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16. Abstract

This report presents the results of an analytical and experimental investigation of mower-thrown-object phenomena.

A mower-thrown-object analysis was conducted by developing a simple mathematical model to simulate the rotary action of the mower. A computer program that will compute the object velocities when provided with input for the mower and mowing conditions was also used to aid in the analysis of the mower-thrown-object phenomena. The experimental analysis was conducted to test the theoretical model by using a scale size mower model. High speed photography was used to determine the discharge characteristics of the mower-thrown object. The results show that (1) a higher value of the coefficient of restitution increased the velocity of the thrown object, (2) an increase in the blad rpm caused the object velocity to increase proportionally, (3) an increase in the inertia of the blade significantly increased the object velocities, (4) an increase in the distance of the point of contact of the object with the blade, from the center of rotation, increased the object velocities, and (5) the location of the pivot point played a major role in the object velocity after impact.

The comparison of a straight bar blade and a pivoted bar blade (pivot located at the mid point) indicated that the pivoted blade construction resulted in approximately 28 percent smaller magnitudes for the object velocities.

17. Key Words
mower-thrown objects, discharge velocities, straight blade, pivoted blade, coefficient of restitution, blade rpm, blade mass

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AN EXPERIMENTAL AND ANALYTICAL INVESTIGATION OF MOWER-THROWN-OBJECT PHENOMENA

by

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Kurt M. Marshek
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Research Report Number 445-1

Mower-Thrown-Object Accidents
Research Project 3-20-86-445

conducted for

Texas State Department of Highways and
Public Transportation

in cooperation with the
U. S. Department of Transportation
Federal Highway Administration

by the

Center for Transportation Research
The University of Texas at Austin

August 1986

PREFACE

This is the first of two reports which describe work done on Project 445, "Mower-Thrown-Object Accidents." This study was conducted at the Center for Transportation Research (CTR), The University of Texas at Austin, as part of a cooperative research program sponsored by the Texas State Department of Highways and Public Transportation and the Federal Highway Administration.

Many people have contributed their help toward the completion of this report. Thanks are extended to Dr. B. F. McCullough for his help and guidance and to all the CTR personnel, expecially Lyn Gabbert, Monica Gonzalez, Loretta McFadden, Art Frakes, John Ermis, and Kitty Collins, and to Dr. E. P. Fahrenthold for reading this report. We would also like to thank Rick Connell, Sao-Dung Lu, Lei Rao, and Tsen-loong Peng for their work in fabricating the experimental mower model and to Hank Franklin, Benny Benningfield and Jon Bolander at the Mechanical Engineering Department machine shop for their cooperation and support. Invaluable comments were provided by Quinner F. Williams and Bruce Barber from the Texas State Department of Highways and Public Transporation and by Byron N. Lord from the Federal Highway Adminstration, and by John R. Fisher, Chief Engineer, Terrain King.

LIST OF REPORTS

Report No. 445-1, "An Experimental and Analytical Investigation of Mower-Thrown-Object Phenomena," by Rowan E. DaSilva, Kurt M. Marshek and Srikanth M. Kannapan, presents data on mower-thrown-object phenomena. August 1986.

Report No. 445-2F, "Study and Recommendations for the Reduction of Mower-Thrown-Object Accidents," by Kurt M. Marshek, Rowan E. DaSilva and Srikanth M. Kannapan, presents an experimental evaluation of mowers and mower-thrown-object accidents. August 1986.

ABSTRACT

This report presents the results of an analytical and experimental investigation of mower-thrown-object phenomena.

A mower-thrown-object analysis was conducted by developing a simple mathematical model to simulate the rotary action of the mower. A computer program that will compute the object velocities when provided with input for the mower and mowing conditions was also used to aid in the analysis of the mower-thrown-object phenomena. The experimental analysis was conducted to test the theoretical model by using a scale size mower model. High speed photography was used to determine the discharge characteristics of the mower-thrown-object. The results show that (1) a higher value of the coefficient of restitution increased the velocity of the thrown object, (2) an increase in the blade rpm caused the object velocity to increase proportionally, (3) an increase in the inertia of the blade significantly increased the object velocities, (4) an increase in the distance of the point of contact of the object with the blade, from the center of rotation increased the object velocities, and (5) the location of the pivot point played a major role in the object velocity after impact.

The comparison of a straight bar blade and a pivoted bar blade (pivot located at the mid point) indicated that the pivoted blade construction resulted in approximately 28 percent smaller magnitudes for the object velocities.

Key words: mower-thrown-objects, discharge velocities, straight blade, pivoted blade, coefficient of restitution, blade rpm, blade mass, distance of point of contact.

SUMMARY

The main contributions of this report are results showing the effect of the blade speed, blade inertia or blade mass, coefficient of restitution, blade length or the distance of the point of contact from the center of rotation and the location of the pivot along the length of the blade. The results can be summarized as follows: (1) an increase in the blade speed causes the object velocity to increase proportionately, (2) the mass of the blade significantly affects the object velocity when the mass of the object is large, (3) an increase in the coefficient of restitution increases the object velocity, (4) an increase in the distance of the point of contact from the center of rotation (or in effect an increase in the blade length) produces higher object velocities, and (5) the location of the pivot point (for the same blade length) plays a major role in determining the object velocities.

IMPLEMENTATION STATEMENT

The work carried out under this project provides mowing equipment design engineers with information useful for evaluating the effect of the blade speed, blade inertia and the blade length on mower-thrown-object (M.T.O.) velocities. It also provides useful infomation to evaluate the performance from an M.T.O. stand point, for a straight bar blade, pivoted blade and a multi-pivoted blade. Such information will hopefully result in a decrease in the frequency and severity of M.T.O. accidents.

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CHAPTER 1. INTRODUCTION

The Texas State Department of Highways and Public Transportation (SDHPT) extensively deploys tractor driven rotary power lawn mowers for road and adjoining right of way maintenance in the State of Texas. Over the years this activity has resulted in significant personal injuries and financial losses as a consequence of mower-thrown-object (M.T.O.) accidents. In an effort to curb mower-thrown-object accident frequency and severity, and improve public relations, the SDHPT has decided to investigate the mower-thrown-object phenomenon.

BACKGROUND

A variety of factors are known to contribute to mower-thrown-object accidents, including high rotary mower speeds, height of cut, rotary blade inertia and the length of the rotary blade. The subject of mower-thrown-object accident investigation is difficult to study experimentally because of the short interval of time during which the impact between the blade and the object takes place, and also because of the safety requirements of experimenting with such equipment.

Broadly speaking past efforts to reduce the M.T.O. frequency and severity involved either a modification in the blade or a provision of a suitable guard or a shield. The authors did not find any organization or agency that maintains records or other information on highway mowers.

OBJECTIVES

The first objective of this report was to develop a simple theoretical model to simulate the rotary action of the mower and a computer program that will aid in the analysis of the M.T.O. problem. The second stage was to build an experimental model to test the theoretical model. High speed photography was used to determine the discharge characteristics of the mower-thrown-object. A mower model built out of polycarbonate and installed with a 1/20 HP DC motor was used to simulate the rotary mower in the experimental analysis. The third stage was to simulate the rotary mower in the experimental analysis. The fourth and final stage was to discuss and recommend areas for future research on M.T.O. accidents.

SCOPE AND ORGANIZATION

Chapter 1 serves as an introduction to this report.

Chapter 2 deals with the analysis of the mower-thrown-object phenomena. A simple mathematical model of the mower and the object is developed using the impulse momentum laws. A computer program is developed for this model and the design parameters that effect mower thrown phenomena are studied [see Appendix A]. Analytical results are presented, discussed and conclusions drawn.

Chapter 3 verifies the theoretical analysis of Chapter 2 using high speed photography to measure the discharge characteristics of the test objects discharged from a physical model built of polycarbonate and powered by a 1/20 HP AC motor.

Chapter 4 presents the conclusions of this report and gives recommendations for future investigations of M.T.O. accidents.

CHAPTER 2. ANALYSIS OF MOWER THROWN OBJECTS

This chapter deals with the analysis of mower-thrown-objects. In the first stage, the impulse momentum laws, general impact theory and the theory of the coefficient of restitution will be reviewed. In the second stage of this chapter, two simple mathematical models will be developed to investigate the mower-thrown-object phenomena caused by a straight blade and a pivoted blade impacting an object. The analytical model for the pivoted blade will be extended to cover a multipivoted blade. In the third stage, a general computer program (see Appendix A) to predict the discharge characteristics of mower-thrown-objects will be discussed. This stage will also include the results predicted by the computer program when applied to the existing mowing conditions, namely the results predicted when the blade mass, blade speed and mower travel speed are in the same range as in present day mowers. A discussion of the results will also be included. The end of the chapter lists conclusions based on the analysis of the mower-thrown-object phenomena.

IMPULSE MOMENTUM PHENOMENA

Impulse Momentum Laws

The classical theory of impact, called stereomechanics (Ref 1) is based primarily on the impulse momentum law for rigid bodies. The advantage of this theory lies in the fact that the acceleration of the bodies does not have to be determined. However, the theory is incapable of describing the transient stresses, forces, or deformation produced in the rigid bodies and this theory is limited to determination of the terminal velocity states and the determination of the linear and/or angular impulse of the bodies. The linear and angular impulse-momentum laws for a rigid body are expressed by the vector equations:

$$\Delta\, m\mathbf{v} = m\mathbf{v}_f - m\mathbf{v}_i = \int \mathbf{F}\, dt = \mathbf{P} \qquad (2.1)$$

$$\text{and } \Delta\, I\mathbf{w} = I\mathbf{w}_f - I\mathbf{w}_i = \int \mathbf{F}\,\mathbf{r}\, dt \qquad (2.2)$$

In the above equations (Ref 1) m is the mass and I is the moment of inertia about the axis of rotation of the body, **v** and **w** are the linear and angular velocities, **r** is the moment arm, **F** is the external force, **P** is the impulse of the force **F**, and t is the time.

These equations express that when a rigid body is acted upon by a force **F** during a given time interval, the final momentum $m\mathbf{v}_f$ of the rigid body may be obtained by adding vectorially its initial momentum $m\mathbf{v}_i$ and the impulse **P** of the force **F** during the time interval.

When a problem involves two or more rigid bodies the momenta of all the rigid bodies and the impulses of all the forces involved are vectorially added (Ref 1). From Eq 2.1 it follows that

$$\Sigma m\mathbf{v}_i + \Sigma\mathbf{P} = \Sigma m\mathbf{v}_f \qquad (2.3)$$

Since the internal forces are equal and opposite (action and reaction) they cancel out and only the impulses of the external forces are considered. If however no external forces are exerted on the rigid bodies, or more generally, the sum of the external forces is zero then

$$\Sigma m v_i = \Sigma m v_f \qquad (2.4)$$

Eq 2.4 expresses that the total momentum of the system of rigid bodies is conserved (Ref 1). In addition to a momentum exchange among the rigid bodies an appreciable energy loss may result since the transfer of energy between the colliding bodies may generate dissipative losses incurred in plastic deformation. Nonimpulsive forces include the weight of the body, the force exerted by a spring or any other force which is known to be small relative to an impulsive force. Unknown reaction forces may or may not be impulsive (Ref 1).

Impact Phenomena

A collision between two bodies which occurs in a very small interval of time (less than half the lowest fundamental natural period of the two bodies (Ref 2)), and during which the two bodies exert on each other relatively large forces, is called an impact. The common normal to the surfaces in contact during impact is called the line of impact. If the mass centers of the two colliding bodies are located on this line, the impact is a central impact. Otherwise the impact is said to be an eccentric impact.

In the past, impact theory has been exceedingly difficult to verify experimentally by virtue of the short time intervals available for measurements. However, with the advent of modern electronic instrumentation, reliable data for many impact problems has become available (Ref 1, Ref 3).

As an introducton to impact, consider the collinear motion of two bodies of masses m_1 and m_2, travelling with velocities v_1 and v_2. In Fig 2.1a, v_1 is greater than v_2, collision occurs with the contact forces directed along the line of centers. This condition, as mentioned previously, is called direct central impact (Ref 4). Following contact, a short period of deformation takes place until the contact area between the bodies ceases to increase. At this instant, both bodies (see Fig 2.1b) are moving with the same velocity v_0. During the remainder of contact a period of restoration occurs during which the contact area reduces to zero. The final condition is shown in Fig 2.1c, where the bodies now have new velocities v_1' and v_2', and v_1' is less than v_2'. All velocities are arbitrarily assumed positive to the right so that with this scalar notation, a velocity to the left would carry a negative sign. If the impact is not overly severe, and if the bodies are highly elastic, they will regain their original shape following the restoration. With a more severe impact, and with less elastic bodies, a permanent deformation may result (Ref 4).

Inasmuch as the contact forces are equal and opposite during impact, the linear momentum of the system of the two bodies remains unchanged, as discussed in Chapter 2 under Impulse Momentum Laws. Thus, the momentum before and after impact is conserved. The equation for conservation of momentum for this case may be written as

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \qquad (2.5)$$

where subscripts 1 and 2 represent the conditions before and after impact respectively.

For given initial conditions, the momentum Eq 2.5 contains two unknowns, v_1' and v_2'. Clearly an additional relationship is required before the final velocities can be found. This relationship must reflect the capacity of the contacting bodies to recover from the impact and can be expressed by the ratio, e, of the magnitude of the restoration impulse to the magnitude of the deformation impulse. This ratio is called the coefficient of restitution. If F_r and F_d represent the magnitudes of the contact forces during the restoration and deformation periods respectively, as shown in Fig 2.2, for body 1 the definition of e together with the impulse momentum equation gives (Ref 4),

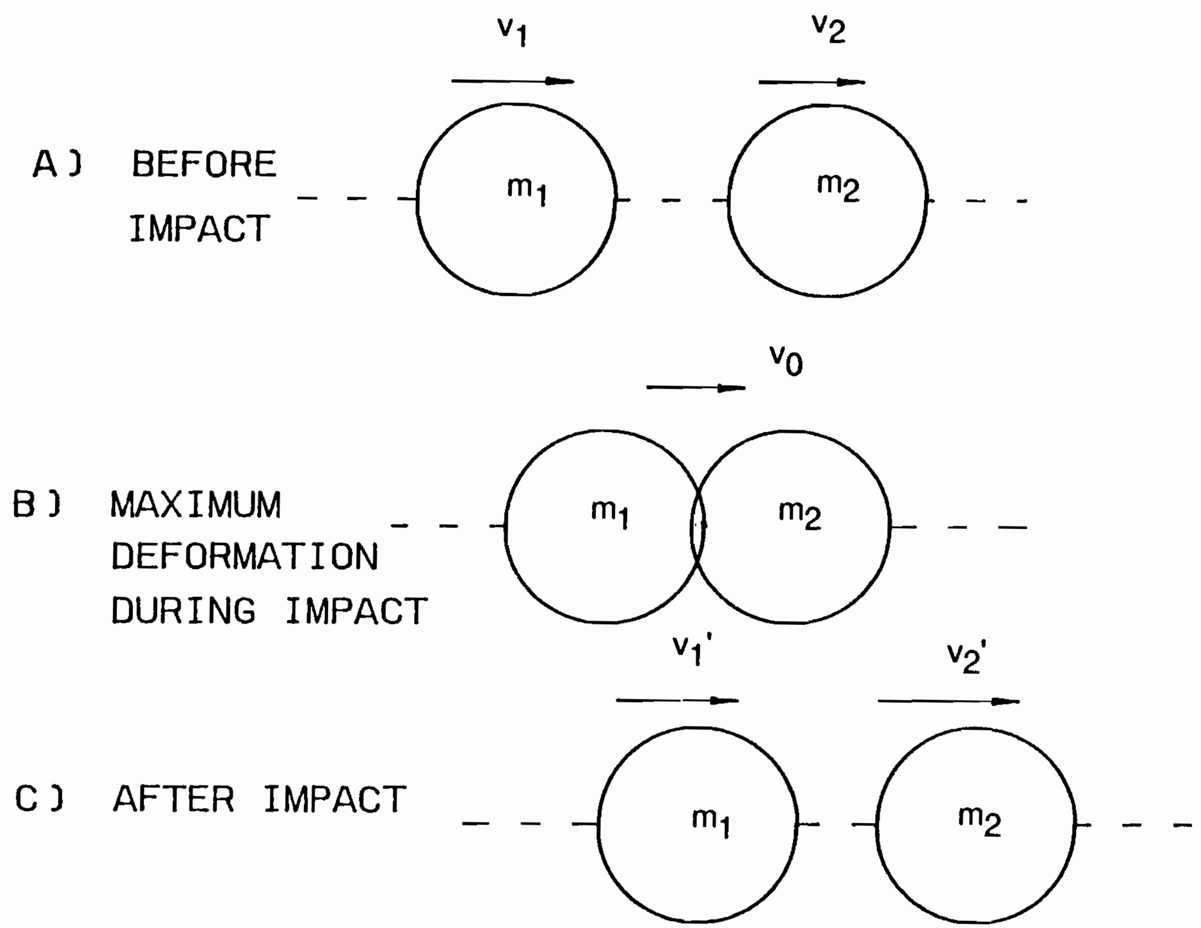


Fig. 2.1. Direct central impact of two moving bodies.

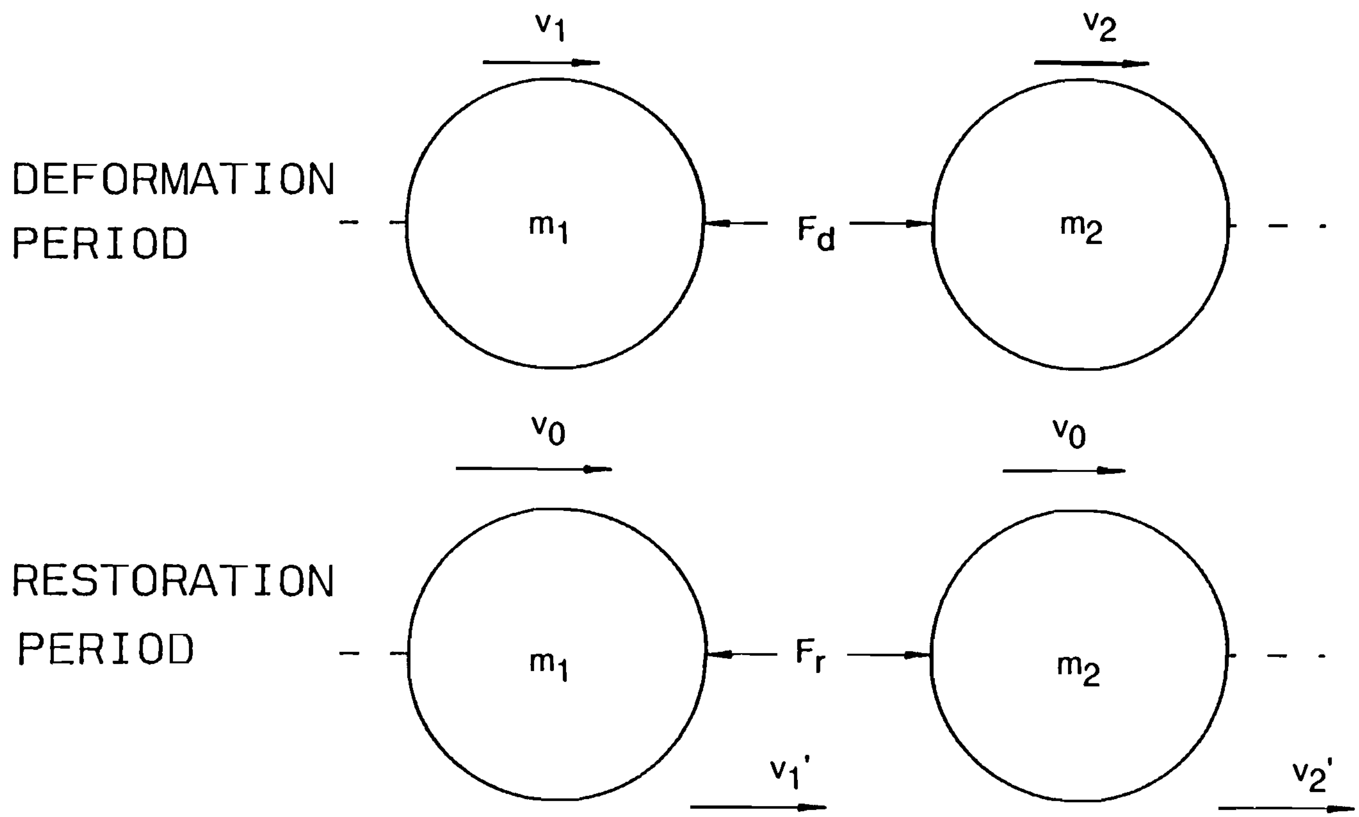


Fig. 2.2. Contact forces during restoration and deformation period

$$e = \frac{\int_{t_0}^{t} F_r dt}{\int_{0}^{t_0} F_r dt} = \frac{m_1(-v_1' - [-v_0])}{m_1(-v_0 - [-v_1])} = \frac{v_0 - v_1'}{v_1 - v_0'} \tag{2.6}$$

Similarly for body 2

$$e = \frac{\int_{t_0}^{t} F_r dt}{\int_{0}^{t} F_r dt} = \frac{m_2(v_2' - v_0)}{m_2(v_0 - v_2)} = \frac{v_2' - v_0}{v_0 - v_2} \tag{2.7}$$

The change of momentum must be in the same direction as the direction of the impulse or the force causing the impulse. The time for the deformation is taken as t_0 and the total time of contact is t. Eliminating v_0 in Eqs 2.6 and 2.7 gives

$$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{|\text{ relative velocity of separation }|}{|\text{ relative velocity of approach }|} \tag{2.8}$$

In addition to the initial conditions, if e is known for the impact condition at hand, then Eqs 2.5 and 2.8 are two equations in two unknown final velocities (Ref 4).

Impact phenomena are almost always accompanied by energy loss, which may be calculated by subtracting the kinetic energy of the system just after impact from that just before impact. Energy is lost through the generation and dissipation of elastic stress waves within the bodies, and through the generation of sound energy.

According to the classical theory of impact, the value e = 1 means that the capacity of the two bodies to recover equals their tendency to deform. This condition is one of elastic impact with no energy loss. The value e = 0, on the other hand, describes inelastic or plastic impact where the bodies cling together after collision and the loss of energy is a maximum. All impact conditions lie somewhere between these two extremes. A coefficient of restitution must be associated with a pair of contacting bodies (Ref 4).

The coefficient of restitution is frequently considered a constant for given geometries and given combinations of contacting materials. Actually the coefficient depends upon the impact velocity, and approaches unity as the impact velocity approaches zero as shown schematically in Fig 2.3. A handbook value for e is generally unreliable (Ref 4).

The relationship developed for direct central impact is extended to the case where the initial and final velocities are not parallel (see Fig.2.4). Here the bodies of masses m_1 and m_2 have initial velocities v_1 and v_2 in the same plane and approach each other on a collision course, as shown in Fig 2.4a. The directions of velocity are measured arbitrarily from the direction tangent to the contacting surfaces (see Fig 2.4b). The final rebound conditions are shown in Fig 2.4c. The impact forces F and -F, as seen in Fig 2.4d, vary from zero to their peak value during the deformation portion of impact and return to zero during the restoration period, as seen in Fig 2.4e, where t is the duration of the impact

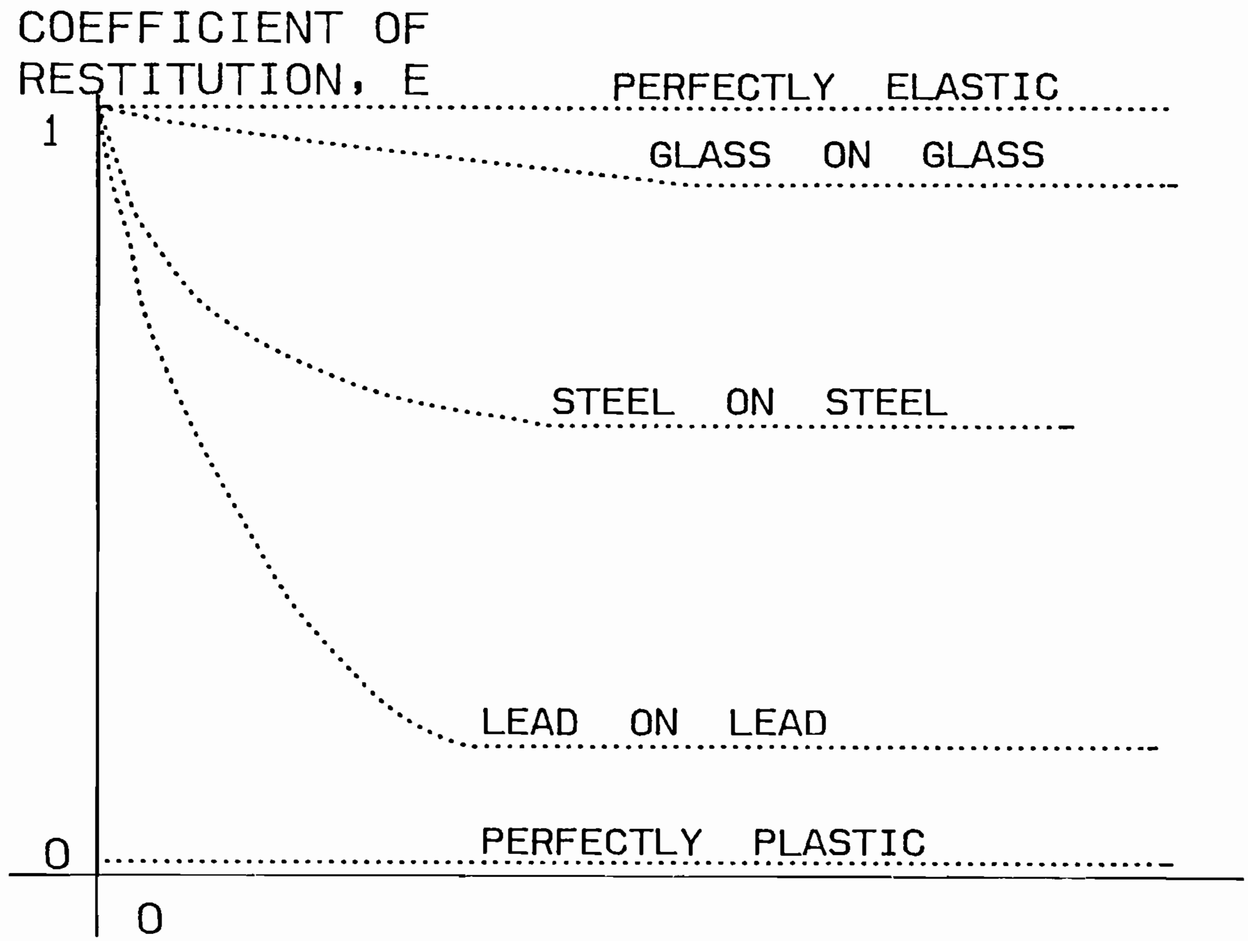


Fig. 2.3 Variation of e with the impact velocity

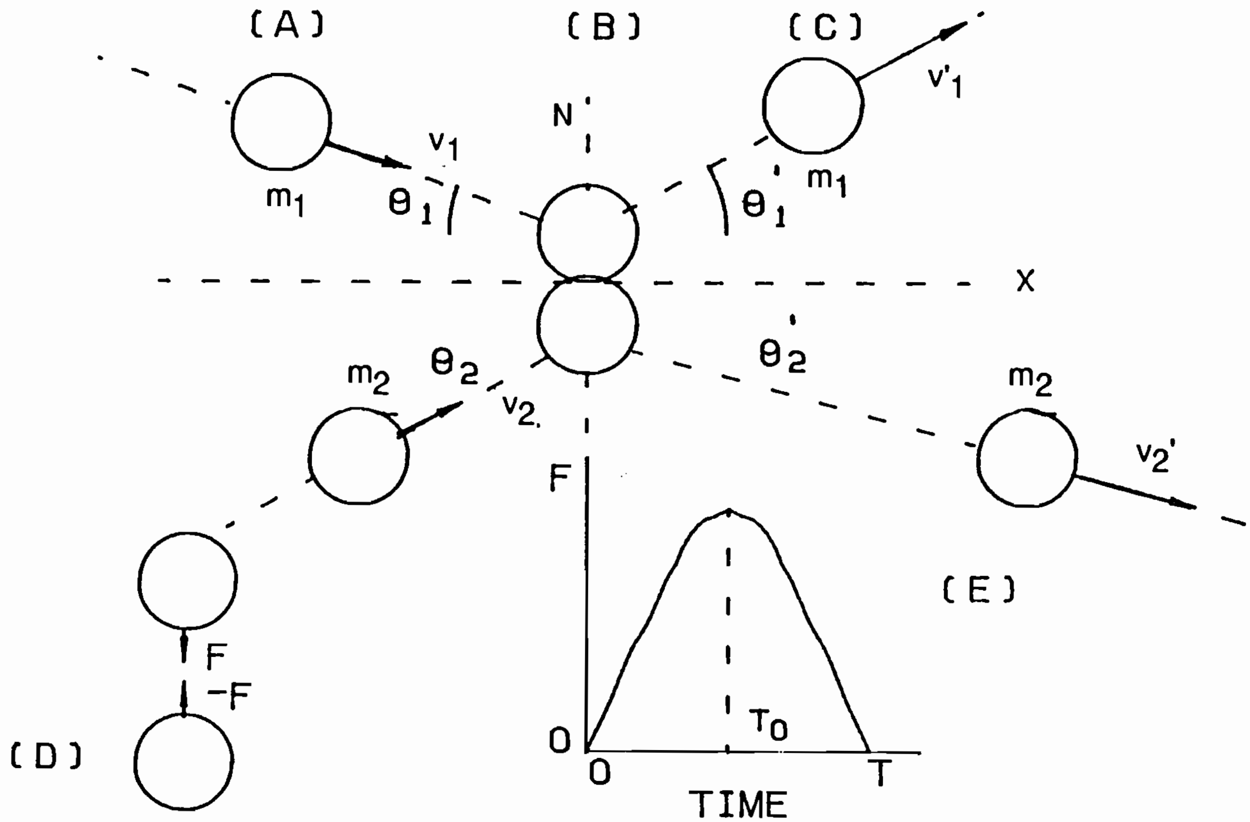


Fig. 2.4 Variation of contact forces during the deformation and restitution periods of a oblique central impact

interval.

For the given initial conditions of m_1, m_2, v_1, v_2, θ_1, and θ_2, there will be four unknowns, namely, v_1', v_2', θ_1' and θ_2'. The four needed equations are :

(1) The equation for the conservation of linear momentum in the n-direction given by

$$- m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2 = m_1 v_1' \sin\theta_1' - m_2 v_2' \sin\theta_2' \qquad (2.9)$$

(2) There is no impulse on body 1 in the x-direction and hence the conservation of momentum in the x-direction gives

$$m_1 v_1 \cos\theta_1 = m_1 v_1' \cos\theta_1' \qquad (2.10)$$

(3) There is no impulse on body 2 in the x-direction and hence the conservation of momentum in the x-direction gives

$$m_2 v_2 \cos\theta_2 = m_2 v_2' \cos\theta_2' \qquad (2.11)$$

(4) The coefficient of restitution, as in the case of direct central impact, is the ratio of the magnitude of the recovery impulse to the magnitude of the deformation impulse. Equation 2.8 can be applied for the velocity components in the n-direction (shown in Fig 2.4). Substituting $v_2'=v_1'\sin\theta_1'$, $v_1'=v_2'\sin\theta_2'$, $v_1=v_1\sin\theta_1$, $v_2=v_2\sin\theta_2$ in Eq 2.8 gives

$$e = \frac{v_1'\sin\theta_1' + v_2'\sin\theta_2'}{v_1\sin\theta_1 + v_2\sin\theta_2} \qquad (2.12)$$

With the above four equations, (2.9) through (2.12), the four unknowns v_1', v_2', θ_1', and θ_2' can be calculated (Ref 4).

When no external forces act on a rigid body or on a system of rigid bodies, the impulses of the external forces are zero and the system of the momenta at time t_1 is equipollent (same in effect or signification) to the system of the momenta at time t_2. Summing and equating successively the x-components, y-components, and the moments of the momenta at times t_1 and t_2, it can be found that the total linear momentum of the system is conserved in any direction and that the total angular momentum of the system is conserved about any point.

There are many engineering applications, however, in which the linear momentum is not conserved, yet in which the angular momentum H_o of the system about a given point O is conserved. Such cases occur when the lines of action of all the external forces pass through O or more generally when the sum of the angular impulses of the external forces about O is zero (Ref 4). For this case, the conservation of angular momentum about point O is given by

$$(Ho)_1 = (Ho)_2 \qquad (2.13)$$

where the subscripts 1 and 2 represent the initial and final conditions.

Coefficient of Restitution

Although the impulse momentum theory does not evaluate deformations, the existence of deformations during a finite period of contact is tacitly recognized. Consider two bodies which collide and denote by v_1 and v_2 the velocities before impact of the two points of contact 1 and 2 on each body. The deformation history is envisaged as consisting of two sub-intervals (Ref 1), as shown in Fig 2.5.

1. The approach period extends from the point of contact to the point of maximum deformation at the end of which the two bodies have the same velocity (Ref 1).
2. This is followed by a period of restitution lasting to the point of separation. Thereafter the bodies will return to their original shape or will stay permanently deformed depending on the magnitude of the impact forces and upon the materials involved. In the case of complete elasticity, an axis of symmetry exists about the point of maximum indentation, while an unsymmetrical curve is obtained for the case of partial restoration (Ref 1). The second sub-interval vanishes in the event of a plastic impact and the bodies do not separate.

Assuming the bodies are completely smooth, it is found that the forces they exert on each other are directed along the line of impact. Denoting respectively, by $\int P dt$ and $\int R dt$ the magnitude of the impulse of one of these impact forces during the period of deformation (approach) and during the period of restitution, the ratio is denoted as the coefficient of restitution, e, and is given by

$$e = \frac{\int R dt}{\int P dt} \qquad \text{(Ref 4)} \qquad (2.14)$$

It can be proved mathematically (Ref 4) that the coefficient of restitution is

$$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{|\text{ relative velocity of separation }|}{|\text{ relative velocity of approach }|} \qquad (2.15)$$

In order to determine the velocities of the two colliding bodies after impact, Eq 2.15 should be used in conjunction with one or several other equations such as Eqs 2.9, 2.10 and 2.11 obtained by the principle of impulse and momentum (Ref 4). When the impact produces a permanent deformation the coefficient of restitution is introduced to determine the final velocities. This coefficient purports to describe the degree of plasticity of the collision. The value of e = 1 and e = 0 denote the idealized concepts, respectively, of perfectly elastic and perfectly plastic impacts (Ref 4).

ANALYSIS

Mathematical Model

Models representing physical systems are idealized to render them amenable to theoretical treatment. As a consequence of this idealization a complete solution is obtained for a simple geometrical configuration utilizing the laws of conservation of mass, conservation of momentum, and a mechanical energy balance (Ref 1). In this report, the physical system is a rotary blade of a mower in operation that strikes a stationary object. The analytical model of the mower blade is a slender rod of

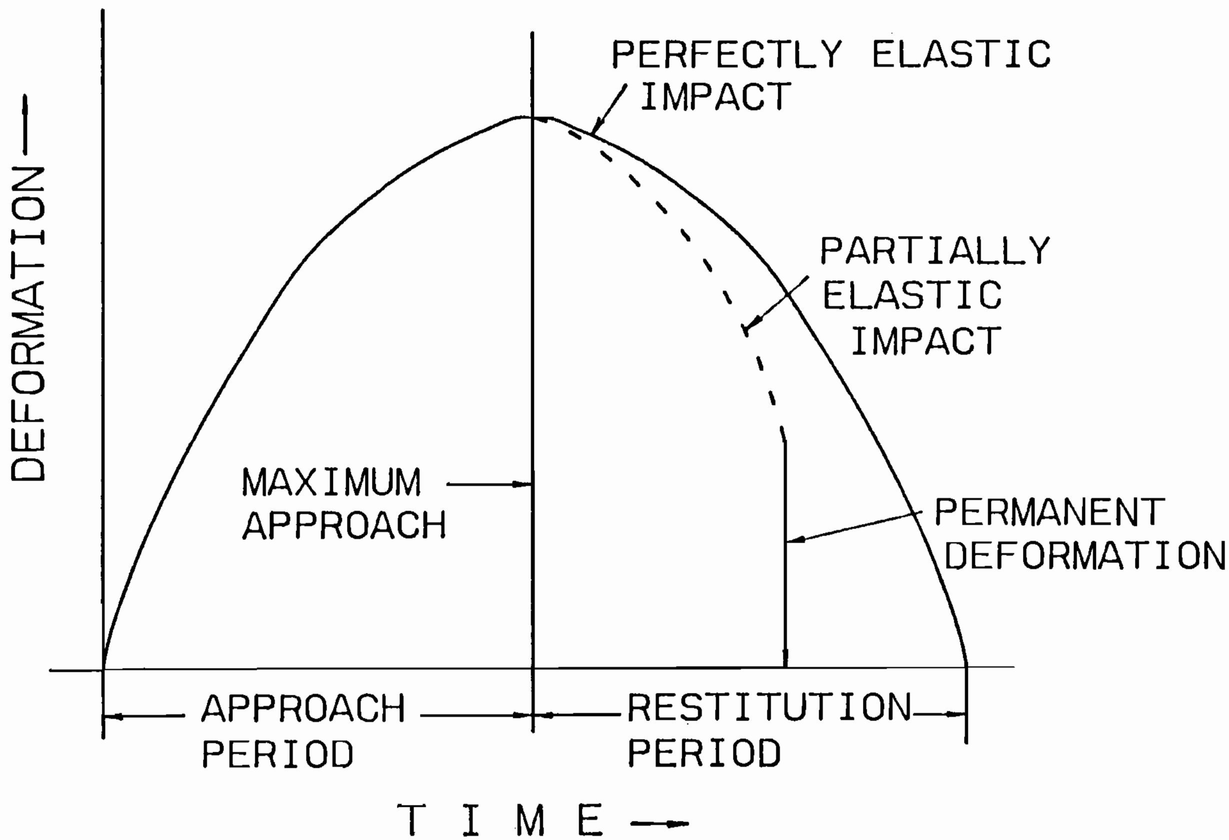


Fig. 2.5 Deformation of a body under impact

length, mass and angular velocity equal to the length, mass and angular velocity of the mower blade. In order to simulate relative velocity between the traveling mower and a stationary thrown object, the thrown object is fed to the rotating rod with a certain velocity. The assumptions made in this analytical model development are as follows.

1. Assume that the impact between the blade and the thrown object is a central impact and that the object does not spin after impact. This is assumed because the coefficient of restitution e is defined only for a central impact.
2. Neglect the resistance of the grass and air on the blade and on the thrown object.
3. Neglect the air flow around the blade.
4. Assume frictionless joints.
5. Assume that the moment of inertia of the blade is equivalent to the moment of inertia of a slender rod. This is a good assumption since the width of the rectangular blade is small compared to its length.

In the following part of this chapter the mower-thrown-object phenomena for the straight and pivoted blade types is investigated. The straight bar blade and the pivoted blade are being used today by most highway departments. The analysis for a single pivoted blade is then extended for analyzing a multi-pivoted blade.

Case I - Straight Blade

Consider a slender rod of mass m_b and length l that is rotating counterclockwise (arbitrary assumption) at a constant velocity ω about a fixed point O. Another object of known mass m_c is moving towards the rotating rod with a velocity v_c as seen in Fig 2.6. It is required to calculate the velocity of the object, v_c' after impact.

Consider the rod and the object as a single system and express that the initial momenta of the rod and the object and the impulses of the external forces are together equipollent to the final momenta of the system.

Table 2.1
LIST OF VARIABLES IMPORTANT IN STRAIGHT BLADE ANALYSIS
(see Fig 2.6)

1. m_b = mass of the rod
2. m_c = mass of the object
3. v_{cm} = velocity of the center of mass of the rod before impact
4. v'_{cm} = velocity of the center of mass of the rod after impact
5. v_c = velocity of the object before impact
6. v_c' = velocity of the object after impact
7. v_p = velocity of the rod at the point of contact before impact
8. v_p' = velocity of the rod at the point of contact after impact
9. ℓ= length of the rod
10. ω = angular velocity of the rod before impact
11. ω' = angular velocity of the rod after impact
12. r = distance of the point of impact from the center of rotation
13. I = moment of inertia of the rod about its center of mass

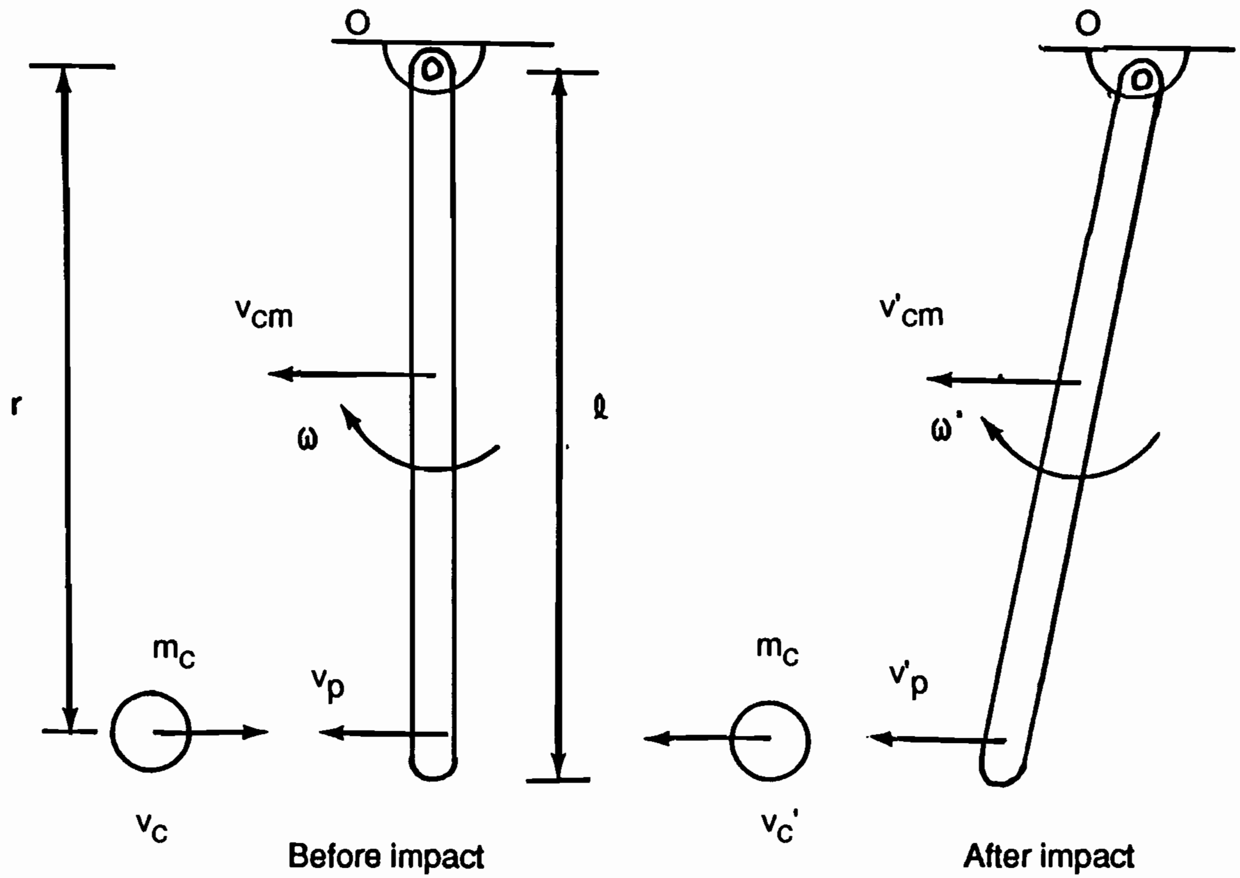


Fig. 2.6. Case - I, impact phenomena of a straight blade

As there are no impulsive external forces acting on the system, as seen in Fig 2.6, and the reaction forces at O being internal, the principle of conservation of angular momentum about O can be applied. Taking the clockwise direction as the positive direction, the angular momentum of the system before impact, about point O, is calculated as follows (Ref 4):

(1) The angular momentum of the rod about O is $-I\omega - (m_b v_{cm})\, \ell/2$

(2) The angular momentum of the object about O is $(m_c v_c)\, r$

(3) The total angular momentum of the rod and the object before impact, about point O is
$-I\omega - (m_b v_{cm})\, \ell/2 + (m_c v_c)\, r$

The angular momentum of the system after impact, about the point the O is calculated as

(1) The angular momentum of the rod about O is $-I\omega' - (m_b v_{cm}')\, \ell/2$

(2) The angular momentum of the object about point O is $-(m_c v_c')\, r$

(3) The total angular momentum of the rod and the object after impact, about point O is
$-I\omega' - (m_b v_{cm}')\, \ell/2 - (m_c v_c')\, r$

By the principle of conservation of angular momentum, the angular momentum of the system about point O before impact is equal to the angular momentum of the system about point O after impact, which gives

$$-I\omega - (m_b v_{cm})\, \ell/2 + (m_c v_c)\, r = -I\omega' - (m_b v_{cm}')\, \ell/2 - (m_c v_c') \qquad (2.16)$$

As explained in Chapter 2 under Coefficient of Restitution and based on our assumption of a central impact between the blade and the object (the coefficient of restitution is defined for a central impact only) a coefficient of restitution is associated with the impacting rod and the object. A second equation based on the coefficient of restitution is given by applying Eq 2.15 or

$$e = \frac{|\text{ relative velocity of separation }|}{|\text{ relative velocity of approach }|} \qquad (2.17)$$

wherein the relative velocity of approach between the rod and object is $|v_p - v_c|$ and the relative velocity of separation between the rod and the object is $|v_c' - v_p'|$.

Thus
$$e = \frac{|v_p' - v_c'|}{|v_p - v_c|} \qquad (2.18)$$

From the kinematics of the rod, the linear velocity at any point in the rod is equal to the radius of rotation times angular velocity; i.e.,

$$v_p = r \times \omega$$

$v_p' = r \times \omega'$

Equations (2.16) and (2.18) have two unknowns and can be solved to determine the velocity v_c' of the object after impact.

<u>Case II - Pivoted Blade</u>

This section is devoted to the analysis of M.T.O.'s discharged by the pivoted blade. The pivoted blade is used by many Highway Departments to reduce the frequency and severity of M.T.O. accidents. The pivoted blade is modelled as two slender rods each of mass m_1 and m_2 and of length ℓ_1 and ℓ_2, respectively, that are connected by a smooth pivot (see Fig 2.7). The assembly rotates in the counterclockwise direction (direction assumed arbitrarily) at a constant angular velocity ω about a fixed point O. Another object of known mass m_c strikes the rod assembly at a velocity v_c. It is desired to calculate the velocity of the object v_c' after impact.

Consider the rod assembly and the object as a single system and express that the initial momenta of the rod assembly and the object and the impulses of the external forces are together equipollent to the final momenta of the system.

Table 2.2
LIST OF VARIABLES IMPORTANT IN PIVOTED BLADE ANALYSIS
(see Fig 2.7)

1. m_c = mass of the object
2. v_c = velocity of the object before impact
3. ω_1 = angular velocity of rod 1 before impact
4. ω_2 = angular velocity of rod 2 before impact
5. $\omega_1 = \omega_2$ (before impact the rod assembly rotates as a straight rigid link and the rods 1 and 2 have the same angular velocities)
6. ω_1' = absolute angular velocity rod 1 after impact
7. ω_2' = angular velocity rod 2 with respect to pivot after impact
8. $\omega_1' + \omega_2'$ = absolute angular velocity rod 2 after impact
9. m_1 = mass of rod 1
10. m_2 = mass of rod 2
11. I = moment of inertia of the blade assembly about 0
12. I_1 = moment of inertia of rod 1 about its center of mass
13. I_2 = moment of inertia of rod 2 about its center of mass
14. ℓ_1 = length of rod 1
15. ℓ_2= length of rod 2
16. $\ell_1 + \ell_2$ = length of rod assembly
17. v_1 = velocity of center of mass of rod 1 before impact
18. v_1' = velocity of center of mass of rod 1 after impact
19. v_2 = velocity of center of mass of rod 2 before impact

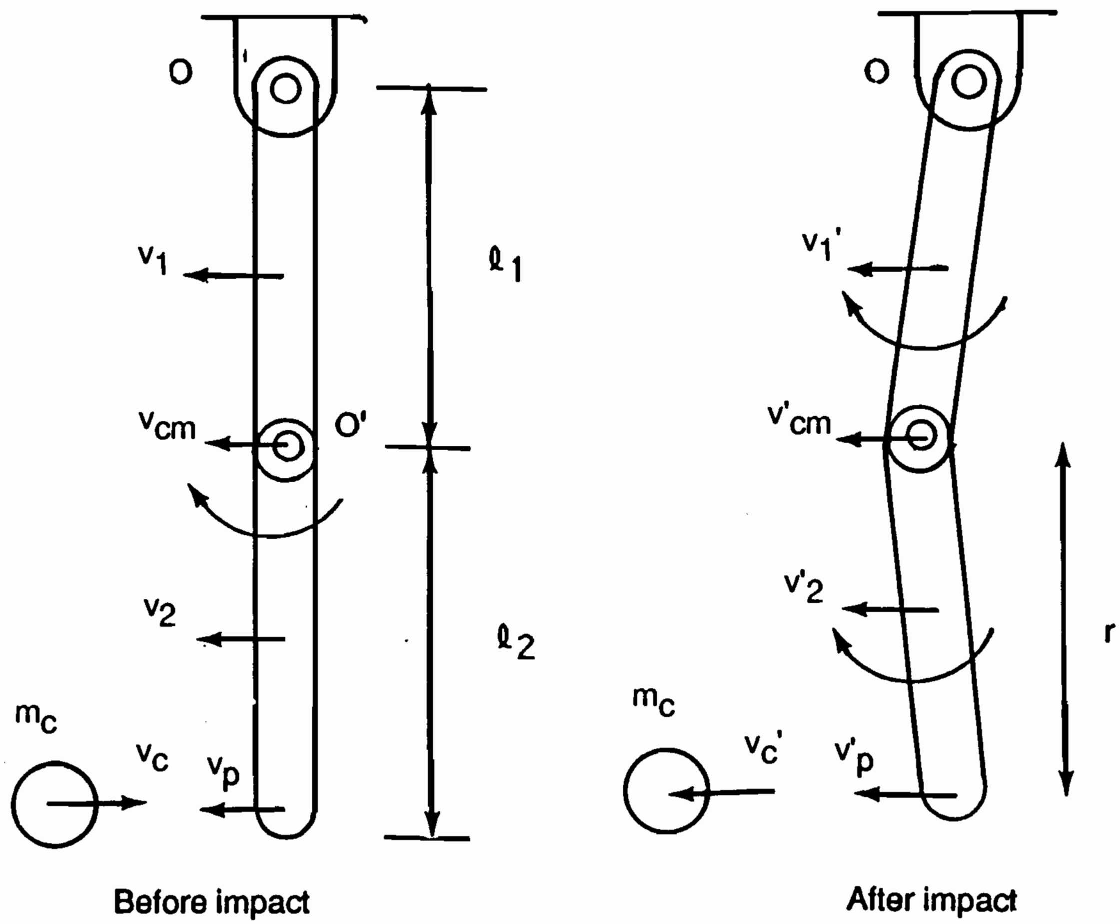


Fig. 2.7 Case - II, impact phenomena of a pivoted blade.

20. v_2' = velocity of center of mass of rod 2 after impact
21. v_{cm} = velocity of the center of mass of the rod assembly before impact
22. v_{cm}' = velocity of the center of mass of the rod assembly after impact
23. R = distance of point of impact from fixed point
24. v_p = velocity of the rod assembly at the point of contact before impact
25. v_p' = velocity of the rod assembly at the point of contact after impact
26. $I_1 = m_1 \ell_1^2 / 12$
27. $I_2 = m_2 \ell_2^2/12$

As there are no impulsive external forces acting on the system, as seen in Fig 2.7 and the reaction forces at O and O' being internal, the principle of conservation of angular momentum can be applied about O and O'. Taking the clockwise direction as the positive direction, the angular momentum of the system before impact, about point O, is calculated as follows (Ref 4):

(1) The angular momentum of the rod 1 about O is $-I_1\omega_1 - m_1v_1(\ell_1/2)$

(2) The angular momentum of the rod 2 about O is $-I_2\omega_2 - m_2v_2(\ell_1+\ell_2/2)$

(3) The angular momentum of the object about O is $(m_cv_c)R$

(4) The total angular momentum of the rod and the object before impact, about point O is
$-I_1\omega_1 - m_1v_1(\ell_1/2) - I_2\omega_2 - m_2v_2(\ell_1+\ell_2/2) + (m_cv_c)R$

The angular momentum of the system after impact, about the point the O is calculated as (Ref 4):

(1) The angular momentum of the rod 1 about O is $-I_1\omega_1' - m_1v_1'(\ell_1/2)$

(2) The angular momentum of the rod 2 about O is $-I_2\omega_2' - m_2v_2'(\ell_1+\ell_2/2)$

(3) The angular momentum of the object about O is $-(m_cv_c')R$

(4) The total angular momentum of the rod assembly and the object after impact, about point O is
$-I_1\omega_1' - m_1v_1'(\ell_1/2) - I_2\omega_2' - m_2v_2'(\ell_1 + \ell_2/2) - (m_cv_c')R$.

By the principle of conservation of angular momentum, the angular momentum of the system about point O before impact is equal to the angular momentum of the system about point O after impact, which gives

$$- I_1\omega_1 - m_1v_1(\ell_1/2) - I_2\omega_2 - m_2v_2(\ell_1+\ell_2/2) + (m_cv_c)R = - I_1\omega_1' - m_1v_1'(\ell_1/2) - I_2\omega_2' - m_2v_2'(\ell_1+\ell_2/2) - (m_cv_c')R \tag{2.19}$$

As explained in Chapter 2 under Coefficient of Restitution, and based on our assumption of a central impact between the blade and the object (the coefficient of restitution is defined for a central impact only) a coefficient of restitution is associated with the impacting rod and the object. A second equation based on the coefficient of restitution is given by applying Eq 2.15 or

$$e = \frac{|\text{ relative velocity of separation }|}{|\text{ relative velocity of approach }|} \tag{2.20}$$

wherein the relative velocity of approach between the rod and object is $|v_p - v_c|$ and the relative velocity of separation between the rod and the object is $|v_c' - v_p'|$.

Thus
$$e = \frac{|v_p' - v_c'|}{|v_p - v_c|} \tag{2.21}$$

From the kinematics of the rod, the linear velocity at any point in the rod is equal to the radius of rotation times the angular velocity, i.e.,

$$v_p = \ell_1\omega_1 + (R - \ell_1)\omega_2$$

$$v_1 = (\ell_1/2)\omega_1$$

$$v_2 = (\ell_1 + \ell_2/2)\,\omega_2$$

$$v_p' = \ell_1\omega_1' + (R - \ell_1)\omega_2'$$

$$v_1' = (\ell_1/2)\omega_1'$$

$$v_2' = \ell_1\omega_1' + (\ell_2/2)\omega_2'$$

Conservation of angular momentum about the pivot point O' (see Fig 2.7) gives the third equation. The angular momenta of the rod and object, before and after impact, about the point O' are written in the same manner as the angular momenta of the rod and object about point O, as written earlier. However, in Eq 2.19 the term $(\ell_1+\ell_2/2)$ changes to $\ell_2/2$ and R changes to $(R - \ell_1)$. This is because the point about which the moments are calculated has changed from O to O'.

$$- I_1\omega_1 + m_1v_1(\ell_1/2) - I_2\omega_2 - m_2v_2(\ell_2/2) + (m_cv_c)(R - \ell_1) =$$

$$- I_1\omega_1' + m_1v_1'(\ell_1/2) - I_2\omega_2' - m_2v_2'(\ell_2/2) - (m_cv_c')(R - \ell_1) \tag{2.22}$$

Thus three Eqs 2.19, 2.21 and 2.22 for the three unknowns ω_1', ω_2', and v_c' are obtained. The system of equations can be solved to determine the velocity v_c' of the object after impact.

Case III - Multi-Pivoted Blade

The analysis for a pivoted blade is extended to a multi-pivoted blade assembly. Consider a rod assembly made up of N slender rods each connected by a smooth pivot. The assembly rotates at an angular velocity ω about the fixed point O. The j^{th} rod of the assembly is struck by a mass m_c moving towards it with a velocity v_c. The variables important in the analysis are listed in Table 2.3.

Table 2.3
LIST OF VARIABLES IMPORTANT IN MULTI-PIVOTED BLADE ANALYSIS

1. m_i = mass of the i^{th} rod
2. N = number of rods in the assembly
3. ℓ = length of each rod
4. I = moment of Inertia of the assembly about its center of mass
5. I_i = moment of Inertia of the i th rod about its center of mass
6. ω_i = absolute angular velocity of the i^{th} rod assembly before impact
7. ω_i' = absolute angular velocity of the i^{th} rod after impact
8. v_{cm} = velocity of the center of mass of the rod assembly before impact
9. v_i' = velocity of the center of mass of the i^{th} rod after impact
10. v_p = velocity of the rod assembly at the point of contact before impact
11. v_p' = velocity of the rod assembly at the point of contact before impact
12. j = rod struck by the object
13. i=1 denotes the fixed pivot point about which the rod assembly rotates
14. r = distance of the point of contact from the j^{th} pivot

---

As seen in case I and II, the only impulsive forces acting on the system of the rod assembly and the object are the reaction forces at the fixed point of rotation and at each pivot connecting the rod assembly. Since the reaction forces are internal to the system, angular momentum is conserved about the fixed point of rotation and about each individual pivot. An assembly of N rods will have N-1 pivots, and about each pivot the angular momentum of the rod assembly is conserved. The angular momentum is also conserved about the fixed pivot point. The law of conservation of angular momentum applied to the N pivot points gives a total of N equations. The coefficient of restitution applied to the point of contact gives an additional equation. Thus a system of N + 1 equations has been obtained for a rod assembly of N rods. The unknowns in the system of N+1 equations are the angular velocities of the N rods as well as the velocity of the object after impact, a total of N+1 unknowns. This system of N + 1 linear equations can be solved to obtain the velocity of the object after impact.

$$M = \text{mass of the assembly} = \sum_{i=1}^{N} m_i$$

ℓ = length of each rod

L = length of assembly = $N\ell$

v_{cm} = velocity of center of mass (c.m.) of the assembly = $1/2(N\ell)\omega$

I_i = moment of inertia of each rod about its c.m.

$v_i = (i-1/2)\ell\omega$

$v_p = (j-1)\ell\omega + r\omega$

Before impact the rods all have the same angular velocity. The angular momentum of the assembly about the k^{th} pivot is given by

$$-\sum_{i=1}^{N} I_i\omega - \sum_{i=1}^{N} m_i v_i\{(i-1/2)\ell - (k-1)\ell\} - m_c v_c[(k-1)\ell - \{(j-1)\ell + r\}]$$

From kinematics of the rod assembly, linear velocity is equal to distance times the angular velocity

$$v_i' = \sum_{k=1}^{i-1} \ell\omega_k' + 1/2(\ell\omega_i')$$

Similarly the velocity of the rod assembly at the point of contact, on the jth link, is given by

$$v_p' = \ell \sum_{k=1}^{j-1} \omega_k' + r\omega_j'$$

The angular momentum of the system about each of the N pivot points is conserved before and after impact. The general formulation of the angular momentum of the assembly and the object about any point k, is given by

$$-\sum_{i=1}^{N} I_i\omega_i' - \sum_{i=1}^{N} m_i v_i'\{(i-1/2)\ell - (k-1)\ell\} - m_c v_c'[(k-1)\ell - \{(j-1)\ell + r\}]$$

Applying the law of conservation of angular momentum about each pivot point, we obtain N equations (k varying from k=1 to k=N in Eq 2.23).

$$-\sum_{i=1}^{N} I_i\omega_i - \sum_{i=1}^{N} m_i v_i\{(i-1/2)\ell - (k-1)\ell\} - m_c v_c[(k-1)\ell - \{(j-1)\ell + r\}]$$

$$= -\sum_{i=1}^{N} I_i\omega_i' - \sum_{i=1}^{N} m_i v_i'\{(i-1/2)\ell - (k-1)\ell\} - m_c v_c'[(k-1)\ell - \{(j-1)\ell + r\}] \qquad (2.23)$$

The definition for the coefficient of restitution gives one additional equation, given by

$$e = \frac{|v_p' - v_c'|}{|v_c - v_p|} \qquad (2.24)$$

The above system of N+1 equations (N equations obtained from Eq 2.23 and the additional Eq 2.24 can be solved to obtain the velocity of the object after impact.

RESULTS

This section presents the results obtained from running a computer program (see Appendix A) written for the analysis of Chapter 2 under Impact Phenomena. The program is a general program that will compute the M.T.O. velocities when provided with input for the mower and mowing conditions. Input values for average mowing conditions, for blade speed, blade mass, relative velocity between the blade and the M.T.O. mass (see Table 2.4 for inputs to the computer program) have been provided to the program, and the results predicted by the program have been plotted in Figs. 2.8 through 2.15. The mass of the blade is given as 22 lbs, blade rpm as 1000, blade length as 2 ft and relative velocity between the blade and the M.T.O. as 5 mph (440 ft/min). These values are for average mowing conditions and have been obtained from a mower manufacturer's catalog (Ref 7). From the analysis it is apparent that the major factors that contribute to the M.T.O. velocity are the blade mass, blade speed, coefficient of restitution and the distance from the center of rotation to the point of contact (these major factors are hereafter referred to as the parameters).

In order to obtain general results the parameters are varied in the computer program and the results are plotted in a nondimensional form in Fig 2.8 through Fig 2.11. The abscissa is the dimensionless mass of the object and the ordinate is the dimensionless object velocity. Additionally the effect of a variation in the blade mass and blade speeds is plotted in Fig 2.12 through 2.15 in dimensional form. The plots in Fig 2.8 through Fig 2.15 show the variation of the M.T.O. discharge velocity with a change in one of the above parameters.

In Fig 2.8, the coefficient of restitution takes the values 0.3, 0.5, 0.7, and 0.9 in order to represent a wide range of thrown objects. The blade rpm is 700, blade mass is 16 lbs, mower speed is 5 mph and the distance to the point of contact is 22 inches (this length represents the distance of the point of M.T.O. impact with the blade from the fixed point of rotation). The variation of dimensionless M.T.O. velocity with dimensionless M.T.O. mass is plotted in Fig 2.8 for each of the four values of e. There is a significant increase in velocity with an increase in the value of e.

In Fig 2.9, the rpm of the blade is 700, 800, 900 and 1000. The blade mass for the top curve is 22 lbs and the blade mass for the bottom curve is 16 lbs, coefficient of restitution is 0.9, mower speed is 5 mph and the point of contact is 22 inches. A curve is plotted for each of the four speeds and each dimensionless blade mass. It is observed from the two sets of curves that the M.T.O. has a higher discharge velocity when struck by the heavier 22 lb blade. Also, the difference in discharge velocities increases as the dimensionless mass of the M.T.O. increases.

In Fig 2.10, the blade mass varies as 16, 18, 20 and 22 lbs. The blade rpm is 1000, the distance to the point of contact is 22 inches and the coefficient of restitution is 0.9. It is observed that when the dimensionless mass of the M.T.O. is small there is no significant change in the dimensionless M.T.O. velocity with change in the blade mass. However, as the dimensionless mass of the M.T.O. increases, an increase in the mass of the blade significantly increases the dimensionless M.T.O. velocity.

In Fig 2.11, the distance from the pivot to the point of contact varies as 1.33, 1.5, 1.75 and 1.84 ft respectively, from the pivot point. The blade speed is 1000 rpm, coefficient of restitution is 0.9 and the blade mass is 22 lbs. A significant change in the dimensionless M.T.O. velocity is observed with a variation in the point of contact when the dimensionless mass of the object is small. However, as the dimensionless M.T.O. mass increases, the effect of the change in the point of contact is minor.

In Fig 2.12, the rpm of the blade varies as 700, 800, 900 and 1000. The blade mass is 22 lbs, coefficient of restitution is 0.9, mower speed is 5 mph and the point of contact is 22 inches. A curve showing the variation of the M.T.O. velocity versus the object mass (dimensional form) is plotted for each of the four speeds. It is observed that the change in M.T.O. speed is approximately proportional to the change in the blade speed and that the effect of the variation in blade speed on the M.T.O. velocity is significant.

In Fig 2.13, the rpm of the blade is varied as in Fig 2.12 but the blade mass is reduced to 16 lbs, other parameters remaining the same as in Fig 2.12. It is observed from Fig 2.13 that an increase in the blade speed significantly increases the M.T.O. velocity. On comparing Fig 2.12 and Fig 2.13, it is observed that though the blade mass is reduced by about 27% there is a marginal decrease in the M.T.O. velocity. In particular, as the M.T.O. mass decreases, the effect of a variation in the blade mass on the M.T.O. velocity becomes smaller and smaller. However, the effect of the change in the blade mass is significant as the M.T.O. mass increases.

In Fig 2.14, the blade mass varies as 16, 18, 20 and 22 lbs. The blade rpm is 1000, the distance to the point of contact is 22 inches and the coefficient of restitution is 0.9. The plots for the different blade masses are clustered together and indicate a minor effect on the M.T.O. velocity with a variation in the blade mass. As the mass of the M.T.O. increases, an increase in mass of the blade significantly increases the M.T.O. velocity.

In Fig 2.15, the blade mass varies as in Fig 2.14 but the blade rpm is reduced to 700, all other parameters remaining the same as in Fig 2.14. On comparing Fig 2.15 and Fig 2.14, it is observed that there is a significant reduction in M.T.O. velocities. Clearly the reduction in M.T.O. velocities is a direct consequence of the reduction in the blade speed. As the mass of the M.T.O. increases, an increase in mass of the blade significantly increases the M.T.O. velocity.

In Fig 2.16, S/L is plotted against Object velocity (Final - Initial)/Blade tip speed for a pivoted blade, S is the distance of the pivot from the blade tip circle, L is the length measured from the center of rotation to the blade tip circle, Final velocity is the object velocity after it has been struck by the mower blade and Initial velocity is the relative velocity before impacting of the object (or the mower speed), Blade tip speed is the linear velocity of the mower blade tip. The blade mass is 20 lbs, blade rpm is 600, blade length is 24 inches, the coefficient is restitution is 0.7, the initial object velocity is 440 ft/min and the distance to the point of contact is 22 inches. The figure shows that when the ratio S/L lies in the range of 55 percent to 80 percent, the discharge velocities rise rapidly. When the pivot is located halfway between the blade tip circle and the center of rotation, as in most blades used today, an approximate average of the highest and lowest discharge velocities is obtained.

Table 2.4

SUMMARY OF INPUTS TO THE COMPUTER PROGRAM

Figure	Blade speed, rpm	Blade mass, lbs	Distance from center of rotation to the point of contact, inches	Coefficient of restitution
2.8	700	16	22	0.3, 0.5. 0.7, 0.9
2.9	700, 800 900, 1000	16, 22	22	0.9
2.10	1000	16, 18 20, 22	22	0.9
2.11	1000	22	16, 18, 20, 22	0 . 9
2.12	700, 800 900, 1000	22	22	0.9
2.13	700, 800 900, 1000	16	22	0.9
2.14	1000	16, 18 20, 22	22	0.9
2.15	700	16, 18 20, 22	22	0.9

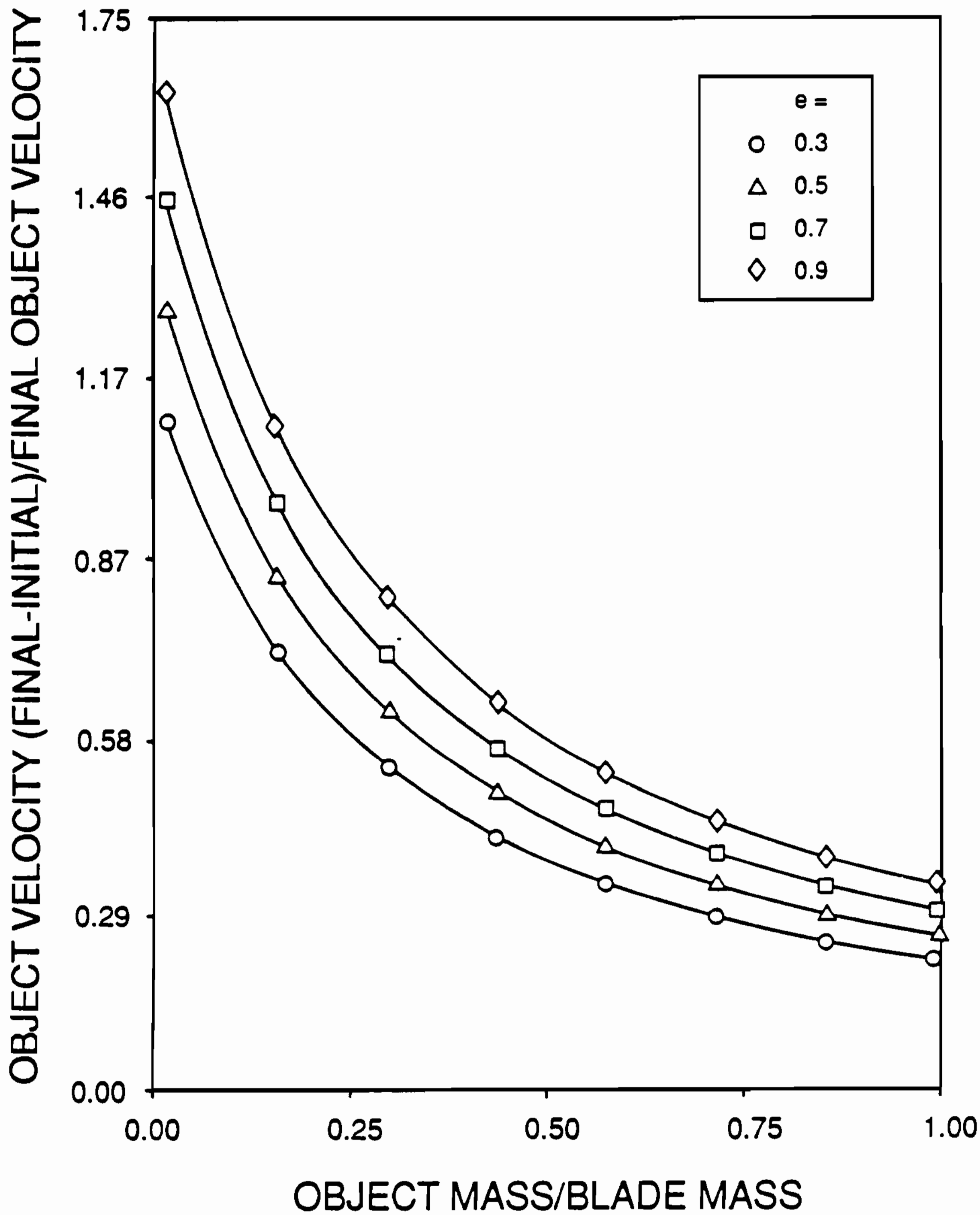


Fig. 2.8 Effect on dimensionless MTO velocity with varying coefficient of restitution and a blade mass of 16 lbs, blade rpm of 700 and the point of contact at 22 inches

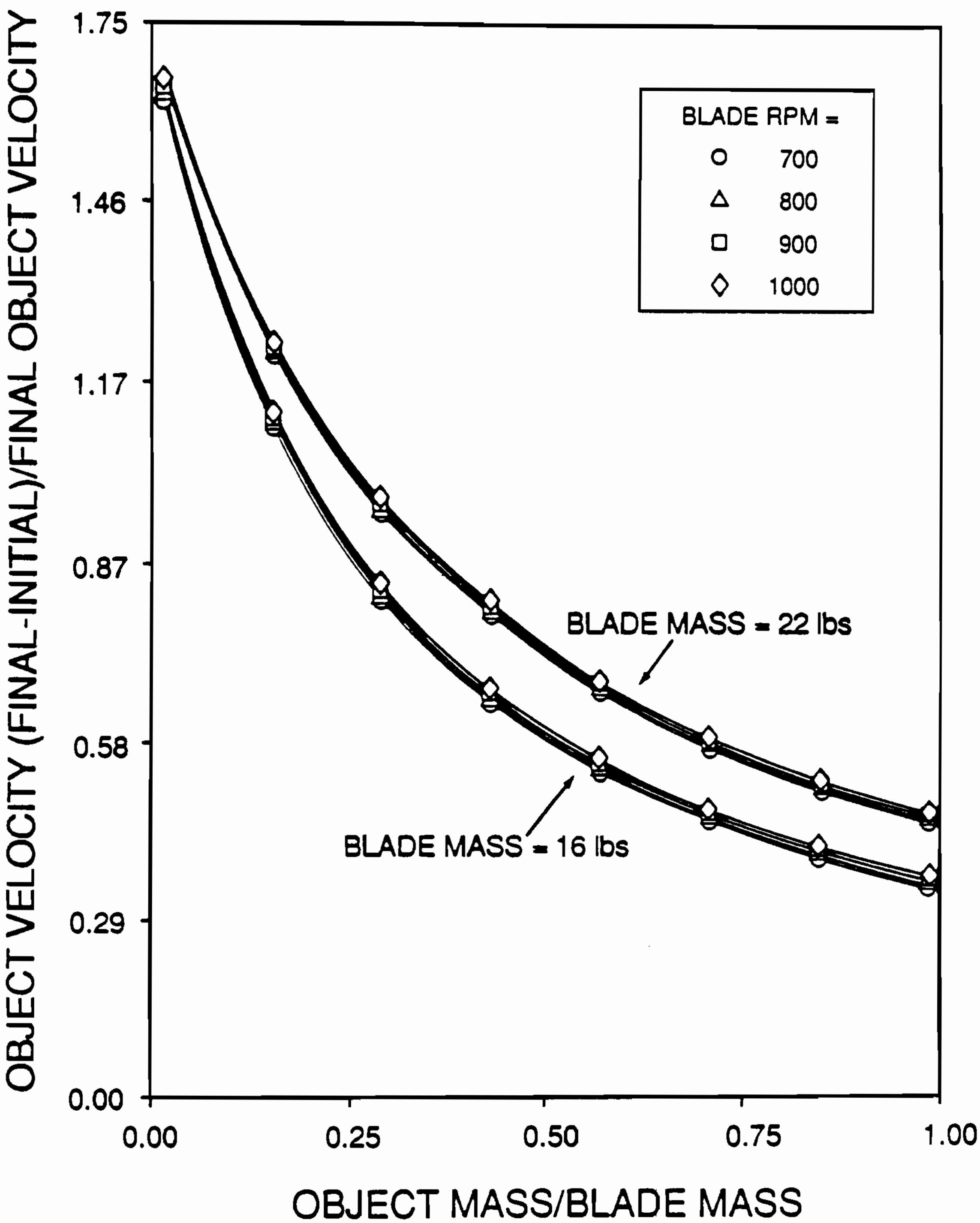


Fig. 2.9 Effect on dimensionless MTO velocity with different blade speeds blade masses of 16 lbs and 22 lbs, coefficient of restitution of 0.9 and the point of contact at 22 inches.

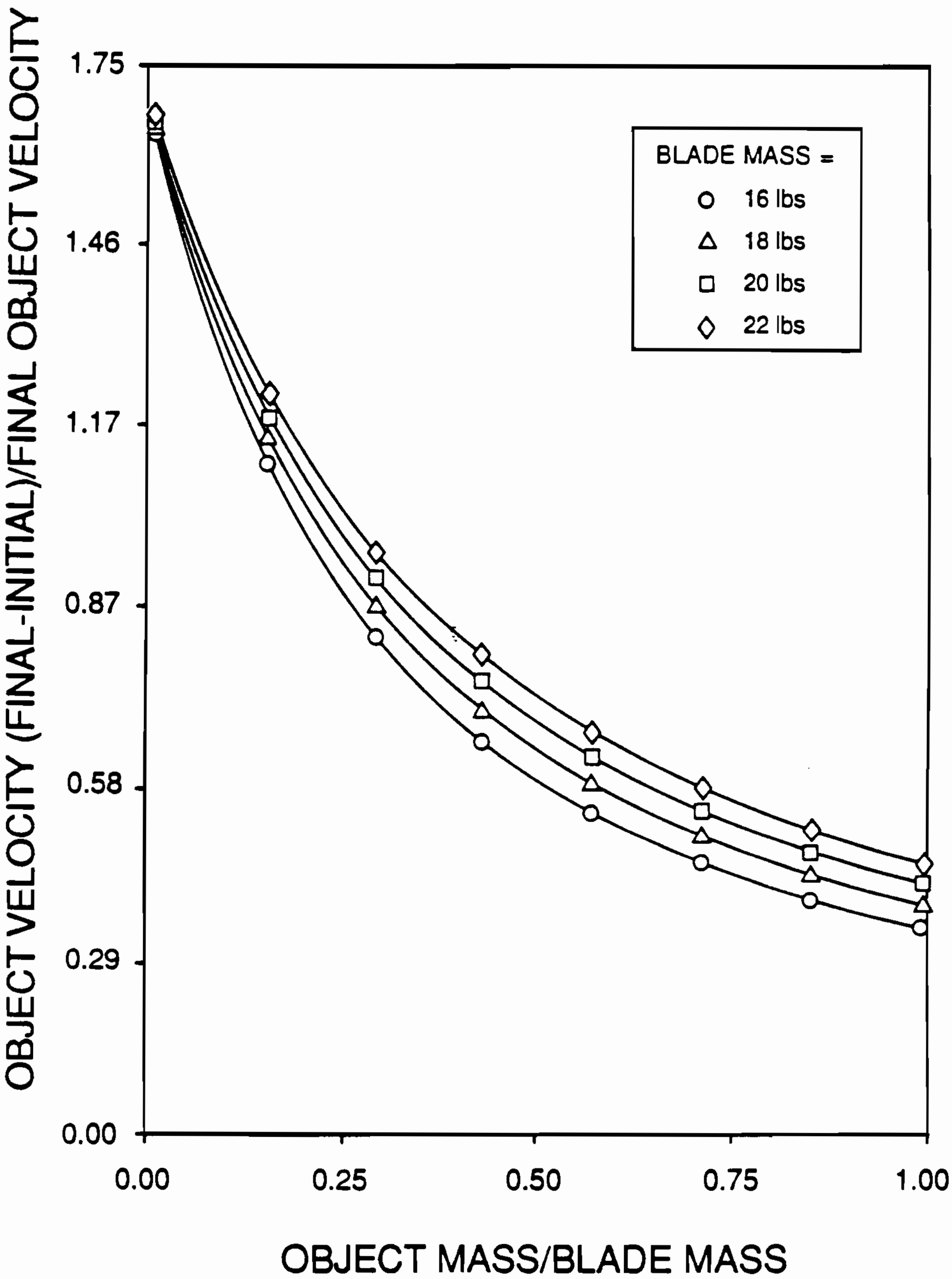


Fig. 2.10 Effect on dimensionless MTO velocity with different blade masses, blade speed of 1000 rpm, coefficient of restitution of 0.9 and the point of contact at 22 inches

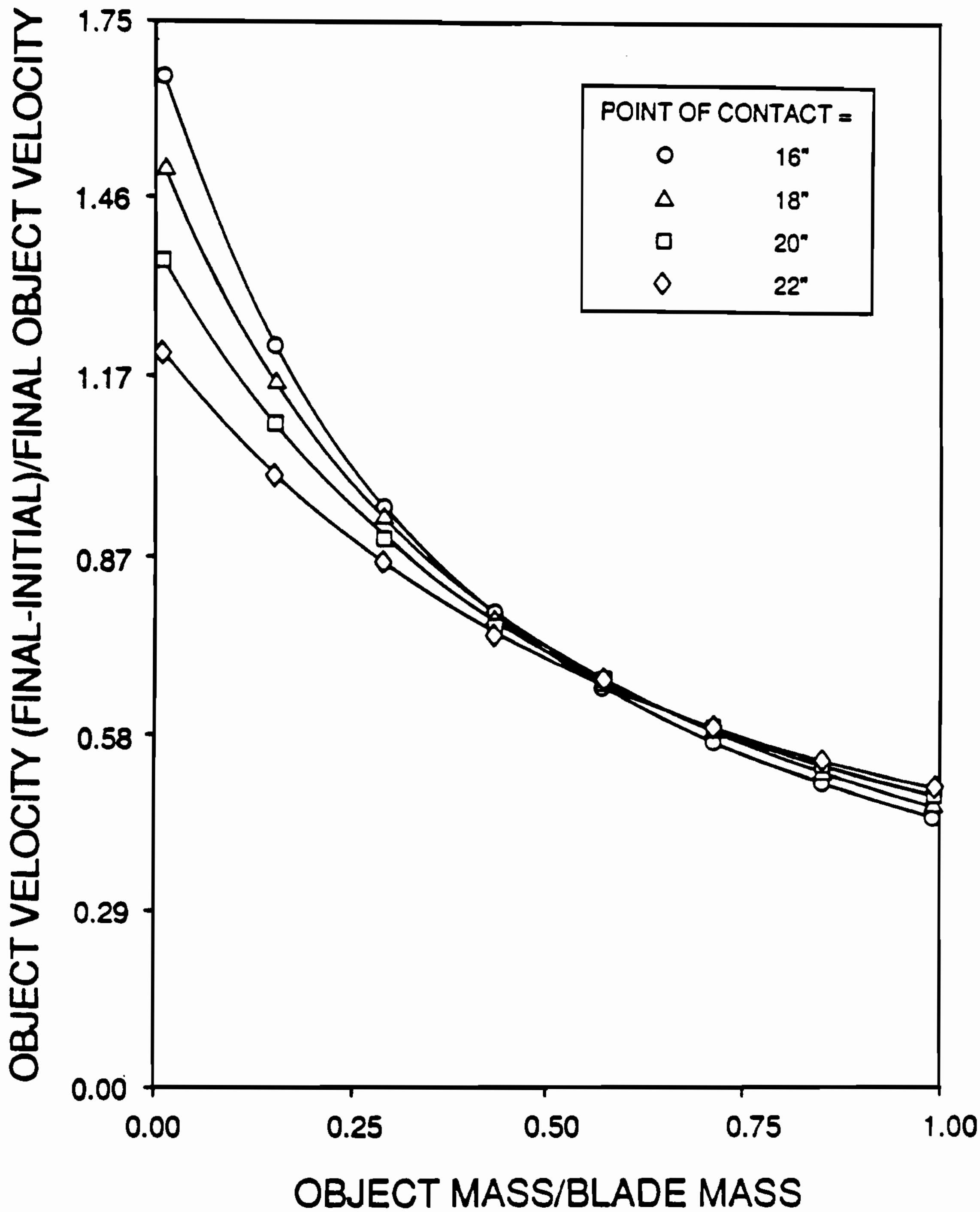


Fig. 2.11 Effect on dimensionless MTO velocity with varying point of contact, blade rpm of 1000, blade mass of 22 lbs and a coefficient of restitutionof 0.9

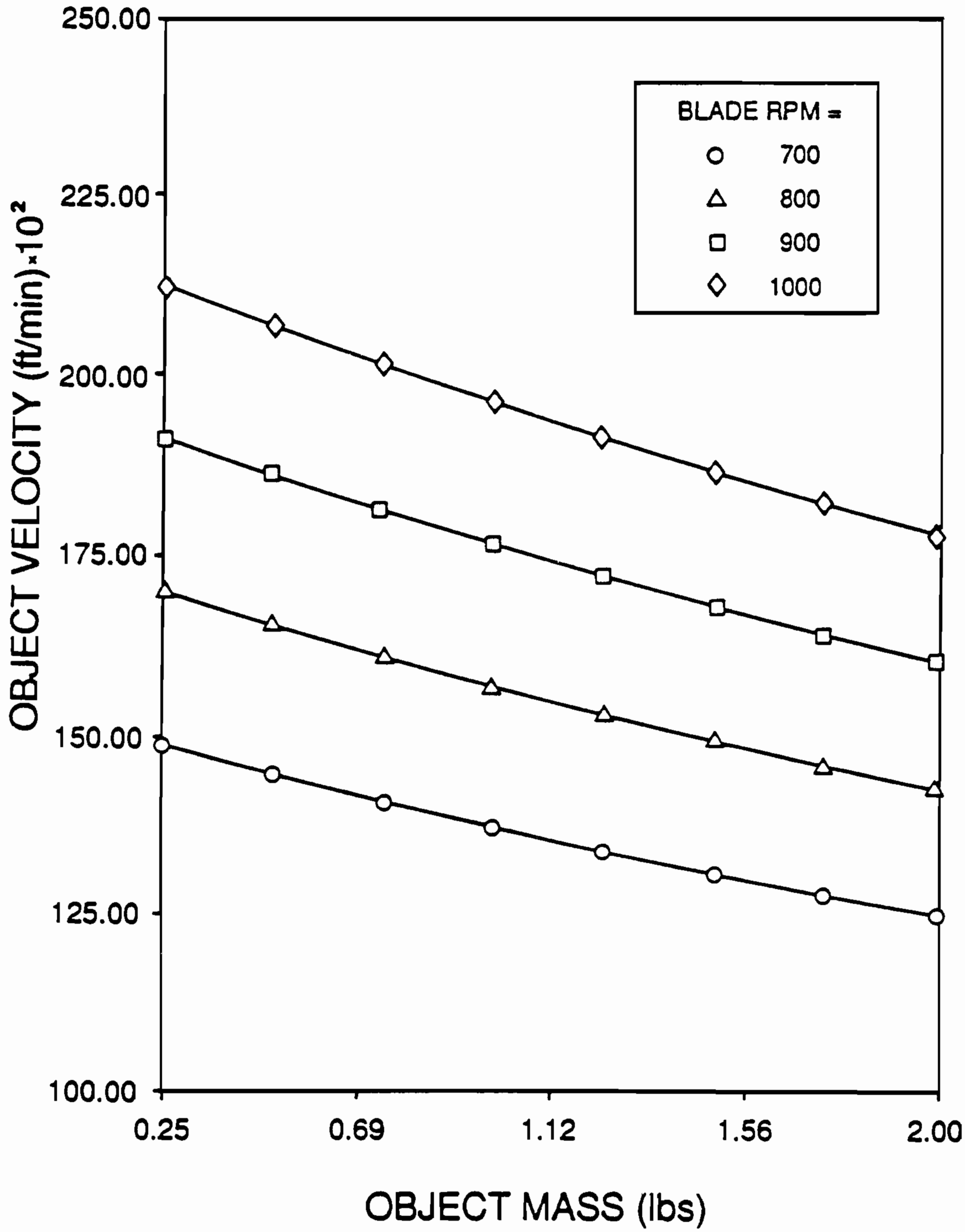


Fig. 2.12 Effect on the MTO velocity with varying blade rpm, blade mass of 22 lbs, coefficient of restitution of 0.9 and the point of contact at 22 inches.

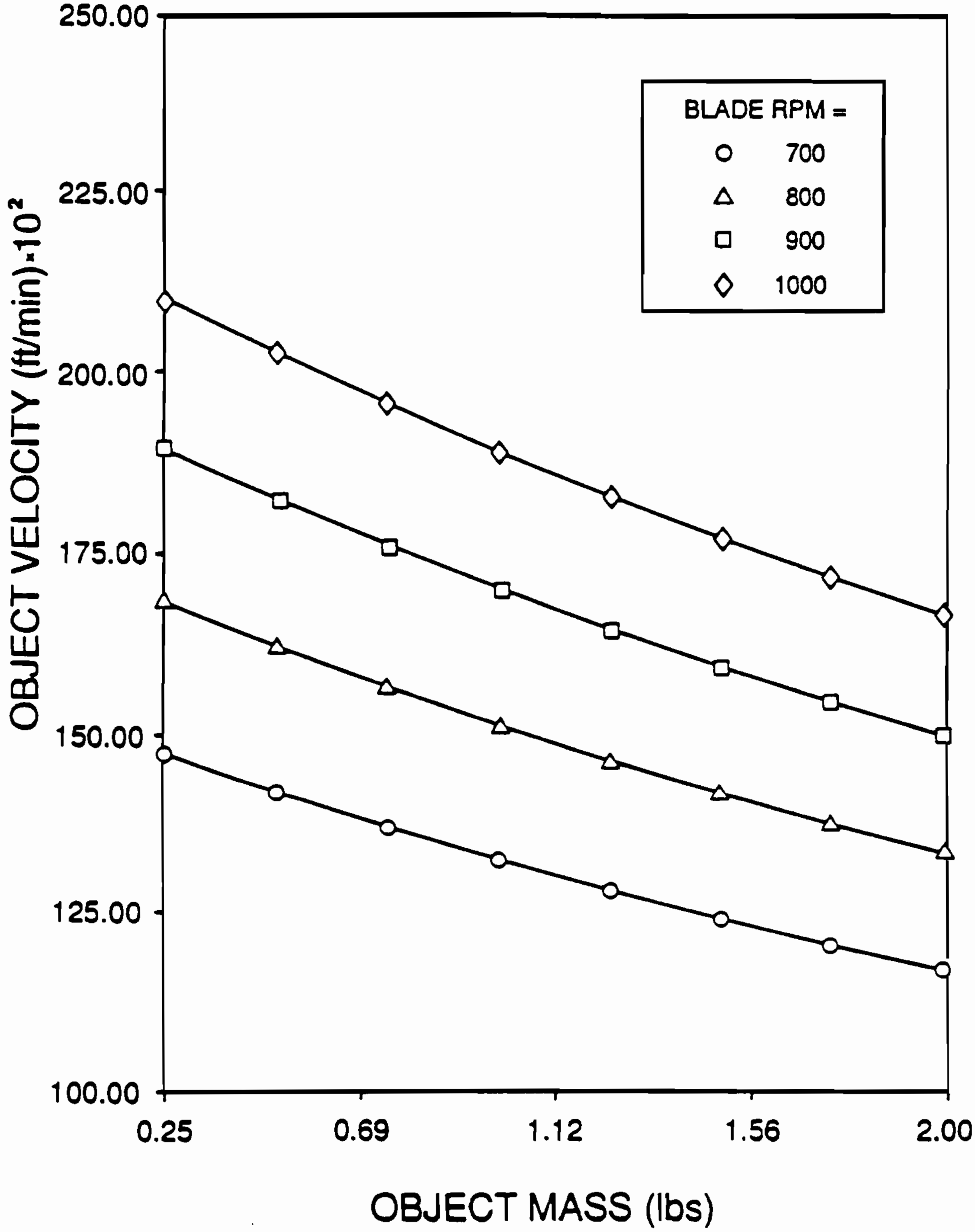


Fig. 2.13 Effect on the MTO velocity with different blade speeds, blade mass of 16 lbs, coefficient of restitution of 0.9 and the point of contact at 22 inches

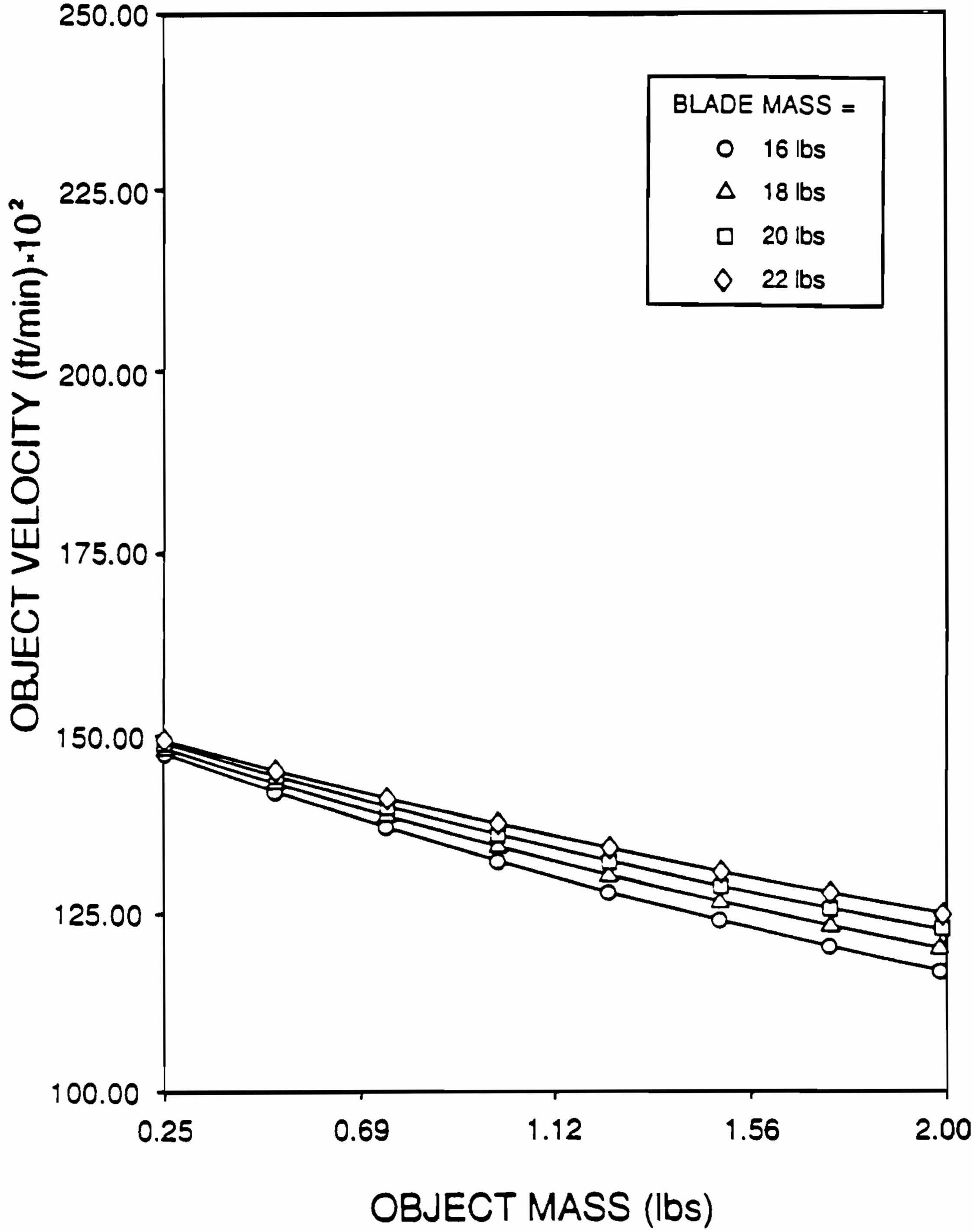
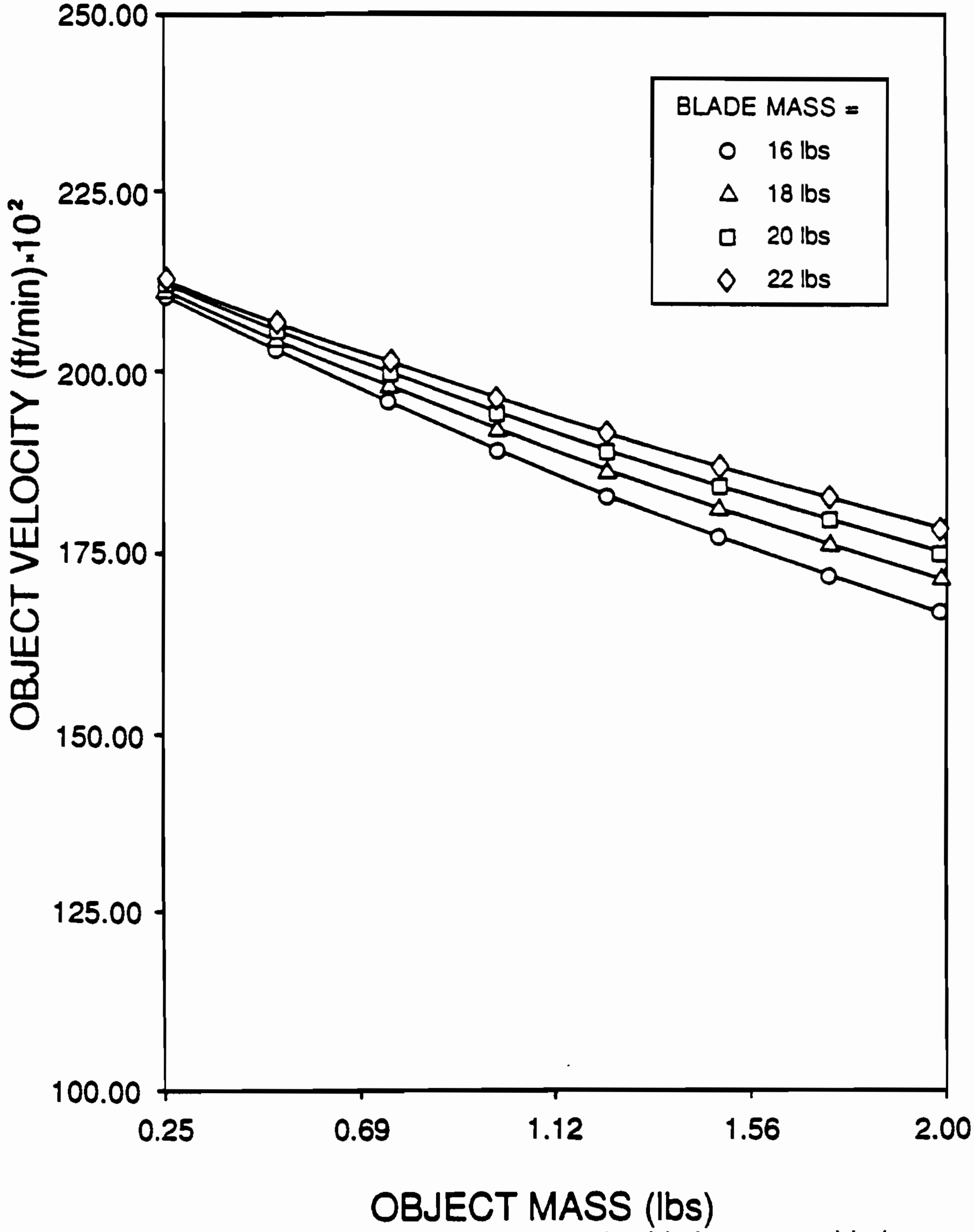


Fig. 2.14 Effect on the MTO velocity with varying blade masses, blade rpm of 1000, coefficient of restitution of 0.9 and the point of contact at 22 inches



Fig, 2.15 Effect on the MTO velocity with varying blade masses, blade rpm of 700, coefficient of restitution of 0.9 and the point of contact at 22 inches

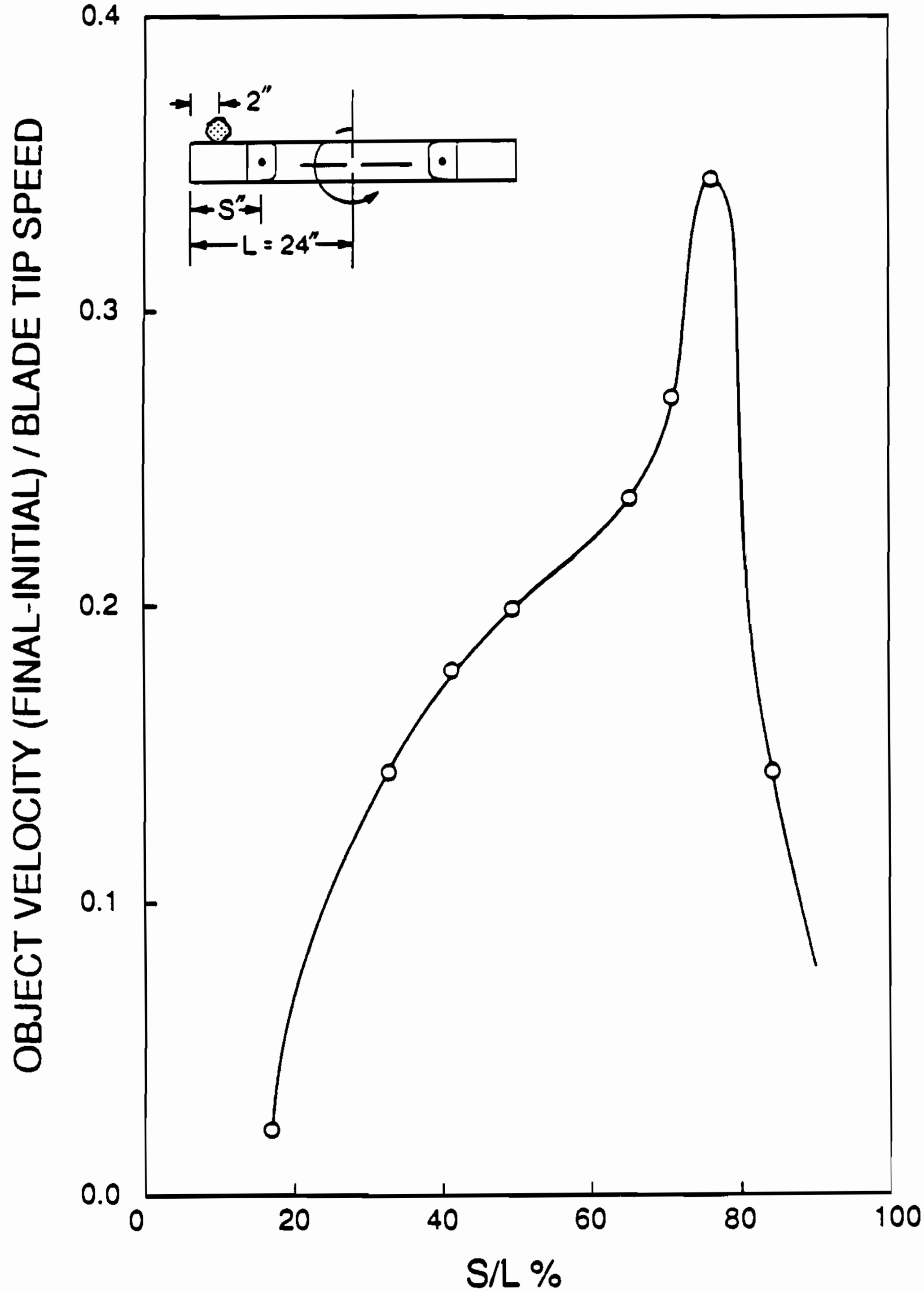


Fig. 2.16 Effect of pivot location on the MTO velocity after impact for a pivoted blade of mass 20 lbs, blade rpm of 600, blade length of 24 inches, coefficient of restitution of 0.7, initial velocity of 440 ft/min and the point of contact at 22 inches.

DISCUSSION OF RESULTS

From Fig 2.8 it is evident that a change in the value of e causes a significant change in the M.T.O. velocity. This result indicates that a harder M.T.O. will have a higher discharge velocity. Thus a piece of steel will go much further than a chunk of wood of the same mass.

From Fig 2.9, it is observed that when the dimensionless mass of the object is large, an increase or decrease in the blade mass results in a significant increase or decrease in the dimensionless M.T.O. velocity. For a dimensionless object mass of 0.5, a reduction of 27 percent in the blade mass causes a 20 percent reduction in the dimensionless M.T.O. velocity. This reduction in the velocity becomes smaller as the dimensionless mass of the object reduces. From Fig 2.10 it is clear that a change in the blade mass does not significantly effect the dimensionless M.T.O. velocity when the dimensionless object mass is small. However, as the dimensionless object mass increases, a change in the blade mass causes a significant variation in the M.T.O. discharge velocity. From Fig 2.11, it is observed that an increase in the distance of the point of contact from the center of rotation, or in effect the length of the blade, results in a significant increase in the M.T.O. velocity.

From Fig 2.12 and Fig 2.13, is observed that an increase or decrease in the blade speed causes a significant increase or decrease in the M.T.O. velocity. However, in Fig 2.13 the blade mass is reduced by approximately 27 percent compared to that in Fig 2.12, but this reduction in the blade mass effects only about a 6 percent reduction in the M.T.O. velocity. This reduction in velocity becomes even smaller as the mass of the M.T.O. reduces. From Fig 2.14 and Fig 2.15, it is clear that a change in the blade mass does not significantly effect the M.T.O. velocities as the object mass becomes small. However, a reduction in the blade speed causes a significant reduction in the M.T.O. velocities.

From Fig 2.16, it is observed that the location of the pivot point is a significant factor in obtaining lower discharge velocites. When the pivot point is located at a distance equal to one fourth the total blade length from the center of rotation maximum discharge velocities are obtained. The location of the pivot point away from the center of rotation for a constant length blade results in lower discharge velocities.

CONCLUSIONS

Inspection of Eqs 2.18 to 2.26 reveals that in order to reduce the discharge velocity v_C' of the M.T.O., there must be a reduction in the blade tip speed, blade mass and the length of the blade. Reduction in the blade tip speed is often proposed as a safety measure but is generally resisted by mower manufacturers as it increases the HP requirements and also reduces the air turbulence which prevents grass buildup and plugging of the discharge port (Ref 5).

Results from the computer program show that a decrease in the blade mass helps to reduce the M.T.O. velocities when the mass of the M.T.O. is greater than at least 50 percent of the blade mass. To achieve a reduction in the blade mass, the alloy steel blade presently being used, could be redesigned to reduce its mass or a Kevlar blade could be used. The conventional rectangular cross-section of the blade could be changed so as to reduce the moment of inertia, thereby reducing the rotational energy of the blade.

Reduction in the blade length would contribute to the reduction of M.T.O. velocities as seen from the analysis and the results of Fig 2.11. However a reduction of blade length would also result in a decrease in the width of cut. This problem could be overcome by the use of smaller multiple blades to achieve the desired width of cut. Also the results of Fig 2.16 show that moving the pivot point away from the center of rotation for a constant length blade results in lower discharge velocities.

CHAPTER 3. EXPERIMENTAL STUDY OF MOWER THROWN OBJECT PHENOMENA

This chapter deals with the experimental verification of the analysis carried out in Chapter 2. The objective of this chapter was to conduct an experiment to measure the discharge characteristics of mower-thrown-objects.

The only experimental test procedure available for industrial lawn mowers is the proposed, SAE J232, "Industrial Rotary Mower Recommended Practice." As far as M.T.O.'s are concerned, this experimental procedure is only qualitative in nature. It does not enforce any numerical bounds on the velocity or the energy level of the M.T.O.. In this test, 150 uncoated six (6) penny steel box nails are introduced vertically downwards, through a tube and funnel arrangement, onto the rotating blades of a full size mower. The marks on a double wall 350# corrugated board (target material) above the plane of the blade tip circle are recorded for hits (rupture of the first layer of the board) inside the operator zone (the area into which the extremities of a 95th percentile male can reach from the normal operator position) and outside the operator zone. The number of punctures (rupture of all the layers of the board) inside and outside the operator zone are also recorded (see Fig 3.1). The number of hits inside the operator zone, number of hits outside the operator zone, number of punctures inside the operator zone and number of punctures outside the operator zone constitute four different categories that are recorded in the SAE J232 test. Each of these four categories is divided by the total number of hits plus punctures in order to calculate individual percentages. The mower is said to have met the test requirements if the below mentioned maximum criteria are satisfied.

1. 20% hits in the operator zone
2. 0.5% punctures in the operator zone
3. 15% hits outside the operator zone
4. 5% punctures outside the operator zone

The test does not indicate the severity of the hit or puncture. It does not indicate which thrown object caused the maximum damage to the board; i.e., which thrown object is the most dangerous from a M.T.O. point of view.

EQUIPMENT AND THROWN OBJECTS

Equipment Parts List

1. **Mower case with chain guard**. The mower case is made of colorless one half inch thick polycarbonate sheet material. The sides are bolted together for ease of assembly and disassembly (see Fig 3.2). The model occupies an approximate volume of 2' x 3 1/4' x 0.5' cu. ft. A hinge is provided along one side to enable the user to open the top cover to clean the interior, change the blades and perform routine maintenance. The bottom of the case is covered with flat ultra black art quality paper to provide photographic contrast.

2. The power unit is a **1/20 HP AC motor** with an electronic speed control unit. The motor is mounted vertically on the top cover with the shaft protruding into the mower case. For accurate results the motor shaft must be balanced and eccentricity minimized (see Fig 3.2).

3. **The NAC HSV-200 high speed video system** (see Fig 3.2) is specifically designed for analyzing fast moving objects and is the first system in the world that can record and playback clear images in color or black and white at a framing rate of 200 frames per second. The system components are:

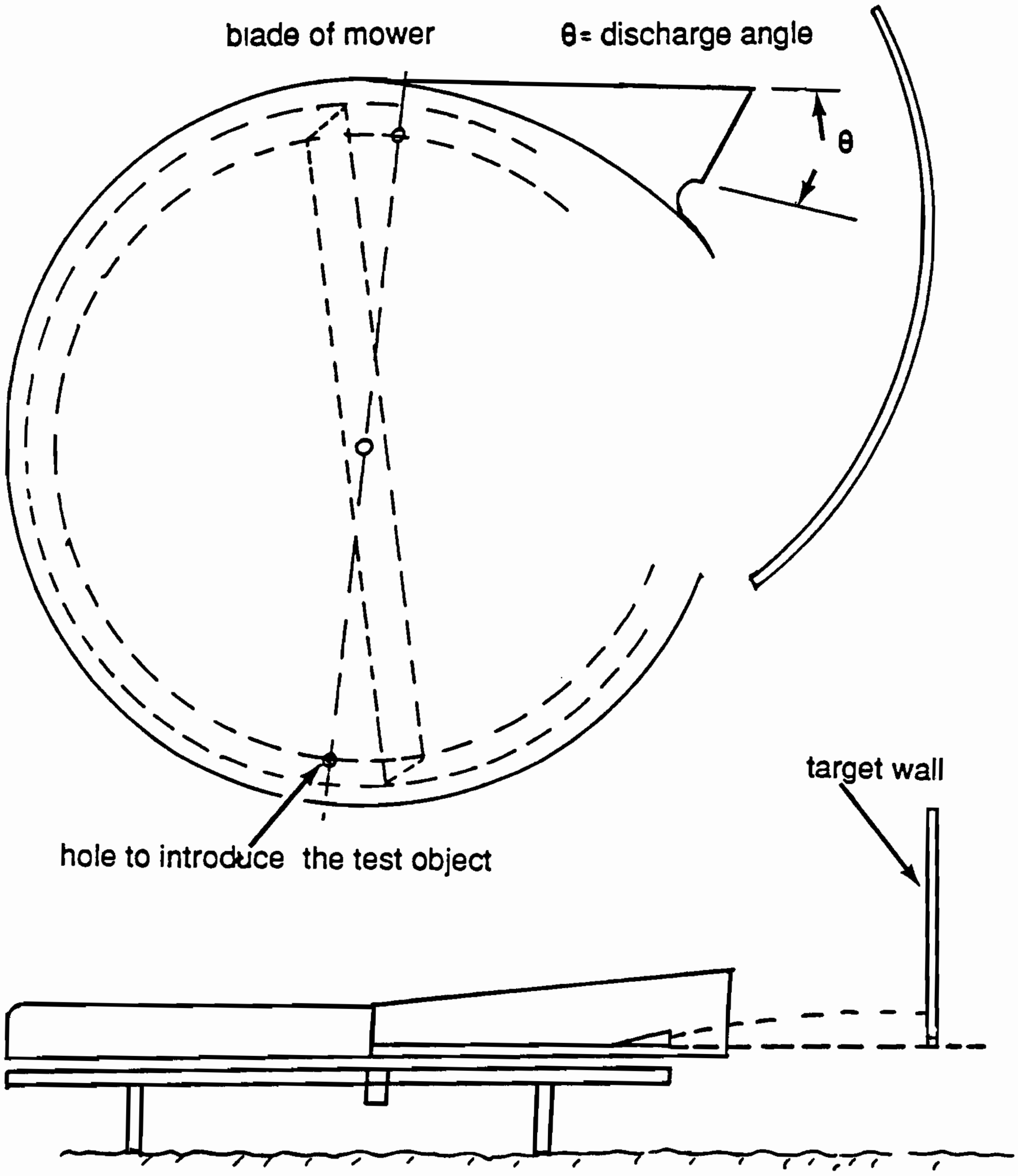


Fig. 3.1 Proposed SAE-J232 industrial rotary mower test

1. Video tape recorder
2. High resolution color monitor
3. High performance video camera
4. Zoom lens
5. 3' x 3' x 1/8" glass mirror
6. Floodlights
7. Steel table
8. Mower model
9. Feeder to introduce test object
10. 120 volt AC supply

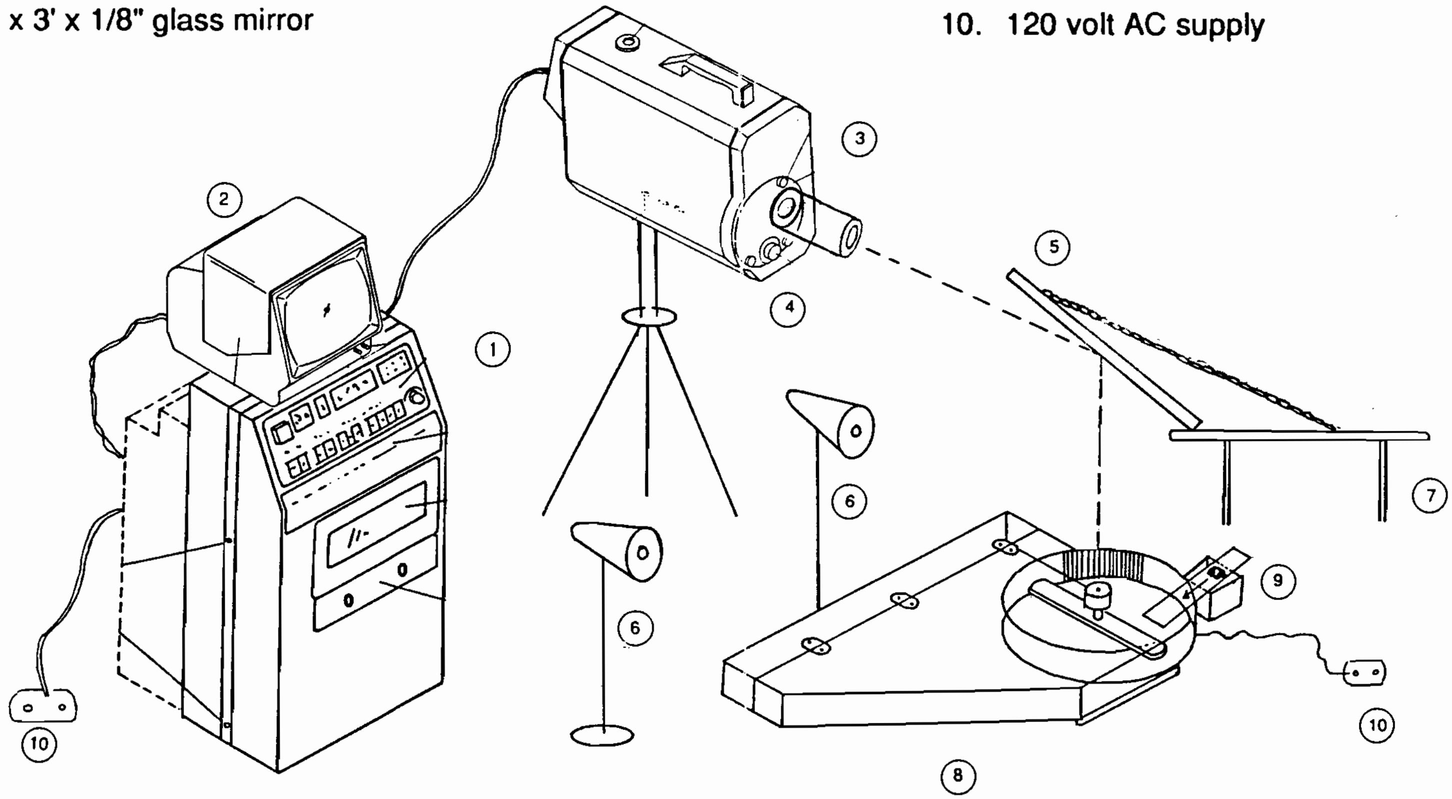


Fig. 3.2. Experimental layout.

a. National Panasonic high-performance **color video camera with tripod.**

b. **Zoom lens.**

c. **Video tape recorder** with remote control unit. Playback is possible at the recorded rate, in slow motion from 1 to 15 frames per second in addition to the still (one frame at a time) and reverse modes. In addition the system displays in the top right comer of the video monitor the **time code** which is the time in seconds, at intervals of 1/200 of a second. In the top left corner the system displays the **scene code**, a three digit number, which makes it possible to identify the scene or display during playback (see Fig 3.3).

d. **14" Color high Resolution video monitor.** The monitor was used to view the impact of the mower blade and the thrown object during playback. The time and distance travelled by the object were measured on the monitor in order to compute the velocities before and after impact.

4. **A glass mirror 3' x 3' x 1/8", mounted at 45° degrees to the horizontal**. The camera is mounted horizontally on a tripod and can rotate about a vertical axis. The glass mirror was used to photograph the blade perpendicular to its horizontal plane of rotation.

5. **Adequate lighting arrangements** with 5 floodlights. The additional lighting was provided to obtain clear and well defined images on the video monitor. It was observed that during playback, with the zoom lens in operation, there was some loss in image definition.

6. A 20 in. x 1 in. x 1/8 in. **straight aluminum blade** and a 20 in. x 1 in. x 1/8 in. **pivoted aluminum blade**, with a smooth pivot at the blade center. The lengths indicated are tip to tip lengths.

7. **A heavy steel frame table** with a wooden top, 6' x 5' x 4' high. The table was used to position the mirror above the mower model.

8. 1/2 inch VHS Maxell **video cassette** was used to record the velocities before and after impact.

9. **Framework of double wall cardboard panels.** The panels were used for safety from flying objects

10. **A heavy wooden pallet** (25-25 lbs). The pallet was used as a counterweight for the mirror at a 45 degree orientation, to prevent the mirror from falling over (see Fig 3.3).

Thrown Objects

The recording rate of the HSV-200 system (maximum of 200 frames per second) and the required clarity in the film during playback imposed limitations on the selection of the thrown objects used in the experiment. Initially 1/2", 3/4" and 1" diameter lucite balls were used in the experiment. The discharge velocities of the lucite balls were observed to be too high for the camera to record, primarily due to the light weight of the lucite balls. Steel and aluminum cylindrical bars of size 1/2" to 1 1/8" weighing 14 gms to 84 gms were thereafter used in the experiment. To enhance the picture clarity for measurement purposes the objects were polished to a bright silver metallic color so that they could be distinguished in the video film, against the black background of the art quality paper covering the mowercase bottom. Compared to carbon steel and lucite, aluminum has a brighter surface and is easily visible on the monitor before and after impact. The reason that cylindrical and spherical objects were selected is that it is easier to roll these along an inclined surface for gravity feeding of the test objects.

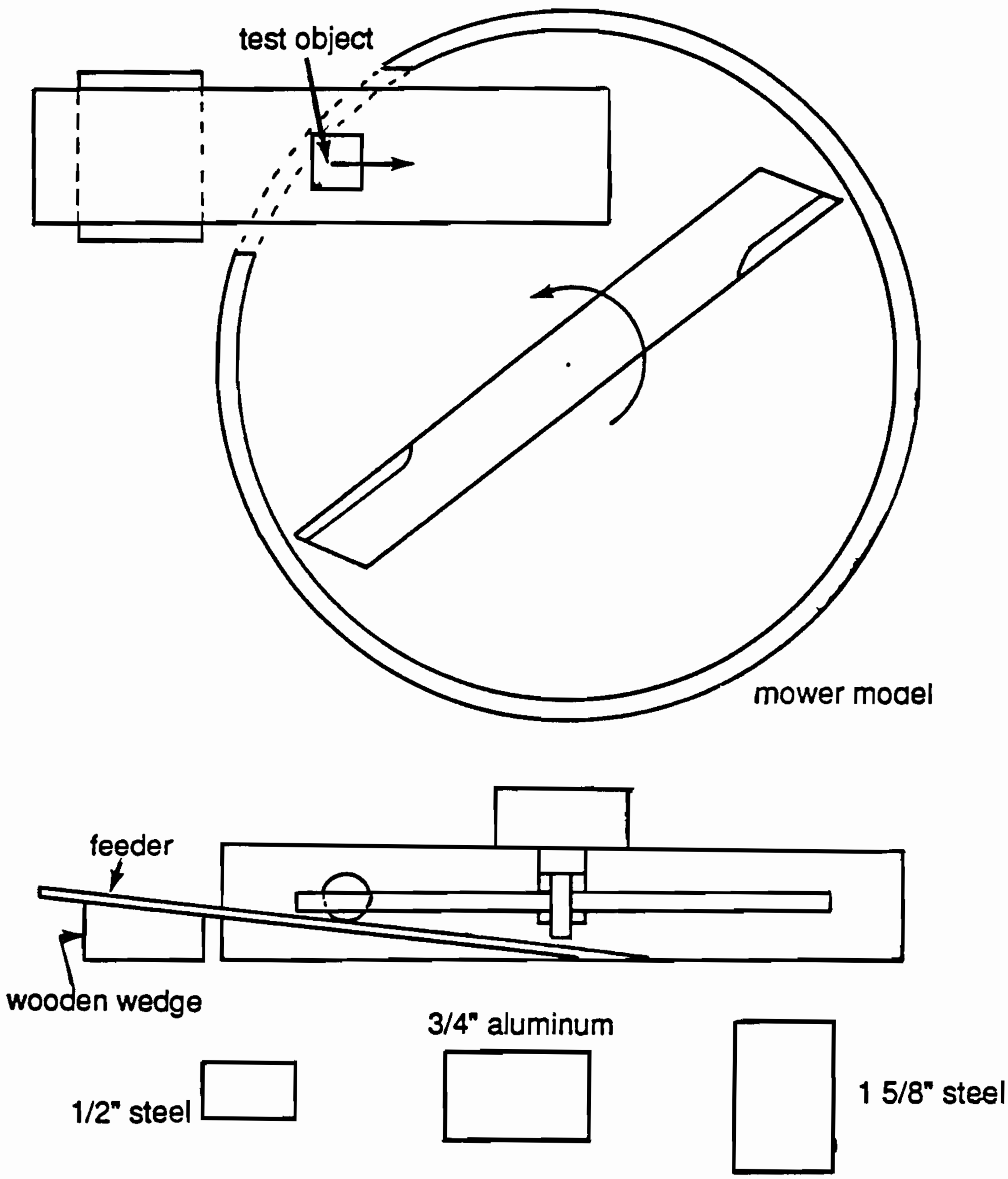


Fig. 3.3 Introduction of the test object

EXPERIMENTAL APPARATUS AND PROCEDURE

Calibration of the Video Monitor

The velocity of the thrown object is determined in this experiment by measuring the distance travelled by the thrown object on the monitor and dividing the distance by time in seconds indicated by the time code (see Chapter 3 under Equipment Parts List) given on the video monitor. Since the distance is measured on the monitor during playback, the relationship between the actual distance travelled by the object and the corresponding distance on the monitor can be determined.

To determine this relationship, a 3 foot wooden scale was taped to the top of the mower case. At the center of the scale, a 6 inch length was prominently marked off. This 6 inch section was filmed and then observed on the monitor and found to be only 2.05 inches long. Hence 2.05 inches on the monitor corresponded to 6 inches in the mower case. The ratio 6/2.05 was denoted as the "scaling factor". In order to maintain this value constant the distances between the camera, mower case and the glass mirror were maintained constant during successive experiments. Any change in this distance would mean that the monitor would have to be recalibrated. In order to make measurements on the monitor screen easier, a one inch square grid, hand drawn on an acetate sheet, was taped to the monitor screen (see Fig 3.3). The number of frames taken to travel a measured distance on the screen were recorded. Each frame corresponds to 0.005 secs. The time taken by the object to travel the distance can be calculated, and the average velocity immediately after impact can be computed.

Introduction of the Thrown Object

In an actual mowing operation, a mower travelling at 3 to 5 mph strikes a stationary object. In order to simulate this relative velocity between the mower and the object, the object is fed with an initial velocity to the stationary mower model.

The cylindrical thrown object is fed by gravity to the rotating blade. The object is rolled down a 3 inch wide strip of polycarbonate called the 'feeder'. The feeder is inserted into the mower case through a 4 inch wide slot cut into the housing of the model at the upper left corner (see Fig 3.3). The feeder rests on a wooden wedge on one end and rests on the base of the mower case at the other end (see Fig 3.4). By moving the wedge horizontally along the base of the mower, the inclination of the feeder is adjusted so that the blade will make contact with the object. The feeder is positioned so that the object contacts the blade within one inch of the blade tip. The cylindrical object is rolled down the feeder to feed it to the rotating blade.

Experimental Procedure

The mower is positioned on a 6 inch high wooden pallet placed on the ground. Another heavy wooden pallet is clamped to the top of the steel table by means of two heavy C-clamps (see Fig 3.2). Thereafter the mirror is positioned on the clamped pallet and tilted so as to make a 45 $\pm$ 5 degree angle with the horizontal. The mirror is held in position by means of two chains, as seen in Fig 3.2. The angle of orientation of the mirror can be altered by changing the number of links in the chain.

In the next step, the video recording system is set up and the necessary connections between the individual components and the external 120 volt power supply are completed. The floodlights are installed and the light is focussed on the mower model. Care is taken to avoid directing the light at the mirror or into the camera lens. Cardboard panels around the feeder are installed for safety reasons.

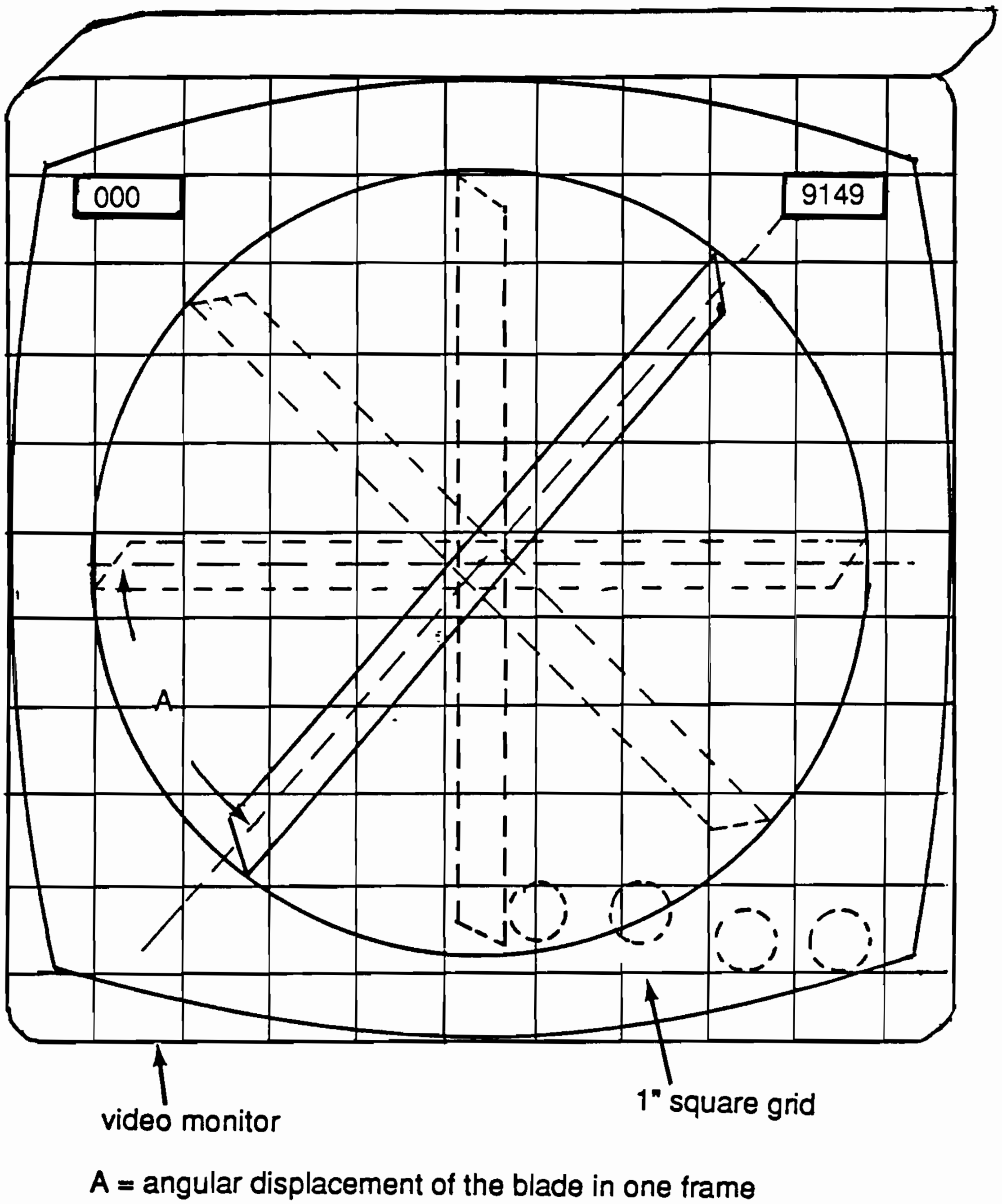


Fig. 3.4 Measurement of the blade and object velocities on the video monitor

The distances between the center of blade rotation and the mirror, and the camera are recorded. These distances are kept constant when the set up is disassembled and assembled again.

To carry out a trial run, the mower model cover is lifted and the straight blade installed. The scene code is adjusted to ØØØ. The scene number identifies the display (or scene) during playback. This number can be manually adjusted, and by recording this number and the event it represents the display is identified at any given time. The feeder is installed, floodlights turned on, the motor supply connection completed and the motor is started. After the blades have picked up speed the video system is put in the play mode and checked to see if the model is in the field of view on the screen. The clarity and definition of the display are checked by zooming in on the blade or on the thrown object. Compensation for poor image quality is made by either adjusting the height of the camera, the lighting or the camera distance. After obtaining good image quality the experiment is started.

To measure the speed of the blade, the scene code is adjusted to a new number and the number is recorded on the data sheet (see Appendix B). The rotation of the blade is recorded for about 5 seconds and played back in the still mode (only one frame at the click of the remote control unit button). The number of frames (each frame is 0.005 secs) to complete one revolution of the blade is recorded (see Appendix B). From this data the number of revolutions per second (rps) or revolutions per minute (rpm) is calculated. An average of three readings is considered for the experimental calculation of the blade speed before impact.

Next the feeder is inserted and positioned such that the feeder center line is within 1 inch from the blade tip (see Fig 3.3). The scene code is changed using the remote control unit. The video system is put in the record mode, and the test object is introduced as explained in Chapter 3 under Thrown Objects. The recording is continued for 2 to 3 seconds after impact.

In order to measure the velocity of the thrown object before impact, the portion of the tape before impact is played back and the speed of the object is measured as it moves along the feeder. The number of frames (each frame is 0.005 seconds) the thrown object requires to travel one inch or two inches on the monitor grid is recorded. This distance travelled on the monitor by the thrown object is multiplied by the scaling factor and thus the actual distance travelled by the object along the feeder in that time interval is obtained. With the knowledge of the distance travelled in the mower case and the time taken in travelling that distance the velocity of the thrown object before impact is computed. To measure the thrown object velocity after impact, the distance travelled by the thrown object on the monitor grid immediately after impact is recorded (see Appendix B). The time taken to travel this distance, as indicated by the time code, is also recorded in the data sheet (see Appendix B). With the knowledge of time and distance, the velocity of the object after impact can be computed (see Fig 3.4).

The above procedure which gives the speed of the blade before impact, velocity of the object before impact and the velocity of the object after impact is repeated for different blades and different test objects. The experimental data is recorded in Appendix B and the experimental results are plotted in Fig 3.6.

Experimental Estimation of the Coefficient of Restitution

The value of e is estimated by a simple drop test. A steel ball bearing about 3/8" in diameter is dropped from a measured height h (varying from 9" to 12") on to an aluminum plate. The rebound height h_1, is recorded by visual observation, as seen in Fig 3.5. When the height h is large (greater than 9"), the value of e ranges from 0.45 to 0.6. This value of the coefficent of restitution less than unity is a reduction greater than can be justified by the transformation of energy into vibrations of the colliding bodies and indicates the presence of permanent deformation. The theoretical and experimental estimation of e at high impact velocities (large values of h) should be modified to account for strain-rate effects in the material behavior and the estimation of e becomes very complex (Ref 1).

The value of e decreases monotonously with impact velocity (Ref 1) or in this case the height h. The drop test is performed again with the 3/8" steel bearing and the height h in the 1" to 2" range. The value of e is observed to be in the range of 0.9 to 0.96. This is more representative of perfect elasticity (e = 1). This higher range for e is closer to the value in (Ref 1) for metals colliding against metals. Hence, it was decided to choose e in the 0.9 to 0.96 range for steel against aluminum. The same drop test was conducted with an aluminum spherical object and e is estimated to be in the 0.90 to 0.92 range for aluminium against aluminium.

The above experimental estimation of e does not give accurate results, but is the only method for a simple estimate of e other than from handbooks (see Fig 3.5). An expression for e in terms of the drop height h and the rebound height h_1 is derived below (see Fig 3.5).

The velocity v of an object (at rest) released from a height h, just before it strikes the ground is v = $(2gh)^{1/2}$ (Ref 3). Alternatively, an object that leaves the ground with a initial velocity v_1, travels a height h_1 before coming to rest, where h_1 is given by $v_1 = (2gh_1)^{1/2}$ (Ref 3).

$$v_1 = (2gh_1)^{1/2}$$

where v_1 is the velocity of the object after it strikes the plate at O

$$v_2 = (2gh_2)^{1/2}$$

where v_2 is the velocity of the object after it strikes the plate at O'. The velocity of the plate is zero at all times. From the definition of the coefficient of restitution we have

$$e = \frac{|\text{ Relative velocity of separation }|}{|\text{ Relative velocity of approach }|} = \frac{v_1 - 0}{v - 0} = \frac{v_2 - 0}{v_1 - 0}$$

$$\text{so } \quad v_1^2 = w_2 = [(v_2)^{1/2}/ v] = (h_2/h)^{1/4}$$

$$e = \frac{v_1}{v} = \left(\frac{h_1}{h}\right)^{1/2}$$

$$e = \left(\frac{h_1}{h}\right)^{1/2}$$

EXPERIMENTAL RESULTS

The results from the experiment conducted to measure the discharge characteristics of a mower model, in a laboratory, are presented in Table 3.1.

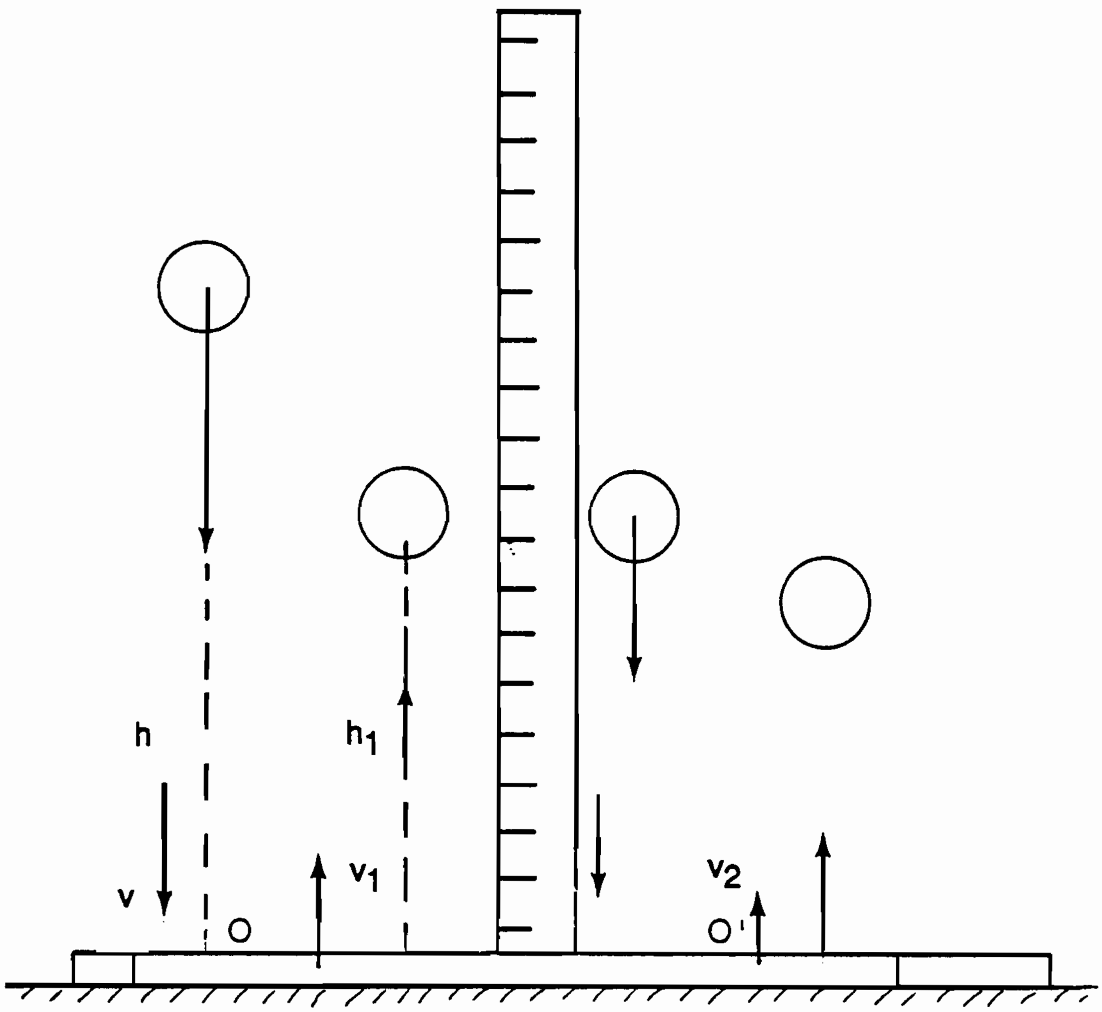


Fig. 3.5 Measurement of the coefficient of restitution

Table 3.1
EXPERIMENTAL RESULTS FROM MOWER MODEL

Object Mass	Blade Mass	Blade Rpm	Straight Blade Object Velocity Before Impact	Straight Blade Object Velocity After Impact	Pivoted Blade Object Velocity Before Impact	Pivoted Blade Object Velocity After Impact
14 gms	170 gms	413.5 RPM	65 ft/min	2760 ft/min	63.5 ft/min	2325 ft/min
19 gms	170 gms	413 RPM	88 ft/min	2615 ft/min	60.5 ft/min	1937 ft/min
86 gms	170 gms	413 RPM	75 ft/min	1453 ft/min	75 ft/min	872 ft/min
42.5 gms	170 gms	413 RPM	71 ft/min	1960 ft/min	75 ft/min	1260 ft/min

The results given in Table 3.1 are plotted in Fig 3.6. The M.T.O. discharge velocity (ft/min) is plotted on the Y-axis and the M.T.O. mass is plotted on the X-axis. The results are plotted for the straight blade and the pivoted blade (broken line) in Fig 3.6. The results from Fig 3.6 indicate that as the M.T.O. mass (less than 20 percent of the blade mass) decreases the M.T.O. velocity rises rapidly for both the blade types. The difference in the discharge velocities of both blade types diminishes in the low mass region (less than 20 percent of the blade mass) of the plot. The results for the straight blade are located above that of the pivoted blade indicating lower magnitudes of discharge velocities for the pivoted blade. The numerical results shown above indicate that on an average the pivoted blade construction resulted in approximately 29 percent smaller magnitudes of the object discharge velocities. The calculations for the numerical results are shown in Appendix C.

THEORETICAL RESULTS

The discharge velocities of the thrown objects, as predicted by the analysis in Chapter 2 under Case I-Straight Blade and Case II-Pivoted Blade, are computed in Appendix C and are tabulated as follows:

Table 3.2
THEORETICAL RESULTS FROM MOWER MODEL

Object Mass	Blade Mass	Blade Rpm	Straight Blade Object Velocity Before Impact	Straight Blade Object Velocity After Impact	Pivoted Blade Object Velocity Before Impact	Pivoted Blade Object Velocity After Impact
14gms	170 gms	413 RPM	65 ft/min	3273 ft/min	63.5 ft/min	2677 ft/min
19 gms	170 gms	413 RPM	88 ft/min	3039 ft/min	60.5 ft/min	2331 ft/min
84 gms	170 gms	413 RPM	75 ft/min	1680 ft/min	73 ft/min	1026 ft/min
42.5 gms	170 gms	413 RPM	71 ft/min	2356 ft/min	75 ft/min	1621 ft/min

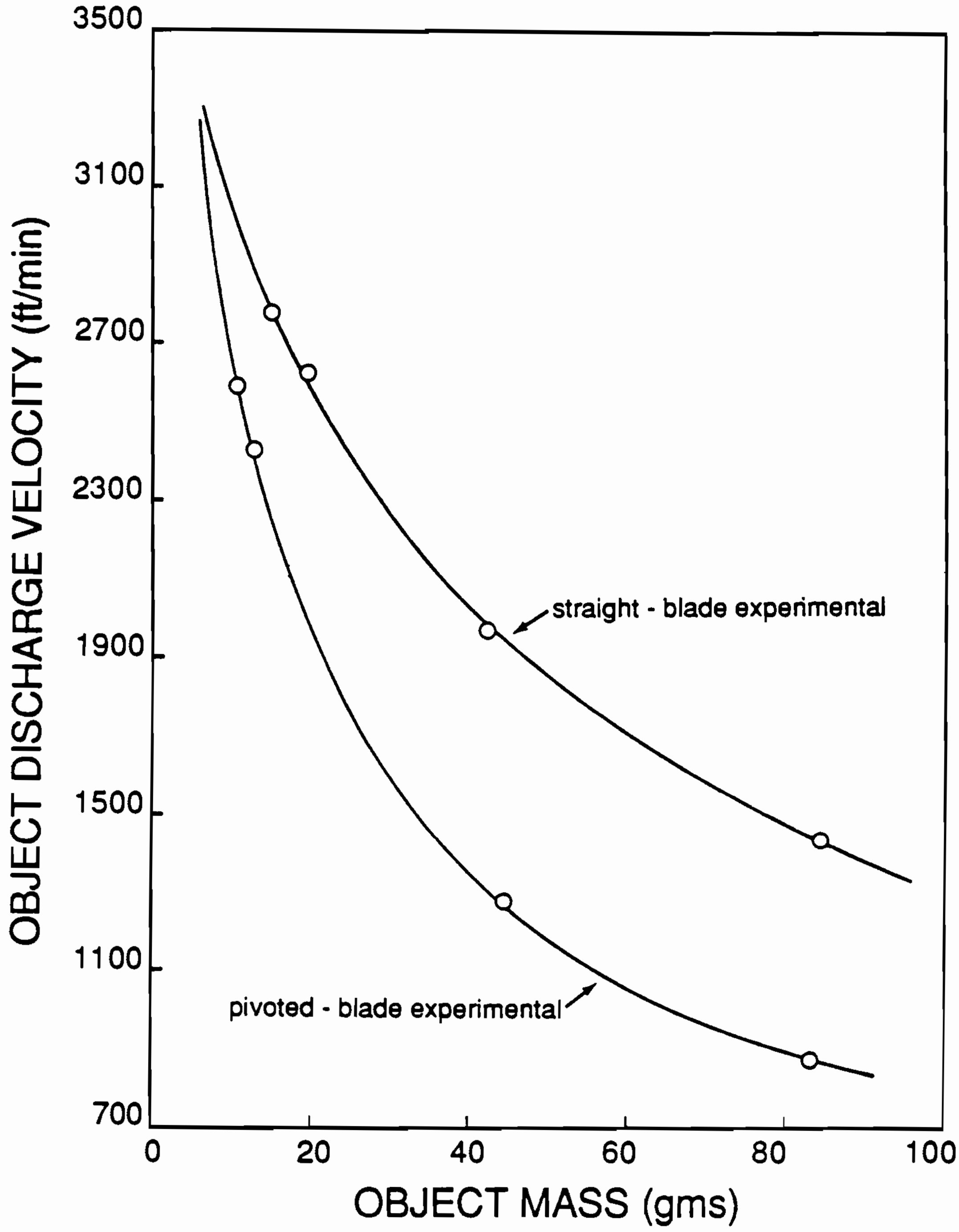


Fig. 3.6 Experimental results of the straight and pivoted blades

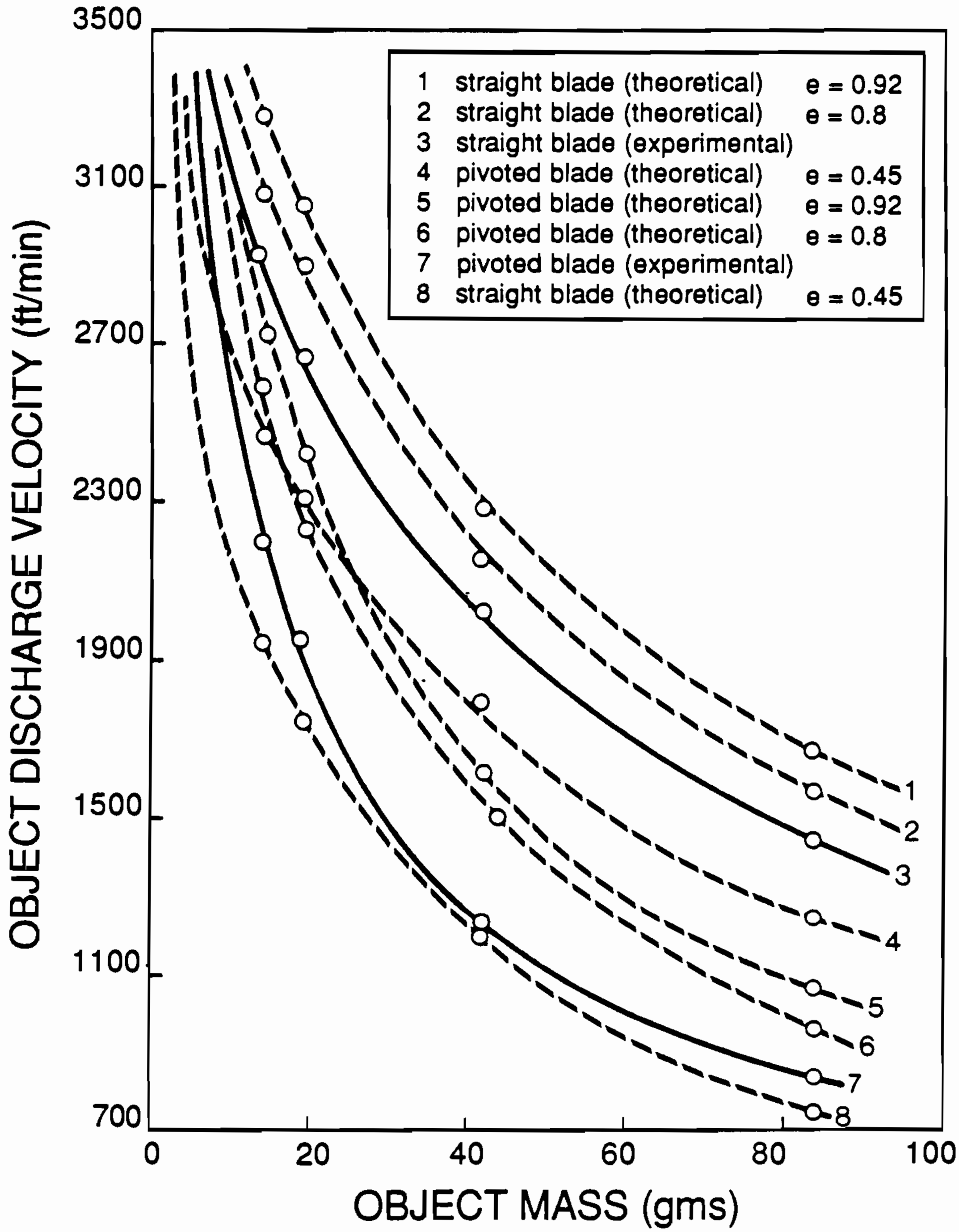


Fig. 3.7 Comparison of the experimental and theoretical results

The results in Table 3.2 are plotted in Fig 3.7, with the M.T.O. mass (lbs) plotted along the X-axis and the M.T.O. velocity (ft/min) plotted along the Y-axis. The theoretical results are presented for the straight and pivoted blade types. The straight blade is modelled as a single rod, rotating about a fixed point and having the same mass, angular velocity and length as that of the experimental model. The pivoted blade is modelled in a similar manner except that a pivot located at the center of the rod has been considered as in the analysis of Chapter 2 under Case II-Pivoted Blade. The theoretical results in Fig 3.7 indicate that the magnitudes of M.T.O. discharge velocities for a pivoted blade are lower than that for a straight blade. The results also indicate that as the M.T.O. mass decreases (less than approximately 20 percent of the blade mass) the M.T.O. velocities increase rapidly and the difference in the discharge velocities for the two blade types diminishes. The numerical results shown in Table 3.2 indicate that on an average the pivoted blade construction resulted in approximately 28 percent smaller magnitudes for the object discharge velocities.

DISCUSSION OF THE RESULTS

The experimental and theoretical results are plotted in Fig 3.7. The results indicate that in the low mass region the discharge velocity of the M.T.O. is high and the difference in the discharge velocities for the two blade types diminishes. The plots for the theoretical and experimental results indicate lower discharge velocities for the pivoted blade. The results also indicate that the pivoted blade is effective in reducing the severity of M.T.O. accidents when the mass of the M.T.O. is large (at least about 50 percent of the mass of the blade). This result is of particular significance since it is the large M.T.O.'s that are usually responsible for breaking windshields of motorists along highways (Ref 6) .

CONCLUSIONS

Based on the experimental results for the four different thrown objects, for the two different blade types, and a comparison with the theoretical results, the following conclusions are drawn:

1. The mower-thrown-object phenomena for a straight bar blade or a pivoted bar blade can be modelled by a rod (straight or pivoted) rotating about a fixed point and striking a moving object.
2. M.T.O. discharge velocities tend to be high when the mass of the M.T.O. is small and tend to be small when the mass of the M.T.O. is large. Also when the mass of the M.T.O. is small the use of either type of blade does not significantly change the discharge velocity. However the blade selection does make a difference when the mass of the M.T.O. exceeds 50 percent of the mass of the blade.

CHAPTER 4. CONCLUSIONS AND RECOMMENDATIONS

This report presented (1) a mathematical model of the mower-thrown-object phenomena; and (2) an experimental prototype mower model to estimate the discharge characteristics of M.T.O.'s in a laboratory environment. Additionally a computer program was developed to aid in analytically predicting the discharge characteristics of M.T.O.'s. Conclusions of the experimental and analytical study and recommendations for future investigations are included in this chapter.

CONCLUSIONS

A mathematical model for the mower thrown object phenomena was developed in Chapter 2. This analysis showed that efforts to reduce M.T.O. discharge velocities should require a reduction in the inertia of the blade and the rpm of the blade. Figs. 2.9 through 2.12 indicate that the rpm of the blade is a key factor in the cause of M.T.O. accidents. The computer program results also indicate that the coefficient of restitution, the point of contact of the M.T.O. along the blade length and the mass of the blade contribute to M.T.O. accidents. As the distance of the point of contact from the center of rotation increases the M.T.O. will have a higher velocity, this implies that as the blade length increases the M.T.O. velocity increases. Therefore, mowers having short cutting blades, like the rotary disc mower, are likely to have less severe M.T.O. accidents than mowers that use the long bar blade.

Chapter 3 is devoted to the experimental verification of the analysis in Chapter 2. An experiment to estimate the discharge velocities of M.T.O.'s has been conducted on a mower model. It has been shown in Fig 3.7 that the pivoted blade causes a reduction in M.T.O. discharge velocities. However, as the mass of the M.T.O. decreases, Fig 3.7 shows that it does not make a significant difference as to which blade is being used because the objects discharged by both blade types have very high velocities. Figure 3.7 also shows that the pivoted blade results in approximately 28 percent lower M.T.O. discharge velocities.

RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

1. In order to reduce the moment of inertia of the blade different cross sections for the mower blade must be investigated. Blades of lower cross sections must be fabricated and experiments conducted in the field to test the performance of the new blades. Blades made of lighter materials like Kevlar must be fabricated and field experiments must be conducted to test the performance of these light weight blades.
2. An investigation of grass cutting power requirements must be performed to determine the minimum required power to cut grass. For a given blade, a reduction in the cutting power implies a reduction in the operating speed of the blade.

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APPENDIX A

MOWER-THROWN OBJECT ANALYSIS COMPUTER PROGRAM

APPENDIX A: MOWER-THROWN-OBJECT ANALYSIS COMPUTER PROGRAM

The program in this Appendix was used to predict the discharge characteristics of M.T.O.'s. Each step of the computer program has extensive comments that are self explanatory. In addition to computing the discharge velocities the program also plots the M.T.O. velocities (ft/min) against the M.T.O. (mass).

```
C       MOWER THROWN OBJECTS
C       ****************************************************************
C       THIS PROGRAM PREDICTS THE DISCHARGE VELOCITIES OF MOWER
C       THROWN OBJECTS ( MTO ), ENCOUNTERED IN HIGHWAY LAWN MOWING
C       OPERATIONS.  IT GIVES THE USER THE NUMERICAL AND GRAPHICAL
C       REPRESENTATION OF THE VELOCITIES FOR A VARIATION IN THE INERTIA
C       OF THE BLADE, SPEED OF ROTATION OF THE BLADE, COEFFICIENT OF
C       RESTITUTION, AND THE POINT OF CONTACT ALONG THE BLADE.
C       ****************************************************************
C                               ++++++ CAUTION +++++++
C               BEFORE PLOTTING THE RESULTS CHECK THE OUTPUT.  IF THE
C               OUTPUT STATES " OUT OF RANGE CALL IN NEGATIVE Y-AXIS"
C               IMPLIES THAT THE RANGE(3) AND RANGE(4) IN THE PLOTTING
C               ROUTINE MUST BE CHANGED TO ACCOMODATE THE MINIMUM AND
C               MAXIMUM VALUES IN THE OUTPUT
C                               ++++++++++++++++++++++
C       ***************************************************************
        DIMENSION IRPM(4),IW2(4),R(4),E(4),W1(8),ZZ(8),
     1  VA2(4,4,4,4,8),RANGE(4),X(10),Y(10,8),DTS(4),
     2  XAXIS(5),YAXIS(5),Z(4,4,4,4,8)
C       *******************************************************
C       THE BLADE DIMENSIONS OF MOST MOWERS IN USE TODAY VARIES FROM
C       20" X 4" TO 30" X 4" DEPENDING ON THE MOWER AND MOWING CONDIIONS.
C       A FIXED LENGTH OF 2 FT IS RECOMMENDED FOR THIS PROGRAM.
        WRITE(6,*) 'GIVE THE BLADE LENGTH D1 IN FT, RECOMMENDED RANGE IS
     1  1.5 TO 3 FT.'
        READ(5,*) D1
C       ***************************************************************
C       IRPM = RPM OF THE BLADE.
C       THE RPM IN THIS PROGRAM TAKES 4 VALUES.  THE USER OF THE PROGRAM
C       GIVES THE FIRST VALUE AND THE REMAINING THREE VALUES ARE COMPUTED
C       IN INCREMENTS OF 100 RPM.
C       BATWING TYPE ROTARY MOWERS USED BY NEARLY ALL HIGHWAY DEPARTMENTS
C       OPERATE IN THE 600 TO 1100 RPM RANGE AN HENCE IT IS RECOMMENDED THAT
C       THE PROGRAM USER MAINTAIN THE 4 VALUES OF THE RPM IN THIS RANGE.
        WRITE(6,*)'GIVE THE RPM (SPEED) OF THE MOWER, RECOMMENDED RANGE IS
     1  500 TO 800 RPM, BASED ON MANUFACTURER CATALOGS'
        READ(5,*) SPEED
        DO I=1,4
        IRPM(I)=100*(I-1)+SPEED
        END DO
C       ***************************************************************
C       E = THE COEFFICIENT OF RESTITUTION
C       THE COEFFICIENT OF RESTITUTION IN THIS PROGRAM HAS 4 VALUES.  THE USER
C       SPECIFIES THE FIRST VALUE AND THE REMAINING THREE VALUES ARE COMPUTED
C       IN INCREMENTS OF 0.2
C       SELECT A WIDE RANGE TO COVER SOFT DEBRIS LIKE CANS AND HARD
```

```
C       DEBRIS LIKE STEEL RODS AND AUTO PARTS. THE VALUES OF THE COEFFICIENT
C       MUST LIE BETWEEN 0 AND 1.

        WRITE(6,*)'GIVE THE COEFFICIENT OF RESTITUTION (EINT), RECOMMENDED
     1  RANGE IS 0.1 TO 0.4'
        READ(5,*) EINT
        DO J=1,4
        E(J)=J*0.2+ EINT
        END DO

C       ****************************************************************

C       IW2 = MASS OF THE BLADE (LBS).
C       THE MASS OF THE BLADE IN THIS PROGRAM HAS FOUR VALUES. THE USER
C       SPECIFIES THE FIRST VALUE BLMASS AND THE REMAINING ARE COMPUTED IN
C       INCREMENTS OF 2 LBS.
C       ALLOY STEEL CASE HARDENED BLADES OF THE DIMENSIONS CONSIDERED IN
C       THIS PROGRAM ON AN AVERAGE WEIGH ACCORDING TO MANUFACTURER CATALOGS
C       ABOUT 20 TO 22 LBS.

        WRITE(6,*)'GIVE THE MASS OF THE BLADE (BLMASS) IN LBS, RECOMMENDED
     1  RANGE IS 10 TO 18 LBS, BASED ON MANUFACTURER CATALOGS'
        READ(5,*) BLMASS
        DO K=1,4
        IW2(K)=2*(K-1)+BLMASS
        END DO

C       ****************************************************************

C       VA1 = SPEED OF THE OBJECT BEFORE STRIKING THE BLADE.
C       THE AVERAGE SPEED OF THE MOWING OPERATION IS IN THE 3 - 5 MPH
C       RANGE. THE TRAVELLING MOWER STRIKES THE STATIONERY OBJECT. IN THE
C       PROGRAM THE OBJECT IS TRAVELLING AT 5 MPH ( 440 FT/MIN) BUT THE MOWER
C       IS STATIONERY, HOWEVER WE STILL HAVE A RELATIVE VELOCITY OF 440 FT/MIN
C       BETWEEN THE MOWER AND THE OBJECT.

        WRITE(6,*)'GIVE THE VELOCITY OF THE OBJECT(OBJVEL) IN FT/MIN, THIS
     1  VALUE MUST BE REPRESENTATIVE OF MOWER TRAVELLING SPEEDS.
     2  RECOMMENDED RANGE IS 300 TO 500 FT/MIN'
        READ(5,*) OBJVEL

C       ******************************************************************

C       R = THE DISTANCE(FT)OF THE POINT OF CONTACT (BETWEEN THE BLADE AND THE
C       OBJECT) FROM THE CENTER OF ROTATION
C       THE GREATER THE DISTANCE HIGHER THE VELOCITY OF THE MTO.

        WRITE(6,*)'GIVE THE DISTANCE (DINIT) OF THE POINT OF CONTACT (INCHES),
     1  THE VALUE SHOULD BE SUCH THAT DINT+6 INCHES MUST NOT EXCEED HALF THE
     2  THE BLADE LENGTH'
        READ(5,*) DINIT
        DO M=1,4
        R(M)=(2*(M-1)+DINIT)/12.0
        END DO

C       ******************************************************************

C       W1 = MASS OF THE OBJECT (LBS).
C       THIS VALUE IS PLOTTED ON THE X-AXIAND VARIES IN INCREMENTS OF 0.25 LBS.
```

```
C        THESE VALUES ARE REPRESENTATIVE OF MTO'S LIKE SMALL PEBBLES TO HEAVIER
C        MTO'S LIKE TREE STUMPS AND ROCKS. THE USER GIVES THE FIRST VALUE OBJMAS
C        AND THE REMAINING 7 VALUES ARE COMPUTED BY THE PROGRAM.

         WRITE(6,*)'GIVE THE OBJECT MASS (OBJMAS) IN LBS, THIS VALUE MUST BE
      1  REPRESENTATIVE OF THE MASS OF MTO ENCOUNTERED IN THE FIELD '
         READ(5,*) OBJMAS
         DO N=1,8
         W1(N)=3.1071429*(N-1)+OBJMAS
         END DO
C        ****************************************************************************
         DO K=1,4
         DO J=1,4
         DO I=1,4
         DO M=1,4
         DO N=1,8

C        CW1 = ANGULAR VELOCITY OF THE BLADE IN RAD/MIN
         CW1=IRPM(I)*2*3.1416

C        VI = MOMENT OF INERTIA OF THE BLADE ABOUT ONE END. THE WIDTH OF THE
C        BLADE IS NEGLECTED AND THE BLADE IS MODELLED AS A ROD
C        VI = M X L**2 / 3.0
         VI=IW2(K)*D1**2/3.0

C        VB1 = VELOCITY (FT/MIN) OF THE POINT OF CONTACT BEFOR IMPACT.
         VB1=R(M)*(IRPM(I))*2.0*3.1416
         BTS(I)=4.*3.14*IRPM(I)
C        A AND B ARE INTERMEDIATE CONSTANTS
         A=VI/R(M)+W1(N)*R(M)
         B=VI*((E(J)*(VA1+VB1))/R(M))
         VA2(I,J,K,M,N)=(B+VI*CW1-W1(N)*VA1*R(M))/A
         Z(I,J,K,M,N)=(VA2(I,J,K,M,N)-440.0)/BTS(I)
         ZZ(N)=(W1(N))/IW2(K)
         END DO
         END DO
         END DO
         END DO
         END DO

C        PLOT THE MTO VELOCITY V/S OBJECT MASS CURVES WITH VARIATION OF ANY
C        ONE OF THE ABOVE PARAMETERS.

         DO N=1,8
           X(N) =ZZ(N)
         END DO
         DO N=1,8
         DO J=1,4
           Y(N,J) = Z(J,4,1,4,N)
         END DO
         DO J=1,4
         Y(N,J+4)=Z(J,4,4,4,N)
         END DO
         END DO
         WRITE(6,20)
   20    FORMAT('1',15X,'TEST  RESULT'//2X,' MASS '
      +    25X,'OBJECT  VELOCITY')
         DO N=1,8
```

```
        WRITE(6,30)X(N),(Y(N,J),J=1,8)
  30    FORMAT(2X,F5.2,2X,8(2X,F7.2)/)
        END DO
C   LET ROUTINE DO SCALING

          RANGE(1) = 0.0
          RANGE(2) = 1.0
          RANGE(3) =0.0
          RANGE(4) =1.75


C   SET TITLES IN HOLERITH FORMAT

       NXAXIS = 1
       ENCODE( NXAXIS, 100, XAXIS )
 100    FORMAT ( ' ' )
       NYAXIS = 1
       ENCODE( NYAXIS, 200, YAXIS )
 200    FORMAT ( ' ')
C       PLOT CURVES

       CALL ZETAPLT ( 1, 8, 8, RANGE, X, Y, 10, XAXIS, NXAXIS,
      1     YAXIS, NYAXIS )

       STOP
       END
       SUBROUTINE ZETAPLT ( NPLOT, NCVS, NPTS, RANGE, X, Y, NY,
      1     XAXIS, NXAXIS, YAXIS, NYAXIS )

C   SUBROUTINE PLOTS DATA BY CALLS TO ZETA PLOTTING SUBROUTINES
C
C   INPUT:   NPLOT = NO. OF PLOT FILE TO CREATE
C            NCVS = NO. OF CURVES TO PLOT
C            NPTS = NO. OF POINTS TO PLOT FOR EACH CURVE
C            RANGE(1),RANGE(2) = MIN. AND MAX. FOR X ARRAY
C            RANGE(3),RANGE(4) = MIN. AND MAX. FOR Yi ARRAYS
C                 SET RANGE VALUES FOR EACH AXIS TO ZERO FOR AUTOMATIC SCALING
C            X = ARRAY OF INDEPENDENT POINTS (DIMENSIONED NPTS+2)
C            Y = ARRAY OF DEPENDENT POINTS (DIMENSIONED NPTS+2 BY NCVS)
C            NY = ROW DIMENSION OF Y AS GIVEN IN CALLING PROGRAM
C            XAXIS = X AXIS LABEL IN HOLLERITH FORMAT
C            NXAXIS = NUMBER OF CHARACTERS IN X AXIS (NXAXIS>0)
C            YAXIS = Y AXIS LABEL IN HOLLERITH FORMAT
C            NYAXIS = NUMBER OF CHARACTERS IN Y AXIS (NYAXIS>0)
C   OUTPUT:  ZETAnn.PLT, FILE CONTAINING ZETA PLOTTING COMMANDS (nn=NPLOT)

       REAL RANGE(1),X(1),Y(NY,1),XAXIS(1),YAXIS(1)

       DATA AXLENX,AXLENY / 4.0, 6.0 /

C   INITALIZE PLOTTING

       CALL PLOTS ( 0, 0, NPLOT )
       CALL PLOT ( 0.5, 0.5, -3 )

C   COMPUTE PLOTTING SCALE FROM FIRST CURVE AND DRAW PLOTTING AXIS
```

```
      IF ( (RANGE(1).EQ.0.0) .AND. (RANGE(2).EQ.0.0) ) THEN
              CALL SCALE ( X, AXLENX, NPTS, 1 )
          ELSE
              X(NPTS+1) = RANGE(1)
              X(NPTS+2) = (RANGE(2) - RANGE(1))/AXLENX
      ENDIF

      IF ( (RANGE(3).EQ.0.0) .AND. (RANGE(4).EQ.0.0) ) THEN
              CALL SCALE ( Y, AXLENY, NPTS, 1 )
          ELSE
              Y(NPTS+1,1) = RANGE(3)
              Y(NPTS+2,1) = (RANGE(4) - RANGE(3))/AXLENY
      ENDIF

      IF ( NCVS .GE. 2 ) THEN
          DO N=2,NCVS
              Y(NPTS+1,N) = Y(NPTS+1,1)
              Y(NPTS+2,N) = Y(NPTS+2,1)
          END DO
      ENDIF

      CALL AXIS ( 0.0, 0.0, XAXIS, -NXAXIS, AXLENX, 0.0,
     1    X(NPTS+1), X(NPTS+2) )
      CALL AXIS ( 0.0, 0.0, YAXIS, NYAXIS, AXLENY, 90.0,
     1    Y(NPTS+1,1), Y(NPTS+2,1) )

C  PLOT EACH CURVE

      DO N=1,NCVS
          CALL LINE ( X, Y(1,N), NPTS, 1, 1, N )
      END DO
      CALL PLOT ( 0.0, 0.0, 999)

      RETURN
      END
```

```
C       MOWER THROWN OBJECTS
C
C       ****************************************************************
C       THIS PROGRAM IS A INTERACTIVE PROGRAM THAT REQUESTS THE USER
C       FOR INPUT LIKE THE BLADE DIMENSIONS, BLADE MASSS, MOWER SPEED,
C       COEFFICIENT OF RESTITUTION AND THE POINT OF CONTACT. THIS PROGRAM
C       PREDICTS THE MTO EXIT VELOCITIES FOR THE STRAIGHT AND THE PIVOTED
C       BLADE TYPES
C       ****************************************************************
C

        REAL*8 A(3,3),B(3),WKAREA(20)
        INTEGER M,N,IA,IDGT,IER


        WRITE (6,*)' DO YOU WANT TO SOLVE FOR THE STRAIGHT BLADE OR THE
     1  PIVOTED BLADE ??????? '



  10    WRITE (6,*) 'PRESS KEY 1 ON KEYBOARD FOR STRAIGHT BLADE'


        WRITE (6,*) ' OR                                              '


        WRITE (6,*) 'PRESS KEY 2 ON KEYBOARD FOR PIVOTED BLADE '

        READ (5,*) J
        IF(J.EQ.1)GOTO 25
        IF(J.EQ.2)GOTO 50
        GOTO 10

  25    WRITE (6,*)' GIVE THE RPM OF THE BLADE (RPM)'
        READ (5,*) RPM
        WRITE (6,*) ' GIVE THE LENGTH OF THE BLADE (FT) '
        READ (5,*) D1
        WRITE(6,*) 'GIVE THE MASS OF THE BLADE (LBS)'
        READ (5,*) W2
        WRITE (6,*) 'GIVE THE VALUE OF THE COEFFICIENT OF RESTITUTION'
        READ (5,*) E
        WRITE (6,*) 'GIVE THE VELOCITY OF THE OBJECT, FT/MIN (SAME AS THE TRA
     1  SPEED OF THE MOWER) BEFORE IMPACT'
        READ (5,*) VA1
        WRITE (6,*)' GIVE THE DISTANCE OF THE POINT OF CONTACT FROM THE CENTE
     1  OF ROTATION (FT) '
        READ(5,*) R
        WRITE (6,*) 'GIVE THE MASS OF THE OBJECT (LBS)'
        READ (5,*) W1
        CW1 = RPM*2*3.1416
        VI = W2*D1**2/3.0
        VB1 = R*CW1
        AX = VI/R + W1*R
        BX = VI*((E*(VA1+VB1))/R)
        VA2 = (BX+VI*CW1-W1*VA1*R)/AX
        WRITE(6,30) VA2
   30   FORMAT(' THE VELOCITY OF THE MTO IS =  ',F8.2,' FT/MIN')
        GOTO 100

   50   WRITE (6,*)'                    PIVOTED BLADE                 '
```

```
      WRITE (6,*)'          LEGEND:    * - PIVOT                    '
      WRITE (6,*)'                     O - CENTER OF ROTATION       '
      WRITE (6,*)'                    XL1,XL2 - DISTANCE            '
      WRITE (6,*)'                  R - DISTANCE OF PT  OF CONTACT'
      WRITE (6,*)'                                                  '
      WRITE (6,*)'                                                  '
      WRITE (6,*)'                                                  '

      WRITE (6,*)'                       |-LINK 1--|-LINK 2-|  '
      WRITE (6,*)'           ____________________________________|  '
      WRITE (6,*)'          /                                    \  '
      WRITE (6,*)'          |         *        O...XL1...*...XL2..|  '
      WRITE (6,*)'          \____________________________________/  '
      WRITE (6,*)'                             ........R......^     '
      WRITE (6,*)'                                                  '
      WRUTE (6,*)'                                                  '

      WRITE (6,*)' MASS OF SECTION XL1 & XL2 = XM1 & XM2 RESPECTIVELY'
      WRITE (6,*) ' GIVE THE VALUE OF XL1 AND XL2(INCH) RESPECTIVELY. '

      READ (5,*) XL1,XL2

      WRITE (6,*)' GIVE THE VALUE OF XM1 AND XM2 RESPECTIVELY'

      READ (5,*) XM1,XM2

      WRITE (6,*) ' GIVE THE VALUE OF THE OBJECT VELOCITY (FT/MIN) BEFORE
     1 IMPACT, RECOMMENDED RANGE IS 300 TO 500 FT/MIN. '

      READ (5,*)VC

      WRITE (6,*)' GIVE THE VALUE OF THE COEFFICIENT OF RESTITUTION FOR THE
     1 IMPACT CONDITION.VALUE IS TO BE LESS THAN 1 BUT GREATER THAN 0'

      READ (5,*)E

      WRITE (6,*)'GIVE THE OBJECT MASS, RECOMMENDED RANGE IS 0.25 TO
     1 20 LBS '

      READ(5,*)XMC

      WRITE (6,*)'GIVE THE VALUE OF THE MOWER RPM, RECOMMENDED RANGE IS
     1 500 TO 1100 RPM'

      READ (5,*)RPM

      WRITE (6,*)'GIVE THE VALUE OF THE DISTANCE (INCHES) OF THE POINT
     1 OF CONTACT FROM THE POINT OF ROTATION'

      READ (5,*)R

      W=2*3.142*RPM
      V1=(XL1/24.)*W
      V2=(XL2+XL1/2.)/12.*W
      VP=(XL1/12.)*W+((R-XL1)/12.)*W
      XI1=XM1*(XL1**2)/1728.
      XI2=XM2*(XL2**2)/1728.
      B(3)=E*(VC+VP)
```

```
      B(1)=XI1*W + XI2*W + XM1*V1*(XL1/24.) + XM2*V2*(XL1/12.+XL2/24.)
     1 - XMC*VC*R/12.
      B(2)=XI1*W + XI2*W - XM1*V1*(XL1/24.) + XM2*V2*(XL2/24.) - XMC*VC*
     1 (R-XL1)/12.
      A(1,1)=XI1+XI2+XM1*(XL1**2)/576.+XM2*XL1/12.*(XL1/12.+XL2/24.)
      A(1,2)=XI2+XM2*(XL2/24.)*(XL1/12.+ XL2/24.)
      A(1,3)=XMC*R/12.
      A(2,1)=XI1+XI2-XM1*(XL1**2)/576.+XM2*XL1*XL2/288.
      A(2,2)=XI2+XM2*(XL2**2)/576.
      A(2,3)=XMC*(R-XL1)/12.
      A(3,I)= -0.42
      A(3,2)= -0.38
      A(3,3)= 1.0
      M=1
      N=3
      IA=3
      IDGT=6
      CALL LEQT1F(A,M,N,IA,B,IDGT,WKAREA,IER)
      WRITE (6,21) B(3)
 21   FORMAT (' THE VELOCITY OF THE MTO IS=  ',F8.2,' FT/MIN')
100   CONTINUE
      STOP
      END
```

APPENDIX B

DATA FROM MOWER THROWN OBJECT EXPERIMENTS

APPENDIX B: DATA FROM MOWER-THROWN-OBJECT EXPERIMENTS

Appendix B contains the experimental data of the laboratory experiment conducted to measure the discharge velocities of mower-thrown-objects. The experiment was conducted with the mower model and the experimental setup shown in Fig. 3.2, using both the straight and pivoted blade types. The thrown objects were a 3/4" diameter aluminum cylindrical bar weighing 19 gms, a 1/2" diameter steel cylindrical bar weighing 14 gms and a 1 1/8 " diameter steel cylindrical bar weighing 84 gms and a 3/4" diameter steel cylinder weighing 42.5 gms.

Each experiment contains the velocity of the blade before impact and the velocities of the object before and after impact. The distance was measured on the grid of the video monitor and the time in seconds was measured from the time code displayed on the monitor. The scaling factor (see Chapter 3, under Calibration of the Video Monitor) was 2.906 inches. The blade speed and the initial object velocity were maintained almost constant (413 rpm and 65 to 85 ft/min range) for the straight and pivoted blades in order to eliminate the variation in the discharge velocity of the object due to these two parameters. The observed variation in the discharge velocity for the two types of blades was then due to the difference in the construction of the two blades. The experimental data is plotted in Fig. 3.6.

B.1 CASE I - STRAIGHT BLADE

TEST #1:

MOTOR HP	1/20	SHIELDING TYPE	1/8 chain guard
BLADE TYPE	STRAIGHT	CONDUCTED BY	R. da Silva
TEST OBJECT	3/4" AL	DATE	02-11-86
MASS OF TEST OBJ:	19 gms	TIME	10:30 P.M.
SCALE FACTOR	2.906	LOCATION	ETC 6.106

A. BLADE VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFFERENCE	VELOCITY(RPM)
003	4590	4735	145	413
003	4780	4925	145	413
003	5970	5115	145	413

AVERAGE VELOCITY 413 RPM

B. OBJECT VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ALONG MONITOR GRID (inches)	DISTANCE MOVED BY OBJECT ALONG FEEDER (inches)	VELOCITY (ft/min)
003	5260	5435	175	1	2.906	83
003	5450	5605	155	1	2.906	93

AVERAGE VELOCITY BEFORE IMPACT = 88 ft/min

C. OBJECT VELOCITY AFTER IMPACT

Scene code	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ON MONITOR GRID (inches)	ACTUAL DISTANCE MOVED (inches)	VELOCITY (ft/min)
003	5645	5655	10	1.8	5.23	2615

AVERAGE VELOCITY AFTER IMPACT = 2615 FT/MIN

TEST #2:

MOTOR HP	1/20	SHIELDING TYPE 1/8 chain guard	
BLADE TYPE	STRAIGHT	CONDUCTED BY	R. da Silva
TEST OBJECT	1/2" DIA STEEL	DATE	02-16-86
MASS OF TEST OBJ:	14 gms	TIME	9:00 P.M.
SCALE FACTOR	2.906	LOCATION	ETC 6.106

A. BLADE VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER			
	START	END	DIFFERENCE	VELOCITY(RPM)
510	1910	2060	150	400
510	2090	2230	140	427
510	2370	2515	145	413

AVERAGE VELOCITY = 413.5 RPM

B. OBJECT VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ALONG MONITOR GRID (inches)	DISTANCE MOVED BY OBJECT ALONG FEEDER (inches)	VELOCITY (f t/min)
510	9230	4120	215	1	2.906	68
510	9470	9705	235	1	2.906	62

AVERAGE VELOCITY BEFORE IMPACT = 65 ft/min

C. OBJECT VELOCITY AFTER IMPACT

Scene code	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ON THE MONITOR (inches)	ACTUAL DISTANCE MOVED (inches)	VELOCITY (ft/min)
510	9785	9795	10	1.9	5.52	2760

AVERAGE VELOCITY AFTER IMPACT = 2760 ft/min

TEST #3:

MOTOR HP	1/20	SHIELDING TYPE	1/8 chain gaurd
BLADE TYPE	STRAIGHT	CONDUCTED BY	R. da Silva
TEST OBJECT	1 1/8"DIASTEEL	DATE	02-27-86
MASS OF TEST OBJ:	84 gms	TIME	11:00 P.M.
SCALE FACTOR	2.906	LOCATION	ETC 6.106

A. BLADE VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFFERENCE	VELOCITY(RPM)
001	2015	2160	145	413
001	2160	2305	145	413
001	2305	2450	145	413

AVERAGE VELOCITY = 413 RPM

B. OBJECT VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ALONG MONITOR GRID (inches)	DISTANCE MOVED BY OBJECT ALONG FEEDER (inches)	VELOCITY (ft/min)
001	2550	2740	190	1	2.906	77
001	2780	2980	200	1	2.906	73

AVERAGE VELOCITY BEFORE IMPACT = 75 FT/MIN

C. OBJECT VELOCITY AFTER IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ON MONITOR GRID (inches)	ACTUAL DISTANCE MOVED (inches)	VELOCITY (ft/min)
001	3075	3090	15	1.5	4.34	1453

AVERAGE VELOCITY AFTER IMPACT = 1453 FT/MIN

TEST #4:

MOTOR HP	1/20	SHIELDING T	1/8 chain gaurd
BLADE TYPE	STRAIGHT	CONDUCTED BY	R. da Silva
TEST OBJECT	3/4 "DIASTEEL	DATE	06-27-86
MASS OF TEST OBJ:	42.5 gms	TIME	10.30:00 P.M.
SCALE FACTOR	2.906	LOCATION	ETC 6.106

A. BLADE VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFFERENCE	VELOCITY(RPM)
222	5455	5600	145	413
222	5600	5745	145	413
222	5745	5890	145	413

AVERAGE VELOCITY = 413 RPM

B. OBJECT VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ALONG MONITOR GRID (inches)	DISTANCE MOVED BY OBJECT ALONG FEEDER (inches)	VELOCITY (ft/min)
222	5940	6165	215	1	2.906	68
222	6165	6365	200	1	2.906	73

AVERAGE VELOCITY BEFORE IMPACT = 71 FT/MIN

C. OBJECT VELOCITY AFTER IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ON MONITOR GRID (inches)	ACTUAL DISTANCE MOVED (inches)	VELOCITY (ft/min)
222	6475	6485	10	1.35	3.92	1960

AVERAGE VELOCITY AFTER IMPACT = 1960 FT/MIN

B. 2 CASE II - PIVOTED BLADE

TEST #1

MOTOR HP	1/20	SHIELDING TYPE	1/8" chain guard
BLADE TYPE	PIVOTED	CONDUCTED BY	R. da Silva
TEST OBJECT	3/4" AL	DATE	03-01-86
MASS OF TEST OBJ:	19 gms	TIME	9:30 P. M.
SCALE FACTOR	2.906	LOCATION	ETC 6.106

A. BLADE VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFFERENCE	VELOCITY(RPM)
500	3390	3540	140	400
500	3575	3720	145	413
500	3730	3870	140	427

AVERAGE VELOCITY = 413.5 RPM

B. OBJECT VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ALONG MONITOR GRID (inches)	DISTANCE MOVED BY OBJECT ALONG FEEDER (inches)	VELOCITY (ft/min)
500	3950	4190	240	1	2.906	61
500	4215	4460	245	1	2.906	60

AVERAGE VELOCITY BEFORE IMPACT = 60.5 ft/min

C. OBJECT VELOCITY AFTER IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ON THE MONITOR (inches)	ACTUAL DISTANCE MOVED (inches)	VELOCITY (ft/min)
500	4515	4530	15	2.0	5.81	1937

AVERAGE VELOCITY AFTER IMPACT = 1937 FT/MIN

TEST #2

MOTOR HP	1/20	SHIELDING TYPE	1/8" guard
BLADE TYPE	PIVOTED	CONDUCTED BY	R. da Silva
TEST OBJECT	1/2" DIA STEEL	DATE	03-03-86
MASS OF TEST OBJ:	14 gms	TIME	10:30 P.M.
SCALE FACTOR	2.906	LOCATION	ETC 6.106

A. BLADE VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER			
	START	END	DIFFERENCE	VELOCITY(RPM)
950	6220	6365	145	413
950	6365	6500	145	413
950	6500	6645	145	413

AVERAGE VELOCITY = 413 RPM

B. OBJECT VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ALONG MONITOR GRID (inches)	DISTANCE MOVED BY OBJECT ALONG FEEDER (inches)	VELOCITY (ft/min)
950	6680	6790	120	0.5	1.45	66
950	6860	6980	110	0.5	1.45	61

AVERAGE VELOCITY BEFORE IMPACT = 63.5 ft/min

C. OBJECT VELOCITY AFTER IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ON THE MONITOR (inches)	ACTUAL DISTANCE MOVED (inches)	VELOCITY (ft/min)
950	7040	7050	10	1.6	4.65	2325

AVERAGE VELOCITY AFTER IMPACT = 2325 FT/MIN

TEST #3:

MOTOR HP	1/20	SHIELDING TYPE	1/8"chainguard
BLADE TYPE	PIVOTED	CONDUCTED BY	R. da Silva
TEST OBJECT	1 1/8" DIA STEEL	DATE	05-03-86
MASS OF TEST OBJ:	84 gms	TIME	11:15 P.M.
SCALE FACTOR	2.906	LOCATION	ETC 6.106

A. BLADE VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFFERENCE	VELOCITY(RPM)
750	3985	4135	150	400
750	4130	4270	140	427
750	4280	4425	145	413

AVERAGE VELOCITY = 413.5 RPM

B. OBJECT VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ALONG MONITOR GRID (inches)	DISTANCE MOVED BY OBJECT ALONG FEEDER (inches)	VELOCITY (ft/min)
750	4450	4650	200	1	2.906	73
750	4700	4890	190	1	2.906	77

AVERAGE VELOCITY BEFORE IMPACT = 75 ft/min

C. OBJECT VELOCITY AFTER IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ON THE MONITOR (inches)	ACTUAL DISTANCE MOVED (inches)	VELOCITY (ft/min)
750	4945	4980	35	2.1	6.1	872

AVERAGE VELOCITY AFTER IMPACT = 872 FT/MIN

TEST #4:

MOTOR HP	1/20	SHIELDING TYPE	1/8"chainguard
BLADE TYPE	PIVOTED	CONDUCTED BY	R. da Silva
TEST OBJECT	3/4" DIA STEEL	DATE	06-27-86
MASS OF TEST OBJ:	42.5 gms	TIME	12:00 A.M.
SCALE FACTOR	2.906	LOCATION	ETC 6.106

A. BLADE VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFFERENCE	VELOCITY(RPM)
999	4395	4545	150	400
999	4540	4680	140	427
999	4725	4870	145	413

AVERAGE VELOCITY = 413.5 RPM

B. OBJECT VELOCITY BEFORE IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ALONG MONITOR GRID (inches)	DISTANCE MOVED BY OBJECT ALONG FEEDER (inches)	VELOCITY (ft/min)
999	5500	5690	190	1	2.906	76.5
999	5690	5990	200	1	2.906	73

AVERAGE VELOCITY BEFORE IMPACT = 75 ft/min

C. OBJECT VELOCITY AFTER IMPACT

Scene code #	FIELD COUNTER START	END	DIFF	DISTANCE MOVED BY OBJECT ON THE MONITOR (inches)	ACTUAL DISTANCE MOVED (inches)	VELOCITY (ft/min)
999	6030	6045	15	1.3	3.8	1260

AVERAGE VELOCITY AFTER IMPACT = 1260 FT/MIN

APPENDIX C

THEORETICAL CALCULATIONS FOR STRAIGHT AND PIVOTED BLADES

APPENDIX C: THEORETICAL CALCULATIONS FOR STRAIGHT AND PIVOTED BLADES

Appendix C contains the theoretical calculations of the discharge velocities of the test objects, as predicted by the analysis in Chapter 2. The calculations are worked out for both straight and pivoted blade types. In the calculations the angular velocity of the blade before impact, mass of the test object and the velocity of the object before impact are the same as the values recorded in Appendix B, for the experimental model. The mass of the blade and distance of the point of impact are taken from the experimental model. The coefficient of restitution e was estimated as explained in Chapter 3, under Experimental Estimation of the Coefficient of Restitution. The results of the theoretical calculations are plotted in Fig 3.7 and compared with the experimental results from Appendix B.

C. 1 CASE I - STRAIGHT BLADE

TEST #1

Test blade: Straight blade
Test object: 3/4" diameter aluminum cylinder
Given Parameters:
m_b = mass of the blade = 170 gms = 0.187 lbs.
m_c = mass of test object = 19 gms = 0.042 lbs.
ω = angular velocity of blade before impact = 413 x 2π = 2595 rad/min
v_c = velocity of object before impact = 88 ft/min
r = distance of point of impact from center of rotation = 9.5 inches
ℓ = length of blade = 10 inches
Computed Parameters:
ω' = angular velocity of the blade after impact
v_p = velocity of the blade at the point of impact, before impact = r x ω = 9.5/12 x 2595 = 2054 ft/min
v_p' = velocity of the blade at the point of impact after impact = 9.5/12 x ω' = 0.79ω
v_c' = velocity of the blade after impact
v_b = velocity of the center of mass of the blade before impact = ℓ /2 x ω = 5/12 x 2595 = 1081 ft/min
v_b'= velocity of the center of mass of a the blade after impact = ℓ /2 x ω' = 5/12 ω
I = moment of Inertia of blade about its mass center

$$I = \frac{m_b \ell^2}{12} = 0.374 \times (10/12)^2 \times 1/12 = 0.022$$

$$e = 0.90 = \frac{v_p' - v_c'}{v_c - v_p} = \frac{0.79\omega' - v_c'}{-88 - 2054}$$

$$\omega' = \frac{v'_c - 1927}{0.79}$$

Using the principle of conservation of angular momentum about the fixed point ,

$$- I\omega - (m_b v_b)\ \ell /2 + (m_c v_c)\ r = -I\omega' - (m_b v_b')\ \ell /2 - (m_c v_c')\ r$$

LHS = -0.022 x 2595 - 0.374 x 1081 x 5/12 + 0.042 x 88 x 9.5/12 = -222.6

$$RHS = -0.022 \times \left(\frac{v_C' - 1927}{0.79}\right) - 0.374 \left(\frac{v_C' - 1927}{0.79}\right) \frac{25}{12 \times 12} - 0.042 x v_C' \times \frac{9.5}{12}$$

$= 0.143v_C' + 212$

$-434.6 = 0.143v_C'$

$v_C' = 3039$ ft/min

Test object velocity = 3039 ft/min

TEST #2

Test blade: Straight blade
Test object: 1/2" diameter steel cylinder
Given Parameters:
m_c = 14 gms = 0.03 lbs
m_b = 0.374 lbs
ω = 413 x 2π = 2595 rad/min
v_c = 65 ft/min
r = 9.5 inches
l = length of the blade = 10 inches
Calculated Parameters:
I = 0.022
v_p = 2054 ft/min
v_p' = 0.79 ω'

$$e = 0.92 = \frac{v_p' - v_c'}{v_c - v_p} = \frac{0.79\omega' - v_c'}{-65 - 2054}$$

$$\omega' = \left(\frac{v_c - 1950}{0.79}\right)$$

All other variables are the same as in Test # 1 and have the same values. Using the principle of conservation of angular momentum about the fixedpoint ,

$$- I\omega - (m_b v_b)\,\ell/2 + (m_c v_c)\,r = -I\omega' - (m_b v_b')\,\ell/2 - (m_c v_c')\,r$$

LHS = -0.022 x 2595 - 0.374 x 1081 x 5/12 + 0.03 x 65 x 9.5/12 = -224.0

$$RHS = -0.022 \times \left(\frac{v_c' - 1950}{0.79}\right) - 0.374\left(\frac{v_c' - 1950}{0.79}\right)\frac{25}{144} - 0.03 \times v_c' \times \frac{9.5}{12}$$

= 0.134v_c'+ 214.5
- 438.5 = 0.134 v_c'
v_c' = 3273 ft/min
Test object velocity after impact = 3273 ft/min

TEST #3

Test blade: Straight blade
Test object: 1 1/8" diameter steel cylinder
Given Parameters:
m_c = 84 gms = 0.185 lbs
v_c = 75 ft/min
m_b = mass of the blade = 170 gms = 0.374 lbs.
ω = angular velocity of blade before impact = 413 x 2π = 2595 rad/min
r = 9.5 inches
ℓ = length of blade = 10 inches

Computed Parameters:

$$e = 0.92 = \frac{0.79\omega' - v_c'}{-75 - 2054}$$

v_p = 2054 ft/min
v_p' = $0.79\omega'$
ω = 2595 rad/min

$$\omega' = \left(\frac{v_c' - 1959}{0.79}\right)$$

All other variables are the same as in Test #1 and have the same values
Using the principle of conservation of angular momentum about the fixed point ,

$$- I\omega - (m_b v_b)\,\ell/2 + (m_c v_c)\, r = -I\omega' - (m_b v_b')\,\ell/2 - (m_c v_c')\, r$$

LHS = -0.022 x 2595 - 0.374 x 1081 x 5/12 + 0.185 x 75 x 9.5/12 = -214

$$RHS = -0.022 \times \left(\frac{v_c' - 1959}{0.79}\right) - 0.374 \left(\frac{v_c' - 1959}{0.79}\right) \times \frac{25}{144} - 0.185 \times v_c' \times \frac{9.5}{12}$$

$= 0.256 v_c' + 215.6$

v_c' = 1680 ft/min

Test object velocity after impact = 1680 ft/min

TEST #4

Test blade: Straight blade
Test object: 3/4" diameter steel cylinder
Given Parameters:
m_c = 42.5 gms = 0.094 lbs
v_c = 71 ft/min
m_b = mass of the blade = 170 gms = 0.374 lbs.
ω = angular velocity of blade before impact = 413 x 2π = 2595 rad/min
r = 9.5 inches
ℓ = length of blade = 10 inches
Computed Parameters:

$$e = 0.92 = \frac{0.79\omega' - v_c'}{-75 - 2054}$$

v_p = 2054 ft/min

v_p' = 0.79w'

ω = 2595 rad/min

$$\omega' = \left(\frac{v_c' - 1959}{0.79}\right)$$

All other variables are the same as in Test #1 and have the same values. Using the principle of conservation of angular momentum about the fixed point ,

$$-I\omega - (m_b v_b)\,\ell/2 + (m_c v_c)\,r = -I\omega' - (m_b v_b')\,\ell/2 - (m_c v_c')\,r$$

LHS = -0.022 x 2595 - 0.374 x1081 x 5/12 + 0.094 x71 x 9.5/12 = -220.3

$$RHS = -0.022 \times \left(\frac{v_c' - 1959}{0.79}\right) - 0.374 \left(\frac{v_c' - 1959}{0.79}\right) \times \frac{25}{144} - 0.094 \times v_c' \times \frac{9.5}{12}$$

$= 0.185v_c' + 215.6$

v_c' = 2356 ft/min

Test object velocity after impact = 2356 ft/min

C.2 CASE II - PIVOTED BLADE

TEST #1
Test blade: Pivoted blade
Test object: 3/4" diameter aluminum cylinder
Given Parameters:
m_c = 14 gms = 0.042 lbs
v_c = 60.5 ft/min
ℓ = length of link 1 = 5"
ℓ = length of link 2 = 5"
2ℓ = length of blade assembly = 10"
m_1 = mass of link 1, assumed to be half that of the blade assembly
= 0.0187 lbs
m_2 = mass of link 2, assumed to be half that of the blade assembly = 0.0187 lbs
$\omega = \omega_1$ = angular velocity of link 1 before impact
$\omega = \omega_2$ = angular velocity of link 2 before impact
$\omega_1 = \omega_2$ = 2595 rad/min, before impact
R = distance of point of impact from fixed point = 9.5 inches
Calculated Parameters:
ω_1' = absolute angular velocity of link 1 after impact
ω_2' = angular velocity of link 2 with respect to pivot after impact
$\omega_1' + \omega_2'$ = absolute angular vlocity of link 2 after impact
I_1 = Moment of inertia of link 1 about center of mass
I_2 = Moment of inertia of link 2 about center of mass
v_1 = velocity of center of mass of link 1 before impact
v_1' = velocity of center of mass of link 1 after impact
v_2 = velocity of center of mass of link 2 before impact
v_2' = velocity of center of mass of link 2 after impact
$I_1 = m_1(\ell^2/12) = (0.374/2) \times (1/12) \times (5/12)^2 = 0.0026$
$I_2 = I_1$, as the dimensions of the two links are equal
$v_1' = (\ell/2)\omega_1' = (2.5/12)\omega_1' = 0.208\omega_1'$
$v_2' = \ell\omega_1' = (\ell/2)\omega_2' = 0.416\omega_1' + 0.208\omega_2'$
$v_1 = (\ell/2)\omega = (2.5/12) \times 2595 = 540$ ft/min
$v_2 = (\ell + \ell/2)\omega = (7.5/12) \times 2595 = 1620$ ft/min
Using the principle of conservation of angular momentum about fixed the point 0. (clockwise positive)
$-I_1\omega_1 - I_2\omega_2 - m_1v_1(\ell/2) - m_2v_2(\ell + \ell/2) + m_cv_cR =$
$-I_1\omega_1' - I_2(\omega_1' + \omega_2') - m_1v_1'(l/2) - m_2v_2'(l + l/2) - m_cv_c'R$
LHS = -0.0026 x 2595 - 0.0026 x 2595 - 0.187 x 540 x (2.5/12)
- 0.187 x 1620 x (7.5/12) + 0.042 x 60.5 x (9.5/12) = - 221.9
RHS = $-0.0026\omega_1' - 0.0026\omega_1' - 0.0026\,\omega_2' - 0.187 \times 0.208\omega_1' \times (2.5/12)$

$- 0.187x(0.416\omega_1' + 0.208\omega_2') \times (7.5/12) - 0.042 \times (9.5/12)V_C'$

$221.9 = +0.062\omega_1' + 0.027\omega_2' + 0.033V_C'$

from kinematics of the assembly

$v_p' = \ell\omega_1' + (\ell/2)\omega_2' = (5/12)\omega_1' + (2.5/12)\omega_2' = 0.42\omega_1' + 0.38\omega_2'$

$$e = \frac{v_c' - v_p'}{v_p - v_c} = \frac{v_c' - 0.42\omega_1' - 0.38\omega_2'}{2054 - (-60.5)}$$

$e = 0.90$

$-0.42\omega_1' - 0.38\omega_2' + v_c' = 1903$

Conservation of angular momentum about the pivot point gives us the third equation

$-I_1\omega_1 - I_2\omega_2 + m_1v_1(\ell/2) - m_2v_2(\ell/2) + m_cv_c(R - \ell)$

$= -I_1\omega_1' - I_2(\omega_1' + \omega_2') + m_1v_1'(\ell/2) - m_2v_2'(\ell/2) - m_cv_c'(R - \ell)$

LHS = $-0.0026 \times 2595 - 0.0026 \times 2595 + 0.187 \times (2.5/12) \times 540 - 0.187 \times (2.5/12) \times 1620 + 0.042 \times 60.5 \times (9.5\text{-}5/12) = -54.6$

RHS = $-0.0026\omega_1' - 0.0026(\omega_1' + \omega_2') + (2.5/12) \times 0.187 \times (2.5/12)\omega_1'$

$- (2.5/12) \times 0.187 \times (0.416\omega_1' + 0.208\omega_2') - 0.042 \times (4.5/12)v_c'$

$0.0133\omega_1' + 0.0107\omega_2' + 0.016\, v_c' = 54.6$

The system of equations we have now obtained is as follows:

$0.062\,\omega_1' + 0.027\omega_2' + 0.033v_c' = 221.9$

$0.0133\,\omega_1' + 0.0107\,\omega_2' + 0.016v_c' = 54.6$

$-0.42\,\omega_1' - 0.38\,\omega_2' + v_c' = 1903$

Solve using Kramer's Rule

$$v_c' = \frac{\begin{vmatrix} 0.062 & 0.027 & 221.9 \\ 0.0133 & 0.0107 & 54.6 \\ -0.42 & -0.38 & 1903 \end{vmatrix}}{\begin{vmatrix} 0.062 & 0.027 & 0.033 \\ 0.0133 & 0.0107 & 0.016 \\ -0.42 & -0.38 & 1.00 \end{vmatrix}} = 2331 \text{ ft/min}$$

Test object velocity after impact = 2331 ft/ min.

TEST #2

Test blade: Pivoted blade
Test object: 1/2" diameter steel cylinder
Given Parameters:
m_C = 14 gms = 0.03 lbs
v_C = 63.5 ft/min
m_1 = mass of link 1, assumed to be half that of the blade assembly = 0.0187 lbs
m_2 = mass of link 2, assumed to be half that of the blade assembly = 0.0187 lbs
ℓ = length of link 1 = 5"
ℓ = length of link 2 = 5"
2ℓ = length of blade assembly = 10"
ω_1 = angular velocity of link 1 before impact
ω_2 = angular velocity of link 2 before impact
$\omega_1 = \omega_2$ = 2595 rad/min, before impact
R = distanct of point of impact from fixed point = 9.5 inches
Calculated Parameters:
ω_1' = absolute angular velocity of link 1 after impact
ω_2' = angular velocity of link 2 with respact to pivot after impact
$\omega_1' + \omega_2'$ = absolute angular vlocity of link 2 after impact
I_1 = Moment of inertia of link 1 about center of mass
I_2 = Moment of inertia of link 2 about center of mass
v_1 = velocity of center of mass of link 1 before impact
v_1' = velocity of center of mass of link 1 after impact
v_2 = velocity of center of mass of link 2 before impact
v_2' = velocity of center of mass of link 2 after impact
$I_1 = m_1(\ell^2/12) = (0.374/2) \times (1/12) \times (5/12)^2 = 0.0026$
$I_2 = I_1$, as the dimensions of the two links are equal
$v_1' = (\ell/2)\omega_1' = (2.5/12)\omega_1' = 0.208\omega_1'$
$v_2' = \ell\omega_1' = (\ell/2)\omega_2' = 0.416\omega_1' + 0.208\omega_2'$
$v_1 = (\ell/2)\omega = (2.5/12) \times 2595 = 540$ ft/min
$v_2 = (\ell + \ell/2)w = (7.5/12) \times 2595 = 1620$ ft/min
Using the principle of conservation of angular momentum about fixed point O (clockwise positive)
$-I_1\omega 1 - I_2\omega 2 - m_1v_1(\ell/2) - m_2v_2(\ell + \ell/2) + m_Cv_CR$
$-I_1\omega_1 - I_2(\omega_1' + \omega_2') - m_1v_1'(l/2) - m_2v_2'(l + l/2) - m_Cv_C'R$
LHS = -0.0026 x 2595 - 0.0026 x 2595 - 0.187 x 540 x (2.5/12)
- 0.187 x 1620 x (7.5/12) + 0.03 x 63.5 x (9.5/12) = -222.3
RHS = $-0.0026\omega_1' - 0.0026\omega_1' - 0.0026\,\omega_2' - 0.187 \times 0.208\omega_1' \times (2.5/12)$
$- 0.187\times(0.416\omega_1' + 0.208\omega_2') \times (7.5/12) - 0.03\times(9.5/12)v_C'$
$222.3 = +0.062\omega_1' + 0.027\omega_2' + 0.024V_C'$

from kinematics of the assembly

$v_p' = \ell\omega_1' + (\ell/2)\omega_2' = (5/12)\omega_1' + (2.5/12)\omega_2' = 0.42\omega_1' + 0.38\omega_2'$

$$e = \frac{v_c' - v_p'}{v_p - v_c} = \frac{v_c' - 0.42\omega_1' - 0.38\omega_2'}{2054 - (-63.5)}$$

$e = 0.92$

$-0.42\omega_1' - 0.38\omega_2' + v_c' = 1948$

Conservation of angular momentum about the pivot point gives us the third equation

$-I_1\omega_1 - I_2\omega_2 + m_1v_1(\ell/2) - m_2v_2(\ell/2) + m_cv_c(R - \ell)$

$= -I_1\omega_1' - I_2(\omega_1' + \omega_2') + m_1v_1'(\ell/2) - m_2v_2'(\ell/2) + m_cv_c'(R - \ell)$

LHS = -0.0026 x 2595 - 0.0026 x 2595 + 0.187 x (2.5/12) x 540 - 0.187 x (2.5/12) x 1620 + 0.03 x 63.5 x (9.5-5/12) = - 54.9

RHS = $-0.0026\omega_1' - 0.0026(\omega_1' + \omega_2') + (2.5/12) \times 0.187 \times (2.5/12)\omega_1' - (2.5/12) \times 0.187 \times (0.416\omega_1' + 0.208\omega_2') - 0.03 \times (4.5/12)v_c'$

$0.0133\omega_1' + 0.0107\omega_2' + 0.011v_c' = 54.9$

The system of equations we have now obtained is as follows:

$0.062\,\omega_1' + 0.027\omega_2' + 0.024\,v_c' = 222.3$

$0.0133\,\omega_1' + 0.0107\,\omega_2' + 0.011v_c' = 54.9$

$-0.42\,\omega_1' - 0.38\,\omega_2' + v_c' = 1948$

Solve using Kramer's Rule

$$v_c' = \frac{\begin{vmatrix} 0.062 & 0.027 & 222.3 \\ 0.0133 & 0.0107 & 54.9 \\ -0.42 & -0.38 & 1948 \end{vmatrix}}{\begin{vmatrix} 0.062 & 0.027 & 0.024 \\ 0.0133 & 0.0107 & 0.011 \\ -0.42 & -0.38 & 1.00 \end{vmatrix}} = 2677 \text{ ft/min}$$

Test object velocity after impact = 2677 ft/min

TEST #3

Test blade: Pivoted blade
Test object: 1 1/8" diameter steel cylinder
Given Parameters:
m_c = 84 gms = 0.185 lbs
v_c = 73 ft/min
$\omega = \omega_1$ = angular velocity of link 1 before impact
$\omega = \omega_2$ = angular velocity of link 2 before impact
$\omega_1 = \omega_2$ = 2595 rad/min, before impact
ℓ = length of link 1 = 5"
ℓ = length of link 2 = 5"
2ℓ = length of blade assembly = 10"
m_1 = mass of link 1, assumed to be half that of the blade assembly=0.187
m_2 = mass of link 2, assumed to be half that of the blade assembly=0.187
R = distanct of point of impact from fixed point = 9.5 inches
Calculated Parameters:
ω_1' = absolute angular velocity of link 1 after impact
ω_2' = angular velocity of link 2 with respact to pivot after impact
$\omega_1' + \omega_2'$ = absolute angular vlocity of link 2 after impact
I_1 = Moment of inertia of link 1 about center of mass
I_2 = Moment of inertia of link 2 about center of mass
v_1 = velocity of center of mass of link 1 before impact
v_1' = velocity of center of mass of link 1 after impact
v_2 = velocity of center of mass of link 2 before impact
v_2' = velocity of center of mass of link 2 after impact
$I_1 = m_1(\ell^2/12) = (0.374/2) \times (1/12) \times (5/12)^2 = 0.0026$
$I_2 = I_1$, as the dimensions of the two links are equal
$v_1' = (\ell/2)\omega_1' = (2.5/12)\omega_1' = 0.208\omega_1'$
$v_2' = \ell\omega_1' = (\ell/2)\omega_2' = 0.416\omega_1' + 0.208\omega_2'$
$v_1 = (\ell/2)\omega = (2.5/12) \times 2595 = 540$ ft/min
$v_2 = (\ell + \ell/2)w = (7.5/12) \times 2595 = 1620$ ft/min
Using the principle of conservation of angular momentum about fixed point 0. (clockwise positive)
$-I_1\omega - I_2\omega - m_1v_1(\ell/2) - m_2v_2(\ell + \ell/2) + m_cv_cR$
$-I_1\omega_1 - I_2(\omega_1' + \omega_2') - m_1v_1'(l/2) - m_2v_2'(l + l/2) - m_cv_c'R$
LHS = -0.0026 x 2595 - 0.0026 x 2595 - 0.187 x 540 x (2.5/12)
 - 0.187 x 1620 x (7.5/12) + 0.185 x 73 x (9.5/12) = -213.2
RHS = $-0.0026\omega_1' - 0.0026\omega_1' - 0.0026\,\omega_2' - 0.187 \times 0.208\omega_1' \times (2.5/12)$
$- 0.187\times (0.416\omega_1' + 0.208\omega_2') \times (7.5/12) - 0.185 \times (9.5/12)v_c'$
$213.2 = +0.062\omega_1' + 0.027\omega_2' + 0.146V_c'$

from kinematics of the rotating rod assembly

$v_p' = \ell\omega_1' + (\ell/2)\omega_2' = (5/12)\omega_1' + (2.5/12)\omega_2' = 0.42\omega_1' + 0.38\omega_2'$

$$e = \frac{v_c' - v_p'}{v_p - v_c} = \frac{v_c' - 0.42\omega_1' - 0.38\omega_2'}{2054 - (-73)}$$

$e = 0.92$

$-0.42\omega_1' - 0.38\omega_2' + v_c' = 1959$

Conservation of angular momentum about the pivot point gives us the third equation

$-I_1\omega_1 - I_2\omega_2 + m_1v_1(\ell/2) - m_2v_2(\ell/2) + m_cv_c(R - \ell)$

$= -I_1\omega_1' - I_2(\omega_1' + \omega_2') + m_1v_1'(\ell/2) - m_2v_2'(\ell/2) + m_cv_c'(R - \ell)$

LHS = -0.0026 x 2595 - 0.0026 x 2595 + 0.187 x (2.5/12) x 540 - 0.187 x (2.5/12) x 1620 + 0.185 x 13.6 x (9.5-5/12) = -50.5

RHS = $-0.0026\omega_1' - 0.0026(\omega_1' + \omega_2') + (2.5/12) \times 0.187 \times (2.5/12)\omega_1' - (2.5/12) \times 0.187 \times (0.416\omega_1' + 0.208\omega_2') - 0.185 \times (4.5/12)v_c'$

$= -0.0133\omega_1' - 0.0107\omega_2' - 0.069v_c'$

$0.0133\omega_1' + 0.0107\omega_2' + 0.069v_c' = 50.5$

The system of equations we have now obtained is as follows:

$0.062\,\omega_1' + 0.027\omega_2' + 0.146v_c' = 213.2$

$0.0133\,\omega_1' + 0.0107\,\omega_2' + 0.069v_c' = 50.5$

$-.42\,\omega_1' - 0.38\,\omega_2' + v_c' = 1959$

Solve using Kramer's Rule

$$v_c' = \frac{\begin{vmatrix} 0.062 & 0.027 & 213.2 \\ 0.0133 & 0.0107 & 50.5 \\ -0.42 & -0.38 & 1959 \end{vmatrix}}{\begin{vmatrix} 0.062 & 0.027 & 0.146 \\ 0.0133 & 0.0107 & 0.069 \\ -0.42 & -0.38 & 1.00 \end{vmatrix}} = 1026 \text{ ft/min}$$

Test object velocity after impact = 1026 ft/ min.

TEST #4

Test blade: Pivoted blade
Test object: 3/4" diameter steel cylinder
Given Parameters:
m_C = 42.5 gms = 0.094 lbs
v_C = 75 ft/min
$\omega = \omega_1$ = angular velocity of link 1 before impact
$\omega = \omega_2$ = angular velocity of link 2 before impact
$\omega_1 = \omega_2$ = 2595 rad/min, before impact
ℓ = length of link 1 = 5"
ℓ = length of link 2 = 5"
2ℓ = length of blade assembly = 10"
m_1 = mass of link 1, assumed to be half that of the blade assembly=0.187
m_2 = mass of link 2, assumed to be half that of the blade assembly=0.187
R = distanct of point of impact from fixed point = 9.5 inches
Calculated Parameters:
ω_1' = absolute angular velocity of link 1 after impact
ω_2' = angular velocity of link 2 with respect to pivot after impact
$\omega_1' + \omega_2'$ = absolute angular vlocity of link 2 after impact
I_1 = Moment of inertia of link 1 about center of mass
I_2 = Moment of inertia of link 2 about center of mass
v_1 = velocity of center of mass of link 1 before impact
v_1' = velocity of center of mass of link 1 after impact
v_2 = velocity of center of mass of link 2 before impact
v_2' = velocity of center of mass of link 2 after impact
$I_1 = m_1(\ell^2/12) = (0.374/2) \times (1/12) \times (5/12)^2 = 0.0026$
$I_2 = I_1$, as the dimensions of the two links are equal
$v_1' = (\ell/2)\omega_1' = (2.5/12)\omega_1' = 0.208\omega_1'$
$v_2' = \ell\omega_1' = (\ell/2)\omega_2' = 0.416\omega_1' + 0.208\omega_2'$
$v_1 = (\ell/2)\omega = (2.5/12) \times 2595 = 540$ ft/min
$v_2 = (\ell + \ell/2)w = (7.5/12) \times 2595 = 1620$ ft/min
Using the principle of conservation of angular momentum about fixed point 0. (clockwise positive)
$-I_1\omega - I_2\omega - m_1v_1(\ell/2) - m_2v_2(\ell + \ell/2) + m_Cv_CR$
$-I_1\omega_1 - I_2(\omega_1' + \omega_2') - m_1v_1'(l/2) - m_2v_2'(l + l/2) - m_Cv_C'R$
LHS = -0.0026 x 2595 - 0.0026 x 2595 - 0.187 x 540 x (2.5/12)
- 0.187 x 1620 x (7.5/12) + 0.094 x 75 x (9.5/12) = -218.3
RHS = $-0.0026\omega_1' - 0.0026\omega_1' - 0.0026\,\omega_2' - 0.187 \times 0.208\omega_1' \times (2.5/12)$
$- 0.187 \times (0.416\omega_1' + 0.208\omega_2') \times (7.5/12) - 0094 \times (9.5/12)v_C'$
$218.3 = +0.062\omega_1' + 0.027\omega_2' + 0074v_C'$

from kinematics of the rotating rod assembly

$v_p' = \ell\omega_1' + (\ell/2)\omega_2' = (5/12)\omega_1' + (2.5/12)\omega_2' = 0.42\omega_1' + 0.38\omega_2'$

$$e = \frac{v_c' - v_p'}{v_p - v_c} = \frac{v_c' - 0.42\omega_1' - 0.38\omega_2'}{2054 - (-75)}$$

$e = 0.92$

$-0.42\omega_1' - 0.38\omega_2' + v_c' = 1959$

Conservation of angular momentum about the pivot point gives us the third equation

$-I_1\omega_1 - I_2\omega_2 + m_1v_1(\ell/2) - m_2v_2(\ell/2) + m_cv_c(R - \ell)$

$= -I_1\omega_1' - I_2(\omega_1' + \omega_2') + m_1v_1'(\ell/2) - m_2v_2'(\ell/2) + m_cv_c'(R - \ell)$

LHS = -0.0026 x 2595 - 0.0026 x 2595 + 0.187 x (2.5/12) x 540 - 0.187 x (2.5/12) x 1620 + 0.094 x 75 x (9.5-5/12) = -52.9

RHS = $-0.0026\omega_1' - 0.0026(\omega_1' + \omega_2') + (2.5/12) \times 0.187 \times (2.5/12)\omega_1' - (2.5/12) \times 0.187 \times (0.416\omega_1' + 0.208\omega_2') - 0.094 \times (4.5/12)v_c'$

$0.0133\omega_1' + 0.0107\omega_2' + 0.035v_c' = 52.9$

The system of equations we have now obtained is as follows:

$0.062\,\omega_1' + 0.027\omega_2' + 0.146v_c' = 218.3$

$0.0133\,\omega_1' + 0.0107\,\omega_2' + 0.069v_c' = 52.9$

$-.42\,\omega_1' - 0.38\,\omega_2' + v_c' = 1959$

Solve using Kramer's Rule

$$v_c' = \frac{\begin{vmatrix} 0.062 & 0.027 & 218.3 \\ 0.0133 & 0.0107 & 52.9 \\ -0.42 & -0.38 & 1959 \end{vmatrix}}{\begin{vmatrix} 0.062 & 0.027 & 0.074 \\ 0.0133 & 0.0107 & 0.035 \\ -0.42 & -0.38 & 1.00 \end{vmatrix}} = 1621 \text{ ft/min}$$

Test object velocity after impact = 1621 ft/ min.

APPENDIX D

EFFECT OF INCLUDING THE MOTOR INERTIA IN THE CALCULATION OF BLADE INERTIA

APPENDIX D: EFFECT OF INCLUDING THE MOTOR INERTIA IN THE CALCULATION OF BLADE INERTIA

1. Inertia of the blade assembly

mass of blade assembly = 0.374 lbs

ℓ = length of the blade assembly of the mower model = 20"

I = inertia of the assembly about its center of mass

$I = m\ell^2 / 12$

$I = (0.374/12) \times (20/12) \times (20/12) = 0.0866$ lb-ft^2

2. Inertia of the motor (rotor + shaft)

The measured radius of the rotor shaft (R) is 1", and is cylindrical in shape and made of aluminum.

2a. mass of the rotor assembly = 0.25 lbs

motor inertia = $1/2 \times m \times R^2 = 1/2 \times 0.25 \times (1/12)^2 = 0.000868$ lb-ft^2

motor inertia = 0.0000868 / 0.0866 = 1% of the blade inertia

2b. mass of the rotor assembly = 0.50 lbs

motor inertia = $1/2 \times m \times R^2 = 1/2 \times 0.50 \times (1/12)^2 = 0.001736$ lb-ft^2

motor inertia = 0.001736 / 0.0866 = 2% of the blade inertia

2c. mass of the rotor assembly = 0.75 lbs

motor inertia = $1/2 \times m \times R^2 = 1/2 \times 0.75 \times (1/12)^2 = 0.002604$ lb-ft^2

motor inertia = 0.002604 / 0.0866 = 3% of the blade inertia

Thus the motor inertia is observed to be small when compared to the blade inertia and is therefore neglected in the blade inertia calculation.

APPENDIX E

NON DIMENSIONAL FORM OF EQUATIONS (2.16) AND (2.18)

APPENDIX E: NON DIMENSIONAL FORM OF EQUATIONS (2.16) AND (2.18)

The Eq 2.16 and 2.18 given below, will be expressed in non dimensional form in terms of the dimensionless parameters π_1, π_2, π_3 and π_4, each of which is defined below.

$\pi_1 = m_b/m$
$\pi_2 = r/\ell$

$$\pi_3 = \frac{v_b - v_b'}{v_c' - v_c}$$

π_4 = coefficient of restitution = e

From Chapter 2,

$$-I\omega - (m_b v_{cm})\,\ell/2 + (m_c v_c)\,r = -I\omega' - (m_b v_{cm}')\,\ell/2 - (m_c v_c') \qquad (2.16)$$

$$e = \frac{|v_p' - v_c'|}{|v_p - v_c|} \qquad (2.18)$$

From Chapter 2, under Case I - Straight Blade,
$v_b = (\ell/2)\omega$ and $v_b' = (\ell/2)\omega'$
or $\omega = 2v_b/\ell$ and $\omega' = 2v_b'/\ell$
Substituting ω and ω' in Eq 2.16 and dividing the equation by $m_c r(v_c' - v_c)$ gives

$$-1/6(m_b/m_c)(\ell/r)\{(v_b - v_b')/(v_c' - vc)\} - 1/2(m_b/m_c)(\ell/r)\{(v_b - v_b')/(v_c' - v_c)\} + 1 = 0 \qquad (E.1)$$

Substituting the non dimensional π terms into Eq E.1 gives

$$-1/6(\pi_1)(1/\pi_2)(\pi_3) - 1/2(\pi_1)(1/\pi_2)(\pi_3) + 1 = 0 \qquad (E.2)$$

which reduces to

$$-2/3(\pi_1\pi_3)/\pi_2 + 1 = 0 \qquad (E.3)$$

or $$\pi_1\pi_3/\pi_2 = 3/2 \qquad (E.4)$$

Combining Eq 2.10 and π_4 gives

$$\pi_4 = e \qquad (E.5)$$