PROJECT HPR - 2 (104)

HIGHWAY SIGN SUPPORT RESEARCH

A STUDY OF ECONOMIC AND SAFETY ASPECTS OF A SIGN AS A FUNCTION OF THE FREQUENCY OF OCCURRENCE OF DESIGN WINDS

INTERIM REPORT

TO THE PROJECT POLICY COMMITTEE

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I. Introduction

Many factors should be considered in designing an "optimal" sign. For example, one should consider the different types of signs available, the sign material, the location of the sign, the size and color of the letters and sign background, etc., However, the intent of this study is limited to presenting criteria that can be applied in optimizing a sign design when considering the safety aspects (size or mass of sign) and cost as related to the frequency of occurrence of design winds. The study is constrained to presenting information that can be used in existing AASHO design procedures[1*], with the exception of design wind velocities. In other words, if improvements can be made to the economic and safety aspects of a sign, they will be accomplished by reducing the present design wind velocity rather than changing the sign type, material, location, etc.

A formulation of the problem is included in Section III of this report in order that one may better understand how the equations were developed and may be used. Part A of Section III deals with the relation between sign size and frequency of occurrence of design winds. Part B deals with cost as related to the frequency of occurrence of design winds.

It is important that the meaning of "size" as used in this study is understood. By size is meant the cross-sectional area of those members of the sign whose geometry is determined by wind loads. For example, the cross-sectional areas of the supports,

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[*] Superscripts refer to references at the end of the report.

wind beams, and sign background, depend on the design wind loads. The frontal area of the sign background and the support spacing are determined by factors other than design wind loads.

The present recurrence interval for the design winds on a sign as specified by AASHO is 50 years. Designing to these low probability winds often results in having larger structures than needed, and thus a greater safety hazard. In some cases, signs have to be removed or replaced within 5 to 10 years after installation for reasons other than wind damage. Another factor to consider when designing a sign is its replacement cost after being blown down. If the replacement cost is small compared to the initial cost, it may be advantageous to reduce the recurrence interval of design winds. These and other factors are considered in this study and are discussed in more detail in Sections III and IV.

Obtaining optimum criteria when safety is involved is difficult, if not impossible. However, the information contained in this study will afford the user with data to aid him in arriving at a more satisfactory design within the boundaries of his particular situation.

The study relies heavily on information obtained from a report by Mr. H. C. S. Thom,[2] Chief Climatologist, Office of Climatology, U. S. Weather Bureau. In his report, Mr. Thom took the records of 141 open-country stations, with a cumulative total of about 1,700 years of records averaging about 15 years per station, and, through statistical analysis, arrived at distributions of extreme winds in the United States. The data obtained from Mr. Thom's report were invaluable.

II. Assumptions

1. The assumption is made that the sign's design loads consist of wind forces only. For most roadside signs, this assumption is acceptable. However, in the large overhead bridge signs, the live and dead loads (as defined by AASHO) have a larger influence on the size structure required and the assumption would likely be unacceptable in these cases.

2. As the sign is designed for higher probability winds (lower velocities) its size (or mass) and stiffness is reduced. In so doing, the critical wind speed at which resonance occurs may fall below the recommended value as determined in this study. No provisions are made for that possibility, i. e., it is assumed that resonance will not occur below the recommended design wind velocity. The validity of this assumption will depend to a large degree on a particular sign's geometric configuration, including the support spacing, the type of support used, and the type of sign background.

3. With regard to Assumption 2, it is also assumed that the stiffness is such that fatigue is not a problem. Again, the validity of this assumption depends on the geometric configuration of the sign and the material used.

4. It is assumed that a sign will blow down or experience a structural failure when subjected to the wind velocity for which it was designed. This assumption applies to the present design and to the reduced design sign, with both types being designed

according to AASHO procedures, including safety factors. The validity of this assumption is questionable since the safety factors referred to sometimes reach a value as large as 1.8. However, it is not the purpose of this study to investigate the use of safety factors.

5. It is assumed that the replacement cost, due to a wind load failure, of the currently used sign equals the replacement cost of the reduced design for the same type failure. If a built-in failure mechanism could be incorporated in the reduced design its replacement cost would likely fall below the currently used design and, in turn, enhance the use of the reduced design.

6. It is assumed that the yearly maintenance cost is independent of sign size (as defined previously).

III. FORMULATION OF PROBLEM

A. Size

In Mr. Thom's paper, an equation was presented for determining the probability F(X) of given wind speed X occurring at any given geographical location.

$$F(X) = e^{[-X/B]^{-G}} \qquad (1)$$

In this equation, the value of B and G are found by imposing the boundary conditions.

$$@\ X = V_1,\ F(X) = 0.50$$

and $$@\ X = V_2,\ F(X) = 0.98$$

These boundary conditions render two equations in terms of B and G and upon solving yields

$$B = V_1\ (.694)^{\left(\frac{\ln \frac{V_1}{V_2}}{-3.54}\right)}$$

and $$G = -3.54/\ln\left(\frac{V_1}{V_2}\right)$$

where

V_1 = Extreme mile wind velocity that occurs during an average 2 year period.

V_2 = Extreme mile wind velocity that occurs during an average 50 year period.

The average recurrence interval R can be found by the relationship

$$R = 1/[1-F(X)] \qquad (2)$$

For example, if, for a given ratio V_1/V_2 and a velocity X, the probability $F(X) = 0.90$, the recurrence interval R would be

$$R = 1/(1-.9) = 10 \text{ years}$$

i.e., one could expect the wind to reach the velocity X once in a mean 10 year period.

Taking the natural logarithm of both sides of Equation 1 and solving for X yields:

$$X = B(-\ln F)^{-1/G} \qquad (3)$$

where $F = F(X)$

The widely accepted equation for the force, Q, on a structure due to wind loads is,

$$Q = 1/2\ CpX^2 \qquad (4)$$

where C is a constant determined by the shape factor and gust load factor and p equals the mass density of the air.

Substituting Equation 3 into Equation 4 yields

$$Q = 1/2\ CpB^2\ (-\ln F)^{-2/G} \qquad (5)$$

Thus, for a given structure and V_1/V_2 ratio, it can be seen that the forces or loads on the structures are proportional to $(-\ln F)^{-2/G}$.

The size, or weight, W, of an elastic structure is related to the applied loads and in the present case,

$$W \sim [(-\ln F)^{-2/G}]^{f} \qquad (6)$$

(The symbol "$\sim$" means proportional.)

where the exponent f is dependent on the manner in which the structure resists the loads and the type member used. For most roadside sign configurations, the wind loads are resisted by bending in the supports. Consider a member as shown in Figure 1,

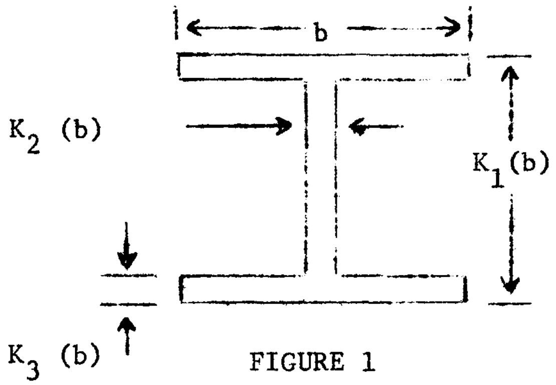


FIGURE 1

The cross sectional area $A = b^2\ (2K_3 + K_1K_2 - 2K_2K_3)$ and section modulus S is

$$S = b^3 \left[\frac{K_3}{K_1} (K_1 - K_3)^2 + \frac{K_2}{6K_1} (K_1 - 7K_3)^3 \right]$$

The bending stress $\sigma_b = \frac{M}{S}$, where M equals the bending moment. Let $\sigma_b = \sigma_D$ where σ_D is the design stress. Then $S = \frac{M}{\sigma_D}$. If the factors, K_1, K_2 and K_3 remain constant as the moment (M) varies, then S and A are proportional to b^3 and b^2, respectively. This is obviously not possible in some cases with available commercial sizes, however, for the purpose of this example, this assumption is made.

The following proportionalities exist:

$$S \sim M$$

therefore,

$$b^3 \sim (-\ln F)^{-2/G}$$

since the moment $M \sim Q = (-\ln F)^{-2/G}$.

Then,

$$b^2 \sim [(-\ln F)^{-2/G}]^{2/3} \sim A$$

Since the weight/unit length W is proportional to the area A,

$$W \sim [(-\ln F)^{-2/G}]^{2/3}$$

Thus,

$$f = 2/3$$

In fact, $f = 2/3$ for any cross section in which the area is proportional to b^2 and the section modulus is proportional to b^3. For instance, a rectangular section in bending as shown in Figure 2,

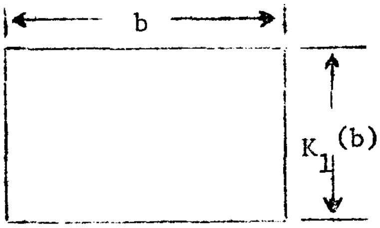


FIGURE 2

would have $f = 2/3$, provided K_1 remains constant as the cross section is varied.

For members subjected to axial loads, $f = 1.0$ since the area is directly proportional to the load.

With the factor "f" and Equation 6, the relative size of a structure as the probability F is varied can now be investigated.

As a general case, let F_p be the probability factor for which the signs in a particular area are being designed. The question is: What percent reduction can one expect in the sign size if the probability factor is reduced to F? If r represents the percent reduction, we get, using Equation 6,

$$r = 100 \left\{ \frac{\left[(-\ln F_p)^{-2/G}\right]^f - \left[(\ln F)^{-2/G}\right]^f}{\left[(-\ln F_p)^{-2/G}\right]^f} \right\}$$

or

$$r = 100\% \left[1 - \left(\frac{\ln F}{\ln F_p} \right)^{-2f/G} \right] \qquad (7)$$

A relationship between F and R can be obtained from Equation 2,

$$F = F(X) = \frac{R-1}{R} \qquad (8)$$

Using this value of F in Equation 7, one obtains a relation between the percent reduction r and the recurrence interval R,

$$r = 100\% \left[1 - \left(\frac{\ln \frac{R-1}{R}}{\ln \frac{R_p - 1}{R_p}} \right)^{-2f/G} \right] \qquad (9)$$

where R_p is the wind recurrence interval for which the signs are presently being designed.

As an example, the curves in Figure 3 represent a plot of Equation 9, with the following values used:

R_p = 50 years (i.e., the present sign's design life is 50 years).

f = 0.667 (bending loads).

and V_1/V_2 = 0.20, (G = 2.19).

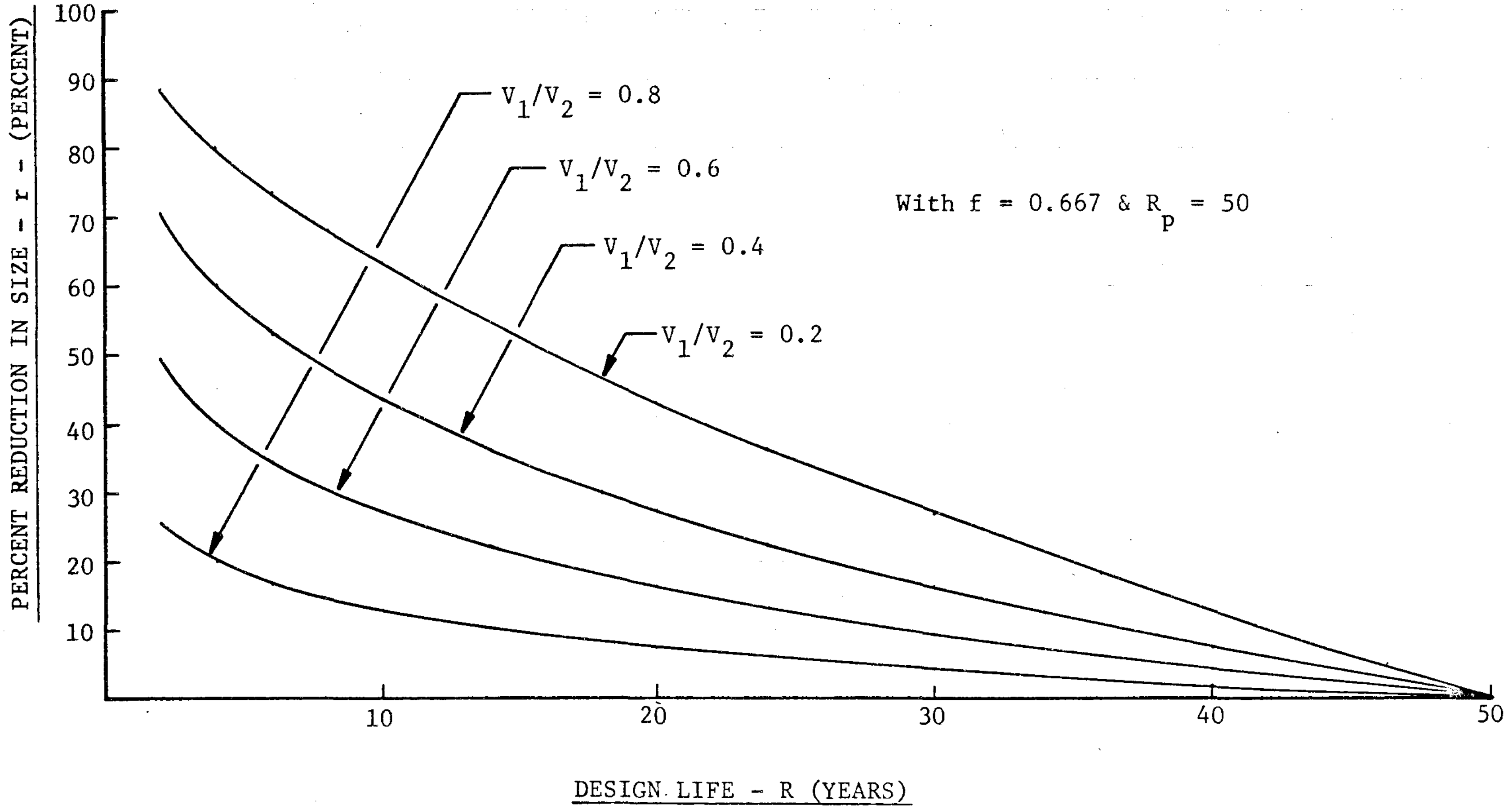
V_1/V_2 = 0.40, (G = 3.86).

V_1/V_2 = 0.60, (G = 6.92).

V_1/V_2 = 0.80, (G = 15.85).

If, for instance, a sign is in an area where the ratio of V_1/V_2 equals 0.60 and one wishes to redesign to a recurrence interval of 10 years, the percent reduction in the 50 year design would be 27.2%. By design is meant the size (refer to page 1 for definition of size).

FIGURE 3. PERCENT CHANGE IN A 50 YEAR DESIGN SIZE VERSUS A REDUCED DESIGN LIFE FOR THE GIVEN PARAMETERS.



B. Cost

It has been shown what reductions in size can be expected as the sign is designed for higher probability winds. The effects on sign cost of designing for higher probability winds will now be considered.

Following is a list of factors considered, in addition to those in part III-A, and an explanation of each:

C_I - Initial cost of the sign with reduced design life, i.e., the cost based on the reduced design life R.

C_{Ip} - Initial cost of the sign now being used, i.e., the cost based on the present design life R_p. Initial cost includes material, fabrication, and installation cost.

A - Useful life of sign - defined as the length of time the sign, whether the present design or the reduced design, will normally remain in place, assuming no wind damage. At the end of its useful life, the sign will either be replaced or removed entirely. Useful life is not to be confused with "design life" or "recurrence interval" R_p as defined previously.

B. - Replacement factor - defined as the ratio of the cost required to replace a blown-down sign to the initial sign cost.

$$B = \frac{\text{Cost to replace}}{\text{Initial Cost}}$$

ϕ - Cost function - a function which relates the initial cost of a sign to the size of the sign.

I - Interest rate - used to discount to the present all costs occurring in future periods.

K_p - Maintenance factor - defined as the ratio of the yearly maintenance cost to the sign's initial cost.

$$K_p = \frac{\text{Maintenance Cost Per Year}}{\text{Initial Cost}}$$

K_1 - Salvage factor - defined as the ratio of the salvage value of the sign with reduced design at end of its useful life to its initial cost.

$$K_1 = \frac{\text{Salvage Value of Reduced Design Sign}}{\text{Initial Cost of Reduced Design Sign}}$$

K_2 - Salvage factor of present design

$$K_2 = \frac{\text{Salvage Value of Present Sign}}{\text{Initial Cost of Present Sign}}$$

In order to have a basis for comparison, total sign cost as used in this report will refer to all sign costs incurred during the useful life of the sign. The total cost related to the present design and the reduced design will be computed and the results compared to show the percent difference.

Computation of Initial Cost:

Consider the cost function as shown in Figure 4. It

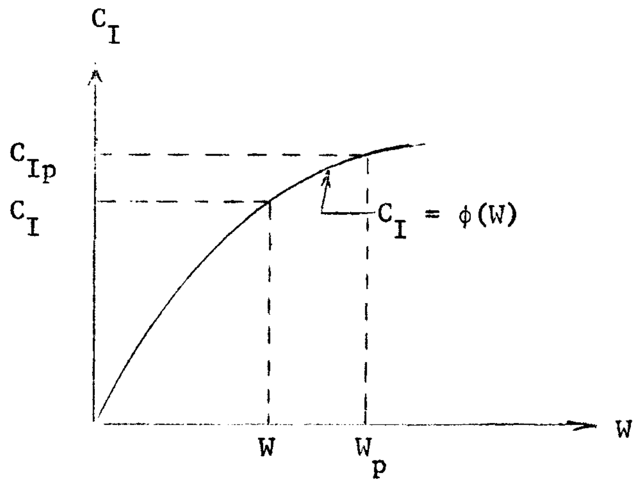


FIGURE 4

can be shown by use of Equation 9 that

$$\frac{W}{W_p} = 1 - \frac{r}{100}$$

where W_p is the present sign's size and W is the reduced design size. Then,

$$W = W_p \left(1 - \frac{r}{100}\right)$$

C_I and C_{Ip} can now be expressed as,

$$C_I = \phi\left[W_p \left(1 - \frac{r}{100}\right)\right] \qquad (10)$$

and

$$C_{Ip} = \phi(W_p) \qquad (11)$$

Computation of Replacement Cost:

C_R and C_{Rp} are now defined as the replacement cost occurring over the useful life of the reduced design and the present design, respectively. Then,

$$C_R = \sum_{J=1}^{A} \frac{P(B)(C_I)}{(1+I)^J} \tag{12}$$

$$C_{Rp} = \sum_{J=1}^{A} \frac{P_p(B)\ (C_{Ip})}{(1+I)^J} \tag{13}$$

where $P = \frac{1}{R}$ = probability of the reduced wind velocity occurring in any one year,

and $P_p = \frac{1}{R}$ = probability of the present design wind velocity occurring in any one year.

If these values of P and P_p are submitted into Equations 12 and 13, respectively, and the equations are simplified, the following equations are determined.

$$C_R = \frac{B\ C_I}{R} \sum_{J=1}^{A} \frac{1}{(1+I)^J} \tag{14}$$

$$C_{Rp} = \frac{B\ C_{Ip}}{R_p} \sum_{J=1}^{A} \frac{1}{(1+I)^J} \tag{15}$$

<u>Computation of Maintenance Cost:</u>

C_{mp} is defined as the maintenance cost of the sign presently being used. As mentioned earlier, the assumption is made that the maintenance cost of the reduced design also equals C_{mp}. C_{mp} is then computed as,

$$C_{mp} = \sum_{J=1}^{A} \frac{K_p\ C_{Ip}}{(1+I)J} = K_p\ C_{Ip} \sum_{J=1}^{A} \frac{1}{(1+I)^J} \tag{16}$$

Computation of Salvage Value:

C_s and C_{sp} are defined, respectively, as the salvage values of the reduced design and the present design, at the end of their useful life. C_s and C_{sp} are then determined by the equations,

$$C_s = K_1 C_I \left[\frac{1}{(1 + I)^A} \right] \qquad (17)$$

and

$$C_{sp} = K_2 C_{Ip} \left[\frac{1}{(1 + I)^A} \right] \qquad (18)$$

Computation of Total Cost:

a. Reduced Design. Let C_t denote the total cost of the reduced design occurring over its useful life. Then,

$$C_t = C_I + C_R + C_{mp} - C_s \qquad (19)$$

Note: The cost involved in salvaging the sign may exceed its salvage value in which case C_s would be negative, which would then result in an additional cost.

If values from Equations 10, 14, 16 and 17 are substituted for their equivalent in Equation 19,

$$C_t = \phi \left[W_p \left(1 - \frac{r}{100}\right) \right] \left[1 + \frac{B}{R} (\Sigma) - \frac{K_1}{(1+I)^A} \right] + K_p C_{Ip} (\Sigma)$$

If the relation for C_{Ip} from Equation 11 is substituted into the above equation, the following relation is obtained.

$$C_t = \phi \left[W_p \left(1 - \frac{r}{100}\right) \right] \left[1 + \frac{B}{R} (\Sigma) - \frac{K_1}{(1+I)^A} \right]$$

$$+ \phi \, (W_p) \, K_p \, (\Sigma) \qquad (20)$$

where

$$\Sigma = \sum_{J=1}^{A} \frac{1}{(1 + I)^J} = \frac{(1 + I)^A - 1}{I(1 + I)^A}$$

b. Present Design. Let C_{tp} denote the total cost of the present design occurring over its useful life. Then,

$$C_{tp} = C_{Ip} + C_{Rp} + C_{mp} - C_{sp} \qquad (21)$$

If values from Equations 11, 15, 16 and 18 are substituted for the equivalent values in Equation 21,

$$C_{tp} = \phi \, (W_p) \left[1 + \frac{B}{R_p} (\Sigma) + K_p \, (\Sigma) - \frac{K_2}{(1 + I)^A} \right] \qquad (22)$$

A ratio of reduced design to present design cost can now be obtained from Equations 20 and 22. If Z denotes this ratio,

$$Z = \frac{C_t}{C_{tp}} \qquad (23)$$

The values of C_t and C_{tp} from Equations 20 and 22, substituted into Equation 23 yeilds

$$Z = \frac{\phi[W_p\,(1 - \frac{r}{100})]\ [1 + \frac{B}{R}\,(\Sigma) - \frac{K_1}{(1 + I)^A}] + \phi(W_p)\ K_p\ (\Sigma)}{\phi(W_p)\ [1 + \frac{B}{R_p}\,(\Sigma) + K_p\ (\Sigma) - \frac{K_2}{(1 + I)^A}\ ]} \qquad (24)$$

Thus, for a given cost function ϕ and values of A, B, I, K_p, K_1, K_2, W_p, f, G, and R_p, the ratio Z can be found as a function of the recurrence interval R of the design wind velocity.

If ϕ was of the form $\phi = KW^m$, Equation 24 could be altered as follows. The term

$$\frac{\phi[W_p\ (1 - \frac{r}{100})]}{\phi(W_p)}$$

would simplify to

$$\frac{K(W_p)^n\ (1 - \frac{r}{100})^n}{K(W_p)^n} = (1 - \frac{r}{100})^n$$

Substituting the above relationship into Equation 24 yeilds:

$$Z = \frac{(1 - \frac{r}{100})^n\ [1 + \frac{B}{R}\ (\Sigma) - \frac{K_1}{(1 + I)^A}\ ] + K_p\ (\Sigma)}{1 + \frac{B}{R_p}\ (\Sigma) + K_p\ (\Sigma) - \frac{K_1}{(1 + I)^A}} \qquad (25)$$

If the value of r from Equation 9 is substituted into Equation 25, the following relation is obtained.

$$Z = \frac{\left(\frac{\ln \frac{R-1}{R}}{\ln \frac{R_p-1}{R_p}}\right)^{-2fn/G} \left[1 + \frac{B}{R}\,(\Sigma) - \frac{K_1}{(1 + I)^A}\right] + K_p\ (\Sigma)}{1 + \frac{B}{R_p}\ (\Sigma) + K_p\ (\Sigma) - \frac{K_2}{(1 + I)^A}} \qquad (26)$$

Thus, Equation 26 is a special form of Equation 24 when ϕ is of the form KW^n.

If the percent change in cost is wanted, the following relationship is used:

$$Z_R = (1 - Z)\ 100\% \qquad (27)$$

where Z is obtained from either Equation 24 or 26. It should be noted that a positive Z_R indicates the reduced design cost is less than the present design, and vice-versa.

Next, some examples will be considered. Assume the following values are given:

$V_1/V_2 = 0.60$ (G = 6.92, from Equation for G on page).

$f = 0.667$

$R_p = 50$ years

$B = 0.25$

$n = 0.50$ (i.e., the initial cost varies as the square root of the size).

$K_1 = K_2 = 0$

$K_p = 0.025$

$I = 0.0$

The curves of Figure 5 represent a plot of Equation 27 (using Z from Equation 26) for the given values of the parameters and for values of A equal to 5, 10, 20 and 50 years. The portions of the curves above the zero axis represent the percent savings possible in the present design cost as the design life is reduced and the portions below it represent a percent increase.

The curves of Figures 6, 7 and 8 are similar to those in Figure 5 with the following exceptions:

Figure 6 $I = 0.05$
Figure 7 $B = 0.50$
Figure 8 $B = 0.50$
$I = 0.05$

Note: "Design Life" as used in the plots and "recurrence interval of design wind velocity" are one and the same.

FIGURE 5. PERCENT CHANGE IN A 50 YEAR DESIGN COST VERSUS A REDUCED DESIGN LIFE FOR THE GIVEN PARAMETERS.

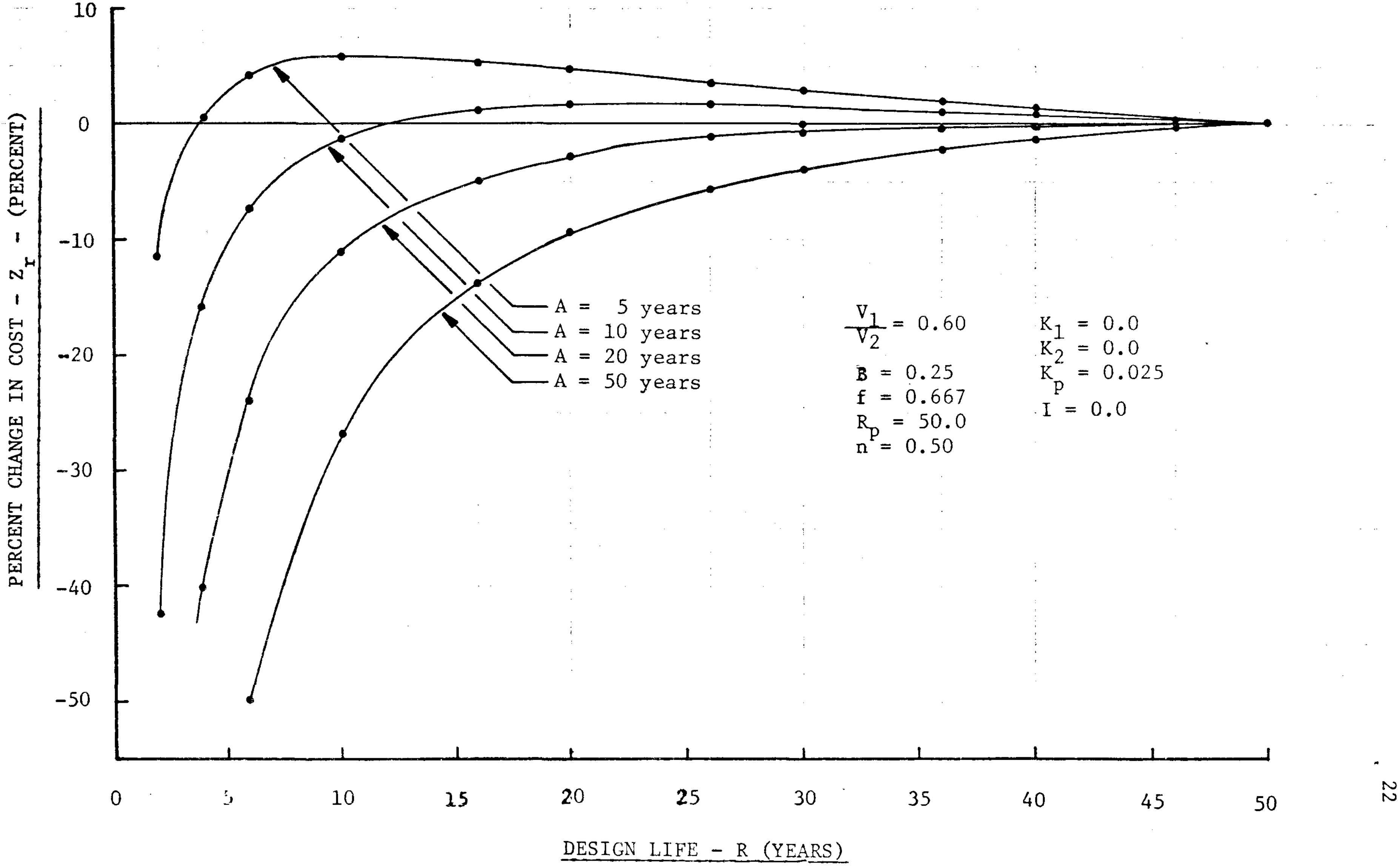


FIGURE 6. PERCENT CHANGE IN A 50 YEAR DESIGN COST VERSUS A REDUCED DESIGN LIFE FOR THE GIVEN PARAMETERS.

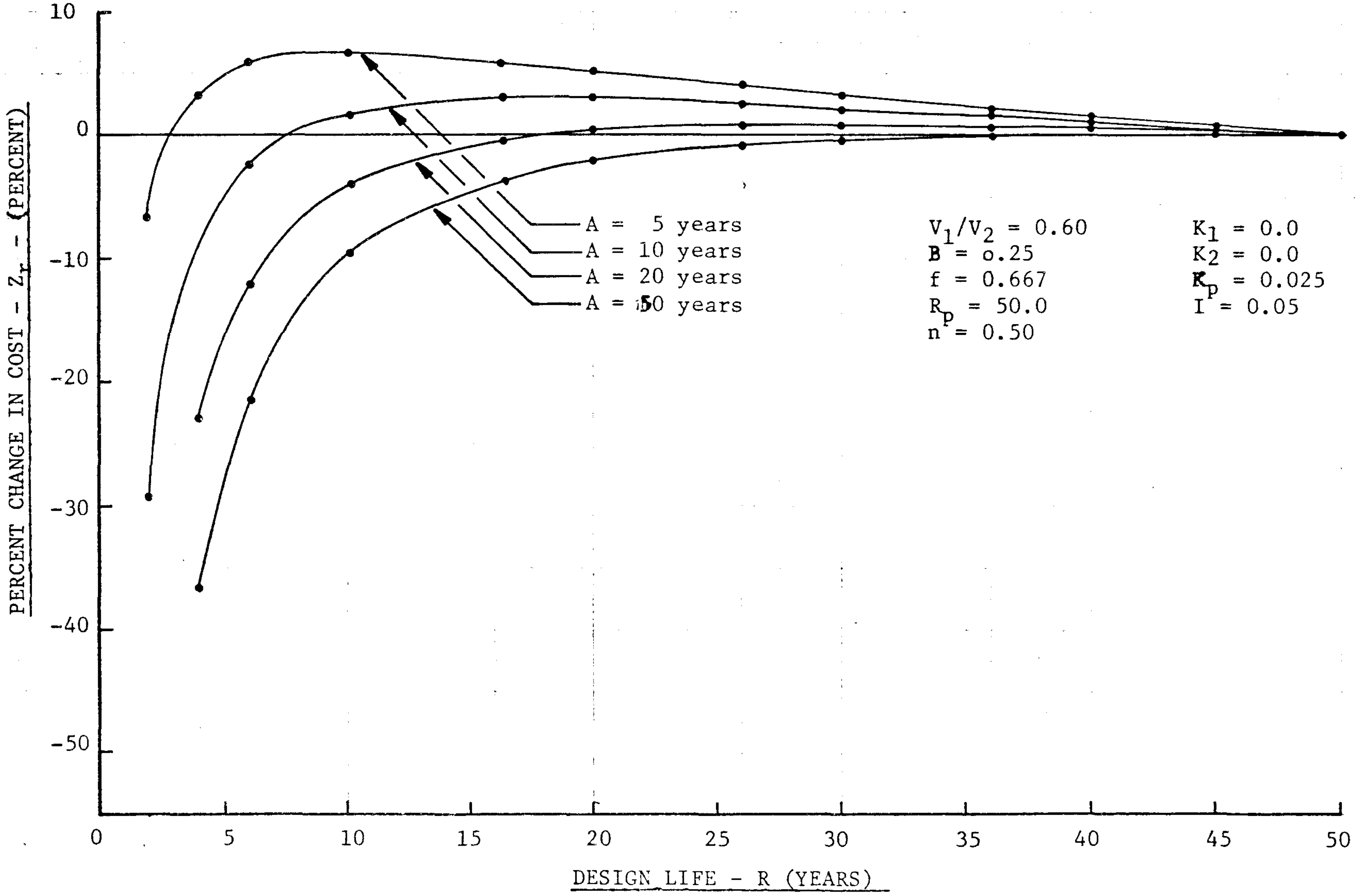


FIGURE 7. PERCENT CHANGE IN A 50 YEAR DESIGN COST VERSUS A REDUCED DESIGN LIFE FOR THE GIVEN PARAMETERS.

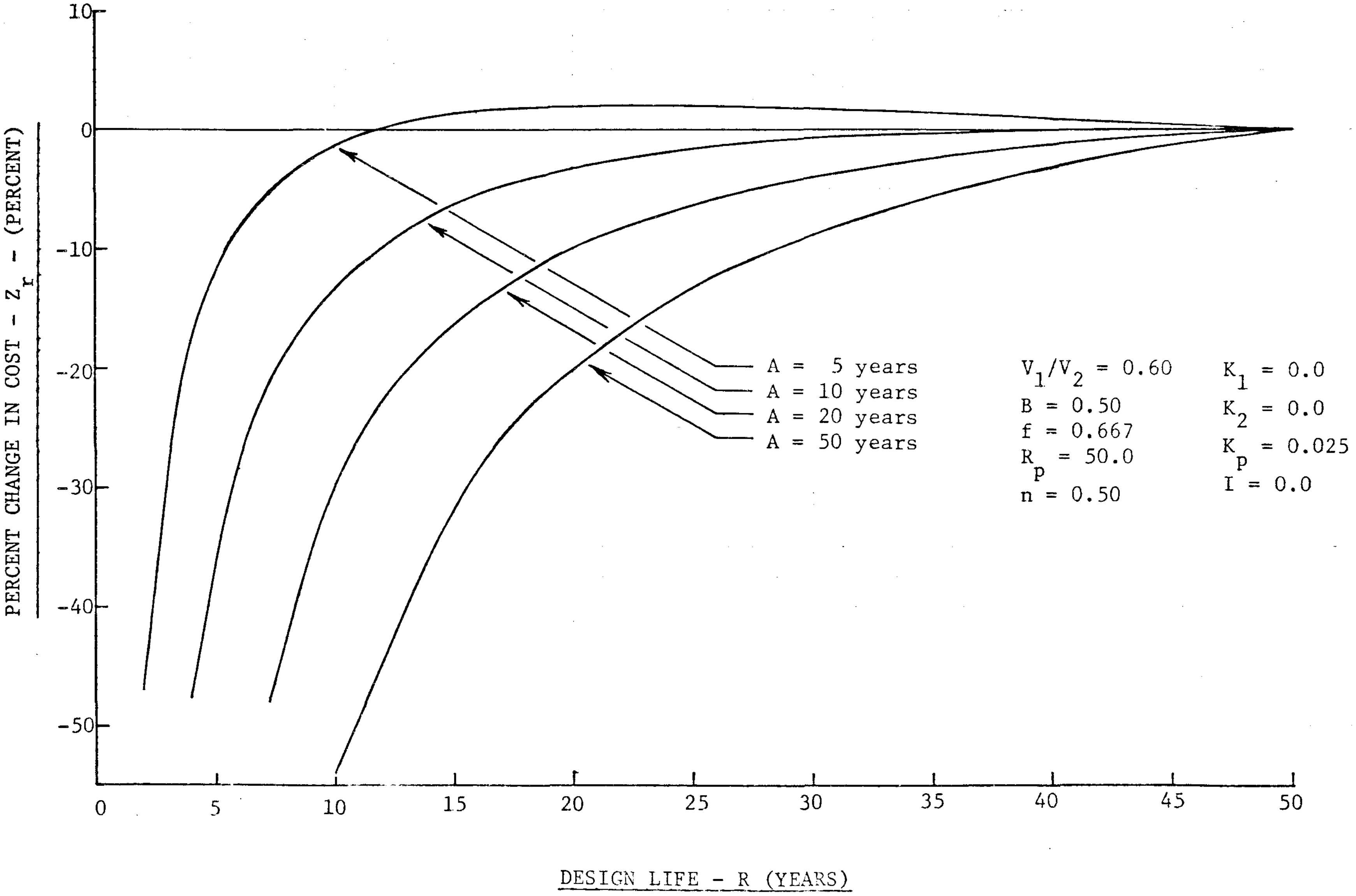
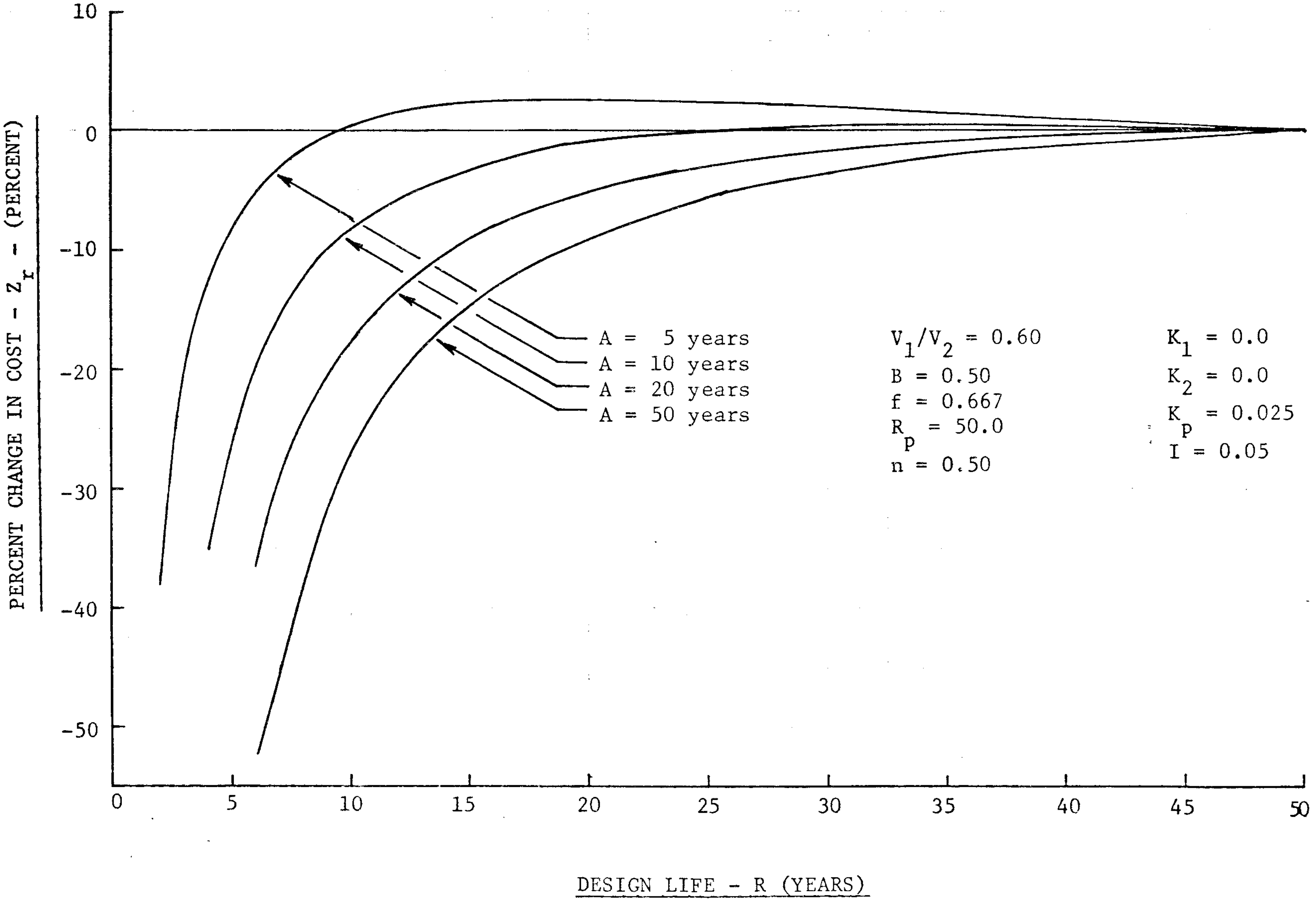


FIGURE 8. PERCENT CHANGE IN A 50 YEAR DESIGN COST VERSUS A REDUCED DESIGN LIFE FOR THE GIVEN PARAMETERS.



IV Discussion of Results

Two basic equations were derived in this study whereby the percent change in size (Equation 9) and cost (Equation 27) can be computed as a function of the design life (which is equal to the recurrence interval of the design wind velocity), provided certain parameters are available and provided the previously listed assumptions are accepted. Examples of each equation were plotted for selected parameters. The parameters used in the examples were chosen for specific reasons:

1. The 50 year value for the present design life R_p was selected because the design windspeed for signs, as specified by AASHO, has a recurrence interval of 50 years.
2. The factor $f = 0.667$ was used because most highway signs resist the wind loads by bending in the supports, i.e., the signs are of the cantilever type.
3. A ratio of V_1/V_2 equal to 0.6 was selected because this is representative of a considerable area of the United States (see Figures 2 and 3 of reference 2).
4. A non-linear relationship between initial cost and size of the form $\phi = KW^{0.5}$ was used since it is believed that this type of relationship will approximate the actual cost function of most highway signs, especially the larger types of roadside signs. For example, if the mass of reduced design is 80% of the present design mass, the percent reduction in initial cost would be computed as follows:

$\phi_1 = KW_p^{0.5}$ = initial cost of present sign

$$\phi_2 = K(.8W_p)^{.5} = \text{initial cost of reduced sign}$$

$$\text{Therefore \% reduction in cost} = \frac{KW_p^{.5} - K(.8W_p)^{.5}}{KW_p^{.5}} \times 100\%$$

$$= [1 - (.8)^{.5}] \times 100\%$$

$$= 10.5\%$$

5. A replacement factor, B, equal to 0.25 was used in the belief that it is a reasonable value for the ratio of cost to replace a blown-down sign to initial cost of the sign. The curves of Figures 7 and 8, based on B = 0.50, were included for comparative purposes.
6. The particular values of K_1, K_2 and K_p were chosen, again, because it is felt that these values are representative of most signs.

The curves of Figure 3 show the percent change in size for four different ratios of V_1/V_2 as the design life is reduced. As is evident, the smaller the ratio of V_1/V_2 the larger the reduction in size. Areas along the coastline, where tropical storms are expected to occur on the average of once in a 40 or 50 year period, have the lower V_1/V_2 ratios and could therefore realize substantial reductions in sign size by reducing the 50-year design life to a lower value. In areas where the average yearly wind speed is not much less than the 50-year wind, the reduction in size would not be as great.

From the curves of Figures 5, 6, 7 and 8 it can be seen, for certain values of the parameters, that reductions in cost can be realized as the design life is reduced. For example, from Figure 5, with the useful life "A" equal to 5 years, a 5.6% reduction in cost can be realized if the 50 year design life is reduced to 12-years, provided the parameters used are acceptable. This 12 year design life would be optimum from an economic standpoint, i.e., this is the point of maximum savings. For a 12 year design life, the corresponding reduction in size would be 24.5% (from Figure 3 with $V_1/V_2 = 0.60$). If safety is of primary concern, the same sign could be designed for a 3 3/4 year life (from Figure 5), with no change in the 50 year design cost, but with a 41% reduction in size (from Figure 3). The choice would be up to the agency concerned.

Figures 5, 6, 7 and 8 also show that for certain values of the parameters, the optimum design life is actually the present design life, i.e., 50 years. In fact, for some cases, the optimum design life from an economic standpoint may go beyond the 50 year value. This possibility was not investigated.

Figure 6 is similar to Figure 5 except the interest rate "I" equals 5% instead of 0%. A comparison of Figures 5 and 6 shows the effect of discounting to the present, at the rate of 5%, all costs which will occur in the future.

Table I summarizes the results of Figures 5 and 6 and in addition includes a summary for interest rates of 1.0, 3.0 and 7.0%.

The curves of Figures 7 and 8 were included for comparative purposes. These curves represent data similar to Figures 5 and 6 except the replacement factor B was set equal to 0.50. The effect of increasing B from 0.25 to 0.50 is quite pronounced. It is therefore important, when economizing, to keep the "blow down" replacement cost small. For more on this matter, see Assumption 4 on page 4.

A computer program was written to use in evaluating Equations 9 and 27. This program and the instructions for its use are included in the Appendix of this report.

TABLE I SUMMARY OF EXAMPLE RESULTS

The values shown are all based on the following conditions:

$V_1/V_2 = 0.60$ $n = 0.50$ $R_p = 50$

$B = 0.25$ $K_1 = K_2 = 0$

$f = 0.667$ $K_p = 0.025$

INTEREST RATE (%)	USEFUL LIFE (YEARS)	OPTIMUM DESIGN					
		ECONOMIC			SAFETY*		
		DESIGN LIFE (YEARS)	COST REDUCTION (%)	SIZE REDUCTION** (%)	DESIGN LIFE (YEARS)	COST REDUCTION (%)	SIZE REDUCTION** (%)
0	5	12	5.7	24.5	3.75	0	41
	10	22	1.7	14.8	12.0	0	24.5
	20	46	0.02	1.6	43	0	3
	50	50	0	0	50	0	0
1.0	5	10	5.9	27.2	3.50	0	41.8
	10	22	2.0	14.8	10.75	0	26.2
	20	42	0.09	3.3	35	0	6.8
	50	50	0	0	50	0	0
3.0	5	10	6.3	27.2	3.25	0	42.8
	10	20	2.5	16.4	9.5	0	28.0
	20	34	0.4	7.2	24	0	13.4
	50	50	0	0	50	0	0
5.0	5	9	6.7	28.7	2.75	0	44.8
	10	18	3.0	18.1	7.75	0	30.9
	20	28	0.9	10.7	18	0	18.1
	50	42	0.08	3.3	36	0	6.2
7.0	5	9	7.1	28.7	2.50	0	46.2
	10	16	3.5	20.0	6.50	0	33.2
	20	24	1.5	13.4	14.0	0	22.1
	50	32	0.6	8.3	21.5	0	15

*Optimum design from safety standpoint is arbitrarily defined as the maximum reduction in mass with <u>no</u> increase in present cost.

**From Figure 3.

V. Conclusions

The value of the information contained in this study will depend, to a large degree, on the accuracy of values assigned the parameters in a particular situation, i.e., the accuracy of estimates for ϕ, B, I, etc., and the degree to which the assumptions that were made approximate the actual conditions. As mentioned previously, the particular parameters chosen to exemplify the derived equations were felt to represent an average situation. If so, it is evident that the safety and economic aspects of a sign can be improved by reducing the design life (recurrence interval of design wind velocity).

In any case, some general conclusions can be drawn from the results of this study which will be useful when considering the costs of a sign as a function of the sign's design life. It can be said, holding all other factors constant, that the sign's cost will decrease as: (1) the ratio of the 2 year wind to the 50 year wind decreases (the same conclusion can also be said of the size decrease); (2) the useful life A decreases; (3) the replacement cost decreases, i.e., as B is reduced; (4) the discount rate I increases; (5) the maintenance cost decreases, i.e., as K_p decreases; and (6) as the salvage value of the reduced design increases with respect to the salvage value of the present design, i.e., as K_1 increases with respect to K_2.

The report has covered the effects of reducing design life from present standards. It may be that in some cases, sign cost could be reduced by increasing design life. However, if design life increases are contemplated, consideration should be given to the effect of such increases on safety.

An example problem is included in the Appendix (page 40) to show how the information contained in this report could be applied to a given situation.

VI Recommendations for Further Research

Several parameters were considered in this study, e.g., cost function ϕ, replacement factor B, and useful life A. It is likely that additional research will be necessary to properly define these parameters.

In this report no consideration was given to those costs incurred by the motorist for the time the sign is down from wind damage or his cost when colliding with the sign. These costs should be considered in future studies. Consideration should also be given to the highway department's costs of repairing signs damaged by auto collisions. As the sign's mass is reduced, damage to the colliding auto will likely decrease, but the cost to repair the sign may increase.

VII References

1. "Specifications for the Design and Construction of Structural Supports for Highway Signs," American Association of State Highway Officials, 917 National Press Building, Washington, D. C., 1961.

2. Thom, H. C. S., "Distributions of Extreme Winds in the United States," Transactions of the American Society of Civil Engineers, Vol. 126, Paper No. 3191, Page 450.

APPENDIX A

Computer Program

This program was written to evaluate Equations 9 and 27 for given values of the various parameters. The flow chart appears on page 37, and the Fortran IV listing appears on page 38.

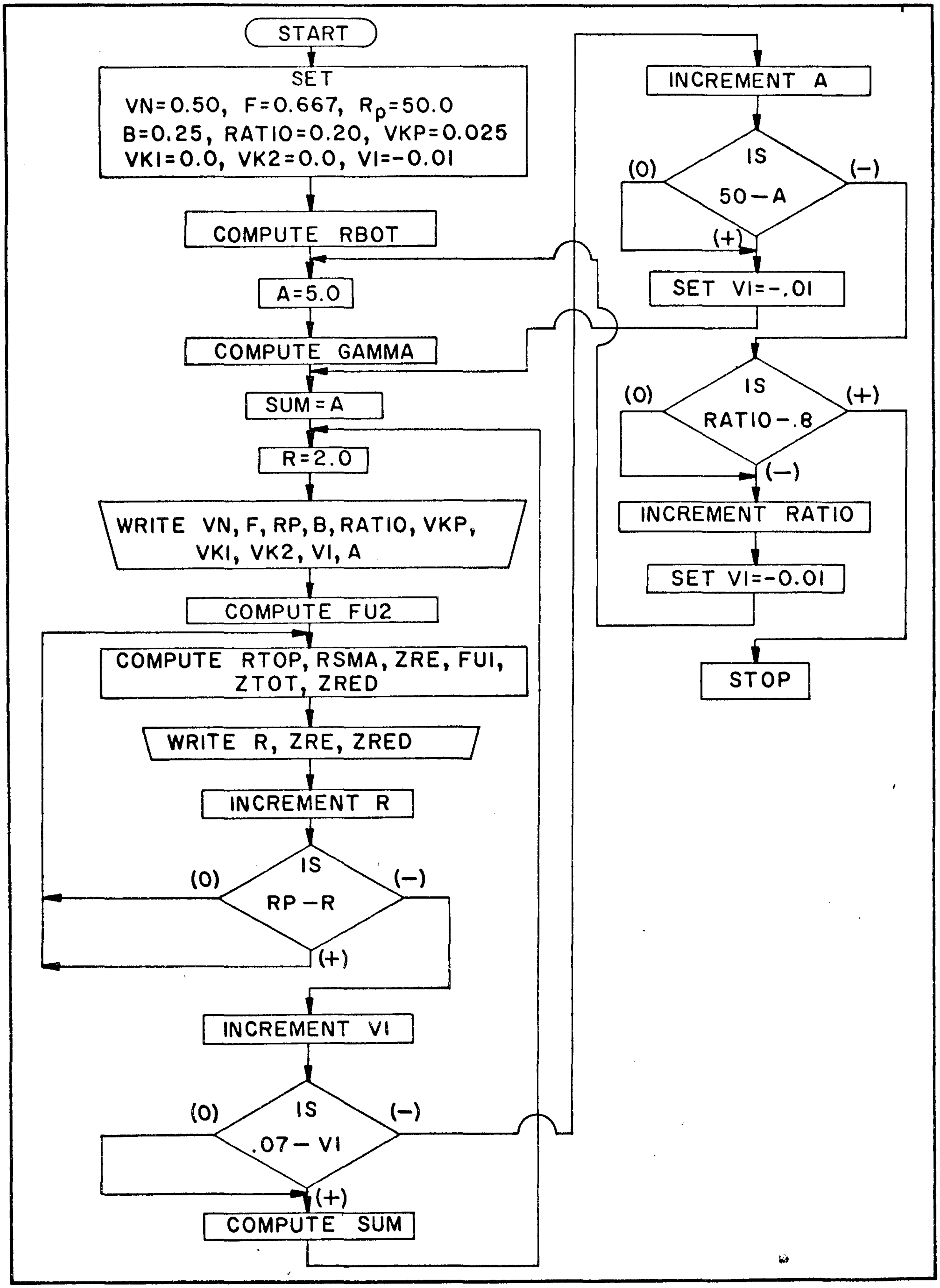
The computer notations are related to the symbols used in the study as follows:

Computer Notation	Study Notation
VN	n
F	f
RP	R_p
B	B
RATIO	V_1/V_2
VKP	K_p
VK1	K_1
VK2	K_2
VI	I
A	A
GAMMA	G
R	R
ZRE	r
ZRED	Z_R

The program, as shown, is set up to furnish the data contained in Figures 3, 5 and 6 and similar data for values of "VI" equal to 1.0%, 3.0% and 7.0%. However, the program can easily be altered for any given situation.

A page of output from this particular program is also included on page 39. Note in the first group of numbers of the print-out, the value of "VI" appears as -0.01. This is not to say that the interest rate was a negative 1.0%. The -0.01 is used for the purposes of incrementing the interest rate in the program and where -0.01 appears in the output the true value is actually zero.

FLOW DIAGRAM



```
$IBFTC MAIN
      WRITE(6,52)
      VN = 0.50
      F = 0.66667
      RP = 50.0
      B=0.25
      RATIO = 0.20
      VKP=0.025
      VK1 = 0.0
      VK2 = 0.0
      VI = -0.01
      RBOT = -ALOG((RP-1.)/RP)
   10 A = 5.0
      GAMMA = -3.54/ALOG(RATIO)
    1 SUM = A
    2 R = 2.0
      WRITE(6,50)VN,F,RP,B,RATIO,VKP,VK1,VK2,A,VI
      FU2 = 1. + (1./RP)*B*SUM + VKP*SUM - VK2*(1./((1.+VI)**A))
    3 RTOP = -ALOG((R-1.)/R)
      RSMA = (RTOP/RBOT)**(-2.*F/GAMMA)
      ZRE=100.*(1.-RSMA)
      FU1 = 1. + (1./R)*B*SUM + (1./((RSMA)**VN))*VKP*SUM - VK1*(1./(((1.
     1+ VI)**A))
      ZTOT = 100.*((RSMA)**VN)*(FU1/FU2)
      ZRED = (100. - ZTOT)
      WRITE(6,51)R,ZRE,ZRED
      R = R + 2.
      IF(RP - R)4,3,3
    4 VI = VI + 0.02
      IF(.07 - VI)6,5,5
    5 SUM = (((1.+VI)**A)-1.)/(VI*(1.+VI)**A)
      GO TO 2
    6 A = A + 5.0
      IF(50.-A)8,7,7
    7 VI = -0.01
      GO TO 1
    8 IF(RATIO-.8)9,9,11
    9 RATIO=RATIO+0.2
      VI=-0.01
      GO TO 10
   50 FORMAT(//20X,3HVN=,F4.1,3X,2HF=,F4.2,3X,3HRP=,F5.1,3X,2HB=,F4.2,
     13X,6HRATIO=,F3.1,3X,4HVKP=,F6.3/20X,4HVK1=,F4.1,3X,4HVK2=,F4.1,3X,
     22HA=,F5.1,3X,3HVI=,F5.2,//)
   51 FORMAT(10X,2HR=,F7.3,15X,5HZRE =,F9.3,15X,5HZRED=,F9.3)
   52 FORMAT(1H1)
   11 STOP
      END
```

```
               VN= 0.5    F=0.67   RP= 50.0    B=0.25   RATIO=0.2    VKP= 0.025
               VK1= 0.     VK2= 0.     A=  5.0    VI=-0.01

R=  2.000                    ZRE =    88.272                  ZRED=   40.738
R=  4.000                    ZRE =    80.013                  ZRED=   38.106
R=  6.000                    ZRE =    73.647                  ZRED=   35.192
R=  8.000                    ZRE =    68.172                  ZRED=   32.407
R= 10.000                    ZRE =    63.255                  ZRED=   29.831
R= 12.000                    ZRE =    58.736                  ZRED=   27.454
R= 14.000                    ZRE =    54.519                  ZRED=   25.251
R= 16.000                    ZRE =    50.543                  ZRED=   23.200
R= 18.000                    ZRE =    46.765                  ZRED=   21.279
R= 20.000                    ZRE =    43.154                  ZRED=   19.471
R= 22.000                    ZRE =    39.686                  ZRED=   17.761
R= 24.000                    ZRE =    36.343                  ZRED=   16.139
R= 26.000                    ZRE =    33.111                  ZRED=   14.594
R= 28.000                    ZRE =    29.978                  ZRED=   13.117
R= 30.000                    ZRE =    26.933                  ZRED=   11.704
R= 32.000                    ZRE =    23.968                  ZRED=   10.346
R= 34.000                    ZRE =    21.077                  ZRED=    9.039
R= 36.000                    ZRE =    18.253                  ZRED=    7.780
R= 38.000                    ZRE =    15.491                  ZRED=    6.563
R= 40.000                    ZRE =    12.787                  ZRED=    5.386
R= 42.000                    ZRE =    10.136                  ZRED=    4.245
R= 44.000                    ZRE =     7.534                  ZRED=    3.138
R= 46.000                    ZRE =     4.980                  ZRED=    2.063
R= 48.000                    ZRE =     2.469                  ZRED=    1.018
R= 50.000                    ZRE =     0.                     ZRED=    0.


               VN= 0.5    F=0.67   RP= 50.0    B=0.25   RATIO=0.2    VKP= 0.025
               VK1= 0.     VK2= 0.     A=  5.0    VI= 0.01

R=  2.000                    ZRE =    88.272                  ZRED=   41.378
R=  4.000                    ZRE =    80.013                  ZRED=   38.546
R=  6.000                    ZRE =    73.647                  ZRED=   35.536
R=  8.000                    ZRE =    68.172                  ZRED=   32.693
R= 10.000                    ZRE =    63.255                  ZRED=   30.075
R= 12.000                    ZRE =    58.736                  ZRED=   27.666
R= 14.000                    ZRE =    54.519                  ZRED=   25.438
R= 16.000                    ZRE =    50.543                  ZRED=   23.366
R= 18.000                    ZRE =    46.765                  ZRED=   21.426
R= 20.000                    ZRE =    43.154                  ZRED=   19.602
R= 22.000                    ZRE =    39.686                  ZRED=   17.878
R= 24.000                    ZRE =    36.343                  ZRED=   16.243
R= 26.000                    ZRE =    33.111                  ZRED=   14.686
R= 28.000                    ZRE =    29.978                  ZRED=   13.199
R= 30.000                    ZRE =    26.933                  ZRED=   11.776
R= 32.000                    ZRE =    23.968                  ZRED=   10.409
R= 34.000                    ZRE =    21.077                  ZRED=    9.094
R= 36.000                    ZRE =    18.253                  ZRED=    7.826
R= 38.000                    ZRE =    15.491                  ZRED=    6.602
R= 40.000                    ZRE =    12.787                  ZRED=    5.417
R= 42.000                    ZRE =    10.136                  ZRED=    4.270
R= 44.000                    ZRE =     7.534                  ZRED=    3.156
R= 46.000                    ZRE =     4.980                  ZRED=    2.075
R= 48.000                    ZRE =     2.469                  ZRED=    1.024
```

APPENDIX B

Example Problem:

Given:

An 8' x 16' frontal area sign, as shown in Figure 9. The sign is located in southeast Texas where the mean **2-year** extreme wind velocity is 50 miles per hour, (from Reference 2, Figure 2), and the mean 50-year extreme wind velocity is 90 miles per hour (from Reference 2, Figure 3, or Reference 1, Figure 4).

Thus

$$V_1/V_2 = 50/90 = 0.556$$

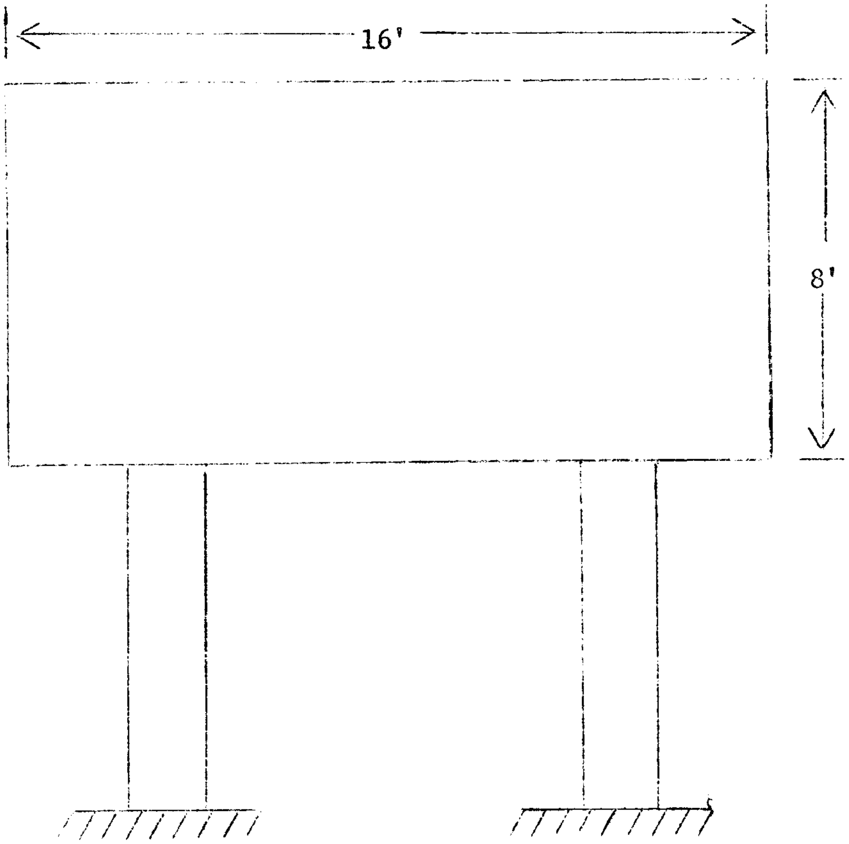


FIGURE 9

The present design life, R_p, is 50 years as specified by AASHO.

Assumptions:

The following assumptions are made on this problem and are based on "best estimates".

1. Initial cost of the present sign C_I including materials, fabrication and installation, is $10.00 per square foot of sign frontal area.
2. The useful life, A, of the sign is lo years, i.e., within 10 years after installation, the entire sign structure will have to be either removed or replaced.
3. The replacement factor, B, is 0.25, i.e., the cost to replace or repair the sign, if blown down, is 25% of its initial cost. This factor is assumed to be applicable in both the present design as well as the reduced design.
4. The cost function, ϕ, which relates the initial cost of the sign to its size (size as defined on page 1), is equal to $KW^{0.5}$.
5. The interest rate, I, used to discount to the present all cost occurring in future periods is 5%.
6. The maintenance factor, K_p, is 0.025, i.e., the yearly maintenance cost of the sign, whether the present size or the reduced size, is equal to 2.5% of the present sign's initial cost. Maintenance cost includes those due to painting and mowing around the sign, repairs to the sign background, etc.

7. The salvage factor of the reduced size sign, K_1, and the present size K_2, is zero, i.e., at the end of the useful life of either type signs, the salvage value is zero.

Required:

Determine an optimum design for this sign based on economic considerations.

Solution:

The values of Table I, (page 30) can be used to obtain the optimum values since they are based on the same conditions that exist in this example. For I = 5% and A = 10 years, the optimum design life (R), from an economic standpoint, is 18 years. The amount of reduction in present cost (Z_R) is 3%, and the reduction in size or mass of the sign structure (r) is 18.1%. In order to realize these reductions in present sign's size and cost, it will be necessary to reduce the size of the members in the sign, whose cross-sectional properties depend upon the magnitude of wind loading, by 18.1%. This will not always be possible with available commercial sizes. However, if consideration is given to the many different sign configurations that now exist, it is believed that, on the average, the results of the study can be used in practical applications. In those cases where the available member sizes do not meet the requirements of the situation, it may be economically feasible to consider an alternate approach. For example, it may be feasible to make a special order for the required size, or consider different type members (e.g., channel section as opposed to a zee section).

It will now be shown how the 3% value for Z_R was determined.

Computation of Initial Cost:

(a) Present Design

The initial cost, C_{Ip}, of the present design is

$$C_{Ip} = (\$10.00) \text{ X } (\text{frontal area of sign})$$

$$C_{Ip} = (10)\ (16)\ (8) = \$1280.00$$

Since the cost function ϕ is of the form $KW^{0.5}$, a relation between C_{Ip} and W_p exists, i.e.,

$$C_{Ip} = KW_p^{0.5}$$

Since C_{Ip} and W_p are known, the value of K can be found.

$$K = \frac{C_{Ip}}{W_p^{0.5}} = \frac{1280.00}{(W_p)^{.5}}$$

(b) Reduced Design

The initial cost, C_I, of the reduced design is

$$C_I = KW^{0.5}$$

The reduced size W is found by,

$$W = (1. - .181)\ (W_p) = .819\ W_p$$

Therefore

$$C_I = \left(\frac{1280.}{W_p^{.05}}\right) (.819\ W_o)^{.5}$$

$$C_I = \$1157.00$$

Computation of Replaced Cost:

The replacement cost of both the present design, C_{Rp}, and the reduced design, C_R, over the useful life A of the sign is

$$C_{Rp} = \frac{BC_{Ip}}{R_p} \sum_{J=1}^{A} \frac{1}{(1+I)^J} = \frac{(.25)\ (1280)}{50} \left[\frac{(1. + .05)^{10} - 1}{.05(1 + .05)^{10}} \right]$$

C_{Rp} = \$49.47

$$C_R = \frac{BC_I}{R} \sum_{J=1}^{A} \frac{1}{(1+I)^J} = \frac{(.25)\ (1157)}{18} \left[\frac{(1. + .05)^{10} - 1}{.05(1 + .05)^{10}} \right]$$

C_R = \$124.22

Computation of Maintenance Cost:

The maintenance cost, C_{mp}, for both the present design and the reduced design, over the useful life of the sign, is computed by,

$$C_{mp} = K_p\ C_{Ip} \sum_{J=1}^{A} \frac{1}{(1+I)^J} = (.025)\ (1280) \left[\frac{(1. + .05)^{10} - 1}{.05(1 + .05)^{10}} \right]$$

C_{mp} = \$247.36

Computation of Salvage Value:

The salvage value of both type signs at the end of their useful life was assumed to be zero.

Computation of Total Cost:

For the present design, the total cost C_{tp} over the useful life of the sign would be,

$$C_{tp} = C_{Ip} + C_{Rp} + C_{mp} - C_{Sp}$$

$$C_{tp} = \$1280.0 + 49.47 + 247.36 - 0.00$$

$$C_{tp} = \$1576.83$$

For the reduced design, the total cost C_t on the useful life of the sign would be,

$$C_t = C_I + C_R + C_{mp} - C_S$$

$$C_t = \$1157.00 + 124.22 + 247.36 - 0.00$$

$$C_t = \$1528.58$$

The cost reduction, Z_R, is determined by

$$Z_R = \frac{C_{tp} - C_t}{C_{tp}} \text{ X } 100\% = \frac{1576.83 - 1528.58}{1576.83} \text{ X } 100\%$$

$$Z_R = 3.0\%$$

In this example, the amount of reduction in cost, over the useful life of the two type signs, was not too great ($1528.58, as opposed to the present cost of $1576.83). However, the reduction in size, or mass, of the sign adds considerably to the safety aspects of the sign. If safety was of more concern than economy, the mass of this sign could be reduced by 30.9% with no change in the present cost (see Table I).