EXPERIMENTAL EVALUATION OF SUBGRADE MODULUS AND
ITS APPLICATION IN SMALL-DIMENSION SLAB STUDIES

by

Qaiser S. Siddiqi
W. Ronald Hudson

Research Report Number 56-16

Development of Methods for Computer Simulation
of Beam-Columns and Grid-Beam and Slab Systems

Research Project 3-5-63-56

conducted for

The Texas Highway Department
Interagency Contract No. 4613-1007

in cooperation with the
U.S. Department of Transportation
Federal Highway Administration

by the

CENTER FOR HIGHWAY RESEARCH

THE UNIVERSITY OF TEXAS AT AUSTIN

April 1970

PREFACE

This report describes an experimental program developed in the laboratory
for the evaluation of the coefficient of subgrade reaction for use in the dis-
crete-element solution of small-dimension slabs on layered foundations (Ref
42). These discrete-element solutions are compared with the experimental slab
test described herein, the testing program for the slab having been developed
earlier (Ref 1).

This is the sixteenth in a series of reports that describe the work in
Research Project No. 3-5-63-56, entitled "Development of Methods for Computer
Simulation of Beam-Columns and Grid-Beam and Slab System." The project is
divided into two parts, one concerned primarily with bridge structures, and
the other with pavement slabs. The reader may find it particularly advanta-
geous to review Research Report No. 56-15 (see List of Reports) to gain further
information in the application of k-value in the discrete-element solution of
slabs on clay soil.

This report is a product of the combined efforts of many people. The
advice and assistance of Messrs. S. L. Agarwal, B. F. McCullough, and Harold
H. Dalrymple are greatly appreciated. The entire staff of the Center for
Highway Research at The University of Texas must be thanked for the coopera-
tion and contribution they provided in the preparation of this report. Thanks
are due to Art Frakes, Joye Linkous, Polly Kitchen, and others who assisted
with the manuscript.

The support of the Federal Highway Administration and the Texas Highway
Department is gratefully acknowledged.

<div align="right">

Qaiser S. Siddiqi

W. Ronald Hudson

</div>

LIST OF REPORTS

Report No. 56-1, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns" by Hudson Matlock and T. Allan Haliburton, presents a finite-element solution for beam-columns that is a basic tool in subsequent reports.

Report No. 56-2, "A Computer Program to Analyze Bending of Bent Caps" by Hudson Matlock and Wayne B. Ingram, describes the application of the beam-column solution to the particular problem of bent caps.

Report No. 56-3, "A Finite-Element Method of Solution for Structural Frames" by Hudson Matlock and Berry Ray Grubbs, describes a solution for frames with no sway.

Report No. 56-4, "A Computer Program to Analyze Beam-Columns under Movable Loads" by Hudson Matlock and Thomas P. Taylor, describes the application of the beam-column solution to problems with any configuration of movable non-dynamic loads.

Report No. 56-5, "A Finite-Element Method for Bending Analysis of Layered Structural Systems" by Wayne B. Ingram and Hudson Matlock, describes an alternating-direction iteration method for solving two-dimensional systems of layered grids-over-beams and plates-over-beams.

Report No. 56-6, "Discontinuous Orthotropic Plates and Pavement Slabs" by W. Ronald Hudson and Hudson Matlock, describes an alternating-direction iteration method for solving complex two-dimensional plate and slab problems with emphasis on pavement slabs.

Report No. 56-7, "A Finite-Element Analysis of Structural Frames" by T. Allan Haliburton and Hudson Matlock, describes a method of analysis for rectangular plane frames with three degrees of freedom at each joint.

Report No. 56-8, "A Finite-Element Method for Transverse Vibrations of Beams and Plates" by Harold Salani and Hudson Matlock, describes an implicit pro-cedure for determining the transient and steady-state vibrations of beams and plates, including pavement slabs.

Report No. 56-9, "A Direct Computer Solution for Plates and Pavement Slabs" by C. Fred Stelzer, Jr., and W. Ronald Hudson, describes a direct method for solving complex two-dimensional plate and slab problems.

Report No. 56-10, "A Finite-Element Method of Analysis for Composite Beams" by Thomas P. Taylor and Hudson Matlock, describes a method of analysis for composite beams with any degree of horizontal shear interaction.

Report No. 56-11, "A Discrete-Element Solution of Plates and Pavement Slabs Using a Variable-Increment-Length Model" by Charles M. Pearre, III, and W. Ronald Hudson, presents a method of solving for the deflected shape of freely discontinuous plates and pavement slabs subjected to a variety of loads.

Report No. 56-12, "A Discrete-Element Method of Analysis for Combined Bending and Shear Deformations of a Beam" by David F. Tankersley and William P. Dawkins, presents a method of analysis for the combined effects of bending and shear deformations.

Report No. 56-13, "A Discrete-Element Method of Multiple-Loading Analysis for Two-Way Bridge Floor Slabs" by John J. Panak and Hudson Matlock, includes a procedure for analysis of two-way bridge floor slabs continuous over many supports.

Report No. 56-14, "A Direct Computer Solution for Plane Frames" by William P. Dawkins and John R. Ruser, Jr., presents a direct method of solution for the computer analysis of plane frame structures.

Report No. 56-15, "Experimental Verification of Discrete-Element Solutions for Plates and Slabs" by Sohan L. Agarwal and W. Ronald Hudson, presents a comparison of discrete-element solutions with the small-dimension test results for plates and slabs, along with some cyclic data on the slab.

Report No. 56-16, "Experimental Evaluation of Subgrade Modulus and Its Application in Model Slab Studies" by Qaiser S. Siddiqi and W. Ronald Hudson, describes an experimental program developed in the laboratory for the evaluation of the coefficient of subgrade reaction for use in the solution of small dimension slabs on layered foundations based on the discrete-element method.

ABSTRACT

In this report available theories of subgrade reaction are briefly reviewed, and a testing program is developed to evaluate the value of soil support k from small plate load tests used in the discrete-element solution of small-dimension slab-on-foundation problems.

The results confirm that the k-value of soil depends not only upon the diameter of the plate, but also on the amount of soil deformation.

A small dimension slab was tested on a thin asphalt-stabilized layer under a center load up to 200 pounds. Deflections and stresses were measured for each solution based on the k-value determined from plate load tests on the layered system. The agreement between the two solutions is within 5 percent in the interior of the slab near the point of load application. The effect of cyclic loading to a constant deflection produce some permanent deformation in the layered foundation. By the tenth cycle the applied load appeared to start stabilizing.

The prediction of the load-deflection characteristics for a layered system based on Burmister's theory by the use of E_1 , the modulus of elasticity of the asphaltic material in the layer determined from the split tensile tests on asphalt concrete specimens, did not provide as effective a solution as the plate load tests on the layered system itself.

The provision of a thin layer of asphalt concrete over clay subgrade increased the composite k-value by 40 percent as compared to clay alone.

A small side study showed that temperature significantly affected the stiffness of the asphalt concrete which in turn affected the composite k-value of the two-layered system investigated.

KEY WORDS: clay (Taylor marl), deflection and stress, discrete-element solution, subgrade support, modulus of subgrade reaction k , plate load tests, small-dimension slab, static and cyclic load tests, temperature, two-layered system, experimental.

SUMMARY

This report briefly examines existing theories of subgrade reaction. A
testing program was used to evaluate the subgrade reaction k-values utilizing
small-dimension plate load tests on a clay subgrade and on clay with an asphalt-
stabilized subbase layer. The relationship between pressure and deflection
was found to be linear for small initial deflections of the plate and nonlinear
for higher deflections of the plate.

By placing a thin layer of asphalt concrete on the clay subgrade, the
k-value of the subgrade was improved by 40 percent, and thus for the same
load the slab was less deformed on the layered foundation than on clay alone.

A small-dimension slab was placed on the thin asphalt-stabilized layer
and tested under a center load up to 200 pounds. Deflections and stresses
were measured at selected points on the slab. Discrete-element slab solutions
were run for the experimental setup. The computed deflections and stresses
in the slab were within 5 percent of the measured values in the interior of
the slab near the point of loading. The major discrepancy in the comparison
was found in deflections at the corners of the slab. Computed deflections
were 20 to 23 percent higher than the measured deflections at the corners of
the slab, showing that a constant value of k is probably not fully repre-
sentative of the actual subgrade conditions existing at any point beneath
the slab.

In the test, the temperature of the asphalt layer significantly affected
the stiffness of the composite system being investigated. This effect was
used to provide support at various stiffness factors. This fact, however,
illstrates the variability of composite support available to a rigid pavement
over its life due to variation in the environment. Such variations must be
taken into account in some way if proper pavement design is to be evolved.

IMPLEMENTATION STATEMENT

This report describes part of an experimental program to study the parameters which affect the support value available to pavement slabs on stabilized subbase layers on clay subgrades. In the past, it has been very difficult to evaluate these so-called composite k-values for use in analysis and design. The results of this study provide additional information for estimating these composite support values. The results will be implemented by providing estimates of k-value for use in discrete-element slab analysis methods and in providing basic information for more complete and definitive studies of the problem in the future. Composite k-values such as this are absolutely essential in the rational design of rigid pavement systems which, according to modern design practice, almost always utilize layered support, usually involving a stabilized layer on a subgrade.

The data from this study are also being used in Research Project 3-8-66-98 to provide experimental data with which to evaluate a pavement design procedure that is being developed. The procedure is based on evaluation of material properties by indirect tensile testing. Such data are important to provide preliminary checks for theories such as these and are also important in designing subsequent, more comprehensive experiments.

TABLE OF CONTENTS

CHAPTER 1. INTRODUCTION

Highway and airport pavements are complex structures supported on foundations of soil. During the useful life of a pavement, materials in and soils beneath the pavement structure are subjected to different intensities of loads by the wheels of moving vehicles. The weight of this traffic is finally carried by the subgrade itself, which in turn provides support to the pavement structure. The behavior of subgrade soils under different loading conditions must be more fully investigated before a rational design for pavements is achieved.

It is important that the soil support for the pavement structure be determined and represented as accurately as possible for any theoretical or experimental approach to the problem.

Pavements or slabs-on-foundation can be represented by a physical model consisting of bars, springs, and torsion bars grouped in a system of orthogonal beams, as described by Hudson and Matlock (Ref 25) and Stelzer and Hudson (Ref 42). In the discrete-element model representation of slabs-on-foundation, the modulus of support k is based on the Winkler foundation and the method is capable of representing the modulus of support by a linear spring (initial tangent modulus) or by a nonlinear spring. Subgrade support for use as a linear k can be directly determined by conventional plate load tests as demonstrated by several investigators (Refs 23, 31, 33, and 47), but the evaluation of nonlinear foundation support needs further investigation.

Agarwal and Hudson (Ref 1) have used a linear value of k for clay, determined from the plate load tests on clay, in obtaining a satisfactory analytical solution for deflections and stresses in a small-dimension slab on clay. They verified the analytical method with the results of a small-dimension slab on clay.

The use of a cement or lime-stabilized layer or an asphaltic concrete layer over the clay soil can provide an increase in the subgrade support, and this increased value of k for the layered system provides more resistance to deformation of the slab as compared to clay alone.

1

A testing program was developed to determine k-values for a clay soil subgrade; a layered foundation for use in the discrete-element solutions for deflections and stresses in slabs on the clay subgrade and on the layered system. The latter consists of an asphalt-stabilized layer over the clay subgrade.

The purpose of this study is to evaluate a k-value for the layered system from the load-deflection data of the plate tests in order to investigate the use of discrete-element analytical methods. These methods would determine deflections and stresses at various points in a slab-on-layer foundation and verify the analytical solution with the results of a small-dimension slab test on the layered system.

A side study was conducted to investigate the effect of temperature on the stiffness of the asphalt-stabilized layer since the k-value of the layered system is directly influenced by the stiffness of the asphaltic material in the layer.

Chapter 2 presents a brief review of the theories of the subgrade reaction, the contributions of many investigators, and methods for determining the subgrade reaction k .

Chapter 3 describes the development of a test program mainly for the plate load tests including the tests conducted, instruments used for measurements of load, deflection, strains, materials used and their preparation, and testing procedures.

Chapter 4 presents the analysis of the plate load test data and values of k for the soil subgrade and the layered system. The chapter describes the prediction of load-deflection curves for layered systems based on Burmister's elastic-layered theory, and a comparison with the test data.

Chapter 5 describes the effects of the stiffness of the asphalt-stabilized layer due to change in temperature.

Chapter 6 presents an analysis of the slab test, including a comparison of the experimental deflections and principal stresses and the analytical solutions. Some observations from the cyclic test are included.

Chapter 7 presents conclusions and some recommendations suggested for future study.

Appendices include data from tests conducted in this study.

CHAPTER 2. THEORIES OF SUBGRADE SUPPORT

A review of the various theories involved in the behavior of subgrade soils acting as foundation support to the pavement structure is presented in this chapter. Various methods are described to determine the modulus of subgrade reaction k .

Theoretical Background

Subgrade soil is usually represented by one of three theories, the Winkler foundation theory, the elastic isotropic half-space theory, or a variation of the latter, the elastic-layered theory.

Winkler's Foundation

To study the behavior of subgrade under an application of load, Winkler (Ref 51) introduced a simplified assumption in 1867. His hypothesis assumes the subgrade to be a dense liquid represented by a bed of closely-spaced discrete springs. A loaded beam or slab resting on subgrade is supported by localized forces, each of which is proportional to the deflection of the spring at that point. By distributing these forces over a unit area, the subgrade support is represented as a unit pressure p , which is equal to a constant times the deflection w :

$$p = k \times w$$

The constant k is called modulus of subgrade reaction. In the expression Hertz, Murphy, Westergaard, Winkler, and Zimmerman (Refs 16, 35, 50, 51, and 52, respectively) assumed that the modulus is constant at every point, independent of the deflection, and the same at all points within the area of consideration. This theory thus assumes a linear relationship between pressure and deflection.

Westergaard (Ref 50) has used the dense liquid concept for representing the subgrade in the development of his equations for the determination of deflection and stresses in concrete pavements. The deflection of a pavement depends not only on its flexural rigidity but also on the stiffness of the support. To facilitate the mathematical treatment, Westergaard introduced the term "radius of relative stiffness," which has a lineal dimension and is a function of subgrade support in the form of modulus of subgrade reaction known as k . The radius of relative stiffness is expressed as

$$\ell = \sqrt[4]{\frac{Eh^3}{12(1 - v^2)k}} \qquad (2)$$

where

ℓ = radius of relative stiffness, inches ;

E = modulus of elasticity of slab, psi ;

v = Poisson's ratio of slab;

h = thickness of slab, inches;

k = modulus of subgrade reaction, lb/cu in.

The Westergaard formulas have certain limitations particularly in view of the difficulties experienced in determining the value of k .

It has been shown by Terzaghi (Ref 44) that a linear pressure-deflection relationship holds good in some soils up to one-half of their ultimate bearing capacities. Based on experimental observations, Terzaghi formulated an empirical expression for the coefficient of subgrade reaction k_{sb} for a beam of width B resting on sand:

$$k_{sb} = k_{sl} \left(\frac{2B}{B + 1} \right)^2 \qquad (3)$$

wherein k_{sl} is the coefficient of subgrade reaction for a beam with a width of one foot. This expression is valid for contact pressure smaller than one-half of the ultimate bearing capacity of the subgrade.

Elastic Isotropic Solid Theory

In other theories (Refs 6 and 45), the soil is regarded as an elastic, isotropic, and homogeneous semi-infinite half-space. With this assumption, those characteristics of the soil which influence the stresses in the pavements are the modulus of elasticity E and Poisson's ratio ν . Boussinesq (Ref 6) developed an expression for deflection w , due to a pressure p, uniformly distributed over a circular area (radius r) and applied to the surface of a semi-infinite body:

$$w = \frac{pr(1 - \nu^2)}{\pi E} \tag{4}$$

In addition to the characteristics of the slab, this vertical deformation of the semi-infinite body is an important factor in determining the distribution of pressure between the slab and the subgrade.

Hogg (Ref 19) and Holl (Ref 20) represented the subgrade as a semi-infinite solid. They independently analyzed for deflection of a thin elastic plate of infinite size resting on a semi-infinite elastic foundation.

Bergstrom (Ref 4) in 1946 formulated equations for deflections of a circular slab of finite size on an elastic solid. Unable to integrate the resulting equations, he used a method of approximation, and obtained numerical results for the case of a circular slab under a centrally applied load.

Biot (Ref 5) presented a theory of bending of beams resting on an elastic isotropic solid. He expressed the subgrade support factor k by the equation

$$k = 1.23 \left[\frac{1}{c(1 - \nu_s^2)} \times \frac{E_s b^4}{E_b I} \right]^{0.11} \times \frac{E_s}{c(1 - \nu_s^2)} \tag{5}$$

where

k = coefficient of subgrade reaction, in lb/cu in;

E_s = modulus of elasticity of subgrade, in psi;

b = half-width of beam, in inches;

E_b = modulus of elasticity of beam, in psi;

6

$$I \quad = \quad \text{moment of inertia of beam, in in.}^{4};$$

$$c \quad = \quad \text{fundamental length of beam, inches;}$$

$$\nu_s \quad = \quad \text{Poisson's ratio of subgrade.}$$

Vesic (Ref 49) extended the well-known Biot solution to include an infinite beam resting on a semi-infinite elastic solid and approximately evaluated the integrals appearing in the resulting equations for the bending of beams on elastic solid. He showed that the Winkler hypothesis is practically satisfied for any determined beam of infinite length resting on a semi-infinite elastic subgrade. He concluded that any problem of bending of an infinite beam having a stiffness $E_b I$ and a width B and resting on a semi-infinite subgrade defined by a Young's modulus E_s and a Poisson's ratio ν_s can be treated with reasonable accuracy by the conventional analysis using a coefficient of subgrade reaction k given by the following expression where the terms are defined in Eq 5:

$$k_\infty B \;=\; K_\infty \;=\; 0.65 \; \sqrt[12]{\frac{E_s B^4}{E_b I}} \times \frac{E_s}{1 - \nu_s} \tag{6}$$

where

$$k_\infty \quad = \quad \text{coefficient of subgrade reaction per unit width of beam of infinite length,}$$

$$K_\infty \quad = \quad \text{coefficient of subgrade reaction of beam (width } B \text{) of infinite length.}$$

Similar empirical equations have been formulated by Benscoter, DeBeer, Habel, and Hetenyni (Refs 3, 11, 13, and 17, respectively) to represent support factor k for a semi-infinite elastic foundation.

Skempton (Ref 40) has developed a procedure for predicting the load-deflection curve in a plate load test on a saturated clay from the results of a laboratory compression test on the same material. He has expressed the equation based on elastic theory for determining the mean settlement of the plate as

$$w \;=\; pBi_p \; \frac{1 - \nu_s^2}{E_s} \tag{7}$$

where

w   =   settlement of plate, inches;

p   =   foundation pressure, psi;

B   =   breadth of foundation (diameter for circular footing), inches;

i_p =   influence value depending upon the shape and rigidity of plate;

ν_s =   Poisson's ratio of soil;

E_s =   modulus of elasticity of soil, psi.

Based on Eq 7, Skempton related the stress σ and strain ϵ of soil in a triaxial compression test to the pressure-deflection curve of soil obtained from plate load tests under the same loading conditions by the expressions:

$$p = 0.29\sigma \tag{8}$$

$$w = 2B\epsilon \tag{9}$$

Seed (Ref 39) has used these correlations to predict successfully the deflection of circular plates on subgrade soils under static and repetitive applications of load.

Elastic-Layered Theory

The theories described previously dealt with the assumption that the subgrade is a homogeneous, isotropic elastic half-space. The thin plate theory used by Westergaard, Hogg, and others neglects normal and shearing stresses in the plate. In reality the subgrade consists of many layers of soil of finite depth and even the pavements are made up of layers of different materials.

In 1943 Burmister (Ref 7) published the first fundamental calculation of deflections due to a uniformly distributed circular and vertical load on the surface of an elastic two-layered system. He assumed that each layer acts as a continuous, isotropic, homogeneous, linearly elastic medium which is infinite in horizontal extent, and is continuously supported by the layer beneath, with the interface conditions between layers either perfectly smooth or extremely rough. Deformations throughout the system are small.

With these assumptions Burmister formulated the following equations for calculating the deflection in an elastic mass of a two-layered system.

Using a flexible plate, the equation for deflection is:

$$w = 1.5 \frac{pa}{E_2} F \tag{10}$$

Using a rigid plate, the equation becomes:

$$w = 1.18 \frac{pa}{E_2} F \tag{11}$$

where

w = deflection of the plate, inches;

p = unit load on a circular plate, psi;

E_2 = modulus of elasticity of lower layer, psi;

a = radius of the plate, inches;

F = deflection factor, which is a function of the ratio of thickness of layer/radius of plate and the modular ratio of the materials in two layers E_1/E_2 .

In addition to the charts prepared by Burmister (Refs 7 and 8), tables and charts have also been developed by Hank and Scrivner (Ref 15), Fox (Ref 12), and Jones (Ref 26) for the determination of deflection and stresses in a two-layered system.

Methods for Determining the k-Value

It is noted from the theoretical background that the modulus of subgrade reaction plays an important role in the evaluation of deflections and stresses in pavement slabs and plates resting on soil. The modulus k is used in the Westergaard formulas for the deflections of the pavements and has a marked influence on the value of deflection. The modulus of subgrade reaction can be determined by both field tests and laboratory tests. The most common tests used are plate load tests and triaxial tests; however, correlations have been developed for California bearing ratio (CBR) tests and cone penetration tests with plate load tests.

Field plate load tests representing actual field conditions are quite reliable ways of determining k , but the data are only applicable to the conditions existing in the subgrade at the time of the test. Moreover, field tests are cumbersome, expensive, and time-consuming. McLeod (Ref 31) carried out an extensive testing program of plate bearing tests on the subgrade, base courses, and flexible surfaces of the runways at ten Canadian airports. He correlated the field load test data with cone bearing, Housel penetrometer, field California bearing ratio, and triaxial compression tests, and thus developed methods for predicting k-value from these other tests.

Plate load tests are the most frequently used tests for finding the value of k . Numerous investigators and highway agencies recommended these tests for the design of pavements, both rigid and flexible. In the light of the Westergaard analysis of stress conditions in concrete pavements, Teller and Sutherland (Ref 43) described the following three methods to measure the modulus of subgrade reaction under field conditions:

(1) Load-deflection tests in which loads are applied at the center of rigid circular plates of relatively small size, the pressure intensity being uniform over the entire area of the plate. The value of k is determined by the ratio of the applied pressure p and its corresponding mean vertical plate deflection w (same as Eq 1):

$$k = \frac{p}{w}$$

(2) Load-deflection tests in which the load is applied at the center of a slightly flexible rectangular or circular plate of relatively large dimensions. In this case some bending of the plate occurs and the pressure intensity under the plate is not uniform throughout the area of its contact with the soil. The load and the vertical deflection of various points throughout the area of the plate are measured. The shape of the deflected plate must be determined precisely and its vertical displacement measured in order to be able to estimate accurately the volumetric displacement of the soil that is affected by the application of the test load on the plate. The modulus of subgrade reaction is then computed by dividing the total applied load (in pounds) by the volume of the displaced soil (in cubic inches).

(3) Load-deflection tests on full size pavement slabs in which the load-deflection data are obtained by measurement and used in Westergaard deflection formulas to predict a value for the modulus of subgrade reaction, where all other factors must be known.

In plate load tests the rigidity and the size of the plate are important factors. It has been shown by various investigators (Refs 9, 31, and 43), that, within limits, the area of the plate had a marked effect on the value of the modulus k as determined from the plate bearing tests. The minimum size of plate that will give satisfactory data depends upon the soil structure being tested. It is important, therefore, to select carefully the size of the plate to be used in the determination of the k-value since plate size has a marked effect on k .

Methods of Approach for Finding k-Value from Load-Deflection Curves

Several approaches may be used to find the k-value from rigid plate tests data. The initial straight-line portion of the load-deflection curve is sometimes used to evaluate the value of k . A tangent is drawn to the initial part of the curve (Fig 1) which gives the value of k as a tangent modulus. It is a linear estimate of the load-deflection relationship identical to Winkler's assumption. This approach is probably realistic for small loads and deflections. Terzaghi (Ref 44) suggests that this approach may be approximately true for values of contact pressures up to one-half the ultimate bearing capacity of the soil.

Another approach uses the secant modulus. Points on a load-deflection curve beyond the initial straight position are selected, depending upon the deflection criteria considered. The ratio of load and deflection at each point gives an estimate of k . This value is always less than the value obtained from the tangent modulus approach because of the increase in deflection with respect to load, which is a nonlinear characteristic of the load-deflection curve, as typified in Fig 1. The secant modulus approach may be useful when the anticipated deflections in the slabs are relatively high with regard to the changing conditions existing in the foundation beneath the pavement and also when the repetitive application of load is being considered.

Research is underway at The University of Texas (Ref 27) to use the nonlinear load-deflection curve as a more realistic approach to subgrade evaluation in solving slabs-on-foundation. Points on a load-deflection curve are connected by straight lines (Fig 2) and they are input as a variable k in a discrete-element solution for deflections and stresses in plates on nonlinear foundations.
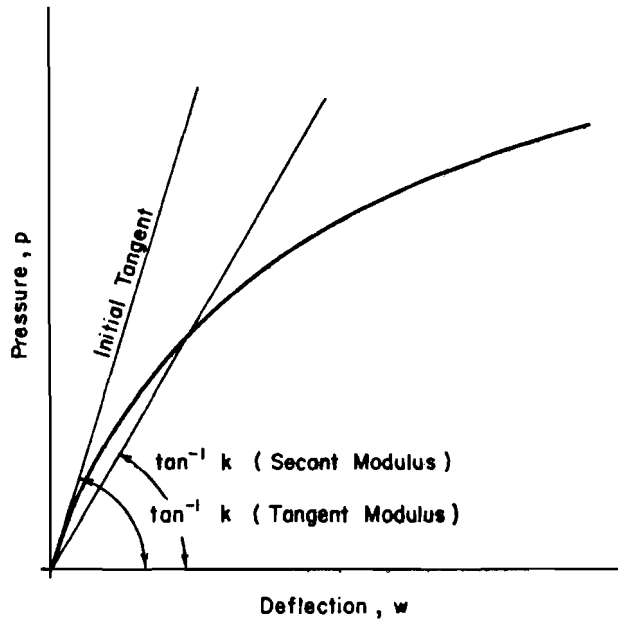
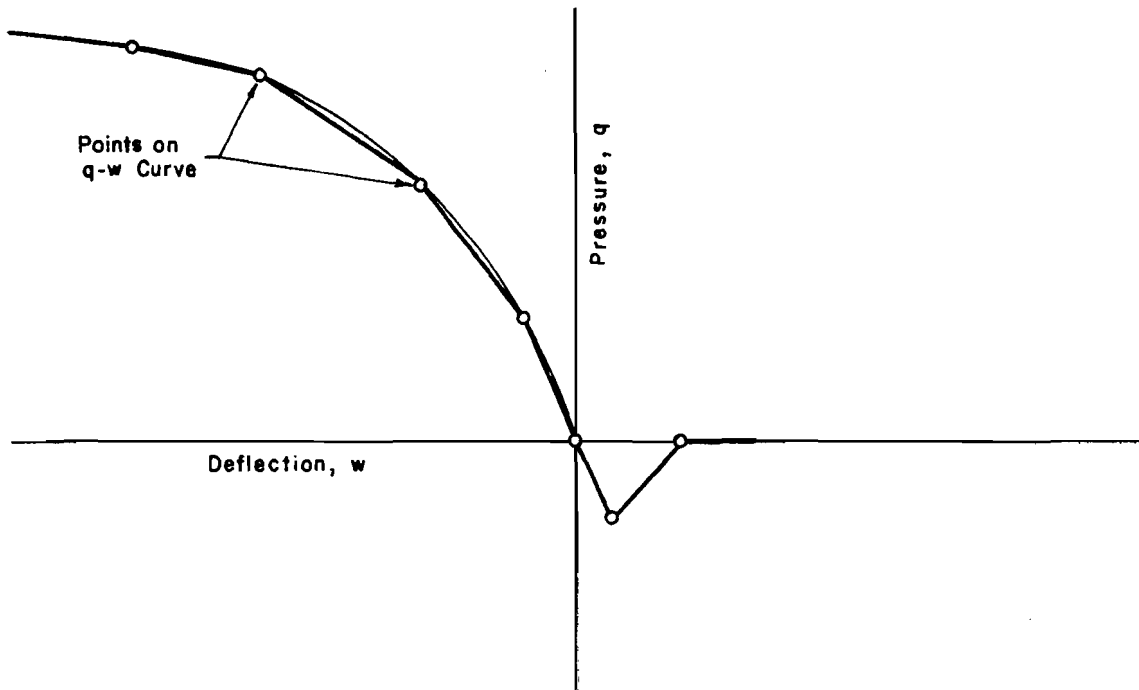Fig 1. Tangent and secant moduli approaches for finding k-value.

Fig 2. Use of load-deflection curve of soil to represent the soil support.

CHAPTER 3. DEVELOPMENT OF TEST PROGRAM

A testing program was developed to evaluate the k-value of a clay subgrade from small-scale plate load tests using circular rigid plates of diameters ranging from 2 to 9 inches. The k-value thus obtained was used in the discrete-element solution for deflections and principal stresses in a small dimension slab resting on the same clay subgrade and the solution was compared with the experimental results of slab tests on clay under static loads as reported by Agarwal and Hudson (Ref 1). Plate load tests were also conducted on the surface of an asphalt-stabilized layer over the clay to evaluate the k-value of the layered system. The small-dimension slab tested on clay (Ref 1) was subsequently tested on the same asphalt-stabilized layered system and results were compared with the analytical solutions using the k-value as determined from the plate load tests.

In this study, the following tests were conducted using local soil (Taylor marl clay) and a thin stabilized layer of asphalt over the clay subgrade:

(1) slab test on asphalt concrete layer with clay subgrade,

(2) plate load tests on clay, and

(3) plate load tests on asphalt-stabilized layer over clay subgrade.

The slab tests on clay are reported in detail by Agarwal and Hudson (Ref 1).

Details of Test Setup

The tests were conducted in a wooden box with inside dimensions of 24 by 24 inches and a height of 21 inches. The box was constructed of 3/4-inch plywood and was securely braced on all sides by five 1-1/2 by 4-inch wood sections. The inside of the box was coated with a waterproof paint. The top and bottom covers of the box were made detachable. Two steel straps with hooks were fixed to the sides of the box to facilitate lifting and overturning. A photograph of the box is shown in Fig 3.

Load Application

The loads were applied through a mechanical screw jack and were measured by a proving ring in the plate load tests and by a load cell in the slab test.
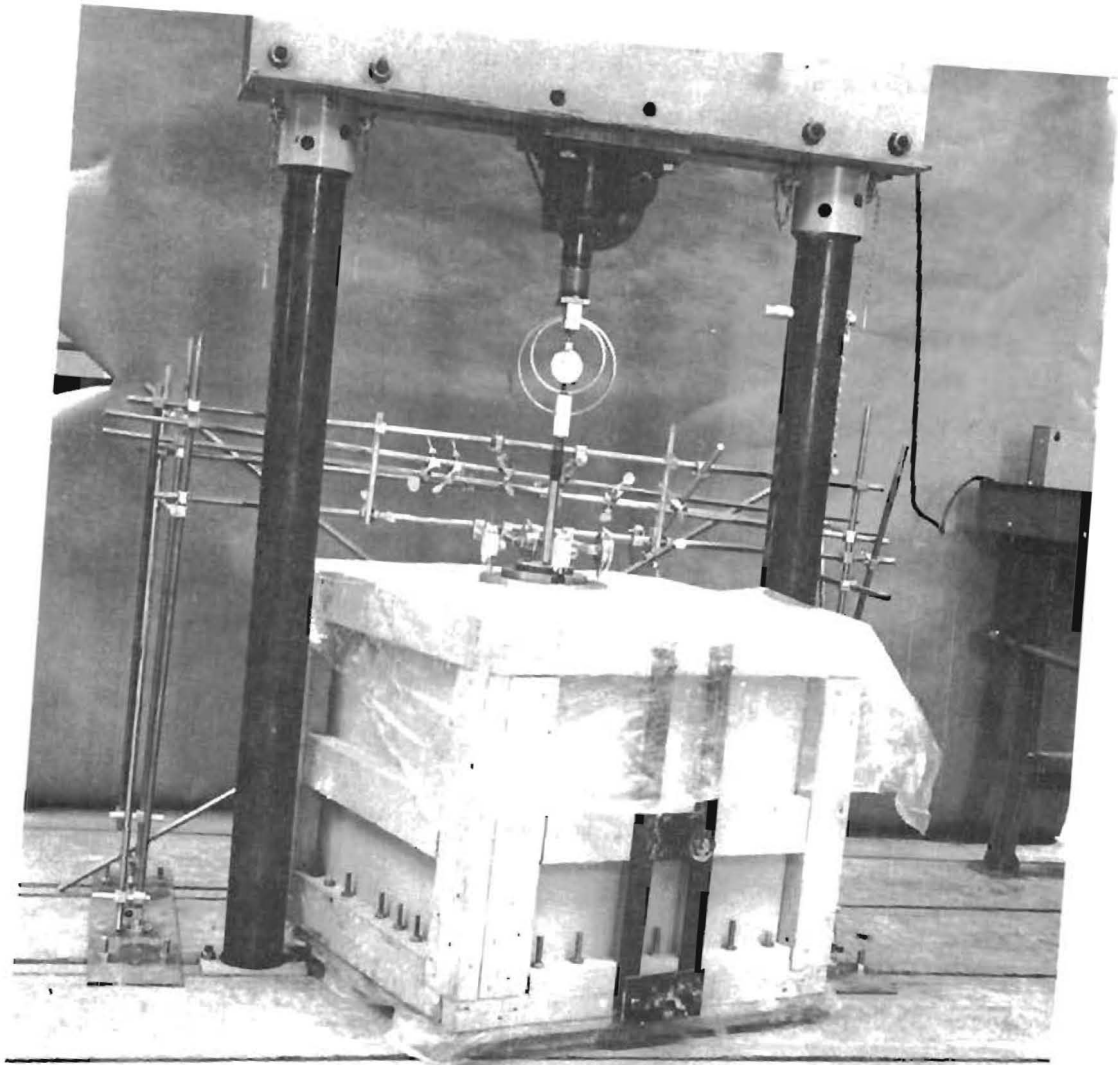
Fig 3.   Arrangement for plate load test on clay.

One proving ring, having a capacity of 500 pounds, was used for the first series of experiments. A second proving ring, having a capacity of 1,500 pounds, was required for measuring loads on top of the asphalt-stabilized layer. Both proving rings were calibrated before use. The proving rings could measure the loads to an accuracy of one pound. In the plate load tests on clay, the plates were first seated by applying a load equivalent to one psi and releasing it after a few seconds. In case of plate load tests on the asphaltic layer, a seating load was applied to produce a deflection of .02 inch. The load was repeated several times (4 to 6) until this deflection was stabilized. The plates were then loaded to a limiting deflection of .25 inch.

For the slab test on a layered system, load was applied through a screw jack, measured by a load cell, and recorded automatically on a digital volt-meter. The capacity of the load cell was 5,000 pounds with a resolution of two pounds. The load was applied through the screw jack at a constant rate of strain of .001 in/min.

Deflection Measurement

Deflections were measured by dial gages having an accuracy of 1/1000 inch and range of 1/2 inch. The dial gages were supported on steel rods and a rigid steel frame. The steel frame was rigidly fixed to the floor with steel plates and was not in contact with the box or the loading frame (Fig 3). In plate load tests, the dial gages were evenly spaced around the periphery of the circular plate. Four gages were used in the 9-inch-diameter plate test. For 4-inch and 6-inch-diameter plates, three dial gages were located at 120° intervals around the edge of the plate. The deflection was taken to be the average of all the dial readings used in one test.

In the slab test, deflections were measured at various points on the surface of the slab. Since load was applied at the center of the plate, it was not possible to measure the maximum plate deflection. However, a deflection gage was located at a point one inch from the center to measure the deflection at the nearest possible point toward the center. Additional dial gages were placed at one corner and at one point along the edge as check points. The schematic location of gages is given in Fig 4.
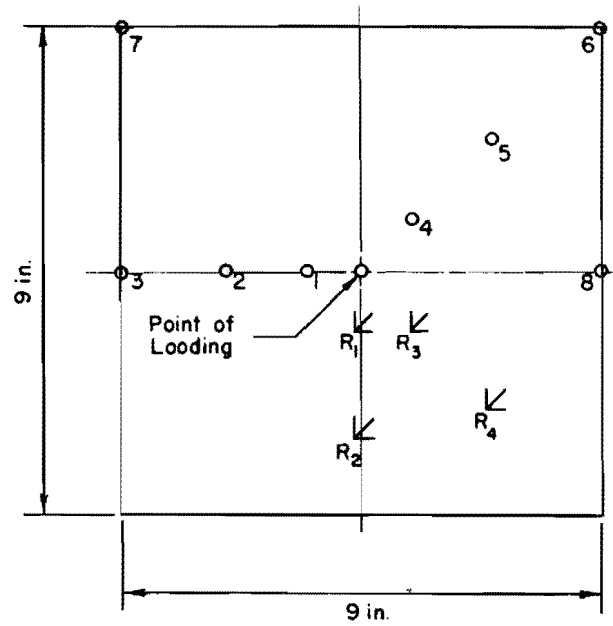
Dial Gage	Distance from Center
1	1-in. on Center Line
2	2.5-in. on Center Line
3	4.5-in. on Center Line
4	1.4-in. on Diagonal
5	3.5-in. on Diagonal
6	6.4-in. on Diagonal
7	6.4-in. on Diagonal
8	4.5-in. on Center Line
Rosettes	
R_1	1-in. on Center Line
R_2	3-in. on Center Line
R_3	1.4-in. on Diagonal
R_4	3.5-in. on Diagonal

Fig 4. Schematic location of dial gages and rosettes on aluminum slab.

Strain Measurement

Strains were measured in the slab test only. Four rosettes, each with gages oriented at 0°, 45°, and 90°, were fixed at four locations (Fig 4) on the slab surface to measure strains near the loaded area.

Data from the slab tests were recorded in digital form using a Honeywell data logging system. The system, described in detail by Agarwal and Hudson (Ref 1), recorded strains and the corresponding load in close time proximity on the 40-channel digital logging system. One example of typical data print-out is given in Appendix 3.

Preparation of the Soil

The clay used in the experiments is geologically classified as Taylor marl I and was obtained near Manor, Texas. A number of tests were performed to investigate its properties and workability. The Atterberg limits and other properties for this material are given in Table A1.1, Appendix 1.

Mixing Procedure

The clay was thoroughly dried in the oven at about 240° F, crushed to small pieces in a chipmunk crusher, and finally pulverized. The clay was then mixed with water, in a mechanical mixer, to a moisture content of 38 ± 1 percent. The mixed soil was placed in cans in plastic bags and stored in a 100 percent humidity room to maintain a fairly uniform moisture content in soil.

Extrusion and Placement of Soil in the Box

The soil was extruded from an extruding device at the soil mechanics laboratory of The University of Texas. The extruded soil came out in the shape of prismatic blocks 3 X 3 inches in section and 6 to 12 inches long (Fig 5). The extruding device provided a thorough mixing of the clay and removed much of the air thus producing a fairly uniform density in the clay. The extruded clay was cut into 6 to 12-inch lengths and placed in the box (Fig 6). Particular care was taken to insure close contact between the prismatic blocks. Kneading was sometimes necessary for close jointing of the blocks. The transverse joints were alternated by placing blocks of different lengths in adjoining rows. After a layer was placed, the soil was compacted by a 5 pound hammer dropping 18 inches and having a compacting surface of 6 by 6 inches.

18

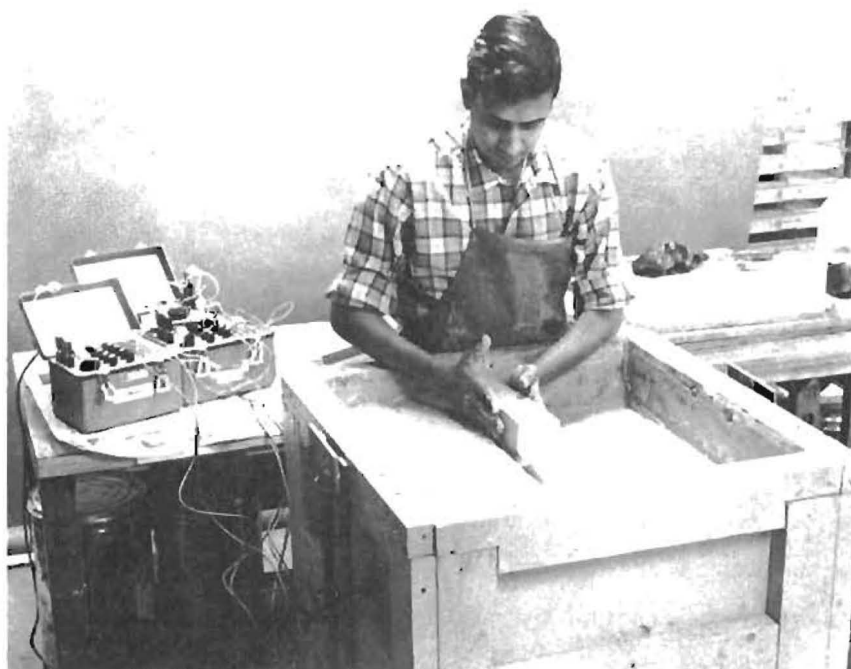Fig 5. Extrusion of soil in the form of prismatic block.

Fig 6. Placement of soil in the test box.

The surface was evenly compacted by two blows of the hammer, and this extra effort provided further consistency within the soil. Alternate layers were rotated 90°. The procedure was repeated after each layer was placed, until the box was filled to its entire depth.

The box was then covered with two polyethylene sheets to prevent moisture evaporation and tightly secured at the top with a wooden cover. On the day before the plate load tests, the surface of the soil in the box was carefully worked with a sharp steel straight edge to make it level. Special care was taken to accurately level the surface where the rigid plate rested. The leveling could be done within a very short time because of the excellent molding properties of clay.

Samples of soil for moisture content determination were taken from each layer of soil placed in the box during the extrusion of soil.

Preparation of the Asphalt Concrete Mix

To study the behavior of a small-dimension slab on a layered system, a thin asphalt-stabilized layer was used over the clay which was used in earlier tests. A slab test was performed on the asphalt-stabilized layer in the same way as the slab test on clay (Ref 1). The k-value for the system was evaluated from the plate load test results reported herein. Experimental results of the slab test conducted in this study were compared with the analytical solution based on the k-value evaluated earlier. Experimental results of the slab tests on clay and on the layered system were also compared for the point of the maximum measured deflections of slab (see Chapter 6).

Selection of the Aggregate

An aggregate with fine gradation was chosen so that a thin layer of the mix could be prepared relative to the thin slab being tested. A crushed limestone aggregate with gradation similar to Texas Highway Department specifications for Type E (Ref 41), fine-graded surface course, was used. The gradations selected are shown in Table A1.3, Appendix I, along with the Texas Highway Department specifications.

Optimum Asphalt Content

In general the strength of a stabilized mixture increases with the addition of asphalt until a maximum stability value is obtained. The asphalt content

at the maximum stability value is generally considered to be the optimum asphalt content.

To find optimum asphalt content for use in this study, specimens were prepared using four different asphalt contents, i.e., 7, 8, 9, and 10 percent by weight of the aggregate. The range of asphalt content was obtained from Texas Highway Department specifications (Ref 41). Asphalt AC-10 was selected. It is readily available and is widely used in hot mix asphalt mixtures. Its properties are given in Table 3, Appendix I.

The procedure for the preparation and testing of asphalt concrete specimens, briefly described in Appendix I, is the same as that recommended by Hudson and Kennedy (Refs 24 and 28) with slight modifications (Ref 14) and is presently in use as a standard procedure at The University of Texas.

Plots of density versus percent asphalt content and stress at failure versus percent asphalt content (Fig 7) gave an optimum asphalt content of 9 percent by weight of aggregate.

Preparation and Laying of the Mix

The properly graded aggregate and 9 percent AC-10 asphalt by weight of aggregate were preheated separately in an oven at 250° F $\pm$ 5° F for 2 to 4 hours and then mixed at 250° F $\pm$ 5° F for a period of 3 minutes in an automatic Hobart mixer (Fig 8). This procedure provided a thorough mixing of aggregate and asphalt.

To provide a better workability for the placement and compaction of asphalt concrete, the surface of the wooden cover of the test box was temporarily bounded by 2 by 2-inch wood sections as shown in Fig 9. An aluminum slab was placed in the center and level with the surface of the bottom cover. This procedure helped not only in achieving a uniform contact of slab with asphalt concrete, but also provided a better compaction of the mix. The hot mix was laid down immediately over the entire surface of the bottom cover of the test box and was compacted uniformly by a 6-inch-square compacting foot (Fig 10). Two layers of the mix were used to achieve the desired 2-inch thickness of the asphalt concrete. Special care was taken to keep the finished surface of the asphalt concrete as level as possible. This prepared layer of asphalt concrete was then cured for two days to gain the required strength. After two days, the wood sections were removed, and the box was placed on the cover with its open
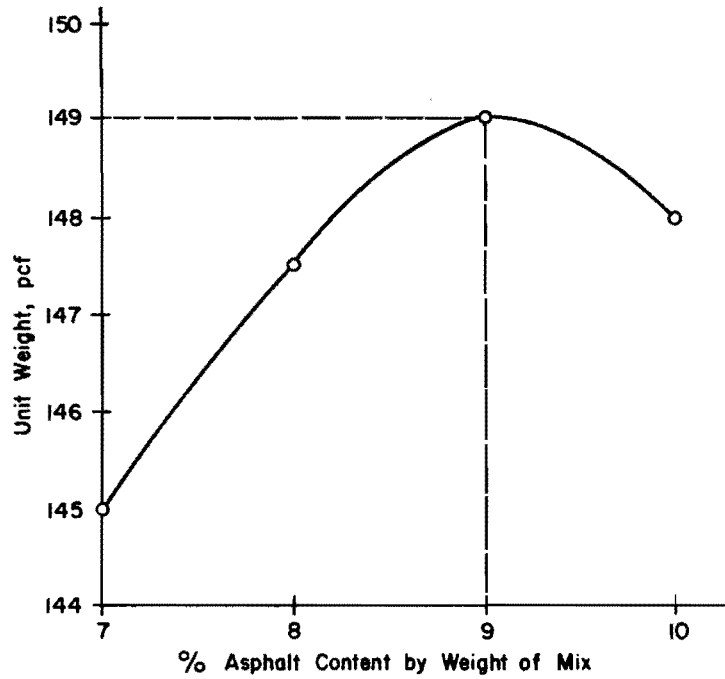
(a) Effect of asphalt content on unit weight of mix.
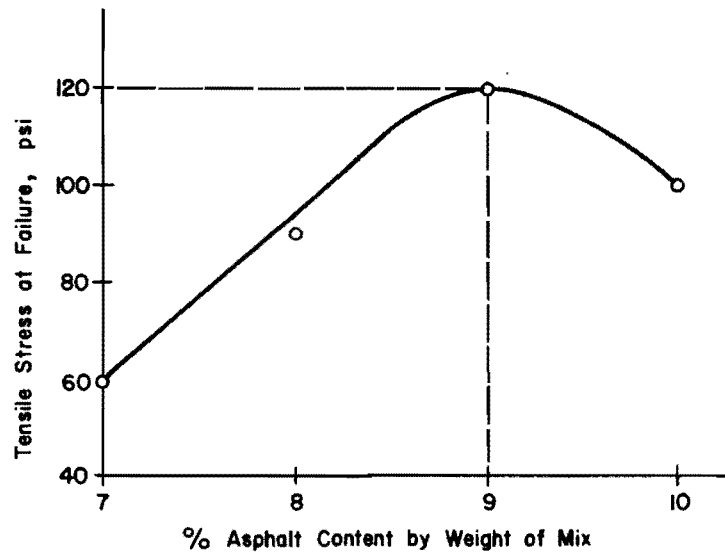
(b) Effect of asphalt content on tensile strength of asphalt concrete.

Fig 7. Determination of optimum asphalt content.
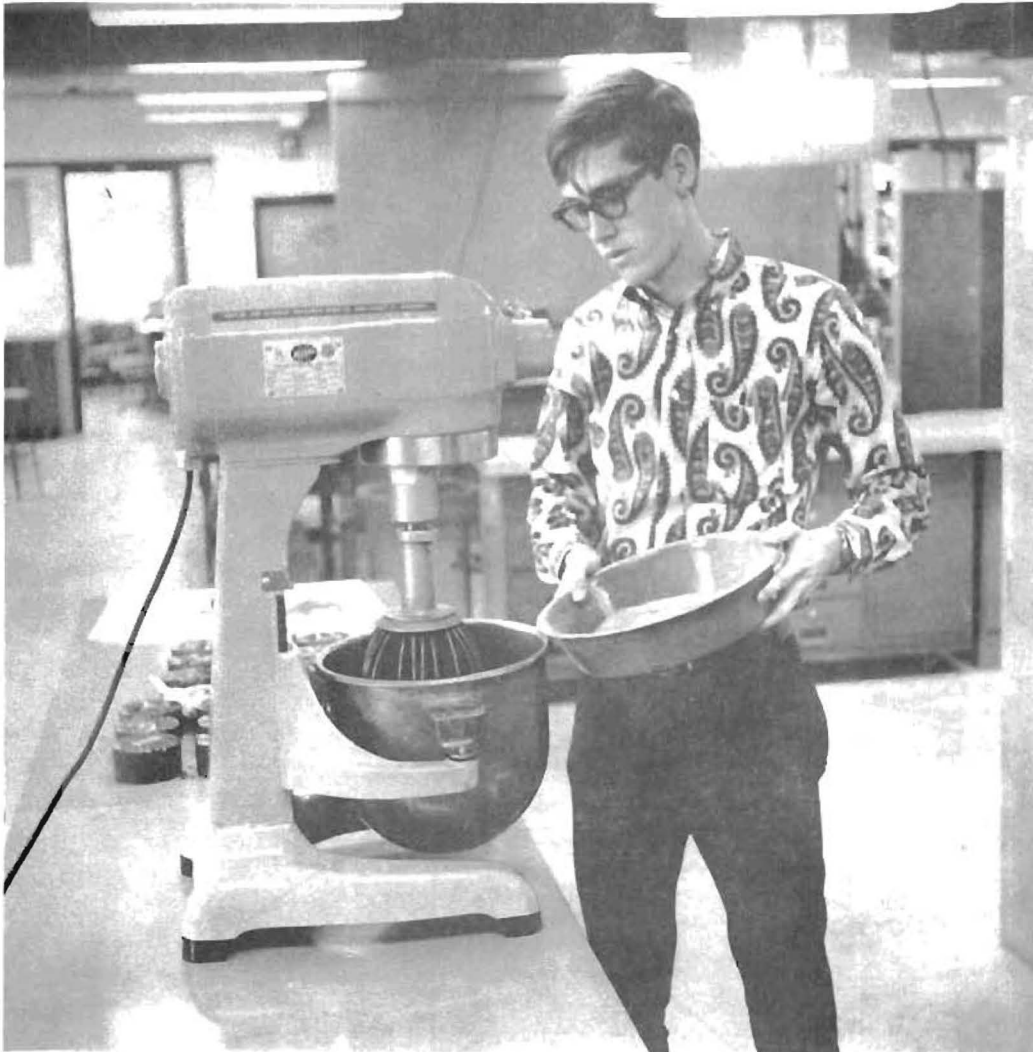
Fig 8.   Hobart mixer used for asphalt concrete.

9-in x 9-in Aluminum Slab
(Top in Level with
Adjoining Wooden Pieces)

2-in x 2-in.
Wood Section

23¾-in. x 23⅞-in. Inside
Clear Dimensions of Box
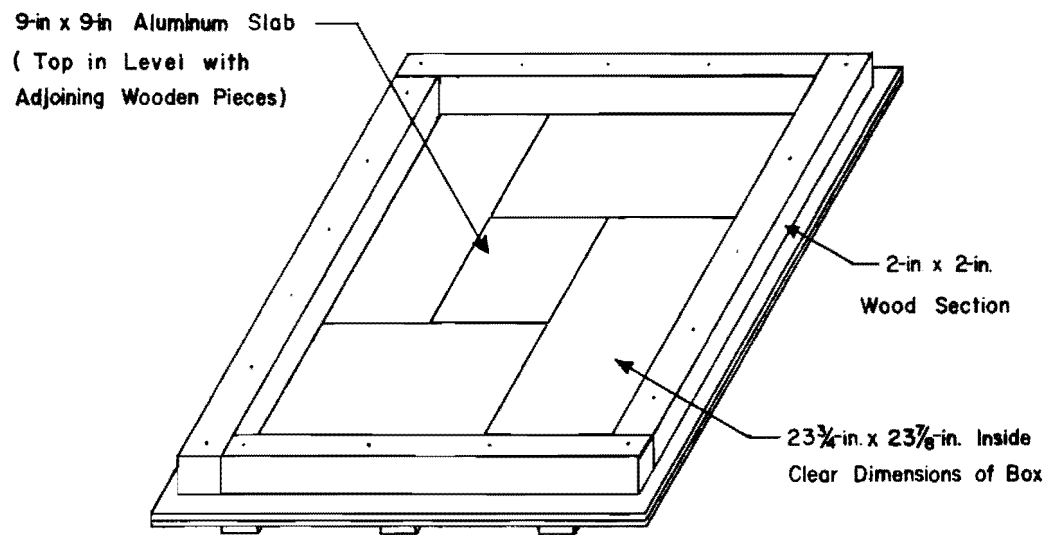
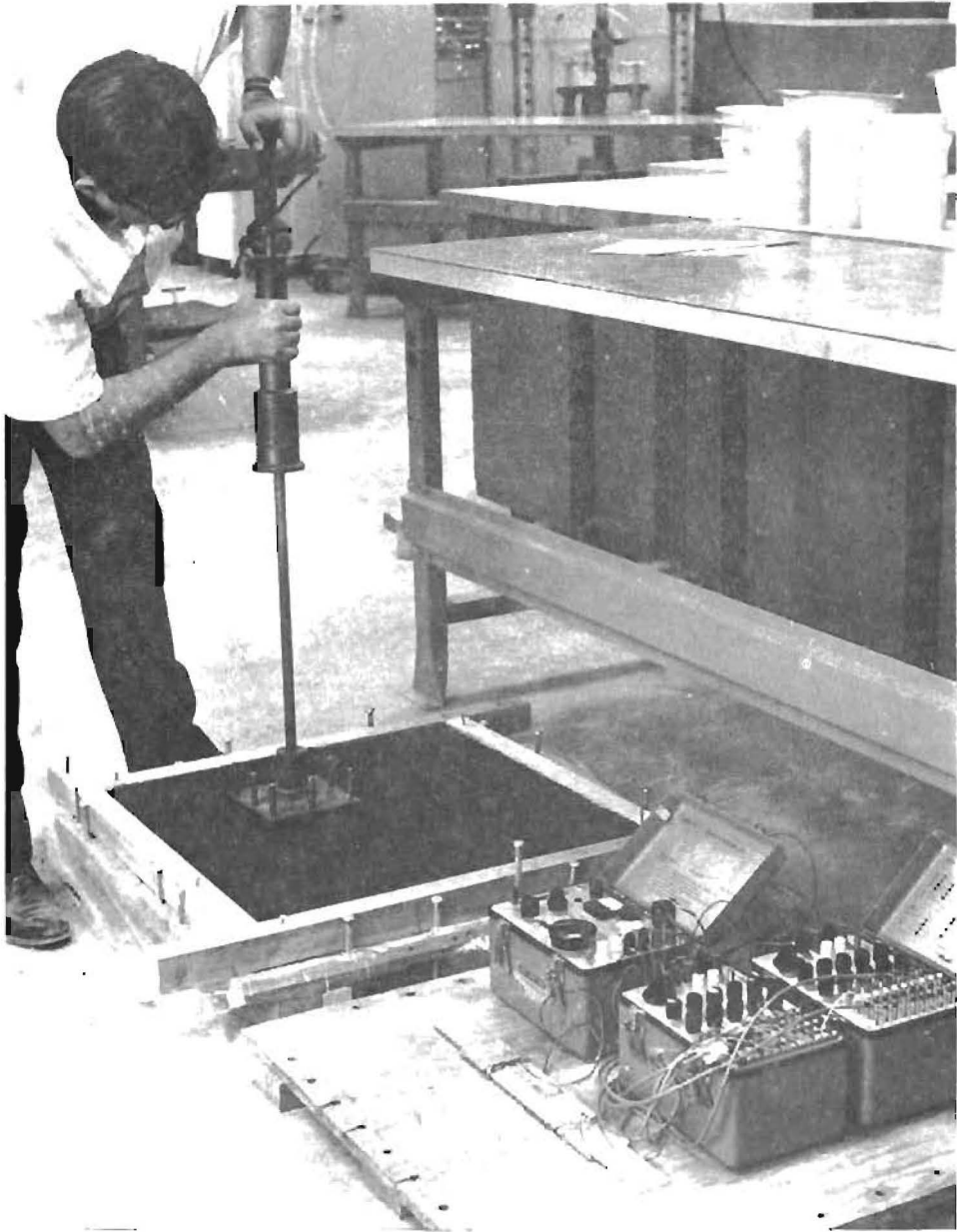Fig 9. Typical arrangement for placing and compacting asphalt concrete.

Fig 10. Compaction of asphaltic concrete.

bottom encasing the asphalt concrete layer. The box was then filled with soil
in the manner described previously, The box was turned over for the slab test
on the surface of the asphalt-stabilized layer and then plate load tests were
conducted on the layer to determine a k-value for the layered system.

Testing Procedure

Plate Load Tests

A series of plate load tests was conducted on the surface of both the clay
and the asphalt-stabilized layer using circular steel plates of 2, 4, 6, and
9-inch diameters (see Figs 11 and 12). A thin layer of fine sand was placed
over the leveled surface of clay and asphalt concrete to insure uniform contact
with the plate surface. Loads were applied under a controlled rate of strain
of .001 inch per minute. The loads were applied to the plate through a ball
to insure that they acted vertically. Plates were stiffened by placing small-
diameter plates over the bigger plates. This arrangement gives a better dis-
tribution of pressure over the entire surface of the plate. It was assumed
that the deflection under the rigid footing plate was equal to the average de-
flection of a uniformly-loaded, flexible circular area (Ref 46). In plate load
tests on clay, each plate was loaded until the soil reached bearing capacity
failure. The soil was considered to fail when there were large increases in
deflection without increase in load. The plate was unloaded and the rebound
recorded. For plate load test on an asphalt concrete layer, a limiting deflec-
tion of .25 inch was established. Each plate was tested at different locations
on the surface of the clay at a distance at least the diameter of the plate from
the sides of the box. The detachable top and bottom covers of the box made it
possible to use both surfaces of the soil after turning the box over. Proving
ring readings were taken at close intervals of time and corresponding dial gage
readings were recorded.

Slab Test

Static and cyclic load tests were conducted on a thin aluminum with
an applied load-acting at the center of the slab (Figs 13 and 14). In order
to achieve a uniform contact of the slab with the asphalt concrete, the slab
was placed over the cover of the box (Fig 9) and asphalt concrete was laid.
The surface of the aluminum slab was lightly greased to break continuity with
the asphalt concrete surface.

26
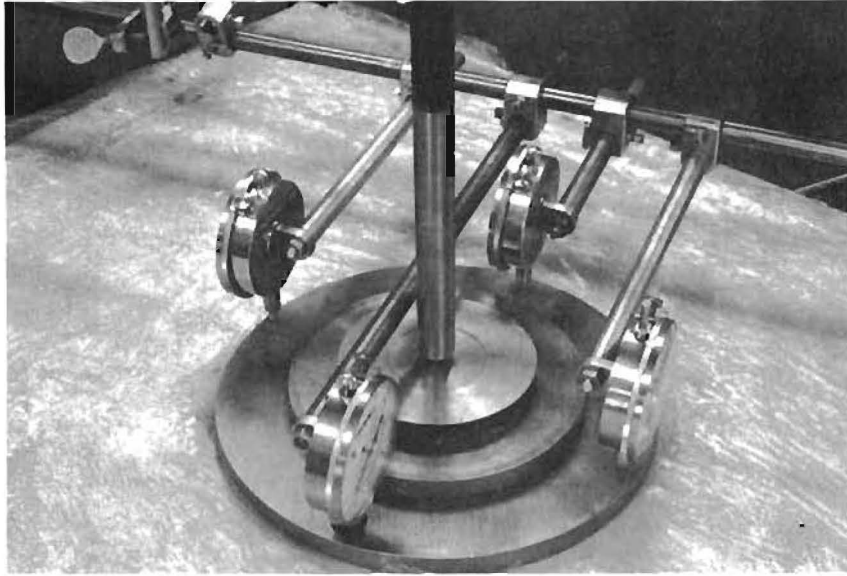
Fig 11. Nine-inch plate load test on clay.
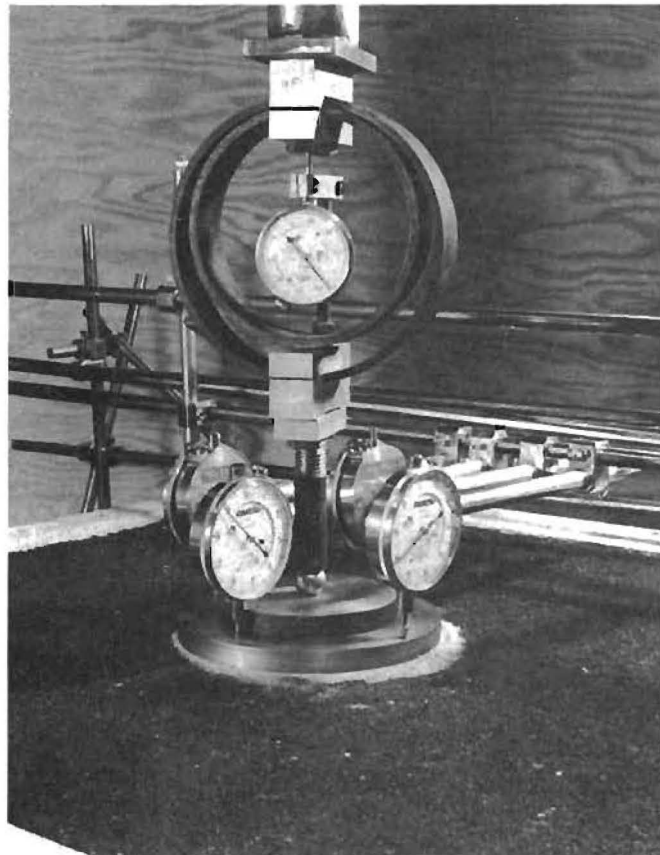
Fig 12. Plate load test on asphalt-stabilized layer with clay subgrade.
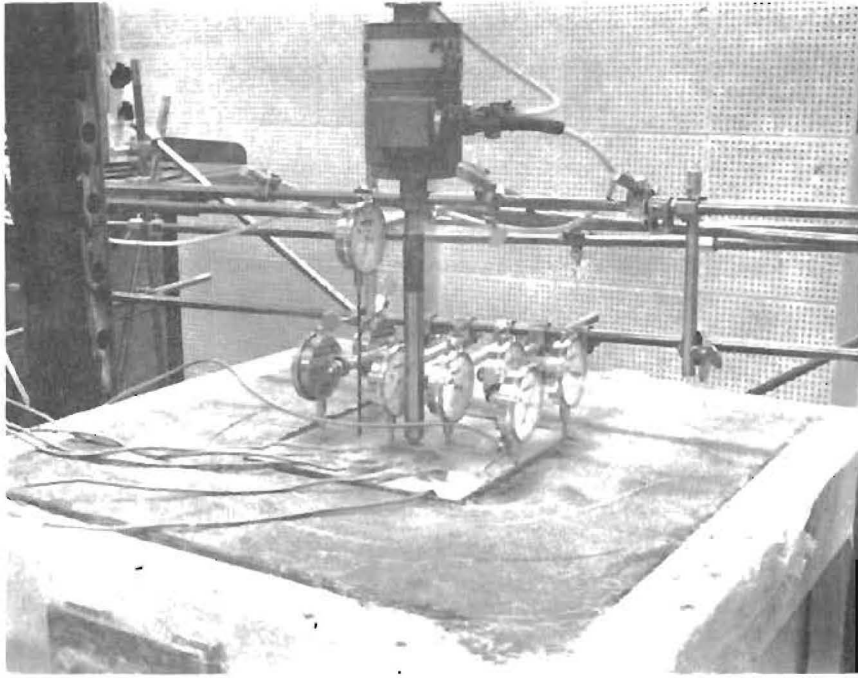
Fig 13. Slab test on asphalt-stabilized layer with clay subgrade.
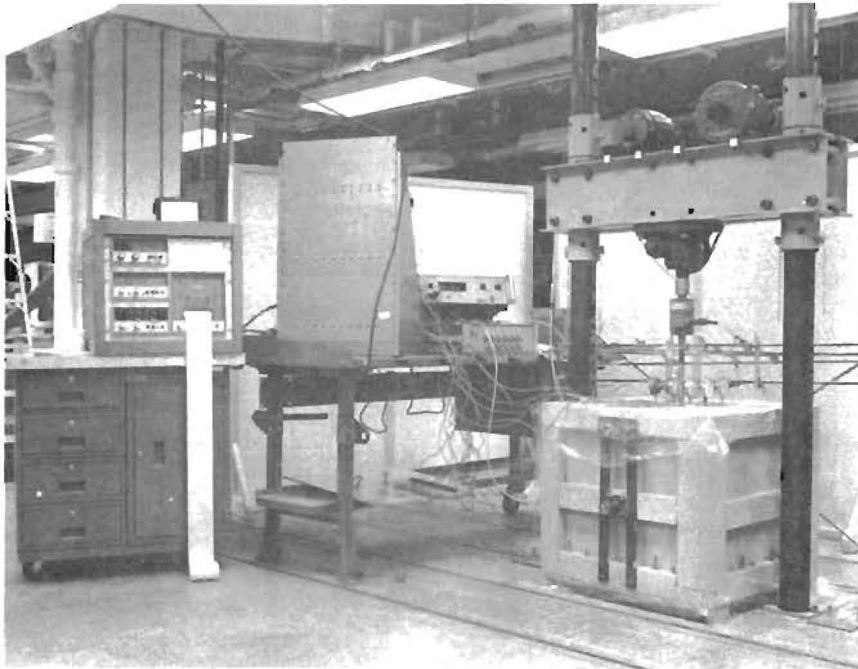
Fig 14. Detailed arrangement for slab test.

Loads were applied at a controlled rate of strain of .001 inch per minute. The plate was loaded to a maximum load of 200 pounds. Readings of dial gages were recorded at intervals along with measurements of the load immediately preceding each of the strain measurements. Load readings were also given by a digital voltmeter to an accuracy of one pound. When a peak load of 200 pounds was reached, the plate was unloaded, and readings were recorded on the unloading cycle in the same manner. Maximum recorded deflection at gage No. 1 was noted for the first cycle (static case). Load was again applied to produce the maximum deflection of the slab recorded in the first cycle. Readings of loads and strains were recorded accordingly. The slab was loaded and unloaded ten times in cyclic fashion, with repetition of the maximum recorded deflection. During the first, fifth, and tenth cycles of loading and unloading, reading of dial gages, load, and strain data were recorded at close intervals of time.

CHAPTER 4. ANALYSIS OF PLATE LOAD TESTS DATA

Plate Load Tests on Clay

Three series of tests, A, B, and C, were conducted using the same type of soil. They were preceded by the slab tests conducted by Agarwal and Hudson (Ref 1). Load-deflection data from the plate tests were used in determining a k-value for the soil. Series A provided a k-value for the soil used in the analytical solution for comparison with the preliminary slab tests. Test Series B and C provided k-values for analytical solutions for comparison with the results of slab tests for center and two-point corner loading, respectively (Ref 1). Unit load-deflection data for 9-inch, 6-inch, 4-inch and 2-inch plate tests, for Series A, B, and C, are given in Appendix 2, Tables A2.1 to A2.4, respectively. Unit load versus deflection curves are plotted as shown in Figs 15, 16, and 17. The average data for the three series of tests are plotted in Fig 18.

The data obtained in these three series of tests afforded two significant comparisons. They showed first the reproducibility of the data. For this comparison, the data showed that for a given plate size (within the 2 to 9-inch-diameter range) essentially the same load-deflection curve for soil was recognized in each of the three tests performed on the Taylor marl clay. To illustrate the similarity, pressure-deflection curves for 9-inch-diameter plate are given in Fig 19 for the three tests.

The second significant observation from the data was the important effect of plate area on the pressure intensity required to produce a given plate-deflection on the soil in question. For the same unit load, the deflection increased as the plate diameter increased (Fig 18). The significance of this observation was that since tire footprint area decreases as tire pressure increases, greater deflection under a given load with a relatively high tire pressure might be expected than with a relatively low tire pressure. From the pressure-deflection curves, it is also observed that ultimate soil pressure varied under different plates tested. The variation in failure pressure is not significant.

Fig 15. Pressure versus deflection curves for rigid plate
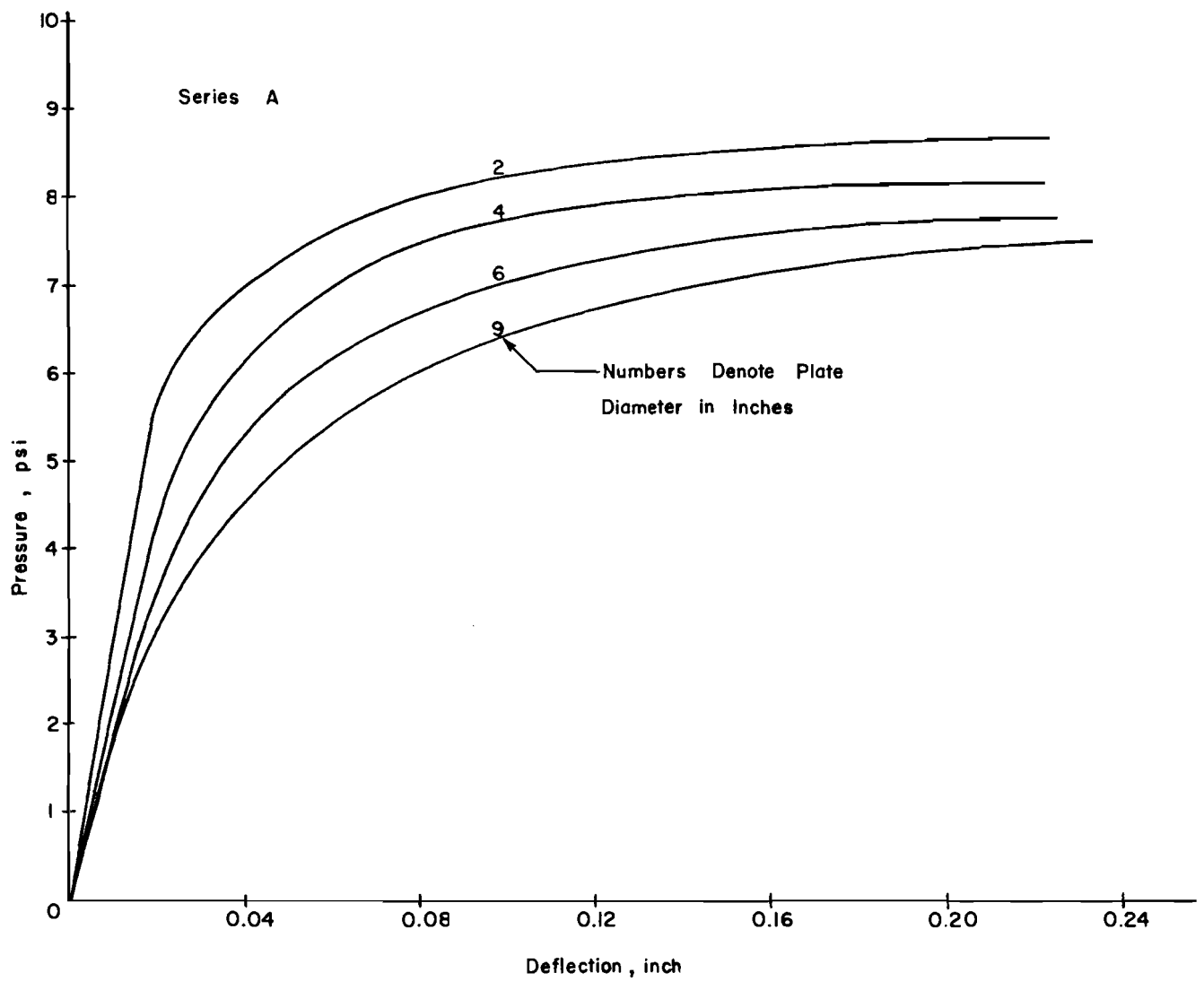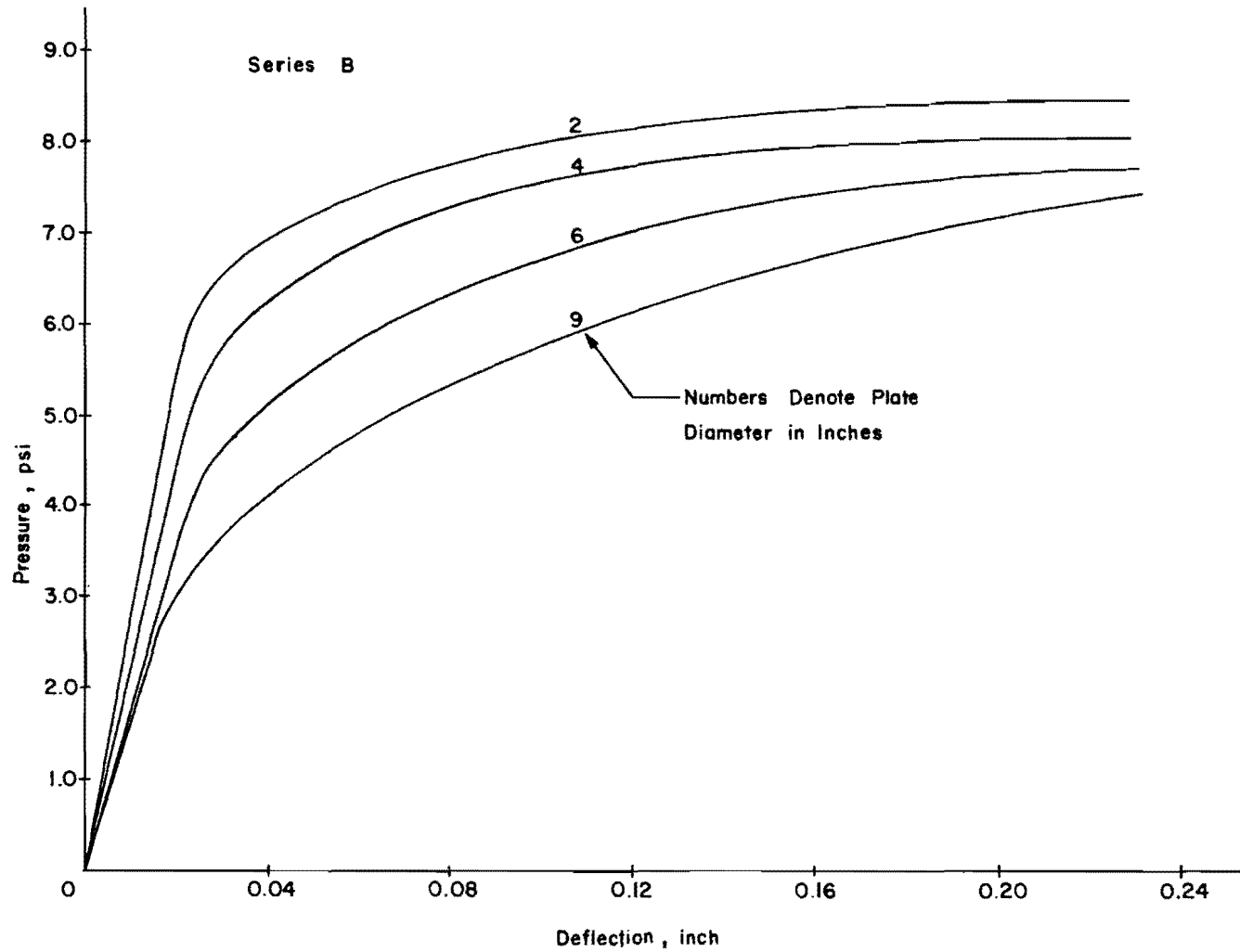tests of the clay subgrade, Series A.

Fig 16. Pressure versus deflection curves for rigid plate
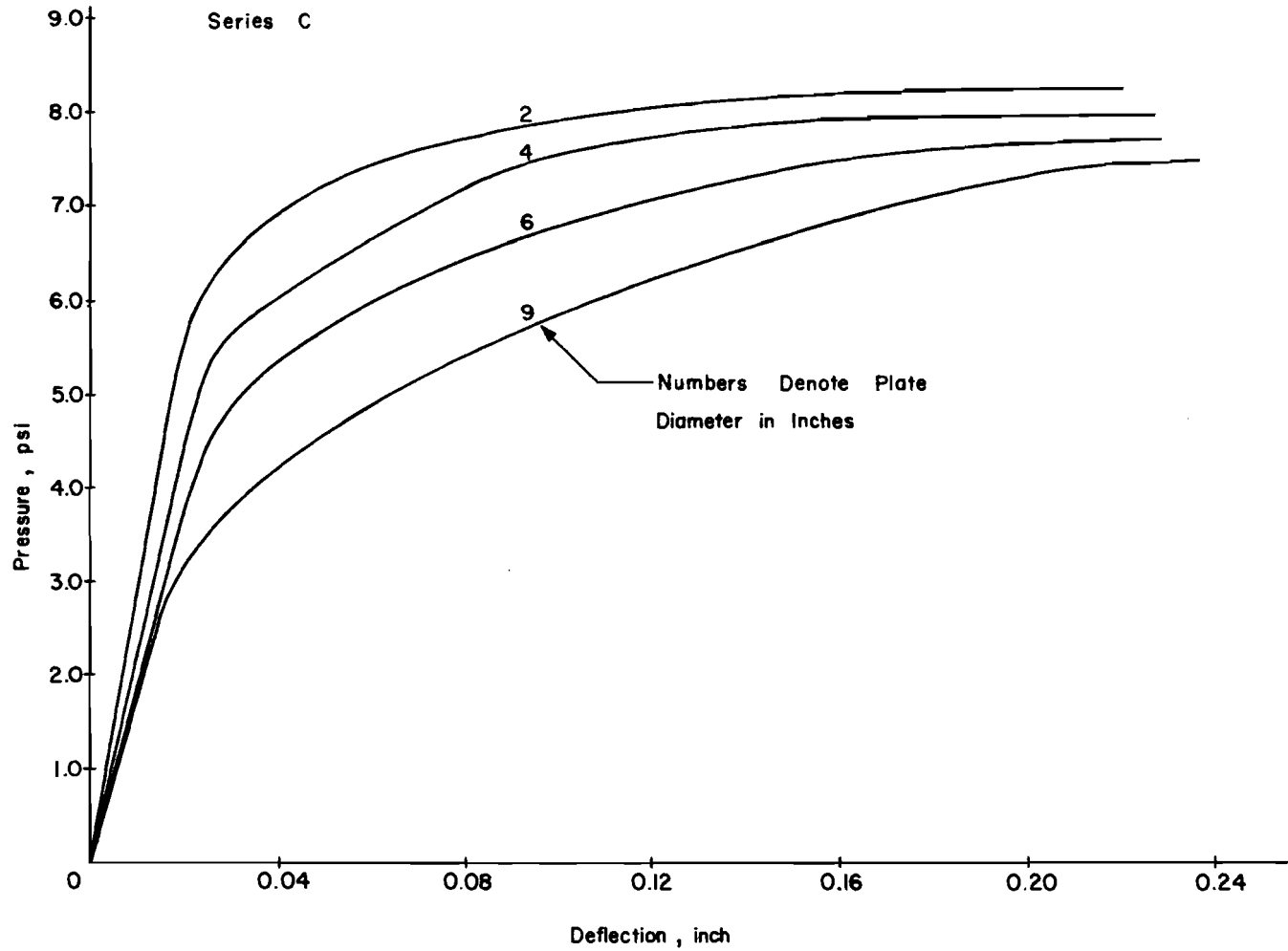tests of the clay subgrade, Series B.

Fig 17. Pressure versus deflection curves for rigid plate
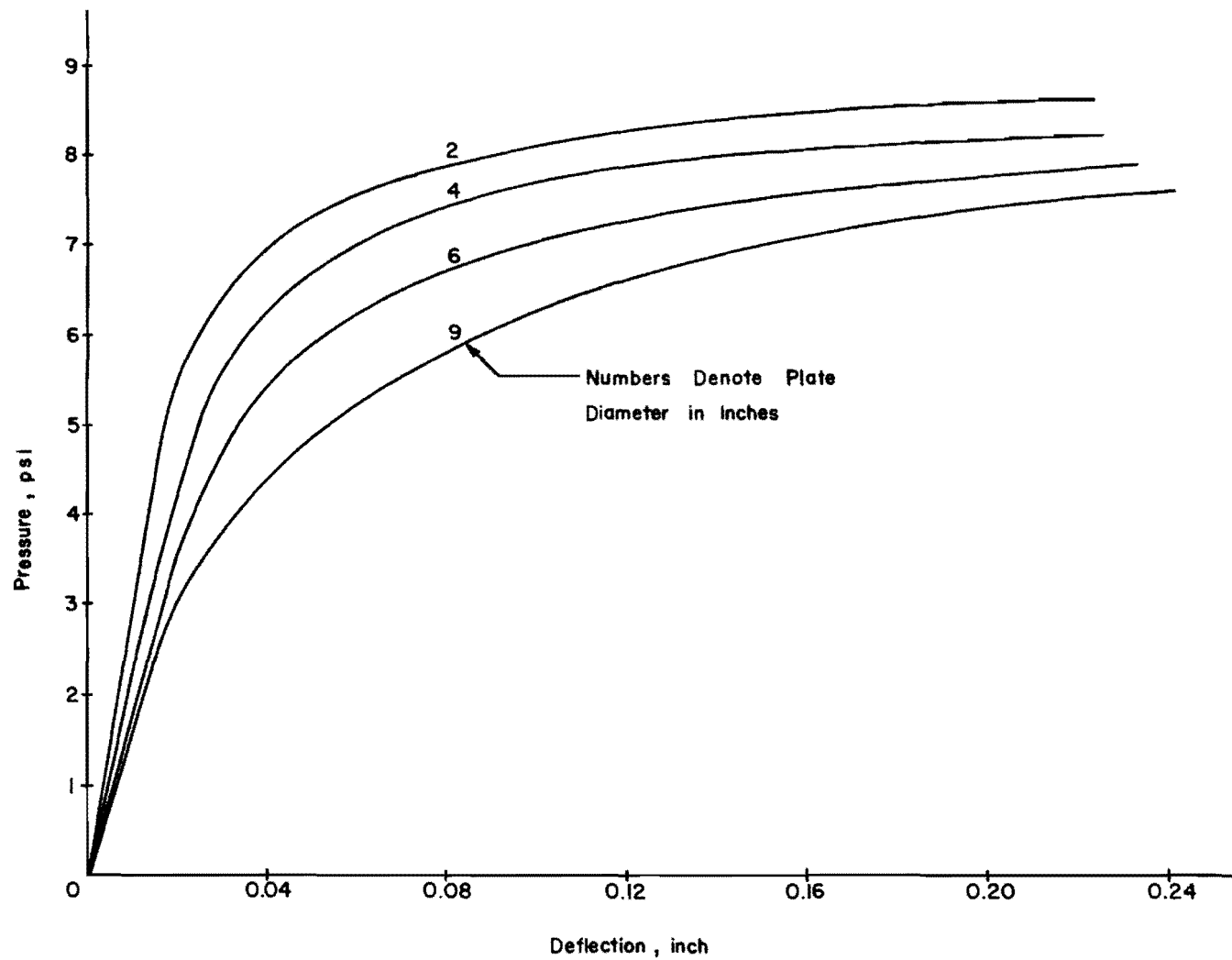tests of the clay subgrade, Series C.

Fig 18. Average pressure versus deflection curves for plate
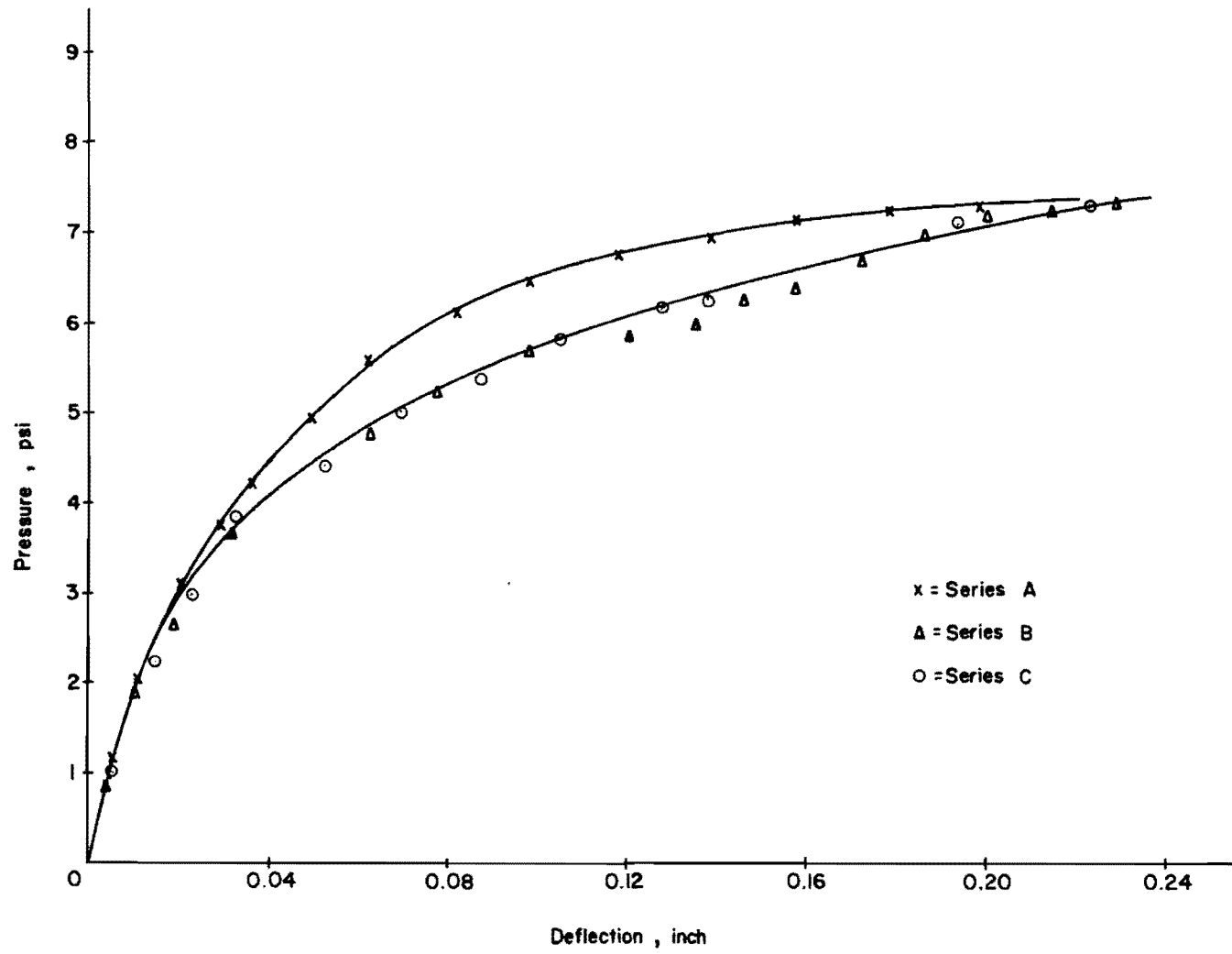load tests on clay, Series A, B, and C.

Fig 19. Pressure versus deflection curves for 9-inch plate load tests on clay.

Using the data shown on Fig 18 and the tangent modulus and secant modulus approaches, the value of k was calculated. The influence of the loaded area on k is shown in Fig 20. It is noted that in using the tangent modulus approach, k-value increases with the decrease in plate size. At higher deflections the effect of plate area on the k-value (secant modulus) ceases.

By plotting unit-load versus perimeter-area ratios for the plates at constant deflections of .01, .02, .04, and .10 inch, it is observed (Fig 21) that a linear relationship holds good for the two parameters, as previously demonstrated by several investigators, including Housel (Ref 22), Hubbard and Field (Ref 23), Campen and Smith (Ref 9), Teller and Sutherland (Ref 43), Middlebrook and Bertram (Ref 33), McLeod (Ref 31), and others.

Attempts to linearize the load-deflection data of the plate load tests on semi-log and log-log plots using various parameters, including a dimensionless parameter, were unsuccessful. The plot of pressure versus the deflection to diameter ratio related all the data, as shown in Fig 22. The data from all the tests fell reasonably close together on a straight line for deflections as low as .01 inch. At higher unit loads and higher deflections, the data fell on the curve shown. This method of plotting deflection to diameter ratio, on a log scale, reduces the scatter of the data, making it possible to predict load-deflections of any size plate within the range of plate sizes used in this study. The data in Fig 20 indicate that when tests to determine the value of soil modulus or soil stiffness coefficient k are made, the deformation must be limited to a magnitude within the range of pavement deflection, and that, therefore it is important to use a bearing plate of adequate size (as large as possible).

Agarwal and Hudson (Ref 1) chose the load-deflection data from the 9-inch-diameter plate load test to determine the soil modulus k . They used this value of k in their comparison of calculated and measured values of deflections and stresses in a test of a small slab (9 × 9 inches) resting on Taylor marl. Their report provides detailed information on these comparisons and the use of the pressure deflection curves of the clay.

Plate Load Tests on Layered System

To evaluate a representative k-value for a two-layered system, a series of load tests was performed on the surface of an asphalt-stabilized layer using rigid circular plates of 2, 4, 6, and 9-inch diameters. The load-deflection data are summarized in Appendix 2, Table A2.5. The procedure for plotting the
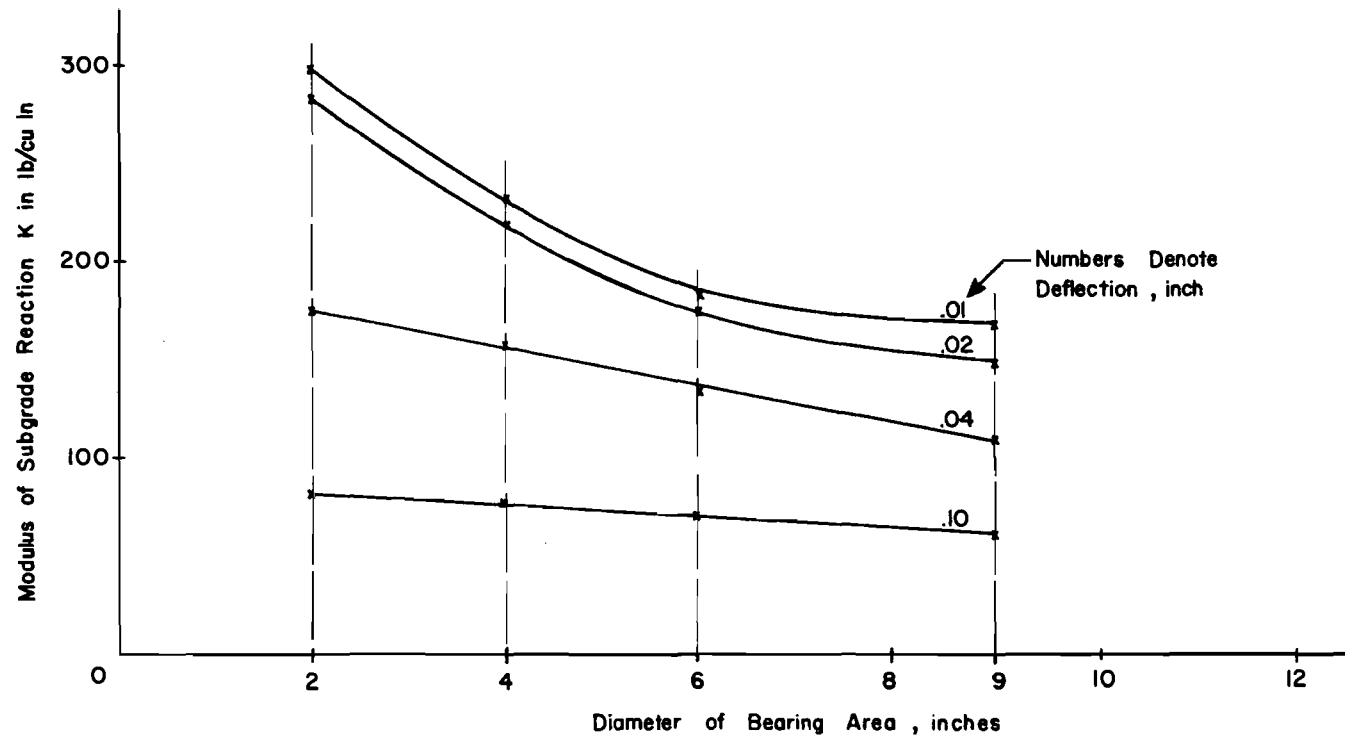
Fig 20. Influence of diameter of bearing area on k-value at constant deflections.
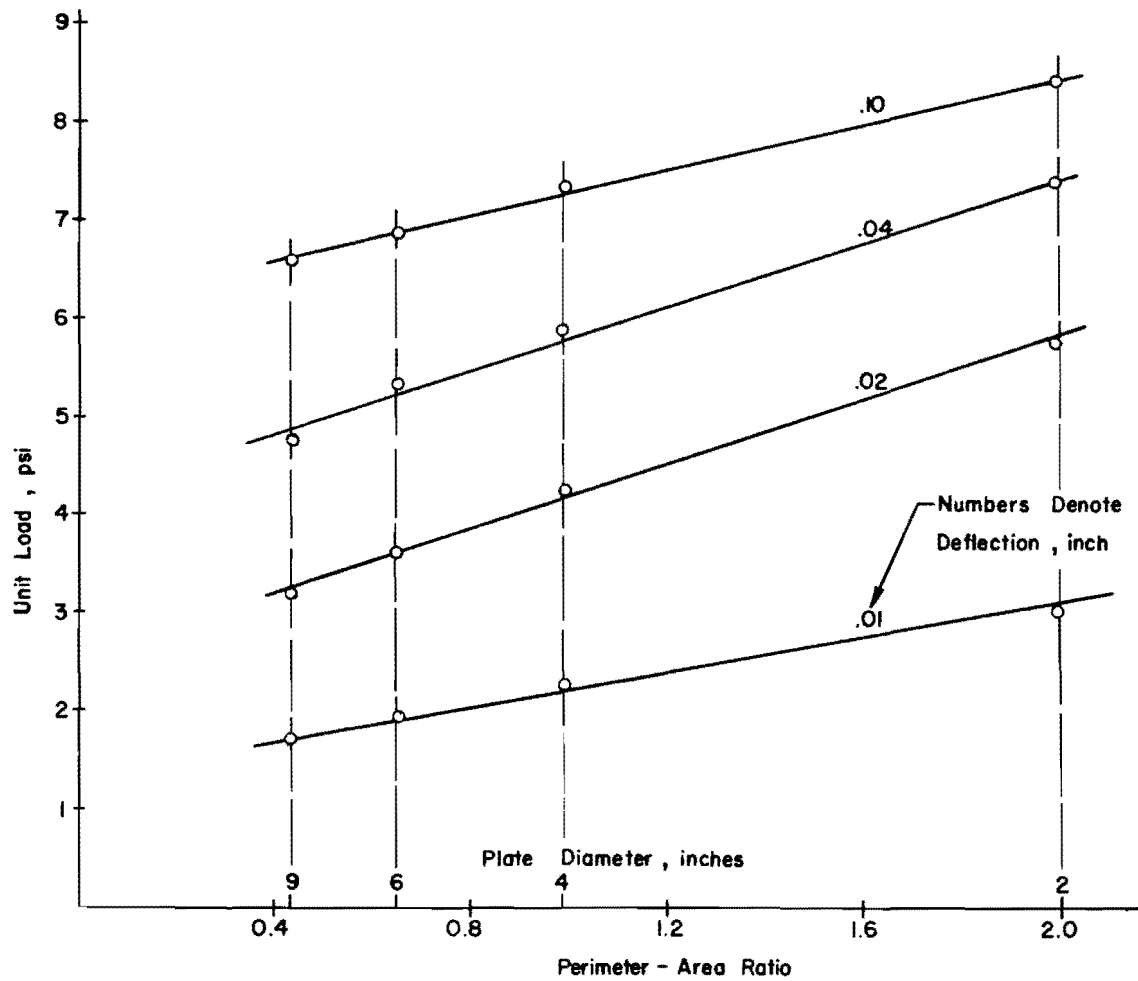
Fig 21. Influence of plate size on unit load at different deflections.
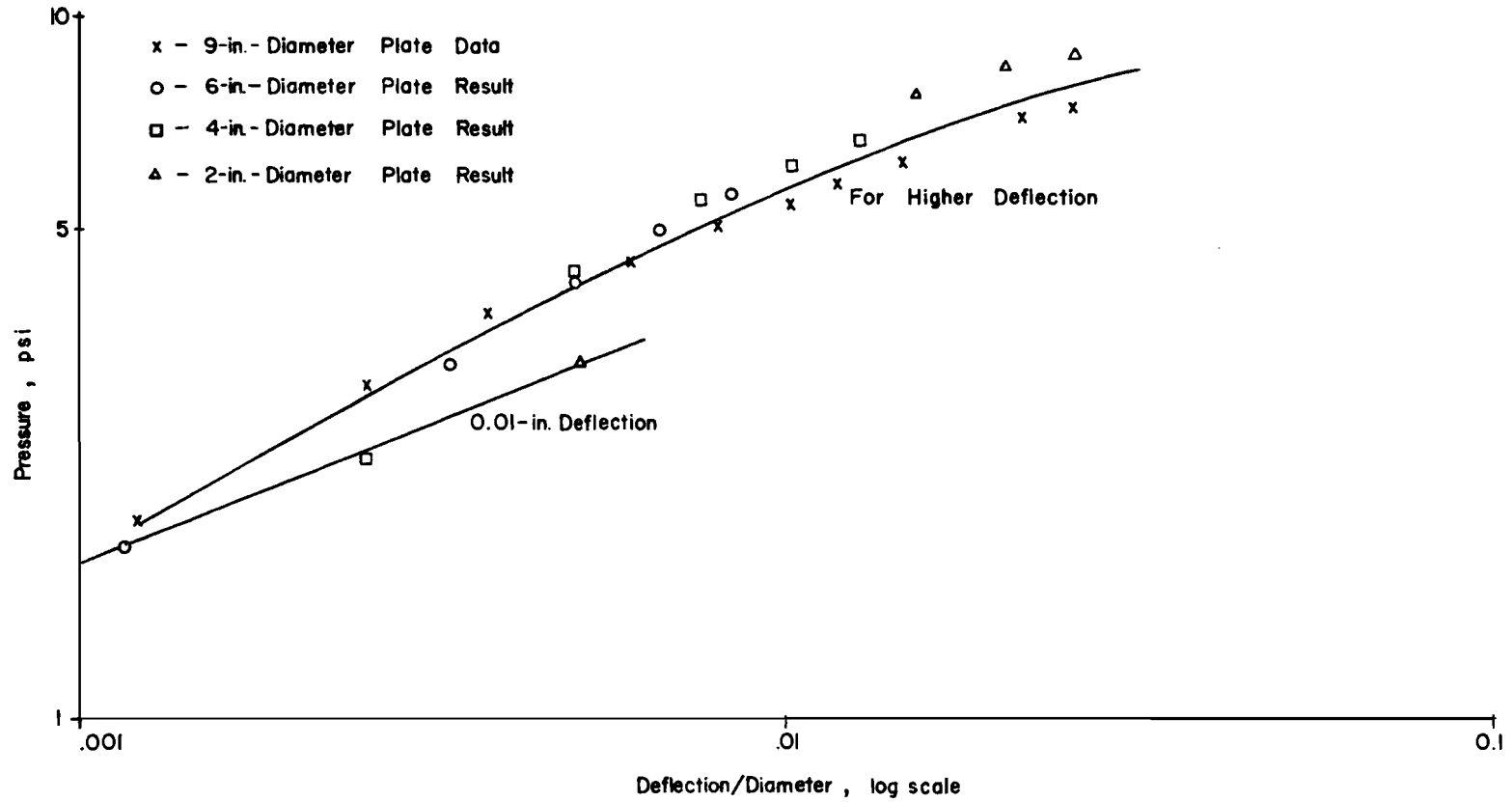
Fig 22. Pressure versus ratio of deflection to diameter of plate for plate tests on clay.

data was the same as used for the load-deflection data for plates tested on clay. In Fig 23 the unit load is plotted against the deflection. The maximum deflection was limited to .25 inch in tests. Almost the same conclusions were derived from these tests as from tests on clay mentioned previously; namely, for the same pressure the deflection increased with plate diameter.

One of the objectives of this study was to compare load-deflection support characteristics of the two systems, i.e., clay subgrade alone, and asphalt-stabilized layer over the clay subgrade, and thus evaluate the increase in the value of k resulting from the addition of the asphalt-stabilized layer. Figure 24 demonstrates the load-deflection curves for clay (average of the three tests) and for the asphalt-stabilized layer for load tests on the 9-inch-diameter plate. Table 1 shows the increase in k-value at several levels of deflection. It is noted that the use of the asphalt-stabilized layer increased the initial tangent modulus value of k by 41 percent. Other values of modulus are increased accordingly. Thus the composite action of the asphalt concrete layer and the clay subgrade provided a significant increase in the supporting strength at a given deformation level.

TABLE 1.   IMPROVEMENT OF k-VALUE BY STIFFENING CLAY SUBGRADE WITH
A LAYER OF ASPHALT CONCRETE FOR THE 9-INCH PLATE TEST

	Tangent Modulus k in lb/in^3	Secant Moduli in lb/in^3			
		at 0.02-inch Deflection	at 0.04-inch Deflection	at 0.08-inch Deflection	at 0.10-inch Deflection
Clay subgrade	170	150	110	75	63
Layered system (Asphalt concrete layer on clay)	240	235	200	135	108
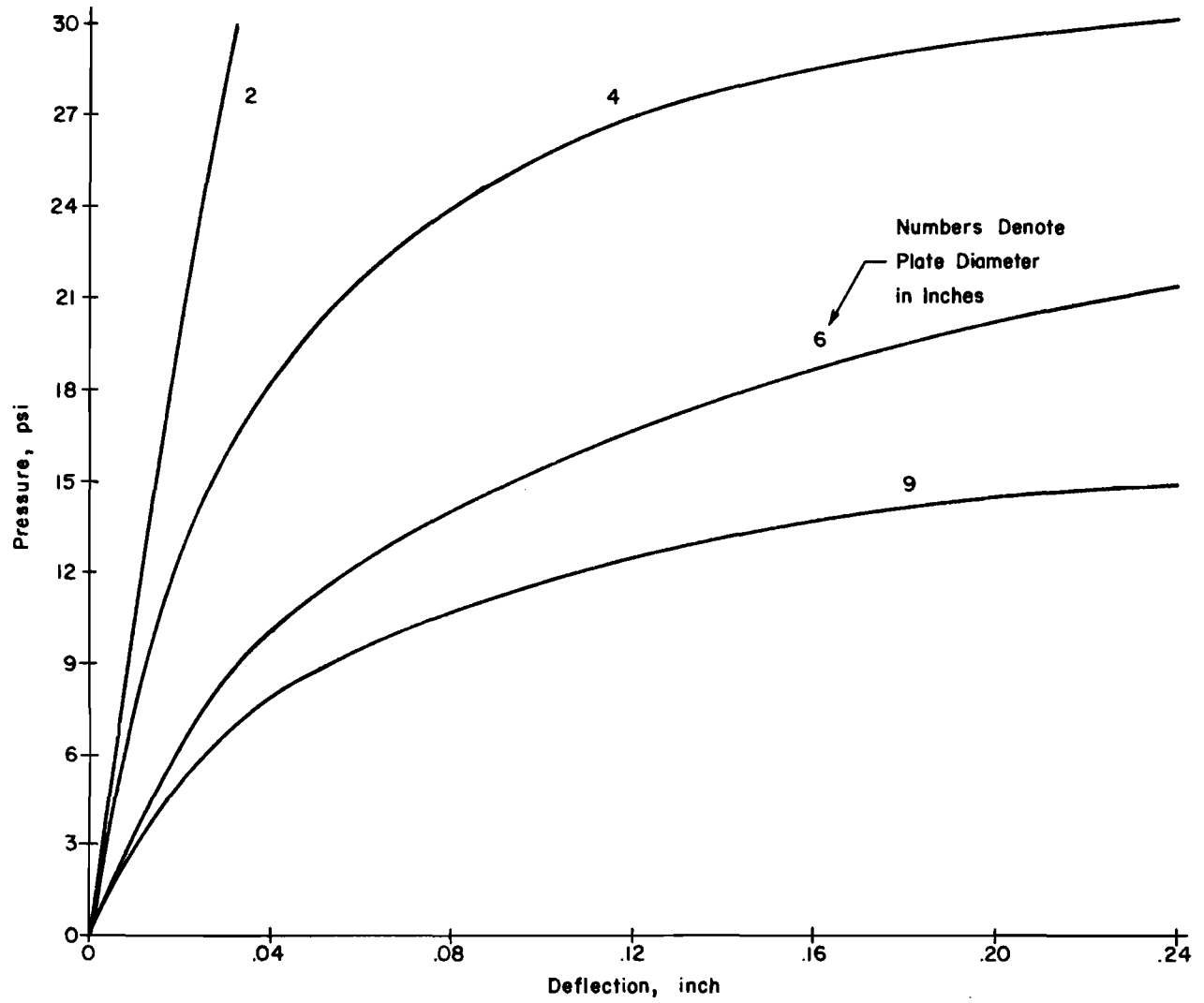
Fig 23. Pressure versus deflection curves of rigid
plate tests of the layered system.
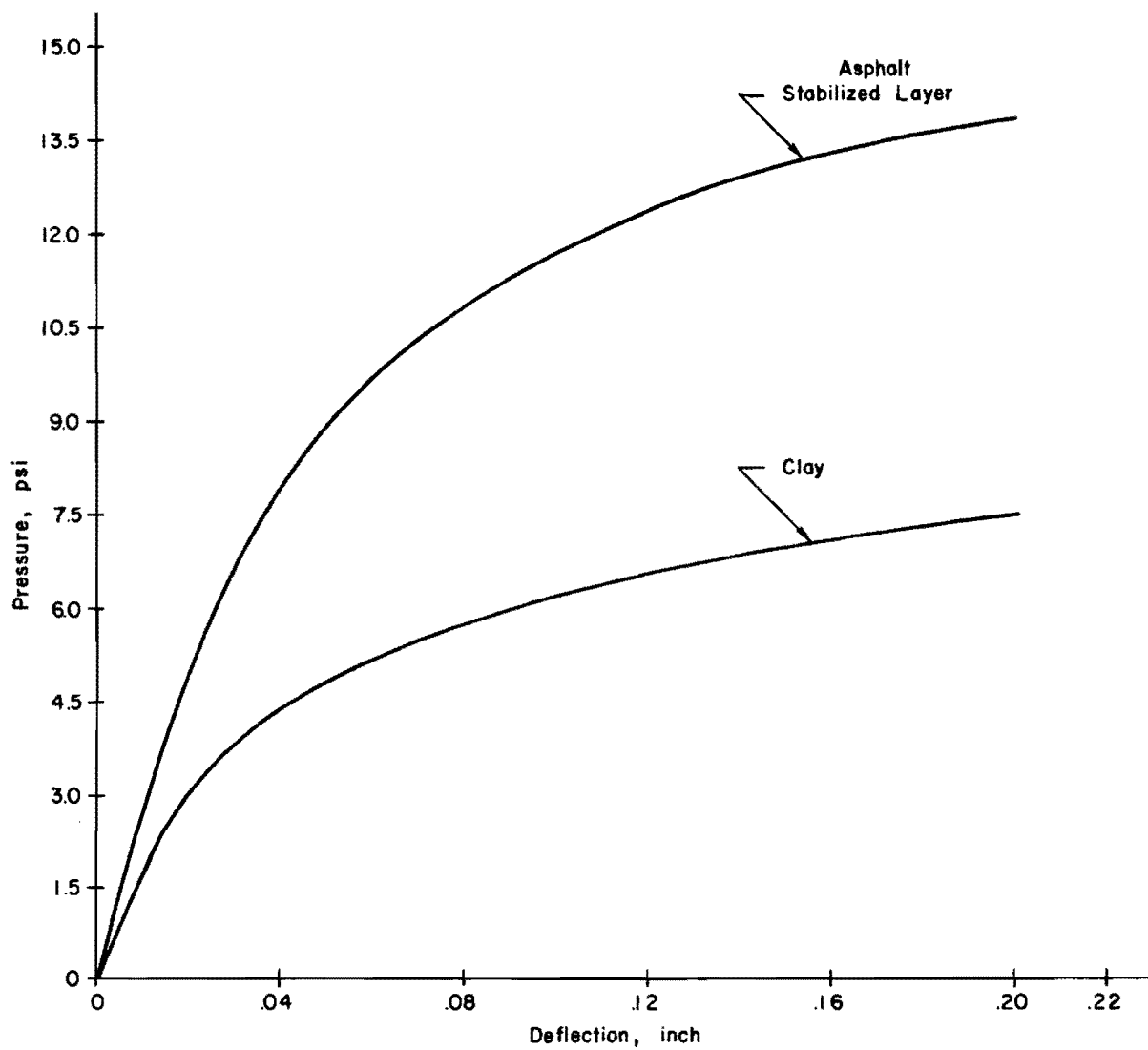
Fig 24. Pressure versus deflection for 9-inch plate tests
on the clay subgrade and on the layered system.

Application of Elastic-Layered Theory

Burmister's theory for a two-layered system was investigated for use in predicting the load-deflection characteristics of the asphalt-stabilized layer and its solution was compared with the plate load test data.

The deflection of a rigid plate on the surface of a two-layered system is determined from Eq 11, Chapter 2 (terms are defined on page 8):

$$w = 1.18 \frac{pa}{E_2} F$$

The factor F is a function of the modular ratios of materials in two layers, E_1/E_2. Thus, to predict deflection from any size of plate, the values of E_1 and E_2 are required.

Determination of E_2

The modulus of elasticity of subgrade soil E_2 is obtained from conventional plate load test data on the subgrade soil (Fig 18) by the use of Eq 12, which is the same as Eq 11, except for the factor F which becomes unity when only the soil mass is under consideration.

$$E_2 = 1.18 \frac{pa}{w} \tag{12}$$

From plate load data (Fig 18) for different values of deflection w, and corresponding values of pressure p, values of E_2 were calculated for 9-inch, 6-inch, and 4-inch-diameter plates, using Eq 12. Since the calculated value of E_2 (Fig 25) varies not only with the size of the plate, but also with the amount of deflection, it is not possible to specify a constant value of E_2 for the soil mass.

Determination of E_1

The modulus of elasticity of the asphaltic material, E_1, used in the tests can be found by various methods, including the indirect tensile test (Ref 37), the plate bearing test, and the methods suggested by Van der Poel (Ref 48). In this study, E_1 is determined from the indirect tensile test and a crude estimate is obtained, using Van der Poel's method.
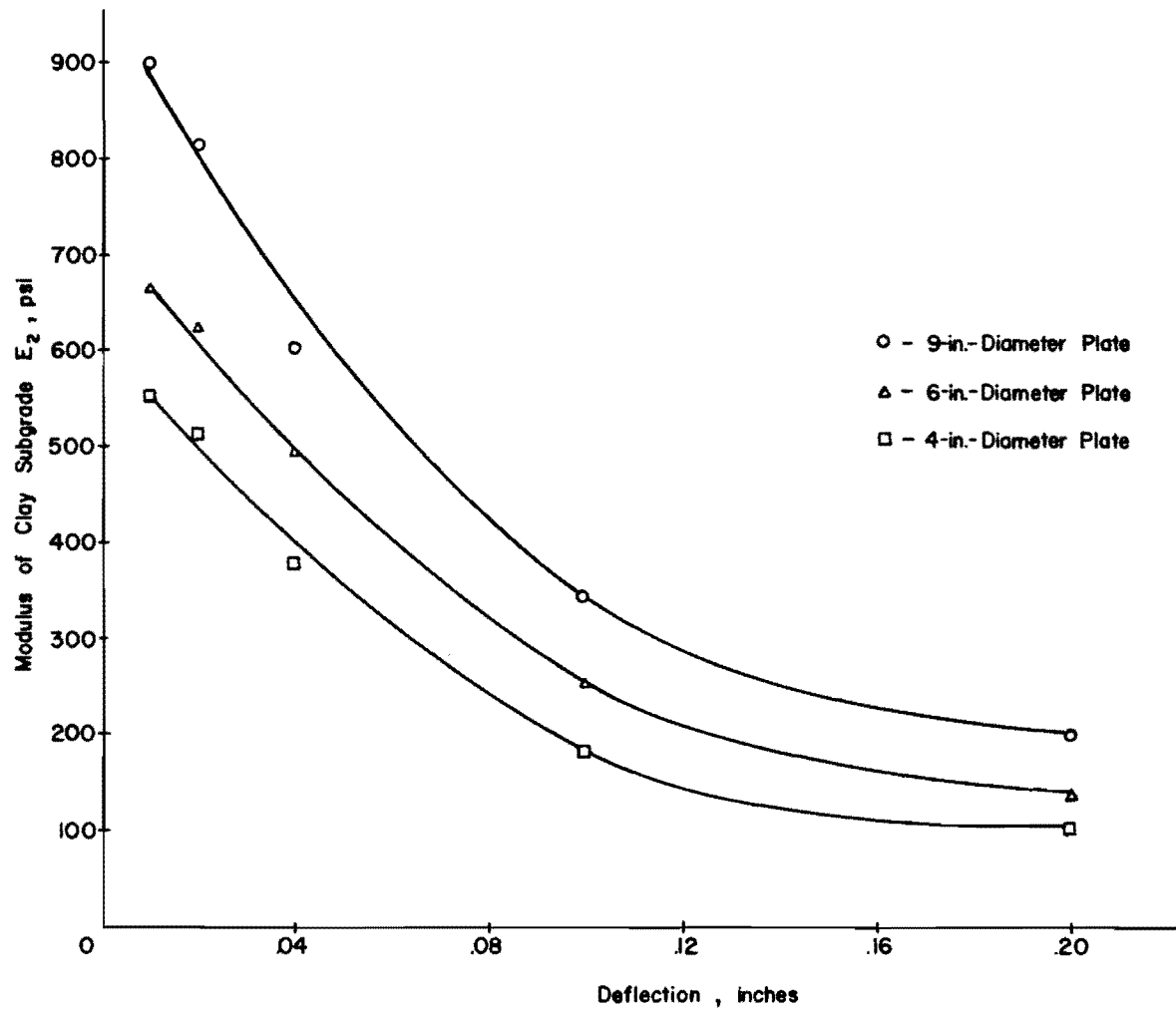
Fig 25. Influence of plate diameter and deflection on
the modulus of elasticity of clay soil.

For the indirect tensile test, cylindrical specimens of asphalt concrete were prepared and tested as described in Appendix 1. Vertical and horizontal deformations of specimens were measured. The theoretical treatment of the test data is based on linear elastic equations derived by Hondros (Ref 21). The total deformation in either direction in terms of modulus of elasticity and Poisson's ratio equals the sum of the strains of all individual elements along the principal axes. These total strain equations are then set equal to the total measured deformation in the two principal directions, leaving two equations and two unknowns. Formulas for E and ν are then obtained by solving the two equations simultaneously. The technique (Ref 37) used for estimating the modulus value involved a closed-form solution of each equation by complicated mathematical integrals through the use of a computer. The value of E_1 was thus computed to be 34,000 psi at 77° F. Asphalt was recovered from a specimen of the asphalt concrete layer and measurements were made for the penetration at 77° F and for the ring and ball softening point temperature. With these data and from the knowledge of the volume concentration of the aggregate used, the stiffness modulus E_1 of asphalt concrete was computed using the method in a nomograph prepared by Heukelom and Klomp (Ref 18). The value of E_1 was computed to be 20,000 psi, which is lower than the value obtained from the indirect tensile tests of the asphalt concrete specimen.

Thus with known values of E_1 and E_2 and the ratio of thickness of the layer to the radius of the plate, the deflection factor F is obtained with the assistance of an influence chart (Fig 26) developed by Burmister (Ref 7). Now, for any value of deflection, the corresponding pressure on any size of plate is calculated from Eq 11, which can be rewritten as

$$p = \frac{wE_2}{1.18\ aF}$$

(Definitions of terms may be found on page 8.)

A comparison of the measured and predicted pressure deflection values for the range of conditions investigated is shown in Fig 27. It is noted that for small deflections the corresponding measured values of pressures follow more closely the predicted values based on $E_1 = 34{,}000$ psi than the values of pressures obtained by using $E_1 = 20{,}000$ psi . At higher deflections the variation in the two curves tends to increase. The computed values for pressure
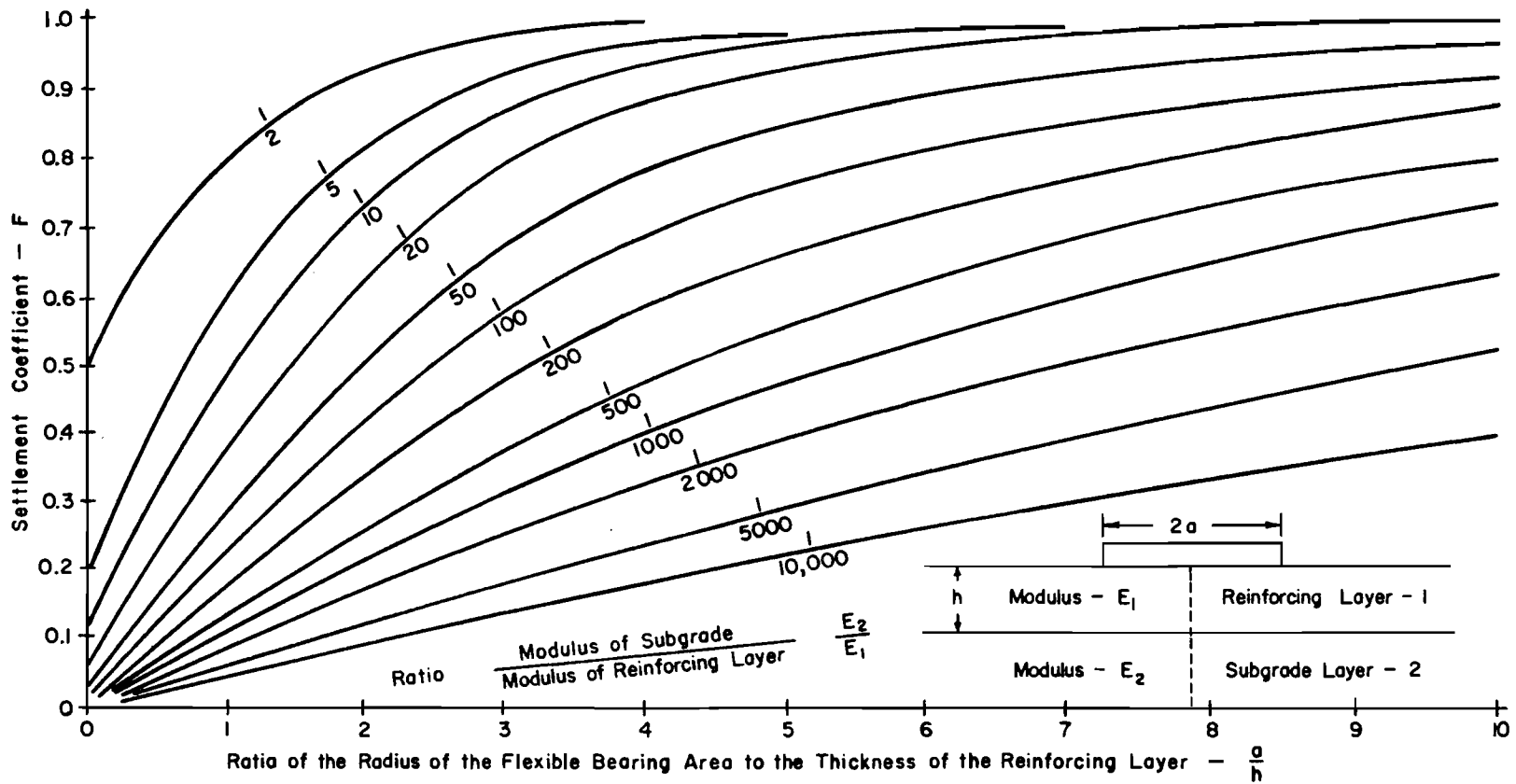
Fig 26. Influence curves of the settlement coefficient -F
for the two-layered system (after Burmister, Ref 7).
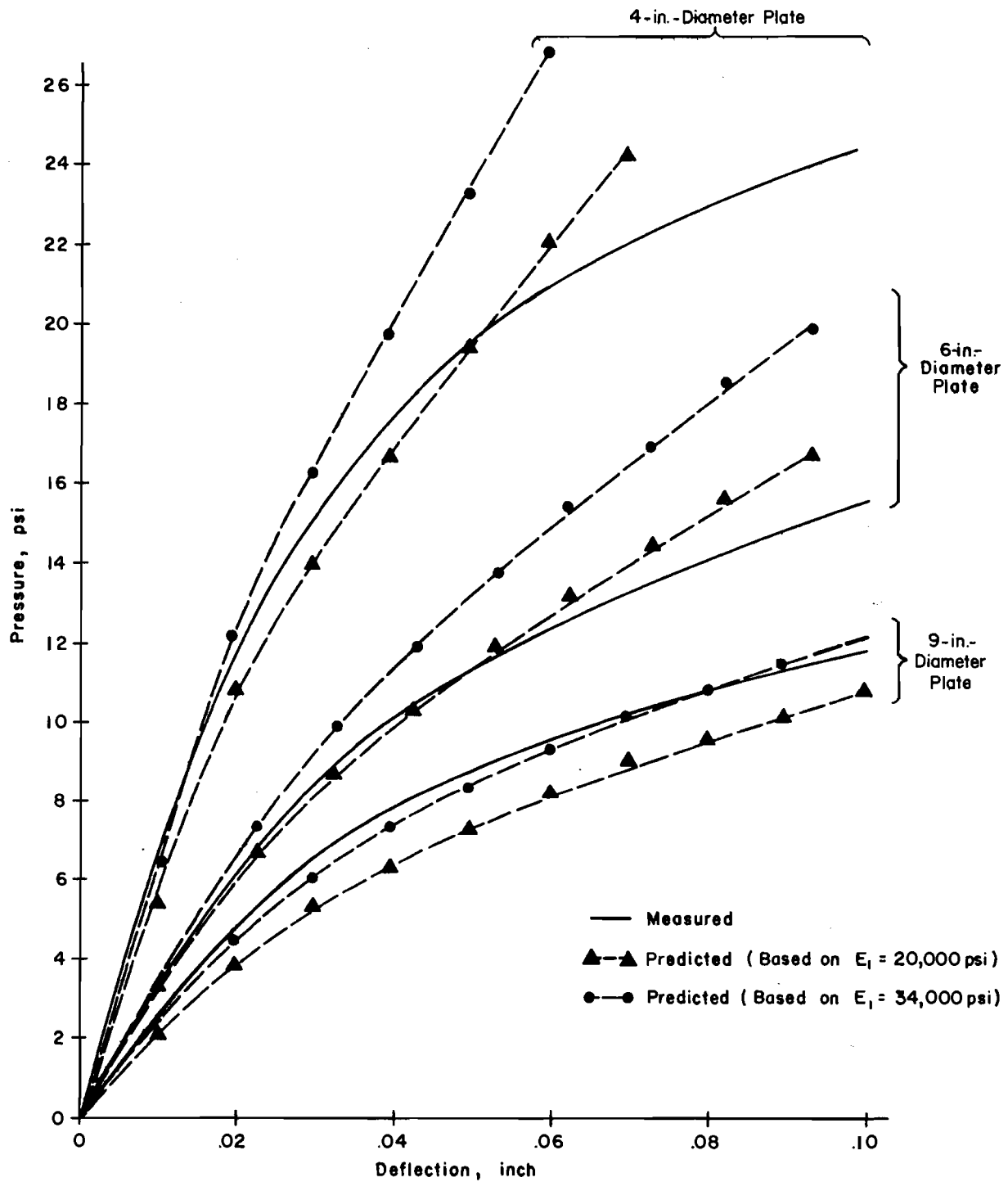
Fig 27. Comparison of pressure deflection data from plate load tests with predictions according to Burmister's theory for two-layered system.

are lower than the measured values for the 9" diameter plate and higher than the measured values obtained from the 4" and 6" plate load tests.

Since it has been shown previously that the value of E_2 is dependent on plate size and amount of deflection, it is quite probable that the same analogy applies to the value of E_1, and further studies are warranted to investigate this relationship.

CHAPTER 5. EFFECT OF STIFFNESS OF ASPHALT-STABILIZED LAYER

A limited study was conducted to investigate the effect of increasing the stiffness of the asphalt-stabilized layer on the load-deflection characteristics of the layered system. This provided an easy method of evaluating the effect of E of the stabilized layer on the improved k-value of the two-layered system investigated herein.

Asphalt is a thermoplastic material and any change in temperature affects the stiffness of the asphaltic material. Recently, extensive investigations (Refs 10, 29, 32, 34, and 38) have been conducted to measure the stiffness of asphalt concrete with time-temperature dependencies. Hudson and Kennedy (Ref 24) showed that temperature and variation of the loading rate have a significant effect on the indirect tensile strength and failure deformations of asphaltic materials.

Development of Test Program

With the test set-up described in Chapter 3, it was relatively simple to conduct a series of plate load tests on the surface of the thin asphalt concrete layer (2.0-inches thick) at temperatures of 40°, 77°, and 100° F, using a 6-inch-diameter plate. This provided a range of stiffness values in the asphaltic layer. The tests were conducted in one of the controlled environment chambers at The University of Texas. This chamber is capable of achieving temperatures ranging from -20° F to $+140^{\circ}$ F, $\pm 2^{\circ}$ F, and maintaining them for long periods of time. The chamber was first cooled down to 40° F and stabilized for 36 hours. Then the plate load tests at this temperature were performed. In a similar manner the temperatures were stabilized for nearly three days at 77° F and 100° F, respectively, to conduct subsequent tests.

Test Results

Unit load-deflection data for the three tests conducted at 40°, 77°, and 100° F are plotted in Fig 28. The plots show that for the same applied load
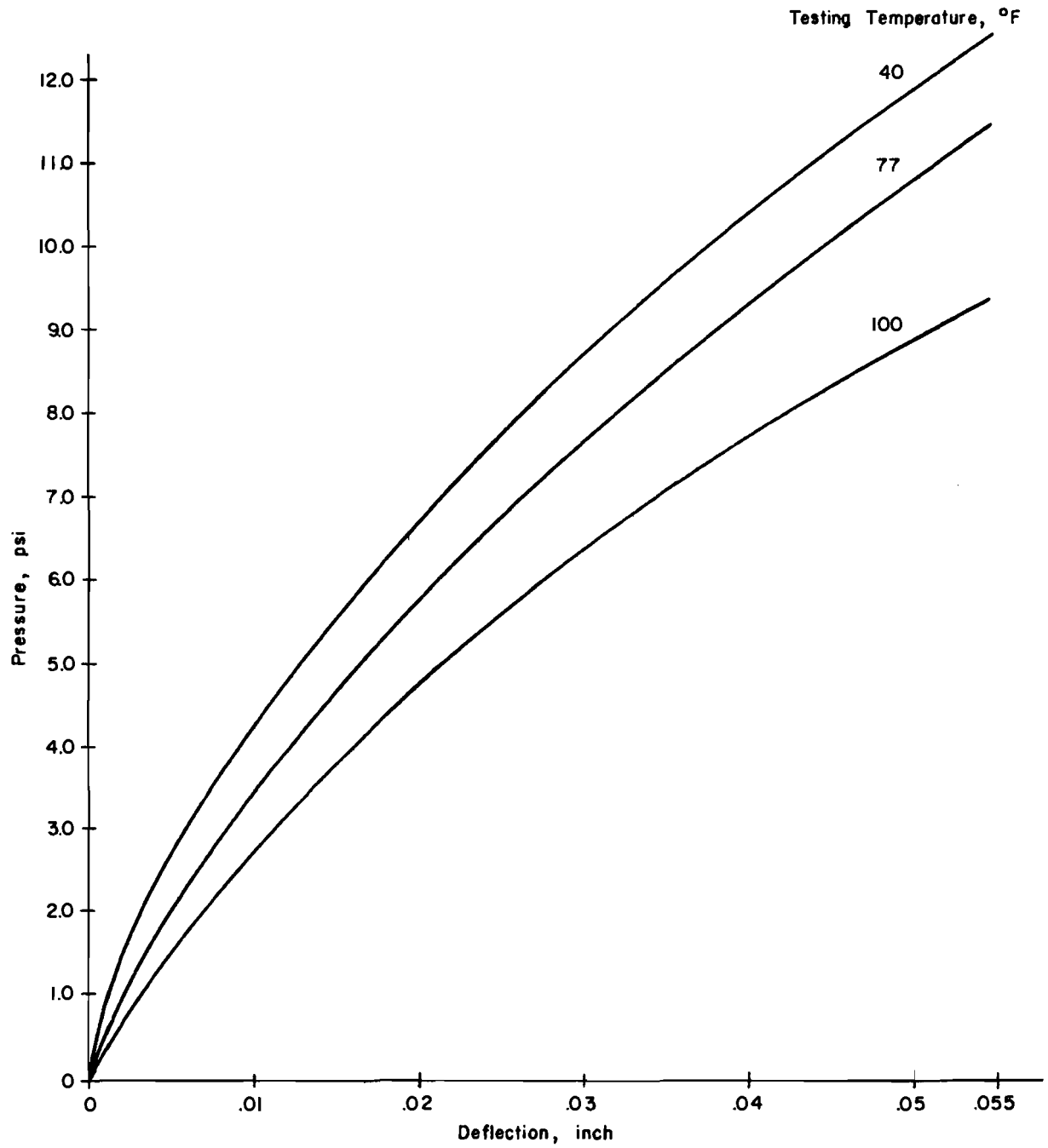
Fig 28.   Pressure versus deflection for a 6-inch plate tested
on layered system at different temperatures.

the deflection of the plate increases with the increase of testing tempera-
ture. In other words, the asphalt-stabilized layer withstood greater loads
at low temperatures than at high temperatures within the range of 40° and
100° F.

From the load-deflection data, the value of k , which is a composite
measure of stiffness and pavement support, was obtained for each testing tem-
perature using tangent and secant moduli approaches. In Fig 29, the value of
k is plotted versus temperature at constant deflections. It can be seen
that the k-value of the system decreases with the increase in temperature,
and the range of variation in k-value is larger at lower deflections than at
higher deflections as shown in the following table.

TABLE 2. EFFECT OF TEMPERATURE ON k-VALUE AT
VARIOUS LEVELS OF DEFLECTION

Temperature, °F	Value of k at Constant Deflection of			
	.005 in.	.01 in.	.02 in.	.05 in.
40	550	430	340	240
77	425	340	280	210
100	310	280	245	180

Calculation of E_1

The Burmister theory for the two-layered system was used in calculating
the modulus of asphalt-stabilized layer E_1 from plate load tests on clay
and on the asphalt-stabilized layer at temperatures of 40°, 77°, and 100° F.
There were two different methods used to calculate the modulus, E_1 , of the
asphalt-stabilized layer. The first method assumed that the load-deflection
characteristics of the clay were insignificantly changed during plate tests
while the second method allowed for changes in both the asphalt and clay
layers. The results of these two methods should provide limits within which
the actual modulus value, E , would fall.

The modulus of elasticity of the clay subgrade, E_2 , was computed from
the load-deflection data from the 6-inch plate test on clay by the use of
Eq 12. The settlement factor F was computed with the calculated value of
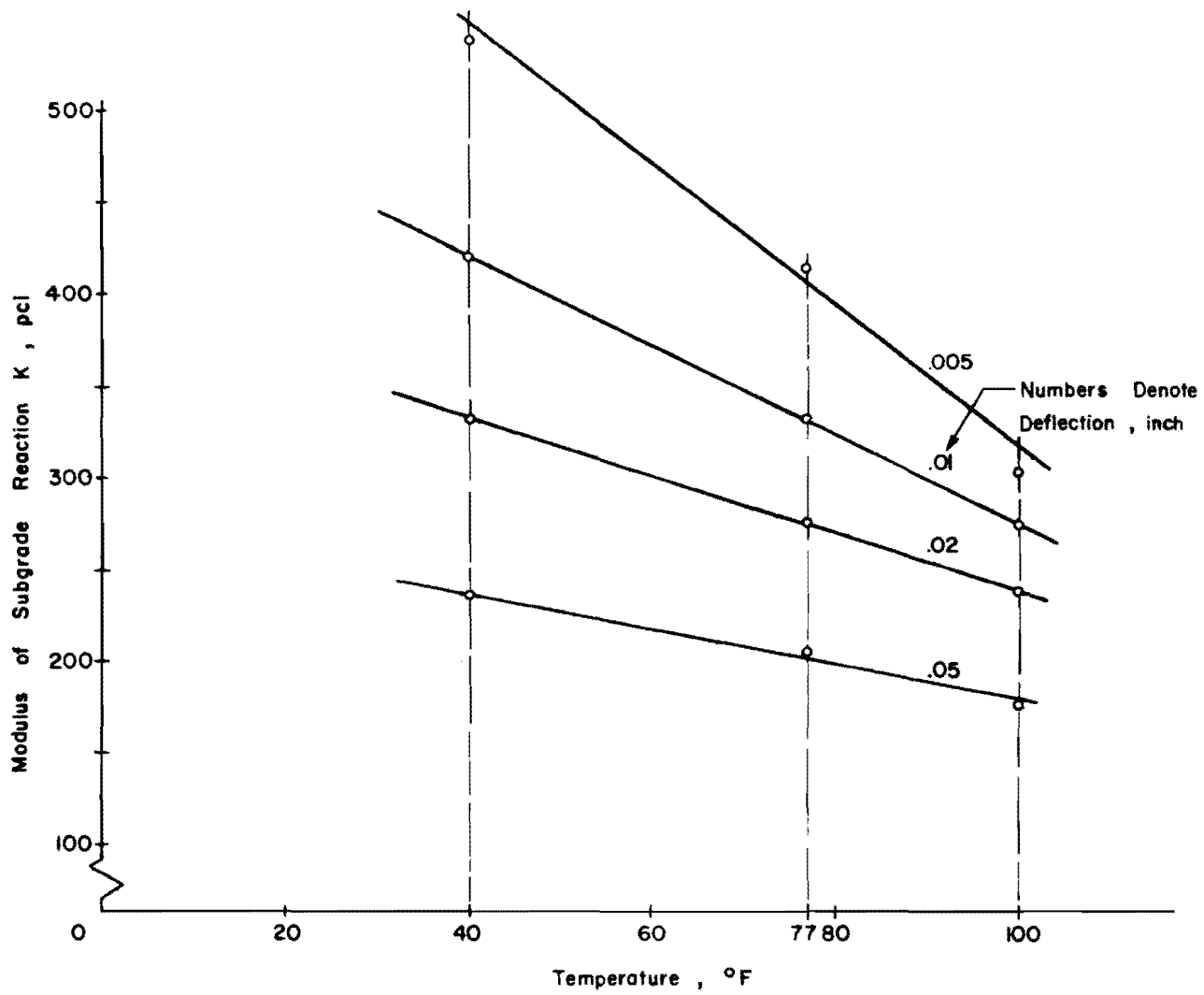
Fig 29. Influence of temperature on composite k-value of
layered systems at various levels of deflection.

E_2 and the data from the 6-inch plate test on an asphalt-stabilized layer by using Eq 11. With the known values of F and $\frac{a}{h}$, E_1 was determined from Fig 26.

In this manner, values of E_1 at different temperatures were determined (Fig 30). Thus, it was observed that variations in temperature had a considerable influence on the pavement modulus E_1.

k versus E_1

It was observed that the modulus of the asphaltic material of layer E_1 directly affected the load-deflection characteristics of the asphaltic layer; or, in other words, k-value, which is a ratio of pressure and deflection $(k = \frac{p}{w})$, is directly related to E_1. For use in studying the influence of E_1 on k, Table 3 was prepared. It shows the corresponding values of E_1 and k for the different levels of deflection and at the different temperatures considered in this study. It can be noted that with the increase in temperature from 40° F to 100° F, the value of E_1 for a constant value of $E_2 = 575$ was reduced from 76,500 psi to 9950 psi (87 percent reduction) while the k-value decreased from 550 psi to 310 psi (44 percent reduction) at a constant deflection of .005 inch. Similarly the value of E_1 for a varying value E_2 for the clay was reduced from 93,000 psi to 13,000 psi (86 percent reduction) for an increase in temperature from 40° F to 100° F. For each level of deflection, the value of E_1 decreased more rapidly than the k-value as temperature increased.

Thus, k-value, which is a measure of pavement deformation, depends upon the modulus of the asphalt-stabilized layer investigated herein. This cursory study shows the variation of supporting capacity of the system with changes in the E_1 value of the reinforcing layer as indicated by the k-value. No firm conclusions are drawn from this preliminary study, but the wide variations of pavement support observed point out the complexity of the problem and warrant further study.
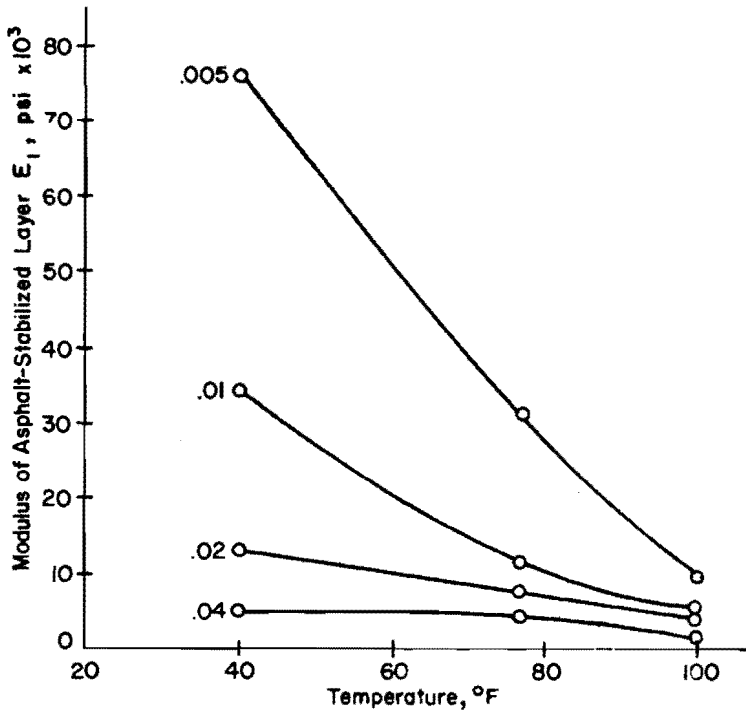
(a) Constant modulus E_2 assumed for clay layer.
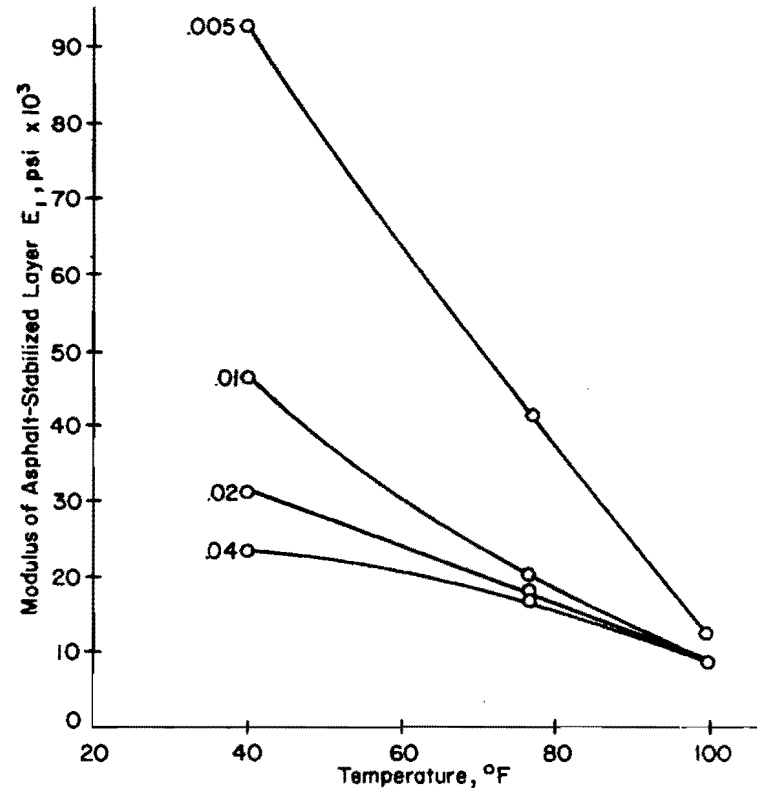
(b) Varying modulus E_2 assumed for clay layer.

Fig 30. Effect of temperature on E_1, the modulus of elasticity of asphaltic material layer.

TABLE 3. VALUES OF E AND k AT CONSTANT TEMPERATURES AND DEFLECTIONS*

	Temperature, °F	$40°$				$77°$				$100°$			
	Deflection, in.	.005	.01	.02	.04	.005	.01	.02	.04	.005	.01	.02	.04
Constant E_2	Modulus of Subgrade E_2 , psi **	575	575	575	575	575	575	575	575	575	575	575	575
	Modulus of Asphalt Stabilized Layer E_1 , psi	76500	34500	13000	5200	31000	11500	7900	4200	9950	5750	4100	1800
Varying E_2	Modulus of Subgrade E_2 , psi	525	500	425	335	525	500	425	335	525	500	425	335
	Modulus of Asphalt Stabilized Layer E_1 , psi	93000	46500	31000	23500	41000	20500	18000	16750	13000	8700	8700	8700
	k-Value for the layer, pci	550	425	335	260	425	340	290	235	310	275	235	200

*Computed from the results of the 6-inch plate test on an asphalt-stabilized layer and clay subgrade.

**Average value of E_2 is used.

CHAPTER 6. ANALYSIS OF SLAB TEST DATA

To evaluate the plate load determination of k for the layered founda-
tion, a small-dimension aluminum slab was tested over a thin layer of asphalt
concrete with clay subgrade, under a static load applied at the center of the
slab. The details of this test are described in Chapter 3. Deflections and
strains were measured at various points on the slab as shown in Fig 31. The
test is similar to the test reported by Agarwal and Hudson (Ref 1), and com-
parisons between the tests are discussed.

Deflections at various points on the slab were measured by dial gages
(Table A3.1, Appendix 3). Loads and corresponding strains were recorded in
digital form by a digital voltmeter. A sample printout from the digital
voltmeter is included in Appendix 3. The recorded data were processed to
obtain loads and strains. A sample calculation of a typical printout for
channels 1 and 2 is included in Appendix 3. These data were recorded for a
maximum load of 205 pounds applied in the first cycle of the slab test.

Load-Deflection Curves

Curves for load versus deflection were plotted for each of the six posi-
tions of deflection measurements as recorded by dial gages (Fig 32). The maxi-
mum applied load was 210 pounds and the maximum measured deflection was
0.0362 inch, recorded by gage No. 1, at a distance of one inch from the center
of the slab.

The plots of load versus deflection indicated a nearly linear relation-
ship for loads up to 120 pounds, but for higher loads the slab deformations
gradually increased. The corners of the slab lifted under applied loads as
measured by gage 6.

The experimental solution for deflections and stresses was compared with
the discrete-element solution developed by Hudson, Matlock, and Stelzer (Refs
25 and 42).

The values of k for the solutions were determined from the load-
deflection data of 9-inch plate tests discussed in Chapter 4. The initial
straight line portion of the load-deflection curve gave a tangent modulus

57

9-in. x 9-in. x 1/8-in.
Aluminum Slab

Point of Loading

1 5/8-in. Asphalt
Concrete Layer

22-in.-Thick Clay
Subgrade
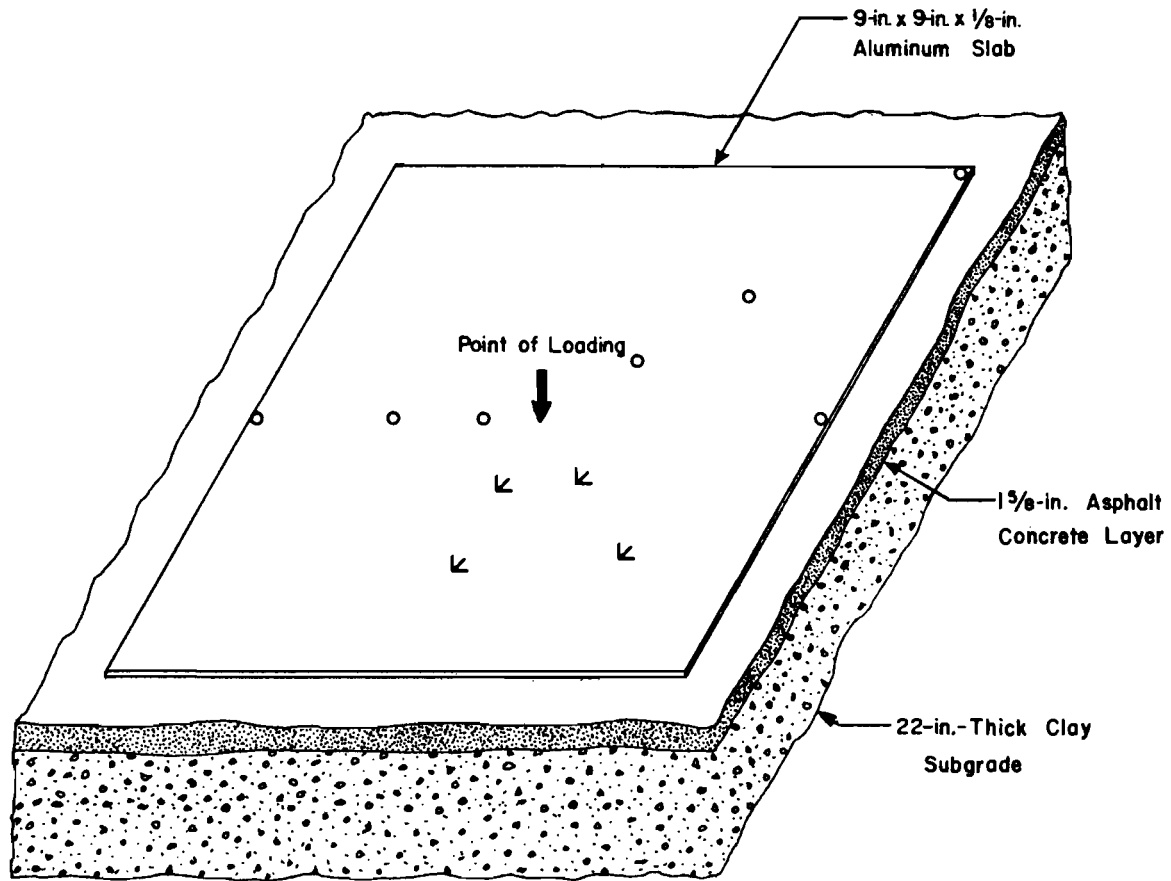
NOTE: For Location of Dial Gages
and Rosettes See Figure 4

o — Dial Gage

k — Rosettes

Fig 31. Aluminum slab on layered system under a center load.
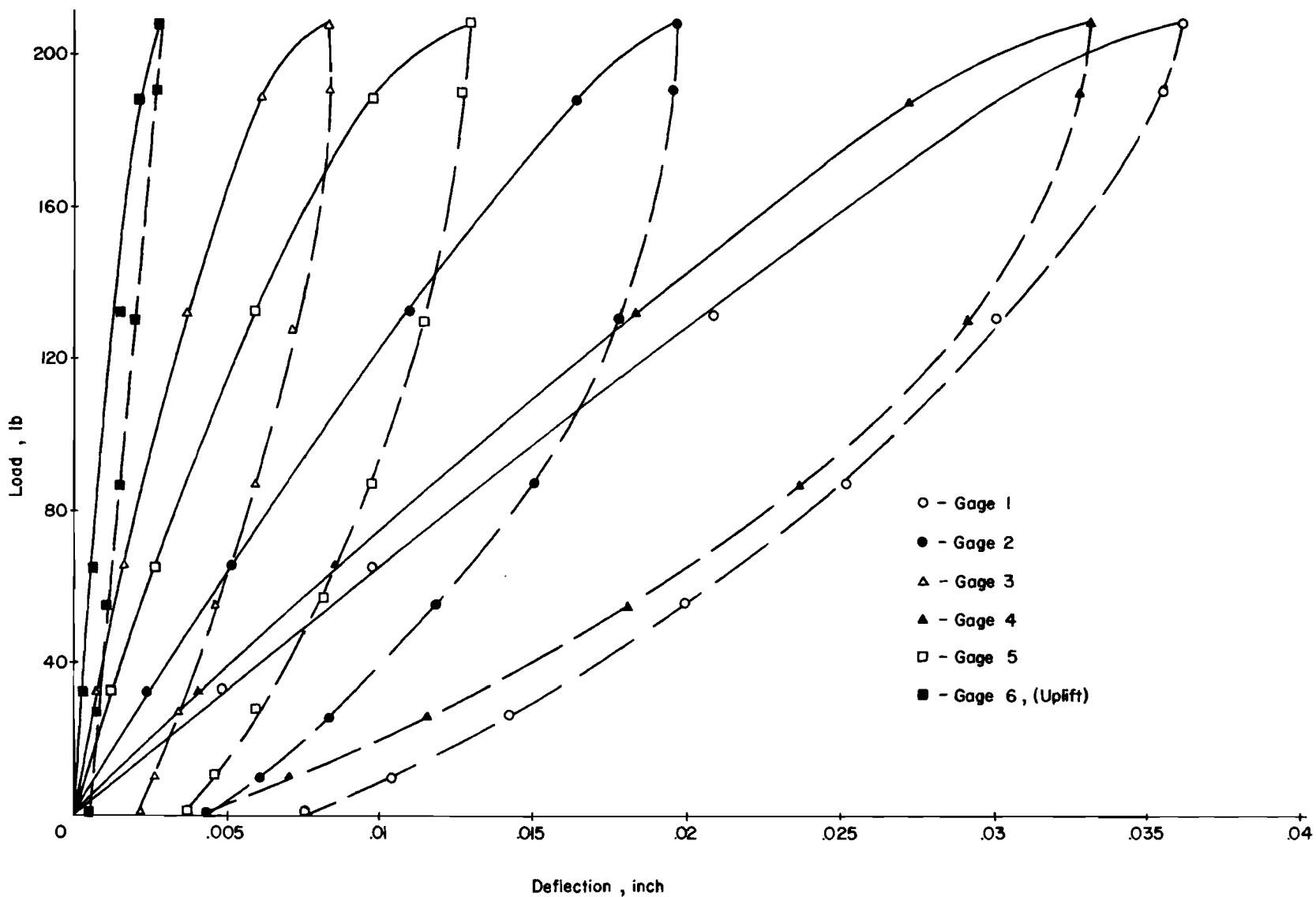
Fig 32. Load versus deflection for gage points located on slab.

value of k equal to 240 lb/cu in. This k-value was used as a linear spring value for the solution of the test slab. A nonlinear q-w curve, developed directly from load-deflection data, was used for the solution of the slab based on nonlinear support.

Comparison of Experimental and Analytical Solutions for Deflections

The experimental and analytical solutions for deflection in slabs are compared in Figs 33, 34, 35, and 36. These figures show the comparison of the solutions for deflections of the slab on points along its center line and the diagonal for loads of 100 and 200 pounds, respectively.

The two solutions were compared and the percentage errors were calculated. The calculation of percentage error was based on the maximum measured deflection in the slab test and was taken equal to:

$$\% \text{ error} = \frac{w_E - w_C}{w_M} \times 100$$

where

w_E = measured deflection of any point in slab,

w_C = corresponding computed deflection of the point in slab,

w_M = maximum measured deflection in slab (measured to be 0.0325 inch for a point 1.0 inch away from the center of the slab).

Tables 4 and 5 show the comparison of deflections of the slab as obtained from the experimental and the analytical solutions. The comparison was good in the interior two-thirds of the slab around the point of application, where the experimental data were within 6 percent of the analytical solution. The analytical solutions with linear and nonlinear springs gave similar results. The similarity of results may be due to the fact that the load-deflection curve of gage 1 which measured the maximum deflection appeared to be linear up to 200 pounds. For 100 pound load, the solutions showed less percentage error on the whole than for the load of 200 pounds.

For deflections at the corner and the edge of the slab, the two solutions, experimental and analytical, differed considerably. Computed deflections were lower than the measured deflection by 12 to 15 percent for points on edge, and
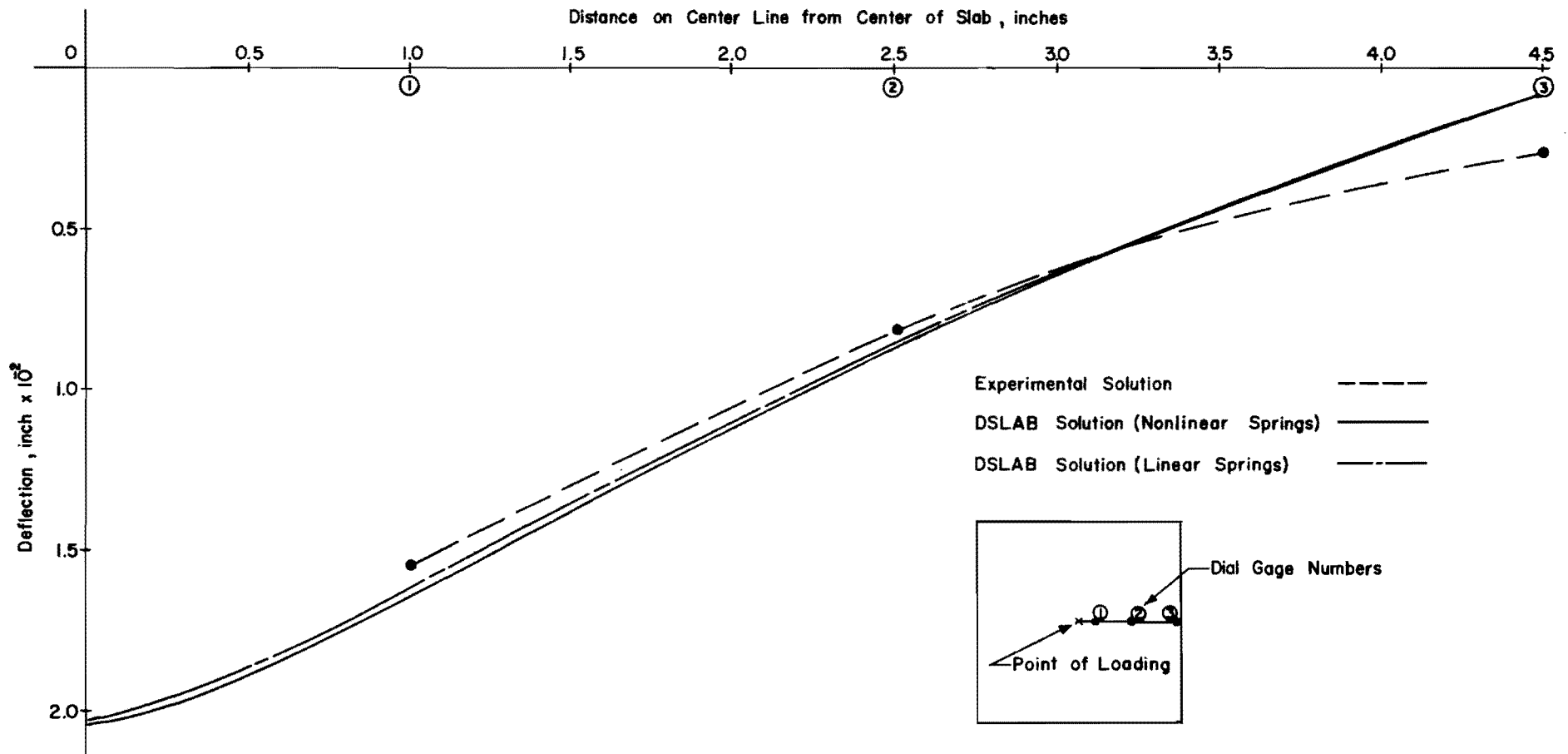
Fig 33. Comparison of experimental data and analytical solutions for deflections of slab on
points along its center line for a load of 100 pounds applied at center.
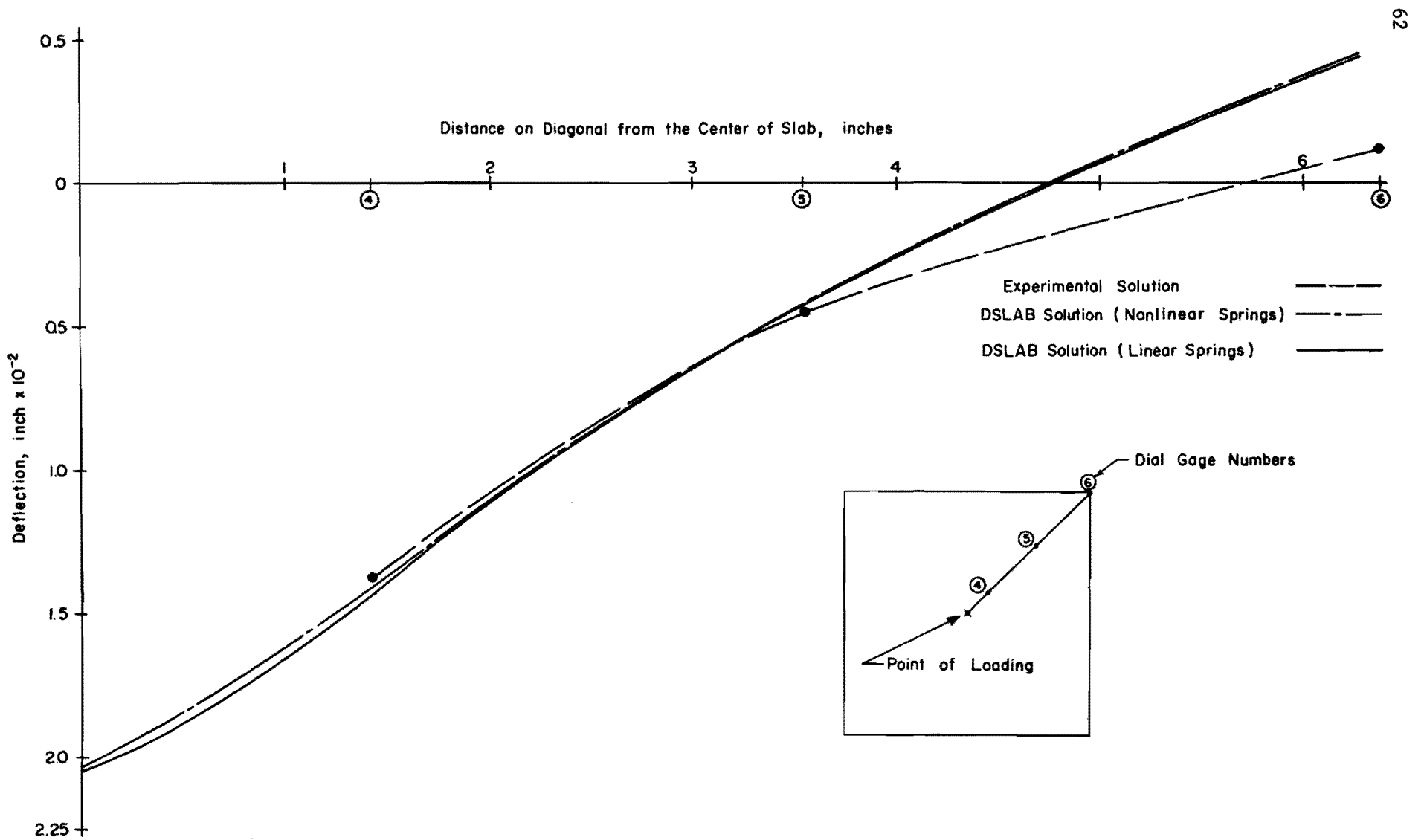
Fig 34. Comparison of experimental and analytical solutions for deflections of slab on points along its diagonal for a load of 100 pounds applied at center.
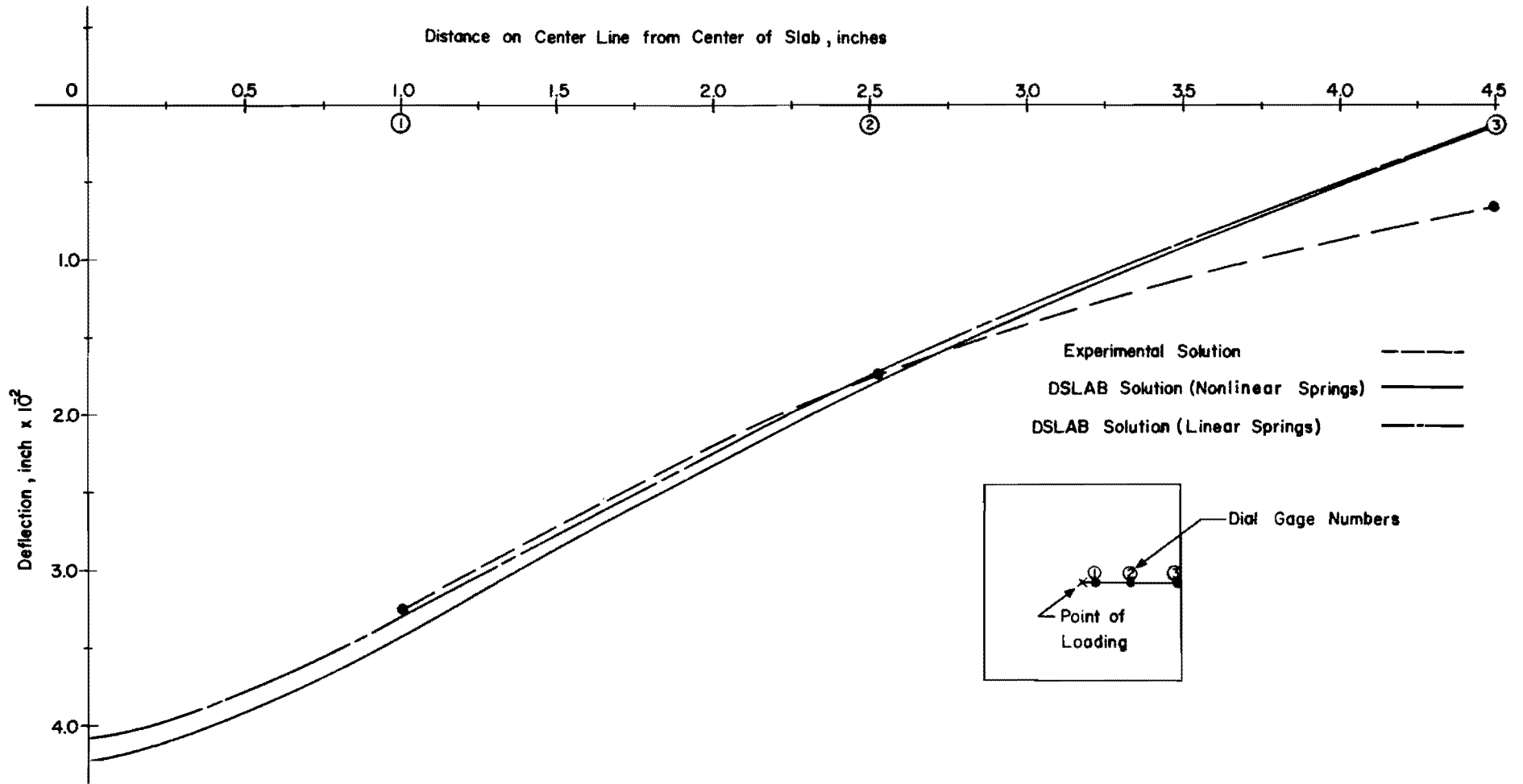
Fig 35. Comparison of experimental and analytical solutions for deflections of slab on points along its center line for a load of 200 pounds applied at center.
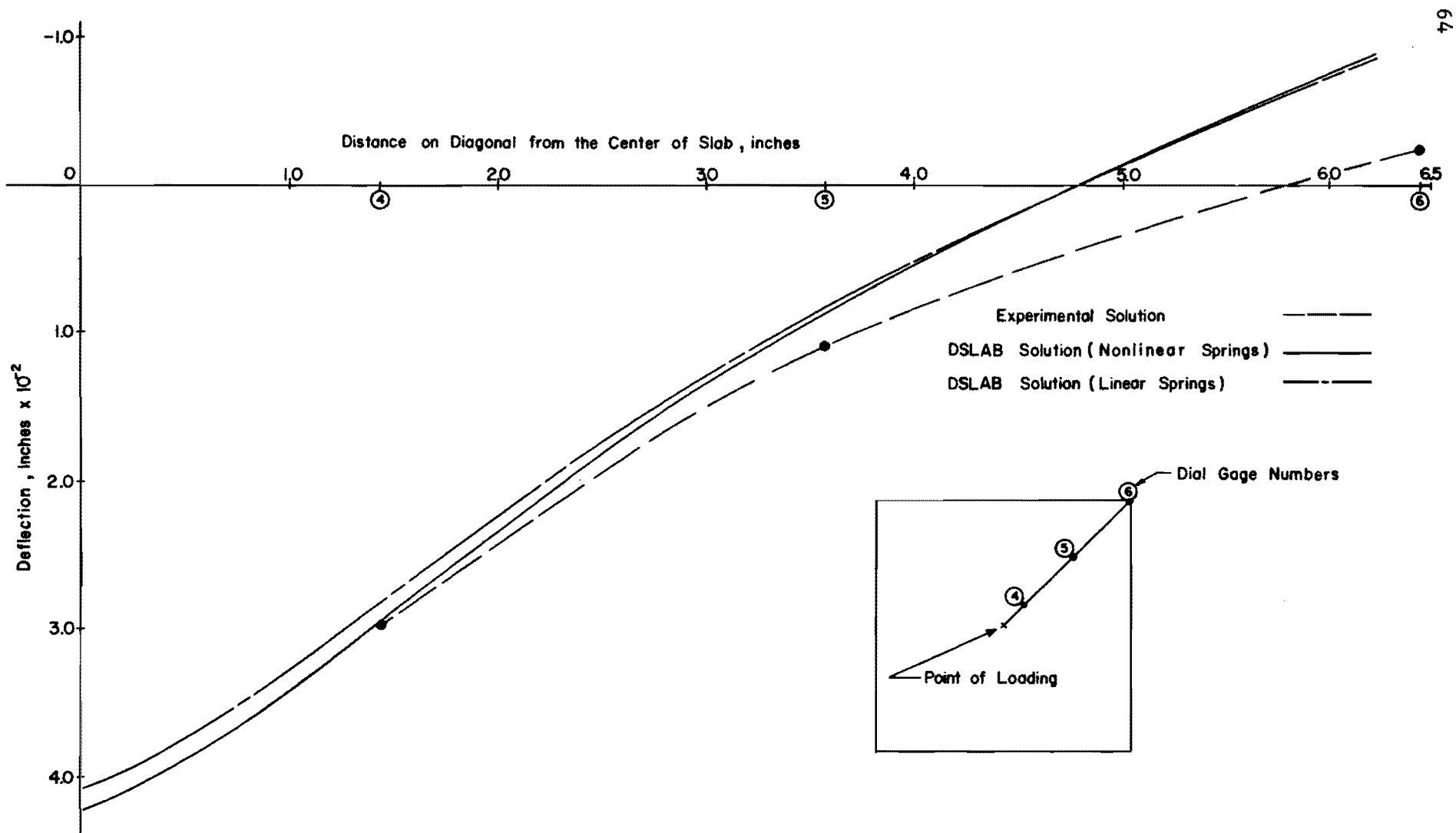
Fig 36. Comparison of experimental and analytical solutions for deflections of slab on points along its diagonal for a load of 200 pounds applied at center.

TABLE 4. COMPARISON OF EXPERIMENTAL AND ANALYTICAL SOLUTIONS FOR DEFLECTIONS FOR SLAB TEST

Load: 10C lb at center

Programs: DSLAB 30 (linear springs)
          DSLAB 26 (nonlinear springs)

(18 × 18 increments)

Solutions	Dial Gages (Deflections measured in inches)					
	1	2	3	4	5	6
	1 inch	2.5 inches	4.5 inches	1.41 inches	3.54 inches	6.36 inches
Experimental	0.01550	0.00825	0.00275	0.01375	0.00450	-0.00120
Linear springs	0.01649	0.00872	0.00082	0.01423	0.00421	-0.00481
% Error	-6.38	-3.03	12.45	-3.09	1.87	-23.2
Nonlinear springs	0.01633	0.00862	0.00080	0.01408	0.00415	-0.00467
% Error	-5.35	-2.39	12.58	-2.12	2.26	-22.4

Note: Plot along the center line is shown in Fig 33.

TABLE 5. COMPARISON OF EXPERIMENTAL AND ANALYTICAL SOLUTIONS FOR DEFLECTIONS FOR SLAB TEST

Load: 200 lb at center

Programs: DSLAB 30 (linear springs)
DSLAB 26 (nonlinear springs)

(18 × 18 increments)

| Solutions | Dial Gages (Deflections measured in inches) | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 |
	1 inch	2.5 inches	4.5 inches	1.41 inches	3.54 inches	6.36 inches
Experimental	0.03250	0.01750	0.00650	0.02985	0.01100	-0.00250
Linear springs	0.03299	0.01743	0.00164	0.02845	0.00841	-0.00963
% Error	-1.51	0.21	14.95	4.31	7.96	-21.93
Nonlinear springs	0.03424	0.01811	0.00170	0.02956	0.00874	-0.01002
% Error	-5.36	-1.88	14.76	-0.89	6.95	-23.13

Note: Plot along the center line is shown in Fig 35.

deflections were higher by 22 to 23 percent in case of corner deflection of the slab. The use of the nonlinear q-w curve in the analytical solution did not provide significant improvements in the comparison of corner deflections with experimental solution of the slab. In the real problem the adjoining soil may be giving an additional restraint to the slab around the edges. Moreover, the absence of shear connections in the discrete spring representation of the foundation may result in unrealistically high computed deflections.

Comparison of Experimental and Analytical Solutions for Principal Stresses

The largest principal stress for a given load on the slab was calculated from the corresponding strains measured by the rosettes during the slab test, as described in Appendix 3. Fig 37 shows the plot of load versus measured principal stresses for the rosettes points R_1, R_2, R_3, and R_4 on the slab.

The measured principal stresses thus obtained correspond to the top of the slab, whereas the stresses obtained from the analytical solutions (DSLAB 26 and 30) correspond to the bottom of the slab. Keeping this in mind, a comparison of the measured and the DSLAB largest principal stress was made for the loads of 100 and 200 pounds, and the percentage errors were calculated as a function of the measured largest principal stress, as given below:

$$\text{Percentage error} = \frac{\sigma_m - \sigma_c}{\sigma_{xm}} \times 100$$

where

σ_m = measured largest principal stress,

σ_c = DSLAB largest principal stress,

σ_{xm} = maximum measured principal stress for the load under consideration.

Tables 6 and 7 show the comparison of experimental and analytical solutions for principal stresses in the slab for loads of 100 and 200 pounds, respectively.

From the comparison, it is observed that the computed stresses are in good agreement with the measured stresses in the slab near the loaded area. The maximum measured principal stress is within 3 percent of the computed
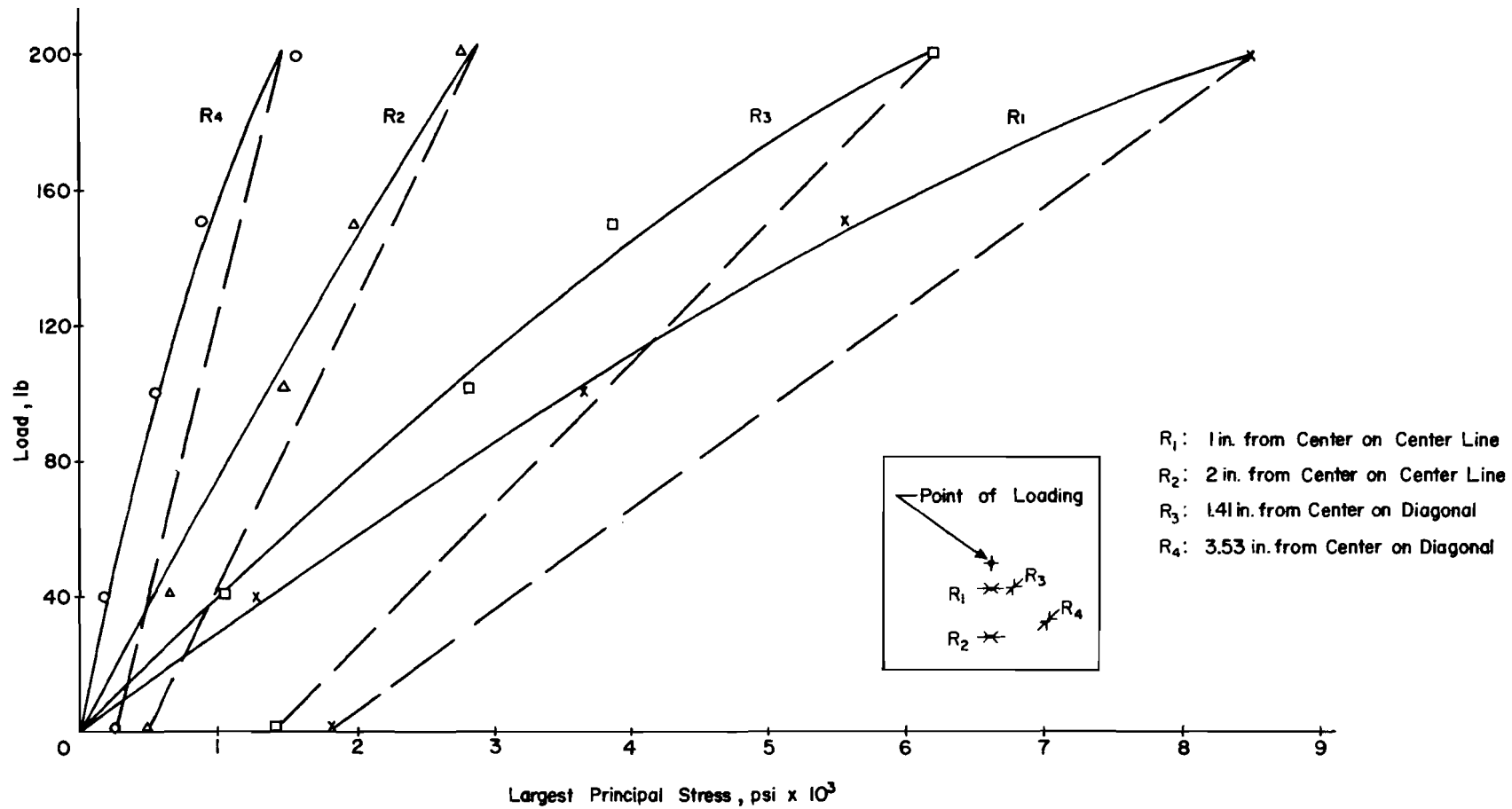
Fig 37. Curves showing load versus largest principal stress for Rosette 1.

TABLE 6. COMPARISON OF EXPERIMENTAL AND ANALYTICAL SOLUTIONS FOR LARGEST PRINCIPAL STRESSES FOR SLAB TEST.

Load: 100 lb at center

Programs: DSLAB 30 (linear springs)
DSLAB 26 (nonlinear springs)

(18 × 18 increment)

Solutions	Rosettes* (Principal stresses measured in psi)			
	R_1	R_2	R_3	R_4
Experimental	3650.00	1060.00	2815.00	515.00
Linear springs	4215.00	878.00	2668.00	588.00
% Error	-15.47	4.98	4.03	-2.00
Nonlinear springs	4190.00	867.00	2640.00	581.00
% Error	-14.79	5.28	4.79	-1.80

*For location of rosettes on slab see Fig 4.

TABLE 7. COMPARISON OF EXPERIMENTAL AND ANALYTICAL SOLUTIONS
FOR LARGEST PRINCIPAL STRESSES FOR SLAB TEST.

Load: 200 lb at center

Programs: DSLAB 30 (linear springs)
            DSLAB 26 (nonlinear springs)

(18 × 18 increment)

Solutions	Rosettes* (Principal stresses measured in psi)			
	R_1	R_2	R_3	R_4
Experimental	8500.00	2700.00	6200.00	1075.00
Linear springs	8250.00	1720.00	5000.00	1350.00
% Error	2.94	11.5	14.11	-3.23
Nonlinear springs	8670.00	1820.00	5500.00	1230.00
% Error	-2.00	10.35	8.23	1.82

*For location of rosettes on slab see Fig 4.

stress. At the other points in the slab, the two solutions for principal stresses differed considerably, particularly for the load of 200 pounds. The maximum percentage error is noted at load of 100 pounds for the stresses in the slab at 1.0 inch from its center where the computed stresses are 15 percent higher than the measured ones. The use of nonlinear springs did not provide results which were significantly better than those derived from the use of linear springs.

Comparison of Load-Deflection Curves for the Slab Tests on Clay and on Layered System

The load-deflection data were compared with the load-deflection data for the same slab tested on clay as reported by Agarwal and Hudson (Ref 1). The comparison was made possible by conducting the two tests under similar loading conditions and test set up, i.e., the measurement of deflections at the same points and over the same range of loads.

For a case study, maximum measured deflection in the two tests were compared (Fig 38). These points were located at a distance of 1.0 inch from the center of the slabs. It is noted that by providing a thin layer of asphaltic material over clay, the maximum measured deflection at a load of 200 pounds decreased from 0.0607 inch to 0.0325 inch (46 percent reduction).

Cyclic Load Test

In order to have a cursory look at the behavior of a slab on layered foundation under cyclic loading, the static slab test described earlier was continued for ten repetitions of load. The load was governed by the maximum measured deflection (.0362 inch) recorded in the static load test (first cycle). In each cycle, the slab loading was stopped at the governing deflection of 0.0362 inch.

Figure 39 shows the load-deflection curve for the point of maximum measured deflection (dial gage 1 located at 1.0 inch from point of application of load) for the first, fifth, and tenth cycles of loading and unloading.

It is observed that the curves for the fifth and tenth cycle follow quite closely the slope of the curve for the first cycle. The effect of cyclic loading of the slab to the maximum measured deflection of 0.0362 inch is to reduce the load from 208 pounds for the first cycle to 189 pounds and 182 pounds for the fifth and tenth cycles, respectively. Presumably, by the tenth cycle the load has begun to stabilize.
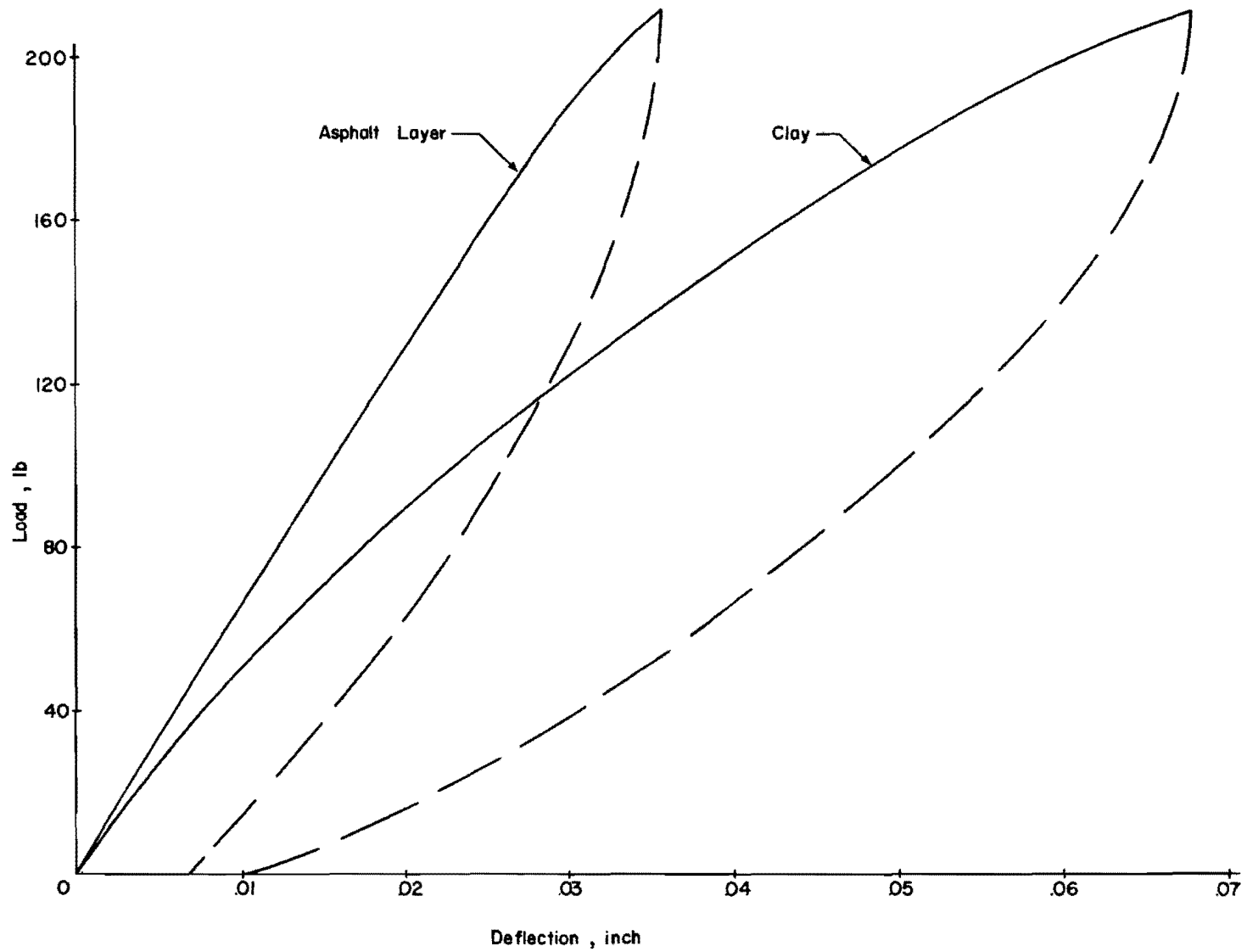
Fig 38. Comparison of load versus deflection curves for Gage 1
for slab test on clay and on layered system.
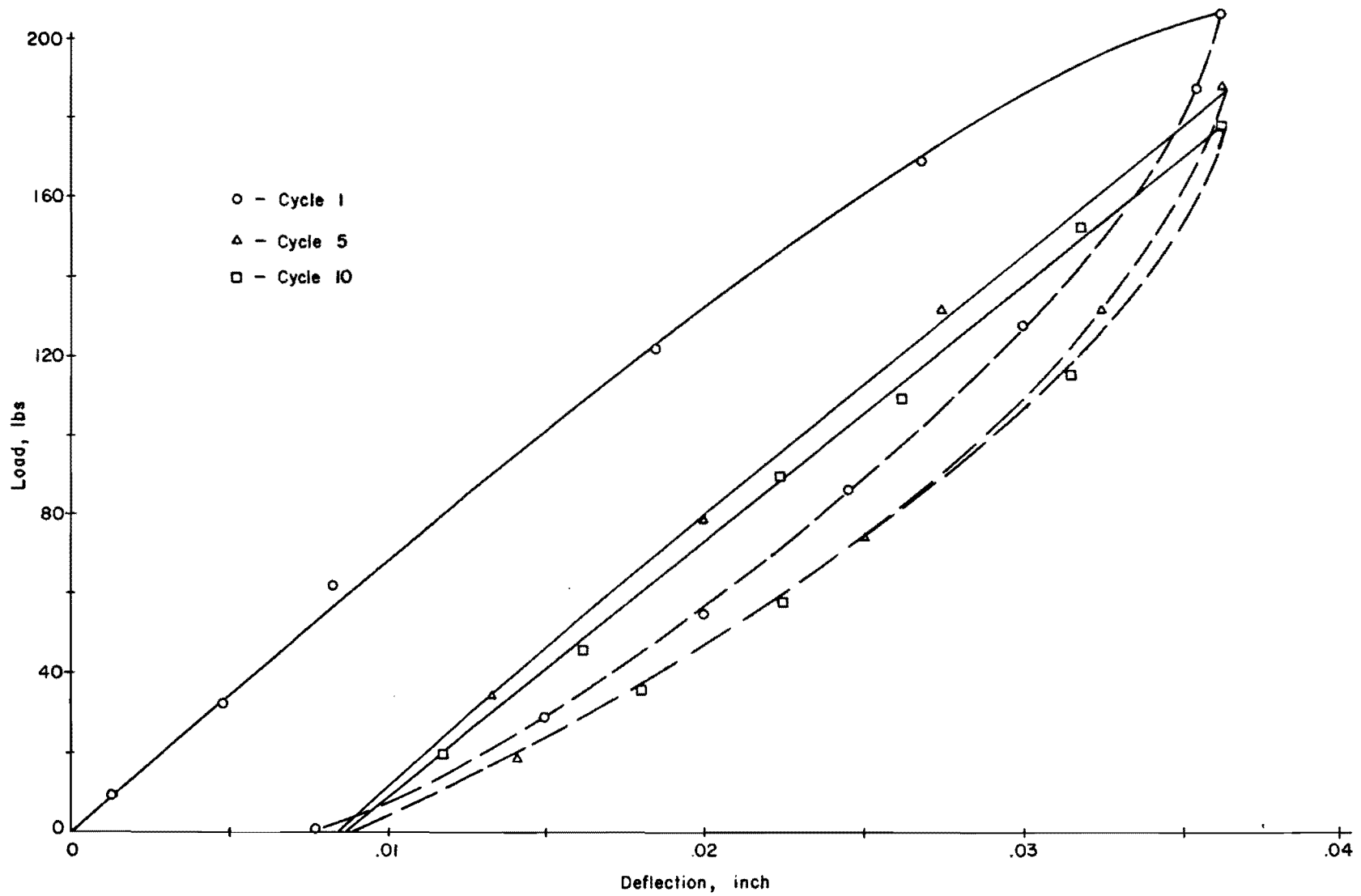
Fig 39. Load versus deflection for Gage 1 under cyclic loading.

In general, deflections after cycling appeared to be greater than those for the first cycle, except for the observation point one inch from the point of load application. This exception was the point of maximum measured deflection which was kept at a constant deflection.

CHAPTER 7. CONCLUSIONS, RECOMMENDATIONS, AND APPLICATIONS

Load-deflection data of the plate load tests described in this study were used in the evaluation of a practical and a representative value of the modulus of subgrade reaction k . This value of k was utilized in the solution of a two-dimensional slab-on-foundation model using the discrete-element method (Refs 27 and 36). The analytical solutions were compared with the experimental results of the slab test conducted in this study using the same method of comparison as Agarwal and Hudson (Ref 1). The following conclusions are drawn:

(1) The relationship between pressure and deflection is linear for a small initial deflection of the plate and nonlinear for higher deflections of the plate; the latter differs from the assumption of the theory of linear elasticity.

(2) The value of soil modulus k varies inversely with the size of the plate used in the load tests and with the magnitude of the soil deformation.

(3) Putting a thin layer of asphalt concrete on the clay subgrade improved the k-value of the system by 40 percent, and thus for the same load the slab deformed less on the layered foundation than on clay alone.

(4) The initial tangent modulus value of k obtained from the pressure-deflection data of the 9-inch plate test provided a good analytical solution of slab-on-layered-foundation for comparison with the experimental solution of the slab test.

(5) Computed deflections and stresses in the slab were within 5 percent of the measured values in the interior of the slab near the point of loading. The major discrepancy in the comparison was found in deflections at the corners of the slab. Computed deflections were 20 to 23 percent higher than the measured deflections at the corners of the slab, showing that a constant value of k is probably not

75

fully representative of the actual conditions of subgrade existing at any point beneath the slab.

(6) On the slab, the cycling of the load to 10 cycles to a constant deflection produced some permanent deformation, and by the tenth cycle, the load appeared to be stabilizing. Moreover, it may be observed that the curves for the fifth and tenth cycles followed the slope of the curve quite closely for the first cycle of loading. The same pattern was noticed for measured strains in slabs.

(7) The value of E_1 , the modulus of elasticity of the asphaltic material in the layered system determined from the split tensile test, predicted the load-deflection curve for a two-layered system. It was based on Burmister's theory which compared reasonably well with the load-deflection data from the plate load tests for small deflections.

(8) Temperature significantly affected the modulus of elasticity of asphalt concrete which in turn affected the composite k-value of the two-layered system investigated in this study. For a change in temperature from 40° F to 100° F, the modulus of elasticity of asphalt concrete reduced from 76,500 psi to 9,950 psi (87 percent reduction) while the k-value (initial tangent modulus) was decreased from 550 lb/cu in to 310 lb/cu in (44 percent decrease).

Recommendations

The following recommendations are suggested for further research into the study described in this report:

(1) In the determination of a k-value for a soil from the plate load tests, the size of the plate should be carefully selected, since the value of k is influenced by the size of the plate used; and it is also advisable to limit the deformation of the plate to a magnitude within the range of pavement deflection.

(2) Plate load tests should also be investigated under cyclic loading to take into account the repetitions of loads on pavement due to moving vehicles. This may simulate the actual loading conditions of pavement.

(3) Further studies on the deformation of the layered subbases are needed to correlate E_1 , the modulus of elasticity of the subbase material and E_2 , the modulus of elasticity of subgrade material with the value of k for the layered system. This will be a significant step in the evaluation of k from inexpensive laboratory tests on the material in the layered system.

(4) For loads up to 200 pounds applied at the center of the slab, the use of linear or nonlinear values of k provides a satisfactory analytical solution in the interior of the slab for comparison with the experimental solution, but for larger loads, the use of nonlinear q-w curve for the soil may yield a satisfactory solution.

(5) The effect of cyclic loading on a slab should be more fully investigated in order to study the behavior of pavement slabs under repetitive loading.

Applications

The research reported herein is an experimental look at parameters affecting the subgrade support of pavement slabs on stabilized subbase layers on clay subgrades. The results of the study are not applicable immediately to highway practice, but will be very useful data in correlating work from Research Project No. 3-8-66-98, entitled "Evaluation of Tensile Properties of Subbases for Use in New Rigid Pavement Design" for use with slab analysis programs developed in Research Project No. 3-5-63-56, entitled "Development of Methods for Computer Simulation of Beam-Columns and Grid-Beam and Slab Systems."

The findings of the study are also quite useful in verifying the discrete-element analysis of slab-on-foundation problems and will ultimately lead to improved pavement design methods for the Texas Highway Department.

REFERENCES

1. Agarwal, Sohan L., and W. Ronald Hudson, "Experimental Verification of Discrete-Element Solutions for Two-Dimensional Soil-Structure Problems," Research Report No. 56-15, Center for Highway Research, The University of Texas at Austin, April 1970.

2. Barber, E. S., "Application of Triaxial Compression Test Results to the Calculation of Flexible Pavement Thickness," Proceedings, Vol 26, Highway Research Board, 1946.

3. Benscoter, S. U., "A Symmetrically Loaded Base Slab on an Elastic Foundation," Proceedings, Vol 109, American Society of Civil Engineers, 1944.

4. Bergstrom, S. G., "Circular Plates with Concentrated Load on an Elastic Foundation," Bulletin No. 6, Swedish Cement and Concrete Research Institute, Stockholm, 1946.

5. Biot, M. A., "Bending of an Infinite Beam on an Elastic Foundation," Journal of Applied Mechanics, Vol 4, 1937.

6. Boussinesq, J., Application des Potentiels, Paris, 1885.

7. Burmister, D. M., "The Theory of Stresses and Displacements in Layered Systems and Applications to Design of Airport Runways," Proceedings, Vol 23, Highway Research Board, 1943.

8. Burmister, D. M., "Stress and Displacement Characteristics of a Two-Layered Rigid Base Soil System: Influence Diagrams and Practical Applications," Proceedings, Vol 36, Highway Research Board, 1956.

9. Campen, W. H., and J. R. Smith, "An Analysis of Field Load Bearing Tests Using Plates," Proceedings, Vol 24, Highway Research Board, 1944.

10. Coffman, B. S., D. C. Kraft, and J. Tamayo, "A Comparison of Calculated and Measured Deflections for the AASHO Road Test," Proceedings of the Association of Asphalt Paving Technologists, Vol 33, 1964.

11. De Beer, E. E., "Computation of Beams Resting on Soil," Proceedings, Second International Conference on Soil Mechanics and Foundation Engineering, Rotterdam, 1948.

12. Fox, L., "Computation of Traffic Stresses in Simple Road Structures," Proceedings, Vol 2, Second International Conference on Soil Mechanics and Foundation Engineering, 1948.

13. Habel, A., "Näherungsberechnung des auf dem elastisch-isotropen Halbraum aufliegenden elastischen Balkens," (An Approximate Calculation of Elastic Beams in an Elastic Isotropic Hemisphere), Der Bauingenieur 19, 1938.

14. Hadley, William O., W. Ronald Hudson, and Thomas W. Kennedy, "An Evaluation of Factors Affecting the Tensile Properties of Asphalt-Treated Materials," Research Report 98-2, Center for Highway Research, The University of Texas at Austin, March 1969.

15. Hank, R. J., and F. H. Scrivner, "Some Numerical Solutions of Stresses in Two and Three-Layered Systems," Proceedings, Vol 28, Highway Research Board, 1948.

16. Hertz, H., "Uber das Gleichgewicht Schwimmender Elasticher Platten," Weidemann's Annalen der Physik und Chemie, Vol 22, 1884.

17. Heteyni, M., "Beams on Elastic Foundations," University of Michigan Press, Ann Arbor, 1946.

18. Heukelom, W., and A. J. G. Klomp, "Road Design and Dynamic Loading," Proceedings of the Association of Asphalt Paving Technologists, Vol 33, 1964.

19. Hogg, A. H. A., "Equilibrium of a Thin Plate, Symmetrically Loaded, Resting on an Elastic Foundation of Infinite Depth," Philosophical Magazine, Series 7, Vol 25, 1938.

20. Holl, D. L., "Thin Plates on Elastic Foundation," Proceedings, Vol 1, Fifth International Conference on Soil Mechanics and Foundation Engineering, 1961.

21. Hondros, G., "The Evaluation of Poisson's Ratio and the Modulus of Materials of a Low Tensile Resistance by the Brazilian (Indirect Tensile) Test with Particular Reference to Concrete," Australian Journal of Applied Science, Vol 10, No. 3.

22. Housel, W. S., "A Practical Method for the Selection of Foundations Based on Fundamental Research in Soil Mechanics," Engineering Research Bulletin No. 13, Department of Engineering Research, University of Michigan, Ann Arbor, October 1929.

23. Hubbard, P., and F. R. Field, Research Series No. 8, The Asphalt Institute.

24. Hudson, W. Ronald, and Thomas W. Kennedy, "An Indirect Tensile Test for Stabilized Materials," Research Report No. 98-1, Center for Highway Research, The University of Texas at Austin, 1968.

25. Hudson, W. Ronald, and Hudson Matlock, "Discontinuous Orthotropic Plates and Pavement Slabs," Research Report No. 56-6, Center for Highway Research, The University of Texas, Austin, 1965.

26. Jones, A., "Calculation of Stresses, Strains, and Displacements in Layered Systems Having Constant and Variable Elastic Parameters," a paper submitted to Second International Conference on the Structural Design of Asphalt Pavements, University of Michigan, Ann Arbor, 1967.

27. Kelly, Allan E., "Dynamic Analysis of Discrete-Element Plates on Nonlinear Foundation," (DSLAB 26), Ph.D. Dissertation, The University of Texas at Austin, January 1970.

28. Kennedy, Thomas W., and W. Ronald Hudson, "Application of the Indirect Tensile Test to Stabilized Materials," Highway Research Record, No. 235, Highway Research Board, January 1968.

29. Ku, A. B., "Stress-Strain Law for Visco-Elastic Flexible Pavement Under Temperature Variations," Proceedings, Second International Conference on the Structural Design of Asphalt Pavements, University of Michigan, Ann Arbor, 1967.

30. Lee, C. E., "The Determination of Pavement Deflections Under Repeated Load Applications," Ph.D. Dissertation, University of California, Berkeley, 1961.

31. McLeod, N. W., "A Canadian Investigation of Load Testing Applied to Pavement Design," Symposium on Load Tests of Bearing Capacity of Soils, Special Technical Publication No. 79, American Society for Testing Materials, 1947.

32. Metcalf, C. T., "Field Measurements of Dynamic Elastic Moduli of Materials in Flexible Pavement Structures," Proceedings, Second International Conference on the Structural Design of Asphalt Pavements, University of Michigan, Ann Arbor, 1967.

33. Middlebrook, T. A., and G. E. Bertram, "Field Investigation for Flexible Pavement Design," Proceedings, Vol 21, Highway Research Board, 1941.

34. Monismith, C. L., G. A. Secor, and K. E. Secor, "Temperature Induced Stresses and Deformations in Asphalt Concrete," Proceedings of the Association of Asphalt Paving Technologists, Vol 34, 1965.

35. Murphy, G., "Stresses and Deflections in Loaded Rectangular Plates on Elastic Foundations," Iowa Engineering Experiment Station Bulletin, No. 135, 1937.

36. Panak, John J., and Hudson Matlock, "A Discrete-Element Method for Multiple-Loading Analysis for Two-Way Bridge Floor Slabs," Research Report No. 56-13, Center for Highway Research, The University of Texas at Austin, August 1968.

37. Hadley, William O., W. Ronald Hudson, and Thomas W. Kennedy, "A Method of Estimating Tensile Properties of Materials Tested in Indirect Tension," Research Report 98-7, Center for Highway Research, The University of Texas at Austin, July 1970.

38. Secor, K. E., and C. L. Monismith, "Analysis and Interrelation of Stress-Strain-Time Data for Asphalt Concrete," *Transactions*, Vol 8, Society of Rhelology, 1964.

39. Seed, H. B., F. G. Mitry, C. L. Monismith, and C. K. Chan, "Prediction of Pavement Deflections from Laboratory Repeated Load Tests," Report Number TE 65-6, University of California, Berkeley, 1965.

40. Skempton, A. W., "The Bearing Capacity of Clays," Building Research Congress, London, 1951.

41. "Standard Specifications for Road and Bridge Construction," Texas Highway Department, 1962.

42. Stelzer, C. Fred, Jr., and W. Ronald Hudson, "A Direct Computer Solution for Plates and Pavement Slabs," Research Report No. 56-9, Center for Highway Research, The University of Texas, Austin, October 1967.

43. Teller, L. W., and E. C. Sutherland, "The Structural Design of Concrete Pavements. V. An Experimental Study of the Westergaard Analysis of Stress Conditions in Concrete Pavement Slabs of Uniform Thickness," *Public Roads*, Vol 23, 1942.

44. Terzaghi, K., "Evaluation of Coefficients of Subgrade Reaction," *Geotechnique*, Vol 5, 1955.

45. Timoshenko, S., *Theory of Elasticity*, McGraw-Hill, New York, 1934.

46. Timoshenko, S., and J. N. Goodier, *Theory of Elasticity*, 2nd Edition, McGraw-Hill, New York, 1951.

47. Ueshita, K., and G. G. Meyerhof, "Deflection of Multilayer Soil Systems," *Journal of the Soil Mechanics and Foundations Division*, Vol 93, No. SM5, Proceedings of the American Society of Civil Engineers, September 1967.

48. Van der Poel, C., "Road Asphalts in Building Materials, Their Elasticity and Inelasticity," edited by M. Reiner, Interscience Publishers, New York, 1954.

49. Vesic, A. B., "Beams on Elastic Subgrade and the Winkler's Hypothesis," *Proceedings*, Vol 1, Fifth International Conference on Soil Mechanics and Foundation Engineering, 1961.

50. Westergaard, H. M., "Stresses in Concrete Pavements Computed by Theoretical Analysis," *Public Roads*, Vol 7, No. 2, 1926.

51. Winkler, E., "Die Lehre von Elasticitaet und Festigkeit" (On Elasticity and Fixity), Prague, 1867.

52. Zimmermann, H., "Die Berechnung des Eisenbahn Oberbaues" (The Calculation of Railway Superstructures), Berlin, 1888.

APPENDICES

APPENDIX 1

PROPERTIES OF MATERIALS USED IN THE EXPERIMENTS

TABLE A1.1   PROPERTIES OF CLAY

Type: Taylor marl I

Liquid Limit (L.L.)   =   53.8

Plastic Limit (P.L.)   =   24.5

Plasticity Index (P.I.)   =   29.3

Optimum Moisture Content   =   17.5%

Maximum Dry Density   =   106.5 lb/cu ft

In situ Properties of Clay in the Test Box:

Average Density   =   116 lb/cu ft

Average Moisture Content   =   38%

Average Degree of Saturation   =   96%

Average Shear Strength   =   180 lb/sq ft

TABLE A1.2   PROPERTIES OF ASPHALT

Type: AC-10

Optimum Asphalt Content	9% by weight of the aggregate
Water, %	Nil
Viscosity at 275° F, Stokes	2.6
Viscosity at 140° F, Stokes	1088
Solubility in CCL_4, %	99.7+
Flash Point C.O.C., $^{\circ}$F	570
Ductility, 77° F, 5 cm/min, cm.	141+
Viscosities Determined at 77° F, Centistokes	4.0
Penetration at 77° F, 100 g, 5 sec	92
Specific Gravity at 77° F	1.006

TABLE A1.3 GRADATION USED IN THE EXPERIMENT

Sieve Size	Texas Highway Department Specification* Percent by weight	Gradation used Percent by weight
Passing No. 4 Retained on No. 10	0 - 5	0
Passing No. 10 Retained on No. 40	15 - 40	30
Passing No. 40 Retained on No. 80	20 - 45	35
Passing No. 80 Retained on No. 200	12 - 32	25
Passing No. 200	7 - 20	10
		100

*Reference 41

MIX DESIGN AND SAMPLE PREPARATION PROCEDURE FOR ASPHALT CONCRETE

Aggregate: Crushed Limestone

Asphalt: AC-10

Design: Asphalt Contents: 7%, 8%, 9%, and 10%
          Gradation: same as given in Table A1.3

Specimen size: 4" diameter $\times$ 2" height

Sample Preparation:

(1)   Mixed at 275° F $\pm$ 5° F for 3 minutes in an automatic 12-quart capacity Hobart food mixer at 107 rpm.

(2)   Cured at 140° F $\pm$ 5° F for 18 to 24 hours.

(3)   Compacted at 250° F $\pm$ 5° F.

Compaction:

Gyratory shear compaction performed according to Texas Highway Department Standard 206-F, Part II.

Testing Procedure:

(1)   Preheat at 180° F for 18 to 24 hours.

(2)   Cool at 75° F for 18 to 24 hours.

(3)   Hold at testing temperature for 18 to 24 hours prior to testing.

The basic testing equipment consists of an adjustable loading frame, a closed loop electrohydraulic loading system, and a loading gear which is a modified, commercially-available shoe-die with upper and lower platens constrained to remain parallel during tests.

Another piece of equipment, a device for measuring the transverse strain in a specimen, was used to obtain a measure of specimen deformation in the direction of the tensile stresses causing failure. This measuring device consisted of two cantilever arms with attached strain gages.

Vertical deformations were measured by a DC linear variable differential transformer which also was used to control the rate of load application

providing an electrical signal related to the relative movements of the upper and lower platens. All measurements were recorded on two x-y plotters.

The maximum load needed for the tensile strength calculation was obtained from the load versus vertical deformation plot. The maximum horizontal deformation value was taken from the load versus horizontal deformation plot and is the deformation recorded at maximum load. These values of horizontal and vertical deformations and loads were input into a computer program developed at The University of Texas (Ref 37) for the calculation of modulus of elasticity and tensile strength of specimens.

APPENDIX 2

DATA OF PLATE LOAD TESTS

TABLE A2.1. PRESSURE VERSUS DEFLECTION DATA OF 9-INCH PLATE LOAD TESTS ON CLAY

Series A		Series B		Series C	
Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch
0	0	0	0	0	0
0.33	.0014	0.08	.0002	0.49	.0018
1.00	.0060	0.36	.0012	0.84	.0040
2.00	.0130	0.52	.0030	1.94	.0110
2.46	.0160	1.21	.0068	2.75	.0153
3.00	.0210	1.97	.0125	3.00	.0230
3.80	.0290	2.65	.0068	3.87	.0331
4.28	.0360	3.71	.0320	4.45	.0534
4.65	.0425	4.82	.0640	5.00	.0710
5.00	.0495	5.50	.0930	5.40	.0890
5.64	.0635	5.90	.1225	5.80	.1040
6.20	.0840	6.05	.1370	6.20	.1306
6.52	.1000	6.35	.1480	6.35	.1400
6.84	.1200	6.48	.1590	6.55	.1360
7.05	.1406	6.82	.1740	6.75	.1481
7.24	.1603	7.10	.1880	7.05	.1695
7.31	.1809	7.30	.2030	7.20	.1950
7.40	.2012	7.40	.2160	7.40	.2240
		7.46	.2310		

TABLE A2.2. PRESSURE VERSUS DEFLECTION DATA OF 6-INCH PLATE LOAD TESTS ON CLAY

Series A		Series B		Series C	
Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch
0	0	0	0	0	0
0.39	.0019	.2	.0012	0.17	.0012
0.81	.0039	.4	.0018	0.34	.0021
1.38	.0071	.5	.0033	0.53	.0032
2.30	.0121	1.10	.0062	0.91	.0051
3.50	.0191	1.82	.0115	1.49	.0083
4.00	.0240	2.65	.0143	2.13	.0121
4.66	.0308	3.54	.0198	2.70	.0147
5.28	.0389	4.42	.0271	3.36	.0185
5.56	.0427	5.30	.0442	4.40	.0223
5.96	.0520	6.00	.0661	4.80	.0303
6.40	.0640	6.54	.0882	4.96	.0319
6.85	.0853	7.08	.1205	6.60	.0563
7.20	.1050	7.30	.1403	7.10	.1160
7.50	.1403	7.55	.1681	7.45	.1450
7.70	.1643	7.71	.2015	7.70	.1805
7.80	.2013	7.78	.2219	7.80	.2095
7.90	.2329	7.80	.2410	7.82	.2225

TABLE A2.3. PRESSURE VERSUS DEFLECTION DATA OF 4-INCH PLATE LOAD TESTS ON CLAY

Series A		Series B		Series C	
Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch
0	0	0	0	0	0
0.16	.0001	0.40	.0017	0.41	.0011
0.24	.0004	0.80	.0041	1.00	.0035
0.72	.0015	1.20	.0058	1.31	.0065
1.40	.0040	1.60	.0079	1.84	.0085
2.07	.0075	2.40	.0127	2.20	.0107
2.55	.0105	3.20	.0144	2.66	.0134
3.82	.0165	4.00	.0182	3.28	.0185
4.54	.0208	4.80	.0218	4.44	.0255
5.50	.0300	5.60	.0307	5.05	.0330
6.13	.0381	6.40	.0461	6.30	.0460
6.90	.0550	7.20	.0771	6.81	.0620
7.20	.0640	7.50	.0942	7.46	.0900
7.40	.0725	7.72	.1207	7.75	.1100
7.73	.0921	7.96	.1412	7.90	.1325
7.97	.1230	8.03	.1662	8.00	.1505
8.11	.1490	8.10	.1827	8.02	.1660
8.31	.1880	8.13	.2105	8.05	.1960
8.35	.2212	8.17	.2213	8.10	.2290

TABLE A2.4.   PRESSURE VERSUS DEFLECTION DATA OF 2-INCH PLATE LOAD TESTS ON CLAY

Series A		Series B		Series C	
Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch
0	0	0	0	0	0
0.32	.0010	0.83	.0017	0.40	.0155
1.27	.0031	1.64	.0057	0.75	.0212
1.91	.0051	2.45	.0091	1.30	.0350
2.55	.0078	3.30	.0121	1.50	.0554
3.00	.0100	4.90	.0180	2.01	.0761
3.66	.0121	5.74	.0213	2.81	.0102
4.30	.0139	6.05	.0231	3.31	.0121
5.10	.0163	6.30	.0262	4.15	.0153
5.57	.0184	6.75	.0332	5.10	.0186
6.05	.0220	7.20	.0481	6.10	.0245
7.00	.0375	7.66	.0665	6.95	.0407
7.65	.0573	7.80	.0805	7.52	.0610
7.96	.0718	8.00	.0952	7.80	.0801
8.27	.0959	8.23	.1207	8.00	.1019
8.43	.1203	8.41	.1610	8.19	.1314
8.55	.1412	8.50	.1813	8.30	.1601
8.70	.1822	8.55	.2102	8.36	.1911
8.80	.2310	8.61	.2307	8.41	.2307

TABLE A2.5. PRESSURE VERSUS DEFLECTION DATA OF PLATE LOAD TESTS ON ASPHALT STABILIZED LAYER

9-Inch-Diameter Plate		6-Inch-Diameter Plate		4-Inch-Diameter Plate		2-Inch-Diameter Plate	
Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch	Pressure (p) psi	Deflection (w) inch
0	0	0	0	0	0	0	0
0.73	.0013	0.53	.0002	1.19	.0002	4.78	.0048
1.21	.0025	1.06	.0007	2.38	.0006	9.56	.0086
1.70	.0045	1.59	.0028	3.57	.0022	14.34	.0144
2.18	.0070	2.12	.0040	4.76	.0040	19.12	.0183
2.66	.0088	3.71	.0093	5.95	.0060	23.90	.0231
3.14	.0116	4.77	.0130	7.14	.0080	28.50	.0288
3.64	.0137	5.85	.0173	8.33	.0103	33.50	.0353
4.12	.0160	6.90	.0227	9.52	.0124	38.50	.0419
5.00	.0203	8.00	.0288	10.71	.0142	43.00	.0499
6.79	.0300	9.06	.0388	11.90	.0175	48.00	.0590
8.08	.0407	12.40	.0620	14.28	.0220	52.50	.0690
8.72	.0498	13.80	.0765	16.66	.0312	57.20	.0815
10.00	.0699	14.90	.0920	19.00	.0420	62.00	.0985
11.30	.0956	16.30	.1120	21.50	.0555	67.00	.1245
12.00	.1120	17.70	.1360	24.60	.0815	67.60	.1600
12.60	.1293	19.00	.1630	27.70	.1280	67.60	.1700
13.22	.1503	20.50	.1867	28.50	.1445		
14.50	.2082	21.60	.2263	29.00	.1740		
15.10	.2494	22.00	.2465	29.70	.2150		

APPENDIX 3

DATA OF SLAB TEST

TABLE A3.1. LOAD-DEFLECTION DATA OF SLAB TEST (CYCLE 1) FOR VARIOUS POINTS ON SLAB

Time	Load	Deflections Measured by Dial Gages Located from Center of Slab, inch							
		1	2	3	4	5	6	7	8
min	lbs	1 inch	2.5 inches	4.5 inches	1.41 inches	3.54 inches	6.36 inches	4.5 inches	6.36 inches
Loading									
0	0	.0000	.0000	.0000	.0000	.0000	-.0000	.0000	-.0000
1/2	3.75	.0006	.0003	.0002	.0038	.0002	-.0002	.0003	-.0002
1	32.50	.0048	.0025	.0012	.0041	.0013	-.0003	.0009	-.0002
2	65.00	.0102	.0052	.0017	.0085	.0028	-.0004	.0015	-.0003
3	132.00	.0185	.0111	.0040	.0182	.0061	-.0010	.0046	-.0008
4	188.00	.0303	.0166	.0062	.0273	.0099	-.0017	.0063	-.0018
5	208.00	.0362	.0205	.0083	.0335	.0131	-.0021	.0092	-.0027
Unloading									
0	190.00	.0355	.0198	.0082	.0330	.0128	-.0021	.0088	-.0026
1/2	130.00	.0305	.0180	.0075	.0282	.0116	-.0020	.0080	-.0024
1	87.50	.0255	.0152	.0061	.0241	.0098	-.0018	.0064	-.0020
1-1/2	55.00	.0200	.0120	.0046	.0182	.0076	-.0012	.0045	-.0008
2	27.50	.0148	.0083	.0033	.0118	.0058	-.0006	.0034	-.0006
2-1/2	10.00	.0125	.0062	.0024	.0067	.0042	-.0004	.0025	-.0004
3-1/2	8.75	.0110	.0053	.0024	.0061	.0040	-.0003	.0023	-.0003
4-1/2	0	.0075	.0035	.0020	.0038	.0033	-.0003	.0021	-.0002

DATA REDUCTION

Fig A3.1 shows a sample print output of digital voltmeter. These data were recorded for the maximum loading in the first cycle after five minutes of loading during slab test. Sample calculations for load and corresponding strain are made as follows:

Odd numbered channels are for loads and even numbered channels are for strains.

Channel 1: (Load)

$$\text{Output} = 81 \times 10^{-5}\text{v} = 81 \times 10^{-2}\text{mv}$$

$$\text{Difference from initial reading} = (81 - (-1)) \times 10^{2}$$
$$= 82 \times 10^{-2}\text{mv}$$

Calibration constant = 250 lb/mv

$$\text{Load applied} = 82 \times 10^{-2} \times 250 = 205 \text{ lb}$$

Channel 2: (Strain)

Rosette 1, Gage A

$$\text{Output} = -209 \times 10^{-5} = -2090\mu\text{v}$$

$$\text{Difference from initial reading} = -2090 - (-50)$$
$$= -2040\mu\text{v}$$

Calibration constant = $.325\mu\epsilon/\mu\text{v}$

$$\text{Strain measured} = -2040 \times 0.325$$
$$= 663\mu\text{-in/in}$$

In a similar way the recorded data were processed for each channel and each cycle. Load-strain data for cycle 1 is included herein (See Table A3.2). Fig A3.2 shows the plot of load-strain of three gages of rosette 1 for the first cycle of loading and unloading (static slab test).

```
        1 0    + ·  0 0 0 3 3    5
        0 9    + ·  0 0 0 7 9    5
        0 8    - ·  0 0 0 6 6    5

        0 7    + ·  0 0 0 8 0    5
        0 6    - ·  0 0 0 4 5    5

        0 5    + ·  0 0 0 8 0    5
        0 4    - ·  0 0 1 1 5    5

        0 3    + ·  0 0 0 8 1    5
Channel 2  0 2    - ·  0 0 2 0 9    5

Channel 1  0 1    + ·  0 0 0 8 1    5
        0 0    - ·  0 0 0 0 1    5
```

Fig A3.1. A sample of printout by digital voltmeter for
load cycle 1 during the slab test.

TABLE A3.2. TEST DATA OF STRAINS FOR ROSETTE 1 FOR SLAB TEST

| | Strain No. 1A | | | | | Strain No. 1B | | | | | Strain No. 1C | | | | |
| Channel | 1 | | 2 | | | 3 | | 4 | | | 5 | | 6 | | |
| | Load | | Strain | | | Load | | Strain | | | Load | | Strain | | |
Time	Output	Load lbs	Output μv	Difference from Zero μv	Strain uε	Output	Load lbs	Output μv	Difference from Zero μv	Strain uε	Output	Load lbs	Output μv	Difference from Zero μv	Strain uε
Loading															
0	0	0	-50	0	0	0	0	-20	0	0	0	0	-40	0	0
1/2 min	14-15	32.5	-410	360	1168	15	37.5	-200	180	59	15	37.5	-80	40	13
1 min	10-11	30.0	-350	300	100	11	27.5	-180	160	56	11	27.5	-90	50	16
2 min	24-25	62.5	-620	370	120	25	62.5	-320	300	98	25	62.5	-110	70	23
3 min	50-51	128.0	-1250	1200	390	51	127.0	-660	640	208	51	127.0	-230	190	64
4 min	72-73	182.0	-1840	1790	582	.72	180.0	-1000	980	320	73	183.0	-380	340	110
5 min	81-82	205.0	-2090	2040	663	81	202.0	-1150	1130	370	80	200.0	-450	410	134
Unloading															
0	74-72	185.0	-2060	2010	650	74-72	180.0	-1130	1110	360	74-72	180.0	-440	400	132
1/2 min	47-45	110.0	-1620	1570	510	46-44	110.0	-980	960	319	46-44	110.0	-420	380	125
1 min	32-30	75.0	-1260	1210	395	32-30	75.0	-800	780	254	31-29	72.0	-400	360	120
1-1/2 min	18-16	40.0	-870	820	266	18-16	40.0	-580	560	182	17-15	37.5	-380	320	106
2 min	9-7	17.5	-560	510	166	8-6	15.0	-390	370	120	7-5	12.5	-330	290	95
2-1/2 min	1-0	2.5	-290	240	78	1-0	2.5	-230	210	68	1-0	2.5	-230	190	69
3-1/2 min	1-0	0	-260	210	68	1-0	0	-200	180	58	1-0	0	-210	170	55
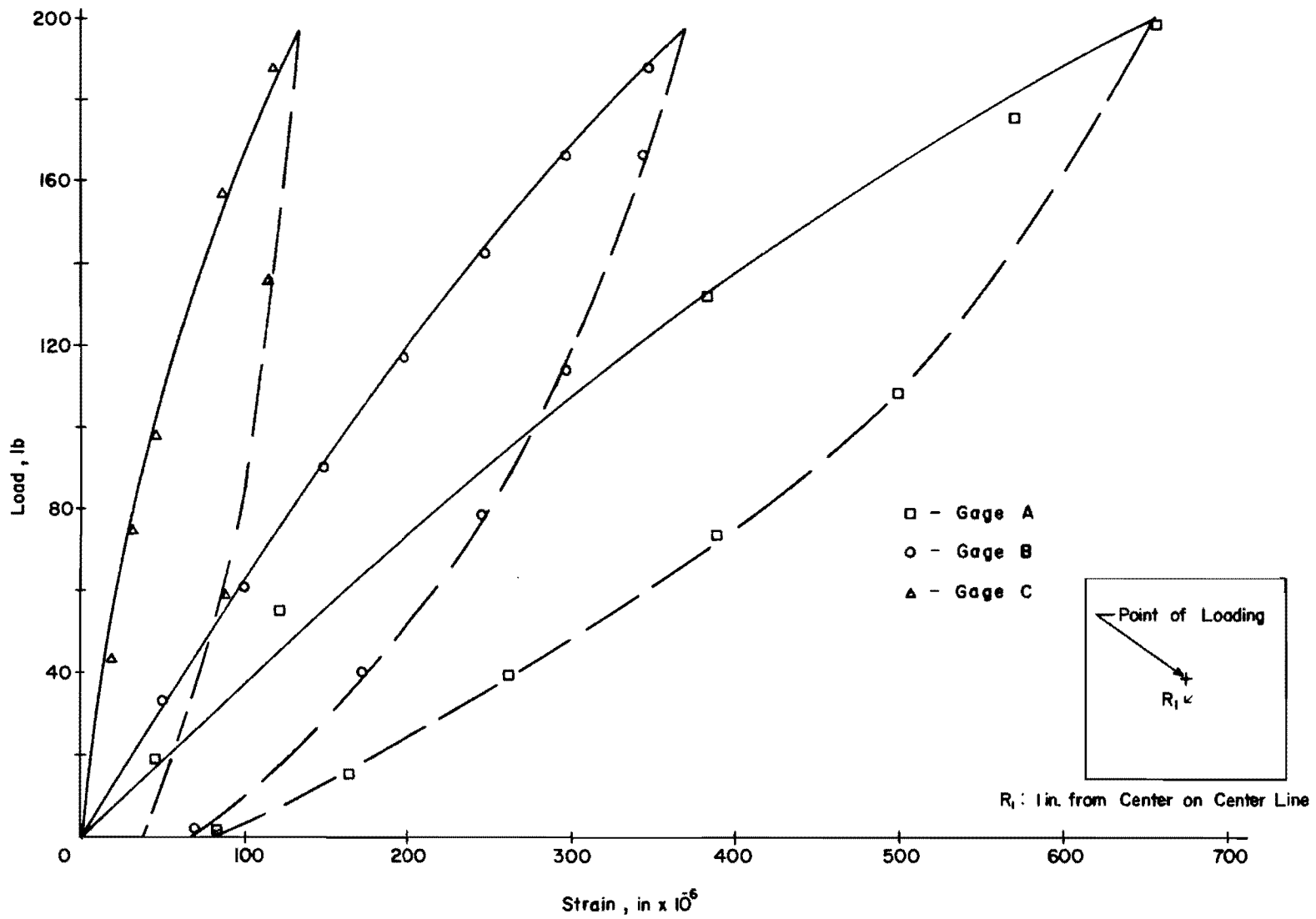
Fig A3.2.   Load versus strain for Rosette 1.

Determination of Largest Principal Stress

The largest principal stress was calculated from the measured strains of the three gages of a rosette at a given load from the following equations:

$$\sigma_{max} = \frac{E}{2}\left[\frac{\epsilon_1 + \epsilon_3}{1 - \nu} + \frac{1}{1 - \nu}\sqrt{(\epsilon_1 - \epsilon_3)^2 + \left[2\epsilon_2 - (\epsilon_1 + \epsilon_3)\right]^2}\right] \quad (A3.1)$$

$$\sigma_{min} = \frac{E}{2}\left[\frac{\epsilon_1 + \epsilon_3}{1 - \nu} - \frac{1}{1 - \nu}\sqrt{(\epsilon_1 - \epsilon_3)^2 + \left[2\epsilon_2 - (\epsilon_1 + \epsilon_3)\right]^2}\right] \quad (A3.2)$$

where

σ_{max} = maximum principal stress, psi

σ_{min} = minimum principal stress, psi

E = modulus of elasticity of slab material, psi

ν = Poisson's ratio of slab material

ϵ_1 , ϵ_2 , ϵ_3 = strain readings of the rosette

The largest principal stress was chosen out of σ_{max} and σ_{min} , corresponding to the larger absolute value. The direction of the largest principal stress was then calculated using the expression:

$$\alpha = \frac{1}{2}\tan^{-1}\frac{2\epsilon_2 - (\epsilon_1 + \epsilon_3)}{\epsilon_1 - \epsilon_3} \quad (A3.3)$$

where

α = the angle made by the largest principal stress in the ϵ_1 direction

For example, Fig A3.2 on page 96 shows the load versus strain for the three gages of Rosette 1 for the slab test.

For a load of 200 pounds

$$\varepsilon_1 \ = \ -640 \ \mu\text{-in/in}$$

$$\varepsilon_2 \ = \ -370 \ \mu\text{-in/in}$$

$$\varepsilon_3 \ = \ -135 \ \mu\text{-in/in}$$

Using Eq A3.1 and A3.2, the largest principal stress is calculated to be 8500 psi. The difference between the direction of stress and strain (ε_1) is 1°.

Similar calculations were made for different loads both during loading and unloading. Load versus largest principal stress thus obtained is shown in Fig 37. For the other rosettes, the largest principal stresses were obtained in the same way.

THE AUTHORS

Qaiser S. Siddiqi is currently an engineer with Brown and Root Engineers, Houston, Texas. While at The University of Texas at Austin (1967-1969), he was a Research Engineer Assistant at the Center for Highway Research, where his research dealt with the development of computer methods of simulating beam columns and slab systems.

W. Ronald Hudson is an Associate Professor of Civil Engineering and Associate Dean of the College of Engineering at The University of Texas at Austin. He has had a wide variety of experience as a research engineer with the Texas Highway Department and the Center for Highway Research at The University of Texas at Austin and was Assistant Chief of the Rigid Pavement Research Branch of the AASHO Road Test. He is the author of numerous publications and was the recipient of the 1967 ASCE J. James R. Croes Medal. He is presently concerned with research in the areas of (1) analysis and design of pavement management systems, (2) measurement of pavement roughness performance, (3) slab analysis and design, and (4) tensile strength of stabilized subbase materials.