END SILLS AND THE FORCED HYDRAULIC JUMP IN CIRCULAR CULVERTS OPERATING AT LOW DISCHARGE FACTORS

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PREFACE

The research reported herein is a study of the performance of a rectangular end sill as a method for restoring the flow leaving a broken-back circular culvert to a condition with an energy level comparable to that of the existing natural channel. With a sill of proper height located between the flared wing walls at the culvert outlet, the supercritical flow usually present in a broken-back culvert can be changed to subcritical through a forced hydraulic jump within the pipe. Thus the flow at the culvert outlet could be distributed uniformly across the downstream channel width within a shorter distance. Experiments with model broken-back culverts were run to determine the sill height to culvert geometry relationship, the most desirable sill location within the wing walls, and the effectiveness of a rectangular sill in distributing the culvert flow.

This study was initiated under an agreement between the Texas Highway Department, the Bureau of Public Roads, and the Center for Highway Research at The University of Texas at Austin. Special acknowledgement is made to personnel of the Texas Highway Department for their valuable suggestions and comments during the investigation.

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ABSTRACT

An experimental investigation was made to determine the effectiveness of a rectangular end sill in stabilizing a hydraulic jump a prescribed distance inside a broken-back culvert and in distributing the flow at the culvert outlet across the downstream channel width. In this way, the supercritical, high velocity flow usually characteristic of a broken-back culvert could be released to the downstream channel so that the flow condition and energy level of the natural channel could more readily be resumed. A major objective of this study was to experimentally determine the relationship between the sill height and the controlling variables associated with forcing a hydraulic jump inside a broken-back culvert without an improved inlet and for a given range of discharge factors, $Q/D^{2.5}$. For this portion of the investigation, the discharge factor, culvert geometry, and sill location were all varied systematically so that each of their effects on the sill height to hydraulic jump relationship could be determined. Velocity and depth measurements across various sections in the channel downstream of the sills were useful in determining the most desirable sill location within the flared wing walls and in estimating the effectiveness of the sills in distributing the flow.

For the data collected during this study, a consistent relationship appears to exist between sill height and culvert geometry for a given discharge factor and a given sill location. Relating sill

height to specific energy upstream of the jump results in a family of curves of constant H/D values for a given discharge factor. For a culvert of given geometry without an improved inlet and closed to the atmosphere except at its two ends, the sill height required to stabilize a jump a given distance inside the pipe increases with discharge for $Q/D^{2.5} \leq 2.5$. At $Q/D^{2.5} > 2.5$ the headwater submerges the culvert entrance, and the air entrained by the jump inside the pipe causes a partial vacuum to develop adding an upstream force to aid the sill in stabilizing the jump. Thus less sill height is required.

Considering the sill locations, the sill at the culvert outlet is probably the least desirable because of the maintenance problems it poses when debris and eroded material are deposited by flood waters inside the culvert. The sill at the mid-point of the flared wing walls appears to be the most desirable since less additional channel protection immediately downstream of such a sill is required. Protection in the form of a concrete apron or rip rap is usually supplied between the wing walls soerosion by the flow over a sill at the mid-point of the wing walls would be limited. Slightly less sill height is required by a sill at the mid-point of the wing walls than one at the end of the wing walls, however, the sill at the end of the wing walls is more effective in spreading the flow across the downstream channel within a shorter distance. At distances of ten to twelve pipe diameters from the sills, the difference in flow patterns does not appear to be appreciable. With the sill located at either the mid-point or end of the wing walls, an average of 80% energy reduction was measured between the end of the sloped portion of the broken-back culvert and a section downstream of the sills.

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LIST OF SYMBOLS

D	Pipe diameter
E_C	Specific energy evaluated at Section C-C of broken-back culvert
E_D	Specific energy at section D-D downstream of sills
ΔE_{C-D}	Difference in specific energies at sections C-C and D-D of broken-back culvert
f	Darcy-Weisbach friction factor
g	Acceleration of gravity
h	Average water depth over top of sill
H	Total elevation fall of culvert
HW	Water depth in headwater tank
L_1, L_2, L_3	Lengths of respective sections of broken-back culvert
L_d	Distance from downstream end of jump to culvert outlet
L_j	Length of jump
L_u	Distance from upstream end of jump to culvert outlet
Q	Volume rate of flow
Rey	Reynold's number based on mean velocity and pipe diameter
s	Sill height

S_1, S_2, S_3	Bottom slopes of respective sections of broken-back culvert
V_a	Average velocity of approach in upstream channel
V_C	Average velocity at section C-C of broken-back culvert
V_D	Average velocity at section D-D of broken-back culvert
V_s	Average velocity of flow over top of sill
w	Maximum distance from any given sill location to the downstream point where flow over the top of the sill strikes the channel bed
x	Distance from culvert outlet to sill
y_C	Average water depth at end of middle section of broken-back culvert
y_D	Average water depth at section D-D downstream of sills
α	Coefficient applied to velocity head in energy balance equation to correct for nonuniform velocity distribution across section of flow
ρ	Density of water

INTRODUCTION

With over three and one-half million miles of highways and rural roads extending across the United States, it is inevitable that portions of such a vast system should cross mountainous or hilly terrain. In these areas drainage facilities along and under roadways must be designed not only with the capability of controlling the rapidly rising, high volume flows characteristic of such regions, but also for releasing these flows to downstream channels such that allowable velocities and shear stresses are not exceeded. Drainage facilities in mountainous or hilly terrain often involve culverts which confine and pass flood flows down steep grades. The broken-back circular culvert used by the Texas State Highway Department is an example of this type of facility. In detail, this culvert is a continuous circular pipe consisting of three constant-slope sections with the middle section set on a much steeper grade than the two mildly-sloped end sections. Often the upstream mild sloped section is omitted all together. Typical of the usage of broken-back culverts is the Austin, Texas, area where there are twenty-nine culverts on steep grades on the east Loop around the city many of which are circular culverts. One of these is a 550 foot long, 60 inch culvert with a total fall to culvert size ratio approximately equal to four.

One problem with the use of broken-back culverts occurs in the natural channel immediately downstream from the culvert outlet. When supercritical flow exists in the natural channel prior to the installation of a broken-back culvert, the grade of the channel is usually classified as a hydraulically steep slope. Likewise the slope of the middle section of a broken-back culvert is also steep since it is usually set near or greater than the grade of the natural channel. In the absence of tail water at the culvert outlet, supercritical free surface flow may exist throughout the length of the culvert with attendant high velocities.

Another effect of any culvert is to confine the natural flow of an existing channel. Since the flow conditions in the case of the broken-back culvert both within the culvert and in the natural channel are usually supercritical, the momentum and erosion potential of the flow leaving the culvert outlet are likely to be even greater than that of the natural channel flow. With the concentration of flow in the central portion of the channel immediately downstream from the culvert outlet, the occurrence of excessively high velocities is possible. Under these conditions, scouring of the channel bed downstream of the culvert outlet, as well as, undermining of the culvert itself may result. Thus a major problem associated with the use of broken-back culverts on steep grades where no tail water exists is one of returning or spreading the culvert flow back to the original condition of the natural channel without an appreciable gain of kinetic energy. It is important from the standpoint of channel stability that the original flow condition and flow width of the natural channel upstream of the culvert also be maintained downstream of the culvert. It is primarily this idea that serves as a basis for the subject of this research study.

One way that the energy level of the flow leaving the culvert can be modified is for a hydraulic jump to be formed within the pipe. The hydraulic jump has long been used as an energy dissipator in open channel flow. With the existence of a jump within the culvert, the flow passes from supercritical to subcritical conditions in the barrel, and a reduction in flow momentum, as well as, a loss of energy results. Ideally, with the passage of this subcritical flow through the flared portion of the culvert outlet and into the downstream channel, the establishment of conditions and flow patterns similar to those of the natural channel can be accomplished within a short distance of the culvert outlet.

In a culvert with a geometric configuration similar to that of the broken-back culvert, the natural formation of a hydraulic jump within the culvert usually does not occur without sufficiently high tail water to force the jump. In the absence of tail water, a jump will form naturally within the pipe only after the energy losses due to friction around the wetted perimeter of the pipe have sufficiently reduced the momentum of the upstream supercritical flow. In order for frictional energy losses within the pipe to be sufficiently high to produce a natural jump, the downstream mildly-sloped section of the broken-back culvert must be either very long or very rough. Usually the physical dimensions of a culvert site and the desirability for setting the steep sloped portion near the natural grade limit the length of this downstream portion of the culvert such that under the design conditions, a natural jump will not form, and supercritical flow will exist throughout the entire length of the culvert. This suggests that some artificial or external means should be used to force the hydraulic jump to form within the prescribed length of culvert so that the flow leaving the culvert will spread and reduce both its velocity and erosion potential.

The placing of a rectangular sill either at the culvert outlet or in the flared wing wall portion of the outlet structure will force a jump to be formed within the pipe if the sill is of sufficient height. This is true since the criterion for the formation and existence of a hydraulic jump is that pressure-plus-momentum be equal on both sides of the jump. The presence of a sill at the pipe outlet provides additional external force in the upstream direction acting against the supercritical pressure-plus-momentum. Thus for a given set of design conditions and culvert geometry, there should be some optimum sill height that would force a jump to form and stabilize within the culvert.

This study was initiated to investigate the relationship between the height of end sills and the forced hydraulic jump in the broken-back culvert. The feasibility of using end sills to force the jump and some of the problems that they might create were also considered. Such information should be valuable to the highway engineer confronted with the design of drainage facilities in areas where steep grades exist.

Objectives

This study was primarily an experimental investigation to establish the relationship between the height of a rectangular end sill and the controlling variables associated with a forced hydraulic jump in a broken-back culvert. Specifically, it was desirous to determine the necessary sill height that would force and stabilize a hydraulic jump for a given discharge factor, $Q/D^{2.5}$, inside a culvert of given geometry at a prescribed distance from the culvert outlet. Also of particular interest during the study was the effect that a sill near the end of a broken-back culvert might have on the flow conditions downstream of the sill. Although the main purpose of the sill is to aid in spreading the culvert flow across the width of the downstream channel, it seems possible that if the sill gets too high, potential energy is converted to kinetic energy, and a problem similar to that of supercritical flow shooting directly out of the culvert outlet can be created. A third phase of the experimental portion of this study was to investigate the effects that different sill locations within the flared wing walls at the culvert outlet have on downstream energy levels, and, if possible, to determine the most desirable location for the sill.

Devising a useful and concise means of presenting the results of this study, particularly the relationship between sill height, discharge factor, and culvert geometry, was an important factor in fulfilling the basic objectives of this investigation. Such presentation schemes could prove useful to the highway engineer as design aids when confronted with drainage problems in hilly or mountainous terrain.

Scope and Limitations

In accordance with the major objective of this study, the height of end sill required to force and stabilize a hydraulic jump inside the culvert was determined for a range of discharge factors, various culvert geometries, and different sill locations. Although the relationships between sill height and the pertinent variables associated with stabilizing a jump inside the culvert were established only for $Q/D^{2.5} \leq 2.5$, additional data were taken at $Q/D^{2.5} = 3$ and 3.5 so that the effect of the subatmospheric pressures developed within culverts without improved inlets at these higher discharge factors could be determined. Tests with and without artificial ventilation of the culvert were run at these higher discharge values. In changing the geometry of the model broken-back culverts, three different lengths of downstream horizontal section were used with each of several differently sloped middle sections of varying length. The total fall over the entire length of the culvert was varied by using different lengths and slopes of the middle unit of culvert since the slopes of the two end sections of culvert were maintained horizontal. The total fall of the culverts was limited to about two feet or a fall to pipe diameter ratio of approximately four. Only one diameter of pipe (six inch) was used to construct the model broken-back culverts.

For each culvert geometry configuration and discharge value, the required sill height was determined experimentally with the sill located at the mid-point and end of the flared wing wall portion of the culvert outlet and at the end of the downstream horizontal section of the culvert. With the sills located at the mid-point and end of the wing walls, tests were run at the higher discharge values for several of the culvert geometries both with and without artificial air ventilation of the pipe.

The culverts were ventilated by means of a 1-1/2 inch hole in the crown of the upstream horizontal section of pipe. Transverse velocity and depth distributions at sections downstream of the sills were also determined at these higher discharge factors and sill locations. These data were useful in determining energy levels in the downstream channel, as well as, the distance from the various sill locations that uniformly distributed flow was re-established in the channel.

Only one type of end sill was used for all of the experimental tests run. A rectangular sill, uniform in both height and thickness, was placed vertically across the entire width of the channel between the flared wing walls and perpendicular to the longitudinal axis of the culvert. Different sills of various heights were used in a trial and elimination process to determine the proper sill height that would force and stabilize the hydraulic jump a given distance upstream from the culvert outlet for a particular discharge factor and culvert geometry.

Throughout the entire experimental portion of the study, visual observations were made of the flow patterns in the vicinity of the sills located at various positions. The water surface profile of the flow immediately upstream of and over the sills, the degree of flow concentration over the sills, the effectiveness of each of the sills in spreading and distributing the flow across the width of the channel, and the horizontal distance traveled by the flow from the top of the sills to the channel floor were all observed during the experiments. All of these factors were important in determining which of the various sill locations might be the most effective in aiding the flow leaving the culvert to resume the normal flow patterns and conditions of the natural channel.

EXPERIMENTAL PROGRAM

The collection of experimental data in the laboratory comprised a major portion of this study. The culvert models were constructed in the Hydraulics Laboratory of the University of Texas with the same test set-up used by Price (1967)* in his experiments with the broken-back culvert. This set-up made use of the pumps, piping system, flow meters, and gages existing in the laboratory.

Experimental Equipment

A schematic sketch of a typical broken-back culvert model is shown in Figure 1. The outlet and sill locations within standard THD (Texas Highway Department) wing walls are shown in detail in Figure 2. Photographs of the test stand and the outlet structure with sill in place are shown in Figures 3 and 4, respectively. The test stand consisted of a 7 foot long by 3 foot wide by 4 foot high smooth plywood storage tank; a 6 foot long by 2 foot wide by 2 foot high smooth plywood table which served as the upstream approach channel; a series of 8 foot long by 2 foot wide steel tables for supporting the plexiglas culvert model; and a 6 foot long by 1 foot-10 1/2 inch wide by 1 foot-6 inch high smooth plywood table which served as an outlet channel for the culvert. Standard THD

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*References are listed alphabetically by author in the Bibliography.

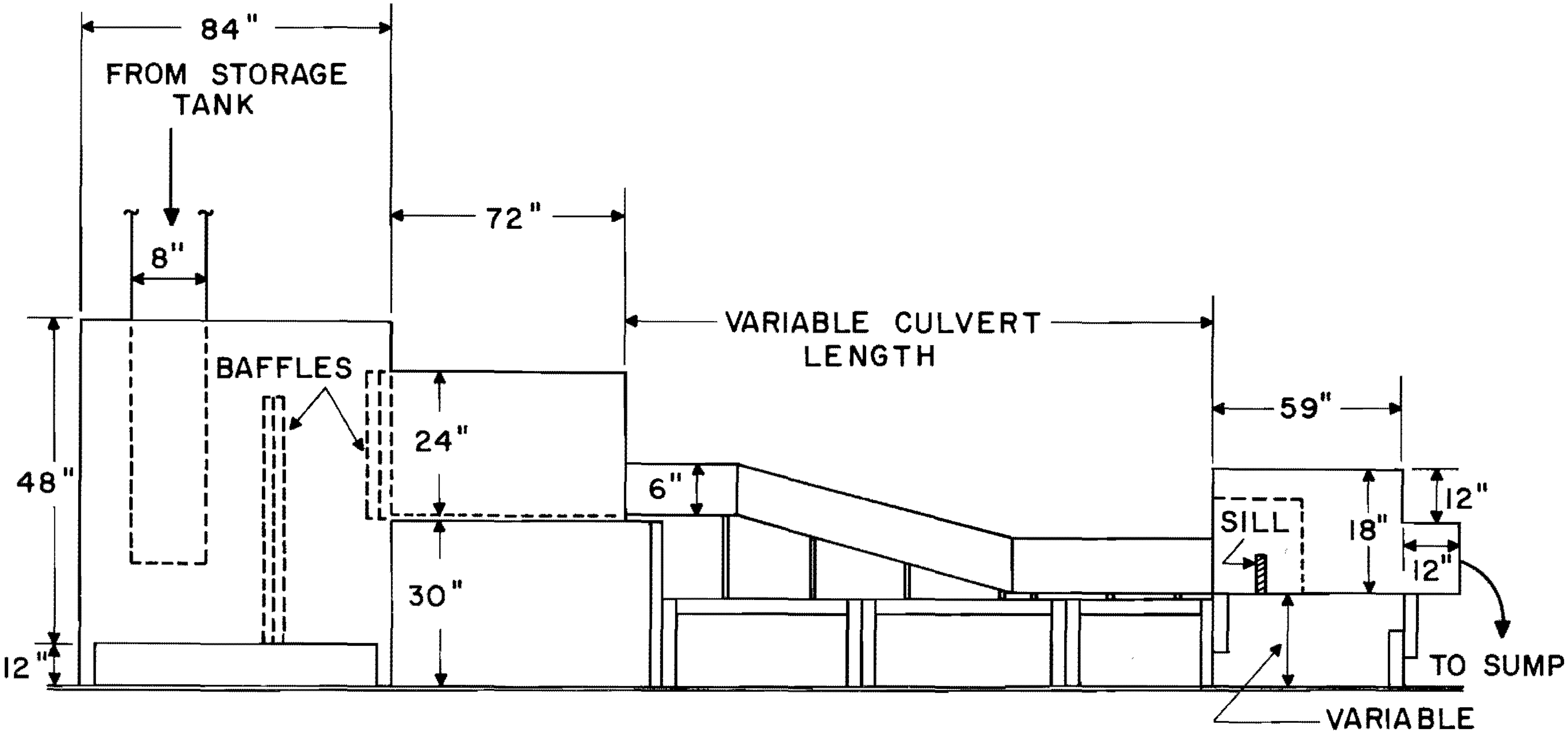
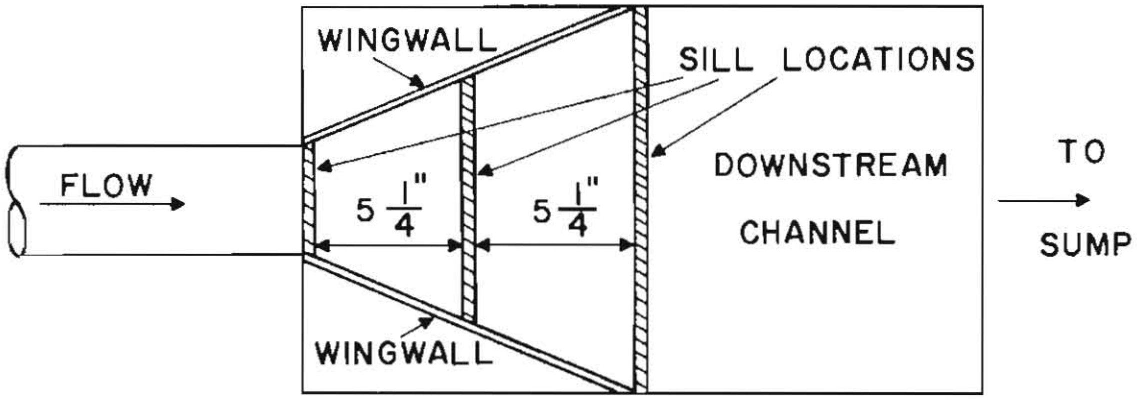
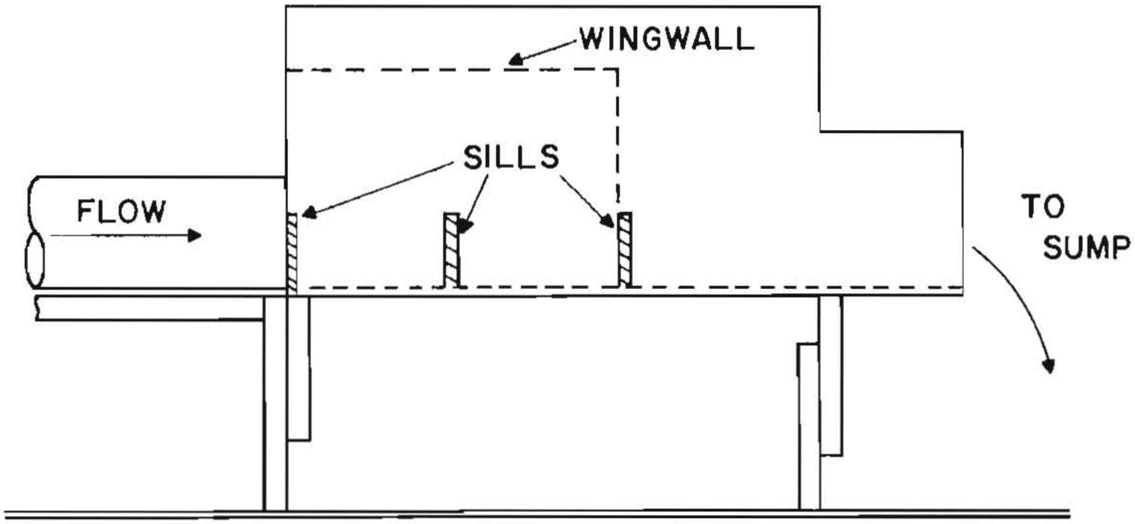


FIGURE I SCHEMATIC SKETCH OF TEST STAND

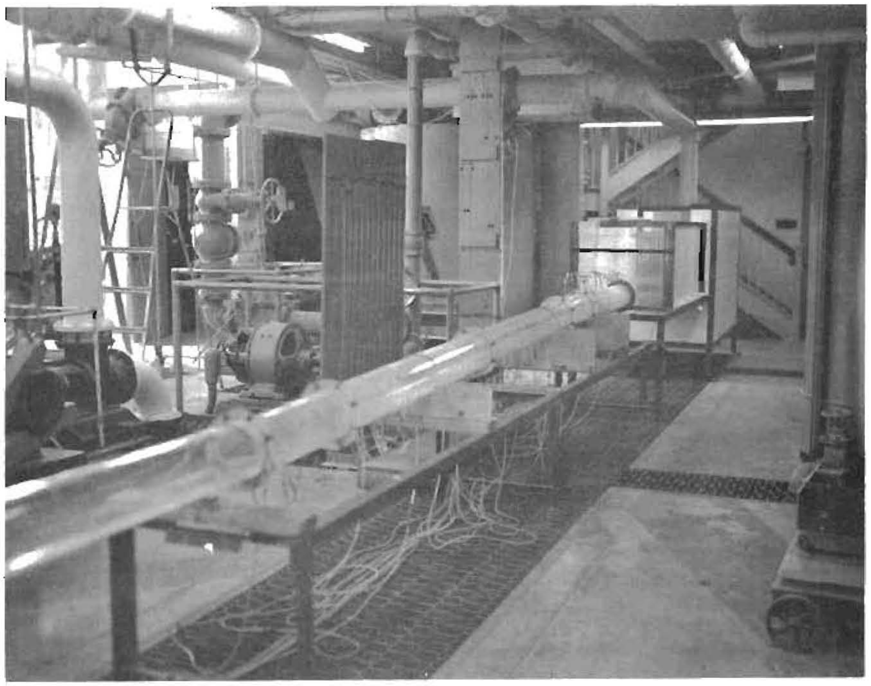


(a) PLAN VIEW

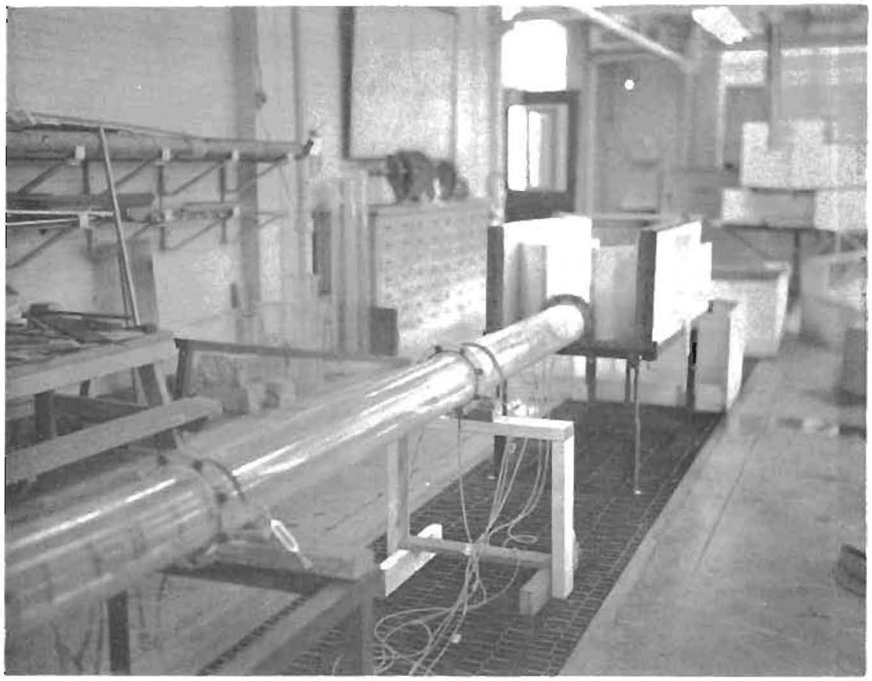


(b) PROFILE VIEW

FIGURE 2 SCHEMATIC SKETCH OF OUTLET CHANNEL SHOWING SILL LOCATIONS USED IN EXPERIMENT

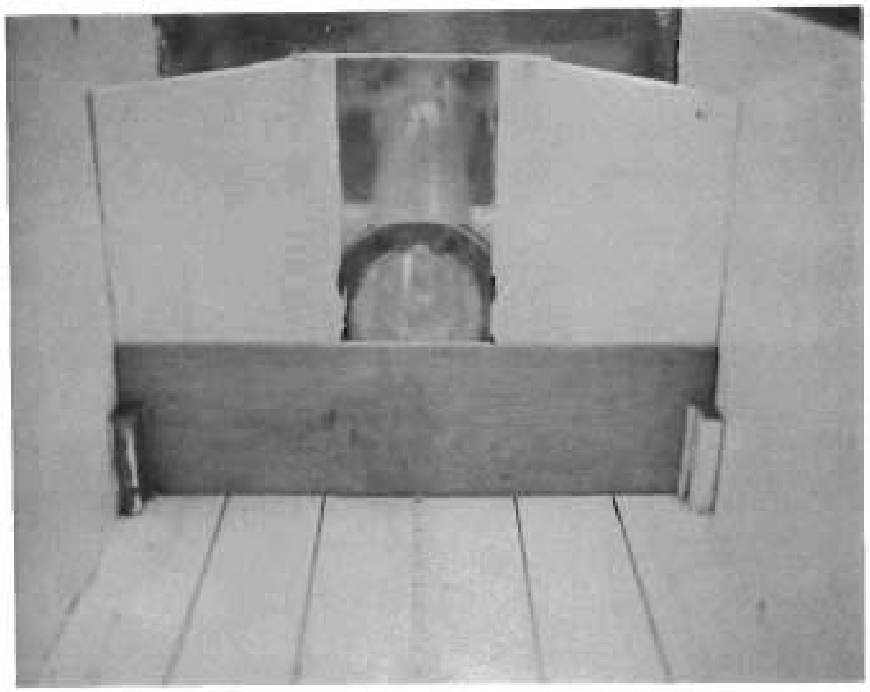


(a) HEAD WATER TABLE AND CULVERT

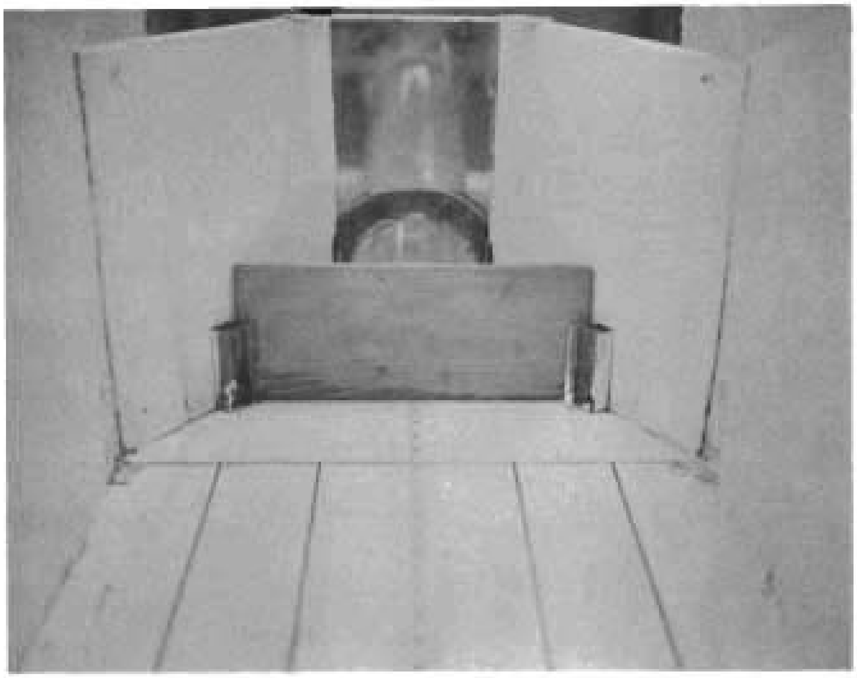


(b) TAIL WATER TABLE AND CULVERT

FIGURE 3. PHOTOGRAPHS OF TEST STAND AND BROKEN-BACK CULVERT MODEL



(a) END OF WING WALLS



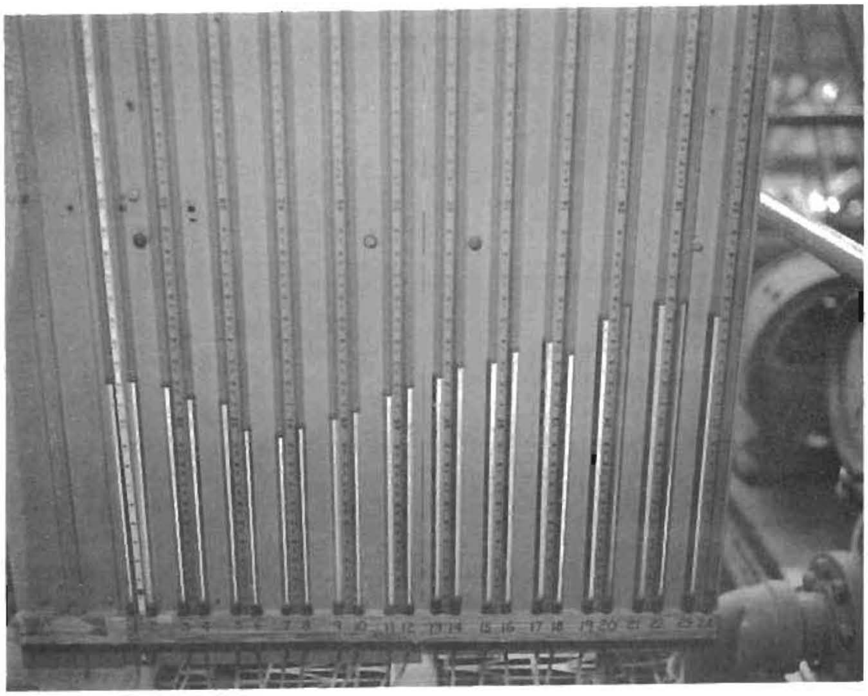
(b) MID-POINT OF WING WALLS

FIGURE 4. PHOTOGRAPHS OF OUTLET CHANNEL WITH SILLS AT TWO DIFFERENT LOCATIONS

wing walls were constructed inside of this outlet channel making an angle of 30° with the longitudinal axis of the culvert. The outlet channel table could also be raised and lowered to vary the total fall of the culvert by adjusting bolt-slot connections on the table supports.

Variations in the slope of the middle section of the culvert were obtained by placing cradle supports of required heights on the supporting tables. When changes in the total length of the culvert were made, supporting tables were either removed or attached as required to provide sufficient support for the plexiglas culvert. In order to obtain greater total fall of the culvert, the supporting tables for both the pipe and the downstream outlet channel were removed for several of the tests. Floor supports were used in this case to achieve desired elevations.

The plexiglas pipe used for the culvert model had a nominal inside diameter of 6.125 inches with a maximum variation of ± 0.004 inches. Pipe sections were bolted together by plexiglas flanges chemically bonded to the ends of the pipe sections. Along the bottom of the entire length of the plexiglas pipe were mounted piezometers which were used to measure hydrostatic water depths and pressure heads. The piezometers were located at intervals not exceeding 6 inches and were connected by "Tygon" tubing to 1/2 inch glass tubes on a manometer board. Strips of engineer's tape graduated to 0.01 feet were attached beside all 1/2 inch tubes, and the water levels were read with a vernier. Desired pipe slopes were easily checked with the piezometers. Figure 5 shows the manometer board with water levels indicating the profile of the water surface in the culvert.



(a) WATER SURFACE PROFILE

(b) BOTTOM OF CULVERT

FIGURE 5. PHOTOGRAPHS SHOWING WATER LEVELS ON MANOMETER BOARD

A recirculating water system with a constant head storage tank supplied water for the test set-ups. Volume rates of flow were regulated by a valve located between the constant head tank and the head water tank of the test stand. Volume rates of flow were measured with a calibrated 4-7/8 inch orifice plate installed in a venturi-orifice meter in the pipe system of the laboratory. The water discharging into the head storage tank of the test stand from an 8 inch supply pipe was stilled and the turbulence level in the approach channel reduced by a baffle constructed of two alternating layers of vertical wood slats placed 6 feet upstream from the culvert entrance.

As was previously mentioned, rectangular sills that spanned the width of the outlet channel between the wing walls were used to force the hydraulic jump in the pipe. The sills were made of 1/4 and 3/8 inch plywood with their length equal to the width of the outlet channel at the required location between the wing walls. No appreciable effect was noted in the position of the jump due to the use of sills of slightly different thicknesses. Availability of materials was the only reason for the difference in sill thickness. Vertical braces were fixed to the wing walls at their mid-point and downstream end for the purpose of holding the sills in an upright position perpendicular to the flow. Sills located at the culvert outlet were held in position with bolts attached to the headwall of the outlet channel. Selection of the proper sill to stabilize a jump at a given distance from the culvert outlet was a trial and error process utilizing an assortment of varying sill heights. Sill heights were measured with a point gage mounted on the tailwater table. The point gage was also used to measure water depths above the sill and in the downstream channel. A Pitot tube was also installed on a sliding frame on the tailwater table such that velocity measurements could be made of the water flowing over the sill and in the downstream channel. Figure 6 shows this apparatus in position over a sill.

Experimental Procedure

Since this study was primarily empirical in nature, data collection was a detailed process. All quantities pertinent to the stabilization of the jump by the sill had to be varied in an organized manner so that their individual effects on the sill to jump location relationship could be determined. In this respect, the discharge, culvert geometry, and sill location were all varied systematically.

The discharge factor, $Q/D^{2.5}$, was varied in increments of 0.5 over the range of 1.0 to 3.5. This gave six different discharges at which various sills could be used to force the jump in the pipe for a given culvert geometry. This discharge range was used for all experiments run with the sills located at the mid-point and end of the wing walls with and without artificial ventilation being supplied to the culvert at the higher $Q/D^{2.5}$ values. With the sill located at the pipe outlet, the maximum discharge factor was limited to 2.5 for several of the tests because of difficulties encountered in selecting the exact sill height that would stabilize the jump only in the downstream horizontal section of culvert.

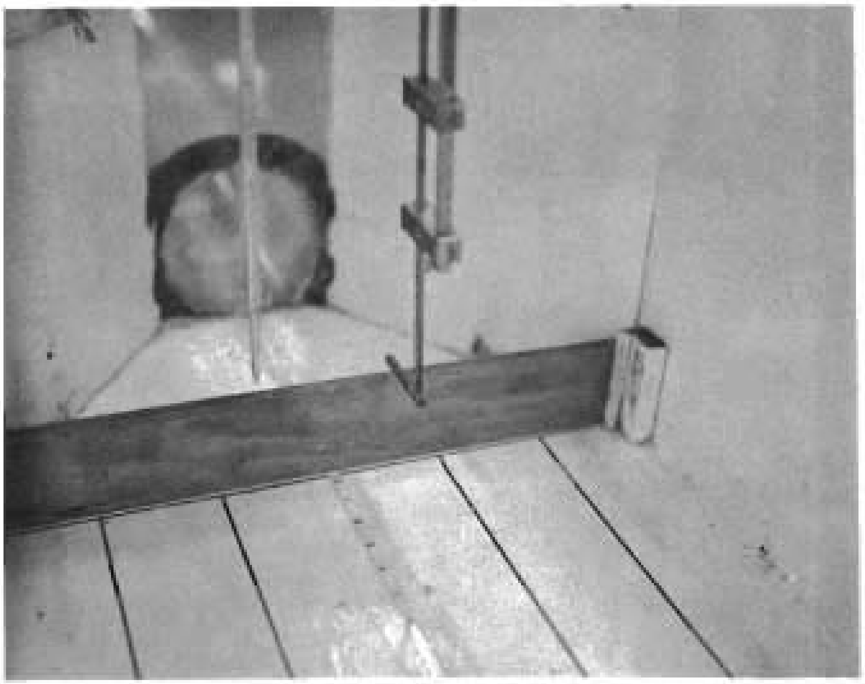


FIGURE 6. PHOTOGRAPH SHOWING POINT GAGE AND PITOT TUBE APPARATUS USED IN EXPERIMENT.

The discharge factors and corresponding discharges for the six inch pipe expressed in units of cubic feet per second are listed in the following table.

Table 1. Discharge Values Used in the Experiment

$Q/D^{2.5}$	Q(cfs)
1.0	0.186
1.5	0.279
2.0	0.372
2.5	0.465
3.0	0.558
3.5	0.651

Although basic data were collected during the experiment for discharge factors ranging up to 3.5, the determination of the relationship between sill height and the controlling variables involved in stabilizing the jump a prescribed distance inside the culvert was limited to $Q/D^{2.5} \leq 2.5$. This upper limit is usually accepted as the point near which the culvert flow at the inlet changes from weir type flow to the slug and mixture type flow as described by Blaisdell (1966). Figure 7 summarizes Blaisdell's results of culvert flow on a steep-sloped section of pipe. It can be noted that $HW/D=1.1$ approximately when $Q/D^{2.5}= 2.5$. With a jump forced into a broken-back culvert operating at a discharge factor greater than 2.5, subatmospheric pressure develops within the culvert when air from the space above the free surface of the supercritical flow is entrained by the jump. Since both the inlet and outlet of the culvert are submerged,

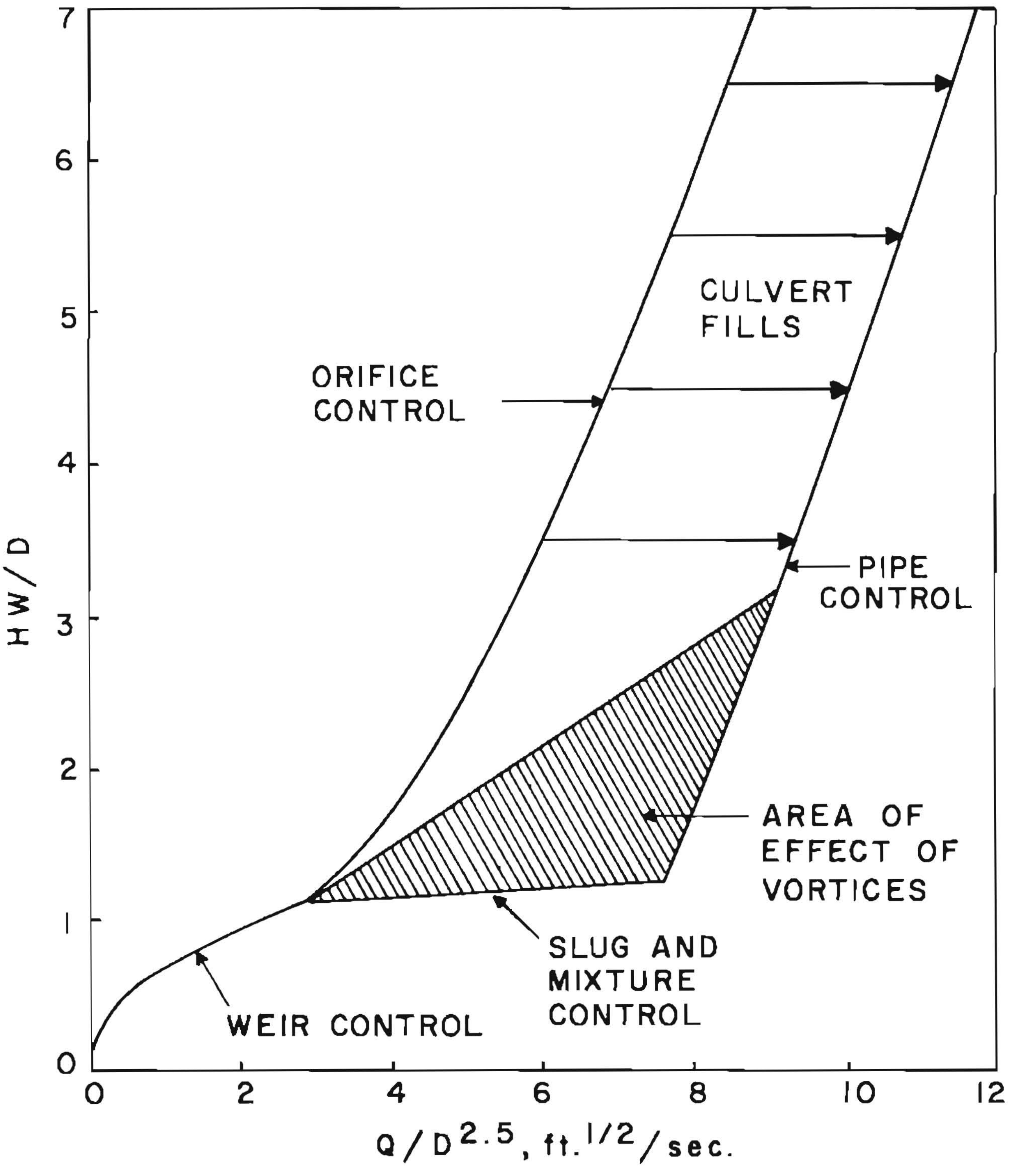


FIGURE 7 HEAD-DISCHARGE RELATIONSHIP FOR A CULVERT ON A STEEP SLOPE (BLAISDELL, 1966)

no air is available to replenish that entrained by the jump, and a partial vacuum is developed within the pipe. Thus to avoid the additional problems associated with the subatmospheric pressure, the relationship between sill height and the hydraulic jump within the pipe was developed only for discharge factors less than or equal to 2.5. Although data were collected at higher discharge factors ($Q/D^{2.5} = 2.5, 3, 3.5$) both with and without ventilation of the pipe, it was used primarily in a qualitative way to better define the sill relations near the discontinuity between atmospheric pressure and low pressure conditions. A culvert operating at a discharge factor greater than 2.5 and without any artificial ventilation will require less sill height to stabilize a jump at a particular location inside the pipe than would a similar culvert that was artificially ventilated. One reason for this is that the subatmospheric pressure developed within the unventilated pipe provides an additional force in the upstream direction that aids the sill in forcing the jump. Although higher velocities in the supercritical flow of the subatmospheric pressure region might result, their effect on increasing the momentum of the supercritical flow is minor when compared to the additional force of the low pressure region.

In order to incorporate into the study the effect of culvert geometry, the lengths of all three sections of the broken-back culvert, the slope of the middle section, and the total fall of the culvert over its entire length were all changed systematically. Pipe diameter was not changed. A summary of all of the test set-ups used in the study appears in Tables 2 and 3. For each of the two different nominal slopes of the

Table 2. Experimental Culvert Set-Ups Used With the Sill Located at the Mid-Point and End of the Flared Wing Walls

Designation	S_2 (%)	L_1 (ft)	L_2 (ft)	L_3 (ft)	H (ft)
A-1	10.85	1.76	5.49	3.17	0.60
A-2	10.85	1.76	5.49	7.42	0.60
A-3	10.85	1.76	5.49	18.70	0.60
B-1	4.00	4.75	11.06	2.90	0.44
B-2	4.00	4.75	11.06	4.99	0.44
B-3	4.00	4.75	11.06	9.45	0.44
C-1	11.29	4.75	11.06	4.99	1.26
C-2	11.29	4.75	11.06	9.12	1.26
C-3	11.29	4.75	11.06	13.46	1.26
D-1	11.29	4.75	14.02	4.99	1.58
D-2	11.29	4.75	14.02	9.12	1.58
D-3	11.29	4.75	14.02	13.46	1.58
E-1	11.29	4.75	18.42	4.99	2.08
E-2	11.29	4.75	18.42	9.12	2.08
E-3	11.29	4.75	18.42	13.46	2.08

Table 3. Experimental Culvert Set-Ups Used With the Sill Located at the End of the Downstream Horizontal Section of the Culvert

Designation	S_2 (%)	L_1 (ft)	L_2 (ft)	L_3 (ft)	H (ft)
F-1	10.85	1.76	5.49	3.17	0.60
F-2	10.85	1.76	5.49	7.42	0.60
F-3	10.85	1.76	5.49	18.70	0.60
G-1	4.00	4.75	10.85	2.17	0.43
G-2	4.00	4.75	10.85	4.33	0.43
G-3	4.00	4.75	10.85	8.67	0.43

middle section of culvert, three different lengths of Unit 3 were used for each of the three sill locations. The slopes of the two end sections of the culvert were always maintained as near horizontal as possible.

The basic data collection procedure was the same for all of the test set-ups. With the sill removed from the outlet channel and no tail-water in the downstream channel, the desired discharge was set and allowed to pass through the culvert until steady state conditions were obtained. Under these conditions, supercritical flow existed throughout the entire length of the culvert. A trial sill of particular height was then selected and placed into the desired location of the outlet channel between wing walls. Depending on the height of the sill, either a jump was formed and forced upstream in the pipe until pressure-plus-momentum conditions were satisfied and the jump stabilized, or no jump at all was formed and supercritical flow conditions remained throughout the entire culvert length. When the total fall of the culvert was not too great, a sill could be selected that was high enough to force a jump all the way back to the head water tank and thus cause the culvert to flow full. The desired condition was to find the exact sill height that would force a jump and stabilize it within the pipe such that its downstream end was approximately a distance of two pipe diameters upstream of the culvert outlet. The use of a distance of two pipe diameters was arbitrary as far as the basic purpose of the experiment was concerned. First of all, it was decided that if what was believed to be the end of the jump could be located in the vicinity of at least two pipe diameters from the pipe outlet, it was unlikely that such a jump could be washed out of the culvert outlet by the normal fluctuations in jump position. Also during the experiments, there was always some uncertainty as to the exact location of the end of the jump, and the distance of two pipe diameters helped assure that the end of the jump was always inside the culvert.

Any sill of greater height than that required to stabilize the end of the jump two pipe diameters from the end of the culvert would cause the jump to move upstream in the pipe. The desired sill height could then be thought of as the minimum sill height that would force and stabilize a hydraulic jump within a broken-back culvert of given geometry and flowing at a given discharge.

When the proper sill height was obtained for a particular culvert geometry and discharge factor, measurements were made of the pressure head before and after the jump, the location of the upstream and downstream ends of the jump, the pressure head in the vicinity of the joint between Units 2 and 3 of the culvert, and the sill height. As was previously mentioned, selection of the proper sill height for a given culvert geometry was carried out over the range of discharge factors, $1.0 \leqslant Q/D^{2.5} \leqslant 3.5$, with the culvert unventilated. For the discharge factors of 2.5, 3.0, and 3.5 similar tests were also run with the culvert ventilated. These tests at the higher discharge factors were run only with the sill located at mid-point and end of the flared wing walls.

In order to measure the degree of flow concentration above the sills and in the channel downstream of the sills, flow patterns were established by taking velocity measurements at these points with a Pitot tube. Vertical velocity profiles were measured above several of the sills at even increments of the sill width, and transverse velocity distributions were measured across the channel at various sections downstream of the sills. Of particular interest during these measurements was the downstream section where uniformly distributed flow was established in the channel. All velocity measurements were taken at discharge factors of 2.5 and 3.5 since these higher discharge values would create the most critical velocity conditions as far as channel erosion is concerned.

THEORETICAL CONSIDERATIONS

Although a major portion of this study was experimental, an examination and understanding of the variables involved was desirable. Interpretation of the data and presentation of the results in a concise and useful form necessitated a complete investigation of all of the variables involved in the experiment. Selection of a final method for presentation of the results was a detailed trial and error process to find a combination of parameters that would yield meaningful plots of the required variables. A complete and detailed analysis of this phase of the study is included in the next chapter; however, the basic theoretical considerations involved in the experiment are discussed here.

One method of solution that often proves useful in open channel hydraulics is to write an energy balance between two sections of the flow such that the variables in question are included. Referring to Figure 8 and assuming that the flow over the sill is uniform and parallel, such an energy balance can be written between sections A-A and B-B of the broken-back culvert system to yield the following equation.

$$HW + H + V_a^2/2g = s + h + \alpha V_s^2/2g + \Sigma \, LOSSES_{A-B} \qquad (1)$$

If the cross-sectional area of the approach channel is large compared to that of the pipe, the velocity of approach, V_a, can be assumed to be small and therefore the velocity head in the approach channel, $V_a^2/2g$,

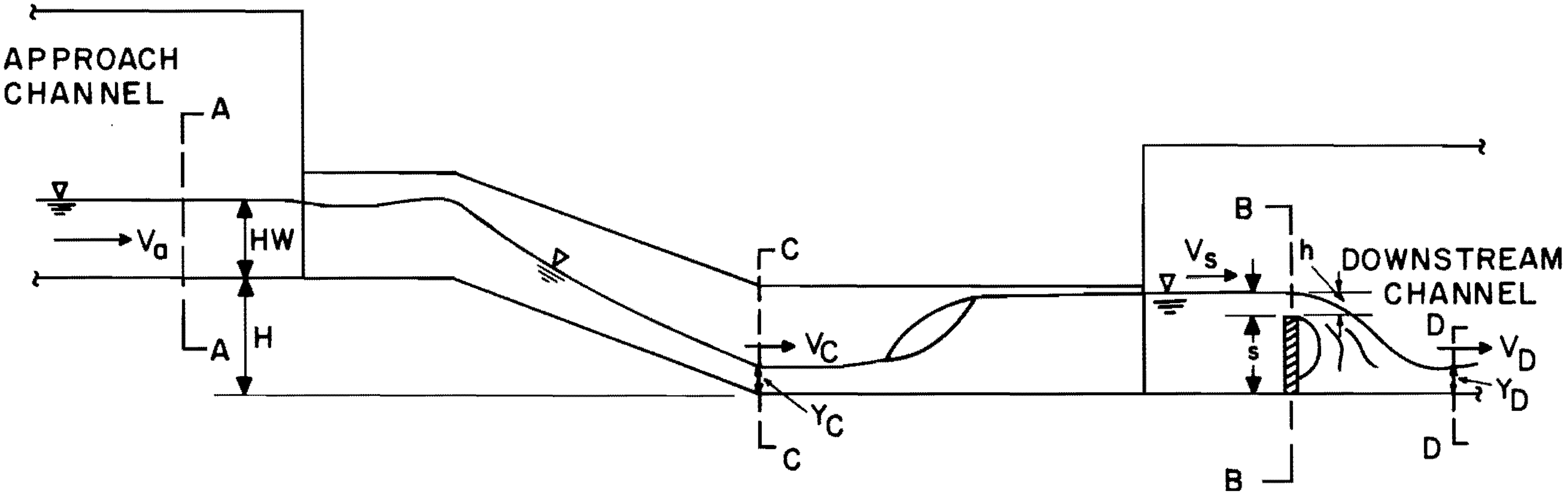


FIGURE 8 DEFINITION SKETCH INCLUDING VARIABLES USED IN ENERGY BALANCE ANALYSIS

can be neglected. An examination of the individual terms in the energy balance equation indicates that the headwater, HW, the total fall of the culvert, H, and the sill height, s, could all easily be determined in the laboratory from linear measurements. The remaining terms, however, could be difficult to evaluate. The water depth, h, and velocity above the sill, V_s, should be considered simultaneously since established uniformly distributed flow does not truly exist over the sill. Some averaging technique would have to be utilized in this situation that would incorporate the effects of acceleration, curvilinear flow, flow concentration to one side or the other of the sill, and varying water depths across the length of the sill.

The most difficult problems arise in the use of the energy balance equation when the individual components of the $[\Sigma \text{LOSSES}_{A-B}]$ term are evaluated. Included in this term are contraction losses at the pipe entrance, expansion losses at the pipe outlet, friction losses throughout the length of the culvert, energy losses in the hydraulic jump, and energy losses due to the turbulent action of the water just upstream of the sill. Pipe entrance and exit losses could be obtained by applying suitable coefficients to the respective velocity heads, however, the determination of the proper coefficients is still a formidable problem. Friction losses could be determined from the Darcy-Weisbach equation or the Manning equation written for the friction slope. The per cent energy loss due to the hydraulic jump could be determined theoretically using an equation that involves the conjugate depths of the jump or from the difference in specific energies upstream and downstream of the jump. In order to determine the upstream depth so that its conjugate depth could be computed, the location of the jump would also have to be established. Thus the determination of the $[\leqslant \mathrm{LOSSES}_{A-B}]$ term of the energy balance equation could involve several additional variables and an investigation beyond the scope of this study.

An analysis of all of the problems involved with the use of the energy balance equation between sections A-A and B-B to describe the flow conditions of the culvert system indicates that it would be much too difficult and may have only limited application as a method of solution. Another possibility for the use of energy balance type analysis that could prove to be more desirable as a measure of the relative effectiveness of the various sill locations in spreading the flow would be to consider the energy levels at sections C-C and D-D in Figure 8. Since both sections are at the same elevation, or at least very near the same elevation for mildly sloped Unit 3 sections, the difference in specific energies at these locations would give a meaningful estimate of the total amount of energy dissipated in the jump and in the turbulent flow in the vicinity of the sill. Section D-D would have to be located downstream of the sill where the flow pattern approaches that of being uniformly distributed. Section D-D would also have to be located a fixed distance from the end of the wing walls for all of the sill locations in order that meaningful comparisons of the energy levels downstream of various sills could be made. Referring to Figure 8, the following expression can be written,

$$y_C + V_C^2/2g = y_D + V_D^2/2g + \Sigma \text{ LOSSES}_{C\text{-}D} \qquad (2)$$

Rewriting in terms of specific energies and letting the change in specific energy, $\Delta E_{C\text{-}D}$, between the two sections be equal to the $[\Sigma \text{ LOSSES}_{C\text{-}D}]$ term, the following form is obtained,

$$E_C = E_D + \Delta E_{C\text{-}D} \qquad (3)$$

Rearranging,

$$\Delta E_{C\text{-}D} = E_C - E_D \qquad (4)$$

and,

$$\Delta E_{C\text{-}D}/E_C = 1 - E_D/E_C \qquad (5)$$

The term $\Delta E_{C\text{-}D}/E_C$ expressed in per cent would be valuable in an analysis of the feasibility of the various sill locations in spreading the flow. This term was evaluated from the data taken during several of the experiments, and a measure of the relative efficiencies of the sills located at the mid-point and end of the flared wing walls was obtained by plotting $\Delta E_{C\text{-}D}/E_C$ versus $Q/D^{2.5}$. A discussion of the results expressed in these plots is included in the next chapter.

Since an energy balance type analysis between sections A-A and B-B did not appear to be a feasible means of representing the relationship between sill height and the variables associated with stabilizing the jump inside the culvert, a different approach was used to try to combine the effects of all of the variables involved. A dimensional analysis of all of the pertinent variables involved in stabilizing the jump by an end sill was performed in order that a set of dimensionless parameters might be developed that could be used to present the results of the study. The dimensional analysis of the variables involved was only a minor portion of this phase of the study when compared to the amount of time and effort that was spent in utilizing the results of the dimensional analysis in trying to develop useful plots to represent the experimental data.

Rand (1965) gives a detailed description of his dimensional analysis of the variables involved in forcing a jump in an open channel using a vertical sill. The fact that the study here is concerned with stabilizing the jump within a circular culvert requires that several additional variables be included in the dimensional analysis. Figure 9 is a schematic of the culvert with a hydraulic jump located within the length of Unit 3. The required sill height, s, is a function of the following quantities:

$$s = s\,[Q, D, f, S_1, S_2, S_3, L_1, L_2, L_3, L_u, L_d, x, g, \rho, H] \quad (6)$$

In this equation, Q is the discharge flowing through the culvert, g is the acceleration of gravity, ρ is the density of the water, and f is the friction factor. All other quantities are defined in Figure 9.

The sill height thus appears to be a function of fifteen different variables. Three basic dimensions, space, mass, and time, are represented among all of the variables. According to the Buckingham Pi Theorem (Henderson, 1966) a total of twelve (15 - 3) dimensionless parameters are needed to represent all of the variables. Of course it

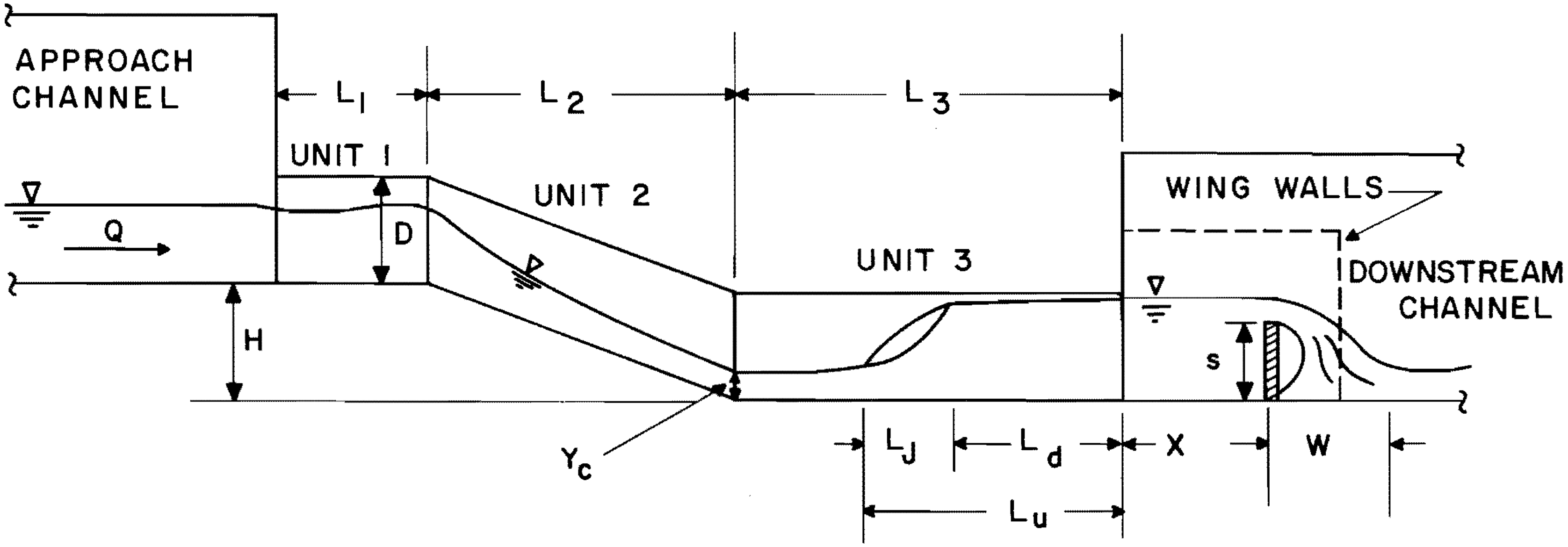


FIGURE 9 DEFINITION SKETCH INCLUDING VARIABLES USED IN DIMENSIONAL ANALYSIS

is impossible to represent that many quantities with one graph. Even if some of the parameters could be related to one another independently of the others, it would still be a difficult task to obtain a concise and meaningful representation of all of the variables. Therefore to reduce the total number of necessary variables, several modifications were tried.

First it was decided that the effects of the entire culvert geometry and flow conditions upstream of the joint between Units 2 and 3 could be represented by the specific energy at the downstream end of Unit 2. There are many different upstream pipe geometry and discharge combinations that could produce the same specific energy value at the beginning of Unit 3. Under many conditions, the flow throughout the lengths of Units 1 and 2 of a broken-back culvert should be supercritical. This is true only when the tail water at the culvert outlet is not sufficiently high to force a jump back upstream into Unit 2 and when the discharge factor, $Q/D^{2.5}$, is low enough such that subatmospheric pressures within the pipe would not move a jump upstream into Unit 2 and cause full pipe flow. Since under these conditions the flow is supercritical, it seems that the upstream conditions producing a certain specific energy at the upstream end of Unit 3 are immaterial to the downstream conditions. The resulting supercritical downstream flow conditions for a given specific energy will be the same regardless of the source of the supercritical specific energy at the end of Unit 2. The components of specific energy, flow depth and velocity head, can easily be computed at the downstream end of Unit 2 using any of the standard nonuniform flow computation procedures for open channel flow discussed in Chow (1959), Henderson (1966), or Morris (1963). Price (1967) has developed and verified a computer program capable of computing the water surface profile throughout the entire length of a broken-back culvert of any geometric configuration. If the specific energy at point C is considered to be representative of upstream conditions and if friction in Unit 3 is neglected because of its short length, Equation 6 becomes

$$s = s[Q, D, E_C, S_3, L_3, L_u, L_d, g, \rho, x, H] \qquad (7)$$

Dimensional analysis of the variables in Equation 7 results in the following dimensionless parameters:

$$s/D, \; Q/D^{2.5}g^{0.5}, \; E_C/D, \; S_3, \; L_3/D, \; L_u/D, \; L_d/D, \; x/D, \; H/D$$

These nine parameters are rearranged in the next chapter until a suitable scheme for presentation of the results of the experiment is arrived at by trial and error. Some of the parameters such as S_3 and L_d/D are held constant for all of the test set-ups, further reducing the number of parameters involved.

All of the variables involved in the nine dimensionless parameters were measured experimentally in the laboratory except for E_C. The values of D, g, and x were known prior to the experimentation process. Although an extrapolated value of the water depth, y_C, at the downstream end of Unit 2 was measured in the experimental procedure, the value of specific energy, E_C, at that point was calculated by a computer program capable of computing the nonuniform water surface profile through the pipe. The reason for using the depth y_C as calculated by the program is that acceleration effects due to the abrupt change in slope at section C-C made depths read with piezometers in the laboratory appear to be too great. Thus in order that consistent values of y_C for corresponding discharge values and culvert geometries could be used in data analysis, the computed value was used. A value for y_C was measured in the laboratory, but not at the exact location of the desired y_C. Usually the reading was made at a distance between 2 and 3 pipe diameters downstream from the end of Unit 2 to minimize the effects of the abrupt change in slope.

The computer program used was developed and written by Price (1967) during his investigation of the broken-back culvert. One change made in Price's program was to calculate the friction slope at a particular point in the culvert by the Darcy-Weisbach equation using a friction factor dependent upon the Reynold's number. Price calculated the friction slope using the Manning equation and a constant roughness "n" value of 0.010, however, better reproduction of the water surface profile measured in the laboratory was obtained in this study using the computed friction factor. In tests run with smooth lucite pipe, French (1956) obtained the following relationship between friction factor and Reynold's number:

$$f = 0.0186\ (10^5/\text{Rey})^{0.2} \qquad (8)$$

This relationship was used in the computer program instead of Manning's "n" when calculating friction slope. Fairly good agreement was obtained between the computed value of y_C and the extrapolated value of y_C measured downstream of Section C-C during the experiments. Figure 10

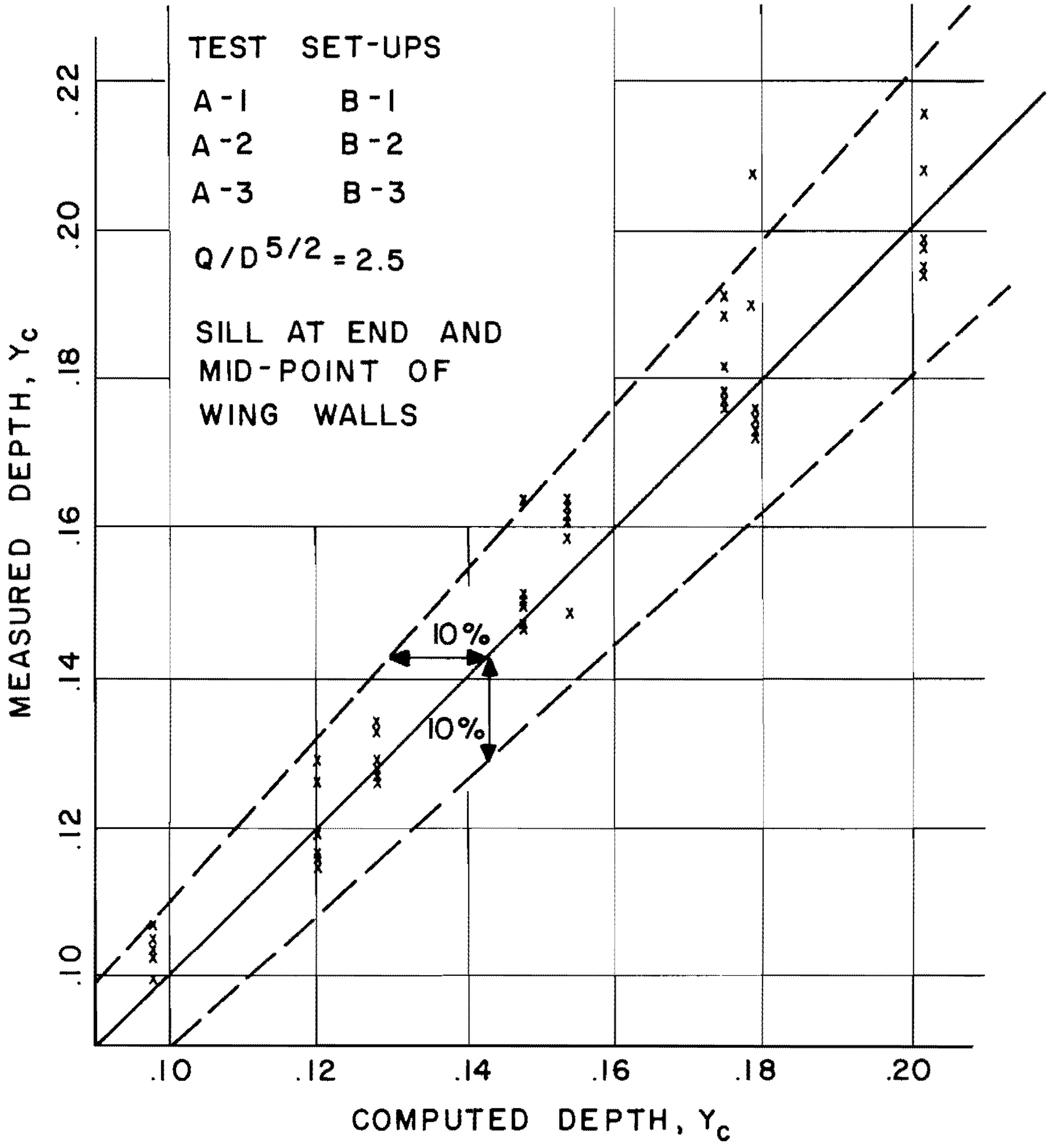


FIGURE 10 CORRELATION PLOT OF COMPUTED AND MEASURED WATER DEPTHS AT END OF UNIT 2

is a correlation plot of the two y_C values which gives some indication of the agreement between the two. The fact that almost all of the computed values of y_C are within at least 10% of their corresponding values measured in the laboratory indicates that the accuracy of the computer program is well within the requirements for this study.

DATA INTERPRETATION AND RESULTS

An empirical study involving the collection of a large amount of data is of little value unless presented in a concise and useful form that can be easily interpreted. With the data collection and experimental portions of this study completed, an analysis of the data was made using the series of dimensionless parameters derived in the previous chapter to develop a desirable presentation scheme.

Beginning with the nine basic parameters listed on page 36, a trial and elimination process was undertaken that involved plotting the data in terms of various combinations of the parameters. This process was repeated until a suitable form for the presentation of the results of the investigation was obtained. Such a presentation scheme should be as general as possible and include the pertinent variables involved in stabilizing a jump with an end sill. This was a time consuming process; however, there were several assumptions and changes made that considerably reduced the difficulty of the task.

The first of these was to further reduce the number of parameters involved by assuming several of them to be constant or nearly constant for all portions of the experiment. Since the slope of Unit 3 of the culvert system was maintained as nearly horizontal as possible for all geometry configurations used in the experiment, it could justifiably be removed from the analysis as a non-varying parameter. The same is true of L_d/D which was maintained at a value of 2 as a necessary condition in the selection of proper sill height. Figure 11 shows how accurately this criterion was satisfied throughout the experiment and indicates the validity of assuming that the distance from the downstream end of the jump to the culvert outlet was a constant for all of the tests run. Although the error in maintaining the value of L_d/D at 2 throughout the experiment appears to be large, a corresponding error in the height of the sill does not exist. This is because of the sensitivity of the location of the jump inside the pipe to the sill height. It was observed throughout the experiment that a slight change in the height of sill on the order of only 4 or 5 thousandths of a foot would cause the hydraulic jump to move as much as 1-1/2 or 2 feet. Another reason for the scatter of points in Figure 11 is that the downstream end of the jump could not always be accurately determined. The point where bubbles of air entrained by the jump separated with approximately half moving upstream and half moving downstream was used as the criterion for the downstream end of the jump. Exact determination of this point was always subjective and not an easy task.

Another aid in analyzing the data was to assume that L_u/D was a constant. Theoretically this is not true since L_u is equal to the sum of L_d plus the jump length. If L_d remains constant as the discharge is varied during a series of tests on a given pipe set-up, the jump length, L_j, will vary as the conjugate depths adjust to the change in

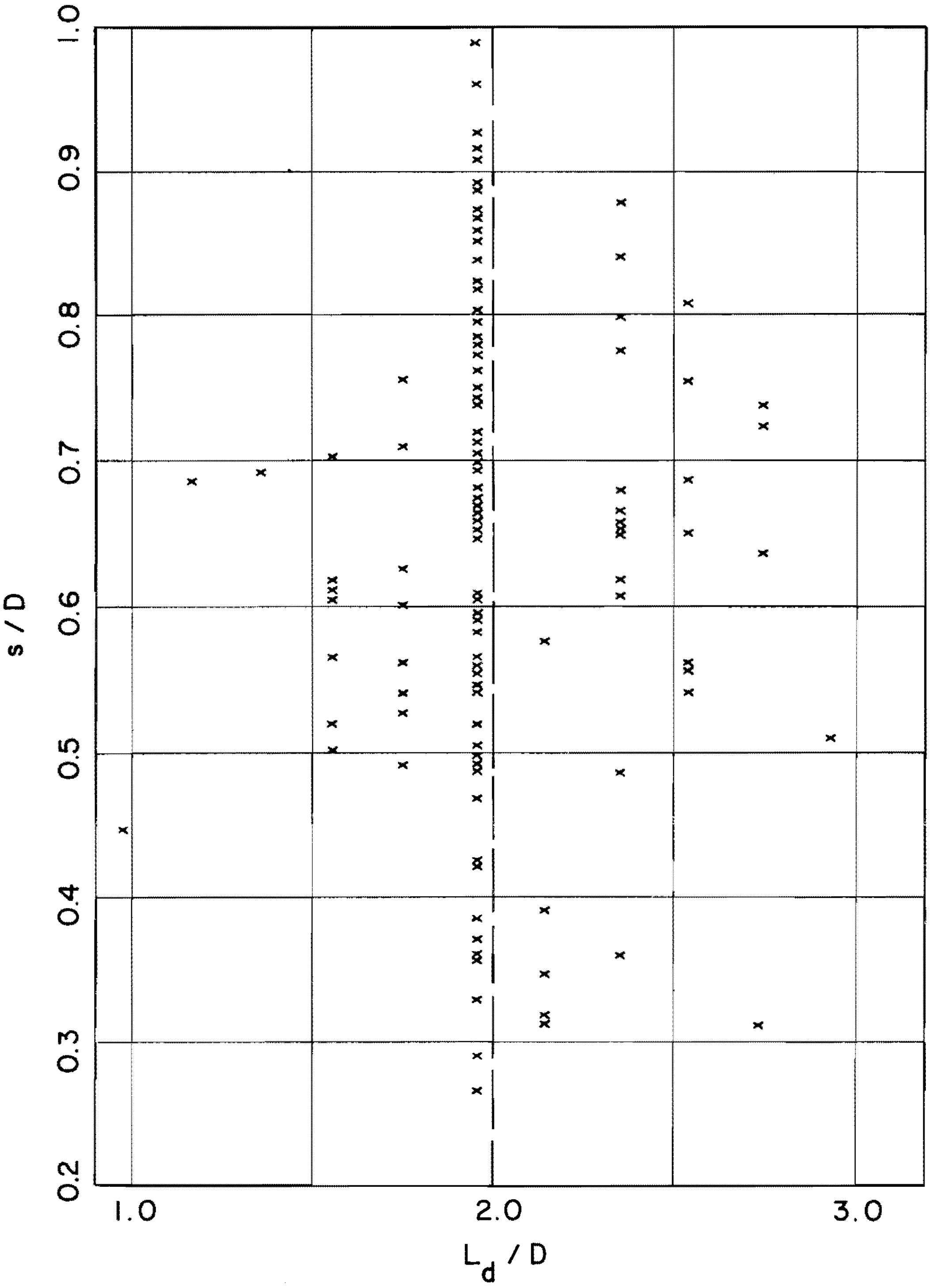


FIGURE II GRAPHICAL RELATIONSHIP SHOWING THE ACCURACY WITH WHICH L_d / D WAS MAINTAINED EQUAL TO 2.0

discharge, thus causing L_u to change. The change in L_u is insignificant, however, when compared to the accuracy with which the downstream end of the jump can be stabilized at a particular location as indicated in Figure 11. Figure 12 shows values of L_u measured in the laboratory for most of the tests run, and it can be seen that its average location is within $\pm$ 0.75 feet. Again, this wide variation in the location of the upstream end of the jump is a direct reflection on the accuracy with which the location of the downstream end of the jump was maintained at a constant value.

The location of the sill parameter, x/D, also remained constant for a given set of tests run. Although the sill was placed at three different locations in the downstream channel, the data collected with the sill at a single location was independent of the data collected at other sill locations. Thus analysis of the data for each sill location could be performed separately with the x/D parameter held constant.

Considering the assumptions discussed in the previous paragraphs, the basic dimensionless parameters involved in stabilizing a jump inside a broken-back culvert reduce to the following terms:

$$s/D, \; Q/D^{2.5}g^{0.5}, \; E_C/D, \; L_3/D, \; H/D$$

Rearrangement of several of these parameters and recognizing that the acceleration of gravity, g, remains constant for all of the experiments, the following forms of the parameters were deduced,

$$s/L_3, \; Q/D^{2.5}, \; L_3/E_C, \; L_3/D, \; H/D$$

In the terms s/D and E_C /D, the quantity L_3 was substituted for D in order that the effects of culvert geometry could be better incorporated

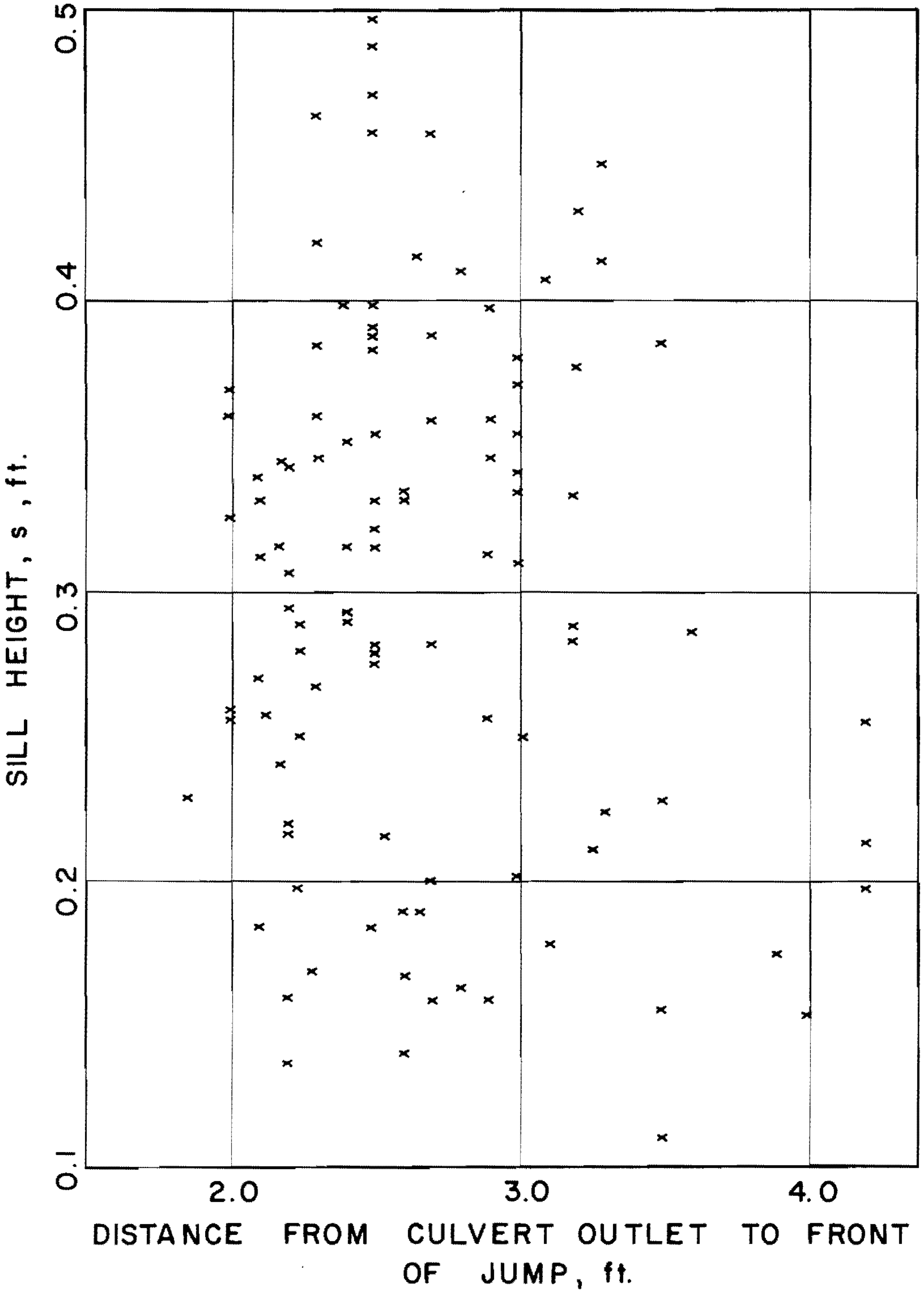


FIGURE 12 RELATIONSHIP SHOWING RANGE OF L_u

in the dimensionless parameters. Thus what once was a problem of relating sill height to fifteen different variables has been reduced to finding the empirical relationship between five dimensionless parameters. It should be recognized here that although the original number of variables involved in this investigation has been considerably reduced, none of them were completely disregarded in the analysis. Those not appearing in the five basic dimensionless parameters were either represented by some other variable or were considered constant for given test runs.

Using the data collected in the laboratory, each of the parameters was determined for every test run during the experiment. Plotting of various combinations of these parameters was then undertaken to find the most desirable means of presentation. One of the first observations made was that a plot s/L_3 versus y_C/L_3, the depth y_C, here being substituted for the specific energy, E_C, at the end of Unit 2, resulted in a unique set of curves for a constant discharge factor, $Q/D^{2.5}$, and sill location. It appears that for constant discharge factors, data plotted as nearly straight lines for a given pipe geometry upstream of Unit 3. Figures 13 through 19 are examples of this type of plot. The family of curves in any one of these plots is associated with one particular sill location, one diameter of unventilated pipe, and one value of total fall over the length of the culvert, H/D. In these figures, solid lines represent constant discharge factors and dashed lines represent constant L_3/D values.

These plots of s/L_3 versus y_C/L_3 were useful qualitative checks on the accuracy of the data taken for a given pipe set-up and sill

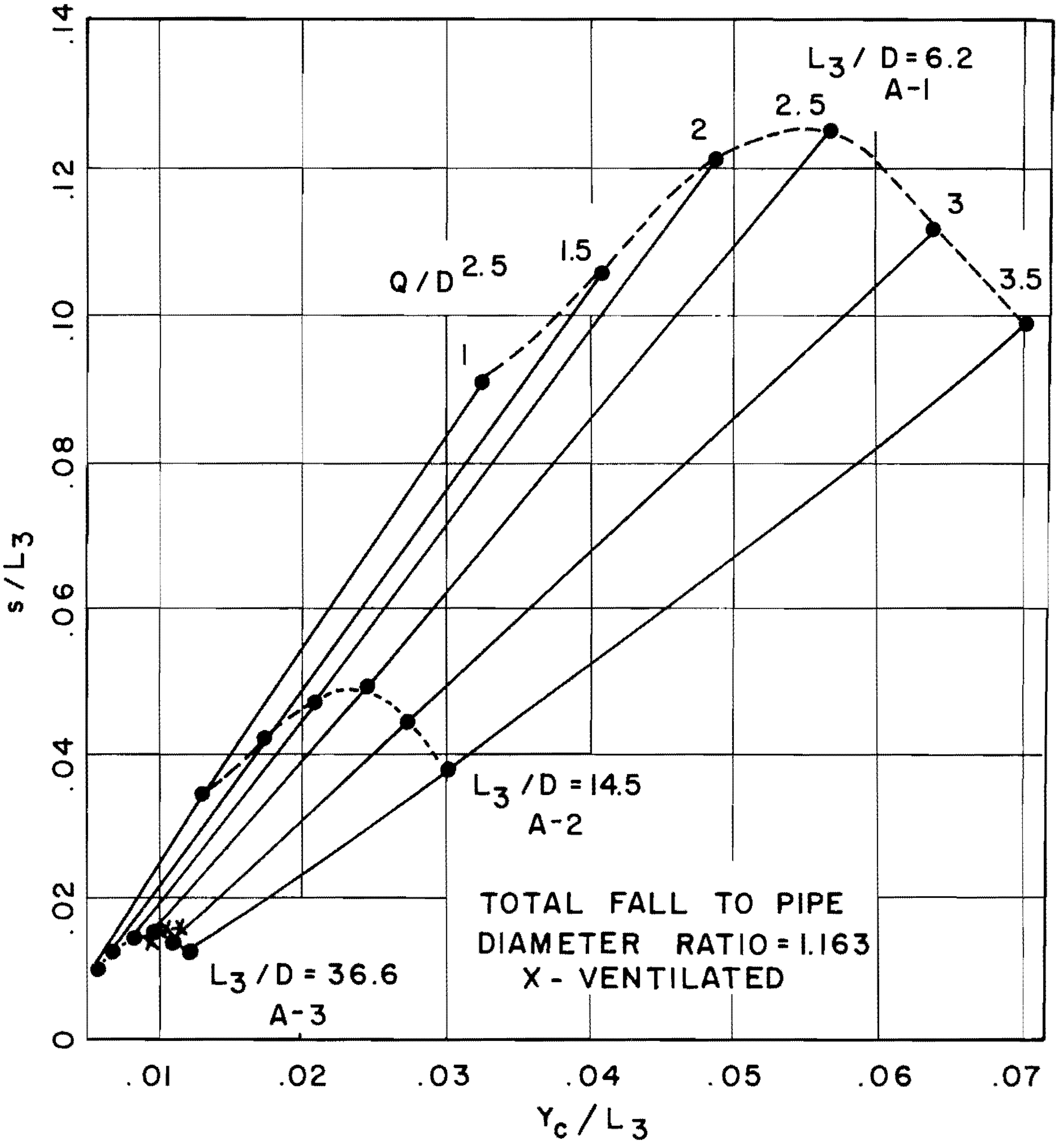


FIGURE 13 s / L_3 vs Y_c / L_3 FOR TEST SET-UP A WITH SILL AT MID-POINT OF WING WALLS

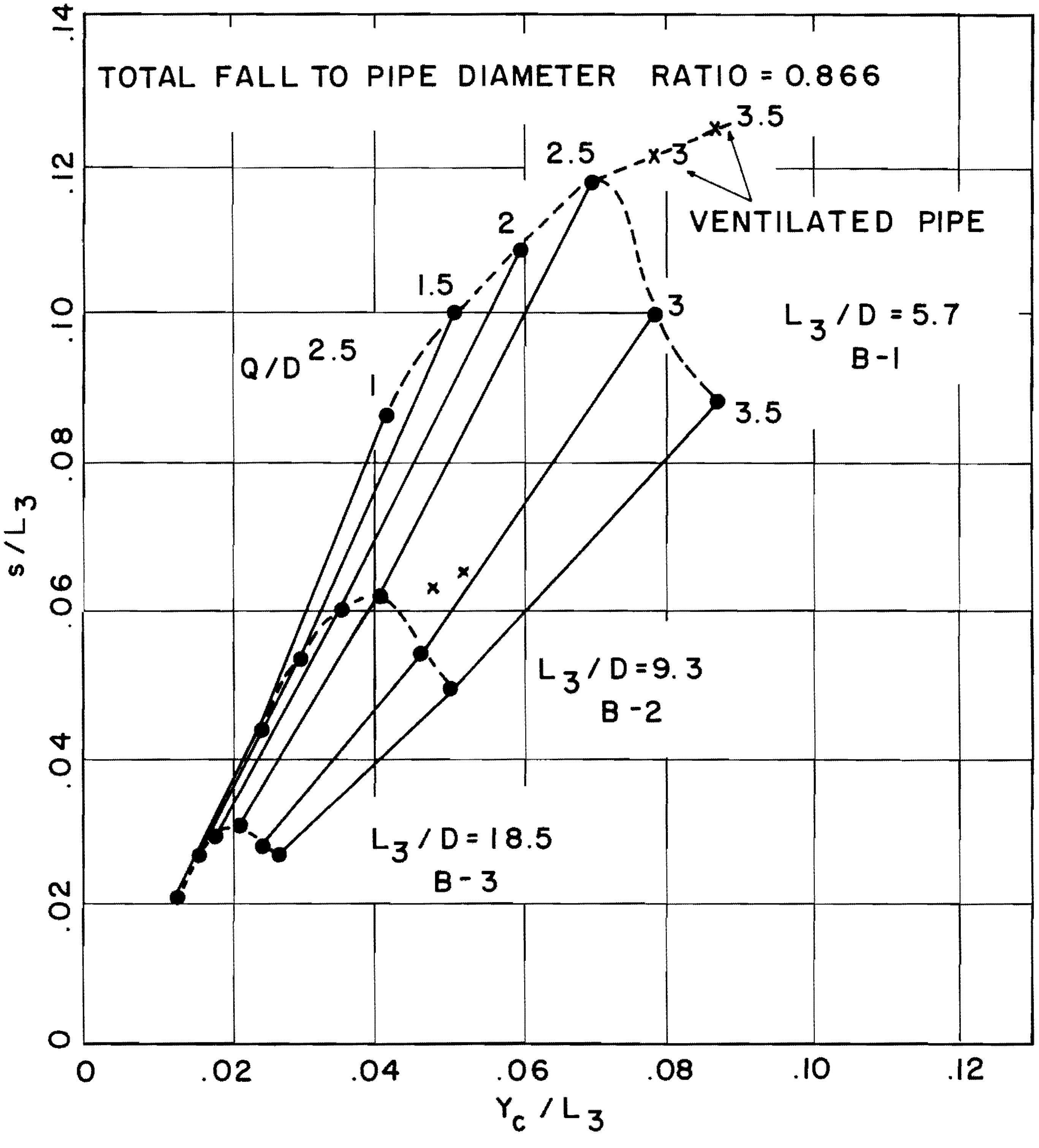


FIGURE 14 s/L_3 vs Y_c/L_3 FOR TEST SET-UP B WITH SILL AT MID-POINT OF WING WALLS

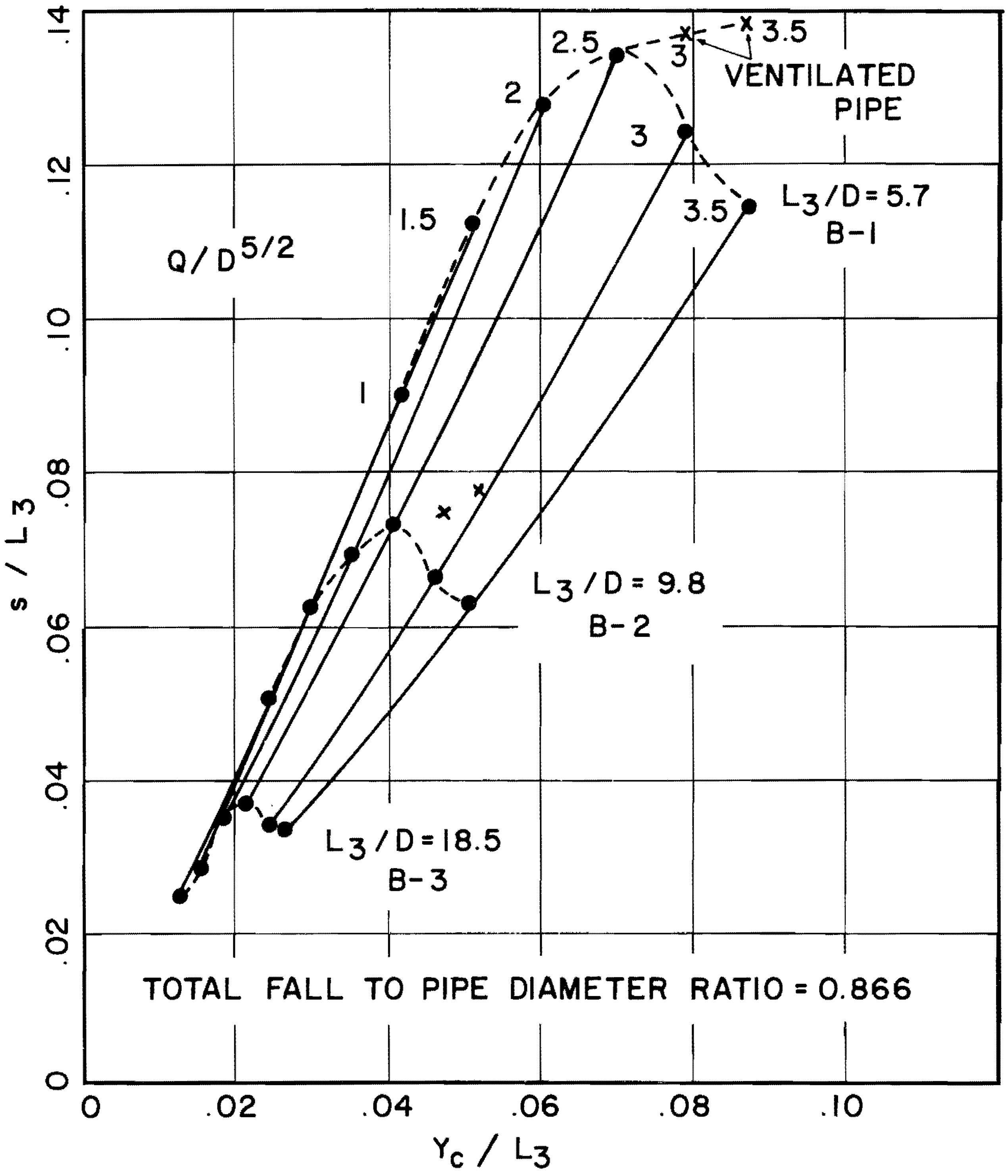


FIGURE 15 s / L_3 vs Y_c / L_3 FOR TEST SET-UP B WITH SILL AT END OF WING WALLS

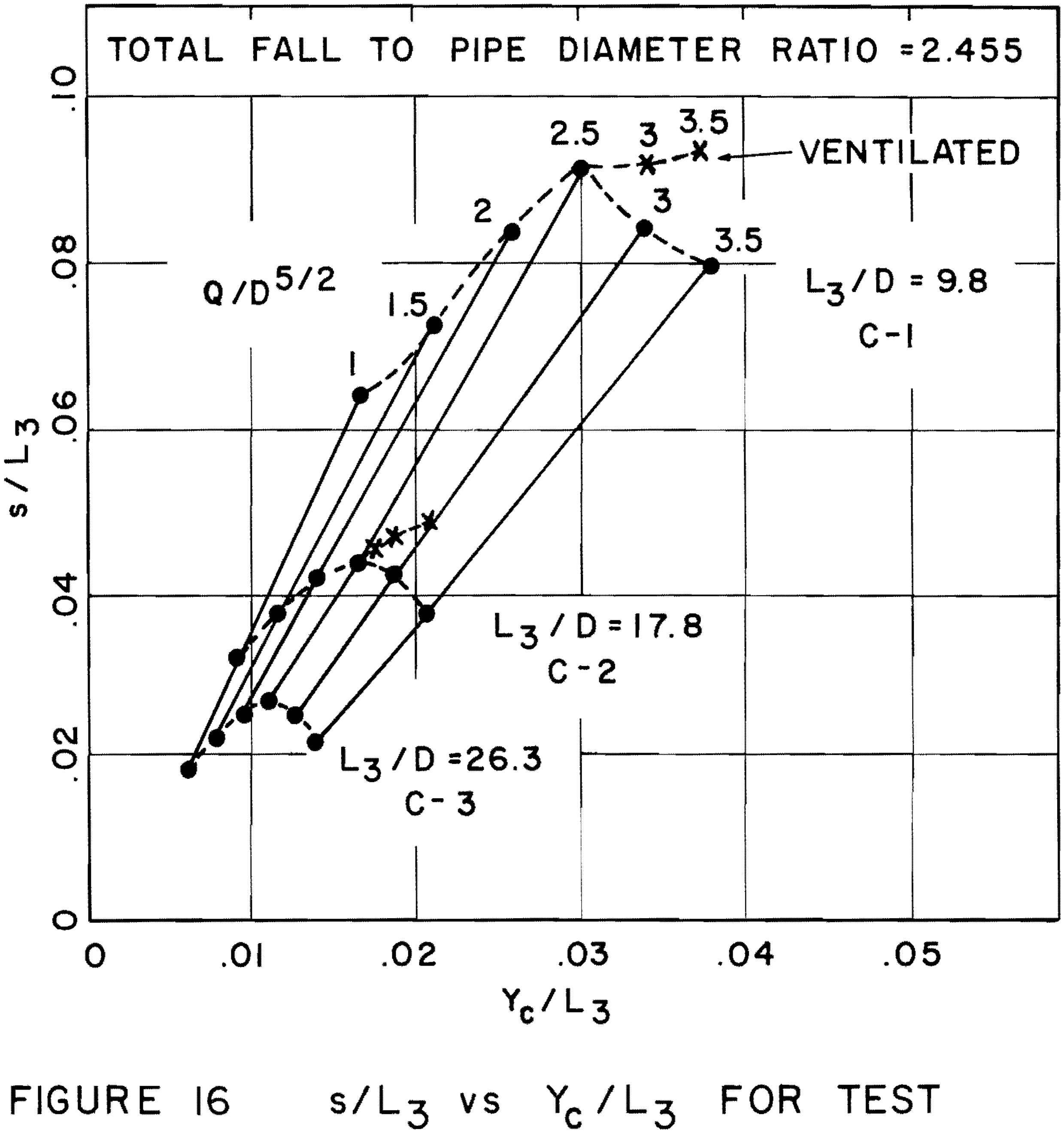


FIGURE 16 s/L_3 vs Y_c/L_3 FOR TEST SET-UP C WITH SILL AT MID-POINT OF WING WALLS

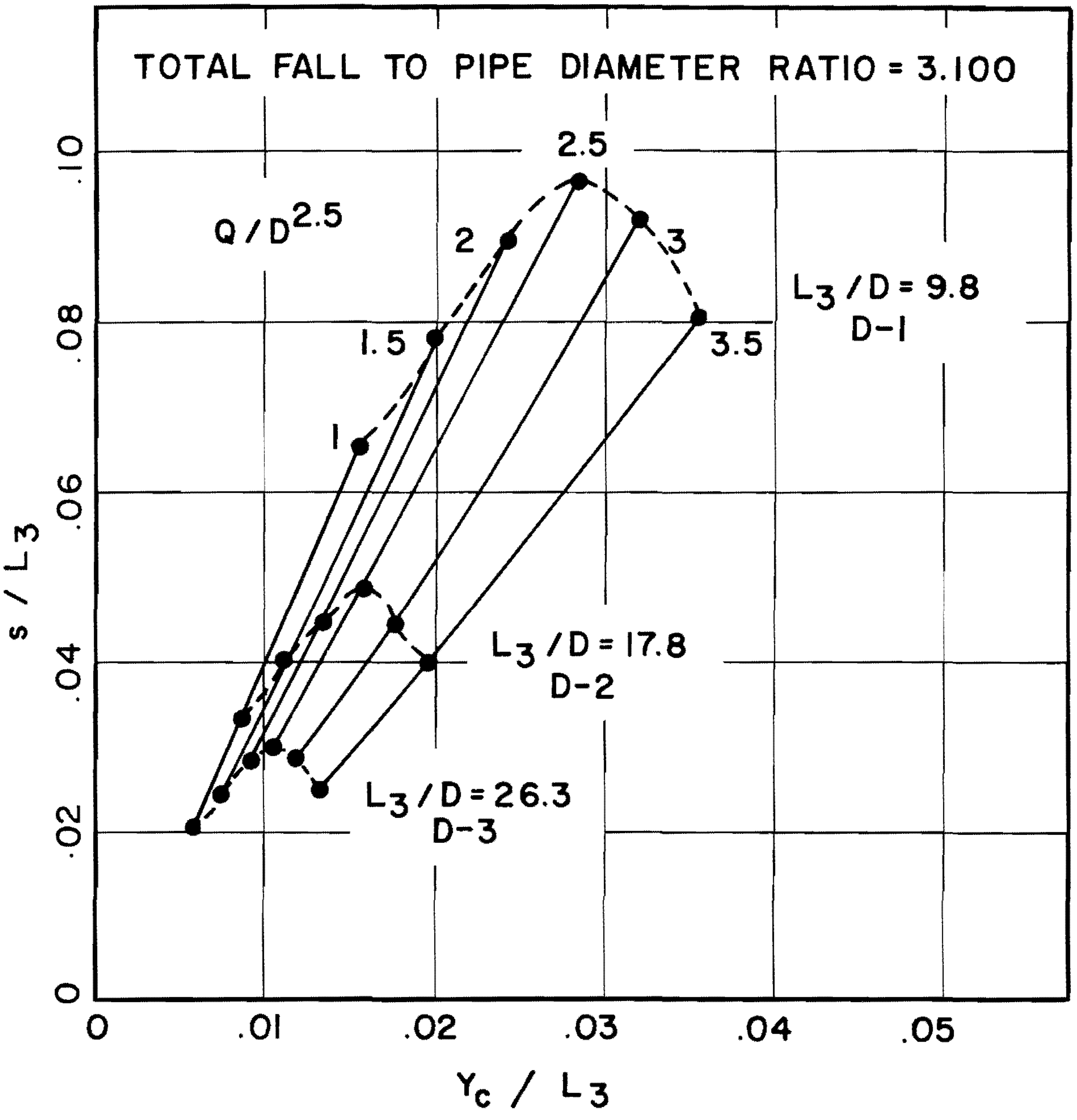


FIGURE 17 s / L_3 vs Y_c / L_3 FOR TEST SET-UP D WITH SILL AT MID-POINT OF WING WALLS

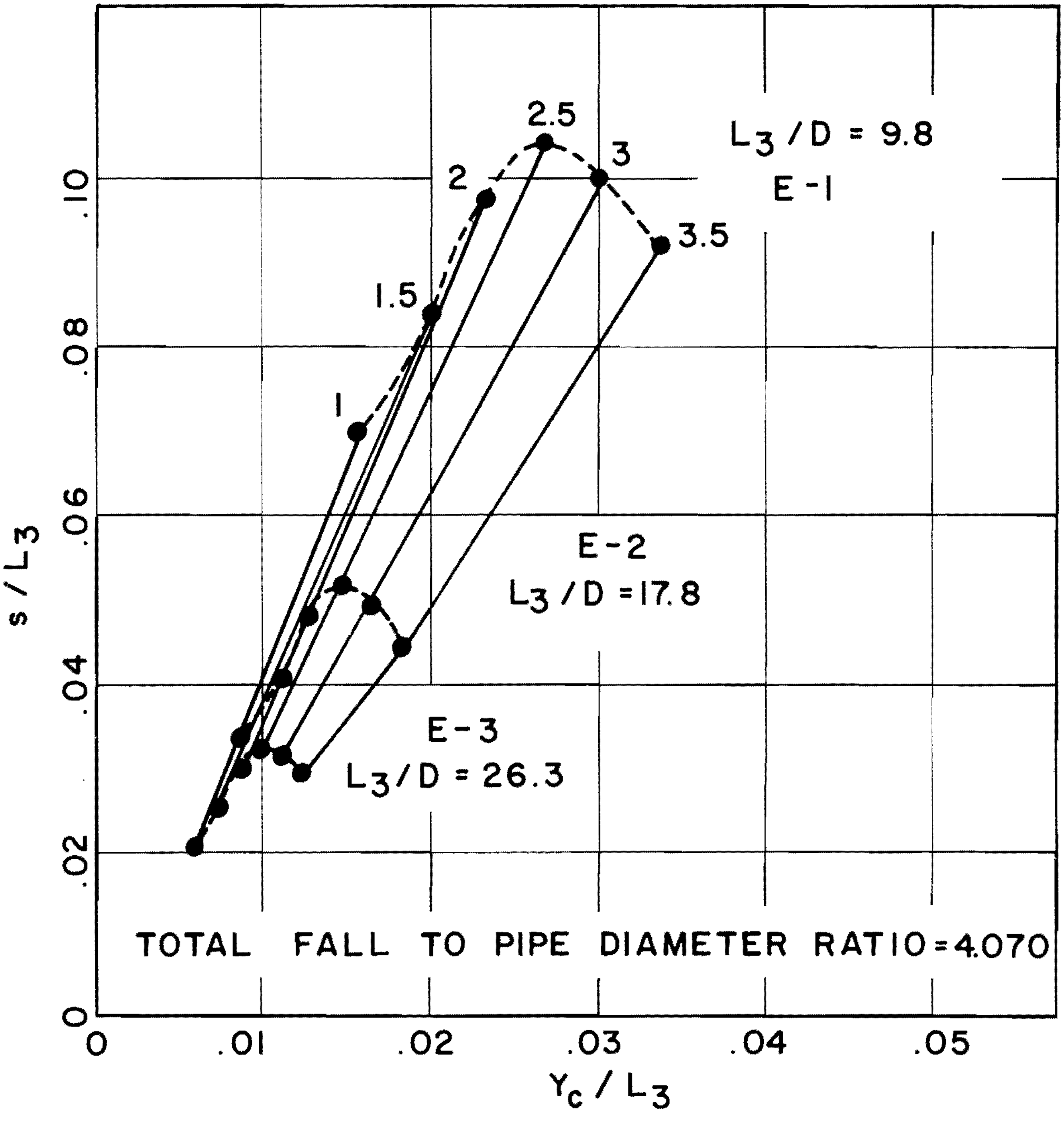


FIGURE 18 s/L_3 vs Y_c/L_3 FOR TEST SET-UP E WITH SILL AT MID-POINT OF WING WALLS

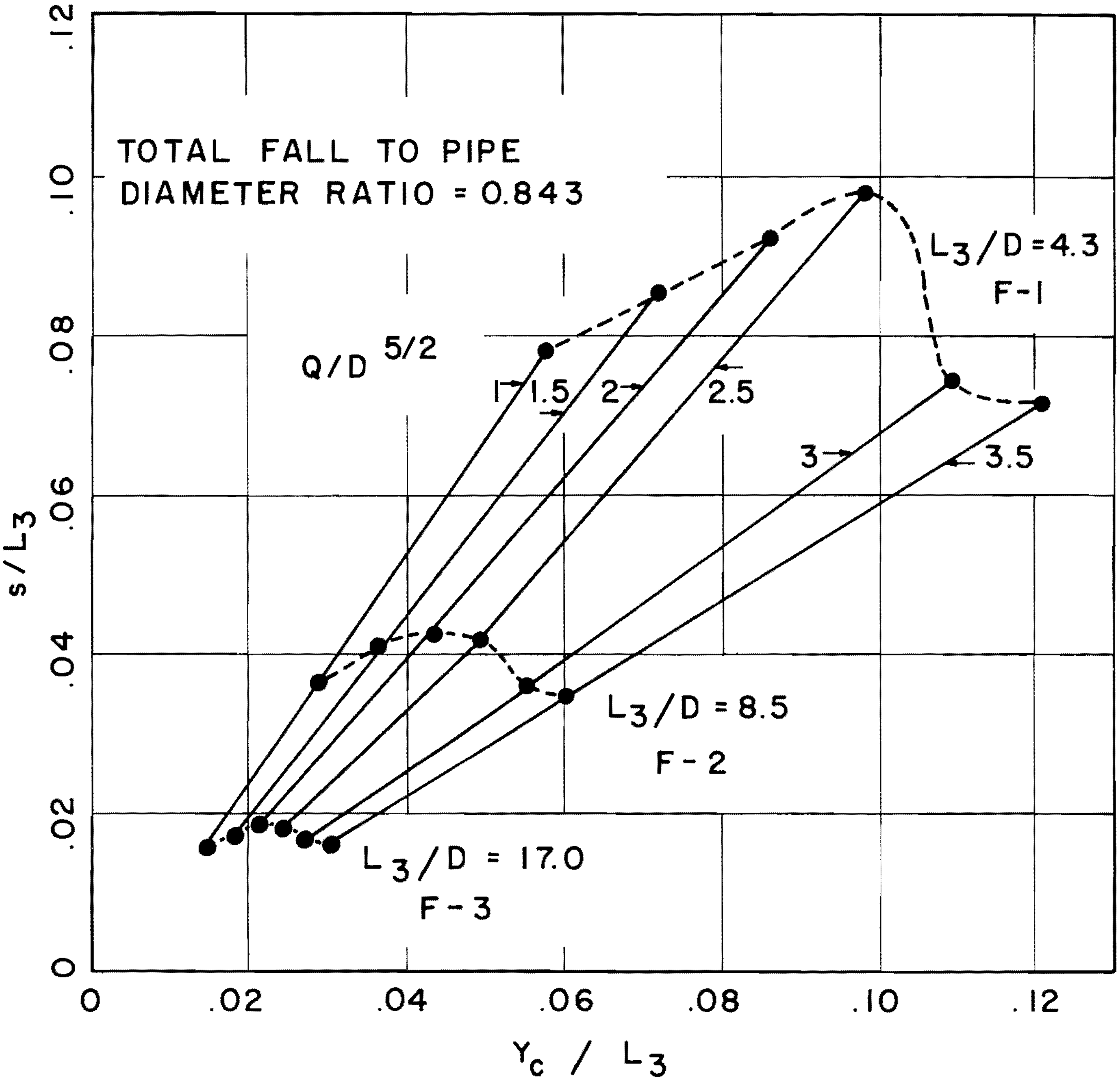


FIGURE 19 s/L_3 vs Y_c/L_3 FOR TEST SET-UP F WITH SILL AT CULVERT OUTLET location. It should be emphasized here that the basic shape of these curves is highly dependent upon the fact that the data were collected with no artificial ventilation supplied to the pipe. Two distinctive characteristics of these particular plots were checked on all of the data collected. The first was that lines of constant discharge factor plotted as nearly straight lines or at least very smooth curves for given x/D ratios. The second check was that the maximum required sill height for any given length of Unit 3 associated with a particular geometry upstream of Unit 3 occurred at $Q/D^{2.5} = 2.5$. This is the point where the flow at the culvert inlet changes from weir type flow to slug and mixture type flow in an unventilated pipe. At discharge factors greater than 2.5, the subatmospheric pressure developed within the culvert aids the sill in stabilizing a jump such that less sill height is required at these higher discharge values than that required at $Q/D^{2.5} = 2.5$. This effect is illustrated in the plots of s/L_3 versus y_C/L_3 by the dashed lines of constant L_3/D values. The effect of artificially ventilated and unventilated culverts on the sill height is also demonstrated in several of the s/L_3 versus y_C/L_3 plots. Figures 13, 14, 15 and 16 show the higher s/L_3 points required at given y_C/L_3 and L_3/D values for Set-Ups A-3, B-1, B-2, C-1 and C-2 operating with artificial ventilation at $Q/D^{2.5} = 3$ and 3.5. If the dashed lines of constant L_3/D values are fitted to these higher s/L_3 values, they appear to continually increase with discharge. There should be some limiting $Q/D^{2.5}$ value, however, where once a jump was forced into the pipe by a sill, the high tail water level produced by the sill would push the jump upstream to the culvert entrance and cause the culvert to flow full. Determination of this point was beyond the scope of this present study. Although the plot of s/L_3 versus y_C/L_3 has unique physical characteristics and is helpful in checking data accuracy, its usefulness in representing the data is limited. The fact that each plot is representative of only one culvert geometry and one total fall limits their usefulness as possible designs aids in selecting the sill height required by a culvert system of any given geometry.

The plot of s/L_3 versus L_3/E_C appears to be much more useful in this capacity than do any other combinations of the dimensionless parameters. Lines of constant H/D values, total fall of the culvert to pipe diameter ratio, can be represented in these plots with one family of H/D curves pertaining to one value of discharge factor and sill location. The data from all of the tests run during the experiment including all culvert geometries and sill locations were plotted in terms of s/L_3 versus L_3/E_C for the range of discharge factors used during the experiments. These plots are shown in Figures 20 through 35. Each plot represents all of the sill heights measured in the laboratory for one particular value of discharge factor and one of the three sill locations with no artificial ventilation of the pipe.

Analysis of the geometric pattern characteristic of these plots indicates that they are in qualitative agreement with the expected theoretical results. The fact that all demonstrate the same generalized pattern illustrates a definite relationship between the height of end sill,

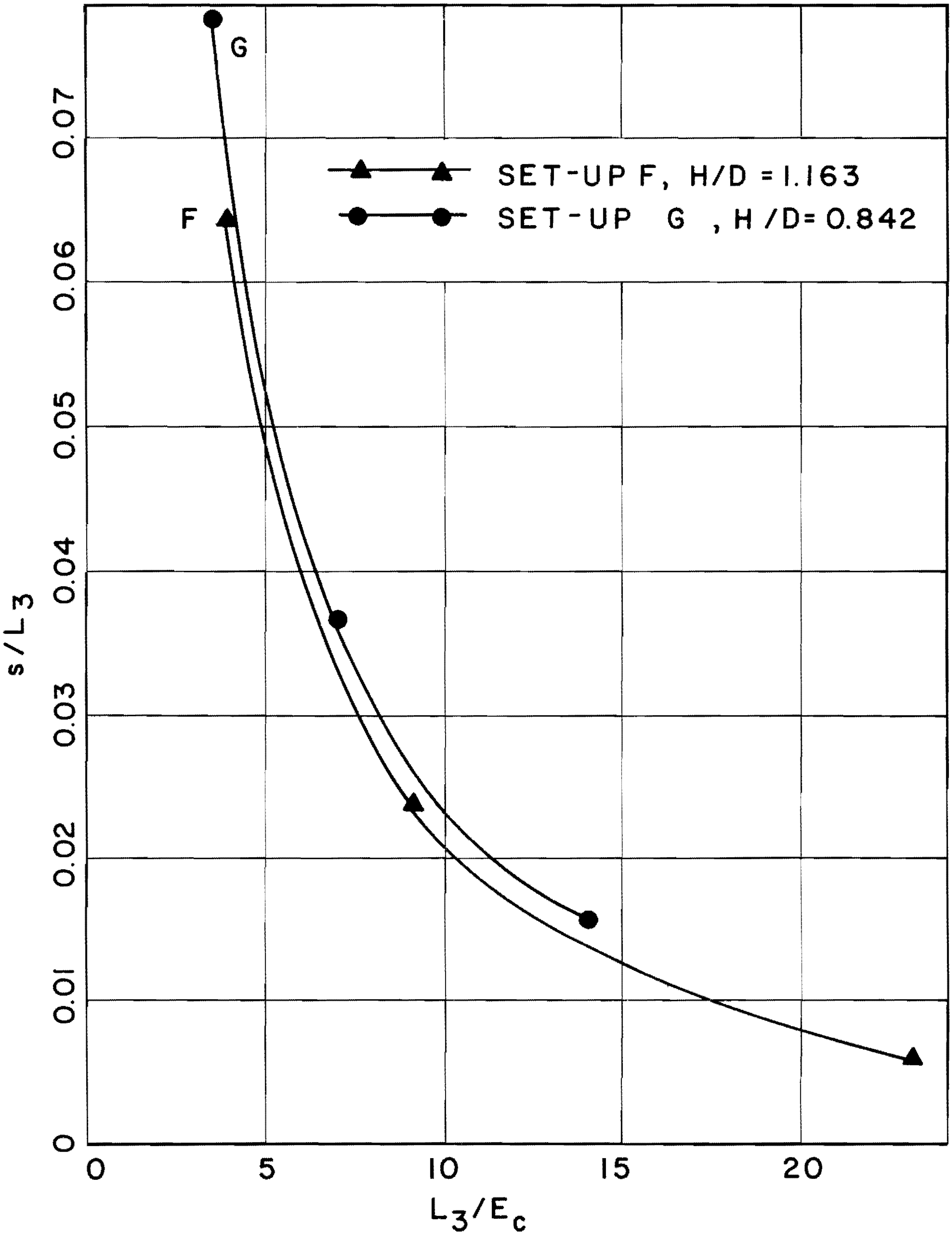


FIGURE 20 s/L_3 vs L_3/E_c WITH SILL AT CULVERT OUTLET AND $Q/D^{2.5} = 1.0$

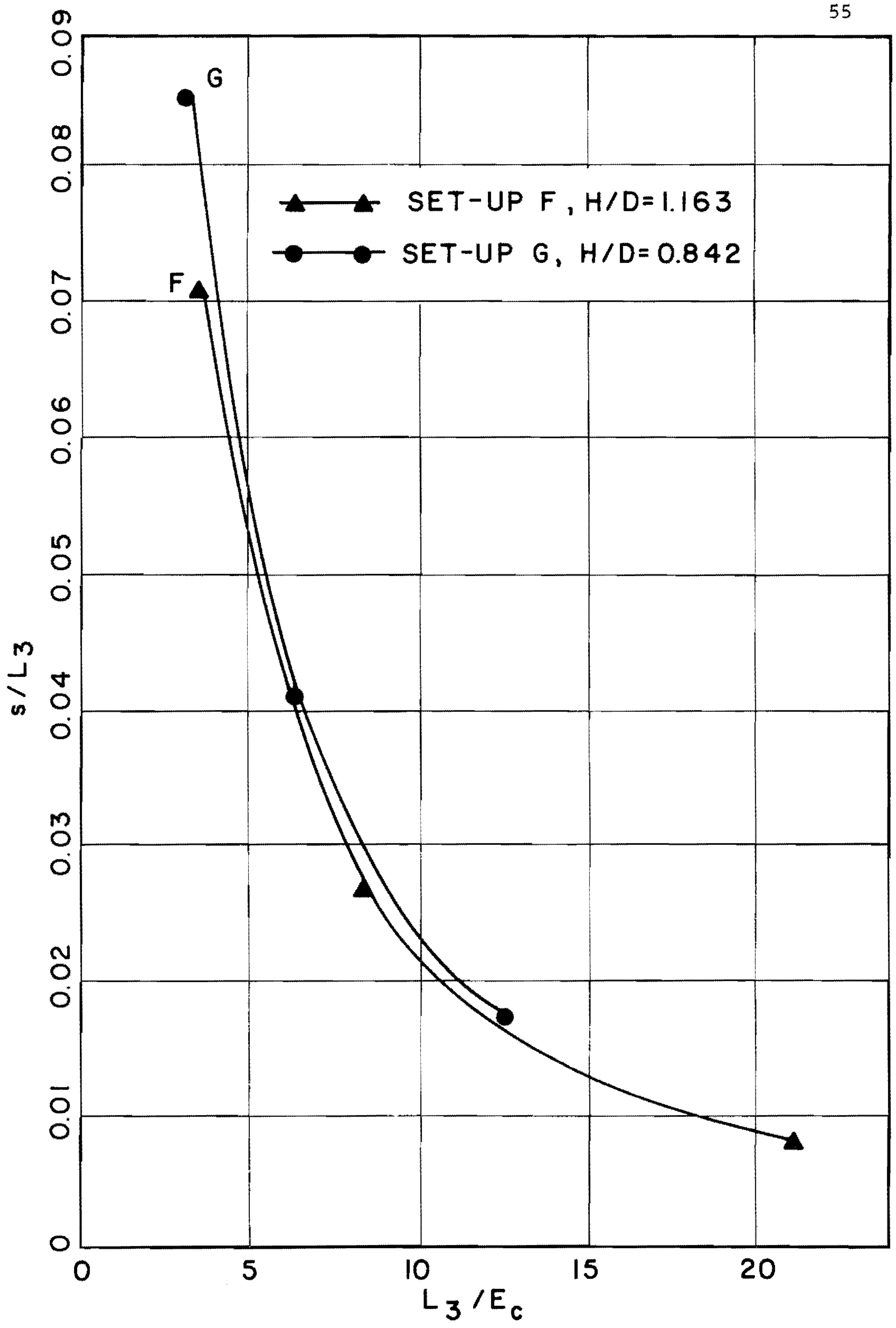


FIGURE 21 s/L_3 vs L_3/E_c WITH SILL AT CULVERT OUTLET AND $Q/D^{5/2} = 1.5$

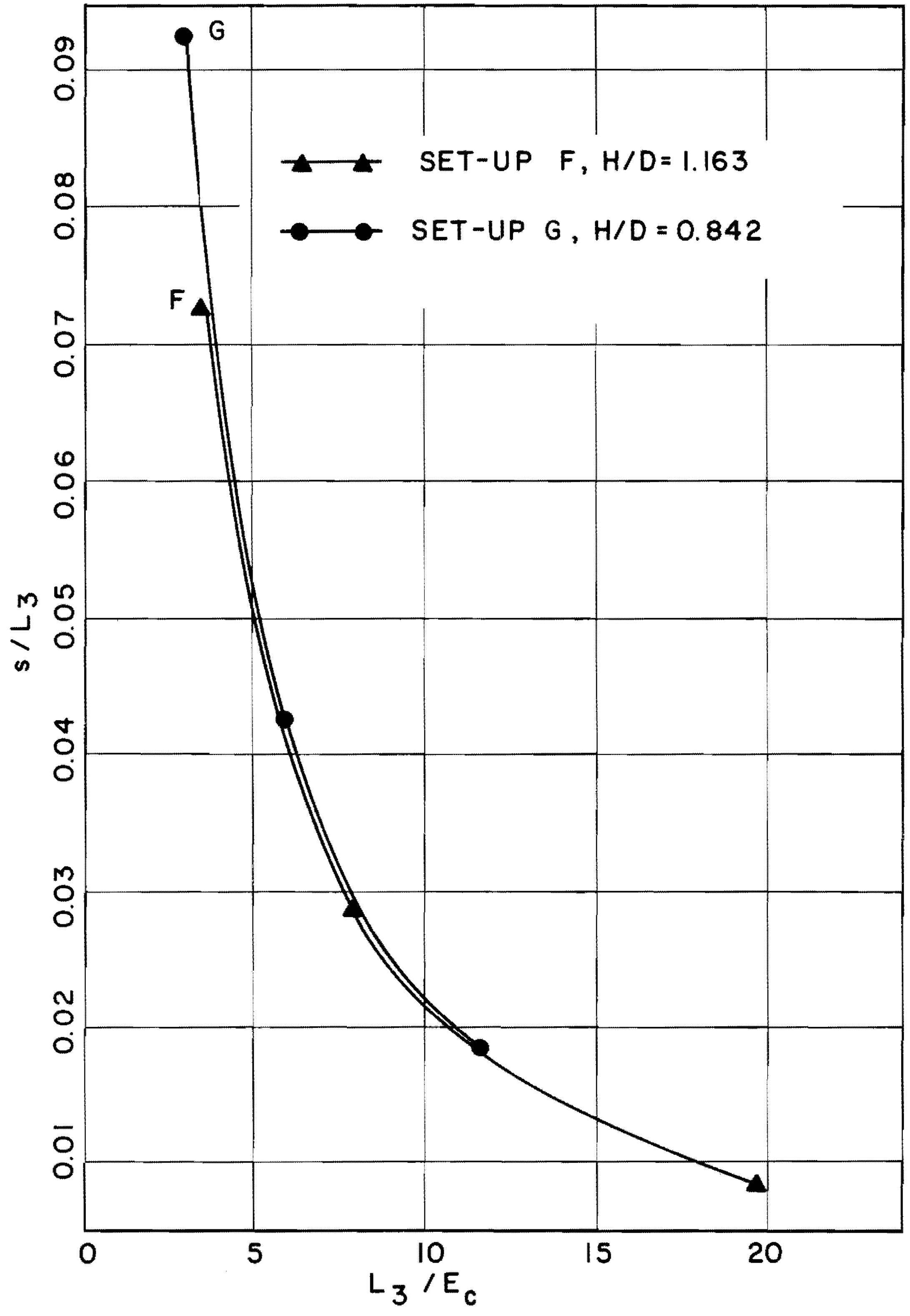


FIGURE 22 s/L_3 vs L_3/E_c WITH SILL AT CULVERT OUTLET AND $Q/D^{5/2} = 2.0$

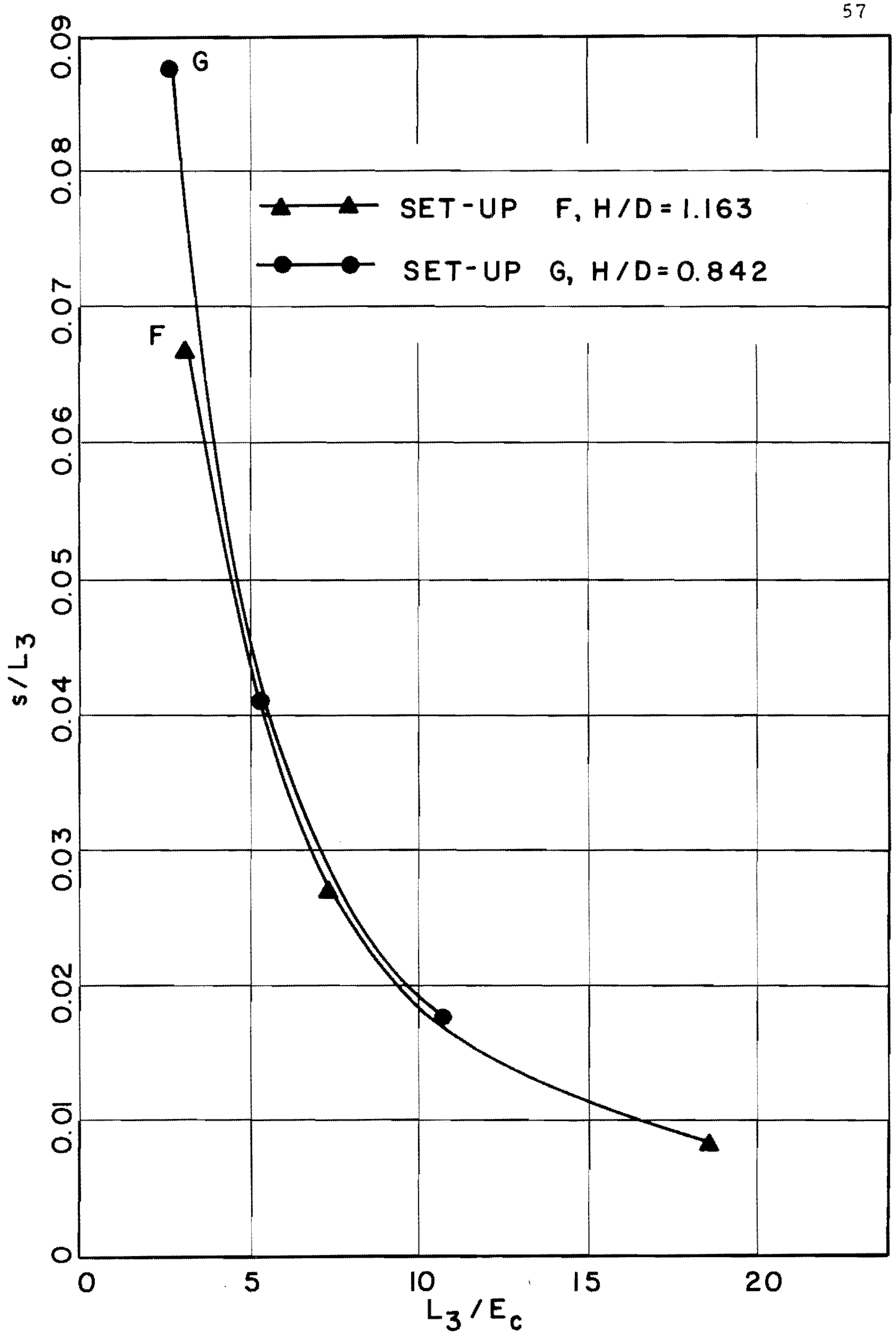


FIGURE 23 s/L_3 vs L_3/E_c WITH SILL AT CULVERT OUTLET AND $Q/D^{5/2} = 2.5$

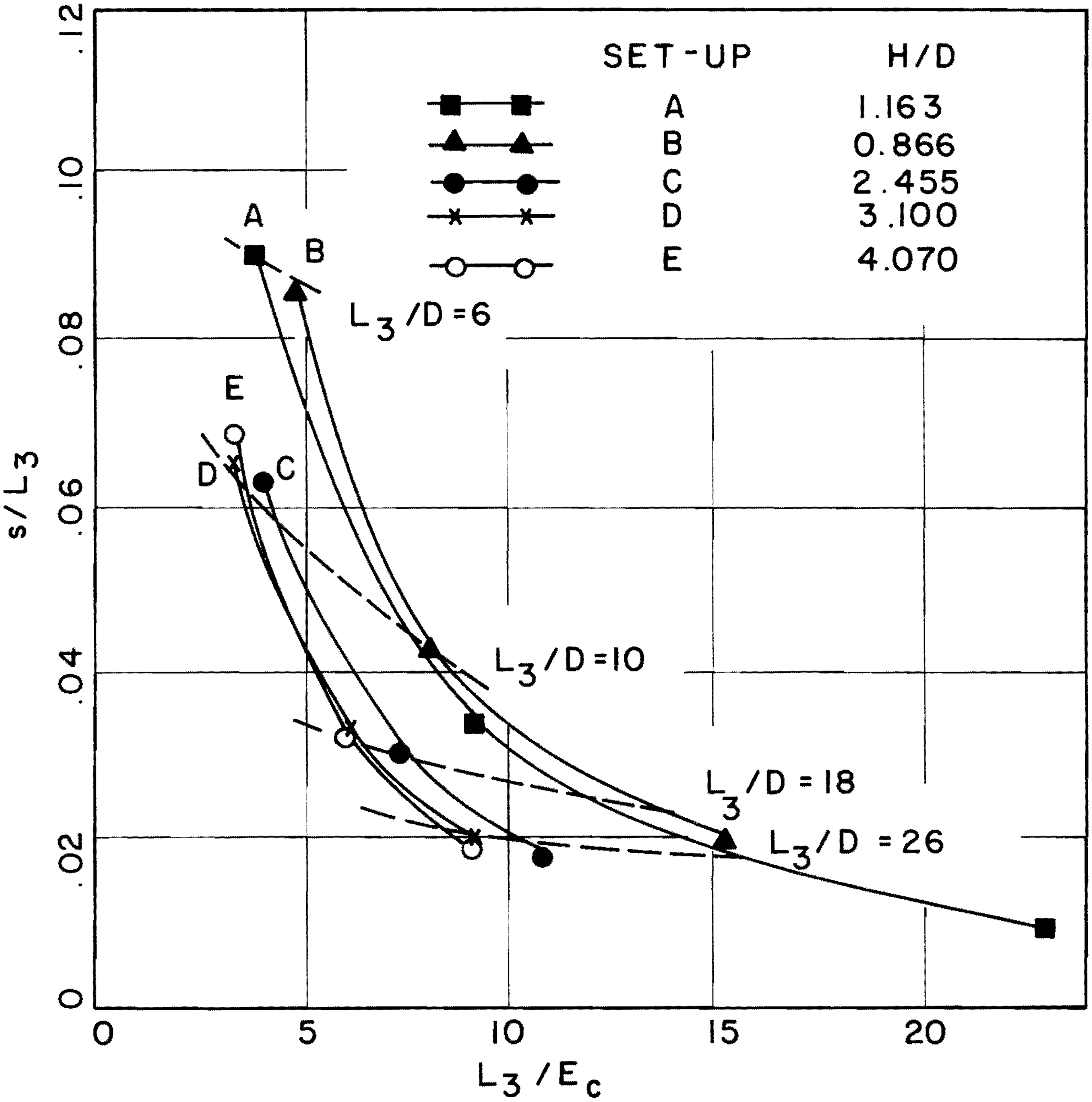


FIGURE 24 s/L_3 vs L_3/E_c WITH SILL AT MID-POINT OF WING WALLS AND $Q/D^{5/2} = 1.0$

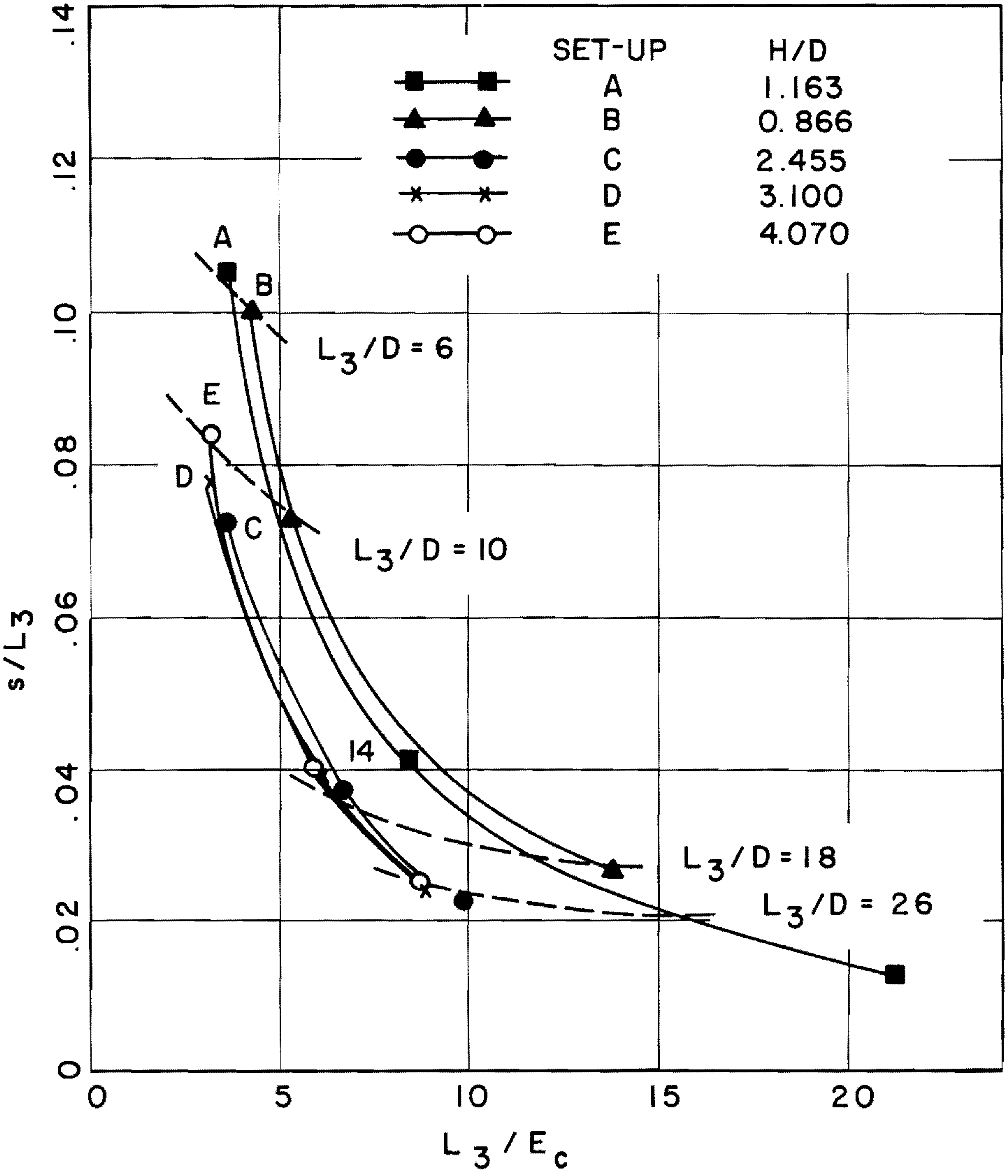


FIGURE 25 s/L_3 vs L_3/E_c WITH SILL AT MID-POINT OF WING WALLS AND $Q/D^{5/2} = 1.5$

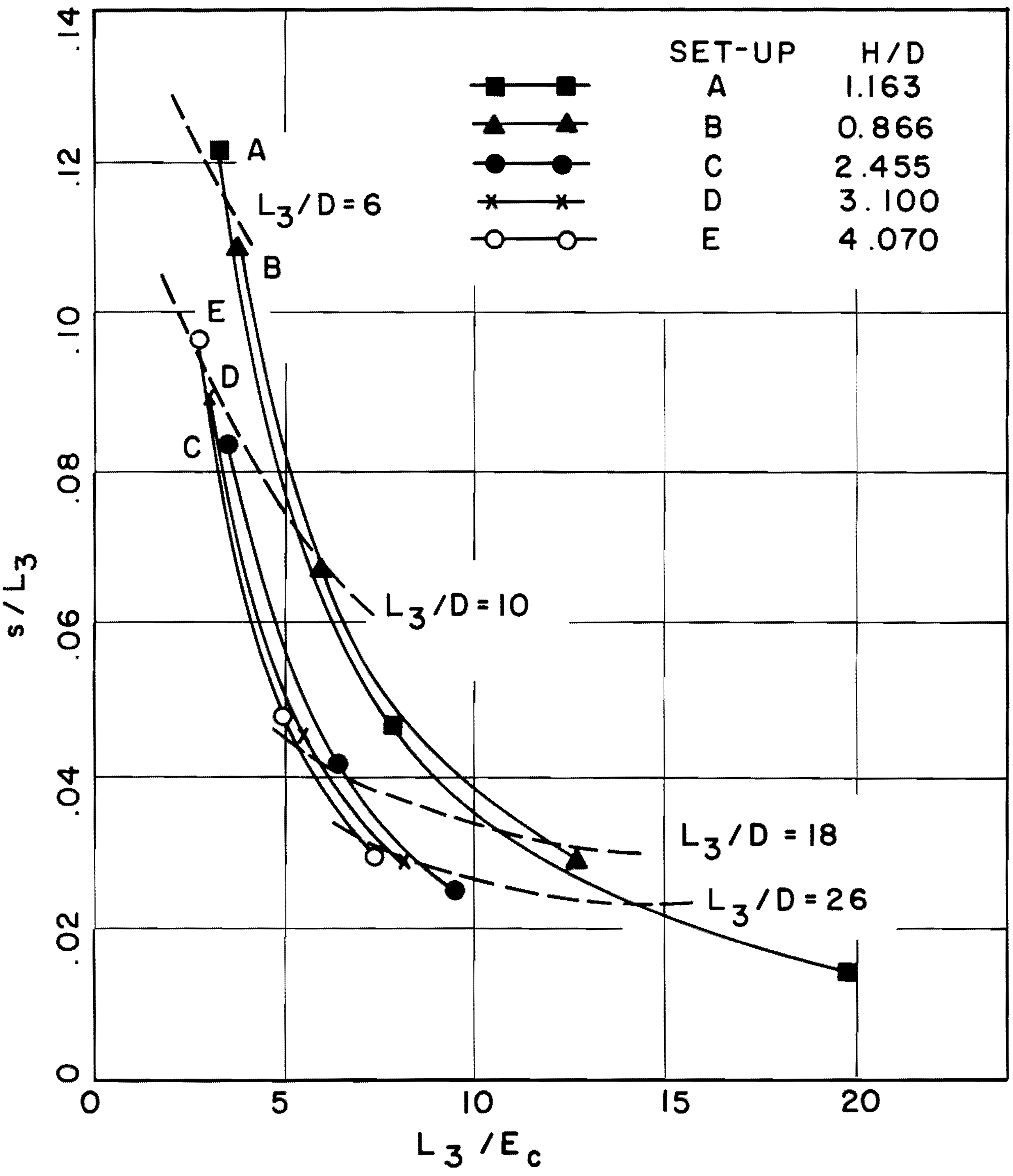


FIGURE 26 s/L_3 vs L_3/E_c WITH SILL AT MID-POINT OF WING WALLS AND $Q/D^{5/2} = 2.0$

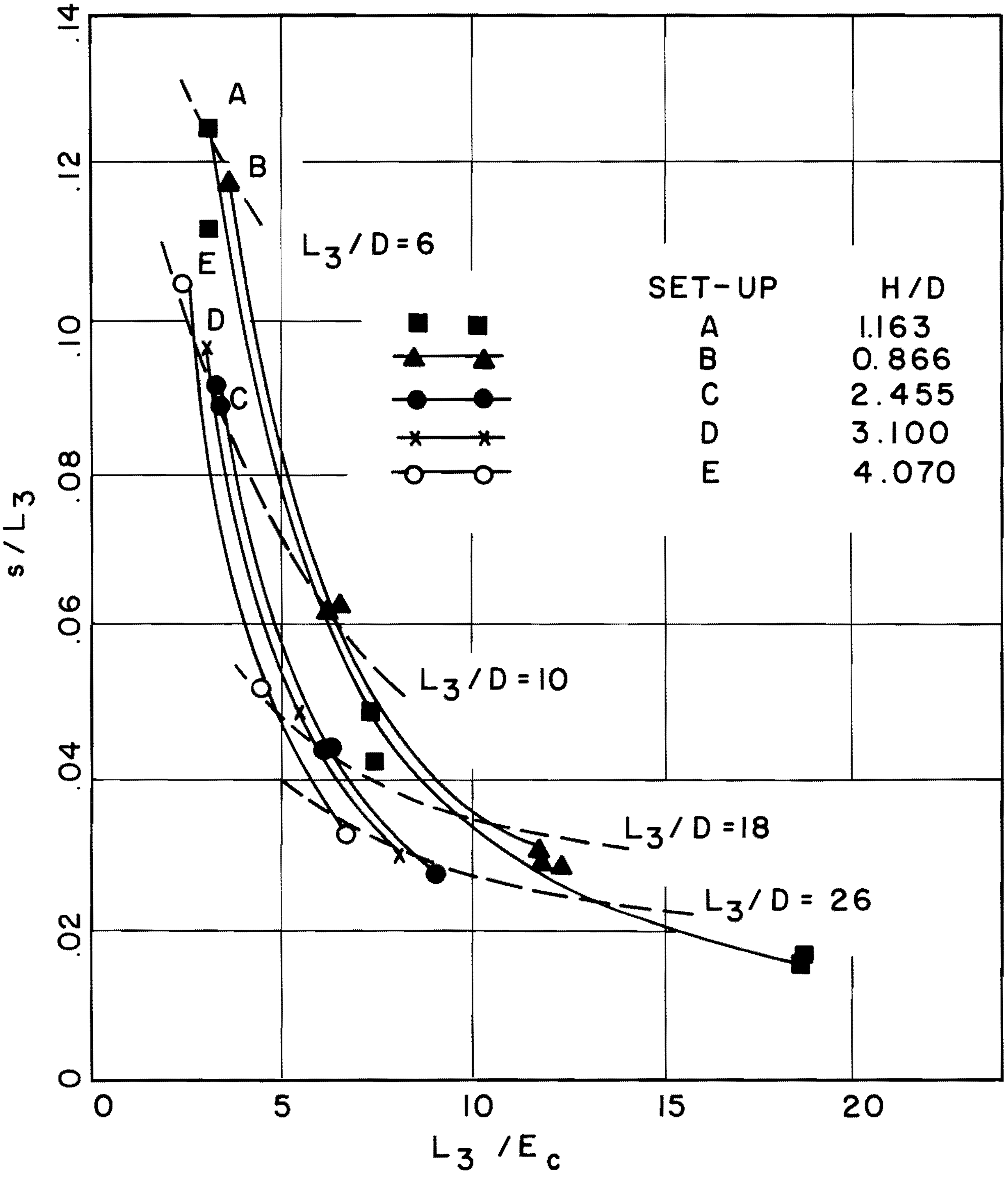


FIGURE 27 s / L_3 vs L_3 / E_c WITH SILL AT MID-POINT OF WING WALLS AND $Q / D^{5/2} = 2.5$

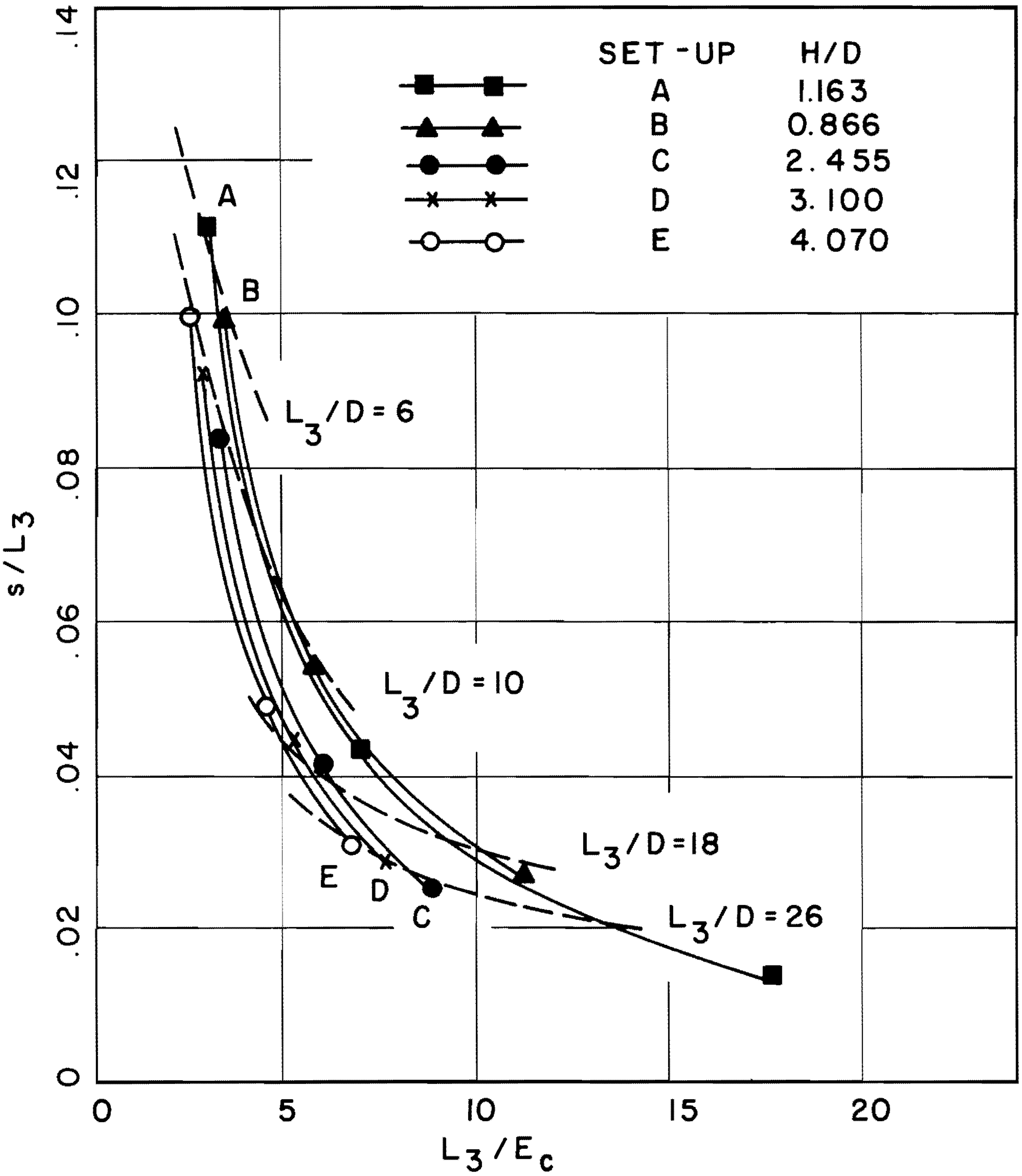


FIGURE 28 s / L_3 vs L_3 / E_c WITH SILL AT MID-POINT OF WING WALLS AND $Q / D^{5/2} = 3.0$

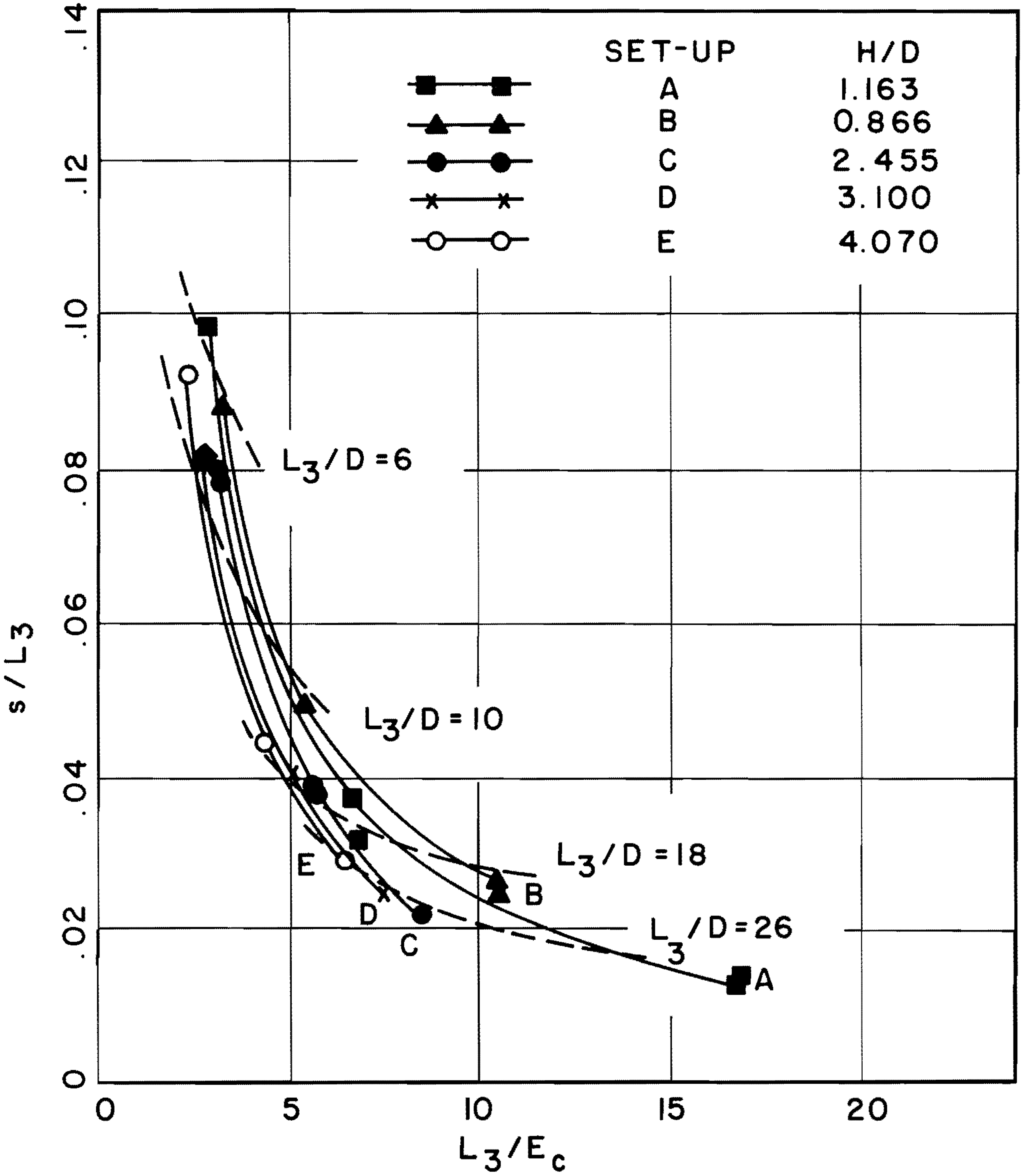


FIGURE 29 s/L_3 vs L_3/E_c WITH SILL AT MID-POINT OF WING WALLS WITH $Q/D^{5/2} = 3.5$

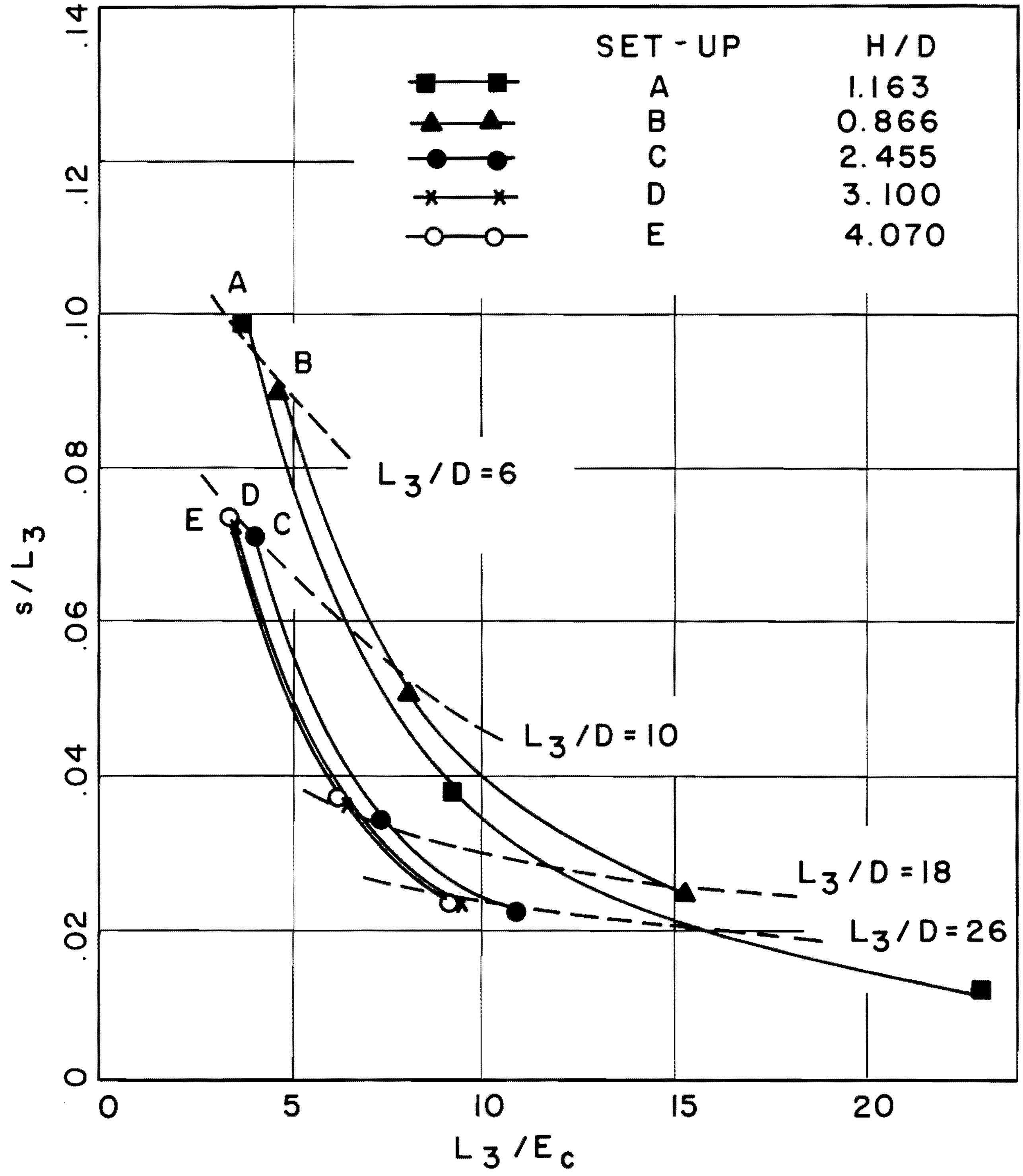


FIGURE 30 s/L_3 vs L_3/E_c WITH SILL AT END OF WING WALLS AND $Q/D^{5/2} = 1.0$

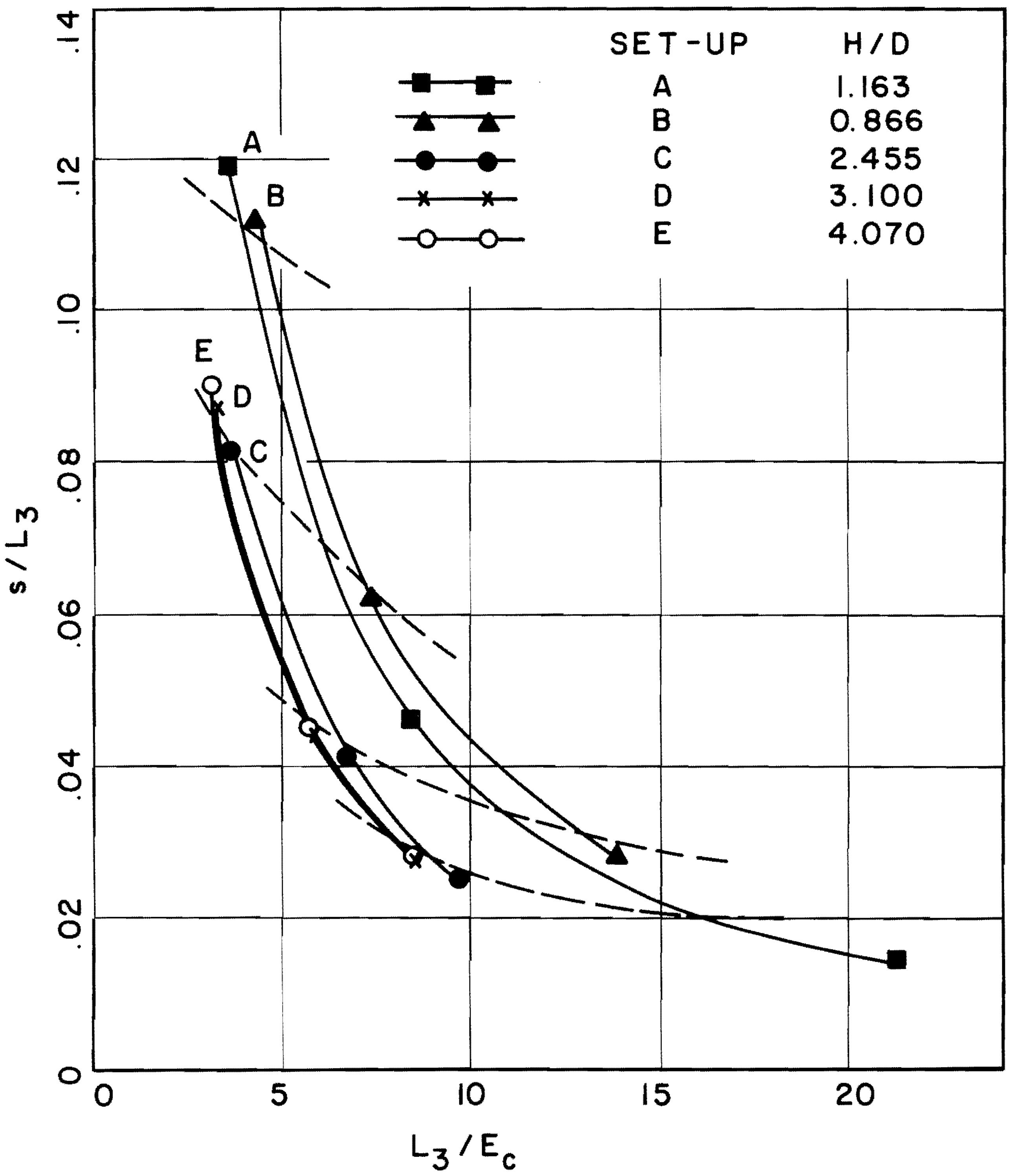


FIGURE 31 s / L_3 vs L_3 / E_c WITH SILL AT END OF WING WALLS AND $Q / D^{5/2} = 1.5$

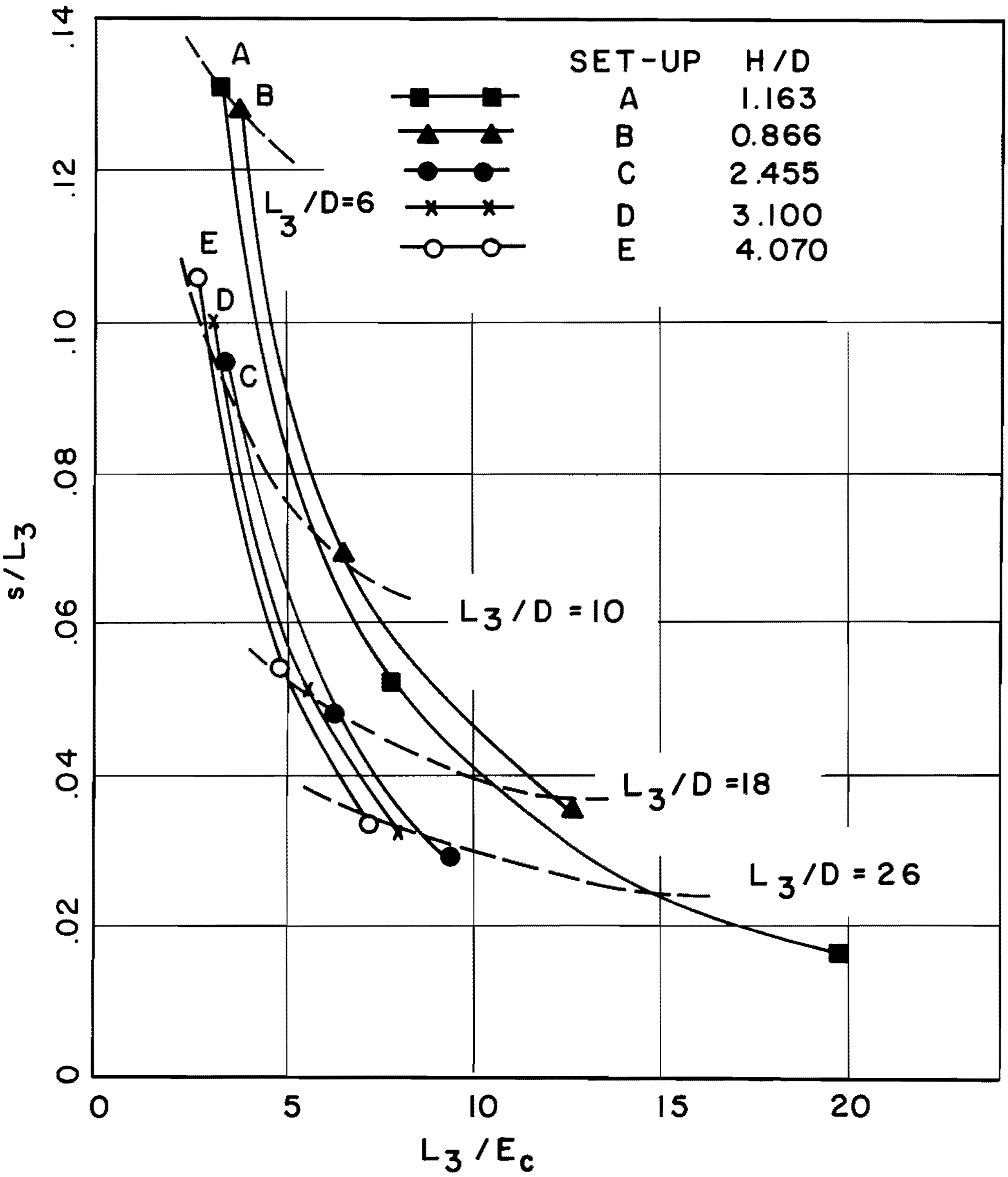


FIGURE 32 s/L_3 vs L_3/E_c WITH SILL AT END OF WING WALLS AND $Q/D^{5/2} = 2.0$

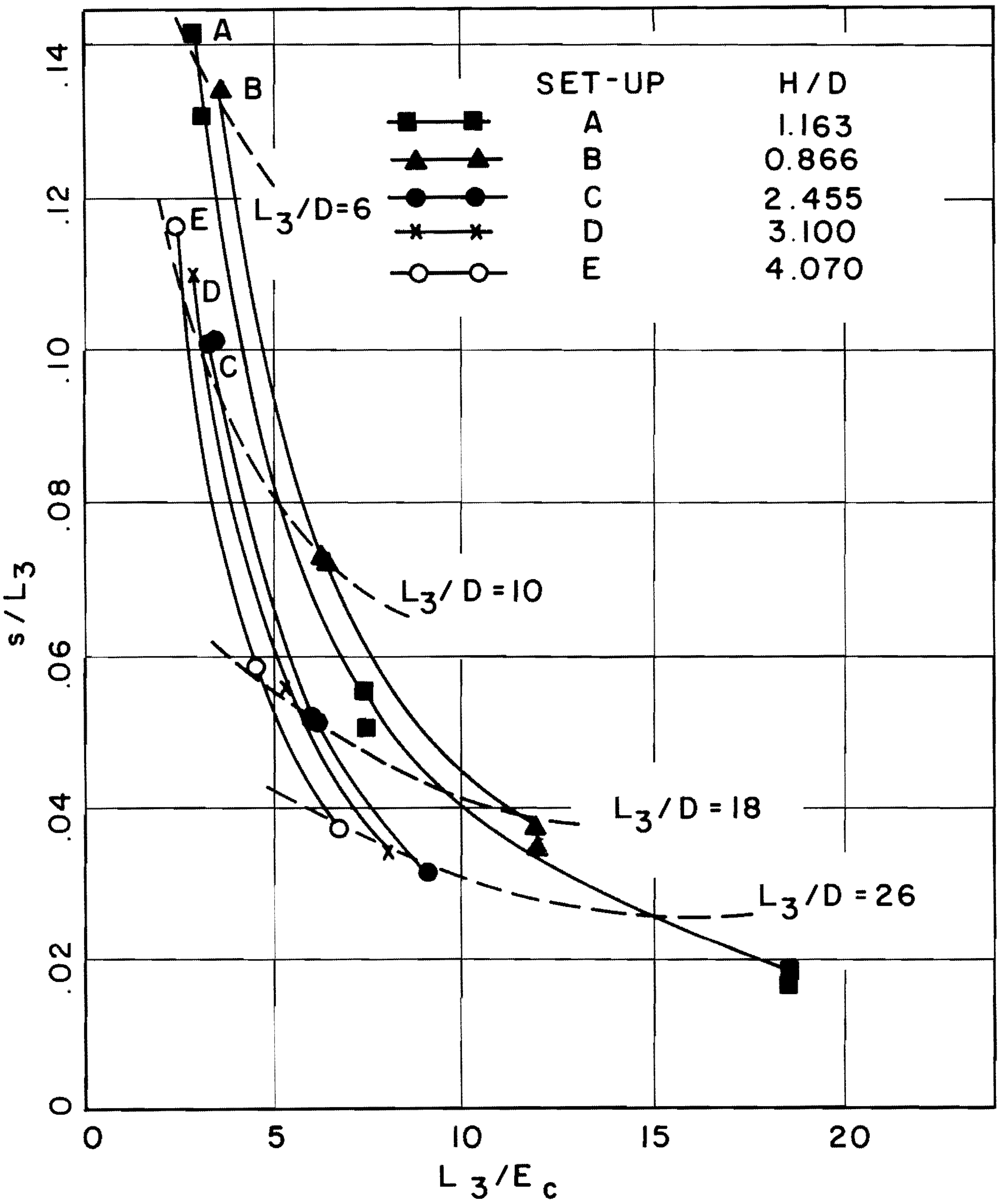


FIGURE 33 s/L_3 vs L_3/E_c WITH SILL AT END OF WING WALLS AND $Q/D^{5/2}=2.5$

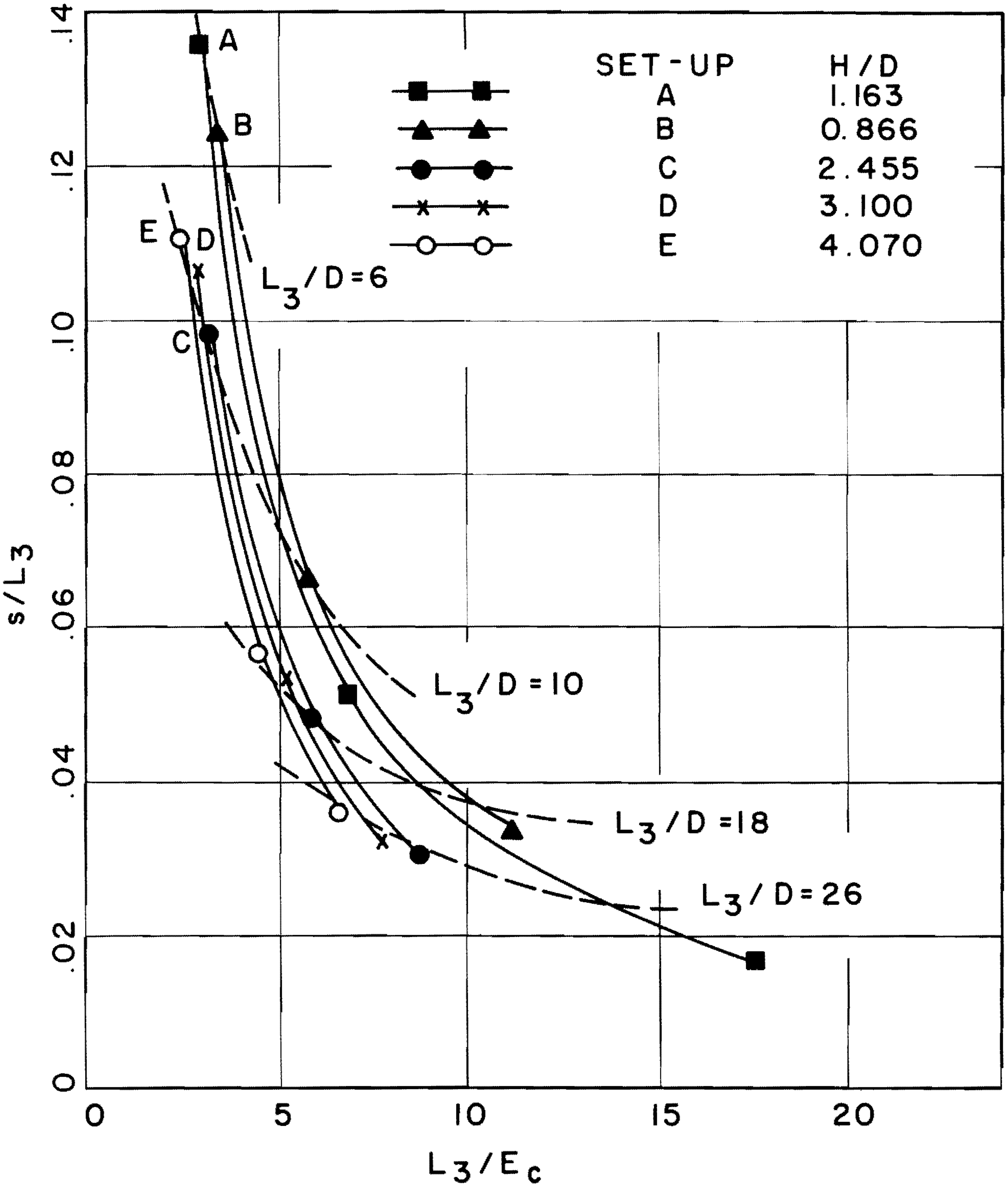


FIGURE 34 s/L_3 vs L_3/E_c WITH SILL AT END OF WING WALLS AND $Q/D^{5/2} = 3.0$

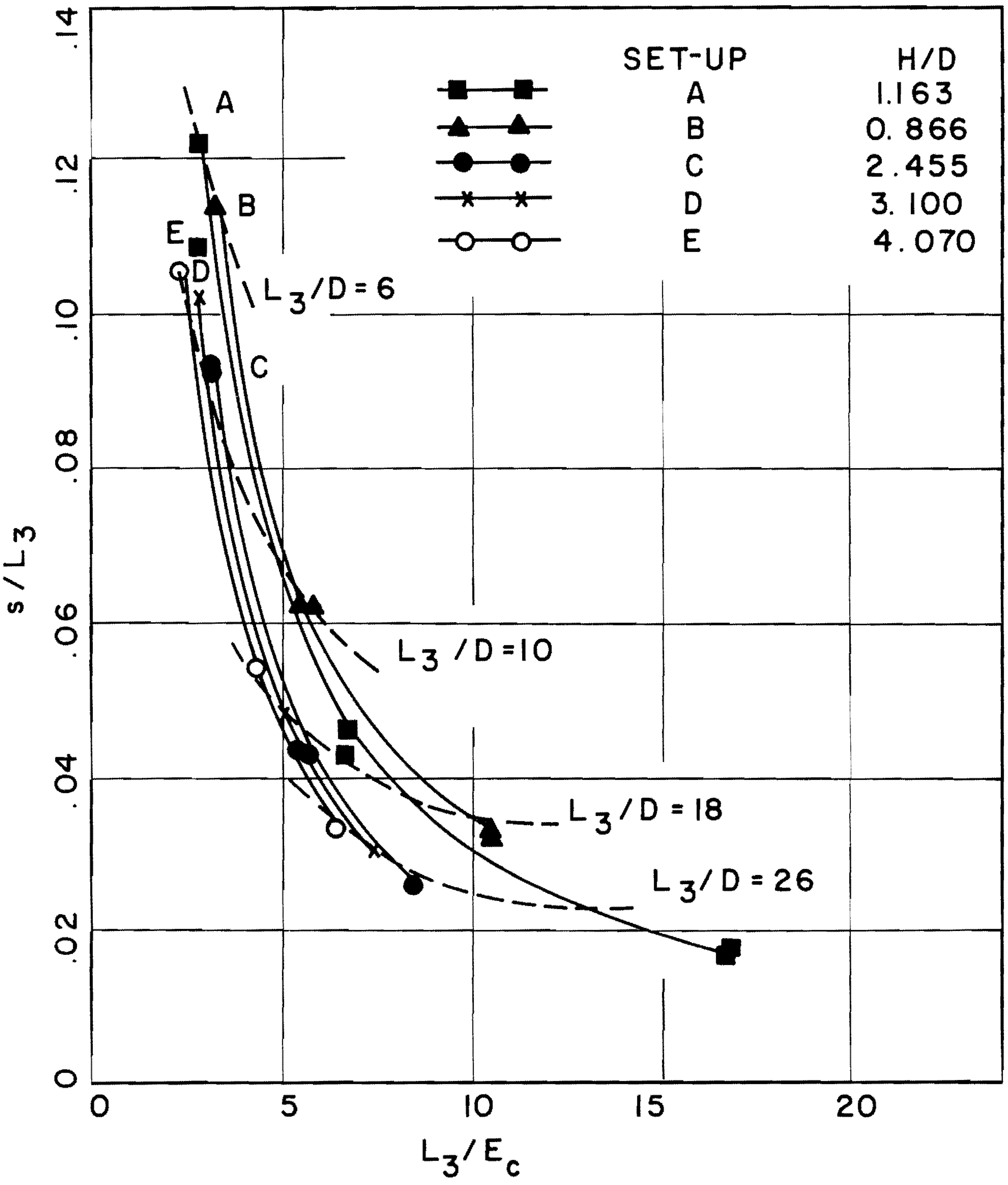


FIGURE 35 s/L_3 vs L_3/E_c WITH SILL AT END OF WING WALLS AND $Q/D^{5/2} = 3.5$

the variables associated with the geometry of a broken-back culvert, and the discharge factor in stabilizing a hydraulic jump inside the culvert. First, considering the two parameters s/L_3 and L_3/E_C at a constant value of E_C, as L_3 decreases to zero, L_3/E_C also goes to zero and s/L_3 approaches infinity. On the other hand, as L_3 increases, the effect of pipe friction becomes more and more significant in helping to force a jump until the length of Unit 3 reaches some finite value long enough such that no sill at all is required to force the jump. This upper limit on the length of Unit 3 for a given discharge factor and upstream culvert geometry can be easily computed using standard backwater calculations. It is approximately equal to the distance through which the flow would have to pass for the water depth in the pipe to increase from the depth at the end of Unit 2 to the critical depth for the given conditions. This distance is usually several magnitudes larger than the actual lengths of Unit 3 used in the experiment, or for that matter, in most field installations; therefore, in presenting the results of the study, the values of L_3/E_C in Figures 20 through 35 were limited to the actual ranges of experimental values. For example, it was computed that Set-Up C operating at $Q/D^{2.5} = 1.0$ required a Unit 3 length of approximately 70 feet, or $L_3/E_C \approx 57$, for a jump to be formed naturally without the aid of a sill. The longest length of Unit 3 actually used was only 13.5 feet, or $L_3/E_C = 11.0$ as shown in Figures 24 and 30. Likewise Set-Up A required nearly 40 feet at a discharge factor of 2.5 although 18 feet was the longest Unit 3 used during the experiment. The effect of increasing the total fall of the culvert also is illustrated consistently in all of the curves. For any given discharge factor and constant L_3/E_C value, increasing the total fall to pipe diameter ratio of the culvert with a constant value of L_3/D being maintained, appears to always increase the sill height required to stabilize the jump within the pipe. Lines of constant L_3/D values are sketched in Figures 24 through 35 as dashed lines crossing the curves of constant H/D values.

It should be emphasized that the results presented in Figures 28, 29, 34, and 35 for $Q/D^{2.5}$ values of 3 and 3.5 correspond to unventilated pipe. These plots for higher discharge factors are included only to substantiate the fact that less sill height is required to stabilize a jump within the culvert at these higher discharge factors than at $Q/D^{2.5} = 2.5$ for unventilated culverts and to help define the discontinuity in the sill height to discharge factor relation. Again, the development of subatmospheric pressures at discharge factors greater than 2.5 inside a culvert with an unimproved inlet is the cause for the discontinuity in the sill height relation. This suggests that if it could be certain that a culvert was completely closed to atmospheric conditions except through its entrance and outlet, then the sill height required by such a culvert operating at a discharge factor of 2.5 should be of sufficient height to force a jump within the pipe at any other value of $Q/D^{2.5}$.

While Figures 20 through 35 summarize all of the basic data taken in the laboratory, other measurements were made for several of the test set-ups and discharge factors in an effort to determine the effectiveness of the end sills in spreading the flow across the downstream channel width and to determine which of the sill locations appeared to be the most desirable. Although the basic data were taken for three different sill locations, it was generally recognized that the sill at the end of the culvert was probably the least desirable to use under actual conditions because of the possible maintenance problems that it posed. Accumulation of eroded material and debris carried by flood waters inside the culvert immediately upstream of such a sill could be difficult to remove. The sills located within the flared portion of the wing walls, however, may be more desirable in this regard. For this reason, velocity and energy measurements taken for the purpose of evaluating the effectiveness of the different sill locations were limited only to the sills located at the mid-point and end of the wing walls.

One problem that might be anticipated with the use of a sill near the end of a broken-back culvert is that too high a sill causes potential energy upstream of the sill to be converted to kinetic energy downstream of the sill thus producing high velocities. Thus some indication of the sill height to sill location relation for given discharge factors and given culvert geometries would prove to be useful. This relationship can be seen from the s/L_3 versus L_3/E_C plots in Figures 20 through 35. However, it appears a plot of s/D versus x/D would prove to be much more useful. Figures 36 and 37 are such plots for a selected number of the culvert geometries used during the experiment. An examination of these plots clearly indicates that the farther downstream from the end of the culvert the sill is located, the higher the sill height

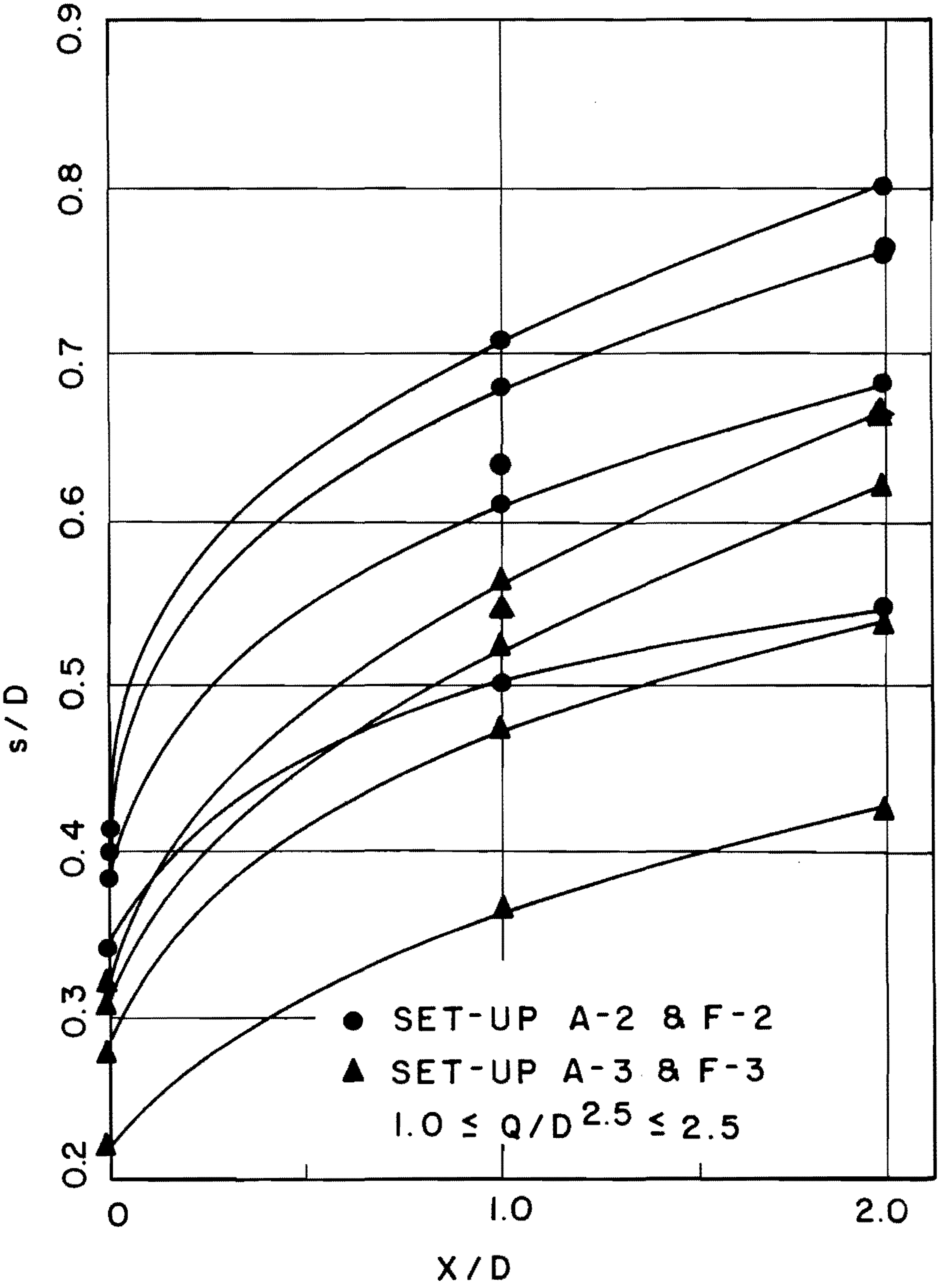


FIGURE 36 RELATIONSHIP BETWEEN SILL HEIGHT AND SILL LOCATION FOR SET-UPS A-2, F-2, A-3 & F-3.

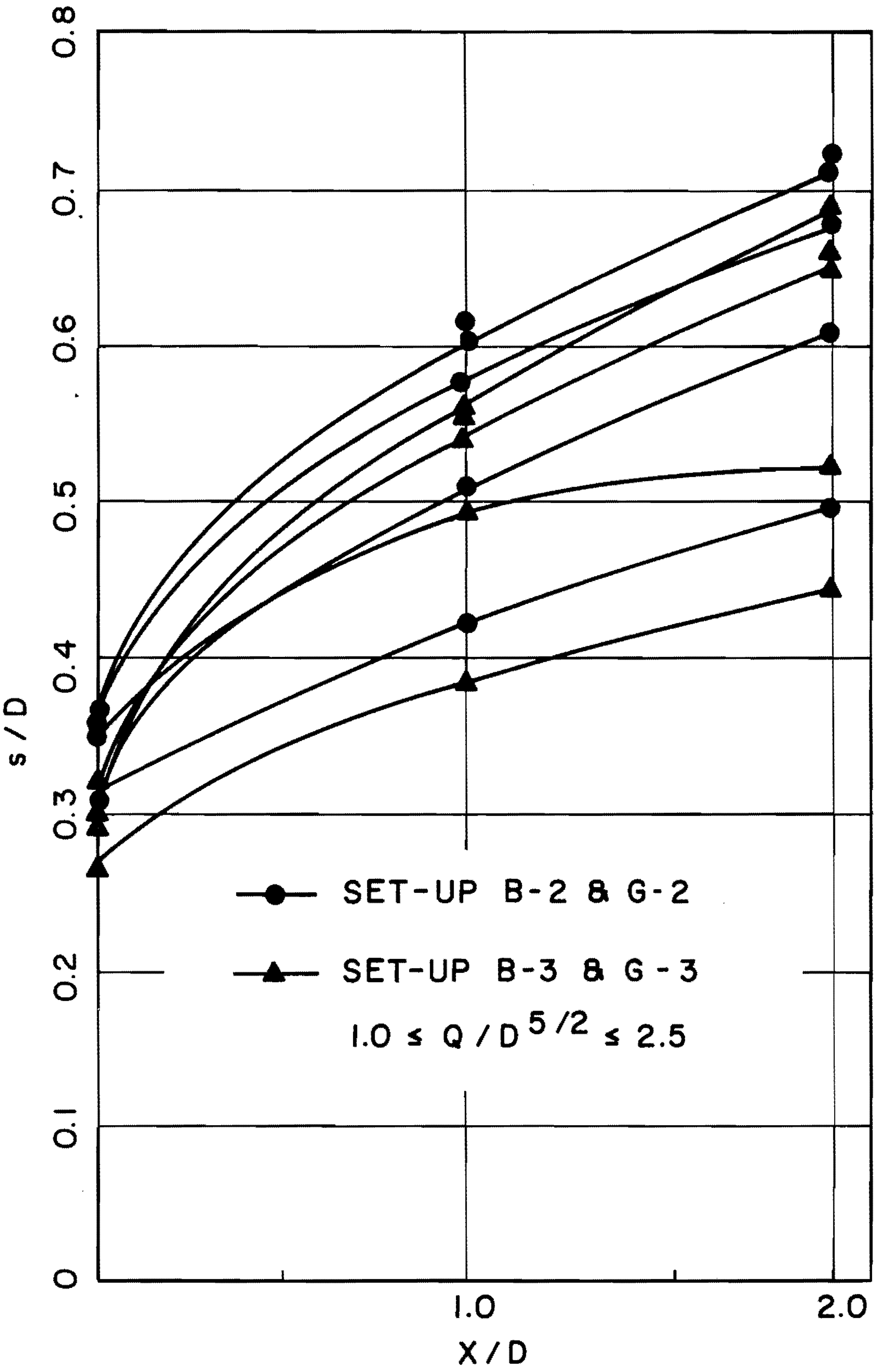


FIGURE 37 RELATIONSHIP BETWEEN SILL HEIGHT AND SILL LOCATION FOR SET-UPS B-2, G-2, B-3, & G-3

required to stabilize the jump for a given test set-up operating at a given $Q/D^{2.5}$ value. With the elimination of the end of the culvert as a possible sill location, these plots suggest that the mid-point of the wing walls would be a more desirable sill location than the end of the wing walls as far as returning the culvert flow back to the original condition of the natural channel is concerned.

With the sill located at either the mid-point or the end of the flared wing walls, cross waves in the flow downstream of the sill were noted for distances up to ten pipe diameters from the sill. The occurrence of cross waves is illustrated by the transverse velocity and corresponding water depth distributions shown in Figures 38 through 45 for Set-Ups B-1 and B-2. These velocity patterns were measured approximately 0.02 feet above the channel bottom for both sill locations at three different sections downstream of the sills, and they serve as useful qualitative measures of the effectiveness of the different sill locations in spreading the flow. Examination of the velocity distributions downstream of sills located at the mid-point and end of the wing walls for corresponding culvert geometries and discharge factors, such as in Figures 38 and 40, indicates that the sill at the end of the wing walls appears to be more effective in spreading the flow across the channel width within a shorter distance from the sill than does the sill at the mid-point of the wing walls. The difference in velocity patterns at an approximate distance of six pipe diameters downstream of the two

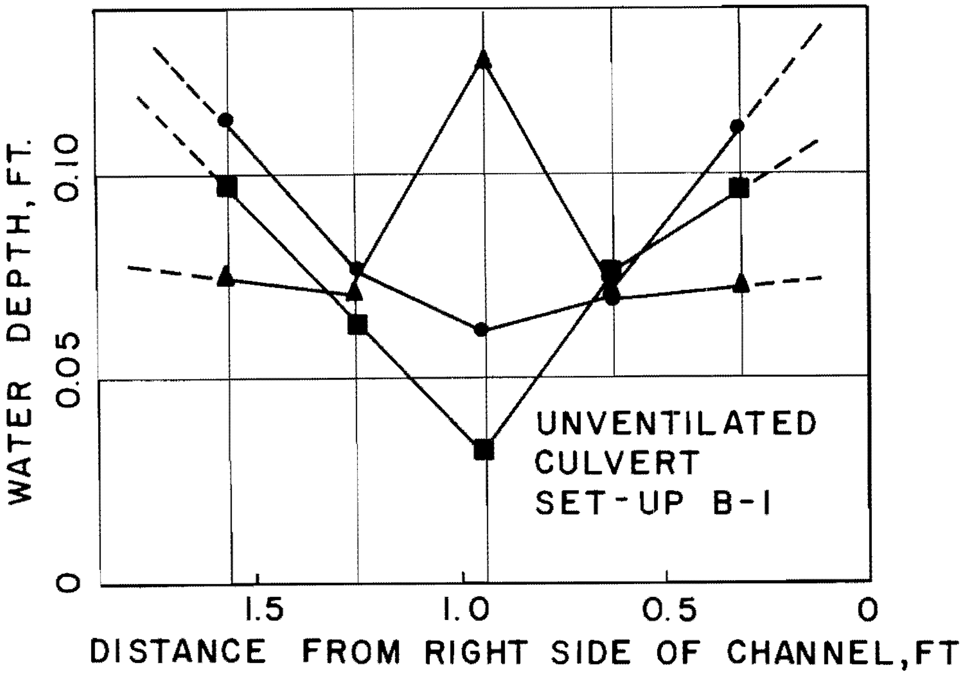


FIGURE 39 DEPTHS ACROSS CHANNEL WIDTH DOWNSTREAM OF A SILL AT MID-POINT OF WING WALLS WITH $Q/D^{5/2} = 2.5$

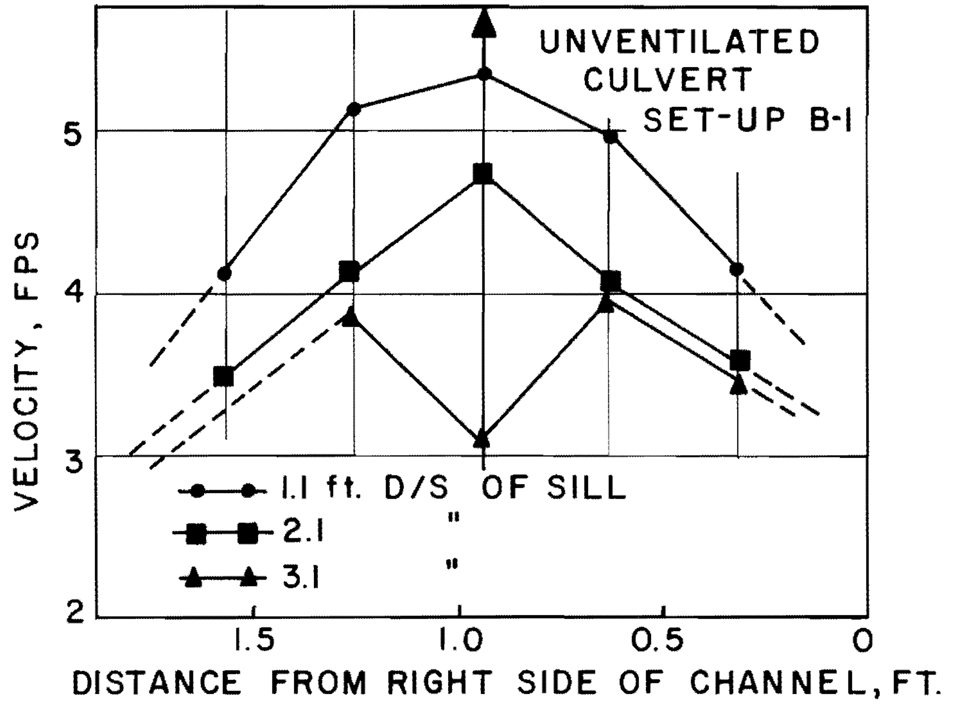


FIGURE 38 TRANSVERSE VELOCITY PATTERNS DOWNSTREAM OF A SILL AT MID-POINT OF WING WALLS WITH $Q/D^{2.5} = 2.5$

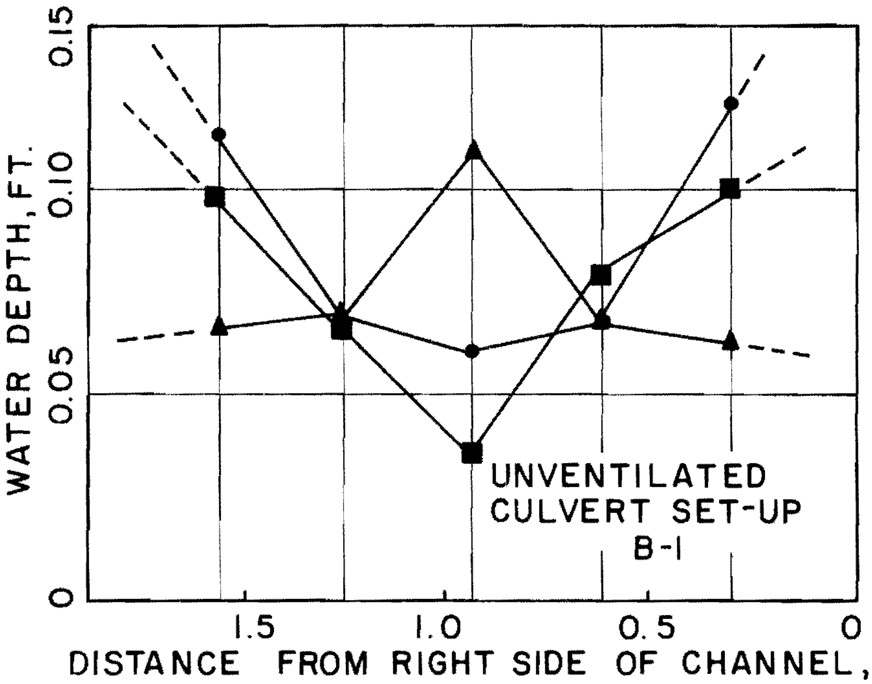


FIGURE 41 DEPTHS ACROSS CHANNEL WIDTH DOWNSTREAM OF A SILL AT END OF WING WALLS WITH $Q/D^{5/2} = 2.5$

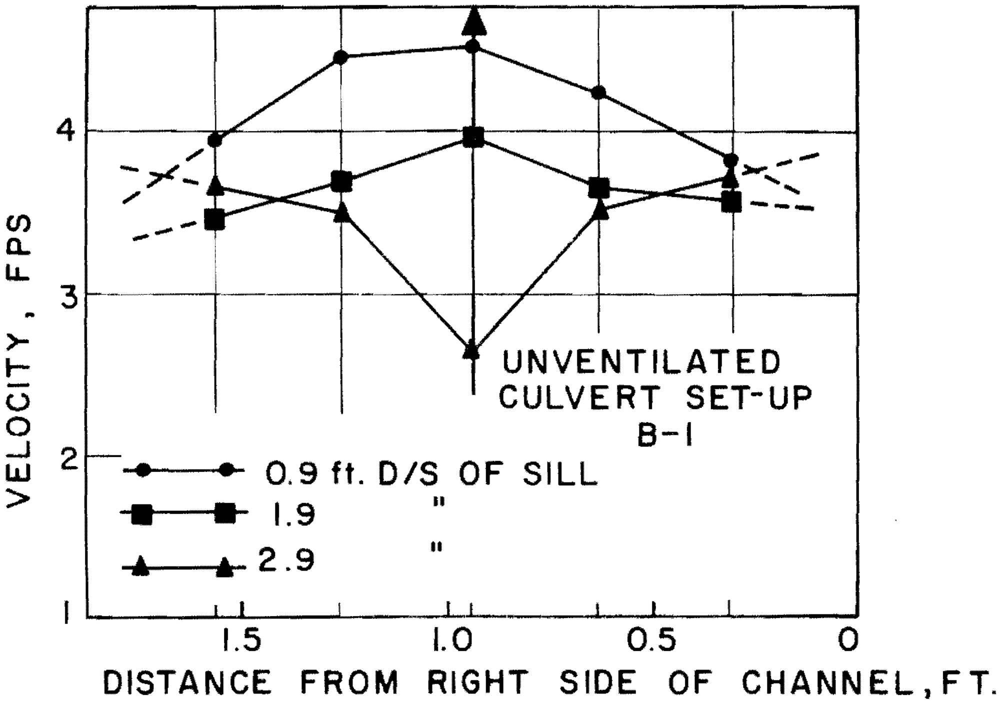


FIGURE 40 TRANSVERSE VELOCITY PATTERNS DOWNSTREAM OF A SILL AT END OF WING WALLS WITH $Q/D^{5/2} = 2.5$

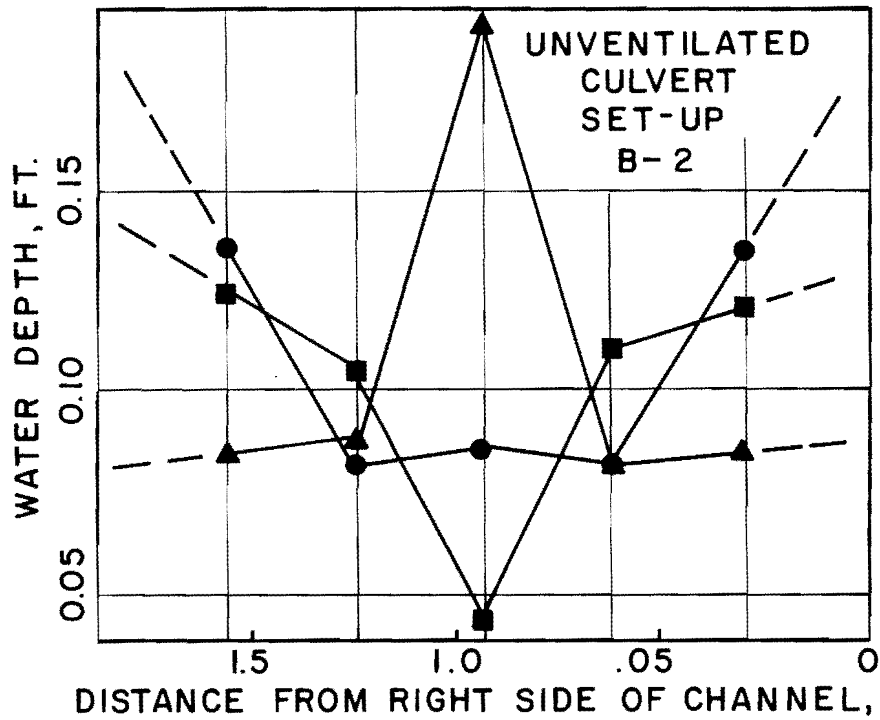


FIGURE 43 DEPTHS ACROSS CHANNEL WIDTH DOWNSTREAM OF A SILL AT MID-POINT OF WING WALLS WITH $Q/D^{5/2}=3.5$

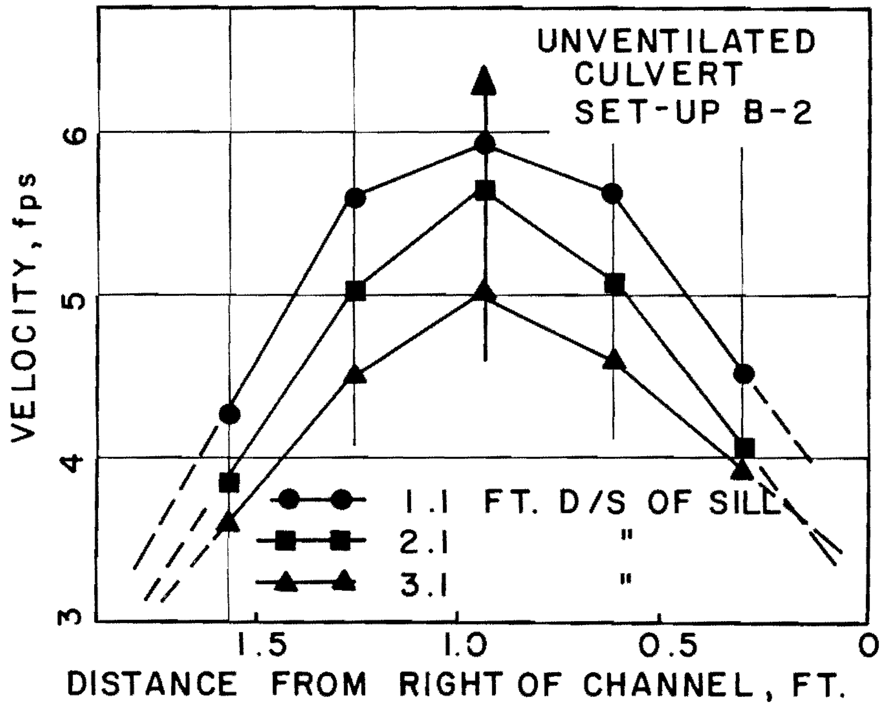


FIGURE 42 TRANSVERSE VELOCITY PATTERN DOWNSTREAM OF A SILL AT MID-POINT OF WING WALLS WITH $Q/D^{5/2}=3.5$

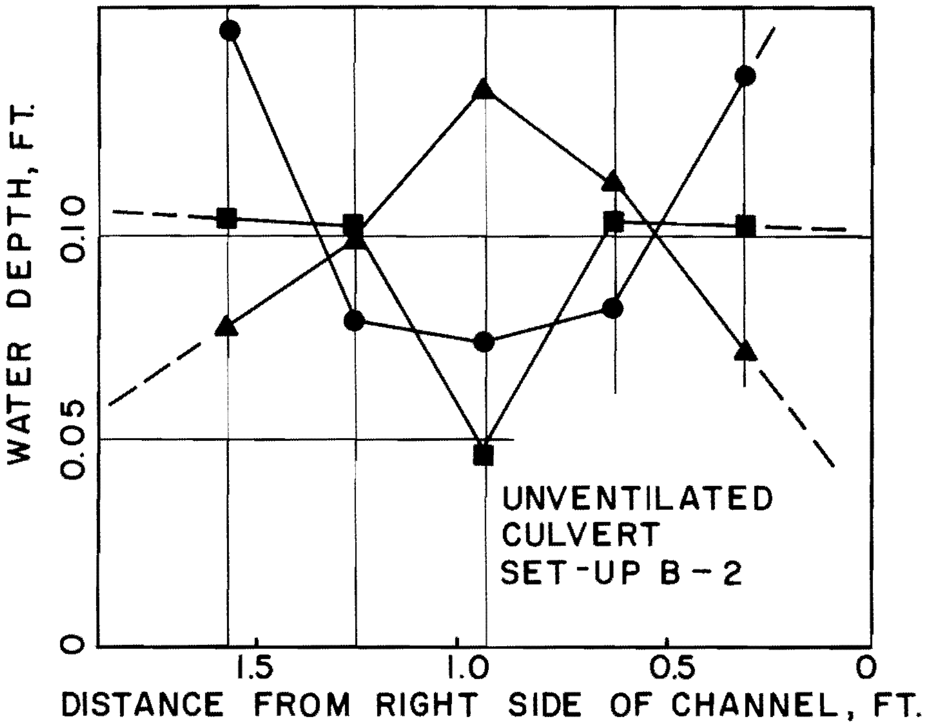


FIGURE 45 DEPTHS ACROSS CHANNEL WIDTH DOWNSTREAM OF A SILL AT END WING WALLS WITH $Q/D^{5/2} = 3.5$

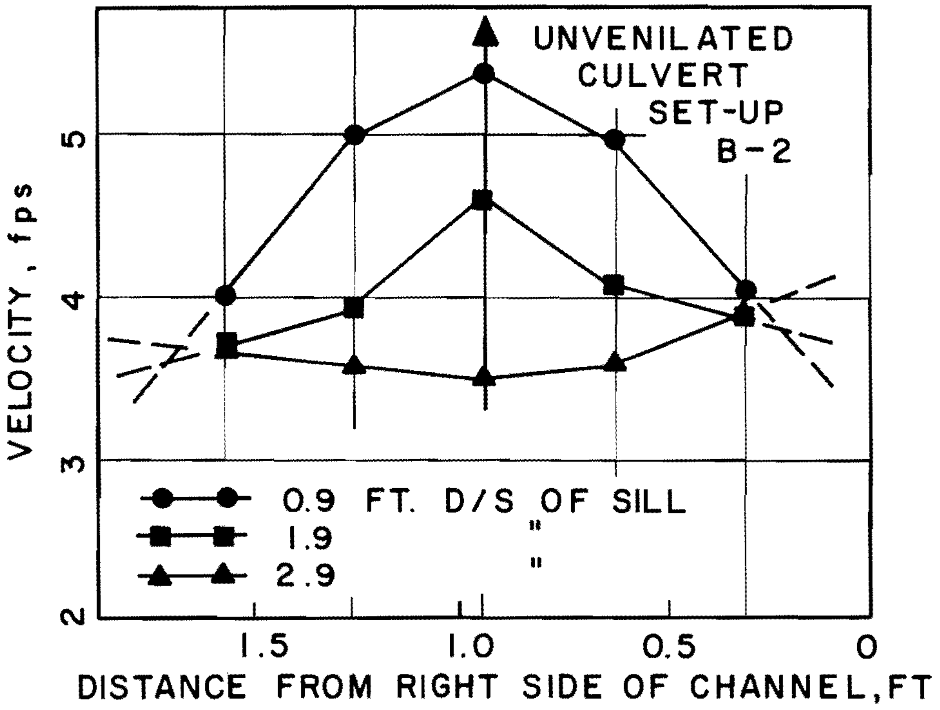


FIGURE 44 TRANVERSE VELOCITY PATTERN DOWNSTREAM OF A SILL AT END OF WING WALL WITH $Q/D^{5/2} = 3.5$

respective sills does not appear to be appreciable; however, the greater effectiveness of the sill at the end of the wing walls in spreading the flow across the channel immediately downstream of the sill is expected since such a sill spans the entire width of the downstream channel.

The effect of the different sill locations on the amount of energy dissipated through the hydraulic jump and the turbulence of the flow in the vicinity of the sills is demonstrated in Figures 46, 47 and 48. These are the plots of $\Delta E_{C-D}/E_C$ versus $Q/D^{2.5}$ for the Set-Ups B-2, C-1 and C-2 respectively with section D-D located at a distance of approximately four pipe diameters from the end of the wing walls. The cross-sectional flow area at section D-D was computed using the average depth across the section as calculated from plots similar to Figures 43 and 45. The average velocity, discharge/area, was then used to determine the velocity head portion of the specific energy. Figures 46, 47 and 48 indicate that for either of the two sill locations, there is approximately an 80% loss of energy between sections C-C and D-D for the range of $Q/D^{2.5}$ values used in the experiment. Neither of the two sill locations appears to be more advantageous than the other in this regard. For the range of Froude numbers encountered in this study, Sylvester (1964) and Price (1967) indicate that as much as 40% theoretical energy loss can be expected through a hydraulic jump in a circular culvert. This means that approximately 40% of the energy level at the end of Unit 2 of the culverts in Figures 46 through 48 must have been lost at the culvert exit and in the turbulent action of the flow in the vicinity upstream and

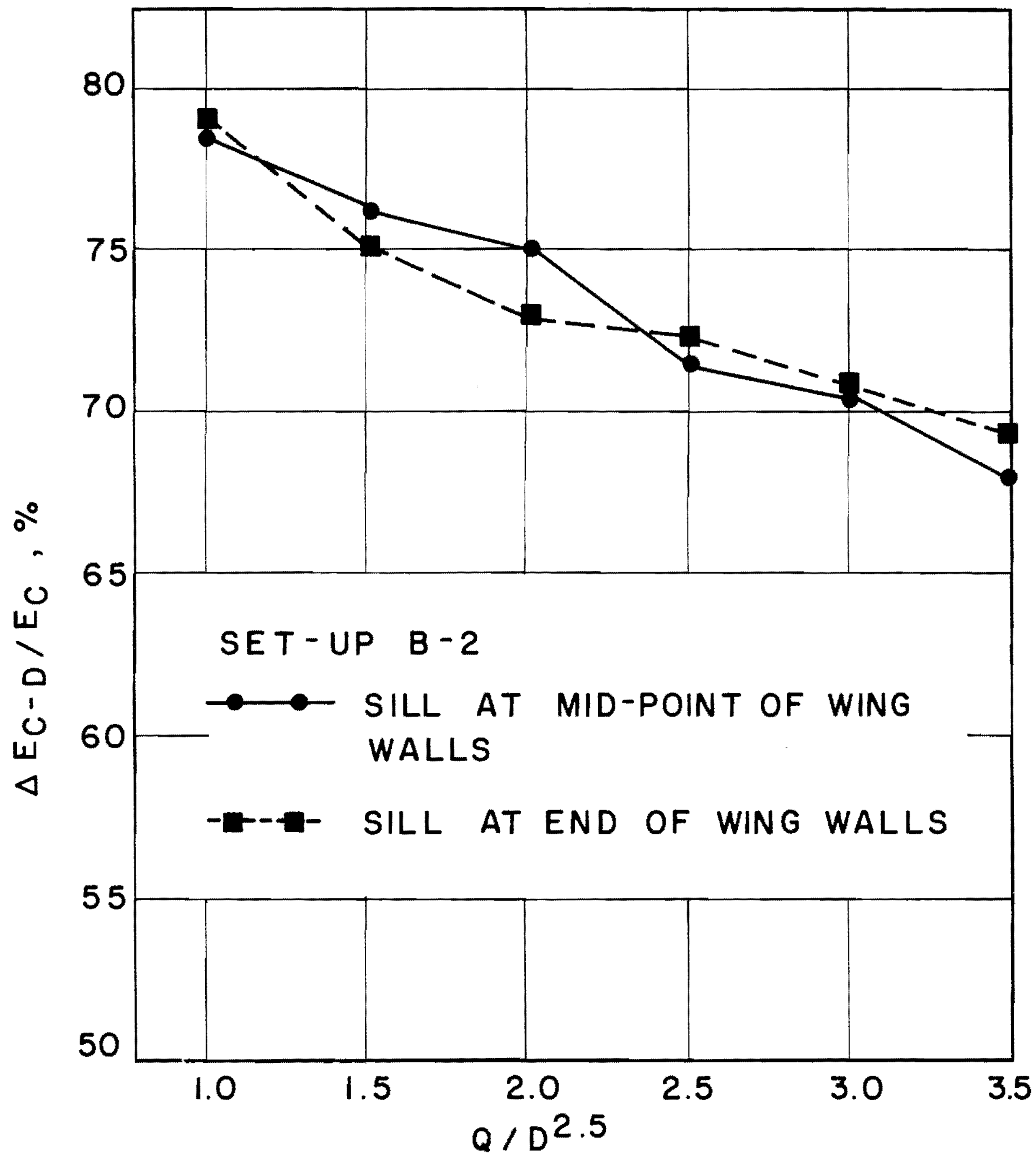


FIGURE 46 RELATIVE ENERGY DISSIPATED WITH SILL AT MID-POINT AND END OF WING WALLS

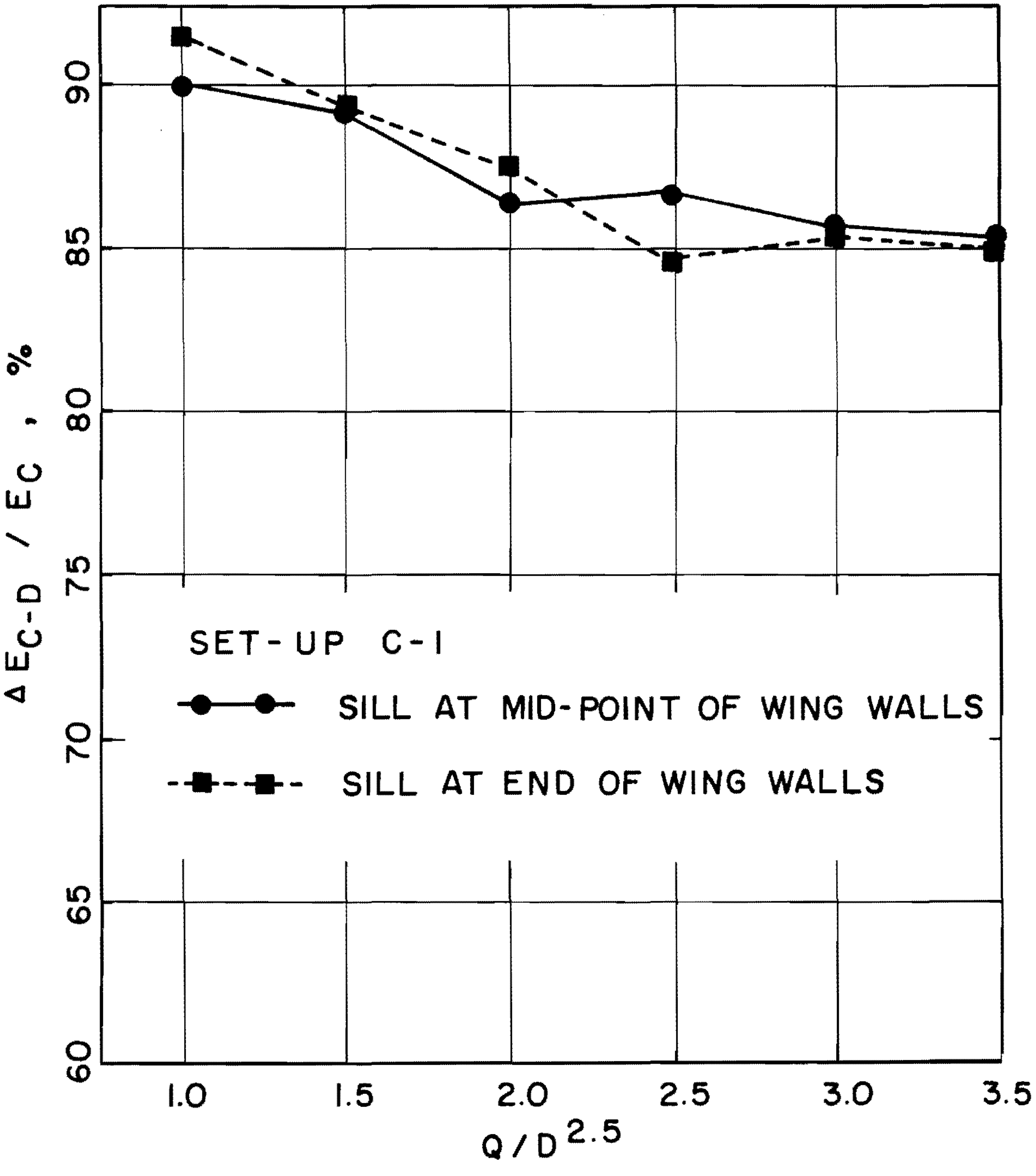


FIGURE 47 RELATIVE ENERGY DISSIPATED WITH SILL AT MID-POINT AND END OF WING WALLS

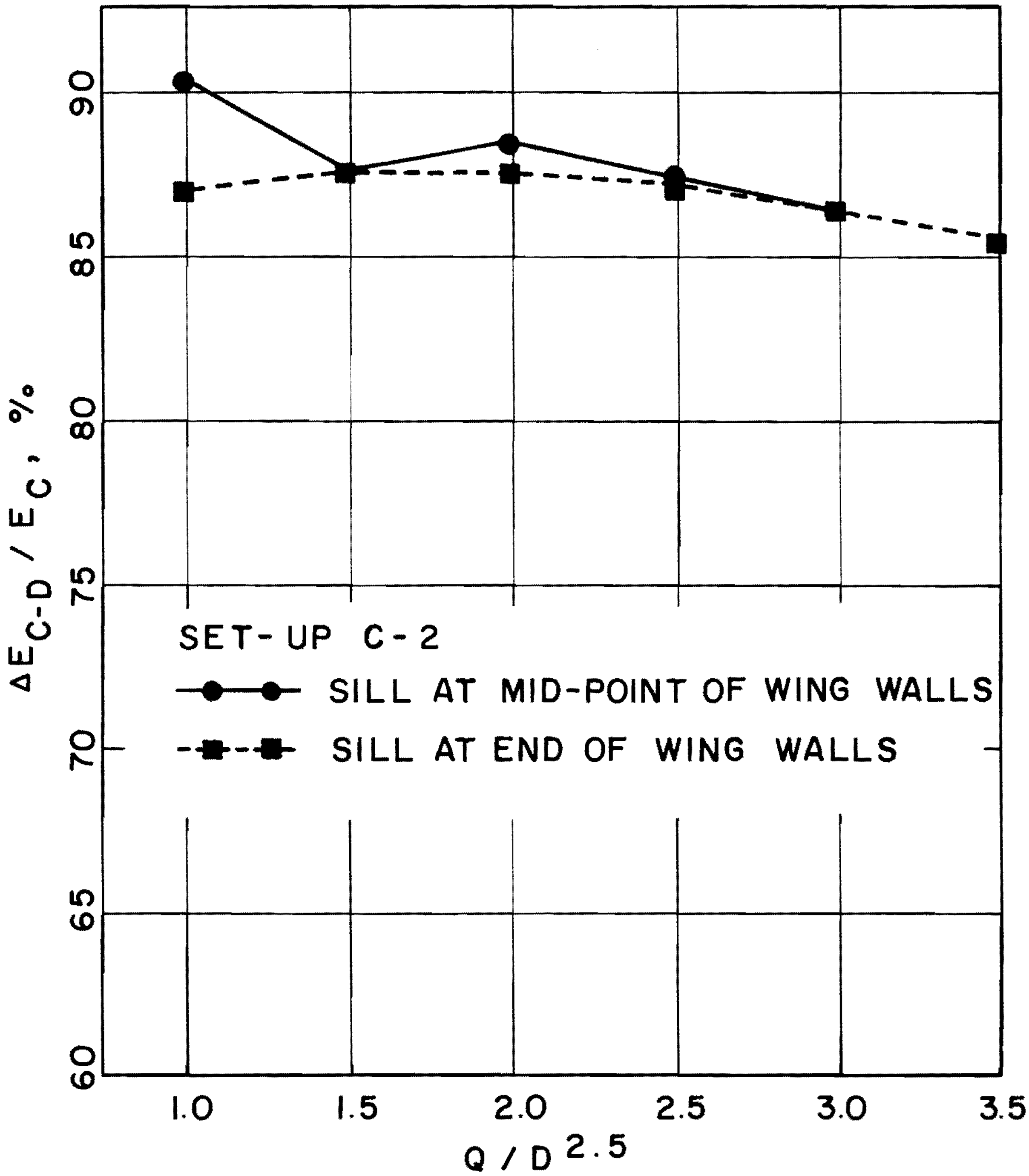


FIGURE 48 RELATIVE ENERGY DISSIPATED WITH SILL AT MID-POINT AND END OF WING WALLS

downstream of the sill since friction losses are very small. One reason for the low energy level in the channel downstream of the sills is that the width of the channel is nearly four times as great as the pipe diameter thus permitting the culvert flow to spread considerably.

The point where the flow over a sill falls to the floor of the downstream channel is important since this area is likely to be subject to highly turbulent action, as well as, impact forces which could cause excessive eroding and scouring of the channel bed. In order to avoid these problems, protection of the channel with erosion-resistant material immediately downstream of the sill would be necessary. An indication of the distance downstream of the sill that such protection might have to be supplied is given in Figure 49. The ratio, w/D, in these plots represents the maximum distance downstream of a given sill where the flow strikes the channel bed for the given discharge factors divided by the pipe diameter. It can be seen that this ratio does not vary appreciably with the sill located at the mid-point or end of the wing walls for the higher range of discharge factors presented in the plots.

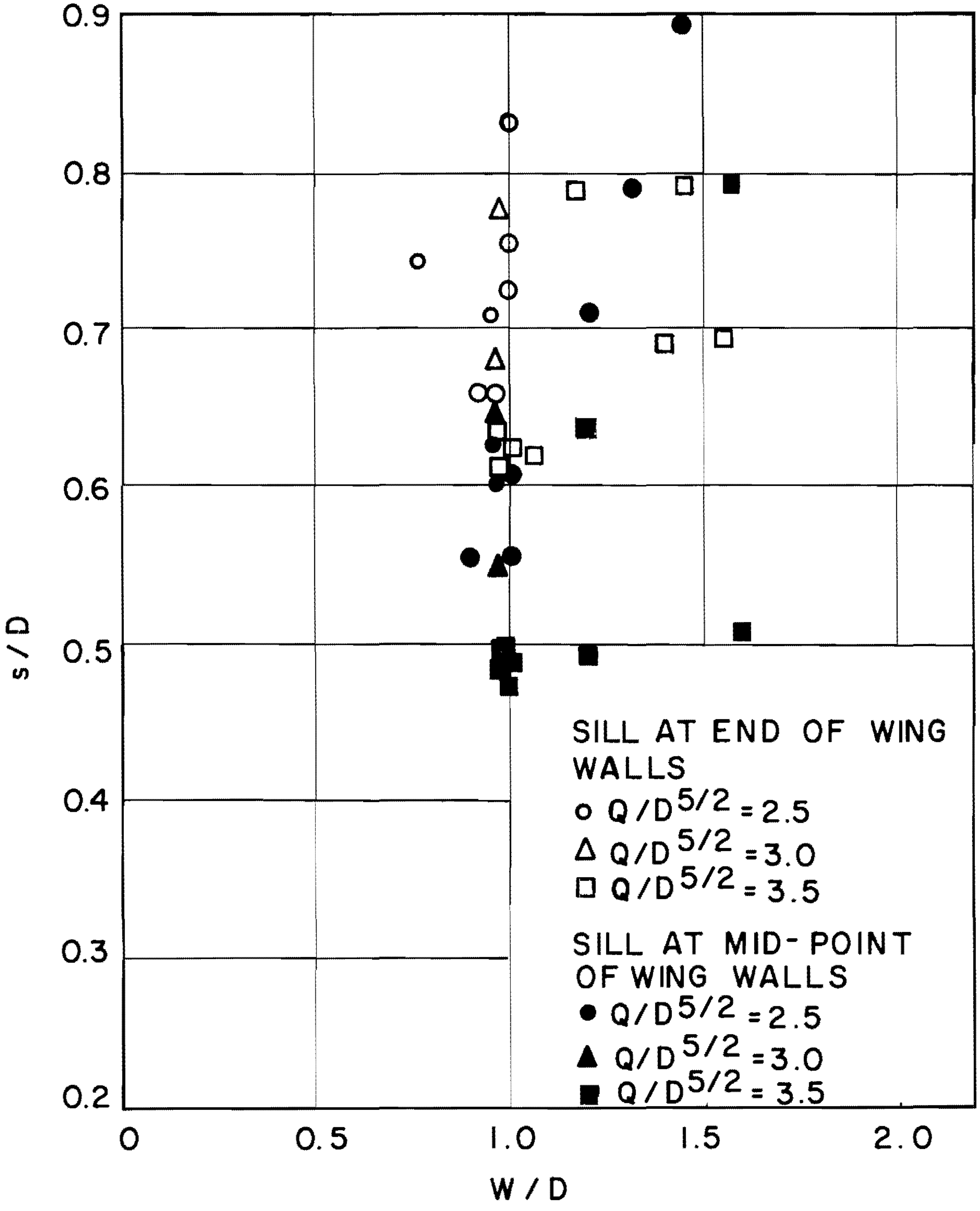


FIGURE 49 RELATIONSHIP BETWEEN SILL HEIGHT AND DISTANCE DOWNSTREAM OF SILL WHERE FLOW STRIKES DOWNSTREAM CHANNEL

CONCLUSIONS AND RECOMMENDATIONS

Having completed the experimental portion of the study and having analyzed the data taken during the investigation, the following conclusions summarize the results of the study:

(1) For all of the basic tests run there exists a consistent relationship between sill height and the pertinent variables involved in stabilizing a hydraulic jump a prescribed distance within the downstream horizontal portion of a broken-back culvert for discharge factors ≤ 2.5. A plot of sill height to length of Unit 3 ratio, s/L_3, versus length of Unit 3 to specific energy ratio, L_3/E_C, appeared to be the most desirable form for graphical representation of the data. This relation assumes that the losses from the end of Unit 2 to the toe of the jump are small and can be neglected if Unit 3 is short.

Such a plot included a family of curves of constant total fall to pipe diameter ratios, H/D, associated with one discharge factor, $Q/D^{2.5}$, and one sill location to pipe diameter ratio, x/D. Data collected for the various culvert geometries used during the experiment with the sills located at the culvert outlet and at the mid-point and end of the flared wing walls plotted with a consistent pattern in terms of s/L_3 versus L_3/E_C.

(2) A rectangular sill placed perpendicular to the flow within the outlet structure of a broken-back culvert does appear to be effective

in forcing and stabilizing a hydraulic jump in the downstream portion of the culvert for the range of discharge factors used. With the jump inside the pipe, the sill does aid in spreading the culvert flow across the width of the downstream channel and returning the flow to energy conditions nearer those of the natural channel. With the sill located at either the mid-point or the end of the flared wing walls and a jump stabilized within Unit 3 of the culvert, total energy loss of approximately 80% was measured between sections C-C and D-D for Set-Ups B-2, C-1 and C-2 and for discharge factors up to 3.5.

(3) For any given culvert geometry and sill location, for the range of $Q/D^{2.5} \leq 3.5$, the maximum sill height required to stabilize the jump a distance of two pipe diameters from the culvert outlet occurred at a discharge factor of 2.5 when the culvert system was not ventilated between its two ends. When the culvert was artificially ventilated, the required sill height increased with discharge. The development of subatmospheric pressures by air entrainment within the jump inside the pipe at discharge factors greater than 2.5 accounted for the lower sill heights of the unventilated culvert.

(4) For any given culvert geometry, sill location and discharge factor, increasing the sill height above the height required to stabilize the jump near the culvert outlet would force the jump farther upstream within the culvert. With enough sill height, the pipe could be made to flow full.

(5) For any given culvert geometry and discharge factor, a higher sill height was required to stabilize the jump at the desired location

within the culvert with the sill located at the end of the flared wing walls than with the sill at the mid-point of the wing walls. The sill located at the culvert outlet required the least sill height of all of the three different locations.

(6) With the sills located at either the mid-point or the end of the wing walls with discharge factors of 2.5, 3.0, and 3.5, no appreciable differences were noted in the position downstream of the sills where the flow over the sills fell on the bottom of the channel. The distance from the sills to this point was usually near one pipe diameter.

(7) Analysis of the transverse velocity distributions at various sections downstream of the sills indicates that the sill located at the end of the wing walls was more effective in spreading the flow within a shorter distance in the downstream channel than the sill located at the mid-point of the wing walls. Such a result is expected since the sill at the end of the wing walls is fartherest from the culvert outlet and it spans the entire width of the downstream channel. At distances of approximately six to eight pipe diameters downstream of the end of the wing walls, the differences in the flow patterns of the two sill locations did not appear to be appreciable.

Considering the results and conclusions of the study, it is recommended that the sill located at the mid-point of the flared wing walls would be most desirable for stabilizing a jump within the culvert and spreading the culvert flow across the width of the natural channel. It was generally recognized throughout the study that because of possible maintenance problems associated with debris that might be deposited inside a culvert immediately upstream of a sill located at the culvert outlet, this would be the least desirable sill location. Also because no appreciable differences could be noted between such relationships as w/D versus x/D, s/D versus x/D, and downstream velocity patterns versus x/D for the sills located at the mid-point and end of the wing walls, the fact that less additional downstream channel protection would be required by a sill located at the mid-point of the wing walls made it the most desirable location of the three. Since protection against erosion and scouring in the form of a concrete or rip rap apron is usually supplied between the flared wing walls of the downstream channel, only a short distance of additional protection would be required by a sill at the mid-point of the wing walls compared to that required by a sill at the end of the wing walls to assure that flow over the sills does not fall on the unprotected natural channel bed.

Considering the further research on the use of end sills in stabilizing a jump within a culvert and spreading the culvert flow, the fact that this study was limited to only one diameter of pipe suggests the need for similar investigations of culverts whose diameters are within the range of those that might be used in practical situations. Such research would be useful in verifying the sill height relationships developed in this study for pipes of larger diameters. Tests involving radically varying culvert geometries and culverts with improved inlets might also be useful in extending the range of such plots as s/L_3 versus L_3/E_C. Use of slopes of Unit 2 within the two limits used in this study would be an effective way of varying the culvert geometry and total fall of the test set-ups.

The use of types of sills different from those used in this study might yield more desirable results with less flow concentration over the sills and in the downstream channel. A sill oriented in a position other than vertical or fixed at an angle other than perpendicular to the culvert flow might aid in this capacity. One observation made during the experiment was that the use of two sills simultaneously, one at the mid-point and one at the end of the wing walls, resulted in flow that appeared to be nearly uniformly distributed across the width of the second sill. Such a combination of sills was also very effective in spreading the flow downstream of the sills across the channel width. An effect similar to that of using two sills simultaneously might also result from placing a rectangular sill within the flared wing walls above a channel bottom that is slightly raised (one to two tenths of the pipe diameter) above the invert of the culvert outlet. The use of guide vanes either upstream or downstream of the sills could also be investigated to determine the effect they might have in spreading the culvert flow. Debris carried by flood water could be a problem in this situation.

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