FINAL REPORT ON NUMERICAL METHODS FOR RADIAL TRIANGULATION

by

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Research Report Number 79-1

Research Project Number 3-8-64-79
Numerical Methods for
Radial Triangulation

Conducted for

The Texas Highway Department

In Cooperation with the
U. S. Department of Transportation
Federal Highway Administration
Bureau of Public Roads

by

CENTER FOR HIGHWAY RESEARCH
THE UNIVERSITY OF TEXAS
AUSTIN, TEXAS
May 1966

PREFACE

This report on numerical radial triangulation contains the results of Research Contract Number 3-8-64-79 performed by The University of Texas for the Texas Highway Department and the U.S. Bureau of Public Roads. The purpose of this report was to investigate and evaluate the use of radial triangulation for extending control, with precise measurements taken directly from the photography and data reduced through a computational procedure which could be programmed for a high-speed digital computer.

The interpretation of aerial triangulation used in this study and the conclusions and recommendations developed represent a departure from the standard approach to the subject. For this reason, extensive background literature on aerial triangulation and errors was examined, and a section based on this theoretical material has been included as an introduction to this report.

The author is indebted to personnel from both the Texas Highway Department and the U.S. Bureau of Public Roads for their assistance throughout the course of this research.

R.D.T.

May 1966
Austin, Texas

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THEORETICAL BACKGROUND

Almost all topographic and planimetric maps now are compiled from aerial photography. Although aerial photography is the basis of many maps, there are other processes and operations involved which are just as important as the photography.

Mapping Procedures

In the process of preparing topographic maps from aerial photographs, the steps listed below are usually followed:

1. Planning the project.
2. Conducting ground surveys to provide control dimensions.
3. Making the aerial photographs of the designated area.
4. Determining supplementary control from the photographs by aerial triangulation.
5. Orienting the photographs in stereoscopic plotting instruments.
6. Compiling the details of the map.
7. Editing and checking the map.

In the use of aerial photography for the preparation of such maps, some ground surveys (usually referred to as control surveys) are needed in order to determine the scale and the orientation of the photography.

In Figure 1 is an illustration of the variations in scale and orientation of aerial photography which result from the relative position to the ground of the aircraft carrying the camera. Since the altitude and attitude of the aircraft can be controlled only within certain limits, the photography necessarily varies in scale and in orientation. Thus, in the compilation of aerial maps from aerial photographs, it is necessary to supplement the photography with ground surveys (between points which are identifiable on the photography), in order to compensate for this variation in scale and in orientation.

Ground Control

There are a number of satisfactory methods of obtaining the necessary ground control. In Figure 2 is illustrated one of the more common procedures, which is called traversing, wherein the distances and directions between successive stations are determined. It is also a common practice to determine horizontal position by triangulation procedures, as well as by traversing. In the traversing process, elevations also may be obtained. In addition, elevations can be secured by differential leveling, by trigonometric leveling, or from barometric-pressure measuring.

These field-control methods, although satisfactory, are usually time consuming and often prove to be quite expensive. Depending upon the particular area and circumstances, the field-control portion of the preparation of topographic maps could be from 25 per cent to 75 per cent of the total cost of the mapping operation. Thus, any procedure whereby some of the ground-control surveys could be eliminated or minimized would be quite desirable from an economy standpoint.

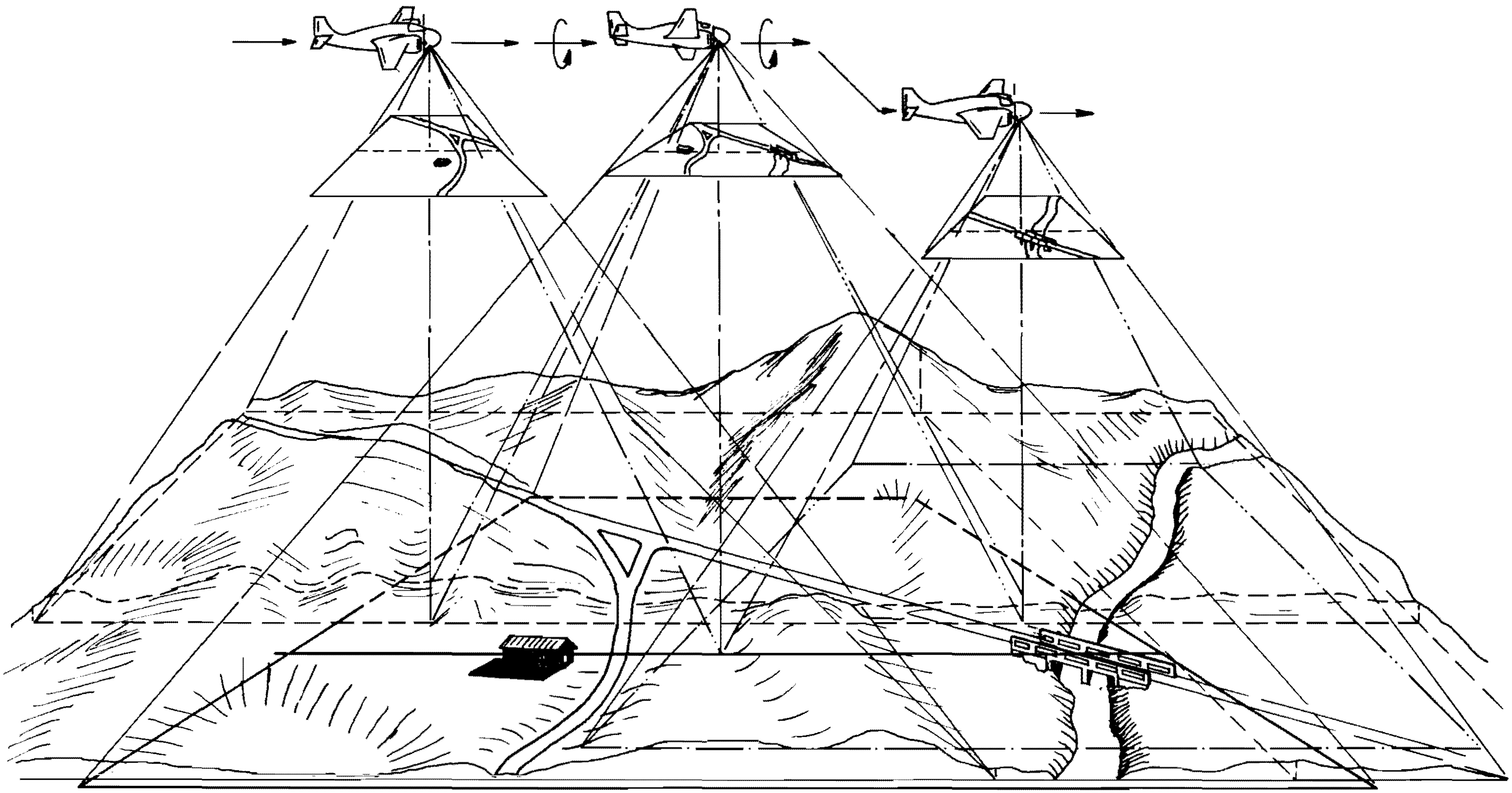

FIGURE 1

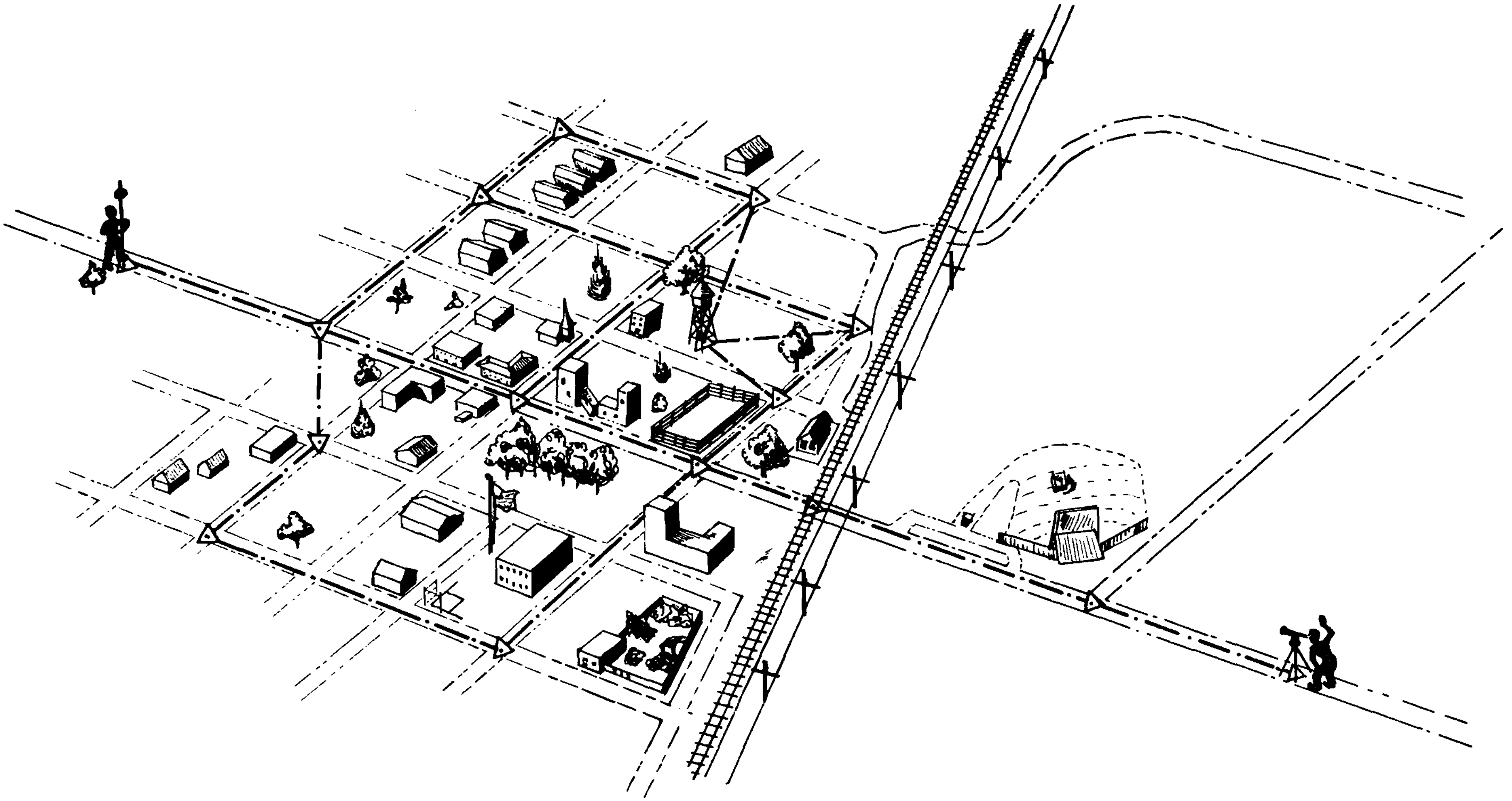

FIGURE 2

Control Extension

After the ground control is completed and the photography has been obtained, it may be desirable to extend the amount of control by processes involving measurements from the photographs.

In this report, those procedures by which ground control is extended and increased, so as to provide three-dimensional coordinates for the selected points, will be referred to as aerial triangulation. Those techniques through which only the horizontal positions (X and Y values) are found will be treated as radial triangulation.

Stereo Plotter Work

After the photography has been obtained and the control work completed, the photographs are placed in a stereo plotting instrument and properly oriented. The details of the map are compiled, including whatever information is desired.

Before the maps are released, they are usually edited, field-checked, and reproduced in a format which is desired for use.

All of these map-compilation procedures are dependent upon a satisfactory control network.

Aerial Triangulation

Since the ground surveys (item 2 in the listing above) are costly and time consuming, it is desirable to reduce the amount of such surveys whenever possible. One means of achieving this saving is by any of the several procedures for extending the ground surveys by aerial triangulation. These processes are based on angles and distances, measured on the photographs, which can be arranged into a spatial-triangulation system to extend from a minimum of control to a more dense control system. Each of the processes has certain limitations, however, which must be considered when the principles are applied to practical cases.

Much research work and considerable investigation have been done on aerial triangulation (see the list of references at the end of this report). This process has been referred to by a variety of names including phototriangulation, spatial triangulation, stereo triangulation, and bridging. Aerial triangulation has been defined by the American Society of Photogrammetry as "the process for the extension of horizontal and/or vertical control whereby the measurements of angles and/or distances on overlapping photographs are related into a spatial solution using the perspective principles of the photographs." Generally, this process, which involves the use of aerial photographs, is called either aerotriangulation or aerial triangulation. (See Figure 3.)

An extensive bibliography on aerial triangulation has been compiled by Professor Gordon Gracie of the University of Illinois; of the more than 700 titles he found devoted to this subject, however, more than half are concerned with errors in aerial triangulation. Even so, a study of some of the publications by several well-known photogrammetrists (including Bertil Hallert, G. C. Tewinkel, G. H. Schut, and R. Roelofs) would lead one to conclude that, notwithstanding these extensive investigations, the effect of the different sources of errors in aerial triangulation upon the final results (ground coordinates) has not been well established.

Aerial triangulation normally involves a complete spatial solution of the problem (using X, Y, and Z coordinates), a procedure which requires either very elaborate stereo photogrammetric instruments or very involved

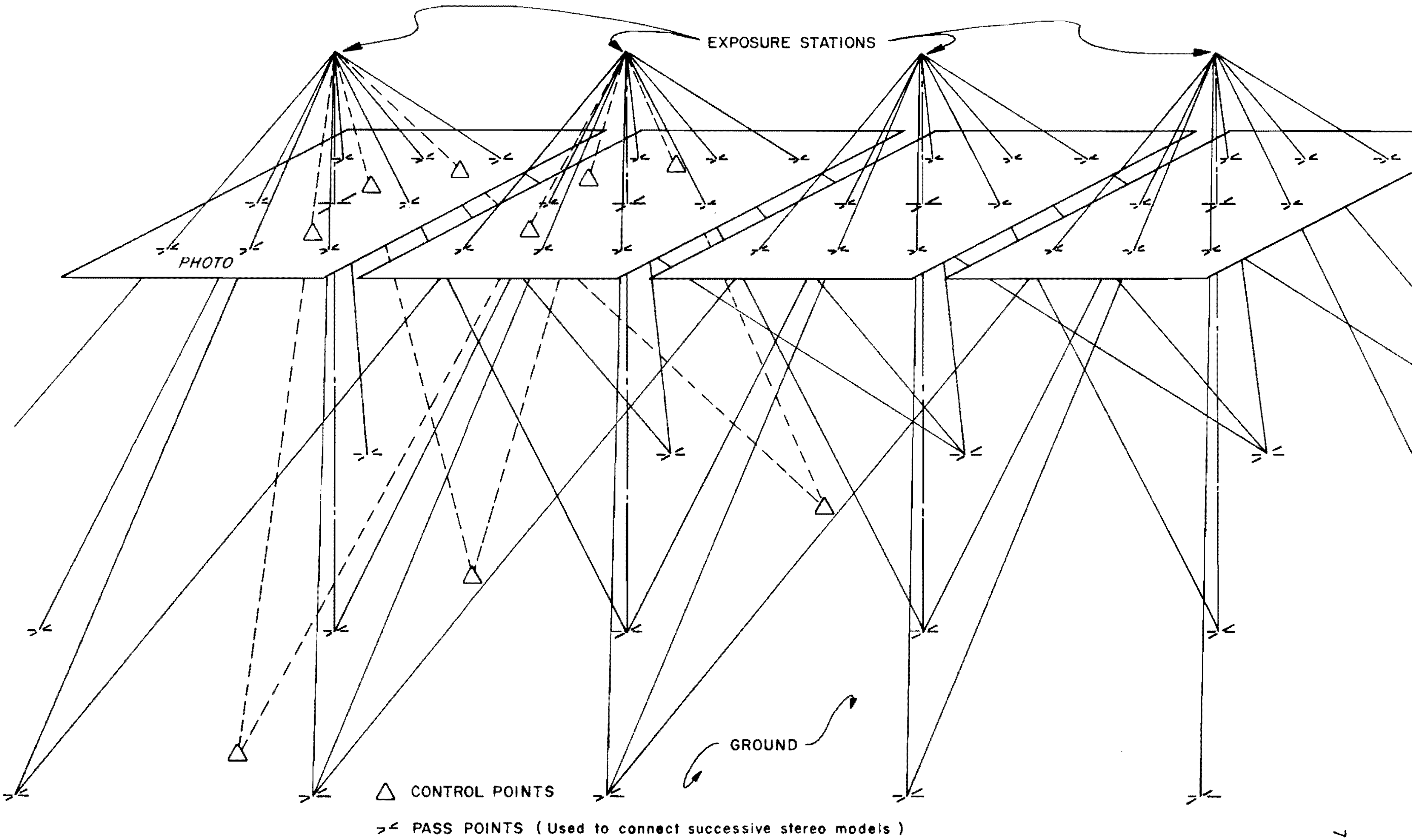
EXPOSURE STATIONS
PHOTO
GROUND
CONTROL POINTS
PASS POINTS (Used to connect successive stereo models)

FIGURE 3

mathematical processes if the data are to be acquired directly from the aerial photographs. (See the illustration of this process in Figure 3.)

The early procedures used in bridging employed a stereo plotter of the multiplex type, in which an analogue process was used; however, in the use of stereo plotters for control extension, the degree of accuracy is a limiting factor, because the quality with which the points may be measured is limited by the dexterity of the stereo plotter's operator, and this is generally approximately equivalent to graphical accuracies.

In any aerial triangulation procedure, the results are positions (generally X and Y coordinates and elevations) of points which can be identified on the photograph but which were not surveyed on the ground. These data are then utilized in the orientation of the stereo plotting instruments and in the compilation of maps, just as though survey data had been obtained for these points in the ground control surveys.

The results of the aerial triangulation may be only horizontal coordinates (X and Y values), when vertical control is not needed or when it can be provided satisfactorily by field methods.

Radial Triangulation

One form of aerial triangulation is termed radial triangulation, wherein the horizontal positions of pass points are determined by angle measurements made on the photograph with the angle vertex at the "center" of the photograph.

Radial triangulation has been used in a graphical or mechanical form for many years. (See the illustration of the process in Figures 4 and 5.) The directions, as measured at the photographic center (principal point) of a

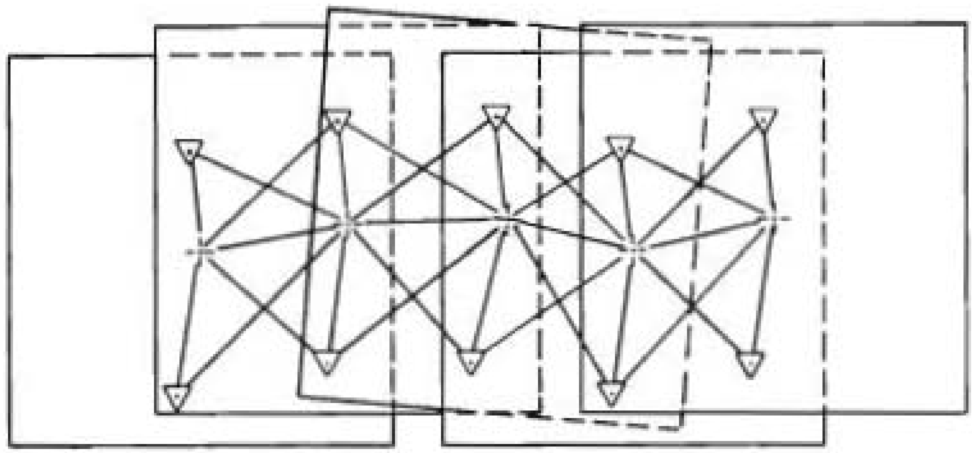

FIGURE 4

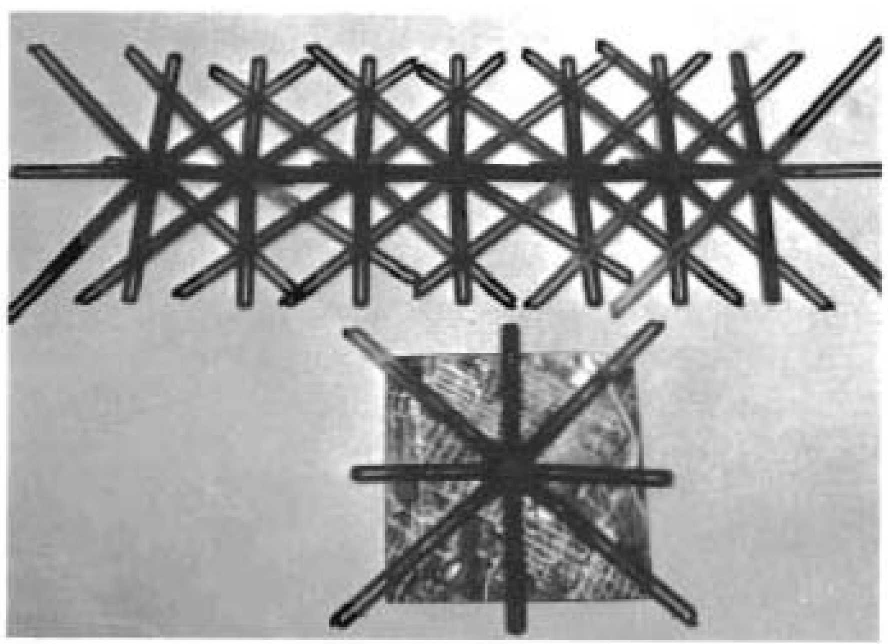

FIGURE 5

truly vertical aerial photograph, are correct or true directions, regardless of (1) the focal length of the camera, (2) the flying height of the aircraft, or (3) the irregularities of the ground being photographed. Thus, it is possible to measure angles from the photographs in the same manner as such angles might be measured on the ground with a surveying instrument. These angles, in turn, can be used in a triangulation process to extend the ground control. Generally speaking, radial triangulation has been used for small-scale photography (and, consequently, for small-scale maps) and for those cases wherein the accuracy desired was relatively low. To accomplish radial triangulation graphically, overlays or radial templets which are limited in accuracy usually have been used.

In the practical use of the radial line plot, the photographs are assumed to be truly vertical, an assumption which may be false in some cases. It is recognized that the photographic angles are not identical to the ground angles if the photographs are not truly vertical, in which case some errors are introduced. When equipment of the type illustrated in Figure 5 is used for radial triangulation, the errors due to the tilting of the photograph and other small errors may be compensated for partially by the fact that the mechanical triangulation equipment is slightly flexible and can be forced to fit the known control points, even if some small errors are present.

Numerical Radial Triangulation

Some attempts have been made for a number of years to use numerical methods for radial triangulation. However, until the development of the high-speed digital computer, the calculations involved in the numerical methods associated with any form of aerial triangulation were so laborious that those who advocated the use of this process soon were discouraged by the time and expense required for reducing the data. Therefore, photogrammetrists turned to other methods for extending control. Since the results obtainable from the stereo plotter are equivalent to (or better than) those obtained from graphical radial triangulation procedures, the question may arise as to whether radial triangulation procedures really are practical. The answer to this question is best summarized in a technical paper (sponsored by Commission 3 of the International Society of Photogrammetry) by Professor R. Roelofs, in which he points out that, when only planimetry and no elevations are to be determined (as, for instance, in the topographic map of a flat area, a cadastral map, or certain other similar requirements), there are some obvious advantages to radial triangulation over aerial triangulation (spatial triangulation). These include the following: (1) relatively inexpensive equipment for obtaining the data, (2) no need for the compensation for radial distortion which may be a problem in some analogue plotters, and (3) a greater speed of operation, which is possible because no relative orientation of the plotter is necessary.

The first consideration should be the accuracy of the data obtained from stereo plotters as compared with the data in numerical methods which may be obtained from either monocular or stereoscopic comparators. For example, if aerial photography at a scale of one inch to five hundred feet (1 in. to 500 ft) is used in a plotter in which the model (as viewed by the operator) would be at a scale of one inch to one hundred feet (1 in. to 100 ft), and if the operator is able to plot a line which is one one-hundredth inch (1/100 in.) in width, this line would represent approximately one foot (1 ft) on the ground. On the other hand, if this same photography at a scale of one inch to five hundred feet (1 in. to 500 ft) is measured in a monocular comparator which is capable of measurements to ± 10 microns, then this value of 10 microns would represent approximately only one-fourth foot (1/4 ft) on the ground. Thus, in some analogue bridging equipment, the width of a plotted line may exceed, by several times, the possible accuracy of measurements which are to be used in numerical procedures. With highly specialized equipment, an accuracy of <u>better</u> than ± 10 microns is possible. Thus, the relative accuracy of the numerical methods over the analogue-plotter methods appears to be significant. In addition to the advantages for radial triangulation cited above, this procedure offers possibilities in other areas besides the extension of control for maps: (1) In the determination of the coordinates at the corners of property surveys which, in turn, may be needed for highway right-of-way acquisition, it is not necessary to have a complete map, but only the horizontal position of the property corner is needed. (2) Likewise, in the preparation of route locations (in the preliminary stages), the location of the center line or references to this line can be obtained by radial triangulation without the preparation of a complete map. (3) A very accurate base map containing relatively few plotted points could be compiled by radial triangulation. The map, in turn, could then be used for many special causes (e.g., locating geological conditions or engineering materials).

Mechanical radial triangulation has been utilized to provide a dense control network for the laying of controlled aerial photographic mosaics. This control network for mosiacs could be determined by numerical methods rather than the mechanical processes, even though this does not appear to be a strong use of the computational process.

Further, as compared to spatial or aerial triangulation, the computations involved in numerical radial triangulation are not as complicated, and the observer needs less experience in order to obtain proficiency.

The photographic measurements for the radial triangulation procedure may be obtained with any of several different types of instruments. Since the data as presented in the calculation are in the form of angles or directions measured from the principal point, the data can be utilized more readily if acquired directly in this form. However, since instruments for measuring angles in this manner usually are not readily available, other types of equipment which can be used to measure coordinates in an orthogonal system are quite satisfactory. The simplest method would be to use a linear measuring device (such as a finely calibrated glass scale), although more sophisticated equipment (such as the heavy plotters and monocular comparators) generally would be more desirable.

Since displacements which result from lens distortion in general and from relief in a vertical photograph are radial from the principal point of the photograph, these displacements have proportional components in any x and y coordinate system in which the origin is at the principal point. Therefore, if x and y coordinate values are measured on a photograph, the angle formed between the axis system and the line from the principal point to the point in question may be computed simply by the tangent equation (see Figure 6).

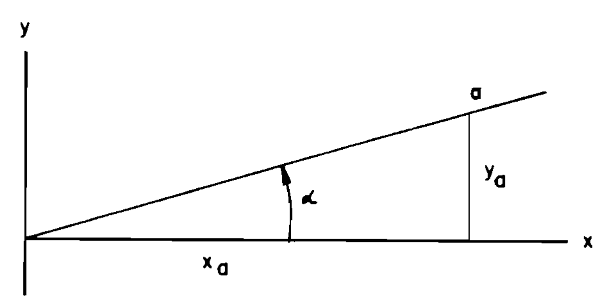

FIGURE 6

Measuring Equipment

For obtaining data to be used in numerical radial triangulation, the instruments listed below should be considered in the following order, depending on the accuracy desired:

1. Glass scale.
2. Coordinatograph.
3. Theodolite coordinate instrument.
4. Monocular comparator.
5. First-order (heavy) stereo plotter.

Glass Scale. A glass scale (or a similar measuring scale) should be considered for obtaining data for numerical radial triangulation because of its relative economy and availability, since more sophisticated measuring equipment often is not readily obtainable. This equipment is limited in accuracy, since the scales normally are calibrated to one-tenth of a millimeter (0.1 mm); however, by utilizing magnifying equipment and repeated measurements, accuracies on the order of ± 10 microns can be obtained. Also, since it is necessary to establish the axis system by drawing lines on the photograph, this procedure is itself a limitation on accuracy.

Coordinatograph. Another type of equipment to be considered is a coordinatograph, which, although designed for drawing, may be used for measuring. This instrument would be particularly valuable if photographic enlargements were available. Some of these instruments are equipped with scales and micrometers to be read to 0.001 of an inch or better and are large enough to accommodate photographs as big as 60 inches by 60 inches. If enlargements are used, the accuracy is effectively increased by the amount of the enlargement.

Theodolite Coordinate Instrument. An intermediate-quality coordinate measuring instrument can be constructed using a one-second (1") theodolite as the principal unit. As is shown in Figure 7, a theodolite is mounted at one end of a supporting frame, and a holder for the photograph is placed at the other end of the frame. The distance from the theodolite to the holder should be slightly more than the minimum focal distance for the theodolite, in order to allow for easy focusing on the photograph. If the photograph holder is so arranged that it can be tilted, then the line of sight from the theodolite to the holder can be made perpendicular to the photograph at any point the operator selects.

Monocular Comparator. A monocular comparator (e.g., the Mann Comparators) is one of the most accurate measuring instruments which can be used in obtaining data for radial triangulation. Accuracies of better than one micron (1 μ) have been reported. This type of equipment is not as commonly used because of its cost and relatively specialized applications.

First-Order (Heavy) Stereo Plotter. A first-order (heavy) stereo plotter (e.g., the Wild A-7 Autograph or the Zeiss C-8 Stereoplanigraph) could be used very effectively for obtaining photograph coordinates for radial triangulation processes. However, since these instruments are designed to be used in aerial (spatial) triangulation and since they are relatively expensive, they are not generally used as data-gathering devices for numerical radial triangulation.

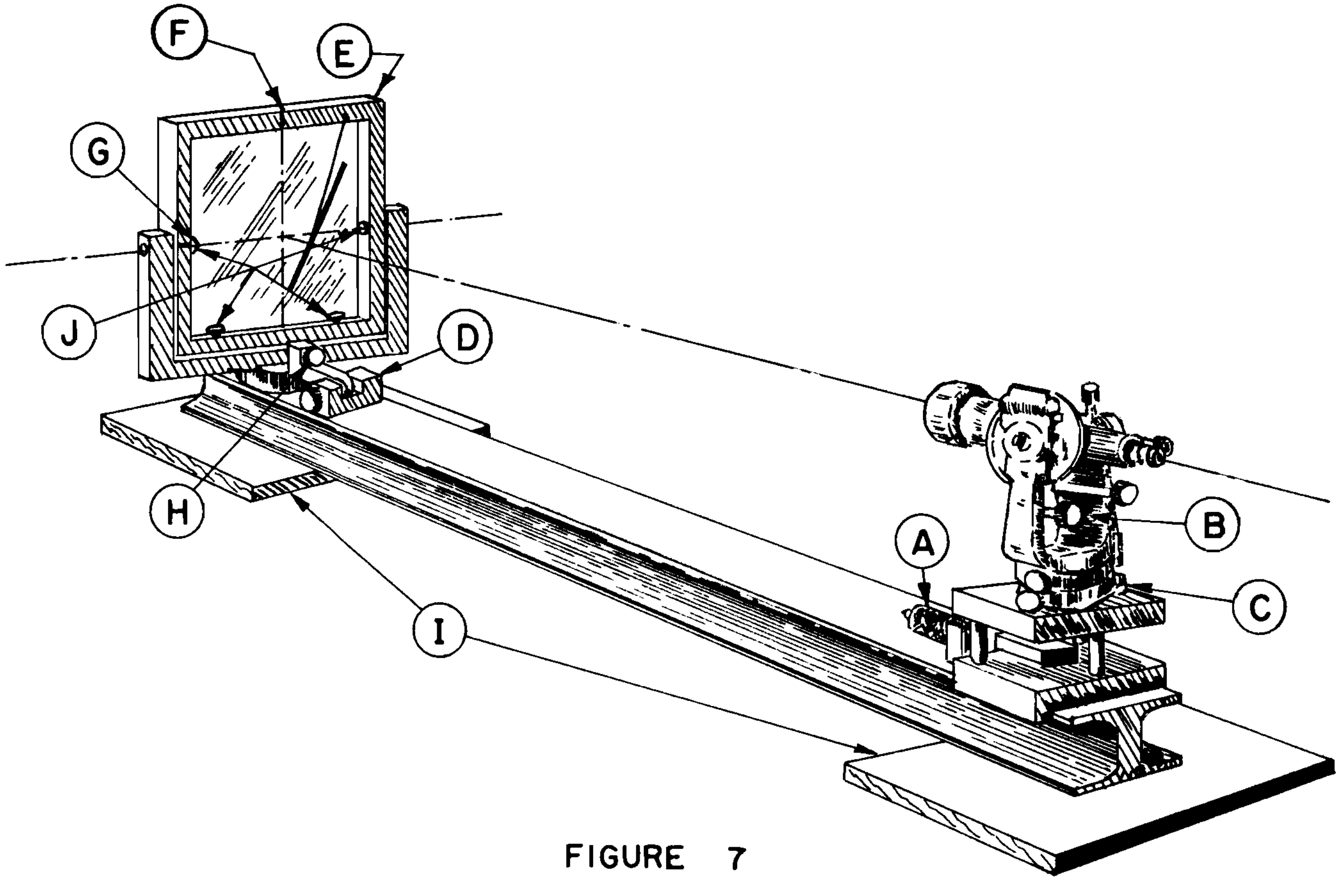

FIGURE 7

RESEARCH PROCEDURE

General Considerations

In the graphical and mechanical procedures for radial triangulation, the principal point of the photograph normally is determined by the intersection of lines connecting opposite fiducial marks. Then, the conjugate principal points of the adjacent photographs are located by stereoscopic transfer of the detail at the principal point. The flight line thus is established on each photograph from the principal point to the conjugate principal point. These flight lines serve as base lines for triangulation from control points to pass points. This procedure requires the transfer of detail by stereoscopic examination. Since it is often difficult to determine the detail at the principal point (for example, if this point happens to lie in the center of a lake), methods have been evolved in this research to eliminate stereoscopic transfer. If three control points are imaged in the overlap area of any two photographs, then the resection principle may be employed to determine the ground coordinates of the principal point of each of these two photographs. One of the basic assumptions underlying this research project has been that the use of the resection principle to determine the coordinates of the principal point is, in general, a more satisfactory procedure than stereoscopic transfer.

Basic Principles

Two basic principles are involved in this radial triangulation process--resection and intersection. If three control points (with horizontal positions known, e.g., the X and Y values in the state plane coordinate system) can be identified on a photograph, then the resection principle can be used to determine the state plane coordinates of the principal point of the photograph by using the angles measured from the principal point to the three control points. By use of control points selected within the overlap area of two photographs, the coordinates of the principal point of each of these two photographs can be determined from these three control points.

Once the ground coordinates of the principal points of the first and second photograph have been determined, these points become the terminals of a base line connecting these two photographs, and, if angles are measured from the principal points to other points identifiable on both photographs 1 and 2, the ground coordinates of these other points then may be computed by the intersection principle. If three such points are selected so that they are in the overlap area of the two photographs and also in the overlap area for a third photograph, the coordinates of these pass points may be used with the resection principle to determine the ground coordinates of the principal point of photograph 3. Then, with the ground coordinates of the principal points of photographs 2 and 3 known, these, in turn, can be used to determine the position of additional pass points which would lie in the overlap area between photographs 2, 3, and 4. This process can be continued indefinitely by successively computing the position of pass points in the overlap area and then extending to the principal point of the next photograph. However, since errors are always present (and possibly mistakes also), checks and compensation are necessary.

Equipment Used in the Research Project

In the preceding section on theoretical background, five types of measuring equipment were described: (1) glass scale, (2) coordinatograph,

(3) theodolite coordinate instrument, (4) monocular comparator, and (5) first-order (heavy) stereo plotter. These were listed approximately in order of increasing accuracy; however, any of those discussed might be used for obtaining data for numerical radial triangulation.

Type of Instrument. For the tests conducted in this research, an instrument capable of an intermediate quality of measuring accuracy was used. This instrument, which had been built by technicians at The University of Texas prior to this research project, could be described as a theodolite coordinate measuring instrument (see Figure 7 for a schematic diagram of this instrument), a detailed description of which is given in the following section.

Description of Instrument. The essential parts of this measuring instrument include a precision theodolite (item B in Figure 7) and a holder for the photograph (item E in Figure 7). These two parts are mounted on a base so that the distance from the theodolite to the holder is slightly greater than the minimum focal distance for the theodolite. This distance may be adjusted with a slow-motion screw (item A in Figure 7), and the orientation of the holder may be adjusted with screws D and H (see Figure 7).

In operation, the theodolite is placed on the stand as shown in the figure. The lines (F and G) are located in the field of view of the telescope, and angular readings are taken for each line. Then, an average of each set of readings is placed on the circles of the instrument, which should give the axes center of the frame (E). A mirror is placed in the center of the frame, and the telescope is focused so that a reflection of the instrument can be seen in the eyepiece. The image in the mirror is moved by the knobs

(D and H) until the image of the telescope is symmetrical about the cross hairs in the telescope. The mirror is removed, and a photographic plate is placed in the frame, so that the two side fiducial marks are on the line (G), and the top and bottom fiducial marks are on the line (F).

With the plate in place, readings are taken, using the fiducial marks as the new axes, and these values are used for the final position of the principal point. Readings then are taken on each control point and each pass point which is imaged on the plate.

It should be recognized that the tangent of the angle measured between the photographic axis and a point is a measure of the coordinate of the point. (A sample data sheet is included as Table 1.)

The theodolite coordinate instrument is potentially very accurate when a procedure is followed which will minimize the errors inherent in its use. Tests conducted in other research projects have shown that the micrometers on some theodolites are subject to a systematic error of several seconds. Thus, for the maximum accuracy, corrections must be applied to compensate for such errors. Also, an observing procedure should be followed which will minimize operator errors and provide for an averaging of several observations.

In the course of this research, a detailed procedure was used with this instrument, and it was found possible to secure measurements with a maximum error of the order of ± 5 microns. To achieve this level of accuracy, it was necessary to carry out the procedure with extreme care. However, since the prime objective in this study was to develop the program rather than to test the refinement of the equipment, the measurements actually utilized were made with a maximum error on the order of ± 40 microns.

Table 1

SAMPLE DATA SHEET

Point	Axis	1st Reading	2nd Reading	3rd Reading	4th Reading	5th Reading	Average	Difference
Reference Point	Horizontal	124°28'57"	124°28'57"	124°28'59"	124°29'01"	124°28'58"	124°28'58"	---
	Vertical	88°21'09"	88°21'08"	88°21'10"	88°21'08"	88°21'08"	88°21'09"	---
P	Horizontal	126°56'48"	126°56'48"	126°56'48"	126°56'49"	126°56'51"	126°56'49"	+2°30'51"
	Vertical	87°03'28"	87°03'27"	87°03'29"	87°03'29"	87°03'27"	87°03'28"	-1°17'41"
Q	Horizontal	121°26'21"	121°26'24"	121°26'19"	121°26'18"	121°26'15"	121°26'19"	-3°02'39"
	Vertical	88°11'23"	88°11'25"	88°11'28"	88°11'26"	88°11'28"	88°11'26"	-0°09'43"
R	Horizontal	127°16'04"	127°16'00"	127°16'04"	127°16'01"	127°16'06"	127°16'03"	+2°47'05"
	Vertical	88°59'34"	88°59'32"	88°59'37"	88°59'40"	88°59'40"	88°59'37"	+0°38'28"
S	Horizontal	125°07'00"	125°07'01"	125°07'02"	125°07'04"	125°07'04"	125°07'02"	+0°38'04"
	Vertical	89°28'22"	89°28'23"	89°28'24"	89°28'27"	89°28'27"	89°28'25"	+1°07'16"

Calculations

An outline of the procedure is given below, followed by a set of sample calculations.

1. Identify and mark the principal point and at least three control points with known horizontal coordinates in photographs 1 and 2 of a flight strip.

2. Identify and mark three pass points in the overlap area between photographs 1, 2, and 3 on the first three photographs of the flight strip.

3. Measure the coordinates on each photograph of the control points and of the pass points using the principal point as the origin of the coordinates on each photograph.

4. Convert the measured coordinates into angles or directions from the principal point to each of the control points and of the pass points.

5. By the resection principle, solve for the ground coordinates of the principal point of photograph 1 and the principal point of photograph 2, using the angles at the principal points and the known horizontal dimensions for the three control points.

6. Using these computed ground coordinates for the principal points of photograph 1 and photograph 2, apply the intersection principle to determine the ground coordinates for the pass points for photographs 1, 2, and 3.

7. Using the ground coordinates for the pass points as determined in step 6, apply the resection principle to determine the ground coordinates of the principal point of photograph 3.

The following explanation and equations have been used in this procedure for extending horizontal control (see Figure 8). In Figure 8, points 1, 2, 3 . . . represent the principal points on successive photographs. Points A, B, and C represent the images of the known control points, and points D, E, F . . . represent the images of the pass points. If the photographic coordinates of A are x_a and y_a, the angle between the y-axis and line $\overline{1A}$ (symbolized as $\angle A$) may be calculated as:

$$\angle A = \tan^{-1} \left(\frac{x_a}{y_a} \right)$$

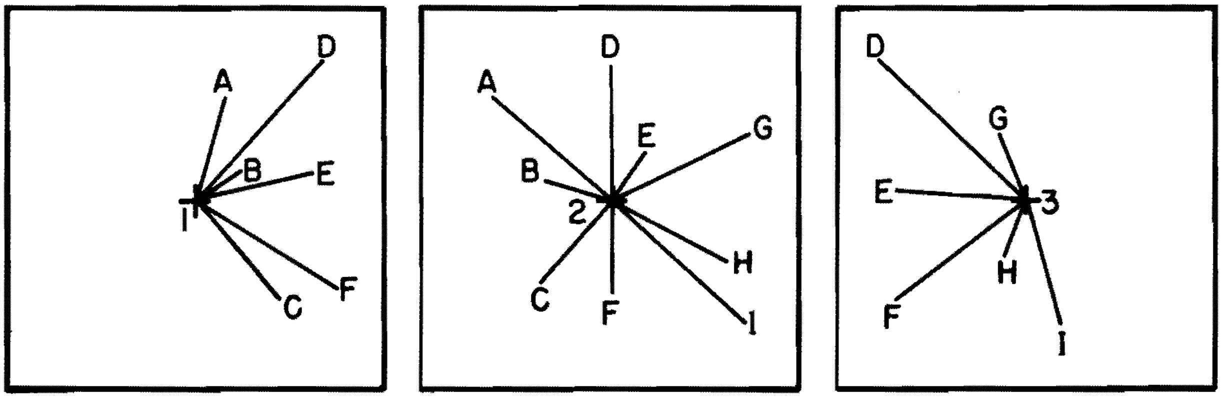

FIGURE 8

Similarly, if the photographic coordinates of B are x_b and y_b, the angle between the y-axis and line $\overline{1B}$ (symbolized as $\angle B$) may be determined as follows:

$$\angle B = \tan^{-1} \left(\frac{x_b}{y_b} \right)$$

Then, $\angle A1B$ can be computed by combining $\angle A$ and $\angle B$. In a similar manner, other angles (such as $\angle B1C$, $\angle A1D$, $\angle A1E$, and $\angle A1F$) may be determined. The same procedure then can be used to find the angles on photograph 2 (for $\angle A2B$, $\angle B2C$, $\angle A2D$, $\angle A2E$, and $\angle A2F$).

Based on known control data, the distances and directions between the control points can be determined as follows:

$$\overline{AB} = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$$

$$\overline{BC} = \sqrt{(X_b - X_c)^2 + (Y_b - Y_c)^2}$$

The angle between the Y-axis (in the ground survey) and the line $\overline{AB}$ symbolized $\angle\alpha$ can be calculated as follows:

$$\angle\alpha = \tan^{-1} \left(\frac{X_a - X_b}{Y_a - Y_b} \right)$$

The angle between the Y-axis (in the ground survey) and the line $\overline{BC}$ symbolized $\angle\beta$ then is found to be:

$$\angle\beta = \tan^{-1} \left(\frac{X_b - X_c}{Y_b - Y_c} \right)$$

The combination of $\angle\alpha$ and $\angle\beta$ then will yield $\angle ABC$.

By means of the angles and directions, as found on the photographs and the dimensions connecting the control points, the coordinates of the ground position of the principal points can be computed as follows:

$$\cot \angle 1AB = \frac{\overline{AB} \sin \angle B1C}{\overline{BC} \sin \angle A1B \sin (\angle 1AB + \angle 1CB)} + \cot (\angle 1AB + \angle 1CB)$$

Where: $(\angle 1AB + \angle 1CB) = 360^{o} - \angle A1B - \angle B1C - \angle ABC$

Then $\overline{1B} = \overline{AB} \dfrac{\sin \angle 1AB}{\sin \angle A1B}$

The azimuth of $\overline{1B}$ (or of $\overline{B1}$) can be found by combining the azimuth of $\overline{AB}$ and $\angle AB1$.

$$X_1 = X_b + \overline{B1} \sin (\text{azimuth of } \overline{1B})$$

$$Y_1 = Y_b + \overline{B1} \cos (\text{azimuth of } \overline{1B})$$

The coordinates of X_2 and Y_2 can be computed in a similar manner, from the data in photograph 2. From the values for X_1, Y_1, X_2, and Y_2 and the azimuths of lines $\overline{1D}$, $\overline{2D}$, $\overline{1E}$, $\overline{2E}$, $\overline{1F}$, and $\overline{2F}$, the coordinates of points D, E, and F can be determined from the following equations:

$$X_1 + \overline{1D} \sin (\text{azimuth of } \overline{1D}) = X_2 + \overline{2D} \sin (\text{azimuth of } \overline{2D})$$

$$Y_1 + \overline{1D} \cos (\text{azimuth of } \overline{1D}) = Y_2 + \overline{2D} \cos (\text{azimuth of } \overline{2D})$$

The simultaneous solution of the above equations will yield values for $\overline{1D}$ and $\overline{2D}$. Then

$$X_d = X_1 + \overline{1D} \sin (\text{azimuth of } \overline{1D})$$

$$Y_d = Y_1 + \overline{1D} \cos (\text{azimuth of } \overline{1D})$$

A similar procedure can be employed to yield values for X_e, Y_e, X_f, and Y_f.

When these steps have been carried out, one cycle of the calculations will have been completed. Then, the values for the ground coordinates of points D, E, and F may be used as control points so that the resection principle can be applied to find X_3 and Y_3.

The procedure outlined and illustrated above may be repeated in order to determine the principal points and the pass points in successive photographs along the flight line. Since the procedure itself is repetitive in nature, the process has been programmed for computation on the CDC-1604 high-speed digital computer.

The ideal spacing of control points and pass points, as shown in the outline, is not always possible, but this outlined procedure may be used if a solution is possible for both the resection and intersection procedures.

The relationship of control points and pass points on photograph 156 and photograph 157 are shown in Figure 9. In the following calculations, principal point 1 is for photograph 157, and principal point 2 is for photograph 156. The sample calculations are for principal point 2.

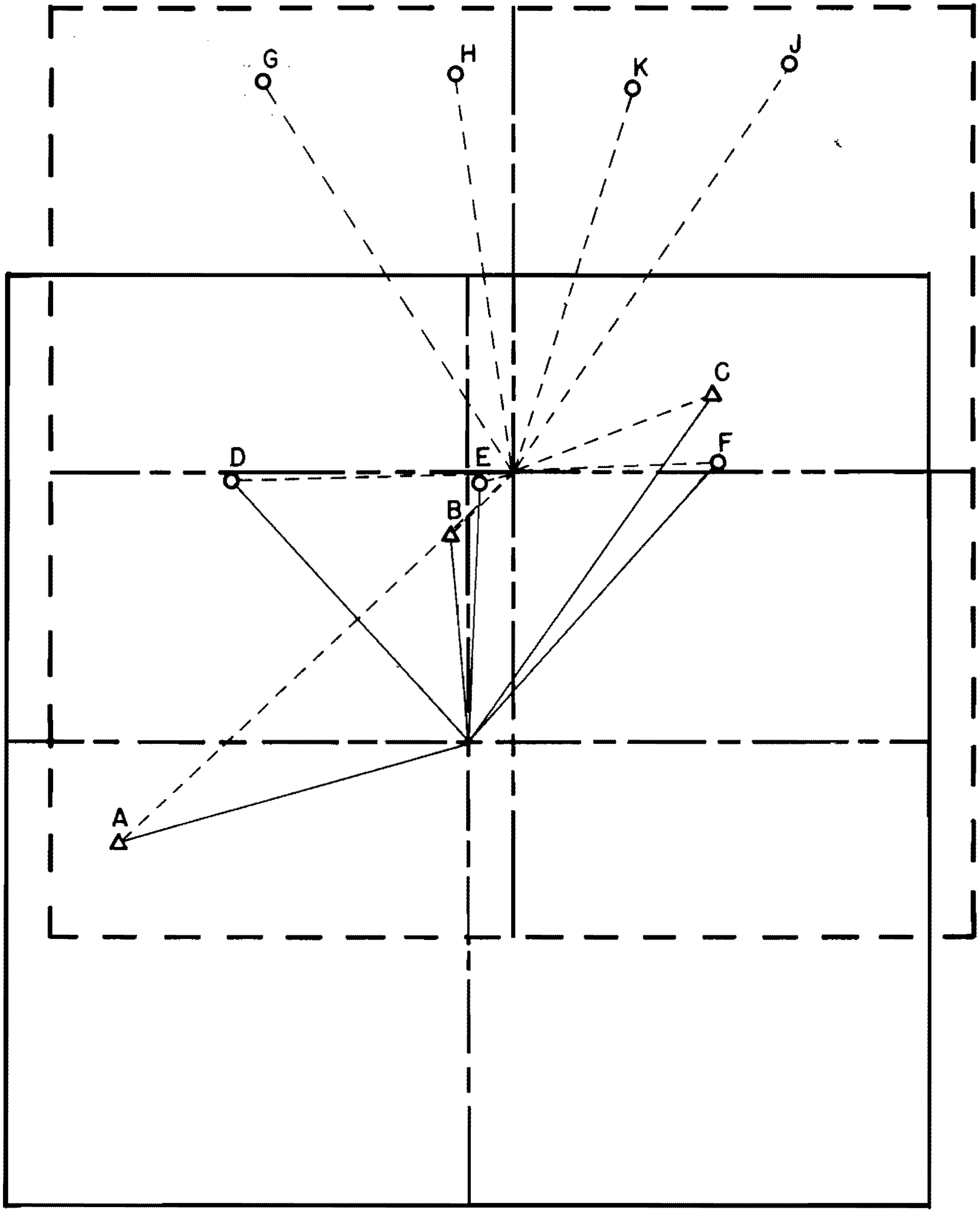

FIGURE 9

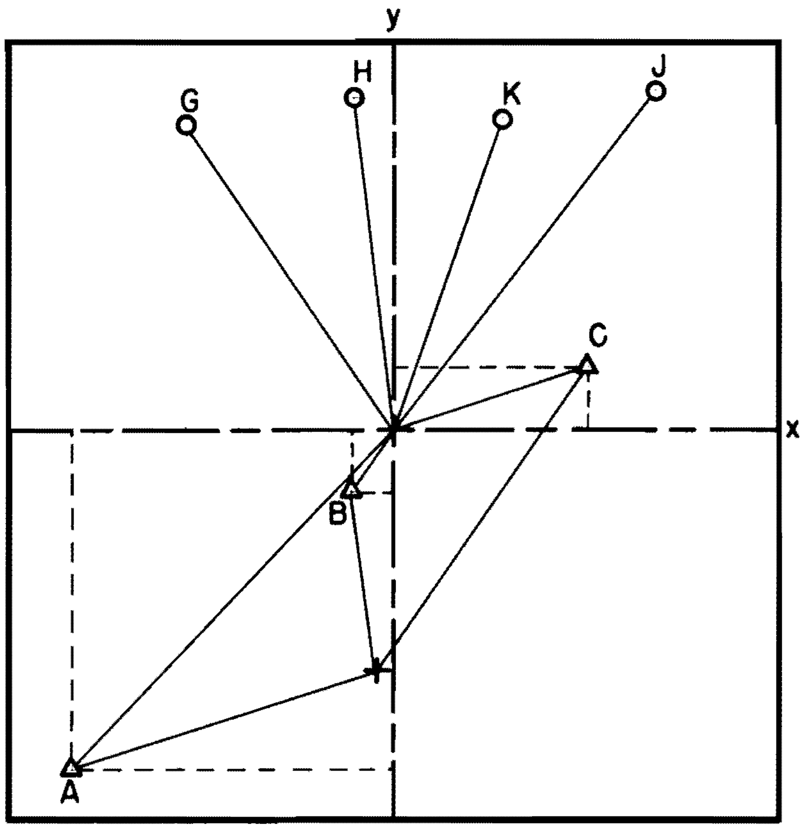

FIGURE 10

Points A, B, and C are control points for which the known coordinates are:

X_A = 815,285.12

Y_A = 227,631.31

X_B = 818,557.76

Y_B = 230,594.42

X_C = 821,026.06

Y_C = 232,041.68

Based on measured photograph coordinates, the angles can be found as follows:

$$\tan \angle A = \frac{95.935}{102.903} = 0.9322857, \text{ where } x_a = 102.903,\; y_a = 95.935$$

$$\angle A = 42^{\circ}59'35''$$

$$\tan \angle B = \frac{18.689}{13.424} = 1.3922080, \text{ where } x_b = 13.424,\; y_b = 18.689$$

$$\angle B = 54^{\circ}18'39''$$

$$\tan \angle C = \frac{18.301}{52.861} = 0.3462099, \text{ where } x_c = 52.861,\; y_c = 18.301$$

$$\angle C = 19^{\circ}05'47''$$

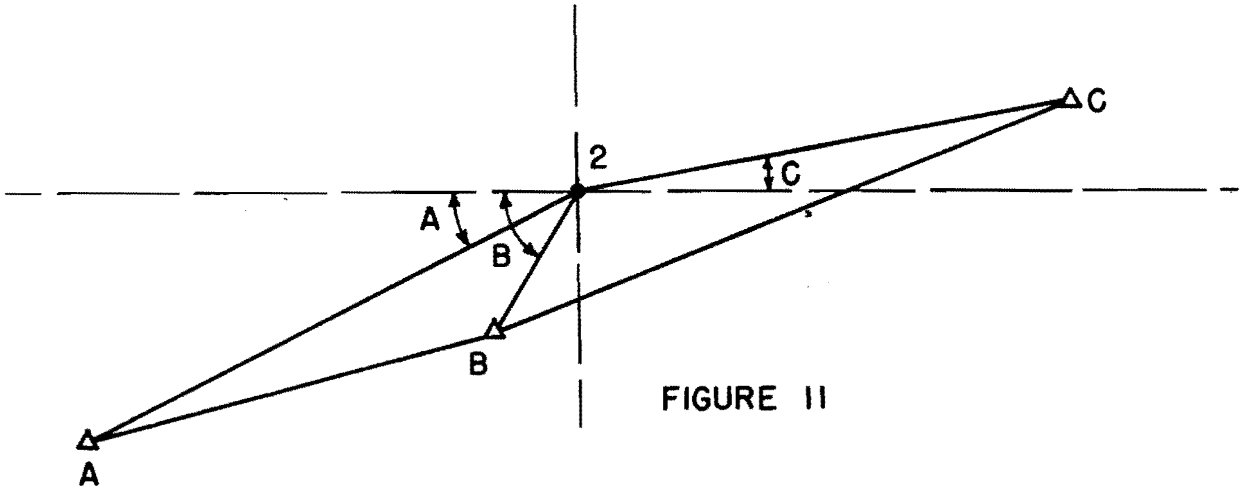

FIGURE 11

$\angle A2B = \angle B - \angle A = 54^\circ 18'39'' - 42^\circ 59'35'' = 11^\circ 19'04''$

$90^\circ - \angle B = 89^\circ 59'60'' - 54^\circ 18'39'' = 35^\circ 41'21''$

$\angle B2C = 19^\circ 05'47'' + 90^\circ + 35^\circ 41'21'' = 144^\circ 47'08''$

Based on the known ground coordinates, the following distances and angles can be computed as follows:

$$AB = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}$$

$$= \sqrt{(815{,}285.12 - 818{,}557.76)^2 + (227{,}631.31 - 230{,}594.42)^2}$$

$$= 4{,}414.77$$

$$BC = \sqrt{(X_B - X_C)^2 + (Y_B - Y_C)^2}$$

$$= \sqrt{(818{,}557.76 - 821{,}026.06)^2 + (230{,}594.42 - 232{,}041.68)^2}$$

$$= 2{,}861.31$$

$$\angle ABC = \sin^{-1} \frac{Y_B - Y_A}{\overline{AB}} + 180^\circ - \sin^{-1} \frac{Y_C - Y_B}{\overline{BC}}$$

$$= \sin^{-1} \frac{2{,}963.11}{4{,}414.77} + 180^\circ - \sin^{-1} \frac{1{,}447.26}{2{,}861.31}$$

$$= 42^\circ 09'30'' + 180^\circ - 30^\circ 23'05'' = 191^\circ 46'25''$$

$$\angle 2AB + \angle 2CB = 359^\circ 59'60'' - 11^\circ 19'04'' - 144^\circ 47'08'' - 191^\circ 46'25''$$

$$= 12^\circ 07'23''$$

Define: $\angle R = \angle 2AB + \angle 2CB$

With these distances and angles known, the resection principle can be applied to determine $\angle 2AB$ and $\angle 2CB$ as follows:

$$\cot \angle 2AB = \frac{\overline{AB} \sin \angle B2C}{\overline{BC} \sin \angle B2A \sin \angle R} + \cot \angle R$$

$$= \frac{(4{,}414.77)\ (0.576638)}{(2{,}861.31)\ (0.196250)\ (0.210012)} + 4.655442$$

$$= 26.24236$$

$$\angle 2AB = 2^\circ 10' 56''$$

$$\cot \angle 2CB = \frac{\overline{BC} \sin \angle A2B}{\overline{AB} \sin \angle B2C \sin \angle R} + \cot \angle R$$

$$= \frac{(2{,}861.31)\ (0.196250)}{(4{,}414.77)\ (0.576638)\ (0.210012)} + 4.655442$$

$$= 5.705757$$

$$\angle 2CB = 9^\circ 56' 27''$$

At this point, an arithmetic check may be applied as $\angle 2AB + \angle 2CB = \angle R$.

$\angle 2AB + \angle 2CB$ should equal $12^\circ 07' 23''$

$2^\circ 10' 56'' + 9^\circ 56' 27'' = 12^\circ 07' 23''$

With all the angles and two of the distances known, the distance from a control point to the principal point can be calculated as follows:

$$\frac{\overline{AB}}{\sin \angle A2B} = \frac{\overline{2B}}{\sin \angle 2AB} \quad \text{or} \quad \frac{\overline{BC}}{\sin \angle B2C} = \frac{\overline{2B}}{\sin \angle 2CB}$$

$$\overline{2B} = 4{,}414.77\ \frac{0.038078}{0.196250} \quad \text{or} \quad \overline{2B} = 2{,}861.31\ \frac{0.172631}{0.576638}$$

$$= 856.58 \qquad\qquad = 856.60$$

Use 856.59

Finally, the coordinates of the principal point can be found as follows:

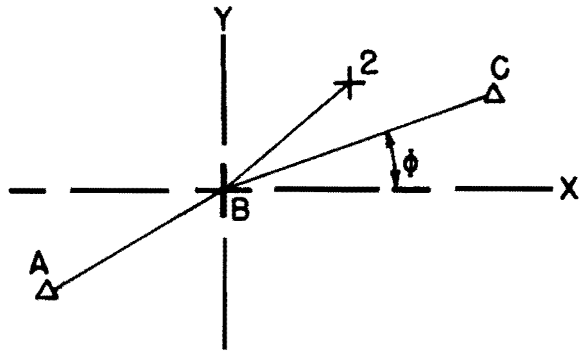

FIGURE 12

$\angle 2BC = 180^{o} - \angle B2C - \angle 2CB = 25^{o}16'25''$

$\angle \phi = 30^{o}23'05''$

(See calculation of $\angle ABC$)

$Az.\overline{B2} = 90^{o} - 55^{o}39'30'' = 34^{o}20'30''$

$X_2 = X_B + \overline{2B} \sin (Az.\overline{B2}) = 818,557.76 + (856.59)\ (0.561267)$

$= 819,038.54$

$Y_2 = Y_B + \overline{2B} \cos (Az.\overline{B2}) = 230,594.42 + (856.59)\ (0.825688)$

$= 231,301.70$

By means of a similar procedure, the ground coordinates of principal point 1 were found to be:

$X_1 = 818,710.65$

$Y_1 = 228,654.15$

With the ground coordinates of the principal point of both photographs 1 and 2 known, the direction from one of the original control points to each of the principal points could be obtained. Also, the angle formed between one of the original control points, the principal point of the photograph, and a pass point selected in the area of overlap between photographs 1, 2, and 3, could be measured from the photograph. (See the sample calculation which follows.) If the coordinates of the principal points of photograph 1 and photograph 2 and the directions to another point were known, the intersection principle could be employed to determine the ground coordinates of this new pass point. If three of these pass points were so selected that they were imaged on both photograph 1 and photograph 2 and so that their coordinates might be determined and also imaged on photograph 3, then they could, in turn, be used again with the resection principle to determine the coordinates of the principal point of photograph 3.

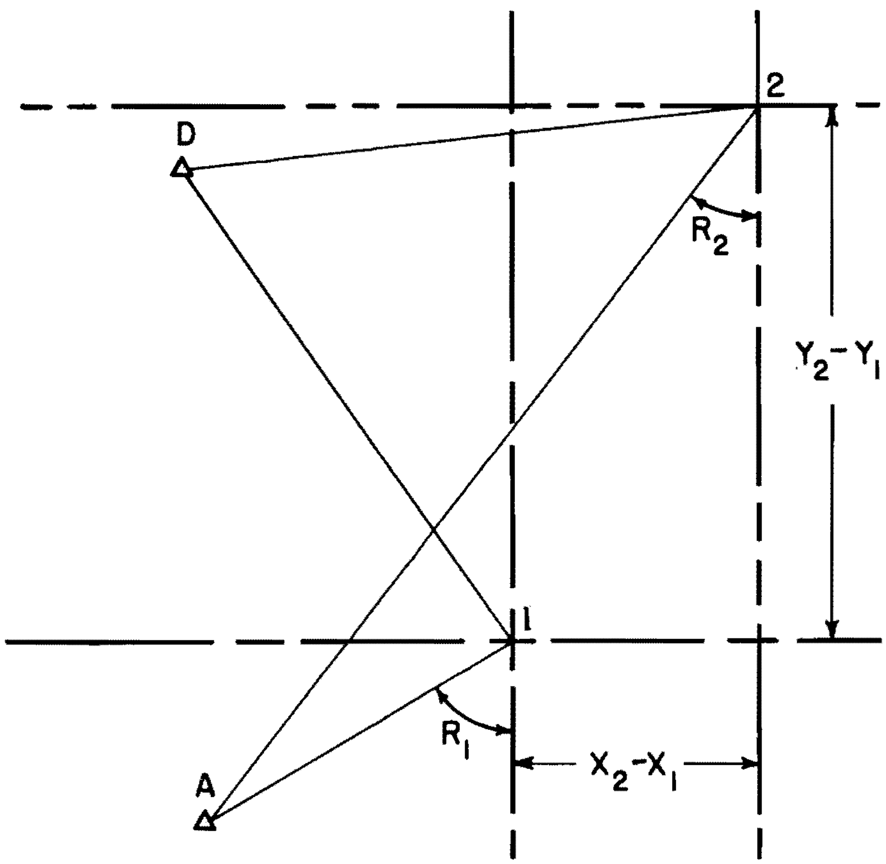

FIGURE 13

If the differences in ground coordinates (as shown in Figure 13) are used, angles R_1 and R_2 can be found as follows:

$$\tan \angle R_1 = \frac{X_1 - X_A}{Y_1 - Y_A} = \frac{3,425.53}{1,022.84} = 3.349037 \qquad \angle R_1 = 73^\circ 22' 30''$$

$$\tan \angle R_2 = \frac{X_2 - X_A}{Y_2 - Y_A} = \frac{3,753.42}{3,670.39} = 1.022621 \qquad \angle R_2 = 45^\circ 38' 30''$$

Then, from photo coordinates, the bearings of lines 1D and 2D can be determined as shown below:

$$\angle A1D = \tan^{-1} \frac{y_1 - y_a}{x_1 - x_a} + \tan^{-1} \frac{y_d - y_1}{x_d - x_1}$$

$$= \tan^{-1} \frac{25.608}{90.571} + \tan^{-1} \frac{67.533}{60.887}$$

$$= \tan^{-1} (0.282739) + \tan^{-1} (1.109153)$$

$$= 15^\circ 47' 16'' + 47^\circ 57' 45'' = 63^\circ 45' 01''$$

$$180^\circ - \angle R_1 - \angle A1D = \text{Bearing of } \overline{1D}$$

$$180^\circ - 73^\circ 22' 30'' - 63^\circ 45' 01'' = \text{N } 42^\circ 53' 29'' \text{ W}$$

Again, from photo coordinates, the following calculations can be made:

$$\angle A2D = \tan^{-1} \frac{y_2 - y_a}{x_2 - x_a} - \tan^{-1} \frac{y_2 - y_d}{x_2 - x_d}$$

$$= \tan^{-1} \frac{95.935}{102.903} - \tan^{-1} \frac{2.100}{70.864}$$

$$= \tan^{-1} (0.932285) - \tan^{-1} (0.029634)$$

$$= 42^\circ 59' 35'' - 1^\circ 41' 51''$$

$$\angle A2D = 41^\circ 17' 44''$$

Bearing of $\overline{2D}$ = $\angle R_2$ + $\angle 2AD$

= $45^{\circ}38'30''$ + $41^{\circ}17'44''$

= S $86^{\circ}56'14''$ W

Finally, the coordinates of the pass points can be found, as follows:

$$X_2 - X_1 = 1_2 \sin 86^{\circ}56'14'' - 1_1 \sin 42^{\circ}53'29''$$

$$Y_2 - Y_1 = 1_2 \cos 86^{\circ}56'14'' + 1_1 \cos 42^{\circ}53'29''$$

$$X_2 - X_1 = 819{,}038.54 - 818{,}710.65 = 327.89$$

$$Y_2 - Y_1 = 231{,}301.70 - 228{,}654.15 = 2{,}647.55$$

$$327.89 = 1_2\ (0.998572) - 1_1\ (0.680611)$$

$$2{,}647.55 = 1_2\ (0.053430) + 1_1\ (0.732645)$$

If these two equations are solved simultaneously, they will yield the following values:

$1_1 = 3{,}419.75$ $\qquad$ $1_2 = 2{,}659.21$

$$X_D = X_1 - 1_1 \sin 42^{\circ}53'29''$$

$$= 818{,}710.65 - 3{,}419.75\ (0.680611) = 816{,}383.13$$

$$Y_D = Y_1 + 1_1 \cos 42^{\circ}53'29''$$

$$= 228{,}654.15 + 3{,}419.75\ (0.732645) = 231{,}159.61$$

or

$$X_D = X_2 - 1_2 \sin 86^{\circ}56'14''$$

$$= 819{,}038.54 - 2{,}659.21\ (0.998572) = 816{,}383.13$$

$$Y_D = Y_2 - 1_2 \cos 86^{\circ}56'14''$$

$$= 231{,}301.70 - 2{,}659.21\ (0.053430) = 231{,}159.62$$

COMPUTER PROGRAM

On the following pages is a copy of the listing of the computer program used to solve the equations employed in this research on numerical radial triangulation.

DEFINITION OF INPUT AND OUTPUT TERMS

IPRJ1 IPRJ2 IPRJ3	Job identification
N	Number of photographs in the strip
PHO	Photograph identification
XIA XIB	Point identification
PX (input)	Known ground x-coordinate
PY (input)	Known ground y-coordinate
X	Photograph x-coordinate*
Y	Photograph y-coordinate*
PPX	Ground x-coordinate of principal point
PPY	Ground y-coordinate of principal point
PX (output)	Computed ground x-coordinate
PY (output)	Computed ground y-coordinate
XSET	Known ground x-coordinate
YSET	Known ground y-coordinate
EX	Error in ground x-coordinate
EY	Error in ground y-coordinate

*If linear x and y photograph coordinates are to be used, one statement before 81 and statement 81 should be deleted and the statement "81 CONTINUE" should be inserted.

```
-COOP,CE171650,TURPIN,S/2S,02,600.
-FTN,L,E,R,N.
      PROGRAM GEODET
      DIMENSION PHO(10),XIA(9,10),XIB(9,10),PX(9,10),PY(9,10),
     1          X(9,10),Y(9,10),ANG(6),CHI(2),XLEN(2),DX(2),
     2          DY(2),PI(2),PEAK(3),SID(3),SED(3),BET(10),
     3          ALPH(3,10),GAM(3,10),XX(3),YY(3),PPX(10),
     4          PPY(10),XSET(9,10),YSET(9,10)
  999 ITEST=4H
      READ 400,IPRJ1,IPRJ2,IPRJ3,N
  400 FORMAT (3A4,5X,I3,9X)
      IF(IPRJ1-ITEST)998,997,998
  998 CALL INDAT(N,PHO,XIA,XIB,PX,PY,X,Y,XSET,YSET)
      NN=0
  117 NN=NN+1
      NM=NN+1
      IF(NM-3)71,70,70
   70 I=NM
      IF(NM-N)72,72,996
   71 IF(NM-N)69,69,996
  996 CALL OUTDAT(IPRJ1,IPRJ2,IPRJ3,PX,PY,XSET,YSET,XIA,XIB,
     1              PHO,N,PPX,PPY)
      GO TO 999
   69 DO 68 I=NN,NM
   72 DO 50 JA=1,3
      XX(JA)=X(JA,I)
   50 YY(JA)=Y(JA,I)
      CALL AZIMUTH(XX,YY,ANG,1,3)
      CALL SWITCH(X,Y,PX,PY,XSET,YSET,XIA,XIB,ANG,1,2,I)
      K=2
      DO 55 J=1,K
      JM=J+1
      CHI(J)=ANG(J)-ANG(JM)
      IF(CHI(J)-0.0523599)129,129,55
  129 IT=J+1
      PRINT 130,XIA(J,I),XIB(J,I),XIA(IT,I),XIB(IT,I),PHO(I)
  130 FORMAT (/ 4X,31HCENTRAL ANGLE BETWEEN STATIONS ,2A4,1X,
     14HAND ,2A4,1X,10HIS SMALLER / 4X,19HTHAN ALLOWABLE FOR ,
     238HCOMPUTING PRINCIPAL POINT, PHOTOGRAPH ,A4,1X,1H. / 4X,
     344HPLEASE SELECT A NEW POINT AND RESUBMIT DATA /)
   55 CONTINUE

C     COMPUTATION FOR INTERIOR ANGLES.

      K=2
      DO 18 J=1,K
      JK=J+1
      XLEN(J)=SQRTF((PX(J,I)-PX(JK,I))**2+(PY(J,I)-PY(JK,I))**2)
      DX(J)=PX(J,I)-PX(JK,I)
   18 DY(J)=PY(J,I)-PY(JK,I)
      CALL AZIMUTH(DX,DY,PI,1,2)
```

```
      IF(PI(1)-PI(2))31,32,31
   31 AKAP=3.1415927+PI(2)-PI(1)
      IF(AKAP-6.2831853)39,38,38
   38 AKAP=AKAP-6.2831853
   39 IF(AKAP)40,33,33
   40 AKAP=6.2831853+AKAP
      GO TO 33
   32 AKAP=3.1415927
   33 RHO=6.2831853-(AKAP+CHI(1)+CHI(2))
      CTRHO=COSF(RHO)/SINF(RHO)
      PRINT 1234,CTRHO
 1234 FORMAT(5X,E14.6)
      TANM=1.0/(XLEN(2)*SINF(CHI(1))/(XLEN(1)*SINF(CHI(2))*SINF
     1     (RHO))+CTRHO)
      TANL=1.0/(XLEN(1)*SINF(CHI(2))/(XLEN(2)*SINF(CHI(1))*SINF
     1     (RHO))+CTRHO)
      ANGM=ATANF(TANM)
      ANGL=ATANF(TANL)
      IF(ANGL)119,120,120
  119 ANGL=3.1415927+ANGL
  120 IF(ANGM)121,122,122
  121 ANGM=3.1415927+ANGM
  122 CONTINUE

C     COMPUTATION FOR STATE PLANE COORDINATES OF PRINCIPAL POINT

      B11=XLEN(1)*SINF(ANGL)/SINF(CHI(1))
      B12=XLEN(2)*SINF(ANGM)/SINF(CHI(2))
      DIX=ABSF(B11-B12)
      IF(DIX-0.0001*B11)34,34,35
   34 B=(B11+B12)/2.0
      GO TO 36
   35 PRINT 106,XIA(2,I),XIB(2,I),I,B11,B12
  106 FORMAT (/ 4X,28HDIFFERENCE IN INTERIOR SIDE ,2A4,1H-,I3,
     11X,18HEXCEEDS ALLOWABLE. / 4X,24HPLEASE CHECK DATA AND RE
     27HSUBMIT. / 4X,F12.2,6X,F12.2 /)
   36 CONTINUE
      ANGN=3.1415927-(CHI(1)+ANGL)+PI(1)
      PPX(I)=PX(2,I)+B*COSF(ANGN)
      PPY(I)=PY(2,I)+B*SINF(ANGN)

C     REVOLUTION OF RELATIVE COORDINATE SYSTEM TO ALIGN WITH
C                          STATE PLANE SYSTEM

      XX(2)=PX(1,I)-PPX(I)
      YY(2)=PY(1,I)-PPY(I)
      CALL AZIMUTH(XX,YY,ANG,2,2)
      ROT=ANG(1)-ANG(2)
      IF(I-1)125,125,123
  123 IF(I-N)126,125,125
  126 DO 124 J=4,9
```

```
      TX=X(J,I)
      TY=Y(J,I)
      X(J,I)=TX*COSF(ROT)+TY*SINF(ROT)
      Y(J,I)=TY*COSF(ROT)-TX*SINF(ROT)
  124 CONTINUE
      GO TO 127
  125 CONTINUE
      DO 67 J=4,6
      TX=X(J,I)
      TY=Y(J,I)
      X(J,I)=TX*COSF(ROT)+TY*SINF(ROT)
   67 Y(J,I)=TY*COSF(ROT)-TX*SINF(ROT)
  127 CONTINUE
   68 CONTINUE
C     COMPUTATION FOR STATE PLANE COORDINATES OF REFERENCE
C                          STATIONS
      XX(1)=PPX(NM)-PPX(NN)
      YY(1)=PPY(NM)-PPY(NN)
      XLENA=SQRTF(XX(1)**2+YY(1)**2)
      CALL AZIMUTH(XX,YY,ANG,1,1)
      BET(NN)=ANG(1)
      IF(BET(NN)-3.1415927)85,86,86
   85 BET(NM)=BET(NN)+3.1415927
      IL=0
      GO TO 87
   86 BET(NM)=BET(NN)-3.1415927
      IL=0
   87 DO 108 IJ=NN,NM
      NK=0
      IF(NN-1)88,88,89
   88 IK1=4
      IK2=6
      GO TO 91
   89 IF(IL)88,90,88
   90 IK1=7
      IK2=9
   91 IL=IL+1
      DO 108 IK=IK1,IK2
      NK=NK+1
      XX(NK)=X(IK,IJ)
      YY(NK)=Y(IK,IJ)
      CALL AZIMUTH(XX,YY,ANG,NK,NK)
      ALPH(NK,IJ)=ANG(NK)
      IF(ABSF(BET(IJ)-ALPH(NK,IJ))-3.1415927)100,100,101
  100 GAM(NK,IJ)=ABSF(BET(IJ)-ALPH(NK,IJ))
      GO TO 104
  101 IF(BET(IJ)-ALPH(NK,IJ))102,100,103
  102 GAM(NK,IJ)=6.2831853-ALPH(NK,IJ)+BET(IJ)
      GO TO 104
```

```
103 GAM(NK,IJ)=ALPH(NK,IJ)+6.2831853-BET(IJ)
104 IF(GAM(NK,IJ)-0.0174533)105,105,108
105 PRINT 107,XIA(IK,IJ),XIB(IK,IJ)
107 FORMAT (//4X,41HTHE ANGLES FOR COMPUTING POSITION OF STAT
   13HION ,1X,2A4,1X,12HARE SMALLER ,//4X,15HTHAN ALLOWABLE,,
   250HPLEASE CHOOSE A NEW LOCATION FOR THIS STATION AND ,//
   34X,14HRESUBMIT DATA //)
108 CONTINUE
    DO 112 J=1,3
    JL=J+3
    PEAK(J)=3.1415927-(GAM(J,NN)+GAM(J,NM))
    SID(J)=XLENA*SINF(GAM(J,NM))/SINF(PEAK(J))
    SED(J)=XLENA*SINF(GAM(J,NN))/SINF(PEAK(J))
    DUMA=SID(J)*SINF(GAM(J,NN))
    DUMB=SED(J)*SINF(GAM(J,NM))
    IF(ABSF(DUMA-DUMB)-0.50)111,111,109
109 PRINT 110,XIA(JL,NM),XIB(JL,NM)
110 FORMAT (//4X,16HDATA FOR STATION ,1X,2A4,1X,10HIS ERRONEO
   130HUS, PLEASE CHECK AND RESUBMIT //)
111 PX(JL,NM)= PPX(NN)+SID(J)*COSF(ALPH(J,NN))
112 PY(JL,NM)= PPY(NN)+SID(J)*SINF(ALPH(J,NN))
    DO 116 J=1,3
    JK=J+3
    IF(NN-1)92,92,93
 92 IK=J+3
    GO TO 94
 93 IK=J+6
 94 PX(IK,NN)=PX(JK,NM)
    PY(IK,NN)=PY(JK,NM)
    IF(NM-N)95,116,116
 95 NI=NM+1
    PX(J,NI)=PX(JK,NM)
    PY(J,NI)=PY(JK,NM)
116 CONTINUE
    GO TO 117
997 CONTINUE
    END

C         SUBROUTINE INDAT

    SUBROUTINE INDAT(N,PHO,XIA,XIB,PX,PY,X,Y,XSET,YSET)
    DIMENSION PHO(10),XIA(9,10),XIB(9,10),PX(9,10),PY(9,10),
   1          X(9,10),Y(9,10),PXA(8),PXB(8),PXC(8),PYA(8),
   2          PYB(8),PYC(8),XP(9,10),YP(9,10),XSET(9,10),
   3          YSET(9,10)
 71 READ 401,PHO(1),XIA(1,1),XIB(1,1),PX(1,1),PY(1,1),X(1,1),
   1         Y(1,1)
401 FORMAT (1X,A4,2X,2A4,5X,F12.2,4X,F12.2,4X,F12.2,4X,F12.2)
    READ 402,(XIA(K,1),XIB(K,1),PX(K,1),PY(K,1),X(K,1),Y(K,1)
   1         ,K=2,6)
402 FORMAT (7X,2A4,5X,F12.2,4X,F12.2,4X,F12.2,4X,F12.2)
```

```
      M=N-1
      DO 118 I=2,M
      READ 403,PHO(I),XIA(1,I),XIB(1,I),PX(1,I),PY(1,I),X(1,I),
     1          Y(1,I)
  403 FORMAT (1X,A4,2X,2A4,5X,F12.2,4X,F12.2,4X,F12.2,4X,F12.2)
      READ 404,(XIA(K,I),XIB(K,I),PX(K,I),PY(K,I),X(K,I),Y(K,I)
     1          ,K=2,9)
  404 FORMAT (7X,2A4,5X,F12.2,4X,F12.2,4X,F12.2,4X,F12.2)
  118 CONTINUE
      READ 401,PHO(N),XIA(1,N),XIB(1,N),PX(1,N),PY(1,N),X(1,N),
     1          Y(1,N)
      READ 402,(XIA(K,N),XIB(K,N),PX(K,N),PY(K,N),X(K,N),Y(K,N)
     1          ,K=2,6)
      DO 81 I=1,N
      IF(I-1)73,72,73
   72 K=6
      GO TO 75
   73 IF(I-N)74,72,72
   74 K=9
   75 CONTINUE
      DO 81 KA=1,K
      XSET(KA,I)=PX(KA,I)
      YSET(KA,I)=PY(KA,I)
      X(KA,I)=TANF(X(KA,I)/(3600.*57.2957795))
   81 Y(KA,I)=TANF(Y(KA,I)/(3600.*57.2957795))
      END

C     SUBROUTINE AZIMUTH

      SUBROUTINE AZIMUTH(X,Y,ANG,J1,J2)
      DIMENSION X(J2),Y(J2),ANG(J2)
      DO 71 I=J1,J2
      IF(X(I))60,64,68
   60 IF(Y(I))61,62,63
   61 ANG(I)=ATANF(ABSF(Y(I)/X(I)))+3.1415927
      GO TO 71
   62 ANG(I)=3.1415927
      GO TO 71
   63 ANG(I)=3.1415927-ATANF(ABSF(Y(I)/X(I)))
      GO TO 71
   64 IF(Y(I))65,66,67
   65 ANG(I)=4.7123890
      GO TO 71
   66 ANG(I)=0.0
      GO TO 71
   67 ANG(I)=1.5707963
      GO TO 71
   68 IF(Y(I))69,66,70
   69 ANG(I)=6.2831853-ATANF(ABSF(Y(I)/X(I)))
      GO TO 71
   70 ANG(I)=ATANF(ABSF(Y(I)/X(I)))
```

```
71 CONTINUE
   END
C       SUBROUTINE SWITCH
   SUBROUTINE SWITCH(X,Y,PX,PY,XSET,YSET,XIA,XIB,ANG,J1,J2,I)
   DIMENSION X(9,10),Y(9,10),PX(9,10),PY(9,10),XIA(9,10),
  1          XIB(9,10),ANG(6),XSET(9,10),YSET(9,10)
   L=1
 5 DO 7 J=J1,J2
   DO 7 M=J,J2
   JK=M+1
   IF(ANG(J)-ANG(JK))6,7,7
 6 TEMP=ANG(J)
   ANG(J)=ANG(JK)
   ANG(JK)=TEMP
   TEMP=X(J,I)
   X(J,I)=X(JK,I)
   X(JK,I)=TEMP
   TEMP=Y(J,I)
   Y(J,I)=Y(JK,I)
   Y(JK,I)=TEMP
   TEMP=PX(J,I)
   PX(J,I)=PX(JK,I)
   PX(JK,I)=TEMP
   TEMP=PY(J,I)
   PY(J,I)=PY(JK,I)
   PY(JK,I)=TEMP
   TEMP=XSET(J,I)
   XSET(J,I)=XSET(JK,I)
   XSET(JK,I)=TEMP
   TEMP=YSET(J,I)
   YSET(J,I)=YSET(JK,I)
   YSET(JK,I)=TEMP
   TEMP=XIA(J,I)
   XIA(J,I)=XIA(JK,I)
   XIA(JK,I)=TEMP
   TEMP=XIB(J,I)
   XIB(J,I)=XIB(JK,I)
   XIB(JK,I)=TEMP
 7 CONTINUE
   IF(L-1)8,8,13
 8 DO 12 J=J1,J2
   JK=J+1
   IF(ABSF(ANG(J)-ANG(JK))-3.1415927)12,9,9
 9 IF(ANG(J)-ANG(JK))10,12,11
10 ANG(J)=ANG(J)+6.2831853
   GO TO 12
11 ANG(JK)=ANG(JK)+6.2831853
12 CONTINUE
   L=L+1
```

```
      GO TO 5
   13 CONTINUE
      END
C     SUBROUTINE OUTDAT
      SUBROUTINE OUTDAT(IPRJ1,IPRJ2,IPRJ3,PX,PY,XSET,YSET,
     1                   XIA,XIB,PHO,N,PPX,PPY)
      DIMENSION PX(9,10),PY(9,10),XSET(9,10),YSET(9,10),EX(9),
     1          EY(9),XIA(9,10),XIB(9,10),PHO(10),PPX(10),
     2          PPY(10)
      PRINT 300
  300 FORMAT (1H1)
      DO 30 I=1,N
      PRINT 301,IPRJ1,IPRJ2,IPRJ3,PPX(I),PPY(I)
  301 FORMAT (     10X,37HGEODETIC LOCATION OF PANEL POINTS ON
     110HPROJECT - ,3A4,6X,F12.2,5X,F12.2)
      PRINT 302,PHO(I)
  302 FORMAT (  /  10X,35HSTATIONS APPEARING ON PLATE NUMBER
     12H- ,A4)
      PRINT 303
  303 FORMAT (   //  10X,7HSTATION ,6X,21HCOMPUTED PLANE COORDI
     16HNATES ,9X,23HKNOWN PLANE COORDINATES,10X,11HCOORDINATE
     26HERRORS /27X,1HX,16X,1HY,16X,1HX,16X,1HY,15X,1HX,10X,
     31HY //)
      IF(I-1)20,20,21
   20 K=6
      GO TO 23
   21 IF(I-N)22,20,20
   22 K=9
   23 DO 26 J=1,K
      IF(XSET(J,I))25,24,25
   24 PRINT 304,XIA(J,I),XIB(J,I),PX(J,I),PY(J,I)
  304 FORMAT ( 10X,2A4,3X,F12.2,5X,F12.2 /)
      GO TO 26
   25 EX(J)=PX(J,I)-XSET(J,I)
      EY(J)=PY(J,I)-YSET(J,I)
      PRINT 305,XIA(J,I),XIB(J,I),PX(J,I),PY(J,I),XSET(J,I),
     1          YSET(J,I),EX(J),EY(J)
  305 FORMAT ( 10X,2A4,3X,4(F12.2,5X),F10.2,1X,F10.2 /)
   26 CONTINUE
      PRINT 300
   30 CONTINUE
      END
         END
         FINIS
-EXECUTE,,,1.
```

INVESTIGATIONS AND TESTS

Purpose

Both analysis and tests were used to evaluate the potential of numerical radial triangulation procedures.

Since the theory of the radial triangulation procedure is based on true angles as determined from vertical photographs, the first analysis considered the effect of tilt on the computations. This condition was investigated by using a hypothetical set of data, in which ground coordinates for both control points and pass points were assumed to have certain coordinates. Then, with these assumed values, the corresponding photographic coordinates were computed for both truly vertical photographs and for tilted photographs. The computations, as discussed in the preceding outline, were applied using these hypothetical data. Thus, a comparison was made between the known (i.e., assumed) ground coordinates and the computed values based on the tilted photographs. Also to be considered in the analysis are the relative positions of the photographic principal points and the control points or pass points.

Procedural Limitations

The relative positions of these points as imaged on the photograph may introduce two critical limitations to the numerical radial triangulation process. If the control points or the pass points fall on a line connecting the principal points of two successive photographs, the intersection process will not give a solution. Therefore, it is necessary that both the control points and the selected pass points be situated in such a position as to give a satisfactory angular relationship. A poor condition has been diagrammed in Figure 14, in which the control points lie along a highway, the center line of which also very nearly coincides with the flight line for the photography. Properly spaced control points and pass points can be seen in Figure 15.

Another limitation occurs when the principal point and the three control points lie on a common circle. This condition is known as the critical circle. In the application of the resection principle, the positions of three points are known, and the position of a fourth point is determined from these known points through the angles measured at the fourth point. In the event that all four of these points lie on a common circle, a solution is not possible. The procedure which will be recommended at the end of this report will, in general, avoid both of these difficulties.

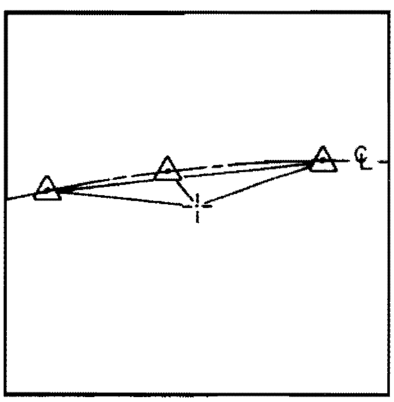

FIGURE 14

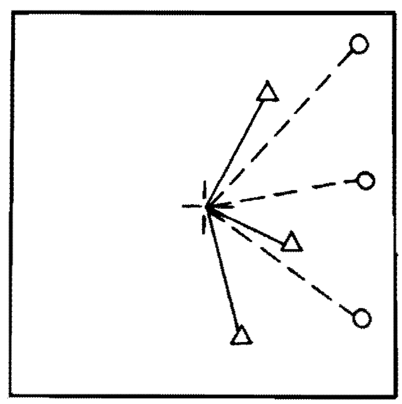

FIGURE 15

Procedural Approach

Four sets of photographic data, each utilizing 6 to 10 photographs in a strip, were used to test the computational procedures outlined previously. Among these sets was a hypothetical set of data generated by assuming a set of 6 photographs which might have the stated conditions (flying height, focal length, camera orientation, and control points) and then calculating the hypothetical photographic coordinates which would fit these conditions. The nominal scale for these conditions was chosen as 1:3,000. The other three sets of real photographs consisted of (1) a series of 6 photographs covering a portion of the City of Austin, Texas, at a nominal scale of 1:12,000; (2) a series of 8 photographs of an area near San Antonio, Texas, along a proposed highway-improvement project, at a nominal scale of 1:2,400; and (3) a series of 9 photographs of an area along State Highway 3, between Galveston and Houston, Texas, at a nominal scale of 1:2,400.

Use of Hypothetical Data

Six hypothetical photographs were used in a strip forming 5 models. Three control points were assumed in the first model (photographs 10 and 11), and the positions of 14 pass points were determined, using the procedures outlined and discussed in this report. The values, as computed, were compared with the known coordinates of these 14 pass points. (See Tables 2 and 3.)

Use of Test Data

A single flight strip of 6 photographs covering a portion of the City of Austin, Texas, at a scale of 1:12,000 was used in the first test. The control points were intersection stations based on surveys by the U.S. Coast and Geodetic Survey. Since there were 5 of these stations imaged within this strip, 3 were used as basic control, and the other 2 served as checks to test the results. These control points included 3 smokestacks for power plants, the dome atop the Texas State Capitol Building, and a water standpipe. The diameter of these points was 20 feet to 40 feet, which, in turn, meant a photographic image size of 0.02 inch to 0.04 inch. Thus, the measuring accuracy was limited to ± 0.01 inch to ± 0.02 inch because of the difficulty in identifying the center of these large objects. The pass points were "targets of opportunity," selected on the photograph without ground identification. Thus, the data from this set of photographs were used to test the general procedure but not to evaluate the quality of the results.

When the computation procedure had been developed and programmed on the CDC-1604 digital computer, it was tested using the data from the Austin photographs. These test runs served as a means of examining the workability of the computer program.

Some data were collected from a strip of aerial photographs covering an area near San Antonio, Texas, along a proposed highway-improvement project. This photography, at a scale of approximately 1:2,400 (1 in. = 200 ft) is typical of that used by the Texas Highway Department for large-scale mapping. The control points were paneled marks on the roadway and were located approximately along the flight line. As has been discussed earlier (see Figures 14 and 15), when control points are in this position with respect to the flight line, the radial triangulation procedure is weak at best, and sometimes it will not work at all. Thus, in this research, as soon as preliminary tests definitely demonstrated that this control arrangement was unsatisfactory, no further tests were performed on this San Antonio photography.

The third test strip was a series of 9 photos at a scale of 1:2,400 (1 in. = 200 ft) of an area along State Highway 3, between Galveston and Houston, Texas. The primary control points were panel markings along the highway, but, since these were not consistently along the flight line, the spacing proved satisfactory for the radial triangulation procedure.

Large amounts of data were taken from these photographs using the theodolite coordinate measuring instrument. Pass points were "targets of opportunity" selected and located on the photograph without field checking. Usually, more than the minimum number of points were selected and measured on each photograph. Thus, data were made available for 10 to 15 pass points and control points on each photograph.

Control-extension calculations were performed using these data, and these have been reported in the following section on results.

RESULTS

In the evaluation of the analyses and tests conducted on numerical radial triangulation procedures, consideration was given to the following:

1. The effect of tilt on this procedure.
2. The importance of the location of control points and pass points with respect to the photographic principal points.
3. The propagation of errors through the calculation procedure. (Also presented are the results of some tests which show the size of errors, propagation of these errors, and the effect of adjustment procedures.)

When analyses were performed using a photographic tilt of 3° to 6°, the errors in the computed position of the pass points varied extensively. In some combinations of the spacing of the control points, of the amount and direction of tilt, and of the location of pass points, the error was as much as 3 feet for photography at a scale of 1:2,400. In other cases, the error would be negligible.

Thus, with the procedure being investigated in this research, tilts of 3° or more would, in general, result in computed values of coordinates which would be unsatisfactory. However, for the conditions investigated, a tilt of 1° or less caused an error of less than 0.5 foot in the computed position of pass points for photography at a scale of 1:2,400. It will be noted that this size of error (0.5 ft) is consistent with the errors found to be the result of measuring.

When control points are spaced properly (see Figures 15 and 16) so that the angles measured at the principal points are 30° to 60°, an error of ± 10 microns in photo coordinate measuring will propagate to an error of ± 0.5 foot to ± 1 foot in the horizontal position of the principal point <u>if</u> the point is imaged on a truly vertical photograph at a scale of 1:2,400.

However, if the control points are spaced poorly, as illustrated in Figures 14 and 17, an error of ± 10 microns in photo coordinate measuring may produce a much larger error in the computed position of the principal point. In fact, under some very undesirable arrangements,* the error may be so large as to render the values meaningless.

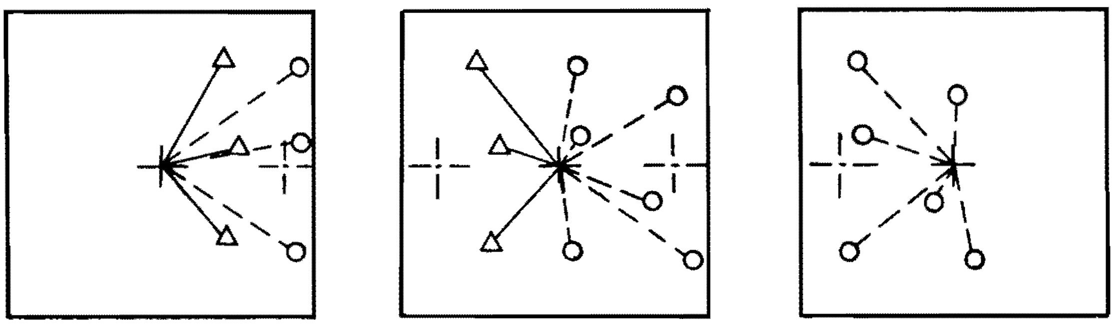

FIGURE 16

Legend:
+ principal points.
-¦- transferred principal points.
△ control points.
○ pass points.

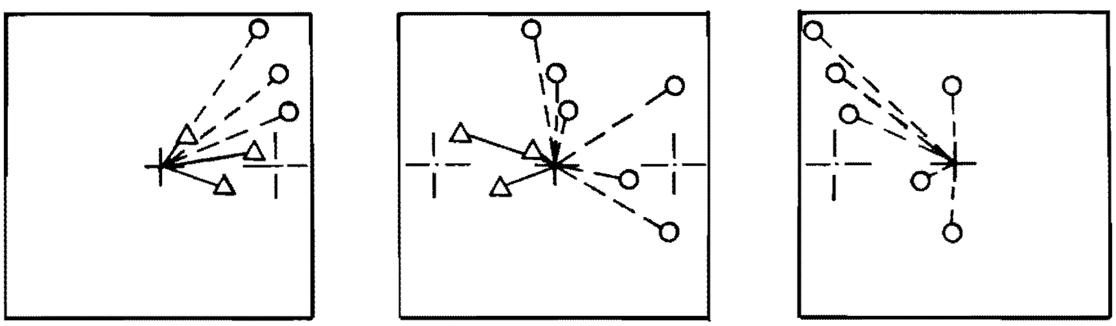

FIGURE 17

*For example, as was pointed out earlier, if the three control points and the principal point all lie on a common circle, no solution is possible.

Thus, to control or reduce the size of errors introduced into the computed positions of principal points (or pass points), the spacing and positioning of the control points (or pass points) must be arranged to provide the strongest possible triangular configuration (i.e., usually involving angles greater than 30^{o}). Then, improved accuracy may be obtained by reducing the measuring errors.

Error Propagation

According to the data gathered in this study, the errors in the horizontal position of the pass points did not appear to increase significantly with increased distance from the initial control points. While this conclusion may seem inconsistent with popular concepts in aerial triangulation, the analysis of the test results indicate strongly that the accidental errors in this research were more significant than the systematic errors. On this assumption, it is logical to conclude that the propagation of the accidental errors would be in a random fashion and might be as large in the early portions of the flight strip as they were in the latter portions. The significant inference to be gained from this conclusion involves the possibility of adjustments. Since the random nature of the propagation of accidental errors makes it difficult, if not impossible, to apply <u>any</u> type of adjustment, improvement in the results would seem dependent upon improved quality in the measuring process and in the location of the control points and the pass points, rather than upon any adjustment procedure to bring the computed values into line.

However, according to some investigators, triangulation extensions have been observed to follow a trend which tends to be systematic and can be adjusted. For example, if the true position of a flight line is defined by a series of straight lines as shown in Figure 18, the computed position of points along this line might be as shown by the dashed line.

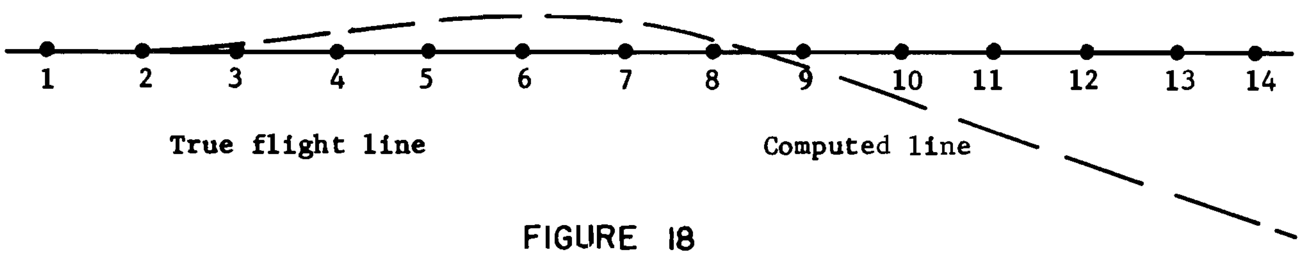

FIGURE 18

If, for example, control data are available at points 1, 2, 7, and 12, the observed discrepancies at these points could be used to apply corrections at other points. These corrections would probably change the values for most points along the line so that they would be more nearly correct.

Most investigators have employed a second- or third-order polynomial of the general form,

$$X = a_1 + b_1x + b_2y + c_1(x^2 + y^2) + 2c_2xy$$

$$Y = a_2 + b_2x + b_1y + c_2(x^2 + y^2) + 2c_1xy$$

to transform the computed coordinates (x, y) to the adjusted ground coordinates (X, Y).

An error equation based on a third-order polynomial was developed and tested using data and calculations from this research. As is shown in the results, however, this adjustment procedure does not appear to be satisfactory when applied to the computations discussed here.

Test Results

The results of the calculations of coordinates (based on the data referred to earlier as "hypothetical") are shown in Tables 2 and 3. Three points (Δ_1, Δ_2, and Δ_3, imaged on photographs 1 and 2) were used as basic control from which coordinates for points A through N were calculated. Since the true coordinates were known for each of these points, the actual error was determined (see the second and third columns of Tables 2 and 3). The adjustment referred to above was then applied to these data. The results, as shown in Table 2, used points C, F, I, and L as additional control to establish the parameters for computing adjustments for other pass points. The results shown in Table 3 used points A, D, G, and J as additional control.

It is interesting to note that, of the 20 adjusted values (see the last two columns of Tables 2 and 3), the errors after adjustment were less in 11 cases and larger in 9.

It is also interesting to note that, in all cases shown in Tables 2 and 3, the errors were erratic (i.e., they did not appear to follow a trend), and they did not necessarily increase with an increased distance from the control. (In Table 2, this erratic trend can be noted in the values of X for points A through I and the values of Y for all points.)

This adjustment does <u>not</u> appear to be valuable since almost as many errors were increased as were reduced; and, in some cases, the adjustment actually introduced an unreasonable error. (See point J in Table 2.)

The results of the calculations utilizing data from six photographs of the third test strip are shown in Table 4. Control data consisted of nine control points which were established by field survey and, for this test, were assumed to be without error. Also, six additional check points were used with data taken from a large-scale map with a maximum position error of ± 1 foot. Control points R-A, O, and PI-8 were used for the

Table 2

ERRORS BEFORE AND AFTER ADJUSTMENT FOR HYPOTHETICAL DATA

	Error before Adjustment (feet)		Amount of Adjustment (feet)		Error after Adjustment (feet)	
Point	X	Y	X	Y	X	Y
Δ_1	0	0	0	0	0	0
Δ_2	0	0	0	0	0	0
Δ_3	0	0	0	0	0	0
A	-0.64	+0.04	+0.11	-0.33	-0.53	-0.29
B	-0.18	-0.17	+0.01	+0.04	-0.17	-0.13
C*	-0.15	+0.08	+0.15	-0.08	0	0
D	-0.08	-0.46	+0.13	-0.04	+0.05	-0.50
E	+0.16	-0.11	-0.08	-0.26	+0.08	-0.37
F*	+0.16	+0.26	-0.16	-0.26	0	0
G	+0.60	-1.32	-0.66	-0.44	-0.06	-1.76
H	+0.85	-0.33	-0.73	-0.51	+0.12	-0.84
I*	+0.79	+0.59	-0.79	-0.59	0	0
J	-1.17	-4.51	-12.35	+12.55	-13.52	+8.04
K	+1.40	-3.62	-6.86	+6.11	-5.46	+2.49
L*	+3.15	-1.80	-3.15	+1.80	0	0
M	+7.84	-3.91	-2.46	+1.01	+5.38	-2.90
N	+7.54	-0.81	-0.59	-1.03	+6.95	-1.84

*Points used as control for adjustment.

Table 3

ERRORS BEFORE AND AFTER ADJUSTMENT FOR HYPOTHETICAL DATA

	Error before Adjustment (feet)		Amount of Adjustment (feet)		Error after Adjustment (feet)	
Point	X	Y	X	Y	X	Y
Δ_1	0	0	0	0	0	0
Δ_2	0	0	0	0	0	0
Δ_3	0	0	0	0	0	0
A*	-0.64	+0.04	+0.64	-0.04	0	0
B	-0.18	-0.17	+0.52	+0.12	+0.34	-0.05
C	-0.15	+0.08	0	0	-0.15	+0.08
D*	-0.08	-0.46	+0.08	+0.46	0	0
E	+0.16	-0.11	-0.42	+0.93	-0.26	+0.82
F	+0.16	+0.26	+0.96	+1.13	+1.12	+1.39
G*	+0.60	-1.32	-0.60	+1.32	0	0
H	+0.85	-0.33	-0.55	+1.58	+0.30	+1.25
I	+0.79	+0.59	-0.33	+1.83	+0.46	+2.42
J*	-1.17	-4.51	+1.17	+4.51	0	0
K	+1.40	-3.62	-0.73	+3.02	+0.67	-0.60
L	+3.15	-1.80	-1.58	+2.03	+1.57	+0.23
M	+7.84	-3.91	-1.51	+2.14	+6.33	-1.77
N	+7.54	-0.81	-1.31	+1.40	+6.23	+0.59

*Points used as control for adjustment.

Table 4

RESULTS FROM THE GALVESTON PHOTOGRAPHS

Photograph Number	Point	Known Coordinates X	Known Coordinates Y	Computed Coordinates X	Computed Coordinates Y	Error X	Error Y	Adjustment X	Adjustment Y	Residual X	Residual Y	Check Data*
1-47	R-A	241,454.5	631,128.5	--	--	--	--	--	--	--	--	
	0	241,119.0	629,917.5	--	--	--	--	--	--	--	--	
	PI8	240,987.3	630,921.8	--	--	--	--	--	--	--	--	
	A-8	241,244.1	630,627.0	--	--	--	--	--	--	--	--	
	0-B	--	--	--	--	--	--	--	--	--	--	
1-46	0-A	241,397.5	630,051.5	241,395.7	630,055.0	-1.8	+3.5	+1.8	-3.5	0	0	
	P	(241,761)	(630,448)	241,755.4	630,449.9	-6	+2	-1.6	+3.4	-8	+5	Low
	M-A	--	--	--	--	--	--	--	--	--	--	
1-45	N-B	242,406.5	629,781.5	242,400.3	629,784.0	-6.2	+2.5	+6.2	-2.5	0	0	
	N-A	(242,580)	(629,960)	242,573.4	629,960.1	-7	0	+11	-10	+4	-10	Good
	K	(242,084)	(628,757)	242,080.6	628,762.7	-3	+6	+3	-5	0	+1	Average
1-44	K-A	242,390.5	629,084.5	242,388.4	629,087.8	-2.1	+3.3	+2.1	-3.3	0	0	
	L-A	(242,728)	(629,289)	242,727.6	629,294.4	0	+5	-1	+6	-1	+1	Average
	I	--	--	--	--	--	--	--	--	--	--	
1-43	E-9	243,019.8	628,592.8	243,022.0	628,584.2	+2.2	-8.6	-2.2	+8.6	0	0	
	J	--	--	--	--	--	--	--	--	--	--	
	G	(243,397)	(627,771)	243,394.9	627,775.1	-2	+4	-1	+4	0	+8	Low
1-42	A-10	243,612.2	627,915.4	243,613.4	627,912.0	+1.2	-3.4	-1.1	+3.4	0	0	
	H	(243,729)	(628,243)	243,722.5	628,246.2	-7	+3	-2	+5	-9	+8	Average

*A qualitative expression of the reliability of the check data.

resection on photograph 1-47; and control points R-A, O, and A-8 were used for the resection on photograph 1-46.

Under the procedure described in this report, coordinates were then computed for the other points, and these values can be seen in Table 4. At eleven of these points, check values were available, and the error in computed position (as compared with the known values) was determined.

It is interesting to note that the errors varied from 0 to 8.6 feet in a random pattern. The size of the error did not increase with distance from the point of beginning. The random nature of this error is further demonstrated in the adjustment process which was applied to these results. With points O-A, N-B, K-A, E-9, and A-10 as additional control points, an adjustment procedure was applied so as to reduce the error at these points to 0 and apply an associated correction to the other points. When these adjustment values were applied to the computed values, errors of 0 to 9 feet were still present. Thus, this adjustment procedure does not conform to the random error distribution found in these tests.

CONCLUSIONS

Three conclusions were developed from this research study:

1. The distribution and spacing of the control points and the pass points constitute very critical considerations in numerical radial triangulation for extending horizontal control.
2. Errors in the horizontal position of the pass points do not increase significantly as the distance is increased between the initial control points and the pass points.
3. Small amounts of tilt in the photograph (i.e., less than 1°) do not cause appreciable errors in the computed position of the pass points.

Since control extension by any triangulation process is dependent upon the strength of the triangular configuration, it is apparent that the position of the control points or the pass points should provide the strongest possible triangulation arrangement. If the location of the points produces triangles with either very small or very large angles (near either 0° or 180°) it will be found that the calculations associated with this type of a triangle are necessarily weak. For example, in the preparation of large-scale maps used in highway design, the horizontal control points are often located along an existing highway, one which may more or less parallel the flight line for the photography. This arrangement, however, is not a satisfactory one to use for the control in radial triangulation, since the location of points resulting in small angles will produce a poor geometric configuration.

The point of view developed in this research--that errors in the horizontal position of pass points do not appear to increase significantly with distance from the initial control points--may be inconsistent with popular concepts in photogrammetric triangulation. However, the results of the tests conducted strongly suggest that the accidental errors are more significant than the systematic errors. If the accidental or random errors are predominant, then it is logical to conclude that the propagation of the errors through the calculations will also be random in nature.

In an article by M. C. Van Wijk, "The Accuracy of Coordinates, Obtained by Radial Triangulation," the following conclusions are reached:

> The remarkable high precision of the radial triangulation is most evident from the last rows of figures in the tabulation. This illustrates once again that the influence of systematic errors has often been overestimated and that radial triangulation is a powerful method of planimetric control-extension not only in flat but even in hilly terrain.

The information reported in the present research is not sufficient either in the extent of analysis or the volume of numerical tests to establish unquestionable conclusions. However, the general conclusion reached by Van Wijk concerning the overestimation of the influence of systematic errors and his emphasis on the strong value of triangulation in planimetric control-extension is supported by the data gathered and conclusions drawn from the analyses in this research.

If truly vertical aerial photographs (with zero tilt) could be used for radial triangulation, one source of systematic error would be eliminated. However, since this is not practical with present-day methods of making aerial photographs, small amounts of tilt must be expected. However, for the condition studied, tilts of up to 1^{o} do not appear to add significantly to the errors in the results of radial triangulation.

RECOMMENDATIONS

Numerical radial triangulation as presented in this research can be used to extend horizontal control for mapping or to determine ground coordinates of specific points for other purposes. The specific recommendations presented below may be helpful:

1. Consideration must be given to the quality or accuracy of the measuring process in order to establish the quality of the computed pass points.
2. Since each measurement is critical to the complete process, it is necessary that the measured values be repeated in an attempt to minimize blunders and to evaluate the size of the error present. Additional control points should be located along the photographic strip so that checks may be made to evaluate the extension and provide for at least a minimum of checks against gross mistakes.
3. Since control extension by any triangulation process is dependent upon the strength of the triangular configuration, the position of the control points (or pass points) must be such as to provide the strongest possible triangulation arrangement. Control points and pass points may be established either before or after the photography has been obtained. If these points are selected <u>before</u> the photography is made, they can be so located as to provide proper spacing; also, these points may be marked with a panel to provide easy identification of the image on the photographs. If control points and pass points are selected <u>after</u> the photo-

graphs have been made, then physical features imaged on the photograph (such as poles, roof lines, fence corners, etc.) must be used, even if the geometric spacing or configuration is poor. For very large scale photographs, the area covered is often such that a significant portion of a photograph will not contain any well-defined pass points. This condition then forces either the use of poorly defined points or a change in point location. When it is necessary to change the position of pass points from their planned positions, the result is often small angles and, thus, a weaker configuration for the triangulation process. For the preparation of large-scale maps to be used in highway design, the horizontal control points are often located along an existing highway which may approximately parallel the flight line for the photography. This arrangement, however, is unsatisfactory for control when radial triangulation is used.

4. For the best results, the control points and the pass points should be so located that the angles measured at the photographic center (and used in the triangulation calculation) have values between 30° and 60°. Ideal spacing and location have been shown in Figure 8 and Figure 15 of this report.

5. Notwithstanding the extensive investigations which have been made on aerial triangulation (as indicated in the literature), the effect of both accidental and systematic errors on control-extension and how these errors should be adjusted have not been well established. Therefore, it is the recommendation of this study that fundamental research be continued in order to help to isolate the

sources of errors (either accidental or systematic) and to show how errors are propagated through control extension.*

6. It is recommended further that research be initiated to consider the distribution of control for use with various methods of aerial triangulation.*

7. Finally, it is recommended that field tests be conducted using a variety of control-point and pass-point conditions with photographs of several different scales, in order to extend the possibilities of using numerical radial triangulation as a method for horizontal control surveys.

*Research is now in progress on these questions under Contract No. 3-8-66-93 entitled, "Evaluation of Computation and Adjustment Procedures for Control Surveys in Photogrammetric Mapping for Highways."

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