A FINITE-ELEMENT METHOD FOR BENDING ANALYSIS OF LAYERED STRUCTURAL SYSTEMS

by

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Development of Methods for Computer Simulation
of Beam-Columns and Grid-Beam and Slab Systems

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PREFACE

This report is the fifth in a series of developments planned to enhance the use of computers in the analysis of highway bridge structures. It is concerned specifically with the bending analysis of a layered structural system, such as a grid overlying a series of parallel beams or a plate over parallel beams. A large group of structures, including portions of highway bridges, falls into this category. This report will provide research background for use in later programs which will consider more layers and other parameters such as shear interaction between layers.

This work is a part of Research Contract No. 3-5-63-56, entitled "Development of Methods for Computer Simulation of Beam-Columns and Grid-Beam and Slab Systems." It forms an extension to methods previously reported in Reports No. 56-1 and 56-4 and the method presented in Ref 29.

The computer programs included in this report are written in FORTRAN 63 language for the CDC 1604 or 6600 computers. With minor changes, the program would be compatible with the IBM 7090 or 360 systems. Duplicate copies of the program decks and test data cards for the example problems in this report may be obtained from the Center for Highway Research at The University of Texas.

The support of this work by the Texas Highway Department and the U. S. Bureau of Public Roads is acknowledged. Particular thanks are due to Mr. Larry G. Walker, who acted as project contact representative. The use of the computer and facilities of The University of Texas Computation Center has contributed significantly to this report.

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LIST OF REPORTS

Report No. 56-1, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns" by Hudson Matlock and T. Allan Haliburton, presents a finite-element solution for beam-columns that is a basic tool in subsequent reports.

Report No. 56-2, "A Computer Program to Analyze Bending of Bent Caps" by Hudson Matlock and Wayne B. Ingram, describes the application of the beam-column solution to the particular problem of bent caps.

Report No. 56-3, "A Finite-Element Method of Solution for Structural Frames" by Hudson Matlock and Berry Ray Grubbs, describes a solution for frames with no sway.

Report No. 56-4, "A Computer Program to Analyze Beam-Columns under Movable Loads" by Hudson Matlock and Thomas P. Taylor, describes the application of the beam-column solution to problems with any configuration of movable non-dynamic loads.

Report No. 56-5, "A Finite-Element Method for Bending Analysis of Layered Structural Systems" by Wayne B. Ingram and Hudson Matlock, describes an alternating-direction iteration method for solving two-dimensional systems of layered grids-over-beams and plates-over-beams.

Report No. 56-6, "Discontinuous Orthotropic Plates and Pavement Slabs" by W. Ronald Hudson and Hudson Matlock, describes an alternating-direction iteration method for solving complex two-dimensional plate and slab problems with emphasis on pavement slabs.

Report No. 56-7, "A Finite-Element Analysis of Structural Frames" by T. Allan Haliburton and Hudson Matlock, describes a method of analysis for rectangular plane frames with three degrees of freedom at each joint.

Report No. 56-8, "A Finite-Element Method for Transverse Vibrations of Beams and Plates" by Harold Salani and Hudson Matlock, describes an implicit procedure for determining the transient and steady-state vibrations of beams and plates, including pavement slabs.

Report No. 56-9, "A Direct Computer Solution for Plates and Pavement Slabs" by C. Fred Stelzer, Jr., and W. Ronald Hudson, describes a direct method for solving complex two-dimensional plate and slab problems.

Report No. 56-10, "A Finite-Element Method of Analysis for Composite Beams" by Thomas P. Taylor and Hudson Matlock, describes a method of analysis for composite beams with any degree of horizontal shear interaction.

ABSTRACT

An efficient numerical method of bending analysis for simply-connected grid-over-beam and orthotropic plate-over-beam layered structural systems is presented. The method relies upon the availability of a high-speed digital computer.

In the method, the real structure is replaced by a finite-element model of bars and springs. The equations for the two types of systems considered then are derived directly from the model to circumvent conventional assumptions of continuity. The model elements are further grouped into individual beams or line members for the three-phase iteration process. An alternating-direction relaxation technique is employed to coordinate the solution of the individual beams.

The alternating-direction relaxation technique uses certain closure parameters for convergence of solution. The proper parameters for each phase of iteration should not be based upon the particular beam being considered but upon the properties of the opposing beams.

Eight example problems demonstrate the applicability of the method to very diverse problems, including variable and discontinuous transverse and axial loads, bending stiffnesses, angular restraints, and support conditions.

TABLE OF CONTENTS

NOMENCLATURE

Symbol	Typical Units	Definition
A	-	A matrix
$A_i \cdots E_i$	-	Continuity Coefficients
$a_i \cdots n_i$	-	Deflection coefficients in finite-difference equations
a	-	A constant or a matrix of load terms
b	-	A constant
C	lb-in^2/in	Torsional stiffness
c_k	-	Constant
D , D′	lb-in^2/in	Plate flexural stiffness per unit width
E	lb/in^2	Modulus of elasticity
e, e_o, e_{km}	in.	Error in deflection
F	lb-in^2	Flexural stiffness
G	lb/in^2	Shear modulus
G_n	-	A matrix
H	-	Horizontal matrix operator
h	in.	Increment length
I	-	Identity matrix
i	-	Station number in x-direction
j	-	Station number in y-direction
K	lb/in	Spring constant
K_e	lb/in	Equivalent spring constant
k	-	A constant
L	in.	Length
M	in-lb -	Bending moment Number of increments

Symbol	Typical Units	Definition
M'	in-lb/in	Bending moment per unit width
m	-	Number of closure parameters
n	-	A constant = M - 1
P	lb	Axial tension (+) or compression (-)
P'	lb/in	Axial force per unit width
ΔP	lb	Change in axial force
Q	lb	Concentrated applied transverse load
q	lb/in	Applied transverse load per unit length
R	in-lb/rad	Concentrated rotational spring restraint
r	in-lb/in/rad	Rotational restraint per unit length
S	lb/in	Concentrated transverse spring restraint
$\overline{SF}$	lb/in	Fictitious closure spring
$\overline{SY}$	lb/in	Natural closure spring
s	lb/in^2	Transverse spring restraint per unit length
T	in-lb	Concentrated applied torque
T_i , T_n	-	A matrix
t	in-lb/in or in.	Applied torque per unit length Thickness of plate
V	lb or -	Shear or Vertical matrix operator
V'	lb/in	Shear per unit width
v	-	Eigenvector
w , $\bar{w}$	in.	Transverse deflection
x, y, z	-	Direction
z	in.	Distance from neutral axis of member
α, β	-	Angles used in derivation of closure equations
βn	degrees	An angle

Symbol	Typical Units	Definition
ϵ	in/in	Strain
θ	-	Central-difference slope
λ	-	An eigenvalue
μ	-	An eigenvalue
ν	-	An eigenvalue or
	-	Poisson's ratio
π	radian	The constant pi 3.14159
$\prod$	-	Infinite product
ρ	-	Closure parameter
σ	lb/in^2	Normal stress
ϕ	-	Simple-difference slope

CHAPTER 1. INTRODUCTION

This study concerns a rational numerical method for the bending analysis of layered structural systems. A layered structural system is a structure composed of members which overlay and are supported by other members which may in turn be supported by still other members. An example of a layered structure is the floor system of a highway bridge. A bridge generally is composed of a plate or slab which is supported by girders that are connected laterally by diaphragms and which rest on a support such as a bent cap. Thus, a bridge can be visualized as a three-deep system of structural elements: a series of bent caps, a gridwork composed of girders and diaphragms, and a plate or slab. Some other layered structures include slabs-on-foundations, building foundations, aircraft structures, ship hulls, and sign boards.

The analysis of such systems usually is simplified so that a single partial differential equation may be used for an approximate analysis. For instance, conventional solutions reduce the system to a simple gridwork or plate for which the equations are well known. These solutions at best contain restrictions which are undesirable. Closed-form solutions of differential equations are restricted to systems which have simple shapes and continuous or very nearly continuous functions for loading, support conditions, and bending stiffness. The numerical approximations to these differential equations are likewise limited. The previously stated example of a layered structural system does not necessarily conform to restrictions of continuity, and thus a better solution for such common systems is desirable.

The layered system is conducive to a difference-equation solution. However, when difference equations are used for the solution of such systems, large numbers of simultaneous algebraic equations may result. The solving of such large numbers of equations by classical methods is difficult if not impossible. Methods such as matrix inversion or relaxation usually are much too slow or require excessive amounts of computer storage capacity.

Presented herein is a numerical method for the analysis of layered structural systems which is efficient. The method contains almost complete generality regarding both transverse and in-plane forces, flexural stiffness, and support conditions.

The method is based on the following four items:

(1) replacement of the layered structural system by a finite-element model of each layer - only vertical connections are considered between layers;

(2) a difference-equation formulation of equations for solving individual beams on elastic foundations;

(3) an alternating-direction relaxation technique for coordination of individual beam solutions - three layers of beams are solved, one layer in each phase of solution; and

(4) a process whereby each individual beam solution considers the response of each opposing beam to obtain information needed for solution of the entire system.

The contributions of this study are in four areas. First, a concept of solving structures more realistically as layered systems, rather than approximating the behavior as that of a single grid or plate, is developed and presented herein. Second, a finite-element model concept is extended to solve the system. Third, the applicability of a relaxation method previously used for grids and simple isotropic plates is extended to the solution of layered systems, even those involving orthotropic plates. Fourth, the concept of substitution of loads and springs for each opposing member in the system is presented as a means of obtaining an efficient solution for very diverse systems.

Chapter 2 discusses previous methods of solution which are applicable to the layered system. Chapter 3 discusses the development of the equations for a simply-connected gridwork over a series of parallel beams. The method of selection of closure parameters for the alternating-direction technique is developed in Chapter 4. Chapter 5 shows the arrangement of the solution and discusses the computer program written to solve the equations developed in Chapter 2. Example problems of simply-connected grids over parallel beams are included in Chapter 6 to indicate the generality and capability of the method.

Chapter 7 indicates the differences between a simply-connected grid and a plate and develops the equations for an orthotropic-plate-over-beam system. Closure parameters for orthotropic-plate-over-beam systems and the computer program which solves the system are discussed in Chapter 8. Example solutions to problems of orthotropic plates over parallel beams are presented in Chapter 9.

Conclusions concerning the method of analysis are presented in Chapter 10 with recommended extensions of the method.

CHAPTER 2. AVAILABLE METHODS OF ANALYSIS

Numerous methods are available for the solution of the individual elements of the layered structural system. The elements consist of a simply-connected gridwork or a plate and a series of parallel beams. Several of these methods will be discussed in this chapter.

Solutions for Bending of Beams

The use of difference equations derived from a finite-element model furnishes one means of solution for the bending of beams which is readily adaptable to digital computer techniques. Such a method has been developed by Matlock (Ref 20) for the solution of beam-columns on elastic foundations. The details of this method have been given by Matlock and Ingram (Ref 23), Matlock and Haliburton (Ref 22) and others (Refs 10, 17). The essentials of this method of beam solution which are used in solving the layered system will be summarized in Chapter 5.

Solution Methods for the Grid

At the present time, iteration techniques appear to be the most efficient means of solving a grid system when the number of mesh points is large. Numerous relaxation methods are available which involve the judicious selection of initial estimates for unknown values and improving these estimates as the solution progresses from mesh point to mesh point in the selected region.

Over-relaxation methods are available which require the determination of relaxation parameters to speed up the solution. Unfortunately the expressions for the parameters are quite complicated and considerable time is required for their determination.

Successful alternating-direction iteration techniques have been presented by Conte and Dames (Ref 6) and Peaceman and Rachford (Ref 25). The alternating-direction scheme involves successive solutions of the ordinary differential equation for a beam. The solution of the grid would be accomplished by solving a system of beams in one direction and then a system of beams in the other direction with a coupling procedure at the points of intersection. Convergence of solution generally is assured and accelerated by appropriate selection of one or more closure parameters. These closure parameters will be discussed in detail in Chapter 4.

Tucker (Ref 29) applied the Conte and Dames procedure for the biharmonic equation to simply connected grids and simple isotropic plates with a good degree of success. Tucker gave a physical interpretation of the closure parameters but used the Conte and Dames procedure of cycling a parameter several times before selection of another for several cycles of solution.

Two unpublished methods (Ref 34) are available for the solution of gridworks. One is concerned with orthogonal systems while the second concerns a randomly-oriented gridwork of beams. Both of these methods consider only transverse loading, spring supports, and varying bending stiffness.

Approximate Methods for the System

Jenson (Ref 18) and Chen, Seiss, and Newmark (Ref 3) have presented specialized equations for skewed plate-beam systems. The equations involve the LaPlace equation as applied to a plate and the conventional fourth order derivative expression of a beam. By judiciously combining the effects of the beams into the plate equation, new equations were developed which represented solutions to the combined system. These equations were quite complicated and other equations were necessary for special considerations such as a free edge near a simply supported boundary.

Ang and Prescott (Ref 2) have given other difference equations for a plate-beam system which involve fictitious deflections in the vertical direction at each intersecting point. Schade (Ref 27), Hoppmann (Ref 14), Huffington (Ref 16), and others (Refs 15, 9) have considered the problem of orthogonally stiffened plates or variable thickness plates. The analysis of such systems involved the combination of the stiffnesses of the stiffening members into the stiffness of the plate and the solution of the simpler problem of an approximately orthotropic plate.

Chu and Krishnamoorthy (Ref 4) have studied the case of the orthotropic-plate bridge. Girders are included as stiffeners which furnish the orthotropic condition. Vitols, Clifton, and Au (Ref 30) and Clifton, Chang, and Au (Ref 5) also have considered the orthotropic-plate bridge but by a "more exact" orthotropic-plate theory involving a single and double sine series solution. "Torsionally soft" and "torsionally stiff" stiffeners are considered.

Summary

Several methods for solving the elements of the layered system have been discussed. There are several needed improvements to any of these methods for a solution of the layered structural system. The method necessarily must properly consider more than two intersecting beams at one common intersection. Torsional and Poisson's ratio effects of the plate must not affect the solution of the supporting beams. A more efficient closure process for the alternating-direction technique must be developed as more intersecting beams are considered. A random orientation of the supporting beams is desirable but not essential for this study.

CHAPTER 3. EQUATION DEVELOPMENT FOR A FINITE-ELEMENT MODEL OF THE GRID-OVER-BEAM SYSTEM

This chapter presents a discussion of very simple layered systems and the application of conventional differential equations to their solution. The equations for a three-beam-deep system are developed from a finite-element model and pertinent assumptions are noted.

Simple Systems

A very simple layered system supporting a downward and horizontal acting loading is shown in Fig 3.1. The system is composed of three sets of beams; the beams in two layers (A and C) are parallel while those in the third layer (B) are orthogonal to the first two. Layers A and B are two orthogonal sets of beams and represent a simply-connected gridwork. The third set of beams are parallel and support the grid. The term "simply-connected" means that each intersection of the beams can be visualized as a ball-and-socket connection. Transverse load is transferred by this connection but otherwise the beams are independent. The differential equation which governs the solution of a simply-connected grid, the first two layers of Fig 3.1, has the form of Eq 3.1. Timoshenko (Ref 28) uses a form of Eq 3.1 that includes beam torsion:

$$F \frac{\partial^4 w}{\partial x^4} + F \frac{\partial^4 w}{\partial y^4} = q \tag{3.1}$$

In Eq 3.1, w represents deflection transverse to the plane of the system, x and y signify two orthogonal directions, q is a uniformly distributed loading over the system, and F is the flexural stiffness (EI) which is temporarily considered to be a constant for all the beams of the grid. The term $F \frac{\partial^4 w}{\partial x^4}$ represents the load resisted by a beam in the x-direction and $F \frac{\partial^4 w}{\partial y^4}$ is the load resisted by a beam in the y-direction.

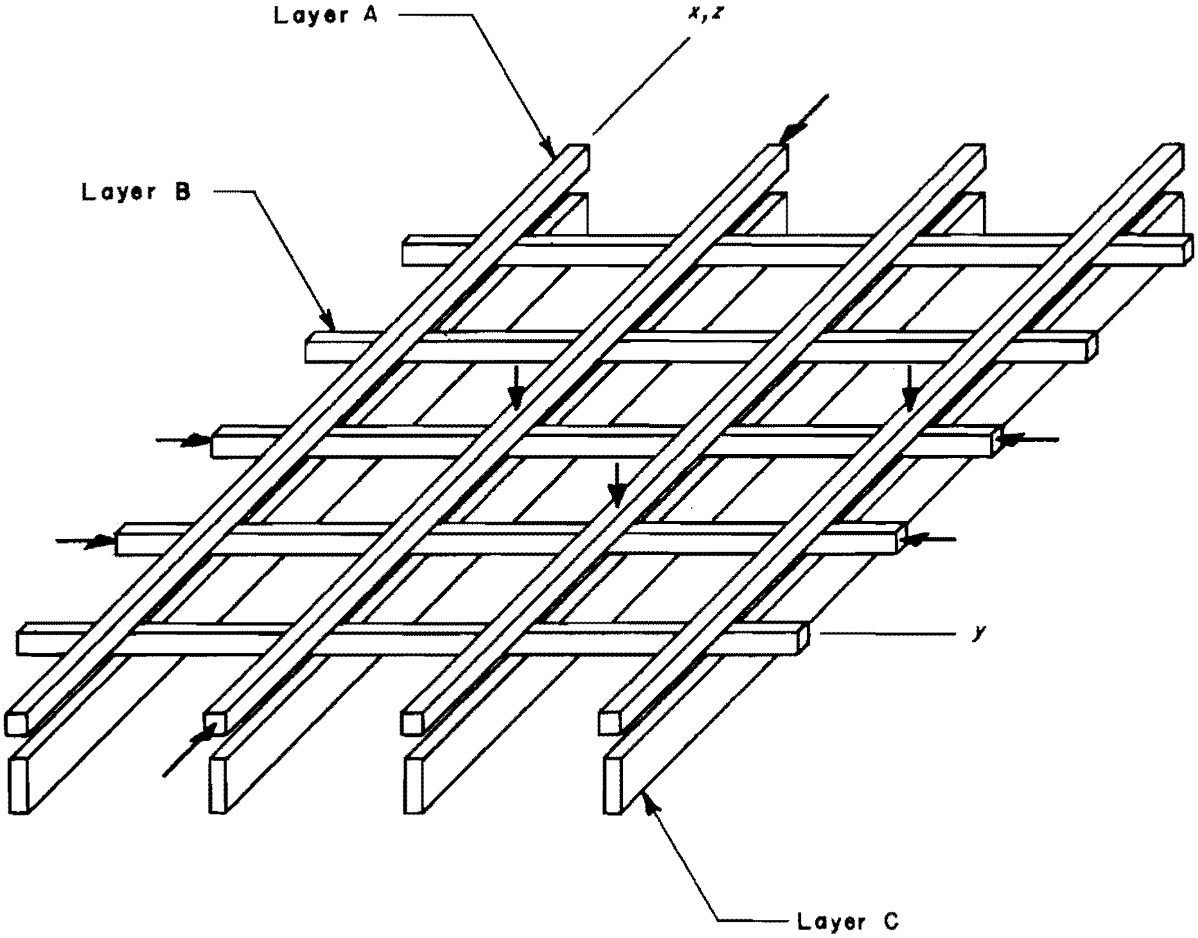


Fig 3.1. Simplified layered system of beams.

Equation 3.2 governs the solution of the third layer (C) of the system shown in Fig 3.1 and the terms are analogous to those in Eq 3.1. The z-direction is the same as the x-direction for this study and any desired spacing of beams is possible. In practice, the spacing of beams in the third layer usually is wider than the spacing of the beams in the gridwork.

$$F^z \frac{\partial^4 w}{\partial z^4} = q \tag{3.2}$$

The solution of the layered system involves solving both Eq 3.1 and Eq 3.2 under the condition that all beams deflect equally at the common intersections. The system then could be described with a differential equation having the form of Eq 3.3 since there is only one loading pattern on the system:

$$F^x \frac{\partial^4 w}{\partial x^4} + F^y \frac{\partial^4 w}{\partial y^4} + F^z \frac{\partial^4 w}{\partial z^4} = q \tag{3.3}$$

Equation 3.3 could be altered slightly because of the assumption of parallelism of layers A and C to have the form

$$(F^x + F^z) \frac{\partial^4 w}{\partial x^4} + F^y \frac{\partial^4 w}{\partial y^4} = q \tag{3.4}$$

The usual procedure for the solution of systems of the layered type is to solve an equation similar to Eq 3.4 with the stiffnesses of the beams combined with the stiffness of the grid. The ability to maintain the use of Eq 3.3, with separate consideration of all variables, enhances the possibility of solving problems of plates-over-beams where it is necessary to include Poisson's ratio and torsional effects on the plate members and not on the supporting beams.

Development of finite-difference equations for the solution of a simply-connected grid-over-beam system will be discussed in the remainder of this chapter.

Finite-Element Representation of the Layered System

The use of finite-element models as representations of physical systems was discussed by Ang and Newmark (Ref 1) for the solution of plates. The derivation of equations based on a discrete model allows the conventional assumptions of continuity of input variable to be circumvented (Ref 2). All approximations of the physical system are in the discrete model, and the derived equations are exact for the finite-element system.

Figure 3.2 shows the forces and members which are pertinent to one joint of the model where all three layers exist. Nine joints are necessary to describe two orthogonal beams, and five other joints are necessary to describe the third beam. The model is composed of rigid bars and joints. The joints are composed of spring-restrained hinges. All pertinent variables are considered as lumped values. The bending stiffness F is represented by a spring-restrained hinge.

The loading system shown in Fig 3.2 is rather general. The Q term represents a concentrated force at the joint. P^x , P^y , and P^z represent axial tension (or compression) on the respective x , y , and z-members. Change in axial tension is assumed to occur at the joints. R^x , R^y , and R^z represent springs which tend to resist rotation of a joint. The T^x , T^y , and T^z terms are representations of externally applied couples and appear as loads at each joint adjacent to the joint in question. The transverse spring S provides an elastic foundation response or a specified amount of deflection by using the proper ratio of load Q and spring S . A very large spring would produce virtually a zero deflection. The Q and S terms refer to the system, while all other terms refer to particular beams.

The quantities F , Q , and S are generally distributed effects over some length or area of the structure. The length over which these effects are

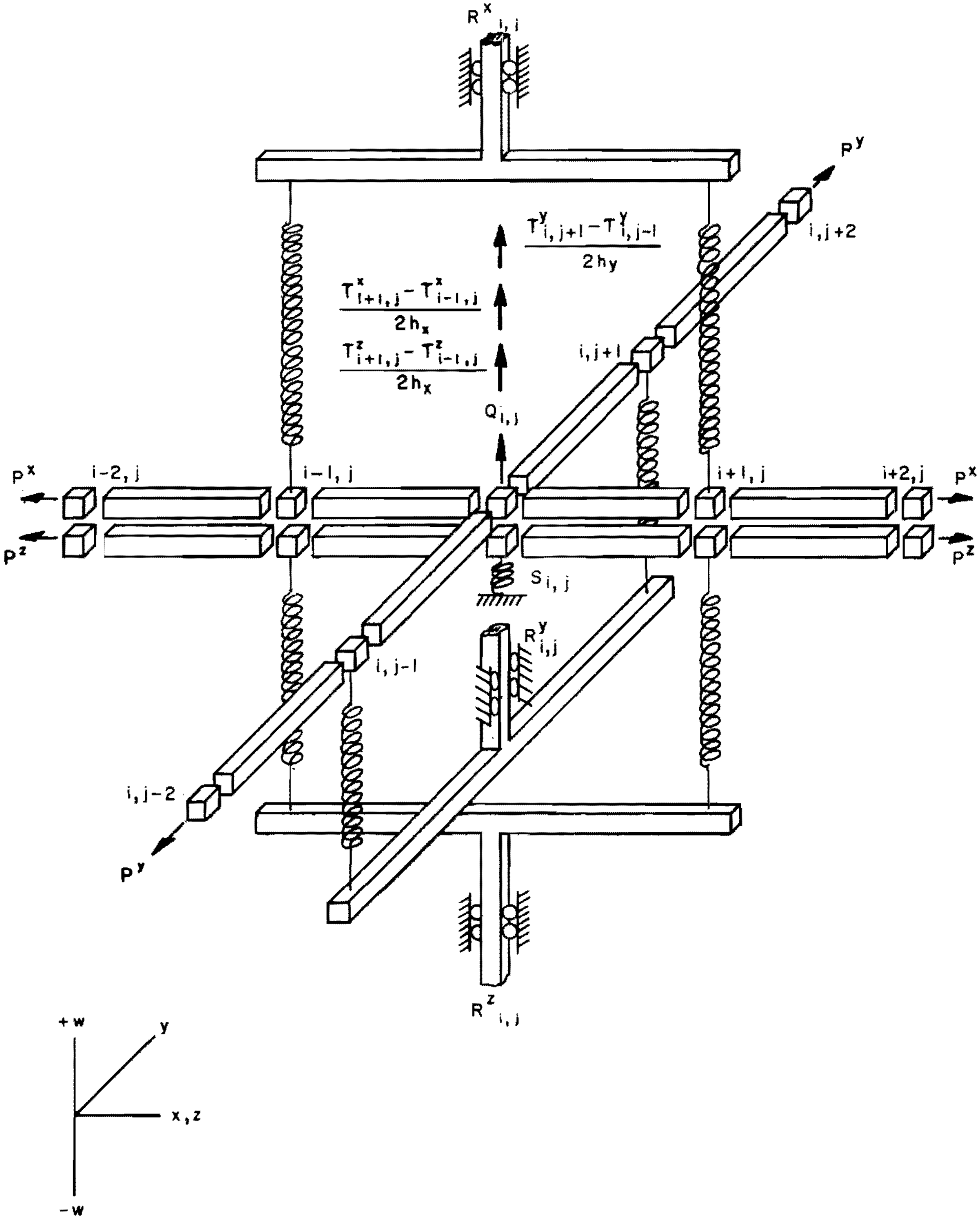


Fig 3.2. External forces and members pertinent to the behavior of one joint of the system.

considered to extend includes a length h/2 on each side of a joint. Thus, the correct usage of distributed effects from one station to another station necessitates the use of half values at end stations and full values at intermediate stations. Axial tension values P refer to bars rather than joints and full values should therefore be used throughout. The bar number for which a P value is described is the same as the joint number ahead of the bar in the increasing direction of station numbers. Thus, when describing P values in the x-direction, the station number i=0 should not be used or the P value will be felt outside of the system. Similarly, the station number j=0 should not be used for P values in the y-direction. Rotational restraints R and torques T usually are needed only as concentrated quantities.

A detailed discussion of the terms just defined is given by Matlock and Haliburton (Ref 21) as they pertain to an individual beam-column solution. A brief discussion of each term is given below as it refers to the simply-connected layered system. Equation 3.5 is the general equation for the simply-connected layered system. Lower-case characters t , r , q , and s are related to the upper-case lumped values T , R , Q , and S by the increment length h .

$$F^x \frac{\partial^4 w}{\partial x^4} + F^y \frac{\partial^4 w}{\partial y^4} + F^z \frac{\partial^4 w}{\partial z^4} + \frac{\partial t^x}{\partial x} + \frac{\partial t^y}{\partial y} + \frac{\partial t^z}{\partial z} + \frac{\partial}{\partial x}\left(r^x \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(r^y \frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(r^z \frac{\partial w}{\partial z}\right) + \frac{\partial}{\partial x}\left(P^x \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(P^y \frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(P^z \frac{\partial w}{\partial z}\right) = q - sw \qquad (3.5)$$

Bending Stiffness

The bending stiffness F , or EI (product of modulus of elasticity and cross-sectional area moment of inertia), may vary in a freely discontinuous manner. A value of F=0 creates a hinge or point of zero moment. Two consecutive hinges completely disconnect a beam. Dummy extensions of each beam, which have no flexural stiffness, act as multiple hinges to isolate the beam from any effects beyond either end.

Transverse Load

The term Q may be considered as concentrated or distributed. The present solution considers only concentrated values so that, if a uniform load is desired, it must be considered as the product of q , h_x , and h_y where h_x and h_y are increment lengths in orthogonal directions.

Axial Tension or Compression

Axial tensions (or compressions) are in the rigid bars. A change in axial tension ΔP occurs only in the joints between rigid bars. The terms in Eq 3.5 representing axial tension could be converted directly to finite-difference form. However, if this were done, the limitation of constant tension would be present. Equation 3.6 shows an appropriate discontinuous finite-difference form for the axial tension. The development of this form is in the general derivation in Appendix 1.

$$\frac{\partial}{\partial x}\left(P\,\frac{\partial w}{\partial x}\right) \quad \left[P_{i-1}w_{i-1} - \left(P_{i-1} + P_{i+1}\right) w_i + P_{i+1}w_{i+1}\right]\frac{1}{h^2} \tag{3.6}$$

External Torque

The external torque is represented as a couple created by forces which act at stations adjacent to Station i with a magnitude of $T_i/2h$. The difference form for external torque is

$$\frac{\partial t}{\partial x} = \frac{t_{i-1} - t_{i+1}}{2h} \tag{3.7}$$

Foundation Support

The foundation support spring S provides both resistance to transverse deflection and a means of deflection control. Virtually zero deflection would occur with a very large S value. This means of specifying zero deflection is used to create simple support conditions. A specified value of deflection can be produced by the proper ratio of Q to S . The term simple support means a hinge or pin-type support which prevents horizontal or vertical movement but does not resist rotation about the hinge.

The finite-difference equations for the joint of Fig 3.2 can be shown to have the form

$$\begin{aligned} & a^x_{i,j}\, w_{i-2,j} + b^x_{i,j}\, w_{i-1,j} + c^x_{i,j}\, w_{i,j} + d^x_{i,j}\, w_{i+1,j} + e^x_{i,j}\, w_{i+2,j} \\ & + f^y_{i,j}\, w_{i,j-2} + g^y_{i,j}\, w_{i,j-1} + h^y_{i,j}\, w_{i,j+1} + i^y_{i,j}\, w_{i,j+2} \\ & + j^z_{i,j}\, w_{i-2,j} + k^z_{i,j}\, w_{i-1,j} + \ell^z_{i,j}\, w_{i+1,j} + m^z_{i,j}\, w_{i+2,j} \\ & = n_{i,j} \end{aligned} \tag{3.8}$$

where the constants $a_{i,j} \ldots n_{i,j}$ are constants which are functions of the properties of the beams, of the loads, and of the restraints on the system. Equation 3.8 is the finite-difference form of Eq 3.3 or Eq 3.5.

Assumptions of the Method of Analysis

The assumptions embodied in the method must be clearly understood in order to recognize the approximations of the real physical problem. Pertinent assumptions are listed below:

(1) There are vertical forces only between the various beams; there is no horizontal shear at the grid-beam interface.

(2) All beams deflect equally at common joints.

(3) The bars between joints are rigid and weightless.

(4) Deflections are small; plane sections remain plane after bending.

(5) All loads, including dead loads, are concentrated at the joints; no external forces act directly on the rigid bars.

(6) Axial forces do not produce axial deformations. Changes in axial forces occur at the joints.

(7) The joint is an elastic, orthotropic, and homogeneous material.

(8) There are no torsional moments nor Poisson's ratio effects at the joints nor in the supporting beams.

(9) The size increment is constant on all parallel beams.

(10) All loads and stiffnesses may vary in a freely discontinuous manner.

Equations for the Grid-over-Beam Problem

Equation 3.3 plus the effect of a foundation spring can be rewritten in symbolic form as Eq 3.9, where Q^x , Q^y , and Q^z represent the loads resisted by the respective x , y , or z-beams and include the effects of special terms such as R , P , and T .

$$Q^x + Q^y + Q^z = Q_i - S_i w_i \tag{3.9}$$

Figure 3.3 shows an expanded view of the joint from the three intersecting beams in Fig 3.2. The forces shown are the same as for Eq 3.9 except that springs K^x , K^y , and K^z have been added to appropriate joints. These springs are temporary representations of the stiffnesses of the beams which oppose deflections of the particular joint. Thus, if the y and z-beams were removed from Fig 3.2, their effects on the x-beam would be shown in the top sketch of Fig 3.3. Summing vertical forces on the three independent joints in Fig 3.3 results in Eq 3.10. Superscripts indicate which beam the variable temporarily represents.

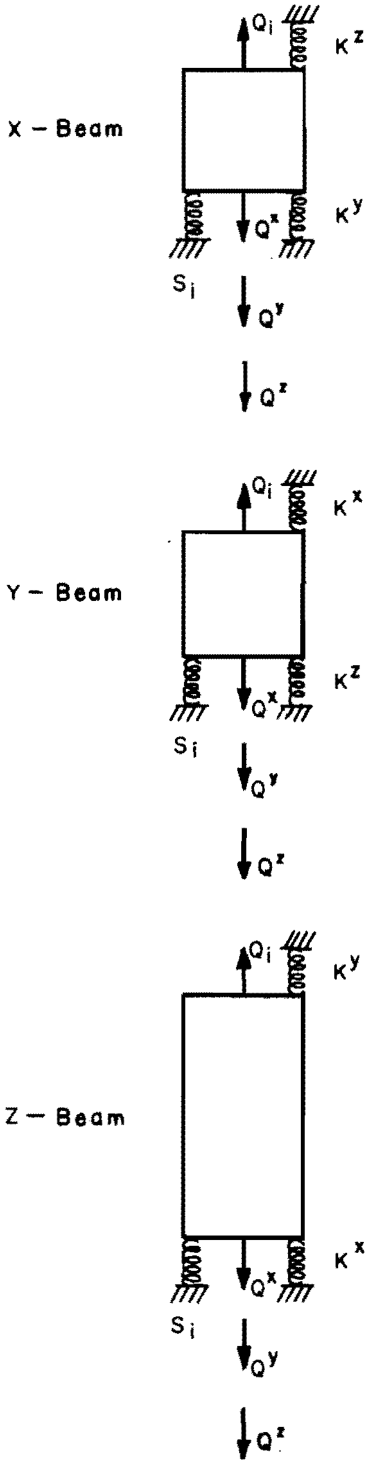


Fig 3.3. Expanded view of a joint where three beams exist.

$$Q^x + S_i w_i^x + K^y w_i^x + K^z w_i^x = Q_i - Q^z - Q^y + K^y w_i^y + K^z w_i^z$$

$$Q^y + S_i w_i^y + K^x w_i^y + K^z w_i^y = Q_i - Q^x - Q^z + K^x w_i^x + K^z w_i^z \qquad (3.10)$$

$$Q^z + S_i w_i^z + K^x w_i^z + K^y w_i^z = Q_i - Q^x - Q^y + K^x w_i^x + K^y w_i^y$$

In Eq 3.10, w_i indicates a deflection at a particular Station i and the superscripts x , y , z designate the beam being considered. This arrangement of terms shows clearly the iterative process to be used since all terms on the left side could be considered as unknowns, and all terms on the right side could be considered as temporary constants or known quantities.

The actual derivation of these equations is in Appendix 1, where the terms Q^x , Q^y , and Q^z are shown in expanded form in terms of deflections and include the R , P , and T terms shown in Fig 3.2. It is also shown that the equations can be rearranged in the form

$$a_i w_{i-2} + b_i w_{i-1} + c_i w_i + d_i w_{i+1} + e_i w_{i+2} = f_i \qquad (3.11)$$

Summary

The need for a simple means of developing the concept of, and tools to solve, the layered structural system has been fulfilled by this chapter. The concept and equations were developed for the simplest possible system. If the concepts and tools can be shown to adequately solve these simple systems, then the extension to include torsional rigidity and Poisson's ratio effects should be relatively straightforward.

Either a two-phase solution, with Eq 3.4 as a basis, or a three-phase solution, with Eq 3.3, will ultimately be equivalent for the simply-connected system. The three-phase solution does, however, present a direct basis for solving plate-over-beam problems where only the beams are unaffected by torsion and Poisson's ratio.

A very important side benefit from the three-phase solution is the acceptability of more than two intersecting beams. Previous work in this area of interest at The University of Texas has been limited to only two intersecting beams at one point (Ref 34). This ability to properly handle more than two intersecting beams will allow solution of grids or plates over nonorthogonal beams even though this application has not been included in this study.

CHAPTER 4. CRITERIA FOR SELECTING CLOSURE PARAMETERS FOR SIMPLY-CONNECTED GRIDS

In Chapter 3, springs K^x , K^y , and K^z were discussed as being representative of the stiffnesses of various beams of the system. These spring stiffnesses also act as closure parameters in the alternating-direction method of solution. These closure parameters add stability to the system by tending to hold the system together. The effects of the parameters are nullified at closure. The structural system considered in this study is generally quite complex and may involve nonsymmetric matrices. The selection and use of these closure springs for diverse systems will be discussed in this chapter.

Previous Closure Parameter Criteria for Alternating-Direction Methods

Two of the most frequently used methods of closure parameter selection are attributed to Wachpress (Ref 31) and Peaceman and Rachford (Ref 25). The mathematical process used in these two references is quite similar. The process is valid for cases involving second-order difference equations such as the LaPlace equation. The two methods assume symmetric matrices and use one closure parameter in each full cycle of solution. A set of m parameters is chosen and used in a cyclic manner. Young and Wheeler (Ref 32) have extended the method of Peaceman and Rachford to consider the case of nonsymmetric matrices that commute. A proof similar to one of Young and Wheeler's (Ref 32) will be presented later in support of the criteria to be presented.

Tucker (Ref 29) considered the work of Peaceman and Rachford when applying the alternating-direction technique to fourth-order difference equations. He concluded that the closure parameters must stay constant during any iteration to assure convergence. The criteria presented by Tucker for selection of fictitious spring stiffnesses predicted a value c_k which could range from 0 to 1.0 in Eq 4.1:

$$K = c_k \frac{6F}{h_x^3} \qquad (4.1)$$

In Appendix 2 it is shown that the product $6c_k$ theoretically ranges from 0 to 16 for simple grid systems but a value of 6 is possibly a practical maximum.

Demonstration of Need for Varying Closure Parameters

The previously discussed criteria were for the case of symmetric matrices that commute, as in Eq 4.2. H and V represent respectively the horizontal (x-direction) and vertical (y-direction) difference operators for the two orthogonal directions of the system (Ref 32).

$$HV = VH \qquad (4.2)$$

The alternating-direction iteration scheme is a process which solves alternately members in one direction and then members in the other direction, from boundary to boundary, until the deflections of the various members at the joints are the same to some specified tolerance. The alternating-direction process is defined by the equations

$$(H + \rho_{n+1} I) w_{n+\frac{1}{2}} = b - (V - \rho_{n+1} I) w_n \qquad (4.3)$$

and

$$(V + \rho_{n+1} I) w_{n+1} = b - (H - \rho_{n+1} I) w_{n+\frac{1}{2}} \qquad (4.4)$$

where ρ_{n+1} is the positive constant closure parameter for both cycles of the iteration. The ρ_{n+1} values are selected to minimize the error in successive iterations and are based on the upper and lower bounds of the eigenvalues of H and V .

The physical structural counterpart for the symmetric case is a structure composed of equal numbers of beams of equal lengths, stiffnesses, and numbers of increments in each of the directions of solution. Loads and support conditions must also be symmetric. In the general structural case the condition of symmetry seldom exists and the eigenvalues for adjacent beams and crossbeams might vary by factors of thousands. The eigenvalue λ for a simply supported beam can be shown to have the form

$$\lambda = \frac{F}{h^4}\left[16\left(\sin\frac{n\pi}{2M}\right)^4\right] . \tag{4.5}$$

This form is developed in Appendix 2 where it also is shown that the closure parameters for simply-connected grids are related to the eigenvalues. The relationship is shown in Eq 4.6. $\overline{SF}$ is the symbol for the fictitious spring stiffness. The springs are termed fictitious since their effect is nullified at closure. Variations in bending stiffness F from beam-to-beam can cause drastic changes in λ :

$$\overline{SF} = h\lambda \tag{4.6}$$

The use of upper and lower bounds for both H and V of Eq 4.2 was an attempt to average the reduction of error for the entire system. A greater reduction in error can be achieved by allowing each individual beam to recognize the effects of connecting beams. This more efficient solution is obtained by using $\overline{SF}$ values for each set of beams in the system rather than one constant $\overline{SF}$ for the system.

The substitution of a load and closure parameter for each opposing beam at each intersection of the beam being solved, temporarily simulates the effects of each replaced beam. This in effect states that the closure parameters to be considered during the solution of the H matrix should be parameters computed from the V matrix. Similarly, when solving the V matrix the parameters should be based on the H matrix. A demonstration that this is possible is in the next section and a step-by-step criteria for the selection of closure parameters will follow.

Special Symmetrical Case

The alternating-direction iterative process is now defined as follows:

$$(H + \rho_{n+\frac{1}{2}} I)\, w_{n+\frac{1}{2}} = k - (V - \rho_{n+\frac{1}{2}} I)\, w_n \tag{4.7}$$

and

$$(V + \rho_{n+1} I)\, w_{n+1} = k - (H - \rho_{n+1} I)\, w_{n+\frac{1}{2}} \tag{4.8}$$

Solving Eq 4.7 for $w_{n+\frac{1}{2}}$ and substituting into Eq 4.8 gives

$$\begin{aligned} w_{n+1} = {} & (V + \rho_{n+1} I)^{-1} k - (V + \rho_{n+1} I)^{-1} (H - \rho_{n+1} I) \\ & \times \left[(H+\rho_{n+\frac{1}{2}} I)^{-1} k - (H+\rho_{n+\frac{1}{2}} I)^{-1} (V-\rho_{n+\frac{1}{2}} I) w_n \right] \end{aligned} \tag{4.9}$$

Or, in notation similar to Young and Wheeler's

$$w_{n+1} = T_{n+1}\, w_n + G_{n+1}\, k \tag{4.10}$$

where

$$T_{n+1} = (V+\rho_{n+1} I)^{-1} (H-\rho_{n+1} I)\, (H+\rho_{n+\frac{1}{2}} I)^{-1} (V-\rho_{n+\frac{1}{2}} I) \tag{4.11}$$

and

$$G_{n+1} = (V+\rho_{n+1} I)^{-1} \left[I - (H-\rho_{n+1} I)\, (H+\rho_{n+\frac{1}{2}} I)^{-1} \right] \tag{4.12}$$

If the error at the n^{th} iteration is defined as

$$e_n = w_n - \bar{w} \tag{4.13}$$

where $\bar{w}$ is the exact solution of Eq 3.1, then, from Eq 4.10, Eq 4.14 can be written.

$$\bar{w} = T_{n+1} \bar{w} + G_{n+1} k \tag{4.14}$$

Substituting Eq 4.10 and 4.14 into Eq 4.13 results in

$$e_n = (T_n w_{n-1} + G_n k) - (T_n \bar{w} + G_n k) \tag{4.15}$$

which simplifies to

$$e_n = T_n (w_{n-1} - \bar{w}) \tag{4.16}$$

or

$$e_n = T_n e_{n-1} \tag{4.17}$$

Equation 4.17 implies that the error after n iterations could be written as

$$e_n = (T_n) (T_{n-1}) (T_{n-2}) \cdots (T_1) e_o \tag{4.18}$$

or

$$e_n = \left(\prod_{i=1}^{n} T_i \right) e_o. \tag{4.19}$$

Now, a set of m parameters is selected (with no requirement that the parameters be constant during each full cycle) and used in cyclic order so that for any integer k representing a cycle of the set of m parameters, Eq 4.20

is true.

$$e_{km} = \left[\left(\prod_{i=1}^{m} T_i\right)^k\right] e_o \tag{4.20}$$

For convergence of solution, e_{km} must approach zero which implies that k must approach infinity. Therefore Eq 4.21 must be satisfied.

$$\left(\prod_{i=1}^{m} T_i\right) < 1 \tag{4.21}$$

According to Young and Wheeler (Ref 33) the H and V matrices have a common basis of eigenvectors. Let v be one of these eigenvectors, ν represent the eigenvalue of V, and μ represent the eigenvalue of H, so that

$$Hv = \mu v \tag{4.22}$$

and

$$Vv = \nu v \tag{4.23}$$

Then using Eq 4.11, we have

$$\prod_{i=1}^{m} T_i = \prod_{i=1}^{m} \Big\{ (V + \rho_i I)^{-1} (H - \rho_i I) (H + \rho_{i-\frac{1}{2}} I)^{-1} \times (V - \rho_{i-\frac{1}{2}} I) \Big\} \tag{4.24}$$

Now, an eigenvalue λ of $\prod_{i=1}^{m} T_i$ is sought so that

$$\lambda v = \left(\prod_{i=1}^{m} T_i\right) v \tag{4.25}$$

Substituting the right side of Eq 4.24 into Eq 4.25 and performing the multiplication by v results in

$$\lambda v = \prod_{i=1}^{m} \Big\{ (Vv + \rho_i vI)^{-1} (Hv - \rho_i vI) \ (Hv + \rho_{i-\frac{1}{2}} vI)^{-1}$$
$$\times \ (Vv - \rho_{i-\frac{1}{2}} vI) \Big\} \qquad (4.26)$$

Substituting μv for Hv and νv for Vv results in

$$\lambda v = \prod_{i=1}^{m} \Big\{ (\nu v + \rho_i vI)^{-1} \ (\mu v - \rho_i vI) \ (\mu v + \rho_{i-\frac{1}{2}} vI)^{-1}$$
$$\times \ (\nu v - \rho_{i-\frac{1}{2}} vI) \Big\} \qquad (4.27)$$

Now by dividing out the eigenvector v , Eq 4.28 results:

$$\lambda = \prod_{i=1}^{m} \frac{(\mu - \rho_i) \ (\nu - \rho_{i-\frac{1}{2}})}{(\nu + \rho_i) \ (\mu + \rho_{i-\frac{1}{2}})} = \prod_{i=1}^{m} \frac{(\mu - \rho_i) \ (\nu - \rho_{i-\frac{1}{2}})}{(\mu + \rho_{i-\frac{1}{2}}) \ (\nu + \rho_i)} \qquad (4.28)$$

It should be recalled that μ is an eigenvalue of H and ν is an eigenvalue of V . Equation 4.28 shows that the closure parameter for any one particular portion of the solution cycle does not affect only its own corresponding matrix H or V , but that the eigenvalues of both H and V and the closure parameter on the particular portion of the cycle must be considered.

Thus, considering one term of the product in Eq 4.28 such as $\mu - \rho_i / \nu + \rho_i$, if μ is much greater than ν and if ρ_i and ν are of the same magnitude, then the ratio will be greater than 1. Thus, divergence is implied where μ is the eigenvalue of H and ρ_i is a parameter used in solution of the V matrix. Consideration of the other term in Eq 4.28 shows that the same relation holds where ν is an eigenvalue of V and $\rho_{i-\frac{1}{2}}$ is the parameter used in solving the H matrix.

Thus to minimize the eigenvalues λ , $\rho_{i-\frac{1}{2}}$ should be chosen to minimize the ν's and ρ_i should minimize the μ's . This implies that one parameter should be chosen for each matrix to be solved and the ρ for the H solution should be based on eigenvalues of V and the ρ for V should be based on eigenvalues of H .

The arrangement of Eq 4.28 in the form discussed was suggested by Haliburton (Ref 11).

Method of Selection of Closure Parameters

It has been shown that the parameters should be approximations of opposing systems of beams and that the parameters are related to the eigenvalues of the opposing systems. The proposed method of selection is outlined below.

(1) Compute eigenvalues from Eq 4.5 for the beams in each layer of the system. If the beams vary in stiffness from end to end, or in the layers, then the highest lower bound and the least upper bound of the eigenvalues should be used for the range of parameters.

(2) Convert eigenvalues to fictitious springs using Eq 4.6.

(3) Select some number of parameters, usually 6 or less, or according to the method outlined in the next section. The selection should cover roughly the range of values from highest lower bound to least upper bound.

(4) Apply the sets of parameters cyclicly and in increasing order.

Number of Parameters

A means of estimating the optimum number of closure parameters to be used cyclicly was proposed by both Wachpress (Ref 31) and Peaceman and Rachford (Ref 25). The Wachpress method, as applicable to fourth-order systems, is as follows.

Solve Eqs 4.29 and 4.30 for a and b where M is the number of mesh points in the longest row or column in the system.

$$a = 16 \sin^4 \frac{\pi}{2M} \qquad (4.29)$$

$$b = 16 \cos^4 \frac{\pi}{2M} \qquad (4.30)$$

The number of parameters to cycle is the smallest integer m which satisfies Eq 4.31.

$$\left(\sqrt{2}-1\right)^{2m-1} \leqq \frac{a}{b} \qquad (4.31)$$

Use of Closure Parameters

The best sequence of parameters for the layered type problem under consideration seems to be a logarithmic increase. The solution has been found to be fairly insensitive to the number of parameters, provided that the lower bound of the eigenvalues is avoided. Usually three to six parameters are a reasonable number for each layer of the system with the same number being used on all layers. The range of parameters used usually should cover the lower one-half of the range from highest lower eigenvalue bound to least upper bound plus one fairly large value from the upper one-half of the range. It is possible to develop divergence or oscillation in the solution if the lower bound limit is not observed. The worst result, for large parameter values, appears to be slow convergence if the parameters are selected in the upper range or beyond the upper bound.

Repeating or cycling sequences of parameters has been recommended by several investigators and appears to be beneficial in reducing the error between successive iterations of solution. In using the cycling procedure it is sometimes advisable to let the final parameter in the sequence be a value between the lowest and highest values of the sequence so that a large change in condition does not occur instantaneously.

The recommended use of eigenvalues for the selection of fictitious springs immediately poses the question of various conditions other than simple supports.

For the condition of a free-free or unsupported beam, which many of the individual beams are during the first cycle of solution, zero eigenvalues would theoretically be correct. This aspect has been checked in a computer program and fortunately, after one cycle of solution, no beams are unsupported; all are beams on elastic foundations. The cycling procedure brings the zero fictitious spring into the system again, at which time the solution generally falters. No extensive checkout has been performed for other conditions of initial beam support.

A Physical Interpretation of Closure Parameters

The use of eigenvalues to compute closure parameters for the alternating-direction scheme gives good results. An alternate procedure may be intuitively seen by considering the physical problem. The previously defined terms Q^x , Q^y , and Q^z are approximations of the loads which are absorbed by the finite-element beam. The values of Q^x , Q^y , and Q^z will vary from station to station depending on the changes in the deflected shape.

If the system is deflected, then, at each station along each of the beams, a value of a spring stiffness $\overline{SY}$ can be computed. This spring is the spring which will exactly duplicate the deflected shape if the loads applied are the predicted Q^x , Q^y , or Q^z . Equation 4.32 shows the form for the calculation of such a spring stiffness for a y-beam.

$$\overline{SY} = \frac{Q^y}{w^y_{i,j}} \tag{4.32}$$

This is a "natural" spring for the model. Studies have indicated that negative values of the spring stiffness $\overline{SY}$ may be computed from Eq 4.32. A negative spring stiffness may have very adverse effects on the solution. It is advisable to use a very small positive value rather than a negative value and to add the computed load directly to the system to maintain complete simulation of the system. Studies have indicated that this "natural" spring concept gives more rapid deflection closure to a fairly loose tolerance than does the use of spring stiffnesses based on eigenvalues. Unfortunately, after just a few cycles of iteration, the further use of the "natural" springs no longer reduces the error by a significant amount per cycle. It is possible that some combination of the natural spring constant and the eigenvalue spring constant might produce rapid closure to very tight tolerance. As yet, this combination has not been determined and the eigenvalue springs will be used to solve the numerical examples in Chapters 6 and 9.

CHAPTER 5. SOLUTION OF SIMPLY-CONNECTED SYSTEMS

This chapter summarizes and extends an available method of solution for the bending of beams to the solution of the layered system. A computer program which solves the equations describing the system is discussed.

Fourth-Order Equation for Beams

Any of the fourth-order difference equations derived in Appendix 1 can be arranged in the form of Eq 3.11. A system of Eqs 3.11 written in matrix form is a five-member-wide diagonally banded matrix. Figure 5.1 shows the arrangement of several such equations when written for successive stations along the beam. This quidiagonal system is equivalent to

$$Aw = b \tag{5.1}$$

In Eq 5.1, A is a matrix of coefficients such as a_i , b_i , c_i , d_i , and e_i ; w is a column matrix of unknown deflections; and b is a column matrix of known loads.

Solution Process for One Beam

Figure 5.2 shows a complete solution of one beam. The finite-element model of a beam is Fig 5.2a and the stiffness and load matrices are Figs 5.2b and c. The solution process is a back-and-forth recursion equation process and the essential steps of the method are as follows:

(1) Write Eq 3.11 at each Station i along the beam.

(2) Calculate continuity coefficients on an initial sweep along the beam from top to bottom as shown in Fig 5.2d. In this phase, two unknown deflections (w_{i-2} and w_{i-1}) of Eq 5.1 are eliminated from each equation. The result is a tridiagonal band matrix with each equation having the form of the deflection equation shown in Fig 5.2d.

$$\begin{array}{lllllllllll}
\ast & \ast & \ast & & & & & & & \\
+a_{i-2}\, w_{i-4} & +\, b_{i-2}\, w_{i-3} & +c_{i-2}\, w_{i-2} & +d_{i-2}\, w_{i-1} & +\, e_{i-2}\, w_{i} & & & & & = f_{i-2} \\
 & +\, a_{i-1}\, w_{i-3} & +b_{i-1}\, w_{i-2} & +c_{i-1}\, w_{i-1} & +\, d_{i-1}\, w_{i} & +\, e_{i-1}\, w_{i+1} & & & & = f_{i-1} \\
 & & +a_{i}\, w_{i-2} & +b_{i}\, w_{i-1} & +\, c_{i}\, w_{i} & +\, d_{i}\, w_{i+1} & +\, e_{i}\, w_{i+2} & & & = f_{i} \\
 & & & +a_{i+1}\, w_{i-1} & +\, b_{i+1}\, w_{i} & +\, c_{i+1}\, w_{i+1} & +\, d_{i+1}\, w_{i+2} & +\, e_{i+1}\, w_{i+3} & & = f_{i+1} \\
 & & & & +\, a_{i+2}\, w_{i} & +\, b_{i+2}\, w_{i+1} & +\, c_{i+2}\, w_{i+2} & +\, d_{i+2}\, w_{i+3} & +\, e_{i+2}\, w_{i+4} & = f_{i+2} \\
 & & & & & \ast & \ast & \ast & &
\end{array}$$

Fig 5.1. Equation arrangement for solution of deflections.
(After Matlock and Haliburton, Ref 21 p 21).

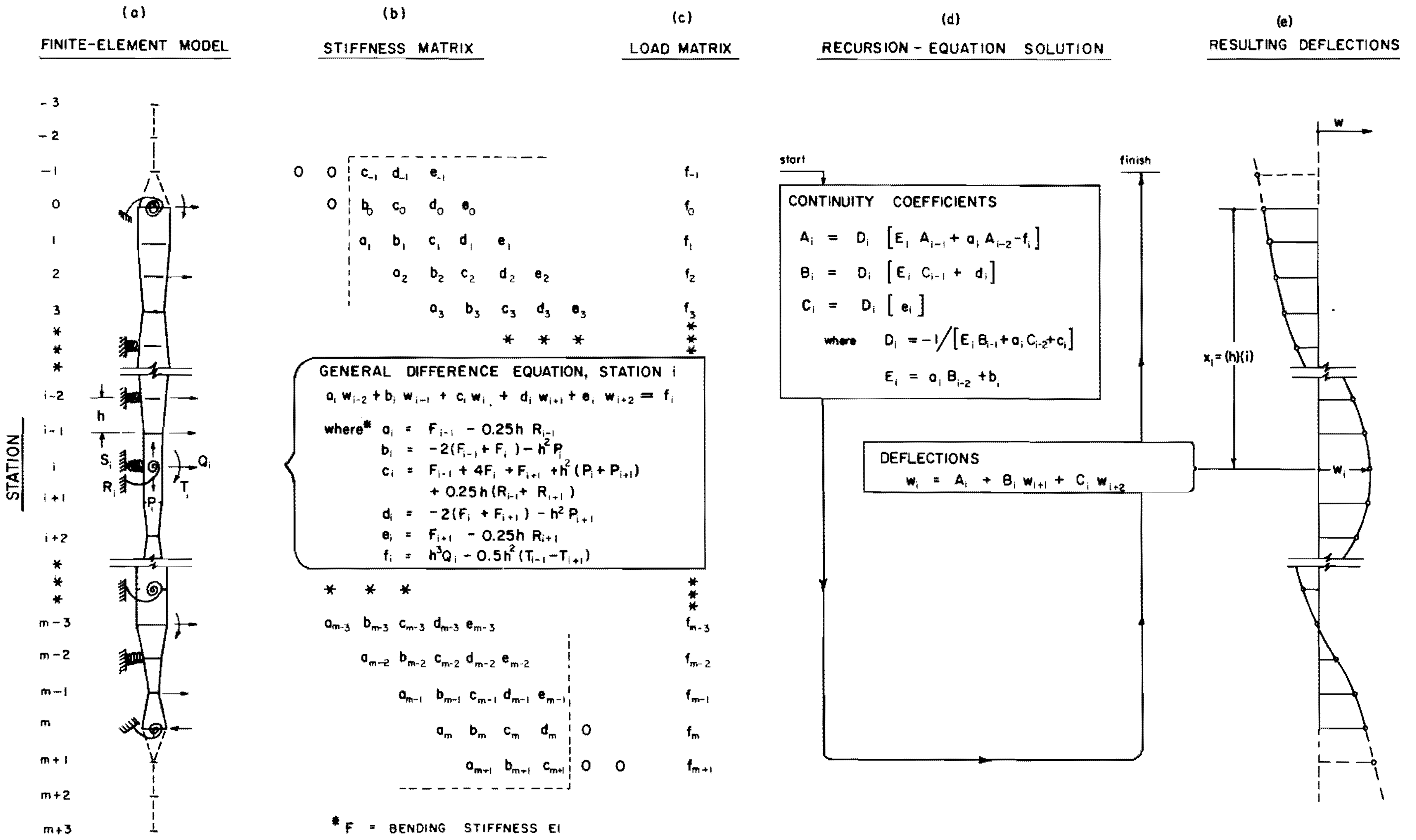


Fig 5.2. Complete solution process for one beam.
(After Matlock and Haliburton, Ref 21 p 37).

(3) Calculate deflections on a second sweep along the beam from bottom to top as also shown in Fig 5.2d.

(4) Calculate any derivatives of deflection that are desired, i.e., slope, moment, shear reaction.

Figure 5.2e illustrates a possible deflection pattern for the beam model of Fig 5.2a.

Repetitive Use of Beam Equations

The finite-element layered system is composed of many beams in each layer. Each beam can be solved by the foregoing procedure. With proper transfer of information to adjacent beams in other layers, the entire system can be solved through repetitive use of the beam equations.

Figure 5.3 is a very simple flow diagram illustrating the general solution process for the system. Each of the three phases of solution comprises one-third of the iteration cycle. Closure parameters (fictitious springs) are adjusted between each beam solution to properly reflect the effects of beams which are temporarily held constant (the beams in the other two phases of solution).

A means of determining when this method has converged to a solution is to have the computed deflections meet some selected degree of accuracy. The closure tests are based on deflections of each beam versus the corresponding deflection of beams in adjacent layers as well as tests on each successive deflection value of each beam from iteration to iteration.

Boundary Conditions

Boundary conditions for the grid-over-beam solution are created automatically by data input to the general purpose finite-element equations. Dummy extensions on each end of the individual beams furnish additional equations needed to solve the system of equations. The extensions of the beams form

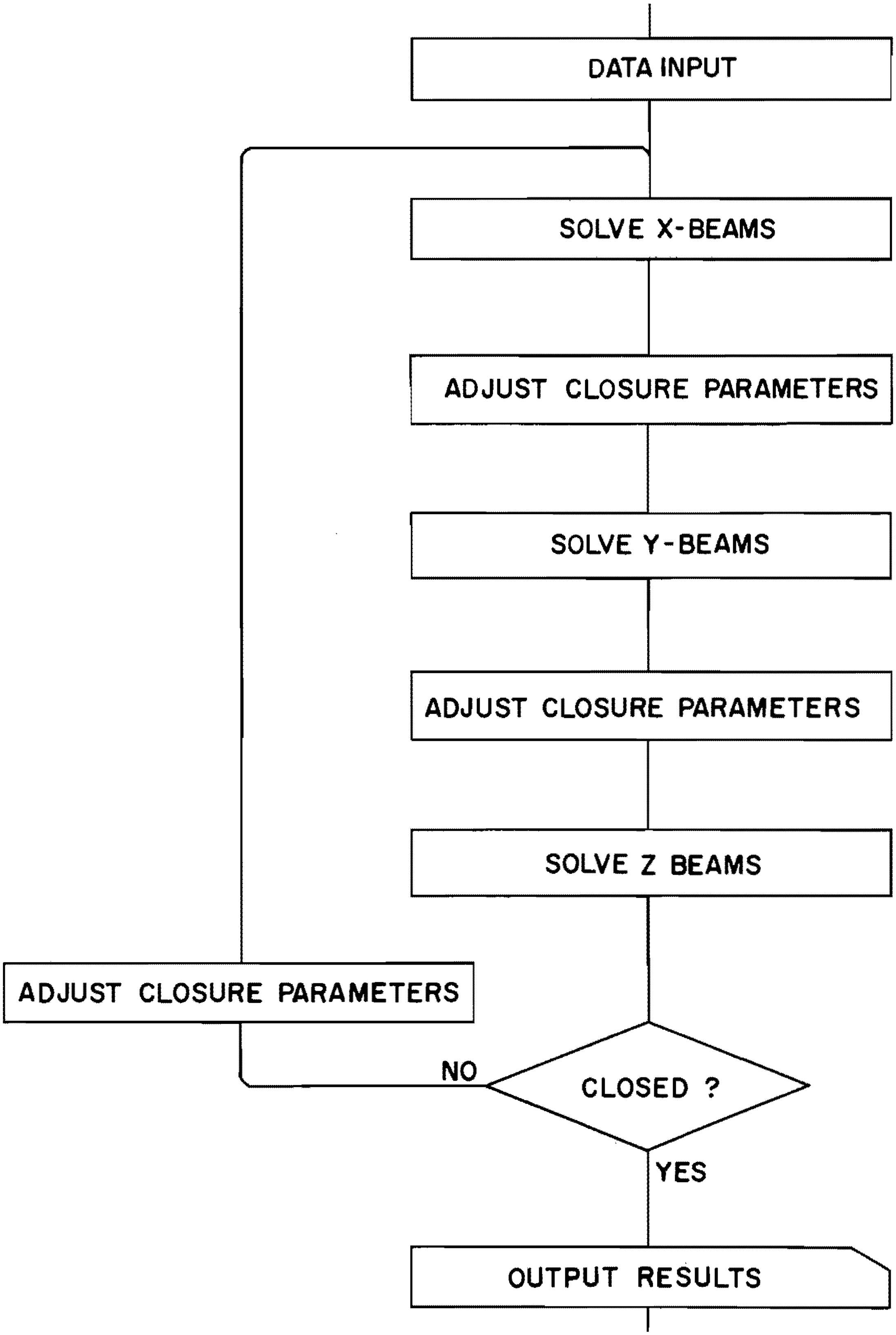


Fig 5.3. Simplified flow diagram showing three solution phases.

hinges which disconnect the model from any other information to right or left of the physical beam.

Program Layer 7

Program LAYER 7 is a FORTRAN 63 language source program. It incorporates all items previously mentioned as necessary for the solution of the simply-connected layered system. It is written especially for the CDC 1604 computer but could be easily adapted for use on other high speed computers with a large storage capacity.

A detailed flow chart of LAYER 7 is in Appendix 3. Pertinent notation and a listing of the FORTRAN program are in Appendix 4. Appendix 5 contains an input form for help in using the program. Pertinent comments and warnings are included.

CHAPTER 6. EXAMPLE GRID-OVER-BEAM PROBLEMS

Four example problems have been selected to show the general applicability and capability of the method of analysis presented in the preceding chapters. Sample input and output data for the first and third examples are in Appendix 12.

Example Problem 6.1. Uniformly Loaded Grid Over Five Beams

A hypothetical layered structural system is shown in Fig 6.1. It is composed of a gridwork of nine members by nine members and is supported by five beams. It is square in plan. The ends of the system, at Station i = 0 and Station i = 8, are simply supported. The grid has a uniform stiffness. The supporting beams have a stiffness equal to 1000 times the stiffness of the grid. The end stations of the beams in all three layers have a one-half value of stiffness to conform with the standard beam column solution procedures in Ref 21. There is a uniform tension load of 1.0×10^4 pounds applied along the axis of each beam and each grid member. Each member is divided into 8 increments, each increment having a length of 6 inches. Three sets of closure spring stiffness, with five values in each set, were used for the x , y , and z-beams. The values of spring stiffnesses are listed in Fig 6.2a, which shows a typical example of the closure process for this hypothetical problem. Thirteen cycles of solution were used to meet a specified tolerance in deflection of 1.0×10^{-6} inch. The maximum error in load equilibrium at a single station was 0.029 pound.

Figures 6.2b and 6.2c show the deflected shape of two selected beams from the system. Figure 6.2b is interesting since the drape of the grid-over-beams is readily apparent. The deflected shape is composed of straight lines since only one increment was used between adjacent beams. The computation time for this problem was approximately 24 seconds.

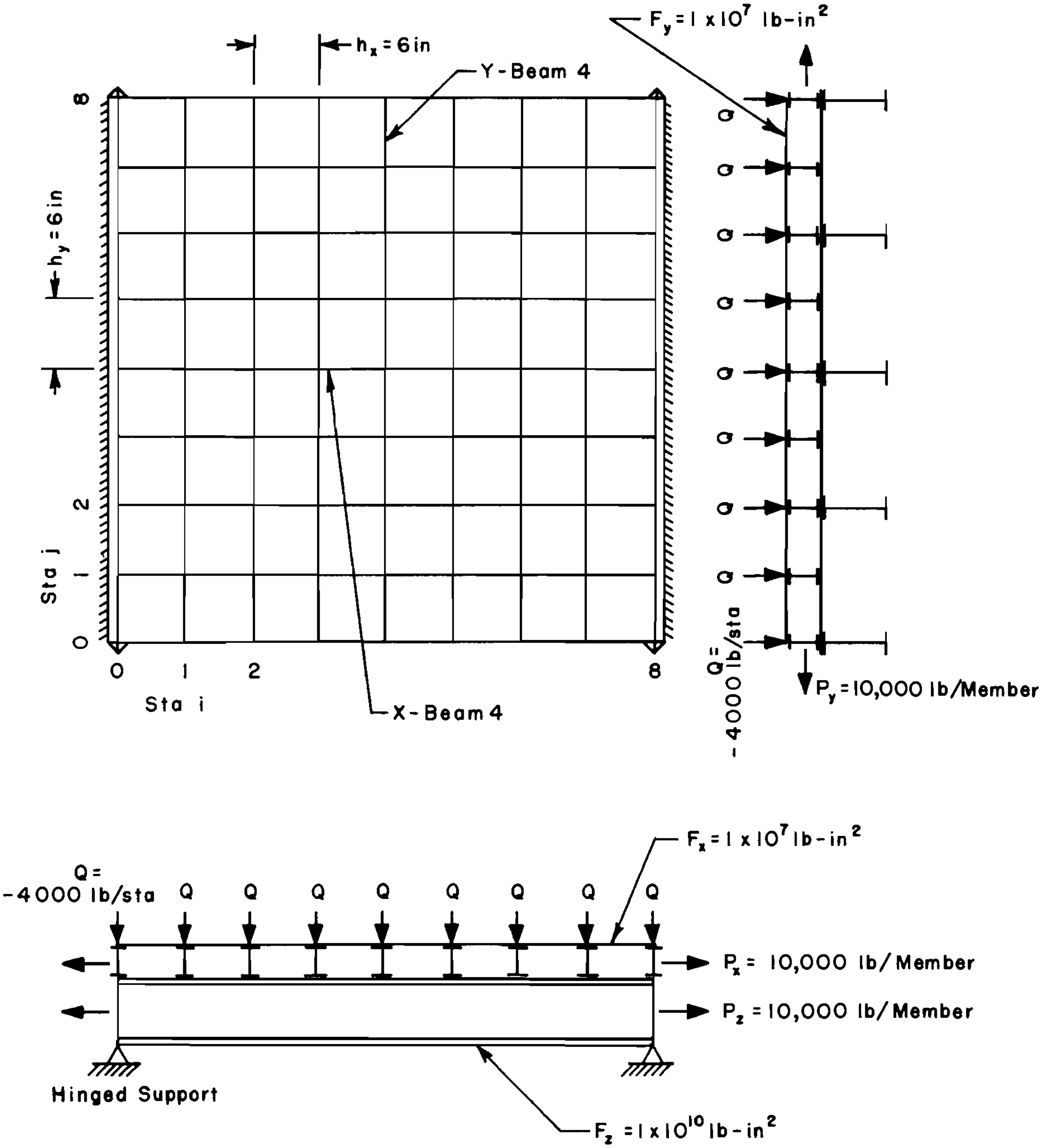
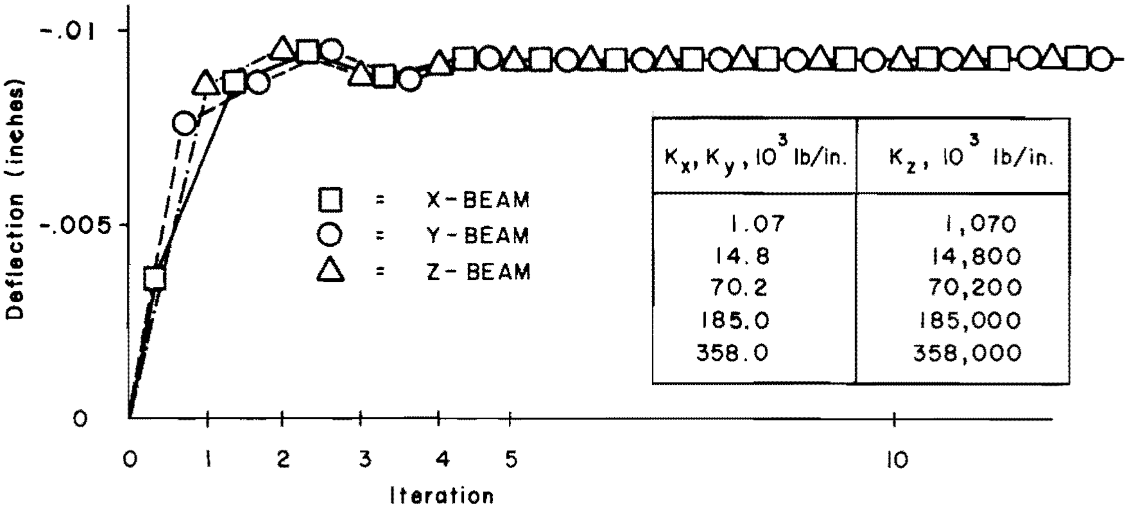
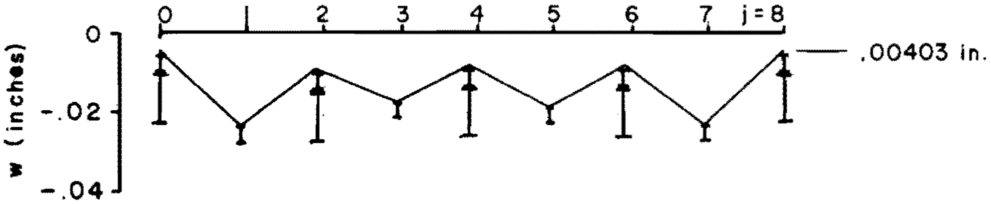


Fig 6.1. Example Problem 6.1. Hypothetical structural system subjected to transverse loads and axial tension.

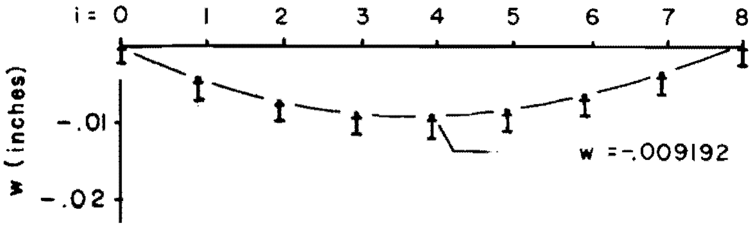


$K_x, K_y, 10^3$ lb/in.	$K_z, 10^3$ lb/in.
1.07	1,070
14.8	14,800
70.2	70,200
185.0	185,000
358.0	358,000

(a) CLOSURE PLOT OF STA 4,4



(b) DEFLECTED SHAPE OF Y-BEAM 4



(c) DEFLECTED SHAPE OF X-BEAM 4

Fig 6.2. Example Problem 6.1. Closure plot and deflected shape of beams for hypothetical grid over five girders.

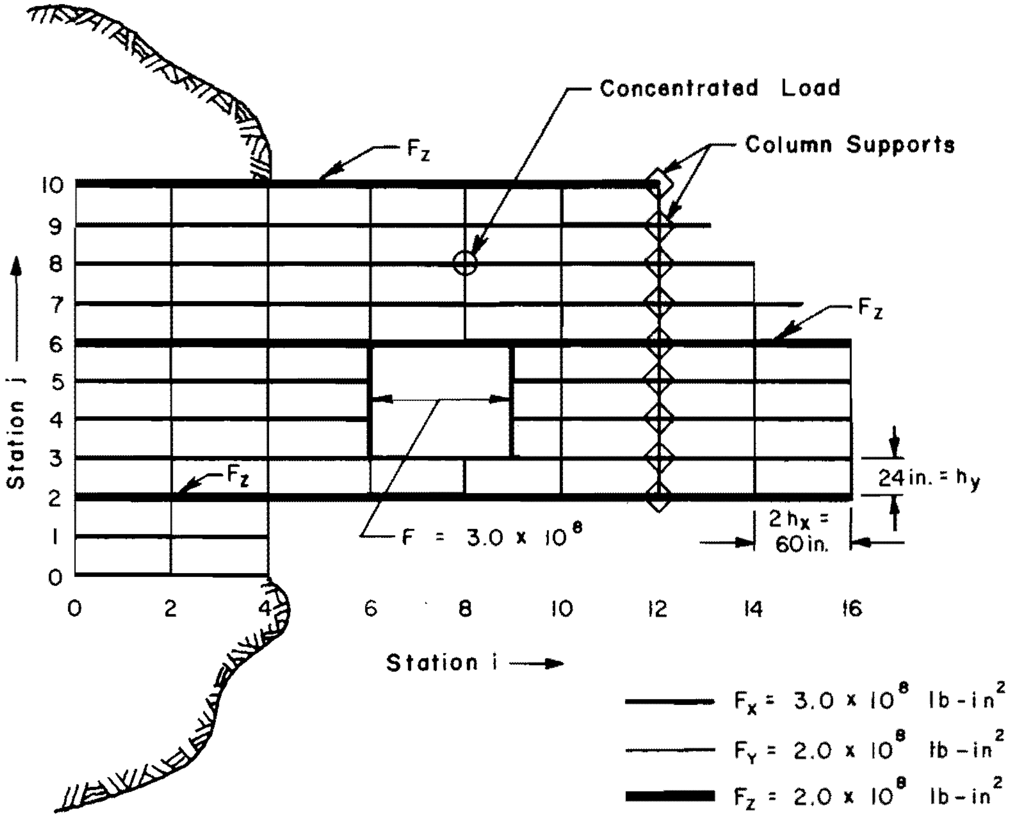
Example Problem 6.2. Support System for a Structure

Figure 6.3 depicts a more complicated structure and is similar to one first studied by Tucker (Ref 29). The present type of analysis solved the system as a gridwork overlying three girders. The girders are assigned a flexural stiffness of 2×10^8 lb-in^2 while the x-beams in the grid are assigned a stiffness of 3×10^8 lb-in^2. This gives a total x-direction stiffness at the girders of 5×10^8 lb-in^2 as compared to Tucker's total of 4×10^8 lb-in^2. The other various characteristics of the structure are outlined in Fig 6.3. Alternate members in the x-direction are omitted and a hole is created by omitting portions of other beams. The soil is represented by linear springs with spring constant $S = 300$ lb/in per station. The column supports are assumed embedded in rock so that no deflection occurs at the columns. Rotational resistance is also furnished by the columns.

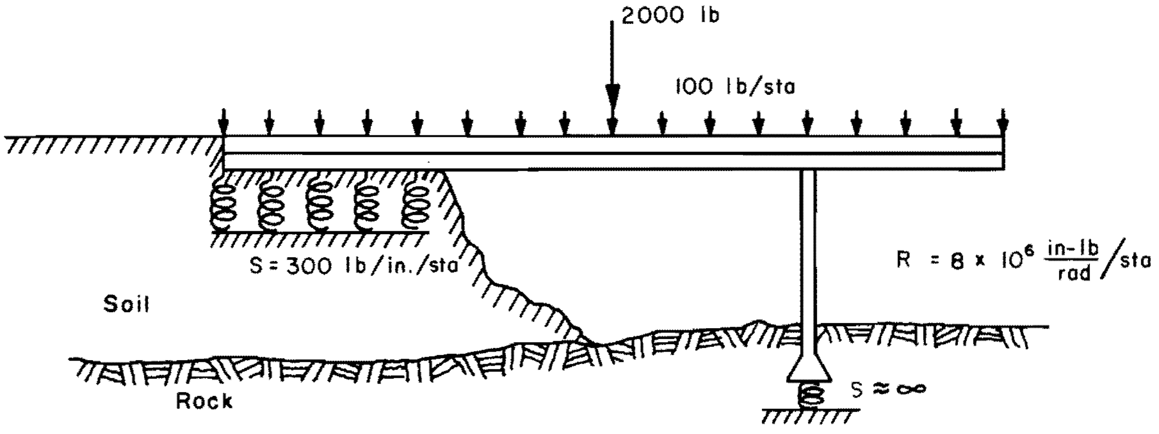
Figure 6.4 shows the deflection contours for this problem. The deflections are greatest in the region near the concentrated load and pass through a zero contour at the line of column supports. A very small amount of positive deflection occurs at the end of the longitudinal beam which carries the concentrated load. Results are in reasonable agreement with those of Tucker. The deflections are slightly less than those given by Tucker due to the increase in girder stiffness in the present problem.

Example Problem 6.3. Girder-Diaphragm System of a Bridge

Example 6.3, shown in Fig 6.5, might represent the beam-girder-diaphragm system of a bridge. It has 16 increments in one direction and 10 in the other. It is supported by two large simply-supported beams. A uniform load of 2000 lb/sta acts over the whole system. An additional uniform load is shown in one section. The load from this section was assumed as 5000 lb/sta.



(a) PLAN



(b) ELEVATION

Fig 6.3. Example Problem 6.2. Plan and elevation of structural support system.

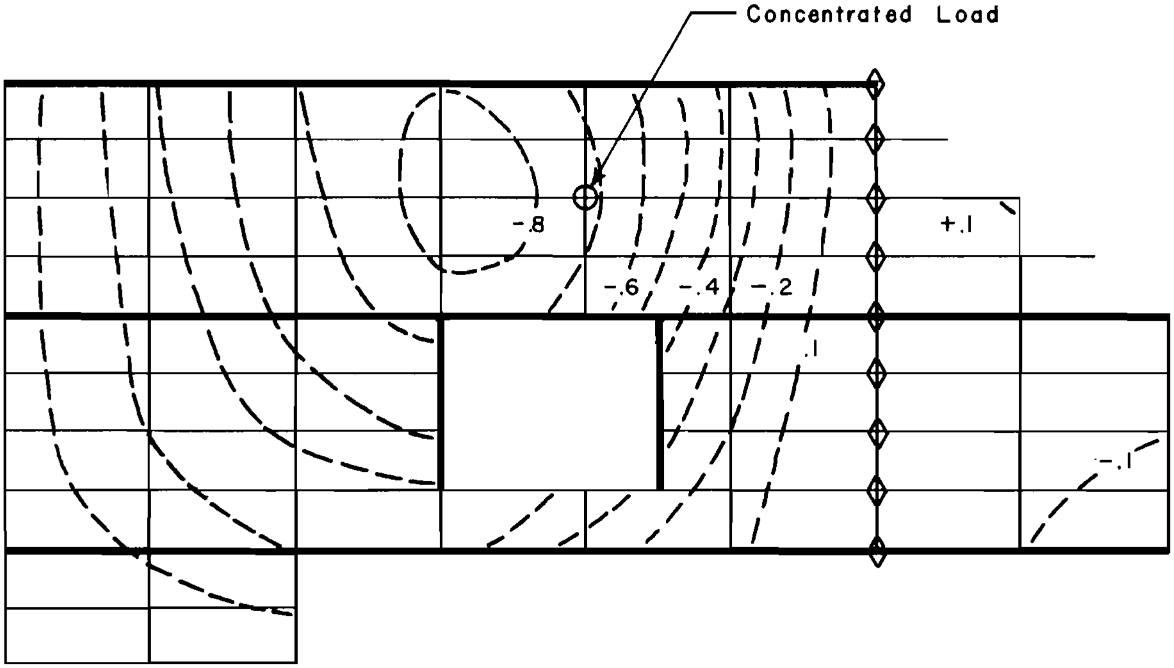


Fig 6.4. Example Problem 6.2. Deflection contours in inches for structural support system.

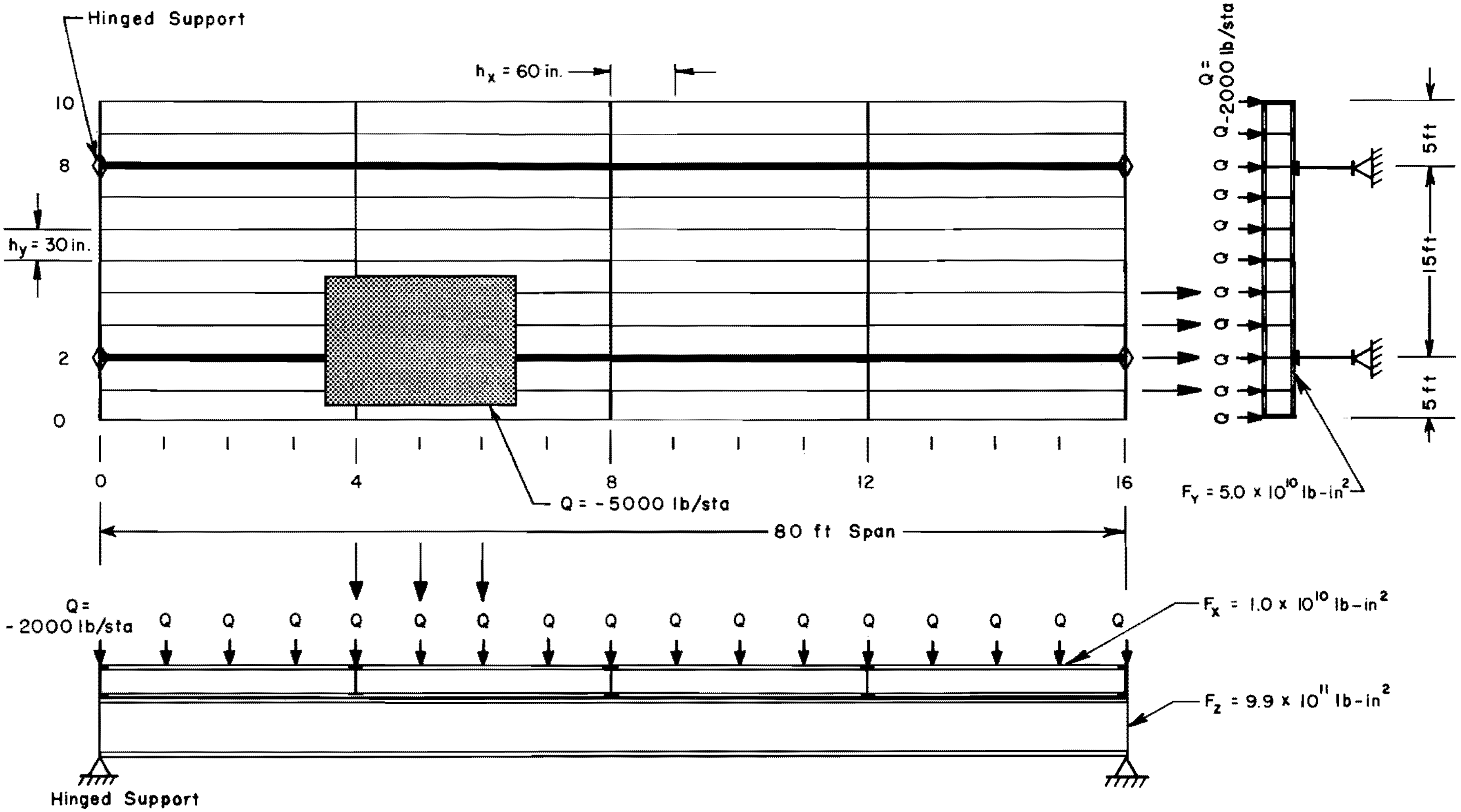


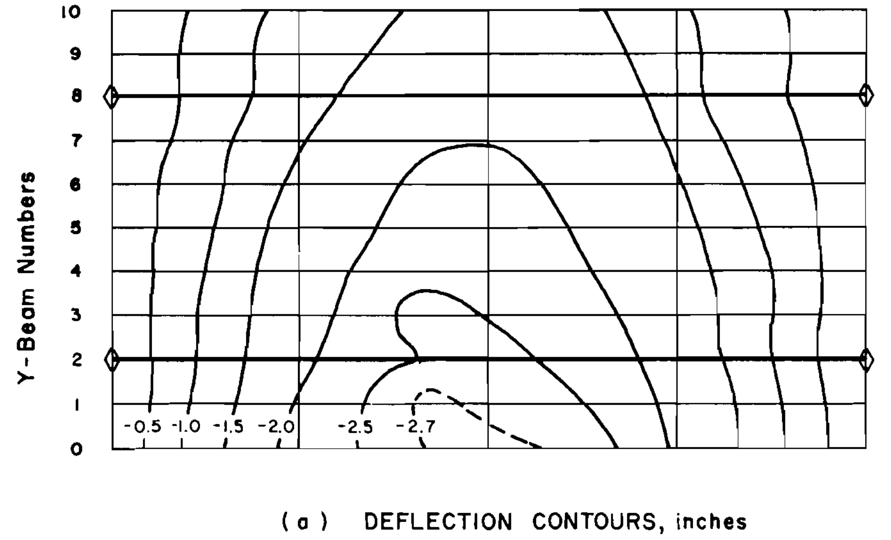
Fig 6.5. Example Problem 6.3. Girder and beam-diaphragm system of a bridge.

Deflection contours for this system are shown in Fig 6.6a. Moments in the x-beams and girders are shown in Fig 6.6b. The moments in the girders approach parabolic form but the magnitude is less than would be expected from a uniformly loaded beam.

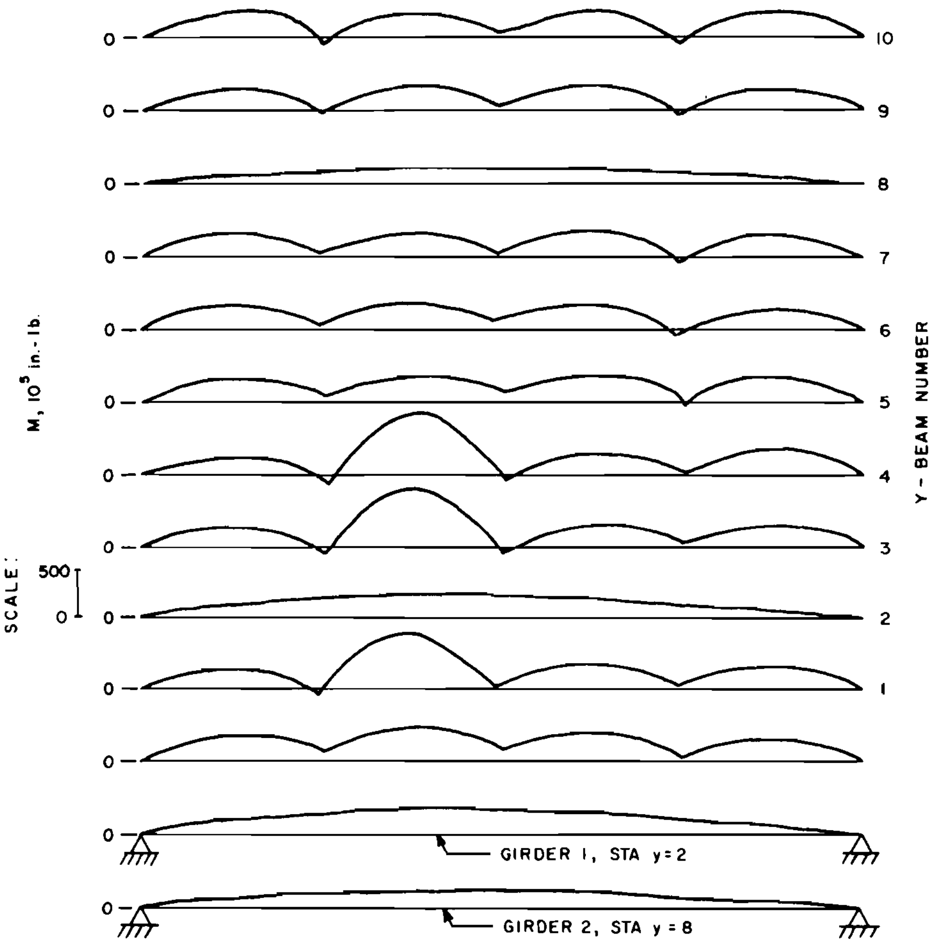
Example Problem 6.4. Cantilever Bridge Substructure

Figure 6.7 shows the structure selected for the fourth example. It represents the beam-girder-diaphragm system of a large bridge, in this case a cantilever-type supported by two piers. Frame action is not considered except for the resistance to rotation furnished by the piers at the points of connection. A dead weight of 5 kips/sta was used and concentrated loads of 200 kips/sta were placed at the cantilever end. The various characteristics of the problem are noted in Fig 6.7. An increment length of 20 feet was used in the x-direction while an increment length of 4 feet was used in the y-direction for a total size of 500 feet by 20 feet. Two longitudinal girders were placed under the grid system 4 feet from each edge as shown in Fig 6.7.

Deflection contours for the system and the deflected shape of one of the beams under the system are shown in Fig 6.8. The contours show that there is a drape of the lighter transverse members over the large girders. Positive (upward) deflections occur between the support points while negative deflections exist at all other places. The dashed lines represent contours of deflection which did not fit regular contour spacing.

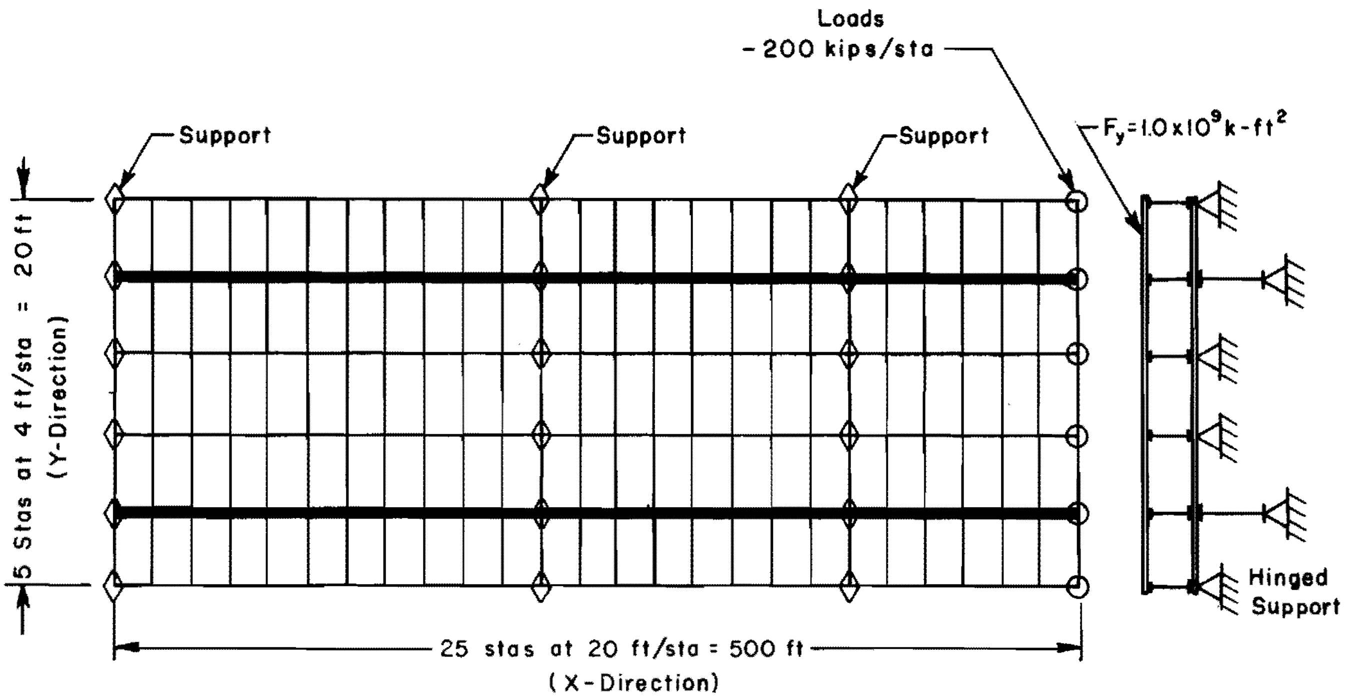


(a) DEFLECTION CONTOURS, inches



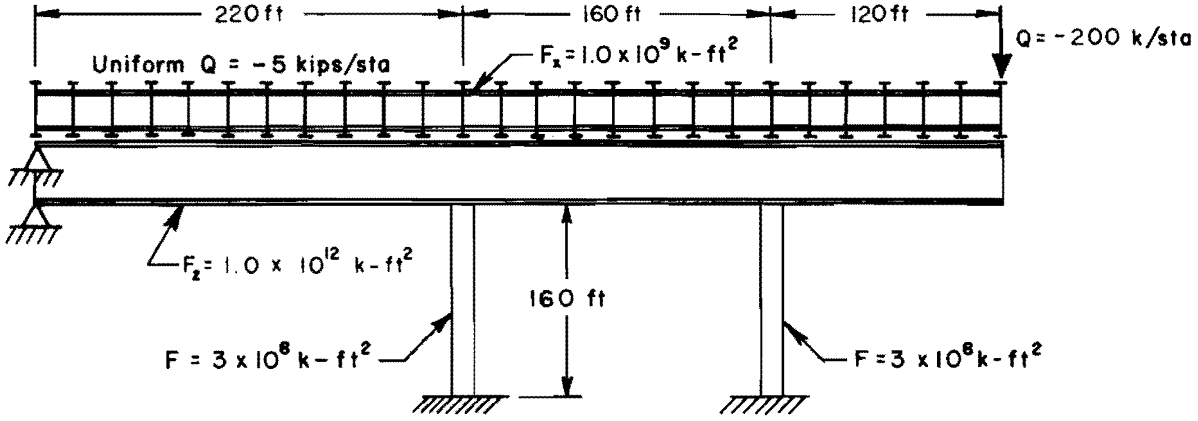
(b) BEAM MOMENT DIAGRAMS

Fig 6.6. Example Problem 6.3. Deflection contours and beam moment diagrams for girder and beam-diaphragm.



(a) PLAN

(b) TYPICAL LATERAL SECTION



(c) TYPICAL LONGITUDINAL SECTION

Fig 6.7. Example Problem 6.4. Substructure of large cantilever-type bridge.

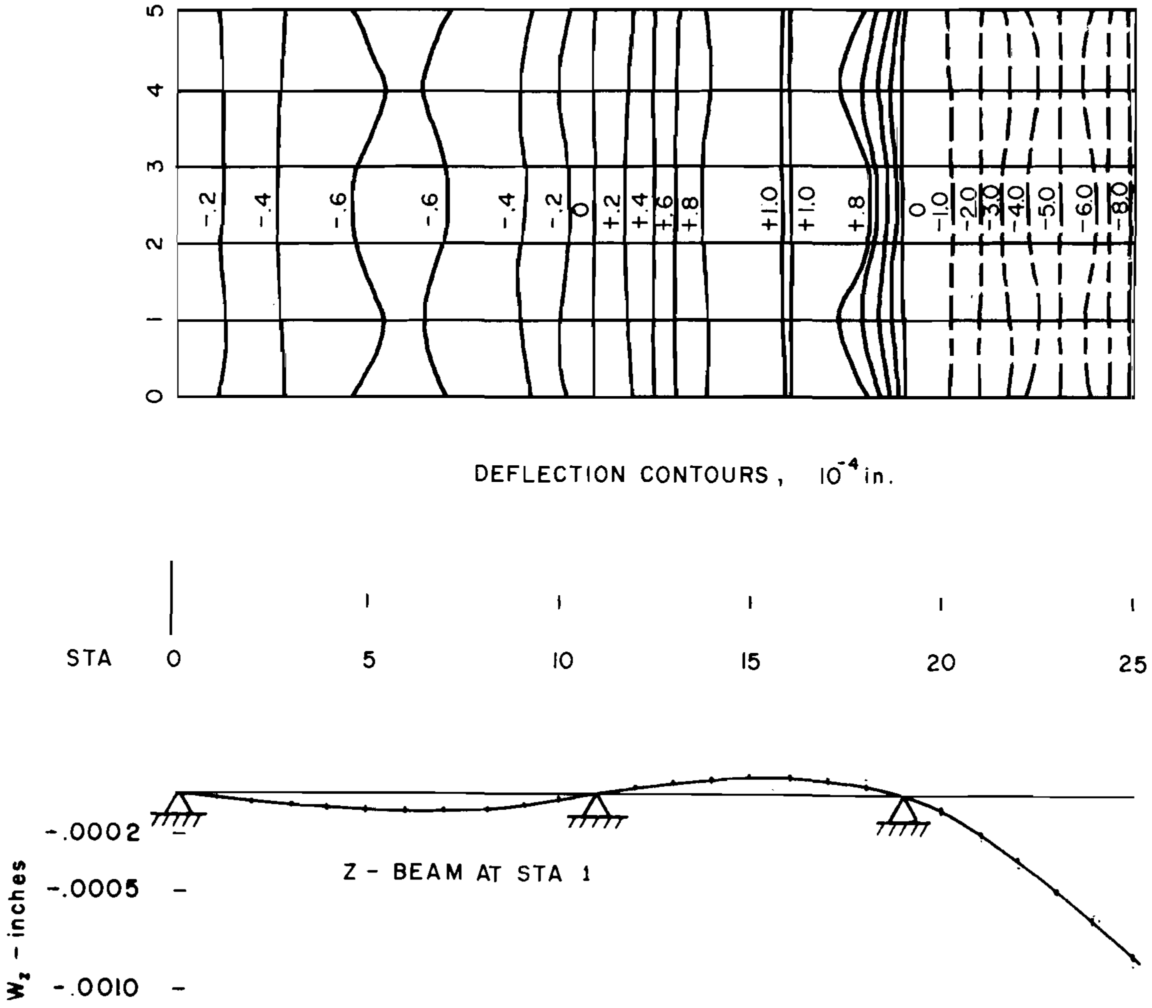


Fig 6.8. Example Problem 6.4. Deflection contours and deflected shape of one girder of cantilever bridge.

CHAPTER 7. EQUATIONS FOR THE PLATE-OVER-BEAM SYSTEM

The two primary differences between simply-connected grid-over-beam systems and orthotropic-plate-over-beam systems are torsional and Poisson's ratio effects which are included in the plate analysis. The supporting beams of each of the two different types of systems are handled in exactly the same manner. An orthotropic plate is a plate whose material and section properties may vary in orthogonal (mutually perpendicular) directions. This chapter presents the model used for the orthotropic-plate-over-beam system and discusses pertinent equations, assumptions, and boundary conditions. The complete derivation of equations for the orthotropic-plate-over-beam system is shown in Appendix 6.

Finite-Element Model of Plate-over-Beam System

The conventional equation (Ref 28) which governs the solution of a thin plate is

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad (7.1)$$

where D is the plate bending stiffness, w is the transverse deflection, and q is the load on the plate.

The equation which governs the solution of the beams supporting the plate, assuming small deflection theory, is

$$F \frac{\partial^4 w}{\partial z^4} = q \qquad (7.2)$$

While a plate is a continuum, the finite-element model is not and it is desirable to allow parameters to be freely discontinuous. It also is desirable to include other effects such as axial tension P , rotational

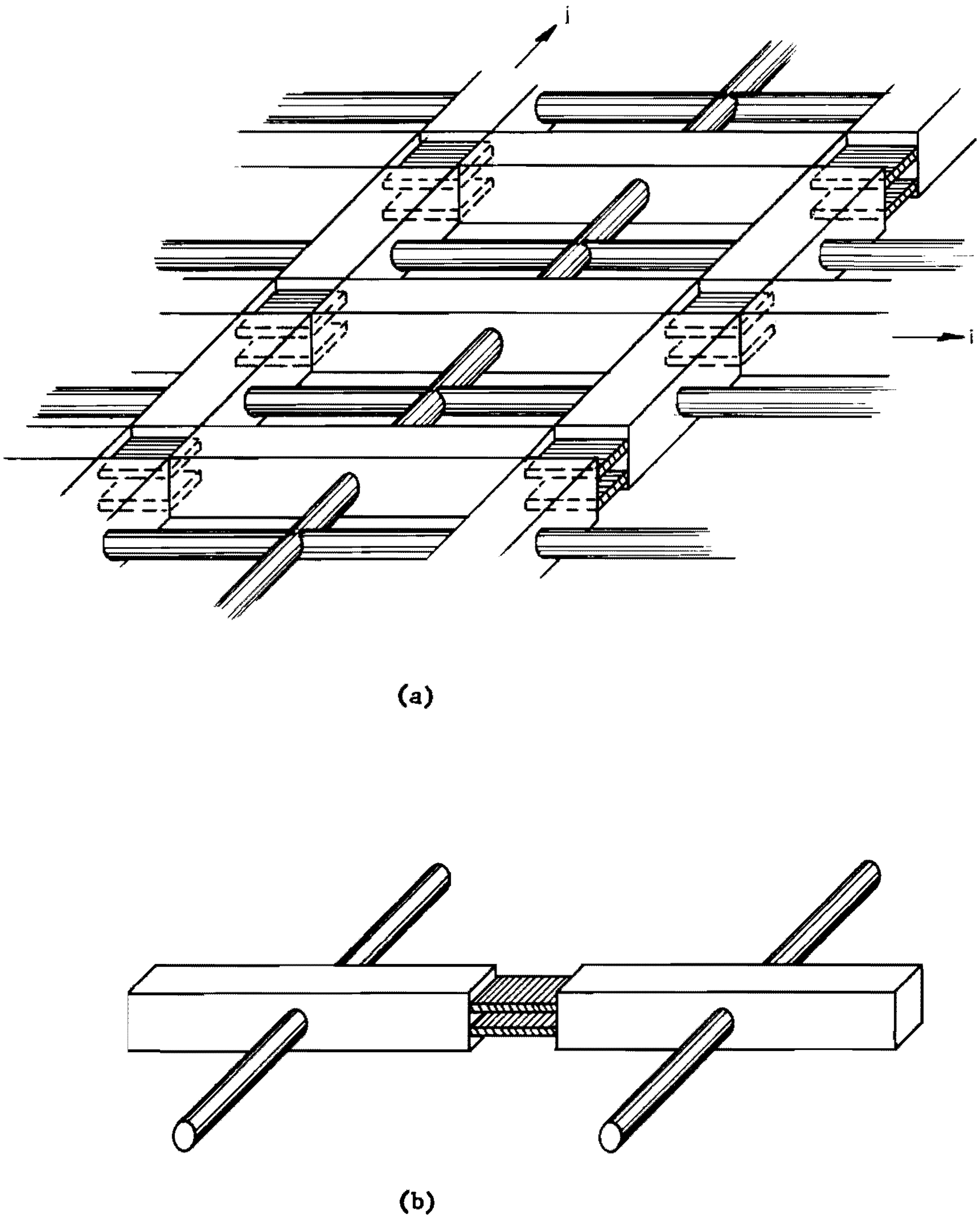


Fig 7.1. Finite-element model of orthotropic plate.

restraints r , external torques t , and elastic spring supports s . These effects are included in the general plate equation

$$D^x \frac{\partial^4 w}{\partial x^4} + D^{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D^y \frac{\partial^4 w}{\partial y^4} + \frac{\partial}{\partial x}\left(r^x \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(r^y \frac{\partial w}{\partial y}\right)$$

$$+ \frac{\partial}{\partial x}\left(P^x \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(P^y \frac{\partial w}{\partial y}\right) + \frac{\partial t^x}{\partial x} + \frac{\partial t^y}{\partial y} = q - sw \tag{7.3}$$

This equation is seen to be similar to the general equation 3.5 for the simple grid-over-beam system, with the z-beam terms omitted. The term $D^{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2}$ is the one which includes Poisson's ratio and torsion effects in the plate.

The finite-element model for the plate is shown in Fig 7.1a. It is composed of joints, rigid bars connecting the joints, and torsion bars between midpoints of rigid bars. The joints are made of a material which induces Poisson's ratio effects at the joints. Torsional stiffness of the plate is represented by the torsion bars connecting the midpoints of each of the four rigid bars which surround one mesh of the plate. Figure 7.1b shows one joint of the plate with torsion bars connected to the rigid bar on each side of the joint. The model for the beams of the plate-over-beam system is composed of rigid bars and spring-restrained hinges (Ref 21). The beams are not affected by Poisson's ratio and beam torsion is neglected.

Equations 7.4 are symbolic expressions which govern the solution of the plate-over-beam system. The first of Eqs 7.4 refers to the joints of the x-direction of the plate, the second refers to the joints of the y-direction of the plate, and the third refers to the joints of the supporting z-beam. The Q^x , Q^y , and Q^z terms include the previously mentioned effects of P , R , and T as well as the torsional stiffnesses of the torsion bars. Each of the equations represents the complete system even though some terms are different in each equation. These equations are derived from the finite-element model. The derivations are included in Appendix 6. The arrangement of Eq 7.4 is convenient for the iterative solution process since all terms on the left side of the equation could be considered as unknowns and all terms on the right side could be considered temporarily as known values in the particular phase of the iterative cycle.

$$\begin{aligned} Q^x + S_i w_i^x + K^z w_i^x + K^y w_i^x &= Q_i - Q^z - Q^y + K^z w_i^z + K^y w_i^y \\ Q^y + S_i w_i^y + K^x w_i^y + K^z w_i^y &= Q_i - Q^z - Q^x + K^x w_i^x + K^z w_i^z \qquad (7.4) \\ Q^z + S_i w_i^z + K^x w_i^z + K^y w_i^z &= Q_i - Q^x - Q^y + K^x w_i^x + K^y w_i^y \end{aligned}$$

Note that Eq 7.4 is identical to Eq 3.10 which was derived previously for the simple grid-over-beam system.

All of the the special terms have been discussed in detail elsewhere (Ref 21). However, a brief discussion is included below as each is used in the plate solution.

Rotational Restraint

The rotational spring r acts on the member in an angular sense and tends to resist rotation of the member. It has typical units of in-lb/in/rad per unit of width for the plate when shown in a distributed fashion, r . The lumped quantity of rotational restraint R will have typical units of in-lb/rad per unit of width. The finite-difference version analogous to the $\frac{\partial}{\partial x}\left(r \frac{\partial w}{\partial x}\right)$ term of Eq 7.3 is

$$\frac{\partial}{\partial x}\left(r \frac{\partial w}{\partial x}\right) = \frac{1}{4h^2}\left[r_{i-1}w_{i-2} - (r_{i-1} + r_{i+1})\, w_i + r_{i+1}w_{i+2}\right] \qquad (7.5)$$

Axial Tensions

The axial tension (or compression) may have considerable effect on the bending of a plate. The tension P has typical units of lb per unit width and is considered to act at the neutral axis of the member. In the present formulation, the change in tension occurs in the joint as shown in Appendix 6, Fig A6.1. Axial tensions are also included in the supporting beam solution.

In-plane tensions in plates may be accompanied by an in-plane shear (Ref 28). The present formulation uses principal directions only for the axial tension since only orthogonal directions are accommodated. Thus no in-plane shear is considered.

External Torques

An external torque is represented as a couple created by forces $T_i/2h$ which act at stations on each side of Station i. This term is included for cases where members frame into the system in such a manner as to apply external moments.

Foundation Spring

The foundation spring S provides an elastic foundation response as well as simple support capabilities. Very large S values restrain the deflection to virtually zero. The proper ratio of Q to S may be used to produce a specified deflection.

Assumptions of the Method

Assumptions pertinent to the use of the model in Fig 7.1 are listed below.

(1) Only vertical connections are considered between the plate and the beams. No consideration is given to horizontal shear at the plate-beam interface.

(2) All layers deflect equally at common joints.

(3) Deformations are small; plane sections remain plane after bending.

(4) All loads, including dead loads, act at the joints; no external forces act directly on the rigid bars.

(5) The bars between joints are rigid and weightless.

(6) Axial tensions do not produce axial deformations. In-plane shear is not considered.

(7) The joint may have the properties of an orthotropic material.

(8) The length of increment is constant on all parallel beams.

(9) All loads and stiffnesses may vary in a freely discontinuous manner.

(10) Torsion in the supporting beams is not considered.

Boundary Conditions

A variety of edge conditions may be considered for the plate-over-beam system. The conventional edge conditions are simply-supported, free, and elastically supported by beams. Other conditions available by use of the present method include soil support and rotationally-restrained edges as well as the conventional conditions. The boundary or edge conditions are created automatically by data input to the general purpose finite-element equations previously formulated in Chapter 3. A discussion follows concerning the conventional edge conditions. The equations which follow are not used directly in the solution process.

Simply-Supported Edge. When an edge of a plate is simply supported, the deflection along that edge must be zero along with the bending moment at that edge (Ref 28). Equations 7.6 state these two conditions. The simple support condition is created by the use of a foundation spring of large numerical magnitude along the edge.

$$w = 0$$

$$\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0 \tag{7.6}$$

Free Edge. The bending moment normal to the free edge of a plate must vanish. Along the free edge there are no twisting moments nor shearing forces. The twisting moment and shearing force conditions can be combined to reduce the total number of conditions at a free edge to two (Ref 28). These two conditions are stated in Eqs 7.7 where the first is the bending moment condition and the second is the twisting moment condition.

$$\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0$$

$$\frac{\partial^3 w}{\partial x^3} + (2 - \nu) \frac{\partial^3 w}{\partial x \partial y^2} = 0 \tag{7.7}$$

Edge Elastically Supported by a Beam. A frequently encountered condition in the plate-over-beam system is an edge supported elastically by a beam. In this case, the bending moment normal to the edge must vanish and pressure from the plate to the beam must equal the reaction in the beam (Ref 28). These two conditions are

$$\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} = 0$$

$$D \left[\frac{\partial^3 w}{\partial x^3} + (2 - \nu) \frac{\partial^3 w}{\partial x \partial y^2}\right] = F \frac{\partial^4 w}{\partial x^4} \tag{7.8}$$

The above conditions assume isotropic conditions for the plate and constant stiffness for the beam.

CHAPTER 8. CLOSURE PARAMETER SELECTION AND COMPUTER SOLUTION FOR PLATES-OVER-BEAMS

The concepts developed in Chapter 4 concerning closure parameters of simply-connected systems extend almost directly to the closure parameters for a plate. This chapter presents the necessary equations for computing closure parameters for a plate and briefly discusses a computer program which solves for the bending of an orthotropic plate over a series of parallel beams.

Closure Parameters for a Plate

An expression for an eigenvalue of a simply-supported isotropic plate is developed in Appendix 8. The equation for calculation of the eigenvalue, λn , is

$$\frac{h^4 \lambda_n}{D} = 4 \; (1 - \cos \frac{n\pi}{M}) \; (2 - \cos \frac{n\pi}{M}) \tag{8.1}$$

On a units basis, the eigenvalues are related to the closure springs needed in the computer solution. This relationship is

$$\overline{SF} = \frac{D}{h^2} (4) \; (1 - \cos \frac{n\pi}{M}) \; (2 - \cos \frac{n\pi}{M}) \tag{8.2}$$

The number of closure springs to use may be determined by the method presented in Chapter 4 for simply-connected layered systems. The closure springs for the supporting beams are the same as presented in Chapter 4.

Program LAYER 8

The method of beam solution described in Chapter 5 is used for the solution of the individual strips of a plate. If the equations developed in Appendix 6 were substituted in Fig 5.2b, then the solution process necessary for the plate-over-beam system would be completely outlined in Fig 5.3.

Program LAYER 8 is written in FORTRAN 63 language for the CDC 1604 or CDC 6600 digital computers. A detailed flow diagram of this computer program is in Appendix 9.

Notation for Program LAYER 8 is in Appendix 10 with a complete listing of the source program. Compilation time for this FORTRAN program is approximately 2 minutes. The maximum number of strips into which the plate may be divided is 25 by 25. Several variables have double meanings as shown in the Notation listing. This is only to aid in the printout of all necessary results. An input form is in Appendix 10 along with additional comments about the program and warnings to the user.

CHAPTER 9. EXAMPLE PLATE-OVER-BEAM PROBLEMS

Four example problems follow which indicate the capabilities of the method of analysis for plate and plate-over-beam problems. Two of the problems selected are compared to theoretical solutions given by Timoshenko and Woinowsky-Krieger (Ref 28). Sample input for all 4 examples and output data for the second and fourth examples are in Appendix 13.

Example Problem 9.1. Isotropic Plate with Concentrated Load

A simple isotropic plate with Poisson's ratio of 0.25 is shown in Fig 9.1. The plate is 48 inches square and 0.98 inches thick, with simply-supported boundaries and has one concentrated load of 10^5 pounds at the center. There is a constant bending stiffness throughout. The plate is divided into sixteen 3-inch strips in the x-direction and eight 6-inch strips in the y-direction. No supporting beams are used. The simple supports are created by specifying very stiff foundation springs in the input data. The modulus of elasticity is 3.0×10^7 lbs/in^2.

Figure 9.2 shows deflection contours resulting from this problem. The deflection at the midpoint of the plate is 1.11 inches which is in very close agreement with 1.07 inches given by the closed-form solution of Timoshenko and Woinowsky-Krieger. Comparison of the corner uplift forces is accomplished by taking the product of the foundation spring stiffness S and the corner deflection w. The computed results show a value of 1.28×10^4 pounds versus 1.219×10^4 pounds from Timoshenko and Woinowsky-Krieger.

Example Problem 9.2. End-Supported Plate with Two Edge Beams

The second example is shown in Fig 9.3. It is a plate 10 feet square and 2.02 inches thick, simply supported along two edges and supported along the

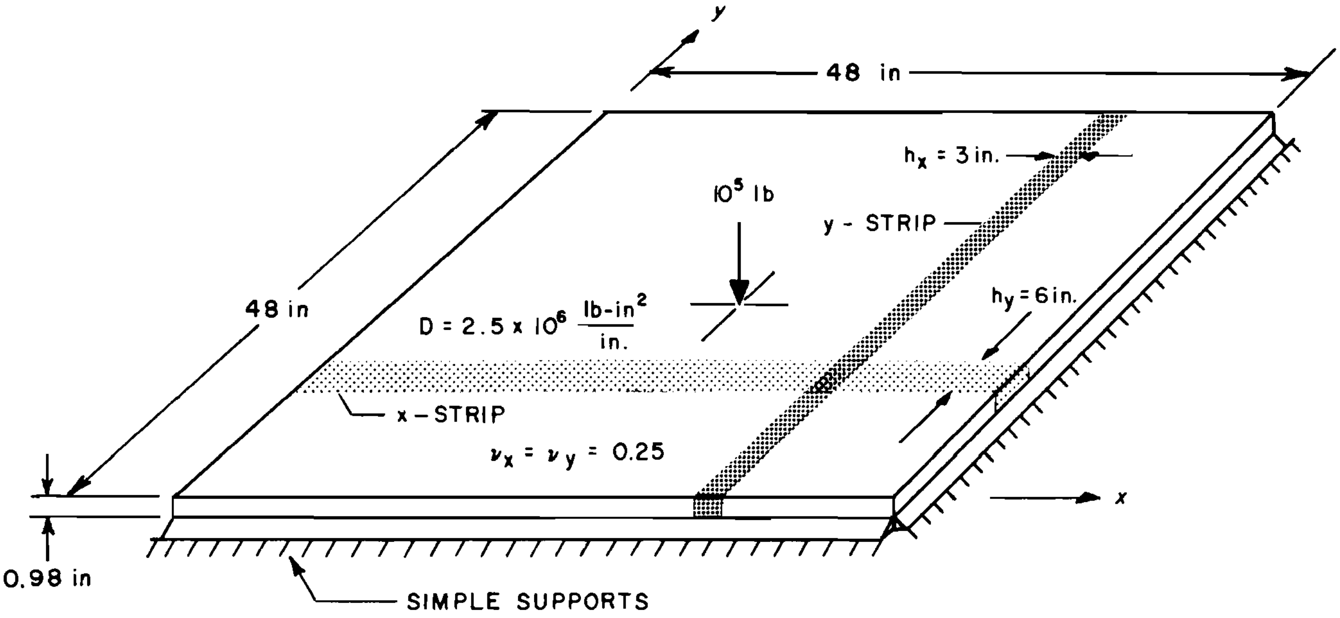


Fig 9.1. Example Problem 9.1. Simply-supported square plate with concentrated load.

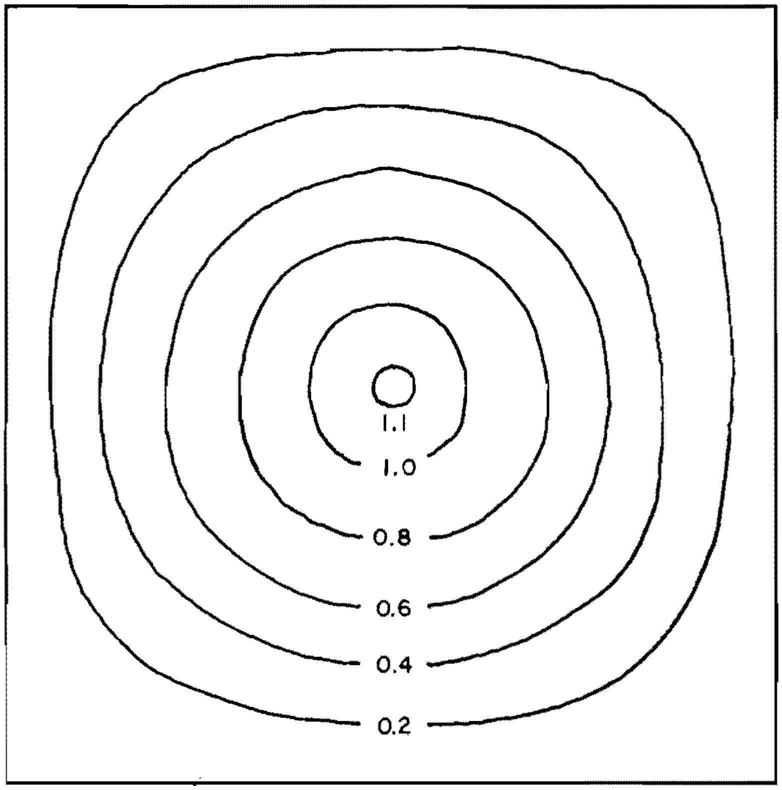


Fig 9.2. Example Problem 9.1. Deflection contours in inches for the simply-supported square plate.

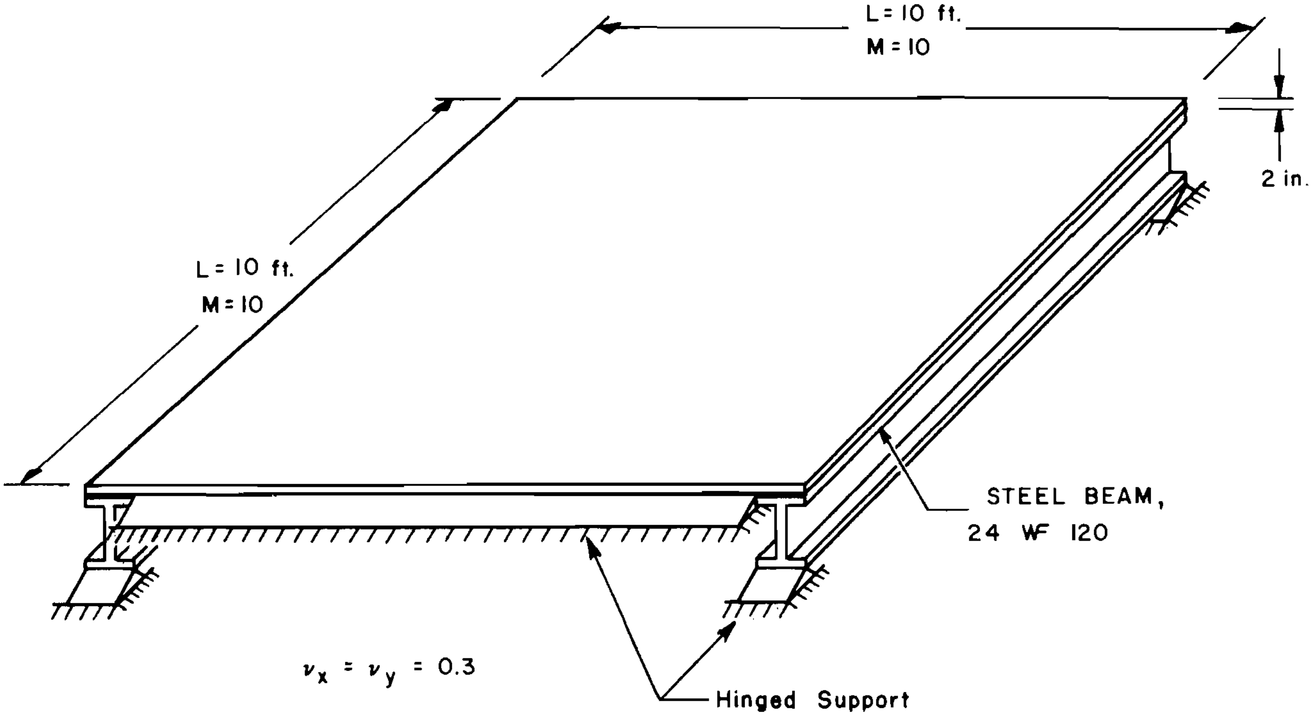


Fig 9.3. Example Problem 9.2. End-supported plate with two flexible edge beams.

remaining two edges by a flexible steel beam. The supporting beams are 24 WF 120 beams with flexural stiffnesses of 1.09 X 10^{11} lb-in^2. The system is divided into 10 increments in each direction with a strip width of 12 inches. Poisson's ratio is 0.3 for the plate, and a uniform transverse load is applied with a magnitude of 1000 lb/sta. Young's modulus for the steel is assumed to be 3.0 X 10^7 lb/in^2.

Figure 9.4 shows contours of the deflected shape of this structure. The maximum deflection at the midpoint of 0.265 inch compares favorably with the theoretical deflection of 0.272 inch found by Timoshenko and Woinowsky-Krieger. The bending moment in the x-direction at the midpoint is computed as 4713 in-lb. The bending moment in the y-direction computes to 4640 in-lb, as compared to a theoretical value of 4730 in-lb.

A smaller increment length would undoubtedly give closer results. A smaller tolerance on deflection would give a smoother deflected shape and probably a slightly better set of bending moments.

Example Problem 9.3. Ribbed-Plate Structure over Three Supports

A ribbed-plate structure over three supports is shown in Fig 9.5. Each span is 20-ft long and the structure is 25-ft wide. There is an assumed dead load of 2000 lb/sta. Two girders, each of bending stiffness 1 X 10^{12} lb-in^2, support the plate. A live load of 5000 lb/sta is used at each of twelve stations, as shown by the dashed-line rectangle in Fig 9.5a. The plate is divided into 16 increments in the x-direction and 10 increments in the y-direction. An increment length of 30 inches is used in each direction. Due to an effective thickness of the plate that is greater in the x-direction, the bending stiffness of the plate in the x-direction is greater than the stiffness in the y-direction. Thus geometry considerations give an "apparent" orthotropic

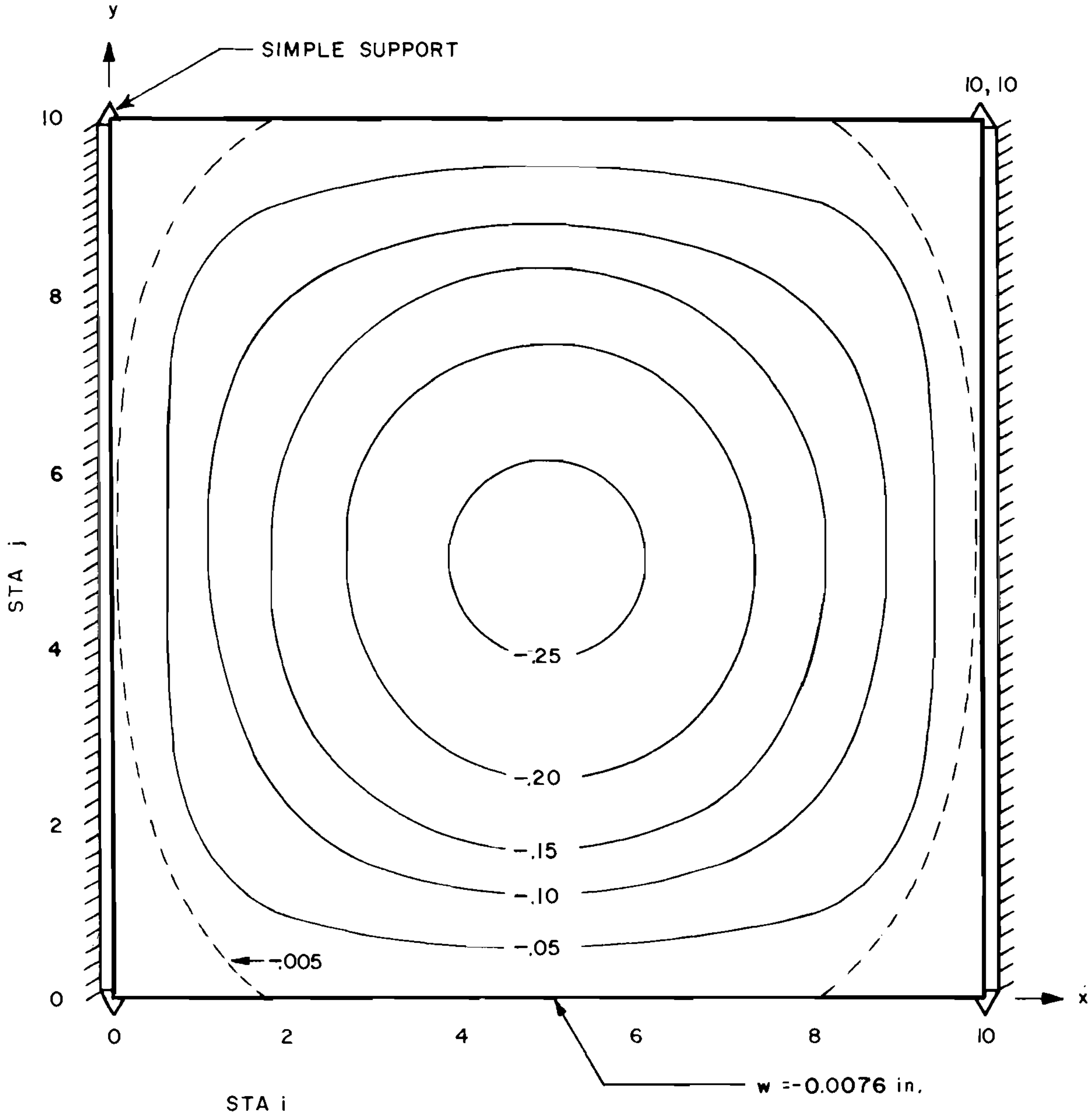
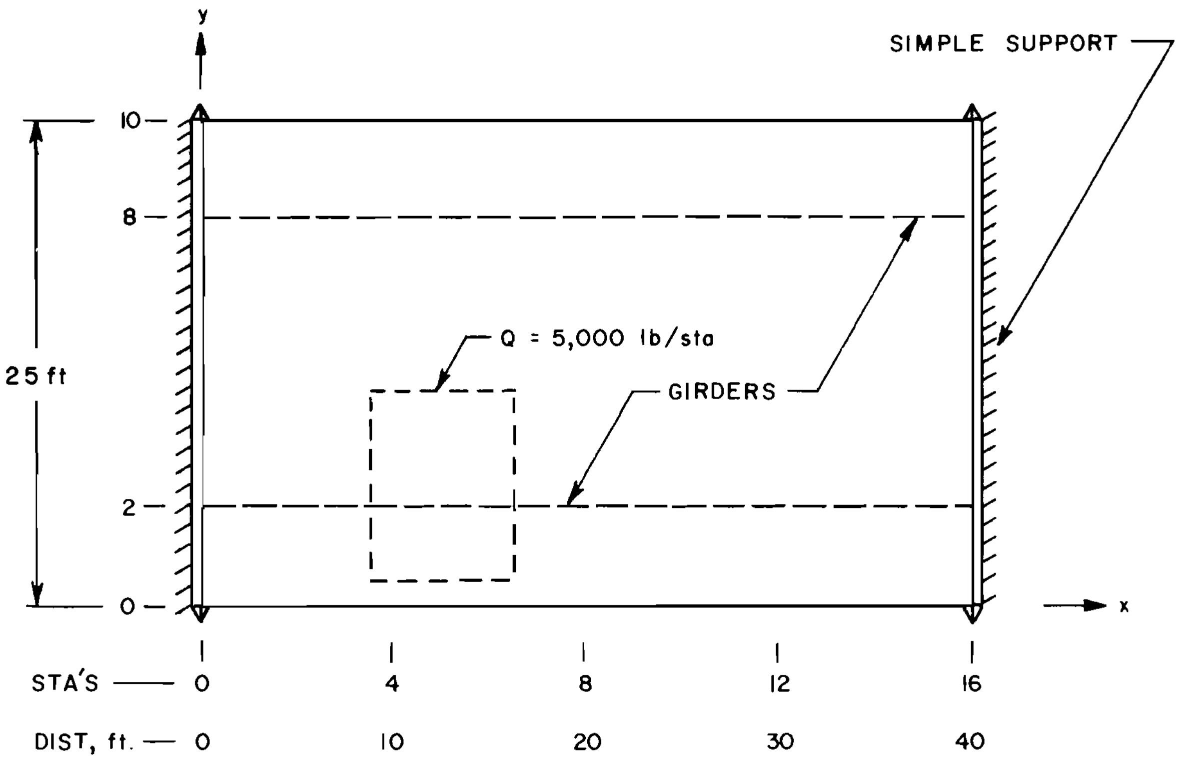
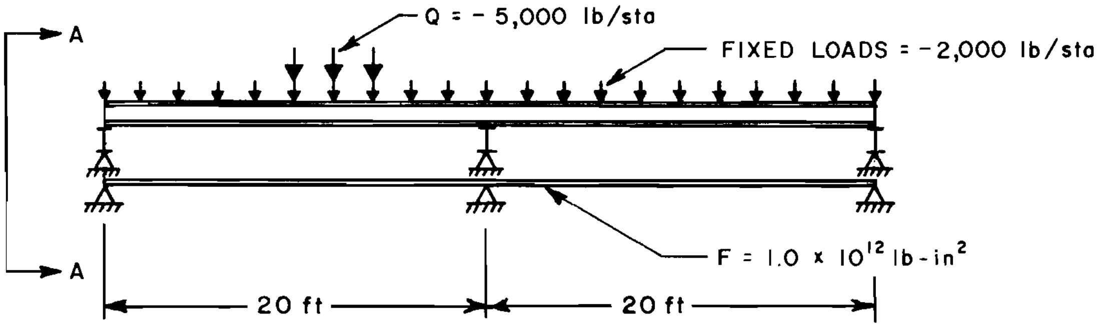


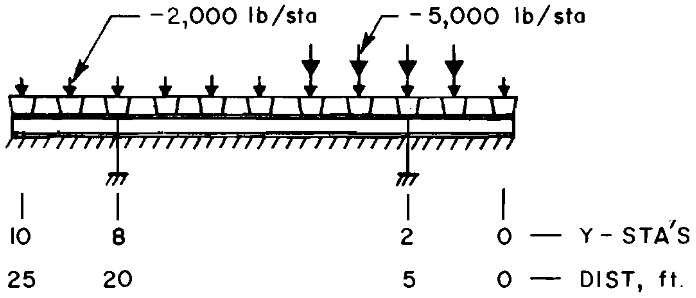
Fig 9.4. Example Problem 9.2. Deflection contours in inches for end-supported plate with edge beams.



(a) PLAN



(b) ELEVATION



(c) END VIEW A-A

Fig 9.5. Example Problem 9.3. Ribbed-plate structure over three supports.

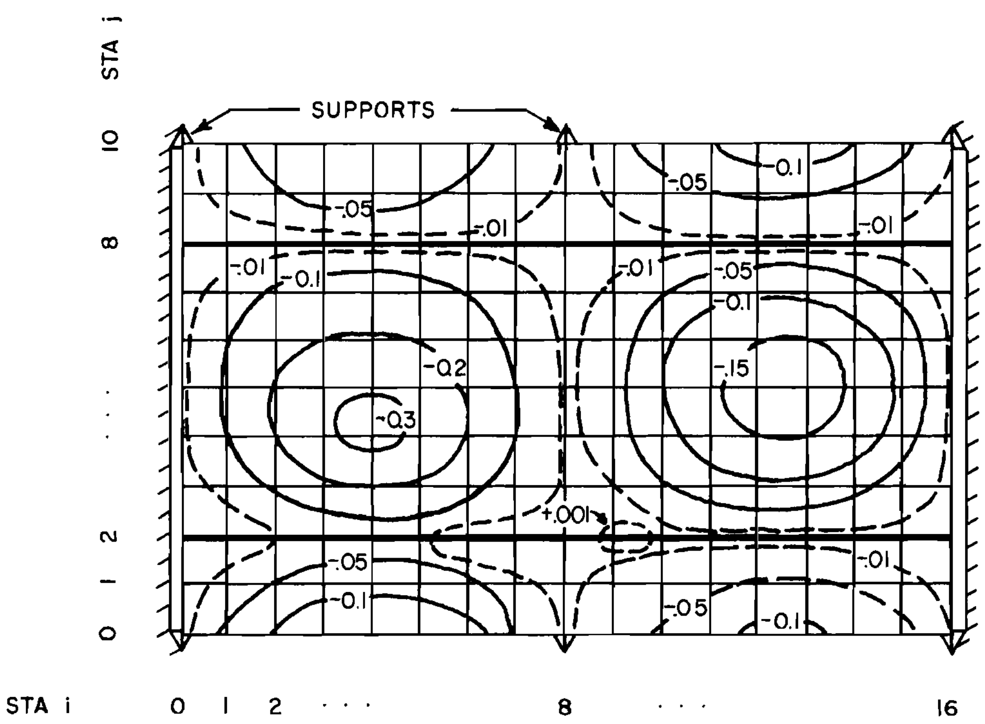
condition even though all material constants are the same. The torsional stiffness is taken as 70 per cent of the average plate stiffness since Poisson's ratio for the steel was assumed to be 0.3.

Results from this problem are in Fig 9.6. The maximum deflection observed was 0.30 in. Contours of deflection are shown in Fig 9.6a. Figure 9.6b shows the deflection of one of the girders. Positive deflections occurred on the girder just past the interior support. Twenty iterations of solution and 73 seconds of computation time were required to achieve a deflection closure tolerance of 0.0001 inch.

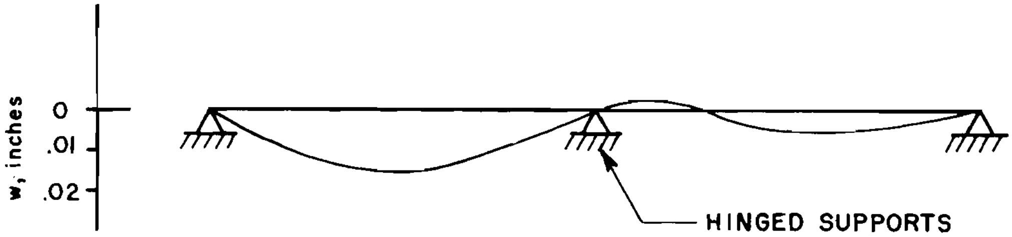
Example Problem 9.4. Concrete Slab over Beams

As shown in Fig 9.7, the structure is a single-span reinforced concrete slab supported by five girders. The span length is 60 ft and the structure width is 30 feet. The slab and the girders are simply supported at the ends. An increment length of 36 inches is used in each direction. Four concentrated loads of 1000 lb each are applied symmetrically about the center of the bridge. Poisson's ratio for the concrete was assumed to be 0.15 and Young's modulus for the concrete was taken as 3.0×10^6 lb/in^2.

Figure 9.8a shows contours of equal deflection for the slab. Figures 9.8b and 9.8c are deflected shapes in the x and y-directions respectively at the center of the slab. The maximum deflection of 0.052 in. occurs at the center. This solution reached a deflection closure tolerance of 1×10^{-5} in. in 44 iterations. Three minutes and thirty seconds of computer time were required.

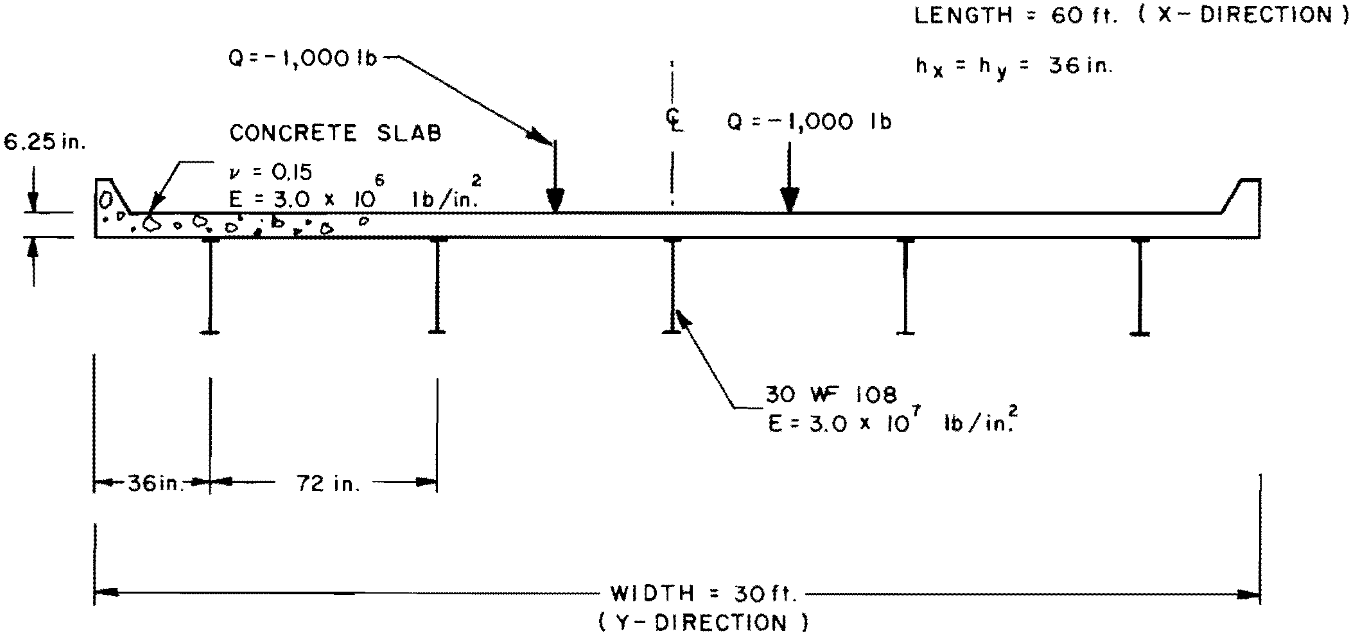


(a) DEFLECTION CONTOURS, inches



(b) DEFLECTION OF GIRDER AT STA j = 2

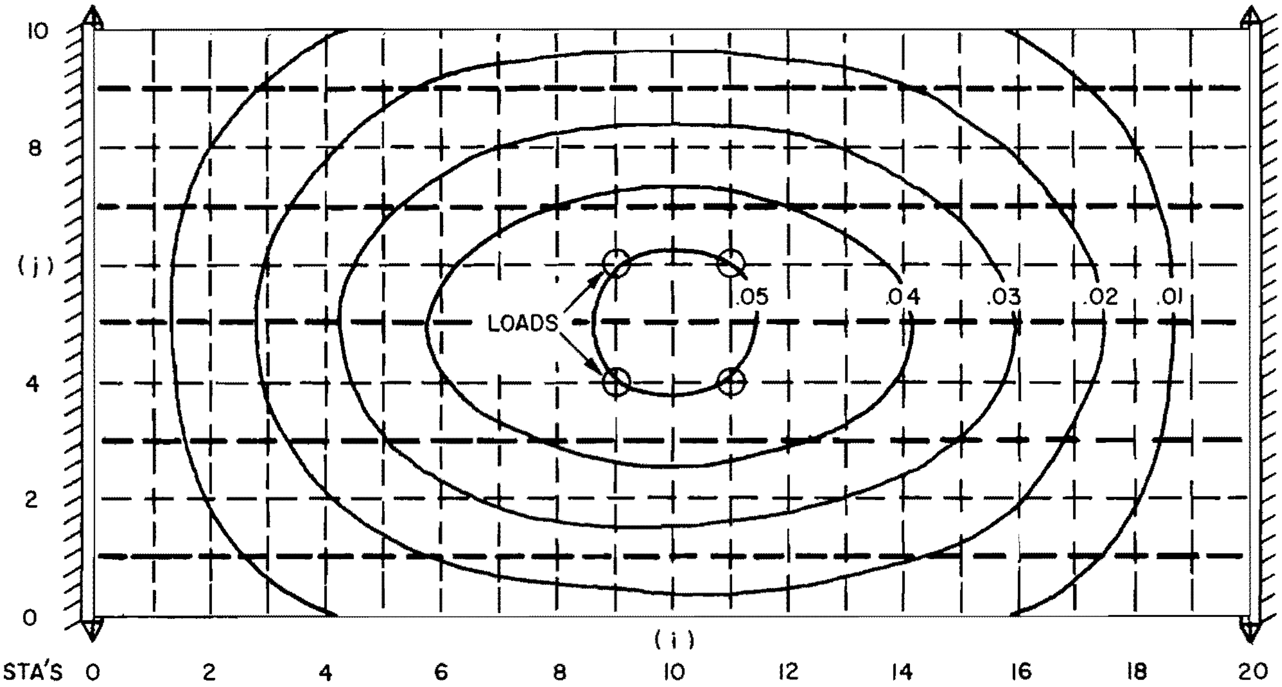
Fig 9.6. Example Problem 9.3. Deflection contours and girder deflection for ribbed-plate structure.



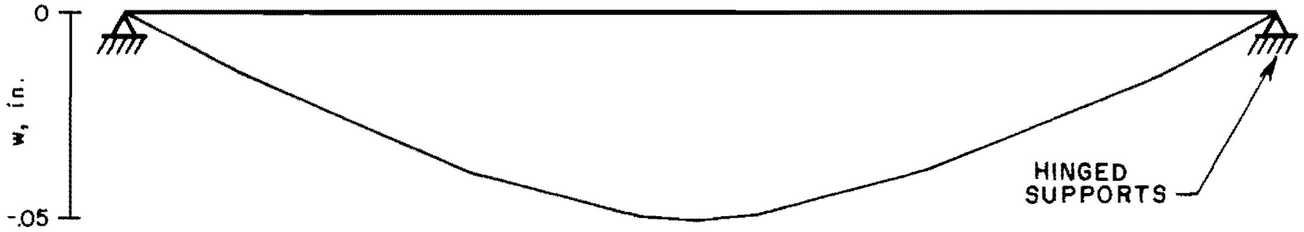
NOTE:

Four loads of 1,000 lb each placed symmetrically about center of structure.

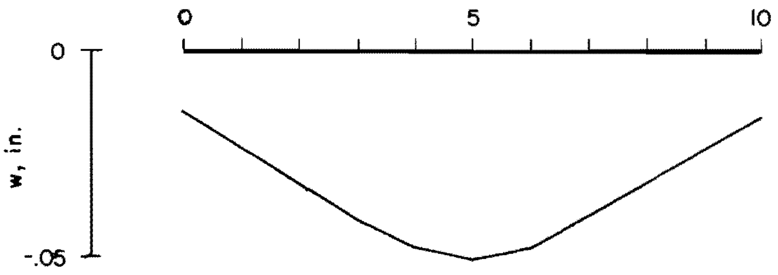
Fig 9.7. Example Problem 9.4. Typical section of end-supported slab-on-girder bridge.



(a) DEFLECTION CONTOURS, inches



(b) DEFLECTION SHAPE OF MIDDLE GIRDER (sta j = 5)



(c) DEFLECTED SHAPE AT MIDSPAN (sta i = 10)

Fig 9.8. Example Problem 9.4. Deflection contours and beam deflected shapes for slab-on-girder bridge.

CHAPTER 10. CONCLUSIONS AND RECOMMENDATIONS

A method of analysis for the bending of structural systems composed of layers of grids or plates over parallel beams has been presented. The numerical procedure has been shown to be rapid and accurate. It has been applied to both simple and complex problems and favorably compared to theoretical solutions. A method for the selection of closure parameters for the alternating-direction iterative solution has also been presented.

The following conclusions have been reached.

(1) The method of analysis is applicable to a wide variety of problems of complex geometry, loading, stiffness, and support conditions. Orthotropic as well as isotropic plate stiffnesses are allowable in the plate-over-beam solution.

(2) The method is accurate. Problems were compared to theoretical answers with close agreement.

(3) The method for selection of the proper range of closure parameters gives adequate results. The optimum number of parameters to use is open to question. The result, if poor selections are made, is that the time to solve the problem will increase.

(4) The concept of closure parameters being based on the opposing or replaced beams is applicable for systems of the type studied. The method presented predicts the same closure parameters for each beam of the system when uniform geometry and stiffness exist. Different values of parameters are predicted when geometry and stiffness conditions are allowed to vary.

(5) The method presented for the solution of the layered structural system also is directly applicable to the cases of simple grids, isotropic plates or orthotropic plates without supporting beams.

The method which has been described furnishes a much more general means of analyzing certain structural problems than has been previously available. It is envisioned that some extension to the method might be desirable in order to consider an even wider class of problems than those presented. The following comments are deemed pertinent to extensions and further uses of this method.

(1) The in-plane or axial tensions acting on such systems need additional study. Hondros and Marsh (Ref 13) point out that the neutral planes of girders do not stay coplanar or stationary and that distributed in-plane forces will be induced due to bending. The present program does not allow a varying neutral surface, but the mechanics of induced in-plane forces could be handled readily. In-plane shear will need to be included in the solution when directions other than principal directions are considered.

(2) A natural closure parameter selection process, such as the one described briefly, offers the possibility of an extremely efficient and straightforward closure mechanism for the alternating-direction techniques. A detailed study of such a closure selection process is recommended.

(3) Extensions of the method to handle non-orthogonal beams and to solve more than three layers of beams may be desirable. The non-orthogonal support system appears to be necessary for certain classes of structures, such as on-off ramps for freeways. A maximum of five layers is envisioned for any one problem that could consist of a plate and a grid over a set of beams.

(4) Horizontal shear at the plate-beam or grid-beam interface should be studied as it pertains to full composite action of the structure. There appear to be ways of approximating this effect, such as using an increased but equivalent stiffness in the beams or by use of iterative solutions which adjust axial tension and induced couples to achieve compatibility.

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APPENDIX 1

DERIVATION OF EQUATIONS FOR A SIMPLE GRID OVER PARALLEL BEAMS

APPENDIX 1. DERIVATION OF EQUATIONS FOR A SIMPLE GRID OVER PARALLEL BEAMS

Equations 3.10 were presented in Chapter 3 as the governing equations for the layered system. These equations involve the assumption that the loads Q and the springs S act on all beams of the system. The fictitious springs or closure parameters K^x, K^y, and K^z are temporary representations of the x y and z-beams respectively and serve to add stability as well as rapid closure to the system. Likewise, the rotational restraints R^x, R^y, R^z, and the external torques T^x, T^y, T^z and the axial forces P^x, P^y, and P^z act on the individual beams.

Consider Fig A1.1 which represents a free-body of a section of an x-beam with the y and z-beams assumed temporarily constant and represented by the terms $Q^y_{i,j}$, $Q^z_{i,j}$, K^y, and K^z. All forces pertinent to the behavior of the system are shown. The external torques T and the rotational restraints R, both of which are considered to act over a distance of two increment lengths, have been reduced to their force counterparts which act at some i^{th} joint of the j^{th} beam. The rigid bars are assumed to have no external forces applied. Thus, the axial tensions P and the shear V will change at the i^{th} joint. A complete derivation of the equations governing the behavior of this layered system follows.

Considering only the one joint of an x-beam shown in Fig A1.1 and summing forces in the vertical direction results in Eq A1.1.

$$Q_{i,j} - S_i w^x_{i,j} - Q^y_{i,j} - Q^z_{i,j} + V^x_{i,j} - V^x_{i+1,j} - K^y(w^x_{i,j} - w^y_{i,j}) - K^z(w^x_{i,j} - w^z_{i,j}) + \frac{R^x_{i+1,j}\,\theta^x_{i+1,j}}{2h_x} - \frac{R^x_{i-1,j}\,\theta^x_{i-1,j}}{2h_x} + \frac{T^x_{i+1,j}}{2h_x} - \frac{T^x_{i-1,j}}{2h_x} = 0 \qquad \text{(A1.1)}$$

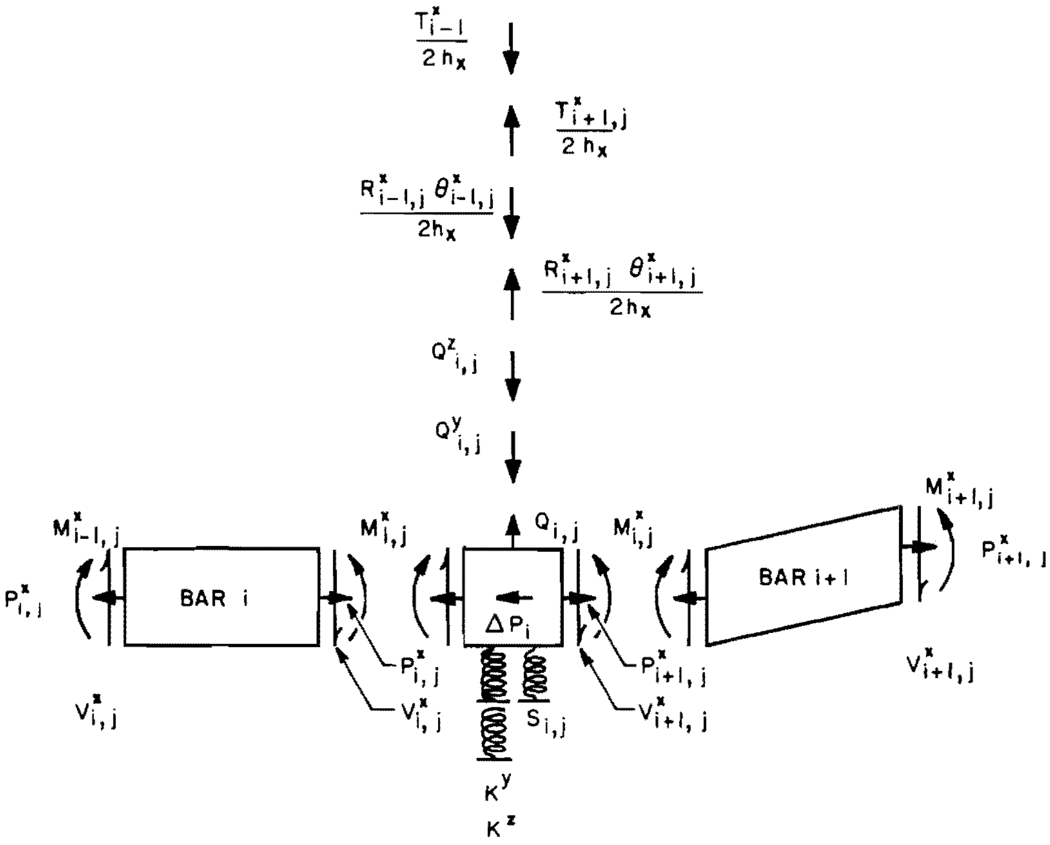


Fig A1.1 An x-direction portion of the grid-over-beam system.

The shear $V^x_{i,j}$ can be obtained by summing moments about the left end of Bar i in Fig A1.1, i.e.,

$$M^x_{i-1,j} + V^x_{i,j}\, h_x + P^x_{i,j}\,(-\, w^x_{i-1,j} + w^x_{i,j}) - M^x_{i,j} = 0 \qquad \text{(A1.2)}$$

and solving for the shear $V^x_{i,j}$

$$V^x_{i,j} = \frac{1}{h_x}\left[-\, M^x_{i-1,j} + M^x_{i,j} - P^x_{i,j}\,(-\, w^x_{i-1,j} + w^x_{i,j})\right] \qquad \text{(A1.3)}$$

A similar expression for the shear $V^x_{i+1,j}$ may be obtained by summing moments about the right end of Bar i+1 in Fig A1.1, i.e.

$$V^x_{i+1,j} = \frac{1}{h_x}\left[-\, M^x_{i,j} + M^x_{i+1,j} - P^x_{i+1,j}\,(-\, w^x_{i,j} + w^x_{i+1,j})\right] \qquad \text{(A1.4)}$$

Substituting Eqs A1.3 and A1.4 into A1.1 shows that

$$\begin{aligned} h_x &\left[Q_{i,j} - Q^y_{i,j} - Q^z_{i,j} - S_{i,j} w^x_{i,j} - K^y(w^x_{i,j} - w^y_{i,j})\right. \\ &\left. -\, K^z(w^x_{i,j} - w^z_{i,j})\right] - M^x_{i-1,j} + 2M^x_{i,j} - M^x_{i+1,j} \\ &-\, P^x_{i,j}\,(-\, w^x_{i-1,j} + w^x_{i,j}) + P^x_{i+1,j}\,(-\, w^x_{i,j} + w^x_{i+1,j}) \\ &+\, \frac{R^x_{i+1,j}\ \theta^x_{i+1,j}}{2} - \frac{R^x_{i-1,j}\ \theta^x_{i-1,j}}{2} + \frac{T^x_{i+1,j}}{2} - \frac{T^x_{i-1,j}}{2} = 0 \qquad \text{(A1.5)} \end{aligned}$$

Central difference expression for the slopes $\theta^x_{i+1,j}$ and $\theta^x_{i-1,j}$ are

$$\theta^x_{i+1,j} = \frac{-\, w^x_{i,j} + w^x_{i+2,j}}{2h_x} \qquad \text{(A1.6)}$$

and

$$\theta^x_{i-1,j} = \frac{-\, w^x_{i-2,j} + w^x_{i,j}}{2h_x} \qquad \text{(A1.7)}$$

It remains only to express the moments M in terms of deflections. For small deflection theory, the conventional approximation relating beam curvature to bending moment is

$$\frac{\partial^2 w}{\partial x^2} = \frac{M}{F} \qquad \text{(A1.8)}$$

where F is the flexural stiffness EI (product of cross-sectional moment of inertia and Young's modulus).

Equation A1.8 written in second-order central difference form in the x-direction about Joint i,j gives the relation

$$M^x_{i,j} = F^x_{i,j} \frac{(w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j})}{h^2_x} \qquad \text{(A1.9)}$$

and in the y and z-direction respectively

$$M^y_{i,j} = F^y_{i,j} \frac{(w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1})}{h^2_y} \qquad \text{(A1.10)}$$

and

$$M^z_{i,j} = F^z_{i,j} \frac{(w^z_{i-1,j} - 2w^z_{i,j} + w^z_{i+1,j})}{h^2_z} \qquad \text{(A1.11)}$$

Writing Eq A1.9 at three adjacent stations and substituting these and the expressions for slopes given by Eqs A1.6 and A1.7 into Eq A1.5 yields

$$\begin{aligned} h_x \Big[Q_{i,j} - Q^y_{i,j} - Q^z_{i,j} - S_{i,j} w^x_{i,j} - K^y(w^x_{i,j} - w^y_{i,j}) \\ - K^z(w^x_{i,j} - w^z_{i,j}) \Big] &= F^x_{i-1,j} \frac{(w^x_{i-2,j} - 2w^x_{i-1,j} + w^x_{i,j})}{h^2_x} \\ &- 2F^x_{i,j} \frac{(w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j})}{h^2_x} \\ &+ F^x_{i+1,j} \frac{(w^x_{i,j} - 2w^x_{i+1,j} + w^x_{i+2,j})}{h^2_x} \end{aligned}$$

(Equation Cont'd.)

$$+ P^x_{i,j} (- w^x_{i-1,j} + w^x_{i,j}) - P^x_{i+1,j} (- w^x_{i,j} + w^x_{i+1,j})$$

$$+ \frac{R^x_{i-1,j}}{2} \frac{(-w^x_{i-2,j} + w^x_{i,j})}{2h_x} - \frac{R^x_{i+1,j}}{2} \frac{(- w^x_{i,j} + w^x_{i+2,j})}{2h_x}$$

$$- \frac{T^x_{i+1,j}}{2} + \frac{T^x_{i-1,j}}{2} \qquad \text{(A1.12)}$$

Multiplying by h_x^2 and rearranging terms results in

$$w^x_{i-2,j} \left[F^x_{i-1,j} - \frac{h_x}{4} (R^x_{i-1,j}) \right] + w^x_{i-1,j} \left[- 2F^x_{i-1,j} - 2F^x_{i,j} - h_x^2 P^x_{i,j} \right]$$

$$+ w^x_{i,j} \left[F^x_{i-1,j} + 4F^x_{i,j} + F^x_{i+1,j} + h_x^2 (P^x_{i,j} + P^x_{i+1,j}) \right.$$

$$\left. + \frac{h_x}{4} (R^x_{i-1,j} + R^x_{i+1,j}) + S_{i,j} h_x^3 + h_x^3 (K^y + K^z) \right]$$

$$+ w^x_{i+1,j} \left[- 2F^x_{i,j} - 2F^x_{i+1,j} - h_x^2 P^x_{i+1,j} \right]$$

$$+ w^x_{i+2,j} \left[F^x_{i+1,j} - \frac{h_x}{4} R^x_{i+1,j} \right]$$

$$= h_x^3 \left[(Q_{i,j} - Q^z_{i,j} - Q^y_{i,j} + K^y w^y_{i,j} + K^z w^z_{i,j}) \right.$$

$$\left. - \frac{1}{2h_x} (T^x_{i-1,j} - T^x_{i+1,j}) \right] \qquad \text{(A1.13)}$$

which is of the form

$$a_i w_{i-2,j} + b_i w_{i-1,j} + c_i w_{i,j} + d_i w_{i+1,j} + e_i w_{i+2,j} = f_i \qquad \text{(A1.14)}$$

Equation A1.13 is of exactly the same form as Eqs 3.10 except that $Q^x_{i,j}$ is now expanded in a more useable form. All terms on the left side of Eq A1.13, excepting the S, K^z, and K^y terms, represent $Q^x_{i,j} \times h_x^3$. The forms of $Q^y_{i,j}$ and $Q^z_{i,j}$ remain to be developed.

A similar derivation for the y-beam layer would show that

$$\begin{aligned}
& w^{y}_{i,j-2} \left[F^{y}_{i,j-1} - \frac{h_y}{4} R^{y}_{i,j-1} \right] + w^{y}_{i,j-1} \left[- 2\, F^{y}_{i,j-1} - 2F^{y}_{i,j} - h^2_y\, P^{y}_{i,j} \right] \\
& \quad + w^{y}_{i,j} \Big[F^{y}_{i,j-1} + 4F^{y}_{i,j} + F^{y}_{i,j+1} + h^2_y\, (P^{y}_{i,j} + P^{y}_{i,j+1}) \\
& \quad + \frac{h_y}{4} (R^{y}_{i,j-1} + R^{y}_{i,j+1}) + S_{i,j}\, h^3_y + h^3_y\, (K^x + K^z) \Big] \\
& \quad + w^{y}_{i,j+1} \left[- 2F^{y}_{i,j+1} - h^2_y P^{y}_{i,j+1} \right] + w^{y}_{i,j+2} \left[F^{y}_{i,j+1} - \frac{h_y}{4} R^{y}_{i,j+1} \right] \\
& \quad = h^3_y \Big[(Q_{i,j} - Q^{x}_{i,j} - Q^{z}_{i,j} + K^x w^{x}_{i,j} + K^z w^{z}_{i,j}) \\
& \quad - \frac{1}{2h_y} (T^{y}_{i,j-1} - T^{y}_{i,j+1}) \Big] \qquad \text{(A1.15)}
\end{aligned}$$

Equation A1.15 is in a very useful form since all terms on the left side of Eq A.1, excluding the S , K^x , and K^z terms, represent $Q^{y}_{i,j} \times h^3_y$. An expression for the z-beam layer would have similar form as follows:

$$\begin{aligned}
& w^{z}_{i-2,j} \left[F^{z}_{i-1,j} - \frac{h_z}{4} R^{z}_{i-1,j} \right] + w^{z}_{i-1,j} \left[- 2F^{z}_{i-1,j} - 2F^{z}_{i,j} - h^2_z\, P^{z}_{i,j} \right] \\
& \quad + w^{z}_{i,j} \Big[F^{z}_{i-1,j} + 4F^{z}_{i,j} + F^{z}_{i+1,j} + h^2_z\, (P^{z}_{i,j} + P^{z}_{i+1,j}) \\
& \quad + \frac{h_z}{4} (R^{z}_{i-1,j} + R^{z}_{i+1,j}) + S_{i,j}\, h^3_z + h^3_z\, (K^x + K^y) \Big] \\
& \quad + w^{z}_{i+1,j} \left[- 2F^{z}_{i,j} - 2F^{z}_{i+1,j} - h^2_z\, P^{z}_{i+1,j} \right] \\
& \quad + w^{z}_{i+2,j} \left[F^{z}_{i+1,j} - \frac{h_z}{4} R^{z}_{i+1,j} \right] \\
& \quad = h^3_z \Big[(Q_{i,j} - Q^{x}_{i,j} - Q^{y}_{i,j} + K^x w^{x}_{i,j} + K^y w^{y}_{i,j}) \\
& \quad - \frac{1}{2h_z} (T^{z}_{i-1,j} - T^{z}_{i+1,j}) \Big] \qquad \text{(A1.16)}
\end{aligned}$$

All terms on the left side of Eq A1.16, excluding the S , K^x , and K^y terms, represent $Q^{z}_{i,j} \times h^3_z$. Substitution of the terms representing $Q^{y}_{i,j}$ and $Q^{z}_{i,j}$ into Eq A1.13 yields Eq A1.17 which is the full iteration expression for the x-beam.

$$
\begin{aligned}
& w^x_{i-2,j}\Bigl[F^x_{i-1,j} - \frac{h_x}{4}(R^x_{i-1,j})\Bigr] + w^x_{i-1,j}\Bigl[-2F^x_{i-1,j} - 2F^x_{i,j} - h_x^2 P^x_{i,j}\Bigr] \\
& + w^x_{i,j}\Bigl[F^x_{i-1,j} + 4F^x_{i,j} + F^x_{i+1,j} + h_x^2 (P^x_{i,j} + P^x_{i+1,j}) \\
& + \frac{h_x}{4}(R^x_{i-1,j} + R^x_{i+1,j}) + S_{i,j}\, h_x^3 + h_x^3 (K^y + K^z)\Bigr] \\
& + w^x_{i+1,j}\Bigl[-2F^x_{i,j} - 2F^x_{i+1,j} - h_x^2 P^x_{i+1,j}\Bigr] \\
& + w^x_{i+2,j}\Bigl[F^x_{i+1,j} - \frac{h_x}{4} R^x_{i+1,j}\Bigr] \\
& = h_x^3 \Bigl\{Q_{i,j} + K^y w^y_{i,j} + K^z w^z_{i,j} - \frac{1}{2h_x}(T^x_{i-1,j} - T^x_{i+1,j}) \\
& - \frac{1}{2h_y}(T^y_{i,j-1} - T^y_{i,j+1}) - \frac{1}{2h_z}(T^z_{i-1,j} - T^z_{i+1,j}) \\
& + \Bigl[- w^y_{i,j-2}(F^y_{i,j-1} - \frac{h_y}{4} R^y_{i,j-1}) - w^y_{i,j-1}(-2F^y_{i,j-1} \\
& - 2F^y_{i,j} - h_y^2 P^y_{i,j}) - w^y_{i,j}(F^y_{i,j-1} + 4F^y_{i,j} + F^y_{i,j+1} \\
& + h_y^2 (P^y_{i,j} + P^y_{i,j+1}) + \frac{h_y}{4}(R^y_{i,j-1} + R^y_{i,j+1})) \\
& - w^y_{i,j+1}(-2F^y_{i,j} - 2F^y_{i,j+1} - h_y^2 P^y_{i,j+1}) \\
& - w^y_{i,j+2}(F^y_{i,j+1} - \frac{h_y}{4} R^y_{i,j+1})\Bigr] \frac{1}{h_y^3} + \Bigl[- w^z_{i-2,j}(F^z_{i-1,j} \\
& - \frac{h_z}{4} R^z_{i-1,j}) - w^z_{i-1,j}(-2F^z_{i-1,j} - 2F^z_{i,j} - h_z^2 P^z_{i,j}) \\
& - w^z_{i,j}(F^z_{i-1,j} + 4F^z_{i,j} + F^z_{i+1,j} + h_z^2 (P^z_{i,j} + P^z_{i+1,j}) \\
& + \frac{h_z}{4}(R^z_{i-1,j} + R^z_{i+1,j})) - w^z_{i+1,j}(-2F^z_{i,j} - 2F^z_{i+1,j} \\
& - h_z^2 P^z_{i+1,j}) - w^z_{i+2,j}(F^z_{i+1,j} - \frac{h_z}{4} R^z_{i+1,j})\Bigr] \frac{1}{h_z^3}\Bigr\}
\end{aligned}
\tag{A1.17}
$$

Equation A1.17 has a five-diagonal banded matrix form which is shown in simplified form in Eq A1.18. The coefficients a_i through e_i represent the terms inside the brackets on the left side of Eq A1.17, and f_i represents all the terms on the right side.

$$a_i\ w_{i-2,j} + b_i\ w_{i-1,j} + c_i\ w_{i,j} + d_i\ w_{i+1,j} + e_i\ w_{i+2,j} = f_i \qquad \text{(A1.18)}$$

Equations similar to A1.17 which are merely combinations of Eqs A1.13, A1.15, and A1.16 represent the total equations for the y and z-beams.

APPENDIX 2

CLOSURE SPRING CALCULATION FOR SIMPLY-CONNECTED SYSTEMS

APPENDIX 2. CLOSURE SPRING CALCULATION FOR SIMPLY-CONNECTED SYSTEMS

The following development for the eigenvalues of a simply-supported beam was suggested by Salani (Ref 26). The eigenvalues λ for an equation such as Eq A2.1, which governs the individual line element solutions for the layered system, can be found from Eq A2.2.

$$F \frac{\partial^4 w}{\partial x^4} = q \tag{A2.1}$$

$$F \frac{\partial^4 w}{\partial x^4} = \lambda_n w \tag{A2.2}$$

Equation A2.2 can be expanded in finite difference form as Eq A2.3.

$$\frac{F}{h^4}\left[w_{i-2} - 4w_{i-1} + 6w_i - 4w_{i+1} + w_{i+2}\right] = \lambda_n w_i \tag{A2.3}$$

If we let Eq A2.4 represent the deflected shape of a simply supported beam,

$$w_i = \sin i\beta_n \tag{A2.4}$$

then Eqs A2.5 can be written for the four other terms of Eq A2.3 as follows

$$\begin{aligned} w_{i-2} &= \sin\,(i-2)\,\beta_n = \sin\,(i\beta_n - 2\beta_n) \\ w_{i-1} &= \sin\,(i-1)\,\beta_n = \sin\,(i\beta_n - \beta_n) \\ w_{i+1} &= \sin\,(i+1)\,\beta_n = \sin\,(i\beta_n + \beta_n) \\ w_{i+2} &= \sin\,(i+2)\,\beta_n = \sin\,(i\beta_n + 2\beta_n) \end{aligned} \tag{A2.5}$$

Using the trigonometric identity in Eq A2.6, and rewriting Eqs A2.5, Eqs A2.7 result.

$$\sin\,(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \tag{A2.6}$$

$$\begin{aligned} w_{i-2} &= \sin i\beta_n \cos 2\beta_n - \cos i\beta_n \sin 2\beta_n \\ w_{i-1} &= \sin i\beta_n \cos \beta_n - \cos i\beta_n \sin \beta_n \\ w_{i+1} &= \sin i\beta_n \cos \beta_n + \cos i\beta_n \sin \beta_n \\ w_{i+2} &= \sin i\beta_n \cos 2\beta_n + \cos i\beta_n \sin 2\beta_n \end{aligned} \tag{A2.7}$$

Substituting Eq A2.7 and Eq A2.4 into A2.3 and simplifying yields Eq A2.8.

$$\frac{h^4}{F} \lambda_n = 2 \left[\cos 2\beta_n - 4 \cos \beta_n + 3\right] \tag{A2.8}$$

Using the identity $\cos 2\beta_n = 2 \cos^2 \beta_n - 1$ results in Eq A2.9.

$$\frac{h^4}{F} \lambda_n = 4 \left[\cos^2 \beta_n - 2 \cos \beta_n + 1\right] \tag{A2.9}$$

Rewriting Eq A2.9 yields

$$\frac{h^4}{F} \lambda_n = 4 (1 - \cos \beta_n)^2 \tag{A2.10}$$

Noting that

$$(\sin \frac{1}{2} \beta_n)^2 = \frac{1 - \cos \beta_n}{2} \tag{A2.11}$$

and substituting into Eq A2.10 yields Eq A2.12.

$$\lambda_n = \frac{F}{h^4} (16) \left(\sin \frac{\beta_n}{2}\right)^4 \tag{A2.12}$$

Boundary conditions for a simply supported beam are zero deflection and moments at the ends. Thus equating Eq A2.4 to zero shows that

$$w_m = \sin M\beta_n = 0 \tag{A2.13}$$

and that

$$M\beta_n = n\pi \;;\; n = 1, 2, 3, \ldots, M - 1 \;. \tag{A2.14}$$

Thus an expression for the eigenvalues of the simply supported beam is Eq A2.15.

$$\lambda_n = \frac{F}{h^4} (16) \left(\sin \frac{n\pi}{2M}\right)^4 \tag{A2.15}$$

The closure springs are related to the eigenvalues of Eq A2.15 by the increment length h . Equation A2.15 has units of lb/in^2 so that Eq A2.16 shows the relationship between the eigenvalues and the closure springs which have units of lb/in.

$$\overline{SF} = \frac{F}{h^3} (16) \left(\sin \frac{n\pi}{2M}\right)^4 = h\lambda_n \tag{A2.16}$$

A good approximation for the closure springs can be determined from a uniformly-loaded simply-supported beam since the deflected shape closely corresponds to a sine curve. The deflection at the center of the beam, using lumped values of load, is expressible as Eq A2.17.

$$w = \frac{5}{384} \frac{Q}{h} \frac{L^4}{F} \tag{A2.17}$$

The equivalent stiffness of the beam which would be felt by any opposing beam would be the applied load divided by the deflection w . Equation A2.18 shows this relation

$$K_E = Q/w \tag{A2.18}$$

where

K_E = equivalent stiffness, lb/in.

Q = load, lb

and

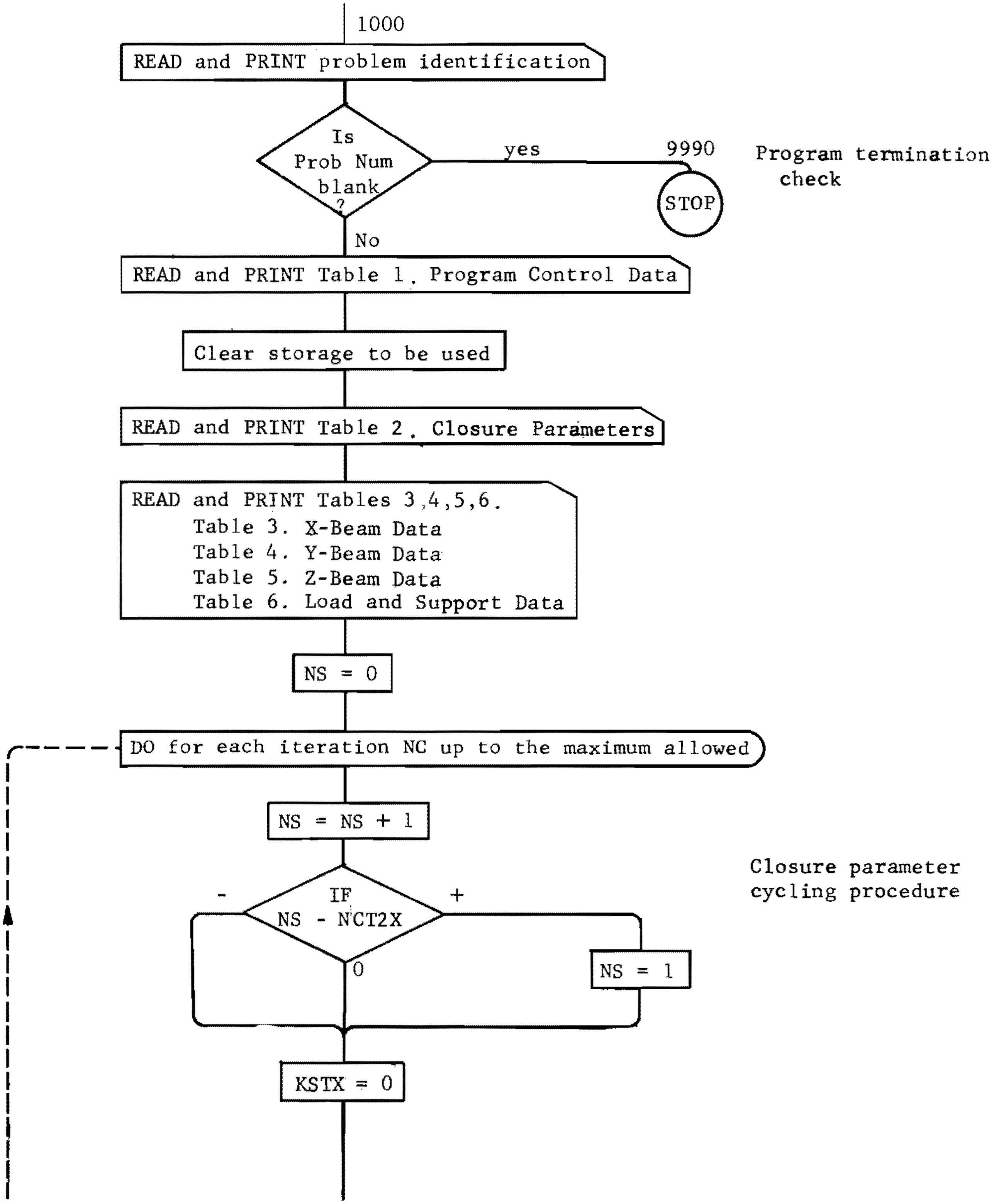
w = deflection, in.

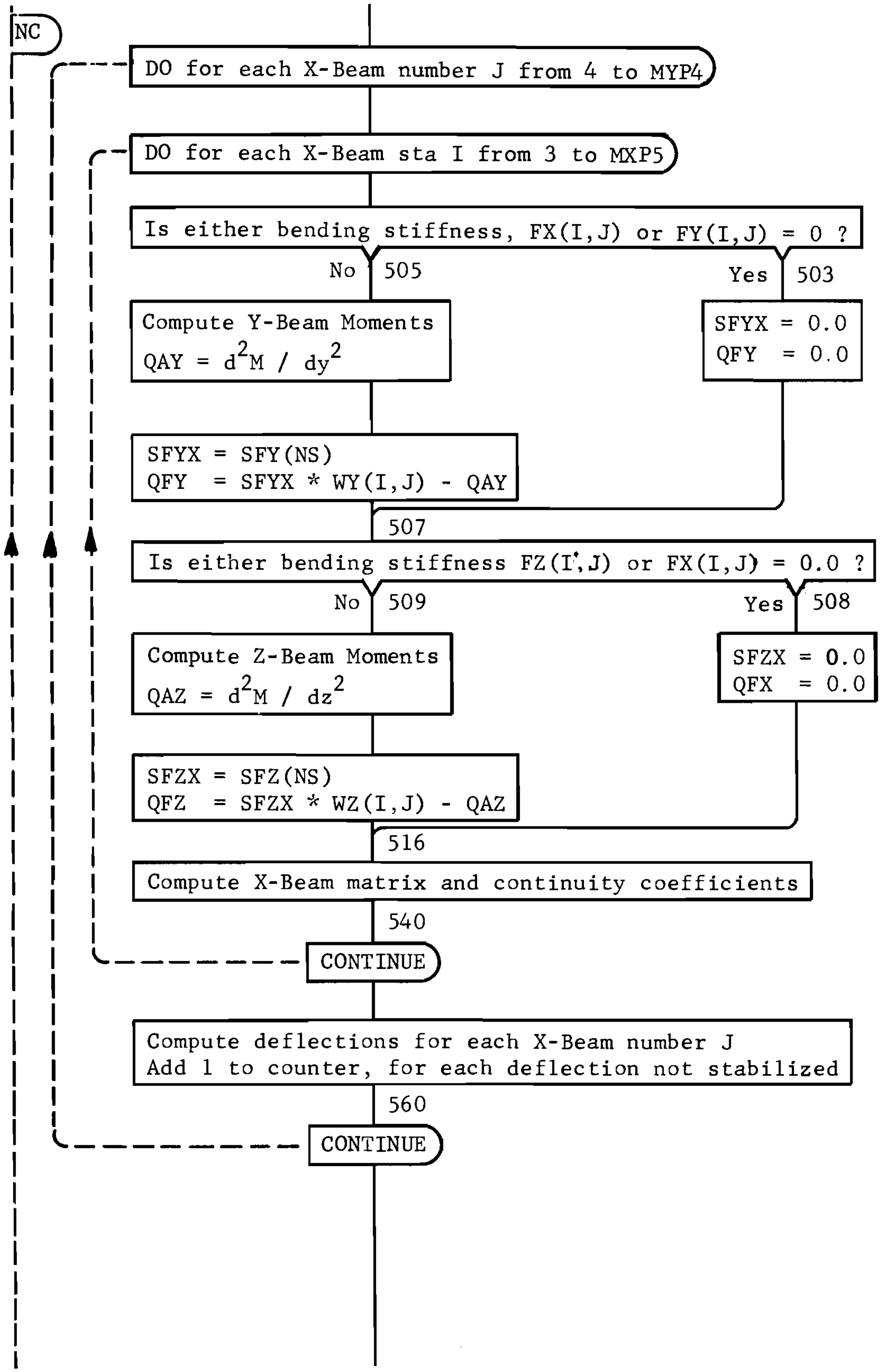
This approximate method for obtaining spring stiffness compares favorably with the eigenvalue method, at least for the lowest mode shape.

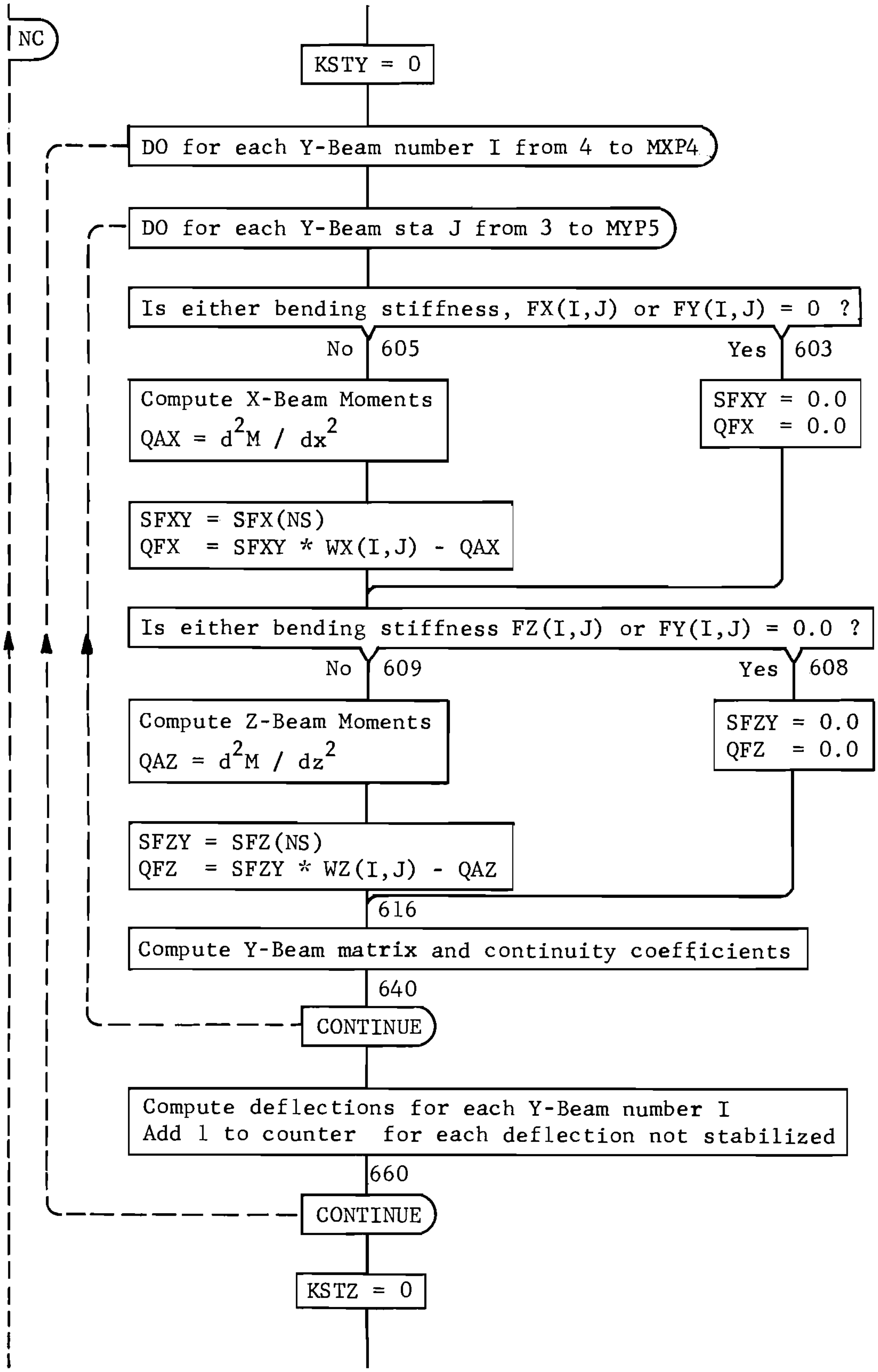
APPENDIX 3

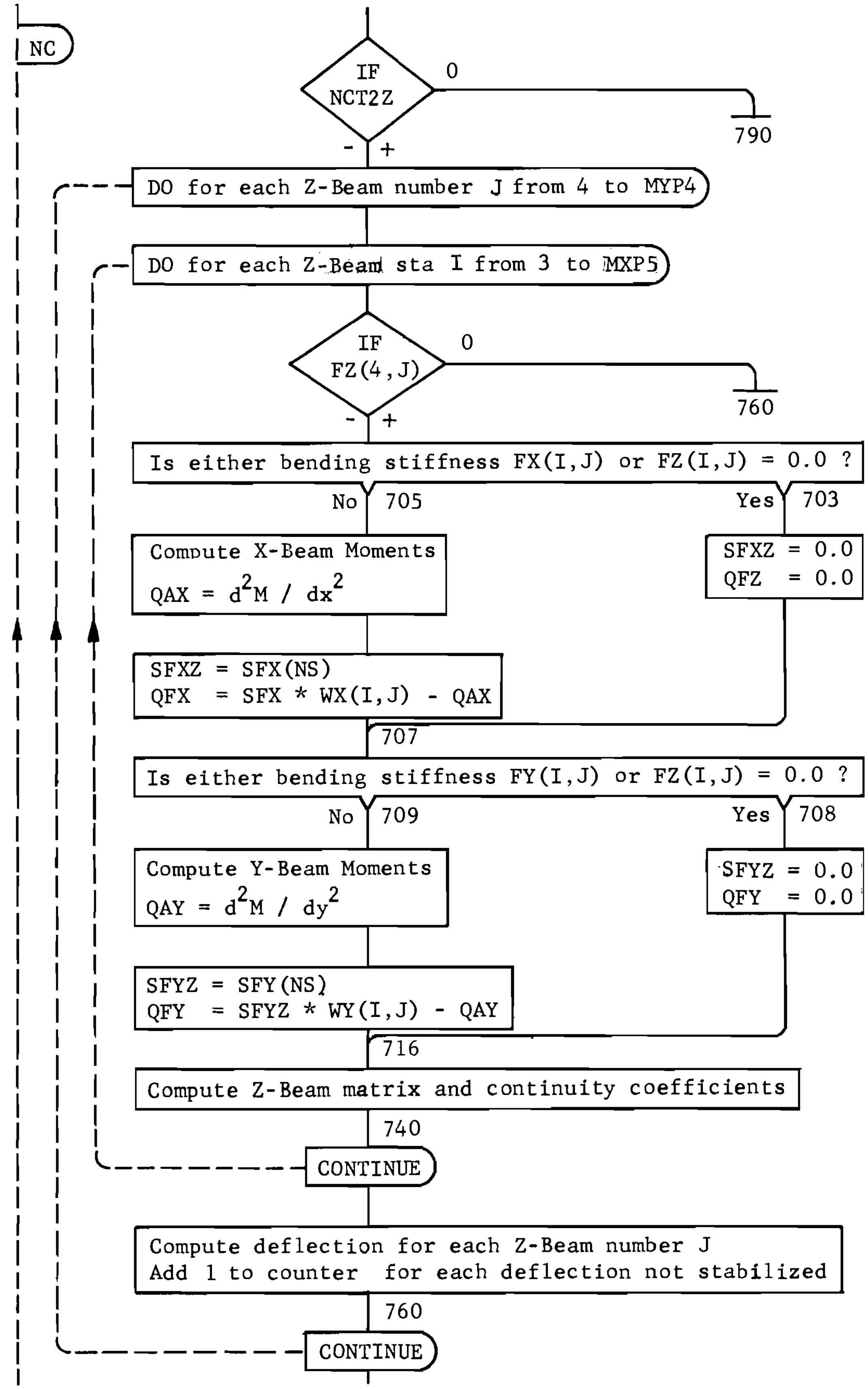
FLOW DIAGRAM FOR PROGRAM LAYER 7

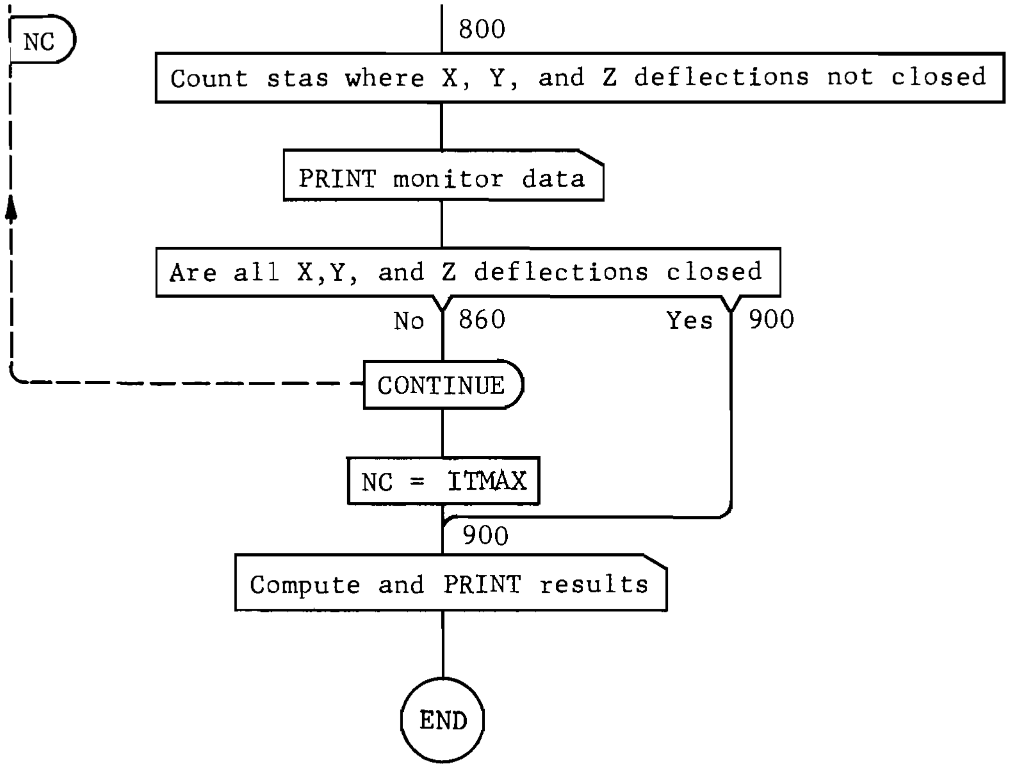
APPENDIX 3. FLOW DIAGRAM FOR PROGRAM LAYER 7.




NC
DO for each X-Beam number J from 4 to MYP4
DO for each X-Beam sta I from 3 to MXP5
Is either bending stiffness, FX(I,J) or FY(I,J) = 0 ?
No
505
Yes
503
Compute Y-Beam Moments
QAY = d2M / dy2
SFYX = 0.0
QFY = 0.0
SFYX = SFY(NS)
QFY = SFYX * WY(I,J) - QAY
507
Is either bending stiffness FZ(I,J) or FX(I,J) = 0.0 ?
No
509
Yes
508
Compute Z-Beam Moments
QAZ = d2M / dz2
SFZX = 0.0
QFX = 0.0
SFZX = SFZ(NS)
QFZ = SFZX * WZ(I,J) - QAZ
516
Compute X-Beam matrix and continuity coefficients
540
CONTINUE
Compute deflections for each X-Beam number J
Add 1 to counter, for each deflection not stabilized
560
CONTINUE


NC
KSTY = 0
DO for each Y-Beam number I from 4 to MXP4
DO for each Y-Beam sta J from 3 to MYP5
Is either bending stiffness, FX(I,J) or FY(I,J) = 0 ?
No
605
Yes
603
Compute X-Beam Moments
QAX = d²M / dx²
SFXY = 0.0
QFX = 0.0
SFXY = SFX(NS)
QFX = SFXY * WX(I,J) - QAX
Is either bending stiffness FZ(I,J) or FY(I,J) = 0.0 ?
No
609
Yes
608
Compute Z-Beam Moments
QAZ = d²M / dz²
SFZY = 0.0
QFZ = 0.0
SFZY = SFZ(NS)
QFZ = SFZY * WZ(I,J) - QAZ
616
Compute Y-Beam matrix and continuity coefficients
640
CONTINUE
Compute deflections for each Y-Beam number I
Add 1 to counter for each deflection not stabilized
660
CONTINUE
KSTZ = 0


NC
IF NCT2Z
0
790
- +
DO for each Z-Beam number J from 4 to MYP4
DO for each Z-Beam sta I from 3 to MXP5
IF FZ(4,J)
0
760
- +
Is either bending stiffness FX(I,J) or FZ(I,J) = 0.0 ?
No 705
Yes 703
Compute X-Beam Moments
QAX = d2M / dx2
SFXZ = 0.0
QFZ = 0.0
SFXZ = SFX(NS)
QFX = SFX * WX(I,J) - QAX
707
Is either bending stiffness FY(I,J) or FZ(I,J) = 0.0 ?
No 709
Yes 708
Compute Y-Beam Moments
QAY = d2M / dy2
SFYZ = 0.0
QFY = 0.0
SFYZ = SFY(NS)
QFY = SFYZ * WY(I,J) - QAY
716
Compute Z-Beam matrix and continuity coefficients
740
CONTINUE
Compute deflection for each Z-Beam number J
Add 1 to counter for each deflection not stabilized
760
CONTINUE


NC
800
Count stas where X, Y, and Z deflections not closed
PRINT monitor data
Are all X,Y, and Z deflections closed
No
860
Yes
900
CONTINUE
NC = ITMAX
900
Compute and PRINT results
END

APPENDIX 4

NOTATION AND LISTING OF PROGRAM LAYER 7

APPENDIX 4. NOTATION AND LISTING OF PROGRAM LAYER 7

```
      PROGRAM LAYER 7
    1 FORMAT (5X,52HPROGRAM LAYER 7 - MASTER DECK - WB INGRAM, H MATLOCK
     1                                  28H     REVISION DATE 27 JUL 65)
C-----PROGRAM LAYER 7 SOLVES A LAYERED GRID-OVER-BEAMS SYSTEM BY A        27JL5
C         THREE-PHASE ALTERNATING-DIRECTION PROCESS.  INDIVIDUAL LAYER    27JL5
C         CLOSURE PARAMETERS ARE INPUT BUT THEY ARE USED ONLY ON THE      27JL5
C         OPPOSING LAYERS IN THE SYSTEM.  SOLUTION IS LIMITED TO THREE    27JL5
C         INTERSECTING BEAMS.                                             27JL5
C-----NOTATION FOR LAYER 7                                                31MY5
C         A( ), B( ), ETC.    RECURSION COEFFICIENTS                      12MY5
C         AA THRU FF          COEFFS IN STIFFNESS AND LOAD MATRICES       12MY5
C         AN1(N), ETC.        ALPHANUMERIC REMARKS, INFORMATION ONLY      12MY5
C         BM(N)               TEMPORARY BENDING MOMENT VALUES (N=1,9)     13MY5
C         CTOL                CLOSURE TOLERANCE, X VS Y DEFLS             12MY5
C         DEN, DENO           DENOMINATOR                                 13MY5
C         ERROR               FINAL ERROR IN SUM OF VERTICAL FORCES       12MY5
C         FX, FY, FZ          BENDING STIFFNESSES                         13MY5
C         HX, HY              INCREMENT LENGTH                            12MY5
C         HXD2, HYD2          ONE HALF OF INCREMENT LENGTH                10MY5
C         HXE2, HYE2          INCREMENT LENGTH SQUARED                    10MY5
C         HXE3, HYE3          INCREMENT LENGTH CUBED                      10MY5
C         I, J                STATION NUMBERS, X,Y DIRECTIONS             12MY5
C         I1, J1, ETC.        TEMPORARY INTERNAL INPUT STATIONS           12MY5
C         IM1, JM1, ETC.      MONITOR STATION USED DURING ITERATION       12MY5
C         IN1, JN1, ETC.      TEMPORARY INPUT STATION VALUES              12MY5
C         ISTA, JSTA          OUTPUT STA NUMBERS                          12MY5
C         ITEST               PROGRAM TERMINATION TEST = 5 BLANK SPACES   13MY5
C         ITMAX               MAXIMUM NUMBER OF ITERATIONS ALLOWED        12MY5
C         KCTOL, KCTOLZ       CLOSURE COUNTERS                            10MY5
C         KSTX, KSTY, KSTZ    COUNT OF BEAM NOT STABILIZED                10MY5
C         KTAB                NUM OF TABLE 3, 4, 5, 6                     12MY5
C         L, K,               TEMPORARY STATION INDEXES                   12MY5
C         MX, MY              NUMBER OF INCREMENTS                        12MY5
C         MXP4, MYP4, ETC.    NUMBER OF INCREMENTS PLUS 4, ETC.           10MY5
C         NC                  NUMBER OF ITERATIONS                        12MY5
C         NCDT                TEMPORARY VALUES OF NCT3, 4, 5, 6           12MY5
C         NCT2X, ETC.         NUM CARDS TABLE 2 FOR X, Y, Z-BEAMS         13MY5
C         NCT3, ETC.          NUM OF CARDS IN TABLE 3, ETC.               10MY5
C         NONE                DUMMY VARIABLE                              13MY5
C         NPROB               NUMBER OF PROBLEM, PROG STOPS IF ZERO       12MY5
C         NS                  COUNTER ON SPRING CYCLE                     12MY5
C         NTABLE1,2           TEMPORARY NUMBER OF TABLE                   31MY5
C         PX( , ), ETC.       AXIAL FORCE, X, Y, Z-BEAMS                  12MY5
C         Q(I,J)              TRANSVERSE FORCE PER MESH POINT             12MY5
C         QAX, QAY, QAZ       H * SECOND DERIV BM, X, Y, Z                13MY5
C         QFX, QFY, QFZ       FICT LOADS IN X, Y, Z-BEAMS                 13MY5
C         REACT               NET TRANSVERSE FORCE                        12MY5
C         RX( , ), ETC.       ROTATIONAL RESTRAINT, X, Y, Z-BEAMS         12MY5
C         S(I,J)              SPRING SUPPORT, VALUE PER MESH POINT        12MY5
C         SFX( ), ETC.        FICT SPRINGS FOR X, Y, Z-BEAMS              12MY5
C         SFYX, ETC.          INTERNAL FICT SPRINGS BETWEEN BEAMS         12MY5
C         TX( , ), ETC.       EXTERNAL TORQUE, X, Y, Z-BEAMS              12MY5
C         WX(I,J), ETC.       VERTICAL DEFLECTION                         10MY5
C         WTEMP               TEMPORARY VALUE OF DEFLECTION               12MY5
C         ZN1 THRU ZN5        TEMPORARY INPUT VARIABLE VALUES             12MY5
      DIMENSION     AN1(32), AN2(14),                                     18FE5 ID
```

```
  1              SFX(6), SFY(6), SFZ(6), S(32,32), Q(32,32),         07AP5
  2              BM(9), A(32), B(32), C(32),                         07AP5
  3              FX(32,32), FY(32,32), FZ(32,32),                    07AP5
  4              WX(32,32), WY(32,32), WZ(32,32),                    07AP5
  5              TX(32,32), TY(32,32), TZ(32,32),                    07AP5
  6              RX(32,32), RY(32,32), RZ(32,32),                    07AP5
  7              PX(32,32), PY(32,32), PZ(32,32)                     07AP5
 6 FORMAT ( )                                                        04MY3
10 FORMAT (  5H       , 80X, 10HI-----TRIM )                         27FE4 ID
11 FORMAT (  5H1      , 80X, 10HI-----TRIM )                         27FE4 ID
12 FORMAT (  16A5  )                                                 04MY3 ID
13 FORMAT (        5X, 16A5 )                                        27FE4 ID
14 FORMAT ( A5, 5X, 14A5 )                                           18FE5 ID
15 FORMAT (///10H     PROB , /5X, A5, 5X, 14A5 )                     18FE5 ID
16 FORMAT (///17H     PROB (CONTD), /5X, A5, 5X, 14A5 )             18FE5 ID
19 FORMAT (///48H     RETURN THIS PAGE TO TIME RECORD FILE -- HM )   12MR5 ID
20 FORMAT  (10( I5 ),   3E10.3 )                                     06AP5
21 FORMAT ( 10X, 8 ( I5 ) )                                          08AP5
22 FORMAT ( 10X, I5, 5X, 6E10.3 )                                    06AP5
23 FORMAT ( 10X, 4( I5 ), 5E10.3 )                                   08AP5
25 FORMAT ( 5X, I5, 5X, 7E10.3 )                                     02JA4
26 FORMAT  ( 5X, I5, 5X, 7E10.3  )                                   21JA5
30 FORMAT  (//30H     TABLE 1.  CONTROL DATA   ,              /      15AP3
  1        /  30H          NUM CARDS TABLE 3   , 40X, I5,     /      04JA5
  2           30H          NUM CARDS TABLE 4   , 40X, I5,     /      07AP5
  3           30H          NUM CARDS TABLE 5   , 40X, I5,     /      07AP5
  4           30H          NUM CARDS TABLE 6   , 40X, I5,     /      07AP5
  5           30H          MAX NUM ITERATIONS  , 40X, I5,     /      06AP5
  6           30H          NUM INCREMENTS  MX  , 40X, I5,     /      06AP5
  7           30H          NUM INCREMENTS  MY  , 40X, I5,     /      06AP5
  8           30H          INCR LENGTH  HX     , 35X,   E10.3,/      06AP5
  9           30H          INCR LENGTH  HY     , 35X,   E10.3,/      06AP5
  1           30H          CLOSURE TOLERANCE   , 35X,   E10.3 )      06AP5
31 FORMAT  (  30H          MONITOR STAS  I,J   ,  5X, 4( I7, I3 ) )  15AP3
32 FORMAT  (//40H     TABLE 2.  RELAXATION CONTROL DATA ,     /      15AP3
  1           52H        NUM       C L O S U R E  P A R A M E T E R S29DE4
  2       ,/, 12H      VALUES  /    )                                29DE4
33 FORMAT  (//51H     TABLE 3. X-BEAM DATA, FULL VALUES ADDED AT ALL 06AP5
  1           22H STAS I,J IN RECTANGLE, /                           06AP5
  2          / 50H          FROM      THRU         FX         TX    , 06AP5
  3           20H     RX           PX                             ,/)07AP5
43 FORMAT  (//51H     TABLE 4. Y-BEAM DATA, FULL VALUES ADDED AT ALL 06AP5
  1           22H STAS I,J IN RECTANGLE, /                           06AP5
  2          / 50H          FROM      THRU         FY         TY    , 06AP5
  3           20H     RY           PY                             ,/)07AP5
53 FORMAT  (//51H     TABLE 5. Z-BEAM DATA, FULL VALUES ADDED AT ALL 08AP5
  1           22H STAS I,J IN RECTANGLE, /                           06AP5
  2          / 50H          FROM      THRU         FZ         TZ    , 06AP5
  3           20H     RZ           PZ                             ,/)07AP5
63 FORMAT (// 48H     TABLE 6. LOAD AND SUPPORT DATA, FULL VALUES    06AP5
  1           35H ADDED AT ALL STAS I,J IN RECTANGLE, /              06AP5
  2          / 48H          FROM      THRU         Q          S   ,/)07AP5
64 FORMAT ( / 33H     NO DATA SPECIFIED FOR TABLE  A2, A1,           10MY5
  1           27HNEW PROBLEM HAS BEEN SOUGHT  )                      10MY5
73 FORMAT  ( 5X,  2( 5X, I2, 1X, I2 ),  5( E12.3))                   06AP5
74 FORMAT (///48H     TABLE 7.  MONITOR TALLY AND DEFLS AT 4 STAS,/  06AP5
```

```
     1         / 46H        ITR   FICT      CYC  NOT  NOT      I,J  ,/  15AP3
     2           42H        NUM   SPRING    NUM  STAB CLOS    ,         15AP3
     3                                              4( I2, 1X, I2, 7X ))15AP3
   75 FORMAT ( /  7H    X  , I3, E12.3, I4, I5, 5X, 4E12.3 /            28JL5
     1            7H    Y  , 3X, E12.3, I4, I5, I5, 4E12.3 /            28JL5
     2            7H    Z  , 3X, E12.3, I4, I5, I5, 4E12.3 )            28JL5
   85 FORMAT (/  51H     TABLE 8. DEFLECTION AND ERROR RESULTS -- ITERA 27MY5
     1            5HTION,  I4      )                                     27MY5
   86 FORMAT  (/ 51H     SOLUTION NOT CLOSED WITHIN SPECIFIED TOLERANCE)06AP5
   87 FORMAT ( / 50H       I,J    X-DEFL      Y-DEFL      Z-DEFL        27MY5
     1           20H REACT    ERROR     )                               27MY5
   88 FORMAT  (  8X, I2, I3,     6E12.3 )                               06AP5
   89 FORMAT ( / 34H     TABLE 9. MOMENTS -- ITERATION , I4  )          27MY5
   90 FORMAT ( / 50H       I,J    X-MOM       Y-MOM       Z-MOM       )27MY5
               ITEST = 5H                                               27JL5
 1000 PRINT 10                                                          12JL3 ID
      CALL TIME                                                         18FE5 ID
C-----PROGRAM AND PROBLEM IDENTIFICATION                                04MY3 ID
      READ  12, ( AN1(N), N = 1, 32 )                                   18FE5 ID
 1010 READ  14,  NPROB, ( AN2(N), N = 1, 14 )                           28AG3 ID
          IF ( NPROB - ITEST ) 1020, 9990, 1020                         26FE5 ID
 1020 PRINT 11                                                          26AG3 ID
      PRINT 1                                                           18FE5 ID
      PRINT 13, ( AN1(N), N = 1, 32 )                                   18FE5 ID
      PRINT 15, NPROB, ( AN2(N), N = I, 14 )                            26AG3 ID
C-----INPUT TABLE 1                                                     27JL5
      READ  20,  NONE, NONE, NCT3, NCT4, NCT5, NCT6, NONE, ITMAX, MX,   06AP5
     1           MY, HX, HY, CTOL                                       06AP5
      PRINT 30,  NCT3, NCT4, NCT5, NCT6, ITMAX, MX, MY, HX, HY, CTOL    06AP5
      READ  21,  IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                 13AP3
      PRINT 31,  IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                 13AP3
C-----COMPUTE FOR CONVENIENCE                                           27JL5
               HXE2 =   HX * HX                                         13AP3
               HYE2 =   HY * HY                                         13AP3
               HXE3 =   HX * HXE2                                       13AP3
               HYE3 =   HY * HYE2                                       13AP3
               MXP4 =   MX + 4                                          13AP3
               MYP4 =   MY + 4                                          13AP3
               MXP5 =   MX + 5                                          13AP3
               MYP5 =   MY + 5                                          13AP3
               MXP7 =   MX + 7                                          13AP3
               MYP7 =   MY + 7                                          13AP3
C-----CLEAR VALUES FROM PRIOR PROBS                                     27JL5
          DO 250  I = 1,MXP7                                            13AP3
          DO 250  J = 1,MYP7                                            13AP3
               FX(I,J) = FY(I,J) = FZ(I,J) = 0.0                        07AP5
               Q(I,J)  = 0.0                                            27JL5
               S(I,J)  = 0.0                                            27JL5
               TX(I,J) = TY(I,J) = TZ(I,J) = 0.0                        06AP5
               RX(I,J) = RY(I,J) = RZ(I,J) = 0.0                        06AP5
               PX(I,J) = PY(I,J) = PZ(I,J) = 0.0                        06AP5
               WX(I,J) = WY(I,J) = WZ(I,J) = 0.0                        07AP5
  250     CONTINUE                                                      13AP3
          DO 280 N =1, 6                                                03MY5
               SFX(N) = 0.0                                             03MY5
               SFY(N) = 0.0                                             03MY5
```

```
               SFZ(N) = 0.0                                             03MY5
  280     CONTINUE                                                      03MY5
C-----INPUT TABLE 2                                                     27JL5
      PRINT 32                                                          27JA5
C-----PROGRAM LAYER 7 MUST HAVE THE SAME NUMBER OF PARAMETERS FOR ALL   27JL5
C         BEAMS                                                         03MY5
      READ   22, NCT2X, ( SFX(N), N = 1, NCT2X )                        207R5
      PRINT  26, NCT2X, ( SFX(N), N = 1, NCT2X )                        207R5
      READ   22, NCT2Y, ( SFY(N), N = 1, NCT2Y )                        207R5
      PRINT  26, NCT2Y, ( SFY(N), N = 1, NCT2Y )                        207R5
      READ   22, NCT2Z, ( SFZ(N), N = 1, NCT2Z )                        207R5
      PRINT  26, NCT2Z, ( SFZ(N), N = 1, NCT2Z )                        207R5
      CALL TIME                                                         08AP5
C-----INPUT TABLE 3, 4, 5, 6                                            27JL5
  300 PRINT 33                                                          10MY5
          IF ( NCT3 )  302, 305, 302                                    10MY5
  302          NCDT = NCT3                                              10MY5
               KTAB = 1                                                 10MY5
          GO TO 325                                                     10MY5
  305          NTABLE1 = 2H3.                                           10MY5
      PRINT 64, NTABLE1                                                 10MY5
          IF ( NCT4 )  307, 310, 307                                    10MY5
  307          NCDT = NCT4                                              10MY5
               KTAB = 2                                                 10MY5
          GO TO 325                                                     10MY5
  310          NTABLE1 = 2H4.                                           10MY5
      PRINT 43                                                          06AP5
      PRINT 64, NTABLE1                                                 11MY5
          IF ( NCT5 )  312, 315, 312                                    10MY5
  312          NCDT = NCT5                                              10MY5
               KTAB = 3                                                 10MY5
          GO TO 325                                                     10MY5
  315          NTABLE1 = 2H5.                                           10MY5
      PRINT 53                                                          06AP5
      PRINT 64, NTABLE1                                                 10MY5
          IF ( NCT6 )  317, 320, 317                                    10MY5
  317          NCDT = NCT6                                              10MY5
               KTAB = 4                                                 10MY5
          GO TO 325                                                     10MY5
  320          NTABLE1 = 2H6.                                           10MY5
               NTABLE2 = 1H                                             10MY5
      PRINT 63                                                          06AP5
      PRINT 64, NTABLE1, NTABLE2                                        11MY5
          GO TO 1000                                                    10MY5
  325     DO 390 N = 1, NCDT                                            10MY5
      READ  23, IN1, JN1, IN2, JN2, ZN1, ZN2, ZN3, ZN4                  08AP5
               I1 =  IN1 + 4                                            03MY5
               J1 =  JN1 + 4                                            03MY5
               I2 =  IN2 + 4                                            03MY5
               J2 =  JN2 + 4                                            03MY5
          GO TO (  330, 340, 350, 360  ), KTAB                          07AP5
  330 PRINT 73, IN1, JN1, IN2, JN2, ZN1, ZN2, ZN3, ZN4                  08AP5
          DO 335 I = I1, I2                                             08AP5
          DO 335 J = J1, J2                                             08AP5
               FX(I,J) = FX(I,J) + ZN1                                  08AP5
               TX(I,J) = TX(I,J) + ZN2                                  08AP5
```

```
               RX(I,J) = RX(I,J) + ZN3                                   08AP5
               PX(I,J) = PX(I,J) + ZN4                                   08AP5
335        CONTINUE                                                      06AP5
336        IF ( N - NCDT )  390, 337, 337                                10MY5
337    PRINT 43                                                          14MY5
           IF ( NCT4 )  338, 339, 338                                    14MY5
338            NCDT = NCT4                                               10MY5
               KTAB = 2                                                  10MY5
           GO TO 325                                                     10MY5
339            NTABLE1 = 2H4.                                            10MY5
       PRINT 64, NTABLE1                                                 10MY5
           GO TO 347                                                     10MY5
340    PRINT 73, IN1, JN1, IN2, JN2, ZN1, ZN2, ZN3, ZN4                  08AP5
           DO 345 I = I1, I2                                             08AP5
           DO 345 J = J1, J2                                             08AP5
               FY(I,J) = FY(I,J) + ZN1                                   08AP5
               TY(I,J) = TY(I,J) + ZN2                                   08AP5
               RY(I,J) = RY(I,J) + ZN3                                   08AP5
               PY(I,J) = PY(I,J) + ZN4                                   08AP5
345        CONTINUE                                                      06AP5
346        IF ( N - NCDT )  390, 347, 347                                10MY5
347    PRINT 53                                                          14MY5
           IF ( NCT5 )  348, 349, 348                                    14MY5
348            NCDT = NCT5                                               10MY5
               KTAB = 3                                                  10MY5
           GO TO 325                                                     10MY5
349            NTABLE1 = 2H5.                                            10MY5
       PRINT 64, NTABLE1                                                 10MY5
           GO TO 357                                                     10MY5
350    PRINT 73, IN1, JN1, IN2, JN2, ZN1, ZN2, ZN3, ZN4                  08AP5
           DO 355 I = I1, I2                                             08AP5
           DO 355 J = J1, J2                                             08AP5
               FZ(I,J) = FZ(I,J) + ZN1                                   08AP5
               TZ(I,J) = TZ(I,J) + ZN2                                   08AP5
               RZ(I,J) = RZ(I,J) + ZN3                                   08AP5
               PZ(I,J) = PZ(I,J) + ZN4                                   08AP5
355        CONTINUE                                                      06AP5
356        IF ( N - NCDT )  390, 357, 357                                10MY5
357    PRINT 63                                                          14MY5
           IF ( NCT6 )  358, 359, 358                                    14MY5
358            NCDT = NCT6                                               10MY5
               KTAB = 4                                                  10MY5
           GO TO 325                                                     10MY5
359            NTABLE1 = 2H6.                                            10MY5
               NTABLE2 = 1H                                              10MY5
       PRINT 64, NTABLE1, NTABLE2                                        10MY5
           GO TO 1000                                                    10MY5
360    PRINT 73, IN1, JN1, IN2, JN2, ZN1, ZN2                            08AP5
           DO 365 I = I1, I2                                             08AP5
           DO 365 J = J1, J2                                             08AP5
               Q(I,J) = Q(I,J) + ZN1                                     08AP5
               S(I,J) = S(I,J) + ZN2                                     08AP5
365        CONTINUE                                                      06AP5
390        CONTINUE                                                      06AP5
       CALL TIME                                                         08AP5
               KTAB = 0                                                  06AP5
```

```
      PRINT 74,  IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                 06AP5
C-----BEGIN MAIN SOLUTION                                               27JL5
               NS = 0                                                   25AG4
          DO 860  NC = 1, ITMAX                                         27AP5
               NS = NS + 1                                              25AG4
          IF ( NS-NCT2X ) 501, 501, 500                                 20MR5
  500          NS = 1                                                   25AG4
C-----SOLVE X-BEAMS                                                     27JL5
  501          KSTX =   0                                               25AG4
          DO 560  J = 4, MYP4                                           01MA3
          DO 540  I = 3,MXP5                                            13AP3
C-----ESTABLISH ITERATION CONTROL PARAMETERS FOR X BEAMS                27JL5
          IF ( FX(I,J) * FY(I,J) )  505, 503, 505                       24FE5
  503          SFYX = QFY = 0.0                                         29MR5
          GO TO 507                                                     17FE5
  505     DO 506 N = 1, 3                                               17FE5
               L   =    J + N - 2                                       01MA3
               BM(N+3) = FY(I,L) * ( WY(I,L-1) - WY(I,L) -WY(I,L)       01MA3
     1                   + WY(I,L+1) ) / HYE2                           01MA3
  506     CONTINUE                                                      07AP5
               QAY = ( BM(4) -2.0 * BM(5) + BM(6) ) / HY                31DE4
     1              + ( - 0.25 * HY * RY(I,J-1) * WY(I,J-2)             27JL5
     2              + 0.25 * HY * ( RY(I,J-1) + RY(I,J+1))* WY(I,J)     21MY5
     3              - 0.25 * HY * RY(I,J+1) * WY(I,J+2)                 21MY5
     4              - HYE2 * PY(I,J) * WY(I,J-1)                        24MY5
     5              + HYE2 * ( PY(I,J) + PY(I,J+1) ) * WY(I,J)          24MY5
     6              - HYE2 * PY(I,J+1) * WY(I,J+1) ) / HYE3             07JE5
               SFYX = SFY(NS)                                           29MR5
               QFY = SFYX * WY(I,J) - QAY                               29MR5
  507     IF(  FZ(I,J)* FX(I,J) )  509, 508, 509                        20MR5
  508          SFZX = QFZ = 0.0                                         29MR5
          GO TO 516                                                     07AP5
  509     DO 510 N = 1, 3                                               17FE5
               K = I + N - 2                                            21JA5
               BM(N+6) = FZ(K,J)*( WZ(K-1,J) - 2.0*WZ(K,J) +            20MR5
     1              WZ(K+1,J) ) / HXE2                                  27JL5
  510     CONTINUE                                                      07AP5
               QAZ  = ( BM(7) - 2.0 *BM(8) + BM(9) )/ HX                20MR5
     1              + ( - 0.25 * HX * RZ(I-1,J) * WZ(I-2,J)             27JL5
     2              + 0.25* HX * ( RZ(I-1,J) + RZ(I+1,J) ) * WZ(I,J)    21MY5
     3              - 0.25* HX * RZ(I+1,J) * WZ(I+2,J)                  21MY5
     4              - HXE2 * PZ(I,J) * WZ(I-1,J)                        24MY5
     5              + HXE2 * ( PZ(I,J) + PZ(I+1,J) ) * WZ(I,J)          24MY5
     6              - HXE2 * PZ(I+1,J) * WZ(I+1,J) ) / HXE3             27MY5
               SFZX = SFZ(NS)                                           29MR5
               QFZ = SFZX*WZ(I,J) - QAZ                                 29MR5
C-----COMPUTE X-BEAM MATRIX COEFFS                                      27JL5
  516          AA    =    FX(I-1,J) - 0.25 * HX * RX(I-1,J)             06AP5
               BB    = - 2.0 * ( FX(I-1,J) + FX(I,J) ) - HXE2 * PX(I,J) 08AP5
               CC    =    FX(I-1,J) + 4.0 * FX(I,J) + FX(I+1,J)         13AP3
     1                  + HXE2 * ( PX(I,J) + PX(I+1,J) )                05MY5
     2                  + 0.25 * HX * ( RX(I-1,J) + RX(I+1,J) )         05MY5
     3                  + HXE3 * S(I,J) + HXE3 * ( SFYX + SFZX )        05MY5
               DD    = - 2.0 * ( FX(I,J) + FX(I+1,J) ) - HXE2 * PX(I+1,J)07AP5
               EE    =    FX(I+1,J) - 0.25 * HX * RX(I+1,J)             06AP5
               FF    =    HXE3 * ( Q(I,J) + QFY + QFZ                   27JL5
```

```
     1               -0.5/HX *(TX(I-1,J)-TX(I+1,J))                         21MY5
     2               -0.5/HY *(TY(I,J-1)-TY(I,J+1))                         21MY5
     3               -0.5/HX *(TZ(I-1,J)-TZ(I+1,J))    )                    21MY5
C-----COMPUTE X-BEAM CONTINUITY COEFFS                                      27JL5
           E     =   AA * B(I-2) + BB                                       13AP3
           DEN   =   E  * B(I-1) + AA * C(I-2) + CC                         13AP3
       IF   (DEN)  531, 521, 531                                            13AP3
  521      D     =   0.0                                                    01MA3
       GO TO  532                                                           13AP3
  531      D     = - 1.0 / DEN                                              13AP3
  532      C(I) =   D  * EE                                                 13AP3
           B(I) =   D  * ( E  * C(I-1) + DD )                               13AP3
           A(I) =   D  * ( E  * A(I-1) + AA * A(I-2) - FF )                 13AP3
  540     CONTINUE                                                          13AP3
C-----COMPUTE X-BEAM DEFLS                                                  27JL5
       DO 550  L = 3,MXP5                                                   13AP3
           I     =   MX + 8 - L                                             13AP3
           WTEMP     =   WX(I,J)                                            13AP3
           WX(I,J)   =   A(I) + B(I) * WX(I+1,J) + C(I) * WX(I+2,J)         13AP3
C-----COUNT STAS WHERE X-BEAMS NDT STABILIZED                               27JL5
       IF   ( FX(I,J) * FY(I,J) )  545, 550, 545                            13AP3
  545  IF   ( ABSF ( WX(I,J) - WTEMP ) - CTOL     )550, 550, 546            25AG4
  546      KSTX =   KSTX + 1                                                13AP3
  550  CONTINUE                                                             13AP3
  560  CONTINUE                                                             13AP3
C
C-----SOLVE Y-BEAMS                                                         27JL5
           KSTY   =  0                                                      29AP3
       DO 660  I = 4, MXP4                                                  01MA3
       DO 640  J = 3,MYP5                                                   13AP3
C-----ESTABLISH ITERATION CONTROL PARAMETERS FOR Y BEAMS                    27JL5
       IF( FX(I,J) * FY(I,J) )  605, 603, 605                               24FE5
  603      SFXY = QFX = 0.0                                                 20MR5
       GO TO 607                                                            17FE5
  605  DO 606 N = 1, 3                                                      17FE5
           K    =    I + N - 2                                              01MA3
           BM(N) = FX(K,J) * ( WX(K-1,J) - WX(K,J) - WX(K,J)                01MA3
     1                + WX(K+1,J) ) / HXE2                                  01MA3
  606  CONTINUE                                                             07AP5
           QAX = ( BM(1) - 2.0 * BM(2) + BM(3)) / HX                        31DE4
     1               + ( - 0.25 * HX * RX(I-1,J) * WX(I-2,J)                27JL5
     2               + 0.25* HX * ( RX(I-1,J) + RX(I+1,J) ) * WX(I,J)       21MY5
     3               - 0.25* HX * RX(I+1,J) * WX(I+2,J)                     21MY5
     4               -HXE2 * PX(I,J) * WX(I-1,J)                            24MY5
     5               + HXE2 * ( PX(I,J) + PX(I+1,J) ) * WX(I,J)             24MY5
     6               - HXE2 * PX(I+1,J) * WX(I+1,J) ) / HXE3                27MY5
           SFXY = SFX(NS)                                                   20MR5
           QFX = SFXY*WX(I,J) - QAX                                         20MR5
  607  IF(  FZ(I,J)* FY(I,J) )  609, 608, 609                               207R5
  608      SFZY = QFZ = 0.0                                                 29MR5
       GO TO 616                                                            07AP5
  609  DO 610 N = 1, 3                                                      17FE5
           K = I + N - 2                                                    19JA5
           BM(N+6) = FZ(K,J)*( WZ(K-1,J) - 2.0*WZ(K,J) +                    20MR5
     1               WZ(K+1,J) ) / HXE2                                     27JL5
  610  CONTINUE                                                             07AP5
```

```
                QAZ = ( BM(7) - 2.0 *BM(8) + BM(9) )/ HX                20MR5
     1              + ( - 0.25 * HX * RZ(I-1,J) * WZ(I-2,J)             27JL5
     2              + 0.25* HX * ( RZ(I-1,J) + RZ(I+1,J) ) * WZ(I,J)    21MY5
     3              - 0.25* HX * RZ(I+1,J) * WZ(I+2,J)                  21MY5
     4              - HXE2 * PZ(I,J) * WZ(I-1,J)                        24MY5
     5              + HXE2 * ( PZ(I,J) + PZ(I+1,J) ) * WZ(I,J)          24MY5
     6              - HXE2 * PZ(I+1,J) * WZ(I+1,J) ) / HXE3             27MY5
               SFZY = SFZ(NS)                                           29MR5
               QFZ = SFZY*WZ(I,J) - QAZ                                 29MR5
C-----COMPUTE Y-BEAM MATRIX COEFFS                                      27JL5
  616          AA   =   FY(I,J-1) - 0.25 * HY * RY(I,J-1)               06AP5
               BB   = - 2.0 * ( FY(I,J-1) + FY(I,J) ) - HYE2 * PY(I,J)  08AP5
               CC   =   FY(I,J-1) + 4.0 * FY(I,J) + FY(I,J+1)           13AP3
     1                + HYE2 * ( PY(I,J) + PY(I,J+1) )                  05MY5
     2                + 0.25 * HY * ( RY(I,J-1) + RY(I,J+1)  )          05MY5
     3                + HYE3 * S(I,J) + HYE3 * ( SFXY + SFZY )          05MY5
               DD   = - 2.0 * ( FY(I,J) + FY(I,J+1) ) - HYE2 * PY(I,J+1)08AP5
               EE   =   FY(I,J+1) - 0.25 * HY * RY(I,J+1)               06AP5
               FF  = HYE3* (  Q(I,J) + QFX + QFZ                        21MY5
     1              -0.5/HX *(TX(I-1,J)- TX(I+1,J))                     21MY5
     2              -0.5/HY *(TY(I,J-1)- TY(I,J+1))                     26MY5
     3              -0.5/HX *(TZ(I-1,J)- TZ(I+1,J))  )                  21MY5
C-----COMPUTE Y-BEAM CONTINUITY COEFFS                                  27JL5
               E    =   AA * B(J-2) + BB                                13AP3
               DEN  =   E  * B(J-1) + AA * C(J-2) + CC                  13AP3
          IF  (DEN)  631, 621, 631                                      13AP3
  621          D    =   0.0                                             01MA3
          GO TO  632                                                    13AP3
  631          D    = - 1.0 / DEN                                       13AP3
  632          C(J) =   D  * EE                                         13AP3
               B(J) =   D  * ( E  * C(J-1) + DD )                       13AP3
               A(J) =   D  * ( E  * A(J-1) + AA * A(J-2) - FF )         13AP3
  640     CONTINUE                                                      13AP3
C-----COMPUTE Y-BEAM DEFLS                                              27JL5
          DO 650  L = 3,MYP5                                            13AP3
               J    = MY + 8 - L                                        13AP3
               WTEMP    =   WY(I,J)                                     13AP3
               WY(I,J)  =   A(J) + B(J) * WY(I,J+1) + C(J) * WY(I,J+2)  13AP3
C-----COUNT STAS WHERE Y-BEAMS NOT STABILIZED                           27JL5
          IF  ( FX(I,J) * FY(I,J) )  645, 650, 645                      13AP3
  645     IF  ( ABSF ( WY(I,J) - WTEMP ) - CTOL    )650, 650, 646       25AG4
  646          KSTY =   KSTY + 1                                        13AP3
  650     CONTINUE                                                      13AP3
  660     CONTINUE                                                      13AP3
C                                                                       03MY5
C-----SOLVE  Z BEAMS                                                    03MY5
               KSTZ = 0                                                 20MR5
          IF ( NCT2Z )  700, 790, 700                                   26AP5
  700     DO 760  J = 4, MYP4                                           26AP5
          DO 740  I = 3,MXP5                                            26AP5
C-----THE FOLLOWING TEST SEEKS ONLY THE END OF A Z-BEAM.  IF FULL LENGTH27JL5
C     Z-BEAMS ARE NOT USED, REMOVE THE FOLLOWING CARD.                  27JL5
          IF ( FZ(4,J) )  702, 760, 702                                 05MY5
C-----ESTABLISH ITERATION CONTROL PARAMETERS FOR Z BEAMS                27JL5
  702     IF ( FX(I,J) * FZ(I,J) ) 705, 703, 705                        03MY5
  703          SFXZ = QFX = 0.0                                         26AP5
```

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          GO TO 707                                                     26AP5
  705     DO 706 N = 1, 3                                               26AP5
               K = I + N - 2                                            21JA5
               BM(N) = FX(K,J)*( WX(K-1,J) - 2.0*WX(K,J) +              21JA5
     1              WX(K+1,J) ) / HXE2                                  21JA5
  706     CONTINUE                                                      26AP5
               QAX = ( BM(1) - 2.0* BM(2) + BM(3) ) / HX                31DE4
     1              + ( - 0.25 * HX * RX(I-1,J) * WX(I-2,J)             27JL5
     2              + 0.25* HX * ( RX(I-1,J) + RX(I+1,J) ) * WX(I,J)    21MY5
     3              - 0.25* HX * RX(I+1,J) * WX(I+2,J)                  21MY5
     4              -HXE2 * PX(I,J) * WX(I-1,J)                         24MY5
     5              + HXE2 * ( PX(I,J) + PX(I+1,J) ) * WX(I,J)          24MY5
     6              - HXE2 * PX(I+1,J) * WX(I+1,J) ) / HXE3             28MY5
               SFXZ = SFX(NS)                                           29MR5
               QFX = SFXZ*WX(I,J) - QAX                                 20MR5
  707     IF ( FY(I,J)*FZ(I,J) ) 709, 708,  709                         26AP5
  708          SFYZ = QFY = 0.0                                         26AP5
          GO TO 716                                                     26AP5
  709     DO 710 N = 1, 3                                               26AP5
               L = J + N - 2                                            31DE4
               BM(N+3) = FY(I,L)* ( WY(I,L-1)-2.0*WY(I,L) + WY(I,L+1))  27JL5
     1                  / HYE2                                          31DE4
  710     CONTINUE                                                      26AP5
               QAY = ( BM(4) -2.0 * BM(5) + BM(6) ) / HY                31DE4
     1              + ( - 0.25 * HY * RY(I,J-1) * WY(I,J-2)             27JL5
     2              + 0.25 * HY * ( RY(I,J-1) + RY(I,J+1))* WY(I,J)     21MY5
     3              - 0.25 * HY * RY(I,J+1) * WY(I,J+2)                 21MY5
     4              - HYE2 * PY(I,J) * WY(I,J-1)                        24MY5
     5              + HYE2 * ( PY(I,J) + PY(I,J+1) ) * WY(I,J)          24MY5
     6              - HYE2 * PY(I,J+1) * WY(I,J+1) ) / HYE3             27MY5
               SFYZ = SFY(NS)                                           20MR5
               QFY = SFYZ*WY(I,J) - QAY                                 20MR5
C-----COMPUTE Z-BEAM MATRIX COEFFS                                      27JL5
  716          AA   =   FZ(I-1,J) - 0.25 * HX * RZ(I-1,J)               26AP5
               BB   = - 2.0 * ( FZ(I-1,J) + FZ(I,J) ) - HXE2 * PZ(I,J)  08AP5
               CC   =   FZ(I-1,J) + 4.0 * FZ(I,J) + FZ(I+1,J)           20MR5
     1                + HXE2 * ( PZ(I,J) + PZ(I+1,J)   )                05MY5
     2                + 0.25 * HX * ( RZ(I-1,J) + RZ(I+1,J)   )         05MY5
     3                + HXE3 * S(I,J) + HXE3 * ( SFXZ + SFYZ )          05MY5
               DD   = - 2.0 * ( FZ(I,J) + FZ(I+1,J) ) - HXE2 * PZ(I+1,J)08AP5
               EE   =   FZ(I+1,J) - 0.25 * HX * RZ(I+1,J)               06AP5
               FF   = HXE3* (  Q(I,J) + QFX + QFY                       21MY5
     1              -0.5/HX *(TX(I-1,J)-TX(I+1,J))                      21MY5
     2              -0.5/HY *(TY(I,J-1)-TY(I,J+1))                      21MY5
     3              -0.5/HX *(TZ(I-1,J)-TZ(I+1,J))    )                 21MY5
C-----COMPUTE Z-BEAM CONTINUITY COEFFS                                  27JL5
               E    =   AA * B(I-2) + BB                                13AP3
               DEN  =   E  * B(I-1) + AA * C(I-2) + CC                  13AP3
          IF   (DEN)  731, 721, 731                                     26AP5
  721          D    =   0.0                                             26AP5
          GO TO  732                                                    26AP5
  731          D    = - 1.0 / DEN                                       26AP5
  732          C(I) =   D  * EE                                         26AP5
               B(I) =   D  * ( E  * C(I-1) + DD )                       13AP3
               A(I) =   D  * ( E  * A(I-1) + AA * A(I-2) - FF )         13AP3
  740     CONTINUE                                                      26AP5
```

```
C-----COMPUTE Z-BEAM DEFLS                                              27JL5
          DO 750  L = 3,MXP5                                            26AP5
               I     =    MX + 8 - L                                    13AP3
               WTEMP = WZ(I,J)                                          20MR5
               WZ(I,J) = A(I) + B(I)*WZ(I+1,J) + C(I)*WZ(I+2,J)         20MR5
C-----COUNT STAS WHERE Z-BEAMS NOT STABILIZED                           27JL5
          IF( FZ(I,J) )  745, 750, 745                                  26AP5
  745     IF  ( ABSF ( WZ(I,J) - WTEMP ) - CTOL    )750, 750, 746       26AP5
  746          KSTZ = KSTZ + 1                                          26AP5
  750     CONTINUE                                                      26AP5
  760     CONTINUE                                                      26AP5
  790     CONTINUE                                                      26AP5
C-----COUNT STAS WHERE DEFLS NOT CLOSED                                 27JL5
               KCTOL  =  0                                              13AP3
               KCTOLZ = 0                                               20MR5
          DO 850 I = 4, MXP4                                            04MY5
          DO 850 J = 4, MYP4                                            03MY5
          IF ( FX(I,J) * FY(I,J) )  810, 820, 810                       03MY5
  810     IF ( ABSF( WX(I,J) - WY(I,J) ) - CTOL )  820, 820, 815        06MY5
  815          KCTOL = KCTOL + 1                                        03MY5
  820     IF ( ( FX(I,J) + FY(I,J) ) * FZ(I,J) )   830, 850, 830        03MY5
  830     IF ( FX(I,J) )  840, 835, 840                                 03MY5
  835     IF ( ABSF( WZ(I,J) - WY(I,J) ) - CTOL )  850, 850, 845        03MY5
  840     IF ( ABSF( WZ(I,J) - WX(I,J) ) - CTOL )  850, 850, 845        03MY5
  845          KCTOLZ = KCTOLZ + 1                                      03MY5
  850     CONTINUE                                                      03MY5
C-----PRINT MONITOR DATA                                                27JL5
      PRINT 75, NC, SFX(NS), NS, KSTX,   WX(IM1+4,JM1+4),               06AP5
     1    WX(IM2+4,JM2+4), WX(IM3+4,JM3+4), WX(IM4+4,JM4+4),            25AP3
     2              SFY(NS), NS,     KSTY, KCTOL, WY(IM1+4,JM1+4),      20MR5
     3    WY(IM2+4,JM2+4), WY(IM3+4,JM3+4), WY(IM4+4,JM4+4),            31DE4
     4          SFZ(NS), NS, KSTZ,KCTOLZ,WZ(IM1+4,JM1+4),               207R5
     5     WZ(IM2+4,JM2+4), WZ(IM3+4,JM3+4), WZ(IM4+4,JM4+4)            20MR5
C
C-----CONTROL ITERATION PROCESS                                         27JL5
          IF ( KCTOL + KCTOLZ )   900, 900, 860                         26AP5
  860     CONTINUE                                                      26AP5
               NC = ITMAX                                               25AG4
C-----COMPUTE AND PRINT RESULTS                                         27JL5
  900     CONTINUE                                                      26AP5
      PRINT 11                                                          08MY3 ID
      PRINT 1                                                           18FE5 ID
      PRINT 13, ( AN1(N), N = 1, 32 )                                   18FE5 ID
      PRINT 16, NPROB, ( AN2(N), N = 1, 14 )                            28AG3 ID
      PRINT 85, NC                                                      06AP5
          IF ( KCTOL + KCTOLZ ) 912, 912, 911                           26AP5
  911 PRINT 86                                                          26AP5
  912 PRINT 87                                                          26AP5
          DO 960  J = 3,MYP5                                            26AP5
      PRINT  6                                                          24AP3
          DO 950  I = 3,MXP5                                            26AP5
               ISTA = I - 4                                             18AP3
               JSTA = J - 4                                             18AP3
          DO 940  N = 1,3                                               26AP5
               K = I + N - 2                                            21JA5
               BM(N) = FX(K,J) * ( WX(K-1,J) - WX(K,J) - WX(K,J)        01MA3
```

```
     1                 + WX(K+1,J) ) / HXE2                             01MA3
               BM(N+6) = FZ(K,J)*( WZ(K-1,J) - 2.0*WZ(K,J) +            20MR5
     1                 WZ(K+1,J) ) / HXE2                               27JL5
               L = J + N - 2                                            21JA5
               BM(N+3) = FY(I,L) * ( WY(I,L-1) - WY(I,L) -WY(I,L)       01MA3
     1                   + WY(I,L+1) ) / HYE2                           01MA3
  940     CONTINUE                                                      26AP5
               QAX = ( BM(1) - 2.0 * BM(2) + BM(3)) / HX                31DE4
     1              + ( - 0.25 * HX * RX(I-1,J) * WX(I-2,J)             27JL5
     2              + 0.25* HX * ( RX(I-1,J) + RX(I+1,J) ) * WX(I,J)    21MY5
     3              - 0.25* HX * RX(I+1,J) * WX(I+2,J)                  21MY5
     4              -HXE2 * PX(I,J) * WX(I-1,J)                         24MY5
     5              + HXE2 * ( PX(I,J) + PX(I+1,J) ) * WX(I,J)          24MY5
     6              - HXE2 * PX(I+1,J) * WX(I+1,J) ) / HXE3             27MY5
               QAY = ( BM(4) - BM(5) - BM(5) + BM(6) ) / HY             19JA5
     1              + ( - 0.25 * HY * RY(I,J-1) * WY(I,J-2)             27JL5
     2              + 0.25 * HY * ( RY(I,J-1) + RY(I,J+1))* WY(I,J)     21MY5
     3              - 0.25 * HY * RY(I,J+1) * WY(I,J+2)                 21MY5
     4              - HYE2 * PY(I,J) * WY(I,J-1)                        24MY5
     5              + HYE2 * ( PY(I,J) + PY(I,J+1) ) * WY(I,J)          24MY5
     6              - HYE2 * PY(I,J+1) * WY(I,J+1) ) / HYE3             27MY5
               QAZ = ( BM(7) - BM(8) - BM(8) + BM(9) ) / HX             20MR5
     1              + ( - 0.25 * HX * RZ(I-1,J) * WZ(I-2,J)             27JL5
     2              + 0.25* HX * ( RZ(I-1,J) + RZ(I+1,J) ) * WZ(I,J)    21MY5
     3              - 0.25* HX * RZ(I+1,J) * WZ(I+2,J)                  21MY5
     4              - HXE2 * PZ(I,J) * WZ(I-1,J)                        24MY5
     5              + HXE2 * ( PZ(I,J) + PZ(I+1,J) ) * WZ(I,J)          24MY5
     6              - HXE2 * PZ(I+1,J) * WZ(I+1,J) ) / HXE3             27MY5
               REACT = QAX + QAY + QAZ                                  21MY5
          IF   ( FX(I,J) * FY(I,J) )  948, 946, 948                     26AP5
  946          ERROR  =   0.0                                           26AP5
          GO TO  949                                                    26AP5
C-----STATEMENT 948 MAY NOT OPERATE PROPERLY ON COMPUTERS OTHER THAN THE27JL5
C     CDC 1604 WHERE ZERO DIVIDED BY ZERO IS ZERO.                      27JL5
C-----THIS STATEMENT SUMS THE NUMBER OF INTERSECTING BEAMS AT A JOINT.  27JL5
  948          DENO    =   WX(I,J) / WX(I,J) + WY(I,J) / WY(I,J) +      26AP5
     1                     WZ(I,J) / WZ(I,J)                            07AP5
               ERROR   =   Q(I,J) - REACT - S(I,J) * ( WX(I,J) +        21MY5
     1                  WZ(I,J) + WY(I,J) ) / DENO                      07AP5
     2                   -0.5/HX * (TX(I-1,J) - TX(I+1,J))              21MY5
     3                   -0.5/HY * (TY(I,J-1) - TY(I,J+1))              21MY5
     4                   -0.5/HX * (TZ(I-1,J) - TZ(I+1,J))              21MY5
  949 PRINT 88, ISTA, JSTA, WX(I,J), WY(I,J), WZ(I,J), REACT, ERROR     27MY5
               WX(I-2,J-2)= BM(2)                                       27MY5
               WY(I-2,J-2)= BM(5)                                       27MY5
               WZ(I-2,J-2)= BM(8)                                       27MY5
  950     CONTINUE                                                      26AP5
  960     CONTINUE                                                      26AP5
      PRINT 89, NC                                                      27MY5
      PRINT 90                                                          27MY5
          DO 980 J = 3, MYP5                                            27MY5
      PRINT 6                                                           27MY5
          DO 970 I = 3, MXP5                                            27MY5
               ISTA = I - 4                                             27MY5
               JSTA = J - 4                                             27MY5
      PRINT 88, ISTA,JSTA,WX(I-2,J-2),WY(I-2,J-2),WZ(I-2,J-2)           27MY5
```

```
 970     CONTINUE                                                    27MY5
 980     CONTINUE                                                    27MY5
     CALL TIME                                                       18FE5 ID
         GO TO 1010                                                  26AG3 ID
9990 CONTINUE                                                        12MR5 ID
9999 CONTINUE                                                        04MY3 ID
     PRINT 11                                                        08MY3 ID
     PRINT 1                                                         18FE5 ID
     PRINT 13, ( AN1(N), N = 1, 32 )                                 18FE5 ID
     PRINT 19                                                        26AG3 ID
     END                                                             04MY3 ID
     END                                                             04MY3 ID
```

APPENDIX 5

PROGRAM LAYER 7 AND GUIDE FOR DATA INPUT

APPENDIX 5. PROGRAM LAYER 7 AND GUIDE FOR DATA INPUT

Program LAYER 7 is a FORTRAN 63 language program for solving the equations for a simply connected layered structural system. The program requires a large capacity digital computer such as the CDC 1604 or CDC 6600. The FORTRAN source program requires approximately one minute and thirty seconds to compile on the 1604, and seven seconds to compile on the 6600. Various items pertinent to the use of this program follow.

Input Parameters

Input to the program is usually in terms of pounds and inches. However, any consistent set of units may be used. Lumped values (i.e. values per station) are used throughout. Input may be described in the form of a rectangle or on a station-by-station basis. If the rectangle method is used, a full value of the variable is stored at each intersection point in the designated rectangle. An example of rectangle input follows.

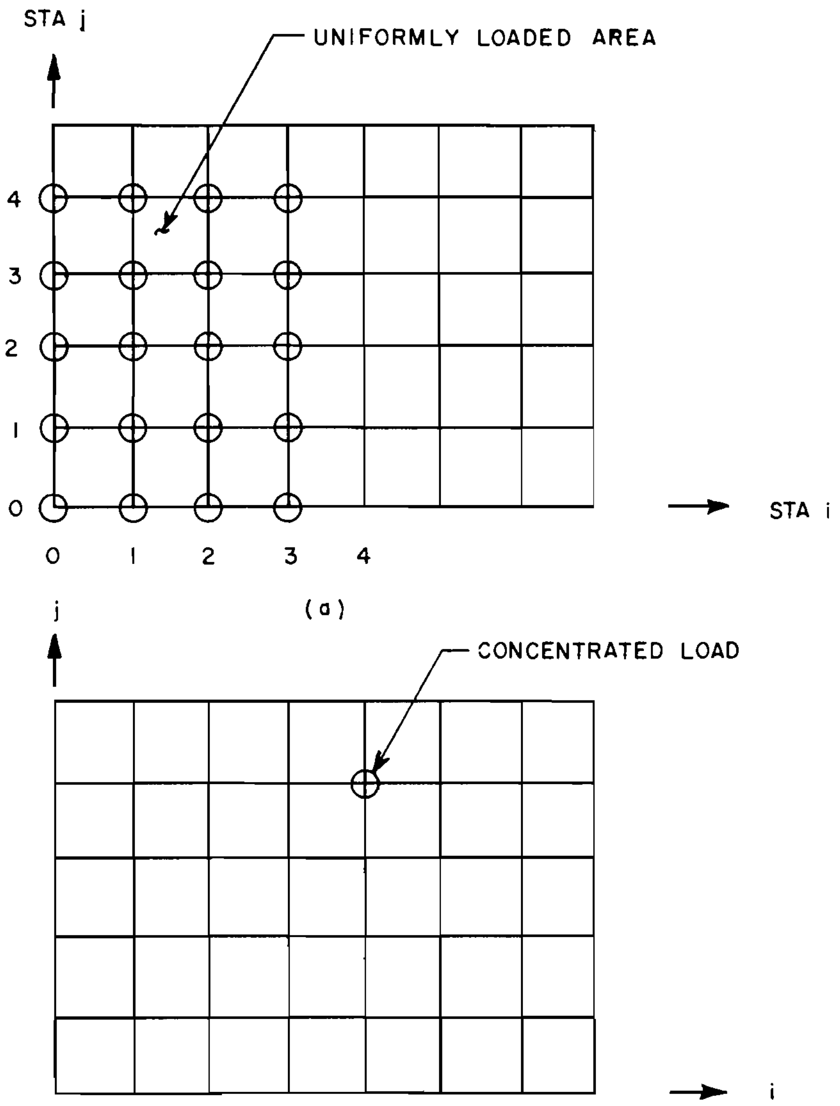
From Sta		to Sta		Value
1	j	i	j	Q
0	0	3	4	1.00E+03

The program would accept into storage a value of transverse load, Q , equal to 1000 lb/sta at each intersection in the rectangle shown in Fig A5.1a. The circles represent stations which would receive load.

The station-by-station method of input is as follows:

From Sta		to Sta		Value
i	j	i	j	Q
4	4	4	4	1.00E+03

A value of transverse load, Q , of 1000 lb/sta would be stored at intersection $i = 4$, $j = 4$ as shown by the circle in Fig A5.1b.



(a)

(b)

Fig A5.1. Methods of data input to computer program LAYER 7.

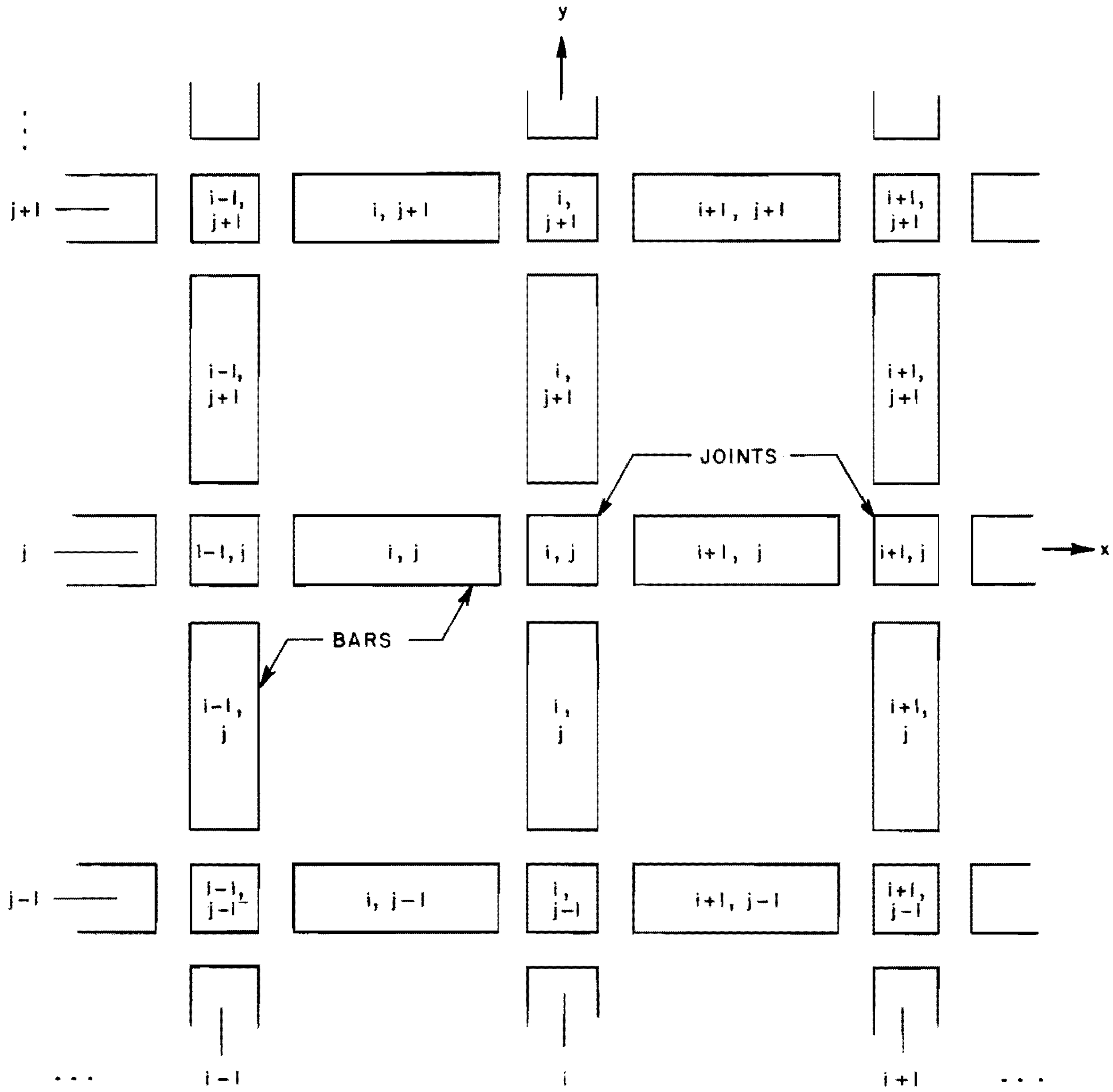


Fig A5.2. Numbering system for the rigid bars and the joints of the finite-element model.

Figure A5.2 shows the manner in which the bars and joints of the model are numbered. Each bar is numbered "before" its joint. In the x-direction this puts the bar to the left of the joint and in the y-direction the bar is shown below its correspondingly numbered joint. It is important to remember that axial tensions refer to bars so that no tension should be numbered that would act outside the real system.

All input to the program is cummulative, i.e., if two data cards such as the one used to create Fig 5.1b were input, the total value of load stored at stations $1 = 4$, $j = 4$ would be 2×10^3 lb . Any amount of adding or subtracting of data may be done to obtain the desired values at each intersection.

Program Results

The results from the program are arranged in tables. Tables 1, 2, 3, 4, 5, and 6 are reflections of the input data. Table 7 lists the deflection versus number of iterations at four selected stations. Table 8 gives all deflections for each x , y , and z-beam plus a reaction term and an error term. The error term refers to the error in load equilibrium at each intersection. The error represents the amount of unbalance between the iteration equation, Eq 3.11, and the equation which must be satisfied, Eq 3.1. The bending moments for each intersecting member are tabulated in Table 9.

GUIDE FOR DATA INPUT FOR LAYER 7

with Supplementary Notes

extract from

A FINITE-ELEMENT METHOD FOR BENDING ANALYSIS OF LAYERED STRUCTURAL SYSTEMS

by

Wayne B. Ingram and Hudson Matlock

June 1967

LAYER 7 GUIDE FOR DATA INPUT -- Card Forms

IDENTIFICATION OF PROGRAM AND RUN (2 alphanumeric cards per run)

1	80

1	80

IDENTIFICATION OF PROBLEM (one card per problem; program stops if NPROB = 0)

NPROB		Description of problem (alphanumeric)	
1	5	11	80

TABLE 1. PROGRAM CONTROL DATA (2 cards per problem)

NUM CARDS IN TABLE					NUM INCRS		INCREMENT LENGTHS		
3	4	5	6	ITERS	MX	MY	X-DIRECTION	Y-DIRECTION	TOLERANCE
15	20	25	30	40	45	50	60	70	80

MONITOR DATA (Specify four intersection in terms of i-sta and j-sta coordinates)

15	20	25	30	35	40	45	50

TABLE 2. CLOSURE PARAMETERS

	NUM VALUES	CLOSURE PARAMETER VALUES (3 cards, all cards must have same number of values)					
X:	15	21	30	40	50	60	70	80
Y:	15	21	30	40	50	60	70	80
Z:	15	21	30	40	50	60	70	80

TABLE 3. X-BEAM DATA

FROM i	STA j	TO i	STA j	F BENDING STIFFNESS	T TRANSVERSE COUPLE	R ROTATIONAL RESTRAINT	P AXIAL TENSION OR COMPRESSION
15	20	25	30	40	50	60	70

TABLE 4. Y-BEAM DATA

FROM i	STA j	TO i	STA j	F BENDING STIFFNESS	T TRANSVERSE COUPLE	R ROTATIONAL RESTRAINT	P AXIAL TENSION OR COMPRESSION
15	20	25	30	40	50	60	70

TABLE 5. Z-BEAM DATA

FROM i	STA j	TO i	STA j	F BENDING STIFFNESS	T TRANSVERSE COUPLE	R ROTATIONAL RESTRAINT	P AXIAL TENSION OR COMPRESSION
15	20	25	30	40	50	60	70

TABLE 6. LOAD AND SUPPORT DATA

FROM i	STA j	TO i	STA j	Q TRANSVERSE FORCE	S SPRING SUPPORT
15	20	25	30	40	50

BLANK CARD TO STOP PROGRAM	
1	80

GENERAL PROGRAM NOTES

Two cards containing any desired alphanumeric information are required (for identification purposes only) at the beginning of the data for each new run.

The data cards must be stacked in proper order for the program to run.

A consistent system of units must be used for all input data; for example: pounds and inches.

All data words must be justified completely to the right in the field provided.

All data words of 5 spaces or less are to be whole integer numbers: . . . . $\boxed{\text{-1234}}$

All data words of 10 spaces are to be entered as floating-point decimal numbers including a multiplier expressed in terms of an exponent of 10: . . . . $\boxed{\text{1.234E+03}}$

Blank data fields are interpreted as zeros.

One card with a problem number in spaces 1-5 is required as the first card of each problem. This number may be alphanumeric. The remainder of the card may be any information desired.

Any number of problems may be stacked in one run.

One card with problem number blank is required to stop the run.

If it is desired to work a two-layer system (a simple grid) it is only necessary to omit the proper tables 3, 4 or 5. Three cards must still appear in Table 2 except one may be blank.

TABLE 1.

The card counts in Table 1 should be rechecked carefully after coding of each problem. The run will be abandoned if the card counts are incorrect.

The number of iterations should equal the product of the number of closure parameters to be cycled and the number of cycles desired.

The maximum allowable number of increments in the x-direction is 25; in the y-direction is 25.

The length of increment may be anything desired by the user. Unreasonably large increments may give too crude an approximation to the problem.

The deflection closure tolerance may be difficult to achieve if specified unreasonably small. A value between 0.001 and 0.00001 is usually satisfactory.

TABLE 1. (Continued)

Four monitor intersections are allowable to follow the convergence of the solution at four points. The program automatically outputs deflections for all intersecting beams at each monitor intersection.

TABLE 2. CLOSURE PARAMETERS

Up to six closure parameters are allowed for each layer. The same number must be used on all layers.

Parameters should be input in the order in which they are to be cycled.

Three cards are required; one may be blank if a two-layer system is to be worked.

TABLES 3, 4, 5. X , Y , AND Z-BEAM DATA

Typical units:

variables:	F	T	R	P
values per station:	lb × in^2	in × lb	in × lb/rad	lb

Axial tension or compression values P are stated at stations in the same manner as any other distributed data. Since the P values refer to bars, no station number should be used which would specify a P value in a bar outside the real system.

Data must not be entered which would express effects at fictitious stations beyond the edges of the real grid system.

There are no restrictions on the order of cards in these tables. Cumulative input, with full values at each station, is used in these tables.

TABLE 6. LOAD AND SUPPORT DATA

Typical units:

variables:	Q	S
values per station:	lb	lb/in

Same comments as for Tables 3, 4, 5.

APPENDIX 6

DERIVATION OF EQUATIONS FOR THE FINITE-ELEMENT MODEL OF AN ORTHOTROPIC PLATE-OVER-BEAM SYSTEM

APPENDIX 6. DERIVATION OF EQUATIONS FOR THE FINITE-ELEMENT MODEL OF AN ORTHOTROPIC PLATE-OVER-BEAM SYSTEM

The primary differences between the grid-over-beam system and the plate-over-beam system are torsional and Poisson's ratio effects. The torsional effects are treated with the torsion bars connecting the midpoints of the various beams in Fig 7.1 and Poisson's ratio is considered as internal moments at each joint. It is conventional to consider internal shears and moments as values per unit of width of the plate perpendicular to the direction being considered. For the finite-element model, however, it is necessary to have lumped values of these parameters for the solution process which involves beam-element approximations to the continuum. A listing of notation to be used in these derivations is in Table A6.1. These derivations will be in terms of values per unit width until an appropriate point at which time a changeover will be apparent.

A free-body representation of a portion of an x-beam is shown in Fig A6.1. Figure A6.2 is a free-body representation of all the forces on some i^{th} joint of the j^{th} beam of the system. For these derivations the y and z-beams of the system will be considered temporarily fixed in position. The effects of the y and z-beam on the x-beam are shown by loads Q^y and Q^z and differential springs K^y and K^z. The joint material is assumed to have properties such that the Poisson's ratio effects are created when the joint is deformed. The C terms represent torsional stiffness of the plate.

Summing forces in the vertical direction on the joint in Fig A6.2 gives

$$Q_{i,j} - Q^y_{i,j} - Q^z_{i,j} - S_{i,j}w^x_{i,j} - K^y (w^x_{i,j} - w^y_{i,j}) \quad \text{(Equation Cont'd.)}$$

TABLE A6.1. NOTATION FOR PLATE-OVER-BEAM DERIVATIONS

Symbol	Unit	Definition
D_x , D_y	lb-in^2/in.	Plate stiffness/unit width.
F	lb-in^2	Beam stiffness.
Q	lb	Transverse load.
Q^x, Q^y	lb	Load resisted by x , y , or z-beam.
θ	radians	Central-difference slope.
ϕ	radians	Simple-difference slope.
C_x , C_y	lb-in^2/rai./in.	Torsional stiffness/unit width.
P_x', P_y'	lb/in.	Axial force/unit width.
S	lb/in.	Spring support.
M_x', M_y'	in-lb/in.	Plate moment/unit width.
ν_{xy}, ν_{yx}	-	Poisson's ratios.
V_x', V_y'	lb/in.	Shear/unit width.
M_x , M_y	in-lb	Moment.
V_x , V_y	lb	Shear.
P_x , P_y	lb	Axial tension.
D_x', D_y'	lb-in^2/in.	Orthotropic stiffness/unit width.
T_x , T_y	in-lb	External torque.
R_x , R_y	in-lb/rad	External rotational restraint.
h	in.	Increment length.
V_A', V_B', V_C', V_O'	lb/in	Temporary representation in derivations of shear/unit width
P_A', P_B', P_C', P_O'	lb/in	Temporary representation in derivations of axial force/unit width

$$- K^z \, (w^x_{i,j} - w^z_{i,j}) - \left[\frac{R^x_{i-1,j} \; \theta^x_{i-1,j}}{2h_x}\right] + \left[\frac{R^x_{i+1,j} \; \theta^x_{i+1,j}}{2h_x}\right]$$

$$- \frac{T^x_{i-1,j}}{2h_x} + \frac{T^x_{i+1,j}}{2h_x} + V'_A \, h_y - V'_B \, h_y \;=\; 0 \qquad \text{(A6.1)}$$

where the values of θ^x are defined by central difference expressions as follows

$$\theta^x_{i-1,j} \;=\; \frac{-w_{i-2,j} + w_{i,j}}{2h_x}$$

$$\theta^x_{i+1,j} \;=\; \frac{-w_{i,j} + w_{i+2,j}}{2h_x} \qquad \text{(A6.2)}$$

An expression for V'_A can be developed from consideration of Bar i as a free-body. Writing the summation of moments expression about V'_B , we find

$$h_y \, M'_{i-1,j} + h_x C^x_{i,j} \left(\frac{\phi^x_{i,j} - \phi^x_{i,j-1}}{h_y}\right) + h_x C^x_{i,j+1} \left(\frac{\phi^x_{i,j} - \phi^x_{i,j+1}}{h_y}\right)$$

$$+ V'_A \, h_x h_y - h_y M'_{i,j} + h_y P'_A \, (-w^x_{i-1,j} + w^x_{i,j}) \;=\; 0 \qquad \text{(A6.3)}$$

where the values of ϕ^x are the slopes of parallel bars expressed by simple differences as follows for Bar i.

$$\phi^x_{i,j} \;=\; \frac{-w^x_{i-1,j} + w^x_{i,j}}{h_x}$$

$$\phi^x_{i,j-1} \;=\; \frac{-w^x_{i-1,j-1} + w^x_{i,j-1}}{h_x} \qquad \text{(A6.4)}$$

$$\phi^x_{i,j+1} \;=\; \frac{-w^x_{i-1,j+1} + w^x_{i,j+1}}{h_x}$$

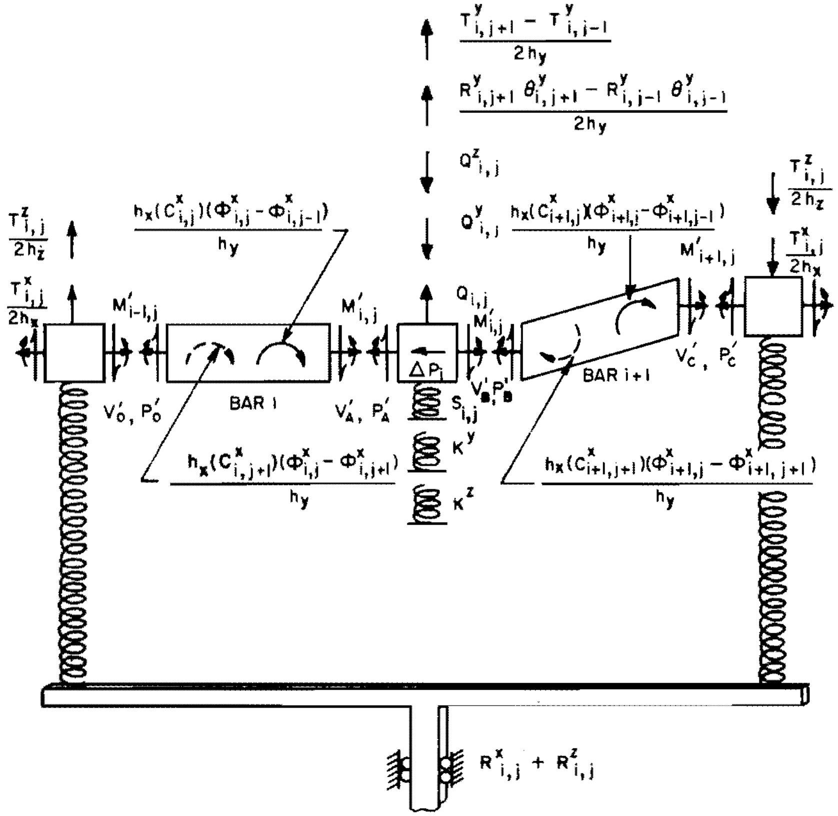


Fig A6.1. An x-direction portion of the plate for the plate-over-beam system.

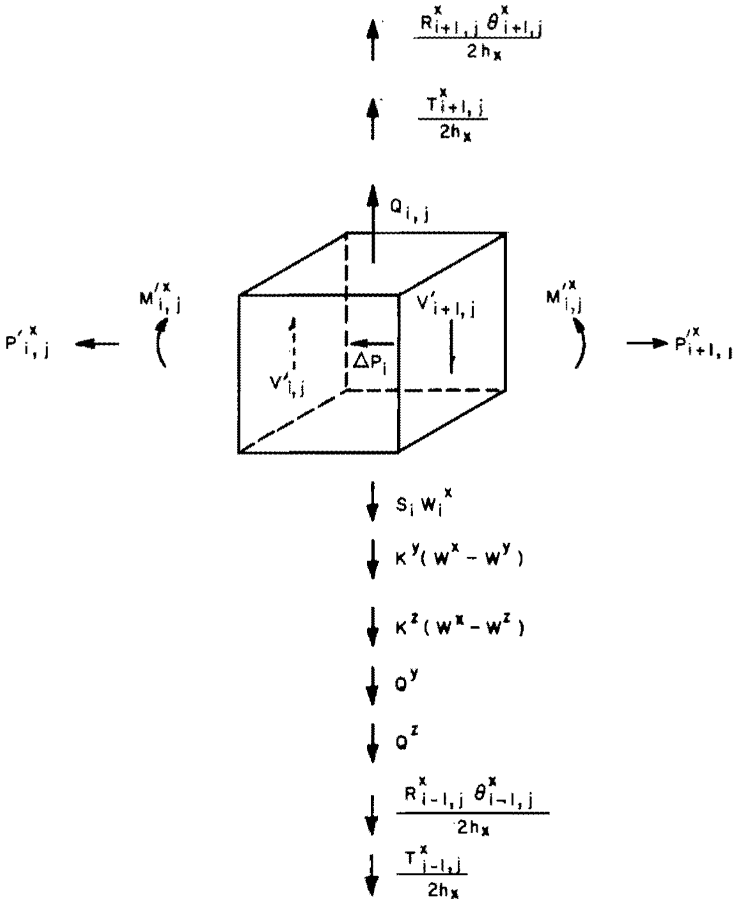


Fig A6.2. Forces on one joint of the plate-over-beam system.

Rearranging Eq A6.3, we have

$$V'_A = \frac{1}{h_x}\Big[-M'_{i-1,j} + M'_{i,j} - \frac{h_x}{h_y} C^x_{i,j} \left(\frac{\phi^x_{i,j} - \phi^x_{i,j-1}}{h_y}\right)$$

$$- \frac{h_x}{h_y} C^x_{i,j+1} \left(\frac{\phi^x_{i,j} - \phi^x_{i,j+1}}{h_y}\right) - P'_A\,(-w^x_{i-1,j} + w^x_{i,j})\Big] \qquad \text{(A6.5)}$$

Since no external horizontal loads are applied to the middle of the bar,

$$P'_A = P'_0 = P'_{i,j} \qquad \text{(A6.6)}$$

and similarly

$$P'_B = P'_C = P'_{i+1,j} \qquad \text{(A6.7)}$$

The shears V' are also applied at the joints so that no vertical forces apply to the middle of the bar, thus

$$V'_A = V'_0 = V'_i \qquad \text{(A6.8)}$$

and

$$V'_B = V'_C = V'_{i+1} \qquad \text{(A6.9)}$$

A similar analysis applied to Bar i+1 will yield an expression for V'_B with moments summed about V'_C, i.e.,

$$h_y M'_{i,j} - h_y M'_{i+1,j} + h_y h_x V'_B + h_x C^x_{i+1,j} \left(\frac{\phi^x_{i+1,j} - \phi^x_{i+1,j-1}}{h_y}\right)$$

$$+ h_x C^x_{i+1,j+1} \left(\frac{\phi^x_{i+1,j} - \phi^x_{i+1,j+1}}{h_y}\right) + h_y P'_B\,(-w^x_{i,j} + w^x_{i+1,j})$$

$$= 0 \qquad \text{(A6.10)}$$

and rearranging to solve for V'_B,

$$
\begin{aligned}
V'_B = \frac{1}{h_x}\Bigg[& -M'_{i,j} + M'_{i+1,j} - \frac{h_x}{h_y} C^x_{i+1,j} \left(\frac{\phi^x_{i+1,j} - \phi^x_{i+1,j-1}}{h_y}\right) \\
& - \frac{h_x}{h_y} C^x_{i+1,j+1} \left(\frac{\phi^x_{i+1,j} - \phi^x_{i+1,j+1}}{h_y}\right) - P'_B \, (-w^x_{i,j} + w^x_{i+1,j})\Bigg] \qquad \text{(A6.11)}
\end{aligned}
$$

Now, substituting Eq A6.11 and A6.5 into Eq A6.1 yields Eq A6.12,

$$
\begin{aligned}
& Q_{i,j} - Q^y_{i,j} - Q^z_{i,j} - S_{i,j} w^x_{i,j} - K^y \, (w^x_{i,j} - w^y_{i,j}) \\
& \quad - K^z \, (w^x_{i,j} - w^z_{i,j}) - R^x_{i-1,j} \left(\frac{-w^x_{i-2,j} + w^x_{i,j}}{4h_x^2}\right) \\
& \quad + R^x_{i+1,j} \left(\frac{-w^x_{i,j} + w^x_{i+2,j}}{4h_x^2}\right) - \frac{T^x_{i-1,j}}{2h_x} + \frac{T^x_{i+1,j}}{2h_x} \\
& \quad + \frac{h_y}{h_x} \Bigg\{ -M'_{i-1,j} + M'_{i,j} - \frac{h_x}{h_y^2} C^x_{i,j} \Bigg[\left(\frac{-w^x_{i-1,j} + w^x_{i,j}}{h_x}\right) \\
& \quad - \left(\frac{-w^x_{i-1,j-1} + w^x_{i,j-1}}{h_x}\right) \Bigg] - \frac{h_x}{h_y^2} C^x_{i,j+1} \Bigg[\left(\frac{-w^x_{i-1,j} + w^x_{i,j}}{h_x}\right) \\
& \quad - \left(\frac{-w^x_{i-1,j+1} + w^x_{i,j+1}}{h_x}\right) \Bigg] - P'_{i,j} \, (-w^x_{i-1,j} + w^x_{i,j}) \Bigg\} \\
& \quad - \frac{h_y}{h_x} \Bigg\{ -M'_{i,j} + M'_{i+1,j} - \frac{h_x}{h_y^2} C^x_{i+1,j} \Bigg[\left(\frac{-w^x_{i,j} + w^x_{i+1,j}}{h_x}\right) \\
& \quad - \left(\frac{-w^x_{i,j-1} + w^x_{i+1,j-1}}{h_x}\right) \Bigg] - \frac{h_x}{h_y^2} C^x_{i+1,j+1} \Bigg[\left(\frac{-w^x_{i,j} + w^x_{i+1,j}}{h_x}\right) \\
& \quad - \left(\frac{-w^x_{i,j+1} + w^x_{i+1,j+1}}{h_x}\right) \Bigg] - P'_{i+1,j} \, (-w^x_{i,j} + w^x_{i+1,j}) \Bigg\} \\
& \quad = \; 0 \qquad \text{(A6.12)}
\end{aligned}
$$

Collecting terms and multiplying by h_x/h_y, we obtain

$$
\begin{aligned}
& M'_{i-1,j} - 2M'_{i,j} + M'_{i+1,j} + \frac{1}{h_y^2}\Big[C^x_{i,j}\,(-w^x_{i-1,j} + w^x_{i,j} + w^x_{i-1,j-1} \\
& \quad - w^x_{i,j-1}) + C^x_{i,j+1}\,(-w^x_{i-1,j} + w^x_{i,j} + w^x_{i-1,j+1} - w^x_{i,j+1}) \\
& \quad - C^x_{i+1,j}\,(-w^x_{i,j} + w^x_{i+1,j} + w^x_{i,j-1} - w^x_{i+1,j-1}) \\
& \quad - C^x_{i+1,j+1}\,(-w^x_{i,j} + w^x_{i+1,j} + w^x_{i,j+1} - w^x_{i+1,j+1})\Big] \\
& \quad + P'_{i,j}\,(-w^x_{i-1,j} + w^x_{i,j}) - P'_{i+1,j}\,(-w^x_{i,j} + w^x_{i+1,j}) \\
& \quad = \frac{h_x}{h_y}\Big[Q_{i,j} - Q^y_{i,j} - Q^z_{i,j} - S_{i,j}w^x_{i,j} - K^y\,(w^x_{i,j} - w^y_{i,j}) \\
& \quad - K^z\,(w^x_{i,j} - w^z_{i,j}) - \frac{R^x_{i-1,j}}{4h_x^2}\,(-w^x_{i-2,j} + w^x_{i,j}) \\
& \quad + \frac{R^x_{i+1,j}}{4h_x^2}\,(-w^x_{i,j} + w^x_{i+2,j}) - \frac{T^x_{i-1,j}}{2h_x} + \frac{T^x_{i+1,j}}{2h_x}\Big]
\end{aligned}
\tag{A6.13}
$$

It is now convenient to convert to lumped values for the plate bending moments. The M' terms have represented plate moments and the relation to lumped values for the model is Eq A6.14.

$$
M_{i,j} = h_y M'_{i,j} = h_y D^x_{i,j}\,\frac{\partial^2 w}{\partial x^2} + h_y D^{x\prime}_{i,j}\,\frac{\partial^2 w}{\partial y^2}
\tag{A6.14}
$$

The second-order simple difference expression for moment in terms of deflection is Eq A6.15.

$$
\begin{aligned}
M_{i,j} = {} & h_y D^x_{i,j}\left(\frac{w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j}}{h_x^2}\right) \\
& + h_y D^{x\prime}_{i,j}\left(\frac{w^x_{i,j-1} - 2w^x_{i,j} + w^x_{i,j+1}}{h_y^2}\right)
\end{aligned}
\tag{A6.15}
$$

In Eqs A6.14 and A6.15, the $D^{x\prime}_{i,j}$ represents the Poisson's ratio effect on the stiffness. $D^{x\prime}_{i,j}$ is defined by Eq A6.16. This definition will be shown in Appendix 7.

$$D^{x\prime}_{i,j} = \nu_y D^x_{i,j} \tag{A6.16}$$

It is also convenient now to substitute a lumped value of axial tension as defined by Eq A6.17

$$P^x_{i,j} = h_y P'_{i,j} \tag{A6.17}$$

Writing Eq A6.15 at each of three successive stations and substituting these and Eq A6.17 into Eq A6.13 yields Eq A6.18 after clearing equations by multiplying by a factor of h_y^2 .

$$\frac{h_y^2}{h_x^2}\Big[D^x_{i-1,j}\,(w^x_{i-2,j} - 2w^x_{i-1,j} + w^x_{i,j}) - 2D^x_{i,j}\,(w^x_{i-1,j} - 2w^x_{i,j}$$

$$+ w^x_{i+1,j}) + D^x_{i+1,j}\,(w^x_{i,j} - 2w^x_{i+1,j} + w^x_{i+2,j})\Big]$$

$$+ \Big[D^{x\prime}_{i-1,j}\,(w^y_{i-1,j-1} - 2w^y_{i-1,j} + w^y_{i-1,j+1})$$

$$- 2D^{x\prime}_{i,j}\,(w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1})$$

$$+ D^{x\prime}_{i+1,j}\,(w^y_{i+1,j-1} - 2w^y_{i+1,j} + w^y_{i+1,j+1})\Big]$$

$$+ \Big[\text{C terms from Eq A6.13}\Big] + h_y P^x_{i,j}\,(-w^x_{i-1,j} + w^x_{i,j})$$

$$- h_y P^x_{i+1,j}\,(-w^x_{i,j} + w^x_{i+1,j})$$

$$= h_y h_x \Big[Q_{i,j} - Q^y_{i,j} - Q^z_{i,j} - S_{i,j} w^x_{i,j} - K^y\,(w^x_{i,j} - w^y_{i,j})$$

(Equation Cont'd.)

$$- K^z (w^x_{i,j} - w^z_{i,j})\Big] - h_y \frac{R^x_{i-1,j}}{4h_x} (-w^x_{i-2,j} + w^x_{i,j})$$

$$+ h_y \frac{R^x_{i+1,j}}{4h_x} (-w^x_{i,j} + w^x_{i+2,j}) - \frac{h_y}{2} (T^x_{i-1,j} - T^x_{i+1,j}) \qquad \text{(A6.18)}$$

Now, rearranging Eq A6.18 into form for a five-diagonal matrix solution results in Eq A6.19.

$$w^x_{i-2,j} \left[\frac{h_y^2}{h_x^2} D^x_{i-1,j} - R^x_{i-1,j} \frac{h_y}{4h_x}\right]$$

$$+ w^x_{i-1,j} \left[-2 \frac{h_y^2}{h_x^2} (D^x_{i-1,j} + D^x_{i,j}) - C^x_{i,j} - C^x_{i,j+1} - h_y P^x_{i,j}\right]$$

$$+ w^x_{i,j} \Big[\frac{h_y^2}{h_x^2} (D^x_{i-1,j} + 4D^x_{i,j} + D^x_{i+1,j}) + C^x_{i,j} + C^x_{i,j+1}$$

$$+ C^x_{i+1,j} + C^x_{i+1,j+1} + h_y (P^x_{i,j} + P^x_{i+1,j})$$

$$+ \frac{h_y}{4h_x} (R^x_{i-1,j} + R^x_{i+1,j}) + h_x h_y (S_{i,j} + K^y + K^z)\Big]$$

$$+ w^x_{i+1,j} \left[-2 \frac{h_y^2}{h_x^2} (D^x_{i,j} + D^x_{i+1,j}) - C^x_{i+1,j} - C^x_{i+1,j+1} - h_y P^x_{i+1,j}\right]$$

$$+ w^x_{i+2,j} \left[\frac{h_y^2}{h_x^2} D^x_{i+1,j} - \frac{h_y}{4h_x} R^x_{i+1,j}\right] + \frac{h_y}{2} (T^x_{i-1,j} - T^x_{i+1,j})$$

$$+ C^x_{i,j} (w^x_{i-1,j-1} - w^x_{i,j-1}) + C^x_{i,j+1} (w^x_{i-1,j+1} - w^x_{i,j+1})$$

$$- C^x_{i+1,j} (w^x_{i,j-1} - w^x_{i+1,j-1}) - C^x_{i+1,j+1} (w^x_{i,j+1} - w^x_{i+1,j+1})$$

$$+ D^{x\prime}_{i-1,j} (w^y_{i-1,j-1} - 2w^y_{i-1,j} + w^y_{i-1,j+1}) \qquad \text{(Equation Cont'd.)}$$

$$- 2D^{x\prime}_{i,j} \; (w^y_{i,j-1} - 2w^y_{i,j} + w^y_{i,j+1})$$

$$+ D^{x\prime}_{i+1,j} \; (w^y_{i+1,j-1} - 2w^y_{i+1,j} + w^y_{i+1,j+1})$$

$$= h_x h_y \; (Q_{i,j} - Q^y_{i,j} - Q^z_{i,j} + K^y w^y_{i,j} + K^z w^z_{i,j}) \qquad \text{(A6.19)}$$

All terms on the left side of Eq A6.19, with the exception of the $S_{i,j}$ and K terms, represent the load $Q^x_{i,j}$ as shown previously in Eq 7.4. A similar derivation for each of the y and z-beams would yield Eqs A6.20 and A6.21 whose left sides represent respectively the $Q^y_{i,j}$ and $Q^z_{i,j}$ terms. No torsion nor Poisson's ratio terms appear in Eq A6.21 since beam torsion is not considered for the z-beam.

$$w^y_{i,j-2} \left[\frac{h_x^2}{h_y^2} D^y_{i,j-1} - \frac{h_x}{4h_y} R^y_{i,j-1}\right]$$

$$+ w^y_{i,j-1} \left[\frac{-2h_x^2}{h_y^2} (D^y_{i,j-1} + D^y_{i,j}) - C^y_{i,j} - C^y_{i+1,j} - h_x P^y_{i,j}\right]$$

$$+ w^y_{i,j} \left[\frac{h_x^2}{h_y^2} (D^y_{i,j-1} + 4D^y_{i,j} + D^y_{i,j+1})\right.$$

$$+ C^y_{i,j} + C^y_{i+1,j} + C^y_{i,j+1} + C^y_{i+1,j+1} + h_x \; (P^y_{i,j} + P^y_{i,j+1})$$

$$\left. + \frac{h_x}{4h_y} (R^y_{i,j-1} + R^y_{i,j+1}) + h_y h_x \; (S_{i,j} + K^x + K^z)\right]$$

$$+ w^y_{i,j+1} \left[\frac{-2h_x^2}{h_y^2} (D^y_{i,j} + D^y_{i,j+1}) - C^y_{i,j+1} - C^y_{i+1,j+1} - h_x P^y_{i,j+1}\right]$$

$$+ w^y_{i,j+2} \left[\frac{h_x^2}{h_y^2} D^x_{i,j+1} - \frac{h_x}{4h_y} R^y_{i,j+1}\right] \qquad \text{(Equation Cont'd.)}$$

$$
\begin{aligned}
&+ \frac{h_x}{2} (T^y_{i,j-1} - T^y_{i,j+1}) + C^y_{i,j} (w^y_{i-1,j-1} - w^y_{i-1,j}) \\
&+ C^y_{i+1,j} (w^y_{i+1,j-1} - w^y_{i+1,j}) - C^y_{i,j+1} (w^y_{i-1,j} - w^y_{i-1,j+1}) \\
&- C^y_{i+1,j+1} (w^y_{i+1,j} - w^y_{i+1,j+1}) + D^{y\prime}_{i,j-1} (w^x_{i-1,j-1} - 2w^x_{i,j-1} \\
&+ w^x_{i+1,j-1}) - 2D^{y\prime}_{i,j} (w^x_{i-1,j} - 2w^x_{i,j} + w^x_{i+1,j}) \\
&+ D^{y\prime}_{i,j+1} (w^x_{i-1,j+1} - 2w^x_{i,j+1} + w^x_{i+1,j+1}) \\
&= h_x h_y (Q_{i,j} + K^x w^x_{i,j} + K^z w^z_{i,j} - Q^x_{i,j} - Q^z_{i,j}) \qquad \text{(A6.20)}
\end{aligned}
$$

$$
\begin{aligned}
& w^z_{i-2,j} [F^z_{i-1,j} - \frac{h_z}{4} R^z_{i-1,j}] \\
&+ w^z_{i-1,j} [-2 (F^z_{i-1,j} + F^z_{i,j}) - h_z^2 P^z_{i,j}] \\
&+ w^z_{i,j} [F^z_{i-1,j} + 4F^z_{i,j} + F^z_{i+1,j} + h_z^2 (P^z_{i,j} + P^z_{i+1,j}) \\
&+ \frac{h_z}{4} (R^z_{i-1,j} + R^z_{i+1,j}) + S_{i,j} h_z^3] + h_z^3 (K^y + K^x) \\
&+ w^z_{i+1,j} [-2 (F^z_{i,j} + F^z_{i+1,j}) - h_z^2 P^z_{i+1,j}] \\
&+ w^z_{i+2,j} [F^z_{i+1,j} - \frac{h_z}{4} R^z_{i+1,j}] + \frac{h_z^2}{2} (T^z_{i-1,j} - T^z_{i+1,j}) \\
&= h_z^3 [Q_{i,j} - Q^y_{i,j} - Q^x_{i,j} + K^x w^x_{i,j} + K^y w^y_{i,j}] \qquad \text{(A6.21)}
\end{aligned}
$$

Eqs A6.19, A6.20 and A6.21 can be written in symbolic form as Eqs A6.22 where all terms with superscripts represent particular layers of the system. Equation A6.22 is arranged in a convenient manner for iterative solution.

$$Q^x + S_i w_i^x + K^y w_i^x + K^z w_i^x = Q_i - Q^z - Q^y + K^y w_i^y + K^z w_i^z$$

$$Q^y + S_i w_i^y + K^x w_i^y + K^z w_i^y = Q_i - Q^x - Q^z + K^x w_i^y + K^z w_i^z \qquad \text{(A6.22)}$$

$$Q^z + S_i w_i^z + K^x w_i^z + K^y w_i^z = Q_i - Q^x - Q^y + K^y w_i^y + K^x w_i^x$$

Eqs A6.19, A6.20 and A6.21 are in the form necessary for the use of the quidiagonal matrix solution process. They can be represented as Eq A6.23 in the same manner that Eq A1.17 is represented.

$$a_o w_{-2} + b_o w_{-1} + c_o w_o + d_o w_{+1} + e_o w_{+2} = f_o \qquad \text{(A6.23)}$$

APPENDIX 7

DEVELOPMENT OF CONSTANTS FOR ORTHOTROPIC PLATE

APPENDIX 7. DEVELOPMENT OF CONSTANTS FOR ORTHOTROPIC PLATE

An orthotropic plate is a plate which may have different material properties in any of its orthogonal directions. The usual example of an orthotropic material is plywood. The independent constants for an orthotropic material will be discussed in this section.

Consider Fig A7.1a, a plane element of some orthotropic material. If some stress σ_x is applied along with a $\sigma_y = 0$, there will be a deformation of the element to the deformed shape shown in Fig A7.1a corresponding to a strain ϵ_{xx} and ϵ_{yx} . The first subscript indicates stress direction and the second indicates strain direction.

Equation A7.1 is Hooke's law as a stress-strain relation. The conventional definition for Poisson's ratio is the ratio of lateral strain to longitudinal strain, or Eq A7.2.

$$E_x = \frac{\sigma_x}{\epsilon_{xx}} \tag{A7.1}$$

$$\nu_{xy} = \frac{-\epsilon_{xy}}{\epsilon_{xx}} \tag{A7.2}$$

A similar stress applied to the Fig A7.1b, but with ϵ_y having a value and $\sigma_x = 0$, would lead to Eqs A7.3 and A7.4.

$$E_y = \frac{\sigma_y}{\epsilon_{yy}} \tag{A7.3}$$

$$\nu_{yx} = \frac{-\epsilon_{yx}}{\epsilon_{yy}} \tag{A7.4}$$

and

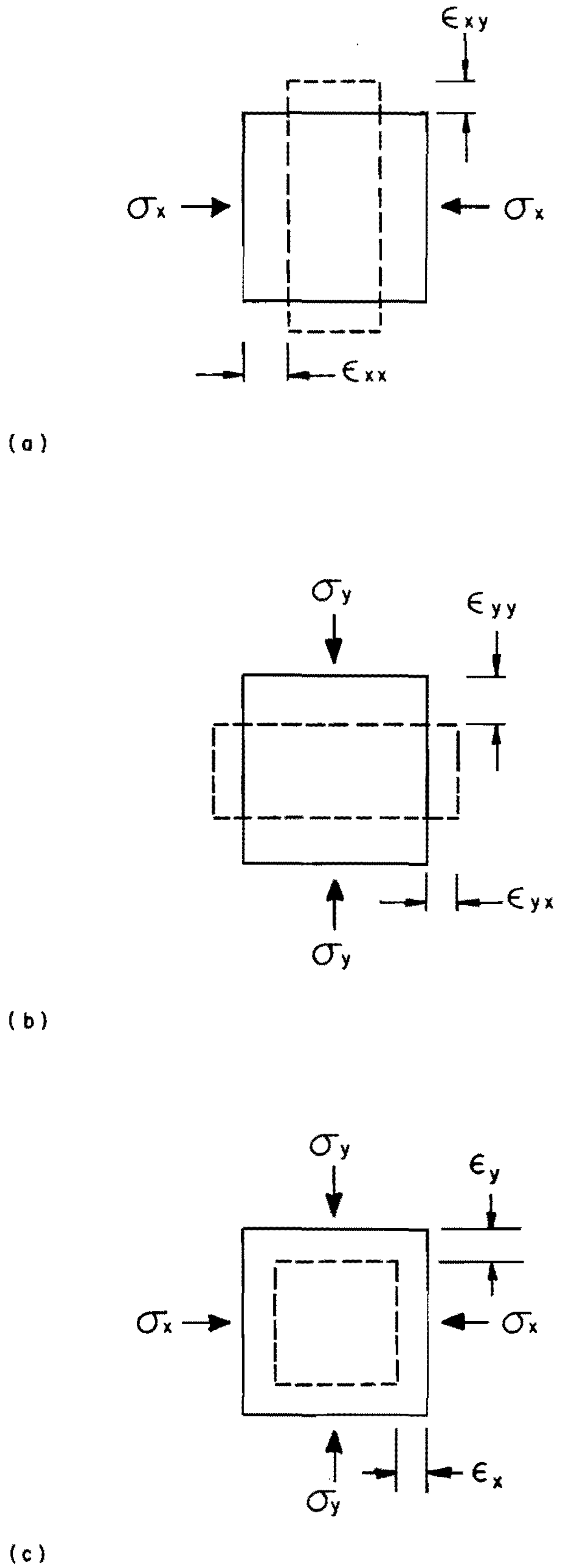


Fig A7.1. Plane stress element.

$$\epsilon_{yx} = -\nu_{yx}\,\epsilon_{yy} \tag{A7.5}$$

$$\epsilon_{yx} = -\nu_{yx}\,\frac{\sigma_y}{E_y} \tag{A7.6}$$

and similarly from Eqs A7.2 and A7.1

$$\epsilon_{xy} = \frac{-\nu_{xy}}{E_x}\,\sigma_x \tag{A7.7}$$

If two stress components were applied simultaneously (Fig A7.1c) then by superposition the components of strain in the respective x and y-directions would be expressed as Eqs A7.8 and A7.9

$$\epsilon_x = \epsilon_{xx} + \epsilon_{yx} \tag{A7.8}$$

$$\epsilon_y = \epsilon_{yy} + \epsilon_{xy} \tag{A7.9}$$

Then, substituting Eqs A7.1, A7.3, A7.6, and A7.7 into Eqs A7.8 and A7.9, it can be shown that

$$\epsilon_x = \frac{\sigma_x}{E_x} - \nu_{yx}\,\frac{\sigma_y}{E_y} \tag{A7.10}$$

and

$$\epsilon_y = -\nu_{xy}\,\frac{\sigma_x}{E_x} + \frac{\sigma_y}{E_y} \tag{A7.11}$$

Equation A7.10 can be rearranged to arrive at Eq A7.12,

$$\sigma_x = \left(\epsilon_x + \nu_{yx}\,\frac{\sigma_y}{E_y}\right) E_x \tag{A7.12}$$

which in turn can be substituted in Eq A7.11 to obtain an expression for the y-strain component, Eq A7.13

$$\epsilon_y = \frac{-\nu_{xy}}{E_x}\left(\epsilon_x + \nu_{yx}\,\frac{\sigma_y}{E_y}\right) E_x + \frac{\sigma_y}{E_y} \tag{A7.13}$$

which in turn rearranged yields Eq A7.14 which is a function of three material constants.

$$\sigma_y = \frac{E_y}{1 - \nu_{xy}\nu_{yx}} (\epsilon_y + \nu_{xy}\epsilon_x) \tag{A7.14}$$

A similar expression, Eq A7.15 can be developed for the x-stress component.

$$\sigma_x = \frac{E_x}{1 - \nu_{xy}\nu_{yx}} (\epsilon_x + \nu_{yx}\epsilon_y) \tag{A7.15}$$

Under the usual assumptions of small deflection theory, the strain may be expressed in the forms of Eq A7.16 (Ref 28, p 364).

$$\begin{aligned} \epsilon_x &= z \frac{\partial^2 w}{\partial x^2} \\ \epsilon_y &= z \frac{\partial^2 w}{\partial y^2} \end{aligned} \tag{A7.16}$$

The external moments M_x and M_y can be defined in terms of normal stresses (Ref 28, p 365) as Eq A7.17 for M_x.

$$M_x dy = \int_{-t/2}^{t/2} \sigma_x z dy dz \tag{A7.17}$$

where t is the plate thickness.

Substituting Eq A7.15 and A7.16 into Eq A7.17, we have

$$M_x = \int_{-t/2}^{t/2} \frac{E_x}{1 - \nu_{xy}\nu_{yx}} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2}\right) z^2 dz \tag{A7.18}$$

and performing the integrations results in Eq A7.19.

$$M_x = \frac{E_x t^3}{12\ (1 - \nu_{xy}\nu_{yx})} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2}\right) \tag{A7.19}$$

It now is convenient to define several new terms in notation similar to

Timoshenko and Woinowsky-Krieger (Ref 28).

$$E_x' = \frac{E_x}{1 - \nu_{xy}\nu_{yx}} \tag{A7.20}$$

$$E_y' = \frac{E_y}{1 - \nu_{xy}\nu_{yx}} \tag{A7.21}$$

$$D_x = \frac{E_x' t^3}{12} \tag{A7.22}$$

$$D_y = \frac{E_y' t^3}{12} \tag{A7.23}$$

$$D' = \nu_{yx} D_x = \nu_{xy} D_y \tag{A7.24}$$

Equation A7.24 implies that there is a relationship between the two Poisson's constants. This relationship is stated in Eq A7.25. The validity of Eq A7.25 is proved by Hearmon (Ref 12), and similar results can be found in Crandall and Dahl (Ref 7) as well as Dow, Libove, and Hubka (Ref 8).

$$\nu_{yx} = \nu_{xy} \frac{E_y}{E_x} \tag{A7.25}$$

Thus, the orthotropic plate is seen to have two values of Poisson's ratio and two stiffness constants.

Torsional Stiffness

The values of stiffness for the torsion bars in Fig A7.2 are clear for isotropic conditions. For orthotropic conditions, howevever, there appears to be considerable debate about the proper manner to express the shear modulus G.

The twisting moments on each face of a plate segment must be such that equilibrium exists at segment corners, or Eq A7.26 must be satisfied (Ref 28).

$$M_{yx} = -M_{xy} \tag{A7.26}$$

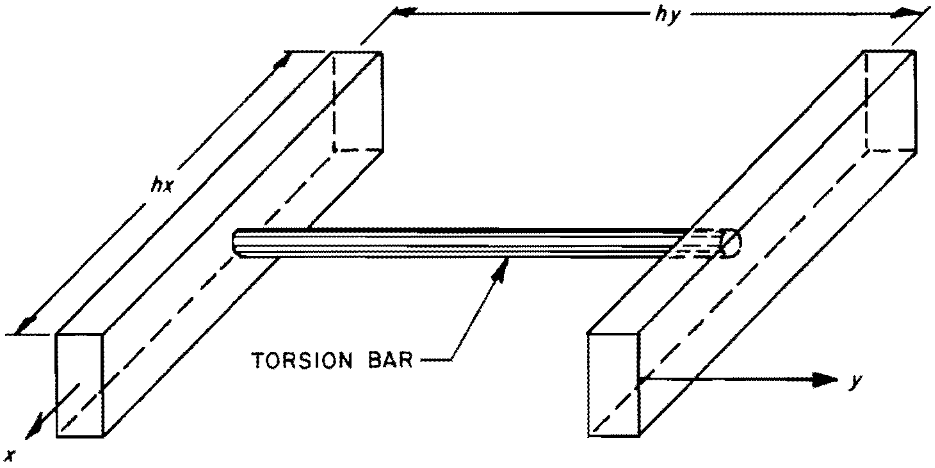


Fig A7.2. Model of plate torsional stiffness.

Furthermore, for isotropic conditions Eq A7.27 relates the twisting moments to the shearing modulus and the plate thickness.

$$M_{yx} = \frac{Gt^3}{6} \frac{\partial^2 w}{\partial x \partial y} = D (1 - \nu) \frac{\partial^2 w}{\partial x \partial y} \qquad (A7.27)$$

The twisting moments have units of in-lb/in. of plate width. Either form of Eq A7.27 is acceptable for the isotropic plate.

If C^x is the resistance to twisting about the x-axis and C^y is the resistance to twisting about the y-axis, then Eq A7.28 and A7.29 relate the stiffnesses to the twisting moments.

$$M_{yx} = C^x \frac{\partial^2 w}{\partial y \partial x} \qquad (A7.28)$$

$$-M_{xy} = C^y \frac{\partial^2 w}{\partial x \partial y} \qquad (A7.29)$$

Equating Eqs A7.28 and A7.29, and noting Eq A7.27, it is apparent that

$$C^x = C^y = \frac{Gt^3}{6} \qquad (A7.30)$$

for the isotropic plate.

For the isotropic plate the shear modulus G is defined by Eq A7.31.

$$G = \frac{E}{2 (1 + \nu)} \qquad (A7.31)$$

For the orthotropic plate the shear modulus has been shown to take the form of Eq A7.32 (Ref 19).

$$G = \frac{E_x E_y}{E_y (1 + \nu_{xy}) + E_x (1 + \nu_{yx})} \qquad (A7.32)$$

Equation A7.32 is approximate but can be used with reasonable reliability. Otherwise the shear modulus must be determined from an independent test.

APPENDIX 8

SELECTION OF CLOSURE PARAMETERS FOR PLATES

APPENDIX 8. SELECTION OF CLOSURE PARAMETERS FOR PLATES

The concepts discussed in Chapter 4 and Appendix 2 for simply-connected gridworks will be extended to the solution of an orthotropic-plate-over-beam layered system. The selection of closure parameters for the beams is identical to the method in Chapter 4. The following developments for eigenvalues of plates are primarily those of Salani (Ref 26).

Eigenvalue for Plate

The finite-difference form for either the horizontal matrix operator or the vertical matrix operator of the plate equation can be written as Eq A8.1.

$$\frac{D}{h^4}\left(w_{i-2} - 6w_{i-1} + 10w_i - 6w_{i+1} + w_{i+2}\right) = \lambda_n w_i \qquad \text{(A8.1)}$$

If Eq A8.2 is assumed to represent the deflected shape of the plate, then four additional equations may be written, Eqs A8.3.

$$w_i = \sin i\beta_n \qquad \text{(A8.2)}$$

$$\begin{aligned} w_{i-2} &= \sin (i-2)\beta_n = \sin (i\beta_n - 2\beta_n) \\ w_{i-1} &= \sin (i-1)\beta_n = \sin (i\beta_n - \beta_n) \\ w_{i+1} &= \sin (i+1)\beta_n = \sin (i\beta_n + \beta_n) \\ w_{i+2} &= \sin (i+2)\beta_n = \sin (i\beta_n + 2\beta_n) \end{aligned} \qquad \text{(A8.3)}$$

Using an appropriate trigonometric identity, Eqs A8.3 can be converted to Eq A8.4.

$$\begin{aligned} w_{i-2} &= \sin (i\beta_n) \cos (2\beta_n) - \cos (i\beta_n) \sin (2\beta_n) \\ w_{i-1} &= \sin (i\beta_n) \cos (\beta_n) - \cos (i\beta_n) \sin (\beta_n) \end{aligned}$$

$$w_{i+1} = \sin (i\beta_n) \cos (\beta_n) + \cos (i\beta_n) \sin (\beta_n) \tag{A8.4}$$

$$w_{i+2} = \sin (i\beta_n) \cos (2\beta_n) + \cos (i\beta_n) \sin (2\beta_n)$$

Then, by substituting Eq A8.4 and Eq A8.2 into Eq A8.1 and simplifying, we have

$$\frac{h^4\lambda_n}{D} = 2 \cos 2\beta_n - 12 \cos \beta_n + 10 \tag{A8.5}$$

Noting that $\cos 2\beta_n = 2 \cos^2 \beta_n - 1$ results in the following change in Eq A8.5

$$\frac{h^4\lambda_n}{D} = 4 \cos^2 \beta_n - 12 \cos \beta_n + 8 \tag{A8.6}$$

which, after rearranging, becomes

$$\frac{h^4\lambda_n}{D} = 4 (1 - \cos \beta_n) (2 - \cos \beta_n) \tag{A8.7}$$

Boundary conditions show that

$$\sin M\beta_n = 0 \tag{A8.8}$$

or that

$$\beta_n = \frac{n\pi}{M} \tag{A8.9}$$

where n is the number of the eigenvalue and ranges from 1 to $M-1$. M is the number of increments into which the strip of the plate is divided. The final expression for the eigenvalues of the line-element strips of plate is Eq A8.10.

$$\frac{h^4\lambda_n}{D} = 4 (1 - \cos \frac{n\pi}{M}) (2 - \cos \frac{n\pi}{M}) \tag{A8.10}$$

Fictitious Springs

The closure spring for a plate has the form

$$\overline{SF} = \frac{D}{h^2} (4) \left(1 - \cos \frac{n\pi}{M}\right) \left(2 - \cos \frac{n\pi}{M}\right) \qquad (A8.11)$$

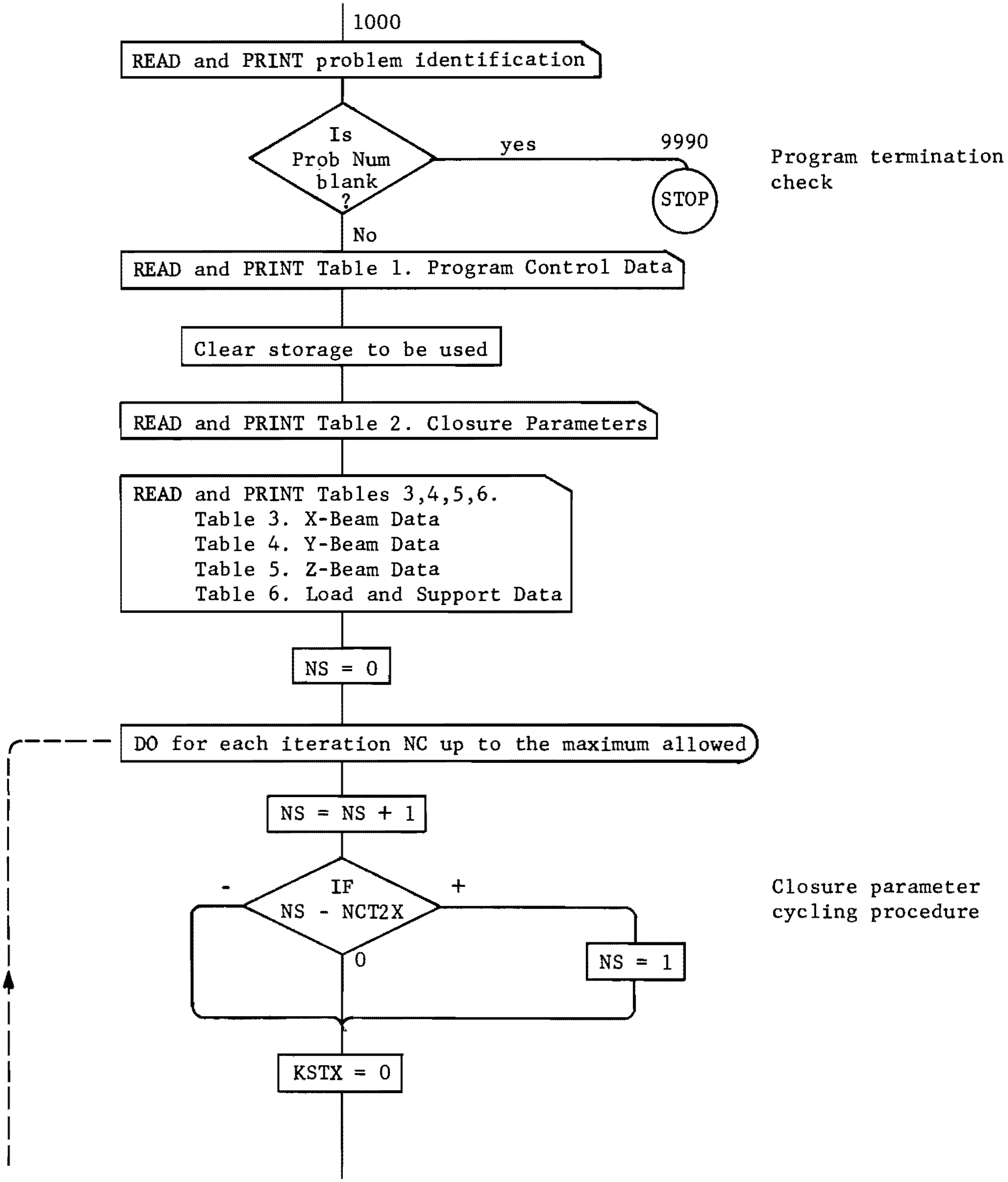
with units of lb/in.

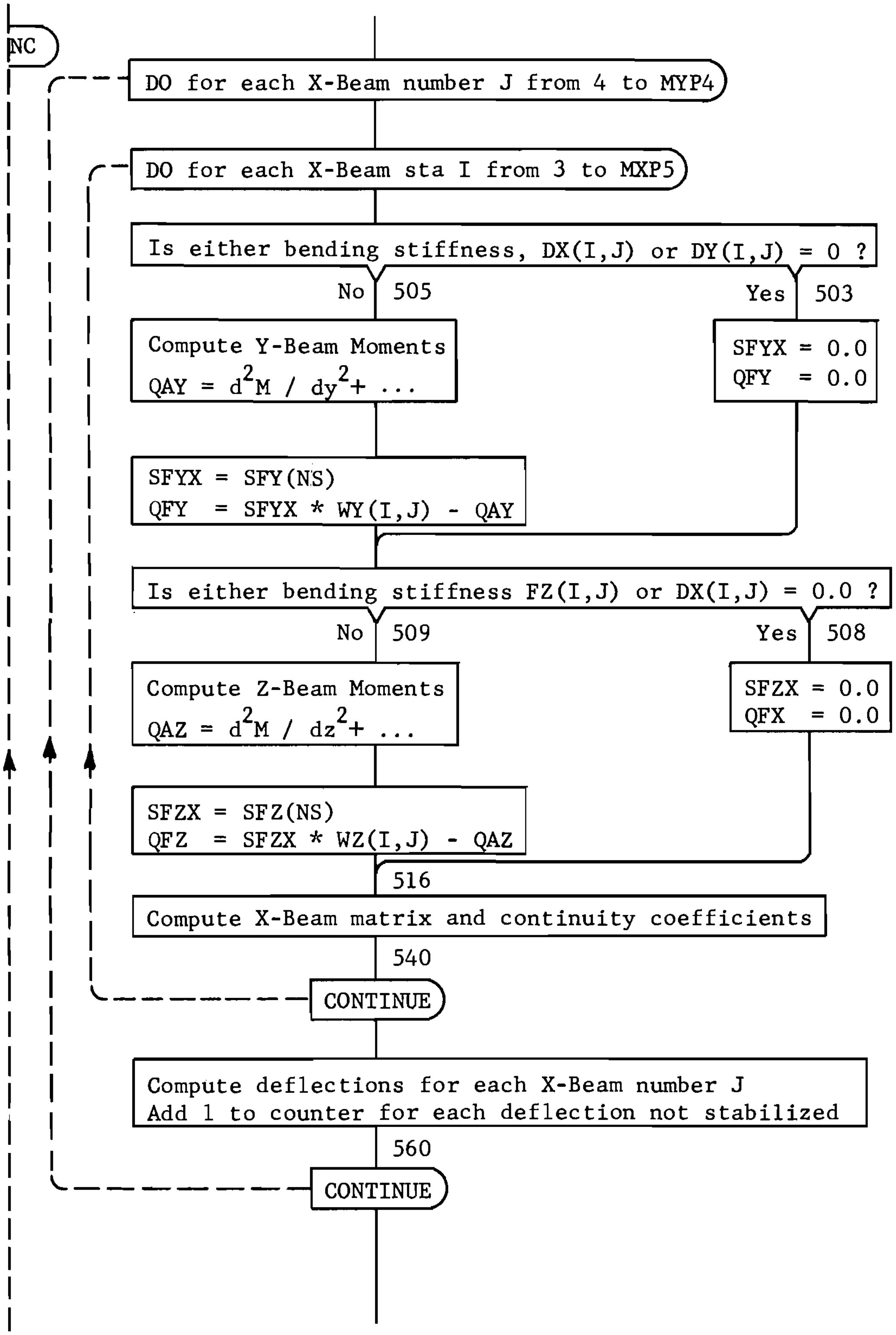
The number of parameters to use may be determined by the method presented in Chapter 4 pertaining to simply-connected layered systems.

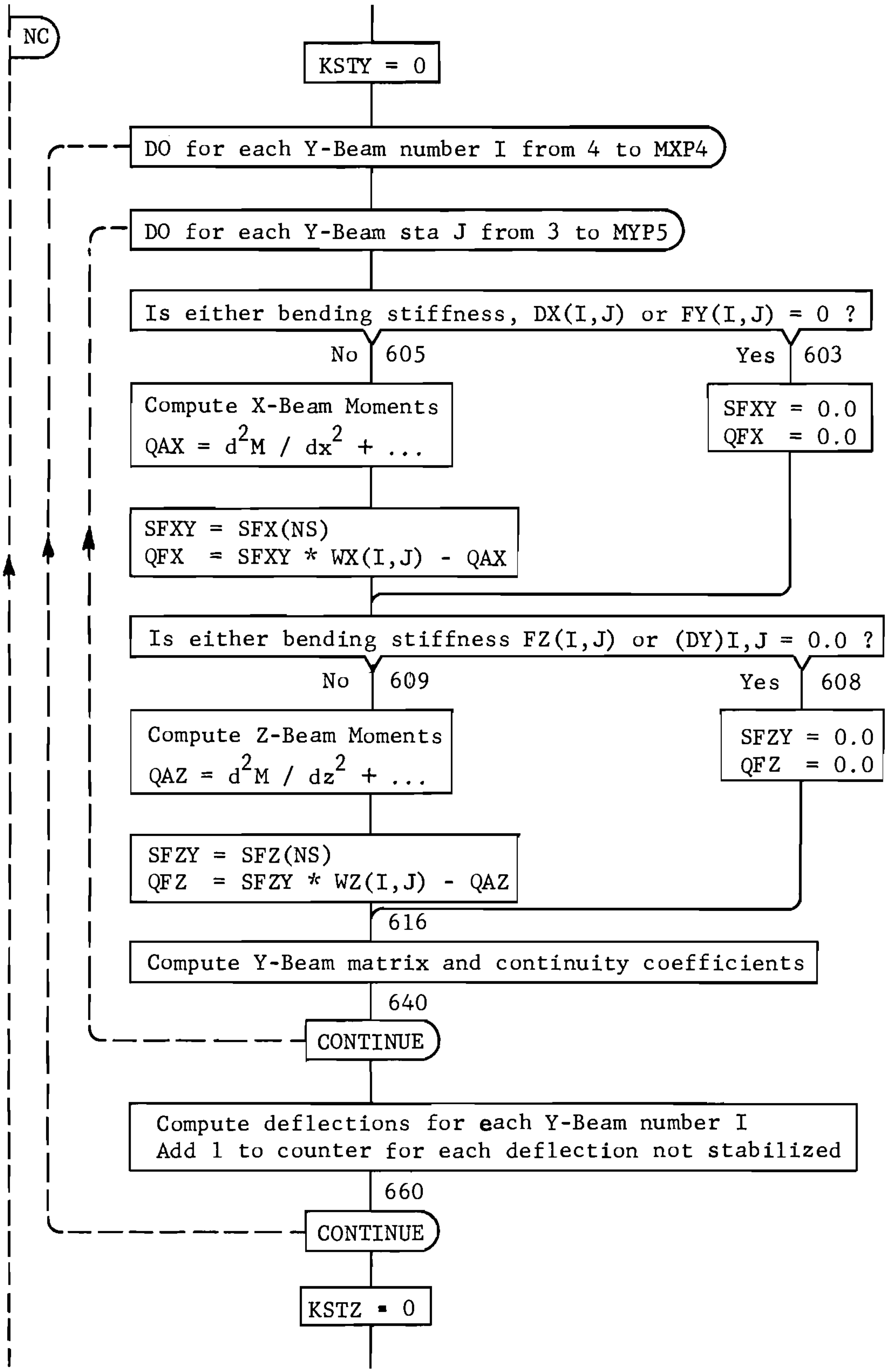
APPENDIX 9

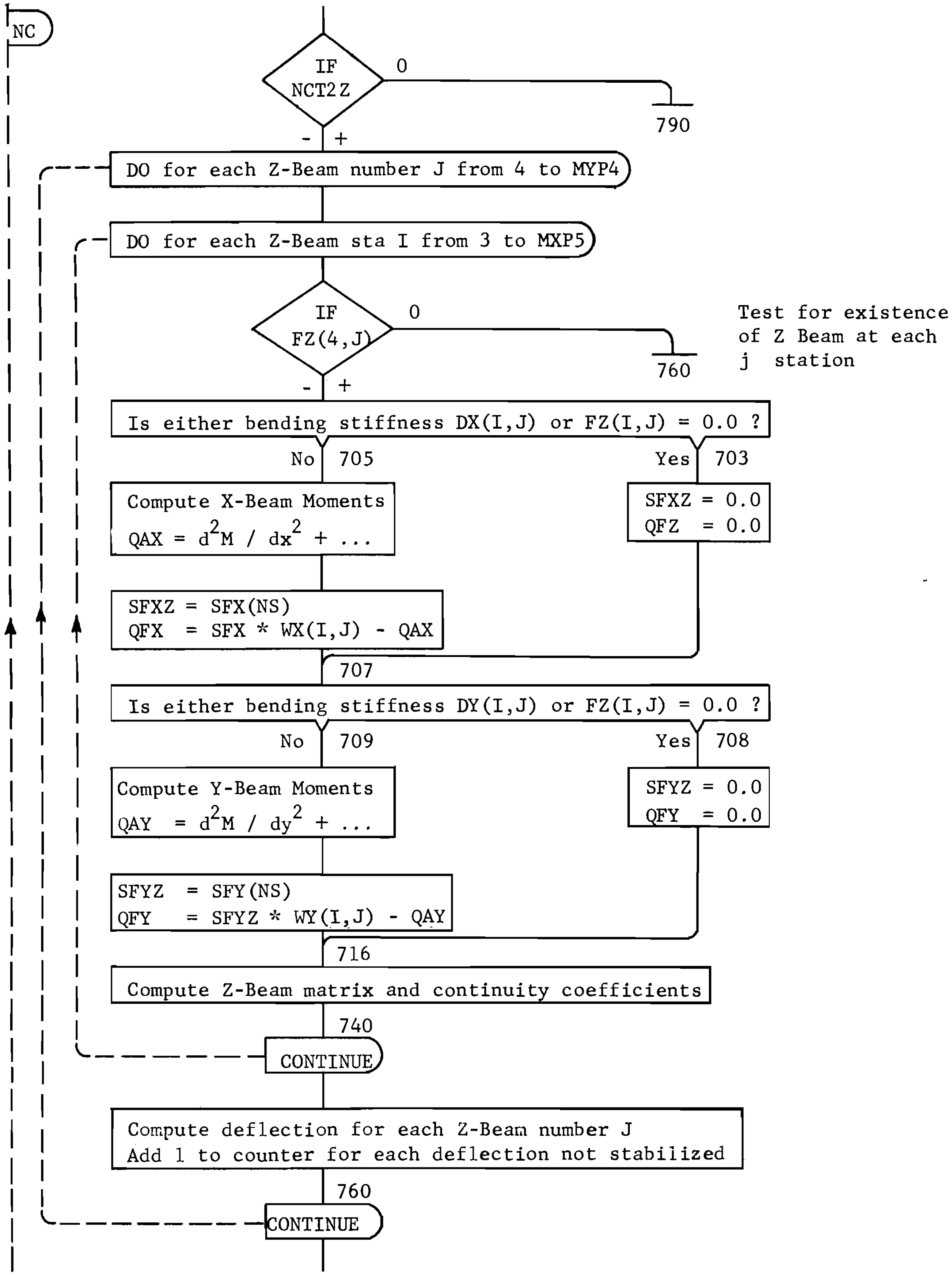
FLOW DIAGRAM FOR PROGRAM LAYER 8

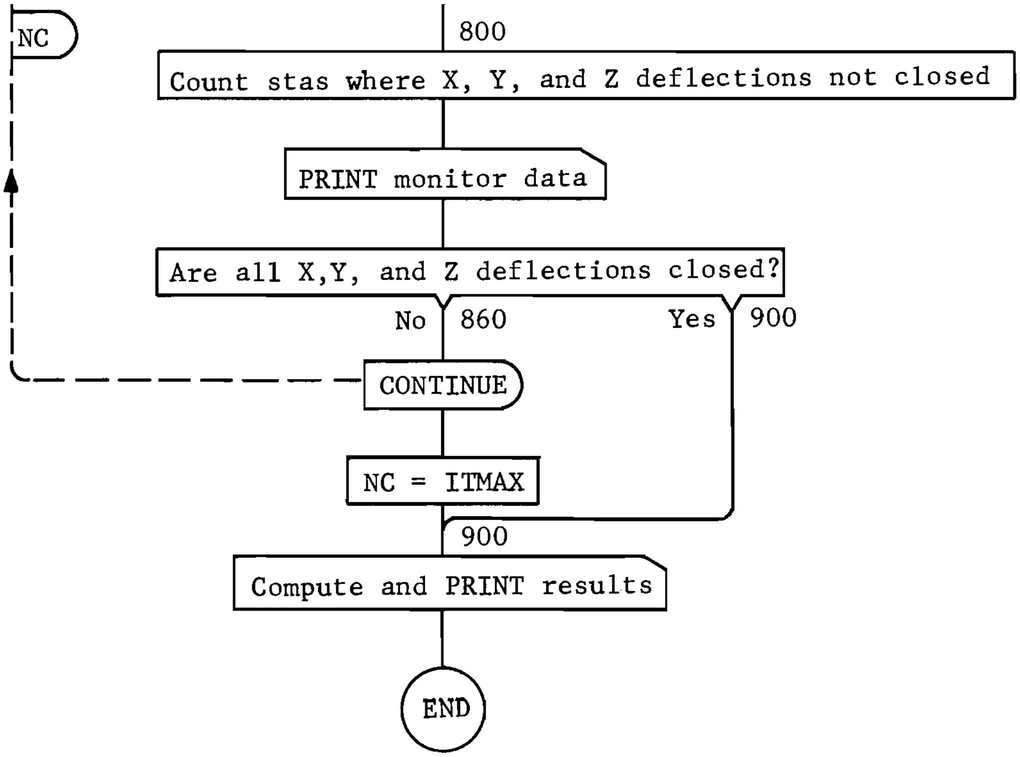
APPENDIX 9. FLOW DIAGRAM FOR PROGRAM LAYER 8.




NC
DO for each X-Beam number J from 4 to MYP4
DO for each X-Beam sta I from 3 to MXP5
Is either bending stiffness, DX(I,J) or DY(I,J) = 0 ?
No
505
Yes
503
Compute Y-Beam Moments
QAY = d^{2}M / dy^2+ ...
SFYX = 0.0
QFY = 0.0
SFYX = SFY(NS)
QFY = SFYX * WY(I,J) - QAY
Is either bending stiffness FZ(I,J) or DX(I,J) = 0.0 ?
No
509
Yes
508
Compute Z-Beam Moments
QAZ = d^{2}M / dz^2+ ...
SFZX = 0.0
QFX = 0.0
SFZX = SFZ(NS)
QFZ = SFZX * WZ(I,J) - QAZ
516
Compute X-Beam matrix and continuity coefficients
540
CONTINUE
Compute deflections for each X-Beam number J
Add 1 to counter for each deflection not stabilized
560
CONTINUE


NC
KSTY = 0
DO for each Y-Beam number I from 4 to MXP4
DO for each Y-Beam sta J from 3 to MYP5
Is either bending stiffness, DX(I,J) or FY(I,J) = 0 ?
No
605
Yes
603
Compute X-Beam Moments
QAX = d^{2}M / dx^2 + ...
SFXY = 0.0
QFX = 0.0
SFXY = SFX(NS)
QFX = SFXY * WX(I,J) - QAX
Is either bending stiffness FZ(I,J) or (DY)I,J = 0.0 ?
No
609
Yes
608
Compute Z-Beam Moments
QAZ = d^{2}M / dz^2 + ...
SFZY = 0.0
QFZ = 0.0
SFZY = SFZ(NS)
QFZ = SFZY * WZ(I,J) - QAZ
616
Compute Y-Beam matrix and continuity coefficients
640
CONTINUE
Compute deflections for each Y-Beam number I
Add 1 to counter for each deflection not stabilized
660
CONTINUE
KSTZ ▪ 0


NC
IF NCT2Z
0
790
- +
DO for each Z-Beam number J from 4 to MYP4
DO for each Z-Beam sta I from 3 to MXP5
IF FZ(4,J)
0
760
Test for existence of Z Beam at each j station
- +
Is either bending stiffness DX(I,J) or FZ(I,J) = 0.0 ?
No 705
Yes 703
Compute X-Beam Moments
QAX = d2M / dx2 + ...
SFXZ = 0.0
QFZ = 0.0
SFXZ = SFX(NS)
QFX = SFX * WX(I,J) - QAX
707
Is either bending stiffness DY(I,J) or FZ(I,J) = 0.0 ?
No 709
Yes 708
Compute Y-Beam Moments
QAY = d2M / dy2 + ...
SFYZ = 0.0
QFY = 0.0
SFYZ = SFY(NS)
QFY = SFYZ * WY(I,J) - QAY
716
Compute Z-Beam matrix and continuity coefficients
740
CONTINUE
Compute deflection for each Z-Beam number J
Add 1 to counter for each deflection not stabilized
760
CONTINUE


NC
800
Count stas where X, Y, and Z deflections not closed
PRINT monitor data
Are all X,Y, and Z deflections closed?
No
860
Yes
900
CONTINUE
NC = ITMAX
900
Compute and PRINT results
END

APPENDIX 10

NOTATION AND LISTING OF PROGRAM LAYER 8

APPENDIX 10. NOTATION AND LISTING OF PROGRAM LAYER 8

```
      PROGRAM LAYER 8
    1 FORMAT (5X,52HPROGRAM LAYER 8 - MASTER DECK - WB INGRAM, H MATLOCK
     1                                 28H     REVISION DATE 27 JUL 65)
C-----PROGRAM LAYER 8 SOLVES A LAYERED ORTHOTROPIC-PLATE-OVER-BEAMS     27JL5
C         SYSTEM BY A THREE-PHASE ALTERNATING-DIRECTION PROCESS.        27JL5
C         INDIVIDUAL LAYER CLOSURE PARAMETERS ARE INPUT BUT THEY ARE    27JL5
C         USED ONLY ON THE OPPOSING LAYERS IN THE SYSTEM.  SOLUTION IS  27JL5
C         LIMITED TO THREE INTERSECTING BEAMS.                          27JL5
C-----NOTATION FOR LAYER 8                                              31MY5
C         A( ), B( ), ETC.    RECURSION COEFFICIENTS                    12MY5
C         AA THRU FF          COEFFS IN STIFFNESS AND LOAD MATRICES     12MY5
C         AN1(N), ETC.        ALPHANUMERIC REMARKS, INFORMATION ONLY    12MY5
C         BM( )               TEMPORARY BENDING MOMENT VALUES (N=1,9)   12MY5
C         CMX( )              TEMPORARY TWISTING MOMENT VALUES          12MY5
C         CTOL                CLOSURE TOLERANCE, X VS Y DEFLS           12MY5
C         CX( , ), CY( , )    TORSIONAL STIFFNESS, X,Y                  12MY5
C         DPX( , ), DPY( , )  PLATE TWISTING STIFFNESS X,Y              12MY5
C         DX( , ), DY( , )    PLATE BENDING STIFFNESS, X, Y             13MY5
C         ERROR               FINAL ERROR IN SUM OF VERTICAL FORCES     12MY5
C         FZ( , )             BENDING STIFFNESS, Z-BEAM                 12MY5
C         HDHY, HDHX          RATIO OF INCREMENT LENGTHS                10MY5
C         HX, HY              INCREMENT LENGTH                          12MY5
C         HX2H2, HY2H2        RATIO OF SQUARES OF INCREMENT LENGTHS     10MY5
C         HXD2, HYD2          ONE HALF OF INCREMENT LENGTH              10MY5
C         HXE2, HYE2          INCREMENT LENGTH SQUARED                  10MY5
C         HXE3, HYE3          INCREMENT LENGTH CUBED                    10MY5
C         HXHY                PRODUCT OF INCREMENT LENGTHS              10MY5
C         I, J                STATION NUMBERS, X,Y DIRECTIONS           12MY5
C         I1, J1, ETC.        TEMPORARY INTERNAL INPUT STATIONS         12MY5
C         IM1, JM1, ETC.      MONITOR STATION USED DURING ITERATION     12MY5
C         IN1, JN1, ETC.      TEMPORARY INPUT STATION VALUES            12MY5
C         ISTA, JSTA          OUTPUT STA NUMBERS                        12MY5
C         ITMAX               MAXIMUM NUMBER OF ITERATIONS ALLOWED      12MY5
C         KCTOL, KCTOLZ       CLOSURE COUNTERS                          10MY5
C         KSTX, KSTY, KSTZ    COUNT OF BEAM NOT STABILIZED              10MY5
C         KTAB                NUM OF TABLE 3, 4, 5, 6                   12MY5
C         L, K, N             TEMPORARY STATION INDEXES                 12MY5
C         MX, MY              NUMBER OF INCREMENTS                      12MY5
C         MXP4, MYP4, ETC.    NUMBER OF INCREMENTS PLUS 4, ETC.         10MY5
C         NC                  NUMBER OF ITERATIONS                      12MY5
C         NCDT                TEMPORARY VALUES OF NCT3, 4, 5, 6         12MY5
C         NCT2X, ETC.         NUM CARDS TABLE 2 FOR X, Y, Z-BEAMS       13MY5
C         NCT3, ETC.          NUM OF CARDS IN TABLE 3, ETC.             10MY5
C         NONE                DUMMY VARIABLE                            13MY5
C         NPROB               NUMBER OF PROBLEM, PROG STOPS IF ZERO     12MY5
C         NS                  COUNTER ON SPRING CYCLE                   12MY5
C         NTABLE1,2           TEMPORARY NUMBER OF TABLE                 31MY5
C         PRPR                PRX * PRY                                 12MY5
C         PRX, PRY            POISSONS RATIO, X AND Y                   10MY5
C         PX( , ), ETC.       AXIAL FORCE, X, Y, Z-BEAMS                12MY5
C         Q(I,J)              TRANSVERSE FORCE PER MESH POINT AND OUTPUT28JE5
C                             TWISTING MOMENT                           28JE5
C         QBMX, QBMY, QAZ     HX*HY * SECOND DERIV BM, X, Y, Z          12MY5
C         QFX, QFY, QFZ       FICT LOADS IN X, Y, Z-BEAMS               13MY5
C         QTMX, QTMY          HX*HY * SECOND DERIV TWIST MOMENT X,Y     12MY5
C         REACT               NET TRANSVERSE FORCE                      12MY5
```

```
C         RX( , ), ETC.       ROTATIONAL RESTRAINT, X, Y, Z-BEAMS       12MY5
C         S(I,J)              SPRING SUPPORT, VALUE PER MESH POINT AND  28JE5
C                             OUTPUT TWISTING MOMENT                    28JE5
C         SFX( ), ETC.        FICT SPRINGS FOR X, Y, Z-BEAMS            12MY5
C         SFYX, ETC.          INTERNAL FICT SPRINGS BETWEEN BEAMS       12MY5
C         TX( , ), ETC.       EXTERNAL TORQUE, X, Y, Z-BEAMS            12MY5
C         WX(I,J), ETC.       VERTICAL DEFLECTION AND OUTPUT BENDING    28JE5
C                             MOMENT                                    28JE5
C         WTEMP               TEMPORARY VALUE OF DEFLECTION             12MY5
C         ZN1 THRU ZN5        TEMPORARY INPUT VARIABLE VALUES           12MY5
      DIMENSION     AN1(32), AN2(14),                                   18FE5 ID
     1              SFX( 6), SFY(6), SFZ(6), S(32,32), Q(32,32),        04MY5
     2              BM(9), CMX(6), CMY(6), A(32), B(32), C(32),         05MY5
     3              DX(32,32), DY(32,32), FZ(32,32),                    04MY5
     4              WX(32,32), WY(32,32), WZ(32,32),                    04MY5
     5              TX(32,32), TY(32,32), TZ(32,32),                    04MY5
     6              RX(32,32), RY(32,32), RZ(32,32),                    04MY5
     7              PX(32,32), PY(32,32), PZ(32,32),                    04MY5
     8              CX(32,32), CY(32,32), DPX(32,32), DPY(32,32)        04MY5
      COMMON   AN1, AN2, SFX, SFY, SFZ, S, Q, BM, CMX, CMY, A, B, C, DX,27JL5
     1         DY, FZ, WX, WY, WZ, TX, TY, TZ, RX, RY, RZ, PX, PY, PZ,  27JL5
     2         CX, CY, DPX, DPY                                         27JL5
    6 FORMAT ( )                                                        04MY3
   10 FORMAT (    5H     , 80X, 10HI-----TRIM )                         03FE4 IC
   11 FORMAT (    5H1    , 80X, 10HI-----TRIM )                         03FE4 IC
   12 FORMAT (  16A5   )                                                04MY3 IC
   13 FORMAT (    5X, 16A5 )                                            26AG3 IC
   14 FORMAT ( A5, 5X, 14A5 )                                           18FE5 IC
   15 FORMAT (///10H     PROB , /5X, A5, 5X, 14A5 )                     10MY5
   16 FORMAT (///17H     PROB (CONTD), /5X, A5, 5X, 14A5 )              10MY5
   19 FORMAT (///50H     RETURN THIS PAGE TO TIME RECORD FILE -- HM   ) 10DE4
   20 FORMAT  (10( I5 ),   3E10.3 )                                     06AP5
   21 FORMAT ( 10X, 8 ( I5 ) )                                          09AP5
   22 FORMAT ( 10X, I5, 5X, 6E10.3 )                                    06AP5
   23 FORMAT ( 10X, 4( I5 ), 5E10.3 )                                   08AP5
   26 FORMAT  ( 5X, I5, 5X, 7E10.3  )                                   21JA5
   28 FORMAT  ( 20X, 2E10.3 )                                           05MY5
   30 FORMAT  (//30H     TABLE I.  CONTROL DATA   ,              /      15AP3
     1         /  30H          NUM CARDS TABLE 3   , 40X, I5,     /     10DE4
     2            30H          NUM CARDS TABLE 4   , 40X, I5,     /     04MY5
     3            30H          NUM CARDS TABLE 5   , 40X, I5,     /     04MY5
     4            30H          NUM CARDS TABLE 6   , 40X, I5,     /     04MY5
     5            30H          MAX NUM ITERATIONS  , 40X, I5,     /     08AP5
     6            30H          NUM INCREMENTS  MX  , 40X, I5,     /     08AP5
     7            30H          NUM INCREMENTS  MY  , 40X, I5,     /     08AP5
     8            30H          INCR LENGTH  HX     , 35X,   E10.3,/     08AP5
     9            30H          INCR LENGTH  HY     , 35X,   E10.3,/     08AP5
     1            30H          CLOSURE TOLERANCE   , 35X,   E10.3,/     04MY5
     2            30H          POISSONS RATIO X    , 35X,    E10.3,/    12MY5
     3            30H          POISSONS RATIO Y    , 35X,    E10.3 /  ) 12MY5
   31 FORMAT  (  30H          MONITOR STAS  I,J   ,  5X, 4( I7, I3 ) )  15AP3
   32 FORMAT  (//40H     TABLE 2.  RELAXATION CONTROL DATA  ,    /      15AP3
     1            52H       NUM       C L O S U R E  P A R A M E T E R S29DE4
     2        ,/, 12H      VALUES  /     )                              29DE4
   33 FORMAT  (//51H     TABLE 3. X-BEAM DATA, FULL VALUES ADDED AT ALL 06AP5
     1            22H STAS I,J IN RECTANGLE, /                          06AP5
```

```
     2        5X,50H       FROM  THRU     DX         CX         TX      08AP5
     3           30H     RX         PX            , /  )                08AP5
   43 FORMAT  (//51H     TABLE 4. Y-BEAM DATA, FULL VALUES ADDED AT ALL 06AP5
     1           22H STAS I,J IN RECTANGLE, /                           06AP5
     2        5X,50H       FROM  THRU     DY         CY         TY      08AP5
     3           30H     RY         PY            , /  )                08AP5
   53 FORMAT  (//51H     TABLE 5. Z-BEAM DATA, FULL VALUES ADDED AT ALL 08AP5
     1           22H STAS I,J IN RECTANGLE, /                           06AP5
     2        5X,50H       FROM  THRU     FZ        TZ        RZ        03MY5
     3           30H     PZ                       , /  )                03MY5
   63 FORMAT (// 48H     TABLE 6. LOAD AND SUPPORT DATA, FULL VALUES    06AP5
     1           35H ADDED AT ALL STAS I,J IN RECTANGLE, /              06AP5
     2         / 48H          FROM      THRU        Q           S    ,/)07AP5
   64 FORMAT ( / 33H     NO DATA SPECIFIED FOR TABLE  A2, A1,           10MY5
     1           27HNEW PROBLEM HAS BEEN SOUGHT  )                      10MY5
   73 FORMAT  (10X,  2(     I3, 1X, I2 ),  5( E12.3))                   14MY5
   74 FORMAT (///48H     TABLE 7.  MONITOR TALLY AND DEFLS AT 4 STAS,/  06AP5
     1         / 46H        ITR   FICT      CYC  NOT  NOT      I,J   ,/ 15AP3
     2           42H        NUM   SPRING    NUM  STAB CLOS    ,         15AP3
     3                                              4( I2, 1X, I2, 7X ))15AP3
   75 FORMAT ( /  7H    X  , I3, E12.3, I4, I5, 5X, 4E12.3 /            28JL5
     1            7H    Y  , 3X, E12.3, I4, I5, I5, 4E12.3 /            28JL5
     2            7H    Z  , 3X, E12.3, I4, I5, I5, 4E12.3 )            28JL5
   85 FORMAT (/  51H     TABLE 8. DEFLECTION AND ERROR RESULTS -- ITERA 27MY5
     1            5HTION,  I4      )                                    27MY5
   86 FORMAT  (/ 51H     SOLUTION NOT CLOSED WITHIN SPECIFIED TOLERANCE)06AP5
   87 FORMAT ( / 50H       I,J  X-DEFL        Y-DEFL      Z-DEFL        27MY5
     1           20H REACT    ERROR     )                               27MY5
   88 FORMAT (   7X,I2, I3,  8E12.3 )                                   03JE5
   89 FORMAT ( / 34H     TABLE 9. MOMENTS -- ITERATION , I4  )          27MY5
   90 FORMAT (   50H       I,J  X-MOM         Y-MOM       Z-MOM         27MY5
     1           20H TX-MOM   TY-MOM     )                              27MY5
               ITEST = 5H                                               28JL5
 1000 PRINT 10                                                          12JL3 IC
      CALL TIME                                                         25MY4 IC
C-----PROGRAM AND PROBLEM IDENTIFICATION                                04MY3 IC
      READ  12, ( AN1(N), N = 1, 32 )                                   18FE5 IC
 1010 READ  14,  NPROB, ( AN2(N), N = 1, 14 )                           28AG3 IC
          IF ( NPROB - ITEST ) 1020, 9990, 1020                         26FE5 IC
 1020 PRINT 11                                                          26AG3 IC
      PRINT 1                                                           18FE5 IC
      PRINT 13, ( AN1(N), N = 1, 32 )                                   18FE5 IC
      PRINT 15, NPROB, ( AN2(N), N = 1, 14 )                            26AG3 IC
C-----INPUT TABLE 1
      READ  20,  NONE, NONE, NCT3, NCT4, NCT5, NCT6, NONE, ITMAX, MX,   06AP5
     1           MY, HX, HY, CTOL                                       06AP5
      READ  28, PRX, PRY                                                03MY5
      PRINT 30,      NCT3, NCT4, NCT5, NCT6, ITMAX, MX, MY, HX, HY,    08AP5
     1         CTOL, PRX, PRY                                           03MY5
      READ  21,  IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                 13AP3
      PRINT 31,  IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                 13AP3
C-----COMPUTE FOR CONVENIENCE                                           26JL5
               HXE2 =    HX * HX                                        13AP3
               HYE2 =    HY * HY                                        13AP3
               HXE3 =    HX * HXE2                                      13AP3
               HY2H2 = HYE2/HXE2                                        03MY5
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```
               HX2H2 = HXE2/HYE2                                        03MY5
               HXHY  = HX*HY                                            03MY5
               HDHY  = HX/HY                                            03MY5
               HDHX  = HY/HX                                            03MY5
               HYD2  = HY/2.0                                           03MY5
               HXD2  = HX/2.0                                           03MY5
               PRPR = PRX*PRY                                           04MY5
               HYE3 =   HY * HYE2                                       13AP3
               MXP4 =   MX + 4                                          13AP3
               MYP4 =   MY + 4                                          13AP3
               MXP5 =   MX + 5                                          13AP3
               MYP5 =   MY + 5                                          13AP3
               MXP7 =   MX + 7                                          13AP3
               MYP7 =   MY + 7                                          13AP3
C-----CLEAR VALUES FROM PRIOR PROBS                                     27JL5
          DO 230  N = 1, 6                                              08AP5
               SFX(N) = 0.0                                             03MY5
               SFY(N) = 0.0                                             03MY5
               SFZ(N) = 0.0                                             03MY5
  230     CONTINUE                                                      12FE5
          DO 250  I = 1,MXP7                                            13AP3
          DO 250  J = 1,MYP7                                            13AP3
               DX(I,J) = DY(I,J) = FZ(I,J) = 0.0                        08AP5
               CX(I,J) = CY(I,J) = 0.0                                  08AP5
               Q(I,J)  = 0.0                                            28JL5
               S(I,J)  = 0.0                                            28JL5
               TX(I,J) = TY(I,J) = TZ(I,J) = 0.0                        03MY5
               RX(I,J) = RY(I,J) = RZ(I,J) = 0.0                        08AP5
               PX(I,J) = PY(I,J) = PZ(I,J) = 0.0                        08AP5
               WX(I,J) = WY(I,J) = WZ(I,J) = 0.0                        08AP5
  250     CONTINUE                                                      13AP3
C-----INPUT TABLE 2                                                     27JL5
      PRINT 32                                                          27JA5
C-----PROGRAM LAYER 8 MUST HAVE THE SAME NUMBER OF PARAMETERS FOR ALL   21MY5
C         BEAMS                                                         03MY5
      READ  22, NCT2X, ( SFX(N), N = 1, NCT2X )                         08AP5
      PRINT 26, NCT2X, ( SFX(N), N = 1, NCT2X )                         08AP5
      READ  22, NCT2Y, ( SFY(N), N = 1, NCT2Y )                         08AP5
      PRINT 26, NCT2Y, ( SFY(N), N = 1, NCT2Y )                         08AP5
      READ  22, NCT2Z, ( SFZ(N), N = 1, NCT2Z )                         08AP5
      PRINT 26, NCT2Z, ( SFZ(N), N = 1, NCT2Z )                         08AP5
C-----INPUT TABLE 3, 4, 5, 6                                            27JL5
  300 PRINT 33                                                          10MY5
          IF ( NCT3 )  302, 305, 302                                    10MY5
  302          NCDT = NCT3                                              10MY5
               KTAB = 1                                                 10MY5
          GO TO 325                                                     10MY5
  305          NTABLE1 = 2H3.                                           10MY5
      PRINT 64, NTABLE1                                                 10MY5
          IF ( NCT4 )  307, 310, 307                                    10MY5
  307          NCDT = NCT4                                              10MY5
               KTAB = 2                                                 10MY5
          GO TO 325                                                     10MY5
  310          NTABLE1 = 2H4.                                           10MY5
      PRINT 43                                                          06AP5
      PRINT 64, NTABLE1                                                 10MY5
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        IF ( NCT5 )  312, 315, 312                                      10MY5
312          NCDT = NCT5                                                10MY5
             KTAB = 3                                                   10MY5
        GO TO 325                                                       10MY5
315          NTABLE1 = 2H5.                                             10MY5
    PRINT 53                                                            06AP5
    PRINT 64, NTABLE1                                                   10MY5
        IF ( NCT6 )  317, 320, 317                                      10MY5
317          NCDT = NCT6                                                10MY5
             KTAB = 4                                                   10MY5
        GO TO 325                                                       10MY5
320          NTABLE1 = 2H6.                                             10MY5
             NTABLE2 = 1H                                               10MY5
    PRINT 63                                                            06AP5
    PRINT 64, NTABLE1, NTABLE2                                          10MY5
        GO TO 1000                                                      10MY5
325     DO 390 N = 1, NCDT                                              10MY5
    READ  23, IN1, JN1, IN2, JN2, ZN1, ZN2, ZN3,ZN4,ZN5                 25MY5
             I1 = IN1 + 4                                               16MY5
             J1 = JN1 + 4                                               16MY5
             I2 = IN2 + 4                                               16MY5
             J2 = JN2 + 4                                               16MY5
        GO TO (  330, 340, 350, 360  ), KTAB                            07AP5
330 PRINT 73, IN1, JN1, IN2, JN2, ZN1, ZN2, ZN3, ZN4, ZN5               03MY5
        DO 335 I = I1, I2                                               08AP5
        DO 335 J = J1, J2                                               08AP5
             DX(I,J) = DX(I,J) + ZN1                                    03MY5
             CX(I,J) = CX(I,J) + ZN2                                    03MY5
             TX(I,J) = TX(I,J) + ZN3                                    03MY5
             RX(I,J) = RX(I,J) + ZN4                                    03MY5
             PX(I,J) = PX(I,J) + ZN5                                    03MY5
335     CONTINUE                                                        06AP5
336     IF ( N - NCDT )  390, 337, 337                                  10MY5
337 PRINT 43                                                            14MY5
        IF ( NCT4 )  338, 339, 338                                      14MY5
338          NCDT = NCT4                                                10MY5
             KTAB = 2                                                   10MY5
        GO TO 325                                                       10MY5
339          NTABLE1 = 2H4.                                             10MY5
    PRINT 64, NTABLE1                                                   10MY5
        GO TO 347                                                       10MY5
340 PRINT 73, IN1, JN1, IN2, JN2, ZN1, ZN2, ZN3, ZN4, ZN5               03MY5
        DO 345 I = I1, I2                                               08AP5
        DO 345 J = J1, J2                                               08AP5
             DY(I,J) = DY(I,J) + ZN1                                    03MY5
             CY(I,J) = CY(I,J) + ZN2                                    03MY5
             TY(I,J) = TY(I,J) + ZN3                                    03MY5
             RY(I,J) = RY(I,J) + ZN4                                    03MY5
             PY(I,J) = PY(I,J) + ZN5                                    03MY5
345     CONTINUE                                                        06AP5
346     IF ( N - NCDT )  390, 347, 347                                  10MY5
347 PRINT 53                                                            14MY5
        IF ( NCT5 )  348, 349, 348                                      14MY5
348          NCDT = NCT5                                                10MY5
             KTAB = 3                                                   10MY5
        GO TO 325                                                       10MY5
```

```
  349          NTABLE1 = 2H5.                                           10MY5
      PRINT 64, NTABLE1                                                 10MY5
          GO TO 357                                                     10MY5
  350 PRINT 73, IN1, JN1, IN2, JN2, ZN1, ZN2, ZN3, ZN4, ZN5             03MY5
          DO 355 I = I1, I2                                             08AP5
          DO 355 J = J1, J2                                             08AP5
               FZ(I,J) = FZ(I,J) + ZN1                                  08AP5
               TZ(I,J) = TZ(I,J) + ZN3                                  03MY5
               RZ(I,J) = RZ(I,J) + ZN4                                  03MY5
               PZ(I,J) = PZ(I,J) + ZN5                                  03MY5
  355     CONTINUE                                                      06AP5
  356     IF ( N - NCDT )  390, 357, 357                                10MY5
  357 PRINT 63                                                          14MY5
          IF ( NCT6 )  358, 359, 358                                    14MY5
  358          NCDT = NCT6                                              10MY5
               KTAB = 4                                                 10MY5
          GO TO 325                                                     10MY5
  359          NTABLE1 = 2H6.                                           10MY5
               NTABLE2 = 1H                                             10MY5
      PRINT 64, NTABLE1, NTABLE2                                        10MY5
          GO TO 1000                                                    10MY5
  360 PRINT 73, IN1, JN1, IN2, JN2, ZN1, ZN2                            08AP5
          DO 365 I = I1, I2                                             08AP5
          DO 365 J = J1, J2                                             08AP5
               Q(I,J) = Q(I,J) + ZN1                                    08AP5
               S(I,J) = S(I,J) + ZN2                                    08AP5
  365     CONTINUE                                                      06AP5
  390     CONTINUE                                                      06AP5
      CALL TIME                                                         08AP5
      PRINT 74,  IM1, JM1, IM2, JM2, IM3, JM3, IM4, JM4                 10MY5
C-----BEGIN MAIN SOLUTION
          DO 280 I = 1, MXP7                                            31MY5
          DO 280 J = 1, MYP7                                            31MY5
               DPY(I,J) = PRY*DY(I,J)                                   19MY5
               DPX(I,J) = PRX*DX(I,J)                                   19MY5
  280     CONTINUE                                                      03MY5
               NS = 0                                                   25AG4
          DO 860  NC = 1, ITMAX                                         04MY5
               NS = NS + 1                                              25AG4
          IF ( NS - NCT2X )  501, 501, 500                              08AP5
  500          NS = 1                                                   27JL5
C
C-----SOLVE X-BEAMS                                                     27JL5
  501          KSTX =   0                                               04DE64
          DO 560  J = 4, MYP4                                           01MA3
          DO 540  I = 3,MXP5                                            13AP3
C-----ESTABLISH ITERATION CONTROL PARAMETERS                            27JL5
          IF ( DX(I,J) * DY(I,J) )  505, 503, 505                       24FE5
  503          SFYX = QFY = 0.0                                         08AP5
      GO TO 507                                                         12FE5
  505     DO 506 N = 1, 3                                               12FE5
               L   =   J + N - 2                                        11JE5
               BM(N+3) =    DY(I,L)*( WY(I,L-1) - 2.0*WY(I,L) + WY(I,   03MY5
     1              L+1) )/ HYE2 +    DPY(I,L)*(WX(I-1,L) - 2.0*        25MY5
     2              WX(I,L) + WX(I+1,L) ) / HXE2                        03MY5
  506     CONTINUE                                                      12FE5
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               QAY  = ( BM(4) - 2.0* BM(5) + BM(6) )* HDHY              24MY5
     1              + ( - 0.25 * HDHY * RY(I,J-1) * WY(I,J-2)           11JE5
     2              + 0.25 *HDHY* ( RY(I,J-1) + RY(I,J+1))* WY(I,J)     21MY5
     3              - 0.25 *HDHY* RY(I,J+1) * WY(I,J+2)                 21MY5
     4              - HX * PY(I,J) * WY(I,J-1)                          25MY5
     5              + HX * ( PY(I,J) + PY(I,J+1) ) * WY(I,J)            25MY5
     6              - HX * PY(I,J+1) * WY(I,J+1)    ) / HXHY            27MY5
               QTMY = ( WY(I-1,J-1) * CY(I,J) - WY(I-1,J) * (CY(I,J)    03MY5
     1              +CY(I,J+1) )  + WY(I-1,J+1) * CY(I,J+1) - WY(I,J-1) 03MY5
     2              * ( CY(I,J) + CY(I+1,J) ) +  WY(I,J)*(CY(I,J) +     03MY5
     3              CY(I,J+1) + CY(I+1,J) + CY(I+1,J+1)) - WY(I,J+1) *  05MY5
     4              (CY(I,J+1)+ CY(I+1,J+1) ) + WY(I+1,J-1) * CY(I+1,J) 03MY5
     5              -WY(I+1,J) * ( CY(I+1,J) + CY(I+1,J+1) ) +          03MY5
     6              WY(I+1,J+1) * CY(I+1,J+1) ) / HXHY                  03MY5
               SFYX = SFY(NS)                                           08AP5
               QFY = SFYX * WY(I,J) - QAY - QTMY                        24MY5
  507     IF(  FZ(I,J)* DX(I,J) )  509, 508, 509                        08AP5
  508          SFZX = QFZ = 0.0                                         04MY5
      GO TO 516                                                         12FE5
  509     DO 510 N = 1, 3                                               24FE5
               K = I + N - 2                                            21JA5
               BM(N+6) =  FZ(K,J)*(  WZ(K-1,J) -2.0* WZ(K,J) +          08AP5
     1               WZ(K+1,J) ) / HXE2                                 08AP5
  510     CONTINUE                                                      11FE5
               QAZ = ( BM(7) - 2.0 *BM(8) + BM(9) )/ HX                 08AP5
     1              + ( - 0.25 * HX * RZ(I-1,J) * WZ(I-2,J)             11JE5
     2              + 0.25* HX * ( RZ(I-1,J) + RZ(I+1,J) ) * WZ(I,J)    21MY5
     3              - 0.25* HX * RZ(I+1,J) * WZ(I+2,J)                  21MY5
     4              -HXE2* PZ(I,J) * WZ(I-1,J)                          25MY5
     5              +HXE2* ( PZ(I,J) + PZ(I+1,J) ) * WZ(I,J)            25MY5
     6              -HXE2* PZ(I+1,J) * WZ(I+1,J)    ) / HXE3            27MY5
               SFZX = SFZ(NS)                                           08AP5
               QFZ = SFZX * WZ(I,J) - QAZ                               08AP5
C-----COMPUTE X-BEAM MATRIX COEFFS                                      27JL5
  516          AA = HY2H2 * DX(I-1,J) - RX(I-1,J) * HDHX / 4.0          06MY5
               BB = -2.0 * HY2H2 *(DX(I-1,J) + DX(I,J) )                06MY5
     1              -        ( CX(I,J) + CX(I,J+1) )                    19MY5
     2              - HY   * PX(I,J)                                    27MY5
               CC = HY2H2 * ( DX(I-1,J) + 4.0 * DX(I,J) + DX(I+1,J) )   06MY5
     1              +        (CX(I,J)+CX(I,J+1)+CX(I+1,J)+CX(I+1,J+1))  19MY5
     2              + HY   * ( PX(I,J) + PX(I+1,J) )                    03JE5
     3              + HDHX * (RX(I-1,J) + RX(I+1,J) ) / 4.0             06MY5
     4              + HXHY * ( S(I,J) + SFYX + SFZX )                   10MY5
               DD = -2.0 * HY2H2 * ( DX(I,J) + DX(I+1,J) )              06MY5
     1              -        ( CX(I+1,J) + CX(I+1,J+1) )                19MY5
     2              - HY   * PX(I+1,J)                                  03JE5
               EE = HY2H2 * DX(I+1,J) - RX(I+1,J) * HDHX / 4.0          06MY5
               FF = HXHY*( Q(I,J) +QFY +QFZ  )                          03MY5
     1              -0.5/HX *(TZ(I-1,J)-TZ(I+1,J)) * HXHY               27MY5
     1              - HXD2 * ( TY(I,J-1) - TY(I,J+1) )                  21MY5
     1              - HYD2*( TX(I-1,J) - TX(I+1,J) ) +      (-CX(I,J)*  21MY5
     2              (WX(I-1,J-1) - WX(I,J-1) ) - CX(I,J+1)*(WX(I-1,J+1) 03MY5
     3              -WX(I,J+1) ) + CX(I+1,J)*( WX(I,J-1) - WX(I+1,J-1)) 03MY5
     4              +CX(I+1,J+1)*( WX(I,J+1)- WX(I+1,J+1) ) )           10MY5
     5              - ( DPX(I-1,J) * ( WY(I-1,J-1) - 2.0 * WY(I-1,J)    11JE5
     6              +WY(I-1,J+1)) - 2.0*DPX(I,J)*( WY(I,J-1) - 2.0*     03MY5
```

```
     7              WY(I,J) + WY(I,J+1)) + DPX(I+1,J)*( WY(I+1,J-1)     03MY5
     8              -2.0* WY(I+1,J) + WY(I+1,J+1) ) )                   03MY5
C-----COMPUTE X-BEAM CONTINUITY COEFFS                                  27JL5
               E   = AA * B(I-2) + BB                                   03MY5
               DEN = E * B(I-1) + AA * C(I-2) + CC                      10MY5
          IF  (DEN)   531, 521, 531                                     13AP3
  521          G    =  0.0                                              08JL4
          GO TO  532                                                    13AP3
  531          G   = - 1.0 / DEN                                        08JL4
  532          C(I) =  G * EE                                           03MY5
               B(I) =  G * ( E * C(I-1) + DD )                          03MY5
               A(I) =  G * ( E * A(I-1) + AA * A(I-2) - FF  )           03MY5
  540    CONTINUE                                                       13AP3
C-----COMPUTE X-BEAM DEFLS                                              27JL5
          DO 550  L = 3,MXP5                                            13AP3
               I    =  MX + 8 - L                                       13AP3
               WTEMP    =    WX(I,J)                                    08AP5
               WX(I,J) = A(I) + B(I) * WX(I+1,J) + C(I) * WX(I+2,J)     08AP5
C-----COUNT STAS WHERE X-BEAMS NOT STABILIZED                           27JL5
          IF  ( DX(I,J) * DY(I,J) )  545, 550, 545                      4SE64
  545     IF  ( ABSF (  WX(I,J) - WTEMP ) - CTOL )  550, 550, 546       08AP5
  546          KSTX =   KSTX + 1                                        13AP3
  550     CONTINUE                                                      13AP3
  560     CONTINUE                                                      13AP3
C                                                                       28JL5
C-----SET EXTERIOR PLATE DEFLECTIONS                                    27JL5
          DO 570 I = 3, MXP5                                            12MA5
               J = 3                                                    12MA5
               WX(I,J) = WY(I,J)                                        12MY5
               J = MYP5                                                 12MA5
               WX(I,J) = WY(I,J)                                        12MY5
  570 CONTINUE                                                          12MA5
               WX(3,3) = 2.0* WX(3,4) - WX(3,5)                         02JE5
               WX(MXP5,3) = 2.0*WX(MXP5,4) - WX(MXP5,5)                 02JE5
               WX(3,MYP5) = 2.0*WX(3,MYP4) - WX(3,MY+3)                 02JE5
               WX(MXP5,MYP5)=2.0*WX(MXP5,MYP4) - WX(MXP5,MY+3)          02JE5
C-----SOLVE Y-BEAMS                                                     27JL5
               KSTY   =  0                                              29AP3
          DO 660  I = 4, MXP4                                           01MA3
          DO 640  J = 3,MYP5                                            13AP3
C-----ESTABLISH ITERATION CONTROL PARAMETERS                            27JL5
          IF( DX(I,J) * DY(I,J) )  605, 603, 605                        24FE5
  603          SFXY = QFX = 0.0                                         08AP5
      GO TO 607                                                         12FE5
  605     DO 606 N = 1, 3                                               12FE5
               K    =  I + N - 2                                        01MA3
               BM(N) =    DX(K,J) *( WX(K-1,J) -2.0*WX(K,J) + WX(K+1,J))03MY5
     1              / HXE2 +    DPX(K,J)*( WY(K,J-1) -2.0*WY(K,J) +     25MY5
     2              WY(K,J+1))/ HYE2                                    03MY5
  606     CONTINUE                                                      12FE5
               QAX  = (BM(1) - 2.0*BM(2) + BM(3) ) * HDHX               24MY5
     1              + (  - 0.25*HDHX* RX(I-1,J) * WX(I-2,J)             27JL5
     2              + 0.25*HDHX* ( RX(I-1,J) + RX(I+1,J) ) * WX(I,J)    27MY5
     3              - 0.25*HDHX* RX(I+1,J) * WX(I+2,J)                  27MY5
     4              - HY * PX(I,J) * WX(I-1,J)                          25MY5
     5              + HY * ( PX(I,J) + PX(I+1,J) ) * WX(I,J)            25MY5
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     6              - HY * PX(I+1,J) * WX(I+1,J)    ) / HXHY            27MY5
               QTMX = ( WX(I-1,J-1)* CX(I,J) - WX(I-1,J)*( CX(I,J)      03MY5
     1              + CX(I,J+1)) + WX(I-1,J+1) * CX(I,J+1) - WX(I,J-1)  04MY5
     2              *(CX(I,J) + CX(I+1,J)) + WX(I,J) * (CX(I,J) +       14MY5
     3              CX(I,J+1) + CX(I+1,J) + CX(I+1,J+1) ) - WX(I,J+1)   04MY5
     4              * ( CX(I,J+1) + CX(I+1,J+1) ) + WX(I+1,J-1) *       04MY5
     5              CX(I+1,J) - WX(I+1,J) * (CX(I+1,J) + CX(I+1,J+1) )  04MY5
     6              + WX(I+1,J+1) * CX(I+1,J+1) ) /  HXHY               04MY5
               SFXY = SFX(NS)                                           08AP5
               QFX = SFXY * WX(I,J) - QAX - QTMX                        24MY5
  607     IF( FZ(I,J)* DY(I,J) )  609, 608, 609                         08AP5
  608          SFZY = QFZ = 0                                           02JE5
      GO TO 616                                                         12FE5
  609     DO 610 N = 1, 3                                               24FE5
               K = I + N - 2                                            19JA5
               BM(N+6) =  FZ(K,J) *( WZ(K-1,J) - 2.0* WZ(K,J) +         08AP5
     1                   WZ(K+1,J) ) / HXE2                             08AP5
  610     CONTINUE                                                      11FE5
               QAZ = ( BM(7) - 2.0 *BM(8) + BM(9) )/ HX                 08AP5
     1              + ( - 0.25*HX * RZ(I-1,J) * WZ(I-2,J)               27JL5
     2              + 0.25* HX * ( RZ(I-1,J) + RZ(I+1,J) ) * WZ(I,J)    21MY5
     3              - 0.25* HX * RZ(I+1,J) * WZ(I+2,J)                  21MY5
     4              -HXE2* PZ(I,J) * WZ(I-1,J)                          25MY5
     5              +HXE2* ( PZ(I,J) + PZ(I+1,J) ) * WZ(I,J)            25MY5
     6              -HXE2* PZ(I+1,J) * WZ(I+1,J)    ) / HXE3            27MY5
               SFZY = SFZ(NS)                                           08AP5
               QFZ = SFZY * WZ(I,J) - QAZ                               08AP5
C-----COMPUTE Y-BEAM MATRIX COEFFS                                      27JL5
  616          AA = HX2H2 * DY(I,J-1) - RY(I,J-1) * HDHY / 4.0          06MY5
               BB = -2.0 * HX2H2 * ( DY(I,J-1) + DY(I,J) )              06MY5
     1              -        ( CY(I,J) + CY(I+1,J) ) - HX  *PY(I,J)     27MY5
               CC = HX2H2 * ( DY(I,J-1) + 4.0 * DY(I,J) + DY(I,J+1) )   06MY5
     1              +      (CY(I,J)+CY(I+1,J)+CY(I,J+1)+CY(I+1,J+1) )   19MY5
     2              + HX   * PY(I,J) + HX  *PY(I,J+1)                   03JE5
     3              +(HDHY/4.0) * ( RY(I,J-1) + RY(I,J+1) )             06MY5
     4              + HXHY * ( S(I,J) + SFXY + SFZY )                   06MY5
               DD = -2.0 * HX2H2 * ( DY(I,J)+DY(I,J+1) )                06MY5
     1              -        ( CY(I,J+1) + CY(I+1,J+1) ) -HX  *PY(I,J+1) 03JE5
               EE = HX2H2 * DY(I,J+1) -(HDHY / 4.0)* RY(I,J+1)          06MY5
               FF =  HXHY*( Q(I,J) + QFX + QFZ )                        04MY5
     1              -0.5/HX *(TZ(I-1,J)-TZ(I+1,J)) * HXHY               27MY5
     1              - HYD2 * ( TX(I-1,J) - TX(I+1,J) )                  27JL5
     1              -HXD2*(TY(I,J-1) - TY(I,J+1) )                      10MY5
     2              + ( - CY(I,J) * ( WY(I-1,J-1) -WY(I-1,J))-CY(I+1,J)*11JE5
     3              (WY(I+1,J-1) - WY(I+1,J) ) + CY(I,J+1)*(WY(I-1,J)   10MY5
     4              - WY(I-1,J+1) ) + CY(I+1,J+1)*(WY(I+1,J)-WY(I+1,J+1)10MY5
     5              )) - ( DPY(I,J-1)*(WX(I-1,J-1)-2.0*WX(I,J-1)        10MY5
     6              + WX(I+1,J-1) ) - 2.0* DPY(I,J)*( WX(I-1,J)         10MY5
     7              - 2.0*WX(I,J) + WX(I+1,J) ) + DPY(I,J+1)*           10MY5
     8              ( WX(I-1,J+1) -2.0* WX(I,J+1) +WX(I+1,J+1)) )       04MY5
C-----COMPUTE Y-BEAM CONTINUITY COEFFS                                  27JL5
               E    =  AA * B(J-2) + BB                                 04MY5
               DEN  =  E * B(J-1) + AA * C(J-2) + CC                    04MY5
          IF  (DEN)  631, 621, 631                                      13AP3
  621          G    =   0.0                                             08JL4
          GO TO  632                                                    13AP3
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  631          G    = - 1.0 / DEN                                       08JL4
  632          C(J) =  G * EE                                           04MY5
               B(J) =  G * ( E * C(J-1) + DD )                          04MY5
               A(J) =  G * ( E * A(J-1) + AA * A(J-2) - FF )            05MY5
  640     CONTINUE                                                      13AP3
C-----COMPUTE Y-BEAM DEFLS                                              27JL5
          DO 650  L = 3,MYP5                                            13AP3
               J    = MY + 8 - L                                        13AP3
               WTEMP = WY(I,J)                                          08AP5
               WY(I,J)  = A(J) + B(J) *  WY(I,J+1) + C(J) *  WY(I,J+2)  08AP5
C-----COUNT STAS WHERE Y-BEAMS NOT STABILIZED                           27JL5
          IF  ( DX(I,J) * DY(I,J) )  645, 650, 645                      4SE64
  645     IF  ( ABSF (  WY(I,J) - WTEMP ) - CTOL )  650, 650, 646       08AP5
  646          KSTY =   KSTY + 1                                        13AP3
  650     CONTINUE                                                      13AP3
  660     CONTINUE                                                      13AP3
C-----SET EXTERIOR PLATE DEFLECTIONS                                    12MY5
          DO 670 J = 3, MYP5                                            12MA5
               I = 3                                                    12MA5
               WY(I,J) = WX(I,J)                                        12MY5
               I = MXP5                                                 12MA5
               WY(I,J) = WX(I,J)                                        12MY5
  670 CONTINUE                                                          12MA5
C
C
C-----SOLVE  Z BEAMS                                                    03MY5
               KSTZ = 0                                                 20MR5
          IF ( NCT2Z )  700, 790, 700                                   26AP5
  700     DO 760  J = 4, MYP4                                           04MY5
          DO 740  I = 3,MXP5                                            04MY5
C-----ESTABLISH ITERATION CONTROL PARAMETERS                             27JL5
          IF ( FZ(4,J) )  702, 760, 702                                 05MY5
  702     IF ( FZ(I,J) * DX(I,J) ) 705, 703, 705                        05MY5
  703          SFXZ = QFX = 0.0                                         04MY5
          GO TO 707                                                     04MY5
  705     DO 706 N = 1, 3                                               04MY5
               K = I + N - 2                                            21JA5
               BM(N)   = DX(K,J) * (   WX(K-1,J)-2.0* WX(K,J) + WX(K+1,J)08AP5
     1                   )/ HXE2+    DPX(K,J)*(WY(K,J-1)-2.0*WY(K,J)    25MY5
     2                   + WY(K,J+1) ) / HYE2                           12MY5
  706 CONTINUE                                                          04MY5
               QAX = ( BM(1) -2.0 * BM(2) + BM(3) ) *HDHX               12JE5
     1              + (  - 0.25*HDHX* RX(I-1,J) * WX(I-2,J)             27JL5
     2              + 0.25*HDHX* ( RX(I-1,J) + RX(I+1,J) ) * WX(I,J)    21MY5
     3              - 0.25*HDHX* RX(I+1,J) * WX(I+2,J)                  21MY5
     4              - HY * PX(I,J) * WX(I-1,J)                          25MY5
     5              + HY * ( PX(I,J) + PX(I+1,J) ) * WX(I,J)            25MY5
     6              - HY * PX(I+1,J) * WX(I+1,J)    ) / HXHY            27MY5
               QTMX = ( WX(I-1,J-1)* CX(I,J) - WX(I-1,J)*( CX(I,J)      21MY5
     1              + CX(I,J+1)) + WX(I-1,J+1) * CX(I,J+1) - WX(I,J-1)  21MY5
     2              *(CX(I,J) + CX(I+1,J)) + WX(I,J) * (CX(I,J) +       21MY5
     3              CX(I,J+1) + CX(I+1,J) + CX(I+1,J+1) ) - WX(I,J+1)   21MY5
     4              * ( CX(I,J+1) + CX(I+1,J+1) ) + WX(I+1,J-1) *       21MY5
     5              CX(I+1,J) - WX(I+1,J) * (CX(I+1,J) + CX(I+1,J+1) )  21MY5
     6              + WX(I+1,J+1) * CX(I+1,J+1) ) /  HXHY               21MY5
               SFXZ = SFX(NS)                                           08AP5
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              QFX = SFXZ * WX(I,J) - QAX - QTMX                         07JE5
  707    IF ( DY(I,J) * FZ(I,J) ) 709, 708, 709                         04MY5
  708         SFYZ = QFY = 0.0                                          04MY5
         GO TO 716                                                      04MY5
  709    DO 710 N = 1, 3                                                04MY5
              L = J + N - 2                                             31DE4
              BM(N+3) = DY(I,L)*(  WY(I,L-1) -2.0* WY(I,L) +  WY(I,L+1)04MY5
     1                  )/ HYE2+    DPY(I,L)*(WX(I-1,L)-2.0*WX(I,L)     12JE5
     2                  + WX(I+1,L) ) / HXE2                            12MY5
  710 CONTINUE                                                          04MY5
              QAY = ( BM(4) - 2.0* BM(5) + BM(6) ) * HDHY               12JE5
     1             + ( - 0.25 *HDHY* RY(I,J-1) * WY(I,J-2)              27JL5
     2             + 0.25 *HDHY* ( RY(I,J-1) + RY(I,J+1))* WY(I,J)      21MY5
     3             - 0.25 *HDHY* RY(I,J+1) * WY(I,J+2)                  21MY5
     4             - HX * PY(I,J) * WY(I,J-1)                           25MY5
     5             + HX * ( PY(I,J) + PY(I,J+1) ) * WY(I,J)             25MY5
     6             - HX * PY(I,J+1) * WY(I,J+1)     ) / HXHY            27MY5
              QTMY = ( WY(I-1,J-1) * CY(I,J) - WY(I-1,J) * (CY(I,J)     21MY5
     1             +CY(I,J+1) )  + WY(I-1,J+1) * CY(I,J+1) - WY(I,J-1)  21MY5
     2             * ( CY(I,J) + CY(I+1,J) ) +  WY(I,J)*(CY(I,J) +      21MY5
     3             CY(I,J+1) + CY(I+1,J) + CY(I+1,J+1)) - WY(I,J+1) *   21MY5
     4             (CY(I,J+1)+ CY(I+1,J+1) ) + WY(I+1,J-1) * CY(I+1,J)  21MY5
     5             -WY(I+1,J) * ( CY(I+1,J) + CY(I+1,J+1) ) +           21MY5
     6             WY(I+1,J+1) * CY(I+1,J+1) ) / HXHY                   21MY5
              SFYZ = SFY(NS)                                            08AP5
              QFY = SFYZ * WY(I,J) - QAY - QTMY                         07JE5
C-----COMPUTE Z-BEAM MATRIX COEFFS                                      27JL5
  716         AA   =   FZ(I-1,J) - 0.25 * HX * RZ(I-1,J)                26AP5
              BB   = - 2.0 * ( FZ(I-1,J) + FZ(I,J) ) - HXE2 * PZ(I,J)   08AP5
              CC   =   FZ(I-1,J) + 4.0 * FZ(I,J) + FZ(I+1,J)            20MR5
     1                + HXE2 * ( PZ(I,J) +                              08AP5
     2                  PZ(I+1,J) ) + 0.25 * HX * ( RZ(I-1,J)           06AP5
     3                + RZ(I+1,J) ) + HXE3 * S(I,J)                     06AP5
     4                + HXE3 * ( SFXZ + SFYZ )                          06AP5
              DD   = - 2.0 * ( FZ(I,J) + FZ(I+1,J) ) - HXE2 * PZ(I+1,J)08AP5
              EE   =   FZ(I+1,J) - 0.25 * HX * RZ(I+1,J)                06AP5
              FF   = HXE3* (  Q(I,J) + QFX + QFY                        21MY5
     1             -HYD2   *(TX(I-1,J)-TX(I+1,J)) / HXHY                27MY5
     2             -HXD2   *(TY(I,J-1)-TY(I,J+1)) / HXHY                27MY5
     3             -0.5/HX *(TZ(I-1,J)-TZ(I+1,J))    )                  25MY5
C-----COMPUTE Z-BEAM CONTINUITY COEFFS                                  27JL5
              E    =   AA * B(I-2) + BB                                 13AP3
              DEN  =   E  * B(I-1) + AA * C(I-2) + CC                   13AP3
         IF   (DEN)  731, 721, 731                                      04MY5
  721         D    =   0.0                                              04MY5
         GO TO  732                                                     04MY5
  731         D    = - 1.0 / DEN                                        04MY5
  732         C(I) =   D  * EE                                          04MY5
              B(I) =   D  * ( E  * C(I-1) + DD )                        13AP3
              A(I) =   D  * ( E  * A(I-1) + AA * A(I-2) - FF )          13AP3
  740   CONTINUE                                                        04MY5
C-----COMPUTE Z-BEAM DEFLS                                               27JL5
         DO 750  L = 3,MXP5                                             04MY5
              I    =   MX + 8 - L                                       13AP3
              WTEMP = WZ(I,J)                                           09MR5
              WZ(I,J) = A(I) + B(I)*WZ(I+1,J) + C(I) * WZ(I+2,J)        09MR5
```

```
C-----COUNT STAS WHERE Z-BEAMS NOT STABILIZED                            27JL5
          IF ( FZ(I,J) ) 745, 750, 745                                   04MY5
  745     IF ( ABSF ( WZ(I,J) - WTEMP ) - CTOL ) 750, 750, 746           04MY5
  746          KSTZ = KSTZ + 1                                           26AP5
  750     CONTINUE                                                       04MY5
  760     CONTINUE                                                       04MY5
  790     CONTINUE                                                       26AP5
C-----COUNT STAS WHERE DEFLS NOT CLOSED                                  27JL5
               KCTOL  =  0                                               13AP3
               KCTOLZ = 0                                                04MY5
          DO 850 I = 4, MXP4                                             04MY5
          DO 850 J = 4, MYP4                                             04MY5
          IF ( DX(I,J) * DY(I,J) ) 810, 820, 810                         04MY5
  810     IF ( ABSF( WX(I,J) - WY(I,J) ) - CTOL )  820, 820, 815         14MY5
  815          KCTOL = KCTOL + 1                                         04MY5
  820     IF ( ( DX(I,J) + DY(I,J) ) * FZ(I,J) )   830, 850, 830         04MY5
  830     IF ( DX(I,J) )  840, 835, 840                                  04MY5
  835     IF ( ABSF( WZ(I,J) - WY(I,J) ) - CTOL )  850, 850, 845         04MY5
  840     IF ( ABSF( WZ(I,J) - WX(I,J) ) - CTOL )  850, 850, 845         04MY5
  845          KCTOLZ = KCTOLZ + 1                                       04MY5
  850     CONTINUE                                                       04MY5
C-----PRINT MONITOR DATA                                                 27JL5
      PRINT 75, NC, SFX(NS), NS, KSTX,   WX(IM1+4,JM1+4),                06AP5
     1    WX(IM2+4,JM2+4), WX(IM3+4,JM3+4), WX(IM4+4,JM4+4),             25AP3
     2    SFY(NS), NS, KSTY, KCTOL, WY(IM1+4,JM1+4),                     11JE5
     3    WY(IM2+4,JM2+4), WY(IM3+4,JM3+4), WY(IM4+4,JM4+4),             31DE4
     4    SFZ(NS), NS, KSTZ, KCTOLZ, WZ(IM1+4,JM1+4),                    11JE5
     5     WZ(IM2+4,JM2+4), WZ(IM3+4,JM3+4), WZ(IM4+4,JM4+4)             20MR5
C
C-----CONTROL ITERATION PROCESS                                          27JL5
          IF ( KCTOL + KCTOLZ )   900, 900, 860                          26AP5
  860     CONTINUE                                                       26AP5
               NC = ITMAX                                                25AG4
C-----COMPUTE AND PRINT RESULTS                                          27ML5
  900     CONTINUE                                                       26AP5
      PRINT 11                                                           08MY3 ID
      PRINT 1                                                            18FE5 ID
      PRINT 13, ( AN1(N), N = 1, 32 )                                    09MR5
      PRINT 16, NPROB, ( AN2(N), N = 1, 14 )                             28AG3 ID
      PRINT 85, NC                                                       06AP5
          IF ( KCTOL + KCTOLZ ) 912, 912, 911                            26AP5
  911 PRINT 86                                                           26AP5
  912 PRINT 87                                                           26AP5
          DO 960  J = 3,MYP5                                             26AP5
      PRINT  6                                                           24AP3
          DO 950  I = 3,MXP5                                             26AP5
               ISTA = I - 4                                              18AP3
               JSTA = J - 4                                              18AP3
          DO 940  N = 1,3                                                26AP5
               K = I + N - 2                                             21JA5
               BM(N)    = DX(K,J) * ( WX(K-1,J)-2.0*WX(K,J) + WX(K+1,J)  09MR5
     1                    )/ HXE2+    DPX(K,J)*(WY(K,J-1)-2.0*WY(K,J)    25MY5
     2                    + WY(K,J+1) ) / HYE2                           12MY5
               BM(N+6) = FZ(K,J) * ( WZ(K-1,J) - WZ(K,J) - WZ(K,J)       09MR5
     2                    +WZ(K+1,J)) / HXE2                             09MR5
               L    =   J + N - 2                                        01MA3
```

```
             BM(N+3) = DY(I,L)*(WY(I,L-1) -2.0*WY(I,L) + WY(I,L+1)  09MR5
   1                   )/ HYE2+    DPY(I,L)*(WX(I-1,L)-2.0*WX(I,L)    12JE5
   2                   + WX(I+1,L) ) / HXE2                           12MY5
940     CONTINUE                                                      26AP5
             QAX = ( BM(1) - BM(2) - BM(2) + BM(3) ) * HDHX           10JE5
   1              + (  - 0.25*HDHX* RX(I-1,J) * WX(I-2,J)             27JL5
   2              + 0.25*HDHX* ( RX(I-1,J) + RX(I+1,J) ) * WX(I,J)    10JE5
   3              - 0.25*HDHX* RX(I+1,J) * WX(I+2,J)                  10JE5
   4              - HY * PX(I,J) * WX(I-1,J)                          25MY5
   5              + HY * ( PX(I,J) + PX(I+1,J) ) * WX(I,J)            25MY5
   6              - HY * PX(I+1,J) * WX(I+1,J)    ) / HXHY            27MY5
             QAY = ( BM(4) - BM(5) - BM(5) + BM(6) ) * HDHY           10JE5
   1              + (  - 0.25 *HDHY* RY(I,J-1) * WY(I,J-2)            27JL5
   2              + 0.25 *HDHY* ( RY(I,J-1) + RY(I,J+1))* WY(I,J)     10JE5
   3              - 0.25 *HDHY* RY(I,J+1) * WY(I,J+2)                 10JE5
   4              - HX * PY(I,J) * WY(I,J-1)                          25MY5
   5              + HX * ( PY(I,J) + PY(I,J+1) ) * WY(I,J)            27JL5
   6              - HX * PY(I,J+1) * WY(I,J+1)    ) / HXHY            27MY5
             QAZ  = ( BM(7) - BM(8) - BM(8) + BM(9) ) / HX            05MY5
   1              + (  -0.25* HX * RZ(I-1,J) * WX(I-2,J)              27JL5
   2              + 0.25* HX * ( RZ(I-1,J) + RZ(I+1,J) ) * WZ(I,J)    21MY5
   3              - 0.25* HX * RZ(I+1,J) * WZ(I+2,J)                  21MY5
   4              -HXE2* PZ(I,J) * WZ(I-1,J)                          25MY5
   5              +HXE2* ( PZ(I,J) + PZ(I+1,J) ) * WZ(I,J)            25MY5
   6              -HXE2* PZ(I+1,J) * WZ(I+1,J)    ) / HXE3            27MY5
             CMX(2) =(CX(I,J)+CX(I,J+1)+CX(I+1,J+1)+CX(I+1,J) )*0.250 12MY5
   1                   * ( WX(I-1,J-1) - WX(I-1,J+1) - WX(I+1,J-1)    09MR5
   2                   + WX(I+1,J+1)) / ( 4.0 * HX * HY )             09MR5
             CMY(5) =-(CY(I,J)+CY(I,J+1)+CY(I+1,J+1)+CY(I+1,J))*0.250 12MY5
   1                   * ( WY(I-1,J-1) - WY(I-1,J+1) - WY(I+1,J-1)    09MR5
   2                   + WY(I+1,J+1)) / ( 4.0 * HY * HX )             09MR5
             QTMX = ( WX(I-1,J-1)* CX(I,J) - WX(I-1,J)*( CX(I,J)      03MY5
   1              + CX(I,J+1)) + WX(I-1,J+1) * CX(I,J+1) - WX(I,J-1)  04MY5
   2              *(CX(I,J) + CX(I+1,J)) + WX(I,J) * (CX(I,J) +       04MY5
   3              CX(I,J+1) + CX(I+1,J) + CX(I+1,J+1) ) - WX(I,J+1)   04MY5
   4              * ( CX(I,J+1) + CX(I+1,J+1) ) + WX(I+1,J-1) *       04MY5
   5              CX(I+1,J) - WX(I+1,J) * (CX(I+1,J) + CX(I+1,J+1) )  04MY5
   6              + WX(I+1,J+1) * CX(I+1,J+1) ) /  HXHY               04MY5
             QTMY = ( WY(I-1,J-1) * CY(I,J) - WY(I-1,J) * (CY(I,J)    03MY5
   1              +CY(I,J+1) )  + WY(I-1,J+1) * CY(I,J+1) - WY(I,J-1) 03MY5
   2              * ( CY(I,J) + CY(I+1,J) ) +  WY(I,J)*(CY(I,J) +     03MY5
   3              CY(I,J+1) + CY(I+1,J) + CY(I+1,J+1)) - WY(I,J+1) *  05MY5
   4              (CY(I,J+1)+ CY(I+1,J+1) ) + WY(I+1,J-1) * CY(I+1,J) 03MY5
   5              -WY(I+1,J) * ( CY(I+1,J) + CY(I+1,J+1) ) +          03MY5
   6              WY(I+1,J+1) * CY(I+1,J+1) ) / HXHY                  03MY5
             REACT = QAX + QAY + QAZ + QTMX + QTMY                    24MY5
        IF  ( DX(I,J) * DY(I,J) )  948, 946, 948                      04MY5
946          ERROR   =     0.0                                        06MY5
        GO TO  949                                                    04MY5
948          DENO    =    WX(I,J) / WX(I,J) + WY(I,J) / WY(I,J) +     21MY5
   1                      WZ(I,J) / WZ(I,J)                           21MY5
             ERROR   =    Q(I,J) - REACT - S(I,J) * ( WX(I,J) +       07AP5
   1                  WZ(I,J) + WY(I,J) ) / DENO                      21MY5
   1                  - HYD2 * ( TX(I-1,J) - TX(I+1,J) ) / HXHY       11JE5
   2                  - HXD2 * ( TY(I,J-1) - TY(I,J+1) ) / HXHY       11JE5
   4                   -0.5/HX * (TZ(I-1,J) - TZ(I+1,J))              21MY5
```

```
 949 PRINT 88, ISTA, JSTA, WX(I,J), WY(I,J), WZ(I,J), REACT, ERROR      27MY5
              WX(I-2,J-2)= BM(2)                                          27MY5
              WY(I-2,J-2)= BM(5)                                          27MY5
              WZ(I-2,J-2)= BM(8)                                          27MY5
              Q(I-2,J-2) = CMX(2)                                         27MY5
              S(I-2,J-2) = CMY(5)                                         27MY5
 950      CONTINUE                                                        04MY5
 960      CONTINUE                                                        04MY5
     PRINT 89, NC                                                         27MY5
     PRINT 90                                                             27MY5
          DO 980 J = 3, MYP5                                              27MY5
     PRINT 6                                                              27MY5
          DO 970 I = 3, MXP5                                              27MY5
               ISTA = I - 4                                               27MY5
               JSTA = J - 4                                               27MY5
     PRINT 88, ISTA, JSTA, WX(I-2,J-2), WY(I-2,J-2), WZ(I-2,J-2),         27MY5
    1           Q(I-2,J-2), S(I-2,J-2)                                    27MY5
 970      CONTINUE                                                        27MY5
 980      CONTINUE                                                        27MY5
     CALL TIME                                                            25MY4 ID
          GO TO 1010                                                      26AG3 ID
9990 CONTINUE                                                             12MR5 ID
9999 CONTINUE                                                             04MY3 ID
     PRINT 11                                                             08MY3 ID
     PRINT 1                                                              18FE5 ID
     PRINT 13, ( AN1(N), N = 1, 32 )                                      18FE5 ID
     PRINT 19                                                             26AG3 ID
     END                                                                  04MY3 ID
     END
```

APPENDIX 11

PROGRAM LAYER 8 AND GUIDE FOR DATA INPUT

APPENDIX 11. PROGRAM LAYER 8

Program LAYER 8 is a FORTRAN 63 language computer program written to solve the equations presented in Chapter 7 for a plate-over-beam system. The program requires a large capacity digital computer such as the CDC 1604. The program requires approximately 26,000 words of core memory and compiles in approximately 2 minutes. A brief explanation of several items pertinent to the use of this program follows. An input form to aid in coding problems for this program is included in this Appendix.

Input Parameters

Data is input to this program by two methods. These two methods have been detailed in Appendix 5 for program LAYER 7. Data input is arranged in tables. Table 1 contains constants and control data. Of interest are the two Poisson constants. Poisson's ratio "x" pertains to a stress application in the x-direction and is defined by Eq A7.2. Poisson's ratio "y" is defined in a similar manner by Eq A7.4. The monitor data allow four intersections of the system to be observed for convergence and closure rate. The monitor stations are designated by the x-y layer coordinates but output includes deflections for all layers existing at the specified station.

Table 2 includes closure parameter data. The same number of values are used on all layers. Tables 3, 4, and 5 pertain to the x , y , and z-beams of the layered system. Table 6 is for all load and support data. A blank card is required as the last card of any data run to stop the program.

Program Results

The results from the program are also arranged in Tables. Tables 1, 2, 3, 4, 5, and 6 are reflections of the input data. Table 7 includes the monitor data output. Table 8 includes the deflections at each station of each of the beams along with a reaction and an error term. The error term is the unbalance of load equilibrium at the intersection and is computed only from statics. Table 9 gives the plate and beam moments as well as the torsional moments in the plate.

GUIDE FOR DATA INPUT FOR LAYER 8

with Supplementary Notes

extract from

A FINITE-ELEMENT METHOD FOR BENDING ANALYSIS OF LAYERED STRUCTURAL SYSTEMS

by

Wayne B. Ingram and Hudson Matlock

August 1965

LAYER 8 GUIDE FOR DATA INPUT -- Card forms

IDENTIFICATION OF PROGRAM AND RUN (2 alphanumeric cards per run)

1	80

1	80

IDENTIFICATION OF PROBLEM (one alphanumeric card each problem; program stops if NPROB = 0)

NPROB	Description of problem (Alphanumeric)
1 5	11 80

TABLE 1. PROGRAM CONTROL DATA (3 cards per problem)

NUM CARDS IN TABLE					NUM INCRS		INCREMENT LENGTHS		
3	4	5	6	ITERS	MX	MY	X-DIRECTION	Y-DIRECTION	TOLERANCE
15	20	25	30	40	45	50	60	70	80

POISSON'S RATIOS (Poisson's ratio X refers to a stress applied in the x-direction. Similarly Poisson's ratio Y refers to a stress applied in the y-direction)

X	Y
21 30	40

MONITOR DATA (specify four intersections in i-sta and j-sta coordinates)

15	20	25	30	35	40	45	50

TABLE 2. CLOSURE PARAMETERS (3 cards, all cards must have same number of values but not necessarily the same values)

	NUM VALUES	VALUES OF PARAMETERS					
X:	15	21 30	40	50	60	70	80
Y:	15	21 30	40	50	60	70	80
Z:	15	21 30	40	50	60	70	80

TABLE 3. X-BEAM DATA

FROM STA		TO STA		D	C	T	R	P
i	j	i	j	BENDING STIFFNESS	TORSIONAL STIFFNESS	TRANSVERSE COUPLE	ROTATIONAL RESTRAINT	AXIAL TENSION OR COMPRESSION
15	20	25	30	40	50	60	70	80

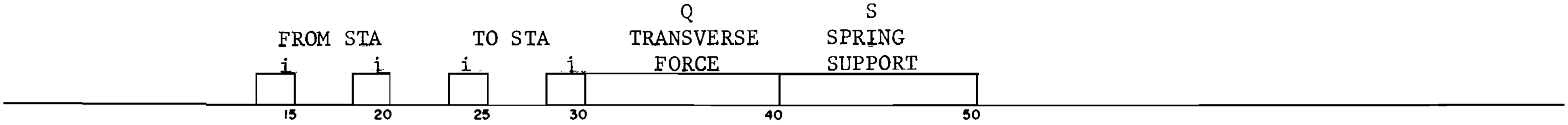
TABLE 4. Y-BEAM DATA

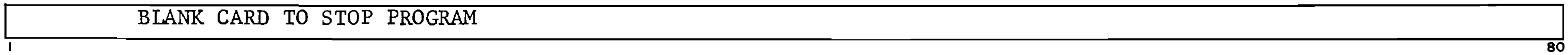
FROM STA		TO STA		D	C	T	R	P
i	j	i	j	BENDING STIFFNESS	TORSIONAL STIFFNESS	TRANSVERSE COUPLE	ROTATIONAL RESTRAINT	AXIAL TENSION OR COMPRESSION
15	20	25	30	40	50	60	70	80

TABLE 5. Z-BEAM DATA

FROM STA		TO STA		F		T	R	P
i	j	i	j	BENDING STIFFNESS		TRANSVERSE COUPLE	ROTATIONAL RESTRAINT	AXIAL TENSION OR COMPRESSION
15	20	25	30	40		51 60	70	80

TABLE 6. LOAD AND SUPPORT DATA





GENERAL PROGRAM NOTES

Two cards containing any desired alphanumeric information are required (for identification purposes only) at the beginning of the data for each new run.

The data cards must be stacked in proper order for the program to run.

A consistent system of units must be used for all input data; for example: pounds and inches.

All data words must be justified completely to the right in the field provided.

All data words of 5 spaces or less are to be whole integer numbers: . . . . -1234

All data words of 10 spaces are to be entered as floating-point decimal numbers including a multiplier expressed in terms of an exponent of 10: . . . . 1.234E+03

Blank data fields are interpreted as zeros.

One card with a problem number in spaces 1-5 is required as the first card of each problem. This number may be alphanumeric. The remainder of the card may be any information desired.

Any number of problems may be stacked in one run.

One card with problem number blank is required to stop the run.

If it is desired to work a two layer system (a plate), it is only necessary to omit the proper tables 3, 4, or 5. Three cards must still appear in Table 2 except one may be blank.

TABLE 1.

The card counts in Table 1 should be rechecked carefully after coding of each problem. The run will be abandoned if the card counts are incorrect.

The number of iterations should equal the product of the number of closure parameters to be cycled and the number of cycles desired.

The maximum allowable number of increments in the x-direction is 25 in the y-direction 25.

The length of increment may be anything desired by the user. Unreasonably large increments may give too crude an approximation to the problem.

The deflection closure tolerance may be difficult to achieve if specified unreasonably small. A value between 0.001 and 0.00001 is usually satisfactory.

Four monitor intersections are allowable to follow the convergence of the solution at four points.

TABLE 1. (Continued)

The program automatically outputs deflections for all intersecting beams at each monitor intersection.

TABLE 2. CLOSURE PARAMETERS

Up to six closure parameters are allowed for each layer. The same number must be used on all layers.

Parameters should be input in the order in which they are to be cycled.

Three cards are required; one may be blank if a simple plate is to be solved.

TABLES 3, 4, 5. X , Y , and Z-BEAM DATA

Typical units:

variables:	D	C	T	R	P
values per station:	lb-in^2/in	lb-in^2/in	in × lb	in × lb/rad	lb

Axial tension or compression values P are stated at stations in the same manner as any other distributed data. Since the P values refer to bars, no station number should be used which would specify a P value in a bar outside the real plate.

Torsional stiffnesses, C , are defined for the mesh of the plate surrounded by four rigid bars and four joints. The mesh is numbered according to the top right corner of the mesh. Thus, the station numbers i=0 and j=0 should never be used.

Data must not be entered which would express effects at fictitious stations beyond the edges of the real plate.

There are no restrictions on the order of cards in these tables. Cumulative input, with full values at each station, is used in these tables.

TABLE 6. LOAD AND SUPPORT DATA

Typical units:

variables:	Q	S
values per station:	lb	lb/in

Same comments as for Tables 3, 4, 5.

APPENDIX 12

SAMPLE INPUT AND OUTPUT FOR PROGRAM LAYER 7

APPENDIX 12. SAMPLE INPUT AND OUTPUT FOR PROGRAM LAYER 7

```
6.1     HYPOTHETICAL GRID OVER FIVE BEAMS
            5    5   15    3        20    8    8 6.000E 00 6.000E 00 1.000E-06
            4    4    2    2    4    2    3    2
            5      1.070E+03 1.480E+04 7.020E+04 1.850E+05 3.580E+05
            5      1.070E+03 1.480E+04 7.020E+04 1.850E+05 3.580E+05
            5      1.070E+06 1.480E+07 7.020E+07 1.850E+08 3.580E+08
            0    0    8    8 2.500E 06
            1    1    7    7 2.500E 06
            1    0    7    8 2.500E 06
            0    1    8    7 2.500E 06
            1    0    8    8                               1.000E 04
            0    0    8    8 2.500E 06
            1    1    7    7 2.500E 06
            1    0    7    8 2.500E 06
            0    1    8    7 2.500E 06
            0    1    8    8                               1.000E 04
            0    0    8    0 5.000E 09
            1    0    7    0 5.000E 09
            0    2    8    2 5.000E 09
            1    2    7    2 5.000E 09
            0    4    8    4 5.000E 09
            1    4    7    4 5.000E 09
            0    6    8    6 5.000E 09
            1    6    7    6 5.000E 09
            0    8    8    8 5.000E 09
            1    8    7    8 5.000E 09
            1    0    8    0                               1.000E+04
            1    2    8    2                               1.000E+04
            1    4    8    4                               1.000E+04
            1    6    8    6                               1.000E+04
            1    8    8    8                               1.000E+04
            0    0    8    8-2.000E 03 1.000E 99
            1    1    7    7-2.000E 03
            1    0    7    8          -1.000E+99
```

```
6.2     SUPPORT SYSTEM FOR STRUCTURE-- INCLUDES HOLE AND FOUNDATION SUPPORT
            15   21    7   14          40   16   10 3.000E 01 2.400E 01 1.000E-04
             8    8   16    2    4    2    6    6
             5      1.000E 03 5.000E 03 1.000E 04 5.000E 04 1.000E 05
             5      1.000E 03 5.000E 03 1.000E 04 5.000E 04 1.000E 05
             5      1.000E 03 5.000E 03 1.000E 04 5.000E 04 1.000E 05
             4    0    4    1-1.500E 08
             0    0    4    1 3.000E+08
             0    0    0   10-1.500E 08
             0    2   16    6 3.000E+08
            16    2   16    6-1.500E 08
             0    7   15    7 3.000E+08
            15    7   15    7-1.500E 08
             0    8   14    8 3.000E+08
            14    8   14    8-1.500E 08
             0    9   13    9 3.000E+08
            13    9   13    9-1.500E 08
             0   10   12   10 3.000E 08
            12   10   12   10-1.500E 08
            12    2   12   10                     8.000E 06
             7    4    8    5-3.000E 08
             0    0    0   10 1.000E 08
             0    1    0    9 1.000E 08
             2    0    2   10 1.000E 08
             2    1    2    9 1.000E 08
             4    0    4   10 1.000E 08
             4    1    4    9 1.000E 08
             6    2    6   10 1.000E 08
             6    3    6    9 1.000E 08
             6    3    6    6 1.000E 08
             8    2    8   10 1.000E 08
             8    3    8    9 1.000E 08
             8    4    8    5-2.000E 08
             9    3    9    6 3.000E+08
            10    2   10   10 1.000E 08
            10    3   10    9 1.000E 08
            12    2   12   10 1.000E 08           8.000E 06
            12    3   12    9 1.000E 08
            14    2   14    8 1.000E 08
            14    3   14    7 1.000E 08
            16    2   16    6 1.000E 08
            16    3   16    5 1.000E 08
             0    2   16    2 1.000E 08
             1    2   15    2 1.000E 08
             0    6   16    6 1.000E 08
             1    6   15    6 1.000E 08
             0   10   12   10 1.000E 08
             1   10   11   10 1.000E 08
            12    2   12   10                     8.000E 06
             0    0    4   10 5.000E 01 3.000E 02
             1    1    4    9 5.000E 01
             5    2   12   10 5.000E 01
             5    3   12    9 5.000E 01
            13    2   16    6 5.000E 01
            13    3   15    5 5.000E 01
```

```
13     6    13     9 5.000E 01
13     7    13     8 5.000E 01
14     6    14     8 5.000E 01
14     7    14     7 5.000E 01
15     6    15     7 5.000E 01
 7     4     8     5-1.000E+02
 8     8     8     8 2.000E+03
12     2    12    10               1.000E+99
```

```
6.3     GIRDER-DIAPHRAGM SYSTEM OF A BRIDGE
            2   10    4    7        40   16   10 6.000E 01 3.000E 01 1.000E-04
            8    6    8    2    4    2    8    0
            4      5.400E+01 2.700E+02 1.350E+03 6.750E 03
            4      2.700E+02 1.350E+03 6.750E+03 3.375E 04
            4      5.400E+03 2.700E+04 1.350E+05 6.750E 05
            0    0   16   10 5.000E 09
            1    1   15    9 5.000E 09
            0    0    0   10 2.500E 10
            0    1    0    9 2.500E 10
            4    0    4   10 2.500E 10
            4    1    4    9 2.500E 10
            8    0    8   10 2.500E 10
            8    1    8    9 2.500E 10
           12    0   12   10 2.500E 10
           12    1   12    9 2.500E 10
           16    0   16   10 2.500E 10
           16    1   16    9 2.500E 10
            0    2   16    2 4.950E 11
            1    2   15    2 4.950E 11
            0    8   16    8 4.950E 11
            1    8   15    8 4.950E 11
            0    2    0    2           1.000E+99
           16    2   16    2           1.000E+99
           16    8   16    8           1.000E+99
            0    8    0    8           1.000E+99
            0    0   16   10-1.000E 03
            1    1   15    9-1.000E 03
            4    1    6    4-5.000E 03
```

```
6.4     LARGE CANTILEVER TYPE BRIDGE
            3    3    6    5         60   25    5 2.000E 01 4.000E 00 1.000E-08
           15    0    7    2   15    3   25    0
            4      7.000E 02 3.500E 03 1.000E 05 3.500E 03
            4      7.000E 02 3.500E 03 1.000E 05 3.500E 03
            4      7.000E 05 3.500E 06 1.000E 07 3.500E 06
            0    0   25    5 1.000E 09
           11    0   11    5                     7.500E 06
           19    0   19    5                     7.500E 06
            0    0   25    5 1.000E 09
           11    0   11    5                     7.500E 06
           19    0   19    5                     7.500E 06
            0    1   25    1 1.000E 12
           11    1   11    1                     7.500E 06
           11    4   11    4                     7.500E 06
            0    4   25    4 1.000E 12
           19    4   19    4                     7.500E 06
           19    1   19    1                     7.500E 06
            0    0    0    5           1.000E 99
           11    0   11    5           1.000E 99
           19    0   19    5           1.000E 99
            0    0   25    5-5.000E+00
           25    0   25    5-2.000E 02
```

```
PROGRAM LAYER 7 - MASTER DECK - WB INGRAM, H MATLOCK     REVISION DATE 27 JUL 65
FINAL RUN ON EXAMPLE PROBLEMS
CHG TO CE 05-1023 --CODED BY WBI-- RUN DATE 27 JL 65


PROB
  6.1     HYPOTHETICAL GRID OVER FIVE BEAMS


TABLE 1.  CONTROL DATA

     NUM CARDS TABLE 3                                                 5
     NUM CARDS TABLE 4                                                 5
     NUM CARDS TABLE 5                                                15
     NUM CARDS TABLE 6                                                 3
     MAX NUM ITERATIONS                                               20
     NUM INCREMENTS  MX                                                8
     NUM INCREMENTS  MY                                                8
     INCR LENGTH  HX                                          6.000E 00
     INCR LENGTH  HY                                          6.000E 00
     CLOSURE TOLERANCE                                        1.000E-06
     MONITOR STAS  I,J              4  4      2  2      4  2      3  2


TABLE 2.  RELAXATION CONTROL DATA
   NUM       C L O S U R E  P A R A M E T E R S
 VALUES

    5      1.070E 03 1.480E 04 7.020E 04 1.850E 05 3.580E 05
    5      1.070E 03 1.480E 04 7.020E 04 1.850E 05 3.580E 05
    5      1.070E 06 1.480E 07 7.020E 07 1.850E 08 3.580E 08

     TIME =     2 MINUTES, 30 AND  6/60 SECONDS


TABLE 3. X-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

     FROM       THRU          FX          TX           RX           PX

      0  0       8  8   2.500E 06           0            0            0
      1  1       7  7   2.500E 06           0            0            0
      1  0       7  8   2.500E 06           0            0            0
      0  1       8  7   2.500E 06           0            0            0
      1  0       8  8           0           0            0    1.000E 04


TABLE 4. Y-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

     FROM       THRU          FY          TY           RY           PY

      0  0       8  8   2.500E 06           0            0            0
      1  1       7  7   2.500E 06           0            0            0
      1  0       7  8   2.500E 06           0            0            0
      0  1       8  7   2.500E 06           0            0            0
      0  1       8  8           0           0            0    1.000E 04


TABLE 5. Z-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
```

```
        FROM        THRU          FZ          TZ           RZ           PZ

         0  0       8  0   5.000E 09           0            0            0
         1  0       7  0   5.000E 09           0            0            0
         0  2       8  2   5.000E 09           0            0            0
         1  2       7  2   5.000E 09           0            0            0
         0  4       8  4   5.000E 09           0            0            0
         1  4       7  4   5.000E 09           0            0            0
         0  6       8  6   5.000E 09           0            0            0
         1  6       7  6   5.000E 09           0            0            0
         0  8       8  8   5.000E 09           0            0            0
         1  8       7  8   5.000E 09           0            0            0
         1  0       8  0           0           0            0    1.000E 04
         1  2       8  2           0           0            0    1.000E 04
         1  4       8  4           0           0            0    1.000E 04
         1  6       8  6           0           0            0    1.000E 04
         1  8       8  8           0           0            0    1.000E 04

   TABLE 6. LOAD AND SUPPORT DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

        FROM        THRU          Q           S

         0  0       8  8  -2.000E 03   1.000E 99
         1  1       7  7  -2.000E 03           0
         1  0       7  8           0  -1.000E 99

        TIME =      2 MINUTES, 35 AND 12/60 SECONDS


   TABLE 7.  MONITOR TALLY AND DEFLS AT 4 STAS

      ITR   FICT        CYC  NOT  NOT         I,J
      NUM   SPRING      NUM  STAB CLOS        4  4          2  2          4  2          3  2

  X     1   1.070E 03    1   63          -3.734E-03  -3.830E-03  -3.734E-03  -3.756E-03
  Y         1.070E 03    1   63    63    -7.779E-03  -7.227E-03  -8.467E-03  -8.147E-03
  Z         1.070E 06    1   35    35    -8.731E-03  -6.680E-03  -9.408E-03  -8.702E-03

  X     2   1.480E 04    2   63          -8.730E-03  -6.673E-03  -9.407E-03  -8.699E-03
  Y         1.480E 04    2   63    63    -8.765E-03  -6.748E-03  -9.448E-03  -8.753E-03
  Z         1.480E 07    2   35    35    -9.535E-03  -7.387E-03  -1.039E-02  -9.614E-03

  X     3   7.020E 04    3   63          -9.535E-03  -7.386E-03  -1.039E-02  -9.614E-03
  Y         7.020E 04    3   63    57    -9.517E-03  -7.385E-03  -1.037E-02  -9.599E-03
  Z         7.020E 07    3   35    35    -8.976E-03  -6.927E-03  -9.719E-03  -8.999E-03

  X     4   1.850E 05    4   63          -8.976E-03  -6.927E-03  -9.719E-03  -8.999E-03
  Y         1.850E 05    4   63    43    -8.979E-03  -6.928E-03  -9.722E-03  -9.001E-03
  Z         1.850E 08    4   35    35    -9.265E-03  -7.132E-03  -1.001E-02  -9.267E-03

  X     5   3.580E 05    5   63          -9.265E-03  -7.132E-03  -1.001E-02  -9.267E-03
  Y         3.580E 05    5   63    28    -9.264E-03  -7.132E-03  -1.001E-02  -9.267E-03
  Z         3.580E 08    5   35    35    -9.189E-03  -7.093E-03  -9.954E-03  -9.216E-03

  X     6   1.070E 03    1   63          -9.139E-03  -7.067E-03  -9.917E-03  -9.181E-03
  Y         1.070E 03    1   63    63    -9.187E-03  -7.091E-03  -9.952E-03  -9.214E-03
  Z         1.070E 06    1   17    35    -9.187E-03  -7.091E-03  -9.952E-03  -9.214E-03

  X     7   1.480E 04    2   63          -9.187E-03  -7.091E-03  -9.952E-03  -9.214E-03
```

```
Y          1.480E 04    2    28    28  -9.187E-03  -7.092E-03  -9.953E-03  -9.214E-03
Z          1.480E 07    2    27    27  -9.196E-03  -7.097E-03  -9.959E-03  -9.220E-03

X     8    7.020E 04    3    55        -9.196E-03  -7.097E-03  -9.959E-03  -9.220E-03
Y          7.020E 04    3    51    28  -9.196E-03  -7.096E-03  -9.959E-03  -9.220E-03
Z          7.020E 07    3    21    21  -9.190E-03  -7.093E-03  -9.955E-03  -9.216E-03

X     9    1.850E 05    4    49        -9.190E-03  -7.093E-03  -9.955E-03  -9.216E-03
Y          1.850E 05    4    49    28  -9.190E-03  -7.093E-03  -9.955E-03  -9.216E-03
Z          1.850E 08    4    17    17  -9.193E-03  -7.095E-03  -9.957E-03  -9.218E-03

X    10    3.580E 05    5    45        -9.193E-03  -7.095E-03  -9.957E-03  -9.218E-03
Y          3.580E 05    5    41     6  -9.193E-03  -7.095E-03  -9.957E-03  -9.218E-03
Z          3.580E 08    5     0     0  -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03

X    11    1.070E 03    1    31        -9.192E-03  -7.094E-03  -9.956E-03  -9.217E-03
Y          1.070E 03    1     0    28  -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03
Z          1.070E 06    1     0     0  -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03

X    12    1.480E 04    2    28        -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03
Y          1.480E 04    2     0    20  -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03
Z          1.480E 07    2     0     0  -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03

X    13    7.020E 04    3    24        -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03
Y          7.020E 04    3     0     0  -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03
Z          7.020E 07    3     0     0  -9.192E-03  -7.094E-03  -9.956E-03  -9.218E-03
```

```
PROGRAM LAYER 7 - MASTER DECK - WB INGRAM, H MATLOCK     REVISION DATE 27 JUL 65
FINAL RUN ON EXAMPLE PROBLEMS
CHG TO CE 05-1023 --CODED BY WBI-- RUN DATE 27 JL 65


PROB (CONTD)
  6.1     HYPOTHETICAL GRID OVER FIVE BEAMS

TABLE 8. DEFLECTION AND ERROR RESULTS -- ITERATION,  13

  I,J    X-DEFL      Y-DEFL      Z-DEFL        REACT    ERROR

   -1 -1           0           0           0           0           0
    0 -1           0  -2.543E-95           0           0           0
    1 -1           0   1.210E-02           0           0           0
    2 -1           0   1.515E-02           0  -1.053E-08           0
    3 -1           0   1.503E-02           0           0           0
    4 -1           0   1.479E-02           0  -1.053E-08           0
    5 -1           0   1.503E-02           0           0           0
    6 -1           0   1.515E-02           0           0           0
    7 -1           0   1.210E-02           0           0           0
    8 -1           0  -2.543E-95           0   1.262-102           0
    9 -1           0           0           0           0           0

   -1  0   1.565E-03           0   1.565E-03  -1.315E-06           0
    0  0  -1.395E-95  -1.395E-95  -1.395E-95   1.195E 04   1.164E-02
    1  0  -1.565E-03  -1.565E-03  -1.565E-03  -2.000E 03   3.422E-04
    2  0  -2.871E-03  -2.871E-03  -2.871E-03  -2.000E 03   6.407E-04
    3  0  -3.731E-03  -3.731E-03  -3.731E-03  -2.000E 03   8.149E-04
    4  0  -4.030E-03  -4.030E-03  -4.030E-03  -2.000E 03   9.001E-04
    5  0  -3.731E-03  -3.731E-03  -3.731E-03  -2.000E 03   8.014E-04
    6  0  -2.871E-03  -2.871E-03  -2.871E-03  -2.000E 03   6.257E-04
    7  0  -1.565E-03  -1.565E-03  -1.565E-03  -2.000E 03   3.380E-04
    8  0  -1.395E-95  -1.395E-95  -1.395E-95   1.195E 04   1.164E-02
    9  0   1.565E-03           0   1.565E-03  -6.582E-07           0

   -1  1   1.523E-02           0           0           0           0
    0  1  -2.468E-96  -2.468E-96           0   4.682E 02  -7.451E-09
    1  1  -1.523E-02  -1.523E-02           0  -4.000E 03  -4.961E-03
    2  1  -2.089E-02  -2.089E-02           0  -4.000E 03  -9.175E-03
    3  1  -2.249E-02  -2.249E-02           0  -4.000E 03  -1.198E-02
    4  1  -2.285E-02  -2.285E-02           0  -4.000E 03  -1.298E-02
    5  1  -2.249E-02  -2.249E-02           0  -4.000E 03  -1.198E-02
    6  1  -2.089E-02  -2.089E-02           0  -4.000E 03  -9.175E-03
    7  1  -1.523E-02  -1.523E-02           0  -4.000E 03  -4.961E-03
    8  1  -2.468E-96  -2.468E-96           0   4.682E 02  -7.451E-08
    9  1   1.523E-02           0           0           0           0

   -1  2   3.866E-03           0   3.866E-03  -2.632E-06           0
    0  2  -3.155E-95  -3.155E-95  -3.155E-95   2.955E 04   7.565E-02
    1  2  -3.866E-03  -3.866E-03  -3.866E-03  -4.000E 03   2.240E-03
    2  2  -7.094E-03  -7.094E-03  -7.094E-03  -4.000E 03   4.084E-03
    3  2  -9.218E-03  -9.218E-03  -9.218E-03  -4.000E 03   5.448E-03
    4  2  -9.956E-03  -9.956E-03  -9.956E-03  -4.000E 03   5.848E-03
    5  2  -9.218E-03  -9.218E-03  -9.218E-03  -4.000E 03   5.299E-03
    6  2  -7.094E-03  -7.094E-03  -7.094E-03  -4.000E 03   4.087E-03
    7  2  -3.866E-03  -3.866E-03  -3.866E-03  -4.000E 03   2.259E-03
    8  2  -3.155E-95  -3.155E-95  -3.155E-95   2.955E 04   7.564E-02
    9  2   3.866E-03           0   3.866E-03  -1.317E-06           0
```

```
 -1   3   1.254E-02           0           0   5.263E-09           0
  0   3  -2.374E-96  -2.374E-96           0   3.738E 02   6.706E-08
  1   3  -1.254E-02  -1.254E-02           0  -4.000E 03  -1.127E-02
  2   3  -1.745E-02  -1.745E-02           0  -4.000E 03  -2.083E-02
  3   3  -1.936E-02  -1.936E-02           0  -4.000E 03  -2.722E-02
  4   3  -1.995E-02  -1.995E-02           0  -4.000E 03  -2.946E-02
  5   3  -1.936E-02  -1.936E-02           0  -4.000E 03  -2.722E-02
  6   3  -1.745E-02  -1.745E-02           0  -4.000E 03  -2.083E-02
  7   3  -1.254E-02  -1.254E-02           0  -4.000E 03  -1.127E-02
  8   3  -2.374E-96  -2.374E-96           0   3.738E 02  -3.725E-08
  9   3   1.254E-02           0           0           0           0

 -1   4   3.570E-03           0   3.570E-03  -2.630E-06           0
  0   4  -2.931E-95  -2.931E-95  -2.931E-95   2.731E 04   1.041E-01
  1   4  -3.570E-03  -3.570E-03  -3.570E-03  -4.000E 03   3.095E-03
  2   4  -6.550E-03  -6.550E-03  -6.550E-03  -4.000E 03   5.617E-03
  3   4  -8.510E-03  -8.510E-03  -8.510E-03  -4.000E 03   7.501E-03
  4   4  -9.192E-03  -9.192E-03  -9.192E-03  -4.000E 03   7.986E-03
  5   4  -8.510E-03  -8.510E-03  -8.510E-03  -4.000E 03   7.423E-03
  6   4  -6.550E-03  -6.550E-03  -6.550E-03  -4.000E 03   5.635E-03
  7   4  -3.570E-03  -3.570E-03  -3.570E-03  -4.000E 03   3.073E-03
  8   4  -2.931E-95  -2.931E-95  -2.931E-95   2.731E 04   1.041E-01
  9   4   3.570E-03           0   3.570E-03  -1.316E-06           0

 -1   5   1.254E-02           0           0   5.263E-09           0
  0   5  -2.374E-96  -2.374E-96           0   3.738E 02   4.470E-08
  1   5  -1.254E-02  -1.254E-02           0  -4.000E 03  -1.127E-02
  2   5  -1.745E-02  -1.745E-02           0  -4.000E 03  -2.083E-02
  3   5  -1.936E-02  -1.936E-02           0  -4.000E 03  -2.722E-02
  4   5  -1.995E-02  -1.995E-02           0  -4.000E 03  -2.946E-02
  5   5  -1.936E-02  -1.936E-02           0  -4.000E 03  -2.722E-02
  6   5  -1.745E-02  -1.745E-02           0  -4.000E 03  -2.083E-02
  7   5  -1.254E-02  -1.254E-02           0  -4.000E 03  -1.127E-02
  8   5  -2.374E-96  -2.374E-96           0   3.738E 02   7.451E-09
  9   5   1.254E-02           0           0           0           0

 -1   6   3.866E-03           0   3.866E-03  -2.632E-06           0
  0   6  -3.155E-95  -3.155E-95  -3.155E-95   2.955E 04   7.566E-02
  1   6  -3.866E-03  -3.866E-03  -3.866E-03  -4.000E 03   2.218E-03
  2   6  -7.094E-03  -7.094E-03  -7.094E-03  -4.000E 03   4.087E-03
  3   6  -9.218E-03  -9.218E-03  -9.218E-03  -4.000E 03   5.417E-03
  4   6  -9.956E-03  -9.956E-03  -9.956E-03  -4.000E 03   5.867E-03
  5   6  -9.218E-03  -9.218E-03  -9.218E-03  -4.000E 03   5.316E-03
  6   6  -7.094E-03  -7.094E-03  -7.094E-03  -4.000E 03   4.102E-03
  7   6  -3.866E-03  -3.866E-03  -3.866E-03  -4.000E 03   2.233E-03
  8   6  -3.155E-95  -3.155E-95  -3.155E-95   2.955E 04   7.564E-02
  9   6   3.866E-03           0   3.866E-03  -1.317E-06           0

 -1   7   1.523E-02           0           0           0           0
  0   7  -2.468E-96  -2.468E-96           0   4.682E 02  -1.490E-08
  1   7  -1.523E-02  -1.523E-02           0  -4.000E 03  -4.961E-03
  2   7  -2.089E-02  -2.089E-02           0  -4.000E 03  -9.175E-03
  3   7  -2.249E-02  -2.249E-02           0  -4.000E 03  -1.198E-02
  4   7  -2.285E-02  -2.285E-02           0  -4.000E 03  -1.298E-02
  5   7  -2.249E-02  -2.249E-02           0  -4.000E 03  -1.198E-02
  6   7  -2.089E-02  -2.089E-02           0  -4.000E 03  -9.175E-03
  7   7  -1.523E-02  -1.523E-02           0  -4.000E 03  -4.961E-03
  8   7  -2.468E-96  -2.468E-96           0   4.682E 02  -1.118E-07
  9   7   1.523E-02           0           0           0           0
```

```
  -1  8   1.565E-03           0   1.565E-03  -1.315E-06           0
   0  8  -1.395E-95  -1.395E-95  -1.395E-95   1.195E 04   1.164E-02
   1  8  -1.565E-03  -1.565E-03  -1.565E-03  -2.000E 03   3.422E-04
   2  8  -2.871E-03  -2.871E-03  -2.871E-03  -2.000E 03   6.406E-04
   3  8  -3.731E-03  -3.731E-03  -3.731E-03  -2.000E 03   8.149E-04
   4  8  -4.030E-03  -4.030E-03  -4.030E-03  -2.000E 03   9.001E-04
   5  8  -3.731E-03  -3.731E-03  -3.731E-03  -2.000E 03   8.014E-04
   6  8  -2.871E-03  -2.871E-03  -2.871E-03  -2.000E 03   6.257E-04
   7  8  -1.565E-03  -1.565E-03  -1.565E-03  -2.000E 03   3.380E-04
   8  8  -1.395E-95  -1.395E-95  -1.395E-95   1.195E 04   1.164E-02
   9  8   1.565E-03           0   1.565E-03  -6.582E-07           0

  -1  9           0           0           0           0           0
   0  9           0  -2.543E-95           0   5.046-102           0
   1  9           0   1.210E-02           0  -1.053E-08           0
   2  9           0   1.515E-02           0  -2.105E-08           0
   3  9           0   1.503E-02           0  -2.105E-08           0
   4  9           0   1.479E-02           0           0           0
   5  9           0   1.503E-02           0  -2.105E-08           0
   6  9           0   1.515E-02           0  -2.105E-08           0
   7  9           0   1.210E-02           0  -1.053E-08           0
   8  9           0  -2.543E-95           0  -5.046-102           0
   9  9           0           0           0           0           0

TABLE 9. MOMENTS -- ITERATION  13

  I,J     X-MOM       Y-MOM       Z-MOM

  -1 -1           0           0           0
   0 -1           0           0           0
   1 -1           0           0           0
   2 -1           0           0           0
   3 -1           0           0           0
   4 -1           0           0           0
   5 -1           0           0           0
   6 -1           0           0           0
   7 -1           0           0           0
   8 -1           0           0           0
   9 -1           0           0           0

  -1  0           0           0           0
   0  0   1.974E-09           0  -7.895E-06
   1  0   3.581E 01           0   7.162E 04
   2  0   6.209E 01  -6.316E-08   1.242E 05
   3  0   7.786E 01           0   1.557E 05
   4  0   8.311E 01  -6.316E-08   1.662E 05
   5  0   7.786E 01           0   1.557E 05
   6  0   6.209E 01           0   1.242E 05
   7  0   3.581E 01           0   7.162E 04
   8  0  -1.974E-09   7.570-102  -3.947E-06
   9  0           0           0           0

  -1  1           0           0           0
   0  1           0  -5.634E-90           0
   1  1   2.657E 03   6.952E 03           0
   2  1   1.129E 03   8.838E 03           0
   3  1   3.456E 02   8.898E 03           0
   4  1   1.974E 02   8.808E 03           0
   5  1   3.456E 02   8.898E 03           0
   6  1   1.129E 03   8.838E 03           0
   7  1   2.657E 03   6.952E 03           0
```

```
   8   1           0  -5.634E-90           0
   9   1           0           0           0

  -1   2           0           0           0
   0   2           0   8.092E-90  -1.579E-05
   1   2   1.771E 02  -5.565E 03   1.771E 05
   2   2   3.070E 02  -6.709E 03   3.070E 05
   3   2   3.846E 02  -6.505E 03   3.846E 05
   4   2   4.103E 02  -6.357E 03   4.103E 05
   5   2   3.846E 02  -6.505E 03   3.846E 05
   6   2   3.070E 02  -6.709E 03   3.070E 05
   7   2   1.771E 02  -5.565E 03   1.771E 05
   8   2  -7.895E-09   8.092E-90  -7.895E-06
   9   2           0           0           0

  -1   3           0           0           0
   0   3   3.158E-08  -7.794E-90           0
   1   3   2.118E 03   4.900E 03           0
   2   3   8.338E 02   5.905E 03           0
   3   3   3.680E 02   5.833E 03           0
   4   3   3.264E 02   5.765E 03           0
   5   3   3.680E 02   5.833E 03           0
   6   3   8.338E 02   5.905E 03           0
   7   3   2.118E 03   4.900E 03           0
   8   3           0  -7.794E-90           0
   9   3           0           0           0

  -1   4           0           0           0
   0   4   7.895E-09   7.483E-90  -1.579E-05
   1   4   1.636E 02  -4.982E 03   1.636E 05
   2   4   2.834E 02  -6.056E 03   2.834E 05
   3   4   3.551E 02  -6.029E 03   3.551E 05
   4   4   3.789E 02  -5.977E 03   3.789E 05
   5   4   3.551E 02  -6.029E 03   3.551E 05
   6   4   2.834E 02  -6.056E 03   2.834E 05
   7   4   1.636E 02  -4.982E 03   1.636E 05
   8   4           0   7.483E-90  -7.895E-06
   9   4           0           0           0

  -1   5           0           0           0
   0   5   3.158E-08  -7.794E-90           0
   1   5   2.118E 03   4.900E 03           0
   2   5   8.338E 02   5.905E 03           0
   3   5   3.680E 02   5.833E 03           0
   4   5   3.264E 02   5.765E 03           0
   5   5   3.680E 02   5.833E 03           0
   6   5   8.338E 02   5.905E 03           0
   7   5   2.118E 03   4.900E 03           0
   8   5           0  -7.794E-90           0
   9   5           0           0           0

  -1   6           0           0           0
   0   6           0   8.092E-90  -1.579E-05
   1   6   1.771E 02  -5.565E 03   1.771E 05
   2   6   3.070E 02  -6.709E 03   3.070E 05
   3   6   3.846E 02  -6.505E 03   3.846E 05
   4   6   4.103E 02  -6.357E 03   4.103E 05
   5   6   3.846E 02  -6.505E 03   3.846E 05
   6   6   3.070E 02  -6.709E 03   3.070E 05
   7   6   1.771E 02  -5.565E 03   1.771E 05
   8   6  -7.895E-09   8.092E-90  -7.895E-06
```

```
  9   6           0           0           0

 -1   7           0           0           0
  0   7           0  -5.634E-90           0
  1   7   2.657E 03   6.952E 03           0
  2   7   1.129E 03   8.838E 03           0
  3   7   3.456E 02   8.898E 03           0
  4   7   1.974E 02   8.808E 03           0
  5   7   3.456E 02   8.898E 03           0
  6   7   1.129E 03   8.838E 03           0
  7   7   2.657E 03   6.952E 03           0
  8   7           0  -5.634E-90           0
  9   7           0           0           0

 -1   8           0           0           0
  0   8   1.974E-09   3.028-101  -7.895E-06
  1   8   3.581E 01  -6.316E-08   7.162E 04
  2   8   6.209E 01  -1.263E-07   1.242E 05
  3   8   7.786E 01  -1.263E-07   1.557E 05
  4   8   8.311E 01           0   1.662E 05
  5   8   7.786E 01  -1.263E-07   1.557E 05
  6   8   6.209E 01  -1.263E-07   1.242E 05
  7   8   3.581E 01  -6.316E-08   7.162E 04
  8   8  -1.974E-09  -3.028-101  -3.947E-06
  9   8           0           0           0

 -1   9           0           0           0
  0   9           0           0           0
  1   9           0           0           0
  2   9           0           0           0
  3   9           0           0           0
  4   9           0           0           0
  5   9           0           0           0
  6   9           0           0           0
  7   9           0           0           0
  8   9           0           0           0
  9   9           0           0           0

   TIME =      2 MINUTES, 56 AND 59/60 SECONDS
```

```
PROGRAM LAYER 7 - MASTER DECK - WB INGRAM, H MATLOCK    REVISION DATE 27 JUL 65
FINAL RUN ON EXAMPLE PROBLEMS
CHG TO CE 05-1023 --CODED BY WBI-- RUN DATE 27 JL 65


PROB
  6.3     GIRDER-DIAPHRAGM SYSTEM OF A BRIDGE


TABLE 1.  CONTROL DATA

     NUM CARDS TABLE 3                                               2
     NUM CARDS TABLE 4                                              10
     NUM CARDS TABLE 5                                               4
     NUM CARDS TABLE 6                                               7
     MAX NUM ITERATIONS                                             40
     NUM INCREMENTS  MX                                             16
     NUM INCREMENTS  MY                                             10
     INCR LENGTH  HX                                         6.000E 01
     INCR LENGTH  HY                                         3.000E 01
     CLOSURE TOLERANCE                                       1.000E-04
     MONITOR STAS  I,J              8  6      8  2      4  2      8  0


TABLE 2.  RELAXATION CONTROL DATA
   NUM       C L O S U R E   P A R A M E T E R S
 VALUES

    4      5.400E 01 2.700E 02 1.350E 03 6.750E 03
    4      2.700E 02 1.350E 03 6.750E 03 3.375E 04
    4      5.400E 03 2.700E 04 1.350E 05 6.750E 05

     TIME =     2 MINUTES, 58 AND  4/60 SECONDS


TABLE 3. X-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

     FROM      THRU         FX         TX          RX          PX

      0  0     16 10    5.000E 09           0           0           0
      1  1     15  9    5.000E 09           0           0           0


TABLE 4. Y-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

     FROM      THRU         FY         TY          RY          PY

      0  0      0 10    2.500E 10           0           0           0
      0  1      0  9    2.500E 10           0           0           0
      4  0      4 10    2.500E 10           0           0           0
      4  1      4  9    2.500E 10           0           0           0
      8  0      8 10    2.500E 10           0           0           0
      8  1      8  9    2.500E 10           0           0           0
     12  0     12 10    2.500E 10           0           0           0
     12  1     12  9    2.500E 10           0           0           0
     16  0     16 10    2.500E 10           0           0           0
     16  1     16  9    2.500E 10           0           0           0
```

```
TABLE 5. Z-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

     FROM        THRU          FZ          TZ          RZ          PZ

      0  2      16  2   4.950E 11           0           0           0
      1  2      15  2   4.950E 11           0           0           0
      0  8      16  8   4.950E 11           0           0           0
      1  8      15  8   4.950E 11           0           0           0

TABLE 6. LOAD AND SUPPORT DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

     FROM        THRU          Q           S

      0  2       0  2           0   1.000E 99
     16  2      16  2           0   1.000E 99
     16  8      16  8           0   1.000E 99
      0  8       0  8           0   1.000E 99
      0  0      16 10  -1.000E 03           0
      1  1      15  9  -1.000E 03           0
      4  1       6  4  -5.000E 03           0

     TIME =      3 MINUTES,  2 AND 16/60 SECONDS


TABLE 7.  MONITOR TALLY AND DEFLS AT 4 STAS

    ITR   FICT        CYC  NOT  NOT        I,J
    NUM   SPRING      NUM  STAB CLOS      8  6          8  2          4  2          8  0

X     1  5.400E 01    1   51        -2.686E 01   -6.050E-01   -8.644E-01   -1.452E 01
Y        2.700E 02    1   51   51   -7.589E 00   -8.962E 00   -9.330E 00   -9.648E 00
Z        5.400E 03    1   30   30            0   -2.584E 00   -1.858E 00            0

X     2  2.700E 02    2   51        -7.468E 00   -2.632E 00   -1.982E 00   -9.967E 00
Y        1.350E 03    2   51   51   -2.223E 00   -2.672E 00   -1.451E 00   -2.906E 00
Z        2.700E 04    2   30   30            0   -2.575E 00   -1.846E 00            0

X     3  1.350E 03    3   51        -1.976E 00   -2.583E 00   -1.854E 00   -2.462E 00
Y        6.750E 03    3   51   51   -2.076E 00   -2.501E 00   -1.976E 00   -2.709E 00
Z        1.350E 05    3   30   30            0   -2.531E 00   -1.845E 00            0

X     4  6.750E 03    4   51        -2.087E 00   -2.532E 00   -1.852E 00   -2.735E 00
Y        3.375E 04    4   48   51   -2.099E 00   -2.534E 00   -1.859E 00   -2.751E 00
Z        6.750E 05    4   30   30            0   -2.561E 00   -1.878E 00            0

X     5  5.400E 01    1   51        -1.313E 00   -2.643E 00   -1.949E 00   -1.743E 00
Y        2.700E 02    1   48   51   -1.988E 00   -2.405E 00   -1.667E 00   -2.612E 00
Z        5.400E 03    1   30   30            0   -2.566E 00   -1.881E 00            0

X     6  2.700E 02    2   51        -1.965E 00   -2.568E 00   -1.884E 00   -2.594E 00
Y        1.350E 03    2   41   51   -2.122E 00   -2.561E 00   -1.911E 00   -2.779E 00
Z        2.700E 04    2   30   30            0   -2.569E 00   -1.885E 00            0

X     7  1.350E 03    3   51        -2.110E 00   -2.572E 00   -1.888E 00   -2.786E 00
Y        6.750E 03    3   39   51   -2.126E 00   -2.568E 00   -1.888E 00   -2.788E 00
Z        1.350E 05    3   30   28            0   -2.571E 00   -1.885E 00            0

X     8  6.750E 03    4   49        -2.120E 00   -2.572E 00   -1.887E 00   -2.786E 00
Y        3.375E 04    4   47   51   -2.126E 00   -2.570E 00   -1.888E 00   -2.790E 00
```

```
Z       6.750E 05   4   29   26           0  -2.570E 00  -1.886E 00           0

X   9   5.400E 01   1   50       -2.119E 00  -2.571E 00  -1.884E 00  -2.790E 00
Y       2.700E 02   1   33   50  -2.125E 00  -2.569E 00  -1.887E 00  -2.790E 00
Z       5.400E 03   1   30   24           0  -2.569E 00  -1.885E 00           0

X  10   2.700E 02   2   51       -2.120E 00  -2.570E 00  -1.885E 00  -2.788E 00
Y       1.350E 03   2   32   50  -2.124E 00  -2.568E 00  -1.886E 00  -2.789E 00
Z       2.700E 04   2    0   26           0  -2.569E 00  -1.885E 00           0

X  11   1.350E 03   3   42       -2.122E 00  -2.569E 00  -1.885E 00  -2.788E 00
Y       6.750E 03   3   23   50  -2.124E 00  -2.568E 00  -1.886E 00  -2.789E 00
Z       1.350E 05   3   13   20           0  -2.569E 00  -1.885E 00           0

X  12   6.750E 03   4   48       -2.123E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       3.375E 04   4   31   47  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       6.750E 05   4   17   17           0  -2.569E 00  -1.885E 00           0

X  13   5.400E 01   1   48       -2.120E 00  -2.569E 00  -1.885E 00  -2.785E 00
Y       2.700E 02   1   33   51  -2.124E 00  -2.568E 00  -1.884E 00  -2.789E 00
Z       5.400E 03   1   24   18           0  -2.569E 00  -1.885E 00           0

X  14   2.700E 02   2   48       -2.123E 00  -2.569E 00  -1.885E 00  -2.788E 00
Y       1.350E 03   2   33   42  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       2.700E 04   2    0   20           0  -2.569E 00  -1.885E 00           0

X  15   1.350E 03   3   33       -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       6.750E 03   3    8   37  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       1.350E 05   3    0   16           0  -2.569E 00  -1.885E 00           0

X  16   6.750E 03   4   26       -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       3.375E 04   4    7   25  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       6.750E 05   4    0    5           0  -2.569E 00  -1.885E 00           0

X  17   5.400E 01   1   45       -2.123E 00  -2.569E 00  -1.885E 00  -2.788E 00
Y       2.700E 02   1   33   51  -2.124E 00  -2.568E 00  -1.885E 00  -2.789E 00
Z       5.400E 03   1    0    8           0  -2.569E 00  -1.885E 00           0

X  18   2.700E 02   2   46       -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       1.350E 03   2   32   11  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       2.700E 04   2    0    3           0  -2.569E 00  -1.885E 00           0

X  19   1.350E 03   3   17       -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       6.750E 03   3    0   13  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       1.350E 05   3    0    1           0  -2.569E 00  -1.885E 00           0

X  20   6.750E 03   4    5       -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       3.375E 04   4    0    1  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       6.750E 05   4    0    0           0  -2.569E 00  -1.885E 00           0

X  21   5.400E 01   1   45       -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       2.700E 02   1    6   47  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       5.400E 03   1    0    0           0  -2.569E 00  -1.885E 00           0

X  22   2.700E 02   2   45       -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       1.350E 03   2    5    5  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       2.700E 04   2    0    0           0  -2.569E 00  -1.885E 00           0

X  23   1.350E 03   3    2       -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Y       6.750E 03   3    0    0  -2.124E 00  -2.569E 00  -1.885E 00  -2.789E 00
Z       1.350E 05   3    0    0           0  -2.569E 00  -1.885E 00           0
```

```
PROGRAM LAYER 7 - MASTER DECK - WB INGRAM, H MATLOCK    REVISION DATE 27 JUL 65
FINAL RUN ON EXAMPLE PROBLEMS
CHG TO CE 05-1023 --CODED BY WBI-- RUN DATE 27 JL 65


PROB (CONTD)
  6.3     GIRDER-DIAPHRAGM SYSTEM OF A BRIDGE

TABLE 8. DEFLECTION AND ERROR RESULTS -- ITERATION,  23

  I,J     X-DEFL       Y-DEFL       Z-DEFL        REACT    ERROR

  -1 -1         0         0         0         0         0
   0 -1         0 1.563E-03         0-5.263E-08         0
   1 -1         0         0         0         0         0
   2 -1         0         0         0         0         0
   3 -1         0         0         0         0         0
   4 -1         0-2.125E 00         0 1.078E-04         0
   5 -1         0         0         0         0         0
   6 -1         0         0         0         0         0
   7 -1         0         0         0         0         0
   8 -1         0-2.901E 00         0 1.078E-04         0
   9 -1         0         0         0         0         0
  10 -1         0         0         0         0         0
  11 -1         0         0         0         0         0
  12 -1         0-1.992E 00         0 5.390E-05         0
  13 -1         0         0         0         0         0
  14 -1         0         0         0         0         0
  15 -1         0         0         0         0         0
  16 -1         0 2.781E-03         0-1.053E-07         0
  17 -1         0         0         0         0         0

  -1  0 6.370E-01         0         0-3.368E-07         0
   0  0 1.513E-03 1.519E-03         0-1.000E 03 7.458E-03
   1  0-6.340E-01         0         0-1.000E 03         0
   2  0-1.198E 00         0         0-1.000E 03         0
   3  0-1.663E 00         0         0-1.000E 03         0
   4  0-2.044E 00-2.044E 00         0-1.000E 03 2.100E-02
   5  0-2.400E 00         0         0-1.000E 03         0
   6  0-2.655E 00         0         0-1.000E 03         0
   7  0-2.781E 00         0         0-1.000E 03         0
   8  0-2.789E 00-2.789E 00         0-1.000E 03-1.191E-03
   9  0-2.736E 00         0         0-1.000E 03         0
  10  0-2.570E 00         0         0-1.000E 03         0
  11  0-2.286E 00         0         0-1.000E 03         0
  12  0-1.919E 00-1.919E 00         0-1.000E 03 1.532E-02
  13  0-1.550E 00         0         0-1.000E 03         0
  14  0-1.113E 00         0         0-1.000E 03         0
  15  0-5.880E-01         0         0-1.000E 03         0
  16  0 2.309E-03 2.307E-03         0-1.000E 03-1.636E-03
  17  0 5.926E-01         0         0-6.737E-07         0

  -1  1 5.761E-01         0         0-3.368E-07         0
   0  1 1.476E-03 1.474E-03         0-1.000E 03-2.651E-03
   1  1-5.731E-01         0         0-2.000E 03         0
   2  1-1.093E 00         0         0-2.000E 03         0
   3  1-1.546E 00         0         0-2.000E 03         0
   4  1-1.964E 00-1.963E 00         0-7.000E 03 5.830E-02
   5  1-2.421E 00         0         0-7.000E 03         0
```

```
  6  1  -2.707E 00          0          0  -7.000E 03          0
  7  1  -2.762E 00          0          0  -2.000E 03          0
  8  1  -2.678E 00 -2.678E 00          0  -2.000E 03 -5.855E-02
  9  1  -2.590E 00          0          0  -2.000E 03          0
 10  1  -2.431E 00          0          0  -2.000E 03          0
 11  1  -2.177E 00          0          0  -2.000E 03          0
 12  1  -1.847E 00 -1.847E 00          0  -2.000E 03  5.045E-02
 13  1  -1.504E 00          0          0  -2.000E 03          0
 14  1  -1.087E 00          0          0  -2.000E 03          0
 15  1  -5.765E-01          0          0  -2.000E 03          0
 16  1   1.840E-03  1.833E-03          0  -1.000E 03 -9.235E-03
 17  1   5.801E-01          0          0   3.368E-07          0

 -1  2   5.256E-01          0  5.256E-01  -3.368E-07          0
  0  2  -1.183E-94 -1.183E-94 -1.183E-94   1.173E 05 -9.282E-02
  1  2  -5.256E-01          0 -5.256E-01  -2.000E 03          0
  2  2  -1.029E 00          0 -1.029E 00  -2.000E 03          0
  3  2  -1.489E 00          0 -1.489E 00  -2.000E 03          0
  4  2  -1.885E 00 -1.885E 00 -1.885E 00  -7.000E 03  2.895E-01
  5  2  -2.195E 00          0 -2.195E 00  -7.000E 03          0
  6  2  -2.415E 00          0 -2.415E 00  -7.000E 03          0
  7  2  -2.540E 00          0 -2.540E 00  -2.000E 03          0
  8  2  -2.569E 00 -2.569E 00 -2.569E 00  -2.000E 03  1.449E-01
  9  2  -2.499E 00          0 -2.499E 00  -2.000E 03          0
 10  2  -2.339E 00          0 -2.339E 00  -2.000E 03          0
 11  2  -2.095E 00          0 -2.095E 00  -2.000E 03          0
 12  2  -1.777E 00 -1.777E 00 -1.777E 00  -2.000E 03  1.355E-01
 13  2  -1.392E 00          0 -1.392E 00  -2.000E 03          0
 14  2  -9.562E-01          0 -9.562E-01  -2.000E 03          0
 15  2  -4.867E-01          0 -4.867E-01  -2.000E 03          0
 16  2  -9.769E-95 -9.769E-95 -9.769E-95   9.669E 04 -1.730E-01
 17  2   4.867E-01          0  4.867E-01  -1.684E-05          0

 -1  3   5.279E-01          0          0  -3.368E-07          0
  0  3  -6.248E-03 -6.248E-03          0  -1.000E 03 -7.533E-04
  1  3  -5.404E-01          0          0  -2.000E 03          0
  2  3  -1.021E 00          0          0  -2.000E 03          0
  3  3  -1.437E 00          0          0  -2.000E 03          0
  4  3  -1.822E 00 -1.822E 00          0  -7.000E 03  4.386E-02
  5  3  -2.251E 00          0          0  -7.000E 03          0
  6  3  -2.515E 00          0          0  -7.000E 03          0
  7  3  -2.557E 00          0          0  -2.000E 03          0
  8  3  -2.469E 00 -2.469E 00          0  -2.000E 03 -4.232E-02
  9  3  -2.388E 00          0          0  -2.000E 03          0
 10  3  -2.246E 00          0          0  -2.000E 03          0
 11  3  -2.016E 00          0          0  -2.000E 03          0
 12  3  -1.715E 00 -1.715E 00          0  -2.000E 03  3.862E-02
 13  3  -1.405E 00          0          0  -2.000E 03          0
 14  3  -1.022E 00          0          0  -2.000E 03          0
 15  3  -5.481E-01          0          0  -2.000E 03          0
 16  3  -6.789E-03 -6.794E-03          0  -1.000E 03 -5.688E-03
 17  3   5.345E-01          0          0   3.368E-07          0

 -1  4   5.014E-01          0          0  -3.368E-07          0
  0  4  -1.223E-02 -1.223E-02          0  -1.000E 03  3.897E-04
  1  4  -5.259E-01          0          0  -2.000E 03          0
  2  4  -9.864E-01          0          0  -2.000E 03          0
  3  4  -1.384E 00          0          0  -2.000E 03          0
  4  4  -1.752E 00 -1.752E 00          0  -7.000E 03  3.593E-02
  5  4  -2.167E 00          0          0  -7.000E 03          0
  6  4  -2.420E 00          0          0  -7.000E 03          0
```

```
  7  4  -2.455E 00           0           0  -2.000E 03           0
  8  4  -2.366E 00  -2.366E 00           0  -2.000E 03  -3.175E-02
  9  4  -2.289E 00           0           0  -2.000E 03           0
 10  4  -2.156E 00           0           0  -2.000E 03           0
 11  4  -1.938E 00           0           0  -2.000E 03           0
 12  4  -1.652E 00  -1.652E 00           0  -2.000E 03   3.161E-02
 13  4  -1.358E 00           0           0  -2.000E 03           0
 14  4  -9.922E-01           0           0  -2.000E 03           0
 15  4  -5.361E-01           0           0  -2.000E 03           0
 16  4  -1.305E-02  -1.305E-02           0  -1.000E 03  -3.631E-03
 17  4   5.100E-01           0           0           0           0

 -1  5   5.122E-01           0           0  -3.368E-07           0
  0  5  -1.478E-02  -1.478E-02           0  -1.000E 03   1.581E-03
  1  5  -5.418E-01           0           0  -2.000E 03           0
  2  5  -1.001E 00           0           0  -2.000E 03           0
  3  5  -1.369E 00           0           0  -2.000E 03           0
  4  5  -1.665E 00  -1.665E 00           0  -2.000E 03   2.827E-02
  5  5  -1.950E 00           0           0  -2.000E 03           0
  6  5  -2.154E 00           0           0  -2.000E 03           0
  7  5  -2.249E 00           0           0  -2.000E 03           0
  8  5  -2.251E 00  -2.251E 00           0  -2.000E 03  -2.216E-02
  9  5  -2.218E 00           0           0  -2.000E 03           0
 10  5  -2.096E 00           0           0  -2.000E 03           0
 11  5  -1.873E 00           0           0  -2.000E 03           0
 12  5  -1.580E 00  -1.580E 00           0  -2.000E 03   2.446E-02
 13  5  -1.293E 00           0           0  -2.000E 03           0
 14  5  -9.439E-01           0           0  -2.000E 03           0
 15  5  -5.115E-01           0           0  -2.000E 03           0
 16  5  -1.550E-02  -1.550E-02           0  -1.000E 03  -1.471E-03
 17  5   4.805E-01           0           0           0           0

 -1  6   4.849E-01           0           0  -3.368E-07           0
  0  6  -1.260E-02  -1.260E-02           0  -1.000E 03   6.487E-04
  1  6  -5.101E-01           0           0  -2.000E 03           0
  2  6  -9.417E-01           0           0  -2.000E 03           0
  3  6  -1.285E 00           0           0  -2.000E 03           0
  4  6  -1.560E 00  -1.560E 00           0  -2.000E 03   2.720E-02
  5  6  -1.832E 00           0           0  -2.000E 03           0
  6  6  -2.028E 00           0           0  -2.000E 03           0
  7  6  -2.120E 00           0           0  -2.000E 03           0
  8  6  -2.124E 00  -2.124E 00           0  -2.000E 03  -2.260E-02
  9  6  -2.098E 00           0           0  -2.000E 03           0
 10  6  -1.985E 00           0           0  -2.000E 03           0
 11  6  -1.775E 00           0           0  -2.000E 03           0
 12  6  -1.498E 00  -1.497E 00           0  -2.000E 03   2.496E-02
 13  6  -1.228E 00           0           0  -2.000E 03           0
 14  6  -8.990E-01           0           0  -2.000E 03           0
 15  6  -4.875E-01           0           0  -2.000E 03           0
 16  6  -1.308E-02  -1.308E-02           0  -1.000E 03  -1.473E-03
 17  6   4.613E-01           0           0           0           0

 -1  7   4.582E-01           0           0  -3.368E-07           0
  0  7  -6.639E-03  -6.639E-03           0  -1.000E 03  -1.168E-03
  1  7  -4.715E-01           0           0  -2.000E 03           0
  2  7  -8.725E-01           0           0  -2.000E 03           0
  3  7  -1.189E 00           0           0  -2.000E 03           0
  4  7  -1.443E 00  -1.443E 00           0  -2.000E 03   2.974E-02
  5  7  -1.701E 00           0           0  -2.000E 03           0
  6  7  -1.890E 00           0           0  -2.000E 03           0
  7  7  -1.981E 00           0           0  -2.000E 03           0
```

```
  8  7  -1.988E 00  -1.988E 00           0  -2.000E 03  -2.770E-02
  9  7  -1.967E 00           0           0  -2.000E 03           0
 10  7  -1.865E 00           0           0  -2.000E 03           0
 11  7  -1.667E 00           0           0  -2.000E 03           0
 12  7  -1.406E 00  -1.406E 00           0  -2.000E 03   2.768E-02
 13  7  -1.155E 00           0           0  -2.000E 03           0
 14  7  -8.472E-01           0           0  -2.000E 03           0
 15  7  -4.581E-01           0           0  -2.000E 03           0
 16  7  -6.849E-03  -6.851E-03           0  -1.000E 03  -2.360E-03
 17  7   4.444E-01           0           0   1.684E-07           0

 -1  8   3.661E-01           0   3.661E-01  -1.684E-07           0
  0  8  -8.394E-95  -8.394E-95  -8.394E-95   8.294E 04  -9.411E-02
  1  8  -3.661E-01           0  -3.661E-01  -2.000E 03           0
  2  8  -7.178E-01           0  -7.178E-01  -2.000E 03           0
  3  8  -1.041E 00           0  -1.041E 00  -2.000E 03           0
  4  8  -1.322E 00  -1.322E 00  -1.322E 00  -2.000E 03   1.450E-01
  5  8  -1.547E 00           0  -1.547E 00  -2.000E 03           0
  6  8  -1.713E 00           0  -1.713E 00  -2.000E 03           0
  7  8  -1.815E 00           0  -1.815E 00  -2.000E 03           0
  8  8  -1.848E 00  -1.848E 00  -1.848E 00  -2.000E 03   9.222E-02
  9  8  -1.811E 00           0  -1.811E 00  -2.000E 03           0
 10  8  -1.707E 00           0  -1.707E 00  -2.000E 03           0
 11  8  -1.539E 00           0  -1.539E 00  -2.000E 03           0
 12  8  -1.312E 00  -1.312E 00  -1.312E 00  -2.000E 03   1.343E-01
 13  8  -1.032E 00           0  -1.032E 00  -2.000E 03           0
 14  8  -7.113E-01           0  -7.113E-01  -2.000E 03           0
 15  8  -3.627E-01           0  -3.627E-01  -2.000E 03           0
 16  8  -8.206E-95  -8.206E-95  -8.206E-95   8.106E 04  -9.637E-02
 17  8   3.627E-01           0   3.627E-01  -1.684E-07           0

 -1  9   4.012E-01           0           0  -3.368E-07           0
  0  9   2.064E-03   2.061E-03           0  -1.000E 03  -4.744E-03
  1  9  -3.971E-01           0           0  -2.000E 03           0
  2  9  -7.362E-01           0           0  -2.000E 03           0
  3  9  -9.984E-01           0           0  -2.000E 03           0
  4  9  -1.210E 00  -1.210E 00           0  -2.000E 03   3.373E-02
  5  9  -1.441E 00           0           0  -2.000E 03           0
  6  9  -1.615E 00           0           0  -2.000E 03           0
  7  9  -1.704E 00           0           0  -2.000E 03           0
  8  9  -1.717E 00  -1.717E 00           0  -2.000E 03  -3.801E-02
  9  9  -1.710E 00           0           0  -2.000E 03           0
 10  9  -1.628E 00           0           0  -2.000E 03           0
 11  9  -1.457E 00           0           0  -2.000E 03           0
 12  9  -1.228E 00  -1.228E 00           0  -2.000E 03   3.398E-02
 13  9  -1.015E 00           0           0  -2.000E 03           0
 14  9  -7.485E-01           0           0  -2.000E 03           0
 15  9  -4.036E-01           0           0  -2.000E 03           0
 16  9   2.202E-03   2.199E-03           0  -1.000E 03  -4.140E-03
 17  9   4.080E-01           0           0           0           0

 -1 10   3.704E-01           0           0  -1.684E-07           0
  0 10   2.852E-03   2.853E-03           0  -1.000E 03   2.019E-04
  1 10  -3.647E-01           0           0  -1.000E 03           0
  2 10  -6.740E-01           0           0  -1.000E 03           0
  3 10  -9.098E-01           0           0  -1.000E 03           0
  4 10  -1.100E 00  -1.100E 00           0  -1.000E 03   8.798E-03
  5 10  -1.317E 00           0           0  -1.000E 03           0
  6 10  -1.485E 00           0           0  -1.000E 03           0
  7 10  -1.571E 00           0           0  -1.000E 03           0
  8 10  -1.587E 00  -1.587E 00           0  -1.000E 03  -2.164E-03
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  9 10   -1.587E 00           0            0  -1.000E 03          0
 10 10   -1.516E 00           0            0  -1.000E 03          0
 11 10   -1.359E 00           0            0  -1.000E 03          0
 12 10   -1.146E 00   -1.146E 00           0  -1.000E 03  9.990E-03
 13 10   -9.512E-01           0            0  -1.000E 03          0
 14 10   -7.049E-01           0            0  -1.000E 03          0
 15 10   -3.811E-01           0            0  -1.000E 03          0
 16 10    3.103E-03    3.104E-03           0  -1.000E 03  1.933E-03
 17 10    3.873E-01           0            0  -1.684E-07          0

 -1 11            0           0            0           0          0
  0 11            0    3.645E-03           0  -5.263E-08          0
  1 11            0           0            0           0          0
  2 11            0           0            0           0          0
  3 11            0           0            0           0          0
  4 11            0   -9.910E-01           0  -2.695E-05          0
  5 11            0           0            0           0          0
  6 11            0           0            0           0          0
  7 11            0           0            0           0          0
  8 11            0   -1.457E 00           0   5.390E-05          0
  9 11            0           0            0           0          0
 10 11            0           0            0           0          0
 11 11            0           0            0           0          0
 12 11            0   -1.064E 00           0   2.695E-05          0
 13 11            0           0            0           0          0
 14 11            0           0            0           0          0
 15 11            0           0            0           0          0
 16 11            0    4.009E-03           0  -2.105E-07          0
 17 11            0           0            0           0          0

TABLE 9. MOMENTS -- ITERATION  23

 I,J     X-MOM        Y-MOM        Z-MOM

 -1 -1            0           0            0
  0 -1            0           0            0
  1 -1            0           0            0
  2 -1            0           0            0
  3 -1            0           0            0
  4 -1            0           0            0
  5 -1            0           0            0
  6 -1            0           0            0
  7 -1            0           0            0
  8 -1            0           0            0
  9 -1            0           0            0
 10 -1            0           0            0
 11 -1            0           0            0
 12 -1            0           0            0
 13 -1            0           0            0
 14 -1            0           0            0
 15 -1            0           0            0
 16 -1            0           0            0
 17 -1            0           0            0

 -1  0            0           0            0
  0  0   -2.021E-05   -1.579E-06           0
  1  0    9.891E 04           0            0
  2  0    1.378E 05           0            0
  3  0    1.167E 05           0            0
  4  0    3.563E 04    3.234E-03           0
  5  0    1.382E 05           0            0
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```
   6   0    1.807E 05            0            0
   7   0    1.632E 05            0            0
   8   0    8.578E 04    3.234E-03            0
   9   0    1.554E 05            0            0
  10   0    1.650E 05            0            0
  11   0    1.146E 05            0            0
  12   0    4.208E 03    1.617E-03            0
  13   0    9.316E 04            0            0
  14   0    1.221E 05            0            0
  15   0    9.105E 04            0            0
  16   0   -4.042E-05   -3.158E-06            0
  17   0            0            0            0

  -1   1            0            0            0
   0   1   -2.021E-05   -7.945E 04            0
   1   1    1.526E 05            0            0
   2   1    1.852E 05            0            0
   3   1    9.782E 04            0            0
   4   1   -1.096E 05   -1.218E 05            0
   5   1    4.757E 05            0            0
   6   1    6.410E 05            0            0
   7   1    3.863E 05            0            0
   8   1    1.154E 04   -1.035E 05            0
   9   1    1.977E 05            0            0
  10   1    2.638E 05            0            0
  11   1    2.100E 05            0            0
  12   1    3.615E 04   -1.297E 05            0
  13   1    2.071E 05            0            0
  14   1    2.581E 05            0            0
  15   1    1.890E 05            0            0
  16   1    2.021E-05   -7.553E 04            0
  17   1            0            0            0

  -1   2            0            0            0
   0   2   -2.021E-05   -2.652E 05            0
   1   2    6.095E 04            0    6.046E 06
   2   2    1.212E 05            0    1.197E 07
   3   2    1.793E 05            0    1.778E 07
   4   2    2.375E 05   -8.500E 05    2.347E 07
   5   2    2.511E 05            0    2.491E 07
   6   2    2.623E 05            0    2.593E 07
   7   2    2.676E 05            0    2.653E 07
   8   2    2.732E 05   -5.475E 05    2.702E 07
   9   2    2.523E 05            0    2.501E 07
  10   2    2.313E 05            0    2.288E 07
  11   2    2.081E 05            0    2.063E 07
  12   2    1.846E 05   -4.917E 05    1.826E 07
  13   2    1.400E 05            0    1.387E 07
  14   2    9.471E 04            0    9.366E 06
  15   2    4.786E 04            0    4.743E 06
  16   2   -1.011E-05   -2.756E 05   -1.000E-03
  17   2            0            0            0

  -1   3            0            0            0
   0   3   -2.021E-05    1.496E 04            0
   1   3    1.492E 05            0            0
   2   3    1.784E 05            0            0
   3   3    8.759E 04            0            0
   4   3   -1.232E 05    3.600E 05            0
   5   3    4.576E 05            0            0
   6   3    6.184E 05            0            0
```

```
  7  3   3.593E 05          0            0
  8  3  -1.989E 04   2.099E 05           0
  9  3   1.719E 05          0            0
 10  3   2.438E 05          0            0
 11  3   1.956E 05          0            0
 12  3   2.743E 04   1.054E 05           0
 13  3   2.006E 05          0            0
 14  3   2.537E 05          0            0
 15  3   1.869E 05          0            0
 16  3   2.021E-05   2.976E 04           0
 17  3          0           0            0

 -1  4          0           0            0
  0  4  -2.021E-05   1.905E 05           0
  1  4   1.475E 05          0            0
  2  4   1.749E 05          0            0
  3  4   8.237E 04          0            0
  4  4  -1.302E 05   9.641E 05           0
  5  4   4.484E 05          0            0
  6  4   6.069E 05          0            0
  7  4   3.454E 05          0            0
  8  4  -3.606E 04   6.219E 05           0
  9  4   1.589E 05          0            0
 10  4   2.338E 05          0            0
 11  4   1.887E 05          0            0
 12  4   2.367E 04   4.718E 05           0
 13  4   1.978E 05          0            0
 14  4   2.518E 05          0            0
 15  4   1.859E 05          0            0
 16  4          0    2.117E 05           0
 17  4          0           0            0

 -1  5          0           0            0
  0  5  -2.021E-05   2.624E 05           0
  1  5   1.872E 05          0            0
  2  5   2.543E 05          0            0
  3  5   2.015E 05          0            0
  4  5   2.866E 04   9.627E 05           0
  5  5   2.255E 05          0            0
  6  5   3.023E 05          0            0
  7  5   2.592E 05          0            0
  8  5   9.600E 04   6.857E 05           0
  9  5   2.484E 05          0            0
 10  5   2.809E 05          0            0
 11  5   1.933E 05          0            0
 12  5  -1.428E 04   6.087E 05           0
 13  5   1.693E 05          0            0
 14  5   2.329E 05          0            0
 15  5   1.764E 05          0            0
 16  5          0    2.706E 05           0
 17  5          0           0            0

 -1  6          0           0            0
  0  6  -2.021E-05   2.106E 05           0
  1  6   1.828E 05          0            0
  2  6   2.456E 05          0            0
  3  6   1.883E 05          0            0
  4  6   1.110E 04   7.164E 05           0
  5  6   2.095E 05          0            0
  6  6   2.879E 05          0            0
  7  6   2.463E 05          0            0
```

```
   8   6    8.464E 04    5.316E 05            0
   9   6    2.383E 05            0            0
  10   6    2.719E 05            0            0
  11   6    1.855E 05            0            0
  12   6   -2.082E 04    4.900E 05            0
  13   6    1.644E 05            0            0
  14   6    2.296E 05            0            0
  15   6    1.748E 05            0            0
  16   6            0    2.113E 05            0
  17   6            0            0            0

  -1   7            0            0            0
   0   7   -2.021E-05    3.750E 04            0
   1   7    1.775E 05            0            0
   2   7    2.350E 05            0            0
   3   7    1.725E 05            0            0
   4   7   -9.996E 03    2.224E 05            0
   5   7    1.913E 05            0            0
   6   7    2.725E 05            0            0
   7   7    2.338E 05            0            0
   8   7    7.507E 04    1.600E 05            0
   9   7    2.291E 05            0            0
  10   7    2.631E 05            0            0
  11   7    1.771E 05            0            0
  12   7   -2.884E 04    1.155E 05            0
  13   7    1.584E 05            0            0
  14   7    2.256E 05            0            0
  15   7    1.728E 05            0            0
  16   7    1.011E-05    3.466E 04            0
  17   7            0            0            0

  -1   8            0            0            0
   0   8   -1.011E-05   -2.544E 05            0
   1   8    4.020E 04            0    3.985E 06
   2   8    7.940E 04            0    7.850E 06
   3   8    1.170E 05            0    1.160E 07
   4   8    1.540E 05   -5.236E 05    1.522E 07
   5   8    1.663E 05            0    1.648E 07
   6   8    1.782E 05            0    1.762E 07
   7   8    1.881E 05            0    1.864E 07
   8   8    1.976E 05   -4.281E 05    1.954E 07
   9   8    1.869E 05            0    1.852E 07
  10   8    1.758E 05            0    1.738E 07
  11   8    1.627E 05            0    1.613E 07
  12   8    1.492E 05   -5.155E 05    1.475E 07
  13   8    1.134E 05            0    1.124E 07
  14   8    7.699E 04            0    7.612E 06
  15   8    3.900E 04            0    3.866E 06
  16   8   -1.011E-05   -2.584E 05            0
  17   8            0            0            0

  -1   9            0            0            0
   0   9   -2.021E-05   -7.048E 04            0
   1   9    1.669E 05            0            0
   2   9    2.137E 05            0            0
   3   9    1.406E 05            0            0
   4   9   -5.258E 04   -1.318E 05            0
   5   9    1.543E 05            0            0
   6   9    2.412E 05            0            0
   7   9    2.081E 05            0            0
   8   9    5.503E 04   -1.069E 05            0
```

```
  9  9   2.104E 05           0           0
 10  9   2.458E 05           0           0
 11  9   1.612E 05           0           0
 12  9  -4.344E 04  -1.289E 05           0
 13  9   1.474E 05           0           0
 14  9   2.183E 05           0           0
 15  9   1.691E 05           0           0
 16  9           0  -7.192E 04           0
 17  9           0           0           0

 -1 10           0           0           0
  0 10  -1.011E-05  -1.579E-06           0
  1 10   8.095E 04           0           0
  2 10   1.019E 05           0           0
  3 10   6.286E 04           0           0
  4 10  -3.619E 04  -8.084E-04           0
  5 10   6.832E 04           0           0
  6 10   1.128E 05           0           0
  7 10   9.735E 04           0           0
  8 10   2.187E 04   1.617E-03           0
  9 10   1.002E 05           0           0
 10 10   1.186E 05           0           0
 11 10   7.697E 04           0           0
 12 10  -2.466E 04   8.084E-04           0
 13 10   7.150E 04           0           0
 14 10   1.077E 05           0           0
 15 10   8.383E 04           0           0
 16 10  -1.011E-05  -6.316E-06           0
 17 10           0           0           0

 -1 11           0           0           0
  0 11           0           0           0
  1 11           0           0           0
  2 11           0           0           0
  3 11           0           0           0
  4 11           0           0           0
  5 11           0           0           0
  6 11           0           0           0
  7 11           0           0           0
  8 11           0           0           0
  9 11           0           0           0
 10 11           0           0           0
 11 11           0           0           0
 12 11           0           0           0
 13 11           0           0           0
 14 11           0           0           0
 15 11           0           0           0
 16 11           0           0           0
 17 11           0           0           0

   TIME =      3 MINUTES, 42 AND 44/60 SECONDS
```

APPENDIX 13

SAMPLE INPUT AND OUTPUT FOR PROGRAM LAYER 8

APPENDIX 13. SAMPLE INPUT AND OUTPUT FOR PROGRAM LAYER 8

```
9.1     SIMPLY SUPPORTFD SQUARE PLATF WITH COPCENTRATFD LOAD
            5    5    0    3        100   16    8 3.000F 00 6.000F 00 1.000E-04
                   0.250E 00 0.250F 00
            8    4    0    4    4    2    8    0
            3      9.000E 02 5.000E 03 5.040E 05
            3      9.000E 02 5.000E 03 5.040E 05

            0    0   16    8 6.250E 05
            1    0   15    8 6.250F 05
            0    1   16    7 6.250E 05
            1    1   15    7 6.250E 05
            1    1   16    8           1.875E 06
            0    0   16    8 6.250F 05
            1    0   15    8 6.250E 05
            0    1   16    7 6.250E 05
            1    1   15    7 6.250E 05
            1    1   16    8           1.875E 06
            8    4    8    4-1.000E 05
            0    0   16    8           1.000F 99
            1    1   15    7          -1.000F 99
```

```
9.2     SQUARE PLATE OVER FLEXIBLE EDGE BEAMS
            5    5    4    4        60   10   10 1.200E 01 1.200E 01 1.000E-04
                   0.300E+00 0.300E+00
            5    5    5    0    0    5   10   10
            3      3.000E+04 5.000E+04 1.000E+05 1.000E+06
            3      3.000E+04 5.000E+04 1.000E+05 1.000E+06
            3      5.000E+05 1.000E+06 5.000E+06 1.000E+07
            0    0   10   10 5.500E 06
            1    0    9   10 5.500E 06
            0    1   10    9 5.500E 06
            1    1    9    9 5.500E 06
            1    1   10   10           1.600E+07
            0    0   10   10 5.500E 06
            1    0    9   10 5.500E 06
            0    1   10    9 5.500E 06
            1    1    9    9 5.500E 06
            1    1   10   10           1.600E+07
            0    0   10    0 5.450E 10
            1    0    9    0 5.450E 10
            0   10   10   10 5.450E 10
            1   10    9   10 5.450E 10
            0    0   10   10-5.000E 02
            1    1    9    9-5.000E 02
            0    0    0   10           1.000E+99
           10    0   10   10           1.000E+99
```

```
9.3     RIBBED PLATE STRUCTURE OVER THREE SUPPORTS
            5    5    4    6         40   16   10 3.000E 01 3.000E 01 1.000E-04
                   0.300E 00 0.300E 00
            4    6    4    2    4    2    8    0
            4      1.000E 04 5.000E 04 1.000E 05 2.000E 06
            4      3.000E 03 1.000E 04 1.000E 05 3.000E 05
            4      1.000E 04 5.000E 04 1.000E 06 1.000E 07
            0    0   16   10 1.850E 07
            1    0   15   10 1.850E 07
            0    1   16    9 1.850E 07
            1    1   15    9 1.850E 07
            1    1   16   10           5.990E 07
            0    0   16   10 2.900E 06
            1    0   15   10 2.900E 06
            0    1   16    9 2.900E 06
            1    1   15    9 2.900E 06
            1    1   16   10           5.990E 07
            0    2   16    2 5.000E 11
            1    2   15    2 5.000E 11
            0    8   16    8 5.000E 11
            1    8   15    8 5.000E 11
            0    0   16   10-1.000E 03
            1    1   15    9-1.000E 03
            0    0    0   10           1.000E 99
            8    0    8   10           1.000E+99
           16    0   16   10           1.000E 99
            4    1    6    4-5.000E 03
```

```
9.4     REINFORCED SLAB OVER FIVE GIRDERS
            5    5   10    6         50   20   10 3.600E 01 3.600E 01 1.000E-05
                   0.150E+00 0.150E+00
           10    2    5    6   10   10   10    5
            5      3.380E 03 3.350E 04 1.000E 05 5.000E 05 3.000E 04
            5      1.425E 04 1.625E 05 5.550E 05 1.100E 06 1.000E 05
            5      1.000E 04 2.000E 05 4.000E 05 1.000E 07 1.000E 05
            0    0   20   10 1.562E 07
            1    0   19   10 1.562E 07
            0    1   20    9 1.562E 07
            1    1   19    9 1.562E 07
            1    1   20   10           2.666E 07
            0    0   20   10 1.562E 07
            1    0   19   10 1.562E 07
            0    1   20    9 1.562E 07
            1    1   19    9 1.562E 07
            1    1   20   10           2.666E 07
            0    1   20    1 6.690E 10
            1    1   19    1 6.690E 10
            0    3   20    3 6.690E 10
            1    3   19    3 6.690E 10
            0    5   20    5 6.690E 10
            1    5   19    5 6.690E 10
            0    7   20    7 6.690E 10
            1    7   19    7 6.690E 10
            0    9   20    9 6.690E 10
            1    9   19    9 6.690E 10
            9    4    9    4-1.000E 03
            9    6    9    6-1.000E 03
           11    4   11    4-1.000E 03
           11    6   11    6-1.000E 03
            0    0    0   10           1.000E 99
           20    0   20   10           1.000E 99
```

```
PROGRAM LAYER 8 - MASTER DECK - WB INGRAM, H MATLOCK     REVISION DATE 27 JUL 65
FINAL RUN ON EXAMPLE PROBLEMS
CHG TO CE 05-1023 --CODED BY WBI-- RUN DATE 27 JL 65


PROB
  9.2     SQUARE PLATE OVER FLEXIBLE EDGE BEAMS


TABLE 1.  CONTROL DATA

     NUM CARDS TABLE 3                                               5
     NUM CARDS TABLE 4                                               5
     NUM CARDS TABLE 5                                               4
     NUM CARDS TABLE 6                                               4
     MAX NUM ITERATIONS                                             60
     NUM INCREMENTS  MX                                             10
     NUM INCREMENTS  MY                                             10
     INCR LENGTH  HX                                         1.200E 01
     INCR LENGTH  HY                                         1.200E 01
     CLOSURE TOLERANCE                                       1.000E-04
     POISSONS RATIO X                                        3.000E-01
     POISSONS RATIO Y                                        3.000E-01

     MONITOR STAS  I,J              5  5      5  0      0  5     10 10


TABLE 2.  RELAXATION CONTROL DATA
   NUM       C L O S U R E   P A R A M E T E R S
 VALUES

    3      3.000E 04 5.000E 04 1.000E 05
    3      3.000E 04 5.000E 04 1.000E 05
    3      5.000E 05 1.000E 06 5.000E 06


TABLE 3. X-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
       FROM  THRU     DX          CX          TX          RX          PX

       0  0 10 10   5.500E 06           0           0           0           0
       1  0  9 10   5.500E 06           0           0           0           0
       0  1 10  9   5.500E 06           0           0           0           0
       1  1  9  9   5.500E 06           0           0           0           0
       1  1 10 10           0   1.600E 07           0           0           0


TABLE 4. Y-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
       FROM  THRU     DY          CY          TY          RY          PY

       0  0 10 10   5.500E 06           0           0           0           0
       1  0  9 10   5.500E 06           0           0           0           0
       0  1 10  9   5.500E 06           0           0           0           0
       1  1  9  9   5.500E 06           0           0           0           0
       1  1 10 10           0   1.600E 07           0           0           0


TABLE 5. Z-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
       FROM  THRU     FZ        TZ        RZ           PZ
```

```
      0  0 10  0    5.450E 10           0           0           0           0
      1  0  9  0    5.450E 10           0           0           0           0
      0 10 10 10    5.450E 10           0           0           0           0
      1 10  9 10    5.450E 10           0           0           0           0

TABLE 6. LOAD AND SUPPORT DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

     FROM       THRU          Q          S

      0  0 10 10  -5.000E 02           0
      1  1  9  9  -5.000E 02           0
      0  0  0 10           0   1.000E 99
     10  0 10 10           0   1.000E 99

     TIME =     2 MINUTES, 16 AND  7/60 SECONDS


TABLE 7.  MONITOR TALLY AND DEFLS AT 4 STAS

    ITR   FICT       CYC  NOT  NOT       I,J
    NUM   SPRING     NUM  STAB CLOS      5  5          5  0          0  5        10 10

X    1   3.000E 04    1   99           -3.144E-02  -9.455E-04  -2.341E-96   6.323E-97
Y        3.000E 04    1   99    99     -6.147E-02  -6.943E-03  -9.743E-97   1.901E-96
Z        5.000E 05    1   18    18              0  -3.223E-03           0  -6.018E-96

X    2   5.000E 04    2   97           -7.954E-02  -3.322E-03  -3.313E-96  -5.661E-96
Y        5.000E 04    2   99    99     -9.742E-02  -4.853E-03  -1.645E-96  -5.495E-96
Z        1.000E 06    2   18    16              0  -3.979E-03           0  -6.911E-96

X    3   1.000E 05    3   97           -1.061E-01  -3.992E-03  -2.792E-96  -6.859E-96
Y        1.000E 05    3   97    91     -1.147E-01  -4.134E-03  -1.989E-96  -6.667E-96
Z        5.000E 06    3   14    14              0  -4.271E-03           0  -7.038E-96

X    4   3.000E 04    1   98           -1.367E-01  -4.335E-03  -3.475E-96  -6.731E-96
Y        3.000E 04    1   99    99     -1.531E-01  -7.226E-03  -2.392E-96  -6.294E-96
Z        5.000E 05    1   18    18              0  -5.027E-03           0  -7.773E-96

X    5   5.000E 04    2   99           -1.628E-01  -5.080E-03  -3.534E-96  -7.458E-96
Y        5.000E 04    2   99    97     -1.724E-01  -5.959E-03  -2.715E-96  -7.321E-96
Z        1.000E 06    2   18    17              0  -5.522E-03           0  -8.154E-96

X    6   1.000E 05    3   98           -1.771E-01  -5.530E-03  -3.414E-96  -8.100E-96
Y        1.000E 05    3   97    80     -1.818E-01  -5.618E-03  -2.951E-96  -8.026E-96
Z        5.000E 06    3   12    10              0  -5.739E-03           0  -8.233E-96

X    7   3.000E 04    1   96           -1.940E-01  -5.789E-03  -3.798E-96  -8.065E-96
Y        3.000E 04    1   99    99     -2.032E-01  -7.426E-03  -3.189E-96  -7.819E-96
Z        5.000E 05    1   18    16              0  -6.188E-03           0  -8.641E-96

X    8   5.000E 04    2   98           -2.086E-01  -6.219E-03  -3.836E-96  -8.465E-96
Y        5.000E 04    2   99    96     -2.140E-01  -6.714E-03  -3.375E-96  -8.387E-96
Z        1.000E 06    2   14    12              0  -6.468E-03           0  -8.849E-96

X    9   1.000E 05    3   94           -2.166E-01  -6.473E-03  -3.765E-96  -8.816E-96
Y        1.000E 05    3   94    80     -2.192E-01  -6.522E-03  -3.507E-96  -8.775E-96
Z        5.000E 06    3    3     3              0  -6.590E-03           0  -8.890E-96

X   10   3.000E 04    1   92           -2.261E-01  -6.618E-03  -3.980E-96  -8.795E-96
```

```
Y          3.000E 04   1   99    99  -2.312E-01  -7.532E-03  -3.640E-96  -8.658E-96
Z          5.000E 05   1   14    11           0  -6.841E-03           0  -9.117E-96

X    11    5.000E 04   2   94        -2.343E-01  -6.858E-03  -4.002E-96  -9.018E-96
Y          5.000E 04   2   97    93  -2.373E-01  -7.135E-03  -3.744E-96  -8.975E-96
Z          1.000E 06   2   10     5           0  -6.997E-03           0  -9.233E-96

X    12    1.000E 05   3   86        -2.387E-01  -7.000E-03  -3.962E-96  -9.213E-96
Y          1.000E 05   3   91    80  -2.402E-01  -7.027E-03  -3.818E-96  -9.191E-96
Z          5.000E 06   3    0     0           0  -7.065E-03           0  -9.255E-96

X    13    3.000E 04   1   81        -2.440E-01  -7.081E-03  -4.082E-96  -9.202E-96
Y          3.000E 04   1   98    97  -2.469E-01  -7.591E-03  -3.892E-96  -9.126E-96
Z          5.000E 05   1   10     5           0  -7.206E-03           0  -9.382E-96

X    14    5.000E 04   2   88        -2.486E-01  -7.215E-03  -4.094E-96  -9.326E-96
Y          5.000E 04   2   93    89  -2.502E-01  -7.369E-03  -3.950E-96  -9.302E-96
Z          1.000E 06   2    0     0           0  -7.293E-03           0  -9.446E-96

X    15    1.000E 05   3   81        -2.510E-01  -7.294E-03  -4.072E-96  -9.435E-96
Y          1.000E 05   3   81    78  -2.519E-01  -7.310E-03  -3.991E-96  -9.423E-96
Z          5.000E 06   3    0     0           0  -7.331E-03           0  -9.459E-96

X    16    3.000E 04   1   81        -2.540E-01  -7.340E-03  -4.139E-96  -9.429E-96
Y          3.000E 04   1   97    96  -2.556E-01  -7.625E-03  -4.033E-96  -9.386E-96
Z          5.000E 05   1    0     0           0  -7.409E-03           0  -9.529E-96

X    17    5.000E 04   2   81        -2.565E-01  -7.414E-03  -4.146E-96  -9.498E-96
Y          5.000E 04   2   87    70  -2.575E-01  -7.501E-03  -4.065E-96  -9.485E-96
Z          1.000E 06   2    0     0           0  -7.458E-03           0  -9.565E-96

X    18    1.000E 05   3   79        -2.579E-01  -7.459E-03  -4.133E-96  -9.559E-96
Y          1.000E 05   3   77    66  -2.584E-01  -7.467E-03  -4.088E-96  -9.552E-96
Z          5.000E 06   3    0     0           0  -7.479E-03           0  -9.572E-96

X    19    3.000E 04   1   81        -2.596E-01  -7.484E-03  -4.171E-96  -9.556E-96
Y          3.000E 04   1   93    90  -2.605E-01  -7.643E-03  -4.112E-96  -9.532E-96
Z          5.000E 05   1    0     0           0  -7.523E-03           0  -9.612E-96

X    20    5.000E 04   2   81        -2.610E-01  -7.526E-03  -4.175E-96  -9.595E-96
Y          5.000E 04   2   79    66  -2.615E-01  -7.574E-03  -4.130E-96  -9.587E-96
Z          1.000E 06   2    0     0           0  -7.550E-03           0  -9.632E-96

X    21    1.000E 05   3   74        -2.618E-01  -7.550E-03  -4.168E-96  -9.629E-96
Y          1.000E 05   3   71    43  -2.620E-01  -7.555E-03  -4.143E-96  -9.625E-96
Z          5.000E 06   3    0     0           0  -7.562E-03           0  -9.636E-96

X    22    3.000E 04   1   78        -2.627E-01  -7.565E-03  -4.189E-96  -9.627E-96
Y          3.000E 04   1   81    70  -2.632E-01  -7.653E-03  -4.156E-96  -9.613E-96
Z          5.000E 05   1    0     0           0  -7.586E-03           0  -9.658E-96

X    23    5.000E 04   2   79        -2.635E-01  -7.588E-03  -4.191E-96  -9.648E-96
Y          5.000E 04   2   71    50  -2.638E-01  -7.615E-03  -4.166E-96  -9.644E-96
Z          1.000E 06   2    0     0           0  -7.602E-03           0  -9.669E-96

X    24    1.000E 05   3   63        -2.639E-01  -7.602E-03  -4.187E-96  -9.667E-96
Y          1.000E 05   3   50    20  -2.641E-01  -7.604E-03  -4.173E-96  -9.665E-96
Z          5.000E 06   3    0     0           0  -7.608E-03           0  -9.671E-96

X    25    3.000E 04   1   71        -2.644E-01  -7.610E-03  -4.199E-96  -9.666E-96
Y          3.000E 04   1   77    57  -2.647E-01  -7.659E-03  -4.180E-96  -9.659E-96
```

```
Z        5.000E 05   1    0    0            0  -7.622E-03           0  -9.684E-96

X   26   5.000E 04   2   71        -2.649E-01  -7.623E-03  -4.200E-96  -9.678E-96
Y        5.000E 04   2   52   28   -2.650E-01  -7.638E-03  -4.186E-96  -9.676E-96
Z        1.000E 06   2    0    0            0  -7.630E-03           0  -9.690E-96

X   27   1.000E 05   3   44        -2.651E-01  -7.630E-03  -4.198E-96  -9.689E-96
Y        1.000E 05   3   25    0   -2.652E-01  -7.632E-03  -4.190E-96  -9.688E-96
Z        5.000E 06   3    0    0            0  -7.634E-03           0  -9.691E-96
```

```
PROGRAM LAYER 8 - MASTER DECK - WB INGRAM, H MATLOCK      REVISION DATE 27 JUL 65
FINAL RUN ON EXAMPLE PROBLEMS
CHG TO CE 05-1023 --CODED BY WBI-- RUN DATE 27 JL 65


PROB (CONTD)
  9.2       SQUARE PLATE OVER FLEXIBLE EDGE BEAMS

TABLE 8. DEFLECTION AND ERROR RESULTS -- ITERATION,  27

  I,J     X-DEFL        Y-DEFL        Z-DEFL          REACT     ERROR

  -1 -1  -2.535E-02  -2.535E-02             0            0              0
   0 -1  -1.705E-14  -1.705E-14             0            0              0
   1 -1   2.525E-02   2.528E-02             0   1.292E-08              0
   2 -1   4.668E-02   4.673E-02             0   1.979E-08              0
   3 -1   6.264E-02   6.270E-02             0   1.458E-07              0
   4 -1   7.241E-02   7.247E-02             0   1.397E-09              0
   5 -1   7.570E-02   7.576E-02             0   3.679E-08              0
   6 -1   7.243E-02   7.248E-02             0   2.175E-07              0
   7 -1   6.267E-02   6.271E-02             0  -2.026E-08              0
   8 -1   4.672E-02   4.674E-02             0   2.980E-08              0
   9 -1   2.529E-02   2.530E-02             0  -1.129E-08              0
  10 -1   1.705E-14  -1.711E-95             0   4.163-102              0
  11 -1  -2.536E-02  -2.536E-02             0            0              0

  -1  0   2.375E-03   2.375E-03   2.376E-03   1.795E-06              0
   0  0  -9.682E-96  -9.679E-96  -9.685E-96   9.185E 03   -2.546E 00
   1  0  -2.375E-03  -2.376E-03  -2.376E-03  -4.999E 02   -1.088E-01
   2  0  -4.507E-03  -4.508E-03  -4.509E-03  -4.997E 02   -2.931E-01
   3  0  -6.188E-03  -6.189E-03  -6.191E-03  -4.996E 02   -4.262E-01
   4  0  -7.261E-03  -7.263E-03  -7.265E-03  -4.995E 02   -5.197E-01
   5  0  -7.630E-03  -7.632E-03  -7.634E-03  -4.994E 02   -5.621E-01
   6  0  -7.262E-03  -7.263E-03  -7.265E-03  -4.995E 02   -5.495E-01
   7  0  -6.188E-03  -6.189E-03  -6.191E-03  -4.995E 02   -4.848E-01
   8  0  -4.507E-03  -4.507E-03  -4.509E-03  -4.996E 02   -3.723E-01
   9  0  -2.375E-03  -2.375E-03  -2.376E-03  -4.998E 02   -2.146E-01
  10  0  -9.683E-96  -9.679E-96  -9.686E-96   9.186E 03   -2.910E 00
  11  0   2.375E-03   2.375E-03   2.376E-03   1.249-102              0

  -1  1   3.010E-02   3.010E-02             0  -1.927E-91              0
   0  1  -2.255E-96  -2.250E-96             0   1.750E 03    2.002E 00
   1  1  -3.010E-02  -3.011E-02             0  -9.992E 02   -8.210E-01
   2  1  -5.586E-02  -5.588E-02             0  -9.977E 02   -2.286E 00
   3  1  -7.524E-02  -7.526E-02             0  -9.973E 02   -2.670E 00
   4  1  -8.718E-02  -8.721E-02             0  -9.970E 02   -3.001E 00
   5  1  -9.122E-02  -9.124E-02             0  -9.969E 02   -3.107E 00
   6  1  -8.719E-02  -8.722E-02             0  -9.971E 02   -2.862E 00
   7  1  -7.525E-02  -7.527E-02             0  -9.977E 02   -2.274E 00
   8  1  -5.588E-02  -5.589E-02             0  -9.986E 02   -1.387E 00
   9  1  -3.011E-02  -3.012E-02             0  -9.988E 02   -1.181E 00
  10  1  -2.255E-96  -2.250E-96             0   1.750E 03    2.424E 00
  11  1   3.011E-02   3.011E-02             0  -3.474E-08              0

  -1  2   5.377E-02   5.377E-02             0   9.381E-93              0
   0  2  -3.235E-96  -3.228E-96             0   2.728E 03    2.851E 00
   1  2  -5.377E-02  -5.378E-02             0  -9.974E 02   -2.611E 00
   2  2  -1.001E-01  -1.001E-01             0  -9.962E 02   -3.792E 00
   3  2  -1.350E-01  -1.351E-01             0  -9.950E 02   -4.983E 00
```

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  4  2  -1.566E-01  -1.567E-01         0  -9.943E 02  -5.707E 00
  5  2  -1.639E-01  -1.640E-01         0  -9.943E 02  -5.726E 00
  6  2  -1.566E-01  -1.567E-01         0  -9.949E 02  -5.145E 00
  7  2  -1.351E-01  -1.351E-01         0  -9.959E 02  -4.080E 00
  8  2  -1.001E-01  -1.001E-01         0  -9.972E 02  -2.842E 00
  9  2  -5.379E-02  -5.380E-02         0  -9.987E 02  -1.319E 00
 10  2  -3.237E-96  -3.231E-96         0   2.731E 03   3.171E 00
 11  2   5.379E-02   5.379E-02         0  -6.948E-08           0

 -1  3   7.151E-02   7.151E-02         0   1.390E-07           0
  0  3  -3.804E-96  -3.796E-96         0   3.297E 03   3.136E 00
  1  3  -7.151E-02  -7.153E-02         0  -9.969E 02  -3.130E 00
  2  3  -1.334E-01  -1.335E-01         0  -9.948E 02  -5.196E 00
  3  3  -1.803E-01  -1.804E-01         0  -9.932E 02  -6.796E 00
  4  3  -2.094E-01  -2.094E-01         0  -9.925E 02  -7.533E 00
  5  3  -2.192E-01  -2.193E-01         0  -9.925E 02  -7.539E 00
  6  3  -2.094E-01  -2.095E-01         0  -9.932E 02  -6.826E 00
  7  3  -1.804E-01  -1.804E-01         0  -9.946E 02  -5.404E 00
  8  3  -1.335E-01  -1.335E-01         0  -9.964E 02  -3.621E 00
  9  3  -7.153E-02  -7.155E-02         0  -9.982E 02  -1.840E 00
 10  3  -3.807E-96  -3.799E-96         0   3.299E 03   3.652E 00
 11  3   7.153E-02   7.153E-02         0  -1.390E-07           0

 -1  4   8.242E-02   8.242E-02         0   1.390E-07           0
  0  4  -4.104E-96  -4.096E-96         0   3.597E 03   3.155E 00
  1  4  -8.242E-02  -8.244E-02         0  -9.965E 02  -3.537E 00
  2  4  -1.540E-01  -1.540E-01         0  -9.940E 02  -6.037E 00
  3  4  -2.084E-01  -2.084E-01         0  -9.924E 02  -7.647E 00
  4  4  -2.421E-01  -2.422E-01         0  -9.915E 02  -8.510E 00
  5  4  -2.535E-01  -2.536E-01         0  -9.914E 02  -8.590E 00
  6  4  -2.421E-01  -2.422E-01         0  -9.922E 02  -7.771E 00
  7  4  -2.084E-01  -2.085E-01         0  -9.939E 02  -6.136E 00
  8  4  -1.540E-01  -1.541E-01         0  -9.960E 02  -4.038E 00
  9  4  -8.245E-02  -8.247E-02         0  -9.980E 02  -2.038E 00
 10  4  -4.107E-96  -4.100E-96         0   3.600E 03   3.719E 00
 11  4   8.245E-02   8.245E-02         0  -1.390E-07           0

 -1  5   8.610E-02   8.610E-02         0   1.390E-07           0
  0  5  -4.198E-96  -4.190E-96         0   3.691E 03   2.879E 00
  1  5  -8.610E-02  -8.612E-02         0  -9.963E 02  -3.665E 00
  2  5  -1.609E-01  -1.610E-01         0  -9.939E 02  -6.135E 00
  3  5  -2.178E-01  -2.179E-01         0  -9.923E 02  -7.737E 00
  4  5  -2.532E-01  -2.532E-01         0  -9.913E 02  -8.693E 00
  5  5  -2.651E-01  -2.652E-01         0  -9.912E 02  -8.762E 00
  6  5  -2.532E-01  -2.533E-01         0  -9.921E 02  -7.878E 00
  7  5  -2.179E-01  -2.179E-01         0  -9.938E 02  -6.208E 00
  8  5  -1.610E-01  -1.610E-01         0  -9.959E 02  -4.136E 00
  9  5  -8.613E-02  -8.615E-02         0  -9.980E 02  -1.994E 00
 10  5  -4.201E-96  -4.194E-96         0   3.694E 03   3.404E 00
 11  5   8.613E-02   8.613E-02         0   4.302E-93           0

 -1  6   8.243E-02   8.243E-02         0   1.390E-07           0
  0  6  -4.103E-96  -4.097E-96         0   3.598E 03   2.277E 00
  1  6  -8.243E-02  -8.245E-02         0  -9.966E 02  -3.406E 00
  2  6  -1.540E-01  -1.540E-01         0  -9.944E 02  -5.594E 00
  3  6  -2.084E-01  -2.084E-01         0  -9.929E 02  -7.119E 00
  4  6  -2.421E-01  -2.422E-01         0  -9.920E 02  -7.983E 00
  5  6  -2.535E-01  -2.536E-01         0  -9.920E 02  -8.009E 00
  6  6  -2.421E-01  -2.422E-01         0  -9.928E 02  -7.181E 00
  7  6  -2.084E-01  -2.085E-01         0  -9.944E 02  -5.645E 00
  8  6  -1.541E-01  -1.541E-01         0  -9.962E 02  -3.774E 00
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  9  6   -8.246E-02   -8.248E-02            0   -9.982E 02   -1.803E 00
 10  6   -4.106E-96   -4.100E-96            0    3.600E 03    2.754E 00
 11  6    8.246E-02    8.246E-02            0   -1.390E-07            0

 -1  7    7.152E-02    7.152E-02            0    1.390E-07            0
  0  7   -3.802E-96   -3.798E-96            0    3.298E 03    1.367E 00
  1  7   -7.152E-02   -7.154E-02            0   -9.972E 02   -2.818E 00
  2  7   -1.334E-01   -1.335E-01            0   -9.955E 02   -4.466E 00
  3  7   -1.804E-01   -1.804E-01            0   -9.942E 02   -5.788E 00
  4  7   -2.094E-01   -2.095E-01            0   -9.936E 02   -6.433E 00
  5  7   -2.192E-01   -2.193E-01            0   -9.936E 02   -6.416E 00
  6  7   -2.094E-01   -2.095E-01            0   -9.943E 02   -5.748E 00
  7  7   -1.804E-01   -1.804E-01            0   -9.955E 02   -4.503E 00
  8  7   -1.335E-01   -1.335E-01            0   -9.970E 02   -2.995E 00
  9  7   -7.155E-02   -7.157E-02            0   -9.985E 02   -1.477E 00
 10  7   -3.804E-96   -3.801E-96            0    3.301E 03    1.771E 00
 11  7    7.155E-02    7.155E-02            0   -2.779E-07            0

 -1  8    5.378E-02    5.378E-02            0    9.409E-93            0
  0  8   -3.231E-96   -3.230E-96            0    2.730E 03    1.427E-01
  1  8   -5.378E-02   -5.380E-02            0   -9.980E 02   -1.956E 00
  2  8   -1.001E-01   -1.001E-01            0   -9.971E 02   -2.876E 00
  3  8   -1.351E-01   -1.351E-01            0   -9.962E 02   -3.839E 00
  4  8   -1.567E-01   -1.567E-01            0   -9.957E 02   -4.295E 00
  5  8   -1.640E-01   -1.640E-01            0   -9.958E 02   -4.243E 00
  6  8   -1.567E-01   -1.567E-01            0   -9.962E 02   -3.787E 00
  7  8   -1.351E-01   -1.351E-01            0   -9.970E 02   -2.975E 00
  8  8   -1.001E-01   -1.002E-01            0   -9.980E 02   -1.986E 00
  9  8   -5.380E-02   -5.381E-02            0   -9.989E 02   -1.120E 00
 10  8   -3.233E-96   -3.232E-96            0    2.732E 03    4.888E-01
 11  8    5.380E-02    5.380E-02            0   -6.948E-08            0

 -1  9    3.011E-02    3.011E-02            0   -1.929E-91            0
  0  9   -2.251E-96   -2.251E-96            0    1.751E 03   -4.180E-01
  1  9   -3.011E-02   -3.011E-02            0   -9.997E 02   -3.410E-01
  2  9   -5.588E-02   -5.590E-02            0   -9.983E 02   -1.650E 00
  3  9   -7.526E-02   -7.528E-02            0   -9.983E 02   -1.663E 00
  4  9   -8.722E-02   -8.723E-02            0   -9.981E 02   -1.889E 00
  5  9   -9.125E-02   -9.127E-02            0   -9.980E 02   -2.018E 00
  6  9   -8.722E-02   -8.724E-02            0   -9.981E 02   -1.852E 00
  7  9   -7.528E-02   -7.529E-02            0   -9.985E 02   -1.483E 00
  8  9   -5.590E-02   -5.591E-02            0   -9.990E 02   -9.873E-01
  9  9   -3.012E-02   -3.013E-02            0   -9.992E 02   -8.408E-01
 10  9   -2.251E-96   -2.251E-96            0    1.751E 03    6.080E-02
 11  9    3.012E-02    3.012E-02            0   -3.474E-08            0

 -1 10    2.376E-03    2.376E-03    2.377E-03    1.795E-06            0
  0 10   -9.688E-96   -9.688E-96   -9.690E-96    9.190E 03   -1.774E 00
  1 10   -2.376E-03   -2.377E-03   -2.377E-03   -5.000E 02   -3.014E-02
  2 10   -4.510E-03   -4.511E-03   -4.511E-03   -4.999E 02   -1.312E-01
  3 10   -6.192E-03   -6.193E-03   -6.194E-03   -4.998E 02   -2.060E-01
  4 10   -7.267E-03   -7.268E-03   -7.269E-03   -4.997E 02   -2.596E-01
  5 10   -7.636E-03   -7.637E-03   -7.638E-03   -4.997E 02   -2.865E-01
  6 10   -7.267E-03   -7.268E-03   -7.269E-03   -4.997E 02   -2.871E-01
  7 10   -6.192E-03   -6.193E-03   -6.194E-03   -4.997E 02   -2.628E-01
  8 10   -4.510E-03   -4.510E-03   -4.511E-03   -4.998E 02   -2.123E-01
  9 10   -2.376E-03   -2.377E-03   -2.377E-03   -4.999E 02   -1.261E-01
 10 10   -9.689E-96   -9.688E-96   -9.691E-96    9.191E 03   -1.918E 00
 11 10    2.376E-03    2.376E-03    2.377E-03   -1.976E-09            0

 -1 11   -2.536E-02   -2.536E-02            0            0            0
```

```
  0 11   -1.710E-95   -1.705E-14            0            0            0
  1 11    2.526E-02    2.529E-02            0   -2.782E-08            0
  2 11    4.670E-02    4.674E-02            0   -3.213E-08            0
  3 11    6.266E-02    6.271E-02            0    5.914E-08            0
  4 11    7.243E-02    7.249E-02            0   -1.029E-07            0
  5 11    7.572E-02    7.578E-02            0   -2.086E-07            0
  6 11    7.245E-02    7.250E-02            0    1.346E-07            0
  7 11    6.269E-02    6.272E-02            0   -1.094E-07            0
  8 11    4.673E-02    4.675E-02            0    6.054E-09            0
  9 11    2.529E-02    2.530E-02            0   -3.807E-08            0
 10 11   -1.710E-95    1.705E-14            0            0            0
 11 11   -2.536E-02   -2.536E-02            0            0            0

TABLE 9. MOMENTS -- ITERATION  27
 I,J    X-MOM        Y-MOM        Z-MOM           TX-MOM   TY-MOM

 -1 -1            0            0            0            0            0
  0 -1            0            0            0            0            0
  1 -1            0            0            0            0            0
  2 -1            0            0            0            0            0
  3 -1            0            0            0            0            0
  4 -1            0            0            0            0            0
  5 -1            0            0            0            0            0
  6 -1            0            0            0            0            0
  7 -1            0            0            0            0            0
  8 -1            0            0            0            0            0
  9 -1            0            0            0            0            0
 10 -1           .0            0            0            0            0
 11 -1            0            0            0            0            0

 -1  0            0            0            0            0            0
  0  0    1.976E-09            0    2.151E-05   -7.695E 02    7.697E 02
  1  0    1.689E 01    1.292E-08    1.840E 05   -1.424E 03    1.425E 03
  2  0    3.132E 01    1.979E-08    3.412E 05   -1.146E 03    1.147E 03
  3  0    4.223E 01    1.458E-07    4.601E 05   -7.923E 02    7.927E 02
  4  0    4.899E 01    1.397E-09    5.337E 05   -4.033E 02    4.035E 02
  5  0    5.128E 01    3.679E-08    5.587E 05   -3.396E-01    2.551E-01
  6  0    4.900E 01    2.175E-07    5.338E 05    4.027E 02   -4.031E 02
  7  0    4.224E 01   -2.026E-08    4.601E 05    7.920E 02   -7.925E 02
  8  0    3.133E 01    2.980E-08    3.413E 05    1.146E 03   -1.147E 03
  9  0    1.689E 01   -1.129E-08    1.840E 05    1.425E 03   -1.425E 03
 10  0    1.249-102    4.163-102            0    7.698E 02   -7.700E 02
 11  0            0            0            0            0            0

 -1  1            0            0            0            0            0
  0  1   -1.927E-91   -6.422E-91            0   -1.428E 03    1.428E 03
  1  1    8.489E 02    8.180E 02            0   -2.655E 03    2.656E 03
  2  1    1.303E 03    1.384E 03            0   -2.151E 03    2.152E 03
  3  1    1.558E 03    1.753E 03            0   -1.494E 03    1.495E 03
  4  1    1.690E 03    1.964E 03            0   -7.623E 02    7.625E 02
  5  1    1.730E 03    2.033E 03            0   -5.007E-01    4.694E-01
  6  1    1.690E 03    1.965E 03            0    7.615E 02   -7.618E 02
  7  1    1.559E 03    1.754E 03            0    1.494E 03   -1.494E 03
  8  1    1.305E 03    1.384E 03            0    2.152E 03   -2.152E 03
  9  1    8.487E 02    8.190E 02            0    2.656E 03   -2.656E 03
 10  1   -3.474E-08   -1.042E-08            0    1.428E 03   -1.428E 03
 11  1            0            0            0            0            0

 -1  2            0            0            0            0            0
  0  2    9.381E-93    3.127E-92            0   -1.150E 03    1.150E 03
  1  2    1.411E 03    1.248E 03            0   -2.154E 03    2.155E 03
```

```
  2   2    2.235E 03    2.184E 03              0   -1.769E 03    1.770E 03
  3   2    2.706E 03    2.829E 03              0   -1.240E 03    1.240E 03
  4   2    2.949E 03    3.206E 03              0   -6.359E 02    6.360E 02
  5   2    3.025E 03    3.330E 03              0   -4.743E-01    4.039E-01
  6   2    2.950E 03    3.206E 03              0    6.351E 02   -6.354E 02
  7   2    2.708E 03    2.830E 03              0    1.240E 03   -1.240E 03
  8   2    2.237E 03    2.185E 03              0    1.769E 03   -1.770E 03
  9   2    1.412E 03    1.248E 03              0    2.155E 03   -2.156E 03
 10   2   -6.948E-08   -2.084E-08              0    1.151E 03   -1.151E 03
 11   2            0            0              0            0            0

 -1   3            0            0              0            0            0
  0   3    1.390E-07    4.169E-08              0   -7.958E 02    7.959E 02
  1   3    1.780E 03    1.483E 03              0   -1.497E 03    1.498E 03
  2   3    2.876E 03    2.638E 03              0   -1.241E 03    1.241E 03
  3   3    3.520E 03    3.457E 03              0   -8.767E 02    8.769E 02
  4   3    3.856E 03    3.944E 03              0   -4.517E 02    4.518E 02
  5   3    3.960E 03    4.106E 03              0   -3.159E-01    3.028E-01
  6   3    3.857E 03    3.945E 03              0    4.511E 02   -4.513E 02
  7   3    3.523E 03    3.458E 03              0    8.764E 02   -8.767E 02
  8   3    2.879E 03    2.640E 03              0    1.241E 03   -1.242E 03
  9   3    1.781E 03    1.484E 03              0    1.498E 03   -1.498E 03
 10   3   -1.390E-07   -4.169E-08              0    7.962E 02   -7.963E 02
 11   3            0            0              0            0            0

 -1   4            0            0              0            0            0
  0   4    1.390E-07    4.169E-08              0   -4.052E 02    4.053E 02
  1   4    1.989E 03    1.603E 03              0   -7.643E 02    7.645E 02
  2   4    3.251E 03    2.870E 03              0   -6.367E 02    6.368E 02
  3   4    4.005E 03    3.782E 03              0   -4.518E 02    4.519E 02
  4   4    4.401E 03    4.330E 03              0   -2.335E 02    2.335E 02
  5   4    4.524E 03    4.513E 03              0   -1.682E-01    1.377E-01
  6   4    4.403E 03    4.331E 03              0    2.332E 02   -2.333E 02
  7   4    4.008E 03    3.784E 03              0    4.517E 02   -4.518E 02
  8   4    3.255E 03    2.872E 03              0    6.367E 02   -6.369E 02
  9   4    1.991E 03    1.604E 03              0    7.646E 02   -7.647E 02
 10   4   -1.390E-07   -4.169E-08              0    4.054E 02   -4.055E 02
 11   4            0            0              0            0            0

 -1   5            0            0              0            0            0
  0   5    1.390E-07    4.169E-08              0   -2.349E-01    2.233E-01
  1   5    2.058E 03    1.639E 03              0   -4.592E-01    3.594E-01
  2   5    3.375E 03    2.942E 03              0   -3.828E-01    2.893E-01
  3   5    4.166E 03    3.883E 03              0   -2.579E-01    2.276E-01
  4   5    4.583E 03    4.450E 03              0   -1.384E-01    1.021E-01
  5   5    4.713E 03    4.640E 03              0   -1.144E-03   -2.297E-02
  6   5    4.585E 03    4.451E 03              0    1.525E-01   -1.280E-01
  7   5    4.169E 03    3.884E 03              0    2.860E-01   -2.176E-01
  8   5    3.378E 03    2.943E 03              0    3.698E-01   -3.079E-01
  9   5    2.060E 03    1.640E 03              0    4.322E-01   -3.464E-01
 10   5    4.302E-93    1.434E-92              0    2.339E-01   -2.006E-01
 11   5            0            0              0            0            0

 -1   6            0            0              0            0            0
  0   6    1.390E-07    4.169E-08              0    4.048E 02   -4.049E 02
  1   6    1.990E 03    1.603E 03              0    7.635E 02   -7.639E 02
  2   6    3.252E 03    2.871E 03              0    6.360E 02   -6.363E 02
  3   6    4.006E 03    3.783E 03              0    4.514E 02   -4.516E 02
  4   6    4.402E 03    4.331E 03              0    2.332E 02   -2.334E 02
  5   6    4.525E 03    4.514E 03              0    1.839E-01   -1.636E-01
  6   6    4.403E 03    4.332E 03              0   -2.329E 02    2.331E 02
```

```
  7  6   4.009E 03   3.785E 03              0  -4.512E 02   4.515E 02
  8  6   3.255E 03   2.873E 03              0  -6.361E 02   6.364E 02
  9  6   1.992E 03   1.604E 03              0  -7.638E 02   7.641E 02
 10  6  -1.390E-07  -4.169E-08              0  -4.050E 02   4.051E 02
 11  6           0           0              0           0           0

 -1  7           0           0              0           0           0
  0  7   1.390E-07   4.169E-08              0   7.956E 02  -7.958E 02
  1  7   1.780E 03   1.484E 03              0   1.497E 03  -1.498E 03
  2  7   2.877E 03   2.640E 03              0   1.241E 03  -1.241E 03
  3  7   3.522E 03   3.459E 03              0   8.764E 02  -8.767E 02
  4  7   3.857E 03   3.946E 03              0   4.515E 02  -4.517E 02
  5  7   3.961E 03   4.108E 03              0   3.380E-01  -2.958E-01
  6  7   3.859E 03   3.947E 03              0  -4.510E 02   4.512E 02
  7  7   3.524E 03   3.460E 03              0  -8.762E 02   8.766E 02
  8  7   2.880E 03   2.641E 03              0  -1.241E 03   1.241E 03
  9  7   1.782E 03   1.485E 03              0  -1.498E 03   1.498E 03
 10  7  -2.779E-07  -8.337E-08              0  -7.960E 02   7.961E 02
 11  7           0           0              0           0           0

 -1  8           0           0              0           0           0
  0  8   9.409E-93   3.136E-92              0   1.150E 03  -1.150E 03
  1  8   1.411E 03   1.249E 03              0   2.154E 03  -2.155E 03
  2  8   2.236E 03   2.185E 03              0   1.769E 03  -1.770E 03
  3  8   2.707E 03   2.831E 03              0   1.240E 03  -1.241E 03
  4  8   2.951E 03   3.208E 03              0   6.360E 02  -6.361E 02
  5  8   3.026E 03   3.332E 03              0   4.860E-01  -3.918E-01
  6  8   2.952E 03   3.208E 03              0  -6.352E 02   6.354E 02
  7  8   2.709E 03   2.832E 03              0  -1.240E 03   1.240E 03
  8  8   2.237E 03   2.186E 03              0  -1.769E 03   1.770E 03
  9  8   1.412E 03   1.249E 03              0  -2.155E 03   2.156E 03
 10  8  -6.948E-08  -2.084E-08              0  -1.151E 03   1.151E 03
 11  8           0           0              0           0           0

 -1  9           0           0              0           0           0
  0  9  -1.929E-91  -6.428E-91              0   1.428E 03  -1.428E 03
  1  9   8.492E 02   8.186E 02              0   2.656E 03  -2.656E 03
  2  9   1.304E 03   1.385E 03              0   2.152E 03  -2.153E 03
  3  9   1.559E 03   1.755E 03              0   1.495E 03  -1.495E 03
  4  9   1.691E 03   1.966E 03              0   7.625E 02  -7.627E 02
  5  9   1.731E 03   2.034E 03              0   4.796E-01  -4.537E-01
  6  9   1.691E 03   1.966E 03              0  -7.617E 02   7.620E 02
  7  9   1.560E 03   1.755E 03              0  -1.494E 03   1.495E 03
  8  9   1.305E 03   1.385E 03              0  -2.152E 03   2.153E 03
  9  9   8.490E 02   8.193E 02              0  -2.656E 03   2.657E 03
 10  9  -3.474E-08  -1.042E-08              0  -1.428E 03   1.429E 03
 11  9           0           0              0           0           0

 -1 10           0           0              0           0           0
  0 10   1.976E-09           0   2.151E-05   7.697E 02  -7.700E 02
  1 10   1.689E 01  -2.782E-08   1.840E 05   1.425E 03  -1.425E 03
  2 10   3.135E 01  -3.213E-08   3.414E 05   1.147E 03  -1.147E 03
  3 10   4.226E 01   5.914E-08   4.603E 05   7.926E 02  -7.929E 02
  4 10   4.903E 01  -1.029E-07   5.340E 05   4.034E 02  -4.036E 02
  5 10   5.132E 01  -2.086E-07   5.590E 05   3.144E-01  -2.415E-01
  6 10   4.903E 01   1.346E-07   5.340E 05  -4.029E 02   4.032E 02
  7 10   4.227E 01  -1.094E-07   4.603E 05  -7.923E 02   7.928E 02
  8 10   3.135E 01   6.054E-09   3.414E 05  -1.147E 03   1.147E 03
  9 10   1.690E 01  -3.807E-08   1.841E 05  -1.425E 03   1.426E 03
 10 10  -1.976E-09           0           0  -7.701E 02   7.702E 02
 11 10           0           0           0           0           0
```

```
-1 11          0           0           0           0           0
 0 11          0           0           0           0           0
 1 11          0           0           0           0           0
 2 11          0           0           0           0           0
 3 11          0           0           0           0           0
 4 11          0           0           0           0           0
 5 11          0           0           0           0           0
 6 11          0           0           0           0           0
 7 11          0           0           0           0           0
 8 11          0           0           0           0           0
 9 11          0           0           0           0           0
10 11          0           0           0           0           0
11 11          0           0           0           0           0

   TIME =     3 MINUTES, 18 AND 10/60 SECONDS
```

```
PROGRAM LAYER 8 - MASTER DECK - WB INGRAM, H MATLOCK     REVISION DATE 27 JUL 65
FINAL RUN ON EXAMPLE PROBLEMS
CHG TO CE 05-1023 --CODED BY WBI-- RUN DATE 27 JL 65


PROB
  9.4     REINFORCED SLAB OVER FIVE GIRDERS


TABLE 1.  CONTROL DATA

     NUM CARDS TABLE 3                                                  5
     NUM CARDS TABLE 4                                                  5
     NUM CARDS TABLE 5                                                 10
     NUM CARDS TABLE 6                                                  6
     MAX NUM ITERATIONS                                                50
     NUM INCREMENTS  MX                                                20
     NUM INCREMENTS  MY                                                10
     INCR LENGTH  HX                                            3.600E 01
     INCR LENGTH  HY                                            3.600E 01
     CLOSURE TOLERANCE                                          1.000E-05
     POISSONS RATIO X                                           1.500E-01
     POISSONS RATIO Y                                           1.500E-01

     MONITOR STAS  I,J              10   2       5  6      10 10      10  5


TABLE 2.  RELAXATION CONTROL DATA
   NUM       C L O S U R E   P A R A M E T E R S
 VALUES

    5      3.380E 03 3.350E 04 1.000E 05 5.000E 05 3.000E 04
    5      1.425E 04 1.625E 05 5.550E 05 1.100E 06 1.000E 05
    5      1.000E 04 2.000E 05 4.000E 05 1.000E 07 1.000E 05


TABLE 3. X-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
       FROM  THRU      DX          CX          TX         RX          PX

       0  0 20 10   1.562E 07           0           0           0            0
       1  0 19 10   1.562E 07           0           0           0            0
       0  1 20  9   1.562E 07           0           0           0            0
       1  1 19  9   1.562E 07           0           0           0            0
       1  1 20 10           0    5.333E 07          0           0            0


TABLE 4. Y-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
       FROM  THRU      DY          CY          TY         RY          PY

       0  0 20 10   1.562E 07           0           0           0            0
       1  0 19 10   1.562E 07           0           0           0            0
       0  1 20  9   1.562E 07           0           0           0            0
       1  1 19  9   1.562E 07           0           0           0            0
       1  1 20 10           0    5.333E 07          0           0            0


TABLE 5. Z-BEAM DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE
       FROM  THRU      FZ        TZ        RZ           PZ
```

```
      0  1 20  1   6.690E 10          0          0          0          0
      1  1 19  1   6.690E 10          0          0          0          0
      0  3 20  3   6.690E 10          0          0          0          0
      1  3 19  3   6.690E 10          0          0          0          0
      0  5 20  5   6.690E 10          0          0          0          0
      1  5 19  5   6.690E 10          0          0          0          0
      0  7 20  7   6.690E 10          0          0          0          0
      1  7 19  7   6.690E 10          0          0          0          0
      0  9 20  9   6.690E 10          0          0          0          0
      1  9 19  9   6.690E 10          0          0          0          0

  TABLE 6. LOAD AND SUPPORT DATA, FULL VALUES ADDED AT ALL STAS I,J IN RECTANGLE

      FROM      THRU          Q          S

      9  4  9  4  -1.000E 03          0
      9  6  9  6  -1.000E 03          0
     11  4 11  4  -1.000E 03          0
     11  6 11  6  -1.000E 03          0
      0  0  0 10           0  1.000E 99
     20  0 20 10           0  1.000E 99

      TIME =     3 MINUTES, 20 AND 52/60 SECONDS


  TABLE 7.  MONITOR TALLY AND DEFLS AT 4 STAS

     ITR   FICT       CYC  NOT  NOT       I,J
     NUM   SPRING     NUM  STAB CLOS     10  2         5  6        10 10         10  5

 X    1   3.380E 03    1  119                  0  -2.780E-03  -4.538E-05  -2.289E-03
 Y        1.425E 04    1  205  206  -9.791E-03  -3.100E-04  -3.820E-03  -1.982E-02
 Z        1.000E 04    1   95   95           0           0           0  -1.604E-02

 X    2   3.350E 04    2  209       -1.047E-02  -2.492E-04  -3.772E-03  -1.790E-02
 Y        1.625E 05    2  209  209  -8.941E-03  -6.632E-03  -2.511E-03  -1.770E-02
 Z        2.000E 05    2   93   95           0           0           0  -1.631E-02

 X    3   1.000E 05    3  209       -7.967E-03  -6.980E-03  -2.123E-03  -1.660E-02
 Y        5.550E 05    3  195  190  -5.706E-03  -7.375E-03  -7.276E-04  -1.486E-02
 Z        4.000E 05    3   93   93           0           0           0  -1.622E-02

 X    4   5.000E 05    4  197       -6.522E-03  -7.420E-03  -9.231E-04  -1.615E-02
 Y        1.100E 06    4  189  134  -7.825E-03  -7.505E-03  -1.308E-03  -1.630E-02
 Z        1.000E 07    4   82   87           0           0           0  -1.624E-02

 X    5   3.000E 04    5  200       -7.434E-03  -7.882E-03  -1.119E-03  -1.625E-02
 Y        1.000E 05    5  208  207  -7.646E-03  -8.576E-03  -9.071E-04  -1.619E-02
 Z        1.000E 05    5   93   94           0           0           0  -1.705E-02

 X    6   3.380E 03    1  208       -7.886E-03  -9.927E-03   3.646E-04  -2.025E-02
 Y        1.425E 04    1  209  209  -1.120E-02  -1.370E-02  -1.876E-03  -2.224E-02
 Z        1.000E 04    1   95   95           0           0           0  -2.577E-02

 X    7   3.350E 04    2  209       -1.129E-02  -1.372E-02  -1.783E-03  -2.444E-02
 Y        1.625E 05    2  208  207  -1.339E-02  -1.495E-02  -2.423E-03  -2.557E-02
 Z        2.000E 05    2   93   92           0           0           0  -2.534E-02

 X    8   1.000E 05    3  206       -1.340E-02  -1.504E-02  -2.426E-03  -2.546E-02
```

```
Y        5.550E 05   3  187  169  -1.330E-02  -1.509E-02  -2.375E-03  -2.542E-02
Z        4.000E 05   3   84   81           0           0           0  -2.563E-02

X    9   5.000E 05   4  180       -1.336E-02  -1.511E-02  -2.363E-03  -2.563E-02
Y        1.100E 06   4  160   98  -1.349E-02  -1.515E-02  -2.350E-03  -2.565E-02
Z        1.000E 07   4   56   56           0           0           0  -2.568E-02

X   10   3.000E 04   5  191       -1.367E-02  -1.527E-02  -2.356E-03  -2.597E-02
Y        1.000E 05   5  201  197  -1.398E-02  -1.550E-02  -2.433E-03  -2.634E-02
Z        1.000E 05   5   95   95           0           0           0  -2.675E-02

X   11   3.380E 03   1  209       -1.455E-02  -1.620E-02  -2.265E-03  -2.912E-02
Y        1.425E 04   1  209  209  -1.737E-02  -1.878E-02  -4.437E-03  -3.116E-02
Z        1.000E 04   1   95   95           0           0           0  -3.324E-02

X   12   3.350E 04   2  209       -1.748E-02  -1.880E-02  -4.414E-03  -3.244E-02
Y        1.625E 05   2  208  209  -1.868E-02  -2.000E-02  -4.825E-03  -3.314E-02
Z        2.000E 05   2   95   95           0           0           0  -3.299E-02

X   13   1.000E 05   3  209       -1.872E-02  -2.009E-02  -4.820E-03  -3.306E-02
Y        5.550E 05   3  184  164  -1.867E-02  -2.017E-02  -4.760E-03  -3.307E-02
Z        4.000E 05   3   95   92           0           0           0  -3.317E-02

X   14   5.000E 05   4  179       -1.870E-02  -2.017E-02  -4.761E-03  -3.317E-02
Y        1.100E 06   4  147   63  -1.876E-02  -2.018E-02  -4.770E-03  -3.318E-02
Z        1.000E 07   4   50   53           0           0           0  -3.320E-02

X   15   3.000E 04   5  198       -1.888E-02  -2.030E-02  -4.774E-03  -3.339E-02
Y        1.000E 05   5  205  209  -1.907E-02  -2.050E-02  -4.835E-03  -3.362E-02
Z        1.000E 05   5   95   95           0           0           0  -3.393E-02

X   16   3.380E 03   1  202       -1.949E-02  -2.103E-02  -4.814E-03  -3.565E-02
Y        1.425E 04   1  209  209  -2.174E-02  -2.299E-02  -6.628E-03  -3.716E-02
Z        1.000E 04   1   95   95           0           0           0  -3.873E-02

X   17   3.350E 04   2  209       -2.181E-02  -2.302E-02  -6.624E-03  -3.812E-02
Y        1.625E 05   2  209  209  -2.281E-02  -2.388E-02  -7.098E-03  -3.864E-02
Z        2.000E 05   2   95   95           0           0           0  -3.854E-02

X   18   1.000E 05   3  209       -2.286E-02  -2.394E-02  -7.107E-03  -3.860E-02
Y        5.550E 05   3  159  137  -2.285E-02  -2.399E-02  -7.098E-03  -3.862E-02
Z        4.000E 05   3   93   91           0           0           0  -3.869E-02

X   19   5.000E 05   4  171       -2.286E-02  -2.399E-02  -7.102E-03  -3.868E-02
Y        1.100E 06   4  142   59  -2.290E-02  -2.400E-02  -7.113E-03  -3.869E-02
Z        1.000E 07   4   41   51           0           0           0  -3.871E-02

X   20   3.000E 04   5  199       -2.300E-02  -2.409E-02  -7.146E-03  -3.885E-02
Y        1.000E 05   5  209  209  -2.315E-02  -2.423E-02  -7.242E-03  -3.902E-02
Z        1.000E 05   5   95   95           0           0           0  -3.925E-02

X   21   3.380E 03   1  209       -2.350E-02  -2.461E-02  -7.391E-03  -4.052E-02
Y        1.425E 04   1  209  209  -2.534E-02  -2.611E-02  -8.936E-03  -4.172E-02
Z        1.000E 04   1   95   95           0           0           0  -4.287E-02

X   22   3.350E 04   2  209       -2.539E-02  -2.613E-02  -8.946E-03  -4.242E-02
Y        1.625E 05   2  209  209  -2.617E-02  -2.679E-02  -9.375E-03  -4.280E-02
Z        2.000E 05   2   95   95           0           0           0  -4.273E-02

X   23   1.000E 05   3  209       -2.621E-02  -2.683E-02  -9.388E-03  -4.277E-02
Y        5.550E 05   3  136  136  -2.621E-02  -2.686E-02  -9.402E-03  -4.279E-02
```

```
Z          4.000E 05   3   93   91           0           0           0  -4.284E-02

X   24     5.000E 05   4  173       -2.622E-02  -2.686E-02  -9.406E-03  -4.284E-02
Y          1.100E 06   4  143   56  -2.624E-02  -2.687E-02  -9.417E-03  -4.284E-02
Z          1.000E 07   4   29   33           0           0           0  -4.285E-02

X   25     3.000E 04   5  205       -2.631E-02  -2.694E-02  -9.459E-03  -4.296E-02
Y          1.000E 05   5  209  209  -2.644E-02  -2.705E-02  -9.560E-03  -4.309E-02
Z          1.000E 05   5   95   95           0           0           0  -4.326E-02

X   26     3.380E 03   1  209       -2.672E-02  -2.734E-02  -9.772E-03  -4.422E-02
Y          1.425E 04   1  209  209  -2.822E-02  -2.849E-02  -1.107E-02  -4.516E-02
Z          1.000E 04   1   95   95           0           0           0  -4.602E-02

X   27     3.350E 04   2  209       -2.825E-02  -2.852E-02  -1.109E-02  -4.568E-02
Y          1.625E 05   2  209  209  -2.888E-02  -2.902E-02  -1.147E-02  -4.597E-02
Z          2.000E 05   2   95   95           0           0           0  -4.592E-02

X   28     1.000E 05   3  209       -2.891E-02  -2.905E-02  -1.149E-02  -4.595E-02
Y          5.550E 05   3  124  127  -2.892E-02  -2.907E-02  -1.151E-02  -4.597E-02
Z          4.000E 05   3   91   83           0           0           0  -4.600E-02

X   29     5.000E 05   4  162       -2.892E-02  -2.907E-02  -1.152E-02  -4.600E-02
Y          1.100E 06   4  136   37  -2.893E-02  -2.908E-02  -1.153E-02  -4.600E-02
Z          1.000E 07   4   15   22           0           0           0  -4.601E-02

X   30     3.000E 04   5  201       -2.899E-02  -2.913E-02  -1.157E-02  -4.609E-02
Y          1.000E 05   5  209  209  -2.909E-02  -2.922E-02  -1.167E-02  -4.619E-02
Z          1.000E 05   5   95   95           0           0           0  -4.633E-02

X   31     3.380E 03   1  209       -2.932E-02  -2.943E-02  -1.190E-02  -4.705E-02
Y          1.425E 04   1  209  209  -3.054E-02  -3.034E-02  -1.299E-02  -4.780E-02
Z          1.000E 04   1   95   95           0           0           0  -4.846E-02

X   32     3.350E 04   2  209       -3.057E-02  -3.035E-02  -1.301E-02  -4.820E-02
Y          1.625E 05   2  209  209  -3.107E-02  -3.074E-02  -1.335E-02  -4.842E-02
Z          2.000E 05   2   91   95           0           0           0  -4.838E-02

X   33     1.000E 05   3  209       -3.110E-02  -3.077E-02  -1.336E-02  -4.840E-02
Y          5.550E 05   3  117  108  -3.111E-02  -3.078E-02  -1.339E-02  -4.842E-02
Z          4.000E 05   3   85   81           0           0           0  -4.844E-02

X   34     5.000E 05   4  149       -3.111E-02  -3.078E-02  -1.339E-02  -4.844E-02
Y          1.100E 06   4  117   20  -3.112E-02  -3.079E-02  -1.340E-02  -4.844E-02
Z          1.000E 07   4    0    0           0           0           0  -4.845E-02

X   35     3.000E 04   5  193       -3.117E-02  -3.083E-02  -1.345E-02  -4.851E-02
Y          1.000E 05   5  209  207  -3.125E-02  -3.089E-02  -1.354E-02  -4.859E-02
Z          1.000E 05   5   95   95           0           0           0  -4.869E-02

X   36     3.380E 03   1  209       -3.143E-02  -3.106E-02  -1.377E-02  -4.925E-02
Y          1.425E 04   1  209  209  -3.243E-02  -3.178E-02  -1.467E-02  -4.985E-02
Z          1.000E 04   1   95   95           0           0           0  -5.036E-02

X   37     3.350E 04   2  209       -3.245E-02  -3.179E-02  -1.469E-02  -5.016E-02
Y          1.625E 05   2  209  209  -3.286E-02  -3.209E-02  -1.499E-02  -5.033E-02
Z          2.000E 05   2   87   95           0           0           0  -5.030E-02

X   38     1.000E 05   3  209       -3.288E-02  -3.211E-02  -1.500E-02  -5.031E-02
Y          5.550E 05   3  109   86  -3.289E-02  -3.212E-02  -1.503E-02  -5.033E-02
Z          4.000E 05   3   85   77           0           0           0  -5.035E-02
```

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X    39   5.000E 05   4  126        -3.289E-02  -3.212E-02  -1.503E-02  -5.035E-02
Y         1.100E 06   4   85      2 -3.290E-02  -3.213E-02  -1.504E-02  -5.035E-02
Z         1.000E 07   4    0      0          0           0           0  -5.035E-02

X    40   3.000E 04   5  187        -3.293E-02  -3.216E-02  -1.508E-02  -5.040E-02
Y         1.000E 05   5  209    197 -3.300E-02  -3.221E-02  -1.516E-02  -5.046E-02
Z         1.000E 05   5   95     95          0           0           0  -5.054E-02

X    41   3.380E 03   1  209        -3.315E-02  -3.233E-02  -1.537E-02  -5.098E-02
Y         1.425E 04   1  209    209 -3.397E-02  -3.291E-02  -1.613E-02  -5.145E-02
Z         1.000E 04   1   95     95          0           0           0  -5.185E-02

X    42   3.350E 04   2  209        -3.398E-02  -3.292E-02  -1.614E-02  -5.170E-02
Y         1.625E 05   2  209    209 -3.431E-02  -3.316E-02  -1.639E-02  -5.183E-02
Z         2.000E 05   2   81     95          0           0           0  -5.181E-02

X    43   1.000E 05   3  209        -3.433E-02  -3.317E-02  -1.640E-02  -5.182E-02
Y         5.550E 05   3   94     62 -3.434E-02  -3.318E-02  -1.643E-02  -5.183E-02
Z         4.000E 05   3   83     71          0           0           0  -5.185E-02

X    44   5.000E 05   4  116        -3.434E-02  -3.318E-02  -1.643E-02  -5.185E-02
Y         1.100E 06   4   49      0 -3.434E-02  -3.318E-02  -1.644E-02  -5.185E-02
Z         1.000E 07   4    0      0          0           0           0  -5.185E-02
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PROGRAM LAYER 8 - MASTER DECK - WB INGRAM, H MATLOCK      REVISION DATE 27 JUL 65
FINAL RUN ON EXAMPLE PROBLEMS
CHG TO CE 05-1023 --CODED BY WBI-- RUN DATE 27 JL 65


PROB (CONTD)
  9.4      REINFORCED SLAB OVER FIVE GIRDERS

TABLE 8. DEFLECTION AND ERROR RESULTS -- ITERATION,  44

  I,J     X-DEFL       Y-DEFL       Z-DEFL        REACT    ERROR

  -1 -1    1.441E-03    1.441E-03             0            0            0
   0 -1    8.527E-15    8.527E-15             0   -3.388E-21            0
   1 -1   -1.454E-03   -1.456E-03             0    9.604E-10            0
   2 -1   -2.835E-03   -2.838E-03             0    6.577E-09            0
   3 -1   -4.074E-03   -4.079E-03             0    4.118E-09            0
   4 -1   -5.121E-03   -5.127E-03             0    2.328E-10            0
   5 -1   -5.950E-03   -5.957E-03             0    1.534E-08            0
   6 -1   -6.565E-03   -6.573E-03             0    5.733E-09            0
   7 -1   -6.999E-03   -7.007E-03             0    7.654E-09            0
   8 -1   -7.296E-03   -7.306E-03             0    2.081E-08            0
   9 -1   -7.477E-03   -7.486E-03             0    3.286E-08            0
  10 -1   -7.529E-03   -7.538E-03             0    1.758E-08            0
  11 -1   -7.475E-03   -7.484E-03             0    2.305E-08            0
  12 -1   -7.292E-03   -7.301E-03             0    8.236E-09            0
  13 -1   -6.993E-03   -7.001E-03             0    1.234E-08            0
  14 -1   -6.557E-03   -6.565E-03             0    1.318E-08            0
  15 -1   -5.942E-03   -5.949E-03             0    1.234E-08            0
  16 -1   -5.113E-03   -5.118E-03             0    1.641E-08            0
  17 -1   -4.066E-03   -4.070E-03             0    6.301E-09            0
  18 -1   -2.828E-03   -2.831E-03             0    1.295E-09            0
  19 -1   -1.452E-03   -1.453E-03             0    1.099E-09            0
  20 -1    8.527E-15    8.527E-15             0   -3.388E-21            0
  21 -1    1.438E-03    1.438E-03             0            0            0

  -1  0    2.705E-03    2.705E-03             0   -6.697E-10            0
   0  0    4.989E-98    4.995E-98             0   -4.995E 01    2.817E-02
   1  0   -2.705E-03   -2.706E-03             0    6.194E-01   -6.194E-01
   2  0   -5.323E-03   -5.325E-03             0    1.068E 00   -1.068E 00
   3  0   -7.773E-03   -7.777E-03             0    1.658E 00   -1.658E 00
   4  0   -9.985E-03   -9.989E-03             0    2.201E 00   -2.201E 00
   5  0   -1.190E-02   -1.191E-02             0    2.571E 00   -2.571E 00
   6  0   -1.349E-02   -1.350E-02             0    2.999E 00   -2.999E 00
   7  0   -1.473E-02   -1.474E-02             0    3.367E 00   -3.367E 00
   8  0   -1.563E-02   -1.564E-02             0    3.497E 00   -3.497E 00
   9  0   -1.617E-02   -1.618E-02             0    3.648E 00   -3.648E 00
  10  0   -1.635E-02   -1.636E-02             0    3.793E 00   -3.793E 00
  11  0   -1.617E-02   -1.618E-02             0    3.679E 00   -3.679E 00
  12  0   -1.563E-02   -1.563E-02             0    3.493E 00   -3.493E 00
  13  0   -1.473E-02   -1.474E-02             0    3.372E 00   -3.372E 00
  14  0   -1.348E-02   -1.349E-02             0    3.067E 00   -3.067E 00
  15  0   -1.190E-02   -1.190E-02             0    2.593E 00   -2.593E 00
  16  0   -9.979E-03   -9.983E-03             0    2.200E 00   -2.200E 00
  17  0   -7.768E-03   -7.771E-03             0    1.762E 00   -1.762E 00
  18  0   -5.319E-03   -5.321E-03             0    1.227E 00   -1.227E 00
  19  0   -2.703E-03   -2.704E-03             0    5.049E-01   -5.049E-01
  20  0    4.989E-98    4.996E-98             0   -4.996E 01    3.710E-02
  21  0    2.703E-03    2.703E-03             0   -6.697E-10            0
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-1  1   3.968E-03   3.968E-03   3.969E-03  -1.658E-07           0
 0  1  -2.894E-97  -2.894E-97  -2.895E-97   2.895E 02  -3.682E-02
 1  1  -3.968E-03  -3.969E-03  -3.969E-03   1.093E 00  -1.093E 00
 2  1  -7.838E-03  -7.838E-03  -7.839E-03   2.161E 00  -2.161E 00
 3  1  -1.151E-02  -1.151E-02  -1.151E-02   3.179E 00  -3.179E 00
 4  1  -1.489E-02  -1.490E-02  -1.490E-02   4.120E 00  -4.120E 00
 5  1  -1.791E-02  -1.791E-02  -1.791E-02   4.966E 00  -4.966E 00
 6  1  -2.047E-02  -2.047E-02  -2.047E-02   5.697E 00  -5.697E 00
 7  1  -2.252E-02  -2.252E-02  -2.253E-02   6.282E 00  -6.282E 00
 8  1  -2.402E-02  -2.402E-02  -2.402E-02   6.707E 00  -6.707E 00
 9  1  -2.493E-02  -2.493E-02  -2.493E-02   6.974E 00  -6.974E 00
10  1  -2.523E-02  -2.523E-02  -2.524E-02   7.069E 00  -7.069E 00
11  1  -2.493E-02  -2.493E-02  -2.493E-02   6.979E 00  -6.979E 00
12  1  -2.402E-02  -2.402E-02  -2.402E-02   6.722E 00  -6.722E 00
13  1  -2.252E-02  -2.252E-02  -2.253E-02   6.312E 00  -6.312E 00
14  1  -2.047E-02  -2.047E-02  -2.047E-02   5.738E 00  -5.738E 00
15  1  -1.790E-02  -1.790E-02  -1.791E-02   5.016E 00  -5.016E 00
16  1  -1.489E-02  -1.489E-02  -1.489E-02   4.180E 00  -4.180E 00
17  1  -1.151E-02  -1.151E-02  -1.151E-02   3.241E 00  -3.241E 00
18  1  -7.836E-03  -7.836E-03  -7.837E-03   2.205E 00  -2.205E 00
19  1  -3.968E-03  -3.968E-03  -3.968E-03   1.104E 00  -1.104E 00
20  1  -2.891E-97  -2.892E-97  -2.892E-97   2.892E 02  -2.659E-02
21  1   3.968E-03   3.968E-03   3.968E-03  -2.740E-09           0

-1  2   5.242E-03   5.242E-03           0  -2.740E-09           0
 0  2  -1.332E-98  -1.324E-98           0   1.321E 01   6.799E-02
 1  2  -5.242E-03  -5.243E-03           0   4.962E-01  -4.962E-01
 2  2  -1.037E-02  -1.038E-02           0   9.229E-01  -9.229E-01
 3  2  -1.529E-02  -1.529E-02           0   1.415E 00  -1.415E 00
 4  2  -1.987E-02  -1.987E-02           0   1.811E 00  -1.811E 00
 5  2  -2.401E-02  -2.401E-02           0   2.062E 00  -2.062E 00
 6  2  -2.759E-02  -2.759E-02           0   2.483E 00  -2.483E 00
 7  2  -3.050E-02  -3.050E-02           0   2.787E 00  -2.787E 00
 8  2  -3.262E-02  -3.262E-02           0   2.796E 00  -2.796E 00
 9  2  -3.388E-02  -3.389E-02           0   2.966E 00  -2.966E 00
10  2  -3.434E-02  -3.434E-02           0   3.151E 00  -3.151E 00
11  2  -3.388E-02  -3.389E-02           0   2.953E 00  -2.953E 00
12  2  -3.262E-02  -3.262E-02           0   2.777E 00  -2.777E 00
13  2  -3.050E-02  -3.050E-02           0   2.793E 00  -2.793E 00
14  2  -2.759E-02  -2.759E-02           0   2.512E 00  -2.512E 00
15  2  -2.400E-02  -2.401E-02           0   2.025E 00  -2.025E 00
16  2  -1.987E-02  -1.987E-02           0   1.787E 00  -1.787E 00
17  2  -1.528E-02  -1.529E-02           0   1.487E 00  -1.487E 00
18  2  -1.037E-02  -1.038E-02           0   8.506E-01  -8.506E-01
19  2  -5.242E-03  -5.243E-03           0   4.539E-01  -4.539E-01
20  2  -1.340E-98  -1.325E-98           0   1.325E 01   7.429E-02
21  2   5.242E-03   5.242E-03           0  -2.240E-93           0

-1  3   6.353E-03   6.353E-03   6.354E-03  -3.315E-07           0
 0  3  -3.566E-97  -3.565E-97  -3.567E-97   3.567E 02  -7.711E-02
 1  3  -6.353E-03  -6.354E-03  -6.354E-03   1.241E 00  -1.241E 00
 2  3  -1.259E-02  -1.259E-02  -1.259E-02   2.439E 00  -2.439E 00
 3  3  -1.859E-02  -1.859E-02  -1.860E-02   3.556E 00  -3.556E 00
 4  3  -2.424E-02  -2.424E-02  -2.424E-02   4.574E 00  -4.574E 00
 5  3  -2.940E-02  -2.940E-02  -2.940E-02   5.485E 00  -5.485E 00
 6  3  -3.394E-02  -3.394E-02  -3.395E-02   6.270E 00  -6.270E 00
 7  3  -3.773E-02  -3.773E-02  -3.773E-02   6.896E 00  -6.896E 00
 8  3  -4.061E-02  -4.061E-02  -4.061E-02   7.350E 00  -7.350E 00
 9  3  -4.242E-02  -4.242E-02  -4.243E-02   7.630E 00  -7.630E 00
10  3  -4.304E-02  -4.304E-02  -4.304E-02   7.722E 00  -7.722E 00
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 11   3  -4.242E-02  -4.243E-02  -4.243E-02   7.615E 00  -7.615E 00
 12   3  -4.061E-02  -4.061E-02  -4.061E-02   7.327E 00  -7.327E 00
 13   3  -3.773E-02  -3.773E-02  -3.773E-02   6.869E 00  -6.869E 00
 14   3  -3.394E-02  -3.395E-02  -3.395E-02   6.232E 00  -6.232E 00
 15   3  -2.940E-02  -2.940E-02  -2.940E-02   5.431E 00  -5.431E 00
 16   3  -2.424E-02  -2.424E-02  -2.424E-02   4.504E 00  -4.504E 00
 17   3  -1.859E-02  -1.859E-02  -1.860E-02   3.462E 00  -3.462E 00
 18   3  -1.259E-02  -1.259E-02  -1.259E-02   2.332E 00  -2.332E 00
 19   3  -6.354E-03  -6.354E-03  -6.355E-03   1.158E 00  -1.158E 00
 20   3  -3.568E-97  -3.568E-97  -3.568E-97   3.568E 02   6.701E-03
 21   3   6.354E-03   6.354E-03   6.355E-03  -1.685E-07           0

 -1   4   7.107E-03   7.107E-03           0  -5.481E-09           0
  0   4  -2.799E-98  -2.791E-98           0   2.789E 01   6.234E-02
  1   4  -7.107E-03  -7.109E-03           0   6.047E-01  -6.047E-01
  2   4  -1.410E-02  -1.410E-02           0   1.108E 00  -1.108E 00
  3   4  -2.086E-02  -2.086E-02           0   1.679E 00  -1.679E 00
  4   4  -2.726E-02  -2.727E-02           0   2.159E 00  -2.159E 00
  5   4  -3.317E-02  -3.317E-02           0   2.504E 00  -2.504E 00
  6   4  -3.844E-02  -3.845E-02           0   2.987E 00  -2.987E 00
  7   4  -4.300E-02  -4.301E-02           0   3.332E 00  -3.332E 00
  8   4  -4.690E-02  -4.691E-02           0   3.400E 00  -3.400E 00
  9   4  -5.020E-02  -5.021E-02           0  -9.964E 02  -3.602E 00
 10   4  -5.040E-02  -5.041E-02           0   3.777E 00  -3.777E 00
 11   4  -5.020E-02  -5.021E-02           0  -9.964E 02  -3.586E 00
 12   4  -4.690E-02  -4.691E-02           0   3.410E 00  -3.410E 00
 13   4  -4.300E-02  -4.301E-02           0   3.372E 00  -3.372E 00
 14   4  -3.844E-02  -3.845E-02           0   3.036E 00  -3.036E 00
 15   4  -3.317E-02  -3.317E-02           0   2.504E 00  -2.504E 00
 16   4  -2.726E-02  -2.727E-02           0   2.183E 00  -2.183E 00
 17   4  -2.086E-02  -2.086E-02           0   1.804E 00  -1.804E 00
 18   4  -1.410E-02  -1.410E-02           0   1.093E 00  -1.093E 00
 19   4  -7.108E-03  -7.109E-03           0   5.081E-01  -5.081E-01
 20   4  -2.809E-98  -2.794E-98           0   2.794E 01   7.498E-02
 21   4   7.108E-03   7.108E-03           0  -2.740E-09           0

 -1   5   7.400E-03   7.400E-03   7.401E-03  -3.315E-07           0
  0   5  -3.534E-97  -3.534E-97  -3.535E-97   3.535E 02  -7.772E-02
  1   5  -7.400E-03  -7.401E-03  -7.401E-03   1.282E 00  -1.282E 00
  2   5  -1.469E-02  -1.469E-02  -1.469E-02   2.521E 00  -2.521E 00
  3   5  -2.175E-02  -2.175E-02  -2.175E-02   3.682E 00  -3.682E 00
  4   5  -2.846E-02  -2.846E-02  -2.846E-02   4.747E 00  -4.747E 00
  5   5  -3.468E-02  -3.468E-02  -3.468E-02   5.708E 00  -5.708E 00
  6   5  -4.025E-02  -4.025E-02  -4.026E-02   6.536E 00  -6.536E 00
  7   5  -4.499E-02  -4.499E-02  -4.500E-02   7.200E 00  -7.200E 00
  8   5  -4.868E-02  -4.868E-02  -4.868E-02   7.685E 00  -7.685E 00
  9   5  -5.105E-02  -5.105E-02  -5.105E-02   7.986E 00  -7.986E 00
 10   5  -5.185E-02  -5.185E-02  -5.185E-02   8.091E 00  -8.091E 00
 11   5  -5.105E-02  -5.105E-02  -5.105E-02   7.989E 00  -7.989E 00
 12   5  -4.868E-02  -4.868E-02  -4.868E-02   7.696E 00  -7.696E 00
 13   5  -4.499E-02  -4.499E-02  -4.500E-02   7.223E 00  -7.223E 00
 14   5  -4.025E-02  -4.025E-02  -4.026E-02   6.563E 00  -6.563E 00
 15   5  -3.468E-02  -3.468E-02  -3.468E-02   5.733E 00  -5.733E 00
 16   5  -2.846E-02  -2.846E-02  -2.846E-02   4.769E 00  -4.769E 00
 17   5  -2.175E-02  -2.175E-02  -2.175E-02   3.679E 00  -3.679E 00
 18   5  -1.469E-02  -1.469E-02  -1.469E-02   2.483E 00  -2.483E 00
 19   5  -7.401E-03  -7.401E-03  -7.401E-03   1.237E 00  -1.237E 00
 20   5  -3.537E-97  -3.537E-97  -3.537E-97   3.537E 02   1.444E-02
 21   5   7.401E-03   7.401E-03   7.401E-03  -2.740E-09           0

 -1   6   7.110E-03   7.110E-03           0  -5.481E-09           0
```

```
   0   6  -2.799E-98  -2.791E-98           0   2.789E 01   6.352E-02
   1   6  -7.110E-03  -7.111E-03           0   6.022E-01  -6.022E-01
   2   6  -1.410E-02  -1.411E-02           0   1.111E 00  -1.111E 00
   3   6  -2.087E-02  -2.087E-02           0   1.673E 00  -1.673E 00
   4   6  -2.727E-02  -2.727E-02           0   2.148E 00  -2.148E 00
   5   6  -3.318E-02  -3.318E-02           0   2.509E 00  -2.509E 00
   6   6  -3.846E-02  -3.846E-02           0   2.983E 00  -2.983E 00
   7   6  -4.301E-02  -4.302E-02           0   3.314E 00  -3.314E 00
   8   6  -4.692E-02  -4.692E-02           0   3.402E 00  -3.402E 00
   9   6  -5.022E-02  -5.023E-02           0  -9.964E 02  -3.605E 00
  10   6  -5.042E-02  -5.042E-02           0   3.757E 00  -3.757E 00
  11   6  -5.022E-02  -5.023E-02           0  -9.964E 02  -3.579E 00
  12   6  -4.692E-02  -4.692E-02           0   3.420E 00  -3.420E 00
  13   6  -4.301E-02  -4.302E-02           0   3.359E 00  -3.359E 00
  14   6  -3.846E-02  -3.846E-02           0   3.021E 00  -3.021E 00
  15   6  -3.318E-02  -3.318E-02           0   2.512E 00  -2.512E 00
  16   6  -2.727E-02  -2.727E-02           0   2.181E 00  -2.181E 00
  17   6  -2.087E-02  -2.087E-02           0   1.797E 00  -1.797E 00
  18   6  -1.411E-02  -1.411E-02           0   1.084E 00  -1.084E 00
  19   6  -7.110E-03  -7.111E-03           0   5.203E-01  -5.203E-01
  20   6  -2.809E-98  -2.794E-98           0   2.794E 01   7.474E-02
  21   6   7.110E-03   7.110E-03           0  -2.368E-93           0

  -1   7   6.358E-03   6.358E-03   6.359E-03  -3.315E-07           0
   0   7  -3.569E-97  -3.568E-97  -3.570E-97   3.570E 02  -7.897E-02
   1   7  -6.358E-03  -6.358E-03  -6.359E-03   1.239E 00  -1.239E 00
   2   7  -1.260E-02  -1.260E-02  -1.260E-02   2.434E 00  -2.434E 00
   3   7  -1.861E-02  -1.861E-02  -1.861E-02   3.549E 00  -3.549E 00
   4   7  -2.426E-02  -2.426E-02  -2.426E-02   4.565E 00  -4.565E 00
   5   7  -2.942E-02  -2.942E-02  -2.942E-02   5.476E 00  -5.476E 00
   6   7  -3.397E-02  -3.397E-02  -3.397E-02   6.259E 00  -6.259E 00
   7   7  -3.776E-02  -3.776E-02  -3.776E-02   6.884E 00  -6.884E 00
   8   7  -4.064E-02  -4.064E-02  -4.064E-02   7.338E 00  -7.338E 00
   9   7  -4.245E-02  -4.245E-02  -4.246E-02   7.617E 00  -7.617E 00
  10   7  -4.306E-02  -4.307E-02  -4.307E-02   7.709E 00  -7.709E 00
  11   7  -4.245E-02  -4.245E-02  -4.246E-02   7.603E 00  -7.603E 00
  12   7  -4.064E-02  -4.064E-02  -4.064E-02   7.317E 00  -7.317E 00
  13   7  -3.776E-02  -3.776E-02  -3.776E-02   6.858E 00  -6.858E 00
  14   7  -3.397E-02  -3.397E-02  -3.397E-02   6.221E 00  -6.221E 00
  15   7  -2.942E-02  -2.942E-02  -2.942E-02   5.424E 00  -5.424E 00
  16   7  -2.426E-02  -2.426E-02  -2.426E-02   4.499E 00  -4.499E 00
  17   7  -1.861E-02  -1.861E-02  -1.861E-02   3.456E 00  -3.456E 00
  18   7  -1.260E-02  -1.260E-02  -1.260E-02   2.328E 00  -2.328E 00
  19   7  -6.358E-03  -6.358E-03  -6.359E-03   1.158E 00  -1.158E 00
  20   7  -3.571E-97  -3.571E-97  -3.571E-97   3.571E 02  -1.506E-02
  21   7   6.358E-03   6.358E-03   6.359E-03  -1.658E-07           0

  -1   8   5.249E-03   5.249E-03           0  -2.740E-09           0
   0   8  -1.330E-98  -1.324E-98           0   1.321E 01   5.816E-02
   1   8  -5.249E-03  -5.250E-03           0   4.839E-01  -4.839E-01
   2   8  -1.039E-02  -1.039E-02           0   9.419E-01  -9.419E-01
   3   8  -1.531E-02  -1.531E-02           0   1.405E 00  -1.405E 00
   4   8  -1.989E-02  -1.990E-02           0   1.787E 00  -1.787E 00
   5   8  -2.404E-02  -2.404E-02           0   2.087E 00  -2.087E 00
   6   8  -2.763E-02  -2.763E-02           0   2.481E 00  -2.481E 00
   7   8  -3.054E-02  -3.054E-02           0   2.745E 00  -2.745E 00
   8   8  -3.266E-02  -3.267E-02           0   2.814E 00  -2.814E 00
   9   8  -3.393E-02  -3.393E-02           0   2.986E 00  -2.986E 00
  10   8  -3.438E-02  -3.439E-02           0   3.103E 00  -3.103E 00
  11   8  -3.393E-02  -3.393E-02           0   2.946E 00  -2.946E 00
  12   8  -3.266E-02  -3.267E-02           0   2.818E 00  -2.818E 00
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13   8  -3.054E-02  -3.054E-02           0   2.764E 00  -2.764E 00
14   8  -2.762E-02  -2.763E-02           0   2.478E 00  -2.478E 00
15   8  -2.404E-02  -2.404E-02           0   2.057E 00  -2.057E 00
16   8  -1.989E-02  -1.990E-02           0   1.793E 00  -1.793E 00
17   8  -1.531E-02  -1.531E-02           0   1.469E 00  -1.469E 00
18   8  -1.039E-02  -1.039E-02           0   8.079E-01  -8.079E-01
19   8  -5.249E-03  -5.250E-03           0   5.155E-01  -5.155E-01
20   8  -1.335E-98  -1.323E-98           0   1.323E 01   6.080E-02
21   8   5.249E-03   5.249E-03           0  -2.243E-93           0

-1   9   3.978E-03   3.978E-03   3.979E-03  -1.658E-07           0
 0   9  -2.902E-97  -2.901E-97  -2.901E-97   2.901E 02  -1.715E-02
 1   9  -3.978E-03  -3.978E-03  -3.979E-03   1.083E 00  -1.083E 00
 2   9  -7.857E-03  -7.857E-03  -7.858E-03   2.143E 00  -2.143E 00
 3   9  -1.154E-02  -1.154E-02  -1.154E-02   3.152E 00  -3.152E 00
 4   9  -1.493E-02  -1.493E-02  -1.493E-02   4.085E 00  -4.085E 00
 5   9  -1.795E-02  -1.795E-02  -1.795E-02   4.925E 00  -4.925E 00
 6   9  -2.052E-02  -2.052E-02  -2.052E-02   5.648E 00  -5.648E 00
 7   9  -2.258E-02  -2.258E-02  -2.258E-02   6.228E 00  -6.228E 00
 8   9  -2.408E-02  -2.408E-02  -2.408E-02   6.652E 00  -6.652E 00
 9   9  -2.499E-02  -2.499E-02  -2.499E-02   6.916E 00  -6.916E 00
10   9  -2.530E-02  -2.530E-02  -2.530E-02   7.008E 00  -7.008E 00
11   9  -2.499E-02  -2.499E-02  -2.499E-02   6.921E 00  -6.921E 00
12   9  -2.408E-02  -2.408E-02  -2.408E-02   6.669E 00  -6.669E 00
13   9  -2.258E-02  -2.258E-02  -2.258E-02   6.259E 00  -6.259E 00
14   9  -2.052E-02  -2.052E-02  -2.052E-02   5.689E 00  -5.689E 00
15   9  -1.795E-02  -1.795E-02  -1.795E-02   4.977E 00  -4.977E 00
16   9  -1.493E-02  -1.493E-02  -1.493E-02   4.148E 00  -4.148E 00
17   9  -1.154E-02  -1.154E-02  -1.154E-02   3.209E 00  -3.209E 00
18   9  -7.855E-03  -7.855E-03  -7.856E-03   2.186E 00  -2.186E 00
19   9  -3.977E-03  -3.977E-03  -3.978E-03   1.100E 00  -1.100E 00
20   9  -2.899E-97  -2.898E-97  -2.899E-97   2.899E 02  -1.906E-02
21   9   3.977E-03   3.977E-03   3.978E-03  -2.740E-09           0

-1  10   2.717E-03   2.717E-03           0  -6.697E-10           0
 0  10   4.982E-98   4.982E-98           0  -4.982E 01  -1.093E-03
 1  10  -2.717E-03  -2.718E-03           0   5.846E-01  -5.846E-01
 2  10  -5.348E-03  -5.350E-03           0   1.069E 00  -1.069E 00
 3  10  -7.810E-03  -7.813E-03           0   1.625E 00  -1.625E 00
 4  10  -1.003E-02  -1.004E-02           0   2.136E 00  -2.136E 00
 5  10  -1.196E-02  -1.196E-02           0   2.539E 00  -2.539E 00
 6  10  -1.355E-02  -1.356E-02           0   2.954E 00  -2.954E 00
 7  10  -1.480E-02  -1.481E-02           0   3.276E 00  -3.276E 00
 8  10  -1.571E-02  -1.571E-02           0   3.439E 00  -3.439E 00
 9  10  -1.625E-02  -1.626E-02           0   3.603E 00  -3.603E 00
10  10  -1.643E-02  -1.644E-02           0   3.698E 00  -3.698E 00
11  10  -1.625E-02  -1.626E-02           0   3.601E 00  -3.601E 00
12  10  -1.570E-02  -1.571E-02           0   3.460E 00  -3.460E 00
13  10  -1.480E-02  -1.481E-02           0   3.300E 00  -3.300E 00
14  10  -1.355E-02  -1.356E-02           0   2.991E 00  -2.991E 00
15  10  -1.195E-02  -1.196E-02           0   2.565E 00  -2.565E 00
16  10  -1.003E-02  -1.003E-02           0   2.154E 00  -2.154E 00
17  10  -7.804E-03  -7.808E-03           0   1.765E 00  -1.765E 00
18  10  -5.344E-03  -5.346E-03           0   1.128E 00  -1.128E 00
19  10  -2.715E-03  -2.716E-03           0   5.686E-01  -5.686E-01
20  10   4.985E-98   4.986E-98           0  -4.986E 01   6.529E-03
21  10   2.715E-03   2.715E-03           0  -1.339E-09           0

-1  11   1.456E-03   1.456E-03           0           0           0
 0  11   8.527E-15   8.527E-15           0  -3.388E-21           0
 1  11  -1.470E-03  -1.472E-03           0   1.506E-09           0
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   2  11  -2.866E-03  -2.869E-03           0   5.763E-09           0
   3  11  -4.119E-03  -4.124E-03           0   1.206E-08           0
   4  11  -5.180E-03  -5.186E-03           0  -7.945E-09           0
   5  11  -6.021E-03  -6.027E-03           0  -5.180E-09           0
   6  11  -6.645E-03  -6.653E-03           0   2.273E-08           0
   7  11  -7.087E-03  -7.096E-03           0   2.029E-08           0
   8  11  -7.391E-03  -7.400E-03           0   6.024E-09           0
   9  11  -7.575E-03  -7.584E-03           0   5.501E-09           0
  10  11  -7.628E-03  -7.637E-03           0   3.507E-08           0
  11  11  -7.573E-03  -7.582E-03           0   1.478E-08           0
  12  11  -7.387E-03  -7.395E-03           0   6.577E-09           0
  13  11  -7.081E-03  -7.089E-03           0   1.068E-08           0
  14  11  -6.638E-03  -6.645E-03           0  -2.416E-09           0
  15  11  -6.012E-03  -6.019E-03           0   9.895E-09           0
  16  11  -5.171E-03  -5.176E-03           0  -7.858E-10           0
  17  11  -4.111E-03  -4.115E-03           0  -2.459E-09           0
  18  11  -2.859E-03  -2.862E-03           0   3.580E-09           0
  19  11  -1.467E-03  -1.468E-03           0   4.664E-09           0
  20  11   3.891E-97   1.705E-14           0  -6.776E-21           0
  21  11   1.453E-03   1.453E-03           0           0           0

TABLE 9. MOMENTS -- ITERATION  44
   I,J    X-MOM       Y-MOM       Z-MOM              TX-MOM    TY-MOM

  -1  -1           0           0           0           0           0
   0  -1           0           0           0           0           0
   1  -1           0           0           0           0           0
   2  -1           0           0           0           0           0
   3  -1           0           0           0           0           0
   4  -1           0           0           0           0           0
   5  -1           0           0           0           0           0
   6  -1           0           0           0           0           0
   7  -1           0           0           0           0           0
   8  -1           0           0           0           0           0
   9  -1           0           0           0           0           0
  10  -1           0           0           0           0           0
  11  -1           0           0           0           0           0
  12  -1           0           0           0           0           0
  13  -1           0           0           0           0           0
  14  -1           0           0           0           0           0
  15  -1           0           0           0           0           0
  16  -1           0           0           0           0           0
  17  -1           0           0           0           0           0
  18  -1           0           0           0           0           0
  19  -1           0           0           0           0           0
  20  -1           0           0           0           0           0
  21  -1           0           0           0           0           0

  -1   0           0           0           0           0           0
   0   0  -6.697E-10  -3.388E-21           0  -6.483E 00   6.481E 00
   1   0   2.026E 00   9.604E-10           0  -1.286E 01   1.286E 01
   2   0   3.968E 00   6.577E-09           0  -1.266E 01   1.265E 01
   3   0   5.630E 00   4.118E-09           0  -1.227E 01   1.226E 01
   4   0   6.924E 00   2.328E-10           0  -1.162E 01   1.161E 01
   5   0   7.773E 00   1.534E-08           0  -1.062E 01   1.062E 01
   6   0   8.112E 00   5.733E-09           0  -9.178E 00   9.175E 00
   7   0   8.162E 00   7.654E-09           0  -7.249E 00   7.245E 00
   8   0   8.326E 00   2.081E-08           0  -4.955E 00   4.953E 00
   9   0   8.606E 00   3.286E-08           0  -2.523E 00   2.523E 00
  10   0   8.442E 00   1.758E-08           0  -3.912E-03   3.946E-03
  11   0   8.598E 00   2.305E-08           0   2.516E 00  -2.516E 00
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12  0   8.325E 00   8.236E-09           0   4.950E 00  -4.947E 00
13  0   8.158E 00   1.234E-08           0   7.243E 00  -7.239E 00
14  0   8.094E 00   1.318E-08           0   9.174E 00  -9.170E 00
15  0   7.766E 00   1.234E-08           0   1.062E 01  -1.062E 01
16  0   6.922E 00   1.641E-08           0   1.162E 01  -1.161E 01
17  0   5.602E 00   6.301E-09           0   1.227E 01  -1.226E 01
18  0   3.930E 00   1.295E-09           0   1.267E 01  -1.266E 01
19  0   2.052E 00   1.099E-09           0   1.288E 01  -1.287E 01
20  0  -6.697E-10  -3.388E-21           0   6.486E 00  -6.485E 00
21  0           0           0           0           0           0

-1  1           0           0           0           0           0
 0  1  -2.740E-09  -4.111E-10  -5.869E-06  -1.305E 01   1.305E 01
 1  1   4.708E 00   1.567E-01   1.027E 04  -2.597E 01   2.597E 01
 2  1   9.285E 00   1.926E-01   2.028E 04  -2.558E 01   2.558E 01
 3  1   1.359E 01  -7.192E-03   2.978E 04  -2.484E 01   2.484E 01
 4  1   1.749E 01  -6.045E-01   3.852E 04  -2.361E 01   2.361E 01
 5  1   2.083E 01  -1.769E 00   4.623E 04  -2.169E 01   2.169E 01
 6  1   2.350E 01  -3.521E 00   5.265E 04  -1.883E 01   1.883E 01
 7  1   2.544E 01  -5.407E 00   5.753E 04  -1.485E 01   1.485E 01
 8  1   2.677E 01  -6.275E 00   6.072E 04  -9.992E 00   9.991E 00
 9  1   2.761E 01  -5.734E 00   6.238E 04  -5.130E 00   5.131E 00
10  1   2.772E 01  -6.879E 00   6.300E 04  -7.361E-03   6.909E-03
11  1   2.760E 01  -5.728E 00   6.237E 04   5.117E 00  -5.118E 00
12  1   2.677E 01  -6.266E 00   6.070E 04   9.982E 00  -9.980E 00
13  1   2.543E 01  -5.391E 00   5.751E 04   1.484E 01  -1.484E 01
14  1   2.349E 01  -3.496E 00   5.262E 04   1.883E 01  -1.883E 01
15  1   2.082E 01  -1.741E 00   4.620E 04   2.169E 01  -2.169E 01
16  1   1.748E 01  -5.688E-01   3.849E 04   2.361E 01  -2.361E 01
17  1   1.358E 01   4.322E-02   2.975E 04   2.485E 01  -2.485E 01
18  1   9.281E 00   2.341E-01   2.026E 04   2.560E 01  -2.560E 01
19  1   4.707E 00   1.653E-01   1.026E 04   2.600E 01  -2.599E 01
20  1  -2.740E-09  -4.111E-10           0   1.306E 01  -1.306E 01
21  1           0           0           0           0           0

-1  2           0           0           0           0           0
 0  2  -2.740E-09  -4.111E-10           0  -1.227E 01   1.227E 01
 1  2   6.498E 00   8.704E 00           0  -2.444E 01   2.444E 01
 2  2   1.297E 01   1.717E 01           0  -2.416E 01   2.416E 01
 3  2   1.935E 01   2.515E 01           0  -2.361E 01   2.361E 01
 4  2   2.565E 01   3.237E 01           0  -2.269E 01   2.269E 01
 5  2   3.194E 01   3.835E 01           0  -2.124E 01   2.124E 01
 6  2   3.812E 01   4.218E 01           0  -1.909E 01   1.909E 01
 7  2   4.356E 01   4.208E 01           0  -1.601E 01   1.601E 01
 8  2   4.566E 01   3.592E 01           0  -1.177E 01   1.177E 01
 9  2   4.208E 01   2.619E 01           0  -6.243E 00   6.243E 00
10  2   4.689E 01   2.666E 01           0  -4.797E-03   4.752E-03
11  2   4.208E 01   2.621E 01           0   6.234E 00  -6.234E 00
12  2   4.566E 01   3.594E 01           0   1.176E 01  -1.176E 01
13  2   4.356E 01   4.210E 01           0   1.601E 01  -1.601E 01
14  2   3.811E 01   4.222E 01           0   1.909E 01  -1.909E 01
15  2   3.195E 01   3.839E 01           0   2.124E 01  -2.124E 01
16  2   2.566E 01   3.241E 01           0   2.269E 01  -2.269E 01
17  2   1.933E 01   2.520E 01           0   2.362E 01  -2.362E 01
18  2   1.300E 01   1.721E 01           0   2.417E 01  -2.417E 01
19  2   6.515E 00   8.727E 00           0   2.445E 01  -2.445E 01
20  2  -2.240E-93  -1.493E-92           0   1.227E 01  -1.227E 01
21  2           0           0           0           0           0

-1  3           0           0           0           0           0
 0  3  -5.481E-09  -8.221E-10  -1.174E-05  -9.594E 00   9.594E 00
```

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 1  3   8.179E 00   1.797E 01   1.202E 04  -1.917E 01   1.917E 01
 2  3   1.640E 01   3.558E 01   2.424E 04  -1.908E 01   1.908E 01
 3  3   2.468E 01   5.246E 01   3.684E 04  -1.887E 01   1.887E 01
 4  3   3.304E 01   6.823E 01   4.998E 04  -1.845E 01   1.845E 01
 5  3   4.148E 01   8.251E 01   6.377E 04  -1.779E 01   1.779E 01
 6  3   4.990E 01   9.445E 01   7.830E 04  -1.718E 01   1.718E 01
 7  3   5.782E 01   1.005E 02   9.366E 04  -1.764E 01   1.764E 01
 8  3   6.343E 01   8.897E 01   1.097E 05  -1.964E 01   1.964E 01
 9  3   6.349E 01   4.489E 01   1.244E 05  -9.153E 00   9.153E 00
10  3   6.850E 01   7.250E 01   1.263E 05  -2.037E-03   1.996E-03
11  3   6.349E 01   4.490E 01   1.244E 05   9.149E 00  -9.149E 00
12  3   6.343E 01   8.900E 01   1.097E 05   1.964E 01  -1.964E 01
13  3   5.783E 01   1.006E 02   9.366E 04   1.764E 01  -1.764E 01
14  3   4.991E 01   9.450E 01   7.830E 04   1.717E 01  -1.718E 01
15  3   4.149E 01   8.256E 01   6.378E 04   1.779E 01  -1.779E 01
16  3   3.306E 01   6.829E 01   4.999E 04   1.845E 01  -1.845E 01
17  3   2.469E 01   5.251E 01   3.685E 04   1.887E 01  -1.887E 01
18  3   1.641E 01   3.562E 01   2.424E 04   1.908E 01  -1.908E 01
19  3   8.186E 00   1.799E 01   1.202E 04   1.917E 01  -1.917E 01
20  3  -5.481E-09  -8.221E-10  -5.869E-06   9.596E 00  -9.597E 00
21  3           0           0           0           0           0

-1  4           0           0           0           0           0
 0  4  -5.481E-09  -8.221E-10           0  -5.384E 00   5.384E 00
 1  4   8.868E 00   2.317E 01           0  -1.079E 01   1.079E 01
 2  4   1.793E 01   4.630E 01           0  -1.086E 01   1.086E 01
 3  4   2.731E 01   6.929E 01           0  -1.092E 01   1.092E 01
 4  4   3.708E 01   9.188E 01           0  -1.091E 01   1.091E 01
 5  4   4.678E 01   1.137E 02           0  -1.073E 01   1.073E 01
 6  4   5.419E 01   1.355E 02           0  -1.019E 01   1.019E 01
 7  4   5.546E 01   1.635E 02           0  -9.051E 00   9.051E 00
 8  4   6.164E 01   2.228E 02           0  -7.002E 00   7.002E 00
 9  4   1.999E 02   3.575E 02           0  -3.818E 00   3.818E 00
10  4   6.192E 01   2.889E 02           0  -1.503E-04   1.506E-04
11  4   1.999E 02   3.575E 02           0   3.818E 00  -3.818E 00
12  4   6.164E 01   2.228E 02           0   7.001E 00  -7.001E 00
13  4   5.546E 01   1.635E 02           0   9.051E 00  -9.051E 00
14  4   5.418E 01   1.355E 02           0   1.019E 01  -1.019E 01
15  4   4.679E 01   1.137E 02           0   1.073E 01  -1.073E 01
16  4   3.709E 01   9.188E 01           0   1.091E 01  -1.091E 01
17  4   2.729E 01   6.929E 01           0   1.092E 01  -1.092E 01
18  4   1.795E 01   4.631E 01           0   1.086E 01  -1.086E 01
19  4   8.903E 00   2.318E 01           0   1.079E 01  -1.079E 01
20  4  -2.740E-09  -4.111E-10           0   5.384E 00  -5.384E 00
21  4           0           0           0           0           0

-1  5           0           0           0           0           0
 0  5  -5.481E-09  -8.221E-10  -1.174E-05  -1.141E-02   1.139E-02
 1  5   9.581E 00   2.883E 01   1.152E 04  -2.248E-02   2.251E-02
 2  5   1.941E 01   5.800E 01   2.348E 04  -2.174E-02   2.171E-02
 3  5   2.975E 01   8.778E 01   3.633E 04  -2.043E-02   2.029E-02
 4  5   4.085E 01   1.184E 02   5.060E 04  -1.828E-02   1.839E-02
 5  5   5.297E 01   1.496E 02   6.690E 04  -1.601E-02   1.610E-02
 6  5   6.614E 01   1.793E 02   8.599E 04  -1.360E-02   1.336E-02
 7  5   7.941E 01   1.983E 02   1.088E 05  -1.024E-02   1.028E-02
 8  5   8.885E 01   1.793E 02   1.357E 05  -6.765E-03   6.994E-03
 9  5   8.776E 01   9.133E 01   1.623E 05  -3.744E-03   3.528E-03
10  5   9.773E 01   1.495E 02   1.650E 05  -5.525E-05  -6.029E-05
11  5   8.776E 01   9.133E 01   1.623E 05   3.935E-03  -3.621E-03
12  5   8.885E 01   1.793E 02   1.357E 05   7.004E-03  -7.059E-03
13  5   7.941E 01   1.983E 02   1.088E 05   1.007E-02  -1.033E-02
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 14   5    6.614E 01    1.793E 02    8.600E 04    1.360E-02   -1.338E-02
 15   5    5.297E 01    1.496E 02    6.690E 04    1.626E-02   -1.612E-02
 16   5    4.085E 01    1.184E 02    5.060E 04    1.821E-02   -1.837E-02
 17   5    2.975E 01    8.778E 01    3.633E 04    2.015E-02   -2.021E-02
 18   5    1.942E 01    5.799E 01    2.348E 04    2.183E-02   -2.163E-02
 19   5    9.585E 00    2.882E 01    1.153E 04    2.250E-02   -2.241E-02
 20   5   -2.740E-09   -4.111E-10            0    1.121E-02   -1.127E-02
 21   5            0            0            0            0            0

 -1   6            0            0            0            0            0
  0   6   -5.481E-09   -8.221E-10            0    5.361E 00   -5.361E 00
  1   6    8.871E 00    2.317E 01            0    1.075E 01   -1.075E 01
  2   6    1.794E 01    4.629E 01            0    1.081E 01   -1.081E 01
  3   6    2.731E 01    6.928E 01            0    1.088E 01   -1.088E 01
  4   6    3.709E 01    9.186E 01            0    1.088E 01   -1.088E 01
  5   6    4.679E 01    1.137E 02            0    1.069E 01   -1.069E 01
  6   6    5.420E 01    1.355E 02            0    1.016E 01   -1.016E 01
  7   6    5.547E 01    1.635E 02            0    9.030E 00   -9.030E 00
  8   6    6.165E 01    2.228E 02            0    6.987E 00   -6.987E 00
  9   6    2.000E 02    3.575E 02            0    3.811E 00   -3.811E 00
 10   6    6.193E 01    2.889E 02            0    2.256E-04   -2.510E-04
 11   6    2.000E 02    3.575E 02            0   -3.811E 00    3.811E 00
 12   6    6.165E 01    2.228E 02            0   -6.987E 00    6.987E 00
 13   6    5.547E 01    1.635E 02            0   -9.030E 00    9.030E 00
 14   6    5.420E 01    1.355E 02            0   -1.016E 01    1.016E 01
 15   6    4.680E 01    1.137E 02            0   -1.069E 01    1.070E 01
 16   6    3.710E 01    9.186E 01            0   -1.088E 01    1.088E 01
 17   6    2.729E 01    6.928E 01            0   -1.088E 01    1.088E 01
 18   6    1.795E 01    4.630E 01            0   -1.081E 01    1.081E 01
 19   6    8.902E 00    2.318E 01            0   -1.075E 01    1.075E 01
 20   6   -2.368E-93   -1.579E-92            0   -5.361E 00    5.361E 00
 21   6            0            0            0            0            0

 -1   7            0            0            0            0            0
  0   7   -5.481E-09   -8.221E-10   -1.174E-05    9.569E 00   -9.569E 00
  1   7    8.183E 00    1.796E 01    1.203E 04    1.912E 01   -1.912E 01
  2   7    1.641E 01    3.557E 01    2.426E 04    1.903E 01   -1.903E 01
  3   7    2.469E 01    5.243E 01    3.687E 04    1.882E 01   -1.883E 01
  4   7    3.306E 01    6.820E 01    5.003E 04    1.841E 01   -1.841E 01
  5   7    4.150E 01    8.247E 01    6.383E 04    1.775E 01   -1.776E 01
  6   7    4.992E 01    9.440E 01    7.836E 04    1.715E 01   -1.715E 01
  7   7    5.784E 01    1.005E 02    9.373E 04    1.762E 01   -1.762E 01
  8   7    6.345E 01    8.892E 01    1.098E 05    1.963E 01   -1.963E 01
  9   7    6.351E 01    4.483E 01    1.244E 05    9.145E 00   -9.146E 00
 10   7    6.852E 01    7.245E 01    1.263E 05    1.963E-03   -2.129E-03
 11   7    6.352E 01    4.485E 01    1.244E 05   -9.141E 00    9.142E 00
 12   7    6.346E 01    8.894E 01    1.098E 05   -1.962E 01    1.962E 01
 13   7    5.785E 01    1.005E 02    9.373E 04   -1.761E 01    1.761E 01
 14   7    4.993E 01    9.445E 01    7.836E 04   -1.715E 01    1.715E 01
 15   7    4.151E 01    8.252E 01    6.383E 04   -1.775E 01    1.776E 01
 16   7    3.307E 01    6.825E 01    5.003E 04   -1.841E 01    1.841E 01
 17   7    2.470E 01    5.248E 01    3.688E 04   -1.883E 01    1.883E 01
 18   7    1.642E 01    3.560E 01    2.427E 04   -1.903E 01    1.904E 01
 19   7    8.190E 00    1.798E 01    1.203E 04   -1.912E 01    1.912E 01
 20   7   -2.740E-09   -4.111E-10   -5.869E-06   -9.572E 00    9.572E 00
 21   7            0            0            0            0            0

 -1   8            0            0            0            0            0
  0   8   -2.740E-09   -4.111E-10            0    1.224E 01   -1.224E 01
  1   8    6.508E 00    8.696E 00            0    2.439E 01   -2.439E 01
  2   8    1.298E 01    1.715E 01            0    2.411E 01   -2.411E 01
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  3  8   1.937E 01   2.513E 01           0   2.356E 01  -2.356E 01
  4  8   2.568E 01   3.234E 01           0   2.264E 01  -2.264E 01
  5  8   3.196E 01   3.831E 01           0   2.120E 01  -2.120E 01
  6  8   3.815E 01   4.214E 01           0   1.906E 01  -1.906E 01
  7  8   4.361E 01   4.203E 01           0   1.599E 01  -1.599E 01
  8  8   4.569E 01   3.587E 01           0   1.176E 01  -1.176E 01
  9  8   4.211E 01   2.614E 01           0   6.235E 00  -6.235E 00
 10  8   4.695E 01   2.661E 01           0   4.918E-03  -4.895E-03
 11  8   4.212E 01   2.616E 01           0  -6.225E 00   6.225E 00
 12  8   4.569E 01   3.589E 01           0  -1.175E 01   1.175E 01
 13  8   4.360E 01   4.206E 01           0  -1.598E 01   1.598E 01
 14  8   3.815E 01   4.218E 01           0  -1.905E 01   1.905E 01
 15  8   3.197E 01   3.836E 01           0  -2.120E 01   2.120E 01
 16  8   2.568E 01   3.238E 01           0  -2.265E 01   2.265E 01
 17  8   1.936E 01   2.518E 01           0  -2.357E 01   2.357E 01
 18  8   1.302E 01   1.720E 01           0  -2.412E 01   2.412E 01
 19  8   6.506E 00   8.717E 00           0  -2.440E 01   2.440E 01
 20  8  -2.243E-93  -1.496E-92           0  -1.224E 01   1.224E 01
 21  8           0           0           0           0           0

 -1  9           0           0           0           0           0
  0  9  -2.740E-09  -4.111E-10  -5.869E-06   1.302E 01  -1.302E 01
  1  9   4.719E 00   1.476E-01   1.029E 04   2.592E 01  -2.592E 01
  2  9   9.304E 00   1.775E-01   2.033E 04   2.553E 01  -2.553E 01
  3  9   1.362E 01  -3.110E-02   2.985E 04   2.479E 01  -2.479E 01
  4  9   1.752E 01  -6.361E-01   3.861E 04   2.357E 01  -2.356E 01
  5  9   2.088E 01  -1.803E 00   4.634E 04   2.165E 01  -2.165E 01
  6  9   2.355E 01  -3.561E 00   5.277E 04   1.880E 01  -1.880E 01
  7  9   2.550E 01  -5.454E 00   5.767E 04   1.483E 01  -1.483E 01
  8  9   2.683E 01  -6.322E 00   6.087E 04   9.975E 00  -9.974E 00
  9  9   2.767E 01  -5.781E 00   6.253E 04   5.122E 00  -5.122E 00
 10  9   2.778E 01  -6.930E 00   6.316E 04   7.233E-03  -6.795E-03
 11  9   2.767E 01  -5.777E 00   6.252E 04  -5.108E 00   5.109E 00
 12  9   2.683E 01  -6.310E 00   6.085E 04  -9.963E 00   9.962E 00
 13  9   2.549E 01  -5.435E 00   5.765E 04  -1.482E 01   1.482E 01
 14  9   2.354E 01  -3.537E 00   5.275E 04  -1.879E 01   1.879E 01
 15  9   2.087E 01  -1.774E 00   4.631E 04  -2.165E 01   2.165E 01
 16  9   1.752E 01  -5.935E-01   3.858E 04  -2.357E 01   2.356E 01
 17  9   1.361E 01   1.949E-02   2.982E 04  -2.480E 01   2.480E 01
 18  9   9.302E 00   2.173E-01   2.031E 04  -2.554E 01   2.554E 01
 19  9   4.715E 00   1.624E-01   1.028E 04  -2.594E 01   2.594E 01
 20  9  -2.740E-09  -4.111E-10           0  -1.303E 01   1.303E 01
 21  9           0           0           0           0           0

 -1 10           0           0           0           0           0
  0 10  -6.697E-10  -3.388E-21           0   6.467E 00  -6.465E 00
  1 10   2.039E 00   1.506E-09           0   1.283E 01  -1.283E 01
  2 10   3.978E 00   5.763E-09           0   1.263E 01  -1.262E 01
  3 10   5.651E 00   1.206E-08           0   1.224E 01  -1.223E 01
  4 10   6.957E 00  -7.945E-09           0   1.160E 01  -1.159E 01
  5 10   7.802E 00  -5.180E-09           0   1.060E 01  -1.060E 01
  6 10   8.147E 00   2.273E-08           0   9.160E 00  -9.156E 00
  7 10   8.210E 00   2.029E-08           0   7.235E 00  -7.232E 00
  8 10   8.368E 00   6.024E-09           0   4.946E 00  -4.944E 00
  9 10   8.646E 00   5.501E-09           0   2.518E 00  -2.518E 00
 10 10   8.495E 00   3.507E-08           0   3.676E-03  -3.654E-03
 11 10   8.646E 00   1.478E-08           0  -2.511E 00   2.511E 00
 12 10   8.362E 00   6.577E-09           0  -4.940E 00   4.937E 00
 13 10   8.202E 00   1.068E-08           0  -7.230E 00   7.226E 00
 14 10   8.136E 00  -2.416E-09           0  -9.156E 00   9.152E 00
 15 10   7.795E 00   9.895E-09           0  -1.060E 01   1.060E 01
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16 10    6.950E 00  -7.858E-10            0  -1.159E 01   1.159E 01
17 10    5.616E 00  -2.459E-09            0  -1.224E 01   1.224E 01
18 10    3.962E 00   3.580E-09            0  -1.264E 01   1.263E 01
19 10    2.043E 00   4.664E-09            0  -1.285E 01   1.284E 01
20 10  -1.339E-09  -6.776E-21            0  -6.472E 00   6.471E 00
21 10           0           0            0           0           0

-1 11           0           0            0           0           0
 0 11           0           0            0           0           0
 1 11           0           0            0           0           0
 2 11           0           0            0           0           0
 3 11           0           0            0           0           0
 4 11           0           0            0           0           0
 5 11           0           0            0           0           0
 6 11           0           0            0           0           0
 7 11           0           0            0           0           0
 8 11           0           0            0           0           0
 9 11           0           0            0           0           0
10 11           0           0            0           0           0
11 11           0           0            0           0           0
12 11           0           0            0           0           0
13 11           0           0            0           0           0
14 11           0           0            0           0           0
15 11           0           0            0           0           0
16 11           0           0            0           0           0
17 11           0           0            0           0           0
18 11           0           0            0           0           0
19 11           0           0            0           0           0
20 11           0           0            0           0           0
21 11           0           0            0           0           0

   TIME =      6 MINUTES, 52 AND  4/60 SECONDS
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