A FINITE-ELEMENT ANALYSIS OF STRUCTURAL FRAMES

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of Beam-Columns and Grid-Beam and Slab Systems

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PREFACE

This report presents the results of an analytical study undertaken to develop an implicit numerical method for determining the deflected shape of a rectangular plane frame with three degrees of freedom at each joint. The study consists of (1) the development of equations describing the behavior of a rectangular plane frame under any reasonable conditions of loading and restraint, (2) the development of an alternating-direction implicit method for the solution of these equations, and (3) the application of the method to the solution of realistic example problems.

Report 56-1 in the List of Reports provides an explanation of some of the basic procedures used in the computer program written to verify the method. The program has been written in FORTRAN 63 for the CDC 1604 digital computer. Copies of the program and data cards for the example problems may be obtained from the Center for Highway Research at The University of Texas.

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LIST OF REPORTS

Report No. 56-1, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns" by Hudson Matlock and T. Allan Haliburton, presents a finite-element solution for beam-columns that is a basic tool in subsequent reports.

Report No. 56-2, "A Computer Program to Analyze Bending of Bent Caps" by Hudson Matlock and Wayne B. Ingram, describes the application of the beam-column solution to the particular problem of bent caps.

Report No. 56-3, "A Finite-Element Method of Solution for Structural Frames" by Hudson Matlock and Berry Ray Grubbs, describes a solution for frames with no sway.

Report No. 56-4, "A Computer Program to Analyze Beam-Columns under Movable Loads" by Hudson Matlock and Thomas P. Taylor, describes the application of the beam-column solution to problems with any configuration of movable non-dynamic loads.

Report No. 56-5, "A Finite-Element Method for Bending Analysis of Layered Structural Systems" by Wayne B. Ingram and Hudson Matlock, describes an alternating-direction iteration method for solving two-dimensional systems of layered grids-over-beams and plates-over-beams.

Report No. 56-6, "Discontinuous Orthotropic Plates and Pavement Slabs" by W. Ronald Hudson and Hudson Matlock, describes an alternating-direction iteration method for solving complex two-dimensional plate and slab problems with emphasis on pavement slabs.

Report No. 56-7, "A Finite-Element Analysis of Structural Frames" by T. Allan Haliburton and Hudson Matlock, describes a method of analysis for rectangular plane frames with three degrees of freedom at each joint.

Report No. 56-8, "A Finite-Element Method for Transverse Vibrations of Beams and Plates" by Harold Salani and Hudson Matlock, describes an implicit procedure for determining the transient and steady-state vibrations of beams and plates, including pavement slabs.

Report No. 56-9, "A Direct Computer Solution for Plates and Pavement Slabs" by C. Fred Stelzer, Jr., and W. Ronald Hudson, describes a direct method for solving complex two-dimensional plate and slab problems.

Report No. 56-10, "A Finite-Element Method of Analysis for Composite Beams" by Thomas P. Taylor and Hudson Matlock, describes a method of analysis for composite beams with any degree of horizontal shear interaction.

ABSTRACT

A rational method for computer analysis of rectangular plane frames is presented. Three degrees of freedom are allowed at each joint. Flexural stiffness, transverse and axial load, and foundation spring restraint are allowed to vary as desired along each frame member. Loads, couples, and restraints can also be specified at each joint.

Equations which mathematically describe a bar-and-spring model of the real frame are formulated. An iterative procedure is used to solve these equations. Each iteration involves a complete solution of the mathematical frame model, consisting of (1) a stiffness matrix solution, using an efficient recursive technique, for the deflected shape of the frame in bending and (2) a solution for the axial tension or compression in each frame member.

Procedures for computer solution of the equations describing frame behavior are developed and convergence of the computer solution is discussed. Comparison is made with results developed by accepted theory and solutions of three example problems are presented.

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NOMENCLATURE

Symbol	Typical Units	Definition
A	-	Stiffness matrix
	-	Coefficient computed in recursive elimination of quidiagonal stiffness matrix
	in^2	Cross-section area of frame member
a	$lb\text{-}in^2$	Coefficient in stiffness matrix
B	-	Coefficient computed in recursive elimination of quidiagonal stiffness matrix
b	$lb\text{-}in^2$	Coefficient in stiffness matrix
C	-	Coefficient computed in recursive elimination of quidiagonal stiffness matrix
	in-lb	External couple applied at frame joint
c	$lb\text{-}in^2$	Coefficient in stiffness matrix
C_x	in-lb	Fraction of external couple C absorbed by horizontal half of frame joint
C_y	in-lb	Fraction of external couple C absorbed by vertical half of frame joint
d	$lb\text{-}in^2$	Coefficient in stiffness matrix
E	lb/in^2	Modulus of elasticity
E_r	in-lb	Error in summation of computed couples acting on frame joint
E_x	lb	Error in summation of computed vertical forces acting on frame joint
E_y	lb	Error in summation of computed horizontal forces acting on frame joint
e	$lb\text{-}in^2$	Coefficient in stiffness matrix
F	$lb\text{-}in^2$	Flexural stiffness EI of frame members

Symbol	Typical Units	Definition
F_x	lb-in^2	Flexural stiffness EI of horizontal frame members
F_y	lb-in^2	Flexural stiffness EI of vertical frame members
f	lb-in^3	Coefficient in load matrix
h	in.	Increment length of horizontal frame members
I	in^4	Moment of inertia
i	-	Station number on horizontal frame members
j	-	Station number on vertical frame members
k	in.	Increment length of vertical frame members
	lb/in	Elastic foundation modulus
ℓ	-	Joint number
M	in-lb	Bending moment in frame members
	-	Total number of joints on horizontal frame members
m_x	-	Total number of stations on horizontal frame members
m_y	-	Total number of stations on vertical frame members
N	-	Total number of joints on vertical frame members
P	lb	Axial tension or compression in frame members
ΔP	lb	Change in axial tension or compression P
P_x	lb	Resultant axial tension or compression in vertical frame members
P_y	lb	Resultant axial tension or compression in horizontal frame members

Symbol	Typical Units	Definition
Q	lb	Externally applied transverse load on frame members, concentrated at each station of the members
q	lb/in	Externally applied transverse load on frame members, distributed between stations on the members
Q_{bx}	lb	Vertical load applied to horizontal half of frame joint, representing load contributed by other joints in the frame
Q_{by}	lb	Horizontal load applied to vertical half of frame joint, representing load contributed by other joints in the frame
Q_{cx}	lb	Total vertical load acting on vertical column
Q_{cy}	lb	Total horizontal load acting on horizontal column
Q_{ix}	lb	Reaction of horizontal beam on vertical column at joint
Q_{iy}	lb	Reaction of vertical beam on horizontal column at joint
Q_r	lb	Load representing the behavior of a frame member at a joint
Q_{rx}	lb	Resultant vertical load on horizontal half of frame joint
Q_{ry}	lb	Resultant horizontal load on vertical half of frame joint
Q_x	lb	Externally applied vertical load on frame joint
Q_y	lb	Externally applied horizontal load on frame joint
R	(in-lb)/rad	Externally applied restraint against frame joint rotation
S	lb/in	Externally applied restraint against frame member deflection

Symbol	Typical Units	Definition
S_A	lb/in	Resistance of an element in a frame member to axial deformation
S_c	lb/in	Externally applied restraint against axial column displacement
S_{bx}	lb/in	Vertical deflection restraint applied to horizontal half of frame joint, representing restraint contributed by other joints in the frame
S_{by}	lb/in	Horizontal deflection restraint applied to vertical half of frame joint, representing restraint contributed by other joints in the frame
S_{cx}	lb/in	Total vertical displacement restraint acting on vertical column
S_{cy}	lb/in	Total horizontal displacement restraint acting on horizontal column
S_{ix}	lb/in	Intrinsic deflection restraint of horizontal frame member at joint
S_{iy}	lb/in	Intrinsic deflection restraint of vertical frame member at joint
S_r	lb/in	Deflection restraint representing the behavior of a frame member at a joint
S_x	lb/in	Externally applied vertical deflection restraint on frame joint
S_y	lb/in	Externally applied horizontal deflection restraint on frame joint
V	lb	Shear
w	in.	Transverse bending deflection of frame members
W_{bx}	in.	Vertical deflection of frame joint
W_{by}	in.	Horizontal deflection of frame joint

Symbol	Typical Units	Definition
W_{cx}	in.	Axial displacement of vertical column
W_{cy}	in.	Axial displacement of horizontal column
x	in.	Distance measured along horizontal frame members
y	in.	Distance measured along vertical frame members
Δ	in.	Joint deflection
	in.	Axial deformation
η	-	Fraction of (F_x/h^3) or (F_y/k^3) used as differential translation restraint
θ	radians	Slope of frame member
θ_x	radians	Slope of horizontal half of frame joint
θ_y	radians	Slope of vertical half of frame joint
ξ	(in-lb)/rad	Differential rotational restraint used to enforce joint rotation compatability during the iterative process of frame analysis
ξ_x	(in-lb)/rad	Differential rotational restraint ξ acting on horizontal half of frame joint
ξ_y	(in-lb)/rad	Differential rotational restraint ξ acting on vertical half of frame joint
ρ	-	Fraction of (F_x/h) or (F_y/k) used as a differential rotational restraint ξ
ϕ	in^{-1}	Curvature
ψ	radians	Change in slope θ

CHAPTER 1. INTRODUCTION

This study is concerned with the development of a rational procedure for the analysis of rectangular plane frames.

Significance of the Problem

The analysis of framed structures is a problem civil engineers have long considered. In recent years, framed structures have become so complex that even the simplest type of frame analysis often requires a large expenditure of time and effort on the part of the engineer.

Before the advent of the digital computer, many simplifying assumptions concerning structural behavior were required to allow complex frame problems to be solved by hand or with a desk calculator. Sets of simultaneous equations describing frame behavior could be formulated, but the time required to solve them was prohibitive. Thus, relaxation methods requiring many assumptions concerning structural behavior became the most widely accepted techniques of frame analysis because they could be solved by hand.

The development of the digital computer, with its ability to perform efficiently large numbers of repetitious computations, opens the way for rapid solution of complex frame problems. However, full benefit of the capabilities made available by the computer can not be realized by simply programming the old hand procedures. New methods of structural analysis, considering so far as possible the aspects of structural behavior neglected or assumed in pre-computer methods, must be developed. One such method is developed in the following chapters for the numerical solution of plane frames.

General Remarks on the Problem and Its Solution

The problem is approached by considering a rectangular plane frame to be a group of connected beam-columns. Flexural stiffness, transverse and axial loads, and elastic deflection restraint are allowed to vary as desired along each frame member. Transverse loads and deflection restraints and applied couples and rotational restraints may be specified as desired at each frame joint. Axial rigidity is assumed for all frame members.

A typical rectangular plane frame is shown in Fig 1.1. Variation of flexural stiffness is indicated by the different sizes and shapes of frame members. Transverse deflection restraints are indicated by coil springs, while joint rotational restraints are simulated by watch-type springs. Transverse loads acting normal to frame members and axial loads acting along the neutral axes of frame members are also shown in Fig 1.1, as are applied couples acting on some of the frame joints.

An iterative procedure is used to solve the problem. Each iteration involves a complete solution of the mathematical frame model and consists of two parts: (1) a solution for the deflected shape of the frame in bending and (2) a solution for the axial displacement and force distribution in each frame member. During the iterative process initial assumptions concerning the effects of member interaction are adjusted, based on previously computed behavior, until a final solution is achieved.

In a physical sense, the proposed iterative process may be visualized as a readjustment procedure. If a frame under specified conditions of loading and restraint is given, the sequence outlined below is followed:

(1) An initial assumption is made concerning the distribution of internal forces and couples in the frame.

(2) The deflected shape of the frame is computed considering the applied loading and assumed distribution of internal forces and couples.

(3) The distribution of internal forces and couples is revised considering the applied loading and the deflected shape of the frame.

Steps 2 and 3 are repeated until the correct deflected shape of the frame is obtained. This distribution, determined by interaction of frame members, is computed using equations derived in the following chapters.

Because of the large number of repetitious calculations involved in a procedure of this type, the structure is simulated and solved on the digital computer.

Scope of the Study

The aims of this study are threefold: (1) the development of equations describing the behavior of a rectangular plane frame supported on an elastic foundation under any reasonable conditions of loading and restraint, (2) the development of an alternating-direction implicit method for the solution of

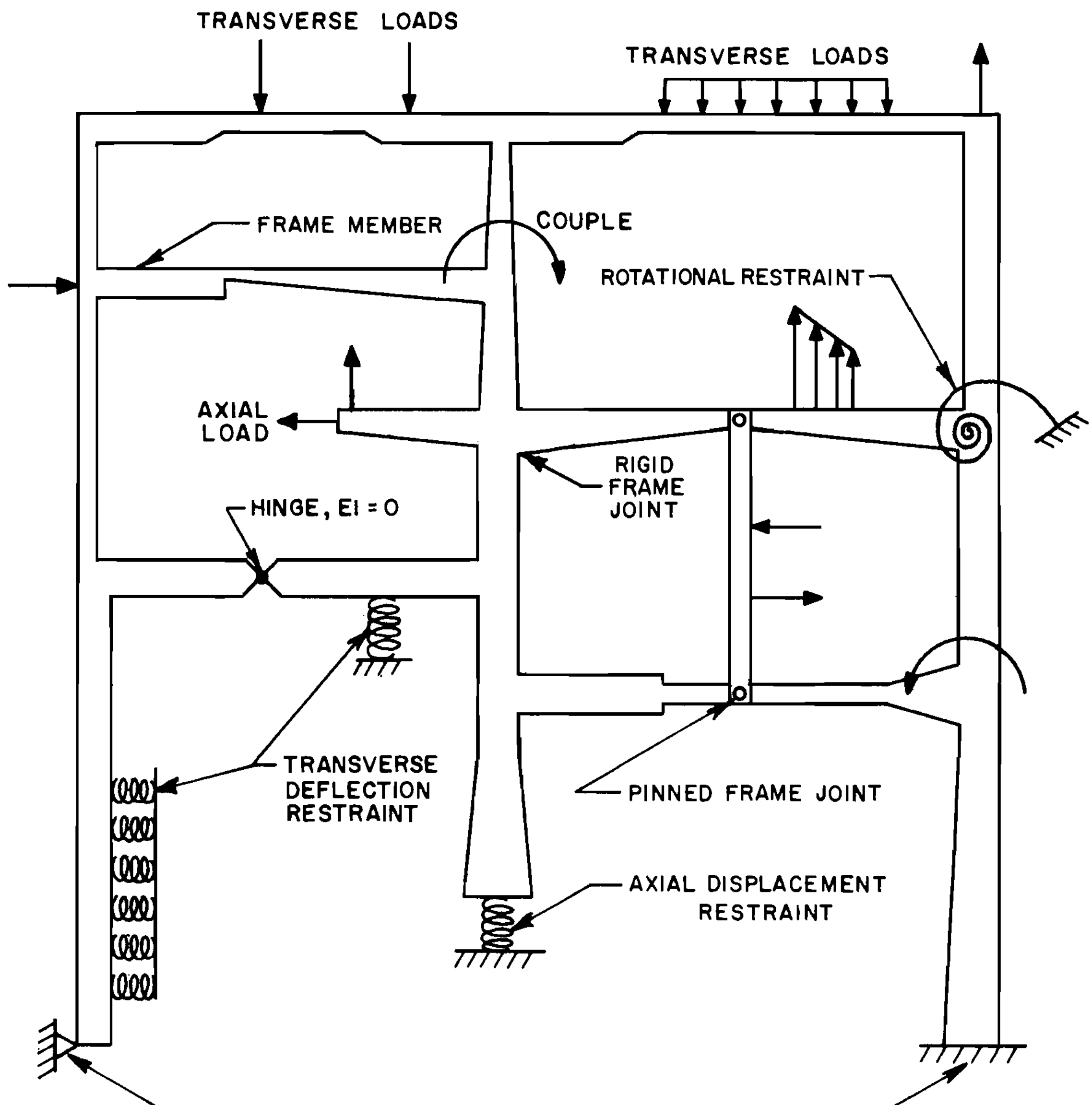


Fig 1.1. A typical rectangular plane frame illustrating variations in geometry, flexural stiffness, and applied loading and restraint considered by the proposed method of frame analysis.

these equations, and (3) the application of the method to the solution of realistic example problems.

Organization of the Study

A summary of previous developments in the solution of related soil structure interaction problems is presented in Chapter 2, as well as a survey of current methods of plane-frame analysis. In Chapter 3, equations describing the behavior of frame members are developed, while Chapter 4 is concerned with equations describing the behavior of a plane frame in bending and Chapter 5 with determination of axial force distribution in frame members. Chapter 6 discusses the procedure for computer solution of the frame equations, and convergence of the programmed method is shown in Chapter 7. Applications of the method to the solution of realistic example problems are shown in Chapter 8. Possible additions to the method are discussed in Chapter 9, while conclusions and recommendations are given in Chapter 10.

CHAPTER 2. SUMMARY OF PERTINENT PREVIOUS DEVELOPMENTS IN STRUCTURAL ANALYSIS

A large number of methods and procedures are presently used to analyze framed structures. These methods may be classified as either (1) hand methods or (2) matrix methods. The majority of these methods allow determination of bending moment distribution, translation, and rotation for each frame joint. Bending moment distribution in each frame member is then determined by a separate analysis. A survey of the most widely used methods is given in the following sections.

Summary of Hand Methods of Frame Analysis

Hand methods are methods or procedures of frame analysis which may be carried out by an individual with the aid of a slide rule or desk calculator. Such methods may be subdivided into classical or closed-form methods and relaxation methods.

The most widely used classical techniques are those of least-work, virtual work, and slope-deflection. These procedures, summarized in any standard text on structural analysis such as Wang and Eckel (Ref 22), require the solution of a set of simultaneous equations to determine frame behavior. For simple frames, requiring only a few simultaneous equations, these procedures are very efficient, but for complex framed structures, the time required for hand solution of the required equations becomes prohibitive.

Relaxation or point iterative methods were developed to surmount the difficulties encountered in the application of classical techniques to the solution of complex frame problems. The most well-known technique is that of moment distribution, developed by Cross (Ref 3) for no-sway frames. This procedure is applicable to all rigid frames, is simple to apply, and is always convergent. Grinter (Ref 8) developed a similar method for balancing end angle changes in frame members.

The moment distribution method of Cross has been modified in various ways to solve frames that sway. Two such methods are the influence-deflection procedure, summarized by Ferguson (Ref 4), which combines several moment distributions in a simultaneous equation procedure, and the statics ratio procedure, developed by Ferguson and White (Ref 5), which combines moment distribution with iterative solution of the equations of statics.

The major limitation of these relaxation methods is defining the required iteration parameters for non-prismatic members and complex conditions of loading and restraint.

Summary of Conventional Matrix Methods of Frame Analysis

The advent of the digital computer has made simultaneous equation methods of frame analysis practical for large and complex structures. Two general approaches, based on classical methods, are normally used to analyze structural frames. These are action or flexibility methods, where redundants are expressed as forces, and displacement or stiffness methods, where redundants are expressed as displacements.

In these procedures the required data concerning frame-joint behavior is found by formulating and solving a set of simultaneous equations. An excellent presentation of conventional matrix methods of frame analysis is given by Hall and Woodhead (Ref 11). The main difficulties encountered in applying these methods are (1) the development of required equations for non-prismatic frame members and for complex conditions of loading and restraint and (2) the inversion of large and sometimes "ill-conditioned" matrices.

Iterative procedures have also been used in determination of frame behavior. Clough, Wilson, and King (Ref 2) have developed and compared iterative and elimination procedures for solving large stiffness matrices describing frame-joint behavior.

Relaxation methods, as described in the previous section, have also been adapted for computer solution. While these methods are still subject to the previously described limitation, a large amount of time is saved by computer solution.

Summary of Related Developments in Structural Analysis

A great deal of work has been done in the field of numerical analysis of structural members. Early procedures for solving beams and beam-columns were developed by Newmark (Ref 20) and Malter (Ref 13). Gleser (Ref 6) suggested a recurring form of difference equation for beam solution that was utilized by Matlock and Reese (Ref 19) in the analysis of laterally loaded piles.

Matlock (Ref 14) developed a more general recursive procedure for solving beam-column problems. This technique was summarized by Matlock and Haliburton (Ref 18).

In related developments, Ingram (Ref 12) and Matlock (Ref 15) revised and extended this method of beam-column solution to include the effects of nonlinear loads and supports. A procedure for solving beam-column problems with nonlinear flexural stiffness was developed by Haliburton (Ref 9) and extended by Haliburton and Matlock (Ref 10).

Tucker (Ref 21), using an alternating-direction implicit method of analysis, applied the beam-column method to the solution of grid-beam and plate problems. Matlock and Grubbs (Ref 17) also used an alternating-direction implicit procedure to solve plane-frame problems with no sidesway.

At the present time, research is underway at The University of Texas to extend present methods for solution of grid and plate systems and to develop methods of analysis for slabs and layered plate-grid systems. Numerical procedures for dynamic analysis of beam-columns, grid systems, and plates are also being developed.

However, little work has been done in direct determination of the complete deflected shape of a plane frame in bending. Such a procedure, based on matrix iterative analysis techniques, is developed in the following chapters.

CHAPTER 3. DEVELOPMENT OF A PROCEDURE FOR THE BENDING ANALYSIS OF FRAME MEMBERS

It has been stated previously that a plane frame is a group of connected beam-columns. Thus, in order to determine the deflected shape of a plane frame, one must be able to determine the deflected shapes of the individual frame members as influenced by the loading and geometry of the frame system.

In this chapter, an efficient numerical procedure will be developed for determining the deflected shape of an individual beam-column under complex conditions of loading and restraint. The behavior of any individual frame member will be influenced by the behavior of all other frame members. The interaction of individual frame members is considered in Chapter 4. It is shown that procedures developed for individual beam-columns are still applicable, subject to slight modification to consider member interaction effects.

Conventional Form of the Differential Equation for a Beam-Column on Elastic Foundation

The well-known differential equation for a beam-column on elastic foundation, from conventional beam mechanics theory, has the form (Ref 7, p 219)

$$EI \left[\frac{d^4 w}{dx^4}\right] - P \left[\frac{d^2 w}{dx^2}\right] + kw = q \tag{3.1}$$

where

EI = constant flexural stiffness of the beam-column,

P = constant axial tension acting along the neutral axis of the beam-column,

k = elastic foundation modulus,

q = applied transverse load per unit length,

w = transverse deflection of the beam-column neutral axis, and

x = distance along the beam-column neutral axis.

Equation 3.1 was derived using the assumptions of conventional beam mechanics theory:

(1) Axial and shear deformations are negligible.

(2) Plane sections normal to the neutral axis of the beam-column before bending are also normal to the neutral axis after bending.

(3) Consideration is limited to straight beam-columns having a vertical axis of symmetry.

(4) Transverse deflections are small compared to original beam-column length,

(5) The material of the beam-column behaves in a linearly elastic manner.

(6) Torsional effects are negligible.

The form of Eq 3.1 requires that the parameters EI and P be constant, and also that k and q be smoothly continuous functions of the dependent variable x . Furthermore, the solution of Eq 3.1 by conventional means is very difficult unless K and q may be described as very simple functions of x . Unfortunately, such simple cases are rarely encountered in the solution of realistic problems.

Some complex problems may be solved by the use of finite-difference approximations, dividing the beam-column into a finite number of equal increments and replacing Eq 3.1 by a corresponding linear difference equation with constant coefficients. If written about some increment point i on the beam-column, substituting appropriate finite-difference relationships directly into Eq 3.1 results in the form:

$$EI\left[\frac{w_{i-2} - 4w_{i-1} + 6w_i - 4w_{i+1} + w_{i+2}}{h^4}\right] - P\left[\frac{w_{i-1} - 2w_i + w_{i+1}}{h^2}\right] + k_i w_i = q_i \qquad (3.2)$$

where

h = increment length or spacing between the increment points or beam-column stations.

If four initial values of deflection are known, Eq 3.2 may be solved explicitly for the deflection of the fifth point, and the deflected shape of the beam-column may be computed by "marching" Eq 3.2 from one end to the other. Alternatively, one could write an equation of the form of Eq 3.2 at every beam-column station. The resulting set of simultaneous equations, including appropriate boundary conditions, could then be solved implicitly for the deflected shape of the beam-column. Once again, however, the form of Eq 3.2 requires that the parameters EI and P be constants.

Thus, a general method of analysis must allow EI and P , as well as k and q , to vary over the length of the beam-column. Such a procedure is developed in the next section.

The Finite-Element Model of a Beam-Column on Elastic Foundation

At least two procedures may be followed to derive a general numerical method for the solution of beam-columns on elastic foundations: (1) approximation of Eq 3.1 by finite-difference equations which allow variation of the parameters EI , P , k , and q along the length of the beam-column or (2) derivation of equations which exactly describe a physical or mechanical model of the real beam-column. Equations derived in either manner have similar forms (Ref 18). The difference is primarily in the point of view.

A derivation based on a physical model is presented because (1) it permits easier visualization of behavior to one not well-versed in numerical techniques and (2) it serves to set the stage for consideration of a frame-joint model in Chapter 4.

Figure 3.1a shows the proposed model of a real beam-column. The model consists of a series of rigid bars of equal length h connected by spring-restrained hinges. The flexural stiffness EI or F of the system is simulated by the springs which restrain the hinges at each increment point or station. An axial tension or compression P is assumed to act along the neutral axis of the model. A transverse load Q and a foundation spring S are applied at each station. Thus the model is in effect a "lumped-parameter" approximation of the real beam-column with the parameters F , Q , S , and P specified at each station. These values may represent either actual concentrated effects or may approximate effects distributed over a distance h/2 on both sides of the station. Because of the finite distance h between stations, this model

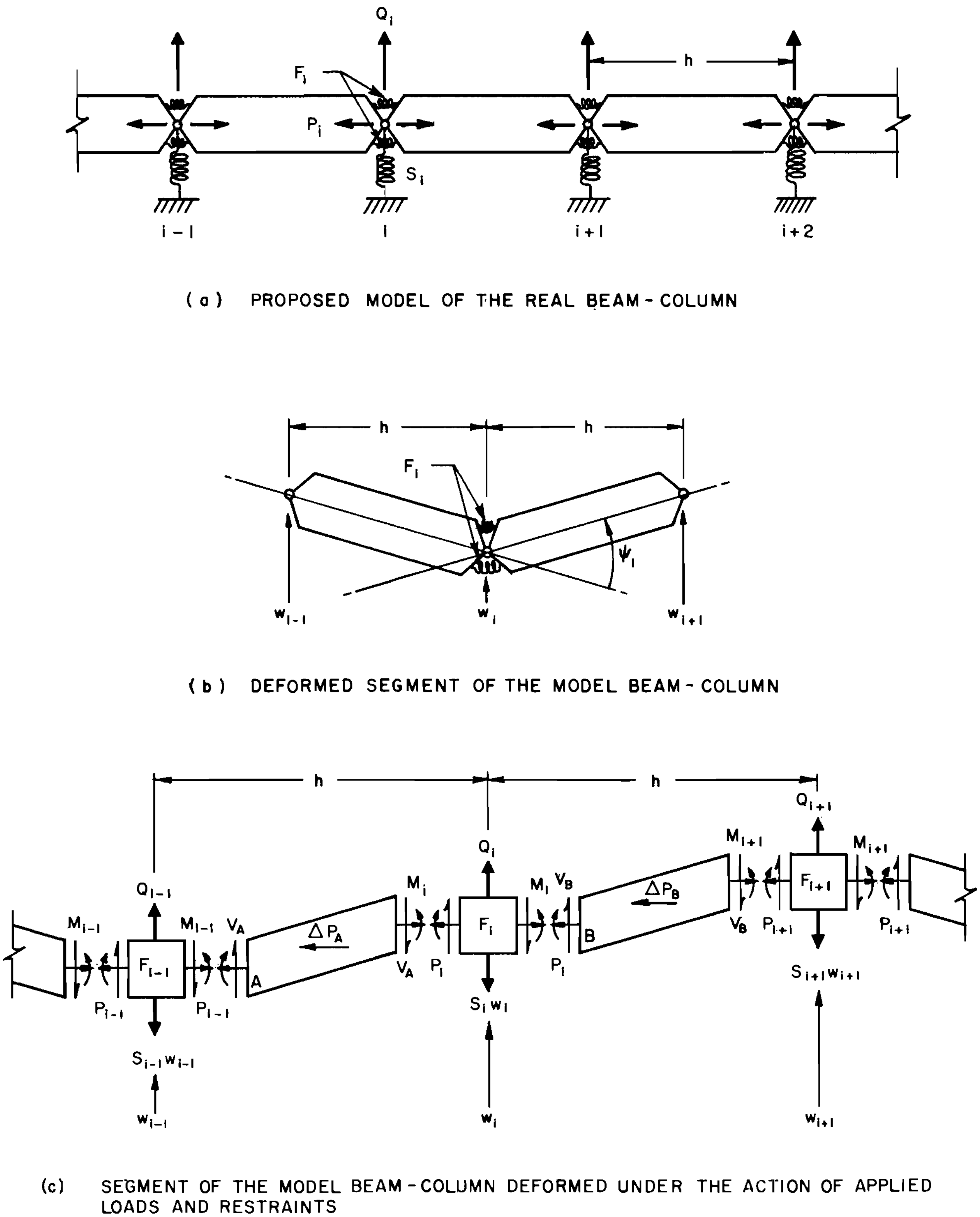


Fig 3.1. Development of a finite-element model beam-column.

will hereafter be referred to as a "finite-element" model of the real beam-column.

Bending Moment as a Function of Model Deformation

Figure 3.1b shows a deformed segment of the finite-element model beam-column. The change in slope between the rigid elements on either side of Station i may be represented by the angle ψ_i . From the figure,

$$\psi_i = \left[\frac{w_{i+1} - w_i}{h}\right] - \left[\frac{w_i - w_{i-1}}{h}\right] \tag{3.3}$$

or

$$\psi_i = \left[\frac{w_{i-1} - 2w_i + w_{i+1}}{h}\right] \tag{3.4}$$

As the angle ψ_i represents the amount of deformation produced in the two springs which simulate the flexural stiffness F_i , the resisting moment produced by this deformation is therefore:

$$M_i = \frac{F_i}{h} \psi_i = \frac{F_i}{h^2} (w_{i-1} - 2w_i + w_{i+1}) \tag{3.5}$$

Equations Defining Model Behavior

Figure 3.1c shows a segment of the finite-element model deformed under the action of applied forces and restraints. These forces and restraints are shown acting in the positive sense. The spring-restrained hinge shown in Fig 3.1a has been replaced by a deformable element containing the concentrated bending stiffness F . The resultant transverse force applied to each element is equal to the applied transverse load less the product of the elastic restraint S and deflection w . A variable axial tension or compression acts along the centroid of the model. The variation in axial tension or compression between increment points is assumed to be linearly distributed across the rigid elements such that the total change ΔP may be concentrated at the centroid of each rigid bar. A similar model has been proposed by Matlock (Ref 16).

The laws of statics may now be applied to the finite-element model of Fig 3.1c to develop equations describing its behavior. The summation of forces (positive upwards) on the deformable element at i gives

$$Q_i - S_i w_i + V_A - V_B = 0 \tag{3.6}$$

while the summation of moments (positive clockwise) about the center of Bar A to eliminate ΔP_A results in the relation

$$M_{i-1} - M_i + V_A h + P_i \left[\frac{w_i - w_{i-1}}{2}\right] + P_{i-1} \left[\frac{w_i - w_{i-1}}{2}\right] = 0 \qquad (3.7)$$

A similar summation of moments about the centroid of Bar B gives

$$M_i - M_{i+1} + V_B h + P_{i+1} \left[\frac{w_{i+1} - w_i}{2}\right] + P_i \left[\frac{w_{i+1} - w_i}{2}\right] = 0 \qquad (3.8)$$

Equations 3.7 and 3.8 may be solved for V_A and V_B . Substituting these values into Eq 3.6 gives

$$M_{i-1} - 2M_i + M_{i+1} = h(Q_i - S_i w_i) - \tfrac{1}{2} (P_{i-1} + P_i) (w_i - w_{i-1})$$

$$+ \tfrac{1}{2} (P_i + P_{i+1}) (w_{i+1} - w_i) \qquad (3.9)$$

If Eq 3.5, the relation between bending moment and model deformation, is substituted three times into the left side of Eq 3.9, collecting terms produces the form

$$a_i w_{i-2} + b_i w_{i-1} + c_i w_i + d_i w_{i+1} + e_i w_{i+2} = f_i \qquad (3.10)$$

where

$$a_i = F_{i-1} \qquad (3.11)$$

$$b_i = -2 \left[F_{i-1} + F_i + \frac{h^2}{4} (P_{i-1} + P_i)\right] \qquad (3.12)$$

$$c_i = F_{i-1} + 4F_i + F_{i+1} + \frac{h^2}{2} (P_{i-1} + 2P_i + P_{i+1}) + h^3 S_i \qquad (3.13)$$

$$d_i = -2 \left[F_i + F_{i+1} + \frac{h^2}{4} (P_i + P_{i+1})\right] \qquad (3.14)$$

$$e_i = F_{i+1} \qquad (3.15)$$

and

$$f_i = h^3 Q_i \tag{3.16}$$

An equation having the form of Eq 3.10 may be written at each station of the model beam-column. It should be noted that no assumptions were made concerning variation of the parameters F , Q , and S ; also, P was assumed only to vary linearly across each rigid bar. Thus, while actual discontinuities in F , Q , S , and P may not be considered, there is no limitation on the increment point by increment point variation of the values F, Q , S , and P which define model behavior. If F and P are considered constant, and if $Q = hq$ and $S = hk$, Eq 3.10 reduces to the conventional finite-difference relationship of Eq 3.2.

Error of Approximation

The error in the use of Eq 3.10 to describe actual beam-column behavior may be thought of as the difference between the finite-element model of Fig 3.1a and the actual beam-column it simulates. If Eq 3.10 had been derived from a differential equation allowing variation of F and P as functions of position x by manipulation of finite-difference relationships (Ref 18), the error in such a procedure would be (1) the error in assuming real beam-column behavior to be described by a differential equation and (2) the error involved in replacing a differential equation by an appropriate difference equation.

The error in defining real beam-column behavior by a differential equation is not completely known, but in classical beam mechanics it is usually assumed to be negligible when beam-column deflections are small compared to beam-column length.

On a comparative basis, for reasonable choices of increment length h , the finite-element model has yielded values of deflection within one per cent of those computed by classical beam mechanics. The relationship describing axial compression has been found to predict buckling within 0.5 per cent of the critical Euler load for both constant and variable axial force.

Solution of the Beam-Column Equations

If Eq 3.10 is written at each Station i of the finite-element model, a set of simultaneous equations is produced. This set of equations may be represented as

$$Aw = f \tag{3.17}$$

where

A = quidiagonal stiffness matrix of the coefficients a_i, b_i, c_i, d_i, and e_i,

w = column matrix of unknown deflections w_i, and

f = column load matrix of the f_i terms.

Any quidiagonal matrix with non-zero diagonal elements may be efficiently solved by a special form of Gaussian elimination using the relation

$$w_i = A_i + B_i w_{i+1} + C_i w_{i+2} \tag{3.18}$$

where

A_i, B_i, C_i = coefficients computed from known stiffness, load, and restraint information.

The derivation of Eq 3.18 and the related coefficients have been presented elsewhere (Ref 18).

The procedure for development of equations at the ends of the model has also been presented elsewhere (Ref 18). In effect, boundary conditions for a free end are produced by the application of Eq 3.10 at the end stations and at an imaginary station with zero F located a distance h from each end of the member. A support may be approximated by specifying a large value of S at a station. Procedures are also available for exact specification of values of slope and deflection at any point along the model beam-column (Ref 18).

The process is summarized in Fig 3.2. Figure 3.2a shows the finite-element model under the action of applied loads and restraints. Equations describing the beam-column are developed which form the quidiagonal stiffness matrix and column load matrix shown in Figs 3.2b and 3.2c. Figure 3.2d describes the recursive-type elimination process and Fig 3.2e shows the desired result: the deflected shape of the beam-column under the applied loads and restraints. Finite-difference relationships may then be used to calculate any of the four derivatives of beam-column deflection.

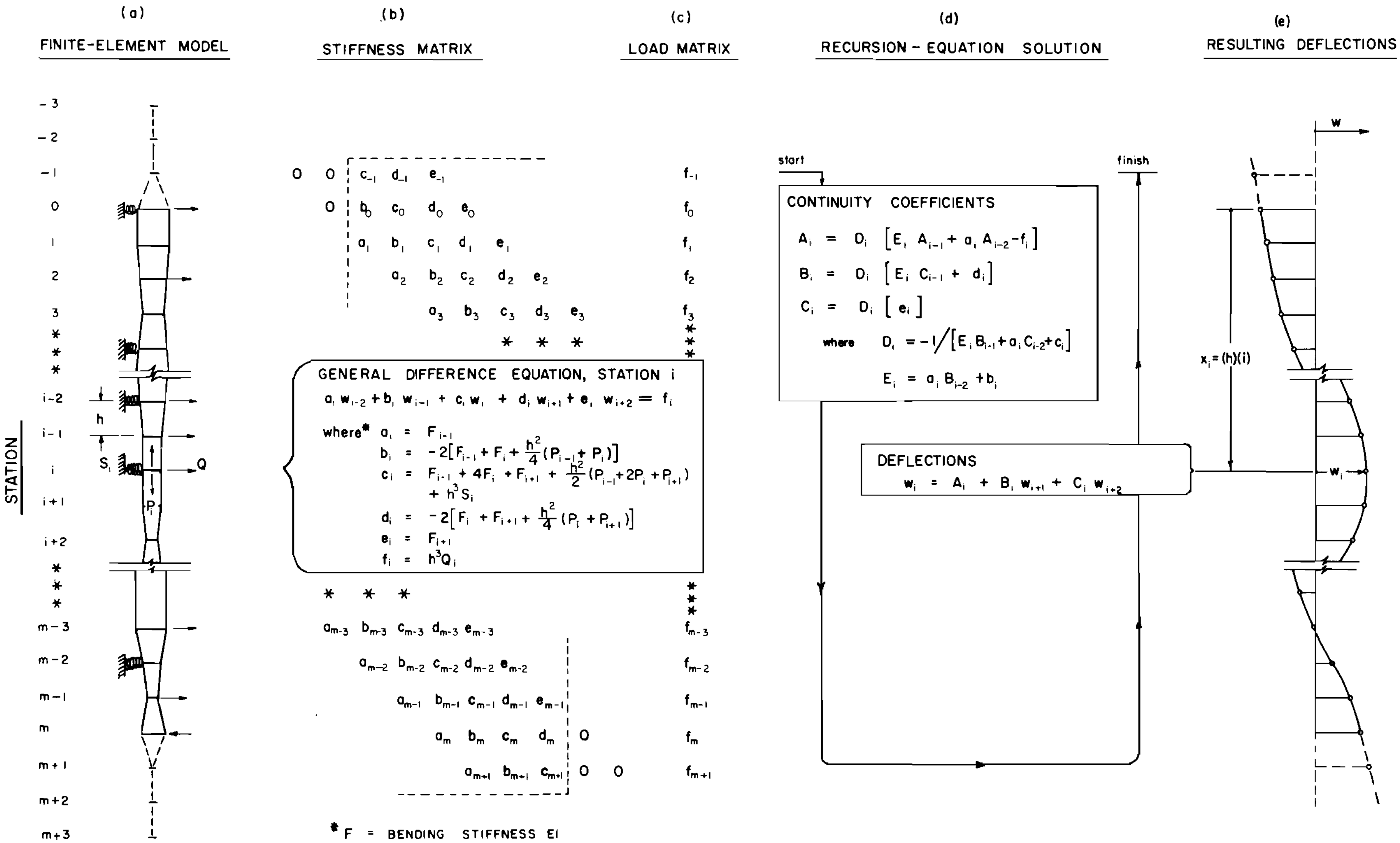


Fig 3.2. Summary of the method of model beam-column solution.

Summary

In this chapter, an efficient numerical procedure for the solution of a finite-element model approximating a real beam-column on elastic foundation has been developed.

This procedure is shown to remain valid for the interior segments of members in a finite-element model of a rectangular plane frame developed in the following chapter. The equations for a member in the vicinity of a frame joint are modified to include the interaction of all frame members.

CHAPTER 4. DEVELOPMENT OF A PROCEDURE FOR THE BENDING ANALYSIS OF A PLANE FRAME

In the previous chapter a numerical procedure for the analysis of a beam-column on an elastic foundation was developed. This chapter is concerned with the development of equations for the iterative analysis of a plane frame in bending. This development is accomplished in three parts: (1) derivation of equations describing a finite-element model of a frame joint, (2) integration of these equations with those describing the members which connect frame joints, and (3) indication of an iterative procedure for the member-by-member solution of the frame system, with member interaction effects being adjusted during each cycle of the iterative process.

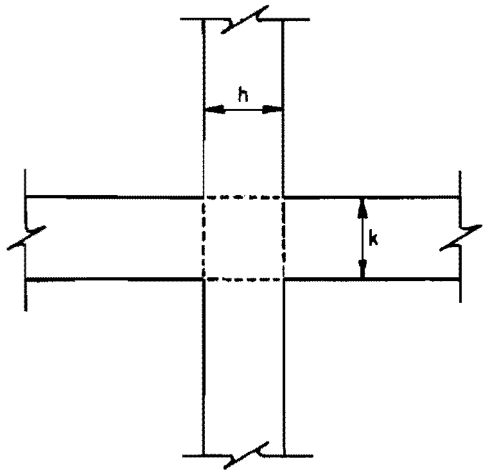
The determination and distribution of axial tension or compression in the frame members is discussed in the following chapter. The "bending" solution developed in this chapter and the "axial" solution to be developed in the following chapter are combined in Chapter 6 to give a complete method of rectangular plane-frame analysis.

Selection of a Finite-Element Frame-Joint Model

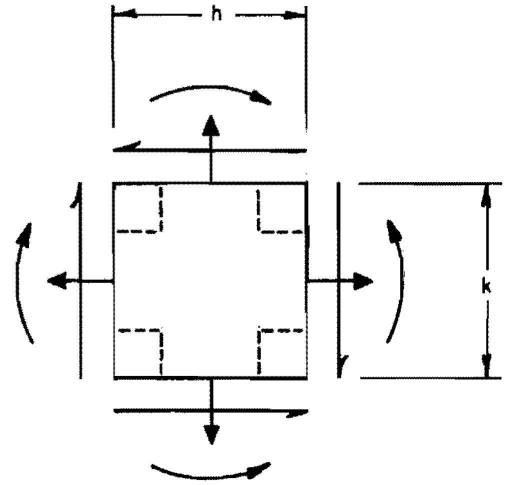
In Chapter 3, a finite-element model of a real beam-column was presented and equations describing its behavior were developed. A similar procedure will be used to develop equations describing model frame-joint behavior.

Figure 4.1a shows a frame joint formed by the right-angle intersection of two frame members. This frame joint obviously has some width in the horizontal and vertical directions. Let h denote the joint width in the horizontal or x-direction and k the joint width in the vertical or y-direction. Figure 4.1b shows the actual frame joint in equilibrium under the action of internal moments and shears. It should be noted that, contrary to the line-member theory of frame analysis, the resisting shears must be considered in equations for joint-moment equilibrium.

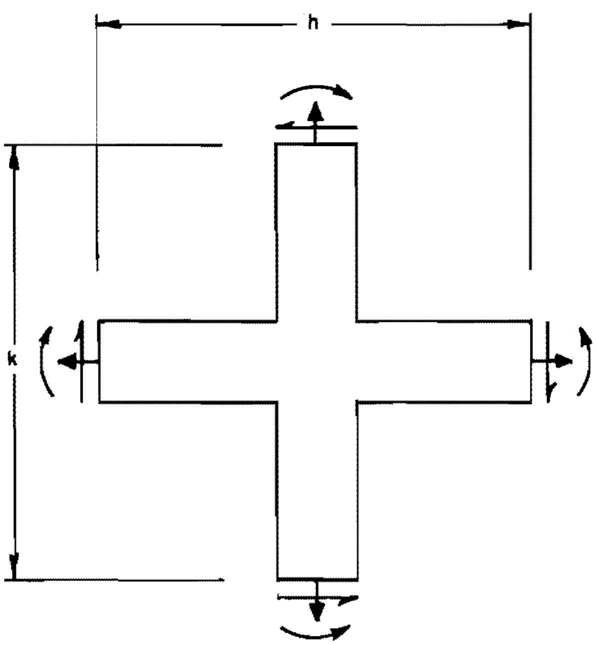
If the joint of Fig 4.1b is assumed to be rigid, one rational finite-element model of the real joint would be that shown in Fig 4.1c, composed of two rigid bars connected at right angles. In effect, only the corners of the joint of Fig 4.1b have been removed.



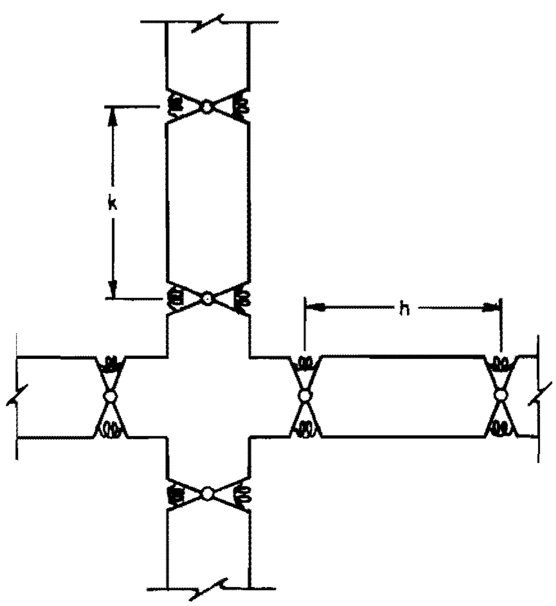
(a) FRAME JOINT FORMED BY RIGHT-ANGLE INTERSECTION OF FOUR FRAME MEMBERS



(b) FRAME JOINT IN EQUILIBRIUM UNDER THE ACTION OF INTERNAL MOMENTS AND SHEARS



(c) PROPOSED FINITE-ELEMENT MODEL OF RIGID FRAME JOINT



(d) PROPOSED FINITE-ELEMENT MODEL OF FRAME JOINT AND CONNECTING MEMBERS

Fig 4.1. Development of a finite-element model frame joint.

The frame has been assumed to consist of connected beam-columns. Thus, if the finite-element model of Chapter 3 is used to simulate these frame members, the model joint can easily connect the beam-columns as shown in Fig 4.1d. The resulting member and joint system may be visualized as two connected beam-columns, with member interaction being transferred through the rigid joint. Figure 4.2 shows the model frame joint and connecting members in greater detail.

With the selected model frame joint of Fig 4.2 in mind, consideration should now be given to the establishment of a consistent sign convention for the externally applied forces, couples, and restraints which may act on this joint.

Establishment of a Consistent Sign Convention

In order to correctly determine the effects of member interaction, a consistent sign convention must be developed for the internal and external forces and couples acting on the frame. The sign convention to be established is similar to that defined in Chapter 3 for a single beam-column.

Let each horizontal line of members in the frame be divided into a finite number of increments numbered from left to right starting with Station 0 and ending with Station m_x. Let each vertical line of members in the frame be divided into a finite number of increments numbered from top to bottom starting with Station 0 and ending with Station m_y.

For the horizontal lines of members, positive load, either internal or external, is defined to act in an upward direction. Positive transverse deflection for the horizontal lines, as well as positive axial displacement for the vertical lines, is also assumed to be positive upward. A positive couple is assumed to act clockwise, while positive slope is measured counterclockwise from the horizontal axis. This convention is shown in Fig 4.3a.

For the vertical lines of members, positive load, either internal or external, is defined to act to the right. Positive transverse deflection for the vertical lines, as well as positive axial displacement for the horizontal lines, is also assumed to be positive to the right. Again, a positive couple is assumed to act clockwise, while positive slope is measured counterclockwise from the vertical axis. This convention is shown in Fig 4.3b.

A different sign convention must be used for the internal axial tension or compression acting on the frame members. For the ordering assumed, Fig 4.3c shows positive axial tension acting on a rigid bar taken from a horizontal line

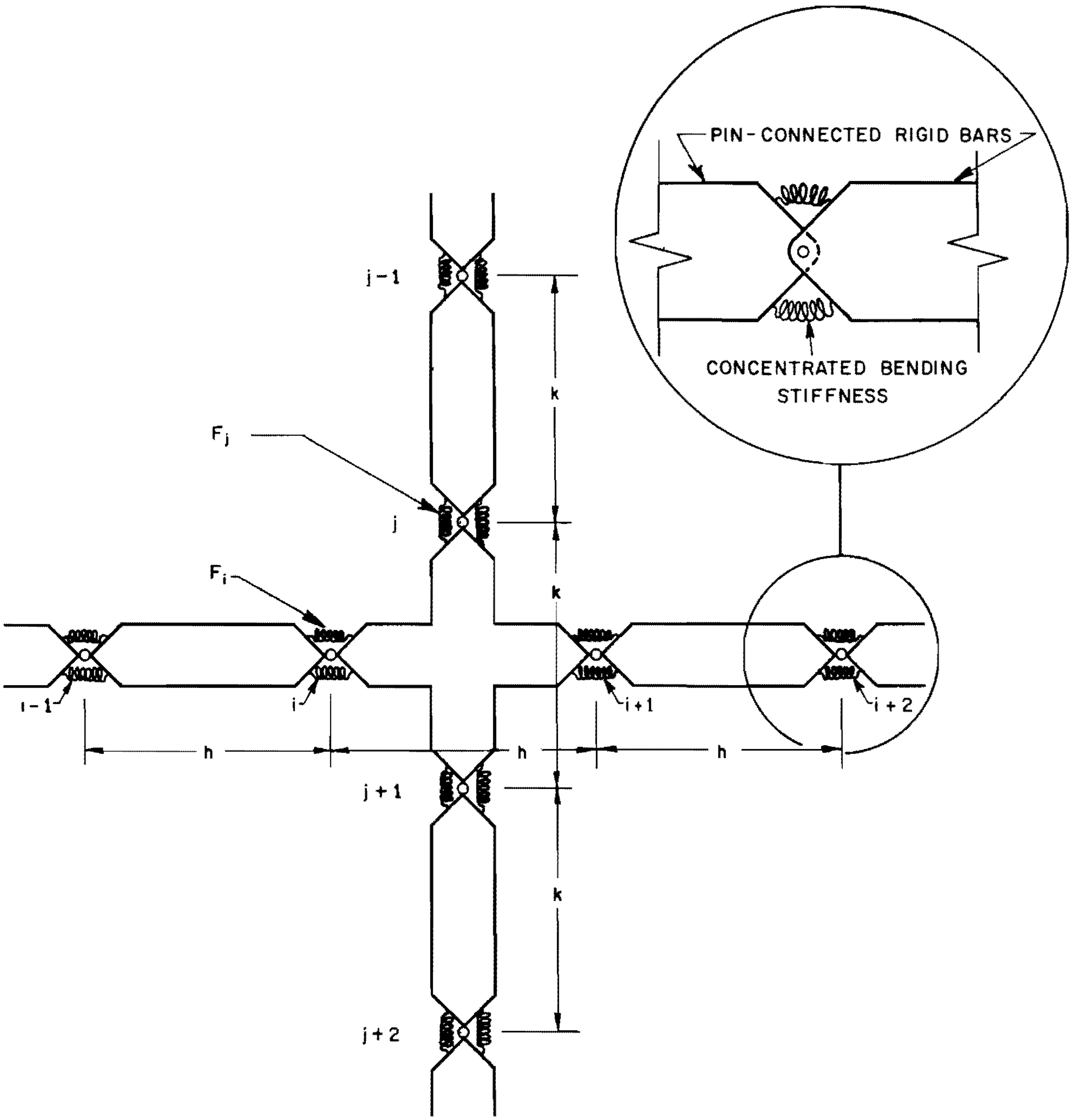
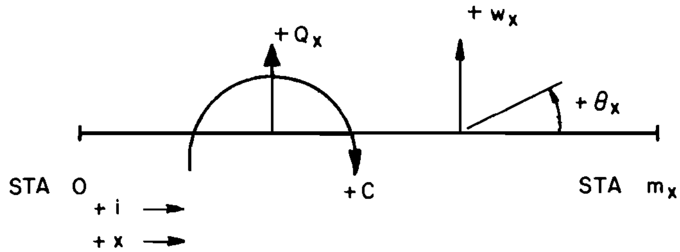
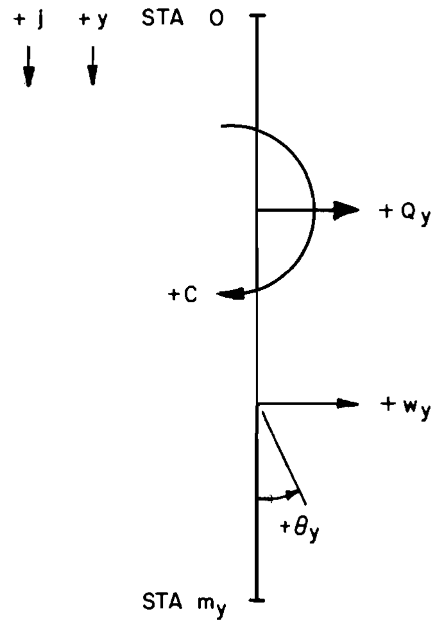


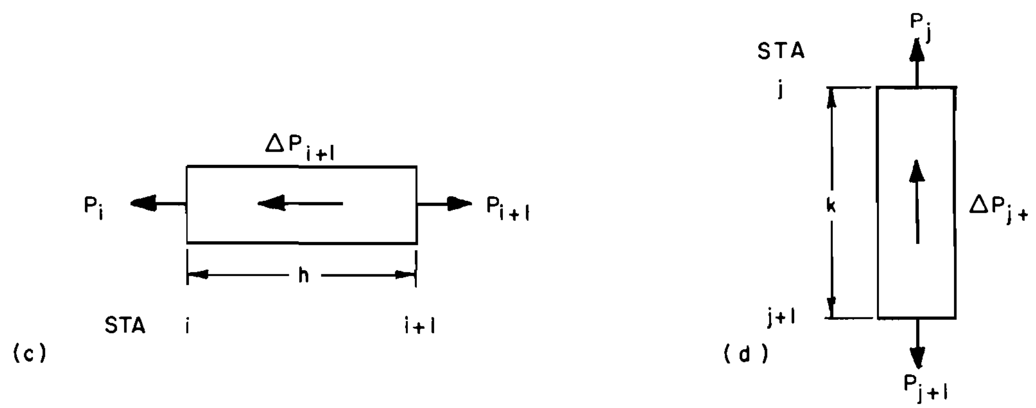
Fig 4.2. The finite-element model frame joint and connecting members.



(a) ORDERING AND SIGN CONVENTION FOR HORIZONTAL FRAME MEMBERS



(b) ORDERING AND SIGN CONVENTION FOR VERTICAL FRAME MEMBERS



SIGN CONVENTION FOR POSITIVE AXIAL TENSION IN HORIZONTAL AND VERTICAL MEMBERS

Fig 4.3. Definition of a consistent sign convention for the frame.

of model frame members. The stations to the left and right of the bar are denoted by i and $i+1$. The positive change in axial tension acting in the bar is denoted by ΔP_{i+1} . It should be noted that this change in axial tension acts in the direction opposite to that assumed for positive horizontal loads.

Figure 4.3d shows positive axial tension acting on a rigid bar taken from a vertical line of model frame members. The axial tension acting on the bar is defined in a manner similar to that of Fig 4.3c. For this case, however, the change in positive axial tension acts in the same direction as that assumed for positive vertical loads.

Possible External Effects Acting on the Model Frame Joint

Figure 4.4a shows the various types of external effects which might act on the model frame joint. Loads Q_x and Q_y , as well as springs S_x and S_y , act normal to the x (horizontal) and y (vertical) parts of the joint. C is an external couple applied to the joint, while R is an external rotational restraint applied to the joint. These effects are shown acting in the positive sense.

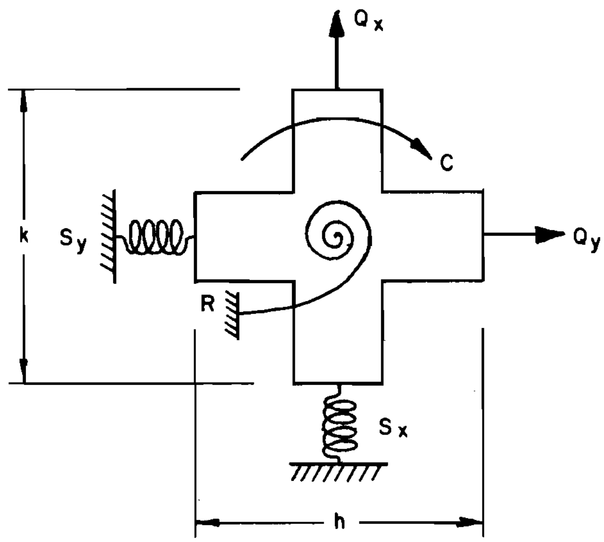
In order to develop equations describing joint behavior, it will be assumed that the joint may be split into x and y-halves. This assumption is valid as long as (1) forces, couples, and restraints acting on the missing half of the joint are applied to the half being considered, (2) the restraint against translation and rotation provided by the other half of the joint is considered, and (3) consistent deformations and rotations are enforced for both halves of the joint. In effect, the joint will be taken apart for efficient iterative analysis, but will be put together in the final solution.

Under this hypothesis, consider the x-half of the joint as shown in Fig 4.4b. The external forces, couples, and restraints acting on this half of the joint are

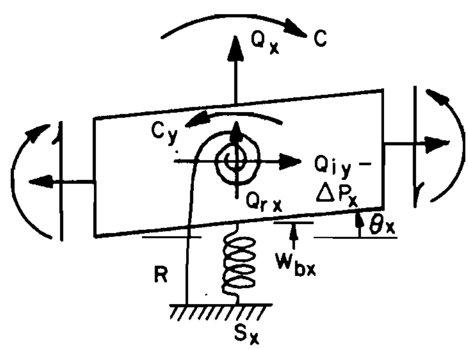
(1) Q_x and S_x , the external load and spring restraint applied normal to the x-half of the joint,

(2) C and R , the external couple and rotational restraint applied to the joint as a whole,

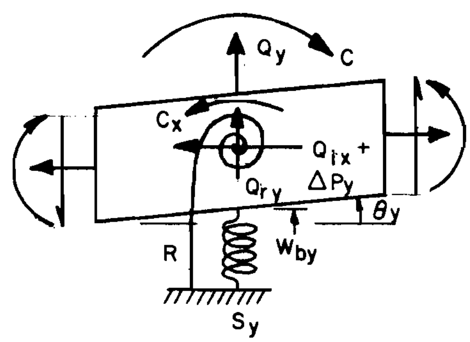
(3) Q_{rx} , a resultant load representing the effect of the missing column and other horizontal members of the frame,



(a) POSSIBLE EXTERNAL TRANSVERSE AND ANGULAR EFFECTS ACTING ON THE MODEL JOINT



(b) FORCES, COUPLES, AND RESTRAINTS ACTING ON THE X-HALF OF THE MODEL JOINT



(c) FORCES, COUPLES, AND RESTRAINTS ACTING ON THE Y-HALF OF THE MODEL JOINT (ROTATED 90° COUNTER-CLOCKWISE)

Fig 4.4. Forces, couples, and restraints acting on the model frame joint.

(4) Q_{iy} , the change in axial tension or compression produced by the crossing y-member, and

(5) C_y , the couple absorbed by the y-half of the joint. Also, W_{bx} and θ_x are respectively the transverse deflection and slope of the x-half of the joint.

Figure 4.4c shows the y-half of the joint under consideration. The forces, couples, and restraints acting on this half of the joint are defined in a manner similar to those above.

Equations describing the behavior of each half of the frame joint may now be derived. In this analysis, it will be assumed that the axial tension or compression distribution in all frame members is known. Procedures for computing this distribution will be developed in Chapter 5.

Resultant Forces Acting on Each Half of the Joint

From Fig 4.4b, the resultant load acting on the x-half of the joint is

$$Q_{rx} = Q_{bx} - S_{bx}W_{bx} \tag{4.1}$$

where Q_{bx} and S_{bx} are values of load and support which represent the missing vertical line of members passing through the joint. These values may be determined from frame stiffness, geometry, and loadings.

Consider the simple frame of Fig 4.5a. The frame is loaded by a constant axial tension P applied along the axis of the axially rigid column. Resistance to column displacement is provided by the three supporting beams. From simple beam mechanics, the resistance furnished by each beam at its joint is given by the ratio of applied load P to resulting displacement Δ :

$$\frac{P}{\Delta} = \frac{48F}{L^3} \tag{4.2}$$

If axial rigidity is assumed, the total resistance to column displacement is

$$S_{cx} = 3\frac{P}{\Delta} = \frac{144F}{L^3} \tag{4.3}$$

The total load acting directly on the column is P plus the sum of load or reaction contributed by each of the crossing beams. In this case, the load

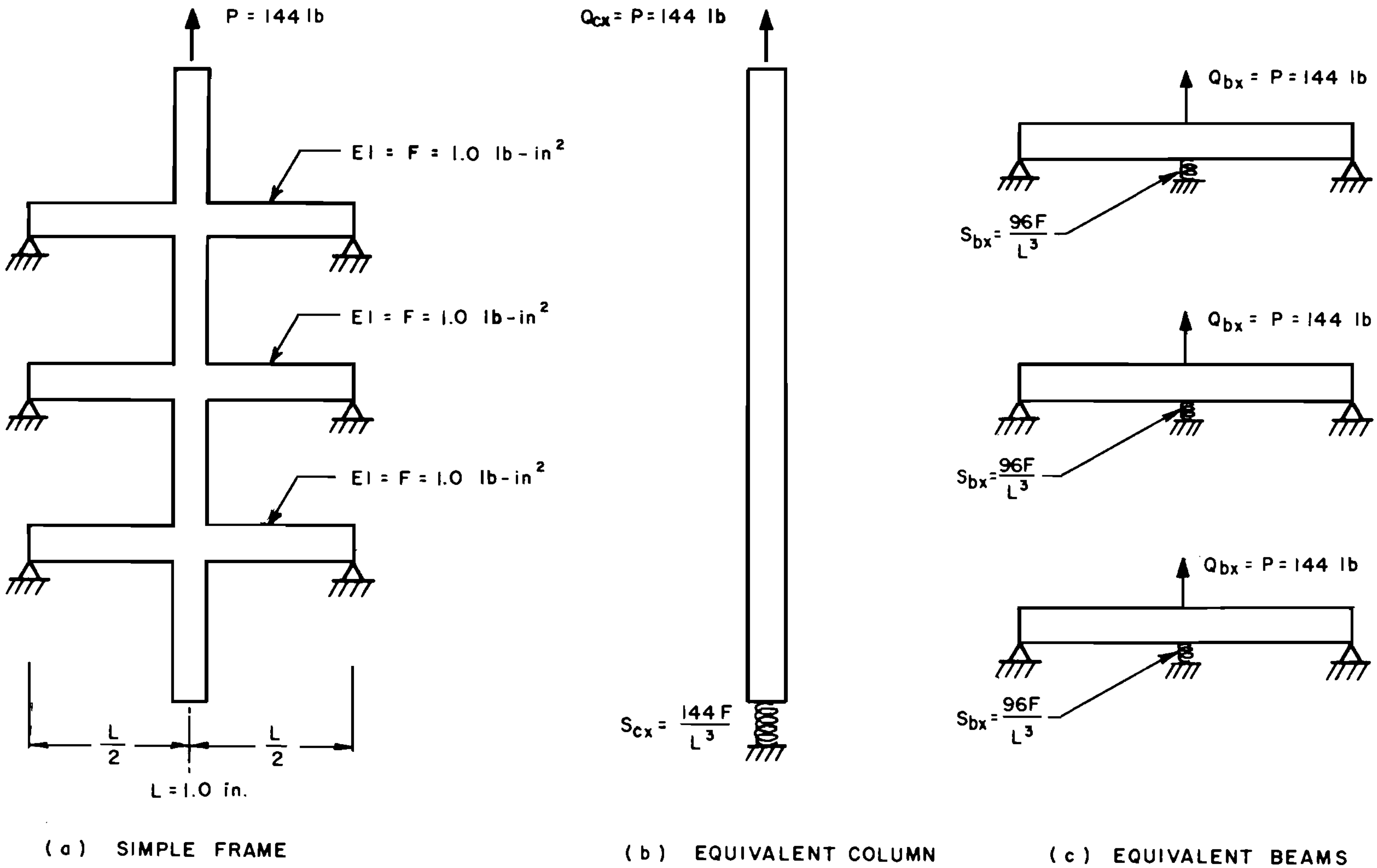


Fig 4.5. Simple frame used to demonstrate the method of translation analysis.

contributed by each beam is zero, and the total load is

$$Q_{cx} = P + 0 + 0 + 0 \tag{4.4}$$

The system of Fig 4.5a may now be divided into components: either a column, as shown in Fig 4.5b, or three individual beams, as shown in Fig 4.5c. In either case, the load and restraint provided by the missing components are applied to the component under consideration. The deflected shape of each beam could now be determined by the procedure developed in Chapter 3. For this simple case the effects of member interaction are expressed by the load and restraint applied at the joint on each beam.

A procedure for the line-by-line solution of the frame, considering translation only, may now begin to be visualized. Each horizontal line of frame members may be solved individually, with the effect of member interaction represented at each joint by a force or load Q_{bx} and a restraint or spring S_{bx} . The load Q_{bx} should represent the total of all vertical loads applied at other joints directly above and below the one being considered plus all loads applied directly to the vertical column passing through the joint. The spring S_{bx} should represent the total restraint provided by other horizontal lines of members at joints above and below the one in question, plus any restraint applied directly to the column. A similar procedure may be followed for vertical members.

The values Q_{bx} and S_{bx} for each joint may be defined in terms of the total load and restraint applied to the axially rigid column. Consider the column of Fig 4.6a. This column is crossed by horizontal members, forming joints, at the locations $\ell = 1, 2, \cdots, N$. Figure 4.6b shows the axially rigid column displaced under the action of applied loads and restraints. These values are

(1) Q_{x_ℓ} , $(\ell = 1, 2, \cdots, N)$, the vertical loads applied directly to each joint,

(2) S_{x_ℓ} , $(\ell = 1, 2, \cdots, N)$, the vertical restraint applied directly to each joint,

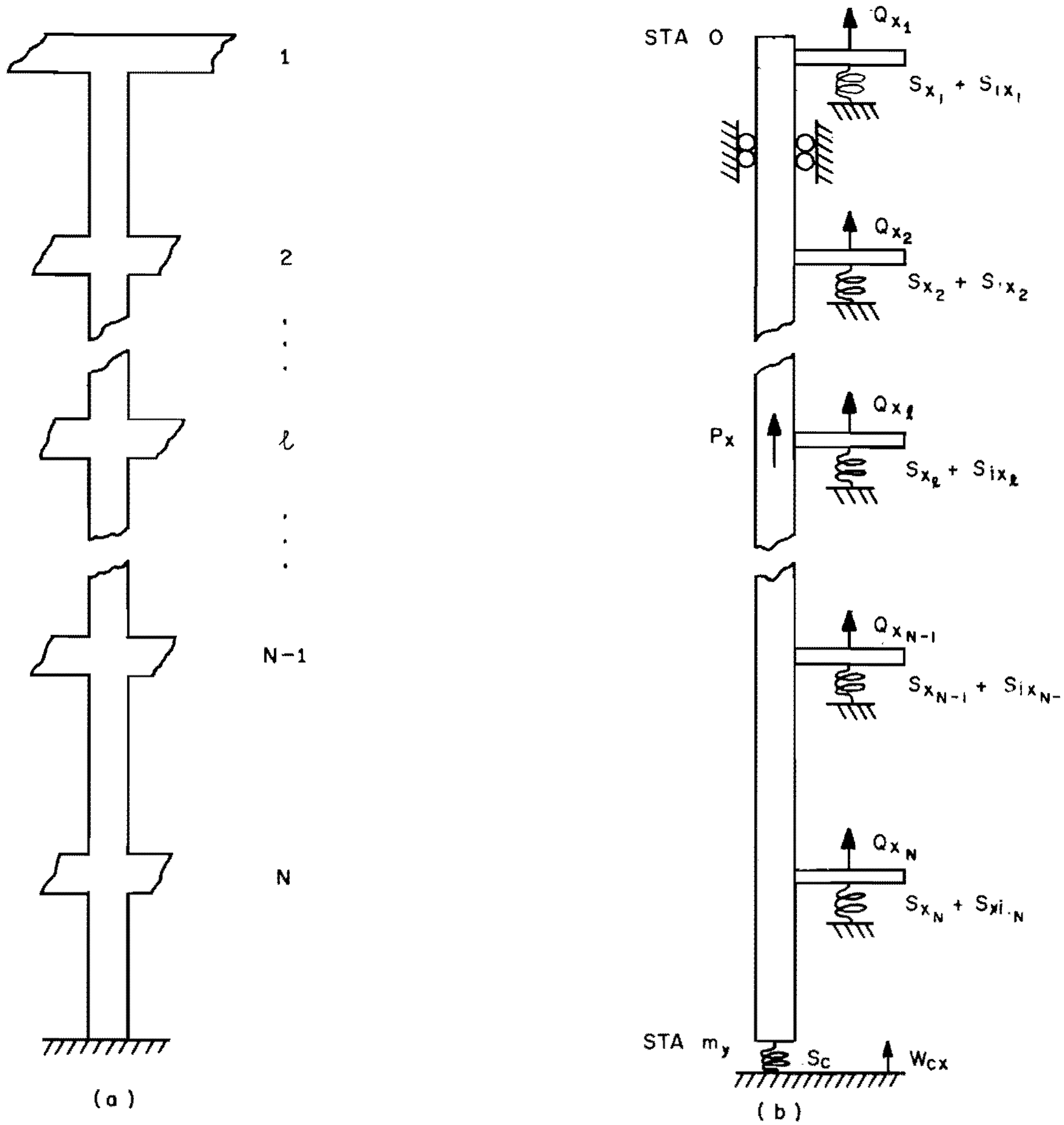


Fig 4.6. Loads and restraints acting on axially rigid vertical members.

(3) S_{ix_ℓ} , (ℓ = 1, 2, ⋯, N) , the intrinsic restraint of the crossing beam at each joint,

(4) P_x , the resultant of internal axial tension or compression acting on the column, and

(5) S_c , the restraint applied at the bottom of the column.

The total load acting on the column is thus

$$Q_{cx} = P_x + \sum_{\ell=1}^{N} Q_{x_\ell} \tag{4.5}$$

where

$$P_x = \sum_{j=0}^{m_y+1} \Delta P_j \tag{4.6}$$

and the total restraint acting on the column is

$$S_{cx} = S_c + \sum_{\ell=1}^{N} (S_{x_\ell} + S_{ix_\ell}) \tag{4.7}$$

such that the displacement of the axially rigid column is given by

$$W_{cx} = \frac{Q_{cx}}{S_{cx}} \tag{4.8}$$

For any joint ℓ , the load and restraint which represent the rest of the system are

$$Q_{bx_\ell} = Q_{cx} - Q_{x_\ell} \tag{4.9}$$

and

$$S_{bx_\ell} = S_{cx} - S_{x_\ell} - S_{ix_\ell} \tag{4.10}$$

The corresponding relations for vertical members are shown in Figs 4.7a and 4.7b. In this case, P_y , the resultant of internal axial tension or compression, acts in a sense opposite to that of the joint loads Q_y . For this case, the total load acting on the axially rigid beam is

$$Q_{cy} = - P_y + \sum_{\ell=1}^{M} Q_{y_\ell} \tag{4.11}$$

where

$$P_y = \sum_{i=0}^{m_x+1} \Delta P_i \tag{4.12}$$

and the total restraint acting on the beam is

$$S_{cy} = S_c + \sum_{\ell=1}^{M} (S_{y_\ell} + S_{iy_\ell}) \tag{4.13}$$

such that the displacement of the axially rigid beam is

$$W_{cy} = \frac{Q_{cy}}{S_{cy}} \tag{4.14}$$

Again, at any joint ℓ , the load and restraint which represent the rest of the system are

$$Q_{by_\ell} = Q_{cy} - Q_{y_\ell} \tag{4.15}$$

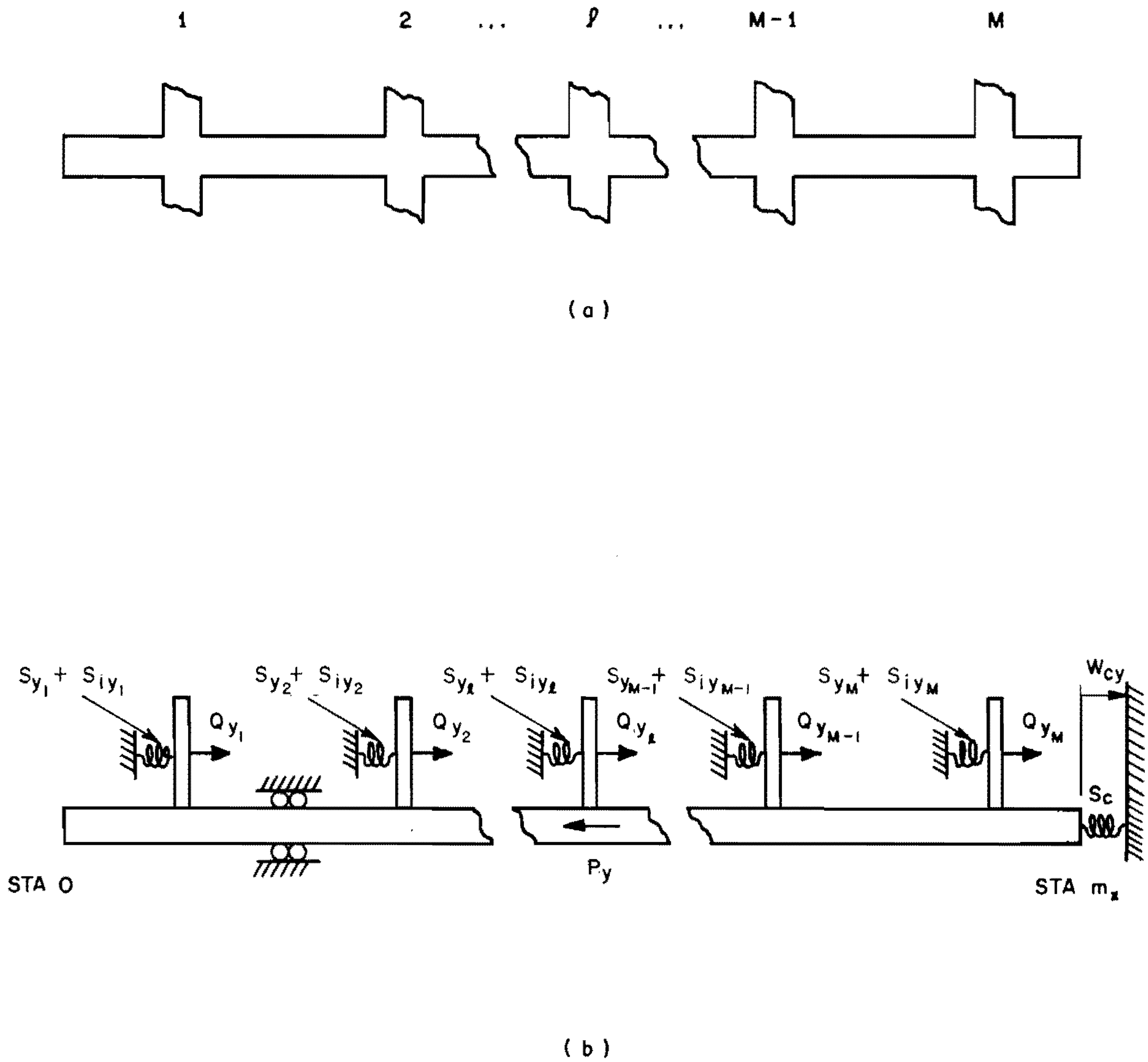


Fig 4.7. Loads and restraints acting on axially rigid horizontal members.

and

$$S_{by_\ell} = S_{cy} - S_{y_\ell} - S_{iy_\ell} \qquad (4.16)$$

All values except the intrinsic spring constants S_{ix} and S_{iy} are known from data describing frame loading and restraint. A method for computing the S_{ix} and S_{iy} values will be discussed later.

Resultant Couples Acting on Each Half of the Joint

From Fig 4.4b, the resultant couple acting on the x-half of the joint is

$$C_x = (C + R\theta_x) - C_y \qquad (4.17)$$

where

C_y = couple absorbed by the missing y-half of the joint.

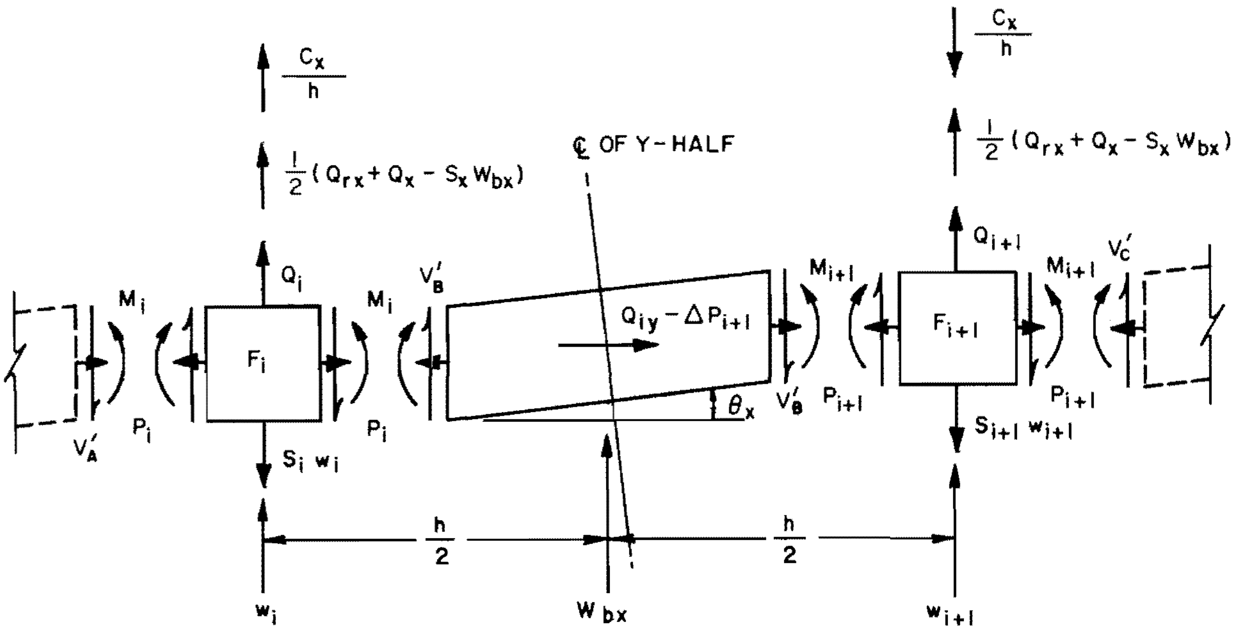
The corresponding relation for the y-half of the joint, from Fig 4.4c is

$$C_y = (C + R\theta_y) - C_x \qquad (4.18)$$

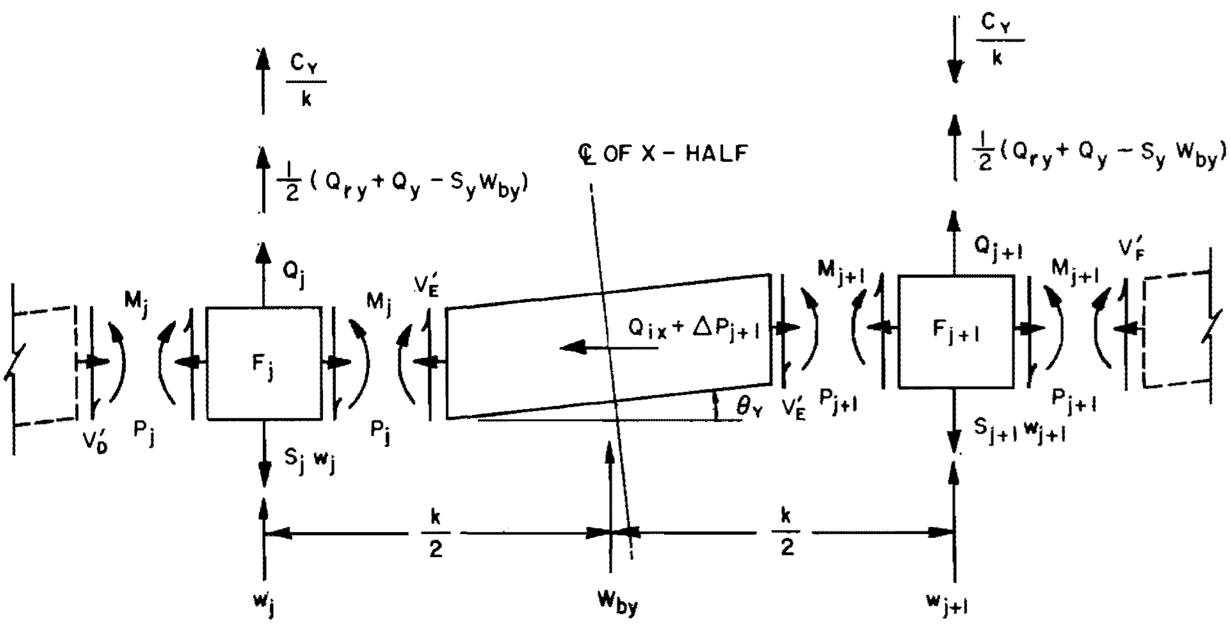
In this case, the values C and R are known from data describing frame loading and restraint. Procedures for computing values of C_x and C_y will be discussed later.

Derivation of Equations from Half-Joint Free Bodies

Figure 4.8a shows a free-body diagram of the x-half of the joint and the stations or increment points i and i+1 on either side of the joint. These stations mark the boundaries between the ends of the joint and the ends of the members which frame into the joint from either side. This free-body diagram is similar to that of Fig 3.1c for the finite-element model beam-column. The resultant couple C_x acting on the x-half of the joint is applied as two equal and opposite loads $\pm \frac{C_x}{h}$ at Stations i and i+1. The reaction Q_{rx} acting on the x-half of the joint has also been split equally to Stations i and i+1, as has the resultant of external load and restraint, $Q_x - S_x W_{bx}$. The direction of the arrows in the figure indicates the sense of the applied positive loadings.



(a) FREE BODY DIAGRAM OF X - HALF OF FRAME JOINT



(b) FREE BODY DIAGRAM OF Y-HALF OF FRAME JOINT (ROTATED 90° COUNTER-CLOCKWISE)

Fig 4.8. Free-body diagrams of the model frame joint.

From Fig 4.8a, it is also apparent that

$$W_{bx} = \frac{1}{2}(w_i + w_{i+1}) \tag{4.19}$$

and

$$\theta_x = \frac{1}{h}(w_{i+1} - w_i) \tag{4.20}$$

Figure 4.8b shows the corresponding free-body diagram for the y-half of the joint. The relations for joint deflection and slope are

$$W_{by} = \frac{1}{2}(w_j + w_{j+1}) \tag{4.21}$$

and

$$\theta_y = \frac{1}{k}(w_{j+1} - w_j) \tag{4.22}$$

If the deflected shape of the x-half of the joint and the members framing into it are known, as they would be from a previous cycle of the assumed iterative process, a finite-difference relationship may be used to compute the resultant forces acting at Stations i and i+1. This relation gives, from Fig 4.8a,

$$\frac{d^2}{dx^2}\left[F\frac{d^2w}{dx^2}\right]_i = \frac{1}{h}\Big[(Q_i - S_i w_i) + \frac{C_x}{h} + \frac{1}{2}(Q_{rx} + Q_x - S_x W_{bx})$$

$$- \frac{1}{2h}(P_{i-1} + P_i)(w_i - w_{i-1}) + \frac{1}{2h}(P_i + P_{i+1})$$

$$(w_{i+1} - w_i)\Big] \tag{4.23}$$

at Station i, and

$$\frac{d^2}{dx^2}\left[F\frac{d^2w}{dx^2}\right]_{i+1} = \frac{1}{h}\Big[(Q_{i+1} - S_{i+1}w_{i+1}) - \frac{C_x}{h} + \frac{1}{2}(Q_{rx} + Q_x - S_x W_{bx})$$

(Equation cont'd)

$$- \frac{1}{2h} (P_i + P_{i+1})(w_{i+1} - w_i) + \frac{1}{2h} (P_{i+1} + P_{i+2})$$

$$(w_{i+2} - w_{i+1})\Big] \qquad (4.24)$$

at Station i+1.

Adding Eqs 4.23 and 4.24 to eliminate $\frac{C_x}{h}$ and solving for the net vertical reaction at the joint gives

$$(Q_{rx} + Q_x - S_x W_{bx}) = h \left\{\frac{d^2}{dx^2}\left[F \frac{d^2w}{dx^2}\right]_i + \frac{d^2}{dx^2}\left[F \frac{d^2w}{dx^2}\right]_{i+1}\right\} - (Q_i - S_i w_i)$$

$$- (Q_{i+1} - S_{i+1} w_{i+1}) + \frac{1}{2h}\Big[(P_{i-1} + P_i)(w_i$$

$$- w_{i-1}) - (P_{i+1} + P_{i+2})(w_{i+2} - w_{i+1})\Big] \qquad (4.25)$$

while subtracting Eq 4.24 from Eq 4.23 to eliminate the net vertical reaction and solving for C_x gives

$$C_x = \frac{h^2}{2} \left\{\frac{d^2}{dx^2}\left[F \frac{d^2w}{dx^2}\right]_i - \frac{d^2}{dx^2}\left[F \frac{d^2w}{dx^2}\right]_{i+1}\right\} - \frac{h}{2}\Big\{(Q_i - S_i w_i) - (Q_{i+1}$$

$$- S_{i+1} w_{i+1})\Big\} + \frac{1}{4}\Big[(P_{i-1} + P_i)(w_i - w_{i-1}) - 2(P_i + P_{i+1})$$

$$(w_{i+1} - w_i) + (P_{i+1} + P_{i+2})(w_{i+2} - w_{i+1})\Big] \qquad (4.26)$$

The corresponding expressions for the y-half of the joint are

$$(Q_{ry} + Q_y - S_y W_{by}) = k\left\{\frac{d^2}{dy^2}\left[F \frac{d^2w}{dy^2}\right]_j + \frac{d^2}{dy^2}\left[F \frac{d^2w}{dy^2}\right]_{j+1}\right\} - (Q_j - S_j w_j)$$

(Equation cont'd)

$$- (Q_{j+1} - S_{j+1}w_{j+1}) + \frac{1}{2k}\Big[(P_{j-1} + P_j)(w_j - w_{j-1}) - (P_{j+1} + P_{j+2})(w_{j+2} - w_{j+1})\quad (4.27)$$

and

$$C_y = \frac{k^2}{2}\left\{\frac{d^2}{dy^2}\left[F\,\frac{d^2w}{dy^2}\right]_j - \frac{d^2}{dy^2}\left[F\,\frac{d^2w}{dy^2}\right]_{j+1}\right\} - \frac{k}{2}\Big\{(Q_j - S_jw_j) - (Q_{j+1} - S_{j+1}w_{j+1})\Big\} + \frac{1}{4}\Big[(P_{j-1} + P_j)(w_j - w_{j-1}) - 2\,(P_j + P_{j+1})(w_{j+1} - w_j) + (P_{j+1} + P_{j+2})(w_{j+2} - w_{j+1})\quad (4.28)$$

<u>Determination of Translational Restraint Provided by Each Half of the Joint</u>

In previous sections, relations were derived for the resultant forces and couples acting on each half of a frame joint. These relations required knowledge of intrinsic values of beam translational and rotational restraint at each joint. Relations for these values are developed in this section.

The intrinsic translational restraint provided by a beam at any particular joint is given by the ratio of net beam reaction to beam deflection. Thus, for a joint on any horizontal member, using the relations of Eqs 4.19 and 4.25,

$$S_{ix} = \frac{(Q_{rx} + Q_x - S_xW_{bx})}{W_{bx}} \qquad (4.29)$$

where

$$W_{bx} \neq 0$$

For vertical members, using Eqs 4.21 and 4.27,

$$S_{iy} = \frac{(Q_{ry} + Q_y - S_yW_{by})}{W_{by}} \qquad (4.30)$$

where

$$W_{by} \neq 0$$

The restraints defined by Eqs 4.29 and 4.30 may be positive, zero, or negative. The concept of a negative translational restraint is difficult to visualize, except in an abstract manner, since its use might create instability under some conditions. This instability may be avoided by substituting the negative of the net reaction $-(Q_{rx} + Q_x - S_x W_{bx})$ or $-(Q_{ry} + Q_y - S_y W_{by})$ as a force representing beam resistance. The negative sign results from the fact that beam and column reactions are equal and opposite. In effect, the replacement of the negative restraint by the negative of the net reaction increases the total load Q_{cx} or Q_{cy} acting on a line of joints instead of reducing the total restraint S_{cx} or S_{cy}.

The above procedure is valid unless at some time during the iterative process all computed restraints for any one line of joints become negative. In such a case, if the column restraint S_c and all joint restraints S_x or S_y are zero, an infinite column displacement would be computed. Furthermore, each joint would be subjected to a large applied load instead of a combination of loads and restraint. Such a condition might also cause instability. These problems may be avoided by introducing at each joint a differential restraint $(\eta F_x/h^3)$ or $(\eta F_y/k^3)$ into the equations for total load Q_{cx} or Q_{cy} and for restraint S_{cx} or S_{cy} acting on the column and line of joints. The revised equations for the total load and restraint provided by all joints and the column are

$$S_{cx} = S_c + \sum_{\ell=1}^{N} (S_{x_\ell} + S_{r_\ell}) \tag{4.31}$$

where

$$S_{r_\ell} = S_{ix_\ell}, \text{ for } S_{ix_\ell} > 0$$

(Equation cont'd)

$$S_{r_\ell} = \eta \frac{F_x}{h^3} , \text{ for } S_{ix_\ell} \le 0$$

and

$$Q_{cx} = P_x + \sum_{\ell=1}^{N} (Q_{x_\ell} + Q_{r_\ell}) \tag{4.32}$$

where

$$Q_{r_\ell} = 0 , \text{ for } S_{ix_\ell} > 0$$

$$Q_{r_\ell} = -(Q_{rx_\ell} + Q_{x_\ell} - S_{x_\ell} W_{bx_\ell}) + \eta \frac{F_x}{h^3} W_{bx_\ell} , \text{ for } S_{ix_\ell} \le 0$$

for horizontal members. The corresponding relations for vertical members are

$$S_{cy} = S_c + \sum_{\ell=1}^{M} (S_{y_\ell} + S_{r_\ell}) \tag{4.33}$$

where

$$S_{r_\ell} = S_{iy_\ell} , \text{ for } S_{iy_\ell} > 0$$

$$S_{r_\ell} = \eta \frac{F_y}{k^3} , \text{ for } S_{iy_\ell} \le 0$$

and

$$Q_{cy} = -P_y + \sum_{\ell=1}^{M} (Q_{y_\ell} + Q_{r_\ell}) \tag{4.34}$$

where

$$Q_{r_\ell} = 0 \text{ , for } S_{iy_\ell} > 0$$

$$Q_{r_\ell} = -(Q_{ry_\ell} + Q_{y_\ell} - S_{y_\ell} W_{by_\ell}) + \eta \frac{F_y}{k^3} W_{by_\ell} \text{ , for } S_{iy_\ell} \leq 0 \quad (4.34)$$

It should be noted that if joint deflection W_{bx} and column displacement W_{cx} are equal, the differential restraint $(\eta F_x/h^3)$ has no effect. If the values are not equal, the differential restraint tends to enforce an equal deformation condition. The same effect occurs for the other differential restraint $(\eta F_y/k^3)$.

The coefficient η determines the relative magnitude of differential restraint to be used. Empirical studies have shown that a reasonable rate of convergence is usually achieved by the following procedure:

(1) If S_{ix} or S_{iy} is negative, and W_{bx} and W_{cx} or W_{by} and W_{cy} have the same sign, choose η such that the resulting differential restraint $(\eta F_x/h^3)$ or $(\eta F_y/k^3)$ is very small (of magnitude 0.01 to 0.001).

(2) If S_{ix} or S_{iy} is negative, but W_{bx} and W_{cx} or W_{by} and W_{cy} have opposite signs, choose η between 1.0 and 0.01, such that the resulting differential restraint $(\eta F_x/h^3)$ or $(\eta F_y/k^3)$ is relatively large.

In following the above procedure, convergence is also accelerated by revising the values of Q_{cx} and S_{cx} or Q_{cy} and S_{cy} acting on a line of joints immediately after each line of members is solved. Thus, the values of total load and restraint acting on each line of joints are always based on the most recent information available concerning member behavior. Also, when computing values of Q_{bx} and S_{bx} or Q_{by} and S_{by} for a particular joint, it is necessary to remember just what was added into the total load and restraint on the previous iteration and to subtract these values from the total load and restraint.

In deriving Eqs 4.29 and 4.30, it was assumed that W_{bx} and W_{by} were nonzero. However, these values might easily be zero, especially if the solution process is started from initial zero deflections.

If joint deflection is zero, Eqs 4.29 and 4.30 correctly predict an infinite resistance to joint translation. In practice, a large value of

S-spring restraint may be substituted for this infinite value without affecting the accuracy of the analysis. A joint fixed against translation in either the x or y-directions may be approximated by a value of S_x or S_y having an order of magnitude equal to the flexural stiffness F of the members on either side of the joint. To prevent numerical roundoff in a digital computer computation, these values should be no more than approximately one order of magnitude greater than the flexural stiffness F at the stations on either side of the joint.

Enforcement of Rotational Compatability for Each Half of the Joint

From Fig 4.4, the resultant couple acting on the x-half of the joint is, as stated previously,

$$C_x = (C + R\theta_x) - C_y \tag{4.35}$$

while the resultant couple acting on the y-half of the joint is

$$C_y = (C + R\theta_y) - C_x \tag{4.36}$$

Equations 4.35 and 4.36 are not valid unless θ_x and θ_y are equal, the condition of a rigid joint. Assume this is not the case. This equal slope condition may be enforced during the proposed iterative process by the introduction of a rotational closure parameter ξ such that Eq 4.35 becomes

$$C_x - \xi\,(\theta_x - \theta_y) = (C + R\theta_x) - C_y \tag{4.37}$$

while Eq 4.36 becomes

$$C_y - \xi\,(\theta_y - \theta_x) = (C + R\theta_y) - C_x \tag{4.38}$$

The closure parameter ξ is a differential rotational restraint which tends to enforce an equal slope during the iterative process. While shown as a constant in Eqs 4.37 and 4.38, ξ may actually vary for each iteration, for each joint throughout the frame, and for each half of each joint. Procedures for selecting values of ξ are given in Chapter 6. When θ_x and θ_y are equal, Eqs 4.37 and 4.38 reduce to Eqs 4.35 and 4.36. A similar procedure, using a constant value of ξ for each iteration, was developed by Matlock and Grubbs for no-sway frames (Ref 17).

It should be noted that Eqs 4.35 and 4.36 are valid only if the joint is rigid. For example, if the joint is pinned, the values C_x and C_y must be defined externally as there is no mechanism for distribution of applied C and R to the respective halves of the joint. If this is done, however, a pinned joint may be considered by simply neglecting to enforce the equal slope condition during the iterative process.

A joint fixed against rotation may be approximated by a rotational restraint R having an order of magnitude equal to the flexural stiffness F of the members which frame into the joint. In order to prevent roundoff in digital computer computation, the maximum values of R chosen for input should be no more than approximately one order of magnitude greater than the flexural stiffness F at the stations on either side of the joint.

Development of Stiffness Matrix Terms Describing Joint Behavior

The laws of statics may now be applied to the joint-free bodies of Fig 4.8 in a manner similar to that used to develop Eq 3.10 for the finite-element beam-column model.

From Fig 4.8a, summing forces in the vertical direction at Station i and taking moments about the center of the bar to the left of Station i to develop equations for V_A, and V_B, gives

$$M_{i-1} - 2M_i + M_{i+1} = h\left[Q_i - S_i w_i + \frac{C_x}{h} + \frac{1}{2}(Q_{rx} + Q_x - S_x W_{bx})\right]$$

$$- \frac{1}{2}(P_{i-1} + P_i)(w_i - w_{i-1})$$

$$+ \frac{1}{2}(P_i + P_{i+1})(W_{i+1} - w_i) \qquad (4.39)$$

Summing forces at Station i+1 and taking moments about the center of the bar to the right of Station i+1 to develop expressions for V_B, and V_C, gives

$$M_i - 2M_{i+1} + M_{i+2} = h \left[Q_{i+1} - S_{i+1}w_{i+1} - \frac{C_x}{h} + \frac{1}{2} (Q_{rx} + Q_x - S_x W_{bx}) \right] - \frac{1}{2} (P_i + P_{i+1})(w_{i+1} - w_i) + \frac{1}{2} (P_{i+1} + P_{i+2})(w_{i+2} - w_{i+1}) \quad (4.40)$$

If Eq 3.5, the relation between bending moment and model deformation, is substituted three times into the left side of Eqs 4.39 and 4.40, collecting terms and substituting

$$Q_{rx} = Q_{bx} - S_{bx}W_{bx} \quad (4.41)$$

$$C_x = (C + R\theta_x) - C_y + \xi (\theta_x - \theta_y) \quad (4.42$$

and Eqs 4.19 and 4.20 for W_{bx} and θ_x gives at Station i

$$a_i w_{i-2} + b_i w_{i-1} + c_i w_i + d_i w_{i+1} + e_i w_{i+2} = f_i \quad (4.43)$$

where

$$a_i = F_{i-1} \quad (4.44)$$

$$b_i = - 2 \left[F_{i-1} + F_i + \frac{h^2}{4} (P_{i-1} + P_i) \right. \quad (4.45)$$

$$c_i = \left[F_{i-1} + 4F_i + F_{i+1} + \frac{h^2}{2} (P_{i-1} + 2P_i + P_{i+1}) + h^3 S_i \right] + \frac{h^3}{4} (S_x + S_{bx}) + h (R + \xi) \quad (4.46)$$

$$d_i = -2\left[F_i + F_{i+1} + \frac{h^2}{4}(P_i + P_{i+1})\right] + \frac{h^3}{4}(S_x + S_{bx})$$

$$- h(R + \xi) \qquad (4.47)$$

$$e_i = F_{i+1} \qquad (4.48)$$

$$f_i = h^3\left\{Q_i + \frac{1}{2}\left[Q_x + Q_{bx}\right] + \frac{1}{h}\left[C - C_y - \xi\theta_y\right]\right\} \qquad (4.49)$$

while at Station i+1,

$$a_{i+1}w_{i-1} + b_{i+1}w_i + c_{i+1}w_{i+1} + d_{i+1}w_{i+2} + e_{i+1}w_{i+3} = f_{i+1} \qquad (4.50)$$

with

$$a_{i+1} = F_i \qquad (4.51)$$

$$b_{i+1} = -2\left[F_i + F_{i+1} + \frac{h^2}{4}(P_i + P_{i+1})\right] + \frac{h^3}{4}(S_x + S_{bx})$$

$$- h(R + \xi) \qquad (4.52)$$

$$c_{i+1} = \left[F_i + 4F_{i+1} + F_{i+2} + \frac{h^2}{2}(P_1 + 2P_{i+1} + P_{i+2}) + h^3 S_{i+1}\right]$$

$$+ \frac{h^3}{4}(S_x + S_{bx}) + h(R + \xi) \qquad (4.53)$$

$$d_{i+1} = -2\left[F_{i+1} + F_{i+2} + \frac{h^2}{4}(P_{i+1} + P_{i+2})\right] \qquad (4.54)$$

$$e_{i+1} = F_{i+2} \qquad (4.55)$$

$$f_{i+1} = h^3\left\{Q_{i+1} + \frac{1}{2}\left[Q_x + Q_{bx}\right] - \frac{1}{h}\left[C - C_y - \xi\theta_y\right]\right. \qquad (4.56)$$

Corresponding forms may be developed for the y-half of the joint from the free-body of Fig 4.8b.

Development of a Procedure for the Bending Analysis of a Plane Frame

It is apparent that Eqs 4.43 and 4.50 have the same form as Eq 3.10 for the finite-element model beam-column. In fact, the equations are the same except for the addition of terms involving external forces, couples, and restraints at the joints.

Under this hypothesis, the coefficients of Eqs 4.43 and 4.50 may be considered as two rows of a quidiagonal stiffness matrix and column load matrix which may be written for each horizontal or vertical line of members and joint halves in a rectangular plane frame. These matrices may be developed for each line by writing Eq 4.43 at all stations to the immediate left of a joint, Eq 4.50 at all stations to the immediate right of a joint, and Eq 3.10 at all other stations along the line.

The resulting stiffness and load matrices may then be solved recursively by Eq 3.18 for the bending deflections of each line of frame members. By solving all lines of members in the rectangular plane frame, the deflected shape of the frame from bending, under the action of applied forces, couples, and restraints, is known. The Q_{rx} , Q_{ry} , C_x , and C_y terms added to the stiffness and load matrices reflect member interaction and are adjusted during each cycle of the iterative process.

The iterative bending solution may be summarized as follows:

(1) Compute Q_{cx} for each vertical line of horizontal joints and Q_{cy} for each horizontal line of vertical joints.

(2) Compute S_{cx} and S_{cy} in similar manner, estimating values of beam restraint if joint deflection is zero.

(3) Solve each line of horizontal members, computing values of Q_{bx} and S_{bx} , and applying the differential restraint ξ at each joint.

(4) Revise the values of Q_{cx} and S_{cx} for the vertical line passing through each joint by computing a new value of S_{1x} and substituting the appropriate values.

(5) Solve each line of vertical members, computing values of Q_{by} and S_{by} , and applying the differential restraint ξ at each joint.

(6) Revise the values of Q_{cy} and S_{cy} for the horizontal line passing through each joint by computing a new value of S_{1y} and substituting the appropriate values.

(7) Repeat steps (2) through (5) until the transverse deflection of each joint equals the associated column displacement and the slope of both halves of each joint are equal.

Enforcement of Consistent Deflections and Rotations in the Frame

During the early stages of the iterative process, the transverse deflections of individual joints in a particular line may not equal the computed value of column displacement or, for that matter, each other. Also, the rotation or slope of each half of each frame joint may not equal that of the other half. The primary reason for these differences is the inaccuracy of initially assumed values of joint restraint.

However, for an elastic system, the translational restraint provided by a particular member at a particular joint is proportional to the magnitude of the forces or couples applied at the joint and the resulting deflections or rotations. The restraint provided by any joint is a function of member stiffness, the behavior of other joints on the line of frame members, and loads and restraints acting directly on the frame members. Thus, a few cycles are required to determine values of translational stiffness at each joint which are very nearly the exact values for the particular conditions of frame loading, stiffness, and geometry being considered. Once this condition is achieved, a final solution is quickly reached.

For the simple frame of Fig 4.5, only two iterations are required to achieve a correct solution. In a complex frame, with translational and rotational interaction to be considered, more iterations will be required to completely eliminate the effects of initially assumed frame translational behavior.

The differential restraint ξ tends to enforce an equal slope condition during the iterative process by representing the missing half of each joint with an appropriate combination of applied couple and rotational restraint. When the correct ratios of applied couple and restraint have been determined for each joint, as well as the correct values of joint translational stiffness, the rotation or slope of both halves of each joint will be equal. Furthermore, as the total load Q_{cx} and the total restraint S_{cx} will be constant for any vertical line of joints in the frame, the transverse bending deflections W_{bx_ℓ} , $\ell = 1, 2, \cdots, N$, will equal the column displacement W_{cx} . Under this condition, the transverse bending deflection of each horizontal line of members at the point where they intersect the vertical column will equal the column displacement. This relation also holds for vertical lines of members intersecting horizontal beams.

Thus, the condition of consistent deformations, that final frame deformations be consistent with original frame geometry, is satisfied.

Summary

In this chapter, equations describing the behavior of a frame joint in bending have been developed. These equations have been combined with those for members between joints and a procedure for the determination of consistent frame-bending deflections has been outlined.

As yet, nothing has been said concerning determination of axial tension or compression distribution in the frame members. This topic is discussed in the next chapter.

CHAPTER 5. DEVELOPMENT OF A PROCEDURE FOR DETERMINING THE AXIAL FORCE DISTRIBUTION IN FRAME MEMBERS

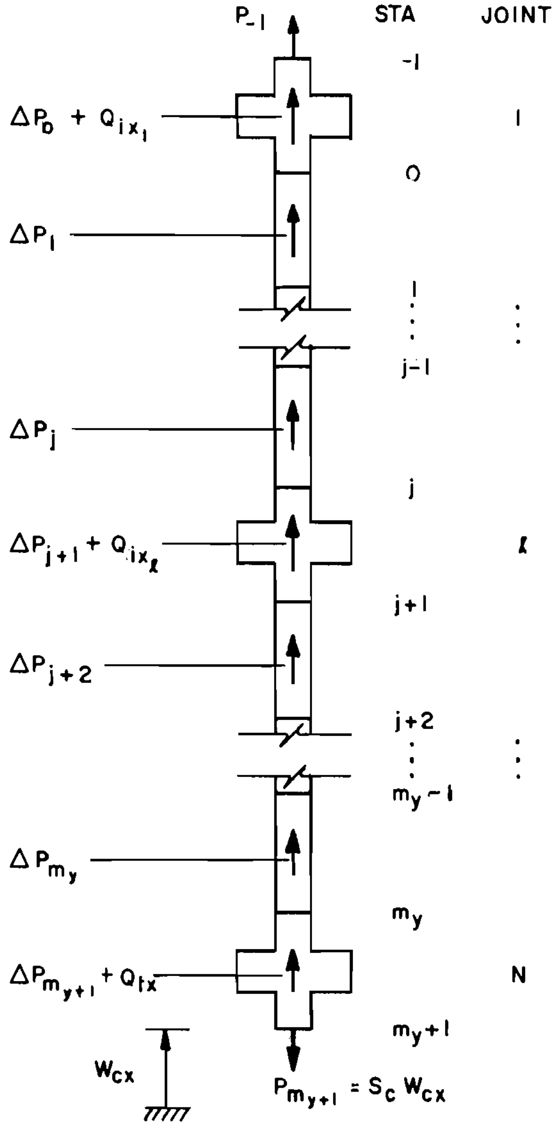
In Chapter 4, an iterative procedure for the bending analysis of a plane frame was developed. This procedure assumed knowledge of the axial tension or compression distribution in all frame members. A method for obtaining these values is developed in this chapter by consideration of each vertical or horizontal line of frame members, as was done in the "bending" analysis of Chapter 4. In this case, information available from a "bending" analysis is used to determine the axial tension or compression distribution in that line of frame members. Thus, the "axial" solution developed in this chapter is dependent on the results of the "bending" solution of Chapter 4 and vice versa.

An iterative method of analysis for the complete frame system may now begin to be visualized. Starting with assumed values from the "axial" solution, (1) a "bending" solution is made and (2) the results of this "bending" solution are used in another "axial" solution. The process is repeated until the desired degree of convergence or closure is obtained. This iterative method of analysis is discussed in Chapter 6.

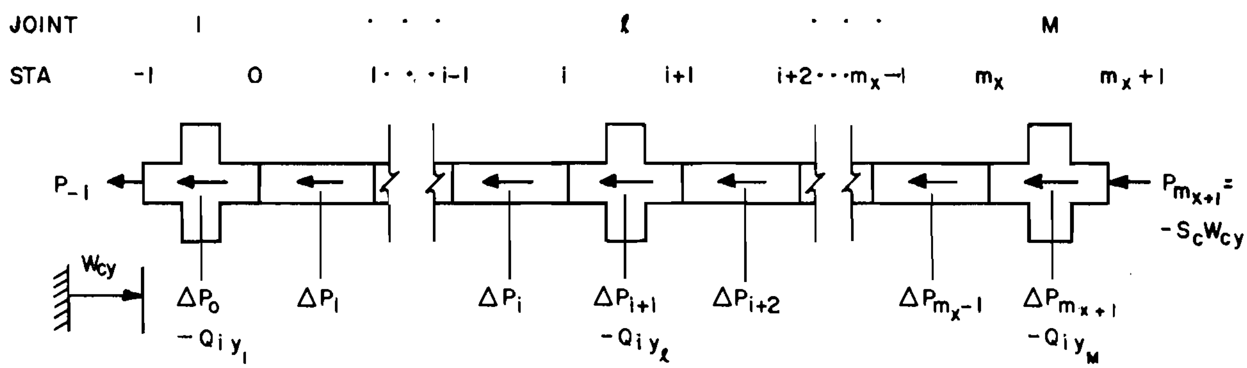
Determination of Axial Tension or Compression Distribution in Vertical Members

Figure 5.1a shows the finite-element model of a line of vertical members in the frame. This model is displaced an amount W_{cx} under the action of applied forces and restraints. Stations or increment points along the model are denoted by j and joints on the model are denoted by ℓ . Each station is separated by a rigid bar. As stated in Chapter 3, the total change in axial tension or compression in each bar may be concentrated at the bar's centroid. Such is the case here where the ΔP_j represent changes in internal axial tension or compression in each bar, caused perhaps by weight forces. At the joints, the reactions acting on the column cause an additional change in axial tension or compression across the bar representing the joint.

In the figure, external forces are shown acting in the positive sense. Using the positive (tensile) sign convention of Fig 4.3d to describe the axial



(a) FINITE - ELEMENT MODEL OF LINE OF VERTICAL FRAME MEMBERS



(b) FINITE - ELEMENT MODEL OF LINE OF HORIZONTAL FRAME MEMBERS

Fig 5.1. Forces which produce changes in axial tension
or compression in frame members.

tension or compression acting on each bar, the relation is either

$$P_{j+1} = P_j + \Delta P_{j+1} \tag{5.1}$$

or

$$P_j = P_{j+1} - \Delta P_{j+1} \tag{5.2}$$

for rigid bars between joints, or

$$P_{j+1} = P_j + \Delta P_{j+1} + Q_{ix_\ell} = P_j + \Delta P_{j+1} - Q_{rx_\ell} \tag{5.3}$$

and

$$P_j = P_{j+1} - \Delta P_{j+1} - Q_{ix_\ell} = P_{j+1} - \Delta P_{j+1} + Q_{rx_\ell} \tag{5.4}$$

for the rigid bars acting as joints. The relation between Q_{rx_ℓ} and Q_{ix_ℓ} is a function of the laws of statics: beam reaction and column reaction are equal and opposite. Thus

$$Q_{ix} = -Q_{rx} \tag{5.5}$$

The axial tension or compression distribution in the line may now be easily computed by integrating from Station -1 to Station m_y+1 using Eqs 5.1 and 5.3 or by integrating from Station m_y+1 to Station -1 using Eqs 5.2 and 5.4. In either case, only an initial condition is required.

If integration from -1 to m_y+1 is desired, the value of P_{-1} must be known. This value is normally zero, but may represent some externally applied force. Conversely, if integration from m_y+1 to -1 is desired, P_{m_y+1} must be known. From Fig 5.1a, considering the previously defined sign conventions, it is apparent that

$$P_{m_y+1} = S_c W_{cx} \tag{5.6}$$

Thus, the integration may be carried out conveniently in either direction. In either case, the resulting axial tension or compression at each station is the same.

Determination of Axial Tension or Compression Distribution in Horizontal Members

Figure 5.1b shows the finite-element model of a line of horizontal members in the frame. The model is displaced an amount W_{cy} under the action of applied forces and restraints. Stations along the model are denoted by i and joints on the model are denoted by ℓ . As is the case for vertical members, the ΔP_i represent changes in internal axial tension or compression in each bar. At the joints, the reaction Q_{iy_ℓ} causes an additional change in axial tension or compression across the bar representing the joint.

Using the positive (tensile) sign convention of Fig 4.3c to describe the axial tension or compression acting on each bar gives either

$$P_{i+1} = P_i + \Delta P_{i+1} \tag{5.7}$$

or

$$P_i = P_{i+1} - \Delta P_{i+1} \tag{5.8}$$

for rigid bars between joints, or

$$P_{i+1} = P_i + \Delta P_{i+1} - Q_{iy_\ell} = P_i + \Delta P_{i+1} + Q_{ry_\ell} \tag{5.9}$$

and

$$P_i = P_{i+1} - \Delta P_{i+1} + Q_{iy_\ell} = P_{i+1} - \Delta P_{i+1} - Q_{ry_\ell} \tag{5.10}$$

for rigid bars acting as joints. The relation between Q_{ry} and Q_{iy} is similar to that of Eq 5.5, namely

$$Q_{iy} = -Q_{ry} \tag{5.11}$$

The axial tension or compression at each station in the line may now be determined by integration from -1 to m_x+1 or m_x+1 to -1 using Eqs 5.7 and 5.9 or 5.8 and 5.10 in a manner similar to that described for vertical members. In this case, the boundary condition at m_x+1 is

$$P_{m_x+1} = -S_c W_{cy} \tag{5.12}$$

<u>Summary</u>

In this chapter, procedures for computing the axial tension or compression distribution in each frame member have been developed. These procedures, when applied to every line of vertical or horizontal members in the frame, are designated as an "axial" solution of the frame.

Chapter 6 discusses a method of combining this "axial" solution with the "bending" solution of Chapter 4 to develop a method for complete analysis of a plane frame.

CHAPTER 6. DEVELOPMENT OF AN ITERATIVE METHOD FOR COMPUTER SOLUTION OF THE FRAME EQUATIONS

In Chapter 4, a numerical procedure for the line-by-line analysis of a plane frame in bending was developed. This procedure required previous knowledge of the axial tension or compression distribution in the frame members. A method for finding the axial tension or compression distribution in the frame members, also on a line-by-line basis, was developed in Chapter 5. However, this procedure required previous knowledge of the deflected shape of the frame in bending.

Initially, the assumption of an iterative method of frame analysis was made. The required iterative procedure may now be defined and procedures may be developed for computer solution of the frame equations.

Definition of the Iterative Method

Each iteration of the required iterative procedure for the analysis of a rectangular plane frame in bending is defined to consist of two parts:

(1) a line-by-line bending solution assuming knowledge of axial force distribution in the frame and

(2) a line-by-line axial force distribution assuming knowledge of the deflected shape of the frame in bending.

These two solutions are repeated in cyclic fashion until the desired degree of convergence, to be discussed later, has occurred. Each cyclic repetition of the two solutions gives, in effect, one complete solution of the frame.

The large number of repetitious calculations required for each cycle of the proposed iterative process make it highly suitable for digital computer solution. In fact, considering the large number of computations involved, digital computer solution is felt to be the only efficient method for using the proposed process. A general flow diagram for computer solution is shown in Fig 6.1.

Specific equations required for implementation of the iterative process

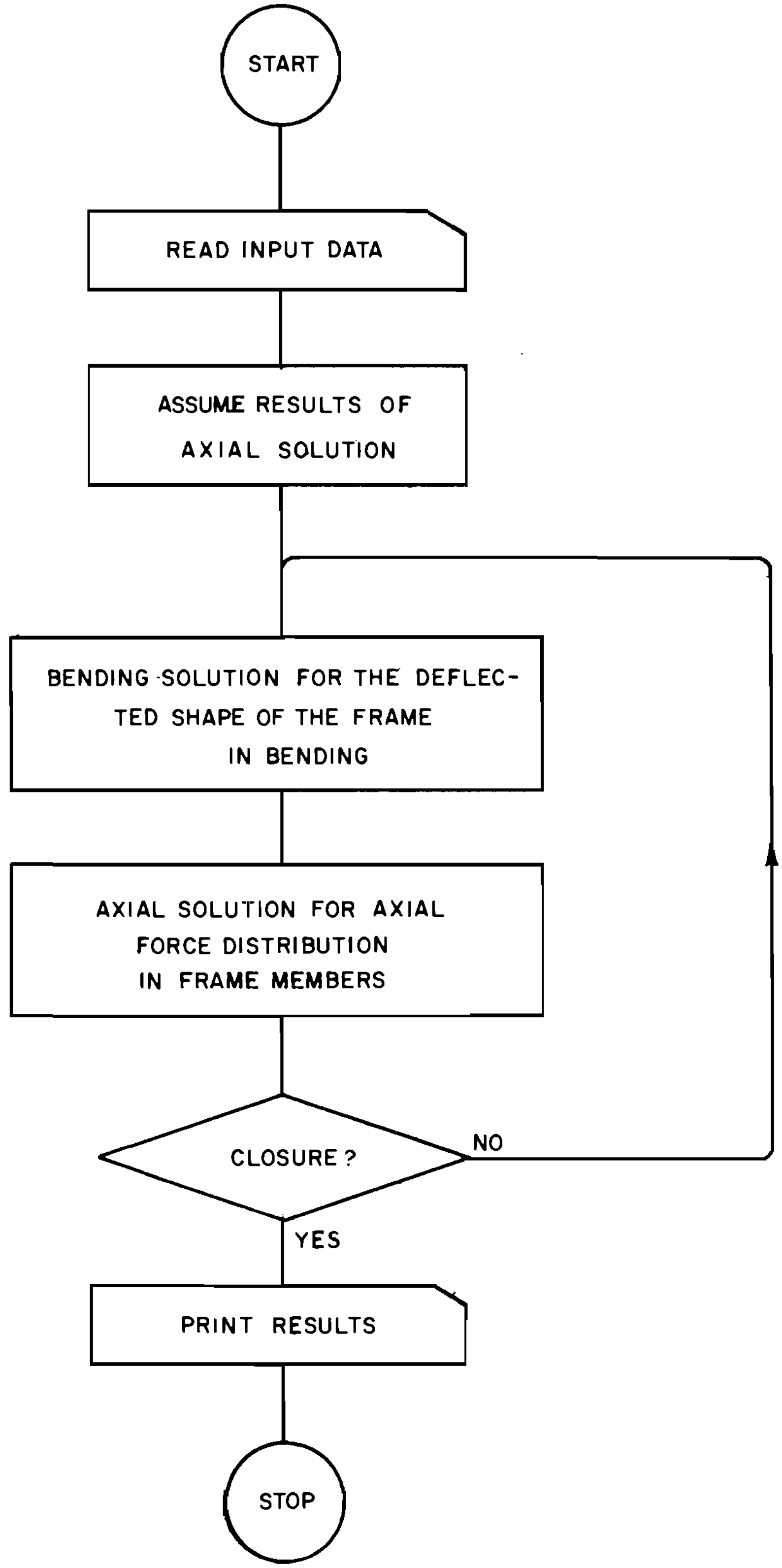


Fig 6.1. Summary flow diagram of the computation process required for frame analysis.

have been developed in previous chapters. However, in order to completely define the proposed method, consideration must be given to (1) a general discussion of the iterative process, (2) selection of rotational closure parameters, and (3) computer solution of the frame equations.

Discussion of the Iterative Method

The iterative method, as defined previously, consists of a series of complete solutions for the deflected shape of a plane frame in bending. Each iteration consists of two half iterations, a bending solution, and an axial solution.

As the final result desired from the iterative process is the correct deflected shape of a plane frame in bending, the bending solution forms the more important part of each iteration. The axial solution, being of secondary nature, is used to generate input data for the bending solution.

Data interchanged during the iterative process consist primarily of resultant forces and couples, with the values of these forces and couples being determined by (1) frame stiffness and geometry, (2) applied loading and restraint, and (3) results of previous iterations.

In a physical sense, the iterative process may be visualized as a readjustment procedure, such that given a frame under specified conditions of loading and restraint, the procedure given below is followed:

(1) An initial assumption is made concerning the distribution of internal forces and couples in the frame.

(2) The deflected shape of the frame is computed considering the applied loading and assumed distribution of internal forces and couples.

(3) The distribution of internal forces and couples is revised considering the applied loading and the deflected shape of the frame.

Steps 2 and 3 are repeated until the correct distribution of internal forces and couples, and thus the correct deflected shape of the frame, is obtained. This distribution, determined by interaction of frame members, is computed using the equations derived in Chapters 4 and 5.

Selection of Rotational Closure Parameters

During the proposed iterative process, each half of each frame joint is considered independently. The differential restraint ξ is used to represent the rotational restraint provided by the missing half of the joint. For example, when solving horizontal members (1) a rotational restraint ξ and (2) a couple $-\xi\theta_y$ are applied to the x-half of the joint. The rotational restraint ξ inhibits total joint rotation while the applied couple $-\xi\theta_y$ tries to rotate the x-half of the joint in the direction taken by the missing y-half of the joint. Thus the ratio of restraint and applied couple represents the effect of the missing y-half of the joint.

The restraint provided by the missing half of the joint must be a function of the flexural stiffness F of the members which frame into the joint and the length of members between joints. Under this hypothesis, a different value of ξ could exist for each half of the joint. Let ξ_x be the restraint applied to the x-half of the joint and ξ_y be the restraint applied to the y-half of the joint where

$$\xi_x = \rho \frac{F_y}{k} \tag{6.1}$$

and

$$\xi_y = \rho \frac{F_x}{h} \tag{6.2}$$

with F_y being the flexural stiffness of the vertical members in the vicinity of the joint, F_x being the flexural stiffness of the horizontal members in the vicinity of the joint, and ρ being a coefficient to be defined later.

The form of the iteration equations (Eqs 4.37 and 4.38) defining rotational compatability for the joint is similar to that used in the generalized alternating-direction implicit method of solving partial differential equations, as summarized by Young and Wheeler (Ref 23). However, several differences are present: (1) the rotational equations (Eqs 4.37 and 4.38) appear only indirectly in the quidiagonal stiffness matrix written for each line of frame members, (2) the quidiagonal stiffness matrix is formulated to solve for the deflected shape of the frame while the rotational equations are functions of the first and second derivatives of frame deflection, and (3) the joints are present on a discrete basis, so that an actual continuum does not exist.

For these reasons, a correct mathematical analysis for closure parameter determination, considering variations in member flexural stiffness, member length, and translational interaction, is felt to be beyond the scope of this study. Instead, a criteria based on actual structural behavior will be presented. While this criteria for closure parameter selection is somewhat empirical, it has been applied successfully to a wide variety of frame problems and has given reasonably good rates of convergence for all cases considered. Some typical convergence or closure plots using the criteria to be presented are shown in Chapters 7 and 8.

The maximum rotational stiffness of any joint is approximately $(4EI/h)$, corresponding to a fixed-end condition one increment away from the joint. If L is the average distance between joints, the minimum rotational stiffness of any joint is somewhere between $(4EI/L)$ with the far end fixed and zero, assuming no negative rotational stiffnesses. Experience has shown that $(1EI/L)$ is a reasonably small value of joint restraint.

Thus the actual value of joint restraint as determined by frame stiffness, loading, and geometry will usually lie between $(4EI/h)$ and $(1EI/L)$. A rational procedure for approximating the actual joint restraint would then be to assume several possible values of joint restraint between these limits and to try these values successively during the iterative process. The coefficient ρ may now be defined. Its maximum value will be approximately four, while its smallest value will be chosen such that $(\rho EI/h)$ will be approximately $(1EI/L)$, that is, its smallest value should be approximately (h/L). Once the upper and lower values of ρ have been established, other values may be selected between these bounds to cover the entire range of possible joint restraint conditions. Usually only a few intermediate values need be selected.

For example, consider a frame with h and k equal to one and L equal to ten. The upper ρ limit would be four while the lower ρ limit would be (h/L) or 0.1. Intermediate values of ρ could be chosen as approximately two and 0.5. This sequence of four ρ values would then be applied in cyclic fashion during the iterative process. These values may be applied in four different cyclic orders: "stairstep" order, "reverse stairstep" order, "hill and dale" order, and "dale and hill" order. The orders are illustrated as follows:

"Stairstep" Order

Iteration	ρ
1	0.1
2	0.5
3	2.0
4	4.0

"Reverse Stairstep" Order

Iteration	ρ
1	4.0
2	2.0
3	0.5
4	0.1

(Table cont'd)

"Stairstep" Order

Iteration	ρ
5	0.1
6	0.5
.	.
.	.
.	.

"Reverse Stairstep" Order

Iteration	ρ
5	4.0
6	2.0
.	.
.	.
.	.

"Hill and Dale" Order

Iteration	ρ
1	4.0
2	2.0
3	0.5
4	0.1
5	0.5
6	2.0
7	4.0
8	2.0
.	.
.	.
.	.

"Dale and Hill" Order

Iteration	ρ
1	0.1
2	0.5
3	2.0
4	4.0
5	2.0
6	0.5
7	0.1
8	0.5
.	.
.	.
.	.

with $\xi_y = \rho \frac{F_x}{h}$ and $\xi_x = \rho \frac{F_y}{k}$.

All orderings have produced reasonable rates of convergence. The "stairstep" and "dale and hill" orderings have produced slightly faster convergence for some problems, but have caused oscillating closure for other problems. The "reverse stairstep" and "hill and dale" orderings have given stable convergence or closure for all problems solved.

Computer Solution of the Frame Equations

The equations derived in Chapters 3, 4, and 5 and the proposed iterative method of analysis described previously are of little practical value unless they may be applied to the solution of actual frame problems. Thus, the actual potential of the method must be demonstrated by programming the derived equations for the digital computer and actually solving realistic example problems.

Chapter 7 demonstrates the closure or convergence of the method as actually programmed for computer solution, while the solution of realistic example problems is shown in Chapter 8. First, however, the development of a computer program to solve the frame equations must be considered. The basic considerations for a generalized computer program are (1) data to be input, (2) equations to be solved, (3) closure techniques, and (4) desired results. These considerations are discussed on the following pages.

Input Data

The data input to a generalized computer program should completely describe the mathematical frame model to be solved. These data may be divided into three classes:

(1) data describing frame geometry with respect to number of lines of horizontal and vertical members, the increment length and number of increments for each line of members, and the intersections or joint locations on each line of members,

(2) data describing the flexural stiffness, lateral loading and spring restraint, and axial tension or compression acting on the frame members, and

(3) data describing the external forces, couples, and restraints acting on each joint.

Frame Geometry. Description of frame geometry requires a consistent ordering system, as defined in Chapter 4. Using such an ordering system, data describing each line of frame members with respect to number of increments, increment length, number of joints, and joint location may be developed. Each joint location must be defined with respect to both lines of members which intersect to form the joint.

Individual Frame Members. Each line of members in the frame is composed of one or more frame members. As noted in Chapter 3, each individual member may be described on a station-by-station basis with respect to flexural stiffness, transverse load and spring restraint, and internal axial tension or compression. The location of initial and final stations on each member would be known from frame geometry considerations.

Frame Joints. All frame joints are assumed to be either rigid or pinned. Thus no data describing joint flexural stiffness is required. Various external transverse and angular effects may be input at each joint, as described in Chapter 4. These effects consist of either horizontal or vertical load and elastic restraint, a rotational restraint, and an applied couple.

Frame joints need not be formed by two intersecting frame members, as shown in Fig 4.2. Some other possible joint configurations are shown in Fig 6.2.

Figure 6.2a shows a three-member joint, which could represent an outside

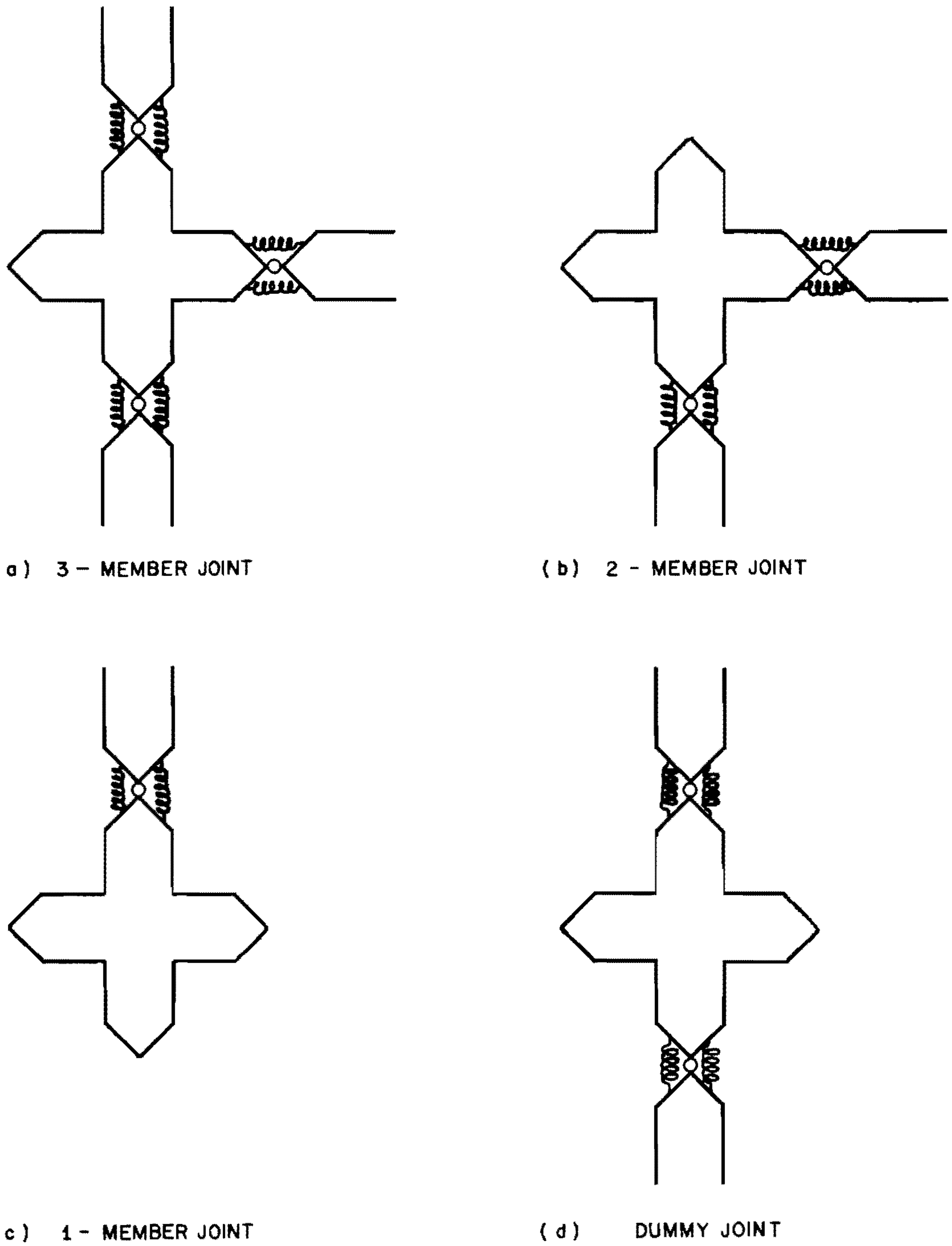


Fig 6.2. Various possible joint configurations desired in developing input data.

joint in a multi-story frame. Here, the horizontal line of members is assumed to begin at the station to the right of the joint.

A method for describing frame corners is shown in Fig 6.2b. Here, the horizontal member is begun at the station to the right of the joint, while the vertical member is begun at the station below the joint.

Figure 6.2c shows a joint used for termination purposes. When a large value of external transverse restraint is specified, this joint will approximate a simply-supported end. Large values of transverse and rotational restraint applied to the joint will approximate a fixed-end condition.

A "dummy" joint configuration is shown in Fig 6.2d. This configuration may be used when it is desired to specify values of either applied transverse load and spring restraint, rotational restraint, or couple at some location between two increment points on a frame member.

Solution of Bending Equations

The equations describing frame bending may be solved exactly as described in Chapter 4. A flow diagram for the computer solution of the equations is shown in Fig 6.3. Using given input data and the results of a previous axial solution, matrix coefficients are computed for each line of frame members. Special coefficients are computed at the stations on either side of a joint. The resulting quidiagonal stiffness and column load matrices are solved for the deflected shape of the line exactly as described in Chapter 3. The resulting deflections of the frame are used in the next axial solution.

Solution of Axial Equations

The equations describing axial frame behavior are solved in two phases, as described in Chapters 4 and 5. First, the axial displacement of each line of frame members is computed; then the axial tension or compression distribution in that line is computed. A flow diagram of the process is shown in Fig 6.4.

Closure or Convergence of the Solution

An iterative method is said to have closed or converged when successive iterations of the method are equal within some prescribed tolerance. Thus, for computer solution of the proposed method, closure is assumed to occur when (1)

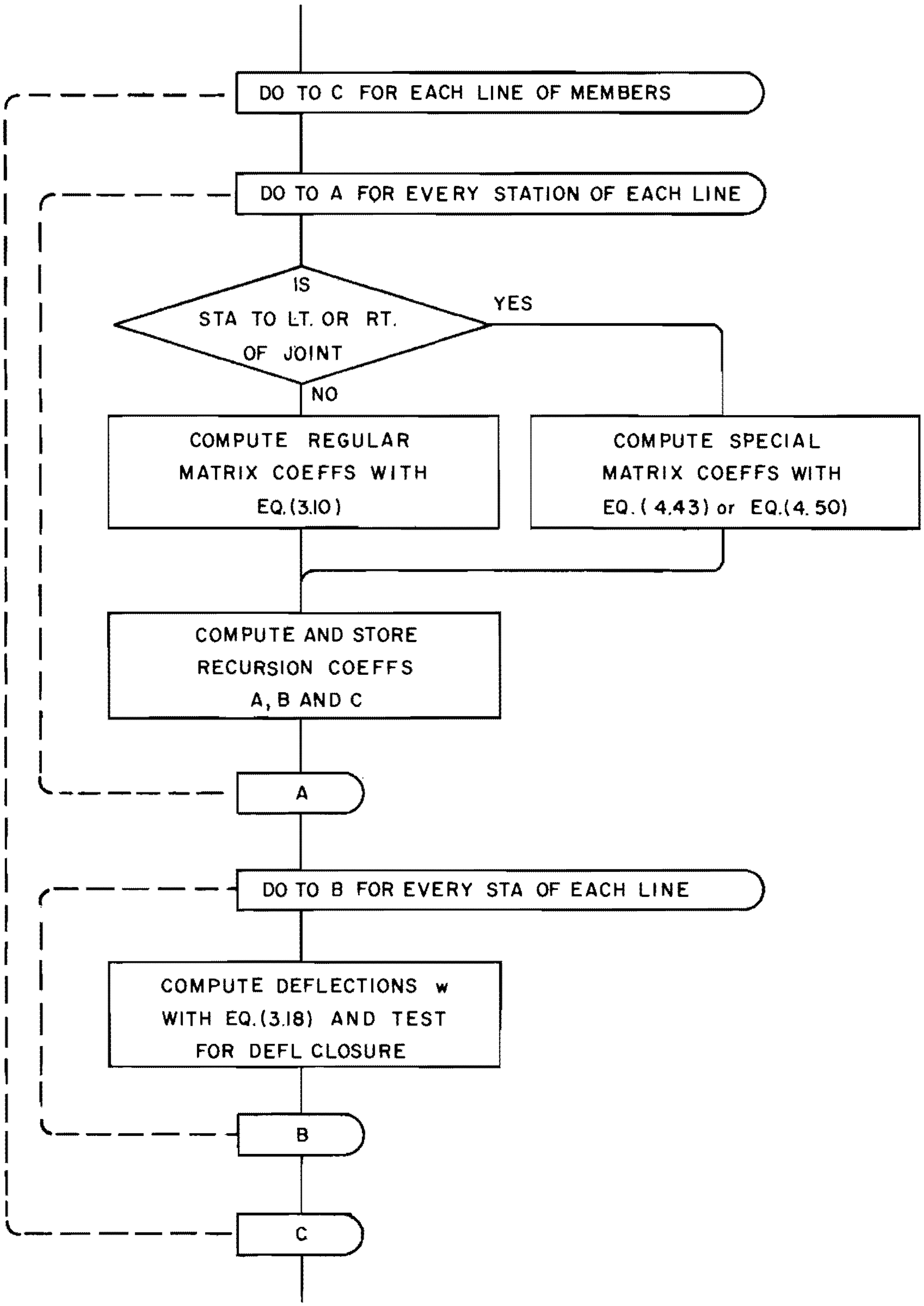


Fig 6.3. General flow diagram for solution of bending equations.

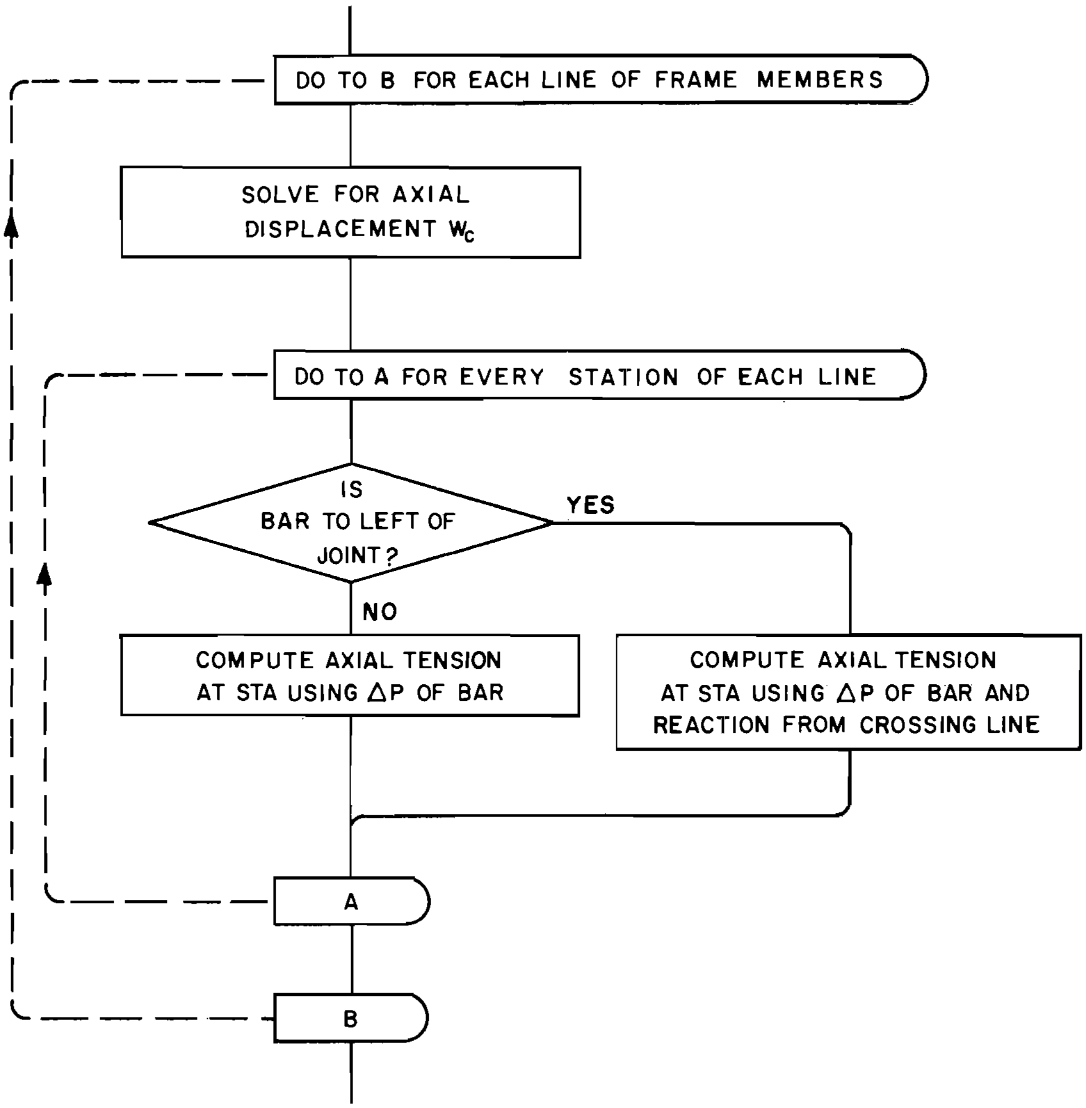


Fig 6.4. General flow diagram for solution of axial equations.

the transverse deflection and corresponding axial displacement for each half of each frame joint, (2) the rotation or slope of both halves of each rigid joint, and (3) the transverse deflections of all frame members are equal within some prescribed tolerance or tolerances for two successive iterations.

Desired Results

The results desired from computer solution of the frame equations consist primarily of data describing the deflected shape of the frame. These data may be organized into two parts, joint data and member data.

The data available for each frame joint consist of three values which define its final position in space with respect to its original or zero position. These three values are vertical joint translation, horizontal joint translation, and joint rotation.

The data describing joint behavior determines the position in space of the members which frame between joints. The deflected shape of the individual members can be differentiated to provide information about the distribution of moment and shear in the frame.

Summary

This chapter has defined a proposed method of plane-frame analysis and outlined the requirements for iterative computer solution of the proposed frame equations.

To show applicability of the method to the solution of realistic problems, a computer program, PLNFRAM 4, was developed, following the requirements outlined in this chapter. This program, written in FORTRAN-63 for a Control Data Corporation 1604 computer, is discussed in detail in Appendices 1, 2, and 3.

CHAPTER 7. VERIFICATION OF THE PROPOSED ITERATIVE METHOD

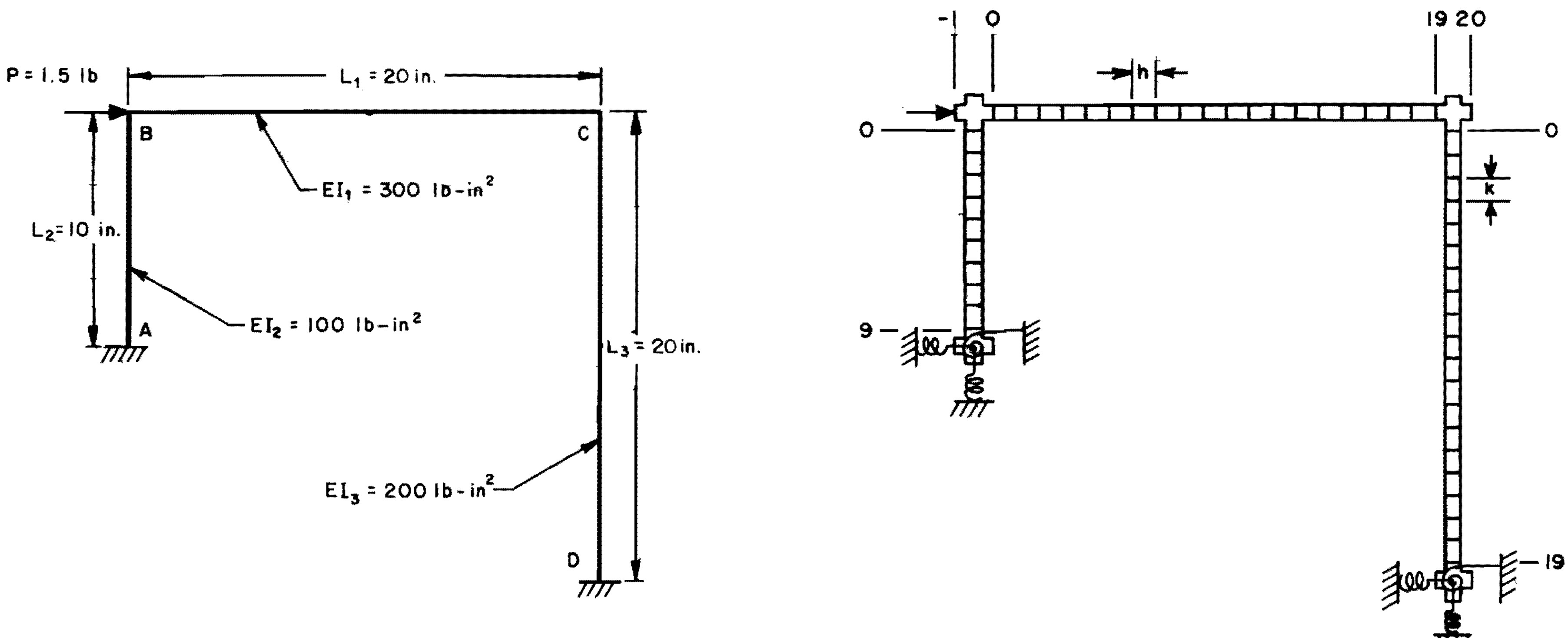
In preceding chapters, equations describing the behavior of a finite-element frame model in bending have been developed. An iterative method for solution of these equations has been proposed and discussed, and procedures for computer solution of the proposed method have been outlined. The last step in the development of an analytical method, verification of results, will be given in this chapter. The generality of the proposed method will be shown by the example problems in Chapter 8.

Comparison of Computed Results with Accepted Theory

The test of any numerical method of analysis is its comparison with the accepted theory it approximates. In this regard, a simple frame problem has been chosen for comparative purposes. While the solution of this simple frame does not completely demonstrate the generality of the method, it nevertheless provides a comparison between results obtained by the method and those produced by accepted theory.

Figure 7.1a shows a simple two-leg bent and Fig 7.1b shows the corresponding finite-element frame model. Values of horizontal translation and rotation for Joints B and C are presented in tabular form. Results obtained using the slope-deflection method of analysis are compared with numerical results for three different increment lengths. As may be seen, good agreement is obtained. The primary cause for difference in results is felt to be caused by the finite joint width used in the frame model, as compared with the infinitesimal joint assumed in the slope-deflection procedure. The effect of joint width will be discussed later.

Perhaps the degree of accuracy available to the method may be better visualized by considering the simple frame of Fig 4.5. This frame was solved in two iterations, with a computed joint translation of 0.9918 inches at all three joints. Each beam was divided into 11 increments. The difference between the computed value and the exact value of one inch was less than one per cent.



(a) IDEALIZED SIMPLE FRAME

(b) FINITE-ELEMENT MODEL FOR h = 1.0 in

VALUE	SLOPE DEFLECTION	FINITE-ELEMENT MODEL		
		h = k = 1.0 in.	h = k = 0.5 in.	h = k = 0.25 in.
$\Delta_B = \Delta_C$, in.	1.335	1.304	1.310	1.312
θ_B, rad.	-7.481×10^{-2}	-7.449×10^{-2}	-7.443×10^{-2}	-7.442×10^{-2}
θ_C, rad.	-1.761×10^{-2}	-1.639×10^{-2}	-1.662×10^{-2}	-1.668×10^{-2}

Fig 7.1. Verification of computed results.

Convergence of the Iterative Method

Figure 7.2a shows computed joint rotations for the x and y-halves of Joint B of Fig 7.1b, plotted against iteration number. Horizontal translation of Joint B, transverse deflection and axial displacement, is plotted against iteration number in Fig 7.2b. The shapes of the closure plots are typical of those produced by the computer program.

Convergence for this simple problem is fairly rapid, representing the relatively small amount of internal force redistribution that must take place during the iterative process. For larger and more complex frames, it will be seen that more iterations are required to achieve reasonable closure.

Justification of One-Increment Finite-Element Joints

Matlock and Grubbs (Ref 17) have proposed an alternate finite-element frame-joint model. This model is two increments in width, such that a station or increment point occurs at the center of the joint, as well as at the ends of the joint. While the two-increment joint concept has been applied only to the solution of plane frames without sway, it is felt that this concept is also applicable to sway problems.

Previously developed techniques (Ref 18) for exact specification of slope and deflection at a station or increment point may be directly applied to a two-increment frame-joint model. This is felt to be the main advantage of the model. At the present time, only procedures for approximating desired slope and deflection at a frame joint have been developed for the one-increment model.

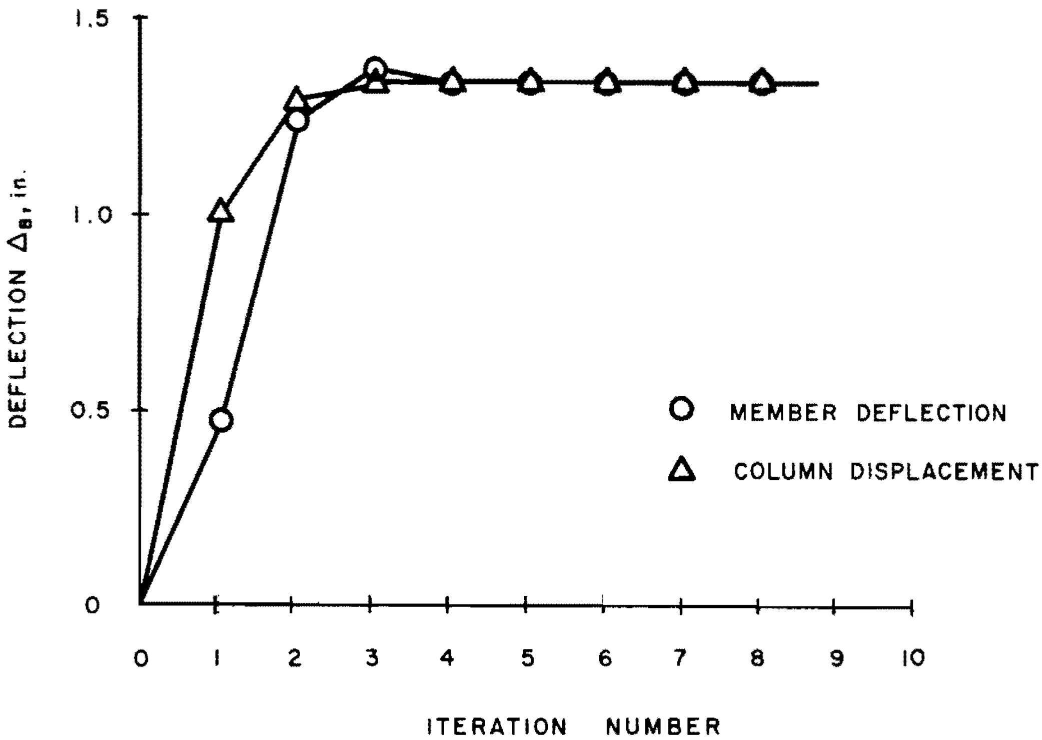
However, one serious disadvantage is felt to be inherent in the two-increment joint concept: the method of computing joint rotation or slope.

For a one-increment joint, with the center of the joint halfway between stations, the slope of the joint is computed by the central-difference relation about $w_{i+\frac{1}{2}}$:

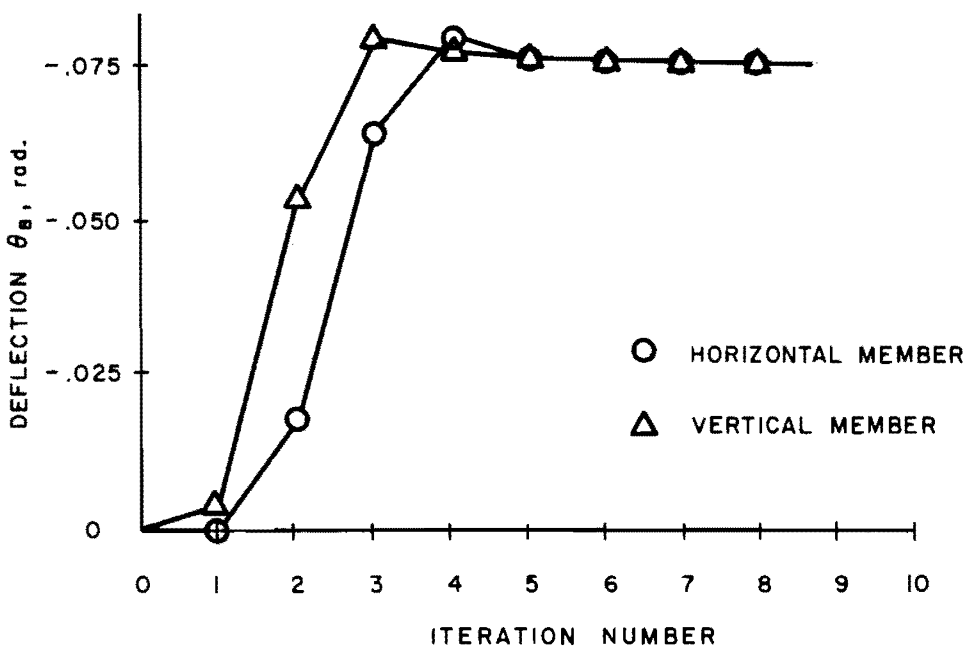
$$\theta_{i+\frac{1}{2}} = \frac{w_{i+1} - w_i}{h} \qquad (7.1)$$

As the bar forming the joint is rigid, Eq 7.1 gives the exact value of joint slope for the finite-element model.

For a two-increment joint, however, the slope of the joint must be computed



(a) DEFLECTION CLOSURE FOR UPPER LEFT JOINT, h = k = 1.0



(b) ROTATION CLOSURE FOR UPPER LEFT JOINT, h = k = 1.0

Fig 7.2. Translational and rotational closure for the problem of Fig 7.1.

by the central-difference relation about w_i :

$$\theta_i = \frac{w_{i+1} - w_{i-1}}{2h} \tag{7.2}$$

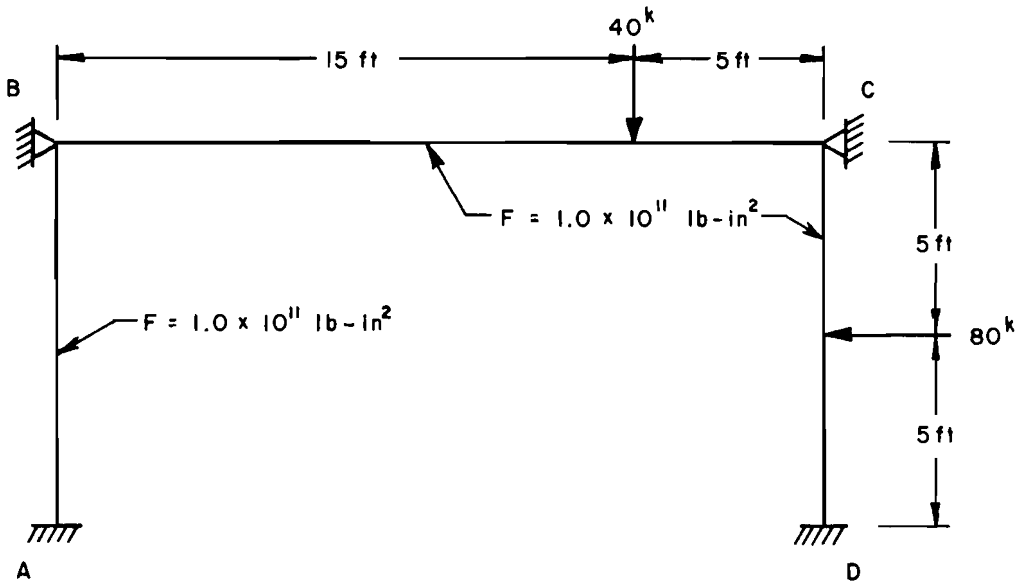
where i is the station in the center of the joint and i-1 and i+1 are the stations at the edges of the joint. As the two-increment joint proposed by Matlock and Grubbs is not completely rigid between i-1 and i+1 , Eq 7.2 gives only an approximation of the slope at the center of the joint. Furthermore, the two-increment joint formulation requires that a value of flexural stiffness be specified at Station i, the center of the joint. This requirement appears to be unrealistic if the joint is to be considered rigid.

For comparative purposes, a problem solved by Matlock and Grubbs (Ref 17, p 31) was re-solved using the one-increment joint concept. The problem and computed results are shown in Fig 7.3. The moment distribution results and those for the two-increment joint model are taken directly from Reference 17. As may be seen, both procedures give good agreement with accepted values. The degree of accuracy obtained by the one-increment joint model with a 12-inch increment length is roughly equal to that obtained for the two-increment model using a three-inch increment length. However, using the one-increment model with a three-inch increment length also gave approximately the degree of accuracy obtained by the two-increment model with a three-inch increment length.

The difference in computed values is felt to be a function of the different procedures used to compute joint slope with Eq 7.1 being a better approximation than Eq 7.2, especially if larger increment lengths are chosen. The difference between computed and theoretical values for both increment lengths is a function of the finite joint widths used in the models. This effect will be discussed in the next section.

Error of Approximation in the Method

The difference in results computed by the method developed in this study and those given by classical theory is a function of the two different procedures used to represent the real structure. In the classical, idealized structure an infinitesimal joint width is assumed, while in the finite-element frame model described exactly by the equations of Chapters 3, 4, and 5 a finite joint width is assumed.



COMPARATIVE RESULTS

SLOPE, RAD.	THEORETICAL VALUES	TWO-INCREMENT MODEL JOINTS	
		h = 12 in.	h = 3 in.
θ_B	-9.740×10^{-5}	-9.923×10^{-5}	-9.781×10^{-5}
θ_C	4.610×10^{-5}	4.684×10^{-5}	4.630×10^{-5}
		ONE-INCREMENT MODEL JOINTS	
		h = 12 in.	h = 3 in.
θ_B	-9.740×10^{-5}	-9.700×10^{-5}	-9.770×10^{-5}
θ_C	4.610×10^{-5}	4.624×10^{-5}	4.632×10^{-5}

Fig 7.3. Comparison of relative accuracy of one and two-increment model frame joints.

One primary difference in the two representations of the real structure is immediately apparent: if the center-to-center distances between joints are the same for both representations, the end-to-end distances for connecting members will be different, with each finite-element member being exactly one increment length shorter than its classical counterpart. For this reason, moments computed at the ends of finite-element members forming joints differ by a distance $h/2$ from classical values. Comparative values of slope and, to a lesser extent, deflection are also affected by the finite joint width.

However, the developed method is intended for solving a real structure, not its classical representation. Thus, comparisons such as that given by Fig 7.1 show only how well the finite-element model compares to an idealized structure. As the increment length is decreased, the difference between the two representations decreases.

Under the above hypothesis, the finite-element frame model is credited with giving at least an equally valid representation of a real structure when compared to classical techniques. If rational choices of increment length based on actual joint width are made, the finite-element model would be expected to give a more valid representation of the real structure.

Errors in the Solution After Closure Has Occurred

Closure for the iterative method, as defined in Chapter 6, is assumed to occur when member deflections, joint deflections and displacements, and joint rotations are the same within specified tolerances for two successive iterations. While such a method of defining closure is the simplest that may be selected, it does not give a true indication of the statical imbalance of forces and couples remaining in the system. This imbalance is a function of the difference between actual values of member restraint and those computed by the method.

The imbalance of forces and couples remaining in the system after deflection and rotation closure to a specified tolerance has occurred may be found by applying the three equations of statics at each joint. Using the notation of Chapter 4, the error in summation of vertical forces at any joint is

$$E_x = Q_{cx} - S_{cx}\left[\frac{1}{2}\left(W_{bx} + W_{cx}\right)\right] \tag{7.3}$$

while the error in summation of horizontal forces is given by

$$E_y = Q_{cy} - S_{cy}\left[\frac{1}{2}\,(W_{by} + W_{cy})\right] \tag{7.4}$$

and the error in summation of applied couples is

$$E_r = C + R\left[\frac{1}{2}\,(\theta_x + \theta_y)\right] - C_x - C_y \tag{7.5}$$

If the iterative process converges within desired tolerances, but the statical errors are excessive, improved values of E_x , E_y , and E_r may usually be obtained by reducing the closure tolerances and allowing the procedure to further refine its computed values of member restraint.

Summary

In this chapter, the proposed method for the iterative analysis of rectangular plane frames has been verified by comparison with accepted theory. In addition, closure of the iterative method has been discussed and justification for the use of a one-increment frame-joint model has been given. The generality of the method will be shown by the example problems to be presented in the following chapter.

CHAPTER 8. EXAMPLE PROBLEMS

Three example problems are selected to illustrate the applicability of the method. These examples are hypothetical and are chosen more to show the generality available from the method than to simulate any particular framed structure.

Example 1

Example 1, a pinned or linkage-type frame shown in Fig 8.1, is chosen to illustrate the translational capabilities of the method. All joints are pinned, a procedure made possible by simply neglecting to enforce the equal slope condition at each joint during the iterative process.

The applied loading and flexural stiffness of the members is shown in the figure. Support for the system is provided by transverse and axial springs located at five joints and also by the springs distributed under the third horizontal member.

The problem was solved to a deflection tolerance of 1.0×10^{-5} inch in 26 iterations, giving the deflected shape shown in Fig 8.2. The maximum error in summation of forces at any joint was approximately 0.04 pound. A closure plot of horizontal translation for the lower left frame joint is plotted in Fig 8.3. By pinning the frame, rotational interaction at each joint is avoided, and only the translational adjustment process may be investigated. As may be seen in Fig 8.2, only seven iterations are required for the method to determine the approximate values of translational restraint acting on the joint. The remaining iterations are used to refine this quickly determined estimate of joint restraint.

Example 2

Example 2, a stepped frame as shown in Fig 8.4, is solved to illustrate the rotational capabilities of the method. No translational forces are applied: the only effects acting on the frame are the opposing couples

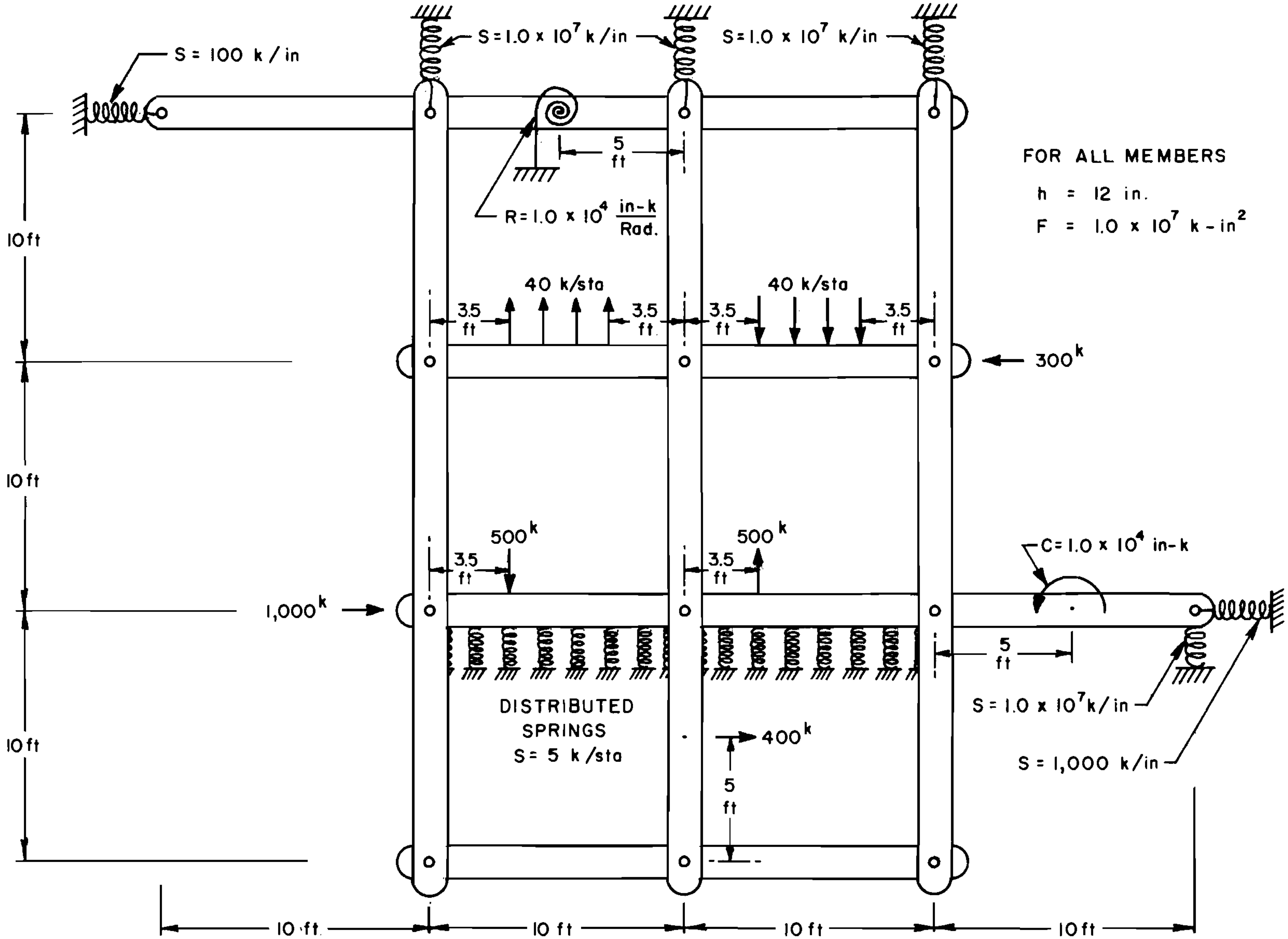


Fig 8.1. Pinned frame of Example 1.

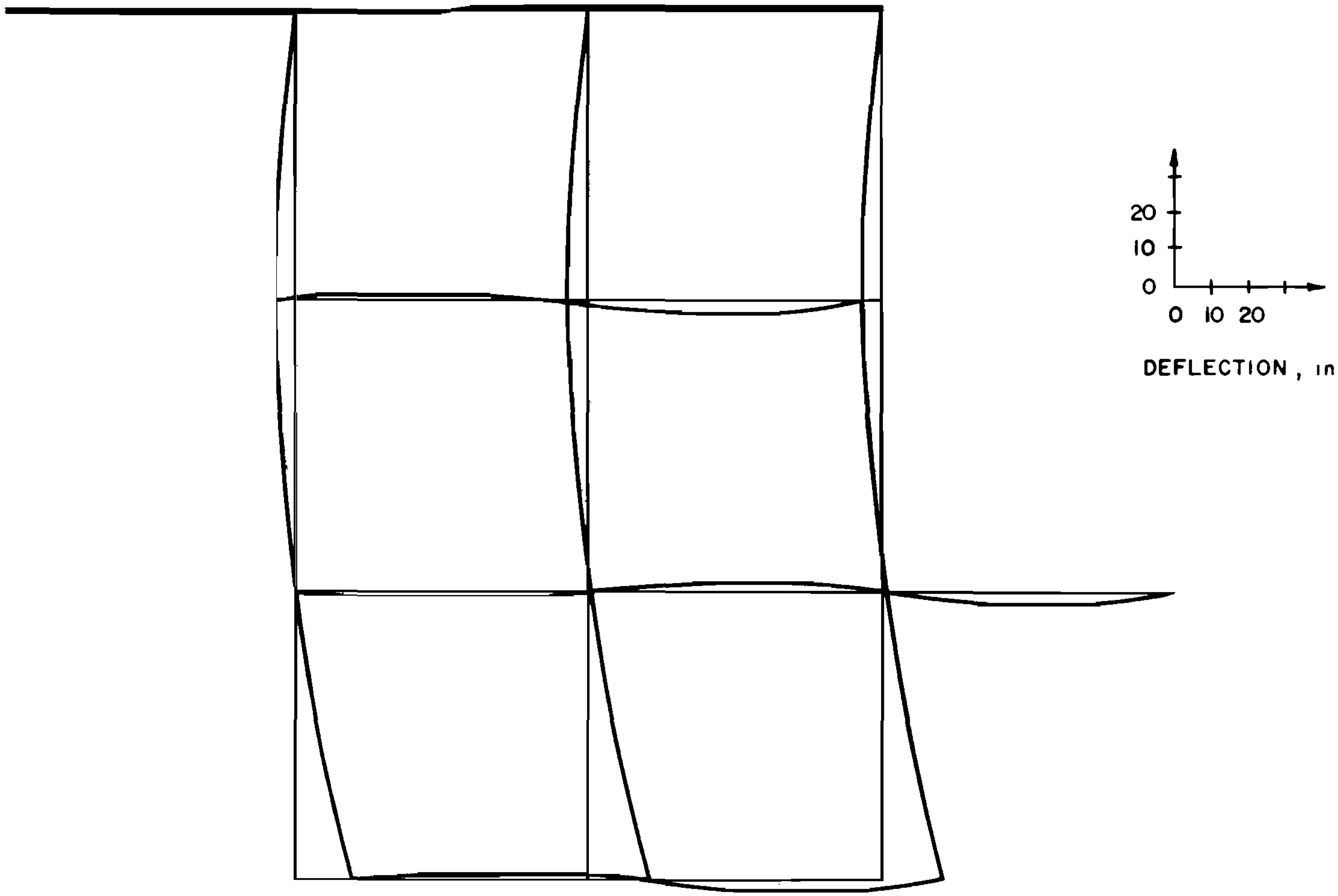


Fig 8.2. Deflected shape of pinned frame of Example 1.

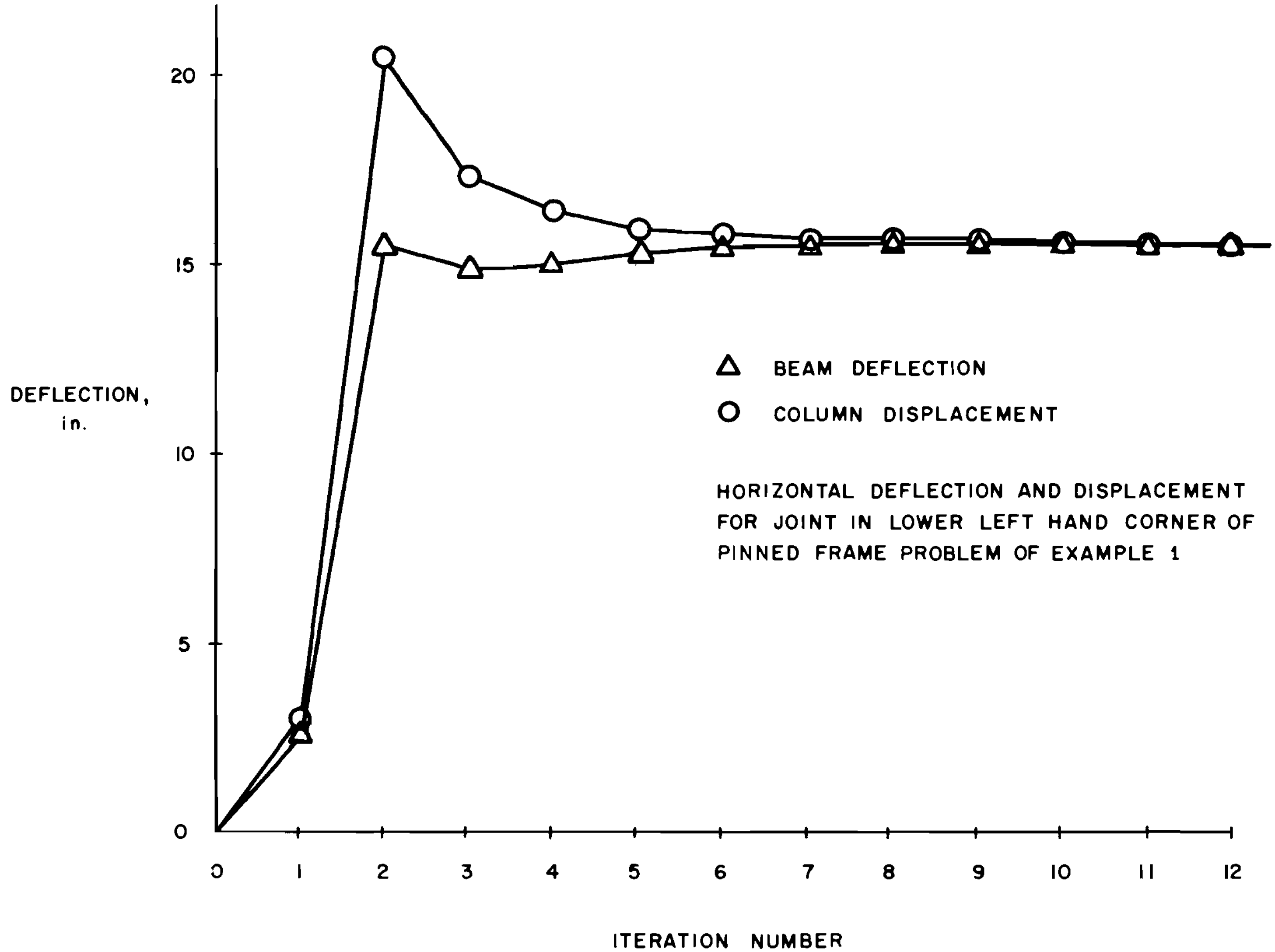


Fig 8.3. Translation closure for lower left joint of pinned frame of Example 1.

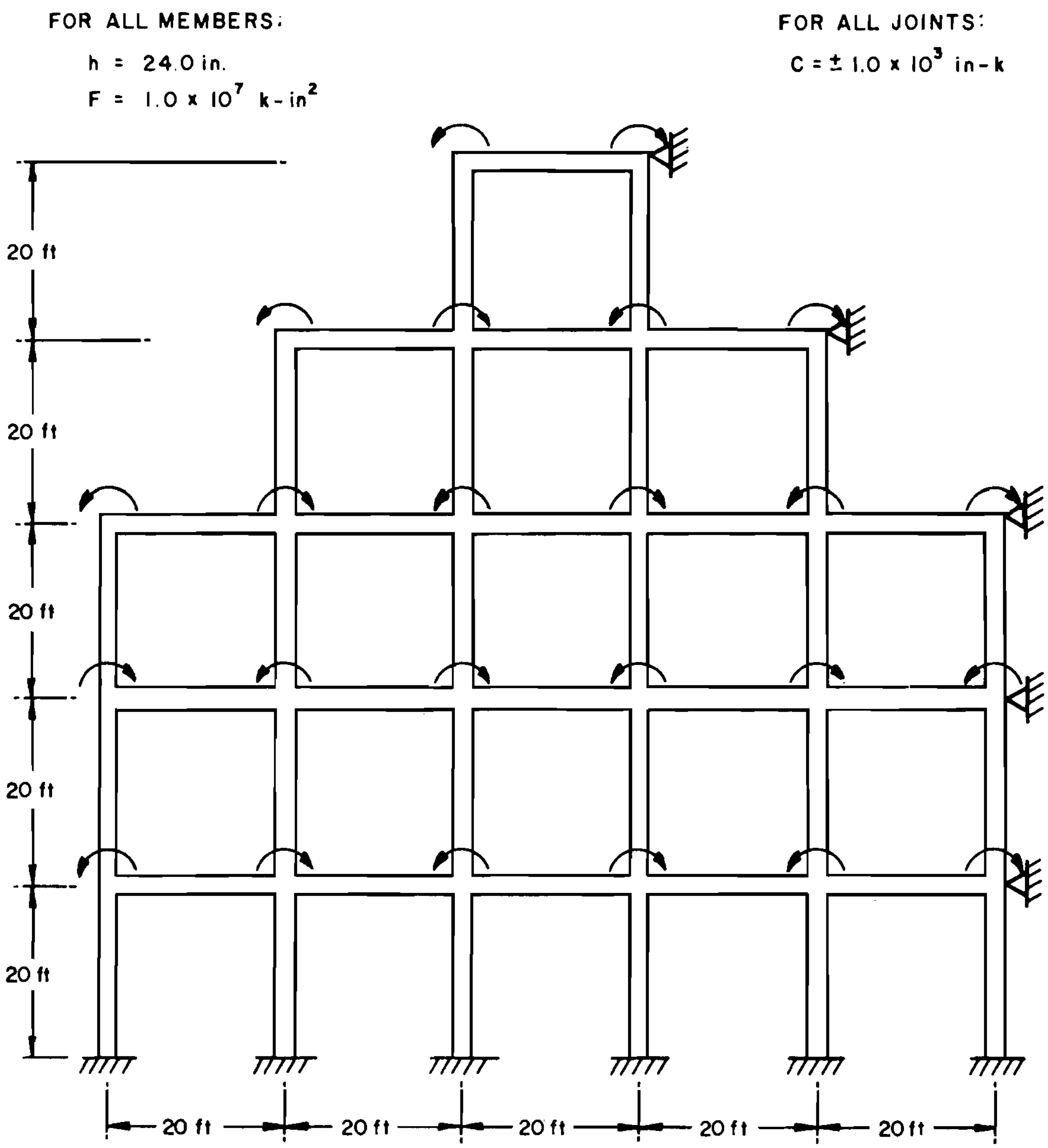


Fig 8.4. Stepped frame of Example 2.

specified at alternate joints. The opposing couples were selected to create a maximum deflection condition for the frame. Fixed ends are approximated in the method by large values of horizontal, vertical, and rotational restraint at the bottom of each vertical column, in the manner shown in Fig 7.1. From the symmetrical loading, the structure should not translate. Thus, in order to consider only rotational effects, each horizontal line of members was restrained by a large value of axial spring, approximating a simple support.

The resulting deflected shape of the frame may be seen in Fig 8.5, and, as expected, it is symmetrical. A total of 11 iterations were required to achieve rotational or equal slope convergence to a tolerance of 1.0×10^{-7} radian. For this tolerance, the maximum error observed in summation of couples at any joint was 0.02 inch-kip.

Rotational closure for the top joint in the left line of vertical members is plotted in Fig 8.6. As seen for translational effects in Example 1, only a few iterations are required for the differential restraint process to correctly estimate the rotational restraint provided by each intersecting member. The rest of the iterations refine this value until the desired degree of closure is achieved.

Example 3

Example 3 is a problem in which translational and rotational interaction must be considered. Shown in Fig 8.7, it is a five-bay-wide frame susceptible to sway.

The frame is subjected to alternate bay loadings plus a linearly decreasing horizontal load intended to simulate wind forces. Not shown in the figure is a constant transverse load of 100 lb/ft applied to all horizontal members and a linearly increasing axial compression of 250 lb/ft applied to all vertical members, simulating weight forces. The lower line of frame members is haunched to give additional resistance to deflection.

Lateral foundation support of constant modulus is assumed to be provided by the transverse springs shown acting on each column, while resistance to column settlement is provided by a spring under each column.

The deflected shape of the frame is shown in Fig 8.8a. The resulting settlement or downward displacement of each column is shown in Fig 8.8b.

Bending moment diagrams for each line of frame members are shown in

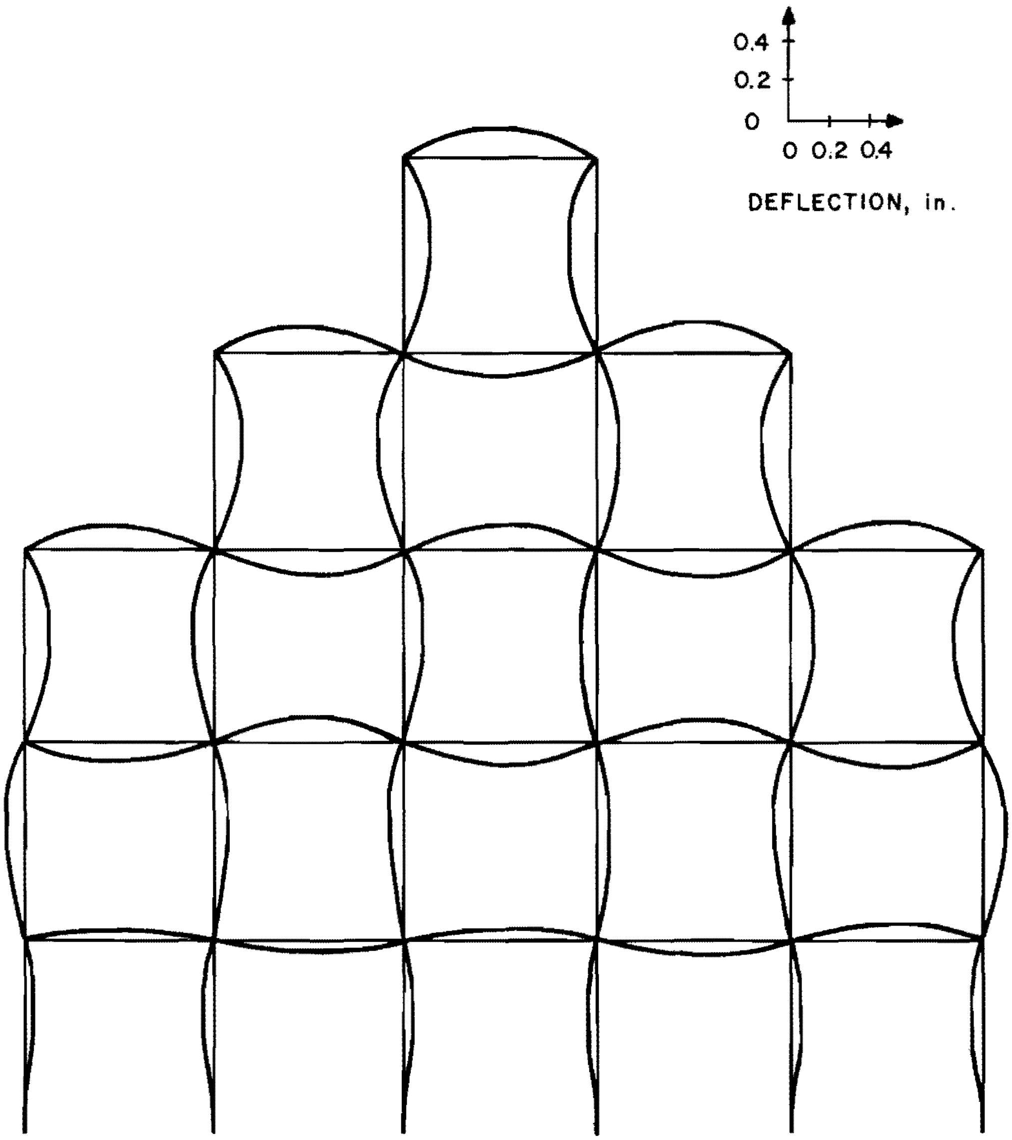


Fig 8.5. Deflected shape of stepped frame of Example 2.

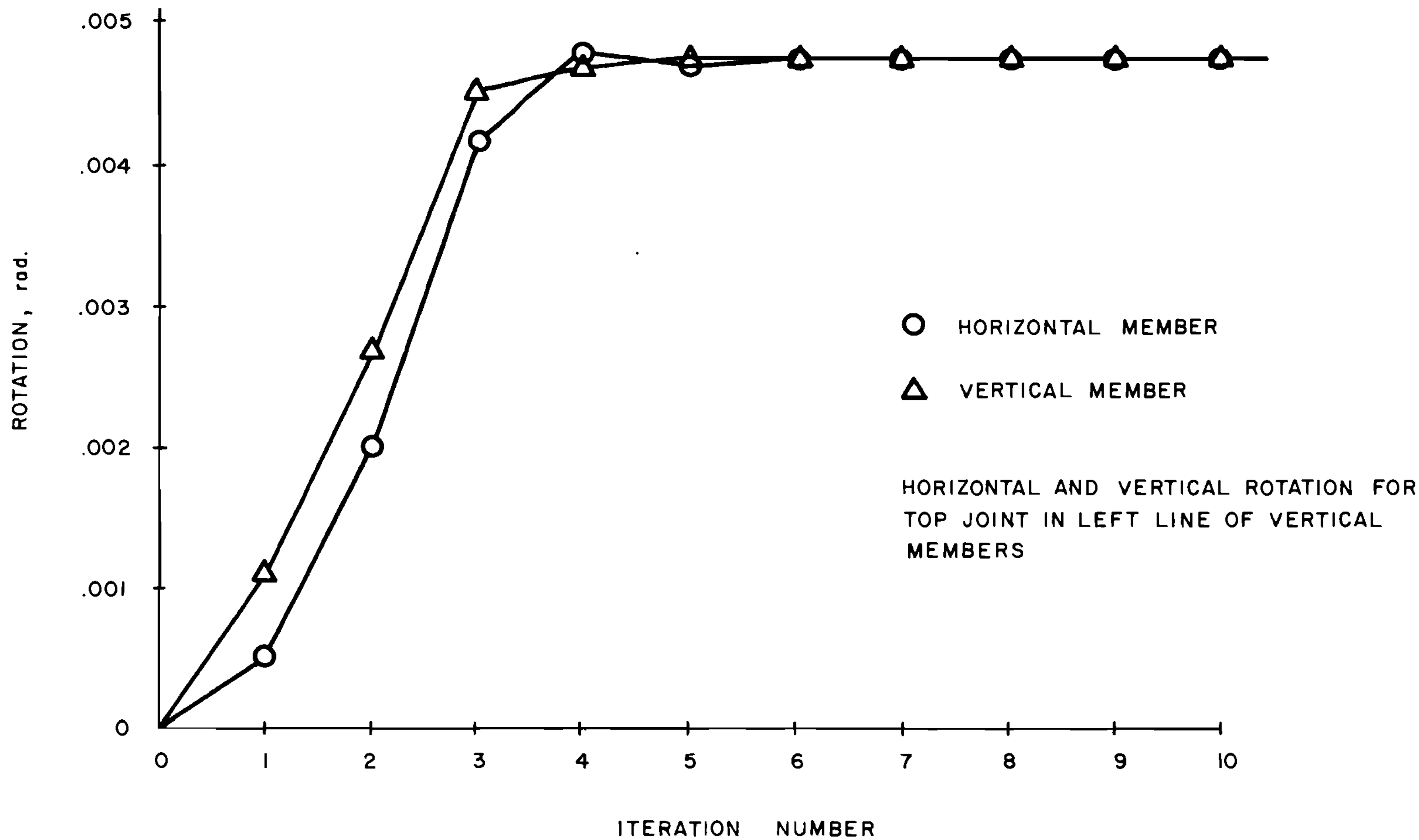
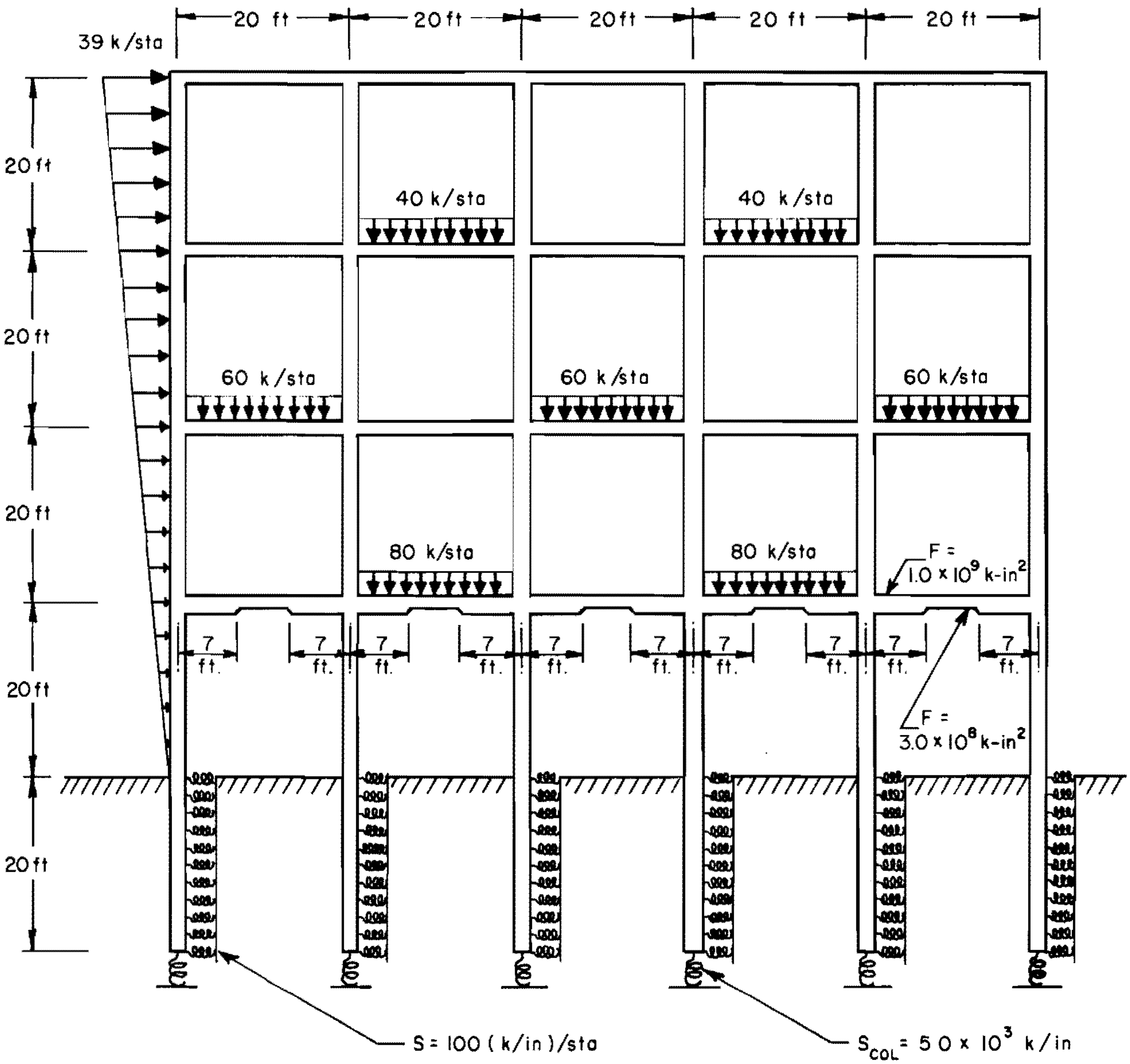


Fig 8.6. Rotational closure of upper joint in left line of vertical members of stepped frame of Example 2.



HORIZONTAL MEMBERS

h = 24.0 in

$F = 3.0 \times 10^{8}$ k-in^2 *

* except as noted

VERTICAL MEMBERS

h = 24.0 in

$F = 1.0 \times 10^{9}$ k-in^2

Fig 8.7. Five-bay frame of Example 3.

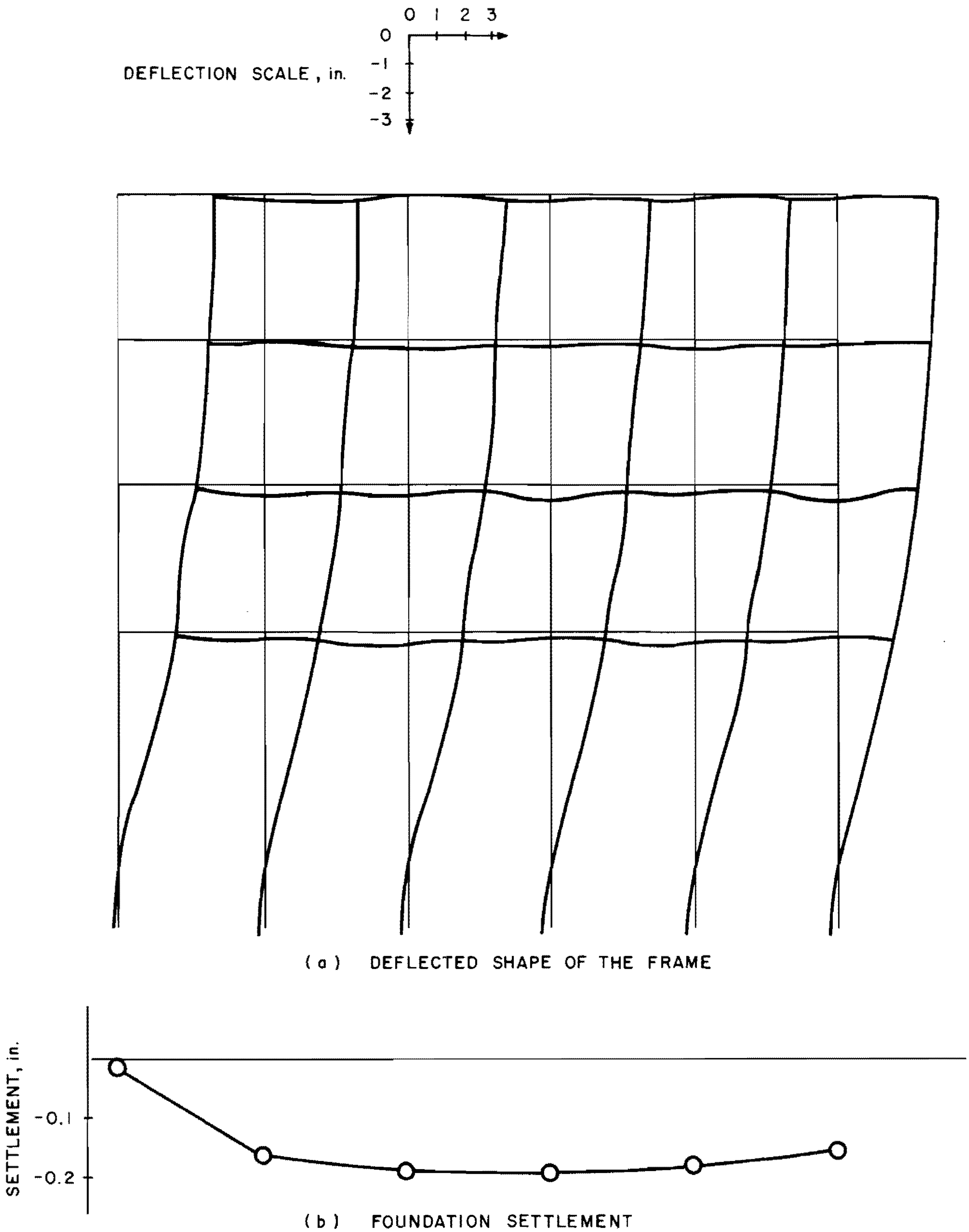


Fig 8.8. Deflected shape of five-bay frame of Example 3.

Fig 8.9. The effect of sway on the moment diagrams for the horizontal lines of members may easily be seen. As the interior vertical members carry no transverse load, a linear variation of moment occurs between joints. The finite joint width prevents a complete discontinuity of moment at each joint: the change in moment must take place across the finite-width joint. This condition is easily seen in the moment diagrams for the vertical members.

More iterations were needed to solve this problem than were required for either Examples 1 or 2. As discussed in Chapter 4, translational and rotational interaction at each joint tends to inhibit immediate self-determination of individual values of translational restraint. Thus, more iterations are required to determine correct estimates of joint restraint and to refine those computed estimates.

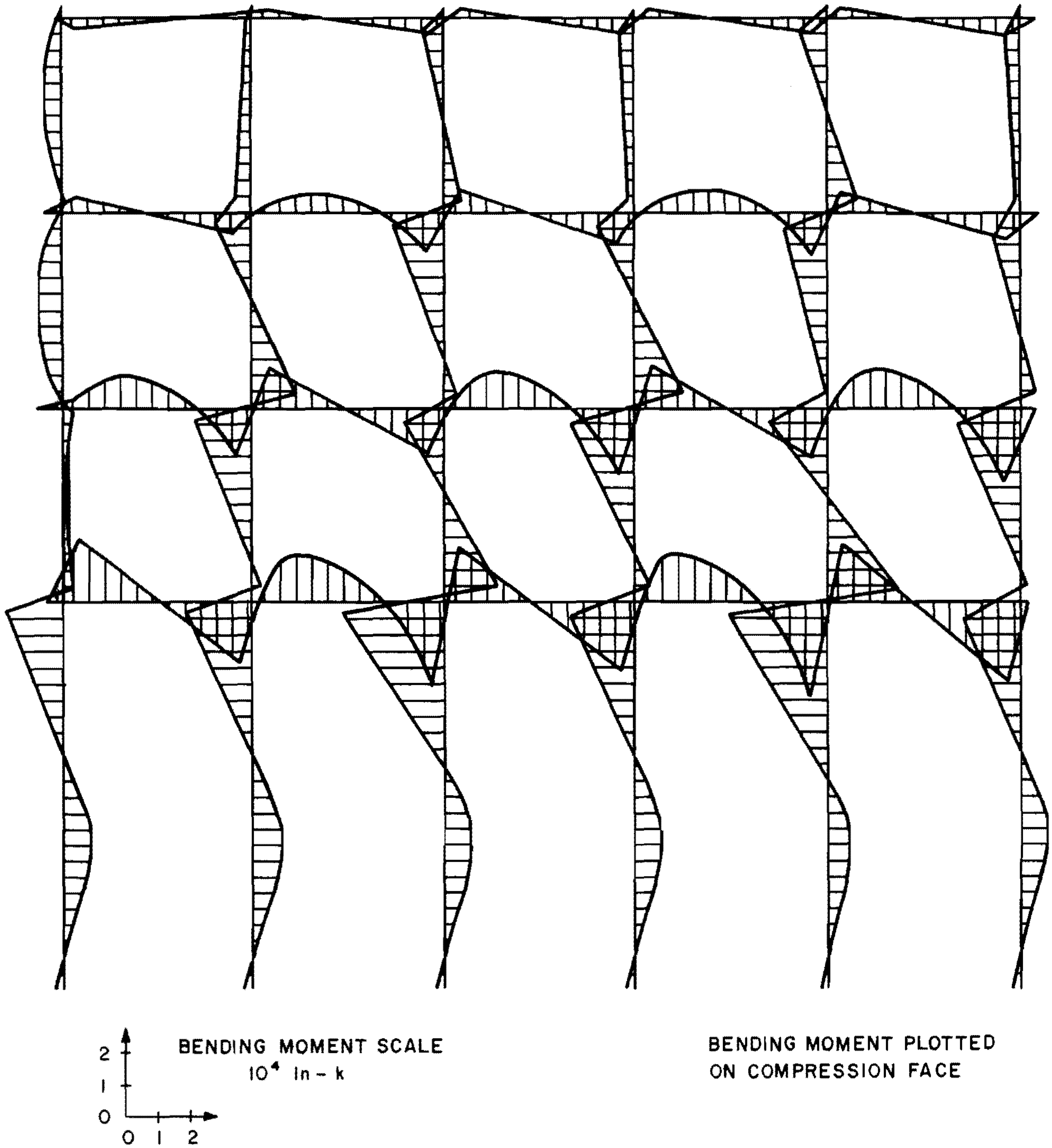


Fig 8.9. Bending moment diagrams for the five-bay frame of Example 3.

CHAPTER 9. POSSIBLE EXTENSIONS OF THE METHOD

The proposed method of frame analysis, as derived and verified in the previous chapters, is a rational procedure for the analysis of rectangular plane frames when constant transverse loading, elastic spring or foundation support, elastic material behavior, and axial rigidity are assumed.

A more general method of frame analysis, then, would consider the effects of nonlinear load and support characteristics, nonlinear frame material behavior, and axial deformations in determining the deflected shape of the frame. Although PLNFRAM 4, the computer program written to verify the proposed method, does not consider these effects, it is felt that they may be easily incorporated into the basic method. Procedures for consideration of these effects are discussed in the following sections.

Nonlinear Load and Support Characteristics

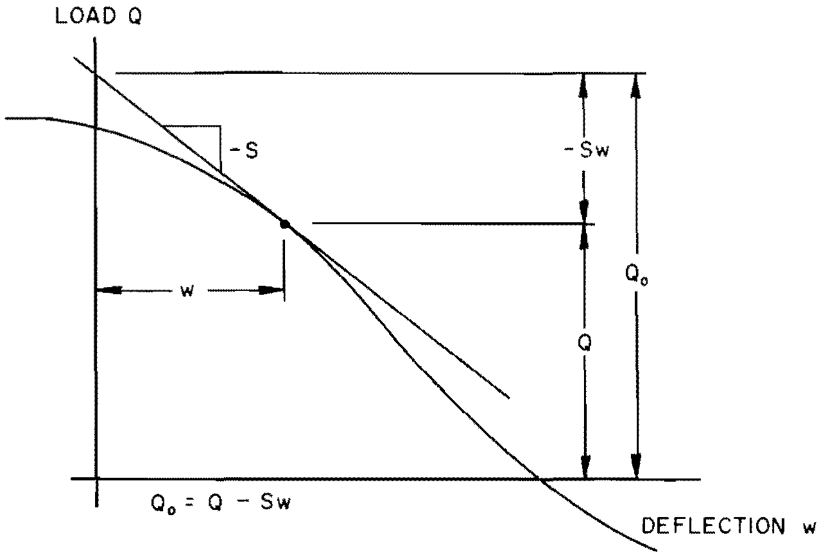
Nonlinear load and support characteristics may be considered in a manner similar to that developed by Ingram (Ref 12). Any single-valued nonlinear force-deformation relationship may be represented by a curve as shown in Fig 9.1a. This curve may be approximated in the computer by a finite series of points.

For any particular deflection w it is possible to temporarily represent the nonlinear relationship of Fig 9.1a by a tangent to the curve. Such a tangent has an intercept Q and a slope $-S$, which correspond respectively to values of transverse load and transverse spring restraint.

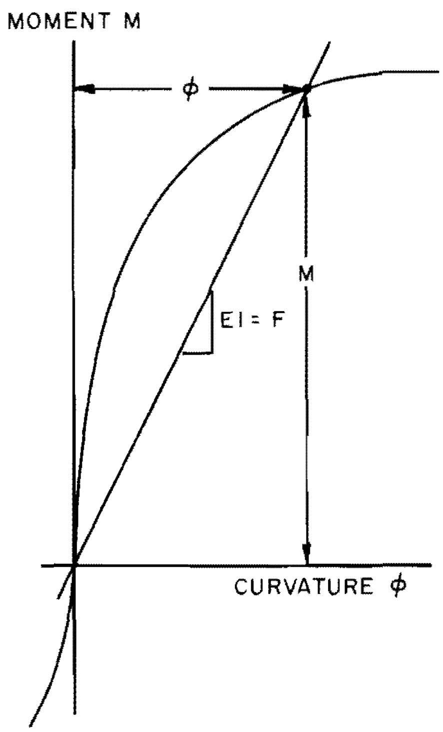
Thus, to solve a nonlinear problem, a series of solutions would be made, with the load and support characteristics at desired stations on the frame members being adjusted, based on computed deflections w, after each solution. This iterative procedure could be carried out in conjunction with the general iterative process of frame solution.

Nonlinear Flexural Stiffness Characteristics

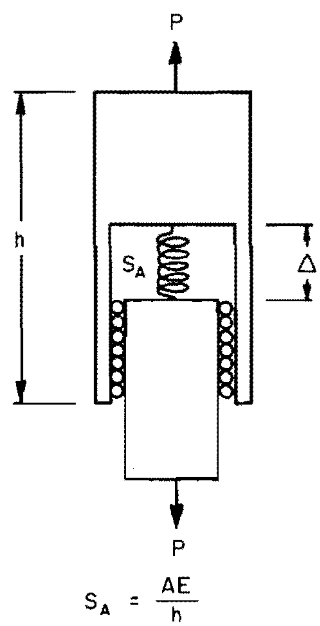
Nonlinear flexural stiffness characteristics may be considered in a manner



(a) REPRESENTATION OF NONLINEAR LOAD-DEFORMATION RELATIONSHIP



(b) REPRESENTATION OF NONLINEAR MOMENT-CURVATURE RELATIONSHIP



(c) RIGID ELEMENT SUBJECT TO AXIAL DEFORMATION

Fig 9.1. Possible additions to the method.

similar to that developed by Haliburton and Matlock (Ref 10). Any single-valued moment-curvature relationship may be represented by a curve as shown in Fig 9.1b. This curve may also be approximated in the computer by a finite series of points.

The secant to any point on the curve has a slope EI or F . Also, the curvature ϕ_i of any station on a frame member may be approximated by the relation

$$\phi_i \doteq \left[\frac{d^2 w}{dx^2}\right]_i \doteq \left[\frac{w_{i-1} - 2w_i + w_{i+1}}{h^2}\right] \tag{9.1}$$

Thus, to solve a nonlinear flexural stiffness problem, a series of solutions would be made with the flexural stiffness at the stations along each frame member being adjusted after each solution. The procedure is very straightforward. Using the previously computed deflections w , the curvature ϕ is computed by Eq 9.1. Interpolation is performed on the given M-ϕ curve for a new value of flexural stiffness F . Another solution is made using these new F values.

In effect, the flexural stiffness is assumed to be temporarily elastic during each trial solution. The effect of axial tension or compression on the relationship may be considered by using a series of M-ϕ curves. In such a case, interpolation is performed between curves as well as between the points on each curve.

As is the case for the proposed nonlinear load and support procedure, the nonlinear flexural stiffness adjustment could also be carried out during the general iterative process of frame analysis.

Axial Deformations

The effect of axial deformation on the bending of a member is probably insignificant. However, the effect of axial deformation on the consistent deformations of a frame may or may not be of particular importance, depending upon the problem to be solved and the degree of accuracy required. One method for considering the effect of axial deformation on the consistent deformations of the frame is proposed.

Consider the element of Fig 9.1c. This element is assumed to be rigid in bending, but is subject to some axial deformation Δ under the action of the axial force P . The resistance of the element to axial deformation is provided by the axial spring S_A , where

$$S_A = \frac{AE}{h} \tag{9.2}$$

with A being the cross-section area of the member approximated by the element and E being the modulus of elasticity of the member.

The effect of axial deformations on a line of model frame members, such as that of Fig 4.6 or 4.7 may be determined for any particular iteration by (1) computing the axial tension or compression distribution in the line of members and (2) computing the displacement Δ_i for each Bar i in the line of model frame members. The axial displacement of any particular point may then be found by integrating the Δ_i from one end of the line toward the other end.

CHAPTER 10. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

A numerical method of analysis for rectangular plane frames has been developed. Results computed by the method have been compared with those from accepted theory. In addition, the method has been applied to the solution of example problems.

It is therefore concluded that the developed method, subject to the previously described assumptions of "small-deflection" theory, elastic support and material behavior, and axial rigidity, is a valid procedure for the bending analysis of rectangular plane frames. Principal features of the method are

(1) A mathematical finite-element model of the real frame is simulated and solved on the digital computer.

(2) Equations describing the finite-element frame model allow point-to-point variation of applied transverse and axial loading, transverse spring restraint, and flexural stiffness along each frame member. Horizontal and vertical loads and restraints, an applied couple, and a rotational restraint may be applied at each frame joint.

(3) An iterative procedure is used to solve the frame equations. Each iteration consists of two parts, a stiffness matrix solution, using an efficient recursive technique, for the deflected shape of the frame in bending, and a solution for the axial force distribution in frame members.

(4) Translational compatibility at each joint is enforced during the iterative process by internally computed values of load and restraint.

(5) Rotational compatability at each joint is enforced during the iterative process by externally defined values of differential rotational restraint.

(6) Results given by the method include the deflected shape of the frame in bending and the horizontal translation, vertical translation, and rotation of each frame joint.

Significance of the Method

The derived method, while subject to previously discussed assumptions, is applicable to a wide range of structural problems. Specifically, any rectangular plane frame having three degrees of freedom at each joint may be analyzed. In contrast with conventional methods of frame analysis, any desired variation of frame-member flexural stiffness, applied loading, and foundation spring restraint may be easily considered. The size of frame to be solved is limited only by available computer storage.

Use of the method in design is facilitated by the fact that the engineer need only describe the system to be solved. Tedious hand computations are avoided and the effect of key parameters may be evaluated by solving several similar problems.

Recommendations for Further Research

Based on the method developed here, the following further research is recommended:

(1) investigation of other procedures for selecting rotational closure parameters,

(2) extension of the present capabilities of the method to consider nonlinear foundation support, nonlinear material behavior, and axial deformation,

(3) extension of presently available procedures for dynamic analysis of beam-columns to consider frame behavior, and

(4) extension of the method to the analysis of three-dimensional frame problems.

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APPENDIX 1

GUIDE FOR DATA INPUT FOR
PROGRAM PLNFRAM 4

GUIDE FOR DATA INPUT FOR PLNFRAM 4

with Supplementary Notes

extract from

A FINITE-ELEMENT ANALYSIS OF STRUCTURAL FRAMES

by

T. Allan Haliburton and Hudson Matlock

July 1967

PLNFRAM 4 GUIDE FOR DATA INPUT -- Card forms

IDENTIFICATION OF PROGRAM AND RUN (2 alphanumeric cards per run)

1 – 80

1 – 80

IDENTIFICATION OF PROBLEM (one card per problem)

PROB NUM (1–5)	(6–10)	Description of problem (alphanumeric) (11–80)

TABLE 1. PROGRAM CONTROL DATA

(A) GENERAL PROBLEM DATA (two cards per problem)

NUMBER OF .....

MEMBERS X (1–5)	MEMBERS Y (6–10)	TOTAL JOINTS (11–15)	MON JOINTS (16–20)	ROT PRMTRS (21–25)	CARDS IN TABLES 3 (26–30)	CARDS IN TABLES 4 (31–35)	CARDS IN TABLES 5 (36–40)	MAX ITERS (41–45)	(46–50)	S-SPRING FACTOR (51–60)	DEFLECTION TOLERANCE (61–70)	ROTATION TOLERANCE (71–80)

JOINTS TO BE MONITORED - ONLY ONE - HALF OF THE JOINT NEED BE DEFINED (8 max)

BEAM NUM (1–5)	STA TO LEFT (6–10)	BEAM NUM (11–15)	STA TO LEFT (16–20)	ETC ..... (21–25)	(26–30)	(31–35)	(36–40)	(41–45)	(46–50)	(51–55)	(56–60)	(61–65)	(66–70)	(71–75)	(76–80)

(B) JOINT - MEMBER INTERSECTION DATA (one card per joint)

INTER NUM	BEAM NUM	JOINT NUM	STA TO LEFT	WITH BEAM NUM	WITH JOINT NUM	WITH STA LEFT	JOINT TYPE
1–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40

Card columns 1 … 80

(C) ROTATIONAL CLOSURE PARAMETERS – coefficients of F/h, input in cyclic order (number of cards as specified)

PRMTR NUM	RHO
1–5	11–20

Card columns 1 … 80

TABLE 2. MEMBER CONSTANTS (one card per member)

MBR NUM	NUM INCRS	INCREMENT LENGTH
1–5	5–10	10–20

Card columns 1 … 80

TABLE 3. MEMBER FIXED INPUT DATA – Full values added to all stas (number of cards as specified in TABLE 1)

MBR NUM	FROM STA	THRU STA	F BENDING STIFFNESS	Q TRANSVERSE LOAD	S SPRING SUPPORT	ΔP CHANGE IN AXIAL FORCE
1–5	5–10	10–15	21–30	30–40	40–50	50–60

Card columns 1 … 80

TABLE 4. JOINT FIXED INPUT DATA (number of cards as specified in TABLE 1)

MBR NUM	JOINT NUM	Q TRANSVERSE LOAD	S SPRING SUPPORT	R ROTATIONAL RESTRAINT	C APPLIED COUPLE	
1–5	10	20	30	40	50	80

TABLE 5. COLUMN FIXED INPUT DATA (number of cards specified in TABLE 1)

MBR NUM		S SPRING SUPPORT	
1–5		11–20	80

TERMINATION CARD – Placed at end of data deck

0		DENOTES TERMINATION OF RUN	
1–5		11	80

GENERAL PROGRAM NOTES

The data cards must be stacked in proper order for the program to run.

A consistent system of units must be used for all input data, for example: pounds and inches.

All five-space words are fixed point integers . . . . | ± 1 2 3 4 |

All 10-space words are floating-point decimal numbers . . . . | ± 1 . 2 3 4 E ± 5 6 |

TABLE 1. PROGRAM CONTROL DATA

Termination and dummy joints must be included in the total number of joints. Each joint need only be counted once.

The S-Spring Factor is the fraction of EI/h^3 assumed for initial member support conditions and also as a differential restraint if the deflection of any joint goes to zero during the iterative process. This value should range from approximately 1.0 if all frame loads are applied at joints to approximately 0.01 if all frame loads are applied to members.

A consistent ordering system must be followed to describe frame geometry. Starting in the upper left-hand corner of the frame, number each line of x-members from top to bottom. Then number each line of y-members from left to right, starting with number of x-members plus one. Number stations on each x-member from left to right and stations on each y-member from top to bottom. The "station to left of the joint" for x-members is the station above the joint for y-members. Then number joints on each individual x-member, including dummy and termination joints, from left to right. Repeat for each individual y-member, counting from top to bottom.

Each joint must be defined at least once as shown in TABLE 1. For dummy or termination joints, describe only the real member, place a zero (0) in column 25 and leave the rest of the card blank.

The rotational closure coefficients ρ are the fractions of EI/h to be used as values of differential rotational restraint during the iterative process. Up to twenty (20) values of ρ may be specified, to be used in cyclic order as input.

Set joint type to zero (0) for rigid joint, to one (1) for pinned joint.

Up to twenty (20) lines of members may be input.

Up to ten (10) joints may be specified on each member.

TABLE 2. MEMBER CONSTANTS

Each member may be divided into a maximum of ninety (90) increments.

Typical units for increment length are inches.

TABLE 3. MEMBER FIXED INPUT DATA

Typical units,

Variables:	F	Q	S	P
Values per Station:	$lb\text{-}in^2$	lb	lb/in	lb

The change in axial tension or compression ΔP is assumed to occur in the bar above or to the left of the specified station. Input of ΔP at the initial station <u>only</u> will result in a constant axial force in the member. Input of a constant ΔP along the member will result in a linearly increasing axial force in the member. Tension is positive (+) while compression is negative (-) .

Station sequencing must be in increasing order.

Data storage is cumulative at each station.

TABLE 4. JOINT FIXED INPUT DATA

Typical units,

Variables:	Q	S	R	C
Values per Joint:	lb	lb/in	(in-lb)/rad	in/lb

Values of applied Q and S are assumed to act normal to the member on which they are applied.

Values of applied couple C and R may be applied to either half (but not both halves) of the joint under consideration.

Data storage is <u>not</u> cumulative at each joint.

TABLE 5. COLUMN FIXED INPUT DATA

Typical units of column restraint S are lb/in .

The column restraint is assumed to be placed at the right end of horizontal members and at the bottom of vertical members.

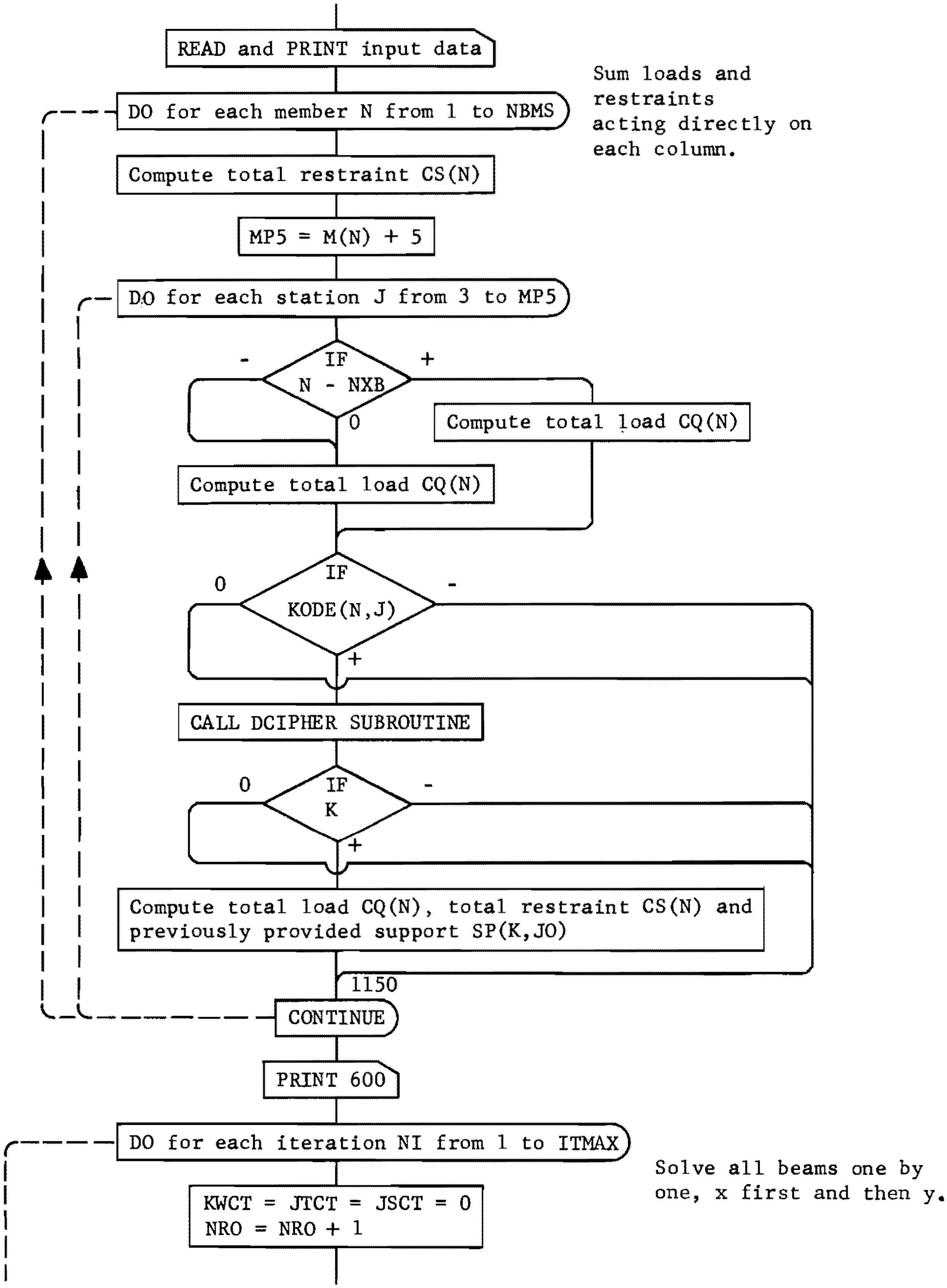
Data storage is <u>not</u> cumulative for each column.

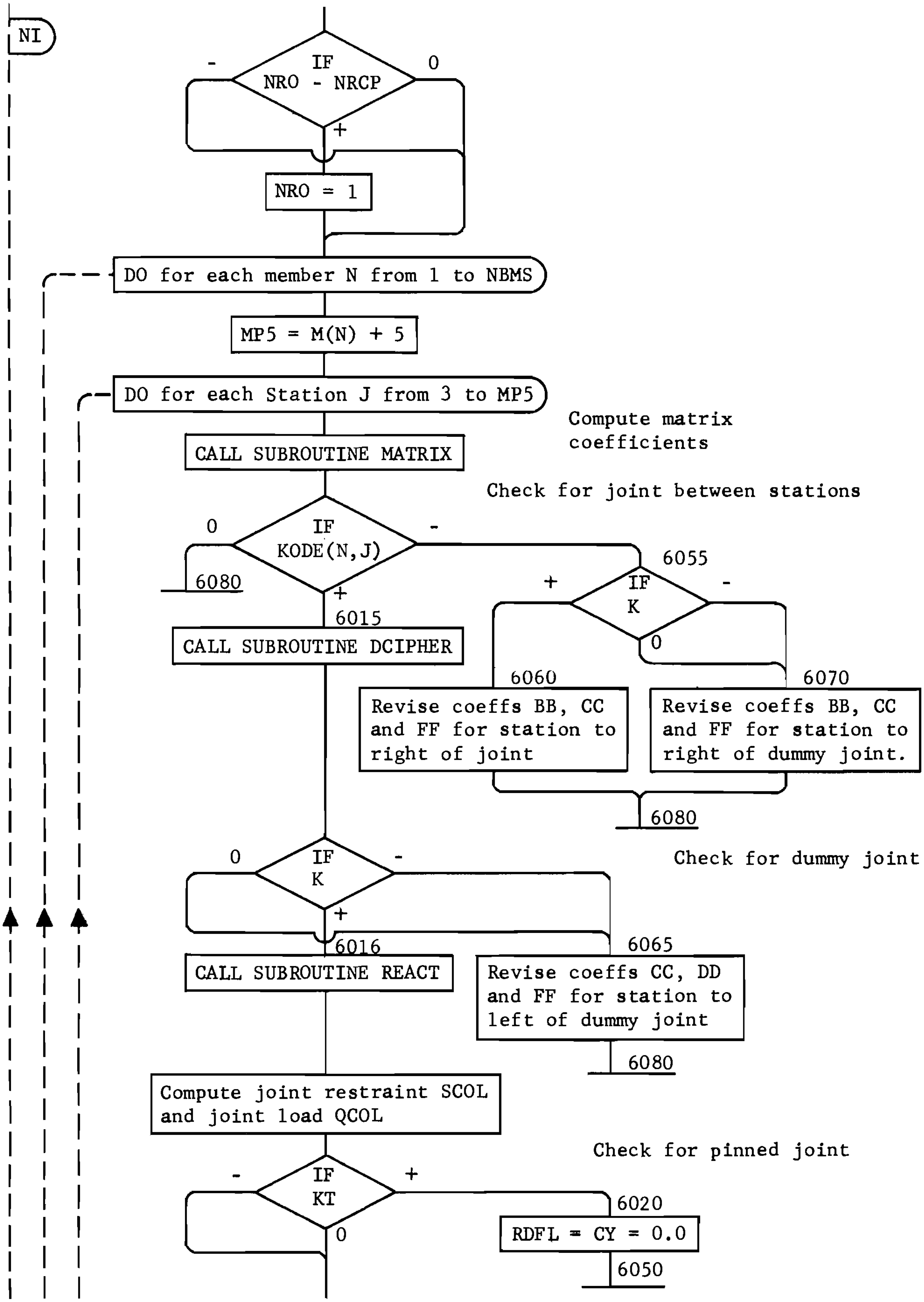
APPENDIX 2

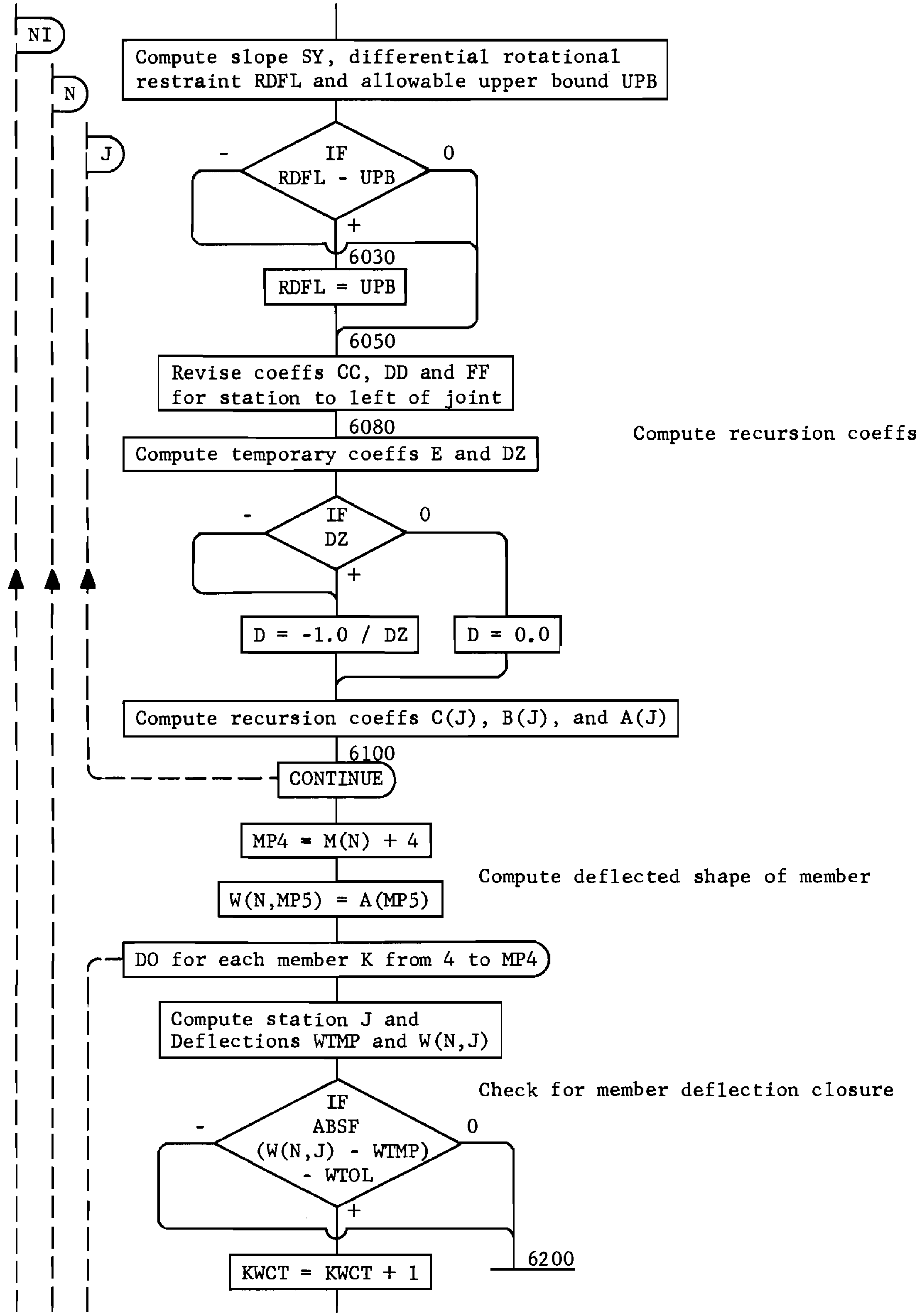
COMPUTATIONAL FLOW DIAGRAM FOR PROGRAM PLNFRAM 4

APPENDIX 2

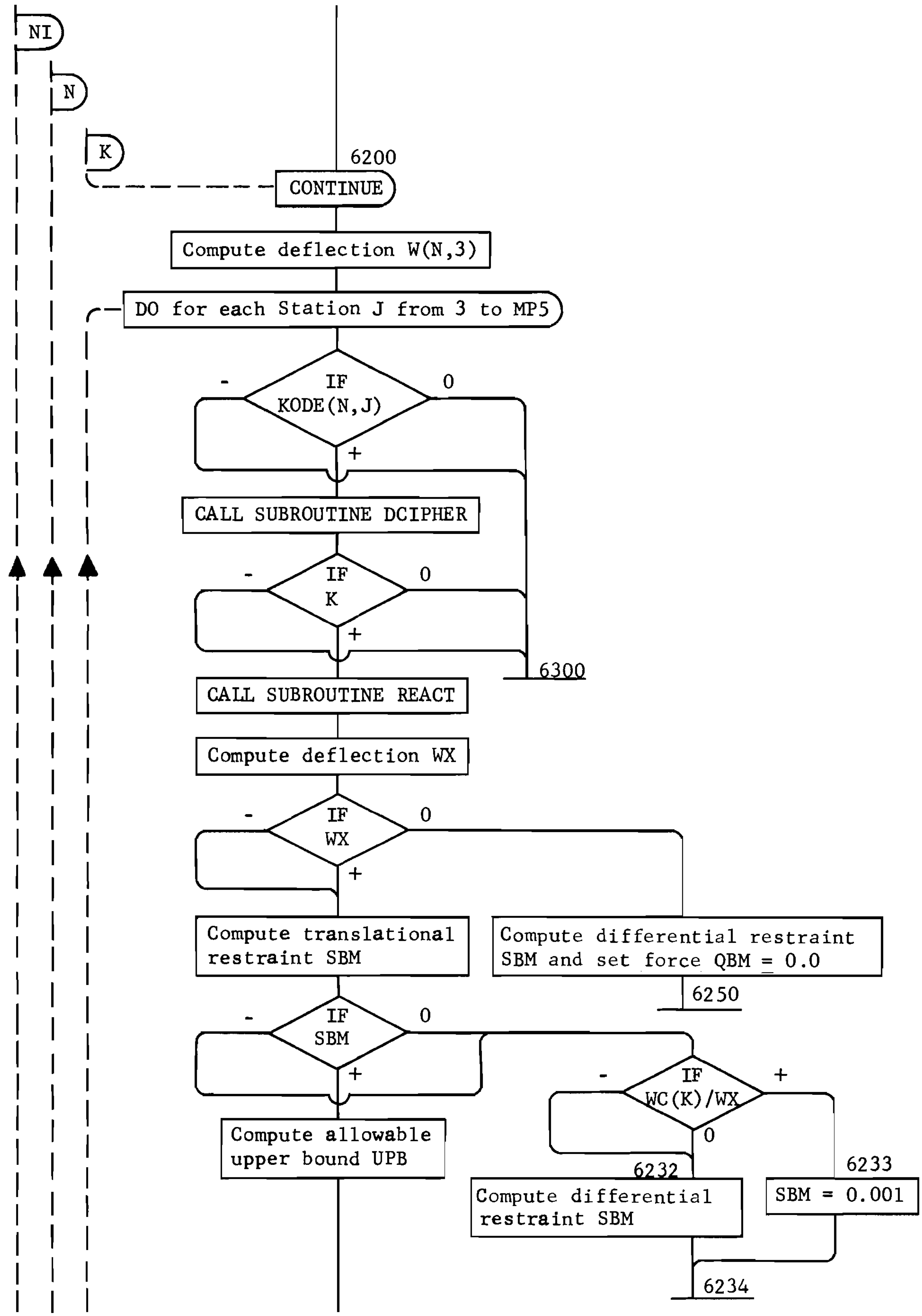
COMPUTATIONAL FLOW DIAGRAM FOR PROGRAM PLNFRAM 4

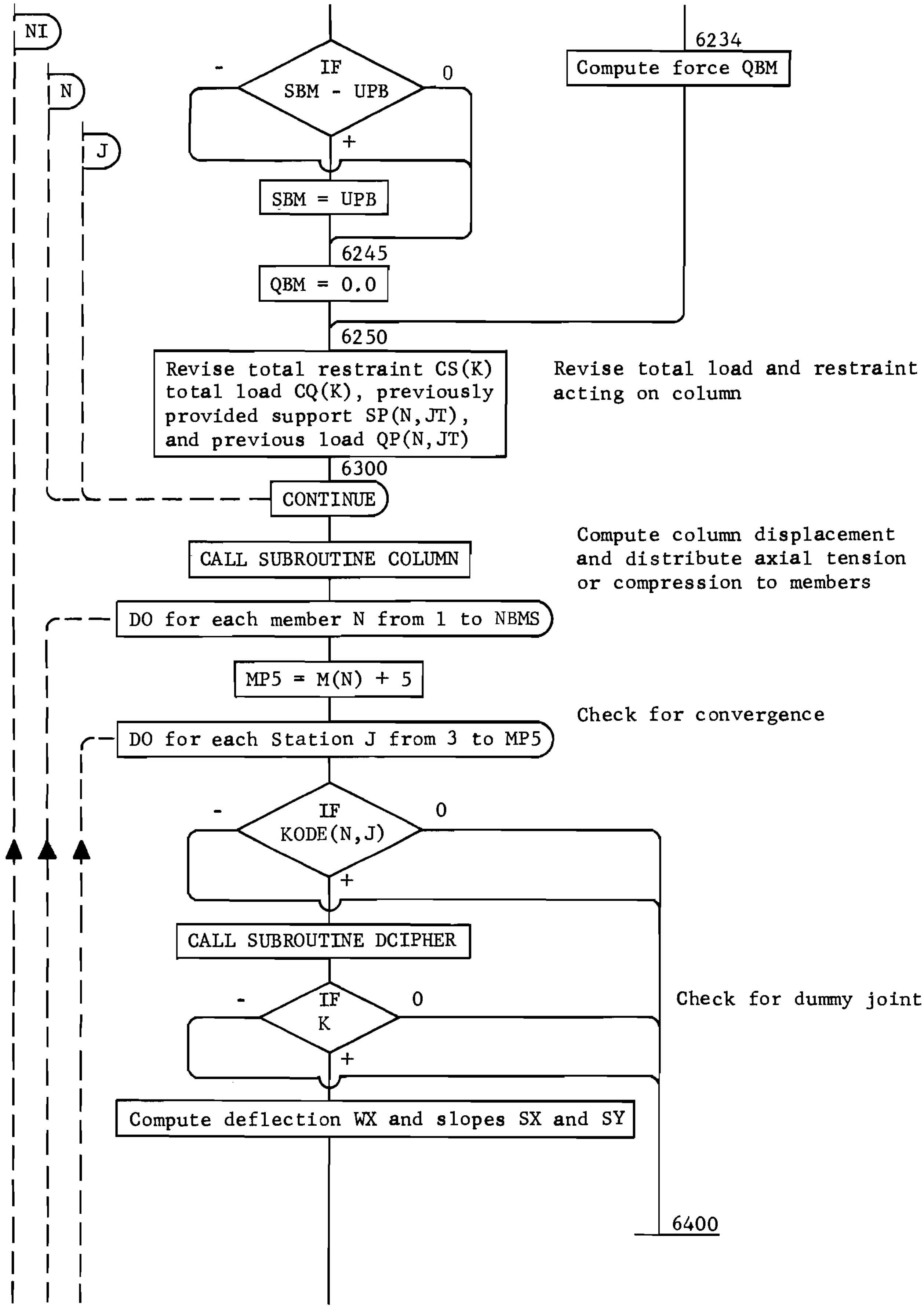


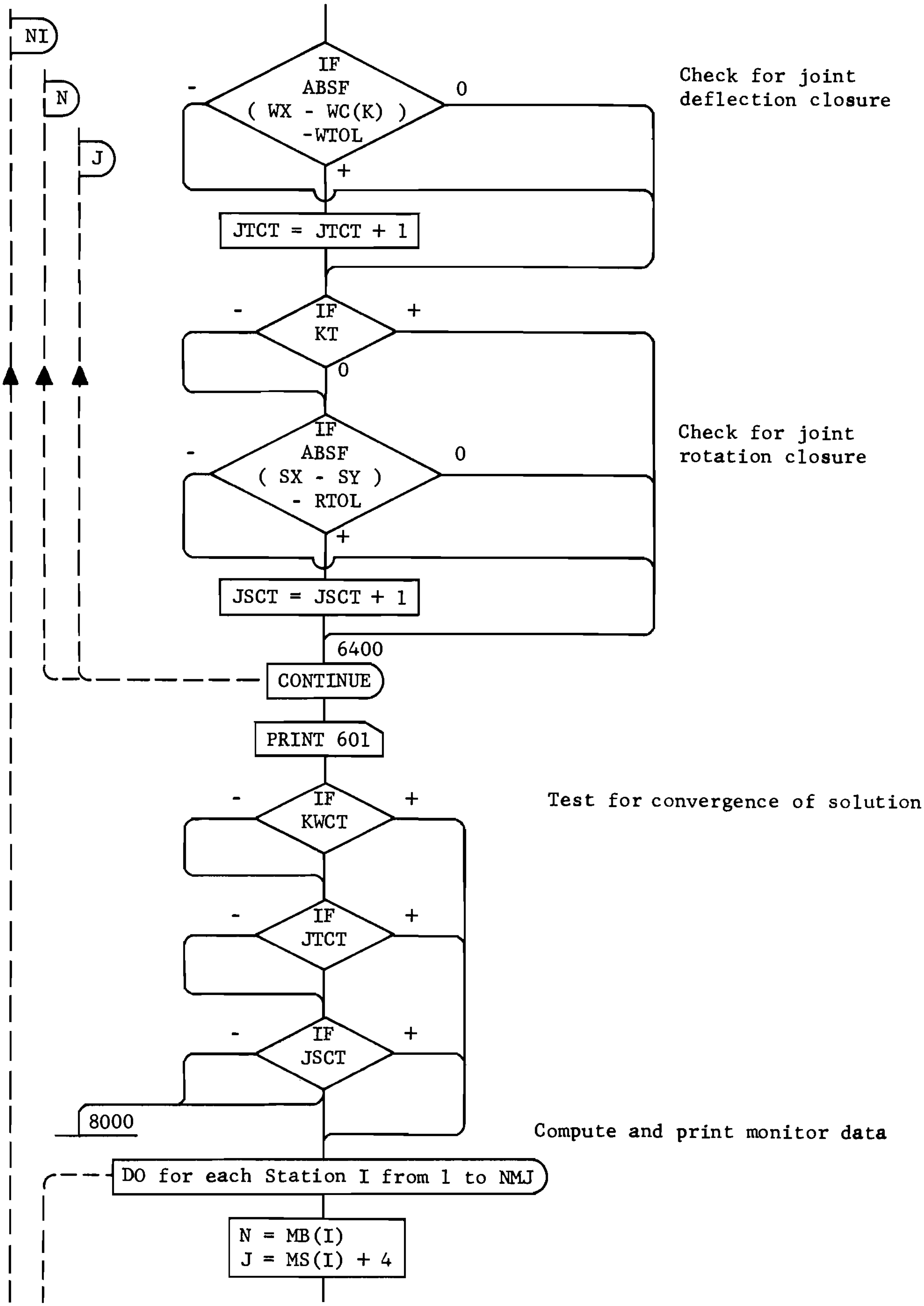

NI
IF NRO - NRCP
-
0
+
NRO = 1
DO for each member N from 1 to NBMS
MP5 = M(N) + 5
DO for each Station J from 3 to MP5
Compute matrix coefficients
CALL SUBROUTINE MATRIX
Check for joint between stations
IF KODE(N,J)
0
-
+
6080
6015
6055
IF K
+
-
0
CALL SUBROUTINE DCIPHER
6060
Revise coeffs BB, CC and FF for station to right of joint
6070
Revise coeffs BB, CC and FF for station to right of dummy joint.
6080
Check for dummy joint
IF K
0
-
+
6016
CALL SUBROUTINE REACT
6065
Revise coeffs CC, DD and FF for station to left of dummy joint
6080
Compute joint restraint SCOL and joint load QCOL
Check for pinned joint
IF KT
-
+
0
6020
RDFL = CY = 0.0
6050



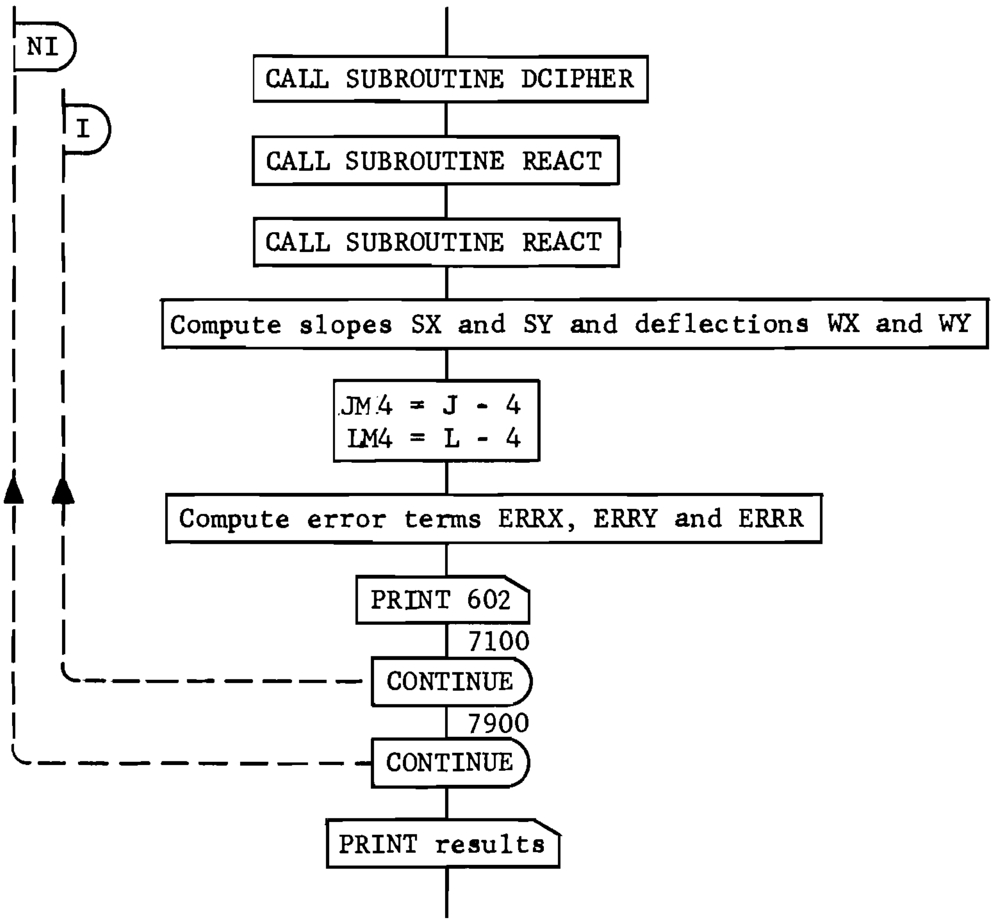
NI
N
J
Compute slope SY, differential rotational restraint RDFL and allowable upper bound UPB
IF RDFL - UPB
-
0
+
6030
RDFL = UPB
6050
Revise coeffs CC, DD and FF for station to left of joint
6080
Compute temporary coeffs E and DZ
Compute recursion coeffs
IF DZ
-
0
+
D = -1.0 / DZ
D = 0.0
Compute recursion coeffs C(J), B(J), and A(J)
6100
CONTINUE
MP4 = M(N) + 4
Compute deflected shape of member
W(N,MP5) = A(MP5)
DO for each member K from 4 to MP4
Compute station J and Deflections WTMP and W(N,J)
Check for member deflection closure
IF ABSF (W(N,J) - WTMP) - WTOL
-
0
+
KWCT = KWCT + 1
6200


NI
N
K
6200
CONTINUE
Compute deflection W(N,3)
DO for each Station J from 3 to MP5
IF KODE(N,J)
-
0
+
CALL SUBROUTINE DCIPHER
IF K
-
0
+
6300
CALL SUBROUTINE REACT
Compute deflection WX
IF WX
-
0
+
Compute translational restraint SBM
Compute differential restraint SBM and set force QBM = 0.0
6250
IF SBM
-
0
+
IF WC(K)/WX
-
+
0
Compute allowable upper bound UPB
6232
Compute differential restraint SBM
6233
SBM = 0.001
6234


NI
N
J
IF
SBM - UPB
-
0
+
6234
Compute force QBM
SBM = UPB
6245
QBM = 0.0
6250
Revise total restraint CS(K) total load CQ(K), previously provided support SP(N,JT), and previous load QP(N,JT)
Revise total load and restraint acting on column
6300
CONTINUE
CALL SUBROUTINE COLUMN
Compute column displacement and distribute axial tension or compression to members
DO for each member N from 1 to NBMS
MP5 = M(N) + 5
Check for convergence
DO for each Station J from 3 to MP5
IF
KODE(N,J)
-
0
+
CALL SUBROUTINE DCIPHER
IF
K
-
0
+
Check for dummy joint
Compute deflection WX and slopes SX and SY
6400



NI
N
J
IF ABSF (WX - WC(K)) -WTOL
-
0
+
Check for joint deflection closure
JTCT = JTCT + 1
IF KT
-
+
0
IF ABSF (SX - SY) - RTOL
-
0
+
Check for joint rotation closure
JSCT = JSCT + 1
6400
CONTINUE
PRINT 601
IF KWCT
-
+
Test for convergence of solution
IF JTCT
-
+
IF JSCT
-
+
8000
Compute and print monitor data
DO for each Station I from 1 to NMJ
N = MB(I)
J = MS(I) + 4


NI
I
CALL SUBROUTINE DCIPHER
CALL SUBROUTINE REACT
CALL SUBROUTINE REACT
Compute slopes SX and SY and deflections WX and WY
JM4 = J - 4
LM4 = L - 4
Compute error terms ERRX, ERRY and ERRR
PRINT 602
7100
CONTINUE
7900
CONTINUE
PRINT results

APPENDIX 3

LISTING OF COMPUTER PROGRAM

PLNFRAM 4

```
'COOP, CE051119, HALIBURTON, S / 2S, 10, 99999.        PLNFRM 4        MASTER DECK
'FTN,E,N,P.
      PROGRAM PLNFRAM 4
    1 FORMAT ( 5X, 48HPROGRAM PLNFRAM 4 - MASTER DECK - T A HALIBURTON  18AG5
     1          9X, 23HREVISION DATE 18 AUG 65 )                        18AG5
C *** PROGRAM PLNFRAM 4 SOLVES A RECTANGULAR PLANE FRAME WITH THREE      30JL5
C *** DEGREES OF FREEDOM AT EACH JOINT USING A LINE-BY-LINE ITERATIVE   14JL5
C *** TECHNIQUE WITH INTERNALLY COMPUTED TRANSLATIONAL AND EXTERNALLY   30JL5
C *** COMPUTED ROTATIONAL ITERATION PARAMETERS                          18AG5
C *** PROGRAM DEVELOPED AND PROGRAMMED BY T. A. HALIBURTON JULY 22, 196530JL5
C *** *** PRESENT CAPACITY OF THE PROGRAM IS TWENTY ( 20 ) LINES OF     14JL5
C *** *** NINETY ( 90 ) INCREMENTS ***** WARNING ***** DO NOT INCREASE  14JL5
C *** *** MEMBER STORAGE CAPACITY TO MORE THAN NINETY-FIVE ( 95 )       14JL5
C *** *** INCREMENTS WITHOUT CHANGING ENCODE AND DCIPHER ROUTINES       14JL5
C *** NOTATION FOR MAIN PROGRAM PLNFRAM 4 --                            30JL5
C ******* ARRAYS IN ALPHABETICAL ORDER --                               14JL5
C ***     A              RECURSION COEFFICIENT                          14JL5
C ***     AN1            ALPHANUMERIC IDENTIFICATION                    14JL5
C ***     AN2            ALPHANUMERIC IDENTIFICATION                    14JL5
C ***     B              RECURSION COEFFICIENT                          14JL5
C ***     C              RECURSION COEFFICIENT                          14JL5
C ***     CJ             COUPLE APPLIED AT JOINT                        14JL5
C ***     CQ             TOTAL LOAD ACTING ON COLUMN                    14JL5
C ***     CS             TOTAL RESTRAINT ACTING ON COLUMN               14JL5
C ***     DP             CHANGE IN AXIAL TENSION OR COMPRESSION         14JL5
C ***     F              MEMBER BENDING STIFFNESS                       14JL5
C ***     H              MEMBER INCREMENT LENGTH                        14JL5
C ***     KODE           CODE FOR JOINT-MEMBER INTERSECTION DATA        14JL5
C ***     M              NUMBER OF MEMBER INCREMENTS                    14JL5
C ***     MB             MONITOR BEAM NUMBER                            14JL5
C ***     MS             MONITOR STATION NUMBER                         14JL5
C ***     P              AXIAL TENSION OR COMPRESSION IN MEMBER         14JL5
C ***     Q              MEMBER TRANSVERSE LOAD                         14JL5
C ***     QJ             JOINT TRANSVERSE LOAD                          14JL5
C ***     QP             LOAD PREVIOUSLY ABSORBED BY MEMBER AT JOINT    14JL5
C ***     RHO            ROTATIONAL CLOSURE COEFFICIENT                 30JL5
C ***     RJ             JOINT ROTATIONAL RESTRAINT                     14JL5
C ***     S              MEMBER TRANSLATIONAL RESTRAINT                 14JL5
C ***     SC             COLUMN DISPLACEMENT RESTRAINT                  14JL5
C ***     SJ             JOINT TRANSLATIONAL RESTRAINT                  14JL5
C ***     SP             SUPPORT PREVIOUSLY PROVIDED BY MEMBER AT JOINT 14JL5
C ***     W              MEMBER TRANSVERSE DEFLECTION                   14JL5
C ***     WC             AXIAL COLUMN DISPLACEMENT                      14JL5
C ******* SINGLE VARIABLES IN ALPHABETICAL ORDER --                     14JL5
C ***     AA             MATRIX COEFFICIENT                             14JL5
C ***     BB             MATRIX COEFFICIENT                             14JL5
C ***     BM1            BENDING MOMENT IN MEMBER                       14JL5
C ***     BM2            BENDING MOMENT IN MEMBER                       14JL5
C ***     BM3            BENDING MOMENT IN MEMBER                       14JL5
C ***     CC             MATRIX COEFFICIENT                             14JL5
C ***     CX             COUPLE ABSORBED BY THIS JOINT HALF             14JL5
C ***     CY             COUPLE ABSORBED BY OTHER JOINT HALF            14JL5
C ***     D              TEMPORARY COEFFICIENT                          14JL5
C ***     DD             MATRIX COEFFICIENT                             14JL5
C ***     DPN            INPUT VALUE OF DP                              14JL5
```

```
C ***     DZ        TEMPORARY COEFFICIENT                            14JL5
C ***     D4W       NET MEMBER REACTION                              14JL5
C ***     E         TEMPORARY COEFFICIENT                            14JL5
C ***     EE        MATRIX COEFFICIENT                               14JL5
C ***     ERRR      ROTATIONAL ERROR FOR THE JOINT                   14JL5
C ***     ERRX      TRANSLATION ERROR FOR THIS JOINT HALF            14JL5
C ***     ERRY      TRANSLATION ERROR FOR OTHER JOINT HALF           14JL5
C ***     FF        MATRIX COEFFICIENT                               14JL5
C ***     FN        INPUT VALUE OF F                                 14JL5
C ***     I         STATION NUMBER - EXTERNAL                        14JL5
C ***     ITMAX     ITERATION LIMIT                                  14JL5
C ***     I1        INITIAL STATION ON MEMBER FOR DATA SEQUENCE      14JL5
C ***     I2        FINAL STATION ON MEMBER FOR DATA SEQUENCE        14JL5
C ***     J         STATION ON THIS MEMBER                           14JL5
C ***     JO        NUMBER OF OTHER JOINT HALF                       14JL5
C ***     JSCT      JOINT ROTATION COUNTER                           14JL5
C ***     JT        NUMBER OF THIS JOINT HALF                        14JL5
C ***     JTCT      JOINT TRANSLATION COUNTER                        14JL5
C ***     K         NUMBER OF INTERSECTING MEMBER                    14JL5
C ***     KT        JOINT TYPE - ZERO FOR RIGID - ONE FOR PINNED     14JL5
C ***     KWCT      MEMBER DEFLECTION COUNTER                        14JL5
C ***     L         STATION ON INTERSECTING MEMBER                   14JL5
C ***     MP3       M PLUS 3                                         14JL5
C ***     MP4       M PLUS 4                                         14JL5
C ***     MP5       M PLUS 5                                         14JL5
C ***     N         NUMBER OF THIS MEMBER                            14JL5
C ***     NB        MEMBER NUMBER                                    14JL5
C ***     NBMS      TOTAL NUMBER OF FRAME MEMBERS                    14JL5
C ***     NCT3      NUM CARDS IN TABLE 3                             14JL5
C ***     NCT4      NUM CARDS IN TABLE 4                             14JL5
C ***     NCT5      NUM CARDS IN TABLE 5                             14JL5
C ***     NI        ITERATION COUNTER                                14JL5
C ***     NJ        JOINT NUMBER                                     14JL5
C ***     NJTS      NUM OF JOINTS                                    14JL5
C ***     NJX       JOINT NUMBER                                     14JL5
C ***     NJY       JOINT NUMBER                                     14JL5
C ***     NMJ       NUM OF MONITOR JOINTS                            14JL5
C ***     NPROB     PROBLEM NUMBER                                   14JL5
C ***     NRCP      NUMBER OF ROTATIONAL PARAMETERS                  14JL5
C ***     NRO       ROTATIONAL CLOSURE COEFF NUMBER                  30JL5
C ***     NSX       STATION NUMBER TO LEFT OF JOINT                  14JL5
C ***     NSY       STATION NUMBER TO LEFT OF JOINT                  14JL5
C ***     NX        BEAM NUMBER                                      14JL5
C ***     NXB       NUM OF X-MEMBERS                                 14JL5
C ***     NY        MEMBER NUMBER                                    14JL5
C ***     NYB       NUM OF Y-MEMBERS                                 14JL5
C ***     QBM       FORCE REPRESENTING MEMBER RESTRAINT AT JOINT     14JL5
C ***     QCOL      JOINT LOAD PROVIDED BY OTHER MEMBERS             14JL5
C ***     QN        INPUT VALUE OF Q                                 14JL5
C ***     QX        LOAD ABSORBED BY THIS JOINT HALF                 14JL5
C ***     QY        LOAD ABSORBED BY OTHER JOINT HALF                14JL5
C ***     RDFL      DIFFERENTIAL ROTATIONAL RESTRAINT                30JL5
C ***     RTOL      ROTATIONAL CLOSURE TOLERANCE                     14JL5
C ***     RX1       HALF OF THIS JOINT HALF REACTION                 14JL5
C ***     RX2       HALF OF THIS JOINT HALF REACTION                 14JL5
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C ***      RY1                 HALF OF OTHER JOINT HALF REACTION                  14JL5
C ***      RY2                 HALF OF OTHER JOINT HALF REACTION                  14JL5
C ***      SBM                 S-SPRING REPRESENTING MEMBER RESTRAINT AT JOINT14JL5
C ***      SCOL                JOINT RESTRAINT PROVIDED BY OTHER MEMBERS          14JL5
C ***      SFTR                S-SPRING FACTOR                                    14JL5
C ***      SN                  INPUT VALUE OF S                                   14JL5
C ***      SX                  SLOPE OF THIS JOINT HALF                           14JL5
C ***      SY                  SLOPE OF OTHER JOINT HALF                          14JL5
C ***      UPB                 ALLOWABLE UPPER BOUND ON SBM OR RINB               14JL5
C ***      WTEMP               TEMPORARY VALUE OF MEMBER DEFLECTION               14JL5
C ***      WTOL                TRANSLATION CLOSURE TOLERANCE                      14JL5
C ***      WX                  DEFLECTION OF THIS JOINT                           14JL5
C ***      WY                  DEFLECTION OF OTHER JOINT HALF                     14JL5
C ***      X                   DISTANCE ALONG MEMBER                              14JL5
C *** ADDITIONAL NOTATION FOR SUBROUTINE COLUMN --                                14JL5
C ***      QCX                 CHANGE IN AXIAL TENSION OR COMPRESSION AT JOINT14JL5
C *** ADDITIONAL NOTATION FOR SUBROUTINE MATRIX                                   14JL5
C ***      Z                   TEMPORARY VALUE OF H                               14JL5
C *** ADDITIONAL NOTATION FOR SUBROUTINE DCIPHER                                  14JL5
C ***      JK                  TEMPORARY COEFFICIENT                              14JL5
C ***      JL                  TEMPORARY COEFFICIENT                              14JL5
C *** ADDITIONAL NOTATION FOR SUBROUTINE REACT                                    14JL5
C ******* ARRAYS --                                                               14JL5
C ***      BM                  BENDING MOMENT                                     14JL5
C ******* SINGLE VARIABLES                                                        14JL5
C ***      CPL                 COUPLE ABSORBED BY JOINT                           14JL5
C ***      D4W1                NET REACTION AT STATION TO LEFT OF JOINT           14JL5
C ***      D4W2                NET REACTION AT STATION TO RIGHT OF JOINT          14JL5
C ***      QJT                 LOAD ABSORBED BY JOINT                             14JL5
C ***      REACT1              HALF OF JOINT NET REACTION                         14JL5
C ***      REACT2              HALF OF JOINT NET REACTION                         14JL5
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   2  / 10X,   40H     ROTATIONAL CLOSURE TOLERANCENCE    , E11.3 )   22JL5
 101 FORMAT( /  60H     MONITOR JOINTS -- MEMBER AND STA TO THE LEFT OF05JL5
   1 JOINT        ( /, 10X, 8I6 ) )                                  30JL5
 102 FORMAT( /  45H          JOINT - MEMBER INTERSECTION DATA   //      07JL5
   1     85H  INTERSECTION  MEMBER  JOINT  STA TO LT  WITH  MEMBER  JOI22JL5
   2NT  STA TO LT JOINT TYPE      )                                  26JE5
 103 FORMAT( /  35H     ROTATIONAL CLOSURE PARAMETERS                22JL5
   1   //      20H     NUM      XI     )                             22JL5
 200 FORMAT(//  59H          TABLE 2. - NUM OF INCREMENTS AND INCREMENT05JL5
   1 LENGTH,// 38H     MEMBER  NUM INC  INCREMENT LENGTH  )          07JL5
 300 FORMAT( // 44H          TABLE 3. - MEMBER FIXED INPUT DATA //      07JL5
   1 73H     MEMBER  FROM STA  TO STA  STIFFNESS F   LOAD Q    SPRING 02AP5
   2S   DELTA P   )                                                  07JL5
 400 FORMAT( // 43H          TABLE 4. - JOINT FIXED INPUT DATA  //      07JL5
   1 66H     MEMBER   JOINT   Q  - LOAD   S - SPRING  R - SPRING   CO02AP5
   2UPLE    )                                                        02AP5
 500 FORMAT(//  44H          TABLE 5. - COLUMN FIXED INPUT DATA //      07JL5
   1 29H     MEMBER    COLUMN SPRING )                               12JL5
 600 FORMAT( // 60H          TABLE 6. - MONITOR JOINT OUTPUT AT SELECTE07JL5
   1D JOINTS / )                                                     02AP5
 601 FORMAT(    15H     ITERATION I3, 26H NOT CLOSED -- BEAM DEFLS I4, 07JL5
   113H JOINT DEFLS I3, 17H JOINT ROTATIONS I3 )                     02AP5
 602 FORMAT( 1X, 2HBM, I2, 3H JT, I2, 7H STA LT, I3, 3H WJ, E10.3,      30JL5
   1 3H WC, E10.3, 4H SLP, E10.3, 5H TERR, E10.3, 3X, 4HRERR, /     30JL5
   2 1X, 2HBM, I2, 3H JT, I2, 7H STA LT, I3, 3H WJ, E10.3, 3H WC,    30JL5
   3 E10.3, 4H SLP, E10.3, 5H TERR, 2E10.3, / )                      30JL5
 603 FORMAT(   / 41H      ***** ITERATION LIMIT EXCEEDED *****   )      07JL5
 604 FORMAT(   / 33H      ***** CLOSURE ACHIEVED ***** )                07JL5
 605 FORMAT(   / 26H           ***** NONE ***** )                       12JL5
 700 FORMAT( // 43H          TABLE 7. - RESULTS FOR EACH JOINT   )      07JL5
 701 FORMAT( // 80H      BEAM JOINT STA TO      BEAM      COLUMN      B30JL5
   1EAM   TRANSLATION ROTATION   , / 77H      NUM  NUM   LEFT   DEFLEC30JL5
   2TION DEFLECTION    SLOPE     ERROR     ERROR  )                  06AG5
 800 FORMAT( // 37H          TABLE 8. - MEMBER RESULTS - //            07JL5
   1            11H     MEMBER I3, //                                07JL5
   2            44H     STAS TO LT OR RT OF JOINT DENOTED BY *  //   07JL5
   3 85H        STA          X            DEFL           MOMENT      01JL5
   4 REACT       AXIAL FORCE     )                                   01JL5
C *** READ PROBLEM CONTROL DATA                                       01JL5
 1000 PRINT 10                                                       12JL3
      CALL TIME                                                      18FE5
      READ  12, ( AN1(N), N = 1, 32 )                                18FE5
 1010 READ 14, NPROB, ( AN2(N), N = 1, 14 )                          18AG5
          IF ( NPROB ) 1020, 9999, 1020                              07JL5
 1020 PRINT 11                                                       26AG3
      PRINT 1                                                        18AG5
      PRINT 13, ( AN1(N), N = 1, 32 )                                18FE5
      PRINT 15, NPROB, ( AN2(N), N = 1, 14 )                         26AG3
      READ 18, NXB, NYB, NJTS, NMJ, NRCP, NCT3, NCT4, NCT5, ITMAX, SFTR,22JL5
     1        WTOL, RTOL                                             22JL5
      PRINT 100, NXB, NYB, NJTS, NMJ, NRCP, NCT3, NCT4, NCT5, ITMAX,    22JL5
     1        SFTR, WTOL, RTOL                                       22JL5
              NBMS = NXB + NYB                                       03MY5
      READ 19, ( MB(N), MS(N), N = 1, NMJ )                          08JL5
      PRINT 101, ( MB(N), MS(N), N = 1, NMJ )                        05AP5
C *** CLEAR ALL STORAGE TO ZERO                                      01JL5
          DO 1030 N = 1, 20                                          01AP5
              SC(N) = WC(N) = H(N) = CQ(N) = CS(N) = RHO(N) = 0.0     30JL5
              M(N) = 0                                               29JE5
          DO 1025 J = 1, 97                                          08JL5
              F(N,J) = Q(N,J) = S(N,J) = P(N,J) = W(N,J) = DP(N,J) =    01AP5
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    1                 A(J) = B(J) = C(J) = 0.0                        01AP5
                KODE(N,J) = 0                                         01AP5
 1025       CONTINUE                                                  05AP5
            DO 1030 J = 1, 10                                         01AP5
                QJ(N,J) = SJ(N,J) = CJ(N,J) = RJ(N,J) = QP(N,J) =    15JL5
    1                 SP(N,J) = 0.0
 1030       CONTINUE                                                  01AP5
C *** READ JOINT-MEMBER INTERSECTION DATA                              01JL5
      PRINT 102                                                       31MR5
            DO 1040 I = 1, NJTS                                       31MR5
      READ 19, NJ, NX, NJX, NSX, NY, NJY, NSY, KT                     08JL5
      PRINT 20, NJ, NX, NJX, NSX, NY, NJY, NSY, KT                    08JL5
C *** ENCODE JOINT - MEMBER INTERSECTION DATA                         08JL5
                KODE(NX,NSX+4) = NJX * 10000000 + NJY * 100000 + NY * 31MR5
    1                        1000 + ( NSY + 4 ) * 10 + KT             26JE5
                KODE(NX,NSX+5) = - 1                                  31MR5
            IF ( NY ) 1040, 1040, 1035                                13MY5
 1035           KODE(NY,NSY+4) = NJY * 10000000 + NJX * 100000 + NX * 13MY5
    1                        1000 + ( NSX + 4 ) * 10 + KT             26JE5
                KODE(NY,NSY+5) = - 1                                  31MR5
 1040       CONTINUE                                                  31MR5
C *** READ ROTATIONAL CLOSURE COEFFICIENTS                            28JL5
      PRINT 103                                                       22JL5
            DO 1045 N = 1, NRCP                                       22JL5
      READ 29, NRO, RHO(NRO)                                          30JL5
      PRINT 35, NRO, RHO(NRO)                                         30JL5
 1045       CONTINUE                                                  22JL5
C *** READ MEMBER INCREMENT DATA                                      01JL5
      PRINT 200                                                       30AP5
            DO 1050 N = 1, NBMS                                       01AP5
      READ 23, NB, M(NB), H(NB)                                       08JL5
      PRINT 24, NB, M(NB), H(NB)                                      08JL5
 1050       CONTINUE                                                  01AP5
C *** READ MEMBER FIXED INPUT DATA                                    01JL5
      PRINT 300                                                       28JL5
            IF ( NCT3 ) 1055, 1055, 1060                              07JL5
 1055 PRINT 605                                                       07JL5
            GO TO 1085                                                07JL5
 1060       DO 1080  N = 1, NCT3                                      28JL5
      READ 25, NB, I1, I2, FN, QN, SN, DPN                            08JL5
      PRINT 26, NB, I1, I2, FN, QN, SN, DPN                           08JL5
                I1 = I1 + 4                                           10FE5
                I2 = I2 + 4                                           16MR5
            DO 1080 J = I1, I2                                        18AP5
                F(NB,J) = F(NB,J) + FN                                16MR5
                Q(NB,J) = Q(NB,J) + QN                                16MR5
                S(NB,J) = S(NB,J) + SN                                16MR5
                DP(NB,J) = DP(NB,J) + DPN                             29JE5
 1080       CONTINUE                                                  01AP5
C *** READ JOINT FIXED INPUT DATA                                     01JL5
 1085 PRINT 400                                                       28JL5
            IF ( NCT4 ) 1086, 1086, 1088                              28JL5
 1086 PRINT 605                                                       07JL5
            GO TO 1092                                                07JL5
 1088       DO 1090 I = 1, NCT4                                       28JL5
      READ 27, N, JT, QJ(N,JT), SJ(N,JT), RJ(N,JT), CJ(N,JT)          08JL5
      PRINT 28, N, JT, QJ(N,JT), SJ(N,JT), RJ(N,JT), CJ(N,JT)         08JL5
 1090       CONTINUE                                                  31MR5
C *** READ COLUMN FIXED INPUT DATA                                    01JL5
 1092 PRINT 500                                                       28JL5
            IF ( NCT5 ) 1094, 1094, 1096                              28JL5
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 1094 PRINT 605                                                          07JL5
         GO TO 1105                                                      07JL5
 1096    DO 1100 I = 1, NCT5                                             28JL5
      READ 29, N, SC(N)                                                  08JL5
      PRINT 30, N, SC(N)                                                 08JL5
 1100    CONTINUE                                                        31MR5
C *** SUM LOADS AND RESTRAINTS ACTING DIRECTLY ON EACH COLUMN             01JL5
 1105    DO 1150 N = 1, NBMS                                             07JL5
              CS(N) = CS(N) + SC(N)                                      18JE5
              MP5 = M(N) + 5                                             18JE5
         DO 1150 J = 3, MP5                                              18JE5
         IF ( N - NXB )  1131, 1131, 1132                                24JE5
 1131         CQ(N) = CQ(N) - DP(N,J+1)                                  24JE5
         GO TO 1133                                                      24JE5
 1132         CQ(N) = CQ(N) + DP(N,J+1)                                  24JE5
 1133    IF ( KODE(N,J) ) 1150, 1150, 1140                               24JE5
 1140 CALL DCIPHER ( N, J, K, L, JT, JO, KT )                            26JE5
         IF ( K ) 1150, 1150, 1145                                       18JE5
 1145         CQ(N) = CQ(N) + QJ(K,JO)                                   26JE5
              CS(N) = CS(N) + ( SFTR * 0.5 * ( F(K,L) + F(K,L+1) ) / ( 01JL5
     1                  H(K) ** 3 ) ) + SJ(K,JO)                         01JL5
              SP(K,JO) = SFTR * 0.5 * ( F(K,L) + F(K,L+1) ) /            16JL5
     1                     ( H(K) ** 3 )                                 20JL5
 1150    CONTINUE                                                        18JE5
      PRINT 11                                                           30JL5
      PRINT 1                                                            18AG5
      PRINT 13, ( AN1(I), I = 1, 32 )                                    30JL5
      PRINT 16, NPROB, ( AN2(I), I = 1, 14 )                             06AG5
      PRINT 600                                                          05AP5
      CALL TIME                                                          05AP5
C *** BEGIN ITERATION LOOP                                               28JL5
         DO 7900 NI = 1, ITMAX                                           19JA5
C *** SOLVE ALL BEAMS ONE BY ONE, X FIRST AND THEN Y                     07JL5
 6000         KWCT = JTCT = JSCT = 0                                     05AP5
              NRO = NRO + 1                                              30JL5
         IF ( NRO - NRCP ) 6005, 6005, 6002                              30JL5
 6002         NRO = 1                                                    30JL5
 6005    DO 6300 N = 1, NBMS                                             28JL5
              MP5 = M(N) + 5                                             27JA5
         DO 6100 J = 3, MP5                                              27JA5
C *** COMPUTE MATRIX COEFFICIENTS                                        01JL5
 6010 CALL MATRIX ( AA, BB, CC, DD, EE, FF, N, J )                       31MR5
C *** CHECK FOR JOINT BETWEEN STATIONS                                   01JL5
         IF ( KODE(N,J) ) 6055, 6080, 6015                               19JL5
 6015 CALL DCIPHER ( N, J, K, L, JT, JO, KT )                            19JL5
C *** CHECK FOR DUMMY JOINT                                              01JL5
         IF ( K )  6065, 6065, 6016                                      19JL5
 6016 CALL REACT ( K, L, JO, CY, QY, RY1, RY2 )                          19JL5
C *** COMPUTE LOAD AND RESISTANCE OF OTHER BEAMS AND COLUMN              01JL5
              SCOL = CS(K) - SP(N,JT) - SJ(N,JT)                         19JL5
              QCOL = CQ(K) - QP(N,JT) - QJ(N,JT)                         19JL5
C *** CHECK FOR PINNED JOINT                                             01JL5
         IF ( KT ) 6022, 6022, 6020                                      19JL5
 6020         RDFL = CY = 0.0                                            28JL5
         GO TO 6050                                                      16JL5
 6022         SY = ( W(K,L+1) - W(K,L) ) / H(K)                          16JL5
C *** COMPUTE DIFFERENTIAL ROTATIONAL RESTRAINT                          28JL5
              RDFL = RHO(NRO) * 0.5 * ( F(K,L) + F(K,L+1) ) / H(K)       30JL5
              UPB = 50.0 * ( F(N,J) + F(N,J+1) )                         22JL5
         IF ( RDFL - UPB ) 6050, 6050, 6030                              22JL5
 6030         RDFL = UPB                                                 22JL5
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C *** REVISE COEFFS FOR STATION TO LEFT OF JOINT                        01JL5
 6050          CC = CC + 0.25 * ( H(N) ** 3 ) * ( SJ(N,JT) + SCOL ) +   16JL5
     1              H(N) * ( RJ(N,JT) + RJ(K,JO) + RDFL )               22JL5
               DD = DD + 0.25 * ( H(N) ** 3 ) * ( SJ(N,JT) + SCOL ) -   07JL5
     1              H(N) * ( RJ(N,JT) + RJ(K,JO) + RDFL )               22JL5
               FF = FF + ( H(N) ** 3 ) * ( 0.5 * ( QJ(N,JT) + QCOL ) +  07JL5
     1              ( 1.0 / H(N) ) * ( CJ(N,JT) + CJ(K,JO) - CY -       22JL5
     2              RDFL * SY ) )                                       19JL5
          GO TO 6080                                                    31MR5
 6055     IF ( K ) 6070, 6070, 6060                                     29AP5
C *** REVISE COEFFS FOR STATION TO RIGHT OF JOINT                       01JL5
 6060          BB = BB + 0.25 * ( H(N) ** 3 ) * ( SJ(N,JT) + SCOL ) -   16JL5
     1              H(N) * ( RJ(N,JT) + RJ(K,JO) + RDFL )               22JL5
               CC = CC + 0.25 * ( H(N) ** 3 ) * ( SJ(N,JT) + SCOL ) +   08JL5
     1              H(N) * ( RJ(N,JT) + RJ(K,JO) + RDFL )               22JL5
               FF = FF + ( H(N) ** 3 ) * ( 0.5 * ( QJ(N,JT) + QCOL ) -  16JL5
     1              ( 1.0 / H(N) ) * ( CJ(N,JT) + CJ(K,JO) - CY -       22JL5
     2              RDFL * SY ) )                                       19JL5
          GO TO 6080                                                    29AP5
C *** REVISE COEFFS FOR STATION TO LEFT OF DUMMY JOINT                  12JL5
 6065          CC = CC + 0.25 * ( H(N) ** 3 ) * SJ(N,JT) + H(N) *       17MY5
     1              RJ(N,JT)                                            29AP5
               DD = DD + 0.25 * ( H(N) ** 3 ) * SJ(N,JT) - H(N) *       12MY5
     1              RJ(N,JT)                                            29AP5
               FF = FF + ( H(N) ** 3 ) * ( 0.5 * QJ(N,JT) +  CJ(N,JT) / 29AP5
     1              H(N) )                                              29AP5
          GO TO 6080                                                    29AP5
C *** REVISE COEFFS FOR STATION TO RIGHT OF DUMMY JOINT                 01JL5
 6070          BB = BB + 0.25 * ( H(N) ** 3 ) * SJ(N,JT) - H(N) *       07JL5
     1              RJ(N,JT)                                            29AP5
               CC = CC + 0.25 * ( H(N) ** 3 ) * SJ(N,JT) + H(N) *       29AP5
     1              RJ(N,JT)                                            29AP5
               FF = FF + ( H(N) ** 3 ) * ( 0.5 * QJ(N,JT) - CJ(N,JT) /  18MY5
     1              H(N) )                                              29AP5
C *** COMPUTE RECURSION COEFFICIENTS                                    22JL5
 6080          E = AA * B(J-2) + BB                                     05MA5
               DZ = E * B(J-1) + AA * C(J-2) + CC                       27JA5
          IF ( DZ ) 6090, 6085, 6090                                    05MA5
 6085          D = 0.0                                                  05MA5
          GO TO 6095                                                    05MA5
 6090          D = - 1.0 / DZ                                           05MA5
 6095          C(J) = D * EE                                            05MA5
               B(J) = D * ( E * C(J-1) + DD )                           27JA5
               A(J) = D * ( E * A(J-1) + AA * A(J-2) - FF )             27JA5
 6100     CONTINUE                                                      27JA5
               MP4 = M(N) + 4                                           27JA5
C *** COMPUTE DEFLECTED SHAPE OF MEMBER                                 01JL5
               W(N,MP5) = A(MP5)                                        27JA5
          DO 6200  K = 4, MP4                                           27JA5
               J = MP4 - K + 4                                          27JA5
               WTMP = W(N,J)                                            10FE5
               W(N,J) = A(J) + B(J) * W(N,J+1) + C(J) * W(N,J+2)        20MY5
C *** CHECK FOR MEMBER DEFLECTION CLOSURE                               01JL5
          IF ( ABSF ( W(N,J) - WTMP )  - WTOL ) 6200, 6200, 6120        05AP5
 6120          KWCT = KWCT + 1                                          27JA5
 6200     CONTINUE                                                      27JA5
               W(N,3) = A(3) + B(3) * W(N,4) + C(3) * W(N,5)            22MY5
C *** COMPUTE MEMBER DEFLECTION RESTRAINT AT ALL JOINTS                 01JL5
          DO 6300 J = 3, MP5                                            30JE5
          IF ( KODE (N,J) ) 6300, 6300, 6210                            30JE5
 6210 CALL DCIPHER ( N, J, K, L, JT, JO, KT )                           30JE5
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      IF ( K ) 6300, 6300, 6215                                         30JE5
 6215 CALL REACT ( N, J, JT, CX, QX, RX1, RX2 )                          30JE5
            WX = 0.5 * ( W(N,J) + W(N,J+1) )                            30JE5
      IF ( WX )  6225, 6220, 6225                                       30JE5
 6220       SBM = SFTR * 0.5 * ( F(N,J) + F(N,J+1) ) / ( H(N) ** 3 ) 01JL5
            QBM = 0.0                                                   30JE5
      GO TO 6250                                                        30JE5
 6225       SBM = ( RX1 + RX2 ) / WX                                    30JE5
      IF ( SBM ) 6230, 6230, 6235                                       30JE5
 6230 IF ( WC(K) / WX ) 6232, 6232, 6233                                16JL5
 6232       SBM = SFTR * 0.5 * ( F(N,J) + F(N,J+1 ) ) / ( H(N) ** 3 )16JL5
      GO TO 6234                                                        16JL5
 6233       SBM = 0.001                                                 16JL5
 6234       QBM = - ( RX1 + RX2 ) + SBM * WX                            16JL5
      GO TO 6250                                                        30JE5
 6235       UPB = 50.0 * ( F(N,J) + F(N,J+1) )                          28JL5
      IF ( SBM - UPB ) 6245, 6245, 6240                                 30JE5
 6240       SBM = UPB                                                   30JE5
 6245       QBM = 0.0                                                   30JE5
C *** REVISE TOTAL LOAD AND RESTRAINT ACTING ON COLUMN                  01JL5
 6250       CS(K) = CS(K) - SP(N,JT) + SBM                              16JL5
            CQ(K) = CQ(K) - QP(N,JT) + QBM                              16JL5
            SP(N,JT) = SBM                                              20JL5
            QP(N,JT) = QBM                                              20JL5
 6300 CONTINUE                                                          27JA5
C *** COMPUTE COLUMN DISPLACEMENT AND DISTRIBUTE AXIAL TENSION OR       22JL5
C *** COMPRESSION TO MEMBERS                                            06JL5
 6305 CALL COLUMN ( NBMS, NXB )                                         30JE5
      DO 6400 N = 1, NBMS                                               01AP5
            MP5 = M(N) + 5                                              30JE5
C *** CHECK FOR CONVERGENCE OF JOINT TRANSLATION AND ROTATION           28JL5
      DO 6400 J = 3, MP5                                                01JL5
      IF ( KODE(N,J) ) 6400, 6400, 6330                                 01JL5
 6330 CALL DCIPHER ( N, J, K, L, JT, JO, KT )                           25JE5
      IF ( K ) 6400, 6400, 6335                                         01JL5
 6335       WX = 0.5 * ( W(N,J) + W(N,J+1) )                            29AP5
            SX = ( W(N,J+1) - W(N,J) ) / H(N)                           01AP5
            SY = ( W(K,L+1) - W(K,L) ) / H(K)                           01AP5
C *** CHECK FOR JOINT DEFLECTION CLOSURE                                01JL5
      IF ( ABSF ( WX - WC(K) ) - WTOL ) 6345, 6345, 6340                01JL5
 6340       JTCT = JTCT + 1                                             01AP5
 6345 IF ( KT ) 6350, 6350, 6400                                        01JL5
C *** CHECK FOR JOINT ROTATION CLOSURE                                  01JL5
 6350 IF ( ABSF ( SX - SY ) - RTOL ) 6400, 6400, 6360                   01JL5
 6360       JSCT = JSCT + 1                                             01AP5
 6400 CONTINUE                                                          01AP5
      PRINT 601, NI, KWCT, JTCT, JSCT                                   13MY5
C *** TEST FOR CONVERGENCE OF SOLUTION                                  01JL5
      IF ( KWCT ) 7015, 7015, 7025                                      03JL5
 7015 IF ( JTCT ) 7020, 7020, 7025                                      01JL5
 7020 IF ( JSCT ) 8000, 8000, 7025                                      01JL5
C *** COMPUTE AND PRINT MONITOR DATA                                    01JL5
 7025 DO 7100 I = 1, NMJ                                                01JL5
            N = MB(I)                                                   01AP5
            J = MS(I) + 4                                               01AP5
 7030 CALL DCIPHER ( N, J, K, L, JT, JO, KT )                           25JE5
 7060 CALL REACT ( N, J, JT, CX, QX, RX1, RX2 )                          14JL5
 7070 CALL REACT ( K, L, JO, CY, QY, RY1, RY2 )                          14JL5
            SX = ( W(N,J+1) - W(N,J) ) / H(N)                           27JA5
            SY = ( W(K,L+1) - W(K,L) ) / H(K)                           19JA5
            WX = 0.5 * ( W(N,J) + W(N,J+1) )                            05FE5
```

```
               WY = 0.5 * ( W(K,L) + W(K,L+1) )                          05FE5
               JM4 = J - 4                                               16JL5
               LM4 = L - 4                                               16JL5
               ERRX = CQ(K) - CS(K) * 0.5 * ( WX + WC(K) )               16JL5
               ERRY = CQ(N) - CS(N) * 0.5 * ( WY + WC(N) )               16JL5
               ERRR = CJ(N,JT) + CJ(K,JO) + ( RJ(N,JT) + RJ(K,JO) ) *    05AP5
     1                  0.5 * ( SX + SY ) - CX - CY                      01AP5
      PRINT 602, N, JT, JM4, WX, WC(K), SX, ERRX, K, JO, LM4, WY, WC(N),16JL5
     1             SY, ERRY, ERRR                                        07JL5
 7100     CONTINUE                                                       01AP5
      CALL TIME                                                          09AP5
 7900     CONTINUE                                                       19JA5
      PRINT 603                                                          05AP5
      CALL TIME                                                          05AP5
          GO TO 8001                                                     05AP5
 8000 PRINT 604                                                          05AP5
      CALL TIME                                                          05AP5
 8001 PRINT 11                                                           08JL5
      PRINT 1                                                            18AG5
      PRINT 13, ( AN1(N), N = 1, 32 )                                    18FE5
      PRINT 16, NPROB, ( AN2(N), N = 1, 14 )                             28AG3
      PRINT 700                                                          13AP5
      PRINT 701                                                          07JL5
C *** COMPUTE AND PRINT JOINT RESULTS                                     01JL5
          DO 8050 N = 1, NXB                                             06AG5
               MP5 = M(N) + 5                                            01JL5
          DO 8050 J = 3, MP5                                             01JL5
          IF ( KODE(N,J) ) 8050, 8050, 8020                              01JL5
 8020 CALL DCIPHER ( N, J, K, L, JT, JO, KT )                            28JE5
          IF ( K ) 8050, 8050, 8035                                      01JL5
 8035 CALL REACT ( N, J, JT, CX, QX, RX1, RX2 )                          14JL5
 8040 CALL REACT ( K, L, JO, CY, QY, RY1, RY2 )                          14JL5
               SX = ( W(N,J+1) - W(N,J) ) / H(N)                         27JA5
               SY = ( W(K,L+1) - W(K,L) ) / H(K)                         27JA5
               WX = 0.5 * ( W(N,J) + W(N,J+1) )                          22JL5
               WY = 0.5 * ( W(K,L) + W(K,L+1) )                          27JA5
               JM4 = J - 4                                               16JL5
               LM4 = L - 4                                               16JL5
               ERRX = CQ(K) - CS(K) * 0.5 * ( WX + WC(K) )               16JL5
               ERRY = CQ(N) - CS(N) * 0.5 * ( WY + WC(N) )               16JL5
               ERRR = CJ(N,JT) + CJ(K,JO) + ( RJ(N,JT) + RJ(K,JO) ) *    05AP5
     1                  0.5 * ( SX + SY ) - CX - CY                      01AP5
      PRINT 33, N, JT, JM4, WX, WC(K), SX, ERRX, K, JO, LM4, WY, WC(N), 16JL5
     1             SY, ERRY, ERRR                                        13JL5
 8050     CONTINUE                                                       01AP5
C *** COMPUTE AND PRINT MEMBER RESULTS                                     08JL5
          DO 8100 N = 1, NBMS                                            27JA5
               MP5 = M(N) + 5                                            27JA5
      PRINT 11                                                           08JL5
      PRINT 1                                                            18AG5
      PRINT 13, ( AN1(I), I = 1, 32 )                                    08JL5
      PRINT 16, NPROB, ( AN2(I), I = 1, 14 )                             08JL5
      PRINT 800, N                                                       29AP5
               BM2 = BM3 = 0.0                                           27JA5
          DO 8100 J = 3, MP5                                             27JA5
               I = J - 4                                                 27JA5
               Z = I                                                     27JA5
               X = Z * H(N)                                              27JA5
               BM1 = BM2                                                 27JA5
               BM2 = BM3                                                 27JA5
               BM3 = F(N,J+1) * ( W(N,J) - 2.0 * W(N,J+1) + W(N,J+2) )   27JA5
```

```
    1             / ( H(N) * H(N) )                                 27JA5
             D4W    = ( BM1 - 2.0 * BM2 + BM3 ) / H(N)              18AP5
         IF ( KODE(N,J) ) 8060, 8070, 8060                          01JL5
 8060 PRINT 34, I, X, W(N,J), BM2, D4W, P(N,J)                      08JL5
         GO TO 8100                                                 01JL5
 8070 PRINT 31, I, X, W(N,J), BM2, D4W, P(N,J)                      08JL5
 8100    CONTINUE                                                   27JA5
         GO TO 1010                                                 08JL5
 9999 PRINT 11                                                      07JL5
      PRINT 1                                                       18AG5
      PRINT 13, ( AN1(N), N = 1, 32 )                               18FE5
      PRINT 17                                                      07JL5
      CALL TIME                                                     09AP5
      END                                                           08JL5
      SUBROUTINE COLUMN ( NBMS, NXB )                               08JL5
      DIMENSION      F(20,97), Q(20,97), S(20,97), P(20,97), W(20,97),   28JE5
    1                KODE(20,97), QJ(20,10), DP(20,97), SJ(20,10),  08JL5
    2                CJ(20,10), RJ(20,10), SC(20), WC(20), H(20), M(20), 08JL5
    3                CS(20), CQ(20)                                 12JL5
      COMMON    F, Q, S, P, W, KODE, QJ, SJ, CJ, RJ, SC, WC, H, DP,  05JL5
    1           M, CS, CQ                                           28JE5
C *** COMPUTE COLUMN DISPLACEMENT AND AXIAL TENSION OR COMPRESSION    28JL5
C *** DISTRIBUTION IN ALL FRAME MEMBERS                             28JL5
         DO 2100 N = 1, NBMS                                        01JL5
        IF ( CS(N) ) 1020, 1010, 1020                               12JL5
 1010        WC(N) = 0.0                                            06JL5
         GO TO 1030                                                 05JL5
 1020        WC(N) = CQ(N) / CS(N)                                  07JL5
 1030        MP3 = M(N) + 3                                         05JL5
         DO 2100 J = 3, MP3                                         30JE5
         IF ( KODE(N,J) ) 1060, 1060, 1050                          21AP5
 1050 CALL DCIPHER ( N, J, K, L, JT, JO, KT )                       28JE5
         IF ( K ) 1060, 1060, 1055                                  29AP5
 1055 CALL REACT ( K, L, JO, CPL, QCX, RX1, RX2 )                   28JE5
         IF ( N - NXB ) 1057, 1057, 1056                            31MY5
 1056        P(N,J+1) = P(N,J) + DP(N,J+1) - QCX                    09MY5
         GO TO 2100                                                 28JE5
 1057        P(N,J+1) = P(N,J) + DP(N,J+1) + QCX                    09MY5
         GO TO 2100                                                 28JE5
 1060        P(N,J+1) = P(N,J) + DP(N,J+1)                          28JE5
 2100    CONTINUE                                                   01JL5
      END                                                           25MR5
      SUBROUTINE MATRIX ( AA, BB, CC, DD, EE, FF, N, J )            01AP5
      DIMENSION      F(20,97), Q(20,97), S(20,97), P(20,97), W(20,97),   28JE5
    1                KODE(20,97), QJ(20,10), DP(20,97), CQ(20), CS(20),  14JL5
    2                SJ(20,10), CJ(20,10), RJ(20,10), SC(20), WC(20),    08JL5
    3                H(20), M(20)                                   14JL5
      COMMON    F, Q, S, P, W, KODE, QJ, SJ, CJ, RJ, SC, WC, H, DP,  05JL5
    1           M, CS, CQ                                           12JL5
C *** COMPUTE MATRIX COEFFICIENTS AT STATION N, J                   28JL5
             Z = H(N)                                               27JA5
             AA = F(N,J-1)                                          27JA5
             BB = - 2.0 * ( F(N,J-1) + F(N,J) + 0.25 * Z * Z * (    27JA5
    1             P(N,J-1) + P(N,J) ) )                             27JA5
             CC = F(N,J-1) + 4.0 * F(N,J) + F(N,J+1) + 0.5 * Z * Z *  27JA5
    1             ( P(N,J-1) + 2.0 * P(N,J) + P(N,J+1) ) + Z * Z * Z  26JA5
    2             * S(N,J)                                          27JA5
             DD = - 2.0 * ( F(N,J) + F(N,J+1) + 0.25 * Z * Z * (    27JA5
    1             P(N,J) + P(N,J+1) ) )                             27JA5
             EE = F(N,J+1)                                          27JA5
             FF = Z * Z * Z * Q(N,J)                                27JA5
```

```
      END                                                                 09AP5
      SUBROUTINE DCIPHER ( N, J, K, L, JT, JO, KT )                       01JL5
      DIMENSION      F(20,97), Q(20,97), S(20,97), P(20,97), W(20,97),    28JE5
     1               KODE(20,97), QJ(20,10), DP(20,97), CQ(20), CS(20),   14JL5
     2               SJ(20,10), CJ(20,10), RJ(20,10), SC(20), WC(20),     08JL5
     3               H(20), M(20)                                         14JL5
      COMMON    F, Q, S, P, W, KODE, QJ, SJ, CJ, RJ, SC, WC, H, DP,       05JL5
     1          M, CS, CQ                                                 05JL5
C *** DECIPHER KODE(N,J) TO FIND JOINT INTERSECTION DATA                   01JL5
                JT = KODE(N,J) / 10000000                                 01AP5
                JK = JT * 10000000                                        01AP5
                JL = KODE(N,J) - JK                                       30AP5
                JO = JL / 100000                                          30AP5
                JK = JO * 100000                                          06AP5
                JL = JL - JK                                              30AP5
                K = JL / 1000                                             30AP5
                JK = K * 1000                                             01AP5
                L  = ( JL - JK ) / 10                                     28JE5
                KT = ( JL - JK ) - L * 10                                 28JE5
      END                                                                 27JA5
      SUBROUTINE REACT ( N, J, JT, CPL, QJT, REACT1, REACT2 )             28JE5
      DIMENSION      F(20,97), Q(20,97), S(20,97), P(20,97), W(20,97),    28JE5
     1               KODE(20,97), QJ(20,10), DP(20,97), CQ(20),           08JL5
     2               SJ(20,10), CJ(20,10), RJ(20,10), SC(20), WC(20),     08JL5
     3               H(20), M(20), BM(4), CS(20)                          08JL5
      COMMON    F, Q, S, P, W, KODE, QJ, SJ, CJ, RJ, SC, WC, H, DP,       05JL5
     1          M, CS, CQ                                                 05JL5
C *** COMPUTE FOURTH DERIVATIVE, ABSORBED LOAD, AND ABSORBED COUPLE AT    14JL5
C *** JOINT N, JT                                                         28JL5
                I = J - 2                                                 01AP5
            DO 1010 II = 1, 4                                             01AP5
                I = I + 1                                                 01AP5
                BM(II) = F(N,I) * ( W(N,I-1) - 2.0 * W(N,I) + W(N,I+1) )  01AP5
     1                   / ( H(N) * H(N) )                                01AP5
 1010       CONTINUE                                                      01AP5
                D4W1 = ( BM(1) - 2.0 * BM(2) + BM(3) ) / H(N)             14AP5
                D4W2 = ( BM(2) - 2.0 * BM(3) + BM(4) ) / H(N)             14AP5
                REACT1 = D4W1 - ( Q(N,J) - S(N,J) * W(N,J) )  + ( 0.5 /   08JE5
     1                   H(N) ) * ( ( P(N,J-1) + P(N,J) ) * ( W(N,J) -    04JE5
     2                   W(N,J-1) ) - ( P(N,J) + P(N,J+1) ) * ( W(N,J+1)  04JE5
     3                   - W(N,J) )  )                                    04JE5
                REACT2 = D4W2 - ( Q(N,J+1) - S(N,J+1) * W(N,J+1) ) + (    08JE5
     1                    0.5 / H(N) ) * ( ( P(N,J) + P(N,J+1) ) * (      04JE5
     2                    W(N,J+1) - W(N,J) ) - ( P(N,J+1) + P(N,J+2) )   04JE5
     3                    * ( W(N,J+2) - W(N,J+1) ) )                     04JE5
                CPL = ( REACT1 - REACT2 ) * 0.5 * H(N)                    04JE5
                QJT = REACT1 + REACT2 - QJ(N,JT) + SJ(N,JT) * 0.5 *       08JE5
     1                ( W(N,J) + W(N,J+1) )                               04JE5
      END                                                                 01AP5
      END                                                                 28JE5
         FINIS                                                            02AP5
-EXECUTE,,,1.
```

APPENDIX 4

LISTING OF INPUT DATA FOR ALL EXAMPLE PROBLEMS

```
1      SIMPLE BENT USED AS COMPARATIVE EXAMPLE - H = 1.0
1   2    4    2    6    3    3    2   25       1.000E-02 1.000E-05 1.000E-06
1  -1    1   19
1   1    1   -1    2    1   -1    0
2   1    2   19    3    1   -1    0
3   2    2    9    0
4   3    2   19    0
1       4.000E+00
2       2.000E+00
3       5.000E-01
4       1.000E-01
5       5.000E-01
6       2.000E+00
1  19 1.000E+00
2   9 1.000E+00
3  19 1.000E+00
1   0   19        3.000E+02
2   0    9        1.000E+02
3   0   19        2.000E+02
2   1 1.500E+00
2   2             1.000E+03 1.000E+10
3   2             1.000E+03 1.000E+10
2       1.000E+03
3       1.000E+03
2      SIMPLE BENT USED AS COMPARATIVE EXAMPLE - H = 0.5
1   2    4    2    6    3    3    2   35       1.000E-02 1.000E-05 1.000E-06
1  -1    1   39
1   1    1   -1    2    1   -1    0
2   1    2   39    3    1   -1    0
3   2    2   19    0
4   3    2   39    0
1       4.000E+00
2       2.000E+00
3       5.000E-01
4       1.000E-01
5       5.000E-01
6       2.000E+00
1  39 5.000E-01
2  19 5.000E-01
3  39 5.000E-01
1   0   39        3.000E+02
2   0   19        1.000E+02
3   0   39        2.000E+02
2   1 1.500E+00
2   2             1.000E+03 1.000E+10
3   2             1.000E+03 1.000E+10
2       1.000E+03
3       1.000E+03
3      SIMPLE BENT USED AS COMPARATIVE EXAMPLE - H = 0.25
1   2    4    2    6    3    3    2   25       1.000E-02 1.000E-05 1.000E-06
1  -1    1   79
1   1    1   -1    2    1   -1    0
2   1    2   79    3    1   -1    0
3   2    2   39    0
4   3    2   79    0
1       4.000E+00
2       2.000E+00
3       5.000E-01
4       1.000E-01
5       5.000E-01
6       2.000E+00
```

```
1   79 2.500E-01
2   39 2.500E-01
3   79 2.500E-01
1    0    79        3.000E+02
2    0    39        1.000E+02
3    0    79        2.000E+02
2    1 1.500E+00
2    2              1.000E+03 1.000E+10
3    2              1.000E+03 1.000E+10
2      1.000E+03
3      1.000E+03
```

```
10      MATLOCK - GRUBBS NO-SWAY FRAME AS COMPARATIVE EXAMPLE H = 12.0 INCHES
 1    2    6    2    6    3    8    3   25       1.000E-02 1.000E-07 1.000E-09
 1    0    1   20
 1    1    1    0    2    1    0
 2    1    2   15    0
 3    1    3   20    3    1    0
 4    2    2   10    0
 5    3    2    5    0
 6    3    3   10    0
 1      4.000E+00
 2      2.000E+00
 3      5.000E-01
 4      1.000E-01
 5      5.000E-01
 6      2.000E+00
 1   21 1.200E+01
 2   11 1.200E+01
 3   11 1.200E+01
 1    0   21      1.000E+11
 2    0   11      1.000E+11
 3    0   11      1.000E+11
 2    2           3.000E+10 1.000E+20
 3    3           3.000E+10 1.000E+20
 1    1           3.000E+10
 1    3           3.000E+10
 2    1           3.000E+10
 3    1           3.000E+10
 1    2-4.000E+04
 3    2-8.000E+04
 1      1.000E+10
 2      1.000E+10
 3      1.000E+10
11      MATLOCK - GRUBBS NO-SWAY FRAME AS COMPARATIVE EXAMPLE H = 3.0 INCHES
 1    2    6    2    6    3    8    3   25       1.000E-02 1.000E-07 1.000E-09
 1    0    1   80
 1    1    1    0    2    1    0
 2    1    2   60    0
 3    1    3   80    3    1    0
 4    2    2   40    0
 5    3    2   20    0
 6    3    3   40    0
 1      4.000E+00
 2      2.000E+00
 3      5.000E-01
 4      1.000E-01
 5      5.000E-01
 6      2.000E+00
 1   81 3.000E+00
 2   41 3.000E+00
 3   41 3.000E+00
 1    0   81      1.000E+11
 2    0   41      1.000E+11
 3    0   41      1.000E+11
 2    2           1.000E+10 1.000E+20
 3    3           1.000E+10 1.000E+20
 1    1           1.000E+10
 1    3           1.000E+10
 2    1           1.000E+10
 3    1           1.000E+10
 1    2-4.000E+04
 3    2-8.000E+04
```

1	1.000E+10
2	1.000E+10
3	1.000E+10

```
  100      CHAPTER 8 EXAMPLE 1 - PINNED FRAME
    4    3   17    4    1   12    6    5   25      1.000E-02 1.000E-05 1.000E+00
    1    9    2    9    3   19    4    9
    1    1    1   -1    0
    2    1    2    9    5    1   -1    1
    3    1    3   14    0    1   -1    1
    4    1    4   19    6    1   -1    1
    5    1    5   29    7    1   -1    1
    6    2    1   -1    5    2    9    1
    7    2    2    9    6    2    9    1
    8    2    3   19    7    2    9    1
    9    3    1   -1    5    3   19    1
   10    3    2    9    6    3   19    1
   11    3    3   19    7    3   19    1
   12    3    4   24    0
   13    3    5   29    0
   14    4    1   -1    5    4   29    1
   15    4    2    9    6    5   29    1
   16    4    3   19    7    4   29    1
   17    6    4   24    0
    1          0.000E+00
    1   29 1.200E+01
    2   19 1.200E+01
    3   29 1.200E+01
    4   19 1.200E+01
    5   29 1.200E+01
    6   29 1.200E+01
    7   29 1.200E+01
    1    0   29      1.000E+10
    2    0   19      1.000E+10
    3    0   29      1.000E+10
    4    0   19      1.000E+10
    5    0   29      1.000E+10
    6    0   29      1.000E+10
    7    0   29      1.000E+10
    2    3    6                 4.000E+04
    2   13   16                -4.000E+04
    3    0   19                 1.000E+03
    3    3    3                -5.000E+05
    3   13   13                 5.000E+05
    1    3                      1.000E+07
    3    4                                1.000E+07
    3    5               1.000E+07
    5    3 1.000E+06
    6    4 4.000E+05
    7    2-3.000E+05
    1      1.000E+06
    3      1.000E+06
    5      1.000E+10
    6      1.000E+10
    7      1.000E+10
```

```
200      CHAPTER 8 EXAMPLE 2 - STEPPED FRAME
  5    6   30    4    6   11   30   11   25       1.000E-02 1.000E-06 1.000E-07
  2   -1    3   19    4   29    5   39
  1    1    1   -1    8    1   -1    0
  2    1    2    9    9    1   -1    0
  3    2    1   -1    7    1   -1    0
  4    2    2    9    8    2    9    0
  5    2    3   19    9    2    9    0
  6    2    4   29   10    1   -1    0
  7    3    1   -1    6    1   -1    0
  8    3    2    9    7    2    9    0
  9    3    3   19    8    3   19    0
 10    3    4   29    9    3   19    0
 11    3    5   39   10    2    9    0
 12    3    6   49   11    1   -1    0
 13    4    1   -1    6    2    9    0
 14    4    2    9    7    3   19    0
 15    4    3   19    8    4   29    0
 16    4    4   29    9    4   29    0
 17    4    5   39   10    3   19    0
 18    4    6   49   11    2    9    0
 19    5    1   -1    6    3   19    0
 20    5    2    9    7    4   29    0
 21    5    3   19    8    5   39    0
 22    5    4   29    9    5   39    0
 23    5    5   39   10    4   29    0
 24    5    6   49   11    3   19    0
 25    6    4   29    0
 26    7    5   39    0
 27    8    6   49    0
 28    9    6   49    0
 29   10    5   39    0
 30   11    4   29    0
  1      4.000E+00
  2      2.000E+00
  3      5.000E-01
  4      1.000E-01
  5      5.000E-01
  6      2.000E+00
  1    9 2.400E+01
  2   29 2.400E+01
  3   49 2.400E+01
  4   49 2.400E+01
  5   49 2.400E+01
  6   29 2.400E+01
  7   39 2.400E+01
  8   49 2.400E+01
  9   49 2.400E+01
 10   39 2.400E+01
 11   29 2.400E+01
  1    0    9        1.000E+10
  2    0   29        1.000E+10
  3    0   49        1.000E+10
  4    0   49        1.000E+10
  5    0   49        1.000E+10
  6    0   29        1.000E+10
  7    0   39        1.000E+10
  8    0   49        1.000E+10
  9    0   49        1.000E+10
 10    0   39        1.000E+10
 11    0   29        1.000E+10
```

1	1				-1.000E+06
1	2				+1.000E+06
2	1				-1.000E+06
2	2				+1.000E+06
2	3				-1.000E+06
2	4				+1.000E+06
3	1				-1.000E+06
3	2				+1.000E+06
3	3				-1.000E+06
3	4				+1.000E+06
3	5				-1.000E+06
3	6				+1.000E+06
4	1				+1.000E+06
4	2				-1.000E+06
4	3				+1.000E+06
4	4				-1.000E+06
4	5				+1.000E+06
4	6				-1.000E+06
5	1				-1.000E+06
5	2				+1.000E+06
5	3				-1.000E+06
5	4				+1.000E+06
5	5				-1.000E+06
5	6				+1.000E+06
6	4		1.000E+10	1.000E+99	
7	5		1.000E+10	1.000E+99	
8	6		1.000E+10	1.000E+99	
9	6		1.000E+10	1.000E+99	
10	5		1.000E+10	1.000E+99	
11	4		1.000E+10	1.000E+99	
1		1.000E+10			
2		1.000E+10			
3		1.000E+10			
4		1.000E+10			
5		1.000E+10			
6		1.000E+10			
7		1.000E+10			
8		1.000E+10			
9		1.000E+10			
10		1.000E+10			
11		1.000E+10			

```
300      CHAPTER 8 EXAMPLE 3 - FIVE BAY FRAME
  4    6   24    4    6   69    0    6  200        1.000E-03 1.000E-03 1.000E-06
  1   -1    2    9    3   19    4   29
  1    1    1   -1    5    1   -1    0
  2    1    2    9    6    1   -1    0
  3    1    3   19    7    1   -1    0
  4    1    4   29    8    1   -1    0
  5    1    5   39    9    1   -1    0
  6    1    6   49   10    1   -1    0
  7    2    1   -1    5    2    9    0
  8    2    2    9    6    2    9    0
  9    2    3   19    7    2    9    0
 10    2    4   29    8    2    9    0
 11    2    5   39    9    2    9    0
 12    2    6   49   10    2    9    0
 13    3    1   -1    5    3   19    0
 14    3    2    9    6    3   19    0
 15    3    3   19    7    3   19    0
 16    3    4   29    8    3   19    0
 17    3    5   39    9    3   19    0
 18    3    6   49   10    3   19    0
 19    4    1   -1    5    4   29    0
 20    4    2    9    6    4   29    0
 21    4    3   19    7    4   29    0
 22    4    4   29    8    4   29    0
 23    4    5   39    9    4   29    0
 24    4    6   49   10    4   29    0
  1      4.000E+00
  2      2.000E+00
  3      5.000E-01
  4      1.000E-01
  5      5.000E-01
  6      2.000E+00
  1   49 2.400E+01
  2   49 2.400E+01
  3   49 2.400E+01
  4   49 2.400E+01
  5   49 2.400E+01
  6   49 2.400E+01
  7   49 2.400E+01
  8   49 2.400E+01
  9   49 2.400E+01
 10   49 2.400E+01
  1    0   49      3.000E+11-2.000E+02
  2    0   49      3.000E+11-2.000E+02
  3    0   49      3.000E+11-2.000E+02
  4    0   49      3.000E+11-2.000E+02
  4    0    3      7.000E+11
  4    6   13      7.000E+11
  4   16   23      7.000E+11
  4   26   33      7.000E+11
  4   36   43      7.000E+11
  4   46   49      7.000E+11
  5    0   49      1.000E+12                      -5.000E+02
  6    0   49      1.000E+12                      -5.000E+02
  7    0   49      1.000E+12                      -5.000E+02
  8    0   49      1.000E+12                      -5.000E+02
  9    0   49      1.000E+12                      -5.000E+02
 10    0   49      1.000E+12                      -5.000E+02
  2   10   19                -4.000E+04
  2   30   39                -4.000E+04
```

3	0	9		-6.000E+04	
3	20	29		-6.000E+04	
3	40	49		-6.000E+04	
4	10	19		-8.000E+04	
4	30	39		-8.000E+04	
5	40	49			1.000E+05
6	40	49			1.000E+05
7	40	49			1.000E+05
8	40	49			1.000E+05
9	40	49			1.000E+05
10	40	49			1.000E+05
5	0	0		3.900E+04	
5	1	1		3.800E+04	
5	2	2		3.700E+04	
5	3	3		3.600E+04	
5	4	4		3.500E+04	
5	5	5		3.400E+04	
5	6	6		3.300E+04	
5	7	7		3.200E+04	
5	8	8		3.100E+04	
5	9	9		3.000E+04	
5	10	10		2.900E+04	
5	11	11		2.800E+04	
5	12	12		2.700E+04	
5	13	13		2.600E+04	
5	14	14		2.500E+04	
5	15	15		2.400E+04	
5	16	16		2.300E+04	
5	17	17		2.200E+04	
5	18	18		2.100E+04	
5	19	19		2.000E+04	
5	20	20		1.900E+04	
5	21	21		1.800E+04	
5	22	22		1.700E+04	
5	23	23		1.600E+04	
5	24	24		1.500E+04	
5	25	25		1.400E+04	
5	26	26		1.300E+04	
5	27	27		1.200E+04	
5	28	28		1.100E+04	
5	29	29		1.000E+04	
5	30	30		9.000E+03	
5	31	31		8.000E+03	
5	32	32		7.000E+03	
5	33	33		6.000E+03	
5	34	34		5.000E+03	
5	35	35		4.000E+03	
5	36	36		3.000E+03	
5	37	37		2.000E+03	
5	38	38		1.000E+03	
5	39	39		0.000E+00	
5			5.000E+06		
6			5.000E+06		
7			5.000E+06		
8			5.000E+06		
9			5.000E+06		
10			5.000E+06		

APPENDIX 5

COMPUTER OUTPUT FOR EXAMPLE OF FIG 7.1

PROGRAM PLNFRAM 4 - MASTER DECK - T A HALIBURTCN REVISICN DATE 18 AUG 65
RUN TO INDICATE FORM OF COMPUTER OUTPUT - DATE RUN 8/18/65 - CHG CE051119
CODED, PRCOFED, AND RUN BY TAH - PUNCHED BY BPF, BW, GB, AND BB - CHG CE051119

PROB
1 SIMPLE BENT USED AS CCMPARATIVE EXAMPLE - H = 1.0

TABLE 1. - CONTROL DATA

NUM OF X-MEMBERS IN THE FRAME	1
NUM OF Y-MEMBERS IN THE FRAME	2
NUM OF INTERSECTIONS IN THE FRAME	4
NUM OF JOINTS TO BE MCNITCRED	2
NUM OF ROTATIONAL PARAMETERS	6
NUM CARDS IN TABLE 3	3
NUM CARDS IN TABLE 4	3
NUM CARDS IN TABLE 5	2
ITERATION LIMIT	25
S-SPRING FACTOR	1.000E-02
TRANSLATICNAL CLCSURE TOLERANCE	1.000E-05
RCTATIONAL CLOSURE TCLERANCENCE	1.000E-06

MCNITOR JOINTS -- MEMBER AND STA TO THE LEFT OF JCINT
1 -1 1 19

JOINT - MEMBER INTERSECTICN DATA

INTERSECTION	MEMBER	JOINT	STA TC LT	WITH	MEMBER	JOINT	STA TC LT	JOINT TYPE
1	1	1	-1		2	1	-1	0
2	1	2	19		3	1	-1	0
3	2	2	9		0	0	0	0
4	3	2	19		0	0	0	0

ROTATICNAL CLOSURE PARAMETERS

NUM	XI
1	4.000E 00
2	2.000E 00
3	5.000E-01
4	1.000E-01
5	5.000E-01
6	2.000E 00

TABLE 2. - NUM OF INCREMENTS AND INCREMENT LENGTH

MEMBER	NUM INC	INCREMENT LENGTH
1	19	1.000E 0C
2	9	1.000E 0C
3	19	1.000E 0C

TABLE 3. - MEMBER FIXED INPUT DATA

MEMBER	FROM STA	TO STA	STIFFNESS F	LOAD Q	SPRING S	DELTA P
1	0	19	3.CCCE C2	0	0	0
2	C	9	1.CCCE C2	0	0	0

3	0	19	2.CCOE C2	C	0	0

TABLE 4. - JOINT FIXED INPLT DATA

MEMBER	JOINT	Q - LOAD	S - SPRING	R - SPRING	COUPLE
2	1	1.5COE 00	0	0	0
2	2	0	1.COOE 03	1.0COE 10	0
3	2	0	1.COOE 03	1.CCOE 10	0

TABLE 5. - COLUMN FIXED INPUT CATA

MEMBER	COLUMN SPRING
2	1.000E 03
3	1.000E 03

```
PROGRAM PLNFRAM 4 - MASTER DECK - T A HALIBURTON          REVISION DATE 18 AUG 65
RUN TO INDICATE FORM OF COMPUTER OUTPUT - DATE RUN 8/18/65 - CHG CE051119
CODED, PROOFED, AND RUN BY TAH - PUNCHED BY BPF, BW, GB, AND BB - CHG CE051119


PROB (CONTD)
    1     SIMPLE BENT USED AS COMPARATIVE EXAMPLE - H = 1.0


     TABLE 6. - MONITOR JOINT OUTPUT AT SELECTED JOINTS


     TIME =      0 MINUTES, 14 AND 46/60 SECONDS
  ITERATION   1 NOT CLOSED -- BEAM DEFLS   30 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ          0 WC          0 SLP          0 TERR          0   RERR
BM 2 JT 1 STA LT -1 WJ 6.966E-01 WC 1.042E 00 SLP-6.586E-03 TERR 2.485E-01 3.951E 00

BM 1 JT 2 STA LT 19 WJ          0 WC          0 SLP          0 TERR          0   RERR
BM 3 JT 1 STA LT -1 WJ 1.042E 00 WC 1.042E 00 SLP-4.893E-03 TERR 9.895E-09 2.936E 00


     TIME =      0 MINUTES, 15 AND 12/60 SECONDS
  ITERATION   2 NOT CLOSED -- BEAM DEFLS   49 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 1.722E-04 WC 1.722E-04 SLP-2.683E-02 TERR-5.675E-10   RERR
BM 2 JT 1 STA LT -1 WJ 1.158E 00 WC 1.183E 00 SLP-3.859E-02 TERR 1.621E-02 3.531E 00

BM 1 JT 2 STA LT 19 WJ-1.722E-04 WC-1.722E-04 SLP-1.199E-02 TERR-2.547E-11   RERR
BM 3 JT 1 STA LT -1 WJ 1.183E 00 WC 1.183E 00 SLP-1.658E-02 TERR 1.024E-08 1.375E 00


     TIME =      0 MINUTES, 15 AND 40/60 SECONDS
  ITERATION   3 NOT CLOSED -- BEAM DEFLS   49 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 3.825E-04 WC 3.825E-04 SLP-7.116E-02 TERR 5.530E-10   RERR
BM 2 JT 1 STA LT -1 WJ 1.313E 00 WC 1.322E 00 SLP-7.501E-02 TERR 5.272E-03 2.939E-01

BM 1 JT 2 STA LT 19 WJ-3.825E-04 WC-3.825E-04 SLP-1.435E-02 TERR-7.276E-11   RERR
BM 3 JT 1 STA LT -1 WJ 1.322E 00 WC 1.322E 00 SLP-1.766E-02 TERR-9.255E-09 2.470E-01


     TIME =      0 MINUTES, 16 AND  8/60 SECONDS
  ITERATION   4 NOT CLOSED -- BEAM DEFLS   50 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 4.100E-04 WC 4.100E-04 SLP-7.493E-02 TERR-9.750E-10   RERR
BM 2 JT 1 STA LT -1 WJ 1.286E 00 WC 1.296E 00 SLP-7.216E-02 TERR 5.354E-03-4.117E-02

BM 1 JT 2 STA LT 19 WJ-4.100E-04 WC-4.100E-04 SLP-1.675E-02 TERR 2.037E-10   RERR
BM 3 JT 1 STA LT -1 WJ 1.296E 00 WC 1.296E 00 SLP-1.546E-02 TERR 6.752E-09-1.959E-02


     TIME =      0 MINUTES, 16 AND 37/60 SECONDS
  ITERATION   5 NOT CLOSED -- BEAM DEFLS   45 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 4.029E-04 WC 4.029E-04 SLP-7.374E-02 TERR-6.912E-10   RERR
BM 2 JT 1 STA LT -1 WJ 1.299E 00 WC 1.304E 00 SLP-7.420E-02 TERR 2.567E-03 3.462E-02

BM 1 JT 2 STA LT 19 WJ-4.029E-04 WC-4.029E-04 SLP-1.631E-02 TERR 2.110E-10   RERR
BM 3 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-1.658E-02 TERR-1.793E-08 2.039E-02


     TIME =      0 MINUTES, 17 AND  5/60 SECONDS
```

```
    ITERATION   6 NOT CLOSED -- BEAM DEFLS   46 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 4.060E-04 WC 4.060E-04 SLP-7.419E-02 TERR-9.677E-10   RERR
BM 2 JT 1 STA LT -1 WJ 1.304E 00 WC 1.303E 00 SLP-7.427E-02 TERR-3.449E-04 2.288E-02

BM 1 JT 2 STA LT 19 WJ-4.060E-04 WC-4.060E-04 SLP-1.655E-02 TERR 4.147E-10   RERR
BM 3 JT 1 STA LT -1 WJ 1.303E 00 WC 1.303E 00 SLP-1.652E-02 TERR 3.510E-08-6.553E-03


         TIME =      0 MINUTES, 17 AND 34/60 SECONDS
    ITERATION   7 NOT CLOSED -- BEAM DEFLS   47 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 4.065E-04 WC 4.065E-04 SLP-7.434E-02 TERR-1.222E-09   RERR
BM 2 JT 1 STA LT -1 WJ 1.303E 00 WC 1.303E 00 SLP-7.435E-02 TERR 6.958E-05 8.414E-03

BM 1 JT 2 STA LT 19 WJ-4.065E-04 WC-4.065E-04 SLP-1.650E-02 TERR 7.276E-11   RERR
BM 3 JT 1 STA LT -1 WJ 1.303E 00 WC 1.303E 00 SLP-1.649E-02 TERR-5.239E-09-6.676E-03


         TIME =      0 MINUTES, 18 AND  2/60 SECONDS
    ITERATION   8 NOT CLOSED -- BEAM DEFLS   46 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 4.067E-04 WC 4.067E-04 SLP-7.441E-02 TERR 1.775E-09   RERR
BM 2 JT 1 STA LT -1 WJ 1.304E 00 WC 1.303E 00 SLP-7.443E-02 TERR-4.863E-05 4.591E-03

BM 1 JT 2 STA LT 19 WJ-4.067E-04 WC-4.067E-04 SLP-1.646E-02 TERR-3.056E-10   RERR
BM 3 JT 1 STA LT -1 WJ 1.303E 00 WC 1.303E 00 SLP-1.645E-02 TERR-1.490E-08-4.262E-03


         TIME =      0 MINUTES, 18 AND 31/60 SECONDS
    ITERATION   9 NOT CLOSED -- BEAM DEFLS   44 JOINT DEFLS   1 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 4.067E-04 WC 4.067E-04 SLP-7.449E-02 TERR 7.640E-10   RERR
BM 2 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-7.450E-02 TERR-8.145E-05 1.422E-04

BM 1 JT 2 STA LT 19 WJ-4.067E-04 WC-4.067E-04 SLP-1.639E-02 TERR 8.004E-11   RERR
BM 3 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-1.639E-02 TERR-1.368E-08-9.792E-05


         TIME =      0 MINUTES, 18 AND 59/60 SECONDS
    ITERATION  10 NOT CLOSED -- BEAM DEFLS   35 JOINT DEFLS   0 JOINT ROTATIONS   4
BM 1 JT 1 STA LT -1 WJ 4.067E-04 WC 4.067E-04 SLP-7.450E-02 TERR 4.366E-10   RERR
BM 2 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-7.447E-02 TERR-3.096E-06-3.561E-04

BM 1 JT 2 STA LT 19 WJ-4.067E-04 WC-4.067E-04 SLP-1.639E-02 TERR-1.455E-10   RERR
BM 3 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-1.639E-02 TERR-1.007E-08-3.831E-05


         TIME =      0 MINUTES, 19 AND 28/60 SECONDS
    ITERATION  11 NOT CLOSED -- BEAM DEFLS   33 JOINT DEFLS   1 JOINT ROTATIONS   2
BM 1 JT 1 STA LT -1 WJ 4.067E-04 WC 4.067E-04 SLP-7.449E-02 TERR-1.375E-09   RERR
BM 2 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-7.449E-02 TERR 8.447E-06 6.754E-05

BM 1 JT 2 STA LT 19 WJ-4.067E-04 WC-4.067E-04 SLP-1.639E-02 TERR-1.965E-10   RERR
BM 3 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-1.639E-02 TERR 1.746E-10 1.666E-04


         TIME =      0 MINUTES, 19 AND 56/60 SECONDS
    ITERATION  12 NOT CLOSED -- BEAM DEFLS    4 JOINT DEFLS   0 JOINT ROTATIONS   0
BM 1 JT 1 STA LT -1 WJ 4.067E-04 WC 4.067E-04 SLP-7.449E-02 TERR 2.176E-09   RERR
BM 2 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-7.449E-02 TERR-1.718E-06 6.817E-05

BM 1 JT 2 STA LT 19 WJ-4.067E-04 WC-4.067E-04 SLP-1.639E-02 TERR 1.528E-10   RERR
BM 3 JT 1 STA LT -1 WJ 1.304E 00 WC 1.304E 00 SLP-1.639E-02 TERR-1.100E-08 9.107E-06
```

```
     TIME =      0 MINUTES, 20 AND 25/60 SECONDS
ITERATION  13 NOT CLOSED -- BEAM DEFLS    0 JOINT DEFLS   0 JOINT ROTATIONS   0

***** CLOSURE ACHIEVED *****

     TIME =      0 MINUTES, 20 AND 46/60 SECONDS
```

PROGRAM PLNFRAM 4 - MASTER DECK - T A HALIBURTON REVISION DATE 18 AUG 65
RUN TO INDICATE FORM OF COMPUTER OUTPUT - DATE RUN 8/18/65 - CHG CE051119
CODED, PROOFED, AND RUN BY TAH - PUNCHED BY BPF, BW, GB, AND BB - CHG CE051119

PROB (CONTD)
1 SIMPLE BENT USED AS COMPARATIVE EXAMPLE - H = 1.0

TABLE 7. - RESULTS FOR EACH JOINT

BEAM NUM	JOINT NUM	STA TO LEFT	BEAM DEFLECTION	COLUMN DEFLECTION	BEAM SLOPE	TRANSLATION ERROR	ROTATION ERROR
1	1	-1	4.067E-04	4.067E-04	-7.449E-02	-6.621E-10	
2	1	-1	1.304E 00	1.304E 00	-7.449E-02	4.529E-07	1.566E-05
1	2	19	-4.067E-04	-4.067E-04	-1.639E-02	-3.638E-11	
3	1	-1	1.304E 00	1.304E 00	-1.639E-02	1.269E-08	-1.565E-06

PROGRAM PLNFRAM 4 - MASTER DECK - T A HALIBURTCN   REVISICN DATE 18 AUG 65
RUN TO INDICATE FORM OF COMPUTER OUTPUT - DATE RUN 8/18/65 - CHG CE051119
CODED, PROCFEC, AND RUN BY TAH - PUNCHED BY BPF, BW, GB, ANC BB - CHG CE051119

PROB (CONTC)
1   SIMPLE BENT USED AS COMPARATIVE EXAMPLE - H = 1.0

TABLE 8. - MEMBER RESULTS -

MEMBER 1

STAS TO LT CR RT OF JOINT DENCTED BY *

STA	X	DEFL	MOMENT	REACT	AXIAL FORCE
-1*	-1.000E 00	3.765E-02	0	4.716E 00	0
0*	0	-3.684E-02	4.716E 00	-5.105E 00	-3.114E-01
1	1.000E 00	-9.560E-02	4.328E 00	-4.492E-03	-3.114E-01
2	2.000E 00	-1.399E-01	3.935E 00	-4.084E-03	-3.114E-01
3	3.000E 00	-1.712E-01	3.538E 00	-3.672E-03	-3.114E-01
4	4.000E 00	-1.906E-01	3.137E 00	-3.256E-03	-3.114E-01
5	5.000E 00	-1.996E-01	2.733E 00	-2.837E-03	-3.114E-01
6	6.000E 00	-1.994E-01	2.327E 00	-2.415E-03	-3.114E-01
7	7.000E 00	-1.916E-01	1.917E 00	-1.990E-03	-3.114E-01
8	8.000E 00	-1.773E-01	1.506E 00	-1.563E-03	-3.114E-01
9	9.000E 00	-1.580E-01	1.093E 00	-1.135E-03	-3.114E-01
10	1.000E 01	-1.350E-01	6.796E-01	-7.053E-04	-3.114E-01
11	1.100E 01	-1.098E-01	2.651E-01	-2.751E-04	-3.114E-01
12	1.200E 01	-8.371E-02	-1.498E-01	1.554E-04	-3.114E-01
13	1.300E 01	-5.812E-02	-5.644E-01	5.858E-04	-3.114E-01
14	1.400E 01	-3.440E-02	-9.785E-01	1.016E-03	-3.114E-01
15	1.500E 01	-1.395E-02	-1.392E 00	1.444E-03	-3.114E-01
16	1.600E 01	1.866E-03	-1.803E 00	1.872E-03	-3.114E-01
17	1.700E 01	1.167E-02	-2.213E 00	2.297E-03	-3.114E-01
18	1.800E 01	1.410E-02	-2.620E 00	2.720E-03	-3.114E-01
19*	1.900E 01	7.789E-03	-3.025E 00	3.430E 00	-3.114E-01
20*	2.000E 01	-8.603E-03	0	-3.025E 00	0

PROGRAM PLNFRAM 4 - MASTER DECK - T A HALIBURTCN   REVISICN DATE 18 AUG 65
RUN TO INDICATE FORM OF COMPUTER OUTPUT - DATE RUN 8/18/65 - CHG CE051119
CODED, PRCOFED, AND RUN BY TAH - PUNCHED BY BPF, BW, GB, AND BB - CHG CE051119

PROB (CCNTC)
1 SIMPLE BENT USED AS CCMPARATIVE EXAMPLE - H = 1.0

TABLE 8. - MEMBER RESULTS -

MEMBER 2

STAS TO LT CR RT OF JOINT DENCTED BY *

STA	X	DEFL	MOMENT	REACT	AXIAL FORCE
-1*	-1.000E 00	1.341E 00	0	-4.329E 00	0
0*	0	1.266E 00	-4.329E 00	5.469E 00	4.067E-01
1	1.000E 00	1.149E 00	-3.188E 00	-1.297E-02	4.067E-01
2	2.000E 00	9.989E-01	-2.060E 00	-8.379E-03	4.067E-01
3	3.000E 00	8.286E-01	-9.408E-01	-3.826E-03	4.067E-01
4	4.000E 00	6.490E-01	1.748E-01	7.108E-04	4.067E-01
5	5.000E 00	4.711E-01	1.291E 00	5.251E-03	4.067E-01
6	6.000E 00	3.061E-01	2.413E 00	9.812E-03	4.067E-01
7	7.000E 00	1.652E-01	3.544E 00	1.441E-02	4.067E-01
8	8.000E 00	5.973E-02	4.690E 00	1.907E-02	4.067E-01
9*	9.000E 00	1.189E-03	5.855E 00	-7.019E 00	4.067E-01
10*	1.000E 01	1.189E-03	0	5.855E 00	0

PROGRAM PLNFRAM 4 - MASTER DECK - T A HALIBURTON REVISION DATE 18 AUG 65
RUN TO INDICATE FORM OF COMPUTER OUTPUT - DATE RUN 8/18/65 - CHG CE051119
CODED, PROOFED, AND RUN BY TAH - PUNCHED BY BPF, BW, GB, AND BB - CHG CE051119

PROB (CONTD)
1 SIMPLE BENT USED AS COMPARATIVE EXAMPLE - H = 1.0

TABLE 8. - MEMBER RESULTS -

MEMBER 3

STAS TO LT OR RT OF JOINT DENOTED BY *

STA	X	DEFL	MOMENT	REACT	AXIAL FORCE
-1*	-1.000E 00	1.312E 00	0	-3.067E 00	0
0*	0	1.295E 00	-3.067E 00	3.391E 00	-4.067E-01
1	1.000E 00	1.264E 00	-2.743E 00	5.577E-03	-4.067E-01
2	2.000E 00	1.218E 00	-2.413E 00	4.907E-03	-4.067E-01
3	3.000E 00	1.161E 00	-2.078E 00	4.226E-03	-4.067E-01
4	4.000E 00	1.093E 00	-1.739E 00	3.537E-03	-4.067E-01
5	5.000E 00	1.016E 00	-1.397E 00	2.840E-03	-4.067E-01
6	6.000E 00	9.326E-01	-1.051E 00	2.138E-03	-4.067E-01
7	7.000E 00	8.438E-01	-7.038E-01	1.431E-03	-4.067E-01
8	8.000E 00	7.515E-01	-3.549E-01	7.217E-04	-4.067E-01
9	9.000E 00	6.573E-01	-5.275E-03	1.071E-05	-4.067E-01
10	1.000E 01	5.632E-01	3.444E-01	-7.003E-04	-4.067E-01
11	1.100E 01	4.708E-01	6.933E-01	-1.410E-03	-4.067E-01
12	1.200E 01	3.818E-01	1.041E 00	-2.117E-03	-4.067E-01
13	1.300E 01	2.980E-01	1.386E 00	-2.819E-03	-4.067E-01
14	1.400E 01	2.212E-01	1.729E 00	-3.516E-03	-4.067E-01
15	1.500E 01	1.530E-01	2.068E 00	-4.205E-03	-4.067E-01
16	1.600E 01	9.518E-02	2.403E 00	-4.886E-03	-4.067E-01
17	1.700E 01	4.935E-02	2.733E 00	-5.557E-03	-4.067E-01
18	1.800E 01	1.719E-02	3.057E 00	-6.217E-03	-4.067E-01
19*	1.900E 01	3.113E-04	3.376E 00	-3.694E 00	-4.067E-01
20*	2.000E 01	3.113E-04	0	3.376E 00	0

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    PROGRAM PLNFRAM 4 - MASTER DECK - T A HALIBURTCN          REVISICN DATE 18 AUG 65
    RUN TO INDICATE FORM CF COMPUTER OUTPUT - DATE RUN 8/18/65 - CHG CE051119
    CODED, PRCCFED, AND RUN BY TAF - PUNCHED BY BPF, BW, GB, ANC BB - CHG CE051119


    RETURN THIS PAGE TO TIME RECORD FILE -- TAH

         TIME =      0 MINUTES, 23 AND 23/60 SECONDS
   00    HOURS,    00  MINUTES,    26  SECONDS.
END JOB 023.   16.05.12

TOTAL NUMBER OF PAGES 011
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