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16. Abstract

The research completed under this project was directed toward the development of basic analytical methods using computer simulation for highway bridges, pavements, and similar structures. This report presents a summary of the major computer developments made during the course of the project. Each development is described briefly and those that are of continuing importance are explained in more detail.

The three major areas of development that are included are for line members, two-dimensional plane frames, and plates and grids. These areas encompass most of the structural problems encountered by design engineers.

Research investigators in other areas and engineers of the Texas Highway Department are currently using a number of the completed developments in their daily work. Experimental and other analytical comparisons have shown that very satisfactory results can be obtained and thus lend confidence to application of the computer programs.

17. Key Words	18. Distribution Statement
discrete-element analysis, bridges, plane frames, beam-columns, pavements, grid systems, computers, soil structure interaction, dynamic behavior, nonlinear analysis	

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FINAL REPORT FOR PROJECT 3-5-63-56

by

John J. Panak

Research Report Number 56-29F

Development of Methods for Computer Simulation
of Beam-Columns and Grid-Beam and Slab Systems

Research Project 3-5-63-56

conducted for

The Texas Highway Department

in cooperation with the
U. S. Department of Transportation
Federal Highway Administration

by the

CENTER FOR HIGHWAY RESEARCH
THE UNIVERSITY OF TEXAS AT AUSTIN

August 1973

PREFACE

This project has been devoted to the development of basic analytical methods using computer simulation for highway bridges, pavements, and similar structures. Over the nearly nine-year duration of the project, a significant series of computer programs has been completed.

This is the twenty-ninth and final report in the series of reports that describe the work during the various phases of the project. This report summarizes the major findings and relative significance of each of the reports.

The work was supported by the Texas Highway Department in cooperation with the U. S. Department of Transportation Federal Highway Administration.

During the course of the project, there were four principal investigators. Hudson Matlock, currently Professor and Chairman of the Civil Engineering Department at The University of Texas at Austin, initiated the project in 1963. In 1965, W. Ronald Hudson, Professor of Civil Engineering at The University of Texas at Austin, became the second principal investigator, and these two were active participants throughout the project. William P. Dawkins, now Professor of Civil Engineering at Oklahoma State University, was a principal investigator during the period from 1967 to 1969. In 1969, John J. Panak, now with the Texas Highway Department Bridge Division, became a principal investigator and continued to be, with Professors Matlock and Hudson, until completion of the project.

Contact agents with the Texas Highway Department during the project were Messrs. Larry G. Walker, H. J. Dunlevy, B. Frank McCullough, Harvey J. Treybig, and Larry G. Buttler. Others from the Texas Highway Department who were instrumental in planning and assistance were Messrs. Wayne Henneberger, Farland C. Bundy, Robert L. Reed, M. D. Shelby, and H. D. Butler.

Numerous graduate students, research associates, engineering assistants, and other staff from the Center for Highway Research gave invaluable help throughout the project. Nine Master of Science students and 10 Doctor of Philosophy students did research on the project as part of their thesis or dissertation topics, and their names have appeared as authors of particular

reports, all of which are shown in the list of project reports on Page v.

The advice and assistance of all those named above and others from the Texas Highway Department and the U. S. Department of Transportation Federal Highway Administration are deeply appreciated.

John J. Panak

August 1973

LIST OF PROJECT REPORTS

Report No. 56-1, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns" by Hudson Matlock and T. Allan Haliburton, presents a solution for beam-columns that is a basic tool in subsequent reports. September 1966.

Report No. 56-2, "A Computer Program to Analyze Bending of Bent Caps" by Hudson Matlock and Wayne B. Ingram, describes the application of the beam-column solution to the particular problem of bridge bent caps. October 1966.

Report No. 56-3, "A Finite-Element Method of Solution for Structural Frames" by Hudson Matlock and Berry Ray Grubbs, describes a solution for frames with no sway. May 1967.

Report No. 56-4, "A Computer Program to Analyze Beam-Columns under Movable Loads" by Hudson Matlock and Thomas P. Taylor, describes the application of the beam-column solution to problems with any configuration of movable non-dynamic loads. June 1968.

Report No. 56-5, "A Finite-Element Method for Bending Analysis of Layered Structural Systems" by Wayne B. Ingram and Hudson Matlock, describes an alternating-direction iteration method for solving two-dimensional systems of layered grids-over-beams and plates-over-beams. June 1967.

Report No. 56-6, "Discontinuous Orthotropic Plates and Pavement Slabs" by W. Ronald Hudson and Hudson Matlock, describes an alternating-direction iteration method for solving complex two-dimensional plate and slab problems with emphasis on pavement slabs. May 1966.

Report No. 56-7, "A Finite-Element Analysis of Structural Frames" by T. Allan Haliburton and Hudson Matlock, describes a method of analysis for rectangular plane frames with three degrees of freedom at each joint. July 1967.

Report No. 56-8, "A Finite-Element Method for Transverse Vibrations of Beams and Plates" by Harold Salani and Hudson Matlock, describes an implicit procedure for determining the transient and steady-state vibrations of beams and plates, including pavement slabs. June 1968.

Report No. 56-9, "A Direct Computer Solution for Plates and Pavement Slabs" by C. Fred Stelzer, Jr., and W. Ronald Hudson, describes a direct method for solving complex two-dimensional plate and slab problems. October 1967.

Report No. 56-10, "A Finite-Element Method of Analysis for Composite Beams" by Thomas P. Taylor and Hudson Matlock, describes a method of analysis for composite beams with any degree of horizontal shear interaction. January 1968.

Report No. 56-11, "A Discrete-Element Solution of Plates and Pavement Slabs Using a Variable-Increment-Length Model" by Charles M. Pearre, III, and W. Ronald Hudson, presents a method for solving freely discontinuous plates and pavement slabs subjected to a variety of loads. April 1969.

Report No. 56-12, "A Discrete-Element Method of Analysis for Combined Bending and Shear Deformations of a Beam" by David F. Tankersley and William P. Dawkins, presents a method of analysis for the combined effects of bending and shear deformations. December 1969.

Report No. 56-13, "A Discrete-Element Method of Multiple-Loading Analysis for Two-Way Bridge Floor Slabs" by John J. Panak and Hudson Matlock, includes a procedure for analysis of two-way bridge floor slabs continuous over many supports. January 1970.

Report No. 56-14, "A Direct Computer Solution for Plane Frames" by William P. Dawkins and John R. Ruser, Jr., presents a direct method of solution for the computer analysis of plane frame structures. May 1969.

Report No. 56-15, "Experimental Verification of Discrete-Element Solutions for Plates and Slabs" by Sohan L. Agarwal and W. Ronald Hudson, presents a comparison of discrete-element solutions with small-dimension test results for plates and slabs, including some cyclic data. April 1970.

Report No. 56-16, "Experimental Evaluation of Subgrade Modulus and Its Application in Model Slab Studies" by Qaiser S. Siddiqi and W. Ronald Hudson, describes a series of experiments to evaluate layered foundation coefficients of subgrade reaction for use in the discrete-element method. January 1970.

Report No. 56-17, "Dynamic Analysis of Discrete-Element Plates on Nonlinear Foundations" by Allen E. Kelly and Hudson Matlock, presents a numerical method for the dynamic analysis of plates on nonlinear foundations. July 1970.

Report No. 56-18, "A Discrete-Element Analysis for Anisotropic Skew Plates and Grids" by Mahendrakumar R. Vora and Hudson Matlock, describes a tridirectional model and a computer program for the analysis of anisotropic skew plates or slabs with grid-beams. August 1970.

Report No. 56-19, "An Algebraic Equation Solution Process Formulated in Anticipation of Banded Linear Equations" by Frank L. Endres and Hudson Matlock, describes a system of equation-solving routines that may be applied to a wide variety of problems by using them within appropriate programs. January 1971.

Report No. 56-20, "Finite-Element Method of Analysis for Plane Curved Girders" by William P. Dawkins, presents a method of analysis that may be applied to plane-curved highway bridge girders and other structural members composed of straight and curved sections. June 1971.

Report No. 56-21, "Linearly Elastic Analysis of Plane Frames Subjected to Complex Loading Conditions" by Clifford O. Hays and Hudson Matlock, presents a design-oriented computer solution for plane frame structures and trusses with variable cross sections randomly loaded and supported. June 1971.

Report No. 56-22, "Analysis of Bending Stiffness Variation at Cracks in Continuous Pavements," by Adnan Abou-Ayyash and W. Ronald Hudson, describes an evaluation of the effect of transverse cracks on the longitudinal bending rigidity of continuously reinforced concrete pavements. April 1972.

Report No. 56-23, "A Nonlinear Analysis of Statically Loaded Plane Frames Using a Discrete Element Model" by Clifford O. Hays and Hudson Matlock, describes a method of analysis which considers support, material, and geometric nonlinearities for plane frames subjected to complex loads and restraints. May 1972.

Report No. 56-24, "A Discrete-Element Method for Transverse Vibrations of Beam-Columns Resting on Linearly Elastic or Inelastic Supports" by Jack Hsiao-Chieh Chan and Hudson Matlock, presents a new approach to predict the hysteretic behavior of inelastic supports in dynamic problems. June 1972.

Report No. 56-25, "A Discrete-Element Method of Analysis for Orthogonal Slab and Grid Bridge Floor Systems" by John J. Panak and Hudson Matlock, presents a computer program particularly suited to highway bridge structures composed of slabs with supporting beam-diaphragm systems. May 1972.

Report No. 56-26, "Application of Slab Analysis Methods to Rigid Pavement Problems" by Harvey J. Treybig, W. Ronald Hudson, and Adnan Abou-Ayyash, illustrates how the program of Report No. 56-25 can be specifically applied to a typical continuously reinforced pavement with shoulders. May 1972.

Report No. 56-27, "Final Summary of Discrete-Element Methods of Analysis for Pavement Slabs" by W. Ronald Hudson, Harvey J. Treybig, and Adnan Abou-Ayyash, presents a summary of the project developments which can be used for pavement slabs. August 1972.

Report No. 56-28, "Finite-Element Analysis of Bridge Decks" by Mohammed R. Abdelraouf and Hudson Matlock, presents a finite-element analysis which is compared with a discrete-element analysis of a typical bridge superstructure. August 1972.

Report No. 56-29F, "Final Report for Project 3-5-63-56" by John J. Panak, summarizes the project history and describes the major developments and findings in concise form. August 1973.

ABSTRACT

The research completed under this project was directed toward the development of basic analytical methods using computer simulation for highway bridges, pavements, and similar structures. This report presents a summary of the major computer developments made during the course of the project. Each development is described briefly and those that are of continuing importance are explained in more detail.

The three major areas of development that are included are for line members, two-dimensional plane frames, and plates and grids. These areas encompass most of the structural problems encountered by design engineers.

Research investigators in other areas and engineers of the Texas Highway Department are currently using a number of the completed developments in their daily work. Experimental and other analytical comparisons have shown that very satisfactory results can be obtained and thus lend confidence to application of the computer programs.

KEY WORDS: discrete-element analysis, bridges, plane frames, beam-columns, pavements, grid systems, computers, soil structure interaction, dynamic behavior, nonlinear analysis.

SUMMARY

This research project was directed toward the development of basic analytical methods using computer simulation for highway bridges, pavements, and similar structures. This report is a final detailed summary of the major developments in the project. Analysis of many highway bridge and pavement structures can be performed by application of the appropriate techniques. Discrete-element modeling is used to represent the actual structures. Computer programs with mathematical formulations based on the models are used to solve the system of resulting equations with a high-speed digital computer.

Input to each program requires simple engineering judgment to provide the appropriate computer model values to represent the stiffnesses, loads, and restraints of the actual structure. Output from most of the programs is presented in tabular and graphical form to allow interpretation by the engineer-user. Two and three-dimensional plotted displays are also available in some of the programs.

IMPLEMENTATION STATEMENT

There are eight final computer programs resulting from this research project which are being used by Texas Highway Department bridge and pavement designers. These eight programs are specifically identified in Chapter 7. No additional program development is necessary to allow their immediate use by the sponsors. Additional user-oriented versions of some of the programs can be developed to make application easier for specific classes of problems, and two of these have been developed by the sponsors and are now being used.

The project programs make it feasible for bridge and pavement investigators to study various design options and parameter relationships to find better and more economical configurations for bridge and pavement facilities.

It is recommended that the sponsors continue application of the computer programs for both day-to-day and certain special problems. In addition, by expending additional effort toward developing more user-oriented versions of some of the programs, even further usage can be attained.

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CHAPTER 1. INTRODUCTION

Background

This report summarizes the major work performed on this project since its beginning in 1963. Numerous significant developments have been made which allow the design engineer to use to the fullest practical extent the capabilities of high-speed digital computers coupled with modern methods of analysis. These structural analysis techniques have demonstrated that the engineer can successfully apply computer methods for the analysis of complex systems of highway bridges, pavements, and other related structures.

For the initial year, project work was on beam-column and grid-beam systems, and then the scope of the project was enlarged to include the development of methods for the solution of pavement slabs. Because of the fundamental similarities between bridge and pavement slab problems, the expanded program was kept within a single project and the duration was extended.

Three Areas of Development

Three distinct areas of development are included in this project. These are concerned with line or beam-column members, two-dimensional plane frames, and plates and grids. Most of these encompass the problems which design engineers encounter in daily practice. The computer modeling and solution techniques that are used are common throughout the three areas.

The many efforts in this project may seem diverse in objective, but they have been demonstrated to be part of an overall systematic study of the behavior of bridge structures, roadway pavements, and other structural systems. By this coordinated approach, better use has been made of available funds and manpower to develop methods to solve complex technical problems. Some of the research is directly applicable to present design problems and other areas of the research include developmental contributions which have been superseded by later studies. The basic objective, which has been met, remains as originally stated: to develop computer solution techniques which permit engineers to study the behavior of structural systems in a more realistic manner.

Each of the next three chapters discusses one of the three major areas described above. Each chapter describes the individual program developments associated with the area.

All references to project reports in this final report are by report number; these are included in the complete list of reports in the beginning pages of this report.

Included in the summary chapter is a list of computer programs and their current status. This will serve as a reference guide so that the appropriate program for a particular application can be selected.

CHAPTER 2. BEAM-COLUMN METHODS

Research Report 56-1, The Basic Beam-Column

The first beam-column development was published in Report 56-1, which introduced the basic analytical procedure that is used for the solution of a wide variety of complex structural problems that can be represented as line members. The basic mechanical model used for this development is shown in Fig 1. Discrete-element difference equations were developed from the beam-column model. The equations are formulated such that deflections at each joint are the unknowns. Changes in bending stiffness, support conditions, loads, and other restraints can vary in a freely discontinuous manner from joint to joint, thus allowing the user to solve a variety of possible problems.

The computer program included with Report 56-1 is known as BMCOL 34, which is an acronym for beam-column together with a number that means that it was the thirty-fourth step in a significant sequence of program changes, some minor and some of major importance. Program BMCOL 34 should be thought of and used as a basic beam-column analysis tool. It has been superseded to a certain extent by later developments on this project but its simplicity makes it advantageous for continued use in the future to form the basis for other work.

Research Report 56-2, Bridge Bent Caps

This report presents a particular application of the beam-column method for the analysis and design of highway bridge bent caps. The program associated with it uses a simplified adaptation of the BMCOL method of Report 56-1.

The various combinations of lane loads and dead loads from the stringers and bridge deck are automatically combined within the program according to AASHO design specifications (Ref 1). These live and dead loads are combined and accumulated to retain the final resulting design maximums of bending moment, shear, and support reaction.

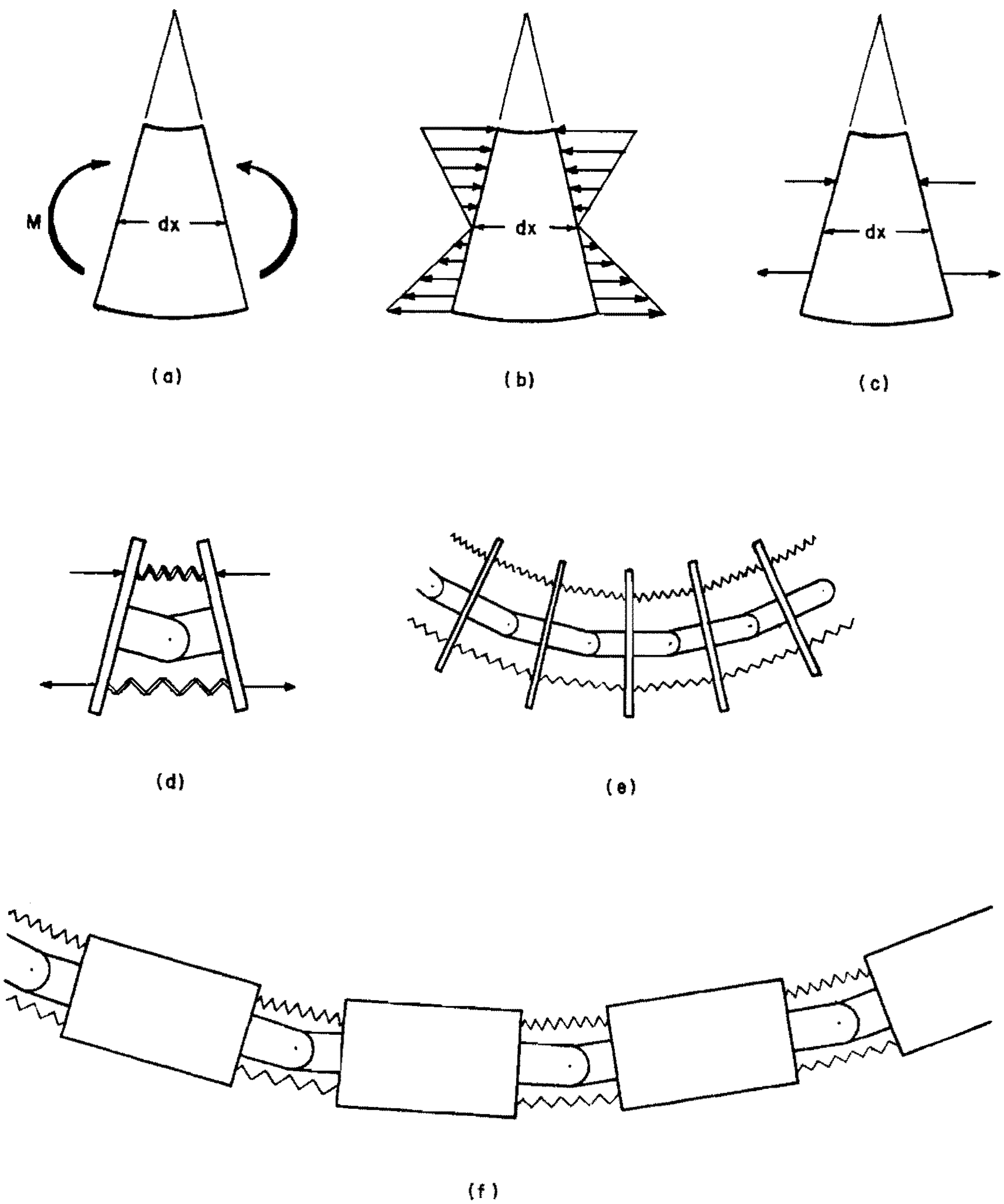


Fig 1. Mechanical representation of a conventional beam.

The specific version of the program described in Report 56-2 is designated as CAP 14. It is formulated to allow for up to 10 traffic lanes, 20 supports, 30 stringers, and 30 design control points for moment and shear.

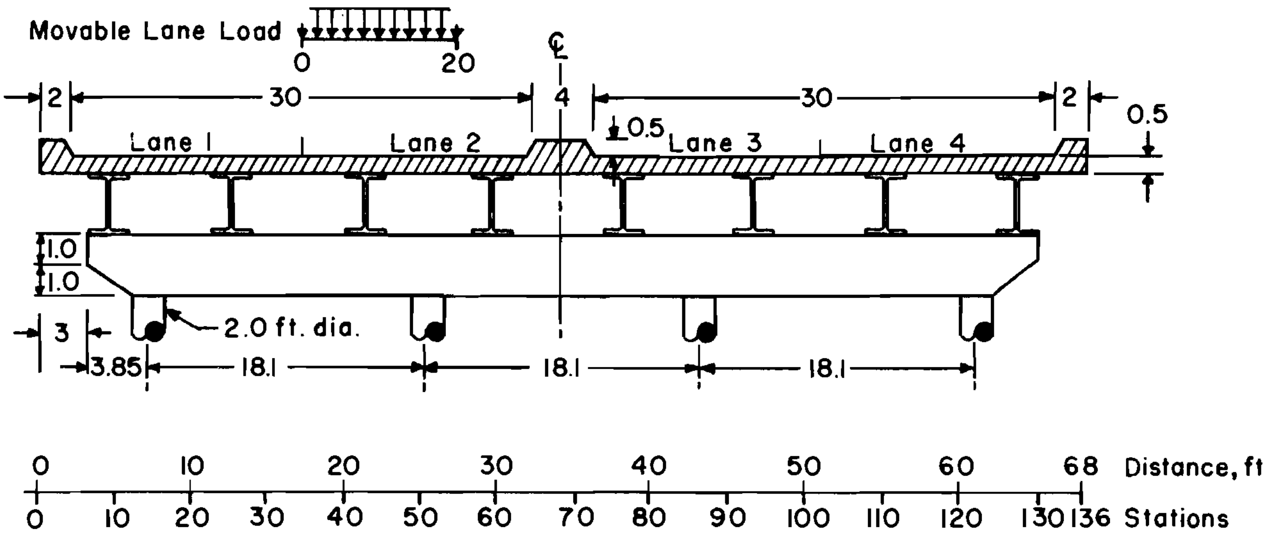
A guide for data input is included in the report, and the designer can refer to it for application and use of the program. In addition, a basic example problem with variations is shown that can also aid in understanding the use of the program. One variation of the example problem for a bent cap is shown in Fig 2.

Program CAP 14 is currently being used successfully by the Texas Highway Department in day-to-day design of highway bridge bent caps.

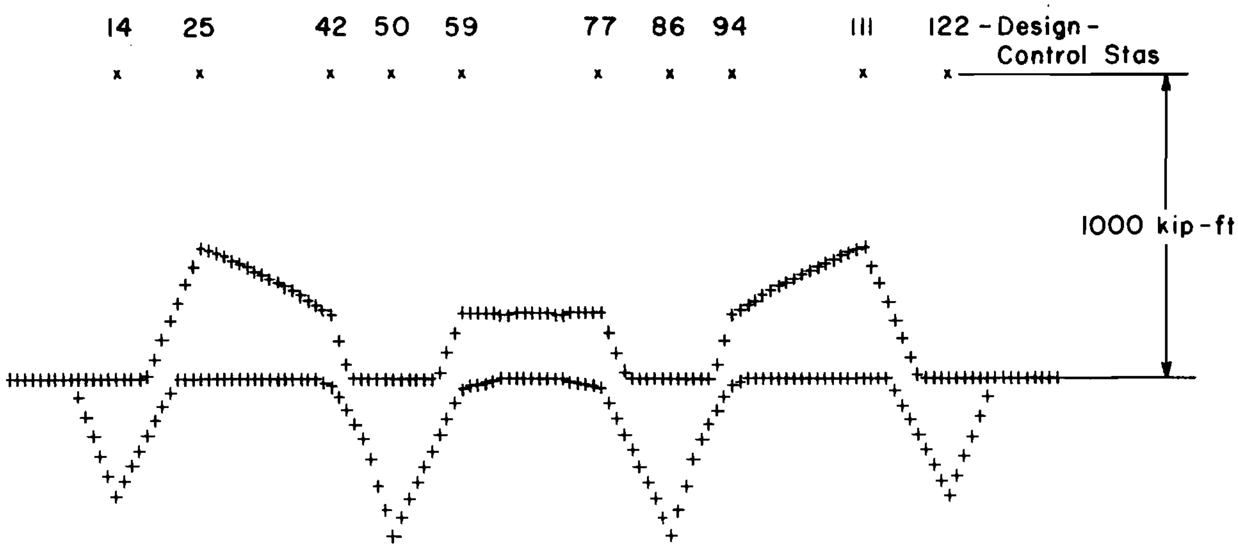
Research Report 56-4, Moving-Load Beam-Column

This report presents a computer program which was developed to analyze beam-columns subjected to fixed or movable static loads. Again the methods used incorporate the discrete-element techniques of Report 56-1. The computer program included with this report can be used by the bridge designer to determine efficiently the critical design parameters at any location along the length of a beam-column, which can represent a girder of a highway bridge. Any pattern of transverse moving loads such as highway trucks, a special overload, or a train on a railroad structure may be used. Envelopes of maximum values of deflection, bending moment, shear, and support reaction are computed and automatically plotted if desired. The effects of fixed loads which usually represent dead loads are also included in the analysis. Changes in support histories, such as settlements or yielding supports, can also be studied. General influence diagrams for any type of movable load can also be produced.

The specific version of the program included in this report is designated as BMCOL 43. Its most important feature is the ability to simulate the movement of a load pattern along a member. The movable-load pattern can be a diverse system of loads; any desired range of movement can be specified by the user by giving initial and final positions for the movable-load pattern. The movable-load analysis does not include dynamic effects; it is a series of static-load solutions. The load pattern is shifted in steps within the designated range of movement, and a beam-column solution is made at each step. A particularly useful aspect of BMCOL 43 is its ability to hold the results in envelopes of maximums from problem to problem in order to simulate past or anticipated loading histories.



(a) The structure.



(b) The envelopes of maximum bending moment.

Fig 2. A typical bent-cap problem.

A typical problem that can be solved by BMCOL 43 is shown in Fig 3. This problem represents one girder of a two-span continuous railroad structure.

Program BMCOL 43 can be used to solve any of the problems which program BMCOL 34 can solve. Thus, it could be thought of as superseding BMCOL 34.

Research Report 56-8, Dynamic Beams and Plates

This report presents a discrete-element method which can be used to determine the transverse linear deflections of a vibrating beam or plate. Two programs are included with this report; the first is DBC 1, which is for beams, and the second is DPI 1, which is for plates. Both of these programs have been superseded by later developments on this project.

Research Report 56-10, Composite Beam-Columns

Report 56-10 describes an analytical tool which can be used to solve composite beam and slab problems. It can be used to study the effect of shear interaction between highway bridge decks and their supporting girders.

The standard approach to analysis of composite structures is the application of customary transformed section property procedures. Many investigators have shown that the transformed section theory is applicable as long as the bond between the steel and concrete remains essentially unbroken. The theory gives good approximation even after bond failure if a sufficient number of fairly stiff shear connectors are included at the interface.

After development of the program, a number of analytical and experimental comparisons were made. One of these is shown in Fig 4. As can be seen, for this particular beam, which was on a span of 20 feet, there was very little change between the solution with complete shear interaction and that with partial shear interaction. Therefore, it can be concluded that the assumption of complete shear interaction will give adequate design results for customary installations of highway bridge structures with shear connectors.

The program name is COMBM 1 and it remains available for future study if desired.

Research Report 56-12, Shearing Deformations in Beams

This report describes a method for the analysis of beams in which shear deformations can be considered. The method replaces the actual beam with a

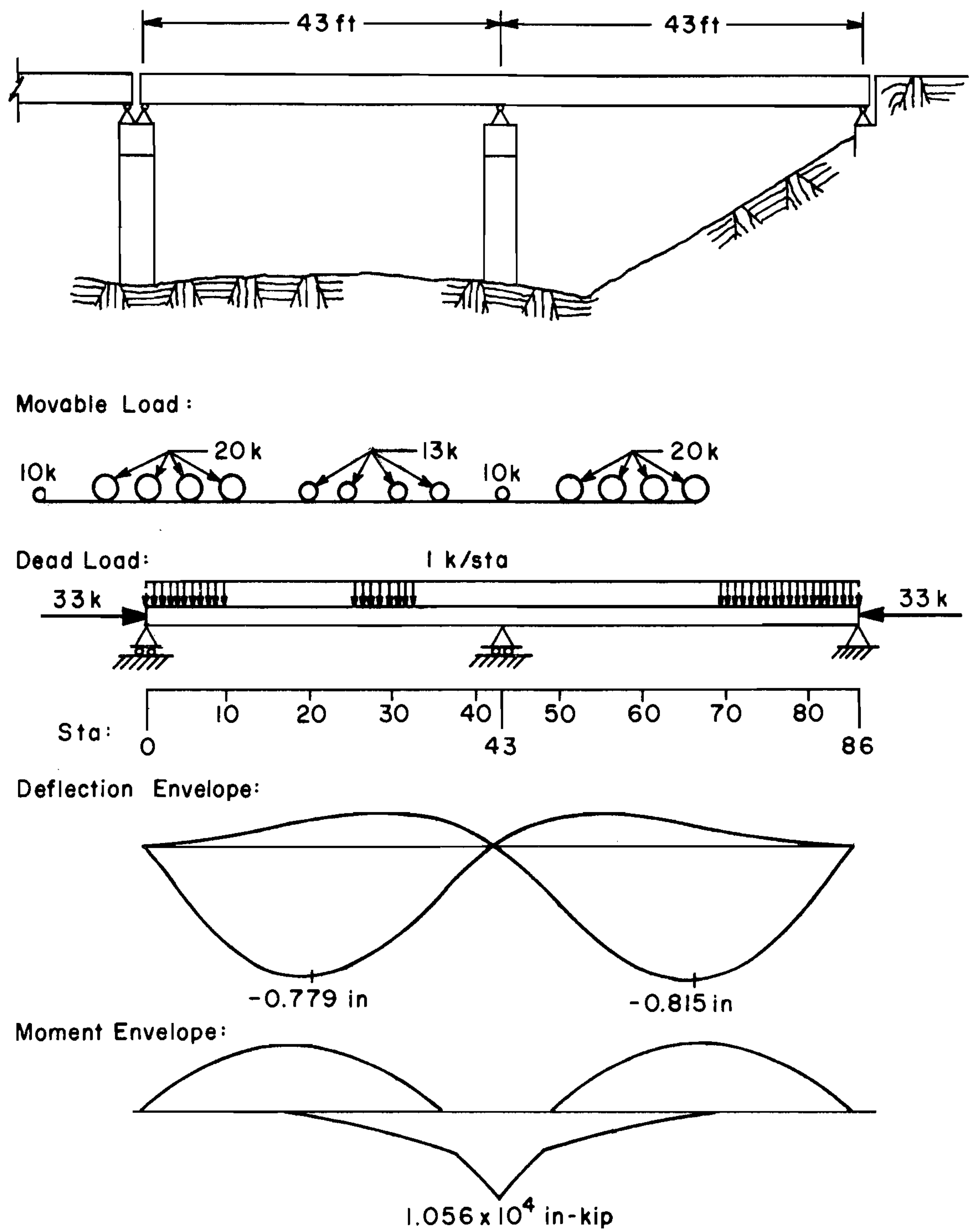
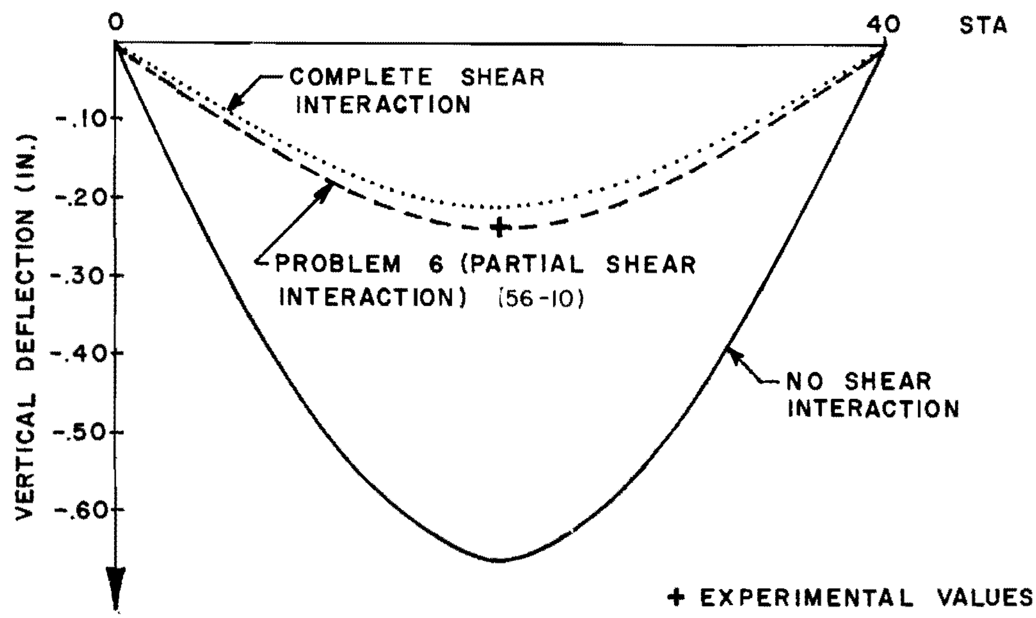
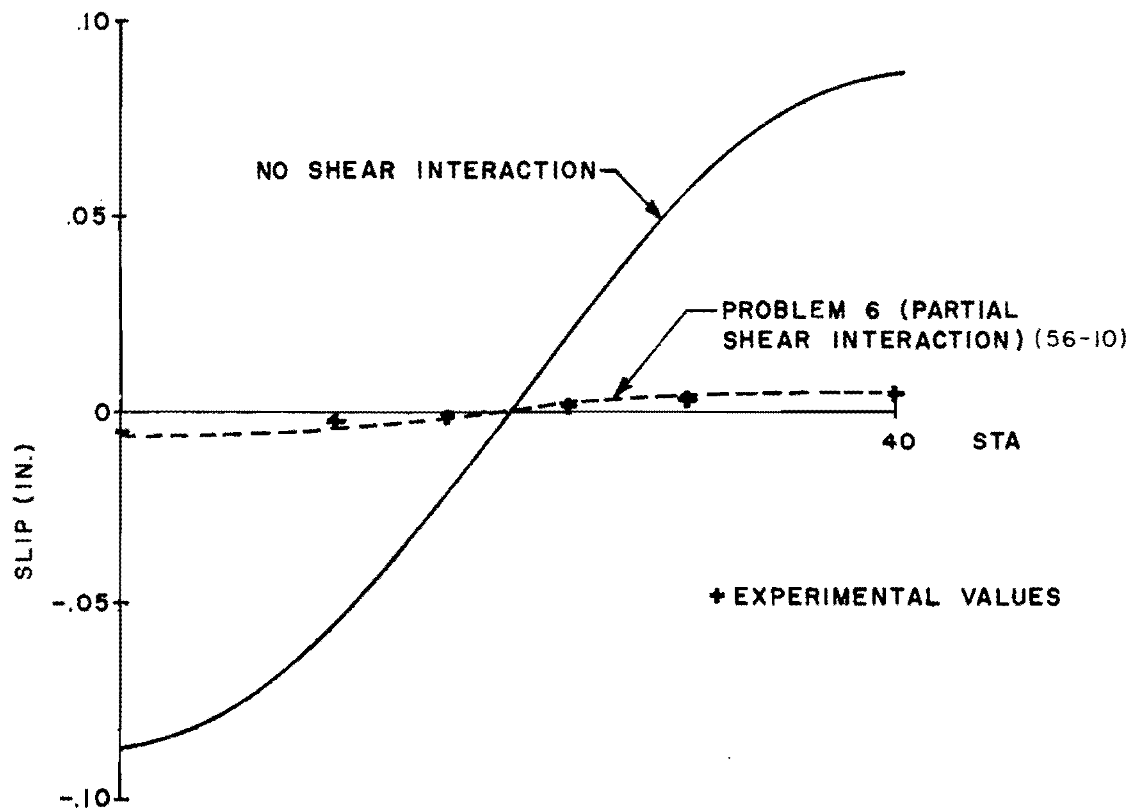


Fig 3. Two-span railroad girder structure.



a. Vertical deflection.



b. Horizontal slip.

Fig 4. Composite beam comparisons.

discrete-element model which is similar to that used in the beam-column developments described above. A shear stiffness coupling is provided in each rigid bar of the model to allow shear deformation effects to take place.

The computer program included with this report is SHRBM 1. The required input information is very similar to that in previous developments. The output information is also the same except that the relative shear deformation is computed at each joint in the system.

A typical bridge diaphragm was analyzed by this computer program, and the shear deformations increased the relative beam deflections by only about 4 to 6 percent. It can be concluded therefore that the shear deformation effects for most bridge structures can be neglected. This is not to say, however, that all structural members should be excluded. The program can be used to investigate shearing deformations if their effect is felt to be significant.

Research Report 56-20, Curved Beam-Columns

A method for analyzing plane curved members is presented in this report. The method combines the versatility of discrete-element modeling with the efficiency of direct structural analysis techniques. The procedure also closely follows the methods used in the beam-column modeling presented in previous reports.

This procedure is the only one in the complete research sequence of this project which specifically deals with any type of curved member.

The computer program included in this report is PCGR 2. The input requirements are in a slightly different form than in previous developments. Provision is made for including variable increment length sections between control points on a curved member. A torsion stiffness constant must be included for the member in addition to the customary beam-column input variables.

The program was tested against an experimental study of a curved I-section and gave results which were very close to the experimental values. A curved highway girder example problem is also included in the report to demonstrate its application to curved girders. It is felt that this program offers the user a versatile tool for analyzing curved members of any type subjected to diverse loads and restraints.

Research Report 56-24, Dynamic Beam-Columns with Nonlinear Supports

Report 56-24 describes a method developed to analyze beam-columns subjected to either static fixed loads or dynamic loads. The supports or other restraints of the beam-column can be either linearly elastic or nonlinear and non-elastic. The method also incorporates the same previously developed discrete-element beam-column techniques. The applied forces can include static fixed loads and time dependent loads. The program can be considered as an extension of programs BMCOL 43 in Report 56-4 and DBC 1 in Report 56-8. Multi-element models are used to simulate inelastic support characteristics which allow the beam to either lift off the support when it deflects or consider the resistance to either upward or downward deflections in a nonlinear and hysteretic fashion. An internal damping factor can also be included, in addition to conventional external viscous damping factors.

The computer program included in the report is DBC 5. Input to the program follows essentially the same format as previous beam-column developments. For problems with nonlinear-elastic or nonlinear-inelastic supports the included iteration process compares successively computed deflections until a desired tolerance is satisfied. Options are available in the program to allow the selection of various appropriate iteration processes for particular problems.

Computed results from the program include solution for the member under static fixed loads, solution under the applied dynamic loads at each time station, and finally, if desired, plots of the computed deflections or moments along either the time or the beam axis.

The report includes a number of theoretical and computed comparative examples which demonstrate the very good accuracy that can be obtained. Other problems demonstrate the validity of the program in predicting the formation of hysteresis loops on nonlinear-inelastic resistance deflection support characteristics. This capability will allow future investigators to study degradation of supporting characteristics for structures subjected to dynamic and vibrating loadings. This should be particularly important to pavement engineers.

Another example problem in the report demonstrates use of the program to analyze a highway bridge structure subjected to dynamic forces from truck loadings. The bridge was a three-span structure with both the dynamic and static loads measured by portable scale transducers installed in the pavement preceding the structure. These measurements were made on Research Project 3-8-67-108, "Dynamics of Highway Loading" (Ref 2). The computed results for the three-span structure showed that the dynamic effect was fairly small compared to the produced static responses. This was because the dynamic forces were generated by the truck moving on a fairly smooth pavement and thus varied smoothly with time. That same research project demonstrated that even small bumps in a pavement or bridge could create dynamic forces up to 150 to 200 percent of the static force on the surface of the bridge deck itself. These forces are not felt directly by the supporting beams due to the mass of the structure.

Future investigation for dynamic bridge behavior should include a variety of vehicle configurations in addition to a range of surface profiles and vehicle speeds. Program DBC 5 when coupled with the results and predicted responses provided by the computer program developed on Project 108 (Ref 2) makes possible for the first time the investigation of bridge structures subjected to actual dynamic loadings.

CHAPTER 3. PLANE FRAME ANALYSES

Many highway structures such as bridge spans and freeway overpasses are designed as plane frames. These structures may be composed of nonprismatic and elastically restrained members subjected to numerous complicated loading conditions. A thorough analysis is economically feasible only with the aid of digital computer programs that are both versatile and convenient.

The frame analysis programs developed on this project incorporate the same versatility as the discrete-element beam-column modeling techniques described in the previous chapter. The first two developments used an alternating-direction iterative solution for the simultaneous equations. Subsequent developments in the direct solution of simultaneous equations have made these ADI solutions relatively less attractive with the present generation of computers.

Research Report 56-3, Plane Frames with No Sway

Research Report 56-3 documents the first discrete-element solution for plane frames. Finite difference equations are written utilizing the computer model and solved by an alternating-direction recursive method. Individual beams of the frame are solved alternately in the two orthogonal directions, and at each joint a relaxation technique is used to adjust the two solutions and achieve rotational compatibility. The program is severely limited by the fact that the joints must be fixed in space. No joint translations are allowed; however, they do rotate. Another limitation is that a rotational closure spring must be chosen to allow the ADI method to proceed to a solution. The selection of this closure spring value necessitates judgement and experience with the particular class of problems being solved.

The computer program in Report 56-3 is FRAME 4, which was written specifically for the CDC 1604 computer. Example problems are included which demonstrate the program. One of these is a three-barrel box culvert, which is shown in Fig 5. The results are as expected and were checked by moment distribution to within 2 percent of the computed values. This is one of the few types of frame problems in which the severe limitation of no joint translations is acceptable.

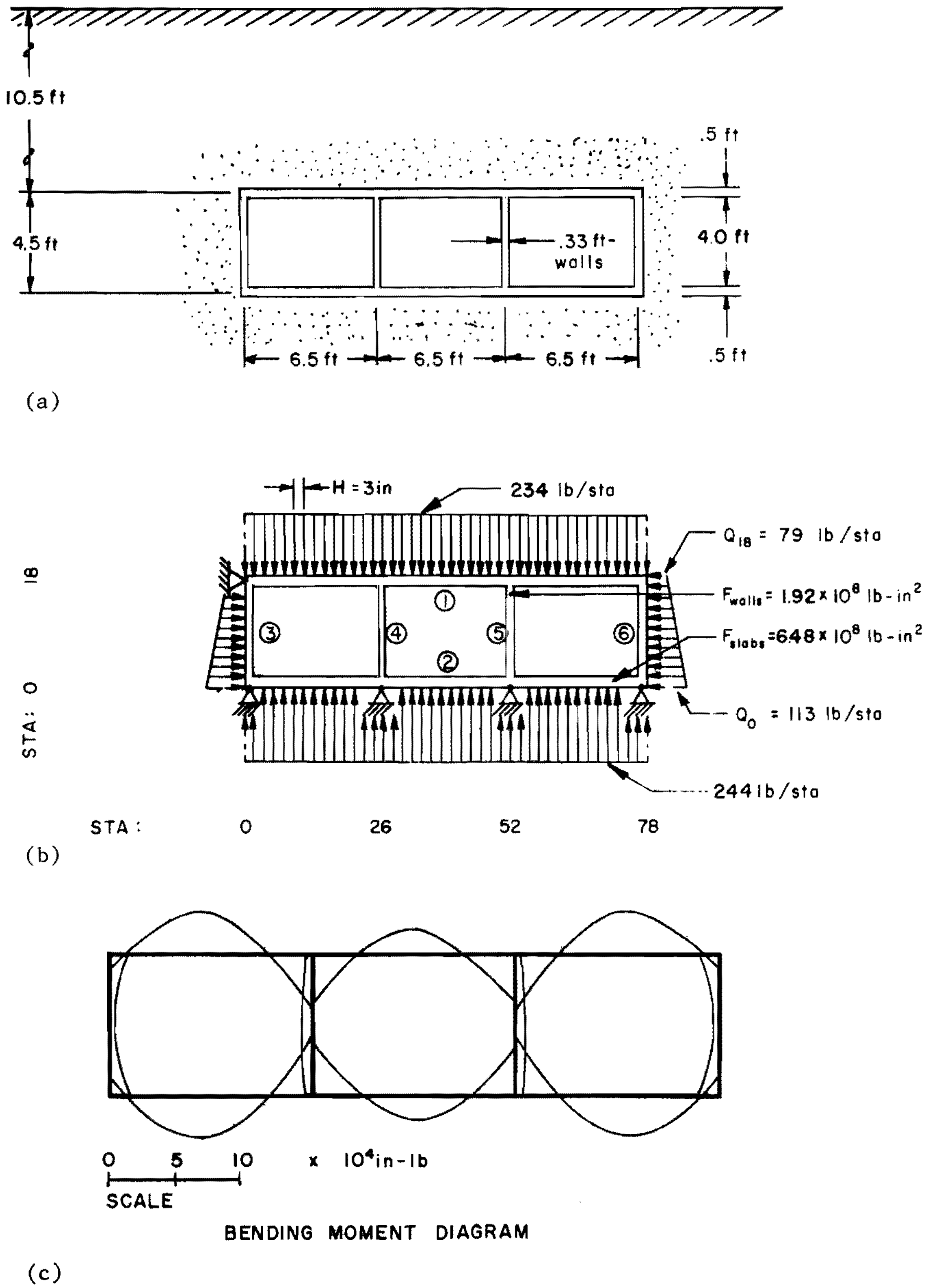


Fig 5. Three-barrel box culvert.

Research Report 56-7, Plane Rectangular Frames Allowing Sway

Research Report 56-7 presents the logical extension of the frame analysis work from Report 56-3 so that it is possible to find the final deflected shape of a rectangular plane frame with a full 3 degrees of freedom at each frame joint. The program is general in that flexural stiffness, transverse loads, axial loads, and foundation restraints may be elastically varied as desired along each frame member.

The computer program included in Report 56-7 is PLNFRAM 4. The individual frame member data are input in a manner very similar to that of the beam-column procedures. As in Report 56-3, the rotational closure parameters required for the solution limit its generality.

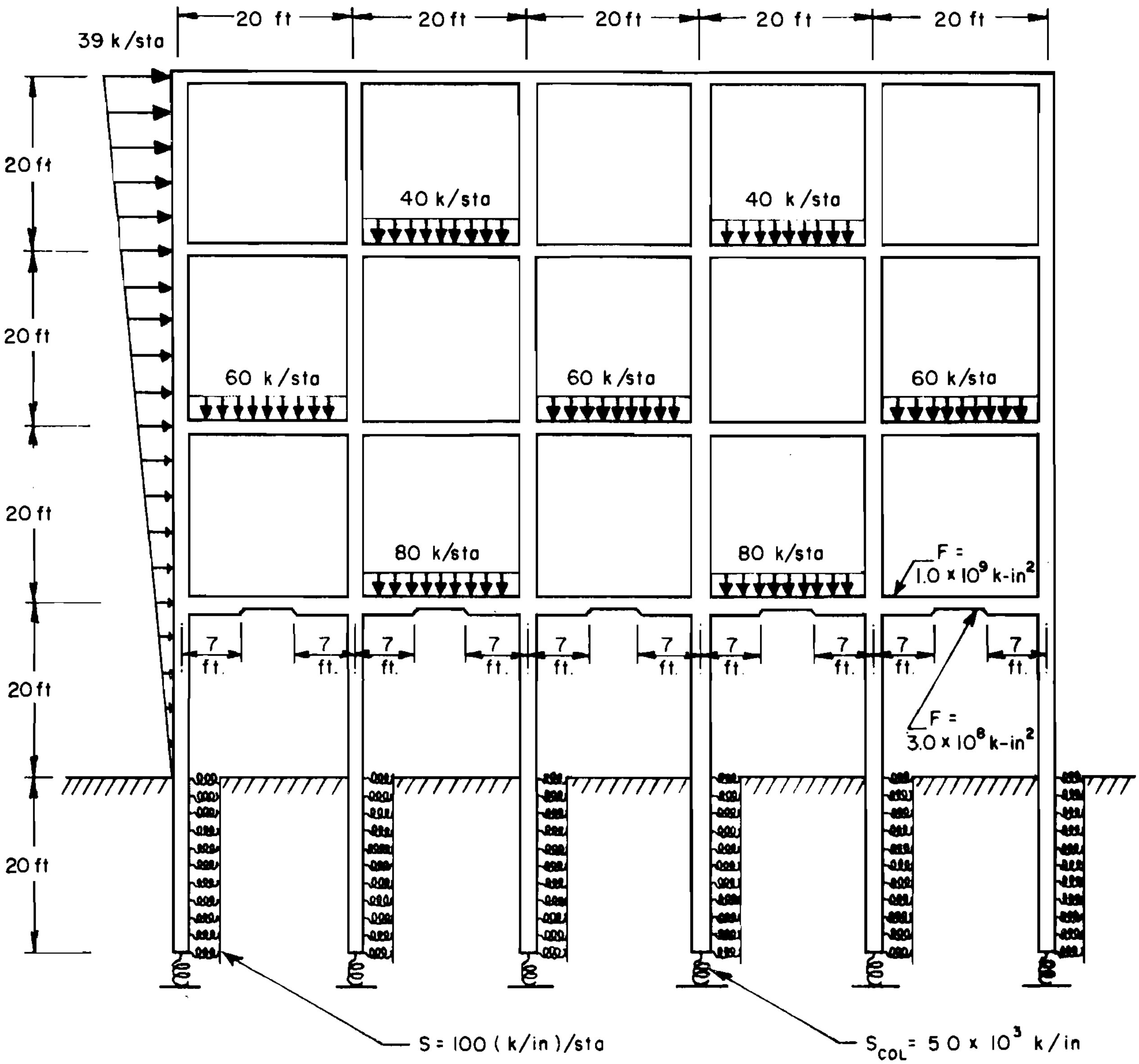
Figure 6 shows a typical problem solved by this program. It is a five-bay, four-story rigid frame in which both the translational and rotational interaction must be considered. The program includes the capability for transverse elastic restraints at the bases of the columns to more closely represent actual field conditions. The results of an analysis are shown in Fig 7.

Research Report 56-14, Direct Solution for Plane Frames

Research Report 56-14 extended the plane frame work one step further and combined the versatility of the discrete-element modeling and direct solution techniques with a conventional frame analysis procedure. In addition, sloping members were also considered. Although the previous ADI methods are efficient in their computer storage requirements, closure of the iterative process is dependent on the more or less arbitrary selection of numerical values for fictitious rotational closure springs. Experience has shown that the selection of the proper values for these closure parameters requires a trial and error procedure which is time consuming and therefore usually prohibitive.

The computer program in the report is PFRM 1. Again, the input data that describe individual members in the frame are similar to those in previous beam-column developments. One severe limitation in this program is the requirement that all joints in the frame must fall on a superimposed grid system; thus the solution is only efficient for frames with regular column and beam spacings.

Axial thrusts (tension or compression) have been shown to affect the flexural stiffness of a beam-column member. Therefore, a level of thrust must



HORIZONTAL MEMBERS

$h = 24.0$ in

$F = 3.0 \times 10^{8}$ k-in^{2} *

* except as noted

VERTICAL MEMBERS

$h = 24.0$ in

$F = 1.0 \times 10^{9}$ k-in^{2}

Fig 6. Five-bay, four-story frame.

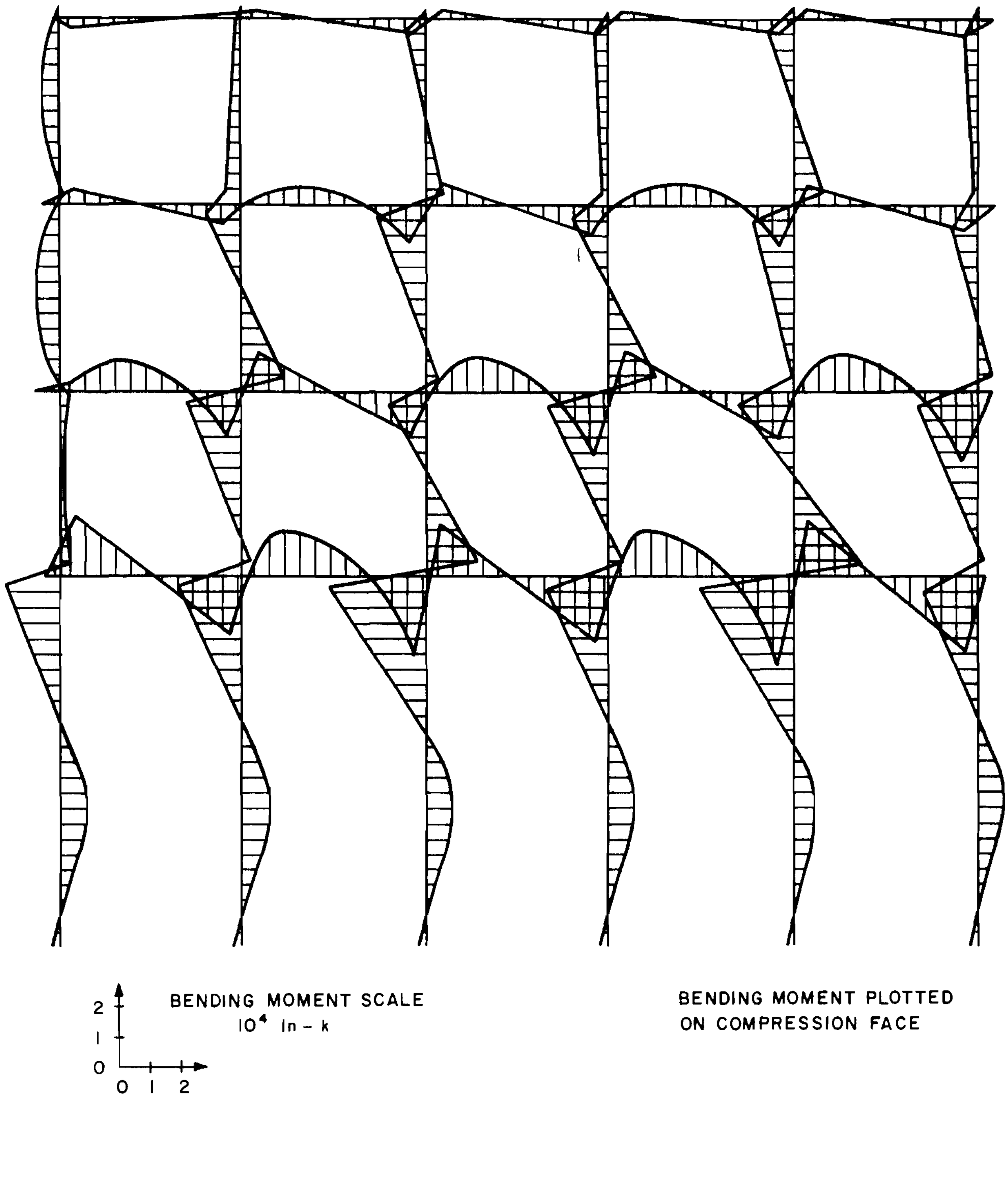


Fig 7. Bending moment diagrams for the five-bay frame.

be known for each member before an exact solution for each member in the frame can be established. In conventional frame analysis procedures the effect of thrust on structural behavior is usually ignored since it is small. Provision has been made in PFRM 1 to allow investigation of axial effects at the user's option. When axial effects are not considered the thrust is assumed zero when each member solution is made. An automatic iterative process is used to include the influence of axial thrust. An initial solution is made with the resulting thrust calculated and the stiffness for each member adjusted. A second solution is made and a new estimate of axial thrust is determined. This process is continued until the joint displacements from two successive trial calculations agree within a desired tolerance.

Research Report 56-21, General Elastic Analysis of Plane Frames

Research Report 56-21 presents a solution for the analysis of plane-frame linear structures. The program is a further extension of previous frame developments and features inputs which greatly reduce the required data preparation. The computer program described is capable of handling large plane-frame structures with randomly located members. Smaller and more regular structures may be quickly input for an economical solution. Rigid frames, beams, and trusses are among the types of structures which may be analyzed.

The solution uses a variation of the basic discrete-element beam-column model for the evaluation of the member solutions. This modified discrete-element model allows flexural rigidity as well as lateral, axial, and rotational values of loading and elastic restraint to vary randomly along the length of the members. This same model is used in the development for nonlinear frame analysis discussed below. Input is not restricted to values lumped at the discrete stations as in previous beam-column developments but may be input in normal engineering values at any point on the member. Internally, the program applies the appropriate values to the jointed discrete-element model. In addition, input options do not require the transforming of loads and dimensions from one axis to another by the user.

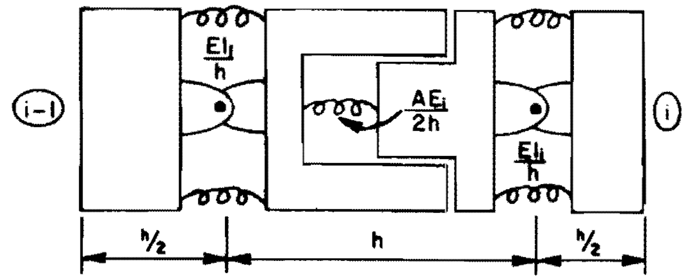
One outstanding design-oriented feature of the program is its ability to superimpose the effects of a large number of loading conditions. A designer may consecutively run a dead-load analysis, a live-load analysis, a wind-load analysis, and so on; a program option then allows him to ask for any linear and fractional combination of these loadings he desires. Designers who have manually checked all of the group loadings required by the AASHO specifications at their various unit stresses will appreciate this feature.

The computer program included with this development is FRAME 11, and it may be considered to be the final plane frame analysis for linear structures on this project. A direct stiffness solution for all of the joint displacements of the frame is used by the program. Each frame member is subdivided into a number of the modified discrete elements as shown in Fig 8. The discrete-element model of each frame member is solved for the member end restraints. After the frame displacements are found, the same model is used to find the individual values of shear, axial force, bending moment, and the corresponding displacements throughout the member. This program does not use an iterative solution to determine the axial force effect on the member stiffnesses, and thus it does not include this beam-column effect, although the axial forces and axial displacements are computed. The model shown in Fig 8 has six degrees of freedom (axial, lateral, and rotational displacements at each end). Members with any form of axial, lateral, and rotational loads and restraints may be conveniently represented by this model. It has been demonstrated to be a very adequate representation of a straight line member based on both theoretical and numerical evidence.

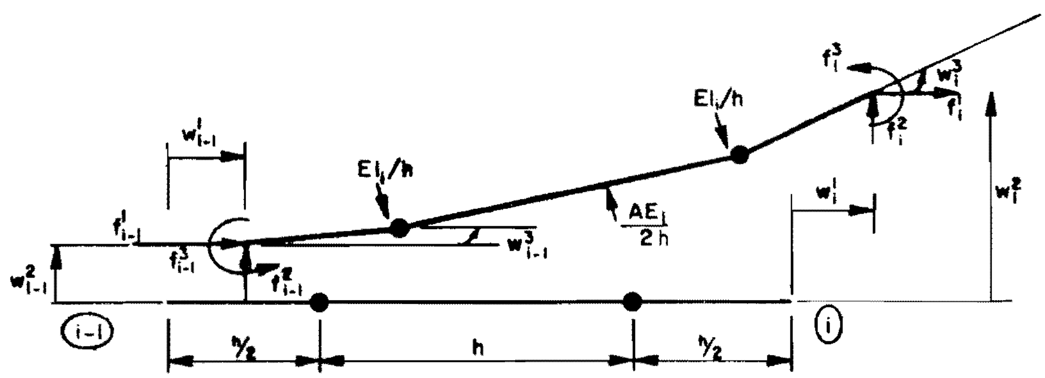
During development of the program, emphasis was placed on maintaining complete generality of input. However, it was recognized that many frames are more regular and often have a large number of members with the same lengths and cross sections and loading. Provisions are therefore made in the program to avoid duplicating the input for such members. The geometry of the frame and directions of the loads may be input in a manner both natural and convenient to the designer.

Research Report 56-23, Nonlinear Frame Analysis

Research Report 56-23 presents the final frame analysis method developed on this project which considers geometric, material, and support nonlinearities of statically loaded plane frames. The frame geometry, loads, cross sections, and support characteristics can be input in the same manner as in the linear frame program of Report 56-21 and is further extended to allow all types of nonlinear variation in the members and support characteristics.



(a) Mechanical model of element i .



(b) Discrete-line-element model of element i .

Fig 8. Discrete-element representation used in programs FRAME 11 and FRAME 51.

The method of analysis uses an iterative procedure in which unbalanced joint forces are applied to a temporarily linear structure whose position is dependent upon the elastic stiffnesses assumed for that iteration. Each frame member is divided into a number of discrete elements and the member solutions are individually made in turn separately from the frame solutions, which reduces computer time and storage requirements.

The load-displacement equations for each individual discrete-element member are valid for large displacements. The numerical technique which is used can determine the force-deformation response of a general cross section composed of several materials, each with any nonlinear stress-strain characteristic. Loads and nonlinear supports are input in normal engineering terms and can be referenced either to the structure or to the member axes.

The computer program included in the development is FRAME 51. The inputs are very similar to those for FRAME 11, in Report 56-21, and allow complete freedom to the user in representing his particular structure. Report 56-23 includes example problems which are compared to existing analytical and experimental solutions. These problems demonstrate the program's ability to predict the general response of frames which undergo large displacements, and include steel frames, reinforced concrete frames, continuous prestressed concrete beams, and frames involving nonlinear soil-structure interaction.

The ability of the solution to input nonlinear stress-strain curves that do not necessarily pass through the origin allows the study of a variety of prestressing effects; one such application is the study of the pretensioned beam, such as shown in Fig 9. This two-span pretensioned beam was tested at the University of Illinois. The cross section and other data are shown in Fig 9. The input stress-strain curve for the pretensioning steel is a typical nonlinear curve for high-strength steels. The axis of the prestressing stress-strain curve is offset to 120,000 psi to represent the initial tensioning of the beam. The input values for the concrete were a typical nonlinear Hogenstaad type of stress-strain curve for the concrete.

The discrete-element solution considers the elastic shortening and bending of the concrete. Thus, the solution finds the new position after the initial prestressing which indicates the stretching and bending of the section due to the initial prestressing.

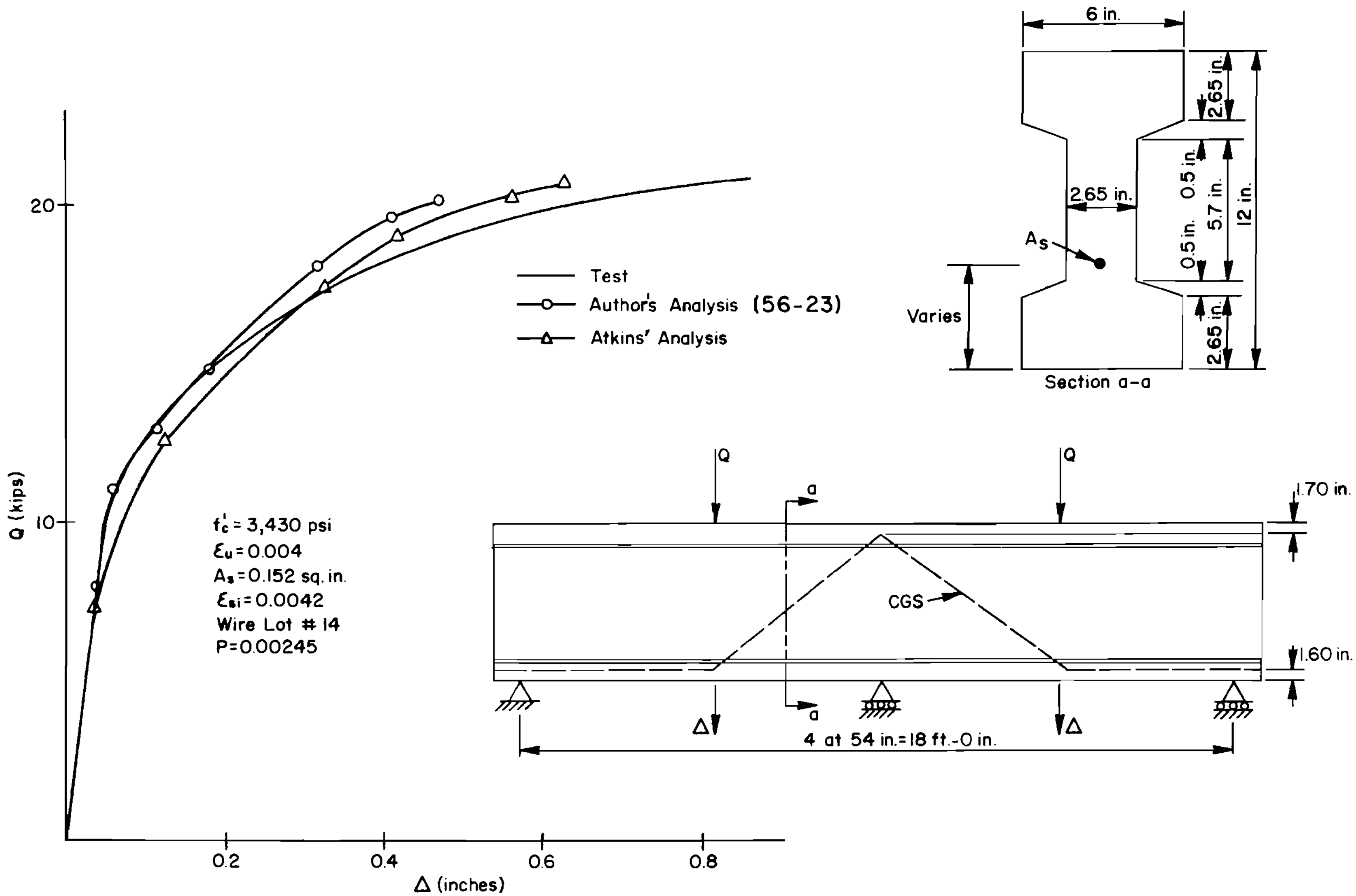


Fig 9. Two-span prestressed beam tested at the University of Illinois.

The results of the analysis as compared to the experiment and to another nonlinear analysis are shown in Fig 9. It can be seen that the computed values are very close to the experiment up to about 75 percent of ultimate, and even after that they agree quite closely.

Another included example problem demonstrates how the program can be applied to the analysis of reinforced concrete bents with battered piles. The bent shown in Fig 10 was a test structure in Romania near the Danube River. The upper layer of the soil in this area was silty, highly compressible clay about fifteen feet thick. The bent consists of three battered piles driven into the soil with a fairly rigid connecting cap. The lateral and axial force-deformation curves needed to define the response of the soil which supported the bent were computed using Matlock's criteria (Ref 3) for establishing the lateral load-displacement curves and Coyle and Reese's criteria (Ref 4) for developing the axial load-displacement curves. Estimates of the shear strength at several levels were made based on available data. The concrete piles and concrete cap were represented by cross sections with nonlinear stress-strain curves for both the concrete and steel.

The computer load-displacement and load-rotation responses for the three-pile bent are shown in Fig 11. The agreement between the observed and calculated curves is quite good up to the load level of approximately 60 kips, particularly considering the possible variation in soil and structural properties. There was an unexplained jump in the experimental data at this point, which might account for some of the variation past this level.

This final result of the frame analysis research has been the development of a method of analysis for predicting the static load-displacement response of plane-frame structures. The analysis has been put into a form for use by engineers which has sufficient generality to work practical frame problems. The program has been verified by working a number of problems and comparing the analyses with existing analytical and experimental results. The nonlinear soil support capabilities available in the program allow the highway designer to realistically model many problems of soil-structure interaction which previously had to be represented in a linear manner. The nonlinear material properties feature of the program allows the designer to specify the cross section as a series of rectangles and thin-walled tubular pieces, each with different nonlinear stress-strain curves, thus allowing a wide variety of real

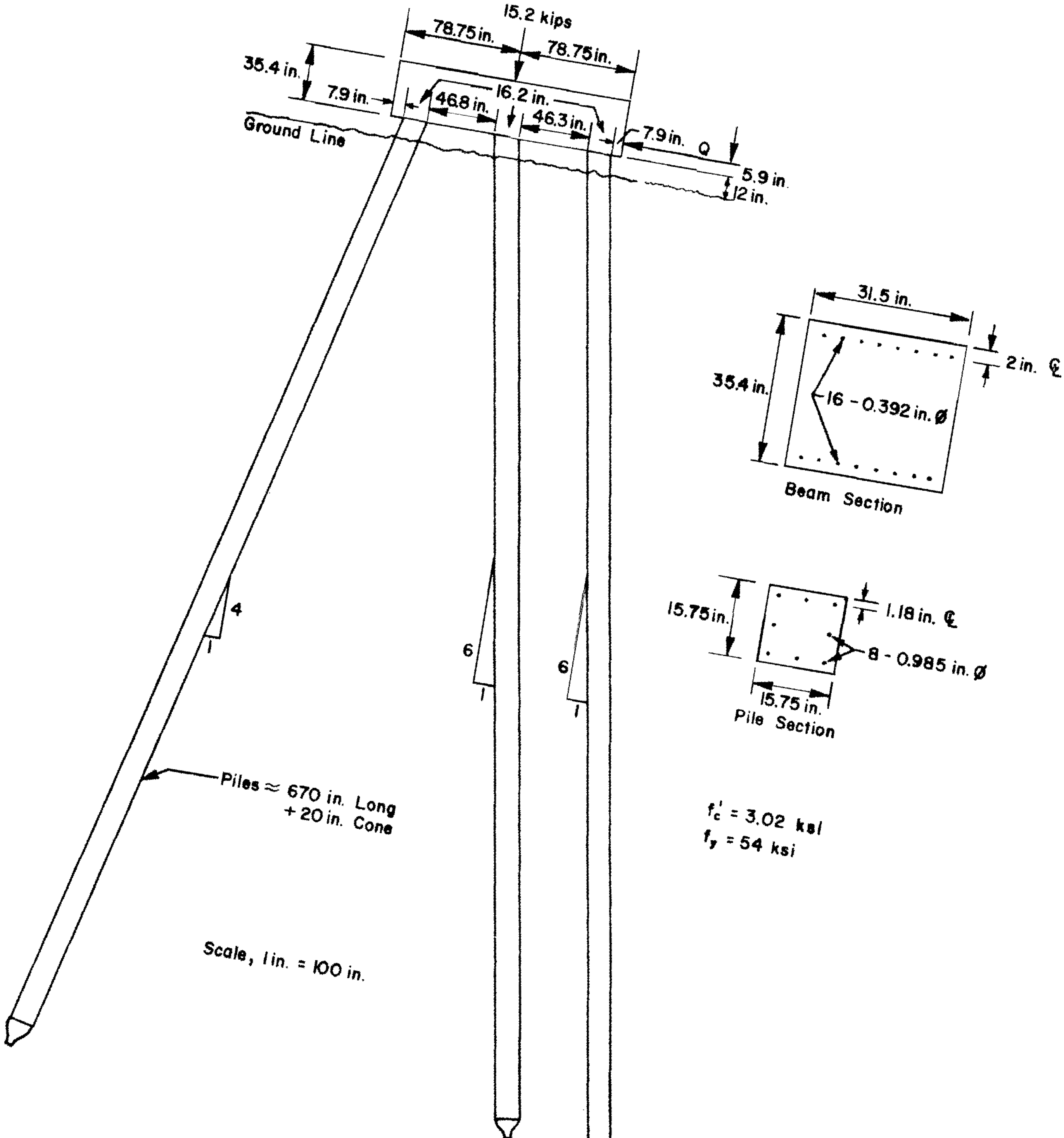
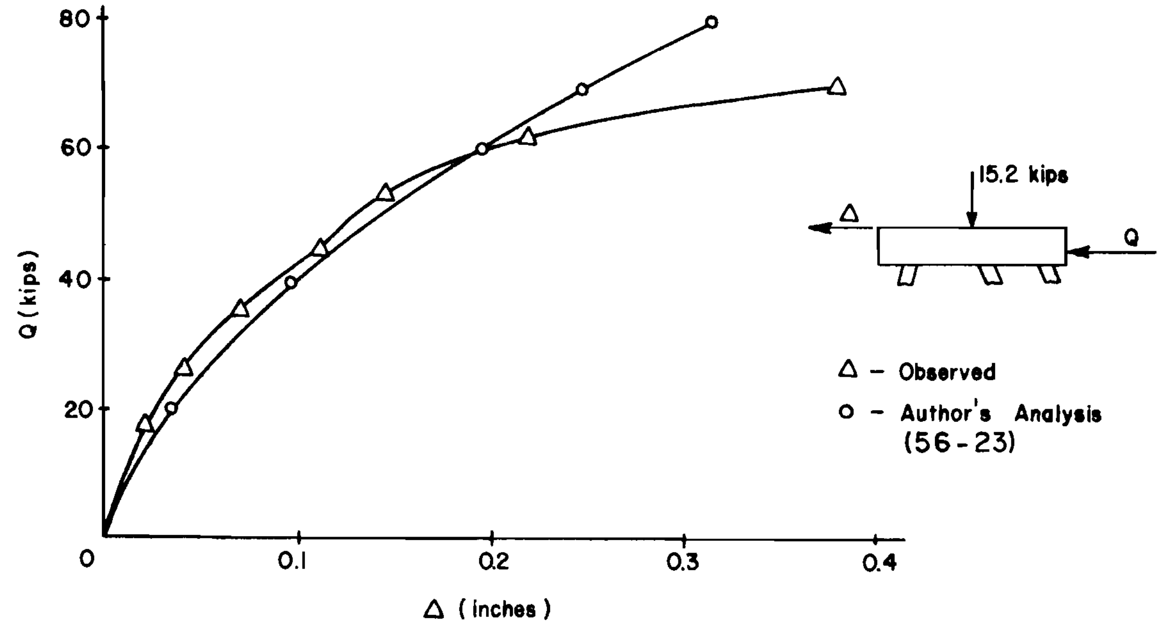


Fig 10. Soil-supported bent tested at Galati, Rumania.



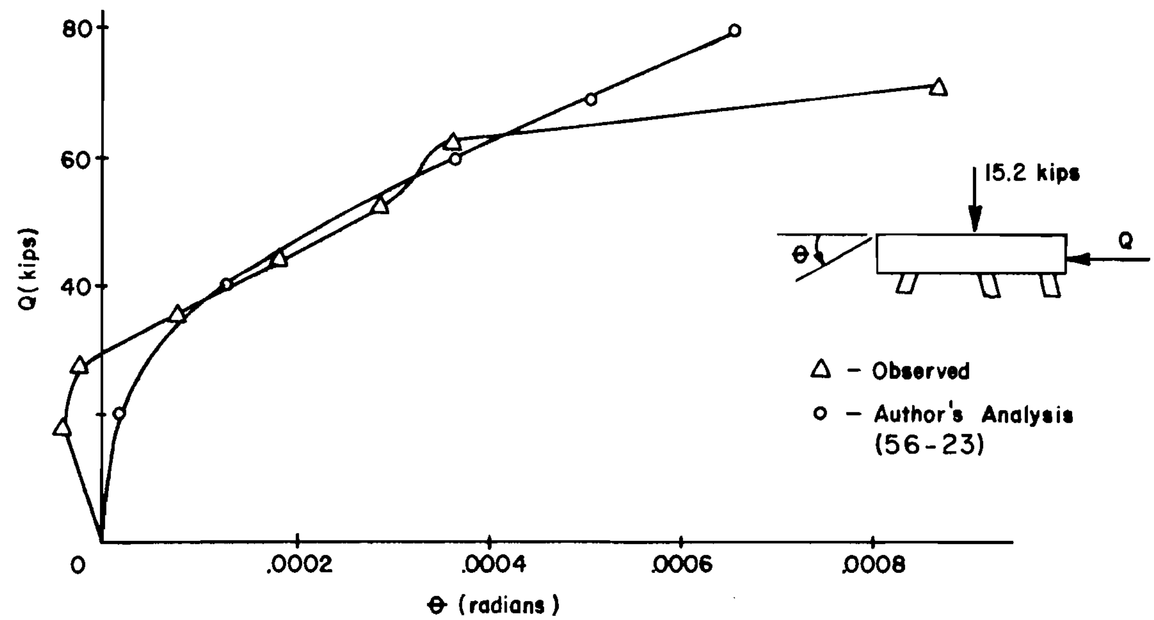


Fig 11. Load-displacement and load-rotation responses for three-pile bent.

structural members to be represented. The yielding of members associated with plastic and limit design may be studied at the discretion of the designer.

The nonlinear geometric effects of axial force and lateral displacement interaction and of the stretching of members due to bending are also considered as a part of a complete large-displacement analysis by the program.

Because of the generality and wide range of application of the program it may be less efficient for a linear type of analysis than the previously described linear analysis program. Therefore, this program should be used primarily for problems which cannot be solved accurately with program FRAME 11 of Report 56-21.

CHAPTER 4. SLABS, PLATES, AND GRIDS

This chapter is intended to summarize all of the project developments that have to do with slab, plate, and grid analysis. The efforts have been directed toward many subjects, but they have been part of an overall systematic study of the behavior of roadway pavements and bridge structural systems. Such a coordinated approach has made better use of available funds and manpower in solving these complex technical problems. Some of the developed computer programs are directly applicable to present design problems and some represent contributions in the continuing research. The basic objective was always to develop solution techniques which permit engineers to study the behavior of structural systems in a realistic manner.

Research Report 56-5, Layered Structural Analysis

Research Report 56-5 contains a numerical method for the analysis of layered structural systems. A layered system is one composed of members which overlay and are supported by other members which may in turn be supported by still other members. An example of such a layered structure is the floor system of a highway bridge. A bridge is generally composed of a slab supported by girders interconnected laterally by diaphragms, all resting on supports such as bent caps. Thus, a bridge can be visualized as a three-deep system of structural elements, a series of bent caps, a gridwork composed of girders and diaphragms, and a slab. Other layered structures might be slabs-on-foundations, grid-type foundations, aircraft assemblages, ship hulls, and signs.

The method of analysis consists of replacing the layered structural system by a discrete-element model of each layer. This model is exactly the same as the beam-column models in the previous developments. Only vertical connections are considered between each layer. The individual beam solutions are solved by an alternating-direction iterative technique for each layer in the system. Closure parameters add stability to the system by tending to hold it together. The effects of the closure parameters are nullified at closure when all members in the network have a set of common deflections.

Two programs are included with the report, LAYER 7 and LAYER 8. LAYER 7 is for solution of orthogonal grid systems in which there are only beams in the network. LAYER 8 is an identical program which has the addition of plate-type twisting elements in the grid network to represent a slab. As was found with the ADI frame solutions, the requirement of closure parameters creates a significant drawback for the use of these ADI methods. Only when a number of problems of a particular type are solved can appropriate closure parameters be chosen with confidence. Both of the programs are set up for only three layers. Two of the layers are parallel to one another and as was found in a later development, this amounted to no more than adding the stiffness of this third layer to that of the layer in the same direction. Therefore, a two-way grid network is all that would have actually been necessary.

Research Report 56-6, Pavement Slab Analysis (ADI)

This work was developed at the same time as the one above and emphasizes application of discrete-element methods to the solution of highway pavements. The report can be considered as an excellent reference for all plate formulations for later developments in this project.

The computer program is SLAB 17. The same drawback of alternating-direction iterative techniques remains in this procedure as in the previous one. The closure process requires a judicious selection of closure parameters. Unfortunately, experience is needed with a particular class of problems prior to obtaining acceptable solutions. ADI methods provide vital techniques and may have promise for other types of work. It has been found with later developments on this project that with increases in computer size, storage, and speed, direct solution techniques are more easily applied, at the sacrifice of only a small amount of additional computer storage space.

A number of example problems are included with the report. Among them are various closed-form solutions for comparison which demonstrate the good accuracy which can be obtained.

One example problem which is included is a bridge approach slab, shown in Fig 12. The problem is somewhat unrealistic, but it does demonstrate the diversity of input parameters that can be used for this type of program.

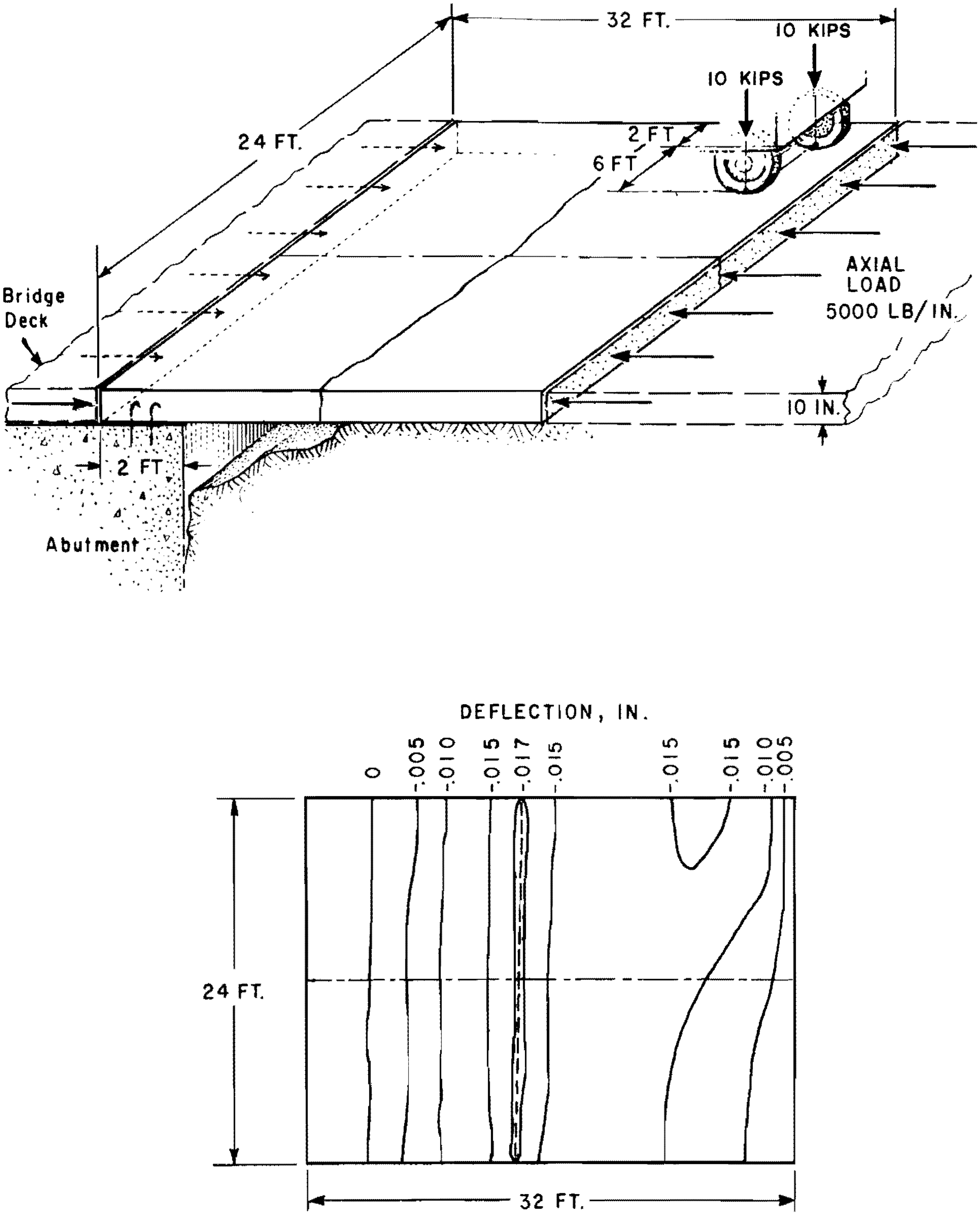


Fig 12. Bridge approach slab.

Research Report 56-9, Direct Solution Process for Slabs

Research Report 56-9 presents a method for the direct computer solution of plates and slabs. The basic procedure is an extension of the same discrete-element methods in Reports 56-5 and 56-6. The formulation of the equations, however, is altogether different. Each line of joints in the transverse direction of a slab or plate is formulated with an integral set of equations. Each following transverse line of joints is also formulated in succession. The solution of the complete set of equations is performed in a manner exactly identical to that previously used for one-dimensional beam-column solutions in Reports 56-1, 56-2, and 56-4. The long direction of a slab or plate problem determines the number of partitions in the overall structural stiffness matrix and is directly analogous to the number of stations in a beam-column solution.

The computer model used for the system of equations is shown in Fig 13. The torsion bars represent the twisting stiffness of the slab or plate with the bending stiffnesses represented by the elastic blocks at the intersecting joints of the rigid bars. Poisson's ratio effects are also included at the joints. The only errors resulting from application of these modeling techniques are caused by approximating the real slab with the model. The solution of the equations is exact for the model within computer accuracy. Therefore, the closer the model duplicates the real slab, the more precise the computed results. Thus, the greater the number of increments used to model a problem, the greater the accuracy of the solution. Reasonable results for most cases can be obtained by using 8 to 15 increments in each direction to define the overall structural properties of the real slab or plate. The number of increments also depends on the dimensions of the problem and the support and load characteristics, as well as the accuracy required.

The computer program included with this report is known as DSLAB 5. The inputs are almost identical to those in the previous two reports except that closure springs or closure parameters are not required. The solution process used in solving the equations, unfortunately, was chosen to manipulate the submatrices of the overall stiffness matrix in a manner which did not take advantage of either the banding of the equations or the many included zeroes in each individual submatrix. Thus, an extraordinary amount of computer storage was required to solve even a small problem. The program is direct in its formulation and easy to understand and should be used as a reference for the other later direct solution developments on this project.

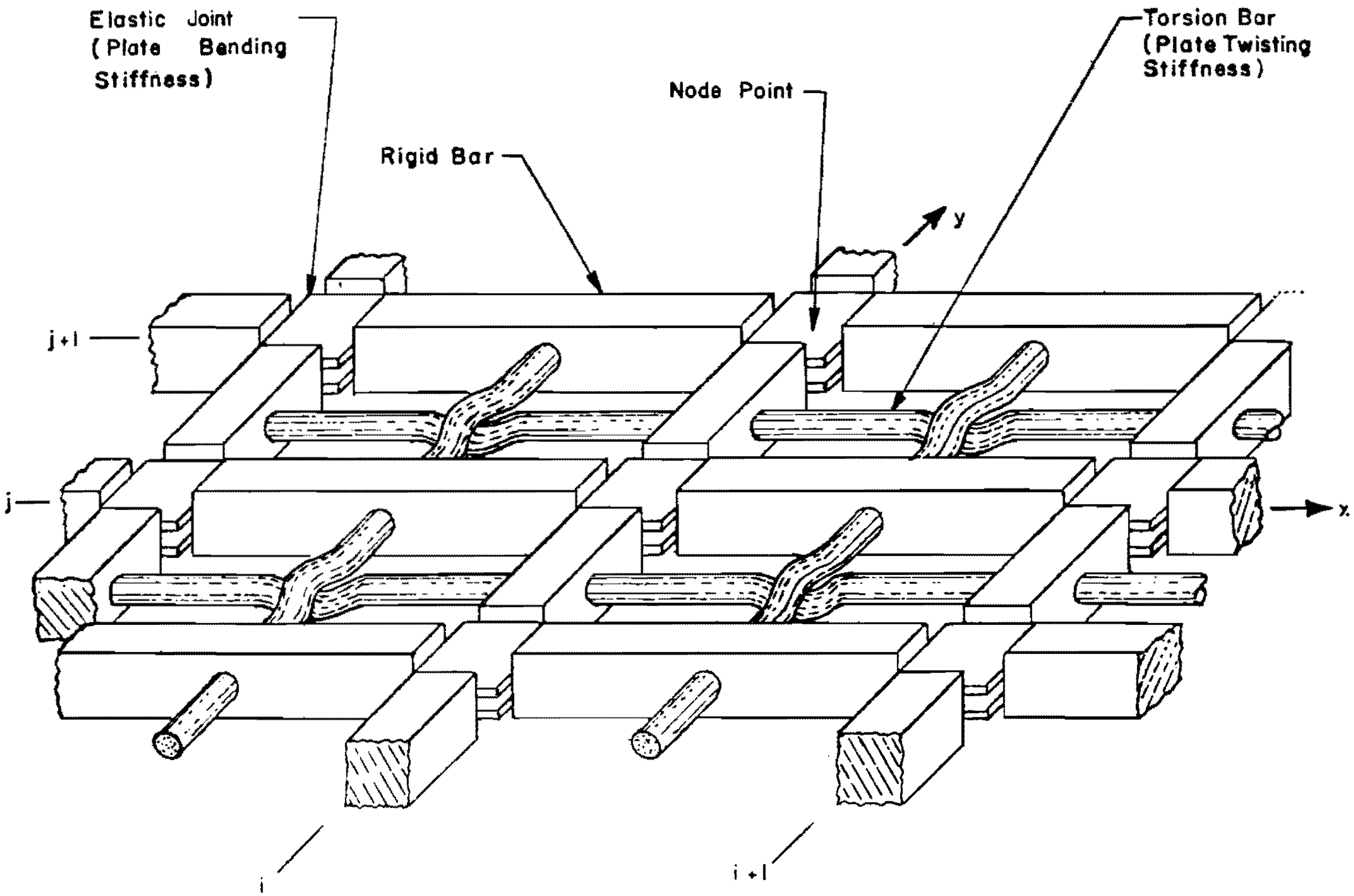


Fig 13. Discrete-element model of a plate or slab.

Following the development of this direct solution analysis technique, a general method was developed to solve this type of equation system. It was found that most of the developments were common in general equation appearance and could be solved by a universal solution technique, which for compactness was documented in a separate report. This is Report 56-19, described in a later section.

Research Report 56-11, Variable Increment Length Slab Analysis

Research Report 56-11 extends the analytical study of the previous report to allow for the inclusion of variable increment lengths in each direction to better represent a slab or plate system. It was later found that the ratio of increment lengths between adjacent increments in a plate or slab model could not be more than one and a half to two times the smaller value. This was found to greatly limit the application of the program. Subsequent developments, which included the ability to allow rotational restraints and specified displacements to be described, let the user look at a small section of a large problem in detail without the necessity or complexity of using variable increment lengths.

The computer program included in Report 56-11 is VISAB 3. It uses essentially the same solution technique as that in Report 56-9. The efficiency of the solution, which was developed in the solution process algorithm described in Report 56-19, was not included with this program. Example problems were included which were essentially the same as those in the previous two reports.

Research Report 56-13, Multiple Loading Technique with Two-Way Slabs

This report was essentially an application of the slab analysis techniques to two-way bridge floor systems. In addition, the program utilizes for the first time the efficient solution techniques developed and documented in Report 56-19.

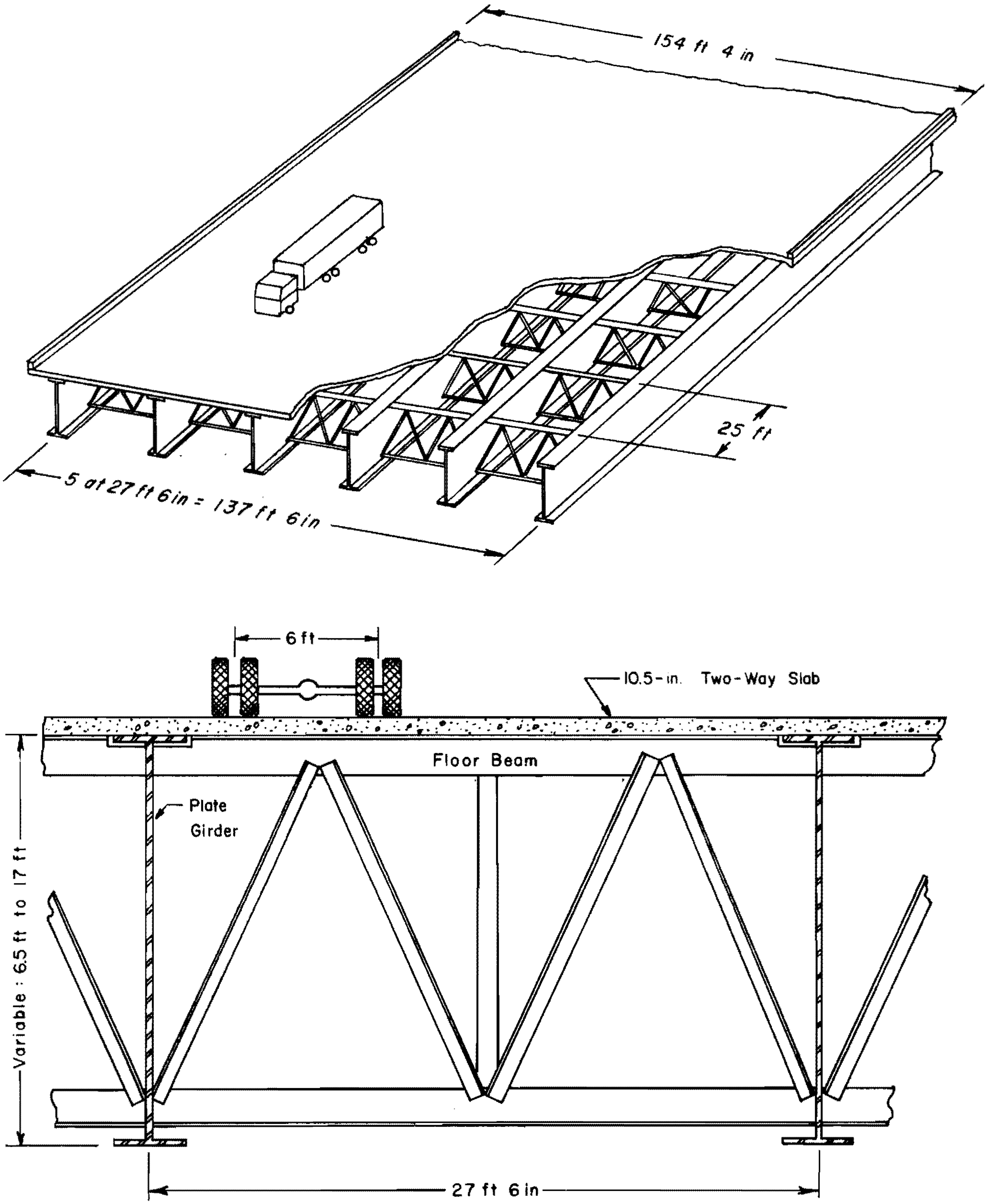
This study was initially concerned with the development of a rational procedure for the analysis of two-way bridge floor slabs continuous over many supports. Most structural systems require several solutions in order to consider different loadings or placements. This is especially true for a bridge floor slab which is subjected to a variety of moving-load patterns. During the study of the two-way floor slab analysis procedure, a further extension to the SLAB computer program series was developed which allows a complete series of problems with different load patterns and positions to be solved in a much more efficient way.

The system of equations is solved by a two-pass recursive technique described previously which is identical in form to those for the previous beam-column developments. The interesting and important property that was observed during the development of the two-way slab analysis procedure was the fact that when the direct two-pass recursive technique is used, the loads appear only in one of the three recursion coefficient equations. The major amount of computer solution time is spent in generating the other two recursion coefficients; thus, the coefficient which includes the loads is simply modified for successive multiple loadings so that any number of back-substitutions can be made for each loading condition. This procedure is called the multiple-loading recursive technique. This same multiple-loading efficiency is included in the basic solution routines of Report 56-19.

The program included with this report is known as SLAB 30. The structure chosen for demonstration of the two-way analysis technique is shown in Fig 14. Due to computer size limitations, a solution with the complete bridge deck was impossible. Therefore, procedures were developed in which only a few panels were analyzed in each area of the deck. An edge restraint spring which represented the continuity of the structure in the areas away from the loaded panels was applied around these panels.

Included in the study is a comparison of one-way and two-way analysis procedures. The AASHO formulas and the specified loads have been shown to give adequate results for one-way slabs when coupled with the specified allowable stresses. It remains to be determined, however, if coupling the same loads, impact factors, and allowable stresses for design of two-way slabs utilizing the developed analysis procedures will also give similar adequate results.

An interesting aspect of the comparative studies was the computed secondary moments which occurred under the load points in the direction transverse to the primary reinforcement. The AASHO specifications give no way of computing this transverse moment but a certain amount of distribution reinforcement is required as a percentage of the main reinforcement. By computing the resisting moment offered by this normal amount of distribution reinforcement,



A TYPICAL SLAB PANEL

Fig 14. Two-way bridge floor slab.

and comparing it to the computed value of the secondary moment, a surprisingly close correlation was observed.

This program can be considered as the basic slab computer development, which can be extended to a number of user-oriented applications. However, its efficiency has been superseded by the developments of Report 56-25.

Research Report 56-17, Dynamic Slabs on Nonlinear Foundations

Research Report 56-17 presents a numerical method for the dynamic analysis of plates and slabs on nonlinear foundations. The method offers the highway engineer a rational approach for the solution of many plate and slab vibration problems, including pavement slabs and highway bridges which can be idealized as orthotropic plates. The effect of vehicle motions on stresses and deflections on highway pavements has long been an unknown factor. An example in the report presents the dynamic analysis of a bridge approach slab (Fig 12). The subgrade nonlinearity used is a bilinear model which represents the loss of foundation support as the slab rises from the foundation.

A rational method for step-by-step dynamic analysis is used in conjunction with a nonlinear analysis procedure in which the load applied to the system is modified to reflect the nonlinearity instead of the customary procedure of altering the stiffness between iterations. The multiple-load procedure originally presented with Report 56-13 was extended for the load-iteration method and coupled with a linear acceleration algorithm for the analysis procedure.

The computer program included with the report is SLAB 35. The required input to describe a slab or plate problem is essentially the same as in the other plate and slab developments. The addition of dynamic behavior requires the definition of time step intervals. Iterative control data are also required if the slab is also supported on a nonlinear foundation. The discrete-element model which is used is essentially the same as in the other plate and slab developments, extended to include dynamic mass and damping characteristics. The mass is lumped at the joints of the model (Fig 13) with each joint connected to a fixed reference plane by a nonlinear spring and dashpot for damping. Solutions to the equations of motion are obtained at each discrete point in time. An algorithm based on the assumption of linear acceleration between time stations is used to advance the solution in a step-by-step fashion. Nonlinear foundation response is analyzed by iteration for equilibrium at each time step. As previously stated, the equivalent loading is modified with the load-iteration technique to produce equilibrium.

The analysis tool presented with this report permits qualitative and quantitative study of some of the effects of dynamic loading on structural pavements, bridges, and other structures. A bridge can be idealized as an orthotropic plate, which is more realistic than idealizing it as a beam. The program is general enough to permit the study of a wide range of structures which contain plate-like substructures such as floors of multistory buildings, certain types of aircraft subassemblages, and grids such as those which make up the deck of an offshore drilling platform.

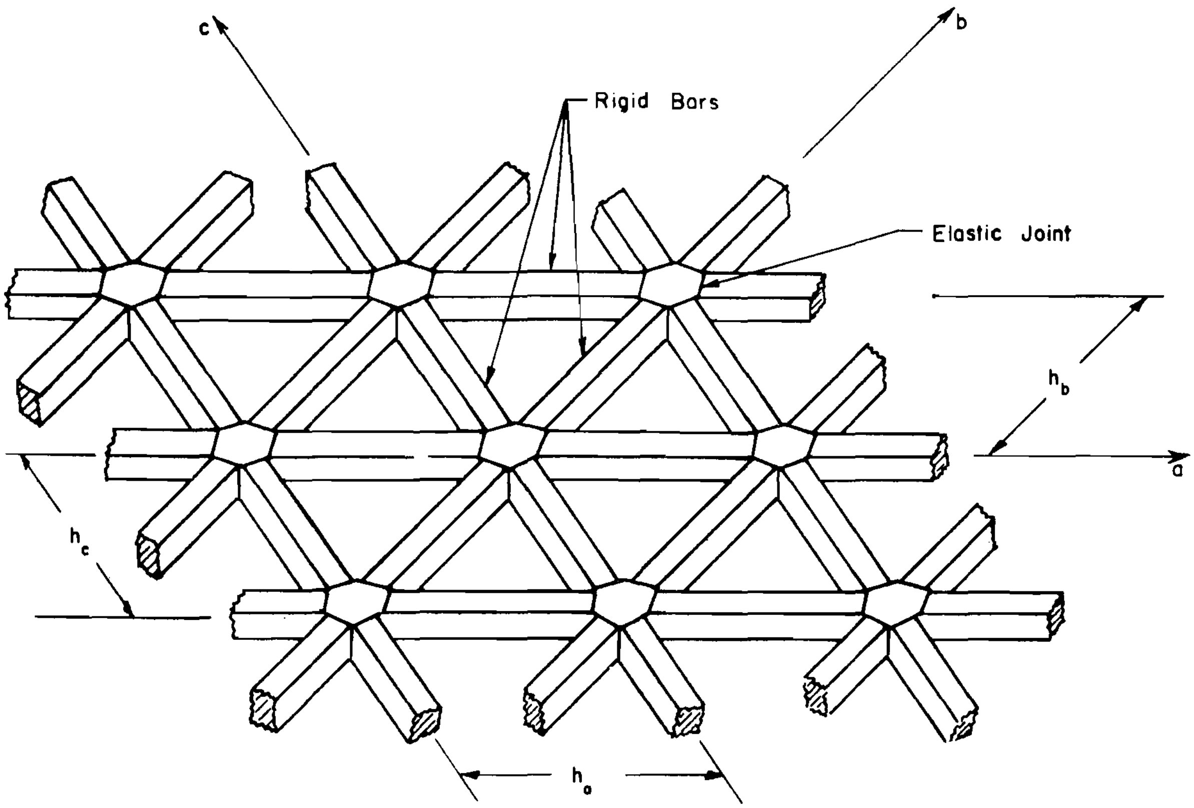
The computer program associated with this development should not be considered as design-oriented. Rather, it should be applied to those types of problems which are considered investigative to determine the basic relationships that occur between a moving vehicle and a pavement or bridge structure supported on a nonlinear foundation.

Research Report 56-18, Skewed, Anisotropic Plates and Slabs

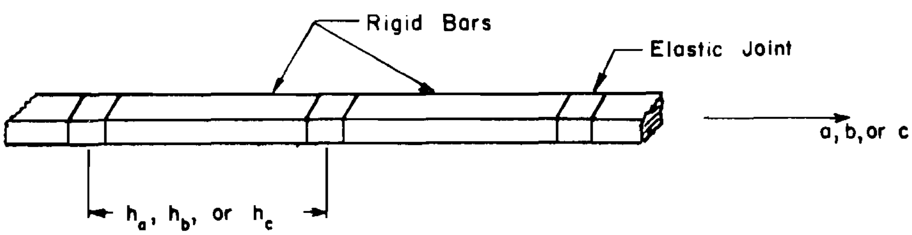
Research Report 56-18 describes a discrete-element method of analysis for anisotropic skew-plate and grid-beam systems. Analysis of skewed slabs or plates has always been difficult since there are no closed form mathematical solutions available for even the simplest cases. The practicing engineer normally has used some approximate procedure for analysis, e.g., considering a strip of slab in the primary direction and analyzing it as a beam. This kind of approximation may be reasonable for a rectangular slab but may be very inappropriate in the case of a skewed slab since there are large twisting effects. The largest principal moments and stresses are generally not in the primary direction.

The report presents the development of relationships for elastic compliances such that the slab or plate may consist of a completely anisotropic material and have embedded grid beams which can run in any three directions.

The slab model has elastic joints connected by rigid bars running in three general directions, as shown in Fig 15. The elastic stiffnesses of each tridirectional beam are also concentrated at the joints as in previous developments. Since stiffness, loads, and restraints are all lumped at the



(a) Skewed plate.



(b) Beam.

Fig 15. Discrete-element model for anisotropic skewed plate and grid beam.

elastic joints, all displacements take place at these joints. The only function of the rigid bars is to transfer bending moments from one elastic joint to another.

For plane stress problems the anisotropic stress-strain relationships require computation of six elastic compliances or elastic stiffness properties. It has been shown that for normal coordinate systems the compliances can be related to six independent elastic constants. These are the moduli of elasticity in two directions, one shear modulus, one Poisson's ratio, and two coefficients of mutual influence of the moduli of elasticity. Experiments are sometimes required to determine all six of these elastic constants.

It is felt that, in this report, for the first time relationships are worked out in which the six elastic constants can be basically related to the three moduli of elasticity with respect to any three directions and three Poisson's ratios related to these same three directions. This simplification is very helpful in understanding and computing the elastic constants. Transformation relationships are included whereby the modulus of elasticity and Poisson's ratio in any desired direction can be obtained from three moduli of elasticity and three Poisson's ratios related to any other three directions.

Thus, the six required constants could be easily experimentally determined for any type of anisotropic plate by testing three specimens from the plate in only unidirectional tension. These three specimens could be taken from any three directions. The measured moduli of elasticity and Poisson's ratios may then be transformed to the required directions.

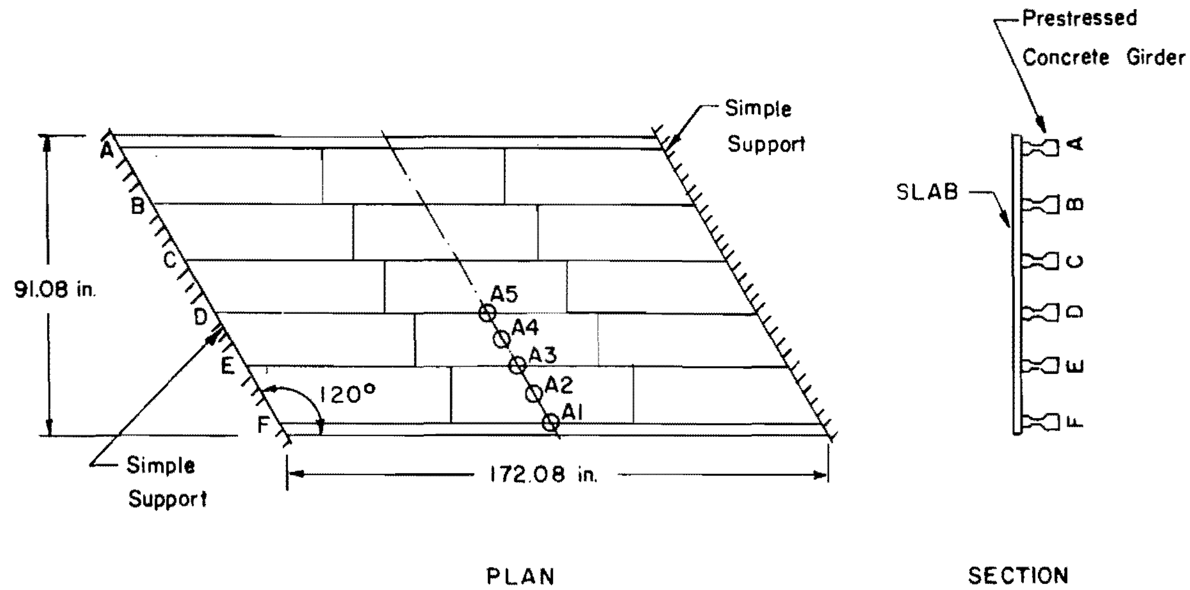
The computer program documented with this report is SLAB 44. The equations of equilibrium are applied at each joint of the discrete-element model and the resulting equations are arranged in a progressive fashion. The same recursion-inversion solution procedure used in other developments on this project is used to solve the equations. The algebraic solution process is documented in Report 56-19. The efficiency of the multiple-load procedure, first presented with Report 56-13, is also included.

Several example problems were solved and included in the report. Since there are no closed form mathematical solutions for skewed plates, the results have been compared with the results from other approximate methods. Some of these are series-type solutions, finite-element, conformal mapping, finite-difference, electrical analogue, and experimental results. It has been observed by others that in the case of simply supported, uniformly loaded plates, the accuracy of finite-difference and finite-element methods of solution drops rapidly as the skew angle increases. This effect has not been observed with the included discrete-element approach. The desired accuracy, however, does depend upon the number of increments selected. In general, the results of the discrete-element model are in very good agreement with the results of other approximate methods and with the limited experimental data.

One of the experimental studies compared to the analytical in this report is shown in Fig 16. This was a prestressed concrete bridge model which was studied under various loading conditions. The model was constructed and tested by another research project, Project 3-5-68-115, "Verification of Computer-Simulation Methods for Slab and Girder Bridge Systems" (Ref 5). The bridge represents a Texas Highway Department standard bridge simply supported with a skew angle of 30 degrees. The bridge consists of precast, prestressed, I-shaped girders with a cast-in-place deck slab. The slab was constructed to act compositely with the precast girders. The dimensions of the model bridge are shown in Fig 16. The scale factor is 5.5. During the experimental investigation, auxiliary tests were made to experimentally determine the bending and twisting stiffnesses of a single precast girder with the appropriate cast-in-place composite slab.

Program SLAB 44 was used to analyze this bridge. The experimentally determined composite girder stiffnesses were used as input. However, as was shown, very good estimates of stiffness can be obtained by simple transformed section calculations. For service loads, an extensive modification of stiffnesses beyond those based on simple calculation is not justified. The design criteria which limit tensile stresses in prestressed concrete to values below those which usually cause cracks insure that at service loads the section will still behave elastically. The torsion stiffness of the girders was also experimentally determined and input as additional twisting stiffness with program SLAB 44.

The analysis was made for five different positions of concentrated load, which were identical to those used in the experimental test. The results of deflection and bending moment at the midspan of the girders are compared with the experimental results in Fig 17.



Number of Increments: 22 along skew by 72 along span

Slab Stiffness: $D_{11} = D_{22} = 7.46 \times 10^5$ lb in^2/in

Poisson's Ratio: $\nu = 0.167$

Girder Stiffness (Composite): 1.73×10^9 lb in^2

Diaphragm Stiffness: 2.34×10^7 lb in^2

Equivalent Girder Twisting Stiffness (distributed over 8.28 in. of slab width):
$D_{33} = 3.625 \times 10^6$ lb in^2/in

Fig 16. Experimental model (Ref 5).

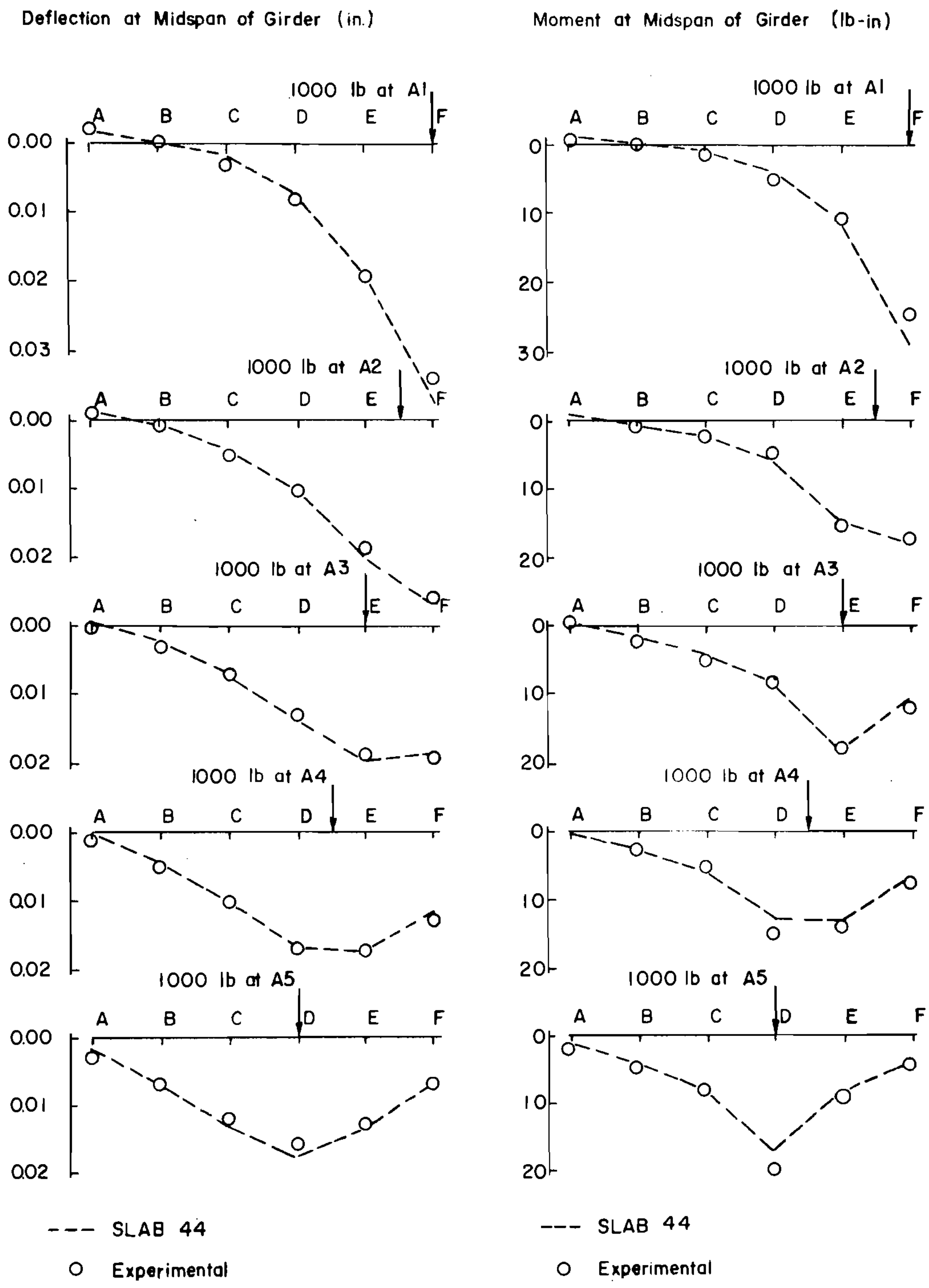


Fig 17. Comparisons with experimental results (Ref 5).

This problem effectively demonstrates the modeling of composite action. It also shows that the diaphragms can be handled very simply even though they run in neither the span nor the skew direction.

A number of other comparative solutions are presented in the report. One of these was for a structure with zero degree skew. The results were compared with those from program SLAB 49, which is described in the next section. The comparison indicates that the difference in maximum deflection between the two solutions was only about one percent, with a very coarse ten by ten increment solution used in both cases. This demonstrates that program SLAB 44 can also be used for orthogonal problems. However, SLAB 49 should be used for that type of problem since computer execution time is more efficient and it has other desirable options not included with SLAB 44.

Research Report 56-25, Slab and Grid Bridge Floor Systems

Research Report 56-25 is the final one in the plate and slab series and is specifically geared toward analysis of orthogonal slab and grid bridge floor systems, such as shown in Fig 18. It is particularly suited for highway bridge structures composed of slabs with supporting beams and diaphragms. The method can also be directly used for highway pavement analysis in which the pavement and subgrade are represented as a slab on elastic supports.

An engineer-user, if he so desires, can apply this program with very little study, using the simplified guide for data input and associating his particular structure with the included example problems. The primary example is a standard Texas Highway Department bridge solved for four different customary load cases.

The computer program included with the report is called SLAB 49 and allows any number of problems to be run at the same time, utilizing the solution and multiple loading techniques described in Reports 56-19 and 56-13. Preliminary versions of the program have been used in Research Project 3-5-68-115, "Experimental Verification of Computer Simulation Methods for Slab and Girder Bridge Systems" (Ref 5), and the method was shown to give extremely good correlations with experimental data for the various bridge structures considered. Comparison with this program was also made in Report 56-28, described in Chapter 6.

The basic computer model used is essentially the same as that in previous developments on this project. The primary difference is that the slab twisting

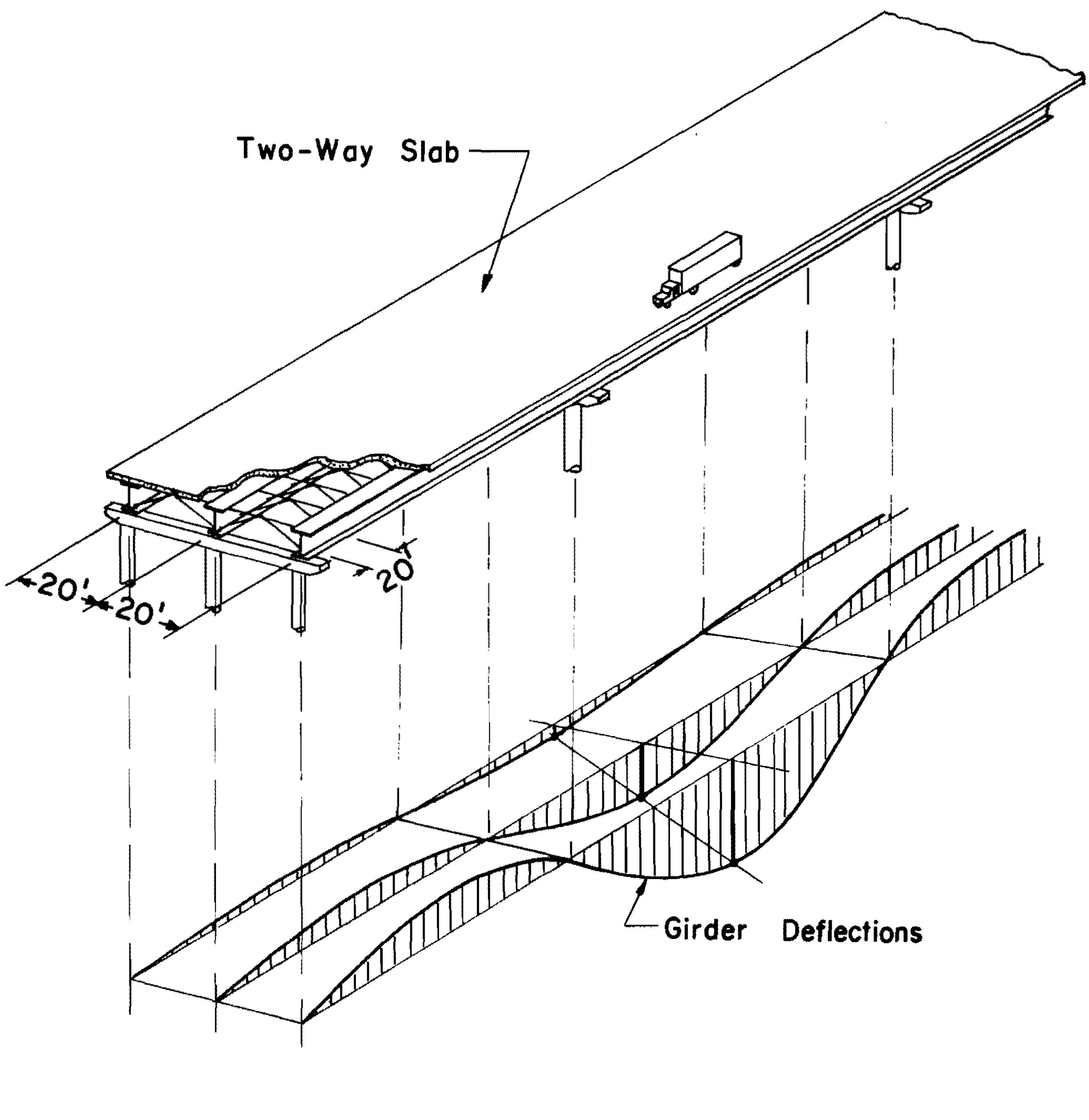


Fig 18. A typical bridge floor system.

stiffness is represented as a twisting element which is directly connected to the joints (Fig 19) instead of rods which frame into the bars (Fig 13). The twist reaction forces which result at the joints are the same as in the previous developments but the model presented in Fig 19 is simpler to visualize. An integral grid system acts in conjunction with the plate element shown in Fig 20. Both models are connected at the joints by ball and socket connections which insure that the deflection will be the same at the common joint locations. These integral grid beams are exactly the same beam-column element as that developed for the first programs on this project (Reports 56-1, 56-2, and 56-4).

Based on the model, the ordered system of equations forms a diagonally banded and symmetrical stiffness matrix in which the recursion-inversion method developed and presented in previous reports and documented in Report 56-19 is used to solve the system of equations for the unknown displacements. An analogous procedure is the recursive process described for solution of beam-columns in the first project report (56-1). The similarity between the recursive equations developed for slabs and grids and those derived for the recursive solution of beam-columns has been shown in Report 56-13. The same efficient multiple-loading technique is also included with this procedure.

A significant feature of the program is the technique by which all data input in a problem series are retained and echo printed for all problems. This procedure was first applied and found useful for the moving-load beam-column program of Report 56-4. The procedure avoids the necessity of recoding and including common data for each problem in a series. The program simply stores the input data card images which are searched at each level in the solution process with only the necessary stiffness and load terms regenerated at each required step. This technique of storing data card images has also been shown to provide the most convenient method of transferring information from data generation routines written for certain specific problem types. The data card images are created by the data generation routine with the basic SLAB 49 program utilized for the solution. By this means any number of special purpose programs can use this one basic orthogonal system computer solution technique.

Several additional features are available with this final program which were not included in previous developments. One of these offers the user the ability to select specific areas of profile output. These profiles are lines

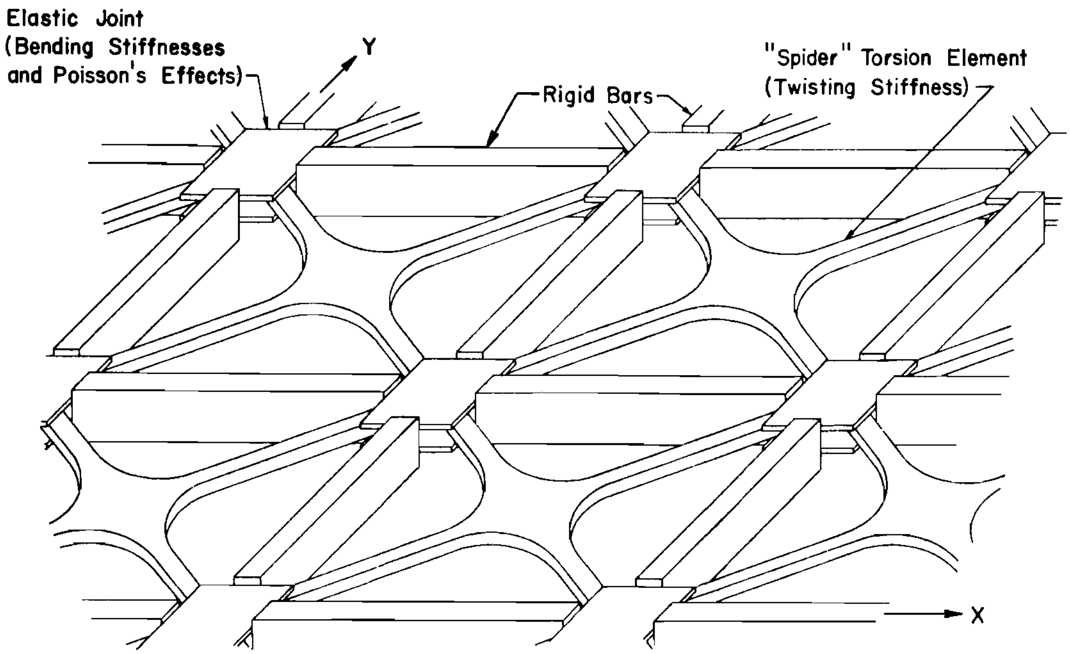


Fig 19. Discrete-element model of slab or plate.

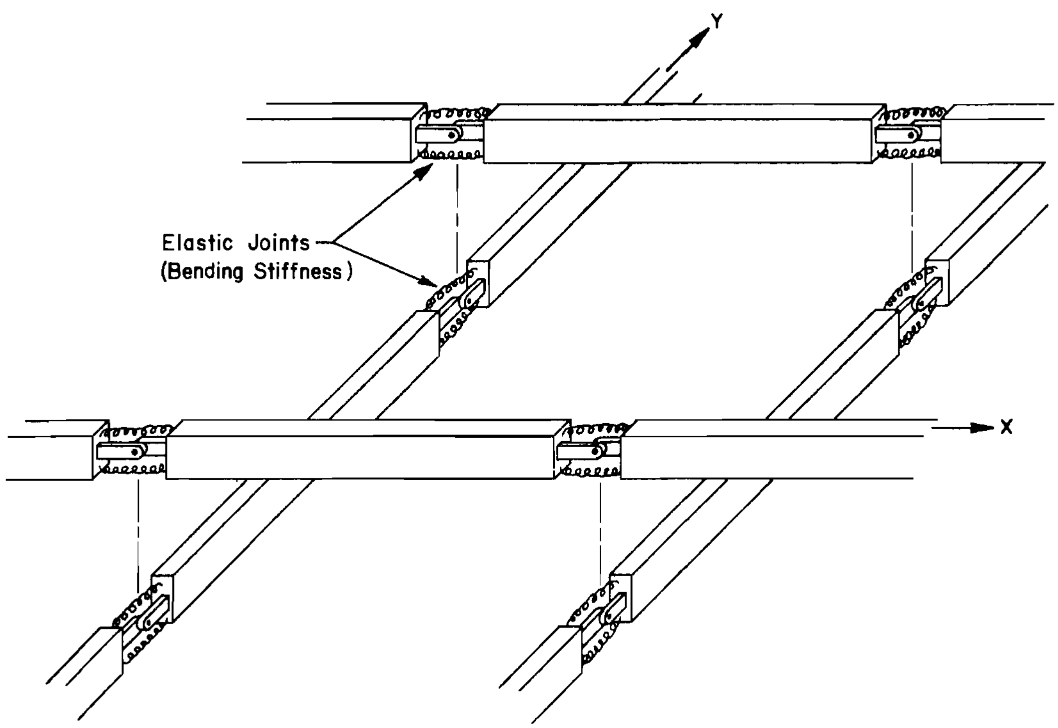


Fig 20. Discrete-element model of grid system.

of bending moment or deflection. Adjacent to the tabulated values is a series of asterisks whose placement relative to one another is based on the numerical value. Thus, a crude printer plot of the output values may be obtained. The plots have been found to be especially valuable because the user obtains them with the rest of his printed output. Time is not spent unnecessarily in waiting for line plotter output or in hand plotting. The plots are also useful in understanding slab behavior for areas adjacent to concentrated wheel loads and supports.

Three-dimensional deflection plots are also obtainable with the program if the appropriate plotter routines and hardware are available. A typical plot is shown in Fig 21, which is taken from one of the example problems. This type of plot allows the user a significant aid in interpreting results from complex structures.

Program SLAB 49 is now being used by Texas Highway Department bridge and pavement designers for analysis problems which up to this time have been difficult or impossible by other conventional methods. It is particularly suited for use on bridge structures since the interconnected grid beam network which is included can represent beams and diaphragms. Composite action of the beam and slab can be considered by appropriate equivalent beam representation. No additional program development is needed to allow immediate use of the program by the sponsors. User-oriented versions of the program could be made to make application easier for specific classes of problems.

The availability of this program makes feasible the study of various design options and their overall effect on highway bridge structural capabilities. Among these parameter studies could be the effect of beam spacing as it relates to span-width aspect ratios to better define the present load distribution coefficients. Diaphragm positioning and spacing could also be studied to aid in defining their effectiveness. Studies can now be conducted to analytically test various schemes for repair or strengthening of bridge decks.

Research Report 56-19, Basic Solution Process

Approximately halfway through the total project it was decided to report the solution process that was being used for the frame and slab type structural analyses separately. Further developments on the project then referred directly

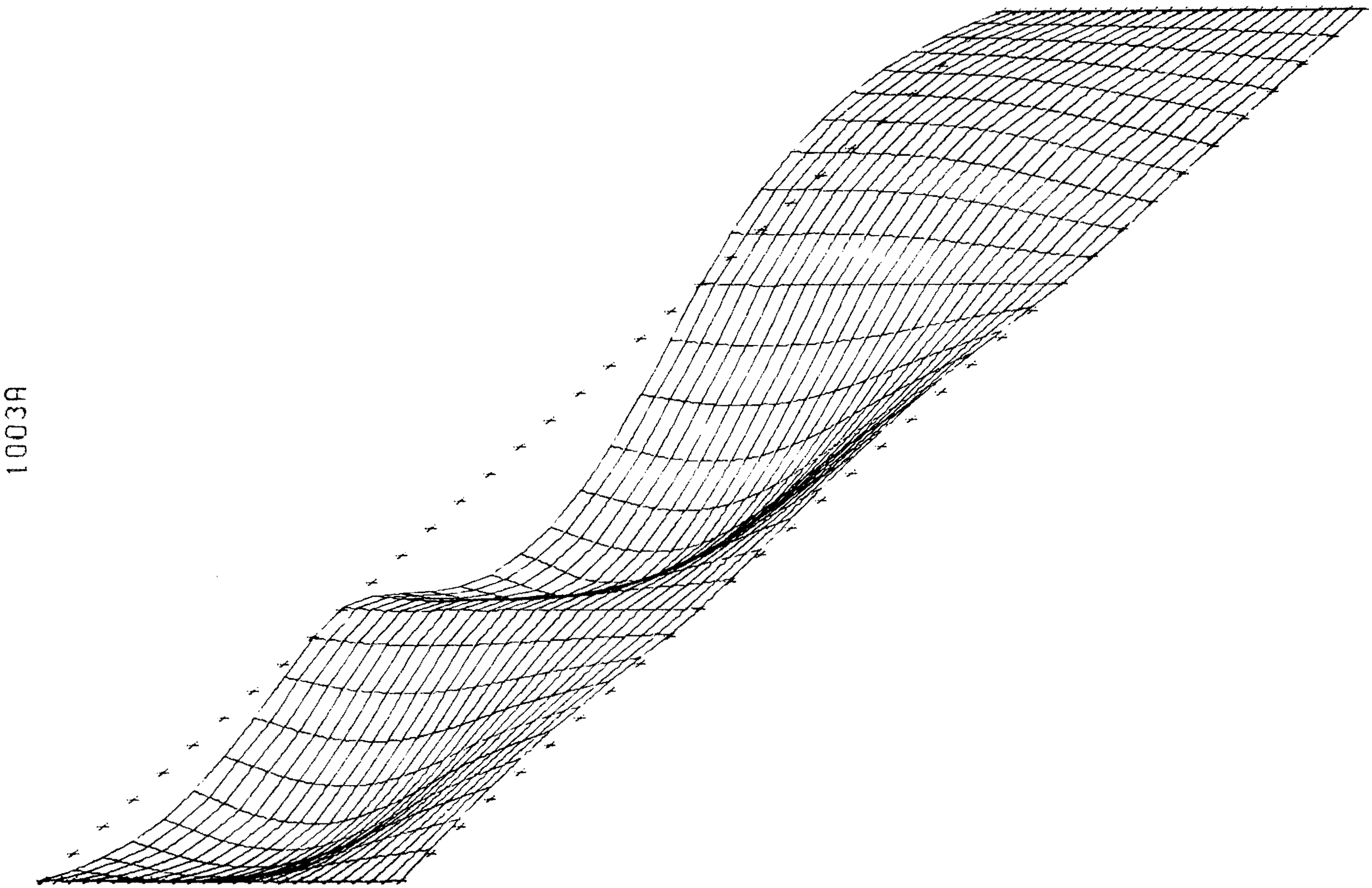


Fig 21. Three-dimensional plot of deflections.

to this solution technique and avoided the necessity of documenting it in each separate development. It is included in this chapter since its development was closely related to the slab and grid analysis techniques.

The report presents the general method for the solution of large, banded, positive definite, structural stiffness matrices. The primary goal was to produce an efficient and reliable solution process and to provide the user-programmer with a package which is problem independent and efficient and easy to use for all types of computers so that program development time could then be spent in problem analysis rather than on solution technique.

Computer storage and time requirements for most problems in structural mechanics often determine whether or not a particular program is feasible to use on an extensive basis. Some of the investigations associated with this project involve nonlinear or time dependent behavior which usually requires some type of incremental or iterative technique. Thus, careful attention must be given to the efficiency of the solution process even with the largest and fastest computers now in use.

To simplify the equation-solving process for the engineer-programmer, a system of general purpose routines has been developed which takes maximum advantage of the properties of usual structural equation systems. The routines provide an efficient two-step recursive elimination process for a diagonally banded system of submatrices. Banding at various levels is considered, depending on the particular type of problem and on the formulation and the arrangement of the equations. The generality of the overall system is enhanced by a general matrix manipulation package.

Options which are provided allow the user to take advantage of symmetry when it exists (as it generally does for structural problems), to select the size of the partitioned submatrices, and to make efficient choices in the use of auxiliary storage equipment.

The method keeps the basic computation effort and indexing requirements at a near minimum while imposing no limitations or burdens on the programmer-user. The core storage that is required for each program has not been minimized beyond the point where inefficiencies in the auxiliary manipulations result, for they would then tend to dominate and result in a greater total cost.

The power of this method is its ability to handle efficiently matrices which exhibit second level banding which lends itself to partitioning into submatrices which are in turn also banded. This is indicated in Fig 22, which shows a typical coefficient matrix generated for a structural problem. Usually the coefficient matrix is symmetrical in addition to being positive definite. Two procedures are included in the routines, one for the symmetric case and one for the nonsymmetric case. The user may choose whichever is appropriate for his particular purposes.

Comparative solution times for a typical problem application are shown in Fig 23. This time graph shows that by utilizing symmetry in an analysis, approximately one-half of the computer time can be saved for any width (K in Fig 23) of problem. In addition, if the multiple-load features are exercised, which are indicated as offspring problems in Fig 23, a very significant decrease in computer time is achieved.

The programs documented in the report are specifically for three and five-wide difference operators. Therefore, four separate routines have been prepared to consider symmetric and nonsymmetric problems. For the nonsymmetric case with a three-wide difference operator, TRIP 3 is used, and TRIP 4 is used for the symmetric case. For the five-wide nonsymmetric case, FRIP 3 is used, and FRIP 4 is used for the symmetric case. Each routine calls a group of secondary matrix manipulation routines referred to as the basic subroutine manipulation package.

The structural analysis user of the solution procedure generates his stiffness matrix and integrates the appropriate routines with his program. The solution routines return the unknowns (usually displacements) for further processing within the main program.

The solution procedure documented in this report should not be thought of as only specifically applicable to developments on this research project. It can be of immediate benefit to any engineer-programmer who has need of solving an ordered system of equations. The routines provide him not only with a ready-to-use efficient package but with one that requires a minimum of input and little or no reorganization of the natural form of his equations.

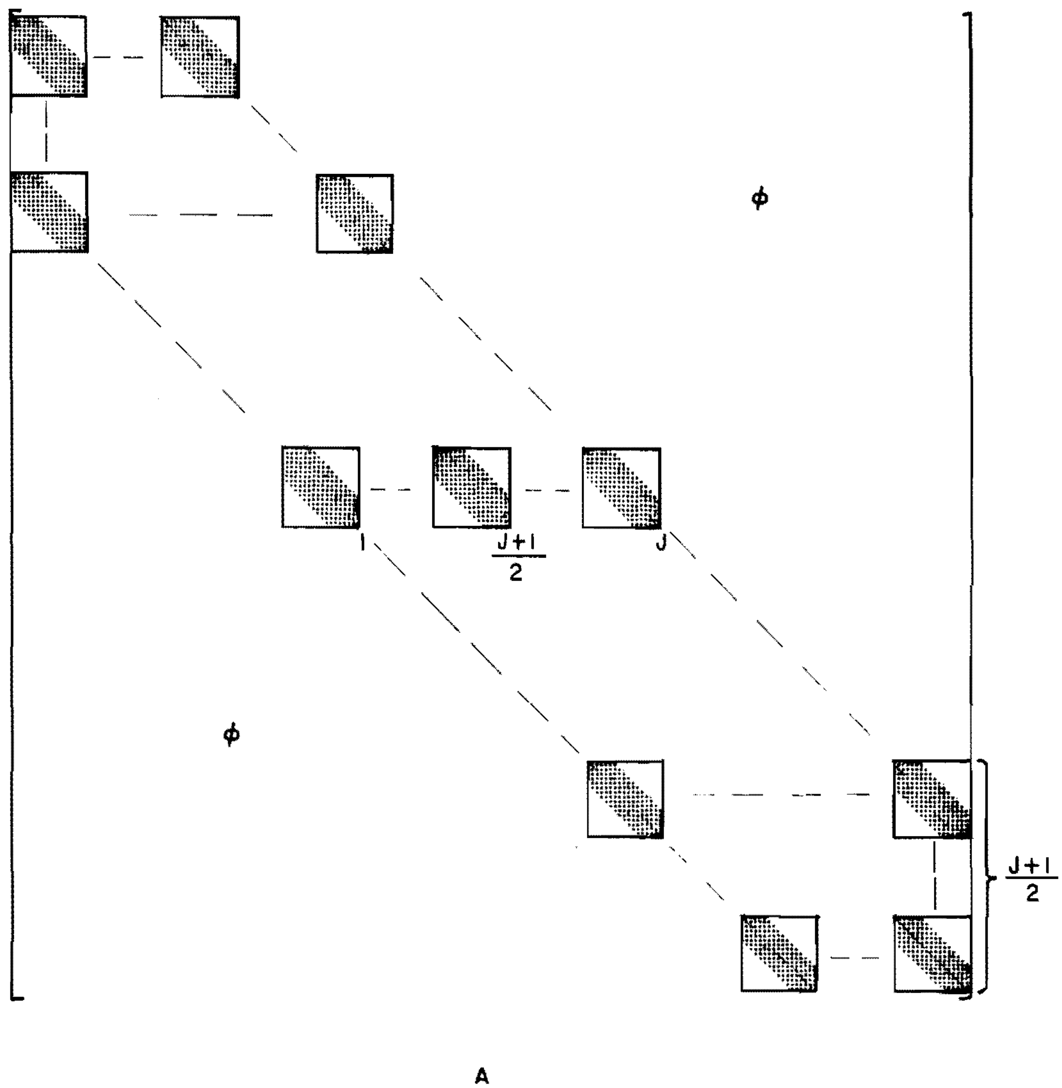


Fig 22. Coefficient matrix for a general J-wide operator.

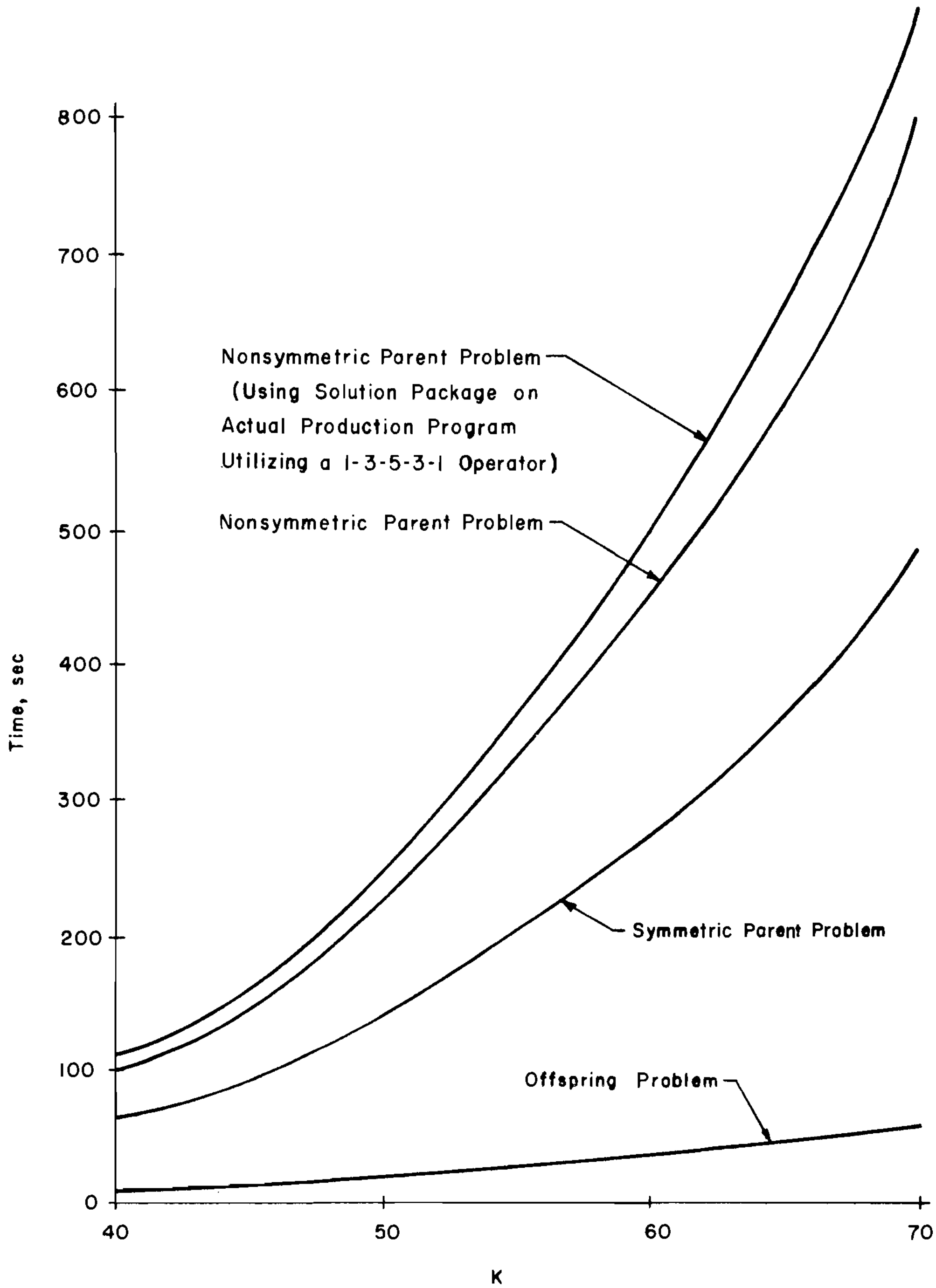


Fig 23. Time graph for five-wide solver.

CHAPTER 5. EXPERIMENTAL VERIFICATIONS AND PAVEMENT APPLICATIONS

One objective of this project as it relates to pavement design has involved the adaptation and extension of the developed discrete-element methods to provide computer programs for the solution of pavement slab problems. Such solutions are valuable for evaluating existing and planned slab designs with fewer simplifying assumption limitations. In addition, available slab performance data can be analyzed by means of a comparison with theory. Available design methods can now be extended to include other parameters, the effects of which are not adequately covered by existing methods.

The validity of any analytical solution can be best demonstrated by comparing solutions with actual experimental results. To obtain these results for use in verifying the basic discrete-element analytical methods for slabs on elastic foundations, a study of small-size plates was made.

Research Report 56-15, Small Dimension Tests with Various Supports

Research Report 56-15 deals with experimental verification based on the results of carefully controlled tests and considers plates on rigid supports under a variety of load and stiffness conditions. In addition, a plate was tested on a simulated clay subgrade under two loading conditions. Results of the static loading on the plates are presented as well as experimental results of some cyclic loadings on the plate supported by clay.

Analytical solutions based on the discrete-element model were found using independently determined plate properties for all cases. A comparison of measured and computed deflections was then made and error was calculated as a function of the maximum measured values.

For small loads, good agreement existed between experimental and analytical solutions using either linear or nonlinear springs for the supports. For larger loads, only the nonlinear soil representation produced good agreement. One of the typical measured sets of deflections compared with the analytical solutions is shown in Fig 24. This set of deflections was measured along the diagonal of

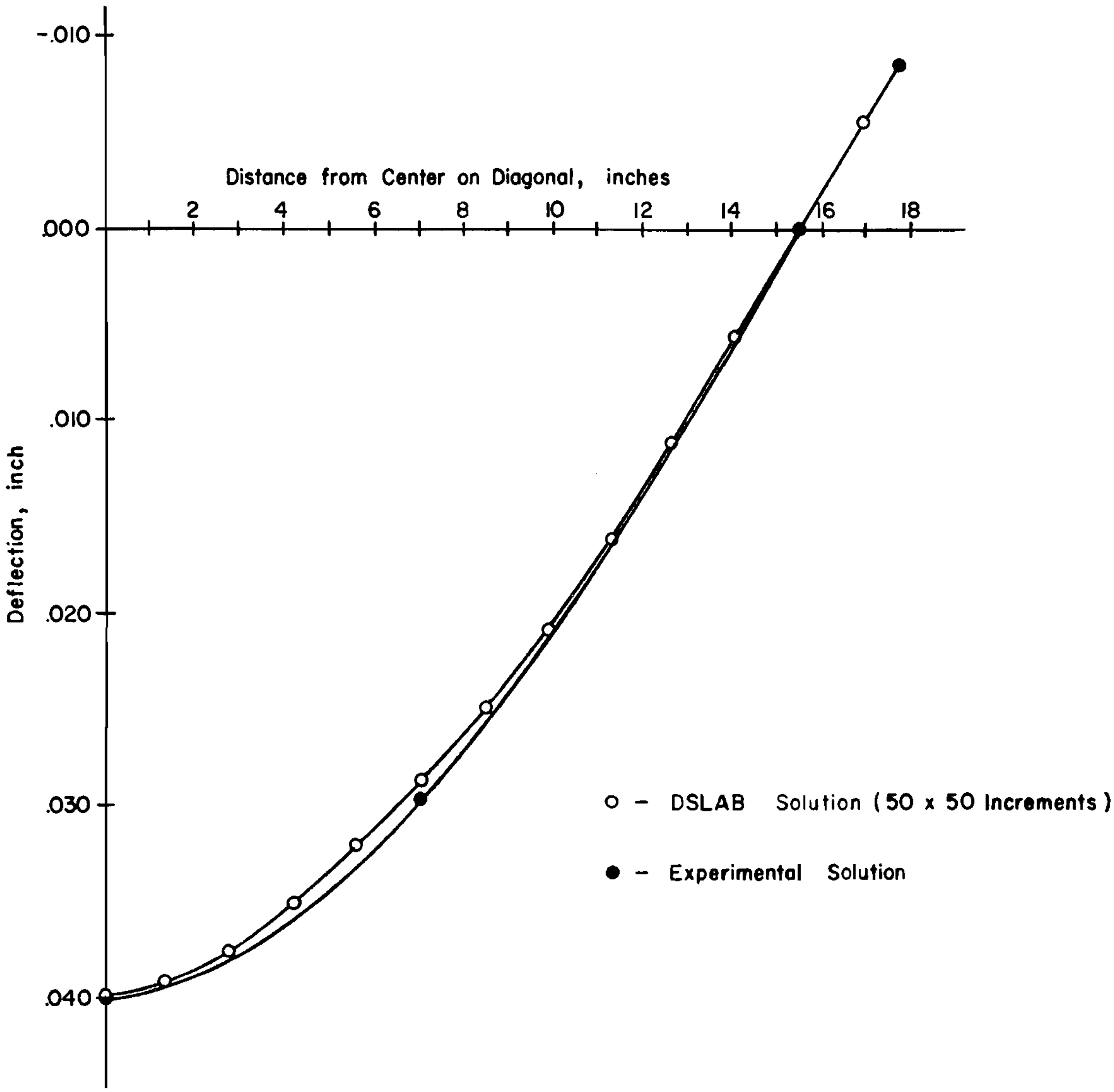


Fig 24. Experimental and analytical deflections.

an aluminum plate supported at four corners and subjected to center loading. For most cases the computed deflections compared well with the experimental values within one percent of the maximum deflections.

The experimental evidence presented in Report 56-15 shows that the discrete-element solution techniques can provide good results in predicting plate and slab deflections and stresses. Therefore, the programs can be confidently applied by highway design engineers and employed as useful tools in the analysis of bridge slabs and pavements.

Research Report 56-16, Small Dimension Tests and Evaluations

The preceding experimental versus analytical evaluation of plates included an experimental measurement of the modulus of effective subgrade reaction for those plate tests on clay. Additional plate load tests were taken with circular plates whose diameters ranged from 2 to 9 inches. The relationship between pressure and deflection was found to be linear for small initial deflections of the plates and nonlinear for higher deflection of the plates.

An additional plate load test was made by placing a thin layer of asphaltic concrete on the clay subgrade to improve the effective k-value of the overall supporting system.

In the past it has been very difficult to evaluate the composite k-values for use in analysis and design. The results of this study provide additional information for estimating these composite support values. Computing a composite k-value is almost essential to the rational design of rigid pavement systems, which, according to modern design practice, almost always utilize some type of layered support system.

Research Report 56-22, Cracked Pavement Study

Research Report 56-22 describes an analytical evaluation of the effect of transverse cracks on the longitudinal bending rigidity of continually reinforced concrete pavements. These cracked pavement systems are analyzed in a rigorous manner in this report by application of the discrete-element method of slab analysis.

A procedure was developed to assist the user in defining the appropriate amount of reduction in bending stiffness to represent the effect of a crack in a continuously reinforced pavement. It was shown that the bending stiffness at crack locations in most pavements should be reduced between 80 and 90 percent from the uncracked stiffness value. This is illustrated in Fig 25. It is important to note how minor the influence of concrete strength was on this stiffness reduction. The report includes a sample calculation of the determination of the average moment of inertia and the corresponding stiffness reduction.

A comprehensive sensitivity study was performed on the parameters usually considered in the design of reinforced concrete pavements. These covered the practical range of each of the following variables: slab bending stiffness, modulus of subgrade reaction, and crack spacing. From an analysis of variance, the most significant variables affecting deflection and principal stresses were bending stiffness and modulus of subgrade reaction. The latter variable showed a higher contribution to deflections than to principal stresses. Crack spacing showed a minor effect on slab behavior, although the inclusion of cracks as opposed to assuming no cracks was significant.

The results of this investigation provide additional information on the effect of discontinuities on the structural behavior of continuously reinforced pavements. The procedures presented will allow highway designers to confidently apply the discrete methods of analysis for particular problems.

Research Report 56-26, Concrete Shoulder Analysis Example

Research Report 56-26 describes how the practicing pavement design engineer can solve or analyze rigid pavement problems by the discrete-element method of slab analysis. The particular design problem used as an example in the report was obtained from the Texas Highway Department and concerns the determination of economics of concrete shoulders.

The use of the computer program associated with Report 56-25, which is known as SLAB 49, is described in detail. Included are all of the necessary inputs, computations, coding instructions, explanation of the data, interpretation of the output, and possible uses of the output in further design analysis. The example problems are coded and explained card by card for the benefit of the practicing pavement engineer.

The primary purpose of this report is not simply to discuss the merits of concrete shoulders. It should be thought of as a user's guide to illustrate

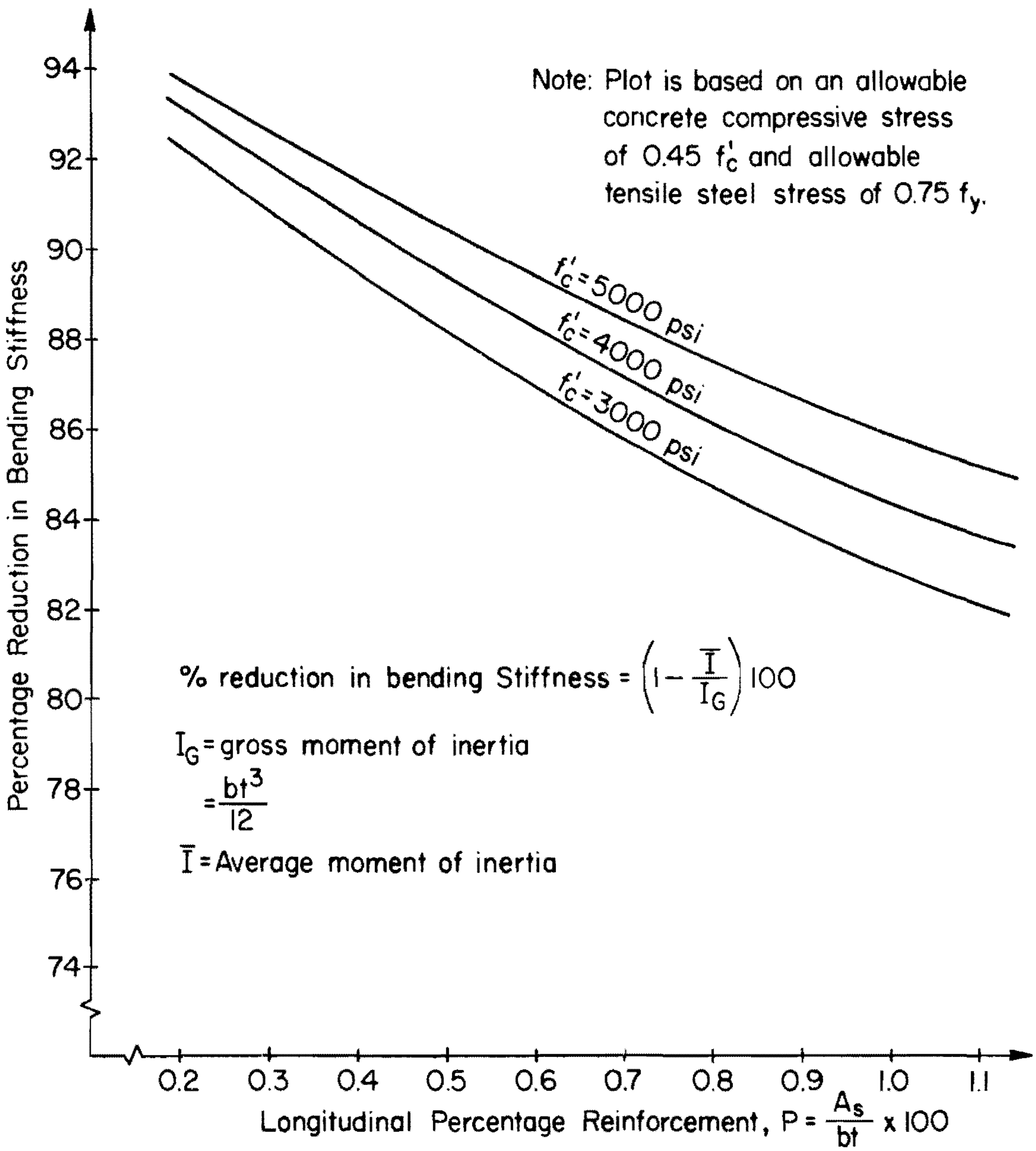


Fig 25. Reduction in bending stiffness at cracks.

in a practical way the use and application of the SLAB 49 program to pavement design problems facing practicing engineers. The coding methods illustrated and the data evaluation techniques used are equally applicable to a wide variety of other pavement problems.

Research Report 56-27, Summary of Pavement Analysis Methods

Research Report 56-27 is a comprehensive summary of all the discrete-element methods of analysis that can be used for pavement slabs. It includes the work related to pavement design and analysis as well as other applications. The details of the discrete-element methods may be found in the individual reports which are referenced within this report. A wide variety of variations of the basic techniques have been developed, including the variable increment length capability of Report 56-11, the multiple-loading analysis technique of Report 56-13, the dynamic loading considerations of Report 56-17 and the consideration of skewed pavement slabs utilizing Report 56-18.

All the methods have been verified by comparison with closed form solutions and various small dimension tests, as described in Reports 56-15 and 56-16. The Texas Highway Department has applied the results in numerous forms, including the development of design charts, the analysis of special problems, and the analysis of field data.

Report 56-27 is essentially a summary itself and can be thought of as a supplement to this final project report.

CHAPTER 6. FINITE-ELEMENT COMPARISON

Throughout the course of this project, interest was taken in how the discrete-element procedures could compare to the classical finite-element procedures which are now in current use by many people. Near the termination of the project the finite-element work had extended to the point where it was feasible to make a comparison between discrete-element and finite-element techniques.

Discrete-element procedures as applied to bridge floor systems are essentially two-dimensional. This plate-type representation has been used throughout this research project for the slab and slab and grid studies. Most finite-element procedures that are currently available from many sources provide for analysis with the structure composed of a series of rectangular or triangular elements which can be oriented in any direction, with connections at their edges. As a comparison of discrete-element and finite-element techniques, the following study was made to demonstrate the effectiveness of the discrete-element procedures as primarily applied to highway bridges. The discrete-element solution techniques provide a much more computer efficient solution than do finite-element procedures.

Research Report 56-28, Finite-Element Analysis of Bridge Decks

Research Report 56-28 presents an analysis of diverse structures by a method completely different from what has been used in all of the other developments of this project. A true finite-element procedure is used for the analysis of several particular types of structures and compared to other means of analysis. The method can be applied successfully for the analysis of several types of bridges. The method is described for application to the analysis of highway bridges. The report includes two typical bridges as example problems and the results obtained are compared with the discrete-element methods developed on this project.

The structures are modeled as an assemblage of flat triangular elements. Four different elements are available for use. Two of these are in the form of a new refined triangular element and a new refined quadrilateral element. The other two elements are a triangular element and a quadrilateral element with less refinement of stiffness evaluation. The refined elements are suitable for analyzing structures with very simple geometry while the other two elements may be used with finer mesh layouts for analysis of structures with complex geometry.

A 6 degree-of-freedom node point displacement system is used for the analysis. Such a system is enough for complete representation of any shell-type problem and at the same time permits refinement for representation of structures with complex geometries.

The method features considerable generality and simplicity in the input, thus enabling highway engineers to perform accurate analyses of three-dimensional type structures with minimum approximations. Other complicated geometries can be represented, as well as material properties and elastic support restraints.

One of the example problems shown in Report 56-28 is the structure which was analyzed and presented with Report 56-25, which is described in Chapter 4. The bridge was loaded with HS20 trucks as shown in Figs 26 and 27. The concentrated wheel loads (Fig 27) are proportioned to adjacent stations for the discrete-element model solution using program SLAB 49. In the finite-element solution, an even more coarse load apportionment was made since the finite-element rectangular elements were of an even larger size than the discrete-element solution. Proportional loads were used at the node points adjacent to each load. This approximation has essentially no effect on the overall bridge deflections, but it has a significant effect on the local deformations of the deck. The discrete-element solution deflections are shown in Fig 21.

Since the structure was symmetrical about two major axes (the first and last spans were of the same length and the girders were uniformly spaced in the transverse direction), the structure was solved using two major lines of symmetry. Because of this convenient symmetry, only one-quarter of the structure was considered for this analysis by the finite-element procedure. This necessitated replacing the applied load case by four separate load cases with known boundary conditions at the planes of symmetry at the deck. Each of the four load conditions was separately analyzed by the finite-element procedure. A hand superposition of the four load conditions was then made to make the final comparison.

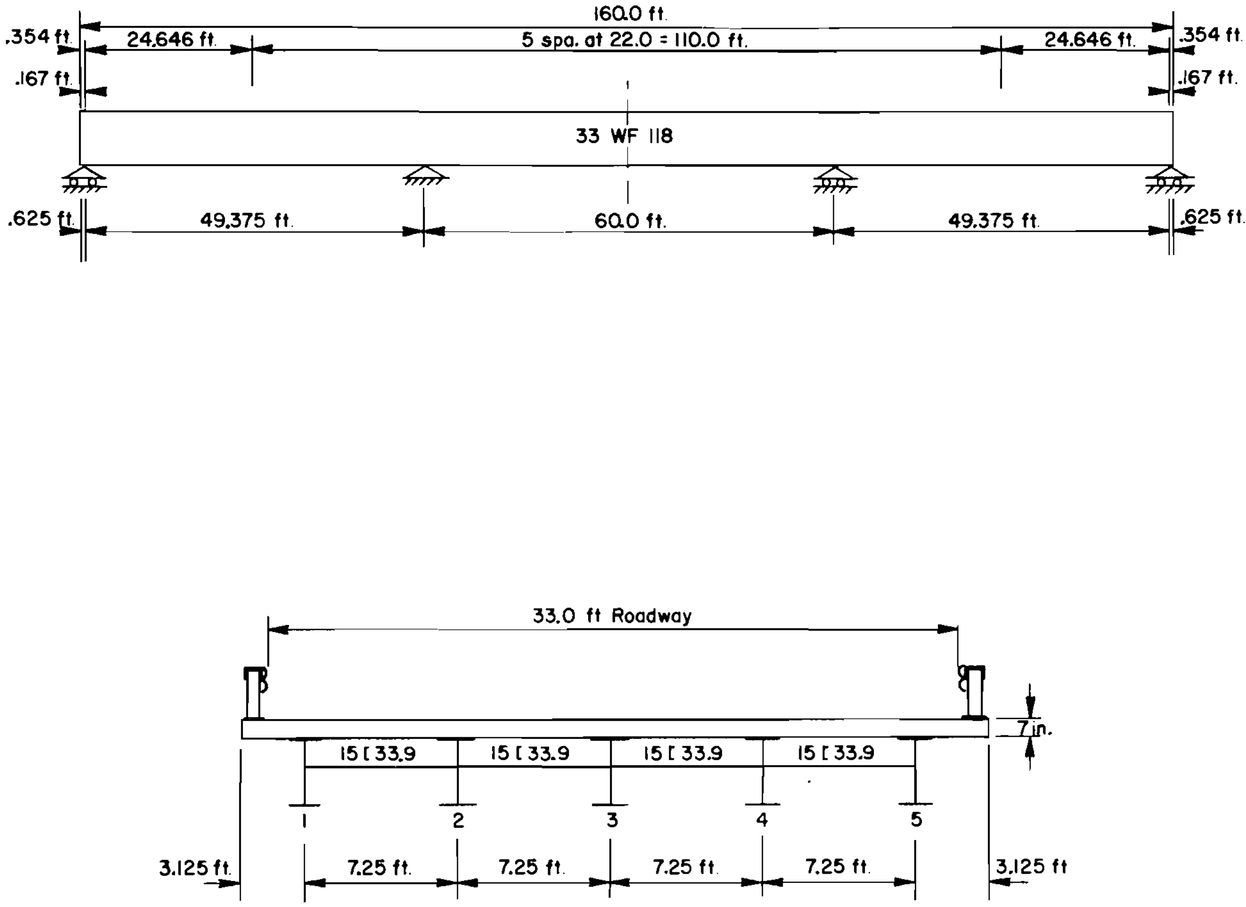
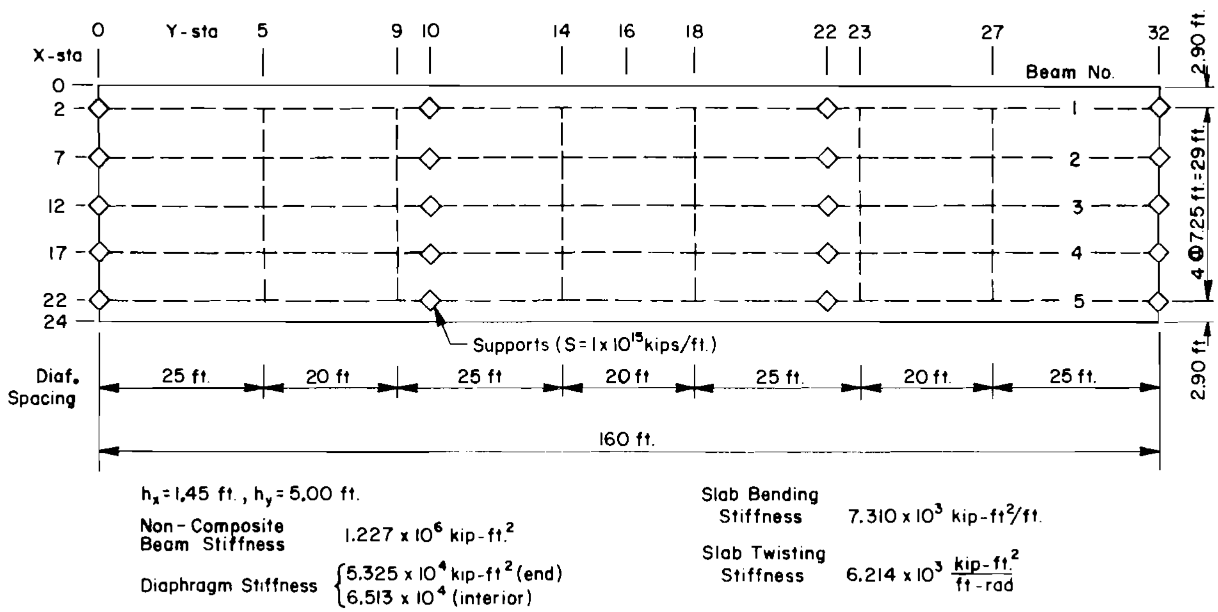
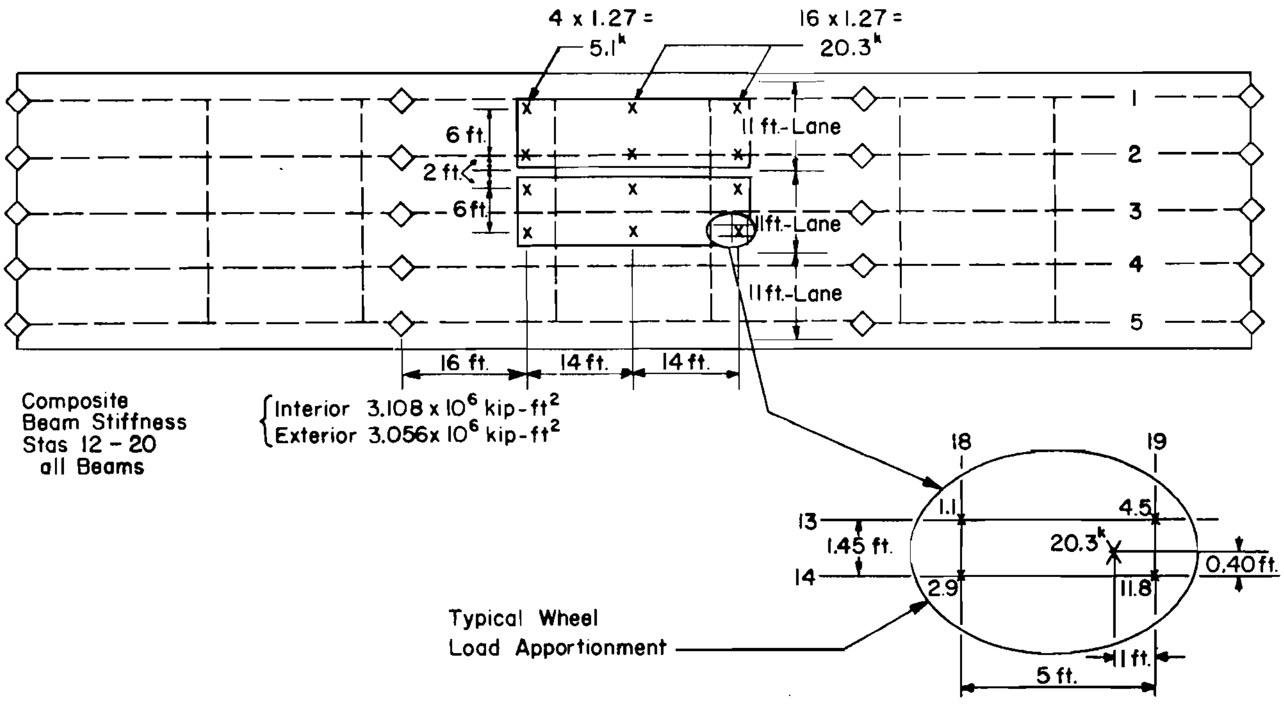


Fig 26. Three-span highway bridge.



(a)



(b)

Fig 27. Three-span structure as modeled for discrete-element analysis.

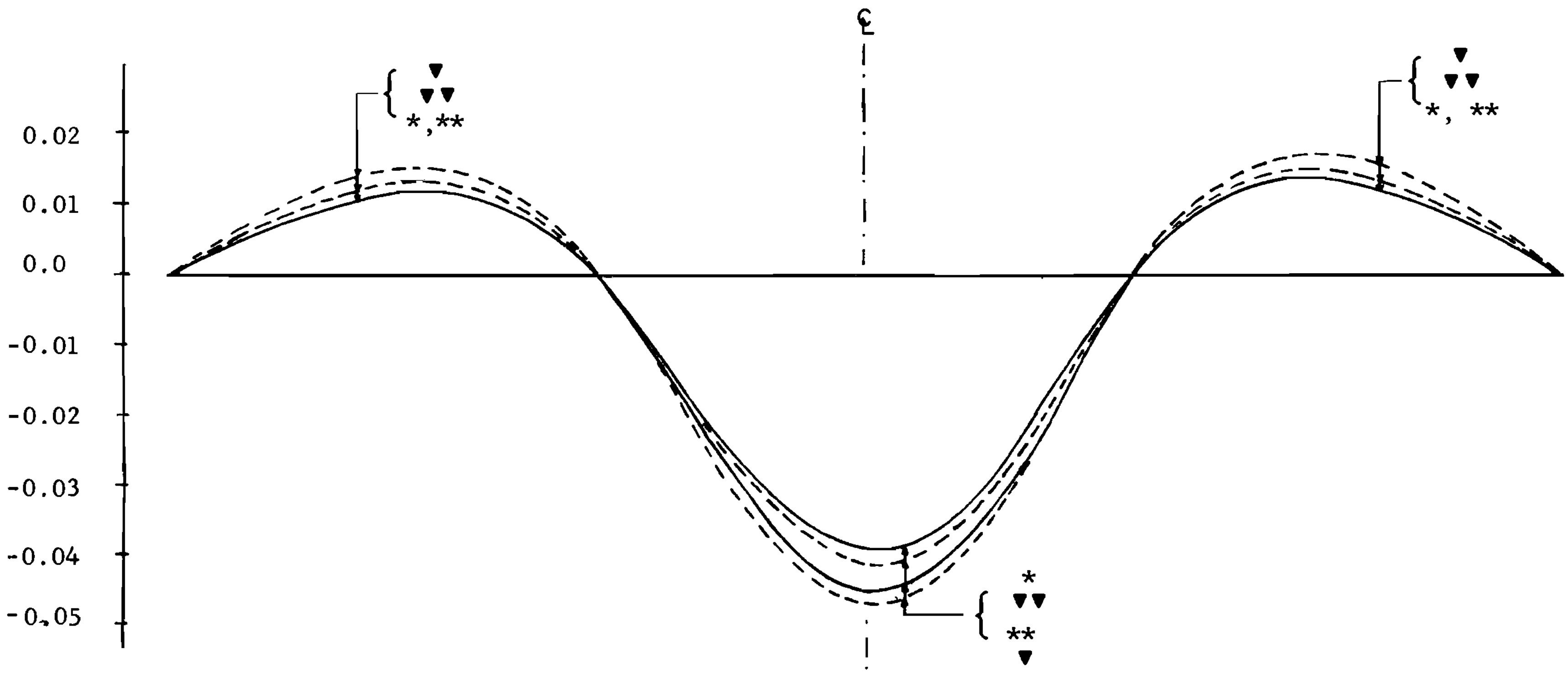
The deflections at girder No. 2 are shown in Fig 28. A comparison of the deflections of the discrete-element analysis with those of the analysis by the finite-element solution indicates that, in general, the magnitudes of the deflections obtained by the discrete-element method are slightly larger than the corresponding finite-element values.

The finite-element solution indicated as solution 1 in Fig 28 was first made with an arbitrary 20 percent composite stiffness of the concrete slab in the zones of expected negative longitudinal bending moment. The slab is considered to act in a full composite sense in the zones of expected positive bending moments. A complete justification cannot be given for including the remaining 20 percent of the composite slab longitudinal stiffness for the finite-element solutions. It did seem logical to the investigator for this study to assume by engineering judgement that some of the cracked slab must contribute to stiffness in the negative moment areas. As was found during the analysis, this 20 percent longitudinal composite stiffness in the negative moment areas did have a significant effect on the results.

Solution 2 by the finite-element method indicated in Fig 28 was of the same finite-element arrangement but the bending stiffness of the beams was represented by a different set of input. Equivalent rectangular sections for the main beams and the diaphragms with equivalent torsional stiffness as well as bending stiffness were used in this second solution. An adjusted shearing modulus of rigidity was used to give the required torsional stiffness in the equivalent rectangular sections. The other elastic constants were the same as for solution 1. The results of solution 2 by the finite-element method indicated a significantly greater twisting deformation of the structure, thus allowing larger vertical deformations to occur in girder No. 2.

A second discrete-element solution was made in which in the zones of negative moment the composite section was modified by the same 20 percent of effective slab considered in the finite-element solutions. This analysis is identified as the discrete-element solution No. 2 in Fig 28. These results are quite close to those computed by the finite-element analysis solution.

Thus, the only real comparison that can be made on Fig 28 is of No. 2 by the finite-element method and No. 2 by the discrete-element method. The other solutions are informative only and indicate the trend of change when other parameters are varied. These comparative solutions do indicate that



* The subject method, solution 1.

** The subject method, solution 2.

▼ The discrete-element method, solution 1.

▼▼ The discrete-element method, solution 2.

Fig 28. Vertical deflections at girder No. 2.

very good agreement can exist between discrete-element solutions which are two-dimensional in nature and finite-element solutions which are three-dimensional assemblages composed of two-dimensional plate elements. The significantly larger computer and user effort for the finite-element solutions as compared to the discrete-element solutions do not therefore justify its usual use for this class of problems. This is not to say, however, that for other structural systems it should not be used. For some types of problems that are very complicated, perhaps with curved members of varying depths and variable tapered girder spacings, it would be necessary to use a finite-element approach.

CHAPTER 7. SUMMARY AND RECOMMENDATIONS

This report is a summary of the total project efforts. The three main areas of interest are

(1) beam columns,

(2) plane frames, and

(3) slabs, plates, and grids.

Research investigators and others currently using or referring to reports and computer programs developed during the course of this lengthy project have encountered difficulty in determining the most up-to-date development for each of the above areas. To summarize and provide a reference guide for this purpose, the following list of reports and programs is given. It recommends the importance to be given to each report and program. Some program numbers were changed when the final version was placed in operation on the Texas Highway Department's computer facility. The eight programs identified by an asterisk are currently being successfully used by the Texas Highway Department.

Beam-Column Developments

Report No. 56-	Program Acronym	Identification and Status
1	BMCOL 34	Linearly elastic beam-columns. Report is current and serves as a basic information text for discrete-element beam-column analysis techniques. Program is superseded by BMCOL 43.
2	* CAP 14	Analysis of bent caps. Report and program are current. THD version is CAP 17.
4	* BMCOL 43	Beam-columns under movable loads. Report and program are current. THD version is BMCOL 51.
8	DBC 1	Vibrations of beams. Report and program are superseded by Report 24.
10	COMBM 1	Composite beams. Report and program are current. Primarily useful for special investigations. No THD program version.

12	SHRBM 1	Combined bending and shear. Report and program are current. Primarily useful for special investigations. No THD program version.
20	* PCGR 2	Plane curved girders. Report and program are current. Useful for curved beam analysis. THD version is also PCGR 2.
23	* FRAME 51	Nonlinear analysis of statically loaded plane frames. Nonlinear analyses of beam-column members may use this current program. THD version is also FRAME 51.
24	* DBC 5	Vibrations of beam-columns resting on linearly elastic or inelastic supports. Report and program are current. THD version is also DBC 5.

Plane-Frame Developments

Report No. 56-	Program Acronym	Identification and Status
3	FRAME 4	Structured frames with no sway. Report and program are superseded by Report 21.
7	PLNFRAM 4	Rectangular plane frames. Report and program are superseded by Report 21.
14	PFRM 1	Direct solution for plane frames. Report and program are superseded by Report 21.
19	TRIP 4 & FRIP 4	Equation solution process. Report and programs are current. Documents the two-dimensional recursion-inversion analysis techniques used for frames. Programs serve as subroutines for other developments.
21	* FRAME 11	Plane frames subjected to complex loading conditions. Report and program are current. THD version is also FRAME 11.
23	* FRAME 51	Nonlinear analysis of statically loaded plane frames. Report and program are current. Useful for nonlinear investigations. THD version is also FRAME 51.

Slab, Plate, and Grid Developments

Report No. 56-	Program Acronym	Identification and Status
5	LAYER 7 & 8	Layered grids-over-beams and plates-over-beams. Report and programs are superseded by Report 25.
6	SLAB 17	Plates and pavement slabs. Report appendices serve as basic reference for discrete-element plate and slab analysis techniques. Program is superseded by SLAB 49.

8	DPI 1	Vibrations of plates. Report and program are superseded by Report 17.
9	DSLAB 5	Direct solution for plates and pavement slabs. Report and program are superseded by Report 25.
11	VISAB 3	Plates and pavement slabs using variable increment length. Report and program are current. Primarily useful for special investigations. Program should be updated if used. No THD program version.
13	SLAB 30	Multiple-loading analysis for two-way bridge floor slabs. Report and program are superseded by Report 25.
15	None	Experimental verification for plates and slabs. Report useful for reference.
16	None	Experimental evaluation of subgrade modulus. Report useful for reference.
17	SLAB 35	Dynamic analysis of plates on nonlinear foundations. Report and program are current. Primarily useful for special dynamic and nonlinear investigations. No THD program version.
18	* SLAB 44	Anisotropic skew plates and grids. Report and program are current. Used for skewed slabs and bridges. THD version is also SLAB 44.
19	TRIP 4 & FRIP 4	Equation solution process. Report and programs are current. Documents the two-dimensional recursion-inversion analysis techniques used for slabs and grids. Programs serve as subroutines for other developments.
22	None	Bending stiffness variation at cracks in continuous pavements. Report useful for reference.
25	* SLAB 49	Orthogonal slab-and-grid floor systems. Report and program are current. THD version is also SLAB 49.
26	None	Applications to rigid pavement problems. Report useful for reference.
27	None	Summary of methods for pavement slabs. Report useful for reference.
28	SHELL 6	Finite-element analysis of bridge decks. Report and program are current. Used for discrete-element to finite-element comparisons of bridge deck analysis. No THD program version.

Recommendations

The developments of this project should never be considered finished. With time and use, each computer program will be modified, improved, and updated as needed by those engineers requiring the analysis methods. All of the programs listed above as current versions are the result of analyses and computer techniques which are felt to be the most efficient and logical at the close of this research project.

The primary sponsors of this project, the Texas Highway Department, should continue to expand their present efforts towards implementation of the developed techniques. One example of this implementation is the CAP 17 program which is a modified version of the documented CAP 14 program. Another example is program ERECT 2 which was developed by Highway Department personnel and is based on program BMCOL 43. It performs the incremental analysis necessary to properly consider the erection stresses in continuous plate girders. The effects of false bents, temporary splices, support jacking, and incremental slab placement can be considered by the program.

In final summary, it is submitted that the basic objective of the overall research has been met; this was to develop computer solution techniques which will permit engineers to study the behavior of structural systems in a more realistic manner. It is now the responsibility of the sponsors to provide sufficient continuing effort to effectively permit engineers to utilize the analytical methods to the fullest practical extent.

REFERENCES

The list of project reports on page v provides the basic reference source for this summary report. Each of the 28 listed reports has its own list of references which should be consulted. Additional references pertinent to this final report are listed here.

1. American Association of State Highway Officials, Standard Specifications for Highway Bridges, Tenth Edition, Washington, D.C., 1969.

2. Al-Rashid, Nasser I., Clyde E. Lee, and William P. Dawkins, "A Theoretical and Experimental Study of Dynamic Highway Loading," Research Report 108-1F, Center for Highway Research, The University of Texas at Austin, May 1972.

3. Matlock, Hudson, "Correlations for Design of Laterally Loaded Piles in Soft Clay," Offshore Technology Conference, OTC 1204, 1970.

4. Coyle, Harry M., and Lymon C. Reese, "Load Transfer for Axially Loaded Piles in Clay," Journal of the Soil Mechanics and Foundations Division, Vol 92, No. SM2, Proceedings of the American Society of Civil Engineers, March 1966.

5. Alani, A. F., and J. E. Breen, "Verification of Computer Simulation Methods for Slab and Girder Bridge Systems," Research Report 115-1F, Center for Highway Research, The University of Texas at Austin, August 1971.

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