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16. Abstract

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Techniques for obtaining these properties for asphalt mixtures are evaluated and a preliminary repeated-load indirect tensile test method is proposed. Relationships between various engineering properties and optimum asphalt contents are evaluated in terms of their application to mixture design. This evaluation indicates that the optimum asphalt contents for various mixture properties are different, and it is recommended that this fact be recognized and considered during the design of the mixture.

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asphalt mixtures, repeated-load indirect tensile test, fatigue, elastic properties, resilient modulus of elasticity, resilient Poisson's ratio, fatigue life

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by

Guillermo Gonzalez
Thomas W. Kennedy
James N. Anagnos

Research Report Number 183-6

Tensile Characterization of Highway Pavement Materials

Research Project 3-9-72-183

conducted for

Texas
State Department of Highways and Public Transportation

in cooperation with
U. S. Department of Transportation
Federal Highway Administration

by the

CENTER FOR HIGHWAY RESEARCH

THE UNIVERSITY OF TEXAS AT AUSTIN

November 1975

PREFACE

This is the sixth in a series of reports dealing with the findings of a research project concerned with the tensile characterization of highway pavement materials for use in mixture and structural design. This report describes and discusses the basic elastic materials properties required as inputs in elastic layer structural analyses and possible test methods for obtaining these elastic properties. The proposed test method for estimating the repeated-load elastic properties, i.e., the resilient modulus of elasticity and Poisson's ratio, is the repeated-load indirect tensile test.

In addition, techniques for obtaining these properties are evaluated and a preliminary test method is proposed. Various relationships between properties and optimum asphalt contents are evaluated in terms of their application to mixture design.

The study was financed by the State Department of Highways and Public Transportation as a part of the Cooperative Highway Research Program. Special appreciation is extended to Messrs. Avery Smith, Gerald Peck, and James L. Brown of the State Department of Highways and Public Transportation, who provided technical liason for the project, and to Mr. Pat S. Hardeman, Mr. Victor N. Toth, and Dr. A. S. Adedimila for their assistance with the testing program.

Guillermo Gonzalez
Thomas W. Kennedy
James N. Anagnos

November 1975

LIST OF REPORTS

Report No. 183-1, "Tensile and Elastic Characteristics of Pavement Materials," by Bryant P. Marshall and Thomas W. Kennedy, summarizes the results of a study on the magnitude of the tensile and elastic properties of highway pavement materials and the variations associated with these properties which might be expected in an actual roadway.

Report No. 183-2, "Fatigue and Repeated-Load Elastic Characteristics of In-service Asphalt-Treated Materials," by Domingo Navarro and Thomas W. Kennedy, summarizes the results of a study of the fatigue response of highway pavement materials and the variation in fatigue life that might be expected in an actual roadway.

Report No. 183-3, "Cumulative Damage of Asphalt Materials under Repeated-Load Indirect Tension," by Calvin E. Cowher and Thomas W. Kennedy, summarizes the results of a study to evaluate the applicability of a linear damage rule, Miner's Hypothesis, to fatigue data obtained utilizing the repeated-load indirect tensile test.

Report No. 183-4, "Comparison of Fatigue Test Methods for Asphalt Materials," by Byron W. Porter and Thomas W. Kennedy, summarizes the results of a study comparing fatigue results of the dynamic indirect tensile test with the results from other commonly used tests and a study comparing creep and fatigue deformations.

Report No. 183-5, "Fatigue and Resilient Characteristics of Asphalt Mixtures by Repeated-Load Indirect Tensile Test," by Adedare S. Adedimila and Thomas W. Kennedy, summarizes the results of a study of the fatigue behavior and the effects of repeated tensile stresses on the resilient characteristics of asphalt mixtures utilizing the repeated-load indirect tensile test.

Report No. 183-6, "Evaluation of the Resilient Elastic Characteristics of Asphalt Mixtures Using the Indirect Tensile Test," by Guillermo Gonzalez, Thomas W. Kennedy, and James N. Anagnos, summarizes the results of a study to evaluate possible test methods for obtaining elastic properties of pavement materials, to recommend a test method and preliminary procedure, and to evaluate properties in terms of mixture design.

ABSTRACT

This report reviews the basic elastic materials properties required as inputs in elastic layer structural analyses and possible test methods for obtaining these elastic properties. The proposed test method for estimating the repeated-load elastic properties, i.e., the resilient modulus of elasticity and Poisson's ratio, is the repeated-load indirect tensile test.

Techniques for obtaining these properties for asphalt mixtures are evaluated and a preliminary repeated-load indirect tensile test method is proposed. Relationships between various engineering properties and optimum asphalt contents are evaluated in terms of their application to mixture design. This evaluation indicates that the optimum asphalt contents for various mixture properties are different, and it is recommended that this fact be recognized and considered during the design of the mixture.

KEY WORDS: asphalt mixtures, repeated-load indirect tensile test, fatigue, elastic properties, resilient modulus of elasticity, resilient Poisson's ratio, fatigue life.

SUMMARY

This report summarizes the findings of a study to develop a technique to estimate the resilient elastic characteristics of asphalt mixtures using the repeated-load indirect tensile test and to evaluate the resilient and static moduli of elasticity and their relationships with fatgiue life for purposes of mixture design. Laboratory prepared specimens of two asphalt mixtures containing either gravel or limestone and various percentages of asphalt cement (AC-10) were tested at either 50, 75, or 100° F. The fundamental elastic properties estimated were the instantaneous resilient modulus of elasticity, the instantaneous resilient Poisson's ratio, static modulus of elasticity, and static Poisson's ratio. In addition, possible relationships involving fatigue life, repeated-load properties, and static properties were also evaluated and comparisons were made between the elastic properties obtained using the indirect tensile tests with the elastic properties obtained using other test methods. Also, a test procedure to obtain these fundamental elastic properties using the repeated-load indirect tensile test was recommended.

The instantaneous modulus of elasticity decreased with increased testing temperature and increased number of load applications but was not affected by the magnitude of the applied stress. No correlations were found between the instantaneous resilient and the static moduli. The resilient moduli, however, were generally larger than the static moduli. The ratios between the resilient moduli and the mean static moduli of elasticity ranged from 0.9 to 10.7.

No definite relationships could be established between fatigue life and either the instantaneous resilient or the static moduli. Thus, it would appear that a repeated-load test must be conducted to estimate resilient moduli of elasticity. Thus, a procedure was established for conducting the test and estimating the modulus values. With respect to mixture design, it was found that the optimum asphalt contents for maximum density, tensile strength, moduli of elasticity, and fatigue life are different. This fact should be recognized and considered in mixture design.

IMPLEMENTATION STATEMENT

The results of this study are of a preliminary nature. A preliminary test procedure for conducting the repeated-load indirect tensile test is recommended. It is recognized that this procedure may need to be modified as additional experience is obtained. Nevertheless, it is felt that the State Department of Highways and Public Transportation should begin to utilize the test method for the evaluation of asphalt mixtures.

In conjunction with this use, it is felt that these mixtures should be designed to obtain a satisfactory value of the various engineering properties important to the performance of asphalt mixtures. Results of this study indicate that the optimum asphalt contents for various mixture properties are different. This fact should be recognized and considered in the design of the mixtures.

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CHAPTER 1. INTRODUCTION

The pavement design field is making increased use of elastic layer structural analyses techniques. One of the essential inputs required for these analyses is the elastic properties of the materials.

This report describes and discusses the basic elastic materials properties required as inputs in the elastic layer structural analyses and possible test methods for obtaining these elastic properties. The proposed test method for estimating the repeated-load elastic properties, i.e., the resilient modulus of elasticity and Poisson's ratio, is the repeated-load indirect tensile test. A series of reports from an earlier research project entitled "Evaluation of the Tensile Properties of Subbases for Use in New Rigid Pavement Design" cover the development of the theory and test equipment used to conduct the static and dynamic testing of stabilized materials. Reports from the current project entitled "Tensile Characterization of Highway Materials" are concerned with both the static, repeated-load, and fatigue characteristics of pavement materials and the application of these characteristics and the indirect tensile test to the design of pavement materials. This report extends the work previously reported by Adedimila and Kennedy (Ref 1).

Chapter 2 briefly describes and discusses currently available methods for obtaining moduli and Poisson's ratios for asphalt mixtures. Chapter 3 includes a summary of the test program, the analysis of the data, and the recommended test procedure, and Chapter 4 summarizes the findings and conclusions of the study. A more detailed description of the experimental program is contained in Ref 1.

CHAPTER 2. REVIEW OF METHODS OF ESTIMATING ELASTIC PROPERTIES

Structural analysis of pavements as elastic layered systems is increasingly becoming a part of working design practice. This is largely due to the ease with which such analyses can be conducted by readily available computer programs and the easy understandability of the results. Moreover, there is growing evidence that the results of these analyses can be directly related to observed field performance.

Examples of operational programs available for layer analysis of flexible pavements include a) BISTRO or BISAR, developed by Shell Oil Co., b) CHEVRON, developed by Chevron Co., c) FEPAVE II or FEPAVE IV, developed at the University of California, Berkeley.

The inputs of these fundamental structural analyses must come from laboratory and field evaluations. Consequently, the pavement designer must have realistic values for materials properties, traffic loads, and temperature before he can conduct the analysis.

Materials testing technology in the pavement field has, for good reason, been largely built on a comparative basis, using index-type tests. Such index testing is useful for within comparisons of materials but it is often inadequate for comparisons between materials, especially when non-conventional materials are being considered. In addition, index-type tests do not provide the fundamental materials properties needed for structural analysis.

These fundamental properties can be evaluated in a number of different ways, both in the field and in the laboratory. Because field testing is usually time consuming and not always practical, laboratory methods have received considerable emphasis. However, even though a fundamental property is being evaluated, different types of tests can give widely different results. It follows then that the predicted structural response of the pavement can also vary widely, depending upon the test results used.

BASIC MATERIALS PROPERTIES REQUIRED

The basic materials properties required as inputs for layer analysis of a pavement structure are

(1) modulus of each layer material and of the subgrade - for bituminous bound layers, the variation of modulus with temperature and rate of loading should be known - and

(2) Poisson's ratio of each layer material, i.e., the ratio of lateral displacement to vertical displacement of the material under the particular test conditions.

The magnitude of the wheel load and the tire pressure are also needed.

The determination of moduli and Poisson's values for the various materials can be accomplished by a wide variety of testing methods, as subsequently discussed in this chapter.

CRITERIA FOR MATERIALS TESTS

Satisfactory design of asphalt concrete pavements requires an understanding of the load-deformation and strength properties of the materials to be used. Strength represents a limiting condition. As such, it is not directly applicable to design, except with respect to thermal or shrinkage cracking (Ref 8), because pavements are not expected to fail under a single application of load. The load-deformation characteristics can, however, relate to a single application or to many applications of load. The type and extent of the testing program used to determine these characteristics relates to the following general criteria:

(1) ease of testing,

(2) reproducibility of test results,

(3) size of project and variability, and

(4) ability to estimate fundamental properties.

Ease of Testing

Ease of testing is one of the more important criteria to be applied to any proposed test method. An imperfect test method may be chosen because it is simple and can be conducted without costly equipment, extensive test time, or extensive training of personnel. Thus, a test which can be readily implemented and used in the field and by design agencies is desirable.

Simplicity and low cost, however, should not be the primary basis for selecting a given test or testing program. In comparison to the total cost of designing, constructing, and maintaining a pavement, the cost of the testing program usually is insignificant.

Reproducible Test Results

A second criterion related to the choice of test is a small error associated with testing. Variation associated with testing and the specimen should be minimized. A test method ideally should be able to reproduce test results for essentially ideal specimens. One measure of this reproducibility is the coefficient of variation for laboratory prepared and tested specimens of a given mixture. This coefficient represents the inherent variation of the mixture and specimen and the testing error.

Size of Project and Variability

The size and cost of the project and the inherent variability of the materials involved must be considered in establishing the type and extent of the materials testing program.

Materials variability must be quantified for meaningful design. It is obvious from even the most cursory evaluation of pavement performance and distress that variation is one of the most significant factors to be considered. If, for example, 10 percent of a pavement fails then the entire pavement has probably failed in terms of performance.

The concept of variability and its relationship to failure is illustrated in Fig 1 which shows the variations of tensile stress and tensile strength for a hypothetical pavement. The area of overlap represents a failure condition in which stress exceeds strength. If the variation in material characteristics increases, the probability of failure increases. Similar examples could be shown involving other properties or a combination of these properties.

Closely related to inherent variability is the question of the extent of testing. It is not practical to conduct an elaborate and extensive testing program on a small sample of material which is quite variable. Such

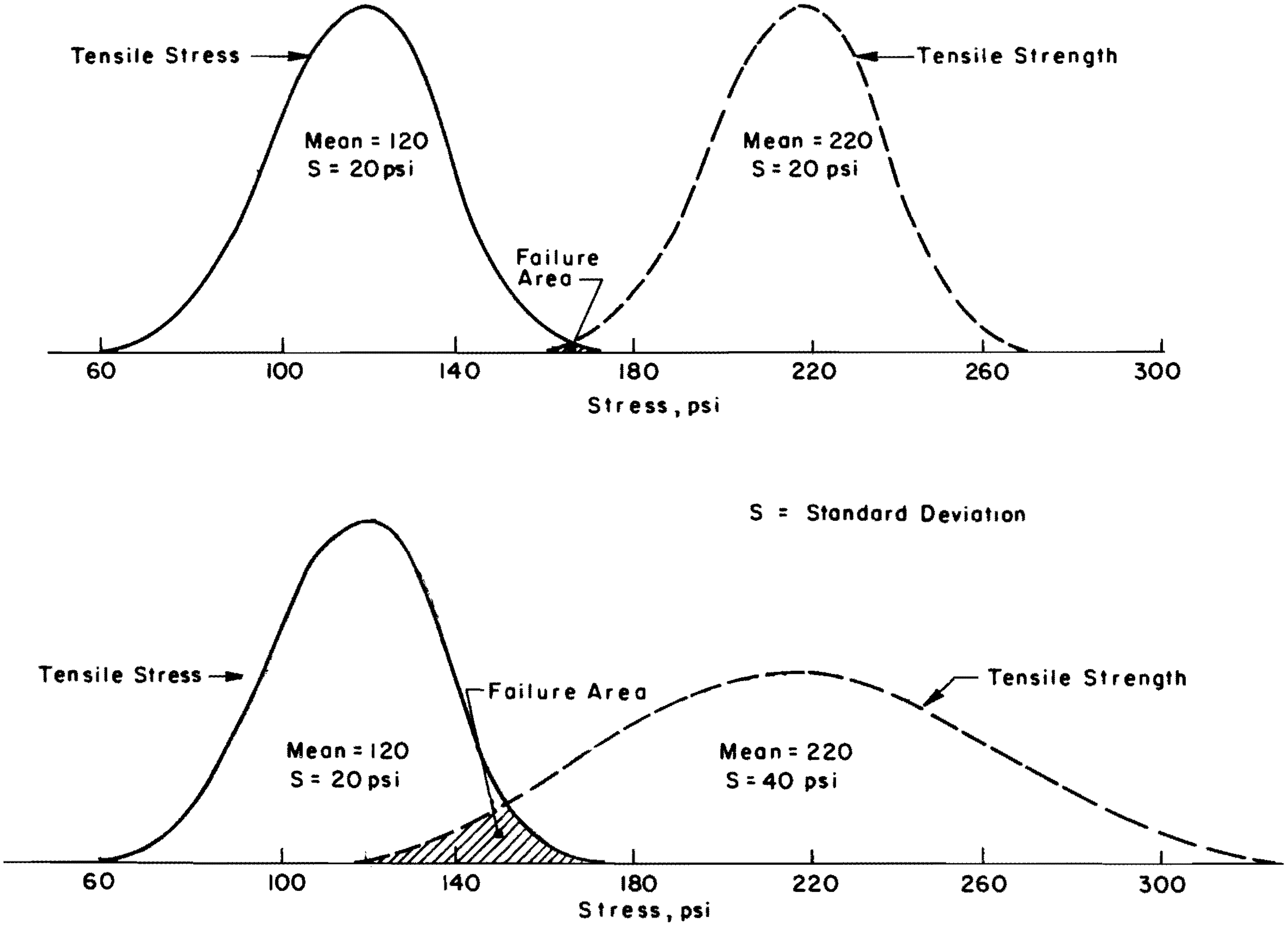


Fig 1. Graphical presentation of failure for two conditions with the same mean values but different variability.

a program would yield a great deal of information which would be meaningful to only a very small portion of the pavement. A very limited program would not provide much useful information, either.

A realistic approach would involve the determination of average values, variation, and significant changes in material properties. For example, where a new pavement is to be constructed, or an existing pavement overlayed, significant changes in subgrade soil support should be identified because of their relationship to required design thickness. Project size and cost are also important. As the size of a project increases, variability will also increase. At the same time as size or cost increases, the need for a more extensive testing program increases. Thus, the extent and nature of the testing program ultimately relate to the variability expected, the cost of the project, and the consequences of failure.

Fundamental Properties

The final criterion relates to the ability of the tests to measure the fundamental or basic properties previously mentioned. In terms of elastic design this means that modulus values and Poisson's ratio as derived from the load-deformation characteristics of the material need to be known. Empirical test results are only of value to an empirical design procedure.

Attempts at using empirical tests to estimate fundamental properties through correlations should be rejected unless better information is not available or cannot be obtained. Such correlations at best are usually very approximate.

CLASSIFICATION AND DESCRIPTION OF MATERIAL CHARACTERIZATION TECHNIQUES

Although the problem of materials characterization has been with us for many years and a great deal of work has been done, it would appear that there is very little agreement with regard to type of test and test procedures required. According to Deacon (Ref 6), this lack of agreement is explainable by the following reasons:

(1) The variety of materials encountered by the pavement designer is unlimited because of the nature and the manner in which they are manufactured.

(2) The nature of the pavement structure in which these materials are used depends upon the intended function of the pavement.

(3) During the service life of a pavement, material properties are altered by such factors as thixotropy, aging, curing, densification, change of moisture content, and temperature.

(4) The response of pavement materials to loading is extremely complex and is characterized by nonlinear, inelastic, rate-dependent, anisotropic behavior which is sensitive to temperature and moisture.

(5) Solutions to the pertinent boundary value problems have been essentially nonexistent until recently.

(6) The approach to the problem has been piecemeal at best and has involved many different researchers from many different agencies, each striving for an optimal solution to a singular problem of limited scope and sometimes prejudiced intent.

It could be added that for the past 50 years or more pavement design agencies have always pressed for an immediate answer to their needs and problems. Long-term, well thought out, sequential efforts have usually been rejected because of the time and expense involved.

Nevertheless, a wide variety of test methods and procedures have been developed over the years, many of which are still being used today. These test methods can be classified as field or laboratory tests, empirical or fundamental, or according to the mode of test, e.g., tension, compression, shear, flexural, torsion, or some method relating empirical results to other more fundamental test parameters.

Empirical tests generally yield index properties related to fundamental materials properties such as strength and stiffness modulus. However, these index properties only have meaning on a comparative basis, i.e., previous tests on similar materials, or in terms of correlations with fundamental properties. An example of a widely used index test is the California Bearing Ratio (CBR) test.

Tests which yield fundamental properties directly are much more useful and a strong emphasis to use them has been apparent in recent years. Examples of such tests include the indirect tensile, triaxial, plate load, Dynaflect, and flexural tests.

Field tests, e.g., plate load, Dynaflect, Benklemen beam, and vibrating tests, basically can only be used to evaluate an existing condition. Thus, they can be used to evaluate the subgrade for a proposed pavement prior to design, an existing pavement to determine its basic structural condition, or design of an overlay. Results generally must be considered in terms of the conditions which exist at the time that the tests are conducted.

Designers are therefore required to rely upon information obtained from laboratory tests, the more applicable and useful of which are

(1) dynamic, complex modulus,
(2) resilient modulus,
(3) flexural stiffness, and
(4) dynamic or static indirect tension.

The following discussion briefly describes and summarizes the basic characteristics of the above tests.

LABORATORY TESTS FOR STIFFNESS OR MODULUS OF ELASTICITY

Two of the tests, dynamic complex modulus and modulus of resilient deformation, are very similar. Papazian (Ref 23) presented the concept and definitions concerning the complex modulus and Seed, Chan, and Lee (Ref 25) introduced the concept of resilient modulus. In both tests, vertical stresses are applied to a specimen and deformations are measured. The modulus value is calculated as the ratio of stress to recoverable, or resilient, strain under repeated loading conditions.

The major differences are that

(1) a confining pressure can be applied in the resilient modulus test,
(2) inelastic, as well as elastic, behavior can be measured in the complex modulus test, and
(3) the resilient modulus test has been used primarily for soils and granular materials.

A third stiffness test has been described by Deacon (Ref 5) in which a beam is subjected to repeated flexure. A flexural stiffness modulus is calculated from the center point deflections under load rather than the recoverable deflection.

In addition, the indirect tensile test has been used, for both repeated-load and single statically applied load, to obtain estimates of modulus and other load-deformation characteristics.

Complex Modulus

Generally, sinusoidal vertical loads are applied to 4-inch-diameter by 8-inch-high specimens, and strains are measured. Typical load-strain-time relationships are shown in Fig 2. Values of the complex modulus and phase lag are calculated using the following equations:

$$E^* = \frac{\sigma}{\epsilon}$$

$$\text{and } \phi = \frac{t_i (360)}{t_p}$$

where

E^* = absolute value of the complex modulus, psi,

ϕ = phase lag, degrees,

σ = amplitude of the sinusoidal vertical stress, psi,

ϵ = amplitude of resulting vertical strain,

t_i = time lag between a cycle of stress and the resulting cycle of stress, seconds, and

t_p = time for a cycle of stress, seconds.

Typical values of the complex modulus and Poisson's ratio for asphalt mixtures tested at 40, 70, and 100°F are shown in Table 1. As shown in these tables, values are dependent on temperature, frequency of loading and stress magnitude. For asphalt concrete and asphalt base, values of modulus and Poisson's ratio ranged from 3.10 to 9.82 x 10^5 psi and 0.28 to 0.49, respectively. Figure 3 illustrates typical relationships between complex modulus and temperature for various loading frequencies.

Resilient Modulus

Recommended procedures for the resilient modulus test for subgrade soils, untreated granular bases, and subbases are described in Refs 18, 25, and 26.

A haversine wave load is applied for 0.1 second and removed for 0.4 second at a frequency of 120 loads per minute, and the resulting axial deformations are recorded. The confining pressures vary from 0 to 50 psi, depending on the type of material. A typical load-deformation-time relationship for a test is shown in Fig 4. Normally for granular base and subbase materials specimens are 6 inches in diameter and 12 inches high while for soils a 4-inch-diameter and 8-inch-high specimen is used.

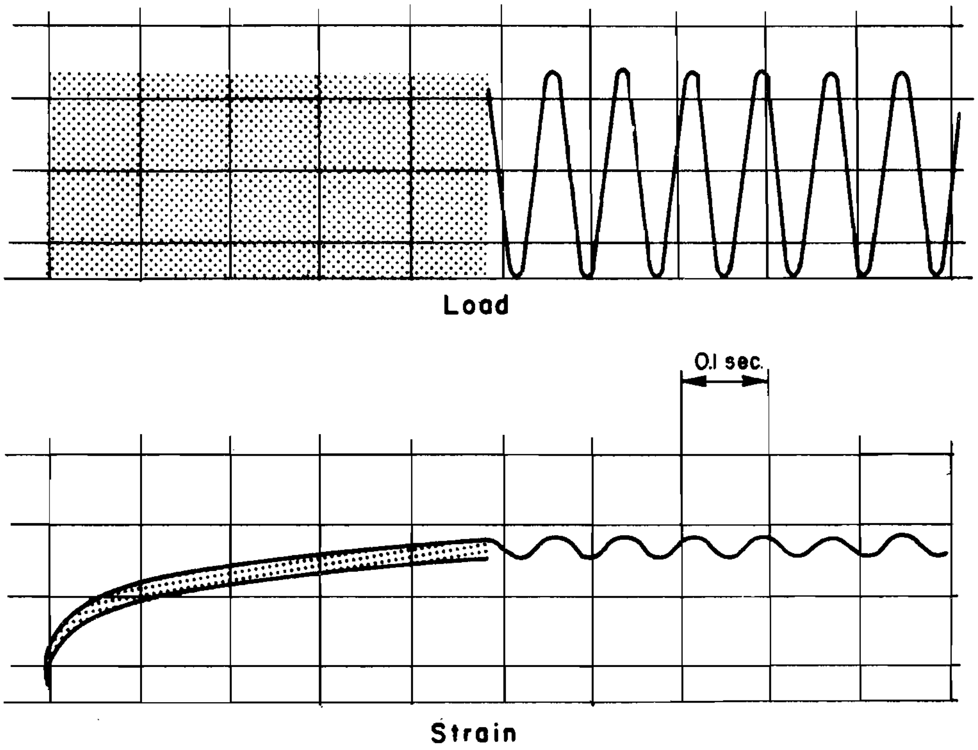


Fig 2. Typical load-strain-time relationships from a dynamic complex modulus test (Ref 26).

TABLE 1a. MEAN VALUES OF $|E^*|^1$ AVERAGED OVER FREQUENCY (REF 26)

Pavement Course	Temperature, °F	Vertical Stress 17.5 psi	35 psi	70 psi
Asphalt	40	18.51	19.73	20.04
Concrete	70	6.62	7.09	7.36
Surface	100	1.63	1.68	1.87
Asphalt	40	22.74	22.60	22.56
Concrete	70	7.08	7.65	8.07
Base	100	1.45	1.79	1.45

[1] in psi X 10^5

TABLE 1b. POISSON'S RATIO DETERMINED BY DYNAMIC COMPLEX MODULUS TEST PROCEDURES (REF 15)

Pavement Course	Temperature, °F	Poisson's Ratio Loading Frequency 1 cps	4 cps	14 cps
Asphalt	40		0.282	
Concrete	70	0.492	0.494	0.375
Surface	100		0.374	
Asphalt	40		0.362	
Concrete	70	0.470	0.445	0.366
Base	100		0.433	
High stability	40		0.374	
Sand asphalt	70	0.374	0.493	0.386
Base	100		0.386	
Low stability	40		0.383	
Sand asphalt	70	0.436	0.440	0.440
Base	100		0.324	

TABLE 1c. |E*| AND ϕ FOR PAVEMENT CORES (REF 15)

		Loading Frequency					
		1 cps		4 cps		14 cps	
Pavement Course	Temperature, °F	|E*|, 10^5 psi	ϕ, deg	|E*|, 10^5 psi	ϕ, deg	|E*|, 10^5 psi	ϕ, deg
Asphalt	40	11.35	16	13.87	16	16.03	21
Concrete	70	3.10	33	5.41	24	6.47	28
Base	100	0.60	30	1.07	32	1.52	43
High Stability	40	8.76	18	11.05	13	13.40	22
Sand	70	1.37	39	2.29	36	4.24	44
Asphalt Base	100	-	-	0.86	31	1.10	43
Low Stability	40	7.78	19	9.84	16	12.13	21
Sand	70	1.61	37	2.58	34	4.38	37
Asphalt Base	100	0.27	32	0.72	29	0.78	53

TABLE 1d. |E*| AND ϕ FOR AASHO ROAD TEST BITUMINOUS TREATED GRAVEL BASE COURSE (REF 15)

	Loading Frequency					
	1 cps		4 cps		14 cps	
Temperature, °F	|E*|, 10^5 psi	ϕ, deg	|E*|, 10^5 psi	ϕ, deg	|E*|, 10^5 psi	ϕ, deg
40	15.82	17	21.29	16	23.32	17
70	6.50	26	8.61	21	9.82	25
100	1.18	34	2.18	33	2.94	38

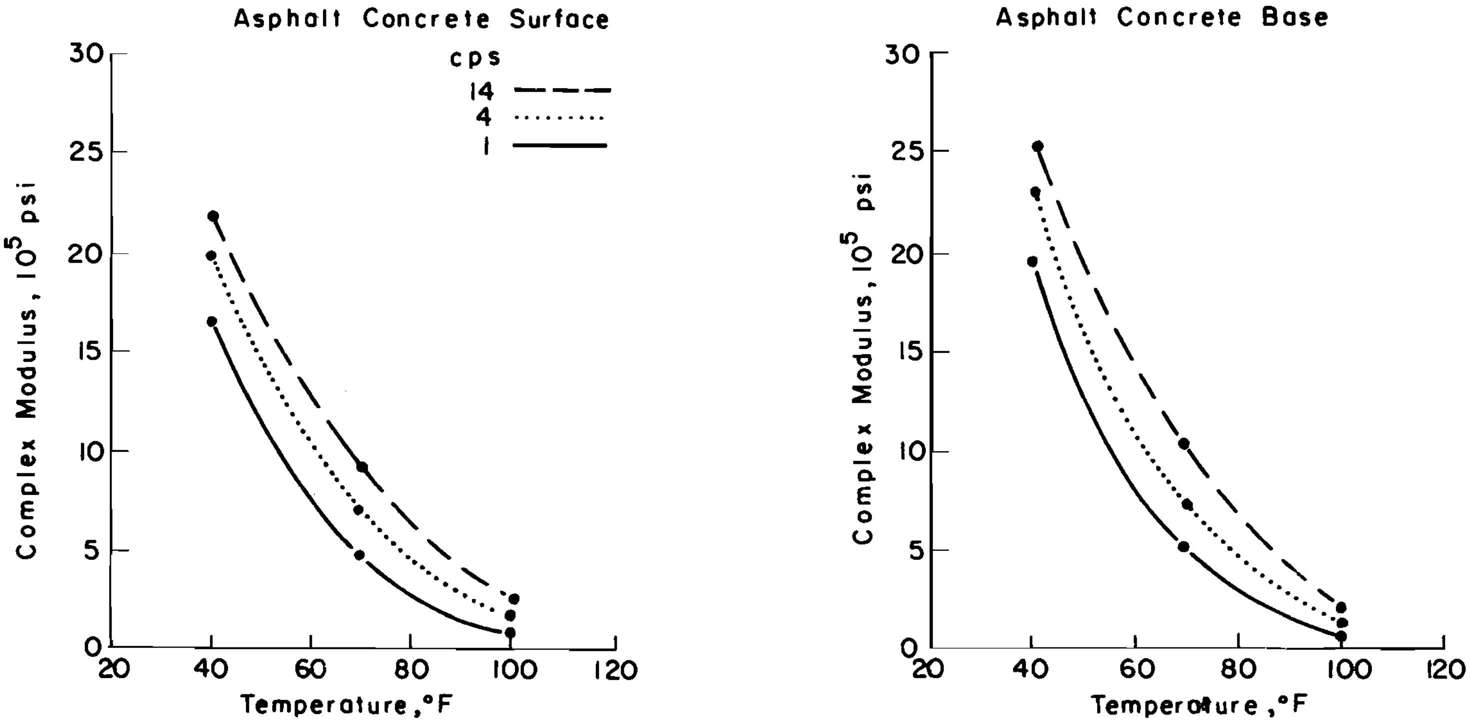
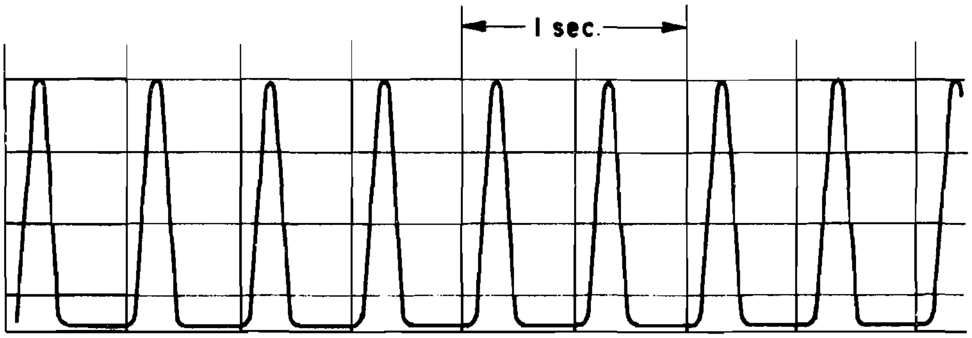
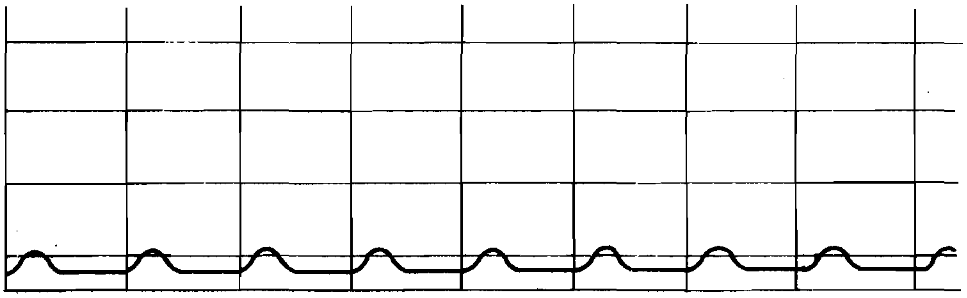


Fig 3. Relationships between complex modulus and temperature for various loading frequencies (Ref 15).



Load



Deformation

Fig 4. Typical load-deformation-time relationships from a modulus of resilient deformation test (Ref 26).

The modulus of resilience M_R is calculated from the following equation:

$$M_R = \frac{\sigma_d}{\epsilon_r}$$

where

M_R = modulus of resilient deformation, psi
σ_d = repeated deviator stress (stress difference), and
ϵ_r = repeated recoverable strain.

Values of M_R are determined after a specified number of repeated applications of the load, after which the specimen generally exhibits essentially constant recoverable strain, i.e., after conditioning. Generally this test is conducted on soils and granular materials; however, there is no reason that the test could not be used for asphalt mixtures.

Kallas and Riley (Ref 15) reported typical values of resilient modulus for gravel base course ranging from 11.9×10^3 psi to 64.6×10^3 psi, values for gravel subbase course ranging from 11.9×10^3 psi to 58.5×10^3 psi, and values for subgrade soil ranging from 3.7×10^3 psi to 33.5×10^3 psi.

For cohesive materials the resilient modulus is affected by the number of stress applications, the magnitude of repeated stress, thixotropy, the method of compaction, the compaction density and water content, and increase in water content after compaction.

Flexural Stiffness Modulus

Repeated loads are applied at the third points of a beam specimen in the form of a haversine. The duration of the load is 0.1 second, which is followed by a 0.4-second rest period, resulting in a frequency of 120 loads per minute. This produces an essentially constant bending moment over the center point of the beam. Normally, a load is applied in the opposite direction, forcing the beam to return to its original position and maintaining it in that position during the rest period. The deflection under the load is measured at the center of the beam. A schematic of the apparatus and a typical load pulse are shown in Fig 5.

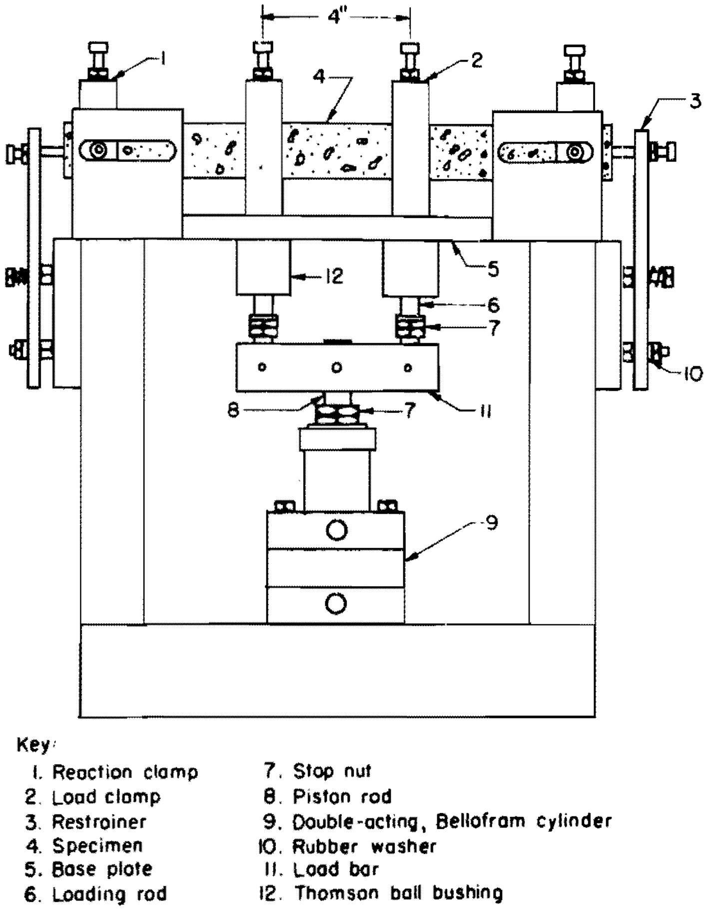


Fig 5(a). Repeated flexure apparatus (Ref 26).

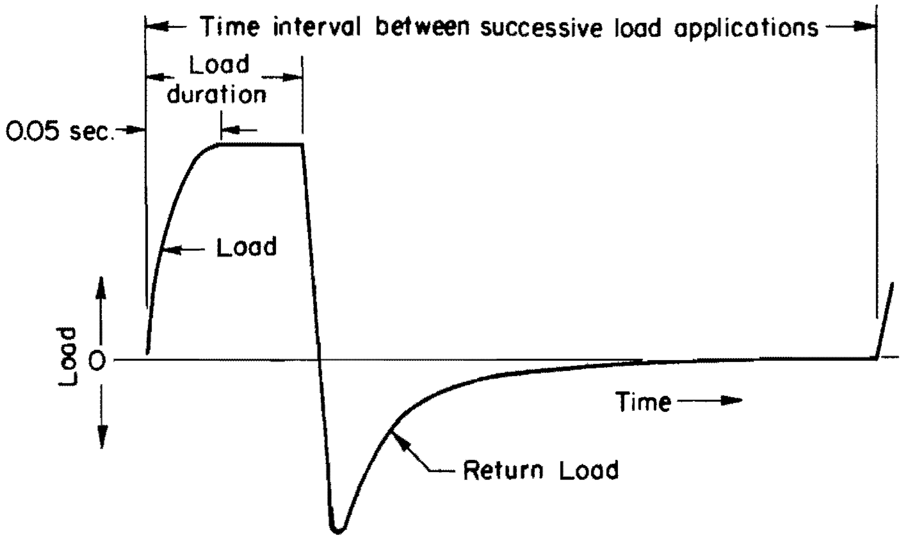


Fig 5(b). Load pulse for constant stress flexure apparatus (Ref 5).

The stress and strain at the outer fibers and the stiffness moduli after about 200 load applications are calculated from the following equations:

$$\sigma = \frac{3aP}{bh^2}$$

$$\epsilon = \frac{12hd}{3\ell^2-4a^2}$$

$$E_s = \frac{Pa(3\ell^2-4a^2)}{48Id}$$

where

σ = stress in the outer fibers, psi,
ϵ = strain in the outer fibers,
E_s = flexural stiffness modulus, psi,
a = $(\ell - 4)/2$, inches,
P = dynamic load applied at third points, pounds,
b = specimen width, inches,
h = specimen depth, inches,
ℓ = reaction span length, inches,
I = moment of inertia of specimen, inches^4, and
d = dynamic deflection of beam at the center, inches.

Using the repeated flexure apparatus, Deacon (Ref 5) reported typical values for the flexural stiffness modulus ranging from 237 × 10^3 psi to 263 × 10^3 psi (Table 2a).

Finn (Ref 7) obtained values of flexural stiffness for specimens of weathered and unweathered asphalt cement at test temperatures of 68°F ranging from 200 × 10^3 psi to 624 × 10^3 psi. The flexural stiffness modulus for field cores ranged from 440 × 10^3 psi to 712 × 10^3 psi at test temperatures of 40°F and from 119 × 10^3 psi to 179 × 10^3 psi at test temperatures of 68°F (Tables 2b and 2c).

Monismith et al (Ref 19), using the flexural test apparatus, obtained values of flexural stiffness moduli that ranged from 440 × 10^3 psi to 712 × 10^3 psi at test temperatures of 40°F and 147 × 10^3 psi to 152 × 10^3 psi at test temperatures of 68°F (Table 2d).

TABLE 2a. COMPARISON OF MEAN FRACTURE LIVES - RANDOM VS REPEATED-BLOCK LOADING (Ref 5)

	Random		Repeated-Block	
Test[a]	Mean Fracture Life	Mean Initial Stiffness Modulus, 10^3 psi	Mean Fracture Life	Mean Initial Stiffness Modulus, 10^3 psi
A	26,500	263	15,800	245
B	13,500	250	9,600	241
C	8,600	237	11,200	258

[a] Approximate applied percentages:
Test A: 10% of 128.5 psi, 30% of 113.5 psi, 60% of 98.5 psi
Test B: 25% of 128.5 psi, 50% of 113.5 psi, 25% of 98.5 psi
Test C: 60% of 128.5 psi, 30% of 113.5 psi, 10% of 98.5 psi

TABLE 2b. VARIATION IN FATIGUE LIFE, INITIAL STRAIN, AND FLEXURAL STIFFNESS OF WEATHERED AND UNWEATHERED ASPHALT CEMENT AT 68°F (Ref 7)

Specimen Description	Stress, psi	No. of Specimens	Fatigue Life			Initial Strain, microunits			Stiffness		
			Mean, cycles	Std. Dev., cycles	Coeff. of Var., %	Mean	Std. Dev.	Coeff. of Var., %	Mean, psi	Std. Dev., psi	Coeff. of Var., %
Unweathered asphalt	200	4	6,474	4,909	75.8	972	200	20.6	213,000	44,600	20.9
	150	4	17,997	11,438	63.6	717	88	12.3	200,000	42,800	21.4
	90	4	363,025	194,385	53.5	431	77	17.8	211,000	39,400	18.6
Weathered asphalt	375	4	1,665	1,108	66.5	662	109	16.4	582,000	112,000	19.2
	280	4	16,172	9,238	57.1	464	106	22.8	624,000	131,000	20.9
	120	3[1]	252,926	142,348	56.3	253	24	9.5	476,000	34,000	7.2

[1] Test of fourth specimen discontinued after approximately 70,000 cycles of loading.

TABLE 2c. FLEXURAL STIFFNESS MEASUREMENTS ON FIELD SAMPLES OF ASPHALTIC CONCRETE USING PULSE LOADING METHOD (Ref 7)

		Measured Stiffness, 10^5 psi					
		68° F			40° F		
Sample Group	No. of Spec-imens	Mean	Std. Dev.	CV, %	Mean	Std. Dev.	CV, %
Specimens From Surface Course							
1	19	1.79	0.42	23.5	6.80	1.53	22.5
2	20	1.65	0.39	23.6	7.03	1.91	27.2
3	20	1.52	0.41	26.9	7.12	1.41	19.8
4	19	1.34	0.37	27.6	5.90	1.11	18.8
Lab compacted	26	1.29	0.22	17.0	5.76	0.74	12.8
Specimens From Base Course							
1	12	1.57	0.42	26.6	5.95	1.54	25.9
2	12	1.39	0.26	18.7	5.66	3.14	55.5
3	8	1.47	0.41	27.9	4.40	0.90	20.4
4	10	1.42	0.42	29.6	4.96	1.22	24.6
Lab compacted	29	1.19	0.19	16.0	5.31	1.50	28.2

TABLE 2d. FLEXURAL STIFFNESS MEASUREMENTS ON PAVEMENT SAMPLES FROM STA 308 + 10 (Ref 19)

		Measured Stiffness, 10^5 psi					
		68° F			40° F		
Location	No. of Specimens	Mean	Standard Deviation	CV, %	Mean	Standard Deviation	CV, %
Surface course	20	1.52	0.41	27.0	7.12	1.41	19.8
Base course	8	1.47	0.41	27.9	4.40	0.90	20.4

Indirect Tension

The indirect tensile test is conducted by loading a cylindrical specimen with a single or repeated compressive load which acts parallel to and along the vertical diametral plane (Fig 6). This loading configuration develops a relatively uniform tensile stress perpendicular to the direction of the applied load and along the vertical diametral plane, which ultimately causes the specimen to fail by splitting along the vertical diameter. The development of stresses within the cylindrical specimen subjected to load is reported by Kennedy and Hudson (Refs 14 and 16).

Most of the work in this area has been done at The University of Texas at Austin as part of two research projects entitled "Evaluation of Tensile Properties of Subbases for Use in New Rigid Pavement Design" and "Tensile Characterization of Highway Materials." The series of reports from the initial project cover the static and fatigue testing of stabilized materials. Reports from the second project are concerned with both the static and the repeated-load characteristics of pavement materials.

A 0.5-inch curved loading strip is used at the interface of the loading head and specimen because the stress distributions are not altered significantly and because calculations of modulus of elasticity and Poisson's ratio are facilitated by maintaining a constant loading width rather than having a constantly changing loading width, which would occur with a flat loading strip.

The development of equations, based on work reported by Hondros (Ref 12), that permit the computation of the tensile strain, the modulus of elasticity, and Poisson's ratio are reported in Refs 2, 9, and 10. The equations are as follows:

$$S_T = \frac{2P_{Fail}}{\pi a h} \left(\sin 2\alpha - \frac{a}{2R}\right)$$

$$E = \frac{P}{X} \left[\int_{-r}^{+r} \frac{\sigma_{rx}}{P} - \nu \int_{-r}^{+r} \frac{\sigma_{\theta x}}{P} \right]$$

$$\nu = \frac{\left[\int_{-r}^{+r} \sigma_{ry} + R \int_{-r}^{+r} \sigma_{rx} \right]}{\left[R' \int_{-r}^{+r} \sigma_{\theta x} + \int_{-r}^{+r} \sigma_{\theta y} \right]}$$

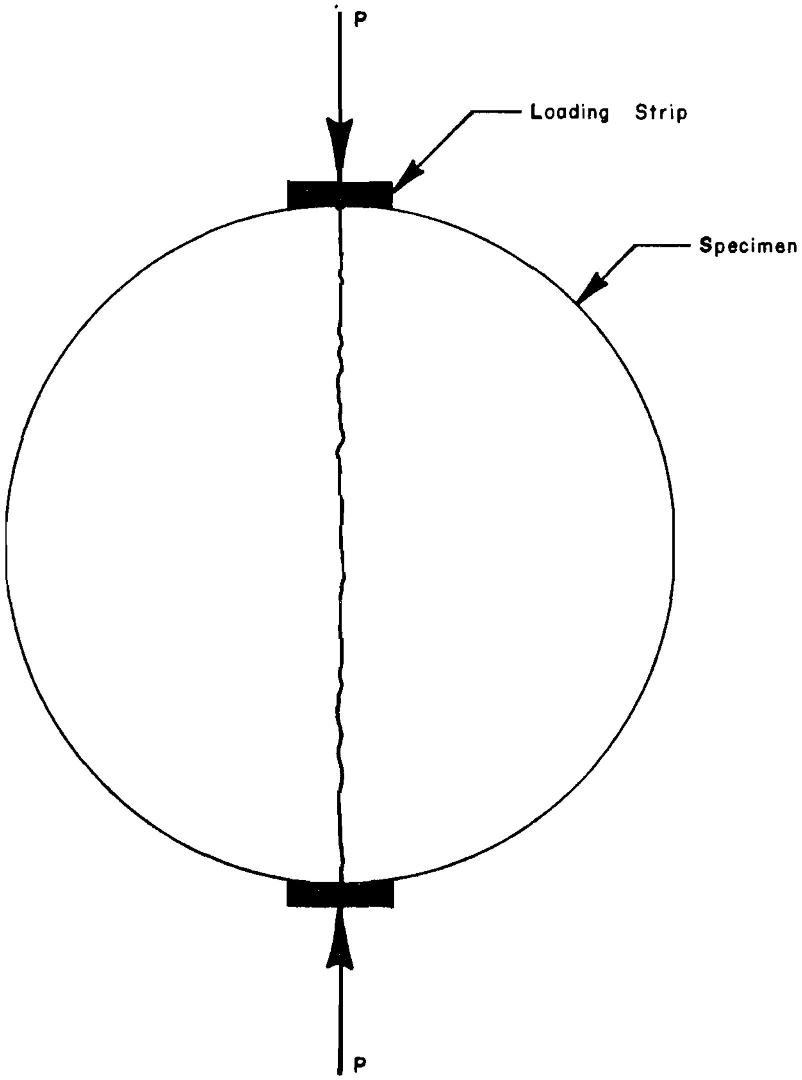


Fig 6. The static or repeated-load indirect tensile test.

$$\varepsilon_T = \frac{X}{\ell} \frac{\left[\int_{-\frac{\ell}{2}}^{+\frac{\ell}{2}} \frac{\sigma_{rx}}{P} - \nu \int_{-\frac{\ell}{2}}^{+\frac{\ell}{2}} \frac{\sigma_{\theta x}}{P} \right]}{\left[\int_{-r}^{+r} \frac{\sigma_{rx}}{P} - \nu \int_{-r}^{+r} \frac{\sigma_{\theta x}}{P} \right]}$$

where

S_T = tensile strength,

E = modulus of elasticity,

ν = Poisson's ratio,

ε_T = tensile strain at any given load,

P_{Fail} = applied load at failure,

P = applied load,

a = width of loading strip,

h = height of specimen,

α = angle (radians) subtended by one-half the width of the loading strip,

$\frac{P}{X}$ = slope of line of best fit between load P and total horizontal deformation X, for loads up to 50 percent of the maximum load,

X = total horizontal deformation at any given load,

T = total vertical deformation at any given load,

ℓ = length over which strain is estimated,

R = radius of specimen,

$R' = \frac{Y}{X}$ = slope of the line of best fit between vertical deformations Y and the corresponding horizontal deformation X,

$\int \frac{\sigma_{rx}}{P}$, $\int \frac{\sigma_{\theta x}}{P}$, $\int \frac{\sigma_{ry}}{P}$, and $\int \frac{\sigma_{\theta y}}{P}$ = integrals of unit stresses,

$\int_{-r}^{+r} \sigma_{ry}$ and $\int_{-r}^{+r} \sigma_{rx}$ = integrals of radial stresses in the y and x directions, respectively, and

$\int_{-r}^{+r} \sigma_{\theta x}$ and $\int_{-r}^{+r} \sigma_{\theta y}$ = integrals of tangential stresses in the x and y-directions, respectively.

These equations require that the integrations be carried out using a computer and a computer program, MODLAS. However, for a given diameter and width of loading strip these equations can be simplified and used without the aid of a computer. Table 3 presents these equations for 4 and 6-inch-diameter specimens and a 0.5-inch loading strip (Ref 2).

In the static test method a cylindrical specimen is loaded at a relatively slow rate (generally 2 inches per minute). The testing temperature normally has been 75°F, although other temperatures can be used to characterize behavior if desired. A special transducer is used to measure the total horizontal (tensile) deformation while the vertical deformations are measured using an LVDT.

In the dynamic, or repeated-load, indirect tensile test method the same equations are used except that it is not necessary to characterize the entire load-deformation relationships. A resilient indirect tensile modulus can be obtained by measuring the recoverable vertical and horizontal deformations and assuming a linear relationship between load and deformation. In addition, this method also provides an estimate of permanent deformation which occurs under repeated loads. Any level of stress less than the static strength can be used and applied generally in the form of a haversine. A typical load pulse and deformation-time relationship is shown in Figs 7 and 8.

Work using the repeated-load indirect tensile test has been conducted by Kennedy et al (Refs 1, 4, 20, 21, 22, and 24) at The University of Texas at Austin and Schmidt at Chevron Oil Corporation, California. From repeated-load tests conducted at the Center for Highway Research, The University of Texas at Austin, on laboratory prepared specimens of blackbase (Ref 1), instantaneous resilient moduli of elasticity ranged from 631×10^3 psi to 1100×10^3 psi at 50°F, from 136×10^3 psi to 647×10^3 psi at 75°F, and from 121×10^3 psi to 258×10^3 psi at 100°F. For in-service blackbase and asphalt concrete cores tested at 75°F (Ref 22) moduli ranged from 221×10^3 psi to 615×10^3 psi. These values are comparable to the values reported for the complex modulus and flexural modulus.

CHOICE OF TEST METHOD

From an evaluation of these test methods, it is felt that the indirect tensile test is the best test method for use by operating agencies such as

TABLE 3. EQUATIONS FOR CALCULATION OF TENSILE PROPERTIES (REF 2)

Tensile Property	Diameter of Specimen	
	4 inches	6 inches
Tensile strength S_T, psi	$0.156 \frac{P_{Fail}}{h}$	$0.105 \frac{P_{Fail}}{h}$
Poisson's ratio ν	$\frac{0.0673DR - 0.8954}{-0.2494DR - 0.0156}$	$\frac{0.04524DR - 0.6804}{-0.16648DR - 0.00694}$
Modulus of elasticity E, psi	$\frac{S_H}{h}\left[0.9976\nu + 0.2692\right]$	$\frac{S_H}{h}\left[0.9990\nu + 0.2712\right]$
Total tensile strain ε_T, microunits	$X_T \left[\frac{0.1185\nu + 0.03896}{0.2494\nu + 0.0673}\right]$	$X_T \left[\frac{0.0529\nu + 0.0175}{0.1665\nu + 0.0452}\right]$

P_{Fail} = total load at failure (maximum load P_{max} or load at first inflection point), pounds,

h = height of specimen, inches,

X_T = total horizontal deformation, inches,

DR = deformation ratio $\frac{Y_T}{X_T}$ (the slope of line of best fit* between vertical deformation Y_T and the corresponding horizontal deformation X_T up to failure load P_{Fail}),

S_H = horizontal tangent modulus $\frac{P}{X_T}$ (the slope of the line of best fit* between load P and total horizontal deformation X_T for loads up to failure load).

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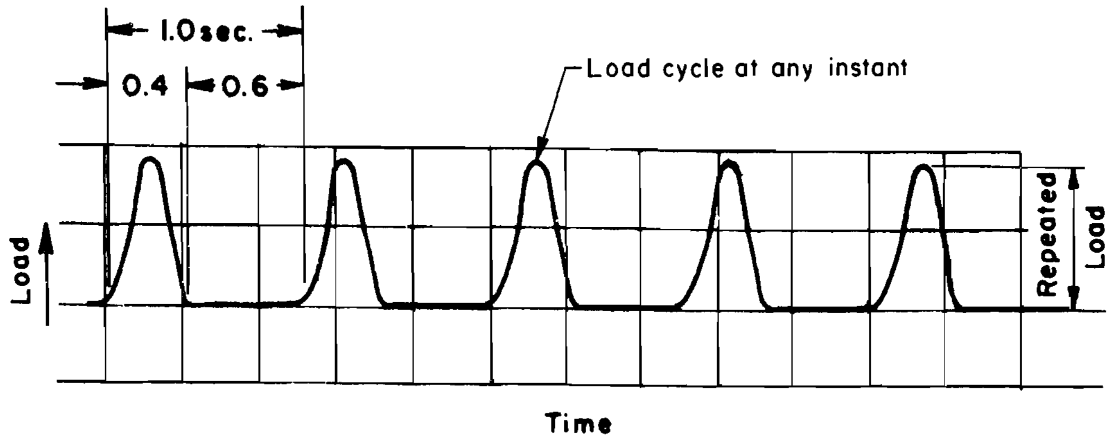
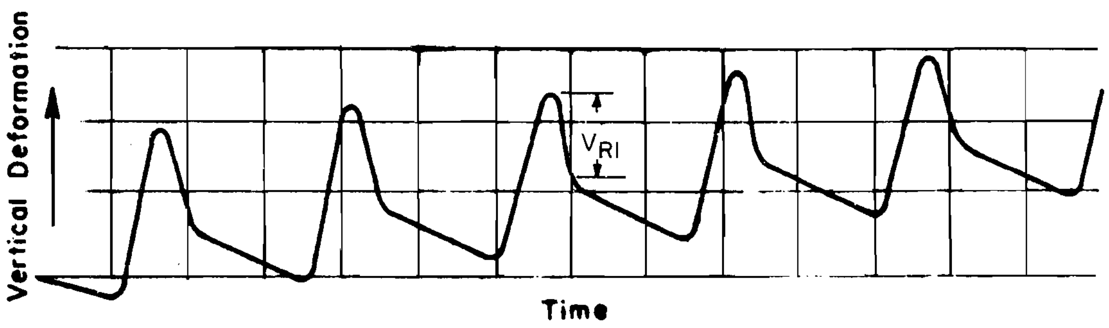
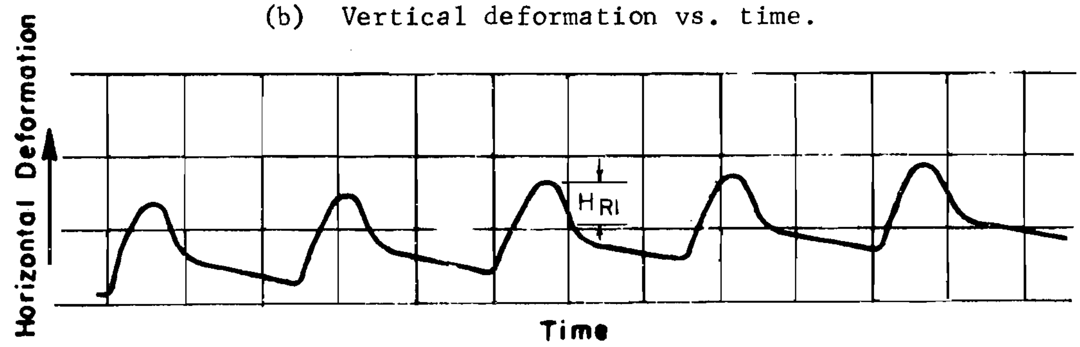
* It is recommended that the line of best fit be determined by the method of least squares.



(a) Load-time pulse.



(b) Vertical deformation vs. time.



(c) Horizontal deformation vs. time.

Fig 7. Typical load and deformation versus time relationships for repeated-load indirect tensile test.

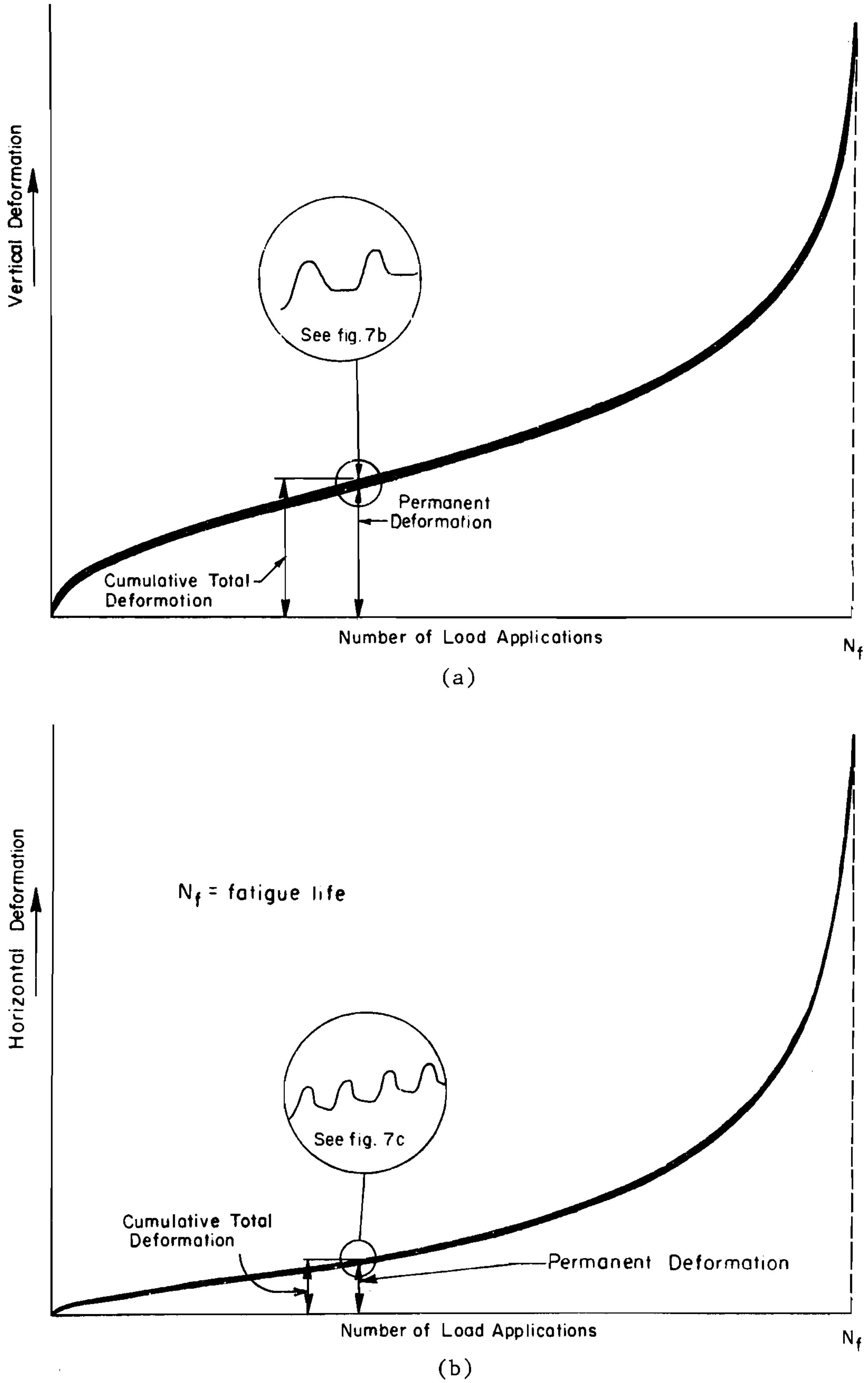


Fig 8. Relationship between number of load applications and vertical and horizontal deformation for the repeated-load indirect tensile test.

the State Department of Highways and Public Transportation. The basic reasons for this are:

(1) The test is relatively simple to conduct.

(2) The type of specimen and the equipment are the same as that used for compression testing.

(3) Failure is not seriously affected by surface conditions.

(4) Failure is initiated in a region of relatively uniform tensile stress.

(5) The coefficient of variation of test results is low compared to other test methods.

(6) The test can be used to test under a static load, i.e., a single load to failure, and repeated loads.

In addition, the test provides information on the tensile strength, modulus of elasticity, and Poisson's ratio for both static and repeated loads, fatigue characteristics, and permanent deformation characteristics of pavement materials. Static tests can be conducted at a sustained rate of 25 specimens per hour and repeated-load tests can be conducted more rapidly than in other test methods since failure occurs more quickly.

A recommended procedure for conducting the static indirect tensile test was previously developed and reported (Ref 2). However, a procedure for conducting the repeated-load test to obtain the modulus of elasticity and Poisson's ratio has not been developed, although it has been shown that realistic values can be obtained.

CHAPTER 3. EXPERIMENTAL PROGRAM, ANALYSIS, AND DISCUSSION

The basic data utilized in this study were obtained from an experimental program which was described in a previous report (Ref 1). These data were analyzed further in an attempt to establish a technique for estimating the modulus of elasticity and Poisson's ratio from the repeated-load indirect tensile test and to further investigate the repeated-load elastic characteristics and fatigue characteristics for purposes of mixture design of asphalt mixtures.

EXPERIMENTAL PROGRAM

The experimental program is summarized below and a detailed description of the experimental program is contained in Ref 1.

Materials

Two types of aggregate were included in the test program, an angular and relatively porous crushed limestone and a relatively nonporous gravel, with a medium gradation basically conforming to the State Department of Highways and Public Transportation standard specification (Ref 27) for hot mix asphalt concrete Class A, Type B (fine graded base or leveling-up course) and Type C (coarse graded surface course).

The asphalt was an AC-10 asphalt cement. The properties of the asphalt are summarized in Ref 1. Asphalt contents varied from 4 to 8 percent by weight of the total mixture.

Preparation of Specimens

Aggregates were batched by dry weight to meet the specified gradation. The aggregates and the required quantity of asphalt cement were then heated to 275°F ± 5°F, subsequent to which both materials were mixed for approximately three minutes using a Hobart automatic mixer.

The mixtures were then compacted at 250°F ± 5°F, using the Texas gyratory-shear compactor, according to test method Tex-206-F, Part II (Ref 17). Specimens to be tested at 75°F were cured for two days at 75°F before testing.

Specimens to be tested at 50 or 100°F were cured for 24 hours at 75°F and then an additional 24 hours at either 50°F ± 2°F or 100°F ± 2°F.

Specimen Characteristics

All specimens were approximately 4 inches in diameter by 2 inches high. The bulk specific gravity and bulk density of each specimen were determined. These values, along with the bulk specific gravity of the aggregates and the apparent specific gravity of the asphalt cement, were utilized to determine the percent air voids. Values for the individual specimens are contained in Ref 1.

Maximum density of the limestone mixtures was 146 pcf at the optimum asphalt content of 6.7 percent. The maximum density and the optimum asphalt content for the gravel mixtures were 144 pcf and 6.5 percent.

Test Equipment

The test equipment utilized for this study was basically the same as that previously used at the Center for Highway Research (Figs 9 and 10), the only difference being the recording system used for the repeated-load tests.

Both the vertical and horizontal deformations of the specimens were required in order to estimate the modulus of elasticity and Poisson's ratio. These deformations were measured using DC-LVDT's. In the static tests, the load-horizontal deformation and load-vertical deformation relationships were recorded on a pair of X-Y plotters. In the repeated-load tests, the horizontal and vertical deformations for individual load cycles were recorded on a 2-channel strip chart. Typical horizontal and vertical deformation versus time relationships are illustrated in Fig 7, along with the corresponding load-time pulse.

Experiment Design

The basic experiment design (Refs 3, 11, and 13) which was developed and then modified during the testing phase of the study is shown in Fig 11. The number of specimens tested for each cell, or each combination of variables, is indicated.

Test Procedure

In the static test, a preload of 20 pounds was applied to the specimen to prevent impact loading and to minimize the effect of deformations occurring

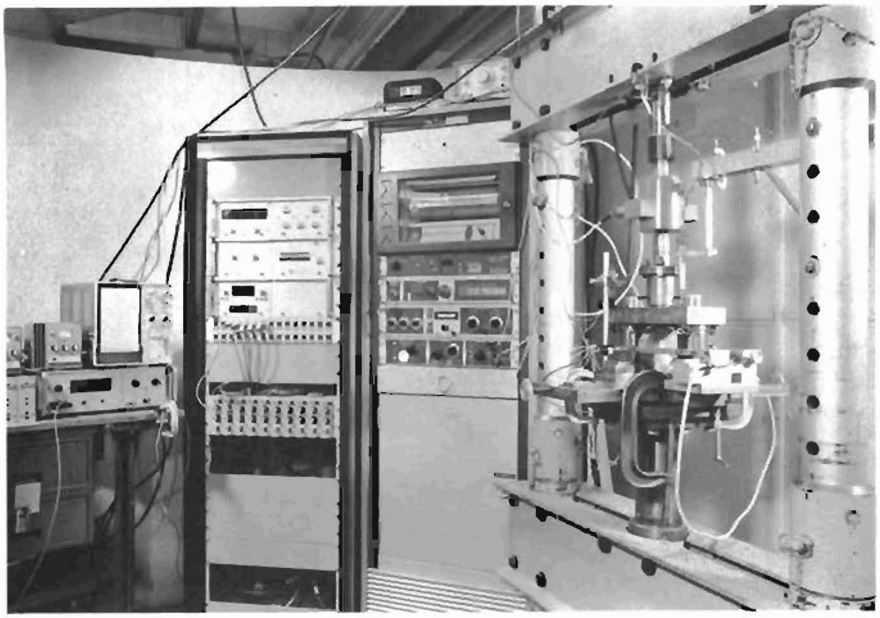


Fig 9. Basic testing equipment.

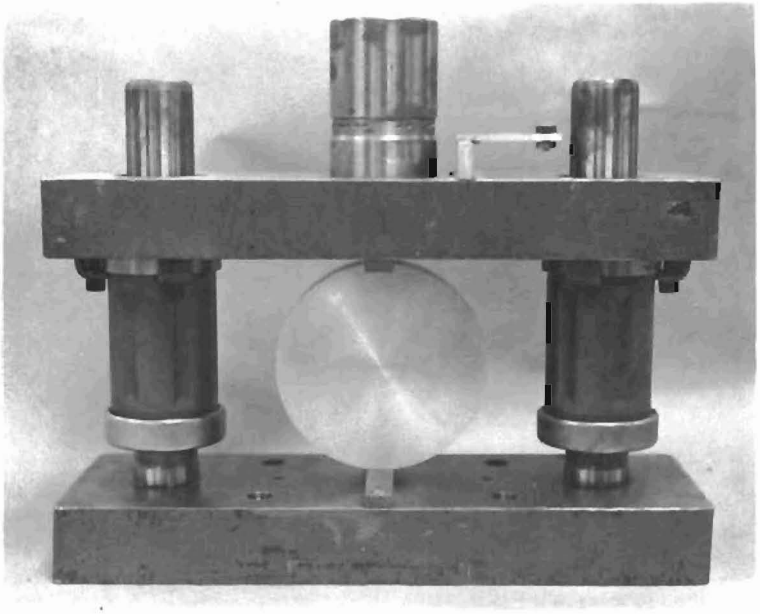


Fig 10. Loading head with rigid parallel platens.

Aggregate Type		Limestone*					Gravel*				
Asphalt Content, %		4	5	6	7	8	4	5	6	7	8
Test Temp., °F	Stress Level, psi										
50	72			3					3		
	96			3					3		
	120			3					3		
75	8				5					5	
	16	3	5	6	8	3	3	5	7	9	3
	24	5	5	7	5	5	5	5	7	9	5
	32	3	5	7	6	3	3	6	7	9	3
	40	4	5	7	8	3	3	5	7	8	3
100	8			3					3		
	16			3					3		
	24			3					3		

*Number of specimens tested indicated in each cell.

Fig 11. Graphical representation of factorial experiment design for repeated-load tests.

during the seating of the load strip. The specimens were then loaded at a rate of two inches per minute and the resulting load-deformation relationships were recorded on the X-Y plotters.

In the repeated-load tests, a preload of 20 pounds was also applied. Then the additional amount of load required to produce the prescribed stress level was applied at a frequency of one cycle per second (1 Hz) with a 0.4-second load duration and a 0.6-second rest period. The vertical and horizontal deformations were recorded either continuously or at selected intervals using a 2-channel strip chart recorder.

Variables Analyzed

Properties, or variables, which were analyzed are static modulus of elasticity, static Poisson's ratio, instantaneous resilient modulus of elasticity, instantaneous resilient Poisson's ratio, and fatigue life.

The static modulus of elasticity E_S and Poisson's ratio ν_S were estimated from the slopes of load-deformation relationships using the computer program MODLAS, which solves the basic equations.

The instantaneous resilient modulus of elasticity E_{RI} and Poisson's ratio ν_{RI} were calculated from the instantaneous resilient horizontal and vertical deformations H_{RI} and V_{RI} (Fig 7) and the applied stress. The same basic equations and computer program were used to calculate the static values except that it was assumed that the relationship between load and deformation was linear and thus only the maximum and minimum deformations were required.

Fatigue life was defined as the number of cycles required to produce complete fracture of the specimen and is illustrated in Fig 8.

ANALYSIS AND DISCUSSION

The analysis of the data has been subdivided into the following categories:

(1) relationships between the instantaneous resilient and the static moduli and Poisson's ratio,

(2) a test procedure to determine the instantaneous resilient modulus and Poisson's ratio, and

(3) relationships between properties and optimum asphalt contents.

Relationships with Static Modulus and Poisson's Ratio

In previous studies Navarro and Kennedy (Ref 22) and Adedimila and Kennedy (Ref 1) found no correlation between the resilient modulus of elasticity and the static modulus of elasticity. Nevertheless, since the static modulus of elasticity can be obtained quickly and easily, it was felt that the possibility of correlations between the instantaneous resilient modulus and static modulus should be investigated further.

The average static modulus of elasticity and the static Poisson's ratio for duplicate specimens corresponding to those used to obtain the instantaneous resilient modulus and Poisson's ratio are contained in Ref 1.

Instantaneous Resilient versus Static Modulus. A comparison of the mean static moduli of elasticity and the instantaneous resilient moduli of elasticity for each specimen is illustrated in Fig 12.

As shown in Fig 12, the instantaneous resilient moduli were significantly larger than the static moduli and it is obvious that no correlation existed. Instantaneous moduli ranged from 116×10^3 to 1100×10^3 psi, depending on the temperature. The average values at 50, 75, and 100°F were approximately 800×10^3 psi, 320×10^3 psi, and 190×10^3 psi.

Figure 13 compares the ratio of the instantaneous resilient modulus and the mean static modulus to the static modulus of elasticity. The ratio ranged from 0.9 to 5.1 for gravel mixtures and from 1.0 to 10.7 for limestone mixtures, with higher values occurring for materials with the lower static moduli. It should be noted that the ratios for the limestone aggregate were approximately twice the value for gravel mixtures.

These ratios are approximately the same as those obtained for inservice blackbase and asphalt concrete (Ref 22).

Instantaneous Resilient versus Static Poisson's Ratio. The instantaneous resilient and static Poisson's ratios are contained in Ref 1. Figure 14 illustrates the relative values of the two Poisson's ratios and indicates that the instantaneous resilient Poisson's ratios tend to be larger than the static values; however, the scatter for the instantaneous resilient values was quite large, with values ranging from -0.06 to 0.90. The majority of the values for the gravel and limestone specimens were in the range of 0.11 to 0.54 and 0.10 to 0.70, respectively, while for the static Poisson's

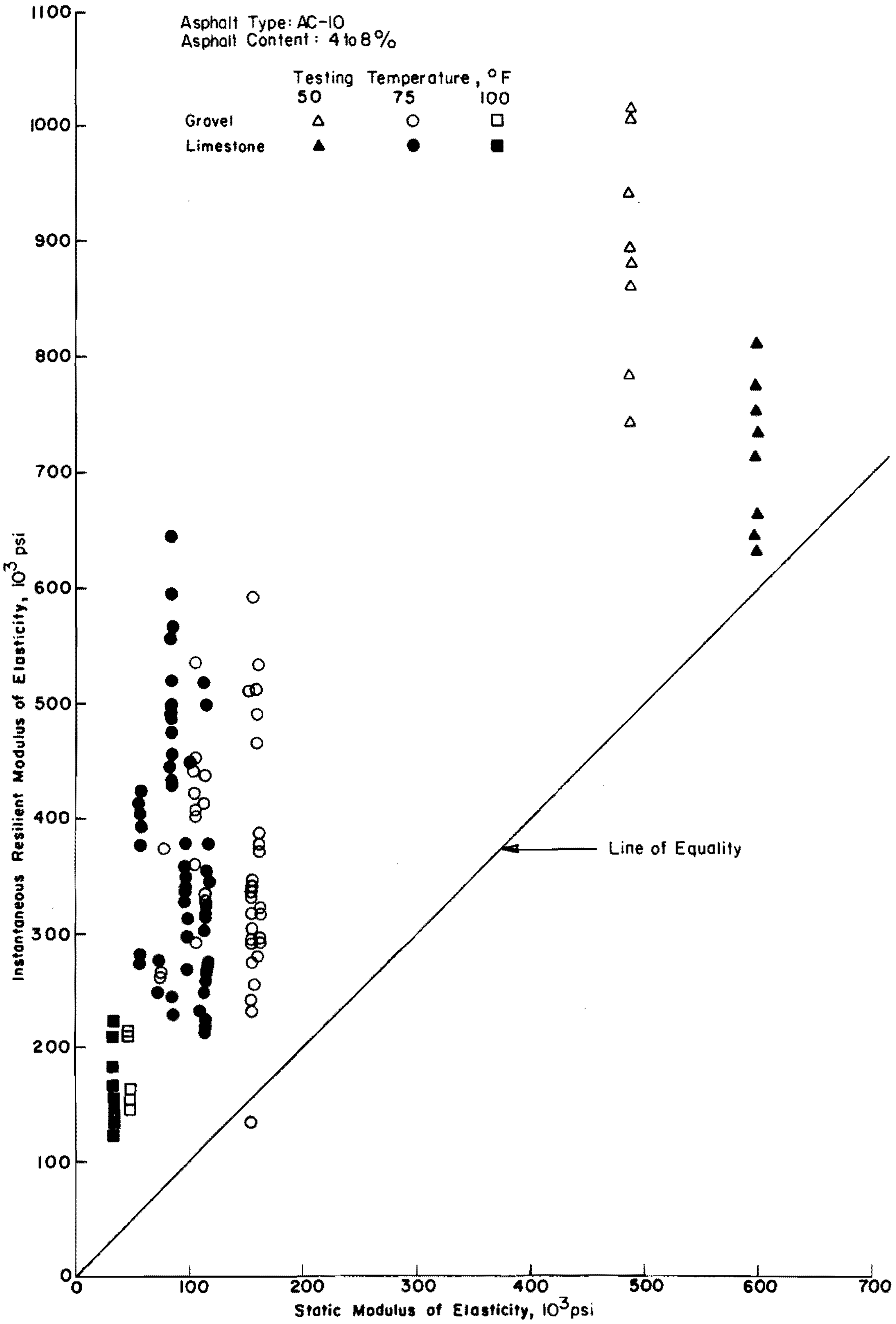


Fig 12. Relationship between static and instantaneous resilient moduli of elasticity.

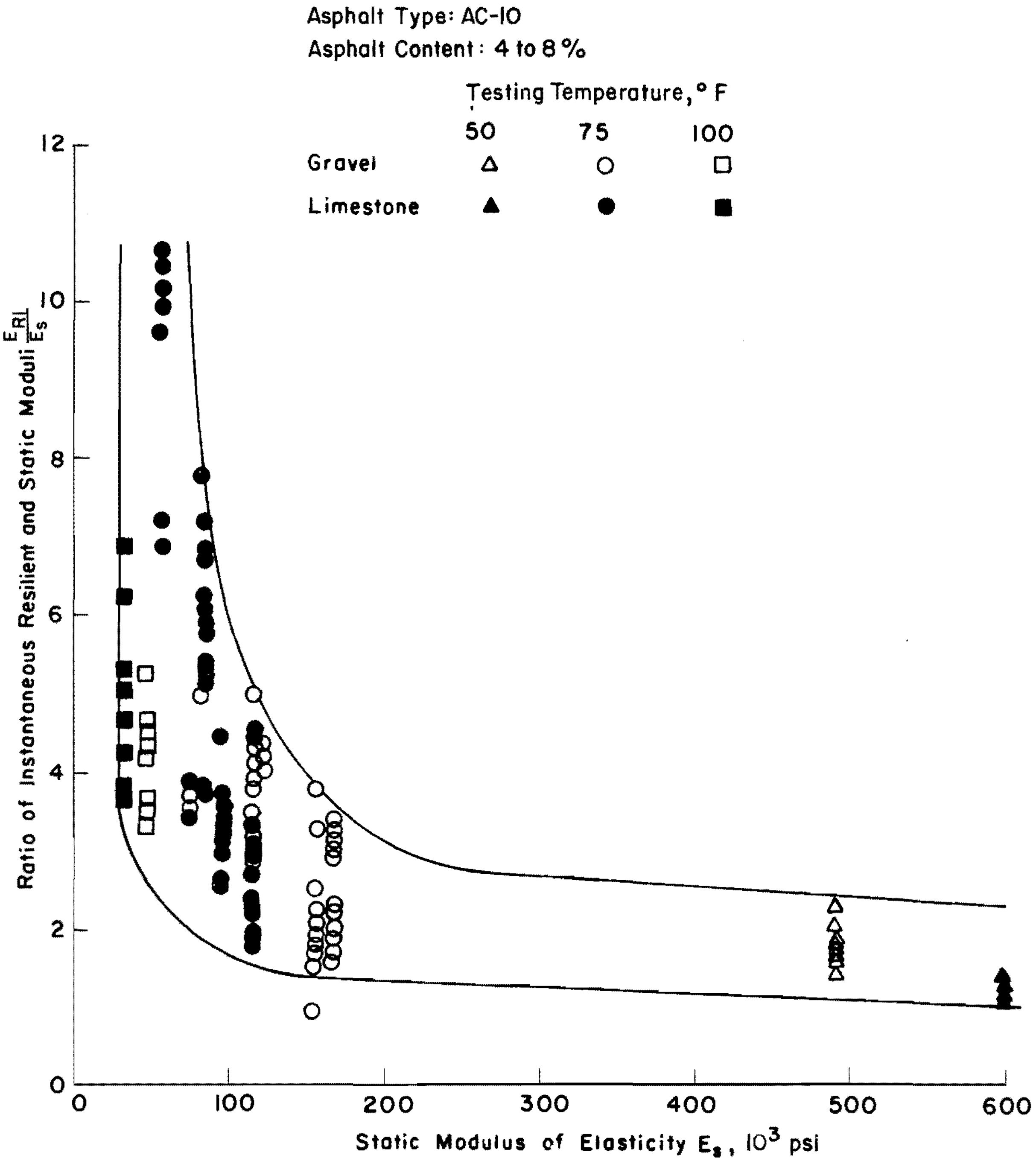


Fig 13. Relationship between the ratio of the instantaneous resilient static moduli and the static moduli.

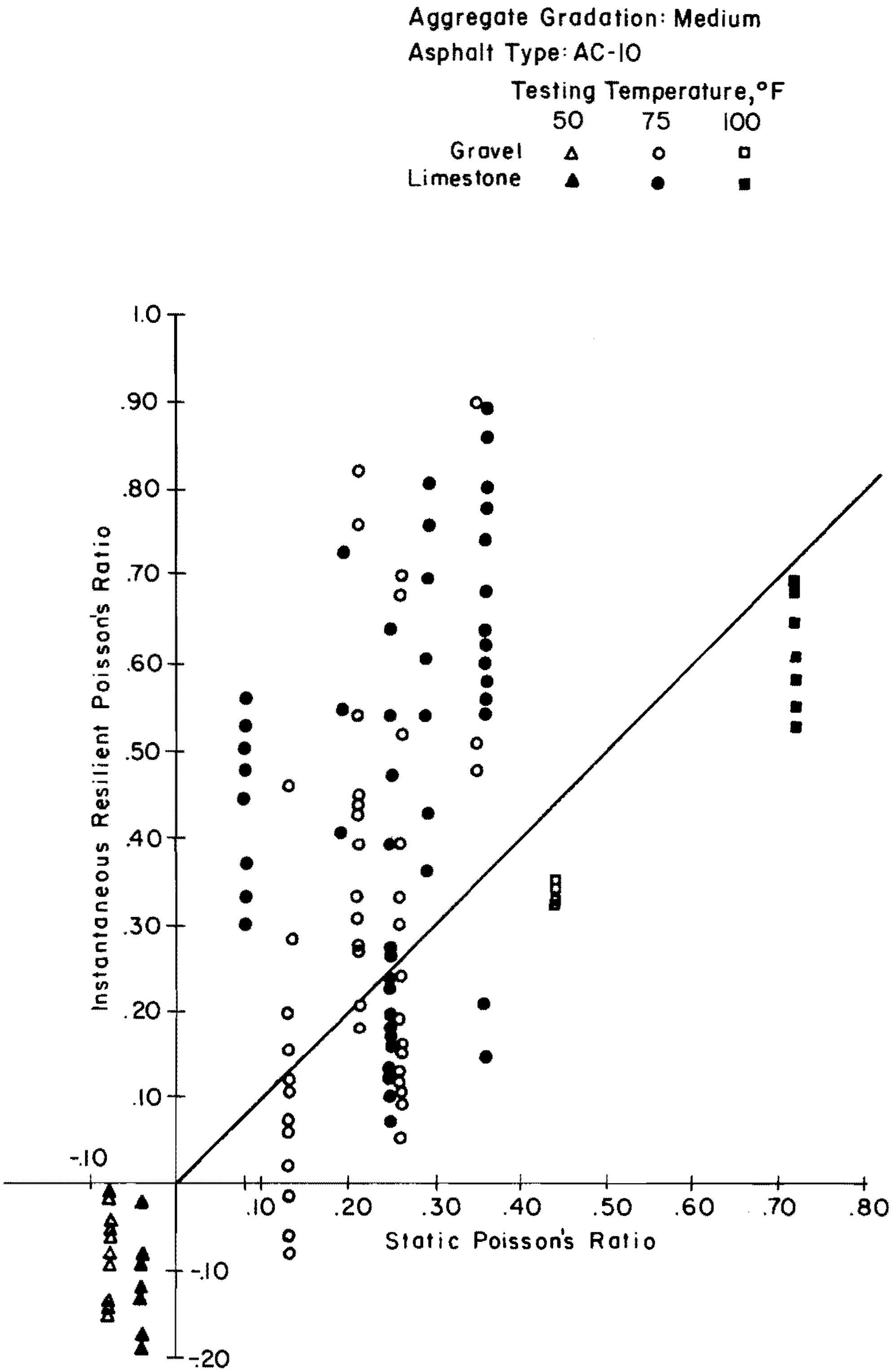


Fig 14. Relationship between static and instantaneous resilient Poisson's ratio values.

ratio the range was 0.13 to 0.35 for gravel and 0.08 to 0.36 for limestone.

Previously Adedimila and Kennedy (Ref 1) reported that Poisson's ratio increased with increased number of load applications until, at about 70 percent to 80 percent of the fracture life, the increase in Poisson's ratio is very rapid. This instance corresponds approximately to the instance at which there is a sharp decrease in resilient modulus.

Instantaneous Resilient versus Static Modulus at the Same Stress Level. The static modulus represents the entire load-deformation relationship to failure while the instantaneous resilient modulus characterizes the load-deformation relationship over a much smaller and lower stress range. Thus, the instantaneous resilient modulus would be expected to be larger since the relationship between load and deformation tends to be steeper at low stress levels. Therefore, the two moduli were compared by characterizing the load-deformation relationship for the static tests for the stress range used in the repeated-load tests at 75°F.

Since the load-deformation relationships were fairly linear for the static tests, no significant improvements in the correlation between the two moduli were obtained, as shown in Fig 15.

Poisson's ratios at the same stress level were not analyzed since the modulus is the more important variable.

Test Procedure to Determine the Instantaneous Resilient Modulus

One of the principal objectives of this investigation was to develop a method to obtain a representative value of the instantaneous resilient modulus of elasticity of an asphalt mixture without conducting long-term repeated-load tests. It appears that there is no definite correlation between the modulus values obtained from the static tests and the instantaneous resilient modulus of elasticity obtained from the repeated-load tests. Thus, it is necessary to establish a test procedure for conducting the repeated-load test to obtain estimates of the modulus of elasticity.

To do this, it was necessary to determine the range of load applications for which the relationship between modulus and the number of load applications was linear and relatively stable. A previous study had assumed, on the basis of the relationship between permanent deformation and the number of

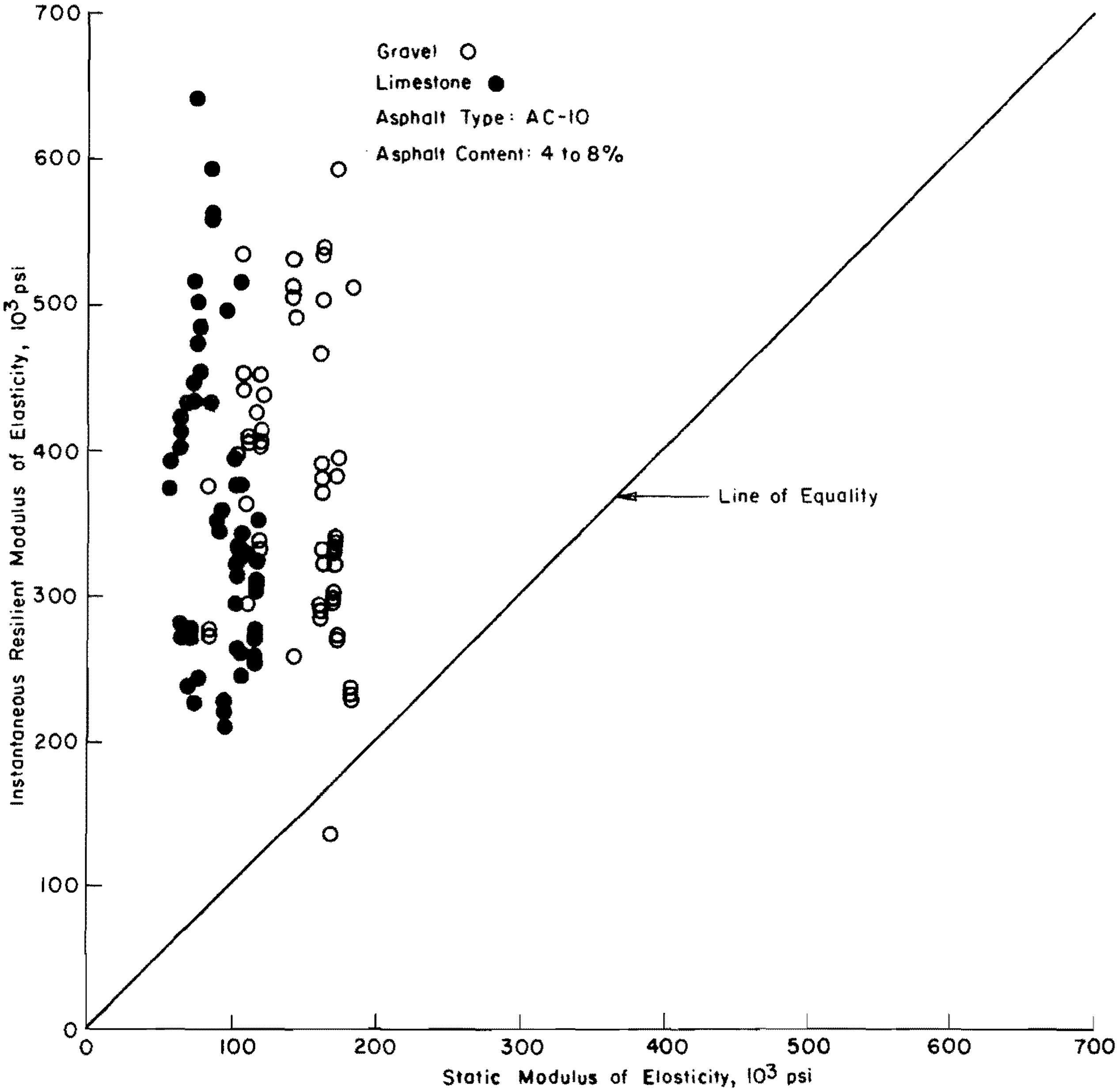


Fig 15. Relationship between the instantaneous resilient modulus of elasticity and the static modulus for the same stress level.

load applications, that this relationship was linear and stable between 15 and 85 percent of the fatigue life (Ref 22). A more recent study (Ref 1) indicated that the actual range was 10 to 80 percent and that after an initial conditioning period the instantaneous resilient modulus of elasticity began to decrease linearly with an increasing number of load applications until failure occurred with a very rapid decrease in modulus (Fig 16). Thus, the modulus changes continuously throughout the life of the specimen and is subject to large variations during the first 10 percent of the fatigue life of the specimen. In order to evaluate the possible error associated with estimating the instantaneous resilient modulus at a lower percent of the fatigue life, estimates of the instantaneous resilient modulus were made at approximately 0.1, 0.5, 1.0, 5.0, 10, 30, 50, and 70 percent of the fatigue life. However, it was not always possible to obtain estimates of modulus for every specimen at the lower percentages of fatigue life.

Test Procedure. Average relationships for both aggregates at 6.0 percent asphalt content and test temperatures of 50, 75, and 100°F are shown in Fig 17. These relationships indicate, as reported by Adedimila and Kennedy (Ref 1), that the modulus decreases with an increase in the number of load applications and that the modulus values are not as well defined during the first 10 percent of the fatigue life. However, it can also be seen that the values generally are not significantly different from the values obtained after additional load applications.

Thus, the instantaneous resilient moduli at any given percentage of the fatigue life were expressed in terms of a ratio with the modulus at 0.5 N_f, which is assumed to be the average modulus during the life of the specimen. The relationships between this ratio and the logarithm of percent fatigue life are shown in Figs 18 through 23. The average relationships indicate that at one percent of the fatigue life the estimated instantaneous resilient modulus would be between 1.01 to 1.51 times as large as the modulus value at 50 percent of the fatigue life. The upper values, however, are associated with a test temperature of 100°F, for which there were very few specimens. Without the values at 100°F, the average instantaneous resilient modulus at 0.01 N_f would be 1.01 to 1.16 times the modulus at 0.5 N_f.

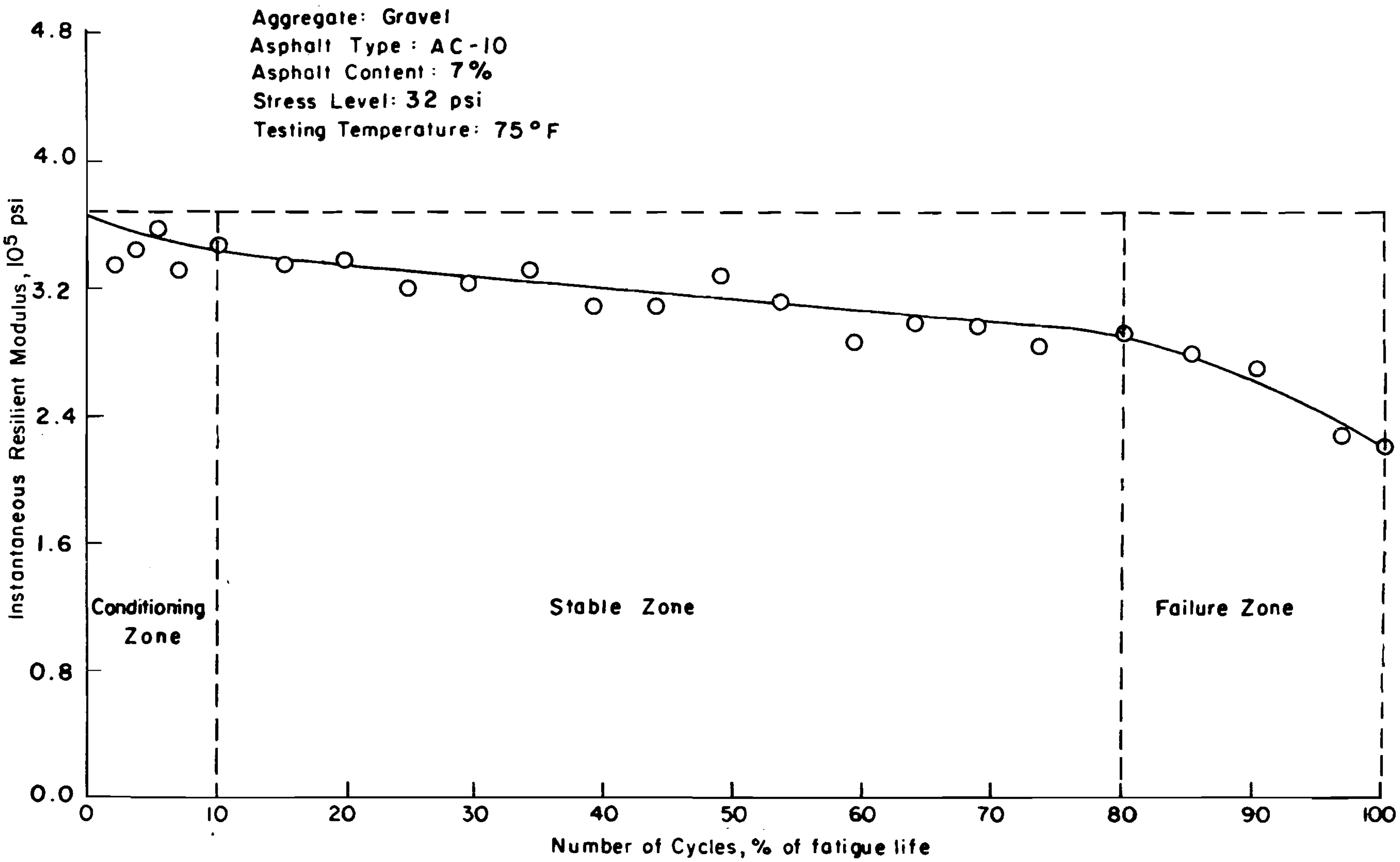


Fig 16. Typical relationship between instantaneous resilient modulus and number of load cycles (Ref 1).

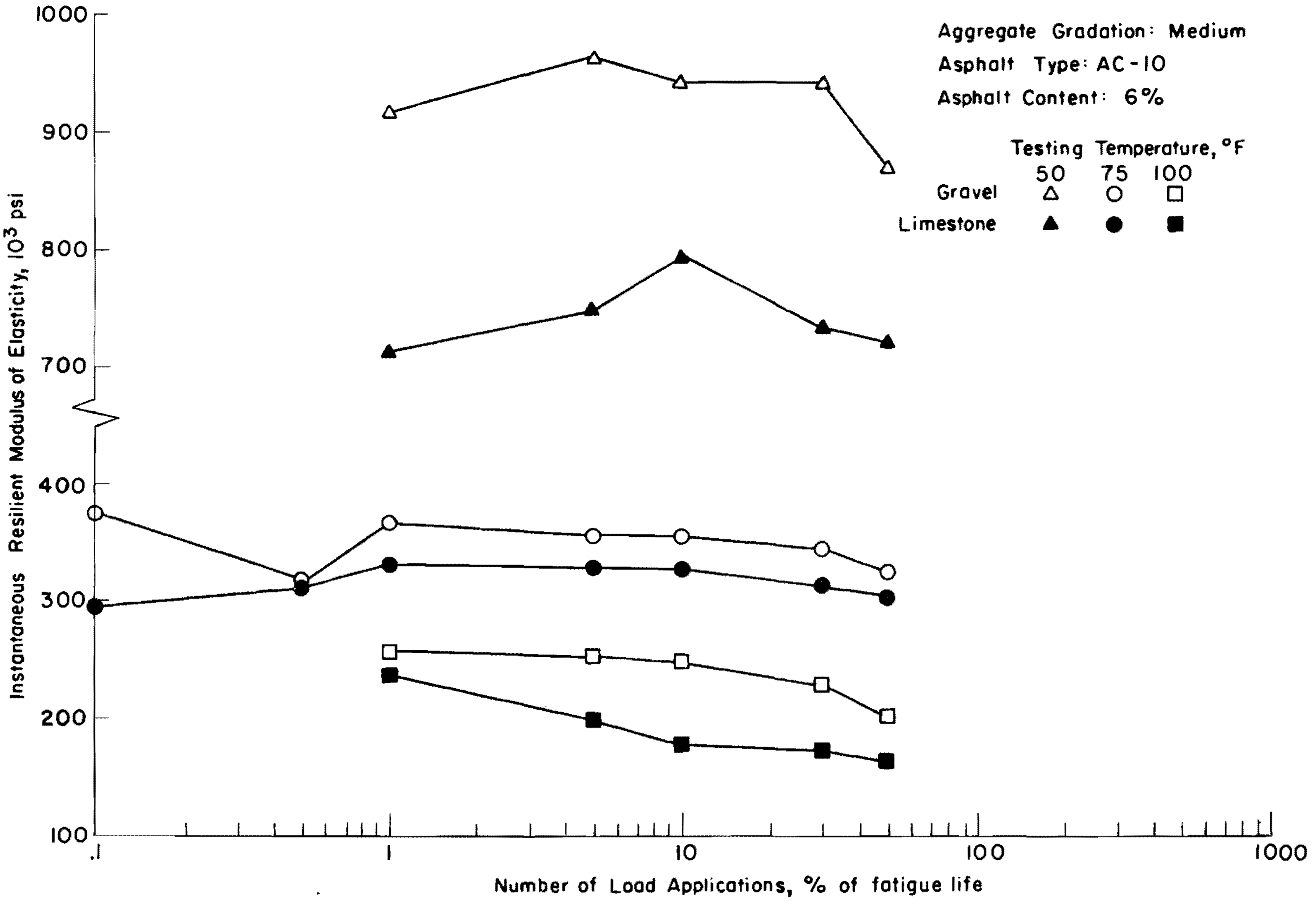


Fig 17. Average relationships between instantaneous resilient modulus of elasticity and number of load applications.

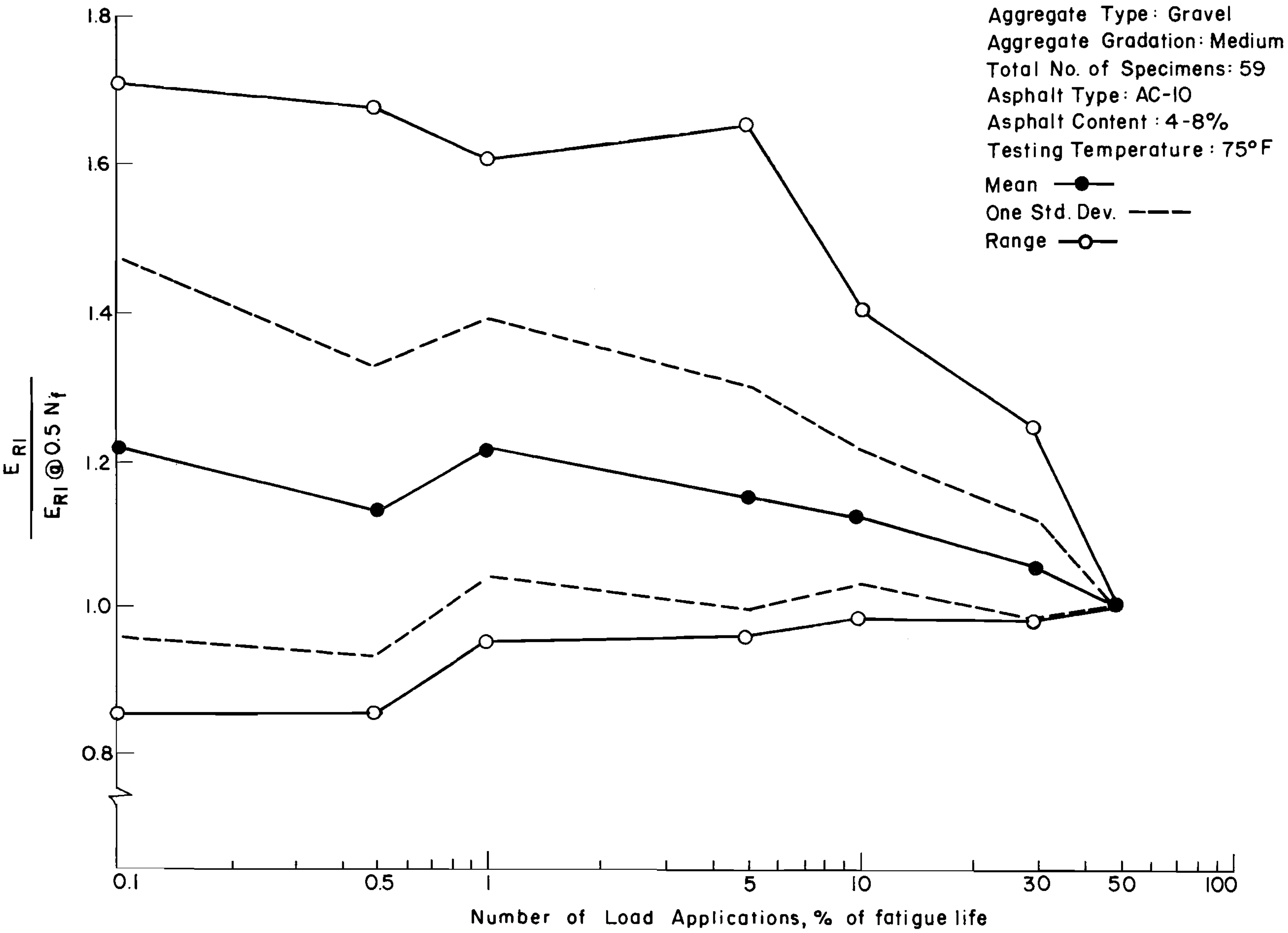


Fig 18. Relationship between resilient modulus and number of load applications for gravel mixture tested at 75° F.

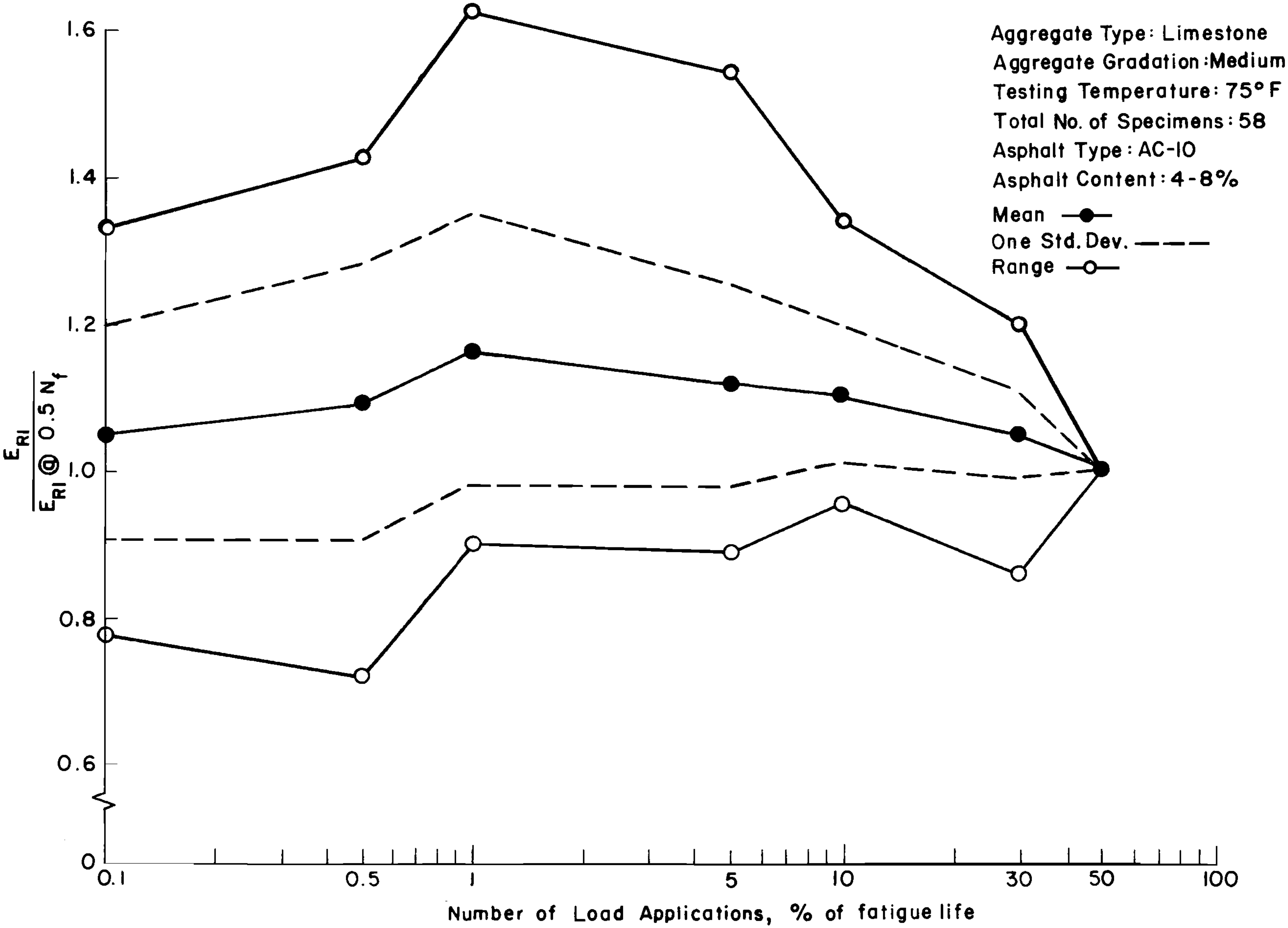


Fig 19. Relationship between resilient modulus and number of load applications for limestone mixture tested at 75° F.

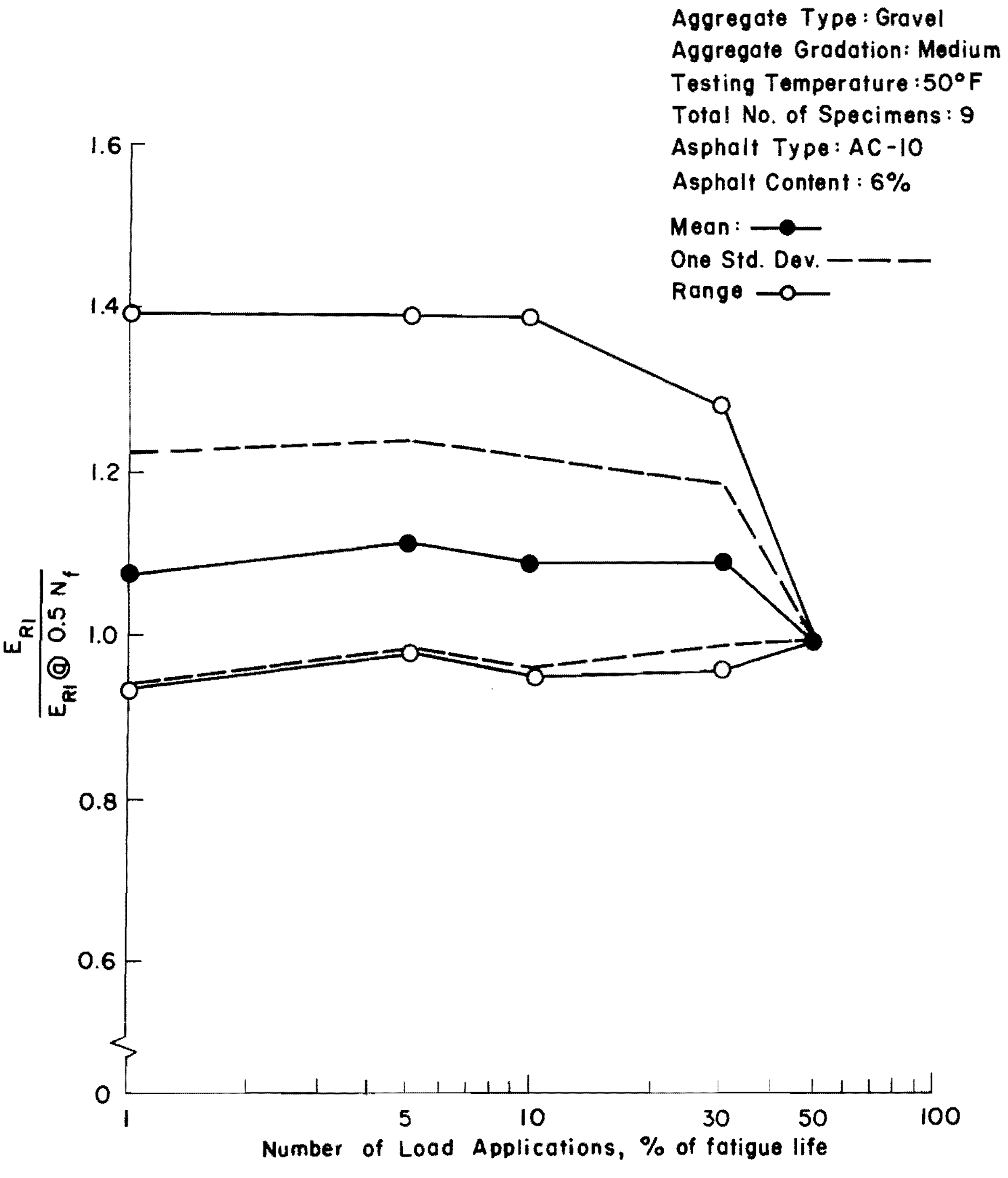


Fig 20. Relationship between resilient modulus and number of load applications for gravel mixture tested at 50° F.

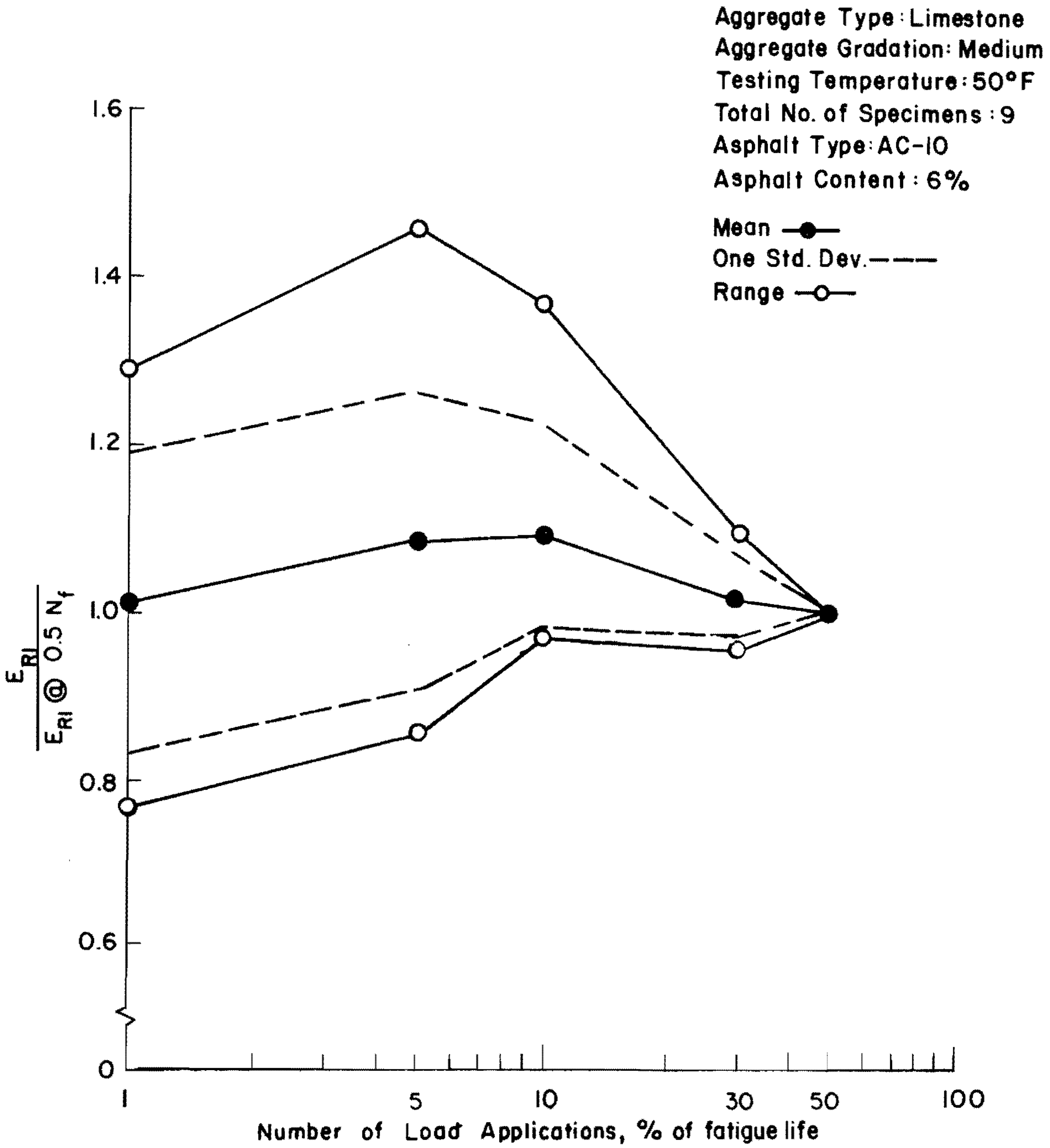


Fig 21. Relationship between resilient modulus and number of load applications for limestone mixture tested at 50° F.

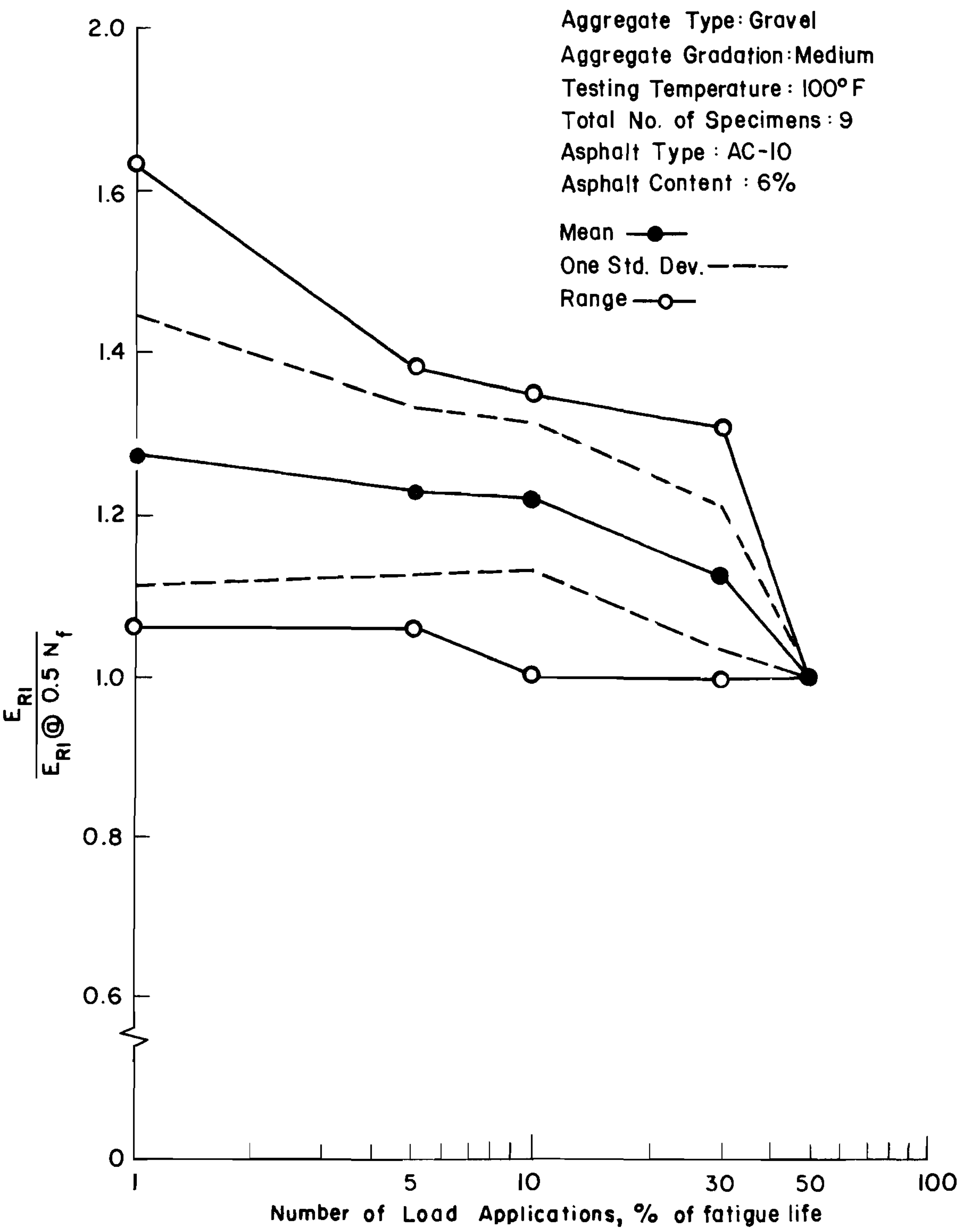


Fig 22. Relationship between resilient modulus and number of load applications for gravel mixture tested at 100° F.

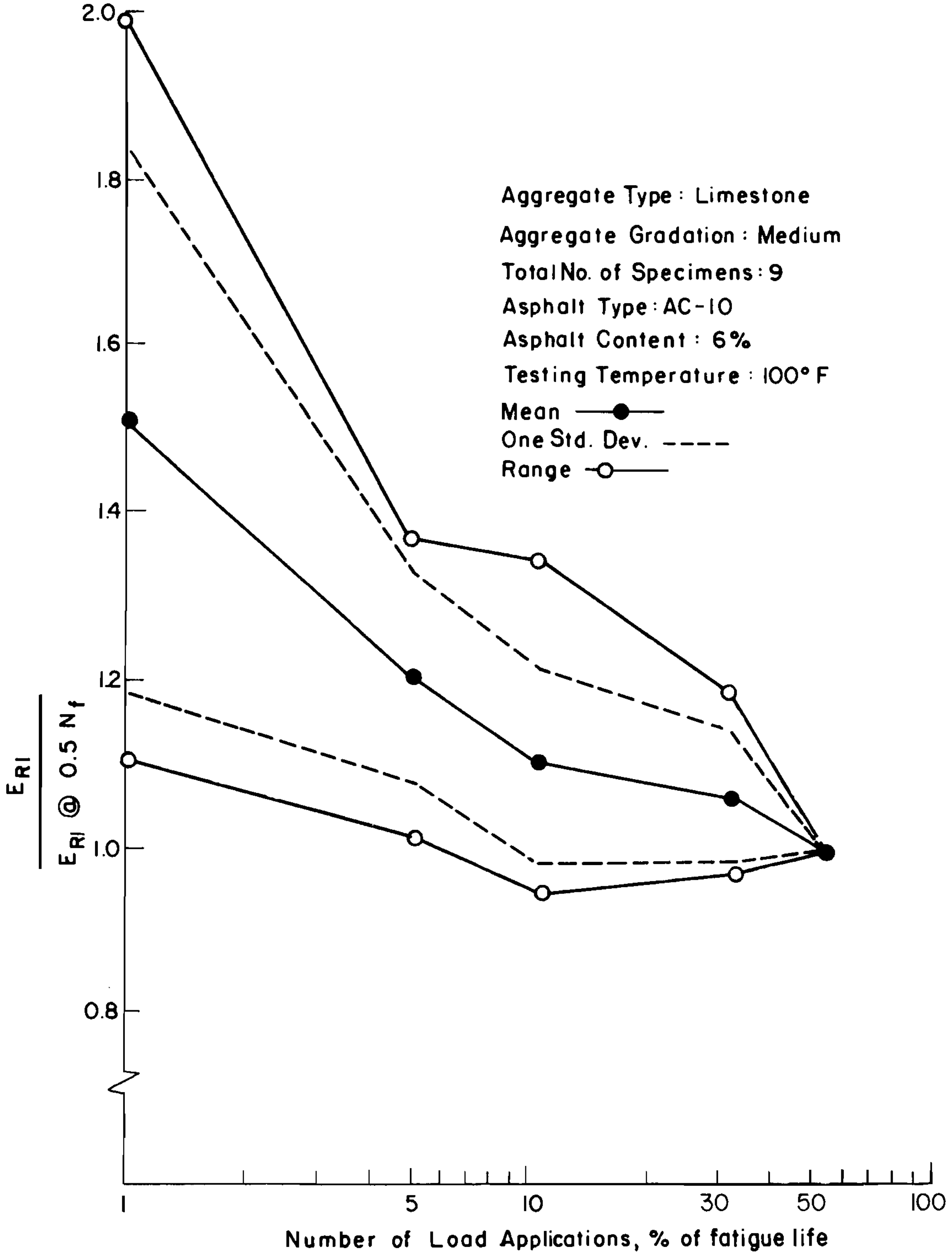


Fig 23. Relationship between resilient modulus and number of load applications for limestone mixture tested at 100° F.

At 75°F, the average modulus at .001 N_f would be 1.22 and 1.05 times the modulus at 0.5 N_f for the gravel and limestone mixtures, respectively.

Thus, it would appear that a reasonable estimate of the modulus could be obtained after 0.1 to 1.0 percent of the fatigue life. However, the amount of scatter increased significantly as the number of load applications was reduced which could be a problem especially at high test temperatures.

Based on the fact that it was difficult to estimate the instantaneous resilient modulus at 0.001 N_f at 50°F and 100°F, it was concluded that the resilient modulus should be estimated at .01 N_f or greater. However, since the actual number of cycles will vary with the fatigue life, which is a function of stress as well as other mixture construction variables, it was necessary to obtain an estimate of the required number of load applications.

Adedimila and Kennedy (Ref 1) and Moore and Kennedy (Refs 20 and 21) concluded that fatigue life could be estimated in terms of stress-strength ratio with reasonable accuracy and that such a relationship minimized the effects of test temperature. The relationship between stress-strength ratio and fatigue life as reported in Ref 1 is shown in Fig 24 along with the relationships for various percentages of fatigue life. From an evaluation of this figure, it was concluded that the specimen should be subjected to a minimum of 25 load applications before estimating the instantaneous modulus of elasticity. As shown in Fig 24, 25 cycles exceeded 0.01 N_f in all cases except for relatively small stress-strength ratios.

However, should the specimen fail prior to the application of 25 loads, the test should be repeated at a lower stress level. If, however, lower stresses are not desired or acceptable, then the load-deformation-time characteristics should be monitored during the first 25 cycles to insure that the specimen is not in the failure zone and that the desired information at 0.1 N_f is available. In fact, if practical, the first 25 cycles should be monitored in all tests.

The recommendation of 25 cycles should be considered as a preliminary estimate of the minimum number of cycles to which the specimen should be subjected prior to making measurements for the calculation of elastic properties and is subject to change as additional experience is obtained.

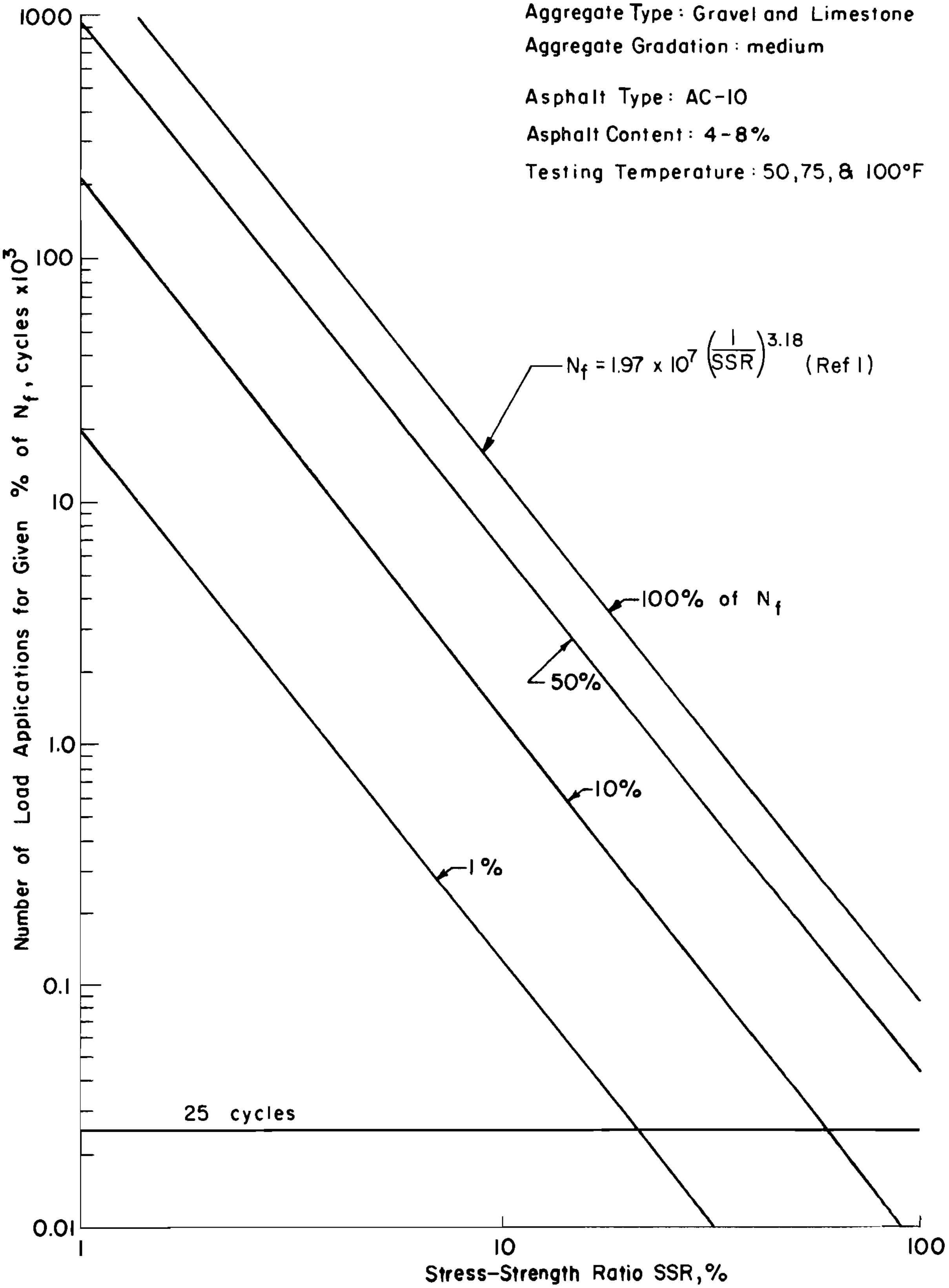


Fig 24. Relationship between number of load applications for a given percent of fatigue life and stress-strength ratio.

Calculation of Elastic Properties. To calculate the instantaneous resilient modulus of elasticity and Poisson's ratio the vertical and horizontal resilient deformations corresponding to V_{RI} and H_{RI} (Fig 7) should be input into the general relationships (numbers 6 and 7) shown in Table 4 along with the appropriate coefficients from Table 5 .

Relationships 1 through 5 in Table 4 can be used to calculate the static properties. The coefficients in Table 5 eliminate the need for a computer program.

Relationship Between Properties and Optimum Asphalt Contents

Previous work (Refs 1, 20, 21, and 22) have indicated that stress-strength ratio could be used to estimate fatigue life with relatively satisfactory accuracy. These same studies, however, have shown no correlation between fatigue life and the static modulus of elasticity or the instantaneous resilient modulus of elasticity. Navarro and Kennedy (Ref 22) investigated the possibility of such a relationship for cores from inservice pavements while Adedimila and Kennedy (Ref 1) evaluated the relationship for the laboratory prepared specimens used in this study.

Further evaluation in this study confirms the fact that there was no definite relationship which would allow fatigue life to be estimated from a single value of either the static or the instantaneous resilient modulus of elasticity. However, the relationship does have significance if the points are connected according to increasing asphalt content.

The relationships between fatigue life and static modulus of elasticity for the gravel and limestone mixtures are shown in Figs 25 and 26, respectively. Similar relationships between fatigue life and instantaneous resilient modulus of elasticity are shown in Figs 27 and 28. In addition to the actual data points, these figures include estimated points at the optimum asphalt contents for maximum fatigue life and the maximum static modulus of elasticity.

Optimum Asphalt Content for Maximum Fatigue Life. The relationships between asphalt content and fatigue life are shown in Fig 29 which indicates a definite optimum asphalt content for maximum fatigue life. For the mixtures and stress levels used in this study, stress level had no apparent affect on the optimum asphalt content although the optimum for the limestone appeared to increase slightly with increased stress. The optimum asphalt

TABLE 4. EQUATIONS FOR CALCULATING TENSILE PROPERTIES

Static Properties

(1) Tensile strength S_T, psi $= \frac{P_{Fail}}{h} \cdot A_0$

(2) Poisson's ratio ν $= \frac{DR \cdot A_1 + B_1}{DR \cdot A_2 + B_2}$

(3) Modulus of elasticity E, psi $= \frac{S_H}{h} (A_3 - \nu \cdot A_4)$

(4) Tensile strain ϵ_T, microunits $= X_T \left[\frac{A_5 - \nu \cdot A_6}{A_1 - \nu \cdot A_2} \right]$

(5) Compressive strain ϵ_C, microunits $= Y_T \left[\frac{B_3 - \nu \cdot B_4}{B_1 - \nu \cdot B_2} \right]$

Repeated-Load Properties

(6) Instantaneous resilient Poisson's ratio ν_{RI} $= \frac{\frac{V_{RI}}{H_{RI}} A_1 + B_1}{\frac{V_{RI}}{H_{RI}} A_2 + B_2}$

(7) Instantaneous resilient modulus of elasticity E_{RI}, psi $= \frac{P}{H_{RI} h} (A_3 - \nu_{RI} \cdot A_4)$

P_{Fail} = total load at failure (maximum load P_{max} or load at first inflection point), pounds

P = applied load or repeated load, pounds

h = height of specimen, inches

DR = deformation ratio $\frac{Y_T}{X_T}$ (the slope of line of best fit* between vertical deformation Y_T and the corresponding horizontal deformation X_T up to failure load)

X_T = total horizontal deformation, inches

Y_T = total vertical deformation, inches

S_H = horizontal tangent modulus $\frac{P}{X_T}$ (the slope of the line of best fit* between load P and horizontal deformation X_T for loads up to failure load)

H_{RI}, V_{RI} = instantaneous resilient horizontal and vertical deformations, respectively

A_0, A_1, A_2, A_3, A_4, A_5, A_6, B_1, B_2, B_3, B_4 = constants (see Table 5).

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*It is recommended that the line of best fit be determined by the method of least squares.

TABLE 5. CONSTANTS FOR EQUATIONS FOR INDIRECT TENSILE PROPERTIES

Diameter, inches	A_0	A_1	A_2	A_3	A_4	A_5	A_6	B_1	B_2	B_3	B_4
3.5	.177	.0766	-.2847	.2680	-.9966	.05056	-.1545	-.9765	-.0204	-.1545	.05056
3.6	.172	.0745	-.2769	.2683	-.9968	.04786	-.1461	-.9590	-.0193	-.1461	.04786
3.7	.168	.0726	-.2694	.2685	-.9970	.04537	-.1384	-.9422	-.0183	-.1384	.04537
3.8	.164	.0707	-.2624	.2688	-.9971	.04307	-.1312	-.9260	-.0173	-.1312	.04307
3.9	.160	.0690	-.2557	.2690	-.9973	.04094	-.1246	-.9104	-.0165	-.1247	.04094
4.0	.156	.0673	-.2494	.2692	-.9974	.03896	-.1185	-.8954	-.0156	-.1185	.03896
4.1	.152	.0657	-.2433	.2694	-.9975	.03712	-.1129	-.8810	-.0149	-.1129	.03712
4.2	.149	.0642	-.2375	.2696	-.9976	.03541	-.1076	-.8671	-.0142	-.1076	.03541
4.3	.145	.0627	-.2320	.2698	-.9977	.03381	-.1027	-.8537	-.0136	-.1027	.03381
4.4	.142	.0613	-.2268	.2699	-.9978	.03232	-.9808	-.8407	-.0130	-.9809	.03232
4.5	.139	.0600	-.2218	.2701	-.9979	.03092	-.9379	-.8282	-.0124	-.9380	.03092
4.6	.136	.0587	-.2170	.2702	-.9980	.02961	-.8978	-.8161	-.0118	-.8979	.02961
4.7	.133	.0575	-.2124	.2703	-.9981	.02838	-.8602	-.8043	-.0114	-.8603	.02839
4.8	.131	.0563	-.2080	.2704	-.9982	.02723	-.8249	-.7930	-.0109	-.8250	.02723
4.9	.128	.0552	-.2037	.2706	-.9983	.02615	-.7917	-.7820	-.0105	-.7918	.02615
5.0	.126	.0541	-.1997	.2707	-.9983	.02512	-.7605	-.7714	-.0100	-.7606	.02513
5.1	.123	.0531	-.1958	.2708	-.9984	.02416	-.7311	-.7610	-.0966	-.7312	.02416
5.2	.121	.0521	-.1920	.2709	-.9985	.02325	-.7034	-.7510	-.0929	-.7034	.02325
5.3	.119	.0511	-.1884	.2709	-.9985	.02239	-.6772	-.7413	-.0895	-.6772	.02240
5.4	.116	.0502	-.1849	.2710	-.9986	.02158	-.6524	-.7319	-.0862	-.6525	.02158
5.5	.114	.0493	-.1816	.2711	-.9986	.02081	-.6290	-.7227	-.0832	-.6291	.02081
5.6	.112	.0484	-.1783	.2712	-.9987	.02008	-.6068	-.7138	-.0803	-.6069	.02008
5.7	.110	.0476	-.1752	.2713	-.9987	.01939	-.5858	-.7051	-.0775	-.5858	.01939
5.8	.109	.0468	-.1722	.2713	-.9988	.01874	-.5658	-.6967	-.0749	-.5659	.01874
5.9	.107	.0460	-.1693	.2714	-.9988	.01811	-.5469	-.6884	-.0724	-.5469	.01811
6.0	.105	.0452	-.1665	.2714	-.9988	.01752	-.5289	-.6804	-.0700	-.5289	.01752
6.1	.103	.0445	-.1638	.2715	-.9989	.01695	-.5117	-.6727	-.0678	-.5117	.01696
6.2	.102	.0438	-.1611	.2716	-.9989	.01642	-.4954	-.6651	-.0657	-.4954	.01642
6.3	.100	.0431	-.1586	.2716	-.9989	.01590	-.4798	-.6577	-.0636	-.4799	.01591
6.4	.099	.0424	-.1561	.2717	-.9990	.01542	-.4650	-.6504	-.0617	-.4650	.01542
6.5	.097	.0418	-.1537	.2717	-.9990	.01495	-.4508	-.6434	-.0598	-.4509	.01495

Strip width a = 0.5 in.

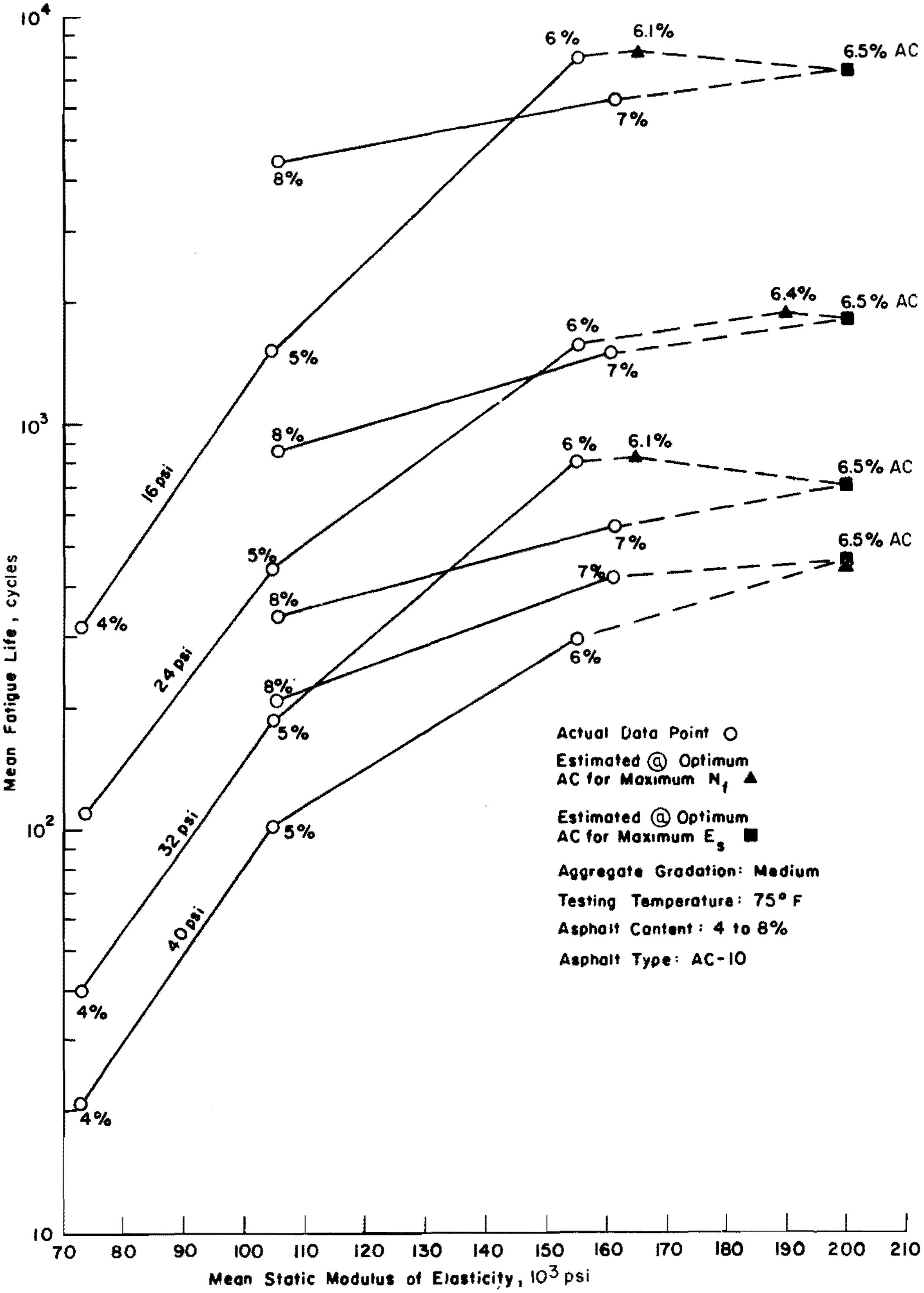

Fig 25. Relationship between static modulus of elasticity and fatigue life for gravel mixtures.

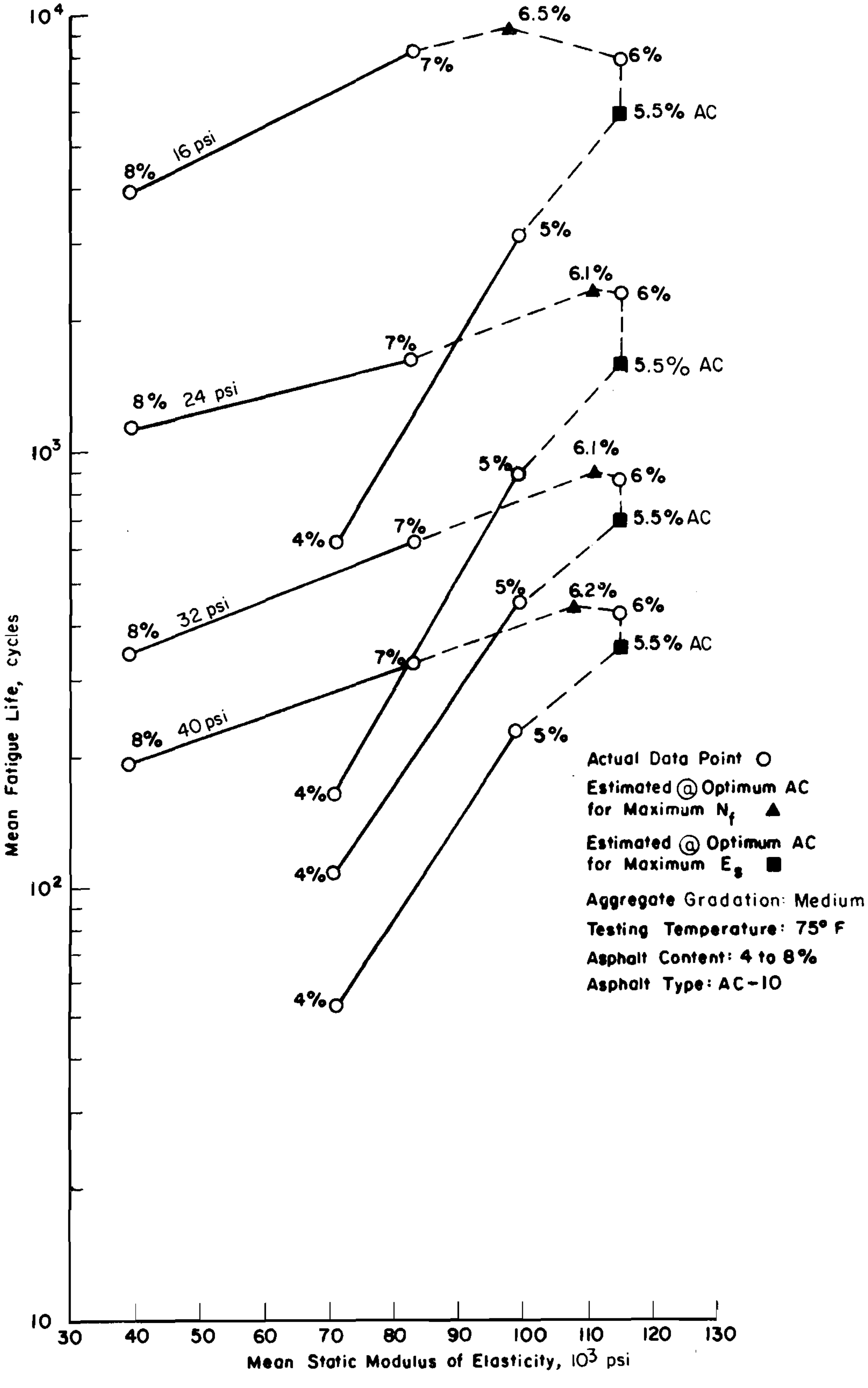

Fig 26. Relationship between static modulus of elasticity and fatigue life for limestone mixtures.

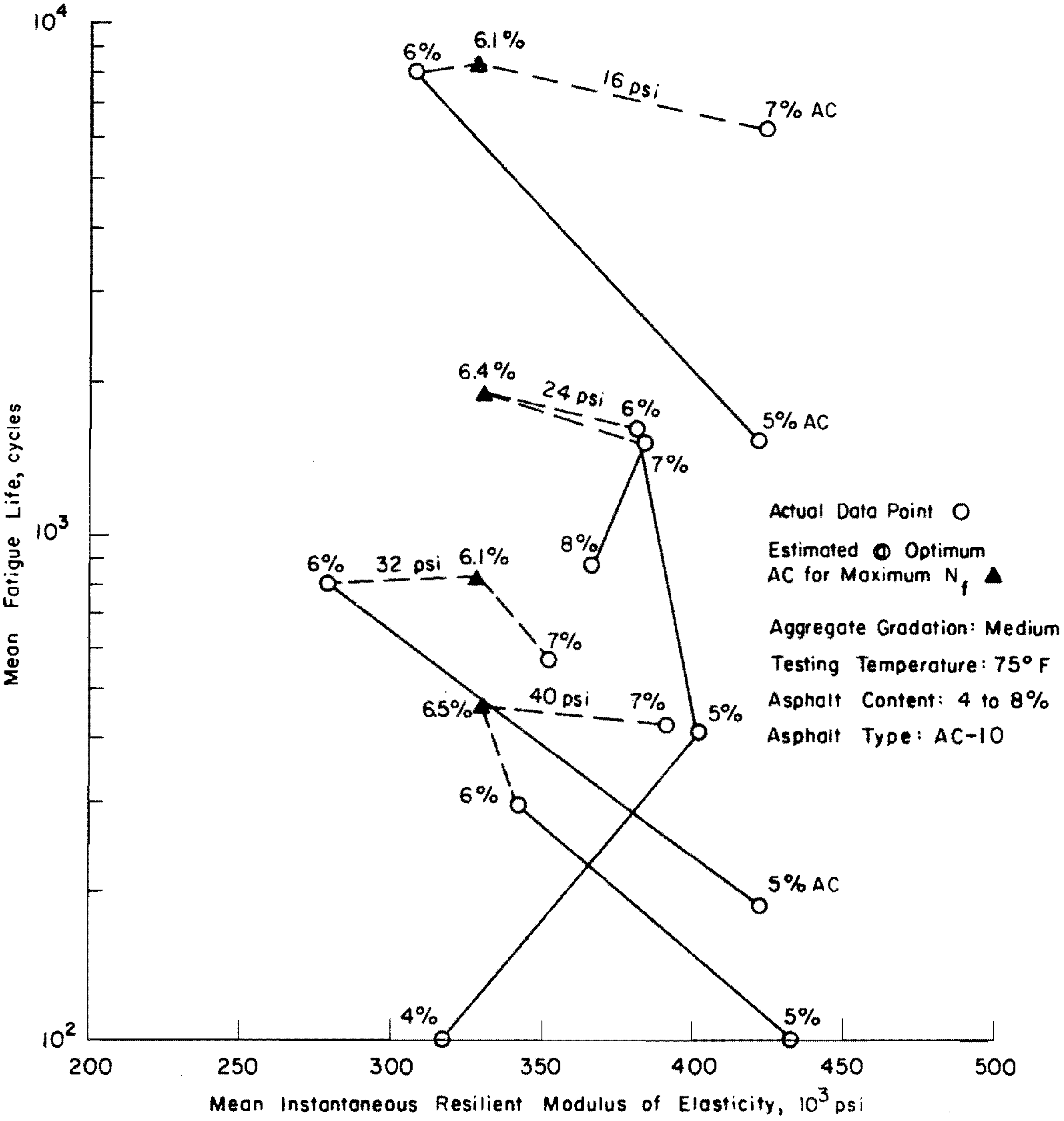

Fig 27. Relationship between instantaneous resilient modulus of elasticity and fatigue life for gravel mixtures.

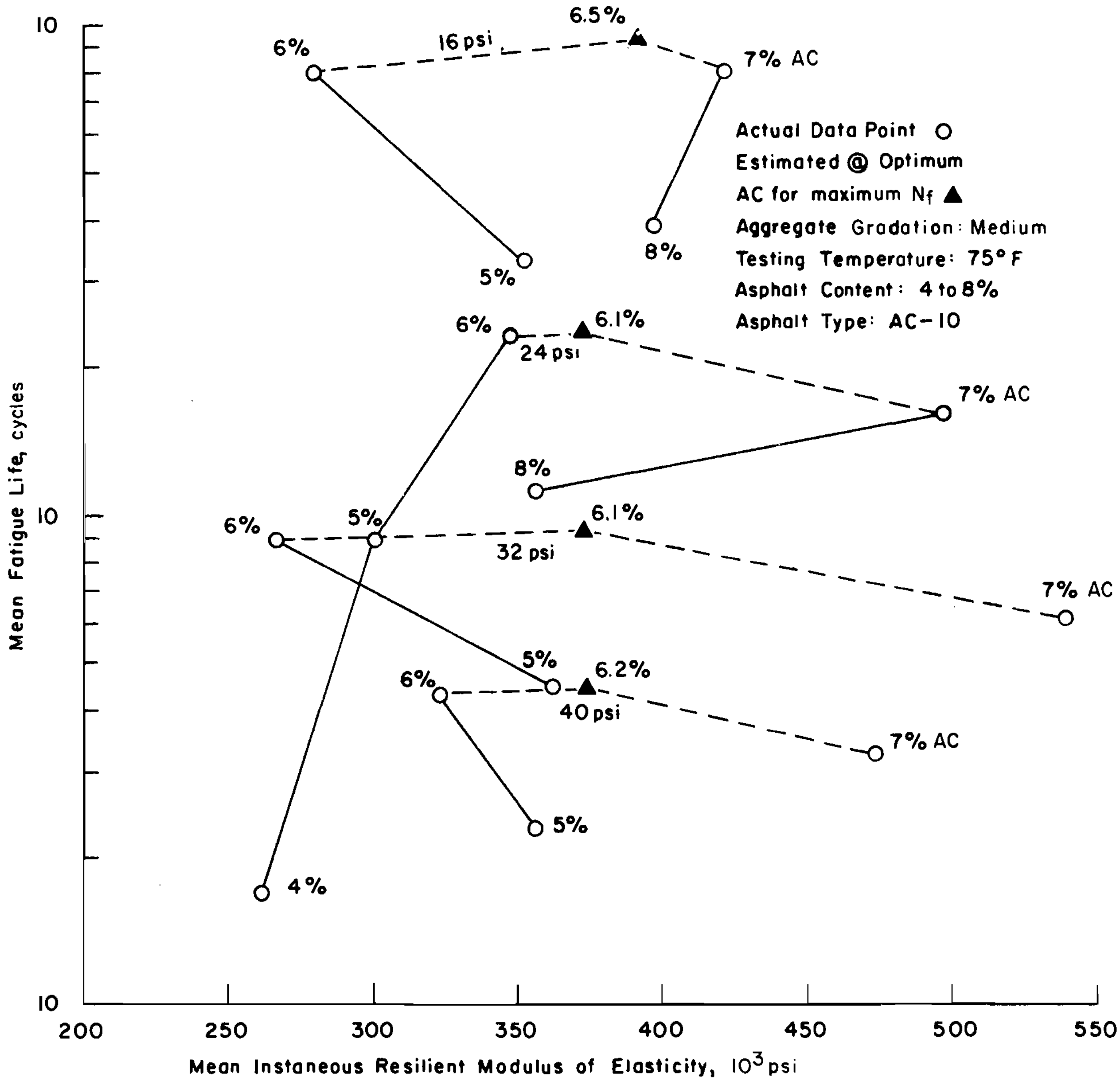

Fig 28. Relationship between instantaneous resilient modulus of elasticity and fatigue life for limestone mixtures.

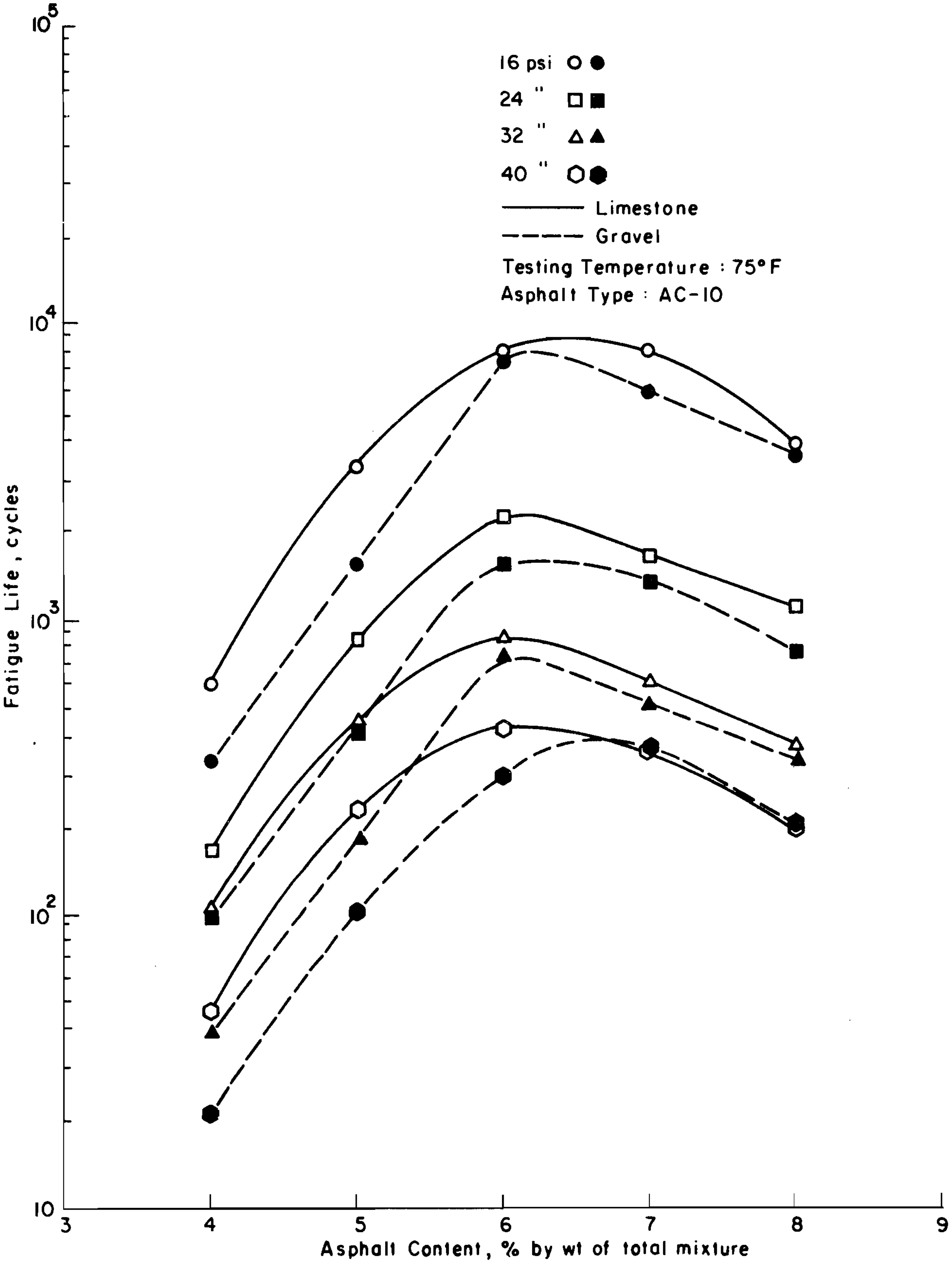

Fig 29. Effect of aggregate and asphalt content on fatigue life (Ref 1).

contents were generally in the range of 6 to 6.5 percent for a test temperature of 75°F. These values were less than the optimum for maximum bulk density, 6.7 percent for limestone and 6.5 percent for gravel (Ref 1).

Optimum Asphalt Content for Maximum Static Modulus of Elasticity. The relationships between static modulus of elasticity and asphalt content is shown in Fig 30 for both aggregate types and testing temperatures of 50, 75, and 100°F. The optimums at 50, 75, and 100°F were 7.0, 6.5 and 6.3 for the gravel mixtures, while for the limestone mixtures the optimums were less at 6.2, 5.8, and 5.75. Thus, the optimum tended to increase slightly with decreased testing temperature. In addition, at the higher temperatures the optimum was not well defined indicating that the actual choice of the optimum is much more critical at lower temperatures.

From the above it can be seen that the optimum asphalt contents for maximum static modulus generally were slightly less than the optimum asphalt content for maximum fatigue life. However, a more detailed comparison, Figs 25 and 26, indicate that for limestone the above statement is true; however, for gravel the reverse is true, although the optimums are much closer. In addition, according to Adedimila and Kennedy (Ref 1), the optimum for maximum static modulus was essentially the same as the optimum for maximum tensile strength.

Optimum Asphalt Content for Maximum Instantaneous Resilient Modulus. The relationship between the instantaneous resilient modulus of elasticity and asphalt content for the gravel and limestone mixtures subjected to a repeated tensile stress of 24 psi at 75°F is shown in Fig 31. Although maximum moduli did occur, there was no well defined optimum asphalt content. It would appear that for the mixtures and test conditions used in this study, the instantaneous resilient modulus was relatively insensitive to asphalt content. Similar relationships were also obtained for other stress levels (Ref 1).

Adedimila and Kennedy (Ref 1) reported that Schmidt detected an optimum asphalt content for maximum resilient modulus but that the optimum occurred on a plateau, thus indicating that the choice of asphalt content was not critical.

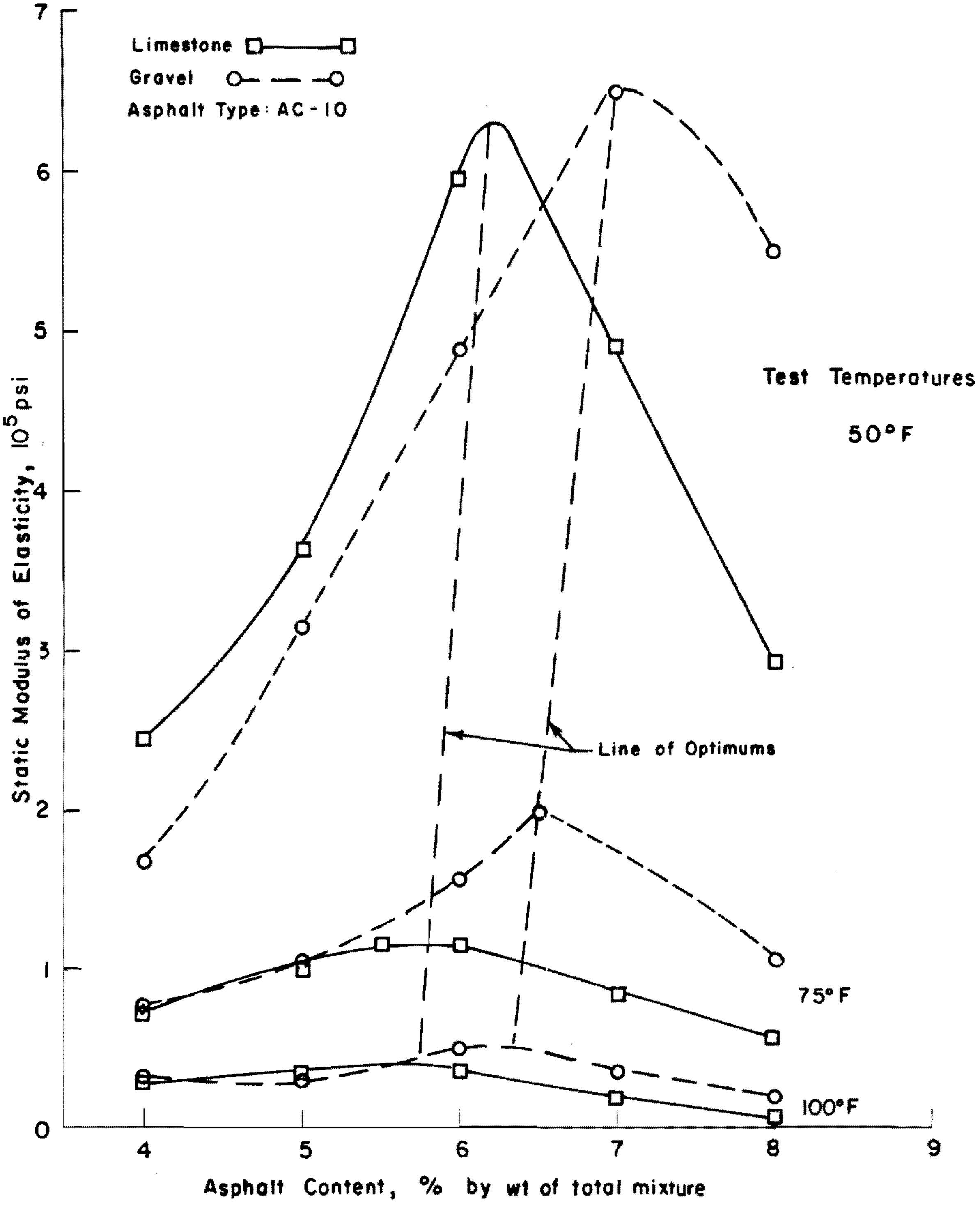

Fig 30. Relationships between average static modulus of elasticity and asphalt content for limestone and gravel mixtures (Ref 1).

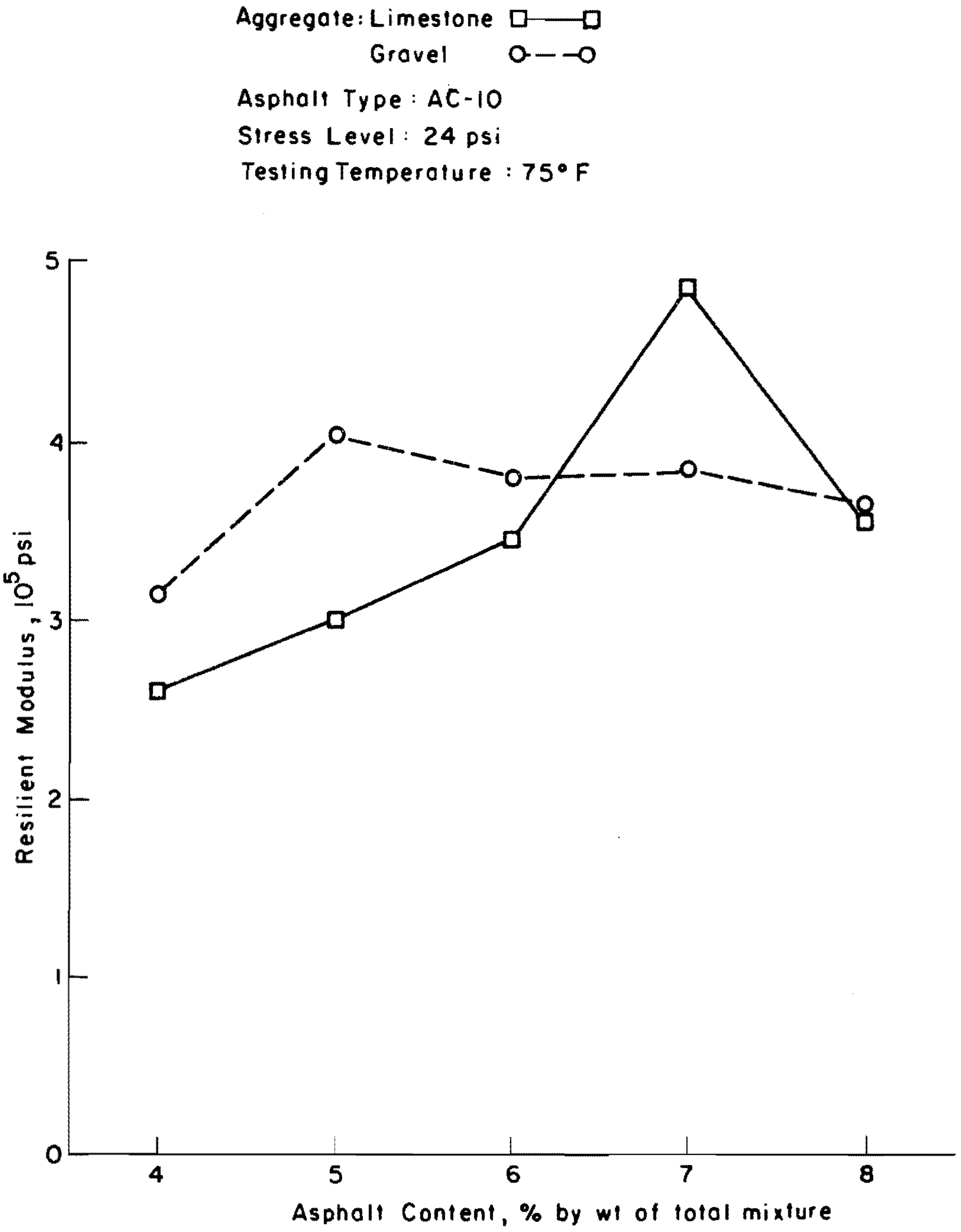

Fig 31. Effect of aggregate type and asphalt content on instantaneous and resilient modulus for limestone and gravel mixtures.

Evaluation of Relationships Between Fatigue Life and Static Modulus. Figures 25 and 26 indicate that for the materials and conditions in this study maximum fatigue life for limestone mixtures occurred at an asphalt content which was larger than the asphalt content for maximum static modulus. However, for gravel mixtures the optimum asphalt contents for maximum fatigue life were slightly less than or equal to the optimums for maximum static modulus. Since, according to Adedimila and Kennedy (Ref 1), the optimums for maximum tensile strength and static modulus are essentially equal, it would appear that the optimum asphalt content for maximum fatigue life generally was larger than for maximum tensile strength. Hence, these relationships indicate that the final choice of the optimum asphalt content for fatigue life was not overly critical. Nevertheless, if a large error in asphalt content can be expected, then the error should be on the wet side of the optimum for maximum fatigue life since the effect is much less for a change in asphalt content on the wet side.

The relationships between fatigue life and instantaneous resilient modulus (Figs 27 and 28) have a slightly different shape which can be attributed to the fact that the resilient modulus was relatively insensitive to changes in asphalt content and, therefore, the actual value is probably determined by other more important factors. However, these figures do indicate that small changes in asphalt content, near the optimum asphalt content for maximum fatigue life, did not produce large reductions in fatigue life. However, for large or small changes, the losses were generally less on the wet side. Probably more important is the fact that the instantaneous resilient modulus increased with an increase in asphalt content above the optimum for maximum fatigue life.

Choice of Asphalt Content. From the above discussion, it would appear that mixtures similar to the ones used in this study should be designed at or above the optimum asphalt content for maximum fatigue life. If the design is to be based on static tests, the asphalt content should be slightly above the optimum for maximum modulus of elasticity or maximum tensile strength.

Additional mixtures involving other aggregates, gradations, and asphalt types need to be studied before a definite conclusion can be made. In addition, consideration of other characteristics such as permanent deformation, or rutting, may require that the asphalt content be altered.

Nevertheless, it is apparent that the optimum asphalt contents for various properties are different and that this fact should be recognized and considered in the design of asphalt mixtures.

CHAPTER 4. CONCLUSIONS AND RECOMMENDATIONS

This report summarizes the findings of a study to develop a technique to estimate the resilient elastic characteristics of asphalt mixtures using the repeated-load indirect tensile test and to evaluate the resilient and static moduli of elasticity and their relationships with fatigue life for purposes of mixture design. Laboratory prepared specimens of two asphalt mixtures containing either gravel or limestone aggregates, and various percentages of asphalt cement (AC-10), were subjected to repeated indirect tensile stresses. Duplicates of the repeated-load specimens were tested statically. All tests were conducted at either 50, 75, or 100°F. Estimates of the instantaneous resilient modulus of elasticity, instantaneous resilient Poisson's ratio, static modulus of elasticity, static Poisson's ratio, and fatigue life were determined.

In order to estimate the stiffness modulus without conducting long-term fatigue or load tests, recommendations were made concerning the procedure for conducting the repeated-load indirect tensile test to obtain this property. Possible relationships involving fatigue life, repeated-load properties, and static properties were also evaluated. In addition, a comparison with the results of some other common tests that are used today was made. The conclusions and recommendations based on the findings of this investigation are stated below.

CONCLUSIONS

Instantaneous Resilient Modulus of Elasticity and Poisson's Ratios

(1) The average instantaneous resilient modulus of elasticity for the three test temperatures of 50, 75, and 100°F were 800×10^3 psi, 320×10^3 psi, and 190×10^3 psi. The total range for all conditions and specimens was 116×10^3 psi to 1100×10^3 psi.

(2) Applied stress level had essentially no effect on the magnitude of the instantaneous resilient modulus values.

(3) The instantaneous resilient modulus tended to decrease with increased number of load applications.

(4) No correlation was found to exist between the instantaneous resilient and static modulus of elasticity.

(5) The instantaneous resilient moduli of elasticity were significantly larger than the static moduli.

(6) The ratios between the resilient moduli and the mean static moduli of elasticity ranged from 0.9 to 5.1 for gravel mixtures and from 1.0 to 10.7 for limestone mixtures.

(7) The instantaneous resilient Poisson's ratios tended to be slightly larger than the static Poisson's ratio. The majority of instantaneous resilient Poisson's ratios were in the range of 0.11 to 0.54 for gravel mixtures and 0.10 to 0.70 for the limestone mixtures. For the static Poisson's ratios the range was 0.13 to 0.35 for gravel and 0.08 to 0.36 for limestone.

(8) No correlation was found to exist between the instantaneous resilient modulus of elasticity and the static modulus over the same stress range used in the repeated-load tests.

Practical Procedure to Determine the Instantaneous Resilient Modulus

(1) An estimate of resilient modulus can be obtained without conducting a long-term repeated-load test.

(2) Reasonable estimates of the modulus could be obtained after about 1.0 percent of the fatigue life.

(3) A test specimen should be subjected to a minimum of 25 load applications before estimating the modulus.

(4) Caution should be exercised to insure that the load-deformation-time measurements used to determine the modulus values are not taken in the failure zone.

Relationships Between Modulus Values and Fatigue Life

(1) No definite relationship could be established which would allow fatigue life to be estimated from a single value of either the static or the instantaneous resilient modulus.

(2) The relationship does have some significance if the points are connected according to increasing asphalt content.

Optimum Asphalt Contents

(1) A definite optimum asphalt content for maximum fatigue life existed. These optimum asphalt contents were in the range of 6 to 6.5 percent which were less than the optimum for maximum bulk density.

(2) An optimum asphalt content for maximum static modulus of elasticity existed but was not well defined at the higher test temperature.

(3) The optimum asphalt contents for static modulus were higher for the gravel mixtures than for the limestone mixtures.

(4) For limestone mixtures the optimum asphalt contents for fatigue life were larger than the optimums for static modulus and tensile strength. For gravel mixtures the optimums for fatigue life were slightly less than or equal to the optimums for static modulus and generally less than the optimums for maximum tensile strength.

(5) The optimum for maximum static modulus was essentially the same as for maximum tensile strength.

(6) There was no well defined optimum asphalt content for a maximum instantaneous resilient modulus relationship indicating that the instantaneous resilient modulus was relatively insensitive to asphalt content.

Choice of Asphalt Content

(1) The optimum asphalt contents for various mixture properties were different. This fact should be recognized and considered in design. Additional mixtures must be evaluated before any definite recommendations can be made.

(2) Mixtures similar to the ones used in this study probably should be designed at an asphalt content equal to or greater than the optimum asphalt content for maximum fatigue life.

(3) If the design is based on static tests, the asphalt content probably should be slightly above the optimum for maximum tensile strength or modulus of elasticity.

RECOMMENDATIONS

(1) The resilient elastic properties of asphalt mixtures tested using the repeated-load indirect tensile test should be used in the pavement design procedures requiring elastic properties.

(2) Additional research should be conducted involving additional asphalt mixtures composed of other aggregates, gradations, and asphalt types to develop definite mixture design procedures. Initially these efforts should be related to the methods currently used by the Texas State Department of Highways and Public Transportation.

(3) Consideration of other characteristics such as permanent deformation or rutting in relation to asphalt content, fatigue life, and elastic properties should be investigated.

(4) The Texas State Department of Highways and Public Transportation should begin to use the repeated-load indirect tensile test to obtain information and to obtain estimates of the resilient modulus of elasticity and Poisson's ratio.

(5) The Texas State Department of Highways and Public Transportation should develop the capability to make and record deformations for test specimens in order to estimate the load-deformation characteristics of pavement materials. This is necessary regardless of the type of test used.

(6) A very definite test procedure should be established for the repeated-load indirect tensile test.

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