EVALUATION OF CONTROL EXTENSION

by

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PREFACE

The following report on "Evaluation of Control Extension" is considered preliminary because the project (3-8-66-93) was terminated after one year of work on a two-year project. This termination is necessary because the principal investigator is leaving The University of Texas. Individuals who have been engaged in this work plan to continue the investigation but not necessarily through a sponsored project. However, additional reports will be prepared concerning this research and will be available to the U.S. Bureau of Public Roads and the Texas Highway Department.

This report contains the results of Research Contract Number 3-8-66-93 performed by the Center for Highway Research, The University of Texas, for the Texas Highway Department and the U.S. Bureau of Public Roads. The purpose of this research was to analyze in detail the processes involved in aerial triangulation so that the various sources of errors could be evaluated and procedures recommended for improving the quality of control extension through aerial triangulation. The report includes work completed through August 31, 1966.

The authors are indebted to personnel from both the Texas Highway Department and U. S. Bureau of Public Roads for their assistance throughout the course of this research.

September 1966

Austin, Texas

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I. INTRODUCTION

The primary reasons for using aerial triangulation are to reduce the time and cost of control survey data which are necessary for the compilation of topographic maps from aerial photographs. Some control surveys are always needed in topographic mapping. The field work for these surveys usually consists of traverse, triangulation, or trilateration for horizontal position determination and differential leveling, trigonometric leveling, or barometric pressure measurements for vertical positions (elevation).

The cost of field work for topographic mapping may be 25% to 75% of the total cost for preparing a map. Furthermore, any blunders in the field work usually result in costly delays or in a very poor map. Therefore, extension of field surveys by aerial triangulation is frequently desirable.

For each stereoscopic model* at least two horizontal control points and at least three vertical control points must be available. Because of the difficulty in coordinating the exposure stations of the photography and the position of the control points, it is usually necessary to have more than the minimum amount of control. For example, in Figure 1 is a sketch of the control distribution commonly used by the Texas Highway Department in their mapping program.

The principle of aerial triangulation may be outlined as follows:

(1) Obtain high-quality photographic measurements of the images of control

*A stereoscopic model is formed by combining the information from two overlapping aerial photographs. This may be accomplished either optically or analytically.

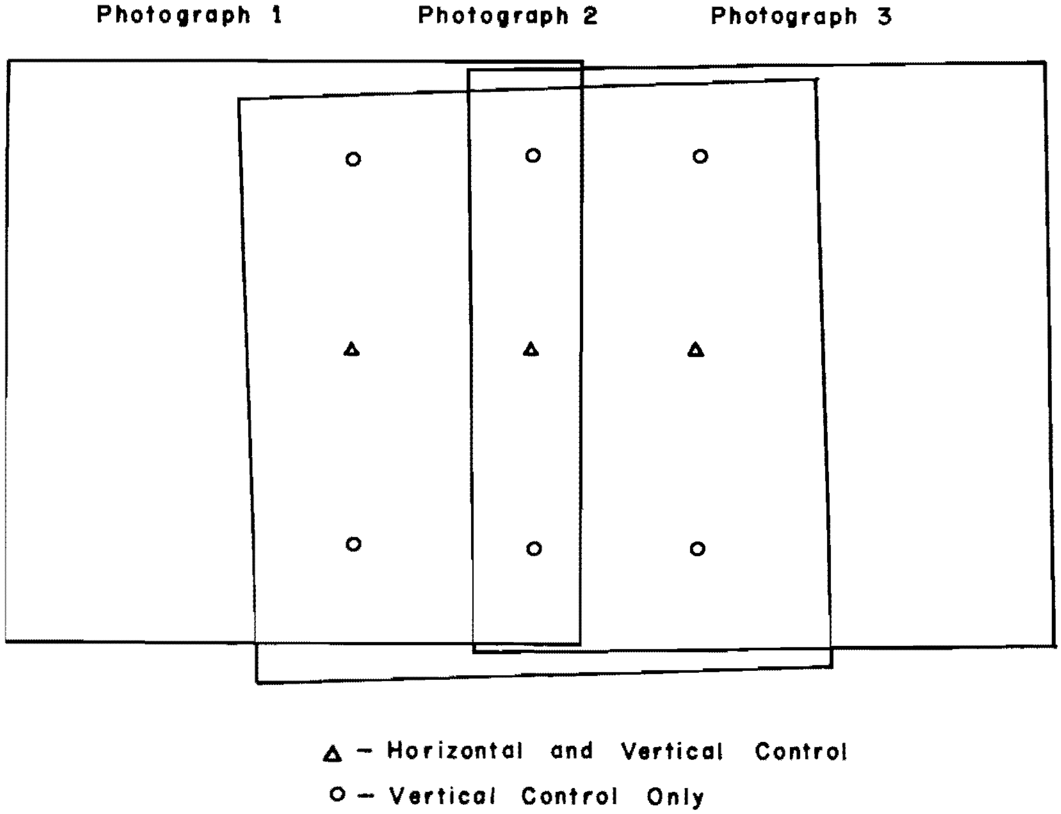

Fig 1. Photograph control distribution.

points.

(2) Solve for the orientation of the photographs with respect to the ground.

(3) Solve for the ground coordinates of the points imaged on the photographs based on the photographic data and the orientation values.

These values found in (3) are used in preparing a map just as though the data had been collected by ground survey methods.

When using aerial triangulation, errors and blunders must be considered, evaluated, and, where possible, corrected. More photographic data may be acquired than the minimum which is needed for the solution of the aerial triangulation. These extra data are used to estimate the quality of measurements and to predict the presence of blunders. Blunders in the photographic data or in the orientation processes usually cause values for the orientation and/or for the ground coordinates to be meaningless. On the other hand, systematic errors (e.g., camera calibration or basic orientation) follow a trend and result in systematic discrepancies in the ground coordinates. Thus, the effects of systematic errors may be reduced by a systematic adjustment. Accidental errors are random in nature and result in a random effect on the computed ground coordinates. These effects of random errors generally will not "fit" a systematic adjustment.

The Texas Highway Department has been compiling topographic maps from aerial photography for approximately 10 years. Their experience in obtaining control data (when the project and the survey crew were located many miles from the central office where the maps are being compiled) has demonstrated a strong need for reduction in the amount of field work. Thus, about 1962 the Texas Highway Department began considering aerial triangulation in their operation. After consideration of procedures being used by other mapping agencies, they elected to establish extensive control points along a short section of Interstate Highway 35 near Austin, Texas, and to photograph this area for experimental work in aerial triangulation. Several commercial firms performed an aerial triangulation computation and adjustment using this photography and a portion of the ground control. In 1964, the photogrammetry section of the Texas Highway Department evaluated the aerial triangulation work which had been performed by commerical firms and evolved a procedure which the Highway Department has been following since then. In January 1965, Mr. Tommie F. Howell reported to the 44th Annual Meeting of the Highway Research Board the results of these investigations in a paper entitled "An Analysis of Stereotriangulation for Highway Engineering Mapping."

The Photogrammetry Section of the Texas Highway Department recognized that the successes which they were experiencing in aerial triangulation were somewhat accidental as the processes they are using have been evolved in somewhat of a piecemeal fashion. Because of the very large scale photography (1 in. = 200 ft) and corresponding large-scale maps (1 in. = 40 ft) commonly used by the Texas Highway Department, it was not possible to adopt a procedure developed by someone else which was ideally suited to their needs. The procedure currently in use by the Texas Highway Department may be outlined as follows:

(1) Use of C-8 Stereoplanigraph to accomplish relative orientation of successive models in the stereoscopic strip.

(2) Print out of machine coordinates of the control points and pass points in the photographic strip.

(3) Apply an adjustment procedure to transform machine coordinates to ground coordinates.

The results of this procedure have generally been satisfactory for horizontal control extension but not for vertical control extension. This condition is understandable when one considers that the Texas Highway Department is compiling topographic maps at a scale of 1 in. = 40 ft with a contour interval of 1 ft which generally requires a vertical accuracy greater than the horizontal accuracy. In using aerial triangulation an error of ±0.5 ft in horizontal position can be accepted, but to accomplish the contour accuracy the errors in vertical position must be somewhat less than this.

Thus, this research has been developed to study and evaluate aerial triangulation procedures specifically for the large-scale topographic mapping being done by the Highway Department.

II. PROCEDURE FOR THIS RESEARCH

A. Literature Study

Aerial triangulation has been investigated extensively and reported in many publications. For example, Professor Gordon Gracie of the University of Illinois found more than 700 titles devoted to this subject. Of these, more than half were concerned with errors in aerial triangulation. In studying the literature, one notes that many of the reports are very specialized and, furthermore, most are concerned with small-scale photography. Thus, many reports although generally devoted to aerial triangulation are not applicable to the process that must be used for the large-scale highway mapping.

Analog aerial triangulation has been used rather extensively since World War II, but until the development of high speed digital computers analytical aerial triangulation was not feasible. The most encouraging accomplishments in analytical aerial triangulation have been reported by the U. S. Coast and Geodetic Survey. This organization uses the Wild PUG point transfer device and the Mann Comparator for measuring photographic coordinates. In one investigation using aerial photography at a scale of 1:40,000 the standard error in elevation determination through aerial triangulation was from 1:5,000 to 1:8,000 of the flying height. If this quality of results could be extended to the large-scale photography with a flying height of approximately 1,200 ft, the standard error of elevation would be less than 1/4 ft. However, the experimental work by the U. S. C. & G. S. on the relatively small-scale photography is not necessarily applicable to the large-scale photography used by the highway department.

The California Highway Department has been using the U. S. C. & G. S. method for their large scale mapping with photography at a scale of 1:3,000. They have reported that using the same type of equipment that is used by U. S. C.

& G. S., they obtain a standard error in elevations of about 1:3,000 of the flying height or about 0.5 ft. This means that the expected error in elevation for one in every ten points will exceed 0.8 ft which is unsatisfactory for compiling maps with a one or two foot contour interval. Due to the results obtained by the California Highway Department, it appeared that there was an inconsistency between results obtained from large-scale and small-scale photography and that the U. S. C. & G. S. method was not applicable to large-scale photography.

After careful study of the U. S. C. & G. S. method, it was discovered that in the relative orientation procedure it is assumed that differences in elevation for points used in relative orientation are small compared to the flying height and the resulting errors are negligible compared to other errors such as in measuring photograph coordinates; therefore, the differences in elevation are neglected. For large-scale photography, this assumption is not applicable. For example, for 1:40,000 photography taken with a six-inch focal length camera, the flying height would be 20,000 ft above the average ground elevation. If there is a 100-ft difference in elevation between the low and high points, the maximum neglected elevation would be 50 ft. This elevation would be 1:400 of the flying height. For 1:3,000 photography taken with the same camera of the same area, this neglected elevation would be 1:30 of the flying height. Also, for low flying heights, the air is less stable causing larger orientation motions and thus compounding the effect of neglecting the elevation differences.

A literature search has been continued throughout the research program because of the extensive publications on the subject and the wide variety of sources for these publications. New reports are being produced all the time. A listing of the most significant publications on aerial triangulation so far as this report is concerned is included in the bibliography.

B. Basic Considerations

A fundamental question to be considered is, "Can aerial triangulation results be improved by an 'improved' adjustment procedure?" There is considerable evidence that other adjustment procedures would not materially improve the results presently being obtained by the Highway Department. For example, the residual errors as reported by Howell[20] are plotted in Figure 2. These errors appear to be random in nature with an equal number of plus and minus values. The results of numerical radial triangulation as reported by Turpin[41] showed that accidental errors are more significant than systematic errors. In Knowles' report to the Texas Surveyor's Association entitled "A Study of the Relationship of the Position Errors to Linear Error of Closure and the Different Methods of Adjustment for Closed Traverses"[22] he points out that even though a systematic adjustment applied to randomly introduced traverse errors gives results that are slightly better than the unadjusted values, any one of several adjustment procedures appears to give approximately equally satisfactory results.

Several adjustment procedures have been considered and studied in this research. The adjustment program used by the Texas Highway Department is an adaptation of one developed for the IBM 650 Computer by Perks.[31] This procedure takes into account errors due to curvature of the earth and scale variation due to elevation, and utilizes a coordinate transformation to change from measured machine coordinates to ground or survey coordinates. Furthermore, the adjustment procedure utilizes a least squares fit to all available control data, both horizontal and vertical.

Although this program, which is designated as SURMAP 17, has worked well for most cases tried by the Texas Highway Department, in some instances the

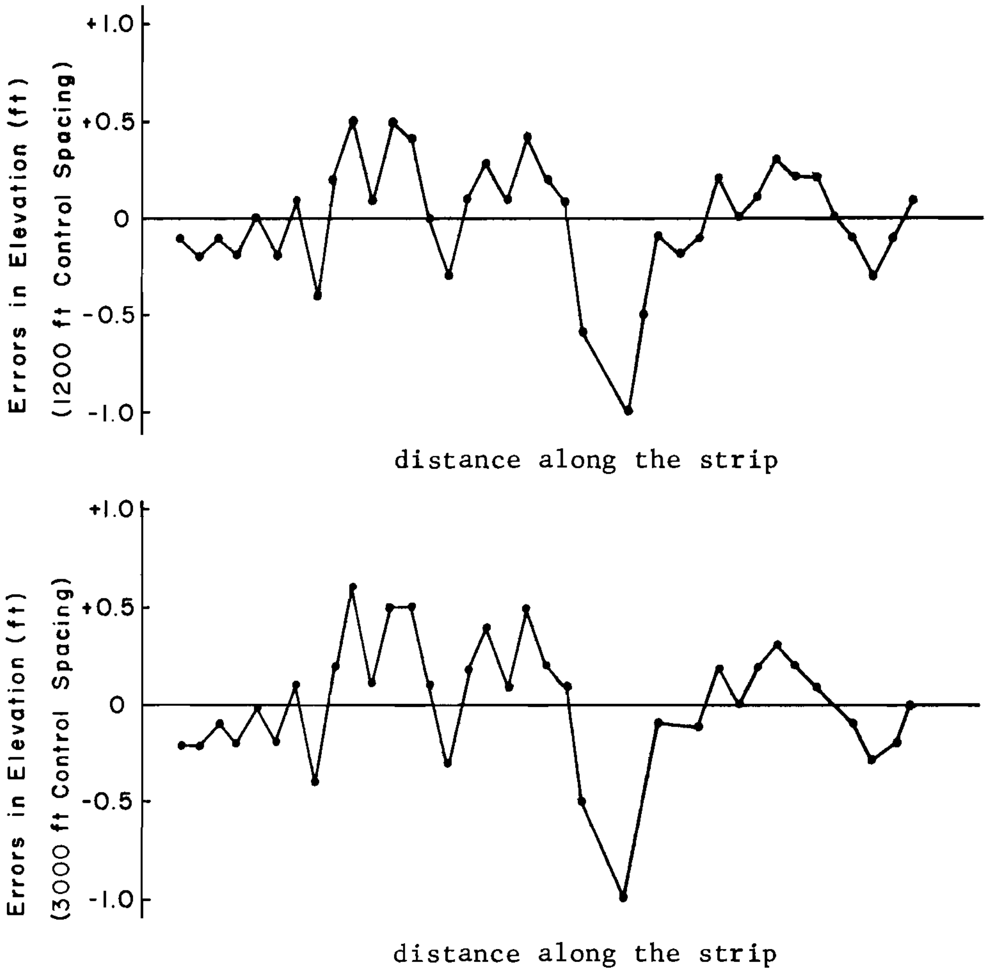

Fig 2. Residual errors reported by Howell.

output has not been satisfactory and, furthermore, no explanation has been evolved to explain a few relatively poor transformations.

Therefore, as an adjunct to this research, a separate linear transformation program (LINTRAN) was developed and tested during June, July and August 1966. A brief explanation of this program with a program listing is included as Appendix A to this report.

Another aerial triangulation procedure described by Harris, Tewinkel, and Whitten of the U.S. Coast and Geodetic Survey, entitled "Analytic Aerial Triangulation,"(19) is probably as nearly analytical or computational as is possible. Significant features of this procedure are outlined as follows:

(1) Transfer of image points from one photograph to the next in a stereo pair is accomplished by the use of a point transfer device.

(2) The photo coordinates of the image points and the fiducial marks are measured with a monocular comparator.

(3) With these photo coordinate measurements and a minimum of ground control data, all elements of orientation and the ground coordinates of additional points are computed and adjusted through an extensive computational program.

Included in the U.S.C. & G.S. Procedure are provisions for such refinements as:

1. Translation to the perspective center.
2. Radial lens distortion.
3. Correction for atmospheric refraction.
4. Computation of relative orientation including a least squares adjustment.
5. Transformation of control data to a secant plane system.

Although the accuracies obtained by the U. S. C. & G. S. are very good, most of their test results are for photography at a scale of 1:40,000 with a corresponding flying height of 20,000 ft. The photography most often used by the Texas Highway Department is flown at a height of 1,000 to 1,500 ft.

Another method to be considered is reported by Anderson, Elum, and McNair of Cornell University entitled "Analytic Aerotriangulation Using Triplets in Strips"(1). This procedure may be outlined as follows:

1. Triplet relative orientation.
2. Triplet assembly.
3. Transformation and adjustment to ground control.

A stereotriplet is used rather than a stereopair as is normally used in aerial triangulation. By using monocular measurements of coordinates of points on each of three photographs data may be obtained which can be fit together without the usual limitation of stereovision. In effect this use of triplets makes possible 100% overlap in one common photograph. The authors state, "Presence of 100% overlap between the one common photograph in successive basic units permits more reliable unit assembly and reduces the propagation of error through the strip."

A study of the technical reports cited above has resulted in the following hypothesis for this research. Accidental errors are more significant than systematic errors in aerial triangulation. Thus, improved quality will require procedures that will improve on the measuring quality or the quality and amount of data used in the orientation calculation.

In order to investigate this hypothesis and to test the associated calculations, it was necessary to develop an analytical orientation procedure through which the cause, effect and behavior of accidental and systematic errors could be studied and evaluated. Since this report contains work completed through August 31, 1966, the following explanations include only the orientation computations and the computer programs. Included in the results section are some preliminary tests of this procedure.

C. Relative Orientation

If two aerial photographs having overlapping coverage are taken such that the second photograph lies in the same plane as the first with its x-axis coinciding with a line defined by the x-axis of the first photograph, then a point in the overlapping area will have the same y photo coordinate on both photographs. Any deviation of this ideal orientation of the second photograph with respect to the first will cause a difference in y photo coordinates. These differences are referred to as y-parallaxes. If y-parallaxes at five points are known it is possible to determine five orientation elements, dY, dZ, dk, dw and dϕ, that will define this deviation of the second photograph (see Figure 3). Such a determination is known as relative orientation. Once the orientation elements have been determined it is possible to rectify the second photograph (i.e., change the x and y photo coordinates of the photo images) so as to produce a hypothetical photograph that would be identical with one that was taken with ideal orientation to the first photograph.

In order to determine the relationship between y-parallax at a point and the orientation elements, each orientation element is first treated independently. Y-parallax (Py) is defined so as to also be a correction to be applied to the y coordinate of the right photograph (i.e., Py=y' - y").

TIP (dϕ) From Figure 4

$$Py_{\phi} = y' - y'' \quad \text{and} \quad r = \frac{f}{\cos \alpha}$$

where $$\alpha = \arctan\left(- \frac{x''}{f} \right)$$

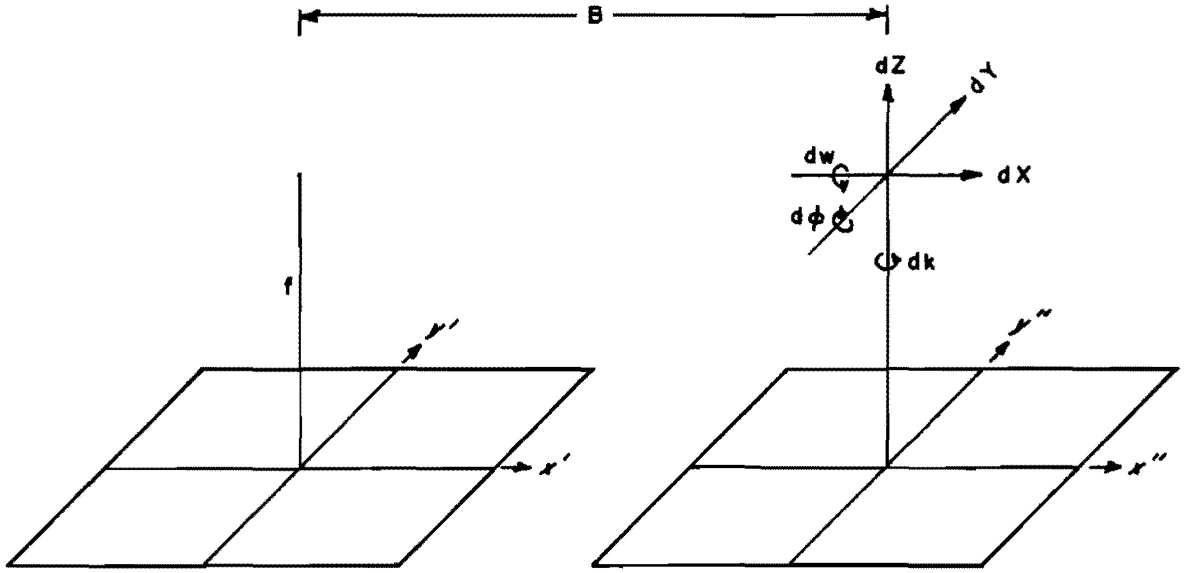

Fig 3. Orientation terms.

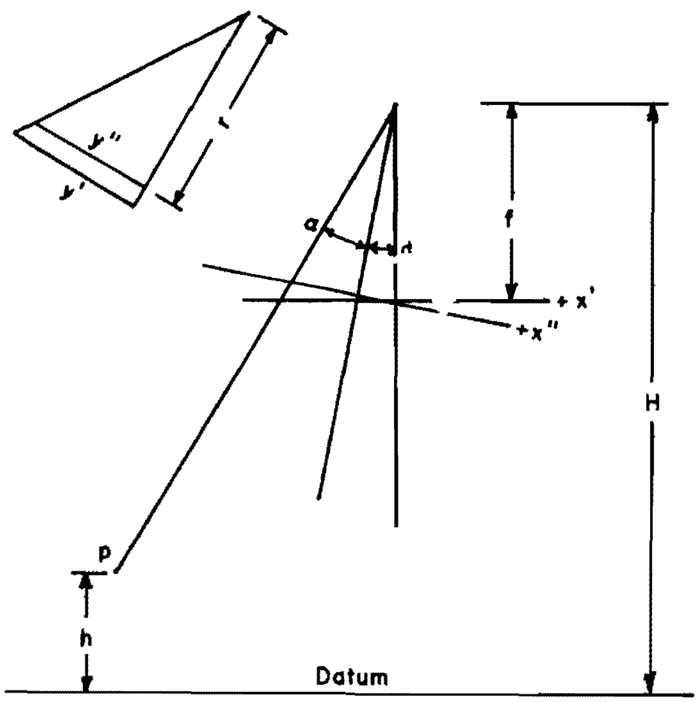

Fig 4. Orientation element $d\phi$.

$$r + \Delta r = \frac{f}{\cos\ (\alpha + d\phi)}$$

$$\Delta r = \frac{f}{\cos\ (\alpha + d\phi)} - r$$

$$= f \left[\frac{1}{\cos(\alpha + d\phi)} - \frac{1}{\cos\ \alpha} \right]$$

From similar triangles

$$\frac{Py_\phi}{\Delta r} = \frac{y''}{r}$$

$$Py_\phi = \frac{y''}{r} \Delta r$$

$$Py_\phi = \frac{y''\ \cos\alpha}{f}\ f \left[\frac{1}{\cos\ (\alpha + d\phi)} - \frac{1}{\cos\alpha} \right]$$

$$Py_\phi = y'' \left[\frac{\cos\ \alpha}{\cos\ (\alpha + d\phi)} - 1 \right] \tag{1}$$

TILT ($d\omega$)

From Figure 5

$$Py_\omega = y' - y''$$

$$y' = f\ \tan\ (\beta + d\omega) \qquad \text{where } \beta = \arctan \left(\frac{y''}{f} \right)$$

$$y'' = f \tan \beta$$

$$Py_\omega = f \left[\tan\ (\beta + d\omega) - \tan\ \beta \right] \tag{2}$$

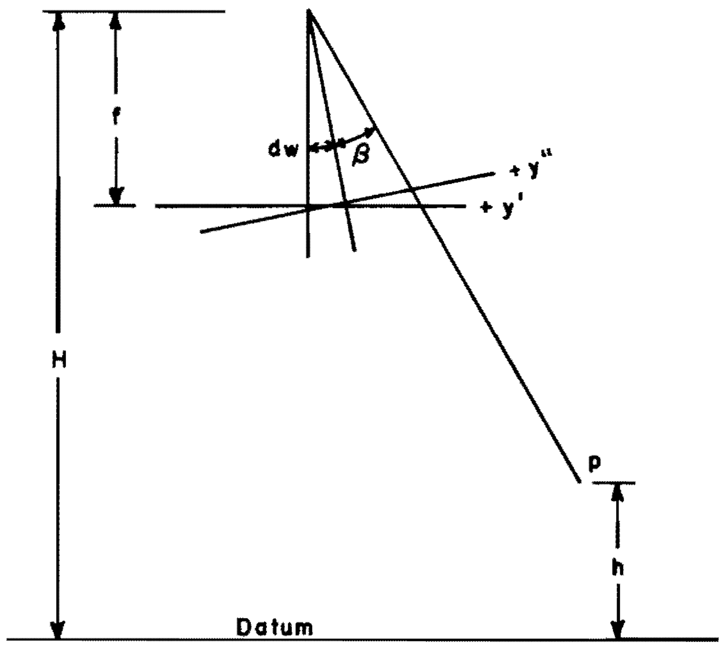

Fig 5. Orientation element dw.

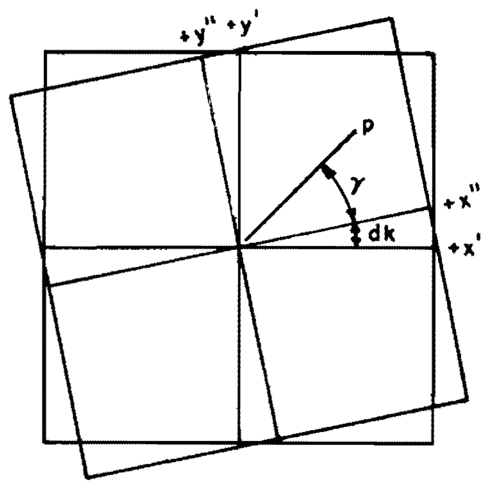

Fig 6. Orientation element dk.

SWING $(d\kappa)$

From Figure 6

$$Py_{\kappa} = y' - y''$$

$$r = \sqrt{y''^2 + x''^2}$$

$$y' = r \sin (\gamma + d\kappa) \qquad \text{where } \gamma = \arcsin \left(\frac{y''}{r}\right)$$

$$y'' = r \sin \gamma$$

$$Py_{\kappa} = \sqrt{y''^2 + x''^2} \left[\sin (\gamma + d\kappa) - \sin \gamma\right] \tag{3}$$

dZ

From similar triangles (see Figure 7)

$$\frac{Y}{y'} = \frac{H - h}{f}$$

$$\frac{Y}{y''} = \frac{H - h + dZ}{f}$$

$$Y = \frac{y'(H-h)}{f} = \frac{y''(H-h + dZ)}{f}$$

$$y'(H-h) = y''(H-h) + y''dZ$$

$$(y' - y'') \ (H-h) = y''dZ$$

$$Py_Z = y' - y''$$

$$Py_Z = \frac{y'' \ dZ}{H-h} \tag{4}$$

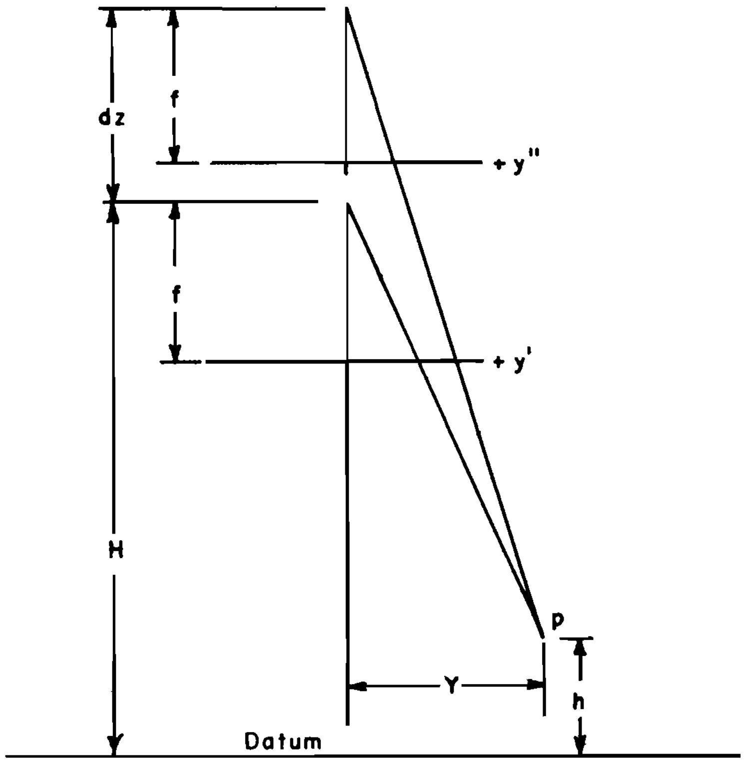

Fig 7. Orientation element dz.

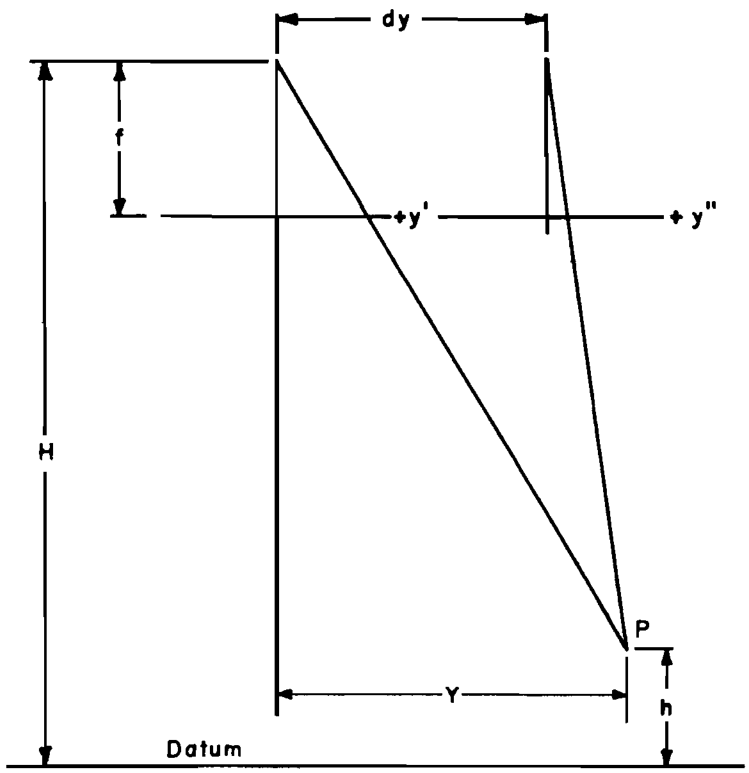

Fig 8. Orientation element dy.

dY

From similar triangles (see Figure 8)

$$\frac{Y}{y'} = \frac{H-h}{f}$$

$$\frac{Y + dY}{y''} = \frac{H-h}{f}$$

$$y' = \frac{f\ Y}{H-h}$$

$$y'' = \frac{f\ (Y - dY)}{H-h}$$

$$Py_Y = y' - y''$$

$$Py_Y = \frac{f\ Y}{H-h} - \frac{f\ Y - fdY}{H-h}$$

$$Py_Y = \frac{fdY}{H-h} \tag{5}$$

The above relationships between y-parallax and the five relative orientation motions are derived assuming no interdependency. In actuality interdependency between the orientation motions must be taken into account. That is to say, if a photograph is moved through an angle $d\phi$ the correction in y due to $d\phi$ must be taken into account when computing the change in y due to a rotation through $d\omega$. A sequence of relative orientation motions is taken to be $d\phi$, $d\omega$, $d\kappa$, dZ, and dY. This sequence makes calculations much simpler because the first two motions render a rectified photograph that lies in a plane parallel to the plane of the photograph to which it is being oriented.

The new set of relationships between y-parallaxes and the relative orientation motions, taking into account interdependency, are as follows:

TIP ($d\phi$)

$$Py_\phi = y'' \left[\frac{\cos \alpha}{\cos (\alpha + d\phi)} - 1\right] \qquad \text{where } \alpha = \arctan\left(-\frac{x''}{f}\right) \tag{6}$$

TILT $(d\omega)$

$$Py_{\omega} = f\left[\tan(\beta + d\omega) - \tan\beta\right]$$

$$\text{where } \beta = \arctan\left(\frac{y'' + Py_{\phi}}{f}\right) \tag{7}$$

SWING $(d\kappa)$

$$Py_{\kappa} = r\left[\sin(\gamma + d\kappa) - \sin\gamma\right] \tag{8}$$

$$\text{where } r = \sqrt{(y'' + Py_{\phi} + Py_{\omega})^2 + (x'' + Cx_{\phi} + Cx_{\omega})^2}$$

$$\gamma = \arcsin\left(\frac{y'' + Py_{\phi} + Py_{\omega}}{r}\right)$$

$$Cx_{\phi} = \text{correction to x due to } d\phi$$

$$Cx_{\omega} = \text{correction to x due to } d\omega$$

dZ

$$Py_{Z} = \frac{(y'' + Py_{\phi} + Py_{\omega} + Py_{\kappa})\ dZ}{H-h} \tag{9}$$

dY

$$Py_{Y} = \frac{f\ dY}{H-h} \tag{10}$$

The y-parallax at any one point due to the five orientation motions is equal to the sum of the y-parallaxes due to each motion. Therefore:

$$Py_{t} = Py_{\phi} + Py_{\omega} + Py_{\kappa} + Py_{z} + Py_{y} \tag{11}$$

In order to rectify a photograph it is also necessary to correct the x photo coordinates. A relationship between the x photo coordinate correction at a point and the five relative orientation motions is obtained in a similar fashion as for the y photo coordinate correction. This analysis yields the following:

TIP $(d\phi)$

$$cx_{\phi} = f\left[\tan\alpha - \tan(\alpha + d\phi)\right] \tag{12}$$

TILT $(d\omega)$

$$cx_{\omega} = (x'' + cx_{\phi})\left[\frac{\cos\beta}{\cos(\beta + d\omega)} - 1\right] \tag{13}$$

SWING $(d\kappa)$

$$cx_{\kappa} = r\left[\cos(\gamma + d\kappa) - \cos\gamma\right] \tag{14}$$

dZ

$$cx_{Z} = \frac{(x'' + Cx_{\phi} + Cx_{\omega} + Cx_{\kappa})dZ}{H-h} \tag{15}$$

dY

$$cx_{Y} = 0 \tag{16}$$

where α, β, γ, and r are the same values as used in the y photo coordinate correction equation.

The total correction for the x photo coordinate of a point due to the five relative orientation motions is:

$$Cx_{t} = Cx_{\phi} + Cx_{\omega} + Cx_{\kappa} + Cx_{z} \tag{17}$$

If a change in the air base is taken into consideration where

$$Cx_{x} = \frac{f\ dX}{H - h} \tag{18}$$

then

$$Cx_{t} = Cx_{\phi} + Cx_{\omega} + Cx_{\kappa} + Cx_{z} + Cx_{x} \tag{19}$$

For the purpose of studying only an analytical relative orientation procedure it is assumed that the flying height of an initial photograph and the distance between any two adjacent photographs are known.* In the expanded form of Equation 11, $d\phi$, $d\omega$, $d\kappa$, dZ, dY, and h are unknown with Py_t being the measured y-parallax at a point. Six observation equations obtained by measuring the x and y photo coordinates of six points on the overlapping area of two photographs will not yield a solution of the orientation elements because each point may have a different value of h. This means there will always be five more unknowns than observation equations. This problem can be surmounted by using five observation equations with assumed elevations and iterating, computing elevations after each iteration.

The expanded form of Equation 11 is non-linear making a direct solution of a set of observation equations impossible. In order to obtain a solution, a linear equation is developed that will approximate Equation 11 and will approach exactness when the orientation motions approach zero.

The derivation of this linear equation is as follows:

TIP $(d\phi)$

$$Py_\phi = y''\left[\frac{\cos\alpha}{\cos(\alpha + d\phi)} - 1\right] \qquad (6)$$

$$= y''\left[\frac{\cos\alpha}{\cos\alpha\cos d\phi - \sin\alpha\sin d\phi} - 1\right]$$

$$= y''\left[\frac{f/r}{(f/r)\cos d\phi - (-x''/r)\sin d\phi} - 1\right]$$

Assuming $\cos d\phi = 1$; $\sin d\phi = d\phi$

$$Py_\phi = y''\left[\frac{f/r}{f/r + (x''/r)\,d\phi} - 1\right]$$

$$= y''\left[\frac{f - (f + x''\,d\phi)}{f + x''\,d\phi}\right]$$

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*In subsequent research absolute orientation will be studied where the flying height and air base are unknown.

Assuming $x''d\phi$ is small compared to f

$$Py_{\phi} = \frac{y''\ x''}{f}\ d\phi \tag{20}$$

TILT (dω)

$$Py_{\omega} = f\left[\tan(\beta + d\omega) - \tan\beta\right] \tag{7}$$

$$= f\left[\frac{\tan\beta + \tan d\omega}{1 - \tan\beta\ \tan d\omega} - \tan\beta\right]$$

Assuming $Py_{\phi} = 0$ then $f \tan\beta = y''$ and

$$Py_{\omega} = \frac{y'' + f\ \tan d\omega}{1-(y''/f)\ \tan d\omega} - y''$$

$$= \frac{fy'' + f^2\ \tan d\omega - fy'' + y''^2\ \tan d\omega}{f - y''\ \tan d\omega}$$

Assuming $\tan d\omega = d\omega$

$$Py_{\omega} = \frac{f^2 d\omega + y''^2 d\omega}{f - y'' d\omega}$$

Assuming $y''d\omega$ is small compared to f

$$Py_{\omega} = \frac{f^2 d\omega + y''^2 d\omega}{f}$$

$$Py_{\omega} = \left[f + \frac{y''^2}{f}\right] d\omega \tag{21}$$

SWING ($d\kappa$)

$$Py_{\kappa} = r\left[\sin\,(\gamma + d\kappa) - \sin\,\gamma\right] \qquad (8)$$

$$= r\left[\sin\,\gamma\,\cos\,d\kappa + \cos\gamma\,\sin\,d\kappa - \sin\,\gamma\right]$$

Assuming $Py_{\phi} = 0$, $Py_{\omega} = 0$, $Cx_{\phi} = 0$, $Cx_{\omega} = 0$

then $r\,\sin\,\gamma = y''$ and $r\,\cos\,\gamma = x''$ and

$$Py_{\kappa} = y''\,\cos\,d\kappa + x''\,\sin\,d\kappa - y''$$

Assuming $\cos\,d\kappa = 1$ and $\sin\,d\kappa = d\kappa$

$$Py_{\kappa} = x''\,d\kappa \qquad (22)$$

dZ

$$Py_{Z} \quad \frac{(y'' + Py_{\phi} + Py_{\omega} + Py_{\kappa})}{H - h}\,dZ \qquad (9)$$

Assuming $Py_{\phi} = 0$, $Py_{\omega} = 0$, $Py_{\kappa} = 0$

$$Py_{Z} = \frac{y''}{H - h}\,dZ \qquad (23)$$

dY

$$Py_{Y} = \frac{f}{H - h}\,dY \qquad (10)$$

The sum of the changes in y due to each orientation motion yields

$$Py_{t} = -\frac{y''x''}{f}\,d\phi + \left[f + \frac{y''^2}{f}\right]\,d\omega + x''\,d\kappa + \frac{y''}{H - h}\,dZ + \frac{f}{H - h}\,dY \qquad (23)$$

An iteration procedure is used with five linear observation equations and their corresponding exact non-linear observation equations. Assuming values for elevations the linear equations yield approximate values of the orientation motions. These motions are used in the exact equations to obtain computed values of y-parallaxes. These parallaxes are compared with the measured y-parallaxes and the differences are used in the linear equations to obtain corrections to the originally computed orientation motions. As the differences in computed and measured parallaxes become small, the orientation corrections become small, making the linear equations approach the exact equations, and convergence is reached. Corrections are then computed for the x photo coordinates and these corrected coordinates are used to compute corrected elevations. The iterating process from approximate to exact equations is then repeated with the corrected elevations, computing more nearly correct elevations after convergence. This process is continued until values of elevations converge.

The linear observation equations enable one to use a least squares adjustment procedure where more than five relative orientation points in a model can be used. Using matrix notation the least squares solution to the observation equations is as follows:

$$x = (A^t A)^{-1} A^t b$$

where

$$x = \begin{bmatrix} d\phi \\ d\omega \\ d\kappa \\ dZ \\ dY \end{bmatrix} \qquad b = \begin{bmatrix} Py_1 \\ Py_2 \\ \cdot \\ \cdot \\ \cdot \\ Py_n \end{bmatrix}$$

A = the coefficient matrix.

III. EVALUATING THE RELATIVE ORIENTATION PROCEDURE

In order to evaluate the analytical relative orientation procedure developed in this report a FORTRAN 63 computer program was written for the CDC 1604 computer located in the Computer Center at The Unviersity of Texas. This program was used to cantilever a strip of eleven models where the twelve photographs were constructed from hypothetical data.

The first photograph in the strip was oriented so that the optical axis was vertical. The flying height was assigned a value and the ground nadir point location was known. The air base between adjacent photographs was constant. By setting these initial conditions, the relative orientation in conjunction with a bx correction for changes in azimuth of flight lines automatically accomplished absolute orientation. This resulted in a convenient means of isolating the errors caused by the relative orientation procedure since procedures for absolute orientation and air base determination were not necessary to compute ground coordinates in X, Y, and Z. A comparison of the computed ground coordinates and the known hypothetical ground coordinates resulted in errors produced only by the relative orientation procedure.

There follows an explanation of the hypothetical data used and the results obtained in a cantilever extension.

A. Generating Hypothetical Data

Using a U.S.C. & G.S. topographic map as a guide, a series of points was selected in such a manner that an equal number of points would appear on each of twelve hypothetical photographs. The photographs were assumed to be taken with a 6-inch focal length camera at about 1200 feet above the ground with 60% overlap. Exact ground coordinates, X, Y, and Z, were assigned to each point. Also, X and Y coordinates were assigned to the ground nadir point of the first photograph.

The orientation in space of each photograph was assumed to be random to the same degree as one might expect the orientation of actual photographs to be when taken under normal flying conditions. These random orientations were computed using normal error distributions for flying height, direction of flight line, and tilt, and using a square distribution for direction of tilt. Random deviates and random numbers were taken from _1,000,000 Random Numbers and 100,000 Random Deviates_, Rand Corporation. The flying height of an exposure station was assumed to be within ±50 feet of the previous exposure station 90% of the time. The direction of flight line between exposure stations was assumed to be within 2° of the direction of the previous flight line 90% of the time. The positive x photo axis at an exposure station was assumed to have the same direction as the flight line to that exposure station. The angle of tilt was assumed to be within 2° 30' of the vertical 90% of the time with the direction of tilt varying from 0° to 360° with respect to the positive y photo axis. The air base between exposure stations was assumed to be constant.

After the random orientations of each photograph were determined, the X and Y coordinates of the ground nadir points of the second through the twelfth photograph were computed from the following equations:

$$X_{N_i} = X_{N_{i-1}} + B \sin Az_i$$

$$Y_{N_i} = Y_{N_{i-1}} + B \cos Az_i$$

where

B = air base

Az_i = azimuth of the ith flight line

The photo coordinates of the selected ground points were computed using the following equations (see Figures 9 and 10):

$$X' = (X - X_N) \cos \alpha + (Y - Y_N) \sin \alpha$$

$$Y' = (Y - Y_N) \cos \alpha - (X - X_N) \sin \alpha$$

$$y' = \frac{Y' f \sec t}{(H-h) \cos t + Y' \sin t}$$

$$x' = \frac{X' (f \sec t - y' \sin t)}{(H-h)}$$

$$x = x' \cos \theta + (y' - f \tan t) \sin \theta$$

$$y = (y' - f \tan t) \cos \theta - x' \sin \theta$$

where

$$\alpha = 270 - (S + Az)$$

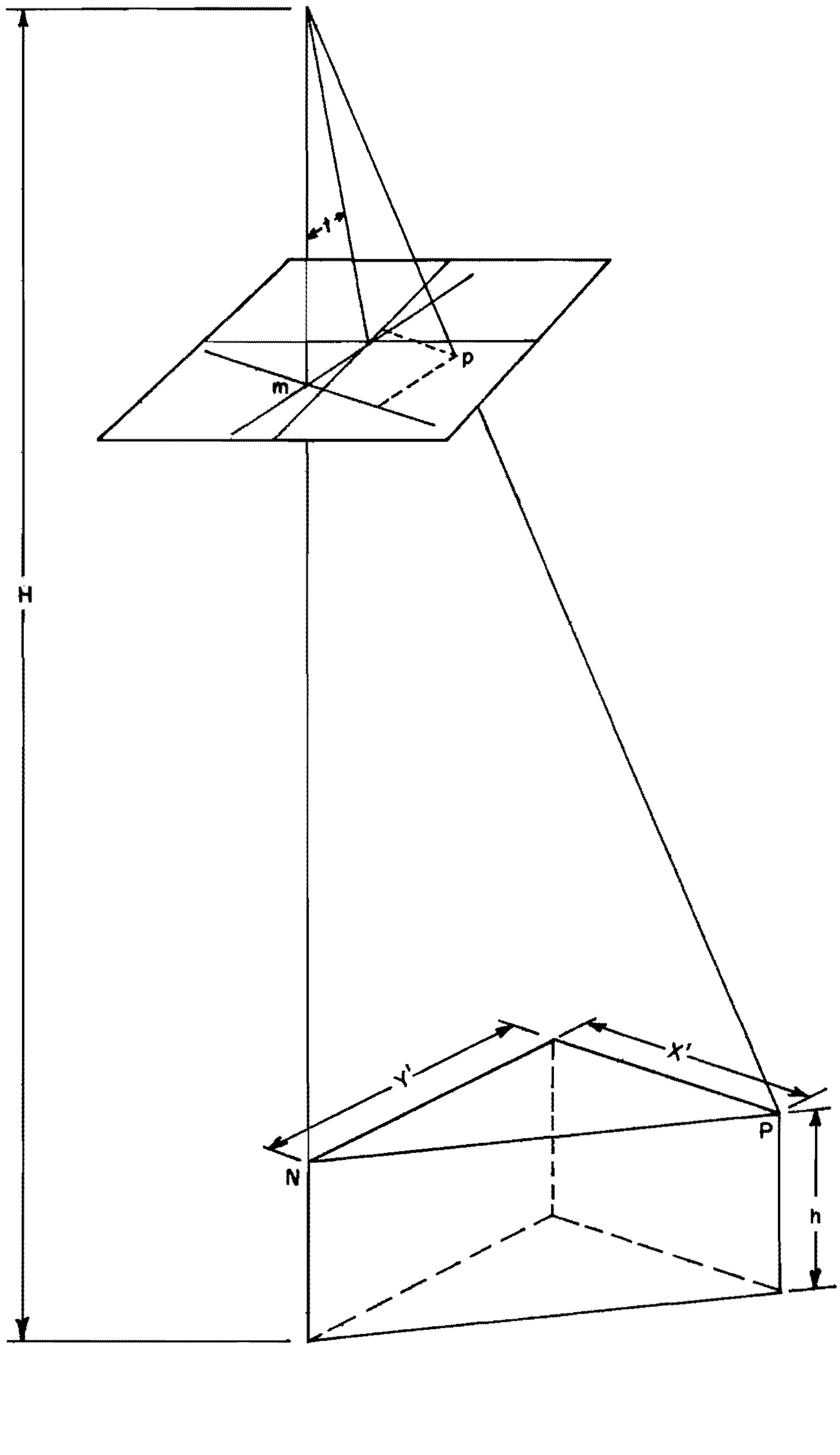

Fig 9. Photo coordinates.

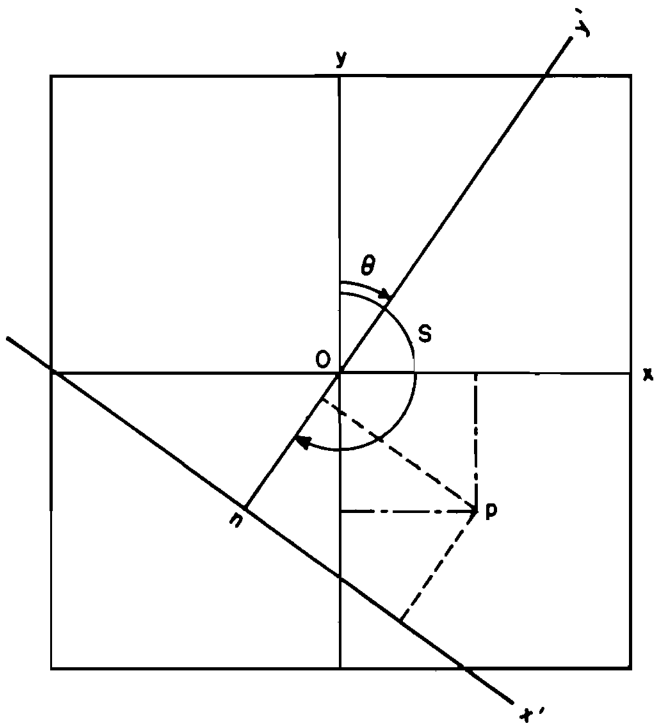

Fig 10. Photo coordinates.

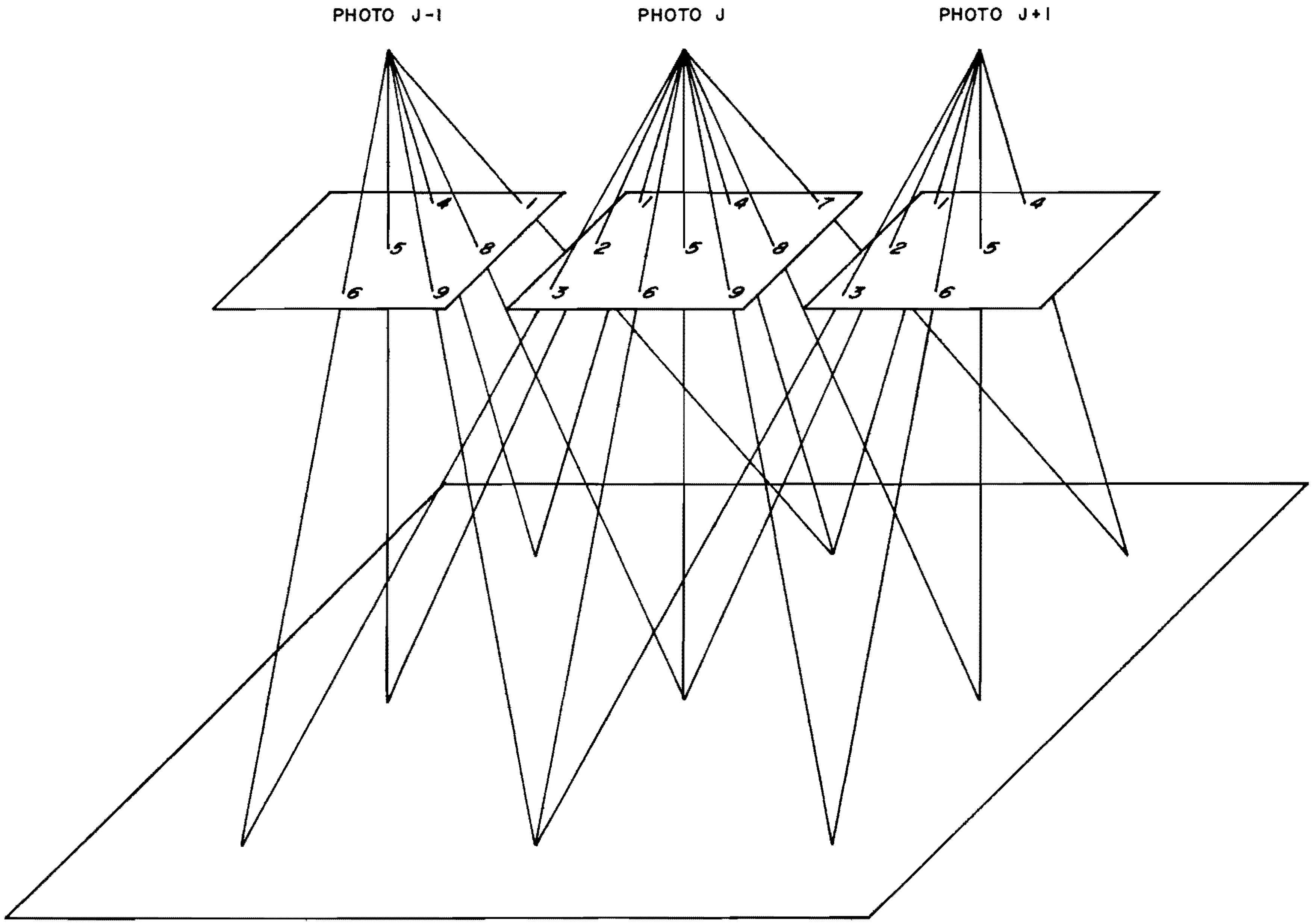

Fig 11. Aerial triangulation scheme.

B. Relative Orientation Procedure

For the results shown in this report hypothetical photographs were developed with nine points per photograph, six of these points being in any one model area (see Figure 11). The photo coordinates computed were exact to as many digits as permitted by the CDC 1604 Computer.

After the hypothetical data had been generated, relative orientation was accomplished by rectifying the second photograph with respect to the first photograph, rectifying the third photograph with respect to the rectified second photograph, etc., to the end of the strip. Due to the repetition involved, only the rectification of the second photograph is explained.

The difference in the y photo coordinates (y-parallaxes) for the six model points on the first and second photographs (points 4 thru 9 on the first photograph and points 1 thru 6 on the second photograph) were computed. These y-parallaxes or y-corrections along with assumed values of elevations were used in the linear approximate equations to obtain a least squares solution of the orientation motions. The orientation motions obtained were substituted into the exact equations solving for x and y photo coordinate corrections. Differences in these computed y corrections and the initial y-parallaxes were re-entered into the linear equations to compute corrections to the originally computed orientation motions. This process was repeated for seven iterations with the final x photo coordinate corrections being applied to the x photo coordinates of points 1 thru 6 on the second photograph. Seven iterations were used to assure convergence. Also, an x photo coordinate correction was made for the change in the air base due to the deviation in the flight line to the second exposure station.

Corrected elevations for the six model points were computed from the x photo coordinates of points 4 thru 9 on the first photograph and the corrected x photo coordinates of points 1 thru 6 on the second photograph. These corrected elevations were used to repeat the iteration process of solving for the relative orientation motions and the x and y photo coordinate corrections. The linear equations, however, were revised to take into account interdependency among the orientation motions. Approximations of these interdependency terms were the computed corrections in the photo coordinates for each orientation motion obtained during the previous set of iterations. This process was repeated for seven iterations with the final x and y photo coordinate corrections being applied to the original x and y photo coordinates of points 1 thru 6 on the second photograph.

In order to correct the photo coordinates of the three points 7, 8, and 9 on the second photograph it was necessary to know the elevations of these points as well as the correct orientation motions of the second photograph. Since these three points do not appear on the first photograph, their elevations could not be computed in the same manner as for the six model points. Assumed elevations along with the correct orientation motions of the second photograph were used in the exact equations to compute approximate corrections for the x and y photo coordinates of the three points. The approximate photo coordinates of these three points and the correct photo coordinates of points 4, 5 and 6 on the second photograph were used to obtain an approximate rectification of the points 1 thru 6 on the third photograph using the same procedure as for the rectification of points 1 thru 6 on the second photograph. Corrected elevations for the points 7, 8 and 9 on the second photograph were obtained from the approximately correct x photo coordinates of points 7, 8, and 9 on the second photograph and 4, 5, and 6 on the third photograph. These corrected elevations were substituted for the assumed elevations of the three points and the process was iterated, computing more accurate values of elevations and x and y photo coordinates of the three points after each iteration. After seven iterations the x and y photo coordinates of the three points 7, 8, and 9 on the second photograph were assumed correct.

This concluded the complete rectification of the second photograph. The entire relative orientation procedure was then repeated for the third photograph and so on until the entire strip of twelve photographs was rectified.

Using the known flying height of the first photograph and the known air base between exposure stations, the ground coordinates in X, Y and Z were computed for all of the hypothetical points. The results of these computations and the residual errors are shown in Table 1. It should be noted that these errors listed in Table 1 are errors resulting from cantilevering out eleven photographs from the first photograph with no ground control after the first model and with no adjustment procedure being applied.

TABLE 1

MODEL	COMPUTED ELEVATION	RESIDUAL	COMPUTED GROUND X	RESIDUAL	COMPUTED GROUND Y	RESIDUAL
1	727.00000	.00000	1340.00000	.00000	3340.00000	.00000
	723.00000	.00000	1760.00000	.00000	2810.00000	.00000
	718.00000	.00000	2110.00000	.00000	2220.00000	.00000
	710.00000	.00000	1900.00000	.00000	3720.00000	.00000
	716.00000	.00000	2240.00000	.00000	3170.00000	.00000
	700.00000	.00000	2660.00000	.00000	2520.00000	.00000
2	750.00000	.00000	2500.00000	.00000	4180.00000	.00000
	720.00000	.00000	2830.00000	.00000	3580.00000	.00000
	720.00000	.00000	3210.00000	.00000	3090.00000	.00000
3	749.99999	-.00001	3100.00000	.00000	4350.00000	.00000
	739.99999	-.00001	3570.00000	.00000	3890.00000	.00000
	719.99999	-.00001	3790.00001	+.00001	3400.00000	.00000
4	749.99999	-.00001	3790.00000	.00000	4700.00001	+.00001
	739.99999	-.00001	4080.00001	+.00001	4300.00001	+.00001
	719.99999	-.00001	4370.00002	+.00002	3800.00000	.00000
5	730.00000	.00000	4330.00000	.00000	5200.00004	+.00004
	719.99998	-.00002	4900.00004	+.00004	4580.00004	+.00004
	709.99998	-.00002	5170.00006	+.00006	4100.00002	+.00002
6	710.00005	+.00005	5000.00001	+.00001	5630.00004	+.00004
	705.00006	+.00005	5320.00002	+.00002	5000.00005	+.00005
	700.00007	+.00007	5800.00003	+.00003	4470.00006	+.00006
7	735.00006	+.00006	5610.00001	+.00001	6020.00004	+.00004
	720.00007	+.00007	5870.00001	+.00001	5550.00003	+.00003
	720.00008	+.00008	6300.00000	.00000	5000.00004	+.00004
8	749.99998	-.00002	6220.00003	+.00003	6380.00012	+.00012
	729.99994	-.00006	6510.00007	+.00007	5800.00006	+.00006
	739.99991	-.00009	6880.00013	+.00013	5199.99998	-.00002
9	760.00002	+.00002	6810.00008	+.00008	6790.00015	+.00015
	759.99995	-.00005	7110.00014	+.00014	6190.00008	+.00008
	762.99991	-.00009	7450.00021	+.00021	5750.00000	.00000
10	750.00041	+.00041	7420.00002	+.00002	7080.00009	+.00009
	765.00028	+.00028	7780.00007	+.00007	6560.00025	+.00025
	770.00017	+.00017	8110.00020	+.00020	6100.00027	+.00027
11	740.00384	+.00384	7999.99892	-.00108	7399.99866	-.00134
	750.00362	+.00362	8429.99758	-.00242	6920.00108	+.00108
	760.00332	+.00332	8799.99682	-.00318	6320.00325	+.00325

IV. CONCLUSIONS AND RECOMMENDATIONS

The most significant accomplishment of this research was the development of a computational procedure for relative orientation of a series of photographs (each photograph being referenced to the preceding photograph), which is essentially exact in form. All aspects of interdependency between elements of relative orientation have been included. Also variation in elevation has been included in the relative orientation since elevation variation will influence the relative orientation calculations when the data taken from a "tilted" photograph is rectified to an equivalent vertical. Thus, this analytical relative orientation procedure may be used to evaluate the size and distribution of errors, and the amount and spacing of control without the results reflecting computational troubles.

Since this report is being prepared before all the proposed research has been completed, the basic recommendation is to use the programs thus far developed to evaluate the effect of errors (in photographic measuring) on computed ground coordinates.

Also, the complete analytical procedure, as described in section III, may be very effective as an aerial triangulation bridging process and should be tested for its practical application.

APPENDIX A

LINTRAN

LINTRAN

At the present time the Texas Highway Department procures the intermediate control data for photogrammetric mapping by stereo-triangulation methods. The procedure employs both a Zeiss C-8 stereoplanigraph, which is used for measurement of three-dimensional coordinates at primary and intermediate control points in a series of stereoscopic optical models, and a CDC 1604 high speed digital computer, which is used for automated transformation of the observed "machine coordinates" to geodetic coordinates. The transformation process is coded in a Fortran 63 language computer program named SURMAP 17 which is a modified version of the program oultined by Michael Perks[31] in his paper, "A Numerical Adjustment Procedure for Aerotriangulation Programmed for IBM 650 Computer", published in the May, 1962 issue of <u>The Canadian Surveyor</u>. In both the original and the revised programs adjustments to horizontal position are interrelated with vertical position and vice-versa; a transformation without rather extensive ground control is impossible.

There are indications that a program which could accomplish the transformation independent of vertical information would be of value in certain instances, e.g., vertical positions are not needed in the compilation of a planimetric map.

In addition, some of the involved operations developed by Perks in his original program have been difficult to follow, and the information generated by the derivative program, SURMAP 17, does not appear to be entirely reliable. For instance, horizontal positions have varied considerably (± 4 ft is typical) between computer runs processing theoretically equivalent data; the only changes made between these runs were either to impose a uniform shift on the elevation datum or to vary the number of avaliable control points. The source of this problem has not been located, but a less complex transformation program could provide a convenient check in cases where questionable results occur.

In an attempt to satisfy these needs, students and other personnel engaged in photogrammetric research at The University of Texas developed a computer program which they named LINTRAN. As the acronym suggests, LINTRAN is a Fortran 63 coding of an algorithm which converts horizontal "machine coordinates" to horizontal geodetic coordinates through a simple linear coordinate transformation. No least squares adjustment is attempted since in this procedure no redundant information is made available.

LINTRAN was first tested on data collected from a photo strip in Gray County, Texas, along IH 40 between Stations 860+48 and 930+51. It was in this area that problems with SURMAP 17 first became evident. The Texas Highway Department personnel did obtain satisfactory information with SURMAP 17, after changing all elevation data by a uniform amount. Also, the data was processed by the Bureau of Public Roads using a Coast and Geodetic Survey programmed adjustment procedure. The satisfactory THD results and the BPR results were in good agreement with a maximum deviation of a little more than a foot. The LINTRAN results agreed just as well with each of the other two sets of results.

LINTRAN was further tested in connection with a test strip which was laid out by the Highway Department along IH 35 in Travis County just north of Austin, Texas, during a previous equipment evaluation program. The strip contained 38 usable horizontal ground control points spaced fairly regularly along a line 2 miles in length. These tests were conducted in order to obtain an absolute check on LINTRAN'S capabilities and to establish guide lines for optimum control spacing. The results indicated that a maximum distance of

2,500 ft should be maintained between control points to provide adequate ground control and that, with such suitably spaced control, generated horizontal position will be reliable to within ± 1.0 ft when using photography at a scale of 1:2400.

Preparation of punched data cards for processing by LINTRAN differs but little from SURMAP 17 format. Station name, machine coordinates, and geodetic coordinates all occupy their accustomed card fields. Cards for stations to be used as control points must be placed in front of the remaining station cards. The computer will assign control status to the first station cards read; this process terminates when the first card with a blank Y ground coordinate field is encountered. At this point the computer assigns that card and all the remaining station cards to non-control status and subsequently computes the corresponding horizontal geodetic positions. At the front of all station cards (both control and non-control) a lead or identification card is required. This card signals the computer that data to be processed follow; some sort of information must occupy any or all of the first eight columns of this lead card, although the nature of the information is in no way restricted. The entire image of this lead card appears at the head of each page of output and provides a convenient means of identification. Following the last station card there should be a blank card which signals the computer that the previous set of data is complete. Then results are computed and printed out for that data and the computer checks for another set of data. At this point another lead card (first eight columns non-blank) will initiate another processing run; a card with a blank field in the first eight columns will terminate the job. LINTRAN is designed to process any number of sets of data so long as compatibility with the line and time limits on the MCS card is maintained.

It is well to note that LINTRAN was developed with "right-handed" rectangular coordinate systems in mind, i.e., the +Y axis is assumed to be separated from the +X axis by a counter-clockwise angle of 90°.

DEFINITION OF TERMS FOR LINTRAN

Main Program

IDC(I)	=	Station names of control points
IDENT(I)	=	Locations for storage of heading for each page of output
IDNC(I)	=	Station names of non-control points
NCC	=	Number of ground control points
NMC	=	Number of points for which horizontal ground coordinates is to be computed
XCALV(I)	=	X ground coordinates of non-control points
XG(I)	=	X ground coordinates of control points
XM(I)	=	X machine coordinates of non-control points
XMC(I)	=	X machine coordinates of control points
YCALV(I)	=	Y ground coordinates of non-control points
YG(I)	=	Y ground coordinates of control points
YM(I)	=	Y machine coordinates of non-control points
YMC(I)	=	Y machine coordinates of control points

Subroutine INDAT

ICC(I)	=	IDC(I) in main program
INC(I)	=	IDNC(I) in main program

Subroutine VSTRAN

VSCL(I)	=	Scaling factors
VTHETA(I)	=	Angles of rotation

Subroutine SORTEM

I1	=	(I) station name
I2	=	(I+1) station name
X1	=	(I) X machine coordinates
X2	=	(I+1) X machine coordinates
X3	=	(I) X ground coordinates
X4	=	(I+1) X ground coordinates
Y1	=	(I) Y machine coordinates
Y2	=	(I+1) Y machine coordinates
Y3	=	(I) Y ground coordinates
Y4	=	(I+1) Y ground coordinates

Function ORIENT

ALPHA	=	Angle of inclination from the +X axis of the line between two successive control points in the ground coordinate system
BETA	=	Angle of inclination from the +X axis of the line between two successive control points in the machine coordinate system
XMAJ1	=	X ground coordinate of the (I-1) control point
XMAJ2	=	X ground coordinate of the (I) control point
XMIN1	=	X machine coordinate of the (I-1) control point
XMIN2	=	X machine coordinate of the (I) control point
YMAJ1	=	Y ground coordinate of the (I-1) control point
YMAJ2	=	Y ground coordinate of the (I) control point
YMIN1	=	Y machine coordinate of the (I-2) control point
YMIN2	=	Y machine coordinate of the (I) control point

```
      PROGRAM LINTRAN
      COMMON/BLOCK1/IDENT(10)
      COMMON/B1/IDNC(200),IDC(20)/B2/XMC(20),YMC(20)/B3/XM(200),YM(200)
      COMMON/B4/XG(20),YG(20)/B5/XCALV(200),YCALV(200)
   60 FORMAT  (10A8)
      ITEST = 8H
    1 READ 60, (IDENT(I),I=1,10)
      NCC=NMC=0
      IF (IDENT(1)-ITEST) 11,10
   11 CALL        INDAT (NCC,NMC)
      LS = NCC -1
      DO 27 I=1,LS
      LQ = NCC - I
      DO 27 J=1,LQ
   27 CALL        SORTEM (XMC(J),XMC(J+1),YMC(J),YMC(J+1),
     *                    XG (J),XG (J+1),YG (J),YG (J+1),
     $                    IDC(J),IDC(J+1))
      LS = NMC - 1
      DO 28 I=1,LS
      LQ = NMC - I
      DO 28 J=1,LQ
   28 CALL        SORTEM (XM(J),XM(J+1),YM(J),YM(J+1),
     *                    XM(J),XM(J+1),YM(J),YM(J+1),
     *                    IDNC(J),IDNC(J+1))
      CALL        VSTRAN (NCC,NMC)
      CALL        OUTDAT (NCC,NMC)
      GO TO 1
   10 PRINT 20
   20 FORMAT (1H1)
      END
      SUBROUTINE INDAT (NCC,NMC)
      COMMON/B1/INC (200),ICC(20)/B2/XMC(20),YMC(20)/B3/XM(200),YM(200)
      COMMON/B4/XG(20),YG(20)
  333 FORMAT (8X,A8,F7.2,F6.2,6X,2(F9.2))
      DO 666 I=1,20
      READ 333, ID,X1,Y1,Y2,X2
      IF (Y2) 70,71
   70 ICC(I)=ID
      XMC(I)=X1
      YMC(I)=Y1
      XG (I)=X2
      YG (I)=Y2
  666 NCC = NCC+1
   71 INC(1)=ID
      XM(1)=X1
      YM(1)=Y1
      NMC = NMC+1
      DO 777I=2,200
      READ 333, ID,X1,Y1,Y2,X2
      IF (X1) 72,73
   72 NMC = NMC+1
      XM(I)=X1
      YM(I)=Y1
  777 INC(I)=ID
   73 END
```

```
      SUBROUTINE OUTDAT (NCC,NMC)
      COMMON/BLOCK1/IDENT(10)
      COMMON/B1/IDNC(200),IDC(20)
      COMMON/B4/XG(20),YG(20)/B5/XCALV(200),YCALV(200)
    1 FORMAT (1H1,10A8//)
    2 FORMAT (19X,35HGEODETIC POSITION OF CONTROL POINTS)
    3 FORMAT (1H0,17X,7HSTATION,10X,1HX,16X,1HY)
    4 FORMAT (1H0,16X,A8,5X,F10.2,6X,F10.2)
    5 FORMAT (23X,27HCOMPUTED GEODETIC POSITIONS)
      NO = 24
      IM = 1
      PRINT 1, (IDENT(I),I=1,10)
      PRINT 2  $  PRINT 3
      DO 50 I=1,NCC
   50 PRINT 4, IDC(I),XG(I),YG(I)
   16 PRINT 1, (IDENT(I),I=1,10)
      PRINT 5  $  PRINT 3
      L = 0
      DO 60 I=IM,NMC
      L = L + 1
      PRINT 4,IDNC(I),XCALV(I),YCALV(I)
      IF (NO .EQ.L  ) 55,60
   55 IF ( I .EQ. NMC) 75,26
   26 IM = I+1  $ GO TO 16
   60 CONTINUE
   75 END
      SUBROUTINE VSTRAN (NCC,NMC)
      DIMENSION VSCL(20),VTHETA(20)
      COMMON/                          B2/XMC(20),YMC(20)/B3/XM(200),YM(200)
      COMMON/B4/XG(20),YG(20)/B5/XCALV(200),YCALV(200)
      DO 72 K=2,NCC
      VSCL(K) = SQRTF( (XG (K)-XG (K-1) )**2 + (YG (K)-YG (K-1) )**2 ) /
     *          SQRTF( (XMC(K)-XMC(K-1) )**2 + (YMC(K)-YMC(K-1) )**2 )
   72 VTHETA(K) = ORIENT (XG (K-1),XG (K),YG (K-1),YG (K),
     *                    XMC(K-1),XMC(K),YMC(K-1),YMC(K))
      L = 0
      LSD = 1
      DO 50 J=2,NCC
      DO 90 I=LSD,NMC
      IF (     J .EQ.NCC       ) 20,12
   20 IF (XM(I) .EQ. XMC(J)) 21,40
   21 IF (YM(I) .EQ. YMC(J)) 22,40
   22 LP1=2
      GO TO 18
   12 IF ( XM(I) .LT. XMC(J) ) 11,15
   15 IF ( XM(I) .EQ. XMC(J) ) 16,17
   16 L = L+1
      IF ( YM(I) .EQ. YMC(J) ) 18,19
   19 GO TO 40
   18 XCALV(I) = XG (J)
      YCALV(I) = YG (J)
   17 GO TO (50,90), LP1
   11 L = L+1
   40 XCALV(I) =XG (J-1) + (XM(I)-XMC(J-1))*COSF(VTHETA(J))*VSCL(J)
     *                   - (YM(I)-YMC(J-1))*SINF(VTHETA(J))*VSCL(J)
```

```
      YCALV(I) =YC (J-1) + (XM(I)-XMC(J-1))*SINF(VTHETA(J))*VSCL(J)
     *                   + (YM(I)-YMC(J-1))*COSF(VTHETA(J))*VSCL(J)
   90 CONTINUE
   50 LSD = L+1
      END
      SUBROUTINE SORTEM (X1,X2,Y1,Y2,X3,X4,Y3,Y4,I1,I2)
      IF ( X1 .GT. X2 ) 5,6
    5 SAVEX = X1
      SAVEY = Y1
      HOLDX = X3
      HOLDY = Y3
          X1 = X2
          Y1 = Y2
          X3 = X4
          Y3 = Y4
          X2 = SAVEX
          Y2 = SAVEY
          X4 = HOLDX
          Y4 = HOLDY
          IT = I1
          I1 = I2
          I2 = IT
    6 END
      FUNCTION ORIENT (XMAJ1,XMAJ2,YMAJ1,YMAJ2,XMIN1,XMIN2,YMIN1,YMIN2)
      ALPHA = ASINF ( (YMAJ2 - YMAJ1) / SQRTF ( (XMAJ2 - XMAJ1)**2 +
     *                                          (YMAJ2 - YMAJ1)**2 ) )
      BETA  = ASINF ( (YMIN2 - YMIN1) / SQRTF ( (XMIN2 - XMIN1)**2 +
     *                                          (YMIN2 - YMIN1)**2 ) )
      PI   =   3.141592654
      IF (XMIN2 .GT. XMIN1) 1,2
    1 IF (XMAJ2 .GT. XMAJ1) 3,5
    2 IF (XMAJ2 .GT. XMAJ1) 4,6
    3 ORIENT =  ALPHA - BETA
      RETURN
    4 ORIENT = PI + ALPHA + BETA
      RETURN
    5 ORIENT = PI - ALPHA - BETA
      RETURN
    6 ORIENT = -ALPHA + BETA
      END
      END
```

APPENDIX B

TABLES AND PROGRAM

TABLE 2

PHOTOGRAPH ORIENTATION ELEMENTS

PHOTO NO.	FLYING HT	AZ OF FLT D M S	TILT D M S	SWING D M S
1	1900.0	58 30 00	0 00 00	
2	1936.9	57 20 50	0 54 59	115 18 41
3	1940.4	57 03 23	1 33 55	104 16 00
4	1975.3	57 11 12	0 32 49	270 09 17
5	1986.9	58 18 50	1 05 39	98 06 38
6	1974.3	57 11 32	0 19 30	66 28 26
7	1979.5	58 55 19	3 57 10	358 00 07
8	1966.7	58 09 28	2 04 28	203 43 39
9	1994.0	57 51 38	0 26 10	222 41 32
10	1983.8	57 09 21	1 07 01	38 22 33
11	1933.0	56 04 11	0 12 56	351 11 26
12	1902.0	55 29 46	1 59 10	47 24 04

TABLE 3

TILTED PHOTO COORDINATES

PHOTO NO.	STA NO.	ELEV	X-COORD GROUND	Y-COORD GROUND	X-COORD PHOTO	Y-COORD PHOTO
1	4	727.0	1400.0	3340.0	8.10546	80.36461
	5	723.0	1760.0	2810.0	11.96576	- 2.77683
	6	718.0	2110.0	2220.0	10.64513	-91.20525
	7	710.0	1900.0	3720.0	88.01513	87.25328
	8	716.0	2240.0	3170.0	88.78596	4.46717
	9	700.0	2660.0	2520.0	89.94973	-93.84811
2	1	727.0	1400.0	3340.0	-78.18478	75.85025
	2	723.0	1760.0	2810.0	-76.08470	- 3.94265
	3	718.0	2110.0	2220.0	-78.98468	-89.35188
	4	710.0	1900.0	3720.0	- 0.17378	81.53731
	5	716.0	2240.0	3170.0	- 1.49805	1.55638
	6	700.0	2660.0	2520.0	- 1.10018	-94.19791
	7	750.0	2500.0	4180.0	96.93793	93.26275
	8	720.0	2830.0	3580.0	89.10984	4.97505
	9	720.0	3210.0	3090.0	96.52245	-73.29705
3	1	710.0	1900.0	3720.0	-85.43261	80.10547
	2	716.0	2240.0	3170.0	-87.83098	1.55160
	3	700.0	2660.0	2520.0	-87.15368	-92.47130
	4	750.0	2500.0	4180.0	6.31122	91.76542
	5	720.0	2830.0	3580.0	0.10767	4.49087
	6	720.0	3210.0	3090.0	6.67288	-72.95454
	7	750.0	3100.0	4350.0	83.51947	69.32994
	8	740.0	3570.0	3890.0	102.05453	-13.70956
	9	720.0	3790.0	3400.0	90.12269	-81.05694
4	1	750.0	2500.0	4180.0	-89.47923	90.95720
	2	720.0	2830.0	3580.0	-93.23068	5.39237
	3	720.0	3210.0	3090.0	-86.62027	-70.00839
	4	750.0	3100.0	4350.0	-14.82322	67.84412
	5	740.0	3570.0	3890.0	3.27559	-11.87845
	6	720.0	3790.0	3400.0	- 6.59006	-76.18867
	7	750.0	3790.0	4700.0	80.49089	57.56987
	8	740.0	4080.0	4300.0	83.12297	- 3.44789
	9	720.0	4370.0	3800.0	78.51016	-73.14009

TABLE 3 (CONT.)

PHOTO NO.	STA NO.	ELEV	X-COORD GROUND	Y-COORD GROUND	X-COORD PHOTO	Y-COORD PHOTO
5	1	750.0	3100.0	4350.0	-98.98987	65.55138
	2	740.0	3570.0	3890.0	-79.39700	-11.97416
	3	720.0	3790.0	3400.0	-86.47792	-75.23790
	4	750.0	3790.0	4700.0	- 5.37132	58.38933
	5	740.0	4080.0	4300.0	- 0.83587	- 2.19048
	6	720.0	4370.0	3800.0	- 2.68861	-71.72030
	7	730.0	4330.0	5200.0	83.01554	75.41701
	8	720.0	4900.0	4580.0	102.13588	-25.63822
	9	710.0	5170.0	4100.0	98.72603	-92.03301
6	1	750.0	3790.0	4700.0	-95.78403	59.87127
	2	740.0	4080.0	4300.0	-91.60314	- 1.37443
	3	720.0	4370.0	3800.0	-93.31699	-71.19789
	4	730.0	4330.0	5200.0	- 5.69598	74.75878
	5	720.0	4900.0	4580.0	11.75435	-26.73953
	6	710.0	5170.0	4100.0	7.66488	-92.68452
	7	710.0	5000.0	5630.0	90.75132	73.62753
	8	705.0	5320.0	5000.0	81.54164	-11.38839
	9	700.0	5800.0	4470.0	95.09152	-95.86332
7	1	730.0	4330.0	5200.0	-100.61232	87.37906
	2	720.0	4900.0	4580.0	-75.12427	-15.93939
	3	710.0	5170.0	4100.0	-74.31578	-78.77809
	4	710.0	5000.0	5630.0	0.19476	88.89468
	5	705.0	5320.0	5000.0	- 5.92602	1.28951
	6	700.0	5800.0	4470.0	10.05808	-78.91377
	7	735.0	5610.0	6020.0	92.21809	92.92895
	8	720.0	5870.0	5550.0	85.95281	23.94845
	9	720.0	6300.0	5000.0	92.67372	-58.07173
8	1	710.0	5000.0	5630.0	-87.30657	70.48066
	2	705.0	5320.0	5000.0	-96.03625	-14.43696
	3	700.0	5800.0	4470.0	-81.44033	-101.72202
	4	735.0	5610.0	6020.0	- 0.14721	72.50897
	5	720.0	5870.0	5550.0	- 3.46348	7.21580
	6	720.0	6300.0	5000.0	5.82397	-78.81177
	7	750.0	6220.0	6380.0	86.41313	70.89717
	8	730.0	6510.0	5800.0	79.25362	- 8.06838
	9	740.0	6880.0	5200.0	81.19723	-96.88630

TABLE 3 (CONT.)

PHOTO NO.	STA NO.	ELEV	X-COORD GROUND	Y-COORD GROUND	X-COORD PHOTO	Y-COORD PHOTO
9	1	735.0	5610.0	6020.0	-85.50675	76.24477
	2	720.0	5870.0	5550.0	-88.30074	11.20605
	3	720.0	6300.0	5000.0	-79.94351	-72.28248
	4	750.0	6220.0	6380.0	0.21930	74.55035
	5	730.0	6510.0	5800.0	- 7.39227	- 4.25698
	6	740.0	6880.0	5200.0	- 8.18536	-90.27076
	7	760.0	6810.0	6790.0	88.34056	79.01161
	8	760.0	7110.0	6190.0	80.60635	- 2.96693
	9	763.0	7450.0	5750.0	87.65176	-71.47276
10	1	750.0	6220.0	6380.0	-85.21878	78.61837
	2	730.0	6510.0	5800.0	-91.66612	- 1.00245
	3	740.0	6880.0	5200.0	-93.35873	-85.96407
	4	760.0	6810.0	6790.0	3.59422	82.89862
	5	760.0	7110.0	6190.0	- 5.56019	- 0.81999
	6	763.0	7450.0	5750.0	0.29082	-69.49079
	7	750.0	7420.0	7080.0	87.48252	71.77086
	8	765.0	7780.0	6560.0	90.44069	- 7.45544
	9	770.0	8110.0	6100.0	93.67737	-78.39046
11	1	760.0	6810.0	6790.0	-90.46849	84.06468
	2	760.0	7110.0	6190.0	-101.45756	- 2.57370
	3	763.0	7450.0	5750.0	-96.78841	-74.74594
	4	750.0	7420.0	7080.0	- 3.43970	70.41549
	5	765.0	7780.0	6560.0	- 2.37428	-11.32341
	6	770.0	8110.0	6100.0	- 0.15102	-85.35171
	7	740.0	8000.0	7400.0	80.98702	62.34578
	8	750.0	8430.0	6920.0	92.94280	-19.41961
	9	760.0	8800.0	6320.0	89.91208	-110.78086
12	1	750.0	7420.0	7080.0	-93.62292	74.86058
	2	765.0	7780.0	6560.0	-93.38686	- 8.43479
	3	770.0	8110.0	6100.0	-91.25326	-82.39732
	4	740.0	8000.0	7400.0	- 6.84920	66.63659

DEFINITION OF INPUT TERMS FOR ERRANT

ALPHK	=	The standard deviation for the percentage of times that deviation limits are not to be exceeded
ALT(1)	=	Flying altitude for the first photograph
B	=	Distance between exposure stations
DELALT	=	Limit for the change in flying altitude from one photograph to the next
EEL(I)	=	Elevation of a point
FOC	=	Focal length in millimeters
LAZD	=	Degrees portion of the limit for the change in aximuth of the direction of flight from one photograph to the next
LAZM	=	Minutes portion of the limit for the change in azimuth of the direction of flight from one photograph to the next
LAZS	=	Seconds portion of the limit for the change in azimuth of the direction of flight from one photograph to the next
LAZFLD	=	Degrees portion of the azimuth of the flight line for the first photograph
LAZFLM	=	Minutes portion of the azimuth of the flight line for the first photograph
LAZFLS	=	Seconds portion of the azimuth of the flight line for the first photograph
ITD	=	Degrees portion of the limit of the tilt of a photograph
ITM	=	Minutes portion of the limit of the tilt of a photograph
ITS	=	Seconds portion of the limit of the tilt of a photograph
N	=	Number of photographs in the strip
NM	=	Number of points per model
NNN	=	Number of points per photograph
NO	=	Dummy variable which can be left blank
XGN(1)	=	Ground X coordinate of the nodir point of the first photograph
XX(I)	=	Ground X coordinate of a point
YGN(1)	=	Ground Y coordinate of the nodir point of the first photograph
YY(I)	=	Ground Y coordinate of a point
ZALT(I)	=	Deviation from the normal distribution for change in flying altitude from one photograph to the next
ZAZFL(I)	=	Deviation from the normal distribution for change in flight direction from one photograph to the next
ZS(I)	=	Deviation from the square distribution for swing of a photograph
ZT(I)	=	Deviation from the normal distribution for tilt of a photograph

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      PROGRAM ERRANT
      DIMENSION ALT(20),ZALT(20),AZFL(20),ZAZFL(20),T(20),ZT(20),
     1          S(20),ZS(20),THETA(20),XGN(20),YGN(20),XXG(20,40)
     2          ,YYG(20,40),XXP(20,40),YYP(20,40),XG(20,40),
     3          YG(20,40),EL(20,40),XP(20,40),YP(20,40),DUMM(20),
     4          D(40),KD(20),XD(40),AM(40),M(20),XM(20),KS(20),
     5          KSD(20),KSM(20),KSS(20),KTD(20),KTM(20),KTS(20),
     6          KAD(20),KAM(20),KAS(20),PHI(40),XX(20,40),PX(40),
     7          YY(20,40),EEL(20,40),PY(40),AA(35,10),CEL(20,40),
     8          CX(40),CY(40),BB(35,10),CXG(20,40),CYG(20,40),
     9          PPP(35),PT(40)
      DIMENSION SDAW(10),SDAK(10),SDAP(10),SDDY(10),SDDZ(11),
     1          AWD(10),AKD(10),APD(10),YDW(10,35),YDK(10,35),
     2          YDP(10,35),XDW(10,35),XDK(10,35),XDP(10,35),
     3          DYDY(35),DYDZ(35),DYDW(35),DYDK(35),DYDP(35),
     4          DXDZ(35),DXDW(35),DXDK(35),DXDP(35),EM(35),AW(35)
     5          ,AP(35),AK(35),DUMMXL(35),DUMMXR(35),
     6          DUMMYL(35),DUMMYR(35),C(20)

C     GENERATION OF HYPOTHETICAL PHOTOGRAPHS

   99 READ 98, N,NNN,NM
   98 FORMAT (3I20)
      IF (N) 400,100,400
  400 READ 39,(ZALT(I),I=1,N)
      READ 39,(ZAZFL(I),I=1,N)
      READ 39,(ZT(I),I=1,N)
   39 FORMAT (8F10.4)
      READ 27, (ZS(I),I=1,N)
   27 FORMAT (8F10.5)
      IMAX = (N-3)*(NNN-NM)+NNN
      READ 38,(XX(I),YY(I),EEL(I),I=1,IMAX)
   38 FORMAT (2F20.2,F40.2)
      READ 37,FOC,ALT(1),IAZFLD,IAZFLM,IAZFLS,XGN(1),YGN(1),B
   37 FORMAT (F9.6,F6.0,3I5,3F10.2)
      READ 36,DELALT,IAZD,IAZM,IAZS,ITD,ITM,ITS,ALPHK
   36 FORMAT (F6.2,6I5,F7.5)
      PI=3.1415926536
      AZFLD=IAZFLD
      AZFLM=IAZFLM
      AZFLS=IAZFLS
      DELAZD=IAZD
      DELAZM=IAZM
      DELAZS=IAZS
      DELTD=ITD
      DELTM=ITM
      DELTS=ITS
      AZFL(1)=(AZFLD+AZFLM/60.+AZFLS/3600.)*2.*PI/360.
      DELAZ=(DELAZD+DELAZM/60.+DELAZS/3600.)*2.*PI/360.
      DELT=(DELTD+DELTM/60.+DELTS/3600.)*2.*PI/360.
      S(1)=ZS(1)*2.*PI
```

```
      T(1) = ABSF(ZT(1))*DELT/ALPHK
      DO 1 I=2,N
      ALT(I)=ALT(I-1)+ZALT(I-1)*DELALT/ALPHK
      AZFL(I) = AZFL(I-1) + ZAZFL(I-1)*DELAZ/ALPHK
      T(I) = ABSF(ZT(I))*DELT/ALPHK
    1 S(I)=ZS(I)*2.*PI
      DO 3 I=1,N
    3 DUMM(I)=AZFL(I)
      KK=0
    7 KK=KK+1
      DO 50 I=1,N
    6 IF(ABSF(DUMM(I))-2.*PI) 50,50,60
   60 IF(DUMM(I)) 70,50,80
   70 DUMM(I)=DUMM(I)+2.*PI
      GO TO 6
   80 DUMM(I)=DUMM(I)-2.*PI
      GO TO 6
   50 CONTINUE
      GO TO (101,102,103),KK
  101 DO 10 I=1,N
      AZFL(I)=DUMM(I)
      THETA(I)=3.*PI/2.-S(I)-AZFL(I)
   10 DUMM(I)=THETA(I)
      GO TO 7
  102 DO 11 I=1,N
      THETA(I)=DUMM(I)
   11 DUMM(I)=S(I)
      GO TO 7
  103 DO 19 I=1,N
      S(I) = DUMM(I)
   19 PHI(I) = S(I)-PI
      MM=0
   25 MM=MM+1
      DO 49 I=1,N
      D(I)=DUMM(I)*360./(2.*PI)
      KD(I)=DUMM(I)*360./(2.*PI)
      XD(I)=KD(I)
      AM(I)=(D(I)-XD(I))*60.
      M(I)=(D(I)-XD(I))*60.
      XM(I)=M(I)
   49 KS(I)=(AM(I)-XM(I))*60.
      GO TO (500,501,502),MM
  500 DO 550 I=1,N
      KSD(I)=KD(I)
      KSM(I)=M(I)
      KSS(I)=KS(I)
  550 DUMM(I)=T(I)
      GO TO 25
  501 DO 551 I=1,N
      KTD(I)=KD(I)
      KTM(I)=M(I)
      KTS(I)=KS(I)
  551 DUMM(I)=AZFL(I)
```

```
      GO TO 25
  502 DO 552 I=1,N
      KAD(I)=KD(I)
      KAM(I)=M(I)
  552 KAS(I)=KS(I)
      DO 12 I=2,N
      XGN(I)=XGN(I-1)+B*SINF(AZFL(I))
   12 YGN(I)=YGN(I-1)+B*COSF(AZFL(I))
      DO 15 J = 1,NNN
      DO 15 I = 1,N
      XG(I,J) = 0.0
      YG(I,J) = 0.0
   15 EL(I,J) = 0.0
      KP1 = 1+NM-NNN
      KP2 = NM
      DO 801 IJ=1,N
      IF (IJ-1) 802,802,803
  803 IF (IJ-N) 804,805,805
  802 KL = 1
      JP1 = 1+NNN-NM
      DO 806 JI=JP1,NNN
      XG(IJ,JI) = XX(KL)
      YG(IJ,JI) = YY(KL)
      EL(IJ,JI) = EEL(KL)
  806 KL = KL+1
      GO TO 801
  805 KM1 = IMAX-NM+1
      JI = 1
      DO 808 KL=KM1,IMAX
      XG(IJ,JI) = XX(KL)
      YG(IJ,JI) = YY(KL)
      EL(IJ,JI) = EEL(KL)
  808 JI = JI+1
      GO TO 801
  804 KP1 = KP1+NNN-NM
      KP2 = KP2+NNN-NM
      JI = 1
      DO 809 KL=KP1,KP2
      XG(IJ,JI) = XX(KL)
      YG(IJ,JI) = YY(KL)
      EL(IJ,JI) = EEL(KL)
  809 JI = JI+1
  801 CONTINUE
      DO 14 L=1,N
      DO 14 K=1,NNN
      IF (XG(L,K)) 778,777,778
  777 IF (YG(L,K)) 778,779,778
  779 IF (EL(L,K)) 778,780,778
  780 GO TO 14
  778 CONTINUE
      XXG(L,K)=(XG(L,K)-XGN(L))*COSF(THETA(L))+(YG(L,K)-YGN(L))*
     1          SINF(THETA(L))
      YYG(L,K)=(YG(L,K)-YGN(L))*COSF(THETA(L))-(XG(L,K)-XGN(L))*
```

```
   1           SINF(THETA(L))
      YYP(L,K)=YYG(L,K)*FOC/(((ALT(L)-EL(L,K))*COSF(T(L))+
   1           YYG(L,K)*SINF(T(L)))*COSF(T(L)))
      XXP(L,K)=XXG(L,K)*(FOC/COSF(T(L))-YYP(L,K)*SINF(T(L)))/
   1           (ALT(L)-EL(L,K))
      XP(L,K)=XXP(L,K)*COSF(PHI(L))+(YYP(L,K)-FOC*TANF(T(L)))
   1          *SINF(PHI(L))
      YP(L,K)=(YYP(L,K)-FOC*TANF(T(L)))*COSF(PHI(L))-
   1          XXP(L,K)*SINF(PHI(L))
   14 CONTINUE

C      RELATIVE ORIENTATION

      N1 = N-1
      MM = NNN-NM
      BM = B/200.*25.400051
      C(1)=0.0
      NM1 = NM + 1
      MM1=MM+1
      M3=NM-MM
      M31=M3+1
      FHG=1200.0
      DO 201 J=1,N1
      II=0
  310 II=II+1
      IF (II-1) 311,311,312
  311 C(J+1)=C(J)-B*(1.0-COSF(AZFL(1)-AZFL(J+1)))
      DO 313 I=1,NM
        IM=I+MM
        DUMMXL(I)=XP(J,IM)
        DUMMXR(I)=XP(J+1,I)
        DUMMYL(I)=YP(J,IM)
  313   DUMMYR(I)=YP(J+1,I)
      GO TO 314
  312 C(J+2)=C(J+1)-B*(1.0-COSF(AZFL(1)-AZFL(J+2)))
      DO 315 I=1,M3
        IM=I+MM
        DUMMXL(I)=XP(J+1,IM)
  315   DUMMYL(I)=YP(J+1,IM)
      DO 316 I=M31,NM
        IM=I+MM
        DUMMXL(I)=XXP(J+1,IM)
  316   DUMMYL(I)=YYP(J+1,IM)
      DO 317 I=1,NM
        DUMMXR(I)=XP(J+2,I)
  317   DUMMYR(I)=YP(J+2,I)
  314 CONTINUE
      DO 285 K=1,11
  285   SDDZ(K)=0.0
      DO 286 I=1,NNN
        DXDK(I)=0.0
```

```
      DXDZ(I)=0.0
      DXDP(I)=0.0
      DXDW(I)=0.0
      DYDY(I)=0.0
      DYDZ(I)=0.0
      DYDK(I)=0.0
      DYDW(I)=0.0
286   DYDP(I)=0.0
      DO 301 K=1,10
        SDAW(K)=0.0
        SDAK(K)=0.0
        SDAP(K)=0.0
        SDDY(K)=0.0
        AWD (K)=0.0
        AKD (K)=0.0
        APD (K) =0.0
       DO 301 I=1,NNN
         YDW(K,I)=0.0
         YDK(K,I)=0.0
         YDP(K,I)=0.0
         XDW(K,I)=0.0
         XDK(K,I)=0.0
301      XDP(K,I)=0.0
    DO 265 I=1,NM
265   EM(I)=0.0
    IID=0
    L=7
261 IID=IID+1
    IF (IID-6) 262,262,263
262 IF(IID-2) 266,267,267
267 DO 268 I=1,NM
      IM=I+MM
    PP=DUMMXL(I)-DUMM(I)
268 EM(I)=FHG-FOC*B/PP
266 DO 202 I=1,NM
      IM=I+MM
      CY(I)=0.0
      CX(I)=0.0
    PY(I)=DUMMYL(I)-DUMMYR(I)
    PPP(I)=PY(I)
202   CONTINUE
    ID = 0
208 CONTINUE
    ID = ID + 1
    IF (ID-L) 209,209,261
209 NC = 5
    DO 280 I=1,NM
      IM=I+MM
      AA(I,1)=FOC/(FHG-EM(I))
      AA(I,2)=(DUMMYR(I)+CY(I)+YDP(ID,I)+YDW(ID,I)+YDK(ID,I))/
   1          (FHG-EM(I))
      AA(I,3)=DUMMXR(I)+CX(I)+XDP(ID,I)+XDW(ID,I)
      AA(I,4)=-(DUMMXR(I)+CX(I))*(DUMMYR(I)+CY(I))/FOC
```

```
      AA(I,5)=(1.0+((DUMMYR(I)+CY(I)+YDP(ID,I))/FOC)**2)*FOC
      BB(I,2)=(DUMMXR(I)+CX(I)+XDP(ID,I)+XDW(ID,I)+XDK(ID,I))/
     1        (FHG-EM(I))
      BB(I,3)=-(DUMMYR(I)+CY(I)+YDP(ID,I)+YDW(ID,I))
      BB(I,4)=-(1.+((DUMMXR(I)+CX(I))/(FOC          ))**2)*(FOC)
      BB(I,5)=(DUMMXR(I)+CX(I)+XDP(ID,I))*(DUMMYR(I)+CY(I)+
     1        YDP(ID,I))/FOC
280   CONTINUE
      CALL LSQMA (NM,NC,AA,PPP,DY,DZ,DAK,DAP,DAW)
      AWD(ID)=DAW
      AKD(ID)=DAK
      APD(ID)=DAP
      IF (ID-1) 281,281,282
281   SDAW(ID)=DAW
      SDAK(ID)=DAK
      SDAP(ID)=DAP
      SDDY(ID)=DY
      SDDZ(ID+1)=DZ
      GO TO 283
282   SDAW(ID)=SDAW(ID-1)+DAW
      SDAK(ID)=SDAK(ID-1)+DAK
      SDAP(ID)=SDAP(ID-1)+DAP
      SDDY(ID)=SDDY(ID-1)+DY
      SDDZ(ID+1)=SDDZ(ID)+DZ
283   CONTINUE
      DO 211 I=1,NM
      AP(I)=-ATANF(DUMMXR(I)/FOC)
      DYDP(I)=DUMMYR(I)*(COSF(AP(I))/COSF(AP(I)+SDAP(ID))-1.0)
      DXDP(I)=FOC*(TANF(AP(I))-TANF(AP(I)+SDAP(ID)))
      AW(I)=ATANF((DUMMYR(I)+DYDP(I))/FOC)
      DYDW(I)=FOC*(TANF(AW(I)+SDAW(ID))-TANF(AW(I)))
      DXDW(I)=(DUMMXR(I)+DXDP(I))*(COSF(AW(I))/COSF(AW(I)+
     1        SDAW(ID))-1.0)
      RD=SQRTF((DUMMYR(I)+DYDP(I)+DYDW(I))**2+(DUMMXR(I)+DXDP(I)+
     1   DXDW(I))**2)
      AK(I)=ASINF((DUMMYR(I)+DYDP(I)+DYDW(I))/RD)
      IF(DUMMYR(I)+DYDP(I)+DYDW(I)) 203,204,204
203   IF(DUMMXR(I)+DXDP(I)+DXDW(I)) 205,205,206
204   IF (DUMMXR(I)+DXDP(I)+DXDW(I)) 205,205,327
205   AK(I)=-AK(I)+PI
      GO TO 327
206   AK(I)=AK(I)+2.*PI
327   CONTINUE
      DYDK(I)=SQRTF((DUMMYR(I)+DYDP(I)+DYDW(I))**2+(DUMMXR(I)+
     1        DXDP(I)+DXDW(I))**2)*(SINF(AK(I)+SDAK(ID))-
     2        SINF(AK(I)))
      DXDK(I)=SQRTF((DUMMYR(I)+DYDP(I)+DYDW(I))**2+(DUMMXR(I)+
     1        DXDP(I)+DXDW(I))**2)*(COSF(AK(I)+SDAK(ID))-
     2        COSF(AK(I)))
      DYDZ(I)=((DUMMYR(I)+DYDP(I)+DYDW(I)+DYDK(I))/(FHG-
     1        EM(I)))*SDDZ(ID+1)
      DXDZ(I)=((DUMMXR(I)+DXDP(I)+DXDW(I)+DXDK(I))/(FHG-
     1        EM(I)))*SDDZ(ID+1)
```

```
      DYDY(I)=(FOC/(FHG-EM(I)))*SDDY(ID)
        PT(I)=DYDY(I)+DYDZ(I)+DYDK(I)+DYDW(I)+DYDP(I)
      IF(II-1) 320,320,321
320     PX(I)=(FOC/(FHG-EM(I)))*C(J+1)+DXDZ(I)+DXDK(I)+DXDW(I)+
     1          DXDP(I)
      GO TO 322
321     PX(I)=(FOC/(FHG-EM(I)))*C(J+2)+DXDZ(I)+DXDK(I)+DXDW(I)+
     1           DXDP(I)
322   CONTINUE
        YDK(ID,I)=AA(I,3)*AKD(ID)
        YDP(ID,I)=AA(I,4)*APD(ID)
        YDW(ID,I)=AA(I,5)*AWD(ID)
        XDK(ID,I)=BB(I,3)*AKD(ID)
        XDP(ID,I)=BB(I,4)*APD(ID)
        XDW(ID,I)=BB(I,5)*AWD(ID)
        CY(I)=PT(I)
        CX(I)=PX(I)
      DUMM(I)=DUMMXR(I)+PX(I)
211     PPP(I)=PY(I)-PT(I)
      GO TO 208
263   CONTINUE
      IF(II-1) 318,318,319
318   DO 264 I=1,NM
      EM(I)=0.0
        YP(J+1,I)=YP(J+1,I)+PT(I)
264     XP(J+1,I)=XP(J+1,I)+PX(I)
      DSDAW=SDAW(L)
      DSDAK=SDAK(L)
      DSDAP=SDAP(L)
      DSDDY=SDDY(L)
      DSDDZ=SDDZ(L+1)
      IF (J-N1) 319,326,326
319   CONTINUE
      DO 213 I=NM1,NNN
      K=I-MM
      AP(I)=-ATANF(XP(J+1,I)/FOC)
      DYDP(I)=YP(J+1,I)*(COSF(AP(I))/COSF(AP(I)+DSDAP)-1.0)
      DXDP(I)=FOC*(TANF(AP(I))-TANF(AP(I)+DSDAP))
      AW(I)=ATANF((YP(J+1,I)+DYDP(I))/FOC)
      DYDW(I)=FOC*(TANF(AW(I)+DSDAW)-TANF(AW(I)))
      DXDW(I)=(XP(J+1,I)+DXDP(I))*(COSF(AW(I))/COSF(AW(I)+DSDAW)-
     1          1.0)
      RD=SQRTF((YP(J+1,I)+DYDP(I)+DYDW(I))**2+(XP(J+1,I)+DXDP(I)+
     1    DXDW(I))**2)
      AK(I)=ASINF((YP(J+1,I)+DYDP(I)+DYDW(I))/RD)
      IF(YP(J+1,I)+DYDP(   I)+DYDW(   I)) 253,254,254
253   IF(XP(J+1,I)+DXDP(   I)+DXDW(   I)) 255,255,256
254   IF(XP(J+1,I)+DXDP(   I)+DXDW(   I)) 255,255,257
255     AK(I)=-AK(I)+PI
        GO TO 257
256     AK(I)=AK(I)+2.*PI
257     CONTINUE
      DYDK(I)=SQRTF((YP(J+1,I)+DYDP(I)+DYDW(I))**2+(XP(J+1,I)+
```

```
     1          DXDP(I)+DXDW(I))**2)*(SINF(AK(I)+DSDAK)-
     2          SINF(AK(I)))
      DXDK(I)=SQRTF((YP(J+1,I)+DYDP(I)+DYDW(I))**2+(XP(J+1,I)+
     1          DXDP(I)+DXDW(I))**2)*(COSF(AK(I)+DSDAK)-
     2          COSF(AK(I)))
      DYDZ(I)=((YP(J+1,I)+DYDP(I)+DYDW(I)+DYDK(I))/(FHG-
     1          EM(K)))*DSDDZ
      DXDZ(I)=((XP(J+1,I)+DXDP(I)+DXDW(I)+DXDK(I))/(FHG-
     1          EM(K)))*DSDDZ
      DYDY(I)=(FOC/(FHG-EM(K)))*DSDDY
        PY(I)=DYDY(I)+DYDZ(I)+DYDK(I)+DYDW(I)+DYDP(I)
      PX(I)=(FOC/(FHG-EM(K)))*C(J+1)+DXDZ(I)+DXDK(I)+DXDW(I)+
     1        DXDP(I)
      YYP(J+1,I)=YP(J+1,I)+PY(I)
  213 XXP(J+1,I)=XP(J+1,I)+PX(I)
      IF (II-6) 310,310,323
  323 DO 324 I=NM1,NNN
        YP(J+1,I)=YYP(J+1,I)
  324   XP(J+1,I)=XXP(J+1,I)
  201 CONTINUE
  326 CONTINUE

C     COMPUTATION OF GEODETIC COORDINATES FROM VERTICAL
C     PHOTOGRAPHS

      TT=PI/2.0-AZFL(1)
      PA=XP(1,MM+1)-XP(2,1)
      DO 216 J=1,N1
      DO 216 I=1,NM
         IM=I+MM
         PP=XP(J,IM)-XP(J+1,1)
         DP=PP-PA
      CEL(J,I)=ALT(1)-B*FOC/PP
         FJ=J
      XX(J,I)=XP(J+1,I)*(ALT(1)-CEL(J,I))/FOC+B*FJ
      YY(J,I)=YP(J+1,I)*(ALT(1)-CEL(J,I))/FOC
      CXG(J,I)=XGN(1)+XX(J,I)*COSF(TT)-YY(J,I)*SINF(TT)
  216 CYG(J,I)=YGN(1)+YY(J,I)*COSF(TT)+XX(J,I)*SINF(TT)
      PRINT 230
  230 FORMAT (50H      ELEVATION          X COORD           Y COORD
      PRINT 231,((CEL(J,I),CXG(J,I),CYG(J,I),I=1,NM),J=1,N1)
  231 FORMAT (2X,3F25.10)
      GO TO 99
  100 END
      SUBROUTINE LSQMA (M,N,A,B,ZA,ZB,ZC,ZD,ZE)
      DIMENSION A(35,10),B(35),AT(10,35),ATA(10,10),C(10,35),
     1           ATAI(10,10),X(10)
      N1 = N+1
      N2 = 2 * N
      DO 401 I = 1,M
```

```
      DO 401 J = 1,N
      C(J,I) = 0.0
  401 AT(J,I) = A(I,J)
      DO 402 I = 1,N
      DO 402 J = 1,N2
  402 ATA(I,J)=0.0
      DO 403 I = 1,N
      DO 403 J = 1,N
      DO 403 K = 1,M
  403 ATA(I,J) = ATA(I,J) + AT(I,K) * A(K,J)
      DO 404 I = 1,N
      I2 = N+I
  404 ATA(I,I2) = 1.0
      DO 604 I = 1,N
  604 X(I) = 0.0
      DO 405 I = 1, N
      DO 405 J = 1,N2
  405 ATAI(I,J) = ATA(I,J)
      DO 408 K = 1,N
      K1 = K + 1
      DO 406 J = K1,N2
      IF (ATAI(K,K)) 502, 501, 502
  501 PRINT 200
  200 FORMAT (10H ERROR ONE)
      PRINT 100, I, J, K
  100 FORMAT (3I10)
      GO TO 415
  502 ATAI(K,J) = ATAI(K,J)/ATAI(K,K)
  406 CONTINUE
      DO 408 I = 1,N
      IF (I-K) 407, 408, 407
  407 CONTINUE
      DO 408 J = K1,N2
       ATAI(I,J) = ATAI(I,J) - ATAI(K,J) * ATAI(I,K)
  408 CONTINUE
      DO 510 I = 1,N
      DO 510 J = 1,N
      J2 = N+J
  510 ATAI(I,J) = ATAI(I,J2)
      DO 411 I = 1,N
      DO 411 J = 1,M
      DO 411 K = 1,N
  411 C(I,J) = C(I,J) + ATAI(I,K)*AT(K,J)
  415 DO 412 I=1,N
      DO 412 K = 1,M
  412 X(I) = X(I) + C(I,K) * B(K)
      ZA=X(1)
      ZB=X(2)
      ZC=X(3)
      ZD=X(4)
      ZE=X(5)
      END
          END
```

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