Report 55-1

RELATIONSHIP BETWEEN CRITICAL MECHANICAL PROPERTIES AND AGE FOR STRUCTURAL LIGHTWEIGHT CONCRETE

by

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PREFACE

This report is the first of two reports on the study of the critical mechanical properties of structural lightweight concrete and the effects of these properties upon the design of pavement structures. The initial phase of this investigation which was concerned with the development of a technique to measure the tensile stress-strain characteristics of lightweight aggregate concrete and how this property is affected by restraint from volume change is described in this first report.

This study was carried out at The University of Texas under the auspices of the Structural Mechanics Research Laboratory and under the general administration of The University of Texas Center for Highway Research.

The authors are grateful to the U. S. Bureau of Public Roads and the Texas Highway Department for their sponsorship in this program. Appreciation is also extended to the staff of these two organizations for their suggestions and comments during this undertaking.

To the following undergraduate and graduate students who are listed in the staff, the authors wish to express their gratitude for their contributions.

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TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

Page

TABLE OF CONTENTS (Cont'd)

LIST OF FIGURES

LIST OF FIGURES (Cont'd)

LIST OF FIGURES (Cont'd)

LIST OF FIGURES (Cont'd)

ABSTRACT

The necessity to use structural lightweight concrete has created a need for investigations into its critical mechanical properties that affect the design and performance of structures. The primary critical properties were found to be direct tensile stress-strain characteristics and the restrained volume changes. These two properties were experimentally determined and analyzed, along with the compressive, indirect tensile (split cylinder), and flexural strengths. With age as a fundamental variable, these properties were determined for several curing-condition and mix-design parameters.

A primary finding from this study was the good correlation exhibited between direct tensile strength and both compressive strength and indirect tensile strength. A direct tensile test method was developed which yields consistently reliable results and from which valuable information on fundamental concrete tensile behavior can be obtained. Along with the concrete direct tensile properties, this test yielded information on the effects of restraining the concrete from changing in volume as a result of hydration and changes in moisture. This restraint was developed by the inclusion of a deformed bar through the specimen. The concrete stresses developed from this restraint yielded important clues toward a better understanding of concrete behavior. It was found that this type of structural lightweight concrete, when oven dried at

110F, attempts to shrink resulting in restrained tensile concrete stresses as high as 70 per cent of the concrete's tensile strength.

Additional primary results included the finding that the per cent of 28-day compressive and indirect tensile strengths versus age relationship was constant over a wide range of mix designs, strengths, and curing conditions; indicating that a designer, by knowing the compressive or indirect tensile strength of a given concrete mix design at only one age, can determine, with a relatively high degree of confidence, the strength of that mix at any other age. Also, the study indicated, for the types of concrete and test parameters investigated, that an excellent correlation existed between indirect tensile strength and compressive strength, while almost no correlation existed between flexural strength and compressive strength. From these results, it was recommended that the indirect tensile strength test be used instead of the flexural strength test as a quality control test for structural lightweight concrete.

1. INTRODUCTION

1.1 Purpose

In order to adequately design an engineering structure, an engineer must possess not only knowledge of the structural analyses, but also must have a thorough understanding of the material, or materials, comprising the structure. This understanding must extend beyond a knowledge of those properties which are accounted for directly in the design formulations to those properties which influence the behavior of the structure. There are three basic reasons why design formulations do not generally include the effects of all critical properties. These reasons are (1) the resulting formulations may be too complex and cumbersome to be utilized by engineers; (2) the design problems may not have been completely defined; and (3) certain critical properties may not have been accurately and reliably determined. As a consequence, existing design formulations are predicated on the assumption that only certain structural materials will be used which, through use, have proven to have suitable properties.

Rigid pavement-design formulations, incomplete for all three of the above reasons, become even more doubtful when applied to materials which have not been proven through use. Recent interest has been generated in the use of structural lightweight concrete in rigid pavements. This, in turn, has created a need for extensive investigations into the determination of mechanical properties of structural lightweight concrete that are known to be critical and of the effects of these properties upon design and performance of concrete pavement structures. Thus the purpose of this study is to answer this need through experimental investigation and analysis.

The primary critical properties of structural lightweight concrete for pavement structures are the direct tensile stress-strain characteristics, and the effects of restrained volume changes. In a pavement structure, these properties control the stresses and deflections from external loads, the loss in tensile capacity from restrained volume changes, and thus the resulting performance of the structure. Also of importance are the relationships between direct tensile strength and other strength measurements such as compressive, indirect tensile (split cylinder), and flexural strengths. In this study, these properties were determined with age as a fundamental test parameter.

1.2 Scope and Limitations

The scope of this research study was to (1) develop a method whereby the direct tensile properties of concrete could reliably and consistently be determined, (2) explore the direct tensile as well as other critical mechanical properties of structural lightweight concrete which affect the design and performance of pavement structures, (3) analyze concrete properties in terms of age as a test parameter, and (4) develop interrelationships between these critical concrete properties.

The critical mechanical properties of structural lightweight concrete include the direct tensile stress-strain characteristics, direct tensile strength, compressive strength, flexural strength, indirect tensile (split cylinder) strength, and restrained volume changes. The investigation of these properties and their relatedness comprise a major portion of this study.

These properties were determined for a total of 48 combinations of 5 different test parameters, which were:

1. Coarse aggregate type--one structural lightweight coarse aggregate and one regular weight coarse aggregate were chosen for comparison purposes.

2. Cement factor--4 sacks per cubic yard and 5 sacks per cubic yard were selected as representative of the range of cement factors currently used for highway pavement structures in Texas.

3. Air content--two air contents, 2 per cent (no air entrainment) and 6 per cent (using an air entrainment additive), were chosen.

4. Curing conditions--three different curing conditions were employed. The condition of moist curing sealed in a polyethylene plastic bag at 100 per cent relative humidity and 75F (termed bag curing in this report) was chosen to represent essentially a 100 per cent moist curing condition. The oven-dried condition at low humidity and 110F was chosen to represent the extreme curing condition that could be expected to occur in Texas. The air-dried condition at 50 per cent humidity and 75F was selected to represent a medium curing condition between the two extreme conditions.

5. Age--4 ages were selected--1/2 day, 2 days, 7 days, and 28 days. These were selected to give an indication of concrete properties during the early "life" of the concrete as well as a correlation of these properties with properties after 28 days.

In order to isolate the interrelationships between the 48 test parameters, the following variables were held constant throughout this study.

1. Mixing time and sequence
2. Cement type
3. Batch size
4. Air-entrainment type
5. Consistency
6. Test procedure (specimen size, rate of loading, etc.)
7. Fine aggregate

1.3 Conclusions

From the results obtained thus far in this study, with a concrete that contains one brand of cement and a lightweight aggregate from one source, the following conclusions for structural lightweight concrete of the type investigated in this study can be drawn:

1. The true and direct tensile properties of concrete can now be accurately and reliably determined by the use of a test method in which a steel bar, instrumented with SR-4 strain gages, is cast in a concrete cylinder and tested in uniaxial tension.

2. A good correlation was found to exist between the direct tensile strength (f_t) and compressive strength (f'_c). The average

ratio of f_t/f'_c for all parameters studied was 0.083, which means that the direct tensile strength of the concrete was some 8.3 per cent of the compressive strength.

3. A good correlation exists between the direct tensile strength (f_t) and the indirect or split cylinder tensile strength (f_{sp}). The average ratio of f_t/f_{sp} for all parameters studied was 0.685. This indicates that the indirect tensile test yields consistently somewhat higher results than the direct tensile test. As the indirect tensile test creates a biaxial stress condition, it is plausible that the concrete exhibits somewhat different behavior than that exhibited in an uniaxial stress condition, and indicates a need for a more thorough understanding of the stress conditions occurring in concrete structures before resulting material properties of the structure may accurately be predicted.

4. The extreme effects of oven drying structural lightweight concrete for a period of 28 days, during which time the concrete is restrained from changing in volume, creates tensile stresses in the concrete of up to 70 per cent or more of the concrete's true and direct tensile strength. Two specimens failed in tension prior to applying any external load from these restrained-concrete, volume-change stresses. This indicates that this type of concrete, if restrained and allowed to dry out during hydration, can develop surface crazing and cracks due to these high stresses and, as a result, create detrimental effects to the concrete performance.

5. Structural lightweight concrete has approximately the same strength-age relationship as regular weight concrete used in the investigation for the report.

6. By converting compressive and indirect tensile strength values at any given age to per cent of 7 or 28-day-strength values, a general relationship results between per cent of 7 or 28-day strength and age which is constant over a wide range of mix designs, cement factors, air contents, and curing conditions. Thus, for a given curing condition, knowing the compressive or indirect tensile strengths for a given mix design at only one age, a designer can reliably predict the strength of the concrete at any other age from 1/2 day to 28 days. For example, assuming the following mix design of structural lightweight concrete:

Cement Factor - 5 sacks per cu yd

Air Content - 6 per cent (using entrained air)

Curing - 3 days moist (under curing paper) followed by air drying

7-Day Compressive Strength - 3550 psi (experimentally determined)

If the expected compressive strengths at 3 days, 14 days, and 28 days are desired, they can be computed from Fig. 5-11:

3-day strength = (.84) (3550) = 2980 psi

14-day strength = (1.06) (3550) = 3760 psi

28-day strength = (1.24) (3550) = 4400 psi

7. Structural lightweight concrete exhibits no rational relationships between direct tensile strength and flexural strength nor between flexural strength and age.

8. Volume changes resulting from hydration and moisture-content changes appear to be greater for the structural lightweight concrete than for regular weight concrete.

9. Structural lightweight concrete, when bag cured at 75F and 100 per cent relative humidity, actually expands during hydration up to at least 28 days of age. The regular weight concrete of the same curing condition, while initially expanding, quickly begins to shrink, continuing the shrinkage process for at least 28 days,

10. For structural lightweight concrete, an excellent correlation exists between indirect tensile or split cylinder strength (f_{sp}) and compressive strength (f'_c). For all the parameters of age, cement factor, curing, and air content investigated, the average ratio of f_{sp}/f'_c was 0.121 with a coefficient of variation of 10.6 per cent. This means that the indirect tensile test can be used to determine the compressive strength, and thus is an excellent quality control test. For example, if an indirect tensile strength test value was 430 psi, the compressive strength of the concrete would be:

$$f'_c = \frac{430}{0.121} = 3550 \text{ psi}$$

11. No correlation exists between flexural strength (f_f) and compressive strength (f'_c). For all the parameters investigated, the average ratio of f_f/f'_c exhibited a 43.4 per cent coefficient of variation, and even the bag-cured specimens

exhibited a 38.3 per cent coefficient of variation.

1.4 Recommendations

On the basis of the conclusions drawn from this study, the following recommendations are offered:

1. Now that a suitable test method for the determination of the direct tensile properties of concrete has been developed and verified, these elusive properties can be properly evaluated which will further the fundamental knowledge of critical concrete properties.

2. The direct tensile properties of structural lightweight concrete should be thoroughly investigated for all types of concretes, curing conditions, air contents, etc.

3. The effects of volume changes occurring in structural concrete should be investigated in detail.

4. The relationships between per cent strength at a specified age and age for structural concrete should be determined for a wide range of lightweight aggregates to see if this approach of analysis in terms of a dimensionless parameter will yield generalized results in all cases.

5. It is tentatively recommended that the flexural strength test should <u>not</u> be used as a quality control test for structural lightweight concrete. It certainly should not be used for the type of concrete investigated in this study, which casts doubt on its value with other types of structural lightweight concrete.

6. In conjunction with recommendation No. 5 above, since the indirect tensile (split cylinder) test appears to be more reliable, the indirect tensile test should be used instead of the flexural strength test to control concrete quality.

2. BACKGROUND

2.1 General

Structural lightweight aggregate concrete, a relatively new engineering material, was first manufactured as it is known today in 1917 when Stephen J. Hayde developed a process for expanding shale and clay into sound, hard, lightweight particles suitable for use as aggregates in structural concrete.[1*] This type of concrete was used during the first World War for concrete ships.[2] In the second World War, more than one hundred concrete barges and ships were built. This concrete developed compressive strengths in excess of 5000 psi and weighed in the neighborhood of 100 pcf. Its early use demonstrated the engineering feasibility of this new material for use in structures where dead-load was an important design factor.

From this beginning, the use of structural lightweight concrete has increased rapidly. Extensive research studies, investigating the properties of this new material, have contributed to its early acceptance by engineers. One such research study, by Richart and Jenson[3] in 1931 presented some of the first engineering data. In more recent times, two reports written by Shideler[4, 5] give an excellent over-all look at properties, developments, and use of structural lightweight concrete.

2.2 Manufacture of Lightweight Aggregate

The manufacture of the majority of the lightweight aggregates in this country consists, in general, of heating clays, shales, or slates

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[*] Numbers indicate references as listed in the Bibliography.

almost to the point of fusion, whereupon gases liberated from impurities in these raw materials cause the material to expand or "bloat"; the resultant product being a hard, durable aggregate with a relatively low specific gravity. Although lightweight aggregates have been manufactured for over forty years now, the exact mechanism and the reactions involved in this bloating process have not been accurately determined until fairly recently.[6, 7]

Several extensive investigations have been conducted to determine the exact nature and process of the thermodynamic and chemical reactions involved in lightweight aggregate manufacture.

The principal bloating agents are thought to be pyrite (FeS_2), hematite (Fe_2O_3), dolomite ($CaCO_3 \cdot MgCO_3$), calcite ($CaCO_3$), or combinations of these.[8] The gas contributing most to this expansion appears to be carbon dioxide (CO_2), along with traces of sulphur dioxide (SO_2).[9] According to Everhard,[10] this bloating action can be explained as follows:

> Bloating is an expansion process in which two conditions occur simultaneously. During firing, the sample partially fuses to a viscous state while at the same time a gas, released as a result of mineral decompositions, becomes entrapped within the viscous mass.

These two conditions must occur simultaneously. If the sample becomes too fluid prior to the release of the gas, it will not possess the strength necessary to contain the released gas; and if the sample is still in a solid state at the time the gas is released, this gas may escape through pores in the sample. The temperature at which most bloating occurs is between 2100F and 2400F.[11] This bloating action can sometimes be improved by the addition of additives such as pyrites, hematites, coal, coke, etc.

There are two processes used extensively in this country for the production of lightweight aggregates. These are the rotary kiln and the sintering processes. In the rotary-kiln process, the raw material is introduced into the upper end of a slanting rotary kiln, similar to those used in the manufacture of portland cement.[12] Expansion occurs in the burning zone and the expanded material is discharged into a cooling pit, after which it is crushed and screened to size. In the sintering or moving-grate process, the material sample, while on a moving grate, is sprinkled with a small quantity of fine coal or coke. This grate passes under an ignition hood where the fuel is ignited and the bloating action occurs,[13] after which the aggregate is crushed and screened to size.

The manufacture of lightweight aggregates is a complicated process and varies for different raw materials. Each manufacturer determines, for each material source, such factors as burning temperatures, rates of heating and burning, rates of cooling, amount and type of additives, etc., required to produce the best product.

2.3 Summary of Known Information

As interest develops in structural lightweight concrete, more and more research is being conducted. In recent years, notable breakthroughs have been made in supplying the answers to some major questions concerning the properties and use of this material. In particular, such studies as reported by Richart and Jenson,[14] Price and Cordon,[15] Shideler,[16,17] Hanson,[18,19] Kluge,[20] Lewis,[21] and others have contributed some significant information about this material.

Mix design. In the mix design of structural lightweight concrete, it was found that various lightweight concretes, even those which contain similar aggregates, exhibit rather large variations in structural properties.[22] This has created problems in evaluating this material and using it in engineering structures. The reasons for this variation in properties have not fully been explained, but it is believed that the high and varied moisture absorption of lightweight aggregates causes the "effective" specific gravity to change from batch to batch, and therefore makes the effective specific gravity for use in mix design difficult to "determine accurately."[23] The term "effective" specific gravity is used here to contrast this type of specific gravity with the "saturated-surface-dried" (SSD) specific gravity used extensively in the design of regular weight concrete batches. It is believed that, due to the relatively high absorption, and hence rather large and/or numerous fissures and pores in the aggregate particles, the SSD condition may not necessarily represent the moisture condition of the aggregate after the concrete has been mixed.

Testing procedures at the present time are not adequate to establish the effective w/c ratio because of the tendency for the paste to enter the larger pores of the aggregates having relatively high absorption. For this reason, the effective water-cement ratio may change with each batch. Therefore, lightweight concrete is proportioned on the basis of cement content rather than water-cement ratio,[24] and the mixing water content is allowed to vary from batch to batch in order to maintain a uniform consistency.

Strength. It has been reported that the strength of lightweight concretes are comparable with regular weight concretes. These include compressive, tensile, bond, and flexural strengths. Nine thousand psi concrete has been made with comparative ease in the laboratory using structural lightweight aggregates[25] and similar high-strength lightweight concretes have been produced commercially to some extent.

Volume changes. In general, volume changes of lightweight concrete have been found to be slightly greater than corresponding regular weight concrete. These volume changes include changes in volume due to thermal excitation and moisture-content changes, and in creep from sustained loads.[26] This complex phenomena of volume change has created difficulties for the design engineer. For example, losses in prestress due to creep is difficult to determine accurately due to the wide variation of data; and moisture loss causing volume change may result in a loss of tensile capacity of the concrete. This loss of tensile capacity of the concrete could result in tensile cracks developing from hydration shrinkage which would be extremely detrimental to the performance of the structural material.

Modulus of elasticity. The modulus of elasticity of lightweight aggregate concrete, which is a measure of the relative stiffness of the material, has been found to be much less than the modulus of elasticity of regular weight concrete. This means that, other factors being equal, a structural lightweight concrete beam would undergo greater deflection for a given load than a comparable regular weight concrete beam.

As reported by Pauw,[27] the modulus of elasticity E_c of both regular weight and lightweight concretes may be approximately determined by the empirical formula

$$E_c = 33.6\,\omega^{3/2}\sqrt{f'_c}$$

where

ω = unit weight of the concrete in pcf

f'_c = compressive strength of the concrete in psi

From this formula it can be seen that for structural lightweight concrete with a unit weight of approximately two-thirds that of similar strength, regular weight concrete will have an E_c of around $(2/3)^{3/2}$ or 55 per cent of regular weight concrete.

Experience in Texas. An extensive study has been reported by Jones, Hirsch, and Stephenson,[28] in which structural lightweight concrete using aggregates and cement manufactured in Texas were investigated. The results of this study substantiate findings reported elsewhere and reveal some very significant additional results. Detailed volume-change studies were made to determine the effects of shrinkage and creep. Strength-versus-age curves showed significant variations in strength for different aggregate types. Table 2-1 contains a tabulation of some of the data taken from this report. By selecting certain mix

TABLE 2-1

Properties of Various Structural Lightweight Concrete Mixes as Reported in "The Physical Properties of Structural Quality Lightweight Concrete," by T. R. Jones, Jr., T. J. Hirsch, and H. K. Stephenson, Report of the Texas Transportation Institute
August 1959

Batch Number*	Cement Factor	Air Content	Slump	3-Day Compressive Strength		7-Day Compressive Strength		14-Day Compressive Strength		28-Day Compressive Strength
	sks/cu yd	%	in.	psi	% 28 Day	psi	% 28 Day	psi	% 28 Day	psi
ST-15	4.01	5.0	2.00	2010	35	3540	62	5050	89	5690
ST-16	3.93	5.0	2.00	2550	41	4030	65	5340	86	6220
ST-17	3.84	4.5	2.00	2200	41	4020	76	4950	93	5310
ST-18	5.59	5.0	2.00	4210	57	5730	77	6640	90	7400
ST-19	5.70	5.1	2.00	3030	42	4590	64	6330	88	7210
ST-20	5.79	5.2	2.00	3100	49	4640	73	5400	85	6340
D-15	5.41	7.0	0.50	2790	54	4230	81	5050	97	5210
D-16	5.83	6.6	2.25	2330	54	3200	75	3990	93	4300
D-17	5.80	7.5	5.00	1870	54	2510	73	2930	86	3430
D-18	5.77	7.2	0.50	2550	49	3570	68	4270	81	5250
D-19	5.67	7.2	2.00	2490	56	3500	78	3690	82	4480
D-20	5.41	7.2	5.00	1720	49	2400	68	3190	91	3520
D-21	5.62	7.9	0.50	2540	51	3660	73	4730	95	4990
D-22	5.82	7.5	2.00	2540	63	3310	83	3650	91	4010
D-23	5.68	6.6	5.00	1670	44	2460	64	3150	82	3840
SG-1	6.02	4.5	4.00	2050	50	3080	75	3680	90	4080

*ST - Concrete made with an expanded clay aggregate produced in Stafford, Texas.

D - Concrete made with an uncoated expanded shale aggregate produced near Dallas, Texas.

SG - Concrete made with a regular weight sand and gravel.

designs with similar cement factors and plotting the compressive strength-age relationship, Fig. 2-1 results. As expected, a significant variation in strength is achieved between different lightweight aggregate types. However, by converting the strength values to per cent 28-day strength for each mix as seen in Table 2-1 and plotting the per cent 28-day strength versus age values, a general relationship appears between these two parameters as is shown in Fig. 2-2, even though the cement factor, air content, and slump all varied widely, Of course, some data scatter is present, but a definite trend is exhibited, suggesting a method whereby a designer could, for a particular lightweight aggregate, knowing the concrete mix design strength at one age, predict with some degree of confidence the strength of this particular concrete mix design at any other age. This approach to data analysis is presented in this report.

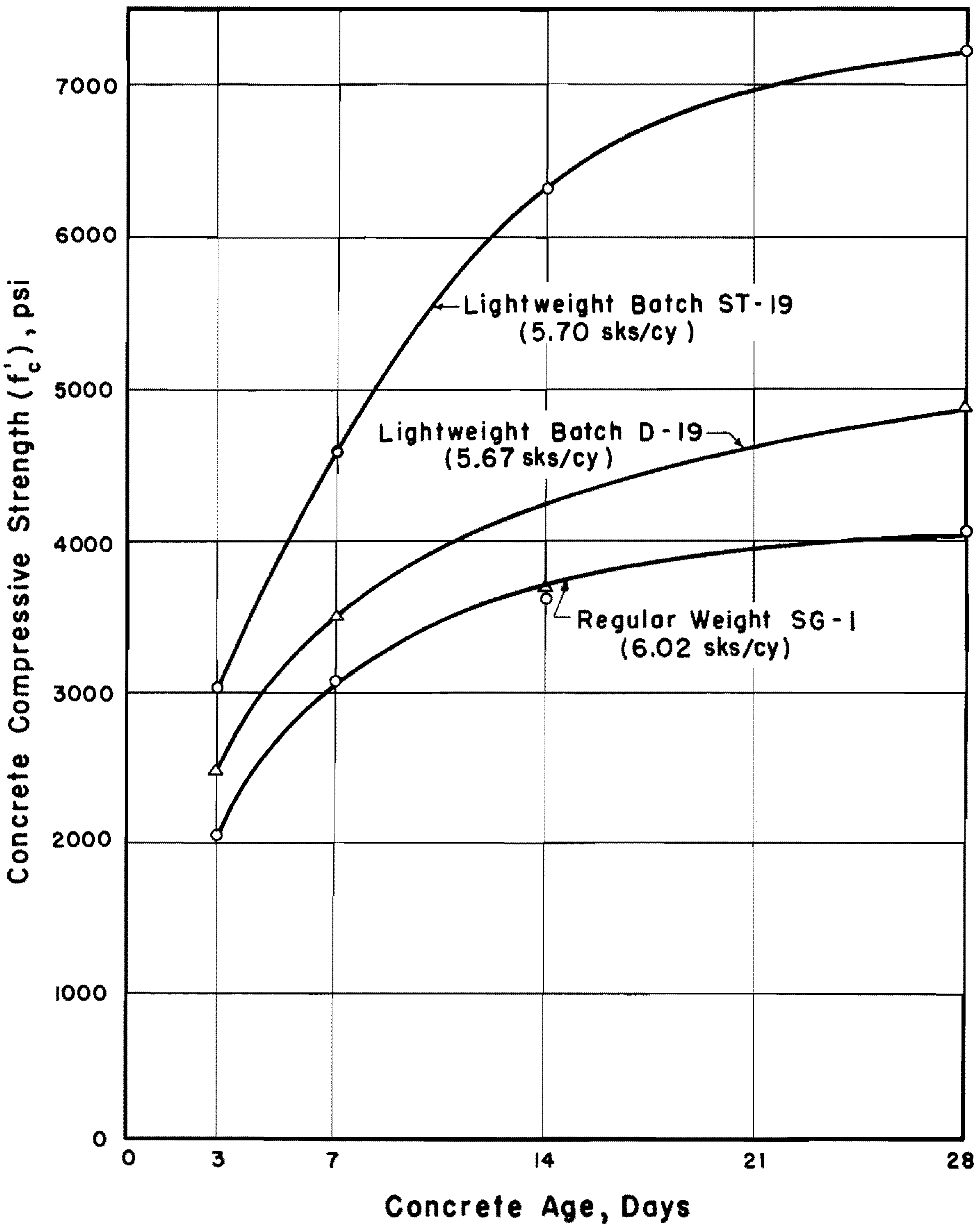


Figure 2-1. Compressive Strength vs Age for Concrete Made With Different Aggregates, Data Taken from Table 2-1.

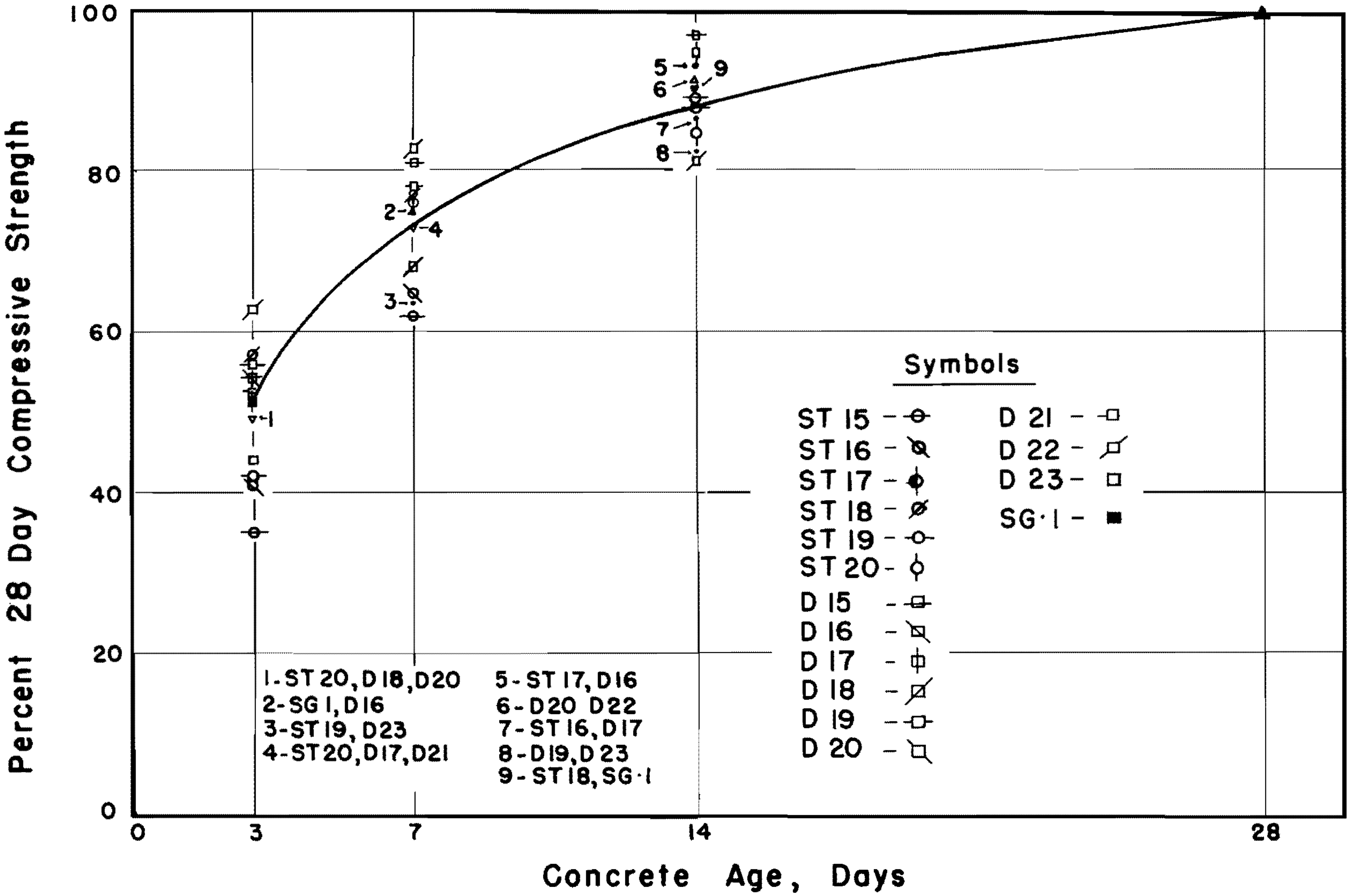


Figure 2-2. Percent of 28 Day Compressive Strength vs Age, Wet Storage, Data Taken From Table 2-1.

3. DIRECT TENSILE TEST

3.1 General

The property of concrete tensile strength has long been of significance to design engineers. This property has been very difficult, if not impossible, to accurately and reliably determine. Many methods for determining this property have been developed and tried, but to this writer's knowledge, none of them experimentally evaluate this important property with any high degree of certainty.[29] The main reasons for this difficulty lie in the nature of the material. For example, concrete, being relatively weak in tension, is significantly influenced by small eccentricities in applied tensile loads, stress concentrations, and variations in paste-aggregate ratios throughout the specimen. These, in turn, make accurate determination of tensile strength extremely difficult to obtain experimentally.

In 1955, a method was reported by Todd,[30] which offered a new technique to determine the tensile strength, the tensile stress-strain relationship, and the effects of restrained volume changes on the tensile strength and stress-strain characteristics of structural concrete. This idea, presented by Mr. Todd, has been amplified and developed at The University of Texas under the direction of Professor J. Neils Thompson. Several studies have been made into the use of this method and into applications of the information obtained toward solving structural problems.[31, 32, 33] This method has been further modified and used extensively in this study. A brief description of the concept and method of test is given in the following paragraphs.

3.2 Direct Tensile Test Specimen

The test specimen consists of a thick-walled steel tube upon which electrical SR-4 strain gages are mounted and protected by a brass sleeve around the tube; and the tube is encased in a specimen of concrete. Figure 3-1 gives an over-all view of the steel tube and the brass sleeve. It can be seen from this figure that the tube surface has deformations which aid in bonding the concrete to the steel. The brass sleeve, in addition to moisture proofing the gages, serves to reduce the cross-sectional area of the concrete, and thereby insures concrete failure at the point where the gages are mounted.

An over-all view of the completed tube assembly is shown in Fig. 3-2. Petrosene wax is sloped around the ends of the sleeves in a gradual taper to reduce stress concentrations in the concrete surrounding the steel specimen. Figure 3-3 gives a view of a tensile specimen that has been loaded to concrete failure. The failure crack has been painted to indicate its position. A schematic drawing of the entire assembly is shown in Fig. 3-4. Note the "0" rings in the sleeve which prevent moisture in the concrete from entering the cavity around the gages.

Two etched-foil type 90° rosette SR-4 strain gages are mounted on the steel tube, one grid of each being parallel to the longitudinal axis of the tube and the other grid perpendicular to the axis. A close-up view of

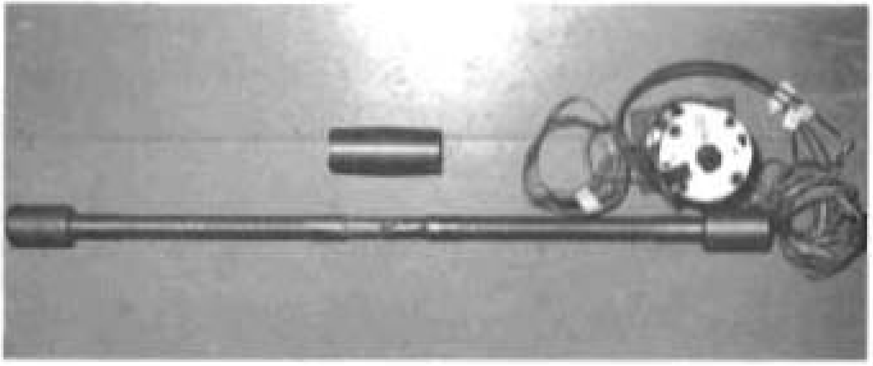


Fig. 3-1. Steel Tube and Brass Sleeve, Disassembled

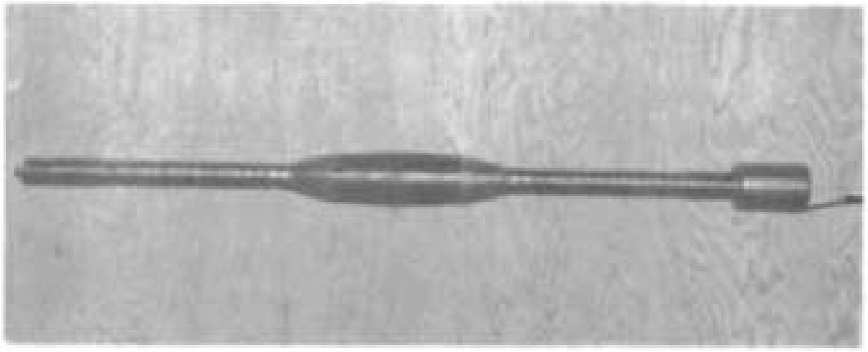


Fig. 3-2. Steel Tube and Brass Sleeve, Assembled

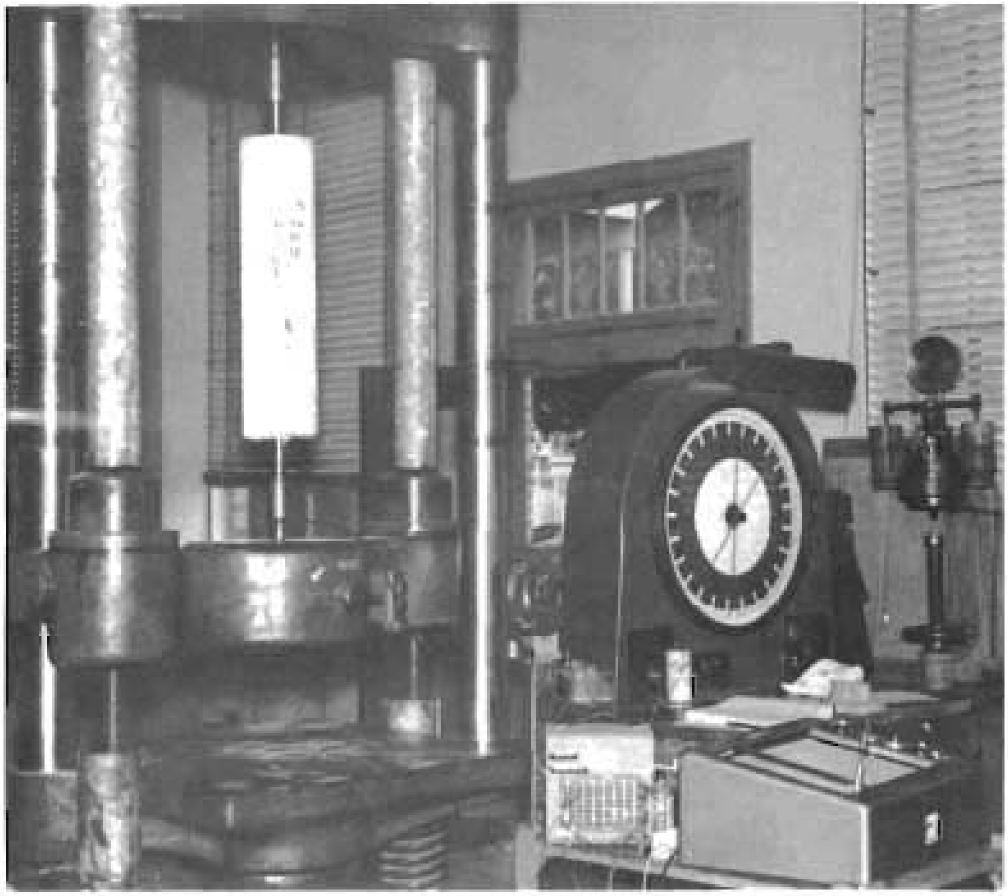
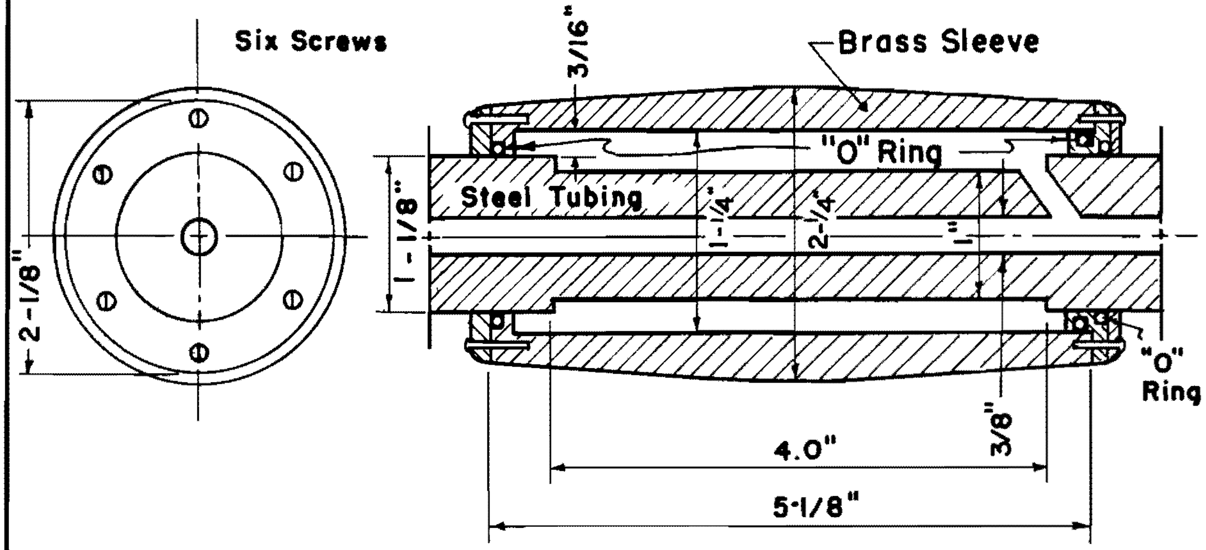
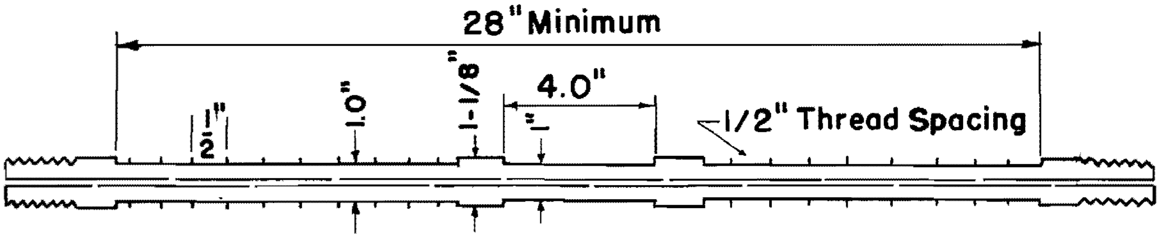


Fig. 3-3. Over-All View of a Direct Tensile Test Concrete Specimen Loaded to Failure



Brass Sleeve for Tensile Test Specimen



Tensile Test Steel Tube

Figure 3-4. Concrete Direct Tensile Test Specimen.

the strain gages is shown in Fig. 3-5. These two gages (a total of four grids) are wired together to form a full Wheatstone bridge circuit. The lead wires are carried from the gages along the inside of the tube to the end of the specimen and then to a switch unit. A schematic drawing showing the leads and gage orientation can be seen in Fig. 3-6. With this type of circuit, the effects of any slight deviation from true uniaxial loading are cancelled, temperature changes affecting changes in apparent strain are minimized, and an output equal to approximately 2.6 times the actual strain in the rod is achieved. (From both longitudinal grids plus Poisson's effect measured on the perpendicular grids.)

3.3 Load Calibration

With the strain-gage circuit shown in Fig. 3-6, the indicated strain from any given loading can be determined by using a Baldwin Type N portable strain indicator. As the strain output is approximately 2.6 times the actual strain on the steel tube, it is necessary to experimentally determine the true strain on the rod. To do this, another specimen was fabricated and, using mechanical strain devices, the true strain for any given load (ϵ^t_{sp}) and hence the modulus of elasticity of the steel (E_s) was determined. For the steel used to make the six specimens fabricated for this study, E_s was found to be 29.6×10^6 psi. Each of the six specimens were then load calibrated.

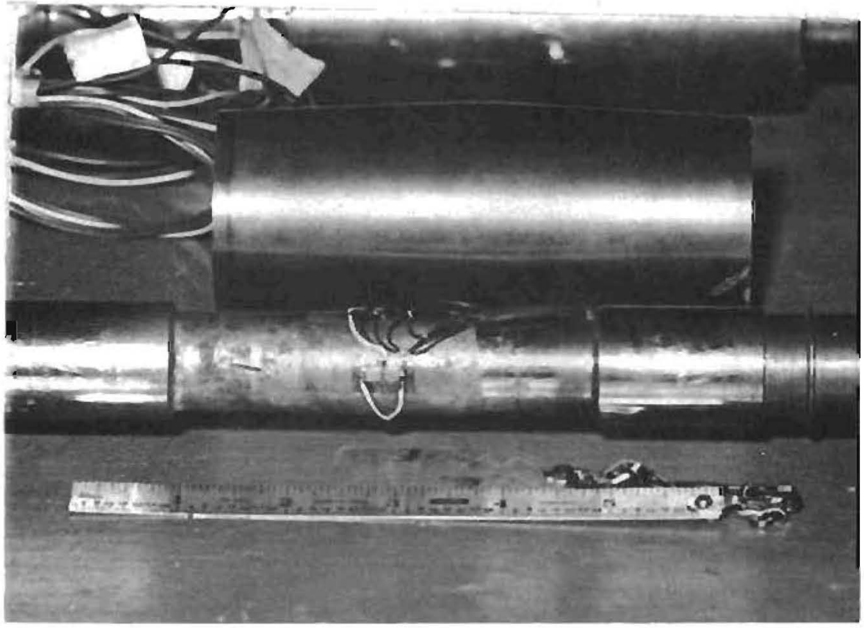


Fig. 3-5. Close-Up of a Direct Tension Steel Tube Showing 90-Degree Rosette Etched Foil SR-4 Strain Gage

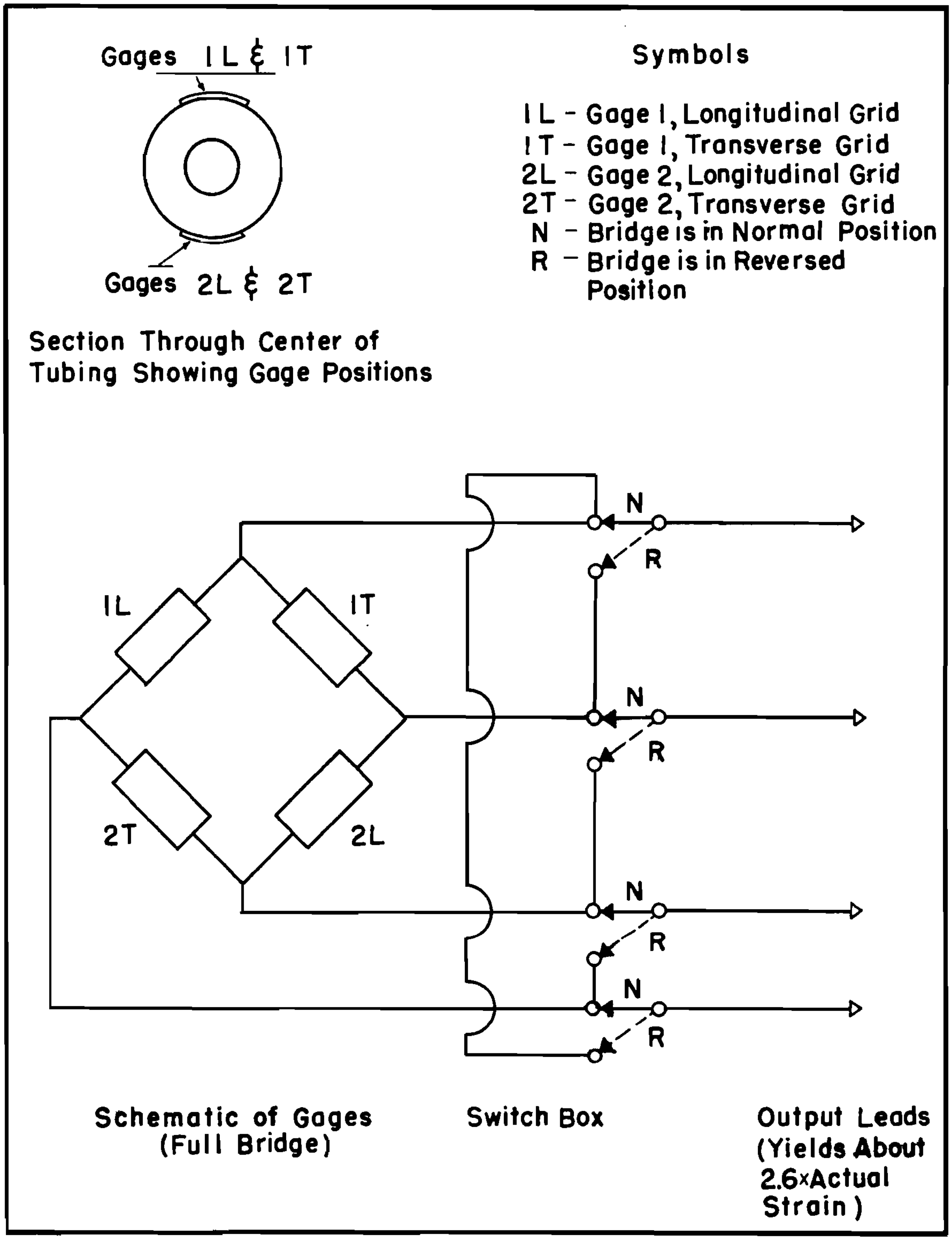


Figure 3-6. Schematic Drawing of Direct Tensile Specimen Strain Gage Hook-up.

Since

$$E_s = \frac{\sigma_s}{\varepsilon^t_{sp}} = \frac{P_s}{A_s \varepsilon^t_{sp}}$$

then

$$\varepsilon^t_{sp} = \frac{P_s}{A_s E_s}$$

where

σ_s = stress in the steel in psi

ε^t_{sp} = true steel strain due to an external load

P_s = external load in the steel, lb

A_s = cross-sectional area of the steel, sq in.

For a given P_s, the indicated steel strain due to an external load ε^i_{sp} was measured for each bar and the relationship between true and indicated strain, called here the strain factor S_f was determined by the following relationship:

$$S_f = \frac{\varepsilon^i_{sp}}{\varepsilon^t_{sp}} \qquad \text{- - - - -} \quad (3\text{-}1)$$

therefore:

$$S_f = \frac{\varepsilon^i_{sp} A_s E_s}{P_s} \qquad \text{- - - - -} \quad (3\text{-}2)$$

The strain factors for each of the six specimens are given in the following table.

Specimen Number	Strain Factor S_f
1	2.61
2	2.59
3	2.58
4	2.50
5	2.52
6	2.50

3.4 Temperature Calibration

With the strain-gage circuit as described earlier, the effects of changes in temperature changing the apparent strain in the specimen were minimized. However, due to the relatively small strains experienced for concrete in tension prior to failure, these effects could not be completely ignored; and accordingly, strain readings were taken on each bare bar over the range of temperatures which would be encountered during the investigation. This was accomplished by placing the bare tube in an electric oven and recording strain readings for each tube at different temperatures. Over a 40F range, the readings were plotted along the abscissa with the specimen temperature as the ordinate. Some slight data scatter occurred, but sufficient data was obtained to plot a straight line temperature calibration curve for each specimen. The slope of the lines was determined and is called the temperature calibration factor S_t. This factor is given in the following table.

Temperature Calibration
Factor (S_t) in Terms of Indicated
Steel Strain (ε_s^i)

Specimen Number	S_t (in./in./°F) x 10^6
1	1.00
2	0.25
3	1.00
4	0.25
5	0.20
6	0.75

It is interesting to note that although each specimen came from the same steel tube, and all gages came from the same manufacturer, the strain factors are not identical, although all the factors are relatively low, especially if they are converted into terms of true steel strain. This variation in S_t can be explained, perhaps, by the fact that the steel in actuality is not uniform. Also, all the gages may not be exactly alike, and they would not have to differ very much to cause a small but measurable difference in strain for these temperature changes. Furthermore, the thicknesses of the epoxy adhesive used and the gage application pressures are certainly not exactly the same for all gages, which could account for some slight deviations in measured strain between gages. Any one or all of the above contribute to the slight differences found between gage resistances, and hence strain readings, at different temperatures.

3.5 Restrained Hydration Shrinkage Test

With the strain-gage arrangement on the steel specimen, the strains occurring on the tube surface can be accurately measured from the time the concrete is molded around the steel specimen. As the concrete hardens from a viscous pseudo-fluid into a pseudo-crystalline solid, extremely complicated chemical reactions and changes of state occur, which cause volume changes to occur in the concrete mass. The steel, being relatively stable dimensionally, partially restrains this volume change in the concrete, thereby resulting in strains in both the steel and the concrete.

An idealized schematic of the effects of a restrained concrete shrinkage is shown in Fig. 3-7. Due to a shrinkage phenomenon occurring in the concrete, the concrete of original length ℓ_o would, if unrestrained, shrink to an unrestrained length ℓ_u. But, due to the presence of the steel, only partial shrinkage can take place to a restrained length ℓ_r somewhere between ℓ_o and ℓ_u. Ideally, this results in a net tensile strain in the concrete ε_{cz} due to restrained concrete volume changes of:

$$\varepsilon_{cz} = \frac{\ell_r - \ell_u}{\ell_o} = \frac{\Delta\ell_c}{\ell_o}$$

and, ideally, a net compressive strain in the steel ε_{sz}^t of:

$$\varepsilon_{sz}^t = \frac{\ell_o - \ell_r}{\ell_o} = \frac{\Delta\ell_s}{\ell_o}$$

if it is assumed that no slippage occurs between the steel and the concrete. During this hydration volume change period, there are no external forces

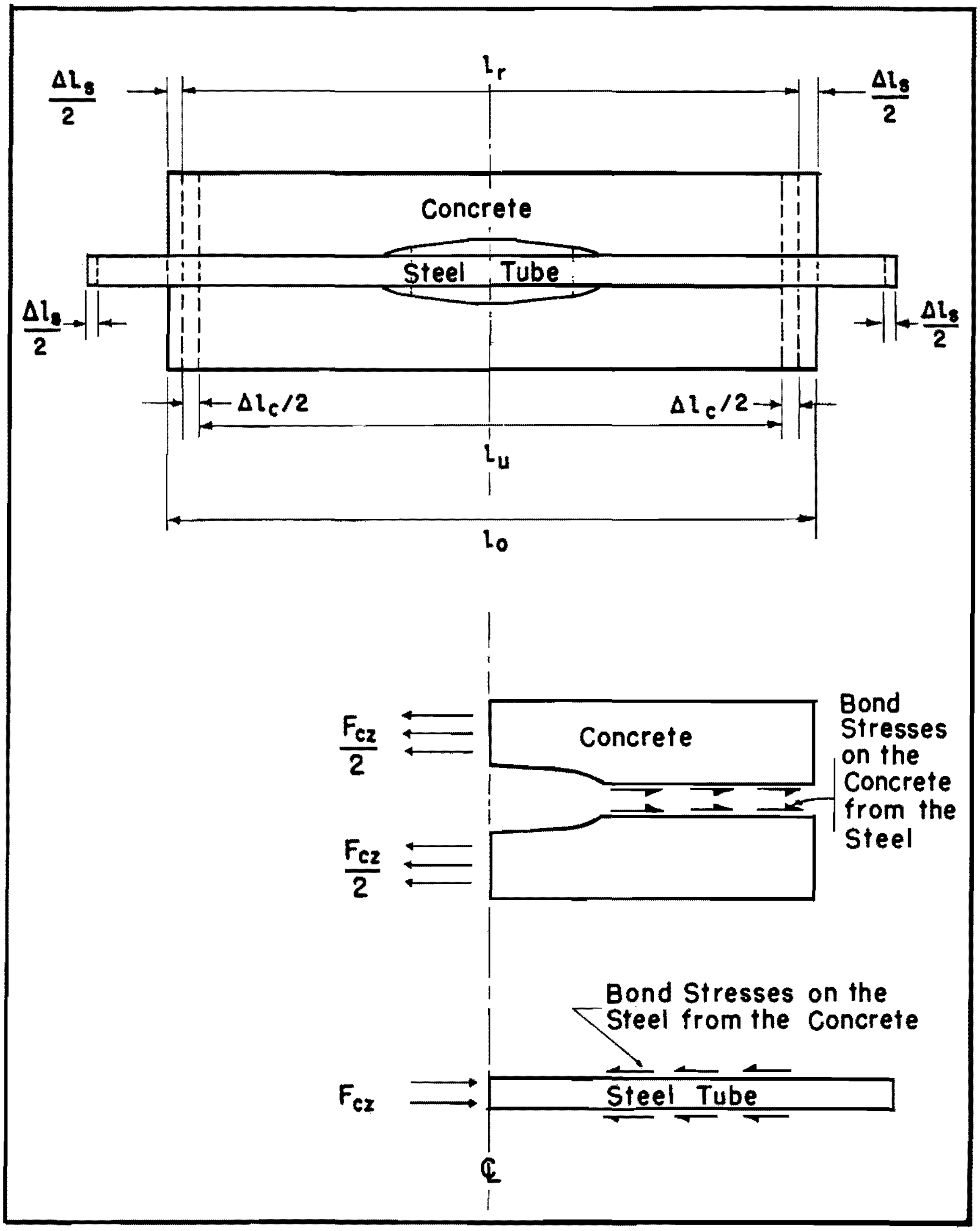


Figure 3-7. Direct Tensile Test Specimen Showing a Simplification of the Forces Resulting from Restrained Hydration Shrinkage in the Concrete.

on the specimen; therefore, since equilibrium exists, at the center line of the specimen, the total tensile force in the concrete must equal the total compressive force in the steel from the restrained shrinkage. Or,

$$F_{cz} = -F_{sz} \qquad \text{- - - - -} \quad (3\text{-}3)$$

where

F_{cz} = the total force in the concrete due to restrained volume changes, lb

F_{sz} = the force in the steel due to restrained volume changes, lb

It follows then, if the true steel strain in the steel ε^t_{sz} is measured, that the force in the concrete, and hence the stress in the concrete, can be computed by:

$$\sigma_{cz} = \frac{F_{cz}}{A_c} = \frac{-F_{sz}}{A_c} = \frac{-\varepsilon^t_{sz} E_s A_s}{A_c} \qquad \text{- - - - -} \quad (3\text{-}4)$$

where

σ_{cz} = concrete stress due to restrained volume changes, psi

A_c = concrete cross-sectional area, sq in.

Therefore, by measuring the strain in the steel at the center line of the specimen at given time intervals, or continuously from the time the concrete is poured, a concrete stress vs. concrete age can be accurately determined throughout the hydration period.

An important point in the foregoing analysis is that no assumption was made as to the relationship between the true steel strain due to restrained volume changes ε^t_{sz} and the concrete strain due to restrained volume changes ε_{cz} at the center line during this restrained hydration shrinkage process. Obviously, since the forces in the steel and the concrete are equal as given in Eq. 3-3, then

$$\varepsilon_{sz}^{t} \neq \varepsilon_{cz}$$

at the center line unless the relationship between A_s/A_c and E_c/E_s are such that the strains could be equal without violating Eq. 3-3. This presents no difficulties since the steel and concrete are unbonded at the center line of the specimen and slip at the interface can easily take place.

3.6 Direct Tensile Load Test

During the tensile load test, a uniaxial tensile load is applied to the ends of the steel tube. The composite specimen of steel and concrete combines to carry the applied external load. In this study, a Southwark-Emery 120,000-lb-capacity hydraulic machine was used to apply the load (see Fig. 3-3 for a view of the machine). Using a slide-wire potentiometer bridge circuit attached to the load dial on the machine, the change in resistance corresponding to the change in the applied load was fed into the Y-axis of a Mosely Autograph X-Y plotter. The full external bridge strain-gage circuit in the specimen was fed into the X-axis of this same plotter. A schematic drawing of this test setup is shown in Fig. 3-8. After suitable load and strain calibration, external applied load vs indicated steel strain can be accurately traced on a sheet of graph paper by this plotter. A reproduction of a test trace of external load vs indicated steel strain is shown in Fig. 3-9.

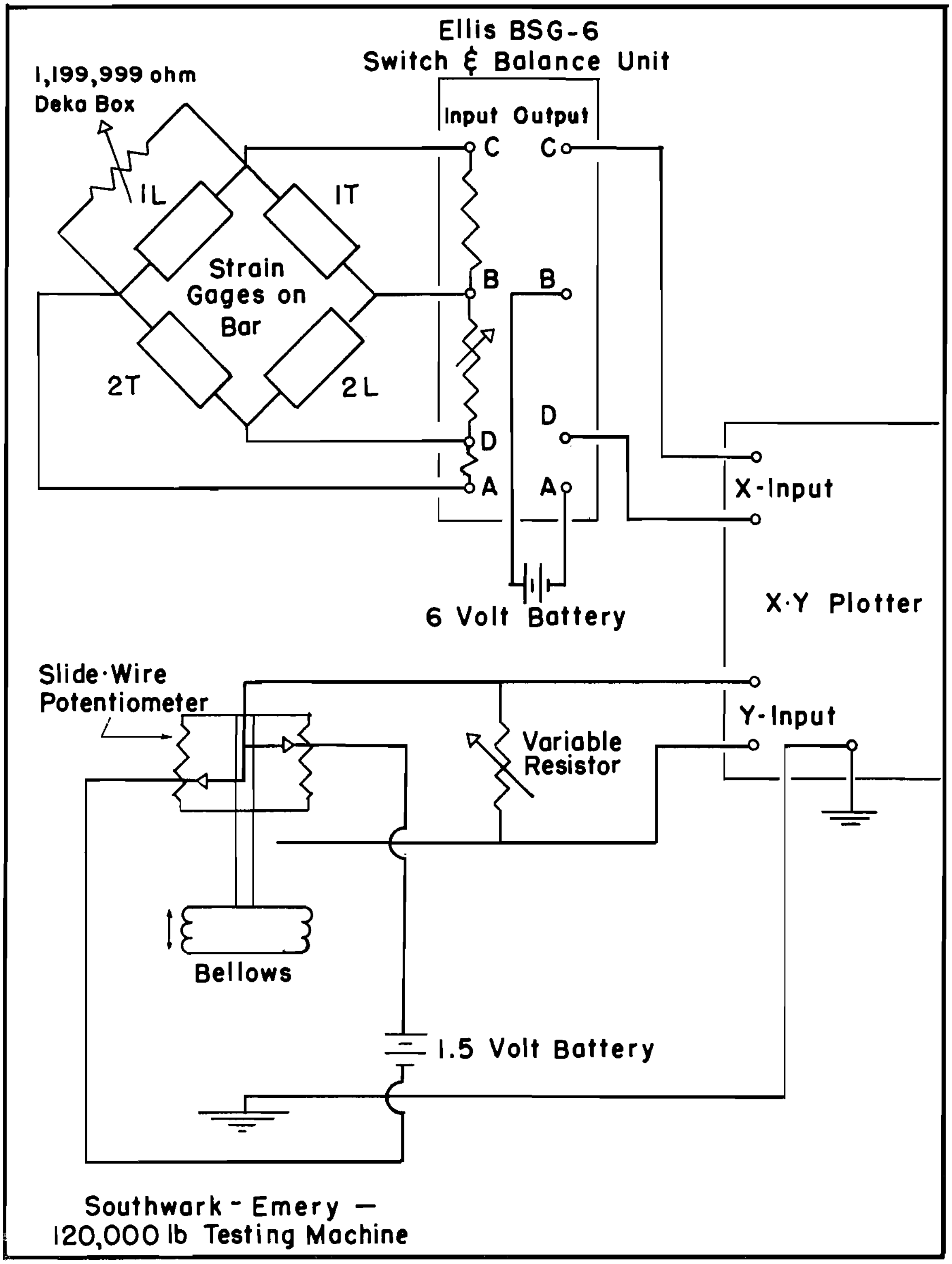


Figure 3-8. Schematic Drawing of Tensile Test Bar X-Y Plotter Hook-up

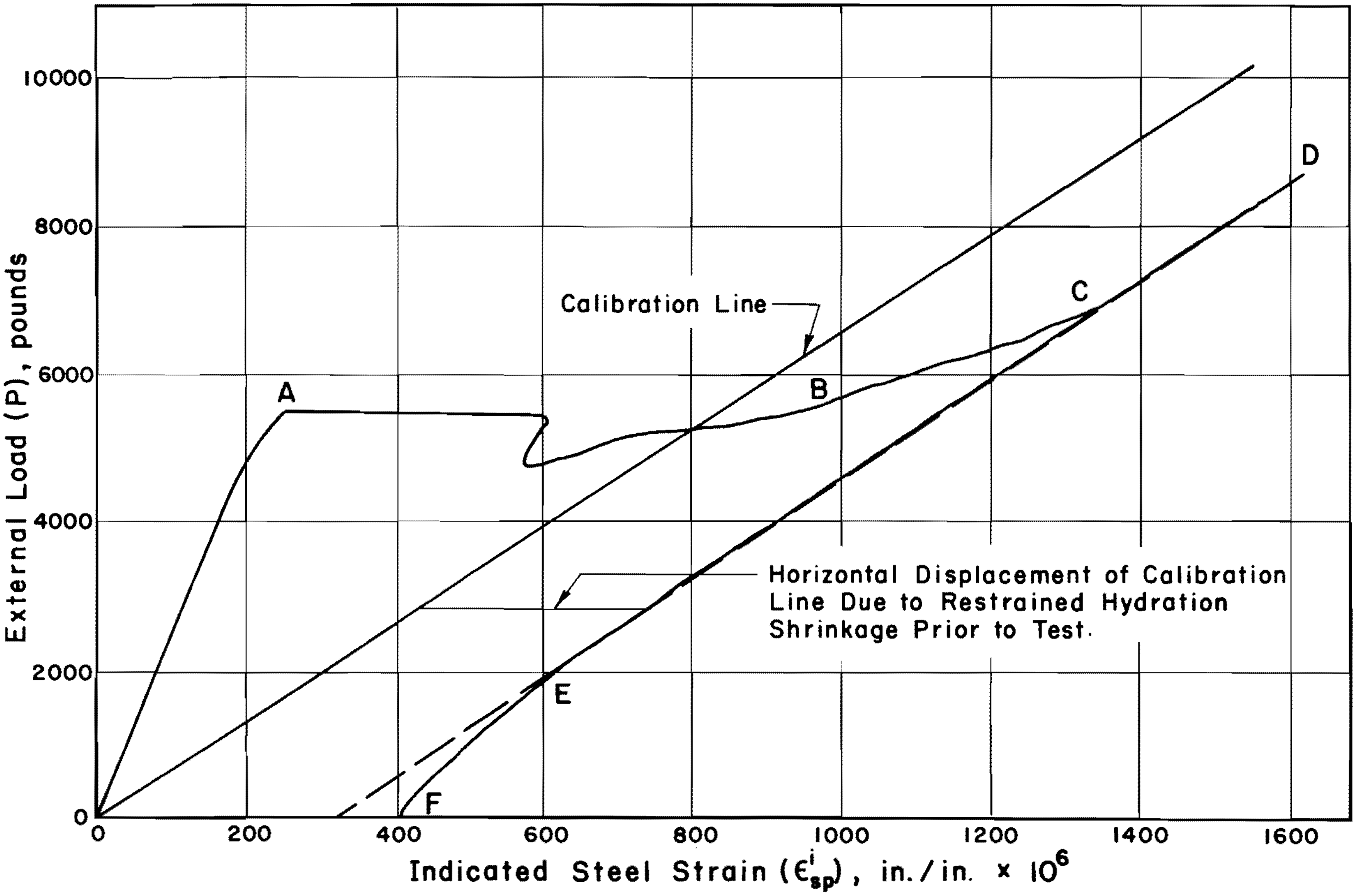


Figure 3-9. External Load-Indicated Steel Strain Test Trace for Direct Tensile Test of Concrete.

The load is carried by the composite action of concrete and steel from point 0 to point A. At point A the concrete has reached its full tensile capacity and upon application of the additional load, the concrete fails in tension. As this occurs, the load being carried by the concrete is suddenly shifted to the steel with a corresponding sudden increase in indicated steel strain.

In order to determine the net load being carried by the concrete, it is necessary to plot on the graph the results of the load vs. indicated-steel-strain-calibration line of the bare specimen (without the concrete) which is determined by running a separate load test with the bare specimen. Then the external load carried by the concrete P_c is simply the vertical distance between the OA curve and the steel calibration line.

$$P_c = P - P_s$$

and

$$\sigma_{cp} = \frac{P_c}{A_c} \qquad \text{- - - - -} \quad (3\text{-}5)$$

where

P = total external load, lb

P_s = the external load steel, lb

σ_{cp} = concrete stress due to the external load, psi

Often the initial concrete crack does not extend throughout the cross section of the concrete and, after initial cracking at point A, the reduced concrete section may carry some small additional load before completely failing as is shown between points B and C in the figure.

The drop in the applied load between points A and B is due to the fact that the machine head is not displacing fast enough to fully counteract the rapid changes in strain at concrete failure thus causing the applied load to drop off momentarily. There is also a slight time lag in plotting time due to the relatively slow dynamic response capability of the X-Y plotter. Between points C and D the steel at the center line of the specimen is carrying the entire external load, and the line is parallel to the steel calibration line. The test trace from C to D is horizontally separated from the steel calibration line by an amount equal to the initial compressive strain ε^t_{sz} in the steel. As unloading occurs, the test trace unloads to a point E, parallel to the steel calibration line, then deviates from this parallel line as the concrete crack does not close completely.

During an early portion of the curve up to point A, the external load is being carried by both the steel and the concrete at the center line of the specimen. Assuming there is sufficient bond to insure that no slippage is occurring, in order for compatibility to exist at this center line,

$$\varepsilon^t_{sp} = \varepsilon_{cp} \qquad \text{- - - - -} \quad (3\text{-}6)$$

where

ε_{cp} = the concrete strain due to an external load

This strain condition is quite different than that existing from strain volume changes prior to testing and thus:

$$P_c \neq P_s$$

which is reversed from the condition existing prior to loading.

Equation 3-6 was verified experimentally by placing two concrete embedment gages inside the concrete adjacent to the steel specimen gages on opposite sides of the cylinder to measure ε_{cp} directly. A comparison between ε_{cp} and ε^t_{sp} during a test are given in Fig. 3-10. Note the very close agreement.

With Eq. 3-6, the true concrete strain from the external applied load ε_{cp} can be determined from the test trace.

$$\varepsilon_{cp} = \frac{\varepsilon^t_s}{S_f} \qquad \text{- - - - - (3-7)}$$

Now, using Eqs. 3-5 and 3-7, the concrete stress-strain curve from applied external loads can be calculated and plotted. From this curve, the modulus of elasticity of the concrete in tension E_{ct} can be calculated. Referring to Eq. 3-4, and assuming the E_{ct} during testing is the same as the E_{ct} just prior to testing, ε_{cz} can be determined by

$$\varepsilon_{cz} = \frac{\sigma_{cz}}{E_{ct}} \qquad \text{- - - - - (3-8)}$$

Therefore, combining Eq. 3-4 and 3-5, the total corrected tensile stress σ_{ct} in the concrete can be calculated from

$$\sigma_{ct} = \sigma_{cz} + \sigma_{cp} \qquad \text{- - - - - (3-9)}$$

And, combining Eq. 3-7 and 3-8, the total corrected tensile strain in the concrete ε_{ct} can be calculated from

$$\varepsilon_{ct} = \varepsilon_{cz} + \varepsilon_{cp} \qquad \text{- - - - - (3-10)}$$

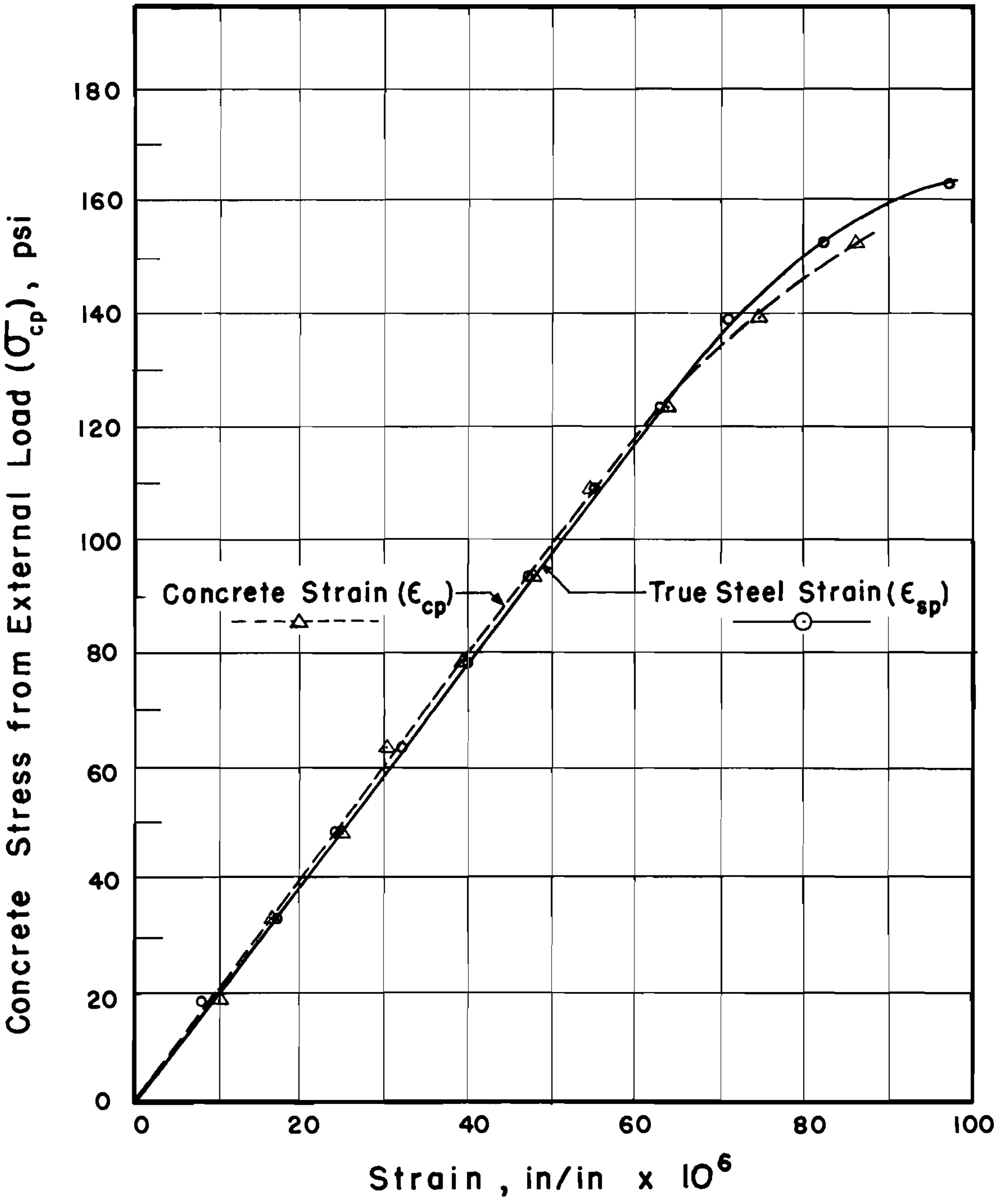


Figure 3-10. Concrete Tensile Stress-Strain Curves Showing Comparison Between Strains Measured in the Concrete and Steel Specimen.

Figure 3-11 portrays a sample concrete direct tensile stress-strain curve on which is shown the coordinate axes for the total stress-strain and the stress-strain from the external applied load. This figure is a graphical representation of Eqs. 3-9 and 3-10 and summarizes the information which may be obtained from the direct tensile test method.

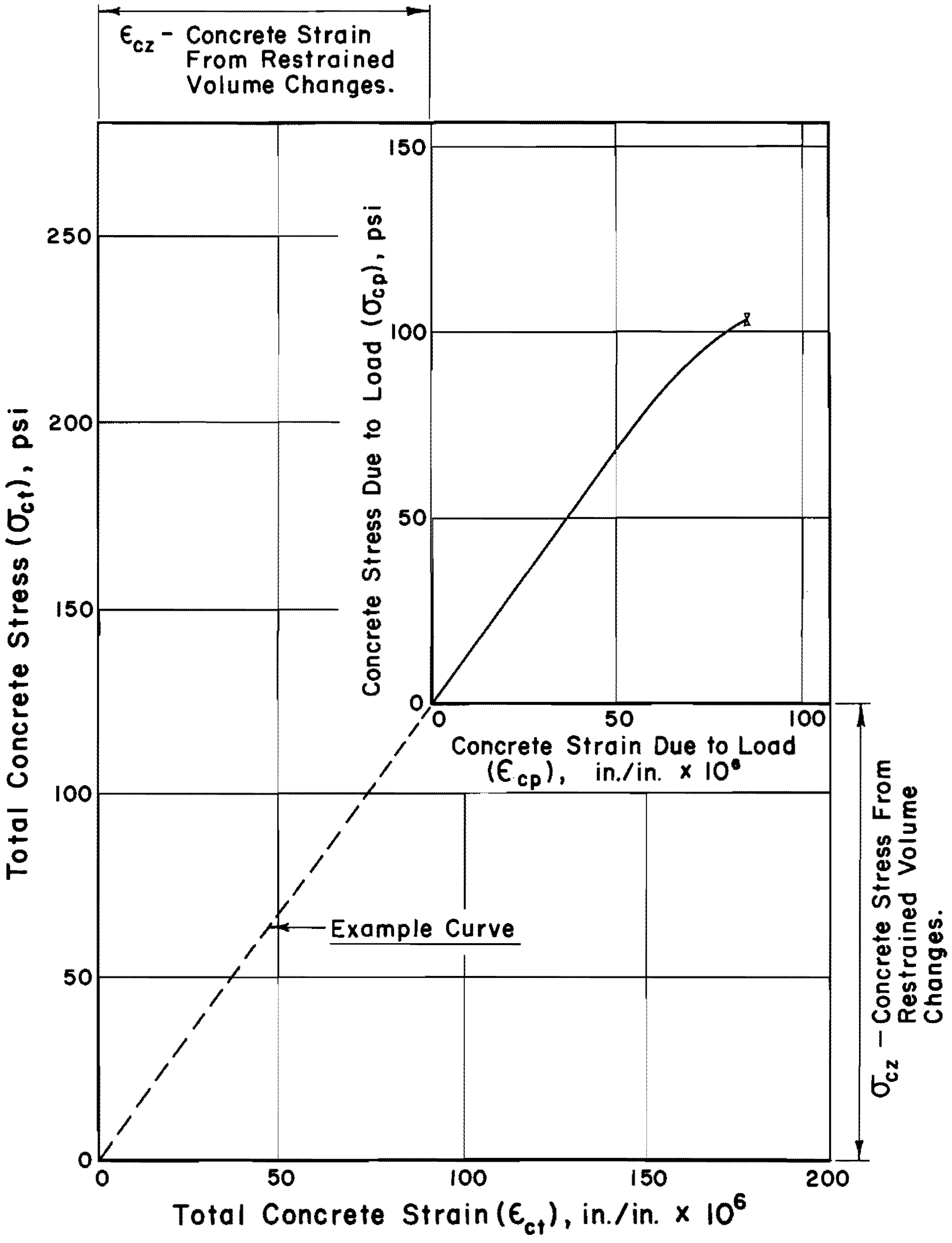


Figure 3-11. Example Structural Lightweight Concrete Direct Tensile Stress—Strain Curve.

4. DISCUSSION OF RESULTS

4.1 General

The direct tensile properties of structural lightweight concrete, being of primary importance in this investigation, are discussed first in this chapter. Following this is the section on restrained concrete volume changes, which were determined in conjunction with the direct tensile properties. In order to compare the direct tensile properties with various standard properties now being determined in this country; in the next three sections of this chapter are presented the indirect tensile strengths, compressive strengths, and flexural strengths, for the various design parameters selected in this investigation.

In order to analyze these data, replication of data values were performed. By so doing, random or unaccountable error was reduced to a minimum. A complete discussion of the reproducibility of results is given in Section 6.1 of this report.

4.2 Direct Tensile Properties

In this investigation, the direct tensile strength of the concrete was accurately determined using a unique test method. From this test the direct tensile strength, the direct tensile stress-strain characteristics, and the effects of restrained concrete volume changes were determined. The development of this test at the University for the past several years has proved its value. Inasmuch as a significant portion of this investigation involved the use and interpretation of this test, the entire method has been described in detail in Chapter 3.

For the direct tensile properties, the parameters as discussed in Chapter 1 were investigated. Concrete properties at two ages, 7 and 28 days, were determined. Figure 4-1 summarizes the stress-strain results of this test at an age of 7 days for 5 sack per cubic yard structural lightweight concrete with 6 per cent air. Direct tensile stress-strain curves for each of the three curing conditions are shown, as well as the curve for regular weight 5 sack per cubic yard concrete for comparison purposes. Each curve represents a single test specimen. The dashed portion of each of the curves, whose ordinate is labeled σ_{cz} represents the amount of restrained concrete volume change stress present in the concrete prior to testing. The solid portion represents the stress-strain characteristics obtained from the tension test. The dashed portion of the curve is drawn as a straight line for simplicity in presentation and does not represent the stress-strain behavior during the hydration period prior to testing. As discussed in Chapter 3, the strains in the steel specimen were measured during this hydration period, from which the concrete restrained volume change stresses can be calculated. However, the concrete strains during this period are not known as the relationship between concrete stress and strain is not known until the time of test. When the specimen is tested, the relationship between concrete stress and strain is determined and, therefore, the strain existent in the concrete prior to testing can be calculated. Thus the solid portion of the tensile stress-strain curve is positioned correctly on the graph and portrays the influence of restrained volume changes on concrete tensile properties.

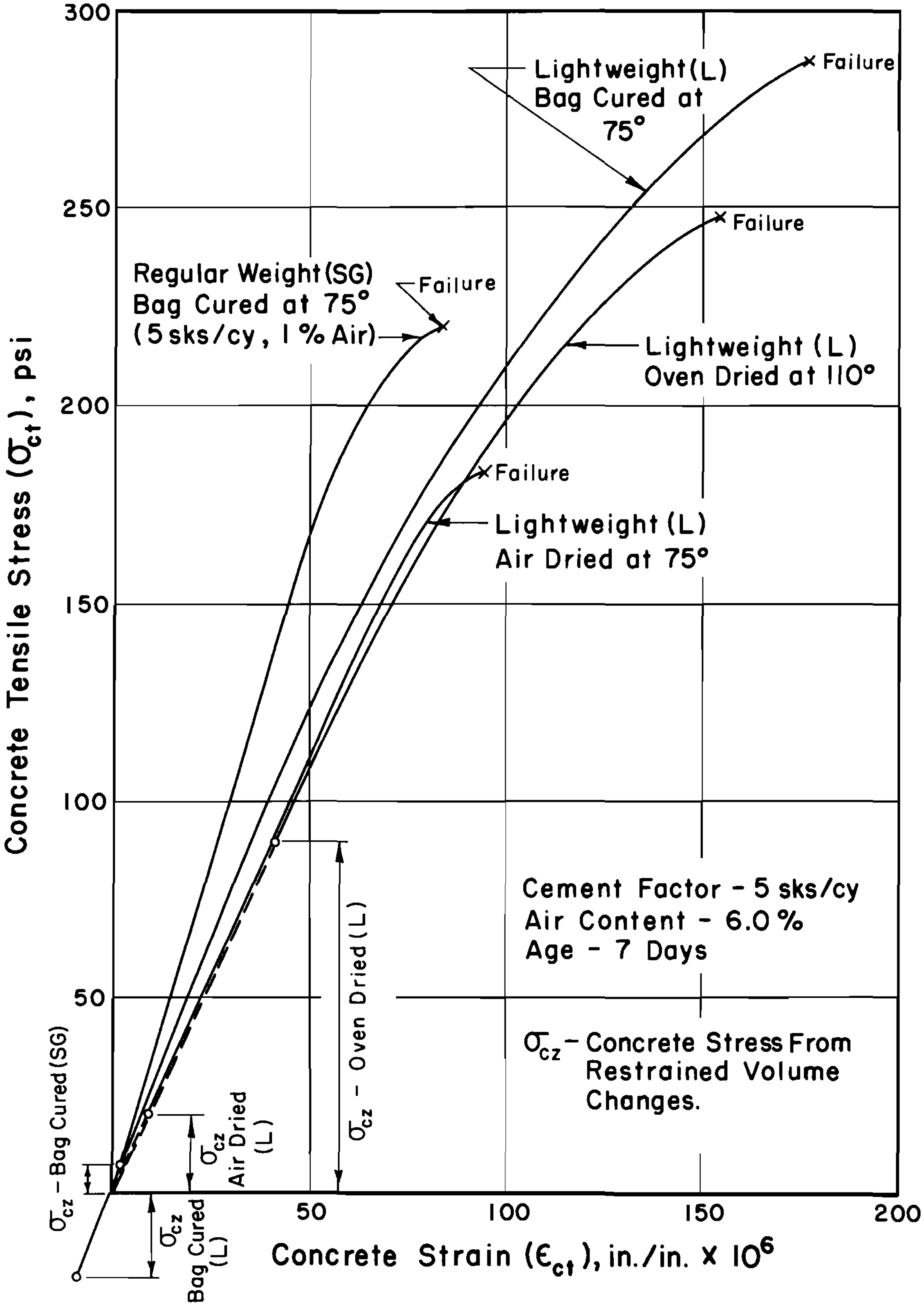


Figure 4-1. Structural Lightweight Concrete Direct Tensile Stress-Strain Curves for Three Curing Conditions at an Age of 7 Days.

Several important factors are readily apparent from an examination of Fig. 4-1. Looking at the ordinate values, it can be seen that the oven-dried concrete specimen had a residual tensile stress of 89 psi prior to testing, and a tensile strength of 249 psi. This 89 psi represents some 36 per cent of the tensile capacity of the concrete. On the other hand, the bag-cured structural lightweight concrete actually expanded during hydration causing a restrained compressive stress, σ_{cz}, from the restrained concrete volume changes. The σ_{cz} stress for the air-dried curing condition lies in between the σ_{cz} stresses of the bag-cured and oven-dried curing, as expected. The slope of the stress-strain curve changes for each curing condition, and thus the bag-cured concrete exhibits the highest modulus of elasticity. For comparison purposes, a direct tensile stress-strain curve for regular weight concrete is also shown in this figure. Note that the σ_{cz} stress for this bag-cured condition results in a small residual tensile stress indicating some slight shrinkage during hydration. This indicates that the structural lightweight aggregate concrete, when moist cured, exhibits a very unusual phenomenon by actually expanding throughout the hydration period, which is not exhibited by regular weight concrete.

The low tensile strength exhibited by the air-dried concrete appears inconsistent with the other curing conditions and other properties. This inconsistency is one of the factors which has made direct tensile strengths very difficult to measure accurately.

Figure 4-2 exhibits the direct tensile stress-strain results at an age of 28 days for 5 sack per cubic yard, 6 per cent air, structural lightweight concrete for the three curing conditions. For comparison

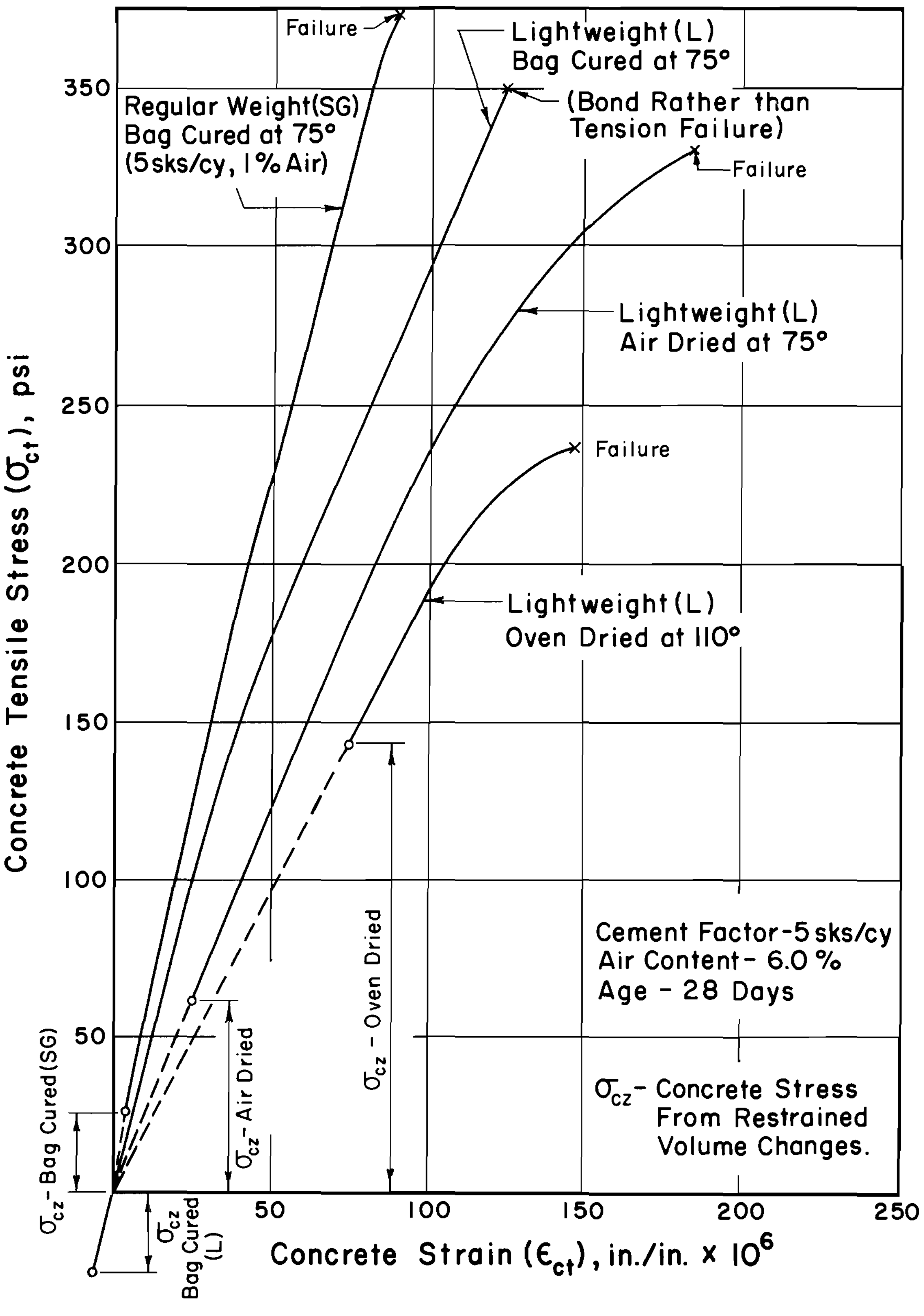


Figure 4-2. Structural Lightweight Concrete Direct Tensile Stress-Strain Curves for Three Curing Conditions at an Age of 28 Days.

purposes, the tensile stress-strain results for regular weight concrete of the same age is also shown. Here, the same comments as made about Fig. 4-1 can be offered. Note the similar resulting compressive σ_{cz} for bag-cured lightweight concrete, and the large tensile σ_{cz} stress for oven-dried lightweight concrete (here representing some 61 per cent, or well over one-half, of the concrete's tensile capacity). Again, the air-dried condition results in a median condition between the two extreme curing conditions.

One factor occurred in this series of 28-day tests (Fig. 4-2) that did not occur in the 7-day test (Fig. 4-1), and that is the failure of the bag-cured lightweight concrete in bond rather than in tension. The steel specimens were designed to have at least two times the strength in bond as in direct tension, yet here a bond failure occurred. This slippage, or bond failure, occurred on all bag-cured lightweight concrete specimens until the specimens were modified. Where time permitted, these tests resulting in bond failure were rerun with the bond area increased by welding steel lugs along the deformed portion of the steel tube. In subsequent tests, the bond was strengthened by the addition of epoxy glue along a part of the deformed portion of the tube just prior to concrete molding.

This phenomenon was observed only for the moist-cured structural lightweight concrete, and thus raises some interesting questions concerning the interrelationships between aggregate type, curing conditions, volume changes, and bond development. This was outside the scope of this study and was not investigated.

Figures 4-3 and 4-4 portray similar tensile stress-strain curves

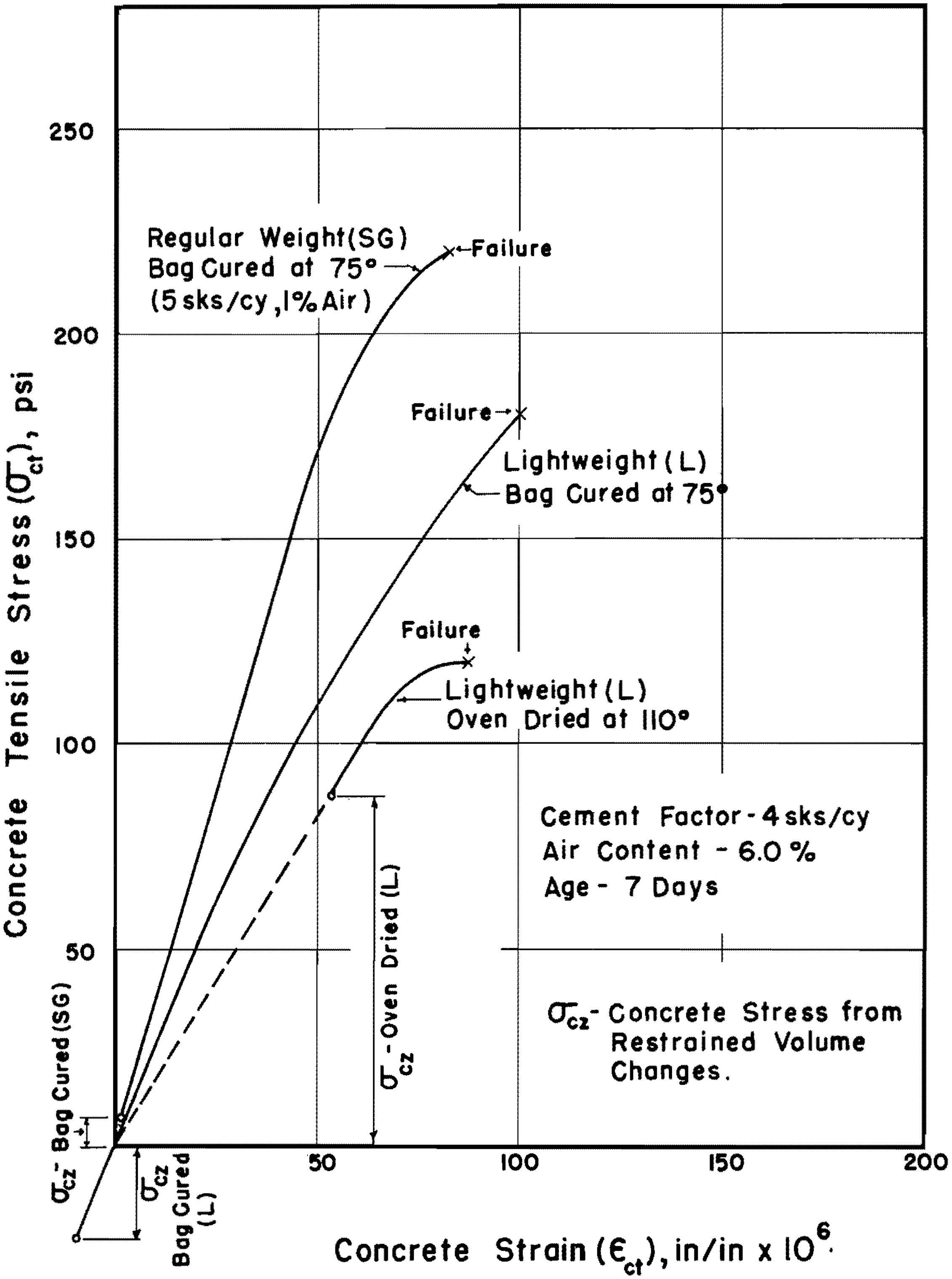


Figure 4-3. Structural Lightweight Concrete Direct Tensile Stress—Strain Curves for Two Curing Conditions at an Age of 7 Days.

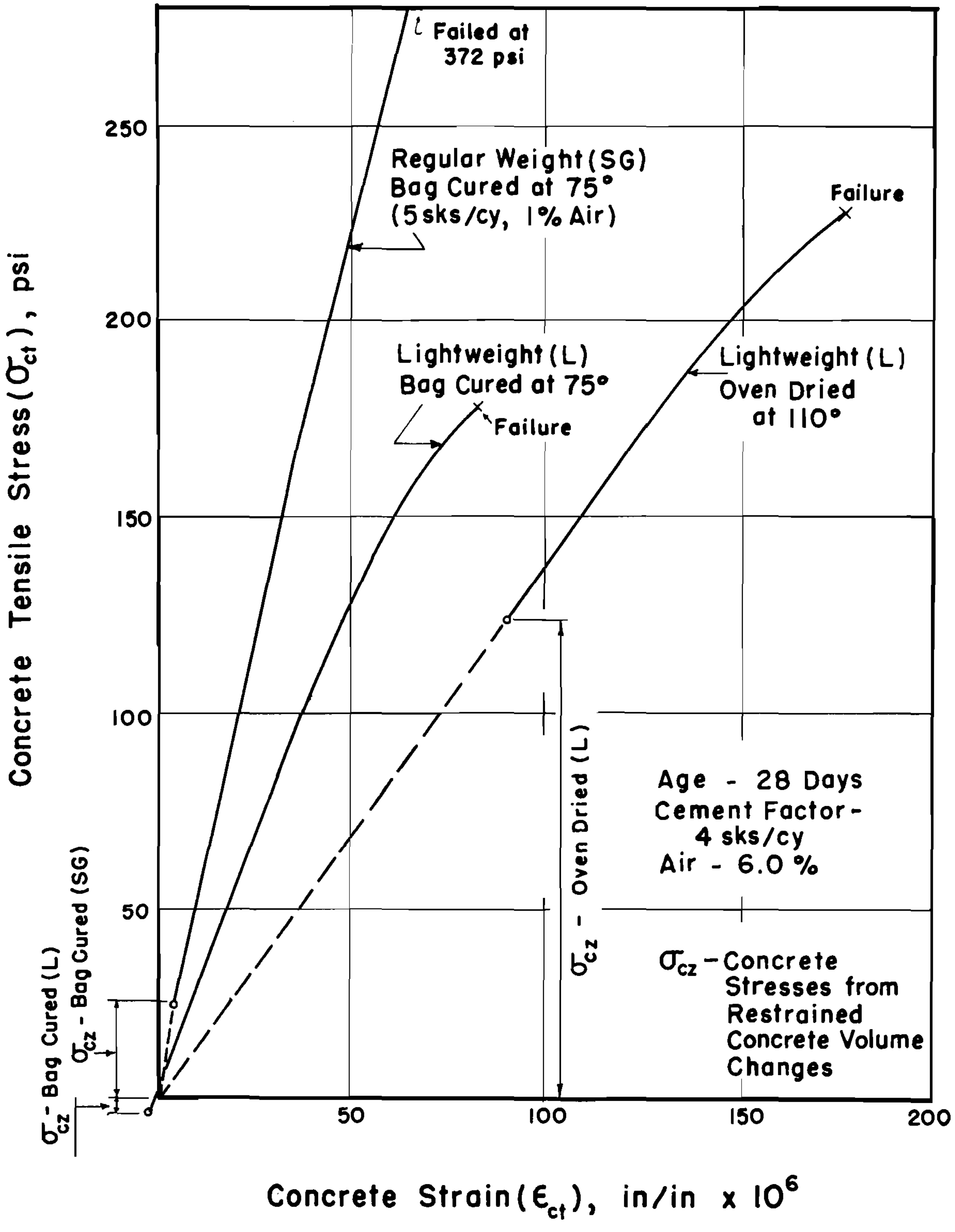


Figure 4-4. Structural Lightweight Concrete Direct Tensile Stress-Strain Curves for Two Curing Conditions at an Age of 28 Days.

for 4 sack per cubic yard, 6 per cent air content lightweight concrete at ages of 7 and 28 days respectively, and for the two extreme curing conditions of bag cured and oven dried. The same general phenomenon of structural lightweight concrete expansion when bag cured is seen, as well as a significant loss in tensile capacity from restrained volume changes in the oven-dried concrete. Again, for comparison purposes, the regular weight tensile stress-strain curves are shown for 5-sk-per-cu-yd concrete. Looking at these four direct tensile stress-strain figures, some interesting and important properties emerge which are analyzed in the next chapter of this report.

4.3 Restrained Concrete Volume Changes

As discussed in the previous section, the stresses resulting from restraining concrete volume changes can be determined with the direct-tension test specimen. Strains occurring in the steel bar are measured and, knowing the properties of the steel, area of the steel, and area of the concrete, the resultant stress in the center portion of the concrete can be calculated. (For a complete description of this method see Chapter 3 and, in particular, Eq. (3-4)). From this test, the resultant restrained stresses in the concrete can be accurately determined at any time after the concrete is poured.

In connection with the direct tensile strength investigation, the concrete stresses resulting from the concrete restrained volume changes have been determined during the hydration period of the concrete prior to testing.

Figure 4-5 portrays the effects of three curing conditions on the

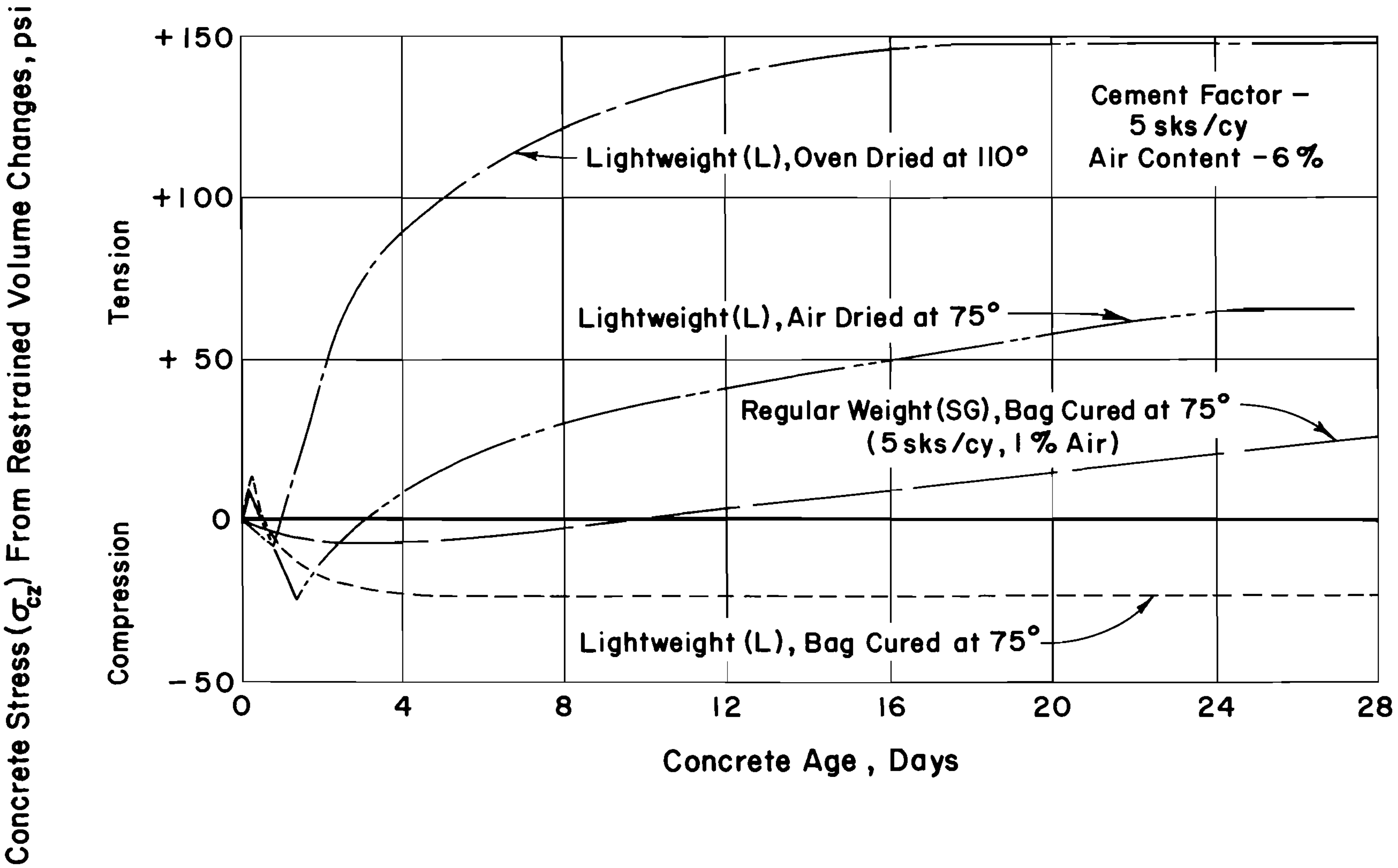


Figure 4-5. Effect of Curing Conditions on Structural Lightweight Concrete Stress-Age Relationship for Restrained Concrete Volume Changes.

restrained stress-age relationship of single specimens of structural lightweight concrete with a cement factor of 5 sacks per cubic yard and an air content of 6 per cent. For comparison, the restrained stress-age relationship for regular weight concrete is shown. Note that at very early ages from 0 to 2 or 3 days, the structural lightweight concrete undergoes shifts from compressive to tensile stress conditions or from tensile to compressive stress conditions depending upon the curing conditions. Then, from this early period throughout the remainder of the test age to 28 days, the structural lightweight concretes behave in a definite and predictable manner. The bag-cured structural lightweight concrete attempts to expand and thus goes into restrained compression, reaching dimensional equilibrium at a fairly early age. The oven-dried structural lightweight concrete attempts to shrink and thus goes into restrained tension; this tension stress increasing very rapidly at first, then finally reaching a dimensional equilibrium, of sorts, after around 20 days. The air-dried curing condition represents an intermediate condition and does not reach a dimensional equilibrium condition for the duration of the test.

It is interesting to note that the bag-cured, regular weight concrete, while initially attempting to expand slightly, eventually contracts as hydration continues, and thus goes into a restrained tension without reaching a dimensional equilibrium condition. Thus it can be seen that the behavior patterns of these two types of concrete are not similar, emphasizing the unique expansion phenomena exhibited by bag-cured structural lightweight concrete.

The same type of information can be obtained from Fig. 4-6 for structural lightweight concrete 4 sack per cubic yard with a cement factor of 4 sacks/cu yd and an air content of 6 per cent. Here the bag-cured and oven-dried conditions were measured and the same general trend was observed. The bag-cured specimen attempts to expand, thus going into restrained compression reaching a maximum value relatively soon. The gradual loss in compressive stress on this particular bag-cured specimen is due to leakage of moisture from within the bag and does not appear to be a property of 4 sack per cubic yard lightweight concrete. If the 100 per cent humidity condition had been maintained, it is believed that the curve would have remained horizontal for the test duration.

Again, for comparison purposes, a regular weight concrete restrained stress-age relationship is given.

4. 4 Indirect Tensile Strength

The indirect tensile, or split cylinder, strength gives a measure of the tensile capacity of the concrete, much the same as does the flexural strength; with the exception that the indirect tensile strength measurement determines the failure stress through the diameter of the specimen, thereby causing concrete to fail initially inside the specimen, and thus is not influenced to any significant extent by concrete surface conditions. A complete description of this test is given in section 7. 1. As with the direct tensile strength, indirect tensile strength values were determined at various ages for two cement factors (5 sacks per cubic yard and 4 sacks per cubic yard), three curing conditions, and two air contents. Each value reported represents the average of three

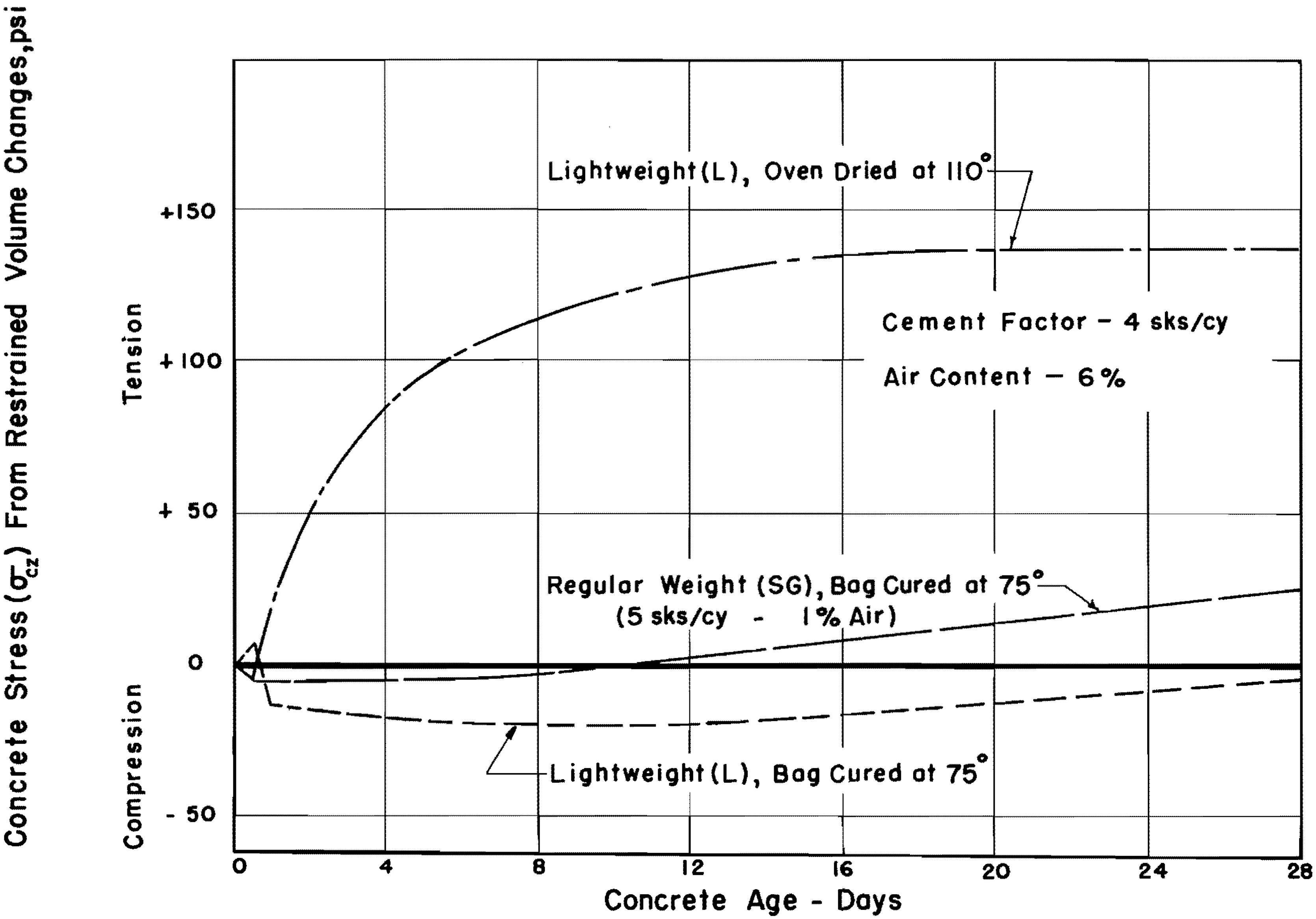


Figure 4-6. Effect of Curing Conditions on Structural Lightweight Concrete Stress - Age Relationships for Restrained Concrete Volume Changes.

cylinder tests. The effects of cement factor and curing conditions on the indirect tensile strength-age relationship are shown in Fig. 4-7. The five sack per cubic yard concretes attained higher strengths than the four sack per cubic yard concretes; the bag-cured strengths were higher than the oven-dried strengths at 28 days, but for this particular concrete the reverse was true at early ages.

The effects of curing conditions and air content on the indirect tensile strength-age relationship of lightweight concrete with a cement factor of 5 sacks per cubic yard is given in Fig. 4-8. The indirect tensile strengths do not seem to be materially affected by air contents in the range investigated for either the bag-cured or the air-cured concrete. However, for oven-dried curing, the concrete with 6 per cent air content loses strength at later ages, whereas the concrete with 2 per cent air content continues to gain strength slowly.

4.5 Compressive Strength

The compressive strength is considered by many to be the best indicator of quality and suitability for structural concrete. For this reason the compressive strength-age relationship for structural lightweight concrete was investigated in order to compare the direct tensile strength with the compressive strengths over a wide range of parameters. The specimens were standard 6 x 12-in. cylinders molded in accordance with ASTM Designation C-192[34] and tested in accordance with ASTM Designation C-39.[35] Figure 4-9 illustrates the compressive strength-age relationship for two lightweight concrete mix designs (5 sack per cubic yard and 4 sack per cubic yard) of the

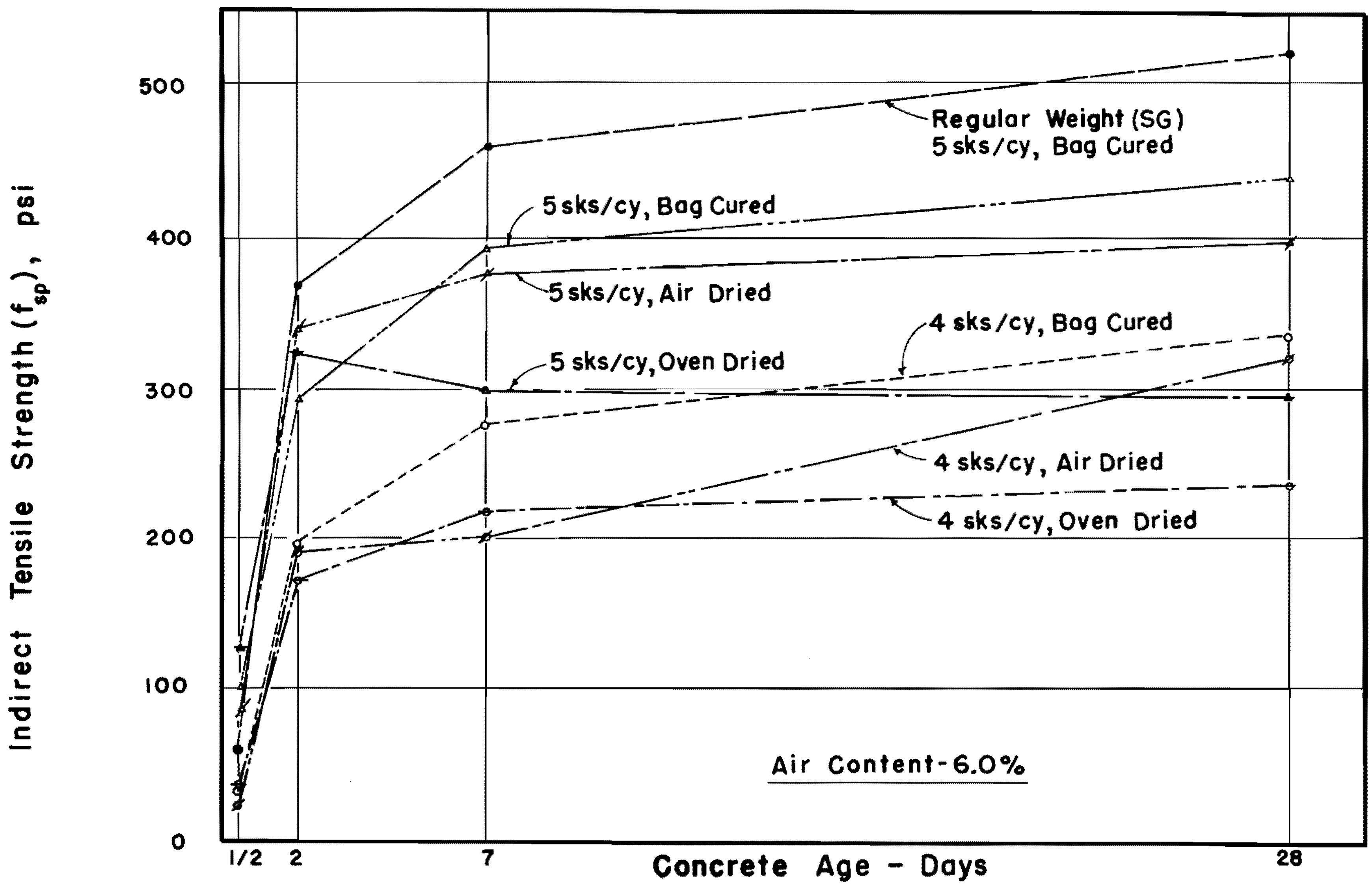


Figure 4-7. Effect of Cement Factor and Curing Conditions on the Indirect Tensile (Split Cylinder) Strength—Age Relationship of Structural Lightweight Concrete.

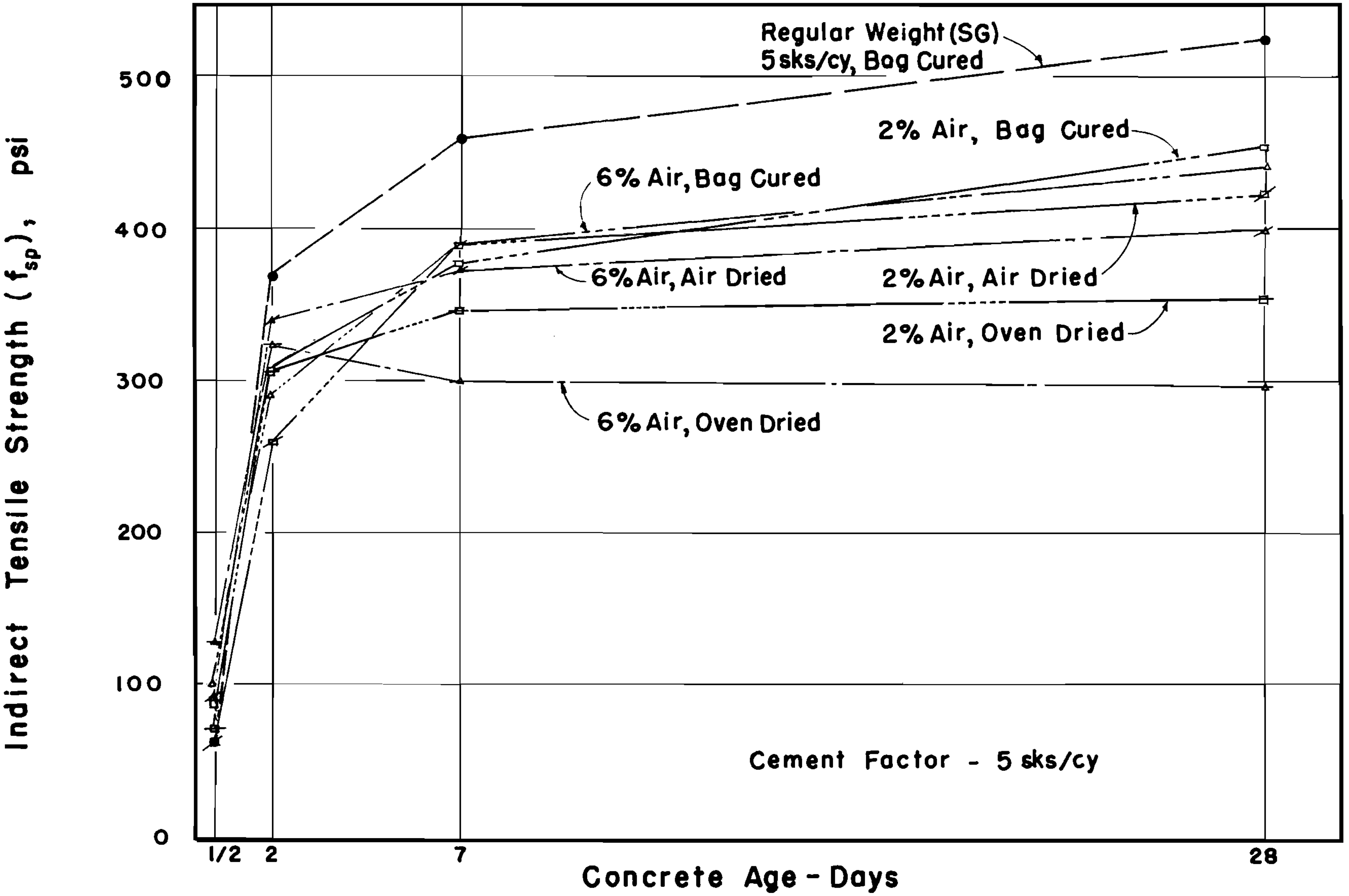


Figure 4-8. Effect of Curing Conditions and Air Content on the Indirect Tensile (Split Cylinder) Strength-Age Relationship of Structural Lightweight Concrete.

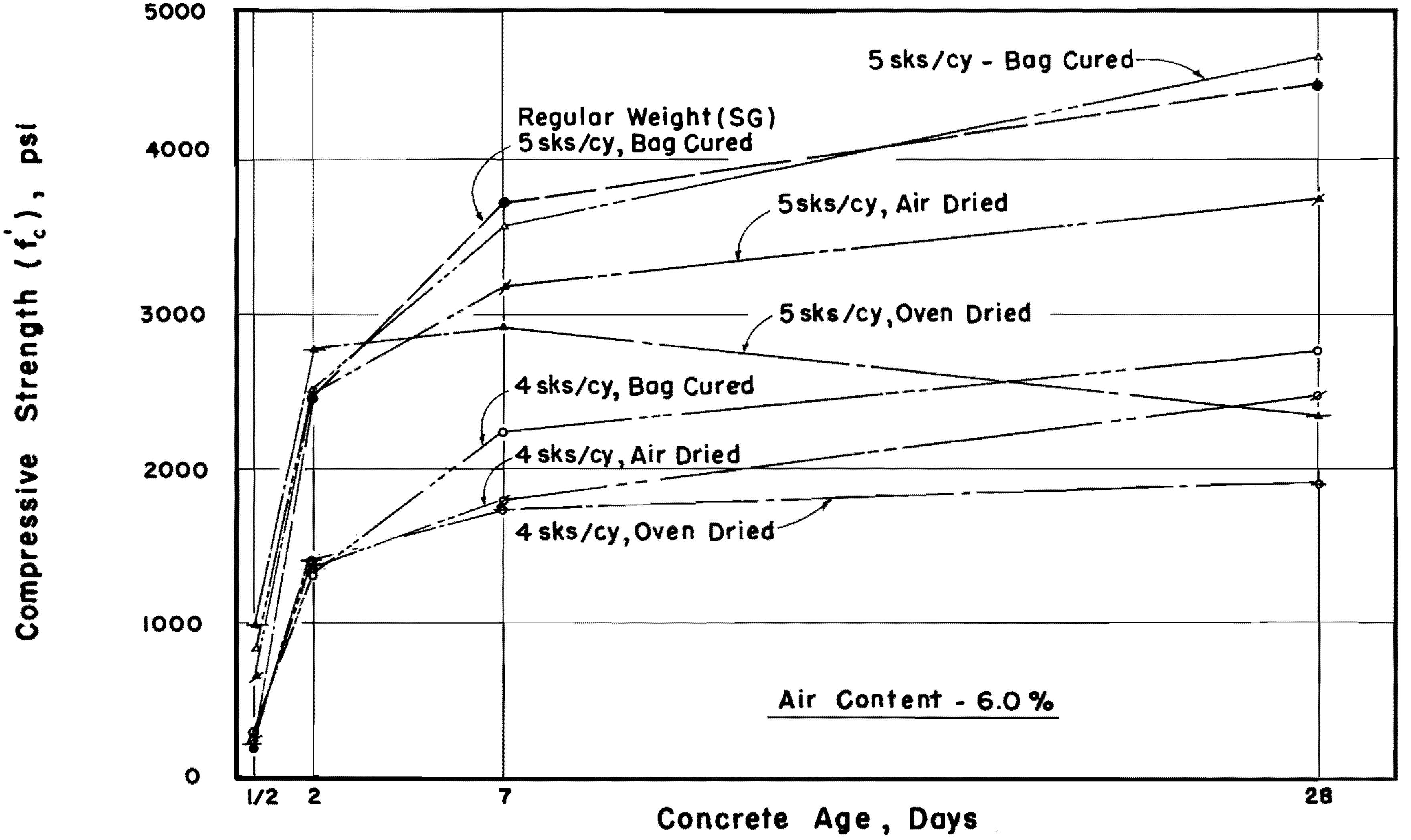


Figure 4-9. Effect of Cement Factor and Curing Conditions on the Compressive Strength-Age Relationship of Structural Lightweight Concrete.

same air content subjected to three curing conditions. Each value on the curve represents the average of three cylinder tests. In all cases, for a given cement factor, the bag-cured concrete attained the highest strength at 28 days and the oven-dried concrete attained the lowest strength. Also, as expected, the higher the cement factor, the greater the compressive strength. However, notice that at the early ages of 1/2 day to 2 days, the oven-dried concrete attained higher strengths than the bag-cured concrete. This is significant to the designer in terms of predicting the ability of the concrete to carry stresses at early ages. Also note the regular weight concrete strength, designated as SG. These strengths are included for comparison purposes, and as can be seen, closely parallel the lightweight concrete strengths, indicating the compressive strength-age relationships for these two particular concretes are very similar. It was also interesting to note that the loss in strength during oven drying experienced by the lightweight concrete with a cement factor of 5 sacks per cubic yard between 7 and 28 days was not experienced by the concrete with a cement factor of 4 sacks per cubic yard; indicating that perhaps there might be some interrelationship between cement factor and resultant loss of strength for the extreme condition of high temperature and low humidity. Of course the data are too limited to draw any conclusions.

Referring now to Fig. 4-10, the effects of 2 air contents on the compressive strength-age relationship of the 5 sack per cubic yard concrete, cured 3 different ways, can be seen. Although some scattering of results occurred, it is evident that air contents of the ranges

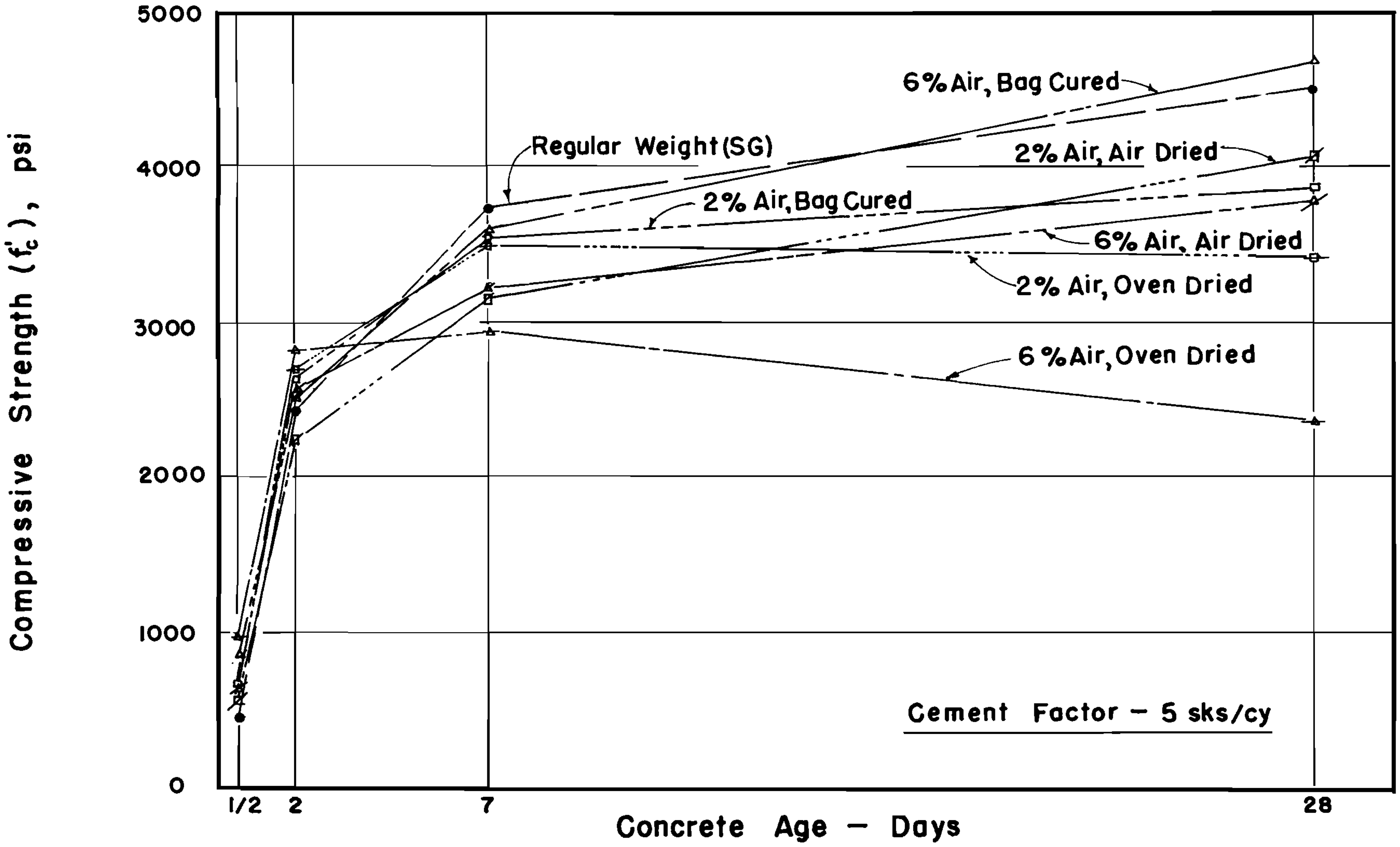


Figure 4-10. Effect of Curing Conditions and Air Content on the Compressive Strength — Age Relationship of Structural Lightweight Concrete.

investigated play a very minor role in influencing compressive strength at early ages up to seven days. Beyond seven days, however, there was a significant difference for oven-dried concrete between the 2 per cent air and the 6 per cent air entrained mixes. The 6 per cent air content concrete developed less strength than the 2 per cent air content concrete. Here again the data are too limited too form any firm conclusions regarding the observed phenomena, but it should be kept in mind. In Fig. 4-10 the regular weight concrete (SG) strengths are included for comparison purposes. The effects of curing upon strengths are very noticeable in this figure, as well as in the preceding figure, emphasizing the importance of curing upon concrete strength.

4.6 Flexural Strength

The flexural strength, or modulus of rupture, as it is called, is based on the well-known formula:[36]

$$S = \frac{MC}{I} \qquad \text{- - - - - (4-1)}$$

where

S = Stress in the fiber farthest from the neutral axis, psi

M = Bending moment in the section, in. lb

I = Moment of inertia of the cross section, in.^{4}

C = Distance from the neutral axis to the farthest fiber, in.

This test is used extensively in the field. For this investigation, concrete flexure specimens were molded in accordance with ASTM Designation C-192[37] and tested under third-point loading in accordance with ASTM Designation C-78.[38] It should be pointed out that the Texas Highway Department uses the center-point loading flexural test and thus the results reported herein are not correlatible to Texas Highway Department results without conversion between the two types of flexure tests. The same parameters as discussed in the compressive-strength section previously were studied for flexural strength. Figure 4-11 portrays the effects of cement factor and curing conditions upon the flexural strength-age relationship of structural lightweight concrete. Each point on the figure represents the average of two breaks on the same beam. Perhaps the most noticeable feature of this figure is the wide variation in results, making it difficult to interpret the graph. The 5 sack per cubic yard concrete appears to be stronger than the 4 sack per cubic yard concrete; but at 28 days, the highest strength is attained by the oven-dried concrete, which is contrary to any of the other strength results obtained in this study. What actually is occurring here has occurred many times, and is the result of the fact that the flexural strength of concrete depends to a large extent on the condition existent in the outermost surface fibers of the concrete beam, which, in turn, is greatly influenced by moisture conditions in the air, temperature, etc. Thus, as surface conditions between different test specimens may vary widely, significant data scatter occurs, making any attempt to predict properties from a flexural strength

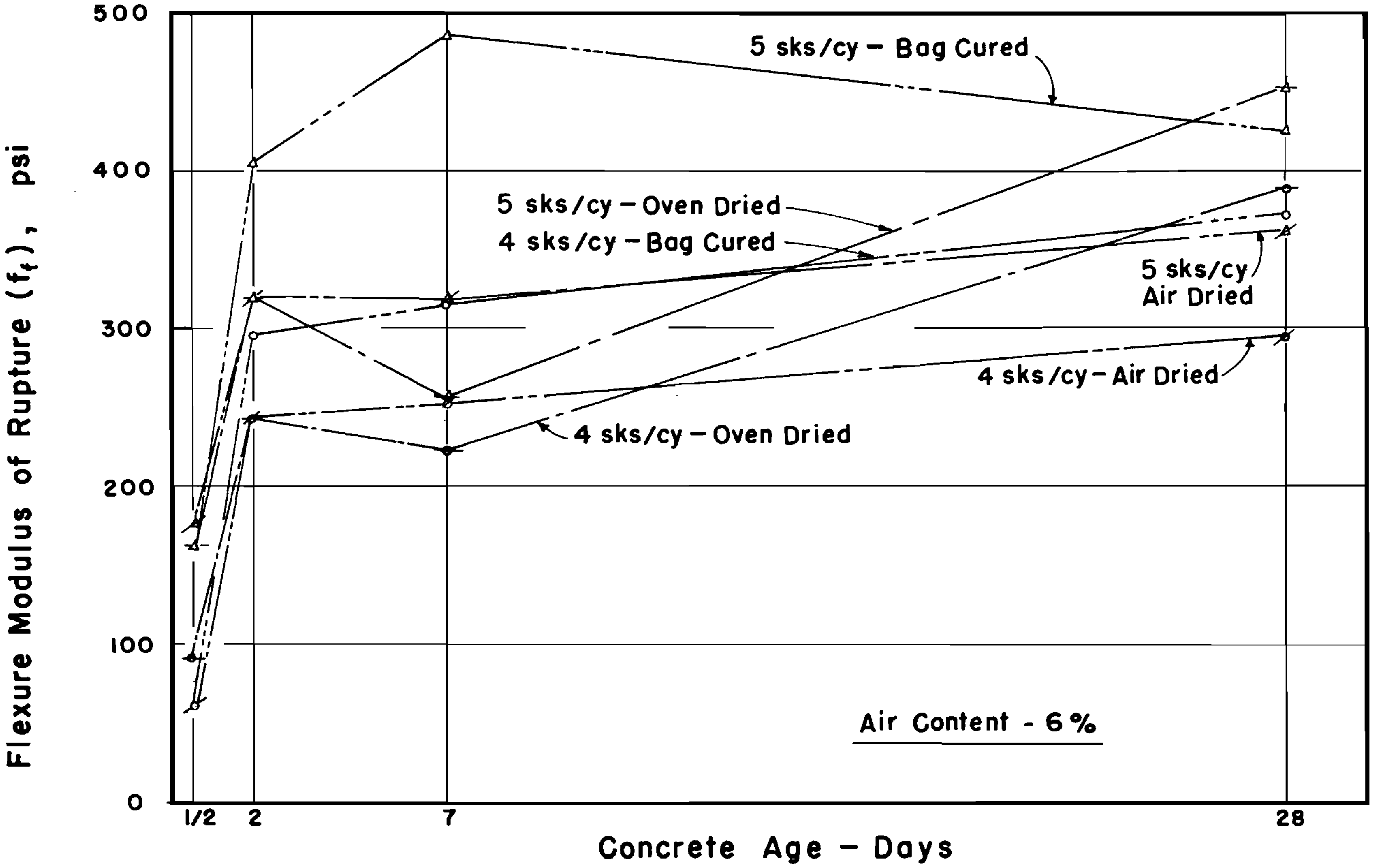


Figure 4-11. Effect of Cement Factor and Curing Conditions on the Flexural Strength – Age Relationship of Structural Lightweight Concrete.

determination very difficult and highly questionable.

The effects of air content upon the flexural strength-age relationship for concrete with a cement factor of 5 sacks/cu yd for the three curing conditions is given in Fig. 4-12. Here again it is difficult to evaluate and discuss the data due to the wide variation in test results.

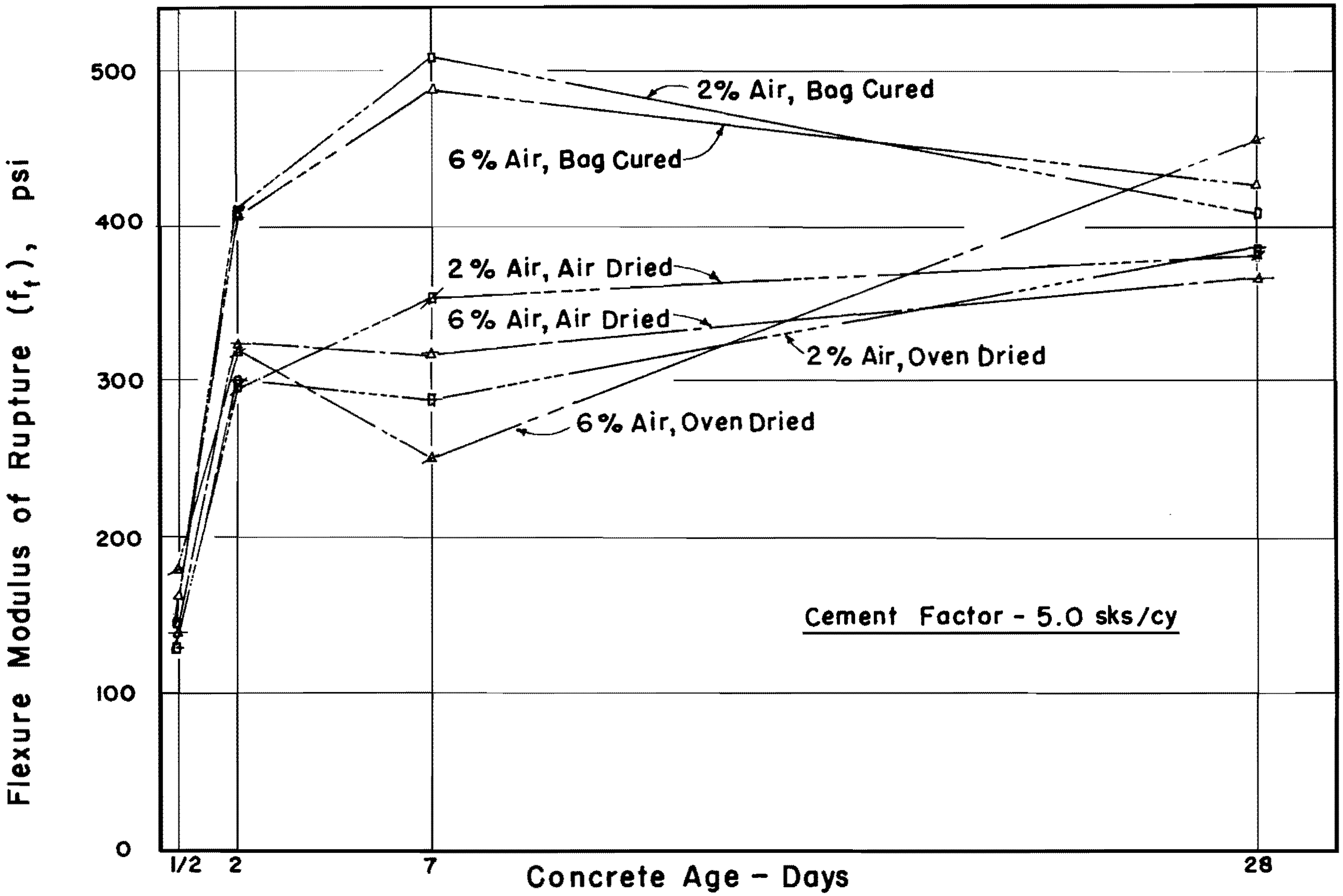


Figure 4-12. Effect of Curing Conditions and Air Content on the Flexural Strength–Age Relationship of Structural Lightweight Concrete.

5. ANALYSIS OF RESULTS

5.1 Introduction

The results discussed in Chapter 4 are analyzed in two general ways. First, comparisons are made between certain properties to include:

1. Direct tensile and compressive strength
2. Direct tensile and indirect tensile (split cylinder) strength
3. Restrained concrete stress and direct tensile strength
4. Indirect tensile and compressive strength
5. Flexural and compressive strength
6. Indirect tensile and flexural strength

Second, strength values at various ages are made dimensionless by converting them to per cent of 28 or per cent of 7-day strength and then are plotted on a per cent strength versus age graph. Various parameters and variables are lumped together in one graph and analyzed statistically (see section 6.5 for complete description of the statistical approach used).

5.2 Relationships Between Tensile Strength and Various Other Strength Measurements

Relationship between direct tensile and compressive strength. The relationship between the direct tensile strength (f_t) and the compressive strength (f'_c) is given in Fig. 5-1. Each point represents a single test value. The various test conditions encountered can be seen on this figure and, as indicated by the straight line, an average correlation of

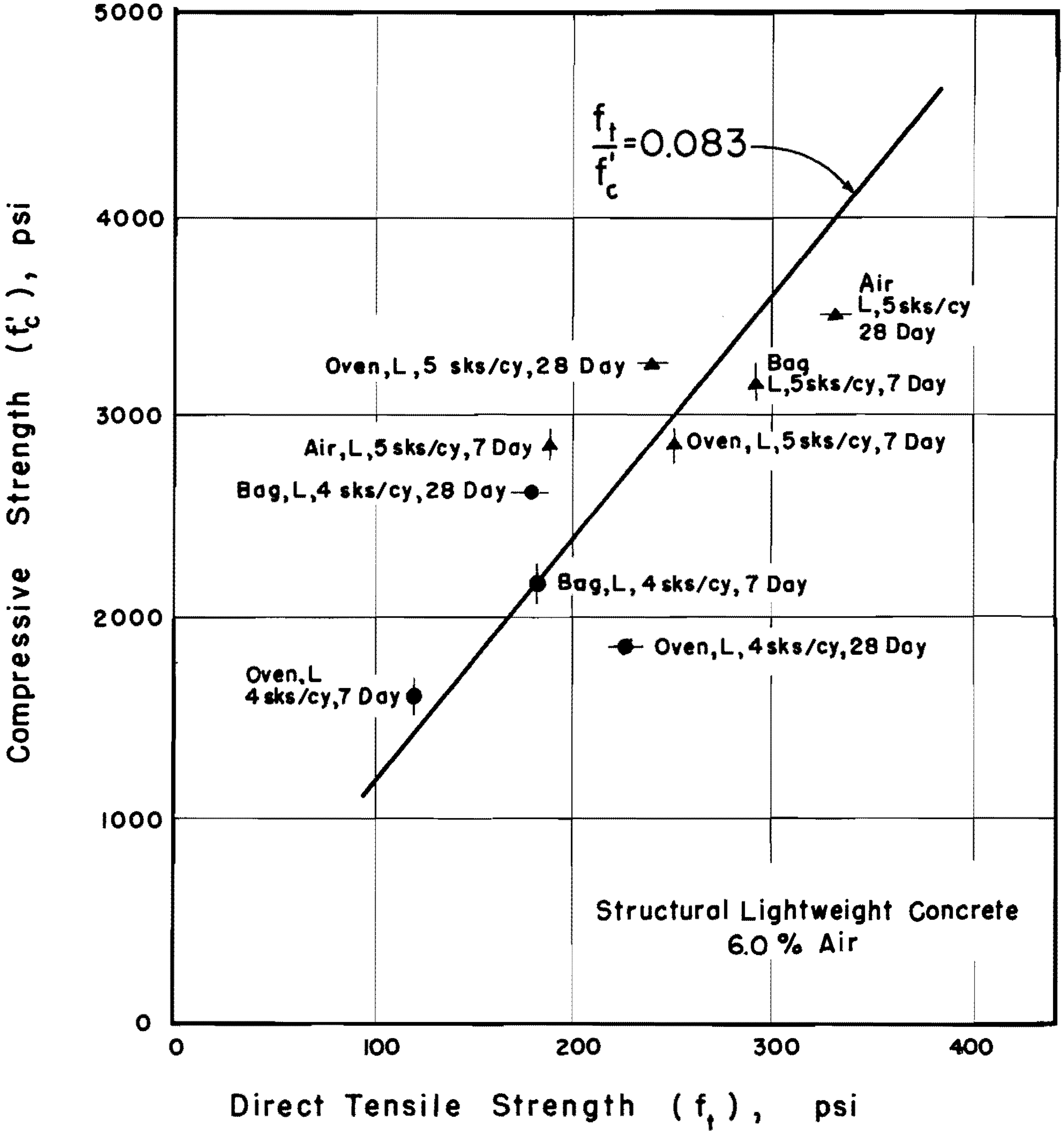


Figure 5-1. Relationship Between Direct Tensile Strength (f_t), and Compressive Strength (f'_c).

0.083 exists for the ratio f_t/f'_c. The data are too limited to run a regression analysis, but it appears as if the data scatter is not too severe. Of course, the direct tensile strength is an elusive property at best and is subject to a great deal of influence from various factors. The degree of correlation exhibited in this figure for the wide range of strength and conditions investigated indicates that this method of direct tensile strength determination is sound and offers a way whereby this elusive property can be explored in detail.

Relationship between direct and indirect tensile strength. The relationship between the direct tensile strength (f_t) and indirect tensile (split cylinder) strength (f_{sp}) is graphically portrayed in Fig. 5-2. Here again the data are too limited to apply a regression analysis, but a definite straight-line correlation is seen. The ratio of f_t/f_{sp} of 0.685 suggests that the indirect tensile strength yields consistently somewhat higher values for the direct tensile strength than actually exist.

While at first glance this ratio appears questionable in that both tests measure tensile strength and thus should yield approximately the same values, it should be kept in mind that the indirect tensile test subjects the concrete to a biaxial stress condition (section 7.1). Inasmuch as the direct tensile test subjects the concrete to a uniaxial tension stress, it then becomes apparent that the two tests should yield different results as the total stress conditions are quite different between the two tests. However, since the data seem to indicate a straight-line correlation between these two tests, the indirect tensile test appears to give a reliable measurement of the tensile capacity of the type of concrete investigated in this study.

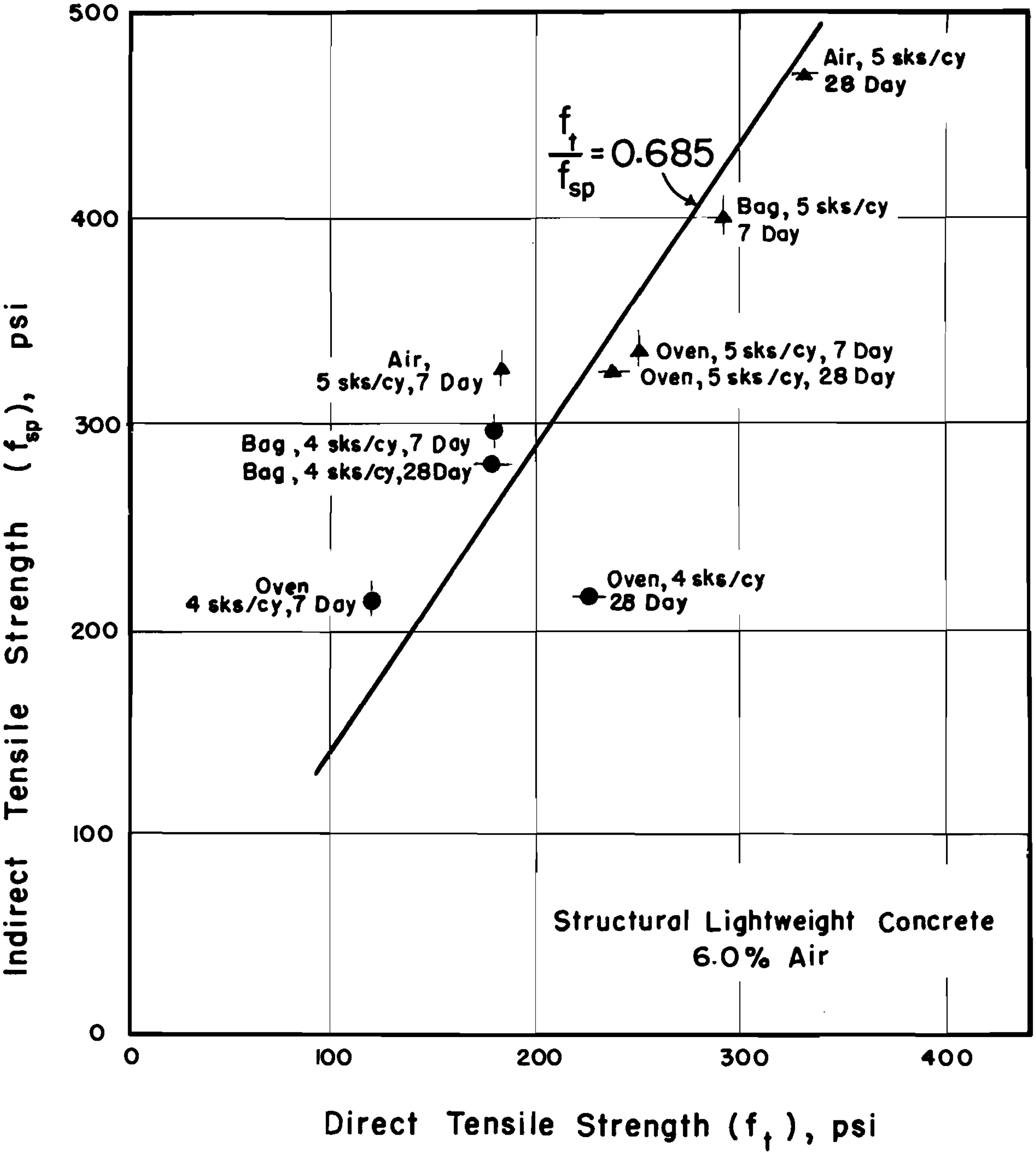


Figure 5-2. Relationship Between Direct Tensile Strength (f_t), and Indirect Tensile Strength (f_{sp}).

Relationship between restrained concrete stress and direct tensile strength. An important aspect of the direct tensile test method used in this study is the ability to determine the effects of restraining concrete volume changes prior to determination of direct tensile strength. This is an extremely important factor in the evaluation of concrete tensile strength capacity in reinforced concrete structures as the steel reinforcement effectively restrains the concrete from changing in volume, thereby creating restrained stresses in the concrete that may add or detract from the concrete tensile capacity. The development of stresses in restrained concrete has been presented in the discussion in section 4.2 previously and in Fig. 4-1 through Fig. 4-4. In order to present the data in perhaps more easily analyzed form, the stresses in the restrained concrete are expressed as a per cent of the direct tensile concrete strength and presented in bar graph form in Fig. 5-3. The important factors to note are:

1. The bag-cured specimens of structural lightweight concrete attempted to expand thus increasing the direct tensile capacity of the concrete slightly. This was not observed in the regular weight concrete specimens.
2. For the extreme curing condition of oven dried at 110F, the restrained concrete volume change stresses may be as high as 70 per cent or more of the direct tensile strength of the concrete. In fact, on two specimens cured in the oven, the specimen failed in tension prior to applying external loads as a result of these restrained volume change stresses.

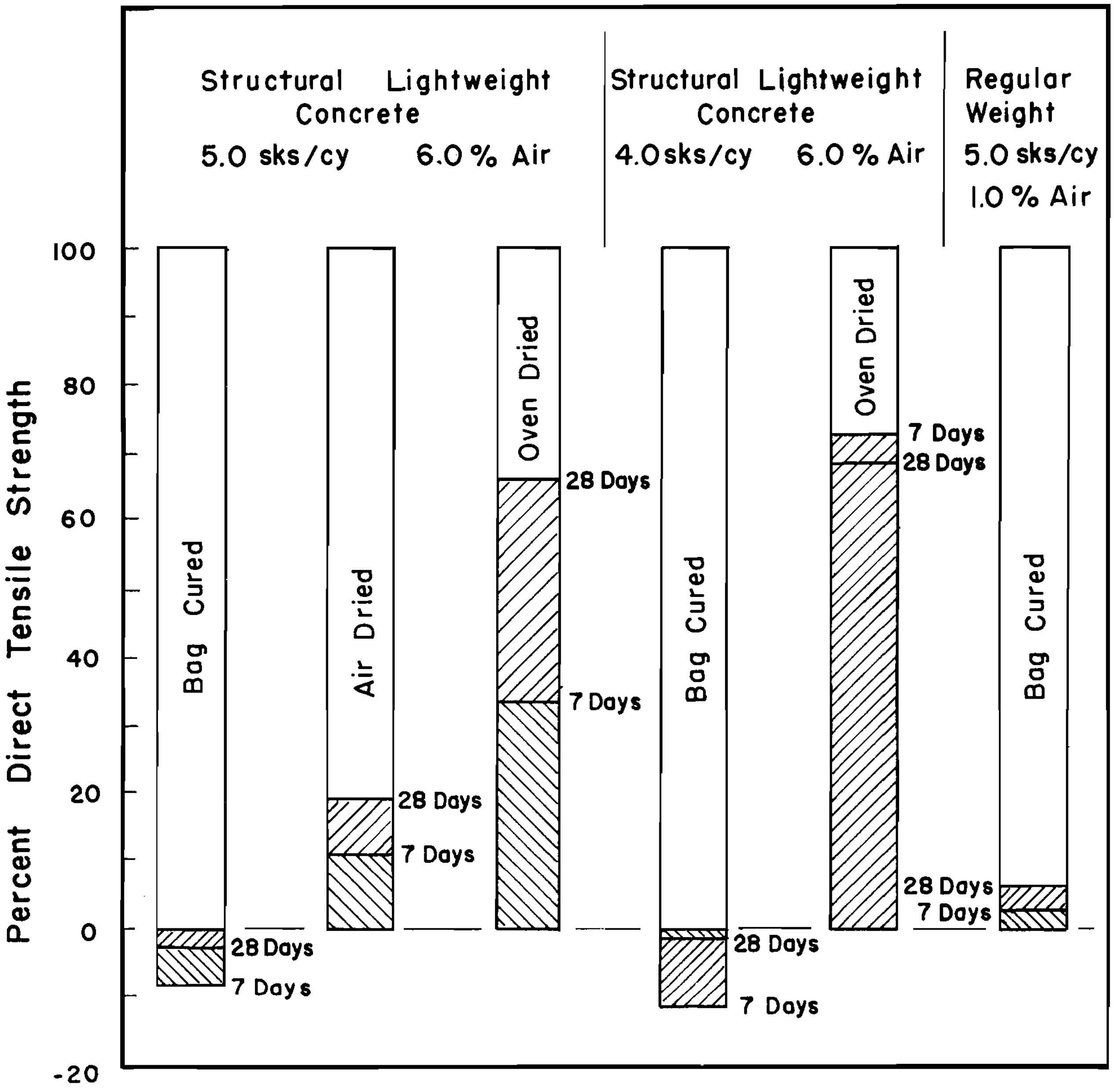


Figure 5-3. Restrained Concrete Volume Change Stresses Expressed As Percent Direct Tensile Strength of the Concrete.

Looking at the 4 sack per cubic yard oven-dried structural lightweight concrete values, it appears as if the concrete volume change stresses reduced from 7 to 28 days. It must be remembered, however, that at each age the concrete was tested to destruction and thus the figure is composed of results taken from different specimens. The 7-day direct tensile strength was lower than expected which accounts for the high per cent restrained stress present, and the seemingly resulting paradox is actually the result of variations in data making the results difficult to accurately analyze. Thus, while the exact per cent reduction in effective tensile strengths is questionable without further data, certainly the order of magnitude exhibited represents general findings which are of importance to the designer.

Relationship between indirect tensile strength and compressive strength. The relationship between the indirect tensile strength f_{sp} and compressive strength f'_c is statistically analyzed in Table 5-1 for the various conditions encountered in this study. The average ratio of f_{sp}/f'_c varies from 0.115 to 0.124 with an average over-all ratio of 0.121; or the indirect tensile strength is 12.1 per cent of the compressive strength. This value is for 36 tests of three cylinders each. The standard deviation is very low, resulting in a low coefficient of variation. The over-all per cent coefficient of variation is 10.6 which indicates extremely good correlation between these two properties. If it is assumed that the compressive strength is one of the most significant and important strength properties of concrete, then the indirect tensile

TABLE 5-1

Statistical Analysis of the Ratio f_{sp}/f'_c for Structural Lightweight Concrete

Cement Factor sks/cu yd	Air Content %	Curing	Arith. Mean $\overline{x}$	Standard Dev.	Coef. of Var. %	Max. Pos. Dev. %	Max. Neg. Dev. %	No. of Test Values n
5	2	All	.115	.009	8.3	19.4	-13.7	12
5	6	All	.118	.014	11.6	16.3	-20.2	12
4	6	All	.130	.010	7.8	12.2	-12.4	12
All	All	Bag	.120	.014	11.3	21.5	-21.8	12
All	All	Air	.124	.013	10.4	16.4	-14.3	12
All	All	Oven	.119	.013	10.6	18.8	-16.6	12
All	All	All	.121	.013	10.6	20.8	-22.2	36

strength property must also be of extreme value as it correlates so very well with compressive strength. Thus the indirect tensile strength can be used to determine the compressive strength with a high degree of reliability.

Relationship between flexural strength and compressive strength. The relationship between flexural strength and compressive strength is statistically analyzed in Table 5-2 for the various conditions encountered in this study. The average ratio of f_t/f'_c varies from 0.146 to 0.207 with over-all average of 0.166. While at first glance this does not appear to be too wide a spread, a look at the large standard deviations and coefficients of variations leads to the obvious conclusion that almost no correlation between these properties exists for the type of structural lightweight concrete used in this study. The over-all per cent coefficient of variation is 43.4, or one standard deviation varies over the range of plus or minus 43.4 per cent of the average value of the ratio. Therefore, since almost no correlation exists between flexural and compressive strengths, the flexural strength property is seen to be a very nebulous and unpredictable property, and one which cannot evaluate this type of concrete strength with any degree of reliability. Note that this poor correlation exists even for the bag-cured specimens, which are widely used for concrete control purposes. As discussed in section 4.6, the flexural strength is greatly influenced by surface moisture conditions, crazing, etc., and thus it is not surprising that extensive data scatter occurs.

TABLE 5-2

Statistical Analysis of the Ratio f_f/f'_c for Structural Lightweight Concrete

Cement Factor sks/cu yd	Air Content %	Curing	Arith. Mean $\bar{x}$	Standard Dev.	Coef. of Dev. %	Max. Pos. Dev. %	Max. Neg. Dev. %	No. of Test Values n
5	2	All	.146	.055	37.6	60.8	-44.3	12
5	6	All	.146	.055	37.4	78.2	-41.1	12
4	6	All	.207	.089	42.9	79.4	-42.4	12
All	All	Bags	.172	.066	38.3	92.1	-47.0	12
All	All	Air	.161	.075	46.4	105.0	-40.5	12
All	All	Oven	.165	.081	49.0	125.0	-50.9	12
All	All	All	.166	.072	43.4	123.0	-51.2	36

Relationship between indirect tensile strength and flexural strength. The relationship between indirect tensile strength and flexural strength is statistically analyzed in Table 5-3 for the various conditions encountered in this study. Here again, almost no correlation between these two properties exists as the over-all per cent coefficient of variation is seen to be in the neighborhood of 30.0. The same comments as made in the previous paragraph can be repeated here.

5.3 Relationships Between Per Cent Specified Strength and Age

A close examination of the curves in section 4.4 for indirect tensile (split cylinder) strengths and section 4.5 for compressive strengths reveals a close similarity between the general shape of the curves, even though the actual values differ markedly. This, then, suggests that perhaps by considering the strength at a given age in terms of its 28-day strength some over-all generalized curves might result. Figures 5-4, 5-5, and 5-6, represent the per cent 28-day compressive and indirect tensile strengths versus age relationship for bag, air, and oven-dried curing conditions, respectively. Here, for a given curing condition, all other parameters were lumped together; *i.e.*, both compressive and indirect tensile strengths, various cement factors, and various air contents (a total of six values for each age). The band on each curve represents the total range of data scatter. For the bag-cured and oven-dried conditions, the range of data scatter is very narrow. This indicates that over a wide variation of test parameters, for a given curing condition, the per cent 28-day strength factor can be considered to be a constant at any

TABLE 5-3

Statistical Analysis of the Ratio f_{sp}/f_f for Structural Lightweight Concrete

Cement Factor sks/cu yd	Air Content %	Curing	Arith. Mean $\bar{x}$	Standard Dev.	Coef. of Dev. %	Max. Pos. Dev. %	Max. Neg. Dev. %	No. of Test Values n
5	2	All	.874	.253	28.9	38.4	-43.7	12
5	6	All	.884	.237	26.8	34.6	-40.6	12
4	6	All	.715	.235	32.8	52.5	-48.1	12
All	All	Bag	.766	.201	26.2	46.2	-50.0	12
All	All	Air	.880	.273	31.0	34.1	-51.8	12
All	All	Oven	.827	.270	32.6	46.3	-55.1	12
All	All	All	.824	.247	30.0	46.8	-55.0	36

Note: For explanation of symbols used, see section 7.3

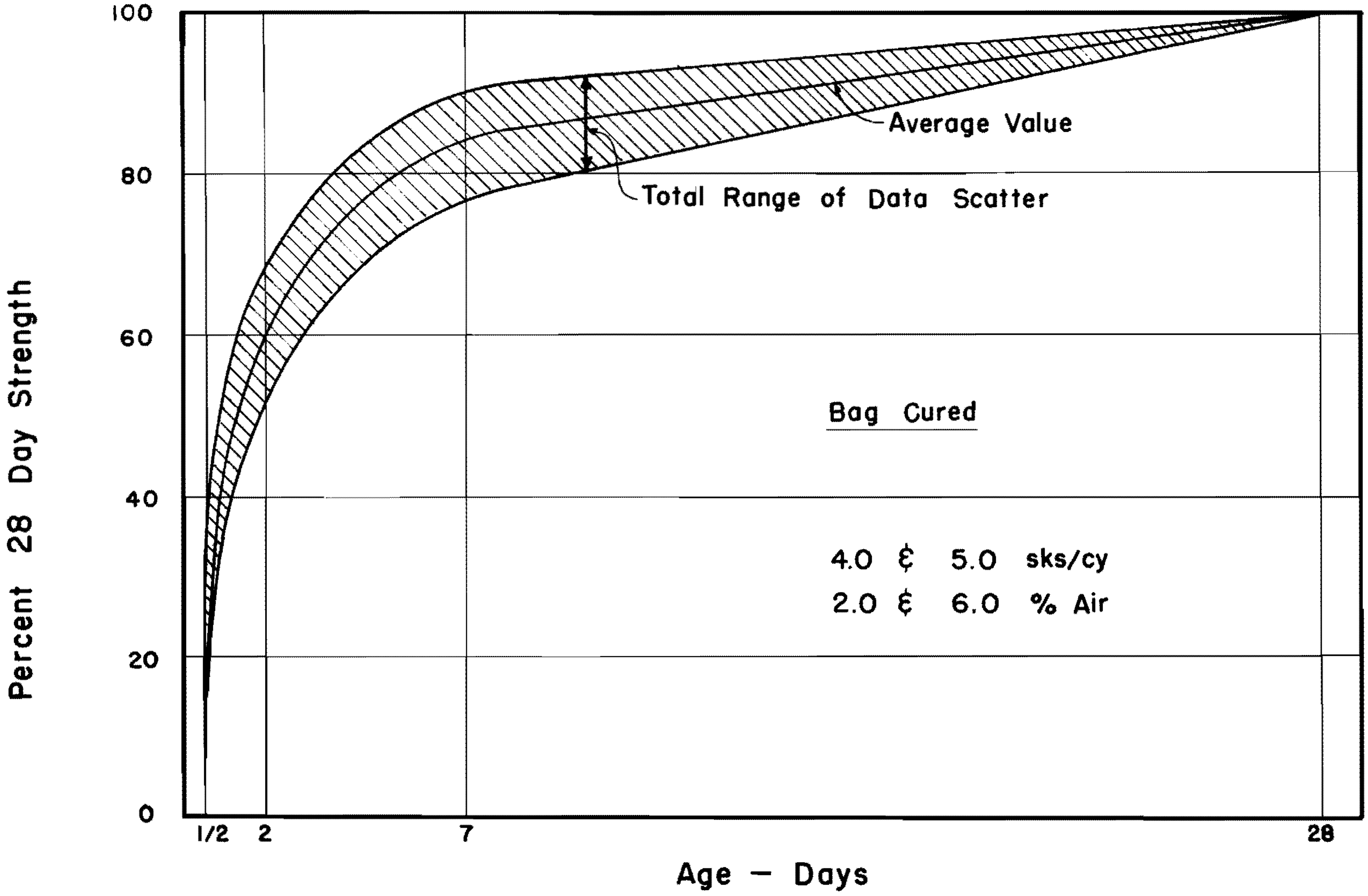


Figure 5-4. Percent 28 Day Strength vs Age for Compressive and Indirect Tension Strengths, Bag Cured Conditions, Showing Data Scatter, for Structural Lightweight Concrete.

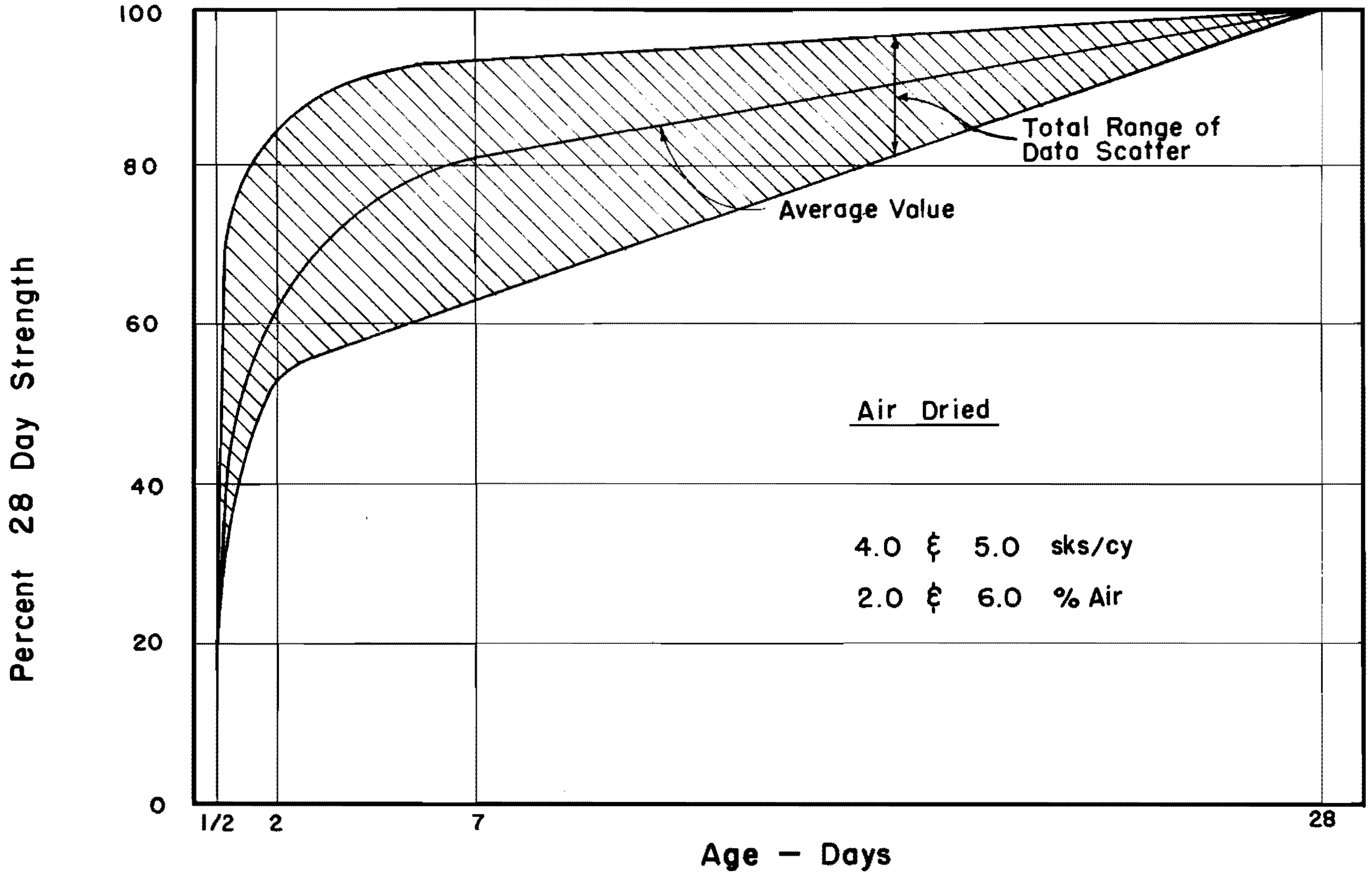


Figure 5-5. Percent 28 Day Strength vs Age for Compressive and Indirect Tension Strengths, Air Dried Conditions, Showing Data Scatter, for Structural Lightweight Concrete.

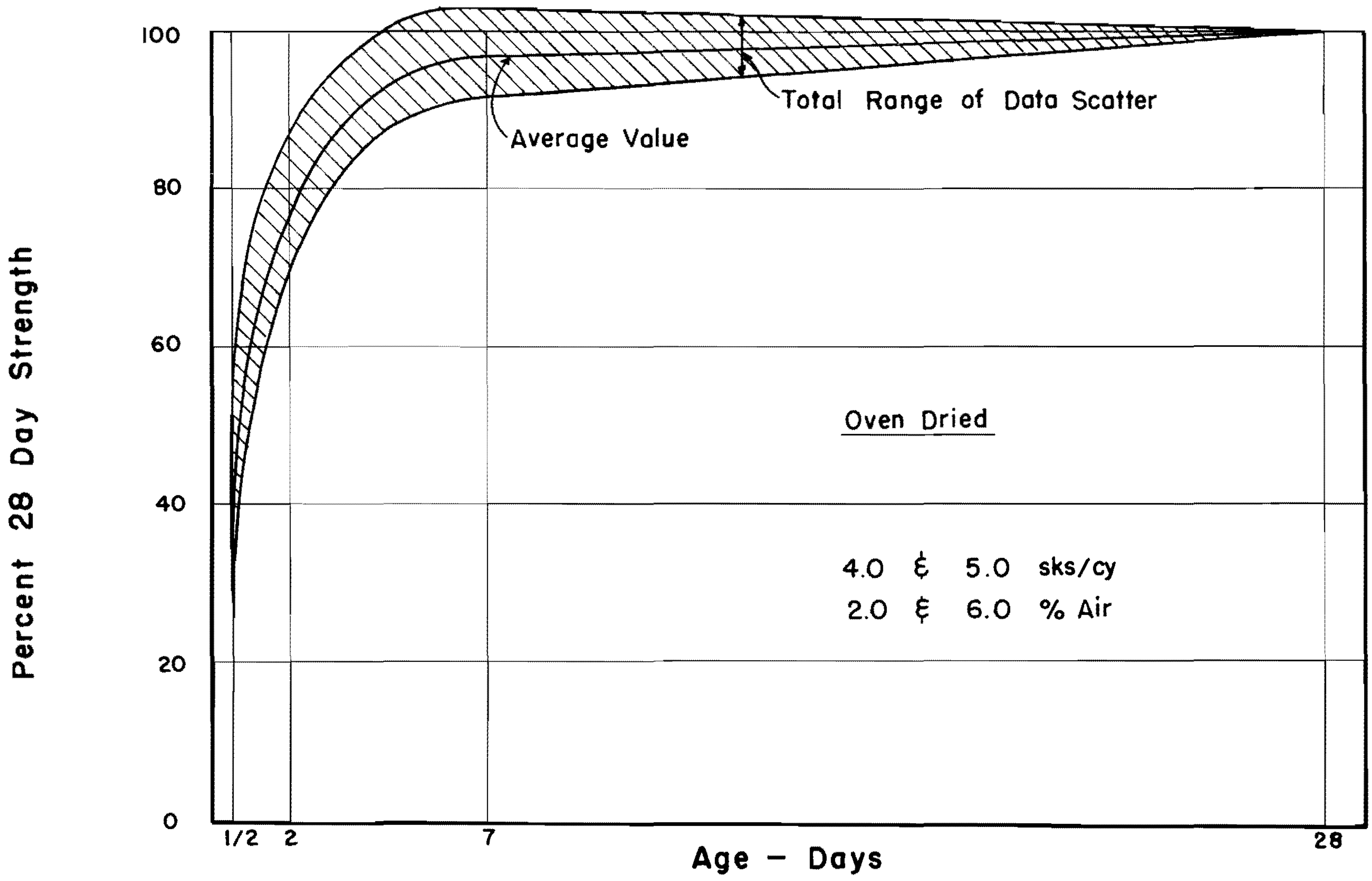


Figure 5-6. Percent 28 Day Strength vs Age for Compressive and Indirect Tension Strengths, Oven Dried Conditions, Showing Data Scatter, for Structural Lightweight Concrete.

given age. Thus for a given mix design and curing condition, a designer can, by knowing this general relationship, determine the compressive and indirect tensile strength of concrete at any age knowing the strength at only one age. The data scatter exhibited by the air-cured concrete is due to the variation in humidity in the curing process, changing the strength, and thereby increasing random error. If the relative humidity were maintained relatively constant, as in the oven- and bag-cured conditions, the data scatter would be much less.

By combining all three average curves, Fig. 5-7 results. Note that the average curves for bag-cured and air-dried per cent 28-day strengths lie very close together. This suggests that for normal curing conditions, which should be somewhere between these two conditions, the per cent 28-day strength versus age relationship should approximate these two curves. Again, once establishing this fact, the designer can reliably predict strength properties for any given set of conditions at any time up to 28 days, knowing the strength at only one age. The oven-dried concrete, representing an extreme curing condition, does not exhibit a similar per cent strength-age relationship. This is due to the fact that under such an extreme curing condition, although the ultimate strength is very low, the strength gained is very rapid and almost 100 per cent of its average 28-day strength is achieved in 7 days.

One other phenomenon should be analyzed here. An examination of Figs. 4-8 and 4-9 reveals that for oven-dried concrete, that concrete made with 5 sacks per cubic yard, actually lost strength from 7 to 28 days, while that concrete made with 4 sacks per cubic yard continued to

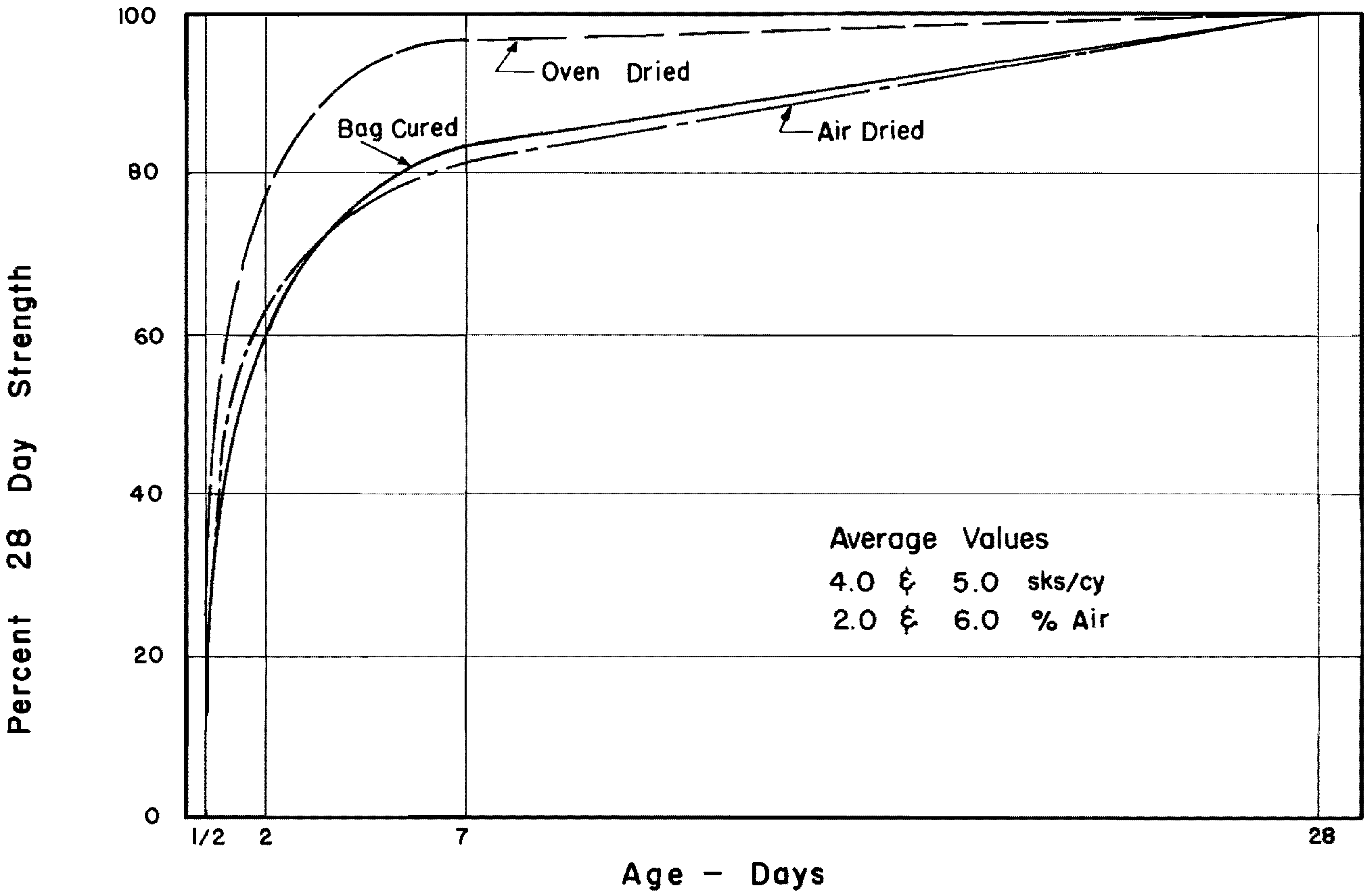


Figure 5-7. Percent 28 Day Strengths vs Age for Compressive and Indirect Tension Strengths, Bag, Air, and Oven Cured Conditions, Showing Average Data, for Structural Lightweight Concrete.

gain strength slowly from 7 to 28 days. This suggests that, since the higher the cement factor the faster the hydration, the increased per cent cement causes the concrete to complete its limited hydration at an early age and, perhaps, due to the increased shrinkage, looses strength slightly with further aging. It is not known whether this observed phenomenon is unique with this particular aggregate type and cement type, or generally occurs for all types of concretes. It is important, however, and should not be overlooked. Further investigation of this phenomenon is needed in order to properly evaluate its effects.

Inasmuch as many organizations such as the Texas Highway Department employ 7-day control tests during construction, the question arises as to whether a per cent 7-day strength-age relationship would yield significant results. In order to answer this question, the per cent 7-day compressive and indirect tensile strength versus age relationships are shown in Figs. 5-8, 5-9, and 5-10 for bag, air, and oven-dried curing conditions, respectively. Again, the band on each curve represents the total range of data scatter. An examination of these figures reveals that the same comments concerning the per cent 28-day curves can be made for the per cent 7-day curves. The average curves for per cent 7-day strength-age relationships for all three curing conditions are shown in Fig. 5-11, which is similar in every respect to Fig. 5-7. The choice between analyzing the strength data in terms of per cent 28-day strengths or in terms of per cent 7-day strengths is purely arbitrary and makes little or no difference in the final results.

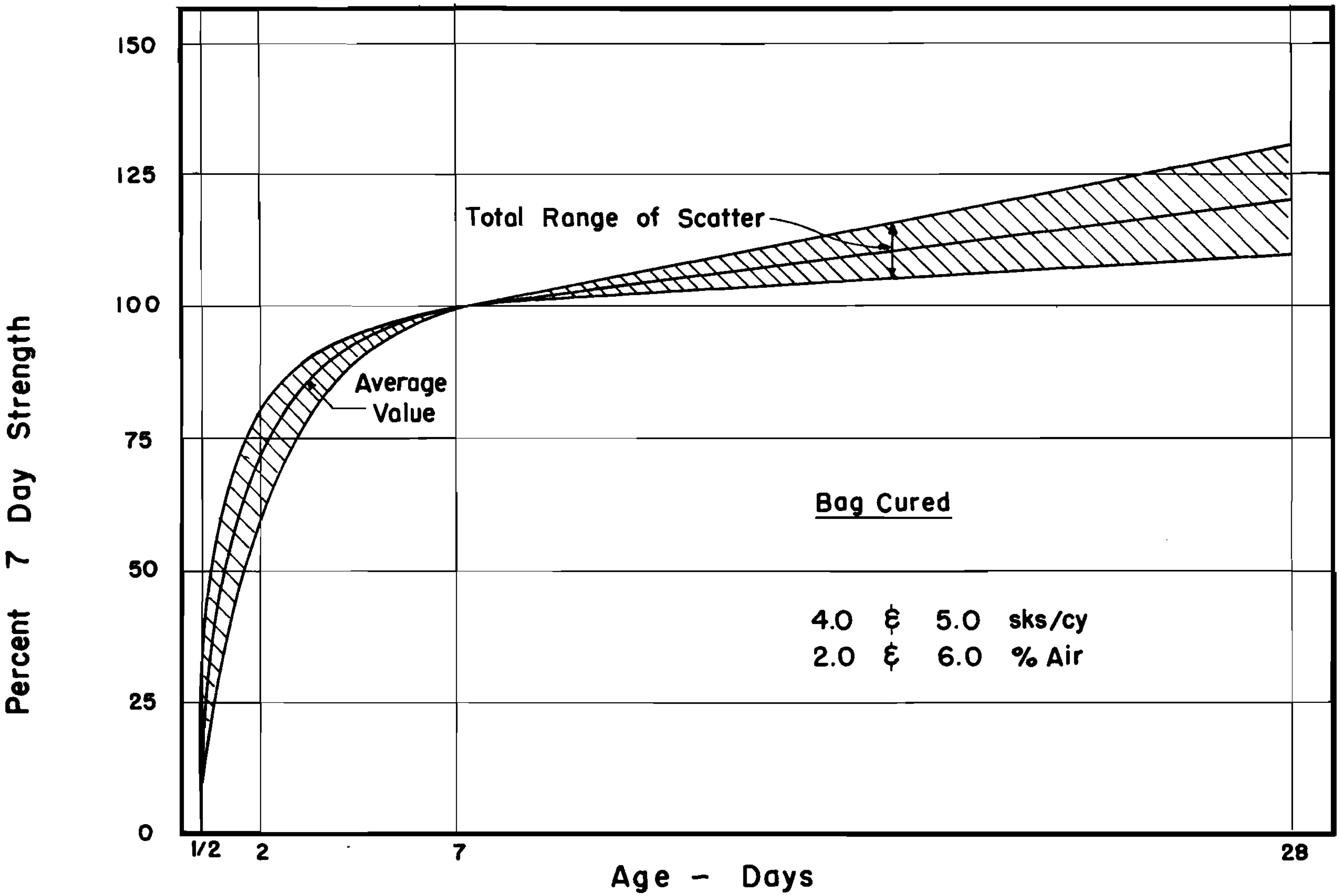


Figure 5-8. Percent 7 Day Strengths vs Age for Compressive and Indirect Tension Strengths, Bag Cured Conditions, Showing Data Scatter, for Structural Lightweight Concrete.

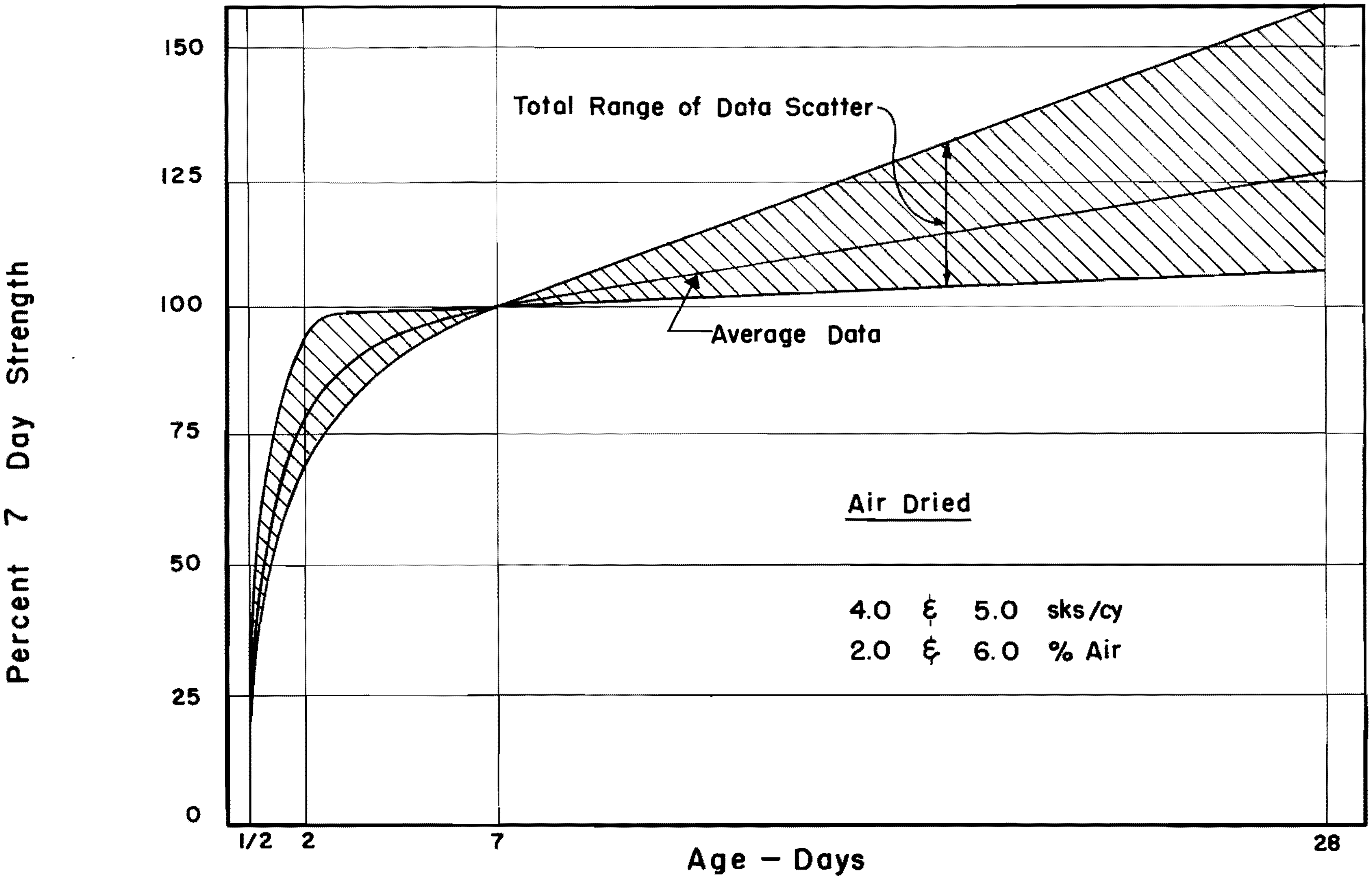


Figure 5-9. Percent 7 Day Strengths vs Age for Compressive and Indirect Tension Strengths, Air Cured Conditions, Showing Data Scatter, for Structural Lightweight Concrete.

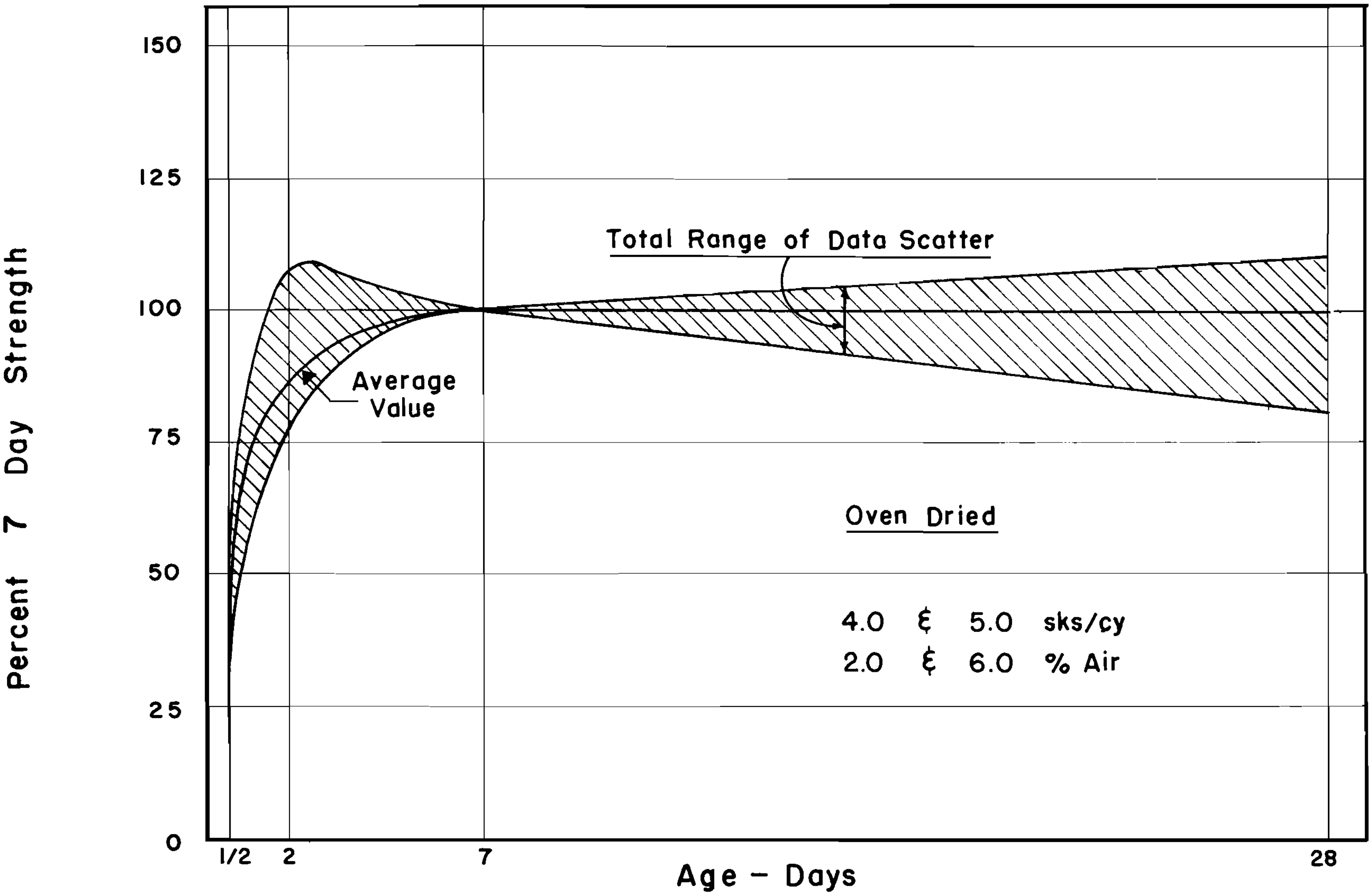


Figure 5-10. Percent 7 Day Strengths vs Age for Compressive and Indirect Tension Strengths, Oven Cured Conditions, Showing Data Scatter, for Structural Lightweight Concrete.

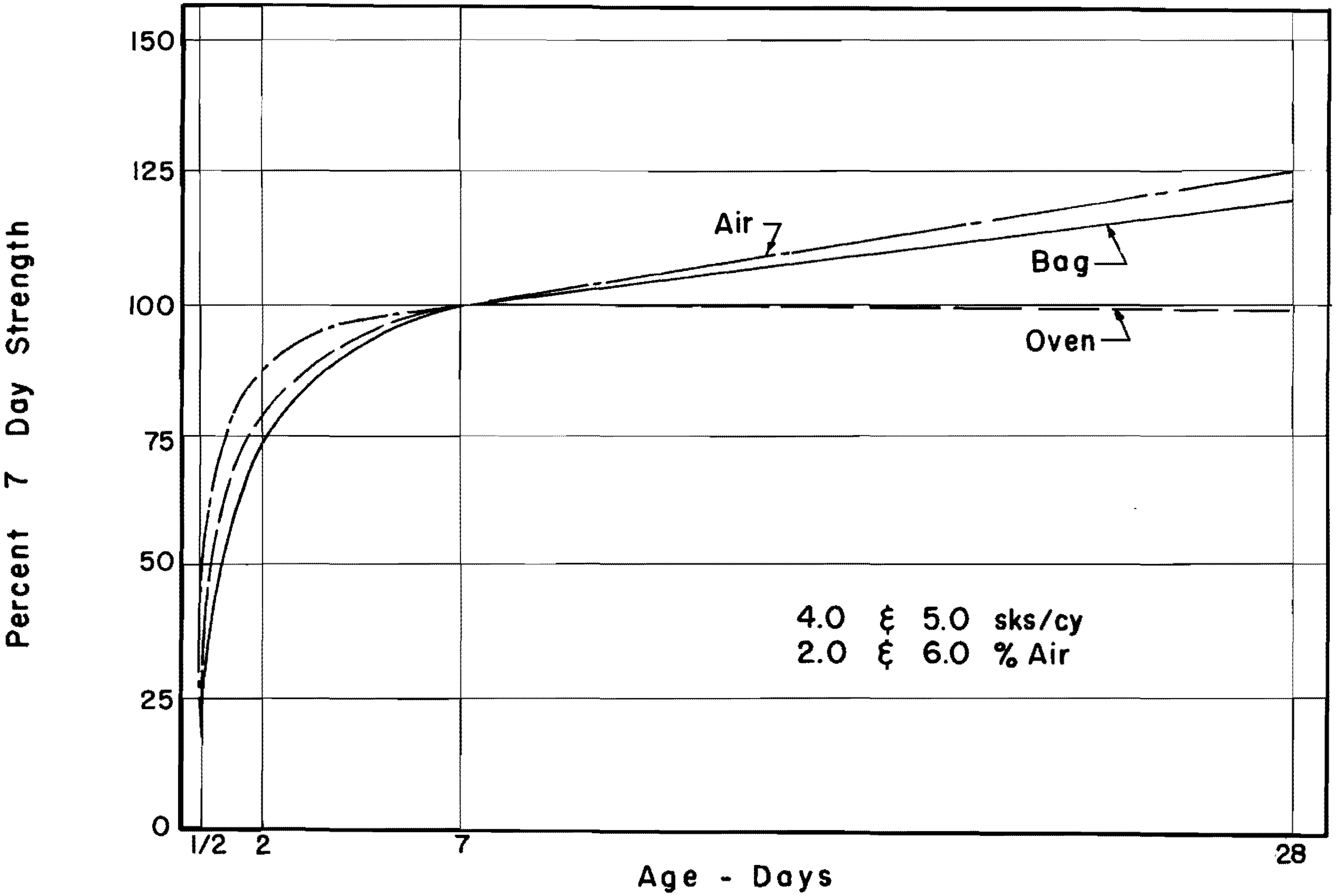


Figure 5-11. Percent 7 Day Strengths vs. Age for Compressive and Indirect Tension Strengths; Bag, Air, and Oven Cured Conditions, Showing Average Data, for Structural Lightweight Concrete.

In the preceding paragraphs of this section, no mention was made of the flexural strengths. Due to the extremely wide data scatter of these strength values, no per cent strength-age relationship could be reliably made with flexural strength determinations. The large coefficient of variation exhibited with flexural strength tests indicates that the flexural strength as a measure of concrete strength properties is of extremely doubtful value.

6. APPENDIX I - MATERIALS AND LABORATORY PROCEDURES

6.1 Concrete Materials

Two types of coarse aggregate, each combined with one type of fine aggregate, were used in this study. Of the two coarse aggregate types used, one was a structural lightweight semicoated expanded shale with a nominal maximum size of 3/4 in.; and the other a regular-weight, river-run gravel with a nominal maximum size of 1 inch. The fine aggregate used was a regular-weight, river-run sand. For purposes of mix identification, the mixes made with the structural lightweight coarse aggregate have the letter L in the code designation, while the mixes made with the regular-weight coarse aggregate have the letters SG in the code designation. The aggregate properties are given in Table 6-1. All of these aggregates are commercially produced in Texas.

Type I (portland cement), purchased from one manufacturer, and conforming to ASTM designation C-150,[39] was used throughout this study. The specific surface area, from the Wagner Turbimeter, of the cement was 1620 sq cm per gm.

The air-entraining agent used was of the neutralized vinsol resin type, approved by the Texas Highway Department, meeting the Texas Highway Department 1962 Standard Item No. 437.[40] Throughout this study, only one brand of the air-entraining agent was used.

City of Austin water was used throughout this study.

TABLE 6-1

Aggregate Properties

Property	Lightweight Coarse Aggregate L	Regular Wt. Coarse Aggregate SG	Regular Wt. Fine Aggregate SG
Sieve Analysis Cumulative % Retained			
1"	0	0	-
3/4"	1	9	-
3/8"	60	48	-
No. 4	98	93	0
No. 8	-	99	3
No. 16	-	-	20
No. 30	-	-	60
No. 50	-	-	92
No. 100	-	-	98
Pan	100	100	100
Specific Gravity (SSD)	1.51	2.60	2.60
Absorption (% of SSD Wt)	4.10	2.00	0.70
Unit Weight (pcf - dry loose)	54.00	99.00	100.00

6.2 Parameters Investigated

Many variables or factors influence the behavior, and hence properties, of concrete. Thus, concrete properties are rather individual in nature, in that properties determined for one particular mix design of a given aggregate, cement, cement factor, etc., may be entirely different from properties determined for a similar mix design in which only one of the factors was altered. This variation in properties is more noticeable in structural lightweight concrete than in regular-weight concrete, with the result that acceptance of structural lightweight concrete has been cautious. Keeping this in mind, certain parameters were selected for investigation, and all other influencing parameters were held constant insofar as practicable.

Two concrete types were selected, a structural lightweight concrete using lightweight coarse aggregate L and the regular-weight fine aggregate; and a regular-weight concrete using regular-weight coarse aggregate SG and the regular-weight fine aggregate. The majority of the research has been conducted with the structural lightweight concrete, with the regular-weight concrete used only for comparison purposes and for correlation with other research studies.

As discussed in Chapter 1, cement factor is, for lightweight concrete, the most directly correlatable parameter with the resulting strength of the concrete. A more direct correlation exists between cement factor and strength than between water-cement ratio and strength. Therefore, in order to investigate the critical properties for concretes with cement factors in the neighborhood of those specified by the Texas

Highway Department on their concrete pavement structures, two cement factors were selected--5 sacks per cubic yard and 4 sacks per cubic yard.

Entrained air, introduced into the fresh concrete in a portion of the mixing water, has been considered to be almost essential to produce workable, homogeneous, easily finished, structural, lightweight concrete. Two air contents have been studied with structural lightweight concrete in this report, and are (1) a 2 per-cent air content in which no air entrainment is added to the mix, and (2) a 6 per-cent air content where enough air-entraining agent is added to entrain and entrap 6 per-cent air in the mix as measured by a concrete pressure meter. The 2 per-cent air content represents air entrapped in the mix from the porus lightweight coarse aggregate. For the regular-weight concrete, no air entrainment was used, resulting in around 1 per-cent entrapped air in the mix.

Three curing conditions were investigated - moist cured at 100 per-cent humidity in plastic bags at 75F, air-dried at approximately 50 per-cent humidity at 75F, and oven-dried at a low humidity at 110F. Two of these curing conditions, the bag-cured and the oven-dried, represent the extreme conditions which would be reasonably encountered by a concrete pavement structure in Texas and, as such, represent the boundary values between which lie normal field curing conditions. The air-dried condition gives a median value between the two boundary extremes which aids in analyzing results. Properties for all of these three curing conditions have been studied for the structural lightweight concrete, and again, for comparison, regular-weight concrete properties have been determined for the bag-cured condition. These curing conditions were begun after the specimens were removed from the molds. This ranged from 10 to 26 hours after mixing.

As stated in Chapter 1, age is one of the most important of the test parameters, and is the factor with which all other parameters are related in this study. As the early concrete ages are of importance to the highway designer, properties at early ages are determined. Specific properties have been determined at ages of 1/2 day, 2 days, 7 days, and 28 days for the structural lightweight concrete. Again, for comparison purposes, certain regular-weight concrete properties were investigated.

6.3 Parameters Held Constant

There are a very large number of parameters which directly influence the behavior and performance of concrete, and, of necessity, several of these parameters had to be isolated and their influencing effects removed from this study. Consequently, the following parameters were held as constant as practicable in this test.

1. Mixing procedures
2. Testing procedures
3. Cement type
4. Air entrainment type
5. Batch size
6. Concrete mix consistency and texture
7. Fine aggregate type

All of the above factors significantly influence concrete behavior and properties, especially for structural lightweight concrete and thus should not be considered as unimportant.

6.4 Laboratory Procedures

General. The mixing procedures followed in the design of a lightweight concrete vary, depending upon the volume of the batch, type of aggregate, design placement conditions, and preference of the engineer. For purposes of this study, the mixing procedures employed were held, insofar as possible, constant. As testing procedures were not designated a variable to be investigated, they also were held constant. The specific procedural details are given in the following paragraphs.

Mixing. In selecting the proportions for the mix, ACI Standard 613 A-59 was followed.[41] A 3-cu-ft-capacity, rotary, tilting drum mixer was used which permitted the mixing of many batches with consistent results. As the slump is generally considered to be the primary mix control device, a 3-in. slump was achieved on every mix made. A rigidly fixed mixing procedure was used and is given briefly as follows:

1. The lightweight coarse aggregate was stored inside the laboratory and allowed to air dry thoroughly before use. In this way, the moisture condition of the aggregate was accurately known.
2. On the day the mix was to be made, the moisture content of the fine aggregate was closely determined by the drying method.
3. After coating the sides of the mixer with a small charge of concrete, the coarse aggregate was introduced into the mixer along with two-thirds of the mixing water. This was mixed slowly for ten minutes in order to allow sufficient time for the relatively porous lightweight aggregate to absorb enough water to develop its

"effective" moisture condition existent in the fresh concrete mixture.

4. The fine aggregate was then added and mixed briefly.

5. The cement was then added, together with almost all of the remaining water in which the air-entraining agent had been mixed. The entire mixture was then thoroughly mixed.

6. The remaining design amount of water was slowly added until the desired slump was reached. In any given mix, the total water used may vary slightly from the design total, depending upon the variations present in the amount of water absorbed by the lightweight coarse aggregate, but this variation was usually insignificant and has been disregarded.

Testing. Tests employed included the compressive (6 x 12 in.), flexural (6 x 6 x 18 in.), indirect tensile (split cylinder 6 x 12 in.), and direct tensile, and shrinkage stresses. Whenever applicable, Standard ASTM testing procedures were followed. In order to insure uniformity of test procedures, all tests were of the static rather than the dynamic type, and thus the loading rates were all 500 psi per minute or less, depending upon the type of test.

6.5 Reproducibility of Results

In order to analyze the different classes of data gathered in this study, close control of all variables was exercised throughout the experimental program. This control consisted of replication of data values where applicable, and statistical analysis of control cylinders cast with each batch of concrete to determine over-all project control.

The replication of data values were determined from an analysis of data scatter on the early testing portions of the project. It was found that the tensile stress-strain characteristics for a given set of conditions were almost identical on four different specimens and therefore, in subsequent tests, only one specimen was tested for each parameter investigated. All indirect tensile (split cylinder) strength values and compressive strength values represent the average of three cylinder strengths. All flexural strength values represent an average of two beam strengths made on one beam. For all strengths, excellent control was achieved.

The statistical analysis employed was the determination of the arithmatic mean, standard deviation, coefficient of variation, and range of the control cylinders cast with each batch. The normal or Gaussian distribution was used to compute these quantities.

The arithmetic mean or average value $\bar{x}$ is a measure of the central value of an array of data x_i, and is computed by:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \text{- - - - -} \quad (6\text{-}1)$$

where

$\bar{x}$ = arithmetic mean or average value

x_i = data value

n = number of data values in the array

To measure the dispersion or scatter of the data, three formulations are used. The standard deviation SD is one of the most useful measurements of data scatter as it gives an indication of the degree of data scatter without being unduly influenced by a few extremely scattered values, should they exist. The standard deviation SD, or root-mean-square deviation as it is sometimes called, is computed by:

$$SD = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}} \qquad \text{- - - - - (6-2)}$$

where

SD = standard deviation

Often it is advantageous to describe the standard deviation in terms of the average value. The coefficient of variation V is used for this purpose, and is usually expressed as a percentage.

$$V = \frac{SD}{\bar{x}} (100) \qquad \text{- - - - - (6-3)}$$

where

V = coefficient of variation, per cent

The other formulation employed to analyze data scatter is the maximum per cent deviation from the average, which gives an indication of the extreme range of values within the array of data.

$$DP = \frac{x_{i\,max} - \bar{x}}{\bar{x}} (100) \qquad \text{- - - - - (6-4)}$$

$$DN = \frac{x_{i\,min} - \bar{x}}{\bar{x}} (100) \qquad \text{- - - - - (6-5)}$$

where

DP = maximum positive deviation from the average

DN = maximum negative deviation from the average

$x_{i\ max}$ = maximum data value in the array

$x_{i\ min}$ = minimum data value in the array

In order to hold random and systematic errors to a minimum for each batch of concrete, two control cylinders were molded, cured 7 days in bags at 75F, and tested in compression. The resulting test strengths were then analyzed statistically. If the control cylinders for any batch deviated excessively from the average or norm, the results from this batch were discarded and another batch was poured and tested. Only five batches out of the more than fifty poured had to be discarded and remixed. The five statistical indices based on 7-day controlled compressive strength for all mixes are given in Table 6-2.

Note that the coefficient of variation for all mixes is 8.5 or less, with the 5 sacks per cubic yard, 6 per cent air mixes having the best quality control and the 4 sacks per cubic yard, 6 per cent air mixes having the worst quality control. This appears reasonable since the greater the cement factor, the more workable and, hence, homogeneous the mixture is; which, in turn, reduces random error or strength variations.

TABLE 6-2

Statistical Analysis of Seven-Day Bag-Cured Control Strengths for All Mixes

Cement Factor sks/cu yd	Air Content (%)	Curing	Arith. Mean $\bar{x}$ (psi)	Standard Dev. (psi)	Coef. of Var. (%)	Max. Pos. Dev. (%)	Max. Neg. Dev. (%)	No. of Data Values n
5	6	Bag	3549	120	3.5	6.6	-5.0	12
5	2	Bag	3556	177	5.0	3.7	-4.2	12
4	6	Bag	2088	178	8.5	14.7	-10.4	12

7. APPENDIX II - MISCELLANEOUS

7.1 Indirect Tension Test

The indirect tention test, or the split-cylinder test as it is more commonly called, for concrete was developed about 17 years ago in Brazil[42] and independently in Japan.[43] The test consists of loading a standard concrete cylinder in compression with its long axis horizontal so that the vertical load acts through the center of the specimen as shown in Fig. 7-1. Failure of the concrete occurs along the loaded axis and starts in the center portion of the specimen, thus, failure is not affected by surface conditions, such as surface residual tensile strain, crazing, moisture differentials, etc. This test, therefore, is free from those conditions which ordinarily cause premature failure of a test specimen.[44] A view of a specimen tested to failure by this method is given in Fig. 7-2.

The theoretical basis for this test is found in many textbooks on theory of elasticity or advanced strength of materials. From one such text by Hartog,[45] he reports that in 1900 Michell in England found a solution of a circular disk or cylinder subjected to a pair of compressive line loads along the diameter by an ingenuous superposition of two Boussinesq solutions and a hydrostatic stress, assuming plain stress conditions exist. The result of this approach is a biaxial stress condition throughout the cross section of the cylinder. Of primary interest are the maximum tensile stresses which act across the loaded diameter

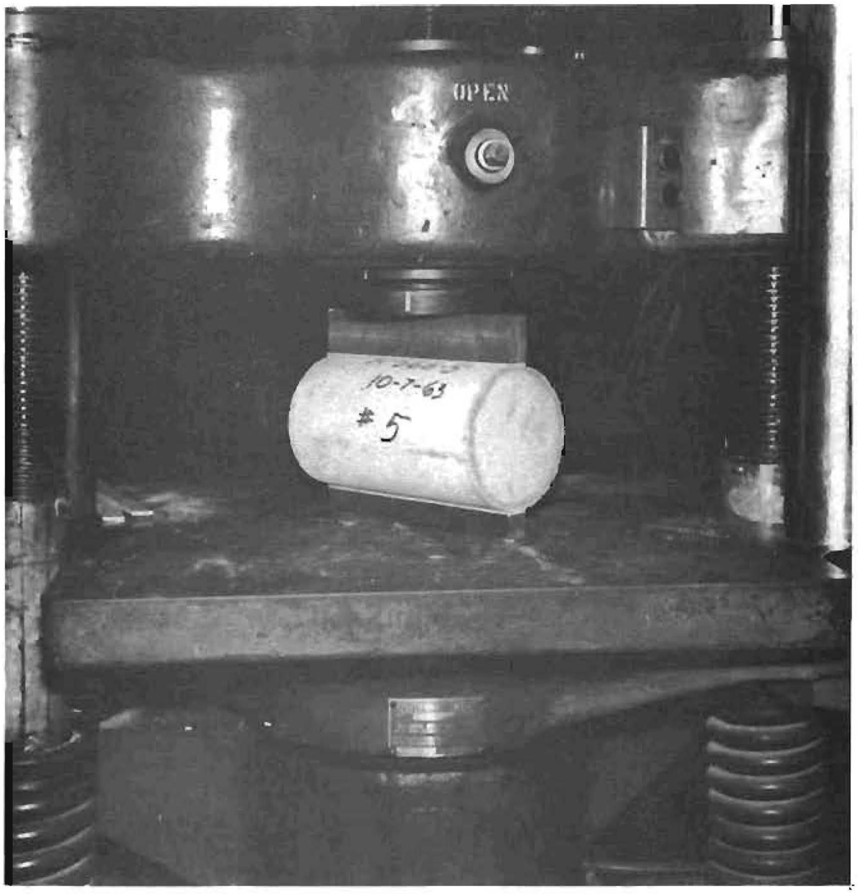


Fig. 7-1. Indirect Tension Test of a Concrete Cylinder

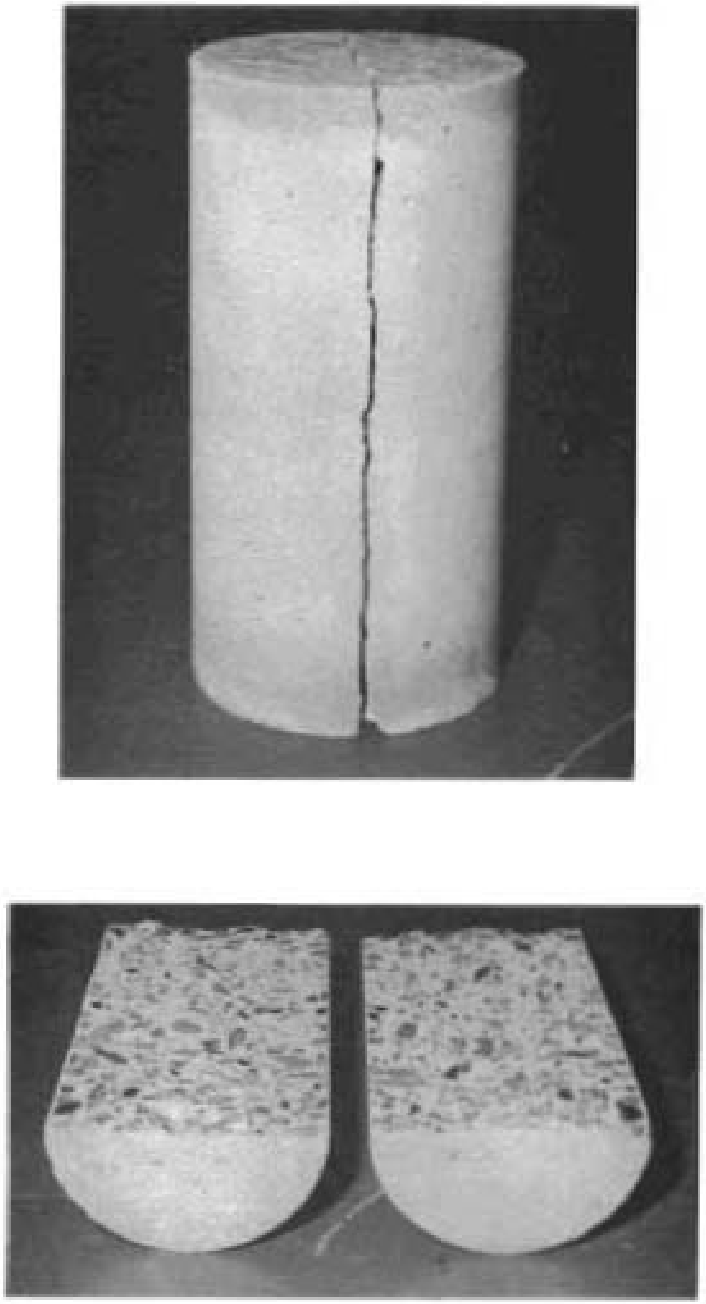


Fig. 7-2. Concrete Indirect Tension Specimen After Failure

and which have the constant magnitude:

$$\sigma_{ct} = \frac{2P}{\pi DT} \qquad \text{- - - - -} \quad (7\text{-}1)$$

where:

σ_{ct} = Maximum tensile stress, psi

P = Applied load, lb

D = Specimen diameter, in.

T = Specimen thickness or length, in.

Also acting across the loaded diameter are the maximum compressive stresses and at the center of the specimen this compressive stress is:

$$\sigma_{cc} = \frac{6P}{\pi DT} \qquad \text{- - - - -} \quad (7\text{-}2)$$

where:

σ_{cc} = Maximum compressive stress, psi

or three times the maximum tensile stress. For the hypothetical line load, this compressive stress would vary in magnitude from three times σ_{ct} in the center to infinitely high values directly under the loads. However, any real loading device will distribute the compressive load over finite area. Rudnik, Hunter, and Holden[46] have worked out a solution of the resultant stresses for a load distributed over an area acting diametrically through a cylinder. The theoretical stress distribution and the stress distribution resulting across a diameter for a cylinder compressed between deformable plates are shown in Fig. 7-3. As can be seen, for certain loading conditions, the tensile stresses are uniform over a large fraction of the diameter, and the compressive stresses are reduced to realistic values. For concrete, whose tensile capacity is in the neighborhood of one-tenth of the compressive capacity, such a test would always cause a tension type of failure and give an indication of concrete tensile strength. In one report[47] the indirect tension test was thoroughly investigated and the findings reported. On page 780 the author states:

> These results give a strong indication that the indirect tension test results are a reasonable measure of tensile strength.

Thus, using Eq. (7-1) and determining the maximum load P, the tensile capacity can be calculated.

This test is gradually being used more and more by designers and researchers. For structural lightweight concrete, this test has been evaluated and reported by Hanson[48, 49] in which it is shown that the indirect tensile strength offers a reliable measure of strength and correlates well with unit shear strength of concrete beams.

This test has now been tentatively standardized by the American Society for Testing and Materials (ASTM)[50] and is incorporated in the

ASTM Test Designation C-496. This specification was followed for all indirect tension tests performed in this investigation.

7.2 Data Tabulation

This section contains the tabulations for data used to prepare the figures in this report. The data are presented in tabular form on the following pages.

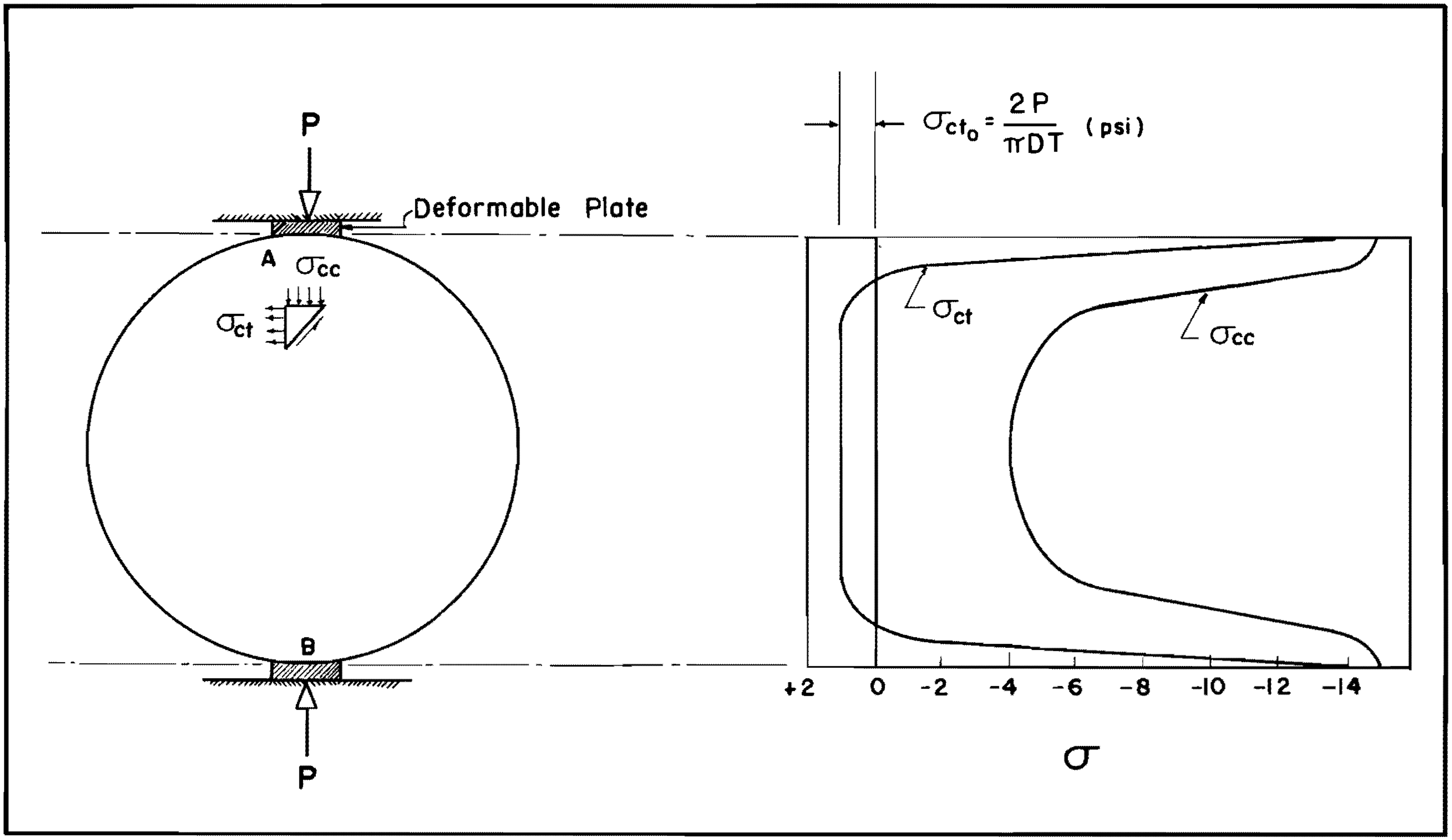


Figure 7-3. Stress Distribution Across Loaded Diameter of a Concrete Cylinder Compressed Between Deformable Plates.

TABLE 7-1

Compressive, Flexural, and Indirect Tensile Strength Values

Mix Type	C/F sks/cu yd	Air %	Cure	Age	Compressive		Flexural		Indirect Tensile	
					f'_c psi	%28 Day f'_c	f_f psi	%28 Day f_f	f_{sp} psi	%28 Day f_{sp}
L	5.0	2	B	1/2	612	15.8	143	35.2	84	18.5
				2	2645	68.5	410	101.0	308	67.7
				7	3516	91.0	507	124.9	379	83.3
				28	3863	100.0	406	100.0	455	100.0
			A	1/2	557	13.8	128	33.6	63	14.0
				2	2223	54.6	296	77.7	261	61.4
				7	3244	79.7	354	93.1	391	92.0
				28	4070	100.0	381	100.0	425	100.0
			O	1/2	607	17.7	138	36.0	71	20.0
				2	2690	78.5	297	77.6	310	87.3
				7	3539	103.0	287	74.9	348	98.0
				28	3424	100.0	383	100.0	355	100.0
L	5.0	6	B	1/2	813	17.4	164	38.6	101	22.9
				2	2470	52.8	406	95.6	293	66.4
				7	3590	76.8	486	114.5	391	88.6
				28	4685	100.0	425	100.0	442	100.0
			A	1/2	630	16.7	164	45.0	86	21.5
				2	2475	65.5	320	88.0	339	84.7
				7	3285	87.0	317	87.3	376	94.0
				28	3780	100.0	364	100.0	400	100.0
			O	1/2	970	41.0	176	38.6	126	42.4
				2	2765	117.0	320	70.4	323	108.5
				7	2945	125.0	252	55.5	299	101.0
				28	2365	100.0	455	100.0	297	100.0

TABLE 7-1 (Cont'd)

Compressive, Flexural, and Indirect Tensile Strength Values

Mix Type	C/F sks/cu yd	Air %	Cure	Age	Compressive f'_c psi	Compressive %28 Day f'_c	Flexural f_f psi	Flexural %28 Day f_f	Indirect Tensile f_{sp} psi	Indirect Tensile %28 Day f_{sp}
L	4.0	6	B	1/2	182	6.6	60	16.1	23	6.8
				2	1342	48.5	296	79.2	196	58.2
				7	2220	80.1	320	85.8	276	81.9
				28	2770	100.0	374	100.0	337	100.0
			A	1/2	236	95.0	78	26.4	33	10.2
				2	1325	53.4	243	82.3	191	59.3
				7	1775	71.6	253	85.8	203	63.0
				28	2480	100.0	295	100.0	322	100.0
			O	1/2	248	12.9	92	23.6	35	14.7
				2	1370	71.0	244	62.6	172	72.3
				7	1750	90.8	223	59.7	218	91.7
				28	1930	100.0	390	100.0	238	100.0
SG	5.0	1	B	1/2	450	10.0	---	-----	60.5	11.5
				2	2420	53.6	---	-----	369	70.0
				7	3720	82.5	---	-----	460	88.0
				28	4510	100.0	---	-----	525	100.0

Note: For explanation of symbols, see section 7.3

TABLE 7-2

Structural Lightweight Concrete Direct Tensile Test Data Tabulation (Average Values)

Cement Factor sks/cu yd	Air %	Age Days	Curing	f'_c psi	f_{sp} psi	f_t psi	σ_{cz} psi	ε_{ct} x 10^6	ε_{cz} x 10^6	E_{ct} psi x 10^{-6}
5.0	6.0	7	Bag	3160	398	289	- 21	177	-10	2.51
			Air	2860	326	183	20	93	11	2.53
			Oven	2870	336	249	89	148	40	2.48
5.0	6.0	28	Bag	4920	505	>375[1]	- 24	---[1]	- 6	4.29
			Air	3500	468	331	61	184	25	2.42
			Oven	3260	326	237	143	146	74	1.92
4.0	6.0	7	Bag	2170	296	180	- 22	99	- 9	2.28
			Oven	1600	214	119	87	88	54	----
4.0	6.0	28	Bag	2620	280	178	- 3	84	- 1	3.00
			Oven	1860	216	225	123	177	91	1.36
5.0[2]	1.0	7	Bag	4780	456	221	7	84	2	3.64
5.0[2]	1.0	28	Bag	4750	496	372	26	92	5	4.84

Notes:
1. Bond, rather than tensile, failure occurred in this specimen
2. This mix is regular weight concrete (SG)
3. For explanation of symbols used, see section 7.3

TABLE 7-3

Restrained Concrete Volume Change Stresses for Various Ages (Average Values)

	Mix L - Air Cured 5.0 sks/cu yd - 6 Per Cent Air			Mix L - Oven Dried 5.0 sks/cu yd - 6 Per Cent Air		
Age Days	ε^{i}_{sz} in./in. x 10^{6}	ε^{t}_{sz} in./in. x 10^{6}	σ_{cz} psi	ε^{i}_{sz} in./in. x 10^{6}	ε^{t}_{sz} in./in. x 10^{6}	σ_{cz} psi
1/2	----	-----	-----	----	------	------
3/4	----	-----	-----	----	------	------
7/8	- 30	-12.0	+ 8.3	----	------	------
1	----	-----	-----	+ 12	+ 4.6	- 3.3
1-1/4	----	-----	-----	+ 26	+ 10.0	- 7.2
1-1/2	----	-----	-----	----	------	------
1-3/4	+ 92	+35.0	-25.0	----	------	------
3	+ 24	+ 9.2	- 6.7	-230	- 89.0	+ 65.0
4	- 25	- 9.6	+ 7.0	-292	-112.0	+ 81.0
5	- 46	-18.0	+13.0	-322	-124.0	+ 90.0
6	- 71	-27.0	+20.0	-365	-140.0	+101.0
7	- 89	-34.0	+25.0	-392	-151.0	+109.0
8	-102	-39.0	+29.0	-416	-160.0	+116.0
9	-112	-43.0	+32.0	-436	-168.0	+122.0
10	-122	-47.0	+35.0	-455	-175.0	+127.0
11	-133	-51.0	+37.0	-472	-181.0	+131.0

TABLE 7-3 (Cont'd)

Restrained Concrete Volume Change Stresses for Various Ages (Average Values)

	Mix L - Air Cured 5.0 sks/cu yd - 6 Per Cent Air			Mix L - Oven Dried 5.0 sks/cu yd - 6 Per Cent Air		
Age Days	ε^{i}_{sz} in./in. x 10^6	ε^{t}_{sz} in./in. x 10^6	σ_{cz} psi	ε^{i}_{sz} in./in. x 10^6	ε^{t}_{sz} in./in. x 10^6	σ_{cz} psi
12	-142	-55.0	+40.0	-485	-186.0	+135.0
13	----	-----	-----	-498	-192.0	+139.0
14	-160	-62.0	+45.0	-508	-195.0	+141.0
16	-177	-68.0	+49.0	-522	-200.0	+145.0
18	-191	-74.0	+54.0	-532	-202.0	+146.0
20	-200	-77.0	+56.0	-536	-204.0	+148.0
23	-218	-84.0	+61.0	----	------	------
24	----	-----	-----	-536	-204.0	+148.0
26	-235	-90.0	+65.0	----	------	------
27	-236	-91.0	+66.0	----	------	------
28	-235	-90.0	+65.0	-534	-203.0	+147.0

Note: (1) "+" indicates tension, "-" indicates compression
(2) For explanation of symbols used, see section 7.3

TABLE 7-4

Restrained Concrete Volume Change Stresses for Various Ages

	Mix L - Bag Cured 4.0 sks/cu yd - 6 Per Cent Air			Mix L - Oven Dried 4.0 sks/cu yd - 6 Per Cent Air		
Age Days	ε^i_{sz} in./in. x 10^6	ε^t_{sz} in./in. x 10^6	σ_{cz} psi	ε^i_{sz} in./in. x 10^6	ε^t_{sz} in./in. x 10^6	σ_{cz} psi
1/2	+22	8.7	- 6.9	+ 14	+ 5.6	- 4.5
1	+46	+18.0	-14.0	- 10	- 4.0	+ 3.2
2	+45	+18.0	-14.0	-166	- 42.0	+ 53.0
3	---	-----	-----	-218	- 87.0	+ 69.0
4	+54	+21.0	-17.0	-256	-102.0	+ 82.0
5	+57	+23.0	-18.0	-288	-115.0	+ 92.0
6	+59	+23.0	-18.0	-300	-120.0	+ 96.0
7	+61	+24.0	-19.0	-325	-130.0	+104.0
8	+48	+19.0	-15.0	-365	-146.0	+116.0
10	+60	+24.0	-19.0	-390	-156.0	+124.0
12	+60	+24.0	-19.0	-415	-166.0	+132.0
14	+55	+22.0	-18.0	-405	-162.0	+129.0
16	+48	+19.0	-15.0	-423	-169.0	+135.0
18	---	-----	-----	-415	-166.0	+132.0
20	+43	+17.0	-14.0	-424	-170.0	+135.0
22	+34	+14.0	-11.0	-432	-173.0	+138.0
24	---	-----	-----	-448	-179.0	+143.0
26	+12	+ 4.8	- 3.8	----	------	------
28	+10	+ 4.0	- 3.2	-414	-166.0	+132.0

Note: (1) "+" indicates tension, "-" indicates compression
(2) For explanation of symbols used, see section 7.3

TABLE 7-5

Restrained Concrete Volume Change Stresses for Various Ages

	Mix L - Bag Cured 5.0 sks/cu yd - 6% Air			Mix SG - Bag Cured 5.0 sks/cu yd - 1% Air		
Age Days	ϵ^{i}_{sz} in./in.x10^6	ϵ^{t}_{sz} in./in.x10^6	σ_{cz} psi	ϵ^{i}_{sz} in./in.x10^6	ϵ^{t}_{sz} in./in.x10^6	σ_{cz} psi
0	0	0	0	0	0	0
3/4	-48	-18	+13	+18	+ 7.2	- 5.7
1-1/2	+40	+15	-11	+26	+10.0	- 8.0
2	+58	+22	-16	+23	+ 9.2	- 7.3
3	+71	+27	-20	+27	+11.0	- 8.8
4	+79	+30	-22	+24	+ 9.6	- 7.7
5	+84	+32	-23	+20	+ 8.0	- 6.4
6	+86	+33	-24	- 4	- 1.6	+ 1.2
7	+88	+34	-25	+10	+ 4.0	- 3.2
8	+86	+33	-24	+ 6	+ 2.4	- 1.9
10	+86	+33	-24	+ 1	+ 0.4	- 0.3
12	+88	+34	-25	- 8	- 3.2	+ 2.6
14	+86	+33	-24	-17	- 6.8	+ 5.4
16	+87	+33	-24	-	-	-
18	+87	+33	-24	-29	-12.0	+ 9.6
20	+87	+33	-24	-37	-15.0	+12.0
22	+86	+33	-24	-	-	-
24	+86	+33	-24	-63	-25.0	+20.0
26	+81	+31	-23	-70	-28.0	+22.0
28	+86	+33	-24	-81	-32.0	+26.0

Note: (1) "+" indicates tension, "-" indicates compression
(2) For explanation of symbols used, see section 7.3

TABLE 7-6

Statistical Analysis of Per Cent 28-Day Compressive and Indirect Tensile Strength Values for Structural Lightweight Concrete

Age	Curing	$\bar{x}$	S. D.	V	DP	DN
1/2	Bag	14. 7	6. 61	45. 0	56. 1	-55. 0
2	Bag	60. 4	8. 47	14. 0	13. 5	-19. 6
7	Bag	83. 6	5. 31	6. 36	8. 83	- 8. 15
28	Bag	100. 0	-	-	-	-
1/2	Air	14. 4	4. 42	30. 7	49. 1	-34. 1
2	Air	63. 2	11. 5	18. 1	34. 1	-15. 4
7	Air	81. 2	12. 2	15. 0	15. 7	-22. 4
28	Air	100. 0	-	-	-	-
1/2	Oven	24. 8	13. 3	53. 8	71. 1	-47. 9
2	Oven	77. 3	7. 44	9. 63	13. 0	- 8. 12
7	Oven	96. 9	5. 47	5. 64	6. 3	- 6. 30
28	Oven	100. 0	-	-	-	-

Note: For explanation of symbols used, see section 7. 3

TABLE 7-7

Statistical Analysis of Per Cent 7-Day Compressive and Indirect Tensile Strength Values for Structural Lightweight Concrete

Age	Curing	$\bar{x}$	S. D.	V	DP	DN
1/2	Bag	17.4	7.57	43.4	48.0	-53.0
2	Bag	71.9	7.12	9.90	13.1	-16.0
7	Bag	100.0	-	-	-	-
28	Bag	120.0	7.55	6.29	8.72	- 8.36
1/2	Air	17.5	3.26	18.6	30.9	-24.0
2	Air	78.6	11.0	13.9	19.6	-12.8
7	Air	100.0	-	-	-	-
28	Air	126.0	20.2	16.1	26.0	-15.4
1/2	Oven	23.9	11.2	47.1	77.4	-40.5
2	Oven	87.3	12.3	14.0	23.7	-12.9
7	Oven	100.0	-	-	-	-
28	Oven	99.7	10.8	10.9	10.5	-19.4

Note: For explanation of Symbols used, see section 7.3

TABLE 7-8

Analysis of the Ratios f_t/f'_c and f_t/f_{sp} for Structural Lightweight Concrete

Cement Factor sks/cu yd	Air Content (%)	Curing	Age (Days)	f_t/f'_c	f_t/f_{sp}
5	6	Bag	7	.091	.726
			28	(1)	(1)
		Air	7	.064	.564
			28	.095	.707
		Oven	7	.087	.741
			28	.073	.727
4	6	Bag	7	.083	.608
			28	.068	.636
		Oven	7	.074	.556
			28	.097	.838
5(3)	1	Bag	7	.046(4)	.485(4)
			28	.078	.750
All	All	All	All	.083	.685

Note: (1) Failed in bond, rather than tension
(2) For explanation of symbols used, see section 7.3
(3) Regular Weight Concrete (SG)
(4) Omitted from Averages

7.3 List of Symbols

A	Air dried at 50 per cent relative humidity, 75F
A_c	Cross-sectional concrete area, sq in.
A_s	Cross-sectional steel area, sq in.
B	Bag cured at 100 per cent relative humidity, 75F
C	Distance from the neutral axis to the farthest fiber, in.
D	Concrete cylindrical diameter, in.
DP	Maximum positive deviation from the average
DN	Maximum negative deviation from the average
$\Delta \ell_c$	Change in concrete length, in.
$\Delta \ell_s$	Change in steel length, in.
E_{cc}	Concrete compressive modulus of elasticity, psi
E_{ct}	Concrete tensile modulus of elasticity, psi
E_s	Steel modulus of elasticity, psi
ε_{cp}	Concrete strain due to an external load P
ε_{ct}	Concrete tensile strain (total)
ε_{cz}	Concrete strain from restrained volume changes
ε^t_{sz}	True steel strain from restrained volume changes
ε^i_{sp}	Indicated steel strain due to an external load P
ε^t_{sp}	True steel strain due to an external load P
F_{cz}	Force in the concrete due to restrained volume changes, lb
F_{sz}	Force in the steel due to restrained volume changes, lb

f'_c	Concrete compressive strength, psi
f_{sp}	Concrete indirect tensile strength, psi
f_t	Concrete direct tensile strength, psi
I	Moment of inertia of the cross section, in.^{4}
L	Concrete made with structural lightweight coarse aggregate
ℓ	Length, in.
ℓ_o	Original length, in.
ℓ_u	Unrestrained length, in.
ℓ_r	Restrained length, in.
M	Bending moment in the section, in.-lb
N	Number of data values in the array
O	Oven dried at low humidity, 110F
ω	Unit weight of concrete, pcf
P	Applied load, lb
P_c	External concrete load, lb
P_s	External steel load, lb
S	Stress in the fiber farthest from the neutral axis
S_f	Steel strain factor $= \varepsilon^i_{sp}/\varepsilon^t_{sp}$
SD	Standard deviation
SG	Concrete made with regular weight coarse aggregate
S_t	Steel temperature calibration factor, in./in. x $10^6/{}^oF$
σ_{cc}	Concrete compressive stress, psi
σ_{cp}	Concrete stress due to an external load P, psi
σ_{ct}	Concrete tensile stress, psi
σ_{cz}	Concrete stress due to restrained volume changes, psi

σ_s	Steel stress, psi
T	Concrete cylinder length (or thickness), in.
V	Coefficient of variation, per cent
$\bar{x}$	Arithmetic mean or average value
x'_i	Data value, i = 1, 2, 3, etc.
$x'_{i\ max}$	Maximum data value in the array
$x'_{i\ min}$	Minimum data value in the array

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