

FINITE-ELEMENT ANALYSIS OF BRIDGE DECKS

by

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Development of Methods for Computer Simulation  
of Beam-Columns and Grid-Beam and Slab Systems  
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## PREFACE

A refined finite element method for the analysis of bridge decks as shell-type structures is presented. The method can be applied successfully for the analysis of several types of bridges. This report describes the method and its application for the analysis of highway bridges. Two typical bridges are analyzed, and the results obtained are compared with other existing methods of analysis.

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## LIST OF REPORTS

Report No. 56-1, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns" by Hudson Matlock and T. Allan Haliburton, presents a finite-element solution for beam-columns that is a basic tool in subsequent reports.

Report No. 56-2, "A Computer Program to Analyze Bending of Bent Caps" by Hudson Matlock and Wayne B. Ingram, describes the application of the beam-column solution to the particular problem of bent caps.

Report No. 56-3, "A Finite-Element Method of Solution for Structural Frames" by Hudson Matlock and Berry Ray Grubbs, describes a solution for frames with no sway.

Report No. 56-4, "A Computer Program to Analyze Beam-Columns under Movable Loads" by Hudson Matlock and Thomas P. Taylor, describes the application of the beam-column solution to problems with any configuration of movable non-dynamic loads.

Report No. 56-5, "A Finite-Element Method for Bending Analysis of Layered Structural Systems" by Wayne B. Ingram and Hudson Matlock, describes an alternating-direction iteration method for solving two-dimensional systems of layered grids-over-beams and plates-over-beams.

Report No. 56-6, "Discontinuous Orthotropic Plates and Pavement Slabs" by W. Ronald Hudson and Hudson Matlock, describes an alternating-direction iteration method for solving complex two-dimensional plate and slab problems with emphasis on pavement slabs.

Report No. 56-7, "A Finite-Element Analysis of Structural Frames" by T. Allan Haliburton and Hudson Matlock, describes a method of analysis for rectangular plane frames with three degrees of freedom at each joint.

Report No. 56-8, "A Finite-Element Method for Transverse Vibrations of Beams and Plates" by Harold Salani and Hudson Matlock, describes an implicit procedure for determining the transient and steady-state vibrations of beams and plates, including pavement slabs.

Report No. 56-9, "A Direct Computer Solution for Plates and Pavement Slabs" by C. Fred Stelzer, Jr., and W. Ronald Hudson, describes a direct method for solving complex two-dimensional plate and slab problems.

Report No. 56-10, "A Finite-Element Method of Analysis for Composite Beams" by Thomas P. Taylor and Hudson Matlock, describes a method of analysis for composite beams with any degree of horizontal shear interaction.

Report No. 56-11, "A Discrete-Element Solution of Plates and Pavement Slabs Using a Variable-Increment-Length Model" by Charles M. Pearre, III, and W. Ronald Hudson, presents a method for solving freely discontinuous plates and pavement slabs subjected to a variety of loads. April 1969.

Report No. 56-12, "A Discrete-Element Method of Analysis for Combined Bending and Shear Deformations of a Beam" by David F. Tankersley and William P. Dawkins, presents a method of analysis for the combined effects of bending and shear deformations. December 1969.

Report No. 56-13, "A Discrete-Element Method of Multiple-Loading Analysis for Two-Way Bridge Floor Slabs" by John J. Panak and Hudson Matlock, includes a procedure for analysis of two-way bridge floor slabs continuous over many supports. January 1970.

Report No. 56-14, "A Direct Computer Solution for Plane Frames" by William P. Dawkins and John R. Ruser, Jr., presents a direct method of solution for the computer analysis of plane frame structures. May 1969.

Report No. 56-15, "Experimental Verification of Discrete-Element Solutions for Plates and Slabs" by Sohan L. Agarwal and W. Ronald Hudson, presents a comparison of discrete-element solutions with small-dimension test results for plates and slabs, including some cyclic data. April 1970.

Report No. 56-16, "Experimental Evaluation of Subgrade Modulus and Its Application in Model Slab Studies" by Qaiser S. Siddiqi and W. Ronald Hudson, describes a series of experiments to evaluate layered foundation coefficients of subgrade reaction for use in the discrete-element method. January 1970.

Report No. 56-17, "Dynamic Analysis of Discrete-Element Plates on Nonlinear Foundations" by Allen E. Kelly and Hudson Matlock, presents a numerical method for the dynamic analysis of plates on nonlinear foundations. July 1970.

Report No. 56-18, "A Discrete-Element Analysis for Anisotropic Skew Plates and Grids" by Mahendrakumar R. Vora and Hudson Matlock, describes a tridirectional model and a computer program for the analysis of anisotropic skew plates or slabs with grid-beams. August 1970.

Report No. 56-19, "An Algebraic Equation Solution Process Formulated in Anticipation of Banded Linear Equations" by Frank L. Endres and Hudson Matlock, describes a system of equation-solving routines that may be applied to a wide variety of problems by using them within appropriate programs. January 1971.

Report No. 56-20, "Finite-Element Method of Analysis for Plane Curved Girders" by William P. Dawkins, presents a method of analysis that may be applied to plane-curved highway bridge girders and other structural members composed of straight and curved sections. June 1971.

Report No. 56-21, "Linearly Elastic Analysis of Plane Frames Subjected to Complex Loading Conditions" by Clifford O. Hays and Hudson Matlock, presents a design-oriented computer solution for plane frames structures and trusses that can analyze with a large number of loading conditions. June 1971.

Report No. 56-22, "Analysis of Bending Stiffness Variation at Cracks in Continuous Pavements," by Adnan Abou-Ayyash and W. Ronald Hudson, describes an evaluation of the effect of transverse cracks on the longitudinal bending rigidity of continuously reinforced concrete pavements. April 1972.

Report No. 56-23, "A Nonlinear Analysis of Statically Loaded Plane Frames Using a Discrete Element Model" by Clifford O. Hays and Hudson Matlock, describes a method of analysis which considers support, material, and geometric nonlinearities for plane frames subjected to complex loads and restraints. May 1972.

Report No. 56-24, "A Discrete-Element Method for Transverse Vibrations of Beam-Columns Resting on Linearly Elastic or Inelastic Supports" by Jack Hsiao-Chieh Chan and Hudson Matlock, presents a new approach to predict the hysteretic behavior of inelastic supports in dynamic problems. June 1972.

Report No. 56-25, "A Discrete-Element Method of Analysis for Orthogonal Slab and Grid Bridge Floor Systems" by John J. Panak and Hudson Matlock, presents a computer program particularly suited to highway bridge structures composed of slabs with supporting beam-diaphragm systems. May 1972.

Report No. 56-26, "Application of Slab Analysis Methods to Rigid Pavement Problems" by Harvey J. Treybig, W. Ronald Hudson, and Adnan Abou-Ayyash, illustrates how the program of Report No. 56-25 can be specifically applied to a typical continuously reinforced pavement with shoulders. May 1972.

Report No. 56-27, "Final Summary of Discrete-Element Methods of Analysis for Pavement Slabs" by W. Ronald Hudson, Harvey J. Treybig, and Adnan Abou-Ayyash, presents a summary of the project developments which can be used for pavement slabs. August 1972.

Report No. 56-28, "Finite-Element Analysis of Bridge Decks" by Mohammed R. Abdelraouf and Hudson Matlock, presents a finite-element analysis which is compared with a discrete-element analysis of a typical bridge superstructure. August 1972.

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## ABSTRACT

A finite element method is presented for the analysis of bridge decks treated as shell-type structures. The method can be used successfully for the analysis of a wide variety of highway bridges such as slab-type bridges, beam slab bridges, box girder bridges, and curved bridges.

The deck is discretized as an assemblage of flat triangular elements. Four different elements are available for use in the analysis. Two of these are in the forms of a new, refined triangular element and a new, refined non-planar quadrilateral element, and the other two elements are a triangular element and a quadrilateral element with less refinement of stiffness evaluation. The refined elements are suitable for use in coarse meshes to analyze decks with simple geometry while the other two elements may be used in fine meshes for the analysis of decks with complex geometry.

A six-degree-of-freedom nodal point displacement system is used in the analysis. Such a system is enough for complete representation of the shell problem and at the same time permits mesh refinement for representation of structures with complex geometries.

Orthotropic material properties as well as elastic supports are considered. The method offers considerable flexibility in expressing practical cases of loads and support conditions together with simplicity of the input data.

A continuous five girder bridge and a continuous box girder bridge were analyzed, and the results are compared with four existing solutions.

**KEY WORDS:** finite element, bridge decks, shell-type structures, geometric idealization, element, nodal point, mesh, orthotropic material, curved bridges, elastic supports.

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## SUMMARY

A finite element method is presented for the analysis of bridge decks. This is a general method which can be used successfully for the analysis of a wide variety of highway bridges as well as other highway constructions such as culverts and retaining walls.

Orthotropic material properties as well as elastic supports are considered. The method offers considerable flexibility in expressing practical cases of loads and support conditions together with simplicity of the input data. The method has been tested and gave excellent results in the analysis of typical straight and curved bridges.

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## IMPLEMENTATION STATEMENT

An extremely useful method for the analysis of bridge decks is reported herein. The method features considerable generality and simplicity in the input, thus enabling highway engineers to perform accurate analyses with minimum approximations for modern bridges. Complicated geometries can be easily represented as well as elastic supports and orthotropic material properties.

The program offers considerable economy in analyzing bridges for various load cases if all the load cases are solved in the same computer run. It includes various output options and is constructed in such a way to facilitate future developments.

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## NOMENCLATURE

<u>Symbol</u>	<u>Typical Units</u>	<u>Definition</u>
$X, Y, Z$	-	Global coordinates
$\bar{X}, \bar{Y}, \bar{Z}$	-	Element coordinates
$\alpha_1, \alpha_2, \dots$	-	Generalized coordinates
$u$	in	In-plane displacement component
$v$	in	In-plane displacement component
$w$	in	Lateral displacement component
$\theta_x, \theta_y$	rad	Angles of rotation
$\{R\}$	lb	Force vector
$\{r\}$	in	Displacement vector
$[\bar{T}]$	-	Direction cosines matrix
$[T]$	-	Transformation matrix
$[K]$	-	Stiffness matrix
$\{Q\}$	-	Force vector
$\{d\}$	-	Displacement vector
$[J]$	-	Stress transformation matrix
$\{S\}$	-	In-plane stress vector at a point
$\{M\}$	-	Bending vector at a point
$\psi$	rad	Angle of in-plane stress or bending transformation
$[C]$	-	Moduli matrix
$[C_p]$	-	The moduli matrix w.r.t. the principal axes
$C_{ij}$	$\text{lb/in}^2$	Element in $C_p$ matrix

<u>Symbol</u>	<u>Typical Units</u>	<u>Definition</u>
$E_x, E_y, E$	lb/in <sup>2</sup>	Moduli of elasticity
$\nu_{xy}, \nu_{yx}, \nu$	-	Poisson's ratio
$G_o$	lb/in <sup>2</sup>	Orthotropic shear modulus
$G$	lb/in <sup>2</sup>	Isotropic shear modulus
$[K_Q]$	-	Stiffness matrix of a quadrilateral element
$\{r_Q\}$	-	Displacement vector of a quadrilateral element
$\{R_Q\}$	-	Force vector of a quadrilateral element
$[K_e]$	-	Condensed quadrilateral element stiffness matrix
$\{R_e\}$	-	Condensed quadrilateral element load vector
$\{r_e\}$	-	Displacement vector corresponding to the external nodes of the quadrilateral element

## CHAPTER 1. INTRODUCTION

In recent years, an increasing importance has been given to the design and construction of bridge decks. This is due primarily to (1) the increase in magnitude of the moving loads, (2) the need for an economical use of construction materials, and (3) the complicated geometries of modern bridges and their longer spans. To meet these requirements, materials of higher strength are being used, and accurate methods of analysis must be developed.

Several methods are available for the analysis of bridge decks. In each method some simplifying assumptions usually exist in order to facilitate analysis. The accuracy of the solution obtained by a given method depends on the accuracy of representation of the structure and the extent of the approximations involved in the method.

Early methods of analysis required a considerable amount of approximation in representing the structure. A typical example of such approximation is the division of a bridge deck composed of a slab and a system of girders into one or more levels of secondary structures supported by main structures. Another example is the discrete-element modeling of slab type and beam-slab type bridge decks (Refs 1, 2, and 3). While such simple methods furnish a good and fast solution for some types of simple bridge decks, it is difficult to apply such methods to modern bridges with complicated geometries.

For such modern bridges, the recent trend is to treat the whole deck as a shell-type structure. This procedure makes it feasible to represent most of the details of the bridge, such as single and double curvature, variable girder depth, girder-slab interaction, boundary details, and variations of material properties. A finite element method for the analysis of shell-type structures was recently presented by the authors (Ref 4). This method can be used successfully for the analysis of a wide variety of bridge decks, and it is the primary subject of the following discussion.

## Finite Element Analysis of Shell-Type Structures

Shell-type structures are those which have small thickness compared to the other dimensions of the structure or those composed of a group of such relatively thin parts. Within this category are a variety of important building and industrial constructions such as thin curved shells, folded plate structures, silos, bunkers, aqueducts, culverts, liquid tanks, docks, and several kinds of bridges. For these structures, the shell behavior assumption can be considered as a good approximation for the actual behavior of the structure. In other words, the structure or its component parts can be treated as two-dimensional surfaces rather than as three-dimensional solids. Consequently, the case of plane stress can be assumed in which the stresses in the direction normal to the middle surfaces of the shell are considered of negligible effect. Also, Kirchhoff's assumption that straight lines normal to the middle surface of the shell remain straight and normal to that surface during deformation can be considered valid for these structures.

Many procedures for the analysis of shell structures are available. Their capabilities range from limitations to particular kinds of structures, such as the mathematical analysis of cylindrical shells and spherical domes, to generality of handling any structure with any geometry, material properties, or loading variations. Usually, the more general method of solution is desirable; however, the special purpose solutions provide ease in application and interpretation of the results.

Recently, the rapid development of the finite element method of structural analysis and the fantastic progress in the speed and memory size of digital computers have made the general purpose solutions a practical approach to the analysis of shell-type structures.

### The Finite Element Method

The finite element method is a general approach to structural analysis. Its generality makes it the most suitable one for analysis of structures with arbitrary properties or geometries. Practical application of this method for the analysis of shell type structures or similar structural problems requires the use of a digital computer of suitable size and speed. The finite element

method is a numerical procedure for the approximate solution of problems in continuum mechanics in which the actual structure is represented as an assemblage of finite elements interconnected at a finite number of nodal points. In each element of the assemblage, the displacement patterns are assumed to vary in such a way as to approximate the actual displacements. The stiffness of each element corresponding to the assumed degrees of freedom at the element nodes is evaluated using the assumed displacement function. The degree of accuracy of the resulting element stiffness, which usually has the most significant effect on the final solution, depends on the degree of reality in the chosen displacement variation. The element stiffnesses are assembled in the proper manner to form the structure stiffness matrix. After proper modifications for the boundary conditions have been made, the rows of the structure stiffness matrix provide the coefficients relating the nodal point displacements to the applied nodal point loads. Thus, for the given nodal point loads, the displacements corresponding to the assumed degrees of freedom at the nodes can be evaluated. The element strains and stresses can then be obtained using the calculated nodal point displacements, the assumed displacement variations, and the material elastic properties of the element. The theory and application of the finite element method in structural and continuum mechanics is discussed in detail in Ref 5, which also contains an extensive list of additional references.

The earliest use of the finite element method for the computer analysis of elastic continua was in problems of plane stress and plane strain (Refs 6 and 7). The method was then applied to the plate bending problem (Ref 8). The success obtained with these two applications was a motivation for applying the method to the analysis of thin shell problems. The earliest programs for the analysis of thin shells using the finite element method utilized flat elements in which the membrane and bending stiffnesses were evaluated independently (Ref 9). Flat rectangles were used in analyzing cylindrical shells, and flat triangles were proposed for approximating doubly curved surfaces (Ref 10). Later, Johnson (Ref 11) used flat triangles in the analysis of thin shells. After that, several achievements were obtained which materially improved and added to the flexibility of the method. Refined elements were developed and used in the analysis (Refs 12 and 13). Curved elements were developed as a

better way for representing curved surfaces (Refs 14, 15, and 16). Thick shells were also analyzed using thick finite elements (Ref 17). Also, compound elements were built by assembling groups of the simple elements. Reference 18 contains a more detailed discussion of the developments of the method and a comparison of the results that were obtained using some of the different types of elements.

The two properties of the finite element method which make it a valuable method of structural analysis are its generality and the arrangement of the resulting equilibrium simultaneous equations. It can be easily seen that the method is a systematic, uniform procedure for the analysis of structures regardless of the types or shapes of the finite elements used in the structural idealization. The procedures for analysis of space trusses using bar type elements or space frames using beam type elements or a shell structure using flat or curved plate elements are essentially the same. The only place where a difference may exist is in the evaluation of the stiffness properties of the constituting finite elements. The resulting total stiffness matrix generally contains only a few non-zero terms; and, by a suitable arrangement of the nodal point numbering of the structure, these non-zero terms can be located in a narrow strip around the major diagonal of the stiffness matrix. The resulting symmetric banded stiffness matrix saves considerably in the time required for solution of the simultaneous equations by making possible the use of special efficient methods for solving such kinds of simultaneous equations.

#### Approximations in Finite Element Analysis of Shell-Type Structures

Finite element analysis of shell-type structures involves some approximations which can be divided according to their nature into two groups. The first group comprises those approximations which exist in any thin shell or thin plate analysis method while the second group includes those approximations which exist specifically in the finite element method. The first group of approximations arises from the assumption that the stresses normal to the middle surfaces of the structure are of negligible effect. This justifies the treatment of the structure or of its component parts as a two-dimensional surface rather than as a three-dimensional solid. This simplifying assumption

is good only for very thin-walled structures. For structures of intermediate or large thickness this approximation could be the main source of error. As mentioned before, this is one of the approximations in the finite element analysis of shell-type structures, but it is not a special feature of the method. The second group of approximations can be considered as special features of the finite element method and can be divided according to source into the two main approximations described below.

The Geometric Approximation. The first step in the analysis of a structure using the finite element method is to idealize the structure as an assemblage of finite elements. The finite elements should have a simple geometry in order to make the evaluation of their stiffnesses feasible. Common examples of such elements are flat or curved triangles, flat or curved rectangles, flat quadrilaterals, and nonplanar quadrilaterals composed of planar triangles. The use of such simple elements to idealize structures of arbitrary geometry usually results in a difference between the actual structure and the idealization. A common example is the representation of curved boundaries by segments of straight lines. The effect of the geometric approximation can be reduced by reducing the size of the elements. Furthermore, a wide variety of shell-type structures, such as folded plate structures and most kinds of bridges, can be easily idealized without any approximations.

The Displacement Field Approximation. In calculating a finite element stiffness matrix, the variation of the displacements within the element must be assumed. The degree of approximation in this assumption is usually the most significant factor affecting the accuracy of the solution. Theoretically, in order to represent a general variation of displacements exactly, a polynomial of infinite degree is required; but practically, only the first few terms of such a polynomial are enough. The size of the element stiffness matrix, the computational effort required for its evaluation, and the accuracy of the results obtained all increase with an increase in the number of terms considered from the general polynomial representation of the displacements. A compromise must be reached here to achieve the required degree of accuracy with an acceptable amount of computation.

A usual trend is to assign to the lateral displacement of a flat element a displacement function of higher order than that assumed for the in-plane displacement function. This assumption may give rise to a kind of incompatibility in solutions using such elements. This incompatibility exists at element interfaces when they meet at a non-zero angle.

The effect of displacement approximation in most of the finite elements used in structural analysis (including the elements used here) decreases as the element size decreases. Also, in most cases, the incompatibility due to the difference in the displacement functions within flat elements is insignificant, and its effect can be reduced by reducing the element size.

### Finite Element Approaches

The analysis of shell-type structures by the finite element method can be approached in two different ways:

- (1) by using coarse meshes composed of refined finite elements, and
- (2) by using fine meshes composed of relatively less refined finite elements.

In the first approach the amount of input data is reduced, and consequently the effect of data preparation and the probability of data error are relatively smaller. The main disadvantage of this approach is its limited flexibility in idealizing complex geometries such as the details of complex, doubly curved surfaces. In such cases the use of fine meshes of refined elements to physically represent the complex geometry may result in an unacceptable increase in the solution time. The main advantage of the second approach is its flexibility in representing complex geometries. This requires more data than the first approach. In structures with simple geometries this approach may have no advantage, and the solution time required for a certain degree of accuracy may not favor its use.

Within either of these two approaches the number of nodal point degrees of freedom considered in the analysis may vary from five to twelve or more. Generally, the minimum number of nodal degrees of freedom for shell analysis purposes is six. Usually, these six degrees of freedom are chosen as three translation components and three rotation components.



The five-degree-of-freedom nodal point system may give good representation in many cases if it is used in such a way as to minimize the effect of the omitted degree of freedom, as in the analysis presented in Ref 11. This analysis considers the five degrees of freedom as three translation components in the directions of the three global axes and two rotation components about two axes in a plane tangential to the structure surface at the node considered. This system is equivalent to specifying the bending rotation component about the axis normal to the surface at each node to be zero. This constraint has proved to have little effect in cases of intersecting surfaces, such as box girder bridges; however, the tangent plane becomes undefined. In such cases, improper choice of this plane may result in considerable error. The necessity of defining the direction cosines of the two tangent axes as input data at each point is inconvenient; and the method may not be applicable for cases in which all three rotation components are of the same order of magnitude.

Reference 19 analyzes continuous box girder bridges using a six-degree-of-freedom nodal point system and a rectangular element which gives good results with relatively fine meshes. The main disadvantage of this method is its limitation to rectangular geometries.

Reference 13 uses a nine-degree-of-freedom nodal point system for the analysis of thin shell structures. This analysis uses a refined triangular element and has considerable flexibility and generality. The main disadvantage of this analysis results from the nine degrees of freedom and especially from the fact that six of these nine degrees are associated with the triangle membrane stiffness. In other words, more refinement was devoted to the membrane stiffness (complete cubic variation) than was given to the bending stiffness (constrained cubic variation), an arrangement which generally may not be necessary in the analysis of plates and shells.

#### The Present Method of Analysis

The purpose of the present work was to develop a finite element method for analysis of shell-type structures that would include most of the finite element generalities and would keep the input as simple as possible for the purpose of users' convenience. Both of the approaches described in the previous

section were included as alternative options in the solution. The six-degree-of-freedom nodal point system is used in the solution because it is generally considered to be the minimum complete representation of the problem. Some limitations exist in the analysis. These limitations arise primarily from the adopted trend of simplifying and minimizing the input data. Even with its limitations, however, the method can be used for analyzing most of the shell-type structures in practical use.

The present method of analysis of shell-type structures is a finite element procedure using a six-degree-of-freedom nodal point system. The method has the capability of analyzing problems of arbitrary geometry and support conditions. Linearly elastic supports are included as well as orthotropic and isotropic materials with arbitrary variations throughout the structure. The method has considerable capabilities and various output options.

The structure is represented as an assemblage of flat triangular elements. The stiffness properties of each triangle are derived from assumed truncated polynomials for the displacement variations. The method provides four different kinds of finite elements in the form of two triangular elements and two nonplanar quadrilaterals (as assemblages of four triangular elements). The stiffness properties of the four elements and their nodal point systems are different, thereby providing considerable flexibility in the choice of a suitable mesh to achieve the required accuracy with minimum computational effort.

The bending stiffness of all the elements is evaluated by using one or more of the fully compatible triangular elements (HCT) after Hsieh, Clough, and Tocher (Ref 20). Three types of plane stress elements are employed in evaluating the membrane stiffnesses. They are

- (1) the linear strain triangle (Ref 12),
- (2) a linear strain triangle with the displacement along one side of the triangle constrained to vary linearly (Ref 12), and
- (3) the constant strain triangle (Ref 6).

The stiffness matrix of the complete assemblage is evaluated using the direct stiffness procedure.

A direct solution procedure based on the Gaussian elimination method is used to solve the equilibrium equations for the nodal point displacements.

The equations are solved using rectangular blocks to perform the required operations. The number of the rows in a block is variable and thus provides added generality and flexibility in the solution. All the load cases are solved together, thus saving considerably in the solution time required for problems with more than one load case.

The thickness and the material properties are assumed constant for each finite element of the assemblage. The material is characterized as either isotropic or orthotropic. Thus, structures with smooth variations in thickness are approximated with abrupt steps in thickness. A similar approximation must be made for complicated variations of material properties. These limitations are not significant in most practical cases, and their effect can be reduced by reducing the element size. Some other limitations concerning the boundary conditions and the application of load are mentioned in Chapter 2.

The method was tested and gave good results when compared to the results of existing methods of analysis. It can be efficiently applied for the analysis of a wide variety of shell-type structures, such as shells with single and double curvatures, folded plate structures, bridges, liquid containers, bunkers, and similar structures.

In this report, the application of the method to the analysis of highway bridges is illustrated by analyzing two typical bridges in Chapter 3. The results of the analysis are compared with previous solutions for the same bridges, and the methods are discussed briefly. For purposes of comparison, the analysis here was done for two straight bridges with existing solutions. No curved bridge analysis was included here; however, a curved prestressed railway bridge was recently analyzed in cooperation with the Texas Highway Department (Research Project 3-5-71-155, "Static and Buckling Analysis of Highway Bridges by Finite Element Procedures") using this method. Some other curved structures were analyzed, and the results are included in Ref 4, which contains more details of the method and of the computer program.

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## CHAPTER 2. THE PRESENT METHOD OF ANALYSIS

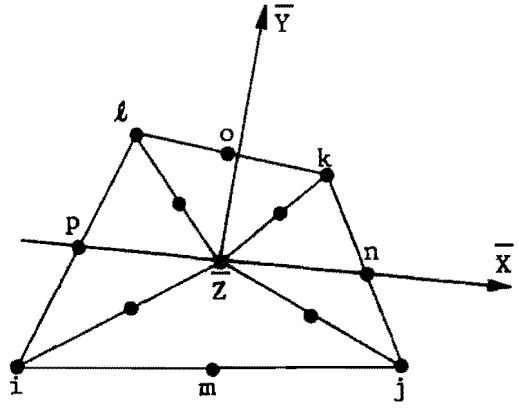
The present method of analysis of shell-type structures is a direct stiffness solution using the finite element method with a six-degree-of-freedom nodal point system. Two main idealizations, or approximations, are present in the analysis, namely the geometric idealization and the displacement field idealization.

### Geometric Idealization

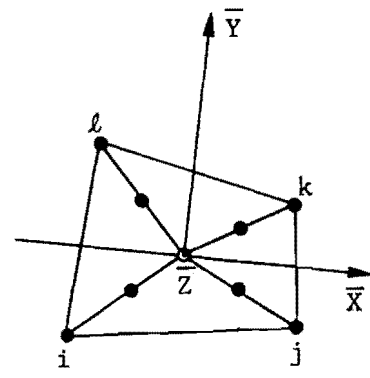
The actual surface of the structure is discretized to an assemblage of planar triangles. Four different finite elements are available for use in the analysis as follows:

- (1) The quadrilateral element with eight external nodes, Fig 1(a), may be a nonplanar or a planar assemblage of four planar triangles. The external nodes are the four corner nodes and the four mid-side nodes. The coordinates of the center of this element are computed as the average of the coordinates of its corner points. This center is the common node of the four composing triangles.
- (2) The quadrilateral element with four external nodes, Fig 1(b), of which the geometry is the same as for the previous element but without the mid-side nodes.
- (3) The planar triangular element with six nodes, shown in Fig 1(c), which has three mid-side nodes as well as the three corner nodes.
- (4) The planar triangular element with three nodes which is shown in Fig 1(d).

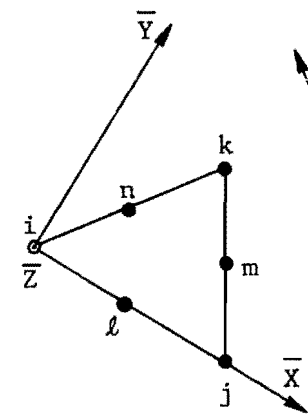
The geometric idealization of a typical shell surface using quadrilateral and triangular elements is shown in Fig 1(e). Figure 1 also shows the two main systems of coordinates used in the analysis. The coordinate systems are described later in this chapter. The shape and the size of each element are determined from the global coordinates of its corner nodes, which should lie in the middle surface of the structure. In general, any one of the four elements can be used for idealizing the structure. Combination of these elements may be necessary in some cases to fit the geometry of the structure. It will be seen later that for the same computational effort, the quadrilateral element



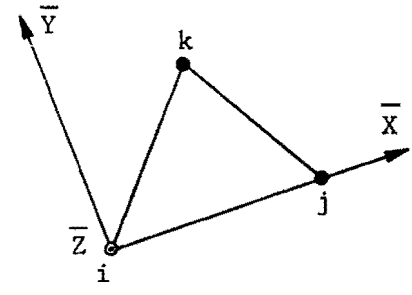
(a)



(b)



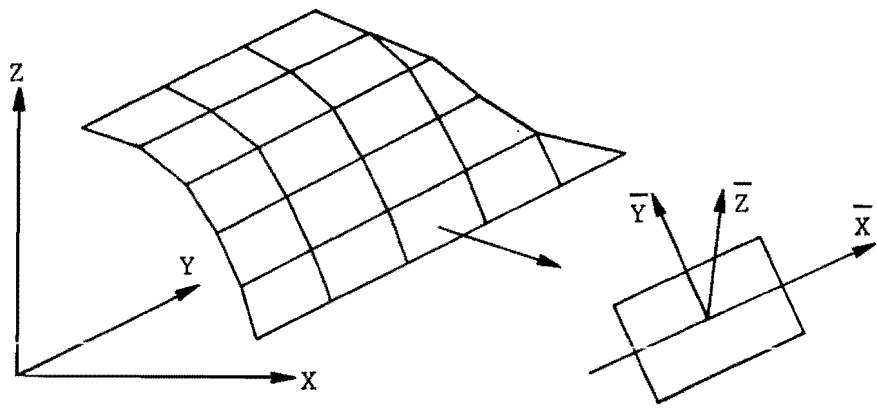
(c)



(d)

Quadrilateral elements.

Triangular elements.



(e) Idealized shell surface.

Coordinate definitions:

$X, Y, Z$  = global coordinates

$\bar{X}, \bar{Y}, \bar{Z}$  = element coordinates

Fig 1. Elements used, coordinate systems, and typical idealized shell.

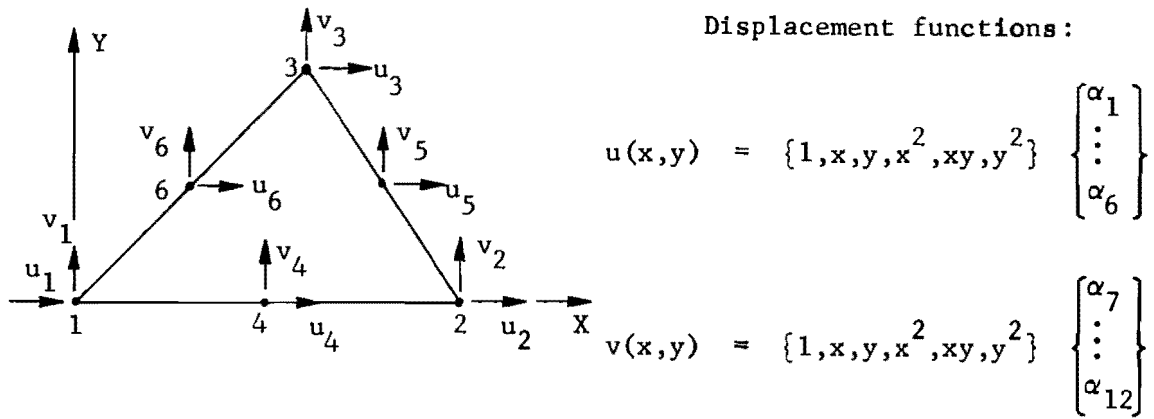
with mid-side nodes gives better results than the triangular element with mid-side nodes. Also, the simple quadrilateral element possesses similar superiority over the simple triangle. Although it is possible to use any combination of the four element types, it is usually most practical to (1) combine the quadrilateral and the triangular elements having mid-side nodes or (2) combine the two elements without mid-side nodes. The relative stiffness superiority of the quadrilaterals in either of the two combinations usually necessitates their use in regions of steep strain variations while the triangles may be used in regions of smaller strain variations.

#### Displacement Field Idealization

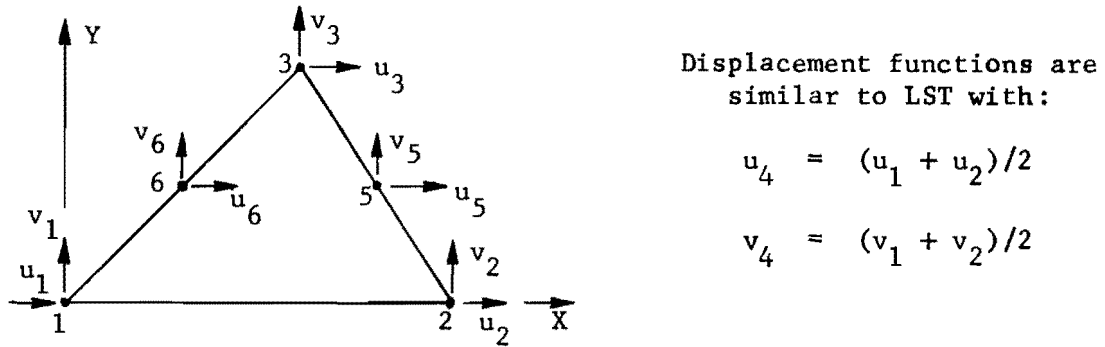
The basic element in the displacement field idealization is the planar triangle. The membrane and bending displacements of this basic triangle are discretized to vary according to certain displacement functions. Three types of membrane displacement discretizations and membrane stiffnesses are available while one basic bending displacement discretization is used in all cases.

Membrane Stiffnesses of the Basic Triangular Elements. The triangular elements used in evaluating the membrane stiffnesses and their displacement functions (expressed in generalized coordinates  $\alpha_1$ ,  $\alpha_2$ , . . . , etc.) are shown in Fig 2. Two degrees of freedom in the form of two perpendicular in-plane displacement components,  $u$  and  $v$ , are assumed at each nodal point. The variation of these displacements is assumed to be quadratic in the case of the linear strain triangle (Fig 2(a)) and linear in the case of the constant strain triangle (Fig 2(c)). For the constrained linear strain triangle, the displacements are assumed to have quadratic variations which gradually change to linear variations toward the constrained side (side 1-2 in Fig 2(b)).

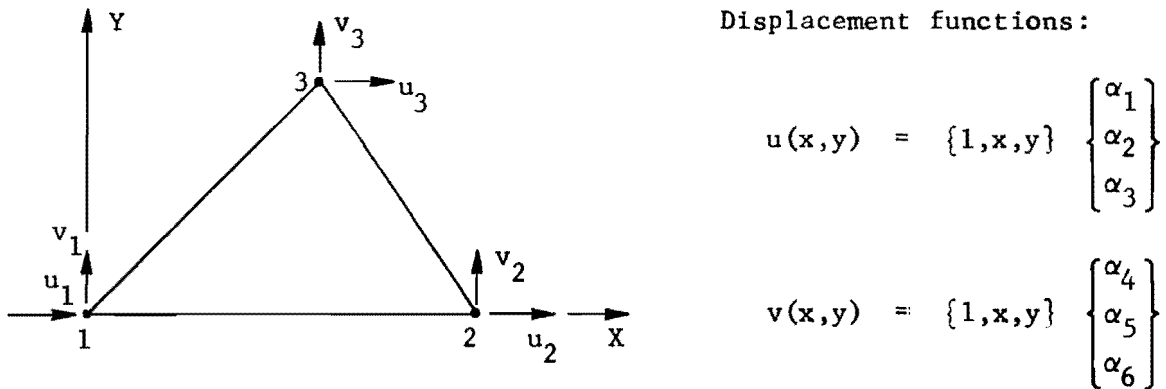
Bending Stiffnesses of the Basic Triangular Element. The fully compatible triangular bending element is used in all cases as a basic element for evaluating the bending stiffness. This element has a total of nine degrees of freedom in the form of the lateral translation and two perpendicular rotation components at each corner of the triangle. In order to achieve full compatibility, the element is divided into three sub-elements, and three displacement



(a) Linear strain triangle (LST).



(b) Constrained linear strain triangle.



(c) Constant strain triangle.

Fig 2. Triangular elements and displacement functions for membrane stiffness.



functions are assumed for the three sub-elements, as shown in Fig 3 where the displacement functions are expressed in generalized coordinates  $\alpha_1$ ,  $\alpha_2$ , . . . , etc. This subdivision results in twenty-seven displacement modes; but only nine of them, corresponding to the corner degrees of freedom, are independent displacement modes. The internal compatibility requirements provide the eighteen conditions needed for reducing the total twenty-seven displacement modes to the independent nine modes. This discretization expresses a cubic variation of the lateral deflection,  $w$ , within the element and a linear variation of the three curvature components over each sub-element except at the edge of the sub-element where the twisting curvature is constrained to be uniform.

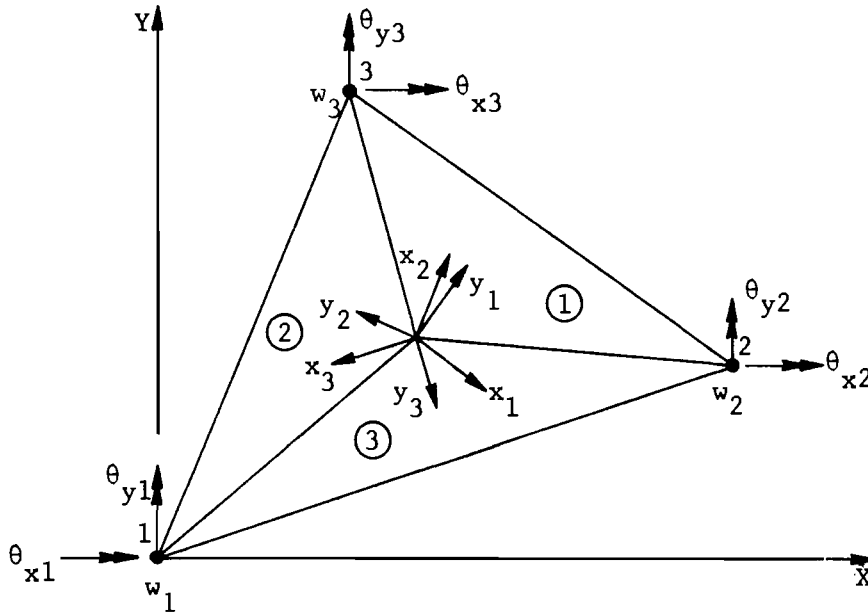
The derivation of the membrane stiffness and the bending stiffness of the basic triangles is given in Ref 4, Appendix 1.

#### Construction of the Element Stiffnesses

The element stiffness of each of the four element types is constructed by using one or more of the basic membrane triangles together with one or more of the basic bending triangles.

The Triangular Element with Three Nodes. The membrane stiffness of this element is that of a constant strain triangle, and its bending stiffness is that of the basic bending triangle. Thus, this element has a total of fifteen degrees of freedom, five at each node, as shown in Fig 4(a).

The Triangular Element with Six Nodes. The membrane stiffness of this element is that of the linear strain triangle, and its bending stiffness is the sum of four triangular bending elements, each triangle representing one of the sub-triangles shown in Fig 4(b). It should be noted that these four sub-triangles are coincident; therefore, for the constant thickness case treated here, the bending stiffness of only one of these sub-triangles, for example, sub-triangle 1, is to be evaluated. The bending stiffnesses of sub-triangles 2 and 3 are identical to that of sub-triangle 1, and that of sub-triangle 4 can be obtained by transforming the bending stiffness matrix of sub-triangle 1 in its plane through an angle of  $180^\circ$ . This coincidence of



Displacement functions:

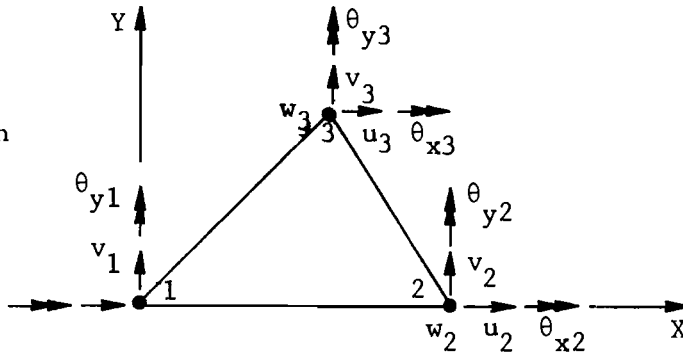
$$w^{(3)} = \{1, x_3, y_3, x_3^2, x_3 y_3, y_3^2, x_3^3, x_3^2 y_3, y_3^3\} \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_9 \end{Bmatrix}$$

$$w^{(1)} = \{1, x_1, y_1, x_1^2, x_1 y_1, y_1^2, x_1^3, x_1^2 y_1, y_1^3\} \begin{Bmatrix} \alpha_{10} \\ \vdots \\ \alpha_{18} \end{Bmatrix}$$

$$w^{(2)} = \{1, x_2, y_2, x_2^2, x_2 y_2, y_2^2, x_2^3, x_2^2 y_2, y_2^3\} \begin{Bmatrix} \alpha_{19} \\ \vdots \\ \alpha_{27} \end{Bmatrix}$$

Fig 3. Triangular element and displacement functions for bending stiffness.

(a) Triangular element with 3 nodes and 15 DOF.



(b) Triangular element with 6 nodes and 30 DOF.

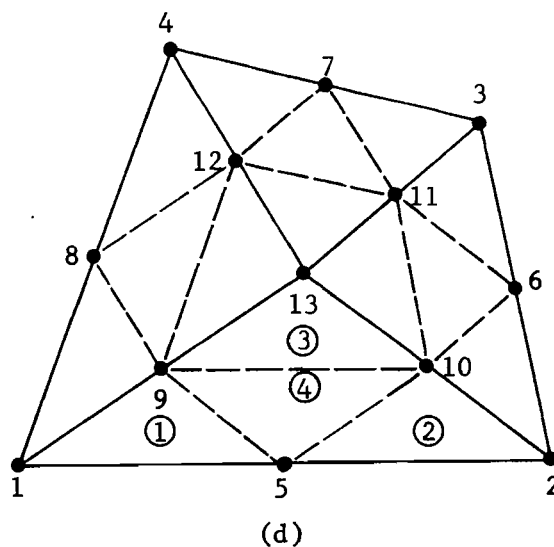
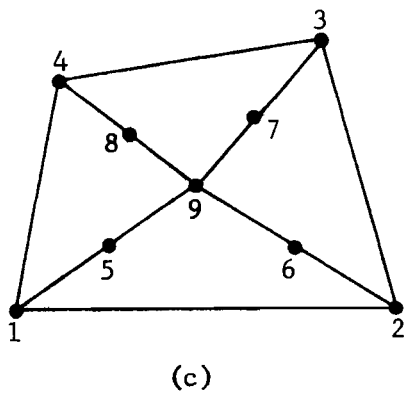
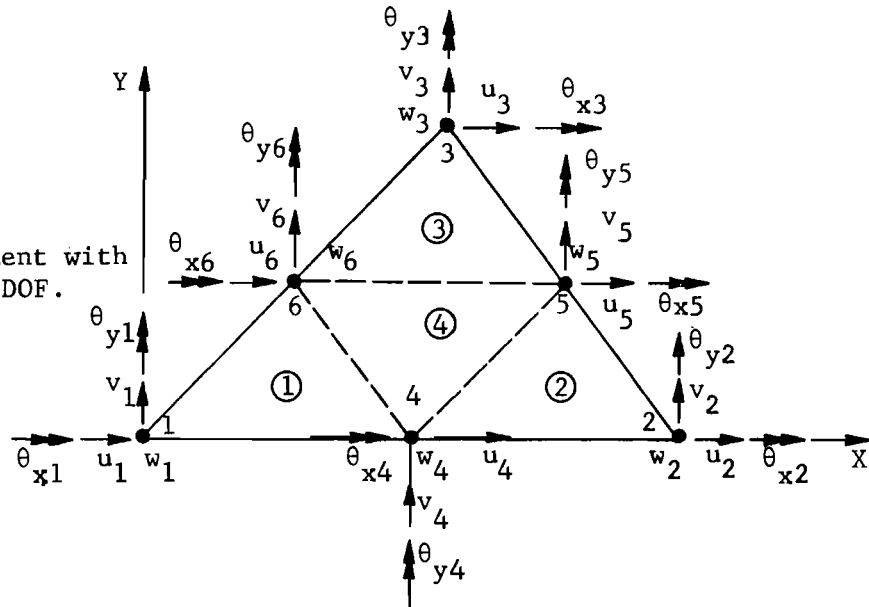


Fig 4. Triangular elements and quadrilateral elements composed of four triangular elements each.

the four sub-triangles makes it possible to obtain refined bending stiffness with only slight extra computational effort beyond evaluating the bending stiffness of one triangle. This is true only in the case of constant thickness. In cases of variable thicknesses it may be necessary to compute four triangular element stiffnesses in order to achieve similar bending stiffness refinement. This element has 30 degrees of freedom.

The Quadrilateral Element with Four External Nodes. This element, Fig 4(c), is composed of four triangles. The membrane stiffness of each composing triangle is that of the constrained linear strain triangle, and its bending stiffness is that of one triangular bending element. Therefore, the corner nodes of each composing triangle have five degrees of freedom each while the mid-side nodes each have only two in-plane translation degrees of freedom.

The Quadrilateral Element with Eight External Nodes. This element, Fig 4(d), is composed of four triangular elements with six nodes similar to the element described earlier and shown in Fig 4(b).

### Coordinate Systems and Transformations

The Global Coordinate System. This is a right-hand Cartesian system, Fig 1(e), which is independent of the mesh. The choice of the global coordinate axes is generally arbitrary, although in some cases the geometry of the structure, the loading, or the boundary conditions indicate the suitable choice. The following information is described in this coordinate system:

- (1) the input nodal point coordinates,
- (2) the input nodal point loads,
- (3) the input boundary conditions, and
- (4) the computed nodal point displacements.

The Element Coordinate System. This is also a right-hand Cartesian system which is associated with the element as shown in Fig 1. The element coordinate axes are set for each element according to the order of input of the element nodes in the element numbering.

For triangular elements, the order of numbering should follow the alphabetical sequences shown in Figs 1(c) and 1(d). The first corner node is

considered as the origin of the element coordinates, and the  $\bar{X}$  axis coincides with the first triangle side (side  $i-j$ ). The  $\bar{Y}$  axis is perpendicular to the  $\bar{X}$  axis in the plane of the triangle; and the  $\bar{Z}$  axis is normal to the  $\bar{X}-\bar{Y}$  plane following the right-hand rule.

The numbering sequence for quadrilateral elements is shown in Figs 1(a) and 1(b). The coordinates of the center of the element are computed as the average of the coordinates of the four corner nodes; and this center is the origin of the element coordinates. The  $\bar{X}-\bar{Y}$  plane is taken as the average element plane. This average plane is defined by the two straight lines joining the mid-side points of each pair of facing sides and intersecting at the element center. The  $\bar{X}$  axis is taken as the line through the mid-side nodes of the fourth and the second sides (sides  $l-i$  and  $j-k$ , respectively); the  $\bar{Y}$  axis is constructed perpendicular to the  $\bar{X}$  axis in the average plane; and the  $\bar{Z}$  axis is normal to the  $\bar{X}-\bar{Y}$  plane following the right-hand rule. In the case of a planar quadrilateral, the average plane will be the same as the plane of the element.

The following information is expressed in the element coordinates:

- (1) the input orientation of the orthotropic material axes
- (2) the computed element forces.

Local Coordinates for Quadrilateral Elements. Local coordinate axes are established for each of the four composing triangles of the quadrilateral element. These local coordinates are similar to the element coordinates of the triangular elements, Figs 1(c) and 1(d). The establishment of these local coordinates is done internally in the program for the purpose of computing and transforming the triangle stiffness matrix and the equivalent nodal point loads for the distributed loads. None of the input or output information is expressed in these coordinates.

Triangular Coordinates. This local coordinate system is described in Ref 4, Appendix 1. It is used to simplify the derivation of the stiffness matrices of the triangular elements.

Coordinate Transformations. In the solution, several coordinate transformations are carried out to express in a common coordinate system all the

variables appearing in a single computation process. All the transformations of this kind are carried out between two Cartesian coordinate systems. Another transformation between the local Cartesian coordinate system and the local triangular coordinate system is described in Ref 4, Appendix 1. The first kind of transformation is summarized below. Detailed derivations may be obtained from textbooks on the subject.

Let the two Cartesian systems of coordinates be denoted as the X-Y-Z system and the X'-Y'-Z' system. If the matrix  $\bar{T}$  represents the direction cosines of system X'-Y'-Z' with respect to system X-Y-Z, then the transformation relations described below can be easily proved.

Linear Transformations.

$$\begin{array}{r} R_{x'} \\ R_{y'} \\ R_{z'} \end{array} = \begin{bmatrix} \bar{T} \end{bmatrix} \begin{array}{r} R_x \\ R_y \\ R_z \end{array} \quad \text{and} \quad \begin{array}{r} r_{x'} \\ r_{y'} \\ r_{z'} \end{array} = \begin{bmatrix} \bar{T} \end{bmatrix} \begin{array}{r} r_x \\ r_y \\ r_z \end{array}$$

or

$$R' = \bar{T} \cdot R \quad \text{and} \quad r' = \bar{T} \cdot r \quad (2.1)$$

where R and r are force and displacement vectors, respectively, at a certain point in the X-Y-Z system, and R', r' are the corresponding vectors in the X'-Y'-Z' system. It should be noted here that force means a general force which may be moment and that displacement may actually be rotation.

The inverse relations are

$$R = \bar{T}^T \cdot R' \quad \text{and} \quad r = \bar{T}^T \cdot r' \quad (2.2)$$

where the matrix  $\bar{T}^T$  is the transpose (or the inverse) of the matrix  $\bar{T}$ .

If all the force and displacement components at the nodal points of a finite element are grouped in the same manner as the vectors above, then the linear transformation relations for the element forces and displacements would have the forms

$$Q' = T \cdot Q \quad \text{and} \quad d' = T \cdot d \quad (2.3)$$

$Q'$ ,  $Q$ ,  $d'$ , and  $d$  are, respectively, the vectors containing  $n$  subvectors  $R'$ ,  $R$ ,  $r'$ , and  $r$  at all the nodal points of the element.  $T$  is the transformation matrix composed of  $n$  repetitions of the matrix  $\bar{T}$  on the diagonal strip as follows:

$$T = \begin{matrix} \bar{T}_1 & & & \\ & \bar{T}_2 & & \\ & & \dots & \\ & & & \bar{T}_n \end{matrix} \quad (2.4)$$

Similarly, the inverse relations are

$$Q = T^T \cdot Q' \quad \text{and} \quad d = T^T \cdot d' \quad (2.5)$$

Stiffness Transformation. If the element stiffness matrix is arranged to correspond to similar degrees of freedom as the vectors  $d$  and  $d'$  in the previous case, then the stiffness transformation relations would be

$$K' = T \cdot K \cdot T^T \quad \text{and} \quad K = T^T \cdot K' \cdot T \quad (2.6)$$

where  $K'$  and  $K$  are the element stiffness matrices in the  $X'-Y'-Z'$  system and the  $X-Y-Z$  system, respectively, and  $T$  and  $T^T$  are the transformation matrix defined by Eq 2.4 and its transpose, respectively.

Planar Transformation of Stresses. To transform plane stress components at a point expressed in  $X-Y$  coordinates to the corresponding components in  $X'-Y'$  coordinates, the transformation relation is

$$\begin{array}{rcccc} \sigma_{x'} & & \cos^2 \psi & \sin^2 \psi & 2 \sin \psi \cos \psi & \sigma_x \\ \sigma_{y'} & = & \sin^2 \psi & \cos^2 \psi & - 2 \sin \psi \cos \psi & \sigma_y \\ \tau_{x'y'} & & - \sin \psi \cos \psi & \sin \psi \cos \psi & \cos^2 \psi - \sin^2 \psi & \tau_{xy} \end{array}$$

or

$$\{S'\} = [J] \{S\} \quad (2.7)$$

where  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$  are the membrane stress components defined in Fig 5(a), which shows their positive directions;  $\sigma_{x'}$ ,  $\sigma_{y'}$ , and  $\tau_{x'y'}$  are the corresponding transformed stresses; and  $\psi$  is the angle between the  $X$  axis and the  $X'$  axis measured from the former to the latter and positive in a counterclockwise direction.



A similar relation can be written for the transformation of the plate bending moment components defined as shown in Fig 5(b).

$$\begin{matrix} M_{x'} \\ M_{y'} \\ M_{x'y'} \end{matrix} = [J] \begin{matrix} M_x \\ M_y \\ M_{xy} \end{matrix}$$

or

$$\{M'\} = [J] \{M\} \quad (2.8)$$

The inverse relations to those of (2.7) and (2.8) are

$$\{S\} = [J]^{-1} \{S'\} \quad \text{and} \quad \{M\} = [J]^{-1} \{M'\} \quad (2.9)$$

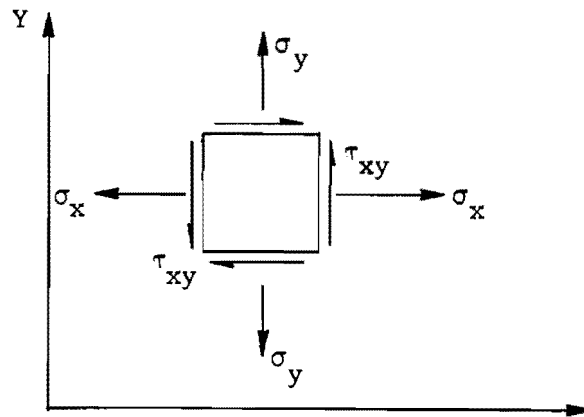
where the matrix  $[J]^{-1}$  is the inverse of the matrix  $[J]$  and can be easily obtained by substituting a negative value for  $\Psi$  in the previous expression for  $[J]$ . Thus,

$$[J]^{-1} = \begin{matrix} \cos^2 \Psi & \sin^2 \Psi & -2 \sin \Psi \cos \Psi \\ \sin^2 \Psi & \cos^2 \Psi & 2 \sin \Psi \cos \Psi \\ \sin \Psi \cos \Psi & -\sin \Psi \cos \Psi & \cos^2 \Psi - \sin^2 \Psi \end{matrix} \quad (2.10)$$

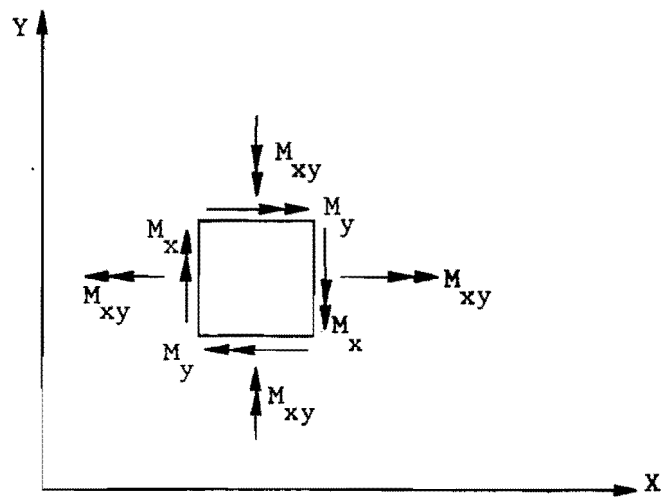
Planar Transformation of Moduli. For orthotropic material in the case of plane stress, if the principal material axes are taken as axes X and Y, then any case of plane stress can be related to the corresponding strains in the X-Y system as

$$\begin{matrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{matrix} = [C_P] \begin{matrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{matrix} \quad (2.11)$$

where  $[C_P]$  is the moduli matrix in the principal axes X and Y.



(a) Membrane stress components.



(b) Plate bending moment components.

Fig 5. Definition of membrane stresses and plate bending moments.

The moduli matrix  $[C]$  in axes  $X'$ ,  $Y'$  in the  $X$ - $Y$  plane and making an angle  $\Psi$  with the principal axes, as defined before in the case of stress transformation, would be obtained from the relation

$$[C] = [J]^{-1} [C_P] [[J]^{-1}]^T \quad (2.12)$$

where the matrix  $[J]^{-1}$  is as defined in Eq 2.10.

### Representation of Material Properties and Loads

Material Properties. In computing the stiffness matrix and the stresses of each element, the elastic properties of the element appear in the form of the moduli matrix,  $[C]$ , in the stress-strain relation

$$\begin{array}{ccc} \sigma_{\bar{x}} & & \epsilon_{\bar{x}} \\ \sigma_{\bar{y}} & = [C] & \epsilon_{\bar{y}} \\ \tau_{\bar{x} \bar{y}} & & \gamma_{\bar{x} \bar{y}} \end{array} \quad (2.13)$$

All the values in this relation are in element coordinates in the case of triangular elements or in the local coordinates of each of the composing triangles in the case of quadrilateral elements.

The moduli matrix,  $[C]$ , is obtained from the principal moduli matrix,  $[C_P]$ , by the transformation explained in Eq 2.12.

Let

$$[C_P] = \begin{array}{ccc} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{array} \quad (2.14)$$

It can then be proved (Ref 21) that for orthotropic materials,

$$C_{11} = \frac{E_x}{1 - \nu_{xy}\nu_{yx}}, \quad C_{22} = \frac{E_y}{1 - \nu_{xy}\nu_{yx}},$$

$$C_{12} = C_{21} = \frac{\nu_{yx}E_x}{1 - \nu_{xy}\nu_{yx}} = \frac{\nu_{xy}E_y}{1 - \nu_{xy}\nu_{yx}},$$

$$C_{33} = G_0, \text{ and } C_{13} = C_{23} = C_{31} = C_{32} = 0$$

where

$E_x$  is the modulus of elasticity in the principal direction X ;

$E_y$  is the modulus of elasticity in the principal direction Y ;

$\nu_{xy}$  is Poisson's ratio which results in strain in the Y direction when stress is applied in the X direction;

$\nu_{yx}$  is Poisson's ratio which results in strain in the X direction when stress is applied in the Y direction; and

$G_0$  is the orthotropic shear modulus.

This orthotropic shear modulus is either measured as described in Ref 22 or is assumed approximately as

$$G_0 = \frac{E_x E_y}{E_y(1 + \nu_{xy}) + E_x(1 + \nu_{yx})} \quad (2.15)$$

For isotropic materials,

$$C_{11} = C_{22} = \frac{E}{1 - \nu^2}, \quad C_{12} = C_{21} = \frac{\nu E}{1 - \nu^2},$$

$$C_{33} = G = \frac{E}{2(1 + \nu)}, \text{ and } C_{13} = C_{23} = C_{31} = C_{32} = 0$$

where  $E$  and  $\nu$  are the modulus of elasticity and Poisson's ratio.

The input values required to represent these two material cases are described in detail in the guide for data input of Ref 4.

Applied Loads. The solution accepts two load types.

Concentrated loads at the mesh nodes are concentrated forces or moments in the global coordinates at the mesh nodal points.

Distributed loads (element loads) are distributed on the surface of the structure. Two types of distributed loads are accepted by the program.

- (1) The element weight which is considered of constant intensity over each element and acting in the negative direction of the global Z coordinate. Therefore, whenever a problem includes this kind of load, the global Z coordinate should take the gravity direction. This may appear as a limitation; however, it is always possible to represent the weight by equivalent nodal point loads.
- (2) Element pressure of linear variation of intensity over each triangle of the idealization and normal to it.

The equivalent nodal point loads for the element loads are computed as described in Ref 4. Any number of load cases can be solved for the same structure. In each load case the concentrated loads can be varied while the element loads are either retained or omitted as specified by the user. The description of loads input is given in Ref 4, Appendix 3.

### Construction of the Total Stiffness Matrix

The stiffness matrix of the assemblage is constructed from the stiffnesses of the composing elements after these element stiffnesses are transformed into a form suitable for assembling into the final six-degree-of-freedom system.

Triangular Elements. The membrane and bending stiffness matrices of the composing triangles are assembled to correspond to five degrees of freedom at each nodal point. The order of these degrees of freedom at node  $i$  is

$$u_i, v_i, w_i, \theta_{xi}, \theta_{yi} \quad (2.16)$$

The stiffness terms corresponding to these five degrees of freedom are in the element coordinates. When transformed to the global system of coordinates,

these five terms at each node yield six stiffness components corresponding to the following global degrees of freedom:

$$\delta_{xi} , \delta_{yi} , \delta_{zi} , \theta_{xi} , \theta_{yi} , \theta_{zi} \quad (2.17)$$

Therefore, for the systematic use of the form 2.6 in transforming the element stiffness matrix, the stiffness terms in element coordinates must be assembled into a six degree nodal point stiffness matrix in which the values corresponding to the sixth term at each node are equal to zeros. The transformation should then be performed, and the resulting stiffness terms would correspond to the six global degrees of freedom of 2.17. The element stiffness matrix transformed in this manner is then assembled in the structure stiffness matrix according to the element location in the mesh as defined by its nodal point numbers.

Quadrilateral Elements. The following steps are carried out in computing and assembling the stiffness matrix of quadrilateral elements:

- (1) The stiffness matrix of each of the four composing triangles is first constructed in local coordinates, transformed to the global coordinate system, then assembled in the proper locations of the quadrilateral stiffness matrix. Similar operations are performed to evaluate the equivalent nodal force loads for the distributed loads on each triangle.
- (2) The stiffness terms and the loads corresponding to the degrees of freedom of the internal points are condensed by an inverse Gaussian elimination procedure described below. The resulting condensed stiffness and loads correspond to the degrees of freedom of the external nodes and are the element's contributions to the equilibrium equations of the assemblage.
- (3) The condensed stiffness and loads are assembled in the structure stiffness matrix and loads according to the external nodal point numbering.

Condensation of the Quadrilateral Stiffness Matrix. The connectivity of the internal nodes of each quadrilateral element is local to that element. Therefore, according to the law of superposition, the effect of these internal nodes may be included in the stiffness and loads of the external nodes. By this process, known as condensation, the equilibrium equations of the internal

nodes are excluded from the total system of simultaneous equations of the assemblage. The condensation process is illustrated by the following partitioning of the matrix form of the equilibrium relation of the quadrilateral element:

$$\begin{array}{cccc}
 K_e & & K_o & & r_e & & R_e \\
 & & & & & & \\
 & & & & & = & \\
 & & K_i & & r_i & & R_i \\
 & & & & & & \\
 & & & & K_Q & & r_Q & & R_Q
 \end{array}
 \quad (2.18)$$

where  $K_Q$ ,  $r_Q$ , and  $R_Q$  are the stiffness matrix, the displacement vector, and the load vector, respectively, of the quadrilateral element and correspond to all its degrees of freedom;  $K_e$ ,  $r_e$ , and  $R_e$  are similar matrices corresponding to the external degrees of freedom only; and  $K_o$  is that part of the stiffness matrix which relates the external degrees of freedom to the internal degrees of freedom in the equilibrium equations of the external degrees of freedom.

Therefore,

$$[K_e] \{r_e\} + [K_o] \{r_i\} = \{R_e\} \quad (2.19)$$

If all the coefficients of the matrix  $[K_o]$  are zeros, the relation 2.19 reduces to

$$[K_e] \{r_e\} = \{R_e\} \quad (2.20)$$

From Eq 2.20, it is clear that the matrix  $\begin{bmatrix} K_e \end{bmatrix}$  represents the condensed stiffness matrix and that the load vector  $\begin{Bmatrix} R_e \end{Bmatrix}$  represents the condensed load vector.

The process of converting all the coefficients of the matrix  $\begin{bmatrix} K_o \end{bmatrix}$  to zeros is carried out by the inverse Gaussian elimination method. As this elimination is performed, the load vector is simultaneously reduced so that the condensed stiffness matrix,  $\begin{bmatrix} K_e \end{bmatrix}$ , and the condensed load vector,  $R_e$ , are obtained upon completion of the elimination operation. The matrix  $\begin{bmatrix} K_i \end{bmatrix}$  is thus transformed to the trapezoidal form indicated with all the coefficients above the major diagonal equal to zero. After the external displacements are calculated from the solution of the equilibrium equations of the assemblage, the matrix  $\begin{bmatrix} K_i \end{bmatrix}$  is used to calculate the internal displacements,  $\begin{Bmatrix} r_i \end{Bmatrix}$ , by a direct forward substitution in the relation

$$\begin{bmatrix} K_i \end{bmatrix} \begin{Bmatrix} r_i \end{Bmatrix} = R_i \quad (2.21)$$

in which  $\begin{Bmatrix} R_i \end{Bmatrix}$  should be in its converted form after condensation.

Inclusion of the Elastic Spring Supports. The elastic spring support stiffnesses are expressed as the force (or moment) required to produce a unit displacement in a particular global direction at a particular nodal point and can be included directly in the structure stiffness matrix. This is done by adding the spring stiffness to the diagonal term of the stiffness matrix which corresponds to the appropriate degree of freedom at the nodal point where the spring is.

Modifying the Stiffness Matrix for the Boundary Conditions. The structure stiffness matrix and the load vector must be modified to represent the desired boundary conditions in the equilibrium relation

$$K \cdot r = R \quad (2.22)$$

where  $K$ ,  $r$ , and  $R$  are the stiffness matrix, the displacement vector, and the load vector, respectively, of the total structure.



The boundary conditions are either in the form of elastic spring supports or in the form of specified displacement values in the global directions. The method for including elastic spring supports in the structure stiffness matrix has been described in the preceding section. To specify the value of a displacement (or rotation) component, the stiffness matrix diagonal which corresponds to the degree of freedom concerned is set equal to unity; all the other stiffness terms in the row are set equal to zero; and the corresponding load value is set equal to the specified displacement value. The displacement value is thus specified, but the symmetry of the stiffness matrix has been destroyed. To maintain symmetry, it is necessary to set equal to zero the stiffness terms above and below the diagonal in the column corresponding to the certain degree of freedom. In order to do this without changing the equilibrium relation, each term on this column must be multiplied by the specified displacement value and the result subtracted from the corresponding load value before the stiffness term is set equal to zero.

Omitting the Dependent Equations. It is important to notice that this method is an analysis of a six-degree-of-freedom system using five-degree-of-freedom finite elements. At nodes where the elements intersect at a non-zero angle, there are six degrees of freedom, and no precautions are required. At nodes where the adjacent elements lie in the same plane or at mid-side nodes on the edges, there will be an extra dependent equation (expressing rotation equilibrium) corresponding to each of these nodes. In such cases it is important to omit or neutralize this extra equation in order to have a true solution. Failure to do so would stop the solution of the equations; or if a solution is obtained, it will be a false solution. The omitted equation should satisfy the following two conditions:

- (1) It should not correspond to the rotation about any axis parallel to the plane at the nodal point.
- (2) The omission should be overridden by any suitable boundary condition at the point that satisfies the independency condition at the nodal point.

If no such boundary condition exists, the best choice for reducing numerical errors is to omit or neutralize the equation corresponding to the rotation about the global axis which is most nearly normal to the plane at the node.

The process of omitting the dependent equations for a single node is illustrated below. It can be seen that this process is valid for any linearly elastic system.

Consider the bending stiffness matrix,  $[\bar{K}]$ , at the node corresponding to the rotation components about two arbitrary local axes,  $\bar{X}$  and  $\bar{Y}$ , which is expressed as follows:

$$\bar{K} = \begin{matrix} k_{\bar{x} \bar{x}} & k_{\bar{y} \bar{x}} \\ k_{\bar{x} \bar{y}} & k_{\bar{y} \bar{y}} \end{matrix} \quad (2.23)$$

Rewriting the above expression in terms of three rotation degrees of freedom and setting the terms which correspond to the rotation about the local normal axis,  $\bar{Z}$ , equal to zero, we have

$$\bar{K}' = \begin{matrix} k_{\bar{x} \bar{x}} & k_{\bar{y} \bar{x}} & 0 \\ k_{\bar{x} \bar{y}} & k_{\bar{y} \bar{y}} & 0 \\ 0 & 0 & 0 \end{matrix} \equiv \begin{matrix} \bar{K} & 0 \\ 0 & 0 \end{matrix} \quad (2.24)$$

The transformation matrix,  $T$ , expressing the relation between any displacement vector,  $\{\bar{r}\}$ , in the local coordinate system ( $\bar{X} - \bar{Y} - \bar{Z}$ ) and the corresponding vector,  $\{r\}$ , in the global coordinate system ( $X - Y - Z$ ) (Eq 2.1) can be written as follows:

$$T = \begin{matrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{matrix} \equiv \begin{matrix} C_1 & S_1 \\ S_2 & C_2 \end{matrix}$$

The stiffness matrix  $[K]$  in the global axes is then obtained as

$$\begin{aligned}
 K &= \begin{matrix} c_1^T & s_2^T & \bar{K} & 0 & c_1 & s_1 \\ s_1^T & c_2^T & 0 & 0 & s_2 & c_2 \end{matrix} \\
 &= \begin{matrix} c_1^T \bar{K} c_1 & c_1^T \bar{K} s_1 & & & k_{xy} & k_{(xy)z} \\ s_1^T \bar{K} c_1 & s_1^T \bar{K} s_1 & & & k_{z(xy)} & k_z \end{matrix} \equiv \quad (2.25)
 \end{aligned}$$

Similarly, the corresponding moments,  $\{\bar{M}\}$ , and the rotations,  $\{\bar{\theta}\}$ , in local coordinates can be written as

$$\bar{M} = \begin{matrix} M_x \\ M_y \\ 0 \end{matrix} \equiv \begin{matrix} M_x \bar{y} \\ \\ 0 \end{matrix}$$

and

$$\bar{\theta} = \begin{matrix} \theta_x \\ \theta_y \\ 0 \end{matrix} \equiv \begin{matrix} \theta_x \bar{y} \\ \\ 0 \end{matrix}$$

The moments,  $\{M\}$ , in global coordinates are

$$\begin{aligned}
 \{M\} &= T^T \cdot \bar{M} = \begin{matrix} c_1^T & s_2^T & M_x \bar{y} \\ s_1^T & c_2 & 0 \end{matrix} \\
 &= \begin{matrix} c_1^T M_x \bar{y} & & M_x y \\ s_1^T M_x \bar{y} & & s_1^T M_x \bar{y} \end{matrix} \equiv \quad (2.26)
 \end{aligned}$$

Consider the following relation:

$$\begin{bmatrix} K_{x y} \end{bmatrix} \{ \theta^* \} = \{ M_{x y} \} \quad (2.27)$$

In this relation,  $\begin{bmatrix} K_{x y} \end{bmatrix}$  is as defined in Eq 2.25 and  $\{ M_{x y} \}$  is as defined in Eq 2.26. We can therefore solve for  $\{ \theta^* \}$ .

Substituting for  $\begin{bmatrix} K_{x y} \end{bmatrix}$  and  $\{ M_{x y} \}$  in Eq 2.27, we have

$$\begin{bmatrix} C_1^T \end{bmatrix} \begin{bmatrix} \bar{K} \end{bmatrix} \begin{bmatrix} C_1 \end{bmatrix} \{ \theta^* \} = \begin{bmatrix} C_1^T \end{bmatrix} \{ M_{x \bar{y}} \}$$

Premultiplying both sides by  $\begin{bmatrix} C_1^T \end{bmatrix}^{-1}$  we get

$$\begin{bmatrix} \bar{K} \end{bmatrix} \begin{bmatrix} C_1 \end{bmatrix} \{ \theta^* \} = \{ M_{x \bar{y}} \}$$

Therefore,

$$\begin{bmatrix} C_1 \end{bmatrix} \{ \theta^* \} = \begin{bmatrix} \bar{K} \end{bmatrix}^{-1} \{ M_{x \bar{y}} \} = \{ \theta_{x \bar{y}} \} \quad (2.28)$$

Writing the rotations  $\{ \theta \}$  in global coordinates, as

$$\theta = \begin{bmatrix} \theta_x \\ \theta_y \\ 0 \end{bmatrix} \equiv \begin{bmatrix} \theta^* \\ 0 \end{bmatrix},$$

and then transforming these rotations to local axes, we get

$$\bar{\theta} = T \cdot \theta = \begin{bmatrix} C_1 \theta^* \\ S_2 \theta^* \end{bmatrix} = \begin{bmatrix} \theta_x \bar{y} \\ S_2 \theta^* \end{bmatrix} \quad (2.29)$$

By this procedure we get the global rotations,  $\{\theta^*\}$ , that are related to the local rotations,  $\{\theta_{\bar{x}\bar{y}}\}$ , by Eq 2.28. When transformed to local coordinates as described above, these global rotations give the correct local rotations,  $\{\theta_{\underline{x}\underline{y}}\}$ , which are required to compute the bending moments at the node. The value of the rotation about the  $\bar{Z}$  axis as obtained in Eq 2.29 is not correct, but it does not affect the computation of the bending moment. The correct value of this component could be computed from the in-plane translations at the node and at adjacent nodes. It is important to notice the following energy equivalence:

$$\{M_{\underline{x}\underline{y}}\}^T \{\theta^*\} = \{M_{\underline{x}\underline{y}}\}^T [C_1] \{\theta^*\} = \{M_{\underline{x}\underline{y}}\}^T \{\theta_{\underline{x}\underline{y}}\}$$

It should be clear that the global rotations,  $\{\theta^*\}$ , obtained are not the total global rotation components at such a node. The correct three global components of rotation could be computed as follows:

- (1) Transform the rotations  $\{\theta^*\}$  as in Eq 2.28 to compute the correct local rotation components,  $\{\theta_{\underline{x}\underline{y}}\}$ .
- (2) Compute the local rotation component about the axis normal to the surface  $\bar{Z}$  from the in-plane translation components at the node and at the adjacent nodes.
- (3) Transform the three local rotation components which are obtained to the global system in the usual manner.

The procedure outlined above results in the omission of the equation corresponding to the rotation about the global Z axis. As mentioned before, the program omits the equation corresponding to the global axis which is most nearly normal to the plane at the node. In the output of the nodal point displacements, the symbol \* \* \* replaces the rotation about the axis for which the equation was omitted.

### The Structure Stiffness Matrix and Solution of Equations

The structure stiffness matrix, as constructed in the preceding section, is generally (1) symmetric, (2) positive definite, and (3) banded. These three properties are very helpful when considered in the solution of the

simultaneous equations of equilibrium of Eq 2.22. Special, efficient equation solvers exist for solving such equations. In these equation solvers, the required solution time is approximately proportional to the number of equations and to the square of the band width; therefore, it is always desirable to have the band width at a minimum. This can be achieved by a suitable choice of the mesh numbering system.

Mesh Numbering and Band Width. The band width of the structure stiffness matrix is the width of the zone that includes all the stiffness terms on each side of the diagonal. This is the shaded zone in Fig 6(a). Because of the symmetry of the stiffness matrix, only one half of this banded zone (including the diagonal) is necessary for solution of the simultaneous equations of 2.22. The upper half is used in the solution described below, and this half-width, (including the diagonal) will be referred to as the band width. This band width  $m$  is shown in Fig 6(a) and can be calculated for a certain mesh from its element nodal point numbering as

$$m = 6 \times (D + 1) \quad (2.30)$$

where  $D$  = the absolute maximum difference in the nodal point numbers of any element in the assemblage. For example, the element shown in Fig 6(b) gives

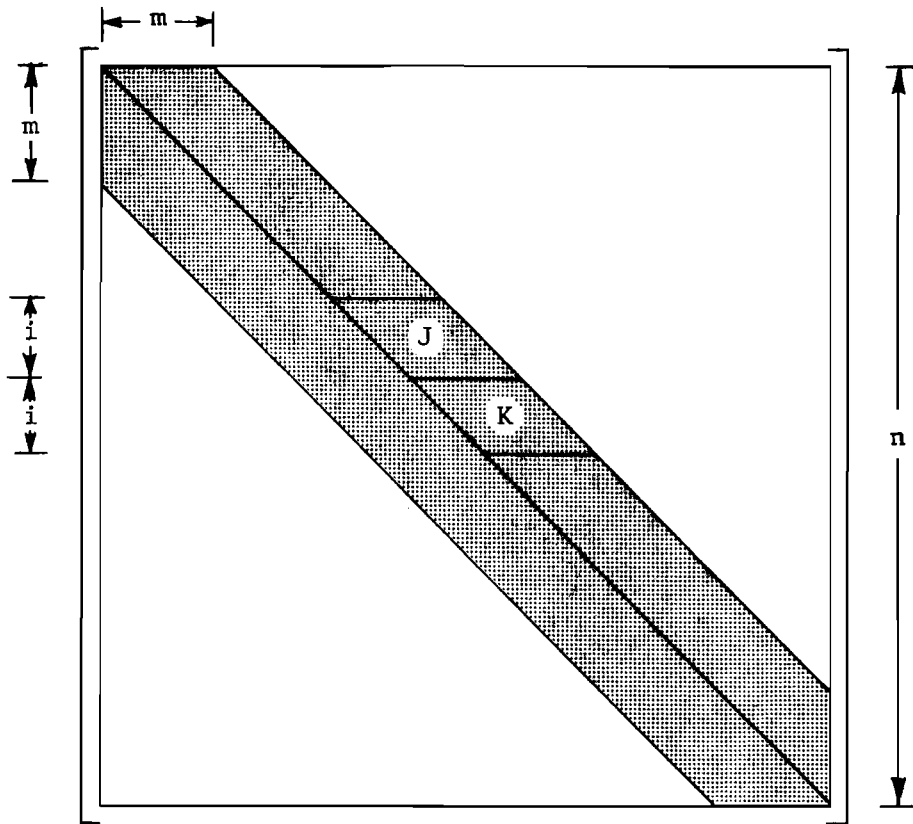
$$D = 28 - 12 = 16$$

Therefore,

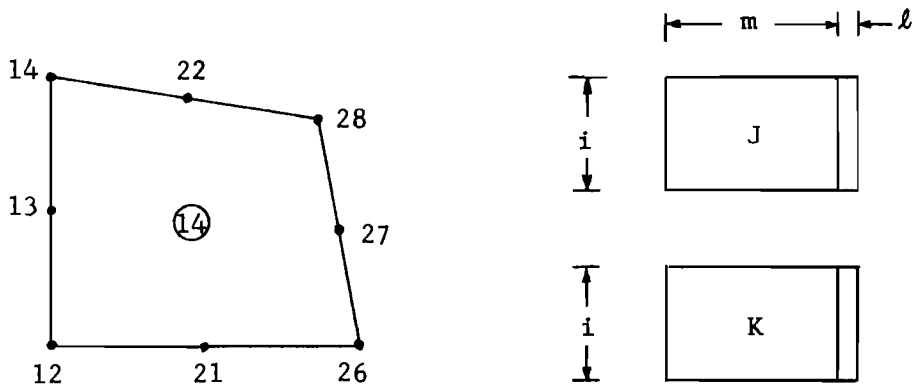
$$m = 6 (16 + 1) = 102$$

The maximum value of  $m$  for all the elements of the assemblage is the band width of the structure stiffness matrix.

The mesh numbering should be chosen in such a way as to result in minimum band width and therefore minimum solution time for the equilibrium equations. Although no restrictions concerning the mesh layout exist, the recommendations included later in this chapter are generally helpful in achieving an economical solution.



(a) Structure stiffness matrix.



(b) Example of element numbering.

(c) Solution blocks.

Fig 6. Structure stiffness matrix, typical element numbering, and solution blocks.

Solution of the Simultaneous Equations of Equilibrium. A direct solution method based on the Gaussian elimination procedure is used to solve the simultaneous equations of equilibrium. In the solution, the upper half of the banded stiffness matrix is used. This half is divided into blocks each of which contains a certain number of rows of the stiffness matrix. The number of rows,  $i$ , in each block is determined by the core storage available for the equation solution as well as by the band width,  $m$ , and by the number of load cases analyzed. These blocks are handled in the rectangular form shown by Fig 6(c) in which the two-dimensional array containing a particular block at a particular stage in the solution process has the diagonal of the stiffness matrix as its first column and the load vectors of all the load cases as its last columns. All the blocks are of the same size except the last block which may have fewer rows. More details of the equation solution are included in Ref 4. The solution of the simultaneous equations yields the global nodal point displacements of the structure for each load case.

#### Element Forces

After the nodal point displacements are computed, the stresses of each element at some of its nodal points are calculated, if required, for all the load cases. The element stress computation for each load case can be summarized as follows:

- (1) The nodal point displacements at the nodes of triangular elements or of composing triangles of quadrilateral elements are transformed to element coordinates for triangular elements or to local coordinates for quadrilateral elements.
- (2) The membrane and the bending stresses for each basic triangle are computed according to the displacement function assigned to it (Ref 4, Appendix 1). If more than one basic triangle is used in constructing the stiffness matrix of the triangular element or of the composing triangle of a quadrilateral element, the stresses at the common nodes are averaged. This step gives the element stresses in the case of triangular elements or the composing triangle stresses in the case of quadrilateral elements.
- (3) For quadrilateral elements, the stresses of each composing triangle are computed as in (2) and then transformed to element coordinates. The stresses of the four composing triangles are added and averaged at the common nodal points.



The stresses are output in the form of forces or moments per unit length at the element nodes. A local numbering is used in the tabulated output of these stresses to refer to the element nodes. This local numbering is shown in Fig 7 for the four element types used in the analysis.

### Solution of Problems, Remarks, and Limitations

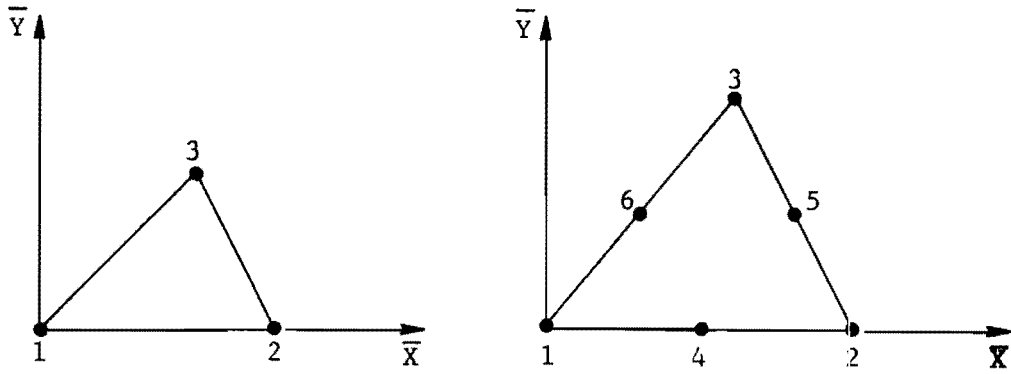
The steps of coding a problem for analysis by this method are summarized below. Any limitation of the solution which has not been mentioned before is described at the step when it arises.

The mesh divisions used should be chosen to approximate the geometry of the structure and the types of elements represented by these divisions should be selected to give the most favorable representation of the expected deformations as described earlier in this chapter. The nodal points as well as the elements are numbered, and a global system of coordinates should be chosen. Certain points must be observed in the mesh numbering and in the choice of the global coordinates.

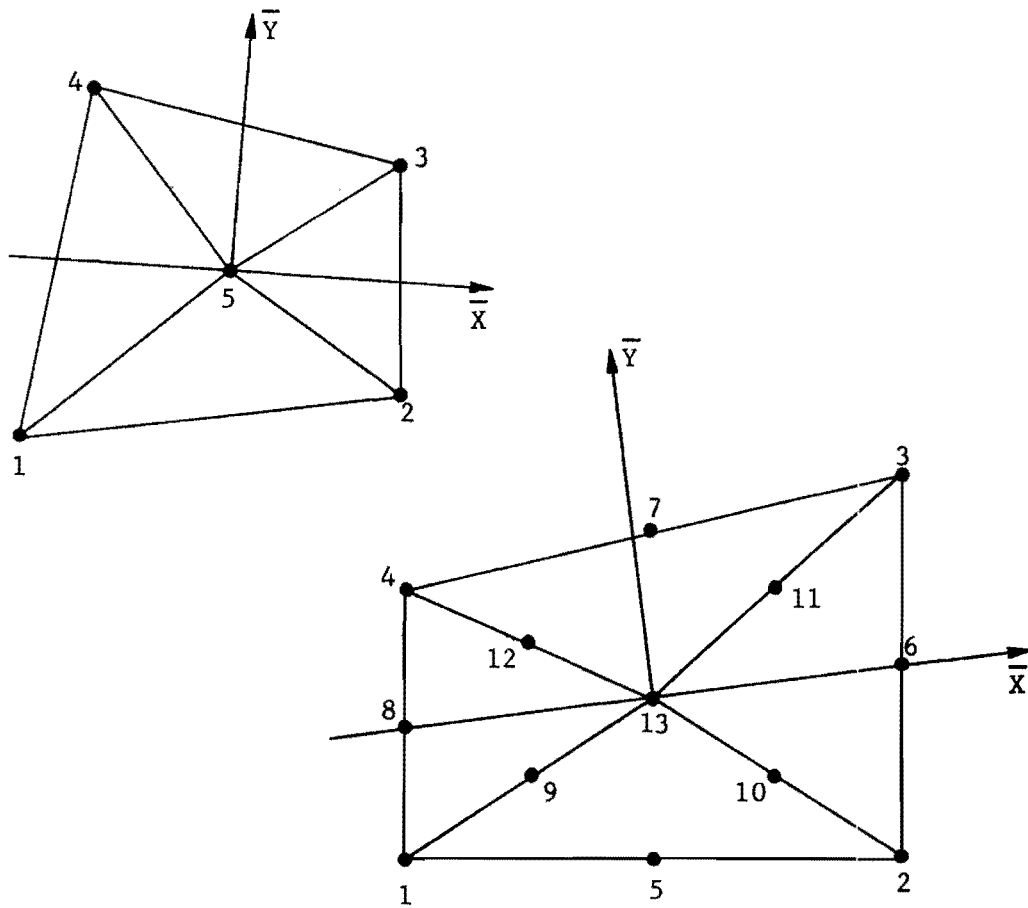
Mesh Numbering. The nodal point numbering should start with the number 1 and increase in a continuous manner. Element numbering should be done similarly. Only the external nodes of quadrilateral elements are to be numbered. No limitations exist concerning mesh numbering; however, to reduce the solution time required, two guidelines can be followed:

- (1) The direction of element numbering should follow the direction of nodal point numbering. This considerably reduces the time required to assemble the structure stiffness matrix in the rectangular blocks described above.
- (2) The band width should be kept at a minimum.

These two rules are observed in the example problems in Chapter 3. Some flexibility in applying these rules may be desirable when the mesh generation described in Ref 4, Appendix 3 ceases to be fully applicable as these rules are applied. In such a case, the man-hours that can be saved by using mesh generation may be more valuable than the computer time that can be saved by following these two rules.



(a) Triangular elements.



(b) Quadrilateral elements.

Fig 7. Local point numbering for element forces.

Choosing the Global Coordinate System. In addition to the limiting relation between the global Z-direction and the structure weight as described previously, one other important limitation may dictate the choice of the global coordinate axes in some problems.

In order to simplify the input and minimize coding errors, all boundary conditions are expressed in global coordinates. In most practical cases, this limitation causes no difficulty in the coding or in the choice of the global axes; however, in a few cases it may be impossible to represent the given arbitrary boundary conditions. In some cases of complicated boundary conditions, it may be possible to represent such boundaries simply by setting the global coordinate axes such that one or all of them lie in the directions of the specified boundary. The following special cases can be indirectly represented:

- (1) Specifying any three independent translation (or rotation) components at a point can be achieved by specifying their global components. This situation often exists in practical cases as a fixed boundary with arbitrary inclination to the axes.
- (2) Specifying any two translation components at a point in a plane parallel to one of three global planes is equivalent to specifying their two global components in that plane. Specifying two rotation components at a point achieves a similar result.

After the coordinate axes and the mesh numbering are chosen, the nodal point coordinates can be calculated. Only the coordinates of the element corner points must be necessarily input or generated. The rest of the problem coding can be carried out as described in the guide for data input of Ref 4, Appendix 3. There is one limitation which should be observed here.

No moment can be applied which is about an axis normal to the surface of the structure at the point of application. Such a moment must be replaced by equivalent in-plane concentrated loads at the adjacent points. Similarly, specifying in-plane rotation can be done only by specifying in-plane displacement components.

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## CHAPTER 3. EXAMPLE PROBLEMS

In this chapter, the solutions of some example problems are presented to illustrate the application of the finite-element analysis of shell-type structures (Ref 4) to the analysis of highway bridges. The input data of all these problems is shown in the Appendix, which also includes complete output for one problem.

The results of the present method of analysis are compared with the following previously available solutions which used different methods of analysis:

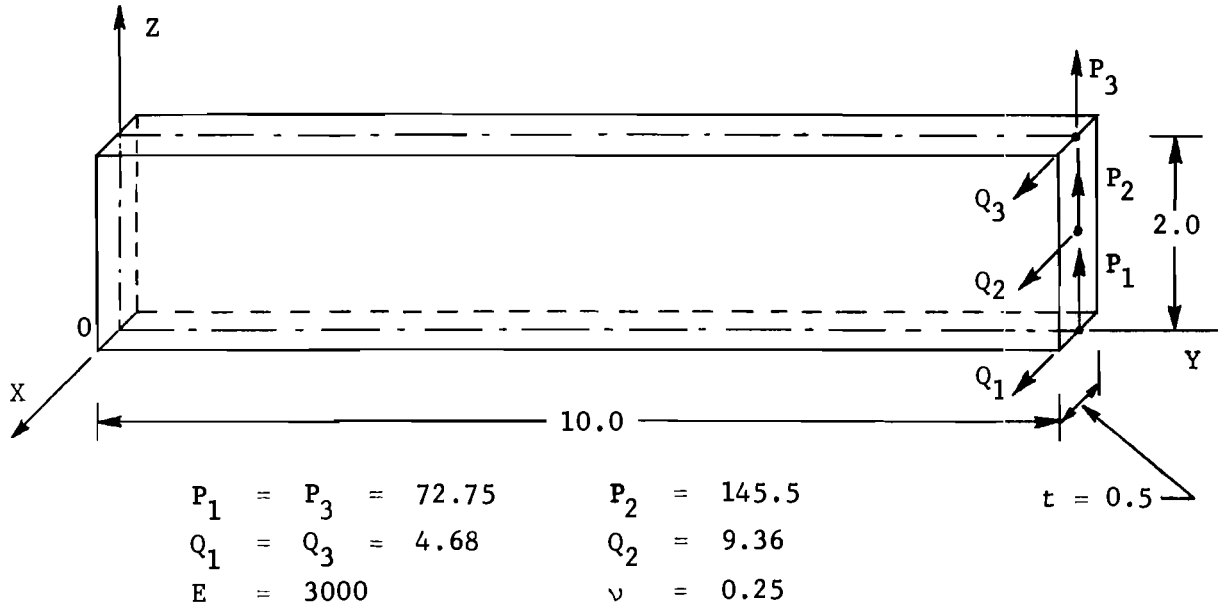
- (1) the folded-plate method (Ref 19),
- (2) the finite-segment method (Ref 19),
- (3) the finite element method (Ref 19),
- (4) the discrete-element method of analysis of slabs and bridge decks (Refs 1, 2, and 3), and
- (5) the theory of bending of shallow beams including shear deformation.

In these comparisons, the solutions are identified by the method names listed above. If more than one solution is shown using the same method of analysis, the solutions will be identified by the name of the method together with a mesh number. It should be clear here that the word mesh stands for a certain selection of element types and geometries for idealizing the analyzed structure.

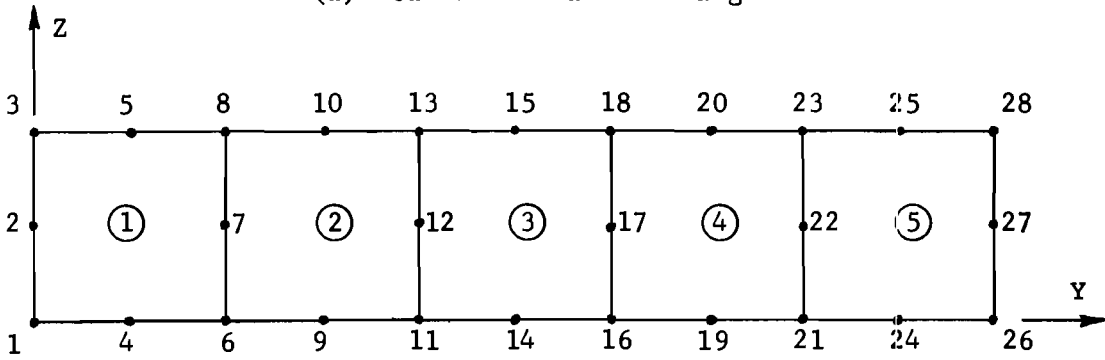
### Example 1. Cantilever Beam

The cantilever beam shown in Fig 8(a) is used to compare the results that can be obtained by the present method to the theoretical results obtained from beam theory. The problem was solved using four different meshes, each of which utilizes one of the four elements available in the method. The four meshes are shown in Figs 8(b), 8(c), 8(d), and 8(e), respectively. It should be noted that the aspect ratios and the number of the constituting plate bending elements (HCT's) for the two meshes of quadrilaterals are the same. This is also the case for the two meshes of triangles.

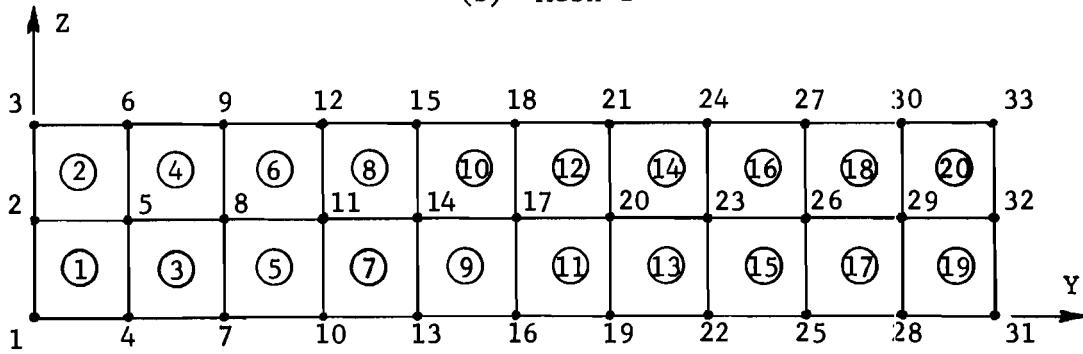
The results are tabulated in Fig 8, together with the half band width of the stiffness matrix, the number of simultaneous equations solved, and the



(a) Cantilever and loading

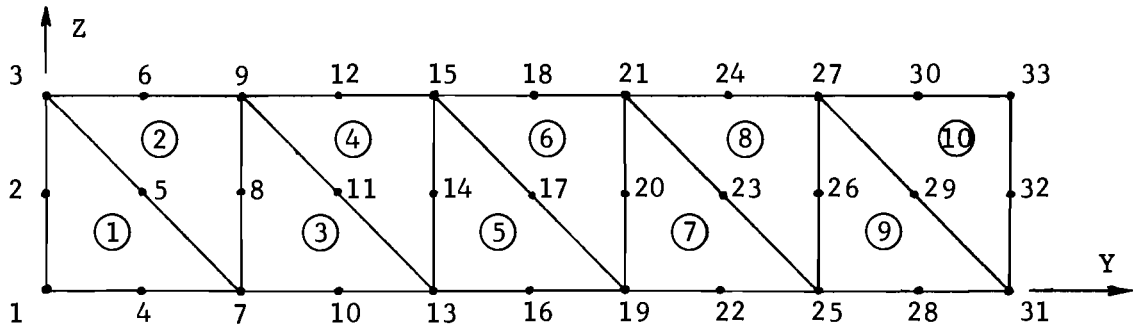


(b) Mesh 1

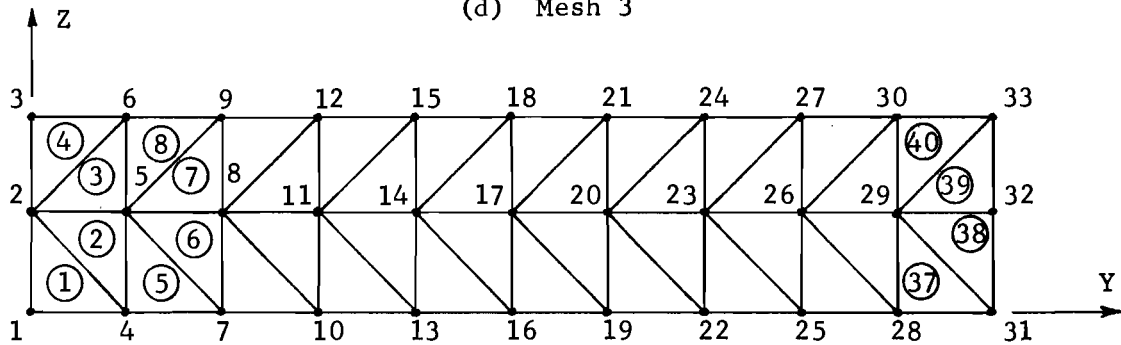


(c) Mesh 2

Fig 8. Example 1. Cantilever beam.



(d) Mesh 3



(e) Mesh 4

## Comparison of Results:

Mesh	Tip Deflection		$\sigma_y$ (max.)	Half Band Width	Number of Equations	Solution Time (sec.)
	$\delta_x$	$\delta_z$				
(1)	98.1	99.6	8478	48	168	15
(2)	98.1	94.3	7638	30	198	23
(3)	97.9	99.2	8246	42	198	9
(4)	97.9	57.1	4840	30	198	14
Beam Theory	100.0	100.0	8730			

Fig 8. (Continued).

solution time for each mesh. The following points should be noted about the results:

- (1) The stiffness properties of the quadrilateral element with eight nodes and the triangular element with six nodes are superior to the stiffness of the other two types of elements. This is clear from comparing the tip deflections given by mesh 1 and mesh 3 to the tip deflections obtained in the beam theory solution.
- (2) The membrane stiffness properties of the triangular element with three nodes and the quadrilateral element with four nodes are deficient in comparison to the membrane stiffness properties of the other two elements. This results from the assumption of linear displacements along the sides of the quadrilateral element or over the whole area of the triangular element. This deficiency in the membrane stiffness properties is also reflected in the membrane axial stresses obtained and is especially severe in the case of the triangular element.
- (3) The solution time required for mesh 1 is less than that for mesh 2. Also, the solution time for mesh 3 is less than that for mesh 4 despite the fact that the latter has a smaller band width. This is due to the fact that other factors in addition to the equation solution may considerably affect the total solution time of a certain problem. It is clear here that the other significant factor is the number of elements. Evaluating the element stiffnesses and assembling them into the total stiffness matrix takes a considerable amount of the solution time.

All four elements are suitable for plate bending representation, with the two elements which have mid-side nodes being more suitable for coarse meshes. The elements with mid-side nodes offer the best tool for representation of in-plane deformation while the triangular element with three nodes provides the least accurate representation. The quadrilateral element with four nodes usually gives good representation of the in-plane deformation if it is used (a) in a relatively fine mesh and (b) with aspect ratios not far from the range of 1.0 to 2.0. For economical analysis of highway bridge decks, the use of the elements with mid-side nodes may be limited to regions such as main girders which undergo considerable in-plane strain variations while the other two elements may be used in regions of lesser in-plane deformations such as slabs or slab-type decks. If the quadrilateral element with four nodes is used to idealize bending members such as main girders, there should be at least two layers of elements on the beam depth.



### Example 2. Beam-Slab Type Bridge

A highway bridge deck similar to those currently being designed by Texas Highway Department is analyzed in this example. The deck is shown in Fig 9 and consists of a concrete slab resting on a system of longitudinal main beams which are continuous over two intermediate supports and have transverse diaphragms between them. The structure is analyzed for the maximum positive moment in the center spans of the main beams from HS20 truck loadings (Ref 23). The same bridge was analyzed in Ref 3 by the discrete-element method.

The discrete-element method of analysis of slabs and bridge decks is described in Refs 1, 2, and 3. In this method the actual structure is replaced by a mechanical model which has the form of a grid. The stiffness of the grid, both in bending and torsion, represents the actual structure's stiffness. The mechanical model is then analyzed for the given loading. Use of a relatively fine grid for the analysis of the model usually provides good accuracy. The main reason for error is generally uncertainty in representing the structure stiffness or geometry.

Slab stiffnesses can be accurately represented by the discrete elements, and good results are obtained. The usual practice is to represent the stiffness of beam-slab combinations as an assumed T-section or L-section composed of the beam and a certain width of the slab. This assumption was used here in the discrete-element analysis where the slab width considered was 7.25 ft for the interior girders and  $(3.625 + 3.125)$  ft for the exterior girders.

Analysis by the discrete-element method. The geometry of the structure shown in Fig 9 was modified slightly for use with the SLAB 49 program (Ref 3). The changes in dimensions were required to fit the actual geometry to suitable increment lengths in both directions. The schematic plan of the structure as modeled is shown in Fig 10(a) which is taken from Ref 3.

The structural system, loaded with two HS20 trucks (Ref 23) as shown in Fig 10(b), was analyzed for the maximum positive moment in the center spans of the main beams. The wheel loads were apportioned to adjacent stations, as depicted in Fig 10(b) because the wheel spacing and the lane boundaries did not exactly fit the stationing and the increment lengths chosen for the analysis. The wheel loads were increased by an impact factor of 27.0 percent computed from the length of the loaded span under consideration (Ref 23).

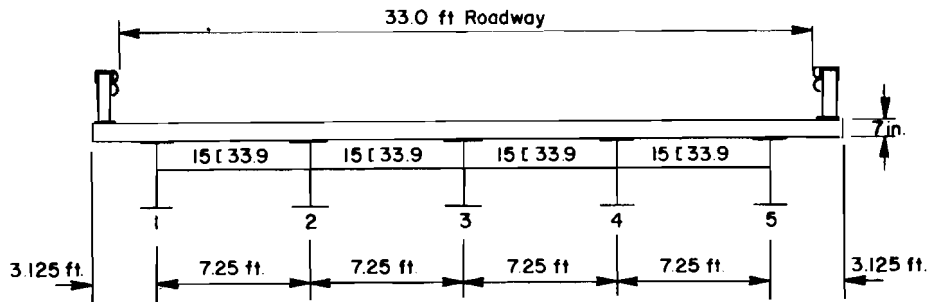
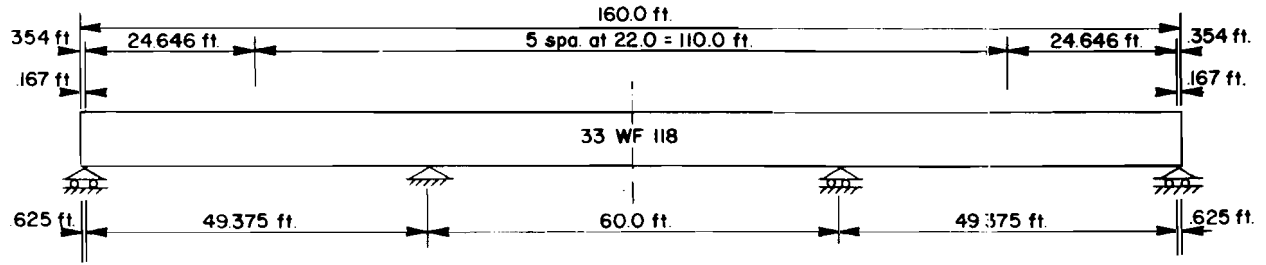
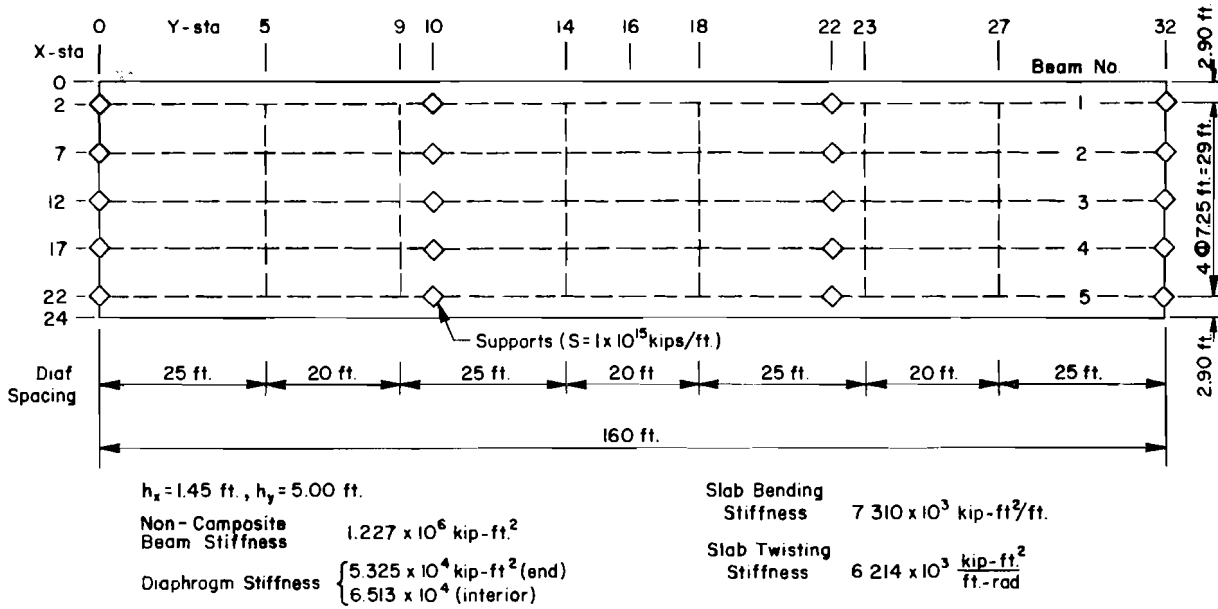
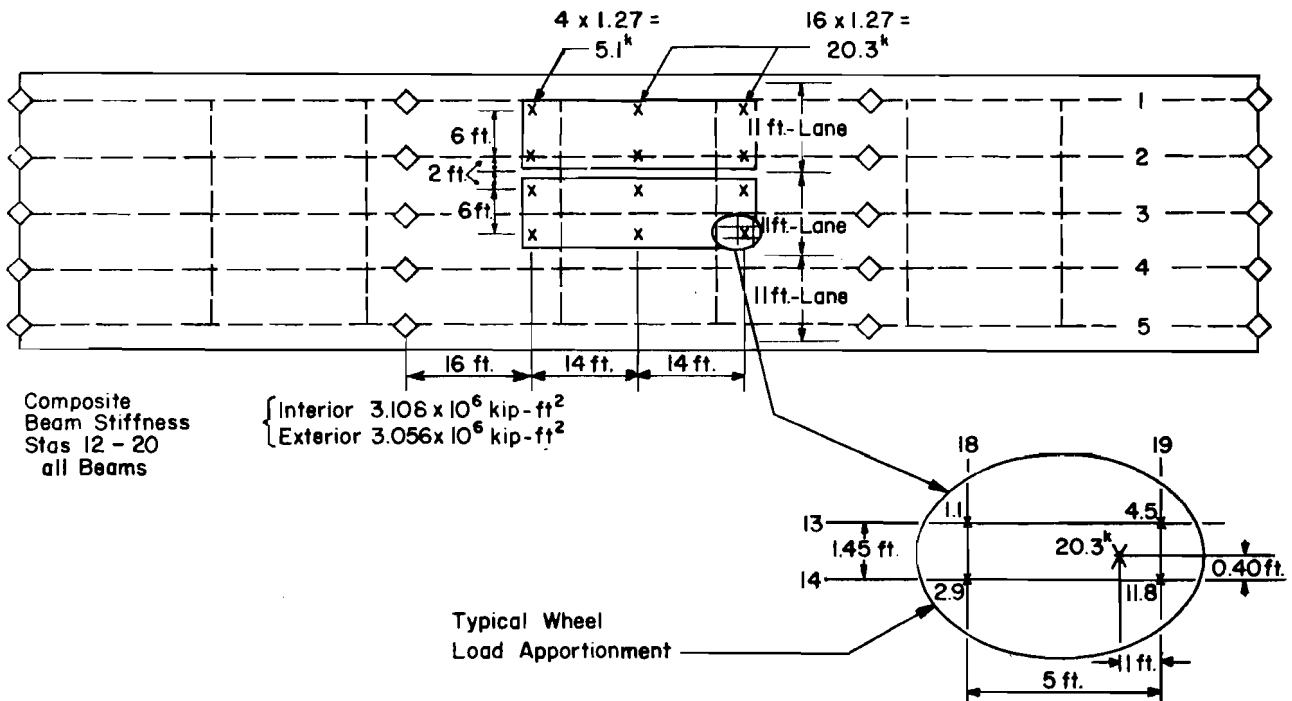


Fig 9. Three-span highway bridge (Ref 3).



(a)



(b)

Fig 10. Example 3. Three-span structure as modeled for discrete-element analysis with HS20 truck loading for maximum positive moment in center spans (Ref 3).

In addition to the bending stiffnesses which were computed and input for the slab and the diaphragms, a composite-beam stiffness was described for the main beams and the overlying slab. After the concrete deck has hardened, the beams and the slab act compositely within an appropriate effective width when subjected to positive bending. The effective slab width for composite action for this structure was assumed equal to the beam spacing of 7.25 ft. The composite stiffness was defined at locations within the approximate positive moment areas as shown in Fig 10(b).

The discrete-element analysis using these dimensions and stiffness values is identified as solution 1 in all the comparisons which follow. Another discrete-element analysis which will be described later was performed with a different assumed stiffness for the structure in the regions of negative moment. This second discrete-element analysis is identified as solution 2.

Analysis by the Present Method. In the analysis by the present finite element method, a geometric approximation was used to represent the steel main girders and the cross diaphragms. This approximation was necessary because the number of elements, the number of nodal points, and the band width of the structure stiffness matrix all increase unreasonably if the details of the flanges are represented exactly. Furthermore, the aspect ratios of such elements on the flanges would be unacceptable even for relatively fine divisions in the longitudinal direction. For these reasons the main girders and the diaphragms were replaced in the analysis by equivalent rectangular sections. The deck was analyzed twice using different properties for the equivalent rectangular sections. In the first analysis, identified as solution 1, the equivalence was chosen only with respect to the bending stiffness as such beams are usually acting mainly to support bending forces. Therefore, for the main girders with a given moment of inertia of  $5886.9 \text{ in}^4$  and a given girder depth of 32.86 in, the depth of the equivalent rectangular section may be chosen as the depth of the girder plus half the slab depth, or  $32.86 \text{ in} + 3.5 \text{ in} = 36.36 \text{ in}$ . Thus, the equivalent rectangular section thickness,  $t$ , is equal to

$$\frac{12 \times 5886.9}{(36.36)^3} = 1.471 \text{ in.}$$

To facilitate the mesh layout, the depth of the equivalent rectangular section for the diaphragms was chosen as one half that of the main girders. Thus, the diaphragm depth was equal to 18.18 in, and the thicknesses were calculated as above so that

$$t = 0.625 \text{ in for the interior diaphragms,}$$

and

$$t = 0.510 \text{ in for the end diaphragms.}$$

It should be noted that the axial stiffness and the torsional stiffness of the equivalent rectangular sections are different from the corresponding stiffnesses of the actual beams. The second analysis, identified as solution 2, considered equivalence of the torsional stiffness as well as bending stiffnesses and will be described later. The concrete slab is considered isotropic with  $E = 3 \times 10^6$  psi and  $\nu = 0.15$  in the zone of expected positive longitudinal bending moment. In the zone of expected negative bending moment, the slab is assumed to have a reduced modulus of elasticity in the longitudinal directions as a result of the transverse tension cracks. An 80% reduction is assumed. A complete justification cannot be given for including the remaining 20% of the composite slab longitudinal stiffness. It seems logical to assume by engineering judgment that some of the cracked slab must contribute to the stiffness.

As will be seen later, this 20% longitudinal composite stiffness in the negative moment areas has a significant effect on the results. An orthotropic slab was thus considered with  $E = 3 \times 10^6$  psi and  $\nu = 0.15$  in the transverse direction, and with  $E = 6 \times 10^5$  psi and  $\nu = 0.03$  in the longitudinal direction.

In the finite element analysis, the zone of positive bending moment was assumed to be the middle 38.0 ft of the slab while in the discrete-element analysis, it was taken as the middle 40.0 ft. No change is needed in the plan dimensions of the deck or in the spacing of the diaphragms. Because the deck

is symmetric about two planes, only one quadrant was considered for analysis. Figure 11 shows the quadrant which was analyzed and the details of the mesh used. The loading case considered was replaced by four load cases with known boundary conditions at the planes of symmetry of the deck. Figure 12(a) shows the HS20 truck loading positions relative to the mesh used. All the loads are increased by an impact factor of 27% as in the discrete-element analysis. The four equivalent load cases are (1) symmetric about both planes of symmetry of the deck (Fig 12(b)); (2) symmetric about the longitudinal plane and anti-symmetric about the transverse plane (Fig 12(c)); (3) symmetric about the transverse plane and anti-symmetric about the longitudinal plane (Fig 12(d)); and (4) anti-symmetric about both planes (Fig 12(e)).

An explanation of the equivalent load replacements follows. Any load case,  $P$ , on a structure symmetric about a given plane can be replaced by a symmetric load case,  $P_s$ , and an anti-symmetric load case,  $P_a$ , where

$$P_s = \frac{1}{2} (P + P')$$

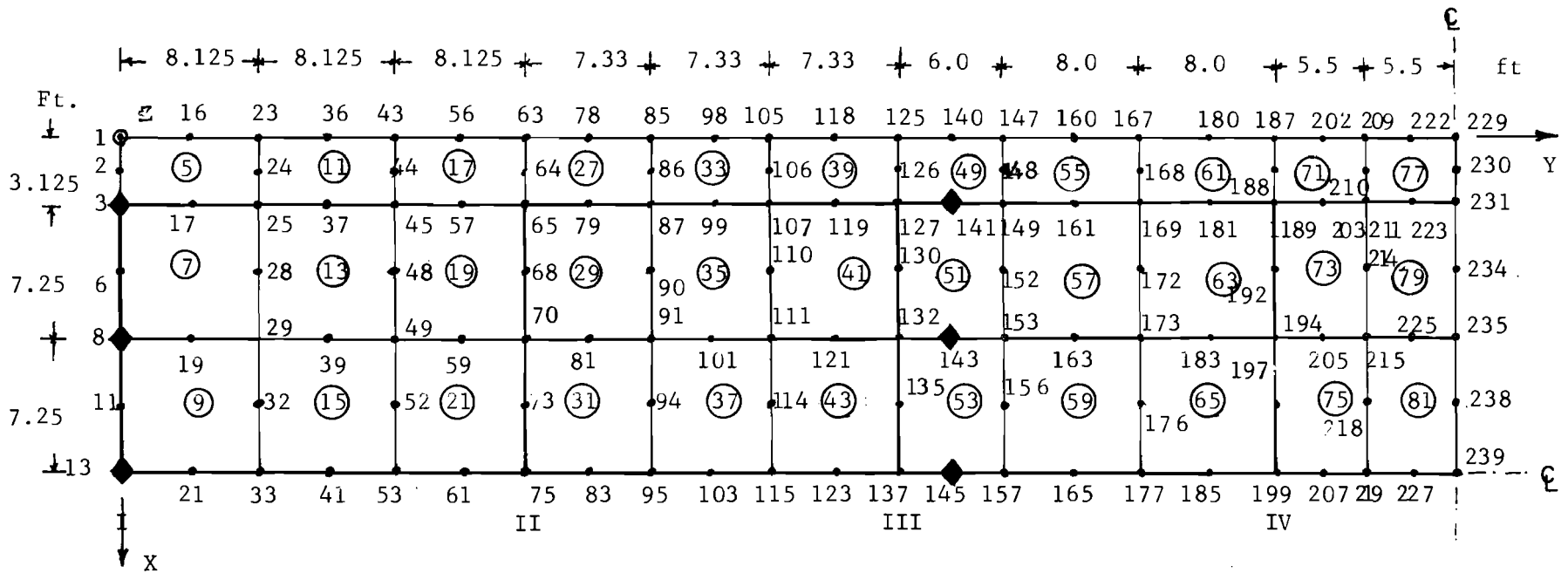
and

$$P_a = \frac{1}{2} (P - P')$$

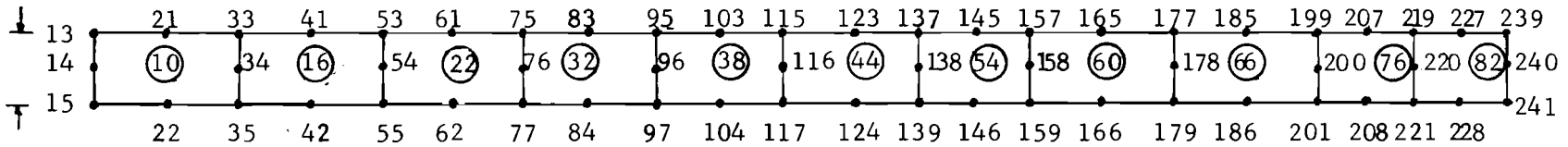
and where  $P'$  is another load obtained by inverting the positions of loading  $P$  with respect to the plane of symmetry of the structure.  $P'$  can be thought of as the image of  $P$  with respect to the plane of symmetry.

Consideration of one of the two planes of symmetry of the structure resulted in the replacement of the original load case by two load cases as described above. On consideration of the other plane of symmetry, each of the first two load cases was replaced by two more load cases with the result of four load cases in which the boundary conditions are known along the two planes of symmetry of the structure.

In each load case the concentrated loads are not at the nodal points. Proportional loads were used at the nodal points adjacent to each load. This approximation is expected to have a very negligible effect on the overall

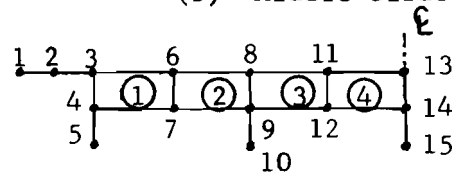


(a) Concrete slab,  $t = 7.0$  inches



(b) Middle Girder,  $2t = 1.471$  inch

(c) Diaphragm I,  
 $t = 0.510$  in.



(d) Diaphragm II,  
 $t = .625$  in.

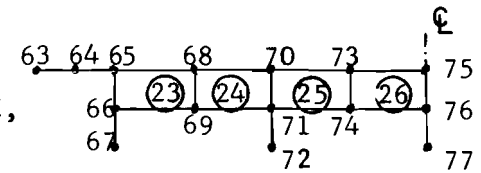


Fig 11. Details of the mesh for the quadrant which was analyzed.

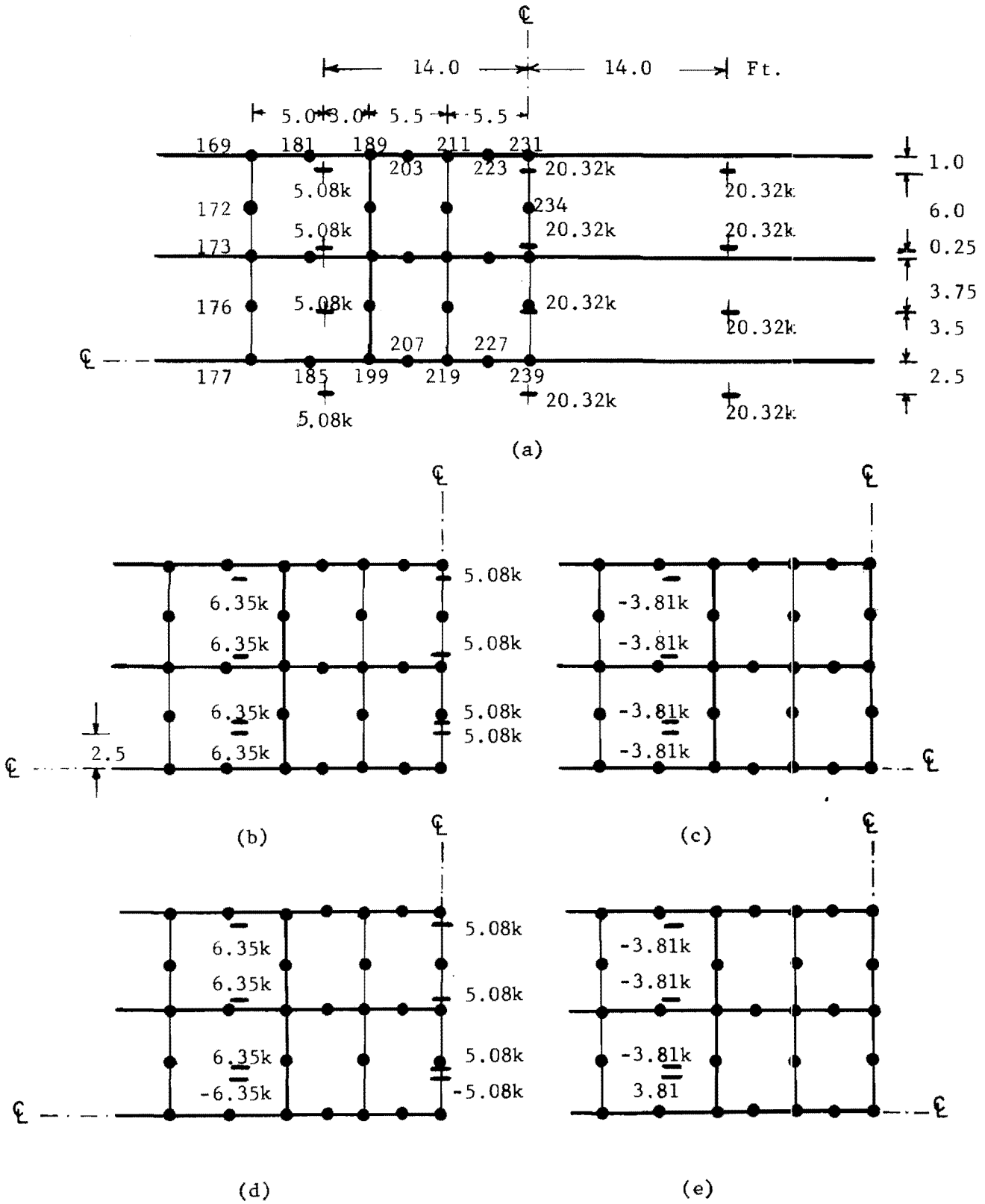


Fig 12. HS20 truck loading and the equivalent 4 loading cases.



bridge deflections and the longitudinal stresses. However, it may have a significant effect on the transverse bending moments of the concrete slab. Practically, this is not significant since this loading case is not expected to be the one which produces the maximum transverse slab moment. Such maximum moment which may govern the design of the slab usually occurs locally under the heaviest single wheel load when it is located at the middle of the spacing between the main girders.

A computer listing of the input data for the four load cases is included in the Appendix under problem numbers 301, 302, 303, and 304. The results on the portions of the deck which were not considered in the analysis were obtained from the results given on the analyzed quadrant by using the conditions of symmetry and anti-symmetry tabulated in the summary. This procedure is outlined below.

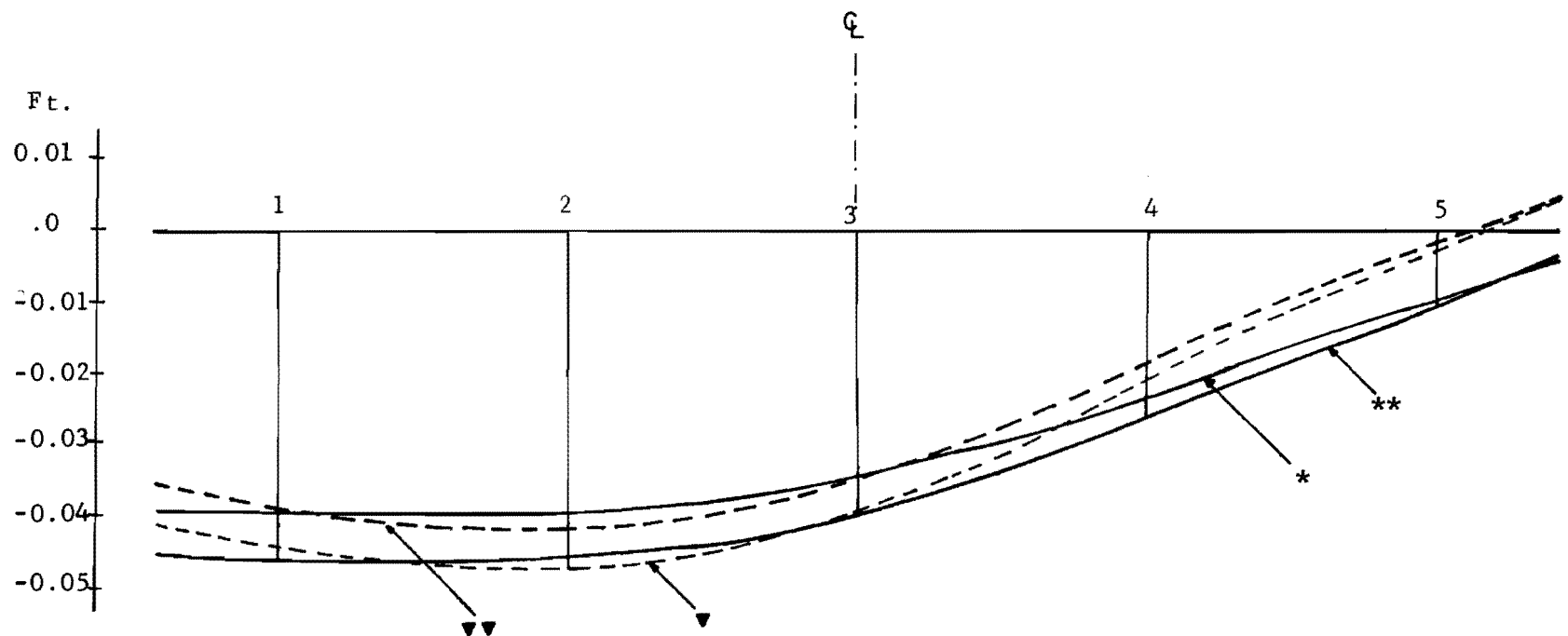
From the results obtained for the quadrant analyzed under one of the four applied load cases, the corresponding results on an adjacent quadrant can be obtained by applying the boundary condition at the plane separating the two quadrants and by noting that

- (1) for symmetric load cases, deflections, normal stresses, and bending moments are symmetric while shearing forces are anti-symmetric, and
- (2) for anti-symmetric load cases, deflections, normal stresses, and bending moments are anti-symmetric while shearing forces are symmetric.

By this procedure the results on one half of the structure are obtained.

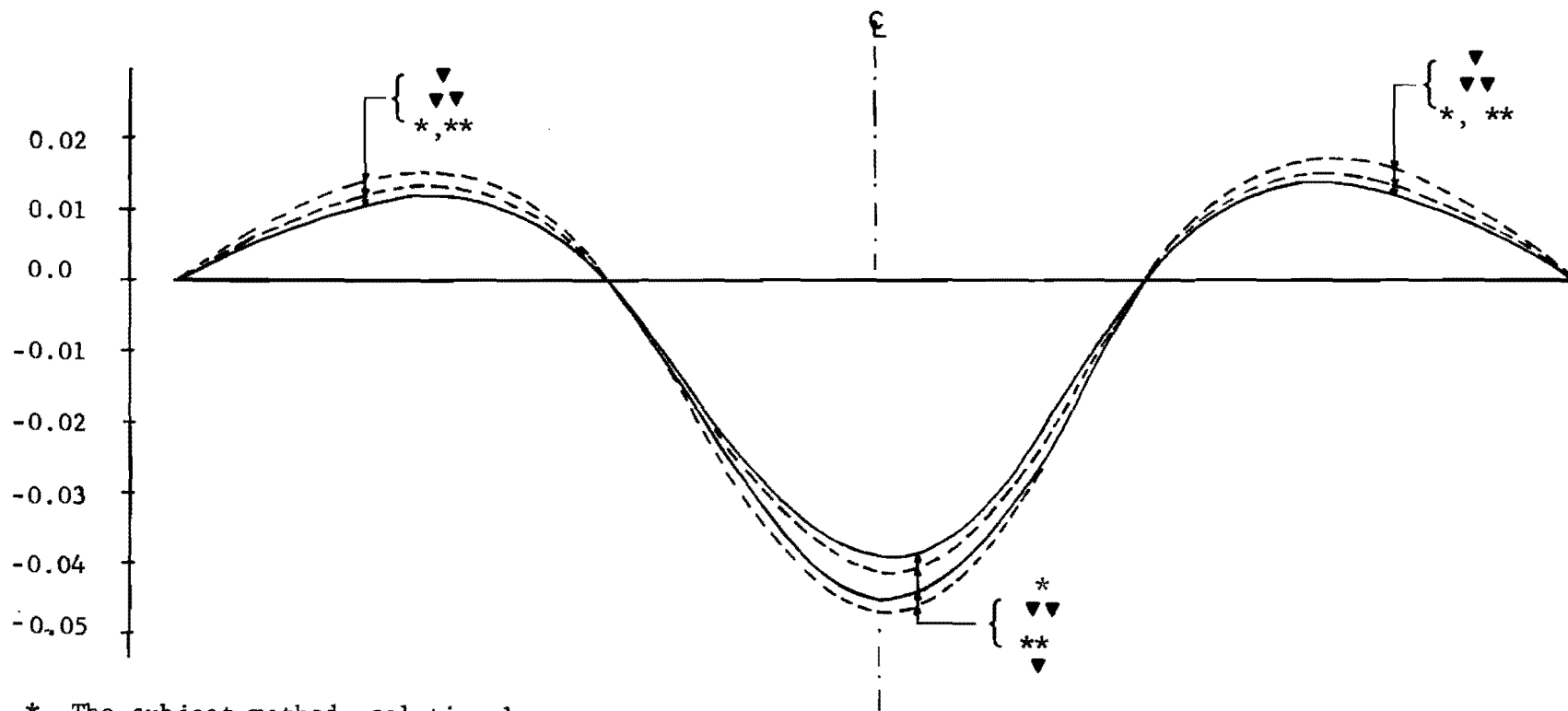
The results on the remaining half of the structure can be obtained from the known results on the first half by again considering the symmetric and anti-symmetric conditions at the plane separating the two halves.

Discussion of the Results. The deflections at the central transverse section of the middle span are shown in Fig 13, and the deflections along the top of girder No. 2 are shown in Fig 14. A comparison of the deflections of the discrete-element analysis (solution 1) with those of the analysis by the present method (solution 1) indicates that, in general, the magnitudes of the deflections obtained by the discrete-element method are greater than the corresponding values obtained by the present method of analysis. The twisting



- \* The subject method, solution 1.
- \*\* The subject method, solution 2.
- ▼ The discrete-element method, solution 1.
- ▼▼ The discrete-element method, solution 2.

Fig 13. Vertical deflection at mid-span.



- \* The subject method, solution 1.
- \*\* The subject method, solution 2.
- ▼ The discrete-element method, solution 1.
- ▼▼ The discrete-element method, solution 2.

Fig 14. Vertical deflections at girder No. 2.

deformations are also greater in the discrete-element analysis than in the finite element analysis. These differences may be due to

- (1) completely neglecting the longitudinal slab stiffness in the zones of negative bending moment in the discrete-element analysis (solution 1),
- (2) neglecting the torsional stiffness of the longitudinal beams and of the diaphragms in the discrete-element analysis,
- (3) including excessive torsional stiffness in the equivalent rectangular sections in the finite element analysis (solution 1), or
- (4) the basic difference between the discrete-element model and the idealized finite element structure as well as the differences in the techniques and assumptions in each case.

To evaluate the effect of the first of these factors, the deck was re-analyzed by the discrete-element method with a composite section stiffness in the zones of negative bending moment as well as in the zone of positive bending moment. The slab modulus of elasticity in the negative moment zones was assumed to be reduced by 80% as in the finite element analysis so that a 20% effective composite slab was considered in these zones. This analysis is identified as the discrete-element solution 2, and it results in the deflections shown in Figs 13 and 14. The results are very close to those computed by the finite element analysis (solution 1) in the vicinity of the loading, but the twisting deformations are still greater than those obtained by the finite element method.

To evaluate the effect of the second or third factors, the deck was re-analyzed by the present method using equivalent rectangular sections for the main beams and the diaphragms with equivalent torsional stiffness as well as the bending stiffness. This analysis is identified as the finite element analysis (solution 2) and uses orthotropic materials with a modulus of rigidity  $G_o$  to give the required torsional stiffness in the equivalent rectangular sections. The modulus of elasticity and Poisson's ratio in the longitudinal direction (X-direction) of the beam are the same as for solution 1. The modulus of elasticity and Poisson's ratio in the Y-direction are calculated from the relations

$$G_o = \frac{E_x E_y}{E_x + E_y (1 + 2\nu_{xy})} \quad \text{and} \quad \nu_{yx} \frac{E_x}{x} = \nu_{xy} \frac{E_y}{y}$$

in which all the terms are as previously defined in Chapter 2. The required value of  $G_o$  is computed from the relation

$$G_o K_e = GK_a$$

in which  $G$  is the modulus of rigidity of the actual material and  $K_e$  and  $K_a$  are the torsional constants for the equivalent rectangular section and for the actual section, respectively.

An approximate value for  $K_e$  and  $K_a$  can be computed for each section as

$$\frac{1}{3} \sum bt^3$$

in which  $b$  and  $t$  are the length and the thickness, respectively, of the composing parts of the cross section.

The values used in the analysis for the original material were

$$E = 4.32 (10)^9 \text{ lb/ft}^2$$

$$\nu = 0.3$$

$$G = \frac{E}{2(1 + \nu)} = 1.662 (10)^9 \text{ lb/ft}^2$$

For the main girders,

$$K_a = 5.6 \text{ in}^3$$

$$K_e = 38.6 \text{ in}^3$$

Therefore,

$$G_o = 5.6 \times 1.662 (10)^9 / 38.6 = 2.413 (10)^8 \text{ lb/ft}^2$$

$$E_y = 2.650 (10)^8 \text{ lb/ft}^2$$

$$\nu_{yx} = 0.0184$$

for the diaphragms,

$$K_a = 1.43 \text{ in}^3$$

$$K_e = 1.00 \text{ in}^3$$

Therefore,

$$G_o = 1.115 (10)^9 \text{ lb/ft}^2$$

$$E_y = 1.900 (10)^9 \text{ lb/ft}^2$$

$$\nu_{yx} = 0.1319$$

A computer listing of the input data for the finite element analysis (solution 2) is included in the Appendix under problem numbers 301-A, 302-A, 303-A, and 304-A.

As shown in Figs 13 and 14, the results of solution 2 by the finite element method indicate greater twisting deformations and vertical deflections in the central span than those observed in solution 1. In the outer spans the two solutions give approximately the same deflections. From the deflections shown in Figs 13 and 14, the following points can be made:

- (1) The discrete-element analysis (solution 2) gives slightly different twisting deformations from those of solution 2 by the present method and considerably different vertical deflections in the central span. The difference in the twisting deformations is consistent with the omission of the torsional stiffness of the beams in the discrete-element analysis. The differences in the vertical deflections may be the result of the basic differences in the two methods which were described earlier. In addition to the increased twisting deformations observed in solution 2 by the present method, a significant increase in the vertical deflections under the loads is observed even though the bending stiffness of the composing members is the same for both solutions. This indicates that the torsional stiffnesses of the beams affect not only the twisting deformations of the deck but also its overall stiffness.
- (2) The two finite element solutions give approximately the same deflections in the outer spans, which indicates that the effect of twisting deformations is rapidly being damped out, possibly due to the effect of the diaphragms. This is emphasized by the fact that the stresses given by the two solutions at the interior supports and at the outer

spans are only slightly different. Another contributory factor is the shear deformations of the main beams in the central span where relatively heavy shearing forces exist. This factor may be partially the cause of the larger vertical deflections given by solution 2 in the loaded zone.

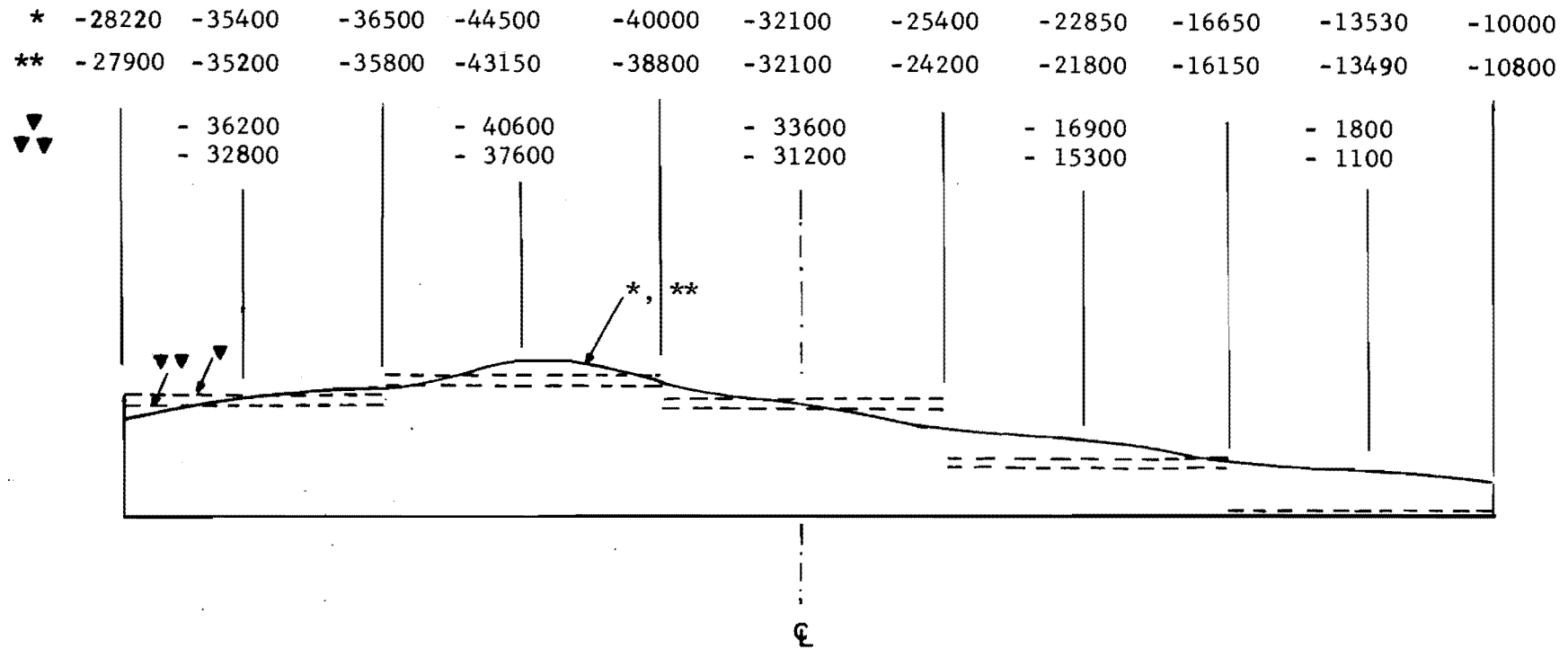
The average slab longitudinal stresses at the central section obtained by the four solutions are shown in Fig 15. There is good agreement between the four solutions in the vicinity of the load; however, at the unloaded exterior beam, stresses of much smaller magnitude are given by the discrete-element solutions than by the finite element solutions. This difference could be attributed to the twisting deformations. The average slab longitudinal stresses of the two finite element solutions show very small differences (the maximum difference is 3% of the largest value).

The longitudinal slab bending moment at the central section is shown in Fig 16. The two discrete-element solutions and solution 1 by the present method give results that differ within a small range. The differences between these three solutions are less visible here than in the cases of deflections and average longitudinal axial stresses. Solution 2 by the present method gives considerably higher values for the bending moments than those given by solution 1 by the same method. At the section of maximum bending moment, an increase of 36% is shown. However, the increase of the maximum total longitudinal slab stress (including slab bending effect) at the same section is only 16%. This is due to the small variation of the average slab stress.

### Example 3. Box Girder Bridge

A box girder bridge continuous over two spans was analyzed by Scordelis (Ref 19) using three different methods for the analysis of such structures. The same box girder is analyzed here to compare the results with those obtained in Ref 19. The three methods described and used in the analysis of Ref 19 are summarized here.

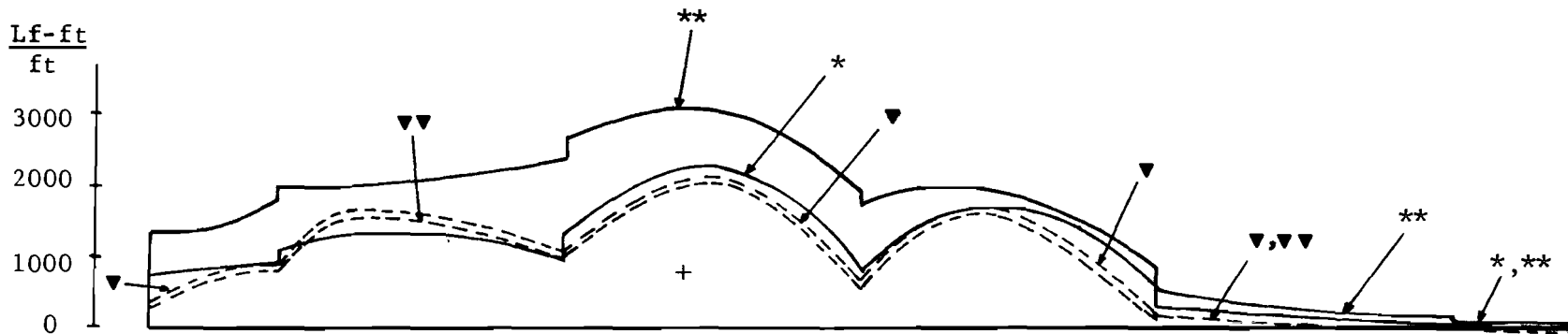
The Folded-Plate Method. This method is based on the elasticity analysis of folded plates and is described in detail in Refs 25 and 26 and summarized in Ref 19. It is a combination of a displacement (stiffness) and a force (flexibility) method and is limited to box girders or folded plates which are simply



- \* The subject method, solution 1.
- \*\* The subject method, solution 2.
- ▼ The discrete-element method, solution 1.
- ▼▼ The discrete-element method, solution 2.

Fig 15. Average slab longitudinal stresses ( $\text{lb/ft}^2$ ) at the central section.





- \* The subject method, solution 1.
- \*\* The subject method, solution 2.
- ▼ The discrete-element method, solution 1.
- ▼▼ The discrete-element method, solution 2.

Fig 16. Longitudinal slab bending moment at the central section.

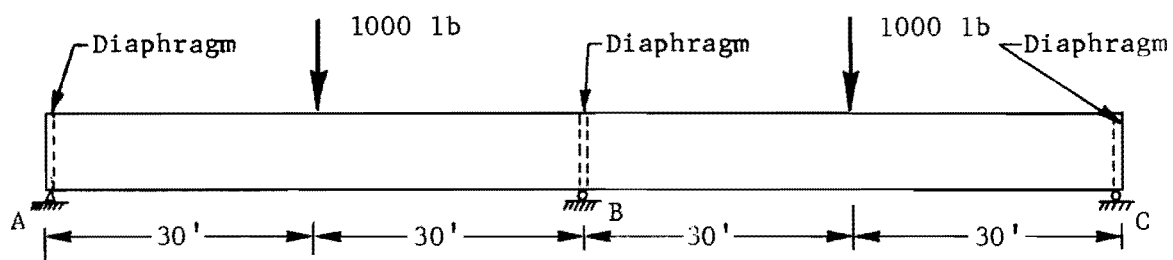
supported at their extreme ends. All the plates are assumed to be rectangular and to have the same length and support conditions. Transverse diaphragms with infinite in-plane stiffness and complete out-of-plane flexibility are assumed at all the supports. Each plate is assumed to be of constant thickness and of the same isotropic, homogeneous material throughout its length.

This method may be considered the best solution for this kind of problem. However, its application is limited to rectangular configurations and special boundary conditions. There are also thickness and material limitations.

The Finite-Segment Method. This method is described in Ref 19 and is based on the ordinary theory of folded plates. In the solution, each plate element is divided longitudinally into a finite number of rectangular segments. Compatibility and equilibrium conditions are satisfied at points along the four edges of each segment. Thus, the accuracy of the results increases as the number of segments increases. This method has all the limitations of the folded-plate method except the condition of simply supported extreme ends.

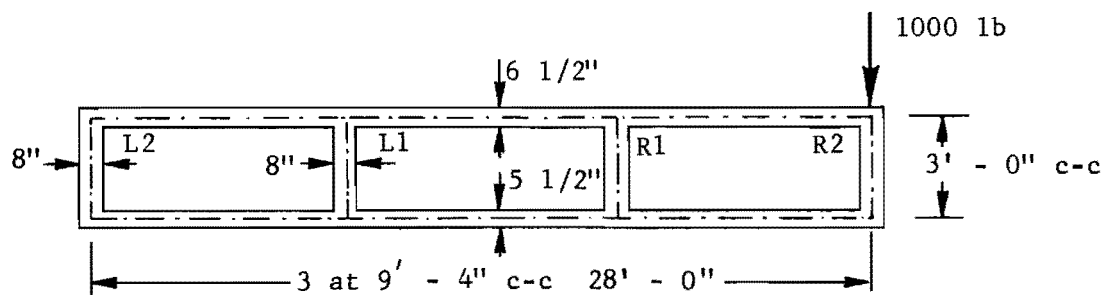
The Finite Element Method. The finite element method used is described in Ref 19. It uses a rectangular element with six degrees of freedom at each corner. These degrees of freedom are three translational and three rotational degrees. The solution has good capabilities and flexibility of support conditions. It is limited, however, to rectangular assemblages with the assumption that, within each element, the thickness is constant and the material is isotropic and homogeneous.

The box girder bridge is shown in Fig 17(a). It is symmetric about the vertical plane through the middle support, and the loads are also symmetric about this plane. Transversely, the bridge consists of three cells, as shown in Fig 17(b), and the loads are concentrated on the outer right girder. In the solutions by Scordelis, each load of 1000 pounds was assumed to be distributed over a length of 1.0 ft in the longitudinal direction. Because of its symmetry, only one-half of the bridge was considered for analysis. Fig 17(c) shows the half which was analyzed and the boundary conditions. Longitudinally, six equal divisions were used, while transversely, each plate was taken as one division. Thus, the total number of elements used is 60. The details of this mesh, designated as mesh 1, are shown in Fig 18. Another finer mesh, designated

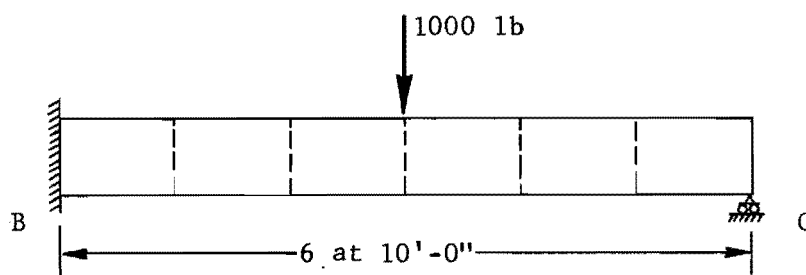


(a) Longitudinal elevation

$$E = 3 \times 10^6 \text{ psi} \quad \nu = 0.15$$



(b) Transverse section



(c) Analyzed half, boundary conditions and longitudinal divisions (mesh 1).

Fig 17. Example 5. Box Girder Bridge.

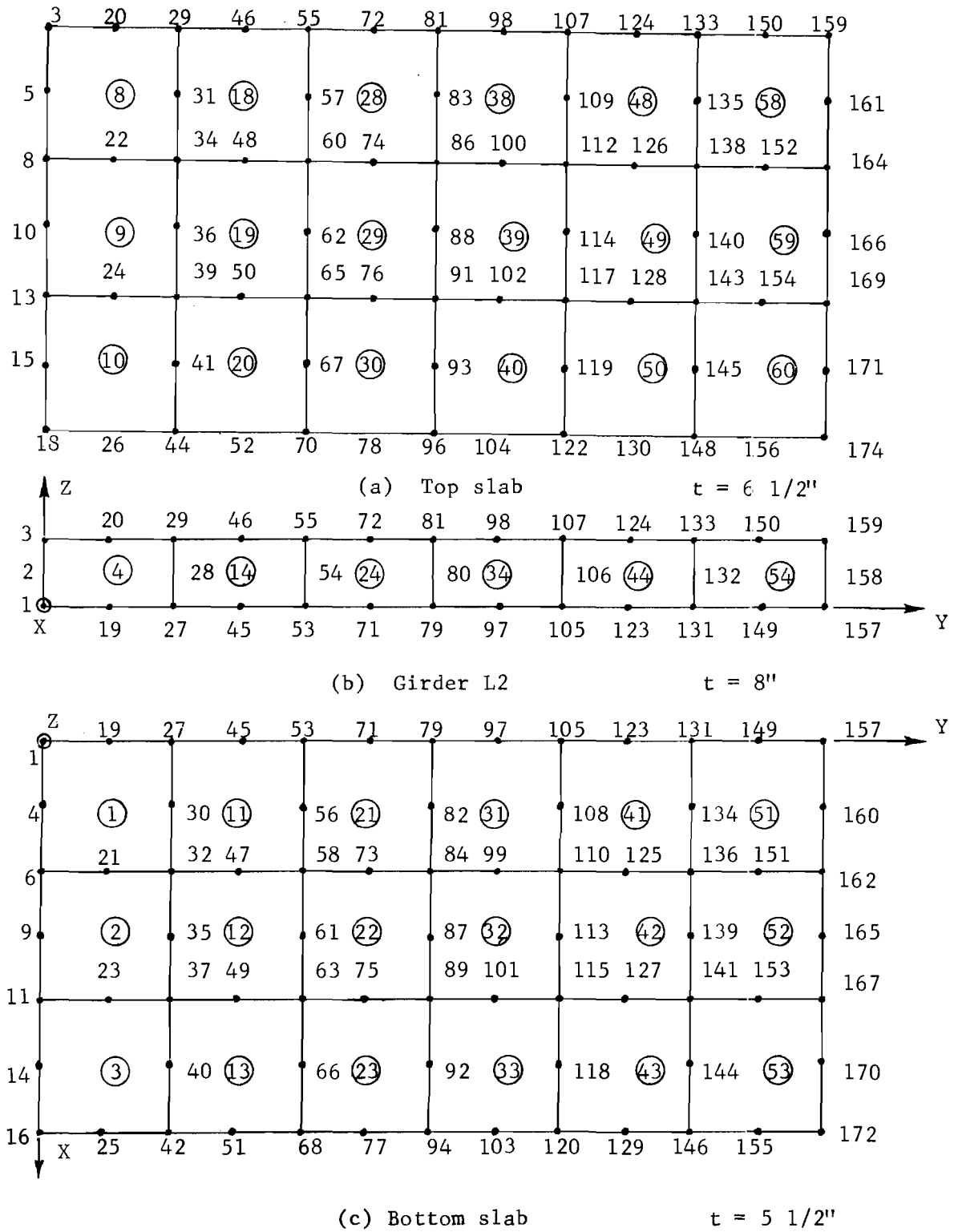


Fig 18. Details of the mesh used (mesh 1).

as mesh 2, was used in the vicinity of the load to study the stresses with more precision. The details of mesh 2 are shown in Fig 19. This local study of stress was conducted by using the displacements determined by mesh 1 on the boundaries of mesh 2 and applying them as boundary conditions for the portion of the bridge shown in Fig 19(a). The displacements of some of the boundary points of mesh 2 were interpolated from the output displacements of mesh 1.

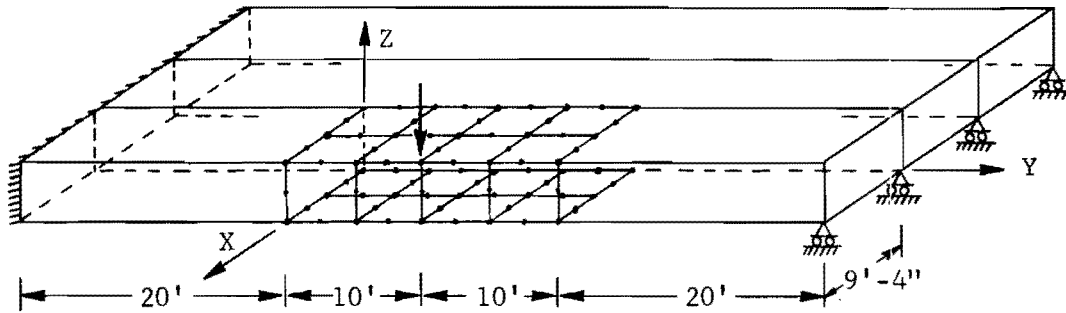
The analysis of Ref 19 used two meshes for the finite element method and two meshes for the finite-segment method. In the finite element method, mesh 1 used ten elements transversely (one on each plate) and 14 divisions longitudinally, thus using a total of 140 elements on the analyzed half of the bridge. Mesh 2 was longitudinally the same as mesh 1 but transversely more refined in the loaded cell, where two elements were used on the outer right girder and three elements on both the top slab and the bottom slab. Thus, the total number of elements of mesh 2 was 210. In the finite-segment method, two meshes were used, transversely similar to the finite element meshes but using 13 divisions longitudinally instead of 14. Therefore, 130 segments and 195 segments were used for mesh 1 and 2, respectively.

In order to represent the load distribution over a 1.0-ft length, the element widths on both sides of the midspan section were taken equal to 0.5 ft in the finite element solutions. In the finite-segment solutions, the width of the central segments was taken equal to 1.0 ft. In the present analysis, this distribution over a 1.0-ft length was ignored, and the load was treated as a concentrated force. The data for both meshes are included in the Appendix, together with a complete output for mesh 1.

### Comparison of Results

Some of the results given by Ref 19 are presented here, together with the corresponding values obtained by the present analysis. The folded-plate method, the best available solution, is taken as the basis for comparison. The results of the other two methods of Ref 19 are listed to show their accuracy relative to that of the present method.

Deflections. The vertical deflections obtained by the present method, mesh 1, along the top of the four girders are plotted in Fig 20. All the



(a) Analyzed portion and coordinate axes.

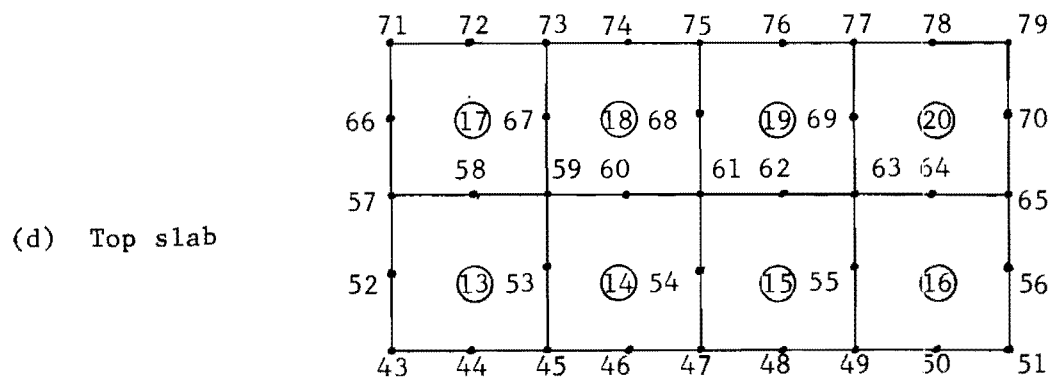
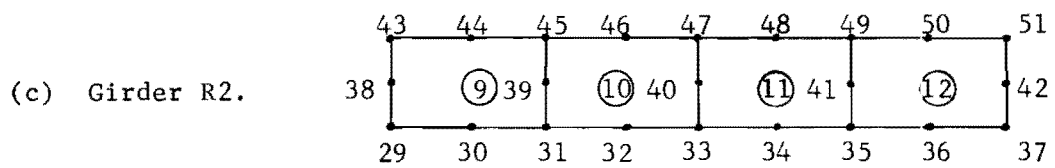
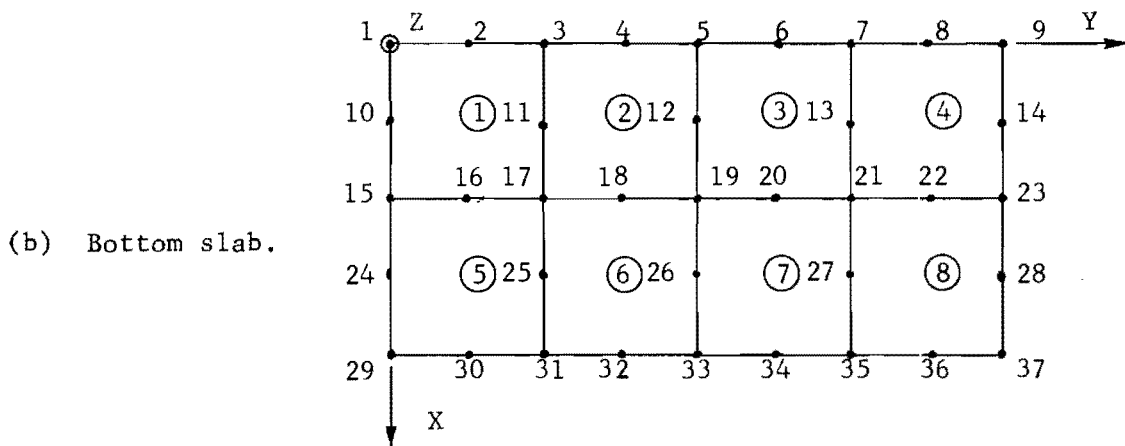
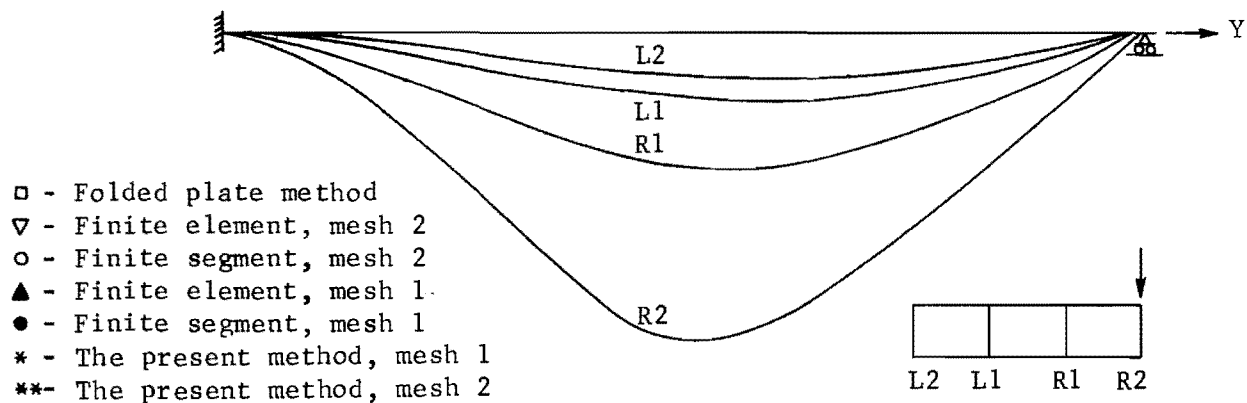


Fig 19. Local mesh 2 for studying the stresses in the vicinity of load.



Girder	Method	Distance y in Feet										
		7.5	12.5	17.5	22.5	27.25	30.0	32.75	37.5	42.5	47.5	52.5
R2	□	0.31	0.69	1.13	1.57	1.93	2.04	2.01	1.79	1.46	1.07	0.65
	▽	0.30	0.66	1.08	1.52	1.85	1.98	2.02	1.77	1.42	1.04	0.63
	○	0.32	0.71	1.17	1.63	1.99	2.16	2.08	1.85	1.50	1.09	0.66
	▲	0.30	0.65	1.07	1.50	1.84	1.95	1.92	1.72	1.39	1.01	0.63
	●	0.30	0.68	1.12	1.56	1.90	2.05	1.99	1.79	1.47	1.08	0.66
	*	0.32	0.68	1.11	1.56	1.89	2.02	1.97	1.78	1.44	1.06	0.65
	**				1.56	1.90	2.05	2.00	1.77			
R1	□	0.14	0.32	0.51	0.69	0.81	0.85	0.87	0.85	0.75	0.58	0.36
	▽	0.13	0.30	0.49	0.65	0.77	0.80	0.82	0.80	0.71	0.55	0.35
	○	0.15	0.32	0.51	0.68	0.80	0.85	0.87	0.84	0.74	0.57	0.36
	▲	0.13	0.28	0.47	0.63	0.75	0.79	0.80	0.78	0.68	0.53	0.34
	●	0.13	0.30	0.48	0.64	0.75	0.79	0.81	0.80	0.71	0.56	0.35
	*	0.14	0.32	0.51	0.69	0.81	0.85	0.87	0.84	0.74	0.58	0.36
L1	□	0.07	0.15	0.24	0.33	0.39	0.41	0.43	0.42	0.38	0.30	0.19
	▽	0.06	0.13	0.21	0.28	0.34	0.37	0.38	0.37	0.34	0.27	0.17
	○	0.06	0.14	0.22	0.30	0.36	0.38	0.39	0.39	0.35	0.28	0.18
	▲	0.06	0.13	0.21	0.28	0.34	0.37	0.38	0.37	0.34	0.27	0.17
	●	0.06	0.13	0.21	0.28	0.34	0.36	0.37	0.37	0.34	0.27	0.17
	*	0.07	0.15	0.24	0.33	0.39	0.41	0.43	0.42	0.38	0.30	0.19
L2	□	0.05	0.11	0.17	0.22	0.26	0.28	0.29	0.29	0.26	0.21	0.14
	▽	0.04	0.09	0.14	0.19	0.22	0.24	0.25	0.26	0.23	0.18	0.12
	○	0.04	0.09	0.14	0.19	0.23	0.24	0.25	0.25	0.23	0.19	0.12
	▲	0.04	0.09	0.14	0.19	0.22	0.24	0.25	0.26	0.23	0.18	0.12
	●	0.04	0.09	0.14	0.18	0.22	0.23	0.24	0.24	0.22	0.18	0.12
	*	0.05	0.11	0.17	0.22	0.26	0.28	0.29	0.29	0.26	0.21	0.14

Fig 20. Vertical deflections ( $\times 10^4$  Ft) along tops of girders.

results are tabulated at the same locations used in Ref 19. It is to be noted that these locations do not match the nodal points of mesh 1 except at mid-span. Thus, the values at locations between the nodal points of mesh 1 are obtained by interpolation. The interpolation process is easy and accurate, since the displacements in the plane of the girders for the element used should have parabolic variation. The deflection shapes of all the girders are similar to those of a beam fixed at one end and hinged at the other. A comparison of the tabulated values reveals that, in general, the present method gives the closest agreement with the folded-plate method. It can be noted that the local mesh gives better agreement with the folded-plate method in the central part of the mesh. As expected, this improvement in the deflections of the local mesh does not exist near the boundaries.

Stresses. The longitudinal distributions of the longitudinal axial stresses at the top of girder R2, at the right edge of the top slab, and at the right edge of the bottom slab are shown in Figs 21, 22, and 23, respectively. The plotted values are those of the present method, and the results of the other three methods are tabulated for comparison. A procedure of averaging and extrapolation similar to that used by Ref 19 was used in plotting these curves. In the present method, this was done simply by drawing a parabola between three points representing stress values obtained from the two elements adjacent to the location in question. The two end values on such a parabola are the stresses at the mid-side nodes of the adjacent elements, and the middle value is the average of the values of the two elements at the common node. After such a parabola was drawn, the values at the locations in question were measured. Such parabolas were extended towards the support sides to extrapolate the values at the supports and near them. The values at the central section are the averages of the extrapolated values from both sides. Finally, the values obtained at the required locations were plotted, and the curves shown were drawn through them. The easiest way to do such interpolation and extrapolation is the graphical procedure which was used here, although better ways may exist to reduce such results. Complicated methods of reduction of results may prove to be of limited advantage.



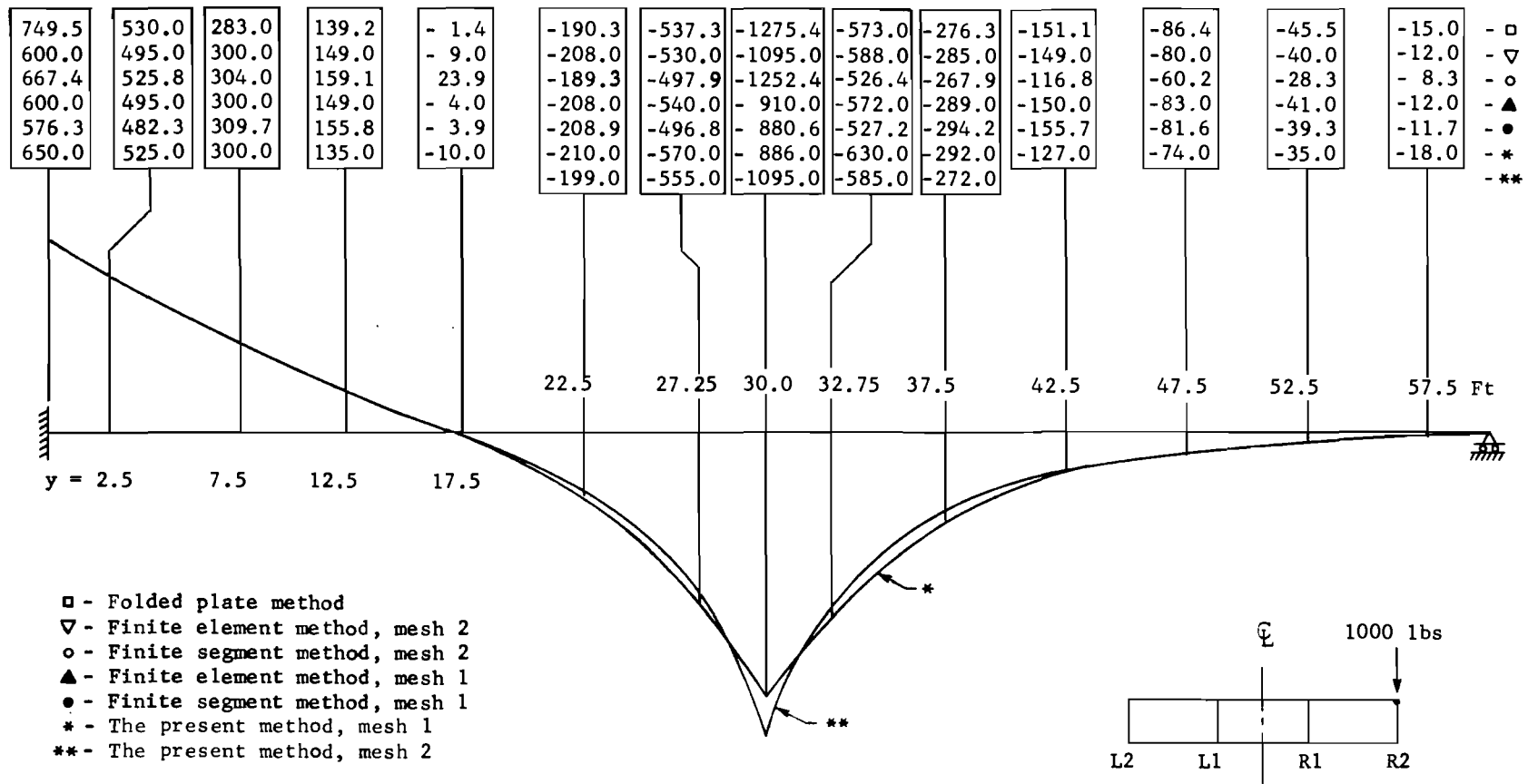


Fig 21. Longitudinal distribution of longitudinal stress  $\sigma_y$  (PSF) in top of girder R2.

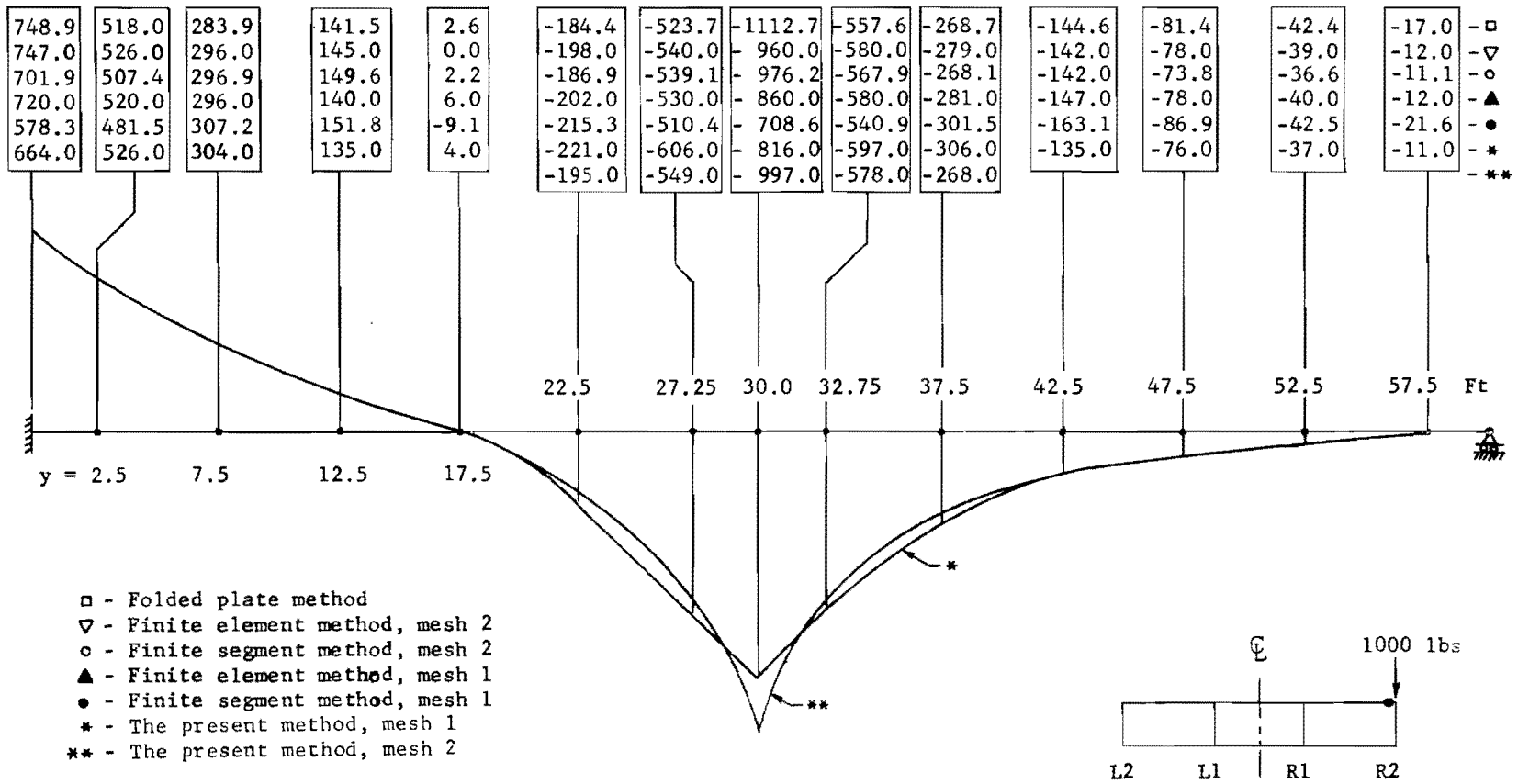


Fig 22. Longitudinal distribution of longitudinal stress  $\sigma_y$  (PSF) in the right edge of the top slab.

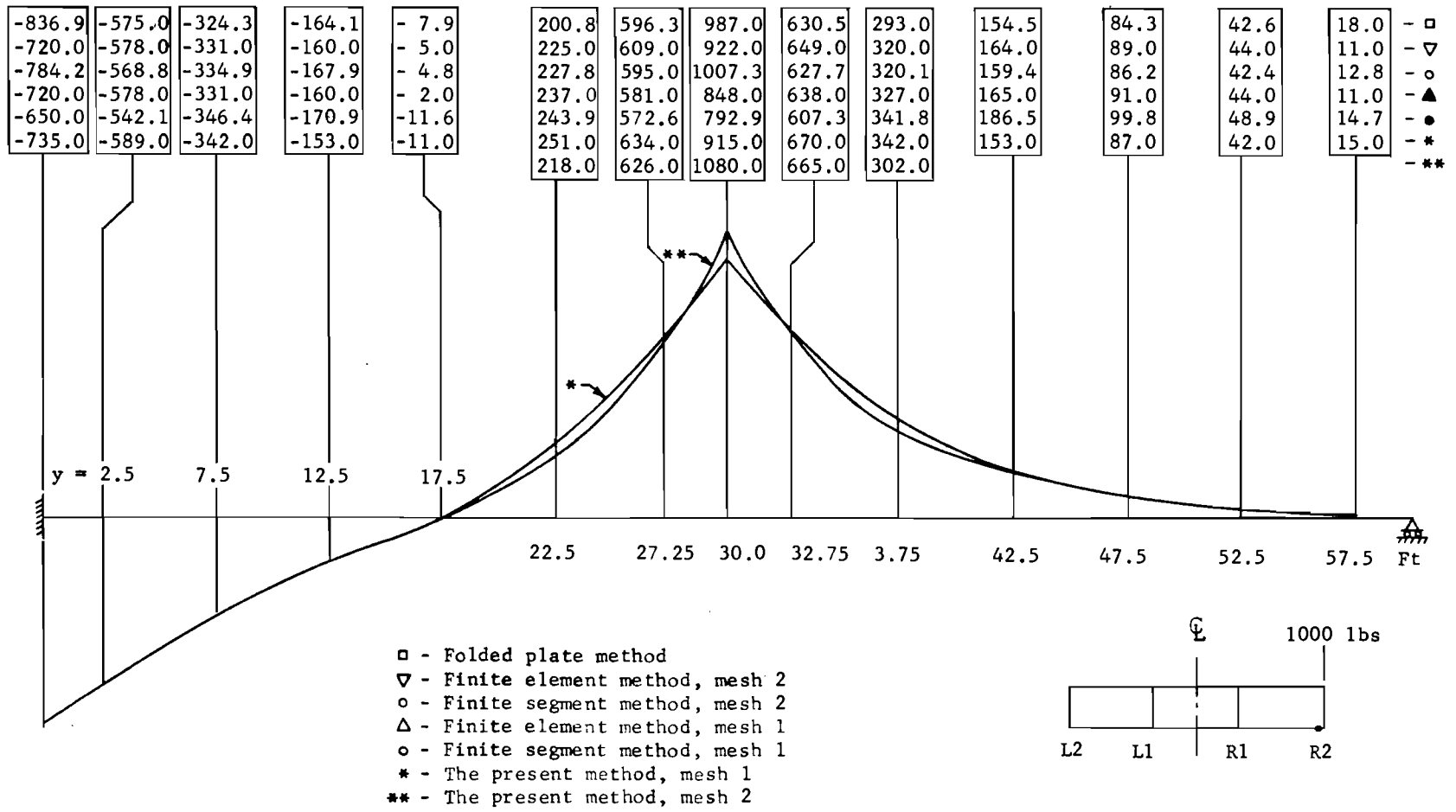


Fig 23. Longitudinal distribution of longitudinal stress  $\sigma_y$  (PSF) in the right edge of the bottom slab.

A comparison of results of stresses reveals that mesh 1 of the present method gives good agreement with the folded-plate method except under the load and at the central support where concentrated forces exist. The local mesh under the load results in a significant improvement in the accuracy of stresses.

Transverse Bending Moments. The transverse bending moment diagrams obtained by the present method at midspan are shown in Figs 24, 25, and 26. The moments were obtained by averaging the values obtained from the adjacent elements at the common points. It should be noted that only the folded-plate method satisfies the condition of moment equilibrium at the joints. Here, the present method gives fair agreement with the folded-plate method. The improvement in accuracy by using a local fine mesh is also notable.

General Remarks. The excellent results for the displacements which were obtained by the present method using a coarse mesh for the severe loading condition demonstrates the superiority in displacement calculations of the quadrilateral element with eight nodes. This is also expected to be the case if the triangular element with six nodes were used. This property provides some flexibility in the solution of large problems where very fine meshes are not desirable. In other words, a coarse mesh can be used to give fairly good values of displacements in the whole structure; then, local study of stresses at the positions of special importance to the designer can be carried out using finer meshes, as demonstrated in this example.

The case of a single concentrated load without distributed loads is a theoretical assumption in the case of most shell-type structures. Practically, distributed loads constitute a major percentage of the total loads on such structures. It is also known that the accuracy of the stresses obtained by the finite element solutions for distributed loads is better than that of the solutions for concentrated loads, especially near the heavily concentrated loads. Therefore, even the fact that the stress results of mesh 1 are only fair is, practically, not a serious problem. The problem may still exist in cases of shells or similar structures supported by a few slender supports. For such cases, local refinement of the mesh at such supports either within the solution of the whole structure or separately is required to obtain accurate stress evaluation.

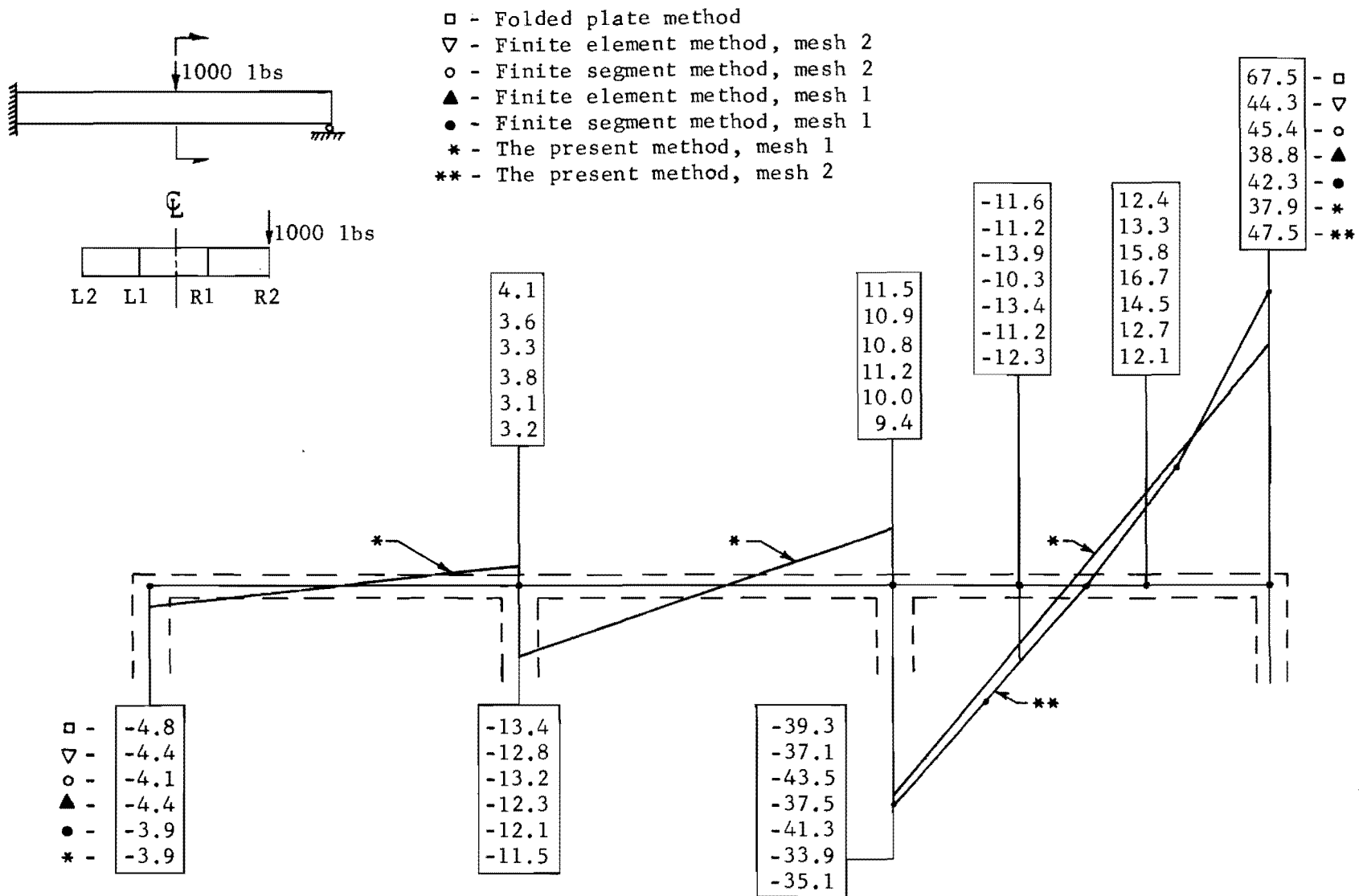


Fig 24. Transverse distribution of transverse bending moment (lb-ft/ft) in top slab at midspan.

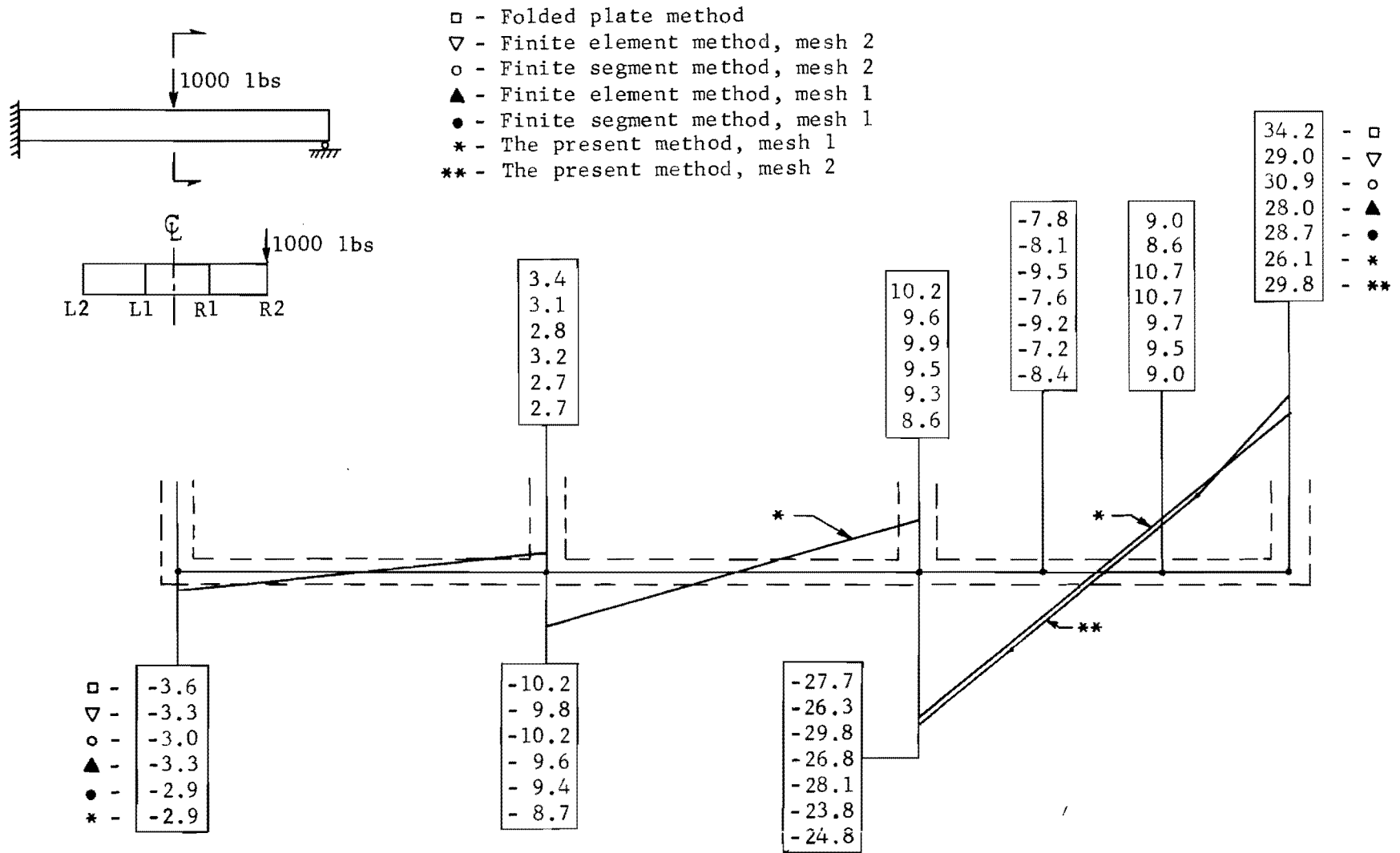


Fig 25. Transverse distribution of transverse bending moment (lb-ft/ft) in bottom slab at midspan.

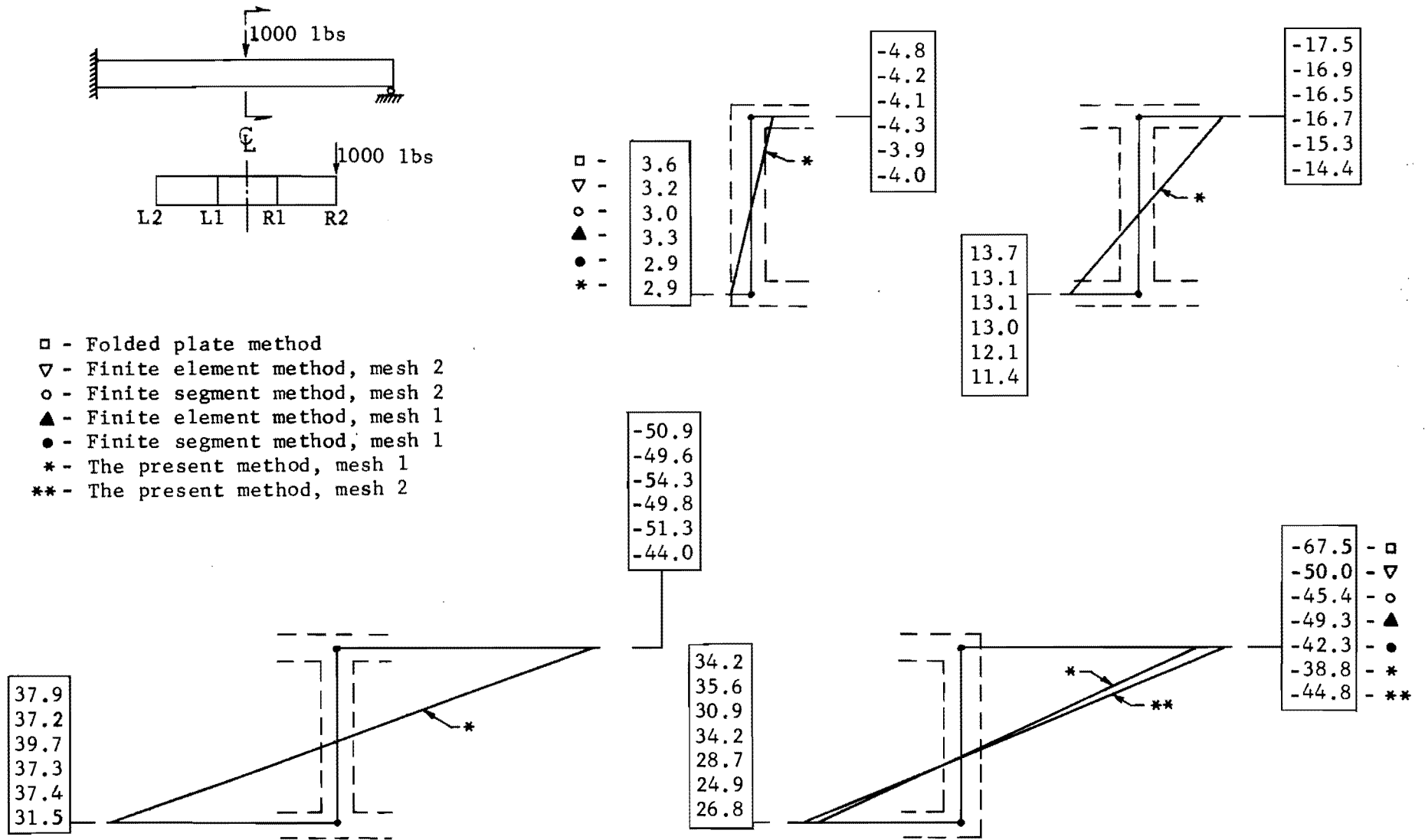


Fig 26. Transverse distribution of transverse bending moment (lb-ft/ft) in girders at midspan.

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## CHAPTER 4. SUMMARY AND CONCLUSIONS

The present method provides the generalities and capabilities of the finite element for the analysis of bridge decks. In the analysis of such structures by the finite element method, there are two approaches. The first approach uses coarse meshes composed of elements with refined stiffness evaluation. This approach has the advantage of reducing the amount of input data and consequently, the effort of preparing and checking such data. The second approach uses fine meshes composed of elements with relatively less refined stiffness evaluation and has the advantage of flexibility in geometric idealization which enables the representation of complicated geometry without unreasonable increase in the solution time. If such complicated geometries are idealized with refined elements, the increase in solution time counteracts the advantage gained by using such refined elements. Both approaches are available in the present method. The refined elements are the quadrilateral and the triangular elements with mid-side nodes while the less refined elements are the quadrilateral and the triangular elements without mid-side nodes. Therefore, for economical use of the program, the refined elements can be used with coarse meshes to idealize bridge decks with simple geometries, such as box girder bridges, slab type bridges, and beam-slab type bridges, while the other elements may be used with fine meshes to idealize complicated geometries, such as bridges with single or double curvature. Combinations of the refined and the less refined elements may also be used, as shown in Example 2 of Chapter 3.

The refined elements have superior membrane stiffness properties. This property makes them most suitable for representing parts of bridge decks such as the girders which mainly support in-plane loadings.

The computer program provides various capabilities for treating load cases and various output options. The user has the option either to stop the solution after computing nodal point displacements or to let it continue and compute the element forces (stresses). This is helpful in cases where a coarse mesh that gives satisfactory accuracy in displacement computation would

not have the desirable accuracy in stress computation. In such cases, use of this option can avoid the time required for computing the stresses of the coarse mesh, and a local, more precise study of the stresses at particular locations may be made as illustrated by Example 3 in Chapter 3. However, it may be possible to have the local mesh refinement within the solution of the whole structure without major increase in the total number of the mesh nodal points.

The present method of analysis considerably reduces and simplifies the required input data by expressing most of such data in the global coordinate system. With the simplified data input, the data preparation time and the chances of error are minimized. However, as a result of the global representation of the data, it may be impossible to represent the boundary conditions of certain complicated cases of support conditions. Although realistic representations of most boundary cases in practice can be made either directly or indirectly, this is the most serious limitation of the solution.

The present use of the six-degree-of-freedom nodal point system is essential in general solutions. The five-degree-of-freedom analysis (Ref 11) may give good results, especially in cases of smoothly curved shells; but in general, it is an insufficient representation of the shell problem and must be used with full understanding of its limitations. This is particularly important at plate intersections, such as the slab-girder junctions of bridges, where the constraint of one rotation component in the five-degree analysis may cause considerable error. The problem arising from the use of five-degree-of-freedom elements in a six-degree-of-freedom system was solved by omitting the dependent equations. The new procedure described for such omission is a general one and can be used in similar problems in structural analysis.

The present method gives good results when compared to existing methods of analysis of bridge decks. In addition, it has more general features than any of the other methods discussed.

Scordelis' finite element method (Ref 19) offers a good solution for bridge decks with rectangular configuration. The accuracy of this method of analysis may be slightly better than the analysis using the less-refined quadrilateral element of the present method. The two other methods of Ref 19 are limited to box girder type bridges.

The discrete-element method of analysis (Refs 1, 2, and 3) is a quick and accurate solution for slab-type bridge decks. For beam and slab-type bridges, the method still gives a good approximation if proper stiffness values are assigned to the beams. In cases of decks with narrowly spaced main beams and enough diaphragms to prevent excessive beam rotations, the common practice for selecting the composite section stiffness may be an accurate representation. In cases of widely spaced beams where considerable beam rotations exist, however, such composite section stiffness may not be a proper representation. One of the most serious limitations of the discrete-element analysis is the requirement of using a regular mesh which usually requires changes in the deck dimensions and in locations of supports and diaphragms.

In conclusion, it can be safely stated that the present method of analysis of bridge decks as shell-type structures offers a relatively good general solution with considerable flexibility. It can be used successfully to analyze a wide variety of bridges on rigid or elastic supports.

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APPENDIX  
SELECTED COMPUTER OUTPUT

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PROGRAM SHELL 6 - MASTER DECK - ABDELRAOUF, MATLOCK REVISION DATE 6 MAY 1971  
 SHELL 6 REPORT CODED BY ABDELRAOUF APRIL 16, 1971  
 EXAMPLE PROBLEMS

I---

PT	PT	*	X	Y	Z	X	Y	Z
1	3	2	1	1	1	0	-0.	-0.
2	-0	-0	1	1	1	1	-0.	-0.
1	3	1	2	1	1	1	-0.	-0.

PROB  
 1- A CANTILEVER BEAM, MESH (1)

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS 5  
 NUM OF POINTS 28  
 NUM OF LOAD CASES 1  
 ELEMENT FORCES REQUIRED ( 1 = YES ) 1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 1

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	3.000E+03	2.500E-01	3.000E+03*	2.500E-01*	1.200E+03*

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 2

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	26	5	0.	0.	0.	0.	1.000E+01	0.
3	28	5	0.	0.	2.000E+00	0.	1.000E+01	2.000E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 2

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES							
1	5	1	1	0.	5.000E-01	1	6	8	3	4	7	5	2
						21	26	28	23	24	27	25	22

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 3

FROM PT	THRU PT	INCR	CASE	COND. **	BOUNDARY VALUES
---------	---------	------	------	----------	-----------------

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
 CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
 CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
 CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 2

ELEMENT LOADS TO BE ADDED ( 1 = YES ) -0

FROM PT	THRU PT	INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
			X	Y	Z	X	Y	Z
26	28	1	4.680E+00	0.	7.275E+01	0.	0.	0.
27	27	-0	4.680E+00	0.	7.275E+01	0.	0.	0.

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 SHELL 6 REPORT CODED BY ABDELRAOUF APRIL 16, 1971  
 EXAMPLE PROBLEMS

PROB 1- B CANTILEVER BEAM, MESH (2)

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS	20
NUM OF POINTS	33
NUM OF LOAD CASES	1
ELEMENT FORCES REQUIRED ( 1 = YES )	1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 1

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	3.000E+03	2.500E-01	3.000E+03*	2.500E-01*	1.200E+03*

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 3

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	31	3	0.	0.	0.	1.000E+01	0.	0.
2	32	3	0.	0.	1.000E+00	1.000E+01	1.000E+00	0.
3	33	3	0.	0.	2.000E+00	1.000E+01	2.000E+00	0.

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 4

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES
1	19	2	1	0.	5.000E-01	1 4 5 2
2	20	2	1	0.	5.000E-01	28 31 32 29
						2 5 6 3
						29 32 33 30

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 3

FROM PT	THRU PT	INCR	CASE *	COND. **			BOUNDARY VALUES		
				X	Y	Z	X	Y	Z
1	3	2	1	1	1	0	-0.	-0.	-0.
2	-0	-0	1	1	1	1	-0.	-0.	-0.
1	3	1	2	1	1	1	-0.	-0.	-0.

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
 CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
 CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
 CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 2

ELEMENT LOADS TO BE ADDED ( 1 = YES ) -0

FROM PT	THRU PT	INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES			
			X	Y	Z	X	Y	Z	
31	33	1	4.680E+00	0.	0.	7.275E+01	0.	0.	0.
32	32	-0	4.680E+00	0.	0.	7.275E+01	0.	0.	0.

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 SHELL 6 REPORT CODED BY ABDELRAOUF APRIL 16, 1971  
 EXAMPLE PROBLEMS

I---

PROB  
 1- C CANTILEVER BEAM, MESH (3)

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS 10  
 NUM OF POINTS 33  
 NUM OF LOAD CASES 1  
 ELEMENT FORCES REQUIRED ( 1 = YES ) 1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 1

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	3.000E+03	2.500E-01	3.000E+03*	2.500E-01*	1.200E+03*

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 2

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	31	6	0.	0.	0.	0.	1.000E+01	0.
3	33	6	0.	0.	2.000E+00	0.	1.000E+01	2.000E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 4

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES					
1	9	2	1	0.	5.000E-01	1	7	3	4	5	2
2	10	2	1	0.	5.000E-01	25	31	27	28	29	26
						7	9	3	8	6	5
						31	33	27	32	30	29

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 3

FROM PT	THRU PT	INCR	CASE *	COND. **			BOUNDARY VALUES		
				X	Y	Z	X	Y	Z
1	3	2	1	1	1	0	-0.	-0.	-0.
2	-0	-0	1	1	1	1	-0.	-0.	-0.
1	3	1	2	1	1	1	-0.	-0.	-0.

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
 CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
 CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
 CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 2

ELEMENT LOADS TO BE ADDED ( 1 = YES ) -0

FROM PT	THRU PT	INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
			X	Y	Z	X	Y	Z
31	33	1	4.680E+00	0.	7.275E+01	0.	0.	0.
32	32	-0	4.680E+00	0.	7.275E+01	0.	0.	0.

PROB 1- D CANTILEVER BEAM, MESH (4)

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS	40
NUM OF POINTS	33
NUM OF LOAD CASES	1
ELEMENT FORCES REQUIRED ( 1 = YES )	1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 1

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	3.000E+03	2.500E-01	3.000E+03*	2.500E-01*	1.200E+03*

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 3

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	31	3	0.	0.	0.	0.	1.000E+01	0.
2	32	3	0.	0.	1.000E+00	0.	1.000E+01	1.000E+00
3	33	3	0.	0.	2.000E+00	0.	1.000E+01	2.000E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 8

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES
1	4	1	0.	5.000E-01	1 4 2	
2	37	4	1 0.	5.000E-01	28 31 29	
	38	4	1 0.	5.000E-01	4 5 2	
3	39	4	1 0.	5.000E-01	31 32 29	
				5.000E-01	2 5 6	
4	40	4	1 0.	5.000E-01	29 32 33	
				5.000E-01	3 2 6	
				5.000E-01	30 29 33	

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 3

FROM PT	THRU PT	INCR	CASE *	COND. **	BOUNDARY VALUES		
				X Y Z	X	Y	Z
1	3	2	1	1 1 0	-0.	-0.	-0.
2	-0	-0	1	1 1 1	-0.	-0.	-0.
1	3	1	2	1 1 1	-0.	-0.	-0.

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
 CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
 CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
 CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 2

ELEMENT LOADS TO BE ADDED ( 1 = YES ) -0

FROM PT	THRU PT	INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
			X	Y	Z	X	Y	Z
31	33	1	4.680E+00	0.	7.275E+01	0.	0.	0.
32	32	-0	4.680E+00	0.	7.275E+01	0.	0.	0.

PROGRAM SHELL 6 - MASTER DECK - ABDELRAOUF, MATLOCK REVISION DATE 6 MAY 1971  
 SHELL 6 REPORT CODED BY ABDELRAOUF MAY 28, 1971  
 3 - SPAN CONTINUOUS HIGHWAY BRIDGE, M520 TRUCK LOADINGS (LB-FT UNITS)

PROB  
 301 LOADING SYMMETRICAL ABOUT THE TWO AXES OF SYMMETRY OF THE BRIDGE

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS	#2
NUM OF POINTS	241
NUM OF LOAD CASES	1
ELEMENT FORCES REQUIRED (1 = YES)	1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 3

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	4.320E+09	3.000E-01	4.320E+09	3.000E-01	1.662E+09
2	4.320E+08	1.500E-01	4.320E+08	1.500E-01	1.878E+08
3	4.320E+08	1.500E-01	6.640E+07	3.000E-02	6.857E+07

(\* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 41

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	-C	-0	0.	0.	0.030E+00	-0.	-0.	-0.
23	63	20	0.	8.125E+00	0.030E+00	0.	2.438E+01	3.030E+00
85	125	20	0.	3.171E+01	0.030E+00	0.	4.437E+01	3.030E+00
147	187	20	0.	5.237E+01	0.030E+00	0.	6.437E+01	3.030E+00
209	229	20	0.	7.387E+01	0.030E+00	0.	7.938E+01	3.030E+00
3	13	5	3.125E+00	0.	0.030E+00	1.762E+01	0.	3.030E+00
5	15	5	3.125E+00	0.	0.	1.762E+01	0.	0.
25	33	4	3.125E+00	8.125E+00	0.030E+00	1.762E+01	8.125E+00	3.030E+00
27	35	4	3.125E+00	8.125E+00	0.	1.762E+01	8.125E+00	0.
45	53	4	3.125E+00	1.625E+01	0.030E+00	1.762E+01	1.625E+01	3.030E+00
47	55	4	3.125E+00	1.625E+01	0.	1.762E+01	1.625E+01	0.
65	75	5	3.125E+00	2.438E+01	0.030E+00	1.762E+01	2.438E+01	3.030E+00
67	77	5	3.125E+00	2.438E+01	0.	1.762E+01	2.438E+01	0.
87	95	4	3.125E+00	3.171E+01	0.030E+00	1.762E+01	3.171E+01	-3.030E+00
89	97	4	3.125E+00	3.171E+01	0.	1.762E+01	3.171E+01	0.
107	115	4	3.125E+00	3.904E+01	0.030E+00	1.762E+01	3.904E+01	3.030E+00
109	117	4	3.125E+00	3.904E+01	0.	1.762E+01	3.904E+01	0.
127	137	5	3.125E+00	4.637E+01	0.030E+00	1.762E+01	4.637E+01	3.030E+00
129	139	5	3.125E+00	4.637E+01	0.	1.762E+01	4.637E+01	0.
149	157	4	3.125E+00	5.237E+01	0.030E+00	1.762E+01	5.237E+01	3.030E+00
151	159	4	3.125E+00	5.237E+01	0.	1.762E+01	5.237E+01	0.
169	177	4	3.125E+00	6.037E+01	0.030E+00	1.762E+01	6.037E+01	3.030E+00
171	179	4	3.125E+00	6.037E+01	0.	1.762E+01	6.037E+01	0.

189	199	5	3.125E+00	6.837E+01	0.030E+00	1.762E+01	6.837E+01	3.030E+00
191	201	5	3.125E+00	6.837E+01	0.	1.762E+01	6.837E+01	0.
211	219	4	3.125E+00	7.387E+01	0.030E+00	1.762E+01	7.387E+01	3.030E+00
213	221	4	3.125E+00	7.387E+01	0.	1.762E+01	7.387E+01	0.
231	239	4	3.125E+00	7.938E+01	0.030E+00	1.762E+01	7.938E+01	3.030E+00
233	241	4	3.125E+00	7.938E+01	0.	1.762E+01	7.938E+01	0.
6	11	5	6.750E+00	0.	0.030E+00	1.400E+01	0.	3.030E+00
7	12	5	6.750E+00	0.	1.515E+00	1.400E+01	0.	1.515E+00
4	14	5	3.125E+00	0.	1.515E+00	1.762E+01	0.	1.515E+00
68	73	5	6.750E+00	2.438E+01	0.030E+00	1.400E+01	2.438E+01	3.030E+00
69	74	5	6.750E+00	2.438E+01	1.515E+00	1.400E+01	2.438E+01	1.515E+00
66	76	5	3.125E+00	2.438E+01	1.515E+00	1.762E+01	2.438E+01	1.515E+00
130	135	5	6.750E+00	4.637E+01	0.030E+00	1.400E+01	4.637E+01	3.030E+00
131	136	5	6.750E+00	4.637E+01	1.515E+00	1.400E+01	4.637E+01	1.515E+00
128	138	5	3.125E+00	4.637E+01	1.515E+00	1.762E+01	4.637E+01	1.515E+00
192	197	5	6.750E+00	6.837E+01	0.030E+00	1.400E+01	6.837E+01	3.030E+00
193	198	5	6.750E+00	6.837E+01	1.515E+00	1.400E+01	6.837E+01	1.515E+00
190	200	5	3.125E+00	6.837E+01	1.515E+00	1.762E+01	6.837E+01	1.515E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 82

FROM ELMT	THRU ELMT	INCH	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES			
1	-0	1	-0.		4.200E-02	4	7	6	3
2	-0	1	-0.		4.200E-02	7	9	8	6
3	-0	1	-0.		4.200E-02	9	12	11	8
4	-0	1	-0.		4.200E-02	12	14	13	11
23	-0	1	-0.		5.210E-02	66	69	68	65
24	-0	1	-0.		5.210E-02	69	71	70	68
25	-0	1	-0.		5.210E-02	71	74	73	70
26	-0	1	-0.		5.210E-02	74	76	75	73
45	-0	1	-0.		5.210E-02	128	131	130	127
46	-0	1	-0.		5.210E-02	131	133	132	130
47	-0	1	-0.		5.210E-02	133	136	135	132
48	-0	1	-0.		5.210E-02	136	138	137	135
67	-0	1	-0.		5.210E-02	190	193	192	189
68	-0	1	-0.		5.210E-02	193	195	194	192
69	-0	1	-0.		5.210E-02	195	198	197	194
70	-0	1	-0.		5.210E-02	198	200	199	197
6	-0	1	-0.		1.246E-01	5	27	25	3
12	-0	1	-0.		1.246E-01	27	47	45	25
18	-0	1	-0.		1.246E-01	47	67	65	45
28	-0	1	-0.		1.246E-01	47	89	87	65
34	-0	1	-0.		1.246E-01	49	109	107	87
40	-0	1	-0.		1.246E-01	109	129	127	107
50	-0	1	-0.		1.246E-01	129	151	149	127
56	-0	1	-0.		1.246E-01	151	171	169	149
62	-0	1	-0.		1.246E-01	171	191	189	169
72	-0	1	-0.		1.246E-01	191	213	211	189
78	-0	1	-0.		1.246E-01	213	233	231	211
8	-0	1	-0.		1.246E-01	10	31	29	8
14	-0	1	-0.		1.246E-01	31	51	49	29
20	-0	1	-0.		1.246E-01	51	72	70	49
30	-0	1	-0.		1.246E-01	72	93	91	70
34	-0	1	-0.		1.246E-01	93	113	111	91
42	-0	1	-0.		1.246E-01	113	134	132	111
52	-0	1	-0.		1.246E-01	134	155	153	132
58	-0	1	-0.		1.246E-01	155	175	173	153

64	-0	1	-0.	1.226E-01	175	196	194	173	184	195	183	174
74	-0	1	-0.	1.226E-01	196	217	215	194	206	216	205	195
80	-0	1	-0.	1.226E-01	217	237	235	215	226	236	225	216
10	-0	1	-0.	6.130E-02	15	35	33	13	22	34	21	14
16	-0	1	-0.	6.130E-02	35	55	53	33	42	54	41	34
22	-0	1	-0.	6.130E-02	55	77	75	53	62	76	61	54
32	-0	1	-0.	6.130E-02	77	97	95	75	84	96	83	76
38	-0	1	-0.	6.130E-02	97	117	115	95	104	116	103	96
44	-0	1	-0.	6.130E-02	117	139	137	115	124	138	123	116
54	-0	1	-0.	6.130E-02	139	159	157	137	146	158	145	138
60	-0	1	-0.	6.130E-02	159	179	177	157	166	178	165	158
66	-0	1	-0.	6.130E-02	179	201	199	177	186	200	185	178
76	-0	1	-0.	6.130E-02	201	221	219	199	208	220	207	200
82	-0	1	-0.	6.130E-02	221	241	239	219	228	240	227	220
5	-0	3	9.000E+01	5.833E-01	3	25	23	1	17	24	16	2
7	-0	3	9.000E+01	5.833E-01	8	29	25	3	19	28	17	6
9	-0	3	9.000E+01	5.833E-01	13	33	29	8	21	32	19	11
11	-0	3	9.000E+01	5.833E-01	25	45	43	23	37	44	36	24
13	-0	3	9.000E+01	5.833E-01	29	49	45	25	39	48	37	28
15	-0	3	9.000E+01	5.833E-01	33	53	49	29	41	52	39	32
17	-0	3	9.000E+01	5.833E-01	45	65	63	43	57	64	56	44
19	-0	3	9.000E+01	5.833E-01	49	70	65	45	59	68	57	48
21	-0	3	9.000E+01	5.833E-01	53	75	70	49	61	73	59	52
27	-0	3	9.000E+01	5.833E-01	65	87	85	63	79	86	78	64
29	-0	3	9.000E+01	5.833E-01	70	91	87	65	81	90	79	68
31	-0	3	9.000E+01	5.833E-01	75	95	91	70	83	94	81	73
33	-0	3	9.000E+01	5.833E-01	87	107	105	85	99	106	98	86
35	-0	3	9.000E+01	5.833E-01	91	111	107	87	101	110	99	90
37	-0	3	9.000E+01	5.833E-01	95	115	111	91	103	114	101	94
39	-0	3	9.000E+01	5.833E-01	107	127	125	105	119	126	118	106
41	-0	3	9.000E+01	5.833E-01	111	132	127	107	121	130	119	110
43	-0	3	9.000E+01	5.833E-01	115	137	132	111	123	135	121	114
49	-0	3	9.000E+01	5.833E-01	127	149	147	125	141	148	140	126
51	-0	3	9.000E+01	5.833E-01	132	153	149	127	143	152	141	130
53	-0	3	9.000E+01	5.833E-01	137	157	153	132	145	156	143	135
55	-0	3	9.000E+01	5.833E-01	149	169	167	147	161	168	160	148
57	-0	3	9.000E+01	5.833E-01	153	173	169	149	163	172	161	152
59	-0	3	9.000E+01	5.833E-01	157	177	173	153	165	176	163	156
61	-0	3	9.000E+01	5.833E-01	169	189	187	167	181	188	180	168
63	-0	3	9.000E+01	5.833E-01	173	194	189	169	183	192	181	172
65	-0	3	9.000E+01	5.833E-01	177	199	194	173	185	197	183	176
71	-0	2	-0.	5.833E-01	189	211	209	187	203	210	202	188
73	-0	2	-0.	5.833E-01	194	215	211	189	205	214	203	192
75	-0	2	-0.	5.833E-01	199	219	215	194	207	218	205	197
77	-0	2	-0.	5.833E-01	211	231	229	209	223	230	222	210
79	-0	2	-0.	5.833E-01	215	235	231	211	225	234	223	214
81	-0	2	-0.	5.833E-01	219	239	235	215	227	238	225	218

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 51

FROM	THRU	INCR	CASE	COND.	**	BOUNDARY	VALUES
PT	PT			X	Y	Z	
3	5	1	1	0	0	1	-0.
							-0.
							-0.

8	10	1	1	0	0	1	-0.	-0.	-0.
13	15	1	1	1	0	1	-0.	-0.	-0.
21	22	1	1	1	0	0	-0.	-0.	-0.
33	35	1	1	1	0	0	-0.	-0.	-0.
41	42	1	1	1	0	0	-0.	-0.	-0.
53	55	1	1	1	0	0	-0.	-0.	-0.
61	62	1	1	1	0	0	-0.	-0.	-0.
75	77	1	1	1	0	0	-0.	-0.	-0.
83	84	1	1	1	0	0	-0.	-0.	-0.
95	97	1	1	1	0	0	-0.	-0.	-0.
103	104	1	1	1	0	0	-0.	-0.	-0.
115	117	1	1	1	0	0	-0.	-0.	-0.
123	124	1	1	1	0	0	-0.	-0.	-0.
137	139	1	1	1	0	0	-0.	-0.	-0.
145	146	1	1	1	0	1	-0.	-0.	-0.
141	144	1	1	0	0	1	-0.	-0.	-0.
157	159	1	1	1	0	0	-0.	-0.	-0.
165	166	1	1	1	0	0	-0.	-0.	-0.
177	179	1	1	1	0	0	-0.	-0.	-0.
185	186	1	1	1	0	0	-0.	-0.	-0.
199	201	1	1	1	0	0	-0.	-0.	-0.
207	208	1	1	1	0	0	-0.	-0.	-0.
219	221	1	1	1	0	0	-0.	-0.	-0.
227	228	1	1	1	0	0	-0.	-0.	-0.
229	238	1	1	0	1	0	-0.	-0.	-0.
239	241	1	1	1	1	0	-0.	-0.	-0.
13	15	1	2	0	1	1	-0.	-0.	-0.
21	22	1	2	0	1	1	-0.	-0.	-0.
33	35	1	2	0	1	1	-0.	-0.	-0.
41	42	1	2	0	1	1	-0.	-0.	-0.
53	55	1	2	0	1	1	-0.	-0.	-0.
61	62	1	2	0	1	1	-0.	-0.	-0.
75	77	1	2	0	1	1	-0.	-0.	-0.
83	84	1	2	0	1	1	-0.	-0.	-0.
95	97	1	2	0	1	1	-0.	-0.	-0.
103	104	1	2	0	1	1	-0.	-0.	-0.
115	117	1	2	0	1	1	-0.	-0.	-0.
123	124	1	2	0	1	1	-0.	-0.	-0.
137	139	1	2	0	1	1	-0.	-0.	-0.
145	146	1	2	0	1	1	-0.	-0.	-0.
141	144	1	2	0	1	1	-0.	-0.	-0.
157	159	1	2	0	1	1	-0.	-0.	-0.
165	166	1	2	0	1	1	-0.	-0.	-0.
177	179	1	2	0	1	1	-0.	-0.	-0.
185	186	1	2	0	1	1	-0.	-0.	-0.
199	201	1	2	0	1	1	-0.	-0.	-0.
207	208	1	2	0	1	1	-0.	-0.	-0.
219	221	1	2	0	1	1	-0.	-0.	-0.
227	228	1	2	0	1	1	-0.	-0.	-0.
229	238	1	2	1	0	1	-0.	-0.	-0.
239	241	1	2	1	1	1	-0.	-0.	-0.

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS



CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 11

ELEMENT LOADS TO BE ADDED ( 1 = YES ) 0

FROM THRU		INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
PT	PT		X	Y	Z	X	Y	Z
181	-0	-0	-0.	-0.	-4.270E+03	-0.	-0.	-0.
183	-0	-0	-0.	-0.	-9.280E+03	-0.	-0.	-0.
185	-0	-0	-0.	-0.	-5.380E+03	-0.	-0.	-0.
189	-0	-0	-0.	-0.	-1.420E+03	-0.	-0.	-0.
194	-0	-0	-0.	-0.	-3.070E+03	-0.	-0.	-0.
199	-0	-0	-0.	-0.	-1.860E+03	-0.	-0.	-0.
211	-0	-0	-0.	-0.	-3.680E+03	-0.	-0.	-0.
214	-0	-0	-0.	-0.	-1.740E+03	-0.	-0.	-0.
235	-0	-0	-0.	-0.	-4.740E+03	-0.	-0.	-0.
238	-0	-0	-0.	-0.	-8.400E+03	-0.	-0.	-0.
239	-0	-0	-0.	-0.	-1.760E+03	-0.	-0.	-0.

PROGRAM SHELL 6 - MASTER DECK - ABDELRAOUF, MATLOCK REVISION DATE 6 MAY 1971  
 SHELL 6 REPORT CONF'D BY ABDELRAOUF MAY 28, 1971  
 3 - SPAN CONTINUOUS HIGHWAY BRIDGE, HS20 TRUCK LOADINGS (LR-FT UNITS)

PROB 302 SYMMETRY ABOUT THE LONGITUDINAL AXIS, ANTISYMMETRY TRANSVERSALLY

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS	82
NUM OF POINTS	241
NUM OF LOAD CASES	1
ELEMENT FORCES REQUIRED ( 1 = YES )	1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 3

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EK	VXY	EY	VYX	
1	4.320E+09	3.000E-01	4.320E+09	3.000E-01	1.662E+09
2	4.320E+08	1.500E-01	4.320E+08	1.500E-01	1.878E+09
3	4.320E+08	1.500E-01	8.640E+07	3.000E-02	8.857E+07

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 41

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	-0	-0	0.	0.	1.030E+00	-0.	-0.	-0.
23	63	20	0.	8.125E+00	1.030E+00	0.	2.438E+01	3.030E+00
85	125	20	0.	3.171E+01	1.030E+00	0.	4.637E+01	3.030E+00
147	187	20	0.	5.237E+01	1.030E+00	0.	4.637E+01	3.030E+00
209	229	20	0.	7.387E+01	1.030E+00	0.	7.978E+01	3.030E+00
3	11	5	3.125E+00	0.	1.030E+00	1.762E+01	0.	3.030E+00
5	15	5	3.125E+00	0.	0.	1.762E+01	0.	0.
25	31	4	3.125E+00	8.125E+00	1.030E+00	1.762E+01	8.125E+00	3.030E+00
27	35	4	3.125E+00	8.125E+00	0.	1.762E+01	8.125E+00	0.
45	51	4	3.125E+00	1.625E+01	1.030E+00	1.762E+01	1.625E+01	3.030E+00
47	55	4	3.125E+00	1.625E+01	0.	1.762E+01	1.625E+01	0.
65	75	5	3.125E+00	2.438E+01	1.030E+00	1.762E+01	2.438E+01	3.030E+00
67	77	5	3.125E+00	2.438E+01	0.	1.762E+01	2.438E+01	0.
87	95	4	3.125E+00	3.171E+01	1.030E+00	1.762E+01	3.171E+01	3.030E+00
69	77	4	3.125E+00	3.171E+01	0.	1.762E+01	3.171E+01	0.
107	115	4	3.125E+00	3.904E+01	1.030E+00	1.762E+01	3.904E+01	3.030E+00
109	117	4	3.125E+00	3.904E+01	0.	1.762E+01	3.904E+01	0.
127	137	5	3.125E+00	4.637E+01	1.030E+00	1.762E+01	4.637E+01	3.030E+00
129	139	5	3.125E+00	4.637E+01	0.	1.762E+01	4.637E+01	0.
149	157	4	3.125E+00	5.237E+01	1.030E+00	1.762E+01	5.237E+01	3.030E+00
151	159	4	3.125E+00	5.237E+01	0.	1.762E+01	5.237E+01	0.
169	177	4	3.125E+00	6.037E+01	1.030E+00	1.762E+01	6.037E+01	3.030E+00
171	179	4	3.125E+00	6.037E+01	0.	1.762E+01	6.037E+01	0.

149	199	5	3.125E+00	6.837E+01	1.030E+00	1.762E+01	6.837E+01	3.030E+00
171	201	5	3.125E+00	6.837E+01	0.	1.762E+01	6.837E+01	0.
211	219	4	3.125E+00	7.387E+01	1.030E+00	1.762E+01	7.387E+01	3.030E+00
213	221	4	3.125E+00	7.387E+01	0.	1.762E+01	7.387E+01	0.
231	239	4	3.125E+00	7.978E+01	1.030E+00	1.762E+01	7.978E+01	3.030E+00
233	241	4	3.125E+00	7.978E+01	0.	1.762E+01	7.978E+01	0.
6	11	5	6.750E+00	0.	1.030E+00	1.400E+01	0.	3.030E+00
7	12	5	6.750E+00	0.	1.515E+00	1.400E+01	0.	1.515E+00
4	14	5	3.125E+00	0.	1.515E+00	1.762E+01	0.	1.515E+00
6A	73	5	6.750E+00	2.438E+01	1.030E+00	1.400E+01	2.438E+01	3.030E+00
69	74	5	6.750E+00	2.438E+01	1.515E+00	1.400E+01	2.438E+01	1.515E+00
66	76	5	3.125E+00	2.438E+01	1.515E+00	1.762E+01	2.438E+01	1.515E+00
130	135	5	6.750E+00	4.637E+01	1.030E+00	1.400E+01	4.637E+01	3.030E+00
131	136	5	6.750E+00	4.637E+01	1.515E+00	1.400E+01	4.637E+01	1.515E+00
128	138	5	3.125E+00	4.637E+01	1.515E+00	1.762E+01	4.637E+01	1.515E+00
192	197	5	6.750E+00	6.837E+01	1.030E+00	1.400E+01	6.837E+01	3.030E+00
193	198	5	6.750E+00	6.837E+01	1.515E+00	1.400E+01	6.837E+01	1.515E+00
190	200	5	3.125E+00	6.837E+01	1.515E+00	1.762E+01	6.837E+01	1.515E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 82

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES
1	-0	1	-0.		4.250E+02	4 7 4 3
2	-0	1	-0.		4.250E+02	7 9 4 4
3	-0	1	-0.		4.250E+02	9 12 11 8
4	-0	1	-0.		4.250E+02	12 14 13 11
23	-0	1	-0.		5.210E+02	46 69 68 65
24	-0	1	-0.		5.210E+02	69 71 70 68
25	-0	1	-0.		5.210E+02	71 74 73 70
26	-0	1	-0.		5.210E+02	74 76 75 73
45	-0	1	-0.		5.210E+02	128 131 130 127
46	-0	1	-0.		5.210E+02	131 133 132 130
47	-0	1	-0.		5.210E+02	133 136 135 132
48	-0	1	-0.		5.210E+02	136 138 137 135
57	-0	1	-0.		5.210E+02	190 193 192 189
68	-0	1	-0.		5.210E+02	193 195 194 192
49	-0	1	-0.		5.210E+02	195 198 197 194
70	-0	1	-0.		5.210E+02	198 200 199 197
6	-0	1	-0.		1.226E+01	5 27 25 3 18 26 17 4
12	-0	1	-0.		1.226E+01	27 47 45 25 18 46 37 24
18	-0	1	-0.		1.226E+01	47 47 45 45 48 46 57 44
28	-0	1	-0.		1.226E+01	67 89 87 45 80 88 79 66
34	-0	1	-0.		1.226E+01	89 109 107 47 100 108 99 88
44	-0	1	-0.		1.226E+01	109 129 127 107 120 128 119 108
54	-0	1	-0.		1.226E+01	129 151 149 127 142 150 141 128
56	-0	1	-0.		1.226E+01	151 171 169 149 162 170 161 150
62	-0	1	-0.		1.226E+01	171 191 189 169 182 190 181 170
72	-0	1	-0.		1.226E+01	191 213 211 189 205 212 203 190
78	-0	1	-0.		1.226E+01	213 233 231 211 224 232 223 212
8	-0	1	-0.		1.226E+01	10 31 29 8 29 30 19 9
14	-0	1	-0.		1.226E+01	31 51 49 29 49 50 39 30
20	-0	1	-0.		1.226E+01	51 72 70 49 60 71 59 50
30	-0	1	-0.		1.226E+01	72 93 91 70 82 92 81 71
74	-0	1	-0.		1.226E+01	93 113 111 91 102 112 101 92
42	-0	1	-0.		1.226E+01	113 134 132 111 122 133 121 112
52	-0	1	-0.		1.226E+01	134 155 153 132 144 154 143 133
58	-0	1	-0.		1.226E+01	155 175 173 153 164 174 163 154

64	-0	1	-0.	1.226E-01	175	196	194	173	184	195	183	174
74	-0	1	-0.	1.226E-01	196	217	215	194	205	216	205	195
80	-0	1	-0.	1.226E-01	217	237	235	215	226	236	225	216
10	-0	1	-0.	6.130E-02	15	35	33	13	22	34	21	14
16	-0	1	-0.	6.130E-02	35	55	53	33	42	54	41	34
22	-0	1	-0.	6.130E-02	55	77	75	53	62	76	61	54
32	-0	1	-0.	6.130E-02	77	97	95	75	84	96	83	76
38	-0	1	-0.	6.130E-02	97	117	115	95	104	116	103	96
44	-0	1	-0.	6.130E-02	117	139	137	115	124	138	123	118
54	-0	1	-0.	6.130E-02	139	159	157	137	144	158	145	138
60	-0	1	-0.	6.130E-02	159	179	177	157	144	178	165	158
66	-0	1	-0.	6.130E-02	179	201	199	177	184	200	185	178
76	-0	1	-0.	6.130E-02	201	221	219	199	204	220	207	200
82	-0	1	-0.	6.130E-02	221	241	239	219	226	240	227	220
5	-0	3	9.000E+01	5.833E-01	3	25	23	1	17	24	16	2
7	-0	3	9.000E+01	5.833E-01	8	29	25	3	19	28	17	6
9	-0	3	9.000E+01	5.833E-01	13	33	29	8	21	32	19	11
11	-0	3	9.000E+01	5.833E-01	25	45	43	23	37	44	36	24
13	-0	3	9.000E+01	5.833E-01	29	49	45	25	39	48	37	28
15	-0	3	9.000E+01	5.833E-01	33	53	49	29	41	52	39	32
17	-0	3	9.000E+01	5.833E-01	45	65	63	43	57	64	56	44
19	-0	3	9.000E+01	5.833E-01	49	70	65	45	59	68	57	48
21	-0	3	9.000E+01	5.833E-01	53	75	70	49	61	73	59	52
27	-0	3	9.000E+01	5.833E-01	65	87	85	63	79	86	78	64
29	-0	3	9.000E+01	5.833E-01	70	91	87	65	81	90	79	68
31	-0	3	9.000E+01	5.833E-01	75	95	91	70	83	94	81	73
33	-0	3	9.000E+01	5.833E-01	87	107	105	85	99	106	98	86
35	-0	3	9.000E+01	5.833E-01	91	111	107	87	101	110	99	90
37	-0	3	9.000E+01	5.833E-01	95	115	111	91	103	114	101	94
39	-0	3	9.000E+01	5.833E-01	107	127	125	105	119	126	118	106
41	-0	3	9.000E+01	5.833E-01	111	132	127	107	121	130	119	110
43	-0	3	9.000E+01	5.833E-01	115	137	132	111	123	135	121	114
49	-0	3	9.000E+01	5.833E-01	127	149	147	125	141	148	140	126
51	-0	3	9.000E+01	5.833E-01	132	153	149	127	147	152	141	130
53	-0	3	9.000E+01	5.833E-01	137	157	153	132	145	156	143	135
55	-0	3	9.000E+01	5.833E-01	149	169	167	147	161	168	160	148
57	-0	3	9.000E+01	5.833E-01	153	173	169	149	163	172	161	152
59	-0	3	9.000E+01	5.833E-01	157	177	173	153	165	176	163	156
61	-0	3	9.000E+01	5.833E-01	169	189	187	167	181	188	180	168
63	-0	3	9.000E+01	5.833E-01	173	194	189	169	183	192	181	172
65	-0	3	9.000E+01	5.833E-01	177	199	194	173	185	197	183	176
71	-0	2	-0.	5.833E-01	189	211	209	187	203	210	202	188
73	-0	2	-0.	5.833E-01	194	215	211	189	205	214	203	192
75	-0	2	-0.	5.833E-01	199	219	215	194	207	216	205	197
77	-0	2	-0.	5.833E-01	211	231	229	209	223	230	222	210
79	-0	2	-0.	5.833E-01	215	235	231	211	225	234	223	214
81	-0	2	-0.	5.833E-01	219	239	235	215	227	236	225	218

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 55

FROM PT	THRU PT	INCR	CASE	COND. **	BOUNDARY VALUES		
PT	PT			X Y Z	X	Y	Z
3	5	1	1	0 0 1	-0.	-0.	-0.

9	10	1	1	0 0 1	-0.	-0.	-0.
13	15	1	1	1 0 1	-0.	-0.	-0.
21	22	1	1	1 0 0	-0.	-0.	-0.
33	35	1	1	1 0 0	-0.	-0.	-0.
41	42	1	1	1 0 0	-0.	-0.	-0.
53	55	1	1	1 0 0	-0.	-0.	-0.
61	62	1	1	1 0 0	-0.	-0.	-0.
75	77	1	1	1 0 0	-0.	-0.	-0.
83	84	1	1	1 0 0	-0.	-0.	-0.
95	97	1	1	1 0 0	-0.	-0.	-0.
103	104	1	1	1 0 0	-0.	-0.	-0.
115	117	1	1	1 0 0	-0.	-0.	-0.
123	124	1	1	1 0 0	-0.	-0.	-0.
137	139	1	1	1 0 0	-0.	-0.	-0.
145	146	1	1	1 0 1	-0.	-0.	-0.
141	144	1	1	0 0 1	-0.	-0.	-0.
157	159	1	1	1 0 0	-0.	-0.	-0.
165	166	1	1	1 0 0	-0.	-0.	-0.
177	179	1	1	1 0 0	-0.	-0.	-0.
195	196	1	1	1 0 0	-0.	-0.	-0.
199	201	1	1	1 0 0	-0.	-0.	-0.
207	208	1	1	1 0 0	-0.	-0.	-0.
219	221	1	1	1 0 0	-0.	-0.	-0.
227	228	1	1	1 0 0	-0.	-0.	-0.
229	238	1	1	0 1 0	-0.	-0.	-0.
239	241	2	1	1 0 1	-0.	-0.	-0.
229	231	1	1	0 2 1	-0.	-0.	-0.
233	235	1	1	0 0 1	-0.	-0.	-0.
237	238	1	1	0 0 1	-0.	-0.	-0.
232	236	4	1	0 1 1	-0.	-0.	-0.
240	-0	-0	1	1 1 1	-0.	-0.	-0.
13	15	1	2	0 1 1	-0.	-0.	-0.
21	22	1	2	0 1 1	-0.	-0.	-0.
33	35	1	2	0 1 1	-0.	-0.	-0.
41	42	1	2	0 1 1	-0.	-0.	-0.
53	55	1	2	0 1 1	-0.	-0.	-0.
61	62	1	2	0 1 1	-0.	-0.	-0.
75	77	1	2	0 1 1	-0.	-0.	-0.
83	84	1	2	0 1 1	-0.	-0.	-0.
95	97	1	2	0 1 1	-0.	-0.	-0.
103	104	1	2	0 1 1	-0.	-0.	-0.
115	117	1	2	0 1 1	-0.	-0.	-0.
123	124	1	2	0 1 1	-0.	-0.	-0.
137	139	1	2	0 1 1	-0.	-0.	-0.
145	146	1	2	0 1 1	-0.	-0.	-0.
157	159	1	2	0 1 1	-0.	-0.	-0.
165	166	1	2	0 1 1	-0.	-0.	-0.
177	179	1	2	0 1 1	-0.	-0.	-0.
195	196	1	2	0 1 1	-0.	-0.	-0.
199	201	1	2	0 1 1	-0.	-0.	-0.
207	208	1	2	0 1 1	-0.	-0.	-0.
219	221	1	2	0 1 1	-0.	-0.	-0.
227	228	1	2	0 1 1	-0.	-0.	-0.
239	241	1	2	0 1 1	-0.	-0.	-0.

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 6

ELEMENT LOADS TO BE ADDED ( 1 = YES ) 0

FROM THRU INCR			FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
PT	PT	INCR	A	Y	Z	X	Y	Z
181	-0	-0	-0.	-0.	2.560E+03	-0.	-0.	-0.
183	-0	-0	-0.	-0.	5.520E+03	-0.	-0.	-0.
185	-0	-0	-0.	-0.	3.350E+03	-0.	-0.	-0.
189	-0	-0	-0.	-0.	4.500E+02	-0.	-0.	-0.
194	-0	-0	-0.	-0.	1.840E+03	-0.	-0.	-0.
199	-0	-0	-0.	-0.	1.120E+03	-0.	-0.	-0.

PROGRAM SHELL 6 - MASTER DECK - ABDELRAOUF, MATLOCK REVISION DATE 6 MAY 1971  
 SHELL 6 REPORT CODED BY ABDELRAOUF MAY 28, 1971  
 3 - SPAN CONTINUOUS HIGHWAY BRIDGE, MS20 TRUCK LOADINGS (LB-FT UNITS)

PROB 303 SYMMETRY ABOUT THE TRANSVERSE AXIS AND ANTISYMMETRY LONGITUDINALLY

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS 82  
 NUM OF POINTS 241  
 NUM OF LCAD CASES 1  
 ELEMENT FORCES REQUIRED ( 1 = YES ) 1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 3

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	4.320E+09	3.000E-01	4.320E+09	3.000E-01	1.662E+09
2	4.320E+08	1.500E-01	4.320E+08	1.500E-01	1.878E+08
3	4.320E+08	1.500E-01	8.640E+07	3.000E-02	6.857E+07

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 41

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	-0	-0	0.	0.	3.030E+00	-0.	-0.	-0.
23	63	20	0.	8.125E+00	3.030E+00	0.	2.438E+01	3.030E+00
85	125	20	0.	3.171E+01	3.030E+00	0.	4.637E+01	3.030E+00
147	187	20	0.	5.237E+01	3.030E+00	0.	6.837E+01	3.030E+00
209	229	20	0.	7.387E+01	3.030E+00	0.	7.938E+01	3.030E+00
3	13	5	3.125E+00	0.	3.030E+00	1.762E+01	0.	3.030E+00
5	15	5	3.125E+00	0.	0.	1.762E+01	0.	0.
25	33	4	3.125E+00	8.125E+00	3.030E+00	1.762E+01	8.125E+00	3.030E+00
27	35	4	3.125E+00	8.125E+00	0.	1.762E+01	8.125E+00	0.
45	53	4	3.125E+00	1.625E+01	3.030E+00	1.762E+01	1.625E+01	3.030E+00
47	55	4	3.125E+00	1.625E+01	0.	1.762E+01	1.625E+01	0.
65	75	5	3.125E+00	2.438E+01	3.030E+00	1.762E+01	2.438E+01	3.030E+00
67	77	5	3.125E+00	2.438E+01	0.	1.762E+01	2.438E+01	0.
87	95	4	3.125E+00	3.171E+01	3.030E+00	1.762E+01	3.171E+01	3.030E+00
89	97	4	3.125E+00	3.171E+01	0.	1.762E+01	3.171E+01	0.
107	115	4	3.125E+00	3.904E+01	3.030E+00	1.762E+01	3.904E+01	3.030E+00
109	117	4	3.125E+00	3.904E+01	0.	1.762E+01	3.904E+01	0.
127	137	5	3.125E+00	4.637E+01	3.030E+00	1.762E+01	4.637E+01	3.030E+00
129	139	5	3.125E+00	4.637E+01	0.	1.762E+01	4.637E+01	0.
149	157	4	3.125E+00	5.237E+01	3.030E+00	1.762E+01	5.237E+01	3.030E+00
151	159	4	3.125E+00	5.237E+01	0.	1.762E+01	5.237E+01	0.
169	177	4	3.125E+00	6.037E+01	3.030E+00	1.762E+01	6.037E+01	3.030E+00
171	179	4	3.125E+00	6.037E+01	0.	1.762E+01	6.037E+01	0.

189	199	5	3.125E+00	6.837E+01	3.030E+00	1.762E+01	6.837E+01	3.030E+00
191	201	5	3.125E+00	6.837E+01	0.	1.762E+01	6.837E+01	0.
211	219	4	3.125E+00	7.387E+01	3.030E+00	1.762E+01	7.387E+01	3.030E+00
213	221	4	3.125E+00	7.387E+01	0.	1.762E+01	7.387E+01	0.
231	239	4	3.125E+00	7.938E+01	3.030E+00	1.762E+01	7.938E+01	3.030E+00
233	241	4	3.125E+00	7.938E+01	0.	1.762E+01	7.938E+01	0.
6	11	5	6.750E+00	0.	3.030E+00	1.400E+01	0.	3.030E+00
7	12	5	6.750E+00	0.	1.515E+00	1.400E+01	0.	1.515E+00
4	14	5	3.125E+00	0.	1.515E+00	1.762E+01	0.	1.515E+00
68	73	5	6.750E+00	2.438E+01	3.030E+00	1.400E+01	2.438E+01	3.030E+00
69	74	5	6.750E+00	2.438E+01	1.515E+00	1.400E+01	2.438E+01	1.515E+00
66	76	5	3.125E+00	2.438E+01	1.515E+00	1.762E+01	2.438E+01	1.515E+00
130	135	5	6.750E+00	4.637E+01	3.030E+00	1.400E+01	4.637E+01	3.030E+00
131	136	5	6.750E+00	4.637E+01	1.515E+00	1.400E+01	4.637E+01	1.515E+00
128	138	5	3.125E+00	4.637E+01	1.515E+00	1.762E+01	4.637E+01	1.515E+00
192	197	5	6.750E+00	6.837E+01	3.030E+00	1.400E+01	6.837E+01	3.030E+00
193	198	5	6.750E+00	6.837E+01	1.515E+00	1.400E+01	6.837E+01	1.515E+00
190	200	5	3.125E+00	6.837E+01	1.515E+00	1.762E+01	6.837E+01	1.515E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 82

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES
1	-0	-0	1	-0.	4.250E-02	4 7 6 3
2	-0	-0	1	-0.	4.250E-02	7 9 8 6
3	-0	-0	1	-0.	4.250E-02	9 12 11 8
4	-0	-0	1	-0.	4.250E-02	12 14 13 11
23	-0	-0	1	-0.	5.210E-02	66 69 68 65
24	-0	-0	1	-0.	5.210E-02	69 71 70 68
25	-0	-0	1	-0.	5.210E-02	71 74 73 70
26	-0	-0	1	-0.	5.210E-02	74 76 75 73
45	-0	-0	1	-0.	5.210E-02	128 131 130 127
46	-0	-0	1	-0.	5.210E-02	131 133 132 130
47	-0	-0	1	-0.	5.210E-02	133 136 135 132
48	-0	-0	1	-0.	5.210E-02	136 138 137 135
67	-0	-0	1	-0.	5.210E-02	190 193 192 189
68	-0	-0	1	-0.	5.210E-02	193 195 194 192
69	-0	-0	1	-0.	5.210E-02	195 198 197 194
70	-0	-0	1	-0.	5.210E-02	198 200 199 197
6	-0	-0	1	-0.	1.226E-01	5 27 25 3 18 26 17 4
12	-0	-0	1	-0.	1.226E-01	27 47 45 25 38 46 37 26
18	-0	-0	1	-0.	1.226E-01	47 67 65 45 58 66 57 45
28	-0	-0	1	-0.	1.226E-01	47 89 87 65 80 88 79 66
34	-0	-0	1	-0.	1.226E-01	89 109 107 87 100 108 99 88
40	-0	-0	1	-0.	1.226E-01	109 129 127 107 120 128 119 108
50	-0	-0	1	-0.	1.226E-01	129 151 149 127 142 150 141 128
56	-0	-0	1	-0.	1.226E-01	151 171 169 149 162 170 161 150
62	-0	-0	1	-0.	1.226E-01	171 191 189 169 182 190 181 170
72	-0	-0	1	-0.	1.226E-01	191 213 211 189 204 212 203 190
78	-0	-0	1	-0.	1.226E-01	213 233 231 211 224 232 223 212
8	-0	-0	1	-0.	1.226E-01	10 31 29 8 20 30 19 9
14	-0	-0	1	-0.	1.226E-01	31 51 49 29 40 50 39 30
20	-0	-0	1	-0.	1.226E-01	51 72 70 49 60 71 59 50
30	-0	-0	1	-0.	1.226E-01	72 93 91 70 82 92 81 71
36	-0	-0	1	-0.	1.226E-01	93 113 111 91 102 112 101 92
42	-0	-0	1	-0.	1.226E-01	113 134 132 111 122 133 121 112
52	-0	-0	1	-0.	1.226E-01	134 155 153 132 144 154 143 133
58	-0	-0	1	-0.	1.226E-01	155 175 173 153 164 174 163 154

64	-0	1	-0.	1.226E-01	175	196	194	173	184	195	183	174
74	-0	1	-0.	1.226E-01	196	217	215	194	206	216	205	195
80	-0	1	-0.	1.226E-01	217	237	235	215	226	236	225	216
10	-0	1	-0.	6.130E-02	15	35	33	13	22	34	21	1*
16	-0	1	-0.	6.130E-02	35	55	53	33	42	54	41	34
22	-0	1	-0.	6.130E-02	55	77	75	53	62	76	61	54
32	-0	1	-0.	6.130E-02	77	97	95	75	84	96	83	76
38	-0	1	-0.	6.130E-02	97	117	115	95	104	116	103	96
44	-0	1	-0.	6.130E-02	117	139	137	115	124	138	123	116
54	-0	1	-0.	6.130E-02	139	159	157	137	146	158	145	138
60	-0	1	-0.	6.130E-02	159	179	177	157	166	178	165	158
66	-0	1	-0.	6.130E-02	179	201	199	177	186	200	185	178
76	-0	1	-0.	6.130E-02	201	221	219	199	208	220	207	200
82	-0	1	-0.	6.130E-02	221	241	239	219	228	240	227	220
5	-0	3	9.000E+01	5.833E-01	3	25	23	1	17	24	16	2
7	-0	3	9.000E+01	5.833E-01	8	29	25	3	19	28	17	6
9	-0	3	9.000E+01	5.833E-01	13	33	29	8	21	32	19	11
11	-0	3	9.000E+01	5.833E-01	25	45	43	23	37	44	36	24
13	-0	3	9.000E+01	5.833E-01	29	49	45	25	39	48	37	28
15	-0	3	9.000E+01	5.833E-01	33	53	49	29	41	52	39	32
17	-0	3	9.000E+01	5.833E-01	45	65	63	43	57	64	56	44
19	-0	3	9.000E+01	5.833E-01	49	70	65	45	59	68	57	48
21	-0	3	9.000E+01	5.833E-01	53	75	70	49	61	73	59	52
27	-0	3	9.000E+01	5.833E-01	65	87	85	63	79	86	78	64
29	-0	3	9.000E+01	5.833E-01	70	91	87	65	81	90	79	68
31	-0	3	9.000E+01	5.833E-01	75	95	91	70	83	94	81	73
33	-0	3	9.000E+01	5.833E-01	87	107	105	85	99	106	98	86
35	-0	3	9.000E+01	5.833E-01	91	111	107	87	101	110	99	90
37	-0	3	9.000E+01	5.833E-01	95	115	111	91	103	114	101	94
39	-0	3	9.000E+01	5.833E-01	107	127	125	105	119	126	118	106
41	-0	3	9.000E+01	5.833E-01	111	132	127	107	121	130	119	110
43	-0	3	9.000E+01	5.833E-01	115	137	132	111	123	135	121	114
49	-0	3	9.000E+01	5.833E-01	127	149	147	125	141	148	140	126
51	-0	3	9.000E+01	5.833E-01	132	153	149	127	143	152	141	130
53	-0	3	9.000E+01	5.833E-01	137	157	153	132	145	156	143	135
55	-0	3	9.000E+01	5.833E-01	149	169	167	147	161	168	160	148
57	-0	3	9.000E+01	5.833E-01	153	173	169	149	163	172	161	152
59	-0	3	9.000E+01	5.833E-01	157	177	173	153	165	176	163	156
61	-0	3	9.000E+01	5.833E-01	169	189	187	167	181	188	180	168
63	-0	3	9.000E+01	5.833E-01	173	194	189	169	183	192	181	172
65	-0	3	9.000E+01	5.833E-01	177	199	194	173	185	197	183	176
71	-0	2	-0.	5.833E-01	189	211	209	187	203	210	202	188
73	-0	2	-0.	5.833E-01	194	215	211	189	205	214	203	192
75	-0	2	-0.	5.833E-01	199	219	215	194	207	216	205	197
77	-0	2	-0.	5.833E-01	211	231	229	209	223	230	222	210
79	-0	2	-0.	5.833E-01	215	235	231	211	225	234	223	214
81	-0	2	-0.	5.833E-01	219	239	235	215	227	238	225	218

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 27

FROM	THRU	INCR	CASE	COND. **	BOUNDARY VALUES		
PT	PT			X Y Z	X	Y	Z
3	5	1	1	0 0 1	-0.	-0.	-0.

8	10	1	1	0 0 1	-0.	-0.	-0.
141	144	1	1	0 0 1	-0.	-0.	-0.
13	53	20	1	0 0 1	-0.	-0.	-0.
75	115	20	1	0 0 1	-0.	-0.	-0.
137	177	20	1	0 0 1	-0.	-0.	-0.
199	219	20	1	0 0 1	-0.	-0.	-0.
15	55	20	1	0 0 1	-0.	-0.	-0.
77	117	20	1	0 0 1	-0.	-0.	-0.
139	179	20	1	0 0 1	-0.	-0.	-0.
201	221	20	1	0 0 1	-0.	-0.	-0.
21	61	20	1	0 0 1	-0.	-0.	-0.
83	123	20	1	0 0 1	-0.	-0.	-0.
145	185	20	1	0 0 1	-0.	-0.	-0.
207	227	20	1	0 0 1	-0.	-0.	-0.
22	62	20	1	0 0 1	-0.	-0.	-0.
84	124	20	1	0 0 1	-0.	-0.	-0.
146	186	20	1	0 0 1	-0.	-0.	-0.
208	228	20	1	0 0 1	-0.	-0.	-0.
14	54	20	1	1 0 1	-0.	-0.	-0.
76	116	20	1	1 0 1	-0.	-0.	-0.
138	178	20	1	1 0 1	-0.	-0.	-0.
200	220	20	1	1 0 1	-0.	-0.	-0.
239	241	2	1	0 1 1	-0.	-0.	-0.
229	238	1	1	0 1 0	-0.	-0.	-0.
240	-0	-0	1	1 1 1	-0.	-0.	-0.
229	241	1	2	1 0 1	-0.	-0.	-0.

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS = GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 8

ELEMENT LOADS TO BE ADDED (1 = YES) 0

FROM	THRU	INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
PT	PT		X	Y	Z	X	Y	Z
181	-0	-0	-0.	-0.	-0.	-0.	-0.	-0.
183	-0	-0	-0.	-0.	-0.	-0.	-0.	-0.
189	-0	-0	-0.	-0.	-0.	-0.	-0.	-0.
194	-0	-0	-0.	-0.	-0.	-0.	-0.	-0.
231	-0	-0	-0.	-0.	-0.	-0.	-0.	-0.
234	-0	-0	-0.	-0.	-0.	-0.	-0.	-0.
235	-0	-0	-0.	-0.	-0.	-0.	-0.	-0.
238	-0	-0	-0.	-0.	-0.	-0.	-0.	-0.



64	-0	1	-0.	1.226E-01	175	196	194	173	184	195	183	174
74	-0	1	-0.	1.226E-01	196	217	215	194	206	216	205	195
80	-0	1	-0.	1.226E-01	217	237	235	215	226	236	225	216
10	-0	1	-0.	6.130E-02	15	35	33	13	22	34	21	14
16	-0	1	-0.	6.130E-02	35	55	53	33	42	54	41	34
22	-0	1	-0.	6.130E-02	55	77	75	53	62	76	61	54
32	-0	1	-0.	6.130E-02	77	97	95	75	84	96	83	76
38	-0	1	-0.	6.130E-02	97	117	115	95	104	116	103	96
44	-0	1	-0.	6.130E-02	117	139	137	115	124	138	123	116
54	-0	1	-0.	6.130E-02	139	159	157	137	146	158	145	138
60	-0	1	-0.	6.130E-02	159	179	177	157	166	178	165	158
66	-0	1	-0.	6.130E-02	179	201	199	177	186	200	185	178
76	-0	1	-0.	6.130E-02	201	221	219	199	208	220	207	200
82	-0	1	-0.	6.130E-02	221	241	239	219	228	240	227	220
5	-0	3	9.000E+01	5.833E-01	3	25	23	1	17	24	16	2
7	-0	3	9.000E+01	5.833E-01	8	29	25	3	19	28	17	6
9	-0	3	9.000E+01	5.833E-01	13	33	29	8	21	32	19	11
11	-0	3	9.000E+01	5.833E-01	25	45	43	23	37	44	36	24
13	-0	3	9.000E+01	5.833E-01	29	49	45	25	39	48	37	28
15	-0	3	9.000E+01	5.833E-01	33	53	49	29	41	52	39	32
17	-0	3	9.000E+01	5.833E-01	45	65	63	43	57	64	56	44
19	-0	3	9.000E+01	5.833E-01	49	70	65	45	59	68	57	48
21	-0	3	9.000E+01	5.833E-01	53	75	70	49	61	73	59	52
27	-0	3	9.000E+01	5.833E-01	65	87	85	63	79	86	78	64
29	-0	3	9.000E+01	5.833E-01	70	91	87	65	81	90	79	68
31	-0	3	9.000E+01	5.833E-01	75	95	91	70	83	94	81	73
33	-0	3	9.000E+01	5.833E-01	87	107	105	85	99	106	98	86
35	-0	3	9.000E+01	5.833E-01	91	111	107	87	101	110	99	90
37	-0	3	9.000E+01	5.833E-01	95	115	111	91	103	114	101	94
39	-0	3	9.000E+01	5.833E-01	107	127	125	105	119	126	118	106
41	-0	3	9.000E+01	5.833E-01	111	132	127	107	121	130	119	110
43	-0	3	9.000E+01	5.833E-01	115	137	132	111	123	135	121	114
49	-0	3	9.000E+01	5.833E-01	127	149	147	125	141	148	140	126
51	-0	3	9.000E+01	5.833E-01	132	153	149	127	143	152	141	130
53	-0	3	9.000E+01	5.833E-01	137	157	153	132	145	156	143	135
55	-0	3	9.000E+01	5.833E-01	149	169	167	147	161	168	160	148
57	-0	3	9.000E+01	5.833E-01	153	173	169	149	163	172	161	152
59	-0	3	9.000E+01	5.833E-01	157	177	173	153	165	176	163	156
61	-0	3	9.000E+01	5.833E-01	169	189	187	167	181	188	180	168
63	-0	3	9.000E+01	5.833E-01	173	194	189	169	183	192	181	172
65	-0	3	9.000E+01	5.833E-01	177	199	194	173	185	197	183	176
71	-0	2	-0.	5.833E-01	189	211	209	187	203	210	202	188
73	-0	2	-0.	5.833E-01	194	215	211	189	205	214	203	192
75	-0	2	-0.	5.833E-01	199	219	215	194	207	216	205	197
77	-0	2	-0.	5.833E-01	211	231	229	209	223	230	222	218
79	-0	2	-0.	5.833E-01	215	235	231	211	225	234	223	214
81	-0	2	-0.	5.833E-01	219	239	235	215	227	236	225	218

8	10	1	1	0	0	1	-0.	-0.	-0.
141	144	1	1	0	0	1	-0.	-0.	-0.
13	53	20	1	0	0	1	-0.	-0.	-0.
75	115	20	1	0	0	1	-0.	-0.	-0.
137	177	20	1	0	0	1	-0.	-0.	-0.
199	219	20	1	0	0	1	-0.	-0.	-0.
15	55	20	1	0	0	1	-0.	-0.	-0.
77	117	20	1	0	0	1	-0.	-0.	-0.
139	179	20	1	0	0	1	-0.	-0.	-0.
201	241	20	1	0	0	1	-0.	-0.	-0.
21	61	20	1	0	0	1	-0.	-0.	-0.
83	123	20	1	0	0	1	-0.	-0.	-0.
145	185	20	1	0	0	1	-0.	-0.	-0.
22	62	20	1	0	0	1	-0.	-0.	-0.
84	124	20	1	0	0	1	-0.	-0.	-0.
146	186	20	1	0	0	1	-0.	-0.	-0.
200	228	20	1	0	0	1	-0.	-0.	-0.
14	54	20	1	1	0	1	-0.	-0.	-0.
76	116	20	1	1	0	1	-0.	-0.	-0.
138	178	20	1	1	0	1	-0.	-0.	-0.
200	220	20	1	1	0	1	-0.	-0.	-0.
239	241	2	1	0	0	1	-0.	-0.	-0.
229	231	1	1	0	0	1	-0.	-0.	-0.
234	235	1	1	0	0	1	-0.	-0.	-0.
237	238	1	1	0	0	1	-0.	-0.	-0.
240	-0	-0	1	1	1	1	-0.	-0.	-0.
232	236	*	1	0	1	1	-0.	-0.	-0.

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 4

ELEMENT LOADS TO BE ADDED ( 1 = YES ) 0

FROM THRU	INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES			
		X	Y	Z	X	Y	Z	
181	-0	-0	-0.	-0.	2.560E+03	-0.	-0.	-0.
183	-0	-0	-0.	-0.	3.550E+03	-0.	-0.	-0.
189	-0	-0	-0.	-0.	8.500E+02	-0.	-0.	-0.
194	-0	-0	-0.	-0.	1.170E+03	-0.	-0.	-0.

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 29

FROM THRU	INCR	CASE	COND. **	BOUNDARY VALUES					
PT	PT		X Y Z	X	Y	Z			
3	5	1	1	0	0	1	-0.	-0.	-0.







CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 11

ELEMENT LOADS TO BE ADDED ( 1 = YES ) 0

FROM THRU INCH			FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
PT	PT		X	Y	Z	X	Y	Z
181	-0	-0 -0.	-0.		-4.270E+03	-0.	-0.	-0.
183	-0	-0 -0.	-0.		-9.200E+03	-0.	-0.	-0.
185	-0	-0 -0.	-0.		-5.580E+03	-0.	-0.	-0.
189	-0	-0 -0.	-0.		-1.420E+03	-0.	-0.	-0.
194	-0	-0 -0.	-0.		-3.070E+03	-0.	-0.	-0.
199	-0	-0 -0.	-0.		-1.860E+03	-0.	-0.	-0.
231	-0	-0 -0.	-0.		-3.680E+03	-0.	-0.	-0.
234	-0	-0 -0.	-0.		-1.740E+03	-0.	-0.	-0.
235	-0	-0 -0.	-0.		-4.740E+03	-0.	-0.	-0.
238	-0	-0 -0.	-0.		-8.400E+03	-0.	-0.	-0.
239	-0	-0 -0.	-0.		-1.760E+03	-0.	-0.	-0.





APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS = GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARS FOR THIS TABLE 6

ELEMENT LOADS TO BE ADDED ( 1 = YES ) 0

FROM THRU INCR			FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
PT	PT	INCR	X	Y	Z	X	Y	Z
181	-0	-0	-0.	-0.	2.560E+03	-0.	-0.	-0.
183	-0	-0	-0.	-0.	5.320E+03	-0.	-0.	-0.
185	-0	-0	-0.	-0.	3.350E+03	-0.	-0.	-0.
189	-0	-0	-0.	-0.	8.500E+02	-0.	-0.	-0.
194	-0	-0	-0.	-0.	1.640E+03	-0.	-0.	-0.
199	-0	-0	-0.	-0.	1.120E+03	-0.	-0.	-0.

PROGRAM SMELL 6 - MASTER DECK - ABDELRAOUF, MATLOCK REVISION DATE 6 MAY 1971  
 SMELL 6 REPORT CODED BY ABDELRAOUF NOVEMBER 9, 1971  
 3 - SPAN CONTINUOUS HIGHWAY BRIDGE, MS20 TRUCK LOADINGS (LB-FT UNITS)

PROB 303-A SYMMETRY ABOUT THE TRANSVERSE AXIS AND ANTISYMMETRY LONGITUOINALLY

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS 82  
 NUM OF POINTS 241  
 NUM OF LOAD CASES 1  
 ELEMENT FORCES REQUIRED 1 1 = YES 1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 4

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	4.320E+09	3.000E-01	1.900E+09	1.319E-01	1.115E+09
2	4.320E+08	1.500E-01	4.320E+08	1.500E-01	1.878E+08
3	4.320E+08	1.500E-01	8.640E+07	3.000E-02	6.857E+07
4	4.320E+09	3.000E-01	2.650E+08	1.840E-02	2.413E+08

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 41

FROM THRU INCR		STARTING POINT COORDINATES			END POINT COORDINATES		
PT	PT	X	Y	Z	X	Y	Z
1	-0	-0	0.	0.	3.030E+00	-0.	-0.
23	63	20	0.	0.	3.030E+00	0.	0.
85	125	20	0.	0.	3.030E+00	0.	0.
147	187	20	0.	0.	3.030E+00	0.	0.
209	229	20	0.	0.	3.030E+00	0.	0.
3	15	5	3.125E+00	0.	3.030E+00	1.762E+01	0.
5	15	5	3.125E+00	0.	3.030E+00	1.762E+01	0.
25	33	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
27	35	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
45	53	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
47	55	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
65	75	5	3.125E+00	0.	3.030E+00	1.762E+01	0.
67	77	5	3.125E+00	0.	3.030E+00	1.762E+01	0.
87	95	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
89	97	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
107	115	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
109	117	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
127	137	5	3.125E+00	0.	3.030E+00	1.762E+01	0.
129	139	5	3.125E+00	0.	3.030E+00	1.762E+01	0.
149	157	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
151	159	4	3.125E+00	0.	3.030E+00	1.762E+01	0.
169	177	4	3.125E+00	0.	3.030E+00	1.762E+01	0.

171	179	4	3.125E+00	6.037E+01	0.	1.762E+01	6.037E+01	0.
189	199	5	3.125E+00	6.037E+01	3.030E+00	1.762E+01	6.037E+01	3.030E+00
191	201	5	3.125E+00	6.037E+01	0.	1.762E+01	6.037E+01	0.
211	219	4	3.125E+00	7.387E+01	3.030E+00	1.762E+01	7.387E+01	3.030E+00
213	221	4	3.125E+00	7.387E+01	0.	1.762E+01	7.387E+01	0.
231	239	4	3.125E+00	7.938E+01	3.030E+00	1.762E+01	7.938E+01	3.030E+00
233	241	4	3.125E+00	7.938E+01	0.	1.762E+01	7.938E+01	0.
6	11	5	6.750E+00	0.	3.030E+00	1.400E+01	0.	3.030E+00
7	12	5	6.750E+00	0.	1.515E+00	1.400E+01	0.	1.515E+00
4	14	5	3.125E+00	0.	1.515E+00	1.762E+01	0.	1.515E+00
68	73	5	6.750E+00	2.438E+01	3.030E+00	1.400E+01	2.438E+01	3.030E+00
69	74	5	6.750E+00	2.438E+01	1.515E+00	1.400E+01	2.438E+01	1.515E+00
66	76	5	3.125E+00	2.438E+01	1.515E+00	1.762E+01	2.438E+01	1.515E+00
130	135	5	6.750E+00	4.637E+01	3.030E+00	1.400E+01	4.637E+01	3.030E+00
131	136	5	6.750E+00	4.637E+01	1.515E+00	1.400E+01	4.637E+01	1.515E+00
120	138	5	3.125E+00	4.637E+01	1.515E+00	1.762E+01	4.637E+01	1.515E+00
192	197	5	6.750E+00	6.037E+01	3.030E+00	1.400E+01	6.037E+01	3.030E+00
193	198	5	6.750E+00	6.037E+01	1.515E+00	1.400E+01	6.037E+01	1.515E+00
190	200	8	3.125E+00	6.037E+01	1.515E+00	1.762E+01	6.037E+01	1.515E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 82

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES		
1	-0	1	-0.		4.250E-02	4	7	6 3
2	-0	1	-0.		4.250E-02	7	9	8 6
3	-0	1	-0.		4.250E-02	9	12	11 8
4	-0	1	-0.		4.250E-02	12	14	13 11
23	-0	1	-0.		5.210E-02	66	69	68 65
24	-0	1	-0.		5.210E-02	69	71	70 68
25	-0	1	-0.		5.210E-02	71	74	73 70
26	-0	1	-0.		5.210E-02	74	76	75 73
45	-0	1	-0.		5.210E-02	128	131	130 127
46	-0	1	-0.		5.210E-02	131	133	132 130
47	-0	1	-0.		5.210E-02	133	136	135 132
48	-0	1	-0.		5.210E-02	136	138	137 135
67	-0	1	-0.		5.210E-02	190	193	192 189
68	-0	1	-0.		5.210E-02	193	195	194 192
69	-0	1	-0.		5.210E-02	195	198	197 194
70	-0	1	-0.		5.210E-02	198	200	199 197
6	-0	4	-0.		1.226E-01	5	27	25 3 18 26 17 4
12	-0	4	-0.		1.226E-01	27	47	45 25 38 46 37 26
18	-0	4	-0.		1.226E-01	47	67	65 45 58 66 57 46
28	-0	4	-0.		1.226E-01	67	89	87 65 80 88 79 66
34	-0	4	-0.		1.226E-01	89	109	107 87 100 108 99 88
40	-0	4	-0.		1.226E-01	109	129	127 107 120 128 119 108
50	-0	4	-0.		1.226E-01	129	151	149 127 142 150 141 128
56	-0	4	-0.		1.226E-01	151	171	169 149 162 170 161 150
62	-0	4	-0.		1.226E-01	171	191	189 169 182 190 181 170
72	-0	4	-0.		1.226E-01	191	213	211 189 204 212 203 190
78	-0	4	-0.		1.226E-01	213	233	231 211 224 232 223 212
8	-0	4	-0.		1.226E-01	10	31	29 8 20 30 19 9
14	-0	4	-0.		1.226E-01	31	51	49 29 40 50 39 30
20	-0	4	-0.		1.226E-01	51	72	70 49 60 71 59 50
30	-0	4	-0.		1.226E-01	72	93	91 70 82 92 81 71
36	-0	4	-0.		1.226E-01	93	113	111 91 102 112 101 92
42	-0	4	-0.		1.226E-01	113	134	132 111 122 133 121 112
52	-0	4	-0.		1.226E-01	134	155	153 132 144 154 143 133





PROGRAM SHELL 6 - MASTER DECK - ABDELRAOUF, MAILLOCK REVISION DATE 6 MAY 1971  
SHELL 6 REPORT CODED BY ABDELRAOUF NOVEMBER 9, 1971  
3 - SPAN CONTINUOUS HIGHWAY BRIDGE, HS20 TRUCK LOADINGS (LB-FT UNITS)

PROB  
304-A ANTISYMMETRY ABOUT THE TWO AXES

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS	82
NUM OF POINTS	241
NUM OF LOAD CASES	1
ELEMENT FORCES REQUIRED ( 1 = YES )	1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 4

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	4.320E+09	3.000E-01	1.900E+09	1.310E+01*	1.115E+09*
2	4.320E+09	1.500E-01	4.320E+08*	1.500E+01*	1.678E+08*
3	4.320E+09	1.500E-01	8.640E+07	3.000E-02*	6.057E+07*
4	4.320E+09	3.000E-01	2.650E+08	1.840E-02*	2.413E+08*

(\* ) ASSUMED VALUES

TABLE 3- NOJAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 41

FROM THRU		INCH	STARTING POINT COORDINATES			END POINT COORDINATES		
PT	PT		A	Y	Z	A	Y	Z
1	-0	-0	0.	0.	3.030E+00	-0.	-0.	-0.
23	83	20	0.	8.125E+00	3.030E+00	0.	0.	0.
85	125	20	0.	3.171E+01	3.030E+00	0.	0.	0.
147	187	20	0.	5.237E+01	3.030E+00	0.	0.	0.
209	229	20	0.	7.304E+01	3.030E+00	0.	0.	0.
3	13	5	3.125E+00	0.	3.030E+00	1.762E+01	0.	3.030E+00
5	15	5	3.125E+00	0.	3.030E+00	1.762E+01	0.	3.030E+00
25	33	4	3.125E+00	8.125E+00	3.030E+00	1.762E+01	8.125E+00	3.030E+00
27	35	4	3.125E+00	8.125E+00	0.	1.762E+01	8.125E+00	0.
45	53	4	3.125E+00	1.625E+01	3.030E+00	1.762E+01	1.625E+01	3.030E+00
47	55	4	3.125E+00	1.625E+01	0.	1.762E+01	1.625E+01	0.
65	75	5	3.125E+00	2.438E+01	3.030E+00	1.762E+01	2.438E+01	3.030E+00
67	77	5	3.125E+00	2.438E+01	0.	1.762E+01	2.438E+01	0.
87	95	4	3.125E+00	3.171E+01	3.030E+00	1.762E+01	3.171E+01	3.030E+00
89	97	4	3.125E+00	3.171E+01	0.	1.762E+01	3.171E+01	0.
107	115	4	3.125E+00	3.904E+01	3.030E+00	1.762E+01	3.904E+01	3.030E+00
109	117	4	3.125E+00	3.904E+01	0.	1.762E+01	3.904E+01	0.
127	137	5	3.125E+00	4.637E+01	3.030E+00	1.762E+01	4.637E+01	3.030E+00
129	139	5	3.125E+00	4.637E+01	0.	1.762E+01	4.637E+01	0.
149	157	4	3.125E+00	5.237E+01	3.030E+00	1.762E+01	5.237E+01	3.030E+00
151	159	4	3.125E+00	5.237E+01	0.	1.762E+01	5.237E+01	0.
169	177	4	3.125E+00	6.937E+01	3.030E+00	1.762E+01	6.937E+01	3.030E+00

171	179	4	3.125E+00	6.937E+01	0.	1.762E+01	6.937E+01	0.
189	199	5	3.125E+00	6.937E+01	3.030E+00	1.762E+01	6.937E+01	3.030E+00
191	201	5	3.125E+00	6.937E+01	0.	1.762E+01	6.937E+01	0.
211	219	4	3.125E+00	7.304E+01	3.030E+00	1.762E+01	7.304E+01	3.030E+00
213	221	4	3.125E+00	7.304E+01	0.	1.762E+01	7.304E+01	0.
231	239	4	3.125E+00	7.938E+01	3.030E+00	1.762E+01	7.938E+01	3.030E+00
233	241	4	3.125E+00	7.938E+01	0.	1.762E+01	7.938E+01	0.
6	11	5	6.750E+00	0.	3.030E+00	1.400E+01	0.	3.030E+00
7	12	5	6.750E+00	0.	1.515E+00	1.400E+01	0.	1.515E+00
4	14	5	3.125E+00	0.	1.515E+00	1.762E+01	0.	1.515E+00
68	73	5	6.750E+00	2.438E+01	3.030E+00	1.400E+01	2.438E+01	3.030E+00
69	74	5	6.750E+00	2.438E+01	1.515E+00	1.400E+01	2.438E+01	1.515E+00
66	76	5	3.125E+00	2.438E+01	1.515E+00	1.762E+01	2.438E+01	1.515E+00
130	135	5	6.750E+00	4.637E+01	3.030E+00	1.400E+01	4.637E+01	3.030E+00
131	136	5	6.750E+00	4.637E+01	1.515E+00	1.400E+01	4.637E+01	1.515E+00
128	138	5	3.125E+00	4.637E+01	1.515E+00	1.762E+01	4.637E+01	1.515E+00
192	197	5	6.750E+00	6.937E+01	3.030E+00	1.400E+01	6.937E+01	3.030E+00
193	198	5	6.750E+00	6.937E+01	1.515E+00	1.400E+01	6.937E+01	1.515E+00
190	200	5	3.125E+00	6.937E+01	1.515E+00	1.762E+01	6.937E+01	1.515E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 82

FROM THRU		INCH	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES			
ELM1	ELM2					4	7	8	9
1	-0	-0	1	-0.	4.250E-02	4	7	8	9
2	-0	-0	1	-0.	4.250E-02	7	9	8	6
3	-0	-0	1	-0.	4.250E-02	9	12	11	8
4	-0	-0	1	-0.	4.250E-02	12	14	13	11
23	-0	-0	1	-0.	5.210E-02	66	69	68	65
24	-0	-0	1	-0.	5.210E-02	69	71	70	68
25	-0	-0	1	-0.	5.210E-02	71	74	73	70
26	-0	-0	1	-0.	5.210E-02	74	76	75	73
43	-0	-0	1	-0.	5.210E-02	128	131	130	127
46	-0	-0	1	-0.	5.210E-02	131	133	132	130
47	-0	-0	1	-0.	5.210E-02	133	136	135	132
48	-0	-0	1	-0.	5.210E-02	136	138	137	135
67	-0	-0	1	-0.	5.210E-02	190	193	192	189
68	-0	-0	1	-0.	5.210E-02	193	195	194	192
69	-0	-0	1	-0.	5.210E-02	195	198	197	194
70	-0	-0	1	-0.	5.210E-02	198	200	199	197
6	-0	-0	4	-0.	1.226E-01	5	27	25	3
12	-0	-0	4	-0.	1.226E-01	27	47	45	25
18	-0	-0	4	-0.	1.226E-01	47	67	65	45
26	-0	-0	4	-0.	1.226E-01	67	89	87	65
34	-0	-0	4	-0.	1.226E-01	89	109	107	87
40	-0	-0	4	-0.	1.226E-01	109	129	127	107
50	-0	-0	4	-0.	1.226E-01	129	151	149	127
56	-0	-0	4	-0.	1.226E-01	151	171	169	149
62	-0	-0	4	-0.	1.226E-01	171	191	189	169
72	-0	-0	4	-0.	1.226E-01	191	213	211	189
76	-0	-0	4	-0.	1.226E-01	213	233	231	211
8	-0	-0	4	-0.	1.226E-01	10	31	29	8
14	-0	-0	4	-0.	1.226E-01	31	51	49	29
20	-0	-0	4	-0.	1.226E-01	51	72	70	49
30	-0	-0	4	-0.	1.226E-01	72	93	91	70
36	-0	-0	4	-0.	1.226E-01	93	113	111	91
42	-0	-0	4	-0.	1.226E-01	113	134	132	111
52	-0	-0	4	-0.	1.226E-01	134	155	153	132

58	-0	4	-0.	1.220E-01	155	175	173	153	164	174	163	154
64	-0	4	-0.	1.220E-01	175	196	194	173	184	195	183	174
74	-0	4	-0.	1.220E-01	196	217	215	194	206	210	205	195
80	-0	4	-0.	1.225E-01	217	237	235	215	226	230	225	216
10	-0	4	-0.	6.130E-02	15	35	33	13	22	34	21	14
10	-0	4	-0.	6.130E-02	35	55	53	33	42	54	41	34
22	-0	4	-0.	6.130E-02	55	77	75	53	62	70	61	54
32	-0	4	-0.	6.130E-02	77	97	95	75	84	90	83	76
38	-0	4	-0.	6.131E-02	97	117	115	95	104	110	103	96
44	-0	4	-0.	6.130E-02	117	139	137	115	124	134	123	116
54	-0	4	-0.	6.130E-02	139	159	157	137	146	156	145	138
60	-0	4	-0.	6.130E-02	159	179	177	157	166	176	165	158
66	-0	4	-0.	6.130E-02	179	201	199	177	186	200	185	178
76	-0	4	-0.	6.130E-02	201	221	219	199	208	220	207	200
82	-0	4	-0.	6.130E-02	221	241	239	219	228	240	227	220
5	-0	3	9.000E+01	5.833E-01	3	25	23	1	17	24	16	2
7	-0	3	9.000E+01	5.833E-01	8	29	25	3	19	28	17	6
9	-0	3	9.000E+01	5.833E-01	13	33	29	8	21	32	19	11
11	-0	3	9.000E+01	5.833E-01	25	45	43	23	37	44	36	24
13	-0	3	9.000E+01	5.833E-01	29	49	45	25	39	48	37	28
15	-0	3	9.000E+01	5.833E-01	33	53	49	29	41	52	39	32
17	-0	3	9.000E+01	5.833E-01	45	65	63	43	57	64	56	44
19	-0	3	9.000E+01	5.833E-01	49	70	65	45	59	68	57	48
21	-0	3	9.000E+01	5.833E-01	53	75	70	49	61	73	59	52
27	-0	3	9.000E+01	5.833E-01	65	87	85	63	79	86	78	64
29	-0	3	9.000E+01	5.833E-01	70	91	87	65	81	90	79	68
31	-0	3	9.000E+01	5.833E-01	75	96	91	70	83	94	81	73
33	-0	3	9.000E+01	5.833E-01	87	107	105	85	99	106	98	86
35	-0	3	9.000E+01	5.833E-01	91	111	107	87	101	110	99	90
37	-0	3	9.000E+01	5.833E-01	95	115	111	91	103	114	101	94
39	-0	3	9.000E+01	5.833E-01	107	127	125	105	119	126	118	106
41	-0	3	9.000E+01	5.833E-01	111	132	127	107	121	130	119	110
43	-0	3	9.000E+01	5.833E-01	115	137	132	111	123	135	121	114
49	-0	3	9.000E+01	5.833E-01	127	149	147	125	141	148	140	126
51	-0	3	9.000E+01	5.833E-01	132	153	149	127	143	152	141	130
53	-0	3	9.000E+01	5.833E-01	137	157	153	132	145	156	143	135
55	-0	3	9.000E+01	5.833E-01	149	169	167	147	161	168	160	148
57	-0	3	9.000E+01	5.833E-01	153	173	169	149	163	172	161	152
59	-0	3	9.000E+01	5.833E-01	157	177	173	153	165	176	163	156
61	-0	3	9.000E+01	5.833E-01	169	189	187	167	181	188	180	168
63	-0	3	9.000E+01	5.833E-01	173	194	189	169	183	192	181	172
65	-0	3	9.000E+01	5.833E-01	177	199	194	173	185	197	183	176
71	-0	2	-0.	5.833E-01	189	211	209	187	203	210	202	188
73	-0	2	-0.	5.833E-01	194	215	211	189	205	214	203	192
75	-0	2	-0.	5.833E-01	199	219	215	194	207	218	205	197
77	-0	2	-0.	5.833E-01	211	231	229	209	223	230	222	210
79	-0	2	-0.	5.833E-01	215	235	231	211	225	234	223	214
81	-0	2	-0.	5.833E-01	219	239	235	215	227	236	225	218

TABLE 5- ELEMENT LOADS  
NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 29

FROM	THRU	INCR	CASE	COND.	BOUNDARY VALUES		
PT	PT		*	X Y Z	X	Y	Z

3	5	1	1	0	0	1	-0.	-0.	-0.
6	10	1	1	0	0	1	-0.	-0.	-0.
141	144	1	1	0	0	1	-0.	-0.	-0.
13	53	20	1	0	0	1	-0.	-0.	-0.
75	115	20	1	0	0	1	-0.	-0.	-0.
137	177	20	1	0	0	1	-0.	-0.	-0.
199	219	20	1	0	0	1	-0.	-0.	-0.
15	55	20	1	0	0	1	-0.	-0.	-0.
77	117	20	1	0	0	1	-0.	-0.	-0.
139	179	20	1	0	0	1	-0.	-0.	-0.
201	241	20	1	0	0	1	-0.	-0.	-0.
21	61	20	1	0	0	1	-0.	-0.	-0.
83	123	20	1	0	0	1	-0.	-0.	-0.
145	185	20	1	0	0	1	-0.	-0.	-0.
22	62	20	1	0	0	1	-0.	-0.	-0.
22	62	20	1	0	0	1	-0.	-0.	-0.
84	124	20	1	0	0	1	-0.	-0.	-0.
146	186	20	1	0	0	1	-0.	-0.	-0.
208	228	20	1	0	0	1	-0.	-0.	-0.
14	54	20	1	1	0	1	-0.	-0.	-0.
76	116	20	1	1	0	1	-0.	-0.	-0.
138	178	20	1	1	0	1	-0.	-0.	-0.
200	220	20	1	1	0	1	-0.	-0.	-0.
239	241	2	1	0	0	1	-0.	-0.	-0.
229	231	1	1	0	0	1	-0.	-0.	-0.
233	235	1	1	0	0	1	-0.	-0.	-0.
237	238	1	1	0	0	1	-0.	-0.	-0.
240	-0	-0	1	1	1	1	-0.	-0.	-0.
232	236	4	1	0	1	1	-0.	-0.	-0.

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS = GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 4

ELEMENT LOADS TO BE ADDED (1 = YES) 0

FROM	THRU	INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
PT	PT		X	Y	Z	X	Y	Z
181	-0	-0	-0.	-0.	2.560E+03	-0.	-0.	-0.
183	-0	-0	-0.	-0.	3.550E+03	-0.	-0.	-0.
189	-0	-0	-0.	-0.	8.500E+02	-0.	-0.	-0.
194	-0	-0	-0.	-0.	1.190E+03	-0.	-0.	-0.

PROGRAM SHELL 6 - MASTER DECK - ABDELRAOUF, MATLOCK REVISION DATE 6 MAY 1971  
 SHELL 6 REPORT CODED BY ABDELRAOUF APRIL 4, 1971  
 EXAMPLE PROBLEMS

PROB 201 BOX GIRDER BRIDGE. ( LB - FT UNITS )

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS 60  
 NUM OF POINTS 174  
 NUM OF LOAD CASES 1  
 ELEMENT FORCES REQUIRED ( 1 = YES ) 1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 1

MAT TYPE	X DIRECTION		Y DIRECTION		SHEAR MODULUS G
	EX	VXY	EY	VYX	
1	4.320E+08	1.500E-01	4.320E+08*	1.500E-01*	1.878E+08*

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 8

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	157	26	0.	0.	0.	0.	6.000E+01	0.
3	159	26	0.	0.	3.000E+00	0.	6.000E+01	3.000E+00
6	162	26	9.333E+00	0.	0.	9.333E+00	6.000E+01	0.
8	164	26	9.333E+00	0.	3.000E+00	9.333E+00	6.000E+01	3.000E+00
11	167	26	1.867E+01	0.	0.	1.867E+01	6.000E+01	0.
13	169	26	1.867E+01	0.	3.000E+00	1.867E+01	6.000E+01	3.000E+00
16	172	26	2.800E+01	0.	0.	2.800E+01	6.000E+01	0.
18	174	26	2.800E+01	0.	3.000E+00	2.800E+01	6.000E+01	3.000E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 20

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES															
						1	6	32	27	4	21	30	19	131	136	162	157	134	151	160	149
1	51	10	1	-0.	4.583E-01	1	6	32	27	4	21	30	19	131	136	162	157	134	151	160	149
2	52	10	1	-0.	4.583E-01	6	11	37	32	9	23	35	21	136	141	167	162	139	153	165	151
3	53	10	1	-0.	4.583E-01	11	16	42	37	14	25	40	23	141	146	172	167	144	155	170	153
4	54	10	1	-0.	6.667E-01	1	27	29	3	19	28	20	2	131	157	159	133	149	158	150	132

5	10	1	-0.	6.667E-01	6	32	34	8	21	33	22	7	
6	55	10	1	-0.	6.667E-01	136	162	164	138	151	163	152	137
7	56	10	1	-0.	6.667E-01	11	37	39	13	23	38	24	12
8	57	10	1	-0.	6.667E-01	141	167	169	143	153	168	154	142
9	58	10	1	-0.	5.417E-01	16	42	44	18	25	43	26	17
10	59	10	1	-0.	5.417E-01	146	172	174	148	155	173	156	147
	60	10	1	-0.	5.417E-01	3	8	34	29	5	22	31	20
						133	138	164	159	135	152	161	150
						8	13	39	34	10	24	36	22
						138	143	169	164	140	154	166	152
						13	18	44	39	15	26	41	24
						143	148	174	169	145	156	171	154

TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 4

FROM PT	THRU PT	INCR	CASE *	COND. **			X	BOUNDARY VALUES		
				X	Y	Z		X	Y	Z
1	18	1	1	1	1	1	-0.	-0.	-0.	
1	18	1	2	1	1	1	-0.	-0.	-0.	
157	174	1	1	1	0	1	-0.	-0.	-0.	
157	174	1	2	0	1	0	-0.	-0.	-0.	

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
 CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
 CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
 CONDITION = 2 FOR ELASTIC RESTRAINTS

APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 1

ELEMENT LOADS TO BE ADDED ( 1 = YES ) 0

FROM PT	THRU PT	INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
			X	Y	Z	X	Y	Z
96	-0	-0	-0.	-0.	-1.000E+03	-0.	-0.	-0.

PROGRAM SHELL 6 - MASTER DECK - ABDELRAOUF, MATLOCK REVISION DATE 6 MAY 1971  
 SHELL 6 REPORT CODED BY ABDELRAOUF APRIL 4, 1971  
 EXAMPLE PROBLEMS

PROB CONTD  
 201 BOX GIRDER BRIDGE. ( LB - FT UNITS )

LOAD CASE 1  
 COMPUTED OR SPECIFIED NODAL POINT DISPLACEMENTS  
 ( GLOBAL COORDINATES )

NODE	DISPLACEMENTS IN DIRECTIONS			ROTATIONS ABOUT AXES		
	X	Y	Z	X	Y	Z
1	0.	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.
15	0.	0.	0.	0.	0.	0.
16	0.	0.	0.	0.	0.	0.
17	0.	0.	0.	0.	0.	0.
18	0.	0.	0.	0.	0.	0.
19	-7.342E-07	-1.201E-06	-2.436E-06	-8.642E-07	4.309E-07	1.673E-07
20	5.913E-07	1.039E-06	-2.429E-06	-8.656E-07	3.806E-07	-1.308E-07
21	-8.791E-07	-1.457E-06	-3.538E-06	-1.224E-06	4.917E-07	1.913E-07
22	7.173E-07	1.284E-06	-3.529E-06	-1.224E-06	4.216E-07	-1.498E-07
23	-1.271E-06	-3.009E-06	-7.399E-06	-2.553E-06	7.423E-07	2.685E-07
24	1.060E-06	2.663E-06	-7.382E-06	-2.552E-06	7.015E-07	-2.095E-07
25	-1.075E-06	-6.423E-06	-1.577E-05	-5.346E-06	6.629E-07	2.472E-07
26	8.839E-07	5.718E-06	-1.574E-05	-5.348E-06	6.772E-07	-1.895E-07
27	-1.115E-06	-1.370E-06	-7.688E-06	-1.193E-06	6.575E-07	-2.063E-08
28	-1.066E-07	-1.231E-07	-7.675E-06	***	6.812E-07	2.972E-09
29	8.586E-07	1.553E-06	-7.686E-06	-1.193E-06	5.975E-07	2.253E-08
30	-1.118E-06	-1.596E-06	-9.100E-06	-1.423E-06	1.490E-07	***
31	8.603E-07	1.390E-06	-9.050E-06	-1.405E-06	1.674E-07	***
32	-1.257E-06	-2.365E-06	-1.081E-05	-1.680E-06	7.644E-07	-3.420E-08
33	-1.311E-07	-1.370E-07	-1.079E-05	***	7.388E-07	5.500E-09
34	9.787E-07	2.083E-06	-1.081E-05	-1.677E-06	7.395E-07	3.085E-08
35	-1.467E-06	-2.822E-06	-1.626E-05	-2.574E-06	1.465E-06	***
36	1.161E-06	2.493E-06	-1.593E-05	-2.517E-06	1.423E-06	***
37	-1.744E-06	4.921E-06	-2.257E-05	-3.523E-06	1.190E-06	-6.357E-08
38	-2.479E-07	-2.741E-07	-2.254E-05	***	8.838E-07	1.355E-08
39	1.398E-06	4.357E-06	-2.257E-05	-3.522E-06	1.384E-06	4.638E-08
40	-1.819E-06	-6.170E-06	-3.536E-05	-5.543E-06	3.768E-06	***
41	1.460E-06	5.474E-06	-3.534E-05	-5.555E-06	3.633E-06	***
42	-1.682E-06	-1.060E-05	-4.809E-05	-7.469E-06	1.111E-06	-1.076E-08
43	-2.555E-07	-5.744E-07	-4.802E-05	***	8.665E-07	1.343E-08
44	1.332E-06	9.412E-06	-4.810E-05	-7.460E-06	1.323E-06	2.324E-09

45	-9.838E-07	-1.833E-06	-1.384E-05	-1.255E-06	5.662E-07	-4.408E-08
46	6.724E-07	1.562E-06	-1.384E-05	-1.253E-06	5.497E-07	5.901E-08
47	-1.103E-06	-2.613E-06	-1.978E-05	-1.874E-06	7.378E-07	-4.718E-08
48	7.708E-07	2.300E-06	-1.978E-05	-1.874E-06	8.455E-07	6.802E-08
49	-1.500E-06	-5.464E-06	-4.142E-05	-3.929E-06	1.217E-06	-5.701E-08
50	1.107E-06	4.838E-06	-4.142E-05	-3.934E-06	1.787E-06	9.345E-08
51	-1.743E-06	-1.220E-05	-8.941E-05	-8.738E-06	1.253E-06	2.365E-08
52	1.313E-06	1.082E-05	-8.939E-05	-8.799E-06	1.761E-06	1.880E-08
53	-6.233E-07	-1.656E-06	-1.977E-05	-1.121E-06	3.477E-07	-1.002E-07
54	-1.860E-07	-1.197E-07	-1.977E-05	***	2.610E-07	1.686E-09
55	2.870E-07	1.412E-06	-1.977E-05	-1.118E-06	3.950E-07	9.398E-08
56	-6.370E-07	-1.712E-06	-2.400E-05	-1.392E-06	1.268E-06	***
57	2.973E-07	1.492E-06	-2.376E-05	-1.369E-06	1.176E-06	***
58	-7.253E-07	-2.461E-06	-2.884E-05	-1.720E-06	6.085E-07	-1.096E-07
59	-2.783E-07	-1.429E-07	-2.885E-05	***	1.230E-07	4.990E-09
60	3.680E-07	2.169E-06	-2.885E-05	-1.716E-06	8.743E-07	9.781E-08
61	-7.409E-07	-3.025E-06	-4.395E-05	-2.627E-06	4.852E-06	***
62	3.759E-07	2.675E-06	-4.308E-05	-2.545E-06	4.471E-06	***
63	-1.046E-06	-5.098E-06	-6.043E-05	-3.585E-06	1.209E-06	-1.619E-07
64	-5.903E-07	-2.835E-07	-6.044E-05	***	-1.742E-07	1.754E-08
65	6.306E-07	4.517E-06	-6.044E-05	-3.582E-06	2.227E-06	1.333E-07
66	-1.073E-06	-6.888E-06	-9.708E-05	-6.251E-06	1.182E-05	***
67	6.542E-07	6.110E-06	-9.726E-05	-6.227E-06	1.126E-05	***
68	-1.617E-06	-1.246E-05	-1.337E-04	-8.758E-06	1.356E-06	-1.606E-07
69	-5.934E-07	-6.568E-07	-1.339E-04	***	3.744E-07	1.378E-08
70	1.126E-06	1.106E-05	-1.339E-04	-8.646E-06	2.289E-06	1.357E-07
71	-1.741E-07	-1.224E-06	-2.469E-05	-8.574E-07	7.473E-08	-7.318E-08
72	-1.567E-07	1.052E-06	-2.470E-05	-8.569E-07	1.897E-07	7.623E-08
73	-1.954E-07	-1.821E-06	-3.642E-05	-1.304E-06	3.724E-07	-8.103E-08
74	-1.476E-07	1.608E-06	-3.643E-05	-1.302E-06	7.897E-07	8.745E-08
75	-1.065E-07	-3.565E-06	-7.590E-05	-2.650E-06	8.878E-07	-1.340E-07
76	-2.474E-07	3.160E-06	-7.591E-05	-2.659E-06	2.383E-06	1.476E-07
77	-4.101E-07	-9.241E-06	-1.750E-04	-7.216E-06	7.297E-07	-1.948E-07
78	4.908E-09	8.200E-06	-1.748E-04	-7.492E-06	2.044E-06	2.061E-07
79	1.086E-07	-6.658E-07	-2.805E-05	-4.831E-07	-8.669E-08	-4.727E-08
80	-2.185E-07	-3.558E-08	-2.806E-05	***	-2.905E-07	-1.476E-09
81	-4.307E-07	5.923E-07	-2.806E-05	-4.815E-07	6.740E-08	4.044E-08
82	1.094E-07	-6.280E-07	-3.433E-05	-5.676E-07	2.215E-06	***
83	-4.337E-07	5.625E-07	-3.394E-05	-5.603E-07	2.025E-06	***
84	5.293E-08	-9.290E-07	-4.146E-05	-6.837E-07	2.773E-07	-3.978E-08
85	-3.564E-07	-4.861E-08	-4.147E-05	***	-5.860E-07	-1.731E-11
86	-3.927E-07	8.289E-07	-4.147E-05	-6.835E-07	7.767E-07	3.218E-08
87	2.396E-07	-9.713E-07	-6.269E-05	-8.880E-07	7.116E-06	***
88	-5.631E-07	8.687E-07	-6.133E-05	-8.874E-07	6.472E-06	***
89	1.527E-07	-1.511E-06	-8.531E-05	-1.115E-06	8.622E-07	-2.875E-08
90	-8.279E-07	-8.014E-08	-8.534E-05	***	-1.491E-06	4.725E-10
91	-5.076E-07	1.346E-06	-8.534E-05	-1.115E-06	2.617E-06	2.171E-08
92	6.420E-07	-1.518E-06	-1.424E-04	-1.303E-06	1.884E-05	***
93	-9.391E-07	1.363E-06	-1.432E-04	-1.311E-06	1.808E-05	***
94	1.303E-08	-2.181E-06	-1.999E-04	-1.506E-06	5.632E-07	-9.903E-09
95	-8.953E-07	-1.102E-07	-2.005E-04	***	-1.240E-06	7.456E-10
96	-4.028E-07	1.955E-06	-2.018E-04	-1.509E-06	2.504E-06	7.156E-09
97	-2.616E-07	6.756E-08	-2.928E-05	-9.554E-09	-1.782E-07	-5.263E-09
98	-5.572E-07	-2.054E-08	-2.929E-05	-9.953E-09	-8.798E-09	8.148E-10
99	1.861E-07	1.948E-07	-4.301E-05	6.374E-08	2.000E-07	1.162E-08
100	-4.994E-07	-1.555E-07	-4.302E-05	6.177E-08	7.267E-07	-1.516E-08
101	1.680E-07	8.866E-07	-8.670E-05	6.102E-07	7.888E-07	8.442E-08
102	-5.007E-07	-7.707E-07	-8.672E-05	6.183E-07	2.400E-06	-9.296E-08
103	-3.378E-07	5.301E-06	-1.898E-04	4.401E-06	7.127E-07	1.861E-07
104	-6.829E-08	-4.661E-06	-1.896E-04	4.676E-06	2.088E-06	-1.920E-07
105	1.833E-07	8.746E-07	-2.807E-05	5.151E-07	-1.186E-07	2.151E-08
106	-1.908E-07	8.574E-08	-2.808E-05	***	-3.178E-07	-5.080E-09

107	-4.545E-07	-7.024E-07	-2.807E-05	5.136E-07	2.867E-08	-2.803E-08	169	0.	-6.794E-06	0.	4.995E-06	0.	9.349E-0P
108	1.429E-07	1.080E-06	-3.395E-05	6.741E-07	2.114E-06	***	170	0.	9.857E-06	0.	6.768E-06	0.	***
109	-4.206E-07	-9.109E-07	-3.359E-05	6.612E-07	1.933E-06	***	171	0.	-8.666E-06	0.	6.787E-06	0.	***
110	-1.867E-08	1.387E-06	-4.079E-05	8.512E-07	2.902E-07	3.534E-08	172	0.	1.285E-05	0.	8.543E-06	0.	-1.324E-07
111	-3.270E-07	9.189E-08	-4.080E-05	***	-4.792E-07	-6.434E-09	173	0.	7.810E-07	0.	***	0.	-1.800E-08
112	-2.843E-07	-1.202E-06	-4.079E-05	8.473E-07	7.594E-07	-3.802E-08	174	0.	-1.128E-05	0.	8.551E-06	0.	1.236E-07
113	-6.218E-08	2.055E-06	-5.966E-05	1.458E-06	6.327E-06	***							
114	-2.472E-07	-1.796E-06	-5.877E-05	1.380E-06	5.822E-06	***							
115	-5.170E-07	3.258E-06	-8.019E-05	2.097E-06	1.018E-06	9.042E-08							
116	-6.532E-07	1.926E-07	-8.023E-05	***	-6.738E-07	-1.806E-08							
117	1.428E-07	-2.867E-06	-8.021E-05	2.092E-06	2.259E-06	-7.682E-08							
118	-6.942E-07	5.205E-06	-1.205E-04	4.434E-06	1.319E-05	***							
119	3.036E-07	-4.575E-06	-1.208E-04	4.398E-06	1.253E-05	***							
120	-1.485E-06	9.458E-06	-1.611E-04	6.583E-06	1.332E-06	1.301E-07							
121	-6.391E-07	5.230E-07	-1.614E-04	***	2.119E-07	-1.303E-08							
122	9.922E-07	-8.343E-06	-1.614E-04	6.466E-06	2.385E-06	-1.117E-07							
123	1.034E-07	1.697E-06	-2.416E-05	1.048E-06	-7.017E-08	2.049E-08							
124	-3.258E-07	-1.407E-06	-2.417E-05	1.047E-06	4.609E-08	-3.340E-08							
125	-1.143E-07	2.498E-06	-3.465E-05	1.588E-06	2.838E-07	2.519E-08							
126	-1.383E-07	-2.179E-06	-3.466E-05	1.589E-06	6.612E-07	-4.290E-08							
127	-6.734E-07	5.152E-06	-6.644E-05	3.354E-06	8.962E-07	2.914E-08							
128	3.512E-07	-4.538E-06	-6.645E-05	3.359E-06	1.780E-06	-6.285E-08							
129	-1.489E-06	1.114E-05	-1.252E-04	7.684E-06	1.178E-06	-2.928E-08							
130	1.069E-06	-9.802E-06	-1.252E-04	7.744E-06	1.862E-06	-1.110E-08							
131	-1.148E-08	2.437E-06	-1.778E-05	1.513E-06	6.166E-09	1.431E-08							
132	-1.100E-07	1.921E-07	-1.779E-05	***	-1.009E-07	-8.240E-09							
133	-1.534E-07	-2.049E-06	-1.778E-05	1.510E-06	7.928E-08	-2.534E-08							
134	-6.597E-08	2.674E-06	-2.117E-05	1.811E-06	1.187E-06	***							
135	-1.062E-07	-2.298E-06	-2.097E-05	1.788E-06	1.091E-06	***							
136	-2.281E-07	3.437E-06	-2.521E-05	2.190E-06	2.847E-07	-3.278E-09							
137	-1.911E-07	2.131E-07	-2.522E-05	***	-1.144E-07	-1.237E-08							
138	3.429E-08	-3.006E-06	-2.521E-05	2.186E-06	5.353E-07	-4.497E-09							
139	-3.766E-07	4.606E-06	-3.570E-05	3.169E-06	3.468E-06	***							
140	1.662E-07	-4.046E-06	-3.535E-05	3.120E-06	3.228E-06	***							
141	-7.604E-07	6.603E-06	-4.732E-05	4.286E-06	7.845E-07	-3.985E-08							
142	-3.369E-07	3.889E-07	-4.735E-05	***	2.018E-08	-1.938E-08							
143	5.015E-07	-5.817E-06	-4.733E-05	4.282E-06	1.335E-06	4.247E-08							
144	-1.002E-06	8.839E-06	-6.594E-05	6.240E-06	5.957E-06	***							
145	7.177E-07	-7.774E-06	-6.610E-05	6.252E-06	5.663E-06	***							
146	-1.340E-06	1.222E-05	-8.501E-05	8.191E-06	9.992E-07	-7.342E-08							
147	-3.230E-07	7.346E-07	-8.505E-05	***	5.284E-07	-1.687E-08							
148	1.012E-06	-1.074E-05	-8.503E-05	8.181E-06	1.423E-06	7.258E-08							
149	-1.916E-08	2.894E-06	-9.422E-06	1.816E-06	9.891E-09	-2.961E-09							
150	-6.641E-08	-2.445E-06	-9.424E-06	1.817E-06	4.654E-08	-1.678E-08							
151	-1.347E-07	3.999E-06	-1.325E-05	2.568E-06	1.527E-07	-2.156E-08							
152	3.426E-08	-3.500E-06	-1.325E-05	2.570E-06	2.795E-07	-7.163E-09							
153	-4.121E-07	7.396E-06	-2.454E-05	4.788E-06	3.984E-07	-7.486E-08							
154	2.785E-07	-6.514E-06	-2.455E-05	4.791E-06	6.645E-07	2.661E-08							
155	-6.931E-07	1.265E-05	-4.296E-05	8.543E-06	4.987E-07	-1.450E-07							
156	5.252E-07	-1.111E-05	-4.296E-05	8.551E-06	6.957E-07	9.092E-08							
157	0.	3.110E-06	0.	1.940E-06	0.	-4.828E-09							
158	0.	2.343E-07	0.	***	0.	-9.518E-09							
159	0.	-2.636E-06	0.	1.936E-06	0.	-9.193E-09							
160	0.	3.325E-06	0.	2.273E-06	0.	***							
161	0.	-2.867E-06	0.	2.248E-06	0.	***							
162	0.	4.238E-06	0.	2.711E-06	0.	-3.632E-08							
163	0.	2.602E-07	0.	***	0.	-1.425E-08							
164	0.	-3.711E-06	0.	2.707E-06	0.	2.493E-08							
165	0.	5.549E-06	0.	3.771E-06	0.	***							
166	0.	-4.877E-06	0.	3.734E-06	0.	***							
167	0.	7.714E-06	0.	4.997E-06	0.	-9.715E-08							
168	0.	4.559E-07	0.	***	0.	-1.982E-08							

















TABLE 5- ELEMENT LOADS

NONE

TABLE 6- BOUNDARY CONDITIONS

NUMBER OF CARDS FOR THIS TABLE 72

FROM PT	THRU PT	INCR	CASE *	COND. ** X Y Z	X	Y	Z
1	-0	-0	1	1 1 1	-1.046E-06	-5.098E-06	-6.043E-05
1	-0	-0	2	1 1 1	-3.586E-06	1.209E-06	-1.619E-07
2	-0	-0	1	1 1 1	-4.912E-07	-4.397E-06	-6.892E-05
2	-0	-0	2	1 1 1	-3.194E-06	1.062E-06	-1.583E-07
3	-0	-0	1	1 1 1	-1.064E-07	-3.565E-06	-7.590E-05
3	-0	-0	2	1 1 1	-2.651E-06	8.878E-07	-1.340E-07
4	-0	-0	1	1 1 1	1.083E-07	-2.603E-06	-8.136E-05
4	-0	-0	2	1 1 1	-1.958E-06	8.880E-07	-9.190E-08
5	-0	-0	1	1 1 1	1.529E-07	-1.511E-06	-8.531E-05
5	-0	-0	2	1 1 1	-1.115E-06	8.622E-07	-2.876E-08
6	-0	-0	1	1 1 1	3.390E-07	-3.084E-07	-8.699E-05
6	-0	-0	2	1 1 1	-2.225E-07	7.876E-07	4.223E-08
7	-0	-0	1	1 1 1	1.681E-07	8.866E-07	-8.670E-05
7	-0	-0	2	1 1 1	6.103E-07	7.887E-07	8.442E-08
8	-0	-0	1	1 1 1	-4.150E-09	2.076E-06	-8.443E-05
8	-0	-0	2	1 1 1	1.384E-06	8.655E-07	9.148E-08
9	-0	-0	1	1 1 1	-5.168E-07	3.258E-06	-8.019E-05
9	-0	-0	2	1 1 1	2.097E-06	1.018E-06	9.043E-08
10	-0	-0	1	1 1 1	-9.950E-07	-5.519E-06	-7.876E-05
10	-0	-0	2	1 1 0	-4.919E-06	9.146E-06	-0.
14	-0	-0	1	1 1 1	-5.288E-07	3.943E-06	-1.004E-04
14	-0	-0	2	1 1 0	3.289E-06	1.021E-05	-0.
15	-0	-0	1	1 1 1	-1.073E-06	-6.888E-06	-9.708E-05
15	-0	-0	2	1 1 0	-6.251E-06	1.182E-05	-0.
23	-0	-0	1	1 1 1	-6.941E-07	5.205E-06	-1.205E-04
23	-0	-0	2	1 1 0	4.434E-06	1.319E-05	-0.
24	-0	-0	1	1 1 1	-1.280E-06	-9.200E-06	-1.154E-04
24	-0	-0	2	1 1 0	-7.505E-06	9.220E-06	-0.
28	-0	-0	1	1 1 1	-1.013E-06	7.043E-06	-1.408E-04
28	-0	-0	2	1 1 0	5.532E-06	1.036E-05	-0.
29	-0	-0	1	1 1 1	-1.617E-06	-1.246E-05	-1.337E-04
29	-0	-0	2	1 1 1	-8.758E-06	1.356E-06	-1.606E-07
37	-0	-0	1	1 1 1	-1.485E-06	9.458E-06	-1.611E-04
37	-0	-0	2	1 1 1	6.583E-06	1.332E-06	1.301E-07
38	-0	-0	1	1 1 1	-5.934E-07	-6.568E-07	-1.339E-04
38	-0	-0	2	0 1 1	-0.	3.744E-07	1.378E-08
42	-0	-0	1	1 1 1	-6.391E-07	5.230E-07	-1.614E-04
42	-0	-0	2	0 1 1	-0.	2.119E-07	-1.303E-08
43	-0	-0	1	1 1 1	1.126E-06	1.106E-05	-1.339E-04
43	-0	-0	2	1 1 1	-8.646E-06	2.289E-06	1.357E-07
51	-0	-0	1	1 1 1	9.921E-07	-8.343E-06	-1.614E-04
51	-0	-0	2	1 1 1	6.466E-06	2.385E-06	-1.117E-07
52	-0	-0	1	1 1 1	8.341E-07	8.164E-06	-1.156E-04
52	-0	-0	2	1 1 0	-7.465E-06	9.024E-06	-0.
56	-0	-0	1	1 1 1	5.818E-07	-6.202E-06	-1.411E-04
56	-0	-0	2	1 1 0	5.462E-06	1.007E-05	-0.
57	-0	-0	1	1 1 1	6.542E-07	6.110E-06	-9.726E-05
57	-0	-0	2	1 1 0	-6.227E-06	1.126E-05	-0.

PROB 202 STUDY OF THE STRESSES IN THE VICINITY OF THE LOAD, PROBLEM 201.

TABLE 1- GENERAL PROBLEM INFORMATION

NUM OF ELEMENTS	20
NUM OF POINTS	79
NUM OF LOAD CASES	1
ELEMENT FORCES REQUIRED ( 1 = YES )	1

TABLE 2- MATERIAL ELASTIC PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 1

MAT TYPE	X DIRECTION EX	VXY	Y DIRECTION EY	VYX	SHEAR MODULUS G
1	4.320E+08	1.500E-01	4.320E+08*	1.500E-01*	1.878E+08*

( \* ) ASSUMED VALUES

TABLE 3- NODAL POINT COORDINATES

NUMBER OF CARDS FOR THIS TABLE 6

FROM PT	THRU PT	INCR	STARTING POINT COORDINATES			END POINT COORDINATES		
			X	Y	Z	X	Y	Z
1	9	2	0.	0.	0.	0.	0.	2.000E+01
15	23	2	4.667E+00	0.	0.	4.667E+00	2.000E+01	0.
29	37	2	9.333E+00	0.	0.	9.333E+00	2.000E+01	0.
43	51	2	9.333E+00	0.	3.000E+00	9.333E+00	2.000E+01	3.000E+00
57	65	2	4.667E+00	0.	3.000E+00	4.667E+00	2.000E+01	3.000E+00
71	79	2	0.	0.	3.000E+00	0.	2.000E+01	3.000E+00

TABLE 4 - ELEMENT PROPERTIES

NUMBER OF CARDS FOR THIS TABLE 10

FROM ELMT	THRU ELMT	INCR	MAT TYPE	ANGLE	THICKNESS	ELEMENT NODES							
1		1	1	-0.	4.583E-01	1	15	17	3	10	16	11	2
	4	1	1	-0.	4.583E-01	7	21	23	9	13	22	14	8
5		1	1	-0.	4.583E-01	15	29	31	17	24	30	25	16
	8	1	1	-0.	6.667E-01	21	35	37	23	27	36	28	27
9		1	1	-0.	6.667E-01	29	31	45	43	30	39	44	38
	12	1	1	-0.	5.417E-01	35	37	51	49	36	42	50	41
13		1	1	-0.	5.417E-01	57	43	45	59	52	44	53	58
	16	1	1	-0.	5.417E-01	63	49	51	65	55	50	56	64
17		1	1	-0.	5.417E-01	71	57	59	73	66	58	67	72
	20					77	63	65	79	69	64	70	78

65	-0	-0	1	1	1	1	3.035E-07	-4.575E-06	-1.208E-04
65	-0	-0	2	1	1	0	4.398E-06	1.253E-05	-0.
66	-0	-0	1	1	1	1	5.864E-07	4.893E-06	-7.885E-05
66	-0	-0	2	1	1	0	-4.933E-06	8.992E-06	-0.
70	-0	-0	1	1	1	1	1.570E-07	-3.463E-06	-1.005E-04
70	-0	-0	2	1	1	0	3.275E-06	1.001E-05	-0.
71	-0	-0	1	1	1	1	6.306E-07	4.517E-06	-6.045E-05
71	-0	-0	2	1	1	1	-3.582E-06	2.227E-06	1.333E-07
72	-0	-0	1	1	1	1	1.143E-07	3.896E-06	-6.894E-05
72	-0	-0	2	1	1	1	-3.198E-06	2.295E-06	1.580E-07
73	-0	-0	1	1	1	1	-2.475E-07	3.160E-06	-7.591E-05
73	-0	-0	2	1	1	1	-2.659E-06	2.383E-06	1.476E-07
74	-0	-0	1	1	1	1	-4.549E-07	2.310E-06	-8.138E-05
74	-0	-0	2	1	1	1	-1.965E-06	2.490E-06	1.022E-07
75	-0	-0	1	1	1	1	-5.078E-07	1.346E-06	-8.534E-05
75	-0	-0	2	1	1	1	-1.115E-06	2.616E-06	2.172E-08
76	-0	-0	1	1	1	1	-5.839E-07	2.851E-07	-8.702E-05
76	-0	-0	2	1	1	1	-2.159E-07	2.499E-06	-5.200E-08
77	-0	-0	1	1	1	1	-5.009E-07	-7.707E-07	-8.672E-05
77	-0	-0	2	1	1	1	6.183E-07	2.400E-06	-9.296E-08
78	-0	-0	1	1	1	1	-2.587E-07	-1.821E-06	-8.445E-05
78	-0	-0	2	1	1	1	1.388E-06	2.320E-06	-1.012E-07
79	-0	-0	1	1	1	1	1.427E-07	-2.867E-06	-8.021E-05
79	-0	-0	2	1	1	1	2.092E-06	2.259E-06	-7.682E-08

\* CASE = 1 FOR SPECIFIED DISPLACEMENTS OR SPRING RESTRAINTS IN DIR. OF AXES  
CASE = 2 FOR SPECIFIED SLOPES OR ROTATIONAL RESTRAINTS ABOUT THE AXES

\*\* CONDITION = 0 FOR NO SPECIFICATION  
CONDITION = 1 FOR SPEC. DISPLACEMENT OR SLOPE  
CONDITION = 2 FOR ELASTIC RESTRAINTS

#### APPLIED LOADINGS

CONCENTRATED FORCES OR MOMENTS - GLOBAL COORDINATES

LOAD CASE 1

NUMBER OF CARDS FOR THIS TABLE 1

ELEMENT LOADS TO BE ADDED ( 1 = YES ) 0

FROM THRU		INCR	FORCES IN DIRECTIONS			MOMENTS ABOUT AXES		
PT	PT		X	Y	Z	X	Y	Z
47	-0	-0	-0.	-0.	-1.000E+03	-0.	-0.	-0.

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