

STRENGTH EFFECT OF CUTTING OFF TENSION BARS
IN CONCRETE BEAMS

by

Phil M. Ferguson

and

Syed I. Husain

Research Report Number 80-1F

Research Project Number 3-5-64-80
Strength Effect of Cutting Off Tension
Bars in Concrete Beams

Conducted for

The Texas Highway Department

In Cooperation with the
U. S. Department of Transportation
Federal Highway Administration
Bureau of Public Roads

by

CENTER FOR HIGHWAY RESEARCH
THE UNIVERSITY OF TEXAS
AUSTIN, TEXAS

June 1967

A C K N O W L E D G M E N T S

This research has been sponsored jointly by the Texas Highway Department and the Bureau of Public Roads, under an interagency contract between The University of Texas and the Texas Highway Department. Liaison with the Texas Highway Department was maintained through Mr. Larry G. Walker, their contact representative, and through Mr. R. E. Stanford of the Bureau of Public Roads.

The first phase of this work, as covered by the interim report of August 23, 1965, was performed under the immediate direction of Dr. Rao J. Tatikonda.

The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Bureau of Public Roads.

A B S T R A C T

Sixty-four concrete beams reinforced with #8 or #11 bars (largely A432) were tested as simple beams. Most of these had bars cut off at the minimum point for moment or at 12D or 15D beyond such points. Some, for comparison, contained full length bars or bars bent up. The majority of the beams with bars cut off were designed to be balanced at ultimate in flexural strength, shear strength, and bond strength.

With bars cut off and no remedial steps, only 2 beams out of 33 developed the design ultimate strength. The losses ranged in the order of 15 to 25 percent, with one more than 40 percent. With bars bent up no deficiencies occurred.

The addition of extra stirrups improved the weak beams, but stirrup effectiveness in replacing strength was only about half the normal rf_y evaluation.

Design recommendations based on the use of the ACI Building Code allowables for bond and shear stresses are made for beams with bars cut off. Designs should be based on a typical 30 percent loss of shear strength where bars are cut off, a 20 percent loss where heavy stirrups are already provided ($rf_y \geq 130$ psi), and a 10 percent loss for slabs 12" or less in thickness. If remedial stirrups are used, they should provide an rf_y adequate to care for twice the indicated shear deficiency and in no case less than 100 psi.

*r is the ratio of stirrup area to the product of beam width by stirrup spacing.

f_y is the yield strength of the stirrup steel, in psi.

STRENGTH EFFECT OF CUTTING OFF TENSION BARS

IN CONCRETE BEAMS

I N T R O D U C T I O N

When a beam contains multiple bars it is common practice to stop or bend some of these where no longer needed for moment. For example, if three bars are needed in Fig. 1a, one each might theoretically be stopped at distances L_1 and $2L_1$ from the load, as indicated by the solid lines.

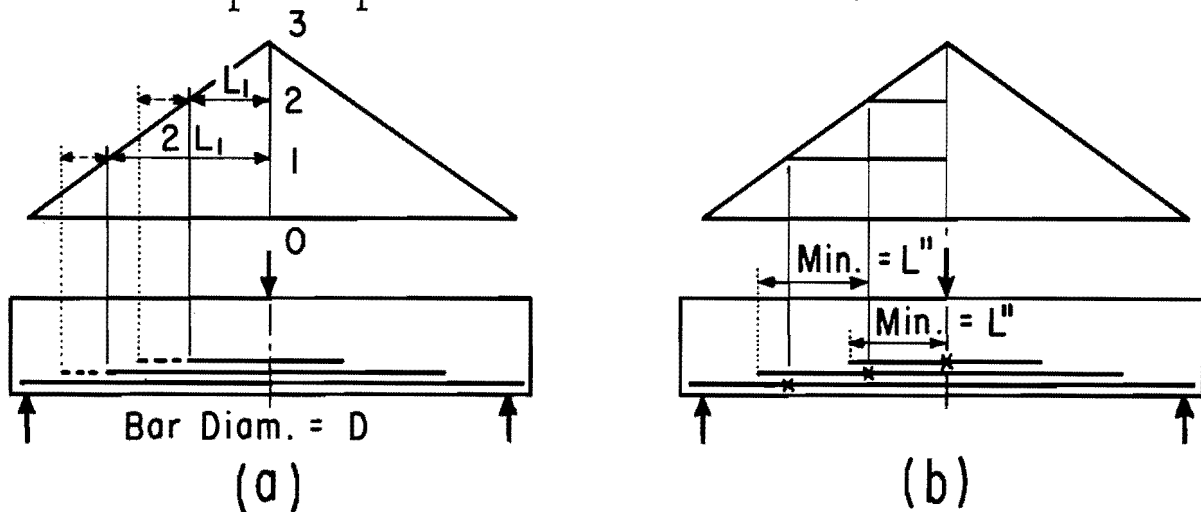


Fig. 1. Bar lengths for moment and bar development length.

Codes and specifications usually add two supplementary requirements:

- (1) Bars must be extended an added distance (end anchorage) as indicated by the dotted dimension lines. This extension in the AASHO specification is $15D$ or in the ACI Building Code the larger of $12D$ or the beam depth d .
- (2) The total length from a point of maximum stress to a cutoff point as in Fig. 1b must be at least the minimum development length, which for ultimate strength design is

$$L'' = \frac{f_y}{4 u_u} D.$$

For $f'_c = 3500$ psi, which is used as the reference strength in this investigation, the minimum development lengths L'' are A or B, as follows, with the length A based on ACI Code bond values and B based on AASHO specification bond values (doubled to refer to USD and f_y):

| <u>Bars</u> | <u>$f_y = 60$ ksi</u> | | <u>$f_y = 40$ ksi</u> | |
|-------------|----------------------------------|----------|----------------------------------|-----------|
| | <u>A</u> | <u>B</u> | <u>A'</u> | <u>B'</u> |
| #8 | 26.7" | 21.4" | 17.8" | 14.3" |
| #11 | 53.0" | 30.2" | 35.3" | 20.2" |

It will be noted that the shorter lengths required for 40 ksi steel (less by a third) have been noted as A' and B' for clarity.

The development length in some cases, as indicated in Fig. 1b, will automatically be greater than the requirement for moment and thus may provide some or all of the end anchorage usually specified.

In this investigation the beam lengths were such that L_1 (Fig. 1a) was normally either A or B; that is, the half span in Fig. 1a was normally either 3A or 3B (or 3A' or 3B'). The end anchorage was often made zero in order to bring out certain comparisons. In other cases end anchorage was 12D or 15D. Variations in these quantities will also be noted, since one objective was to include a wide spread in the layouts used. Stirrups were generally provided to balance the beams in shear and flexural resistance. However, some excess stirrups were used where needed to meet minimum stirrup ratios or a maximum spacing of $d/2$; or, alternately, stirrups were omitted totally in a few such cases.

Objective

The initial objective of this investigation was to establish whether bar cutoffs were always injurious to beam shear strength. An interim report on August 23, 1965, reported that only 2 out of 31 beams

with bars cut off had developed their full shear capacity. The majority showed deficiencies of from 15 to 25 percent and one more than 40 percent. The investigation was then extended to include possible remedial measures.

The present report deals primarily with the deficiencies found in certain specific steel arrangements in 24" and 18" beams and remedial measures which were tried. These remedial measures included extra stirrups, bending instead of stopping bars, and the combination of the two. Deficiencies found in other sizes of beams or other reinforcement patterns are next reported, but without special remedial measures.

Finally, the report makes design recommendations.

T E S T P R O G R A M

The Specimens

A total of 64 simple beam specimens, as listed in Table I, were loaded to failure under a single concentrated load. The rectangular beams tested included 24", 18" 15", 13", and 12" deep specimens made with #11 or #8 deformed bars, some of A432 steel and some of intermediate grade. One 24" deep T-beam was tested with the 42" by 8" flange in tension. The exact dimensions of these beams, as measured after casting, are listed in Table II in the Appendix. Clear cover was approximately 2", except for the 1.5" used for the 12" slab.

The beams were designated in code as follows: 24-8H-a. The first number is the overall beam depth, the second the bar size number, the capital letter is H for A432 steel or I for intermediate grade steel, and the second (lower case) letter is a serial designation for a particular beam. The last designation is used again with extra notations when an identical beam is used with the bars bent up (marked Bt), or with extra stirrups (r), or with both. All bars were cast in the bottom of the beam except for two specimens, noted with a final letter T, where the bars were in the top.

The second column in Table I shows by a sketch the number of bars and their arrangement, the third column tabulates the half-span length (to center of reaction), and the fourth gives the distance from the load to the first cutoff point. Where this cutoff is shown as $A/2$, A , B , A' , or B' , the other cutoffs are at multiples of these distances, unless specially noted. Where the first cutoff is increased by $12D$ or $15D$, all other cutoff points are displaced by this same amount, leaving the distance between first and second cutoff points as the base length A , B , $0.5A$, or $0.75A$ (without the increase). Bent bars were bent at 45 degrees and offset into the compression face of the beam, where they were extended 6 inches or more. This extension was not critical; no bent-up bar showed any distress in the compression depth.

Stirrups were used in most beams, sufficient in amount to develop the beam flexural strength without exceeding ACI design stresses* in shear. The quantity of stirrups has been indicated in the sixth column in terms of rf_y , the stirrup yield stress f_y actually varying from 46 ksi to 58 ksi. (The quantity r is the ratio of stirrup area to the product of beam width by stirrup spacing.) In some cases the listed rf_y is in excess of the design requirement because of reducing a stirrup spacing to $d/2$ or because of an actual f_y greater than assumed in design. Some of the 18" beams did not require stirrups and four of the 13" and 12" beams which needed much less than the minimum r of 0.0015 were made without any stirrups. Two of these four had companion specimens with stirrups.

The fifth column of data is blank except where some remedial measures were taken. If a zero shows in this column it means bars have been bent up instead of cut off, without any extra stirrups. Where a definite value of rf_y shows in this column it describes the extra stirrups added over the development length of the bars cut off.

(Text next on page 10)

*ACI design stresses on the concrete (ult. $v_c = 2\sqrt{f'_c}$) are lower than the AASHTO values current in 1965 (WSD $v_c = 0.03 f'_c$, equivalent roughly to $0.06 f'_c$ by USD). The ACI values agree more closely with test specimens. Stirrups are evaluated as rf_y , the same way under both specifications.

TABLE I. TEST RESULTS

| Beam | Bars | Half span | First cutoff or bend | b in. | Extra r_{fy} psi | r_{fy} used psi | $2\sqrt{f'_c}$ psi | v_u psi | $\frac{v_u}{\text{compt.}}$ | f_{su} ksi | f_{su}/f_y | Failure mode | Complications | |
|-----------------------|------------|-----------|-------------------------------|-------------------|--------------------|-------------------|--------------------|-----------|-----------------------------|--------------|--------------|--------------|-----------------------------|------------|
| 24-8H-a (2 layers) | 2nd 1st | | 3A = A/2+15D 80.1" = 28.4" | 12.1 | - | 175 | 134 | 235 | 0.76 | 53.9 | 0.86 | Shear | 1st c.o. and 2nd c.o. | |
| 24-8H-b | | | 3A | None | 9.7 | - | 61 | 120 | 188 | 1.04 | 68.6 | 1.09 | Flexure | D.T. |
| 24-8H-c | | | 3A | A | 9.2 | - | 64 | 122 | 109 | 0.59 | 38.2 | 0.61 | Shear | Outer c.o. |
| 24-8H-cT | | | 3A* | A | 9.7 | - | 71 | 113 | 125 | 0.68 | 46.3 | 0.74 | Shear | Inner c.o. |
| 24-8H-d | | | 3A | A+12D | 9.2 | - | 65 | 109 | 130 | 0.75 | 46.2 | 0.74 | Shear | Outer c.o. |
| 24-8H-d-Bt | | | 3A | A+12D | 9.6 | 0 | 77 | 134 | 206 | 0.98 | 75.3 | 1.06 | Flex. | -- |
| 24-8H-d-r | | | 3A | A+12D | 9.2 | 52 | 81 | 98 | 178 | 0.99 | 68.0 | 1.08 | Flex. | D.T. |
| 24-8H-d-Bt-r | | | 3A | A+12D | 9.1 | 52 | 81 | 117 | 216 | 1.08 | 75.4 | 1.07 | Flex. | D.T. |
| 24-8H-e | | | 3A | A+12D (1 only) | 9.2 | - | 65 | 125 | 157 | 0.82 | 54.4 | 0.87 | Shear | -- |
| 24-8H-f | | | 3A | 2A (pair) | 9.1 | - | 65 | 113 | 145 | 0.81 | 51.0 | 0.81 | Shear | -- |
| 24-11H-g-Bt | | | 3B | B | 12.5 | 0 | 119 | 127 | 244 | 0.99 | 63.5 | 1.03 | Flex. | -- |
| 24-8H-gT ⁺ | | | 3A | A+15D | 14.0 | - | 164 | 113 | 212 | 0.77 | 63.1 | 1.01 | Splitting Flexure | |

*Not increased to offset low allowable bond on top bars.

⁺T-bm. with flg. 42" wide by 8" thick. Bars distributed in flg. width.

TABLE I. (CONTINUED)

| Beam | Bars | Half span | First cutoff or bend | b in. | Extra r_{fy} psi | r_{fy} used psi | $2\sqrt{f'_c}$ psi | v_u psi | $\frac{v_u}{\text{compt.}}$ | f_{su} ksi | f_{su}/f_y | Failure mode | Complications |
|---------------------------|------|-----------|----------------------|-------|--------------------|-------------------|--------------------|-----------|-----------------------------|--------------|--------------|--------------|--------------------|
| 18-8H-h | | 3A +16" | None | 12.0 | - | 82 | 128 | 169 | 0.80 | 70.7 | 1.13 | Flexure | -- |
| 18-8H-a | | 3A | None | 9.4 | - | 63 | 131 | 177 | 0.91 | 65.0 | 1.03 | Flexure | D.T. |
| 18-8H-b | | 3A | A | 9.2 | - | 65 | 130 | 128 | 0.66 | 46.2 | 0.74 | Shear | Outer c.o. |
| 18-8H-c | | 3A | A+12D | 9.2 | - | 64 | 124 | 160 | 0.85 | 58.6 | 0.94 | Shear | Inner c.o. |
| 18-8H-c-r ₁ | do | 3A | A+12D | 9.0 | 33 | 82 | 106 | 153.5 | 0.81 | 52.7 | 0.84 | Shear | Flexure |
| 18-8H-c-r ₂ | do | 3A | A+12D | 9.3 | 46 | 80 | 121 | 184 | 0.91 | 65.2 | 0.98 | Flexure | D.T., both cutoffs |
| 18-8H-c-Bt | | 3A | A+12D | 9.2 | 0 | 80 | 127 | 209 | 1.01 | 73.8 | 1.06 | Flexure | -- |
| 18-8H-c-Bt-r ₁ | do | 3A | A+12D | 9.2 | 32 | 80 | 129 | 208 | 0.99 | 72.5 | 1.05 | Flexure | D.T. |
| 18-8H-d | | 3A | 2A (pair) | 9.2 | - | 65 | 127 | 135 | 0.71 | 49.0 | 0.78 | Shear | -- |
| 18-8H-e | | 3A | A (pairs) | 18.1 | - | 86 | 117 | 132 | 0.65 | 48.5 | 0.77 | Shear | Inner c.o. |
| 18-8H-f | | 3A | A+12D (pairs) | 18.1 | - | 87 | 117 | 157 | 0.77 | 57.3 | 0.91 | Shear | Outer c.o. |
| 18-8H-g | | 3A | A | 9.3 | - | 85 | 120 | 152 | 0.74 | 56.2 | 0.89 | Splitting | Flexure |
| 18-8H-s-Bt | | 3B | B+15D | 9.2 | 0 | 108 | 124 | 244 | 1.05 | 68.4 | 1.09 | Flexure | D.T. |

TABLE I. (CONTINUED)

| Beam | Bars | Half span | First cutoff or bend | b in. | Extra r_{fy} psi | r_{fy} used psi | $2 \sqrt{f'_c}$ psi | v_u psi | $\frac{v_u}{\text{compt.}}$ | f_{su} ksi | f_{su}/f_y | Failure mode | Complications |
|-------------------------|------|-----------|----------------------|-------|--------------------|-------------------|---------------------|-----------|-----------------------------|--------------|--------------|--------------|--------------------|
| 18-11H-n | | 3A | None | 12.1 | - | 0 | 113 | 126 | 1.11 | 60.0 | 1.05 | Flexure | D.T. final |
| 18-11H-m | | 3A | A+15D | 12.1 | - | 0 | 118 | 88 | 0.74 | 41.4 | 0.73 | Shear | Inner c.o. |
| 18-11H-m-r ₁ | do | 3A | A+15D | 12.2 | 61 | 0 | 118 | 118 | 1.00 | 55.5 | 0.97 | Flexure | Splitting and D.T. |
| 18-11H-m-r ₂ | do | 3A | A+15D | 12.4 | 98 | 0 | 119 | 122 | 1.02 | 57.5 | 1.01 | Flexure | do |
| 18-11H-m-Bt | | 3A | A+15D | 12.1 | 0 | 0 | 120 | 130 | 1.08 | 60.1 | 1.05 | Flexure | - |
| 18-11H-l | | 3A | A | 12.0 | - | 0 | 119 | 82 | 0.69 | 38.3 | 0.67 | Shear | Outer c.o. |
| 18-11H-o | | 3A | 0.75A +12D | 15.1 | - | 0 | 114 | 104 | 0.91 | 46.2 | 0.81 | Shear | Outer c.o. |
| 18-11H-p-Bt | | 3B | B+15D | 12.3 | 0 | 106 | 120 | 221 | 0.98 | 60.5 | 1.06 | Flexure | -- |
| 18-11H-q-Bt | | 3B | B | 12.3 | 0 | 106 | 110 | 225 | 1.04 | 62.4 | 1.01 | Flexure | -- |
| 18-11H-i | | 3B | B | 11.9 | - | 138 | 127 | 179 | 0.68 | 47.4 | 0.83 | Splitting | Flexure |
| 18-11H-j | | 2A | A (pair) | 15.1 | - | 90 | 121 | 158 | 0.75 | 46.5 | 0.82 | Splitting | Flexure |
| 18-11H-k | | 2A | A+15D (pair) | 15.0 | - | 90 | 124 | 183 | 0.85 | 53.4 | 0.84 | Splitting | Flexure |
| 18-11H-t-r | | 3B | B+15D | 12.1 | 191 | 109 | 129 | 238 | 1.00 | 64.5 | 1.03 | Flexure | -- |

TABLE I. (CONTINUED)

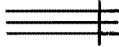
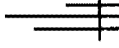
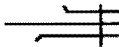
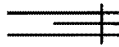
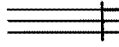
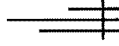
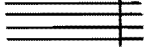
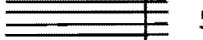
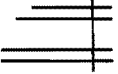


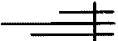
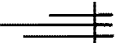
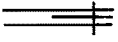


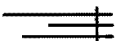

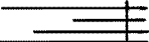

| Beam | Bars | Half span | First cutoff or bend | b in. | Extra r_{fy} psi | r_{fy} used psi | $2\sqrt{f'_c}$ psi | v_u psi | $\frac{v_u}{\text{compt.}}$ | f_{su} ksi | f_{su}/f_y | Failure mode | Complications |
|----------------------------|---|-----------|----------------------|-------|--------------------|-------------------|--------------------|-----------|-----------------------------|--------------|--------------|--------------|---------------|
| 18-11I-a |  | 3B' | None | 12.2 | - | 122 | 135 | 266 | 1.03 | 48.7 | 1.17 | Flexure | -- |
| 18-11I-b |  | 3B' | B' | 12.1 | - | 124 | 134 | 168 | 0.65 | 30.8 | 0.72 | Flexure | D.T. |
| 18-11I-b-r ₁ | do | 3B' | B' | 12.1 | 84 | 133 | 104 | 164 | 0.69 | 28.6 | 0.68 | Flexure | D.T. |
| 18-11I-b-r ₂ | do | 3B' | B' | 12.2 | 103 | 138 | 131 | 206 | 0.77 | 36.9 | 0.88 | Flexure | D.T. |
| 18-11I-b-r ₃ | do | 3B' | B' | 12.3 | 184 | 130 | 114 | 213 | 0.87 | 38.2 | 0.91 | Flexure | D.T. |
| 18-11I-b-Bt |  | 3B' | B' | 12.1 | 0 | 126 | 125 | 279 | 1.11 | 48.8 | 1.16 | Flexure | -- |
| 18-11I-b-Bt-r ₁ | do | 3B' | B' | 12.2 | 90 | 121 | 124 | 264 | 1.08 | 46.4 | 1.10 | Flexure | -- |
| 18-11I-c |  | 3B' | B+15D (1 only) | 12.2 | - | 142 | 102 | 227 | 0.93 | 39.8 | 0.95 | Flexure | D.T. |
| 18-8I-c |  | 3A' | None | 9.1 | - | 77 | 108 | 204 | 1.10 | 49.7 | 1.07 | Flexure | D.T. |
| 18-8I-d |  | 3A' | A' | 9.1 | - | 77 | 108 | 131 | 0.71 | 31.6 | 0.68 | Shear | Flexure |
| 18-8I-e |  | 4A' | None | 12.0 | - | 82 | 134 | 170 | 0.79 | 51.5 | 1.06 | Flexure | -- |
| 18-8I-f |  | 5.2A' | None | 12.0 | - | 0 | 123 | 126 | 1.02 | 51.2 | 1.05 | Flexure | -- |

TABLE I. (CONTINUED)

| Beam | Bars | Half span | First cutoff or bend | b in. | Extra r_{fy} psi | r_{fy} used psi | $2\sqrt{f'_c}$ psi | v_u psi | $\frac{v_u}{\text{compt.}}$ | f_{su} ksi | f_{su}/f_y | Failure mode | Complications |
|-----------------------|--|-----------|----------------------|-------|--------------------|-------------------|--------------------|-----------|-----------------------------|--------------|--------------|--------------|---------------|
| 15-8H-a (2 layers) | 2nd 1st  | 3A | A+12D | 7.1 | - | 153 | 110 | 268 | 1.02 | 63.4 | 1.01 | Flexure | -- |
| 15-8H-c (2 layers) | do | 3A | A+12D | 7.0 | - | 153 | 116 | 294 | 1.09 | 66.8 | 1.06 | Flexure | -- |
| 15-8H-b (2 layers) | 2nd 1st  | 2A | A+12D | 7.0 | - | 298 | 107 | 385 | 0.95 | 61.0 | 0.97 | Shear | -- |
| 13-8H-a |  | 3A | Full | 9.2 | - | 117 | 132 | 191 | 0.76 | 71.7 | 1.14 | Flexure | -- |
| 13-8H-b | do | 3A | Full | 9.0 | - | 0 | 133 | 138 | 1.04 | 51.9 | 0.83 | Shear | -- |
| 13-8H-c |  | 3A | A | 8.9 | - | 121 | 115 | 176 | 0.74 | 65.5 | 1.04 | Flexure | Splitting |
| 13-8H-d | do | 3A | A | 9.0 | - | 0 | 116 | 90 | 0.78 | 34.8 | 0.55 | Shear | Outer c.o. |
| 13-8H-e |  | 3A | A+12D | 9.0 | - | 120 | 121 | 181 | 0.75 | 66.7 | 1.06 | Flexure | Splitting |
| 13-8H-f |  | 3A | A+12D (1 only) | 8.9 | - | 121 | 124 | 192 | 0.78 | 71.5 | 1.14 | Flexure | Splitting |
| 13-8H-g |  | 3A | 2A (pair) | 9.0 | - | 120 | 118 | 174 | 0.73 | 65.5 | 1.04 | Flexure | D.T. |
| 13-8H-h |  | 3A | A 1 only | 9.0 | - | 120 | 118 | 166 | 0.70 | 61.8 | 0.99 | Flexure | D.T.+Split. |
| 13-8H-i |  | 3A | A+12D (pair) | 12.0 | - | 91 | 125 | 185 | 0.86 | 70.5 | 1.13 | Flexure | Splitting |
| 13-8H-j |  | 2A | A+12D (pair) | 12.1 | - | 138 | 125 | 248 | 0.94 | 62.8 | 1.00 | Splitting | Flexure |
| 13-8H-k |  | 4A | A+12D | 12.0 | - | 0 | 126 | 108 | 0.86 | 54.6 | 0.87 | Shear | -- |
| 12-11H-Slab |  | 2A | A+15D (pair) | 24.1 | - | 0 | 116 | 109 | 0.94 | 51.6 | 0.91 | Shear | -- |

Test Data

Also tabulated in Table I are significant test data, reduced to unit stresses, especially the ultimate unit shear stress ($v_u = V/bd$) and ultimate steel stress ($f_{su} = M_u / (A_s 0.9d)$), and the mode of failure.

The shear ratio ($v_u/\text{compt.}$) is the ultimate unit shear stress v_u relative to the computed stress capacity, i.e., to the sum of $2\sqrt{f'_c}$ and rf_y . The extra stirrups shown in the separate column are ignored in this calculation. A ratio of less than unity indicates unsatisfactory shear strength. Likewise the ultimate steel stress ratio of f_{su}/f_y of less than unity indicates lack of full flexural strength.

Types of Failure

Failures were of several types, but a specific type of failure was only occasionally uncomplicated by other factors. The last column lists some of these complications. (When a clean cut shear failure occurred, this column has been used to indicate at which cutoff the failure started.)

Among the 24" beams several with flexural failures have been noted as complicated by diagonal tension (D.T.). This means either (1) a failure which could be classified nearly as accurately as a shear-compression failure, (2) a final failure in diagonal tension but after reaching the yield moment, or (3) a diagonal crack which increased the flexural tension just beyond the first cutoff point, this crack running back to a compression failure near the load without really incorporating much of the strength of the bar first cut off.

Some beams have been designated as splitting failures but in the last column as flexure failures, or the reverse, dependent upon the relative apparent importance. This notation may indicate splitting which seemed to lower the effectiveness of at least one bar at the critical section, thus developing a poor steel stress ratio of 1.00 or 1.01 (compared to the 1.05 to 1.16 normal for uncomplicated flexure) or even as low as 0.82 in some of

the 18" beams with #11 bars of A432 steel. Alternatively, it may simply indicate the appearance of significant splitting without any noticeable deficiency in the steel stress ratio (as in 13-8H-f).

A N A L Y S I S O F R E S U L T S

Order of Presentation

The first phase of the investigation indicated that only 2 out of 31 specimens with bars cut off had reached their nominal shear strength before failure. It may thus be stated with confidence that cutting off bars does lower shear strength in almost all cases. In fact, another investigator has shown that even the addition of an extra bar of short length to a beam already adequately reinforced resulted in a lower strength.

This report will first discuss the weakening effect of several specific cutoff patterns (and variations thereof) and the effect of remedial measures, such as bending up the bars or adding extra stirrups, or both. Other detailing patterns or differing sizes of beams are discussed later and also indicate that the cutting off of bars generally lowers the ultimate beam shear stress attained before failure.

It was noted early in these studies that diagonal cracking was influenced by the bar cutoff points selected, this cracking sometimes being critical from the first cutoff from the load, sometimes from the second. It was suspected that the absolute beam depth could be a factor of significance. However, analysis of this factor was complicated since the ratio of beam depth to development length (distance to cutoff point) changed because of the steel grade as much as because of beam depth; the development length for A432 bars was 50 percent greater than for intermediate grade bars. Beam depths of 24", 18", 15", and 13" were used, but no systematic variation with depth could initially be identified.* The study of remedial measures were made only on 24" and 18" beams.

*A later section of this report indicates that lesser depths did generally show relatively stronger than the deeper sections.

Beams 24" Deep

a. Effect of Cutoff Length

The general deleterious effect of cutting off #8 bars of A432 grade in a 24" beam of half length $3A = 80"$ is shown in Fig. 2 in terms of the shear ratio attained. All these beams were reinforced with stirrups, typically just adequate to resist the shears necessary to develop a moment stress of 60 ksi in the main steel. The typical abscissa value of 60 to 70 psi for rf_y indicates that essentially minimum stirrups were required for most of these beams.

In the upper left of Fig. 2 is a square symbol representing the strength of a beam with three full length bars; it failed in flexure at a relative shear of 1.04 and a steel stress ratio of 1.13. The other specimens with bars cut off in various patterns without extra stirrups all failed in shear at ratios from 0.59 to 0.82. The length from load to first cutoff point is indicated alongside each point. Where bars were cut off singly from a group of three, each increase in distance to the first cutoff improved beam strength:

| <u>First cutoff length</u> | <u>Shear ratio</u> |
|----------------------------|--------------------|
| A = 26.7" | 0.59 |
| A+12D = 38.7 | 0.75 |
| 2A(pair)= 53.4 | 0.82 |
| Full length = 81.0 | 1.04 |

Cutting only a first bar and running the two others full length raised the shear ratio from 0.75 (shown above) to 0.81 (not plotted). The top cast bar specimen performed some 16 percent better than its bottom cast companion. Both had the same length A calculated on the basis of the bond stress normally used for bottom bars. Although the top cast specimen had an f'_c of 3180 psi, compared to 3750 psi for the bottom cast, it developed 125 psi shear compared to 109 psi with the bottom cast bar. The cracking with the bottom cast bar was much less extensive with failure developing

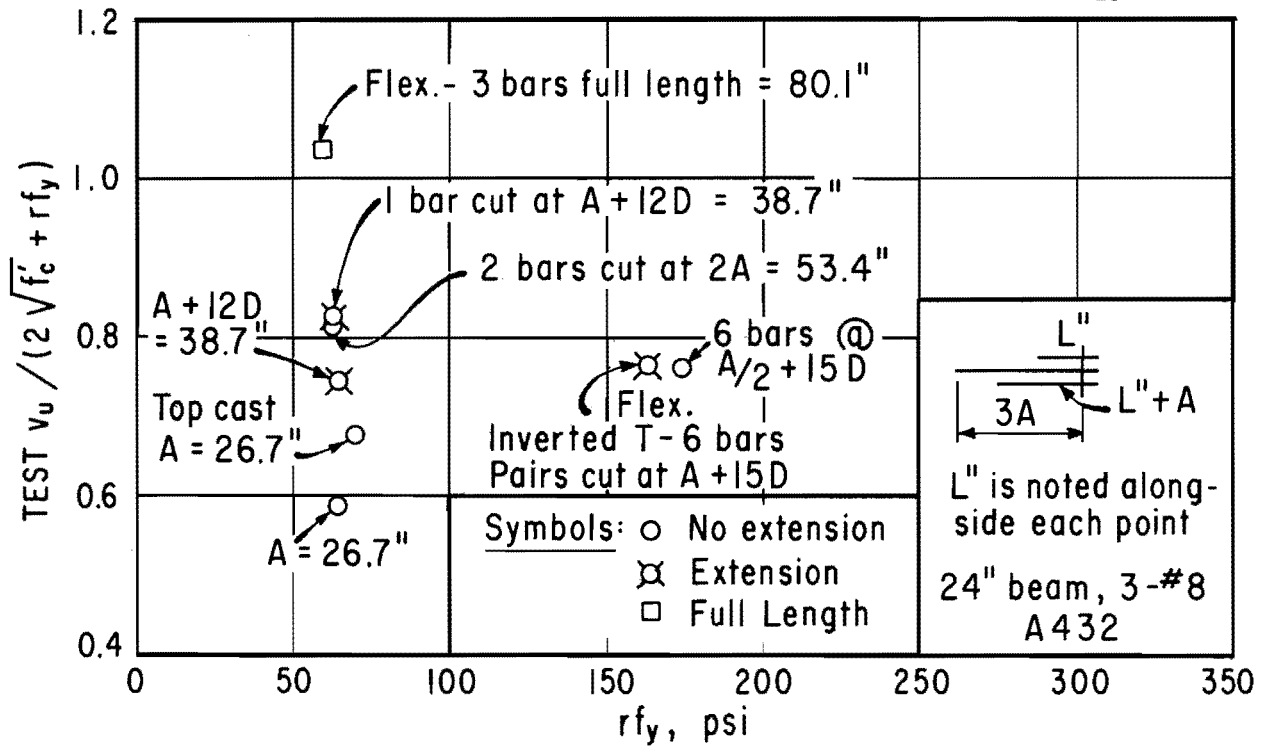


Fig.2. Influence of various cutoff lengths on shear strength.

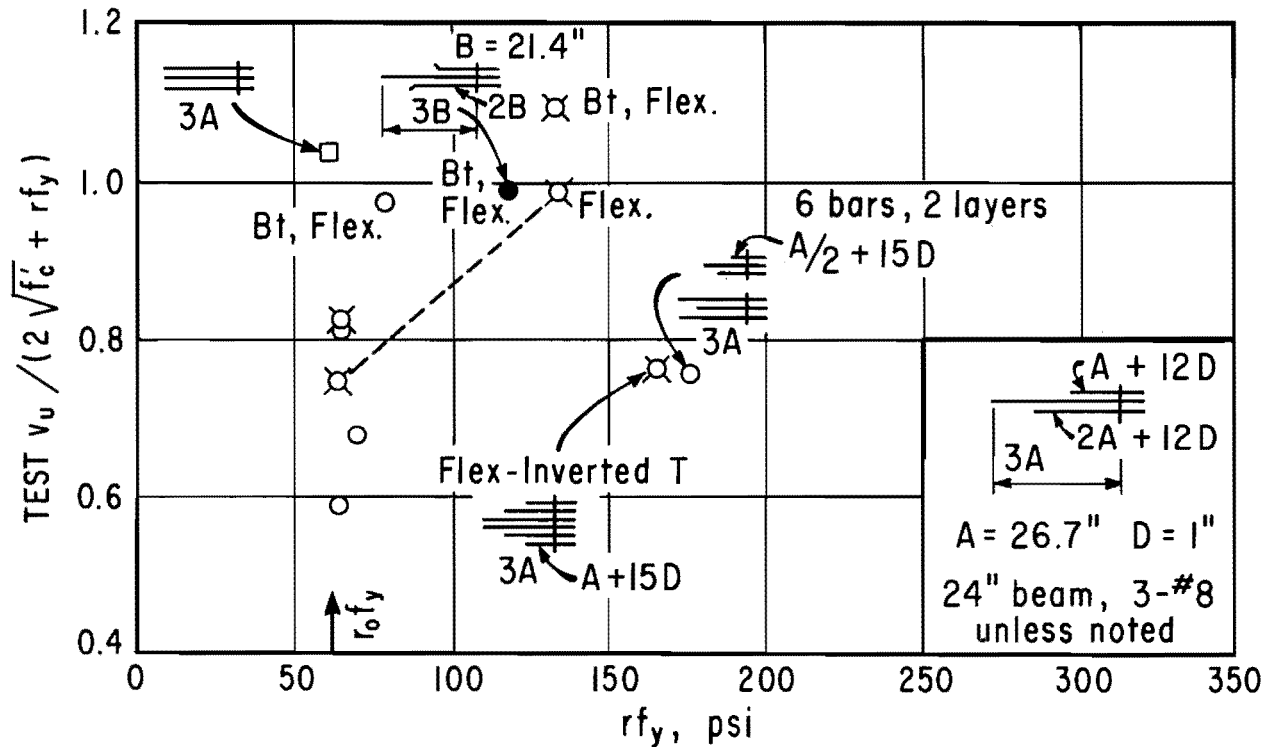


Fig.3. Comparable 24" beams with #8 A 432 bars.

from the second* cutoff point. With the top cast specimen, the cracking up to possibly half the ultimate load appeared quite similar to the other specimen, but with increased load bond splitting (both on tension face and sides) developed quite seriously, both beyond the second cutoff and back to the first cutoff. This produced a final failure almost like a diagonal tension failure from the first cutoff point. This failure, in spite of appearances, was judged to be in flexure, but premature because the middle bar had slipped too much to be fully effective at and beyond the first cutoff point.

At rf_y of 175 and 164 psi two special beams are shown. One is a slightly wider beam with two layers of steel, six #8 bars in all, each bar cut at its minimum $A/2$ for moment plus 15D end anchorage. Although it clearly failed in shear, the failure was not sudden and was a shear compression type. The crack widths indicated the probable yielding of the longitudinal steel near the quarter point as a result of splitting along the bars and the resulting loss of effective steel area. The second of these beams was an inverted T-beam with a wider web and some surplus of stirrups. The low shear value obtained is not really significant since the actual failure was in flexure. Nevertheless, the cracking pattern and crack widths indicate that the extra stirrups were probably necessary to avoid a shear failure.

b. Remedial Measures

The points of Fig. 2 are repeated in Fig. 3 with three added points indicating improved behavior when bars were bent up. At $rf_y = 77$ psi the bent bar specimen otherwise matching the one which failed at a shear ratio of 0.75 reached 0.98 (and this would have been 1.00 except that extra high strength concrete and a slight excess of stirrups raised the nominal shear strength above that required to develop the flexural capacity). At

*Measured from the load.

$rf_y = 133$ the nearly doubled stirrups raised the strength to a shear ratio of 0.99* and a flexural failure (the shear a little low because f'_c was low, only 2370 psi). The dotted line connects specimens alike except in the amount of stirrups. Those same increased stirrups with bent bars gave a shear ratio of 1.08, more satisfactory because f'_c was normal and the steel stress reached 75 ksi. The shorter development length B with bars bent, at $rf_y = 119$ psi, also reached a shear ratio of 0.99 but failed in flexure; the concrete strength was 3990 psi, just enough above the design value to raise the allowable shear and thereby lower the shear ratio below unity. (It might be noted that the shorter B lengths called for more stirrups than the A lengths; both the necessary bond stress and shear stress to develop the given f_y became larger as the development length was reduced.)

Beams 18" Deep

a. With #8 Bars, A432 Grade

When the three #8 bars were used in an 18" deep beam, the use of #2 U-stirrups at maximum spacing provided slightly over 1.6 times the r_o required for the actual f'_c and f_y values of specimens. At the time the specimens were designed all the implications of this divergence were not clear enough to cause a shift in the design. In Fig. 4 it thus becomes necessary to note that the smallest plotted rf_y values are already "over-reinforced" for shear compared to the $r_o f_y$ indicated. This placed the potential shear strength above the flexural strength and caused the full length bars to plot below unity.

This figure is similar to Fig. 2 in that it indicates full length bars plotting highest and the shortest cutoffs plotting lowest. When a double width beam with 6 bars was made and bars cut off by pairs,

*In all cases where extra stirrups have been arbitrarily added over the L" length to raise beam strength closer to normal, the shear ratio is calculated on the original stirrup ratio, but the plotting is in terms of the increased rf_y .

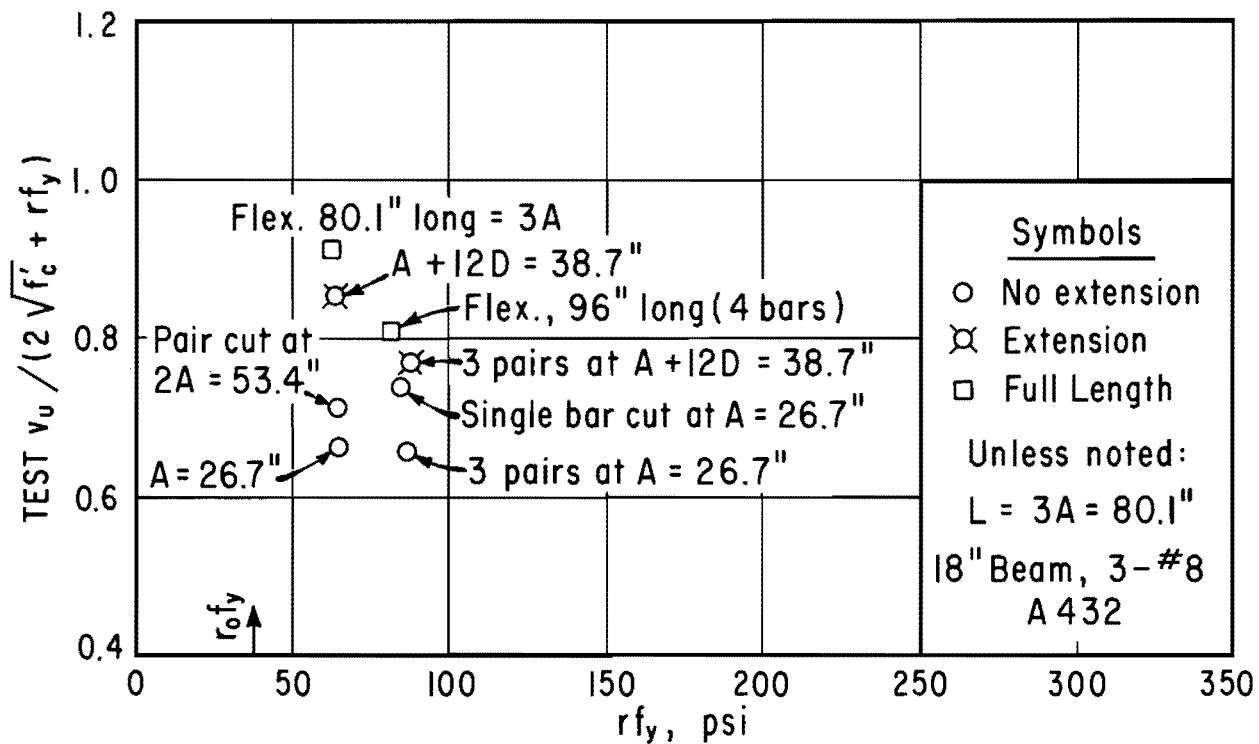


Fig.4. Influence of various cutoff lengths on shear strength.

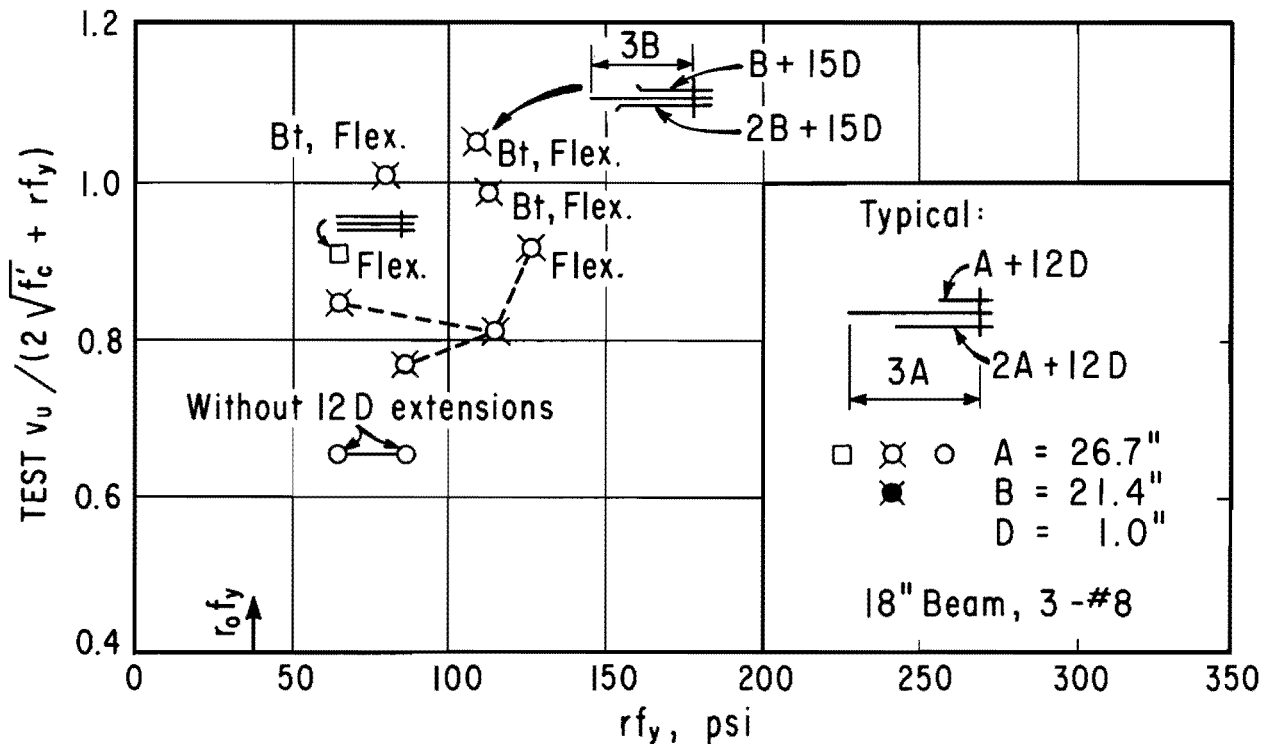


Fig.5. Comparable 18" beams with #8 A 432 bars.

the strength almost duplicated that of the narrower unsymmetrical beam. Because of the overstrength in stirrups, the flexural failure of a still longer beam (96" for the half length) plots with a lower shear ratio.

Several of the specimens of Fig. 4 are replotted in Fig. 5 with specimens strengthened by adding more stirrups or by bending up bars. In general the three specimens with bent bars plot above the full length bar specimens, four comparable specimens with cutoff bars plot lower, and two specimens with bars cut off at the bare moment requirements (without end anchorage) plot the lowest of all. Because a better reference (with the excess stirrups originally present) is flexural strength rather than the inflated nominal shear strength, some of these data are replotted in Fig. 6 in terms of relative ultimate steel stress, f_{su}/f_y . This shows the relationships better, although only a few points shift much relative to each other. In general this figure shows the effect of further added stirrups as rather erratic, with doubled stirrups (over three times the nominal required) bringing a flexural type failure but with diagonal tension complications and only 0.98 of computed strength. On the other hand, bent bars were uniformly adequate and bars without end anchorage (the lowest pair joined with solid line) were slightly improved by a small increase in stirrups.

b. With #11 Bars, A432 Grade

With #11 bars of A432 grade the principal tests were with 18" beams. The relatively poor results wherever bars were cut off are shown in Fig. 7. The rf_y values vary only because the shorter beam lengths, shown on the right, require more stirrups to develop their flexural strengths. The beams on the extreme left, having a half-length of 159", did not require any stirrups under the ACI Code (although the AASHTO specification calls for minimum stirrups in all beams). The full length bars developed 11 percent of excess shear strength. In contrast, no arrangement of cutoff bars was satisfactory. In general, the shorter the bars cut off, the weaker the specimen. It should be noted that with two bars full length and two cut off

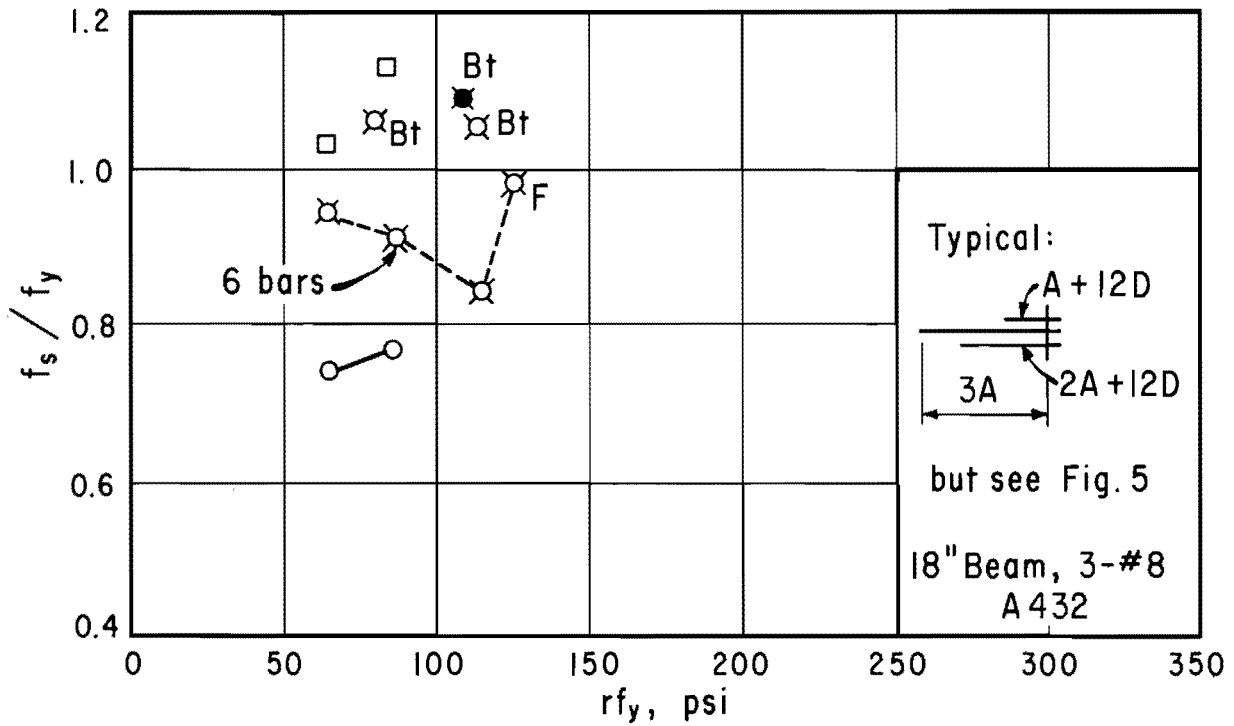


Fig. 6. Developed f_s in 18" beams. Beam details are noted in Fig. 5.

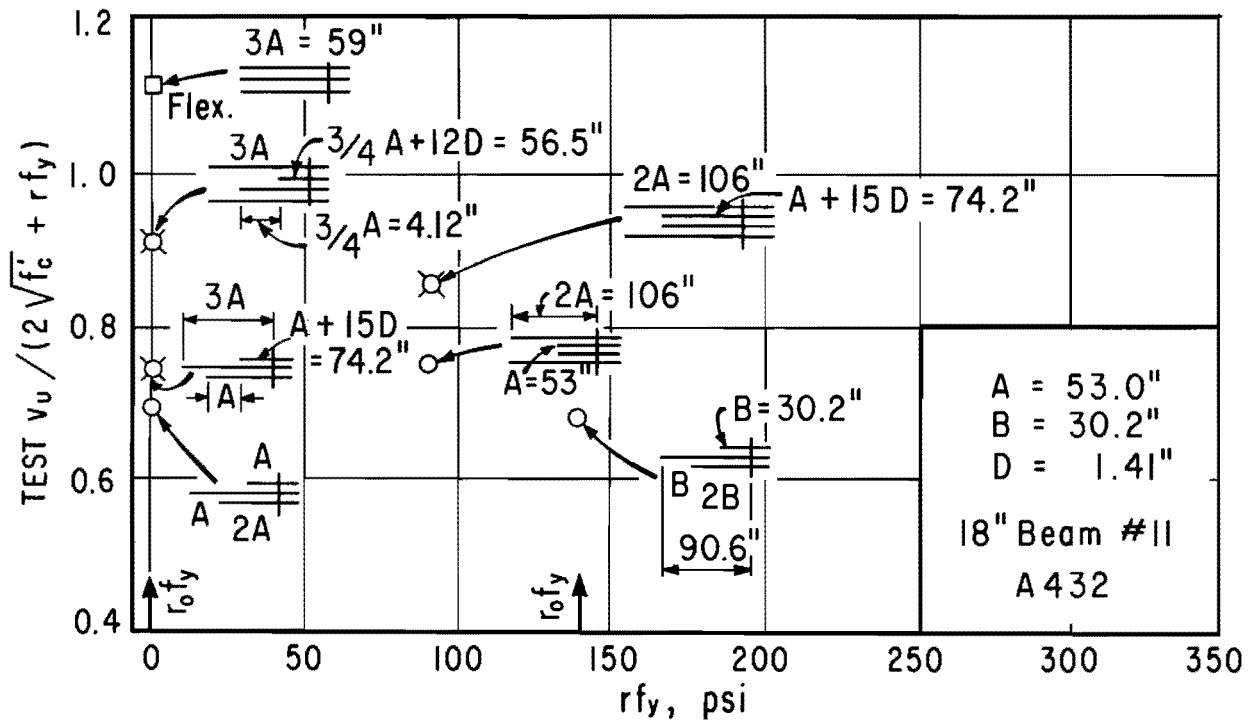


Fig. 7. Various L" with A 432 bars.

rather short (a total of four bars), a little over 91 percent of nominal shear strength was attained.

Remedial measures were studied for the specimen having a development length of $A + 15D$ and requiring no stirrups under the ACI Code. This specimen had failed at 74 percent of nominal shear strength. The addition of minimum stirrups (Fig. 8 at $r = 0.0015$, $rf_y = 61$ psi) brought the beam just up to the limit expected without stirrups, a shear ratio of unity and a flexural stress of unity. This failure also involved splitting and diagonal tension. This type of failure will be marked simply "F" (not "Flex.") in the figures to represent an unsatisfactory flexural failure.

Some comparisons are also available in Fig. 8 for beams with shorter development lengths. Where marked with solid circles the development length is B and where a cross is added to the solid circle, it is $B + 15D$. In both cases the bent bars, even with lightened stirrups, show up reasonably well at ratios of 1.06 and 1.01. Without bent bars, more than double the stirrups (and a 15D extension) led only to a shear ratio of unity without any excess. When one considers the better behavior of the longer development lengths ($A + 15D$) shown at the left side of Fig. 8, he must obviously question whether the specimen at the extreme right might not have behaved as well with fewer stirrups. No data are available and the comparison may actually be invalid since B distances are only 0.57 those of A . Shear at a given f_y is thus some 175 percent as much ($100/0.57$) with the beams having B lengths.

c. With #11 Bars, Intermediate Grade

Intermediate grade #11 bars in an 18" beam are studied in Fig. 9, with AASHTO development lengths to the cutoff points but no end anchorage beyond the theoretical points (except for the one case noted as $B' + 15D$). Full length bars failed in flexure at a very satisfactory value, but with bars cut off without extension for end anchorage the shear ratio dropped to 0.65. In this case it was obvious that a diagonal tension crack developed from the outer cutoff point and gradually worked back over the

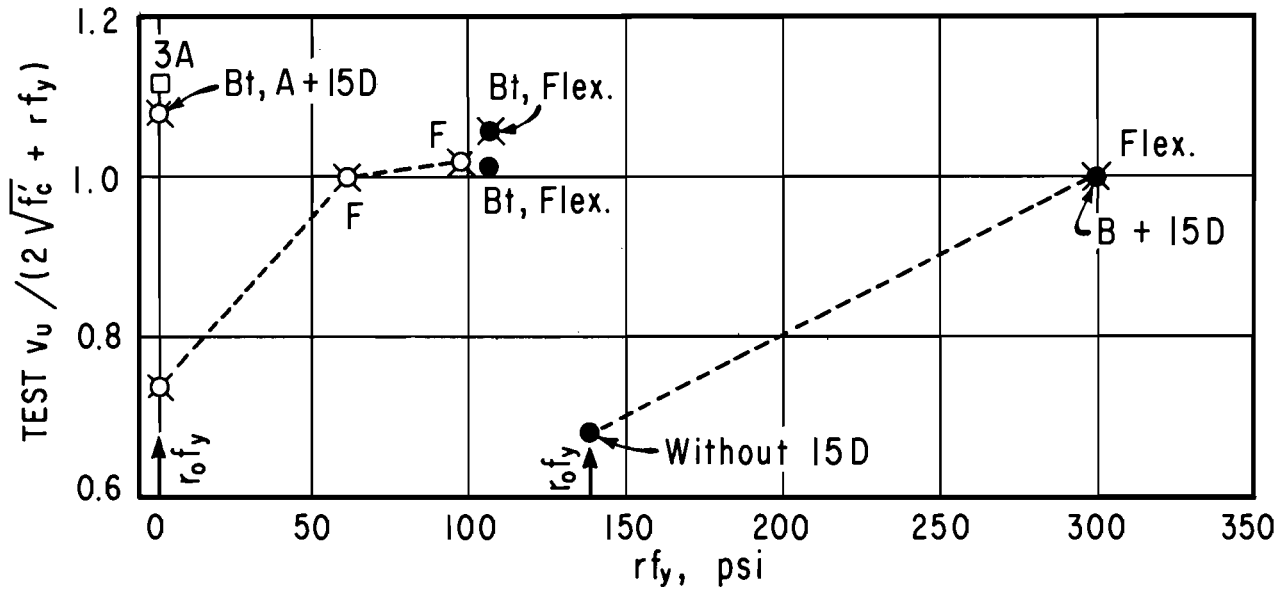


Fig. 8. Bent bars and extra stirrups with 3-#11 A 432 bars in 18" beam.

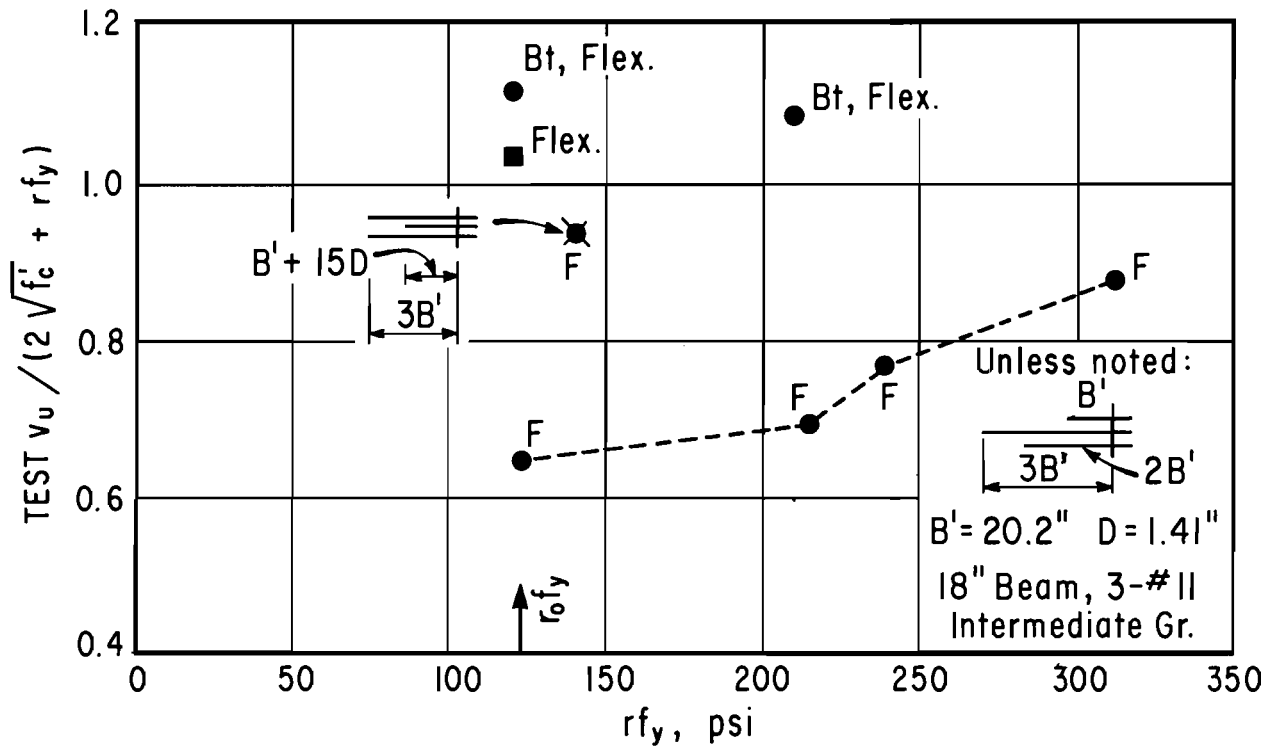


Fig 9. Bent bars and extra stirrups with #11 intermediate grade bars.

inner cutoff point. A calculation of the steel stress at the outer cutoff, considering the stirrups to have been working at yield stress, indicates that the single bar there passed well beyond its yield point, to an estimated $1.27 f_y$; crack width measurements confirm the yielding. On this basis the following mechanism can be postulated. The bar cutoff initiated a premature diagonal crack. The diagonal crack raised the bar stress just beyond the cutoff. After an allowance for the help of stirrups (at yield stress), this particular bar stress increased some 49 percent and resulted in an unsatisfactory flexural failure where only one bar existed, at a shear ratio of 0.65. Additional stirrups raising rf_y from 124 to 241 psi raised the shear ratio from 0.65 to 0.77, and a total rf_y of 314 psi developed a shear ratio of 0.87. The addition of 15D, which is 21.5", to each cutoff point (essentially the same as stopping the first bar at 2B' and running two full length) improved the shear ratio to 0.93 with an rf_y of only 142 psi, indicating the great effectiveness of this large end anchorage; it is more effective than an added rf_y of 172 psi as extra stirrups. The bending of bars, without any other end anchorage, was fully effective in bringing about a flexural failure in all cases.

d. With #8 Bars, Intermediate Grade

When #8 intermediate grade bars were used in a few 18" beams with development length A' set by the ACI Code, again without an end anchorage, the results shown in Fig. 10, were quite consistent with those just discussed for #11 bars. For full length bars, whether in half spans of $3A'$, $4A'$, or $5.4A'$, flexural failures were obtained. When photographs of the $3A'$ half span with full length bars were compared with photographs of a similar beam containing bars cut off, it was noted that the full length bars led (1) to vertical flexural cracks which rose much higher into the beam before inclining, and (2) to shear cracks which were much later in developing.

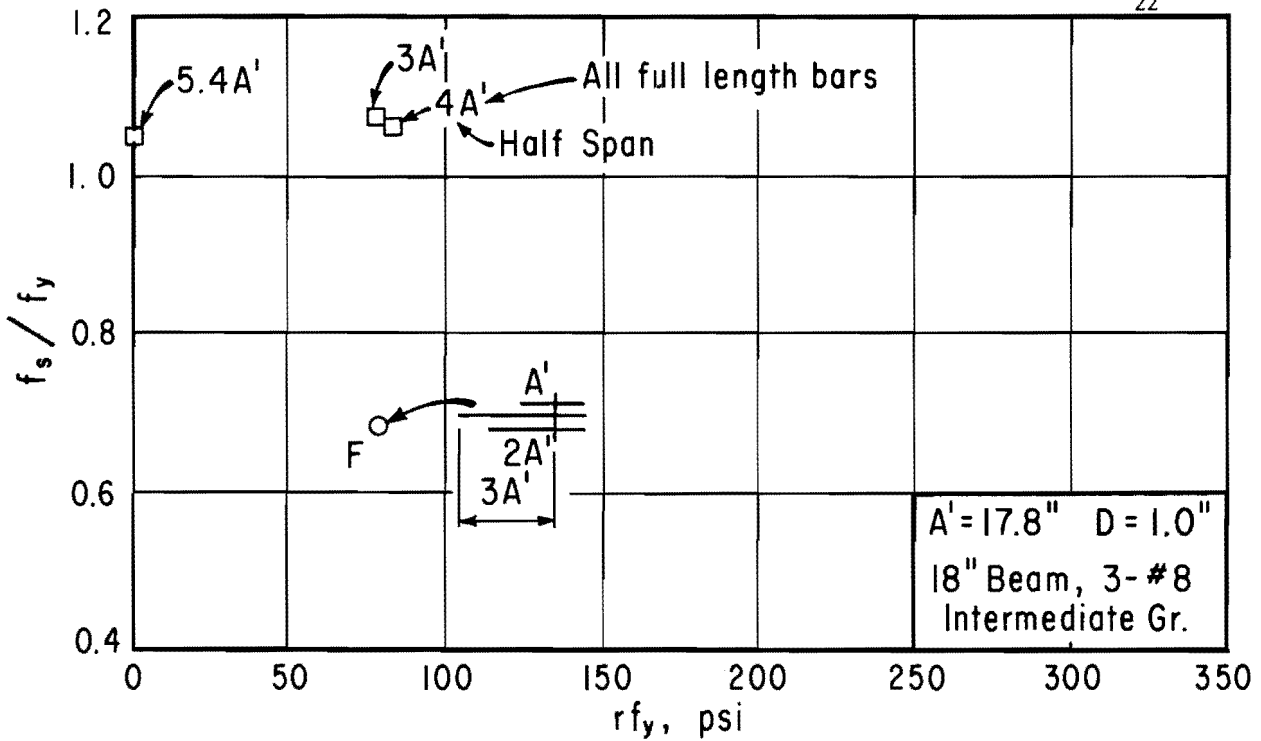


Fig.10. Intermediate grade steel in 18" beams.

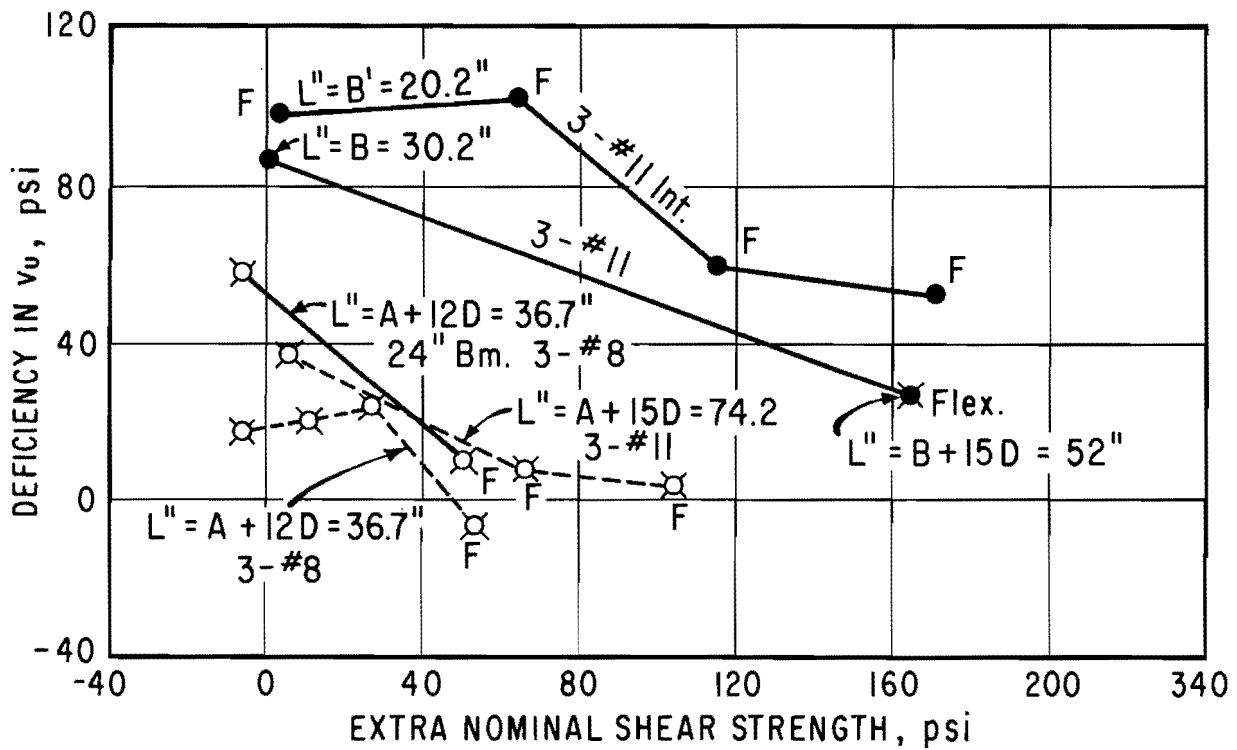


Fig.11. Improvement from adding to nominal shear strength. Unless noted, 18" beams and A 432 steel.

e. Stirrup Efficiency as a Remedial Measure

The general efficiency of extra stirrups is indicated in Fig. 11, which displays the data already discussed in terms of deficiency rather than remaining strength. The plotted values of extra shear strength are not exactly the extra rf_y values added over the L'' length as listed in Table I, but reflect also the effect of an f'_c which varied from specimen to specimen. For the specimen with full length bars, the quantity $rf_y + 2\sqrt{f'_c}$ was computed as a reference value. When added stirrups were used, the extra shear strength was taken as the difference between the new $rf_y + 2\sqrt{f'_c}$ and the reference quantity. Likewise the shear deficiency itself was taken as the difference between the actual v_u and that of the reference specimen.

It is noted that the observed deficiency in Fig. 11 is generally decreased by an increase in either development length or development plus anchorage length. Since there is no reference beam with full length bars based on the B development length of 30.2" for #11 bars, the deficiency in that case was based on the actual v_u compared to the theoretical value of $rf_y + 2\sqrt{f'_c}$ for that beam without any extra stirrups. Since this beam failed in splitting and flexure, associated with diagonal cracking, this may not be entirely proper as a basis for a shear rating. It is possible this curve should be lower as a whole, say, such as to bring the right hand end down to zero.

Beams of Less than 18" Depth

Beams of 15" and 13" depths were also explored to a lesser extent, and one 12" slab-type specimen, but without any attempt to define remedies in these cases. In Fig. 12 three 15" beams with two layers of #8 bars are shown, the cutoff bars being only in the upper (inner) layer. The two beams of length 3A at $rf_y = 153$ psi developed their full flexural strength with some bars cut, but the shorter beam with $L = 2A$ at $rf_y = 298$ psi lacked about 5 percent of doing so. It is not known whether the heavy stirrups

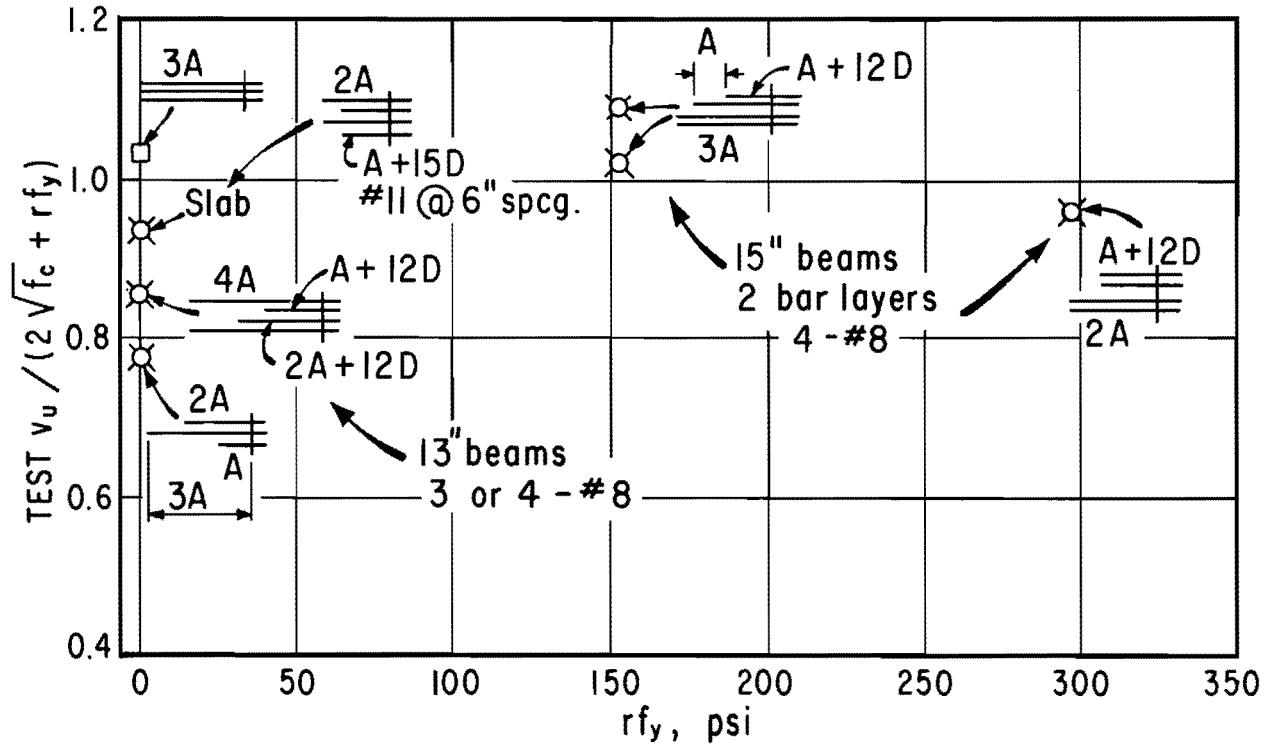


Fig. 12. Shallow beams with A 432 bars.

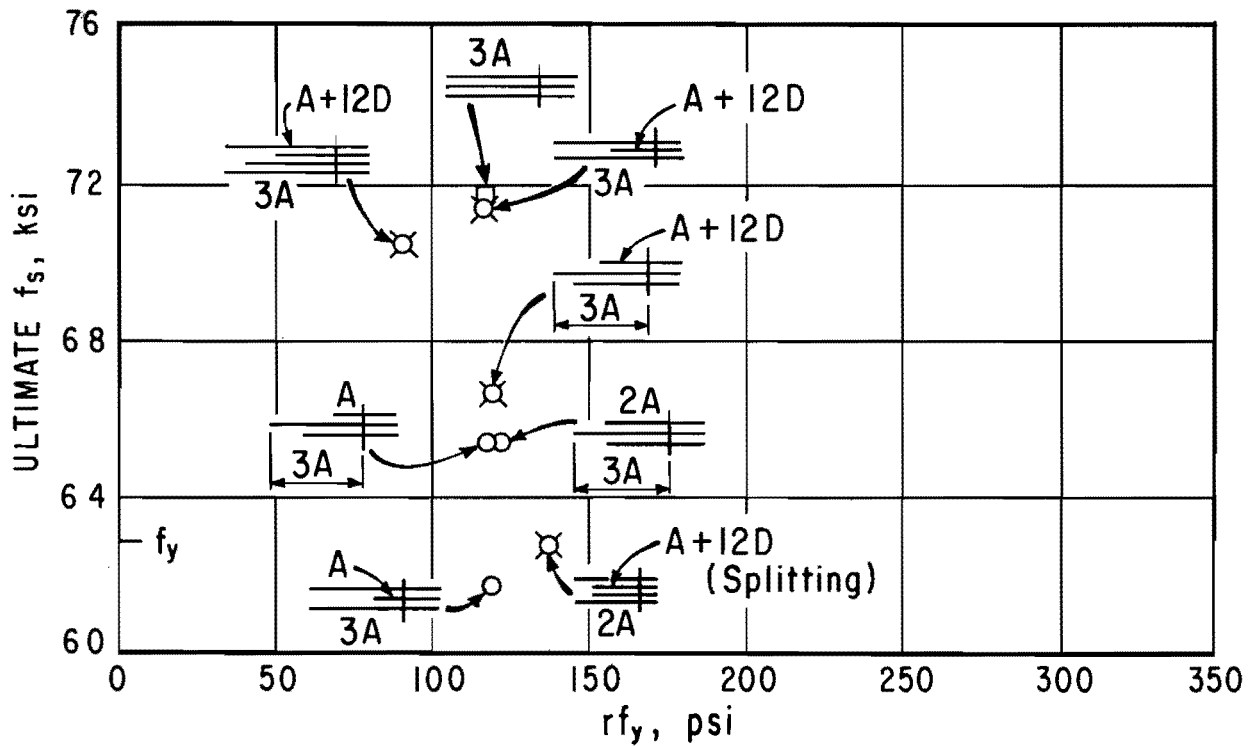


Fig. 13. Flexural failures of 13" beams, #8 bars A 432.

(required for normal expected shear) made the bar cutoffs less critical in these three beams or whether it was the two-layer effect (outer bars running full length) which was beneficial. Probably it was a combination.

Also in Fig. 12 on the extreme left, are three points for 13" beams with #8 bars. These three beams carried no stirrups. The longer (4A length) beam with four bars had a 12D end anchorage and theoretically required no stirrups, but it failed at a shear ratio of 0.85. The other two beams needed stirrups in the amount $rf_y = 46$ psi, but none were actually used because the minimum r of 0.0015 and the $d/2$ maximum spacing could not be satisfied without doubling the required stirrup ratio. As a result of this stirrup deficiency even the full length bars did not reach a flexural failure, but in a shear failure developed all the expected shear strength. With bars cut at the minimum A development length, without end anchorage, a shear ratio of only 0.78 was reached.

In this same 13" beam series, eight other beams with #8 bars were made with stirrups, as diagrammed in Fig. 13. At rf_y of approximately 120 psi the stirrups had a surplus of about 75 psi to meet the maximum spacing rule. At rf_y of 91 the surplus rf_y was 42 psi for a beam of 12" width and at rf_y of 138 the surplus was 13 psi. All of these beams did exceed their flexural yield strength (except for two at ratios of 0.99 and 1.00) giving at least a vague measure of the extra web steel probably needed to offset the ill effects of cutting off bars. This would indicate that at this depth an extra rf_y of about 75 psi (plus in some cases a 10 to 13 percent surplus of f'_c above the design value) was adequate for the cutoff patterns used. This, incidentally, is several times the amount of stirrups one would estimate simply to pick up the deficiencies of 26 and 18 psi in shear indicated by the two low beams on the left of Fig. 12. The adequacy of the specimen at rf_y of 91 psi indicates that less extra stirrups may actually be required. The strength at rf_y of 138 psi is greater than would be expected on the basis of beams of greater depth.

Also shown on the left in Fig. 12 is a point noted as a slab. This was a 12" thick slab 24" wide and reinforced with #11 bars (A432 grade) at

6" spacing. Half of these were stopped at $A + 15D$, which was 74" from the load, in a half span of 106". This seems once again to indicate that the shallow members are less harmed by cutting off their bars. However, in addition to a loss of 5 percent in shear strength, all ductility was lost and the failure was violent, breaking the slab into two separate pieces.

The effect of cutting off bars was not only a loss of strength but also often a reduced toughness at failure. This shows in the deflection curves of Fig. 14. Three of four cases with bars cut off failed with small deflections, while one was somewhat tougher in behavior. As to cutoff patterns, it can be seen that longer bar extensions or a reduced portion of the steel cut off added to strength. Even the full length bars failed to give a large ductility because of a secondary shear failure at f_{su}/f_y of 1.09.

Effect of Beam Depth

Since all beams were designed to develop nearly simultaneously f_y flexure and the ACI allowable shear stress, the ratio f_{su}/f_y is a measure* of how closely any beam approached its potential shear strength. In Fig. 15 beams of 24", 18", and 13" depth using four cutoff patterns are compared in terms of this flexural stress ratio. The two groups on the left included no end anchorage and thus represent deficient designs. The two right-hand groups satisfied the ACI Code and show higher strengths. Three out of four bar patterns show a progressive trend to lower ratios with greater beam depths. In this comparison the 13" beams must be considerably discounted because these carried surplus stirrups equivalent to an rf_y of 75 psi. Still, the indication is that lesser beam depths perform better when bars must be cut off. The greater weakness of the deep beams

*The ratio of $v_u/v_{calc.}$ is nearly the same, but variations in f'_c and practical adjustments to the design requirement for stirrups make for minor differences.

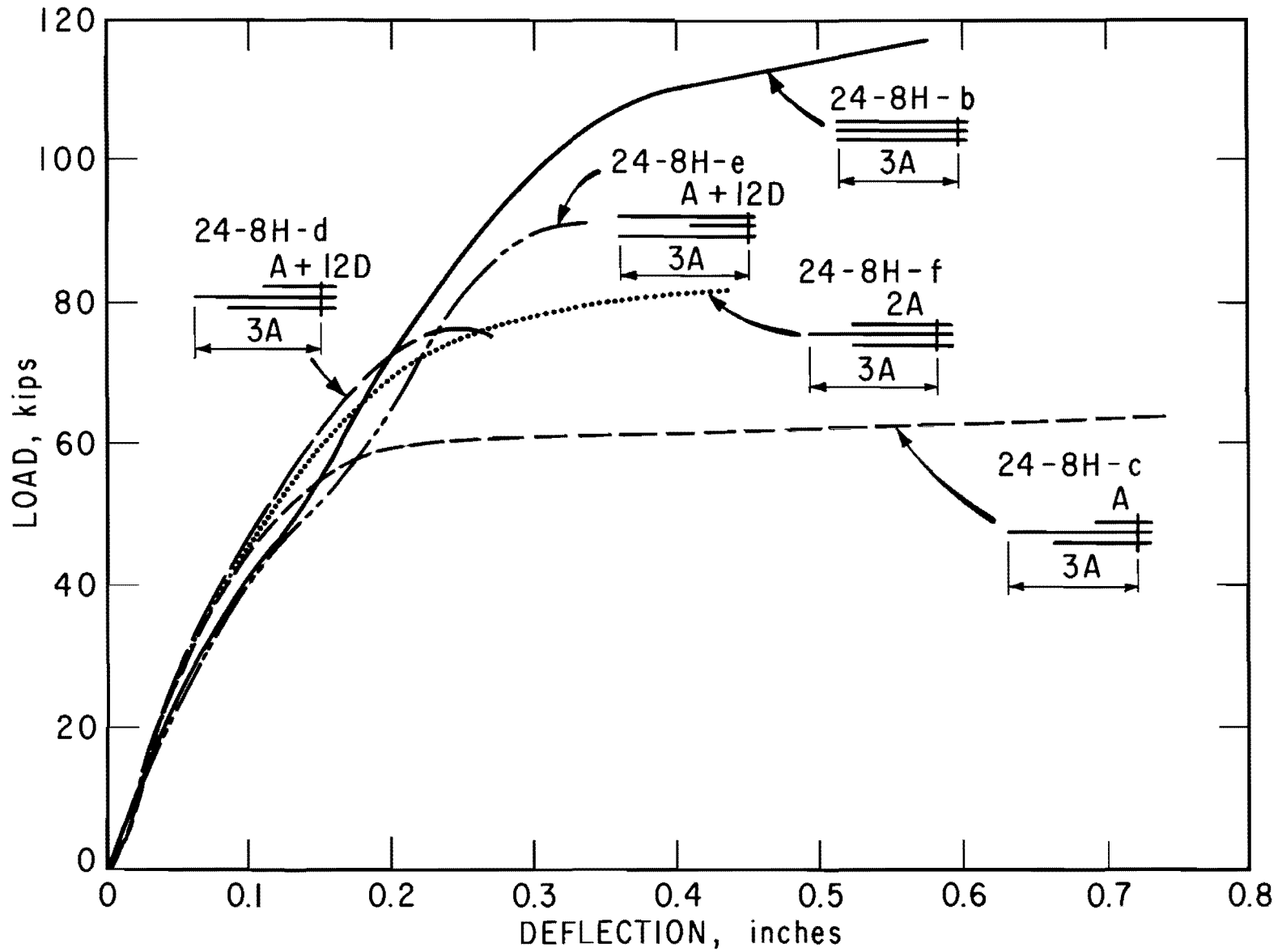


Fig. 14. Deflections of 24" Beams.

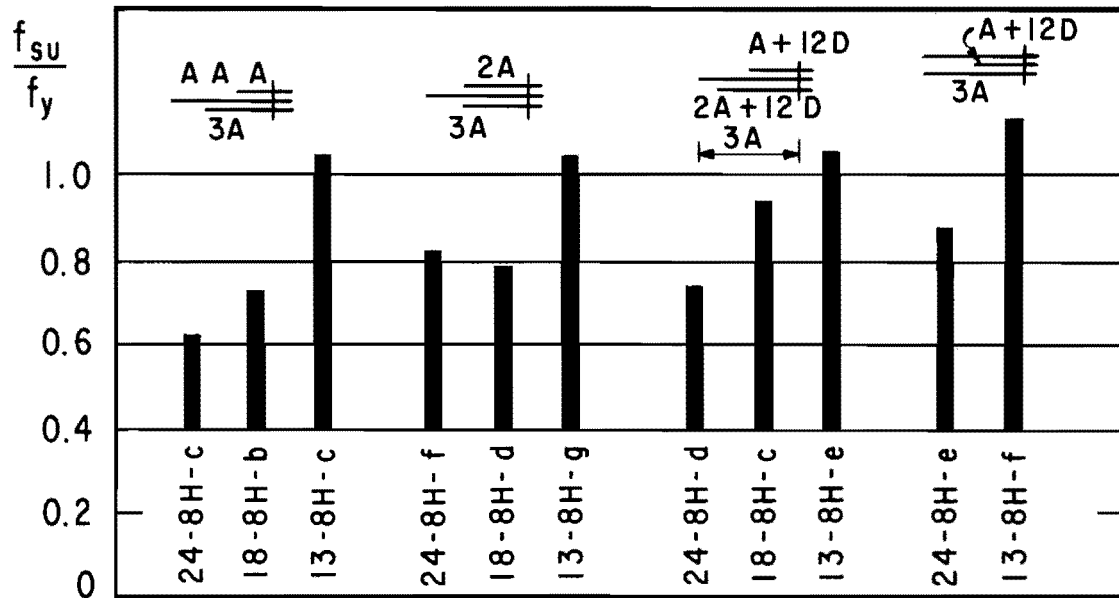


Fig.15. Effect of depth. Depth is first number in code.

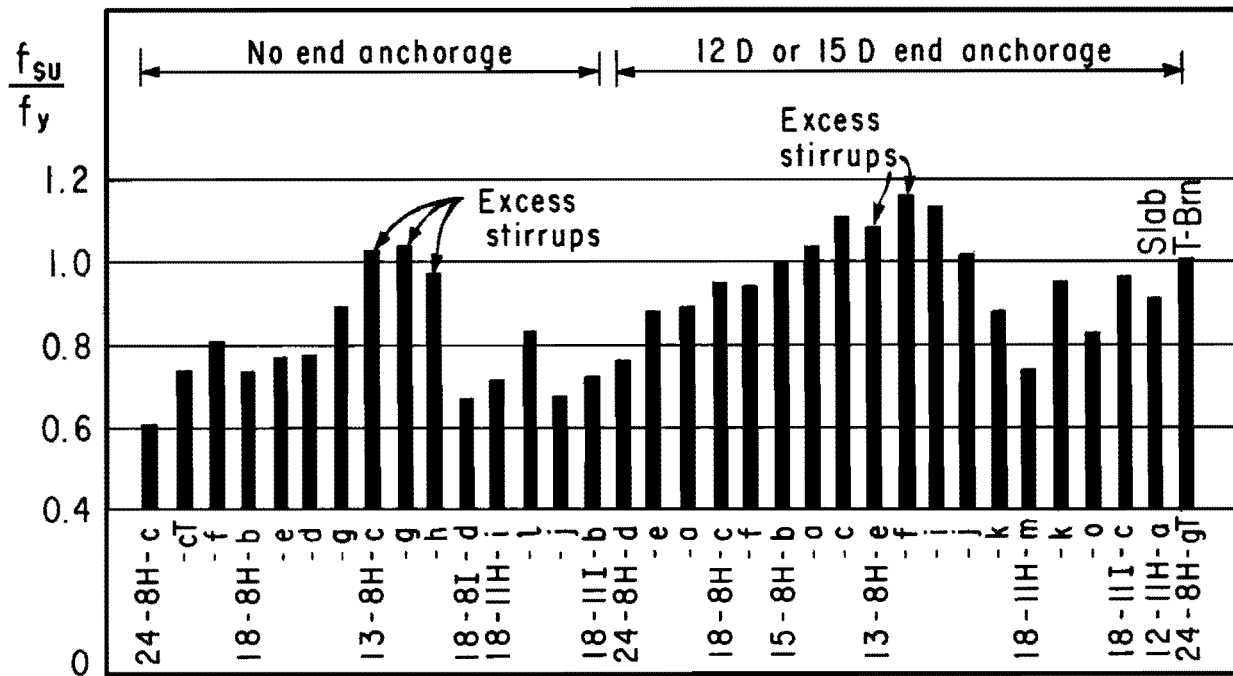


Fig.16. Summary of all beams with bars cut off and without remedial measures. (The five 13" beams had excess r_{fy} of 75 psi to satisfy minimum spacing rule.)

seemed to arise because diagonal cracks from two adjacent cutoffs in deep beams more readily merged to produce earlier failure.

No depth effect showed for full length bars because such beams of all depths reached their full yield moment.

Effect of Symmetry of Bar Pattern

Although a lack of bar symmetry showed so prominently in the crack pattern as to indicate that it might be a significant variable, this possibility was disproved when the beam width and number of bars were doubled to give a symmetrical pattern. In one such case (18-8H-e vs. 18-8H-b) the symmetrical was 4 percent stronger and in another (18-8H-f vs. 18-8H-c) 2 percent weaker, neither difference probably being significant.

Overall Effect of Cutting Off Bars

Bars cut off to both the exact moment diagram and to a minimum ACI development length A gave the minimum ratios of f_{su}/f_y . For these designs the ratio of f_{su}/f_y is a good measurement also of the ultimate developed shear relative to that needed for flexural strength. This ratio varied from 0.61 to 0.73 with an average of 0.67.

| | | |
|----------|---------------|---------------------|
| 24-8H-c | $rf_y = 64.4$ | $f_{su}/f_y = 0.61$ |
| 18-8H-b | 64.4 | 0.74 |
| 18-11H-l | 0 | 0.67 |
| 18-8I-d | 124.3 | 0.68 |
| | | Average 0.67 |

The 13" beams of this pattern (13-8H-c,d) were excluded from this average: one carried no stirrups and failed in shear at a ratio of 0.78; the other carried more than double the necessary stirrups and failed at a steel stress ratio of 1.04. When the end anchorage of 12D or 15D was added to each bar cut off, the steel stress ratio f_{su}/f_y increased to the 0.73 to 0.74 range (18-8H-b, 18-11H-m, 24-8H-d).

It might seem logical that beams using the net AASHO specification length B would be still lower in strength because of this shorter B length. However, this design required more stirrups and the net result was a slight improvement compared to beams with the longer A lengths:

| | | | |
|----------------------|--------------------|------------------|---------------------|
| 18-11H-i | $L'' = B = 30.2''$ | $rf_y = 138$ psi | $f_{su}/f_y = 0.81$ |
| 18-11I-b | $B' = 20.2$ | 124 | 0.72 |
| 18-11H- l | $A = 53.0$ | 0 | 0.67 |

The bar chart of Fig. 16 shows the range of steel stress ratios f_{su}/f_y found for all 34 specimens with other than full length bars, separated into specimens with or without the end anchorage of 12D or 15D. Since in practice this end anchorage will be provided, it appears that the designer can conservatively be assured of 70 percent of the expected flexural strength, but not much more unless he adds extra stirrups. A closer study indicates ratios of 0.83 to 1.06 when stirrups provide an rf_y in excess of 130 psi. Thus, for such cases an 80 percent appears justified. Since many beams did better than this, the limits might be further refined with a longer investigation. The number developing full flexural strength was too small to define cases where no loss of strength would be expected.

The design suggestions recognize the better performance of the 13" beams and the 12" slab specimen by assuming a 90 percent efficiency.

C O N C L U S I O N S

For the considerable range of specimens tested, several conclusions seem to be justified:

(1) Whenever a bar is cut off in a tension zone, a lowered shear strength will normally result. The loss in strength appears to be larger in deep beams than in shallow beams or slabs.

(2) The provision of a 12D to 15D end anchorage, as specifications require, adds 15 to 30 percent to the reduced strength associated with bars cut off exactly to their moment lengths.

(3) Where bars are bent across the beam and continued on the compression face (or anchored there) no shear strength is lost.

(4) Extra stirrups over the development length of bars cut off can rebuild the strength lost by cutting the bars off. However, such an arrangement was not stronger than bent bars without any extra stirrups.

- (a) With #8 bars of A432 steel cut off singly at 12D beyond the longer development length (A) used in the ACI Code (Fig. 3), added stirrups in the amount of $rf_y = 70$ psi nearly restored the strength loss caused by cutting off bars. (This is to be compared with an original required rf_y of 60 psi.) There is a hint in Fig. 5 that even a small deficiency in the extra rf_y provided may drop the overall strength 15 percent.
- (b) With #11 intermediate grade bars cut at the minimum AASHO development length (Fig. 9) without the required 15D end anchorage, one-third of the shear strength was lost in spite of the presence of a normally required rf_y of 125 psi. An extra rf_y of 190 psi brought the strength back to 87 percent of normal. The addition of the required 15D end anchorage with an extra rf_y of only 9 psi increased the 0.65 ratio to 0.93.
- (c) With A432 #11 bars cut at the minimum AASHO development length, without the required 15D end anchorage, the loss was 32 percent (Figs. 7 and 8). With 15D added, plus an extra rf_y of 167 psi, full strength was regained. A smaller amount of corrective stirrups was not tried.

(d) Although the response was somewhat irregular, a given extra rf_y seems to add back roughly half its computed strength when used over the development length of bars cut off.

(5) The case of moment lengths which were in excess of the A or B length required for development length was not seriously investigated.

(6) Modified design recommendations follow which should be safe with bars cut off.

Design Recommendations

Based on this investigation, the following rules are recommended for design (on an ultimate strength basis), although these provisions go a little beyond fully proven ground.

(1) Bond stresses used in calculating development length should be changed from the AASHTO values to the lower ACI Code values:

$$\begin{array}{ll} \text{For bars \#3 to \#11: } & \begin{array}{ll} \text{USD} & \text{WSD} \\ u = 6.7\sqrt{f'_c}/D \leq 560 \text{ for top bars} & 3.4\sqrt{f'_c}/D \leq 350 \\ u = 9.5\sqrt{f'_c}/D \leq 800 \text{ for other bars} & 4.8\sqrt{f'_c}/D \leq 500 \end{array} \end{array}$$

(2) Bars should be extended beyond the theoretical moment cutoff point at least $15D$ or the effective depth of the beam, whichever is larger. The $15D$ agrees with the AASHTO specification, while the distance d is one of the ACI requirements.

(3) With full length bars or with bars bent and run well into a compression area, ultimate shear strength may be assumed at the full normal value in the ACI Building Code, namely, $v_u = 2\sqrt{f'_c} + rf_y$.

(4) When bars are cut off in a tension zone, the total shear strength should be considered lowered by 30 percent.

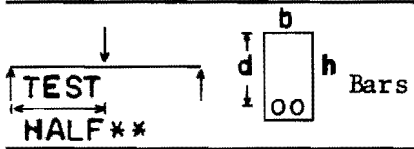
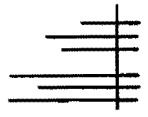
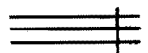
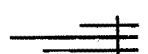
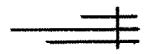
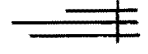
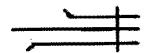
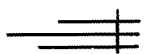
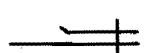

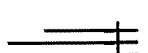
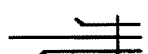
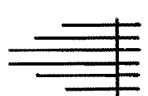
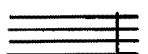
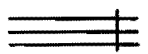
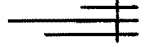
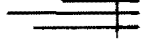
(a) When bars are cut off in a tension zone already requiring stirrups in the amount of $rf_y \geq 130$ psi, total shear strength may be assumed lowered only 20 percent.

(b) When extra stirrups are used to make up the deficiency thus found, the extra rf_y should be at least twice the amount of the deficiency and in no case less than $rf_y = 100$ psi. The resulting stirrup spacing should not exceed $d/4$.

(5) In slabs of 12" or lesser thicknesses with bars cut off in the tension zone, the shear capacity should be assumed lowered by 10 percent. If greater shear strength is needed, only the extension of the bars or bending them into a compression zone should be assumed a proper remedy.

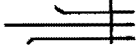
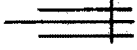
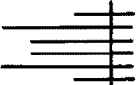

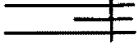
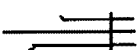
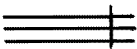
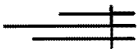

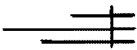
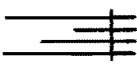
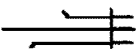
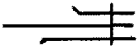
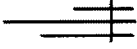
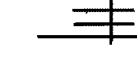
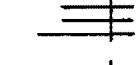

A P P E N D I X

TABLE II. DETAIL SPECIMEN DIMENSIONS

| | |  | b in. | h in. | d in. | f'_c psi | f_y ksi |
|------------------------|------------|---|----------|----------|----------|---------------|--------------|
| 24-8H-a (2 layers) | 2nd 1st |  | 12.1 | 24.1 | 20.2 | 4470 | 62.7 |
| 24-8H-b | |  | 9.7 | 24.3 | 21.4 | 3610 | 62.7 |
| 24-8H-c | |  | 9.2 | 24.0 | 21.1 | 3750 | 62.7 |
| 24-8H-cT | |  | 9.7 | 24.4 | 21.0 | 3180 | 62.7 |
| 24-8H-d | |  | 9.2 | 24.1 | 21.2 | 2950 | 62.7 |
| 24-8H-d-Bt | |  | 9.6 | 24.2 | 21.4 | 4480 | 70.8 |
| 24-8H-d-r | |  | 9.2 | 24.0 | 21.3 | 2370 | 62.7 |
| 24-8H-d-Bt-r | |  | 9.1 | 24.3 | 21.5 | 3430 | 70.3 |
| 24-8H-e | |  | 9.2 | 24.0 | 21.2 | 3940 | 62.7 |
| 24-8H-f | |  | 9.1 | 24.1 | 21.2 | 3180 | 62.7 |
| 24-11H-g-Bt | |  | 12.5 | 24.4 | 21.9 | 3990 | 61.8 |
| 24-18H-gT* | |  | 14.0 | 24.1 | 21.4 | 3180 | 62.7 |
| 18-8H-h | |  | 12.0 | 18.1 | 15.2 | 4090 | 62.7 |
| 18-8H-a | |  | 9.4 | 18.2 | 15.3 | 4300 | 62.7 |
| 18-8H-b | |  | 9.2 | 18.0 | 15.2 | 4260 | 62.7 |
| 18-8H-c | |  | 9.2 | 18.1 | 15.3 | 3870 | 62.7 |
| 18-8H-c-r ₁ | do | | 9.0 | 18.2 | 15.2 | 2780 | 62.7 |
| 18-8H-c-r ₂ | do | | 9.3 | 18.3 | 15.5 | 3620 | 66.0 |

*T-bm. with flg. 42" wide by 8" thick. Bars distributed in flg. width.
 **The test half span is tabulated in the third column of Table I.

TABLE II. (CONTINUED)

| Beam | Bars | b in. | h in. | d in. | f'_c psi | f_y ksi |
|---------------------------|---|----------|----------|----------|---------------|--------------|
| 18-8H-c-Bt |  | 9.2 | 18.0 | 15.8 | 4050 | 69.9 |
| 18-8H-c-Bt-r ₁ | do | 9.2 | 18.1 | 15.2 | 4180 | 68.8 |
| 18-8H-d |  | 9.2 | 18.0 | 15.1 | 4010 | 62.7 |
| 18-8H-e |  | 18.1 | 18.1 | 15.2 | 3420 | 62.7 |
| 18-8H-f |  | 18.1 | 18.1 | 15.2 | 3420 | 62.7 |
| 18-8H-g |  | 9.3 | 18.1 | 15.2 | 3610 | 62.7 |
| 18-8H-s-Bt |  | 9.2 | 18.3 | 15.6 | 3800 | 62.7 |
| 18-11H-n |  | 12.1 | 18.1 | 15.0 | 3190 | 57.0 |
| 18-11H-m |  | 12.1 | 18.1 | 15.0 | 3470 | 57.0 |
| 18-11H-m-r ₁ | do | 12.2 | 18.2 | 14.2 | 3480 | 57.0 |
| 11-11H-m-r ₂ | do | 12.4 | 18.3 | 15.2 | 3510 | 57.0 |
| 18-11H-m-Bt |  | 12.1 | 18.3 | 15.3 | 3590 | 57.0 |
| 18-11H-l |  | 12.0 | 18.1 | 15.0 | 3540 | 57.0 |
| 18-11H-o |  | 15.1 | 18.1 | 15.0 | 3240 | 57.0 |
| 18-11H-p-Bt |  | 12.3 | 18.2 | 15.3 | 3570 | 57.3* |
| 18-11H-q-Bt |  | 12.3 | 18.3 | 15.7 | 3020 | 61.8 |
| 18-11H-i |  | 11.9 | 18.1 | 15.0 | 4020 | 57.0 |
| 18-11H-j |  | 15.1 | 18.1 | 15.0 | 3650 | 57.0 |
| 18-11H-k |  | 15.0 | 18.1 | 15.0 | 3840 | 57.0 |
| 18-11H-t-r |  | 12.1 | 18.2 | 15.2 | 4160 | 62.7 |

*1 bar @ 61.8
2 bars @ 55.0

TABLE II. (CONTINUED)

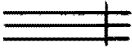
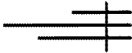
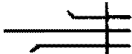
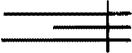
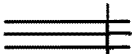
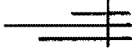

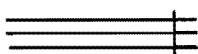

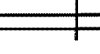

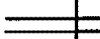

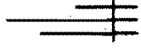
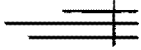
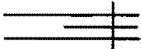
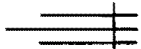
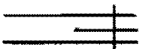
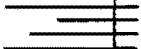
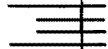
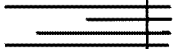
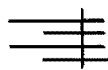
| Beam | Bars | b in. | h in. | d in. | f'_c psi | f_y ksi |
|----------------------------|--|----------|----------|----------|---------------|--------------|
| 18-11I-a |  | 12.2 | 18.0 | 14.9 | 4580 | 42.0 |
| 18-11I-b |  | 12.1 | 18.0 | 14.9 | 4460 | 42.0 |
| 18-11I-b-r ₁ | do | 12.1 | 18.1 | 15.6 | 2720 | 42.0 |
| 18-11I-b-r ₂ | do | 12.2 | 18.2 | 15.3 | 4290 | 42.0 |
| 18-11I-b-r ₃ | do | 12.3 | 18.2 | 14.8 | 3260 | 42.0 |
| 18-11I-b-Bt |  | 12.1 | 18.2 | 15.2 | 3890 | 42.0 |
| 18-11I-b-Bt-r ₁ | do | 12.2 | 18.3 | 15.3 | 3800 | 42.0 |
| 18-11I-c |  | 12.2 | 18.2 | 15.6 | 2620 | 42.0 |
| 18-8I-c |  | 9.1 | 17.9 | 15.1 | 2900 | 46.3 |
| 18-8I-d |  | 9.1 | 18.0 | 15.2 | 2900 | 46.3 |
| 18-8I-e |  | 12.0 | 18.2 | 15.3 | 4470 | 48.6 |
| 18-8I-f |  | 12.0 | 18.2 | 15.3 | 3790 | 48.6 |
| 15-8H-a (2 layers) | 2nd  1st  | 7.1 | 15.1 | 11.3 | 3010 | 62.7 |
| 15-8H-c (2 layers) | do | 7.0 | 15.0 | 11.2 | 3340 | 62.7 |
| 15-8H-b (2 layers) | 2nd  1st  | 7.0 | 15.0 | 11.1 | 2880 | 62.7 |

TABLE II. (CONTINUED)

| Beam | Bars | b in. | h in. | d in. | f'_c psi | f_y ksi |
|-------------|---|----------|----------|----------|---------------|--------------|
| 13-8H-a |  | 9.2 | 12.9 | 10.0 | 4330 | 62.7 |
| 13-8H-b | do | 9.0 | 13.0 | 10.1 | 4400 | 62.7 |
| 13-8H-c |  | 8.9 | 13.1 | 9.9 | 3300 | 62.7 |
| 13-8H-d | do | 9.0 | 13.1 | 10.1 | 3350 | 62.7 |
| 13-8H-e |  | 9.0 | 13.1 | 10.2 | 3640 | 62.7 |
| 13-8H-f |  | 8.9 | 13.0 | 10.1 | 3860 | 62.7 |
| 13-8H-g |  | 9.0 | 13.0 | 10.0 | 3460 | 62.7 |
| 13-8H-h |  | 9.0 | 13.1 | 10.1 | 3460 | 62.7 |
| 13-8H-i |  | 12.0 | 13.0 | 10.2 | 3920 | 62.7 |
| 13-8H-j |  | 12.1 | 13.1 | 10.1 | 3920 | 62.7 |
| 13-8H-k |  | 12.0 | 13.0 | 10.1 | 3950 | 62.7 |
| 12-11H-slab |  | 24.1 | 12.1 | 9.8 | 3345 | 57.0 |