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STRUCTURAL SYSTEMS IN SUPPORT OF SAFETY: NEW HIGHWAY STRUCTURES DESIGN CONCEPTS. VOLUME III. SUPPORTING DATA

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San Antonio, Texas

September 1969

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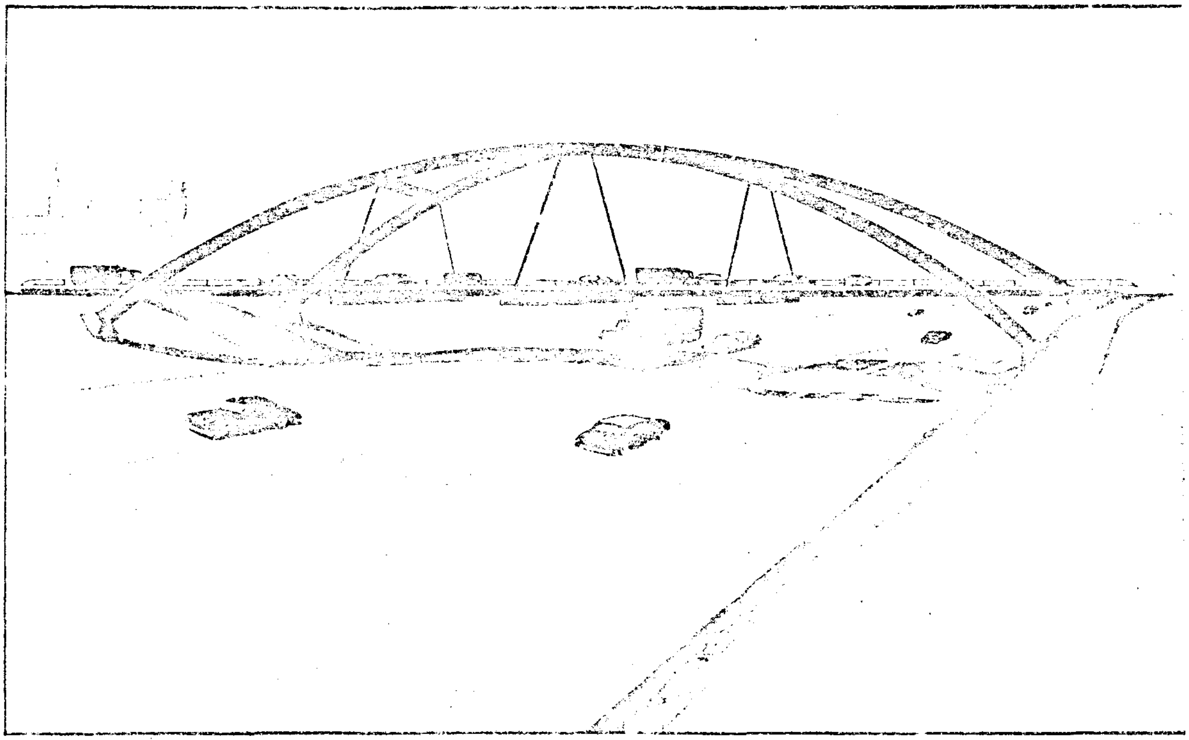

NEW STRUCTURES CONCEPTS FOR HIGHWAY SAFETY

VOLUME III: SUPPORTING DATA

STRUCTURAL SYSTEMS IN SUPPORT OF SAFETY: NEW HIGHWAY STRUCTURES DESIGN CONCEPTS

FINAL REPORT
SwRI Project No. 03-2173

VOLUME III. SUPPORTING DATA

Prepared under
Contract FH-11-6638
for
The Bureau of Public Roads
Federal Highway Administration
Department of Transportation

September 1969

FOREWORD

The investigation reported herein was conducted by Southwest Research Institute in the Department of Structural Research. Joseph F. Minor and Maurice E. Bronstad served as the project Principal Investigators. This report was prepared under Contract No. FH-11-6638 with the Bureau of Public Roads, Federal Highway Administration, Department of Transportation. The scope of work required development of imaginative concepts for highway structures which are responsive to new safety requirements; however, it was specified that these concepts be limited to structural schemes employing structural cable systems in applications which differ from those used in conventional suspension bridges.

The report is presented in three separate volumes:

- Volume I - Research Information
- Volume II - Preliminary Designs and Engineering Data
- Volume III - Supporting Data

Each volume is responsive to different information requirements and is essentially complete within itself. For example, those concerned with study methodology and concept development will be interested in Volume I, while practicing engineers responsible for implementation will find information in Volume II more applicable.

Individuals in both categories who wish to pursue their interests in more detail will find the supporting data contained in Volume III useful. Included are a bibliography, detailed methods of analysis, calculations for preliminary bridge designs, computer program summaries, and supporting data for sign and lighting system support structures preliminary designs.

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ACKNOWLEDGMENTS

The assistance and information provided by many persons in state highway departments and the Bureau of Public Roads are gratefully acknowledged. Direct contributions in the concept selection and review process by Mr. F. J. Tamanini and Mr. Lloyd Cayes of the Bureau of Public Roads, Office of Research and Development, are acknowledged. Suggestions and information provided by Messrs. Odis H. Clark, Jr., Leroy A. Surtees, Jr., and Malcolm L. Steinberg, of the Texas Highway Department, are also acknowledged. Direct assistance provided by Institute staff members is also noted as follows: Dr. Thein Wah, who formulated the theoretical methods, Mr. Raymond E. Kirksey, who supervised computer programming and computer operation activities, and Messrs. Leonard U. Rastrelli, Robert Sherman, Norman I. Turner, Lowell F. Creimann, and R. Kirk Gregory, who contributed concepts and ideas.

ABSTRACT

Volume III of the three volume report contains supporting data for both Volume I (Research Information) and Volume II (Preliminary Designs and Engineering Data). A bibliography of literature reviewed during the concept identification process, detailed methods of analysis for eight bridge concepts given design attention, and engineering data for two concepts (not selected for detailed attention) are included. Calculations for bridge concept preliminary designs, computer programs, and sign and lighting system analysis methods, including comments on dynamic analysis, are included as augmentive information for the highway engineer who wishes to pursue, in detail, one or more of the design concepts presented in Volume II. Methods of analysis, computer program listings, and computer program printouts for the preliminary designs presented in Volume II are also contained herein.

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I. INTRODUCTION

The research information (Volume I) and the preliminary designs and engineering data (Volume II) are syntheses of investigative activities conducted in considerable detail during the accomplishment of the program. In order to not unnecessarily burden these summary documents with presentations of detailed data and calculations, supporting data of the report are contained in this volume.

Information pertinent to the first volume of the report (Research Information) are presented in the first three appendixes. Appendix A is a bibliography of reference material employed in the concept identification and method of analysis review processes. Appendix B contains detailed presentations of methods of analysis for the eight bridge concepts considered in the concept design evaluation portion of Volume I. (These analyses also serve as basis methods of analysis for the four preliminary designs presented in Volume II.) Concept design calculations and discussions for the two bridge concepts which were not given design consideration in Volume II are included in Appendix C.

Supporting data for the second volume of the report (Preliminary Designs and Engineering Data) are presented in the final three appendixes. Analysis and design calculations for the three bridge concepts which received preliminary design consideration are contained in Appendix D; computer program listings and data printouts for these concepts are contained in Appendix E. Thus, complete sets of data for the three bridge concepts which received preliminary design attention can be gained by referring to the appropriate methods of analysis in Appendix B, design calculations in Appendix D, and computer oriented results in Appendix E. Finally, supporting data for sign and light structures are included in Appendix F. This final appendix also includes comments regarding the dynamic analysis of sign support structures.

APPENDIX A

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APPENDIX B

METHODS OF ANALYSIS OF CABLE SUPPORTED BRIDGE CONCEPTS

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APPENDIX B

METHODS OF ANALYSIS FOR CABLE SUPPORTED BRIDGE CONCEPTS

B.1. "A" Frame Bridge Analysis

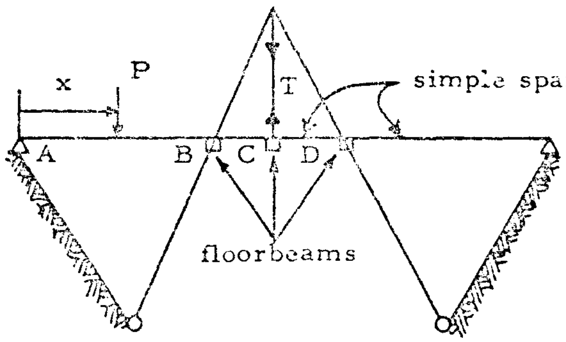


FIGURE B.1. "A" FRAME BRIDGE, ELEVATION VIEW

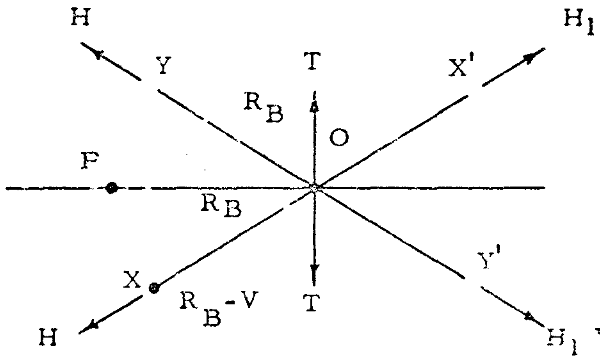


FIGURE B.2. "A" FRAME BRIDGE, PLAN VIEW

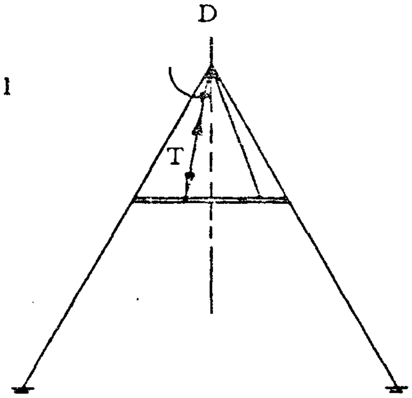


FIGURE B.3. "A" FRAME BRIDGE, TRANSVERSE SECTION AT CENTERLINE

We assume that the frame members are hinged at top and bottom. The floor beams impose concentrated loads only on the frame. It is sufficient to consider the equilibrium of the frame XOX', just as if it were independent of the frame YOY'.

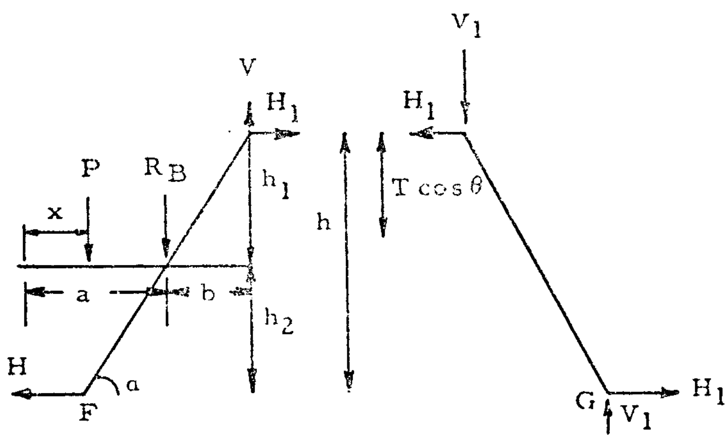


FIGURE B.4. FREE-BODY DIAGRAM OF INCLINED MEMBER

The tension, T, in each suspender has horizontal components which are self-balancing. The vertical components are carried equally by the two frames, XOX' and YOY'. The inclination of the tension, T, to the vertical is θ.

B.1.a. <u>P Between A and B (Fig. B.1)</u>

From Figure B.4, $\sum M_F = 0$ gives

$$Vh \cot \alpha - Hh - R_B h_2 \cot \alpha = 0 \quad (1)$$

Since the reaction at D is zero, $\sum M_G = 0$ gives

$$V_1 h \cot \alpha + H_1 h = 0$$

and (2)

$$H_1 = - V_1 \cot \alpha$$

Equilibrium of the "joint" gives

$$V = V_1, \quad H = H_1 \quad (3)$$

Substituting (2) and (3) into (1), we have

$$Vh \cot \alpha + Vh \cot \alpha - R_B h_2 \cot \alpha = 0$$

$$V = \frac{R_B h_2}{2h} \quad (4)$$

and

$$H = -\frac{R_B h_2}{2h} \cot \alpha$$

But,

$$R_B = \frac{Px}{2a}$$

So, for $0 \leq x \leq a$,

$$\left.\begin{aligned} H &= -\frac{PxH_2}{4ah} \cot \alpha \\ V &= \frac{Ph_2x}{4ah} \\ T &= 0 \end{aligned}\right\} \tag{5}$$

B.1.b <u>P Between B and C</u>

$$R_B = \frac{P}{2}\frac{(a + b - x)}{b} \tag{6}$$

$$T \cos \theta = \frac{P}{2}\frac{(x - a)}{b} \tag{7}$$

Equilibrium of the "joint" gives (Fig. B.4) $\Sigma F_y = 0$,

$$V + T \cos \theta - V_1 = 0 \tag{8}$$

$$\sum F_x = 0, \quad H_1 = H \tag{9}$$

$$\sum M_g = 0, \quad H_1 = - V_1 \cot \alpha \tag{10}$$

$$\sum M_f = 0, \quad Vh \cot \alpha - Hh - \frac{P}{2}\frac{(a + b - x)}{b} h_2 \cot \alpha = 0 \tag{11}$$

From (9) and (10),

$$V_1 = - H \tan \alpha \tag{12}$$

Substituting from (7) and (12) into (8), we have

$$V + \frac{P}{2}\frac{(x - a)}{b} + H \tan \alpha = 0 \tag{13}$$

Multiplying (13) by h cot α, we have

$$Vh \cot \alpha + \frac{P}{2}\frac{(x - a)}{b} h \cot \alpha + Hh = 0 \tag{14}$$

Subtracting (14) from (11), we find

$$-2Hh - \frac{P}{2b}\left\{(x - a)h + (a + b - x)h_2\right\} \cot \alpha = 0$$

So,

$$H = -\frac{P}{4bh} \cot \alpha \left\{(x - a)h + (a + b - x)h_2\right\} \tag{15}$$

and

$$V = -\frac{P}{2}\frac{(x - a)}{b} - \frac{P}{4bh}\left\{(x - a)h + (a + b - x)h_2\right\}$$

or

$$V = \frac{P}{4bh}\left\{(a + b - x)h_2 - (x - a)h\right\} \tag{16}$$

and

$$T = \frac{P(x - a)}{2b} \sec \theta, \quad R_B = \frac{P}{2}\frac{(a + b - x)}{b} \tag{17}$$

B. 2. Leaning Piers Bridge Analysis

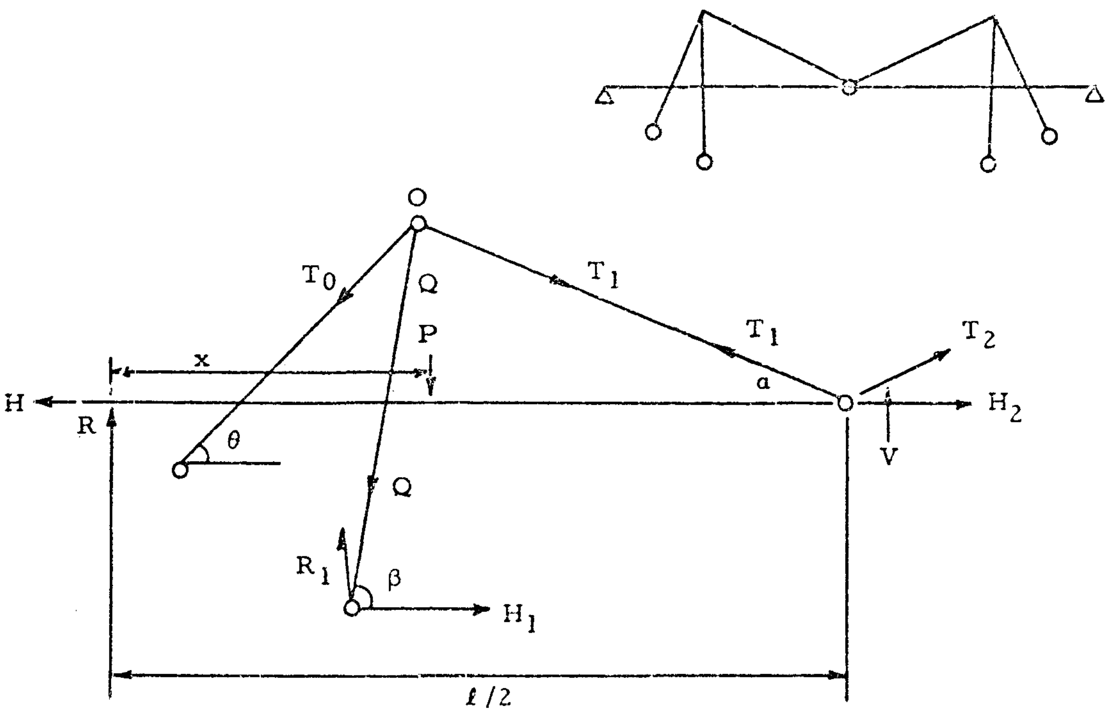


FIGURE B. 5. LEANING PIERS BRIDGE, ELEVATION VIEW

Consider the equilibrium of joint O as the strut is assumed to take direct force only:

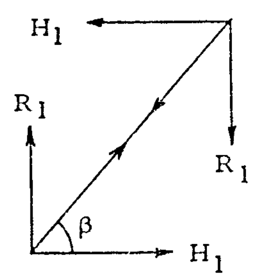


FIGURE B. 6. FREE-BODY DIAGRAM
OF LEANING PIER

$$T_0 \sin \theta + T_1 \sin \alpha - R_1 = 0 \tag{1}$$

and

$$T_0 \cos \theta - T_1 \cos \alpha - H_1 = 0 \tag{2}$$

We have

$$R_1 = Q \sin \beta, \qquad H_1 = Q \cos \beta \tag{3}$$

where Q is the force in the strut.

If we substitute from (3) into (1) and (2) and solve for T_0 and T_1 in terms of Q, we get

$$T_0 = Q \sin (\beta + \alpha) / \sin (\theta + \alpha)$$

and (4)

$$T_1 = Q \sin (\beta - \theta) / \sin (\theta + \alpha)$$

Taking moments of the forces acting on the bridge about the center hinge, we have

$$R \frac{\ell}{2} - P\left(\frac{\ell}{2} - x\right) = 0$$

and

$$R = P \;\; 1 - \frac{2x}{\ell} \tag{5}$$

It is evident that the right-hand reaction $R_2 = 0$, since the shear $V = 0$. The vertical equilibrium of the bridge then gives

$$(T_1 + T_2) = \frac{Px}{\ell} \operatorname{cosec} \alpha \tag{6}$$

Let us assume that the bridge is free to move horizontally. Then $H = H_2 = 0$. The horizontal equilibrium of the <u>hinge</u> then gives

$$T_1 = T_2$$

So,

$$T_1 = \frac{Px}{2\ell} \operatorname{cosec} \alpha \tag{7}$$

$$\frac{Q \sin(\beta - \theta)}{\sin(\theta + \alpha)} = \frac{Px}{2\ell} \operatorname{cosec} \alpha$$

$$Q = \frac{Px}{2\ell} \frac{\operatorname{cosec} \alpha \sin(\theta + \alpha)}{\sin(\beta - \theta)} \tag{8}$$

$$T_0 = \frac{Px}{2\ell} \frac{\operatorname{cosec} \alpha \sin(\theta + \alpha)}{\sin(\beta - \theta)} \frac{\sin(\beta + \alpha)}{\sin(\theta + \alpha)}$$

$$= \frac{Px}{2\ell} \frac{\operatorname{cosec} \alpha \sin(\beta + \alpha)}{\sin(\beta - \theta)} \tag{9}$$

B. 3. Braced Arch Bridge Analysis

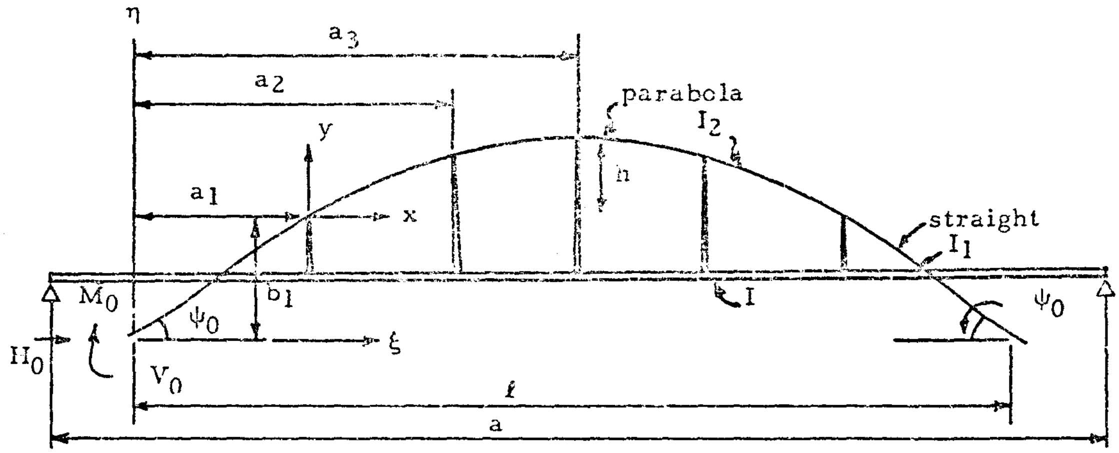


FIGURE B. 7. BRACED ARCH BRIDGE, ELEVATION VIEW

$\xi = x + a_1, \qquad \eta = y + b_1$

$\xi_1^2 = a_1^2 + 2a_1 x + x^2, \qquad \eta^2 = y^2 + 2b_1 y + b_1^2$

$\xi \ \eta = xy + a_1 y + b_1 x$

The equations for determining M_0, H_0, and V_0 are

$$-M_0 \int_0^s \eta \frac{ds}{I} - V_0 \int_0^s \xi\eta \frac{ds}{I} + H_0 \int_0^s \eta^2 \frac{ds}{I} + \int_0^s \frac{M'\eta ds}{I} = 0 \tag{1}$$

$$M_0 \int_0^s \xi \frac{ds}{I} + V_0 \int_0^s \xi^2 \frac{ds}{I} - H_0 \int_0^s \xi\eta \frac{ds}{I} - \int_0^s \frac{M'\xi ds}{I} = 0 \tag{2}$$

$$M_0 \int_0^s \frac{ds}{I} + V_0 \int_0^s \xi \frac{ds}{I} - H_0 \int_0^s \eta \frac{ds}{I} - \int_0^s \frac{M'ds}{I} = 0 \tag{3}$$

We assume that the vertical component in each suspender is T_1, T_2, etc., then the bending moment M' is

$$M' = 2 \left\{ T_1(\xi - a_1) + T_2(\xi - a_2) + \ldots \right\} \tag{4}$$

In Equation (4), $\xi - a_k = 0$ if negative.

We take the equation of the arch to be

$$y = a_0 x + \frac{x^2}{c} \quad \text{(see Fig. B.7)}$$

Let height of arch = h and span = $\ell - 2a_1$

$$h = a_0 \frac{(\ell - 2a_1)}{2} + \frac{(\ell - 2a_1)^2}{4c}$$

$a_0 = \tan \psi_0$. Therefore,

$$\frac{(\ell - 2a_1)^2}{4c} = h - \frac{(\ell - 2a_1)}{2} \tan \psi_0$$

$$c = \frac{\dfrac{(\ell - 2a_1)^2}{4}}{\left\{ h - \dfrac{(\ell - 2a_1)}{2} \tan \psi_0 \right\}}, \quad a_0 = \tan \psi_0 \tag{5}$$

where

$$\tan \psi = a_0 + \frac{2x}{c}$$

$$\tan \psi_k = a_0 + 2\frac{a_k}{c}$$

$$\psi_k = \text{arc tan} \left\{ a_0 + 2\frac{a_k}{c} \right\} \tag{6}$$

These equations determine all properties of the arch.

Equations (1), (2), and (3) may be written in the form

$$-M_0 k_1 - V_0 c k_2 + 4H_0 c k_3 + \sum_{k=1}^{5} c T_k \left\{ h_{2k} - \frac{a_k}{c} h_{1k} \right\} = 0 \tag{7}$$

$$M_0 k_4 + V_0 c k_5 - \frac{H_0 c}{4} k_2 - \sum_{k=1}^{5} T_k c \left\{ h_{5k} - \frac{a_k}{c} h_{3k} \right\} = 0 \tag{8}$$

$$M_0 k_6 + V_0 c k_4 - \frac{H_0 c}{4} k_1 - \sum_{k=1}^{5} T_k c \left\{ h_{3k} - \frac{a_k}{c} h_{4k} \right\} = 0 \tag{9}$$

where T_R is the sum of the two suspenders.

We define the following symbols

$$\phi_1(\psi) = \tan\psi \sec\psi + \ln(\tan\psi + \sec\psi)$$

$$\phi_2(\psi) = \sec^3\psi$$

$$\phi_3(\psi) = \sin\psi \sec^4\psi - \frac{1}{2}\sin\psi \sec^2\psi - \frac{1}{2}\ln(\sec\psi + \tan\psi) \tag{10}$$

$$\phi_4(\psi) = \sec^3\psi\ (3\sec^2\psi - 5)$$

$$\phi_5(\psi) = \frac{1}{3}\sin\psi \sec^6\psi - \frac{7}{12}\sin\psi \sec^4\psi + \frac{1}{8}\sin\psi \sec^2\psi + \frac{1}{8}\ln(\sec\psi + \tan\psi)$$

$$f_1(\psi) = \phi_5 - \phi_3 \tan^2\psi_0 + \phi_1 \tan^4\psi_0$$

$$f_2(\psi) = \frac{\phi_4}{15} - \frac{\phi_2}{3}\tan^2\psi_0 - \frac{\phi_3}{4}\tan\psi_0 + \frac{\phi_1}{2}\tan^3\psi_0$$

$$f_3(\psi) = \frac{\phi_2}{3} - \frac{\phi_1}{2}\tan\psi_0 \tag{11}$$

$$f_4(\psi) = \frac{\phi_3}{2} - \phi_1 \tan^2\psi_0$$

$$f_5(\psi) = \frac{\phi_3}{8} - \frac{\phi_2}{3}\tan\psi_0 + \phi_1 \tan^2\psi_0$$

$$k_1 = f_4(-\psi_0) - f_4(\psi_0) + \frac{16 I_2}{I_1}\frac{a_1^2}{c^2} g_4 + \frac{4b_1}{c}\left\{\phi_1(-\psi_0) - \phi_1(\psi_0)\right\}$$

$$k_2 = f_2(-\psi_0) - f_2(\psi_0) + \frac{a_1}{c}\left\{f_4(-\psi_0) - f_4(\psi_0)\right\} + \frac{4b_1}{c}\left\{f_3(-\psi_0) - f_3(\psi_0)\right\} + \frac{16}{3}\frac{a_1^3}{c^3}\frac{I_2}{I_1} g_2 \tag{12 Cont'd}$$

$$k_3 = f_1(-\psi_0) - f_1(\psi_0) + 8\frac{b_1}{c_1}\left\{f_4(-\psi_0) - f_4(\psi_0)\right\}$$

$$+ 16\frac{b_1^2}{c^2}\left\{\phi_1(-\psi_0) - \phi_1(\psi_0)\right\} + \frac{64}{3}\frac{I_2}{I_1}\frac{a_1^3}{a^3}g_1$$

$$k_4 = f_3(-\psi_0) - f_3(\psi_0) + \frac{a_1}{c}\left\{\phi_1(-\psi_0) - \phi_1(\psi_0)\right\} + \frac{2a_1^2}{c^2}\frac{I_2}{I_1}g_3$$

$$k_5 = f_5(-\psi_0) - f_5(\psi_0) + \frac{2a_1}{c}\left\{f_3(-\psi_0) - f_3(\psi_0)\right\} \qquad \text{(12 Concl)}$$

$$+ \frac{a_1^2}{c^2}\left\{\phi_1(-\psi_0) - \phi_1(\psi_0)\right\} + \frac{4}{3}\frac{a_1^3}{c^3}\frac{I_2}{I_1}g_5$$

$$k_6 = \phi_1(-\psi_0) - \phi_1(\psi_0) + \frac{4a_1}{c}\frac{I_2}{I_1}g_0$$

$$g_0 = 2\sec\psi_0$$

$$g_1 = 2\tan^2\psi_0 \operatorname{cosec}\psi_0$$

$$g_2 = \tan\psi_0 \sec\psi_0 \left(2 + \frac{3}{2}\frac{a_5}{a_1}\right)$$

$$g_3 = \left(1 + \frac{\ell + a_5}{a_1}\right)\sec\psi_0 \qquad (13)$$

$$g_4 = \tan^2\psi_0 \operatorname{cosec}\psi_0$$

$$g_5 = \sec\psi_0\left\{2 + \frac{3a_5}{a_1}\left(1 + \frac{2a_5}{a_1}\right)\right\}$$

$$h_{1k} = f_4(-\psi_0) - f_4(\psi_k) + \frac{4b_1}{c}\left\{\phi_1(-\psi_0) - \phi_1(\psi_k)\right\}$$

$$+ \frac{8a_1^2}{c^2}\frac{I_2}{I_1}\tan^2\psi_0 \operatorname{cosec}\psi_0$$

$$h_{2k} = f_2(-\psi_0) - f_2(\psi_k) + \frac{a_1}{c}\left\{f_4(-\psi_0) - f_4(\psi_k)\right\}$$

$$+ \frac{4b_1}{c}\left\{f_3(-\psi_0) - f_3(\psi_k)\right\} + \frac{16a^3}{3c^3}\frac{I_2}{I_1}\tan\psi_0 \sec\psi_0$$

$$\times \left(1 + \frac{3}{2}\frac{a_5}{a_1}\right)$$

$$h_{3k} = f_3(-\psi_0) - f_3(\psi_k) + \frac{a_1}{c}\left\{\phi_1(-\psi_0) - \phi_1(\psi_k)\right\} \qquad (14)$$

$$+ \frac{2a_1}{c}\frac{I_2}{I_1}\sec\psi_0 \frac{(\ell + a_5)}{c}$$

$$h_{4k} = \phi_1(-\psi_0) - \phi_1(\psi_k) + \frac{a_1}{c}\frac{4I_2}{I_1}\sec\psi_0$$

$$h_{5k} = f_5(-\psi_0) - f_5(\psi_k) + \frac{2a_1}{c}\left\{f_3(-\psi_0) - f_3(\psi_k)\right\}$$

$$+ \frac{a_1^2}{c^2}\left\{\phi_1(-\psi_0) - \phi_1(\psi_k)\right\} + \frac{a_1^3}{c^3}\frac{4}{3}\frac{I_2}{I_1}\sec\psi_0$$

$$\times\left\{1 + \frac{3a_5}{a_1}\left(1 + \frac{2a_5}{a_1}\right)\right\}$$

The next step is to solve Equations (7), (8), and (9) in the form

$$\begin{Bmatrix} M_0 \\ V_0 c \\ H_0 c \end{Bmatrix} = [c_y] \begin{Bmatrix} T_1 c \\ T_2 c \\ T_5 c \end{Bmatrix} \qquad (15)$$

where

$$[c_{ij}] = [A_{ij}]^{-1}_{3\times 3}\,[B_{ij}]_{3\times 5} \tag{16}$$

$$[A_{ij}] = \begin{bmatrix} -k_1 & -k_2 & 4k_3 \\ k_4 & k_5 & -\frac{k_2}{4} \\ k_6 & k_4 & -\frac{k_1}{4} \end{bmatrix} \tag{17}$$

$$[B_{ij}] = \begin{bmatrix} \left(h_{2,1} - \frac{a_1}{c}h_{1,1}\right) & \left(h_{2,2} - \frac{a_2}{c}h_{1,2}\right) \text{-----} \left(h_{2,5} - \frac{a_5}{c}h_{1,5}\right) \\ \left(h_{5,1} - \frac{a_1}{c}h_{3,1}\right) & \left(h_{5,2} - \frac{a_2}{c}h_{3,2}\right) \text{-----} \left(h_{5,5} - \frac{a_5}{c}h_{3,5}\right) \\ \left(h_{3,1} - \frac{a_1}{c}h_{4,1}\right) & \left(h_{3,2} - \frac{a_2}{c}h_{4,2}\right) \text{-----} \left(h_{3,5} - \frac{a_5}{c}h_{4,5}\right) \end{bmatrix} \tag{18}$$

We then obtain

$$\begin{aligned} M_0 &= \sum c T_k \alpha_k \\ V_0 c &= \sum c T_k \beta_k \\ H_0 c &= \sum c T_k \gamma_k \end{aligned} \tag{19}$$

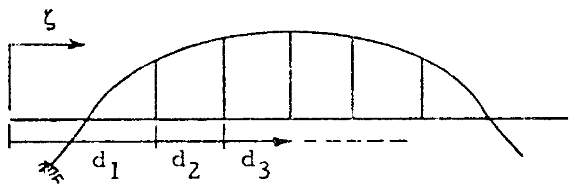


FIGURE B.8. SPAN NOTATION

The deflection w at a point ζ due to a load at z in a simply supported beam is

$$w = \frac{Pa^3}{6EI}\, f(\zeta, z, \eta_{\zeta, z})$$

$$w|_{d = d_j} = \frac{a^3}{6EI}\left\{Pf\left(d_j, z, \eta_{d_j, z}\right) - \sum_k T_k f\left(d_j, d_k, \eta_{d_j, d_k}\right)\right\}$$

For the compatibility of deformation of arch and beam, we must have

$$\frac{c^2}{4I_2}\sum_{k=1}^{5} cT_k q_{kj} + \frac{T_j \ell_j}{2a_j} = \frac{a^3}{6I}\left\{Pf\left(d_j, z, \eta_{d_j, z}\right) - \sum_k T_k f\left(d_j, d_k, \eta_{d_j, d_k}\right)\right\}$$

$$\frac{3}{2}\frac{c^2}{a^3}\frac{I}{I_2}\sum T_k q_{kj} + \sum T_k f\left(d_j, d_k, \eta_{d_j}, d_k\right) + \frac{T_j \ell_j 6I}{2A_j a^3} = Pf\left(d_j, z, \eta_{d_j z}\right) \qquad (20)$$

These equations determine T_k for any value of z

where

$$f(x, \xi, \eta_{x,\xi}) = \frac{x\xi}{a^2}\left\{2\left(1 - \frac{x}{a}\right)\left(1 - \frac{\xi}{a}\right) + \left(\frac{x}{a} - \frac{\xi}{a}\right)^2\right\} - \frac{\eta}{a}\left(\frac{x}{a} - \frac{\xi}{a}\right)^2 \qquad (21)$$

$\eta = x, \qquad x \leq \xi$

$\eta = \xi, \qquad x \geq \xi$

and

$$q_{kj} = \alpha_k\left\{\frac{a_j}{c}h_{4j} - h_{3j}\right\} + \beta_k\left\{\frac{a_j}{c}h_{3j} - h_{5j}\right\} + \frac{\gamma_k}{4}\left\{h_{2j} - \frac{a_j}{c}h_{1j}\right\} + \left\{h_{5p} - \frac{(a_k + a_j)}{c}h_{3p} + \frac{a_k a_j}{c^2}h_{4p}\right\}$$

where

$$p = j, \qquad j \geq k$$

$$p = k, \qquad k > j \tag{22}$$

$$j = 1, 2, \ldots 5, \qquad k = 1, 2, 3, 4, 5$$

We next solve the set of five equations:

$$\sum_{k=1}^{5} T_k \left\{ f \left(d_j, d_k, \eta_{d_j, d_j} \right) + \frac{3}{2} \frac{c^3}{a^3} \frac{I}{I_2} q_{kj} \right\} + \frac{3T_j \ell_j I}{A_j a^3}$$

$$= f \left(d_j, z, \eta_{d_j, z} \right) \tag{23}$$

$(j = 1, 2, 3, \ldots 5)$

for any chosen value of z. This gives T_k for a given z. Thus, the tensions are determined for any position of the load, and all quantities required in the arch and beam can be determined.

B.4. Bridle Bridge Analysis

B.4.a. Bridle Bridge with Hinge

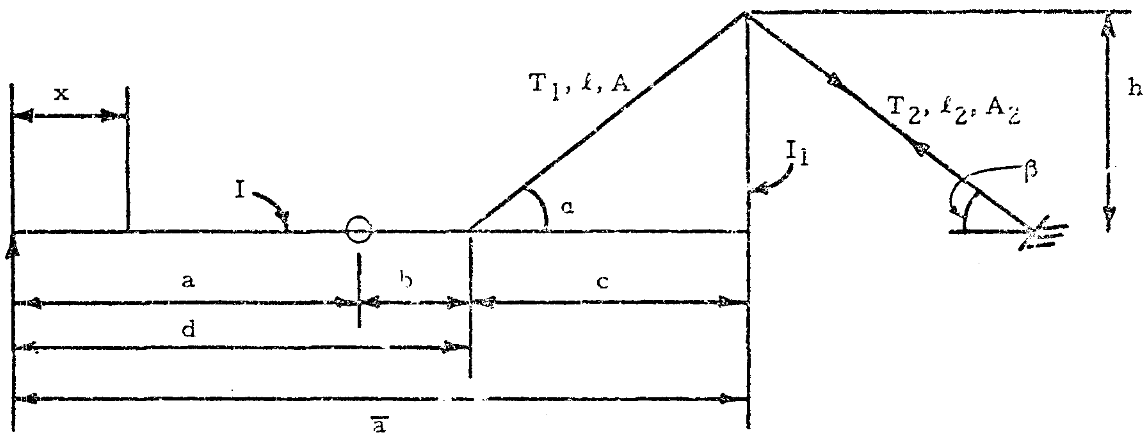


FIGURE B.9. BRIDLE BRIDGE, ELEVATION VIEW

x ≤ a

$$R_1 a - P(a - x) = 0$$

$$R_1 = \frac{P(a - x)}{a} \qquad (1)$$

$$R_1 - P + 2T_1 \sin\alpha + R_2 = 0 \qquad (2)$$

$$R_2 \times c - R_1(a + b) + P(a + b - x) = 0$$

$$R_2 \times c - P(a - x)\frac{(a + b)}{a} + P(a + b - x) = 0$$

$$R_2 c = -\frac{Pbx}{a}$$

$$R_2 = -\frac{Pbx}{ac} \qquad (3)$$

$$\frac{P(a - x)}{a} - P + 2T_1 \sin\alpha - \frac{Pbx}{ac} = 0$$

$$2T_1 \sin\alpha = P\left\{\frac{bx}{ac} + \frac{x}{a}\right\}$$

$$T_1 = \frac{xP \operatorname{cosec}\alpha}{2a}\left(1 + \frac{b}{c}\right) \qquad (4)$$

$$T_1 \cos\alpha = T_2 \cos\beta \qquad (5)$$

$$T_2 = \frac{T_1 \cos\alpha}{\cos\beta} \qquad (6)$$

The compression in the tower is

$$F = T_1 \sin\alpha + T_2 \sin\beta$$

$$F = T_1\left\{\sin\alpha + \cos\alpha \tan\beta\right\}$$

$$F_1 = \frac{xP}{2a}\left(1 + \frac{b}{c}\right)(1 + \cot\alpha \tan\beta) \qquad (7)$$

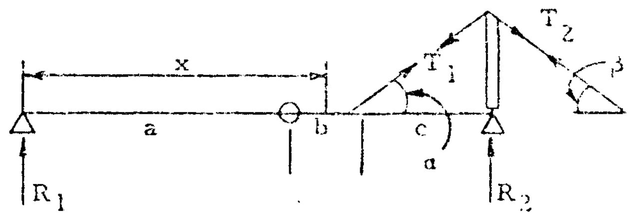


FIGURE B.10. FREE-BODY DIAGRAM OF VERTICAL MEMBER

<u>$x \geq a$</u>

$$R_1 = 0$$

$$R_2 \times c + P(a + b - x) = 0$$

$$R_2 = -\frac{P(a + b - x)}{c}$$

$$R_2 + 2T_1 \sin\alpha - P = 0$$

$$2T_1 \sin\alpha = P\left\{1 + \frac{a + b - x}{c}\right\} \qquad (8)$$

$$T_1 = \frac{P}{2} \operatorname{cosec} \alpha \left(1 + \frac{a + b - x}{c}\right) \tag{9}$$

$$F = \frac{P}{2}\left(1 + \frac{a + b - x}{c}\right)(1 + \cot \alpha \tan \beta) \tag{10}$$

If the horizontal components of the tension, T_1 and T_2, are not equal, there will be bending in the tower.

Equations (4) and (9) are always valid. If the bending of the tower is taken into account, T_2 is to be calculated from the formula

$$T_2 = \frac{T_1 \mu_2 \cos \alpha}{1 + \mu_2 \cos \beta} \quad , \quad \mu_2 = \frac{A_2 h^3}{3 I_1 \ell_2} \cos \beta \tag{11}$$

B. 4. b. Bridle Bridge Without Hinge

This is the same as in Figure B. 9, except that the hinge at the point x = a is removed.

The equation for determining the tension T_1 is

$$T_1 = f(d, x, \eta) \operatorname{cosec} \alpha / \lambda_1 \tag{12}$$

$$\lambda_1 = \frac{\ell I}{A \bar{a}^3}\left(1 - \frac{\mu_1 \cos \alpha}{1 + \mu_2 \cos \beta}\right) + \frac{d^2}{3 \bar{a}^2}\left(1 - \frac{d}{\bar{a}}\right)^2$$

$$\mu_1 = \frac{A h^3}{3 I_1 \ell} \cos \alpha \quad , \quad \mu_2 = \frac{A_2 h^3}{3 I_1 \ell_2} \cos \beta$$

$$f(d, x, \eta) = \frac{1}{6} \frac{dx}{a^2} \left\{ 2 \left(1 - \frac{d}{a}\right) \left(1 - \frac{x}{a}\right) + \left(\frac{d}{a} - \frac{x}{a}\right)^2 \right\} - \frac{\eta}{a} \left(\frac{d}{a} - \frac{x}{a}\right)^2$$

$\eta = d \quad , \quad d \leq x$

$\eta = x \quad , \quad d \geq x$

$$T_2 = \frac{T_1 \mu_2 \cos \alpha}{1 + \mu_2 \cos \beta} \tag{13}$$

B.5. Stayed Girder Bridge Analysis

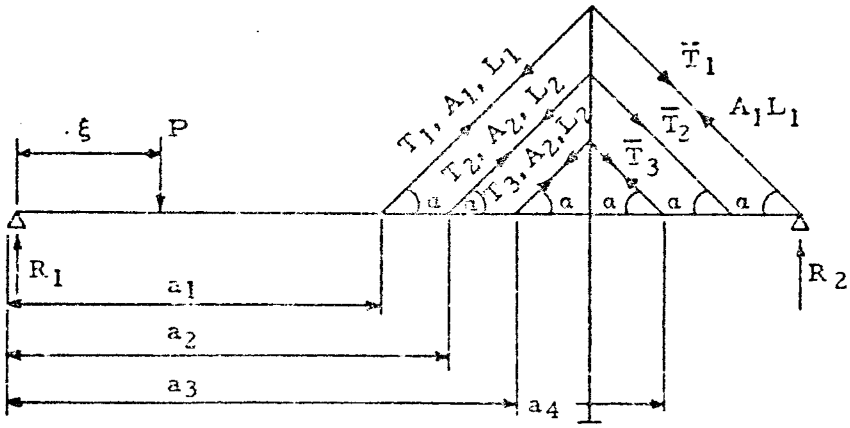


FIGURE B.11. STAYED GIRDER BRIDGE, ELEVATION VIEW

This is a statically indeterminate structure. We apply the statically indeterminate tensions as loads on the beam. The deflection at each point must be compatible with the stretching of the cable.

The basic equations are

$$\frac{T_j A_j L_j}{E_j} \sin a = w_{T_j}, \qquad \text{for } j = 1, 2, \ldots 6 \tag{1}$$

where the right-hand side is the deflection of the beam and the left-hand side is the stretching of the tie.

The deflection at x due to a load P at ξ on a simply supported beam of length a is

$$W = \frac{Pa^3}{6EI}\left[\frac{x\xi}{a^2}\left\{2\left(1-\frac{x}{a}\right)\left(1-\frac{\xi}{a}\right)+\left(\frac{x}{a}-\frac{\xi}{a}\right)^2\right\}-\frac{\eta}{a}\left(\frac{x}{a}-\frac{\xi}{a}\right)^2\right] \tag{2}$$

where

$\eta_{x\xi} = x, \qquad x \leq \xi$

$\eta = \xi, \qquad x \geq \xi$

or

$$w = \frac{Pa^3}{6EI} f(x, \xi, \eta_{x, \xi}) \tag{3}$$

The deflection at $x = a_s$ is

$$w_{x=a_1} = \frac{a^3}{6EI}\Big[Pf\left(a_s, \xi, \eta_{a_s\xi}\right) - 2T_1 f\left(a_s, a_1, \eta_{a_s a_1}\right)\sin\alpha$$

$$- 2T_2 f\left(a_s, a_2, \eta_{a_s a_2}\right)\sin\alpha \ldots$$

$$- 2T_s f\left(a_s, a_s, \eta_{a_s a_s}\right)\sin\alpha \ldots\Big] = \frac{2T_s a_s L_s}{E_s}\sin\alpha \tag{4}$$

or

$$Pf\left(a_s, \xi, \eta_{a_s, \xi}\right) - 2\sum_{j}^{J} T_j f\left(a_s, a_j, \eta_{a_s a_j}\right)\sin\alpha$$

$$- 2T_s f\left(a_s, a_s, \eta_{a_s, a_s}\right)\sin\alpha = 2T_s \sin\alpha k_s, \qquad j \neq s$$

$$k_s = \frac{a_s L_s}{E_s}\cdot\frac{6EI}{a^3} \tag{5}$$

Taking $P = 1$,

$$T_s\left\{f\left(a_s, a_s, \eta_{a_s, a_s}\right) + k_s\right\} + \sum_{j=1}^{J} T_j f\left(a_s, a_j, \eta_{a_s, a_j}\right)$$

$$= \frac{\operatorname{cosec}\alpha}{2} f\left(a_s, \xi, \eta_{a_s, \xi}\right), \qquad j \neq s \tag{6}$$

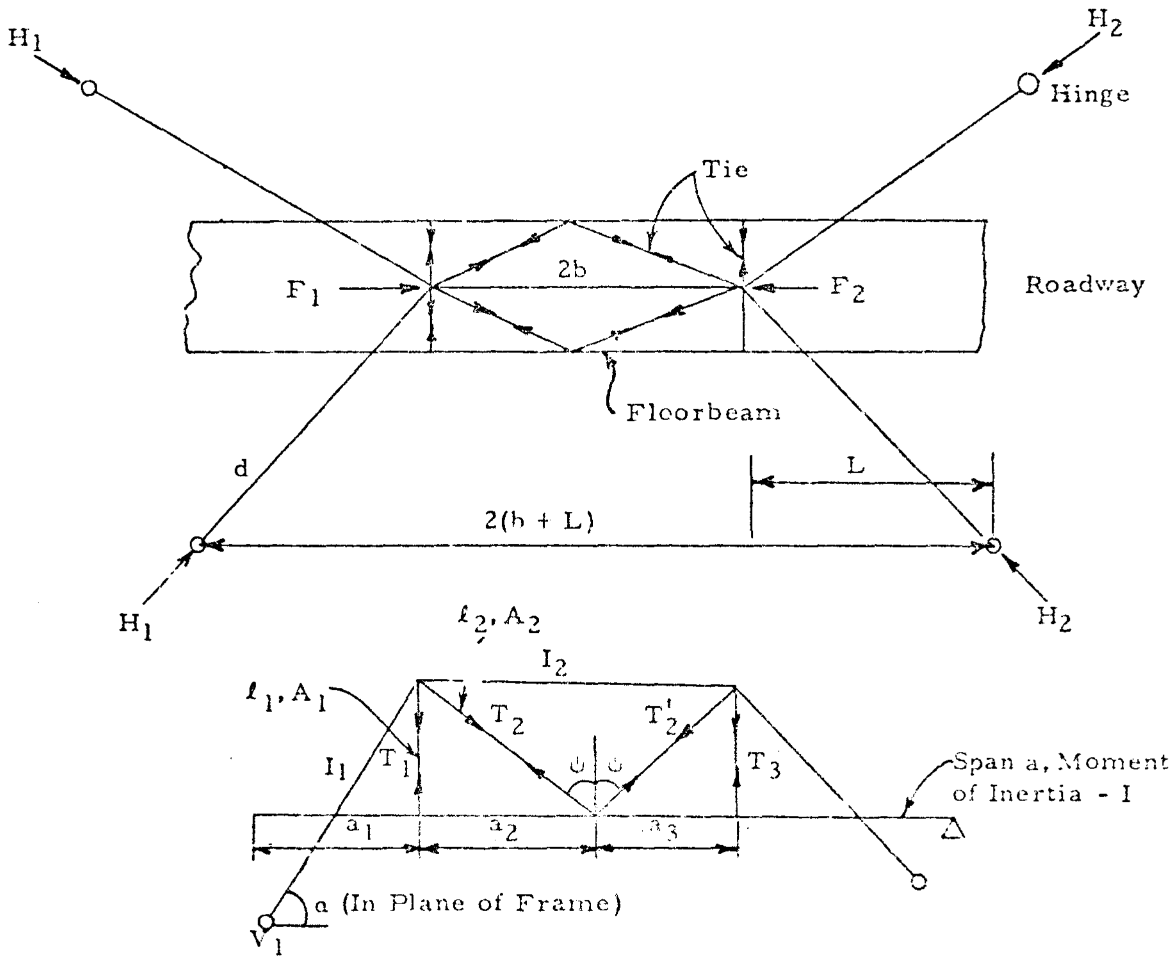
$s = 1, 2, \ldots J$

$j = 1, 2, \ldots J$

There are as many equations as there are tension members, so the statically indeterminate quantities can be determined. Each solution is valid for a given position $x = \xi$ of the unit load.

B.6. Frame Bridge Analysis

B.6.a. Frame Bridge with Continuous Girder



NOTE: Inclination of plane of frame to vertical = ϕ

FIGURE B.12. FRAME BRIDGE, PLAN AND ELEVATION VIEWS

We shall assume that all the forces act in the planes of the frames. This means that the tension members are in the planes of the frames, or, equivalently, the floor beams are in the planes of the frames. This, in fact, is a desirable feature. Obvious modifications may be made if the ties are not in the plane of the frame.

Solution of Rigid Frame

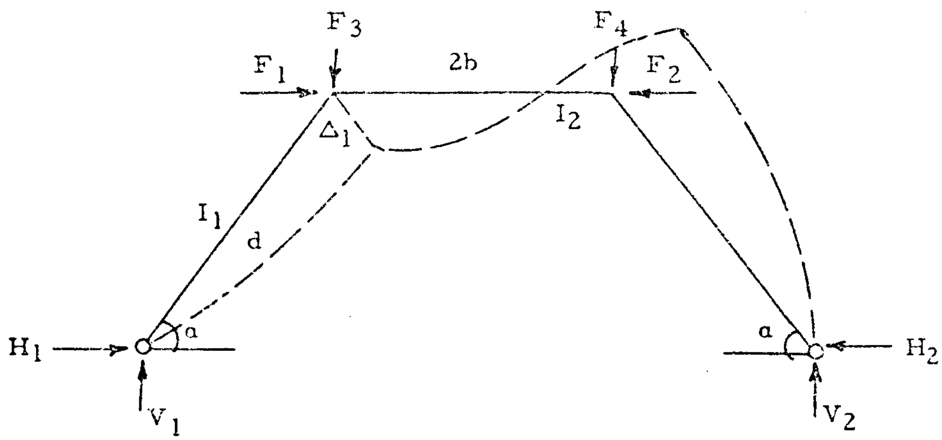


FIGURE B.13. FREE-BODY DIAGRAM OF RIGID FRAME

All forces acting in the plane of the frame are produced by the tensions in the plane of the frame.

$$V_1 = F_3 \frac{(2b + d \cos a)}{2c} + F_4 \frac{d \cos a}{2c} + (F_2 - F_1) \frac{d \sin a}{2c} \qquad (1)$$

$$V_2 = F_4 \frac{(2b + d \cos a)}{2c} + F_3 \frac{d \cos a}{2c} - (F_2 - F_1) \frac{d \sin a}{2c} \qquad (2)$$

$$c = b + d \cos a$$

$$H_1 = \frac{(F_3 + F_4)}{2} \cot a + \frac{F_2 - F_1}{2} \qquad (3)$$

$$H_2 = \frac{(F_3 + F_4)}{2} \cot a - \frac{F_2 - F_1}{2} \qquad (4)$$

The deflection Δ_1 at the top of the frame, in a direction normal to the inclined leg, is given by

$$\Delta_1 = \frac{(F_3 - F_4)\, d^3 \beta_1}{EI_1} + \frac{(F_1 - F_2)\, d^3 \beta_2}{EI_1} \qquad (5)$$

$$\beta_1 = \frac{1}{6}\left\{\frac{b^2}{c^2}\left(1 + \frac{b}{d}\frac{I_1}{I_2}\right)\cos a\right\}$$

$$\beta_2 = \frac{1}{6}\left\{\frac{bd}{c^2}\sin a\left(\frac{c}{d} + \frac{b^2}{d^2}\frac{I_1}{I_2} - \cos a\right)\right\} \qquad (6)$$

Relations between tensions, T, and forces, F, are

$$F_3 = T_1 + T_2 \cos\psi \qquad F_1 = T_2 \sin\psi$$

$$F_4 = T_3 + T_2' \cos\psi \qquad F_2 = T_2' \sin\psi \qquad (7)$$

The displacements downward, in the plane of the frame, of the lower extremity of the tension members, in the direction of their lengths, are as follows:

T_1

$$\delta_1 = \frac{d^3}{EI_1}\left[(T_1 - T_3)\beta_1 + (T_2 - T_2')(\beta_1 \cos\psi + \beta_2 \sin\psi)\right] \cos a + \frac{T_1 \ell_1}{A_1 E} \qquad (8)$$

T_2

$$\delta_2 = \frac{d^3}{EI_1}\left[(T_1 - T_3)\beta_1 + (T_2 - T_2')(\beta_1 \cos\psi + \beta_2 \sin\psi)\right] \cos(a-\psi) + \frac{T_2 \ell_2}{A_2 E} \qquad (9)$$

T_2'

$$\delta_2' = -\frac{d^3}{EI_1}\left[(T_1 - T_3)\beta_1 + (T_2 - T_2')(\beta_1 \cos\psi + \beta_2 \sin\psi)\right] \cos(\alpha-\psi) + \frac{T_2' \ell_2}{A_2 E} \quad (10)$$

T_3

$$\delta_3 = -\frac{d^3}{EI_1}\left[(T_1 - T_3)\beta_1 + (T_2 - T_2')(\beta_1 \cos\psi + \beta_2 \sin\psi)\right] \cos\alpha + \frac{T_3 \ell_1}{A_1 E} \quad (11)$$

The condition $\delta_2 = \delta_2$ leads to

$$(T_2 - T_2') = -\beta_3(T_1 - T_3) \quad (12)$$

with

$$\beta_3 = \beta_1 \Bigg/ \left[(\beta_1 \cos\psi + \beta_2 \sin\psi) + \frac{\ell_2 I_1}{2A_2 d^3} \sec(\alpha-\psi)\right] \quad (13)$$

If Equation (12) is used, T_2' may be eliminated from the equations and we get

$$\begin{aligned} \delta_1 &= (T_1 - T_3)\frac{d^3}{EI_1}\beta_4 \cos\alpha + \frac{T_1 \ell_1}{A_1 E} \\ \delta_2 &= (T_1 - T_3)\frac{d^3}{EI_1}\beta_4 \cos(\alpha-\psi) + \frac{T_2 \ell_2}{A_2 E} \end{aligned} \quad (14)$$

$$\begin{aligned} \delta_3 &= -(T_1 - T_3)\frac{d^3}{EI_1}\beta_4 \cos\alpha + \frac{T_3 \ell_1}{A_1 E} \\ \beta_4 &= \beta_1 - \beta_3(\beta_1 \cos\psi + \beta_2 \sin\psi) \end{aligned} \quad (15)$$

Assume a load P on each bridge girder. The vertical deflection of the beam at any point x when the unit load is at $x = \xi$ is given by

$$w = \frac{Pa^3}{EI} f(x, \xi, \eta) \tag{16}$$

$$f(x, \xi, \eta) = \frac{1}{6} \frac{x\xi}{a^2} \left\{ 2\left(1 - \frac{x}{a}\right)\left(1 - \frac{\xi}{a}\right) + \left(\frac{x}{a} - \frac{\xi}{a}\right)^2 \right\} - \frac{\eta}{a}\left(\frac{x}{a} - \frac{\xi}{a}\right)^2 \Big]$$

$\eta = x \quad , \quad x \leq \xi$

$\eta = \xi \quad , \quad x > \xi.$

a is the bridge span and x and ξ are measured from the left end.

The bridge girder is subjected to the following loads:

(1) The tensions $T_1 \cos \phi$ and $T_3 \cos \phi$ at $x = a_1$ and a_3

(2) The tensions $(T_2 + T_2^{'}) \cos \phi \cos \psi$ at $x = a/2 = a_2$

(3) The travelling load unity at $x = \xi$.

Equating the deflection of the girder, at each of the points $x = a_1$, a_2, and a_3, to the deflections of the cables, one obtains the following equations:

$$T_1 \alpha_{11} + T_2 \alpha_{12} + T_3 \alpha_{13} = f(a_1, \xi, \eta) \cos \phi \tag{17}$$

$$T_1 \alpha_{21} + T_2 \alpha_{22} + T_3 \alpha_{23} = f(a_2, \xi, \eta) \cos \phi \cos \psi \tag{18}$$

$$T_1 \alpha_{31} + T_2 \alpha_{32} + T_3 \alpha_{33} = f(a_3, \xi, \eta) \cos \phi \tag{19}$$

with

$$a_{11} = \frac{d^3}{a^3}\frac{I}{I_1}\beta_4 \cos\alpha + \frac{\ell_1}{A_1}\frac{I}{a^3} + (f_{11} + \beta_3 f_{12} \cos\psi) \cos^2\phi$$

$$a_{12} = 2 f_{12} \cos\psi \cos^2\phi$$

$$a_{13} = -\frac{d^3}{a^3}\frac{I}{I_1}\beta_4 \cos\alpha - (\beta_3 f_{12} \cos\psi - f_{13}) \cos^2\phi$$

$$a_{21} = \frac{d^3}{a^3}\frac{I}{I_1}\beta_4 \cos(\alpha - \psi) + (f_{12} \cos\psi + \beta_3 f_{22} \cos^2\psi) \cos^2\phi$$

$$a_{22} = \frac{\ell_2 I}{a^3 A_2} + 2 f_{22} \cos^2\psi \cos^2\phi \tag{20}$$

$$a_{23} = -\left[\frac{d^3}{a^3}\frac{I}{I_1}\beta_4 \cos(\alpha - \psi) + (\beta_3 f_{22} \cos^2\psi - f_{23} \cos\psi) \cos^2\phi\right]$$

$$a_{31} = -\left[\frac{d^3}{a^3}\frac{I}{I_1}\beta_4 \cos\alpha - (f_{13} + \beta_3 f_{23} \cos\psi) \cos^2\phi\right]$$

$$a_{32} = 2 f_{23} \cos\psi \cos^2\phi$$

$$a_{33} = \frac{d^3}{a^3}\frac{I}{I_1}\beta_4 \cos\alpha + \frac{\ell_1 I}{a^3 A_1} + (f_{33} - \beta_3 f_{23} \cos\psi) \cos^2\phi$$

$$f_{11} = f(a_1, a_1, a_1)$$

$$f_{12} = f_{21} = f(a_1, a_2, a_1)$$

$$f_{13} = f_{31} = f(a_1, a_3, a_1)$$

Equations (17), (18), and (19) are to be solved for the tensions T_1, T_2, and T_3.

Assuming that the T_i are known, the bending moments and shears in the frame and the bridge girder can be calculated.

$$V_1 = \frac{(F_2 - F_1)\, d \sin\alpha}{2c} + \frac{(F_3 + F_4)\, d \cos\alpha + 2F_3 b}{2c}$$

$$H_1 = \frac{(F_2 - F_1)}{2} + \frac{(F_3 - F_4)}{2} \cot\alpha$$

$$H_2 = -\frac{(F_2 - F_1)}{2} + \frac{(F_3 - F_4)}{2} \cot\alpha$$

$$V_2 = \frac{(F_3 + F_4)\, d \cos\alpha + 2F_4 b}{2c} - \frac{(F_2 - F_1)\, d \sin\alpha}{2c}$$

$$V_1 = \frac{(T_2' - T_2)\, d \sin\alpha \sin\psi}{2c} + \frac{[T_1' + T_3 + (T_2 + T_2') \cos\psi]\, d \cos\alpha + 2b(T_1 + T_2 \cos\psi)}{2c} \qquad (21)$$

$$V_2 = \frac{(T_2 - T_2')\, d \sin\alpha \sin\psi}{2c} + \frac{[T_1 + T_3 + (T_2 + T_2') \cos\psi]\, d \cos\alpha + 2b(T_3 + T_2' \cos\psi)}{2c}$$

$$H_1 = \frac{(T_2' - T_2)}{2} \sin\psi + \frac{[(T_1 - T_3) + (T_2 - T_2') \cos\psi]}{2} \cot\alpha$$

$$H_2 = \frac{(T_2 - T_2')}{2} \sin\psi + \frac{[(T_1 - T_3) + (T_2 - T_2') \cos\psi]}{2} \cot\alpha$$

$$M_1 = (V_1 \cos\alpha - H_1 \sin\alpha) x$$

$$M_2 = -(V_2 \cos\alpha - H_2 \sin\alpha) x$$

In the last two equations, x is measured along the leg of the rigid frame.

B. 6. b. Frame Bridge With Hinged Girder

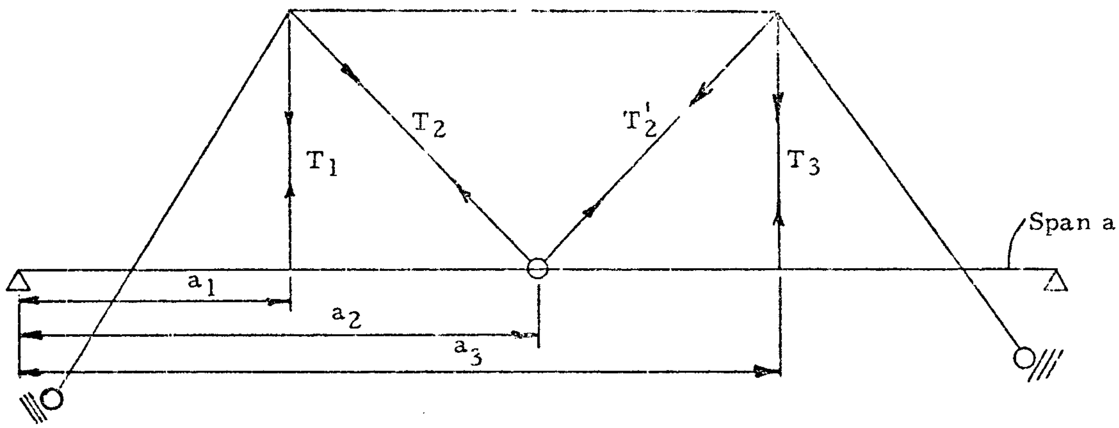


FIGURE B. 14. FRAME BRIDGE WITH HINGE, ELEVATION VIEW

In this case, an additional equation is available from the condition that there is no bending moment at the hinge.

The two equations to be solved for T_1 and T_3 are

$$T_1 \left(\beta_4 \cos \alpha \sec \phi + \frac{\ell I_1}{A_1 d^3} \sec \phi + \lambda \beta_5\right)$$

$$-T_3 \left(\beta_4 \sec \phi \cos \alpha - \lambda \beta_6\right) = \lambda \left\{ f(a_1, \xi, \eta) + \beta_7 \frac{\xi}{a} \right\} \qquad (22)$$

$$T_1(-\beta_4 \sec \phi \cos \alpha + \lambda \beta_9) + T_3\left(\beta_4 \sec \phi \cos \alpha + \frac{\ell I_1}{A_1 d^3} \sec \phi + \lambda \beta_{10}\right) = \lambda \beta_8 \frac{\xi}{a} \qquad (23)$$

β_1, β_2, β_3, and β_4 have been defined in Paragraph B. 6. a.

$$\beta_5 = \frac{\cos \phi}{3} \frac{a_1^2}{g^2} \left(1 - \frac{a_1}{g}\right)^2 + \frac{\ell_2 I}{g^3 A_2} \frac{a_1}{g} \sec \psi \left(\frac{a_1}{a} \sec \psi + \frac{\beta_3}{2}\right) \qquad (24)$$

$$\beta_6 = \frac{\ell_2 I}{g^3 A_2} \frac{a_1}{g} \sec\psi \left(\frac{a_1}{a} \sec\psi - \frac{\beta_3}{2}\right)$$

$$\beta_7 = \frac{\ell_2 I}{A_2 g^3} \frac{a_1}{g} \sec^2\psi \sec\phi$$

$$\beta_8 = \left(1 - \frac{h}{g}\right) \frac{\ell_2 I}{A_2 g^3} \sec^2\psi \sec\phi \qquad \text{(24 Cont'd)}$$

$$\beta_9 = \left(1 - \frac{h}{g}\right) \frac{\ell_2 I}{A_2 g^3} \sec\psi \left(\frac{a_1}{a} \sec\psi + \frac{\beta_3}{2}\right)$$

$$\beta_{10} = \left(1 - \frac{h}{g}\right)^2 \frac{\cos\phi}{3} \frac{h^2}{g^2} + \frac{\ell_2 I}{A_2 g^3} \left(1 - \frac{h}{g}\right) \sec\psi \left(\frac{a_1}{a} \sec\psi - \frac{\beta_3}{2}\right)$$

$$g = \frac{a}{2} \quad , \quad h = a_3 - g \quad , \quad \lambda = \frac{I_1 g^3}{I d^3} \qquad (25)$$

$$\frac{\xi}{a} \leq \frac{1}{2}$$

$$f(a_1, \xi, \eta) = \frac{a_1 \xi}{g^2} \left\{ 2\left(1 - \frac{a_1}{g}\right)\left(1 - \frac{\xi}{g}\right) + \left(\frac{a_1}{g} - \frac{\xi}{g}\right)^2 - \frac{\eta}{g}\left(\frac{a_1}{g} - \frac{\xi}{g}\right)^2 \right\} \qquad (26)$$

$$\eta = a_1 \quad , \quad a_1 \leq \xi$$

$$\eta = \xi \quad , \quad a_1 \geq \xi$$

For calculating T_2 and T_2', we have

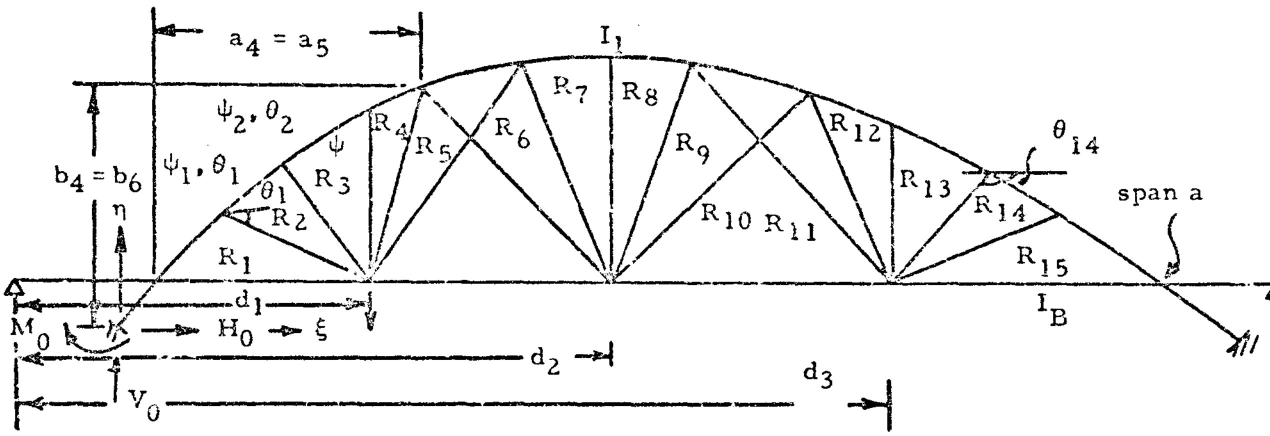
$$T_2 = \frac{\xi}{a} \sec\phi \sec\psi - T_1\left(\frac{a_1}{a} \sec\psi + \frac{\beta_3}{2}\right) - T_3\left(\frac{a_1}{a} \sec\psi - \frac{\beta_3}{2}\right) \qquad (27)$$

and

$$T_2' = T_2 + \beta_3(T_1 - T_3) \qquad (28)$$

Once T_1, T_2, T_2', and T_3 are known, Equations (21) may be used for calculating moments and shears in the rigid frame.

B.7 Leaning Arches Bridge Analysis



$\psi_4 = \psi_6, \quad \psi_5 = \psi_7, \quad \psi_9 = \psi_{11}, \quad \psi_{10} = \psi_{12}$

FIGURE B.15. LEANING ARCHES BRIDGE, ELEVATION VIEW

The aim of the analysis is to obtain the tensions in the cables and the bending moments and shears, in the arch rib as well as in the bridge girder, for a unit load at any point on the span.

The analysis is carried out for two types of arch rib: (1) constant arch section, and (2) moment of inertia varying as the secant of the angle of slope of the arch (i.e., $I(s) = I_c \sec\psi$, where I_c is the moment of inertia at the crown).

The basic equations to be derived express the condition that the deflection of the arch rib plus the stretching of the cable equals the deflection of the bridge girder at the appropriate point in the appropriate direction.

Equations for determining M_0, V_0, and H_0 (Figure B.15) are

$$- M_0 \int \eta \frac{ds}{I} - V_0 \int \xi \eta \frac{ds}{I} + H_0 \int \eta^2 \frac{ds}{I} + \int \frac{M' \eta \, ds}{I} = 0 \qquad (1)$$

$$M_0 \int \xi \frac{ds}{I} + V_0 \int \xi^2 \frac{ds}{I} - H_0 \int \xi \eta \frac{ds}{I} - \int \frac{M' \xi \, ds}{I} = 0 \qquad (2)$$

$$M_0 \int \frac{ds}{I} + V_0 \int \xi \frac{ds}{I} - H_0 \int \eta \frac{ds}{I} - \int M' \frac{ds}{I} = 0 \qquad (3)$$

In Equations (1), (2), and (3), the integrals are to extend over the entire arch length. ds is the differential length of arch rib and M' is the bending moment due to the external loads.

In evaluating the integrals involved in Equations (1), (2), and (3), it will be noted that if

$$I(s) = (\sec \psi)\, I_c$$

then

$$\frac{ds}{I(s)} = \frac{d\xi}{I_c}, \text{ since } ds = d\xi \sec \psi.$$

Thus, the integrals can be readily evaluated. This is the case of "variable moment of inertia" for the arch rib.

The arch rib is assumed to be a parabola defined by

$$\eta = \xi \tan \psi_0 + \xi^2 / c \qquad (4)$$

or

$$\eta = \frac{4h}{L}\,\xi - \frac{4h}{L^2}\,\xi^2 \qquad (5)$$

In Equation (4), ψ_0 is the slope of the arch at the left abutment. In Equation (5), L is the arch span and h is the arch rise. We also have the relation

$$\tan\psi = \frac{d\eta}{d\xi} = \frac{4h}{L}\left(1 - \frac{2\xi}{L}\right) \qquad (6)$$

The integrals in Equations (1), (2), and (3) may be evaluated for the two cases and defined as follows:

Constant Moment of Inertia

$$\begin{aligned}
\int y\,ds &= \frac{L^2}{4}\left[\left(\frac{L}{4h}\right)^2 f_2 - f_0\right] = L^2\,g_0(\psi)\\
\int xy\,ds &= -\frac{L^3}{8}\left[\left(\frac{L}{4h}\right)^3 f_3 - \left(\frac{L}{4h}\right)^2 f_2 - \left(\frac{L}{4h}\right) f_1 + f_0\right] = L^3\,g_1(\psi)\\
\int y^2\,ds &= -\frac{L^3}{16}\left[\left(\frac{L}{4h}\right)^3 f_4 - 2\left(\frac{L}{4h}\right) f_2 + \left(\frac{L}{4h}\right)^{-1} f_0\right] = L^3\,g_2(\psi)\\
\int x\,ds &= \frac{L^2}{2}\left[\left(\frac{L}{4h}\right)^2 f_1 - \left(\frac{L}{4h}\right) f_0\right] = L^2\,g_3(\psi)\\
\int x^2\,ds &= -\frac{L^3}{4}\left[\left(\frac{L}{4h}\right)^3 f_2 - 2\left(\frac{L}{4h}\right)^2 f_1 + \left(\frac{L}{4h}\right) f_0\right] = L^3\,g_4(\psi)\\
\int ds &= -L\left(\frac{L}{4h}\right) f_0 = L\,g_5(\psi)
\end{aligned} \qquad (7)$$

In Equations (7),

$$f_0 = \frac{1}{4}\left\{\tan\psi\,\sec\psi + \ln(\tan\psi + \sec\psi)\right\}$$

$$f_1 = \frac{1}{6} \sec^3 \psi$$

$$f_2 = \frac{1}{8} \left\{ \sin \psi \sec^4 \psi - \frac{1}{2} \sin \psi \sec^2 \psi - \frac{1}{2} \ln (\sec \psi + \tan \psi) \right\} \tag{8}$$

$$f_3 = \frac{\sec^3 \psi}{30} (3 \sec^2 \psi - 5)$$

$$f_4 = \frac{1}{4} \left\{ \frac{1}{3} \sin \psi \sec^6 \psi - \frac{7}{12} \sin \psi \sec^4 \psi + \frac{1}{8} \sin \psi \sec^2 \psi + \frac{1}{8} \ln (\sec \psi + \tan \psi) \right\}$$

We also define

$$g_{2,0} = g_2(-\psi_0) - g_2(\psi_0)$$
$$g_{2,k} = g_2(-\psi_0) - g_2(\psi_k) \tag{9}$$

and similar expressions.

For varying moments of inertia such that $I(s) = I_c \sec \psi$, the expressions for g_1, g_2, etc., are as follows:

$$g_0(\xi) = \frac{4h}{L} \left\{ \frac{1}{2} \frac{\xi^2}{L^2} - \frac{1}{3} \frac{\xi^3}{L^3} \right\}$$

$$g_1(\xi) = \frac{4h}{L} \left\{ \frac{1}{3} \frac{\xi^3}{L^3} - \frac{1}{4} \frac{\xi^4}{L^4} \right\}$$

$$g_2(\xi) = \frac{1}{4} \left(\frac{4h}{L} \right)^2 \left\{ \frac{1}{3} \frac{\xi^3}{L^3} - \frac{1}{2} \frac{\xi^4}{L^4} + \frac{1}{5} \frac{\xi^5}{L^5} \right\} \tag{10}$$

$$g_3(\xi) = \frac{1}{2} \frac{\xi^2}{L^2}, \quad g_4(\xi) = \frac{1}{3} \frac{\xi^3}{L^3}, \quad g_5(\xi) = \frac{\xi}{L}$$

For both constant and varying moments of inertia, the definitions in Equation (9) hold, with appropriate interpretation of the term g_k.

Equations (1), (2), and (3) may now be put in the form

$$[G_{ij}]\begin{Bmatrix} M_0/L \\ V_0 \\ H_0 \end{Bmatrix} = \begin{Bmatrix} \sum_{k=1}^{15} R_k h_{2k} \\ \sum_{k=1}^{15} R_k h_{3k} \\ \sum_{k=1}^{15} R_k h_{1k} \end{Bmatrix} \tag{11}$$

where

$$[G_{ij}] = \begin{bmatrix} -g_{0,0} & -g_{1,0} & g_{2,0} \\ g_{3,0} & g_{4,0} & -g_{1,0} \\ g_{5,0} & g_{3,0} & -g_{0,0} \end{bmatrix} \tag{12}$$

$$\begin{aligned} h_{1k} &= \sin\theta_k \left\{ g_{3,k} - \frac{\xi_k}{L} g_{5,k} \right\} + \cos\theta_k \left\{ g_{0,k} - \frac{\eta_k}{L} g_{5,k} \right\} \\ h_{2k} &= \sin\theta_k \left\{ g_{1,k} - \frac{\xi_k}{L} g_{0,k} \right\} + \cos\theta_k \left\{ g_{2,k} - \frac{\eta_k}{L} g_{0,k} \right\} \\ h_{3k} &= \sin\theta_k \left\{ g_{4,k} - \frac{\xi_k}{L} g_{3,k} \right\} + \cos\theta_k \left\{ g_{1,k} - \frac{\eta_k}{L} g_{3,k} \right\} \end{aligned} \tag{13}$$

Let

$$[G_{ij}]^{-1} = [F_{ij}]$$

$$\begin{aligned} \alpha_k &= \left\{ -h_{2k}F_{11} + h_{3k}F_{12} + h_{1k}F_{13} \right\} \\ \beta_k &= \left\{ -h_{2k}F_{21} + h_{3k}F_{22} + h_{1k}F_{23} \right\} \\ \gamma_k &= \left\{ -h_{2k}F_{31} + h_{3k}F_{32} + h_{1k}F_{33} \right\} \end{aligned} \tag{14}$$

Then

$$\frac{M_0}{L} = \sum_{k=1}^{15} R_k \alpha_k$$
$$V_0 = \sum_{k=1}^{15} R_k \beta_k \qquad (15)$$
$$H_0 = \sum_{k=1}^{15} R_k \gamma_k$$

By use of Castigliano's theorem, it may be shown that the deflection of the arch rib in the direction of the tension R_j is given by

$$\delta_{Rj} = \frac{L^3}{EI}\left[-\frac{M_0}{L} h_{1,j} - V_0 h_{3,j} + H_0 h_{2,j} + \sum_{k=1}^{15} R_k Z(j,k)\right] \qquad (16)$$

Here, I is either the constant moment of inertia or, in the case of varying I, the moment of inertia I_c at the crown, and

$$Z(j,k) = (\sin\theta_j) h_{3,k} - \left(\frac{\xi_j}{L}\sin\theta_j + \frac{\eta_j}{L}\cos\theta_j\right) h_{1,k} + (\cos\theta_j) h_{2,k} \qquad k > j \qquad (17)$$

$$Z(j,k) = (\sin\theta_j) h_{3,j} - \left(\frac{\xi_k}{L}\sin\theta_k + \frac{\eta_k}{L}\cos\theta_k\right) h_{1,j} + (\cos\theta_k) h_{2,j} \qquad j \geq k \qquad (18)$$

Substituting Equation (15) into Equation (16), we may write:

$$\delta_{Rj} = \frac{L^3}{EI} \sum_{k=1}^{15} R_k q(j,k)$$
$$q_{j,k} = Z(j,k) - h_{1j}\alpha_k - h_{3j}\beta_k + h_{2j}\gamma_k \tag{19}$$

The cable tensions act on the beam. In the plane of the arches, the components normal to the beam are

$$T_1 = \sum_{k=1}^{5} R_k \sin\theta_k$$
$$T_2 = \sum_{k=6}^{10} R_k \sin\theta_k \tag{20}$$
$$T_3 = \sum_{k=11}^{15} R_k \sin\theta_k$$

The deflection of the beam at any point x, due to a unit load at x = x', is

$$w = \frac{a^3}{6EI_B} f(x, x', \zeta) \tag{21}$$

where

$$f = \frac{xx'}{a^2}\left\{2\left(1 - \frac{x}{a}\right)\left(1 - \frac{x'}{a}\right) + \left(\frac{x}{a} - \frac{x'}{a}\right)^2\right\} - \frac{\zeta}{a}\left(\frac{x}{a} - \frac{x'}{a}\right)^2$$
$$\zeta = x, \quad x \leq x'$$
$$\zeta = x', \quad x \geq x'$$

The vertical deflection of the beam at the resultant tension T_j, for a unit load on each floor beam, is:

$$w_j = \frac{a^3}{6EI_B}\left[f(d_j,\ x',\ \zeta_{d_j,x'}) - \cos\phi \sum_{k=1}^{15} R_k \sin\theta_k\, f(d_j,\ d_k,\ \zeta_{d_j,d_k}) \right]$$

It is to be noted that there are only 3 values of d, viz, d_1, d_2, and d_3, whereas there are 15 values of R_k and θ_k. ϕ is the inclination to the vertical plane of the plane of the arches.

Compatibility of deformation requires that the component of the deflection of the beam in the plane of the arch equal the deflection of the arch rib plus the stretching of the cable. This condition must be satisfied for every cable.

$$\delta R_j + \frac{R_j \ell_j}{a_j E} = W_j \cos\varphi \sin\theta_j \tag{22}$$

Note that w_j has only 3 values, whereas R_j and θ_j have 15 values each. Thus, there are 15 equations for determining the R_j.

Equation (22) may be put in the following form

$$\mu_j R_j + \sum_{k=1}^{15} R_k \left\{ \lambda q(j,k) + \cos^2\phi \sin\theta_j \sin\theta_k\, f(d_j,\ d_k,\ \zeta_{d_j,d_k}) \right\}$$

$$= f(d_j,\ x',\ \zeta_{d_j,x'}) \sin\theta_j \tag{23}$$

$$\frac{6I_B}{I}\frac{L^3}{a^3} = \lambda, \quad \frac{6\ell_j I_B}{A_j a^3} = \mu_j$$

In applying Equation (23), it is to be noted that

(1) The value of j for R_j on the left must be consistent with the value of d_j on the right.

(2) Since the cables cannot carry compression, if any R_j turns out negative, the solution must be revised by setting that $R_j = 0$.*

(3) $I = I_c$ in the case of varying moment of inertia of arch rib

Additionally, we need the values of

(1) Bending moments at crown and springing

(2) Thrusts at crown and springing

(3) Bending moments in bridge girder.

Arch Rib

At Springing

$$\text{Moment} = M_0$$

$$\text{Thrust} = H_0 \cos \psi_0 + V_0 \sin \psi_0 \tag{24}$$

At Crown

$$\text{Moment} = M_0 + V_0 \frac{L}{2} - H_0 h - \sum_{k=1}^{7} R_k \left\{ (\sin \theta_k) \left(\frac{L}{2} - \xi_k \right) + \cos \theta_k (h - \eta_k) \right\} \tag{25}$$

$$\text{Thrust} = H_0 + \sum_{k=1}^{7} R_k \cos \theta_k \tag{26}$$

Bridge Girder

It would be sufficient to calculate the bending moments at 2 points, say at 1/4- and 1/2-span. The tension T_1 is [see Equation (20)]

$$T_1 = \sum_{k=1}^{5} R_k \sin \theta_k$$

*Note: The signs of the R_j are dependent not only on the cross sectional area of the cables but also the flexinal rigidies of the beam and the arch.

Let

$$\bar{R} = \frac{a - x'}{a} - \sum_{k=1}^{3} T_k \frac{(a - d_k)}{a} \tag{27}$$

Then

$$\begin{aligned} M_1 &= \bar{R} d_1 - (d_1 - x')\delta \\ M_2 &= \bar{R} d_2 + T_1(d_2 - d_1) - (d_2 - x')\delta \end{aligned} \tag{28}$$

where $\delta = 0$ if $\begin{matrix} (d_1 - x') < 0 \\ (d_2 - x') < 0 \end{matrix}$

B. 8 Dome Bridge Analysis

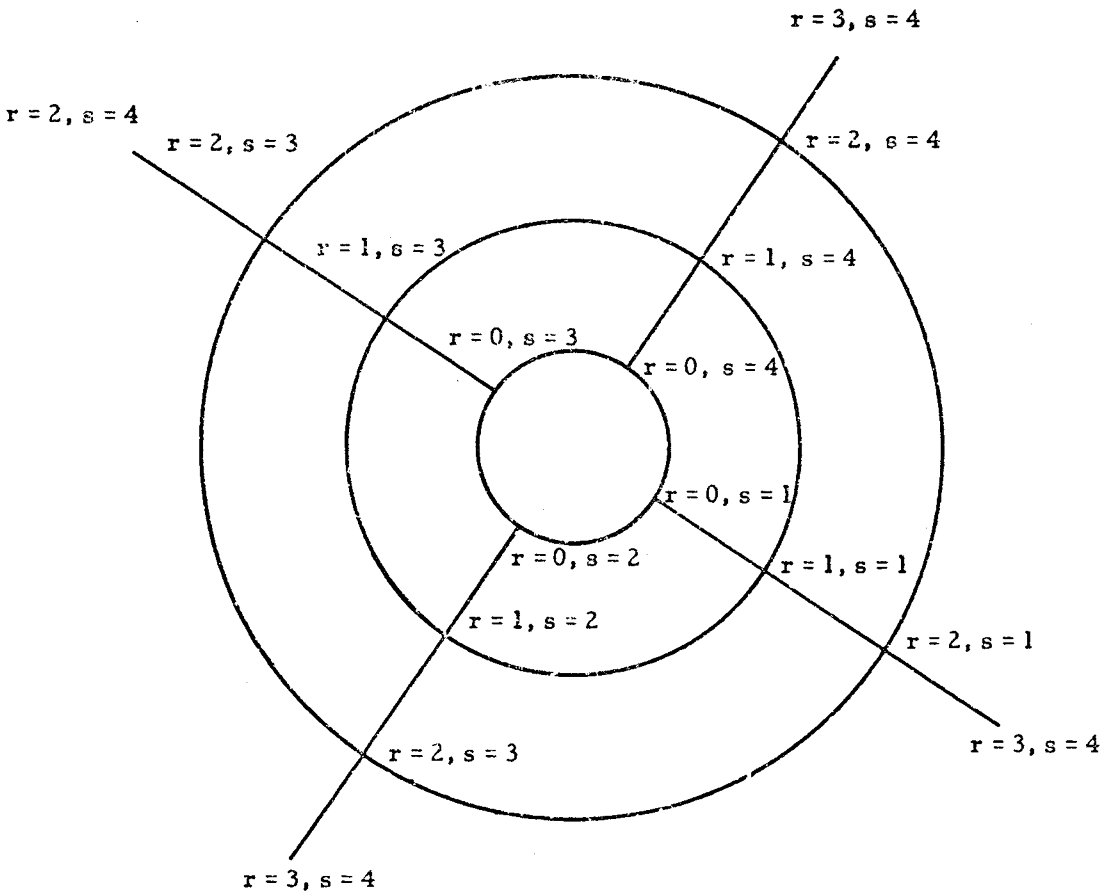


FIGURE B. 16. DOME BRIDGE, PLAN VIEW

The plan view of the gridwork dome is shown in Figure B. 16. We assume that the radial legs are fixed at the lower extremeties. The coordinates (r, s) are shown in the figure. Loads are applied at ay fo the intersections. of the grid.

There are six unknown displacement components at each node which we denote by the column matrix

$$\underset{\sim}{d}rs = \left\{\theta_{rs,x}\theta_{rs,y}\theta_{rs,z}\delta_{rs,x}\delta_{rs,y}\delta_{rs,z}\right\} \quad (1)$$

where x, y, and z are local coordinates. x is taken in the radial direction positive outwards; y is vertically up, and z is in the positive s direction. The θ are the rotations, and the δ are the deflections.

Similarly, we define the force-column matrix as

$$\underset{\sim}{F}^{R}_{rs} = \left\{ M_{rs,x} M_{rs,y} M_{rs,z} f_{rs,x} f_{rs,y} f_{rs,z} \right\} \tag{2}$$

The force matrix (2) consists of all the forces at the (r, s) end of an arc extending for r to (r + 1, s). $M_{rs,x}$, for example, is the moment whose vector is in the positive x direction at the node (r, s). The superscript R denotes that the forces are to the right of the node (r, s) as we proceed in the positive radial direction.

The force-displacement relation for an arc extending from (r, s) to (r + 1, s) may be written in the form:

$$\begin{Bmatrix} F^{R}_{rs} \\ \\ F^{L}_{r+1,\ s} \end{Bmatrix} = [K_{rs}] \begin{Bmatrix} d_{rs} \\ \\ d_{r+1,\ s} \end{Bmatrix} \tag{3}$$

$F^{L}_{r+1,\ s}$ are the generalized forces at the end (r + 1, s) of the arc, and $d_{r+1,\ s}$ are the six displacement components at the same end. The superscript L denotes that the forces are at the left of the node.

The stiffness matrix $[K_{rs}]$ is a 12 X 12 matrix and is given, for example, in "Curved Beam Stiffness Coefficients" by D. L. Morris, proceedings of ASCE Structural Division, May 1968.

For a horizontal arc extending from (r, s) to (r, s +1) with stiffness matrix $[\overline{K}_{rs}]$, we may similarly write

$$\begin{Bmatrix} \overline{F}^{R}_{rs} \\ \\ \overline{F}^{L}_{r,\ s+1} \end{Bmatrix} = [\overline{K}_{rs}] \begin{Bmatrix} d_{rs} \\ \\ d_{r,\ s+1} \end{Bmatrix} \tag{4}$$

To compute $[\overline{K}_{rs}]$ from $[K_{rs}]$, we have to take into account the change in the direction of x, y, and z axes. Initially, we treat a horizontal arc as if it were a radial one, and write the $[K_{rs}]$ matrix. Then

$$[\overline{K}_{rs}] = [T][K_{rs}][T]^{-1} \tag{5}$$

where [T] is the coordinate transformation matrix, $[K_{rs}]$ is a 12 X 12 matrix, and the transformation matrix [T] is

$$[T] = \begin{bmatrix} [T_1] & 0 & 0 & 0 \\ 0 & [T_1] & 0 & 0 \\ 0 & 0 & [T_1] & 0 \\ 0 & 0 & 0 & [T_1] \end{bmatrix} \qquad (6)$$

where

$$[T_1] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad (7)$$

and each of the zeros in (6) stand for a 3×3 matrix of zeros.

By suitable superposition of the matrices, we may generate a system stiffness matrix equation. The left-hand side of the equation then consists only of the external forces applied at the nodes. For this purpose, it is useful to partition the stiffness matrices thus:

$$[K_{rs}] \equiv \begin{bmatrix} a_{rs} & B_{rs} \\ c_{rs} & D_{rs} \end{bmatrix} \qquad (8)$$

$$[\overline{K}_{rs}] = \begin{bmatrix} \overline{a}_{rs} & \overline{B}_{rs} \\ \overline{c}_{rs} & \overline{D}_{rs} \end{bmatrix}$$

If we denote the external forces at each node by the column matrix,

$$\underset{\sim}{P_{rs}} = \left\{ 0_1 \; 0 \; 0 \; P_{rs,x} \; P_{rs,y} \; P_{rs,z} \right\} \qquad (10)$$

we specify that no concentrated moments are applied at the nodes, but only forces in the x, y, and z directions.

The final equations may be displayed in tabular form as shown in Table B.I.

TABLE B.I EQUATION COEFFICIENT MATRIX

	d_{01}	d_{02}	d_{03}	d_{04}	d_{11}	d_{12}	d_{13}	d_{14}	d_{21}	d_{22}	d_{23}	d_{24}	d_{31}	d_{32}	d_{33}	d_{34}
P_{01}	$a_{01}+\bar{a}_{01}+\bar{D}_{04}$	$\bar{B}_{01}$	0	$\bar{c}_{04}$	B_{01}											
P_{02}	$\bar{c}_{01}$	$a_{02}+\bar{H}_{02}+\bar{D}_{01}$	$\bar{B}_{02}$			B_{02}										
P_{03}	0	$\bar{c}_{02}$	$a_{03}+\bar{a}_{03}+\bar{D}_{02}$	$\bar{B}_{03}$			B_{03}									
P_{04}	$\bar{B}_{04}$		$\bar{c}_{03}$	$a_{04}+\bar{a}_{04}+\bar{D}_{03}$				B_{04}								
P_{11}	c_{01}				$a_{11}+D_{01}+\bar{a}_{11}+\bar{D}_{14}$	$\bar{B}_{11}$		$\bar{c}_{14}$	B_{11}							
P_{12}		c_{02}			$\bar{c}_{11}$	$a_{12}+D_{02}+\bar{a}_{12}+\bar{D}_{11}$	$\bar{B}_{12}$			B_{12}						
P_{13}			c_{03}			$\bar{c}_{12}$	$a_{13}+D_{03}+\bar{a}_{13}+\bar{D}_{12}$	$\bar{B}_{13}$			D_{13}					
P_{14}				c_{04}	$\bar{B}_{14}$		$\bar{c}_{13}$	$a_{14}+D_{04}+\bar{a}_{14}+\bar{D}_{13}$				B_{14}				
P_{21}					c_{11}				$a_{21}+D_{11}+\bar{a}_{21}+\bar{D}_{24}$	$\bar{B}_{21}$		$\bar{c}_{24}$	B_{21}			
P_{22}						c_{12}			$\bar{c}_{21}$	$a_{22}+D_{12}+\bar{a}_{22}+\bar{D}_{21}$	$\bar{B}_{22}$			B_{22}		
P_{23}							c_{13}			$\bar{c}_{22}$	$a_{23}+D_{13}+\bar{a}_{23}+\bar{D}_{22}$	$\bar{B}_{23}$			B_{23}	
P_{24}								c_{14}	$\bar{B}_{24}$		$\bar{c}_{23}$	$a_{24}+D_{14}+\bar{a}_{24}+\bar{D}_{23}$				B_{24}

It should be noted in Table B.I that P_{01} stands for six force components at the node $r = 0$, $s = 1$, and so on. Thus, the column matrix on the left has seventy-two force components, six at each of the twelve nodes shown in Figure D. 8. 1. Since the radial beams are fixed at the bottom, all the displacement components vanish at these points. Thus, $d_{31} = d_{32} = d_{33} = d_{34} = 0$.

The displacement components d_{01}, d_{02} are shown horizontally at the top in Table B.I, instead of vertically to the right. Each symbol d stands for six displacement components, the three rotations, and the three deflections at each node.

The system stiffness matrix is a 72 X 96 matrix connecting the seventy-two force components to the ninety-six displacement components of which twenty-four are zero.

In actual practice, for calculating influence lines, all external forces with the exception of a chosen one will be taken as equal to zero so that the left-hand side will have only one nonzero element. The displacements can then be solved for by matrix inversion.

It is convenient to note that:

(1) For the radial beams, the radius a_{rs} is a constant, only the angle β_{rs} varies; and

(2) For the circumferential beams, the radius a_{rs} varies with r only. That is,

$$[\overline{K}_{r,s}] = [\overline{K}_{r+1,s}] = [\overline{K}_{r+k,s}]$$

APPENDIX C

CONCEPT DESIGN SUMMARY FOR "A" FRAME BRIDGE AND DOME BRIDGE

APPENDIX C

CONCEPT DESIGN SUMMARY FOR "A" FRAME BRIDGE AND DOME BRIDGE

Two of the eight bridge concepts evaluated during the research phase of the program (summarized in Volume I) are not given preliminary design or concept design consideration in the summary of results portion of the report (Volume II). These two bridge concepts, the "A" Frame Bridge and the Dome Bridge , were judged to be infeasible as effective methods for eliminating massive support structures adjacent to the roadway. Engineering data and conceptual designs developed for purposes of the bridge concept evaluation exercise* are recorded in this appendix for reference.

C.1 "A" Frame Bridge

This bridge concept, described in Figure C.1, was conceived as a method for permitting removal of the median pier while retaining an existing floor system. The main structure consists of four inclined members which form a pyramid; the four members are pin connected at their apex and are pin connected to their supports. These members support three floor beams that form the supports for four simple spans. The center floor beam is cable supported from the apex while the two outside floor beams are pin connected to the inclined members. It is important to note that the roadway girders or floor beams do not provide tension ties between the inclined members.

The analysis of this bridge concept considered the structure as being statically determinate. The structure is a space frame and is axisymmetric about a vertical axis through the apex. The analysis methodology employed laws of statics in three dimensions, as summarized in Appendix B.

Influence diagrams for major structural members and floor system members are shown in Figure C.2. A significant observation resulting from the analysis of this structure concerns the bending of the principal structure members while a load is on the two spans immediately adjacent to the floor beam/main member connection point. Calculations of design values for forces, moments, and shears are presented in Table C.I. Note that the length of the principal structure main members results in a relatively large bending moment in these components.

Concept design calculations for the "A" Frame Bridge are included in Table C.II. The main frame members are configured as box girders capable

*Summarized in Table II of Volume I.

of carrying the bending and axial forces computed in Table C.I. The beam-column action of these members, in conjunction with a 157-foot length, resulted in the design of a relatively heavy section. The cable supports attached to the center floor beam contribute very little weight to the principal structural system.

The floor system design is dictated by the geometry of the main frame; simple spans of 34, 85, 85, and 34 feet are required. The stringer system within the 85-foot spans consists of 30WF130 beams and, thus, contributes significantly to the weight of the bridge. An alternate plate girder design or continuous girder design could save some weight; however, the interspan relationship dictated by main frame geometry will not permit a high degree of design optimization.

The total weight of the bridge does not compare favorably with bridge concepts responsive to identical load and geometric requirements. The design scheme is not particularly efficient because of the bending present in the relatively long frame members. Finally, the scheme is not as aesthetically pleasing as some other bridge concepts.

C.2 Dome Bridge

The Dome Bridge, described schematically in Figure C.3, is a unique concept which employs two circular arches which intersect at right angles over the centers of the crossed and crossing roadways. The crossing roadway is suspended from cables that connect to the arches at joints formed by stiffeneing rings; these rings intersect the arches in two horizontal planes. The floor system is a stringer and floor-beam system spanning four simple spans.

The Dome Bridge may be analyzed as two circular arches connected by horizontal rings; the complete analysis is included in Appendix B. For preliminary analysis purposes, however, the arches were considered parabolic, and the hanger loads were considered to enter the arches in the plane of the arches at intersections with rings. Uniform dead and live loads were considered evenly distributed on the arches, although it is recognized that these loads must enter the arches at discrete points through the hangers. The floor system in this structure consists of four simple spans, thus simplifying the analysis of the floor system. Influence diagrams for the Dome Bridge are presented in Figure C.4. Table C.III contains computations of design forces, moments, and shears.

Concept design computations are summarized for the Dome Bridge concept in Table C.IV. To accomplish the preliminary design of this structure, it was necessary to make certain assumptions concerning the manner in which the arch is loaded. Uniform dead loads and live loads on the floor system were assumed to load the arches in a uniform manner. As may be noted in the conceptual sketch, the arches are actually loaded at discrete points where the hangers join the arches. If these loads were injected at the hanger locations, as would actually be the case, more severe moments could be realized. To further detract from design effectiveness, the arches and rings which comprise the principal structural system in this concept are relatively long in span. Further, the attempt to keep loads within the planes of the arches, insofar as possible, results in two relatively long floor beams which contribute significantly to the weight of the total system.

The above observations suggest that the advantages of this bridge concept are outweighed by the disadvantages. The disadvantages result in a relatively heavy system; furthermore, the effectiveness of the arch design is compromised by a discrete point loading scheme.

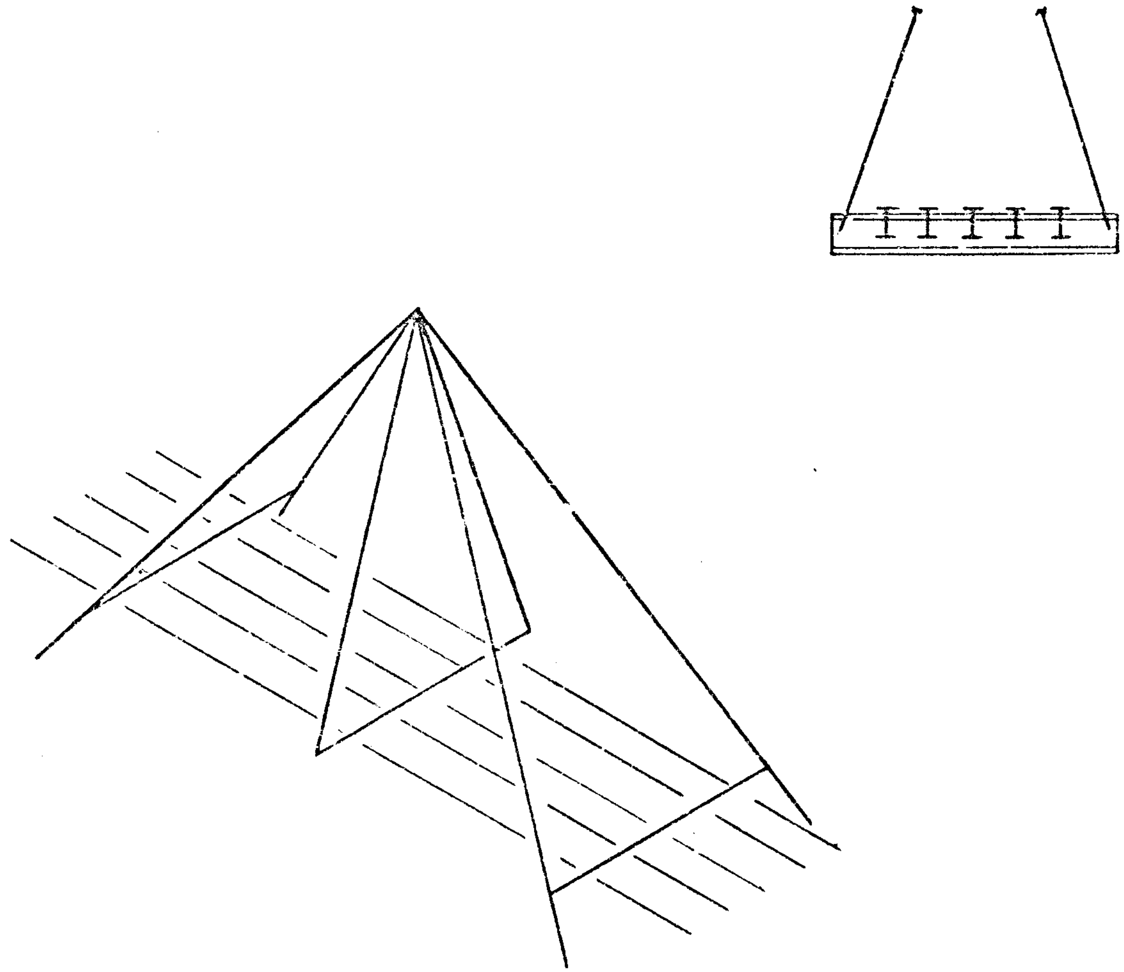


FIGURE C.1. ISOMETRIC SCHEMATIC DIAGRAM OF "A" FRAME BRIDGE CONCEPT

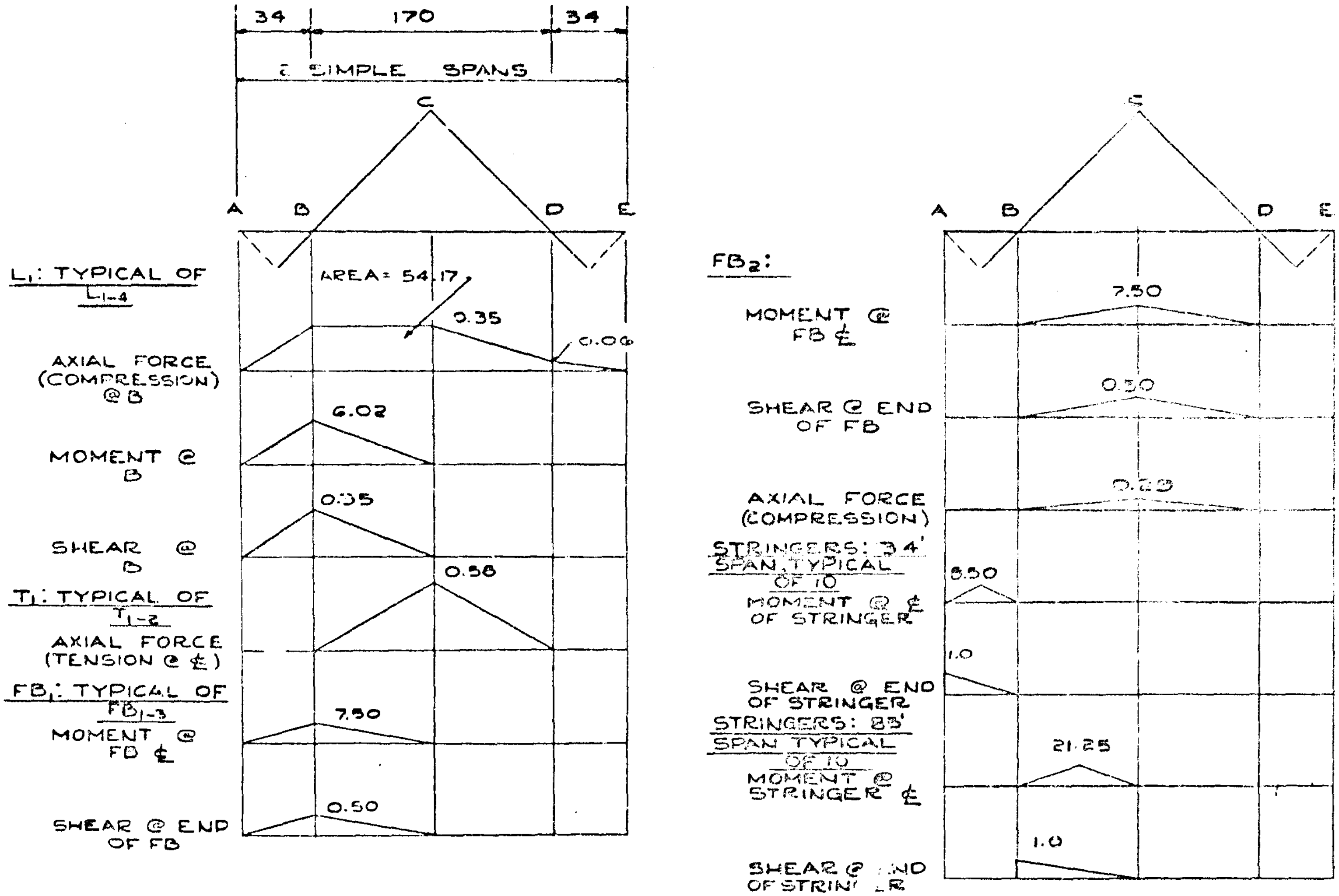


FIGURE C.2. INFLUENCE DIAGRAMS FOR "A" FRAME BRIDGE

TABLE C.1. DESIGN VALUES: "A" FRAME BRIDGE

(a) Principal Structural Members

Member	Concentrated Loads: Equation for Concentrated Live Load Effect	Concentrated Loads: Concentrated Live Load Effect	Concentrated Loads: Impact	Concentrated Loads: Concentrated Live Load Plus Impact	Uniform Loads: Equation for Uniform Load Effect	Uniform Loads: Dead Load Force Effect	Uniform Loads: Live Load Force Effect	Uniform Loads: Live Load Effect Plus Impact Effect	Total Effect for Design
L_{1-4}	Axial Force: $F = -0.35P$[1]	-12.6 K	1.14[2]	-14.4 K	$F = -(54.17)w$[3]	-65 K	-69.4 K	-79.1 K	-158.5K
	Moment: $F = 6.02P$[1]	220 K-ft	1.20[2]	264 K-ft	$F = \frac{1}{2}(6.02)(119)w$[3]	430 K-ft	458 K-ft	550 K-ft	1244 K-ft
	Shear: $F = 0.35P$[1]	12.6 K	1.20[2]	15.1 K	$F = \frac{1}{2}(0.35)(119)w$[3]	25.0 K	26.7 K	32.1 K	72.2 K
$T_{1,2}$	Axial Force: $F = 0.58P$[1]	20.9 K	1.17[4]	24.5 K	$F = \frac{1}{2}(0.58)(170)w$[3]	59.1 K	63.0 K	74.4 K	158.0 K

(1) P is concentrated load for two lanes acting at centerline of bridge. P = 36 K.
(2) Span = 238 ft for impact factor computation, axial force; span = 119 ft for impact factor computation, moment and shear.
(3) w_{DL} = 1200 plf (est.), w_{LL} = 1240; uniform loads for two lanes.
(4) Span = 170 ft for impact factor computation.

(b) Floor System Members

Member	Concentrated Loads: Equation for Concentrated Live Load Effect	Concentrated Loads: Concentrated Live Load Effect	Concentrated Loads: Impact	Concentrated Loads: Concentrated Live Load Plus Impact	Uniform Loads: Equation for Uniform Load Effect	Uniform Loads: Dead Load Force Effect	Uniform Loads: Live Load Force Effect	Uniform Loads: Live Load Effect Plus Impact Effect	Total Effect for Design
$FB_{1,3}$	Moment: $F = 7.50P$[1]	270 K-ft	1.21[2]	327 K-ft	$F = \frac{1}{2}(119)(7.5)w$[3]	535 K-ft	570 K-ft	691 K-ft	1553 K-ft
	Shear: $F = 0.50P$[1]	18 K	1.21[2]	21.8 K	$F = \frac{1}{2}(119)(0.50)w$[3]	35.8 K	38.2 K	46.2 K-ft	103.8 K
FB_2	Moment: $F = 7.50P$[1]	270 K-ft	1.17[4]	316 K-ft	$F = \frac{1}{2}(170)(7.5)w$[3]	755 K-ft	816 K-ft	955 K-ft	2026 K-ft
	Shear: $F = 0.50P$[1]	18 K	1.17[4]	21.1 K	$F = \frac{1}{2}(170)(0.50)w$[3]	51.0 K	54.5 K	63.8 K	135.9 K
	Axial Force: $F = -0.29P$[1]	-10.4 K	1.17	-12.2 K	$F = \frac{1}{2}(170)(0.29)w$[3]	-29.6 K	-31.5 K	-36.8 K	-76.8 K
Stringers (34)	Moment: $F = 8.50P$[5]	61.2 K-ft	1.3[6]	79.5 K-ft	$F = \frac{1}{2}(34)(8.50)w$[7]	35.7 K-ft	37.0 K-ft	48.1 K-ft	163.3 K-ft
(Typical of 10)	Shear: $F = 1.0P$[5]	10.4 K	1.3[6]	13.5 K	$F = \frac{1}{2}(34)(1.00)w$[7]	4.1 K	4.4 K	5.7 K	23.3 K
Stringers (85)	Moment: $F = 21.25P$	153 K-ft	1.23[6]	188 K-ft	$F = \frac{1}{2}(85)(21.25)w$[7]	217 K-ft	232 K-ft	286 K-ft	691 K-ft
(Typical of 10)	Shear: $F = 1.0P$	10.4 K	1.23[6]	12.8 K	$F = \frac{1}{2}(85)(1.00)w$[7]	10.4 K	10.7 K	13.2 K	36.4 K

(1) P is concentrated live load for two lanes acting at center of floor beam; P = 36 K.
(2) Span = 119 ft for $FB_{1,3}$ impact computations.
(3) w_{DL} = 1200 plf and w_{LL} = 1280 plf as est DL and LL for two lanes; considered to act at center of floor bean.
(4) Span = 170 ft for FB_2 impact computation.
(5) P is concentrated load assigned to one of five stringers; P = 7.2 K for moment, 10.4 K for shear.
(6) Impact computed for 34-ft span = 1.30 (max); 85-ft span = 1.23.
(7) w is uniform load assigned to one of five stringers; w_{DL} = 240 plf; w_{LL} = 256 plf.

TABLE C.II. CONCEPT DESIGN, "A" FRAME BRIDGE

Member	Design Value	Design Notes	Section	Area	Unit Weight	Length	Weight	Quantity	Total Weights	
Principal Structural Members										
L_1: Axial Force Moment Shear	 -158.5 K 1244 K-ft 72.2 K	Moment is critical parameter; box girder design appropriate. S req'd = 679 cu in., use box girder designed for bending and compression.	Box girder: 24 X 36 in., 1-in. flanges, 1/2-in. webs, I = 18,874	82 sq in.	279 plf	157 ft	43.8 K	4	174 K	
T_1: Axial Force	 158.0 K	Tensile member, use cable with allowable stress 80 ksi. A req'd = 1.98 sq in., use 1-5/8-dia rod.	1-5/8-in.-dia rod	2.1 sq in.	7.1 plf	85 ft	0.8 K	2	1.6 K	
					Total principal structure				175.6 K	
					Additional bracing, appurtenances (20%)				35.2 K	
					Total estimated weight of principal structure					210.8 K
Floor System Members										
FB_1: Moment Shear	 1553 K-ft 103.8 K	Short deep beam, span = 30 ft; plate girder design. S req'd = 848 cu in., use welded plate girder, 49-in. deep.	Plate girder: 49 X 16 in., 16 X 1-in. flanges, 3/8-in. web, I = 22,667	50 sq in.	170 plf	30 ft	5.1 K	2	10.2 K	
FB_2: Moment Shear Axial Force	 2026 K-ft 135.9 K 76.8 K	Short deep beam, span = 30 ft; plate girder design with axial load capability, S req'd = 1110 cu in.	Plate girder: 49 X 16 in., 16 X 1-1/2-in. flanges, 3/8-in. web, I = 32,868	66 sq in.	224 plf	30 ft	6.7 K	1	6.7 K	
Stringers (34): Moment Shear	 163.3 K-ft 23.3 K	For 34-ft stringer system, use WF beams. S req'd = 89.4, use 18 WF 50.	18 WF 50 beam	--	50 plf	34	1.7 K	10	17.0 K	
Stringers (85): Moment Shear	 691 K-ft 36.4 K	For 85-ft stringer system, use WF beams. S req'd = 377, use 33 WF 130.	33 WF 130 beam	--	130 plf	85	11.5	10	115.0 K	
					Total floor system				148.9 K	
					Lateral bracing stiffeners, etc. (15%)				30.0 K	
					Total estimated weight of floor system					179.9 K
					Total Bridge Weight					390.7K

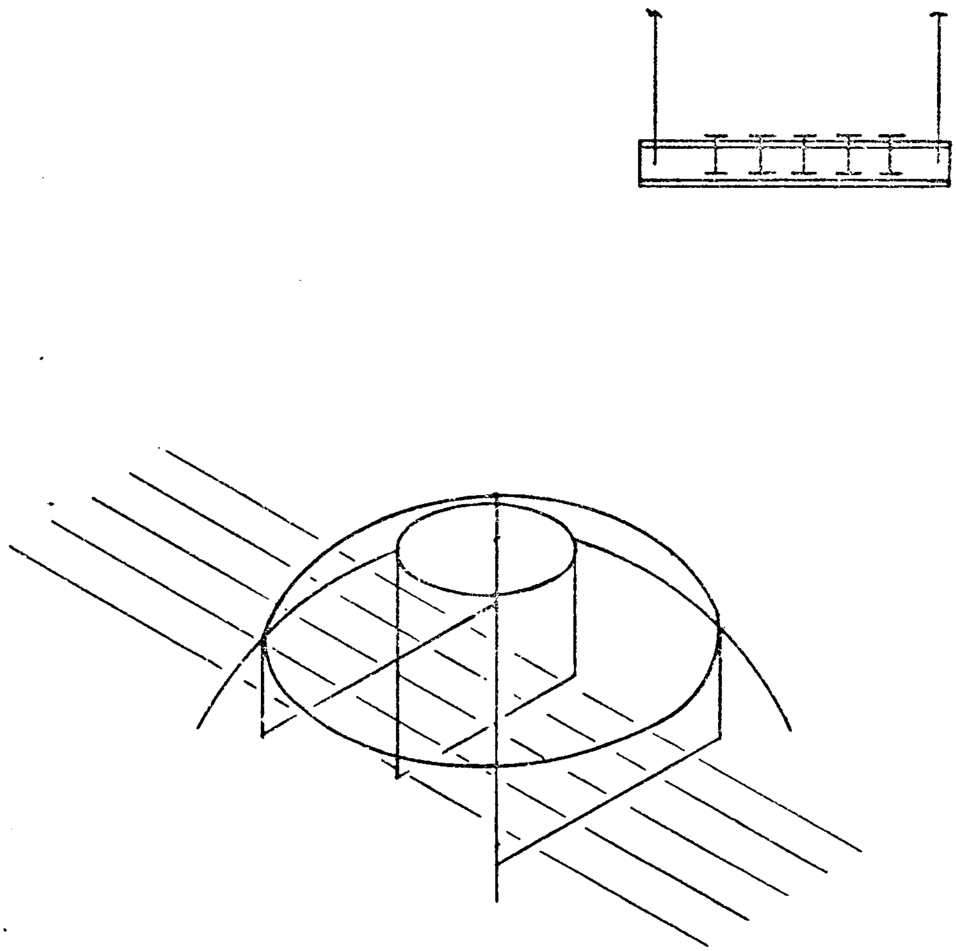


FIGURE C.3. ISOMETRIC SCHEMATIC DIAGRAM OF DOME BRIDGE CONCEPT

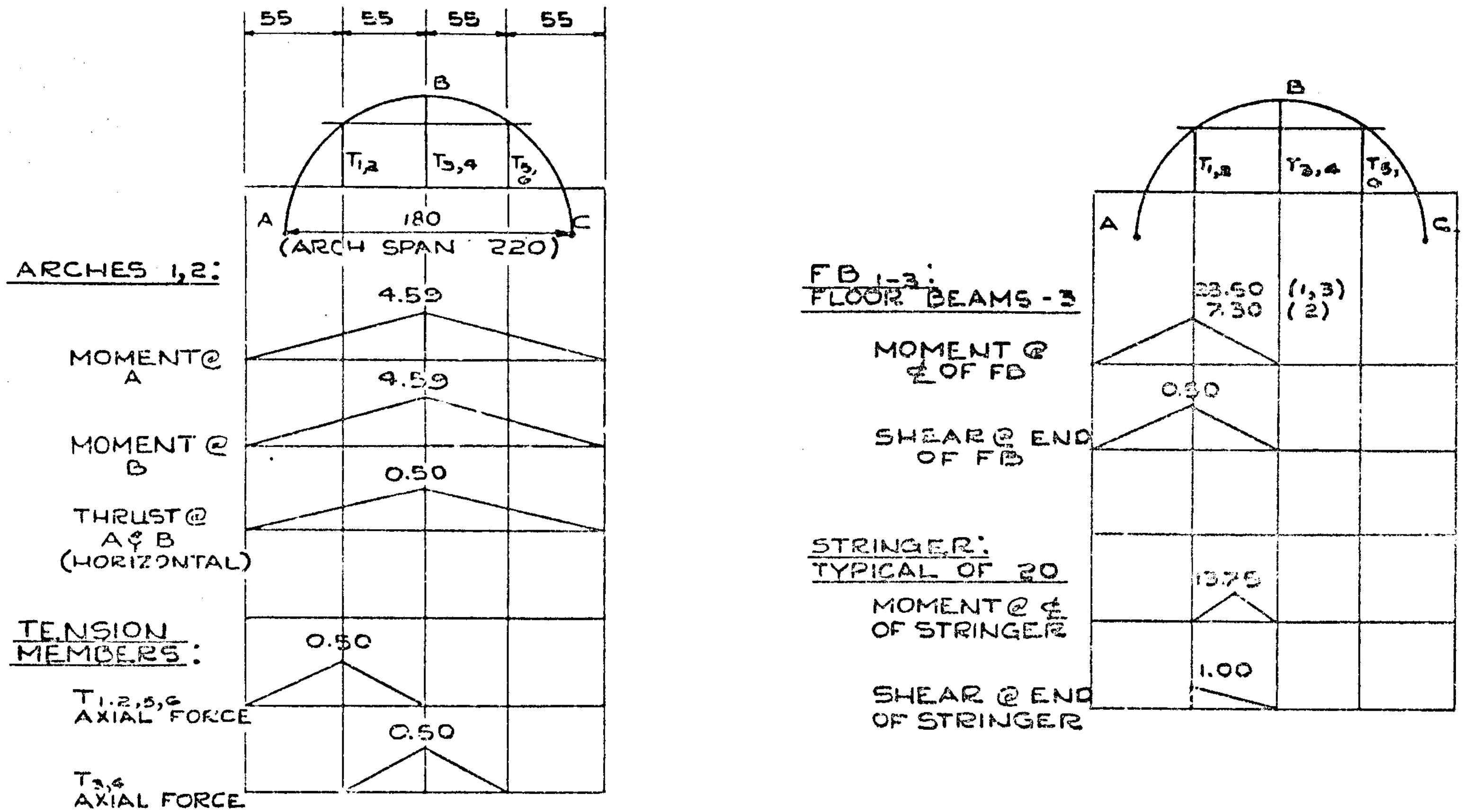


FIGURE C.4. INFLUENCE DIAGRAMS FOR DOME BRIDGE

TABLE C. III. DESIGN VALUES: DOME BRIDGE

(a) Main Structural Members

Member	Concentrated Loads: Equation for Concentrated Live Load Effect	Concentrated Live Load Effect	Impact	Concentrated Live Load Plus Impact	Uniform Loads: Equation for Uniform Load Effect	Dead Load Force Effect	Live Load Force Effect	Live Load Effect Plus Impact Effect	Total Effect for Design
Arch	Moments:								
	Crown: $F = 4.59P$[1,2]	166 K-ft	1.15[3]	190 K-ft	$F = \pm \frac{1}{225}(220)^2 w$[2,4] $= 215w$	129 K-ft	138 K-ft	159 K-ft	478 K-ft
	Spring: $F = 4.59P$[1,2]	166 K-ft	1.15	190 K-ft	$F = \pm \frac{1}{45}(220)^2 w = 1075w$	646 K-ft	689 K-ft	792 K-ft	1628 K-ft
	Thrusts:								
	Crown: $F = 0.50P$[1]	13 K	1.15	20.7 K	$F = \frac{1}{8}(220)^2/25\ w$[2,4] $= 242w$	145 K	155 K	178 K	343.7 K
	Spring: $F = 0.50P$[1]	18 K	1.15	20.7 K	$F = \frac{1}{8}(220)^2/25\ w = 242w$	145 K	155 K	178 K	343.7 K
Tension Members									
$T_{1,2,5,6}$	Axial Force: $F = 0.50P$	18 K	1.21[5]	21.8 K	$F = \frac{1}{2}(0.50)(110)w$[4]	27.0 K	28.8 K	34.8 K	83.6 K
$T_{3,4}$	Axial Force: $F = 0.50P$	18 K	1.21	21.8 K	$F = \frac{1}{2}(0.50)(110)w$[4]	27.0 K	28.8 K	34.8 K	83.6 K

(1) P = 36 K as two lane concentrated Live Load effects.
(2) From Hardy Cross, "Statically Indeterminate Structures," The College Publishing Co. (Champaign), 1926.
(3) Arch span = 220 ft and bridge span = 220 ft, use 220 ft for impact factor computation.
(4) w_{DL} = 1200 plf (est), and w_{LL} = 1280 plf as uniform loads, for preliminary design purposes only, uniform lane loads are considered to be applied uniformly to arches.
(5) Span = 110 ft for impact factor computation.

(b) Floor System Members

Member	Concentrated Loads: Equation for Concentrated Live Load Effect	Concentrated Live Load Effect	Impact	Concentrated Live Load Plus Impact	Uniform Loads: Equation for Uniform Load Effect	Dead Load Force Effect	Live Load Force Effect	Live Load Effect Plus Impact Effect	Total Effect for Design
FB_2	Moment: $F = 7.50P$[1]	270 K-ft	1.21[2]	327 K-ft	$F = \frac{1}{2}(7.50)(110)w$[3]	495 K-ft	529 K-ft	640 K-ft	1405 K-ft
	Shear: $F = 0.50P$[1]	18 K	1.21[2]	21.8 K	$F = \frac{1}{2}(0.50)(110)w$[3]	33 K	35.2 K	42.6 K	97.4 K
$FB_{1,3}$	Moment: $F = 28.5P$[4]	1050 K-ft	1.21	1240 K-ft	$F = \frac{1}{2}(28.5)(110)w$	1860 K-ft	2000 K-ft	2420 K-ft	5520 K-ft
	Shear: $F = 0.50P$[1]	18 K	1.21	21.8 K	$F = \frac{1}{2}(0.50)(110)w$[3]	33 K	35.2 K	42.5 K	97.4 K
Stringers (typical of 20)	Moment: $F = 13.75P$[5]	99 K-ft	1.28[6]	127 K-ft	$F = \frac{1}{2}(13.75)(55)w$[7]	90.9 K-ft	97 K-ft	124 K-ft	341.9 K-ft
	Shear: $F = 1.00P$[5]	10.4 K	1.28[6]	13.3 K	$F = \frac{1}{2}(1.00)(55)w$[7]	6.5 K	7.0 K	9.0 K	28.9 K

(1) P is concentrated load for two lanes acting at center of floor beam, P = 36 K.
(2) Span = 110 ft for floor beam impact considerations.
(3) w_{DL} = 1200 plf and w_{LL} = 1280 plf as est Dead Load and Live Load for two lanes; acting at center of floor beam.
(4) Floor beams 1, 3 extend to vertical hanger, floor beam span is 110 ft.
(5) P is concentrated load assigned to one of five stringers; P = 7.2 K for moment, 10.4 K for shear.
(6) Span = 55 ft for impact computation.
(7) Uniform load assigned to one of five stringers, w_{DL} = 240 plf (est), w_{LL} = 256 plf.

TABLE C.IV. CONCEPT DESIGN, DOME BRIDGE

Member	Design Value	Design Notes	Section	Area	Unit Weight	Length	Weight	Quantity	Total Weight	
Principal Structural Members										
Arches 1, 2										
Moments: Crown Spring Thrusts: Crown Spring	 478 K-ft 1628 K-ft 343.7 K 343.7 K	Design as two plate girders intersecting 30-ft-dia ring 90° apart, assume linear weight change from crown to spring for weight estimate purposes. Crown: S req'd = 262 cu in. × 1.25 (allow for crown thrust) = 328 cu in.; Spring: S req'd = 887 cu in. × 1.15 (allow for spring thrust) = 1020 cu in.	Spring: plate girder 49 × 16 in., 16 × 1-1/4-in. flanges, 3/8-in. web Crown: plate girder 30 × 10-1/2 in., 10-1/2 × 7/8-in. flanges, 5/-16-in. web	58 sq in. 30 sq in.	197.2 plf (156.6 plf avg) 116. plf	346 ft	53.9 K	2	107.8 K	
Tension Members										
$T_{1,2,5,6}$:	83.6K	Use rod with allowable stress = 80 ksi; A req'd = 1.05 sq in.	1-1/4-in.-dia rod	1.23 sq in.	4.2 plf	40 ft	1.7 K	4	6.8 K	
$T_{3,4}$:	83.6 K	Use rod with allowable stress = 80 ksi; A req'd = 1.05 sq in.	1-1/4-in.-dia rod	1.23 sq in.	4.2 plf	55 ft	2.3 K	2	4.6 K	
Compression										
Rings	(No load due to vertical load condition)	Top ring is 30-ft dia, estimate as 24 WF 100 Lower ring is 110-ft dia, estimate as 24 WF 68	24 WF 100 24 WF 68	-- --	100 plf 68 plf	94 ft 346 ft	9.4 K 23.3K	1 1	9.4 K 23.3 K	
					Total main structure				151.9 K	
					Bracing, appurtenances (15%)				22.6	
										174.5 K
Floor System Members										
FB_2: Moment Shear	 1405 K-ft 97.4 K	Plate girder design, S req'd = 768 cu in.	41 × 14 in., 14 × 1-1/4-in. flanges, 5/16-in. web	47.50 sq in.	162 plf	30 ft	4.9 K	1	4.9 K	
Stringers: Moment Shear	 3[illegible] K-ft 28.9 K	Design as WF beam, S req'd = 185 cu in.	24 WF 84	--	84 plf	55 ft	4.6 K	20	92.3 K	
FB_2: Moment Shear	 5520 K-ft 97.4 K	Plate girder design, S req'd = 3020 cu in.	74 × 24 in., 24 × 1-1/2-in. flanges, 9/16-in. web	--	382 plf	110 ft	42.0 K	2	84.0 K	
					Total floor system				191.2 K	
					Appurtenances (15%)				28.6 K	
										219.8 K
					Total Bridge Weight					394.3 K

APPENDIX D

SUPPORTING DATA FOR BRIDGE PRELIMINARY DESIGNS

APPENDIX D

SUPPORTING DATA FOR BRIDGE PRELIMINARY DESIGNS

Preliminary designs for the Leaning Arches Bridge, the Bridle Bridge, and the Frame Bridge are presented in Volume II. Although tabulations of key engineering data are included with the design presentations, certain of the supporting data were not included so as to not burden the design summaries with extensive detail. In this appendix, supporting data for each of the three bridge preliminary designs are recorded for the engineer or researcher who wishes to pursue the design computations in more detail.

D. 1. Leaning Arches Bridge Supporting Data

The two configurations of the Leaning Arches Bridge (new bridge and modified bridge) were designed using the elastic center method outlined by Borg and Gennaro*. Figure D. 1 and Table D. I describe the arch geometry and the method for finding the elastic center for the arch configuration employed in the two preliminary designs. Table D. II carries the analysis procedure to the point where η_H, η_V, and η_M are determined. From these constants, the horizontal and vertical reactions and the moments at the fixed end of the arch are determined for unit loads according to the expressions given below.

$$H_B = \frac{-\eta_H}{\delta_{XH}} \tag{1}$$

$$V_B = \frac{-\eta_V}{\delta_{YV}} \tag{2}$$

$$M = \frac{-\eta_M}{\theta_M} \tag{3}$$

where

$$\delta_{YV} = \Sigma_{x'}2 \frac{\Delta s}{I} \qquad \theta_M = \frac{\Sigma \Delta s}{I}$$

$$\delta_{XH} = \Sigma_{y'}2 \frac{\Delta s}{I} + \frac{\Sigma \Delta s}{A}$$

*Borg, S. F. and Gennaro, J. J., Advanced Structural Analysis, D. Van Nostrand Company, Inc, Princeton, New Jersey, 1959, 368 pp.

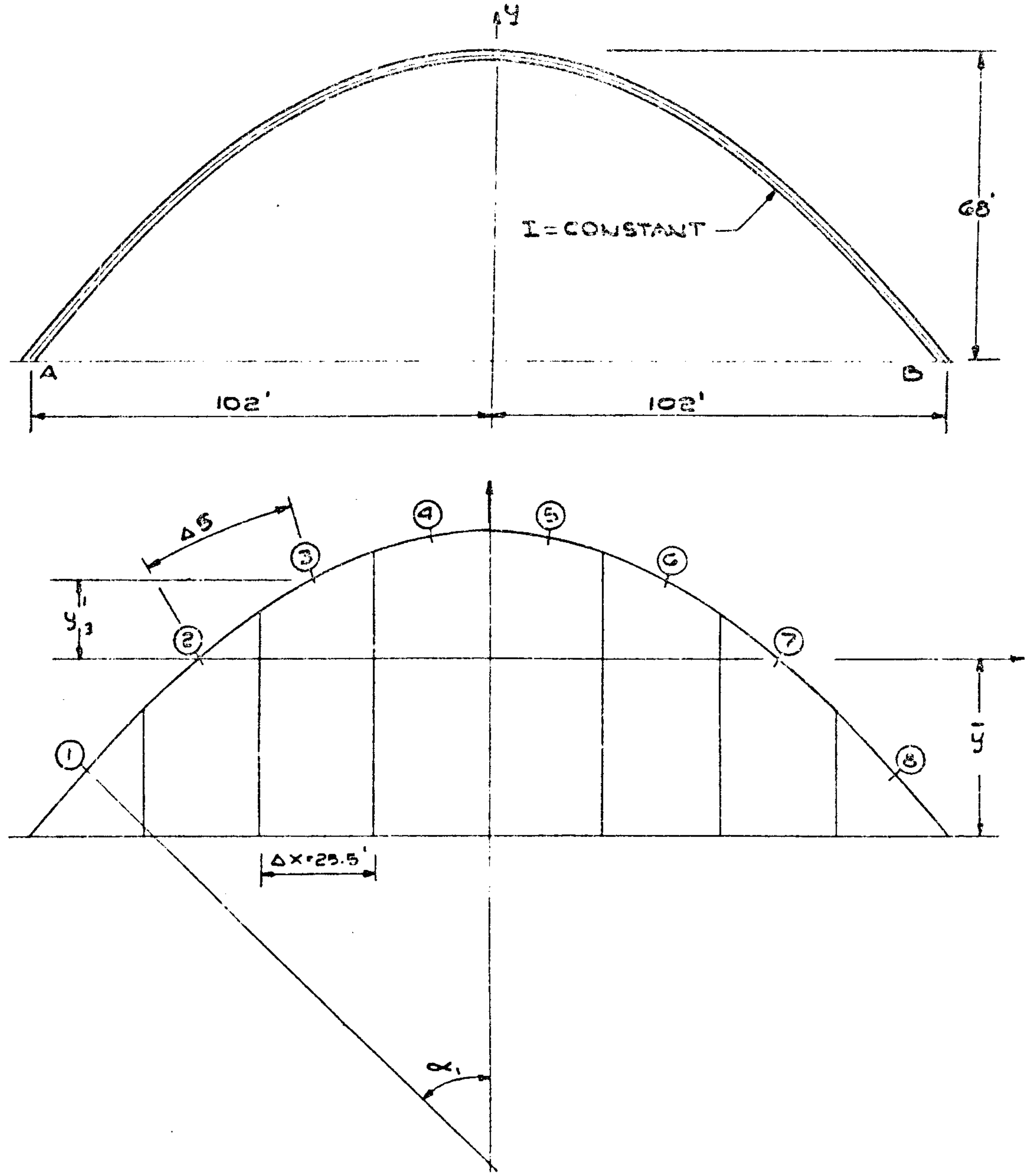


FIGURE D. 1. GEOMETRY OF ARCH ILLUSTRATING METHOD FOR DETERMINING ELASTIC CENTER (LEANING ARCHES BRIDGE)

TABLE D.I. DETERMINATION OF ELASTIC CENTER FOR ARCH

Col / Point	1: $x = x'$ (ft)	2: y (ft)	3: α	4: $I(ft^4)$	5: $\frac{\Delta x}{I \cos \alpha} = \frac{\Delta s}{I}$	6: $y\left(\frac{\Delta s}{I}\right)$	7: $y' = y - \bar{y}$	8: $y'^2\left(\frac{\Delta s}{I}\right)$	9: $x'^2\left(\frac{\Delta s}{I}\right)$	10: A (ft^2)	11: $\frac{\Delta s}{A}$
1	-89.25	15.0	9.5°	2.55	15.1	226.0	-27.3	11,200	12,600	0.865	44.5
2	-63.75	41.0	33°	2.55	13.1	540.0	-1.3	19	5,620	0.865	38.7
3	-38.25	58.0	56.8°	2.55	11.3	658.0	15.7	2,790	1,740	0.865	33.5
4	-12.75	67.0	79.3°	2.55	10.2	684.0	24.7	6,230	174	0.865	[illegible].0
5	12.75	67.0	-79.3°	2.55	10.2	684.0	24.7	6,230	174	0.865	30.0
6	38.25	58.0	-56.8°	2.55	11.3	658.0	15.7	2,790	1,740	0.865	33.5
7	63.75	41.0	-33°	2.55	13.1	540.0	-1.3	19	5,620	0.865	38.7
8	89.25	15.0	-9.5°	2.55	15.1	226.0	-27.3	11,200	12,600	0.865	44.5
					99.4	4216.0		40,600	38,400		293.4

$$\bar{y} = \frac{4216.0}{99.4} = 42.3$$

TABLE D.II. DETERMINATION OF η_H, η_V, AND η_M (LEANING ARCHES BRIDGE)

(1) Point	(2) $-y' \frac{\Delta s}{I}$	(3) $\phi_i = \Sigma(2)$	(4)* $\Delta \eta_H = \phi_i \Delta x$	(5) $\eta_H = \Sigma(4)$	(6) $x \frac{\Delta s}{I}$	(7) $\phi_i = \Sigma(6)$	(8)* $\Delta \eta_V = \phi_i \Delta x$	(9) $\eta_V = \Sigma(8)$	(10) $\frac{\Delta s}{I}$	(11) $\phi_i = \Sigma(10)$	(12)* $\Delta \eta_M = \phi_i \Delta x$	(13) $\eta_M = \Sigma(12)$
8	411		5,250		1347.7	0	17,183	0	15.1	0	192.5	0
		411		5,240		1347.7		17,183		15.1		192.5
7	17.2		11,100		835.12		45,000		13.1		552.0	
		428		16,300		2182.8		62,183		28.2		744.5
6	-178		8,620		432.22		61,200		11.3		864	
		250		25,000		2615.0		123,383		39.5		1608.5
5	-252		3,180		130.05		68,400		10.2		1135	
		0		28,200		2745.1		191,783		49.7		2743.5

*Incremental areas calculated by trapezoidal rule.

After the values for η_H, η_V, and η_M have been determined, influence line values can be determined in the manner illustrated by calculations in Table D.III.

The Leaning Arches Bridge, new bridge configuration, utilizes seven pairs of cables to support the roadway, as shown in Figure D.2. Influence diagrams were constructed to determine the maximum design moment--the influence diagrams are shown in Figure D.3. for this cable configuration. By placing one lane load per arch, the maximum design moment can be calculated; in this specific case, the maximum moment occurs at the crown of the arch. Due to the inclined cable geometry, this moment must be multiplied by a factor which takes into account the angle of inclination of the cables. For the purposes of achieving the preliminary design, the bridge spans were considered to be simply supported in the calculation of cable reactions and, in turn, arch loads. A similar process was employed in designing the Leaning Arches Bridge, modified bridge configuration, which employs three pairs of supporting cables.

D.2. Bridle Bridge Supporting Data

D.2.a. Bridle Bridge with Hinged Girder

The Bridle Bridge, hinged girder configuration, structure is statically determinate, and the method of analysis for this structural configuration is presented in paragraph B.4. of Appendix B. The basic load relationships are determined from influence line data developed in the discussion below:

(1) Cables

- Concentrated Load (Unit Load)

  The maximum cable reaction due to a concentrated load is produced when the load is at the hinge point. By taking moments about the left abutment, the tension in cable T_2 is determined

$$\Sigma M_{L.\,Abut} = 100(1) - T_2 \sin \alpha \, (75)$$

$$\alpha = 23°45'$$

$$T_2 = 1.67$$

From Appendix B.4.

$$T_2 = \frac{T_1 \mu_2 \cos \alpha}{1 + \mu_2 \cos \beta} \quad , \quad \mu_2 = \frac{A_2 h^3}{3 I_1 \ell_2 \cos \beta}$$

TABLE D.III. INFLUENCE LINE VALUES FOR REACTIONS AND MOMENT AT FIXED END OF ARCH (LEANING ARCHES BRIDGE)

Point	a - x	$H = H_B$	V	V_B	M	$+ V_a$	$+ H\bar{y}$	M_B
	0	0	0	-1.00	0			
8								
	25.5	-0.128	-0.044	-0.956	-1.94	-4.49	-5.42	13.66
7								
	51.0	-0.399	-0.163	-0.837	-7.45	-16.6	-16.90	10.05
6								
	76.5	-0.611	-0.323	-0.677	-16.10	-32.9	-25.9	1.60
5								
	102.0	-0.690	-0.500	-0.500	-26.70	+51.0	-29.2	-4.90
4								
	127.5	-0.611	-0.323	-0.323	-16.10	+32.9	-25.9	-9.10
3								
	153.0	-0.399	-0.163	-0.163	-7.45	+16.6	-16.9	-7.75
2								
	178.5	-0.128	-0.044	-0.044	-1.94	+4.49	-5.42	-2.89
1								
	204.0	0	0	0	0			0

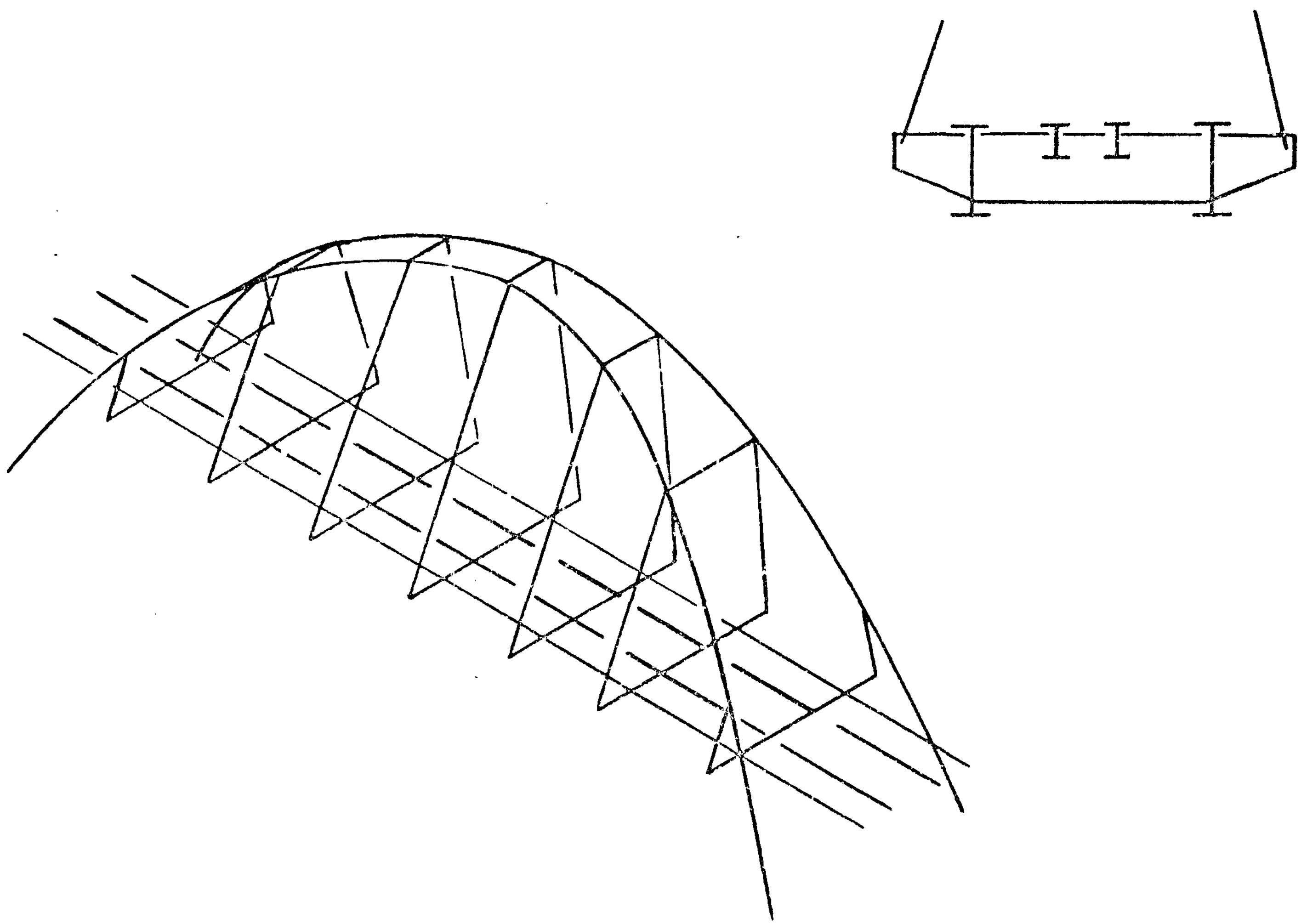


FIGURE D. 2. CONFIGURATION OF LEANING ARCHES BRIDGE STRUCTURE
(New Bridge Configuration)

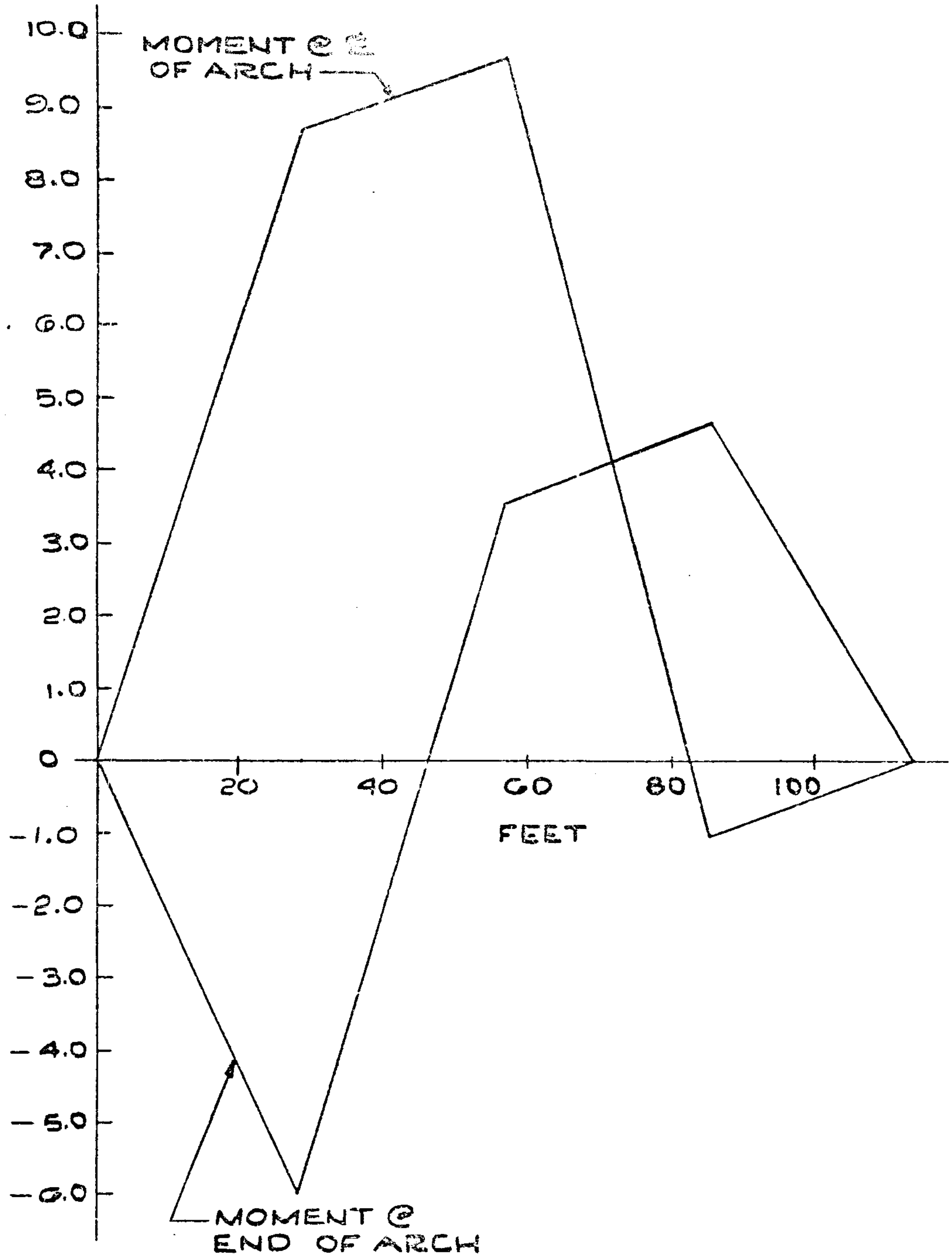


FIGURE D. 3. INFLUENCE DIAGRAMS FOR BENDING MOMENTS AT CROWN AND SPRINGING OF ARCH EMPLOYED IN PRELIMINARY DESIGN (LEANING ARCHES BRIDGE, NEW BRIDGE CONFIGURATION)

Solving for T_2 in the preliminary design bridge configuration

$T_2 = 1.28 \quad T_1 = 2.14$

- Uniform Load

  The cable forces due to a uniform load in the structure are found by computing the area under the influence line. In this case, this area is a triangle with the height being the maximum cable force and the base being the span of the bridge (175 feet)

  $$T_1 = (1.67)\left(\frac{175}{2}\right) = 146w$$

  $$T_2 = (2.14)\left(\frac{175}{2}\right) = 188w$$

  where w = uniform load in lb/ft

(2) Member L

- Concentrated Load (Unit Load)

  Axial Load

  $$F = T_1 \sin \alpha + T_2 \sin \beta$$

  $$F = 0.399\ T_1 + 0.707\ T_2$$

  $$F_{max} = 0.399\ (1.67) + 0.707\ (214) = 2.175$$

  Shear

  $$V = T_1 \cos \alpha - T_2 \cos \beta$$

  $$V = 0.917\ T_1 - 0.707\ T_2$$

  V = 0 for all locations of unit load

- Uniform Load

  $$\text{Axial Force } F = 2.175\left(\frac{175}{2}\right) = 190w$$

(3) Girder $G_{1,3}$

- Concentrated Load

  Axial Force

  This force is produced by horizontal component of T_1

  $F = T_1 \cos \alpha = 1.5P$

  Moment

  The maximum positive moment occurs at $\frac{c}{2}$ when the unit load is at $\frac{c}{2}$.

  $M_+ = 9.38P$

  The maximum negative moment occurs at $x = d$ when the unit load is at the hinge.

  $M_- = -12.5P$

  Shear

  $V = 0.5P$

- Uniform Load

  Forces, moments, and shear effects are determined by calculating areas under the appropriate influence lines.

(4) Girder Grid

This is a simple span beam.

(5) Floor Beams and Stringers

These members are designed in a conventional manner as shown in Table V of Volume II. The floor beam was designed by assuming the concentrated load acting at the centerline of the roadway.

D.2.b. Bridle Bridge with Continuous Girder

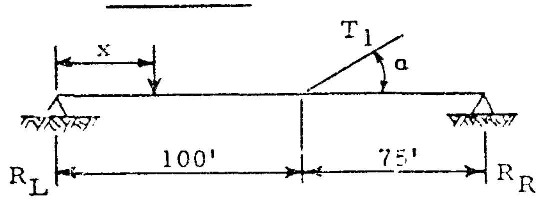
A computer solution was utilized to determine forces in this indeterminate structure. The computer program and data printouts are included in Appendix E. The following discussion is concerned with the use of this computer output in determining forces, moments, and shears for preliminary design purposes. The geometry of this bridge configuration is identical to the Bridle Bridge with hinge, with the hinge removed from the structure. Forces in the cables ($T_{1,2}$) and the vertical members ($L_{1,2}$) are calculated in Table D.IV; design values for the main girders ($G_{1,2}$), floor beams, and stringers are discussed in the following:

TABLE D.IV. DESIGN VALUES FOR BRIDLE BRIDGE COMPONENTS

Member	Type of Load	Concentrated Load Effects	Uniform Load Effects
T_1	Axial Force	$F = \frac{2.5P}{2*} = 1.25P$	$F = (2/3)^{\dagger}\left(\frac{2.5w}{2*}\right)(1.75) = 146.0w$
T_2	Axial Force	$F = \frac{3.08P}{2*} = 1.54P$	$F = 2/3^{\dagger}\left(\frac{3.08w}{2*}\right)(175) = 180.0w$
$L_{1,2}$	Axial Force	$F = T_1 \sin\alpha + T_2 \sin\beta$, $F = 1.59P$	$F = 184.8w$
$G_{1,2}$, Floorbeams and Stringers	(See Discussion)		

*Two cables.
†Parabolic area assumed.

(1) Girder G_1



$\Sigma M_L = -T_1 \sin\alpha\,(100) + x - 175\,(R_R)$

$R_R = 0.00572\,x - 0.226\,T_1$ (1)

$R_L = 0.1 - 0.396\,T_1 - R_R$ (by summation of vertical forces) (2)

Equations (1) and (2) were plotted and are illustrated in Figures D.4 and D.5. These plots were used to determine the maximum moments for the girder for both concentrated and uniform loadings.

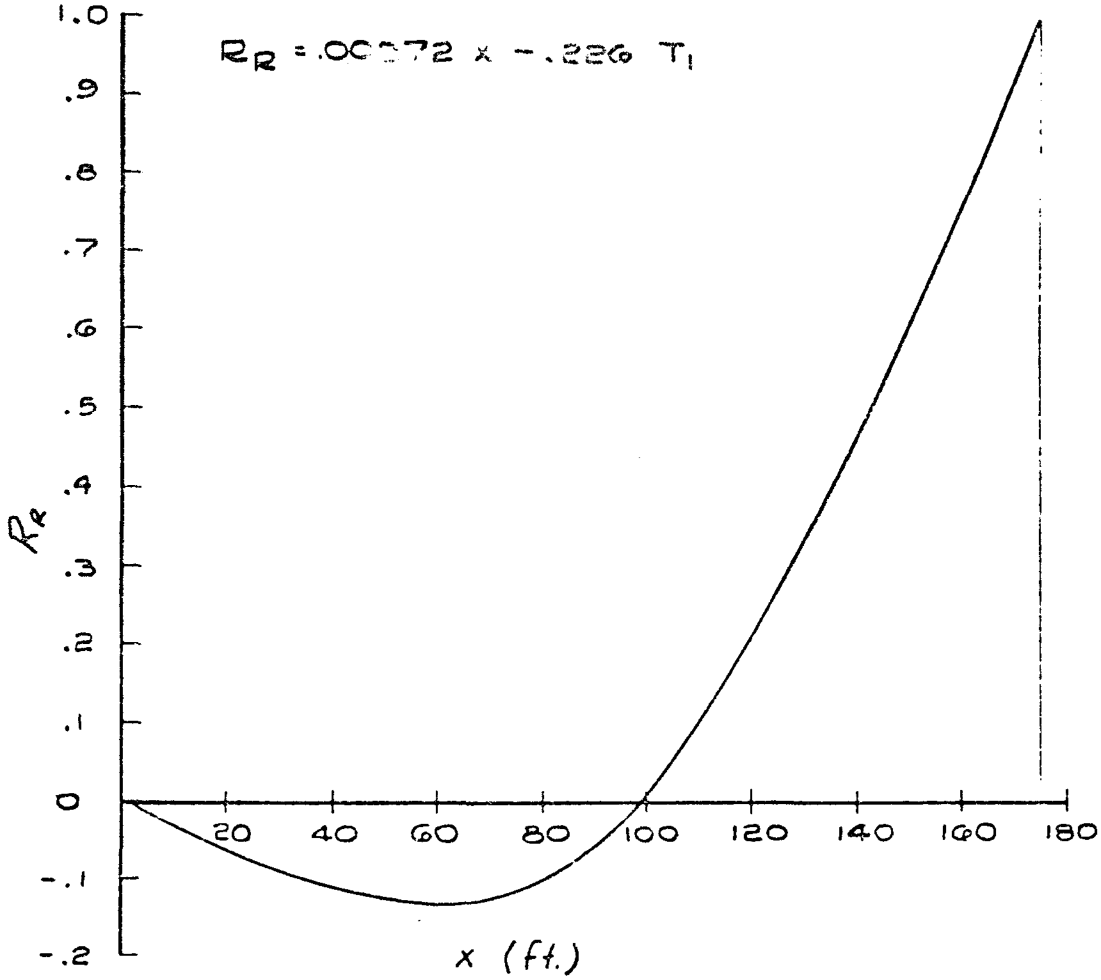


FIGURE D.4. INFLUENCE LINE FOR RIGHT REACTION (BRIDLE BRIDGE, CONTINUOUS GIRDER CONFIGURATION)

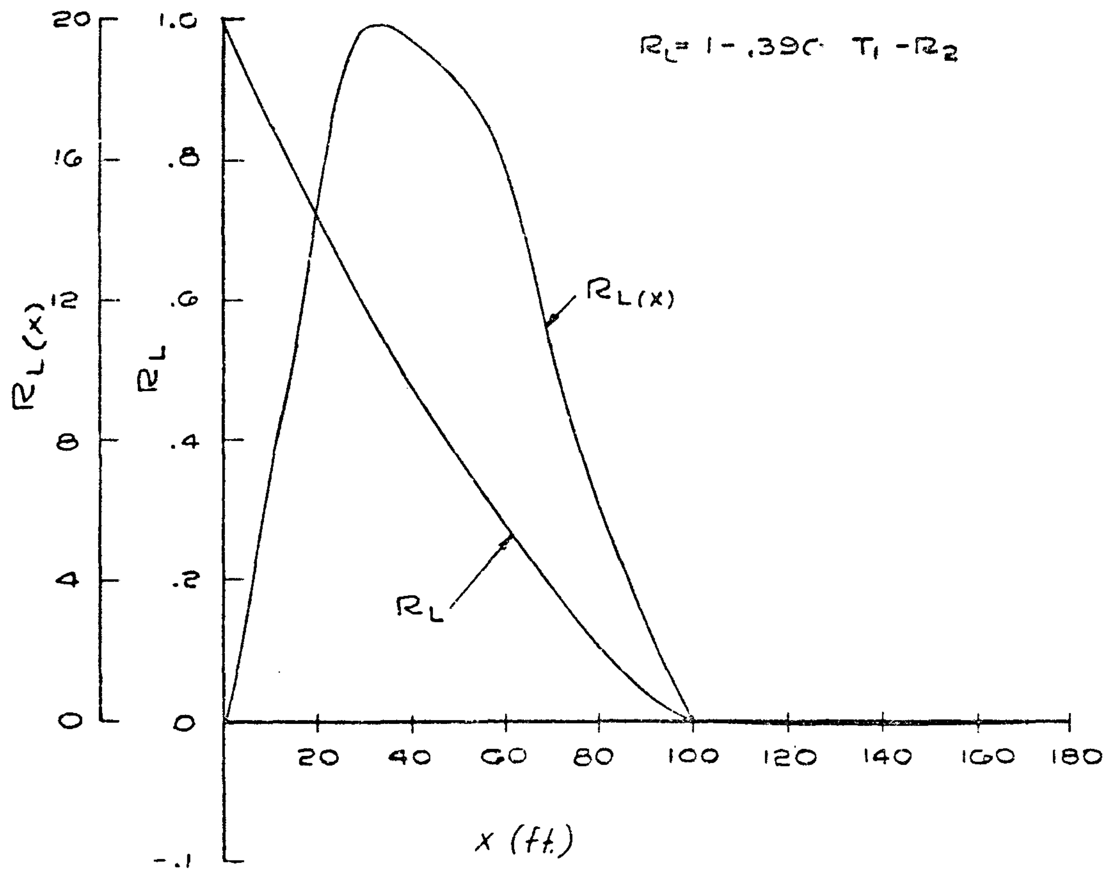


FIGURE D.5. INFLUENCE LINE FOR LEFT REACTION (BRIDLE BRIDGE, CONTINUOUS GIRDER CONFIGURATION)

- Concentrated Loads

Maximum positive moment occurs at x = 40 ft when the load is at x = 40 ft:

$+ M_{max} = 0.4923/2^* (40) = 9.85P$

- Uniform Load

$$R_L = \frac{80}{2^*}\frac{w}{2} = 20w$$

$$\underset{(x = 40)}{+M} = R_L (40) - \frac{w}{2^*}\left(\frac{40}{2}\right)^2 = 400w$$

$$\underset{(x = 100)}{-M} = R_L (100) - \frac{w}{2^*}\left(\frac{100}{2}\right)^2 = -500w$$

(2) Floor Beams and Stringers

These members are designed as simple span beams using conventional analysis techniques. The concentrated load is assumed to act at mid-span of the floor beams.

D.3. Frame Bridge Supporting Data

Computer output presented in Appendix E lists forces and reactions in the cables and frame (due to unit loads). Internal forces in the frame and floor beams are calculated in accordance with the analysis outlined in the following discussion:

(1) Cables

- Concentrated Load Effects

Select maximum ordinate from influence line data:

$T_1, T_3 = 0.550P$ $\qquad T_2, T_2' = 0.405P$

- Uniform Load Effects

Areas under influence lines required:

*Two cables.

Quantity	Total Area, A (ft-lb)	Quantity	Total Area, A (ft-lb)
T_1, T_3	79.2	V_1, V_2	118.6
T_2, T_2'	70.4	H_1, H_2	34.4

T_1, T_3:

$$F = \frac{A}{2^*} w = \frac{79.2}{2} = 39.6w$$

T_2, T_2':

$$F = \frac{A}{2^*} = \frac{70.4}{2} w = 35.2w$$

(2) Legs

- Concentrated Load Effects

Select ordinates for maximum effect

Axial Force

$$F = V_1 \sin \alpha + H_1 \cos \alpha$$

$$F = \frac{0.77}{2^*} (0.811) = 0.313P$$

Moment

$$M = V_1 \cos \alpha \, d$$

where d = 80.41 ft

$$M = \frac{0.77}{2^*} (0.583)(80.41) = 18P$$

- Uniform Load Effects

Axial Force

$$F = V_1 \sin \alpha + H_1 \cos \alpha$$

$$F = \frac{118.6}{2^*} (0.811) + \frac{34.4}{2^*} (0.584) = 58.2w$$

*Two cables.

Moment

$M = (V_1 \cos \alpha - H_2 \sin \alpha)\ d$

$$\left[\frac{118.6}{2*} (0.584) - \frac{34.4}{2*} (0.811)\right] 80.41 = 1660w$$

Shear

$V = V_1 \cos \alpha - H_1 \sin \alpha$

$$V = \frac{118.6}{2*} (0.584) - \frac{34.4}{2*} (0.811) = 20.7w$$

(3) Member F

- Concentrated Load Effects

Moment

$M = V_1 (47) \sin \phi - H_1 (47)$

$M = 32.7\ V_1 - 47\ H_1$

Maximum moment occurs when load is at x = 110 ft

$V_1 = 0.77P \quad , \quad H_1 = 0$

$M = 32.7\ (0.77)P = 25.2P$

Axial Force

$F = H_1 + T_2 \sin \psi$ (solve for load at x = 110 ft)

$H_1 = 0 \quad , \quad T_2 = 0.798$

$F = T_2 \sin \psi = 0.798P(0.32) = 0.255P$

- Uniform Load

Moment

$M = V_1 (47) \cos \phi - H_1 (47)$

$$M = \frac{118.6}{2*} (47)\ (0.695) - \frac{34.4}{2*} (47) = 2250w$$

*Two cables.

Axial Force

$F = H_1 + T_2 \sin \phi$

$F = 34.4 + 70.4(0.798) = 90.6w$

(4) Exterior Floor Beam

To determine the design loads for the transverse floor beams, the concentrated load and the lane load are equally divided among the four stringers. In order to simplify the analysis, rigid supports (at the floor beams) are assumed for the stringers and standard influence line values for a four-equal span structure are employed.

- Concentrated Load Effects

Moment

$$M = \frac{R\ell}{2} - \frac{R\ell}{4}(1.5S)$$

where

R = maximum stringer reaction = 1.004P

ℓ = length of floor beam between cable supports (57 ft)

S = stringer spacing (7 ft 4 in.)

$M = 10.6P$

Shear

$$V = \frac{R}{2} = 0.502P$$

Axial Force

$F = V \tan \phi$

$\phi = 43°55'$

$F = (0.502)P\ (0.963) = 0.484P$

Uniform Load Effects

The load imposed by each stringer acting on the transverse floor beam is computed using standard influence line coefficients for continuous four-span structures on rigid supports:

$$R_s = \frac{kwL}{4}$$

where

R_s = max reaction of individual stringer at transverse floor beam

k = influence line reaction coefficient

w = uniform load (lb/ft)

L = stringer span between cable supports (55 ft)

$$+ R_s = 1.2232\,(w)\left(\frac{55}{4}\right) = 16.8w$$

$$- R_s = (-\,0.0804)(w)\left(\frac{55}{4}\right) = 1.1w$$ (for determining D.L. reaction)

Moment (at ℄ of floor beam)

$$M = R_s\,(2.5)\,\frac{\ell}{2} - R_s(1.5S)$$

where

ℓ = length of floor beam between stringers (57.0 ft)

S = stringer spacing (7 ft 4 in.)

$$+ M = 712.0w$$

$$- M = -\,47.0w$$ (for determining D.L. moments)

(5) Floor Beam, Interior

Design values for the interior floor beams are the same as for the two exterior floor beams, with the addition of shear and moment due to the unbalance in forces in cables T_2 and T_2'.

Unbalanced Force

$$F = (T_2 - T_2') L \sin \psi$$

$$F = 0.796 (T_2 - T_2')$$

This force produces a secondary bending moment:

$$M_{yy} = F\left(\frac{\ell}{2}\right)$$

ℓ = length of floor beam between cables (57 ft)

$$M_{yy} = 0.796 (T_2 - T_2') \frac{57}{2} = 22.7(T_2 - T_2')$$

APPENDIX E

COMPUTER PROCEDURES FOR STRUCTURAL ANALYSES OF BRIDGE STRUCTURES

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APPENDIX E

COMPUTER PROCEDURES FOR STRUCTURAL ANALYSES OF BRIDGE STRUCTURES

The three bridge concepts selected for detailed applications consideration (Leaning Arches Bridge, Bridle Bridge, and Frame Bridge) were subjected to analysis/design iterations which resulted in the preliminary design presentations contained in Volume II. The analysis procedure for these three bridges consisted of refining appropriate systems of equations for the three concepts as presented in Appendix B and, subsequently, developing computer solutions for the systems of equations representing the indeterminate structures. Computer programs are described and presented in the paragraphs which follow for the bridge concepts requiring this type of solution method.

Described in this appendix are the computer programs used to obtain the influence lines for three cable-supported bridge concepts. The programs are written in FORTRAN for the CDC-6600 computer and use the methods of analysis presented in Appendix B. The bridge concepts presented are as follows:

Bridge Concept	Computer Program Name	Analysis Method Reference
Leaning Arches Bridge	LAB	Appendix B.7
Bridle Bridge without hinge*	BRIDL 1	Appendix B.4
Frame Bridge without hinge	RIGID 1	Appendix B.6

*The Bridle Bridge with hinge is a statically determinate configuration which does not require a computer-oriented solution.

E.1 Leaning Arches Bridge

The system of equations which characterizes behavior of the Leaning Arches Bridge was developed for nonvertical cable configurations as described in Paragraph B.7. A computer program for solving these equations was developed to assist in the analysis of specific Leaning Arches Bridge configurations. During the ensuing design/analysis iterations employing the computer program, it became apparent that the most effective structural configuration employs parallel cables in the plane of the arch. Thus, the specific cable configuration selected for the preliminary design presentation represents a special case of the general analysis and general computer program. Figure E.1 illustrates the notations used; Table E.1 relates notations employed in the text with computer symbology. The

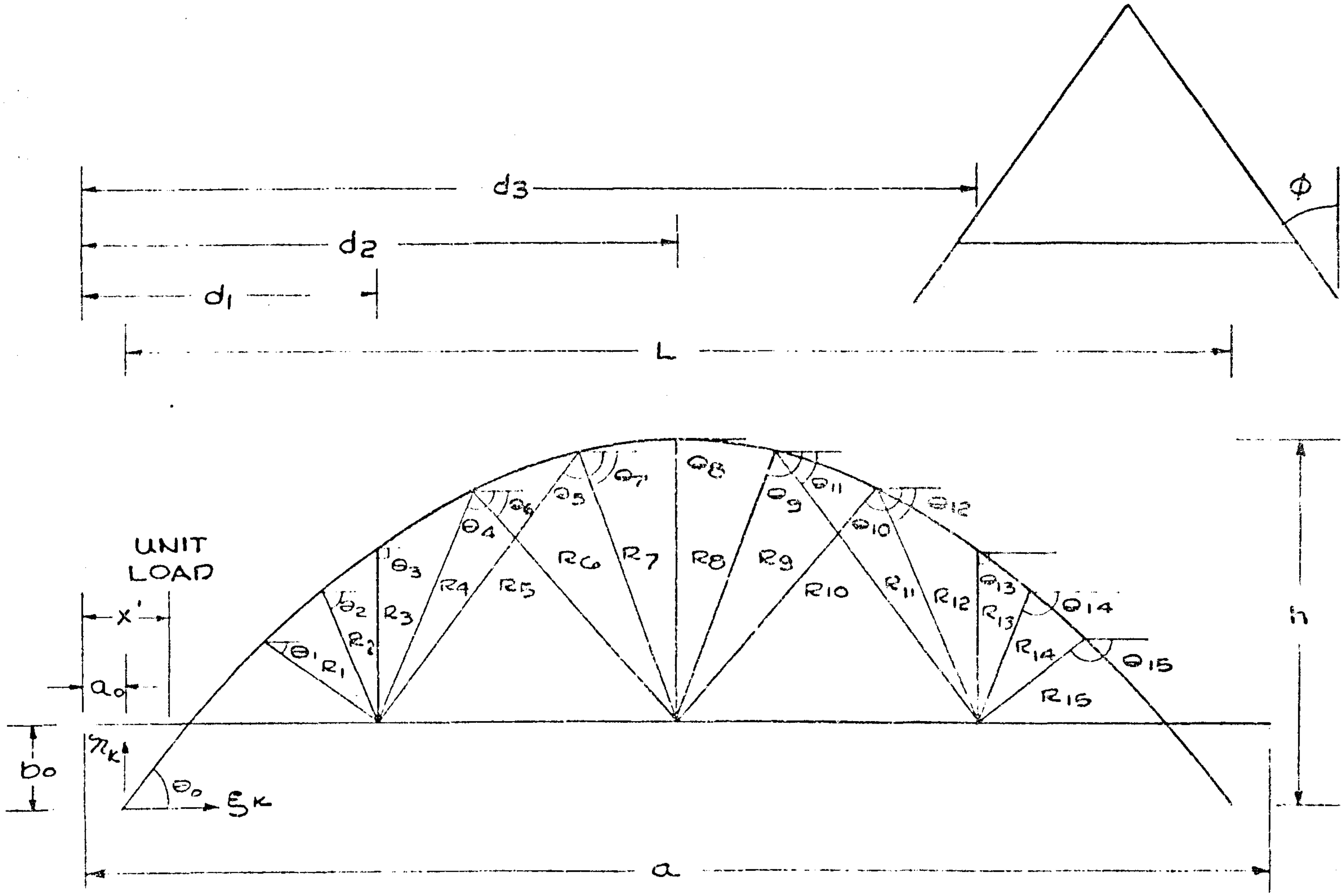

FIGURE E. 1. DEFINITION SKETCH. LEANING ARCHES BRIDGE

TABLE E.I. NOTATION RELATIONSHIPS BETWEEN TEXT AND COMPUTER PROGRAM, LEANING ARCHES BRIDGE

Program "LAB"	Text (Par B.7)
AL(K)	α_K
BE(K)	β_K
CA(KK)	A_J
CL	L
CM1	M_1 (Bridge Girder)
CM2	M_2 (Bridge Girder)
CMC	Moment at Crown
CMO	M_O, Moment at Spring
CMODL	M_O/L
CTH(K)	θ_K
D(KK)	d_K
D1, D2, D3	d_1, d_2, d_3
G0(K), ..., G5(K)	$g_{0,K}, \ldots, g_{5,K}$
GA(K)	γ_K
H	h
HO	H_O
PHI	ϕ
PSI(K)	ψ_K
Q(J, K)	$q_{J,K}$
R(J)	R_j
RBAR	$\overline{R}$
SA	a
SA0	a_0
SB0	b_0
SF0(K), ..., SF4(K)	$f_0, \ldots, f_4$ (Constant I)
SG0(k), ..., SG5(K)	$g_0(\psi), \ldots, g_5(\psi)$ (Constant I)
SG0(K), ..., SG5(K)	$g_0(\xi), \ldots, g_5(\xi)$ (Varying I)
SH1(K), ..., SH3(K)	$h_{1,K}, \ldots, h_{3,K}$
SL(K)	ℓ_j
T1, T2, T3	T_1, T_2, T_3
TC	Thrust at Crown
TS	Thrust at Spring
VO	V_0
XI(KK)	ξ_K
XIB	I_B
XII	I
XMU(J)	μ_j
XNU(K)	η_K
XP(I)	x'_i
Z(J, K)	$Z_{J,K}$

computer program is presented in Table E.II; subroutines written to support the main program are contained in Table E.III. Input data descriptions re contained in Table E.II. Output data are in the form of cable forces, arch shears and moments, and girder shears and moments.

E.2. Bridle Bridge

Two configurations of the Bridle Bridge were considered in the development of preliminary designs: Bridle Bridge with hinge and Bridle Bridge with continuous girder. The former configuration is statically determinate, therefore, the analysis solution does not require a computer-oriented solution (see Paragraph B.4.a. in Appendix B). The Bridle Bridge with continuous girder is an indeterminate structural configuration which requires a computer-oriented solution. A sketch which defines the Bridle Bridge notation is included as Figure E.2. Table E.IV relates text notations with computer symbology. A computer program to solve the system of equations presented in Paragraph B.4.b. of Appendix B was developed and is presented in Table E.V. Input data are described and defined in this table. Output data are presented in Table E.VI for two vertical column conditions. In the first condition, the column is considered pinned at both ends, thus making the horizontal components of the cable forces equal. In the second condition, the column is considered fixed at its lower end, thus causing bending of the column and unequal cable forces. These printouts were employed to develop the influence diagrams presented in Volume II for the Bridle Bridge, continuous girder configuration.

E.3. Frame Bridge

Although two configurations of the Frame Bridge were considered in developing analytical expressions in Paragraph B.6, only the continuous girder configuration was determined to be feasible. A notation definition sketch is presented as Figure E.3. Table E.VII relates text notations with computer symbology. Solution of the statically indeterminate system of equations is represented by the computer program listing in Table E.VIII. Subroutines to the program are contained in Table E.IX. Input data descriptions are contained in Table E.VIII. Output data are presented in Table E.X. These output data were employed to develop the influence diagrams presented in Volume II for the Frame Bridge.

TABLE E. II. COMPUTER PROGRAM: LEANING ARCHES BRIDGE

```
      PROGRAM LAB(INPUT,OUTPUT,TAPE60=INPUT)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C          THIS PROGRAM IS FOR THE SOLUTION OF THE EQUATIONS GIVEN IN
C     APPENDIX D/,METHODS OF ANALYSIS OF CABLE SUPPORTED BRIDGE CON-
C     CEPTS, FOR THE LEANING ARCHES BRIDGE.
C
C          INPUT DATA (LENGTHS ARE IN INCHES, AND ANGLES IN RADIANS)
C
C     CL      = ARCH SPAN
C     H       = ARCH RISE
C     SA      = BRIDGE SPAN
C     SAO     = DISTANCE BETWEEN START OF BRIDGE SPAN AND ARCH SPAN
C     SBO     = HEIGHT OF BRIDGE SPAN ABOVE THE START OF THE ARCH RISE
C     D1      = DISTANCE ALONG THE BRIDGE SPAN TO THE CONNECTION OF
C                THE FIRST CABLE SYSTEM. (1/4 BRIDGE SPAN)
C     D2      = DISTANCE ALONG THE BRIDGE SPAN TO THE CONNECTION OF
C                THE SECOND CABLE SYSTEM. (1/2 BRIDGE SPAN)
C     D3      = DISTANCE ALONG THE BRIDGE SPAN TO THE CONNECTION OF
C                THE THIRD CABLE SYSTEM. (3/4 BRIDGE SPAN)
C     II      = 0 , FOR VARYING MOMENT OF INERTIA OF THE ARCH
C             = 1 , FOR CONSTANT MOMENT OF INERTIA OF THE ARCH
C     KMAX    = NUMBER OF CABLES PLUS ONE
C     IMAX    = NUMBER OF POINTS WHERE THE LOAD IS PLACED
C     XII     = MOMENT OF INERTIA OF THE ARCH
C     XIB     = MOMENT OF INERTIA OF THE FLOOR BEAMS
C     PHI     = THE INCLINATION OF THE VERTICAL PLANE TO THE PLANE OF
C                THE ARCH
C     XI(KK) = HORIZONTAL DISTANCE FROM THE BEGINNING OF THE ARCH
C                SPAN TO THE CONNECTION POINTS OF THE CABLES TO THE
C                ARCH
C     CA(KK) = CROSS-SECTIONAL AREA OF EACH CABLE
C     D(KK)   = HORIZONTAL DISTANCE FROM THE BEGINNING OF THE BRIDGE
C                SPAN TO THE CONNECTION POINTS OF THE CABLES TO THE
C                BRIDGE SPAN
C     XP(I)   = LOCATION OF THE LOAD
C     ISET(N)= 0 , REMOVES THE N-TH CABLE FROM THE SYSTEM
C             = 1 , THE N-TH CABLE IS IN THE SYSTEM
C     IPRINT = 1 , FOR PRINT OUT OF ALL INTERMEDIATE RESULTS
C             = 0 , FOR PRINT OUT OF ONLY THE FINAL RESULTS
C
C                 * * * * * * * * * * * * * * * * *
C
C              OUTPUT
C       (VARIABLES ARE DEFINED BY THE EQUATION INDICATED)
C
C                                  ****
C     CL,H,SA,SAO                     *
C     SBO,D1,D2,D3                    *     INPUT VALUES
C     II,KMAX,IMAX                    *** (SEE INPUT DATA FOR DEFINITION)
C     XII,XIB,PHI                     *
C     TYPE OF MOMENT OF THE ARCH      *
C                                  ****
C
C     XNU(K) -- EQUATION 5
C     PSI(K) -- EQUATION 6
C     CTH(K) = THE ANGLES AT THE CONNECTION OF THE ARCH AND CABLES,
C                MEASURED C.W FROM A HORISONTAL LINE PASSING THROUGH
C                THE CONNECTING POINT AND THE CABLES
C     SF1(K),....,SF4(K)   (CONSTANT I) -- EQUATION 8
C     SG0(K),....,SG5(K)   (CONSTANT I) -- EQUATION 7
C     SG0(K),....,SG5(K)   (VARYING I) -- EQUATION 10
C     G0(K),....,G5(K) -- EQUATION 9
C     SH1(K),....,SH3(K) -- EQUATION 13
C     AL(K),BE(K),GA(K) -- EQUATION 14
C     Z(J,K) -- EQUATIONS 17, AND 18
C     O(J,K) -- EQUATION 19
C     SL(K) = LENGTH OF THE CABLES
C     XMU(J) -- EQUATION 23
C     C(J,K) -- COEFFICIENTS OF R(J) FOM THE LT. SIDE OF EQUATION 23
C     XP(I) -- SEE INPUT DATA
C     CK(I) -- RIGHT SIDE OF EQUATION 23
C     R(J) -- TENSION IN THE CABLES, FROM EQUATION 23
C     SM -- SEE CMODL BELOW
C     SV -- SEE VO BELOW
C     SH -- SEE VO BELOW
C     CMODL -- EQUATION 15
C     VO -- EQUATION 15
C     HO -- EQUATION 15
C     T1 -- EQUATION 20
C     T2 -- EQUATION 20
C     T3 -- EQUATION 20
C     CMO -- EQUATION 24
C     TS -- EQUATION 24
C     CMC -- EQUATION 25
C     TC -- EQUATION 26
C     RBAR -- EQUATION 27
C     CM1 -- EQUATION 28
C     CM2 -- EQUATION 28
C
C                 * * * * * * * * * * * * * * * * *
C
C             THE FOLLOWING SUBPROGRAMS ARE CALLED
C     FUNCTION SF
C     FUNCTION ANGLE
C     SUBROUTINE MATINV
C     SUBROUTINE SOLVE
C     SUBROUTINE MATMPY
C     SUBROUTINE SET
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DIMENSION XI(20),XNU(20),PSI(20),SF0(20),SF1(20),SF2(20),SF3(20),
     1 SF4(20),SG0(20),SG1(20),SG2(20),SG3(20),SG4(20),SG5(20),G0(20),
     2 G1(20),G2(20),G3(20),G4(20),G5(20),SH1(20),SH2(20),SH3(20),AL(20)
     3 ,BE(20),GA(20),CA(20),SL(20),XMU(20),D(20),XP(25),R(20),ISET(15),
     4 Z(20,20),O(20,20),C(20,20),CK(20),CTH(20),F(3,3),IROW(4),ICOL(3)
      READ 500,CL,H,SA,SAO,SBO,D1,D2,D3
      READ 501,II,KMAX,IMAX,XII,XIB,PHI
      KM = KMAX + 1
      KB=KMAX+1
      READ 502,(XI(KK),KK=1,KM)
      READ 502,(CA(KK),KK=2,KMAX)
```

TABLE E. II. COMPUTER PROGRAM: LEANING ARCHES BRIDGE (Cont'd)

```
      READ 502,(D(KK),KK=2,KMAX)
      READ 502,(XP(I),I=1,IMAX)
      READ 503,(ISET(N),N=1,15),IPRINT
  500 FORMAT(8F10.5)
  501 FORMAT(3I5,2F10,1,F10.5)
  502 FORMAT(8F10.5)
  503 FORMAT(15I2,6X,I2)
      PRINT 1000
 1000 FORMAT(1H1)
      PRINT 1001,CL,H,SA,SA0,SB0,D1,D2,D3,II,KMAX,IMAX,XII,XIB,PHI
 1001 FORMAT(5X,3HCL=,F12.5,5X,3HH =,F12.5,5X,3HSA=,F12.5,5X,4HSA0=,F12.
     15/4X,4HSB0=,F12,5,5X,3HD1=,F12.5,5X,3HD2=,F12.5,5X,4HD3 =,F12.5/
     2 5X,3HII=I10,5X,5HKMAX=,I10,5X,5HIMAX=,I10/
     3 4X,4HXII=,F12.5,4X,4HXIB=,F12,5,4X,4HPHI=,F12.5)
      PI=3.1415926536
      DO 1 KK=1,KM,1
C     K=KK-1
      C1=(4.*H)/CL
      C2=XI(KK)/CL
      C3=C1*(1.-2.*C2)
      IF(XI(KK) .EQ. 0.0) GO TO 2
      XNU(KK)=C1*XI(KK)-(C1/CL)*XI(KK)*XI(KK)
      GO TO 3
    2 XNU(KK)=0.0
      GO TO 3
    3 IF(ABS(C3) .LE. 1E-8) GO TO 4
      PSI(KK)=ATAN(C3)
      GO TO 1
    4 PSI(KK)=0.0
      GO TO 1
    1 CONTINUE
      DO 6 KK=1,6,1
C     K=KK-1
      C1=XNU(KK)-SB0
      C2=D1-SA0-XI(KK)
      CTH(KK) = ANGLE(C1,C2,PI)
    6 CONTINUE
      DO 12 KK=7,11,1
C     K=KK-1
      C1=XNU(KK)-SB0
      C2=D2-SA0-XI(KK)
      CTH(KK) = ANGLE(C1,C2,PI)
   12 CONTINUE
      DO 18 KK=12,KM,1
C     K=KK-1
      C1=XNU(KK)-SB0
      C2=D3-SA0-XI(KK)
      CTH(KK) = ANGLE(C1,C2,PI)
   18 CONTINUE
      IF(II .EQ. 1)GO TO 26
C     II=0,VARYING I
      PRINT 1115
 1115 FORMAT(1H0,17X,9HVARYING I//)
      PRINT 1002
      PRINT 1101
 1101 FORMAT(40X,1HK,5X,6HXNU(K),8X,6HPSI(K),8X,6HCTH(K)//)
      DO 2101 KK=1,KM,1
      K=KK-1
      PRINT 1111,K,XNU(KK),PSI(KK),CTH(KK)
 1111 FORMAT(39X,I2,3(2X,E12.5))
 2101 CONTINUE
      DO 27 KK=1,KM,1
C     K=KK-1
      C1=XI(KK)/CL
      C2=(4.*H)/CL
      SG0(KK)=C2*C1*C1*(3.-2.*C1)/6.
      SG1(KK)=C2*C1*C1*C1*(4.-3.*C1)/12.
      SG2(KK)=0.25*C2*C2*C1*C1*C1*(10.-15.*C1+6.*C1*C1)/30.
      SG3(KK)=0.5*C1*C1
      SG4(KK)=(C1*C1*C1)/3.
      SG5(KK)=C1
   27 CONTINUE
      GO TO 28
C     II=1, CONSTANT I
   26 PRINT 1116
 1116 FORMAT(1H0,17X,10HCONSTANT I//)
      DO 25 KK=1,KM,1
C     K=KK-1
      C1=PSI(KK)
      C2=TAN(C1)
      C3=1./COS(C1)
      C4=SIN(C1)
      SF0(KK)=0.25*(C2*C3+ALOG(C2+C3))
      SF1(KK)=(C3**3)/6.
      SF2(KK)=0.125*(C4*C3*C3*(C3*C3-0.5)-0.5*ALOG(C2+C3))
      SF3(KK)=((3.*C3*C3-5.)*C3**3)/30.
      SF4(KK)=0.25*(C4*C3*C3*((C3**4)/3.-(C3**2)*7./12.+0.125)+0.125*
     1 ALOG(C2+C3))
   25 CONTINUE
      PRINT 1002
 1002 FORMAT(1H0)
      PRINT 1100
 1100 FORMAT(3X,1HK,5X,6HXNU(K),8X,6HPSI(K),8X,6HCTH(K),8X,6HSF0(K),8X,
     1 6HSF1(K),8X,6HSF2(K),8X,6HSF3(K),8X,6HSF4(K)//)
      DO 2100 KK=1,KM,1
      K = KK - 1
      PRINT 1110,K,XNU(KK),PSI(KK),CTH(KK),SF0(KK),SF1(KK),SF2(KK),
     1 SF3(KK),SF4(KK)
 1110 FORMAT(I4,8(2X,E12.5))
 2100 CONTINUE
      DO 29 KK=1,KM,1
      C1=CL/(4.*H)
      C2=C1**2
      C3=C1**3
      SG0(KK)=0.25*(C2*SF2(KK)-SF0(KK))
      SG1(KK)=-0.125*(C3*SF3(KK)-C2*SF2(KK)-C1*SF1(KK)+SF0(KK))
      SG2(KK)=-0.0625*(C3*SF4(KK)-2.*C1*SF2(KK)+SF0(K)/C1)
      SG3(KK)=0.5*(C2*SF1(KK)-C1*SF0(KK))
      SG4(KK)=-0.25*(C3*SF2(KK)-2.*C2*SF1(KK)+C1*SF0(KK))
      SG5(KK)=-C1*SF0(KK)
   29 CONTINUE
      GO TO 28
```

TABLE E. II. COMPUTER PROGRAM: LEANING ARCHES BRIDGE (Cont'd)

```
   28 IF(IPRINT .EQ. 0) GO TO 3500
      PRINT 1002
      PRINT 1120
 1120 FORMAT(17X,1HK,5X,6HSG0(K),8X,6HSG1(K),8X,6HSG2(K),8X,6HSG3(K),8X,
     1 6HSG4(K),8X,6HSG5(K)//)
      DO 2120 KK=1,KM,1
      K = KK -1
      PRINT 1130,K,SG0(KK),SG1(KK),SG2(KK),SG3(KK),SG4(KK),SG5(KK)
 1130 FORMAT(13X,I7,6(2X,E12.5))
 2120 CONTINUE
 3500 DO 30 KK=1,KM,1
C     K=KK-1
      G0(KK)=SG0(KM)-SG0(KK)
      G1(KK)=SG1(KM)-SG1(KK)
      G2(KK)=SG2(KM)-SG2(KK)
      G3(KK)=SG3(KM)-SG3(KK)
      G4(KK)=SG4(KM)-SG4(KK)
      G5(KK)=SG5(KM)-SG5(KK)
   30 CONTINUE
      IF(IPRINT .EQ. 0) GO TO 3501
      PRINT 1000
      PRINT 1140
 1140 FORMAT(17X,1HK,6X,5HG0(K),9X,5HG1(K),9X,5HG2(K),9X,5HG3(K),9X,
     1 5HG4(K),9X,5HG5(K)//)
      DO 2140 KK = 1,KM,1
      K = KK - 1
      PRINT 1130,K,G0(KK),G1(KK),G2(KK),G3(KK),G4(KK),G5(KK)
 2140 CONTINUE
 3501 DO 31 KK=2,KMAX,1
C     K=KK-1
      C1=CTH(KK)
      C2=SIN(C1)
      C3=COS(C1)
      C4=XI(KK)/CL
      C5=XNU(KK)/CL
      SH1(KK)=C2*(G3(KK)-C4*G5(KK))+C3*(G0(KK)-C5*G5(KK))
      SH2(KK)=C2*(G1(KK)-C4*G0(KK))+C3*(G2(KK)-C5*G0(KK))
      SH3(KK)=C2*(G4(KK)-C4*G3(KK))+C3*(G1(KK)-C5*G3(KK))
   31 CONTINUE
      N=3
      F(1,1)=-G0(1)
      F(1,2)=-G1(1)
      F(1,3)= G2(1)
      F(2,1)= G3(1)
      F(2,2)= G4(1)
      F(2,3)=-G1(1)
      F(3,1)= G5(1)
      F(3,2)= G3(1)
      F(3,3)=-G0(1)
      CALL MATINV(F,IROW,ICOL,N,3,1.0E-4)
      DO 33 KK=2,KMAX,1
      AL(KK)= -SH2(KK)*F(1,1)+SH3(KK)*F(1,2)+SH1(KK)*F(1,3)
      BE(KK)= -SH2(KK)*F(2,1)+SH3(KK)*F(2,2)+SH1(KK)*F(2,3)
      GA(KK)= -SH2(KK)*F(3,1)+SH3(KK)*F(3,2)+SH1(KK)*F(3,3)
   33 CONTINUE
      IF(IPRINT .EQ. 0) GO TO 3502
      PRINT 1002
      PRINT 1150
 1150 FORMAT(17X,1HK,9X,6HSH1(K),8X,6HSH2(K),8X,6HSH3(K),9X,5HAL(K),
     1 9X,5HBE(K),9X,5HGA(K)//)
      DO 2150 KK=2,KMAX,1
      K = KK - 1
      PRINT 1130,K,SH1(KK),SH2(KK),SH3(KK),AL(KK),BE(KK),GA(KK)
 2150 CONTINUE
 3502 DO 34 K=1,KB,1
      KK = K + 1
      DO 34 J=1,KB,1
      JJ = J + 1
      IF (KK-JJ) 35,35,36
   35 C1=CTH(KK)
      C2=SIN(C1)
      C3=COS(C1)
      C4=XI(KK)/CL
      C5=XNU(KK)/CL
      Z(JJ,KK)=C2*SH3(JJ)-(C4*C2+C5*C3)*SH1(JJ)+C3*SH2(JJ)
      GO TO 37
   36 C1=CTH(JJ)
      C2=SIN(C1)
      C3=COS(C1)
      C4=XI(JJ)/CL
      C5=XNU(JJ)/CL
      Z(JJ,KK)=C2*SH3(KK)-(C4*C2+C5*C3)*SH1(KK)+C3*SH2(KK)
      GO TO 37
   37 Q(JJ,KK)=Z(JJ,KK)-SH1(JJ)*AL(KK)-SH3(JJ)*BE(KK)+SH2(JJ)*GA(KK)
   34 CONTINUE
      IF(IPRINT .EQ. 0) GO TO 3503
      PRINT 1000
      PRINT 1160
 1160 FORMAT(6H  J  K,5X,6HZ(J,K),7X,8HZ(J,K+1),6X,8HZ(J,K+2),6X,8HZ(J,K
     1+3),6X,8HZ(J,K+4),6X,8HZ(J,K+5),6X,8HZ(J,K+6),6X,8HZ(J,K+7)/10X,
     2 8HZ(J,K+8),6X,8HZ(J,K+9),6X,9HZ(J,K+10),5X,9HZ(J,K+11),5X,9HZ(J,K
     3+12),5X,9HZ(J,K+13),5X,9HZ(J,K+14)//)
      DO 3000 J=1,KB,1
      K=1
      JJ = J + 1
      PRINT 1170,J,K,(Z(JJ,KK),KK=2,KMAX)
 3000 CONTINUE
 1170 FORMAT(2I3,8(2X,E12.5)/(6X,7(2X,E12.5))/)
      PRINT 1000
      PRINT 1180
 1180 FORMAT(6H  J  K,5X,6HQ(J,K),7X,8HQ(J,K+1),6X,8HQ(J,K+2),5X,8HQ(J,K
     1+3),6X,8HQ(J,K+4),6X,8HQ(J,K+5),6X,8HQ(J,K+6),6X,8HQ(J,K+7)/10X,
     2 8HQ(J,K+8),6X,8HQ(J,K+9),6X,9HQ(J,K+10),5X,9HQ(J,K+11),5X,9HQ(J,K
     3+12),5X,9HQ(J,K+13),5X,9HQ(J,K+14)//)
      DO 3010 J=1,KB,1
      K=1
      JJ = J + 1
      PRINT 1170,J,K,(Q(JJ,KK),KK=2,KMAX)
 3010 CONTINUE
 3503 C1=(6.*XIB)/XII
      C2=(CL/SA)**3
      XLA=C1*C2
```

TABLE E.II. COMPUTER PROGRAM: LEANING ARCHES BRIDGE (Cont'd)

```
      DO 38 KK=2,6,1
      C1=(XNU(KK)-SHO)**2
      C2=(D1-SAO-XI(KK))**2
      SL(KK)=SQRT(C1+C2)
   38 CONTINUE
      DO 39 KK=7,11,1
      C1=(XNU(KK)-SHO)**2
      C2=(D2-SAO-XI(KK))**2
      SL(KK)=SQRT(C1+C2)
   39 CONTINUE
      DO 40 KK=12,16,1
      C1=(XNU(KK)-SHO)**2
      C2=(D3-SAO-XI(KK))**2
      SL(KK)=SQRT(C1+C2)
   40 CONTINUE
      DO 41 KK=2,16,1
      C1=6.*SL(KK)*XIB
      C2=CA(KK)*SA*SA*SA
      XMU(KK)=C1/C2
   41 CONTINUE
      IF(IPRINT .EQ. 0) GO TO 3504
      PRINT 1002
      PRINT 1190
 1190 FORMAT(4X,1HK,6X,5HSL(K),8X,6HXMU(K)//)
      DO 2000 KK= 2,16,1
      K = KK -1
      PRINT 2010,K,SL(KK),XMU(KK)
 2010 FORMAT(I5,2(2X,E12.5))
 2000 CONTINUE
 3504 DO 43 I=1,IMAX,1
      DO 42 JJ=2,KMAX,1
      CK(JJ) = SF(D(JJ),XP(I),SA) * SIN(CTH(JJ))
      DO 42 KK=2,6,1
      C1=COS(PHI)
      C(JJ,KK)=XLA*G(JJ,KK)+C1*C1*SIN(CTH(KK))*SF(D(JJ),D1,SA)
     1  * SIN(CTH(JJ))
      C(JJ,KK+5)=XLA*G(JJ,KK+5)+C1*C1*SIN(CTH(KK+5))*SF(D(JJ),D2,SA)
     1  * SIN(CTH(JJ))
      C(JJ,KK+10)=XLA*G(JJ,KK+10)+C1*C1*SIN(CTH(KK+10))*SF(D(JJ),D3,SA)
     1  * SIN(CTH(JJ))
   42 CONTINUE
      DO 44 KK=2,KMAX,1
      C(KK,KK)=C(KK,KK)+XMU(KK)
   44 CONTINUE
      DO 45 J=1,KB,1
      JJ=J+1
      CR(J)=CK(JJ)
      DO 45 K=1,KB,1
      KK=K+1
      C(J,K)=C(JJ,KK)
   45 CONTINUE
      CALL SET(C,CR,ISET)
      CALL SOLVE(C,CR,R)
      PRINT 1000
      IF(IPRINT .EQ. 0) GO TO 3505
      PRINT 2020
 2020 FORMAT(6H  J  K,5X,6HC(J,K),7X,8HC(J,K+1),6X,8HC(J,K+2),6X,8HC(J,K
     1+3),6X,8HC(J,K+4),6X,8HC(J,K+5),6X,8HC(J,K+6),6X,8HC(J,K+7)/10X,
     2 8HC(J,K+8),6X,8HC(J,K+9),6X,9HC(J,K+10),5X,9HC(J,K+11),5X,9HC(J,K
     3+12),5X,9HC(J,K+13),5X,9HC(J,K+14)/)
      DO 49 J=1,KB,1
      KK=1
      PRINT 2030,J,KK,(C(J,K),K=1,KB)
 2030 FORMAT(2I3,8(2X,E12.5)/(6X,7(2X,E12.5))/)
   49 CONTINUE
 3505 PRINT 1002
      PRINT 2031,XP(I)
 2031 FORMAT(4X,6HXP(I)=,F9.4/)
      PRINT 2040
 2040 FORMAT(4X,1HJ,6X,5HCR(J),9X,4HR(J))
      DO 2240 J= 1,KB,1
      PRINT 2010,J,CR(J),R(J)
 2240 CONTINUE
      SH=SV=SM=0.0
      DO 46 K=1,KB,1
      KK=K+1
      SH=SH+R(K)*AL(KK)
      SV=SV+R(K)*BE(KK)
      SM=SM+R(K)*GA(KK)
   46 CONTINUE
      CMODL=SM
      VO=SV
      HO=SH
      T1=T2=T3=0.0
      DO 47 K=1,5,1
      KK=K+1
      C1=CTH(KK)
      C2=CTH(KK+5)
      C3=CTH(KK+10)
      T1 =T1 + R(K) * SIN(C1)
      T2 = T2 + R(K+5) * SIN(C2)
      T3 = T3 + R(K+10) * SIN(C3)
   47 CONTINUE
C     AT SPRINGING
      CMO=CL*CMODL
      C1=PSI(1)
      C2=COS(C1)
      C3=SIN(C1)
      TS=HO*C2+VO*C3
C     AT CROWN
      SM=SH=0.0
      DO 48 K=1,7,1
      KK=K+1
      C1=CTH(KK)
      C2=SIN(C1)
      C3=COS(C1)
      SM=SM+R(K)*(C2*(0.5*CL-XI(KK))+C3*(H-XNU(KK)))
      SH=SH+R(K)*C3
   48 CONTINUE
      CMC=CMO+0.5*VO*CL-HO*H-SM
      TC=HO+SH
      RBAR=(SA-XP(I))/SA-(T1*(SA-D1)/SA+T2*(SA-D2)/SA+T3*(SA-D3)/SA)
      CM1=RBAR*D1
      IF(D1-XP(I))50,50,51
   51 CM1=CM1-(D1-XP(I))
   50 CM2=RBAR*D2-T1*(D2-D1)
      IF(D2-XP(I))52,52,53
   53 CM2=CM2-(D2-XP(I))
   52 PRINT 1002
      PRINT 2050,SH,SV,SM,CMODL,VO,HO,T1,T2,T3,CMO,TS, CMC,TC,
     1 RBAR,CM1,CM2
 2050 FORMAT(8X,7HSH     =,E12.5,8X,7HSV     =,E12.5,8X,7HSM     =,E12.5/
     1       8X,7HCMODL =,E12.5,8X,7HVO     =,E12.5,8X,7HHO     =,E12.5/
     2       8X,7HT1     =,E12.5,8X,7HT2     =,E12.5,8X,7HT3     =,E12.5/
     3       8X,7HCMO    =,E12.5,8X,7HTS     =,E12.5,8X,7HCMC    =,E12.5/
     4       8X,7HTC     =,E12.5,8X,7HRBAR   =,E12.5,8X,7HCM1    =,E12.5/
     5       8X,7HCM2    =,E12.5)
   43 CONTINUE
      END
```

TABLE E. III. COMPUTER PROGRAM SUBROUTINES: LEANING ARCHES BRIDGE

```
      FUNCTION SF(X,XP,SA)
      IF(X-XP)1,1,2
    1 XNU=X
      GO TO 3
    2 XNU=XP
      GO TO 3
    3 C1=(X*XP)/(SA*SA)
      C2=X/SA
      C3=XP/SA
      C4=XNU/SA
      SF=C1*(2.*(1.-C2)*(1.-C3)*(C2-C3)**2)-.4*(C2-C3)**2
      RETURN
      END

      FUNCTION ANGLE(Y,X,PI)
      C = Y/X
      IF(Y)1,2,3
    1 IF(X)4,5,6
    3 IF(X)4,7,8
    2 ANGLE = 0.0
      RETURN
    4 ANGLE = PI + ATAN(C)
      RETURN
    5 ANGLE = 1.5 * PI
      RETURN
    6 ANGLE = 2. * PI + ATAN(C)
      RETURN
    7 ANGLE = 0.5 * PI
      RETURN
    8 ANGLE = ATAN(C)
      RETURN
      END

      SUBROUTINE SOLVE(C,CK,R)
      DIMENSION C(20,20),CK(20),R(20),CCK(20),CR(20),A(20,20),D(15,15),
     1IROW(16),ICOL(15)
      N=15
      DO 1 M=1,N
      DO 1 K=1,N
      D(M,K)=C(M,K)
    1 CONTINUE
      CALL MATINV(D,IROW,ICOL, N,15,1.0E-05)
      DO 2 M=1,N
      DO 2  K=1,N
      A(M,K)=D(M,K)
    2 CONTINUE
      DO 3 M=1,N
      CCK(M)=CK(M)
    3 CONTINUE
      CALL MATMPY(A,CCK,CR,N)
      DO 4 M=1,N
      R(M)=CR(M)
    4 CONTINUE
      RETURN
      END

      SUBROUTINE MATMPY(A,B,C,M)
      DIMENSION A(20,20),B(20),C(20)
      DO 1 I=1,M,1
      C(I) = 0.0
      DO 1 J=1,M,1
      C(I) = C(I) + A(I,J) * B(J)
    1 CONTINUE
      RETURN
      END

      SUBROUTINE SET(C,CK,ISET)
C     IF ISET(N)=0, REMOVE THE CABLE R(N),
C     IF ISET(N)=1, CABLE R(N) IS IN THE SYSTEM
      DIMENSION C(20,20),CK(20),ISET(15)
      DO 1 M=1,15
      DO 1 N=1,15
      IF(ISET(N) .EQ. 1)GO TO 1
      C(M,N)=0.0
      C(N,M)=0.0
    1 CONTINUE
      DO 2 N=1,15
      IF(ISET(N) .EQ. 1)GO TO 2
      CK(N)=0.0
      C(N,N)=1.0
    2 CONTINUE
      RETURN
      END
```

TABLE E.III. COMPUTER PROGRAM SUBROUTINES: LEANING ARCHES BRIDGE (Cont'd)

```
      SUBROUTINE MATINV ( A , IROW , ICOL , N , NDIM , SMLST )
      DIMENSION A ( 1 ) , IROW ( 1 ) , ICOL ( 1 )
C     709-10865
C     709-10865  SUBROUTINE MATINV - MATRIX INVERSION ROUTINE
C
C         A = ARRAY NAME OF MATRIX
C      IROW = DIMENSIONED AT N+1 OR GREATER
C      ICOL = DIMENSIONED AT N OR GREATER
C        N  = NUMBER OF EQUATIONS
C      NDIM = VALUE OF I IN DIMENSION A(I,J) , I AND J MAY DIFFER
C     SMLST = SMALLEST LEADING ELEMENT ALLOWED BEFORE CALLING THE
C             SYSTEM SINGULAR , USUALLY = 1.0 E-04 OR 1.0 E-05
C
      NP1 = N + 1
      DO  100 I = 1, N
      ICOL ( I ) = I
  100 IROW ( I ) = I
      DO  245 ITER = 1, N
      MAXR = ITER
      MAXC = 1
      TEMP = ABSF ( A ( MAXR ) )
      LIMITC = NP1 - ITER
      DO  120 I = ITER, N
      DO  120 J = 1, LIMITC
      IJ = ( J - 1 ) * NDIM + I
      IF ( TEMP - ( ABSF ( A ( IJ ) ) ) )  110, 120, 120
C
  110 MAXR = I
      MAXC = J
      TEMP = ABSF ( A ( IJ ) )
  120 CONTINUE
      IF ( TEMP - SMLST )  130, 140, 140
C
  130 IROW ( NP1 ) = ITER
      PRINT 1, ITER , SMLST
      RETURN
C
  140 IF ( MAXR - ITER )  150, 170, 150
C
  150 DO  160 J = 1, N
      MAXRJ = ( J - 1 ) * NDIM + MAXR
      ITJ = ( J - 1 ) * NDIM + ITER
      TEMP = A ( MAXRJ )
      A ( MAXRJ ) = A ( ITJ )
  160 A ( ITJ ) = TEMP
      ITEMP = IROW ( MAXR )
      IROW ( MAXR ) = IROW ( ITER )
      IROW ( ITER ) = ITEMP
  170 IF ( MAXC - 1 )  180, 200, 180
C
  180 DO  190 I = 1, N
      IMAXC = ( MAXC - 1 ) * NDIM + I
      TEMP = A ( I )
      A ( I ) = A ( IMAXC )
  190 A ( IMAXC ) = TEMP
      ITEMP = ICOL ( MAXC )

      ICOL ( MAXC ) = ICOL ( 1 )
      ICOL ( 1 ) = ITEMP
  200 TEMP = A ( ITER )
      ITEMP = ICOL ( 1 )
      DO  210 J = 2, N
      ITJM1 = ( J - 2 ) * NDIM + ITER
      ITJ = ( J - 1 ) * NDIM + ITER
      A ( ITJM1 ) = A ( ITJ ) / TEMP
  210 ICOL ( J - 1 ) = ICOL ( J )
      ITN = ( N - 1 ) * NDIM + ITER
      A ( ITN ) = 1 . 0 / TEMP
      ICOL ( N ) = ITEMP
      DO  240 I = 1, N
      IF ( I - ITER )  220, 240, 220
C
  220 TEMP = A ( I )
      DO  230 J = 2, N
      IJM1 = ( J - 2 ) * NDIM + I
      IJ = ( J - 1 ) * NDIM + I
      ITJM1 = ( J - 2 ) * NDIM + ITER
      A ( IJM1 ) = A ( IJ ) - A ( ITJM1 ) * TEMP
  230 CONTINUE
      IN = ( N - 1 ) * NDIM + I
      ITN = ( N - 1 ) * NDIM + ITER
      A ( IN ) = - ( TEMP * A ( ITN ) )
  240 CONTINUE
      DO  290 I = 1, N
      DO  250 J = I, N
      IF ( IROW ( J ) - I )  250, 260, 250
C
  250 CONTINUE
  260 IF ( I - J )  270, 290, 270
C
  270 DO  280 L = 1, N
      LI = ( I - 1 ) * NDIM + L
      LJ = ( J - 1 ) * NDIM + L
      TEMP = A ( LI )
      A ( LI ) = A ( LJ )
  280 A ( LJ ) = TEMP
      IROW ( J ) = IROW ( I )
  290 CONTINUE
      DO  340 I = 1, N
      DO  300 J = I, N
      IF ( ICOL ( J ) - I )  300, 310, 300
C
  300 CONTINUE
  310 IF ( I - J )  320, 340, 320
C
  320 DO  330 L = 1, N
      IL = ( L - 1 ) * NDIM + I
      JL = ( L - 1 ) * NDIM + J
      TEMP = A ( IL )
      A ( IL ) = A ( JL )
  330 A ( JL ) = TEMP
      ICOL ( J ) = ICOL ( I )
  340 CONTINUE

      IROW ( NP1 ) = 0
      RETURN
C
    1 FORMAT (7H0ON THEI3,63HTH ITERATION ALL THE REMAINING TERMS WERE L
     *ESS THAN OR EQUAL TO E11.4,10H IN ABSOLUTE VALUE)
      END
```

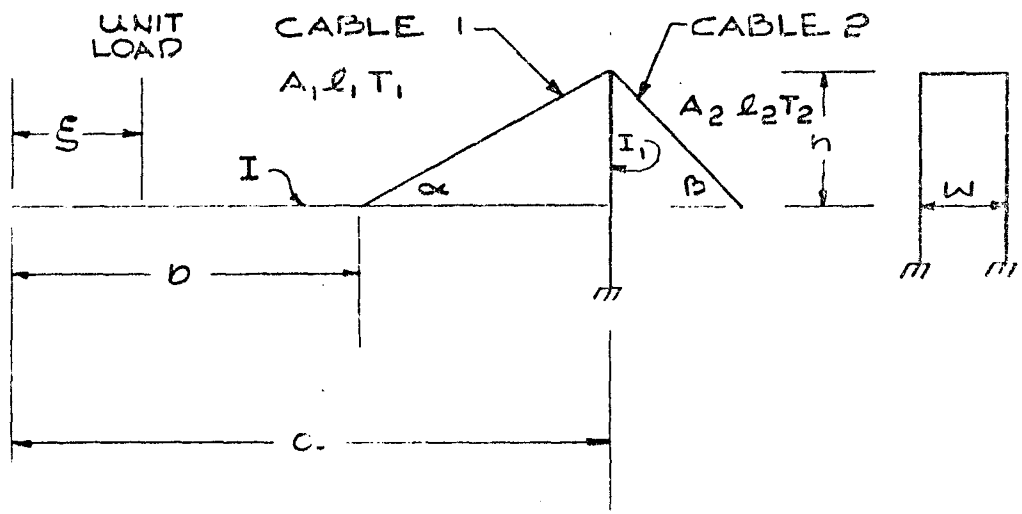

FIGURE E.2. DEFINITION SKETCH: BRIDLE BRIDGE

TABLE E.IV. NOTATION RELATIONSHIPS BETWEEN TEXT AND COMPUTER PROGRAM, BRIDLE BRIDGE

Program "BRIDL 1"	Text (Par. B.4.b)
A1	A_1
A2	A_2
AL	α
BE	β
CI	I
CI1	I_1
H	h
SA	a
SB	b
SL1	ℓ_1
SL2	ℓ_2
T1	T_1
T2	T_2
XI	ξ
XLA	λ
XLA1	λ_1
XMU1	μ_1
XMU2	μ_2

TABLE E.V. COMPUTER PROGRAM: BRIDLE BRIDGE WITH CONTINUOUS GIRDER

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C        THIS PROGRAM IS FOR THE SOLUTION OF THE EQUATIONS GIVEN IN
C     APPENDIX D4,"METHODS OF ANALYSIS OF CABLE SUPPORTED BRIDGE CON-
C     CEPTS, FOR THE BRIDLE BRIDGE WITHOUT HINGE
C
C         INPUT DATA (LENGTHS ARE IN INCHES, ALL ANGLES IN RADIANS)
C
C     SA  = DISTANCE FROM BEGINNING OF BRIDGE SPAN TO THE COLUMN
C     SB  = DISTANCE FROM BEGINNING OF BRIDGE SPAN TO CABLE ONE
C     CI  = MOMENT OF INERTIA OF THE FLOOR BEAMS
C     CI1 = MOMENT OF INERTIA OF THE COLUMN
C     A1  = CROSS-SECTIONAL AREA OF CABLE ONE
C     A2  = CROSS-SECTIONAL AREA OF CABLE TWO
C     SL1 = LENGTH OF CABLE ONE
C     SL2 = LENGTH OF CABLE TWO
C     AL  = ANGLE BETWEEN CABLE ONE AND THE BRIDGE SPAN
C     BE  = ANGLE BETWEEN CABLE TWO AND THE BRIDGE SPAN
C     H   = HEIGHT OF COLUMN ABOVE BRIDGE SPAN
C
C                 * * * * * * * * * * * * * * * * * *
C
C            OUTPUT
C       (VARIABLES ARE DEFINED BY THE EQUATION INDICATED)
C
C         ****
C     SA     *
C     SB     *
C     CI     *
C     CI1    *
C     A1     ***   INPUT VALUES
C     A2     *** (SEE INPUT DATA FOR DEFINITION)
C     SL1    *
C     SL2    *
C     AL     *
C     BE     *
C     H      *
C         ****
C
C            FOR COLUMN DOES NOT BEND
C
C     X1 -- DISTANCE FROM BEGINNING OF BRIDGE SPAN TO THE LOAD
C     T1 -- EQUATION 12
C
C            FOR COLUMN DOES BEND
C
C     X1 -- (SAME AS ABOVE)
C     T1 -- EQUATION 12
C     T2 -- EQUATION 13
C
C                 * * * * * * * * * * * * * * * * * *
C
C            THE FOLLOWING SUBPROGRAM IS CALLED
C
C     FUNCTION SF
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     BRIDLE BRIDGE WITHOUT HINGE
C
      READ 100,SA,SB,CI,CI1,A1
      READ 100,A2,SL1,SL2,AL,BE
      READ 101,H
  100 FORMAT(5F15.7)
  101 FORMAT(F15.7)
      PRINT 200
  200 FORMAT(1H1,33X,46HBRIDLE BRIDGE WITHOUT HINGE,COLUMN DOES NOT BEND
     1//24X,5HINPUT//)
      PRINT 201,SA,SB,CI,CI1,A1,A2,SL1,SL2,AL,BE,H
  201 FORMAT(10X,4HSA =,F15.7//10X,4HSB =,F15.7//10X,4HCI =,F15.7//
     1        10X,4HCI1=,F15.7//10X,4HA1 =,F15.7//10X,4HA2 =,F15.7//
     2        10X,4HSL1=,F15.7//10X,4HSL2=,F15.7//10X,4HAL =,F15.7//
     3        10X,4HBE =,F15.7//10X,4HH  =,F15.7///24X,6HOUTPUT/)
      P=1.0
C     COLUMN DOES NOT BEND
      XLA=(SL1*CI)/(A1*SA*SA)+(SB*SB)/(3.*SA*SA)*(1.0-SB/SA)**2
      CAL=1./SIN(AL)
      X1=0.0
      T1=0.0
      PRINT 400
  400 FORMAT( / ,13X,2HX1,21X,2HT1)
      PRINT 401,X1,T1
  401 FORMAT(1 X,F6.1,13X,F15.7)
    1 X1=X1+6.
      T1=(P*SF(SB,X1,SA)*CAL)/XLA
      PRINT 401,X1,T1
      IF(X1 .LT. SA)GO TO 1
C     COLUMN DOES BEND
      PRINT 202
  202 FORMAT(1H1,30X,46HBRIDLE BRIDGE WITHOUT HINGE,COLUMN DOES BEND
     1//24X,5HINPUT//)
      PRINT 201,SA,SB,CI,CI1,A1,A2,SL1,SL2,AL,BE,H
      PRINT 502
  502 FORMAT(13X,2HX1,21X,2HT1,23X,2HT2//)
      XMU1=(A1*H*H*H)/(3.*CI1*SL1*COS(AL))
      XMU2=(A2*H*H*H)/(3.*CI1*SL2*COS(BE))
      XLA1=(SL1*CI)/(A1*SA*SA)*(1.-(XMU1*COS(AL))/(1.+XMU2*COS(BE)))
     1 + (SB*SB)/(3.*SA*SA)*(1.-SB/SA)**2
      X1=0.0
      T1=0.0
      T2=0.0
      PRINT 501,X1,T1,T2
  501 FORMAT(1 X,F6.1,2(10X,F15.7))
    2 X1=X1+6.
      T1=(P*SF(SB,X1,SA)*CAL)/XLA1
      T2=(T1*[illegible]*COS(AL))/(1.+XMU2*COS(BE))
      PRINT 501,X1,T1,T2
      IF(X1 .LT. SA) GO TO 2
      END
```

```
      FUNCTION SF(A,X1,SA)
      IF(A .GE. X1) GO TO 1
      [illegible]
      GO TO 2
    1 [illegible]
      GO TO 2
    2 [illegible]/(S*SA)
      [illegible]/SA
      [illegible]/SA
      [illegible]/SA
      SF=[illegible]
      RETURN
      END
```

TABLE E.VI. COMPUTER PROGRAM OUTPUT: BRIDLE BRIDGE WITH CONTINUOUS GIRDER

BRIDLE BRIDGE WITHOUT HINGE,COLUMN DOES NOT BEND

INPUT

```
SA =  2100.0000000
SB =  1200.0000000
CI = 55027.0000000
CI'=  3034.0000000
A1 =     4.9100000
A2 =     5.9500000
SL1=  1420.0000000
SL2=   678.7200000
AL =      .4086980
PF =      .7853982
H  =
```

X1	T1
0.0	0.0000000
60.0	.0016032
120.0	.0031968
180.0	.0047712
240.0	.0063166
300.0	.0078236
360.0	.0092824
420.0	.0106834
480.0	.0120170
540.0	.0132736
600.0	.0144435
660.0	.0155172
720.0	.0164849
780.0	.0173371
840.0	.0180641
900.0	.0186567
960.0	.0191040
1020.0	.0193977
1080.0	.0195277
1140.0	.0194843
1200.0	.0192581
1260.0	.0188429
1320.0	.0182481
1380.0	.0174463
1440.0	.0165705
1500.0	.0155134
1560.0	.0143280
1620.0	.0130270
1680.0	.0116233
1740.0	.0101297
1800.0	.0085591
1860.0	.0069243
1920.0	.0052387
1980.0	.0035135
2040.0	.0017632
2100.0	0.0000000

BRIDLE BRIDGE WITHOUT HINGE,COLUMN DOES BEND

INPUT

```
SA =  2100.0000000
SB =  1200.0000000
CI = 55027.0000000
CI'=  3034.0000000
A1 =     4.9100000
A2 =     5.9500000
SL1=  1420.0000000
SL2=   678.7200000
AL =      .4086980
PF =      .7853982
H  =
```

X1	T1	T2
0.0	0.0000000	0.0000000
60.0	.2082022	.2653264
120.0	.4153330	.5293593
180.0	.6198719	.7846353
240.0	.8206579	1.0453721
300.0	1.0164396	1.2947618
360.0	1.2059468	1.5361842
420.0	1.3879877	1.7680483
480.0	1.5612816	1.9887529
540.0	1.7245075	2.1907115
600.0	1.8745043	2.3923268
660.0	2.0159912	2.5681391
720.0	2.1417169	2.7201811
780.0	2.2524307	2.8691024
840.0	2.3468814	2.9895236
900.0	2.4235181	3.0875072
960.0	2.4819497	3.1618372
1020.0	2.5201453	3.2112126
1080.0	2.5370339	3.2317225
1140.0	2.5314143	3.2245525
1200.0	2.5020956	3.1871040
1260.0	2.4480737	3.1164142
1320.0	2.3707895	3.0194581
1380.0	2.2718212	2.8939915
1440.0	2.1578372	2.7423298
1500.0	2.0155847	2.5673394
1560.0	1.8814923	2.3712254
1620.0	1.6924479	2.1554978
1680.0	1.5190095	1.9235044
1740.0	1.3166557	1.6764187
1800.0	1.1123026	1.4164917
1860.0	[illegible]	1.1454410
1920.0	.6875456	.8688623
1980.0	.4564771	.5814694
2040.0	.2240725	.2617071
2100.0	0.0000000	0.0000000

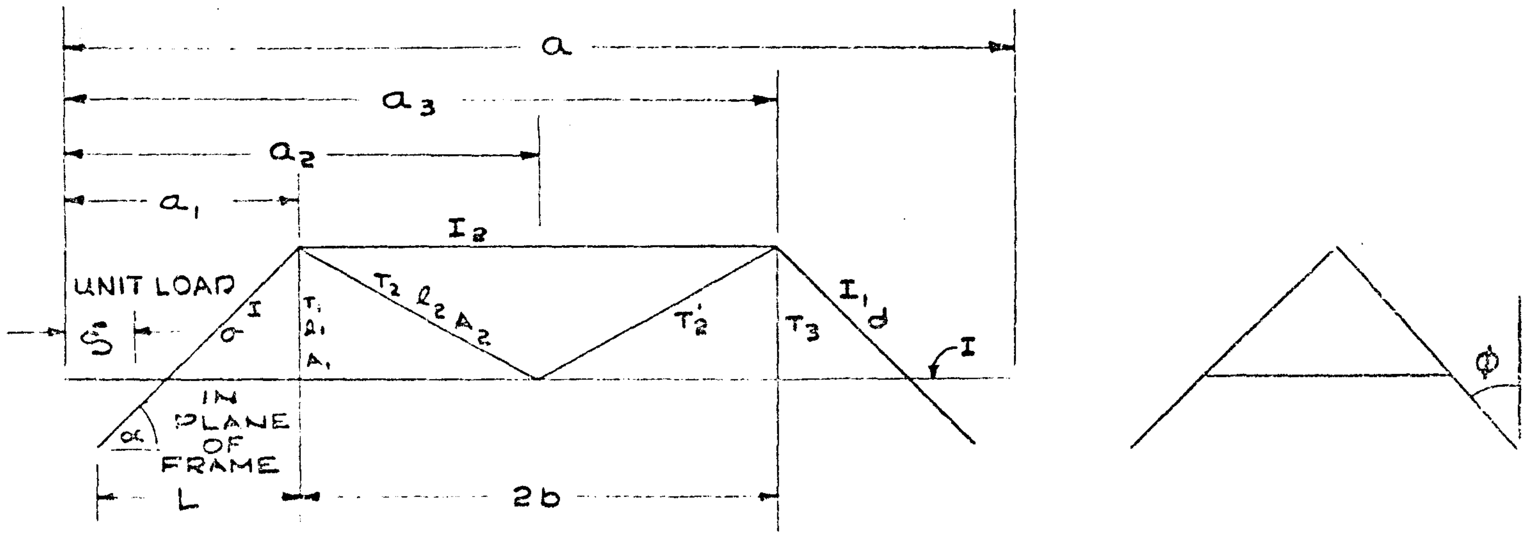

FIGURE E. 3. DEFINITION SKETCH: FRAME BRIDGE

TABLE E.VII. NOTATION RELATIONSHIPS BETWEEN TEXT AND COMPUTER PROGRAM, FRAME BRIDGE

Program "RIGID 1"	Text (Par B.6)
A1, A2	A_1, A_2
AL	α
AL11, . . . , AL33	$\alpha_{11}, \ldots, \alpha_{33}$
BE1, . . . , BE4	$\beta_1, \ldots, \beta_4$
C	c
CI, CI1, CI2	I, I_1, I_2
CL	L
D	d
F1, . . . , F4	$F_1, \ldots, F_4$
H1, H2	H_1, H_2
PHI	ϕ
PSI	ψ
SA, SA1, SA2, SA3	a, a_1, a_2, a_3
SB	b
SF11, . . . , SF33	$f_{11}, \ldots, f_{33}$
SL1, SL2	ℓ_1, ℓ_2
T1, T2, T2P, T3	T_1, T_2, T_2', T_3
V1, V2	V_1, V_2
XI	ξ
XM1, XM2	M_1, M_2

TABLE E. VIII. COMPUTER PROGRAM: FRAME BRIDGE WITH CONTINUOUS GIRDER

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC RIG10010
C                                                                   C RIG10020
C          THIS PROGRAM IS FOR THE SOLUTION OF THE EQUATIONS GIVEN IN C RIG10030
C     APPENDIX D6,METHODS OF ANALYSIS OF CABLE SUPPORTED BRIDGE CON-  C RIG10040
C     CEPTS, FOR THE RIGID FRAME BRIDGE WITHOUT HINGE                 C RIG10050
C                                                                   C RIG10060
C           INPUT DATA (LENGTHS ARE IN INCHES, AND ANGLES IN RADIANS) C RIG10070
C                                                                   C RIG10080
C     SA   = BRIDGE SPAN                                              C RIG10090
C     SA1  = DISTANCE FROM BEGINNING OF BRIDGE SPAN TO CABLE ONE      C RIG10100
C     SA2  = DISTANCE FROM BEGINNING OF BRIDGE SPAN TO MID-SPAN       C RIG10110
C     SA3  = DISTANCE FROM BEGINNING OF BRIDGE SPAN TO CABLE THREE    C RIG10120
C     SB  = ONE HALF THE DISTANCE FROM CABLE ONE TO CABLE THREE       C RIG10130
C     CL   = DISTANCE FROM BEGINNING OF FRAME LEG ONE TO CABLE ONE    C RIG10140
C     D    = LENGTH OF FRAME LEGS                                     C RIG10150
C     SL1  = LENGTH OF CABLES ONE AND THREE                           C RIG10160
C     SL2  = LENGTH OF CABLES TWO AND TWO PRIME                       C RIG10170
C     A1   = CROSS-SECTIONAL AREA OF CABLES ONE AND THREE             C RIG10180
C     A2   = CROSS-SECTIONAL AREA OF CABLES TWO AND TWO PRIME         C RIG10190
C     CI1  = MOMENT OF INERTIA OF FRAME LEGS                          C RIG10200
C     CI2  = MOMENT OF INERTIA OF THE TOP FRAME MEMBER                C RIG10210
C     CI   = MOMENT OF INERTIA OF THE SPAN                            C RIG10220
C     AL   = ANGLE BETWEEN THE HORIZONTAL AND THE FRAME LEG           C RIG10230
C            (IN THE PLANE OF THE FRAME)                              C RIG10240
C     PSI  = ONE HALF THE ANGLE BETWEEN CABLES TWO AND TWO PRIME      C RIG10250
C     PHI  = THE INCLINATION OF THE VERTICAL PLANE TO THE PLANE OF    C RIG10260
C              THE FRAME                                              C RIG10270
C                                                                   C RIG10280
C                 * * * * * * * * * * * * * * * * *                   C RIG10290
C                                                                   C RIG10300
C             OUTPUT                                                  C RIG10310
C       (VARIABLES ARE DEFINED BY THE EQUATION INDICATED)             C RIG10320
C                                                                   C RIG10330
C                   ****                                              C RIG10340
C     SA,SA1,SA2,SA3   *                                              C RIG10350
C     SB,CL,D,SL1      *          INPUT VALUES                        C RIG10360
C     SL2,A1,A2,CI1    ***  (SEE INPUT DATA FOR DEFINITION)           C RIG10370
C     CI2,CI,AL,PSI    *                                              C RIG10380
C     PHI              *                                              C RIG10390
C                   ****                                              C RIG10400
C                                                                   C RIG10410
C     BE1,...,BE4 -- EQUATIONS 6,13, AND 15                           C RIG10420
C     SF11,...,SF33 -- EQUATION 20                                    C RIG10430
C     AL11,...,AL33 -- EQUATION 20                                    C RIG10440
C     X1 = DISTANCE FROM THE BEGINNING OF THE BRIDGE SPAN AND LOAD    C RIG10450
C     T1,T2,T3,T2P -- TENSION IN THE CABLES, FROM EQUATIONS 17,18,19, C RIG10460
C                         AND 12                                      C RIG10470
C     R1 = REACTION AT THE LEFT OF BRIDGE SPAN                        C RIG10480
C     R2 = REACTION AT THE RIGHT OF BRIDGE SPAN                       C RIG10490
C     MT1 = MOMENT AT SA1                                             C RIG10500
C     MT2 = MOMENT AT SA2                                             C RIG10510
C     F1,...,F4 -- EQUATION 7                                         C RIG10520
C     V1,V2 -- EQUATION 21                                            C RIG10530
C     H1,H2 -- EQUATION 21                                            C RIG10540
C     XM1,XM2 -- EQUATION 21                                          C RIG10550
C                                                                   C RIG10560
C                 * * * * * * * * * * * * * * * * *                   C RIG10570
C                                                                   C RIG10580
C               THE FOLLOWING SUBPROGRAMS ARE CALLED                  C RIG10590
C     FUNCTION SF                                                     C RIG10600
C     FUNCTION DET                                                    C RIG10610
C     SUBROUTINE SOLVE                                                C RIG10620
C                                                                   C RIG10630
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC RIG10640
      COMMON/A/AL11,AL12,AL13,AL21,AL22,AL23,AL31,AL32,AL33,CK1,CK2,CK3 RIG10650
     1,BE3                                                              RIG10660
      REAL MT1,MT2                                                      RIG10670
      READ 500,SA,SA1,SA2,SA3                                           RIG10680
      READ 500,SB,CL,D,SL1                                              RIG10690
      READ 500,SL2,A1,A2,CI1                                            RIG10700
      READ 500,CI2,CI,AL,PSI                                            RIG10710
      READ 501,PHI                                                      RIG10720
  500 FORMAT(4F15.7)                                                    RIG10730
  501 FORMAT(F15.7)                                                     RIG10740
      PRINT 1000                                                        RIG10750
 1000 FORMAT(1H1)                                                       RIG10760
      PRINT 1001,SA,SA1,SA2,SA3,SB,CL,D,SL1,SL2,A1,A2,CI1,CI2,CI,AL,    RIG10770
     1PSI,PHI                                                           RIG10780
 1001 FORMAT(4X,4HSA =,F14.7,5X,4HSA1=,F14.7,5X,4HSA2=,F14.7,5X,4HSA3=, RIG10790
     1 F14.7/4X,4HSB =,F14.7,5X,4HCL =,F14.7,5X,4HD  =,F14.7,5X,4HSL1=, RIG10800
     2 F14.7/4X,4HSL2=,F14.7,5X,4HA1 =,F14.7,5X,4HA2 =,F14.7,5X,4HCI1=, RIG10810
     3 F14.7/4X,4HCI2=,F14.7,5X,4HCI =,F14.7,5X,4HAL =,F14.7,5X,4HPSI=, RIG10820
     4 F14.7/4X,4HPHI=,F14.7///)                                        RIG10830
      C1=COS(AL)                                                        RIG10840
      C2=SIN(AL)                                                        RIG10850
      C3=COS(PSI)                                                       RIG10860
      C4=SIN(PSI)                                                       RIG10870
      C=SB+D*C1                                                         RIG10880
      C5=(SB*SB)/(C*C)*C1                                               RIG10890
      C6=1.+(SB/D)*(CI1/CI2)                                            RIG10900
      BE1=(C5*C6)/6.                                                    RIG10910
      C5=(SB*D)/(C*C)*C2                                                RIG10920
      C6=C/D+(SB*SB)/(D*D)*(CI1/CI2)-C1                                 RIG10930
      BE2=(C5*C6)/6.                                                    RIG10940
      C5=BE1*C3+BE2*C4                                                  RIG10950
      C6 = (SL2 * CI1)/(2. * A2 * D**3)                                 RIG10960
      C7=1./COS(AL+PSI)                                                 RIG10970
      BE3=BE1/(C5+C6*C7)                                                RIG10980
      BE4=BE1-BE3*C5                                                    RIG10990
      PRINT 1002,BE1,BE2,BE3,BE4                                        RIG11000
 1002 FORMAT(4X,4HBE1=,F14.7,5X,4HBE2=,F14.7,5X,4HBE3=,F14.7,5X,4HBE4=, RIG11010
     1 F14.7)                                                           RIG11020
      SF11=SF(SA1,SA1,SA)                                               RIG11030
      SF12=SF(SA1,SA2,SA)                                               RIG11040
      SF13=SF(SA1,SA3,SA)                                               RIG11050
      SF21=SF12                                                         RIG11060
      SF22=SF(SA2,SA2,SA)                                               RIG11070
      SF23=SF(SA2,SA3,SA)                                               RIG11080
      SF31=SF13                                                         RIG11090
      SF32=SF23                                                         RIG11100
      SF33=SF(SA3,SA3,SA)                                               RIG11110
      PRINT 1003,SF11,SF12,SF13,SF21,SF22,SF23,SF31,SF32,SF33           RIG11120
```

TABLE E. VIII. COMPUTER PROGRAM: FRAME BRIDGE WITH CONTINUOUS GIRDER (Cont'd)

```
1003 FORMAT(3X,5HSF11=,F14.7,4X,5HSF12=,F14.7,4X,5HSF13=,F14.7//
    1        3X,5HSF21=,F14.7,4X,5HSF22=,F14.7,4X,5HSF23=,F14.7//
    2        3X,5HSF31=,F14.7,4X,5HSF32=,F14.7,4X,5HSF33=,F14.7/)
     C1=COS(AL)
     C2=COS(PSI)
     C3=COS(AL+PSI)
     C4=(CI/CI1)*(D/SA)**3
     C5=(SC1*CI)/(A1*SA*SA*SA)
     C6=(SC2*CI)/(A2*SA*SA*SA)
     C8=COS(PHI)*COS(PHI)
     AL11=C4*BE*C1+C5+(SF11+BE3*SF12*C2)*C8
     AL12=2.*SF12*C2*C8
     AL13=-C4*BE4*C1+(SF13-BE3*SF12*C2)*C8
     AL21=C4*BE4*C3+(SF12 +BE3*SF22*C2)*C2*C8
     AL22=C6+2.*SF22*C2*C2*C8
     AL23=-(C4*BE4*C3+(BE3*SF22*C2-SF23)*C2*C8)
     AL31=-(C4*BE**C1-(SF13+BE3*SF23*C2)*C8)
     AL32=2.*SF23*C2*C8
     AL33=C4*BE**C1+C5+(SF33-BE3*SF23*C2)*C8
     PRINT 1004,AL11,AL12,AL13,AL21,AL22,AL23,AL31,AL32,AL33
1004 FORMAT(3X,5HAL11=,F14.7,4X,5HAL12=,F14.7,4X,5HAL13=,F14.7//
    1        3X,5HAL21=,F14.7,4X,5HAL22=,F14.7,4X,5HAL23=,F14.7//
    2        3X,5HAL31=,F14.7,4X,5HAL32=,F14.7,4X,5HAL33=,F14.7//)
     PRINT 1000
     IPAGE = 1
     X1 = 0.0
     CR1=CR2=CR3=0.0
     T1=T2=T3=T2P=0.0
     R1=R2=MT1=MT2=0.0
     F1=F2=F3=F4=0.0
     V1=V2=0.0
     H1=H2=0.0
     XM1=XM2=0.0
     GO TO 3
   1 X1=X1+60.
     CR1=SF(SA1,X1,SA)*COS(PHI)
     CR2=SF(SA2,X1,SA)*COS(PHI)*COS(PSI)
     CR3=SF(SA3,X1,SA)*COS(PHI)
     CALL SOLVE(T1,T2,T3,T2P)
     R2=(X1-(SA1*T1+SA2*(T2+T2P)*COS(PSI)+SA3*T3)*COS(PHI))/SA
     R1=1.-(T1+(T2+T2P)*COS(PSI)+T3)*COS(PHI)-R2
     F1=T2*SIN(PSI)
     F2=T2P*SIN(PSI)
     F3=T1+T2*COS(PSI)
     F4=T3+T2P*COS(PSI)
     C1=(F2-F1)*D*SIN(AL)
     C2=(F3-F4)*D*COS(AL)
     V1=C1/(2.*C)+(C2+2.*F3*SM)/(2.*L)
     V2=(C2+2.*F4*SM)/(2.*C)-C1/(2.*L)
     C3=(F2-F1)/2.
     C4=(F3-F4)/2.
     COT=COS(AL)/SIN(AL)
     H1=C3+C4*COT
     H2=-C3+C4*COT
     XM1=(V1*COS(AL)-H1*SIN(AL))*D
     XM2=-(V2*COS(AL)-H2*SIN(AL))*D
     IF(X1 .GT. SA1)GO TO 6
     MT1=SA1*(R1-1.)+X1
     GO TO 7
   6 MT1=SA1*R1
     GO TO 7
   7 MT2=SA2*R1-(SA2-X1)+(SA2-SA1)*T1*COS(PHI)
   3 PRINT 1005,X1,R1,R2,MT1,MT2
1005 FORMAT(3X,5HX1 = ,F14.7,4X,5HR1 = ,F14.7,4X,5HR2 = ,F14.7,4X,
    1 5HMT1= ,F14.7,4X,5HMT2= ,F14.7)
     PRINT 1006,T1,T2,T2P,T3
1006 FORMAT(26X,5HT1 = ,F14.7,4X,5HT2 = ,F14.7,4X,5HT2P= ,F14.7,4X,
    1 5HT3 = ,F14.7)
     PRINT 1007,F1,F2,F3,F4
1007 FORMAT(26X,5HF1 = ,F14.7,4X,5HF2 = ,F14.7,4X,5HF3 = ,F14.7,4X,
    1 5HF4 = ,F14.7)
     PRINT 1008,V1,V2,H1,H2,XM1,XM2
1008 FORMAT(26X,5HV1 = ,F14.7,4X,5HV2 = ,F14.7,4X,5HH1 = ,F14.7,4X,
    1 5HH2 = ,F14.7//26X,5HXM1= ,F14.7,4X,5HXM2= ,F14.7/)
     IF(IPAGE .GT. 8) GO TO 4
     IPAGE = IPAGE +1
     GO TO 5
   4 PRINT 1000
     IPAGE = 1
     GO TO 5
   5 IF(SA2 - X1)2,2,1
   2 STOP
     END
```

TABLE E.IX. COMPUTER PROGRAM SUBROUTINES: FRAME BRIDGE (REF. TABLE E.VIII)

```
      FUNCTION SF(X,XI,SA)
      IF(X .LE. XI) GO TO 1
      XNU=XI
      GO TO 2
    1 XNU=X
      GO TO 2
    2 C1=(X*XI)/(SA*SA)
      C2=X/SA
      C3=XI/SA
      C4=XNU/SA
      SF=(C1*(2,*(1,-C2)*(1,-C3)+(C2-C3)**2)-C4*(C2-C3)**2)/6,
      RETURN
      END
```

```
      FUNCTION DET(A11,A12,A13,A21,A22,A23,A31,A32,A33)
      B11 = A11 * A22 * A33 + A12 * A23 * A31 + A13 * A21 * A32
      B21 = A13 * A22 * A31 + A12 * A21 * A33 + A11 * A23 * A32
      DET = B11 - B21
      RETURN
      END
```

```
      SUBROUTINE SOLVE(T1,T2,T3,T2P)
      COMMON/A/A11,A12,A13,A21,A22,A23,A31,A32,A33,C1,C2,C3,BE3
      B11=A11
      B12=A12
      B13=A13
      B21=A21
      B22=A22
      B23=A23
      B31=A31
      B32=A32
      B33=A33
    5 DEL=DET(B11,B12,B13,B21,B22,B23,B31,B32,B33)
      T1=DET(C1,B12,B13,C2,B22,B23,C3,B32,B33)/DEL
      IF(T1 .LT. 0,0) GO TO 1
      T2=DET(B11,C1,B13,B21,C2,B23,B31,C3,B33)/DEL
      IF(T2 .LT. 0,0) GO TO 2
      T3=DET(B11,B12,C1,B21,B22,C2,B31,B32,C3)/DEL
      IF(T3 .LT. 0,0) GO TO 3
      GO TO 4
    1 B11=1,0
      B12=B13=B21=B31=C1=0,0
      GO TO 5
    2 B22=1,0
      B12=B21=B23=B32=C2=0,0
      GO TO 5
    3 B33=1,0
      B13=B23=B31=B32=C3=0,0
      GO TO 5
    4 T2P = T2 + BE3 * (T1 - T3)
      RETURN
      END
```

TABLE E.X. COMPUTER PROGRAM OUTPUT: FRAME BRIDGE WITH CONTINUOUS GIRDER

SA =	2640.0000000	SA1=	660.0000000	SA2=	1320.0000000	SA3=	1980.0000000
SB =	660.0000000	CL =	564.0000000	L =	984.9000000	SL1=	490.0000000
SL2=	827.9000000	A1 =	3.1400000	A2 =	2.7600000	CI1=	214000.0000000
CI2=	373000.0000000	CI =	4200.0000000	AL =	.9465503	PSI=	.9226974
PHI=	.7564904						

HE1=	.0394406	HE2=	.0547528	HE3=	.3621842	HE4=	.0136565
SF11=	.0117148	SF12=	.0143229	SF13=	.0091146		
SF21=	.0143220	SF22=	.0208333	SF23=	.0143229		
SF31=	.0091144	SF32=	.0143229	SF33=	.0117168		
AL11=	.0078396	AL12=	.0089732	AL13=	.0030072		
AL21=	.0060055	AL22=	.0029476	AL23=	.0029679		
AL31=	.0064367	AL32=	.0089732	AL33=	.0044102		

XI =	0.0000000	R1 =	0.0000000	R2 =	0.0000000	MT1=	0.0000000	MT2=	0.0000000
		T1 =	0.0000000	T2 =	0.0000000	T2P=	0.0000000	T3 =	0.0000000
		F1 =	0.0000000	F2 =	0.0000000	F3 =	0.0000003	F4 =	0.0000000
		V1 =	0.0000000	V2 =	0.0000000	H1 =	0.0000000	H2 =	0.0000000
		XM1=	0.0000000	XM2=	0.0000000				
XI =	60.0000000	R1 =	.9061660	R2 =	-.0072942	MT1=	-1.9304143	MT2=	-9.6283923
		T1 =	.1140701	T2 =	0.0000000	T2P=	.0435958	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.0347560	F3 =	.1140701	F4 =	.0263176
		V1 =	.1049692	V2 =	.0354185	H1 =	.0489858	H2 =	.9142276
		XM1=	20.8489355	XM2=	-8.8357061				
XI =	120.0000000	R1 =	.8128364	R2 =	-.0143756	MT1=	-3.5279838	MT2=	-18.9757321
		T1 =	.2273313	T2 =	0.0000000	T2P=	.0868824	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.0692655	F3 =	.2273313	F4 =	.0524485
		V1 =	.2091940	V2 =	.0705858	H1 =	.0976202	H2 =	.0263547
		XM1=	41.5500063	XM2=	-17.6087476				
XI =	180.0000000	R1 =	.7205153	R2 =	-.0210310	MT1=	-4.4598836	MT2=	-27.7609671
		T1 =	.3389744	T2 =	0.0000000	T2P=	.1295506	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.1032820	F3 =	.3389744	F4 =	.0782061
		V1 =	.3119298	V2 =	.1052507	H1 =	.1455618	H2 =	.0422707
		XM1=	61.9553474	XM2=	-26.2564599				
XI =	240.0000000	R1 =	.6297072	R2 =	-.0270478	MT1=	-4.3932590	MT2=	-35.7030448
		T1 =	.4481905	T2 =	0.0000000	T2P=	.1712913	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.1365591	F3 =	.4481935	F4 =	.1034038
		V1 =	.4124322	V2 =	.1391620	H1 =	.1924611	H2 =	.0559021
		XM1=	81.9170442	XM2=	-34.7161765				
XI =	300.0000000	R1 =	.5409162	R2 =	-.0322128	MT1=	-2.9952751	MT2=	-42.5209128
		T1 =	.5541705	T2 =	0.0000000	T2P=	.2117952	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.1688501	F3 =	.5541705	F4 =	.1278548
		V1 =	.5099568	V2 =	.1720686	H1 =	.2379709	H2 =	.0691208
		XM1=	101.2873818	XM2=	-42.9252387				
XI =	360.0000000	R1 =	.4546468	R2 =	-.0363133	MT1=	.8669079	MT2=	-47.9335187
		T1 =	.6561056	T2 =	0.0000000	T2P=	.2507532	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.1999087	F3 =	.6561056	F4 =	.1513726
		V1 =	.6037591	V2 =	.2037191	H1 =	.2817436	H2 =	.0818350
		XM1=	119.9183455	XM2=	-50.8209761				
XI =	420.0000000	R1 =	.3714032	R2 =	-.0391362	MT1=	5.1261298	MT2=	-51.6596100
		T1 =	.7531865	T2 =	0.0000000	T2P=	.2878560	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.2294882	F3 =	.7531865	F4 =	.1737706
		V1 =	.6930946	V2 =	.2338625	H1 =	.3234320	H2 =	.0939437
		XM1=	137.6621204	XM2=	-58.3407259				
XI =	480.0000000	R1 =	.2916697	R2 =	-.0404687	MT1=	12.5152304	MT2=	-53.4187343
		T1 =	.8446045	T2 =	0.0000000	T2P=	.3227945	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.2573424	F3 =	.8446045	F4 =	.1948619
		V1 =	.7772168	V2 =	.2622476	H1 =	.3626885	H2 =	.1053461
		XM1=	154.3768417	XM2=	-65.4216237				

TABLE E.X. COMPUTER PROGRAM OUTPUT: FRAME BRIDGE WITH CONTINUOUS GIRDER (Cont'd)

XI =	540.0000000	R1 =	.2160107	R2 =	-.0400979	MT1=	22.5670495	MT2=	-52.7292392
		T1 =	.9265501	T2 =	0.0000000	T2P=	.3552594	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.2832245	F3 =	.9295533	F4 =	.2144601
		V1 =	.8553874	V2 =	.2886230	M1 =	.3991658	M2 =	.1159413
		XM1=	159.8986448	XM2=	-72.0316048				
XI =	600.0000000	R1 =	.1448703	R2 =	-.0378158	MT1=	35.6144270	MT2=	-49.9102722
		T1 =	1.0072151	T2 =	0.0000000	T2P=	.3649417	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.3066881	F3 =	1.0072151	F4 =	.2323365
		V1 =	.9268558	V2 =	.3127378	M1 =	.4325164	M2 =	.1256283
		XM1=	184.0916647	XM2=	-78.0174047				
XI =	660.0000000	R1 =	.0787730	R2 =	-.0333945	MT1=	51.9502027	MT2=	-44.0807810
		T1 =	1.0767898	T2 =	0.0000000	T2P=	.4119320	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.3280868	F3 =	1.0767898	F4 =	.2484333
		V1 =	.9908796	V2 =	.3343405	M1 =	.4623931	M2 =	.1343162
		XM1=	196.6580367	XM2=	-83.4965588				
XI =	720.0000000	R1 =	.0181110	R2 =	-.0266835	MT1=	11.9532520	MT2=	-35.2221691
		T1 =	1.1376452	T2 =	0.0000000	T2P=	.4347500	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.3460289	F3 =	1.1376452	F4 =	.2624785
		V1 =	1.0468707	V2 =	.3532360	M1 =	.4885255	M2 =	.1418966
		XM1=	207.9367549	XM2=	-88.1203279				
XI =	780.0000000	R1 =	-.0371718	R2 =	-.0177013	MT1=	-24.5334873	MT2=	-23.3656647
		T1 =	1.1698711	T2 =	0.0000000	T2P=	.4547499	T3 =	0.0000000
		F1 =	0.0000000	F2 =	.3625416	F3 =	1.1698711	F4 =	.2745197
		V1 =	1.0946389	V2 =	.3694520	M1 =	.5109523	M2 =	.1484187
		XM1=	217.4762456	XM2=	-92.1656748				
XI =	840.0000000	R1 =	-.0687281	R2 =	-.0155972	MT1=	-45.3605370	MT2=	-20.3243060
		T1 =	1.1576806	T2 =	.0864481	T2P=	.5088953	T3 =	0.0000000
		F1 =	.0529746	F2 =	.4057281	F3 =	1.1977934	F4 =	.3072258
		V1 =	1.1054295	V2 =	.3595678	M1 =	.4971290	M2 =	.1443956
		XM1=	234.2151450	XM2=	-112.2547025				
XI =	900.0000000	R1 =	-.0788791	R2 =	-.0188097	MT1=	-52.0603343	MT2=	-24.8287624
		T1 =	1.0501888	T2 =	.1916162	T2P=	.5929818	T3 =	0.0000000
		F1 =	.1527628	F2 =	.4727446	F3 =	1.1658621	F4 =	.3579664
		V1 =	1.0820742	V2 =	.4417543	M1 =	.4509701	M2 =	.1309683
		XM1=	257.1824098	XM2=	-146.9824000				
XI =	960.0000000	R1 =	-.0819039	R2 =	-.0212436	MT1=	-54.0565795	MT2=	-28.0416187
		T1 =	.9256273	T2 =	.3248651	T2P=	.6786252	T3 =	0.0000000
		F1 =	.2589931	F2 =	.5410223	F3 =	1.1217392	F4 =	.4096670
		V1 =	1.0478893	V2 =	.4835169	M1 =	.3974812	M2 =	.1154520
		XM1=	279.7859108	XM2=	-182.2990281				
XI =	1020.0000000	R1 =	-.0790526	R2 =	-.0222510	MT1=	-52.1746915	MT2=	-29.3712871
		T1 =	.7887126	T2 =	.4623398	T2P=	.7637432	T3 =	0.0000000
		F1 =	.3685685	F2 =	.6065812	F3 =	1.0677960	F4 =	.4610503
		V1 =	1.0048696	V2 =	.5239767	M1 =	.3386875	M2 =	.0963748
		XM1=	301.5590563	XM2=	-218.4872582				
XI =	1080.0000000	R1 =	-.0715759	R2 =	-.0213836	MT1=	-47.2400893	MT2=	-23.2263058
		T1 =	.6441610	T2 =	.6000652	T2P=	.8462534	T3 =	0.0000000
		F1 =	.4783917	F2 =	.6746613	F3 =	1.0064036	F4 =	.5108594
		V1 =	.9550108	V2 =	.5822929	M1 =	.2766144	M2 =	.0803452
		XM1=	322.8292088	XM2=	-254.1897618				
XI =	1140.0000000	R1 =	-.0697245	R2 =	-.0181933	MT1=	-40.0761918	MT2=	-24.0151580
		T1 =	.4966892	T2 =	.7342465	T2P=	.9240732	T3 =	0.0000000
		F1 =	.5853654	F2 =	.7367015	F3 =	.9399331	F4 =	.5578370
		V1 =	.9063055	V2 =	.5974647	M1 =	.2132873	M2 =	.0619512
		XM1=	340.7577340	XM2=	-288.4492104				
XI =	1200.0000000	R1 =	-.0477491	R2 =	-.0122321	MT1=	-34.5144179	MT2=	-16.1463396
		T1 =	.3510134	T2 =	.8609685	T2P=	.9951203	T3 =	0.0000000
		F1 =	.6853924	F2 =	.7933426	F3 =	.8707559	F4 =	.6007262
		V1 =	.8427507	V2 =	.6287313	M1 =	.1507315	M2 =	.0437813
		XM1=	357.2750403	XM2=	-320.3382752				
XI =	1260.0000000	R1 =	-.0352441	R2 =	-.0152070	MT1=	-23.2611027	MT2=	1.0521325
		T1 =	.2262672	T2 =	.9488593	T2P=	1.0183530	T3 =	.0444343
		F1 =	.7544618	F2 =	.8116645	F3 =	.7990669	F4 =	.6591654
		V1 =	.7845596	V2 =	.6736927	M1 =	.0780823	M2 =	.0226797
		XM1=	381.3357290	XM2=	-362.1861085				
XI =	1320.0000000	R1 =	-.0241854	R2 =	-.0241854	MT1=	-15.9623685	MT2=	27.1332529
		T1 =	.1242200	T2 =	.9996526	T2P=	.9996526	T3 =	.1242200
		F1 =	.7569559	F2 =	.7569559	F3 =	.7276822	F4 =	.7276822
		V1 =	.7276822	V2 =	.7276822	M1 =	.0000000	M2 =	.0000000
		XM1=	410.3910644	XM2=	-410.3910644				

APPENDIX F

SUPPORTING DATA FOR PRELIMINARY SIGN AND LIGHTING SYSTEM SUPPORTING STRUCTURE DESIGNS

APPENDIX F

SUPPORTING DATA FOR PRELIMINARY SIGN AND LIGHTING SYSTEM SUPPORTING STRUCTURE DESIGNS

F.1. Analysis of Sign and Lighting System Support Structures

The analyses of the cable supported sign and luminaire supports can be treated in basically the same manner. There are several loading conditions to be considered in the design of these structures and, for simplicity, these loading conditions will be treated separately. The loads are divided into three categories: (1) loads due to the "dead weight" of the structure, (2) loads due to wind loads acting horizontally in any direction, and (3) loads due to the "preloading" of the cable supports. The analysis is based on the following assumptions:

(1) The compression member is installed without cable support, i.e., it supports its own weight in the initial condition.

(2) The amount of preload applied to the cables is ascertained by determining the force required to restore the upper end of the compression member to its original, undeflected position, or determining the force required to assure that neither cable becomes slack (zero force) for all conditions of loading.

(3) By applying a preload that assures both cables of remaining in tension, the cables can be assumed to carry compression forces which, in effect, are a reduction in the amount of tension in the cable.

The stability of the compression member during the pretensioning of the cables and during system responses to severe wind loadings is treated in a stability analysis discussion, Paragraph F.2.

Figure F.1 is a description of the geometry and sign convention used in the analysis. Figure F.2 describes the applied loads and defines the reactions in this structural system. Equilibrium equations written for these applied loads are summarized in Table F.I, along with basic deflection compatibility equations for the cables and compression members.

F.2. Stability Analysis of Sign and Lighting System Support Structures

F.2.a. Cable Pretension and Dead Load

For purposes of a stability analysis, the compression member - cable system is idealized as shown in Figure F.3. The stability of the

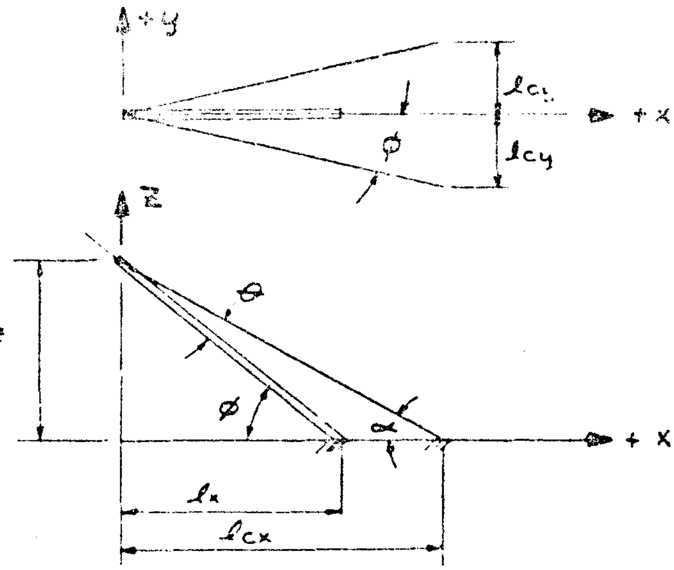

FIGURE F. 1 SIGN AND LIGHTING SYSTEM SUPPORT STRUCTURE GEOMETRY

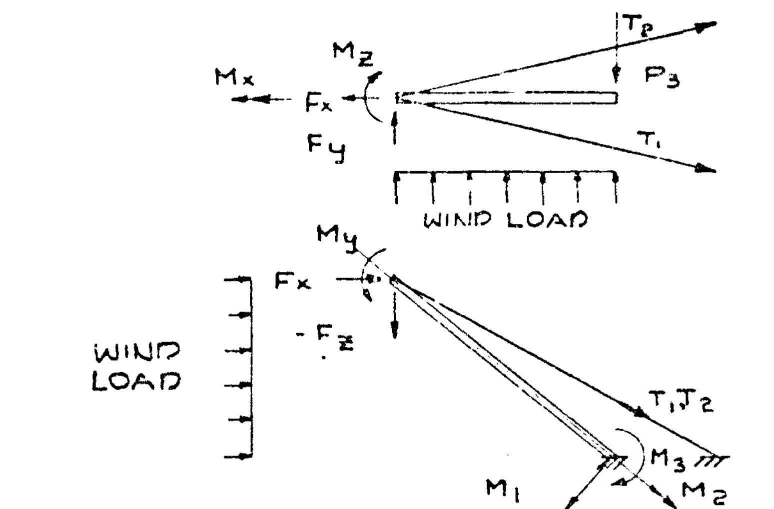

FIGURE F. 2. SIGN AND LIGHTING SYSTEM SUPPORT STRUCTURE LOADS AND REACTIONS

TABLE F.I. LOAD ANALYSIS OF COMPRESSION MEMBER IN SIGN AND LIGHTING SYSTEM SUPPORT STRUCTURES

Applied Load	Compression Member Deflection Equation	Cable Elongation Equation	Equations of Equilibrium
F_x	$\delta_1 = P_1L^3/3E_pI_p$	$\delta_1 = T_{xz}\ell_{c_{xz}}/A_cE_c \sin\theta$	(1) $F_x(L_z) - 2T_{xz}L\sin\theta - M_3 = 0$, (2) $- 2T_{xz}\sin\alpha - P_1\cos\beta + P_2\sin\beta = 0$ (3) $F_x - 2T_{xz}\cos\alpha - P_1\sin\beta - P\ \cos\beta = 0$, (4) $P_1L - M_3 = 0$
F_y	$\delta_3 = P_1L^3/3E_pI_p$	$\delta_3 = T_{xy}\ell_{c_{xy}}/A_cE_c \sin\phi$	(1) $F_yL_x - T_{1_{xy}}L_x\sin\phi - T_{2_{xy}}L\sin\phi - M_1 = 0$, (2) $- T_1 + T_2 = 0$
$-F_z$	$\delta_1 = P_1L^3/3E_pI_p$	$\delta_1 = T_{xz}\ell_{c_{xz}}/A_cE_c \sin\theta$	(1) $- F_zL_x + 2T_{xz}L\sin\theta + M_3 = 0$, (2) $- 2T_{xz}\sin\alpha + P_1\cos\beta + P_2\sin\beta - F_z = 0$
	Deflection Equation		(3) $2T_{xz}\cos\alpha + P_1\sin\beta - P_2\cos\beta = 0$, (4) $- P_1L + M_3 = 0$
M_x	$\delta_3 = \frac{M_x}{2}\frac{\cos\beta\, L^2}{E_pI_p} - \frac{2T_{xy}\sin\phi\, L^3}{3E_pI_p}$		(1) $M_z\cos\beta - 2T_{xy}L\sin\phi - M_1 = 0$
M_y	$\frac{M_yL^2}{2E_pI_p} - \frac{2T_{xz}\sin\theta L^3}{3E_pI_p} = \frac{T_{xz}\ell_{c_{xz}}}{A_cE_c\sin\theta}$		....
M_z	$\delta_3 = \frac{M_z}{2}\frac{\cos\beta L^2}{E_pI_p} - \frac{2T_{xy}\sin\phi L^3}{3E_pI_p}$		(1) $M_z\cos\beta - 2T_{xy}L\sin\phi - M_1 = 0$
w_{w_x}	$\delta_1 = \frac{w_{w_x}L^4}{8E_pI_p} - \frac{2T_{xz}\sin\theta L^3}{3E_pI_p} = \frac{T_{xz}\ell_{c_{xz}}}{A_cE_c\sin\theta}$		....
w_{w_y}	$\delta_3 = \frac{w_{w_y}L^4}{8E_pI_p} - \frac{2T_{xy}\sin\phi L^3}{3E_pI_p} = \frac{T_{xy}\ell_{c_{xy}}}{A_cE_c\sin\phi}$		....
$w_{D.L.}$	$\delta_1 = \frac{w_{D.L.}\cos\beta L^4}{8E_pI_p}$		....
PRELOAD	$\delta_3 = \frac{PRELOAD\sin\theta\cos\phi L^3}{3E_pI_p}$		(1) $2T_{xz}L\sin\theta - M_3 = 0$, (2) $M_3 = P_1L + 2T_{xz}L\sin\theta$, (3) $-2T_{xz}\sin\alpha + P_2\sin\beta - P_1\cos\beta$

Symbols - See Figures F.1 and F.2 for other symbol definitions.

I_p = Moment of inertia of compression member (in.^{4})
E_p = Modulus of Elasticity of compression member (lb/in.^{2})
A_c = Area of cable (in.^{2})
E_c = Modulus of Elasticity of cable (psi)
} for 1/2-in.-dia cable - E_cA_c = 3.59 X 10^6 lb
w_{w_x} = Uniform load due to wind acting in y direction on compression member (lb/ft)
w_{w_y} = Uniform load due to wind acting in x direction on compression member (lb/ft)
$w_{D.L.}$ = "Dead Load" of compression member (lb/ft)
PRELOAD = Load applied to each cable.

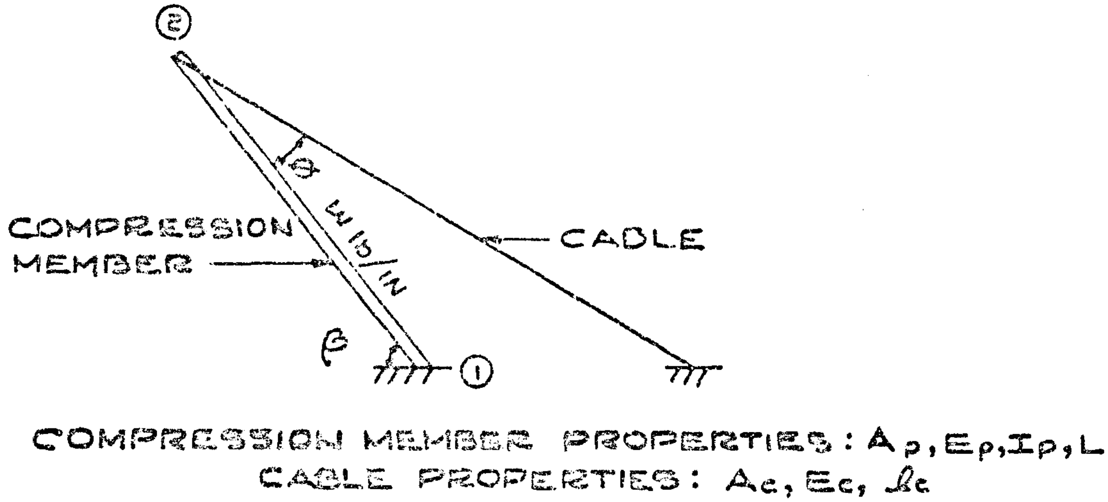

FIGURE F. 3. IDEALIZATION OF COMPRESSION MEMBER-CABLE SYSTEM

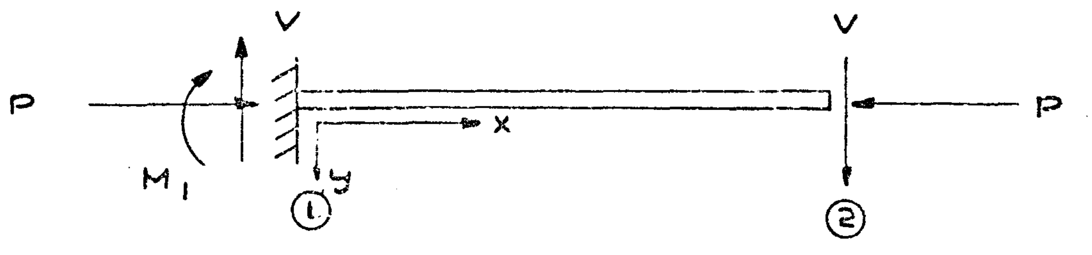

FIGURE F.4. FREE-BODY DIAGRAM OF COMPRESSION MEMBER system will be analyzed by considering only the cable forces. The effect of the compression member's dead weight will be considered later. The differential equation governing the behavior of the compression member, as shown in Figure F.4, is*

$$EIy'''' + Py'' = 0 \tag{1}$$

where the prime refers to derivatives with respect to x. A solution to Equation (1) is

$$y = A \sin kx + B \cos kx + Cx + D \tag{2}$$

where

$$k^2 = P/E_pI_p$$

and A, B, C, and D are arbitrary constants. By substitution of the boundary conditions, as shown in Figure F.4,

$$\begin{aligned} &y = y' = 0 \quad \text{at} \quad x = 0 \\ &M = 0 \quad , \quad Q = V \quad \text{at} \quad x = L \end{aligned} \tag{3}$$

expressions are obtained for A, B, C, and D. Equation (2) can then be rewritten as

$$y = \frac{V}{Pk} [\sin kx - kx - \tan kL (\cos kx - 1)] \tag{4}$$

A buckled configuration for the compression member-cable system is shown in Figure F.5. The quantity δ is the buckled deflection at the top of the pole. By resolving the cable force, one obtains (neglecting second order terms).

$$\begin{aligned} &P = T \cos \theta \\ &V = T \left(\sin \theta + \frac{S}{\ell_c} \cos^2\theta\right) \end{aligned} \tag{5}$$

where T is the cable tension. Noting that

$$y = -\delta \quad \text{at} \quad x = L$$

and substituting Equation (5) into (4), the following expression is obtained for δ:

*S. Timoshenko and J. Gere, Theory of Elastic Stability, McGraw-Hill, 1961.

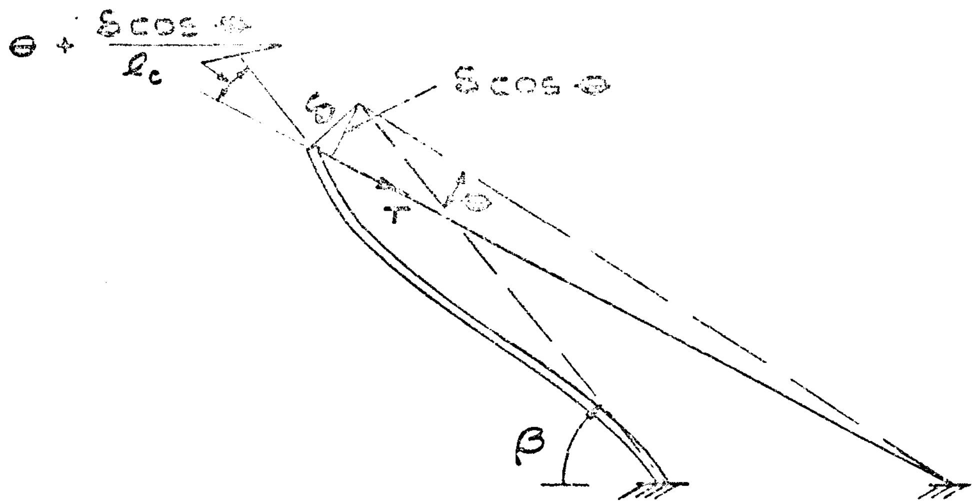

FIGURE F.5. BUCKLED CONFIGURATION OF COMPRESSION MEMBER-CABLE SYSTEM (Pretension Case)

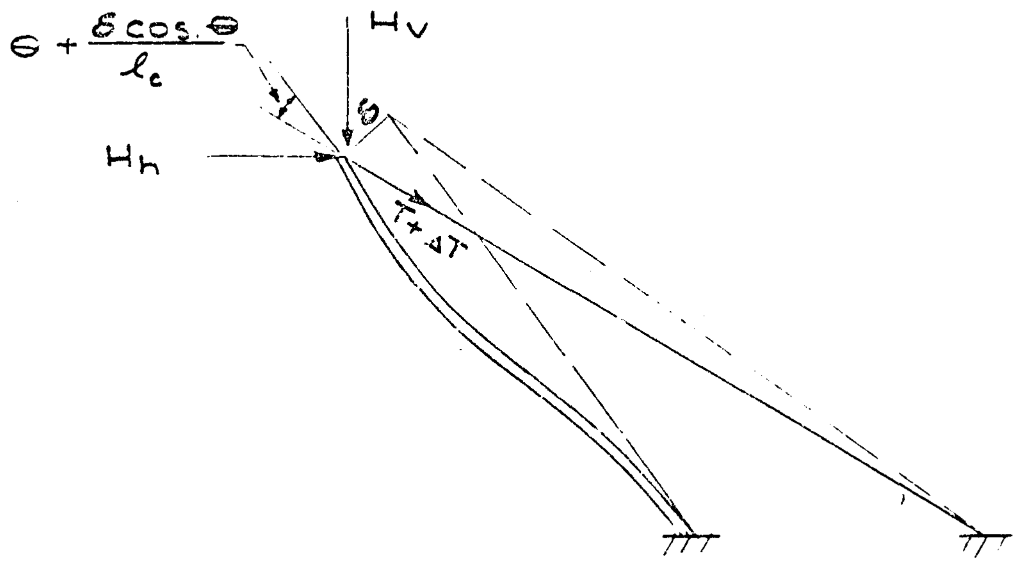

FIGURE F.6. BUCKLED CONFIGURATION OF COMPRESSION MEMBER-CABLE SYSTEM (Live-Load Case)

$$\delta = -\frac{\ell_c \tan \theta (\tan kL - kL)}{k\ell_c + \cos \theta (\tan kL - kL)} \tag{6}$$

The buckling load is defined as the load at which the deflections are unbounded, i.e., δ goes to infinity. Therefore, the critical load is reached when

$$k\ell_c + \cos \theta (\tan kL - kL) = 0$$

or

$$\tan u = u\left(1 - \frac{\ell_c}{L \cos \theta}\right) \tag{7}$$

where $u = kL$. The values of u for which this equation is satisfied are given in the following tabular form by Timoshenko*:

TABLE F.II. SOLUTIONS TO EQUATION (7)

$\ell_c/L \cos \theta$	$(u)_{cr}$
1	π
1.2	2.654
1.5	2.289
2	2.029
3	1.837
4	1.758
5	1.716
8	1.657
10	1.638
20	1.602
∞	$\pi/2$

The critical value of P is found as

$$P_{cr} = (u_{cr})^2 \frac{E_p I_p}{L^2} \tag{8}$$

where $(u)_{cr}$ is tabulated in Table F.II for particular values of $\ell_c/L \cos \theta$. (Note: $P_{cr} = T_{cr} \cos \theta$.)

*Ibid. pg 122.

An approximate (conservative) expression for the critical load which includes the effect of the distributed dead load is given by Timoshenko*

$$\overline{P}_{cr} = \overline{P}_{cr} - \frac{qL}{2} \tag{9}$$

where $\overline{P}_{cr}$ is the corrected critical buckling load, P_{cr} is obtained from Equation (8), and q is the axial component of the distributed dead load

F.2.b. Live Load Effects

In this section, the stability of the total system subject to external loads will be considered. Equations (1), (2), (3), and (4) also apply in this case. The additional applied live loads are shown in Figure F.6. Resolution of the live load and cable forces yields (to the first order)

$$\begin{aligned} P &= T \cos\theta + H_h \cos\beta + H_v \sin\beta \\ V &= (T + \Delta T)\left(\sin\theta + \frac{\delta}{\ell_c}\cos^2\theta\right) + H_h \sin\beta - H_v \cos\beta \end{aligned} \tag{10}$$

where

$$\Delta T = \frac{\delta}{\sin\theta}\,\frac{A_c E_c}{\ell_c}$$

(In this case, the effect of the change in cable tension ΔT must be included. Here the cable is actively resisting the applied loads, whereas, in the pretension case, the cable is acting merely as a means of applying the load to the compression number and the elongation of the cable is immaterial.) Proceeding in a manner similar to that used in the preceeding section, one obtains the following expression to be solved for the critical load: (neglecting second order effects):

$$\tan u = u\,(1 - \epsilon u^2) \tag{11}$$

where

$$u = kL$$

$$k^2 = P/E_pI_p$$

$$\epsilon = \frac{E_pI_p\ell_c}{L^3(T\cos^2\theta + A_cE_c\sin^2\theta)}$$

*Ibid. pg. 122.

with a given value of ϵ, the value of u for which Equation (11) is satisfied can be obtained by trial and error. Let this value of u be u_{cr}, then

$$P_{cr} = (u_{cr})^2 \frac{E_p I_p}{L^2} \quad (12)$$

This value of P_{cr} can be (conservatively) corrected for the effects of the distributed load as was done in the previous section:

$$\overline{P}_{cr} = P_{cr} - \frac{qL}{2} \quad (13)$$

where $\overline{P}_{cr}$ is the corrected critical buckling load, P_{cr} is obtained from Equation (12), and q is the <u>axial</u> component of the distributed dead plus live load.

If the cable is quite stiff relative to the compression member, ϵ is small and may be neglected in Equation (11). Thus, one has

$$\tan u = u \quad (14)$$

which is identical to the expression obtained for a pinned-fixed column. The corresponding critical load is

$$P_{cr} = 20.19 \frac{E_p I_p}{L^2} \quad (15)$$

F.3. Comments on the Dynamic Analysis of Sign Support Structures

F.3.a. Introduction

Dynamic analysis of a structural system of any complexity is usually long and tedious. For preliminary design purposes it is desirable, if possible, to reduce the system to one having a single degree of freedom, i.e., to a single mass and a "spring". If this can be done, it is next necessary to assume the type of dynamic disturbance, which may be one or several of an infinite number of forms. Having decided upon a particular form, one can then determine the "dynamic load factor," either analytically or from prepared curves in the published literature. A good reference is "Effects of Impact on Simple Elastic Structures," by J. M. Frankland in the Proceedings of the Society for Experimental Stress Analysis, Vol. 6, pp. 7-27 (1948). This will be referred to hereafter as "Frankland". The value of the dynamic load factor depends on two parameters: (a) the duration of the pulse, and (b) the natural period of vibration of the structural system.

F. 3. b. Analysis of Sign Support Structure

For purposes of this discussion, the dynamic disturbance will be assumed to be a rectangular pulse of duration t_1 (Figure F. 7). The dynamic load factor is given in Figure F. 8. The period T of the structure depends on the "spring constant," which may be obtained for various types of load on the guyed cantilever. Three types of loads have been considered: (a) load acting vertically in the plane of the sign panel, (b) load acting normal to the plane of the sign panel and (c) torsional moment acting on the structure in a horizontal plane. Each of these loading conditions is discussed in the paragraphs which follow.

- Load Acting in Plane of Sign Panel (Figure F.9)

If there is no pretension in the cables, the tension T_o in the cables is effective only when the end of the cantilever is moving downwards. When the cantilever is moving up, the cables are ineffective. This makes the analysis somewhat complicated. A simplified approach is to omit the effect of the tension. Another would be to take the average value of the periods with and without tension. In this case, the period T may be given by

$$T = 2\pi f_W \tag{1}$$

Static deflection of sign per unit load

$$\Delta = \frac{g}{W} f_W^2 \tag{2}$$

where

$$f_W = \left[\frac{Wb^3}{3gEI_1}\left\{\frac{a^3}{b^3} + \left(3\frac{a^2}{b^2} + 3\frac{a}{b}\cos\alpha + \cos^2\alpha\right)\frac{I_1}{I_2} + \mu\left[\frac{3cI_1}{Ab^3} - \frac{I_1}{I_2}\sin(\alpha - \beta)\left(\frac{3}{2}\frac{a}{b} + \cos\alpha\right)\right]\right\}\right]^{1/2} \tag{3}$$

and:

E = modulus of elasticity (lb-in.$^{-2}$)

μ = $\sin(\alpha - \beta)(3a/2b + \cos\alpha)/\left\{\sin^2(\alpha - \beta) + 3I_2c/Ab^3\right\}$

W = weight of sign (lb)

g = acceleration due to gravity (in. sec^{-2})

c = $d \sin\phi \operatorname{cosec}\beta$, (in.)

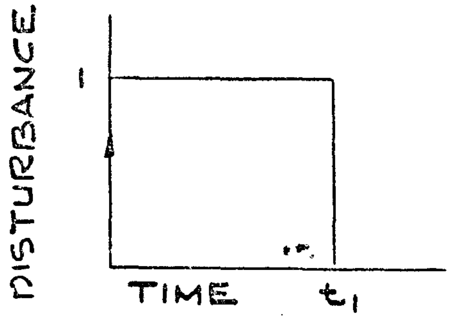

FIGURE F. 7. RECTANGULAR LOAD PULSE ON SIGN PANEL

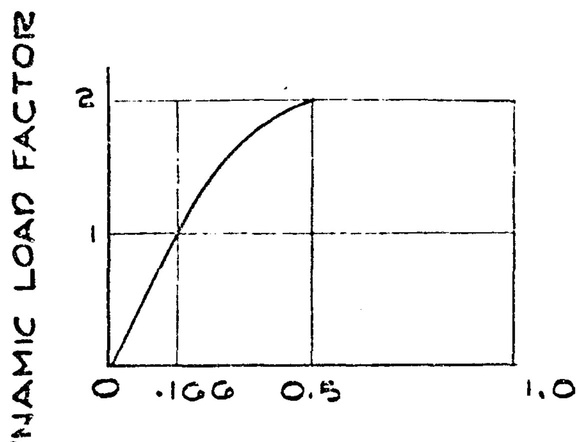

FIGURE F. 8. DYNAMIC LOAD FACTOR (From Frankland)

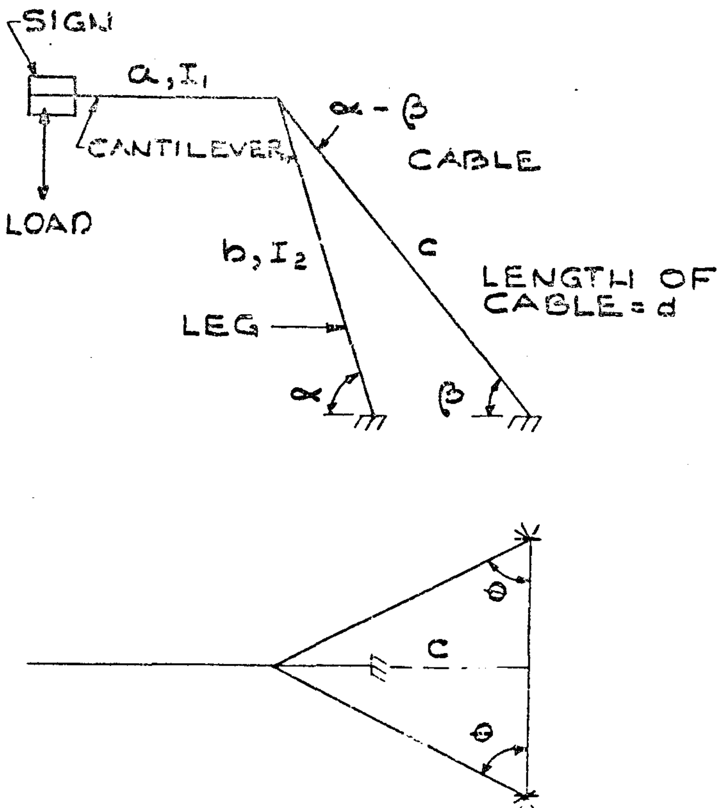

FIGURE F.9. DEFINITION SKETCH FOR SIGN SUPPORT STRUCTURE: IN-PLANE LOADS

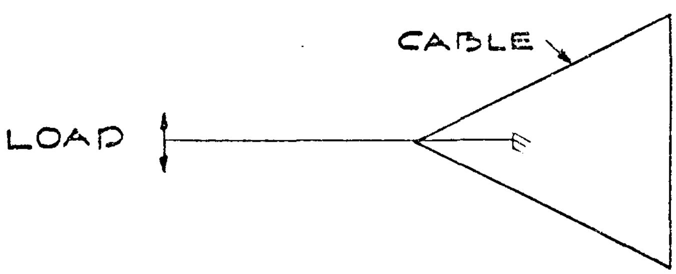

FIGURE F.10. DEFINITION SKETCH FOR SIGN SUPPORT STRUCTURE: LOADS NORMAL TO PLANE OF SIGN PANEL

I_1	=	moment of inertia of cantilever (in.^{4})
I_2	=	moment of inertia of leg (in.^{4})
$\sin\beta$	=	$\sin\phi/\sin\theta$
A	=	cross sectional area of cable (in.^{2})
d	=	length of cables (in.)
α	=	angle of inclination of leg (Fig. F.9)
ϕ	=	angle of inclination of cables to horizontal plane
θ	=	angle of inclination of cables as seen in plan (Fig. F.9)
a	=	length of cantilever (in.), b length of leg (in.)

Equation (2) is valid only when the cable tension is effective. When cable tension is ineffective, set $\mu = 0$.

- <u>Load Acting Normal to Plane of Sign Panel (Figure F.10)</u>

We have assumed in this analysis that only one of the two cables is effective in resisting motion due to load applied normal to plane of sign panel. The period T is given by

$$T = 2\pi f_V \qquad (4)$$

Static deflection of sign per unit load is

$$\Delta_V = \frac{g}{W} f_V^2 \qquad (5)$$

where:

$$f_V = \left[\frac{Wb^3}{3gE\bar{I}_1}\left\{\frac{a^3}{b^3} + \left(3\frac{a^2}{b^2} + 3\frac{a}{b} + 1\right)\frac{\bar{I}_1}{\bar{I}_2} - \left(1 + \frac{3}{2}\frac{a}{b}\right)^2 \frac{\bar{I}_1}{\bar{I}_2}\frac{\cos^2\theta}{\lambda}\right\}\right]^{1/2}$$

$$\lambda = \cos^2\theta + \frac{\bar{I}_2}{I_2}\sin^2\theta\,\sin^2(\alpha - \beta) + \frac{3d\bar{I}_2}{Ab^3}$$

$\bar{I}_1$	=	moment of inertia of cantilever about axis in plane of frame (in.^{4})
$\bar{I}_2$	=	moment of inertia of leg about axis in plane of frame (in.^{4})

Other symbols are the same as in the previous paragraph.

- Torsional Moment Acting at Mass

Period T = $2\pi f_\theta$ (6)

Angle of twist of end of cantilever per unit moment is

$$\theta = \frac{f_\theta^2}{I_m} \tag{7}$$

$$f_\theta = \left[\frac{I_m a}{GJ_1}\left(1 + \frac{b}{a}\frac{J_1}{J_2}\cos\alpha\right)\right]^{1/2}$$

I_m = mass moment of inertia of sign about axis in plane of frame (lb in. sec^2)

G = shear modulus (lb-in.$^{-2}$)

J_1 = torsion constant for cantilever (in.^{4})

J_2 = torsion constant for leg (in.^{4})

In this motion, there will be both a torsional motion of the frame and an out of plane motion of the end of the cantilever.