.1	14.9	14.7	12.0	8.7	3.4	1.9	0.1
14	8.2	11.0	13.0	14.9	14.4	12.4	7.3	4.6	0.2
15	5.3	7.6	10.2	13.2	14.9	14.7	11.6	8.4	1.0
16	3.1	4.7	7.1	10.4	13.4	14.9	14.7	12.0	2.7
17	1.6	2.6	4.5	7.4	10.7	13.2	15.9	14.4	5.4
18	0.8	1.4	2.6	4.8	7.7	10.4	14.5	14.9	8.8
19	0.4	0.7	1.4	2.8	5.0	7.4	11.6	13.4	11.9
20	0.2	0.3	0.7	1.5	3.0	4.8	8.2	10.7	13.8
21	0.1	0.1	0.4	0.8	1.7	2.8	5.3	7.7	13.9
22	—	0.1	0.1	0.4	0.9	1.5	3.1	5.0	12.6
23	—	—	0.1	0.2	0.4	0.8	1.6	3.0	10.2
24	—	—	—	0.1	0.2	0.4	0.8	1.7	7.5
25	—	—	—	—	0.1	0.2	0.4	0.9	5.1
26	—	—	—	—	0.1	0.1	0.2	0.4	3.1
27	—	—	—	—	—	—	0.1	0.2	1.8
28	—	—	—	—	—	—	—	0.1	1.0
29	—	—	—	—	—	—	—	0.1	0.5
30	—	—	—	—	—	—	—	—	0.2
31	—	—	—	—	—	—	—	—	0.1
32	—	—	—	—	—	—	—	—	0.1
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Max. H Truck	21	22	23	24	26	26	27	29	32
Avg. H Truck	11.4	12.2	13.0	14.1	15.2	16.1	17.4	18.2	21.1
Min. H Truck	5	6	6	7	8	9	11	11	13
Poisson's Coeff, Z	6.4	6.2	7.0	7.1	7.2	7.1	6.4	7.2	8.1

[1] Equivalent H truck loadings which occur less than 1 in 1000, or account for less than 0.1 percent of total heavy truck traffic, not shown.
[2] Equivalent H truck loadings based on moments produced by gross vehicle weights.

heavy vehicles would pass over these bridges during that period. Most of these heavy vehicle passages would occur on these spans one at a time.

The numbers of repetitions of various levels of stress, with and without impact, in an interior stringer of a 50-ft bridge of H 15 design resulting from these 11,000,000 heavy vehicles being on the span one at a time are given in Table 6. Similar data for the 11,000,000 heavy vehicles on the 100-ft span, one at a time, are given in Table 7. The numbers of stress repetitions resulting from two or more heavy vehicles occurring on these spans at the same time are shown in Tables 8 and 9 and will be discussed later in more detail. The design stress ratios, with and without allowance for impact, corresponding with each of the several H-equivalencies may be calculated by Eq. 12. The appropriate design stress ratio equations may be found also in Figures 10 and 13.

In Table 6 it should be noted that the Poisson distribution for $Z = 7.2$ is given to six decimal places, which is somewhat more refined than the distributions for $Z = 7.2$ shown in Table 5. In Table 6 it will also be noted that the highest stress level indicated, with full allowance for impact, corresponds with a design stress ratio $Q = 1.309$, or an overstress of 30.9 percent; and this amount of overstress would occur only 22 times in the passing of 11,000,000 heavy vehicles during the 50-year estimated life of this bridge. With no allowance for impact, the highest overstress only amounts to about 10 percent; and this would be expected about 78 times in 50 years. In Table 7, for the 100-ft span, it will be seen that the highest stress level indicated, with full allowance for impact, corresponds with a de-

TABLE 6

NUMBER OF STRESS REPETITIONS [1] PRODUCED IN A 50-FT SIMPLE SPAN BRIDGE OF H 15-44 DESIGN DURING AN ASSUMED USEFUL LIFE OF 50 YEARS; A TOTAL OF 11 MILLION HEAVY VEHICLES OCCUR ON THIS SPAN ONE AT A TIME

Equivalent H Truck Loading	Poisson Distribution for $Z = 7.2$	Number of Vehicles	Design Stress Ratio [2]: With Full Allowance for Impact [3]	Design Stress Ratio [2]: With No Allowance for Impact [4]
8	0.000747	8,218	0.676	0.628
9	0.005375	59,124	0.703	0.649
10	0.019352	212,872	0.730	0.670
11	0.046444	510,884	0.757	0.691
12	0.083598	919,578	0.784	0.712
13	0.120382	1,324,202	0.811	0.733
14	0.144458	1,589,039	0.838	0.754
15	0.148586	1,634,447	0.865	0.775
16	0.133727	1,470,996	0.892	0.796
17	0.106982	1,176,802	0.919	0.817
18	0.077027	847,296	0.946	0.838
19	0.050418	554,598	0.973	0.859
20	0.030251	332,760	1.000	0.880
21	0.016754	184,294	1.027	0.901
22	0.008616	94,776	1.054	0.922
23	0.004136	45,496	1.081	0.943
24	0.001861	20,472	1.108	0.964
25	0.000788	8,668	1.135	0.985
26	0.000315	3,466	1.162	1.006
27	0.000119	1,298	1.189	1.027
28	0.000043	472	1.216	1.048
29	0.000015	164	1.243	1.069
30	0.000005	56	1.270	1.090
31	0.000002	22	1.309	1.111
Total		11,000,000		

[1] Stress effects based on continuous traffic volume of 500 vehicles per hour (12,000 per day) containing 5 percent heavy vehicles (in excess of 13 tons gross weight).
[2] See Figure 10 for design stress ratio equations.
[3] $Q = 0.0270H + 0.460$.
[4] $Q = 0.0210H + 0.460$.

sign stress ratio $Q = 1.113$ or an overstress of about 11.3 percent; and this amount of overstress would occur about 22 times in the 50-year estimated life of the bridge. If these 11,000,000 heavy vehicles passed over this 100-ft bridge one at a time and without impact, the greatest overstress indicated would be about 2.1 percent.

Two or More Heavy Vehicles on Span Simultaneously

In discussing the Poisson distribution given by Eq. 13, it was pointed out that the distribution coefficient, Z, represented the numerical difference between the average and smallest H-equivalencies for a given distribution. And in the case of the distribution shown in Table 5 for the 50-ft span it will be seen that $Z = 15.2 - 8.0 = 7.2$. If the average H-equivalency for these vehicles taken one at a time is 15.2 tons, the average for such a population taken two at a time would be 30.4 tons.

Similarly, the lowest possible H-equivalency for this population taken two at a time would be 16 tons. The Poisson coefficient for such a distribution (equivalent H trucks taken two at a time) would be the difference between the average and smallest cells, or $Z = 30.4 - 16.0 = 14.4$ tons. Tables 8 and 9 show the frequency distribution of average H-equivalencies (vehicles taken two at a time) for this value of Z.

The next problem is that of determining how many times two heavy vehicles would occur simultaneously, one in each of two adjacent lanes, on the 50-ft span, within a critical 10- or 12-ft length at or near mid-span, during the 50-year life of the bridge. Figure 17 shows, for the

TABLE 7

NUMBER OF STRESS REPETITIONS [1] PRODUCED IN A 100-FT SIMPLE SPAN BRIDGE OF H 15-44 DESIGN DURING AN ASSUMED USEFUL LIFE OF 50 YEARS; A TOTAL OF 11 MILLION HEAVY VEHICLES OCCUR ON THIS SPAN ONE AT A TIME

Equivalent H Truck Loading	Poisson Distribution for $Z = 7.2$	Number of Vehicles	Design Stress Ratio [2]: With Full Allowance for Impact [3]	Design Stress Ratio [2]: With No Allowance for Impact [4]
8	0.000747	8,218	0.725	0.704
9	0.005375	59,124	0.740	0.716
10	0.019352	212,872	0.755	0.728
11	0.046444	510,884	0.770	0.740
12	0.083598	919,578	0.785	0.752
13	0.120382	1,324,202	0.800	0.766
14	0.144458	1,589,039	0.815	0.777
15	0.148586	1,634,447	0.829	0.789
16	0.133727	1,470,996	0.844	0.801
17	0.106982	1,176,802	0.859	0.813
18	0.077027	847,296	0.874	0.826
19	0.050418	554,598	0.889	0.838
20	0.030251	332,760	0.904	0.850
21	0.016754	184,294	0.919	0.862
22	0.008616	94,776	0.934	0.874
23	0.004136	45,496	0.949	0.887
24	0.001861	20,472	0.964	0.899
25	0.000788	8,668	0.979	0.911
26	0.000315	3,466	0.993	0.923
27	0.000119	1,298	1.008	0.935
28	0.000043	472	1.023	0.948
29	0.000015	164	1.038	0.960
30	0.000005	56	1.053	0.972
31	0.000002	22	1.068	0.984
Total		11,000,000		

[1] Stress effects based on continuous traffic volume of 500 vehicles per hour (12,000 per day) containing 5 percent heavy vehicles (in excess of 13 tons gross weight).
[2] See Figure 13 for design stress ratio equations.
[3] $Q = 0.0149H + 0.606$.
[4] $Q = 0.0122H + 0.606$.

TABLE 8

NUMBER OF STRESS REPETITIONS[1] PRODUCED IN A 50-FT SIMPLE SPAN BRIDGE OF H 15-44 DESIGN, DURING AN ASSUMED USEFUL LIFE OF 50 YEARS, RESULTING FROM 4,000 OCCURRENCES OF ONE HEAVY VEHICLE IN EACH OF TWO ADJACENT LANES SIMULTANEOUSLY

			Design Stress Ratio[2]	
Equivalent H Truck Loadings	Poisson Distribution for $Z = 14.4$	Number of Occurrences	With Full Allowance for Impact[3]	With No Allowance for Impact[4]
10	0.0013	5	0.820	0.740
11	0.0098	39	0.856	0.768
12	0.0397	159	0.892	0.796
13	0.0998	399	0.928	0.824
14	0.1700	680	0.964	0.852
15	0.2077	831	1.000	0.880
16	0.1921	768	1.036	0.908
17	0.1388	555	1.072	0.936
18	0.0804	322	1.108	0.964
19	0.0382	153	1.144	0.992
20	0.0151	60	1.180	1.020
21	0.0051	20	1.216	1.048
22	0.0015	6	1.252	1.076
23	0.0004	2	1.288	1.104
24	0.0001	1	1.324	1.132
Total		4000		

[1] Stress effects based on continuous traffic volume of 500 vehicles per hour (12,000 per day) containing 5 percent heavy vehicles (in excess of 13 tons gross weight).
[2] See Figure 4 for design stress ratio equations.
[3] $Q = 0.0360H + 0.460$.
[4] $Q = 0.0280H + 0.460$.

traffic conditions assumed, that two heavy vehicles would be expected to occur as defined about 80 times a year, or 4,000 times in 50 years.

The frequency distribution of these 4,000 loadings shows that the highest design stress ratio (with full allowance for impact), $Q=1.324$, occurs but once in the 50-year life of the bridge. And without impact a design stress ratio of $Q=1.132$ would be reached but once in the 50 years.

Table 9 gives similar data for the 100-ft span. In this case the highest design stress ratio is $Q=1.081$, or an 8 percent overstress, which includes full allowance for impact. Without impact, the loadings resulting from one vehicle in each of the adjacent lanes would probably not result in stresses in excess of the basic design stresses.

Simple Span Steel Stringer Bridges of H 15-44 Design

If one considers the simple situation of an ordinary bridge on a main rural highway where the traffic may be considered distributed at random, it will be found that two or more heavy vehicles (those weighing in excess of 13 tons) in each of the two directions of travel would occur so seldom on bridges of 500 ft or less in length that the effects of such loadings might be neglected insofar as their effects on design stresses are concerned.

For ordinary highway bridges, therefore, the most severe loading condition that need be considered (at normal service load allowance stresses) is for one heavy vehicle to occur in each of the two directions of travel at the same time. For example, if one considered a traffic volume of 500 vehicles per hour (or 12,000 vehicles per day) containing 5 percent heavy vehicles, it will be found from Figure 17 that one heavy vehicle would occur in each of the two directions of a 50-ft span, within a critical 10- or 12-ft length at or near the mid-span, about 80 times per year; and, for this same traffic, one heavy vehicle would occur in each of the two directions of a 100-ft span, within a critical 20- to 25-ft length at or near the mid-span, about 120 times per year.

TABLE 9

NUMBER OF STRESS REPETITIONS[1] PRODUCED IN A 100-FT SIMPLE SPAN BRIDGE OF H 15-44 DESIGN, DURING AN ASSUMED USEFUL LIFE OF 50 YEARS, RESULTING FROM 6,000 OCCURRENCES OF ONE HEAVY VEHICLE IN EACH OF TWO ADJACENT LANES SIMULTANEOUSLY

			Design Stress Ratio[2]	
Equivalent H Truck Loadings	Poisson Distribution for $Z = 14.4$	Number of Occurrences	With Full Allowance for Impact[3]	With No Allowance for Impact[4]
10	0.0013	8	0.804	0.768
11	0.0098	59	0.824	0.784
12	0.0397	238	0.844	0.800
13	0.0998	599	0.864	0.817
14	0.1700	1020	0.883	0.833
15	0.2077	1245	0.903	0.849
16	0.1921	1153	0.923	0.865
17	0.1388	833	0.943	0.881
18	0.0804	482	0.962	0.898
19	0.0382	229	0.982	0.914
20	0.0151	91	1.000	0.930
21	0.0051	31	1.022	0.946
22	0.0015	9	1.042	0.962
23	0.0004	2	1.061	0.979
24	0.0001	1	1.081	0.995
Total		6000		

[1] Stress effects based on continuous traffic volume of 500 vehicles per hour (12,000 per day) containing 5 percent heavy vehicles (in excess of 13 tons gross weight).
[2] See Figure 7 for design stress ratio equations.
[3] $Q = 0.0198H + 0.606$.
[4] $Q = 0.0162H + 0.606$.

Similarly, if a traffic volume of 250 vehicles per hour (or 6000 vehicles per day) containing 5 percent heavy vehicles is considered (which is a very high volume for main rural roads and also an extremely high concentration of heavy vehicles) it will be found from Figure 16 that one heavy vehicle would occur in each of the two directions on a 50-ft span, within a critical 10- or 12-ft length at or near the mid-span, about 20 times a year. For this same traffic, one heavy vehicle would occur in each of the two directions of a 100-ft span, within a critical 20- or 25-ft length at or near the mid-span, 35 times a year.

But even though two heavy vehicles do occur within a critical distance at or near the mid-span of a given bridge several times a year, the probability that both vehicles would either be the least or the greatest H-equivalency encountered in such traffic is so remote that it may be neglected. In fact it has been shown in Tables 8 and 9 that the two heaviest vehicles likely to occur on a 50-ft bridge at the same time would produce less stress than a single vehicle with one of the higher H-equivalencies.

If the numbers of stress repetitions in a typical interior stringer of the 50- and 100-ft spans are accumulated in 5 percentile groups from Tables 6, 7, 8 and 9, the results would be as shown in Figures 18 and 19, respectively. The amazing thing about these figures is that, even

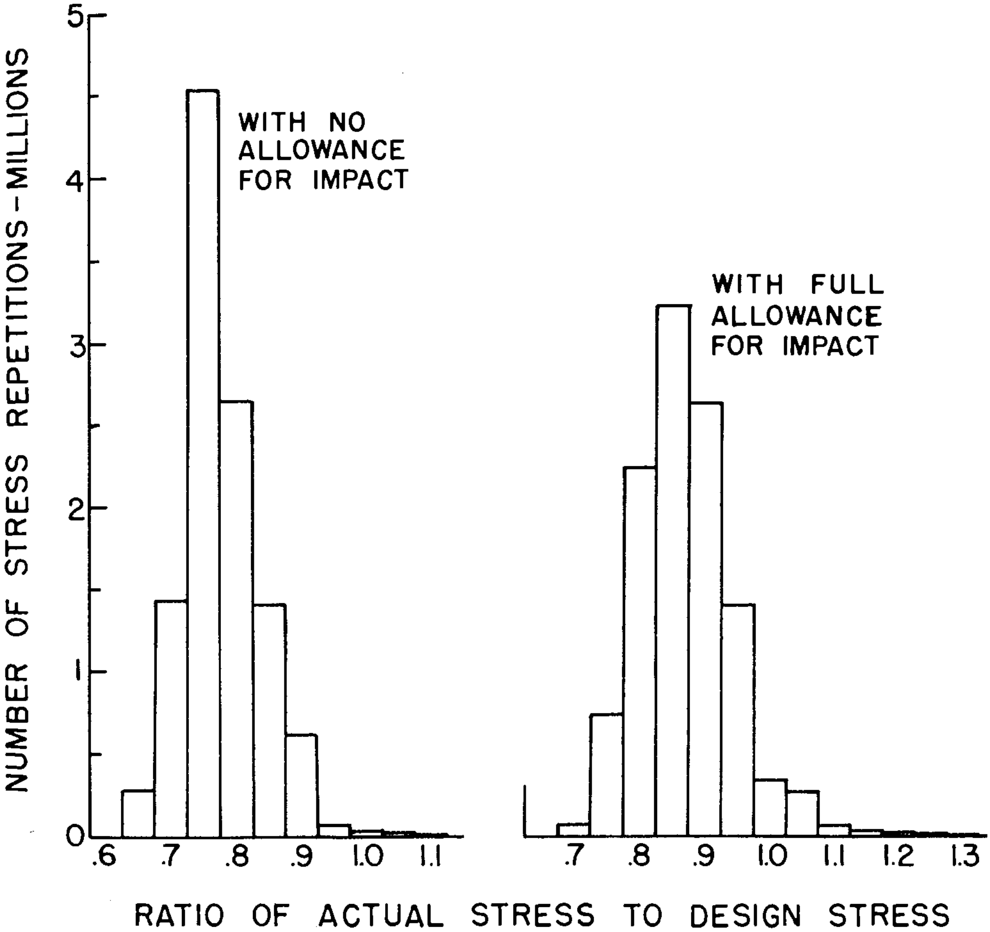

Figure 18. Number of stress repetitions produced in a 50-ft simple span bridge of H 15-44 design during an assumed useful life of 50 years (stress effects based on continuous traffic volume of 500 vph containing 5 percent heavy vehicles).

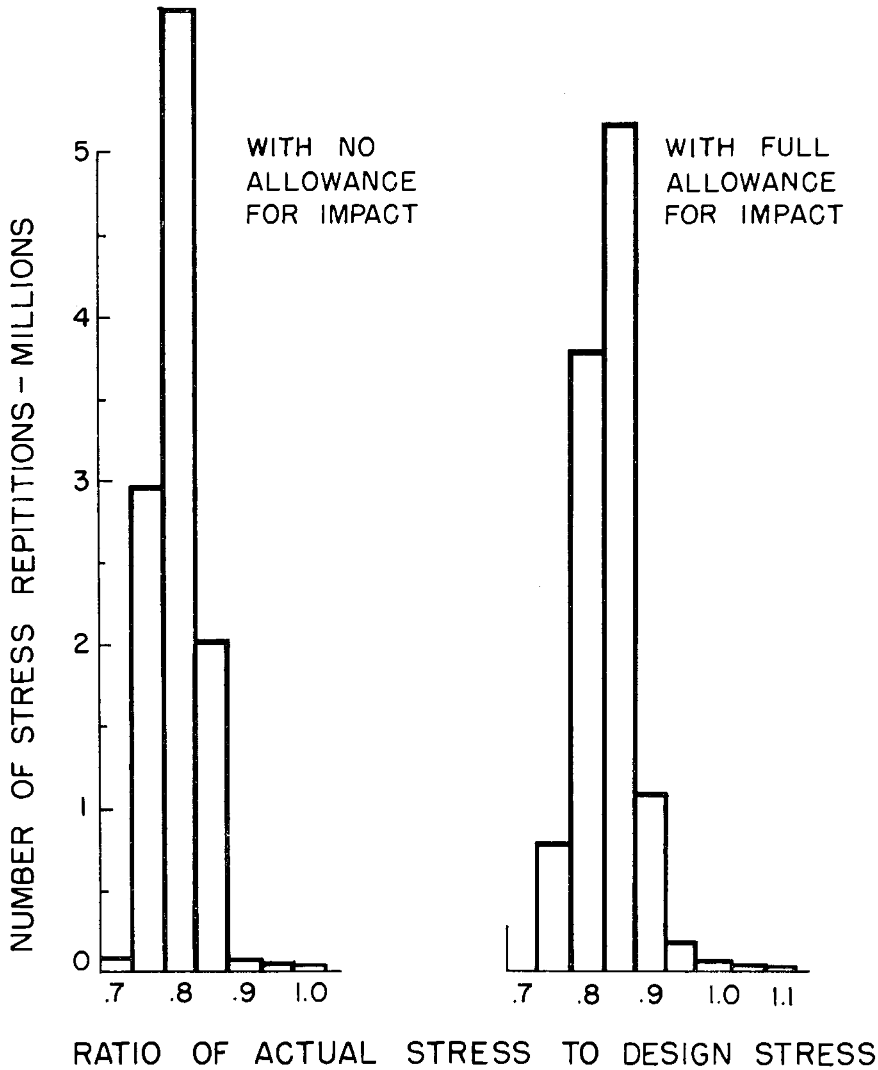

Figure 19. Number of stress repetitions produced in a 100-ft simple span bridge of H 15-44 design during an assumed useful life of 50 years (stress effects based on continuous traffic volume of 500 vph containing 5 percent heavy vehicles).

with full allowance for impact, there is such a small number of stress repetitions in excess of the allowable design stresses that would result from a continuously flowing traffic volume of 500 vehicles per hour or 12,000 vehicles per day containing 5 percent heavy trucks for the full 50-year useful life of each bridge.

Much more could be said about Figures 18 and 19 of course, but it is believed that the implications are sufficiently clear without further explanation or discussion. It might be pointed out, however, that in no case do the maximum bending stresses produced by legal loads approach values that would be considered critical.

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APPENDIX A

Nomenclature and Definitions

A = average number of vehicles per hour in any one designated direction, or total traffic in both directions, as may be specified.

C = coefficient representing the fractional part of the total liveload stress in a given member produced by one or more lanes loaded. $C = 1.00$ if a stringer bridge is loaded with identical vehicles, one in each lane and so placed as to produce maximum stress. For a steel stringer bridge, if one vehicle in one lane only would produce 75 percent as much stress in an interior stringer as identical vehicles in each lane it would mean that $C = 0.75$.

D = average speed of traffic in designated direction.

E = number of events or trials between occurrences of vehicle groups, as may be defined.

G = group of vehicles, as may be defined.

H = equivalent H truck in tons. For example, if a given vehicle produces the same maximum moment (or other stress function) in a given member as a standard H truck weighing 23.6 tons, it would be rated as an equivalent H 23.6 truck loading, in which case $H = 23.6$ tons. H also represents one heavy freight vehicle.

I = impact fraction (maximum 0.30, or 30 percent) as determined by the AASHO formula $I = 50\ (S + 125)$ in which S = length in feet of the portion of the span which is loaded to produce the maximum stress in the member.

I' = impact fraction assumed in connection with the determination of the stress-producing effects of any given vehicle under consideration. For example, if the speed of a given vehicle were limited, to say 5 mph, this impact fraction might be considered so small as to be negligible, in which case I' might be assumed equal to zero. Depending on traffic and conditions, therefore, the impact fraction could be assumed at any reasonable value between zero and the full impact allowance, I, as defined by the AASHO design specifications.

$K = (1.00 + I)$ = coefficient by which the design live load moment (shear, or other stress function) is multiplied to obtain the live load plus impact moment (shear, or other stress function) used for design. Thus, $K\,M_L$ would be equal to the live load plus impact moment used for design; similarly, $K\,V_L$ would be equal to the live load plus impact shear used for design.

$K' = (1.00 + I')$ = coefficient by which the live load moment (shear, or other stress function) produced by a given vehicle is multiplied to obtain the live load plus impact moment (shear, or other stress function) produced on a given span or in a given member by the vehicle under consideration. Thus, $K'\,M_H$ would be equal to the live load plus impact moment produced on a given span by any particular vehicle having an H-equivalency of H tons.

M_D = dead load moment, as included in total design moment.

M_L = live load moment, as included in total design moment.

M_T = moment used for design, or total design moment.

M_H = moment in an interior stringer (or other member) resulting from equivalent H trucks weighing H tons each. Likewise, M_H represents the moment for one lane produced by equivalent H truck weighing H tons.

$M_{H(1)}$ = moment for one lane produced by a standard H truck weighing 1 ton.

P = general term indicating probability that an event (to be defined) will occur as specified.

Q = design stress ratio. This term refers to the ratio of total actual stress to total design stress in any particular member or part of a given highway bridge.

$R_D = (M_D/M_T)$ = ratio of dead load moment M_D (shear, or other stress function) to total moment M_T used for design. In terms of shear this ratio would be $R_D = (V_D/V_T)$, and for other stress functions it would be similar.

$R_L = (K\,M_L/M_T)$ = ratio of live load plus impact moment, $K\,M_L$ (shear, or other stress function), used for design to the total design moment, M_T, or total moment (shear, or other stress function) used for design. In terms of shear, this ratio would be $R_L = (K\,V_L/V_T)$, and for other stress functions it would be similar.

S = span length, or that portion of span which is loaded to produce maximum stress in the member under consideration, in feet.

T = time interval between occurrences of certain specified events, to be defined.

V = vehicle interval between occurrences of certain specified events, to be defined. V may also be used to describe shear as a stress function.

X = length of section or distance along highway (distance interval), in feet within which the grouping of vehicles is to occur.

Z = average number of vehicles expected within a specified length of X feet or a specified time of t seconds, based on total traffic in both directions. For a specified length of X feet, Z=

$AX/5280D$; for a specified time of t seconds, $Z = At/3600$.

$P(2H,X;2) =$
probability of the group, 2H, occurring within X feet in each of the two directions.

$P(G,X;a/2) =$
probability of the group, G, occurring within X feet in any manner in either or both directions.

$E(n,X;2) =$
number of events between occurrences of n vehicles in each of two directions within X feet.

$V(G,X;a/2) =$
vehicle interval between occurrences of the group, G, in any manner in either or both directions within X feet.

$T(G,X;a/2) =$
time interval between occurrences of the group, G, within X feet in either or both directions.

NOTE: The terms given do not show all the possible combinations of symbols for describing conditions associated with vehicle groups on a two or more lane highway. Those shown, however, are typical: other combinations can be selected suitable for describing the particular operation under consideration.

e = exponential base, 2.718,281 . . .

f = unit stress, in psi, or unit stress as may be defined. f_D = unit stress resulting from dead load; f_L = unit stress resulting from live load; f_T = maximum total design stress; f_H = stress resulting from vehicle or vehicles weighing H tons each.

n = number of vehicles in a group or sequence but unassigned as to class or type.

t = time interval, in seconds, within which the grouping of vehicles is to occur.

z = average number of vehicles expected within a length of X feet or a time of t seconds in one designated lane, based on the number of vehicles per hour, (R_1), and average speed of vehicles, D, in that lane.

APPENDIX B

CONVERSION COEFFICIENTS FOR EQUIVALENT LOADINGS ON SIMPLE SPANS

Owing to the fact that an H truck, an H-S truck, or a single concentrated load weighing 1 kip each produce maximum moments and shears on a given span which are definite values, their relative magnitudes may be fully described by the ratios that each bears to the other two. Thus, if these ratios are known for a given span, they may be thought of as coefficients which may be used for converting any one of the foregoing loadings into equivalent loadings measured in terms of either or both of the other two. These ratios or coefficients, based on maximum moments for certain selected spans up to 100 ft in length, are given in Table 10.

In Table 10, for example, it will be seen that the coefficient based on maximum moment, for converting an equivalent H truck loading into an equivalent H-S truck loading on a 50-ft span is given as 1.28. This means that an H truck of given weight will produce 1.28 times as much moment as an H-S truck of equal weight on a 50-ft span. It also means that an H truck of given weight will produce as much moment as an H-S truck weighing 1.28 times as much on a 50-ft span. More specifically, suppose a given heavy vehicle has been found to produce the same moment on a 50-ft span as an H 20 truck and rated accordingly as an equivalent H 20 truck loading. Now suppose it is desired to convert the given heavy vehicle into an equivalent H-S truck loading. This may be done by noting that $1.28 \times 20 = 25.6$ tons would be required on an H-S truck to produce the same moment as the given vehicle on a

TABLE 10

CONVERSION COEFFICIENTS BASED ON MOMENTS FOR EQUIVALENT LOADINGS ON SIMPLE SPANS OF VARIOUS LENGTHS

For Converting[1]	Span 10 ft	20 ft	30 ft	40 ft	50 ft	60 ft	70 ft	80 ft	90 ft	100 ft
EHT to EHST	1.80	1.80	1.57	1.38	1.28	1.22	1.18	1.15	1.13	1.12
EHST to EHT	0.56	0.56	0.64	0.72	0.78	0.82	0.85	0.87	0.88	0.90
EHT to ECL	0.80	0.80	0.82	0.86	0.89	0.91	0.92	0.93	0.94	0.94
ECL to EHT	1.25	1.25	1.22	1.16	1.12	1.10	1.09	1.07	1.07	1.06
EHT to EHD	1.00	1.00	1.00	1.00	1.00	0.98	0.91	0.85	0.80	0.76
EHD to EHT	1.00	1.00	1.00	1.00	1.00	1.02	1.10	1.17	1.25	1.32
EHT to EHSD	1.80	1.80	1.57	1.38	1.28	1.22	1.18	1.15	1.13	1.12
EHSD to EHT	0.56	0.56	0.64	0.72	0.78	0.82	0.85	0.87	0.88	0.90
EHST to ECL	0.44	0.44	0.52	0.62	0.70	0.75	0.78	0.81	0.83	0.85
ECL to EHST	2.25	2.25	1.91	1.60	1.43	1.34	1.28	1.24	1.21	1.18
EHST to EHD	0.56	0.56	0.64	0.72	0.78	0.80	0.77	0.74	0.71	0.68
EHD to EHST	1.80	1.80	1.57	1.38	1.28	1.25	1.29	1.35	1.41	1.48
EHST to EHSD	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
EHSD to EHST	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ECL to EHD	1.25	1.25	1.22	1.16	1.12	1.08	0.99	0.92	0.85	0.80
EHD to ECL	0.80	0.80	0.82	0.86	0.89	0.93	1.01	1.09	1.17	1.25
ECL to EHSD	2.25	2.25	1.91	1.60	1.43	1.34	1.28	1.24	1.21	1.18
EHSD to ECL	0.44	0.44	0.52	0.62	0.70	0.75	0.78	0.81	0.83	0.85
EHD to EHSD	1.80	1.80	1.57	1.38	1.28	1.25	1.29	1.35	1.41	1.48
EHSD to EHD	0.56	0.56	0.64	0.72	0.78	0.80	0.77	0.74	0.71	0.67

[1] EHT = Equivalent H truck loading;
EHD = Equivalent H design loading;
EHST = Equivalent H-S truck loading;
EHSD = Equivalent H-S design loading;
ECL = Equivalent concentrated load.

50-ft span. The given vehicle, therefore, would be rated as an equivalent 25.6 (ton) H-S truck loading or an equivalent 51.2 (kip) H-S truck loading.

In a similar manner, if it were desired to convert an equivalent 51.2 (kip) H-S truck loading into an equivalent H truck loading on a 50-ft span it would be done by multiplying the H-S truck rating by the coefficient 0.78 as shown in Col. 6, Table 10, or $51.2 \times 0.78 = 40.0$ kips. This means that the given vehicle could be rated as either an equivalent 51.2 (kip) H-S truck loading, or an equivalent 40.0 (kip) H truck loading on a 50-ft span.

Similarly, an equivalent 40.0 (kip) H truck loading may be converted into an equivalent concentrated load on a 50-ft span by multiplying the H truck rating by the coefficient 0.89 as shown in Col. 6, Table 10, or $40.0 \times 0.89 = 35.6$ kips. This means that the given vehicle would be rated as an equivalent 35.6 (kip) concentrated load on a 50-ft span.