DISCONTINUOUS ORTHOTROPIC PLATES AND PAVEMENT SLABS

by

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Development of Methods for Computer Simulation
of Beam-Columns and Slab Systems

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PREFACE

This report describes an analytical tool for solving problems involving plates and slabs under static load. The background and theory are presented and a computer program is described which makes the method useful to highway engineers. The method provides the ability to solve complex problems which could not heretofore be solved.

This is the sixth in a series of reports that describe the work in Research Project No. 3-5-63-56, entitled "Development of Methods for Computer Simulation of Beam-Columns and Grid-Beam and Slab Systems." The project is divided into two parts. Part I is concerned primarily with bridge structures. Part II deals with pavement slabs. The reader may find it advantageous to review Report No. 56-1 (See List of Reports) as it will provide background for this report.

This is the first report in the series that deals directly with pavement slabs. Several subsequent reports concerning pavements are planned for submission in the near future.

Although the computer program presented here is written for the CDC 1604 computer, it is in FORTRAN language and only minor changes are required to make it compatible with IBM 7090 systems. Duplicate copies of the program deck and test data cards for the example problems in this report may be obtained from the Center for Highway Research, The University of Texas.

This report is a product of the combined efforts of many people. The advice and assistance of the Texas Highway Department contact representatives, Messrs. M. D. Shelby, B. F. McCullough, and L. G. Walker, are greatly appreciated. The support of the U. S. Bureau of Public Roads is gratefully acknowledged.

The excellent facilities of the Computation Center of The University of Texas and the cooperation of its staff have contributed significantly to this report. Thanks are due to Evangeline Emory, Barbara Powell, Joni McKnight, Jimmy Holmes, Sam Jones, and all others who assisted with the manuscript.

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LIST OF REPORTS

Report No. 56-1, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns" by Hudson Matlock and T. Allan Haliburton, presents a finite-element solution for beam-columns that is a basic tool in subsequent reports.

Report No. 56-2, "A Computer Program to Analyze Bending of Bent-Caps" by Hudson Matlock and Wayne B. Ingram, describes the application of the beam-column solution to the particular problem of bent-caps.

Report No. 56-3, "A Finite-Element Method of Solution for Structural Frames" by Hudson Matlock and Berry Ray Grubbs, describes a solution for frames with no sway.

Report No. 56-4, "A Computer Program to Analyze Beam-Columns under Movable Loads" by Hudson Matlock and Thomas P. Taylor, describes the application of the beam-column solution to problems with any configuration of movable non-dynamic loads.

Report No. 56-5, "A Finite-Element Method for Bending Analysis of Layered Structural Systems" by Wayne B. Ingram and Hudson Matlock, describes an alternating-direction iteration method for solving two-dimensional systems of layered grids-over-beams and plates-over-beams.

Report No. 56-6, "Discontinuous Orthotropic Plates and Pavement Slabs" by W. Ronald Hudson and Hudson Matlock, describes an alternating-direction iteration method for solving complex two-dimensional plate and slab problems with emphasis on pavement slabs.

Report No. 56-7, "A Finite-Element Analysis of Structural Frames" by T. Allan Haliburton and Hudson Matlock, describes a method of analysis for rectangular plane frames with three degrees of freedom at each joint.

Report No. 56-8, "A Finite-Element Method for Transverse Vibrations of Beams and Plates" by Harold Salani and Hudson Matlock, describes an implicit procedure for determining the transient and steady-state vibrations of beams and plates, including pavement slabs.

ABSTRACT

A method of solving for the deflected shape of freely discontinuous orthotropic plates and pavement slabs subjected to a variety of loads including transverse loads, in-plane forces and externally applied couples is presented. The method is applicable for plates and pavement slabs with freely variable foundation support including holes in the subgrade.

Anisotropic elasticity governs the behavior of orthotropic plates and pavement slabs and is used to develop the necessary equations. The method is not limited by discontinuities and uses an efficient alternating-direction iteration means of solving the resulting equations. The method allows considerable freedom in configuration, loading, flexural stiffness and boundary conditions. It solves the problem rapidly and should provide a tool for use in later studies of repetitive stochastic loading. Three principal features are incorporated into the method: (1) the plate is defined by a finite-element model consisting of bars, springs, elastic blocks and torsion bars--these are further grouped for analysis into orthogonal systems of beam-column elements and forces, (2) each individual line-element of the two dimensional system is solved rapidly and directly by recursive techniques, (3) an alternating-direction iterative method is utilized for coordinating the solution of the individual line-elements into the slab solution.

The computer program utilizes the equations and techniques developed and can be used by the reader. Several sample problems illustrate the generality of the method and the use of the computer program. The results compare well with closed-form solutions.

TABLE OF CONTENTS

NOTATION

Symbol	Dimensions	Definition
a , b		Temporary bar numbering used in derivations.
$\alpha_{a,j}$	rad	Angular change across slab element a,j .
β_1		Ratio n/m used in closure parameter calculations.
$c_{i,j}$	lb/in^2	Constants used to relate stress to strain in general anisotropic elasticity (i refers to stress component, j refers to strain component).
$C^x_{i,j}$	$\frac{in\text{-}lb}{rad}$	Torsional stiffness of slab element i,j about the x-axis.
$C^y_{i,j}$	$\frac{in\text{-}lb}{rad}$	Torsional stiffness of slab element i,j about the y-axis.
$C^{x\prime}_{i,j}$	in-lb	Torque exerted on the x-beam due to the relative rotation in torsion bar i,j .
$C^{y\prime}_{i,j}$	in-lb	Torque exerted on the y-beam due to the relative rotation in torsion bar i,j .
$D_{i,j}$	in-lb	Bending stiffness of an isotropic plate.
$D^x_{i,j}$	in-lb	Bending stiffness of an orthotropic plate in the x direction.
$D^y_{i,j}$	in-lb	Bending stiffness of an orthotropic plate in the y direction.
D_{xy}	in-lb	$\frac{Gt^3}{12}$
D_1	in-lb	$\frac{E''t^3}{12}$
E_x	lb/in^2	Modulus of elasticity in x direction.
E_y	lb/in^2	Modulus of elasticity in y direction.
E_x'	lb/in^2	$\frac{E_x}{1 - \nu_{xy}\nu_{yx}}$
E_y'	lb/in^2	$\frac{E_y}{1 - \nu_{yx}\nu_{xy}}$
E''	lb/in^2	$\nu_{yx}E_x' = \nu_{xy}E_y'$
ϵ_x	in/in	Total strain in x direction.
ϵ_y	in/in	Total strain in y direction.
ϵ_z	in/in	Total strain in z direction.

Symbol	Dimensions	Definition
ϵ_{xx}	in/in	Strain in x direction due to stress in x direction.
ϵ_{yy}	in/in	Strain in y direction due to stress in y direction.
ϵ_{xy}	in/in	Strain in y direction due to stress in x direction.
ϵ_{yx}	in/in	Strain in x direction due to stress in y direction.
$F_{v_{i,j}}$	lb	Vertical forces at joint i,j .
ϕ		Angular rotation across a plate element.
G	lb/in^2	Shear modulus $= \dfrac{E}{12(1+\nu)}$.
G_o	lb/in^2	Approximate orthotropic shear modulus $\left(\dfrac{E_x E_y}{E_y\ (1+\nu_{xy}) + E_x\ (1+\nu_{yx})}\right)$
γ	in/in	Shear strain.
H	in-lb	$D_1 + 2D_{xy}$.
h_x	inch	The increment length along the x-beams.
h_y	inch	The increment length along the y-beams.
i		An integer used to number mesh points, stations, and bars in the x direction.
j		An integer used to index parts of the model and stationing in the y direction.
k	$lb/in^2/in$	Support modulus of the foundation.
λ_n		n^{th} eigenvalue.
M_a	in-lb	Moment acting on Bar a about the center of the bar.
M_x	$\dfrac{\text{in-lb}}{\text{in}}$	Bending moment acting on an element of the plate in the x direction.
M_y	$\dfrac{\text{in-lb}}{\text{in}}$	Bending moment acting on an element of the plate in the y direction.
M_{xy}	$\dfrac{\text{in-lb}}{\text{in}}$	Twisting moment tending to rotate the element about the x-axis (clockwise-positive).
M_{yx}	$\dfrac{\text{in-lb}}{\text{in}}$	Twisting moment tending to rotate the element about the the y-axis (clockwise-positive).
$M^{x\prime}_{i,j}$	in-lb	The bending moment in the x-beam at Station i,j (equals $h_y M^x_{i,j}$).
$M^{y\prime}_{i,j}$	in-lb	The bending moment in the y-beam at Station i,j (equals $h_x M^y_{i,j}$).

Symbol	Dimensions	Definition
$M^x_{i,j}$	$\frac{\text{in-lb}}{\text{in}}$	Unit bending moment in the slab in the x direction at Station i,j.
$M^y_{i,j}$	$\frac{\text{in-lb}}{\text{in}}$	Unit bending moment in the slab in the y direction at Station i,j.
m		Counting integer
N_x	lb/in	Unit axial force in the x direction.
N_y	lb/in	Unit axial force in the y direction.
N_{xy}	lb/in^2	Shear stresses in the plane of the plate = N_{yx}
N_{yx}	lb/in^2	Shear stresses in the plane of the plate = N_{xy}
n		Counting integer.
ν_{xy}		Poisson's ratio which results in strain in the y direction if stress is applied in the x direction.
ν_{yx}		Poisson's ratio which results in strain in the x direction if stress is applied in the y direction.
P_x	lb	Axial load per beam in the x direction.
P_y	lb	Axial load per beam in the y direction.
$P^x_{i,j}$	lb	Axial load in the x-beam in Bar i,j (equals $h_y p^x_{i,j}$)
$p^x_{i,j}$	lb/in	Unit axial load in the slab x direction at Station i,j.
$\Delta P^x_{i,j}$	lb	Change in axial load in the x-beam occurring at Station i,j.
q	lb/in^2	Distributed lateral load.
Q	lb	Concentrated lateral load.
$Q_{i,j}$	lb	Externally applied load at point i,j .
$Q^x_{i,j}$	lb	Load absorbed internally by the x-beam system.
$Q^y_{i,j}$	lb	Load absorbed internally by the y-beam system at Station i,j.
$\overline{QBMX}_{i,j}$	lb	Load absorbed by the x-beam in bending.
$\overline{QBMY}_{i,j}$	lb	Load absorbed by the y-beam in bending.
$\overline{QPX}_{i,j}$	lb	Load absorbed by the x-beam due to axial load.
$\overline{QPY}_{i,j}$	lb	Load absorbed by the y-beam due to axial load.

Symbol	Dimensions	Definition
$\overline{QTMX}_{i,j}$	lb	Load absorbed by the x-beam in twisting.
$\overline{QTMY}_{i,j}$	lb	Load absorbed by the y-beam in twisting.
$\frac{1}{r_x}$; $\frac{1}{r_y}$	$\frac{1}{\text{inch}}$	Curvatures in the neutral surface of the slab.
S_f	lb/in	Fictitious spring stiffness or closure parameter.
S_{fx}	lb/in	Fictitious spring representing the x-beams.
S_{fy}	lb/in	Fictitious spring representing the y-beams.
$S_{i,j}$	lb/in	Elastic restraint used to represent the foundation in the finite-element model.
σ_x	lb/in^2	Stress applied in the x direction.
σ_y	lb/in^2	Stress applied in the y direction.
σ_z	lb/in^2	Stress applied in the z direction.
t	inch	Slab thickness.
$T^x_{i,j}$	in-lb	External torque applied to Bar i on the j^{th} x-beam.
τ	lb/in^2	Shear stress.
u	inch	Displacement in the x direction.
v	inch	Displacement in the y direction.
$V^x_{a,j}$	lb	Shear in Bar a of the j^{th} x-beam.
$w_{i,j}$	inch	Lateral deflection.
W	in-lb	Work.
w_{max}	inch	Maximum deflection.
$w^x_{i,j}$	inch	Deflection of the j^{th} x-beam at Station i .
$w^y_{i,j}$	inch	Deflection of the i^{th} y-beam at Station j .
x,y,z		Standard Cartesian coordinate directions.

CHAPTER 1. INTRODUCTION

The first extensive concrete road system in the world was constructed in Wayne County, Michigan, in 1909. These pavements were not designed in the usual sense of the word since no rational theory of pavement design existed at that time. In early 1920, Goldbeck and Older independently developed formulas for approximating the stresses in pavement slabs. The best known of these is called the "corner formula." In 1926, Dr. H. M. Westergaard completed his treatise on the analysis of stresses in pavement slabs (Ref 60). The Westergaard equations have become the definitive design equations for pavement slabs in the United States. Many other engineers and mathematicians have attempted to solve this design problem since LaGrange first developed a differential equation which theoretically described the behavior of homogeneous isotropic plates.

Unfortunately, no satisfactory solutions have been developed. Limitations of conventional mathematics, and particularly of hand solutions, have restricted developments. Thus, the Westergaard solution, as well as all others, is subjected to severely limiting assumptions which are not realistic.

Several large-scale road tests have been conducted in attempts to bridge the gap between theory and reality. These include the Bates Test 1922, the Maryland Road Test 1950, and the AASHO Road Test 1958-61. All three of these full-scale experiments have added to the knowledge of pavement design. However, only the AASHO Road Test was large enough to provide significant information. Even it considered only four basic variables: slab thickness, slab length, axle load, and number of axle repetitions. Other variables were considered cursorily, but little information was gained.

Pavement design involves four general classes of variables: (1) load variables, (2) structural variables, (3) regional variables, and (4) performance variables. Each of these classes is important. Major theoretical efforts have been directed toward evaluating structural variables under a single load. The number of actual variables, and the fact that they interact, prohibits the consideration of all variables in any single road test. Work with AASHO Road Test data has shown that a mechanistic model of structural behavior is essential in the study of load, environment, and performance. Unfortunately at the time the AASHO Road Test was completed, no satisfactory method of evaluating the behavior of pavement structure existed; thus the attempts to extend the findings of the Road Test have been slow and have involved many assumptions.

A mechanistic model based on finite-element theory is presented herein which, it is believed, will aid in studying these parameters and parameter levels, and will assist in attempts to extend the findings to other conditions and other environments.

The Problem

Since the Westergaard work, many solutions have been developed for the pavement slab problem. All these solutions involve uniform slab thickness, uniform homogeneous isotropic (or special-case orthotropic) slabs, uniform foundation support, and certain uniform or special-case loading conditions. The problem, then, is to develop complete equations for describing slab behavior and to develop better methods for solving these equations. The method should allow for freely discontinuous variation of input parameters including bending stiffness and load. Combination loading should be provided for and should include lateral loads, in-plane forces, and applied couples or moments. Freely variable foundation conditions are needed. Such a technique should apply not only to the general slab-on-foundation case, but also for orthotropic plates with various configurations of structural support, and could include the uniform isotropic plate with simple supports as a special case. Such a general model for pavement behavior would provide an important tool toward solving the over-all pavement design problem.

Application of the Model

A general mechanistic model which is easily and rapidly solved is discussed herein. One solution of this model describes the slab behavior for a single load pattern just as previous methods do. However, the generality and speed of the method make it a more powerful tool than was formerly available. Studies are already underway to utilize such a model in dynamic analyses of plates (Ref 45). Such analyses will immediately carry past previous methods. It is but a single step from such analytical capability to an over-all study of pavement performance. Such a study involves four steps: (1) evaluation of the roadway roughness profile to ascertain the effects on roadway variations, i.e., the dynamic effect, (2) evaluating the moving loads to determine the pattern of load variations to which the pavement is subjected, (3) determining the pavement response to these dynamic loads, (4) coupling the pavement response information with other available data, particularly information on the load pattern spectrum applied to the pavement, to evaluate the over-all pavement performance under repetitive loads.

Stochastic Load Studies

The development of this method presents another economical and interesting idea. The AASHO Road Test cost approximately $30,000,000 and was no doubt worth the cost since it provided a great deal of information relating to the destructive effects of various loads. Using the model described here, it will be possible in the future to simulate performance by using a digital computer prior to running any full-scale test. Such a study could use the finite-element model to evaluate the effect of wheel loads on the pavements of interest. Many such single loads, varied randomly in size and placement, could then be applied in rapid succession to the model by the computer. Information as to the size of each load and its effect on the pavement structure at various critical points would be accumulated for later use. These accumulations would in effect provide performance data since they would be plots of applied loads versus accumulated structural damage. Such modeling techniques deserve study and can be evaluated by using data from tests such as the AASHO Road Test to compare information from a stochastic study with that from the real world.

Development

The purpose of this report is to describe a rapid method of solving for the deflected shape of orthotropic plates and pavement slabs.[1] From this deflected shape the stresses, deflections, loads, and bending moments can easily be determined. The method developed takes advantage of groundwork laid by others. The finite-element method was developed by Matlock (Refs 35, 36, 56), and variations and extensions of his methods have been made by Tucker, Haliburton, Ingram, and Salani (Refs 56, 35, 23, and 45). Other work has been done in this general area by Clough (Ref 13), Newmark (Ref 39), and Badir (Ref 6). The principal features incorporated into the finite-element method are: (1) representation of structural members by a physical model of bars and springs which are grouped for analyses into systems of orthogonal beams, (2) a rapid method for solution of individual beams that serve as line elements of a two-dimensional slab, and (3) an alternating-direction iteration technique for coordinating the solutions of individual beams which ties the system together.

[1]Throughout this report the term slab is often used as an abbreviation for pavement slab and slab-on-foundation.

Work done by Lekhnitski (Ref 30) and Hearmon (Ref 16) in orthotropic plate theory has been helpful in deriving the equations.

CHAPTER 2. THEORY OF ELASTIC PLATES AND SLABS

A review of the various theories involved in the analysis of plate and slab bending will be helpful in understanding the problem at hand. A brief discussion of the biharmonic equation is presented in this chapter. A discussion of generalized Hooke's Law which leads into the derivation of the equation of bending for orthotropic plates follows. The effect of in-plane forces applied to the plate in combined loadings is presented next. Finally, elastic foundations are discussed as related to pavement support.

The notations used in these developments are numerous; some will be given in the text for convenience. A complete list of notation, however, is given before Chapter 1.

General Plate Theory

The bending of a plate depends greatly on its thickness as compared with its other dimensions. Timoshenko (Ref 55) distinguishes three kinds of plate bending: (1) thin plates with small deflections, (2) thin plates with large deflections, and (3) thick plates.

For thin plates with small deflections (i.e., the deflection is small in comparison with thickness), a satisfactory approximate theory of bending of a plate by lateral loads can be developed by making the following assumptions:

1. There is no deformation in the middle plane of the plate. This plane remains neutral during bending.
2. Planes of the plate lying initially normal to the middle surface of the plate remain normal to the middle surface of the plate after bending.
3. The normal stresses in the direction transverse to the plate can be disregarded. (This assumption is necessary in the analysis of bending of the plate as will be seen later; approximate corrections can be made to account for pressures directly under the transverse load.)

With these assumptions, all components of stress can be expressed in terms of the deflected shape of the plate. This function has to satisfy a linear partial differential equation which, together with the boundary conditions, completely defines the deflection w . The solution of this differential equation gives all necessary information for calculating the stresses at any point in the plate.

The solution of thin plates with large deflections is a more difficult problem since it must account for the strain in the middle plane in violation of the first assumption. Consideration of these supplementary strains (membrane action) produces nonlinear equations, and the solution of these equations becomes quite complicated. Large deflections tend to induce lateral movement of the plate edges, and it becomes necessary to distinguish between immovable edges and edges free to move in the plane of the plate. As a result of edge movement, the applied load is transmitted partly by the flexural rigidity and partly by a membrane action of the plate. Very thin plates tend to behave as membranes except near the edges.

The approximate theories which define the behavior of thin plates become unreliable for plates of considerable thickness, particularly in the vicinity of highly concentrated loads. In these cases, thick plate theory must be applied which considers the problem of the plate as a three-dimensional problem of elasticity. The stress analysis for such cases is complex, and, according to Timoshenko, the problem is completely solved for only a few particular cases. As a usual case, the necessary corrections to thin plate theory are introduced at the points of application of concentrated loads. This is desirable in pavement slabs and has been discussed by Westergaard (Ref 60).

Timoshenko (Ref 55) develops the theory of bending of plates very thoroughly from the simplest problem of bending in a long rectangular plate subjected to transverse load to the very complex problems of thick plates with various boundary conditions. For more detail than provided here, the reader is invited to study this reference.

The Isotropic Plate Equation

Structural plates and pavement slabs are normally subjected to loads applied perpendicular to their surface, i.e., lateral loads. Timoshenko and others have derived a differential equation which describes the deflection surface of such plates, the biharmonic equation. With one minor change, Timoshenko's equation is given below. This change is to reverse the sense of the z-axis and make "up" positive. This new coordinate system is consistent with recent beam-column developments (Ref 35). The equation becomes

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} = q \tag{2.1}$$

in which M_x is the bending moment acting on an element of the plate in the x direction, M_y is the bending moment acting on an element of the plate in the y direction, M_{xy} is a twisting moment tending to rotate the element about the x-axis (clockwise positive), and M_{yx} is a twisting moment tending to rotate the element about the y-axis. Observing that $M_{xy} = -M_{yx}$ for equilibrium $(\tau_{xy} = \tau_{yx})$, the equation can be condensed into the following form.

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - 2\,\frac{\partial^2 M_{xy}}{\partial x \partial y} = q \tag{2.2}$$

In order to evaluate this equation, it is safe to assume that expressions for moment derived for pure bending can also be used for laterally loaded plates. This assumption is equivalent to neglecting the effect on bending of the shearing forces and the compressive stress in the z direction produced by the lateral load. Errors introduced into these solutions by such assumptions are negligible provided the thickness of the plate is small in comparison with the other dimensions of the plate.

The equations for moment are derived in Appendix A for the general case. For the special case of isotropy, they can be stated as follows:

$$M_x = D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \tag{2.3}$$

$$M_y = D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \tag{2.4}$$

$$M_{xy} = - M_{yx} = -D\ (1 - \nu)\ \frac{\partial^2 w}{\partial x \partial y} \tag{2.5}$$

where D is the bending stiffness of the plate, ν is the Poisson's ratio, and other terms have been previously defined.

Substituting these expressions into Eq 2.2 obtains

$$D\left[\frac{\partial^4 w}{\partial x^4} + 2\ \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] = q \tag{2.6}$$

According to Timoshenko (Ref 55), this equation was obtained by LaGrange in 1811. The history of its development is given in Todhunter and Pearson, History of the Theory of Elasticity. It was first solved by Navier using a double trigonometric series in 1820.

The Generalized Hooke's Law

In order to obtain the relations between the components of stress and the components of deformation in an elastic body, it is necessary to choose some mathematical model which reflects the elastic properties of the body. In these derivations it is always assumed that the components of strain are linear functions of the components of stress. In other words, it is assumed that a continuous body satisfies the generalized Hooke's Law.

For the most general case of a homogeneous anisotropic body, the equations which express Hooke's Law in Cartesian coordinates x , y , z have the form

$$\epsilon_x = S_{11}\sigma_x + S_{12}\sigma_y + S_{13}\sigma_z + S_{14}\tau_{yz} + S_{15}\tau_{xz} + S_{16}\tau_{xy}$$

$$\begin{aligned}
\epsilon_y &= S_{21}\sigma_x + S_{22}\sigma_y + S_{23}\sigma_z + S_{24}\tau_{yz} + S_{25}\tau_{xz} + S_{26}\tau_{xy} \\
\epsilon_z &= S_{31}\sigma_x + S_{32}\sigma_y \ . \ . \ . \ . \\
\gamma_{yz} &= S_{41}\sigma_x \ . \ . \ . \ . \ . \ . \ . \ . \ . \\
\gamma_{xz} &= S_{51}\sigma_x \ . \ . \ . \ . \ . \ . \ . \ . \ . \\
\gamma_{xy} &= S_{61}\sigma_x + S_{62}\sigma_y + . \ . \ . + S_{66}\tau_{xy}
\end{aligned} \tag{2.7}$$

These equations contain 36 coefficients $S_{i,j}$, the so-called elastic constants. Solving the above equations for stress components obtains an equivalent form for the equations in terms of stress:

$$\begin{aligned}
\sigma_x &= c_{11}\epsilon_x + c_{12}\epsilon_y + c_{13}\epsilon_z + c_{14}\gamma_{yz} + c_{15}\gamma_{xz} + c_{16}\gamma_{xy} \\
\sigma_y &= c_{21}\epsilon_x + c_{22}\epsilon_y + . \ . \ . + c_{26}\gamma_{xy} \\
\sigma_z &= c_{31}\ \epsilon_x + c_{32}\epsilon_y \ . \ . \ . \ . \\
\tau_{yz} &= c_{41}\ \epsilon_x \ . \ . \ . \ . \ . \ . \ . \ . \\
\tau_{xz} &= c_{51}\ \epsilon_x \ . \ . \ . \ . \ . \ . \ . \ . \\
\tau_{xy} &= c_{61}\epsilon_x + c_{62}\epsilon_y + . \ . \ . + c_{66}\gamma_{xy}
\end{aligned} \tag{2.8}$$

Several authors (Ref 15) have called the constants $S_{i,j}$ the coefficients of deformation, and the constants $c_{i,j}$ the moduli of elasticity. The moduli of elasticity can be uniquely expressed in terms of the coefficients of deformation when the value of their determinants are different from zero. It has been shown by others that the number of elastic constants in the most general case of anisotropy is reduced to 21 if the deformations of the elastic body can be considered to occur isothermally, that is, the temperature of each element remains constant during the deformation process. This proof is shown in Appendix A.

Since

$$\begin{aligned} S_{12} &= S_{21} \\ &\cdots \\ S_{56} &= S_{65} \end{aligned} \tag{2.9}$$

and likewise,

$$\begin{aligned} c_{12} &= c_{21} \\ &\cdots \\ c_{56} &= c_{65} \end{aligned} \tag{2.10}$$

Equation 2.7 can be written in terms of 21 coefficients as follows:

$$\begin{aligned} \epsilon_x &= S_{11}\sigma_x + S_{12}\sigma_y + S_{13}\sigma_z + S_{14}\tau_{yz} + S_{15}\tau_{xz} + S_{16}\tau_{xy} \\ \epsilon_y &= S_{12}\sigma_x + S_{22}\sigma_y + \cdots + S_{26}\tau_{xy} \\ \epsilon_z &= S_{13} \cdots \\ &\cdots \\ \gamma_{xy} &= S_{16}\sigma_x + S_{26}\sigma_y + \cdots + S_{66}\tau_{xy} \end{aligned} \tag{2.11}$$

and, likewise Eq 2.8 can be written in terms of 21 moduli.

$$\begin{aligned} \sigma_x &= c_{11}\epsilon_x + c_{12}\epsilon_y + c_{13}\epsilon_z + c_{14}\gamma_{yz} + c_{15}\gamma_{yz} + c_{16}\gamma_{xy} \\ \sigma_y &= c_{12}\epsilon_x + c_{22}\epsilon_y + \cdots + c_{26}\gamma_{xy} \\ \sigma_z &= c_{13}\epsilon_x + \cdots \\ &\cdots \\ \tau_{xy} &= c_{16}\epsilon_x + c_{26}\epsilon_y + \cdots + c_{66}\gamma_{xy} \end{aligned} \tag{2.12}$$

The problem of determining 21 coefficients to describe the behavior of an elastic body is still formidable. Fortunately, conditions of elastic symmetry permit still further reduction of this number. If the internal structure of a material possesses symmetry of any kind, the same symmetry can be observed in its elastic properties. F. Neumann (Ref 15) set forth a principle for crystals which establishes a connection between symmetry of construction and elastic symmetry. In general, this principle says that a material has the same kind of symmetry with regard to physical properties as it has in its crystallography. This principle can be expanded to include bodies which are not crystalline but which possess a symmetry of structure such as wood, plywood, and reinforced concrete.

If an anisotropic body possesses elastic symmetry, the equations of the generalized Hooke's Law are simplified. The simplifications can be thought of as follows: When viewed from the center of the symmetric coordinate system of the body, equal elastic properties are seen in both the positive and negative directions of any axis of symmetry. As a result, elastic bodies possessing symmetry have a smaller number of independent elastic constants than 21. The final number depends on the number of axes or planes of symmetry present in the body.

Three Planes of Elastic Symmetry

The case of interest involves three planes of elastic symmetry passing through each point of a body orthogonally, that is, the planes occur at right angles to each other. If the axes of the coordinate system are directed perpendicular to these planes, the following equations of the generalized Hooke's Law for an orthotropic body can be derived.

$$\epsilon_x = S_{11}\sigma_x + S_{12}\sigma_y + S_{13}\sigma_z$$

$$\epsilon_y = S_{12}\sigma_x + S_{22}\sigma_y + S_{23}\sigma_z$$

$$\epsilon_z = S_{13}\sigma_x + S_{23}\sigma_y + S_{33}\sigma_z$$

$$\gamma_{yz} = S_{44}\tau_{yz} \qquad (2.13)$$

$$\gamma_{xz} = S_{55}\tau_{xz}$$

$$\gamma_{xy} = S_{66}\tau_{xy}$$

Since the constants $S_{i,j}$ are redundant with $S_{j,i}$, it can be observed that there are nine independent elastic constants remaining.

Plane Stress Case

For the particular case of thin plates in bending, σ_z is taken to be zero (plane stress), and the following equations are obtained:

$$\epsilon_x = S_{11}\sigma_x + S_{12}\sigma_y$$

$$\epsilon_y = S_{12}\sigma_x + S_{22}\sigma_y \qquad (2.14)$$

$$\gamma_{xy} = S_{66}\tau_{xy}$$

These equations are derived directly from an orthotropic plane stress element shown in Appendix A. The corresponding elements for stress in terms of strain are also developed in Appendix A and can be stated as follows:

$$\sigma_x = \frac{E_x}{1 - \nu_{xy}\nu_{yx}} (\epsilon_x - \nu_{yx}\epsilon_y) = E_x'\epsilon_x - E''\epsilon_y \qquad (2.15)$$

$$\sigma_y = \frac{E_y}{1 - \nu_{xy}\nu_{yx}} (\epsilon_y - \nu_{xy}\epsilon_x) = E_y'\epsilon_y - E''\epsilon_x \qquad (2.16)$$

$$\tau_{xy} = G\,\gamma_{xy} \qquad (2.17)$$

where

$$E_x' = \frac{E_x}{1 - \nu_{xy}\nu_{yx}} \tag{2.18}$$

$$E_y' = \frac{E_y}{1 - \nu_{xy}\nu_{yx}} \tag{2.19}$$

$$E'' = \nu_{yx}E_x' = \nu_{xy}E_y' \tag{2.20}$$

Isotropic Elasticity

Hooke's Law for standard isotropic conditions can be stated as follows:

$$\sigma_x = \frac{E}{1 - \nu^2}(\epsilon_x + \nu\epsilon_y) \tag{2.21}$$

$$\sigma_y = \frac{E}{1 - \nu^2}(\nu\epsilon_x + \epsilon_y) \tag{2.22}$$

$$\tau_{xy} = G\,\gamma_{xy} \tag{2.23}$$

where γ_{xy} is the shearing strain, τ_{xy} is the corresponding shearing stress, and the shear modulus is

$$G = \frac{E}{2\,(1 + \nu)} \tag{2.24}$$

Other terms have been previously defined. By comparing these equations with Eq 2.14 above, it can be seen that four elastic constants are required to describe the behavior of thin orthotropic plates, whereas two independent elastic constants are required for isotropic plates. The orthotropic constants are E_x' , E_y' , E'' , and G_0 . The shear modulus, G_0 , is an independent constant, and a method for determining it is discussed in Appendix A. It can be closely approximated by relating the other three constants as follows:

$$G_0 = \frac{E_x E_y}{E_y\,(1 + \nu_{xy}) + E_x\,(1 + \nu_{yx})} \tag{2.25}$$

Orthotropic Elastic Plate Equations

The complete derivation of the differential equation of equilibrium is given in Appendix A. Utilizing the elastic constants described above, this equation can be stated as follows:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2(D_1 + 2D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \tag{2.26}$$

where

$$D_x = \frac{E_x' t^3}{12} \tag{2.27}$$

$$D_y = \frac{E_y' t^3}{12} \tag{2.28}$$

$$D_1 = \frac{E'' t^3}{12} \tag{2.29}$$

$$D_{xy} = \frac{G_o t^3}{12} \tag{2.30}$$

Since

$$H = D_1 + 2D_{xy} \tag{2.31}$$

then Eq 2.26 reduces to

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \tag{2.32}$$

For the particular case of isotropy, this equation collapses to the equation

$$D \left[\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right] = q \tag{2.33}$$

where

$$D = \frac{Et^3}{12 (1 - \nu^2)} \tag{2.34}$$

Since for isotropy

$$E_x' = E_y' = \frac{E}{1 - \nu^2} \tag{2.35}$$

$$E'' = \frac{\nu E}{1 - \nu^2} \tag{2.36}$$

$$G = \frac{E}{2\ (1 + \nu)} \tag{2.37}$$

therefore, it can be seen that

$$D_x = D_y = \frac{Et^3}{12\ (1 - \nu^2)} = D \tag{2.38}$$

$$H = D_1 + 2D_{xy} = \frac{t^3}{12}\ (E'' + 2G)$$

$$= \frac{t^3}{12}\left(\frac{\nu E}{1 - \nu^2} + 2\ \frac{E}{2\ (1 + \nu)}\right)$$

$$= \frac{Et^3}{12\ (1 - \nu^2)}$$

$$= D \tag{2.39}$$

These equations were derived for structurally orthotropic materials. A great deal of work today deals with geometrically orthotropic plates. The same equations are used in such cases, but an equivalent thickness is derived as appropriate to account for the variation in moment of inertia. Hoppmann and Huffington treat this problem in Ref 21.

Bending of Plates Under Combined Lateral Loads and Forces in the Middle Plane

Previous developments in this chapter have assumed that the plate is bent by lateral loads or bending moments only. It is important to evaluate the bending of plates under combined load. Timoshenko treats this subject on p 378 (Ref 55). In his developments, the effect of in-plane forces is shown to relate to the lateral load in proportion to the second derivative of the deflection with respect to the direction in which the load is applied. Timoshenko derived the equation as follows:

$$\frac{\partial^4 w}{\partial x^4} + 2\ \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$$

$$= \frac{1}{D}\left(q + N_x\ \frac{\partial^2 w}{\partial x^2} + N_y\ \frac{\partial^2 w}{\partial y^2} + 2N_{xy}\ \frac{\partial^2 w}{\partial x \partial y}\right) \tag{2.40}$$

All factors in this equation are previously defined except N_x , the axial force per unit length in the x direction; N_y , the axial force in the y direction; and N_{xy} , which equals N_{yx} , the shear forces in the plane of the plate. The units of these forces are pounds per inch of plate width. In the developments to follow, N_x and N_y will be considered as lumped axial forces P_x and P_y in pounds, where

$$P_x = h_y \cdot N_x$$

$$P_y = h_x \cdot N_y \tag{2.41}$$

in which h_x and h_y are widths of the beams over which N_y and N_x respectively are accumulated.

N_{xy} is neglected in the developments in this text but is expected to be considered in later work. These shear forces become significant in axial deformations of plates such as those in a plate girder. Current techniques do not allow the consideration of diagonal buckling.

Closed-form solutions of these problems of combined bending are available in the form of double trigonometric series. Such solutions are used to compare with the finite-element method to be developed in Chapter 7.

Pavement Slabs

Solutions of pavement slabs, or slabs-on-foundation as they are sometimes called, are of particular interest in this report. There are two basic theories concerning the behavior of such slabs. The first assumes that the intensity of the reaction of the foundation on the slab is proportional to the deflection w of the slab. This intensity is then given by the expression kw , where the constant k, expressed in pounds per square inch per inch of deflection, is called the "support modulus of the foundation." Determination of numerical values for this modulus depends largely on the properties of the foundation, but a discussion of these properties is beyond the scope of this report. Work in making such determinations, however, has been done by Terzaghi (Ref 54) and Skempton (Ref 49).

The second theory considers the foundation of the slab as a semi-infinite elastic half-space. Some authors refer to the first foundation case as the "dense liquid" assumption since k can be likened to the unit weight of a dense liquid on which the slab is floating. Thus the expression kw is the buoyancy of the liquid acting against the slab. The first theory shall be used in this work because it lends itself to consideration of nonlinear support characteristics at some later date.

Although a great deal of work has been done on the pavement slab problem, probably the most significant work to date was accomplished by Westergaard (Refs 59-62), particularly with reference to the design problems encountered in concrete pavement. This work was done in the early 1920's and relates to three special-case loadings as follows:

1. Load applied near the corner of a large rectangular slab (corner load).

2. Load applied near the edge of a slab, but at a considerable distance from any corner (edge load).

3. Load applied at the interior of a large slab at considerable distance from any edge (interior load).

In his solution of this problem, Westergaard made the following important assumptions:

1. The concrete slab acts as a homogeneous, isotropic, elastic solid in equilibrium.

2. The reactions of the subgrade are vertical only, and they are proportional to the deflections of the slab.

3. The reaction of the subgrade is equal to the modulus of support multiplied by the deflection at that point. k is assumed to be constant at every point, independent of the deflection, and to be the same at all points within the area of consideration.

4. The thickness of the slab is uniform.

5. For the cases of load at the interior and near the corner of the slab, the load is distributed uniformly over a circular area of contact. For the corner load, the circumference of this circular area is tangent to two edges of the slab.

6. The load at the edge of the slab is distributed uniformly over a semi-circular area of contact; the diameter of the semi-circle occurs at the edge of the slab.

7. The slab is infinite in extent in all directions away from the load (Fig 2.1a).

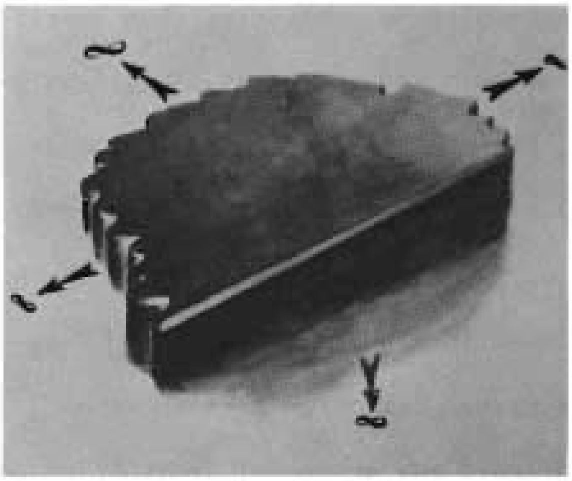
Using these assumptions, Westergaard developed solutions for the deflected shape of the pavement slab, then, the maximum expected moments and stresses. These basic solutions for "thin plate" theory were modified with corrections for stresses due to the concentration of lateral load immediately under the loaded area.

Other theoretical work was done by Gerald Pickett, et al., (Ref 41) in 1951 in studies of deflections and moments for concrete pavements. This study added additional solutions to those available for use by the practicing engineer. Perhaps more significantly, the results of these solutions were made available for practical use by incorporating them into influence charts similar to those developed by Nathan Newmark.

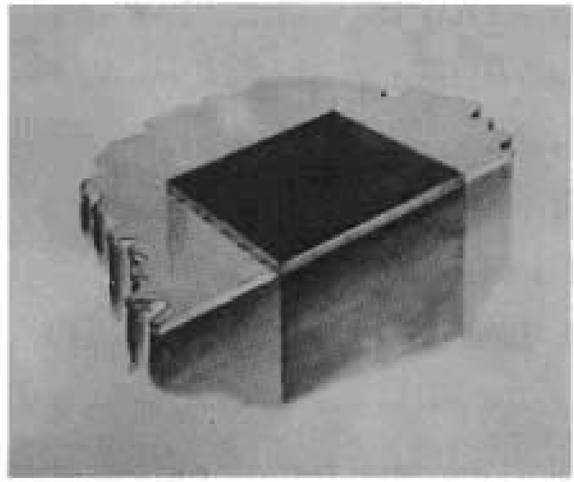
Additional contributions include work by Spangler (Ref 50), Teller and Sutherland (Ref 52), Kelley (Ref 27), and others who have, in the past twenty-five years, conducted experimental studies on pavement slabs to correlate deflections under static load with those predicted by theory. The evaluation of such work is beyond the scope of this paper, but it will be discussed briefly under the heading of needed research.

Non-Uniform Conditions

Unfortunately for the designer, most pavement slabs do not meet the stringent assumptions imposed by Westergaard. First, the slabs must in reality be finite (Fig 2.1b). Second, uniform support is hard to obtain since local loss



(a)



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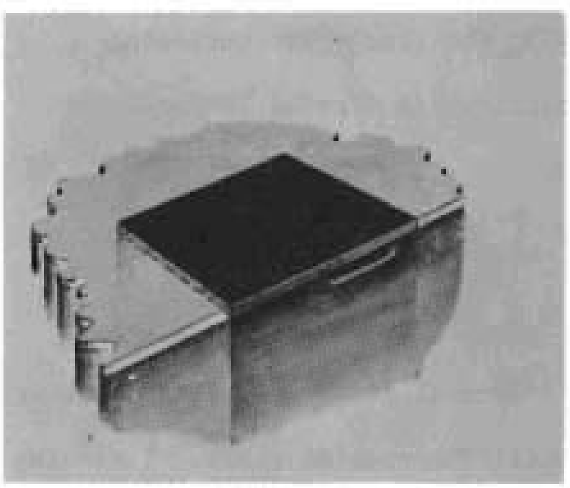

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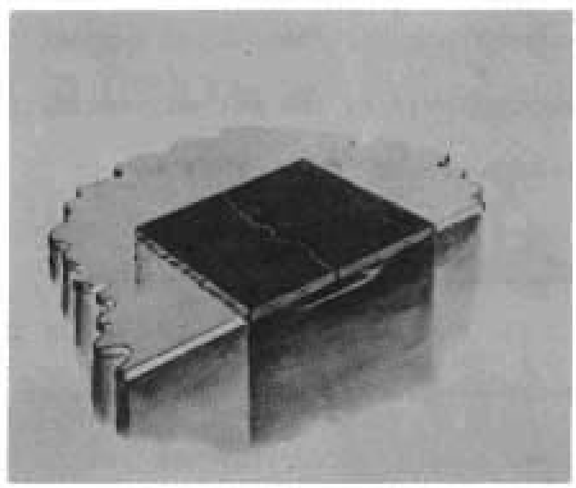


(d)

Fig 2.1. Comparison of real and infinite pavement slabs.

of support under the pavement due to pumping or settlement of the foundation is common (Fig 2.1c). Richart and Zia (Ref 42) have treated this problem by applying a general method developed by Brotchie (Ref 8). Their solution relates specifically to a large slab-on-foundation spanning a circular void. They provide the designer with several curves useful in evaluating this specific condition. They do not treat, however, the more general cases of (1) smaller slabs spanning a void of irregular shape, (2) the problem of random placement of a void near a corner or edge, or (3) in the more general case, several voids under a single slab.

Leonards and Harr (Ref 31) also treat nonuniform subgrade support. They evaluate the effect of curling on a circular slab. The circular slab is not used in pavement design, but the methods developed may be useful in treating differential temperature effects in the future.

In the methods of this paper, the foundation is represented by the modulus of support k . This approach provides the basis for future consideration of nonlinear elastic foundation support (Ref 34). The freely discontinuous inputs allowed by the method provide the capability of varying k anywhere under the slab.

Cracks and Other Discontinuities

The theories described thus far relate to homogeneous materials. No provision has been made for cracks or other discontinuities (Fig 2.1d). Other authors have treated this subject in some detail for special cases. These include Ang (Ref 4, 5), Williams (Ref 64), Reissner (Ref 44), and Knowles and Wang (Ref 29). Many authors including those listed above discuss "Reissner bending of plates." This phrase refers to the equations for bending of elastic plates developed by Eric Reissner of MIT. Classical theory, that discussed at the beginning of this chapter, meets the so-called Kirchhoff boundary conditions at free edges, these being a vanishing bending couple and a vanishing sum of transverse force and edgewise rate-of-change of twisting couple at all free plate edges. These two conditions are actually a compression of three independent conditions: (1) vanishing transverse force, (2) vanishing bending couple, and (3) vanishing twisting couple at free edges. "Reissner bending" includes differential equations fulfilling these three boundary conditions.

Reissner's studies further show that stresses near a finite crack in infinite plates are somewhat greater than those calculated by classical theory. This is accentuated near the base of the crack where extremely high stress concentrations might be expected. Such cracks are beyond the scope or application of this work and will not be treated.

The discontinuities of interest are those which occur across the entire slab cross-section at any particular location. Ang, Williams, and Reissner indicate that stress distribution can be predicted reasonably outside a distance half of the plate thickness from the edge of a crack. This is acceptable for the application of the method discussed herein, since this distance also approximates a half increment length in the finite analogy. Furthermore, such accuracy is quite adequate for structural plates and pavement slabs. Corrections to this theory can be obtained from "thick plate" theory and introduced into any solution where needed (Ref 61).

One qualification of the method to be developed herein should be noted. Cracks will either be treated as hinged discontinuities with no finite width, or as holes in the structure with finite width. It is readily recognized that cracks which are not hinged can in effect be treated as two separate plates, one on either side of the crack, and thus, two separate analyses can be conducted.

Summary of Elastic Theory

The theory described in this chapter is essential to the development of any method for analyzing plate bending. Closed-form solutions of the problems, however, become more difficult as complexities increase. Hand solutions of isotropic plates are readily accomplished, but for solutions of homogeneous orthotropic plates one must usually resort to computers. The addition of elastic support or finite cracks forces the use of approximate methods and limiting assumptions. Furthermore, each solution represents a special case, and a multitude of special-case solutions are required for the problems of interest. A more general, more rapid method would be of great advantage to the engineer. It would also be helpful if these solutions could be accomplished without resorting to higher order functions such as Bessel and Hankel functions. Such a general theory is the object of the research described herein.

CHAPTER 3. FINITE-ELEMENT THEORY

The theories discussed in the preceding chapter are based on infinitesimal calculus (that is, a body is divided into an infinite number of differential elements and any changes occurring in the body can be described by integrating or differentiating their effects as they gradually change over a very large number of small elements). There are many rules governing the use of such calculus. In general, the functions must be continuous, and fourth order systems must have two continuous derivatives. Many complex engineering problems do not properly fulfill these conditions and can not, therefore, be solved by resorting to the calculus. Furthermore, many such classical or analytical methods may not be well adapted for use on high speed digital computers. As a consequence, approximate, or so-called "numerical," methods have been developed. Hardy Cross (Ref 15, p 1) pioneered the use of such methods in civil engineering with moment distribution methods. Newmark (Ref 15, p 138) and Southwell (Ref 15, p 66) have also been instrumental in these developments. In such numerical methods, the differential equation concerned is replaced by its finite difference equivalent. The problem then reduces to solving a large number of simultaneous algebraic equations instead of one complex differential equation.

The method described herein is slightly different and involves breaking a plate or slab into a system of finite elements, each consisting of rigid bars connected by elastic blocks. The algebraic equations describing the system are derived by free-body analysis of the finite model.

Assumptions

It is impossible to develop a completely general theory describing the behavior of any structure. It is often difficult to find solutions for the mathematical equations describing even limited theories; therefore, additional conditions and assumptions are often imposed to permit solution. While many of these assumptions are known, it seems worthwhile to restate the assumptions and conditions relative to the finite-element model describing slab behavior.

(1) Planes of the plate lying initially normal to the middle surface of the plate remain normal to the middle surface of the plate after bending.

(2) Normal stresses in the direction perpendicular to the plate surface can be disregarded for the bending solution.

(3) There is no axial deformation in the middle plane of the plate and, thus, this plane remains "neutral" during bending.

(4) All deformations are small with regard to dimensions of the plate.

(5) The bar elements of the model are infinitely stiff and weightless.

(6) Each joint in the model is composed of an elastic homogenous and orthotropic material which can be described by four independent elastic constants.

(7) Loads, masses, and bending strains occur at the joint.

(8) Torsional stiffness of the plate element can be invested in torsion bars. (See Appendix A).

(9) The neutral axis lies in the same plane for all elements even for nonuniform cross sections.[1]

(10) The spacing of the beam elements, designated by h_x and h_y , need not be equal but must be constant for all parallel beams.

(11) The number of increments into which each beam is divided is equal to the length of the beam divided by the increment length.

The Physical Model

Numerical methods are most often used as mathematical approximations of a governing differential equation by the s bstitution of finite-difference

1 Violation of this assumption, as stated by Ang and Newmark (Ref 5), has been shown to cause little error.

forms for derivatives, or by the approximation of a continuum problem with a discrete nodal system. A second and perhaps preferable method is to model the plate or slab physically by a system of finite elements whose behavior can properly be described with algebraic equations. Newmark (Ref 15) pioneered such models for articulated beams and plates. He states

> "the use of the model (finite-element model) offers certain advantages; there is no ambiguity concerning the boundary conditions; statical checks on the results have a physical meaning and can be made more accurately; variations in dimensions and physical properties can be more easily treated."

For many problems, the finite-difference equations developed by direct substitution for the differential equation and the finite-element model equations developed from a free-body analysis of the model are equivalent. This, however, is not always the case. The physical model seems preferable because it facilitates visualization of the problem and formulation of proper boundary and loading conditions. It is useful, however, to use difference equations to describe the bending moments in the finite-element beams.

Model of a Beam-Column

The basic element in the plate model developed here is the model of a beam subjected to transverse and axial loads (termed a beam-column and developed by Matlock, et al.)(Refs 35, 36). Figure 3.1 shows the development of this model. Figure 3.1a illustrates a beam element deformed by the action of pure bending and subjected to the assumptions of conventional beam theory. For linearly-elastic stress and strain, the stresses acting on the beam element are shown in Fig 3.1b. If these distributed stresses are to be replaced by concentrated forces as shown in Fig 3.1c, as they often are for design purposes, it seems reasonable to develop the mechanical model, Fig 3.1d. Here the deformed beam element is replaced by a pair of hinged plates with linear springs containing the elastic flexural stiffness of the beam restraining movement of the plates, top and bottom. Thus, a beam could be represented by a series of such beam

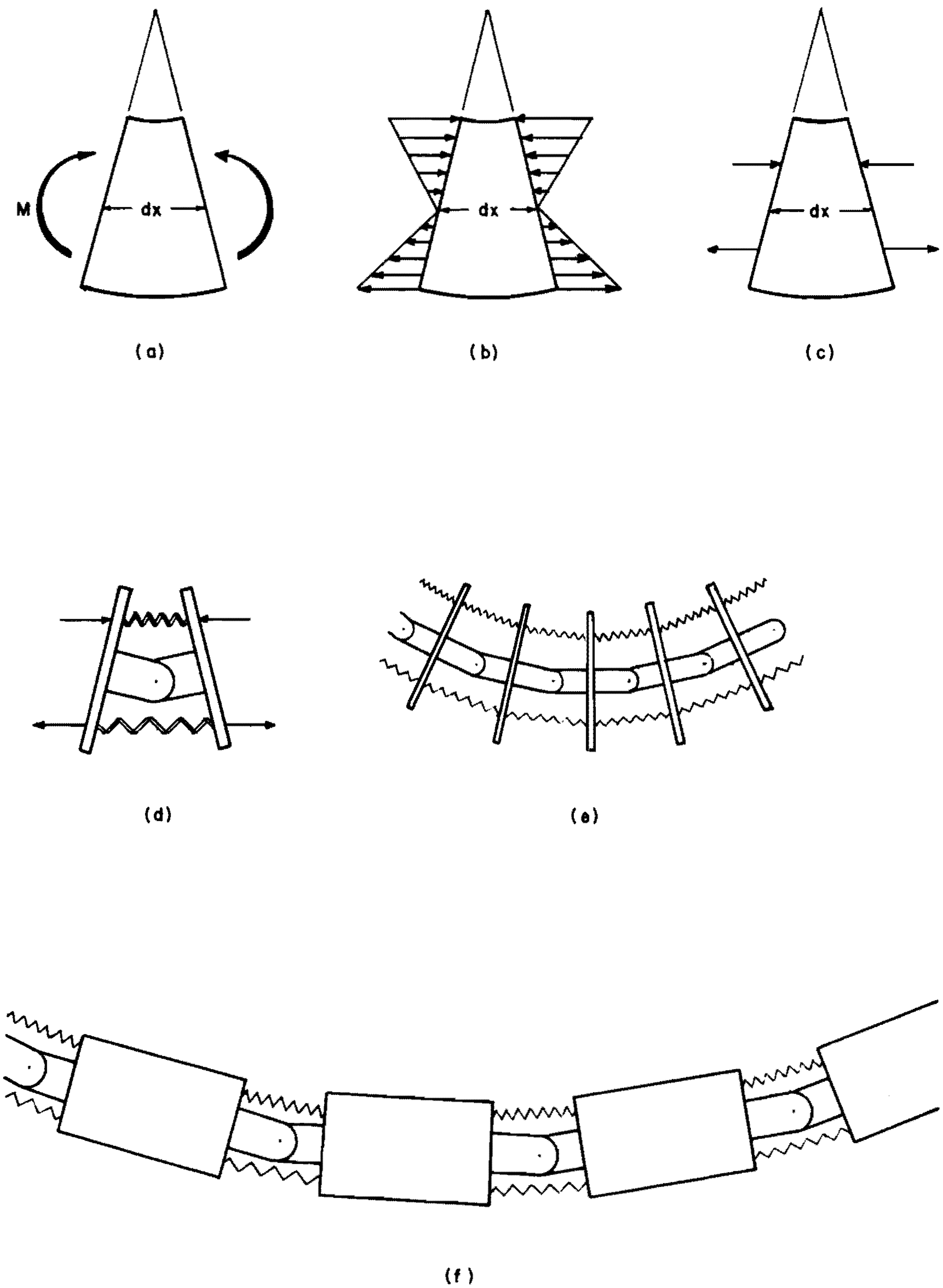


Fig 3.1. Finite mechanical representation of a conventional beam.

element models (Fig 3.1e).

If the thickness of the plates between hinged joints is increased, a cruder representation results (Fig 3.1f). It has been shown, however, that representation of real beams by models containing as few as six elements or increments (as they will be called hereafter) gives satisfactory approximation of real beams. As a specific example of modeling, Fig 3.2 indicates a beam-on-foundation subjected to both lateral and axial loads. Supports may be linearly-elastic, non-linear, or fixed. Figure 3.3 shows these loads and supports depicted in the finite-element model. Many other loads and load combinations are possible. These include distributed or concentrated, transverse loads, transverse couples, axial loads, and bending moments. Elastic restraints are included as linear or non-linear supports, or, as distributed or concentrated rotational restraints. In short, almost any physical combination of loads or restraints can be applied to a beam-column with this method.

Simple Two-Dimensional Systems

If one or more of these beams in each horizontal orthogonal direction are combined, they form a grid-beam system similar to the girder and stiffener system of a bridge deck or similar to the beam system of a waffle floor (Fig 3.4). Tucker and Matlock (Ref 56) extended the use of the beam-column model to such systems. Each of the beams in this grid-beam system can be solved by the beam-column method as a line member. However, the effect of one beam on the next beam is important if the beams act as a monolithic system.

Such systems account for pure bending only. No torsion or Poisson's ratio effect is considered. In a true grid-beam system, these effects are small and do not affect the solution significantly.

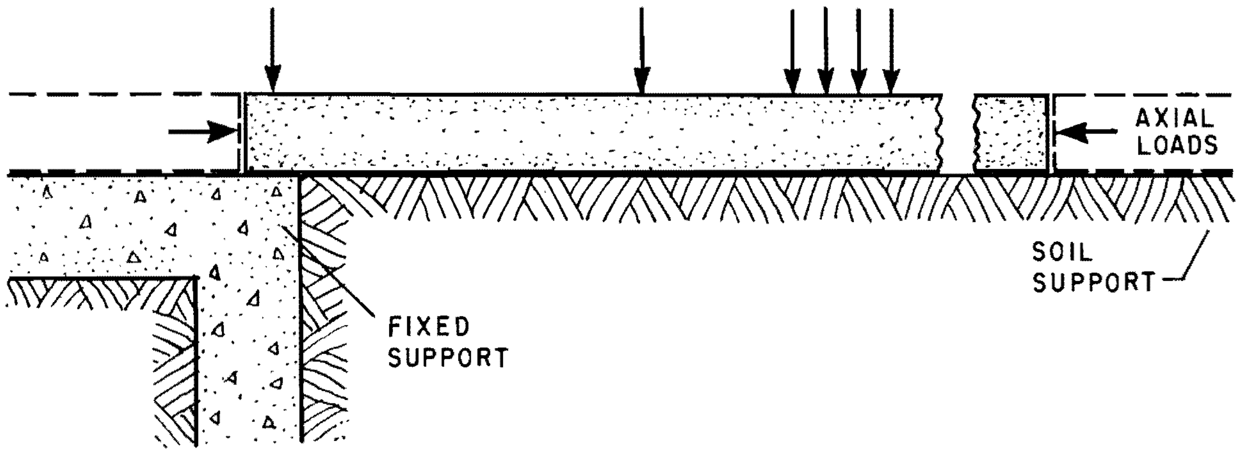


Fig 3.2. Example beam on foundation subjected to lateral and axial loads.

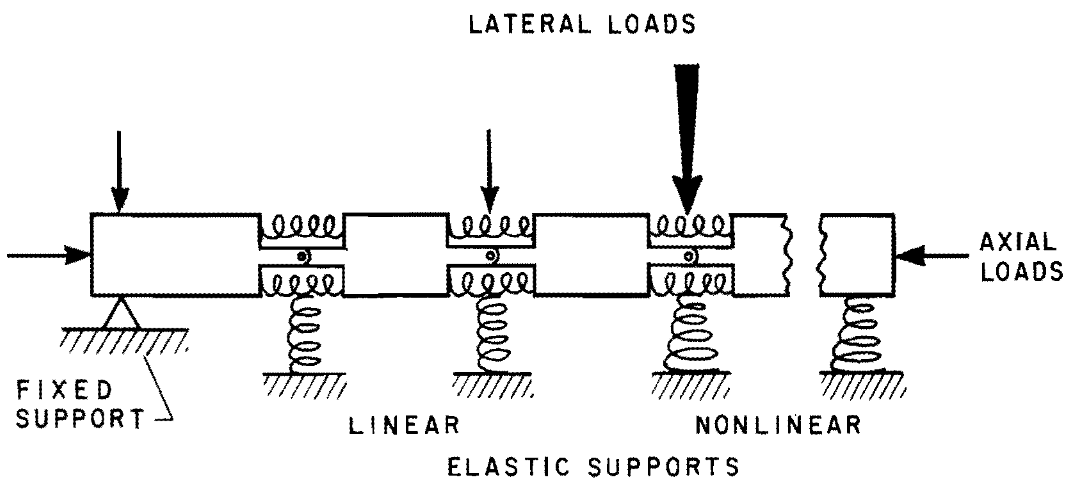


Fig 3.3. Finite-element model of Fig 3.2.

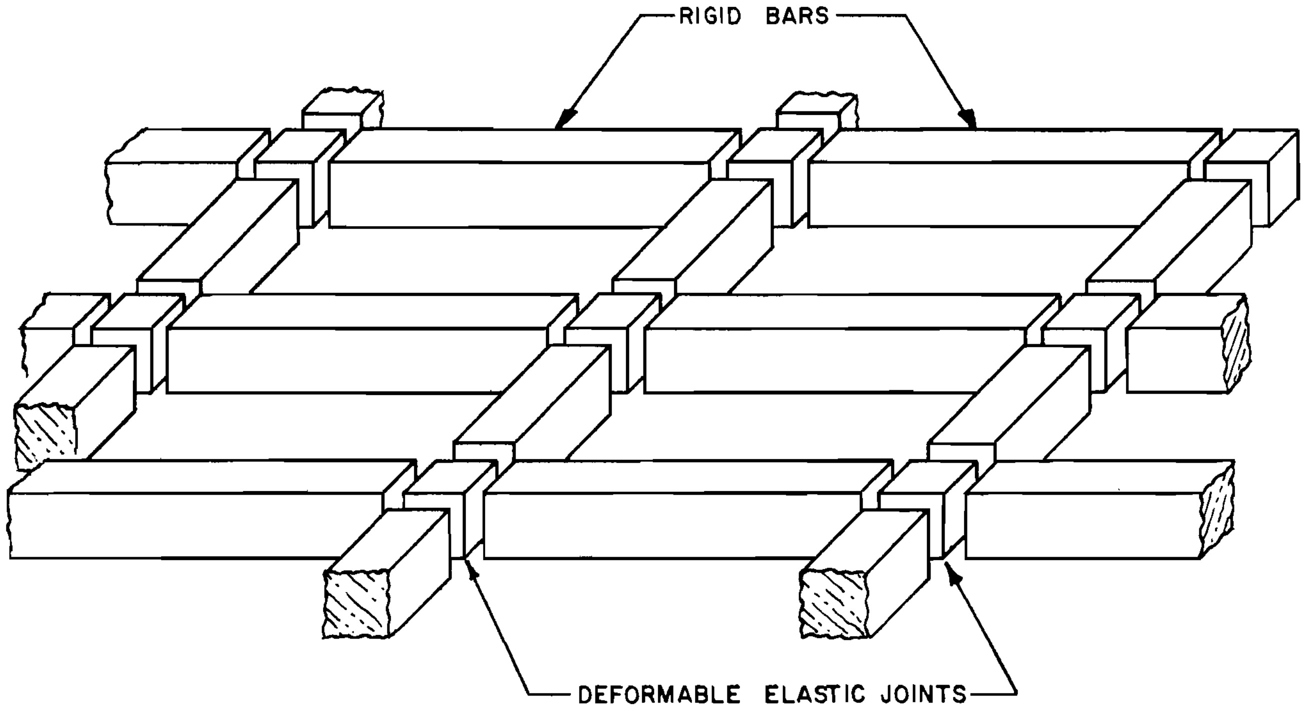


Fig 3.4. Finite-element model of grid-beam system.

Plates and Slabs

For the plate solution, however, the effects of torsion in particular are of significant importance, and the Poisson's ratio effects are more important than for grid-beam problems. Tucker has worked on this problem (Ref 57) as have Ang and Newmark (Ref 5). The next step was to determine some method for including these two factors in the model.

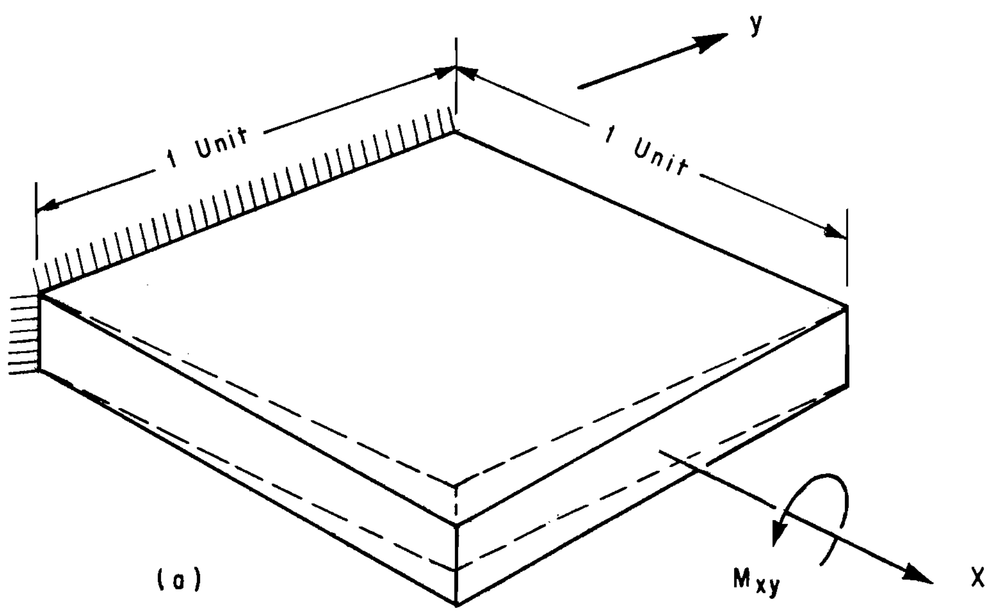
First, consider torsion in Fig 3.5. If a unit element is removed from the slab, a twisting moment M_{xy} can be applied about the x-axis. The torsional stiffness C of the slab is defined as the applied twisting moment divided by the resulting angular rotation, φ, across the element. Then

$$C = \frac{M_{xy}}{\varphi} \tag{3.1}$$

Considering this element as two beams connected by a torsion bar, the bar modulus can be chosen equal to C so that an applied twisting moment will produce the same relative angle change φ as in the real unit element of the plate. Using this technique, torsion bars can be inserted between the adjacent bar elements of all the beams in the y direction of the grid-beam system as shown in Fig 3.6. These torsion bars are also inserted, of course, between the beams in the orthogonal x direction as shown in Fig 3.8 . It is convenient to think of one set of torsion bars acting with each set of beams since the solution proceeds in this manner. It should be emphasized here that these torsion bars represent the real torsional stiffness of the slab and are always active in the system.

The effect of Poisson's ratio is easier to handle than torsion. Remember that the bending stiffness EI of a beam-column is vested in linear springs restraining the movement of the finite-elements at each joint. The analogous bending stiffness of a plate

$$D = \frac{E t^3}{12 (1 - \nu^2)} \tag{3.2}$$



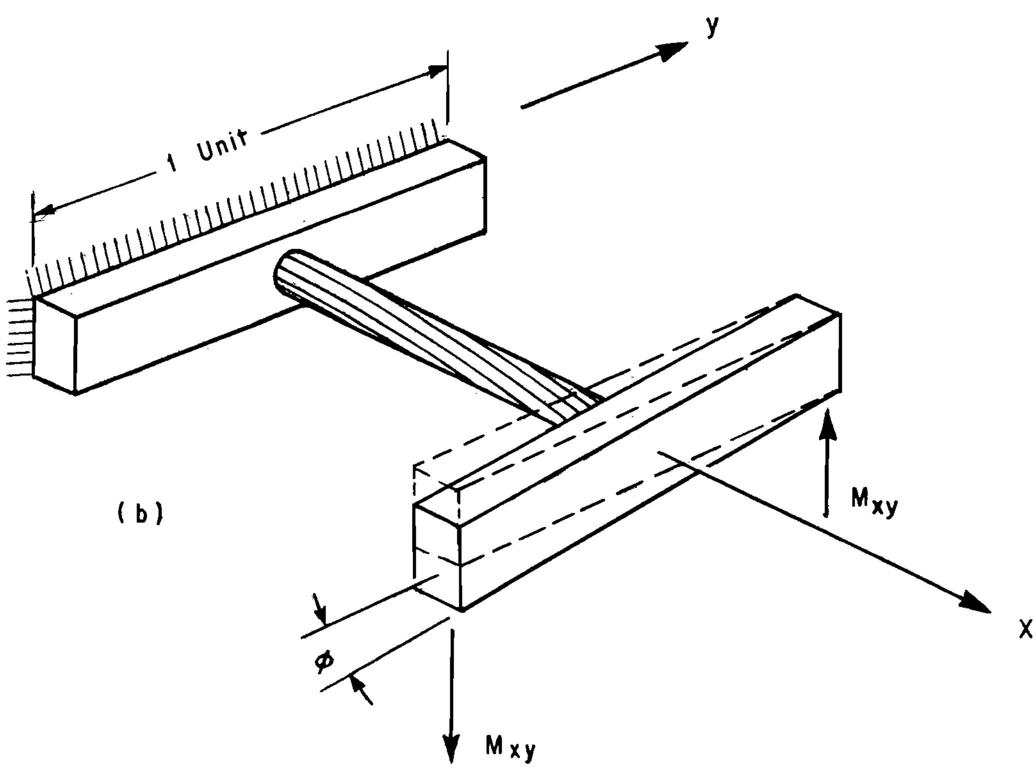


Fig 3.5. Finite-element representation of torsional stiffness.

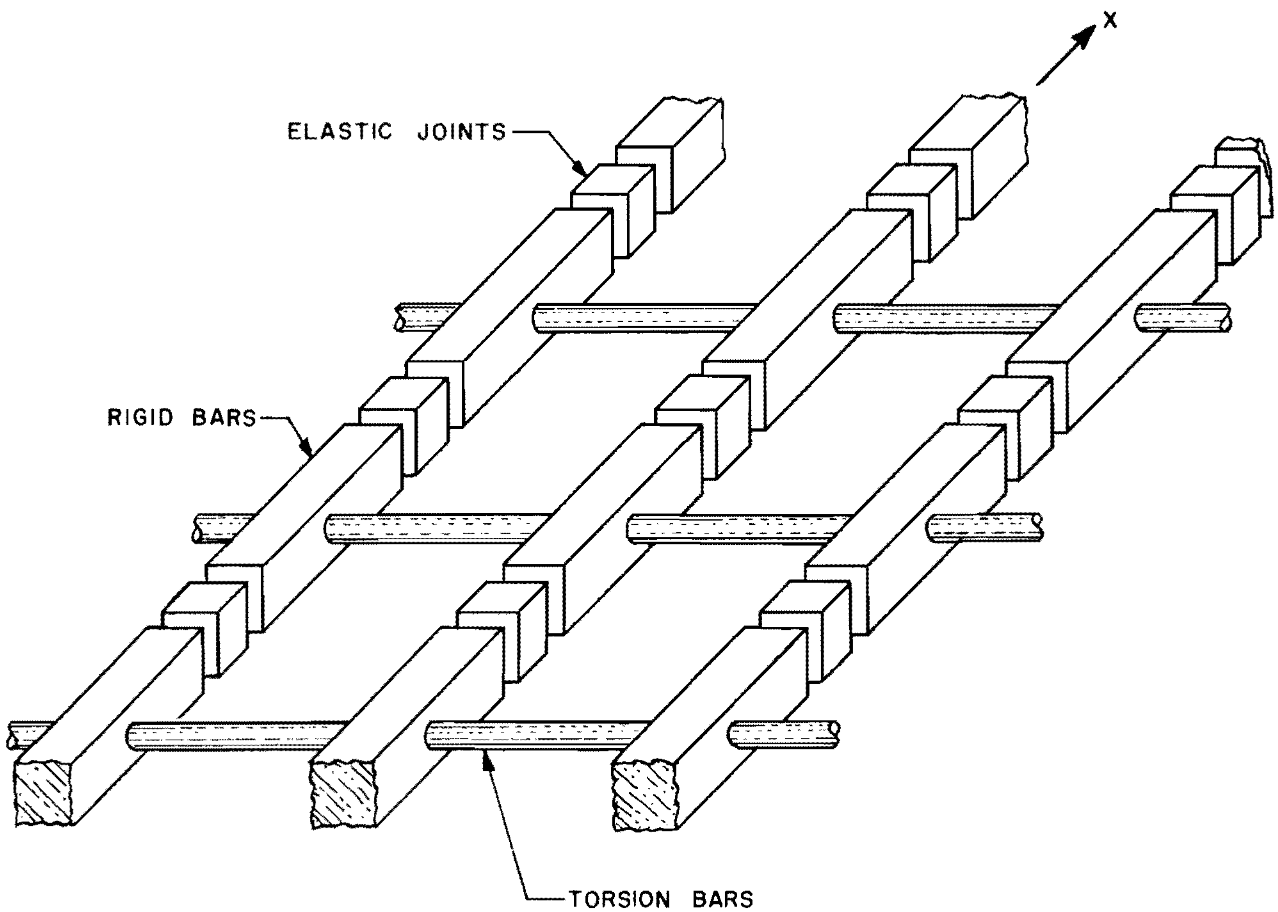


Fig 3.6. Finite-element x-beam system with torsion bars acting between segments.

replaces the EI of the beam and must also be concentrated. A pair of linear springs, however, is not satisfactory for this purpose since they can transfer no Poisson's ratio load. These stiffness springs are, therefore, replaced in the plate model by elastic blocks whose stress-strain relationship is equivalent to that of the real plate and which have Poisson's ratio equal that of the plate. Figure 3.7 illustrates the action of these elastic blocks. The blocks in Fig 3.7b replace the springs in Fig 3.7a. If the beams in the x direction are bent up, the beams in the orthogonal y direction bend down due to Poisson's ratio (unless they are restrained). The force required to restrain them results in an additional bending moment which equals

$$\Delta M_y = \nu D \left(\frac{\partial^2 w}{\partial x^2}\right) \tag{3.3}$$

This is likewise true for the action of the y-beams on the x-beams. As a result, the bending moment in an x-beam becomes

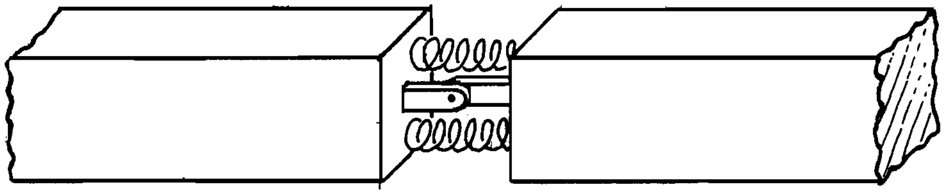
$$M_x = D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \tag{3.4}$$

These developments are fully discussed in Appendix B.

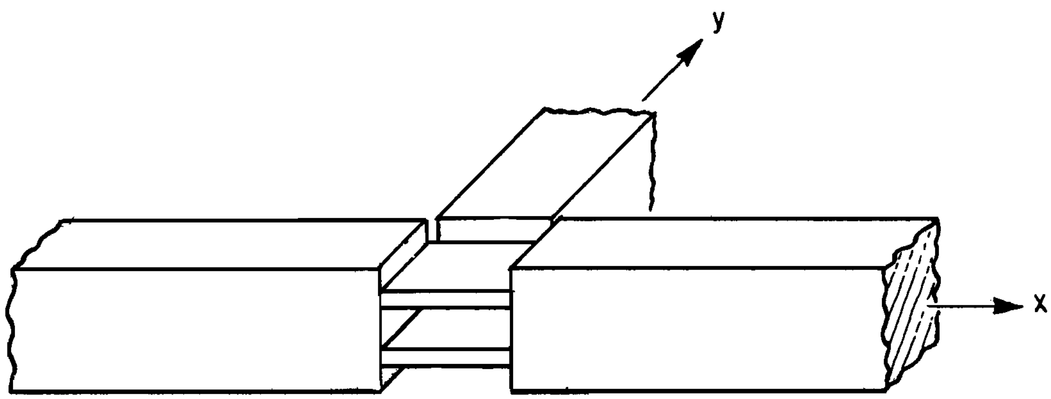
Figure 3.8 shows the assembled slab model. The torsion bars in Fig 3.8 are considered to resist only torsion.

Input Values for Model

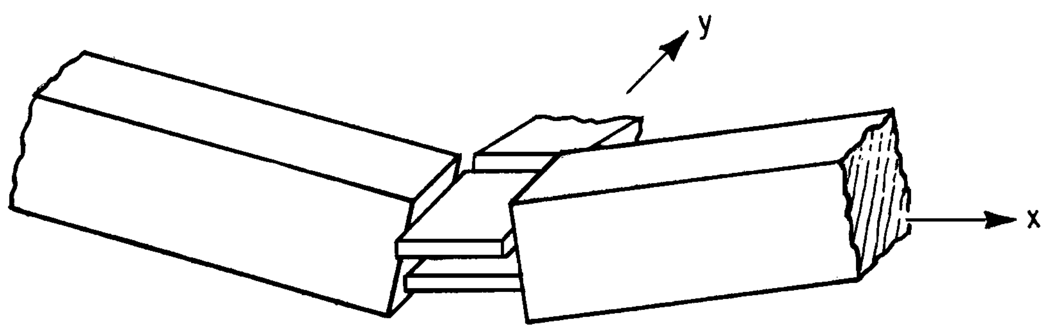
Having developed a model, it is necessary to relate it to a real plate or slab. Figure 3.9 shows the plan view of a segment from a general plate. The plate has been divided into increments in the x and y directions with increment length h_x and h_y respectively. These "beam" increments are designated with i in the x direction and j in the y direction. The mesh point or joint on the positive end of each increment is arbitrarily numbered the same as that increment. This numbering system then gives the i,j grid indicated in this plate segment (Fig 3.9). Figure 3.10 extends the system to a full



(a) Typical joint from the beam-column model.



(b) Typical joint from the plate model (partial cutaway).



(c) Deformed joint from the plate model.

Fig 3.7. Action of Poisson's ratio at a finite joint.

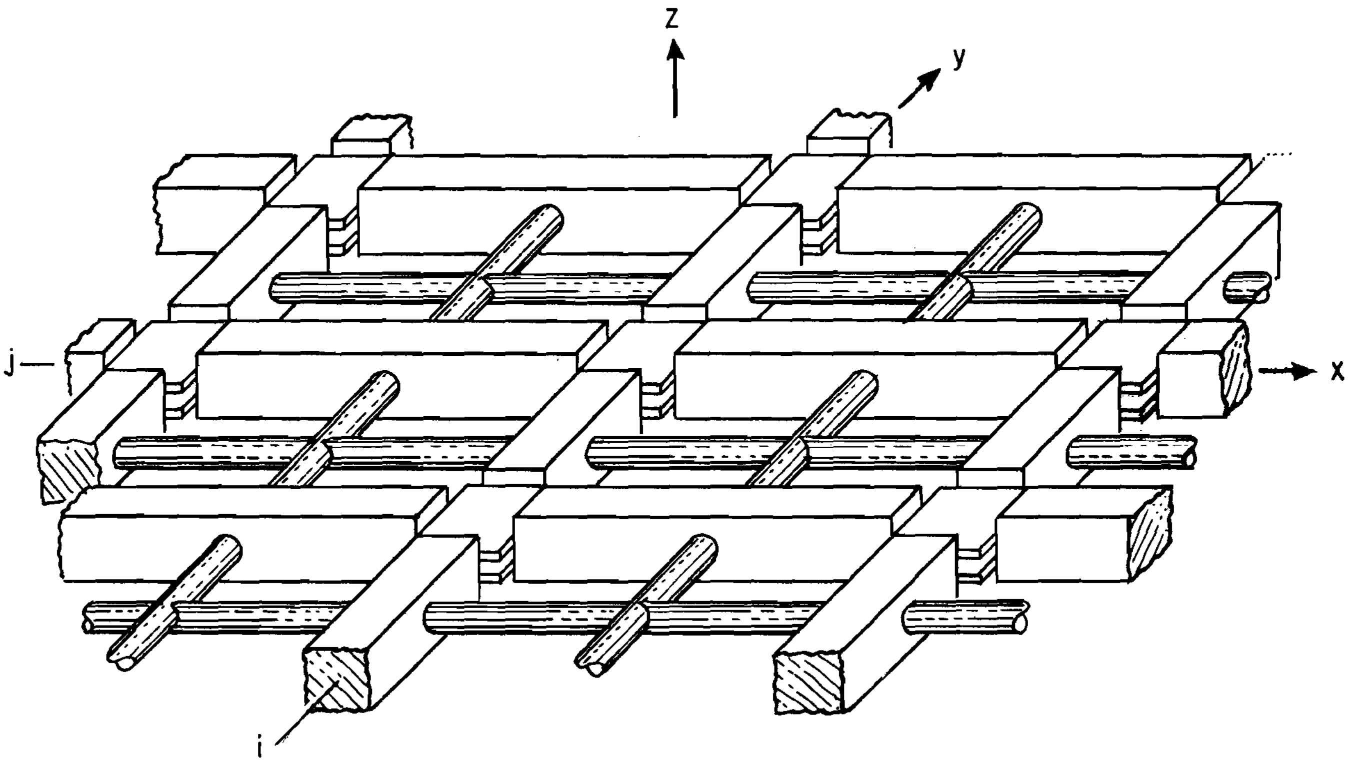


Fig. 3.8. Finite-element model of a plate or slab.

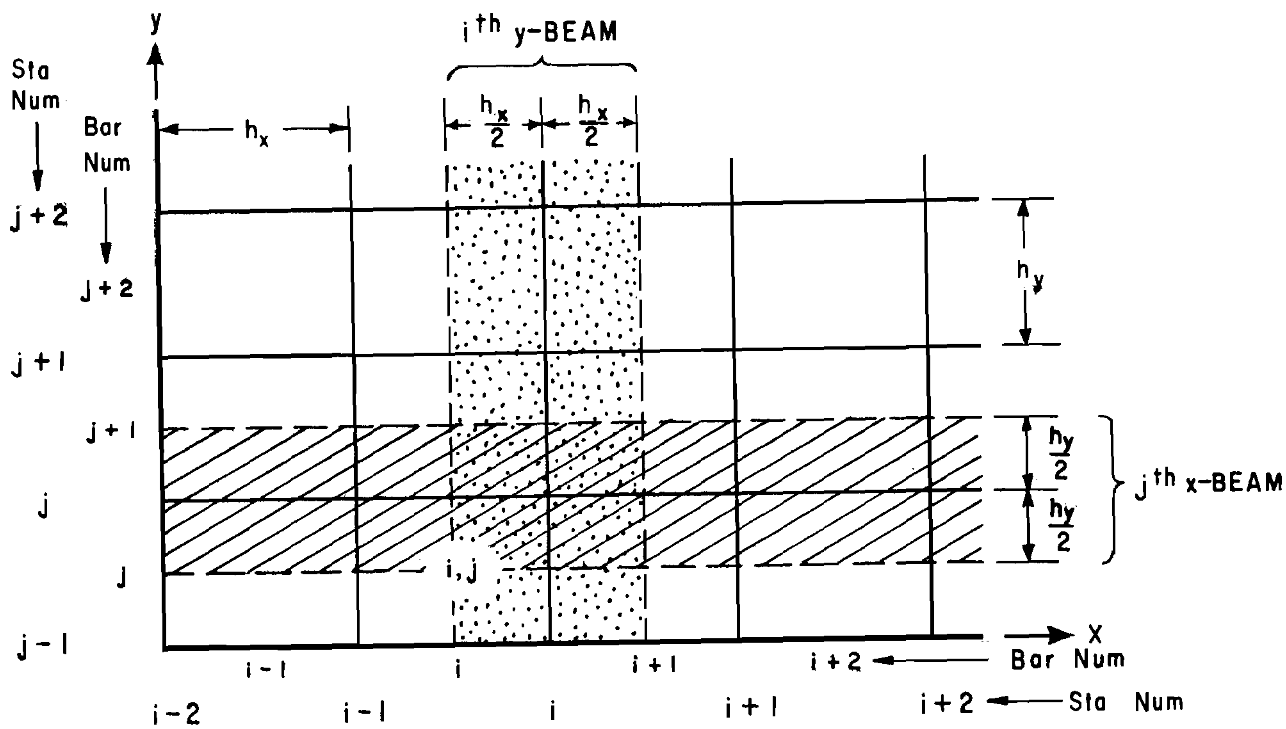


Fig 3.9. Plan view of plate segment divided into x and y beams.

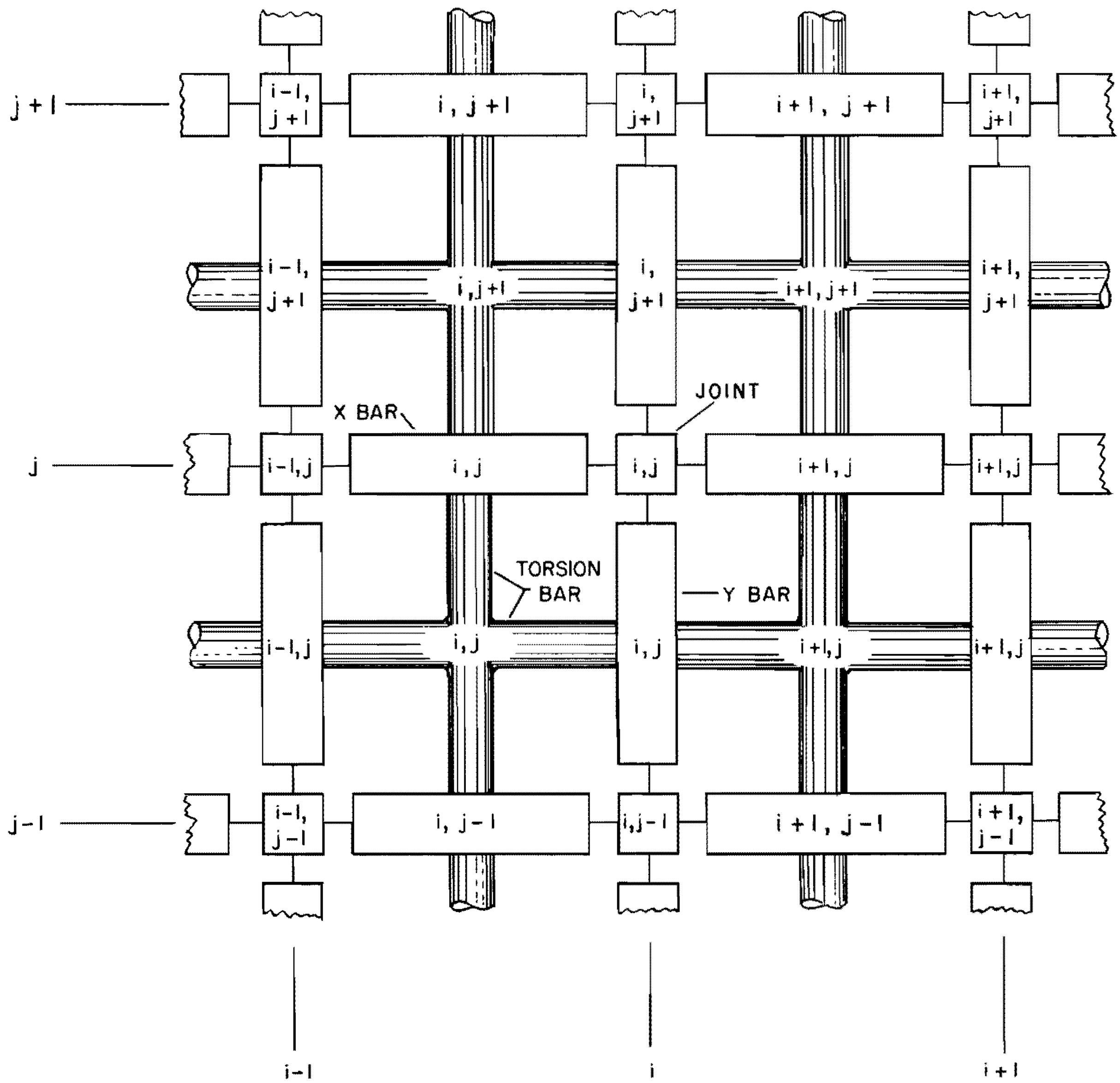


Fig 3.10. Plan view of the slab model showing all parts with generalized numbering system.

grid and designates torsion bar and increment numbers. It is also convenient to denote segments of the plate bounded by increments in both the x and y directions, because these segments correspond to the torsion bars in Fig 3.10.

Stiffness and Lateral Load - To describe the real plate with the model, it is appropriate to look at the j^{th} x-beam. The portion of the plate which is assigned to the j^{th} beam is shown cross-hatched in Fig 3.9. It is one increment wide, centered on the j^{th} line. All stiffnesses and loads lying in this cross-hatched area are considered to act on the j^{th} beam. Figure 3.11 shows a side view of this beam which may be irregular in profile and may be loaded by a varying distributed load q . One increment width of the load, centered on the i^{th} mesh point, is assigned to Station i on the j^{th} x-beam (Station i,j on the model). $Q_{i,j}$ is the lumped load applied at Station i,j. $D_{i,j}$ represents the bending stiffness of the plate segment of which mesh point i,j is the center. The sketch is intended to illustrate that the stiffness may vary. In this case, it decreases from Station i-1 toward Station i+1. The load also varies but increases from Station i-1 toward Station i+1. $Q_{i,j}$ can be expressed by the equation

$$Q_{i,j} = \sum_{i-\frac{1}{2}}^{i+\frac{1}{2}} \sum_{j-\frac{1}{2}}^{j+\frac{1}{2}} q + Q_c \qquad (3.5)$$

where Q_c is any concentrated load which may be present at that station in addition to the distributed load.

The stiffness $D_{i,j}$ for a plate is a unit value per inch of width. It is convenient for use in computations to input average values over a full increment width. If $D^x_{i,j}$ represents the average stiffness in the x direction, it can be calculated as follows:

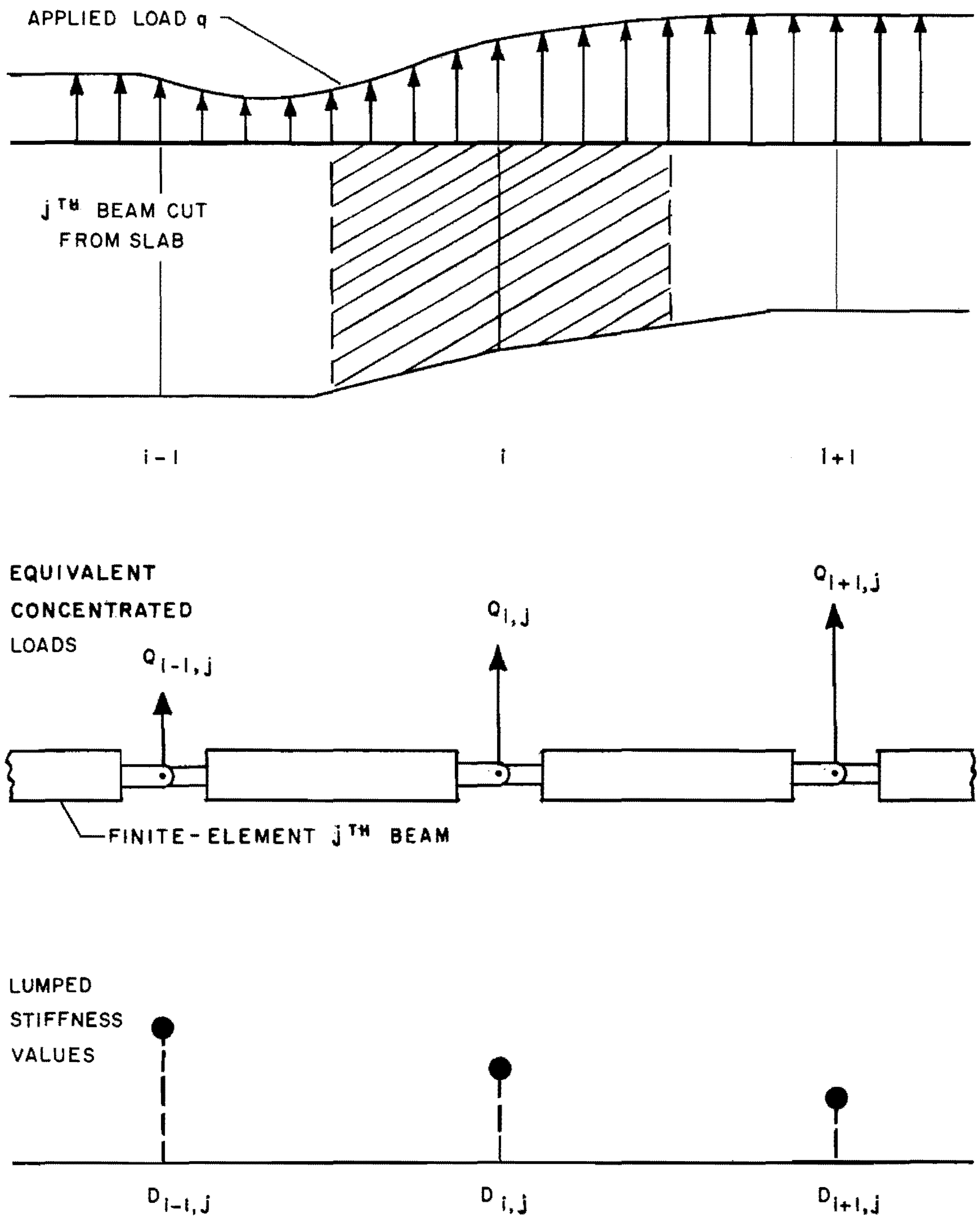


Fig 3.11. Finite-element representation of a beam cut from slab with finite-element loads and stiffness.

$$D^{x}_{i,j} = \int_{i-\frac{1}{2}}^{i+\frac{1}{2}} \int_{j-\frac{1}{2}}^{j+\frac{1}{2}} \frac{D^{x}_{i,j}}{h_x h_y} \tag{3.6}$$

that is, the average bending stiffness of the plate over an area one increment wide and one increment long, centered at Station i,j. Full developments of all input for the model are provided in Appendix B.

Other Input Values - It is convenient to represent the torsional stiffness of plate segment i,j as torsion bars i,j acting at the midpoint of the model elements (Fig 3.10). It is also helpful for external couples or torques applied to the plate to be input into the stiff beam elements. This is properly shown on the free-body in Chapter 4. Axial loads, P , are also input into the bars with the changes, ΔP , considered to occur at mesh points.

Summary

A physical model has been chosen to represent the plate or slab for solution by numerical methods in preference to expressing the differential equation governing slab behavior in finite-difference form. The model is straightforward and assists visualization of the problem. Discontinuities and freely discontinuous changes in load, bending stiffness, torsional stiffness, and other parameters are easily understood with the use of a physical model, but limitations on continuity of the differential equation make direct difference approximations suspect.

Errors - Errors resulting from this method of solution (that is, any differences between the so-called closed-form solutions and solutions by this technique) are due to approximating the real slab with the model. The algebraic solution is exact for the model within computer accuracy. In finite-difference techniques, the errors result from the finite-difference approximation of the differential equation.

Increment Length - The greater the number of increments used to model a particular problem, the greater the accuracy of the solution. All exact solutions are based on infinitesimal changes in the real structure. Experience with this model indicates that reasonable results can be obtained with most problems using 8 to 20 increments in each direction, although the number of increments to be used will certainly depend on the dimensions of the problem as well as the accuracy required and the local complexity to be resolved. This will be discussed in Chapter 7.

CHAPTER 4. FORMULATION OF EQUATIONS

The purpose of this chapter is to formulate from a free-body analysis the equations necessary to solve for the bending of a slab. It is intended here to give a readable and concise account of these developments rather than a complete mathematical treatment. A complete step-by-step development of the equations is included in Appendix B.

Free-Body Analysis

In order to derive the equations for solution of the bending of a plate or slab, it is helpful to refer to a free-body of the model. Consider first a section of the assembled slab model centered at any mesh point i,j (Fig 4.1). For the present, the x-bar to the left of point i,j is called Bar a, and the x-bar to the right of point i,j is called Bar b.

Figure 4.2 shows these same bars as a free-body with other members of the model fixed and replaced by a system of equivalent forces. $Q^y_{i,j}$ represents the load carried by the y-beam at this intersection and the term $\partial^2 w/\partial y^2$ represents the restraint of the y-beam which provides the Poisson's ratio effect in the x-beam moment. The term $S_f\,(w^x_{i,j} - w^y_{i,j})$ represents the load stored in the fictitious spring closure parameter. These closure springs will be discussed fully in the next two chapters. Figure 4.3 shows the external forces which can be applied to these same two bars. Any of these forces may be zero but are considered to be present for generality. Combining the system of equivalent forces and external loads gives the general free-body of the slab model in Fig 4.4. This free-body is for a section of an x-beam. A similar free-body can be developed for the y-beam by changing all x's for y's, and all y's for x's. For clarity the symbols on this free-body are redefined as follows:

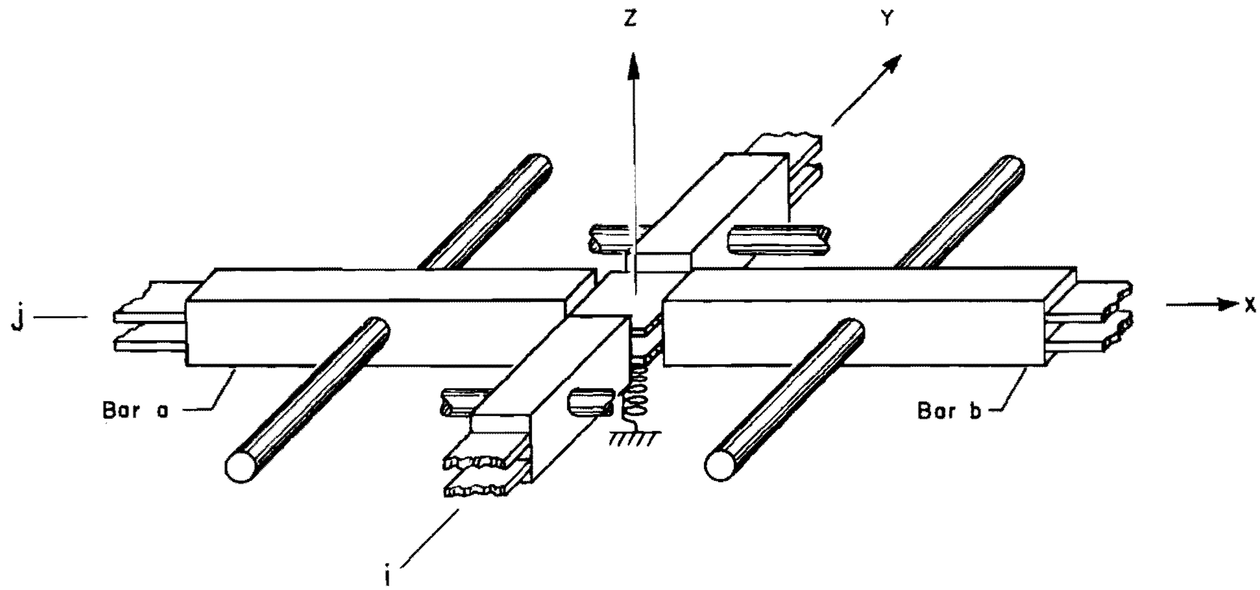


Fig 4.1. Typical joint i,j taken from finite-element slab model.

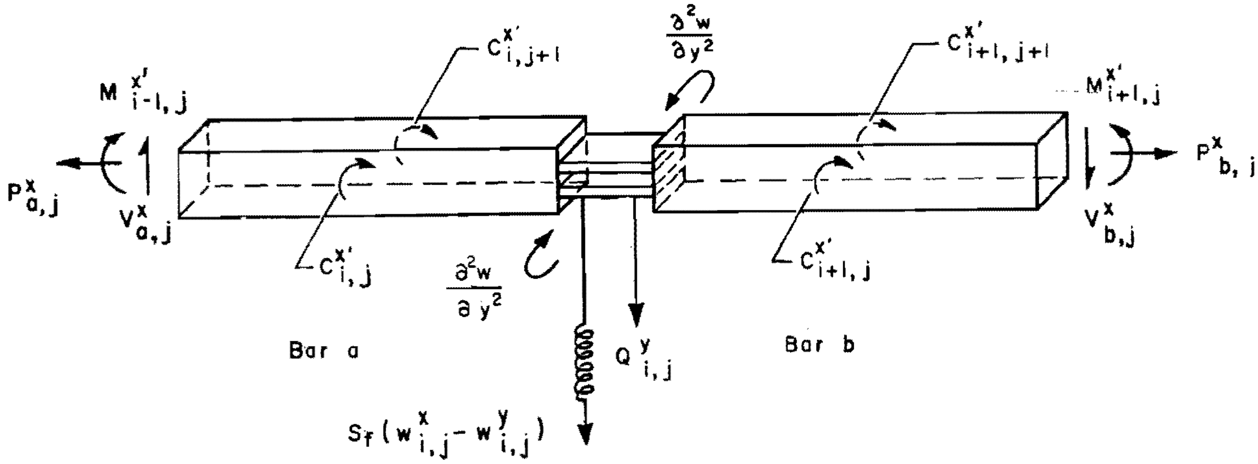


Fig 4.2. Free-body of joint i,j with other members of the model replaced by an equivalent force system.

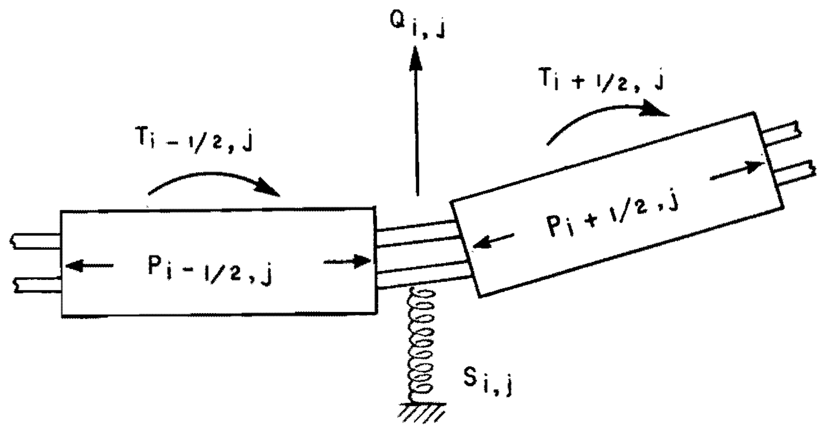


Fig 4.3. Typical joint i,j with force and restraint inputs shown.

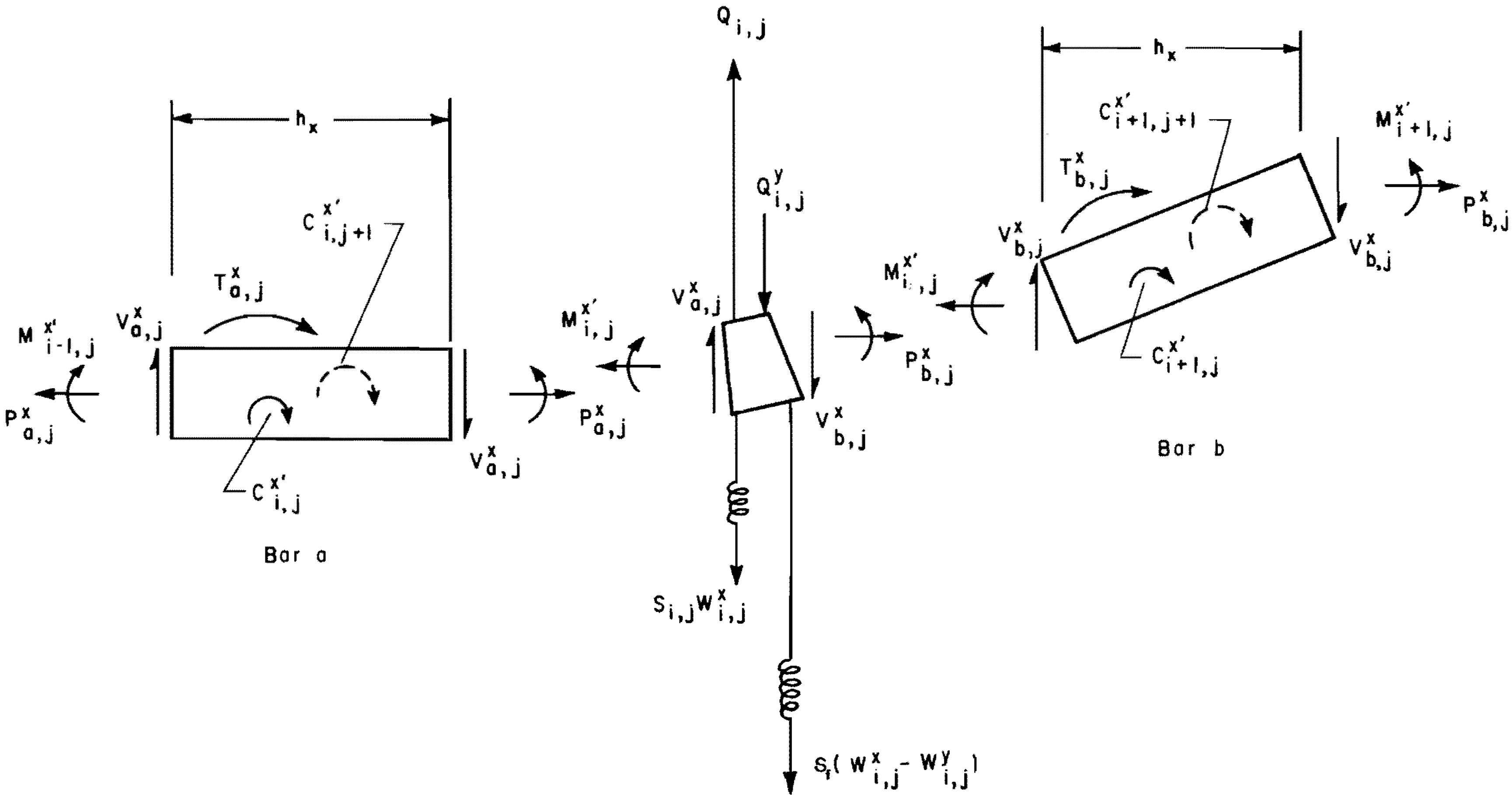


Fig 4.4. Generalized free-body of joint i,j with all forces and restraints shown.

a,b	-	Temporary bar numbering used in derivations to avoid confusion.	(none)
$C'_{a,j}$	-	The torque applied to the a^{th} bar of the j^{th} x-beam by the twist in slab segment a,j .	(inch-lb)
$C_{a,j}$	-	Unit torsional stiffness of slab element a,j .	$\left(\frac{\text{inch-lb}}{\text{rad}}\right)$
$D^x_{i,j}$	-	Average bending stiffness of an orthotropic plate in the x direction at station i,j .	in-lb
$D^y_{i,j}$	-	Average bending stiffness of an orthotropic plate in the y direction at station i,j .	in-lb
i	-	An integer used to number mesh points, Stations and Bars along the x-beams in the x direction.	
h_x	-	The increment length along the x-beams.	(inch)
h_y	-	The increment length along the y-beams.	(inch)
j	-	An integer used to index parts of the model and stationing in the y direction.	
$M^{x\prime}_{i,j}$	-	The bending moment in the x-beam at Station i,j (equals $h_y M^x_{i,j}$).	$\left(\frac{\text{inch-lb}}{\text{inch}}\right)$
$M^x_{i,j}$	-	The unit bending moment in the slab in the x direction at Station i,j .	(lb)
$P^x_{i,j}$	-	The axial load in the x-beam in Bar i,j (equals $h_y p^x_{i,j}$).	(lb)
$p^x_{i,j}$	-	The unit axial load in the x direction at Station i,j .	(lb/inch)
$\Delta P^x_{i,j}$	-	Change in axial load in the x-beam occurring at Station i,j .	(lb)
$Q_{i,j}$	-	The externally applied load at point i,j .	(lb)
$Q^x_{i,j}$	-	The load absorbed internally by the x-beam system at Station i,j .	(lb)
$Q^y_{i,j}$	-	The load absorbed internally by the y-beam system at Station i,j .	(lb)
$\overline{QBMX}_{i,j}$	-	The load absorbed internally by the x-beam in bending.	(lb)
$\overline{QBMY}_{i,j}$	-	The load absorbed internally by the y-beam in bending.	(lb)
$\overline{QTMX}_{i,j}$	-	The load absorbed by the x-beam in twisting.	(lb)

$\overline{QTMY}_{i,j}$ - The load absorbed by the y-beam in twisting. (lb)

$\overline{QPX}_{i,j}$ - The load absorbed by the x-beam system due to axial load. (lb)

$\overline{QPY}_{i,j}$ - The load absorbed by the y-beam system due to axial load. (lb)

S_f - The fictitious spring (closure parameter). (lb/inch)

$S_{i,j}$ - Support value under the slab at station i,j . (lb/inch)

$T^x_{a,j}$ - External torque applied to Bar a on the j^{th} x-beam. (inch-lb)

$V^x_{a,j}$ - Shear in Bar a of the j^{th} x-beam. (lb)

$w^x_{i,j}$ - Deflection of the j^{th} x-beam at Station i . (inch)

$w^y_{i,j}$ - Deflection of the i^{th} y-beam at Station j . (inch)

$\alpha_{a,j}$ - Angular change across slab element a,j . (rad)

ν_{xy} - Poisson's ratio which results in strain in the y direction when a stress is applied in the x direction. (none)

ν_{yx} - Poisson's ratio which results in strain in the x direction when a stress is applied in the y direction. (none)

Summing vertical forces in Fig 4.4 at joint i,j with up taken as positive gives

$$\sum F^x_{V_{i,j}} = Q^y_{i,j} + V^x_{a,j} - V^x_{b,j} - S_{i,j} \ (w^x_{i,j}) - Q_{i,j} - S_f \ (w^x_{i,j} - w^y_{i,j}) = 0 \qquad (4.1)$$

In order to evaluate the shear $V^x_{a,j}$, sum moments acting on Bar a about the center of the bar (clockwise rotations are positive). For equilibrium

$$\sum M_a = 0 = M^{x\,\prime}_{i-1,j} - M^{x\,\prime}_{i,j} + T^x_{a,j} + C^{x\,\prime}_{i,j} + C^{x\,\prime}_{i,j+1} + V^x_{a,j} h_x$$

$$+ 2P^x_{a,j} \left(\frac{-w^x_{i-1,j} + w^x_{i,j}}{2}\right) = 0 \qquad (4.2)$$

Multiplying through by h_x and clearing obtains

$$-h_x \overset{x}{V}_{a,j} = \overset{x\,\prime}{M}_{i-1,j} - \overset{x\,\prime}{M}_{i,j} + \overset{x}{T}_{a,j} + \overset{x\,\prime}{C}_{i,j} + \overset{x\,\prime}{C}_{i,j+1}$$

$$+ \overset{x}{P}_{a,j} \left(- w^x_{i-1,j} + w^x_{i,j}\right) \tag{4.3}$$

Likewise summing moments about Bar b and multiplying through by h_x obtains an expression for the shear $\overset{x}{V}_{b,j}$ as follows:

$$- h_x \overset{x}{V}_{b,j} = \overset{x\,\prime}{M}_{i,j} - \overset{x\,\prime}{M}_{i+1,j} + \overset{x}{T}_{b,j} + \overset{x\,\prime}{C}_{i+1,j}$$

$$+ \overset{x\,\prime}{C}_{i+1,j+1} + \overset{x}{P}_{b,j} \left(- w^x_{i,j} + w^x_{i+1,j}\right) \tag{4.4}$$

Multiplying Eq 4.1 through by h_x and substituting Eqs 4.3 and 4.4 for the shears obtains the equation of interest. After convenient grouping of terms and transfer of all known values to the right hand side of the equation, with a sign change, it becomes

$$\left(\overset{x\,\prime}{M}_{i-1,j} - 2\overset{x\,\prime}{M}_{i,j} + \overset{x\,\prime}{M}_{i+1,j}\right) - \left(- \overset{x\,\prime}{C}_{i,j} - \overset{x\,\prime}{C}_{i,j+1} + \overset{x\,\prime}{C}_{i+1,j} + \overset{x\,\prime}{C}_{i+1,j+1}\right)$$

$$+ \overset{x}{P}_{a,j} \left(- w^x_{i-1,j} + w^x_{i,j}\right) - \overset{x}{P}_{b,j} \left(- w^x_{i,j} + w^x_{i+1,j}\right) + S_{i,j} h_x w^x_{i,j}$$

$$= h_x \left[Q_{i,j} - \overset{y}{Q}_{i,j} - S_f \left(w^x_{i,j} - w^y_{i,j}\right)\right] - \overset{x}{T}_{a,j} + \overset{x}{T}_{b,j} \tag{4.5}$$

This equation relates forces and deflections at point i,j, but all of the prime terms must be evaluated further before the required mathematical manipulations can be performed. It is necessary at this point to substitute the finite-difference formulations of moment developed in Appendices A and B. It is convenient to express these in compressed central difference form. Accordingly, they are written at Stations i-1,j; i,j; and i+1,j and substituted into the equation.

The term $C'_{i,j}$ represents the force exerted on the x-beam due to the relative rotation between this beam and its neighbors. This is fully evaluated in Appendix B. These expressions must be written for $\overset{x\,\prime}{C}$ at Stations i,j; i,j+1; i+1,j; and i+1,j+1.

After making these substitutions, Eq 4.5 becomes

$$
\begin{aligned}
& h_y D^x_{i-1,j} \left[\left(\frac{w^x_{i-2,j} - 2w^x_{i-1,j} + w^x_{i,j}}{h_x^2}\right) + \nu_{yx} \left(\frac{w^y_{i-1,j-1} - 2w^y_{i-1,j} + w^y_{i-1,j+1}}{h_y^2}\right)\right] \\
& - 2h_x D^x_{i,j} \left[\left(\frac{w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j}}{h_x^2}\right) + \nu_{yx} \left(\frac{w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1}}{h_y^2}\right)\right] \\
& + h_y D^x_{i+1,j} \left[\left(\frac{w^x_{i,j} - 2w^x_{i+1,j} + w^x_{i+2,j}}{h_x^2}\right) + \nu_{yx} \left(\frac{w^y_{i+1,j-1} - 2w^y_{i+1,j} + w^y_{i+1,j+1}}{h_y^2}\right)\right] \\
& + \frac{C^x_{i,j}}{h_y} (w^x_{i-1,j-1} - w^x_{i-1,j} - w^x_{i,j-1} + w^x_{i,j}) \\
& + \frac{C^x_{i,j+1}}{h_y} (- w^x_{i-1,j} + w^x_{i,j} + w^x_{i-1,j+1} - w^x_{i,j+1}) \\
& - \frac{C^x_{i+1,j}}{h_y} (- w^x_{i,j} + w^x_{i+1,j} + w^x_{i,j-1} - w^x_{i+1,j-1}) \\
& - \frac{C^x_{i+1,j+1}}{h_y} (- w^x_{i,j} + w^x_{i+1,j} + w^x_{i,j+1} - w^x_{i+1,j+1}) \\
& + P^x_{a,j} (- w^x_{i-1,j} + w^x_{i,j}) - P^x_{b,j} (- w^x_{i,j} + w^x_{i+1,j}) + h_x (S_{i,j} + S_f) w^x_{i,j} \\
& = h_x (Q_{i,j} - Q^y_{i,j} + S_f w^y_{i,j}) - T^x_{a,j} + T^x_{b,j} \qquad (4.6)
\end{aligned}
$$

It is convenient in computation to use the same numbering system for bars, torsion bars, and joints. So far in these developments bars have been referred to as a and b . Referring to the numbering system shown in Fig 3.10, it will be recognized that in reality a becomes i and b becomes i+1 . Therefore, for example, $T^x_{a,j}$ becomes $T^x_{i,j}$, $P^x_{b,j}$ becomes $P^x_{i+1,j}$, etc.

This will be an implicit solution for $w^x_{i,j}$, the deflection of the j^{th} x-beam at Station i . It is convenient for solution, however, to utilize the last estimated values for all deflections, w^x , not falling on the j^{th} beam for a particular iteration, and transfer them to the right hand side of the equation. Furthermore, all of the y-beam deflections $w^y_{i,j}$ will be assumed known from a previous iteration and will also appear on the right hand side of the equation. After making the notation change of a to i and transferring known values to the right hand side, it is helpful to clear fractions and re-arrange terms. The resulting equation is the equation we seek and is most conveniently written in terms of five unknown deflections, i.e.,

$$a_x w^x_{i-2,j} + b_x w^x_{i-1,j} + c_x w^x_{i,j} + d_x w^x_{i+1,j} + e_x w^x_{i+2,j} = f_x \tag{4.7}$$

where

$$a_x = \frac{h_y^2}{h_x^2} D^x_{i-1,j} \tag{4.8}$$

$$b_x = -2.0 \frac{h_y^2}{h_x^2} (D^x_{i-1,j} + D^x_{i,j}) - C^x_{i,j} - C^x_{i,j+1} - h_y P^x_{i,j} \tag{4.9}$$

$$c_x = \frac{h_y^2}{h_x^2} (D^x_{i-1,j} + 4D^x_{i,j} + D^x_{i+1,j}) + C^x_{i,j} + C^x_{i+1,j} + C^x_{i,j+1}$$

$$+ C^x_{i+1,j+1} + h_x h_y (S_{i,j} + S_f) + h_y (P^x_{i,j} + P^x_{i+1,j}) \tag{4.10}$$

$$d_x = -2.0 \frac{h_y^2}{h_x^2} (D^x_{i,j} + D^x_{i+1,j}) - C^x_{i+1,j} - C^x_{i+1,j+1} - h_y P^x_{i+1,j} \tag{4.11}$$

$$e_x = \frac{h_y^2}{h_x^2} D^x_{i+1,j} \tag{4.12}$$

$$f_x = h_x h_y (Q_{i,j} - Q^y_{i,j} + S_f w^y_{i,j}) + h_y (T^x_{i,j} + T^x_{i+1,j})$$

$$-\nu_{yx} \Big[D^x_{i-1,j} (w^y_{i-1,j-1} - 2w^y_{i-1,j} + w^y_{i-1,j+1})$$

$$- 2D^x_{i,j} (w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1}) + D^x_{i+1,j} (w^y_{i+1,j-1} - 2w^y_{i+1,j}$$

$$+ w^y_{i+1,j+1}) \Big] - C^x_{i,j} (w^x_{i-1,j-1} - w^x_{i,j-1}) - C^x_{i,j+1} (w^x_{i-1,j+1} - w^x_{i,j+1})$$

$$+ C^x_{i+1,j} (w^x_{i,j-1} - w^x_{i+1,j-1}) + C^x_{i+1,j+1} (w^x_{i,j+1} - w^x_{i+1,j+1}) \tag{4.13}$$

One term remains to be evaluated, $Q^y_{i,j}$, the load absorbed by the y-beams at any time. This load can be evaluated by numerical differentiation of the deflected pattern of the y-beam system, but it can also be done from the free-body analysis by summing vertical forces in terms of load absorbed by both sets of beams, $Q^x_{i,j}$ and $Q^y_{i,j}$. This summation on the free-body in Fig 4.4. gives

$$Q_{i,j} - Q^y_{i,j} - Q^x_{i,j} - S_{i,j} w^x_{i,j} + S_f \ (w^x_{i,j} - w^y_{i,j}) = 0 \tag{4.14}$$

After necessary algebraic manipulations, the appropriate equation for evaluating $Q^y_{i,j}$ is seen to be as follows (evaluated completely in Appendix B).

$$Q^y_{i,j} = \overline{QBMY}_{i,j} + \overline{QTMY}_{i,j} + \overline{QPY}_{i,j} + \frac{T^y_{i,j} - T^y_{i,j+1}}{h_y} \tag{4.15}$$

If this process is repeated for a segment of y-beam, equations comparable to Equations 4.7 through 4.13 can be developed for the y-beams.

Summary

Equations 4.7 through 4.13 conveniently describe the model at Station i,j and are statically correct since the summation of forces at any time during the solution will equal zero. There are two such sets of equations, one for the x-system and one for the y-system at each mesh point i,j . The number of stations in each direction is equal to the number of increments plus 4. As an example, a problem divided into eight increments in the x direction and eight increments in the y direction would require equations at 12 stations in each direction. Thus the number of equations required to describe the system would be 288; 144 for the y-beams and 144 for the x-beams. This readily explains the need to resort to digital computers to perform the mathematical manipulations. That aspect of the problem is discussed in the next chapter.

CHAPTER 5. SOLUTION OF EQUATIONS

The equations derived in the preceding chapter are formidable. Two sets of such equations are required to describe each mesh point in the system, one for the x-beams and one for the y-beams. In order to make these equations useful, some general technique for solving them rapidly is necessary. Although some hand methods have been developed for small mesh systems, the high speed digital computer offers the desirable approach. This chapter will present several methods available for solution of these equations. A general description of the method chosen for use in this work is included.

Current Methods for Solution of Simultaneous Equations

The methods developed to solve systems of equations like Eq 4.7 fall into about five major categories: (1) simple direct-elimination methods, (2) methods involving iterative techniques similar to moment distribution, (3) general relaxation techniques, (4) successive over-relaxation and (5) alternating-direction-implicit methods. Actually, there are many other methods and many variations of the major methods listed above. The purpose of this writing, however, is not to survey the field of numerical analysis, but to apply a useful method to the solution of plates and slabs.

White and Cottingham (Ref 63) found a simple elimination method to be useful in their solution of plate buckling problems. Such elimination methods, however, are time consuming, requiring time in proportion to the cube of the number of equations involved. Another major drawback of this method is storage space since every term in the matrix must be stored even though many are zero.

Newmark (Ref 39) discusses several methods for solving simultaneous equations including successive approximation and step-by-step methods, as well as the distribution method. Distribution methods are somewhat more formal than relaxation methods and are organized for hand computations by technicians. Such methods are too cumbersome for efficient use in a digital computer.

The relaxation methods, or more specifically the method of "successive relaxation of constraints," is based on the concept that the structure is maintained in a continuous state, but has acting on it residual loads which are not statically consistent with the correct loading. The "residuals" are reduced by introducing arbitrary changes in displacement until convergence or statical balance is obtained. Southwell (Ref 15, p 66) pioneered such methods. These were also originally developed for hand computation but are flexible enough for use in computers. Liebmann (Ref 15, p 147) coded relaxation techniques for use on digital computers and speeded the process up considerably. Even so, he states, "The disadvantages of this procedure are the slow rate of convergence in many cases and the possible lack of convergence." Other work on this technique includes that by Jacobi, Gauss and Seidel, Richardson, and Frankel.

The SOR method, successive over-relaxation, provides still faster and better trial-and-error solutions by applying a complex relaxation factor which over-relaxes or over-compensates the adjustment of the existing data on any given trial. Otherwise, the method is basically that of relaxation.

The alternating-direction method presented by Conte and Dames (Ref 12) appears to offer by far the best techniques for solving the plate equation. Others who have used this method include Griffin and Varga (Ref 67) and Tucker (Ref 57). Because of its applicability, a more complete discussion of this method is warranted.

Alternating-Direction Implicit Solution

Conte and Dames in their paper "An Alternating-Direction Method for Solving the Biharmonic Equation" present an implicit alternating-direction iterative scheme which appears to be more efficient than any of the relaxation methods. The procedure they used is an extension of methods developed by Douglas and Rachford. In their paper, Conte and Dames (Ref 12) present a solution of the partial differential equation which governs slab behavior. In simplest terms, the method divides the partial differential equation into two ordinary differential equations and couples their solution by trial and error in a methodical fashion, proceeding first in the x Cartesian direction, then in the y direction, thus the name alternating direction. The most difficult part of using this method is the selection of proper iteration parameters. Proof of convergence exists for certain parameter selection for regular, well-conditioned systems. For the diverse systems described herein, however, much remains to be done.

Experimentation by Matlock, Tucker, Ingram, Salani, and Haliburton (Refs 35, 57, 23, 45) with these methods has led to the use of the alternating-direction-iterative method in the solutions in this report. This technique has many favorable characteristics which warrant its use.

(1) The method is rapid and well adapted for computer use.

(2) The method fits well with the mechanical model used to describe the system.

(3) The process can be easily visualized as a trial and error solution of the model.

(4) The method is logical and can be understood by practicing engineers.

The concepts developed herein are general in nature. They do not emphasize mathematical rigor and completeness, but are shown to be applicable to many engineering problems. No attempt will be made to prove mathematically absolute convergence, although such proof is available for special-case uniform, homogeneous, isotropic systems. Rather the advantages and capabilities of the method will be demonstrated by examples later in this report. The validity of these diverse examples and their exhibition of closure or convergence to acceptable tolerances (10^{-6} inches for deflection or 1.0 pounds for load) is offered as adequate proof of satisfactory closure.

Use of the alternating-direction iterative method is greatly enhanced by judicious choice of closure parameters. They have been shown to be related to the limiting eigenvalues or characteristic values of the set of equations involved. Many mathematicians maintain that closure parameter values selected for square systems must be used for both halves of any iterative cycle. Ingram (Ref 23) has demonstrated a method, however, which is not troubled by this restriction. Furthermore, diverse problems which prove troublesome to solve with the classical single-iteration control methods are readily solved using the Ingram dual-control techniques.

Details of Solutions

For solving the large number of simultaneous equations which result in each half-cycle of the alternating-direction iterative method, Matlock and Haliburton (Ref 35) used an efficient two-pass method to solve linearly elastic beam-columns. The method involves the elimination of four unknowns, two each in two passes. The first pass from top to bottom eliminates deflections w^x_{i-2} and w^x_{i-1} from each equation (see Eq 4.7). The second pass, in reverse order, eliminates deflections w^x_{i+2} and w^x_{i+1} from each equation, and thus results in the solution for the desired deflection w^x_i. Those readers not familiar with this technique are invited to read Reference 35.

One of the valuable assets of this method is that boundary conditions as normally discussed are automatically provided with two dummy stations specified at each end of each beam in the system. These dummy stations in reality have no bending stiffness, therefore, a bending stiffness equal zero is input for them. Equation 4.7 is then formulated for every station in the beam plus two dummy stations on each end.

To solve for $w^x_{i,j}$ then, the plate is considered to be two systems of orthogonal beams interconnected at Station i,j by S_f, the fictitious closure spring constant. Figure 5.1 shows a view of a grid-beam system with closure springs acting during solution. A comparable view of the slab model with torsion bars present is shown in Fig 5.2.

With the beam-column as a basic tool, the solution of the system of equations for plates and slabs proceeds as follows:

(1) Solve each x-beam successively through the system considering all the y-beams to be held fixed in space. At any particular solution of any x-beam then, the fictitious closure spring acts as restraint on the x-beam of interest.

(2) After all x-beams are solved and their new deflection pattern is known, alternate or change directions and fix the x-beams in this new pattern.

(3) Solve for the deflected shape of each y-beam in turn. The fictitious springs now act as loads or restraints on the y-beam, serving to transfer the load which has been stored in them from the deflected x-beams.

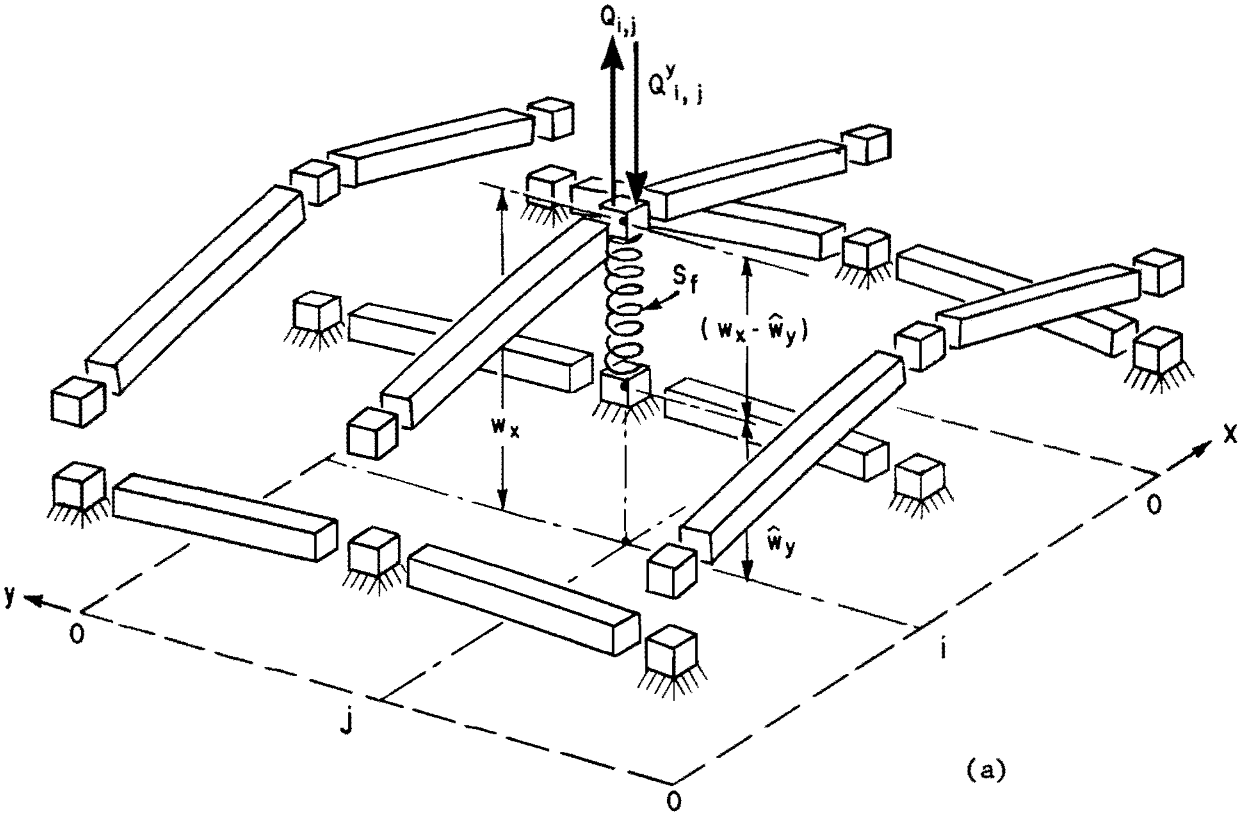
(4) This procedure is repeated alternately until all of the load is properly distributed throughout the system. At this point the summation of static forces at each joint in the system will equal zero within the specified tolerance and the deflection of the x-beam system $w^x_{i,j}$, at any point will equal the deflection of the y-beam system, $w^y_{i,j}$, at the same point within the specified tolerance so that the term $S_f\,(w^x_{i,j} - w^y_{i,j})$ vanishes.

The process described is a rapid one requiring from 5 to 25 iterations for most simple problems with closure to six significant digits.

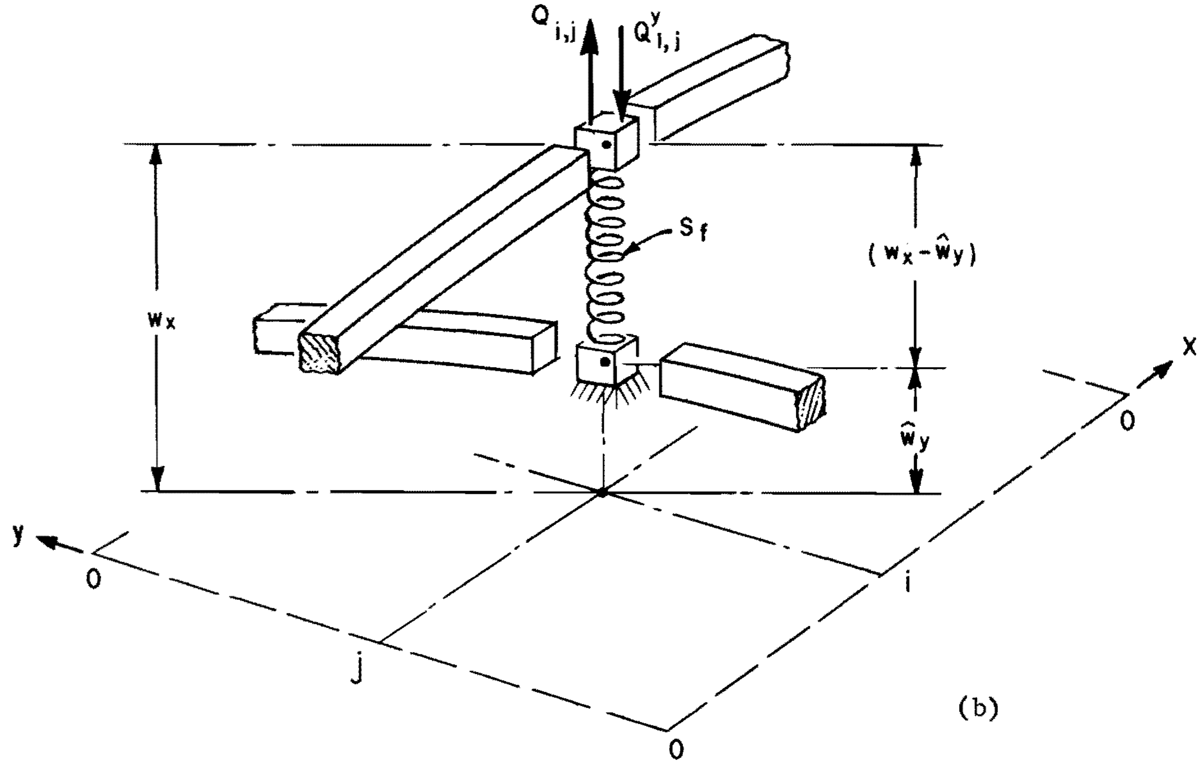
Closure Process

A considerable amount of work has been done on iteration control for the alternating-direction iterative method. In their work with alternating-direction iterative method, Peaceman and Rachford (Ref 40) tried to accelerate convergence by the introduction of constants into the equations. They have developed methods for determining optimum parameters by rational function approximations. These parameters are difficult to compute, however, and can be closely approximated by the easier Wachpress techniques.

Many experimental analyses run with the equations described herein show



(a)



(b)

Fig 5.1. Grid beam system during closure process with fictitious spring acting between the x-beam and y-beam at Station i,j .

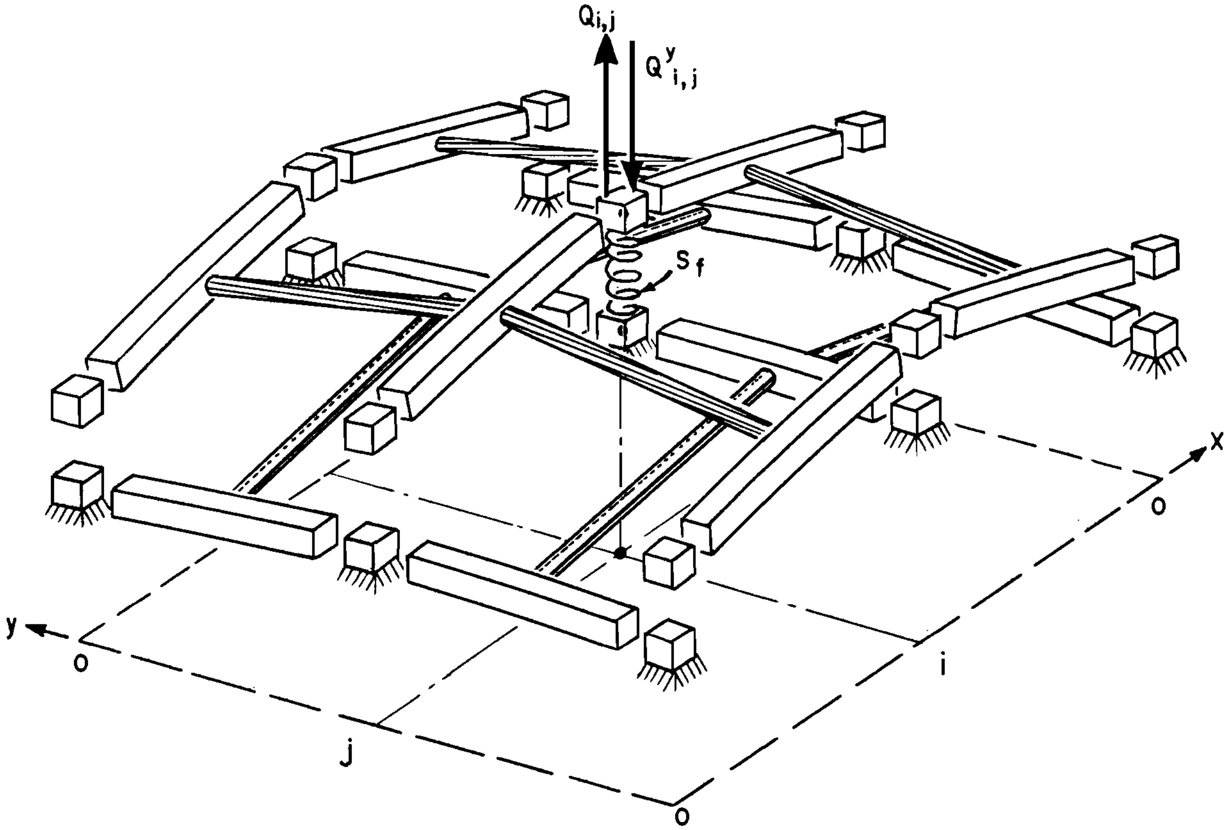


Fig 5.2. Plate represented in the closure process as two orthogonal systems with closure spring acting between them at Station i,j .

interesting results. For square homogeneous isotropic simply supported plates with equal increment length in both directions, almost any choice of iteration parameter or closure spring constant will result in convergence. Certainly, some springs result in faster closure than others and the method proposed by Wachpress and adapted for use by Haliburton (unpublished notes) has proved to be a good method for parameter selection for "regular systems." This method involves the choice of several optimum closure springs to be used in sequence to obtain faster convergence. The number of closure springs to be used and the stiffness size of these springs is determined by the number of increments in the system and the stiffness of the system.

An improved technique for selection of parameters was used by Salani (Ref 45) who suggests that the closure parameter is related to the eigenvalues of the equation matrix. The equation for these eigenvalues is

$$\lambda_n = \frac{4D}{h_x^2 h_y^2} (1 - \cos \beta_n)(2 - \cos \beta_n) \tag{5.1}$$

where

$$\beta_n = \frac{n\pi}{m}$$

m = number of beam increments in the direction being considered

n = 1, 2, ..., m-1

λ_n = n^{th} eigenvalue

h_x = the increment length, x-beams

h_y = the increment length, y-beams

D = the plate stiffness for the direction of input.

The fictitious spring or closure parameter is then equal to

$$S_f = h_x h_y \lambda_n$$

or, directly

$$S_f = \frac{4D}{h_x h_y} (1 - \cos \beta_n)(2 - \cos \beta_n) \tag{5.2}$$

Many schemes have been proposed for selecting optimum numbers of parameters to be used in the solution. In this regard, it is convenient to balance the amount of work required to calculate parameters by hand and the amount of time saved in the computer. Based on the work of Wachpress and the numerous trial solutions obtained in this study, the following rules of thumb are suggested:

No. of Increments	No. of Parameters	Approx. Values of n
Less than 8	4	$2, \frac{m}{2}, m-1, \frac{m}{2}$
8 to 16	5	$3, \frac{m}{3}, \frac{2m}{3}, m-2, \frac{m}{2}$
More than 16	6	$4, \frac{m}{3}, \frac{m}{2}, \frac{2m}{3}, m-2, \frac{m}{2}$

In his work with layered beam and slab systems, Ingram (Ref 23) encountered some difficulty in closure with conventional use of iteration parameters. This led him to the examination of a method for changing iteration parameters on the half cycle. In effect, he selects fictitious springs for use with the x-beams based on the resistance of the y-beams to bending and vice versa. This can be intuitively derived from the physical model and points up another value of the physical model. The program described in the following chapter utilizes the Ingram dual-control iteration process.

The first step is to calculate $S_{f\,x}$ with Eq 5.2 utilizing bending stiffness and number of increments for the x-beams. Then $S_{f\,y}$ is calculated utilizing the same equation but using the bending stiffness and number of increments for the y-beams. The solution process then utilizes $S_{f\,y}$ in calculating deflections for the x-beams, and $S_{f\,x}$ for calculating deflections for the y-beams. This technique has been found to be stable for diverse classes of problems and to date, no plate problems have failed to converge. It can be noted that square isotropic slabs result in equal parameters for x and y directions which is to be expected and is in agreement with Wachpress.

Figure 5.3 illustrates the closure process for an 8×8 increment square plate. The parameters were chosen in accordance with the rules set forth above. Fig 5.4 shows an example of closure for a more diverse problem, an orthotropic plate simply supported at the two ends. Figure 5.5 shows closure for Example Problem 401, a 24-foot-square pavement slab with an edge load of 10,000 pounds.

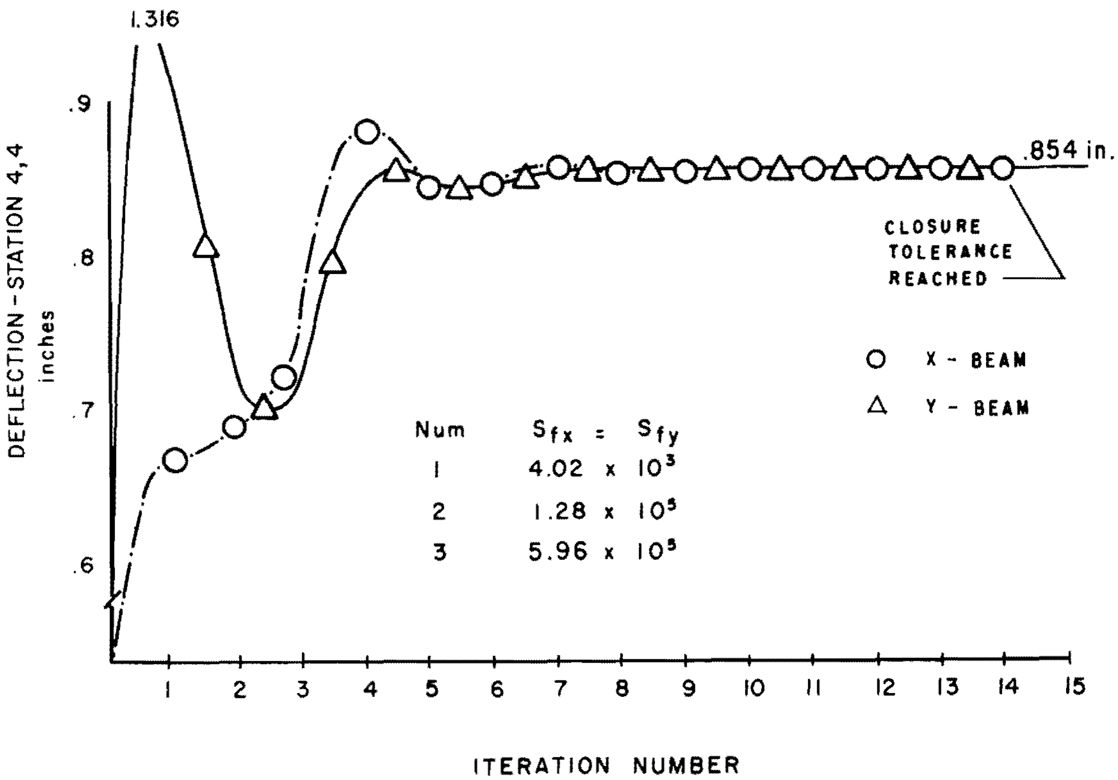


Fig 5.3. Plot of closure for Example Problem 102 (See Chapter 7).

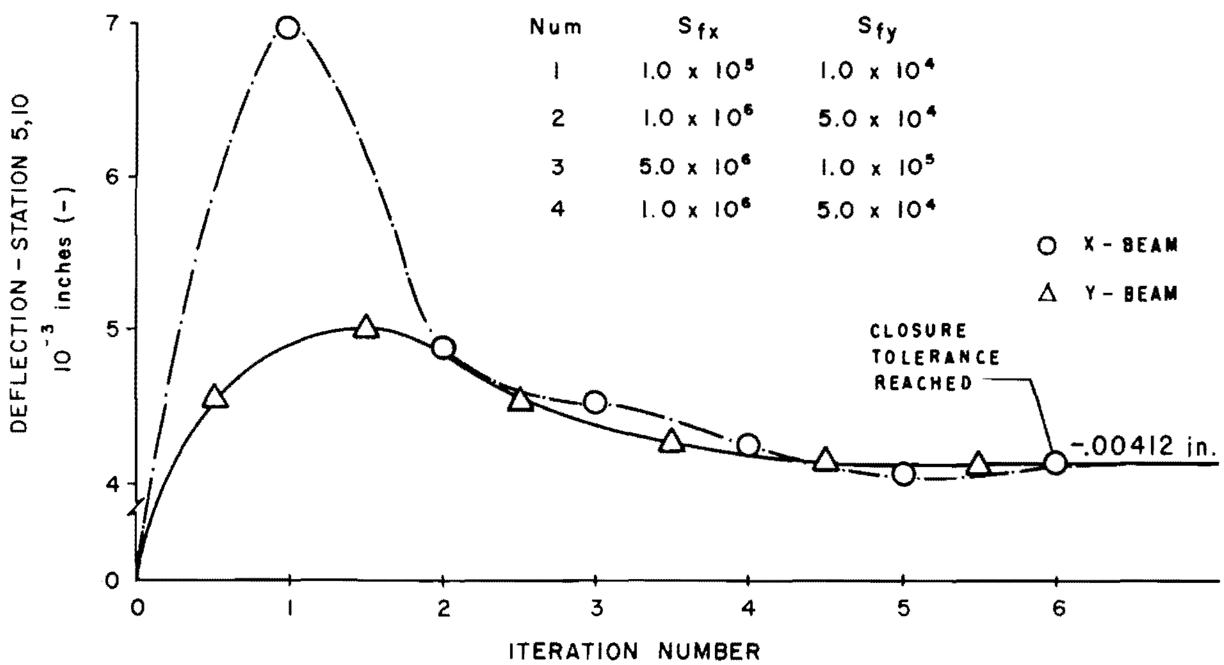


Fig 5.4. Plot of closure for Example Problem 504 (See Chapter 7).

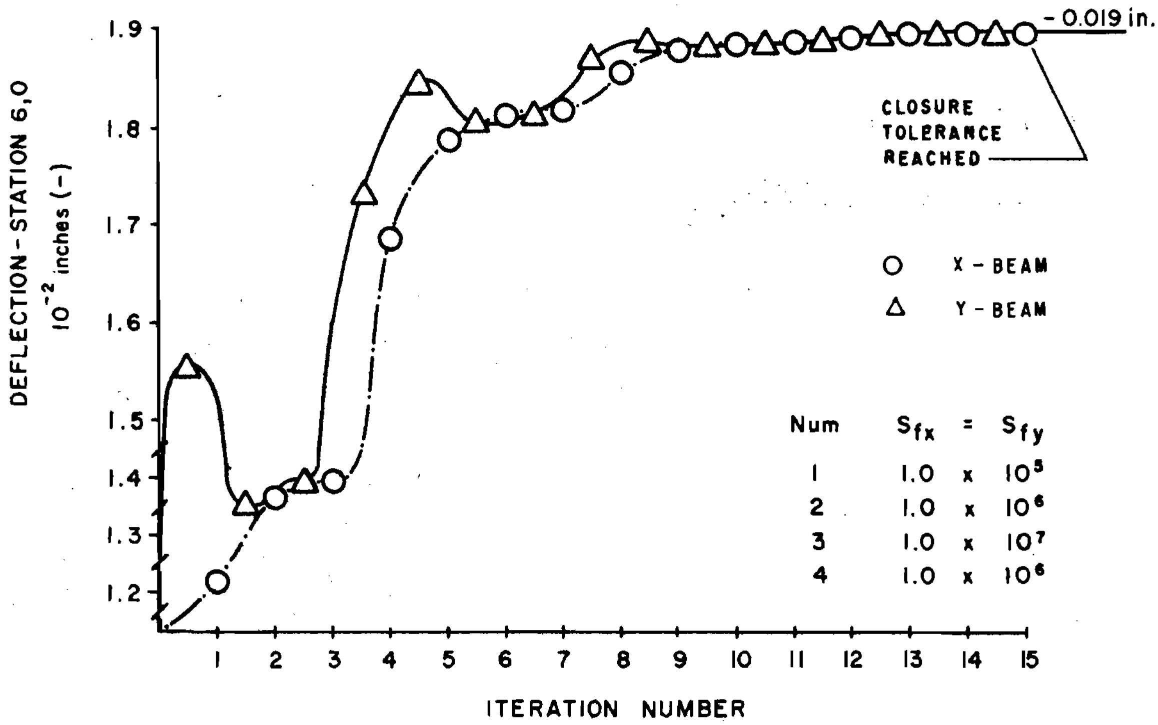


Fig 5.5. Plot of closure for Example Problem 401 (See Chapter 7).

CHAPTER 6. THE COMPUTER PROGRAM

The equations derived in Chapter 4 are not useful for hand calculations, but they are extremely well adapted for digital computer methods. During the eighteen months of this investigation, twelve programs have been developed which are useful for solving slab and plate problems of various types. The earlier programs are simple in format and application. SLAB 3, for example, is a program which solves isotropic slabs subjected only to transverse loads. The most general program is known as SLAB 17. The number 17 signifies that this is the seventeenth version in the chronological sequence of development of SLAB Programs. Not all of these programs are useful; some include only minor variations; others were discarded in the idea stage.

These programs are written in FORTRAN computer language for the Control Data Corporation 1604 Digital Computer which has a 48 bit word length and is operated with a FORTRAN-63 monitor system. The compile time for the basic program is less than two minutes; however, normal operating decks may be compiled on binary cards, thus reducing compile time in the computer to about fifteen seconds. The exact storage requirements of the program as presently dimensioned are undetermined. In general, however, the dimension statements are such that the program will handle as large a problem as practical with present storage capacity. Slight increases in storage can be obtained by using the various space-saving routines available for the CDC equipment. This program can be modified for use with the IBM 7090 computer by the modification of about 12 input-output cards.

The time required to run problems varies, of course, with the complexity and size of the system, i.e., the number of increments involved, and the number of iterations required to obtain the desired accuracy. To give a general idea of operating time, eight-by-eight problems close to a tolerance of 10^{-6} inches in 10 to 60 iterations, and require 30 to 100 seconds for solution. An increase in size to 16 × 16 with fairly uniform stiffnesses in both directions can be closed to similar tolerances in about 100 to 200 seconds computer time. While this may seem high when compared to solution time for simpler problems, the cost of three minutes of computer time ($15 to $30, depending on rental rates) is small compared to three to four days of laborious computation time required to do the problem by hand. More important, perhaps, is the fact that this computer program provides a useful way of making some solutions for the first time. General solutions are not often made with existing methods. In most cases, designs for slabs or plates are based on simplified approximate solutions for two or three points rather than on a complete analysis.

The FORTRAN Program

A summary flow diagram for the SLAB Programs is given in Fig 6.1. This flow diagram describes the program tasks briefly. A detailed flow diagram and listing of the program SLAB 17 is provided in Appendix C. Appendix C is a self-contained instruction and operating manual for SLAB 17. It includes (1) instructions on the operation of the program, (2) detailed input forms and descriptions, (3) program listing and (4) detailed flow chart.

The format used for inputing data into the program is arranged as conveniently as possible. No effort is made to be frugal with the number of cards required to input one problem. Instead, every effort is made to organize the program input logically and concisely. The problem input deck starts with two cover cards used to identify the program and the particular run being made. The information on these cards is alphanumeric and is used to denote projects, coding dates, key puncher, description of the problems being run, etc. After these two alphanumeric cards come the following cards in order:

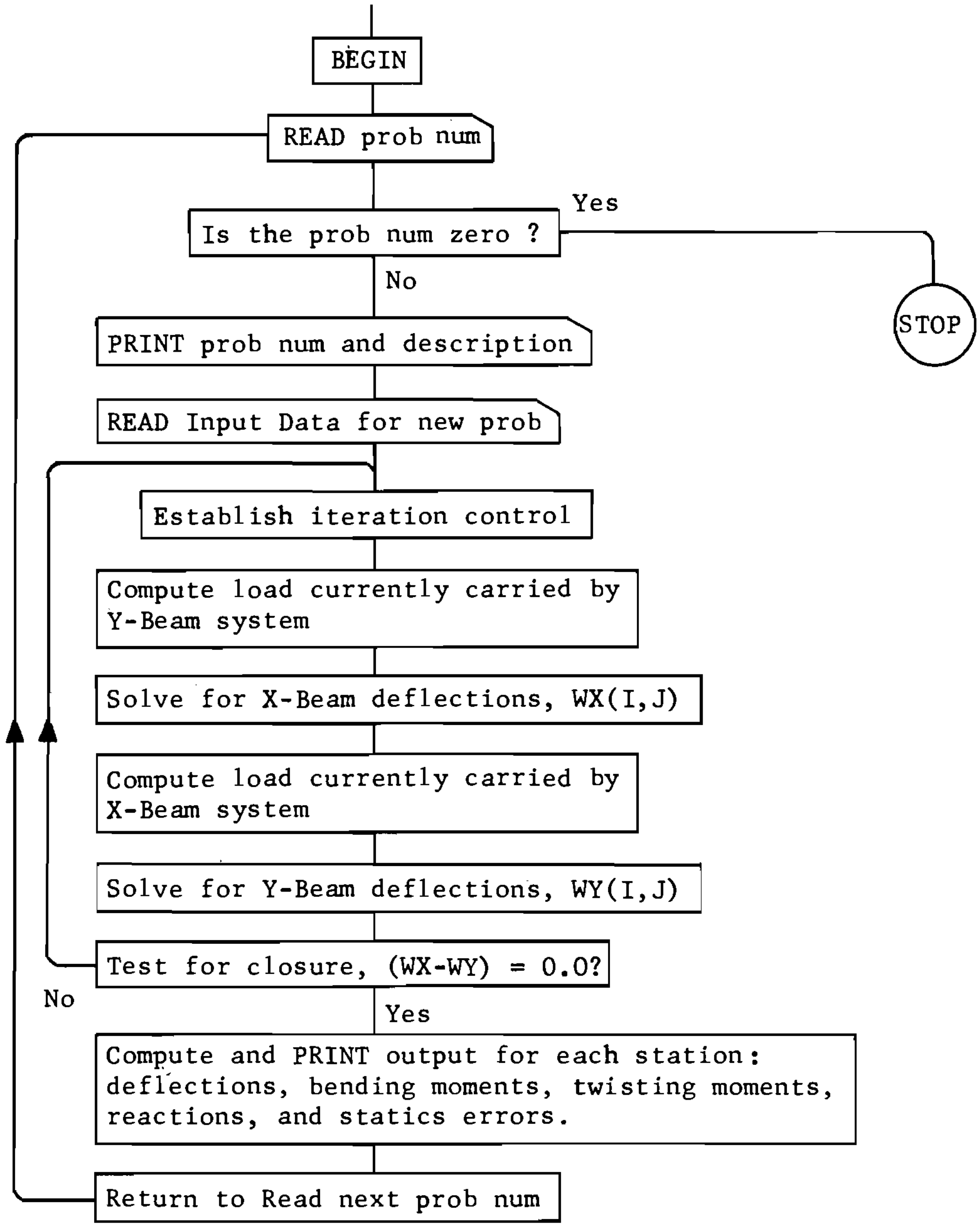


Fig 6.1. Summary flow chart, slab program.

(1) Problem number card with alphanumeric description of the problem.

(2) Table 1 - Input for Data Control and Constants - 2 cards. Information on these cards includes number of cards to be read in Tables 2 and 3, number of iterations, number of increments, increment length, closure tolerance, and Poisson's ratio. The second card gives a list of the monitor stations.

(3) Table 2 - Iteration Control Data - 2 cards with 8 10-digit fields on each card. The first card contains closure springs S_{fx} representing the x-beams. The second card contains closure springs S_{fy} representing the y-beams.

(4) Table 3 - Stiffness and Load Data - The number of cards in this table is variable depending on the number required to specify bending stiffness, load, support springs, torsional stiffness, external couples and axial loads. The number of cards in Table 3A and Table 3B must be properly specified in Table 1 in order to be read properly by the computer. Appendix C gives a complete description of input forms and their use. Appendix D contains numerical examples of input and output for example problems in Chapter 7.

Output Information

The program output is arranged to be useful to the user. A format which can be trimmed to standard 8-½-inch by 11-inch size is provided. For convenience and help in identifying problems, the program prints out all original input data at the beginning of each problem. These values are tabulated and labeled just as they were input. The first output computed by the program itself is Table 4, Monitor Deflections. This table prints out deflections for both the x-beams and y-beams at the four pre-selected monitor stations specified in Table 1. This data can be plotted using other versions of the SLAB Program. This information enables the program user to improve closure processes in an effort to seek optimum closure. It also permits rapid evaluation of the individual closure springs utilized in the system.

The results desired from the program are printed out in Table 5. This table prints in two parts in keeping with the 8-½ x 11-inch format. The first half prints external station numbers, x and y deflections, bending moments in the x and y directions, the external reaction of the slab and the true error in statics as determined by summation of vertical forces at each station. Part 2 of Table 5 prints out station numbers and twisting moments in the x and y directions at each station. Four additional spaces are provided for printing out stresses and direction of principal stress in later programs to be equipped with stress calculating options.

An automatic plot routine can be coupled with SLAB 17 and used to plot any of the variables available at mesh points in the system, although its major use is normally plotting deflection contours.

The bending and twisting moment outputs are calculated by numerical differentiation of the deflected shape. In both cases central differences are used to provide moments at each mesh point in order that these moments may be available for calculation of principal stresses.

As with all finite mathematical techniques, some approximations result in this program. It is not possible, for example, to determine both values of a double-valued function by numerical differentiation. Twisting moments are such double-valued functions, being a maximum just inside the plate boundary, but being zero just outside the plate boundary. A double-value of the maximum and zero fall on the boundary. The best approximation of this in finite-difference techniques is a half-value or the average between maximum and zero. The same half-value approximation results for bending moments at fixed ends for cantilevered structures (Ref 35). The bending moment-stiffness diagram, however, is correct for this case since bending stiffness is input as half-value at edges and ends. Bending moments at free or simply supported edges are calculated correctly by these methods. Third derivatives which are related to the shear forces meet the Kirchoff boundary conditions at free edges (Ref 55, p 84).

Many other investigations of intricate calculations of output values for various discontinuous and orthotropic cases have been made. All other cases studied have been found to calculate correctly. It is quite probable, however, that some diverse cases do exist where minor difficulties will be encountered.

Summary of Program Details

SLAB 17 is the most general and useful program available at the present time for solving the equations developed herein. The program is written in FORTRAN-63 language for the CDC 1604 computer and solves slab and plate problems very rapidly. Appendix C contains all information about the program from flow diagrams to output information, and can be extracted as an operating manual for use with the program. Twelve other programs are available for solving various types of problems. Several of these will be destroyed, but others will be developed to provide special-case solutions which solve more rapidly than the general method.

CHAPTER 7. EXAMPLE PROBLEMS AND VERIFICATION OF THE METHOD

Developments of equations and discussions of techniques are important in analytical work of this kind, however, application of the method and demonstration of technique in solving actual problems is equally important. This chapter provides the solution to several example problems to demonstrate Program SLAB 17 and its use in engineering calculations. Closed-form solutions for some of the problems are provided as a mathematical check to the computed solutions. Computer input data forms for three problems are provided in Appendix D with sample output to provide the reader with a step-by-step example of the program in use.

Problem Series 100 - Simply-Supported Plate with Variations

As a first example, a series of problems illustrating many of the variations possible in the program are applied to a 48-inch square simply-supported steel plate 1 inch thick (Fig 7.1). The modulus of elasticity is 30,000,000 psi. Poisson's ratio is 0.25. Loading variations will be discussed with the individual cases. Once the reader acquaints himself with the physical properties of this plate, it will be possible to evaluate very rapidly six separate cases of load and variations of parameters.

Problem 101 - Concentrated Load - The problem of a simply supported square or rectangular plate with concentrated load is considered by Timoshenko for various load conditions. Several equations for solving this problem are presented using single and double trigonometric series. A consensus value of solutions for maximum deflection, which occurs under the load, is 1.07 inches. Figure 7.2 is a plot obtained automatically from SLAB 17 coupled with a plot routine for the complete deflected shape of the plate when it is divided into

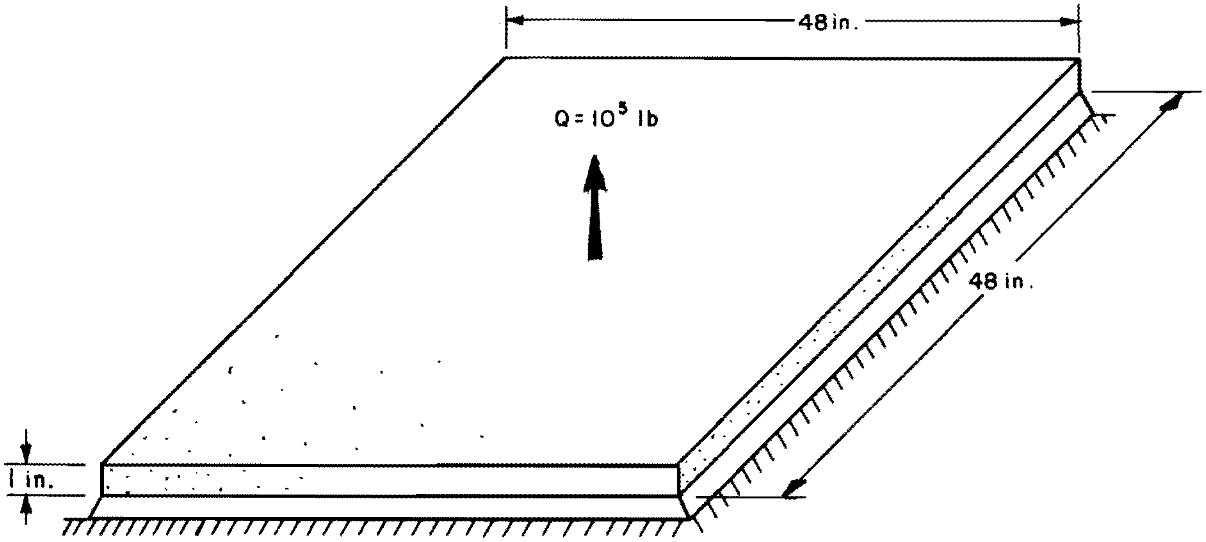


Fig 7.1. Square steel plate simply supported at all edges. (Example Problem 101).

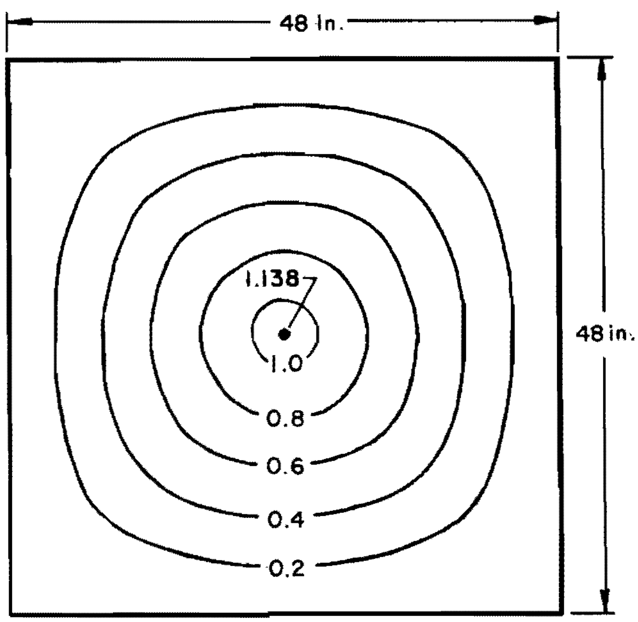


Fig 7.2. Deflection contours for Example Problem 101 with 100-kip concentrated load at the center.

eight 6-inch increments in each direction. The maximum deflection w_{max} is noted to be 1.138 inches. This differs 0.07 inch, or 6 percent, from the closed-form solution. If the number of increments is increased to 16 in each direction, a maximum deflection of 1.08 inches results. Thus, the error is reduced to 1 percent, probably as good as the accuracy of the closed-form solution using a truncated double trigonometric series. Contours of maximum bending moment or twisting moment could have been plotted, if desired, just as easily as the deflections.

Problem 102 - In-Plane Forces - In addition to the concentrated load at the center, add a uniform in-plane force in the y direction of 16,667 pounds per inch of plate width. In the closed-form solution this term appears in the denominator of the series solution and does not have as much effect as might be expected. The maximum deflection occurs under the load and is 0.787 inch. The computed solution for an 8 x 8 grid is 0.854 inch. The difference of 0.067 inch is almost identical with that in Problem 101. Increasing the num- number of increments would reduce the difference accordingly. The computer input and output for this problem are used as examples in Appendix D.

Problem 103 - Two-Way In-Plane Forces - Add to Problem 102 an equal in-plane tensile force in the x direction. The computed solution for maximum deflection reduces to 0.661 inch. If the force in the x direction is tensile or positive but the force in the y direction is compressive or negative, the effects on maximum deflection offset each other as would be expected. This solution gives a maximum deflection of 1.14 inches, the same as Problem 101.

Problem 104 - Uniform Load - If a uniform load of 100 pounds per square inch is substituted for the concentrated load (Fig 7.3), the closed-form solution, as calculated by Irving and Mullineux (Ref 66), is -0.861 inch. The results of SLAB 17 for 8 x 8 increments is 0.861 inch; for 12 x 12 increments, 0.862 inch; and for 16 x 16 increments, 0.860 inch. The average

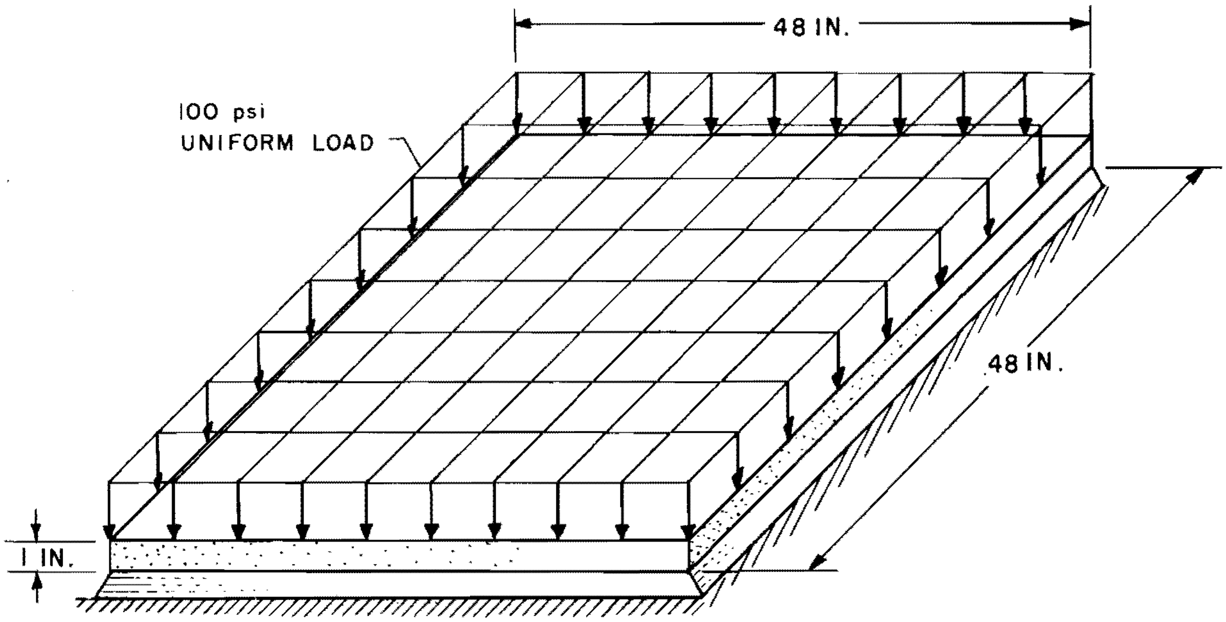


Fig 7.3. Steel plate simply supported all around with 100 psi uniform load. (Example Problem 103).

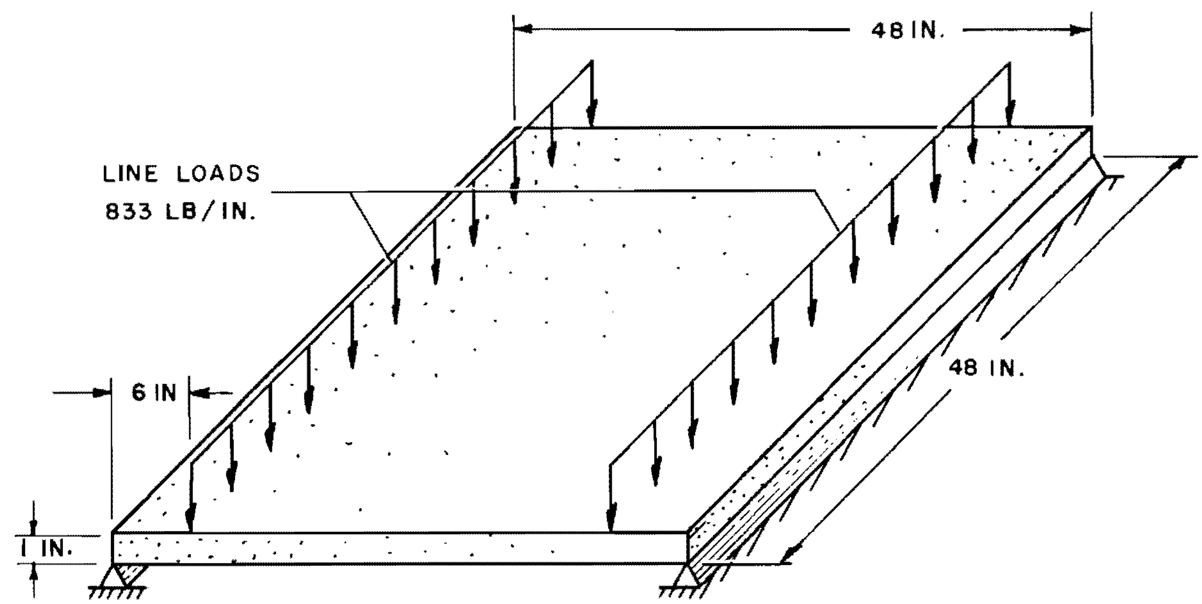


Fig 7.4. Plate simply supported on two edges with line loads. (Example Problem 106).

difference for the three solutions is less than 0.1 percent. A comparison of these solutions seems to indicate that within reasonable ranges the number of increments is not critical for uniform load conditions.

Problem 105 - Interior Foundation Support - If a uniform interior elastic foundation with support k equals 100 pounds per square inch per inch is added to Problem 101 and the load is made negative, Program SLAB 17 calculates a maximum deflection of -0.70 inch compared to the approximate closed-form solution given by Timoshenko of -0.72 inch (Ref 55). The apparent difference between these two solutions is probably due as much to the approximations in the closed-form solution as in the present method.

Problem 106 - End Supports With Line Loads - Modify the basic problem slightly by removing the simple supports under two edges of the plate. This leaves the plate supported as a wide-beam on simple supports (Fig 7.4). Unlike a beam, however, the plate should exhibit Poisson's ratio effects. Poisson's ratio manifests itself in such a structure by anticlastic bending. This may be explained in the following way. If moments are applied to the plate at opposite ends of the x axis, a simple analysis would indicate that a uniform moment in the x direction, M_x, would be present throughout the plate. Two conditions are known from physical equations governing plate behavior. First, the bending moment in the y direction at the free y-edges must be zero. And, second, the bending moment in the y direction may be stated as follows:

$$M_y = D_y \left(\frac{\partial^2 w}{\partial y^2} + \nu_{xy} \frac{\partial^2 w}{\partial x^2}\right) \tag{7.1}$$

The first stated condition requires that the second condition, Eq 7.1, be identically zero. Note that the bending in the x direction is not zero, thus the differential $\partial^2 w/\partial x^2$ can not be equal to zero. Then for Eq.

7.1 to be identically zero,

$$\frac{\partial^2 w}{\partial y^2} = -\nu \frac{\partial^2 w}{\partial x^2} \qquad (7.2)$$

Thus bending in the y direction will be present at the two edges with a sense opposite that in the x direction. This is illustrated in Fig 7.5. Figure 7.6 illustrates the same plate when Poisson's ratio equals zero. This can be recognized as bending equivalent to that of a beam in which Poisson's ratio can be neglected. Brief reference to Eq 7.1 indicates that if Poisson's ratio equals zero, the bending in the y direction is unaffected by bending in the x direction since they are related only through Poisson's ratio.

Two solutions were run, one with Poisson's ratio $\nu = 0.0$, the second with Poisson's ratio $\nu = 0.25$. The hand solution as a beam gives w_{max} at the center of the beam or plate of -0.566 inch. The SLAB 17 solution for eight increments $\nu = 0.0$ gives $w_{max} = -0.576$ inch, a difference of 2 percent. A 16×16 solution reduces this difference to less than one percent. For Poisson's ratio of 0.25 a center deflection of $w = -0.575$ inch results. This increases to -0.640 inch at the two edges due to anticlastic bending. Figure 7.7 compares sections of the two solutions at the middle of the unsupported span to illustrate this anticlastic bending.

Problem 107 - End Supports With Applied Torques - This problem is the equivalent of Problem 106 except the moment due to the applied line loads acting at 6 inches distance from the two simple supports is converted to a uniform moment applied near the ends. It is illustrated in Figure 7.8. The results are exactly comparable to those of Problem 106 as was expected. This indicates that Program SLAB 17 handles applied torques satisfactorily.

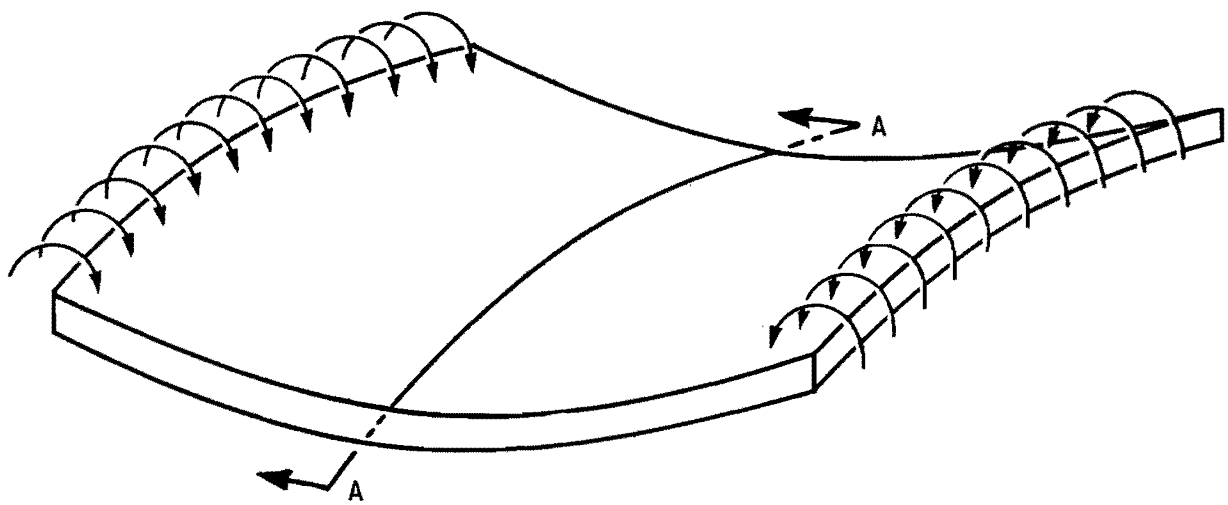


Fig 7.5. Anticlastic bending of plate subjected to uniform bending moment at opposite edges (Example Problems 106a and 107).

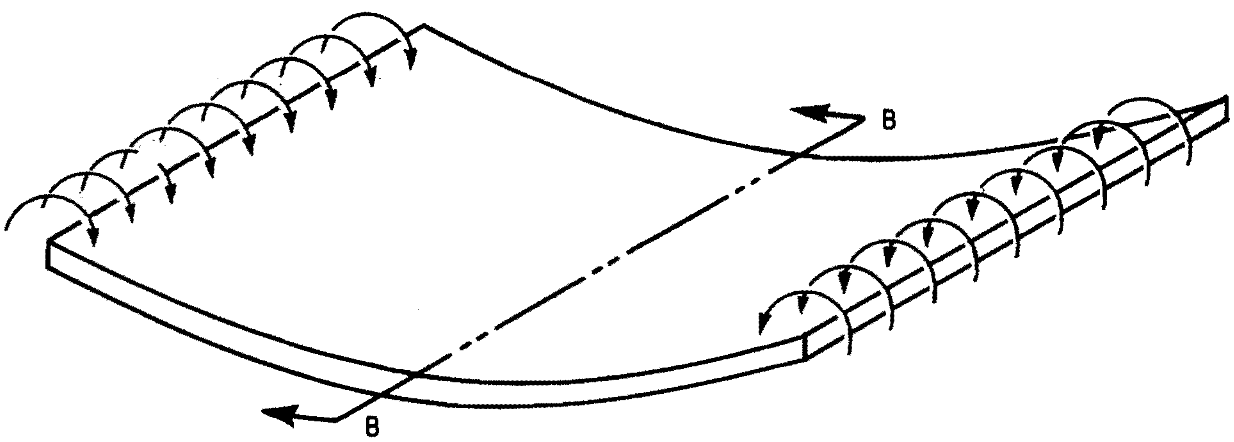


Fig 7.6. A plate bending as a beam when Poisson's ratio is zero (Example Problem 106b).

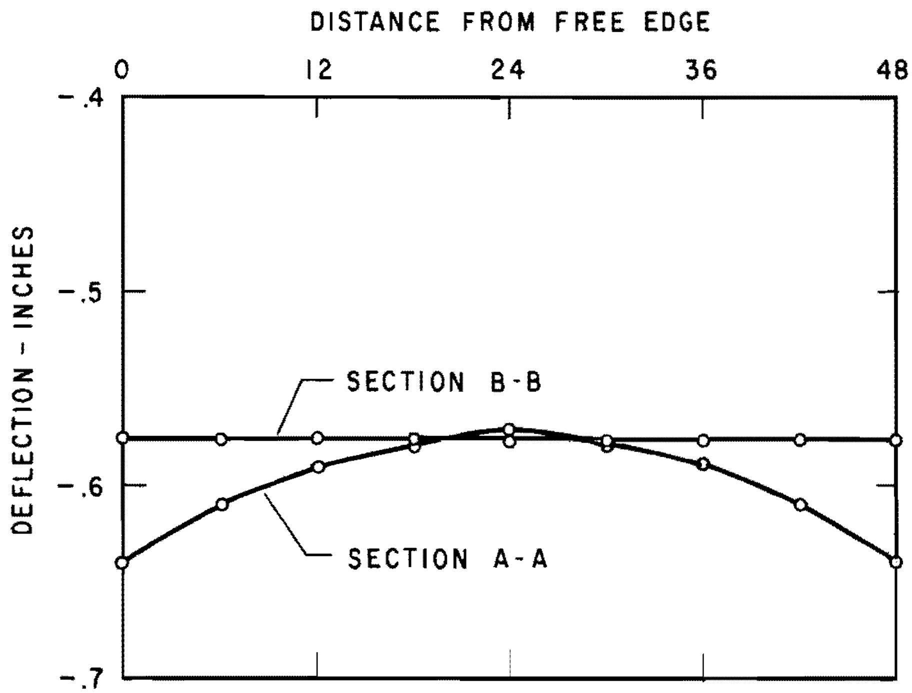


Fig 7.7. Cross sections from Figs. 7.5 and 7.6 to illustrate anticlastic bending.

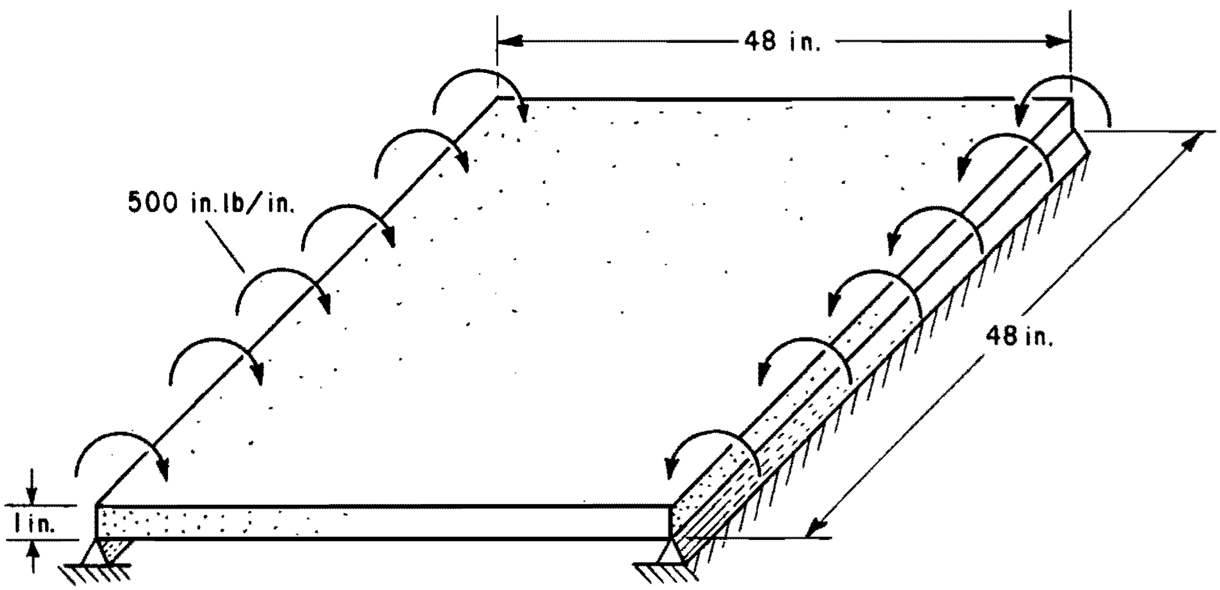


Fig 7.8. Simply supported plate with a bending moment applied at opposite ends (Example Problem 107).

Slabs-on-Foundation - Westergaard Cases, Problem Series 200

For slab-on-foundation problems, the matter of checking theory becomes more complicated because of the lack of closed-form solutions. Three example problems related to the three Westergaard cases are presented here since these solutions are well known and are currently used as a basis for most rigid pavement design. A single pavement slab was chosen for comparison and examined separately for the three Westergaard cases. The closed-form solutions come from Westergaard, page 102 (Ref 60). A standard slab example is used for the computed deflection as shown in Fig 7.9a.

The examples all involve a 10 inch slab thickness, 24 feet square in plan dimension with a modulus of elasticity of 3,000,000 psi and $\nu = 0.20$. The subgrade modulus was assumed to be 200 pounds per square inch per inch of deflection and a single concentrated load of 10 kips was applied in each case.

Problem 201 - Center Load - With these physical constants the Westergaard solution gives the deflection under a load applied at the center of an infinite slab to be -0.0057 inch. The computed results are -0.0060 inch. In addition, you can see from Fig 7.9b that the computed solution gives the complete deflection contours of the slab, whereas the Westergaard equation gives the deflection only under the load. This solution involves 8 increments. A solution using 12 or 16 increments gives deflection results closer to that of Westergaard.

Problem 202 - Edge Load - For the case of edge loading, Westergaard gives -0.019 inch deflection for a point under the load at the edge of a slab and infinitely far from any other boundaries. This is, of course, not a realistic situation since pavements certainly have finite boundaries. In reality, because of cracking or jointing, the load is nearly always relatively close to some boundary in any direction. The finite-element solution based again on the 24-foot square slab, but with the load centered along one

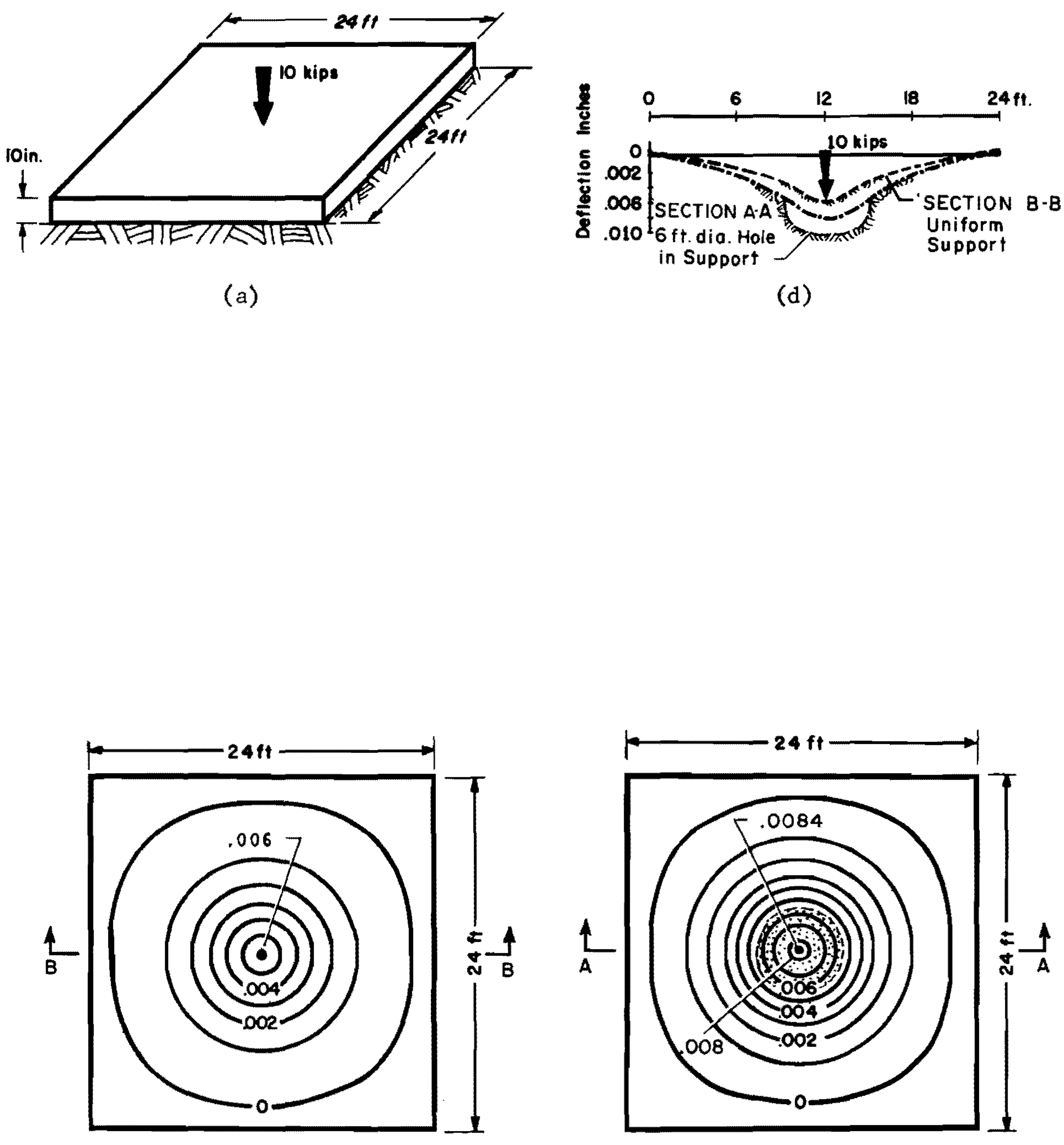


Fig 7.9. Pavement slab subjected to 10-kip wheel load at the center; with and without uniform subgrade support.

edge as shown in Fig 7.10a, gives a deflection of -0.018 inch. The contours are shown in Fig 7.10b. These results compare within 4 percent. Exact comparison need not be expected since one solution is for a real slab and the other for an infinite slab.

Problem 203 - Corner Load - The third Westergaard case is the load applied at a rectangular corner, infinitely far from any other discontinuity. The comparable real slab is shown in Fig 7.11a. The Westergaard solution gives -0.049 inch of deflection under the load. The finite-element solution shown in Fig 7.11b is -0.050 inch. The deflection contours are also of interest and are not easily obtained from the Westergaard solution.

To summarize these comparisons it has been shown that the finite-element method described herein agrees within 2-5 percent with the Westergaard slab-on-foundation solutions which are currently used for pavement design. In addition, the new method readily provides deflection contours. The same is not true with the Westergaard solution although computer programs do exist to solve those equations explicitly.

Real Pavement Slabs

It is helpful to demonstrate that the new method is applicable to real pavement slabs whereas the Westergaard solutions are good only for infinite slabs on uniform foundations. Three examples are included, non-uniform subgrade support, multiple loads, and cracked slabs.

Non-Uniform Subgrade Support - To illustrate non-uniform subgrade support, the three cases described in the paragraph above for center, edge, and corner loadings were rerun with a hole cut in the subgrade centered under the load. For the center load case, the hole 6 feet in diameter cut in the subgrade results in an increased deflection of 40 percent to -0.0084 inch as shown in Fig 7.9c. Figure 7.9d compares the deflected shape for the uniformly and the non-uniformly supported cases. For the non-uniformly supported edge load, a

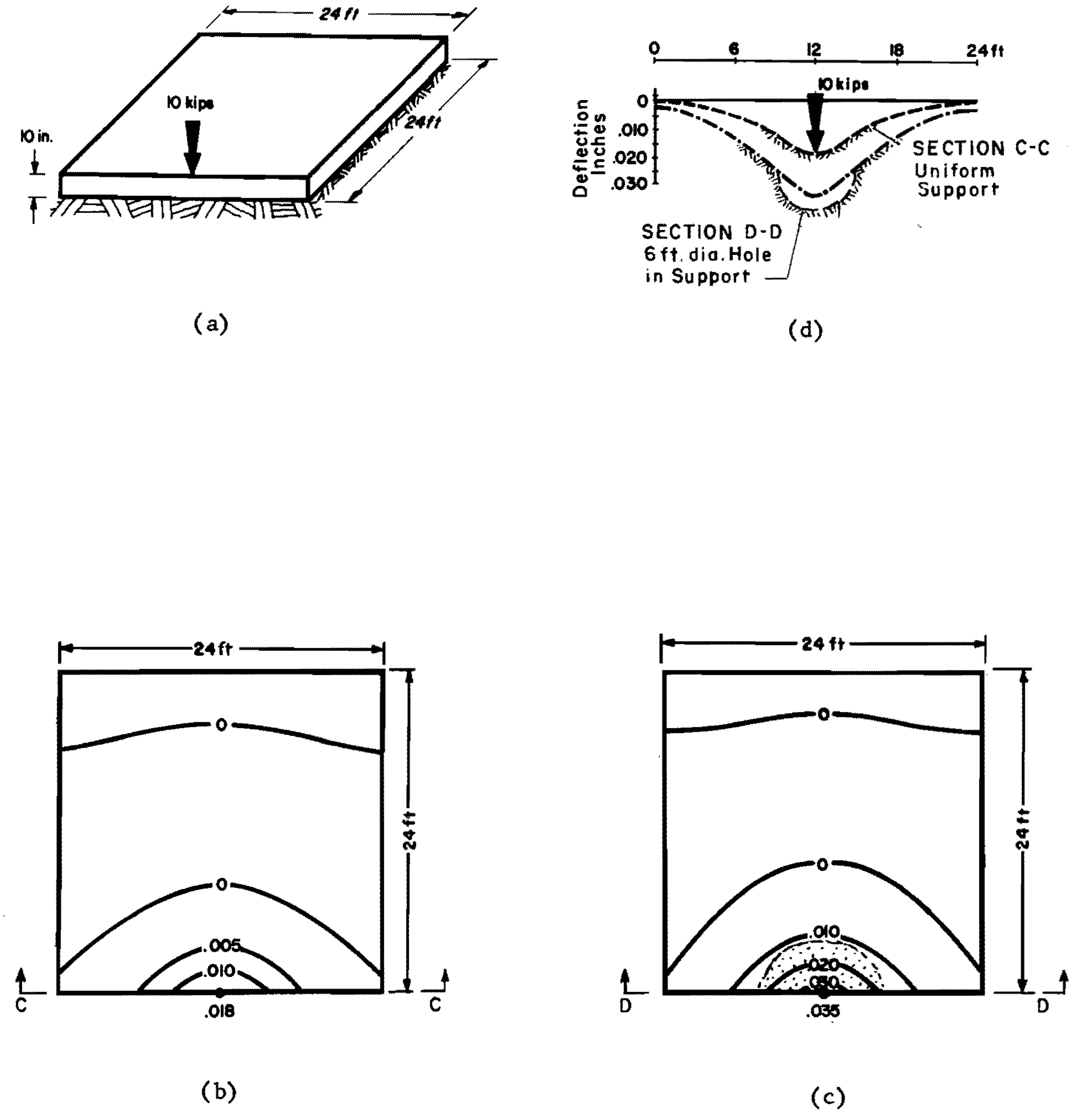


Fig 7.10. Pavement slab subjected to 10-kip wheel load at the edge with and without uniform subgrade support.

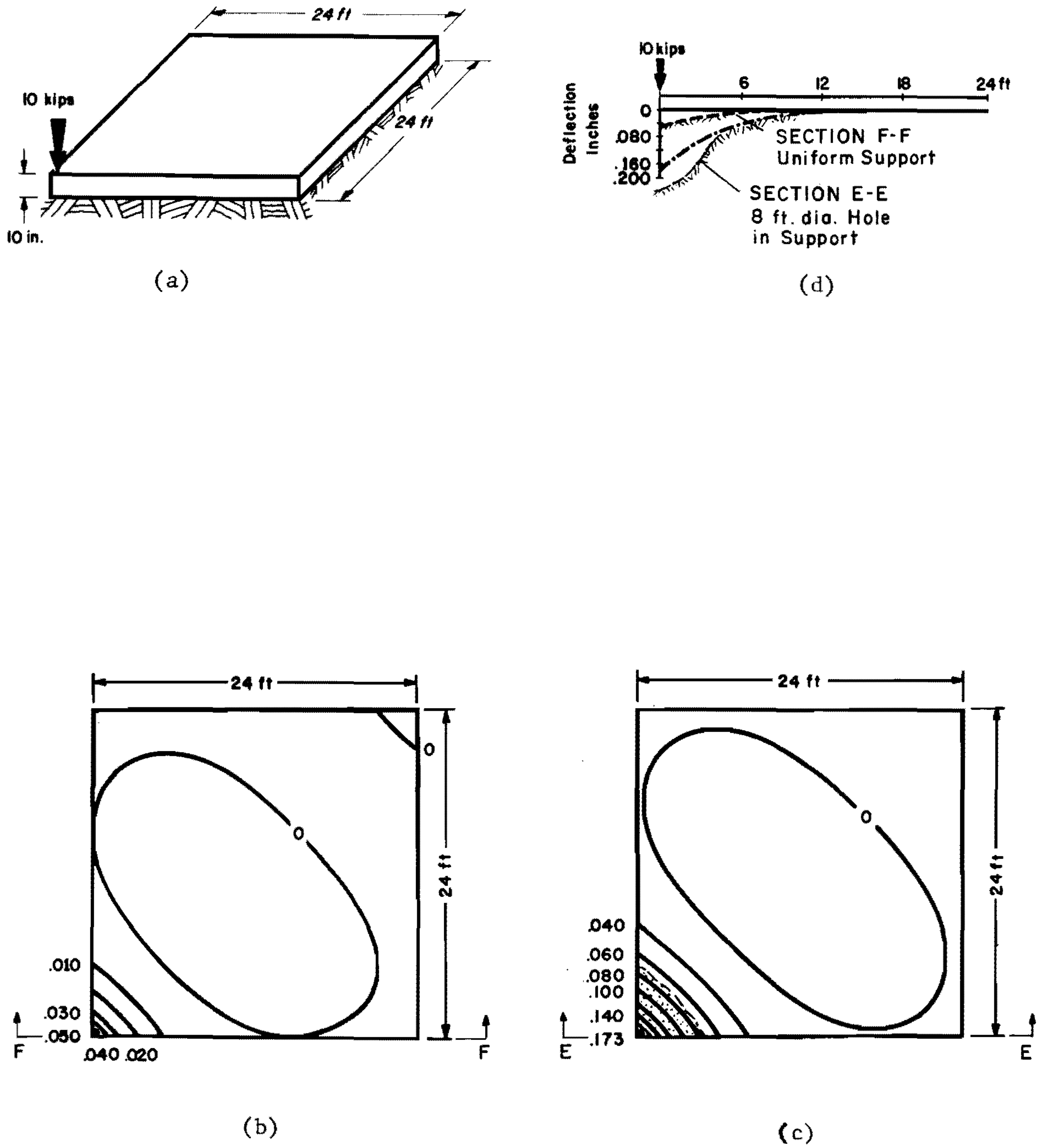
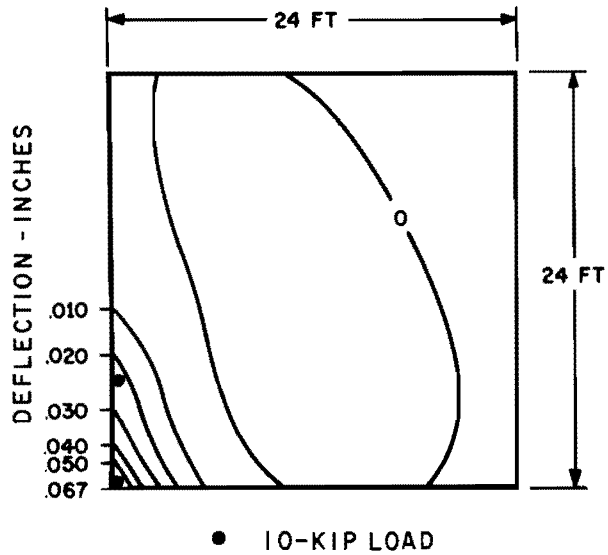


Fig 7.11. Pavement slab subjected to 10-kip wheel load at the corner; with and without uniform subgrade support.

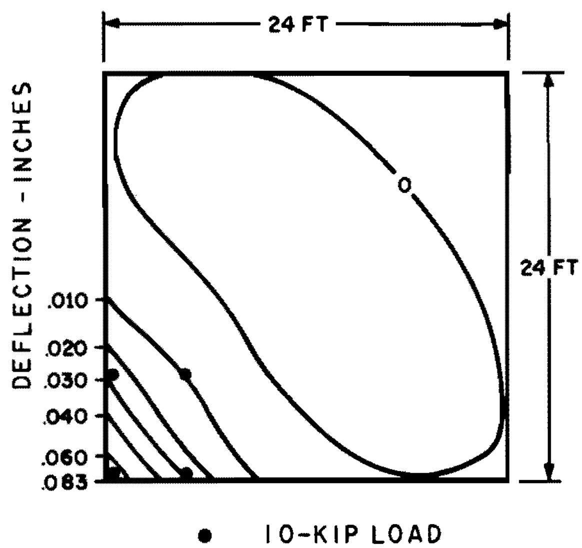
hole 6 feet in diameter is centered under the load. The resulting deflections under the load is -0.35 inch (Fig 7.10c), or nearly double that of the slab with uniform support. Figure 7.10d compares an edge view with and without uniform support. The corner load case has a hole 8 feet in diameter cut in the subgrade centered at the corner. The resulting deflection increased to -0.173 inch, or about 3-1/2 times that of the uniform case as shown in Figs 7.11c and 7.11d. It is not intended to draw conclusions at this time concerning these relative increases in deflection nor their effect on pavement performance. It is merely desired to indicate that the method is easily adaptable to solutions for such non-uniform cases which probably represent a majority of pavement actually in service in the United States.

Multiple Loadings - In order to illustrate the ability of the method to handle multiple loads, the corner load case was re-analyzed (Problem 301) using a pair of 10-kip single wheel loads arranged to form a 20-kip axle located at an edge or joint (see Fig 7.12). The solution was then run (Problem 302) with the addition of a second 20-kip axle to form a 40-kip tandem axle. The results are shown graphically in Fig 7.12. Comparisons of this type can be made with the Westergaard solution although they are extremely difficult and will not be calculated here.

Problem Series 400 - Cracked Slabs - Another problem of interest to pavement designers is that of cracked pavement slabs. The effect of such cracks is dependent upon the open width of the crack, which varies with temperature expansion and contraction of the slab. Such studies are extremely difficult with previous methods but can be made easily with the techniques described here. To illustrate these techniques the pavement slab, 24 feet square and 10 inches thick, described for the Westergaard cases, was used again. This time a crack was introduced through the mid-section of the slab as shown in Fig 7.13 and the load was placed over the crack. It should be recalled that the solu-



(a)



(b)

Fig 7.12. Pavement slab subjected to 20-kip single axle load and 40-kip tandem axle load (Example Problems 301 and 302).

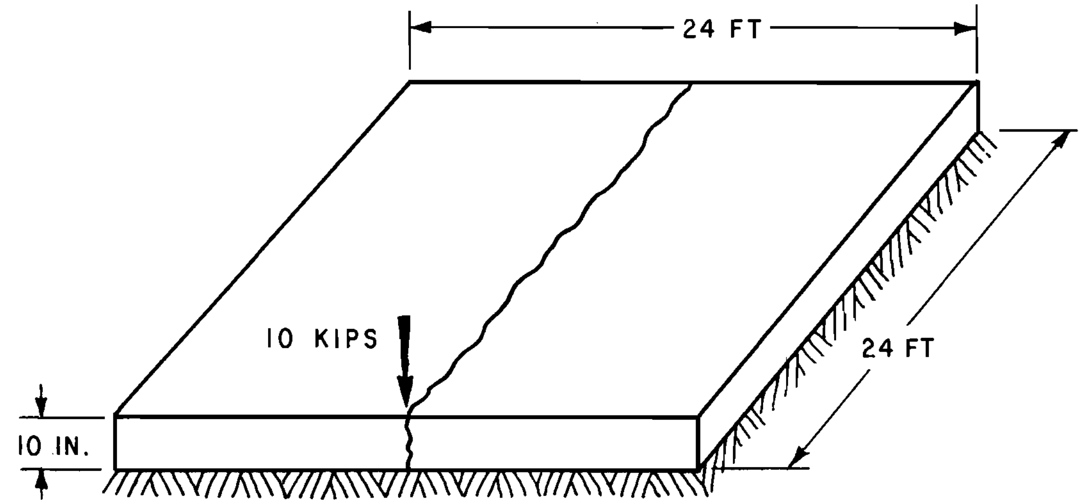


Fig 7.13. Pavement slab containing transverse crack subjected to a 10-kip single wheel load at the edge.

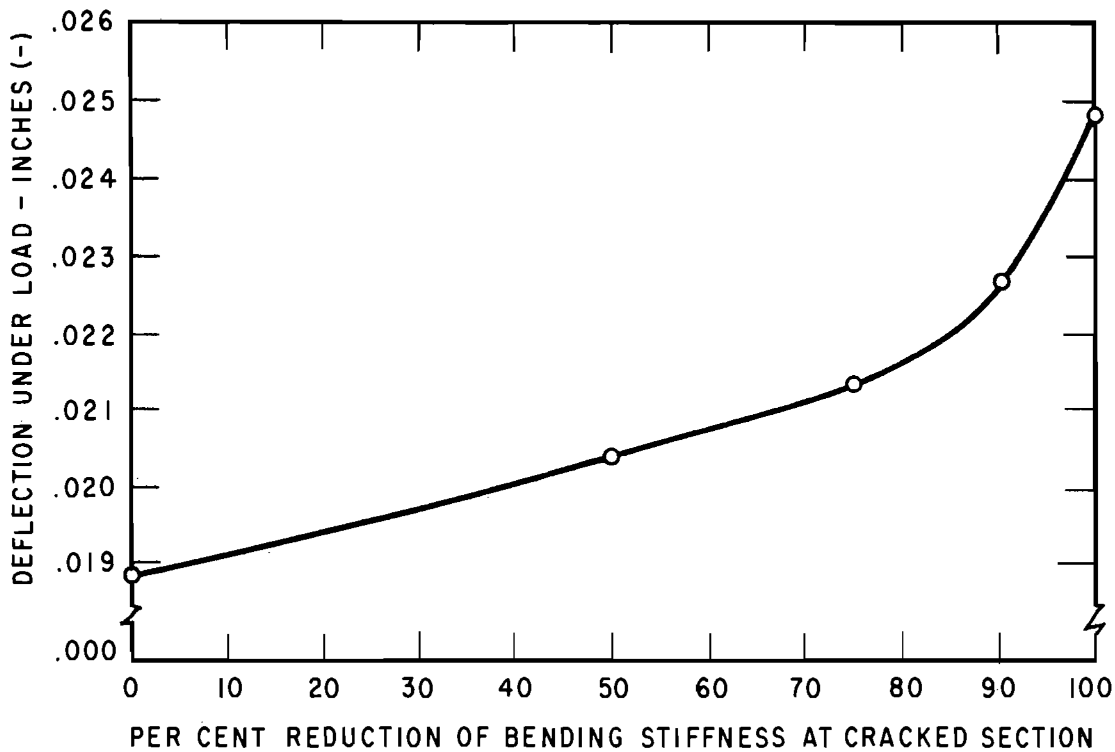


Fig 7.14. Effect of crack severity on deflection under a 10-kip wheel load.

tion for this case uncracked was -0.018 inch deflection under the load. Four subsequent examples were considered. In each case the bending stiffness of the slab across the crack was reduced. The uncracked section had 100 percent stiffness; the completely cracked section had zero bending stiffness. Intermediate cases with 25, 50 and 75 percent of the uncracked stiffness removed were also computed. A plot of the deflection beneath the load versus percent bending stiffness removed is shown in Fig 7.14. Figure 7.15 indicates the trough created at the crack for the zero bending stiffness condition. The ability to analyze such conditions and to compare them with field studies may make it possible to evaluate for the first time load transfer at joints with various jointing systems and dowel assemblies.

Orthotropic Bridge Decks - Problem Series 500

Three examples are included to illustrate the ability to analyze orthotropic plates. First, a simple orthotropic bridge deck was taken from Timoshenko (Ref 55). The problem (Problem 501) is a simple bridge slab supported on caps at each end with I-beam stiffeners included at the opposite edges. The presence of the I-beams at the edges greatly increases the stiffness in the direction parallel to the I-beams which is taken as the x direction in the figure. The I-beams, however, do not substantially increase the stiffness in the y direction since they are very narrow. This results in orthotropic conditions. The deflection at the center of the span under the load is -0.268 inch. The closed-form solution provided by Timoshenko is -0.272 inch (Ref 55, p 214).

The other two examples (Problems 503 and 504) involve more conventional geometric orthotropy. A steel orthotropic bridge deck 10 feet $\times$ 80 feet in plan is shown in Fig 7.16. The deck is 3/8 inch thick. The stiffeners in the x direction are 0.25 inch thick, 9 inches deep, and spaced 12 inches on centers. The y stiffeners are 0.25 inch thick, 4.0 inches deep, and

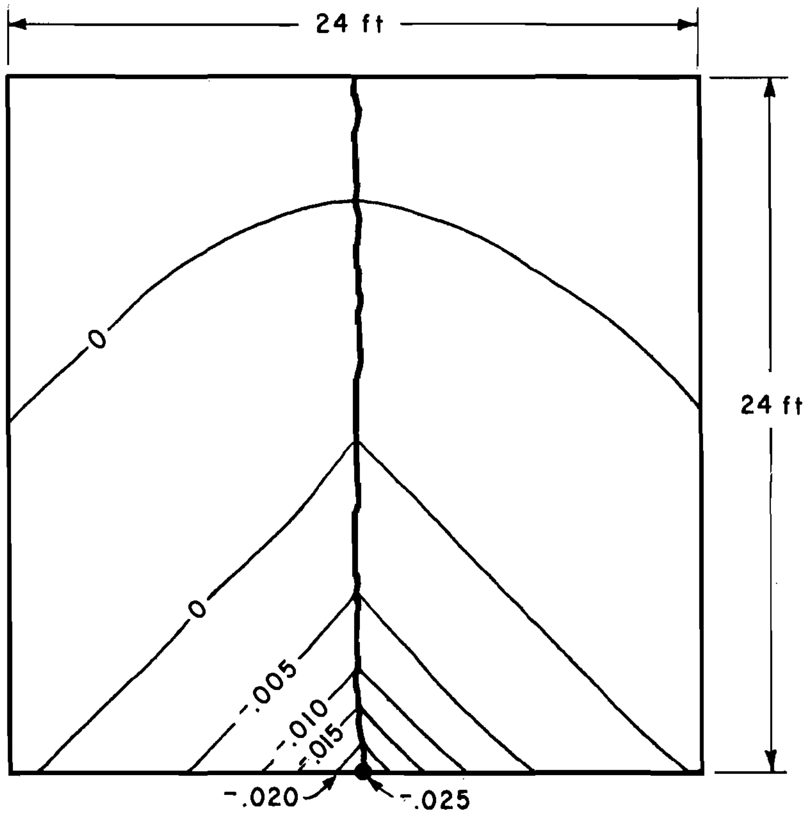


Fig 7.15. Deflection contours of cracked pavement slab under 10-kip wheel load at the edge.

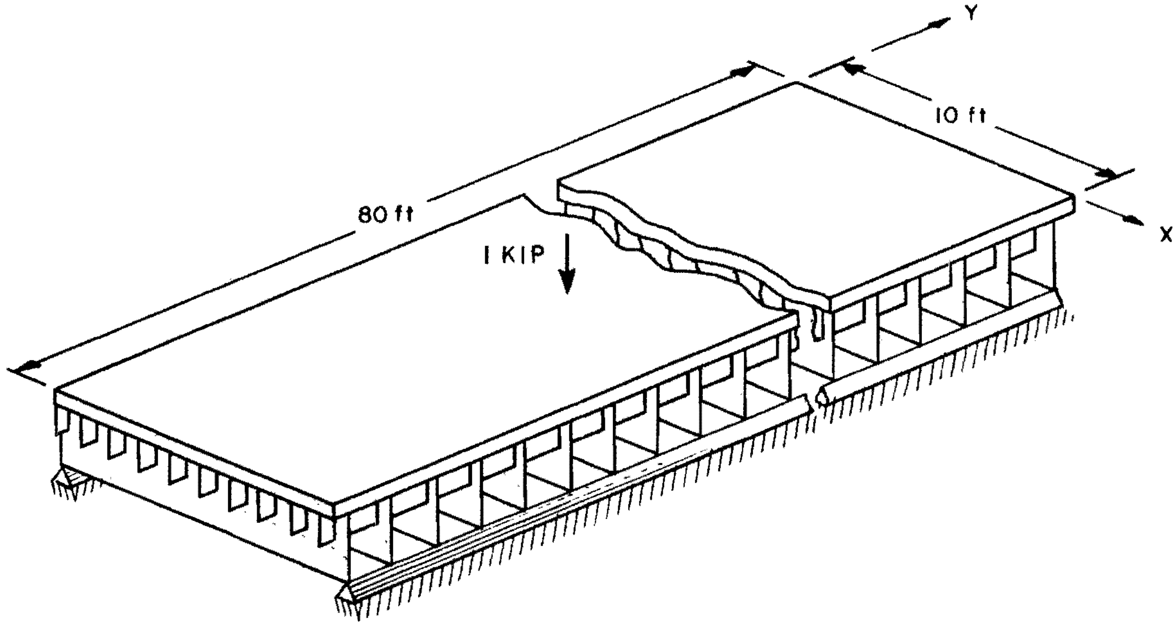


Fig 7.16. Orthotropic plate with 1-kip concentrated load at the center (Example Problems 503 and 504).

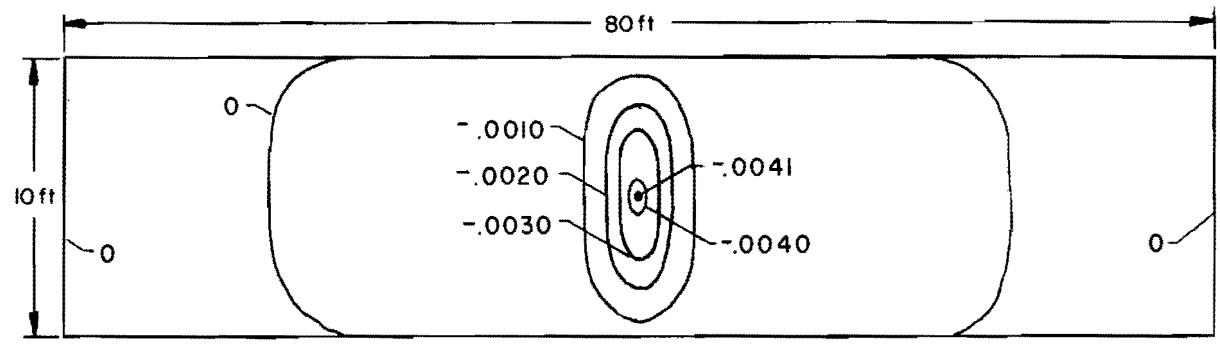


Fig 7.17. Deflection contours for orthotropic plate with 1-kip load (Example Problem 504).

spaced 12 inches on centers. The plate is simply supported along the edges in the y direction. The other two edges are free. Young's modulus is 30 million psi and Poisson's ratio is 0.30. A concentrated load of 1 kip is applied at the center.

In Example Problem 503 the torsional stiffness of the plate was taken as $(1-\nu)D$. This is termed a "torsionally stiff" plate. The maximum deflection for this plate was -0.0009 inch under the load. In reality, thin vertical stiffeners do not offer this much torsional resistance.

Some experimenters (Ref 11) suggest neglecting torsional stiffness for this type of plate. This is often called a "torsionally soft" plate. Problem 504 neglects torsional stiffness. The maximum deflection increases to -0.0041 inch, more than double the stiff case. Deflection contours for this case are shown in Fig 7.17. This might suggest a redesign of the stiffeners for the "torsionally soft" case to take better advantage of torsion. A note of caution is in order at this point. Survey of the literature indicates that the biggest problem with analyzing geometrically orthotropic plates and slabs is the determination of realistic twisting stiffness or twisting moduli for the irregular shape. Appendix A recommends a method of testing to determine such moduli.

Complex Bridge Approach Slab Problem, Problem 601

One of the strengths of the method proposed herein is the ability to handle complex problems with combination loads and a variety of support conditions. Figure 7.18 illustrates such a problem. A 10-inch thick reinforced concrete bridge-approach slab was used. It was supported on one end by the bridge abutment. The other end rests on the embankment. Because of poor compaction which often results in backfill, the soil has settled under the interior of the slab and left a section unsupported. The slab has a centerline joint and a crack which developed from a combination of shrinkage and

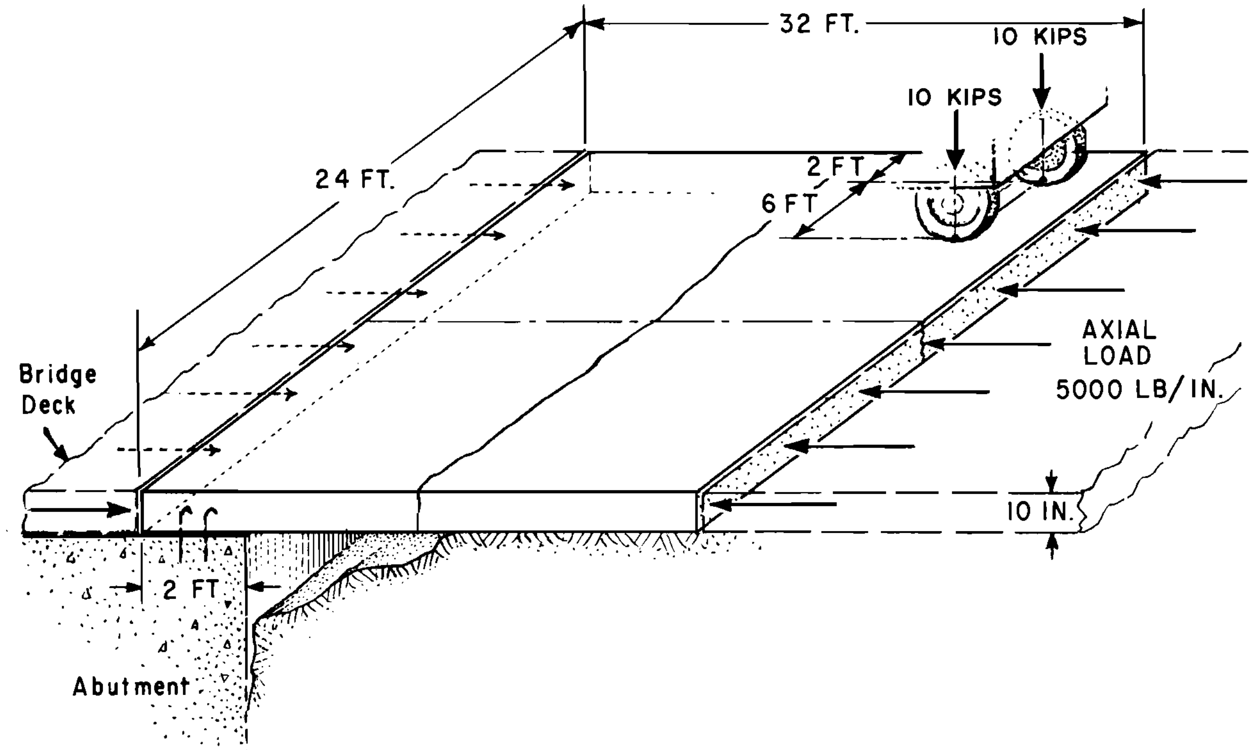


Fig 7.18. Bridge approach slab.

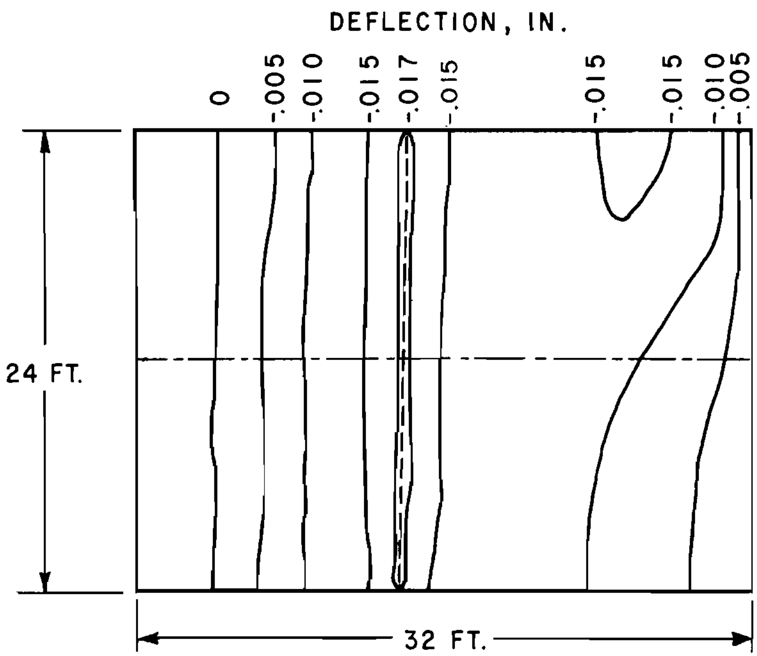


Fig 7.19. Contour of bridge approach slab in Fig 7.18.

and previous overstress. For any non-uniformly supported slab such as this, the dead weight of the slab must be considered when evaluating moment and stresses. This weight acts as a uniform load of 600 pounds per station. Two 10-kip wheel loads were considered in this example. An axial load of 5,000 pounds per inch has been induced by longitudinal expansion of the adjoining pavement. The resulting deflected shape is shown in Fig 7.19. The maximum deflection occurs along the transverse crack and not under the wheel loads. It is virtually impossible to obtain this general solution by any existing methods of analysis.

Summary of Example Problems

A variety of example problems has been solved above to indicate the broad capability of the new method. Two points are worth noting. For those cases having closed-form solutions, the finite-element solution with 8 to 10 increments produced results within 2 to 5 percent of the closed-form solution. If the number of increments was increased to 16, the error comparison reduced to 1-3 percent. Perhaps more important are those cases for which no closed-form solution exists. The finite-element method permits for the first time the evaluation of such cases. It will be helpful if solutions of these new cases can be compared with experimental data obtained from field or model studies.

CHAPTER 8. SUMMARY

This report examines the analysis of plates and pavement slabs. A study of the technical literature resulted in the selection of some sixty helpful references. Many of these papers contain solutions for special-case plates with simple supports and simple load patterns. These solutions are mathematically complex and are often shrouded in jargon not always relatable to real problems. A particular void is noted in the analysis of pavement slabs. The best work available, Westergaard's, is limited by special-case loads and severe assumptions including infinite or semi-infinite plan dimensions and uniform support conditions.

A method has been presented which is not limited by the simplifying assumptions needed for closed-form solutions. The technique is based on a physical model of the problem which is described mathematically. The principal features of the method are:

(1) Representation of the plate or slab by a finite-element model of beam-column elements with freely discontinuous stiffness and load. These line elements are grouped into two systems of orthogonal beams or beam-columns.

(2) A rapid, direct solution of individual beams using recursive techniques.

(3) An alternating-direction iteration method for combining the solutions of the individual beams into a coordinated slab solution.

The finite-element model is helpful in visualizing the problem and forming the solution. The model consists of:

(1) Infinitely stiff and weightless bar elements to connect the joints.

(2) Elastic joints where bending occurs, made of an elastic, homogeneous, and orthotropic material which can be described by four independent elastic constants.

(3) Torsion bars which represent the torsional stiffness of the plate.

(4) Elastic support springs which provide foundation support.

All properties and loads can be freely variable from point to point. Concentrated or distributed loads can be handled including transverse loads, in-plane forces, and external couples. Elastic restraints are provided by vertical support springs.

The alternating-direction iteration method is used to solve the equations describing the behavior of the model because it is well adapted and easy to visualize. The model and method are too complex for hand calculations. A computer program which solves the equations implicitly for the deflection patterns has been developed. The program is written in FORTRAN-63 for the CDC 1604 computer. Minor changes of input formats are required to convert it for use on an IBM 7090. Compile time is 90 to 100 seconds but binary decks are available which compile in about 15 seconds. Automatic plot routines are available for use with the program.

This method has application to a broad variety of complex plate and slab problems which can not be solved by any other existing method. Applications to complex pavement design problems are of particular interest. The use of the method as a tool in stochastic modeling of pavement life and performance studies are of particular interest. Immediate use of the method in developing new pavement design information is suggested.

CHAPTER 9. NEEDED RESEARCH

The finite-element model described herein is a useful tool. The development of such a method opens the door for determination of quantities which heretofore could only be estimated crudely. Such applications of this method are discussed in this chapter. A look into the method and its details pinpoints several areas of study which could lead to improvement.

Improvement of Closure

The closure process is vital in alternating-direction iteration techniques. The methods used here are adequate but far from perfect. Studies are needed to obtain more information about the closure process and the determination of closure parameters.

Study of Material Properties

It would be helpful if the orthotropic properties of materials used in slabs could be determined for exact input into this program. These properties are discussed in Appendix A. In particular, information is needed on the relationship of Poisson's ratio and Young's modulus for orthotropic materials, and torsional rigidity for "torsionally stiff" and "torsionally soft" rib reinforced orthotropic plates. Conventional relationships for such properties are not adequate. Studies in determining an "effective thickness" for geometrically orthotropic slabs would be helpful.

Comparison With Field Measurements

"The proof of any pudding is in the eating." This metaphor is no less true in consideration of mathematical theories of pavement behavior than in other complicated theories. It is desirable that studies of pavement structures be made in the laboratory and in the field, and that corresponding mathematical analyses be made with the finite-element model. Correlation of these data will help in the evaluation of theory and will lead to improved methods for determining some of the unknowns in the field studies. Deflection measurements made under ideal, controlled conditions will be helpful in this regard as will curvature measurements such as those currently being conducted by B. F. McCullough of the Texas Highway Department (Personal communication).

Evaluation of Support Characteristics

Current methods of measuring and specifying pavement support are probably unsatisfactory. It is not adequate to describe a constant k-value for a subgrade or a subbase to be used under a pavement slab. This value is not a linear quantity but is highly dependent upon the deflection of the slab lying immediately above. It is also related to overall slab deflections. The true support value is dependent upon the number of load repetitions, time, and temperature. The first step in such evaluation is the study of non-linear support conditions for the finite-element model.

Time and Temperature Variables

The effect of time and temperature on pavement performance has received some attention in the past few years, although very few people have been successful in evaluating it. It is desirable that viscoelastic and thermoplastic effects be added to this finite-element model.

Dynamic effects are also time-dependent and are important in the study of plate and slab behavior. Harold Salani (Ref 45), one of our colleagues, has previously made strides in applying dynamics to this method. It is desirable that these applications be extended as rapidly as possible.

Design Orientation

It is useful to continue research and development of specific information concerning plate and pavement slab behavior to improve design techniques. It is desirable that the finite-element methods described here be put to use at an early date. Two methods come to mind immediately: (1) The model offers ways of determining realistic stresses for real pavement slabs involving cracks, joints, in-plane forces and other factors not considered by existing methods. This is the information on which design is based. (2) Many special problems such as lug anchors, doweled joints, other load transfer devices, and construction joints can be analyzed. Heretofore, rules of thumb have often been used for such design.

Performance Studies

The finite-element method described here could be used as the basic element to model pavement performance studies under stochastic loads. Such studies are feasible on presently available large computers.

Specific comparisons of various environments can be made with this method. For example, the effect of two environments can be studied by using the same slab structure but applying variations for the two environments. The effect of various loads can then be evaluated, either singly or repetitively on the pavement structure in both environments.

Many other special cases could be cited. As a final example, however, consider criteria for determining load equivalency. The AASHO Road Test (Ref 3) provided a means of comparing performance of pavements constructed exactly alike but subjected to repetitions of different loads. Scrivner (Ref 46) developed a method which compares the effect of these loads by equating the amount of damage each does to the pavement. This technique is based on the assumption that the order of application of the loads has no effect. This is an invalid assumption although it can serve for "normally loaded" pavements. With the finite-element model, various pavements can be evaluated under different loads in ways to determine their relative effects. Such studies would be useful in reinforcing and extending the AASHO Road Test Equivalencies.

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APPENDIX A

DERIVATION OF BENDING MOMENT AND TWISTING MOMENT RELATIONSHIPS FOR THIN ORTHOTROPIC SLABS

APPENDIX A. DERIVATION OF BENDING MOMENT AND TWISTING MOMENT RELATIONSHIPS FOR THIN ORTHOTROPIC SLABS

This report treats problems of plates and slabs which may be of interest to readers in diverse fields ranging from structural mechanics to orthotropic bridge and pavement design. The derivation and formulation of equations used to describe the diverse character of the work may be obscure to some readers. The purpose of this Appendix is to bring together a summary of some of the more important derivations bearing on the problem and from these, to develop the equations which are essential to the over-all solution.

Assumptions

It is often necessary to make assumptions about materials in order to derive usable relationships concerning their behavior. For clarity the assumptions involved in the derivations in this Appendix are stated as follows:

(1) Hooke's Law governs the behavior of the materials.

(2) Plane cross sections of elements lying normal to the middle plane before bending, remain plane and normal to the middle plane during bending.

(3) All deflections are small compared with other dimensions.

(4) The middle plane of the plate before bending is taken to be the xy plane (the neutral surface).

(5) The thickness of the plate is small compared to its plan dimensions.

Notation

The special case of elasticity where the properties of an anisotropic material can be described in three orthogonal planes is called orthotropy. A further condition imposed on orthotropy is that the elastic properties in both the positive and negative directions of a given axis are comparable. The notation used in consideration of the elasticity of such materials under load is diverse and often obscure. The notation used by Hearmon seems to be clear, concise and is used herein (Ref 16).

$c_{i,j}$	lb/in^2	Constants used to relate stress to strain in general anisotropic elasticity; i refers to stress component, j refers to strain component.
C_x	$\frac{in\text{-}lb}{rad}$	Torsional stiffness of slab element about the x-axis.
C_y	$\frac{in\text{-}lb}{rad}$	Torsional stiffness of slab element about the y-axis.
D_x	in-lb	Bending stiffness of an orthotropic plate in the x direction.
D_y	in-lb	Bending stiffness of an orthotropic plate in the y direction.
E_x	lb/in^2	Modulus of Elasticity in the x direction.
E_y	lb/in^2	Modulus of Elasticity in the y direction.
G	lb/in^2	Shear modulus, $\left(\frac{E}{2(1+\nu)}\right)$
G_0	lb/in^2	Approximate orthotropic shear modulus, $\left(\frac{E_x E_y}{E_y(1+\nu_{xy}) + E_x(1+\nu_{yx})}\right)$
M_x	$\frac{in\text{-}lb}{in}$	Bending moment acting on an element of the plate in the x direction.
M_y	$\frac{in\text{-}lb}{in}$	Bending moment acting on an element of the plate in the y direction.
M_{xy}	$\frac{in\text{-}lb}{in}$	Twisting moment tending to rotate the element about the x-axis (clockwise positive).
M_{yx}	$\frac{in\text{-}lb}{in}$	Twisting moment tending to rotate the element about the y-axis (clockwise positive).
$\frac{1}{r_x}$, $\frac{1}{r_y}$	1/in	Curvatures in the neutral surface of the slab.
$S_{i,j}$	in^2/lb	Constants used to relate strain to stress in general anisotropic elasticity; i refers to strain component and j refers to stress component.
x,y		Standard Cartesian coordinate directions.
ϵ_x	in/in	Total strain in x direction.

ϵ_y	in/in	Total strain in y direction.
ϵ_{xx}	in/in	Strain in x direction due to stress applied in x direction.
ϵ_{yy}	in/in	Strain in y direction due to stress applied in y direction.
ϵ_{xy}	in/in	Strain in y direction due to stress applied in x direction.
ϵ_{yx}	in/in	Strain in x direction due to stress applied in y direction.
$\gamma_{xy} = \gamma_{yx}$	in/in	Shear strain in the xy plane.
ν_{xy}		Poisson's ratio which results in strain in the y direction when stress is applied in the x direction.
ν_{yx}		Poisson's ratio which results in strain in the x direction when stress is applied in the y direction.
σ_x	lb/in^2	Stress applied in x direction.
σ_y	lb/in^2	Stress applied in y direction.
$\tau_{xy} = \tau_{yx}$	lb/in^2	Shear stress in the xy plane.

Stress-Strain Relationships

If an element such as Fig A.2 is subjected to a stress σ_x , the following relationships obtain

$$E_x = \frac{\sigma_x}{\epsilon_{xx}} \tag{A.1}$$

$$\epsilon_{xx} = \frac{\sigma_x}{E_x} \tag{A.2}$$

$$\nu_{xy} = -\frac{\epsilon_{xy}}{\epsilon_{xx}} \tag{A.3}$$

$$\epsilon_{xy} = -\nu_{xy}\epsilon_{xx} = \frac{-\nu_{xy}\sigma_x}{E_x} \tag{A.4}$$

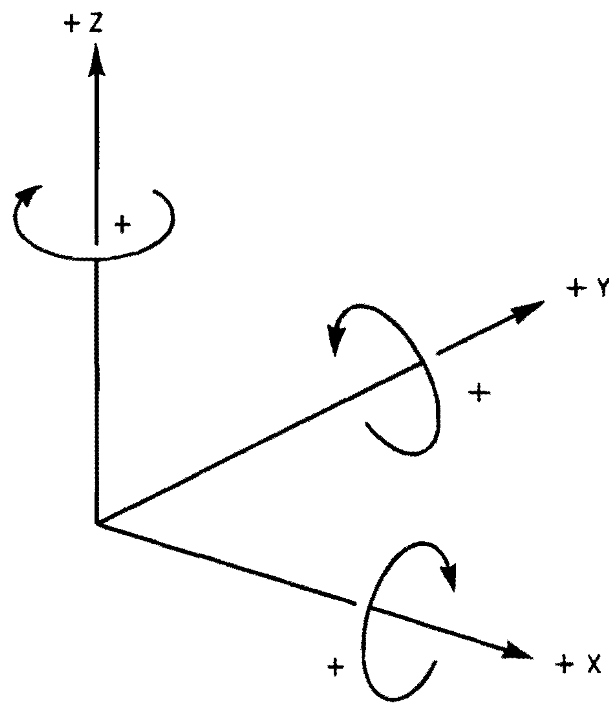


Fig A.1. Sign convention.

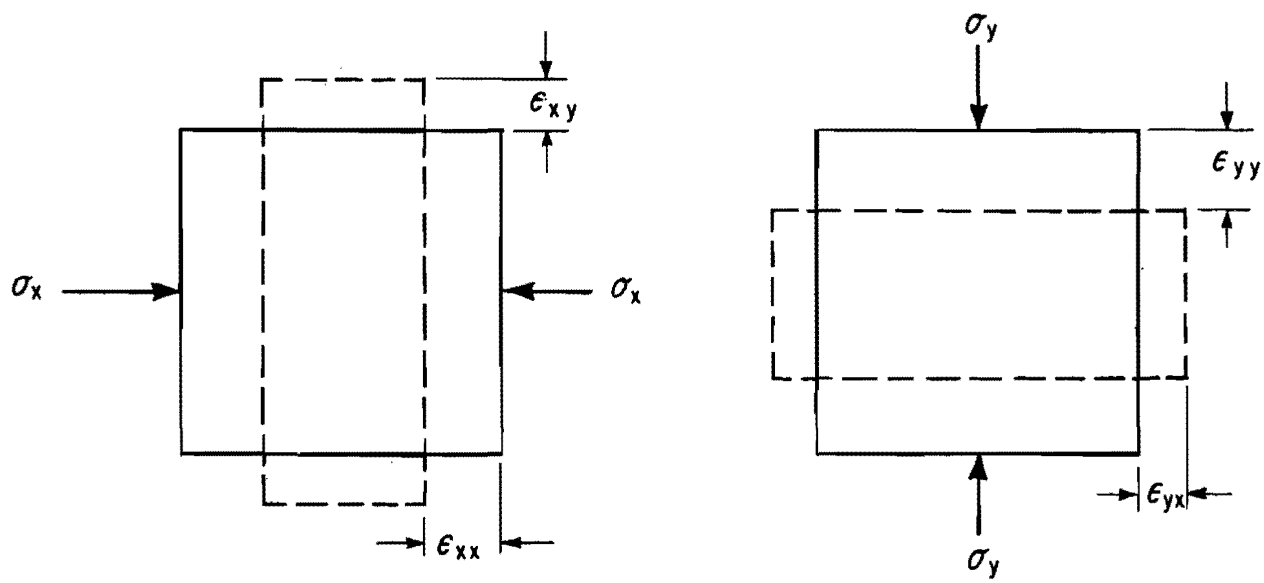


Fig A.2. Plane stress element.

Likewise considering the element subjected to a stress σ_y in the y direction obtains

$$E_y = \frac{\sigma_y}{\epsilon_{yy}} \tag{A.5}$$

$$\epsilon_{yy} = \frac{\sigma_y}{E_y} \tag{A.6}$$

$$\nu_{yx} = -\frac{\epsilon_{yx}}{\epsilon_{yy}} \tag{A.7}$$

$$\epsilon_{yx} = -\nu_{yx}\epsilon_{yy} = \frac{-\nu_{yx}\sigma_y}{E_y} \tag{A.8}$$

Under combined stress by superposition ϵ_x , the total strain in the x direction, is

$$\epsilon_x = \epsilon_{xx} + \epsilon_{yx} \tag{A.9}$$

Likewise,

$$\epsilon_y = \epsilon_{xy} + \epsilon_{yy} \tag{A.10}$$

Then by proper substitution of the equations above,

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}}{E_y}\sigma_y \tag{A.11}$$

$$\epsilon_y = \frac{-\nu_{xy}}{E_x}\sigma_x + \frac{\sigma_y}{E_y} \tag{A.12}$$

Or, in terms of standard orthotropic coefficients (see discussion, Chapter 2),

$$\epsilon_x = S_{11}\sigma_x + S_{12}\sigma_y \tag{A.13}$$

$$\epsilon_y = S_{21}\sigma_x + S_{22}\sigma_y \tag{A.14}$$

where

$$S_{11} = \frac{1}{E_x} \tag{A.15}$$

$$S_{12} = \frac{-\nu_{yx}}{E_y} \tag{A.16}$$

$$S_{21} = \frac{-\nu_{xy}}{E_x} \tag{A.17}$$

$$S_{22} = \frac{1}{E_y} \tag{A.18}$$

$$S_{13} = S_{23} = S_{31} \; . \; . \; . = 0 \tag{A.19}$$

It is helpful to solve Eq A.11 for stress σ_x , σ_y in terms of ϵ_x , ϵ_y ;

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}}{E_y} \sigma_y \tag{A.20}$$

$$\epsilon_y = - \frac{\nu_{xy}}{E_x} \sigma_x + \frac{\sigma_y}{E_y} \tag{A.21}$$

$$\sigma_y = \left(\epsilon_y + \frac{\nu_{xy}}{E_x} \sigma_x \right) E_y \tag{A.22}$$

Substituting Eq A.22 into Eq A.20,

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}}{E_y} \left(\epsilon_y + \frac{\nu_{xy}}{E_x} \sigma_x \right) E_y \tag{A.23}$$

$$\epsilon_x = \frac{\sigma_x}{E_x} - \nu_{yx} \epsilon_y - \frac{\nu_{xy} \nu_{yx}}{E_x} \sigma_x \tag{A.24}$$

$$\epsilon_x + \nu_{yx} \epsilon_y = \sigma_x \left(\frac{1 - \nu_{xy} \nu_{yx}}{E_x} \right) \tag{A.25}$$

$$\sigma_x = \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \left(\epsilon_x + \nu_{yx} \epsilon_y \right) \tag{A.26}$$

At this point it is convenient to define some new orthotropic elastic properties (after Timoshenko, Ref 55) as follows:

$$E_x' = \frac{E_x}{1 - \nu_{xy} \nu_{yx}} \tag{A.27}$$

$$E'' = \nu_{yx} E'_x \tag{A.28}$$

$$\sigma_x = E'_x \epsilon_x + E'' \epsilon_y \tag{A.29}$$

Likewise, it can be shown that

$$\sigma_y = \frac{E_y}{1 - \nu_{xy}\nu_{yx}} \left(\epsilon_y + \nu_{xy}\epsilon_x\right) \tag{A.30}$$

from which other constants of interest can be defined, i.e.,

$$E'_y = \frac{E_y}{1 - \nu_{xy}\nu_{yx}} \tag{A.31}$$

$$E'' = \nu_{xy} E'_y \tag{A.32}$$

$$\sigma_y = E'' \epsilon_x + E'_y \epsilon_y \tag{A.33}$$

Then in terms of these new constants,

$$\sigma_x = E'_x \left(\epsilon_x + \nu_{yx}\epsilon_y\right) \tag{A.34}$$

$$\sigma_y = E'_y \left(\nu_{xy}\epsilon_x + \epsilon_y\right) \tag{A.35}$$

Equations A.28 and A.32 indicate that

$$\nu_{yx} E_x = \nu_{xy} E_y \tag{A.36}$$

because

$$E'' \left(1 - \nu_{xy}\nu_{yx}\right) = \nu_{xy} E_y = \nu_{yx} E_x \tag{A.37}$$

This proof is given later in this appendix. From Eq A.37, given ν_{xy} , E_x and E_y , it may be convenient to calculate ν_{yx} as follows:

$$\nu_{yx} = \nu_{xy} \frac{E_y}{E_x} \tag{A.38}$$

<u>Relation of ν_{xy} and ν_{yx}</u>

In many texts the claim is set forth that

$$c_{rq} = c_{qr} \tag{A.39}$$

so that in the plane stress case

$$c_{12} = c_{21} \tag{A.40}$$

Therefore,

$$\frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x} \tag{A.41}$$

It becomes desirable to prove this claim as follows (due to Hearmon, page 13, Ref 16).

If the strain components in a unit cube of an elastic body are increased from ϵ_q to $\epsilon_q + d\epsilon_q$, the work done is

$$dW = \sigma_q \, d\epsilon_q \tag{A.42}$$

using generalized Hooke's Law,

$$\sigma_1 = c_{11}\epsilon_1 + c_{12}\epsilon_2 + c_{13}\epsilon_3 + c_{14}\epsilon_4 + c_{15}\epsilon_5 + c_{16}\epsilon_6 \tag{A.43}$$

or, in compressed notation,

$$\sigma_q = \sum_{r=1}^{6} c_{qr}\,\epsilon_r \tag{A.44}$$

Substituting,

$$dW = \Sigma\, c_{qr}\,\epsilon_r \;\, d\epsilon_q \tag{A.45}$$

thus

$$\frac{\partial W}{\partial \epsilon_q} = \sigma_q = \Sigma\, c_{qr}\,\epsilon_r \tag{A.46}$$

and

$$\frac{\partial^2 W}{\partial \epsilon_q \,\partial \epsilon_r} = c_{qr} \tag{A.47}$$

one of the general matrix coefficients. Likewise, it can be shown that

$$\frac{\partial^2 W}{\partial \epsilon_r \,\partial \epsilon_q} = c_{rq} \tag{A.48}$$

and therefore

$$c_{rq} = c_{qr} \tag{A.49}$$

since the order of differentiation is immaterial. This proof shows that coefficients with comparable though reversed subscripts are equal.

For plane stress orthotropy this reduces the number of independent constants involved to four, since

$$\sigma_x = c_{11}\epsilon_x + c_{12}\epsilon_y \tag{A.50}$$

$$\sigma_y = c_{21}\epsilon_x + c_{22}\epsilon_y \tag{A.51}$$

It is recognized from Eqs A.26, A.27 and A.28 that

$$c_{11} = \frac{E_x}{1 - \nu_{xy}\nu_{yx}} = E_x' \,* \tag{A.52}$$

$$c_{12} = c_{21} = \frac{\nu_{yx}E_x}{1 - \nu_{xy}\nu_{yx}} = \frac{\nu_{xy}E_y}{1 - \nu_{xy}\nu_{yx}} = E'' \,* \tag{A.53}$$

$$c_{22} = \frac{E_y}{1 - \nu_{yx}\nu_{xy}} = E_y' \,* \tag{A.54}$$

Bending Moment Relationships

For a plate subjected to pure bending as shown in Fig A.3, let $1/r_x$ and $1/r_y$ denote as usual the curvatures in the neutral surface of the slab, a differential element of which is shown in Fig A.4. As in the case of a beam, the unit elongation in the x and y directions respectively of an elemental lamina "a b c d" (Fig A.4) at a distance z from the neutral surface are equal to

$$\epsilon_x = \frac{z}{r_x} \qquad \epsilon_y = \frac{z}{r_y} \tag{A.55}$$

* Timoshenko Notation (Ref 55).

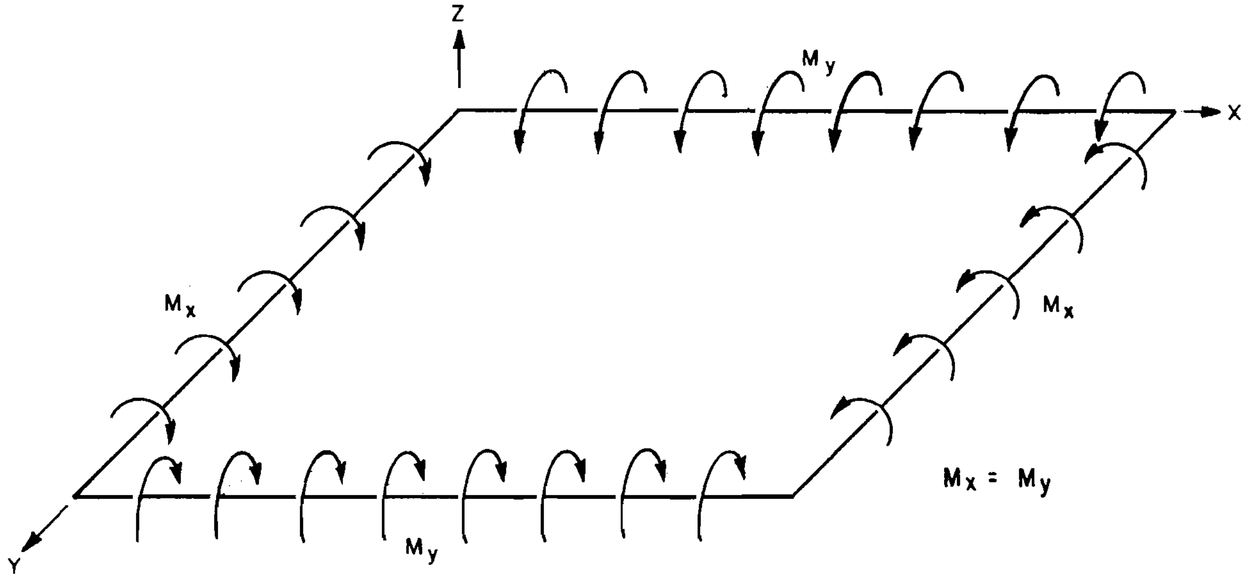


Fig A.3. Plate subjected to uniform bending moment at all edges.

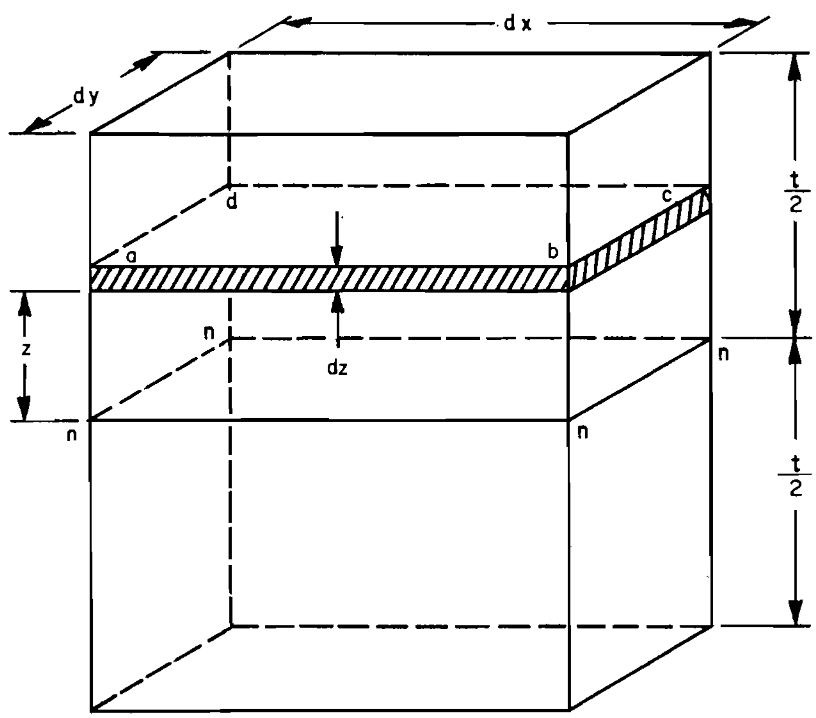


Fig A.4. Differential element from an elastic plate.

where $1/r_x$ and $1/r_y$ are the curvature in the x and y directions respectively and are defined as

$$\frac{1}{r_x} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right) = \frac{\partial^2 w}{\partial x^2} \tag{A.56}$$

$$\frac{1}{r_y} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right) = \frac{\partial^2 w}{\partial y^2} \tag{A.57}$$

It follows then that

$$\epsilon_x = z\,\frac{\partial^2 w}{\partial x^2} \tag{A.58}$$

$$\epsilon_y = z\,\frac{\partial^2 w}{\partial y^2} \tag{A.59}$$

Recognize lamina "a b c d" as the same as the plane stress element in Fig A.2. Equations A.13 and A.14 are seen to represent the corresponding stresses in lamina "a b c d". The stresses are proportional to the distance z of the lamina from the neutral surface.

The applied moments M_x and M_y can be visualized as normal stresses distributed over the lateral sides of the element. Integrating over these sides we obtain the equations

$$\int_{-t/2}^{+t/2} \sigma_x \; z \; dy \; dz = M_x \, dy \tag{A.60}$$

$$M_x = \int_{-t/2}^{+t/2} E_x' \,(\epsilon_x + \nu_{yx}\epsilon_y)\; z \; dz \tag{A.61}$$

$$= \int_{-t/2}^{+t/2} E_x' \left(z\,\frac{\partial^2 w}{\partial x^2} + \nu_{yx}\, z\,\frac{\partial^2 w}{\partial y^2}\right) z \; dz \tag{A.62}$$

$$= E_x' \left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx}\,\frac{\partial^2 w}{\partial y^2}\right) \frac{z^3}{3} \Bigg|_{-t/2}^{+t/2} \tag{A.63}$$

$$= E_x' \frac{\left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2}\right)}{3} \left[\frac{t_x^3}{8} - \left(-\frac{t_x^3}{8}\right)\right] \tag{A.64}$$

$$= \frac{E_x' t^3}{12} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2}\right) \tag{A.65}$$

$$M_x = D_x \left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2}\right) \tag{A.66}$$

where

$$D_x = \frac{E_x' t_x^2}{12} = \frac{E_x t_x^3}{12 \ (1 - \nu_{yx}\nu_{xy})} \tag{A.67}$$

Likewise it can be shown that

$$M_y = D_y \left(\nu_{xy} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{A.68}$$

where

$$D_y = \frac{E_y' t_x^3}{12} = \frac{E_y t_x^3}{12 \ (1 - \nu_{yx}\nu_{xy})} \tag{A.69}$$

Twisting Moments - Derivation of Torsion Bars

In order to examine the twisting of a thin plate, look again at a differential element as shown in Fig A.5. If this element is subjected to twisting moment M_{yx} , then M_{xy} will be present to provide equilibrium. Under these conditions lamina "a b c d" will be subjected to shear stresses as shown in Fig A.6 with the resulting distortions shown in Fig A.7. From geometry and static equilibrium the following relationships can be derived.

By definition from elasticity (Ref 32), shear strain is

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{A.70}$$

Since

$$\tau_{xy} = G \ \gamma_{xy} \tag{A.71}$$

where τ_{xy} is the shearing stress in inch-pounds per inch of plate width for

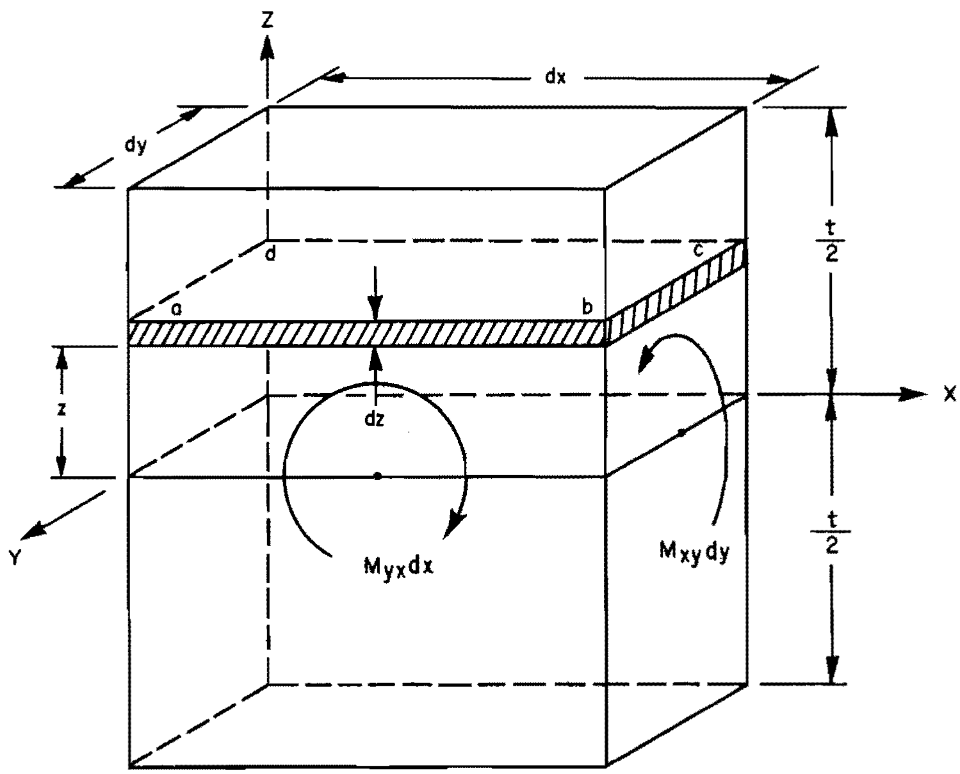


Fig A.5. Differential element of a plate subjected to twisting moments.

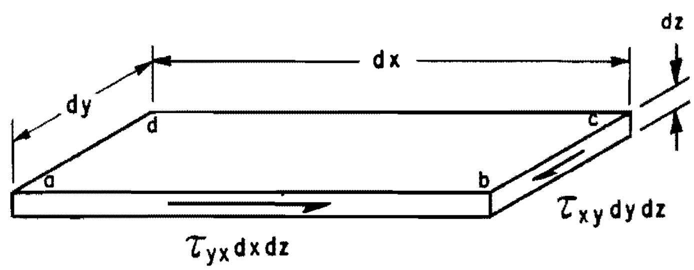


Fig A.6. Lamina "abcd" from Fig A.5.

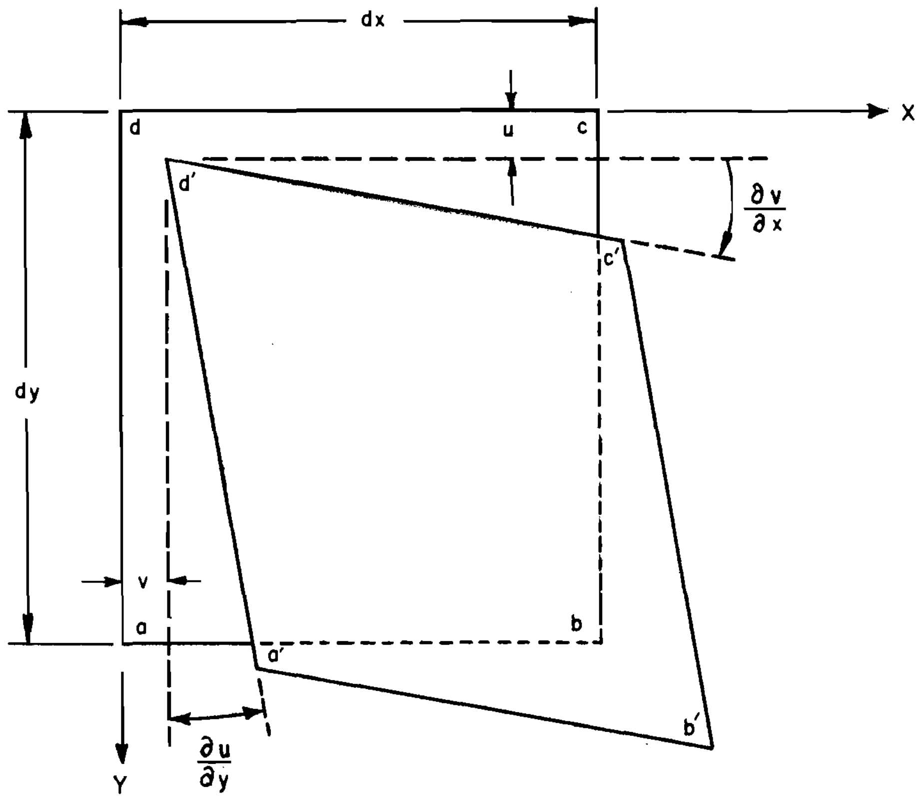


Fig A.7. Plane distortion of lamina "abcd" .

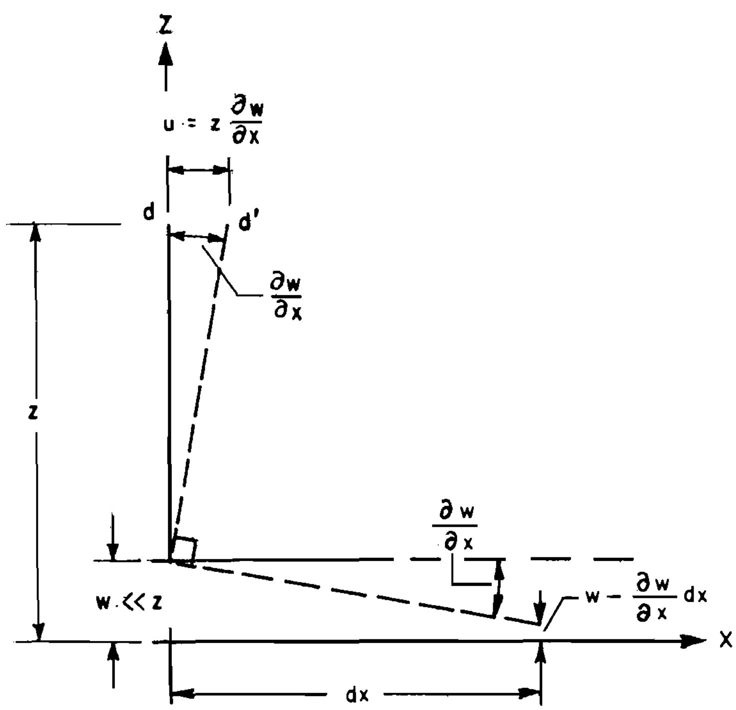


Fig A.8. Rotation of lamina "abcd" in plane xz.

isotropic materials and

$$G = \frac{E}{2\ (1+\nu)} \tag{A.72}$$

is called the shear modulus. Now

$$\tau_{xy} = G\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{A.73}$$

From Figure A.8, assuming that plane cross sections remain plane,

$$u = z\,\frac{\partial w}{\partial x}$$

Similarly for the y-z plane,

$$v = z\,\frac{\partial w}{\partial y}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(z\,\frac{\partial w}{\partial x}\right) = z\,\frac{\partial^2 w}{\partial y \partial x} \tag{A.74}$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}\left(z\,\frac{\partial w}{\partial y}\right) = z\,\frac{\partial^2 w}{\partial x \partial y} \tag{A.75}$$

Since the order of differentiation is immaterial, the shearing stress becomes

$$\tau_{xy} = 2Gz\,\frac{\partial^2 w}{\partial x \partial y}$$

Integrating over the sides of the element from $-t/2$ to $t/2$ obtains the twisting moments,

$$-\,dyM_{xy} = \int_{-t/2}^{+t/2} \tau_{xy}\ dy\ z\ dz \tag{A.76}$$

$$= \int_{-t/2}^{+t/2} 2Gz\,\frac{\partial^2 w}{\partial x \partial y}\ z\ dz\ dy$$

$$-\,\cancel{dy}\,M_{xy} = 2G\,\frac{\partial^2 w}{\partial x \partial y}\,\frac{z^3}{3}\,\Bigg|_{-t/2}^{+t/2}\cancel{dy}$$

$$- M_{xy} = \frac{2G}{3} \frac{\partial^2 w}{\partial x \partial y} \left[\frac{t^3}{8} + \frac{t^3}{8} \right] \tag{A.77}$$

$$- M_{xy} = \frac{Gt^3}{6} \frac{\partial^2 w}{\partial x \partial y} \tag{A.78}$$

or, from Eq A.72,

$$- M_{xy} = D\ (1-\nu) \frac{\partial^2 w}{\partial x \partial y} \tag{A.79}$$

Likewise, except for a reversed sign,

$$M_{yx} = \frac{G\ t^3}{6} \frac{\partial^2 w}{\partial y \partial x} \tag{A.80}$$

$$M_{yx} = D\ (1 - \nu) \frac{\partial^2 w}{\partial y \partial x} \tag{A.81}$$

then

$$- M_{xy} = M_{yx} \tag{A.82}$$

which is correct for the chosen sign convention.

Derivation of Torsional Stiffness for Orthotropic Plates

Looking at Fig A.9 and remembering that it has previously been shown for isotropic slabs that the unit twisting moments are

$$M_{yx} = \frac{G\ t^3}{6} \frac{\partial^2 w}{\partial x \partial y} \tag{A.83}$$

$$- M_{xy} = \frac{G\ t^3}{6} \frac{\partial^2 w}{\partial y \partial x} \tag{A.84}$$

Then, for equilibrium,

$$M_{yx} = - M_{xy} \tag{A.85}$$

where twisting moments are given as unit values with dimensions inch-pound per inch of plate width. Likewise in Fig A.10, observe a segment of an orthotropic slab. For equilibrium

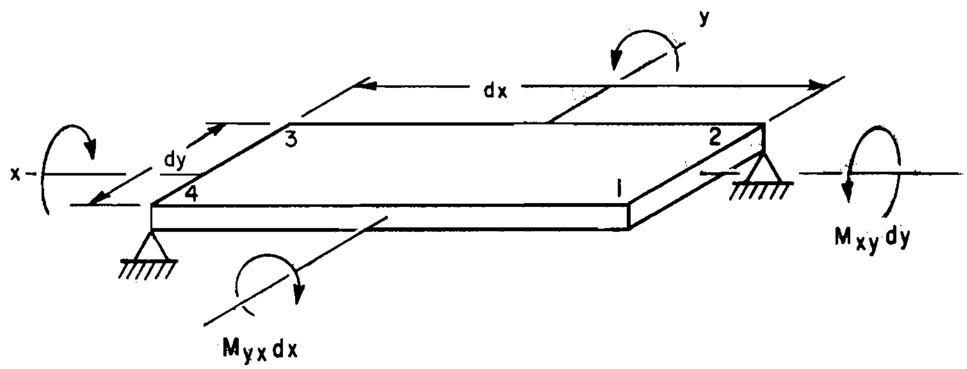


Fig A.9. Isotropic plate segment subjected to pure torsion.

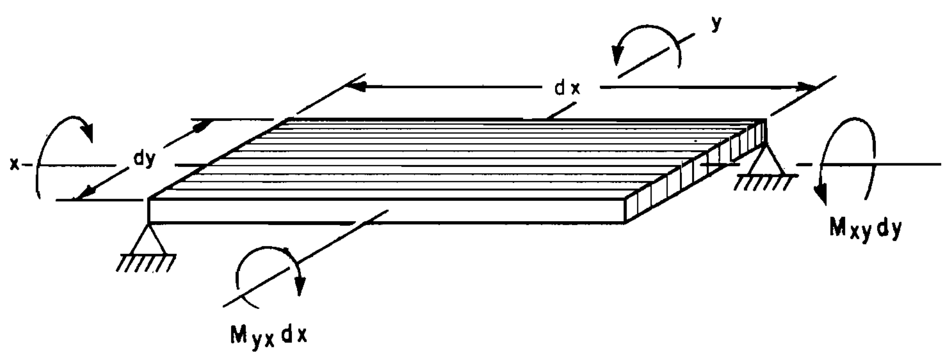


Fig A.10. Orthotropic plate segment subjected to pure torsion.

$$dx\, M_{yx} = -\, dy\, M_{xy} \tag{A.86}$$

For the present define the resistance to twisting about the y-axis as C_x and the resistance to twisting about the x-axis as C_y, then

$$-M_{xy} = C_y \frac{\partial^2 w}{\partial x \partial y} \tag{A.87}$$

$$M_{yx} = C_x \frac{\partial^2 w}{\partial y \partial x} \tag{A.88}$$

Then

$$C_y \frac{\partial^2 w}{\partial x \partial y} = C_x \frac{\partial^2 w}{\partial y \partial x} \tag{A.89}$$

Remembering that the order of differentiation is immaterial, then

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial y \partial x} \tag{A.90}$$

Therefore, from Equations A.83 and A.84, it is seen that

$$C_x = C_y = \frac{G\, t^3}{6} \tag{A.91}$$

which is confirmed by the work of Timoshenko (Ref 55, page 365).

For the isotropic case,

$$G = \frac{E}{2\ (1+\nu)} \tag{A.92}$$

For the orthotropic case it has been suggested that G_0 must satisfy Eq A.93 (Ref 33).

$$G_0 = \frac{E_x E_y}{E_y (1+\nu_{xy}) + E_x (1+\nu_{yx})} \tag{A.93}$$

which is used herein unless determined from an independent test.

Experimental Determination of Torsional Stiffness

If an actual specimen is set up as shown in Fig A.11, G can be determined as follows by neglecting shear strain. Application of load 2β at the

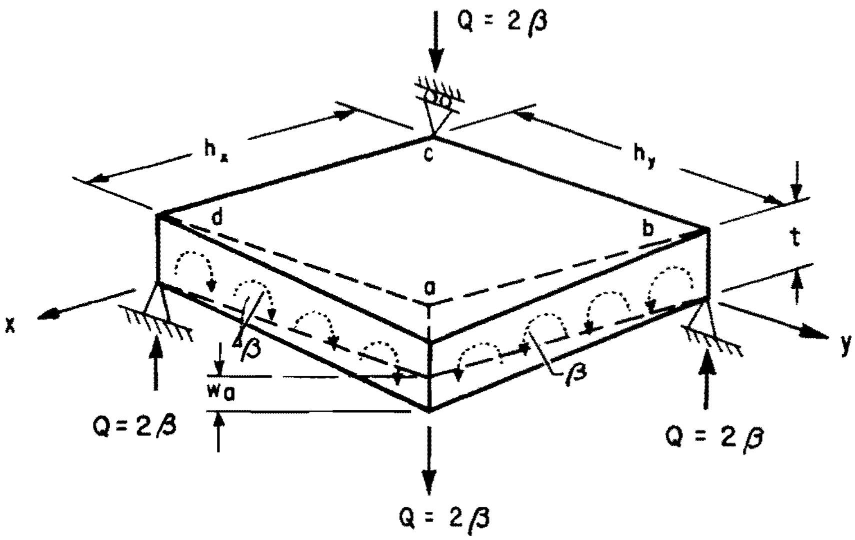


Fig A.11. Small plate subjected to torsion by corner loads.

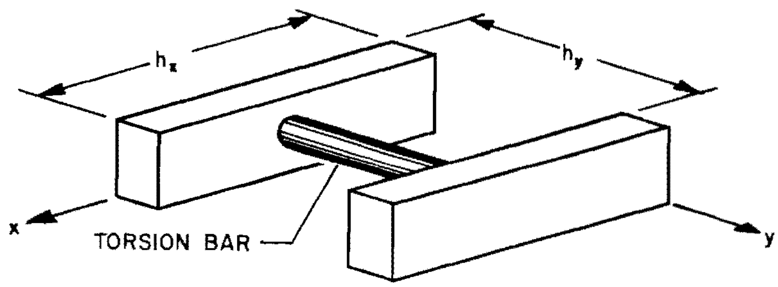


Fig A.12. Partial finite-element representation of Fig A.11 illustrating torsional stiffness in one direction only.

corner is equivalent to applying unit twisting moments β along both edges. Fig A.14 may be helpful in visualizing this.

$$\Sigma M_{xy} = h_y \beta \tag{A.94}$$

The total applied couple on each bar gives a resultant pair of forces at the corners. Looking at the x-axis, the force is shown to be β .

$$\frac{\Sigma M_{xy}}{h_y} = \frac{h_y \beta}{h_y} = \beta \tag{A.95}$$

Likewise, about the x-axis,

$$\frac{h_x \beta}{h_x} = \beta \tag{A.96}$$

thus,

$$Q = \beta + \beta = 2\beta \tag{A.97}$$

at each corner. Since

$$\beta = \frac{G\ t^3}{6} \frac{\partial^2 w}{\partial x \partial y} \tag{A.98}$$

therefore

$$G = \frac{6\beta}{t^3 \frac{\partial^2 w}{\partial x \partial y}} \tag{A.99}$$

Substituting finite differences gives

$$G = \frac{\frac{3Q}{t^3}}{-w_a + w_b - w_c + w_d} \tag{A.100}$$

$$G = \frac{3Qh_x h_y}{t^3(-w_a + w_b - w_c + w_d)} \tag{A.101}$$

but, since b , c and d are restrained,

$$w_b = w_c = w_d = 0 \tag{A.102}$$

Ignoring the sign of w_a and taking absolute values,

$$G = \frac{3\ Q\ h_x\ h_y}{t^3\ w_s} \qquad \left(\frac{\text{lbs} \times \text{in} \times \text{in}}{\text{in}^3 \times \text{in}} = \text{lb/in}^2\right) \tag{A.103}$$

Thus

$$C_y = C_x = \frac{Gt^3}{6} = \frac{Q\ h_x\ h_y}{2\ w_s} \tag{A.104}$$

It is important to remember that plate moments are unit values with dimension, pounds; and, that lines $\overline{ab}$, $\overline{ad}$, $\overline{bc}$, and $\overline{cd}$ will remain straight for this type of anticlastic bending. For proof see Timoshenko (Ref 55).

Finite Model Torsion Bar

The development of G is mathematically rigorous and suitable for use in solution of the biharmonic equation. For easy solution of the finite system in this report, it is desirable to implement the model with a finite element to carry the torque or twisting moments. First, reconsider the slab segment in Fig A.11 as two rigid x-beam elements of length h_x spaced some distance h_y apart and connected by a torsion bar (see Fig A.12) with constant C_y which describes its resistance to twisting about the y-axis. Under load β the system will deform as shown in Fig A.13. It has been shown for this loading condition that the unit moment is

$$M_{yx} = \beta \tag{A.105}$$

The total applied moment is

$$\Sigma M_{yx} = h_x\ M_{yx} = h_x\ \beta \tag{A.106}$$

and C_x , the resistance to twisting about the y-axis per unit of width, must then be

$$h_x\ C_x = \frac{h_x\ M_{yx}}{\frac{d\theta}{dy}} = \frac{h_x\ \beta}{\frac{w_s}{h_x h_y}} \tag{A.107}$$

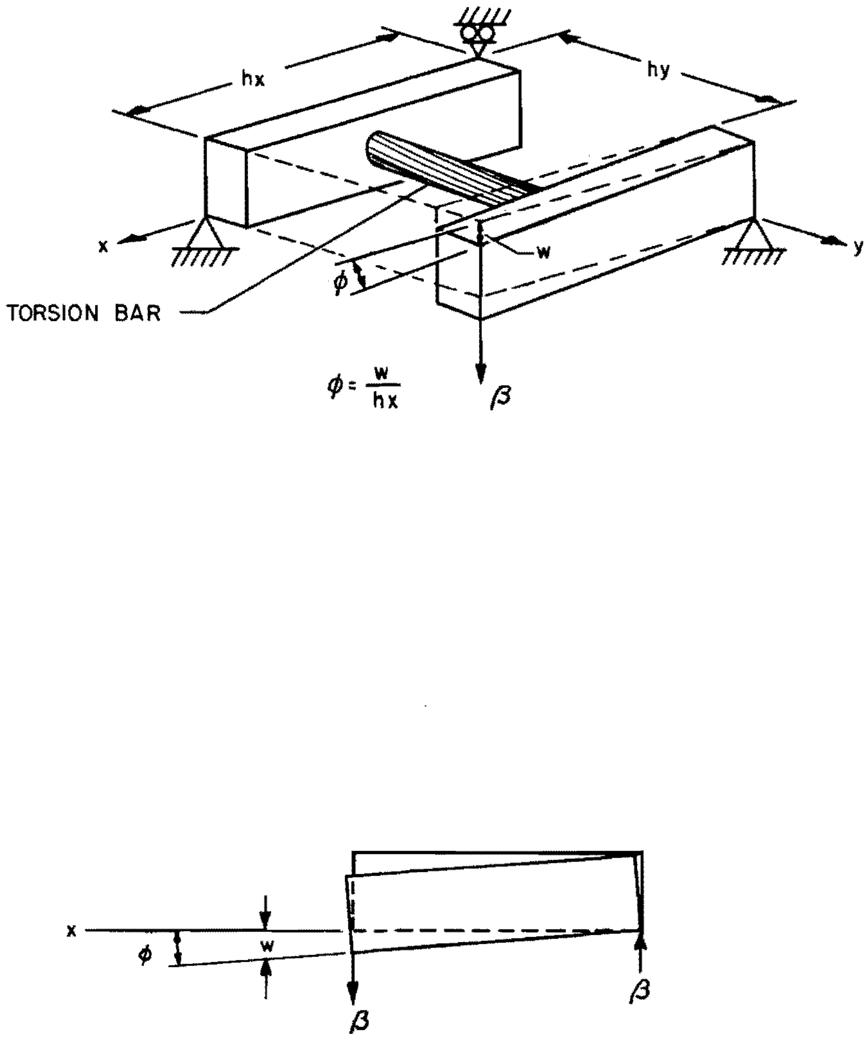


Fig A.13. Partial plate segment distorted by twisting moment about the y-axis.

$$C_y = \frac{\beta h_x h_y}{w_a} \tag{A.108}$$

which is recognized as the same C in Eq A.102 to describe the twisting resistance of the real slab element.

The development is identical for the x torsion bar C_x , thus

$$C_x = C_y = \frac{Gt^3}{6} \tag{A.109}$$

As a final proof, look at an assembled element h_x by h_y in plan with two torsion bars (Fig A.14). Superposing the two torsion bars gives the full load at the corner due to an applied unit torsion M_{xy} on each face, i.e.,

$$M_{yx} = -M_{xy} = \beta \tag{A.110}$$

$$Q = M_{yx} + (-M_{xy}) = \beta + \beta \tag{A.111}$$

$$Q = 2\beta \tag{A.112}$$

where Q is the sum of applied unit torques and thus the total applied load (Ref 55, p 45).

<u>Extract of References used in Appendix A</u>

16. Hearmon, R. F. S., <u>Applied Anisotropic Elasticity</u>, Oxford University Press, Amen House, London, England, 1961.

30. Leknitski, S. K., <u>Theory of Elasticity of an Anisotropic Elastic Body</u>, Translated by P. Fern, Holden-Day, Inc., San Francisco, 1963.

33. Marcus, H., Discussion of Paper by W. H. Hoppmann: "Bending of Orthogonally Stiffened Plates," <u>Journal of Applied Mechanics</u>, ASME Trans, Vol 77, June 1955.

55. Timoshenko, S., and S. Woinowsky-Krieger, <u>Theory of Plates and Shells</u>, 2nd Edition, McGraw-Hill, New York, 1959.

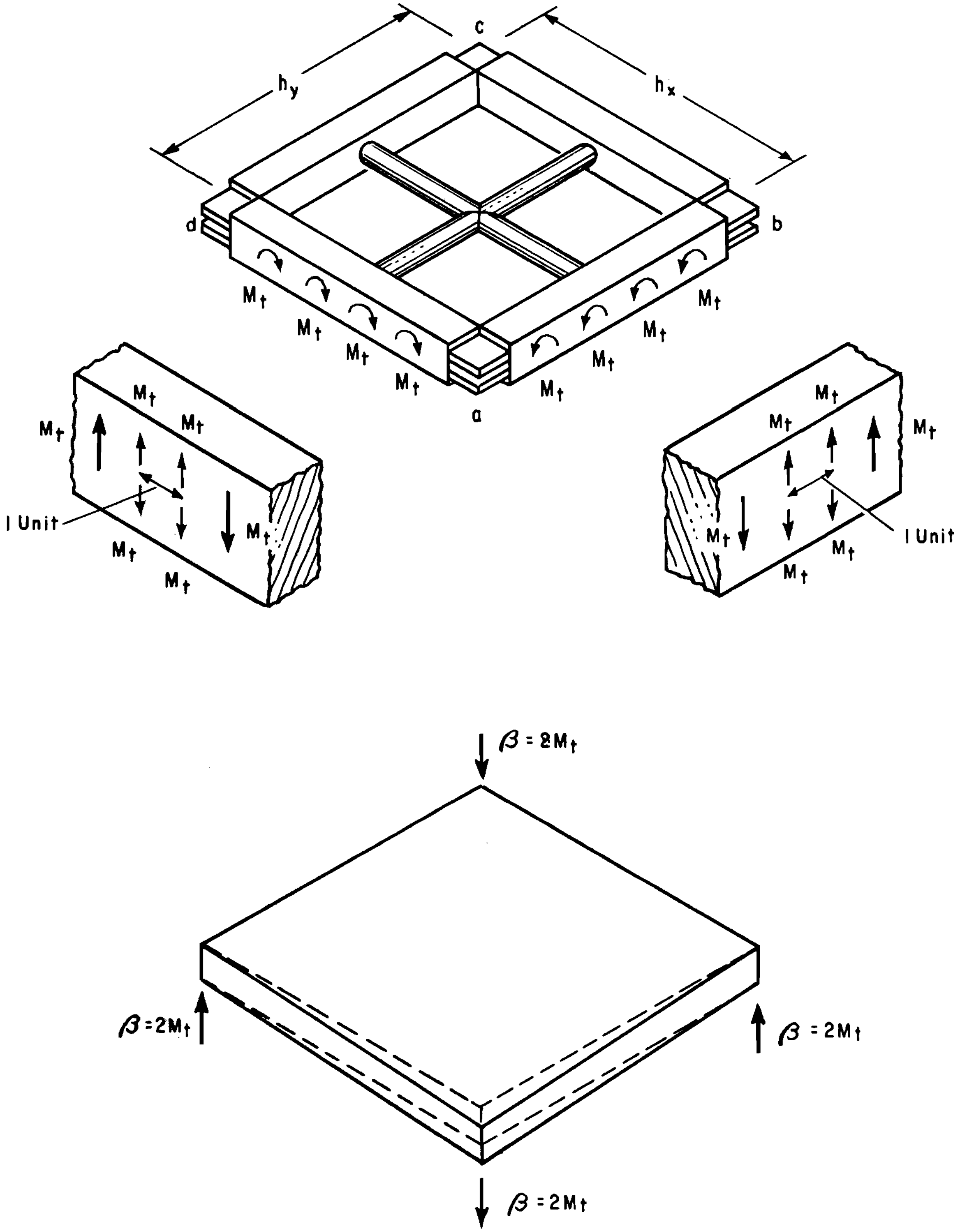


Fig A.14. Finite-element representation of a total slab segment with torsional stiffness in both orthogonal directions.

APPENDIX B

DERIVATION OF PLATE AND SLAB EQUATIONS

APPENDIX B. DERIVATION OF PLATE AND SLAB EQUATIONS

In order to derive the equations for solution of the plate or slab it is helpful to refer to a free-body of the model. Looking first at Fig B.1, a section of the assembled slab model centered at any mesh point i,j , call the x - bar to the left of point i,j , Bar a, and the x-bar to the right of point i,j Bar b.

Figure B.2 shows these same bars as a free body with other members of the model locked and replaced by a system of equivalent forces. $Q^y_{i,j}$ represents the total load carried by the y-beam at this intersection and $\partial^2 w/\partial y^2$ represents the restraint of the y-beam which increases the bending moment in the x direction through Poisson's ratio. Figure B.3 shows the external forces applied to these same two bars. Any of these forces may be equal to zero but can be considered to be present for generality.

Making these substitutions and combining the system of equivalent forces and the external loads gives the general free-body of the slab model in Fig B.4. This free-body is for a segment of x-beam. A similar free-body can be developed for the y-beams by changing all x 's in Fig B.4 to y and all the present y 's to x . All symbols are defined in the list of Notation at the beginning of the text.

Summing vertical forces at joint i,j to evaluate deflections (up is taken as positive)

$$\sum F_{v_{i,j}} = Q_{i,j} + V_{a,j} - V_{b,j} - S_{i,j}(w^x_{i,j}) - Q^y_{i,j} - S_f(w^x_{i,j} - w^y_{i,j}) = 0 \qquad (B.1)$$

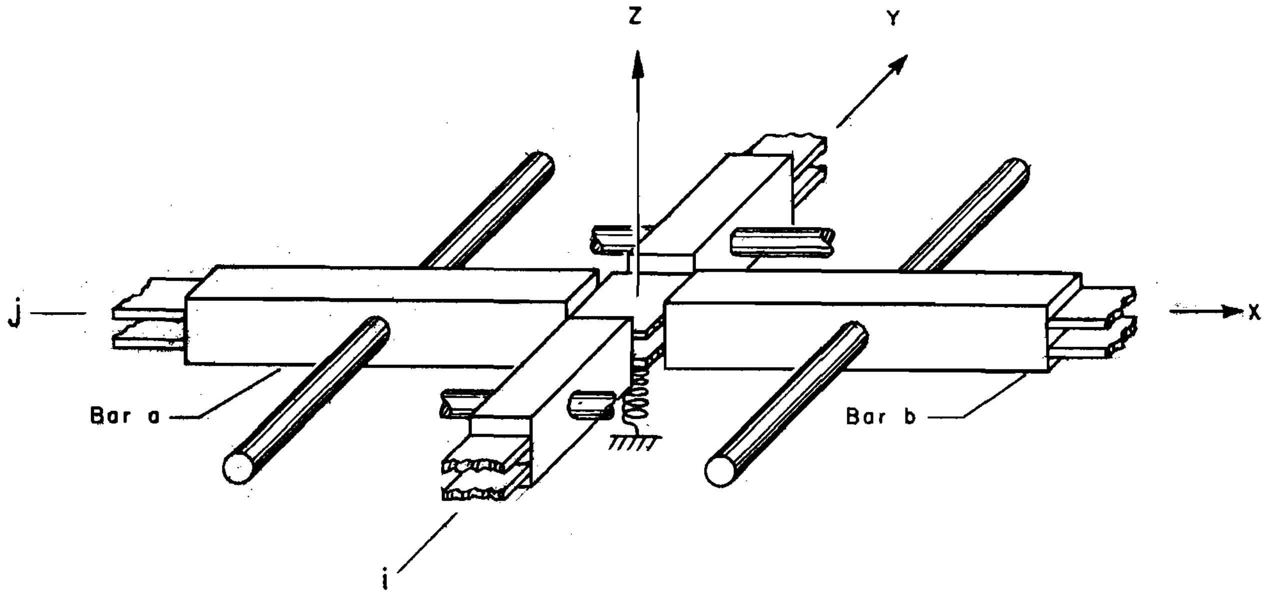


Fig B.1. Typical joint i,j taken from finite-element slab model.

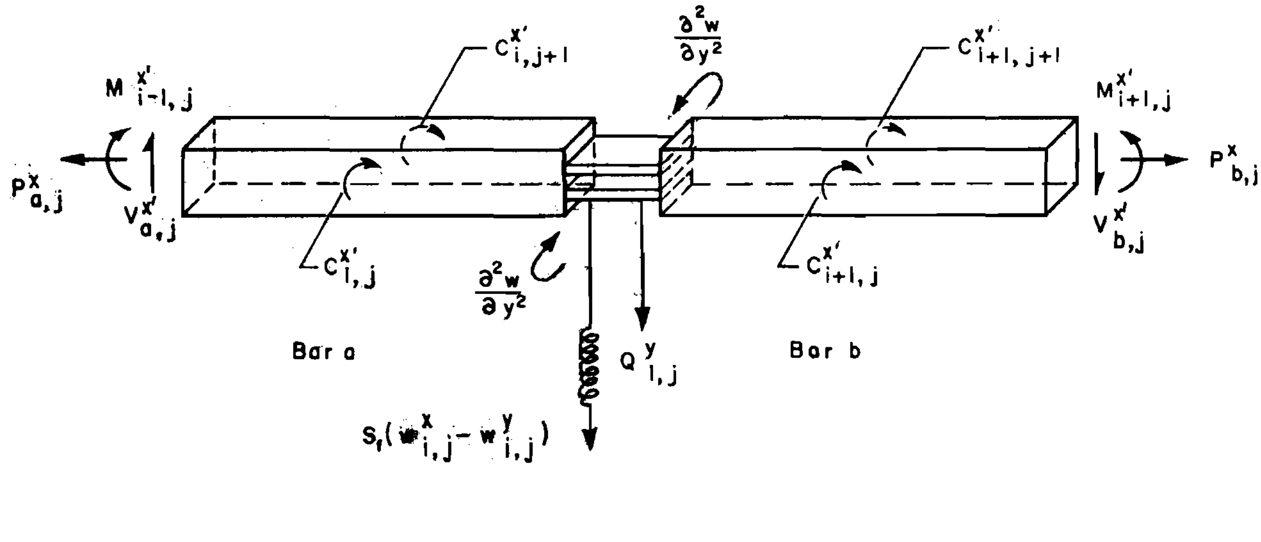


Fig B.2. Free-body of joint i,j with other members of the model replaced by an equivalent force system.

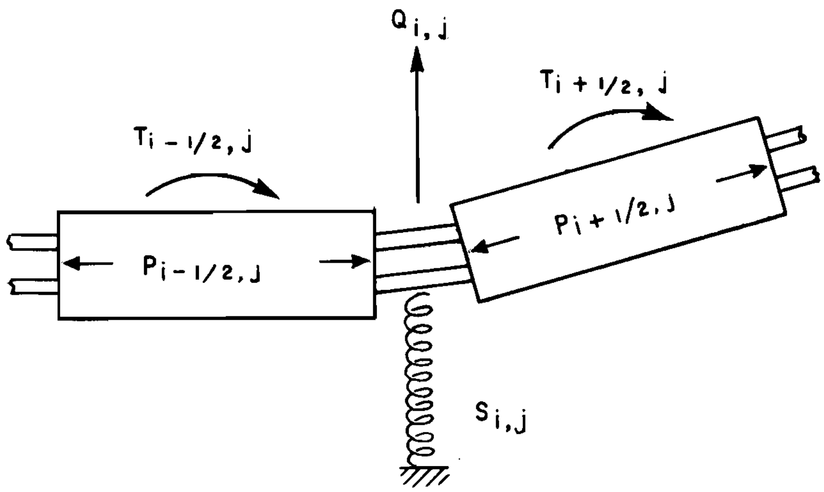


Fig B.3. Typical joint i,j with force and restraint inputs shown.

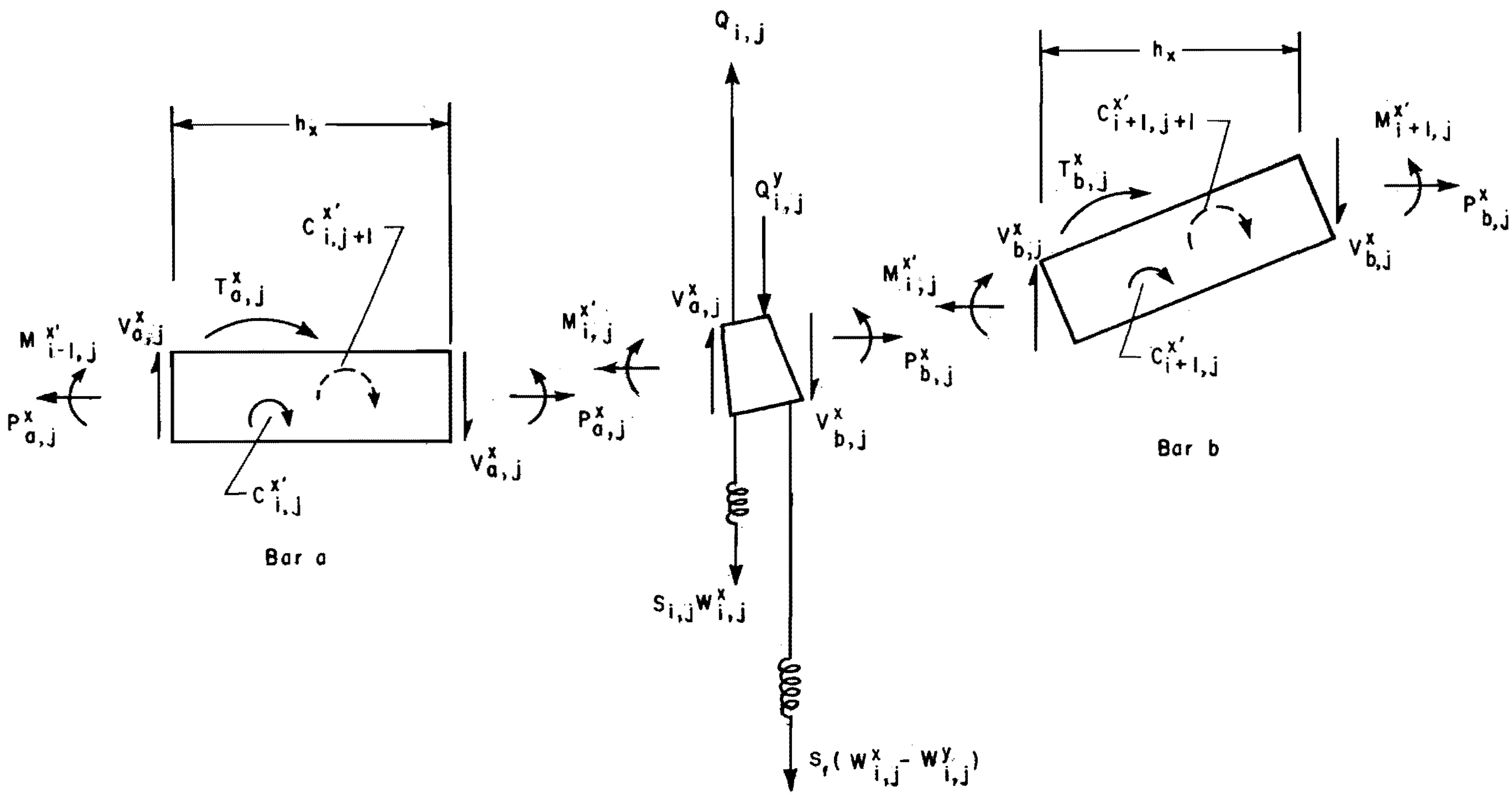


Fig B.4. Generalized free-body of joint i,j with all forces and restraints shown.

In order to derive a relationship for a $V^{x}_{a,j}$ it is helpful to sum moments acting on Bar a about the center of the bar (clockwise rotations are positive), for equilibrium,

$$\Sigma M_a = 0 = M^{x\prime}_{i-1,j} - M^{x\prime}_{i,j} + T^{x}_{a,j} + C^{x\prime}_{i,j} + C^{x\prime}_{i,j+1} + V^{x}_{a,j}\, h_x$$

$$+ P^{x}_{a,j}\left(\frac{-w^{x}_{i-1,j} + w^{x}_{i,j}}{2}\right) + P^{x}_{a,j}\left(\frac{-\, w^{x}_{i-1,j} + w^{x}_{i,j}}{2}\right) \qquad \text{(B.2)}$$

It is convenient to multiply Eq B.1 through by h_x , i. e.,

$$h_x Q_{i,j} + h_x V^{x}_{a,j} - h_x V^{x}_{b,j} - h_x S_{i,j} w^{x}_{i,j} - h_x Q^{y}_{i,j} - h_x S_f (w^{x}_{i,j} - w^{y}_{i,j})$$

$$= 0 \qquad \text{(B.3)}$$

Solving Eq B.2 for $h_x V_{a,j}$,

$$-h_x V^{x}_{a,j} = M^{x\prime}_{i-1,j} - M^{x\prime}_{i,j} + T^{x\prime}_{a,j} + C^{x\prime}_{i,j} + C^{x\prime}_{i,j+1}$$

$$+ P^{x}_{a,j}\ (-w^{x}_{i-1,j} + w^{x}_{i,j}) \qquad \text{(B.4)}$$

Likewise summing moments about Bar b ,

$$\Sigma M_b = 0 = M^{x\prime}_{i,j} - M^{x\prime}_{i+1,j} + T^{x}_{b,j} + C^{x\prime}_{i+1,j} + C^{x\prime}_{i+1,j+1} + h_x V^{x}_{b,j}$$

$$+ P^{x}_{b,j}\left(\frac{-w^{x}_{i,j} + w^{x}_{i+1,j}}{2}\right) + P^{x}_{b,j}\left(\frac{-w^{x}_{i,j} + w^{x}_{i+1,j}}{2}\right) \qquad \text{(B.5)}$$

Solving Eq B.5 for $h_x V^{x}_{b,j}$,

$$-h_x V^{x}_{b,j} = M^{x\prime}_{i,j} - M^{x\prime}_{i+1,j} + T^{x}_{b,j} + C^{x\prime}_{i+1,j} + C^{x\prime}_{i+1,j+1}$$

$$+ P^{x}_{b,j}\ \ (-w^{x}_{i,j} + w^{x}_{i+1,j}) \qquad \text{(B.6)}$$

Equations B.4 and B.6 can now be used to relate the forces on the bars to the joint by substituting into Equation B.3 for $h_x V^{x}_{a,j}$ and $h_x V^{x}_{b,j}$ and noting that it is necessary to change the sign of the term $h_x V^{x}_{a,j}$, then

$$h_x Q_{i,j} = -M^{x\prime}_{i-1,j} + M^{x\prime}_{i,j} - T^{x}_{b,j} - C^{x\prime}_{i,j} - C^{x\prime}_{i,j+1} - P^{x}_{a,j}\ (-w^{x}_{i-1,j} + w^{x}_{i,j})$$

$$+ M^{x\prime}_{i,j} - M^{x\prime}_{i+1,j} + T^{x}_{b,j} + C^{x\prime}_{i+1,j} + C^{x\prime}_{i+1,j+1} + P^{x}_{b,j}\ (-w^{x}_{i,j} + w^{x}_{i+1,j})$$

$$- h_x S_{i,j} w^{x}_{i,j} - h_x Q^{y}_{i,j} - h_x S_f\ (w^{x}_{i,j} - w^{y}_{i,j}) = 0 \qquad \text{(B.7)}$$

At this point it is convenient to group terms and transfer all known terms to the right hand side of the equation, except axial forces, $P^{x}_{i,j}$, which act as stiffness quantities. It is also helpful to multiply through both sides by -1 to change signs. This obtains

$$(M^{x\prime}_{i-1,j} - 2M^{x\prime}_{i,j} + M^{x\prime}_{i+1,j}) - (-C^{x\prime}_{i,j} - C^{x\prime}_{i,j+1} + C^{x\prime}_{i+1,j} + C^{x\prime}_{i+1,j+1})$$

$$+ P^{x}_{a,j}\ (-w^{x}_{i-1,j} + w^{x}_{i,j}) - P^{x}_{b,j}\ (-w^{x}_{i,j} + w^{x}_{i+1,j}) + S_{i,j} h_x w^{x}_{i,j}$$

$$= h_x \left(Q_{i,j} - Q^{y}_{i,j} - S_f\ (w^{x}_{i,j} - w^{y}_{i,j})\right) - T^{x}_{a,j} + T^{x}_{b,j} \qquad \text{(B.8)}$$

This equation relates forces and deflections at point i,j but all of the primed terms must be evaluated before the mathematical manipulations necessary for solution can be performed.

Evaluation of Bending Moments

From Appendix A, it should be recalled that the moment equation for a slab or plate is

$$M^{x}_{i,j} = D^{x}_{i,j}\left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2}\right) \quad \text{(Units: in-lb per in. of plate width)} \qquad \text{(B.9)}$$

$M^{x\prime}_{i,j}$ is the total moment (inch-lb) in a plate section one increment length wide as represented by the j^{th} x-beam.

Therefore

$$M^{x\prime}_{i,j} = h_y M^{x}_{i,j} \qquad \text{(in-lb)} \qquad \text{(B.10)}$$

$$M^{x'}_{i,j} = h_y D^x_{i,j} \left(\frac{\partial^2 w^x}{\partial x^2} + \nu_{yx} \frac{\partial^2 w^y}{\partial y^2}\right) \tag{B.11}$$

Likewise, $M^{y'}_{i,j}$, which will be used later, is

$$M^{y'}_{i,j} = h_x M^y_{i,j} \qquad \text{(in-lb)} \tag{B.12}$$

$$= h_x D^y_{i,j} \left(\frac{\partial^2 w^y}{\partial y^2} + \nu_{xy} \frac{\partial^2 w^x}{\partial x^2}\right) \tag{B.13}$$

For solution in the computer, it is convenient to write Eq B.11 in compressed central finite difference form (Ref 35) as follows:

$$M^{x'}_{i,j} = h_y D^x_{i,j} \left[\left(\frac{w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j}}{h_x^2}\right) + \nu_{yx} \left(\frac{w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1}}{h_y^2}\right)\right] \tag{B.14}$$

A similar pattern governs moment at Station $i-1,j$ and $i+1,j$ which are also required in Eq B.8.

For later use, the moments in the i^{th} y-beam can be written similarly

$$M^{y'}_{i,j} = h_x D^y_{i,j} \left[\left(\frac{w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1}}{h_y^2}\right) + \nu_{xy} \left(\frac{w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j}}{h_x^2}\right)\right] \tag{B.15}$$

Evaluation of Twisting Moments $C'_{i,j}$

Referring to Appendix A, recall that C is a torsional stiffness term per unit of plate width. Then the stiffness of segment i,j in the x direction is equal to $h_x C_{i,j}$ which is the stiffness of the torsion bar. This stiffness multiplied times the unit angular rotation of the bar is equal to the total twisting moment $C^{x'}_{i,j}$.

The unit angular rotations around point i,j can be derived as follows: (In all cases this angle is the slope of the center bar minus the slope of the outer bar.)

$$\alpha_{i,j} = \frac{+\left(\dfrac{-w^x_{i-1,j} + w^x_{i,j}}{h_x}\right) - \left(\dfrac{-w^x_{i-1,j-1} + w^x_{i,j-1}}{h_x}\right)}{h_y}$$

$$= \frac{-w^x_{i-1,j} + w^x_{i,j} + w^x_{i-1,j-1} - w^x_{i,j-1}}{h_x h_y} \qquad \text{(B.16)}$$

$$\alpha_{i,j+1} = \frac{+\left(\dfrac{-w^x_{i-1,j} + w^x_{i,j}}{h_x}\right) - \left(\dfrac{-w^x_{i-1,j+1} + w^x_{i,j+1}}{h_x}\right)}{h_y}$$

$$= \frac{-w^x_{i-1,j} + w^x_{i,j} + w^x_{i-1,j+1} - w^x_{i,j+1}}{h_x h_y} \qquad \text{(B.17)}$$

$$\alpha_{i+1,j} = \frac{+\left(\dfrac{-w^x_{i,j} + w^x_{i+1,j}}{h_x}\right) - \left(\dfrac{-w^x_{i,j-1} + w^x_{i+1,j-1}}{h_x}\right)}{h_y}$$

$$= \frac{-w^x_{i,j} + w^x_{i+1,j} + w^x_{i,j-1} - w^x_{i+1,j-1}}{h_x h_y} \qquad \text{(B.18)}$$

$$\alpha_{i+1,j+1} = \frac{+\left(\dfrac{-w^x_{i,j} + w^x_{i+1,j}}{h_x}\right) - \left(\dfrac{-w^x_{i,j+1} + w^x_{i+1,j+1}}{h_x}\right)}{h_y}$$

$$= \frac{-w^x_{i,j} + w^x_{i+1,j} + w^x_{i,j+1} - w^x_{i+1,j+1}}{h_x h_y} \qquad \text{(B.19)}$$

Then, typically, $C^{x\,\prime}_{i,j}$ becomes

$$C^{x\,\prime}_{i,j} = h_x C^x_{i,j} \alpha_{i,j}$$

$$= h_x C^x_{i,j} \left(\frac{-w^x_{i-1,j} + w^x_{i,j} + w^x_{i-1,j-1} - w^x_{i,j-1}}{h_x h_y} \right) \qquad \text{(B.20)}$$

$$= \frac{C^x_{i,j}}{h_y} \left(-w^x_{i-1,j} + w^x_{i,j} + w^x_{i-1,j-1} - w^x_{i,j-1} \right) \qquad \text{(B.21)}$$

The other C' 's can be derived in the same manner for substitution into Eq B.8. Then writing Eq B.14 at Stations $i-1,j$, i,j , and $i+1,j$; writing Eq B.20 for $C^{x\,\prime}_{i,j}$, $C^{x\,\prime}_{i+1,j}$, $C^{x\,\prime}_{i,j+1}$, $C^{x\,\prime}_{i+1,j+1}$; and substituting into Eq B.8 obtains

$$h_y D^x_{i-1,j} \left[\left(\frac{w^x_{i-2,j} - 2w^x_{i-1,j} + w^x_{i,j}}{h_x^2} \right) + \nu_{yx} \left(\frac{w^y_{i-1,j-1} - 2w^y_{i-1,j} + w^y_{i-1,j+1}}{h_y^2} \right) \right]$$

$$- 2h_y D^x_{i,j} \left[\left(\frac{w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j}}{h_x^2} \right) + \nu_{yx} \left(\frac{w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1}}{h_y^2} \right) \right]$$

$$+ h_y D^x_{i+1,j} \left[\left(\frac{w^x_{i,j} - 2w^x_{i+1,j} + w^x_{i+2,j}}{h_x^2} \right) \right.$$

$$\left. + \nu_{yx} \left(\frac{w^y_{i+1,j-1} - 2w^y_{i+1,j} \quad w^y_{i+1,j+1}}{h_y^2} \right) \right]$$

$$+ \frac{C^x_{i,j}}{h_y} \left(w^x_{i-1,j-1} - w^x_{i-1,j} + w^x_{i,j} - w^x_{i,j-1} \right)$$

$$+ \frac{C^x_{i,j+1}}{h_y} \left(-w^x_{i-1,j} + w^x_{i,j} + w^x_{i-1,j+1} - w^x_{i,j+1} \right)$$

$$- \frac{C^x_{i+1,j}}{h_y} \left(-w^x_{i,j} + w^x_{i+1,j} + w^x_{i,j-1} - w^x_{i+1,j-1} \right)$$

$$- \frac{C^x_{i+1,j+1}}{h_y} \left(-w^x_{i,j} + w^x_{i+1,j} + w^x_{i,j+1} - w^x_{i+1,j+1} \right)$$

$$+ P^x_{a,j}\left(-w^x_{i-1,j} + w^x_{i,j}\right) - P^x_{b,j}\left(-w^x_{i,j} + w^x_{i+1,j}\right) + h_x\left(S_{i,j} + S_f\right) w^x_{i,j}$$

$$= h_x\left(Q_{i,j} - Q^y_{i,j} + S_f w^y_{i,j}\right) - T^x_{a,j} + T^x_{b,j} \qquad \text{(B.22)}$$

At this point an additional note of clarification is helpful. It is convenient in computation to use the same indexing system for bars and torsion bars, as for joints. So far in these developments, bars have been referred to as a and b. Reference to Fig 3.10 shows the numbering system used in the computer. From this it may be seen that a becomes i and b becomes $i+1$. Therefore, for example, $T^x_{a,j}$ becomes $T^x_{i,j}$, $P^x_{b,j}$ becomes $P^x_{i+1,j}$, etc.

This will be an implicit solution for $w^x_{i,j}$, the deflection of the j^{th} x-beam at Station i. For solution, however, all deflections not falling on the j^{th} beam for a particular trial (see Chapter 5 in text for explanation) must be assumed known (the last estimated value is used in these solutions). Furthermore, all the y-beam deflections $w^y_{i,j}$ will be assumed known from a previous iteration. These known deflections should then appear on the right hand side of the equation.

After making the notation change of a to i and transferring all x-beam deflections not on the j^{th} beam and all y-beam deflections to the right hand side, multiply through by h_y to clear fractions. It is also convenient to rearrange terms. Equation B.22 then becomes

$$\frac{h_y^2}{h_x^2}\left[D^x_{i-1,j}\left(w^x_{i-2,j} - 2w^x_{i-1,j} + w^x_{i,j}\right) - 2D^x_{i,j}\left(w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j}\right)\right.$$

$$\left. + D^x_{i+1,j}\left(w^x_{i,j} - 2w^x_{i+1,j} + w^x_{i+2,j}\right)\right] + C^x_{i,j}\left(-w^x_{i-1,j} + w^x_{i,j}\right)$$

$$+ C^x_{i,j+1}\left(-w^x_{i-1,j} + w^x_{i,j}\right) - C^x_{i+1,j}\left(-w^x_{i,j} + w^x_{i+1,j}\right)$$

$$+ C^x_{i+1,j+1}\left(-w^x_{i,j} + w^x_{i+1,j}\right) + h_y P^x_{i,j}\left(- w^x_{i-1,j} + w^x_{i,j}\right)$$

$$
\begin{aligned}
& - h_y P^x_{i+1,j} \left(-w^x_{i,j} + w^x_{i+1,j}\right) + h_x h_y \left(S_{i,j} + S_f\right) w^x_{i,j} \\
& = \; h_y h_x \left(Q_{i,j} - Q^y_{i,j} + S_f w^y_{i,j}\right) + h_y \left(-T^x_{i,j} + T^x_{i+1,j}\right) \\
& - \nu_{yx} \Big[D^x_{i-1,j} \left(w^y_{i-1,j-1} - 2w^y_{i-1,j} + w^y_{i-1,j+1}\right) \\
& - 2D^x_{i,j} \left(w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1}\right) \\
& + D^x_{i+1,j} \left(w^y_{i+1,j-1} - 2w^y_{i+1,j} + w^y_{i+1,j+1}\right)\Big] - C^x_{i,j} \left(w^x_{i-1,j-1} - w^x_{i,j-1}\right) \\
& - C^x_{i,j+1} \left(w^x_{i-1,j+1} - w^x_{i,j+1}\right) + C^x_{i+1,j} \left(w^x_{i,j-1} - w^x_{i+1,j-1}\right) \\
& + C^x_{i+1,j+1} \left(w^x_{i,j+1} - w^x_{i+1,j+1}\right)
\end{aligned}
\tag{B.23}
$$

Sorting the equation in terms of w^x ,

$$
\begin{aligned}
& \frac{h_y{}^2}{h_x{}^2} \Big[w^x_{i-2,j} D^x_{i-1,j} + w^x_{i-1,j} \left(-2D^x_{i-1,j} - 2D^x_{i,j}\right) + w^x_{i,j} \left(D^x_{i-1,j} + 4D^x_{i,j} + D^x_{i+1,j}\right) \\
& + w^x_{i+1,j} \left(-2D^x_{i,j} - 2D^x_{i+1,j}\right) + w^x_{i+2,j} D^x_{i+1,j}\Big] + w^x_{i-1,j} \left(-C^x_{i,j} - C^x_{i,j+1}\right) \\
& + w^x_{i,j} \left(C^x_{i,j} + C^x_{i,j+1} + C^x_{i+1,j} + C^x_{i+1,j+1}\right) + w^x_{i+1,j} \left(-C^x_{i+1,j} - C^x_{i+1,j+1}\right) \\
& - h_y P^x_{i,j} w^x_{i-1,j} + w^x_{i,j} h_y \left(P^x_{i,j} + P^x_{i+1,j}\right) + w^x_{i+1,j} h_y P^x_{i+1,j} \\
& + w^x_{i,j} h_x h_y \left(S_{i,j} + S_f\right) \\
& = \; h_x h_y \left(Q_{i,j} - Q^y_{i,j} + S_f w^y_{i,j}\right) + h_y \left(-T^x_{i,j} + T^x_{i+1,j}\right) \\
& - \nu_{yx} \Big[D^x_{i-1,j} \left(w^y_{i-1,j-1} - 2w^y_{i-1,j} + w^y_{i-1,j+1}\right)
\end{aligned}
$$

$$- 2D^x_{i,j}\left(w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1}\right)$$

$$+ D^x_{i+1,j}\left(w^y_{i+1,j-1} - 2w^y_{i+1,j} + w^y_{i+1,j+1}\right)\Big] - C^x_{i,j}\left(w^x_{i-1,j-1} - w^x_{i,j-1}\right)$$

$$- C^x_{i,j+1}\left(w^x_{i-1,j+1} - w^x_{i,j+1}\right) + C^x_{i+1,j}\left(w^x_{i,j-1} - w^x_{i+1,j-1}\right)$$

$$+ C^x_{i+1,j+1}\left(w^x_{i,j+1} - w^x_{i+1,j+1}\right) \tag{B.24}$$

This equation is the one we seek but it is most conveniently written in terms of 5 unknown deflections:

$$a_x w^x_{i-2,j} + b_x w^x_{i-1,j} + c_x w^x_{i,j} + d_x w^x_{i+1,j} + e_x w^x_{i+2,j} = f_x \tag{B.25}$$

where

$$a_x = \frac{h_y^2}{h_x^2} D^x_{i-1,j} \tag{B.26}$$

$$b_x = -2.0\frac{h_y^2}{h_x^2}\left(D^x_{i-1,j} + D^x_{i,j}\right) - C^x_{i,j} - C^x_{i+1,j} - h^y P^x_{i,j} \tag{B.27}$$

$$c_x = \frac{h_y^2}{h_x^2}\left(D^x_{i-1,j} + 4D^x_{i,j} + D^x_{i+1,j}\right) + C^x_{i,j} + C^x_{i+1,j} + C^x_{i,j+1}$$

$$+ C^x_{i+1,j+1} + h_x h_y\left(S_{i,j} + S_f\right) + h_y\left(P^x_{i,j} + P^x_{i+1,j}\right) \tag{B.28}$$

$$d_x = -2.0\frac{h_y^2}{h_x^2}\left(D^x_{i,j} + D^x_{i+1,j}\right) - C^x_{i+1,j} - C^x_{i+1,j+1} - h_y P^x_{i+1,j} \tag{B.29}$$

$$e_x = \frac{h_y^2}{h_x^2} D^x_{i+1,j} \tag{B.30}$$

$$f_x = h_x h_y\left(Q_{i,j} - Q^y_{i,j} + S_f w^y_{i,j}\right) + h_y\left(-T^x_{i,j} + T^x_{i+1,j}\right)$$

$$- \nu_{yx}\Big[D^x_{i-1,j}\left(w^y_{i-1,j-1} - 2w^y_{i-1,j} + w^y_{i-1,j+1}\right)$$

$$- 2D^x_{i,j} \left(w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1}\right)$$

$$+ D^x_{i+1,j} \left(w^y_{i+1,j-1} - 2w^y_{i+1,j} + w^y_{i+1,j+1}\right)\Big]$$

$$- C^x_{i,j} \left(w^x_{i-1,j-1} - w^x_{i,j-1}\right) - C^x_{i,j+1} \left(w^x_{i-1,j+1} - w^x_{i,j+1}\right)$$

$$+ C^x_{i+1,j} \left(w^x_{i,j-1} - w^x_{i+1,j-1}\right) + C^x_{i+1,j+1} \left(w^x_{i,j+1} - w^x_{i+1,j+1}\right) \qquad \text{(B.31)}$$

A comparable equation can easily be derived for the i^{th} y-beam.

Evaluation of $Q^y_{i,j}$

The external forces acting on any joint i,j must be balanced by net load absorbed in the beams, $Q^y_{i,j}$ and $Q^x_{i,j}$. Summing vertical forces at any joint:

$$Q_{i,j} - Q^y_{i,j} - Q^x_{i,j} - S_{i,j}w^x_{i,j} + S_f \ (w^x_{i,j} - w^y_{i,j}) = 0 \qquad \text{(B.32)}$$

Multiply through by h_x ,

$$h_xQ_{i,j} - h_xQ^y_{i,j} - h_xQ^x_{i,j} - h_xS_{i,j}w^x_{i,j} + h_xS_f \ (w^x_{i,j} - w^y_{i,j}) = 0 \qquad \text{(B.33)}$$

Since the derivation thus far has been for solution of the x-beams, it is helpful to develop $Q^x_{i,j}$ because it can easily be done from the same free-body. $Q^y_{i,j}$ can be developed later using the same reasoning. Referring back to Eq B.7 shows $Q^x_{i,j}$ to be

$$\left(M^{x\prime}_{i-1,j} - 2M^{x\prime}_{i,j} + M^{x\prime}_{i+1,j}\right) + C^{x\prime}_{i,j} + C^{x\prime}_{i,j+1} - C^{x\prime}_{i+1,j} - C^{x\prime}_{i+1,j+1}$$

$$+ P^x_{i,j} \left(-w^x_{i-1,j} + w^x_{i,j}\right) - P^x_{i+1,j} \left(-w^x_{i,j} + w^x_{i+1,j}\right) + T^x_{i,j} - T^x_{i+1,j}$$

$$= h_xQ^x_{i,j} \qquad \text{(B.34)}$$

In this form, using previously developed expressions for $M^{x\prime}_{i,j}$, $C^{x\prime}_{i,j}$, etc., it is possible to arrive at a computational form of $Q^x_{i,j}$. Using the same technique except this time for the y-beams, $Q^y_{i,j}$ can be developed as follows:

$$Q^y_{i,j} = \frac{1}{h_y}\Big[M^{y\,\prime}_{i,j-1} - 2M^{y\,\prime}_{i,j} + 2M^{y\,\prime}_{i,j+1} + C^{y\,\prime}_{i,j} + C^{y\,\prime}_{i+1,j} - C^{y\,\prime}_{i,j+1} - C^{y\,\prime}_{i+1,j+1}$$

$$+ P^y_{i,j}\left(-w^y_{i,j-1} + w^y_{i,j}\right) - P^y_{i,j+1}\left(-w^y_{i,j} + w^y_{i,j+2}\right) + T^y_{i,j} - T^y_{i,j+1}\Big]$$

$$= \frac{1}{h_y}\Big[h_x\left(M^y_{i,j-1} - 2M^y_{i,j} + M^y_{i,j+1}\right)$$

$$+ h_y\left(C^y_{i,j}\alpha_{i,j} + C^y_{i+1,j}\alpha_{i+1,j} - C^y_{i,j+1}\alpha_{i,j+1} - C^y_{i+1,j+1}\alpha_{i+1,j+1}\right)$$

$$+ P^y_{i,j}\left(-w^y_{i,j-1} + w^y_{i,j}\right) - P^y_{i,j+1}\left(-w^y_{i,j} + w^y_{i,j+1}\right) + T^y_{i,j} - T^y_{i,j+1}\Big] \quad (B.35)$$

For use in the computer the loads due to bending, twisting, applied couple and axial load have been defined as follows:

$$\overline{QBMY}_{i,j} = \frac{h_x}{h_y}\left(M^y_{i,j-1} - 2M^y_{i,j} + M^y_{i,j+1}\right) \quad (B.36)$$

$$\overline{QTMY}_{i,j} = C^y_{i,j}\alpha_{i,j} + C^y_{i+1,j}\alpha_{i+1,j} - C^y_{i,j+1}\alpha_{i,j+1} - C^y_{i+1,j+1}\alpha_{i+1,j+1} \quad (B.37)$$

$$\overline{QPY}_{i,j} = \frac{1}{h_y}\left[P^y_{i,j}w^y_{i,j-1} - \left(P^y_{i,j} + P^y_{i,j+1}\right)w^y_{i,j} + P^y_{i,j+1}w^y_{i,j+1}\right] \quad (B.38)$$

Then

$$Q^y_{i,j} = \overline{QBMY}_{i,j} + \overline{QTMY}_{i,j} + \overline{QPY}_{i,j} + \frac{T^y_{i,j} - T^y_{i,j+1}}{h_y} \quad (B.39)$$

Likewise

$$Q^x_{i,j} = \overline{QBMX}_{i,j} + \overline{QTMX}_{i,j} + \overline{QPX}_{i,j} + \frac{T^x_{i,j} - T^x_{i+1,j}}{h_x} \quad (B.40)$$

Thus all the terms needed for solution of this problem have been derived. Solution of these equations is treated in Chapter 5.

APPENDIX C

PROGRAM SLAB 17

APPENDIX C1

PROGRAM SLAB 17

OPERATING MANUAL

Extracted from

DISCONTINUOUS ORTHOTROPIC PLATES AND

PAVEMENT SLABS

by

William Ronald Hudson

and

Hudson Matlock

Report No. 56-6

Center for Highway Research

The University of Texas

SLAB 17 is a computer program written to solve problems involving orthotropic plates and pavement slabs. The development of the equations and the over-all method of solution are discussed in Chapters 4, 5, and 6 of the basic report. The purpose of this Appendix is to provide the program user with a concise manual which can be extracted for daily use with the program.

Program Operation

The general procedures followed in the program are described in the attached flow chart. A problem number card at the beginning of each problem controls the start of the solution. Unless an error occurs because of unacceptable data the program will work any number of problems in sequence, finally stopping when a blank problem number card is encountered.

The data deck starts with two cover cards used to identify the program and the particular run being made. The problems to be solved together in one run are stacked behind the cover cards in sequence as illustrated in Fig C.1. Each problem consists of (a) one problem number card with alphanumeric description of the problem; (b) Table 1, Input for Data Control and Constants, two cards containing necessary control data and constants for the problem; (c) Table 2, Iteration Control Data, 2 cards which contain the fictitious closure springs; and (d) Table 3, Stiffness and Load Data, which contain the number of cards required to properly describe the problem and loads being applied. The number of values on each card in Table 2 and the number of cards in Table 3a and 3b must be properly specified in Table 1 as indicated in the Input Form.

Guide for Input Data

The following pages provide a Guide for Data Input. It should be expected that revisions of these forms and instructions will be developed in the future and may supersede the present versions.

Example problems are discussed in Chapter 7. Appendix D includes example input data for several of these example problems. By comparing these example inputs with the description of the real problem the user can gain practical experience in the preparation of input data. Proficiency in the use of the program can be gained only through actual coding of problems and solution in the computer. Recoding and resolution of the example problems should prove to be helpful.

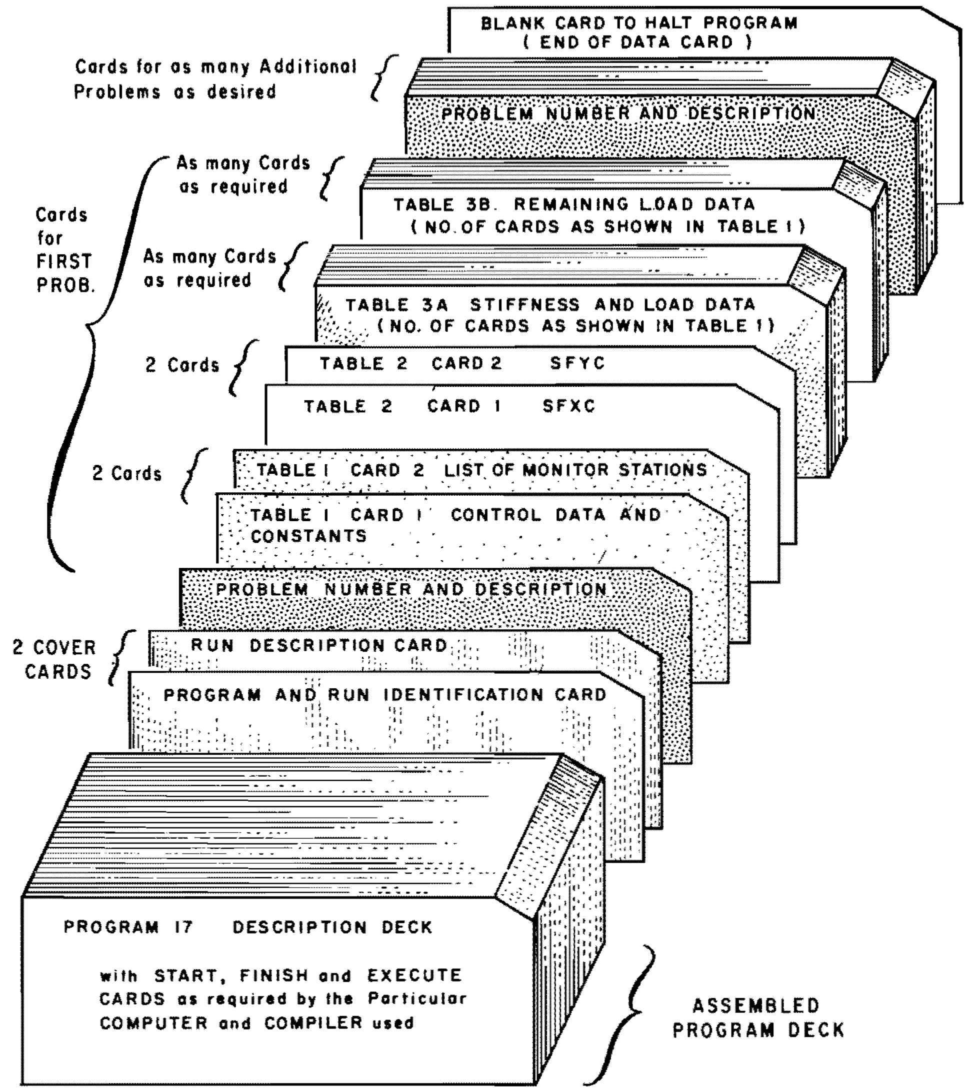


Fig C.1. Assembly order for SLAB 17 program deck with data, ready for run.

SLAB 17 INPUT FORM

W. R. Hudson
Center for Highway Research
9 July 1965

IDENTIFICATION OF PROGRAM AND RUN (2 alphanumeric cards per run)

Account number, project, coding, key punching with dates, etc.	
1	80

Description of run	
1	80

IDENTIFICATION OF PROBLEM (one card each problem; program stops if PROB NUM = 0)

Prob Num.		Description of problem (Alphanumeric)	
1	5	11	80

TABLE 1. PROGRAM-CONTROL DATA AND CONSTANTS (2 cards)

Num Values Tab 2	Num Cards Table 3A	Num Cards Table 3B	Max Num Iters	Num Incrs MX	Num Incrs MY	Increment Length HX	Increment Length HY	Closure Tolerance	Poisson's Ratio
5	10	15	20	25	30	40	50	60	

LIST OF FOUR MONITOR STATIONS

IM1	JM1	IM2	JM2	IM3	JM3	IM4	JM4
5	10	15	20	25	30	35	40

TABLE 2. ITERATION CONTROL DATA (2 cards)

A. Fictitious Springs Representing X-beams, SFXC (num of values specified in Table 1)

10	20	30	40	50	60	70	80

B. Fictitious Springs Representing Y-beams, SFYC (num of values must be same as Table 2A)

10	20	30	40	50	60	70	80

TABLE 3A. STIFFNESS AND LOAD DATA (any number of cards as shown in Table 1)

From Sta.		Thru Sta.		Unit Bending Stiffness		Load	Support Value	Unit Twisting Stiffness	
I1	J1	I2	J2	DX	DY	Q	S	CX	CY
5	10	15	20	30	40	50	60	70	80

TABLE 3B. LOAD DATA CONTINUED (any number of cards as shown in Table 1)

From Sta.		Thru Sta.			External Couple		Axial Load	
I1	J1	I2	J2		TX	TY	PX	PY
5	10	15	20	30	50	60	70	80

Sheet 2 of 2

GENERAL PROGRAM NOTES

The data cards must be stacked in proper order for the program to run.

A consistent system of units must be used for all input data, for example: pounds and inches.

All 2- to 5-space words are understood to be integers or whole decimal numbers . . . . + 2 1

All 10-space words are floating-point decimal numbers . . . . - 4 . 3 2 1 E + 0 3

All numbers must be right justified.

The problem number may be alphanumeric.

Figure C.3 shows the positive sign convention of input and output values.

TABLE 1. PROGRAM-CONTROL DATA AND CONSTANTS

Card 1.

The number of closure spring values input in Table 2 must be the same for both the x and y systems and must be indicated (maximum of eight values for each).

The number of input cards for Table 3A and Table 3B must be shown separately and should be carefully checked (maximum of 99 cards for each).

The maximum number of iterations must be specified to halt the computer if closure is not reached (maximum of 99 iterations).

The number of increments and the increment length to describe the problem must be correctly specified (maximum of 30 increments for each direction).

A desired deflection closure tolerance must be specified. Four to six decimals are usually satisfactory for differentiating load accurately.

Poisson's ratio will be taken as zero unless specified (always positive).

Card 2.

Four monitor stations should be selected by inserting the external station numbers in specified pairs as desired on this card. All monitor stations not specified will be taken to be 0,0.

TABLE 2. ITERATION-CONTROL DATA

Card 1.

The stiffness of closure springs calculated to represent the x-beams, SFXC, must be specified (up to eight values). Chapter 5 suggests methods of calculating the desired values.

Card 2.

The stiffness of closure springs calculated to represent the y-beams, SFYC, must be specified (up to eight values; the number of values must be the same as for SFXC). Chapter 5 suggests methods of calculating the desired values.

TABLE 3A. STIFFNESS AND LOAD DATA

Typical units:

Variables:	DX	DY	Q	S	CX	CY
Input Units:	in-lb	in-lb	lb	lb/in	in-lb	in-lb

To distribute data over a rectangular area, the lower left-hand and the upper right-hand mesh points of the area must be specified. Figure C.2 illustrates this.

The "From Sta" values cannot be greater than the "Thru Sta" values or the computer will not accept any input on that card.

To specify data at a single station, the station numbers (i and j) must be specified in both the "From Sta" and "Thru Sta" columns (see Fig C.2).

Correct input for distributed values of DX , DY , Q , and S results in half-values at mesh points on the edge of the slab and quarter-values at the corners since each mesh point represents the area within 1/2 increment length on all four sides.

There are no restrictions on the order of cards in Table 3A.

Unit stiffness values DX and DY are input at all full value stations. The values are reduced proportionally for edges (half-values) and corners (quarter-values) because 1/2 and 3/4 of the area, respectively, is off the real slab.

Unit torsional stiffness CX and CY are input in appropriate slab segments where full values are required. The values may be reduced as necessary (half segments rarely occur, however). CX and CY values lie in the increment space below and to the left of the mesh point. Care should be taken to keep from placing CX and CY values outside points with real DX and DY values.

S values for any station are determined by multiplying the support value k by the appropriate area of the real slab assigned to that station (half-values for edges, quarter-values for corners). If k is variable, then $S = \Sigma kA$ over the area A of the station. Simple supports are provided by using large S values (1.000E+99).

Q values are input at a mesh point as concentrated loads; distributed loads are input as shown in Fig C.2.

TABLE 3B. LOAD DATA

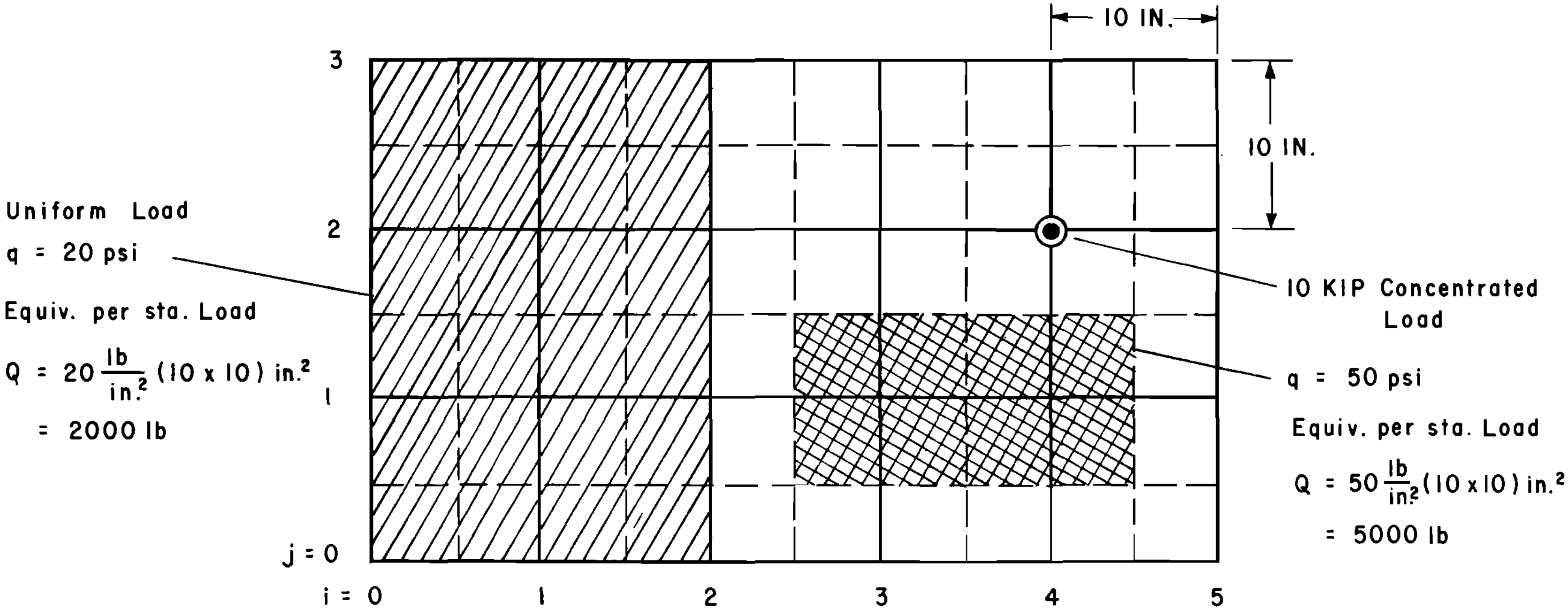
Typical Units:

Variable:	TX	TY	PX	PY
Input Units:	in-lb	in-lb	lb	lb

All inputs in this table are lumped. Distributed data must be summed over the width of the increment involved. Concentrated values are applied directly at the nearest station.

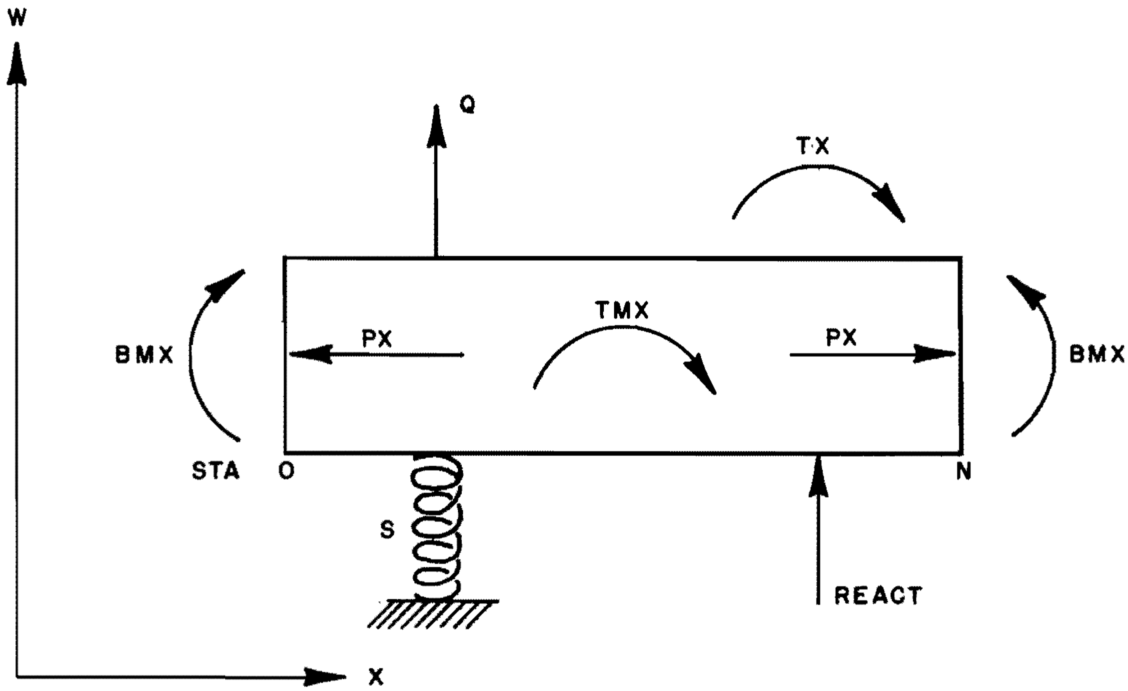
Axial tension (+) or compression (-) values P must be stated at each station in the same manner as indicated above. There is no mechanism in the program to automatically distribute the internal effects of any externally applied axial force.

Torques TX and TY are applied in the bar elements to the left and below the station, not at stations.

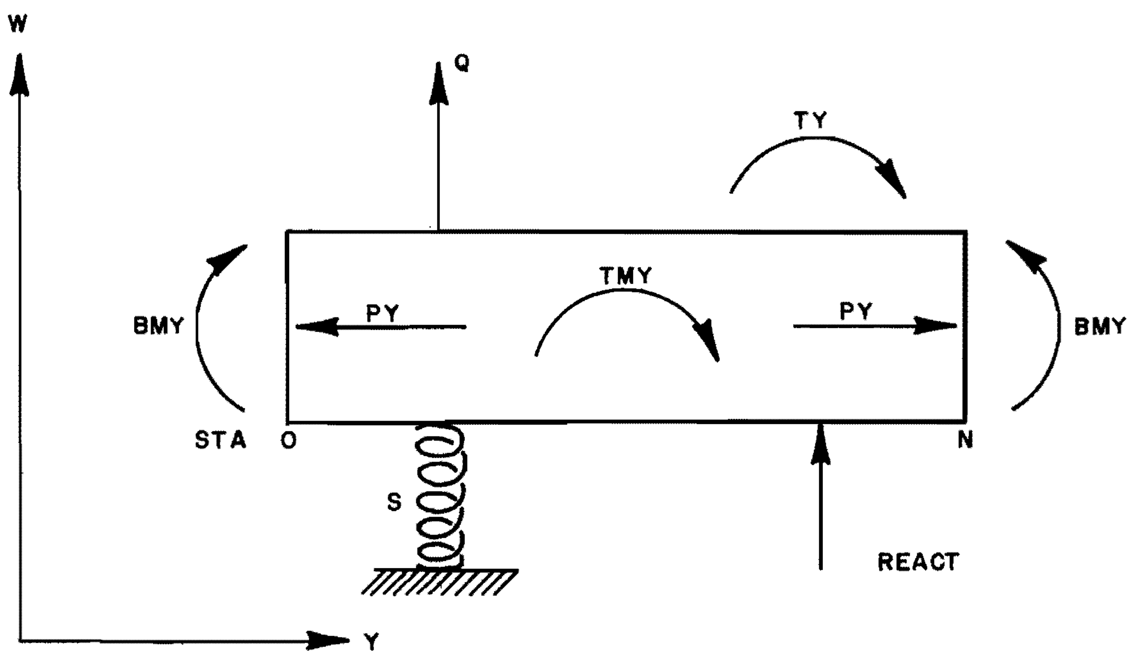


From Sta	Thru. Sta	Load Q	
4 2	4 2	1.000 E + 04	Concentrated value at Station 4,2.
3 1	4 1	5.000 E + 03	Distributed load centered over Stations 3,1 and 4,1 such that each gets a full value.
0 0	2 3	0.500 E + 03	Uniform load in rectangle 0,0 - 2,3, added in one quarter values at a time to provide half values as edges and quarter values at corners of the area.
0 1	2 2	0.500 E + 03	
1 0	1 3	0.500 E + 03	
1 1	1 2	0.500 E + 03	

Fig C.2. Example load input.



(a) Typical section of an x-beam.



(b) Typical section of a y-beam.

Fig C.3. Sign convention for input and output values for Program SLAB 17 (all values shown in positive sense).

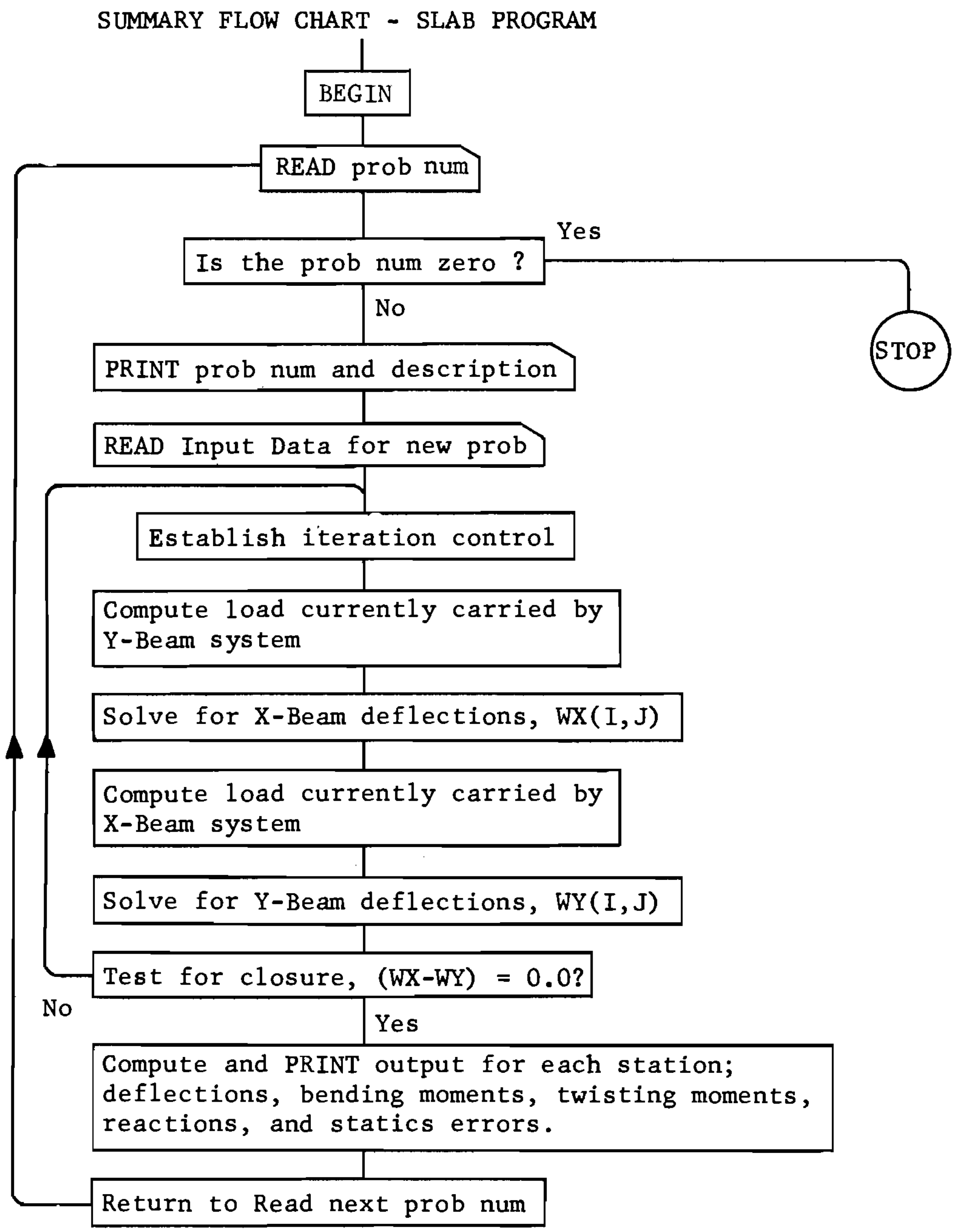
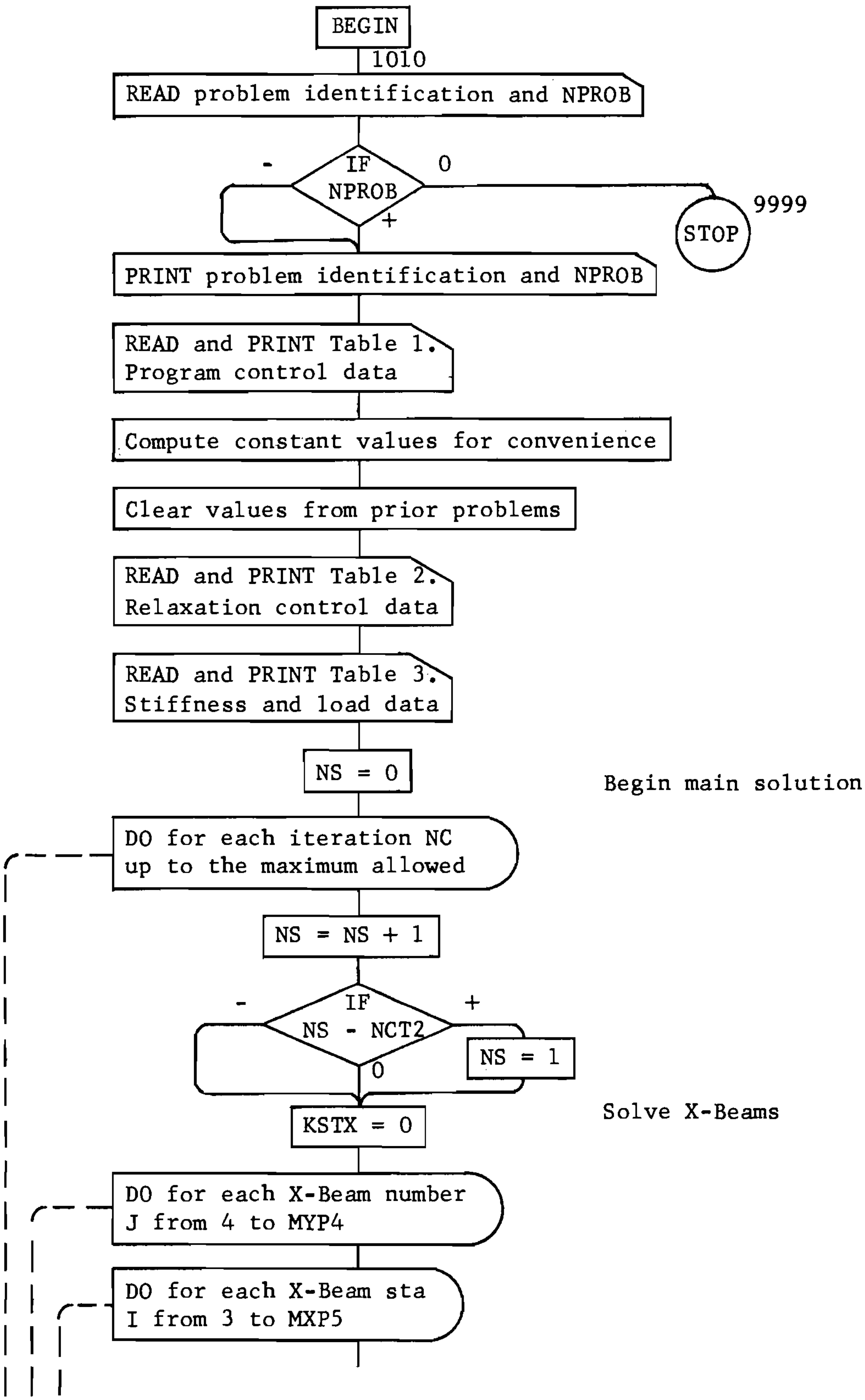


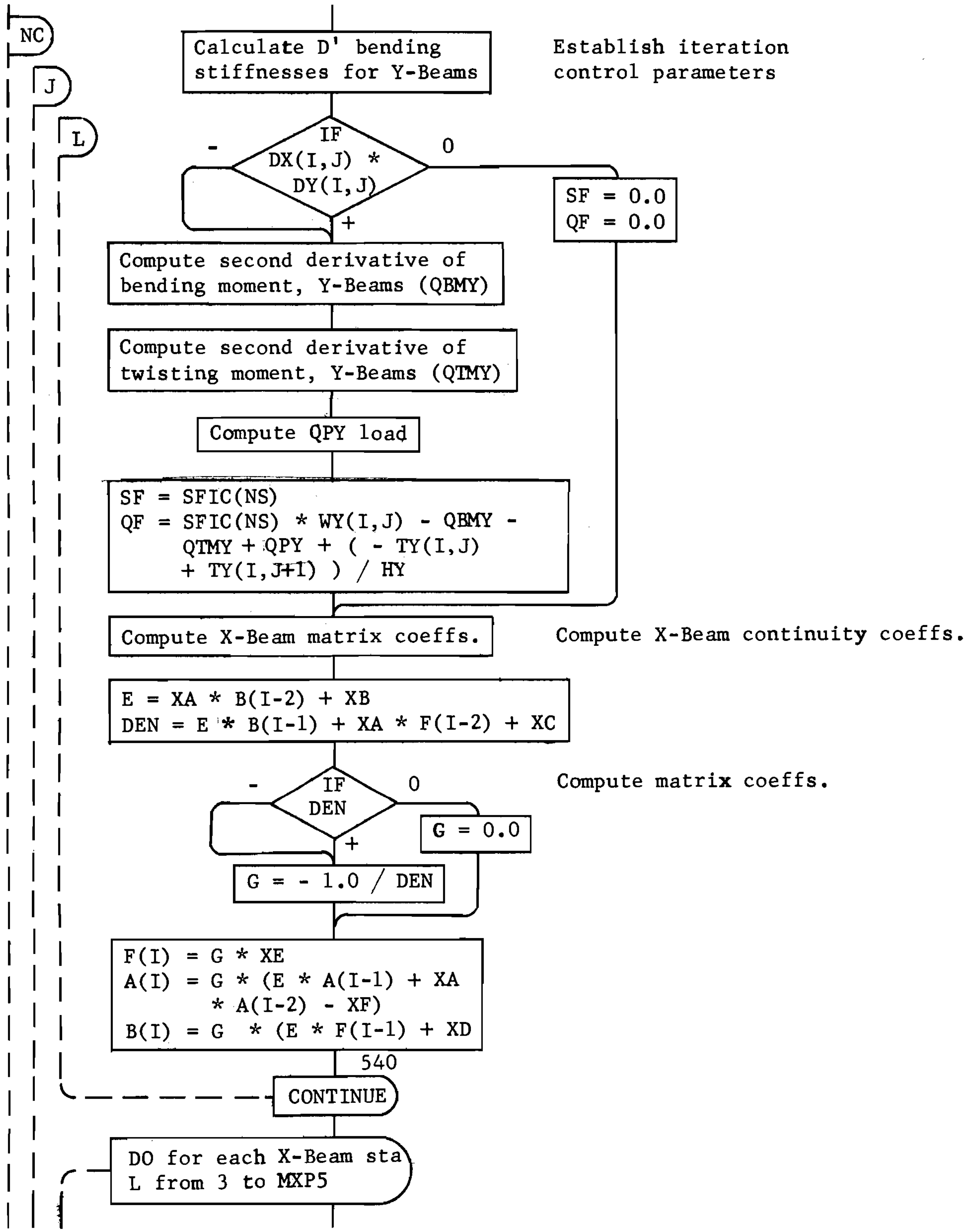
Fig C.4. Summary flow chart, slab program.

APPENDIX C2

FLOW DIAGRAM
PROGRAM SLAB 17

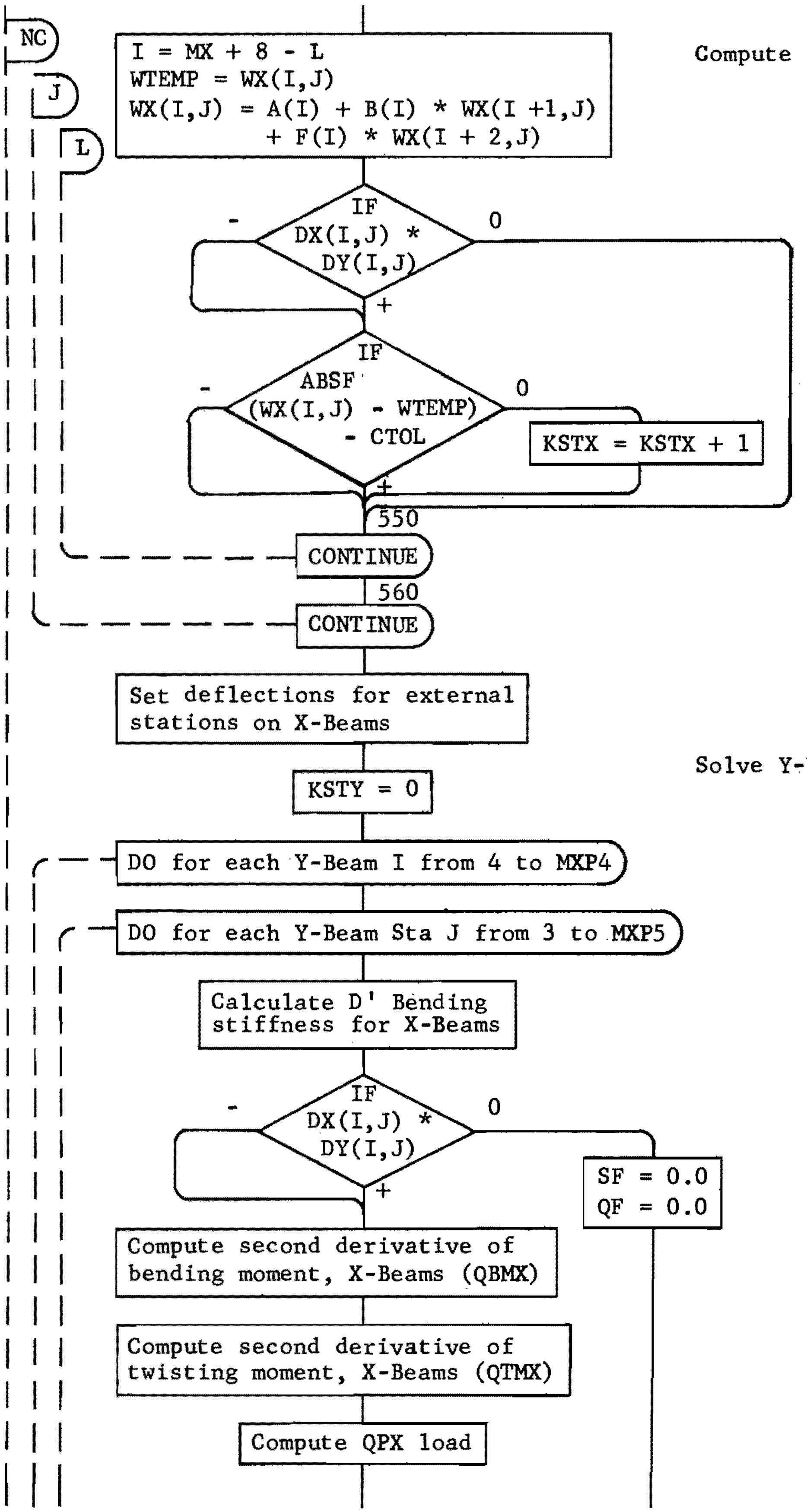
GENERAL FLOW DIAGRAM FOR PROGRAM SLAB 17

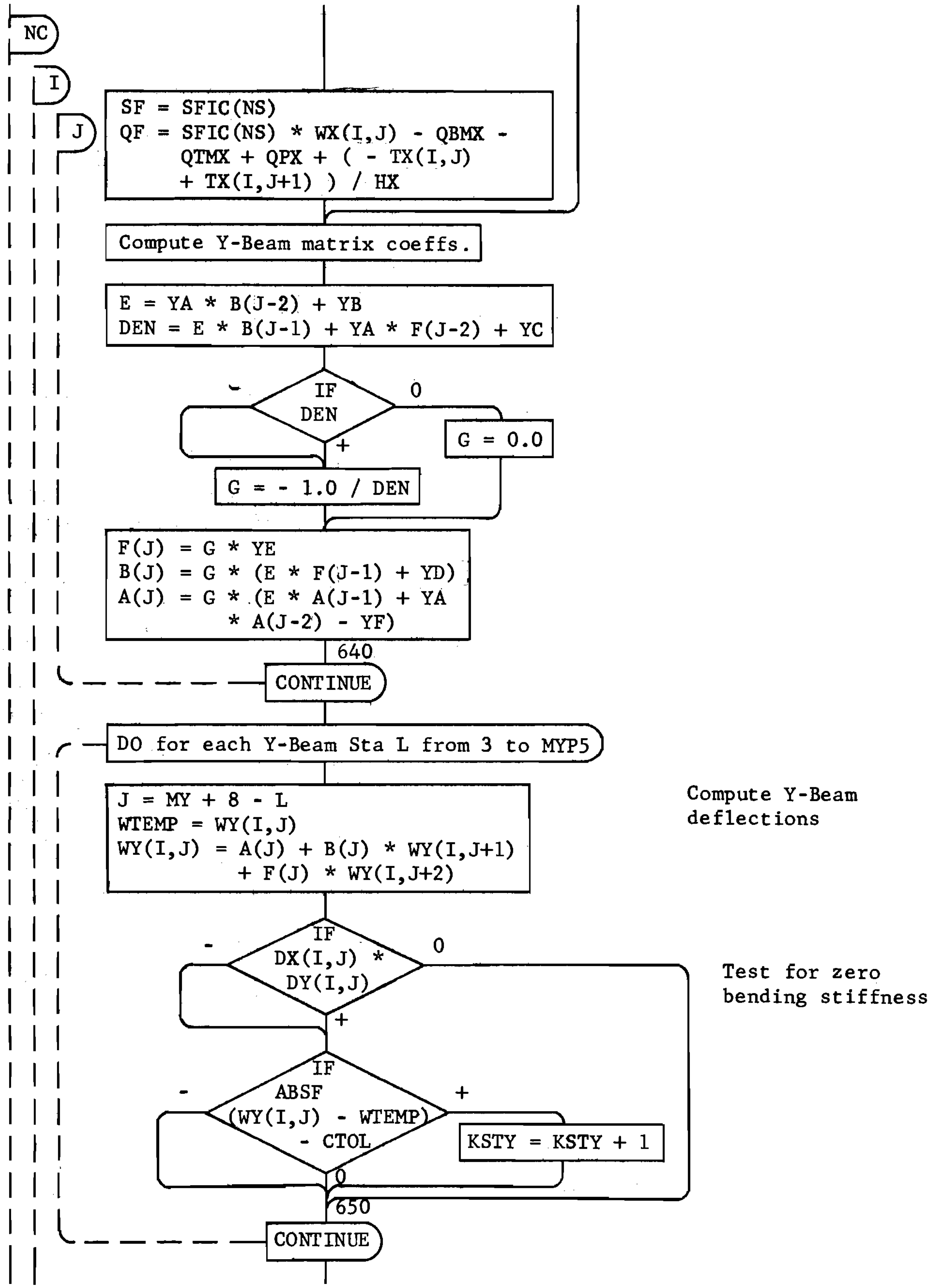


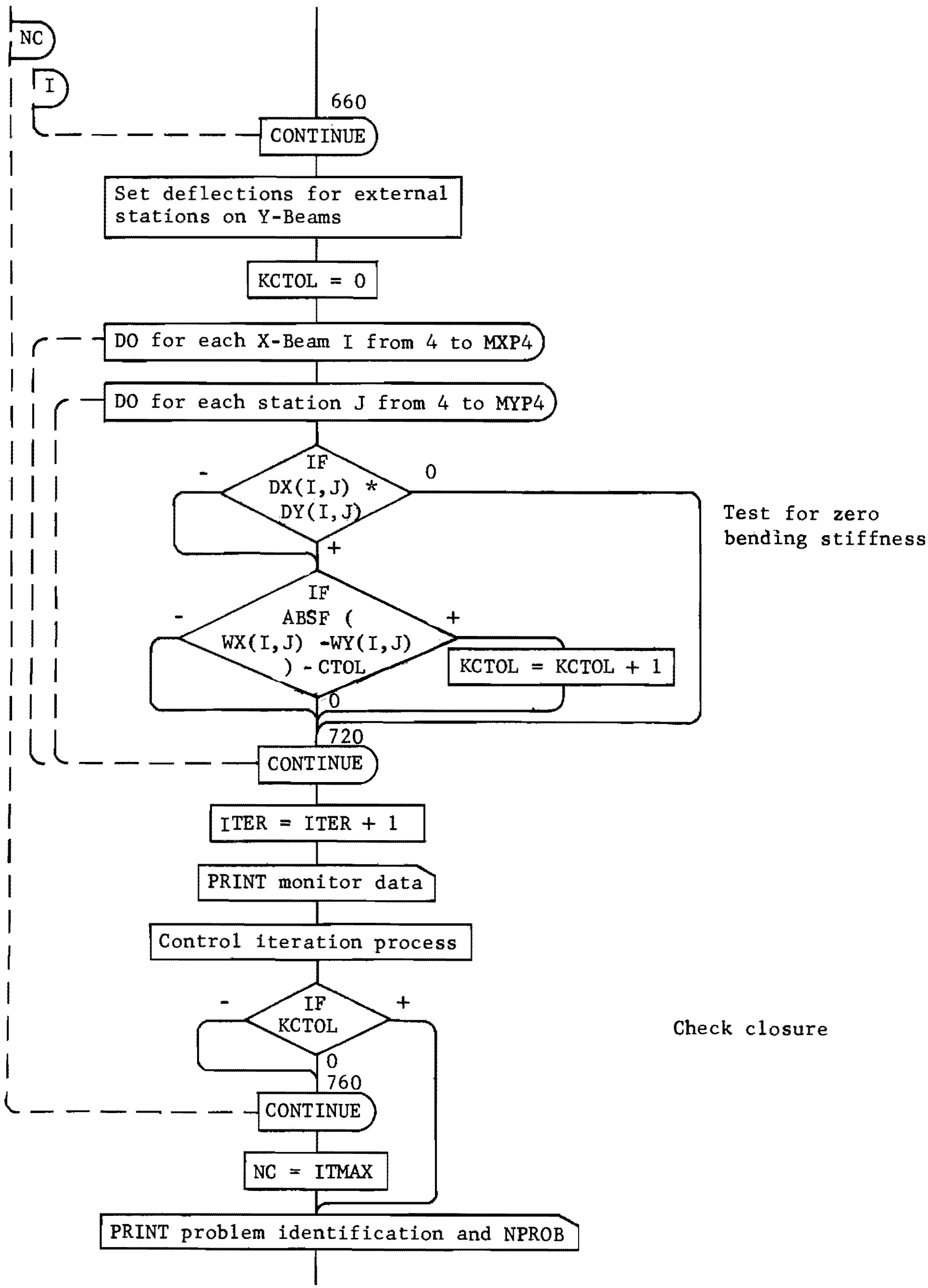

NC
J
L
Calculate D' bending stiffnesses for Y-Beams
Establish iteration control parameters
IF DX(I,J) * DY(I,J)
-
0
+
SF = 0.0
QF = 0.0
Compute second derivative of bending moment, Y-Beams (QBMY)
Compute second derivative of twisting moment, Y-Beams (QTMY)
Compute QPY load
SF = SFIC(NS)
QF = SFIC(NS) * WY(I,J) - QBMY - QTMY + QPY + (- TY(I,J) + TY(I,J+1)) / HY
Compute X-Beam matrix coeffs.
Compute X-Beam continuity coeffs.
E = XA * B(I-2) + XB
DEN = E * B(I-1) + XA * F(I-2) + XC
Compute matrix coeffs.
IF DEN
-
0
+
G = 0.0
G = - 1.0 / DEN
F(I) = G * XE
A(I) = G * (E * A(I-1) + XA * A(I-2) - XF)
B(I) = G * (E * F(I-1) + XD
540
CONTINUE
DO for each X-Beam sta
L from 3 to MXP5

Compute deflections

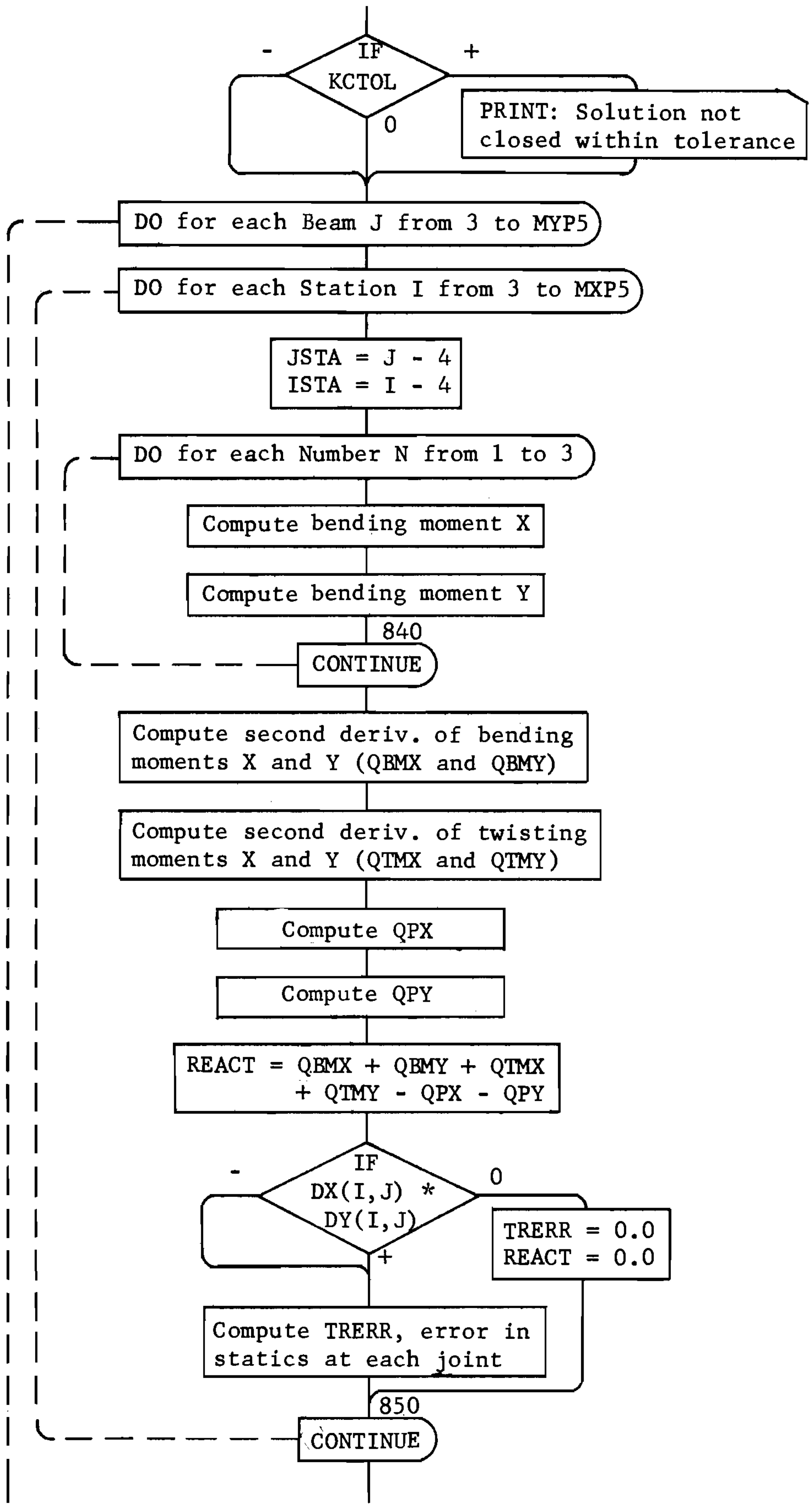
Solve Y-Beams

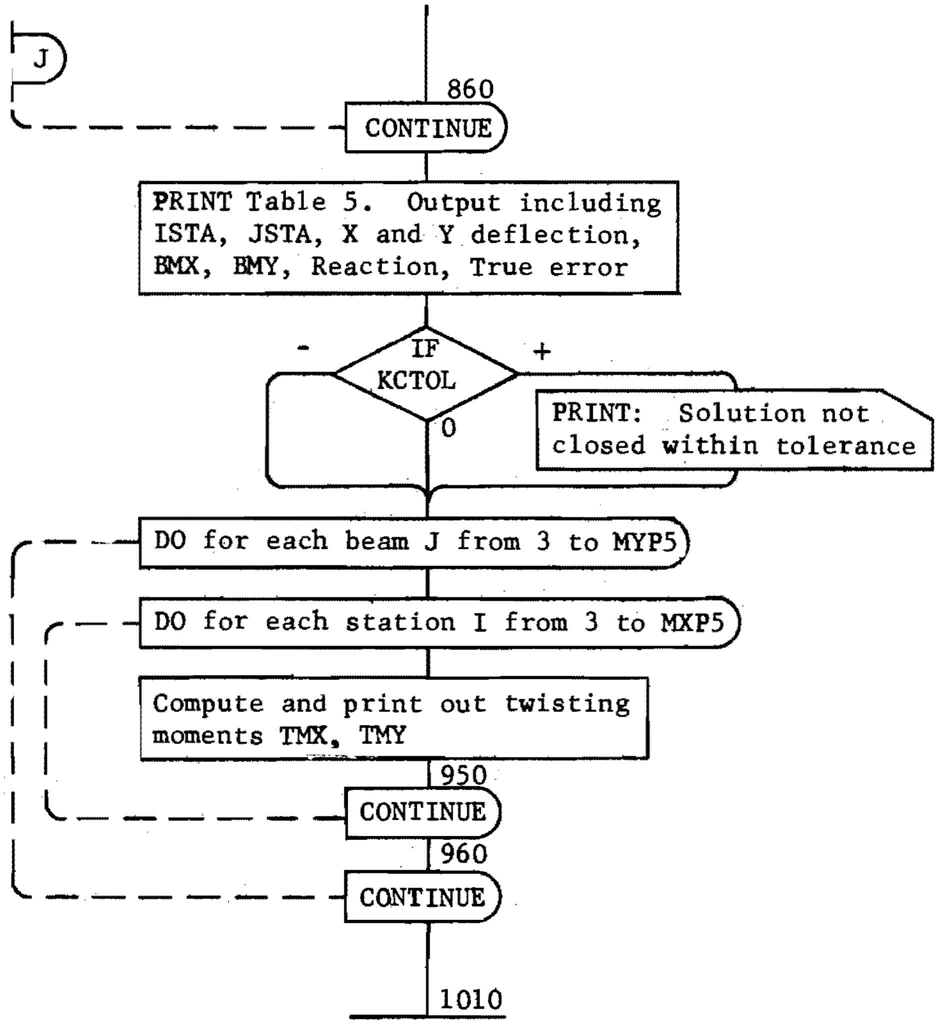



NC
I
J
SF = SFIC(NS)
QF = SFIC(NS) * WX(I,J) - QBMX - QTMX + QPX + (- TX(I,J) + TX(I,J+1)) / HX
Compute Y-Beam matrix coeffs.
E = YA * B(J-2) + YB
DEN = E * B(J-1) + YA * F(J-2) + YC
IF DEN
-
0
+
G = 0.0
G = - 1.0 / DEN
F(J) = G * YE
B(J) = G * (E * F(J-1) + YD)
A(J) = G * (E * A(J-1) + YA * A(J-2) - YF)
640
CONTINUE
DO for each Y-Beam Sta L from 3 to MYP5
J = MY + 8 - L
WTEMP = WY(I,J)
WY(I,J) = A(J) + B(J) * WY(I,J+1) + F(J) * WY(I,J+2)
Compute Y-Beam deflections
IF DX(I,J) * DY(I,J)
-
0
+
Test for zero bending stiffness
IF ABSF (WY(I,J) - WTEMP) - CTOL
-
+
0
KSTY = KSTY + 1
650
CONTINUE



NC
I
660
CONTINUE
Set deflections for external stations on Y-Beams
KCTOL = 0
DO for each X-Beam I from 4 to MXP4
DO for each station J from 4 to MYP4
IF DX(I,J) * DY(I,J)
-
0
+
Test for zero bending stiffness
IF ABSF (WX(I,J) -WY(I,J)) - CTOL
-
+
0
KCTOL = KCTOL + 1
720
CONTINUE
ITER = ITER + 1
PRINT monitor data
Control iteration process
IF KCTOL
-
+
0
Check closure
760
CONTINUE
NC = ITMAX
PRINT problem identification and NPROB


IF
KCTOL
-
+
0
PRINT: Solution not closed within tolerance
DO for each Beam J from 3 to MYP5
DO for each Station I from 3 to MXP5
JSTA = J - 4
ISTA = I - 4
DO for each Number N from 1 to 3
Compute bending moment X
Compute bending moment Y
840
CONTINUE
Compute second deriv. of bending moments X and Y (QBMX and QBMY)
Compute second deriv. of twisting moments X and Y (QTMX and QTMY)
Compute QPX
Compute QPY
REACT = QBMX + QBMY + QTMX + QTMY - QPX - QPY
IF
DX(I,J) * DY(I,J)
-
0
+
TRERR = 0.0
REACT = 0.0
Compute TRERR, error in statics at each joint
850
CONTINUE


J
860
CONTINUE
PRINT Table 5. Output including ISTA, JSTA, X and Y deflection, BMX, BMY, Reaction, True error
IF KCTOL
-
+
0
PRINT: Solution not closed within tolerance
DO for each beam J from 3 to MYP5
DO for each station I from 3 to MXP5
Compute and print out twisting moments TMX, TMY
950
CONTINUE
960
CONTINUE
1010

APPENDIX C3

PROGRAM LISTING
SLAB 17

```
      PROGRAM SLAB 17
    1 FORMAT (5X,52HPROGRAM SLAB 17 - MASTER DECK - WR HUDSON, H MATLOCK
     1                                  28H     REVISION DATE 26 JUL 65)
C
C              THIS PROGRAM SOLVES ORTHOTROPIC PLATES AND PAVEMENT       26JL5
C         SLABS BY AN ALTERNATING-DIRECTION ITERATIVE METHOD.  THE      26JL5
C         THEORY AND EQUATIONS ON WHICH IT IS BASED WERE DEVELOPED      26JL5
C         BY W.R. HUDSON UNDER THE SUPERVISION OF H. MATLOCK.  CLOSURE  26JL5
C         SPRINGS SIMILAR TO THOSE DISCUSSED BY WACHPRESS ARE USED,     26JL5
C         BUT ARE CHANGED ON THE HALF CYCLE AS SUGGESTED BY INGRAM.     26JL5
C
C-----NOTATION                                                          22JL5
C
C         A( ), B( ), F( )    CONTINUITY OR RECURSION COEFFICIENTS      22JL5
C         AN1(N), ETC.        ALPHANUMERIC REMARKS, INFORMATION ONLY    22JL5
C         BM(N)               TEMPORARY BENDING MOMENT VALUES (N = 1, 6)22JL5
C         CTOL                CLOSURE TOLERANCE, X VS Y DEFLS           22JL5
C         CX(I,J)             TORSIONAL STIFFNESS                       22JL5
C         CY(I,J)             TORSIONAL STIFFNESS                       22JL5
C         DEN                 TEMPORARY DENOMINATOR                     22JL5
C         DFERR               FINAL ERROR IN DEFL CLOSURE               22JL5
C         DP(N)               SQUARE ROOT OF PRODUCT OF BENDING         22JL5
C                             STIFFNESS                                 22JL5
C         DX(I,J)             BENDING STIFFNESS (SLAB)                  22JL5
C         DY(I,J)             BENDING STIFFNESS (SLAB)                  22JL5
C         HX, HY              INCREMENT LENGTH                          22JL5
C         I, J                STATION NUMBERS, X,Y DIRECTIONS           22JL5
C         IN1, DXN, QXN, ETC. TEMPORARY INPUT VALUES                    22JL5
C         IM1, JM1, ETC.      MONITOR STATION USED DURING ITERATION     22JL5
C         ISTA, JSTA          EXTERNAL STATION NUMBERS                  22JL5
C         ITER                COUNT OF NUMBER OF ITERATIONS             22JL5
C         ITEST               BLANK FIELD FOR ALPHANUMERIC ZERO         22JL5
C         ITMAX               MAXIMUM NUMBER OF ITERATIONS ALLOWED      22JL5
C         KCTOL               COUNT OF STATIONS NOT CLOSED              22JL5
C         KSTX                COUNT OF X BEAM STAS NOT STABILIZED       22JL5
C         KSTY                COUNT OF Y BEAM STAS NOT STABILIZED       22JL5
C         MX, MY              NUMBER OF INCREMENTS                      22JL5
C         NCT2, NCT3, NCT3C   NUM CARDS IN TABLES 2, 3, 3CONT.          22JL5
C         NCYC(NS)            NUM OF ITERATIONS ALLOWED FOR EACH FICT   22JL5
C                             SPG                                       22JL5
C         NPROB               NUMBER OF PROBLEM, PROG STOPS IF ZERO     22JL5
C         NSPR                SPRING NUM  NS, TO BE RETAINED AFTER      22JL5
C                             DO-LOOP                                   22JL5
C         PR                  POISSONS RATIO                            22JL5
C         PX(I,J)             AXIAL LOAD IN X DIRECTION                 22JL5
C         PY(I,J)             AXIAL LOAD IN Y DIRECTION                 22JL5
C         Q(I,J)              TRANSVERSE FORCE PER MESH POINT           22JL5
C         QBMX                HXHY * SECOND DERIV BEND MOMENT (X)       22JL5
C         QBMY                HYHX * SECOND DERIV BEND MOMENT (Y)       22JL5
C         QPX                 VERTICAL REACTION DUE TO AXIAL FORCES     22JL5
C         QPY                 VERTICAL REACTION DUE TO AXIAL FORCES     22JL5
C         QRX                 VERTICAL REACTION DUE TO ROTATIONAL       22JL5
C                             RESTRAINT                                 22JL5
C         QRY                 VERTICAL REACTION DUE TO ROTATIONAL       22JL5
C                             RESTRAINT                                 22JL5
C         QTMX                HXHY * SECOND DERIV TWIST MOMENT (XY)     22JL5
C         QTMY                HYHX * SECOND DERIV TWIST MOMENT (YX)     22JL5
C         REACT               NET TRANSVERSE FORCE                      22JL5
C         RMO , RNO           DUMMY VARIABLES                           22JL5
C         S(I,J)              SPRING SUPPORT, VALUE PER MESH POINT      22JL5
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C         SFXC(NS)                  FICTITIOUS SPRING REPRESENTING X-BEAMS    22JL5
C         SFYC(NS)                  FICTITIOUS SPRING REPRESENTING Y-BEAMS    22JL5
C         STOL(NS)                  STABILIZATION TOLERANCE                   22JL5
C         TMX                       TWISTING MOMENT (XY)                      22JL5
C         TMY                       TWISTING MOMENT (YX)                      22JL5
C         TRERR                     TRUE ERROR FROM SUM OF FORCES             22JL5
C         TX(I,J)                   EXTERNAL COUPLE IN X DIRECTION            22JL5
C         TY(I,J)                   EXTERNAL COUPLE IN Y DIRECTION            22JL5
C         WTEMP                     TEMPORARY VALUE OF DEFLECTION             22JL5
C         WX(I,J), WY(I,J)          VERTICAL DEFLECTION                       22JL5
C         XA THRU XF                COEFFS IN STIFFNESS AND LOAD MATRICES     22JL5
C         YA THRU YF                COEFFS IN STIFFNESS AND LOAD MATRICES     22JL5
      DIMENSION     AN1(32), AN2(14), PX(37,37), PY(37,37),                   26JL5
     1                      SFXC(9), STOL(20), NCYC(20), BMY(37,37),          26JL5
     2          BMX(37,37), S(37,37),   A(37), B(37), F(37),                  20AP5
     3          DX(37,37), DY(37,37), TX(37,37), TY(37,37), Q(37,37),         20AP5
     4          WX(37,37), WY(37,37), CX(37,37), CY(37,37), DP(6),SFYC(9)20AP5
    6 FORMAT ( )                                                              04MY3
   10 FORMAT (     5H      , 80X, 10HI-----TRIM )                             03FE4 ID
   11 FORMAT (     5H1     , 80X, 10HI-----TRIM )                             03FE4 ID
   12 FORMAT (  16A5   )                                                      04MY3 ID
   13 FORMAT (     5X, 16A5 )                                                 26AG3 ID
   14 FORMAT ( A5, 5X, 14A5 )                                                 19MR5 ID
   15 FORMAT (///10H     PROB , /5X, A5, 5X, 14A5 )                           19MR5 ID
   16 FORMAT (///17H     PROB (CONTD), /5X, A5, 5X, 14A5 )                    19MR5 ID
   19 FORMAT (///48H     RETURN THIS PAGE TO TIME RECORD FILE -- HM    ) 26AG3 ID
   20 FORMAT    ( 6( 3X, I2 ),  5E10.3 )                                      25MY4
   21 FORMAT   ( 8 ( 3X, I2 ) )                                               22AP3
   22 FORMAT   (      6E10.3  )                                               11FE5
   23 FORMAT   ( 4( 3X, I2 ),   6E10.3 )                                      09SE4
   30 FORMAT   (//30H     TABLE 1.  CONTROL DATA    ,              /          15AP3
     1         / 30H          NUM VALUES TABLE 2  , 43X, I2,     /            26JL5
     2           30H          NUM CARDS TABLE 3A  , 43X, I2,     /            26JL5
     2           30H          NUM CARDS TABLE 3B  , 43X, I2,     /            26JL5
     3           30H          MAX NUM ITERATIONS  , 43X, I2,     /            15AP3
     4           30H          NUM INCREMENTS  MX  , 43X, I2,     /            15AP3
     5           30H          NUM INCREMENTS  MY  , 43X, I2,     /            15AP3
     6           30H          INCR LENGTH  HX     , 35X,    E10.3,/           25MY4
     7           30H          INCR LENGTH  HY     , 35X,    E10.3,/           15AP3
     8           30H          CLOSURE TOLERANCE   , 35X,    E10.3,/           01MY3
     9           30H          POISSONS RATIO      , 35X,    E10.3,/           15MR5
     1           30H                              , 35X,    E10.3 )           01JL5
   31 FORMAT   (  30H          MONITOR STAS  I,J   ,  5X, 4( I7, I3 ) )       15AP3
   32 FORMAT   (//40H     TABLE 2A.  ITERATION CONTROL DATA  ,     /          01JL5
     1         / 47H          F. SPRING REPRESENTING X BEAM          /        01JL5
     2         / (5X,            E15.3         ) )                            01JL5
   33 FORMAT   (//51H     TABLE 3A. STIFFNESS AND LOAD DATA, FULL VALUES,01JL5
     1           35H ADDED AT ALL STAS I,J IN RECTANGLE, /                    15AP3
     2         / 50H         FROM    THRU      DX          DY          Q      26AP5
     3           45H         S           CX           CY              ,/)     26AP5
   34 FORMAT (///48H     TABLE 4.  MONITOR TALLY AND DEFLS AT 4 STAS,/        15AP3
     1         / 46H         ITR    FICT      CYC  NOT  NOT       I,J  ,/     15AP3
     2           42H         NUM    SPRING    NUM  STAB CLOS      ,           15AP3
     3                                           4( I2, 1X, I2, 7X ))15AP3
   35 FORMAT   (/ 35H     TABLE 5.  RESULTS -- ITERATION, I4 )                24AP3
   36 FORMAT   (/ 51H     SOLUTION NOT CLOSED WITHIN SPECIFIED TOLERANCE)22AP3
   37 FORMAT   (//44H     TABLE 3B. STIFFNESS AND LOAD DATA, FULL,           01JL5
     1           45H VALUES ADDED AT ALL STAS I,J IN RECT.          , /       26AP5
     2         / 50H         FROM    THRU                             TX      15JL5
     3           45H        TY           PX           PY              ,/)     26AP5
   38 FORMAT   (//40H     TABLE 2B.  ITERATION CONTROL DATA  ,     /          01JL5
     1         / 47H          F. SPRING REPRESENTING Y BEAM          /        01JL5
```

```
    2          / (5X,              E15.3       ) )                         01JL5
   39 FORMAT  (/ 50H          I,J      X-DEFL        Y-DEFL         BMX    08MR5
    1              40H BMY          REACT         TRERR                 ) / 08MR5
   40 FORMAT  (/ 40H          I,J        TMX            TMY        ) /     29MR5
   43 FORMAT  ( 5X,   2( 2X, I2, 1X, I2 ),   6E11.3)                       19NO4
   44 FORMAT  ( / 8H      X    , I3,    E12.3, I4, I5, 5X, 4E12.3 /        30AG4
    1            8H      Y    , 3X,    E12.3, 4X, I5, I5, 4E12.3   )       30AG4
   45 FORMAT  (    7X, I2, I3,     9E12.3 )                                02SE4
   46 FORMAT  ( 8X, I2, I3 )                                               17JL4
   47 FORMAT  (/ 42H     TABLE 5.  RESULTS(CONTD) -- ITERATION, I4    ) 24MY5
                ITEST = 5H                                                 19MR5 ID
 1000 PRINT 10                                                             12JL3 ID
      CALL TIME                                                            25MY4 ID
C-----PROGRAM AND PROBLEM IDENTIFICATION                                   04MY3 ID
      READ  12, ( AN1(N), N = 1, 32 )                                      19MR5 ID
 1010 READ  14,  NPROB, ( AN2(N), N = 1, 14 )                              28AG3 ID
         IF ( NPROB - ITEST ) 1020, 9990, 1020                             19MR5 ID
 1020 PRINT 11                                                             26AG3 ID
      PRINT 1                                                              19MR5 ID
      PRINT 13, ( AN1(N), N = 1, 32 )                                      19MR5 ID
      PRINT 15, NPROB, ( AN2(N), N = 1, 14 )                               26AG3 ID
C-----INPUT TABLE 1
      READ  20,  NCT2, NCT3, NCT3C,ITMAX, MX, MY, HX, HY, CTOL, PR         23JL5
      PRINT 30,  NCT2, NCT3, NCT3C,ITMAX, MX, MY, HX, HY, CTOL, PR         01JL5
      READ  21,  IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                    13AP3
      PRINT 31,  IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                    13AP3
C-----COMPUTE FOR CONVENIENCE
                HXE2 =   HX * HX                                           13AP3
                HYE2 =   HY * HY                                           13AP3
                HXE3 =   HX * HXE2                                         13AP3
                HYE3 =   HY * HYE2                                         13AP3
                HYS  =   HYE2 / HXE2                                       12FE5
                HXS  =   HXE2 / HYE2                                       12FE5
                HXHY =   HX * HY                                           12FE5
                HYHX =   HXHY                                              12FE5
                HHX  =   0.50 * HX                                         05AP5
                HHY  =   0.50 * HY                                         05AP5
                HYI  =  1.0 / HY                                           12JE5
                HXI  =  1.0 / HX                                           12JE5
                MXP4 =   MX + 4                                            13AP3
                MYP4 =   MY + 4                                            13AP3
                MXP5 =   MX + 5                                            13AP3
                MYP5 =   MY + 5                                            13AP3
                MXP7 =   MX + 7                                            13AP3
                MYP7 =   MY + 7                                            13AP3
C-----CLEAR VALUES FROM PRIOR PROBS
            DO 250  I = 1,MXP7                                             13AP3
            DO 250  J = 1,MYP7                                             13AP3
                DX(I,J)   =   0.0                                          28AG4
                DY(I,J)   =   0.0                                          28AG4
                CX(I,J)   =   0.0                                          15AP5
                CY(I,J)   =   0.0                                          15AP5
                PX(I,J)   =   0.0                                          20AP5
                PY(I,J)   =   0.0                                          20AP5
                TX(I,J)   =   0.0                                          28AG4
                TY(I,J)   =   0.0                                          28AG4
                 Q(I,J)   =   0.0                                          13AP3
                 S(I,J)   =   0.0                                          13AP3
                WX(I,J)   =   0.0                                          20AP3
                WY(I,J)   =   0.0                                          20AP3
  250       CONTINUE                                                       13AP3
C-----INPUT TABLE 2
```

```
      READ  22,   ( SFXC(N),                        N = 1,NCT2 )             14MY5
      PRINT 32,   ( SFXC(N),                        N = 1,NCT2 )             14MY5
      PRINT 6                                                                14MY5
      READ  22,   ( SFYC(N),                        N = 1,NCT2 )             14MY5
      PRINT 38,   ( SFYC(N),                        N = 1,NCT2 )             05JL5
C-----INPUT TABLE 3A                                                         01JL5
      PRINT 33                                                               13AP3
          DO 360 N = 1,NCT3                                                  13AP3
      READ  23,   IN1, JN1, IN2, JN2, DXN, DYN, QN, SN, CXN, CYN             22AP5
      PRINT 43,   IN1, JN1, IN2, JN2, DXN, DYN, QN, SN, CXN, CYN             22AP5
               I1   =    IN1 + 4                                             13AP3
               J1   =    JN1 + 4                                             13AP3
               I2   =    IN2 + 4                                             26AP3
               J2   =    JN2 + 4                                             13AP3
          DO 350  I = I1, I2                                                 13AP3
          DO 350  J = J1, J2                                                 13AP3
               DX(I,J) = DX(I,J) + DXN                                       4SE64
               DY(I,J) = DY(I,J) + DYN                                       4SE64
                Q(I,J) =  Q(I,J) + QN                                        15JL5
                S(I,J) =  S(I,J) + SN                                        15JL5
               CX(I,J) = CX(I,J) + CXN                                       15AP5
               CY(I,J) = CY(I,J) + CYN                                       15AP5
  350     CONTINUE                                                           13AP3
  360     CONTINUE                                                           13AP3
C-----INPUT TABLE 3B                                                         01JL5
      PRINT 6                                                                22AP5
      PRINT 37                                                               22AP5
          DO 380 N = 1, NCT3C                                                22AP5
      READ  23,   IN1, JN1, IN2, JN2, RNO, RMO, TXN, TYN, PXN, PYN           15JL5
      PRINT 43,   IN1, JN1, IN2, JN2, RNO, RMO, TXN, TYN, PXN, PYN           15JL5
               I1   =    IN1 + 4                                             20AP5
               J1   =    JN1 + 4                                             20AP5
               I2   =    IN2 + 4                                             20AP5
               J2   =    JN2 + 4                                             20AP5
          DO 370  I = I1, I2                                                 20AP5
          DO 370  J = J1, J2                                                 20AP5
               TX(I,J) = TX(I,J) + TXN                                       20AP5
               TY(I,J) = TY(I,J) + TYN                                       20AP5
               PX(I,J) = PX(I,J) + PXN                                       20AP5
               PY(I,J) = PY(I,J) + PYN                                       20AP5
  370     CONTINUE                                                           20AP5
  380     CONTINUE                                                           20AP5
      PRINT 34,   IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                     26AP3
C
C-----BEGIN MAIN SOLUTION
               NS = 0                                                        25AG4
          DO 760  NC = 1, ITMAX                                              25AG4
               NS = NS + 1                                                   25AG4
          IF ( NS-NCT2 ) 501, 501, 500                                       25AG4
  500          NS = 1                                                        25AG4
C
C-----SOLVE X-BEAMS
  501          KSTX =    0                                                   05FE5
          DO 560  J = 4, MYP4                                                01MA3
          DO 540  I = 3,MXP5                                                 13AP3
C-----ESTABLISH ITERATION CONTROL PARAMETERS SF AND QF
          DO 504  N = 1, 3                                                   15AP5
               L    =    J + N - 2                                           15AP5
               DP(N) = SQRTF (DX(I,L) * DY(I,L))                             15AP5
               K    =    I + N - 2                                           15AP5
               DP(N+3) = SQRTF (DX(K,J) * DY(K,J))                           15AP5
  504     CONTINUE                                                           20AP5
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```
          IF    (DX(I,J) * DY(I,J))  508, 505, 508                       4SE64
  505           SF    =    0.0                                           01MA3
                QF    =    0.0                                           13AP3
          GO TO 516                                                      13AP3
  508     DO 513  N = 1, 3                                               16JE5
                L    =    J + N - 2                                      01MA3
                BMY(I,L) = DY(I,L) * ( WY(I,L-1) - WY(I,L) - WY(I,L)     02MR5
    1                     + WY(I,L+1)) / HYE2 + PR * DP(N) * (WX(I-1,L)  20AP5
    2                     - WX(I,L) - WX(I,L) + WX(I+1,L)) / HXE2        29AP5
  513     CONTINUE                                                       16JE5
                QBMY =(BMY(I,J-1) -BMY(I,J) -BMY(I,J) + BMY(I,J+1) ) *   02MR5
    1                HX / HY                                             26AP5
                QTMY =(WY(I-1,J-1) * CY(I,J) - WY(I-1,J) * (CY(I,J) +    03MY5
    1                CY(I,J+1)) + WY(I-1,J+1) * CY(I,J+1) - WY(I,J-1) *  20AP5
    2                (CY(I,J) + CY(I+1,J)) + WY(I,J) * (CY(I,J) + CY     20AP5
    3                (1,J+1) + CY(I+1,J) + CY(I+1,J+1)) - WY(I,J+1) *    20AP5
    4                (CY(I,J+1) + CY(I+1,J+1)) + WY(I+1,J-1) * CY(I+1,J) 20AP5
    5                - WY(I+1,J) * (CY(I+1,J) + CY(I+1,J+1)) + WY        20AP5
    6                (I+1,J+1) * CY(I+1,J+1)) / HYHX                     26AP5
                QPY  = HYI * ( PY(I,J) * WY(I,J-1) - ( PY(I,J) + PY(I,J  08JE5
    1                     +1) ) * WY(I,J) + PY(I,J+1) * WY(I,J+1) )      03JE5
  514           SF   =  SFYC(NS)                                         14MY5
  515           QF   =  SFYC(NS) * WY(I,J) - QBMY - QTMY + QPY           06JL5
    1                   + ( - TY (I,J) + TY (I,J+1) ) / HY               19MY5
C-----COMPUTE X-BEAM MATRIX COEFFS
  516           XA = HYS * DX(I-1,J)                                     15AP5
                XB = HYS * ( - 2.0 * (DX(I-1,J) + DX(I,J))) - CX(I,J) -  09AP5
    1                CX(I,J+1) -  HY * PX(I,J)                           06JL5
                XC = HYS * (DX(I-1,J) +  4.0 * DX(I,J)  + DX(I+1,J)) +   22AP5
    1                CX(I,J) + CX(I,J+1) + CX(I+1,J) + CX(I+1,J+1) +     09AP5
    2                HY * (PX(I,J) + PX(I+1,J)) +                        31MY5
    3                HXHY * (S(I,J) + SF)                                06JL5
                XD = HYS * (- 2.0 * (DX(I,J) + DX(I+1,J))) - CX(I+1,J) - 09AP5
    1                CX(I+1,J+1) -  HY * PX(I+1,J)                       06JL5
                XE = HYS * DX(I+1,J)                                     05AP5
                XF = HXHY * (Q(I,J) + QF) + HY*(- TX(I,J) + TX(I+1,J)) - 23AP5
    1                WX(I-1,J-1) * CX(I,J) + WX(I,J-1) * (CX(I,J) +      09AP5
    2                CX(I+1,J)) - WX(I+1,J-1) * CX(I+1,J) - WX(I-1,J+1)  09AP5
    3                * CX(I,J+1) + WX(I,J+1) * (CX(I,J+1) + CX(I+1,J+1)) 09AP5
    4                - WX(I+1,J+1) * CX(I+1,J+1) - PR * (DP(4) *         18MY5
    5                (WY(I-1,J-1) - 2.0 * WY(I-1,J) + WY(I-1,            23AP5
    6                J+1)) -  2.0 * DP(5)* (WY(I,J-1) -  2.0 * WY(I,J)   23AP5
    7                + WY(I,J+1))  + DP(6) * (WY(I+1,J-1) -  2.0 * WY(   28AP5
    8                I+1,J)  + WY(I+1,J+1)))                             23AP5
C-----COMPUTE X-BEAM CONTINUITY COEFFS
                E      =    XA * B(I-2) + XB                             29AP5
                DEN    =    E  * B(I-1) + XA * F(I-2) + XC               05AP5
          IF    (DEN)  531, 521, 531                                     13AP3
  521           G      =    0.0                                          08JL4
          GO TO  532                                                     13AP3
  531           G      = - 1.0 / DEN                                     08JL4
  532           F(I) =    G  * XE                                        08AP5
                A(I) =  G * ( E * A(I-1) + XA * A(I-2) - XF )            23AP5
                B(I) =     G  * ( E  * F(I-1) + XD )                     05AP5
  540     CONTINUE                                                       13AP3
C-----COMPUTE X-BEAM DEFLS
          DO 550  L = 3,MXP5                                             13AP3
                I      =    MX + 8 - L                                   13AP3
                WTEMP     =   WX(I,J)                                    13AP3
                WX(I,J)   =   A(I) + B(I) * WX(I+1,J) + F(I) * WX(I+2,J) 30MR5
C-----COUNT STAS WHERE X-BEAMS NOT STABILIZED
          IF  ( DX(I,J) * DY(I,J) )  545, 550, 545                       04SE4
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  545      IF  ( ABSF ( WX(I,J) - WTEMP ) - CTOL    )550, 550, 546         25AG4
  546           KSTX =   KSTX + 1                                          13AP3
  550      CONTINUE                                                        13AP3
  560      CONTINUE                                                        13AP3
C-----SOLVE  EXTERNAL X- BEAMS
           DO 570  I = 3, MXP5                                             15FE5
                J  =  3                                                    15FE5
                WX(I,J)  = WY(I,J)                                         11MY5
                J  =  MYP5                                                 15FE5
                WX(I,J)  = WY(I,J)                                         11MY5
  570     CONTINUE                                                         15FE5
                WX (3,3) = 2.0 * WX (3,4) - WX (3,5)                       19MY5
                WX (MXP5,3) = 2.0 * WX (MXP5,4) - WX (MXP5,5)              19MY5
                WX (3,MYP5) = 2.0 * WX (3,MYP4) - WX (3,MY+3)              19MY5
                WX (MXP5,MYP5) = 2.0 * WX (MXP5, MYP4) - WX (MXP5,MY+3)    19MY5
C
C-----SOLVE Y-BEAMS
                KSTY   =  0                                                29AP3
           DO 660  I = 4, MXP4                                             01MA3
           DO 640  J = 3,MYP5                                              13AP3
C-----ESTABLISH ITERATION CONTROL PARAMETERS SF AND QF
           DO 604  N = 1, 3                                                15AP5
                L    =    J + N - 2                                        15AP5
                DP(N) = SQRTF (DX(I,L) * DY(I,L))                          15AP5
                K    =    I + N - 2                                        15AP5
                DP(N+3) = SQRTF (DX(K,J) * DY(K,J))                        15AP5
  604      CONTINUE                                                        20AP5
           IF  ( DX(I,J) * DY(I,J) )  608, 605, 608                        28AG4
  605           SF   =   0.0                                               01MA3
                QF   =   0.0                                               13AP3
           GO TO 616                                                       13AP3
  608      DO 613  N = 1, 3                                                16JE5
                K    =    I + N - 2                                        21AG4
                BMX(K,J) = DX(K,J) * ( WX(K-1,J) - WX(K,J) - WX(K,J)       02MR5
     1               + WX(K+1,J)) / HXE2 + DP(N+3) * PR * (WY(K,J-1) -     15AP5
     2               WY(K,J) - WY(K,J) + WY(K,J+1)) / HYE2                 15AP5
  613 CONTINUE
                QBMX = ( BMX(I-1,J) - BMX(I,J) -BMX(I,J) + BMX(I+1,J))     02MR5
     1               * ( HY / HX )                                         26AP5
                QTMX = (WX(I-1,J-1) * CX(I,J) - WX(I-1,J) * (CX(I,J) +     20AP5
     1               CX(I,J+1)) + WX(I-1,J+1) * CX(I,J+1) - WX(I,J-1) *    20AP5
     2               (CX(I,J) + CX(I+1,J)) + WX(I,J) * (CX(I,J) + CX       28AP5
     3               (I,J+1) + CX(I+1,J) + CX(I+1,J+1)) - WX(I,J+1) *      20AP5
     4               (CX(I,J+1) + CX(I+1,J+1)) + WX(I+1,J-1) * CX(I+1,J)   20AP5
     5               - WX(I+1,J) * (CX(I+1,J) + CX(I+1,J+1)) + WX          20AP5
     6               (I+1,J+1) * CX(I+1,J+1)) / HYHX                       26AP5
                QPX  = HXI * ( PX(I,J) * WX(I-1,J) - ( PX(I,J) + PX(I+1    03JE5
     1                    ,J) ) * WX(I,J) + PX(I+1,J) * WX(I+1,J) )        03JE5
  614           SF  =  SFXC(NS)                                            14MY5
  615           QF  =  SFXC(NS) * WX(I,J) - QBMX - QTMX + QPX              06JL5
     1                + ( - TX (I,J) + TX (I+1,J) ) / HX                   19MY5
C-----COMPUTE Y-BEAM MATRIX COEFFS
  616           YA = HXS * DY(I,J-1)                                       15AP5
                YB = HXS * (- 2.0 * (DY(I,J-1) + DY(I,J))) - CY(I,J) -     09AP5
     1               CY(I+1,J) -  HX * PY(I,J)                             06JL5
                YC = HXS * (DY(I,J-1) +  4.0 * DY(I,J)  + DY(I,J+1)) +     23AP5
     1               CY(I,J) + CY(I+1,J) + CY(I,J+1) + CY(I+1,J+1) +       23AP5
     2               HX * ( PY(I,J) + PY(I,J+1)) +                         31MY5
     3               HYHX * (S(I,J) + SF)                                  06JL5
                YD = HXS * (- 2.0 * (DY(I,J) + DY(I,J+1))) - CY(I,J+1) - 09AP5
     1               CY(I+1,J+1) -  HX * PY(I,J+1)                         06JL5
                YE = HXS * DY(I,J+1)                                       09AP5
```

```
            YF = HYHX * (Q(I,J) + QF) + HX*(- TY(I,J) + TY(I,J+1)) - 23AP5
    1            WY(I-1,J-1) * CY(I,J) + WY(I-1,J) * (CY(I,J) + CY(  09AP5
    2            I,J+1)) - WY(I-1,J+1) * CY(I,J+1) - WY(I+1,J-1) *   09AP5
    3            CY(I+1,J) + WY(I+1,J) * (CY(I+1,J) + CY(I+1,J+1)) - 13AP5
    4            WY(I+1,J+1) * CY(I+1,J+1) - PR * (DP(1) *           18MY5
    5            (WX(I-1,J-1) - 2.0 * WX(I,J-1) + WX(I+1,            19MY5
    6            J-1)) -  2.0 * DP(2)* (WX(I-1,J) -  2.0 * WX(I,J)   23AP5
    7            + WX(I+1,J))  + DP(3) * (WX(I-1,J+1) -  2.0 * WX(I, 28AP5
    8            J+1)  + WX(I+1,J+1)))                               23AP5
C-----COMPUTE Y-BEAM CONTINUITY COEFFS
            E    =   YA * B(J-2) + YB                                04SE4
            DEN  =   E  * B(J-1) + YA * F(J-2) + YC                  05AP5
        IF  (DEN)  631, 621, 631                                     13AP3
  621       G    =   0.0                                             08JL4
        GO TO  632                                                   13AP3
  631       G    = - 1.0 / DEN                                       08JL4
  632       F(J) =   G  *  YE                                        05AP5
            B(J) =   G  * ( E  * F(J-1) + YD )                       05AP5
            A(J) =   G  * ( E  * A(J-1) + YA * A(J-2) - YF )         23AP5
  640   CONTINUE                                                     13AP3
C-----COMPUTE Y-BEAM DEFLS
        DO 650  L = 3,MYP5                                           13AP3
            J     = MY + 8 - L                                       13AP3
            WTEMP    =   WY(I,J)                                     13AP3
            WY(I,J)  =   A(J) + B(J) * WY(I,J+1) + F(J) * WY(I,J+2)  30MR5
C-----COUNT STAS WHERE Y-BEAMS NOT STABILIZED
        IF  ( DX(I,J) * DY(I,J) )  645, 650, 645                     04SE4
  645   IF  ( ABSF ( WY(I,J) - WTEMP ) - CTOL    )650, 650, 646      25AG4
  646       KSTY =   KSTY + 1                                        13AP3
  650   CONTINUE                                                     13AP3
  660   CONTINUE                                                     13AP3
C-----SOLVE  EXTERNAL Y- BEAMS
        DO 670  J = 3,  MYP5                                         17FE5
            I  =  3                                                  17FE5
            WY(I,J)  = WX(I,J)                                       11MY5
            I  =  MXP5                                               17FE5
            WY(I,J)  = WX(I,J)                                       11MY5
  670   CONTINUE                                                     17FE5
C
C-----COUNT STAS WHERE X AND Y DEFLS NOT CLOSED
            KCTOL  =  0                                              13AP3
        DO 720 I = 4,MXP4                                            13AP3
        DO 720 J = 4,MYP4                                            13AP3
        IF  ( DX(I,J) * DY(I,J) )  711, 720, 711                     28AG4
  711   IF  ( ABSF ( WX(I,J) - WY(I,J) ) - CTOL )  720, 720, 712     13AP3
  712       KCTOL  =   KCTOL + 1                                     13AP3
  720   CONTINUE                                                     13AP3
            ITER   =   ITER + 1                                      13AP3
C-----PRINT MONITOR DATA
      PRINT 44,    NC, SFYC(NS), NS, KSTX, WX(IM1+4,JM1+4),          14MY5
    1   WX(IM2+4,JM2+4), WX(IM3+4,JM3+4), WX(IM4+4,JM4+4),           25AP3
    2   SFXC(NS), KSTY, KCTOL, WY(IM1+4,JM1+4),                      14MY5
    3   WY(IM2+4,JM2+4), WY(IM3+4,JM3+4), WY(IM4+4,JM4+4)            25AP3
C-----CONTROL ITERATION PROCESS
        IF  ( KCTOL )   800, 800, 760                                25AG4
  760   CONTINUE                                                     17AP3
            NC = ITMAX                                               25AG4
C-----COMPUTE AND PRINT RESULTS
  800 PRINT 11                                                       25AG4
      PRINT 1                                                        19MR5 ID
      PRINT 13, ( AN1(N), N = 1, 32 )                                19MR5 ID
      PRINT 16, NPROB, ( AN2(N), N = 1, 14 )                         28AG3 ID
```

```
      PRINT 35, NC                                                      25AG4
           IF  ( KCTOL )  812, 812, 811                                 18AP3
  811 PRINT 36                                                          18AP3
  812 PRINT 39                                                          26MR5
           DO 860  J = 3,MYP5                                           18AP3
      PRINT  6                                                          24AP3
           DO 850  I = 3,MXP5                                           18AP3
                ISTA = I - 4                                            18AP3
                JSTA = J - 4                                            18AP3
           DO 840  N = 1,3                                              22AP3
                K    =    I + N - 2                                     01MA3
                DP(N+3) = SQRTF (DX(K,J) * DY(K,J))                     15AP5
                BMX(K,J) = DX(K,J) * ( WX(K-1,J) - WX(K,J) - WX(K,J)    02MR5
     1               + WX(K+1,J)) / HXE2 + DP(N+3) * PR * (WY(K,J-1) -  15AP5
     2               WY(K,J) - WY(K,J) + WY(K,J+1)) / HYE2              15AP5
                L    =    J + N - 2
                DP(N) = SQRTF (DX(I,L) * DY(I,L))                       15AP5
                BMY(I,L) =  DY(I,L) * ( WY(I,L-1) - WY(I,L) - WY(I,L)   02MR5
     1                    + WY(I,L+1)) / HYE2 + PR * DP(N) * (WX(I-1,L) 20AP5
     2                    - WX(I,L) - WX(I,L) + WX(I+1,L)) / HXE2       29AP5
  840 CONTINUE                                                          01MA3
                QBMX = ( BMX(I-1,J) - BMX(I,J) -BMX(I,J) + BMX(I+1,J))  02MR5
     1               * HY / HX                                          26AP5
                QBMY =(BMY(I,J-1) -BMY(I,J) -BMY(I,J) + BMY(I,J+1) ) *  02MR5
     1               HX / HY                                            26AP5
                QTMX = (WX(I-1,J-1) * CX(I,J) - WX(I-1,J) * (CX(I,J) +  20AP5
     1               CX(I,J+1)) + WX(I-1,J+1) * CX(I,J+1) - WX(I,J-1) * 20AP5
     2               (CX(I,J) + CX(I+1,J)) + WX(I,J) * (CX(I,J) + CX    28AP5
     3               (I,J+1) + CX(I+1,J) + CX(I+1,J+1)) - WX(I,J+1) *   20AP5
     4               (CX(I,J+1) + CX(I+1,J+1)) + WX(I+1,J-1) * CX(I+1,J) 20AP5
     5               - WX(I+1,J) * (CX(I+1,J) + CX(I+1,J+1)) + WX       20AP5
     6               (I+1,J+1) * CX(I+1,J+1)) / HYHX                    26AP5
                QTMY =(WY(I-1,J-1) * CY(I,J) - WY(I-1,J) * (CY(I,J) +   03MY5
     1               CY(I,J+1)) + WY(I-1,J+1) * CY(I,J+1) - WY(I,J-1) * 20AP5
     2               (CY(I,J) + CY(I+1,J)) + WY(I,J) * (CY(I,J) + CY    20AP5
     3               (I,J+1) + CY(I+1,J) + CY(I+1,J+1)) - WY(I,J+1) *   20AP5
     4               (CY(I,J+1) + CY(I+1,J+1)) + WY(I+1,J-1) * CY(I+1,J) 20AP5
     5               - WY(I+1,J) * (CY(I+1,J) + CY(I+1,J+1)) + WY       20AP5
     6               (I+1,J+1) * CY(I+1,J+1)) / HXHY                    26AP5
                QPX  = HXI * ( PX(I,J) * WX(I-1,J) - ( PX(I,J) + PX(I+1 03JE5
     1                    ,J) ) * WX(I,J) + PX(I+1,J) * WX(I+1,J) )     03JE5
                QPY  = HYI * ( PY(I,J) * WY(I,J-1) - ( PY(I,J) + PY(I,J 08JE5
     1                    +1) ) * WY(I,J) + PY(I,J+1) * WY(I,J+1) )     03JE5
                REACT = QBMX + QBMY + QTMX + QTMY - QPX - QPY           04JE5
           IF   ( DX(I,J) * DY(I,J) )  848, 847, 848                    05MR5
  847           TRERR   =    0.0                                        17FE5
                REACT   =  0.0                                          20AP5
           GO TO  849                                                   24AP3
  848           TRERR = Q(I,J) - S(I,J) * ( WX(I,J) + WY(I,J) ) / 2.0   03SE4
     1                 -REACT +(-TX(I,J)+TX(I+1,J) )/HX+(-TY(I,J)       19MY5
     2                 + TY(I,J+1) ) / HY                               06JL5
  849      PRINT 45, ISTA, JSTA, WX(I,J), WY(I,J), BMX(I,J), BMY(I,J),  08MR5
     1               REACT, TRERR                                       20AP5
  850      CONTINUE                                                     18AP3
  860      CONTINUE                                                     18AP3
      PRINT  6                                                          30MR5
      PRINT  6                                                          30MR5
      PRINT 16, NPROB, ( AN2(N), N = 1, 14 )                            30MR5
      PRINT 47,  NC                                                     01JL5
           IF  ( KCTOL )  912, 912, 911                                 30MR5
  911 PRINT 36                                                          30MR5
  912 PRINT 40                                                          30MR5
```

```
          DO 960  J = 3,MYP5                                        30MR5
      PRINT  6                                                      30MR5
          DO 950  I = 3,MXP5                                        30MR5
               ISTA = I - 4                                         30MR5
               JSTA = J - 4                                         30MR5
          TMX  =  (CX(I,J) + CX(I,J+1) + CX(I+1,J) + CX(I+1,J+1)) * 26AP5
     1         0.250 * (WX(I-1,J-1) - WX(I-1,J+1) - WX(I+1,J-1) + WX 26AP5
     2         (I+1,J+1)) / (4.0 * HXHY)                            26AP5
          TMY  =  (CY(I,J) + CY(I,J+1) + CY(I+1,J) + CY(I+1,J+1)) * 26AP5
     1         (-0.250) * (WY(I-1,J-1) - WY(I-1,J+1) - WY(I+1,J-1) + WY 26AP5
     2         (I+1,J+1)) / (4.0 * HXHY)                            26AP5
  949     PRINT 45, ISTA, JSTA, TMX, TMY                            20AP5
  950     CONTINUE                                                  23MR5
  960     CONTINUE                                                  23MR5
      CALL TIME                                                     25MY4 ID
          GO TO 1010                                                26AG3 ID
 9990 CONTINUE                                                      19MR5 ID
 9999 CONTINUE                                                      04MY3 ID
      PRINT 11                                                      08MY3 ID
      PRINT 1                                                       19MR5 ID
      PRINT 13, ( AN1(N), N = 1, 32 )                               19MR5 ID
      PRINT 19                                                      26AG3 ID
      END
      END
         FINIS
9EXECUTE,,,1.
```

APPENDIX D

EXAMPLE INPUT AND OUTPUT

APPENDIX D1

CODED DATA INPUT

EXAMPLE PROBLEMS

```
CE051022   HWY SLAB PROJECT        SLAB 17      W R HUDSON
RUN EXAMPLE PROBLEMS FOR USE IN SLAB REPORT APPENDIX D


  101   48 INCH SQ PLATE SIMPLE SUPPORTS,  Q = 10E5 AT CENTER   WRH      11AP65
    4    6         36    8    8 6.000E+00 6.000E+00 1.000E-06 0.250E+00
    4    4    0    4    2    2    4    0
 5.000E 03 1.000E 05 1.000E 06 5.000E 06
 5.000E 03 1.000E 05 1.000E 06 5.000E 06
    0    0    8    8 0.625E 06 0.625E+06           1.000E 99
    1    0    7    8 0.625E 06 0.625E+06
    0    1    8    7 0.625E 06 0.625E+06
    1    1    7    7 0.625E 06 0.625E+06          -1.000E 99
    4    4    4    4                     1.000E 05
    1    1    8    8                                         1.875E+06 1.875E+06


  102   48 INCH SQ PLATE SIMPLE SUPPORTS, PY = 10E5, Q = 10E5 AT CENTER WRH 01JL5
    3    6    1   30    8    8 6.000E 00 6.000E 00 1.000E-06 0.250E 00
    4    4    0    4    2    2    4    0
 1.000E 03 1.000E 04 5.000E 05
 1.000E 03 1.000E 04 5.000E 05
    0    0    8    8 0.625E 06 0.625E+06           1.000E 99
    1    0    7    8 0.625E 06 0.625E+06
    0    1    8    7 0.625E 06 0.625E+06
    1    1    7    7 0.625E 06 0.625E+06          -1.000E 99
    4    4    4    4                     1.000E 05
    1    1    8    8                                         1.875E+06 1.875E+06
    0    1    8    8                                                   1.000E 05


  103      48 INCH SQ PLATE, SIMPLE SUPPORTS,   PX=PY=10E5, Q=10E5 WRH    2JL5
    3    6    2   36    8    8 6.000E+00 6.000E+00 1.000E-06 0.250E+00
    4    4    0    4    2    2    4    0
 1.000E 03 4.000E 04 6.000E 05
 1.000E 03 4.000E 04 6.000E 05
    0    0    8    8 0.625E 06 0.625E+06           1.000E 99
    1    0    7    8 0.625E 06 0.625E+06
    0    1    8    7 0.625E 06 0.625E+06
    1    1    7    7 0.625E 06 0.625E+06          -1.000E 99
    4    4    4    4                     1.000E 05
    1    1    8    8                                         1.875E+06 1.875E+06
    1    0    8    8                                         1.000E 05
    0    0    8    8                                                   1.000E 05


  104      48 INCH SQ STEEL PLATE, UNIFORM LOAD 1000 LB              WRH    29AP5
    5    5    0   44    8    8 6.000E 00 6.0  E 00 1.0  E-06 0.250E 00
    4    4    2    2    2    4    4    2
 1.000E+03 1.000E 04 1.000E 05 1.000E 06 1.000E 03
 1.000E+03 1.000E 04 1.000E 05 1.000E 06 1.000E 03
    0    0    8    8 6.250E 05 6.250E+05 0.900E 04 1.000E 99
    1    1    8    8                                         1.875E+06 1.875E+06
    1    1    7    7 6.250E 05 6.250E+05 0.900E 04-1.000E 99
    0    1    8    7 6.250E 05 6.250E+05 0.900E 04
    1    0    7    8 6.250E 05 6.250E+05 0.900E 04
```

```
 105      48 INCH SQ STEEL PL, SIMPLE SUPPORT 4 EDGES, K=3600 LBS   WRH  14MY5
   4    7    0   43    8    8 6.000E 00 6.000E 00 1.000E-06 0.250E 00
   4    4    0    4    2    2    4    0
1.000E 04 1.000E 05 1.000E 06 1.000E 05
1.000E 04 1.000E 05 1.000E 06 1.000E 05
   0    0    8    8 0.625E 06 0.625E 06           1.000E 99
   1    0    7    8 0.625E 06 0.625E 06
   0    1    8    7 0.625E 06 0.625E 06
   1    1    7    7 0.625E 06 0.625E 06          -1.000E 99
   1    1    7    7                               3.600E 03
   4    4    4    4                     1.000E 05
   1    1    8    8                                         1.875E 06 1.875E 06


106A      48 INCH SQ STEEL PL, 8 X 8 WITH 5000 LB. LINE LOADS,      WRH 07MY5
   4    9    0   31    8    8 6.000E+00 6.000E+00 1.000E-06 0.250E+00
   4    4    0    4    2    2    0    8
6.583E 03 3.500E 04 3.500E 05 3.500E 04
6.583E 03 3.500E 04 3.500E 05 3.500E 04
   0    0    8    8 0.625E 06 0.625E+06           1.000E 99
   0    1    8    7 0.625E 06 0.625E+06
   1    0    7    8 0.625E 06 0.625E+06          -1.000E 99
   1    1    7    7 0.625E 06 0.625E+06
   1    1    8    8                                         1.875E+06 1.875E+06
   1    0    1    8                     2.500E 03
   7    0    7    8                     2.500E 03
   1    1    1    7                     2.500E 03
   7    1    7    7                     2.500E 03


106B      48 INCH SQ STEEL PL, 8 X 8 WITH 5000 LB LINE LDS          WRH 07MY5
   4    9    0   31    8    8 6.000E+00 6.000E+00 1.000E-06    0.0
   4    4    0    4    2    2    0    8
1.000E 04 1.000E 05 1.000E 06 1.000E 05
1.000E 04 1.000E 05 1.000E 06 1.000E 05
   0    0    8    8 0.625E 06 0.625E+06           1.000E 99
   0    1    8    7 0.625E 06 0.625E+06
   1    0    7    8 0.625E 06 0.625E+06          -1.000E 99
   1    1    7    7 0.625E 06 0.625E+06
   1    1    8    8                                         1.875E+06 1.875E+06
   1    0    1    8                     2.500E 03
   7    0    7    8                     2.500E 03
   1    1    1    7                     2.500E 03
   7    1    7    7                     2.500E 03
```

```
107      48 INCH SQ STEEL PL, 8 X 8 WITH APPLIED  T  = 30000 IN LB WRH 19MY5
   3     5     4    30     8     8 6.000E+00 6.000E+00 1.000E-05 0.250E 00
   4     4     0     4     2     2     4     0
1.600E 03 3.500E 04 1.600E 05
1.600E 02 3.500E 03 5.000E 04
   0     0     8     8 0.625E 06 0.625E+06           1.000E 99
   0     1     8     7 0.625E 06 0.625E+06
   1     0     7     8 0.625E 06 0.625E+06          -1.000E 99
   1     1     7     7 0.625E 06 0.625E+06
   1     1     8     8                                         1.875E+06 1.875E+06
   1     0     1     8                     -15.00E 03
   1     1     1     7                     -15.00E 03
   8     1     8     7                     +15.00E 03
   8     0     8     8                      15.00E 03
```

```
201      8X8 SOF ,   PR=.20   CENTER LOAD, HWY REPORT 6          WRH      29AP5
   4     6     0    24     8     8 3.600E 01 3.600E 01 1.000E-05 0.200E 00
   4     4     0     0     0     4     4     0
4.920E 03 2.460E 04 1.230E 05 6.160E 05
4.920E 03 2.460E 04 1.230E 05 6.160E 05
   0     0     8     8 0.652E 08 0.652E+08           0.650E 05
   0     1     8     7 0.652E 08 0.652E+08           0.650E 05
   1     0     7     8 0.652E 08 0.652E+08           0.650E 05
   1     1     7     7 0.652E 08 0.652E+08           0.650E 05
   1     1     8     8                                         2.080E+08 2.080E+08
   4     4     4     4                      1.000E 04
```

```
202      8X8 SOF ,   PR=.20   EDGE   LOAD, HWY REPORT 6          WRH      29AP5
   4     6          24     8     8 3.600E 01 3.600E 01 1.000E-05 0.200E 00
   4     4     0     0     0     4     4     0
5.000E+03 2.500E+04 1.500E+05 6.000E+05
5.000E+03 2.500E+04 1.500E+05 6.000E+05
   0     0     8     8 0.652E 08 0.652E+08           0.650E 05
   0     1     8     7 0.652E 08 0.652E+08           0.650E 05
   1     0     7     8 0.652E 08 0.652E+08           0.650E 05
   1     1     7     7 0.652E 08 0.652E+08           0.650E 05
   1     1     8     8                                         2.080E+08 2.080E+08
   0     4     0     4                      1.000E 04
```

```
203      8X8 SOF ,   PR=.20   CORNER LOAD, HWY REPORT 6          WRH      15AG5
   4     6          99     8     8  36.0       36.0      1.000E-05 2.000E-01
   4     4     0     0     0     4     4     0
5.000E 03 5.000E 04 5.000E 05 1.000E 05
5.000E 03 5.000E 04 5.000E 05 1.000E 05
   0     0     8     8 0.652E 08 0.652E+08           0.650E 05
   0     1     8     7 0.652E 08 0.652E+08           0.650E 05
   1     0     7     8 0.652E 08 0.652E+08           0.650E 05
   1     1     7     7 0.652E 08 0.652E+08           0.650E 05
   1     1     8     8                                         2.080E+08 2.080E+08
   0     0     0     0                      1.000E 04
```

```
301      16 X 16 SOF, PR=.20 AXLE LOAD (CORNER), HALF D,          .WRH    15AG5
  5     7           58    16    16 1.800E+01 1.800E+01 1.000E-06 2.000E-01
  8     8     0     0     0     8     8     0
4.000E 05 6.000E 05 8.000E 05 5.000E 06 6.000E 05
4.000E 05 6.000E 05 8.000E 05 5.000E 06 6.000E 05
  0     0    16    16 6.520E+07 6.520E+07           1.620E+04
  0     1    16    15 6.520E+07 6.520E+07           1.620E+04
  1     0    15    16 6.520E+07 6.520E+07           1.620E+04
  1     1    15    15 6.520E+07 6.520E+07           1.620E+04
  1     1    16    16                                         2.080E+08 2.080E+08
  0     0     0     0                     1.000E+04
  0     4     0     4                     1.000E+04


302      16 X 16 SOF, TANDEM AXLE LOAD (CORNER), HALF D,           WRH    15AG5
  5     9           98    16    16 1.800E+01 1.800E+01 1.000E-06 2.000E-01
  8     8     0     0     0     8     8     0
1.000E 05 1.000E 06 5.000E 06 1.000E 07 1.000E 06
1.000E 05 1.000E 06 5.000E 06 1.000E 07 1.000E 06
  0     0    16    16 6.520E+07 6.520E+07           1.620E+04
  0     1    16    15 6.520E+07 6.520E+07           1.620E+04
  1     0    15    16 6.520E+07 6.520E+07           1.620E+04
  1     1    15    15 6.520E+07 6.520E+07           1.620E+04
  1     1    16    16                                         2.080E+08 2.080E+08
  0     0     0     0                     1.000E+04
  0     4     0     4                     1.000E+04
  3     0     3     0                     1.000E+04
  3     4     3     4                     1.000E+04


401   SLAB, 24 FT SQ, EDGE LOAD, K = 200 PCI NO CRKS                 WRH    20JE65
  4     6     0    20    12    12 24.00E 00 24.00E 00 1.000E-06 0.200E 00
  0     0     6     0     6     3     6     6
1.000E+05 1.000E+06 1.000E+07 1.000E+06
1.000E+05 1.000E+06 1.000E+07 1.000E+06
  0     0    12    12 6.500E 07 6.500E 07           2.900E 04
  0     1    12    11 6.500E 07 6.500E 07           2.900E 04
  1     0    11    12 6.500E 07 6.500E 07           2.900E 04
  1     1    11    11 6.500E 07 6.500E 07           2.900E 04
  6     0     6     0                     1.000E 04
  1     1    12    12                                         2.080E 08 2.080E 08
```

```
 402      24 FT SQ SLAB, EDGE LOAD  10 KIP   50 PERCENT CRK      WRH 16JE5
   4   10     0    35    12    12 24.00E 00 24.00E 00 1.000E-06 0.200E 00
   0    0     6     0     6     3     6     6
5.000E 04 1.000E 06 1.100E 07 1.000E 06
5.000E 04 1.000E 06 1.100E 07 1.000E 06
   0    0    12    12 6.500E 07 6.500E 07           2.900E 04
   0    1    12    11 6.500E 07 6.500E 07           2.900E 04
   1    0    11    12 6.500E 07 6.500E 07           2.900E 04
   1    1    11    11 6.500E 07 6.500E 07           2.900E 04
   6    0     6     0                     1.000E 04
   1    1    12    12                                         2.080E 08 2.080E 08
   6    0     6    12-13.00E 07
   6    1     6    11-13.00E 07
   6    0     6    12 6.500E 07
   6    1     6    11 6.500E 07
```

```
 403      24 FT SQ   SLAB, EDGE LOAD   10 KIP      75 PERCENT CRK     WRH  16JE5
   4    9     0    50    12    12 24.00E 00 24.00E 00 1.000E-06 0.200E 00
   0    0     6     0     6     3     6     6
6.200E 04 6.000E 05 6.000E 06 6.000E 05
6.200E 04 6.000E 05 6.000E 06 6.000E 05
   0    0    12    12 6.500E 07 6.500E 07           2.900E 04
   0    1    12    11 6.500E 07 6.500E 07           2.900E 04
   1    0    11    12 6.500E 07 6.500E 07           2.900E 04
   1    1    11    11 6.500E 07 6.500E 07           2.900E 04
   6    0     6     0                     1.000E 04
   1    1    12    12                                         2.080E 08 2.080E 08
   6    0     6    12-13.00E 07
   6    1     6    11-13.00E 07
   6    0     6    12 6.500E 07
```

```
 404      24 FT SQ SLAB,   EDGE LOAD   10 KIP    90 PERCENT   CRK    WRH  16JE5
   4    9     0    31    12    12 24.00E 00 24.00E 00 1.000E-06 0.200E 00
   0    0     6     0     6     3     6     6
1.000E 05 1.000E 06 1.100E 07 1.000E 06
1.000E 05 1.000E 06 1.100E 07 1.000E 06
   0    0    12    12 6.500E 07 6.500E 07           2.900E 04
   0    1    12    11 6.500E 07 6.500E 07           2.900E 04
   1    0    11    12 6.500E 07 6.500E 07           2.900E 04
   1    1    11    11 6.500E 07 6.500E 07           2.900E 04
   6    0     6     0                     1.000E 04
   1    1    12    12                                         2.080E 08 2.080E 08
   6    0     6    12-13.00E 07
   6    1     6    11-13.00E 07
   6    0     6    12 2.600E 06
```

```
 405   SLAB, 24 FT SQ, EDGE LOAD, K = 200 FULL CRK UNDER LOAD  WRH      20JE65
   4     9     0    20    12    12 24.00E 00 24.00E 00 1.000E-06 0.200E 00
   0     0     6     0     6     3     6     6
1.000E+05 1.000E+06 1.000E+07 1.000E+06
1.000E+05 1.000E+06 1.000E+07 1.000E+06
   0     0    12    12 6.500E 07 6.500E 07           2.900E 04
   0     1    12    11 6.500E 07 6.500E 07           2.900E 04
   1     0    11    12 6.500E 07 6.500E 07           2.900E 04
   1     1    11    11 6.500E 07 6.500E 07           2.900E 04
   6     0     6     0                     1.000E 04
   1     1    12    12                                         2.080E 08 2.080E 08
   6     0     6    12-13.00E 07
   6     1     6    11-13.00E 07
   6     0     6    12 1.000E-05



 501        ORTHO   EXP     -TWK1 FROM  WBI   - 10JUN 65    WRH
   4     9     0    50    10    10 12.00E 00 12.00E 00 1.000E-06 0.300E 00
   5     5     5     0     0     5    10    10
3.000E 04 5.000E 04 1.000E 05 1.000E 06
3.000E 04 5.000E 04 1.000E 05 1.000E 06
   0     0    10    10 5.500E 06 5.500E 06-1.000E 03
   0     1    10     9 5.500E 06 5.500E 06
   1     0     9    10 5.500E 06 5.500E 06
   1     1     9     9 5.500E 06 5.500E 06
   1     1    10    10                                         1.600E 07 1.600E 07
   0     0    10     0 9.100E 09
   0    10    10    10 9.100E 09
   0     0     0    10                               1.000E 99
  10     0    10    10                               1.000E 99



 503       EXAMPLE - ORTHO PLATE - CLIFTON - ETAL - CENTER LOAD       WRH
   3     5     0    30    10    20 12.00E 00 48.00E 00 1.000E-05 0.300E 00
   5    10     5     0     5     5     0     0
1.000E 04 1.000E 06 4.020E 07
2.260E 03 4.450E 05 3.540E 06
   0     0    10    20 0.660E 08 7.300E 06           1.000E 99
   1     1     9    19 0.660E 08 7.300E 06
   1     0     9    20                              -1.000E 99
   1     1    10    20                                         3.160E 07 3.160E 07
   5    10     5    10                     1.000E 03
```

```
 504      EXAMPLE - ORTHO PLATE - CLIFTON - ETAL - CENTER LOAD CX = CY = 0.0
    4    5    0   20   10   20 12.00E 00 48.00E 00 1.000E-05 0.300E 00
    5   10    5    0    5    5    0    0
1.000E 05 1.000E 06 5.000E 06 1.000E 06
1.000E 04 5.000E 04 1.000E 05 5.000E 04
    1    1    9   19 0.660E 08 7.300E 06
    0    0   10   20 0.660E 08 7.300E 06           1.000E 99
    1    0    9   20                              -1.000E 99
    5   10    5   10                     1.000E 03
    1    1   10   20                                         0.0       0.0
```

```
 601      EXAMPLE PROBLEM   BRIDGE APPROACH SLAB                WRH      10JUN65
    5   17    1   50   16   12 2.400E+01 2.400E+01 1.000E-05 2.500E-01
    7    0    7    6   15    8   15   11
4.773E+04 9.384E+05 4.939E+06 9.817E+06 1.457E+07
8.150E+04 9.384E+05 4.939E+06 9.199E+06 1.439E+07
    0    0   16   12 8.890E+07 8.890E+07 1.500E+02
    0    1   16   11 8.890E+07 8.890E+07 1.500E+02
    1    0   15   12 8.890E+07 8.890E+07 1.500E+02
    1    1   15   11 8.890E+07 8.890E+07 1.500E+02
    1    1   16   12                                         2.667E+08 2.667E+08
    7    0    7   12-1.777E+08
    7    1    7   11-1.777E+08
    0    0    2   12                               1.000E+99
   15    8   15    8                     1.000E+04
   15   11   15   11                     1.000E+04
    7    0   16   12                               1.440E+04
    7    1   16   11                               1.440E+04
    7    0   15   12                               1.440E+04
    7    1   15   11                               1.440E+04
   16    0   16   12                               6.000E+05
    0    6   16    6          -8.890E+07
    1    6   15    6          -1.777E+08
    1    0   16   12                                         1.200E+05
```

APPENDIX D2

SAMPLE COMPUTER SOLUTIONS

EXAMPLE PROBLEMS

```
PROGRAM SLAB 17 - MASTER DECK - WR HUDSON, H MATLOCK     REVISION DATE 26 JUL 65
CE051022    HWY SLAB PROJECT        SLAB 17      W R HUDSON
RUN EXAMPLE PROBLEMS FOR USE IN SLAB REPORT APPENDIX D


PROB
  401      SLAB, 24 FT SQ, EDGE LOAD, K = 200 PCI NO CRKS              WRH     20JE65


TABLE 1.  CONTROL DATA

     NUM VALUES TABLE 2                                                          4
     NUM CARDS TABLE 3A                                                          6
     NUM CARDS TABLE 3B                                                          0
     MAX NUM ITERATIONS                                                         20
     NUM INCREMENTS  MX                                                         12
     NUM INCREMENTS  MY                                                         12
     INCR LENGTH  HX                                                     2.400E 01
     INCR LENGTH  HY                                                     2.400E 01
     CLOSURE TOLERANCE                                                   1.000E-06
     POISSONS RATIO                                                      2.000E-01

     MONITOR STAS  I,J                6  6       0  0       0  6       3  3

TABLE 2A.  ITERATION CONTROL DATA

     F. SPRING REPRESENTING X BEAM

      1.000E 05
      1.000E 06
      1.000E 07
      1.000E 06


TABLE 2B.  ITERATION CONTROL DATA

     F. SPRING REPRESENTING Y BEAM

      1.000E 05
      1.000E 06
      1.000E 07
      1.000E 06

TABLE 3A. STIFFNESS AND LOAD DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
```

FROM		THRU		DX	DY	Q	S	CX	CY
0	0	12	12	6.500E 07	6.500E 07	0	2.900E 04	0	0
0	1	12	11	6.500E 07	6.500E 07	0	2.900E 04	0	0
1	0	11	12	6.500E 07	6.500E 07	0	2.900E 04	0	0
1	1	11	11	6.500E 07	6.500E 07	0	2.900E 04	0	0
6	0	6	0	0	0	1.000E 04	0	0	0
1	1	12	12	0	0	0	0	2.080E 08	2.080E 08

TABLE 3B. STIFFNESS AND LOAD DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECT.

FROM	THRU	TX	TY	PX	PY

TABLE 4. MONITOR TALLY AND DEFLS AT 4 STAS

	ITR NUM	FICT SPRING	CYC NUM	NOT STAB	NOT CLOS	I,J 6 6	0 0	0 6	3 3
X	1	1.000E 05	1	90		4.994E-06	4.833E-04	5.408E-07	-6.852E-05
Y		1.000E 05		158	158	1.597E-04	-1.685E-04	-2.482E-05	1.018E-04
X	2	1.000E 06	2	158		8.663E-05	-1.971E-04	-2.419E-05	1.156E-04
Y		1.000E 06		152	141	-1.817E-05	-2.109E-04	-2.730E-05	1.545E-04
X	3	1.000E 07	3	142		-6.885E-06	-2.149E-04	-2.783E-05	1.555E-04
Y		1.000E 07		119	100	3.658E-06	-2.188E-04	-2.828E-05	1.571E-04
X	4	1.000E 06	4	139		-4.806E-06	-2.611E-04	-3.196E-05	1.039E-04
Y		1.000E 06		150	141	-1.461E-05	-3.083E-04	-3.945E-05	1.091E-04
X	5	1.000E 05	1	152		-1.277E-04	7.030E-05	-5.311E-05	-1.349E-04
Y		1.000E 05		161	162	-1.845E-04	-5.848E-04	-1.145E-04	1.506E-04
X	6	1.000E 06	2	164		-1.949E-04	-5.237E-04	-1.108E-04	1.106E-04
Y		1.000E 06		154	140	-2.114E-04	-5.253E-04	-1.103E-04	8.913E-05
X	7	1.000E 07	3	140		-2.113E-04	-5.246E-04	-1.113E-04	8.749E-05
Y		1.000E 07		86	66	-2.113E-04	-5.246E-04	-1.119E-04	8.557E-05
X	8	1.000E 06	4	130		-2.192E-04	-5.087E-04	-1.157E-04	5.910E-05
Y		1.000E 06		153	122	-2.280E-04	-5.166E-04	-1.204E-04	5.017E-05
X	9	1.000E 05	1	169		-2.699E-04	-1.793E-04	-1.544E-04	-3.546E-05
Y		1.000E 05		168	161	-2.849E-04	-4.664E-04	-1.868E-04	4.725E-05
X	10	1.000E 06	2	160		-2.884E-04	-4.178E-04	-1.877E-04	3.886E-05
Y		1.000E 06		130	105	-2.924E-04	-4.089E-04	-1.902E-04	3.311E-05
X	11	1.000E 07	3	108		-2.927E-04	-4.071E-04	-1.908E-04	3.249E-05
Y		1.000E 07		59	31	-2.931E-04	-4.056E-04	-1.914E-04	3.188E-05
X	12	1.000E 06	4	101		-2.964E-04	-3.855E-04	-1.951E-04	2.340E-05
Y		1.000E 06		132	95	-2.998E-04	-3.801E-04	-1.994E-04	2.008E-05
X	13	1.000E 05	1	158		-3.137E-04	-2.072E-04	-2.215E-04	-1.109E-05
Y		1.000E 05		165	137	-3.191E-04	-3.071E-04	-2.398E-04	1.703E-05
X	14	1.000E 06	2	148		-3.203E-04	-2.841E-04	-2.399E-04	1.473E-05
Y		1.000E 06		102	74	-3.215E-04	-2.778E-04	-2.415E-04	1.303E-05
X	15	1.000E 07	3	76		-3.217E-04	-2.767E-04	-2.419E-04	1.282E-05
Y		1.000E 07		30	8	-3.218E-04	-2.758E-04	-2.422E-04	1.263E-05
X	16	1.000E 06	4	73		-3.229E-04	-2.650E-04	-2.442E-04	9.759E-06
Y		1.000E 06		98	70	-3.241E-04	-2.606E-04	-2.466E-04	8.596E-06
X	17	1.000E 05	1	143		-3.287E-04	-1.855E-04	-2.571E-04	-2.725E-06

Y		1.000E 05		146	112	-3.307E-04	-2.183E-04	-2.652E-04	7.151E-06
X	18	1.000E 06	2	117		-3.311E-04	-2.090E-04	-2.652E-04	6.433E-06
Y		1.000E 06		62	33	-3.315E-04	-2.060E-04	-2.660E-04	5.862E-06
X	19	1.000E 07	3	33		-3.316E-04	-2.055E-04	-2.662E-04	5.793E-06
Y		1.000E 07		5	0	-3.316E-04	-2.050E-04	-2.663E-04	5.731E-06

PROGRAM SLAB 17 - MASTER DECK - WR HUDSON, H MATLOCK   REVISION DATE 26 JUL 65
CE051022   HWY SLAB PROJECT   SLAB 17   W R HUDSON
RUN EXAMPLE PROBLEMS FOR USE IN SLAB REPORT APPENDIX D

PROB (CONTD)
401   SLAB, 24 FT SQ, EDGE LOAD, K = 200 PCI NO CRKS   WRH   20JE65

TABLE 5. RESULTS -- ITERATION 19

I,J		X-DEFL	Y-DEFL	BMX	BMY	REACT	TRERR
-1	-1	-8.673E-04	-8.673E-04	0	0	0	0
0	-1	3.283E-04	3.301E-04	0	0	0	0
1	-1	1.483E-03	1.484E-03	0	0	0	0
2	-1	3.402E-03	3.404E-03	0	0	0	0
3	-1	6.819E-03	6.821E-03	0	0	0	0
4	-1	1.250E-02	1.250E-02	0	0	0	0
5	-1	2.076E-02	2.076E-02	0	0	0	0
6	-1	2.990E-02	2.990E-02	0	0	0	0
7	-1	2.077E-02	2.077E-02	0	0	0	0
8	-1	1.252E-02	1.253E-02	0	0	0	0
9	-1	6.854E-03	6.855E-03	0	0	0	0
10	-1	3.447E-03	3.448E-03	0	0	0	0
11	-1	1.538E-03	1.538E-03	0	0	0	0
12	-1	3.921E-04	3.923E-04	0	0	0	0
13	-1	-7.948E-04	-7.948E-04	0	0	0	0
-1	0	-1.225E-03	-1.225E-03	0	0	0	0
0	0	-2.055E-04	-2.050E-04	0	0	1.264E 00	4.689E 00
1	0	8.144E-04	8.149E-04	1.081E 02	2.328E-09	-5.324E 01	5.993E 00
2	0	2.333E-03	2.333E-03	2.227E 02	-7.451E-09	-1.401E 02	4.750E 00
3	0	4.879E-03	4.880E-03	3.199E 02	-5.588E-09	-2.892E 02	6.142E 00
4	0	8.902E-03	8.902E-03	2.941E 02	1.639E-07	-5.221E 02	5.775E 00
5	0	1.428E-02	1.428E-02	-1.502E 02	-9.080E-08	-8.338E 02	5.431E 00
6	0	1.897E-02	1.897E-02	-2.029E 03	1.714E-07	8.895E 03	5.127E 00
7	0	1.429E-02	1.429E-02	-1.503E 02	5.169E-08	-8.333E 02	4.487E 00
8	0	8.918E-03	8.918E-03	2.939E 02	1.956E-08	-5.212E 02	3.894E 00
9	0	4.902E-03	4.903E-03	3.197E 02	2.794E-08	-2.879E 02	3.549E 00
10	0	2.362E-03	2.363E-03	2.227E 02	1.024E-08	-1.394E 02	2.417E 00
11	0	8.500E-04	8.502E-04	1.077E 02	1.397E-09	-5.095E 01	1.648E 00
12	0	-1.650E-04	-1.650E-04	0	0	4.517E 00	2.671E-01
13	0	-1.180E-03	-1.180E-03	0	0	0	0
-1	1	-1.583E-03	-1.583E-03	0	0	0	0
0	1	-7.403E-04	-7.402E-04	1.849E-02	6.222E 01	4.283E 01	1.023E-01
1	1	4.551E-05	4.580E-05	1.252E 02	1.377E 02	-8.135E 00	2.839E 00
2	1	1.057E-03	1.057E-03	2.900E 02	2.042E 02	-1.232E 02	5.844E-01
3	1	2.643E-03	2.643E-03	4.119E 02	3.205E 02	-3.091E 02	2.534E 00
4	1	5.032E-03	5.032E-03	3.262E 02	4.996E 02	-5.862E 02	2.468E 00
5	1	7.943E-03	7.943E-03	-2.534E 02	7.841E 02	-9.236E 02	2.203E 00
6	1	9.908E-03	9.908E-03	-1.459E 03	1.207E 03	-1.151E 03	2.133E 00
7	1	7.947E-03	7.948E-03	-2.534E 02	7.845E 02	-9.238E 02	1.902E 00
8	1	5.040E-03	5.040E-03	3.261E 02	5.003E 02	-5.865E 02	1.861E 00
9	1	2.655E-03	2.655E-03	4.118E 02	3.216E 02	-3.096E 02	1.678E 00
10	1	1.071E-03	1.071E-03	2.902E 02	2.055E 02	-1.245E 02	2.860E-01
11	1	6.247E-05	6.257E-05	1.249E 02	1.392E 02	-8.338E 00	1.085E 00
12	1	-7.222E-04	-7.222E-04	-4.577E-03	6.311E 01	4.201E 01	-1.183E-01
13	1	-1.565E-03	-1.565E-03	0	0	0	0

-1	2	-1.570E-03	-1.570E-03	0	0	0	0
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1	2	-4.635E-04	-4.634E-04	5.492E 01	1.520E 02	5.319E 01	5.702E-01
2	2	1.176E-04	1.174E-04	1.577E 02	2.306E 02	-1.170E 01	-1.928E 00
3	2	9.562E-04	9.562E-04	2.332E 02	3.658E 02	-1.111E 02	1.681E-01
4	2	2.164E-03	2.164E-03	1.819E 02	5.667E 02	-2.512E 02	1.756E-01
5	2	3.530E-03	3.530E-03	-9.014E 01	8.318E 02	-4.095E 02	-2.763E-02
6	2	4.305E-03	4.305E-03	-4.579E 02	1.063E 03	-4.994E 02	3.878E-02
7	2	3.532E-03	3.532E-03	-9.015E 01	8.323E 02	-4.097E 02	5.327E-02
8	2	2.167E-03	2.167E-03	1.819E 02	5.677E 02	-2.517E 02	3.261E-01
9	2	9.592E-04	9.592E-04	2.332E 02	3.673E 02	-1.115E 02	2.717E-01
10	2	1.204E-04	1.203E-04	1.578E 02	2.326E 02	-1.285E 01	-1.115E 00
11	2	-4.614E-04	-4.614E-04	5.480E 01	1.545E 02	5.276E 01	7.584E-01
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13	2	-1.573E-03	-1.573E-03	0	0	0	0
-1	3	-1.289E-03	-1.289E-03	0	0	0	0
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2	3	-3.623E-04	-3.626E-04	6.991E 01	1.691E 02	4.472E 01	-2.673E 00
3	3	5.793E-06	5.731E-06	1.184E 02	2.749E 02	-5.452E-03	-6.630E-01
4	3	5.202E-04	5.202E-04	1.036E 02	4.227E 02	-5.967E 01	-6.669E-01
5	3	1.079E-03	1.079E-03	1.795E-01	5.911E 02	-1.243E 02	-8.340E-01
6	3	1.365E-03	1.365E-03	-1.087E 02	6.970E 02	-1.576E 02	-7.425E-01
7	3	1.078E-03	1.078E-03	1.988E-01	5.916E 02	-1.244E 02	-6.225E-01
8	3	5.190E-04	5.189E-04	1.036E 02	4.236E 02	-5.993E 01	-2.652E-01
9	3	3.410E-06	3.378E-06	1.183E 02	2.762E 02	-2.333E-02	-3.704E-01
10	3	-3.666E-04	-3.668E-04	6.992E 01	1.709E 02	4.426E 01	-1.722E 00
11	3	-6.542E-04	-6.541E-04	1.251E 01	1.068E 02	7.500E 01	8.764E-01
12	3	-9.622E-04	-9.623E-04	3.599E-03	3.831E 01	5.687E 01	-1.059E 00
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2	4	-4.844E-04	-4.846E-04	2.149E 01	8.949E 01	5.874E 01	-2.541E 00
3	4	-3.649E-04	-3.650E-04	5.265E 01	1.562E 02	4.306E 01	-7.260E-01
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6	4	8.335E-05	8.327E-05	-4.508E 00	3.808E 02	-8.847E 00	-8.169E-01
7	4	-1.039E-05	-1.046E-05	2.579E 01	3.343E 02	1.865E 00	-6.562E-01
8	4	-1.989E-04	-1.990E-04	5.666E 01	2.445E 02	2.339E 01	-3.121E-01
9	4	-3.696E-04	-3.696E-04	5.247E 01	1.571E 02	4.334E 01	-4.712E-01
10	4	-4.916E-04	-4.918E-04	2.139E 01	9.074E 01	5.876E 01	-1.725E 00
11	4	-6.062E-04	-6.061E-04	-7.205E 00	4.782E 01	6.946E 01	8.568E-01
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13	4	-9.234E-04	-9.234E-04	0	0	0	0
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2	5	-4.099E-04	-4.101E-04	-4.350E-01	2.796E 01	4.968E 01	-2.126E 00
3	5	-3.995E-04	-3.996E-04	1.858E 01	6.356E 01	4.692E 01	-5.649E-01
4	5	-3.757E-04	-3.757E-04	2.768E 01	1.080E 02	4.417E 01	-5.836E-01
5	5	-3.378E-04	-3.379E-04	2.265E 01	1.507E 02	3.992E 01	-7.239E-01
6	5	-3.172E-04	-3.173E-04	1.564E 01	1.705E 02	3.750E 01	-6.929E-01
7	5	-3.393E-04	-3.393E-04	2.265E 01	1.508E 02	3.988E 01	-5.253E-01
8	5	-3.786E-04	-3.787E-04	2.762E 01	1.084E 02	4.416E 01	-2.389E-01
9	5	-4.043E-04	-4.043E-04	1.836E 01	6.400E 01	4.729E 01	-3.905E-01
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2	6	-2.707E-04	-2.708E-04	-7.493E 00	-6.395E 00	3.302E 01	-1.613E 00
3	6	-2.961E-04	-2.961E-04	3.116E 00	9.628E 00	3.468E 01	-3.351E-01
4	6	-3.187E-04	-3.187E-04	1.097E 01	2.884E 01	3.735E 01	-3.784E-01
5	6	-3.293E-04	-3.294E-04	1.296E 01	4.654E 01	3.872E 01	-5.116E-01
6	6	-3.316E-04	-3.316E-04	1.227E 01	5.432E 01	3.896E 01	-5.006E-01
7	6	-3.305E-04	-3.305E-04	1.296E 01	4.659E 01	3.869E 01	-3.490E-01
8	6	-3.211E-04	-3.211E-04	1.089E 01	2.893E 01	3.739E 01	-1.478E-01
9	6	-2.998E-04	-2.998E-04	2.913E 00	9.741E 00	3.505E 01	-2.676E-01
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13	6	-2.734E-04	-2.734E-04	0	0	0	0
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1	7	-1.226E-04	-1.226E-04	-7.772E 00	-2.352E 01	1.390E 01	3.208E-01
2	7	-1.427E-04	-1.429E-04	-7.609E 00	-1.872E 01	1.768E 01	-1.112E 00
3	7	-1.718E-04	-1.718E-04	-2.362E 00	-1.311E 01	2.004E 01	-1.149E-01
4	7	-2.002E-04	-2.002E-04	2.669E 00	-6.568E 00	2.340E 01	-1.781E-01
5	7	-2.195E-04	-2.195E-04	5.310E 00	-5.766E-01	2.576E 01	-2.973E-01
6	7	-2.262E-04	-2.262E-04	5.901E 00	2.007E 00	2.654E 01	-2.974E-01
7	7	-2.202E-04	-2.202E-04	5.310E 00	-5.879E-01	2.572E 01	-1.733E-01
8	7	-2.017E-04	-2.017E-04	2.599E 00	-6.607E 00	2.346E 01	-6.004E-02
9	7	-1.742E-04	-1.742E-04	-2.520E 00	-1.318E 01	2.036E 01	-1.487E-01
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11	7	-1.276E-04	-1.275E-04	-8.012E 00	-2.369E 01	1.457E 01	2.290E-01
12	7	-1.163E-04	-1.163E-04	-5.721E-04	-1.443E 01	7.070E 00	-3.252E-01
13	7	-9.167E-05	-9.167E-05	0	0	0	0
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1	8	-3.631E-05	-3.627E-05	-4.307E 00	-1.930E 01	3.749E 00	4.603E-01
2	8	-5.452E-05	-5.459E-05	-5.333E 00	-1.815E 01	7.033E 00	-7.053E-01
3	8	-7.665E-05	-7.665E-05	-3.145E 00	-1.719E 01	8.851E 00	4.046E-02
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5	8	-1.134E-04	-1.134E-04	1.220E 00	-1.489E 01	1.328E 01	-1.307E-01
6	8	-1.189E-04	-1.189E-04	1.777E 00	-1.435E 01	1.393E 01	-1.365E-01
7	8	-1.137E-04	-1.137E-04	1.228E 00	-1.492E 01	1.323E 01	-4.042E-02
8	8	-9.886E-05	-9.886E-05	-5.716E-01	-1.615E 01	1.147E 01	-4.001E-04
9	8	-7.784E-05	-7.784E-05	-3.243E 00	-1.733E 01	9.094E 00	-6.429E-02
10	8	-5.630E-05	-5.631E-05	-5.290E 00	-1.833E 01	6.705E 00	-1.731E-01
11	8	-3.851E-05	-3.850E-05	-4.526E 00	-1.962E 01	4.389E 00	7.747E-02
12	8	-2.211E-05	-2.212E-05	-6.551E-04	-1.087E 01	1.427E 00	-1.437E-01
13	8	4.321E-06	4.321E-06	0	0	0	0
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2	9	-5.692E-06	-5.734E-06	-2.854E 00	-1.235E 01	1.084E 00	-4.209E-01
3	9	-1.974E-05	-1.973E-05	-2.189E 00	-1.300E 01	2.172E 00	1.177E-01
4	9	-3.284E-05	-3.284E-05	-1.094E 00	-1.367E 01	3.755E 00	5.447E-02
5	9	-4.216E-05	-4.216E-05	-2.462E-01	-1.406E 01	4.919E 00	-2.837E-02
6	9	-4.555E-05	-4.556E-05	6.462E-02	-1.418E 01	5.321E 00	-3.653E-02
7	9	-4.225E-05	-4.225E-05	-2.306E-01	-1.409E 01	4.865E 00	3.660E-02

8	9	-3.299E-05	-3.299E-05	-1.122E 00	-1.374E 01	3.803E 00	2.350E-02
9	9	-1.997E-05	-1.997E-05	-2.226E 00	-1.312E 01	2.338E 00	-2.144E-02
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11	9	7.332E-06	7.329E-06	-1.952E 00	-1.232E 01	-8.231E-01	-2.729E-02
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13	9	4.234E-05	4.234E-05	0	0	0	0
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1	10	2.432E-05	2.436E-05	-2.239E-01	-5.395E 00	-3.291E 00	4.676E-01
2	10	1.597E-05	1.594E-05	-9.689E-01	-6.058E 00	-1.643E 00	-2.074E-01
3	10	8.173E-06	8.189E-06	-9.433E-01	-6.921E 00	-1.106E 00	1.574E-01
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5	10	-3.289E-06	-3.284E-06	-3.668E-01	-8.412E 00	3.373E-01	4.397E-02
6	10	-4.940E-06	-4.936E-06	-2.417E-01	-8.632E 00	5.371E-01	3.567E-02
7	10	-3.165E-06	-3.156E-06	-3.451E-01	-8.422E 00	2.805E-01	8.611E-02
8	10	1.701E-06	1.706E-06	-6.617E-01	-7.824E 00	-2.402E-01	4.257E-02
9	10	8.651E-06	8.652E-06	-9.321E-01	-6.969E 00	-1.019E 00	1.528E-02
10	10	1.667E-05	1.669E-05	-8.097E-01	-6.116E 00	-2.117E 00	1.829E-01
11	10	2.564E-05	2.563E-05	-3.483E-01	-5.558E 00	-2.901E 00	-7.213E-02
12	10	3.637E-05	3.637E-05	-1.168E-04	-2.639E 00	-2.168E 00	5.844E-02
13	10	4.953E-05	4.953E-05	0	0	0	0
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0	11	3.548E-05	3.551E-05	8.940E-04	-3.206E-01	-2.301E 00	2.420E-01
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2	11	2.409E-05	2.409E-05	1.984E-01	-1.593E 00	-2.822E 00	2.759E-02
3	11	2.055E-05	2.057E-05	-1.248E-02	-2.105E 00	-2.622E 00	2.371E-01
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5	11	1.633E-05	1.635E-05	-1.003E-01	-2.984E 00	-2.034E 00	1.382E-01
6	11	1.586E-05	1.587E-05	-7.704E-02	-3.115E 00	-1.972E 00	1.309E-01
7	11	1.665E-05	1.666E-05	-7.540E-02	-2.979E 00	-2.088E 00	1.563E-01
8	11	1.864E-05	1.865E-05	-9.211E-02	-2.612E 00	-2.272E 00	1.095E-01
9	11	2.162E-05	2.162E-05	2.960E-02	-2.102E 00	-2.596E 00	8.816E-02
10	11	2.564E-05	2.566E-05	3.818E-01	-1.583E 00	-3.250E 00	2.748E-01
11	11	3.127E-05	3.126E-05	5.369E-01	-1.165E 00	-3.610E 00	-1.699E-02
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13	11	4.640E-05	4.640E-05	0	0	0	0
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2	12	2.845E-05	2.847E-05	3.476E-01	2.001E-11	-1.873E 00	2.225E-01
3	12	2.807E-05	2.811E-05	1.789E-01	1.210E-10	-1.998E 00	3.693E-01
4	12	2.852E-05	2.854E-05	3.355E-02	-2.806E-10	-1.956E 00	3.008E-01
5	12	2.912E-05	2.914E-05	-4.537E-02	4.025E-11	-1.942E 00	2.524E-01
6	12	2.951E-05	2.953E-05	-6.109E-02	-1.603E-10	-1.955E 00	2.433E-01
7	12	2.962E-05	2.964E-05	-3.090E-02	-1.803E-10	-1.973E 00	2.542E-01
8	12	2.958E-05	2.960E-05	3.556E-02	2.024E-11	-1.916E 00	1.994E-01
9	12	2.971E-05	2.973E-05	2.006E-01	1.601E-10	-1.925E 00	2.008E-01
10	12	3.077E-05	3.081E-05	4.618E-01	2.383E-10	-2.196E 00	4.101E-01
11	12	3.396E-05	3.396E-05	4.806E-01	-6.439E-10	-1.988E 00	1.817E-02
12	12	3.937E-05	3.938E-05	-2.205E-10	-1.403E-10	-1.314E 00	1.719E-01
13	12	4.477E-05	4.477E-05	0	0	0	0
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0	13	3.413E-05	3.425E-05	0	0	0	0
1	13	3.160E-05	3.160E-05	0	0	0	0
2	13	3.240E-05	3.253E-05	0	0	0	0
3	13	3.537E-05	3.547E-05	0	0	0	0
4	13	3.902E-05	3.909E-05	0	0	0	0

5 13	4.190E-05	4.197E-05	0	0	0	0
6 13	4.317E-05	4.324E-05	0	0	0	0
7 13	4.258E-05	4.265E-05	0	0	0	0
8 13	4.046E-05	4.053E-05	0	0	0	0
9 13	3.760E-05	3.766E-05	0	0	0	0
10 13	3.547E-05	3.553E-05	0	0	0	0
11 13	3.609E-05	3.621E-05	0	0	0	0
12 13	4.005E-05	4.008E-05	0	0	0	0
13 13	4.314E-05	4.314E-05	0	0	0	0

PROB (CONTD)
401 SLAB, 24 FT SQ, EDGE LOAD, K = 200 PCI NO CRKS WRH 20JE65

TABLE 5. RESULTS(CONTD) -- ITERATION 19

I,J	TMX	TMY
-1 -1	0	0
0 -1	0	0
1 -1	0	0
2 -1	0	0
3 -1	0	0
4 -1	0	0
5 -1	0	0
6 -1	0	0
7 -1	0	0
8 -1	0	0
9 -1	0	0
10 -1	0	0
11 -1	0	0
12 -1	0	0
13 -1	0	0
-1 0	0	0
0 0	-1.628E 01	1.631E 01
1 0	-5.765E 01	5.765E 01
2 0	-1.236E 02	1.236E 02
3 0	-2.312E 02	2.312E 02
4 0	-3.900E 02	3.900E 02
5 0	-5.655E 02	5.655E 02
6 0	-3.665E-01	3.541E-01
7 0	5.648E 02	-5.648E 02
8 0	3.893E 02	-3.893E 02
9 0	2.306E 02	-2.306E 02
10 0	1.229E 02	-1.230E 02
11 0	5.695E 01	-5.699E 01
12 0	1.592E 01	-1.592E 01
13 0	0	0
-1 1	0	0
0 1	-4.215E 01	4.217E 01
1 1	-1.293E 02	1.293E 02
2 1	-2.388E 02	2.388E 02
3 1	-4.083E 02	4.083E 02
4 1	-6.165E 02	6.165E 02
5 1	-7.156E 02	7.156E 02
6 1	-6.308E-01	6.224E-01

7	1	7.144E 02	-7.144E 02
8	1	6.153E 02	-6.153E 02
9	1	4.071E 02	-4.071E 02
10	1	2.376E 02	-2.376E 02
11	1	1.281E 02	-1.281E 02
12	1	4.145E 01	-4.145E 01
13	1	0	0
-1	2	0	0
0	2	-4.454E 01	4.456E 01
1	2	-1.091E 02	1.091E 02
2	2	-1.755E 02	1.755E 02
3	2	-2.792E 02	2.792E 02
4	2	-3.816E 02	3.816E 02
5	2	-3.639E 02	3.639E 02
6	2	-4.438E-01	4.390E-01
7	2	3.631E 02	-3.631E 02
8	2	3.808E 02	-3.808E 02
9	2	2.784E 02	-2.784E 02
10	2	1.747E 02	-1.747E 02
11	2	1.081E 02	-1.081E 02
12	2	4.407E 01	-4.407E 01
13	2	0	0
-1	3	0	0
0	3	-3.605E 01	3.606E 01
1	3	-7.635E 01	7.635E 01
2	3	-1.074E 02	1.074E 02
3	3	-1.587E 02	1.587E 02
4	3	-2.003E 02	2.003E 02
5	3	-1.680E 02	1.680E 02
6	3	-2.406E-01	2.392E-01
7	3	1.675E 02	-1.675E 02
8	3	1.998E 02	-1.998E 02
9	3	1.583E 02	-1.583E 02
10	3	1.069E 02	-1.069E 02
11	3	7.590E 01	-7.590E 01
12	3	3.587E 01	-3.586E 01
13	3	0	0
-1	4	0	0
0	4	-2.432E 01	2.432E 01
1	4	-4.602E 01	4.602E 01
2	4	-5.583E 01	5.583E 01
3	4	-7.659E 01	7.659E 01
4	4	-9.128E 01	9.128E 01
5	4	-7.096E 01	7.096E 01
6	4	-8.119E-02	8.125E-02
7	4	7.080E 01	-7.080E 01
8	4	9.115E 01	-9.115E 01
9	4	7.647E 01	-7.648E 01
10	4	5.574E 01	-5.574E 01
11	4	4.598E 01	-4.598E 01
12	4	2.436E 01	-2.436E 01
13	4	0	0
-1	5	0	0
0	5	-1.388E 01	1.388E 01
1	5	-2.387E 01	2.388E 01
2	5	-2.410E 01	2.411E 01
3	5	-3.036E 01	3.037E 01

4	5	-3.514E 01	3.514E 01
5	5	-2.639E 01	2.639E 01
6	5	1.678E-02	-1.630E-02
7	5	2.643E 01	-2.643E 01
8	5	3.520E 01	-3.520E 01
9	5	3.046E 01	-3.047E 01
10	5	2.427E 01	-2.427E 01
11	5	2.412E 01	-2.411E 01
12	5	1.406E 01	-1.406E 01
13	5	0	0
-1	6	0	0
0	6	-6.437E 00	6.435E 00
1	6	-1.014E 01	1.014E 01
2	6	-7.557E 00	7.558E 00
3	6	-8.274E 00	8.280E 00
4	6	-9.877E 00	9.877E 00
5	6	-7.621E 00	7.621E 00
6	6	6.140E-02	-6.078E-02
7	6	7.751E 00	-7.749E 00
8	6	1.003E 01	-1.003E 01
9	6	8.470E 00	-8.477E 00
10	6	7.839E 00	-7.833E 00
11	6	1.050E 01	-1.050E 01
12	6	6.662E 00	-6.664E 00
13	6	0	0
-1	7	0	0
0	7	-1.988E 00	1.986E 00
1	7	-2.788E 00	2.790E 00
2	7	-3.512E-01	3.513E-01
3	7	4.017E-01	-3.967E-01
4	7	-3.076E-01	3.075E-01
5	7	-7.171E-01	7.169E-01
6	7	6.993E-02	-6.930E-02
7	7	8.625E-01	-8.606E-01
8	7	4.710E-01	-4.700E-01
9	7	-1.933E-01	1.876E-01
10	7	6.396E-01	-6.346E-01
11	7	3.148E 00	-3.144E 00
12	7	2.196E 00	-2.197E 00
13	7	0	0
-1	8	0	0
0	8	1.958E-01	-1.966E-01
1	8	4.857E-01	-4.856E-01
2	8	1.981E 00	-1.981E 00
3	8	2.737E 00	-2.733E 00
4	8	2.281E 00	-2.282E 00
5	8	1.197E 00	-1.198E 00
6	8	6.020E-02	-5.967E-02
7	8	-1.073E 00	1.075E 00
8	8	-2.143E 00	2.144E 00
9	8	-2.563E 00	2.559E 00
10	8	-1.746E 00	1.749E 00
11	8	-1.987E-01	2.009E-01
12	8	-3.981E-02	3.867E-02
13	8	0	0
-1	9	0	0
0	9	9.563E-01	-9.563E-01

1	9	1.507E 00	-1.509E 00
2	9	2.184E 00	-2.185E 00
3	9	2.621E 00	-2.617E 00
4	9	2.279E 00	-2.279E 00
5	9	1.305E 00	-1.306E 00
6	9	4.645E-02	-4.601E-02
7	9	-1.211E 00	1.212E 00
8	9	-2.175E 00	2.175E 00
9	9	-2.491E 00	2.488E 00
10	9	-2.017E 00	2.019E 00
11	9	-1.308E 00	1.309E 00
12	9	-8.546E-01	8.540E-01
13	9	0	0
-1	10	0	0
0	10	9.625E-01	-9.623E-01
1	10	1.478E 00	-1.481E 00
2	10	1.710E 00	-1.711E 00
3	10	1.897E 00	-1.894E 00
4	10	1.643E 00	-1.643E 00
5	10	9.586E-01	-9.590E-01
6	10	3.711E-02	-3.668E-02
7	10	-8.838E-01	8.845E-01
8	10	-1.563E 00	1.563E 00
9	10	-1.801E 00	1.800E 00
10	10	-1.594E 00	1.595E 00
11	10	-1.344E 00	1.344E 00
12	10	-8.968E-01	8.969E-01
13	10	0	0
-1	11	0	0
0	11	6.402E-01	-6.400E-01
1	11	1.109E 00	-1.110E 00
2	11	1.244E 00	-1.246E 00
3	11	1.321E 00	-1.319E 00
4	11	1.129E 00	-1.129E 00
5	11	6.618E-01	-6.619E-01
6	11	3.400E-02	-3.364E-02
7	11	-5.927E-01	5.931E-01
8	11	-1.058E 00	1.058E 00
9	11	-1.244E 00	1.243E 00
10	11	-1.150E 00	1.151E 00
11	11	-1.003E 00	1.004E 00
12	11	-5.907E-01	5.910E-01
13	11	0	0
-1	12	0	0
0	12	2.073E-01	-2.065E-01
1	12	4.366E-01	-4.374E-01
2	12	5.437E-01	-5.488E-01
3	12	5.755E-01	-5.723E-01
4	12	4.850E-01	-4.842E-01
5	12	2.820E-01	-2.823E-01
6	12	1.663E-02	-1.617E-02
7	12	-2.476E-01	2.478E-01
8	12	-4.492E-01	4.493E-01
9	12	-5.412E-01	5.420E-01
10	12	-5.037E-01	5.002E-01
11	12	-3.818E-01	3.826E-01
12	12	-1.824E-01	1.852E-01
13	12	0	0

-1	13	0	0
0	13	0	0
1	13	0	0
2	13	0	0
3	13	0	0
4	13	0	0
5	13	0	0
6	13	0	0
7	13	0	0
8	13	0	0
9	13	0	0
10	13	0	0
11	13	0	0
12	13	0	0
13	13	0	0

PROGRAM SLAB 17 - MASTER DECK - WR HUDSON, H MATLOCK   REVISION DATE 26 JUL 65
CE051022   HWY SLAB PROJECT   SLAB 17   W R HUDSON
RUN EXAMPLE PROBLEMS FOR USE IN SLAB REPORT APPENDIX D

PROB
504   EXAMPLE - ORTHO PLATE - CLIFTON - ETAL - CENTER LOAD CX = CY = 0.0

TABLE 1. CONTROL DATA

NUM VALUES TABLE 2	4
NUM CARDS TABLE 3A	5
NUM CARDS TABLE 3B	0
MAX NUM ITERATIONS	20
NUM INCREMENTS MX	10
NUM INCREMENTS MY	20
INCR LENGTH HX	1.200E 01
INCR LENGTH HY	4.800E 01
CLOSURE TOLERANCE	1.000E-05
POISSONS RATIO	3.000E-01

MONITOR STAS I,J   5 10   5 0   5 5   0 0

TABLE 2A. ITERATION CONTROL DATA

F. SPRING REPRESENTING X BEAM

1.000E 05
1.000E 06
5.000E 06
1.000E 06

TABLE 2B. ITERATION CONTROL DATA

F. SPRING REPRESENTING Y BEAM

1.000E 04
5.000E 04
1.000E 05
5.000E 04

TABLE 3A. STIFFNESS AND LOAD DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

FROM	THRU	DX	DY	Q	S	CX	CY
1 1	9 19	6.600E 07	7.300E 06	0	0	0	0
0 0	10 20	6.600E 07	7.300E 06	0	1.000E 99	0	0
1 0	9 20	0	0	0	-1.000E 99	0	0
5 10	5 10	0	0	1.000E 03	0	0	0
1 1	10 20	0	0	0	0	0	0

TABLE 3B. STIFFNESS AND LOAD DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECT.

FROM THRU TX TY PX PY

TABLE 4. MONITOR TALLY AND DEFLS AT 4 STAS

	ITR NUM	FICT SPRING	CYC NUM	NOT STAB	NOT CLOS	I,J 5 10	5 0	5 5	0 0
X	1	1.000E 04	1	21		4.534E-03	0	0	0
Y		1.000E 05		63	64	6.982E-03	-1.540E-11	-1.586E-06	0
X	2	5.000E 04	2	60		5.042E-03	3.554E-10	-1.543E-06	-9.313-105
Y		1.000E 06		62	32	4.871E-03	3.867E-10	-1.503E-06	-9.313-105
X	3	1.000E 05	3	60		4.521E-03	3.998E-10	-1.208E-06	1.096-104
Y		5.000E 06		60	2	4.510E-03	3.959E-10	-1.203E-06	1.096-104
X	4	5.000E 04	4	60		4.266E-03	2.242E-10	7.129E-07	1.141-104
Y		1.000E 06		61	9	4.243E-03	1.947E-10	9.624E-07	1.141-104
X	5	1.000E 04	1	51		4.135E-03	-9.071E-10	2.594E-06	-4.985-104
Y		1.000E 05		64	52	4.060E-03	-1.130E-09	7.529E-06	-4.985-104
X	6	5.000E 04	2	20		4.116E-03	-3.096E-09	2.083E-06	3.536-104
Y		1.000E 06		46	0	4.121E-03	-2.977E-09	1.572E-06	3.536-104

PROGRAM SLAB 17 - MASTER DECK - WR HUDSON, H MATLOCK REVISION DATE 26 JUL 65
CE051022 HWY SLAB PROJECT SLAB 17 W R HUDSON
RUN EXAMPLE PROBLEMS FOR USE IN SLAB REPORT APPENDIX D

PROB (CONTD)
504 EXAMPLE - ORTHO PLATE - CLIFTON - ETAL - CENTER LOAD CX = CY = 0.0

TABLE 5. RESULTS -- ITERATION 6

I,J		X-DEFL	Y-DEFL	BMX	BMY	REACT	TRERR
-1	-1	7.075E-09	7.075E-09	0	0	0	0
0	-1	-9.780E-20	-1.046-102	0	0	0	0
1	-1	-1.603E-09	-7.069E-09	0	0	0	0
2	-1	-4.875E-09	-1.430E-08	0	0	0	0
3	-1	-1.027E-08	-2.102E-08	0	0	0	0
4	-1	-1.619E-08	-2.718E-08	0	0	0	0
5	-1	-1.918E-08	-3.261E-08	0	0	0	0
6	-1	-1.671E-08	-3.641E-08	0	0	0	0
7	-1	-8.746E-09	-3.703E-08	0	0	0	0
8	-1	2.960E-09	-3.273E-08	0	0	0	0
9	-1	1.597E-08	-2.222E-08	0	0	0	0
10	-1	4.643-102	-9.879-102	0	0	0	0
11	-1	1.216E-08	1.216E-08	0	0	0	0
-1	0	6.180E-10	6.180E-10	0	0	0	0
0	0	3.536-104	3.536-104	7.428-110	8.234-110	-2.818E-05	-7.174E-06
1	0	-6.180E-10	-6.087E-10	-7.046E-06	-1.527E-16	-1.189E-05	1.189E-05
2	0	-1.253E-09	-1.259E-09	-8.397E-06	9.437E-16	-7.828E-06	7.828E-06
3	0	-1.908E-09	-1.921E-09	4.958E-06	-3.053E-15	-8.156E-06	8.156E-06
4	0	-2.551E-09	-2.529E-09	4.073E-05	3.109E-15	-4.212E-05	4.212E-05
5	0	-3.096E-09	-2.977E-09	9.842E-05	-1.998E-15	-1.224E-04	1.224E-04
6	0	-3.406E-09	-3.141E-09	1.654E-04	2.665E-15	-2.364E-04	2.364E-04
7	0	-3.319E-09	-2.892E-09	2.184E-04	-8.882E-16	-3.594E-04	3.594E-04
8	0	-2.708E-09	-2.139E-09	2.280E-04	4.885E-15	-4.663E-04	4.663E-04
9	0	-1.551E-09	-8.931E-10	1.639E-04	8.882E-16	-5.370E-04	5.370E-04
10	0	-7.160-103	-6.558-103	-5.942-109	-6.587-109	6.558E-04	3.009E-05
11	0	1.551E-09	1.551E-09	0	0	0	0
-1	1	-5.839E-09	-5.839E-09	0	0	0	0
0	1	1.116-102	1.116-102	-6.451E-99	-7.151E-99	-1.473E-03	3.566E-04
1	1	5.839E-09	6.096E-09	-3.682E-04	-1.387E-04	-1.163E-04	1.163E-04
2	1	1.139E-08	1.208E-08	-7.535E-04	-2.666E-04	-3.760E-04	3.760E-04
3	1	1.632E-08	1.700E-08	-1.207E-03	-3.913E-04	-4.533E-04	4.533E-04
4	1	2.023E-08	2.072E-08	-1.745E-03	-5.192E-04	-1.106E-04	1.106E-04
5	1	2.261E-08	2.325E-08	-2.285E-03	-6.384E-04	5.111E-04	-5.111E-04
6	1	2.294E-08	2.441E-08	-2.684E-03	-7.219E-04	1.065E-03	-1.065E-03
7	1	2.083E-08	2.369E-08	-2.816E-03	-7.431E-04	1.336E-03	-1.336E-03
8	1	1.615E-08	2.056E-08	-2.627E-03	-6.866E-04	1.300E-03	-1.300E-03
9	1	9.060E-09	1.476E-08	-2.142E-03	-5.501E-04	6.818E-03	-6.818E-03
10	1	2.307-102	8.567-102	-5.282E-98	-5.856E-98	-8.567E-03	3.130E-03
11	1	-9.060E-09	-9.060E-09	0	0	0	0
-1	2	3.959E-09	3.959E-09	0	0	0	0
0	2	-5.962-104	-5.962-104	4.969E-14	4.958E-15	-1.149E-03	1.208E-03
1	2	-3.959E-09	-4.879E-09	-2.872E-04	-4.704E-04	2.040E-03	-2.040E-03
2	2	-7.754E-09	-7.783E-09	-3.515E-04	-9.406E-04	3.416E-03	-3.416E-03
3	2	-1.095E-08	-1.104E-08	-1.236E-04	-1.260E-03	3.169E-03	-3.169E-03

4	2	-1.294E-08	-1.589E-08	1.081E-04	-1.430E-03	2.649E-03	-2.649E-03
5	2	-1.325E-08	-2.169E-08	3.712E-05	-1.493E-03	3.021E-03	-3.021E-03
6	2	-1.190E-08	-2.678E-08	-3.601E-04	-1.456E-03	4.306E-03	-4.306E-03
7	2	-9.405E-09	-2.987E-08	-7.980E-04	-1.282E-03	5.944E-03	-5.944E-03
8	2	-6.482E-09	-3.035E-08	-7.967E-04	-9.186E-04	7.279E-03	-7.279E-03
9	2	-3.522E-09	-2.792E-08	1.729E-04	-3.272E-04	-2.471E-03	2.471E-03
10	2	1.274-101	-6.915-103	-2.934E-97	-3.252E-97	6.915E-04	-6.716E-03
11	2	3.522E-09	3.522E-09	0	0	0	0
-1	3	9.142E-08	9.142E-08	0	0	0	0
0	3	-2.440-101	-2.440-101	3.386E-97	3.754E-97	3.269E-02	-8.287E-03
1	3	-9.142E-08	-9.247E-08	8.173E-03	3.793E-03	-5.095E-03	5.095E-03
2	3	-1.771E-07	-1.847E-07	1.575E-02	7.372E-03	-4.464E-03	4.464E-03
3	3	-2.520E-07	-2.554E-07	2.357E-02	1.049E-02	-6.033E-03	6.033E-03
4	3	-3.099E-07	-3.024E-07	3.190E-02	1.310E-02	-1.307E-02	1.307E-02
5	3	-3.437E-07	-3.262E-07	3.960E-02	1.496E-02	-2.261E-02	2.261E-02
6	3	-3.462E-07	-3.242E-07	4.477E-02	1.568E-02	-3.137E-02	3.137E-02
7	3	-3.120E-07	-2.919E-07	4.548E-02	1.493E-02	-3.717E-02	3.717E-02
8	3	-2.394E-07	-2.268E-07	4.022E-02	1.247E-02	-3.897E-02	3.897E-02
9	3	-1.321E-07	-1.303E-07	2.815E-02	8.289E-03	-7.310E-02	7.310E-02
10	3	-7.694-101	-1.126-100	1.574E-96	1.745E-96	1.126E-01	-1.783E-02
11	3	1.321E-07	1.321E-07	0	0	0	0
-1	4	-3.251E-07	-3.251E-07	0	0	0	0
0	4	6.973-101	6.973-101	3.474E-97	3.851E-97	-7.736E-02	7.626E-03
1	4	3.251E-07	3.364E-07	-1.934E-02	-2.873E-03	-2.085E-02	2.085E-02
2	4	6.302E-07	6.443E-07	-4.203E-02	-5.943E-03	-2.718E-02	2.718E-02
3	4	8.912E-07	9.112E-07	-6.809E-02	-1.007E-02	-1.131E-02	1.131E-02
4	4	1.082E-06	1.131E-06	-9.260E-02	-1.459E-02	8.190E-03	-8.190E-03
5	4	1.177E-06	1.278E-06	-1.103E-01	-1.856E-02	2.211E-02	-2.211E-02
6	4	1.160E-06	1.323E-06	-1.180E-01	-2.120E-02	2.836E-02	-2.836E-02
7	4	1.024E-06	1.248E-06	-1.147E-01	-2.209E-02	2.818E-02	-2.818E-02
8	4	7.748E-07	1.044E-06	-1.017E-01	-2.108E-02	2.401E-02	-2.401E-02
9	4	4.265E-07	7.177E-07	-8.159E-02	-1.817E-02	2.498E-01	-2.498E-01
10	4	5.582-101	3.264-100	-1.358E-96	-1.505E-96	-3.264E-01	1.353E-01
11	4	-4.265E-07	-4.265E-07	0	0	0	0
-1	5	-6.050E-07	-6.050E-07	0	0	0	0
0	5	2.854-100	2.854-100	-6.361E-12	-6.346E-13	-3.305E-01	4.512E-02
1	5	6.050E-07	6.016E-07	-8.264E-02	-3.937E-02	1.048E-01	-1.048E-01
2	5	1.153E-06	1.170E-06	-1.464E-01	-7.407E-02	9.595E-02	-9.595E-02
3	5	1.607E-06	1.510E-06	-2.001E-01	-1.006E-01	1.073E-01	-1.073E-01
4	5	1.928E-06	1.636E-06	-2.465E-01	-1.180E-01	1.458E-01	-1.458E-01
5	5	2.083E-06	1.572E-06	-2.796E-01	-1.249E-01	1.953E-01	-1.953E-01
6	5	2.036E-06	1.335E-06	-2.893E-01	-1.199E-01	2.394E-01	-2.394E-01
7	5	1.773E-06	9.411E-07	-2.639E-01	-1.020E-01	2.664E-01	-2.664E-01
8	5	1.304E-06	4.186E-07	-1.938E-01	-7.135E-02	2.683E-01	-2.683E-01
9	5	6.795E-07	-1.731E-07	-7.258E-02	-2.952E-02	-1.629E-01	1.629E-01
10	5	7.981-100	2.903-100	-7.473E-96	-8.284E-96	-2.903E-01	-2.539E-01
11	5	-6.795E-07	-6.795E-07	0	0	0	0
-1	6	4.461E-06	4.461E-06	0	0	0	0
0	6	-1.290E-99	-1.290E-99	-2.490E-96	-2.760E-96	1.375E 00	-8.452E-02
1	6	-4.461E-06	-4.530E-06	3.437E-01	4.115E-02	1.232E-02	-1.232E-02
2	6	-8.555E-06	-8.617E-06	6.805E-01	8.132E-02	-3.007E-02	3.007E-02
3	6	-1.192E-05	-1.187E-05	9.914E-01	1.196E-01	-1.543E-01	1.543E-01
4	6	-1.423E-05	-1.405E-05	1.240E 00	1.519E-01	-2.979E-01	2.979E-01
5	6	-1.521E-05	-1.496E-05	1.388E 00	1.731E-01	-4.201E-01	4.201E-01
6	6	-1.472E-05	-1.445E-05	1.406E 00	1.790E-01	-5.002E-01	5.002E-01
7	6	-1.274E-05	-1.249E-05	1.278E 00	1.668E-01	-5.301E-01	5.301E-01
8	6	-9.398E-06	-9.227E-06	1.002E 00	1.352E-01	-5.090E-01	5.090E-01

9	6	-5.007E-06	-4.927E-06	5.901E-01	8.531E-02	-7.131E-01	7.131E-01
10	6	-2.044E-99	-2.360E-99	5.631E-96	6.242E-96	2.360E 00	-1.583E-01
11	6	5.007E-06	5.007E-06	0	0	0	0
-1	7	8.492E-06	8.492E-06	0	0	0	0
0	7	-3.737E-99	-3.737E-99	1.018E-10	1.015E-11	3.838E 00	-1.006E-01
1	7	-8.492E-06	-8.472E-06	9.594E-01	2.819E-01	-4.279E-01	4.279E-01
2	7	-1.614E-05	-1.608E-05	1.775E 00	5.301E-01	-3.797E-01	3.797E-01
3	7	-2.223E-05	-2.165E-05	2.426E 00	7.191E-01	-4.412E-01	4.412E-01
4	7	-2.619E-05	-2.486E-05	2.866E 00	8.365E-01	-5.568E-01	5.568E-01
5	7	-2.762E-05	-2.549E-05	3.048E 00	8.720E-01	-6.793E-01	6.793E-01
6	7	-2.634E-05	-2.352E-05	2.933E 00	8.205E-01	-7.720E-01	7.720E-01
7	7	-2.243E-05	-1.913E-05	2.505E 00	6.838E-01	-8.084E-01	8.084E-01
8	7	-1.626E-05	-1.276E-05	1.769E 00	4.714E-01	-7.706E-01	7.706E-01
9	7	-8.477E-06	-5.096E-06	7.602E-01	2.016E-01	1.198E 00	-1.198E 00
10	7	-5.330E-99	-3.041E-99	1.880E-95	2.084E-95	3.041E 00	1.145E 00
11	7	8.477E-06	8.477E-06	0	0	0	0
-1	8	-1.965E-05	-1.965E-05	0	0	0	0
0	8	1.805E-99	1.805E-99	-2.035E-10	-2.031E-11	-2.067E 00	2.623E-01
1	8	1.965E-05	1.987E-05	-5.168E-01	1.107E 00	3.263E-01	-3.263E-01
2	8	3.749E-05	3.768E-05	-9.767E-01	2.111E 00	3.346E-01	-3.346E-01
3	8	5.187E-05	5.131E-05	-1.408E 00	2.913E 00	6.027E-01	-6.027E-01
4	8	6.141E-05	5.981E-05	-1.782E 00	3.427E 00	9.610E-01	-9.610E-01
5	8	6.511E-05	6.246E-05	-2.049E 00	3.596E 00	1.298E 00	-1.298E 00
6	8	6.246E-05	5.895E-05	-2.152E 00	3.403E 00	1.547E 00	-1.547E 00
7	8	5.356E-05	4.948E-05	-2.004E 00	2.880E 00	1.667E 00	-1.667E 00
8	8	3.913E-05	3.486E-05	-1.536E 00	2.094E 00	1.638E 00	-1.638E 00
9	8	2.061E-05	1.654E-05	-7.144E-01	1.132E 00	-3.476E-01	3.476E-01
10	8	5.067E-99	2.857E-99	1.759E-94	1.950E-94	-2.857E 00	-1.105E 00
11	8	-2.061E-05	-2.061E-05	0	0	0	0
-1	9	-2.491E-04	-2.491E-04	0	0	0	0
0	9	7.079E-98	7.079E-98	-3.257E-09	-3.249E-10	-7.084E 01	5.018E-02
1	9	2.491E-04	2.492E-04	-1.771E 01	2.328E 00	3.602E-01	-3.602E-01
2	9	4.745E-04	4.745E-04	-3.387E 01	4.574E 00	1.984E-01	-1.984E-01
3	9	6.544E-04	6.538E-04	-4.707E 01	6.597E 00	2.587E-01	-2.587E-01
4	9	7.707E-04	7.696E-04	-5.595E 01	8.157E 00	3.607E-01	-3.607E-01
5	9	8.111E-04	8.094E-04	-5.926E 01	8.873E 00	4.580E-01	-4.580E-01
6	9	7.708E-04	7.687E-04	-5.601E 01	8.167E 00	5.263E-01	-5.263E-01
7	9	6.546E-04	6.522E-04	-4.714E 01	6.620E 00	5.468E-01	-5.468E-01
8	9	4.747E-04	4.722E-04	-3.388E 01	4.615E 00	5.082E-01	-5.082E-01
9	9	2.492E-04	2.468E-04	-1.758E 01	2.385E 00	-8.071E-01	8.071E-01
10	9	7.188E-98	7.031E-98	6.432E-94	7.130E-94	-7.031E 01	-7.876E-01
11	9	-2.492E-04	-2.492E-04	0	0	0	0
-1	10	-1.189E-03	-1.189E-03	0	0	0	0
0	10	3.634E-97	3.634E-97	-1.303E-08	-1.300E-09	-3.630E 02	-3.524E-01
1	10	1.189E-03	1.189E-03	-9.075E 01	-1.990E 01	-8.053E-01	8.053E-01
2	10	2.291E-03	2.290E-03	-1.845E 02	-3.937E 01	-5.712E-01	5.712E-01
3	10	3.214E-03	3.215E-03	-2.839E 02	-5.788E 01	-9.456E-01	9.456E-01
4	10	3.859E-03	3.862E-03	-3.915E 02	-7.475E 01	-1.459E 00	1.459E 00
5	10	4.116E-03	4.121E-03	-5.099E 02	-8.909E 01	9.981E 02	1.932E 00
6	10	3.858E-03	3.864E-03	-3.911E 02	-7.474E 01	-2.269E 00	2.269E 00
7	10	3.212E-03	3.219E-03	-2.831E 02	-5.787E 01	-2.410E 00	2.410E 00
8	10	2.289E-03	2.296E-03	-1.839E 02	-3.940E 01	-2.321E 00	2.321E 00
9	10	1.188E-03	1.195E-03	-9.070E 01	-1.999E 01	1.306E 00	-1.306E 00
10	10	3.586E-97	3.628E-97	-1.674E-93	-1.856E-93	-3.628E 02	2.080E 00
11	10	-1.188E-03	-1.188E-03	0	0	0	0
-1	11	-2.477E-04	-2.477E-04	0	0	0	0

0	11	7.026E-98	7.026E-98	6.412E-94	7.109E-94	-7.034E 01	8.124E-02
1	11	2.477E-04	2.477E-04	-1.759E 01	2.353E 00	3.316E-01	-3.316E-01
2	11	4.717E-04	4.717E-04	-3.362E 01	4.620E 00	1.454E-01	-1.454E-01
3	11	6.504E-04	6.501E-04	-4.672E 01	6.662E 00	1.422E-01	-1.422E-01
4	11	7.660E-04	7.652E-04	-5.551E 01	8.237E 00	1.853E-01	-1.853E-01
5	11	8.060E-04	8.047E-04	-5.878E 01	8.961E 00	2.406E-01	-2.406E-01
6	11	7.659E-04	7.642E-04	-5.552E 01	8.254E 00	2.902E-01	-2.902E-01
7	11	6.504E-04	6.483E-04	-4.671E 01	6.698E 00	3.190E-01	-3.190E-01
8	11	4.716E-04	4.693E-04	-3.354E 01	4.675E 00	3.173E-01	-3.173E-01
9	11	2.475E-04	2.452E-04	-1.738E 01	2.421E 00	-1.049E 00	1.049E 00
10	11	7.118E-98	6.952E-98	6.469E-94	7.172E-94	-6.952E 01	-8.300E-01
11	11	-2.475E-04	-2.475E-04	0	0	0	0
-1	12	-1.878E-05	-1.878E-05	0	0	0	0
0	12	1.531E-99	1.531E-99	-2.035E-10	-2.031E-11	-1.752E 00	2.205E-01
1	12	1.878E-05	1.898E-05	-4.379E-01	1.124E 00	2.868E-01	-2.868E-01
2	12	3.582E-05	3.598E-05	-8.263E-01	2.144E 00	2.820E-01	-2.820E-01
3	12	4.955E-05	4.903E-05	-1.195E 00	2.959E 00	5.238E-01	-5.238E-01
4	12	5.865E-05	5.720E-05	-1.519E 00	3.480E 00	8.489E-01	-8.489E-01
5	12	6.216E-05	5.980E-05	-1.758E 00	3.651E 00	1.155E 00	-1.155E 00
6	12	5.962E-05	5.652E-05	-1.860E 00	3.454E 00	1.381E 00	-1.381E 00
7	12	5.111E-05	4.755E-05	-1.747E 00	2.922E 00	1.492E 00	-1.492E 00
8	12	3.735E-05	3.366E-05	-1.352E 00	2.121E 00	1.465E 00	-1.465E 00
9	12	1.968E-05	1.619E-05	-6.451E-01	1.138E 00	-1.657E-01	1.657E-01
10	12	4.405E-99	2.580E-99	-2.035E-10	-2.031E-11	-2.580E 00	-9.124E-01
11	12	-1.968E-05	-1.968E-05	0	0	0	0
-1	13	7.161E-06	7.161E-06	0	0	0	0
0	13	-3.224E-99	-3.224E-99	1.966E-95	2.179E-95	3.318E 00	-9.429E-02
1	13	-7.161E-06	-7.157E-06	8.296E-01	2.525E-01	-3.429E-01	3.429E-01
2	13	-1.360E-05	-1.360E-05	1.533E 00	4.750E-01	-2.776E-01	2.776E-01
3	13	-1.872E-05	-1.833E-05	2.087E 00	6.443E-01	-2.985E-01	2.985E-01
4	13	-2.203E-05	-2.105E-05	2.457E 00	7.490E-01	-3.685E-01	3.685E-01
5	13	-2.320E-05	-2.158E-05	2.603E 00	7.803E-01	-4.533E-01	4.533E-01
6	13	-2.210E-05	-1.991E-05	2.496E 00	7.340E-01	-5.253E-01	5.253E-01
7	13	-1.880E-05	-1.620E-05	2.124E 00	6.120E-01	-5.638E-01	5.638E-01
8	13	-1.362E-05	-1.082E-05	1.497E 00	4.232E-01	-5.541E-01	5.541E-01
9	13	-7.094E-06	-4.364E-06	6.440E-01	1.840E-01	1.045E 00	-1.045E 00
10	13	-4.464E-99	-2.576E-99	1.639E-95	1.817E-95	2.576E 00	9.440E-01
11	13	7.094E-06	7.094E-06	0	0	0	0
-1	14	3.790E-06	3.790E-06	0	0	0	0
0	14	-1.101E-99	-1.101E-99	-2.448E-96	-2.713E-96	1.166E 00	-6.519E-02
1	14	-3.790E-06	-3.853E-06	2.915E-01	3.391E-02	1.016E-02	-1.016E-02
2	14	-7.267E-06	-7.323E-06	5.758E-01	6.678E-02	-1.474E-02	1.474E-02
3	14	-1.013E-05	-1.008E-05	8.385E-01	9.826E-02	-1.142E-01	1.142E-01
4	14	-1.209E-05	-1.192E-05	1.049E 00	1.250E-01	-2.344E-01	2.344E-01
5	14	-1.292E-05	-1.268E-05	1.176E 00	1.428E-01	-3.395E-01	3.395E-01
6	14	-1.251E-05	-1.224E-05	1.192E 00	1.481E-01	-4.106E-01	4.106E-01
7	14	-1.082E-05	-1.058E-05	1.083E 00	1.382E-01	-4.395E-01	4.395E-01
8	14	-7.985E-06	-7.803E-06	8.489E-01	1.122E-01	-4.243E-01	4.243E-01
9	14	-4.253E-06	-4.157E-06	4.990E-01	7.079E-02	-5.909E-01	5.909E-01
10	14	-1.749E-99	-1.996E-99	4.469E-96	4.955E-96	1.996E 00	-1.236E-01
11	14	4.253E-06	4.253E-06	0	0	0	0
-1	15	-2.906E-07	-2.906E-07	0	0	0	0
0	15	1.660-100	1.660-100	-3.917E-96	-4.342E-96	-2.011E-01	3.511E-02
1	15	2.906E-07	2.880E-07	-5.027E-02	-2.895E-02	7.996E-02	-7.996E-02
2	15	5.522E-07	5.710E-07	-8.607E-02	-5.434E-02	6.871E-02	-6.871E-02
3	15	7.694E-07	7.093E-07	-1.152E-01	-7.347E-02	7.038E-02	-7.038E-02
4	15	9.279E-07	7.228E-07	-1.414E-01	-8.590E-02	9.365E-02	-9.365E-02

5	15	1.010E-06	6.409E-07	-1.618E-01	-9.074E-02	1.274E-01	-1.274E-01
6	15	9.958E-07	4.818E-07	-1.694E-01	-8.696E-02	1.597E-01	-1.597E-01
7	15	8.729E-07	2.570E-07	-1.559E-01	-7.384E-02	1.815E-01	-1.815E-01
8	15	6.429E-07	-1.705E-08	-1.136E-01	-5.145E-02	1.868E-01	-1.868E-01
9	15	3.322E-07	-3.077E-07	-3.697E-02	-2.098E-02	-1.344E-01	1.344E-01
10	15	5.350-100	1.479-100	-5.805E-96	-6.435E-96	-1.479E-01	-1.936E-01
11	15	-3.322E-07	-3.322E-07	0	0	0	0
-1	16	-2.684E-07	-2.684E-07	0	0	0	0
0	16	6.244-101	6.244-101	7.919E-98	8.779E-98	-6.919E-02	6.752E-03
1	16	2.684E-07	2.776E-07	-1.730E-02	-3.556E-03	-1.262E-02	1.262E-02
2	16	5.199E-07	5.319E-07	-3.655E-02	-7.074E-03	-1.891E-02	1.891E-02
3	16	7.353E-07	7.465E-07	-5.833E-02	-1.114E-02	-6.519E-03	6.519E-03
4	16	8.928E-07	9.183E-07	-7.899E-02	-1.524E-02	1.030E-02	-1.030E-02
5	16	9.720E-07	1.029E-06	-9.416E-02	-1.856E-02	2.343E-02	-2.343E-02
6	16	9.585E-07	1.056E-06	-1.007E-01	-2.044E-02	3.039E-02	-3.039E-02
7	16	8.463E-07	9.857E-07	-9.752E-02	-2.050E-02	3.144E-02	-3.144E-02
8	16	6.394E-07	8.125E-07	-8.501E-02	-1.863E-02	2.802E-02	-2.802E-02
9	16	3.507E-07	5.430E-07	-6.513E-02	-1.491E-02	1.845E-01	-1.845E-01
10	16	7.204-101	2.605-100	-1.266E-96	-1.404E-96	-2.605E-01	9.423E-02
11	16	-3.507E-07	-3.507E-07	0	0	0	0
-1	17	4.949E-08	4.949E-08	0	0	0	0
0	17	-1.339-101	-1.339-101	2.513E-97	2.786E-97	1.919E-02	-5.792E-03
1	17	-4.949E-08	-5.006E-08	4.797E-03	2.608E-03	-4.548E-03	4.548E-03
2	17	-9.604E-08	-1.016E-07	9.013E-03	5.049E-03	-3.927E-03	3.927E-03
3	17	-1.373E-07	-1.395E-07	1.334E-02	7.121E-03	-3.993E-03	3.993E-03
4	17	-1.702E-07	-1.629E-07	1.828E-02	8.839E-03	-8.225E-03	8.225E-03
5	17	-1.907E-07	-1.738E-07	2.324E-02	1.006E-02	-1.480E-02	1.480E-02
6	17	-1.943E-07	-1.715E-07	2.695E-02	1.054E-02	-2.124E-02	2.124E-02
7	17	-1.770E-07	-1.534E-07	2.795E-02	1.003E-02	-2.580E-02	2.580E-02
8	17	-1.370E-07	-1.176E-07	2.505E-02	8.342E-03	-2.753E-02	2.753E-02
9	17	-7.603E-08	-6.436E-08	1.747E-02	5.459E-03	-4.602E-02	4.602E-02
10	17	-5.272-101	-6.989-101	1.127E-96	1.250E-96	6.989E-02	-8.586E-03
11	17	7.603E-08	7.603E-08	0	0	0	0
-1	18	7.701E-09	7.701E-09	0	0	0	0
0	18	-1.312-102	-1.312-102	-2.870E-98	-3.182E-98	8.260E-04	4.863E-04
1	18	-7.701E-09	-8.428E-09	2.065E-04	-1.476E-04	1.266E-03	-1.266E-03
2	18	-1.500E-08	-1.536E-08	5.557E-04	-3.064E-04	2.336E-03	-2.336E-03
3	18	-2.129E-08	-2.147E-08	1.150E-03	-3.762E-04	2.018E-03	-2.018E-03
4	18	-2.580E-08	-2.760E-08	1.777E-03	-3.643E-04	1.195E-03	-1.195E-03
5	18	-2.779E-08	-3.325E-08	2.134E-03	-3.172E-04	9.417E-04	-9.417E-04
6	18	-2.688E-08	-3.697E-08	2.090E-03	-2.613E-04	1.462E-03	-1.462E-03
7	18	-2.317E-08	-3.754E-08	1.757E-03	-1.870E-04	2.448E-03	-2.448E-03
8	18	-1.716E-08	-3.439E-08	1.424E-03	-6.195E-05	3.458E-03	-3.458E-03
9	18	-9.378E-09	-2.747E-08	1.461E-03	1.464E-04	-4.809E-03	4.809E-03
10	18	5.603-102	-5.845-102	-1.482E-97	-1.643E-97	5.845E-03	-5.724E-03
11	18	9.378E-09	9.378E-09	0	0	0	0
-1	19	-3.886E-09	-3.886E-09	0	0	0	0
0	19	7.271-103	7.271-103	-7.739E-99	-8.579E-99	-1.043E-03	3.162E-04
1	19	3.886E-09	4.038E-09	-2.608E-04	-1.211E-04	1.527E-05	-1.527E-05
2	19	7.591E-09	8.112E-09	-5.238E-04	-2.346E-04	-1.249E-04	1.249E-04
3	19	1.092E-08	1.142E-08	-8.281E-04	-3.391E-04	-2.182E-04	2.182E-04
4	19	1.363E-08	1.382E-08	-1.206E-03	-4.407E-04	4.209E-06	-4.209E-06
5	19	1.536E-08	1.538E-08	-1.615E-03	-5.332E-04	4.992E-04	-4.992E-04
6	19	1.574E-08	1.607E-08	-1.946E-03	-5.963E-04	9.960E-04	-9.960E-04
7	19	1.443E-08	1.555E-08	-2.086E-03	-6.077E-04	1.292E-03	-1.292E-03
8	19	1.127E-08	1.345E-08	-1.968E-03	-5.525E-04	1.325E-03	-1.325E-03
9	19	6.353E-09	9.536E-09	-1.583E-03	-4.269E-04	5.045E-03	-5.045E-03

10 19	2.219-102	6.333-102	-5.382E-98	-5.966E-98	-6.333E-03	2.057E-03
11 19	-6.353E-09	-6.353E-09	0	0	0	0
-1 20	1.398E-11	1.398E-11	0	0	0	0
0 20	5.898-104	5.898-104	8.832E-17	0	-5.153E-05	-7.447E-06
1 20	-1.398E-11	1.101E-11	-1.288E-05	5.551E-17	-2.563E-05	2.563E-05
2 20	-5.884E-11	-3.640E-11	-2.460E-05	8.327E-16	-3.284E-05	3.284E-05
3 20	-1.627E-10	-1.526E-10	-2.987E-05	-6.106E-16	-3.030E-05	3.030E-05
4 20	-3.382E-10	-3.222E-10	-2.152E-05	1.055E-15	-4.157E-05	4.157E-05
5 20	-5.652E-10	-5.046E-10	3.982E-06	-9.229E-16	-8.240E-05	8.240E-05
6 20	-7.827E-10	-6.405E-10	4.221E-05	-1.554E-15	-1.493E-04	1.493E-04
7 20	-8.991E-10	-6.565E-10	8.038E-05	-8.882E-16	-2.267E-04	2.267E-04
8 20	-8.227E-10	-4.852E-10	9.987E-05	-6.661E-16	-2.969E-04	2.969E-04
9 20	-5.068E-10	-1.044E-10	7.965E-05	-1.332E-15	-3.445E-04	3.445E-04
10 20	-3.476-103	-3.186-103	8.913-109	9.881-109	3.186E-04	1.449E-05
11 20	5.068E-10	5.068E-10	0	0	0	0
-1 21	3.914E-09	3.914E-09	0	0	0	0
0 21	-4.890E-20	-3.056E-21	0	0	0	0
1 21	-8.841E-10	-3.570E-09	0	0	0	0
2 21	-2.973E-09	-7.334E-09	0	0	0	0
3 21	-7.075E-09	-1.070E-08	0	0	0	0
4 21	-1.247E-08	-1.372E-08	0	0	0	0
5 21	-1.646E-08	-1.653E-08	0	0	0	0
6 21	-1.657E-08	-1.881E-08	0	0	0	0
7 21	-1.224E-08	-1.964E-08	0	0	0	0
8 21	-4.443E-09	-1.787E-08	0	0	0	0
9 21	5.143E-09	-1.250E-08	0	0	0	0
10 21	-4.890E-20	-6.970-102	0	0	0	0
11 21	7.367E-09	7.367E-09	0	0	0	0

PROB (CONTD)
504 EXAMPLE - ORTHO PLATE - CLIFTON - ETAL - CENTER LOAD CX = CY = 0.0

TABLE 5. RESULTS(CONTD) -- ITERATION 6

I,J	TMX	TMY
-1 -1	0	0
0 -1	0	0
1 -1	0	0
2 -1	0	0
3 -1	0	0
4 -1	0	0
5 -1	0	0
6 -1	0	0
7 -1	0	0
8 -1	0	0
9 -1	0	0
10 -1	0	0
11 -1	0	0
-1 0	0	0
0 0	0	0
1 0	0	0
2 0	0	0
3 0	0	0

4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0
9	0	0	0
10	0	0	0
11	0	0	0
-1	1	0	0
0	1	0	0
1	1	0	0
2	1	0	0
3	1	0	0
4	1	0	0
5	1	0	0
6	1	0	0
7	1	0	0
8	1	0	0
9	1	0	0
10	1	0	0
11	1	0	0
-1	2	0	0
0	2	0	0
1	2	0	0
2	2	0	0
3	2	0	0
4	2	0	0
5	2	0	0
6	2	0	0
7	2	0	0
8	2	0	0
9	2	0	0
10	2	0	0
11	2	0	0
-1	3	0	0
0	3	0	0
1	3	0	0
2	3	0	0
3	3	0	0
4	3	0	0
5	3	0	0
6	3	0	0
7	3	0	0
8	3	0	0
9	3	0	0
10	3	0	0
11	3	0	0
-1	4	0	0
0	4	0	0
1	4	0	0
2	4	0	0
3	4	0	0
4	4	0	0
5	4	0	0
6	4	0	0
7	4	0	0
8	4	0	0

9	4	0	0
10	4	0	0
11	4	0	0
-1	5	0	0
0	5	0	0
1	5	0	0
2	5	0	0
3	5	0	0
4	5	0	0
5	5	0	0
6	5	0	0
7	5	0	0
8	5	0	0
9	5	0	0
10	5	0	0
11	5	0	0
-1	6	0	0
0	6	0	0
1	6	0	0
2	6	0	0
3	6	0	0
4	6	0	0
5	6	0	0
6	6	0	0
7	6	0	0
8	6	0	0
9	6	0	0
10	6	0	0
11	6	0	0
-1	7	0	0
0	7	0	0
1	7	0	0
2	7	0	0
3	7	0	0
4	7	0	0
5	7	0	0
6	7	0	0
7	7	0	0
8	7	0	0
9	7	0	0
10	7	0	0
11	7	0	0
-1	8	0	0
0	8	0	0
1	8	0	0
2	8	0	0
3	8	0	0
4	8	0	0
5	8	0	0
6	8	0	0
7	8	0	0
8	8	0	0
9	8	0	0
10	8	0	0
11	8	0	0
-1	9	0	0

0	9	0	0
1	9	0	0
2	9	0	0
3	9	0	0
4	9	0	0
5	9	0	0
6	9	0	0
7	9	0	0
8	9	0	0
9	9	0	0
10	9	0	0
11	9	0	0
-1	10	0	0
0	10	0	0
1	10	0	0
2	10	0	0
3	10	0	0
4	10	0	0
5	10	0	0
6	10	0	0
7	10	0	0
8	10	0	0
9	10	0	0
10	10	0	0
11	10	0	0
-1	11	0	0
0	11	0	0
1	11	0	0
2	11	0	0
3	11	0	0
4	11	0	0
5	11	0	0
6	11	0	0
7	11	0	0
8	11	0	0
9	11	0	0
10	11	0	0
11	11	0	0
-1	12	0	0
0	12	0	0
1	12	0	0
2	12	0	0
3	12	0	0
4	12	0	0
5	12	0	0
6	12	0	0
7	12	0	0
8	12	0	0
9	12	0	0
10	12	0	0
11	12	0	0
-1	13	0	0
0	13	0	0
1	13	0	0
2	13	0	0
3	13	0	0
4	13	0	0

5	13	0	0
6	13	0	0
7	13	0	0
8	13	0	0
9	13	0	0
10	13	0	0
11	13	0	0
-1	14	0	0
0	14	0	0
1	14	0	0
2	14	0	0
3	14	0	0
4	14	0	0
5	14	0	0
6	14	0	0
7	14	0	0
8	14	0	0
9	14	0	0
10	14	0	0
11	14	0	0
-1	15	0	0
0	15	0	0
1	15	0	0
2	15	0	0
3	15	0	0
4	15	0	0
5	15	0	0
6	15	0	0
7	15	0	0
8	15	0	0
9	15	0	0
10	15	0	0
11	15	0	0
-1	16	0	0
0	16	0	0
1	16	0	0
2	16	0	0
3	16	0	0
4	16	0	0
5	16	0	0
6	16	0	0
7	16	0	0
8	16	0	0
9	16	0	0
10	16	0	0
11	16	0	0
-1	17	0	0
0	17	0	0
1	17	0	0
2	17	0	0
3	17	0	0
4	17	0	0
5	17	0	0
6	17	0	0
7	17	0	0
8	17	0	0
9	17	0	0

10	17	0	0
11	17	0	0
-1	18	0	0
0	18	0	0
1	18	0	0
2	18	0	0
3	18	0	0
4	18	0	0
5	18	0	0
6	18	0	0
7	18	0	0
8	18	0	0
9	18	0	0
10	18	0	0
11	18	0	0
-1	19	0	0
0	19	0	0
1	19	0	0
2	19	0	0
3	19	0	0
4	19	0	0
5	19	0	0
6	19	0	0
7	19	0	0
8	19	0	0
9	19	0	0
10	19	0	0
11	19	0	0
-1	20	0	0
0	20	0	0
1	20	0	0
2	20	0	0
3	20	0	0
4	20	0	0
5	20	0	0
6	20	0	0
7	20	0	0
8	20	0	0
9	20	0	0
10	20	0	0
11	20	0	0
-1	21	0	0
0	21	0	0
1	21	0	0
2	21	0	0
3	21	0	0
4	21	0	0
5	21	0	0
6	21	0	0
7	21	0	0
8	21	0	0
9	21	0	0
10	21	0	0
11	21	0	0

```
PROGRAM SLAB 17 - MASTER DECK - WR HUDSON, H MATLOCK     REVISION DATE 26 JUL 65
CE051022   HWY SLAB PROJECT       SLAB 17     W R HUDSON
RUN EXAMPLE PROBLEMS FOR USE IN SLAB REPORT APPENDIX D


PROB
 601     EXAMPLE PROBLEM   BRIDGE APPROACH SLAB                 WRH      10JUN65


TABLE 1.  CONTROL DATA

     NUM VALUES TABLE 2                                                     5
     NUM CARDS TABLE 3A                                                    17
     NUM CARDS TABLE 3B                                                     1
     MAX NUM ITERATIONS                                                    50
     NUM INCREMENTS  MX                                                    16
     NUM INCREMENTS  MY                                                    12
     INCR LENGTH  HX                                                2.400E 01
     INCR LENGTH  HY                                                2.400E 01
     CLOSURE TOLERANCE                                              1.000E-05
     POISSONS RATIO                                                 2.500E-01

     MONITOR STAS  I,J                 7  0       7  6      15  8      15 11

TABLE 2A.  ITERATION CONTROL DATA

     F. SPRING REPRESENTING X BEAM

      4.773E 04
      9.384E 05
      4.939E 06
      9.817E 06
      1.457E 07


TABLE 2B.  ITERATION CONTROL DATA

     F. SPRING REPRESENTING Y BEAM

      8.150E 04
      9.384E 05
      4.939E 06
      9.199E 06
      1.439E 07


TABLE 3A. STIFFNESS AND LOAD DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
```

FROM		THRU		DX	DY	Q	S	CX	CY
0	0	16	12	8.890E 07	8.890E 07	1.500E 02	0	0	0
0	1	16	11	8.890E 07	8.890E 07	1.500E 02	0	0	0
1	0	15	12	8.890E 07	8.890E 07	1.500E 02	0	0	0
1	1	15	11	8.890E 07	8.890E 07	1.500E 02	0	0	0
1	1	16	12	0	0	0	0	2.667E 08	2.667E 08
7	0	7	12	-1.777E 08	0	0	0	0	0
7	1	7	11	-1.777E 08	0	0	0	0	0

0	0	2	12	0	0	0	1.000E 99	0	0
15	8	15	8	0	0	1.000E 04	0	0	0
15	11	15	11	0	0	1.000E 04	0	0	0
7	0	16	12	0	0	0	1.440E 04	0	0
7	1	16	11	0	0	0	1.440E 04	0	0
7	0	15	12	0	0	0	1.440E 04	0	0
7	1	15	11	0	0	0	1.440E 04	0	0
16	0	16	12	0	0	0	6.000E 05	0	0
0	6	16	6	0	-8.890E 07	0	0	0	0
1	6	15	6	0	-1.777E 08	0	0	0	0

TABLE 3B. STIFFNESS AND LOAD DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECT.

FROM		THRU				TX	TY	PX	PY
1	0	16	12	0	0	0	0	1.200E 05	0

TABLE 4. MONITOR TALLY AND DEFLS AT 4 STAS

	ITR NUM	FICT SPRING	CYC NUM	NOT STAB	NOT CLOS	I,J 7 0	7 6	15 8	15 11
X	1	8.150E 04	1	182		2.607E-03	4.640E-03	1.207E-02	1.223E-02
Y		4.773E 04		182	182	8.670E-03	9.540E-03	1.513E-02	1.716E-02
X	2	9.384E 05	2	182		9.592E-03	1.073E-02	1.283E-02	1.414E-02
Y		9.384E 05		182	179	1.039E-02	1.161E-02	9.335E-03	1.085E-02
X	3	4.939E 06	3	180		1.039E-02	1.156E-02	1.037E-02	1.181E-02
Y		4.939E 06		168	172	1.047E-02	1.164E-02	1.081E-02	1.210E-02
X	4	9.199E 06	4	177		1.052E-02	1.168E-02	1.071E-02	1.203E-02
Y		9.817E 06		168	140	1.057E-02	1.173E-02	1.070E-02	1.205E-02
X	5	1.439E 07	5	156		1.060E-02	1.176E-02	1.069E-02	1.204E-02
Y		1.457E 07		160	114	1.063E-02	1.179E-02	1.069E-02	1.204E-02
X	6	8.150E 04	1	178		1.229E-02	1.316E-02	1.053E-02	1.187E-02
Y		4.773E 04		182	182	1.606E-02	1.656E-02	1.024E-02	1.179E-02
X	7	9.384E 05	2	182		1.491E-02	1.539E-02	1.029E-02	1.178E-02
Y		9.384E 05		179	173	1.493E-02	1.531E-02	1.030E-02	1.177E-02
X	8	4.939E 06	3	175		1.492E-02	1.529E-02	1.032E-02	1.179E-02
Y		4.939E 06		165	138	1.490E-02	1.525E-02	1.033E-02	1.181E-02
X	9	9.199E 06	4	142		1.490E-02	1.525E-02	1.034E-02	1.182E-02
Y		9.817E 06		95	53	1.489E-02	1.523E-02	1.035E-02	1.183E-02
X	10	1.439E 07	5	80		1.489E-02	1.523E-02	1.035E-02	1.183E-02
Y		1.457E 07		60	30	1.489E-02	1.523E-02	1.035E-02	1.184E-02
X	11	8.150E 04	1	182		1.537E-02	1.566E-02	1.046E-02	1.201E-02
Y		4.773E 04		175	182	1.521E-02	1.547E-02	1.064E-02	1.226E-02
X	12	9.384E 05	2	170		1.593E-02	1.625E-02	1.058E-02	1.218E-02
Y		9.384E 05		172	152	1.621E-02	1.656E-02	1.055E-02	1.215E-02

X	13	4.939E 06	3	156		1.620E-02	1.651E-02	1.055E-02	1.216E-02
Y		4.939E 06		101	69	1.623E-02	1.652E-02	1.055E-02	1.216E-02
X	14	9.199E 06	4	80		1.624E-02	1.653E-02	1.055E-02	1.216E-02
Y		9.817E 06		48	16	1.625E-02	1.654E-02	1.055E-02	1.216E-02
X	15	1.439E 07	5	46		1.626E-02	1.655E-02	1.055E-02	1.216E-02
Y		1.457E 07		41	2	1.627E-02	1.655E-02	1.055E-02	1.216E-02
X	16	8.150E 04	1	157		1.667E-02	1.664E-02	1.055E-02	1.220E-02
Y		4.773E 04		171	156	1.742E-02	1.738E-02	1.054E-02	1.221E-02
X	17	9.384E 05	2	152		1.704E-02	1.696E-02	1.055E-02	1.221E-02
Y		9.384E 05		128	88	1.702E-02	1.690E-02	1.055E-02	1.222E-02
X	18	4.939E 06	3	86		1.702E-02	1.689E-02	1.055E-02	1.221E-02
Y		4.939E 06		90	67	1.701E-02	1.687E-02	1.055E-02	1.221E-02
X	19	9.199E 06	4	61		1.700E-02	1.687E-02	1.055E-02	1.221E-02
Y		9.817E 06		34	0	1.700E-02	1.686E-02	1.055E-02	1.221E-02

```
PROGRAM SLAB 17 - MASTER DECK - WR HUDSON, H MATLOCK     REVISION DATE 26 JUL 65
CE051022   HWY SLAB PROJECT        SLAB 17     W R HUDSON
RUN EXAMPLE PROBLEMS FOR USE IN SLAB REPORT APPENDIX D


PROB (CONTD)
 601      EXAMPLE PROBLEM   BRIDGE APPROACH SLAB                   WRH       10JUN65
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TABLE 5. RESULTS -- ITERATION 19

I,J	X-DEFL	Y-DEFL	BMX	BMY	REACT	TRERR
-1 -1	-1.761E-97	-1.761E-97	0	0	0	0
0 -1	0	0	0	0	0	0
1 -1	-6.445E-97	-6.394E-97	0	0	0	0
2 -1	-8.821E-04	-8.887E-04	0	0	0	0
3 -1	3.240E-03	3.271E-03	0	0	0	0
4 -1	8.518E-03	8.511E-03	0	0	0	0
5 -1	1.293E-02	1.295E-02	0	0	0	0
6 -1	1.574E-02	1.574E-02	0	0	0	0
7 -1	1.709E-02	1.708E-02	0	0	0	0
8 -1	1.441E-02	1.440E-02	0	0	0	0
9 -1	1.241E-02	1.242E-02	0	0	0	0
10 -1	1.119E-02	1.119E-02	0	0	0	0
11 -1	1.048E-02	1.048E-02	0	0	0	0
12 -1	9.877E-03	9.879E-03	0	0	0	0
13 -1	8.936E-03	8.937E-03	0	0	0	0
14 -1	7.228E-03	7.228E-03	0	0	0	0
15 -1	4.465E-03	4.464E-03	0	0	0	0
16 -1	7.011E-04	7.013E-04	0	0	0	0
17 -1	-2.884E-03	-2.884E-03	0	0	0	0
-1 0	1.029E-96	1.029E-96	0	0	0	0
0 0	1.500E-97	1.500E-97	2.103-102	5.257-103	-8.105E-92	8.105E-92
1 0	-7.293E-97	-7.293E-97	9.006E-91	-1.429-101	1.029E 03	5.105E-01
2 0	1.503E-96	1.487E-96	1.029E 03	-7.451E-09	-1.189E 03	-6.454E 00
3 0	3.555E-03	3.561E-03	3.351E 02	-1.490E-08	2.421E 02	5.791E 01
4 0	8.268E-03	8.272E-03	-1.580E 02	3.446E-08	2.604E 02	3.960E 01
5 0	1.243E-02	1.244E-02	-3.792E 02	-3.353E-08	2.457E 02	5.433E 01
6 0	1.529E-02	1.529E-02	-3.306E 02	-3.353E-08	2.651E 02	3.490E 01
7 0	1.700E-02	1.700E-02	-6.714E-01	-2.719E-07	-1.454E 02	-4.427E 01
8 0	1.459E-02	1.459E-02	1.492E 02	-1.770E-08	-1.159E 02	-4.370E 00
9 0	1.270E-02	1.270E-02	1.802E 02	-1.062E-07	-9.247E 01	2.672E 01
10 0	1.143E-02	1.143E-02	1.258E 02	0	-5.130E 01	2.222E 01
11 0	1.059E-02	1.059E-02	2.363E 01	8.754E-08	-1.482E 01	9.896E 00
12 0	9.831E-03	9.832E-03	-9.357E 01	1.579E-07	9.913E 00	6.948E 00
13 0	8.751E-03	8.751E-03	-2.011E 02	9.686E-08	4.428E 01	3.685E 00
14 0	6.976E-03	6.976E-03	-2.635E 02	6.892E-08	9.884E 01	2.379E-01
15 0	4.291E-03	4.291E-03	-2.243E 02	6.612E-08	1.786E 02	-2.135E 00
16 0	8.306E-04	8.306E-04	-7.677E-09	2.193E-09	-3.606E 02	3.007E-01
17 0	-2.630E-03	-2.630E-03	0	0	0	0
-1 1	2.235E-96	2.235E-96	0	0	0	0
0 1	3.000E-97	3.000E-97	2.054-101	-4.341E-92	6.404E-90	-6.404E-90
1 1	-1.597E-96	-1.597E-96	5.354E-90	1.848E-90	2.197E 03	5.960E-08
2 1	4.958E-96	4.948E-96	2.197E 03	5.493E 02	-4.344E 03	-9.304E 00
3 1	3.559E-03	3.562E-03	6.415E 02	1.480E 02	5.686E 02	3.135E 01
4 1	8.162E-03	8.169E-03	-3.057E 02	-4.394E 01	5.317E 02	6.831E 01
5 1	1.226E-02	1.226E-02	-7.351E 02	-1.173E 02	5.470E 02	5.298E 01

6	1	1.513E-02	1.514E-02	-6.365E 02	-9.985E 01	5.500E 02	5.004E 01
7	1	1.695E-02	1.694E-02	-1.320E 00	3.165E 00	-3.317E 02	-4.443E 01
8	1	1.465E-02	1.465E-02	2.825E 02	5.241E 01	-2.343E 02	-9.728E 00
9	1	1.282E-02	1.282E-02	3.375E 02	5.285E 01	-1.506E 02	1.195E 01
10	1	1.155E-02	1.155E-02	2.370E 02	3.755E 01	-8.588E 01	2.038E 01
11	1	1.068E-02	1.068E-02	4.201E 01	1.508E 01	-2.286E 01	7.912E 00
12	1	9.865E-03	9.865E-03	-1.845E 02	-9.469E 00	2.693E 01	4.841E 00
13	1	8.739E-03	8.740E-03	-3.916E 02	-3.532E 01	9.337E 01	3.240E 00
14	1	6.952E-03	6.952E-03	-5.094E 02	-5.970E 01	2.000E 02	-4.598E-01
15	1	4.311E-03	4.311E-03	-4.285E 02	-7.045E 01	3.535E 02	-1.779E 00
16	1	9.599E-04	9.599E-04	3.210E-03	-1.795E 01	-3.035E 02	-1.100E-01
17	1	-2.376E-03	-2.376E-03	0	0	0	0
-1	2	2.185E-96	2.185E-96	0	0	0	0
0	2	3.000E-97	3.000E-97	1.682-101	4.206-102	4.784E-90	-4.784E-90
1	2	-1.585E-96	-1.585E-96	4.825E-90	1.202E-90	2.185E 03	5.960E-08
2	2	4.346E-96	4.335E-96	2.185E 03	5.463E 02	-3.734E 03	-6.399E 00
3	2	3.540E-03	3.541E-03	6.409E 02	1.676E 02	5.841E 02	1.586E 01
4	2	8.114E-03	8.123E-03	-3.013E 02	-5.702E 01	5.128E 02	8.722E 01
5	2	1.219E-02	1.220E-02	-7.321E 02	-1.568E 02	5.446E 02	5.545E 01
6	2	1.507E-02	1.508E-02	-6.311E 02	-1.353E 02	5.388E 02	6.125E 01
7	2	1.692E-02	1.692E-02	-1.365E 00	-5.559E 00	-3.252E 02	-4.946E 01
8	2	1.468E-02	1.468E-02	2.786E 02	6.688E 01	-2.307E 02	-1.497E 01
9	2	1.289E-02	1.289E-02	3.309E 02	7.577E 01	-1.453E 02	2.687E 00
10	2	1.164E-02	1.164E-02	2.347E 02	5.950E 01	-8.962E 01	1.895E 01
11	2	1.077E-02	1.077E-02	4.244E 01	2.996E 01	-2.639E 01	5.899E 00
12	2	9.962E-03	9.962E-03	-1.840E 02	-3.211E 00	2.334E 01	2.829E 00
13	2	8.836E-03	8.836E-03	-3.933E 02	-3.644E 01	8.854E 01	2.526E 00
14	2	7.045E-03	7.045E-03	-5.141E 02	-6.476E 01	1.958E 02	-1.581E 00
15	2	4.395E-03	4.394E-03	-4.329E 02	-7.193E 01	3.488E 02	-1.960E 00
16	2	1.027E-03	1.027E-03	7.101E-03	-1.028E 01	-3.453E 02	-5.749E-01
17	2	-2.331E-03	-2.331E-03	0	0	0	0
-1	3	2.180E-96	2.180E-96	0	0	0	0
0	3	3.000E-97	3.000E-97	1.682-101	4.206-102	4.815E-90	-4.815E-90
1	3	-1.580E-96	-1.580E-96	4.812E-90	1.199E-90	2.180E 03	0
2	3	4.335E-96	4.325E-96	2.180E 03	5.451E 02	-3.725E 03	-4.782E 00
3	3	3.532E-03	3.533E-03	6.410E 02	1.666E 02	5.862E 02	1.385E 01
4	3	8.099E-03	8.108E-03	-3.009E 02	-6.456E 01	5.091E 02	9.094E 01
5	3	1.217E-02	1.218E-02	-7.335E 02	-1.717E 02	5.449E 02	5.508E 01
6	3	1.506E-02	1.506E-02	-6.309E 02	-1.494E 02	5.353E 02	6.468E 01
7	3	1.691E-02	1.691E-02	-1.404E 00	-1.367E 01	-3.171E 02	-5.690E 01
8	3	1.471E-02	1.470E-02	2.788E 02	6.894E 01	-2.250E 02	-2.204E 01
9	3	1.295E-02	1.295E-02	3.334E 02	8.499E 01	-1.435E 02	-2.373E 00
10	3	1.173E-02	1.174E-02	2.407E 02	7.279E 01	-9.372E 01	1.780E 01
11	3	1.090E-02	1.090E-02	4.787E 01	4.511E 01	-3.224E 01	4.277E 00
12	3	1.013E-02	1.013E-02	-1.850E 02	1.453E 01	1.502E 01	1.299E 00
13	3	9.039E-03	9.039E-03	-4.062E 02	-1.326E 01	7.767E 01	1.689E 00
14	3	7.248E-03	7.248E-03	-5.389E 02	-3.337E 01	1.849E 02	-2.379E 00
15	3	4.541E-03	4.541E-03	-4.565E 02	-3.279E 01	3.409E 02	-2.430E 00
16	3	1.059E-03	1.059E-03	4.557E-03	1.078E 01	-3.653E 02	-5.908E-01
17	3	-2.432E-03	-2.432E-03	0	0	0	0
-1	4	2.182E-96	2.182E-96	0	0	0	0
0	4	3.000E-97	3.000E-97	1.682-101	4.206-102	4.828E-90	-4.828E-90
1	4	-1.582E-96	-1.582E-96	4.822E-90	1.204E-90	2.182E 03	0
2	4	4.347E-96	4.338E-96	2.182E 03	5.456E 02	-3.738E 03	-4.111E 00
3	4	3.535E-03	3.536E-03	6.386E 02	1.628E 02	5.893E 02	1.069E 01
4	4	8.103E-03	8.112E-03	-3.034E 02	-6.915E 01	5.128E 02	8.717E 01
5	4	1.218E-02	1.218E-02	-7.364E 02	-1.760E 02	5.489E 02	5.106E 01
6	4	1.505E-02	1.506E-02	-6.327E 02	-1.538E 02	5.383E 02	6.171E 01

7	4	1.691E-02	1.690E-02	-1.422E 00	-1.862E 01	-3.104E 02	-6.313E 01
8	4	1.473E-02	1.473E-02	2.824E 02	6.684E 01	-2.217E 02	-2.659E 01
9	4	1.301E-02	1.301E-02	3.428E 02	8.690E 01	-1.440E 02	-5.317E 00
10	4	1.185E-02	1.185E-02	2.545E 02	7.932E 01	-9.923E 01	1.681E 01
11	4	1.109E-02	1.109E-02	5.962E 01	5.694E 01	-4.219E 01	3.457E 00
12	4	1.041E-02	1.041E-02	-1.857E 02	3.413E 01	-2.667E-01	6.782E-01
13	4	9.395E-03	9.395E-03	-4.313E 02	1.878E 01	5.764E 01	1.233E 00
14	4	7.626E-03	7.626E-03	-5.923E 02	1.615E 01	1.637E 02	-2.921E 00
15	4	4.828E-03	4.827E-03	-5.133E 02	3.284E 01	3.246E 02	-2.654E 00
16	4	1.128E-03	1.128E-03	-2.003E-03	4.482E 01	-4.090E 02	-3.467E-01
17	4	-2.610E-03	-2.610E-03	0	0	0	0
-1	5	2.188E-96	2.188E-96	0	0	0	0
0	5	3.000E-97	3.000E-97	1.682-101	4.206-102	5.039E-90	-5.039E-90
1	5	-1.588E-96	-1.588E-96	5.021E-90	1.261E-90	2.188E 03	-5.960E-08
2	5	4.656E-96	4.649E-96	2.188E 03	5.469E 02	-4.047E 03	-5.473E 00
3	5	3.544E-03	3.545E-03	6.345E 02	1.508E 02	5.897E 02	1.027E 01
4	5	8.119E-03	8.127E-03	-3.067E 02	-6.896E 01	5.161E 02	8.385E 01
5	5	1.219E-02	1.220E-02	-7.408E 02	-1.661E 02	5.547E 02	4.526E 01
6	5	1.506E-02	1.507E-02	-6.361E 02	-1.451E 02	5.450E 02	5.498E 01
7	5	1.689E-02	1.688E-02	-1.410E 00	-1.950E 01	-3.048E 02	-6.789E 01
8	5	1.474E-02	1.474E-02	2.901E 02	6.047E 01	-2.203E 02	-2.886E 01
9	5	1.307E-02	1.307E-02	3.597E 02	8.099E 01	-1.462E 02	-6.772E 00
10	5	1.199E-02	1.199E-02	2.777E 02	7.634E 01	-1.066E 02	1.614E 01
11	5	1.135E-02	1.135E-02	8.085E 01	5.788E 01	-5.689E 01	3.207E 00
12	5	1.082E-02	1.082E-02	-1.822E 02	4.002E 01	-2.397E 01	4.626E-01
13	5	9.969E-03	9.969E-03	-4.692E 02	3.363E 01	2.503E 01	7.531E-01
14	5	8.288E-03	8.288E-03	-6.906E 02	4.923E 01	1.261E 02	-3.485E 00
15	5	5.393E-03	5.393E-03	-6.396E 02	9.381E 01	2.921E 02	-2.754E 00
16	5	1.352E-03	1.352E-03	-2.930E-05	8.726E 01	-5.499E 02	-1.340E-01
17	5	-2.764E-03	-2.764E-03	0	0	0	0
-1	6	2.184E-96	2.184E-96	0	0	0	0
0	6	3.000E-97	3.000E-97	1.682-101	2.974-102	4.490E-90	-4.490E-90
1	6	-1.584E-96	-1.584E-96	4.481E-90	5.604E-91	2.184E 03	5.960E-08
2	6	3.790E-96	3.776E-96	2.184E 03	2.731E 02	-3.178E 03	-4.755E 00
3	6	3.537E-03	3.540E-03	6.625E 02	8.259E 01	5.817E 02	1.827E 01
4	6	8.148E-03	8.156E-03	-3.261E 02	-5.374E 01	5.264E 02	7.356E 01
5	6	1.224E-02	1.225E-02	-7.818E 02	-1.205E 02	5.516E 02	4.838E 01
6	6	1.509E-02	1.509E-02	-6.712E 02	-1.056E 02	5.449E 02	5.509E 01
7	6	1.687E-02	1.686E-02	-1.505E 00	-1.964E 01	-2.997E 02	-7.170E 01
8	6	1.474E-02	1.474E-02	3.152E 02	4.179E 01	-2.172E 02	-3.168E 01
9	6	1.312E-02	1.312E-02	4.024E 02	6.263E 01	-1.508E 02	-4.745E 00
10	6	1.214E-02	1.214E-02	3.272E 02	6.006E 01	-1.147E 02	1.548E 01
11	6	1.167E-02	1.167E-02	1.251E 02	4.134E 01	-7.485E 01	2.472E 00
12	6	1.139E-02	1.139E-02	-1.645E 02	1.628E 01	-5.628E 01	3.434E-01
13	6	1.080E-02	1.080E-02	-5.161E 02	-5.158E 00	-2.280E 01	4.681E-01
14	6	9.333E-03	9.333E-03	-8.522E 02	-7.479E 00	6.643E 01	-4.017E 00
15	6	6.397E-03	6.396E-03	-9.007E 02	3.024E 01	2.350E 02	-3.443E 00
16	6	1.877E-03	1.877E-03	-6.025E-03	1.128E 02	-8.803E 02	-1.710E-01
17	6	-2.780E-03	-2.780E-03	0	0	0	0
-1	7	2.180E-96	2.180E-96	0	0	0	0
0	7	3.000E-97	3.000E-97	1.682-101	4.206-102	5.021E-90	-5.021E-90
1	7	-1.580E-96	-1.580E-96	5.004E-90	1.256E-90	2.180E 03	0
2	7	4.644E-96	4.634E-96	2.180E 03	5.450E 02	-4.033E 03	-5.604E 00
3	7	3.531E-03	3.533E-03	6.309E 02	1.504E 02	5.826E 02	1.738E 01
4	7	8.087E-03	8.095E-03	-3.114E 02	-6.880E 01	5.206E 02	7.942E 01
5	7	1.213E-02	1.214E-02	-7.430E 02	-1.643E 02	5.520E 02	4.803E 01
6	7	1.497E-02	1.497E-02	-6.377E 02	-1.415E 02	5.442E 02	5.576E 01
7	7	1.676E-02	1.676E-02	-1.317E 00	-1.303E 01	-2.987E 02	-6.665E 01

8	7	1.475E-02	1.475E-02	3.139E 02	6.397E 01	-2.211E 02	-2.838E 01
9	7	1.325E-02	1.325E-02	4.095E 02	7.938E 01	-1.572E 02	-5.959E 00
10	7	1.242E-02	1.242E-02	3.499E 02	6.036E 01	-1.294E 02	1.378E 01
11	7	1.218E-02	1.218E-02	1.631E 02	8.802E 00	-1.028E 02	1.517E 00
12	7	1.221E-02	1.221E-02	-1.266E 02	-7.591E 01	-1.036E 02	5.681E-01
13	7	1.205E-02	1.205E-02	-5.300E 02	-1.965E 02	-9.500E 01	9.449E-01
14	7	1.106E-02	1.106E-02	-1.057E 03	-3.187E 02	-3.344E 01	-3.758E 00
15	7	8.387E-03	8.386E-03	-1.534E 03	-2.842E 02	1.202E 02	-3.254E 00
16	7	3.183E-03	3.183E-03	-2.155E-03	-9.950E 00	-1.701E 03	-5.498E-01
17	7	-2.013E-03	-2.013E-03	0	0	0	0
-1	8	2.168E-96	2.168E-96	0	0	0	0
0	8	3.000E-97	3.000E-97	8.412-102	2.103-102	4.797E-90	-4.797E-90
1	8	-1.568E-96	-1.568E-96	4.791E-90	1.196E-90	2.168E 03	0
2	8	4.326E-96	4.317E-96	2.168E 03	5.420E 02	-3.717E 03	-4.613E 00
3	8	3.511E-03	3.513E-03	6.296E 02	1.611E 02	5.850E 02	1.496E 01
4	8	8.041E-03	8.050E-03	-3.112E 02	-6.977E 01	5.158E 02	8.416E 01
5	8	1.206E-02	1.207E-02	-7.414E 02	-1.737E 02	5.491E 02	5.089E 01
6	8	1.488E-02	1.489E-02	-6.366E 02	-1.478E 02	5.408E 02	5.923E 01
7	8	1.666E-02	1.665E-02	-1.241E 00	-5.825E 00	-3.003E 02	-5.915E 01
8	8	1.473E-02	1.473E-02	3.272E 02	7.636E 01	-2.266E 02	-2.206E 01
9	8	1.334E-02	1.334E-02	4.353E 02	9.036E 01	-1.643E 02	-4.100E 00
10	8	1.266E-02	1.266E-02	3.873E 02	6.256E 01	-1.425E 02	1.321E 01
11	8	1.262E-02	1.262E-02	2.073E 02	-1.026E 01	-1.273E 02	2.867E-01
12	8	1.295E-02	1.295E-02	-8.691E 01	-1.469E 02	-1.461E 02	4.051E-01
13	8	1.318E-02	1.318E-02	-5.213E 02	-4.102E 02	-1.608E 02	1.368E 00
14	8	1.270E-02	1.270E-02	-1.201E 03	-9.532E 02	-1.281E 02	-3.279E 00
15	8	1.055E-02	1.055E-02	-2.799E 03	-2.065E 03	9.996E 03	-3.145E 00
16	8	4.453E-03	4.453E-03	4.517E-03	-4.641E 02	-2.499E 03	-1.191E 00
17	8	-1.240E-03	-1.240E-03	0	0	0	0
-1	9	2.160E-96	2.160E-96	0	0	0	0
0	9	3.000E-97	3.000E-97	8.412-102	2.103-102	4.772E-90	-4.772E-90
1	9	-1.560E-96	-1.560E-96	4.769E-90	1.188E-90	2.160E 03	0
2	9	4.307E-96	4.298E-96	2.160E 03	5.399E 02	-3.698E 03	-4.206E 00
3	9	3.498E-03	3.500E-03	6.272E 02	1.631E 02	5.864E 02	1.357E 01
4	9	8.010E-03	8.018E-03	-3.112E 02	-6.635E 01	5.154E 02	8.464E 01
5	9	1.201E-02	1.202E-02	-7.397E 02	-1.694E 02	5.445E 02	5.554E 01
6	9	1.481E-02	1.482E-02	-6.367E 02	-1.426E 02	5.405E 02	5.948E 01
7	9	1.657E-02	1.656E-02	-1.158E 00	3.355E 00	-3.010E 02	-5.326E 01
8	9	1.471E-02	1.471E-02	3.393E 02	8.565E 01	-2.325E 02	-1.473E 01
9	9	1.340E-02	1.340E-02	4.585E 02	1.010E 02	-1.745E 02	2.649E 00
10	9	1.284E-02	1.284E-02	4.157E 02	7.661E 01	-1.559E 02	1.642E 01
11	9	1.296E-02	1.296E-02	2.275E 02	1.593E 01	-1.470E 02	3.908E-01
12	9	1.347E-02	1.347E-02	-1.038E 02	-7.775E 01	-1.762E 02	2.632E-01
13	9	1.383E-02	1.384E-02	-6.232E 02	-1.933E 02	-1.986E 02	1.697E 00
14	9	1.321E-02	1.321E-02	-1.353E 03	-2.632E 02	-1.582E 02	-2.482E 00
15	9	1.035E-02	1.035E-02	-1.981E 03	-1.061E 02	6.629E 00	-2.861E 00
16	9	4.121E-03	4.121E-03	1.486E-02	1.118E 02	-2.290E 03	-1.512E 00
17	9	-2.207E-03	-2.207E-03	0	0	0	0
-1	10	2.158E-96	2.158E-96	0	0	0	0
0	10	3.000E-97	3.000E-97	8.412-102	2.103-102	4.725E-90	-4.725E-90
1	10	-1.558E-96	-1.558E-96	4.766E-90	1.187E-90	2.158E 03	0
2	10	4.306E-96	4.298E-96	2.158E 03	5.395E 02	-3.696E 03	-6.138E 00
3	10	3.496E-03	3.497E-03	6.238E 02	1.639E 02	5.830E 02	1.703E 01
4	10	7.998E-03	8.007E-03	-3.112E 02	-5.796E 01	5.121E 02	8.786E 01
5	10	1.199E-02	1.199E-02	-7.371E 02	-1.544E 02	5.369E 02	6.314E 01
6	10	1.477E-02	1.478E-02	-6.386E 02	-1.301E 02	5.537E 02	4.633E 01
7	10	1.651E-02	1.650E-02	-1.096E 00	1.031E 01	-2.967E 02	-5.396E 01
8	10	1.469E-02	1.469E-02	3.494E 02	8.612E 01	-2.351E 02	-1.090E 01

9	10	1.343E-02	1.344E-02	4.802E 02	1.015E 02	-1.854E 02	1.158E 01
10	10	1.297E-02	1.297E-02	4.424E 02	8.328E 01	-1.711E 02	2.411E 01
11	10	1.323E-02	1.323E-02	2.490E 02	3.571E 01	-1.667E 02	4.692E 00
12	10	1.391E-02	1.391E-02	-1.028E 02	-3.772E 01	-2.034E 02	2.386E 00
13	10	1.442E-02	1.442E-02	-6.645E 02	-1.263E 02	-2.334E 02	2.704E 00
14	10	1.384E-02	1.384E-02	-1.453E 03	-1.674E 02	-1.959E 02	-1.494E 00
15	10	1.083E-02	1.083E-02	-2.104E 03	1.765E 00	-2.185E 01	-1.766E 00
16	10	4.174E-03	4.174E-03	1.665E-02	1.450E 02	-2.324E 03	-8.299E-01
17	10	-2.604E-03	-2.604E-03	0	0	0	0
-1	11	2.201E-96	2.201E-96	0	0	0	0
0	11	3.000E-97	3.000E-97	2.054-101	-4.341E-92	6.296E-90	-6.296E-90
1	11	-1.564E-96	-1.564E-96	5.267E-90	1.813E-90	2.164E 03	0
2	11	4.889E-96	4.883E-96	2.164E 03	5.410E 02	-4.282E 03	-3.862E 00
3	11	3.505E-03	3.508E-03	6.221E 02	1.449E 02	5.629E 02	3.713E 01
4	11	8.022E-03	8.030E-03	-3.145E 02	-4.399E 01	5.278E 02	7.223E 01
5	11	1.202E-02	1.202E-02	-7.390E 02	-1.164E 02	5.416E 02	5.841E 01
6	11	1.478E-02	1.479E-02	-6.421E 02	-9.693E 01	5.616E 02	3.840E 01
7	11	1.648E-02	1.648E-02	-1.063E 00	1.371E 01	-2.985E 02	-5.063E 01
8	11	1.466E-02	1.466E-02	3.587E 02	6.593E 01	-2.274E 02	-1.718E 01
9	11	1.344E-02	1.344E-02	5.042E 02	7.502E 01	-1.939E 02	1.981E 01
10	11	1.305E-02	1.305E-02	4.717E 02	6.304E 01	-1.821E 02	3.030E 01
11	11	1.345E-02	1.345E-02	2.774E 02	2.711E 01	-1.864E 02	1.157E 01
12	11	1.432E-02	1.432E-02	-7.797E 01	-5.185E 01	-2.329E 02	8.035E 00
13	11	1.508E-02	1.508E-02	-6.462E 02	-2.381E 02	-2.740E 02	5.629E 00
14	11	1.482E-02	1.482E-02	-1.533E 03	-6.983E 02	-2.548E 02	1.223E 00
15	11	1.221E-02	1.221E-02	-3.269E 03	-1.774E 03	9.897E 03	-5.475E-01
16	11	4.728E-03	4.728E-03	-1.322E-02	-3.883E 02	-2.674E 03	6.677E-01
17	11	-2.422E-03	-2.422E-03	0	0	0	0
-1	12	1.013E-96	1.013E-96	0	0	0	0
0	12	1.500E-97	1.500E-97	1.972-102	0	-8.479E-92	8.479E-92
1	12	-7.125E-97	-7.125E-97	8.816E-91	1.072-101	1.012E 03	6.095E-01
2	12	1.471E-96	1.471E-96	1.012E 03	0	-1.171E 03	-3.127E-01
3	12	3.497E-03	3.501E-03	3.220E 02	-1.676E-08	2.558E 02	4.418E 01
4	12	8.106E-03	8.112E-03	-1.600E 02	6.985E-08	2.446E 02	5.535E 01
5	12	1.216E-02	1.217E-02	-3.815E 02	-3.353E-08	2.470E 02	5.300E 01
6	12	1.490E-02	1.491E-02	-3.301E 02	7.078E-08	2.563E 02	4.375E 01
7	12	1.650E-02	1.649E-02	-5.693E-01	9.919E-08	-1.253E 02	-4.982E 01
8	12	1.460E-02	1.460E-02	1.897E 02	1.388E-07	-1.075E 02	-1.287E 01
9	12	1.335E-02	1.336E-02	2.694E 02	1.062E-07	-1.105E 02	2.593E 01
10	12	1.304E-02	1.304E-02	2.534E 02	-1.229E-07	-1.111E 02	3.550E 01
11	12	1.360E-02	1.360E-02	1.491E 02	3.446E-08	-1.099E 02	1.820E 01
12	12	1.468E-02	1.468E-02	-4.534E 01	-1.404E-07	-1.375E 02	1.474E 01
13	12	1.560E-02	1.560E-02	-3.674E 02	1.043E-07	-1.596E 02	1.031E 01
14	12	1.525E-02	1.525E-02	-8.543E 02	1.565E-07	-1.433E 02	4.161E 00
15	12	1.195E-02	1.195E-02	-1.362E 03	4.470E-08	-4.480E 01	7.003E-01
16	12	3.941E-03	3.941E-03	-4.387E-08	2.193E-08	-2.273E 03	1.420E 00
17	12	-4.066E-03	-4.066E-03	0	0	0	0
-1	13	-1.763E-97	-1.763E-97	0	0	0	0
0	13	0	0	0	0	0	0
1	13	-6.117E-97	-6.228E-97	0	0	0	0
2	13	-8.663E-04	-8.742E-04	0	0	0	0
3	13	3.184E-03	3.216E-03	0	0	0	0
4	13	8.347E-03	8.333E-03	0	0	0	0
5	13	1.262E-02	1.264E-02	0	0	0	0
6	13	1.530E-02	1.531E-02	0	0	0	0
7	13	1.654E-02	1.653E-02	0	0	0	0
8	13	1.438E-02	1.437E-02	0	0	0	0
9	13	1.302E-02	1.304E-02	0	0	0	0

10 13	1.281E-02	1.281E-02	0	0	0	0
11 13	1.362E-02	1.362E-02	0	0	0	0
12 13	1.507E-02	1.508E-02	0	0	0	0
13 13	1.644E-02	1.644E-02	0	0	0	0
14 13	1.642E-02	1.642E-02	0	0	0	0
15 13	1.286E-02	1.286E-02	0	0	0	0
16 13	3.153E-03	3.154E-03	0	0	0	0
17 13	-5.710E-03	-5.710E-03	0	0	0	0

PROB (CONTD)
601 EXAMPLE PROBLEM BRIDGE APPROACH SLAB WRH 10JUN65

TABLE 5. RESULTS(CONTD) -- ITERATION 19

I,J	TMX	TMY
-1 -1	0	0
0 -1	0	0
1 -1	0	0
2 -1	0	0
3 -1	0	0
4 -1	0	0
5 -1	0	0
6 -1	0	0
7 -1	0	0
8 -1	0	0
9 -1	0	0
10 -1	0	0
11 -1	0	0
12 -1	0	0
13 -1	0	0
14 -1	0	0
15 -1	0	0
16 -1	0	0
17 -1	0	0
-1 0	0	0
0 0	-9.733E-92	9.748E-92
1 0	5.105E 01	-5.144E 01
2 0	1.845E 01	-1.684E 01
3 0	-7.163E 01	7.120E 01
4 0	-5.728E 01	5.645E 01
5 0	-1.446E 01	1.508E 01
6 0	3.078E 01	-3.175E 01
7 0	4.937E 01	-4.918E 01
8 0	3.197E 01	-3.114E 01
9 0	6.790E 00	-6.671E 00
10 0	-1.243E 01	1.189E 01
11 0	-2.180E 01	2.178E 01
12 0	-2.291E 01	2.283E 01
13 0	-1.527E 01	1.520E 01
14 0	2.493E 00	-2.551E 00
15 0	3.092E 01	-3.094E 01
16 0	1.917E 01	-1.914E 01
17 0	0	0
-1 1	0	0

0	1	-1.164E-91	1.164E-91
1	1	3.117E-91	-3.124E-91
2	1	-1.767E 00	2.298E 00
3	1	-1.777E 01	1.723E 01
4	1	-2.622E 01	2.569E 01
5	1	-7.315E 00	7.542E 00
6	1	1.859E 01	-1.853E 01
7	1	3.530E 01	-3.488E 01
8	1	3.192E 01	-3.170E 01
9	1	1.492E 01	-1.499E 01
10	1	-1.126E 00	8.944E-01
11	1	-9.936E 00	9.947E 00
12	1	-1.162E 01	1.158E 01
13	1	-7.229E 00	7.202E 00
14	1	2.222E 00	-2.237E 00
15	1	1.480E 01	-1.481E 01
16	1	1.128E 01	-1.128E 01
17	1	0	0
-1	2	0	0
0	2	4.113E-93	-4.113E-93
1	2	-7.208E-92	7.210E-92
2	2	-3.147E 00	3.302E 00
3	2	-7.314E 00	7.071E 00
4	2	-6.329E 00	6.143E 00
5	2	-1.288E 00	1.352E 00
6	2	5.337E 00	-5.172E 00
7	2	1.468E 01	-1.434E 01
8	2	1.886E 01	-1.883E 01
9	2	1.488E 01	-1.501E 01
10	2	1.146E 01	-1.158E 01
11	2	1.013E 01	-1.012E 01
12	2	8.496E 00	-8.519E 00
13	2	3.168E 00	-3.189E 00
14	2	-8.072E 00	8.062E 00
15	2	-2.278E 01	2.276E 01
16	2	-1.657E 01	1.656E 01
17	2	0	0
-1	3	0	0
0	3	3.244E-94	-3.244E-94
1	3	4.722E-95	-3.059E-94
2	3	-5.255E-01	5.614E-01
3	3	-1.270E 00	1.277E 00
4	3	-1.291E 00	1.300E 00
5	3	-9.983E-01	9.854E-01
6	3	-3.076E-01	4.199E-01
7	3	7.620E 00	-7.472E 00
8	3	1.571E 01	-1.577E 01
9	3	1.829E 01	-1.840E 01
10	3	2.308E 01	-2.315E 01
11	3	2.814E 01	-2.814E 01
12	3	2.804E 01	-2.805E 01
13	3	1.548E 01	-1.549E 01
14	3	-1.458E 01	1.457E 01
15	3	-5.560E 01	5.558E 01
16	3	-4.122E 01	4.121E 01
17	3	0	0
-1	4	0	0
0	4	-8.562E-94	8.562E-94

1	4	3.714E-92	-3.749E-92
2	4	1.387E 00	-1.338E 00
3	4	2.266E 00	-2.193E 00
4	4	8.149E-01	-7.558E-01
5	4	-1.861E 00	1.908E 00
6	4	-4.848E 00	4.875E 00
7	4	3.978E 00	-4.024E 00
8	4	1.676E 01	-1.684E 01
9	4	2.491E 01	-2.496E 01
10	4	3.757E 01	-3.761E 01
11	4	5.074E 01	-5.075E 01
12	4	5.599E 01	-5.600E 01
13	4	4.035E 01	-4.035E 01
14	4	-9.058E 00	9.051E 00
15	4	-8.648E 01	8.646E 01
16	4	-6.854E 01	6.854E 01
17	4	0	0
-1	5	0	0
0	5	-1.715E-94	1.715E-94
1	5	-6.446E-92	6.500E-92
2	5	2.778E-01	-3.939E-01
3	5	5.235E 00	-5.072E 00
4	5	7.276E 00	-7.134E 00
5	5	-1.207E 00	1.119E 00
6	5	-1.189E 01	1.196E 01
7	5	-2.815E 00	2.808E 00
8	5	1.684E 01	-1.695E 01
9	5	3.260E 01	-3.265E 01
10	5	5.512E 01	-5.510E 01
11	5	7.944E 01	-7.945E 01
12	5	9.556E 01	-9.556E 01
13	5	8.436E 01	-8.435E 01
14	5	1.842E 01	-1.842E 01
15	5	-1.109E 02	1.109E 02
16	5	-1.006E 02	1.006E 02
17	5	0	0
-1	6	0	0
0	6	9.153E-94	-9.153E-94
1	6	-1.443E-93	1.737E-93
2	6	-1.483E 00	1.396E 00
3	6	-3.682E 00	3.737E 00
4	6	-5.361E 00	5.411E 00
5	6	-6.861E 00	6.794E 00
6	6	-7.891E 00	7.910E 00
7	6	1.116E 01	-1.115E 01
8	6	3.527E 01	-3.527E 01
9	6	4.988E 01	-4.984E 01
10	6	7.516E 01	-7.513E 01
11	6	1.093E 02	-1.093E 02
12	6	1.451E 02	-1.451E 02
13	6	1.614E 02	-1.614E 02
14	6	1.057E 02	-1.057E 02
15	6	-1.093E 02	1.093E 02
16	6	-1.298E 02	1.298E 02
17	6	0	0
-1	7	0	0
0	7	1.859E-93	-1.859E-93
1	7	6.209E-92	-6.254E-92

2	7	-3.011E 00	3.079E 00
3	7	-1.240E 01	1.227E 01
4	7	-1.768E 01	1.759E 01
5	7	-1.185E 01	1.194E 01
6	7	-3.351E 00	3.231E 00
7	7	2.375E 01	-2.383E 01
8	7	4.981E 01	-4.968E 01
9	7	6.087E 01	-6.072E 01
10	7	8.407E 01	-8.403E 01
11	7	1.200E 02	-1.201E 02
12	7	1.656E 02	-1.657E 02
13	7	2.091E 02	-2.091E 02
14	7	2.051E 02	-2.051E 02
15	7	-9.129E 01	9.131E 01
16	7	-1.512E 02	1.512E 02
17	7	0	0
-1	8	0	0
0	8	2.331E-93	-2.331E-93
1	8	-3.901E-92	3.884E-92
2	8	-3.776E 00	3.813E 00
3	8	-8.915E 00	8.855E 00
4	8	-1.034E 01	1.024E 01
5	8	-9.488E 00	9.503E 00
6	8	-8.317E 00	8.228E 00
7	8	1.387E 01	-1.399E 01
8	8	3.981E 01	-3.977E 01
9	8	5.255E 01	-5.243E 01
10	8	7.363E 01	-7.351E 01
11	8	9.850E 01	-9.846E 01
12	8	1.157E 02	-1.157E 02
13	8	1.015E 02	-1.015E 02
14	8	2.075E 01	-2.075E 01
15	8	-1.395E 02	1.395E 02
16	8	-1.249E 02	1.249E 02
17	8	0	0
-1	9	0	0
0	9	1.141E-93	-1.141E-93
1	9	-2.335E-93	2.198E-93
2	9	-1.848E 00	1.833E 00
3	9	-5.007E 00	4.961E 00
4	9	-6.916E 00	6.793E 00
5	9	-7.610E 00	7.814E 00
6	9	-8.999E 00	9.091E 00
7	9	7.194E 00	-7.483E 00
8	9	2.864E 01	-2.878E 01
9	9	4.103E 01	-4.103E 01
10	9	5.945E 01	-5.932E 01
11	9	7.549E 01	-7.538E 01
12	9	7.289E 01	-7.285E 01
13	9	2.142E 01	-2.142E 01
14	9	-1.110E 02	1.110E 02
15	9	-1.649E 02	1.649E 02
16	9	-9.505E 01	9.506E 01
17	9	0	0
-1	10	0	0
0	10	-2.652E-93	2.652E-93
1	10	6.745E-92	-6.769E-92
2	10	7.795E-01	-1.003E 00

3	10	1.449E 00	-1.297E 00
4	10	-4.440E-01	5.542E-01
5	10	-4.735E 00	4.852E 00
6	10	-1.060E 01	1.068E 01
7	10	-2.059E 00	1.820E 00
8	10	1.470E 01	-1.485E 01
9	10	2.999E 01	-3.019E 01
10	10	5.231E 01	-5.227E 01
11	10	7.362E 01	-7.355E 01
12	10	8.691E 01	-8.682E 01
13	10	8.842E 01	-8.837E 01
14	10	7.198E 01	-7.196E 01
15	10	-1.163E 02	1.164E 02
16	10	-1.203E 02	1.203E 02
17	10	0	0
-1	11	0	0
0	11	1.152E-91	-1.152E-91
1	11	-3.108E-91	3.098E-91
2	11	1.359E-01	-4.637E-01
3	11	1.252E 01	-1.219E 01
4	11	2.012E 01	-1.964E 01
5	11	2.567E 00	-2.877E 00
6	11	-2.119E 01	2.097E 01
7	11	-2.544E 01	2.544E 01
8	11	-8.523E 00	8.428E 00
9	11	1.859E 01	-1.874E 01
10	11	5.243E 01	-5.243E 01
11	11	8.110E 01	-8.112E 01
12	11	9.328E 01	-9.322E 01
13	11	7.343E 01	-7.336E 01
14	11	-6.489E 00	6.547E 00
15	11	-1.898E 02	1.898E 02
16	11	-1.495E 02	1.495E 02
17	11	0	0
-1	12	0	0
0	12	9.636E-92	-9.604E-92
1	12	-5.014E 01	5.059E 01
2	12	-1.857E 01	1.693E 01
3	12	6.895E 01	-6.815E 01
4	12	5.355E 01	-5.290E 01
5	12	1.123E 01	-1.284E 01
6	12	-3.137E 01	3.271E 01
7	12	-4.655E 01	4.745E 01
8	12	-2.766E 01	2.648E 01
9	12	2.317E 00	-3.069E 00
10	12	3.374E 01	-3.311E 01
11	12	5.771E 01	-5.775E 01
12	12	6.905E 01	-6.901E 01
13	12	4.900E 01	-4.885E 01
14	12	-4.146E 01	4.162E 01
15	12	-1.837E 02	1.838E 02
16	12	-1.138E 02	1.138E 02
17	12	0	0
-1	13	0	0
0	13	0	0
1	13	0	0
2	13	0	0
3	13	0	0

4	13	0	0
5	13	0	0
6	13	0	0
7	13	0	0
8	13	0	0
9	13	0	0
10	13	0	0
11	13	0	0
12	13	0	0
13	13	0	0
14	13	0	0
15	13	0	0
16	13	0	0
17	13	0	0

TIME = 3 MINUTES, 53 AND 15/60 SECONDS