LITERATURE REVIEW ON TIMING PARAMETERS FOR HYDROGRAPHS

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16. Abstract
This report summarizes the literature review for the TxDOT project 0-4696 "Estimating Timing Parameters of Direct Runoff and Unit Hydrograph for Texas Watersheds." Chapter 1 summarizes background information of the NRCS (Natural Resources Conservation Service) dimensionless unit hydrograph, which TxDOT is currently using for its hydrologic design and includes general information about streamflow hydrographs and unit hydrograph. Chapter 2 summarizes various common definitions for the timing parameter of direct runoff and unit hydrograph. Chapter 3 summarizes methods to quantify timing parameters: (1) NRCS velocity method, (2) particle tracking method, and (3) empirical equations developed from earlier studies for several common timing parameters. Chapter 4 summarizes definitions for watershed parameters because many empirical equations are based on correlation between timing parameters and watershed parameters. Chapter 5 summarizes conclusions made based upon literature review findings.

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CHAPTER 1. INTRODUCTION

A literature review for the Texas Department of Transportation (TxDOT) project 0-4696 "Estimating Timing Parameters of Direct Runoff and Unit Hydrograph for Texas Watersheds" is provided in this report. Chapter 1 summarizes the Natural Resources Conservation Service (NRCS) dimensionless unit hydrograph, which TxDOT currently uses for hydrologic design. The chapter also includes general information about streamflow hydrographs and unit hydrographs to provide further context. Chapter 2 summarizes various common definitions for the timing parameters of direct runoff and unit hydrographs. Chapter 3 summarizes methods to quantify timing parameters: (1) NRCS velocity method, (2) particle tracking method, and (3) empirical equations developed from earlier studies for several common timing parameters. Chapter 4 summarizes definitions for watershed parameters because many empirical equations are based on correlation between timing parameters and watershed parameters. Chapter 5 summarizes conclusions made based on literature review findings. References reviewed are listed at the end of this report.

(1.1) NRCS Dimensionless Unit Hydrograph

TxDOT applies rainfall-runoff models for two primary reasons. The first use is to evaluate specific changes or controls within a watershed. The second use is to assess statistical approaches when gage records appear inadequate for a site. TxDOT application of rainfall-runoff modeling is typically used in projects for drainage basins of 10 square miles or less or a time of concentration (T_c) of less than 6 hours (although occasionally basins as large as 100 square miles might be considered) or both.

TxDOT currently uses the NRCS dimensionless unit hydrograph (DUH) procedure for design applications. This DUH developed by Victor Mockus (NRCS, 1972) was derived from a large number of unit hydrographs from watersheds varying widely in size and geographical locations and is shown in Figure 1.1. This unit hydrograph has a point of inflection approximately 1.7 times the time to peak (T_p) and a time to peak approximately 0.2 times the time base (T_b). T_p is equal to the watershed lag time (T_L, defined as the time from the centroid of rainfall excess to the peak discharge of hydrograph) plus one-half the rainfall excess duration or the duration of unit hydrograph (D in Fig. 1.1).

$$T_p = T_L + D/2 \tag{1.1}$$

This dimensionless curvilinear unit hydrograph has 37.5 percent of the total volume in the rising side, which is represented by one unit of time and one unit of discharge. This DUH also can be represented by an equivalent triangular hydrograph (Fig. 1.1). These characteristics of the NRCS unit hydrograph represent values that have been adopted for an "**average**" watershed. For NRCS DUH, the peak discharge (Q_p) is given as

$$Q_p = KA/T_p \tag{1.2}$$

where Q_p is in cubic feet per second, area (A) is in square miles, and T_p is in hours. K is the peak rate factor (PRF) and considered equal to 484 assuming a triangular hydrograph with a time base of 8/3 T_p (Fig. 1.1). K is related to the internal storage characteristic of a basin and can vary considerably depending on watershed characteristics and scale (size) of a basin. For example, K can range from a value of nearly 600 for steep mountainous conditions to a value nearly to 300 in the flat coastal plains (swampy country) of the state (NRCS, 1972). For a very flat, high-water-table watershed, the NRCS peak rate factor of 484 or even 300 likely is too large. The University of Florida (Capece et al., 1986) determined that a peak rate factor of 75–100 is appropriate for Flatwoods watersheds; however it can be as low as 50. Also for NRCS DUH, the time base of 8/3 T_p is based on empirical values for average rural watersheds and should be decreased for steep conditions (causing increased peak flow) or increased for flat conditions (causing decreased peak flow). In addition, the empirical relation for average lag time is assumed to be 0.6 T_c, where T_c is the time of concentration.

$$T_L = 0.6\, T_c \tag{1.3}$$

T_c is the time it takes a water parcel to travel from the hydraulically most distal part of the watershed to the outlet. In hydrograph analysis (Fig. 1.1), T_c is defined as the time difference from the end of excess rainfall to the inflection point of the unit hydrograph (T_{in}).

$$T_c = T_{in} - D = 1.7T_p - D \tag{1.4}$$

Using equations (1.1), (1.3) and (1.4), the duration of unit hydrograph is:

$$D \cong 0.133T_c \text{ and } D < 0.17T_c \text{ , or} \tag{1.5}$$

$$D \cong 0.2T_p \text{ and } D < 0.25T_p \tag{1.6}$$

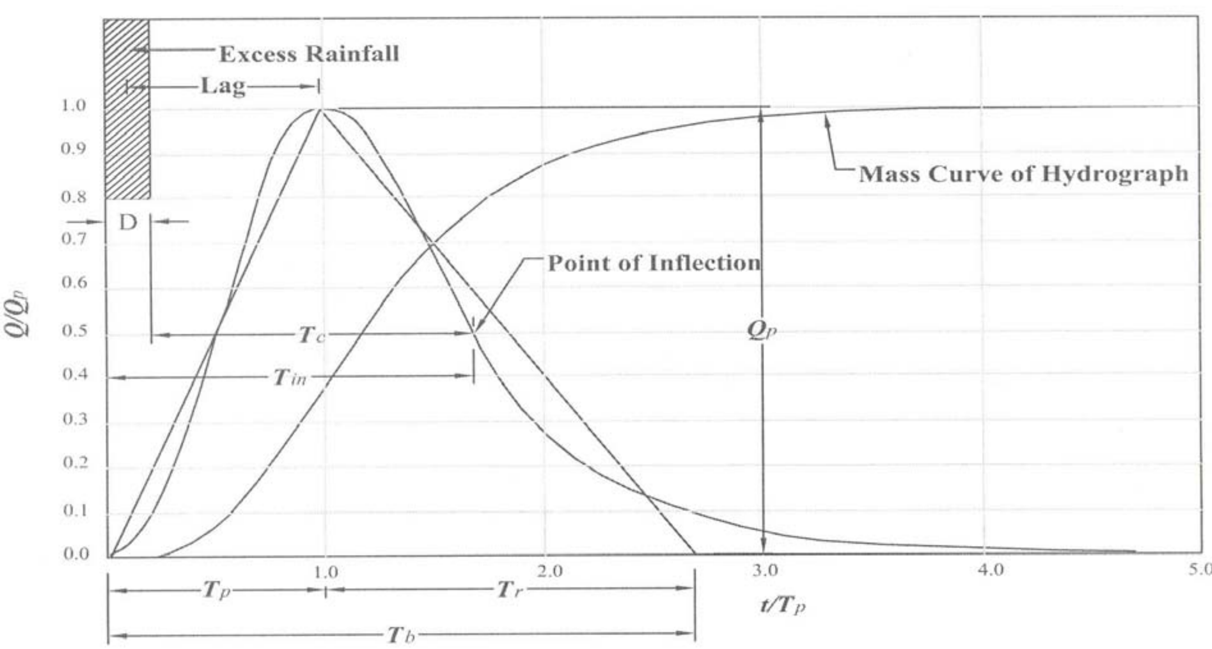

Figure 1.1. NRCS synthetic unit hydrographs including dimensionless unit hydrograph and triangular unit hydrograph.

Based on criteria in equation (1.6), the duration D is typically selected as approximately equal to the rainfall data interval (for example, use 10 minutes as D for T_c = 1.2 hour – 0.133D = 9.6 minutes). Viessman and Lewis (2002) provide two relations of the lag time to size of watershed for two geographic regions but did not provide reference so it is not known to the authors who developed the equations and what data the equations were based on. The equations are:

$$T_L = 1.44\, A^{0.6} \qquad \textit{Texas} \tag{1.7a}$$

$$T_L = 0.54\, A^{0.6} \qquad \textit{Ohio} \tag{1.7b}$$

where A is the watershed area in square miles and T_L is the lag time in hours. Therefore, general use of the NRCS procedure without consideration of actual regional or site characteristics can result in poor correlation with statistical expectation and inadequate design.

(1.2) Streamflow Hydrographs

A streamflow hydrograph is a graphical representation of instantaneous discharge at a given location with respect to time during and after a storm or snowmelt event. An example hydrograph with typical timing parameters and rainfall hyetograph is shown in Figure 1.2.

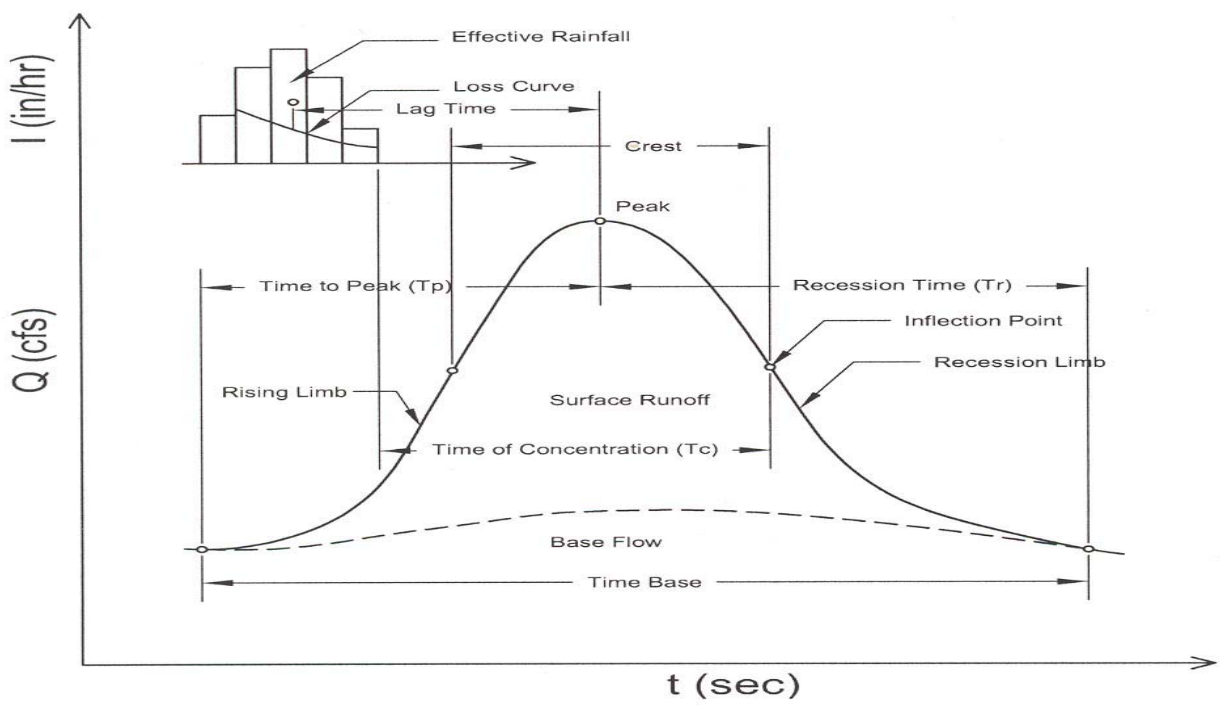

Figure 1.2. Schematic streamflow hydrograph including typical timing parameters and a rainfall hyetograph.

Some important components of a streamflow hydrograph are (1) rising limb, (2) crest segment, (3) recession limb, and (4) base flow. The rising limb of a hydrograph, also known as concentration curve, represents the increase in discharge because of the gradual buildup of storage in channels and over the catchment surface. The initial losses and high infiltration losses during the early period of a storm cause the discharge to rise slowly early in the event. As the storm continues, more and more flow from distant parts of a watershed reach the basin outlet. Simultaneously, the infiltration losses also decrease with time. Thus, under a uniform storm over the catchment, the runoff increases rapidly with time. The crest segment is one of the most important parts of a hydrograph; it contains the peak flow. Peak flow represents the highest concentration of runoff and usually occurs soon after the rainfall has ended.

The recession limb, which extends from the point of inflection at the end of the crest segment to the commencement of the natural groundwater flow, represents the withdrawal of water from the storage built up in the catchment during the earlier phases of the hydrograph. The starting point of the recession limb, i.e. the point of inflection, represents the condition of maximum storage. Because the depletion of storage takes place after the cessation of rainfall, the shape of this part of the hydrograph is much less dependent on storm characteristics and much more dependent on the basin characteristics.

Base flow is the part of flow in the channel that exists even before the occurrence of rainfall. The total streamflow hydrograph has two components: direct runoff hydrograph (DRH) and base flow. Effective rainfall or rainfall excess is the part of precipitation that remains after all the losses like interception, infiltration, evaporation, dead storage and others. The direct runoff hydrograph is the transformation of effective rainfall passing through a watershed. Base flow is the part of precipitation that percolates downward until it reaches the groundwater table and eventually discharges into the stream.

(1.3) Factors Affecting Hydrographs

Before hydrograph timing parameters are examined, it is informative to discuss the factors affecting hydrograph components such as shape and peak discharge. The factors that affect hydrograph shape can generally be grouped into climatic and physiographic factors. Each of the groups contains a host of factors, and the important ones are discussed below:

(a) Size of the basin: Small basins function differently from large basins in terms of the relative importance of various phases of the runoff phenomenon. In small basins, the overland flow phase is predominant over the channel flow. Hence the land use and intensity of rainfall have an important role in affecting peak discharge. In large basins these effects are suppressed as the channel flow phase is more important (Subramanya, 1984).

(b) Slope of the channel and slope of the basin: The slope of the main channel controls the average velocity of flow. The recession limb of the hydrograph represents the depletion of storage; the channel slope has a pronounced effect on this part of the hydrograph. Larger channel slopes give quicker rise to quicker depletion of storage and hence result in steeper recession limbs of hydrographs. This results in smaller time bases. The basin slope is important in small catchments where the overland flow is relatively more important. In such cases the steeper slope of the catchment results in larger peak discharges (Subramanya, 1984).

(c) Shape of the basin: The shape of the basin influences the time required for water to travel from the distal parts of a basin to the outlet of a basin. Thus the occurrence of the peak and hence the shape of the hydrograph are affected by the basin shape. Fan shaped or nearly semi-circular shaped catchments generally yield high-peaked and narrow hydrographs; whereas elongated catchments give low-peaked and broad hydrographs (Subramanya, 1984).

(d) Drainage density: The drainage density is defined as the ratio of the total channel length to the total drainage area. A large drainage density introduces a situation conducive for quick disposal of localized runoff down the channels. This fast response is reflected in a pronounced peak discharge. In basins with smaller drainage densities, the overland flow is predominant and the resulting hydrograph is compressed with a slowly rising limb (Subramanya, 1984).

(e) Land use: Vegetation and forests increase the infiltration and storage capacities of the soils. Further, each causes considerable retardance to overland flow. Thus, vegetative cover reduces peak discharge. This effect is usually very pronounced in small catchments, areas less than about 150 square kilometers (58 square miles). Furthermore, the effect of the vegetative cover is prominent in small storms. For two catchments of equal area, other factors being identical, the peak discharge is higher for a catchment that has a lower density of vegetative cover (Subramanya, 1984).

(f) Intensity of rainfall: Among the climatic factors, intensity, duration and direction of rainfall movement are the three important factors affecting the shape of hydrographs. For a given duration, peak and volume of the surface runoff are essentially proportional to intensity of rainfall. Rainfall intensity has substantial influence on runoff; if the intensity of rain increases, the runoff increases rapidly (Subramanya, 1984).

(g) Duration of rainfall: The duration of rainfall of a given intensity also has a directly proportional effect on the volume of runoff. The effect of duration is reflected in the rising limb and peak discharge (Subramanya, 1984).

(h) Rainfall movement: If the rainfall moves from upstream of the catchment to the downstream end, there will be quicker concentration of flow at the basin outlet. This results in a peaked hydrograph. Conversely, if the storm movement is up the catchment, the resulting hydrograph will have a lower peak and longer time base (Subramanya, 1984).

(1.4) Unit Hydrograph

Unit hydrograph (UH) is the hydrograph resulting from the unit excess rainfall over the watershed at a uniform rate during a given period of time. The UH was first proposed by Sherman (1932). The UH is a simple linear model that can be used as a tool to derive a DRH resulting from any given rainfall excess hyetograph. The basic assumptions for the UH theory are:

- Effective rainfall or rainfall excess has a constant intensity within the effective duration,
- Effective rainfall is uniformly distributed spatially,
- Time base of runoff (period of time that direct runoff exceeds zero) resulting from an effective rainfall of specific duration is constant,
- The ordinates of direct runoff of a constant base time are directly proportional to the total amount of direct runoff represented by the hydrograph (linearity), and

- For a particular watershed, the size of the direct runoff hydrograph for two effective rainfall pulses is in direct proportion to the size of the pulses.

In actuality, these assumptions are often not true, particularly for small watersheds, which have a strong tendency toward non-linear responses. However, the UH approach is usually sufficient to obtain engineering estimates for design purposes. Sherman's UH procedure should be used with watershed drainage areas that are less than about 2,000 square miles. If storm patterns are thought to affect runoff hydrographs, then watersheds can be subdivided into smaller subwatersheds and each of those subjected to a hydrograph analysis.

Unit hydrographs are developed for specific watersheds using two basic approaches. If unit rainfall-runoff data are available, then numerous techniques can be applied to estimate a UH using the data. If no data are available, then methods of synthetic hydrology must be applied. Methods of regionalization are used to transfer known hydrographs (or other hydrologic characteristics) from a location where measurements are available to unmonitored watersheds. The regionalization involves determining timing parameters for the UH procedure, and timing parameters may include time to peak, time base, or time of concentration. Regionalization also involves development of regional regression equations for timing parameters, watershed and/or rainfall characteristics.

Conceptually a UH reflects the effects that both watershed characteristics and watershed conditions have on rainfall excess. An implicit assumption is made that the characteristics and conditions of a watershed are constant for each storm. Additionally, it is assumed that rainfall characteristics are not a factor in the shape of the UH (McCuen, 1998). In practice, watershed conditions show considerable storm-to-storm variation and the UH analysis procedure does not eliminate the effects of rainfall characteristics. Thus, unit hydrographs developed from different storms on the same watershed show considerable variation. Storms that produced a larger portion of rainfall near the watershed outlet typically will have unit hydrographs that peak earlier than expected (McCuen, 1998). A unit storm on a relatively dry watershed can result in a UH with a delayed peak that is lowered than expected. The longer time to peak occurs because a larger time period is needed to fulfill initial abstractions and a lower-than-average soil moisture deficit (McCuen, 1998). These differences in storm characteristics and watershed conditions can produce widely different unit hydrographs. Figure 1.3 shows the unit hydrographs for five storm events on the White Oak Bayou, Texas, and also includes a watershed-average UH for the five events.

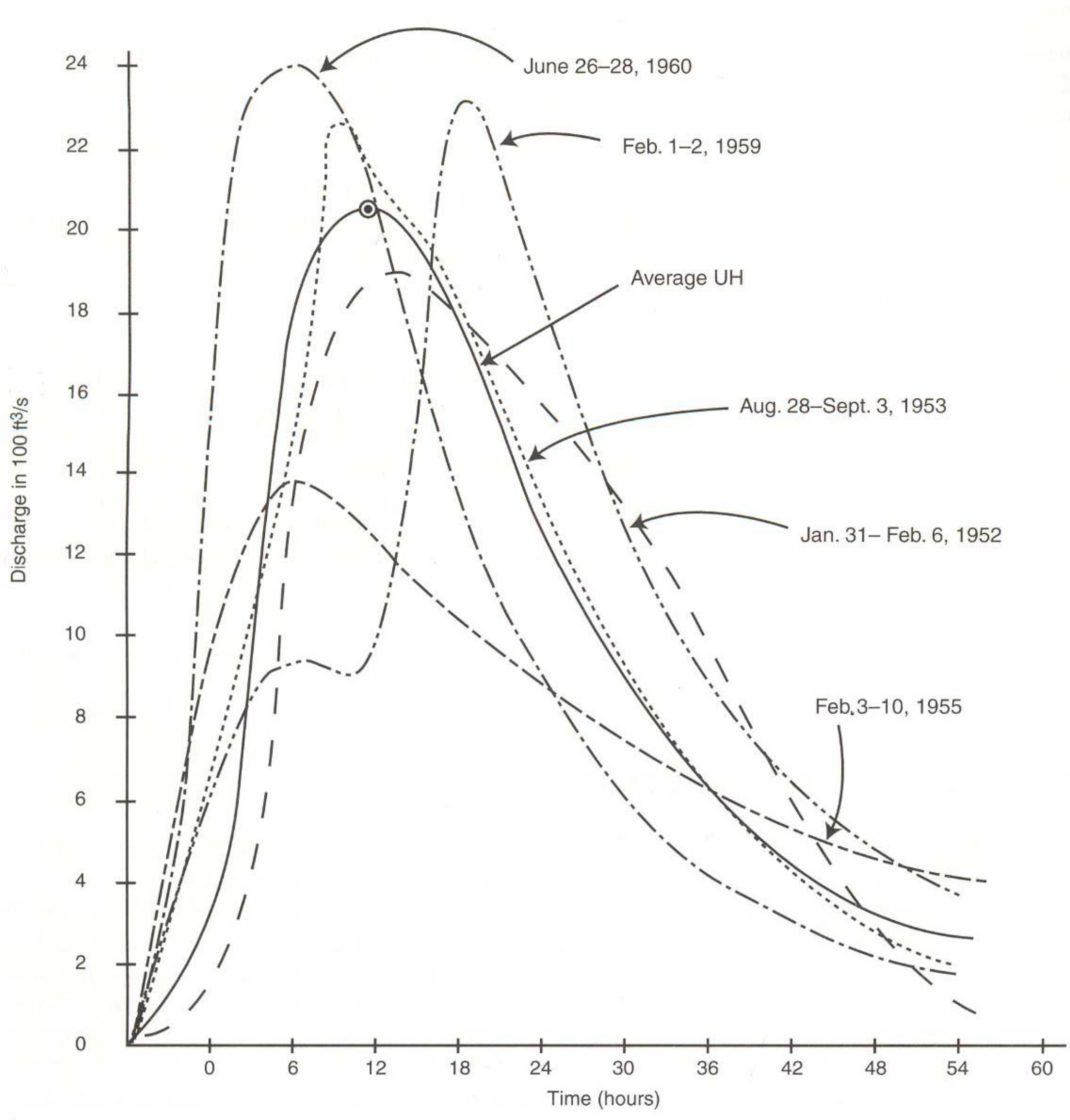

Figure 1.3. Five fitted unit hydrographs from five storm events and watershed-average UH on White Oak Bayou, Texas (from McCuen, 1998).

Chapter 2. Definitions of Hydrograph Timing parameters

When engineers apply rainfall-runoff models for hydrologic design, there is a difficulty in defining and quantifying the timing parameters for unit hydrographs and direct runoff hydrographs. Some of the timing parameters for hydrographs (see Figs. 1.1 and 2.1) are travel time, time of concentration, excess-rainfall release time, wave travel time, time to equilibrium, lag time, time base and time to peak. The same timing parameter, for example, basin lag time, may have different definitions or multiple meanings, and this creates great confusion for applying it in hydrologic design practice. Furthermore, for most timing parameters, there are operational and conceptual definitions. The operational definitions are useful when data are available; the conceptual definition is necessary when trying to synthesize a hydrograph. This chapter summarizes various definitions for common timing parameters for direct runoff hydrographs and unit hydrographs.

(2.1) Travel Time

The conceptual definition is that travel time (T_t) is the time a water parcel takes to travel from one location in a watershed to another location downstream. The travel may occur on the ground surface or below, or in a combination of the two (Kent, 1972). This definition implicitly assumes that the two points are hydraulically connected. The travel time is expected to be a function of the positions of two points in a watershed (NRCS, 1972; Garg, 2001; Viessman and Lewis, 2002). T_t is affected by storage, flow resistance, flow paths, and flow types (overland flow or channel flow).

(2.2) Time of Concentration

The conceptual definition for the time of concentration (T_c) is the time it takes a water parcel to travel from the hydraulically most distal part of the watershed to the outlet or reference point downstream. This definition has been used for many hydrologic studies and applications (Kirpich, 1940; U.S. Army Corps of Engineers (USACE), 1966; Bell and Kar, 1969; NRCS, 1972; Schultz and Lopez, 1974; Ben-Zvi, 1987; Huber, 1987; MacBroom, 1987; McCuen et al., 1984; Subramanya, 1984; McCuen, 1998; and Garg, 2001). This definition is "physically based."

In hydrograph analysis, the time of concentration is the time difference between the end of rainfall excess and the inflection point of a hydrograph where the recession curve begins as shown in Fig. 1.1 (Kirpich, 1940; USACE, 1966; Bell and Kar, 1969; NRCS, 1972; Schultz and Lopez, 1974; and McCuen et al., 1984). The inflection point (Fig. 2.1) is the point on the hydrograph recession limb that the direct runoff ceases (Fig. 2.1). Another slightly different definition uses the time from the centroid of rainfall excess to the inflection point of the hydrograph. This analysis definition has also been used for many hydrologic studies and applications (McCuen et al., 1984; Subramanya, 1984; Huber, 1987; McCuen, 1998; and Garg, 2001). These two definitions are useful to quantify T_c when rainfall-runoff data are available.

Viessman and Lewis (2002) state that the most common definition of time of concentration originates from consideration of overland flow. If a uniform rain is applied to a tract of land, the portions nearest the outlet contribute runoff at the outlet almost immediately. As rain continues, runoff contributions from various points upstream arrive at later times, until flow eventually arrives from all points on the watershed, "concentrating" at the outlet. Thus, time of concentration is the time required, with uniform rain, for 100 percent of a tract of land to contribute to the direct runoff at the outlet.

(2.3) Excess-Rainfall Release Time

The excess-rainfall release time (T_e) is defined as the time required for the last, most hydraulically remote drop of excess rain that fell on the watershed to pass the outlet, signaling the cessation of direct runoff. It is easily determined as the time interval between the end of rainfall and the end of direct runoff (Fig. 2.1). The excess-rainfall release time is often equated with the time of concentration because the time for runoff to arrive at the outlet from the most hydraulically remote point after rain ceases is assumed to be indicative of the time required for 100-percent contribution from all points during any uniform storm having sufficient duration (Viessman and Lewis, 2002). Because few storm durations (especially for large watersheds) exceed the time of concentration, this definition is often preferred and useful in making determination of T_c possible only by examining excess rain recession (Viessman and Lewis, 2002).

It is necessary to point out that the definition of T_c for hydrograph analysis (end of excess to inflection point) contradicts the definition of T_c as the excess-rainfall release time. Excess-rainfall release time as an estimation of T_c is always longer than time to inflection as T_c because the end point of the excess-rainfall release time is always later.

(2.4) Wave Travel Time

The wave travel time (T_w) is the time it takes a shallow wave in a channel to propagate from one location to another. This surface wave celerity is faster than average flow velocity and varies with channel shape and other factors. For rectangular channels, the wave travel time is approximately 5/3 the average velocity of flow (Viessman and Lewis, 2002). This parameter is important for hydrograph routing in streams.

For hydrograph analysis, Viessman and Lewis (2002) point out that the last drop of direct runoff to pass the outlet conceptually travels over the surface at the speed of a small surface wave, rather than a speed equal to the average velocity of flow. They also state that both wave travel time and excess-rainfall release time are often used synonymously with time of concentration.

(2.5) Time to Equilibrium

If an inflow of excess rainfall continues at a steady rate for an indefinite period of time, the outflow continues to increase and its value asymptotically approaches the value of the inflow. The time elapsing before there is no substantial difference between inflow and outflow (usually less than 3 percent) is called the "time to equilibrium" (Bell and Kar, 1969). Even though these conditions rarely occur in nature, and it is not usually possible to determine the time of equilibrium (T_{eq}) from rainfall-runoff data, the concept has been found useful when deriving S-hydrographs (Viessman and Lewis, 2002). The time to equilibrium (maximum discharge of S-hydrograph in Fig. 2.2) is equal to the time base of the UH (T_b) minus the duration of the UH (D) (Viessman and Lewis, 2002).

$$T_{eq} = T_b - D = T_e \tag{2.1}$$

This is the same as the excess-rainfall release time shown in Figure 2.1. When kinematic wave theory is used to model overland flow over planes or different shapes of watersheds, the maximum discharge equals rainfall intensity times plane or watershed area to indicate 100-percent contribution of runoff to the outlet. Therefore, time to equilibrium is treated as synonymous with time of concentration (Holtan and Overton, 1963; Overton and Meadows, 1976). Based on the above definition, for turbulent flow, Overton and Meadows (1976) state that

$$T_c = 1.6\ T_L \text{ (lag time)} \tag{2.2}$$

which is close to the NRCS relation in equation 1.3. Izzard (1946) defined the equilibrium time as the time interval required for the runoff rate to become equal to the supply rate. This definition also was used by Morgali and Linsley (1965), Wei and Larson (1971), and Wong (1996).

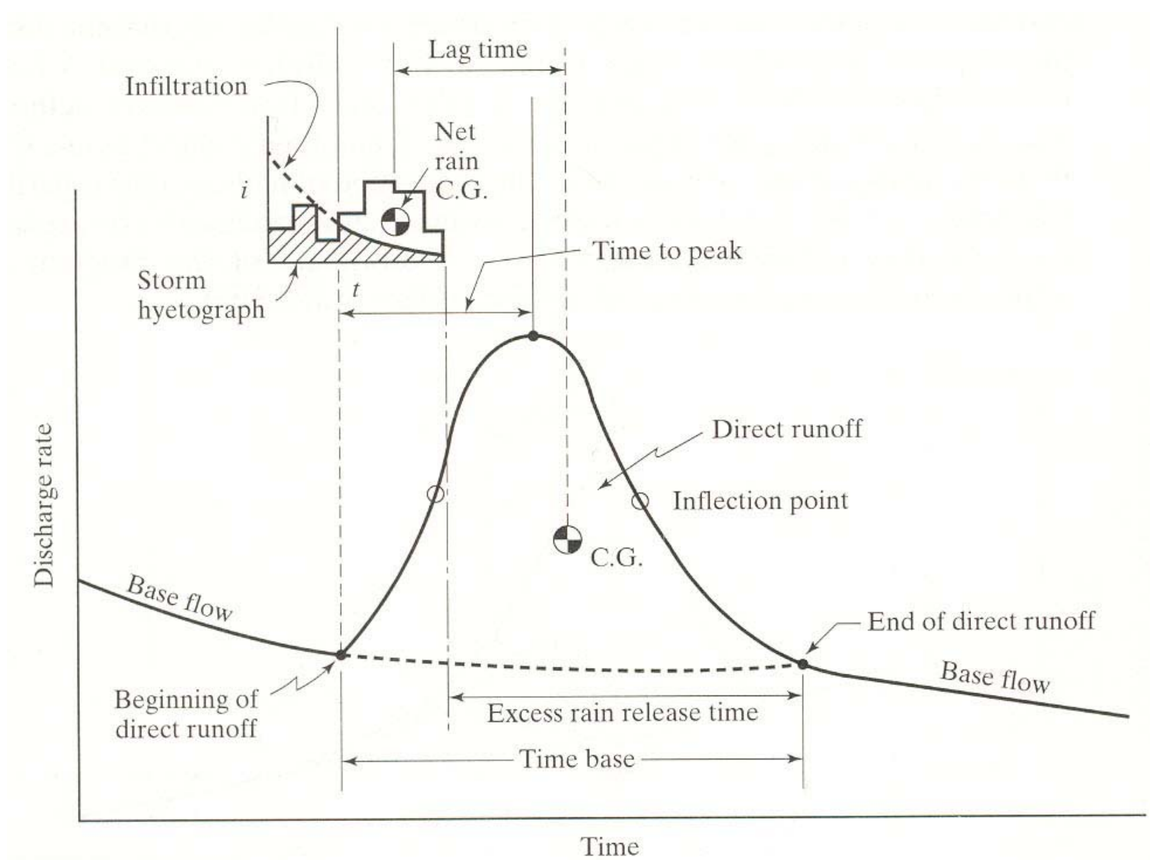

Figure 2.1. Hydrograph time relations and timing parameters of hydrographs (from Viessman and Lewis, 2002).

(2.6) Lag Time

Though direct runoff begins with the commencement of effective rainfall (Fig. 2.1), the largest portion of runoff generally lags the rainfall since it takes time for runoff to travel from any location within the watershed to the outlet. The lag time has been used widely for many hydrologic studies and applications; however several definitions are used to develop different hydrologic procedures (Rao and Delleur, 1973).

The basin lag time is most often defined as the difference in time from the center of mass of rainfall excess to the center of mass of direct runoff produced by the net rain (Carter, 1961; Espey et al., 1966; and Viessman and Lewis, 2002), and it is shown in Figure 2.1 and as T_4 in Figure 2.3. This definition is used for many hydrologic studies and applications (Horner and Flynt, 1936; Mitchell, 1948; Bell and Kar, 1969; Askew, 1970; McCuen, 1998; NRCS, 1972; Schulz and Lopez, 1974; Subramanya, 1984; McCuen et al., 1984; Simas and Hawkins, 1996; and Viessman and Lewis, 2002). The single linear reservoir theory (Chow, 1964) indicates that the reservoir constant K should be equal to the lag time (T_4).

The second definition of the basin lag time is the time from the center of mass of the rainfall excess to the peak discharge rate on the hydrograph (T_1 in Fig. 2.3) and used by Eagleson (1962), Bell and Kar (1969), NRCS (1972), Rao and Delleur (1973), and Schulz and Lopez (1974). The third definition is the time interval from the maximum rainfall rate to the peak rate of runoff (Viessman and Lewis, 2002). U.S. Bureau of Reclamation (USBR, 1965) and Wilson (1972) defined the lag time as the centroid of rainfall excess and the time when 50 percent of the direct runoff has passed the gaging station (T_5 in Fig. 2.3). Wilson (1972) also defined the lag time as the time interval between the beginning of rainfall excess and the centroid of direct runoff hydrograph.

Linsley et al. (1958) used the average lag time T_3 (Fig. 2.3), starting from the beginning to the centroid of the direct runoff, and related it to the length of the main stream (L), the distance along the main stream from the basin outlet to a point nearest the center of the gravity of the basin in miles (L_{ca}), and the mean basin slope S: $T_3 \sim a(L\,L_{ca}/\sqrt{S})^b$.

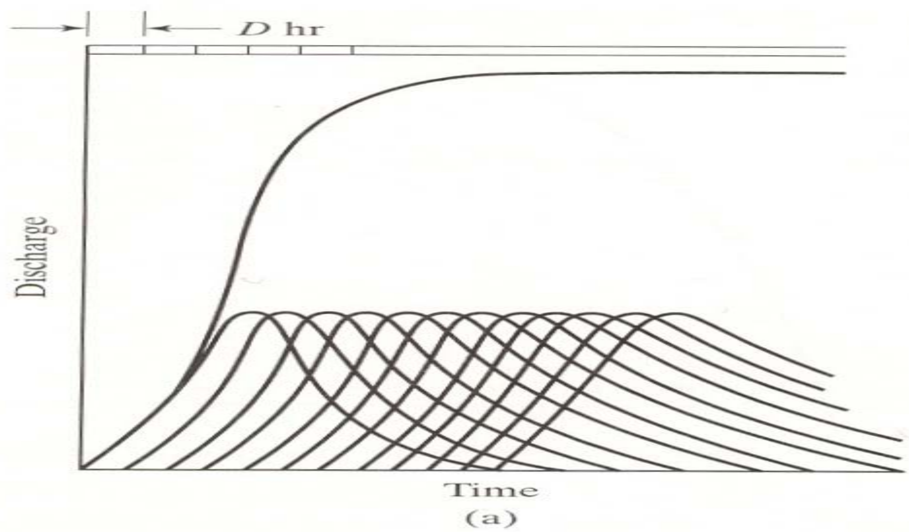

Figure 2.2. S-hydrograph developed by lagging of known D hour UH for infinite times (from Viessman and Lewis, 2002).

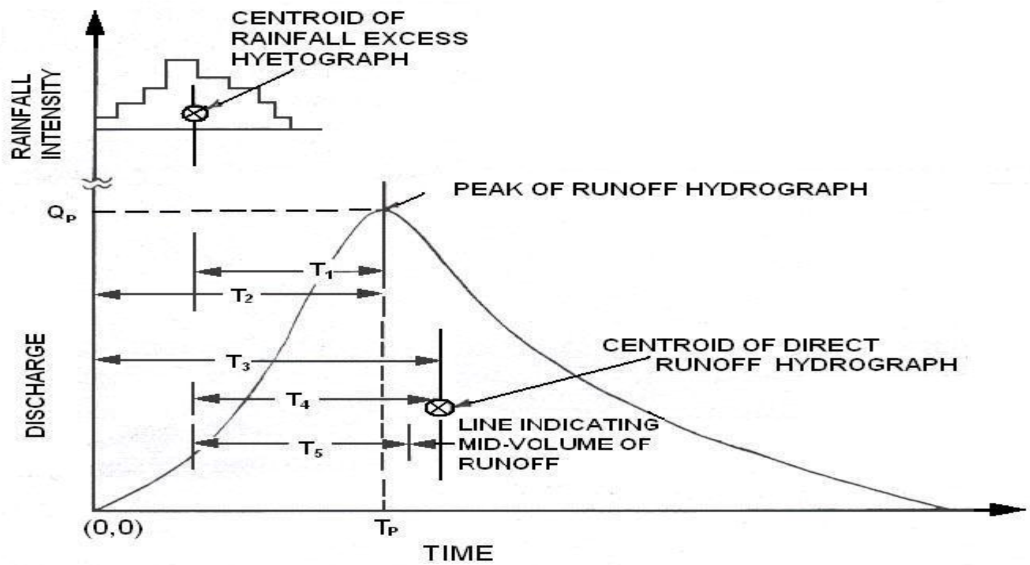

Figure 2.3. Different lag time definitions (after Rao and Delleur, 1973).

(2.7) Time to Peak

The time to peak is the time from the beginning to the peak discharge in a simple (single peak) direct runoff hydrograph (Fig. 2.1 and T_2 in Fig. 2.3), and is used in many hydrologic applications (Linsley et al., 1958; Askew, 1970; NRCS, 1972; Schulz and Lopez, 1974; and McCuen et al., 1984). Sometimes it is called the rise time of the hydrograph (Ramser, 1927; Kirpich, 1940; Gray, 1961; Wu, 1963; and Bell and Kar, 1969). It is also defined as the time interval between the centroid of rainfall excess and the peak of the direct runoff as depicted as T_1 in Figure 2.3 (Snyder, 1938; Taylor and Schwarz, 1952; Eagleson, 1962; and Schulz and Lopez, 1974). Lopez (1973) used the time interval between the beginning of rainfall and the peak discharge of the direct runoff as the time to peak.

(2.8) Duration of Excess Precipitation

The duration of rainfall excess (*D*) is the time from beginning to end of an excess precipitation during a rainfall event. This duration is typically used as the duration of a UH, and is always shorter than the duration of a storm due to initial rainfall abstraction as indicated in Figure 2.1.

(2.9) Time Base of a Runoff Hydrograph

The time base of a runoff hydrograph (T_b) is the elapsed time from the beginning of direct runoff until the return to base flow (the direct-runoff component reaches zero) (Fig. 2.1). Time base for a UH becomes important for some synthetic UH procedures (e.g. Snyder, 1938.

CHAPTER 3. METHODS FOR ESTIMATING TIMING PARAMETERS

In general, it is reasonable to consider three components of flow that can characterize the progression of runoff along a travel path: overland flow (sheet flow), shallow concentrated flow, and open channel and conduit flow (or concentrated channel flow). Methods to estimate timing parameters are related to values of Manning's n because of the hydraulic relation between runoff travel time and velocity, and Manning's equation is often used (in some form) to estimate velocity. The time that it takes water to traverse the watershed is typically assumed to be determined by a linear or non-linear relation among watershed parameters.

McCuen (1998) developed a helpful system to classify timing parameters (Table 3.1) because numerous empirical formulas have been developed. Almost all of the methods are based on four types of input: slope, watershed size, flow resistance, and water input. These characteristics are either for the overland flow portion of the watershed, the pipe system, or the channel system. Methods for overland flow can be further subdivided into sheet flow and concentrated flow methods. The classification system given in Table 3.1 can be used for classifying timing parameter prediction methods. Whereas some prediction methods will fall into one of the four classes based on flow regime, a substantial number of others will have to be identified as a **"mixed"** method (one that includes variables reflecting different flow regimes). For example, designs required for urbanized watersheds that include both a substantial pipe system and overland flow may require a time-parameter model that includes variables that reflect all three flow regimes: sheet flow, concentrated flow, channel and/or pipe flow.

Table 3.1. Criteria for classifying timing parameters and variables commonly used to represent the input type (from McCuen, 1998).

	Input Type			
Flow Regime	Flow Resistance	Watershed Size	Slope	Water Input
Sheet flow	n	L_w	S	i
Concentrated flow	n, C, CN	L_w, A	S	i
Channel	n, ϕ	L_c, L_{10-85}, L_{ca}	S_c, S_{10-85}	R_h, i, Q
Pipe	n	L_w	S	R_h, q_p

Note: n – Manning's roughness coefficient; C – the runoff coefficient of the Rational Method; CN – the NRCS runoff curve number; ϕ – Espey-Winslow channelization factor; L_w – watershed length; A – drainage area; L_c – channel length; L_{10-85} – length of channel within 10 percent and 85 percent points; L_{ca} – length to the center of area of a watershed measured along channel; S – average watershed slope; S_c – channel slope; S_{10-85} – channel slope between 10 percent and 85 percent points; i – rainfall intensity; R_h – hydraulic radius; Q – channel discharge rate; and q_p – peak discharge.

Kent (1972) prepared the Chapter 15 "Travel Time, Time of Concentration and Lag" in the section 4 "Hydrology" of the National Engineering Handbook. Kent (1972) stated that use of stream hydraulics is recommended for the usual case where no usable hydrographs are available. This procedure is most applicable for areas where surface runoff predominates. It can result in a T_c value too short for areas where interflow and groundwater flow are a major portion of the runoff (Kent, 1972). This indicated that it is favorable to use hydrograph analysis to estimate time of concentration in comparison to the velocity method.

(3.1) NRCS Velocity Method For Time of Concentration

The time of concentration (T_c) is used in many procedures to develop runoff hydrographs or estimate peak discharges. TxDOT uses NRCS procedures for hydrologic design. The peak rate of runoff is very sensitive to T_c, particularly for small watersheds (Welle and Woodward, 1986). For example, the peak discharge of the standard NRCS dimensionless UH is inversely related to 0.67 T_c as $D = 0.133\ T_c$. Figure 3.1 shows four unit hydrographs developed from NRCS DUH with four T_c values for a watershed with an area of 7 square miles. The peak discharge decreases from 4,217 cfs to 1,687 cfs as T_c increases from 1.2 hour to 3.0 hour. The duration of UH also changes from 10 minutes to 25 minutes.

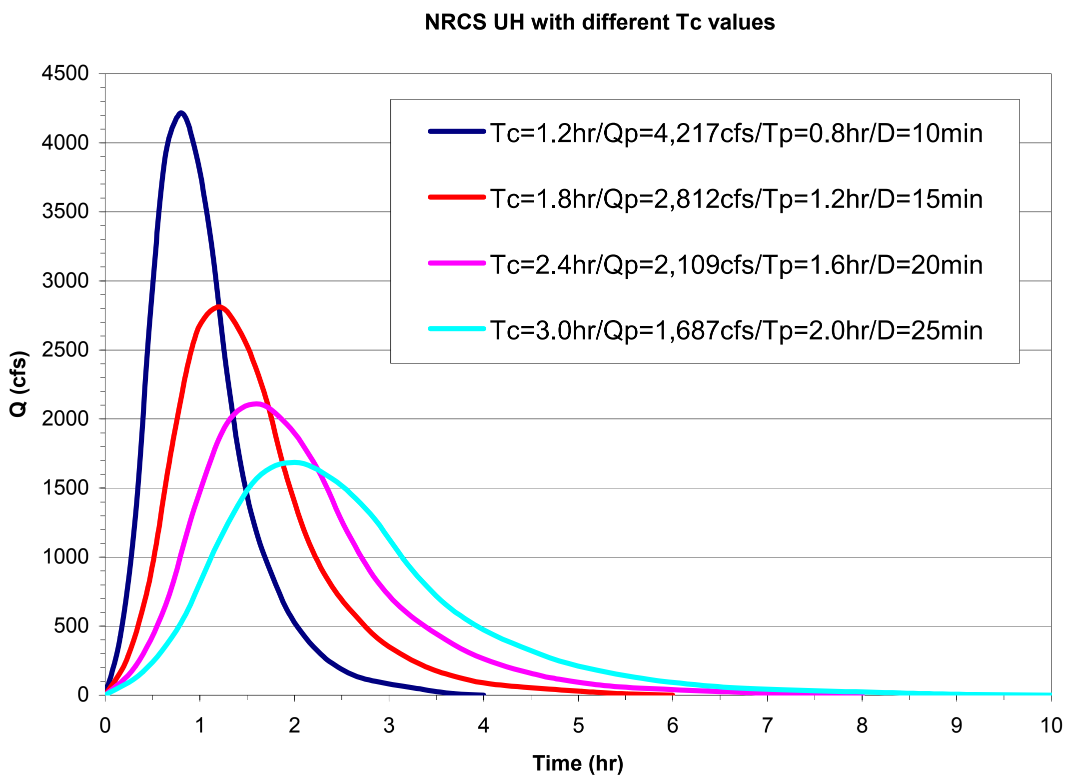

Figure 3.1. Example unit hydrographs developed from NRCS DUH with four T_c values.

The procedures used to estimate T_c depend on several factors including watershed characteristics (especially drainage area), climatic conditions, available data, and available time. For example, to design a small conservation structure such as a grassed waterway, a shortcut procedure that assumes a certain generalized relation between T_c and a few watershed characteristics but no relation between T_c and rainfall intensity, might be acceptable (Welle and Woodward, 1986). However, for the development of a storm-water-management plan, a sophisticated estimate of the peak rate of runoff for a small watershed from at least two storm frequencies for the undeveloped, developing, and fully developed conditions would be required. In this case, all available factors should be considered with particular attention given to the overland flow (Welle and Woodward, 1986).

To sophisticatedly determine the T_c for a watershed, the hydraulics of each part of the flow path is considered separately. This can be achieved by dividing the flow path into overland flow, shallow concentrated flow, and channel and/or pipe flow segments. The travel time (T_r) can then be computed for each segment and totaled to obtain the T_c. This practice is used in the NRCS velocity method to compute the time of concentration, and TxDOT engineers use it for hydrologic design. When runoff flows from the most hydraulically remote point of the drainage area to the point under investigation, there may be a number of possible flow paths; typically the longest flow path is considered in determining the longest travel time as the time of concentration. Identifying the flow path along which the longest travel time is likely to occur is a trial and error process (TxDOT, 2002).

The time of concentration (T_c in minutes) is the sum of travel times for the number of reaches along flow path ($i = 1,2, .., M$):

$$T_c = \sum_{i=1}^{M} T_{ri} = \sum_{i=1}^{M} \frac{L_i}{60\, V_i} = \mathrm{T_{of}} + \mathrm{T_{ch}} \tag{3.1}$$

where T_{ri} is the travel time over the ith reach, L_i (feet) and V_i (feet per second) are estimated length and flow velocity for the ith reach along flow path, respectively. The travel time also can be considered as two general types: (a) overland flow (T_{of}), and (b) channel flow (T_{ch}) that includes shallow concentrated flow and normal channel flow. Kent (1972) stated that runoff is usually concentrated into small gullies or terrace channels within less than a thousand feet of its origin. "A velocity of 1.5 feet per second can be assumed for the average terrace channel (Kent, 1972)." The velocity of equation (3.1) is a function of the type of flow (sheet/overland, concentrated flow, gully flow, channel flow, pipe flow), the roughness of the flow path, and the slope of the flow path.

(3.1.1) Overland Flow

At the upper portion of a basin, runoff is not concentrated, but rather flows as a "sheet," and after some distance the flow becomes concentrated in gullies and channels. Steep slopes can maintain sheet flow (or overland flow) over hundreds of feet (in the case of highly impervious surfaces). Sheet flow exists on gentle slopes over relatively short distances. For overland flow, velocity is estimated by a simplified Manning's equation:

$$V = k\,S^{0.5} \tag{3.2}$$

in which V is the velocity in feet per second and S is the average slope in feet per foot. Thus, k equals $1.486\ R_h^{2/3}\ /\ n$ based on Manning's equation and is a function of the land cover with the effect measured by the value of n and hydraulic radius R_h. Values of n, R_h, and k for selected land covers are given in Table 3.2. Equation (3.2) with values of n and R_h is graphically shown in Figure 3.2 for engineers to use; TxDOT uses the same chart for its T_c computation (TxDOT, 2002). This method is most appropriate for distances up to about 525 feet (160 meters) over open paved and grassed areas such as parking lots, roadways, verges, and landscaped areas (TxDOT, 2002).

Table 3.2. Coefficients of velocity versus slope relation for estimating travel times with the velocity method for overland flow (from McCuen, 1998).

Land Use/Flow Regime	n	R_h (feet)	k
Forest			
Dense underbrush	0.8	0.25	0.7
Light underbrush	0.4	0.22	1.4
Heavy ground litter	0.2	0.20	2.5
Grass			
Bermuda grass	0.41	0.15	1.0
Dense	0.24	0.12	1.5
Short	0.15	0.10	2.1
Short grass pasture	0.025	0.04	7.0
Conventional tillage			
With residue	0.19	0.06	1.2
No residue	0.09	0.05	2.2
Agricultural			
Cultivated straight row	0.04	0.12	9.1
Contour or strip cropped	0.05	0.06	4.6
Trash fallow	0.045	0.05	4.5
Rangeland	0.13	0.04	1.3
Alluvial fans	0.017	0.04	10.3
Grassed waterway	0.095	1.0	15.7
Small upland gullies	0.04	0.5	23.5
Paved area (sheet flow)	0.011	0.06	20.8
Paved area (sheet flow)	0.025	0.2	20.4
Paved gutter	0.011	0.2	46.3

The hydraulic radius should vary with return period, slope, and location along the flow path (Viessman and Lewis, 2002). The second method for evaluating sheet flow uses kinematic wave theory with the assumption that the hydraulic radius is equal to the product of the rainfall intensity and travel time ($R_h = iT_t$). Using Manning's equation with travel time equal to the time of concentration gives:

$$V = \frac{L}{60T_t} = \frac{1.49}{n} R_h^{2/3} S^{1/2} = \frac{1.49}{n} [\frac{iTt}{60(12)}]^{2/3} S^{1/2} \tag{3.3}$$

where T_t is travel time in minutes, i is the rainfall intensity (in per hour) corresponding to the rainfall event with duration of T_t, L is the overland flow length (feet), n is the Manning's roughness coefficient, and S is the slope of the surface (feet per foot). Solving for the travel time yields:

$$T_t = \frac{0.938}{i^{0.4}} \left(\frac{nL}{\sqrt{s}} \right)^{0.6} \tag{3.4}$$

The trial and error procedure can be avoided by using a power model between intensity and duration and substituting the 2-year, 24-hour precipitation depth (inches) for the rainfall intensity (Overton and Meadows, 1976; Welle and Woodward, 1986). This gives sheet flow travel time as:

$$T_t = \frac{0.42}{P_2^{0.5}} \left(\frac{nL}{\sqrt{s}} \right)^{0.8} \tag{3.5}$$

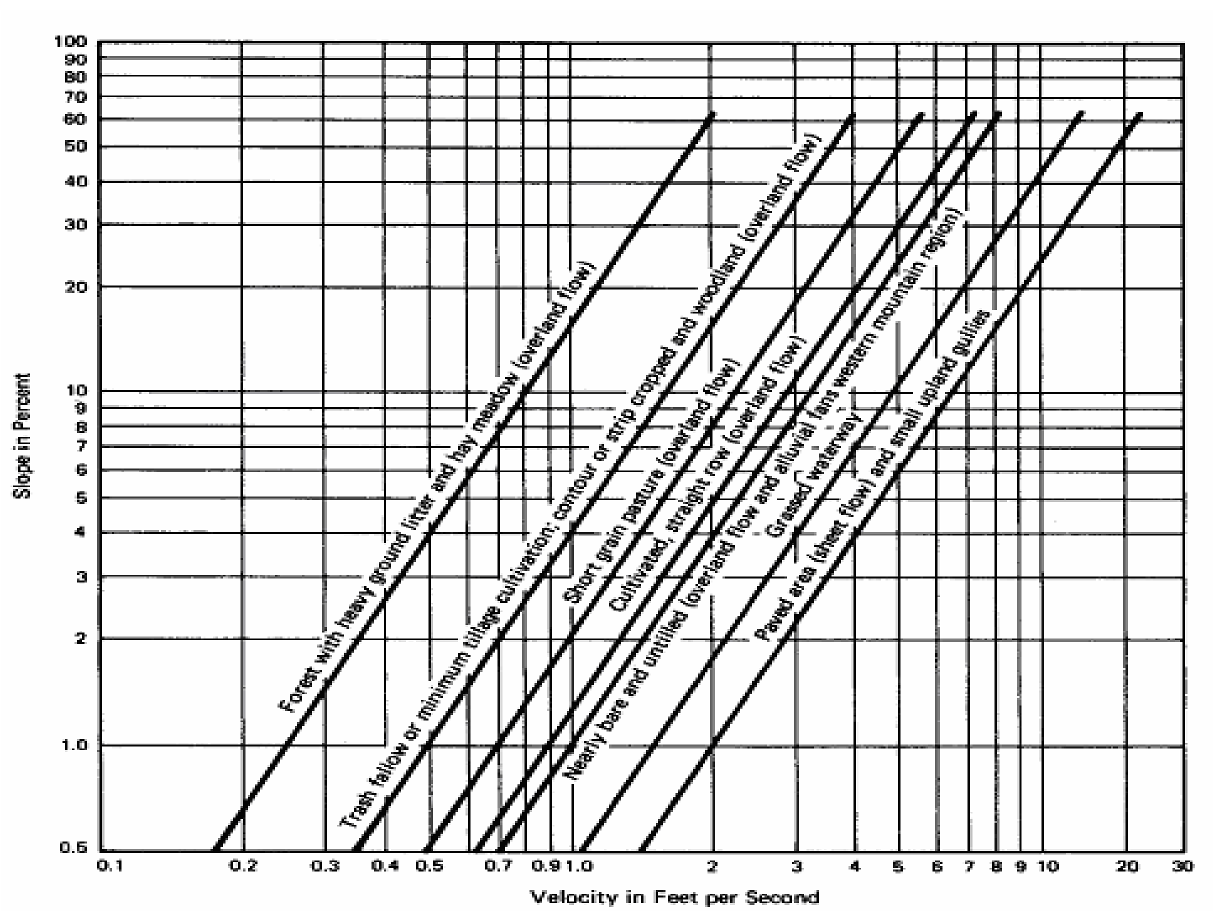

Figure 3.2. Velocities for overland flow method of estimating T_c (from McCuen, 1998).

Equation (3.5) has the advantage of not requiring an iterative solution. Equations (3.4) and (3.5) can yield unusually long times of concentration (McCuen, 1998) when common length limits are <u>from 100 to 300 feet</u>. Ragan and Duru (1972) developed a nomograph to reduce the number of iterations for the Maryland highway drainage study. Overton (1971) studied overland flow by kinematic wave theory to estimate time of concentration for a plane, a cascade plane, V shape watershed and a converging surface and developed various analytical solutions. Overton (1971) equations are not widely used for engineering design.

(3.1.2) Channel Flow

Flow velocities for shallow concentrated and normal open channel flow are typically computed using Manning's equation:

$$V = \frac{1.486}{n} R_h^{2/3} S^{1/2} \tag{3.6}$$

in which V is the mean flow velocity (feet per second), n is the roughness coefficient, R_h is the hydraulic radius (feet) (flow area A divided by the wetted perimeter P), and S is the channel bottom slope (feet per foot). Values of n for channels can be obtained from many reference books of hydraulics (for example, Chow, 1959; McCuen, 1998).

Flow in gullies empties into channels or pipes. Open channels are assumed to begin where either the blue stream shows on U.S. Geological Survey (USGS) quadrangle sheets or the channel is visible on aerial photographs (McCuen, 1998). Cross-section information (depth-area relations and roughness) should be obtained for all channel reaches in the watershed. Manning's equation or water surface profile information then can be used to estimate average flow velocities. The velocity should be computed for normal depth (uniform flow condition) based on bankfull flow conditions. Flow with return periods from 1.5 to 3 years is often assumed to produce bankfull conditions (McCuen, 1998). This assumption avoids the substantial iteration associated with other methods that employ rainfall intensity or discharge (because rainfall intensity and discharge are dependent on time of concentration) (TxDOT, 2002).

Kent (1972) suggested that the 2-year frequency discharge be used for stream hydraulics computation. When this is not feasible, the approximate bankfull discharge of the low flow channel was recommended to use (Kent, 1972). "Use of the low flow channel bankfull discharges with valley lengths is a compromise that gives a T_c for average flood" (Kent, 1972).

(3.2) Particle Tracking Method

Particle tracking refers to a variety of computational approaches for modeling flow of objects in a domain (Harlow, 1963 and 1964; and Amsden, 1966). The fundamental idea is to replace a continuum model with discrete particles (also referred to as parcels) that move according to some kinematics that mimic the continuum behavior. The discrete-parcel-random-walk method (Ahlstron et. al. 1977), USGS-MOC model (Konikow and Bredehoeft, 1978), and marker-and-cell (MAC) (Harlow and Welsh, 1965; Taylor, 1983) methods are examples of particle based methods for modeling the flow of dissolved or suspended materials in porous media (DPRW, USGS-MOC) and the primitive variable Navier-Stokes equations (MAC), respectively. Particle methods present an enormous computational burden because many particles must be tracked, but in many applications they effectively model the physics of the process with relatively simple mathematical constructions and the number of particles selected and the time interval used controls the accuracy of the results.

Appendix A provides preliminary research plans and results of the particle tracking method to estimate timing parameter and instantaneous UH. The particle tracking method as a tool is essentially used to determine a time-area diagram, then the diagram is used to generate a hypothetical UH (for example, Maidment, 1993; Olivera and Maidment, 1999), then the hydrograph is analyzed to determine the relevant timing parameters.

(3.3) Empirical Formulas for Timing parameters

Many empirical formulas have been developed to estimate timing parameters for various watersheds. These equations are based on correlations between various watershed and/or rainfall characteristics and timing parameters. Other equations relate various timing parameters based on correlation between/among timing parameters. For example, it is obvious that large T_p is associated with large T_c. It is worth noting that different definitions of timing parameters were used in many previous studies. Application of empirical equations requires engineering judgment, and the watersheds in which an equation is used should have characteristics comparable to the characteristics of the watersheds on which the equations are based.

(3.3.1) Time of Concentration

Nine empirical methods of estimating T_c were reviewed and summarized by McCuen (1998). Some of these empirical equations are listed in Table 3.4 and are discussed first in this section; additional equations also are discussed. Because not all the methods were originally presented as equations for computing the time of concentration, McCuen (1998) adjusted those empirical equations so that they would compute T_c in minutes. For methods designed to predict the lag time, computed lag values were multiplied by a constant value that varies depending on the definition of the lag (McCuen, 1998). A value of 1.417 was used for the lag (L_H) defined as the time difference between the centers of mass of rainfall excess and direct runoff, determined on the basis of the relation between the time lag and the time of concentration, for a NRCS triangular hydrograph. A conversion factor of 1.67 was used for methods in which the lag (T_L) was defined as the time difference between the center of mass rainfall excess and the peak discharge; this constant was also based on analysis of a triangular hydrograph. These assumptions are probably unimportant because the results of comparisons are insensitive to assumptions (McCuen, 1998).

$$T_c = T_L/0.6 = 1.67\ T_L \quad or \quad T_c = 1.417\ L_H \tag{3.7}$$

Carter (1961) calibrated an equation for predicting the watershed time of concentration (lag) for watersheds that have natural channels and partially sewered land uses. Watersheds studied are in the Washington D.C. area. The length (L_m in miles) and slope (S_m in feet per mile) parameters are measured using the longest channel:

$$T_c = 100\,(L_m / \sqrt{S_m})^{0.6} \qquad (3.8)$$

in which T_c is the time of concentration (minutes). The data that were used to calibrate the equation included watersheds less than 8 square miles with channel lengths less than 7 miles and slopes less than 2 percent. Manning's coefficients for the channels varied between 0.013 and 0.025. The input parameters reflect the channel characteristics; the coefficients reflect a substantial amount of pipe flow because the watersheds were in urbanized areas and the Manning's n suggested a concrete surface (McCuen, 1998).

Eagleson (1962) presented an equation for estimating the time difference between the center of gravity of the rainfall excess and the peak of direct runoff; equation (3.9) includes a factor [1.67 in equation (3.7)] for converting the time lag to a time of concentration:

$$T_c = 0.0111\, L_f\, nR_h^{-2/3}\, S_f^{-1/2} \qquad (3.9)$$

where L_f is the hydraulic length (feet), R_h is the hydraulic radius (feet), S_f is the slope of the principal flow path (feet per foot), and T_c is the time of concentration (minutes). The original equation was calibrated from data for watersheds less than 8 square miles. The parameters that were used in calibrating the model were computed using the characteristics of the sewer system. The length, slope, and n value are for the main sewer, whereas R_h is for the main channel when flowing full.

Espey and Winslow (1968), as reported by Schultz and Lopez (1974) calibrated an equation for predicting the time to peak using data measured by the USGS in Houston from 1964 to 1967. Six of the 17 watersheds were predominately rural; the remaining were urbanized. The watersheds ranged in size from 1 to 35 square miles. The length and slope variables were measured from the channel. The channelization factor ϕ subjectively measured the hydraulic efficiency of the drainage network. The value of ϕ is the sum of two parts, one indicating the amount of channel vegetation and the other indicating the degree of channel improvement. The impervious area factor represents the resistance of the overland flow portion of the travel time. Equation 3.10 includes a coefficient (T_c=1.49T_p) to convert the model from a time to peak to a time of concentration equation:

$$T_c = 31\phi\, L_c^{0.29}\, S_c^{-0.11}\, I^{-0.6} \qquad (3.10)$$

where L_c is the channel length (feet), S_c is the channel slope (feet per foot), I is the percent imperviousness, and T_c is the time of concentration (minutes). Because equation 3.10 includes variables for both overland and channel flow, it is considered to be a "mixed" method.

The **Federal Aviation Administration** (1970) developed the following equation from airfield drainage data:

$$T_c = 1.8(1.1 - C)\, L^{0.5}\, S^{-0.333} \qquad (3.11)$$

where C is the Rational method runoff coefficient, L is the flow length (feet), S is the slope (feet per foot), and T_c is the time of concentration (minutes). Thus it is probably most valid for small watersheds where sheet flow and overland flow dominate. The length, slope, and resistance variables are for the principal flow path.

Kerby (1959) expanded Hathaway's study (1945) on the design of drainage facilities and developed the following equation for computing the time of concentration on very small watersheds in which surface flow dominated:

$$T_c = \left(\frac{2NL}{3S_c^{0.5}} \right)^{0.467} = \left(\frac{0.67NL}{\sqrt{S_c}} \right)^{0.467} \tag{3.12}$$

where T_c is the time of concentration (minutes), L is the flow length (feet), S_c is the surface slope (feet per foot), and N is the average surface retardance value of the overland flow given in Table 3.3. The length used in the equation is the straight-line distance measured from the most distant point of the watershed in a direction parallel to the slope until a well-defined channel is reached. Kerby (1959) stated that overland flow became channel flow within 1,200 feet in all cases and less in most cases. Watersheds of less than 10 acres were used to calibrate the model; the slopes were less than 1 percent, and the retardance coefficient N values were 0.8 and less.

Table 3.3. Average value of retardance coefficient "N" (from Kerby, 1959).

Surface cover type	N
Pavement (smooth impervious surface)	0.02
Smooth bare packed soil	0.10
Poor grass, cultivated row crops, moderately rough bare surface	0.20
Pasture or average grass	0.40
Deciduous timberland	0.60
Conifer timberland, deciduous timberland with deep forest litter or dense grass	0.80

Kirpich (1940) calibrated two equations for computing the time of concentration (minutes) for small watersheds in Pennsylvania:

$$T_c = 0.0013\, L_c^{0.77}\, S_c^{-0.5} \tag{3.13}$$

and Tennessee:

$$T_c = 0.0078\, L_c^{0.77}\, S_c^{-0.385} \tag{3.14}$$

where L_c and S_c are the length (feet) and slope (feet per foot) for the channel. The channel slope S_c was defined as the difference in elevation in feet between the most remote point in the watershed and the outlet divided by the channel length L_c in feet. Kirpich (1940) used the data of seven watersheds from Ramser (1927). The Tennessee watersheds ranged in size from 1.25 to 112 acres, with slopes from 3 to 10 percent. The computed time of concentration should be multiplied by 0.4 and 0.2 for watersheds where the overland flow path is either concrete or asphalt and the channel is concrete lined, respectively (McCuen, 1998). Actually, Kirpich (1940) did not present the equation (3.14) but instead presented a graph with data points. The graph is reproduced in Fig. 3.3 including a new regression equation fitted to the data by using Excel. Fig. 3.3 also shows that the equation (3.14) also fits data well. Kirpich (1940) stated that the curve is applicable to the average small agricultural area ranging in size from 1 to 200 acres.

The **Soil Conservation Service** (SCS) currently NRCS (1972) provides an equation for estimating the watershed lag, which was defined as the time in hours from the center of mass of the excess rainfall to the peak discharge. They also indicate that the time of concentration equals 1.67 times the lag. Thus, T_c (minutes) is:

$$T_c = 0.00526\, L^{0.8} \left(\frac{1000}{CN} - 9 \right)^{0.7} S^{-0.5} \tag{3.15}$$

where L is the watershed length (feet), S is the watershed slope (feet per foot), and CN is the NRCS curve number. Equation 3.15 is intended for use on watersheds where overland flow dominates and was developed for nonurban watersheds. The NRCS (1972) had recommended that the lag equation be used for homogeneous watersheds less than about 2000 acres. The method primarily reflects concentrated flow. However, the NRCS (1986) report known as TR-55 did not include this formula; it was shown by McCuen et al. (1984) that this formula provides good estimates of T_c up to about 4,000 acres.

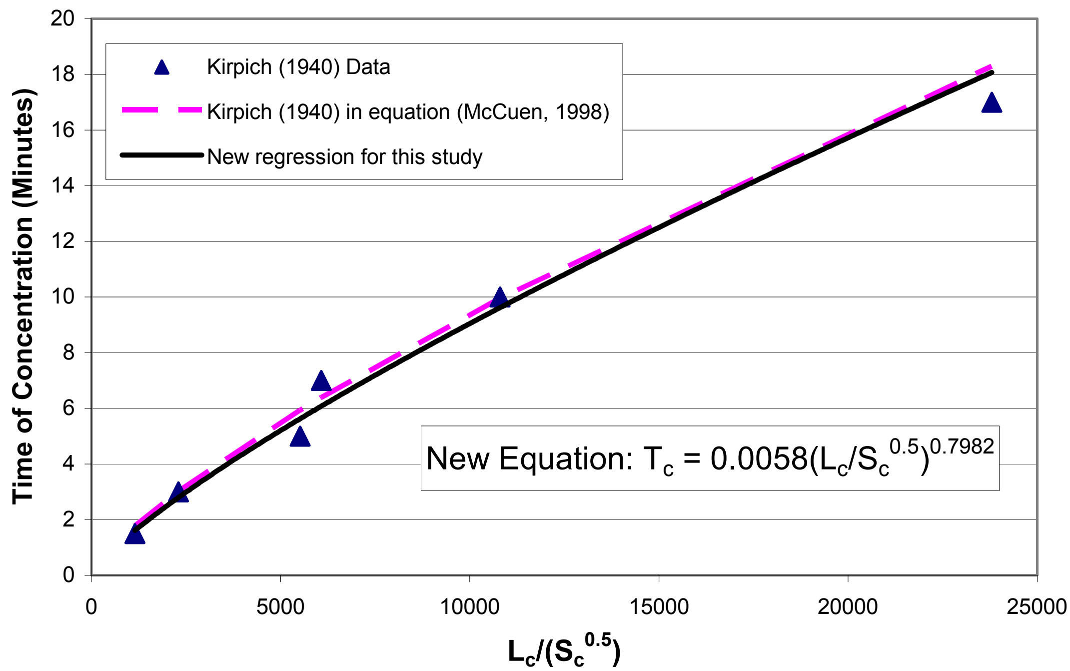

Figure 3.3. Kirpich's data and regression equation comparison (after Kirpich, 1940).

Van Sickle (1962) provided a time to peak equation calibrated from Houston watersheds having drainage areas less than 36 square miles. The equation is based on two length variables: the first, L_t (miles), is the total length of all drainageways and storm sewers greater than 36 inches in diameter, whereas the second, L_m (miles), is the total basin length. The equation reflects both channel and pipe flow; thus it is a "mixed" method. Equation 3.16 includes a factor for converting the time to peak equation to one for predicting the time of concentration:

$$T_c = 0.55 \left(L_t L_m / \sqrt{S_f} \right)^{0.13} \tag{3.16}$$

where S_f is the slope (feet per foot) and T_c is the time of concentration (minutes).

McCuen and others (1984) compared empirical equations 3.1 and 3.7 to 3.16 for NRCS velocity method and used them to compute time of concentration in 48 urban watersheds. Watersheds studied had areas less than 4,000 acres (36 had an area less than 2,000 acres); the average impervious cover was 29.1 percent and the mean time of concentration was 1.49 hour. Thirty-nine out of 48 watersheds had lag times that were determined from an average of three or more measured hydrographs. A time of concentration was computed for each watershed after a field inspection by personnel of the NRCS or the USGS (McCuen et al., 1984). In each case, the watershed was subdivided into flow areas having similar characteristics and the NRCS velocity method was used to compute a time of concentration. This computed value of T_c was treated as the 'true' value (McCuen et al., 1984) and was the basis for comparing the empirical formulas; it was referred to as the 'measured' value of T_c. The mean of the measured T_c was 1.31 hours for watersheds with substantial overland flow and 1.54 hour for watersheds with significant channel flow. The estimated values were then compared to the measured T_c values. The empirical equation methods show substantially different biases. The Eagleson velocity method shows the smallest bias, almost zero; this was expected, as the Eagleson method is a velocity method similar to the one used to compute the measured value of T_c (McCuen et al., 1984).

McCuen and others (1984) stated that the velocity method is widely recognized as being the most accurate method for estimating T_c. From the results of their tests, it was determined that the Kerby-Hathaway method had the smallest bias for watersheds with substantial overland flow and the Kirpich method developed for Tennessee had the smallest bias for watersheds with substantial channel flow (McCuen et al., 1984).

McCuen and others (1984) used the database for 48 urban watersheds to develop two new equations. Stepwise regression was used to select the predictor variables. The following equation that includes three variables is the result of their analysis

$$T_c = 0.01462\, L_f^{0.552}\, i_2^{-0.7164}\, S_{fm}^{-0.2070} \tag{3.17}$$

where T_c is the time of concentration (hour), L_f is the total length of the flow path (feet), i_2 is the 2-year, T_c-hour rainfall intensity (inches per hour) and S_{fm} is the channel slope (feet per mile). The other equation incorporates the flow resistance Φ and is given as:

$$T_c = 0.0469\, L_f^{0.4450}\, i_2^{-0.7231}\, \Phi^{0.5517}\, S_{fm}^{-0.2260} \tag{3.18}$$

When compared with the statistics for equation 3.17, equation 3.18 did not result in a substantial increase in the goodness-of-fit statistics. Equation 3.17 should be used for estimating T_c, except when the channel is undergoing a hydraulic change or is substantially different from the typical channel for an area; in these cases, an estimate of **Φ** should be made and then equation 3.18 should be used to compute T_c (McCuen et al., 1984).

Schulz and Lopez (1974) used rainfall and runoff data from nine watersheds in the Denver metropolitan region as a cooperative program between the USGS and the Urban Drainage Flood Control District to estimate various urban watershed response times. Stepwise multiple regression analysis was used to develop two regression equations for the time of concentrations. T_c from the end of rainfall excess to the point of inflection on recession of the hydrograph was correlated to the volume of rainfall excess (E_{RF}), total volume of rainfall (V_{RF}), the density of paved curbed and guttered streets (D_{CGS}), and hydraulic radius (H_R).

$$T_C = 15.116\ D_{CGS}{}^{1.018} E_{RF}{}^{0.571} H_R{}^{0.783} \tag{3.19a}$$

$$T_C = 218.122\ E_{RF}{}^{0.756} H_R{}^{0.926}\ V_{RF}{}^{-0.751} \tag{3.19b}$$

The **U.S. Bureau of Reclamation** (USBR, 1973), **Izzard** (1946), **Morgali and Linsley** (1965), and **Aron and Erborge** (1973) also developed equations for estimating T_c and are listed in Table 3.4. The USBR equation was developed for California culvert design and was modified from the Kirpich method. Izzard's equation and the kinematic wave formulas (Table 3.4) were primary for overland flow. Su and Fang (2003) used a two-dimensional overland flow model to estimate travel time over flat terrains (rectangular plane). For small slopes or slopes near zero (S < 0.0005), a new formula to estimate T_c (minutes) was developed

$$T_c = C\, n^{0.43} L^{0.33} i^{-0.4} \tag{3.20}$$

where L is plane length (feet), i is rainfall intensity (inches per hour), C is a constant and ranges from 26.3 for 100 percent plane width at the most downstream as open boundary to 52.6 for only 20 percent of width at the most downstream as open boundary. The open boundary is an opening where outflow can leave from the study area.

(3.3.2) Basin Lag Time

As discussed in the previous section, Carter (1961), Eagleson (1962), and NRCS (1972) developed basin lag time equations in correlation to watershed parameters. In Snyder's method (1938) for developing a synthetic UH it is assumed that the lag time is constant for the particular watershed and is not influenced by the variation in the rainfall intensity. The lag time is given by the equation:

$$T_L = C_l C_t\ (L\ L_c)^{0.3} \tag{3.21}$$

where C_t is an empirical coefficient depending on the topography and derived from nearby gaged watersheds. Its values typically range from 1.8 to 2.2 (Snyder, 1938). Viessman and Lewis (2002) summarized typical C_t values at different locations in the United States. C_l is a conversion coefficient (0.75 in SI units and 1.0 in English units). L is the distance along the main stream from the basin outlet to the upstream divide (in kilometers or miles). L_c is the distance along the main stream (in kilometers or miles) from the outlet to a point on the stream nearest the centroid of the watershed area.

Nash (1957) developed one of the earliest formulations of the instantaneous UH or (IUH) as a two-parameter Gamma function. Nash viewed the watershed as a series of n identical linear storage reservoirs, each having the same storage coefficient, K. It was determined that the centroid of the distribution occurs at nK, which is equal to one definition for lag time. Rao et al. (1972) developed relations for urban areas greater than 5 square miles to correlate lag time with watershed area, net rainfall depth and duration, and percent impervious cover. Their equations are presented later by Rao and Delleur (1973).

Table 3.4. List of empirical equations used for the estimation of the time of concentration (after Viessman and Lewis, 2002).

Method and date	Formula for T_c (minutes)	Remarks
Kirpich (1940)	$T_c = 0.0078L^{0.77}S^{-0.385}$ L = length of channel/ditch from headwater to outlet in feet, S = average watershed slope in feet per foot.	Developed from SCS data for seven rural basins in Tennessee with well-defined channel and steep slopes (3 percent to 10 percent); for overland flow on concrete or asphalt surfaces multiply T_c by 0.4; for concrete channels multiply by 0.2; no adjustments for overland flow on bare soil or flow in roadside ditches.
USBR (1973) Design of Small Dams	$T_c = 60(11.9L^3 / H)^{0.385}$ L = Length of longest water course in miles, H = elevation difference between divide and outlet (feet)	Essentially the Kirpich formula; developed from small mountainous basins in California (USBR, 1973, pp. 67-71)."
Izzard (1946)	$T_c = \dfrac{41.025(0.0007i + c)L^{0.33}}{S^{0.333}\, i^{0.667}}$ C = retardance coefficient, L = length of flow path (feet), S =slope of flow path (feet per foot)	Developed in laboratory experiments by Bureau of Public Roads for overland flow on roadway and turf surfaces; values of the retardance coefficient range from 0.0070 for very smooth pavement to 0.012 for concrete pavement to 0.06 for dense turf; solution requires iteration; product i times L should be ≤ 500.
Federal Aviation Administration (1970)	$T_c = 1.8(1.1 - C)L^{0.50} / S^{0.333}$ C = rational method runoff coefficient, L = length of overland flow in feet S = surface slope (percent)	Developed from air field drainage data assembled by the USACE; method is intended for use on airfield drainage problems, but has been used frequently for overland flow in urban basins.
Kinematic Wave Formulas Morgali and Linsley (1965) Aron and Erborge (1973)	$T_c = \dfrac{0.94L^{0.6}\, n^{0.6}}{(i^{0.4}\, S^{0.3})}$ L = length of overland flow, in feet n = Manning roughness coefficient i = rainfall intensity inches per hour S = average overland slope feet per foot	Overland flow equation developed from kinematic wave analysis of surface runoff from developed surfaces; method requires iteration since both i (rainfall intensity) and t, are unknown; superposition of intensity-duration-frequency curve gives direct graphical solution for T_c.

Table 3.4. Continued.

SCS (1972) Lag Equation	$T_c = \frac{1.67L^{0.8}\left[(1000/CN)-9\right]^{0.7}}{1900S^{0.5}}$ L = hydraulic length of watershed (longest flow path), in feet; CN = SCS runoff curve number; S = average watershed slope, in percent	Equation developed by SCS from agricultural watershed data; it has been adapted to small urban basins less than 2,000 acres; found generally good where area is completely paved; for mixed areas it tends to overestimate; adjustment factors are applied to correct for channel improvement and impervious area; the equation assumes that T_c = 1.67 x basin lag
SCS(1972, 1986) Average Velocity Charts	$T_c = \frac{1}{60}\sum\frac{L}{V}$ L = length of flow path in feet; V = average velocity feet per second from Fig. 3-1 of TR 55 for various surfaces.	Overland flow charts in Ref. 20 provide average velocity as function of watercourse slope and surface cover.
Carter (1961) Lag Equation	$T_c=100L_m^{0.6}S_m^{-0.3}$	Using data from the Washington D.C. area, Carter calibrated an equation for predicting the watershed lag for watersheds with natural channels and partially sewered land uses. The length and slope variables in the equation should be measured from the longest channel.
Kerby-Hathaway (1959) Formula	$T_c= 0.83L^{0.47}n^{0.47}S^{-0.235}$	Calibrated by Kerby for computing the time of concentration on very small watersheds in which overland flow is dominated. Length used in this equation is the straight-line distance from the most distant point on the watershed to the outlet and measured parallel to the slope until a well-defined channel is reached.
Van Sickle (1962) Equation	$T_c= 0.55L_t^{0.13}L_m^{0.13}S_f^{-0.065}$	Van Sickle provided this equation calibrated from the data collected in Houston, with drainage area less than 36 square miles. The equation is based on two lengths L_m and L_t.

Bell and Kar (1969) suggested critical lag as a convenient characteristic time that may be related to each of the other characteristic times. Under most circumstances the critical lag is a suitable value for the critical duration of design rainfall. Because lag is supposed to be dependent primarily on the physical characteristics of a catchment, Bell and Kar (1969) suggested that the following modified form of Kirpich equation be used:

$$T_L = M\, L^{0.77} S^{-0.39} \tag{3.22}$$

where T_L is the critical lag (hours), L is the distance from the outlet to the most remote part of catchment along path of flow (miles), S is the slope of flow for L (feet per foot), and M is a constant based on the type of flood zones and is arranged from 1 to 3.4×10^{-4}.

Rao and Delleur (1973) summarized different definitions for the lag time as shown graphically in Figure 2.2. They also summarized various equations for average lag time that depend upon the physiographic characteristics of the watershed. Rao and Delleur (1973) concluded that the average lag time values could not be used for runoff prediction because they depend on various physiographic characteristics. Three new equations were developed by Rao and Delleur from a regression analysis. Each successive equation contains the most important explanatory variables:

$$T_4 = 0.78\, A^{0.496} L^{0.073} S^{-0.075} (1+U)^{-1.289} \tag{3.23a}$$

$$T_4 = 0.78\, A^{0.542} S^{-0.081} (1+U)^{-1.210} \tag{3.23b}$$

$$T_4 = 0.803\, A^{0.512} (1+U)^{-1.433} \tag{3.23c}$$

Rao and Delleur (1973) stated that a regression relation involving only the area of the basin and the urbanization factor U is as effective as the other two relations, which include the length of the main stream and the mean basin slope. The lag time was found to depend on two storm characteristics, the amount of rainfall excess and rainfall duration (Rao and Delleur, 1973), therefore, two more equations were developed to contain the meteorological characteristics as well as the physiographic characteristics:

$$T_4 = 0.831\, A^{0.458} (1+U)^{-1.66}\, P_E^{-0.267}\, T_R^{0.371} \tag{3.24}$$

$$T_4 = 0.731\, A^{0.943} (1+U)^{-4.303}\, P_E^{-2.114}\, T_R^{0.238} \tag{3.25}$$

where P_E is the excess precipitation inches and T_R is rainfall duration (hours). It was determined that the lag time is not a unique characteristic as it varies from storm to storm. To use the single linear reservoir method (Chow, 1964) for regeneration of the direct runoff hydrograph in small basins (< 5 square miles), the reservoir constant should not be taken as the average lag time (Rao and Delleur, 1973). The dependence of the reservoir constant on both basin and storm characteristics must be considered as the reservoir constant varies for different storms.

Schulz and Lopez (1974) developed six regression equations for the lag time, T_{LC} (between the centroid of rainfall excess and the centroid of direct runoff hydrograph). They correlated T_{LC} with duration of total storm rainfall (T_{10}), volume of rainfall excess (E_{RF}), total volume of rainfall (V_{RF}), percent imperviousness (U), density of paved curbed and guttered streets (D_{CGS}), density of paved streets and roads (D_{PSR}), total street and road density (D_{SR}), average capacity of urban area (C_Q), and hydraulic radius (H_R).

$$T_{LC} = 210.233\, T_{10}^{\ 0.031} E_{RF}^{\ 0.323} H_R^{\ 0.80} U^{-0.342} \tag{3.26a}$$

$$T_{LC} = 338.361\, T_{10}^{\ 0.048} E_{RF}^{\ 0.304} H_R^{\ 1.154} C_Q^{\ 0.145} \tag{3.26b}$$

$$T_{LC} = 125.817\, T_{10}^{\ 0.292} D_{CGS}^{\ -0.145} E_{RF}^{\ 0.339} H_R^{\ 0.925} \tag{3.26c}$$

$$T_{LC} = 147.547\, T_{10}^{\ 0.295} D_{PSR}^{\ -0.208} E_{RF}^{\ 0.336} H_R^{\ 0.924} \tag{3.26d}$$

$$T_{LC} = 131.996\, T_{10}^{\ 0.296} D_{SR}^{\ -0.147} E_{RF}^{\ 0.342} H_R^{\ 0.942} \tag{3.26e}$$

$$T_{LC} = 37.851\, T_{10}^{\ 0.467} C_Q^{\ 0.280} H_R^{\ 1.172} V_{RF}^{\ 0.252} \tag{3.26f}$$

Haktanir and Sezen (1990) developed two-parameter Gamma and three-parameter beta distributions as synthetic UHs for 10 watersheds in Anatolia. Regression analyses for peak discharge and lag time of 10 observed UHs were performed to develop the regression equations. For the lag time, the equation was given as

$$T_L = 0.2685 L_m^{0.841} \tag{3.27}$$

where L_m is the length of main channel (kilometers) and T_L is the lag time (hours).

Simas and Hawkins (2002) developed a regression equation of lag time (hours) from more than 3,100 rainfall-runoff events in 168 small watersheds ranging from 0.3 to 3490 acres (5.5 square miles) in the United States.

$$T_L = 0.0051 \times W^{0.594} \times S^{-0.150} \times S_{nat}^{0.313} \tag{3.28}$$

where W is the watershed width (feet) as the watershed area divided by the watershed length, S is the slope (the ratio between the maximum difference in elevation and the watershed longest flow-path length), and S_{nat} is storage coefficient (inches) used in the Curve Number (CN) method. This lag time was defined as the difference between the centroid of effective rainfall and the centroid of direct runoff, which creates some difficulties when the equation is applied.

(3.3.3) Time to Peak

The time to peak (T_p) is an important parameter for developing the synthetic unit hydrograph. In the Snyder (1938) method among others, T_p is equal to half the duration of the UH plus the basin lag time:

$$T_p = \frac{D}{2} + T_L \tag{3.29}$$

Some studies assumed rise time to be synonymous with time to peak and suggested that they are reasonably constant for a particular catchment (Ramser, 1927), while Bell and Kar (1969) concluded that rise times for a particular catchment are far from being constant, in fact, they may commonly vary from about 40 percent to 200 percent of the median value.

Gray (1961) developed synthetic UHs for the geographical area of central Iowa, Missouri, Illinois and Wisconsin; an approximate upper limit for watershed size is 94 mi^2. This method is based on dimensionalizing the incomplete Gamma distribution and results in a dimensionless graph of the equation:

$$Q_{t/P_R} = \frac{25.0(\gamma')^q}{\Gamma(q)}\left(e^{-\gamma' t/P_R}\right)\left(\frac{t}{P_R}\right)^{q-1} \tag{3.30}$$

where Q_{t/P_R} is the percent flow at any given t/P_R value (t is always in 0.25 P_R increment), P_R is the period of rise or time to peak (minutes), t is time (minutes), q is the shape parameter, and γ is the scale parameter with $\gamma' = \gamma P_R$ and $q = 1 + \gamma'$. By using the curves of $L/\sqrt{S_c}$ versus P_R/γ' (Fig. 3.4) and P_R/γ' versus P_R (Fig. 3.5), time to peak from watershed parameters can be estimated.

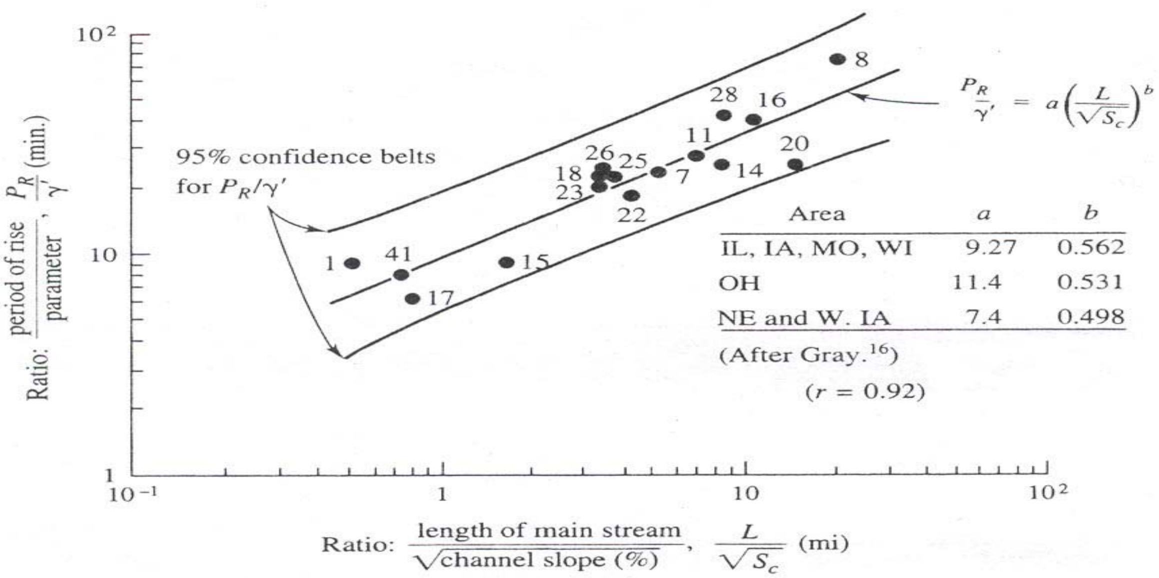

Figure 3.4. Relation of storage factor, P_R/γ', and watershed parameter, $L/\sqrt{S_c}$, for watersheds in Nebraska, Iowa, Missouri, Illinois, and Wisconsin (after Gray, 1961).

Wu (1963) correlated the hydrograph parameters - time to peak (T_p in hours) to the watershed area (A in square miles), length of the mean stream (L in miles) and the mean slope of main stream (Sx10^{-4}) as the regression equation:

$$T_p = \frac{31.42\, A^{1.085}}{L^{1.233}\, S^{0.668}} \tag{3.31}$$

Espey and others (1966) used a regression analysis of data from 24 urban and 11 rural watersheds to derive equations for the rise time to study the effects of urbanization. For rural conditions with data from Texas, New Mexico and Oklahoma, the regression equation was given as

$$T_R = 2.65 L^{0.12} S^{-0.52} \tag{3.32}$$

where T_R is the time of rise (minutes), L is the main channel length (feet), and S is the main channel slope (feet per foot). They also presented a equation of T_R as a function of $L/\sqrt{s}$:

$$T_R = 1.24(L/\sqrt{S})^{0.36} \tag{3.33}$$

For urban watersheds, impervious cover (I) was considered in the regression equation:

$$T_R = 20.8\phi L^{0.29} S^{-0.11} I^{-0.61} \tag{3.34}$$

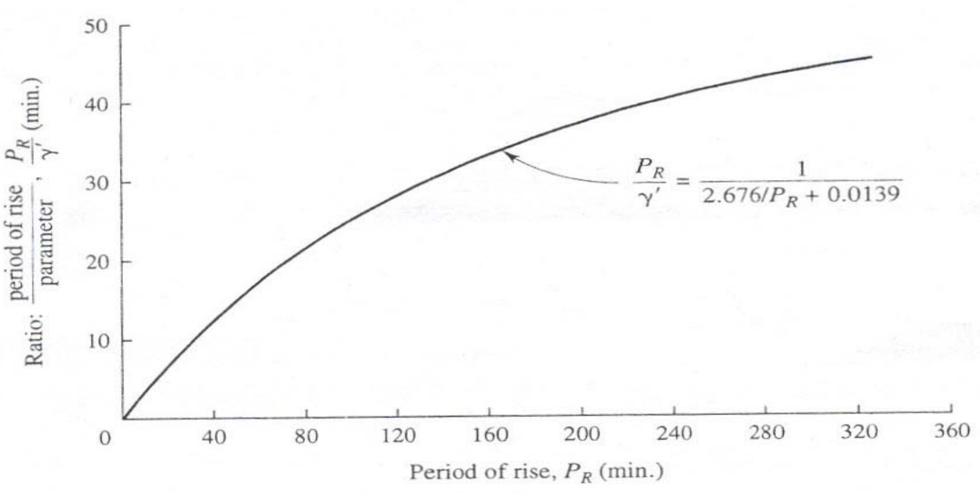

Figure 3.5. Relation for storage factor, P_R/γ', and period of rise P_R (after Gray, 1961).

Espey and Altman (1978) developed a set of regional regression equations to provide seven points representing a 10-minute UH using data from 19 urban watersheds. The entire UH was developed by graphically fitting a smooth curve through the points. The equation for time to peak was given as:

$$T_p = 3.1L^{0.23} S^{-0.25} I^{-.18} \phi^{1.57} \tag{3.35}$$

L is the total distance (feet) along the main channel from the point being considered to the upstream watershed boundary. S is the main channel slope (feet per foot) defined by $H/0.8L$ where H is the difference in elevation between the point on the channel bottom at a distance of $0.2L$ downstream from the upstream watershed boundary and a point on the channel bottom at the downstream point considered. I is the percent impervious area. Φ is the dimensionless watershed conveyance factor.

The **University of Colorado at Denver** (1985) developed the Colorado Urban Hydrograph Procedure (CUHP) and developed a regional 5-minute synthetic UH by using the data from the Denver metropolitan area. The method was based on Snyder's method and was applicable to watersheds with drainage areas ranging from 90 acres to 6,400 acres (10 square miles). W_{50} and W_{75} are two key parameters used to construct Snyder UHs, and are widths of unit hydrographs at discharges equal to 50 and 75 percent of the peak discharge. For plotting W_{50} and W_{75}, the smaller time intervals of 35 percent or 0.6 T_p and 45 percent or 0.424 T_p are placed left of the peak, where T_p is the time from the beginning to the peak discharge of the UH. Because it was supposed that C_t in equation (3.21) and slope S are correlated (Snyder, 1938), the original CUHP procedure was altered to recognize the relation and T_p, rather than lag time T_L in Snyder UH, was used and found to be:

$$T_p = C_t\left(LL_c / \sqrt{S}\right)^{0.48} \tag{3.36}$$

where C_t is obtained as functions of P_a (percent impervious cover) from a graph developed for CHUP. For the small watersheds (< 90 acres), time to peak (in minutes) is given as

$$T_p = 0.39\left(\frac{P_a^2 - 0.36P_a + 0.07}{P_a^2 - 0.49P_a + 0.14}\right)\left(T_c - 6P_a\right) \tag{3.37}$$

Williams and Hann (1973) developed a problem-oriented computer language for **hy**drologic **mo**deling (**HYMO**) to predict surface runoff from watersheds; HYMO was developed for the Agricultural Research Service. The UH for HYMO consists of three parts. From the beginning of the rise to the inflection point (first zone), it is a two-parameter Gamma function with n as a dimensionless shape parameter followed by two exponential decay curves (Williams and Hann, 1973; Viessman and Lewis, 2002).

$$q = q_p\left(T / T_p\right)^{n-1} e^{(1-n)\left(T / T_p - 1\right)} \tag{3.38}$$

To estimate T_p (hours), the Williams and Hann (1973) model uses the following regression equation:

$$T_p = 1.44(A)^{0.422}\left(\frac{L}{W}\right)^{0.133}\left(SLP\right)^{-0.46} \tag{3.39}$$

where L is the watershed length along the main channel from the outlet to the basin divide (kilometers), W is the average watershed width (kilometers), SLP is the slope defined as the difference in elevation divided by the watershed length (feet per mile).

James and others (1987) analyzed 283 storms from 13 states and correlated the physical characteristics of watersheds with time to peak (T_p). The data used were from 85 different watersheds ranging in size from 0.73 to 62.2 square kilometers that represented a wide variety of terrain and climates. The correlation equation developed from 31 test and 17 verification watersheds were

Mild slope (< 5 percent)

$$T_p = 0.85(A)^{0.9}\left(HT\right)^{-0.1}\left(L\right)^{-0.6} \tag{3.40a}$$

Intermediate slope (5 - 10 percent)

$$T_p = 0.92(A)^{0.5}\left(HT\right)^{-0.2}\left(L\right)^{-0.2} \tag{3.40b}$$

Steep slope (>10 percent)

$$T_p = 0.91(A)^{0.2}\left(HT\right)^{-0.3}\left(L\right)^{0.8} \tag{3.40c}$$

where A is the watershed area (square kilometers), L is the main channel length (kilometers), HT is the difference in elevation between divide and outlet (meters).

Schulz and Lopez (1974) developed eight equations for time to peak (T_p) starting from the beginning of rainfall excess to the peak of the UH. The equations are based on the correlation between T_p and lag time T_{LC} (between the centroid of rainfall excess and the centroid of direct runoff hydrograph), duration of total storm rainfall (T_{10}), volume of rainfall excess (E_{RF}), percent imperviousness (U), the slope of curbed and guttered streets (S_{CGS}), and hydraulic radius (H_R).

$$T_p = 3.108\, T_{LC}^{0.554} \tag{3.41a}$$

$$T_p = 2.361\, T_{LC}^{0.594} E_{RF}^{-0.064} \tag{3.41b}$$

$$T_p = 3.533\, T_{LC}^{0.567} E_{RF}^{-0.078} S_{CGS}^{-0.442} \tag{3.41c}$$

$$T_p = 1.461\, T_{LC}^{0.680} E_{RF}^{-0.150} H_R^{-0.217} S_{CGS}^{-0.518} \tag{3.41d}$$

$$T_p = 4.624\, T_{LC}^{0.606} T_{10}^{-0.153} \tag{3.41e}$$

$$T_p = 3.379\, T_{LC}^{-0.625} T_{10}^{-0.115} E_{RF}^{-0.502} \tag{3.41f}$$

$$T_p = 1.855\, T_{LC}^{0.708} T_{10}^{-0.129} E_{RF}^{-0.096} H_R^{-0.146} \tag{3.41g}$$

$$T_p = 2.051\, U^{0.855}\; T_{LC}^{0.675} T_{10}^{-0.114} E_{RF}^{-0.076} \tag{3.41h}$$

Schulz and Lopez (1974) also developed four equations for the time to peak defined as the time interval between the beginning of rainfall excess and the peak of the direct runoff. The authors further developed 16 regression equations for the time to peak defined as the time interval between the beginning of rainfall and the peak of the runoff. These equations were developed to correlate among watershed response timing parameters, not watershed and rainfall characteristics alone, which created difficulties in application of the equations for normal hydrologic design practices.

Meadows and Ramsey (1991) developed a synthetic UH for South Carolina by studying 31 urban watersheds with 10 to 80 events per watershed. Two physiographic provinces were involved in developing UHs: Upper Coastal Plain (UCP) and Lower Coastal Plain (LCP). Meadows and Ramsey (1991) used a two-parameter Gamma function as their regional synthetic UH and developed three sets of regression equations for the peak rate factor (PRF or K in equation 1.2 for NRCS method) and the time to peak (T_p). Several investigators have recognized that PRF is a function only of the shape factor n for the Gamma function (Meadows and Blandford, 1983; Neidrauer, 1988; Meadows and Ramsey, 1991, Khanal, 2004), therefore, it was determined that two parameters, PRF and T_p, can fully define the regional Gamma UH. The equations for the UCP are:

$$PRF = 80\frac{IMP^{0.31}}{A^{0.15}} \tag{3.42a}$$

$$T_p = \frac{1.53}{(1.0 + BDF)^{0.98}} \left(\frac{L_C}{\sqrt{S_C}} \right)^{0.54} \tag{3.42b}$$

where A is watershed area (square miles), L_c and S_c are main channel length (miles) and slope between 10 percent and 85 percent of the channel length, *BDF* is the basin development factor proposed by Sauer et al. (1981), and *IMP* is the percent impervious cover. The equations for the LCP are:

$$PRF = 136 \frac{A^{0.12} IMP^{0.28}}{(1 + BDF)^{0.33}} \tag{3.43a}$$

$$T_p = \frac{0.51}{S_C^{1.29} (1.0 + BDF)^{0.75}} \tag{3.43b}$$

Recognizing the basin development factor is difficult to estimate for some watersheds, therefore, alternative equations were developed that involve only imperviousness, main channel length and slope. The equations for the UCP are:

$$PRF = 127 \frac{IMP^{0.24}}{L_C^{0.47} S_C^{0.12}} \tag{3.44a}$$

$$T_p = \frac{0.41}{IMP^{0.23}} \left(\frac{L_C}{S_C} \right)^{0.36} \tag{3.44b}$$

The equations for the LCP are:

$$PRF = 248 \frac{L_C^{0.24} S_C^{0.20}}{IMP^{0.07}} \tag{3.45a}$$

$$T_p = \frac{0.94}{S_C^{1.39} IMP^{0.67}} \tag{3.45b}$$

(3.3.4) Time Base

There are comparatively few studies on time base (T_B) for the UH. When mathematical functions (such as the Gamma distribution) are used for synthetic UH, the model parameterizations do not utilize time base as a separate parameter from other timing parameters such as T_p. Espey and others (1966) developed the hydrograph width (time base in minutes) for both urban and rural watersheds:

$$\text{Rural area: } T_B = 7.41x10^3 A^{0.64} Q_p^{-0.53} \tag{3.46a}$$

$$\text{Urban area: } T_B = 4.44x10^5 A^{1.17} Q_p^{-1.19} \tag{3.46b}$$

where A is the watershed area (square miles) and Q_p is the peak discharge (cubic feet per second). Espey and Altman (1978) developed a regression equation for time base as:

$$T_B = 125.89 \times 10^3 A\ Q_p^{-0.95} \qquad (3.47)$$

(3.4) Relations Among Timing Parameters

The NRCS (1972) assumed the empirical relation for average lag time to be $0.6T_c$, where T_c is the time of concentration. T_c is the time from the end of rainfall excess to the inflection of the UH (T_{in}). The NRCS UH has a point of inflection approximately 1.7 times the time to peak (T_p); the time to peak is 0.2 times the time base (T_B).

$$T_c = T_{in} - D = 1.7T_p - D \qquad (3.48)$$

Haan et al. (1994) and Singh (2000) determined that the two-parameter Gamma function (equation 3.49) can be used as a good approximation for dimensionless NRCS UH:

$$Q / Q_p = (t / T_p)^n \, e^{(1-t/T_p)n} \qquad (3.49)$$

where n is the shape factor controlling the UH distribution. Haan et al. (1994) used n = 4.77 and Singh (2000) used n = 4.7 for the two-parameter Gamma UH. Figure 3.6 shows a comparison between NRCS DUH and Gamma UH with n = 4.7.

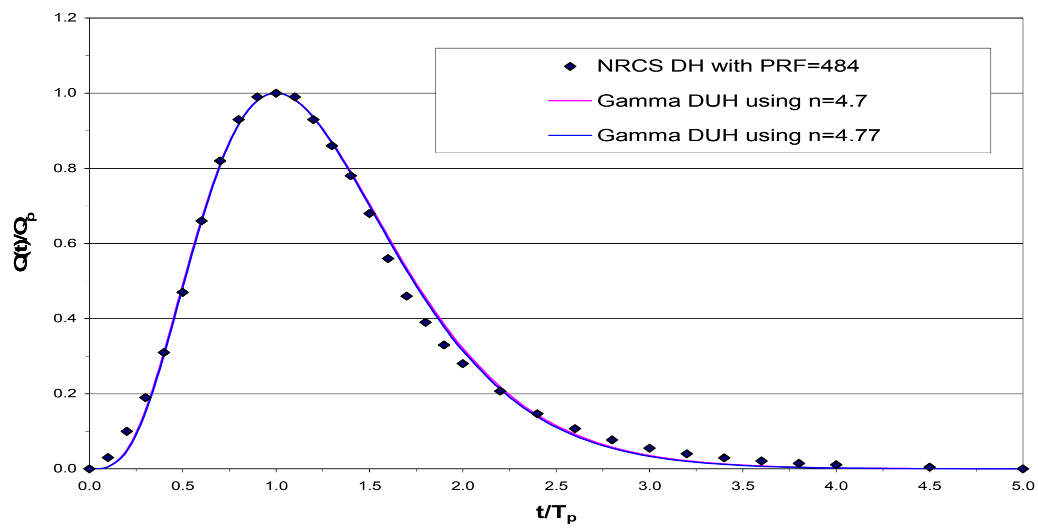

Figure 3.6. Comparison of NRCS UH (stars) with Gamma UH approximation (lines).

The shape parameter n is equivalent to $\alpha+1$. The variable α is used in other publications for Gamma function UHs, for example, Viessman and Lewis (2002). The ordinates or discharges of the Gamma UH at any time t can be calculated by equation (3.50) and by expressing t as a multiple of T_p **(**$t = cT_p$ or c = t/T_p**):**

$$Q_{(t=cT_p)} = Q_p c^{\alpha} e^{(1-c)\alpha} \qquad (3.50)$$

This equation can be used to solve for all the ordinates of UHs.

It is useful to determine the inflection from the above equation by taking the first and second derivatives of $Q(t)$ with respect to time t:

$$\frac{dQ(t)}{dt} = \frac{Q_p}{T_p} \alpha c^{\alpha - 1} e^{(1-c)\alpha} (1 - c) \tag{3.51}$$

$$\frac{d^2 Q(t)}{dt^2} = \frac{\alpha Q_p}{T_p} c^{\alpha - 2} e^{(1-c)\alpha} (\alpha c^2 - 2\alpha c + \alpha - 1) \tag{3.52}$$

If the second derivative is set to zero to determine the time (T_{in}) for the inflection point (a point on a curve at which the sign of the curvature, i.e., the concavity changes) on the Gamma UH, the results are

$$T_{in} = (1 + 1/\sqrt{\alpha})T_p \tag{3.53}$$

The equation gives $T_{in} = 1.52\ T_p$ for the NRCS dimensionless UH if the Gamma approximation has an alpha value of 3.7 (or $n = 4.7$). The NRCS (1972) DUH has the point of inflection at approximately $1.7T_p$. Therefore, the Gamma approximation of the NRCS DUH gives a slightly different inflection point. Table 3.5 lists the relation between the inflection point and time to peak for various alpha values after applying equation (3.53).

Table 3.5. Relation between T_p and T_{in} for Gamma UH.

Gamma Parameter, α	Gamma Parameter, n	Time to Inflection Point
1	2	$T_{in} = 2.00\ T_p$
2	3	$T_{in} = 1.71\ T_p$
2.5	3.5	$T_{in} = 1.63\ T_p$
3	4	$T_{in} = 1.58\ T_p$
3.7	4.7	$T_{in} = 1.52\ T_p$
4	5	$T_{in} = 1.50\ T_p$
5	6	$T_{in} = 1.45\ T_p$

Meadows and Blandford (1983) examined the relation between T_{in} and T_p for Gamma UH and also obtained equation (3.53). The UH duration was assumed to be 0.2 times the lag time T_L (Meadows and Blandford, 1983); actually the NRCS DUH procedures use $D \approx 0.133T_C = 0.133/0.6T_L = 0.22T_L$. By using other NRCS relations ($T_p = T_L + D/2,\ T_c = T_{in} - D$), Meadows and Blandford (1983) developed the following equation:

$$T_c = (0.818 + 1/\sqrt{\alpha})T_p \tag{3.54}$$

T_c ranges from 1.15 to 1.82 times T_p when the Gamma UH shape factor n ranges from 10 to 1 (Meadows and Blandford, 1983). If the UH duration (D) is small, for example, a 5-minute Gamma UH, that is D = 0.08 hour, from equation (3.53), one may take:

$$T_c \cong T_{in} = (1 + 1/\sqrt{\alpha})T_p \quad or \quad T_p = \frac{\sqrt{\alpha}}{1+\sqrt{\alpha}} T_c \tag{3.55}$$

Therefore, for the relation between time of concentration and time to peak ($T_p = X*T_c$), the factor X is not exactly a constant. At values of α ranging from 1 to 5, the factor varies from 0.5 to 0.69.

CHAPTER 4. ESTIMATION OF WATERSHED CHARACTERISTICS

4.1 Introduction

As discussed in previous chapters, timing parameters for hydrographs have been correlated with various watershed and rainfall event parameters. This chapter reviews definitions and methods for computing watershed parameters. This will help researchers to develop relations between time and watershed parameters in the future.

The concept of a watershed is basic to all hydrologic designs (McCuen, 1998). It is necessary to define the watershed in terms of a point – usually the location at which the design is made, referred to as the watershed "outlet." A watershed consists of all land area that contributes water to the outlet during a rainstorm. A watershed also is defined by all points enclosed within an area from which rain falling at these points contributes water to the outlet (McCuen, 1998). Figure 4.1 shows watersheds delineated for design outlets A and B; the watershed for outlet A is larger than for outlet B (dashed line). All points on a stream, tributary, or river have an associated watershed, and small watersheds join to become larger watersheds. It is feasible to delineate watersheds using a topographic map that shows stream channels. Watershed boundaries often follow major ridgelines ("drainage divides") around channels and meet at the outlet, where water flows out of the watershed.

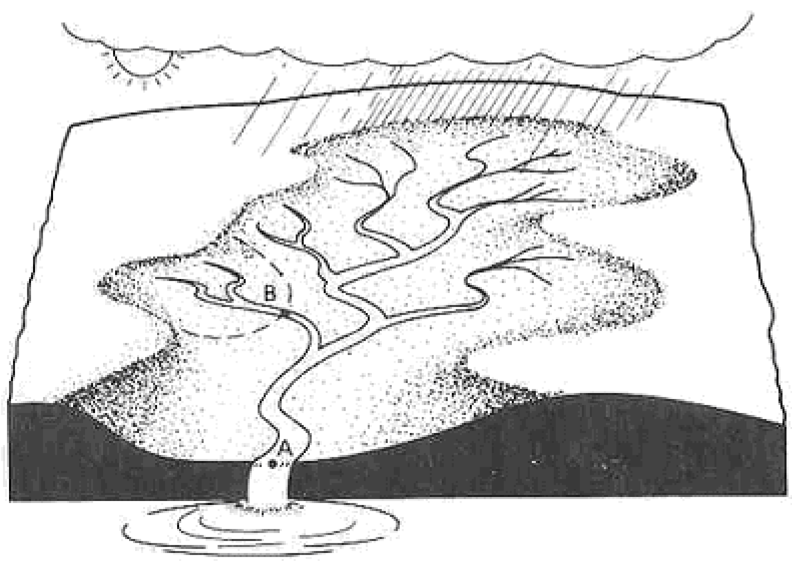

Figure 4.1. Delineation of watershed boundary (from McCuen, 1998).

4.2 Watershed Characteristic Definitions

Watershed characteristics defined and discussed here include drainage area or watershed area, main channel length, channel slope, and parameters depicting watershed (basin) shape. These are watershed characteristics that can be reasonably straightforward to obtain or estimate for the application of hydrologic design by designers or engineers.

4.2.1 Drainage Area

Drainage area (DA) is the most important watershed characteristic for hydrologic design. It reflects the volume of water generated from rainfall. It is common in hydrologic design to assume a constant depth of rainfall occurring uniformly over the watershed. Thus, the volume of water available for runoff is the product of rainfall depth and the drainage area. The drainage area is a required input to models ranging from simple linear prediction equations to complex computer models.

Computing watershed drainage areas requires the delineation of the watershed boundaries. At present, geographic information systems (GIS) commonly are used to delineate watershed boundaries from a digital elevation model (DEM). The methods used for computing drainage areas include: (1) mechanical planimeter method, (2) numerical planimeter method, (3) GIS method, and (4) dot grid method.

4.2.2 Watershed and Channel Lengths

The length of a watershed and a channel is another important parameter for hydrologic design. Watershed length, also referred to as the basin length (BLENG), is commonly defined as the straight line distance intersecting the main channel from the watershed outlet to the basin divide. In hydrologic studies main channel length (*MCL*, *L*, or L_{ch}) or the longest channel length is often used in many empirical equations for estimating hydrologic design parameters. Channel length is commonly defined as the distance measured along the main channel or the center of channel (e.g. blue line on a USGS topographic map) from the watershed outlet to the basin divide.

Hack (1957) studied streams in Pennsylvania and Virginia and later extended his findings to a wide variety of rivers around the world. Hack (1957) found a consistent relation between the longest channel and the drainage area,

$$L = kA^n \tag{4.1}$$

where, L is the longest channel length in miles, A is the watershed drainage area in square miles, k is a coefficient varying from 1 to 2.5 with an average of 1.4 and n is an exponent which varies from 0.6 to 0.7 with an average value of 0.6. These results were based on observations of natural watersheds where the channel systems were free to evolve. In the case of urban watersheds, part or sometimes all of the channel systems are man made and therefore may not be free to evolve into other networks. The channel selected to represent the whole watershed is usually the longest channel in the watershed (Schulz and Lopez, 1974).

Snyder (1938) and McCuen (1998) used the length to the centroid of a watershed (L_{ca}), which was defined as the distance in miles measured along the main channel from the watershed outlet to a point nearest the center (centroid) of the basin.

4.2.3 Watershed and Channel Slopes

Flood magnitudes reflect the momentum of the runoff. Slope is an important factor in the momentum (McCuen, 1998). Both watershed and channel slope are of interest and important parameters for hydrologic design. The contour-band method can be used to estimate an average basin or watershed slope (BS), which is the total length of all selected elevation contours times contour interval, divided by the contributing area (Brown et al., 2000).

McCuen (1998) stated that watershed slope reflects the rate of change of elevation with respect to distance along the principal flow path. Typically, the principal flow path is delineated and watershed slope (*S*) is computed as the difference in elevation (H in Fig. 4.2) between the end points of the principal flow path, the watershed outlet and basin divide, divided by the length of flow path (*L*). The elevation difference is not necessarily the maximum elevation difference within the watershed because a point of higher elevation may occur along a side boundary of the watershed rather than at the end of the principal flow path(McCuen, 1998).

$$S = H / L \tag{4.2}$$

The above definition for the watershed slope by McCuen (1998) is often referred to the main channel slope in other literature. This definition is unambiguous and simple but still correlates with Q_T (T is return period, Asquith and Slade, 1997). Schulz and Lopez (1974) stated that the definition in equation (4.3) may be faulty because too great emphasis may be placed on the steep slopes in the headwater regions, which have substantial hydraulic separation from the outlet. Nash and Shaw (1966) have suggested the equation for determining channel slope as:

$$S = \frac{2\sum L_i Z_i}{\left(\sum L_i\right)^2} \tag{4.3}$$

where L_i is the distance along the main stream between successive contours and Z_i is the average elevation above the outlet for each reach length L_i (Fig. 4.2, Schulz and Lopez, 1974). Reich (1962) and Laurenson et al. (1963) described the slope quantity as the slope of a straight line joining the elevation of the outlet on the profile of the main stream with the average elevation of the actual stream profile. The average main channel slope can be developed by drawing a straight line (Fig. 4.2) such that the area under the line is equal to the area under the profile diagram (hypsometric curve).

$$S_{AVG} = \Delta H / L \tag{4.4}$$

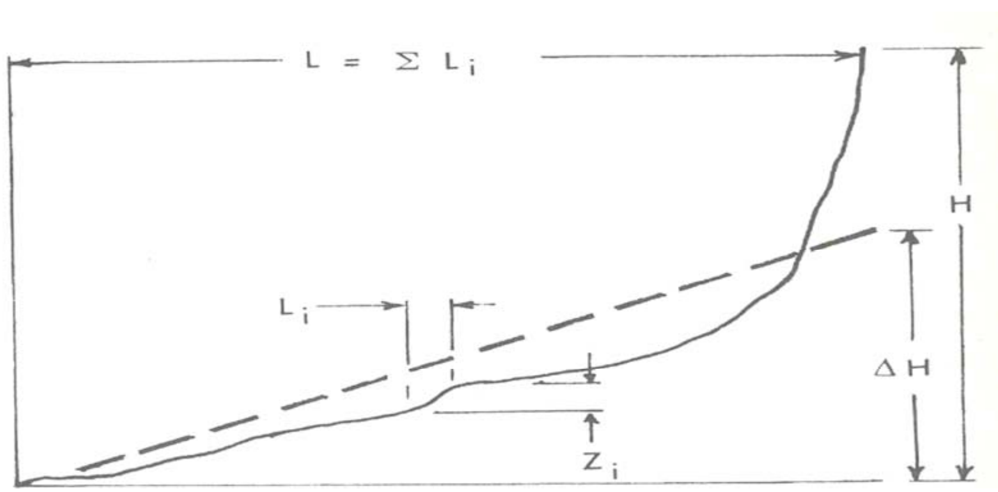

Figure 4.2. Definition of average main channel slope (from Schulz and Lopez, 1974).

A simpler definition of the channel slope than equation (4.3) is in Laurenson et al. (1963), which was previously suggested by Benson (1959). They stated that the greatest emphasis is placed on 75 percent of the main channel length (the longest channel extended to watershed divide), which for most watersheds collects the majority of the flood runoff:

$$S = \frac{Elevation\ at\ 0.85L - Elevation\ at\ 0.1L}{0.75L} \quad (4.5$$

Wu (1963) obtained a mean slope of the main channel by studying topographic maps.

Wu (1963) used the method developed by Taylor and Schwarz (1952) to determine the mean slope as

$$S_{mean} = \left(\frac{1}{1/\sqrt{S_1} + 1/\sqrt{S_2} + 1/\sqrt{S_2} + ... + 1/\sqrt{S_n}} \right)^2 \quad (4.6)$$

where 'n' represents the number of reaches of equal length, and S_1 to S_n are the slopes of each small reach.

4.2.4 Watershed Shape

The characteristics that reflect the basin shape are also used in regional regression equations (for example, Asquith and Slade, 1997); shape typically is not as important as area and slope. Several watershed shape factors that were considered by McCuen (1998), correlated variously to peak discharge. To examine the association, seven hypothetical watersheds with the same drainage area (Fig. 4.3) were evaluated by McCuen (1998).

A number of watershed shape characteristics considered by McCuen (1998) are the following:

(1) Shape factor $L_s = (L\ L_{ca})^{0.3}$, where L_{ca} is the channel length to the center of basin in miles, L is the channel length in miles.

(2) Circularity ratio F_c: $F_c = \frac{P}{(4\pi A)^{0.5}}$, where P and A are the perimeter (feet) and area (square feet) of the watershed, respectively.

(3) Circularity ratio R_c: $R_c = A/A_o$, where A_o is the area of a circle having a perimeter equal to the perimeter of the basin.

(4) Elongation ratio R_e: $R_e = \frac{2}{L_m}\left(\frac{A}{\pi}\right)^{0.5}$, where L_m is the maximum length (feet) of the basin parallel to the principal drainage lines.

McCuen (1998) found that the peak discharge is strongly correlated with the length to the center of area (L_{ca}), the shape factor $L_s = (L\ L_{ca})^{0.3}$, and the elongation ratio R_e. The peak discharge is very weakly correlated with both circularity ratios. McCuen (1998) also suggests that $L\ L_{ca}$ represents a reasonable predictor of peak discharge because it correlates well with peak discharge, and peak discharge correlates with the lag time or the time to peak. This also is the geometric characteristic used by Snyder (1938) for predicting the basin lag time for Snyder's synthetic UH.

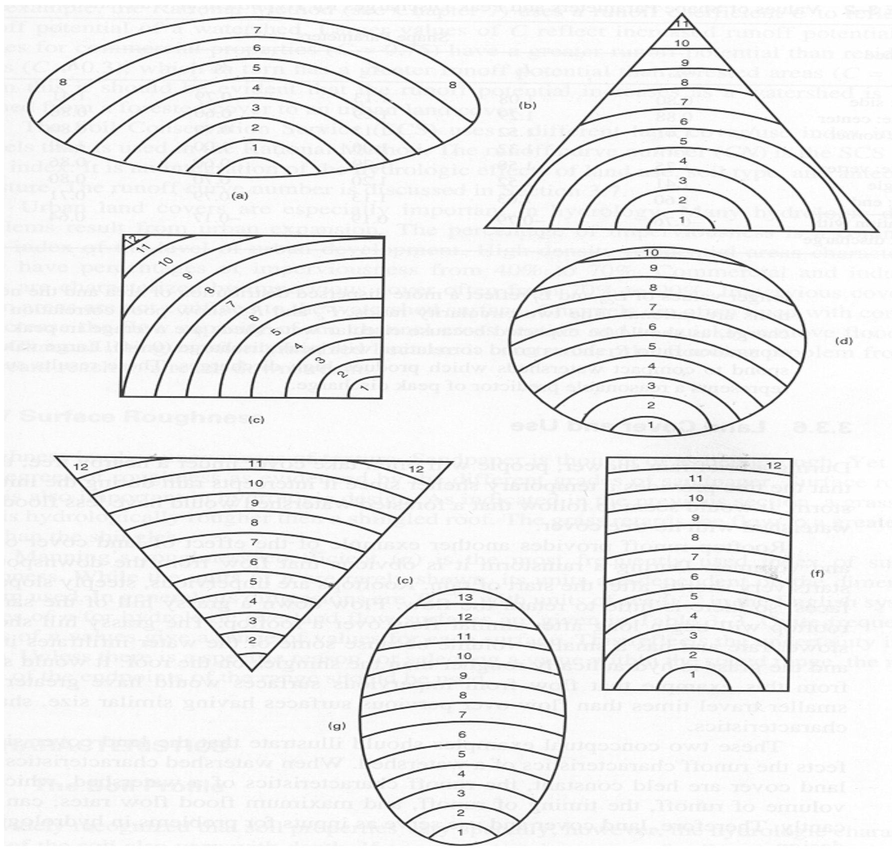

Figure 4.3. Hypothetical watersheds (a) ellipse: side; (b) triangle: center; (c) square: corner; (d) circle; (e) triangle: vertex; (f) rectangle; (g) ellipse: end (McCuen, 1998).

The following five geometric characteristics were evaluated by Wu (1963): (a) drainage area (*A*); (b) length of main stream (*L*); (c) mean slope of main stream (*S*), which was determined using the method introduced by Taylor and Schwarz (1952); (d) watershed shape factor – $f = P/P'$, where *P* is the perimeter of the basin and $P' = 2\sqrt{\pi A}$ is the perimeter of a circle of equal area; and (e) valley shape coefficient – *v* determined by the hypsometric analysis developed by Langbein (1947). Wu (1963) concluded that both the time to peak and the reservoir storage coefficient were weakly correlated to the watershed shape factor (*f*) and the valley shape coefficient (*v*).

4.3 Watershed Characteristics Recently Considered by USGS in Texas

Modern computer technology allows for the generation of preliminary watershed boundaries in a fraction of the time needed for manual methods. The USGS has recently developed an automated process using GIS to reduce development time and improve the quality of watershed boundaries and characteristics (Brown et al., 2000). This process was not available for the basin characteristics used by Asquith and Slade (1997). These watershed data can be compiled in a permanent geodatabase (spatial database) to provide a stable base dataset that offers users greater confidence when further subdividing watersheds.

A standardized dataset of watershed characteristics is a valuable resource that can be used to understand and manage natural resources. The vertical integration of input datasets used to automatically generate watershed boundaries is crucial to the success of such an effort. The optimum situation is to use the digital orthophoto quadrangles as the source of all input datasets (Brown et al., 2000). The hydrographic data from the digital line graphs can be revised to match the digital orthophoto quadrangles. The revised data should then be used to create an updated digital elevation model incorporating the stream channels as revised from the digital orthophoto quadrangles (Brown et al., 2000). Computer-generated, standardized watersheds that are vertically integrated with existing digital line graph hydrographic data continue to be difficult to create. Until revisions can be made to existing source datasets, manual editing is necessary to revise constructed features and changes in the natural landscape not reflected in the DEM.

The following automated watershed characteristics were considered by the USGS Brown and others, (2000) and were revised using improved GIS software for this study.

(1) Basin-Area Computations:

Total drainage area (TDA): Computed upon delineation of watershed boundaries.

Non-contributing drainage area (NCA): NCA is considered to be sinks in the DEM.

Contributing drainage area (CDA): TDA – NCA

(2) Basin-Length Computations:

Basin length (BLENG): The length of the straight centerline of the longest flow path from the watershed outlet to the basin divide.

Basin perimeter (BP): Computed upon delineation of watershed boundaries.

(3) Basin-Relief Computations:

Average basin slope (BS): This is average topographic slope of entire watershed. BS = (total length of all selected elevation contours) (contour interval)/CDA.

Minimum basin elevation (MNELEV): Derived from the DEM.

Maximum basin elevation (MXELEV): Derived from the DEM.

Basin divide elevation (BDELEV): Derived from the DEM at the point where the main channel meets the basin divide.

Outlet elevation (OUELEV): Derived from the DEM at the watershed outlet.

Basin relief (BR): MXELEV-OUELEV, which is the difference between the highest cell value in the elevation grid of the basin and the elevation of the grid cell at the outlet.

(4) Basin-Aspect Computations:

Basin azimuth (BA): The clockwise direction measured from the basin divide at 0 degrees to the watershed outlet.

(5) Basin Computations:

Effective basin width (BW): BW = CDA / BLENG

Basin shape factor (SF): SF = BLENG / BW

Elongation ratio (ER): $ER = [4\ CDA / \pi\ (BLENG)^2]^{0.5} = 1.13\ (1 / SF)^{0.5}$

Rotundity of basin (RB): $RB = [\pi\ (BLENG)^2] / [4\ CDA] = 0.785\ SF$

Compactness ratio (CR): $CR = BP / [2\ (\pi\ CDA)^{0.5}]$

Relative relief (RR): RR = BR / BP

(6) Channel- or Stream-Length Computations:

Main channel length (MCL): The distance measured along the main channel (longest flow path) from the watershed outlet to the basin divide.

Basin factor/characteristic (BFC): $BFC = MCL^2/CDA$.

(7) Channel-Relief Computations:

Main channel slope (MCS) (Brown et al., 2000): Computed as the difference in elevation at 10 percent (E10) and 85 percent (E85) of the distance along the main channel from the outlet to the basin divide. MCS = (E85 – E10) / 0.75 (MCL), and alternate method is MCS = (E85-E10)/(L85-L10), where L85 and L10 are channel lengths up to 85 and 10 percent of the total length.

Main channel slope (MCS2): MCS2 = (BDELEV-OUELEV)/MCL, the ratio of the basin divide elevation minus the outlet elevation to the main channel length (Asquith and Slade, 1997).

(8) Channel or Stream Computation

Main channel sinuosity ratio (MCSR): MCL/BLENG

Slope ratio of main channel slope to basin slope (SR): MCS/BS

CHAPTER 5. SUMMARY AND CONCLUSIONS

The literature dealing with timing parameters of hydrographs is substantial. Many researchers have developed timing parameters for specific hydrologic applications or topical purposes. The literature review was conducted in two major directions: (1) a review and assessment of existing common methods for defining and estimating timing parameters of direct runoff hydrographs and unit hydrographs and (2) a review and assessment of methods for correlating (regionalizing) timing parameters with readily obtainable watershed characteristics and rainfall characteristics. The literature review has provided comprehensive information for the project.

Chapter 2 provided a synthesis of hydrologic timing parameter nomenclature with explanations of terms. Timing parameters with multiple meanings, for example, the time of concentration and basin lag time, have been defined. Table 5.1 provides a glossary and suggested definitions of common timing parameters, which are recommended for the project and for TxDOT hydrologic designs. Chapter 3 summarized the NRCS (1972) velocity method to estimate time of concentration for overland flow and channel flow. The particle tracking method could provide independent methodology to estimate timing parameters and was briefly discussed in Chapter 3. Appendix A provides additional information on preliminary research plans and results for the particle tracking method.

Table 5.1. Glossary of four timing parameters of hydrographs.

Parameters	Suggested Definitions
Time of concentration	(1) The time it takes a water parcel to travel from the hydraulically most distal part of the watershed to the outlet or reference point downstream. (2) The time difference between the end of rainfall excess and the inflection point of a hydrograph where the recession curve begins
Time to peak	The time from the beginning of direct runoff (or rainfall excess) to the peak discharge in a simple hydrograph
Lag time	The time from the center of mass of the rainfall excess to the peak discharge rate on a hydrograph
Time base	The time difference from the beginning of direct runoff until the return to base flow (the direct-runoff component reaches zero).

The literature review on regionalization of timing parameters was important to develop timing parameters for ungaged watersheds. Many empirical formulas have been developed from various databases and watersheds to estimate timing parameters. These equations are based on correlations with various watershed and/or rainfall characteristics. Some investigators also developed equations to compute timing parameters from other timing parameters. Several definitions of timing parameters were used in many previous studies. A comprehensive review of empirical equations encompassing several common timing parameters was given in Chapter 3.

It is important to document how watershed characteristics are conceptually defined and operationally determined from topographic maps during geomorphologic study. The major emphasis should be on those watershed characteristics readily obtainable by design engineers. Therefore, the report also includes a review of common watershed parameter definitions in Chapter 4.

There are typically three methods for estimating timing parameters: (1) NRCS velocity method, (2) rainfall-runoff hydrograph analysis, and (3) application of empirical equations. The velocity method includes the estimation of travel time for overland flow (referred to as upland method in the National Engineering Handbook) and channel flow. Kent (1972) prepared Chapter 15, "Travel time, time of concentration and lag" in section 4, "Hydrology" of the National Engineering Handbook. Kent (1972) stated that the use of stream hydraulics is recommended for the usual case where no usable hydrographs are available. The stream hydraulics procedure is most applicable for areas where surface runoff predominates. It can result in T_c too short for areas where interflow and groundwater flow are a major part of runoff (Kent, 1972). This indicates that it is favorable to use hydrograph analysis to estimate time of concentration in comparison to the velocity method. Kent (1972) also suggested that the 2-year frequency discharge in the stream can be used for stream hydraulics computation. When this cannot be done, Kent (1972) suggested using the approximate bankfull discharge of the low flow channel (Kent, 1972). "Use of the low flow channel bankfull discharges with valley lengths is a compromise that gives a T_c for average flood (Kent, 1972)."

The literature review is documented with several approaches that might be used for evaluation with Texas rainfall-runoff data. The goal of the project is to develop or use more reliable methods (procedures), in consultation with the TxDOT project director and project monitoring committee members, to estimate two key timing parameters of hydrographs: the time to peak and time of concentration, by using recorded rainfall-runoff data from Texas watersheds. In summary, the following four methods might be used to estimate timing parameters for theTxDOT research project 0-4696:

(1) Hydrograph analysis method: Time to peak can be estimated from UH development based on rainfall-runoff data for more than 1,600 events in 90 watersheds in Texas. Time of concentration can be computed from the end of rainfall excess to the point of inflection (T_{in}). T_{in} can be derived from time to peak and the Gamma UH shape factor as given in equation (3.53). This report has not included much discussion on this method because it has been summarized through the TxDOT project 0-4193. Results developed from this method could be shared for both projects 0-4193 and 0-4696. Based on recommendations in the National Engineering Handbook, this is more favorable than the NRCS velocity method.

(2) Particle tracking method: This method uses the digital elevation model (DEM) and provides an independent verification on computation of time of concentration for all other methods. Preliminary research plans and results are presented in Appendix A.

(3) Empirical equations: Two empirical equations might be used to estimate basin time of concentration. The Kerby-Hathaway formula (equation 3.12) can be used to compute travel time for overland flow up to 500 feet of flow path (typically T_c about 0.5 hour for overland flow). The Kirpich method developed for Tennessee watersheds (equation 3.14) can be used to estimate travel time in channels.

(4) The NRCS velocity method: Use of DEMs and GIS automation procedures to implement the NRCS velocity method for estimation of time of concentration.

The NRCS velocity method could be implemented for the current project by the following steps:

(a) Runoff travel time for overland flow for up to a 500 foot flow path could be calculated by using either the NRCS velocity chart (Fig. 3.3) or the simplified Manning's equation $V = k\,S^{0.5}$ (equation 3.2). Land use or vegetation cover, surface slope and overland flow length are required input parameters.

(b) Travel time for shallow concentrated flow is computed by using Manning's equation to estimate flow velocity first. From engineering experience, the velocity could be computed as a triangle channel with a 1:1 side slope, 1 foot water depth, and a roughness coefficient of 0.06. The channel length up to 1,000 feet and channel slope can be estimated from the watershed DEM. Certain sensitivity analyses could be performed to test assumptions of the channel characteristics.

(c) Travel time for main channel flow can also be computed by applying Manning's equation. The channel could be assumed to be a rectangular or trapezoidal channel with a side slope of 1:2 (vertical : horizontal) and a roughness coefficient of 0.04. The bankfull width might likely be estimated from 1-foot or 1-meter USGS DOQQ (digital ortho quarter quads). The 2-year frequency discharge (Q_2) could be estimated from Texas regional equations developed by Asquith and Slade (1997) and adopted by TxDOT (2002). Using Manning's equation, Q_2 and bankfull width, water depth and then average flow velocity could be estimated. Travel time is computed by dividing the main channel length by the average flow velocity. Time of concentration for a watershed is the sum of travel time for overland flow, shallow concentrated flow and main channel flow. The longest flow path could be determined by using ArcMap Spatial Analysis (ESRI, 2001) and DEMs.

In summary, the literature review has provided comprehensive information and basic directions for the research team to work on and complete tasks for the project 0-4696, "Estimating Timing Parameters of Direct Runoff and Unit Hydrograph for Texas Watersheds."

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APPENDIX A. PRELIMINARY RESEARCH PLAN AND RESULTS OF PARTICLE TRACKING METHOD

This appendix outlines preliminary research plans and also provides some preliminary results of the particle tracking method, as investigated in the first year (2004) of the TxDOT research project 0-4696.

(A.1) Time Area Method(s)

Time-area rainfall runoff analysis is a hydrologic watershed routing technique used to derive a discharge hydrograph from a given excess rainfall hyetograph. In the method a watershed is divided into a number of sub-areas separated by isochrones of differing travel times and then a histogram of contributing area versus time is prepared from these isochrones. This histogram is applied as a unit graph to a hyetograph to estimate a discharge hydrograph. The entire procedure is briefly described in the following paragraphs.

Figure A.1 is a plane view sketch of a watershed. In the figure, four areas are indicated, each with a different travel time from anywhere in the area to the outlet. Determining these times would involve using overland flow models to determine the time to move water from anywhere in a sub-area to the channel, then channel routing to move water to the outlet.

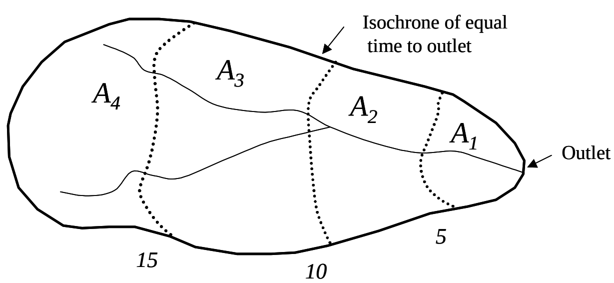


Figure A.1 Watershed sketch.

Once the sub-area travel times are “known” a diagram similar to Figure A.2 is created. This diagram is called the time-area diagram or the contributing area diagram. It is theoretically the translation instantaneous unit hydrograph for the watershed.

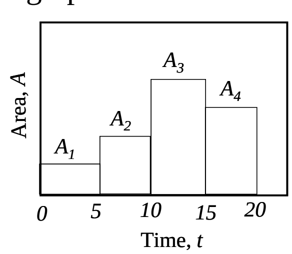


Figure A.2 Time-area diagram.

To produce a runoff hydrograph, the time-area diagram is convolved with a precipitation hyetograph, and the resulting convolution integral is the direct runoff hydrograph. Figure A.3 is a sketch illustrating these relations.

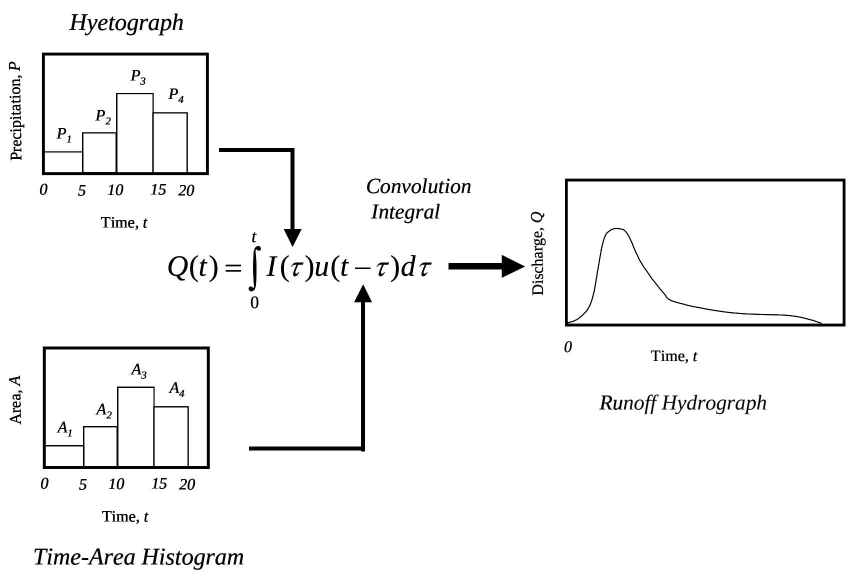


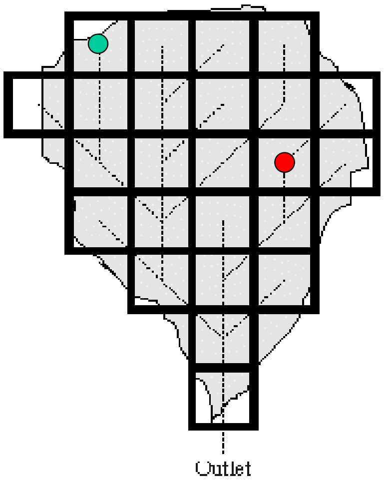
Figure A.3 Time-area-convolution to produce a runoff hydrograph.

The crux of this approach, naturally, is determining the travel times. One method that appears promising is to subdivide the watershed into very small areas, place a set of particles that represent a unit of precipitation onto the small areas, and by some kinematic rules, determine the trajectory of these particles and count them as they leave the watershed through the outlet.

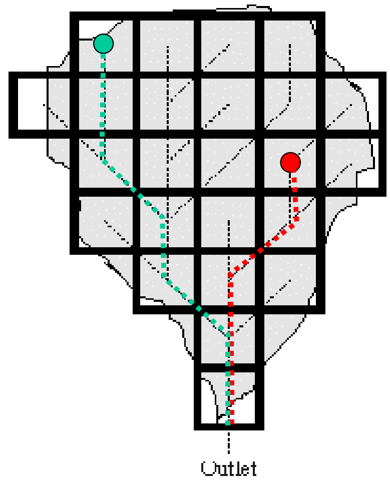
Figure A.4(a) depicts the concept of dividing a watershed into small areas. In the figure the dendritic pattern of the streams is also displayed along with two “particles” at different locations. Figure A.4(b) depicts the two paths that the two different particles would follow. By calculating trajectories, which are the time-position attributes of the particles, the path is determined, as well as the travel time.

In the present work, substantial computational costs exist to track hundreds of particles placed over the watershed and to determine the cumulative instantaneous hydrograph at an outlet. Once these calculations are completed a curvilinear unit hydrograph is fit through this discrete hydrograph, and time parameters are inferred from the curvilinear model.

It was previously stated that the “crux” of the problem is determining travel times; using particle tracking solves this by selecting suitable equations of motion to describe how the particles will travel in the watershed. Once the equation is selected, particles are placed on the watershed and are programmed to move toward the outlet according to this equation of motion. The outlet unit hydrograph is determined by cumulative counting of the particles leaving the watershed as time elapses. Countings ceases when particles stop leaving the watershed. The time series of cumulative particle count versus time, normalized to range from 0 to 1, is a cumulative unit hydrograph.



(a) Dividing watershed into small areas



(b) Tracking movement of two particles

Figure **A.4** Concept diagram dividing a watershed into small areas to track the movement of two particles.

(A.2) Suitable Equations of Motion

The trajectory equations for a particle written as a difference equation are:

$$
\begin{aligned}
x_p(t+\Delta t) &= x_p(t) + u_p(x_p(t), y_p(t), t)\Delta t \\
y_p(t+\Delta t) &= y_p(t) + v_p(x_p(t), y_p(t), t)\Delta t
\end{aligned}
\qquad (A.1)
$$

In the equation, *x* and y are spatial locations, *u* and *v* are *x*- and *y*-components of velocity at a location, *t* is time and the subscript *p* is a particle index (the *p*-th particle). The equations as written represent a first-order Euler model to integrate the displacement rates of the particles. Higher order representations are not warranted as the computation time in particle tracking is already large; using short time steps is not enough of an added burden to justify a more complex difference equation.

The equations require the ability to determine the velocity of a particle at any location, and two general approaches make sense. The first is to assume that the particles move in a velocity field governed by a flow potential (a balance of forces; Darcy's law is an example of a flow model that can be represented by a flow potential). The second, most physically correct and most difficult, is to determine particle accelerations from a force balance, solve the resulting displacements by numerical integration of the acceleration (to recover velocity) and then use the numerical integration of this result to recover position. The following paragraphs only present the relevant equations for the flow potential approaches, but to date only the linear flux law model has been attempted.

(A.3) Flow Potential Approaches

(a) Linear flux law: The first and least complex approach is to assume velocity is proportional to watershed slope and compute the velocity field independent of the particle positions. This assumption is a potential flow approach where the watershed elevation is the flow potential. Equation A.2 represents the formula used to determine the velocity at any location in the watershed. In practice we only have elevations at discrete grid points so a difference equation is used to determine the local watershed slopes.

$$
\begin{aligned}
u(x,y) &= -k * \left.\frac{dz}{dx}\right|_{(x,y)} \\
v(x,y) &= -k * \left.\frac{dz}{dy}\right|_{(x,y)}
\end{aligned}
\qquad \text{(A.2)}
$$

The value of *k* represents the velocity of the particle on a unit slope. As a starting point Figure 3.2 (in the report narrative) can be used to determine a *k* value by extrapolating to the 100-percent slope intercept; this value then can be substituted in the velocity equations (A.2).

This set of kinematics is similar to time-area methods using the isochrone timing derivations of Laurenson, 1964; Muzik, 1995; Kull and Feldman, 1998; or even Clark's (1945) method (ignoring storage).

(b) Quadratic flux law: The second and still relatively straightforward approach is to assume the square of velocity is proportional to watershed slope and to compute the velocity field independent of the particle positions. This assumption is essentially a potential flow approach where the watershed elevation is the flow potential. Equation A.3 represents the formula used to determine the velocity at any location in the watershed.

$$
\begin{aligned}
u(x,y)\cdot\left|u(x,y)\right| &= -k * \left.\frac{dz}{dx}\right|_{(x,y)} \\
v(x,y)\cdot\left|y(x,y)\right| &= -k * \left.\frac{dz}{dy}\right|_{(x,y)}
\end{aligned}
\qquad \text{(A.3)}
$$

The value of *k* represents the square of velocity of the particle on a unit slope. The absolute value formulation is used so that the numerical method preserves correct directional information (we have assumed that flow is always downslope). This approach is similar to the NRCS methods, but no distinction is made between channel and overland flow. Additionally the structure of the formula in any single direction is the same as a Manning's-type formula. As a starting point, Table 3.2 (in the report narrative) can be used to determine k values, which then can be applied in the velocity equations (A.3).

These kinematics appear to be similar to the isochrone derivation technique of Sagafian and Julien (1995) who adapted a kinematic wave theory for distributed rainfall-runoff modeling and presented a single example for a watershed in West Africa (Saghafian and Julien, 1995 and 2002).

(A.4) Watershed Representation

All the methods require information about the spatial distribution of watershed elevation. This information can be obtained manually from USGS topographical maps, by engineering survey, or from USGS DEM maps. Regardless of the original source, the representation will eventually be raster dataset with horizontal and vertical elements representing locations on the surface of the Earth, and the data entries will represent elevation above some datum. The numerical experiments presented below are all based on USGS 30-meter DEM maps downloaded from the Internet. Details of the procedure are illustrated for one watershed, Ash Creek. The other watersheds are handled in the same fashion. Figure A.5 is the entire DEM for White Rock Lake, Texas, the 24-kilometer topographic quad-sheet that contains the Ash Creek watershed.

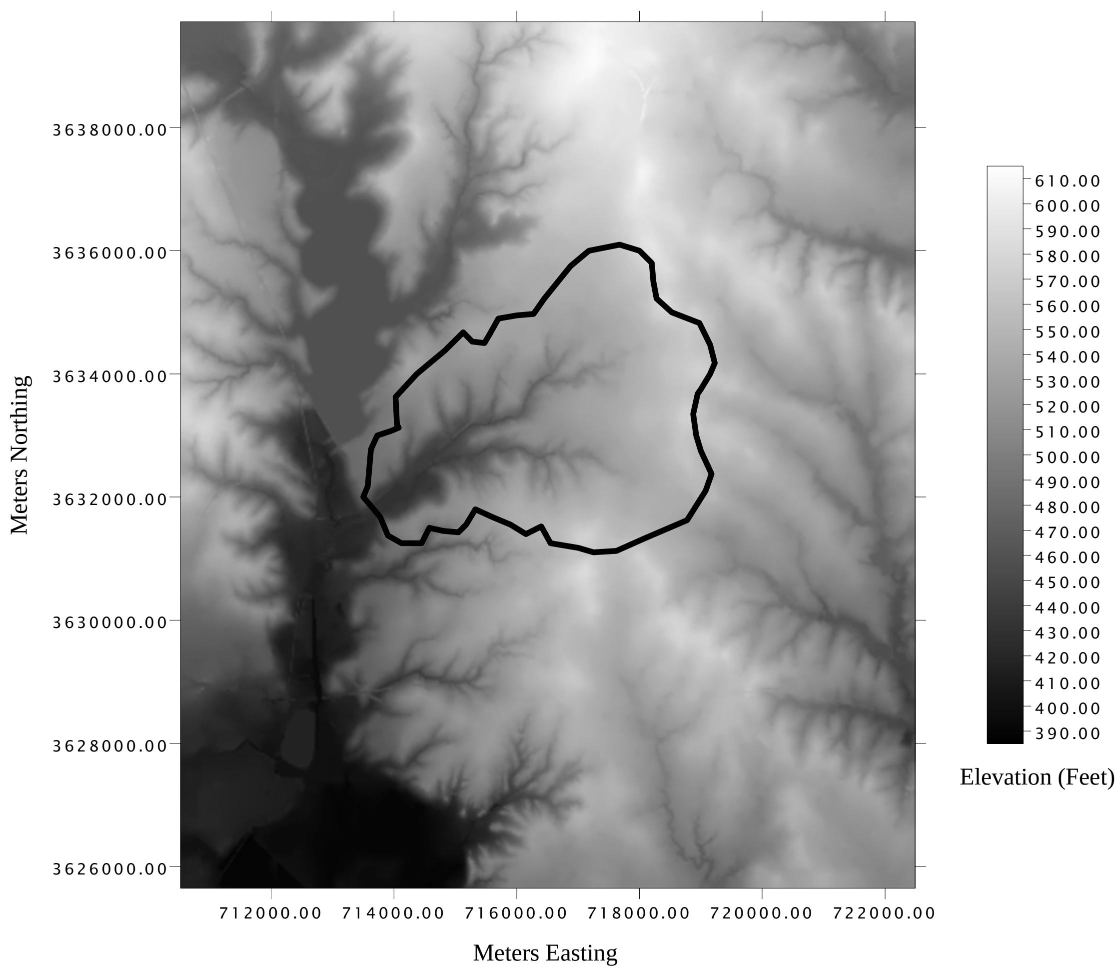


Figure A.5 Ash Creek watershed - manual boundary delineation - sta08057320.

The watershed is depicted on the figure as the bold polygon. The watershed boundary was determined manually using paper-based maps. Figure A.5 was generated using the program

SURFER, which greatly simplifies raster manipulation; the SDTS format files also can be converted into ASCII-XYZ formats for manual manipulation. To reduce the computation burden, we extract the rectangle, or mask, that just encloses the watershed and assign a large value to elevation outside this boundary. The resulting map is rendered in Figure A.6 and requires a far smaller raster. Once the smaller dataset is extracted, the file is converted into a format for the particle tracking model. An excerpt of the ASCII file that was used to create Figure A.2 is displayed below in Table A.1. Table A.1 is unprocessed data as far as the particle tracking code is concerned.

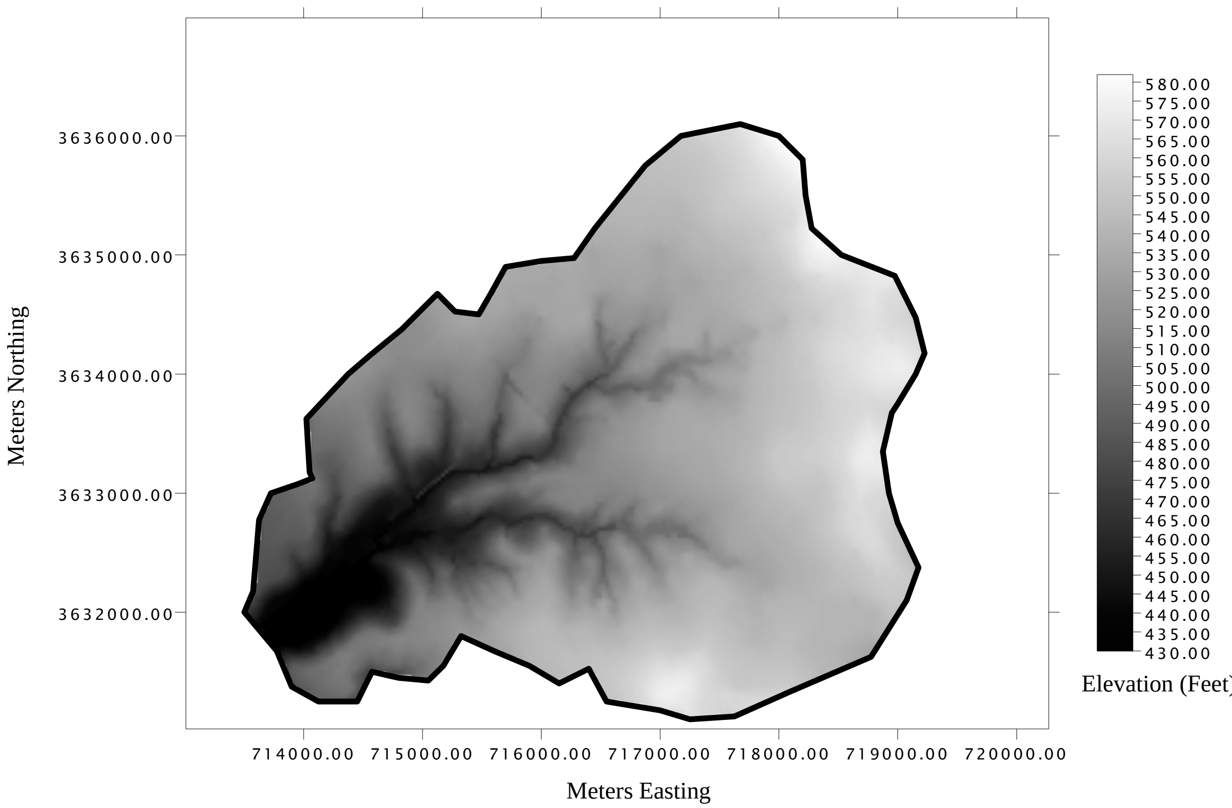


Figure A.6 Ash Creek watershed - manual boundary delineation.

The table entries of importance are as follows: The file format is DSAA which is a mapping format code. The next two entries are the number of rows and number of columns, respectively, in the data grid. The following two entries are the Western-most and Eastern-most UTM coordinates in meters, respectively. The next two entries are the North-most and South-most UTM coordinate in meters, respectively. The next couple of entries are the low and high elevation values in the raster -- it is important to note that the masking elevation value is a very large number by design to prevent any confusion with the useful elevation data.

Table A.1 ASCII-grid file for Ash Creek

```
DSAA

193 169

713490 719250

3.63108e+006 3.63612e+006

429.999 582

1.70141e+038 1.70141e+038 1.70141e+038 1.70141e+038 1.70141e+038 1.70141e+038
1.70141e+038 1.70141e+038 1.70141e+038 1.70141e+038

. . . (many lines of numeric entries)

1.70141e+038 1.70141e+038 1.70141e+038 1.70141e+038 1.70141e+038 566 565.001
564.001 562 561

560 1.70141e+038 1.70141e+038 1.70141e+038 1.70141e+038 1.70141e+038
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. . . (many lines of numeric entries)

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559.999 560 559 558 558 557 557 558 557 1.70141e+038

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. . . (many lines of numeric entries)
```

(A.5) Preliminary Results

Three examples of the approach are presented and represent the work–to-date on this procedure. The three watersheds are: Rush Branch, Station 08057130, with a drainage area of 1.22 square miles; Ash Creek, Station 08057320, with a drainage area of 6.92 square miles; and Slaughter Creek, Station 08158840, with a drainage area of 8.24 square miles.

These stations are each analyzed once by the particle tracking method to produce an empirical cumulative IUH. Then each empirical cumulative IUH is fit to a Weibull based IUH and the three Weibull parameters are extracted. Then two storms from each station were selected at random although we did select storms during the same time of year. The actual precipitation signal was applied to the IUH just determined, and the direct runoff hydrograph was calculated and compared to the observed runoff hydrograph.

The actual particle tracking code used is a modified version of a research code originally developed by Cleveland (1991) and subsequently was used in numerical dye-tracing of the confluence of two streams in Houston, Texas (Wang et. al., 1991 and 1996). The results of the experiments are presented in the order of increasing watershed size.

(A.5.1) Rush Branch

Rush Branch is the smallest watershed modeled with the particle tracking approach. It is located in the Dallas area and has a drainage area of 1.22 square miles. Figures A.7 and A.8 are the renderings of the watershed topography. The renderings suggest that it is mostly uniformly sloped from NE to SW and channelization is obvious only in the lower 1/5 of the watershed. Immediately adjacent is a highly channelized area.

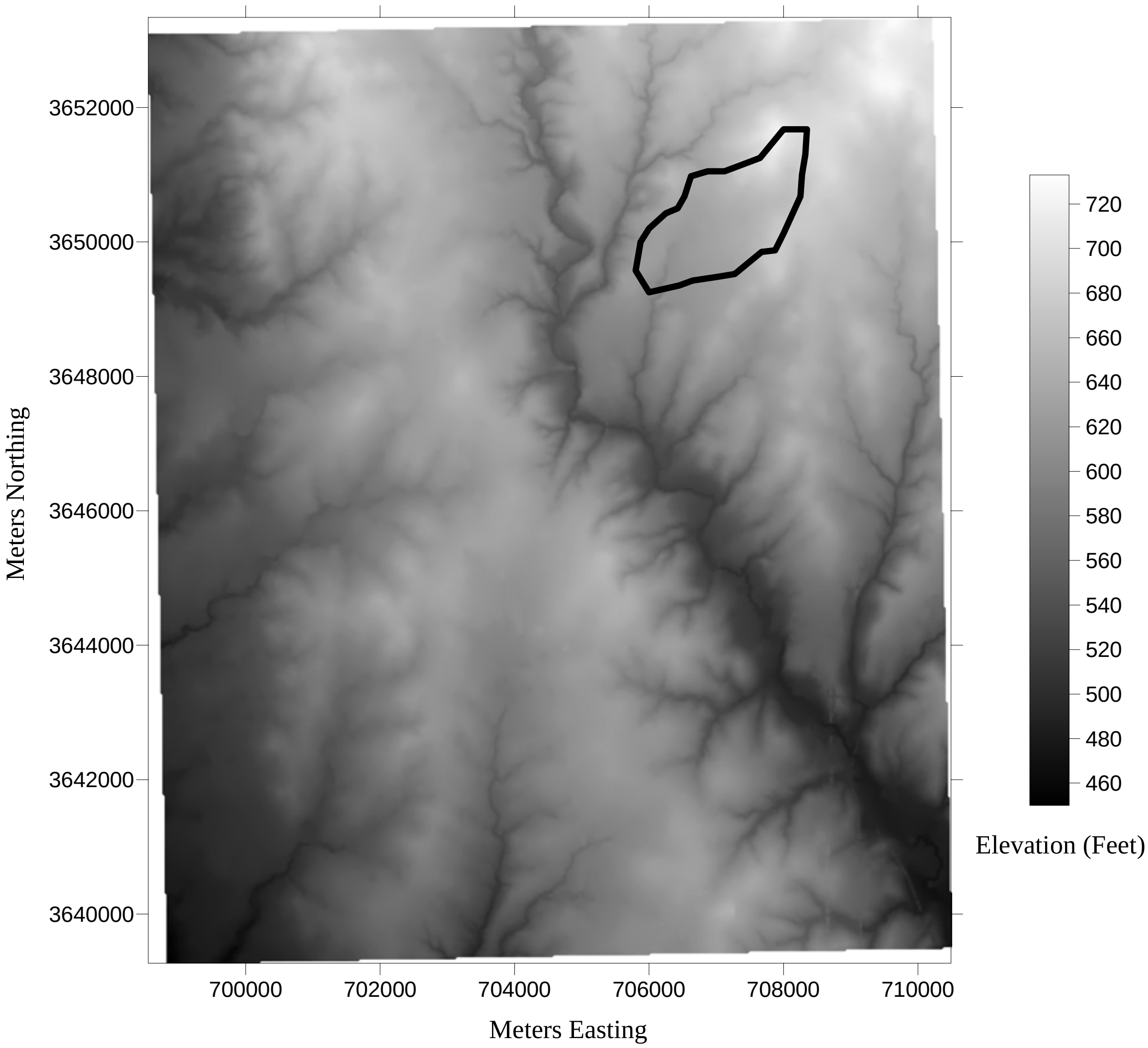

Figure A.7 Rush Branch.

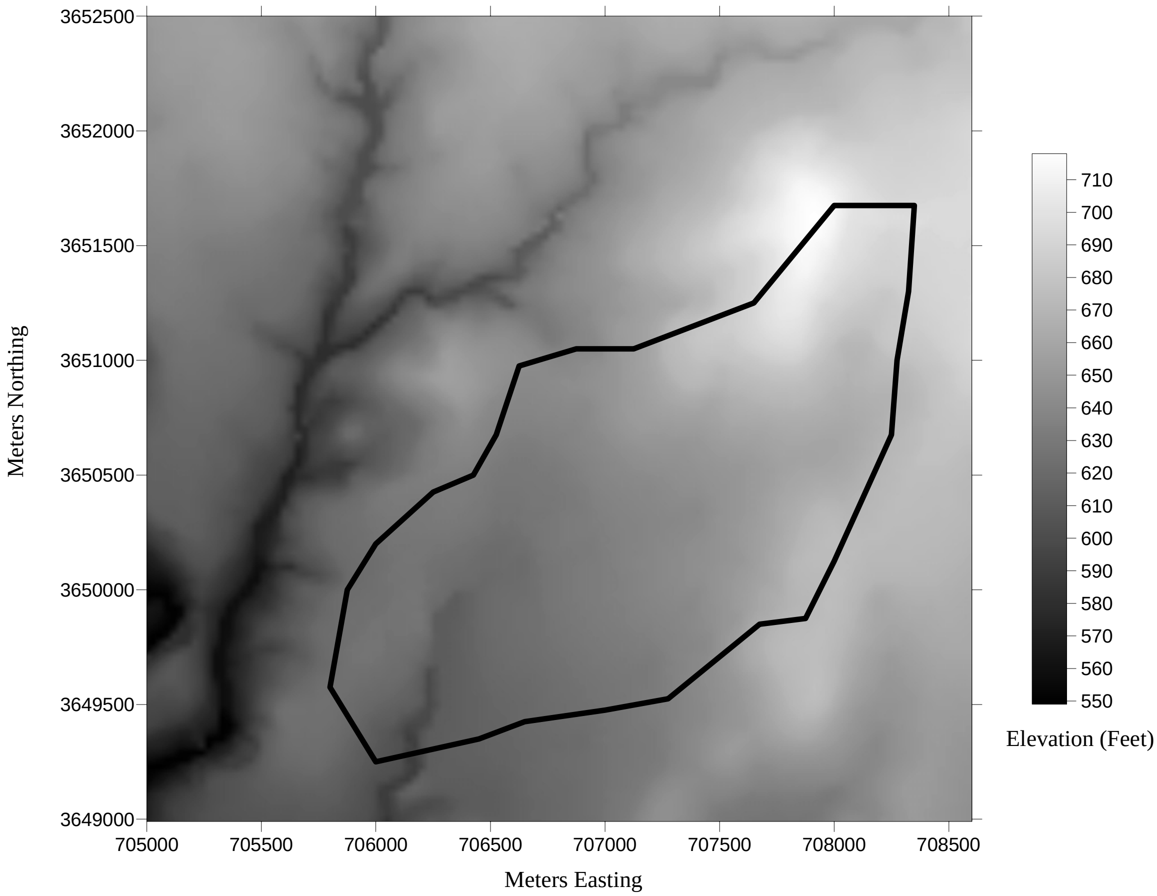

Figure A.8 Rush Branch – Extracted Area.

Once the data are prepared the particle tracking program is executed. The output from the program is a time series that represents the empirical cumulative IUH. This IUH is shown on Figure A.9 as the thick shaded curve. It is monotonically increasing toward its asymptotic value of 1.0 as would be expected with a cumulative hydrograph. The next step is to fit a curvilinear function to this cumulative IUH so that the curvilinear model can be used for simulation of the direct runoff hydrograph. The curvilinear model used is the Weibull model developed and tested by Cleveland et al. (2003) and He (2004). The formula fit to the empirical data is

$$q(t) = pz_0\left(\frac{t^{p-1}}{\bar{t}^{\,p}}\right)\left(\frac{t^{Np-p}}{\bar{t}^{\,Np-p}}\right)\exp(-\left(\frac{t}{\bar{t}}\right)^p); \quad Q(t) = \int_0^t q(\tau)d\tau \tag{A.4}$$

In these equations discharge is represented as L/T; thus to convert to conventional units one needs to multiply the result by the watershed area. The value of z_0 in the equation is 1 depth unit (1 inch).

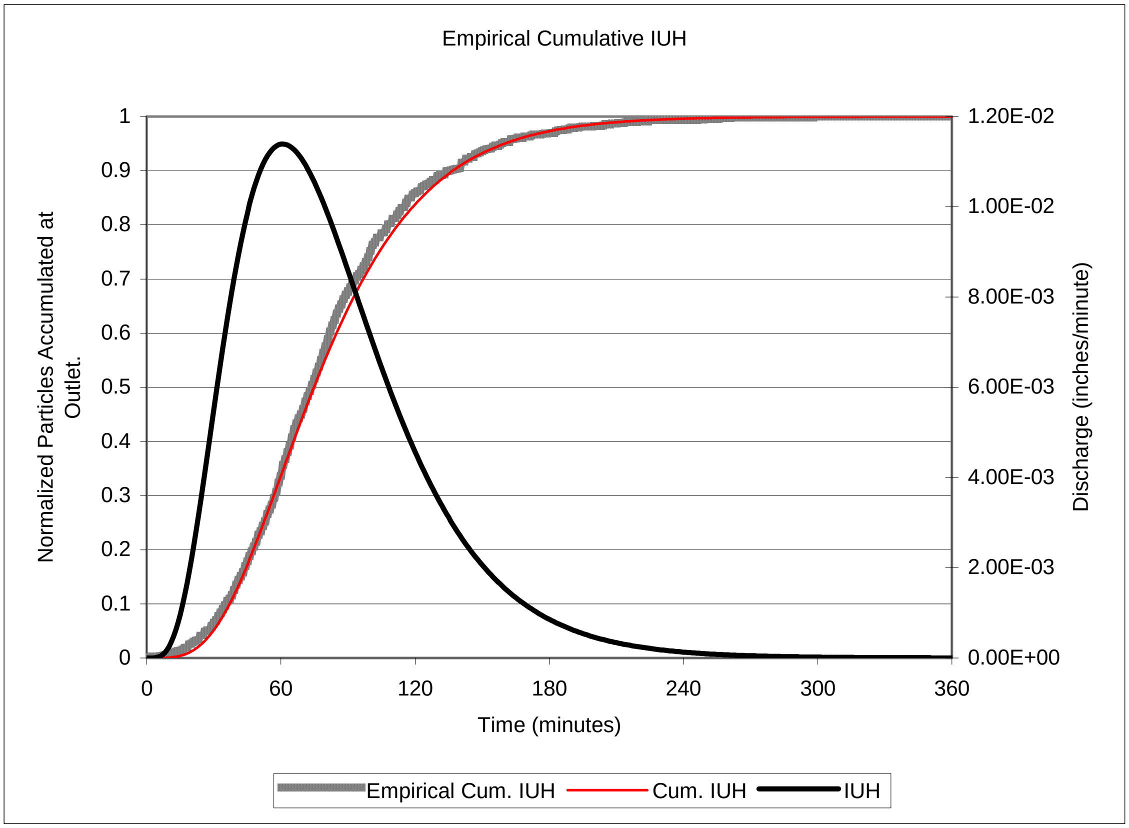

Figure A.9 Empirical cumulative and fitted IUH for Rush Branch.

Figure A.9 also shows the fitted curves and the qualitative agreement is quite good. The fitting values are $\bar{t} = 3.56; N = 8.54; \mathit{and}\ p = 0.69$. These values in theory represent the IUH values for the watershed as determined by the simple linear flux law model. The utility of the approach is to use these values with different historical storms and observe the predicted and actual response. Figures A.10 and A.11 are the responses for two storms on Rush Branch.

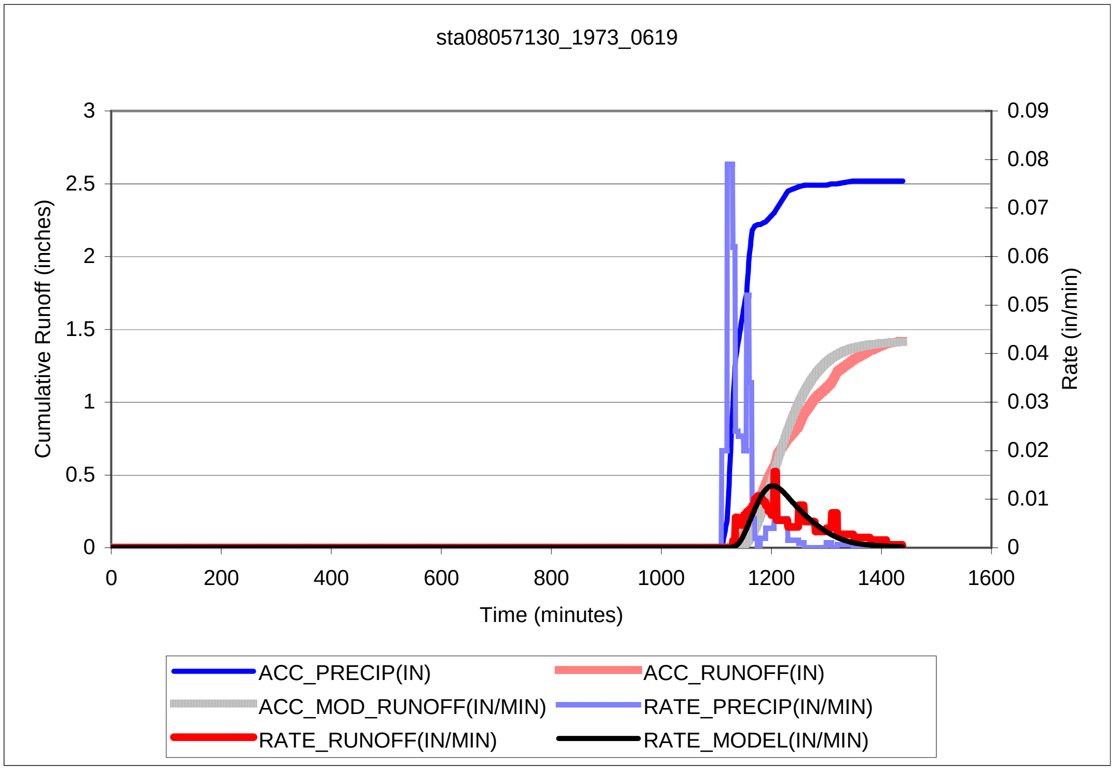

Figure A.10 Rush Branch response under rainfall on 06-19-1973.

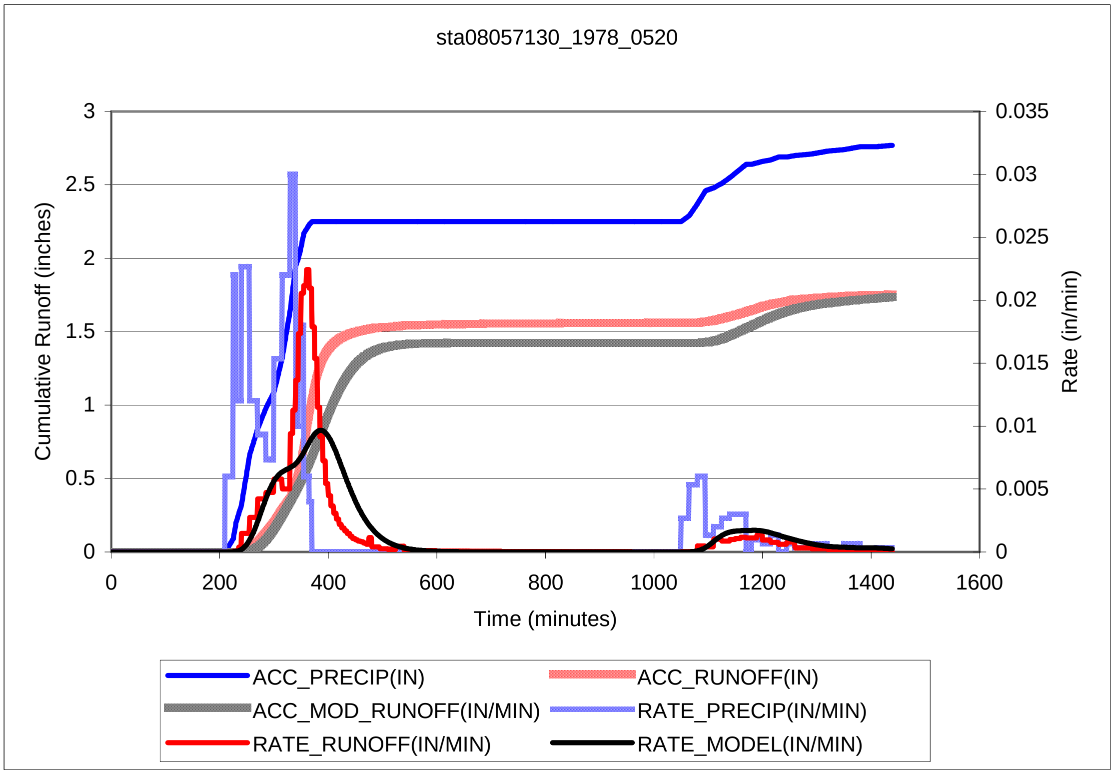

Figure A.11 Rush Branch response under rainfall on 05-20-1978.

Qualitatively the two figures suggest that the particle tracking method's IUH is acceptable, especially at locating the peak time behavior of the watershed. The value of the peak is under-predicted in both cases, but considering that the only input data used to generate the IUH model was a topographic map, the approach is promising.

(A.5.2) Ash Creek

Ash Creek is the second watershed modeled with the particle tracking approach. It is located in the Dallas area and has a drainage area of 6.92 square miles. Figures A.5 and A.6 are the renderings of the watershed topography. The renderings show that the watershed slopes from NE to SW and channelization is present over about 2/3 of the watershed. A strong dendritic pattern is also shown in the renderings.

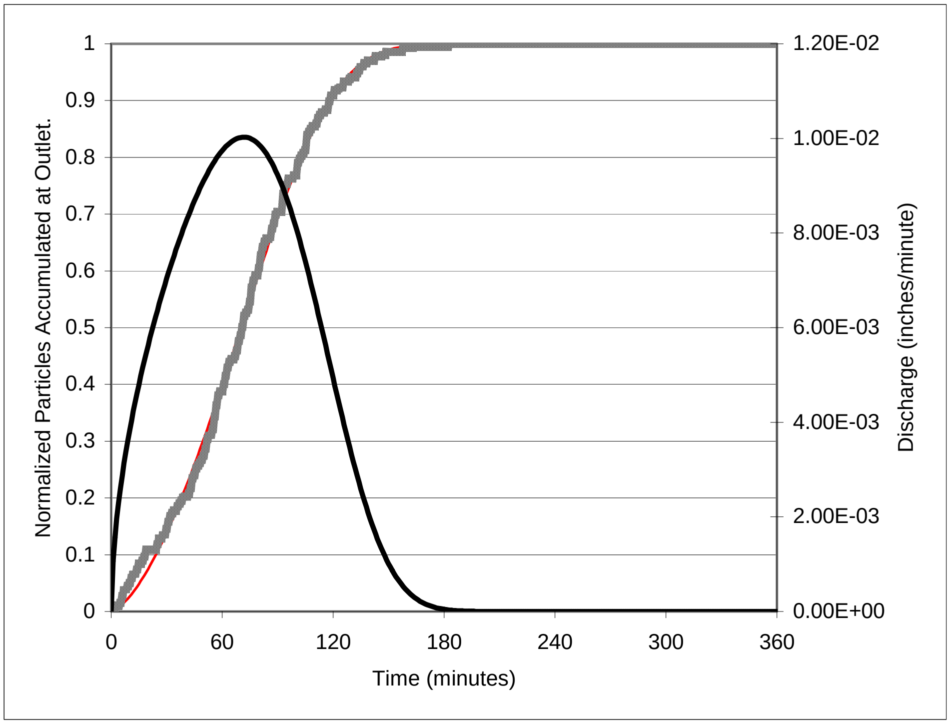

Figure A.12. Empirical and fitted IUH for Ash Creek.

Figure A.12 shows IUH results from the particle tracking and subsequent fitting procedure. The fitting values are $\bar{t} = 116; N = 0.38; p = 4.13$. Using these values as the coefficients for the watershed, observed precipitation for two storms are convolved using the Weibull model and the predicted response is compared to the observed runoff behavior. Figures A.13 and A.14 are plots of the results of these two simulations.

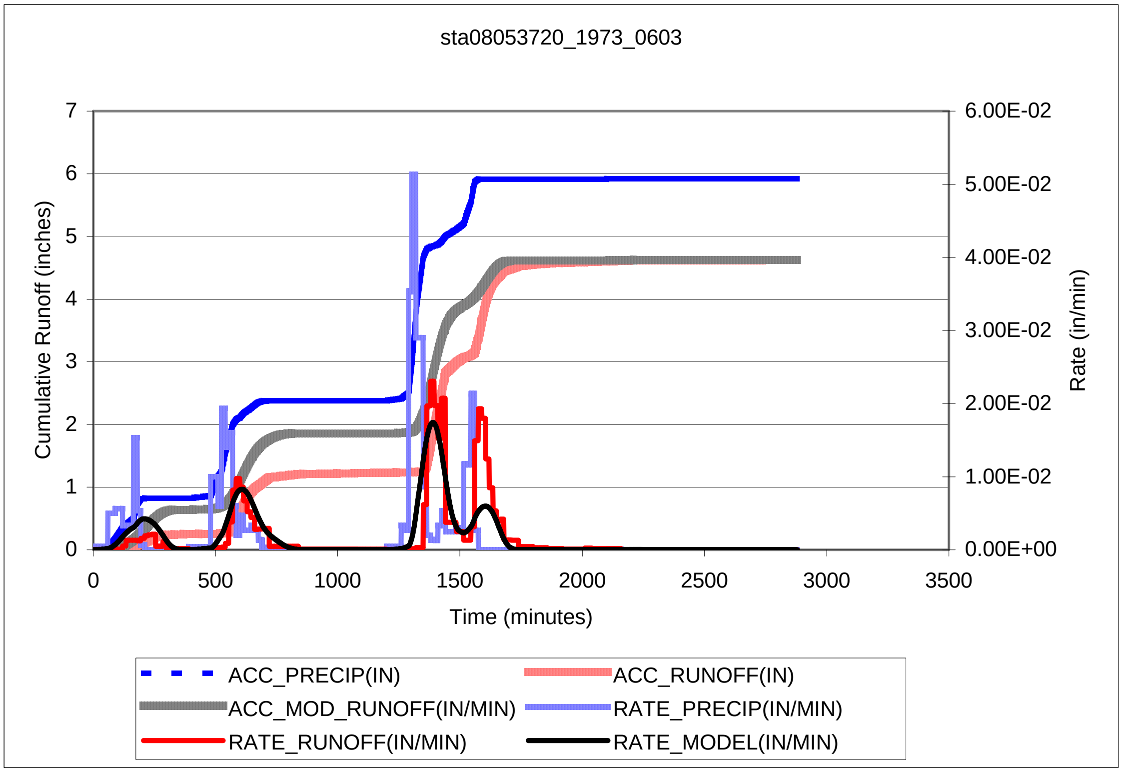

Figure A.13 Ash Creek response under rainfall on 06-03-1973.

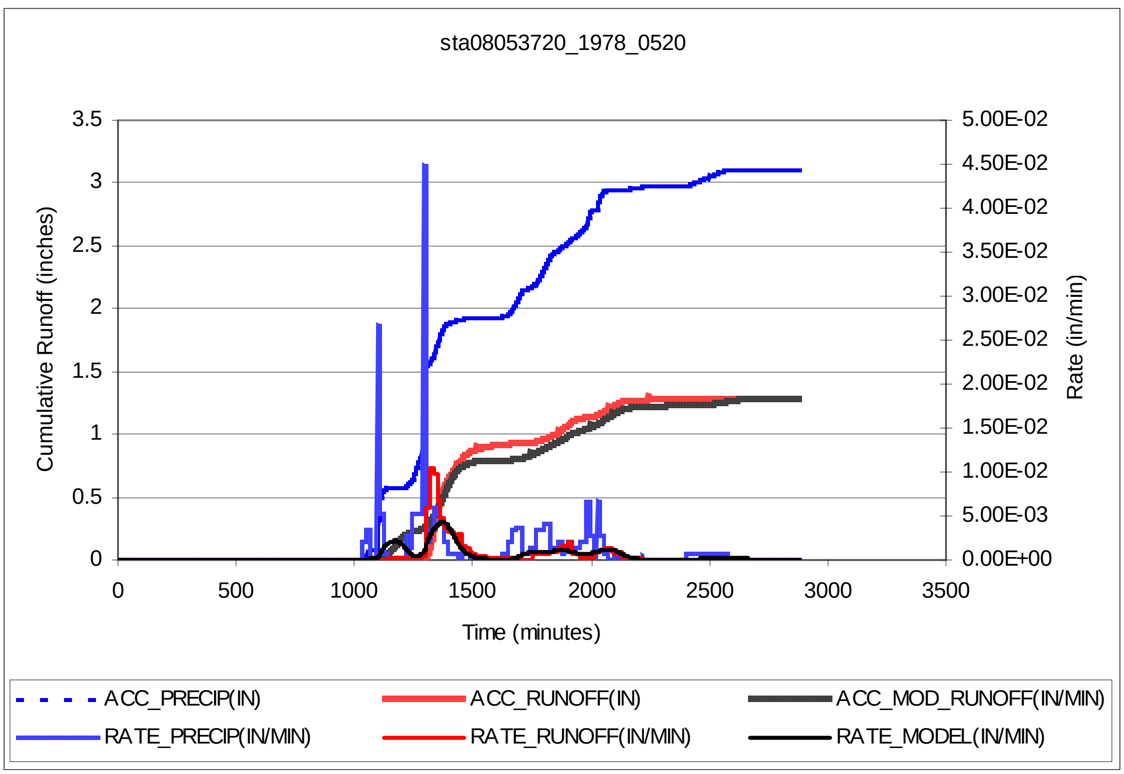

Figure A.14 Ash Creek response under rainfall on 05-20-1978.

Qualitatively these two results are also acceptable. Again there is an inability to predict the magnitude of the peak discharge, but the timing of the model peaks agrees well with the observed peaks.

(A.5.3) Slaughter Creek

Slaughter Creek is the third watershed modeled with the particle tracking approach. It is located in the Austin area and has a drainage area of 8.24 square miles. Figures A.15 and A.16 are the renderings of the watershed topography.

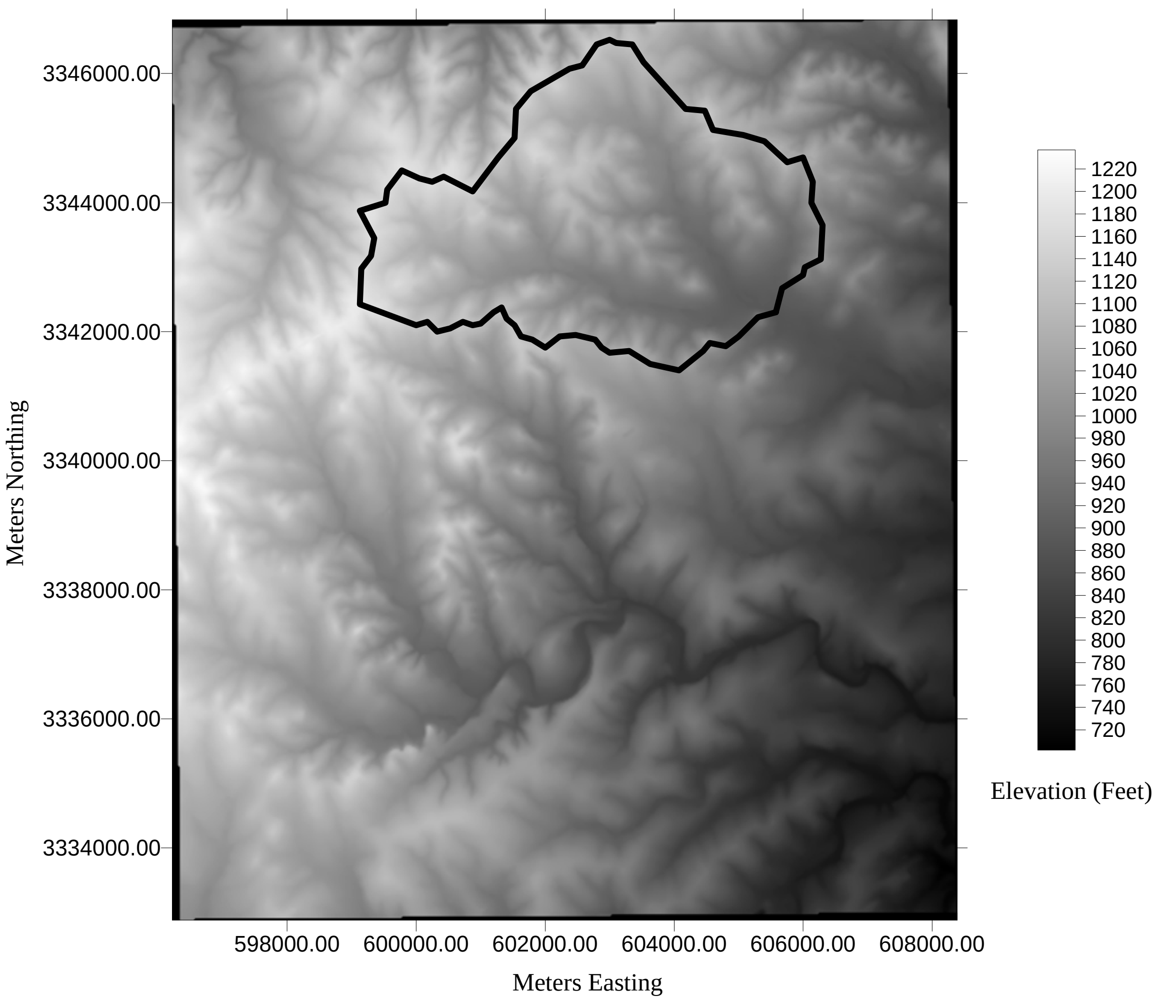

Figure A.15. Slaughter Creek.

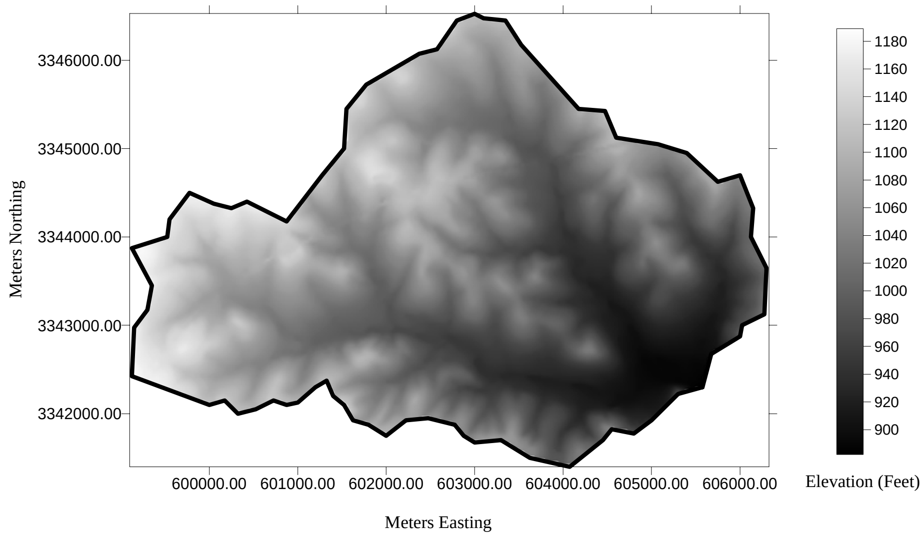

Figure A.16. Slaughter Creek - Extracted Data Region.

The renderings suggest that the watershed slopes from W to E and channelization is present over the entire watershed. The magnitudes of the slopes are substantially larger than in the two Dallas watersheds studied. A strong dendritic pattern is shown in the renderings.

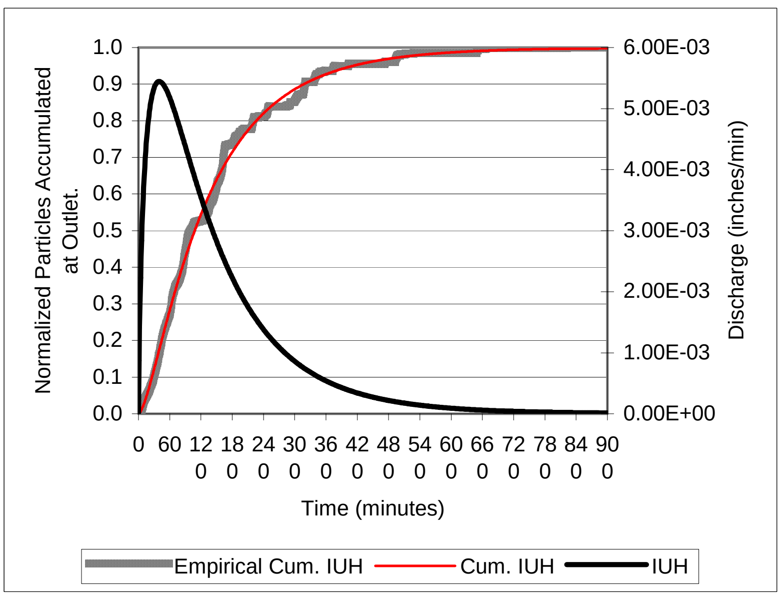

Figure A.17 Empirical and fitted IUH for Slaughter Creek.

Figure A.17 is the IUH model resulting from the particle tracking analysis. In this watershed the empirical distribution is less smooth than the other two examples, which is illustrative of the reason for performing the curvilinear fit. The model parameters for this watershed are $\bar{t} = 33; N = 2.59;$ *and* $p = 0.69$.

Figures A.18 and A.19 are the results of passing two historical precipitation signals through the Weibull model and comparing the predicted to observed responses. Again, as in the previous examples the model response is qualitatively acceptable except for the magnitude of the peaks.

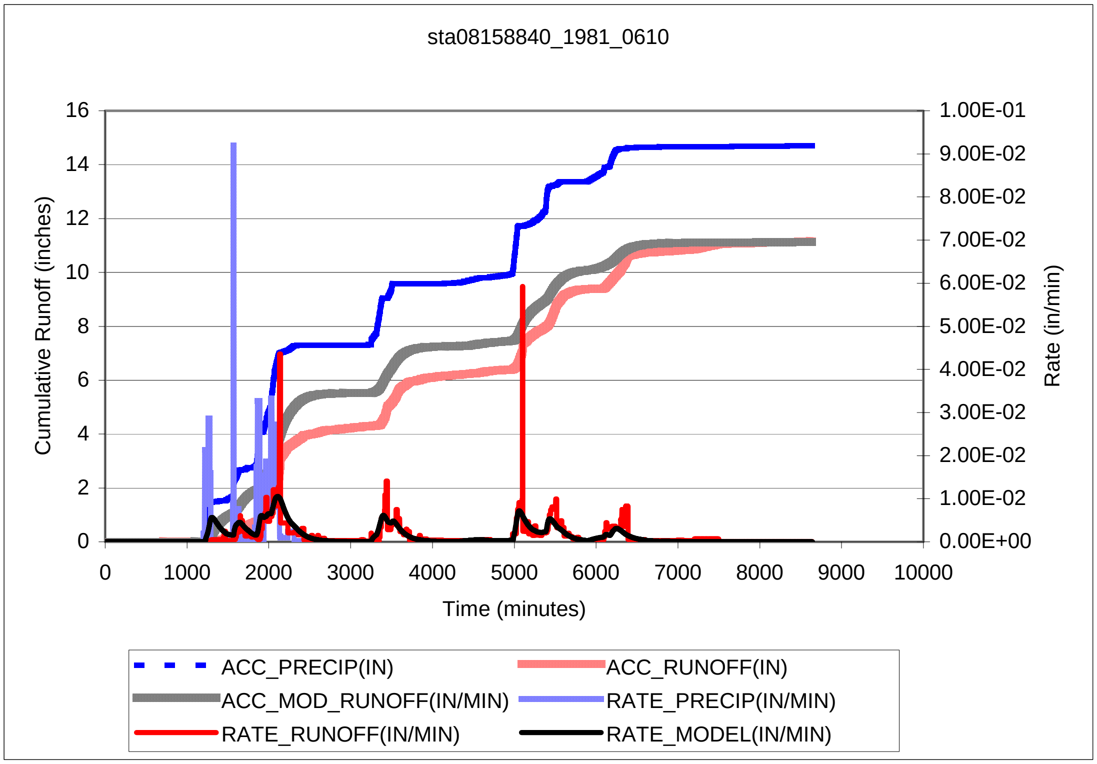

Figure A.18 Slaughter Creek response under rainfall on 06-10-1981.

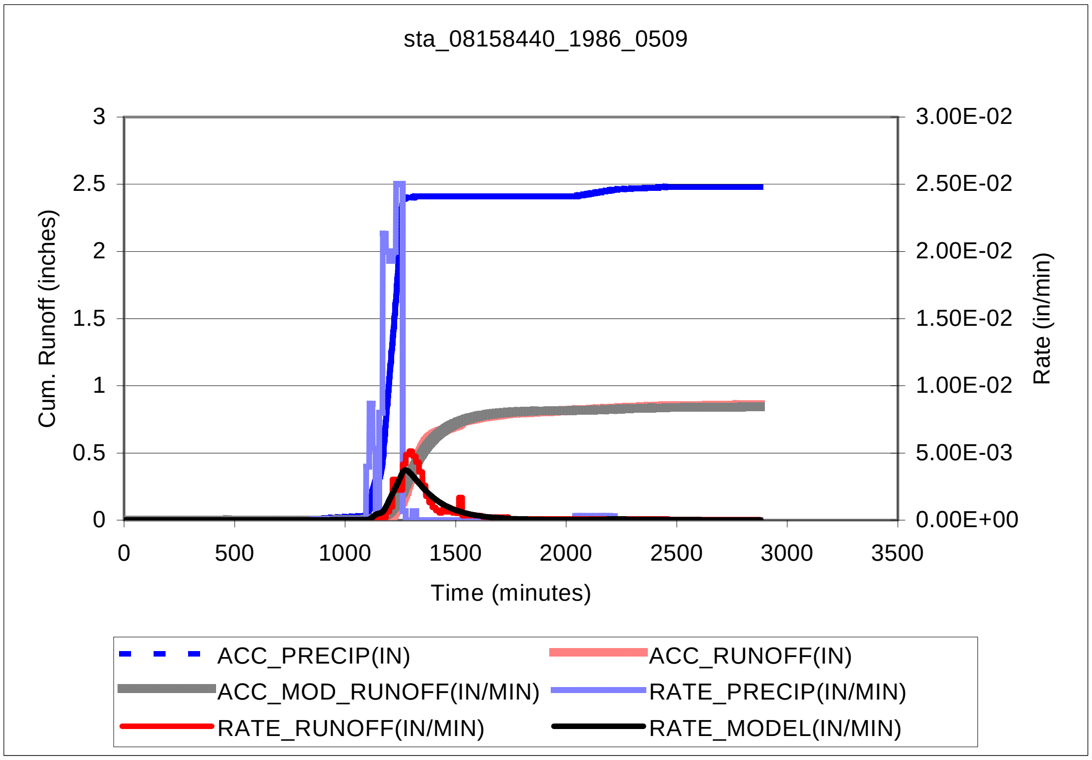

Figure A.19 Slaughter Creek response under rainfall on 05-09-1986.

(A.6) Conclusions

The particle tracking approach to infer hydrologic properties of a watershed response from topological characteristics is briefly explained and three examples are presented. These examples represent watersheds in the middle size range of interest (200 acres – 20 sq.mi.) for the research project. The watersheds differ in size, shape, and geographic location. Two watersheds are from the Dallas area and one is from the Austin area. The particle tracking model was run with only topographic data from DEM maps using the linear flux law model. The resulting empirical cumulative hydrographs are then fit to a curvilinear model, and this model is used to predict responses to actual precipitation data.

The resulting response behavior as compared to the observed behavior proved to be qualitatively notable as the particle tracking model was run with the same unit velocity value regardless of location. These results suggest that this approach may be of great value and further demonstrate that a substantial component of watershed response is controlled by watershed topography. Although the method presented in this appendix is similar to GIS-based techniques, it was developed with the intention of performing the analysis independently of a GIS.