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UNIT HYDROGRAPH ESTIMATION FOR APPLICABLE TEXAS WATERSHEDS

Report 0–4193–4

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16. Abstract

The unit hydrograph is defined as a direct runoff hydrograph resulting from a unit pulse of excess rainfall generated uniformly over the watershed at a constant rate for an effective duration. The unit hydrograph method is a well-known hydrologic-engineering technique for estimation of the runoff hydrograph given an excess rainfall hyetograph. Four separate approaches are used to extract unit hydrographs from the database on a per watershed basis. A large database of more than 1,600 storms with both rainfall and runoff data for 93 watersheds in Texas is used for four unit hydrograph investigation approaches. One approach is based on 1-minute Rayleigh distribution hydrographs; the other three approaches are based on 5-minute gamma-distribution hydrographs. The unit hydrographs by watershed from the approaches are represented by shape and time to peak parameters. Weighted least-squares regression equations to estimate the two unit hydrograph parameters for ungaged watersheds are provided on the basis of the watershed characteristics of main channel length, dimensionless main channel slope, and a binary watershed development classification. The range of watershed area is approximately 0.32 to 167 square miles. The range of main channel length is approximately 1.2 to 49 miles. The range of dimensionless main channel slope is approximately 0.002 to 0.020. The equations provide a framework by which hydrologic engineers can estimate shape and time to peak of the unit hydrograph, and hence the associated peak discharge. Assessment of equation applicability and uncertainty for a given watershed also is provided. The authors explicitly do not identify a preferable approach and hence equations for unit hydrograph estimation. Each equation is associated with a specific analytical approach. Each approach represents the optimal unit hydrograph solution on the basis of the details of approach implementation including unit hydrograph model, unit hydrograph duration, objective functions, loss model assumptions, and other factors.

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No invention or discovery was conceived or first actually reduced to practice in the course of or under this contract, including any art, method, process, machine, manufacture, design, or composition of matter, or any new useful improvement thereof, or any variety of plant, which is or may be patentable under the patent laws of the United States of America or any foreign country.

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ABSTRACT

The unit hydrograph is defined as a direct runoff hydrograph resulting from a unit pulse of excess rainfall generated uniformly over the watershed at a constant rate for an effective duration. The unit hydrograph method is a well-known hydrologic-engineering technique for estimation of the runoff hydrograph given an excess rainfall hyetograph. Four separate approaches are used to extract unit hydrographs from the database on a per watershed basis. A large database of more than 1,600 storms with both rainfall and runoff data for 93 watersheds in Texas is used for four unit hydrograph investigation approaches. One approach is based on 1-minute Rayleigh distribution hydrographs; the other three approaches are based on 5-minute gamma-distribution hydrographs. The unit hydrographs by watershed from the approaches are represented by shape and time to peak parameters. Weighted least-squares regression equations to estimate the two unit hydrograph parameters for ungaged watersheds are provided on the basis of the watershed characteristics of main channel length, dimensionless main channel slope, and a binary watershed development classification. The range of watershed area is approximately 0.32 to 167 square miles. The range of main channel length is approximately 1.2 to 49 miles. The range of dimensionless main channel slope is approximately 0.002 to 0.020. The equations provide a framework by which hydrologic engineers can estimate shape and time to peak of the unit hydrograph, and hence the associated peak discharge. Assessment of equation applicability and uncertainty for a given watershed also is provided. The authors explicitly do not identify a preferable approach and hence equations for unit hydrograph estimation. Each equation is associated with a specific analytical approach. Each approach represents the optimal unit hydrograph solution on the basis of the details of approach implementation including unit hydrograph model, unit hydrograph duration, objective functions, loss model assumptions, and other factors.

INTRODUCTION

A hydrograph is defined as a time series of streamflow at a given location on a stream. The runoff hydrograph is a hydrograph resulting from excess rainfall and is an important component of hydrologic-engineering design as the peak discharge, volume, and time distribution of runoff are represented. The unit hydrograph method is a well-known hydrologic-engineering technique for estimation of the runoff hydrograph given an excess rainfall hyetograph (a time series of excess rainfall). Unit hydrographs are valuable for cost-effective and risk-mitigated hydrologic design of hydraulic structures. The unit hydrograph method is in widespread use by hydrologic engineers and others including engineers with the Texas Department of Transportation (TxDOT, 2004). A unit hydrograph estimate for an arbitrary watershed allows the computation of a direct runoff hydrograph resulting from either measured or design storms.

During 2001–05, a consortium of researchers at Texas Tech University (TTU), Lamar University (LU), the University of Houston (UH), and the U.S. Geological Survey (USGS), in cooperation with TxDOT Research Management Committee No. 3, did a study (TxDOT Research Project 0–4193) of unit hydrographs for Texas. This report describes the results of the unit hydrograph investigation and presents procedures for unit hydrograph estimation for ungaged watersheds.

A unit hydrograph is defined as the direct runoff hydrograph resulting from a unit pulse of excess rainfall (rainfall that is not retained or stored in the watershed) uniformly generated over the watershed at a constant rate for an effective duration (Chow and others, 1988, p. 213). The watershed is assumed to function as a linear system in which the concepts of proportionality and superposition are appropriate. For example, the runoff hydrograph resulting from two simultaneous pulses of unit rainfall of a specific duration would have ordinates that are twice as large as those resulting from a single unit pulse of rainfall of the same duration. The hydrograph resulting from two consecutive pulses, however, is computed using the temporal superposition of two unit hydrographs, separated by the time period of the first pulse. The time step of the unit hydrograph must equal the time step of the rainfall pulses.

The unit hydrograph approach assumes that the watershed characteristics influencing general unit hydrograph shape are invariant. By extension, these assumptions result in similarities among direct runoff hydrograph shape from storms having similar rainfall characteristics. Pertinent watershed characteristics include drainage area, channel slope, and watershed shape. Assumptions (Chow and others, 1988, p. 214) inherent to the unit hydrograph method include:

1. The watershed can be adequately modeled as a linear system with respect to rainfall input and runoff output.

2. Excess rainfall has a constant intensity within the effective duration and is uniformly distributed throughout the entire drainage area.

3. The time base of runoff of the unit hydrograph resulting from an excess rainfall pulse of a given effective time length is invariant.

4. The ordinates of all direct runoff hydrographs of a common time base are directly proportional to the total amount of direct runoff represented by each hydrograph.

Further, unit hydrograph background and the relation of unit hydrographs to hydraulic design are discussed in numerous hydrologic engineering textbooks (for example, Chow and others, 1988; Haan and others, 1994; Dingman, 2002; McCuen, 2005). The relation between excess rainfall, the unit hydrograph, and runoff is algebraically straightforward. The discrete (convolution) equation for a linear system is used to generate direct runoff given excess rainfall and a unit hydrograph. The equation is

$$Q_n^* = \sum_{m=1}^{n \le M} P_m U_{n-m+1}, \tag{1}$$

where Q_n^* is runoff estimated (modeled) from the excess rainfall (P_m) and the unit hydrograph (U_{n-m+1}), M is the number of excess rainfall pulses, and m and n are integers. Various approaches exist to compute U_i given observed runoff (Q_i) and P_i vectors where i represents the ith value in each vector. Four independent approaches are described and implemented in this report.

Purpose and Scope

Methods to estimate the unit hydrograph for ungaged watersheds are useful to hydrologic engineers. Therefore, the three primary objectives of this report are to present procedures or equations for (1) estimation of dimensionless hydrograph shape parameters (K or N), (2) estimation of time to peak (T_p), and (3) evaluation of equation applicability for arbitrary watersheds. These procedures require the common watershed characteristics of main channel length and dimensionless main channel slope. These procedures also account for a binary watershed development classification (undeveloped and developed). The procedures to estimate K or N and T_p are based on four independent research approaches. To clarify, each approach produced independent procedures for estimating dimensionless hydrograph shape and T_p. A secondary objective of this report is a comparison of the results between the independent lines of research.

Rainfall and Runoff Database

A digital database of previously published USGS rainfall and runoff values for more than 1,600 storms from 93 undeveloped and developed watersheds in Texas was used for unit hydrograph research. The database is described and tabulated in Asquith and others (2004); an abbreviated summary of the database is provided here to establish context. The data were derived from more than 220 historical USGS data reports. The locations of all USGS streamflow-gaging stations the stations used in the study are shown in figure 1. Each storm within the database is represented by a single rainfall data file. Digital versions of these data were not available until a distributed team of TTU, LU, UH, and USGS personnel manually entered the data into a database from the printed records. The data and the extent of quality control and quality assurance efforts are described in Asquith and others (2004). Two stations (08111025 Burton Creek at Villa Maria Road, Bryan, Texas, and 08111050 Hudson Creek near Bryan, Texas) were not provided by Asquith and others (2004) as these data manually were added after that report was published. The stations and ancillary information are listed in table 1.

Table 1. U.S. Geological Survey streamflow-gaging stations with rainfall and runoff data used in study

[Development classification (determined on qualitative basis): U, undeveloped watershed; D, development watershed; sub., subwatershed; IH, Interstate Highway; US, United States; SH, State Highway; FM, Farm to Market; --, not available]

Station no.	Station name	Latitude	Longitude	Development classification
08042650	North Creek sub. 28A near Jermyn, Texas	33°14’52"	98°19’19"	U
08042700	North Creek near Jacksboro, Texas	33°16’57"	98°17’53"	U
08048520	Sycamore Creek at IH 35W, Fort Worth, Texas	32°39’55"	97°19’16"	D
08048530	Sycamore Creek tributary above Seminary South Shopping Center, Fort Worth, Texas	32°41’08"	97°19’44"	D
08048540	Sycamore Creek tributary at IH 35W, Fort Worth, Texas	32°41’18"	97°19’11"	D
08048550	Dry Branch at Blandin Street, Fort Worth, Texas	32°47’19"	97°18’22"	D
08048600	Dry Branch at Fain Street, Fort Worth, Texas	32°46’34"	97°17’18"	D
08048820	Little Fossil Creek at IH 820, Fort Worth, Texas	32°50’22"	97°19’20"	D
08048850	Little Fossil Creek at Mesquite Street, Fort Worth, Texas	32°48’33"	97°17’28"	D
08050200	Elm Fork Trinity River sub. 6 near Muenster, Texas	33°37’13"	97°24’15"	U
08052630	Little Elm Creek sub. 10 near Gunter, Texas	33°24’33"	96°48’41"	U
08052700	Little Elm Creek near Aubrey, Texas	33°17’00"	96°53’33"	U
08055580	Joes Creek at Royal Lane, Dallas, Texas	32°53’43"	96°41’36"	D
08055600	Joes Creek at Dallas, Texas	32°51’33"	96°53’00"	D
08055700	Bachman Branch at Dallas, Texas	32°51’37"	96°51’13"	D
08056500	Turtle Creek at Dallas, Texas	32°48’26"	96°48’08"	D
08057020	Coombs Creek at Sylvan Avenue, Dallas, Texas	32°46’01"	96°50’07"	D
08057050	Cedar Creek at Bonnieview Road, Dallas, Texas	32°44’50"	96°47’44"	D
08057120	McKamey Creek at Preston Road, Dallas, Texas	32°57’58"	96°48’11"	U
08057130	Rush Branch at Arapaho Road, Dallas, Texas	32°57’45"	96°47’44"	D
08057140	Cottonwood Creek at Forest Lane, Dallas, Texas	32°54’33"	96°45’54"	D
08057160	Floyd Branch at Forest Lane, Dallas, Texas	32°54’33"	96°45’34"	D
08057320	Ash Creek at Highland Road, Dallas, Texas	32°48’18"	96°43’04"	D
08057415	Elam Creek at Seco Boulevard, Dallas, Texas	32°44’14"	96°41’36"	D
08057418	Fivemile Creek at Kiest Boulevard, Dallas, Texas	32°42’19"	96°51’32"	D
08057420	Fivemile Creek at US Highway 77W, Dallas, Texas	32°41’15"	96°49’22"	D
08057425	Woody Branch at IH 625, Dallas, Texas	32°40’58"	96°49’22"	D
08057435	Newton Creek at IH 635, Dallas, Texas	32°39’19"	96°44’41"	D
08057440	Whites Branch at IH 625, Dallas, Texas	32°39’26"	96°44’25"	D
08057445	Prarie Creek at US Highway 175, Dallas, Texas	32°42’17"	96°40’11"	D
08057500	Honey Creek sub. 11 near McKinney, Texas	33°18’12"	96°41’22"	U
08058000	Honey Creek sub.12 near McKinney, Texas	33°18’20"	96°40’12"	U
08061620	Duck Creek at Buckingham Road, Garland, Texas	32°55’53"	96°39’55"	D
08061920	South Mesquite Creek at SH 352, Mesquite, Texas	32°46’09"	96°37’18"	D
08061950	South Mesquite Creek at Mercury Road, Mesquite, Texas	32°43’32"	96°34’12"	D
08063200	Pin Oak Creek near Hubbard, Texas	31°48’01"	96°43’02"	U
08094000	Green Creek sub. 1 near Dublin, Texas	32°09’57"	98°20’28"	U
08096800	Cow Bayou sub. 4 near Bruceville, Texas	31°19’59"	97°16’02"	U
08098300	Little Pond Creek near Burlington, Texas	31°01’35"	96°59’17"	U
08108200	North Elm Creek near Cameron, Texas	30°55’52"	97°01’13"	U
08111025	Burton Creek at Villa Maria Road, Bryan, Texas	30°38’48’’	96°20’57’’	D
08111050	Hudson Creek near Bryan, Texas	30°39’38’’	96°17’59’’	U
08136900	Mukewater Creek sub. 10A near Trickham, Texas	31°39’01"	99°13’30"	U
08137000	Mukewater Creek sub. 9 near Trickham, Texas	31°41’40"	99°12’18"	U
08137500	Mukewater Creek at Trickham, Texas	31°35’24"	99°13’36"	U
08139000	Deep Creek sub. 3 near Placid,Texas	31°17’25"	99°09’22"	U
08140000	Deep Creek sub. 8 near Mercury, Texas	31°24’08"	99°07’17"	U
08154700	Bull Creek at Loop 360, Austin, Texas	30°22’19"	97°47’04"	U
08155200	Barton Creek at SH 71, Oak Hill, Texas	30°17’46"	97°55’31"	U
08155300	Barton Creek at Loop 360, Austin, Texas	30°14’40"	97°48’07"	U

Station no.	Station name	Latitude	Longitude	Develop-ment classifi-cation
08155550	West Bouldin Creek at Riverside Drive, Austin, Texas	30°15’49"	97°45’17"	D
08156650	Shoal Creek at Steck Avenue, Austin, Texas	30°21’55"	97°44’11"	D
08156700	Shoal Creek at Northwest Park, Austin, Texas	30°20’50"	97°44’41"	D
08156750	Shoal Creek at White Rock Drive, Austin, Texas	30°20’21"	97°44’50"	D
08156800	Shoal Creek at 12th Street, Austin, Texas	30°16’35"	97°45’00"	D
08157000	Waller Creek at 38th Street, Austin, Texas	30°17’49"	97°43’36"	D
08157500	Waller Creek at 23rd Street, Austin, Texas	30°17’08"	97°44’01"	D
08158050	Boggy Creek at US 183, Austin, Texas	30°15’47"	97°40’20"	D
08158100	Walnut Creek at FM 1325, Austin, Texas	30°24’35"	97°42’41"	U
08158200	Walnut Creek at Dessau Road, Austin, Texas	30°22’30"	97°39’37"	U
08158380	Little Walnut Creek at Georgian Drive Austin, Texas	30°21’15"	97°41’52"	D
08158400	Little Walnut Creek at IH 35, Austin, Texas	30°20’57"	97°41’34"	D
08158500	Little Walnut Creek at Manor Road, Austin, Texas	30°18’34"	97°40’04"	D
08158600	Walnut Creek at Webberville Road, Austin, Texas	30°16’59"	97°39’17"	D
08158700	Onion Creek near Driftwood, Texas	30°04’59"	98°00’29"	U
08158800	Onion Creek at Buda, Texas	30°05’09"	97°50’52"	U
08158810	Bear Creek below FM 1826, Driftwood, Texas	30°09’19"	97°56’23"	U
08158820	Bear Creek at FM 1626, Manchaca, Texas	30°08’25"	97°50’50"	U
08158825	Little Bear Creek at FM 1626, Manchaca, Texas	30°07’31"	97°51’43"	U
08158840	Slaughter Creek at FM 1826, Austin, Texas	30°12’32"	97°54’11"	U
08158860	Slaughter Creek at FM 2304, Austin, Texas	30°09’43"	97°49’55"	U
08158880	Boggy Creek (south) at Circle S Road, Austin, Texas	30°10’50"	97°46’55"	U
08158920	Williamson Creek at Oak Hill, Texas	30°14’06"	97°51’36"	D
08158930	Williamson Creek at Manchaca Road, Austin, Texas	30°13’16"	97°47’36"	D
08158970	Williamson Creek at Jimmy Clay Road, Austin, Texas	30°11’21"	97°43’56"	D
08159150	Wilbarger Creek near Pflugerville, Texas	30°27’16"	97°36’02"	U
08177600	Olmos Creek tributary at FM 1535, Shavano Park, Texas	29°34’35"	98°32’45"	D
08177700	Olmos Creek at Dresden Drive, San Antonio, Texas	29°29’56"	98°30’36"	D
08178300	Alazan Creek at St. Cloud Street, San Antonio, Texas	29°27’29"	98°32’59"	D
08178555	Harlendale Creek at West Harding Street, San Antonio, Texas	29°21’05"	98°29’32"	D
08178600	Panther Springs Creek at FM 2696 near San Antonio, Texas	29°37’31"	98°31’06"	U
08178620	Lorence Creek at Thousand Oaks Boulevard, San Antonio, Texas	29°35’24"	98°27’47"	D
08178640	West Elm Creek at San Antonio, Texas	29°37’23"	98°26’29"	U
08178645	East Elm Creek at San Antonio, Texas	29°37’04"	98°25’41"	U
08178690	Salado Creek tributary at Bitters Road, San Antonio, Texas	29°31’36"	98°26’25"	D
08178736	Salado Creek tributary at Bee Street, San Antonio, Texas	29°26’38"	98°27’13"	D
08181000	Leon Creek tributary at FM 1604, San Antonio, Texas	29°35’14"	98°37’40"	U
08181400	Helotes Creek at Helotes, Texas	29°34’42"	98°41’29"	U
08181450	Leon Creek tributary at Kelly Air Force Base, Texas	29°23’12"	98°36’00"	D
08182400	Calaveras Creek sub. 6 near Elmendorf, Texas	29°22’49"	98°17’33"	U
08187000	Escondido Creek sub. 1 near Kenedy, Texas	28°46’41"	97°53’41"	U
08187900	Escondido Creek sub. 11 near Kenedy, Texas	28°51’39"	97°50’39"	U
SSSC	Seminary South Shopping Center drainage, Fort Worth, Texas	--	--	D

The selected watershed characteristics for each station from the 30-meter digital elevation model (DEM) are listed in table 2. The range of watershed area is approximately 0.32 to 167 square miles. The range of main channel length, which is the longest flow path between outlet and basin divide, is approximately 1.2 to 49 miles. The range of dimensionless main channel slope is approximately 0.002 to 0.020. Dimensionless main channel slope is computed as the difference in elevation from outlet to basin divide along the main channel divided by the main channel length.

Table 2. U.S. Geological Survey streamflow-gaging stations and selected watershed charactersisistics

[DA, drainage area; mi^2, square miles; DEM, Digital elevation model; MCL, main channel length; mi, miles; MCS, main channel slope (dimensionless); --, not available]

Station no.	DA (mi^2)		MCL, 30-meter DEM (mi)	MCS, 30-meter DEM
	USGS files	30-meter DEM		
08042650	6.82	6.56	4.632	0.01378
08042700	21.6	23.99	11.57	.006025
08048520	17.7	17.63	7.530	.005081
08048530	.97	.97	1.700	.011813
08048540	1.35	1.29	2.370	.01119
08048550	1.08	1.11	2.017	.004507
08048600	2.15	2.57	3.845	.004729
08048820	5.64	5.66	6.027	.005970
08048850	12.30	12.86	9.397	.005059
08050200	.77	.87	2.643	.01068
08052630	2.10	2.05	3.298	.006489
08052700	75.5	73.10	23.23	.002201
08055580	1.94	1.90	2.997	.007204
08055600	7.51	5.69	6.742	.006012
08055700	10.0	11.04	7.766	.005048
08056500	7.98	6.36	6.365	.006338
08057020	4.75	4.53	5.092	.009707
08057050	9.42	9.48	6.206	.007812
08057120	6.77	6.57	5.187	.007412
08057130	1.22	1.29	2.629	.009077
08057140	8.50	8.64	7.466	.005758
08057160	4.17	4.60	5.343	.006380
08057320	6.92	7.17	5.416	.005595
08057415	1.25	.97	1.884	.006333
08057418	7.65	8.06	5.649	.007879
08057420	13.2	14.39	8.335	.006454
08057425	11.5	10.33	6.155	.007877
08057435	5.91	5.92	4.122	.008684
08057440	2.53	2.62	3.517	.008347
08057445	9.03	8.93	8.416	.003623
08057500	2.14	2.09	2.070	.01061
08058000	1.26	1.21	2.087	.01025
08061620	8.05	7.68	5.522	.003876
08061920	13.4	12.89	7.645	.003890
08061950	23.0	23.31	12.65	.003070
08063200	17.6	18.18	8.730	.004013
08094000	3.34	2.38	3.350	.008705
08096800	5.25	5.07	4.493	.01117
08098300	22.2	22.98	13.73	.002635
08108200	48.6	46.38	19.96	.002524
08111025	1.33	1.35	2.548	.007061
08111050	1.94	1.94	2.453	.005792
08136900	21.8	21.74	12.42	.007657
08137000	4.02	4.09	4.404	.004730
08137500	70.4	69.24	19.39	.005549
08139000	3.42	3.13	3.357	.01518
08140000	5.41	7.32	5.908	.009265
08154700	22.3	22.78	10.04	0.010693
08155200	89.7	89.64	28.50	.004844
08155300	116	116.6	45.07	.004030
08155550	3.12	2.67	3.660	.01258
08156650	2.79	2.71	2.999	.01150
08156700	7.03	6.35	4.527	.009245
08156750	7.56	6.84	5.130	.008750
08156800	12.3	12.75	10.58	.007481
08157000	2.31	2.21	4.119	.009794
08157500	4.13	4.17	5.164	.009425
08158050	13.1	12.63	7.361	.007925
08158100	12.6	12.74	5.669	.009120
08158200	26.2	26.43	10.92	.006628
08158380	5.22	5.26	4.015	.006982
08158400	5.57	5.71	4.477	.006726
08158500	12.1	12.13	8.590	.006769
08158600	51.3	53.58	19.47	.004951
08158700	124	123.7	33.28	.004513
08158800	166	167.3	48.94	.003916
08158810	12.2	12.30	6.287	.011087
08158820	24.0	24.50	14.85	.007462
08158825	21.0	21.02	12.53	.006649
08158840	8.24	8.77	4.960	.01191
08158860	23.10	23.22	12.79	.007875
08158880	3.58	3.57	4.404	.01127
08158920	6.30	6.30	4.974	.01173
08158930	19.0	18.73	10.40	.008850
08158970	27.6	27.38	17.61	.006454
08159150	4.61	4.46	3.739	.008156
08177600	.33	.32	1.305	.01437
08177700	21.2	20.84	10.96	.006584
08178300	3.26	3.27	3.584	.01665
08178555	2.43	1.91	4.052	.002431
08178600	9.54	9.61	7.051	.012544
08178620	4.05	4.05	3.608	.01197
08178640	2.45	2.46	3.044	.01960
08178645	2.33	2.46	3.958	.01627
08178690	.26	.43	1.172	.004040
08178736	.45	.69	1.670	.009415
08181000	5.57	5.55	5.421	.01569
08181400	15.0	14.90	9.821	.01215
08181450	1.19	1.24	3.130	.003207
08182400	7.01	7.15	4.867	.005721
08187000	3.29	3.06	2.780	.009742
08187900	8.43	8.78	4.869	.005251
SSSC	.38	--	--	--

The database is separated into six “modules.” The six modules are *austin*, *bryan*, *dallas*, *fortworth*, *sanantonio*, and *smallruralsheds*. All modules with the exception of *smallruralsheds* are named according to the city or area where the watershed is located. The drainage network for these watersheds generally comprises first- to third-order tributaries and land use ranges from fully “developed” to natural or “undeveloped.” The development classification was made on a qualitative basis. The *smallruralsheds* module contains a cluster of intensively monitored small rural watershed study units within the Brazos River, Colorado River, San Antonio River, and Trinity River basins of Texas.

The storms that comprise the database were chosen by previous USGS analysts. The storms are not inclusive of all rainfall and runoff for the watershed for the period of record. Factors influencing whether a storm was published in the original reports include: instrument operation (data integrity), importance or magnitude of the storm, time of year, or simply the need to have the data for a few storms per year published. The rainfall files contain date-time values and corresponding rainfall for one or more rain gages in the watershed and cumulative rainfall for the storm. The streamflow or hydrograph files for each storm within the database contain date-time values and corresponding aggregate direct runoff and base flow. Further discussion pertinent to caveats and limitations of the database is available in Asquith and others (2004).

Previous Studies

Sherman (1932) introduced the concept of the unit hydrograph. Since the 1930s, the unit hydrograph method has developed into an extremely important and practical tool for applied hydrologic problems. Chow and others (1988) devote a chapter to unit hydrograph methods and give a summary of the various assumptions inherent to a unit hydrograph. Pilgrim and Cordery (1993), Dingman (2002), Viessman and Lewis (2003), and McCuen (2005) provide considerable background and references for the unit hydrograph method. A widely known framework to implement a particular unit hydrograph method is described by Natural Resources Conservation Service (NRCS, 2004). Viessman and Lewis (2003, p. 270–275) describe an approach for unit hydrograph estimation referred to in this report as the traditional unit hydrograph approach (see section “Traditional Unit Hydrograph Analysis” in this report).

Gamma unit hydrographs are unit hydrographs whose shape is defined by the gamma distribution (Evans and others, 2000). Gamma unit hydrographs, considered for three of the four approaches described here, have a long history in the hydrologic engineering community and are thoroughly considered by Edson (1951), Nash (1959), Dooge (1959), Gray (1961), Wu (1963), Haan (1970), Croley (1980), Aron and White (1982), Rosso (1984), James and others (1987), Haktanir and Sezen (1990), Meadows and Ramsey (1991a and b), Haan and others (1994), Singh (2000, 2004), Bhunya and others (2003), and references therein. The specific details of gamma unit hydrograph formulation are provided in section “Gamma Unit Hydrograph Analysis System” in this report.

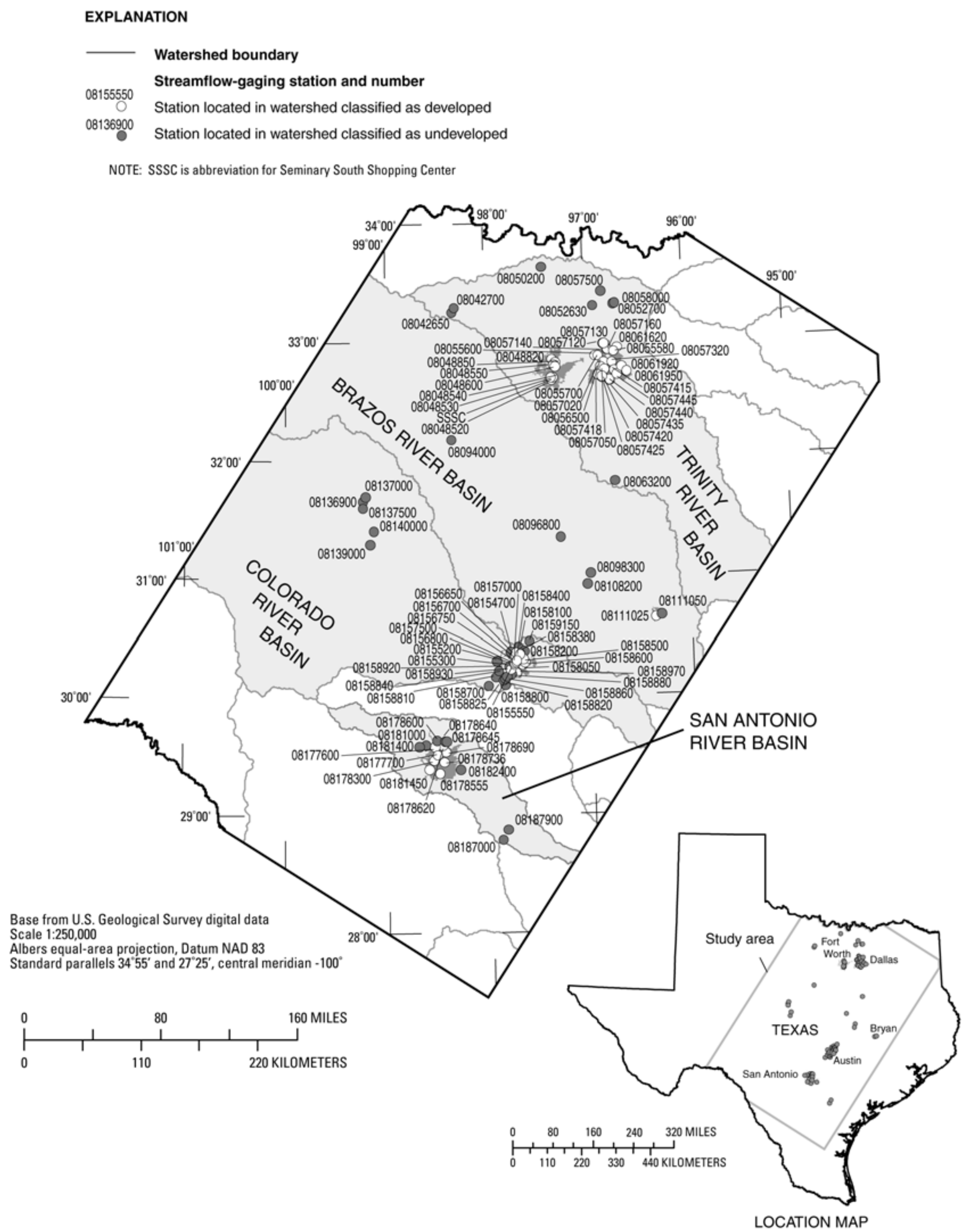


Figure 1. Locations of U.S. Geological Survey streamflow-gaging stations used in the study.

An instantaneous unit hydrograph is the response of the watershed to a unit pulse of excess rainfall with an effective duration of zero (Clarke, 1945). Additional information concerning instantaneous unit hydrographs can be found in Nash (1957; 1959), Rodriguez-Iturbe and Valdes (1979), Valdes and others (1979), Gupta and others (1980), Lee and Yen (1997), Yen and Lee (1997), and references therein. For the instantaneous unit hydrograph analysis described in this report, the Rayleigh distribution (Evans and others, 2000) was used. Use of a Rayleigh distribution in a unit hydrograph context is found in Leinhard and Meyer (1967), Leinhard (1972), and He (2004). Although Leinhard (1972) names the distribution a "hydrograph" distribution, upon close examination, it is the Rayleigh distribution as described by He (2004). The specific details of Rayleigh unit hydrograph formulation are provided in section "Instantaneous Unit Hydrograph based Rayleigh Unit Hydrographs" in this report.

Linear programming is a method of finding solutions to over-determined systems of linear algebraic equations or inequalities by minimization of an objective or merit function. Coincident rainfall and runoff time series can be cast in a form that allows the techniques of linear programming to be used to calculate a mathematically optimized unit hydrograph (Eagleson and others, 1966; Deininger, 1969; and Singh, 1976). Linear programming for unit hydrograph extraction from rainfall and runoff data also has been considered by Mays and Coles (1980). A nonlinear programming method is described by Mays and Taur (1982); nonlinear programming is not considered further. Chow and others (1988) provide a general description of unit hydrograph derivation using linear programming. The linear programming algorithms used for the approach are described in section "Linear Programming Based Gamma Unit Hydrographs" in this report and are derived from Khanal (2004).

UNIT HYDROGRAPH MODELING APPROACHES

Four independent approaches for unit hydrograph estimation from observed rainfall and runoff data are described in this section. Each approach was led by a separate group within the TTU, LU, UH, and USGS research consortium. However, considerable and important cross-communication concerning each approach was made. The communication in turn functions as quality control and quality assurance. Custom computer programs used for each approach were developed independently—that is, source code was mutually exclusive. This is an important observation because the results of the approaches complement each other, and therefore, the complex software required to implement the approaches essentially is confirmed.

Traditional Unit Hydrograph Approach

Researchers at TTU led a comparatively straightforward approach for 5-minute unit hydrograph development with heavy dependence on analyst input—the approach is not automated. Given a record of storm rainfall and runoff, a simple method—the "traditional method" (Viessman and Lewis, 2003)—can be applied to the whole storm to extract the unit hydrograph of the watershed for that storm. The traditional method does not use the sophisticated mathematics described in other sections ("Gamma Unit Hydrograph Analysis System," "Linear Programming Based Gamma Unit Hydrograph Approach," and "Instantaneous Unit Hydrograph Based Rayleigh Unit Hydrograph Approach" in this report). The traditional method is an approach used by analysts prior to development of more computationally sophisticated techniques for computing unit hydrographs from observed rainfall and runoff data for a watershed.

The rainfall and runoff values from the database for each storm described in section "Rainfall Runoff Database" in this report were converted through linear interpolation to 10-minute time intervals. For each watershed, the database was reviewed for candidate storms for traditional unit hydrograph analysis. In particular, desirable storms had rainfall durations substantially less than an estimated lag time of the watershed. Additionally, substantial direct runoff is desirable. Ideally, direct runoff for unit hydrograph analysis should be approximately 1 watershed inch. Viessman and Lewis (2003) suggest that direct runoff should be between 0.5 and 1.75 watershed inches. Because the Texas watersheds considered, in general, have a semiarid climate, storms with about 1 inch of direct runoff are seldom available. Therefore, for each watershed, storms that produced substantial direct runoff preferentially were selected for the traditional analysis. Finally, to make a reliable estimate of the unit hydrograph for a watershed, a sufficient number (on the order of five or more) desirable storms are required.

The traditional unit hydrograph approach comprises four steps. For each desirable event, the following steps are performed.

1. Base flow was abstracted (removed) from the runoff hydrograph to produce the direct runoff hydrograph. Base flow generally is small in comparison to total direct runoff.

2. The area under the direct runoff hydrograph is numerically integrated using the trapezoid rule to compute the total direct runoff.

3. Each ordinate of the direct runoff hydrograph is divided by the total direct runoff. A hydrograph with unit depth is produced—the unit hydrograph.

4. The phi-index method (see section "Gamma Unit Hydrograph Analysis System" in this report) provided a constant-loss rate. This loss rate is applied to the rainfall hyetograph to determine the number of 10-minute pulses of excess rainfall. The number of pulses of excess rainfall multiplied by 10 minutes is the duration of the unit hydrograph produced in step 3.

For each storm, the resulting n-minute unit hydrograph subsequently is converted to a 5-minute unit hydrograph. This conversion is necessary for consistency with two of the other 5-minute gamma unit hydrograph approaches described in this report, and so that a single representative unit hydrograph for each watershed is determined. The representative 5-minute unit hydrograph is computed by averaging the set of 5-minute unit hydrographs available for each watershed.

To facilitate the statistical regionalization of 5-minute unit hydrographs produced by the traditional approach, the Q_p (peak discharge) and T_p of the unit hydrograph are converted to a gamma hydrograph shape parameter (K). The gamma hydrograph is described in section “Gamma Unit Hydrograph Analysis System” in this report. This conversion is made so that an algebraic representation of the unit hydrograph is possible. The regional analysis of the shape parameter and the T_p values from the traditional approach is described in section “Gamma Unit Hydrograph Parameters from Traditional Approach” in this report.

To conclude this section, several observations on the traditional unit hydrograph approach are useful. First, for the analysis of observed rainfall and runoff, the approach does not assume a specific shape of the unit hydrograph for the calculations in contrast to the approaches described in sections “Gamma Unit Hydrograph Analysis System” and “Instantaneous Unit Hydrograph Based Rayleigh Unit Hydrograph Approach” in this report. Second, compared to the three other approaches, the traditional method extracts the unit hydrograph from the direct runoff hydrograph; therefore, the method can be thought of as a “backwards” approach. Third, the traditional approach was applied to storms producing the larger values of total depth of runoff in the database; the other three approaches, in general, used all available or computationally suitable storms. The distinction between the term computationally suitable is approach specific. The conditioning of the database towards the largest events might be expected to yield unit hydrographs having shorter times to peak, larger peak discharges, or both characteristics. Fourth, the authors speculate that errors in the approach are intrinsically attributed to misspecification of the spatial rainfall from the limited number of rainfall stations in the watersheds and to misspecification of the constant-loss rate compared to unknown losses in the watershed.

Gamma Unit Hydrograph Analysis System

Researchers at the USGS led an algebraically straightforward analyst-directed approach for 5-minute unit hydrograph development involving a gamma-distribution based unit hydrograph—a gamma unit hydrograph (GUH). The approach relied on a custom-built software system called the Gamma Unit Hydrograph Analysis System (GUHAS). GUHAS (Trejo, 2004) uses the rainfall and runoff data described in section “Rainfall and Runoff Database” in this report, but both the rainfall and runoff data were converted through linear interpolation to 5-minute intervals. The form of the GUH used by GUHAS is contemporaneously discussed by Haan and others (1994). The GUH provides curvilinear shapes that mimic the general shape of many observed hydrographs (unit or otherwise). The GUH has two parameters that can be expressed variously but considered here in terms of peak discharge (q_p in units of length over time) and T_p. Expression and analysis of unit hydrographs in terms of q_p and T_p are advantageous because of the critical importance of peak discharge and the timing of the peak for hydrologic engineering design.

The GUH model for the GUHAS approach is

$$\frac{q(t)}{q_p} = \left[\frac{t}{T_p}e^{1-\left(\frac{t}{T_p}\right)}\right]^K, \tag{2}$$

where q_p is peak discharge in depth per hour; T_p is time to peak in hours; K is the shape parameter; "*e*" is the natural logarithmic base, 2.71828; and $q(t)$ is the inches per hour discharge at time t. This equation produces the ordinates of the unit hydrograph with a constraint on the shape factor. The shape parameter is a function of q_p and T_p and a function of total runoff volume (V). (V is unity for a unit hydrograph.) K is defined through the following

$$V = 1 = q_p T_p \Gamma(K)\left(\frac{e}{K}\right)^K, \tag{3}$$

where "*e*" is the natural logarithmic base, 2.71828; and $\Gamma(K)$ is the complete gamma function for K. The complete gamma function is described in numerous mathematical texts (Abramowitz and Stegun, 1964). The complete gamma function for a value x is depicted in figure 2; the grid lines are superimposed to facilitate numerical lookup. The time scale of the unit hydrograph is expressed by the T_p value. A numerical root solver is required to compute K in eq. 3.

The GUH becomes increasingly symmetrical for large values of K, unlike the shape of many long recession-limb observed hydrographs. Therefore, large values (greater than about 20) of K are not anticipated for real-world watersheds. The shapes of selected GUH for selected K values are shown in figure 3.

For the GUHAS approach, base flow generally was small (near zero) and assumed to be zero. When the assumption was not attainable on a case-by-case basis, a straight-line base-flow separation was performed. To implement a GUH in practice, a rainfall loss model for the watershed is required because it is necessary to convert observed rainfall to excess rainfall. A constant-loss rate was selected for simplicity in converting the rainfall time series into an excess rainfall time series. Conventionally, a constant-loss rate is determined by

$$L = \frac{P-R}{D}, \tag{4}$$

where L is the loss-rate (depth per time), P is rainfall (depth), R is runoff (depth), and D (time) is duration of the storm. However, the above equation for determining the rate of rainfall abstraction could not be used for the data considered here because the recorded data contained time intervals where $P = 0$ (loss cannot occur) or $(P/D) - L < 0$ (loss rate in excess of rainfall rate) or both.

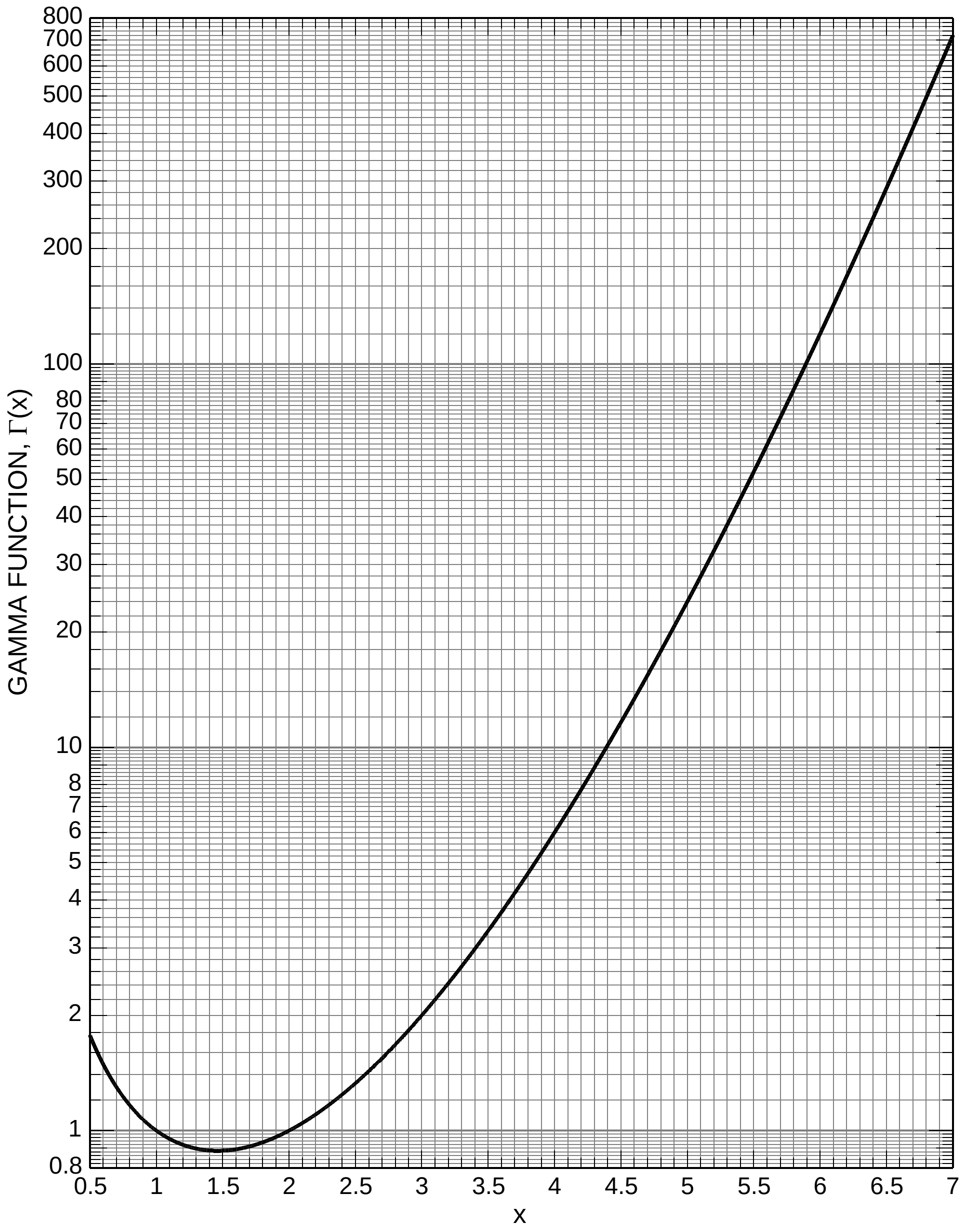


Figure 2. Complete gamma function

For the GUHAS analysis, the phi-index method (Viessman and Lewis, 2003) was implemented instead of eq. 4. With the GUHAS the recorded total rainfall data, minus an analyst-selected initial abstraction, are converted to excess rainfall hyetographs using an iteratively determined loss rate such that the depth of excess rainfall matches the depth of total runoff for the storm with the constraint that incremental excess rainfall is non-negative. The analyst made the judgement on the basis of the position of the initial rise of the modeled hydrograph compared to the observed hydrograph. The analyst manually adjusted the initial abstraction. After the loss rate is determined, it is subtracted from the observed rainfall hyetograph to create an excess rainfall hyetograph.

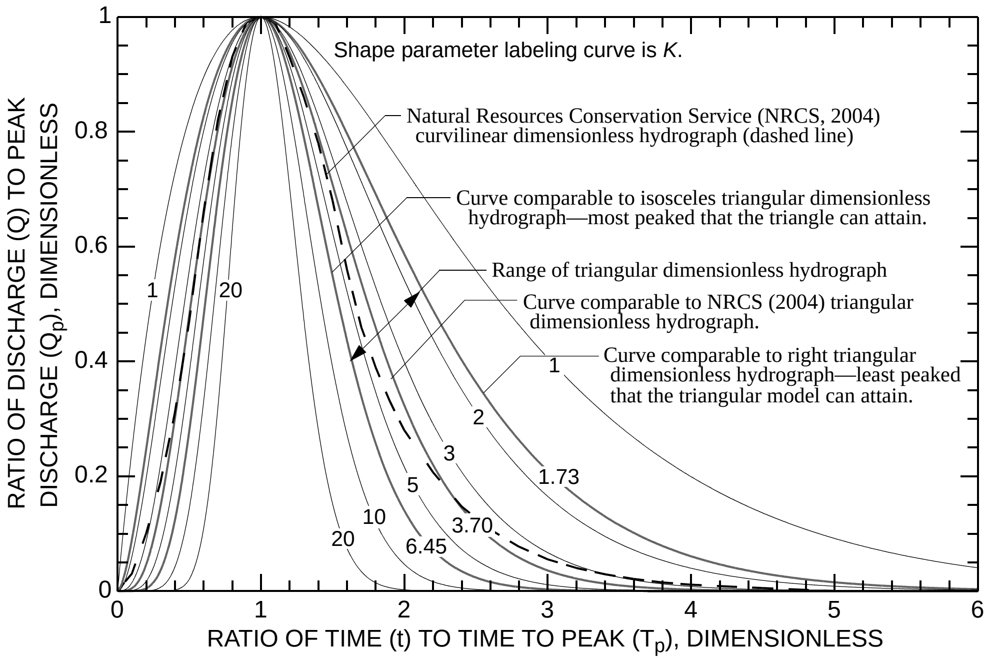


Figure 3. Shape of the gamma dimensionless hydrograph for selected values of shape parameter.

GUHAS computed the excess rainfall hyetograph as described. The resulting excess rainfall hyetograph was successively convolved with analyst-directed GUHs to create simulated streamflow hydrographs for most runoff peaks in the database for each watershed. The optimal GUH for each peak was specified by q_p (depth per hour) and T_p values that produced a simulated runoff hydrograph that matched the Q_p (peak in cubic length per second) and T_p of the observed runoff hydrograph. Many storms in the database have two or more peaks; each peak was analyzed separately. Over a 2-year period, the entire database of more than 1,600 storms (files) for 93 watersheds and some 2,013 analytically suitable peaks was processed. The mean q_p and T_p values of the GUH for each watershed were computed, and K was computed using the two mean values (means of q_p and T_p). The K and mean T_p values provided the basis for statistical analysis described in this report.

To conclude this section, several observations on the GUHAS approach are required. First, GUHAS is analyst-directed; the analyst reviews and manually sets up the analysis, including an initial abstraction, for each suitable storm peak. Second, the approach assumes a specific shape of the 5-minute unit hydrograph for the calculations in contrast to the approach described in section “Traditional Unit Hydrograph Approach” in this report. Third, compared to the traditional unit hydrograph approach, the unit hydrograph is extracted from the excess rainfall hyetograph; therefore, the GUHAS can be thought of as a “forward” approach. Fourth, GUHAS was applied to virtually all analytically suitable peaks contained in the database; the other three whole-storm approaches considered multi-peak storms in totality. Fifth, the authors speculate that errors in the approach are intrinsically attributed to misspecification of the spatial rainfall from the limited number of rainfall stations in the watersheds, to misspecification of the constant-loss rate compared to unknown losses in the watershed, and to lack of fit on the tail of the observed hydrograph. The lack of tail fit exists because GUHAS estimates q_p and T_p by minimizing on observed peak discharge and the time of peak occurrence in contrast to the minimization of objective or merit function approaches described in sections “Linear Programming Based Gamma Unit Hydrographs Approach” and “Instantaneous Unit Hydrograph Based Rayleigh Unit Hydrograph Approach” in this report.

Linear Programming Based Gamma Unit Hydrograph Approach

Researchers at LU led a computationally complex approach for 5-minute unit hydrograph development on the basis of linear programming (LP). Custom FORTRAN computer programs were written to extract unit hydrographs by implementing LP subroutines. The LP subroutines (LPPRIM) were developed by the Division of Information Technology at the University of Wisconsin, Madison. LPPRIM, which is FORTRAN 77 (version 92.05), uses the revised primal phase 1–phase 2 simplex method with inverse explicit form (Gass, 1969).

The main computational program developed uses input data as a form of cumulative rainfall and runoff, in depth, for each storm. The rainfall and runoff data are described in section “Rainfall and Runoff Database” in this report. For the LP approach the rainfall and runoff data were converted through linear interpolation to 5-minute intervals. Cumulative runoff depths (the native data storage unit) were converted into incremental discharge (cubic length per time). Some small discharges at the beginning and end of the hydrograph were truncated as base flow. Because most watersheds studied have small drainage areas and generally have zero or small base flow, a constant base-flow separation from streamflow hydrograph to direct runoff hydrograph was not performed. An initial loss or abstraction and a constant loss rate (see section “Gamma Unit Hydrograph Analysis System” in this report) converted the observed rainfall to excess rainfall. The initial loss was a constant percentage of the total rainfall and was specified by the analyst for all storms and for all watersheds studied. A 5-percent of total rainfall initial loss was used for the results provided in this report. The 5-percent value is arbitrary, but provides a rough approximation to an accepted watershed process. No consideration of antecedent rainfall or percent total runoff was made.

The LP approach selects suitable storms. Because LP provides 5-minute unit hydrographs only with $N - M + 1$ ordinates (see eq. 1), there are some events with too little data for the LP method to work (about 24 percent of the database). Also some unit hydrographs developed by the approach were not suitable for eventual fitting of a GUH. Such unsuitable unit hydrographs include those for which the T_p is greater than one-half of the time base of the unit hydrograph or where more than one-half of the unit hydrograph ordinates are zero.

With the LP approach, a solution for the 5-minute unit hydrograph is sought that minimizes the error between observed and estimated runoff hydrographs ($Q_n - Q_n^*$) through constraints ensuring that unit hydrograph ordinates are positive. The popular least-squares method (Chow and others, 1988, p. 221–222) for unit hydrograph estimation can produce negative unit hydrograph ordinates—a physical inconsistency. The general LP model is stated in the form of a linear objective function to be minimized subject to linear constraints. For this study, four objective functions were evaluated. These are minimization of (1) sum of absolute deviations, (2) largest absolute deviation, (3) range of deviations, and (4) weighted sum of absolute deviation in which weights are proportional to magnitude of peak discharge raised to a power (Zhao and Tung, 1994). Sensitivity analysis (results not reported here) suggested that minimization of the range of deviations produces, in general, the most appropriate LP-derived 5-minute unit hydrograph for each storm. The range of deviations is computed according to

$$Q_n^* - \varepsilon_{max}^+ \le Q_n \text{ and} \tag{5}$$

$$Q_n^* + \varepsilon_{max}^- \ge Q_n , \tag{6}$$

where ε_{max}^+ and ε_{max}^- are the largest positive and negative deviations.

The LP approach initially produces 5-minute unit hydrographs for each watershed having irregular (unsmooth) ordinates. These unit hydrographs were subsequently smoothed by fitting a GUH. The GUH model is the same as that used for the GUHAS approach (see eqs. 2 and 3). For the irregular unit hydrographs, mean values for Q_p and T_p were computed for each watershed. Values for gamma dimensionless hydrograph K were computed, and T_p and K provide the basis for statistical analysis described in this report.

To conclude this section, several observations on the LP approach are useful. First, the LP approach is almost entirely automated, and thus it is between the GUHAS and IUH approaches in analyst involvement. Second, each storm is analyzed in its entirety; multiple peaks in a storm that could potentially serve as subset storms and be analyzed independently are ignored in contrast to the GUHAS approach. Third, the approach produces irregular (unsmooth) 5-minute unit hydrograph ordinates like the traditional approach but not the model-specific instantaneous unit hydrograph (IUH) or GUHAS approaches, which require smoothing by fitting a parametric function (gamma distribution in this case) prior to regional analysis.

Instantaneous Unit Hydrograph Based Rayleigh Unit Hydrograph Approach

Researchers at the UH led a computationally complex approach for 1-minute unit hydrograph development using a Rayleigh-distribution-based unit hydrograph. The approach is referred to as the IUH approach. The IUH approach relies on a set of custom-written FORTRAN programs to de-convolve the rainfall and runoff data and construct the hydrograph parameters. The analysis is based on the method used by Weaver (2003) and described by O'Donnell (1960), where each rainfall increment is treated as an individual storm and the runoff from these individual storms are convolved using a unit hydrograph to produce the model of the observed storm. The IUH approach requires that both the rainfall and runoff data were converted through linear interpolation to a 1-minute interval. The 1-minute interval was selected because it was a small finite interval that approximated the limiting behavior of an instantaneous unit hydrograph.

The IUH approach is conceptualized from a finite interval unit hydrograph as

$$q_T(t) = \frac{S(t) - S(t-T)}{T}, \tag{7}$$

where $q_T(t)$ is the depth per time T and at some time t, T is some finite time interval, and $S(t)$ is the S-hydrograph (a cumulative hydrograph). The S-hydrograph for the IUH approach was inferred from the cumulative runoff data for each station in the database. The basis for linear interpolation was the range between the observed runoff values for T. The limiting case as the duration vanishes is by definition the IUH

$$q(t) = \lim_{T \to 0} \frac{S(t) - S(t-T)}{T} = \frac{d}{dt}[S(t)]. \tag{8}$$

For the IUH approach, T is 1 minute and was selected as being a good approximation to the limiting value; hence the IUH approach results in this report are 1-minute unit hydrographs that are assumed to be valid representations of the instantaneous unit hydrographs.

The assumption was tested by analyzing five storms for station 08057320 using both 1-minute and 5-minute durations. Comparing preliminary Rayleigh-distribution application, these two hydrographs are indistinguishable for all practical purposes. Further, even at T of 15 minutes, the resulting Rayleigh-distribution modeled hydrographs are not distinguishable. Other durations for the IUH approach are not elaborated on further in this report.

The form of the instantaneous unit hydrograph used for the IUH approach is based on a cascade of a hybrid linear and translation reservoirs and is described by He (2004) and similar to one derived using statistical-mechanics by Leinhard (1972). The IUH provides curvilinear shapes that mimic the shape of observed hydrographs. The IUH has two parameters—one controls the time scale $\bar{T}$ (a mean residence time), and the other controls shape N (the reservoir number). He (2004) provides further details.

The Rayleigh distribution was chosen from among several possible gamma-family distributions because it performed marginally better in peak discharge bias (difference between observed peak discharge and modeled peak discharge at the time of observed peak discharge) and in peak time bias (difference between the observed time of peak discharge and modeled time of peak discharge). The equation for the Rayleigh unit hydrograph (with rescalable finite interval) is

$$q(t) = \frac{2}{\bar{T} \times \Gamma(N)}\left(\frac{t}{\bar{T}}\right)^{2N-1} e^{-(t/\bar{T})^2}, \tag{9}$$

where $q(t)$ (depth per time) is runoff at time t; $\bar{T}$ is a time parameter (mean residence time); N is a shape parameter (reservoir number); "*e*" is the natural logarithmic base, 2.71828; and $\Gamma(N)$ is the complete gamma function for N (see fig. 2). The IUH "native" parameters can be transformed into the conventional q_p and T_p formulation by the following transformations. The relation between $\bar{T}$ and T_p is

$$T_p = \bar{T}\sqrt{\frac{2N-1}{2}}, \text{ and} \tag{10}$$

q_p is computed by

$$q_p = \frac{(2N-1)^N}{2^{N-1}\Gamma(N)}\frac{1}{T_p}e^{-(2N-1)/2}. \tag{11}$$

The IUH becomes increasingly symmetrical for larger values of N, and therefore, unlike the shape of many right skewed (longer recession limb) observed hydrographs. As a result, large values (greater than about 5) for N are not anticipated. Leinhard (1972) suggested that values greater than 3 have limited interpretation from arguments of statistical mechanics. The shapes of the Rayleigh dimensionless hydrograph for selected values of N are shown in figure 4.

To implement the IUH, a rainfall loss model is required. A proportional loss model was selected (McCuen, 2005). In the loss model some constant ratio of rainfall becomes runoff. The model was selected for simplicity with regards to automated analysis. The model is represented as

$$L(t) = (1 - C_r)p(t) \text{ and} \tag{12}$$

$$C_r = \frac{\int Q(t)dt}{A\int p(t)dt}, \tag{13}$$

where $L(t)$ is a rainfall loss rate (depth per time); $p(t)$ is observed rainfall rate (depth per time); C_r is fraction of rainfall converted to runoff; $A\int p(t)dt$ is the cumulative rainfall volume for the storm (P) and A is watershed area; and $\int Q(t)dt$ is the cumulative runoff (depth) of the storm.

Using the loss model, the excess rainfall hyetograph was computed. The excess rainfall hyetograph is convolved using a FORTRAN program to generate simulated streamflow hydrographs in the database for each watershed.

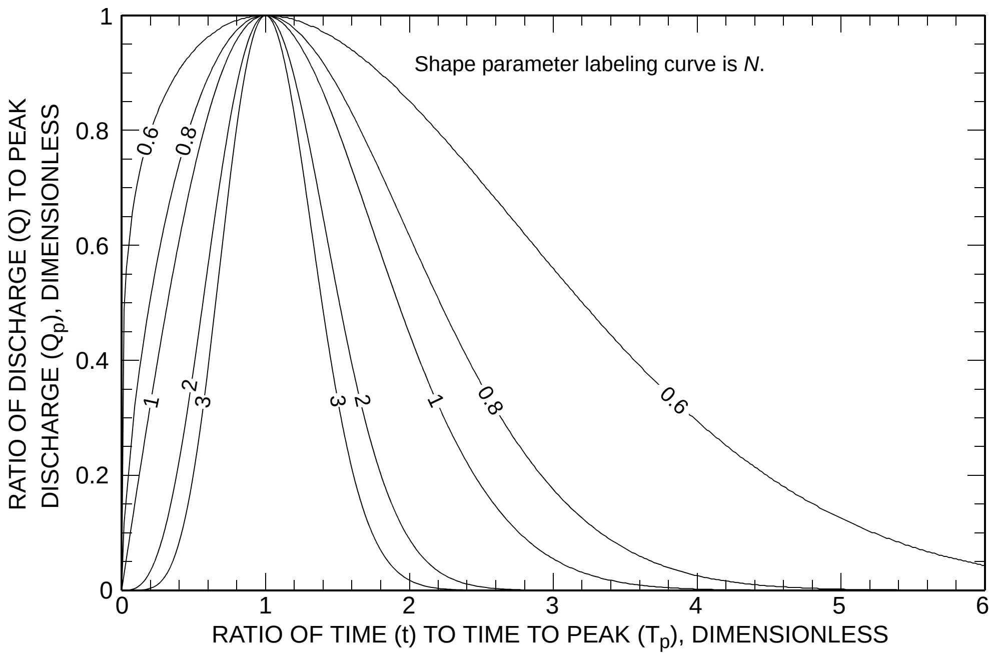


Figure 4. Shape of the Rayleigh dimensionless hydrograph for selected values of shape parameter.

The parameters for each storm are determined by a second FORTRAN program that systematically adjusts parameter values in the IUH until the maximum absolute deviation at peak discharge (Q_pMAD) is minimized—a merit function. The merit function is

$$Q_pMAD = \left|Q_m(t_p) - Q_o(t_p)\right| , \tag{14}$$

where Q is the discharge (cubic length per time); the subscripts m and o represent model and observed discharge, respectively; and t_p is the actual time in the observations when the observed peak discharge occurs. Although a peak time is expressed, the nomenclature of t_p is different from T_p to distinguish between observed peak of the storm (t_p) and time to peak of a unit hydrograph model (T_p). The Q_pMAD merit function is designed to favor matching the peak discharge magnitude with little regard for the rest of the hydrograph. A search technique was used instead of more elegant or adaptive methods such as reduced gradient minimization or simplex minimization, principally to ensure a result for each storm. The search systematically computes the value of a merit function using every permutation of model parameters described by the set-builder notation

$$\bar{T} \in \{1, 2, 3, \ldots, 720\} \text{ and} \tag{15}$$

$$N \in \{1.00, 1.01, 1.02, \ldots, 9.00\} . \tag{16}$$

Fractions of a minute were ignored (hence the 1-minute interval in $\bar{T}$) and less than 0.01 resolution in the shape parameter was considered unnecessary. The set of parameters that produces the smallest Q_pMAD is retained as the optimal set for a storm. This approach, although computationally expensive, is robust. The $\bar{T}$ and N parameters were computed using a purpose-built Linux cluster computer to speed up the computational throughput. In several weeks of computer processing time, the entire database of 93 watersheds and some 1,563 computationally suitable storms can be processed automatically. Finally, $\bar{T}$ and N values were extracted for each storm.

The mean values for $\bar{T}$ and N for each storm subsequently are computed for each watershed. Then T_p is computed, and T_p and N provide the basis for statistical analysis described here.

To conclude this section, several observations on the IUH approach are useful. First, the IUH approach reported here was designed to be entirely automated. Once the database is prepared, the computations are run without analyst intervention in contrast to the GUHAS approach. Second, some of the storms were pathologically unsuitable (peak rainfall rate after peak runoff rate); however, because of program robustness the program still produces a result. These storms were manually removed when detected by graphical data analysis. Third, each storm is analyzed in its entirety; multiple peaks in a storm that could potentially serve as subset storms and analyzed independently are not used in contrast to the GUHAS approach.

REGIONAL ANALYSIS OF UNIT HYDROGRAPH PARAMETERS

Multiple-linear regression analysis (Helsel and Hirsch, 1992; Montgomery and others, 2001; Maindonald and Braun, 2003) is used to establish the statistical relations between one regressor and one or more predictor variables. For the regional analysis of unit hydrograph parameters reported here, a single shape parameter (K or N) and T_p are used as regressor variables, and various watershed characteristics, such as drainage area, watershed perimeter, main channel length, watershed shape, dimensionless main channel slope, and others, are used as candidate predictor variables. An additional predictor variable is a factor or state variable representing a binary classification of watershed development (undeveloped and developed). The classification scheme parallels and accommodates the disparate discussion and conceptualization seen in the reports that provided the data base (see section “Rainfall and Runoff Database” in this report). Logarithmic transformations (base 10, exclusively) of the variables are used to increase the linearity between the variables. Logarithmic transformation was not performed on the watershed development classification. After preliminary analysis (results not provided here) including collinearity assessment, only main channel length, dimensionless main channel slope, and watershed development classification were formally considered.

The regression analysis was performed with the R system software package (R Development Core Team, 2004). Both ordinary-least squares and weighted-least squares regression procedures were evaluated during the iterative development of the equations reported here. Weighted-least squares is preferred because of the differing number of peaks or storms (some storms in the database have multiple peaks) used to compute the mean shape parameter and time to peak for each watershed. Greater confidence, therefore larger statistical weight, is appropriate for watersheds having more peaks. The number of peaks analyzed per watershed provides a convenient basis for generating regression weight factors.

Residual analysis is an important component of regression analysis. The `plot.lm()` function of the R system provided the basis for the residual analysis (results not reported here). The residual analysis considered the standard residual plots, standardized residual plots, and quantile-quantile residual plots. The residual analysis indicated constant variance (homoscedasticity), and the residuals were centered about zero. The residual analysis assisted in identification of outliers.

Analysis of the regression diagnostics also was performed, and the `influence.measures()` function of the R system provided the basis. The analysis of regression diagnostics considered statistics as variance inflation factors (VIF), additional measures of multicollinearity, Akaike Information Criterion (`AIC()` function of the R system), and PRESS as part of evaluating candidate regression equations. Additional regression diagnostics included DFBETAS and DFFITS (`dfbetas()` and `dffits()` functions of the R system) and Cook's distance. The diagnostic analysis enhanced the predictive performance of the regression equations reported here. For example, PRESS statistics are a measure of how well a regression model will perform predicting new data; small values of PRESS are desirable.

The results of each of the four approaches are reported in the following four sections. The collective values of hydrograph shape parameter, T_p, and number of analyzed storms or events by station for each of the approaches are listed in table 3. For the traditional and GUHAS approaches the GUH shape parameter (K) was derived from mean Q_p (traditional approach) or q_p (GUHAS) and T_p from the analysis; hence these K values are not referred to as "mean K" in the table.

Table 3. Unit hydrograph parameters for U.S. Geological Survey streamflow-gaging stations by each of four unit hydrograph approaches

[K, gamma dimensionless hydrograph shape parameter; T_p, time to peak; Count, number of storms (traditional, LP, and IUH approaches) or number of peaks analyzed (GUHAS approach); N, Rayleigh dimensionless hydrograph shape parameter; --, not available]

Station no.	Traditional approach			GUHAS approach			LP approach			IUH approach		
	K	Mean T_p	Count	K	Mean T_p	Count	Mean K	Mean T_p	Count	Mean N	Mean T_p	Count
08042650	10.14	0.67	1	3.807	1.83	18	4.50	1.69	12	2.950	1.60	14
08042700	3.307	1.84	13	4.999	4.08	57	3.11	3.67	48	2.926	3.30	57
08048520	3.267	1.29	14	1.527	1.53	31	1.94	1.71	21	2.023	1.36	22
08048530	1.904	.19	12	1.278	.24	33	2.70	.35	28	2.338	.22	26
08048540	3.760	.23	16	2.246	.32	33	3.26	.37	30	1.818	.26	22
08048550	4.810	.65	10	2.481	.93	27	2.07	.84	22	2.865	.86	23
08048600	5.875	1.44	5	1.169	1.39	28	4.21	1.78	24	2.292	1.29	26
08048820	11.19	2.07	6	1.796	2.55	18	3.04	3.56	13	2.538	2.44	16
08048850	2.478	1.15	9	1.509	2.12	24	3.92	3.70	18	2.664	2.51	23
08050200	13.96	.40	5	8.192	1.62	25	5.50	1.31	15	2.229	1.21	31
08052630	11.66	.30	7	5.564	1.60	31	4.74	1.73	25	2.617	1.12	23
08052700	4.529	6.86	22	4.838	15.3	57	3.31	14.2	37	3.019	8.65	52
08055580	7.109	.32	5	2.043	.39	7	2.43	.43	6	2.200	.32	6
08055600	4.186	.67	5	3.189	.91	9	3.00	1.08	6	2.830	.74	10
08055700	8.267	.65	22	4.769	1.45	56	4.45	1.58	42	2.611	1.39	35
08056500	5.956	.55	36	2.801	1.01	62	4.08	1.21	54	2.753	1.33	38
08057020	3.825	.58	2	2.389	.78	8	3.12	.83	3	2.750	.77	6
08057050	2.626	.17	1	2.823	.67	3	4.43	.75	3	2.650	.51	2
08057120	2.008	.54	2	10.56	1.53	6	.91	1.04	2	1.600	.83	4
08057130	3.414	.22	6	8.194	.84	13	1.82	.74	6	2.350	.61	6
08057140	3.513	.50	2	2.866	1.12	9	2.99	1.53	6	2.250	.94	6
08057160	3.780	.47	6	4.283	1.05	11	5.44	1.23	8	2.329	.83	7
08057320	4.163	.32	7	4.047	.92	11	4.54	.86	6	2.425	.53	4
08057415	1.006	.10	7	1.841	.28	11	3.62	.35	6	1.750	.21	6
08057418	5.742	.62	5	2.081	.91	8	5.65	1.29	7	2.586	.83	7
08057420	6.534	.68	7	3.637	1.46	11	4.23	1.55	9	2.740	.96	10
08057425	4.781	.65	8	2.888	1.03	15	4.55	1.02	7	2.367	.79	9
08057435	5.693	.63	2	2.412	1.16	4	2.36	1.19	4	3.000	1.38	3
08057440	5.197	.36	3	12.18	1.50	7	2.84	1.53	3	4.000	1.52	4
08057445	3.347	1.09	5	1.585	2.38	8	1.12	2.21	8	2.200	2.00	8
08057500	8.049	.44	4	2.102	.97	34	3.25	1.31	23	2.393	.83	28
08058000	15.23	.58	9	8.603	1.08	33	3.12	1.53	9	2.465	.93	26
08061620	5.576	.69	9	2.238	1.04	10	2.60	1.14	8	1.938	1.11	8
08061920	4.343	.94	7	5.183	2.90	15	4.17	2.90	6	3.014	2.64	8
08061950	14.58	1.80	9	4.598	6.08	31	3.63	5.95	25	3.309	5.22	22
08063200	6.178	3.05	11	3.412	5.89	47	5.15	6.99	20	3.177	4.59	31
08094000	22.66	.73	10	5.924	2.22	34	3.41	1.80	16	2.789	1.47	27
08096800	6.059	.57	9	3.597	1.40	46	4.82	1.58	29	2.434	1.13	50
08098300	12.52	1.46	2	5.162	7.44	18	4.15	8.56	10	2.708	4.57	13
08108200	6.370	3.42	5	8.548	10.1	20	4.19	7.99	14	3.242	6.12	19
08111025	--	--	--	8.341	1.47	9	--	--	--	2.400	1.61	7
08111050	--	--	--	2.681	2.83	7	--	--	--	2.286	2.40	6
08136900	2.115	.33	1	4.924	4.97	24	4.95	3.60	19	2.020	4.12	20
08137000	5.110	.31	4	4.292	3.17	53	5.81	2.90	29	2.745	2.78	38
08137500	3.908	4.09	1	8.962	15.9	4	4.26	9.52	5	2.725	9.08	4
08139000	8.425	.31	4	5.565	1.66	35	5.60	1.76	20	2.510	1.37	29
08140000	2.696	.17	3	.648	2.33	3	2.70	3.39	23	2.270	1.41	30
08154700	4.661	.74	6	2.991	2.13	19	4.00	2.24	14	2.971	1.64	14
08155200	2.222	1.71	4	3.029	6.10	6	8.56	6.56	6	2.950	4.70	6
08155300	2.933	1.28	3	10.01	7.90	7	8.43	8.33	7	3.438	7.45	8

Station no.	Traditional approach			GUHAS approach			LP approach			IUH approach		
	K	Mean T_p	Count	K	Mean T_p	Count	Mean K	Mean T_p	Count	Mean N	Mean T_p	Count
08155550	5.520	.50	6	2.716	.79	14	2.45	.81	8	3.233	.67	9
08156650	5.693	.38	7	1.985	.73	23	3.91	.94	12	2.546	.53	13
08156700	7.972	.67	6	1.591	.58	32	1.95	1.17	7	2.994	.78	17
08156750	3.633	.42	8	1.002	.61	24	3.01	1.25	15	2.607	.82	14
08156800	6.267	.96	19	4.119	1.72	33	4.94	1.55	21	2.852	1.21	23
08157000	5.926	.42	8	2.616	.95	66	2.83	.98	39	2.559	.94	41
08157500	7.677	.48	7	2.126	.76	52	2.58	.80	41	2.320	.59	40
08158050	6.878	.90	5	5.523	1.64	14	3.34	1.68	10	3.040	1.54	10
08158100	1.527	1.09	1	3.427	2.31	20	2.76	2.07	11	3.220	2.51	15
08158200	7.195	1.60	11	4.223	2.36	25	3.18	2.57	12	3.506	2.33	17
08158380	5.085	.71	2	5.286	1.43	3	2.48	.71	2	2.800	.92	2
08158400	4.486	.48	9	1.627	.73	18	3.78	.89	12	2.350	.48	10
08158500	5.770	.68	6	1.916	1.44	23	4.02	1.72	13	2.760	.94	15
08158600	3.249	2.09	8	2.730	2.84	29	3.63	3.42	17	3.286	2.54	21
08158700	3.419	1.92	1	3.003	5.96	7	5.35	9.67	4	2.950	4.61	6
08158800	--	--	--	2.076	5.78	3	3.20	13.17	2	2.800	3.96	2
08158810	5.389	.65	5	5.318	2.17	12	2.59	2.97	5	3.550	2.16	8
08158820	--	--	--	8.897	6.50	2	.68	.96	2	4.000	6.17	2
08158825	--	--	--	14.65	2.00	3	--	--	--	4.000	1.56	2
08158840	2.937	.67	12	2.099	1.62	18	3.81	2.05	7	2.636	1.29	11
08158860	4.161	.75	5	7.659	3.24	7	8.58	2.92	1	4.000	2.98	2
08158880	5.542	.54	9	2.287	.70	22	3.94	1.14	15	2.892	1.07	13
08158920	4.812	.69	13	1.375	.80	31	3.87	1.67	13	2.864	.77	14
08158930	8.849	.99	6	5.306	1.72	33	3.33	1.86	11	3.100	1.56	18
08158970	11.31	1.87	6	5.227	3.90	22	5.66	3.18	11	3.400	3.33	16
08159150	3.025	.70	7	2.770	1.73	31	4.31	2.65	17	2.750	1.60	27
08177600	3.194	.48	5	1.471	.83	10	3.24	1.14	8	3.829	2.18	14
08177700	--	--	--	3.176	2.12	21	--	--	--	2.513	2.01	23
08178300	7.918	.45	16	2.909	.51	43	4.11	.57	30	1.100	.53	28
08178555	6.950	.67	4	4.573	2.11	12	2.07	1.79	6	3.175	2.41	8
08178600	3.808	.77	6	7.824	1.49	15	3.16	1.17	11	3.975	2.53	12
08178620	--	--	--	16.32	2.75	3	2.96	.83	1	3.700	4.31	3
08178640	--	--	--	19.81	2.08	8	3.80	1.27	4	4.000	1.41	8
08178645	1.417	.42	1	6.398	2.04	6	1.45	1.02	5	4.000	2.03	6
08178690	5.448	.28	15	.875	.23	49	1.58	.35	28	2.632	.33	34
08178736	2.805	.32	7	1.747	.44	20	.93	.52	4	2.875	.42	12
08181000	2.623	.52	4	4.823	1.48	8	1.99	1.79	6	3.160	1.06	10
08181400	3.491	.69	6	6.324	2.20	15	2.49	2.03	13	3.020	3.53	15
08181450	3.728	.56	7	2.001	1.12	32	1.15	1.05	14	2.800	1.16	28
08182400	8.129	.64	3	5.648	2.82	22	5.90	2.56	18	2.687	2.19	25
08187000	23.56	.96	6	7.236	1.63	35	3.31	1.29	13	2.771	1.11	30
08187900	8.688	.36	3	6.002	2.44	22	14.03	2.77	17	2.543	1.63	21
SSSC	14.21	.27	14	3.558	.23	29	5.45	.29	14	2.217	.17	18

Gamma Unit Hydrograph Parameters from Traditional Unit Hydrograph Approach

The traditional unit hydrograph approach produced mean values of Q_p and T_p for each watershed. Subsequently, the K shape parameter for the GUH was computed. Values for K and T_p are listed in table 3 and were used as regressor variables. The approach produced 5-minute unit hydrographs for 85 of the 93 watersheds. For 92 of the 93 watersheds, 30-meter DEM watershed characteristics were available for regression analysis. The 30-meter DEM watershed characteristics were not available for the Seminary South Shopping Center watershed in Fort Worth, Texas, although this watershed had unit hydrographs derived. Therefore, 84 watersheds were considered for the regression analysis reported here. In total, 602 storms were analyzed for the 84 watersheds. The average number of analyzed peaks per watershed is about seven.

Estimation of Gamma Dimensionless Hydrograph Shape

The relation between the shape parameter (K) and main channel length from the traditional approach is depicted in figure 5. A distinction between undeveloped and developed watersheds is made; there is no statistically significant difference in average or typical K depending on the binary watershed development classification. The weighted mean K (no log transformation) without regard to the development factor is about 6.3, and the undeveloped watershed mean K is not statistically different from the developed watershed mean K according to a one-tailed t-test (undeveloped greater than developed, p-value = .143). The K of 6.3 is larger than the mean values computed from the GUHAS and LP approaches. No statistically significant regression on K from the traditional approach was developed, unlike the other three approaches.

Estimation of Time to Peak

Regression analysis was conducted, as previously described, between T_p and the watershed characteristics for the 84 watersheds having 30-meter DEM watershed characteristics and 5-minute unit hydrographs. For the final T_p regression equation, main channel length, dimensionless main channel slope, and watershed development classification were used. The final steps of the regression computations using the R system are depicted in figure 6 (user inputs shown in bold type).

The most appropriate equation from the analysis for T_p thus is

$$T_p = 10^{(-1.63 - 0.142D)} L^{0.659} S^{-0.497}, \tag{17}$$

where T_p is time to peak in hours; D is 0 for an undeveloped watershed and 1 for a developed watershed; L is main channel length of watershed in miles; and S is the dimensionless main channel slope.

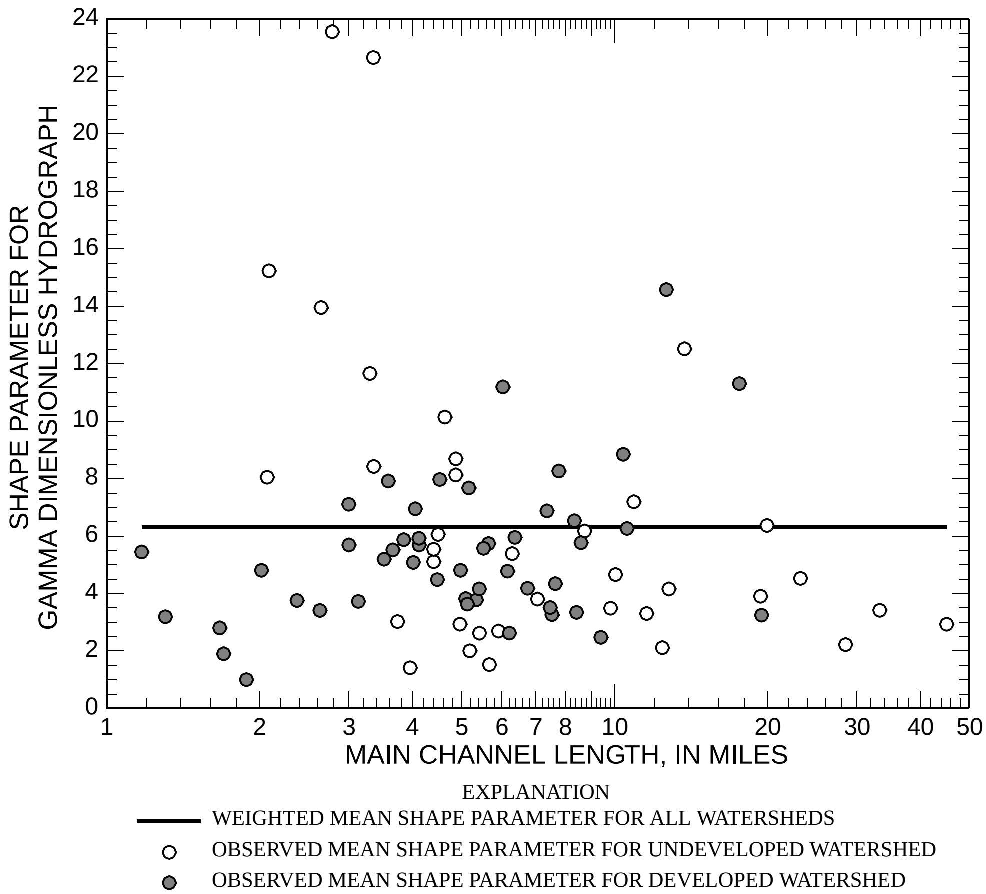


Figure 5. Relation between observed shape parameter of gamma dimensionless hydrograph and main channel length for undeveloped and developed watersheds from traditional approach.

The residual standard error of the equation is 0.1880 $\log(T_p)$ units and the adjusted R-squared is 0.698. The equation has three regressor variables; two represent watershed characteristics that could be a considerable source of multicollinearity. The VIF values, in which values greater than 10 indicate that multicollinearity is a serious problem in the equation, are 1.38, 1.30, and 1.09, for L, S, and D, respectively. These VIF values are small and indicate that multicollinearity between the predictor variables is not of concern. The PRESS statistic is 3.32. The maximum leverage value is about 0.138.

```
R : Copyright 2004, The R Foundation for Statistical Computing
Version 2.0.1  (2004-11-15), ISBN 3-900051-07-0
> WLS1.OUT <- lm(log10(MLR1_MeanTp)~log10(MLR1_MCL)+log10(MLR1_MCS2)+DU,
weights=MLR1_WEIGHTS)
> summary(WLS1.OUT)

Call:
lm(formula = log10(MLR1_MeanTp) ~ log10(MLR1_MCL) + log10(MLR1_MCS2) +
    DU, weights = MLR1_WEIGHTS)

Residuals:
     Min       1Q   Median       3Q      Max
-0.52026 -0.10001 -0.02236  0.07821  0.42547

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      -1.62551    0.22185  -7.327 1.66e-10 ***
log10(MLR1_MCL)   0.65939    0.07812   8.440 1.11e-12 ***
log10(MLR1_MCS2) -0.49696    0.11274  -4.408 3.21e-05 ***
DUU              -0.14242    0.04511  -3.157  0.00225 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.188 on 80 degrees of freedom
Multiple R-Squared: 0.7091,Adjusted R-squared: 0.6982
F-statistic: 64.99 on 3 and 80 DF,  p-value: < 2.2e-16

>
> W <- diag(MLR1_WEIGHTS)
> X <- model.matrix(WLS1.OUT)
> Xt <- t(X)
> invcov <- chol2inv( chol( Xt %*% W %*% X ) )
> invcov
            [,1]       [,2]       [,3]        [,4]
[1,]  1.39315023 0.11004006 0.67384628 -0.01090229
[2,]  0.11004006 0.17275121 0.11918778  0.02846558
[3,]  0.67384628 0.11918778 0.35974839  0.02231067
[4,] -0.01090229 0.02846558 0.02231067  0.05760350
> max(diag(X %*% invcov %*% Xt))
> 0.1380
```

Figure 6. Summary of regression execution and output for final weighted least-squares regression on time to peak of 5-minute gamma unit hydrograph from traditional approach.

The relation between T_p and main channel length is depicted in figure 7. Both main channel length and dimensionless main channel slope are present in the equation for T_p; therefore the lines for the equation cannot be depicted directly in figure 7. The results of the regression analysis on T_p are seen in figure 8, in which the vertical variation in the values is the influence of dimensionless main channel slope on the estimates of T_p. In both figures 7 and 8, a distinction between undeveloped and developed watersheds is made; there is considerable difference in average of typical T_p depending on the watershed development classification. Comparison of the two figures shows less variation in T_p in the estimated values, as expected from a regression equation. Comparison of the figures also indicates that the T_p equation is reliable.

Figure 7. Relation between observed time to peak of 5-minute gamma unit hydrograph and main channel length for undeveloped and developed watersheds from traditional approach.

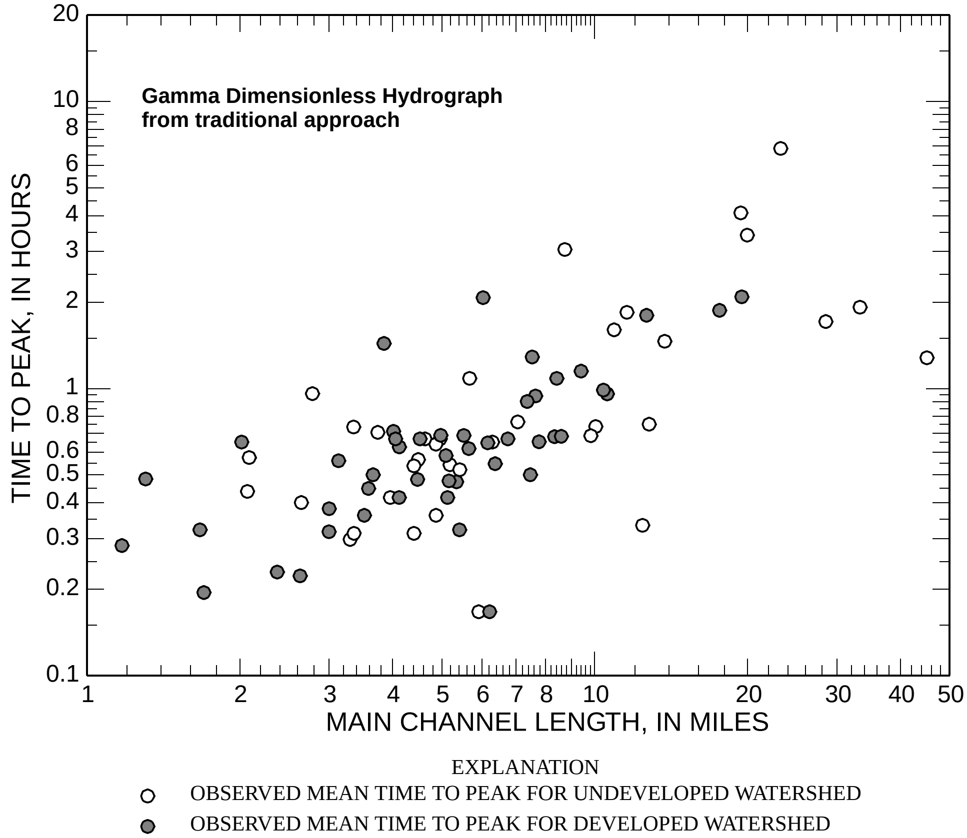
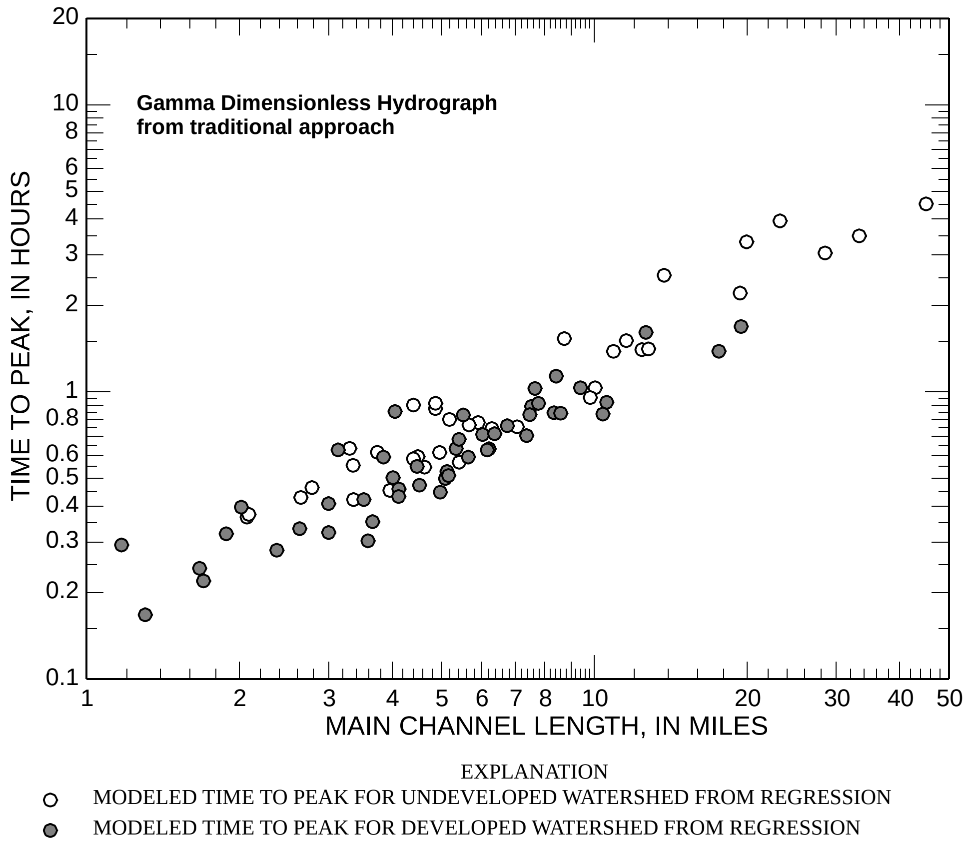


Figure 8. Relation between modeled time to peak of 5-minute gamma unit hydrograph and main channel length for undeveloped and developed watersheds from traditional approach.



The prediction limits of T_p from the equation can be useful for expressing the uncertainty when the equation is used. The prediction limits are computed by

$$10^{\log(T_p) - t_{\alpha/2,df}\,\sigma\sqrt{1+h_o}} \le T_p \le 10^{\log(T_p) + t_{\alpha/2,df}\,\sigma\sqrt{1+h_o}}, \qquad (18)$$

where T_p is the estimate from eq. 17; $t_{\alpha/2,df}$ is the t-distribution with probability $\alpha/2$ and df degrees of freedom; σ is the residual standard error; and h_o is the leverage for the watershed. Leverage for the watershed is computed from the inverted covariance matrix $(X^T WX)^{-1}$ of the regression.

The inverted covariance matrix is shown in figure 6 as `invcov`, where column 1 (row 1) is for regression intercept, column 2 (row 2) is for main channel length, column 3 (row 3) is for dimensionless main channel slope, and column 4 (row 4) is for the watersheds development classification. For brevity, the computer output resolution of the matrix has been rounded; to mitigate for round-off errors, the values in figure 6 should be used. Specifically, h_o is equal to

$$h_o = x_o(X^T WX)^{-1} x_o^T \text{ and} \qquad (19)$$

$$h_o = \begin{bmatrix} 1 & \log(L) & \log(S) & D \end{bmatrix} \begin{bmatrix} 1.3932 & 0.11004 & 0.67385 & -0.01090 \\ 0.11004 & 0.17275 & 0.11919 & 0.02847 \\ 0.67385 & 0.11919 & 0.35975 & 0.02231 \\ -0.01090 & 0.02847 & 0.02231 & 0.05760 \end{bmatrix} \begin{bmatrix} 1 \\ \log(L) \\ \log(S)) \\ D \end{bmatrix}, \qquad (20)$$

where x_o is a row vector of the intercept, L, S; and D for the watershed and x_o^T is a column vector.

It is useful to demonstrate application of the T_p equation. Suppose a hypothetical developed watershed ($D = 1$) has an L of 10 miles and a S of 0.004. The estimate of T_p thus is $T_p = 10^{(-1.63 - 0.142)}10^{0.659}0.004^{-0.497} = 1.20$ hours. The df of the equation is 80 and σ is 0.1880. Suppose the 95th-percentile prediction limits are needed. These limits have an $\alpha/2$ = (1-0.95)/2 = 0.025. The upper tail quantile of the t-distribution for $1 - \alpha/2$ = 0.975 nonexceedance probability with 80 degrees of freedom is 1.990. Finally, the leverage of the hypothetical watershed is

$$h_o = \begin{bmatrix} 1 & \log(10) & \log(0.004) & 1 \end{bmatrix} \begin{bmatrix} 1.3932 & 0.11004 & 0.67385 & -0.01090 \\ 0.11004 & 0.17275 & 0.11919 & 0.02847 \\ 0.67385 & 0.11919 & 0.35975 & 0.02231 \\ -0.01090 & 0.02847 & 0.02231 & 0.05760 \end{bmatrix} \begin{bmatrix} 1 \\ \log(10) \\ \log(0.004) \\ 1 \end{bmatrix} \qquad (21)$$

$$h_o = 0.0339. \qquad (22)$$

Truncation in $(X^T WX)^{-1}$ is shown in eq. 21, but to mitigate round-off errors, the values in figure 6 should be used. Finally, the 95th-percentile prediction limits for T_p are

$$10^{\log(1.20) - 1.990 \times 0.1880\sqrt{1 + 0.0339}} \le T_p \text{ and} \tag{23}$$

$$T_p \le 10^{\log(1.20) + 1.990 \times 0.1880\sqrt{1 + 0.0339}} . \tag{24}$$

Therefore, the 95th-percentile prediction limits of T_p for the watershed are

$$0.50 \le T_p \le 2.88 . \tag{25}$$

Gamma Unit Hydrograph Parameters from GUHAS Approach

The GUHAS approach produced mean values of q_p and T_p of 5-minute GUH for each watershed. Subsequently, the K shape parameter for the GUH was computed. Values for K and T_p, listed in table 3, were used as regressor variables. For 92 of the 93 watersheds, 30-meter DEM watershed characteristics were available for regression analysis. The 30-meter DEM watershed characteristics were not available for the Seminary South Shopping Center watershed in Fort Worth, Texas. Therefore, 92 watersheds were considered for the analysis reported here. In total, 1,984 storm peaks were analyzed for the 92 watersheds. The average number of analyzed peaks per watershed is about 21.

Interpretation of the residual and diagnostic analysis consistently indicated that station 08178690 Salado Creek tributary at Bitters Road, San Antonio, Texas, should be considered an outlier for both K and T_p estimation. This watershed has high leverage and contributes excessive influence on regression coefficients relative to the other 91 watersheds and should be removed from the regression analysis of both K and T_p; PRESS and residual standard errors were substantially reduced without station 08178690. This single station represents a small component of the overall database, and the authors believe that its removal is appropriate.

Estimation of Gamma Dimensionless Hydrograph Shape

The relation between K and main channel length from the GUHAS approach is depicted in figure 9. A distinction between undeveloped and developed watersheds is made; there is considerable difference in average or typical K depending on the binary watershed development classification. Furthermore, a slight increase in K with increasing main channel length also is evident.

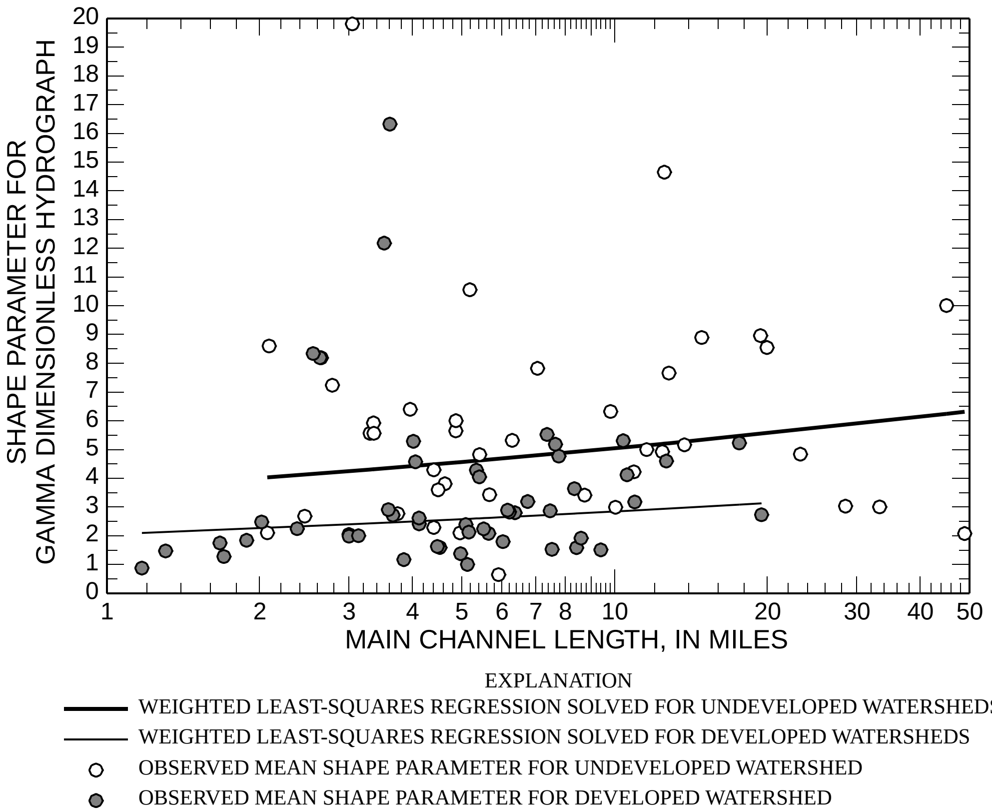


Figure 9. Relation between observed shape parameter of gamma dimensionless hydrograph and main channel length for undeveloped and developed watersheds from GUHAS approach.

Regression analysis was conducted as previously described. The final steps of the regression computations using the R system are depicted in figure 10 (user inputs shown in bold type). The most appropriate equation from the analysis for K is

$$K = 10^{(0.560 - 0.249D)} L^{0.142}, \tag{26}$$

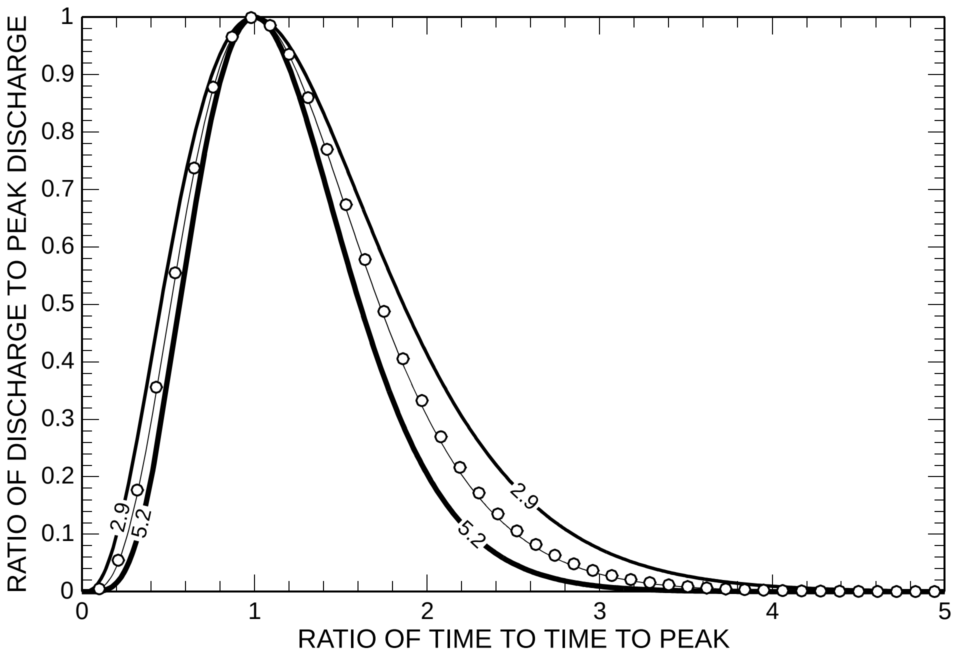
where K is the shape parameter, D is 0 for an undeveloped watershed and 1 for a developed watershed, and L is main channel length of watershed in miles. The residual standard error for the equation is 0.2052 $\log(K)$ units and the adjusted R-squared is 0.292. The maximum leverage value is about 0.132.

Superimposed on the data points in figure 9 are the two lines derived from the K regression equation. The regression was performed in log-log space, but a linear K axis is depicted. Both lines have been terminated at the minimum and maximum values for main channel length for the respective development classification.

```
R : Copyright 2004, The R Foundation for Statistical Computing
Version 2.0.1  (2004-11-15), ISBN 3-900051-07-0

> KWLS3.OUT <- lm(log10(KWLS_MEAN.K)~log10(KWLS_MCL) + KWLS_DU,
weights=KWLS_WEIGHTS)
> summary(KWLS3.OUT)

Call:
lm(formula = log10(KWLS_MEAN.K) ~ log10(KWLS_MCL) + KWLS_DU,
    weights = KWLS_WEIGHTS)

Residuals:
     Min       1Q   Median       3Q      Max
-0.43735 -0.13075  0.01161  0.13980  0.42406

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      0.56016    0.06681   8.385 7.56e-13 ***
log10(KWLS_MCL)  0.14202    0.07443   1.908   0.0597 .
KWLS_DUU        -0.24861    0.04354  -5.710 1.51e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2052 on 88 degrees of freedom
Multiple R-Squared: 0.308, Adjusted R-squared: 0.2923
F-statistic: 19.58 on 2 and 88 DF,  p-value: 9.225e-08
> W <- diag(KWLS_WEIGHTS)
> X <- model.matrix(KWLS3.OUT)
> Xt <- t(X)
> invcov <- chol2inv( chol( Xt %*% W %*% X ) )
> invcov
           [,1]         [,2]         [,3]
[1,]  0.1059896 -0.103490701 -0.031341102
[2,] -0.1034907  0.131564753  0.008592202
[3,] -0.0313411  0.008592202  0.045016142
> max(diag(X %*% invcov Xt))
0.1319
```

Figure 10. Summary of regression execution and output for final weighted least-squares regression on shape parameter of gamma dimensionless hydrograph from GUHAS approach.

The weighted mean K (no log transformation) without regard to the development factor is about 3.9, and the weighted mean values with regard to the development factors are 5.2 and 2.9 for undeveloped and developed, respectively. These values bracket the equivalent K value of 3.77 for a gamma dimensionless hydrograph equivalent to the NRCS (2004) dimensionless hydrograph (Haan and others, 1994, p. 79). The larger shape parameter implies that undeveloped watersheds tend to have a more symmetrical hydrograph in dimensionless space than the developed watershed, and the NRCS dimensionless hydrograph lies between them. A comparison of the gamma dimensionless hydrographs for these specific values of K is shown in figure 11.



EXPLANATION

GAMMA DIMENSIONLESS HYDROGRAPH (K=3.9) DERIVED FROM WEIGHTED MEAN SHAPE PARAMETER

5.2 GAMMA DIMENSIONLESS HYDROGRAPH (K=5.2) DERIVED FROM WEIGHTED MEAN SHAPE PARAMETER FOR UNDEVELOPED WATERSHEDS

2.9 GAMMA DIMENSIONLESS HYDROGRAPH (K=2.9) DERIVED FROM WEIGHTED MEAN SHAPE PARAMETER FOR DEVELOPED WATERSHEDS

NATURAL RESOURCES CONSERVATION SERVICE EQUIVALENT GAMMA DIMENSIONLESS HYDROGRAPH (K=3.77)

Figure 11. Comparison of gamma dimensionless hydrographs from GUHAS approach.

The prediction limits of K from the equation can be useful for expressing the uncertainty when the equation is used. The prediction limits are computed by

$$10^{\log(K) - t_{\alpha/2,df}\,\sigma\sqrt{1+h_o}} \le K \le 10^{\log(K) + t_{\alpha/2,df}\,\sigma\sqrt{1+h_o}}, \tag{27}$$

where K is the estimate from eq. 26; $t_{\alpha/2,df}$ is the t-distribution with probability $\alpha/2$ and df degrees of freedom; σ is the residual standard error; and h_o is the leverage for the watershed. Leverage for the watershed is computed from the inverted covariance matrix $(X^TWX)^{-1}$ of the regression.

The inverted covariance matrix is shown in figure 10 as `invcov`, where column 1 (row 1) is for regression intercept, column 2 (row 2) is for main channel length, column 3 (row 3) is for the watershed development classification. For brevity, the computer output resolution of the matrix has been rounded; to mitigate for round-off errors, the values in figure 10 should be used. Specifically, h_o is equal to

$$h_o = x_o(X^TWX)^{-1}x_o^T \text{ and} \tag{28}$$

$$h_o = \begin{bmatrix} 1 & \log(L) & D \end{bmatrix} \begin{bmatrix} 0.10599 & -0.10349 & -0.03134 \\ -0.10349 & 0.13156 & 0.00859 \\ -0.03134 & 0.00859 & 0.04502 \end{bmatrix} \begin{bmatrix} 1 \\ \log(L) \\ D \end{bmatrix}, \qquad (29)$$

where x_o is a row vector of the intercept, L, and D for the watershed; and x_o^T is a column vector.

It is useful to demonstrate application of the K equation. Suppose a hypothetical undeveloped watershed (D = 0) has an L of 10 miles. The estimate of K thus is $10^{0.560}L^{0.142}$ = 5.04. The df of the equation is 88 and σ is 0.2052. Suppose the 95th-percentile prediction limits are needed. These limits have an $\alpha/2$ = (1-0.95)/2 = 0.025. The upper tail quantile of the t-distribution for $1 - \alpha/2$ = 0.975 nonexceedance probability with 88 degrees of freedom is 1.987 (see `qt()` function in the R system). Finally, the leverage of the hypothetical watershed is

$$h_o = \begin{bmatrix} 1 & \log(10) & 0 \end{bmatrix} \begin{bmatrix} 0.10599 & -0.10349 & -0.03134 \\ -0.10349 & 0.13156 & 0.00859 \\ -0.03134 & 0.00859 & 0.04502 \end{bmatrix} \begin{bmatrix} 1 \\ \log(10) \\ 0 \end{bmatrix} = 0.0306 . \qquad (30)$$

Therefore the 95th-percentile prediction limits of K for the watershed are

$$10^{\log(5.04) - 1.987 \times 0.2052\sqrt{1 + 0.0306}} \le K \text{ and} \qquad (31)$$

$$K \le 10^{\log(5.04) + 1.987 \times 0.2052\sqrt{1 + 0.0306}} . \qquad (32)$$

Therefore the 95th-percentile prediction limits are

$$1.94 \le K \le 13.1 . \qquad (33)$$

Estimation of Time to Peak

Regression analysis was conducted, as previously described, between T_p and the watershed characteristics for the 91 watersheds (not 92 watersheds because station 08178690 was left out). For the final T_p regression equation, main channel length, dimensionless main channel slope, and watershed development classification were used. The final steps of the regression computations using the R system are depicted in figure 12 (user inputs shown in bold type).

```
R : Copyright 2004, The R Foundation for Statistical Computing
Version 2.0.1  (2004-11-15), ISBN 3-900051-07-0
> WLS3.OUT <-lm(log10(MLR3_TP)~log10(MLR3_MCL)+log10(MLR3_MCS2)+MLR3_DU,
weights=MLR3_WEIGHTS)
> summary(WLS3.OUT)

Call:
lm(formula = log10(MLR3_TP) ~ log10(MLR3_MCL) + log10(MLR3_MCS) +
    MLR3_DU, weights = MLR3_WEIGHTS)

Residuals:
     Min       1Q   Median       3Q      Max
-0.36838 -0.10089 -0.00873  0.10355  0.33210

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      -1.49000    0.16056  -9.280 1.20e-14 ***
log10(MLR3_MCL)   0.60180    0.06033   9.976 4.53e-16 ***
log10(MLR3_MCS)  -0.67234    0.08404  -8.001 4.94e-12 ***
MLR3_DUU         -0.35379    0.02942 -12.026  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1383 on 87 degrees of freedom
Multiple R-Squared: 0.8628, Adjusted R-squared: 0.8581
F-statistic: 182.4 on 3 and 87 DF,  p-value: < 2.2e-16

>
> W <- diag(MLR3_WEIGHTS)
> X <- model.matrix(WLS3.OUT)
> Xt <- t(X)
> invcov <- chol2inv( chol( Xt %*% W %*% X ) )
> invcov
            [,1]       [,2]        [,3]         [,4]
[1,]  1.34775208 0.16645692 0.677088876 -0.014512334
[2,]  0.16645692 0.19024886 0.147192830  0.012250620
[3,]  0.67708888 0.14719283 0.369192462  0.009176128
[4,] -0.01451233 0.01225062 0.009176128  0.045244211
> max(diag(X %*% invcov %*% Xt))
> 0.1356
```

Figure 12. Summary of regression execution and output for final weighted least-squares regression on time to peak of 5-minute gamma unit hydrograph from GUHAS approach.

The most appropriate equation from the analysis for T_p thus is

$$T_p = 10^{(-1.49 - 0.354D)} L^{0.602} S^{-0.672}, \tag{34}$$

where T_p is time to peak in hours; D is 0 for an undeveloped watershed and 1 for a developed watershed; L is main channel length of the watershed in miles; and S is the dimensionless main channel slope.

The residual standard error of the equation is 0.1383 $\log(T_p)$ units and the adjusted R-squared is 0.858. The equation has three regressor variables; two represent watershed characteristics that could be a considerable source of multicollinearity. The VIF values, in which values greater than 10 indicate that multicollinearity is a serious problem in the equation, are 1.46, 1.44, and 1.02, for L, S, and D, respectively. These VIF values are small and indicate that multicollinearity between the predictor variables is not of concern. The PRESS statistic is 1.83. The maximum leverage value is about 0.136.

The relation between T_p and main channel length is depicted in figure 13. Unlike the equation for K, dimensionless channel slope is present in the equation for T_p; therefore the lines for the equation can not be depicted directly in figure 13. The results of the regression analysis on T_p are seen in figure 14, in which the vertical variation in the values is the influence of dimensionless main channel slope on the estimates of T_p. In both figures 13 and 14, a distinction between undeveloped and developed watersheds is made; there is considerable difference in average of typical T_p depending on the watershed development classification. Comparison of the two figures shows less variation in T_p in the estimated values, as expected from a regression equation. A comparison of the figures also indicates that the T_p equation is reliable.

The inverted covariance matrix is shown in figure 12 as `invcov`, where column 1 (row 1) is for regression intercept, column 2 (row 2) is for main channel length, column 3 (row 3) is for the dimensionless main channel slope, and column 4 (row 4) is for the watershed development classification.

It is useful to demonstrate application of the T_p equation. Suppose a hypothetical developed watershed ($D = 1$) has an L of 10 miles and a S of 0.004. The estimate of T_p thus is $T_p = 10^{(-1.49 - 0.354)} 10^{0.602} 0.004^{-0.672} = 2.34$ hours. The df of the equation is 87 and σ is 0.1383. Suppose the 95th-percentile prediction limits are needed. These limits have an $\alpha/2 = (1-0.95)/2 = 0.025$. The upper tail quantile of the t-distribution for $1 - \alpha/2 = 0.975$ nonexceedance probability with 87 degrees of freedom is 1.987. Finally, the leverage of the hypothetical watershed is

$$h_o = \begin{bmatrix} 1 & \log(10) & \log(0.004) & 1 \end{bmatrix} \begin{bmatrix} 1.34775 & 0.16646 & 0.67709 & -0.01451 \\ 0.16646 & 0.19025 & 0.14719 & 0.01225 \\ 0.67709 & 0.14719 & 0.36919 & 0.00918 \\ -0.01451 & 0.01225 & 0.00918 & 0.04524 \end{bmatrix} \begin{bmatrix} 1 \\ \log(10) \\ \log(0.004) \\ 1 \end{bmatrix} \quad (35)$$

$$h_o = 0.0374\,. \quad (36)$$

Truncation in $(X^T WX)^{-1}$ is shown in eq. 35, but to mitigate round-off errors, the values in figure 12 should be used. Finally, the 95th-percentile prediction limits for T_p are

$$10^{\log(2.34) - 1.987 \times 0.1383\sqrt{1 + 0.0374}} \le T_p \text{ and} \quad (37)$$

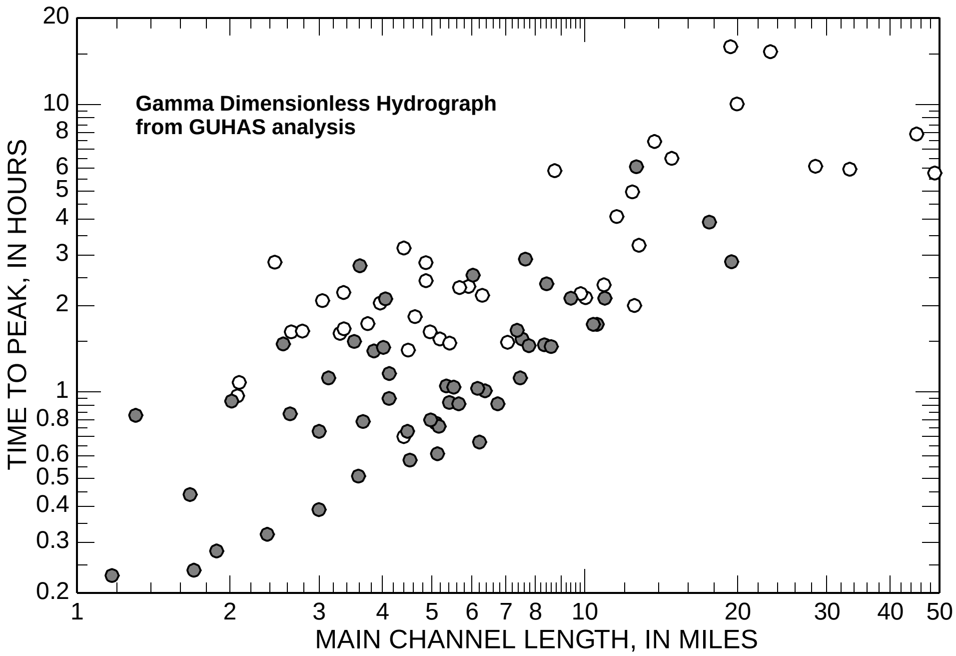
$$T_p \le 10^{\log(2.34) + 1.987 \times 0.1383\sqrt{1 + 0.0374}}\,. \quad (38)$$

Therefore, the 95th-percentile prediction limits of T_p for the watershed are

$$1.23 \le T_p \le 4.46\,. \quad (39)$$

The T_p equation has three parameters and simple two-dimensional representation is problematic. However, it is useful to construct separate diagrams for T_p as a function of main channel length and dimensionless channel slope for both undeveloped and developed watersheds. These diagrams are depicted in figures 15 and 16. The grid lines are specifically included to ease numerical lookup.

Figure 13. Relation between observed time to peak of 5-minute gamma unit hydrograph and main channel length for undeveloped and developed watersheds from GUHAS approach.

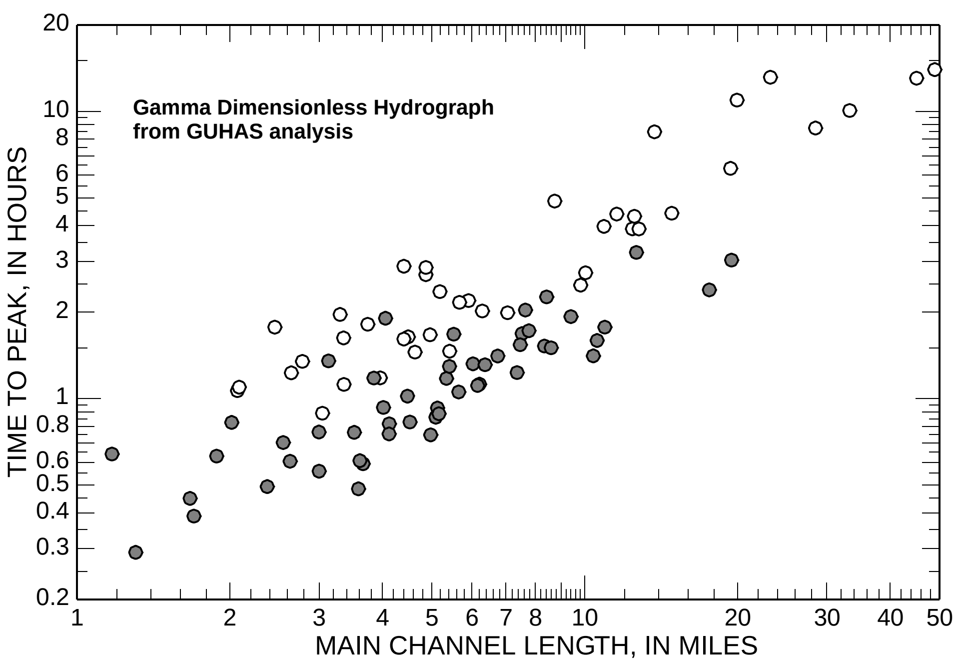


EXPLANATION

○ OBSERVED MEAN TIME TO PEAK FOR UNDEVELOPED WATERSHED

● OBSERVED MEAN TIME TO PEAK FOR DEVELOPED WATERSHED

Figure 14. Relation between modeled time to peak of 5-minute gamma unit hydrograph and main channel length for undeveloped and developed watersheds from GUHAS approach.



EXPLANATION

○ MODELED TIME TO PEAK FOR UNDEVELOPED WATERSHED FROM REGRESSION

● MODELED TIME TO PEAK FOR DEVELOPED WATERSHED FROM REGRESSION

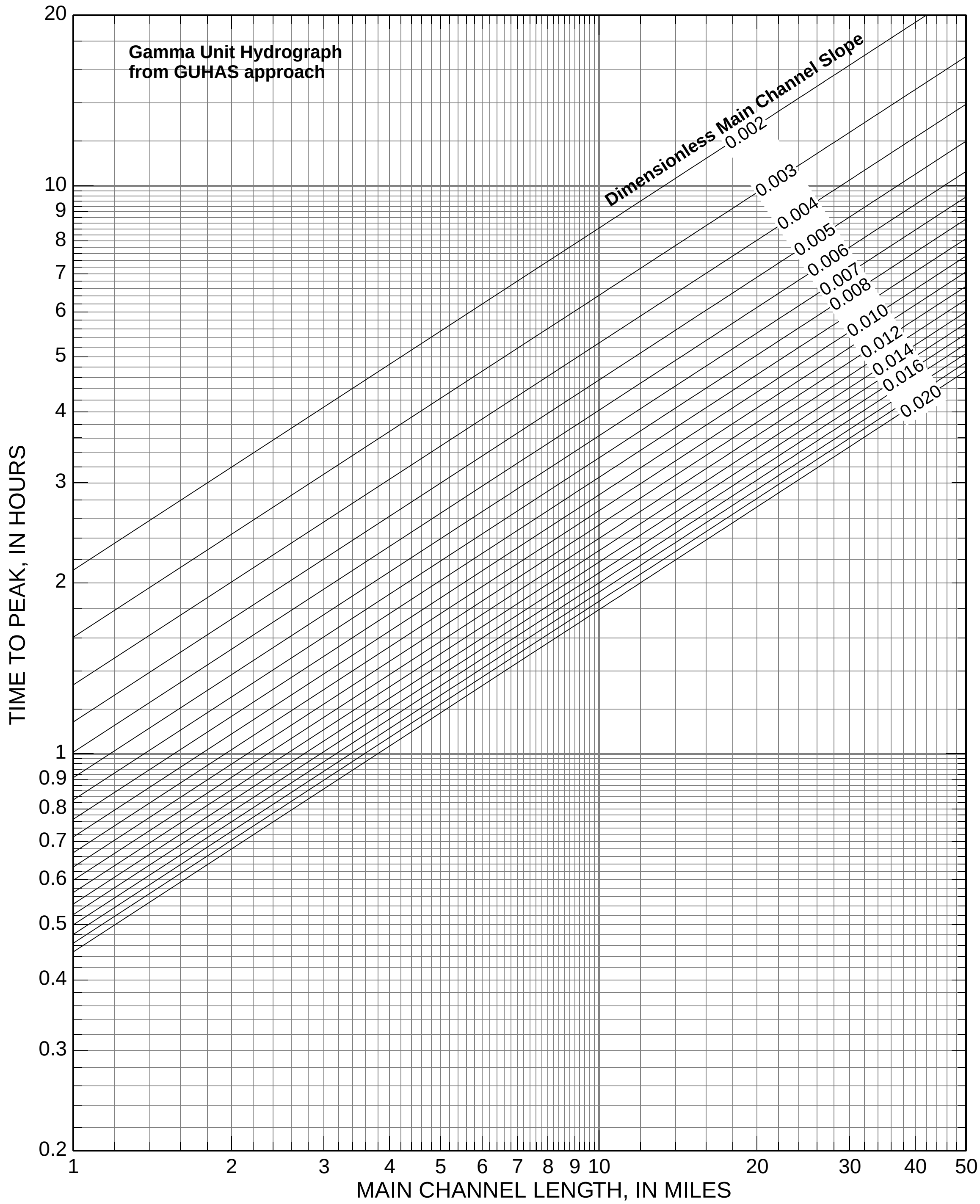


Figure 15. Time to peak of 5-minute gamma unit hydrograph as function of main channel length and dimensionless main channel slope for undeveloped watersheds in Texas from GUHAS approach.

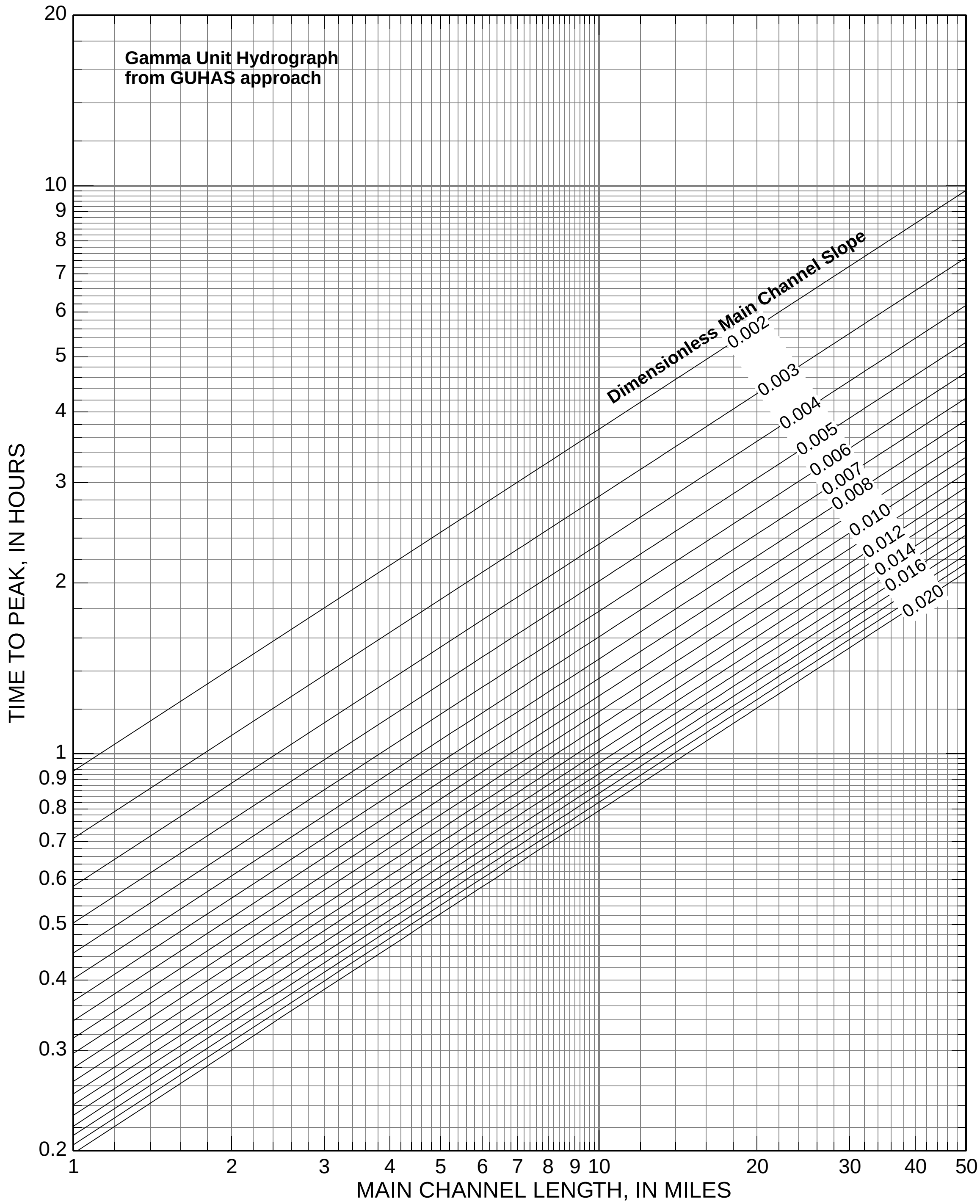


Figure 16. Time to peak of 5-minute gamma unit hydrograph as function of main channel length and dimensionless main channel slope for developed watersheds in Texas from GUHAS approach.

Gamma Unit Hydrograph Parameters from LP Approach

The LP approach used linear programming to compute constrained (positive ordinates) 5-minute unit hydrographs for each watershed. Subsequently, a GUH (eq. 2) was fit to the data. The analysis produced mean values of gamma dimensionless hydrograph shape parameter (K) and time to peak (T_p) for each watershed. These were used as regressor variables.

For 92 of the 93 watersheds, 30-meter DEM watershed characteristics were available for regression analysis. The 30-meter DEM watershed characteristics were not available for the Seminary South Shopping Center watershed in Fort Worth, Texas. Furthermore, LP approach results were not available for four watersheds: 08111025 Burton Creek at Villa Maria Road, Bryan, Texas; 08111050 Hudson Creek near Bryan, Texas; 08158825 Little Bear Creek at FM 1626, Manchaca, Texas; and 08177700 Olmos Creek at Dresden Drive, San Antonio, Texas. Therefore, 88 watersheds were considered for the regression analysis reported here. In total, 1,248 storms were analyzed for the 88 watersheds. The average number of storms per watershed is about 14.

Estimation of Gamma Dimensionless Hydrograph Shape

The relation between K and main channel length is depicted in figure 17. A distinction between undeveloped and developed watersheds is made; there is considerable difference in average or typical K depending on the watershed development classification.

Regression analysis was conducted as previously described. Stations 08158820 Bear Creek at FM 1626, Manchaca, Texas, and 08187900 Escondido Creek subwatershed 11 near Kenedy, Texas, were determined as outliers and were removed from the regression. The final steps of the regression computations using the R system are depicted in figure 18 (user inputs shown in bold type). The most appropriate equation from the analysis for K is

$$K = 10^{(0.481 - 0.0782D)} L^{0.140}, \tag{40}$$

where K is the shape parameter, D is 0 for an undeveloped watershed and 1 for a developed watershed, and L is main channel length of the watershed in miles. The residual standard error for the equation is 0.1460 $\log(K)$ units and the adjusted R-squared is 0.145. The maximum leverage value is about 0.124.

Superimposed on the data points in figure 17 are the two lines derived from the K regression equation. The regression was performed in linear-linear space. Both lines have been terminated at the minimum and maximum values for main channel length for the respective development classification.

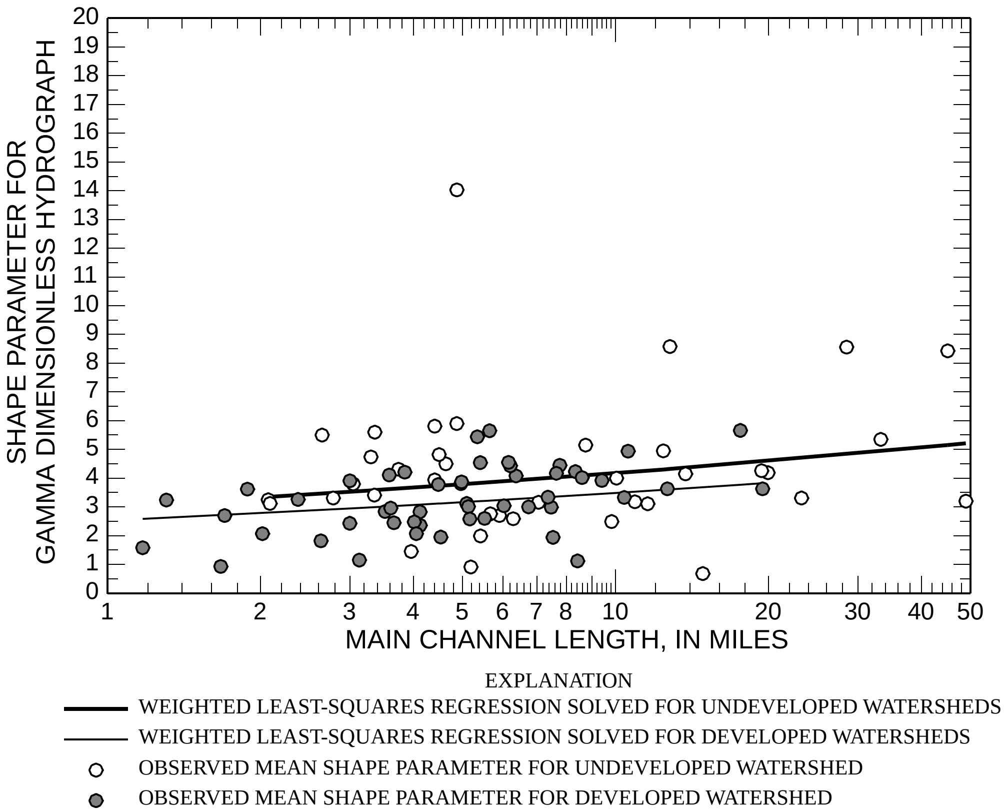


Figure 17. Relation between observed shape parameter of gamma dimensionless hydrograph and main channel length for undeveloped and developed watersheds from LP approach.

The weighted mean K without regard to the development factor is about 3.8, and the weighted mean values with regard to the development classification are about 4.4 and 3.4 for undeveloped and developed, respectively. These values bracket the equivalent K value of 3.77 for a gamma dimensionless hydrograph equivalent to the NRCS dimensionless hydrograph (Haan and others, 1994, p. 79). Thus, the undeveloped watersheds tend to have a more symmetrical hydrograph in dimensionless space than the developed watershed, and the NRCS dimensionless hydrograph lies between them. A comparison of the gamma dimensionless hydrographs for these specific values of K is shown in figure 19.

The prediction limits K from the equation can be useful for expressing the uncertainty when the equation is used. The prediction limits are computed by

$$10^{\log(K) - t_{\alpha/2,df}\,\sigma\sqrt{1+h_o}} \le K \le 10^{\log(K) + t_{\alpha/2,df}\,\sigma\sqrt{1+h_o}}, \qquad (41)$$

where K is the estimate from eq. 40; $t_{\alpha/2,df}$ is the t-distribution with probability $\alpha/2$ and df degrees of freedom; σ is the residual standard error; and h_o is the leverage for the watershed. Leverage for the watershed is computed from the inverted covariance matrix $(X^TWX)^{-1}$ of the regression.

```
R : Copyright 2004, The R Foundation for Statistical Computing
Version 2.0.1  (2004-11-15), ISBN 3-900051-07-0
> KWLS2.OUT <- lm(log10(KWLS_MEAN.K) ~ log10(KWLS_MCL) + KWLS_DU,
weights=KWLS_WEIGHTS)
> summary(KWLS2.OUT)

Call:
lm(formula = log10(KWLS_MEAN.K) ~ log10(KWLS_MCL) + KWLS_DU,
    weights = KWLS_WEIGHTS)

Residuals:
      Min        1Q    Median        3Q       Max
-0.406926 -0.076629 -0.008258  0.082876  0.275517

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      0.48084    0.04936   9.741 2.15e-15 ***
log10(KWLS_MCL)  0.13966    0.05236   2.667   0.0092 **
KWLS_DUU        -0.07822    0.03257  -2.402   0.0185 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.146 on 83 degrees of freedom
Multiple R-Squared: 0.165,Adjusted R-squared: 0.1449
F-statistic: 8.201 on 2 and 83 DF,  p-value: 0.0005623
> W <- diag(KWLS_WEIGHTS)
> X <- model.matrix(KWLS2.OUT)
> Xt <- t(X)
> invcov <- chol2inv( chol( Xt %*% W %*% X ) )
> invcov
            [,1]        [,2]        [,3]
[1,]  0.11436882 -0.10596862 -0.04130258
[2,] -0.10596862  0.12870056  0.01722852
[3,] -0.04130258  0.01722852  0.04978049
> max(diag(X %*% invcov %*% Xt))
[1] 0.1237099
```

Figure 18. Summary of regression execution and output for final weighted least-squares regression on shape parameter of gamma dimensionless hydrograph from LP approach.

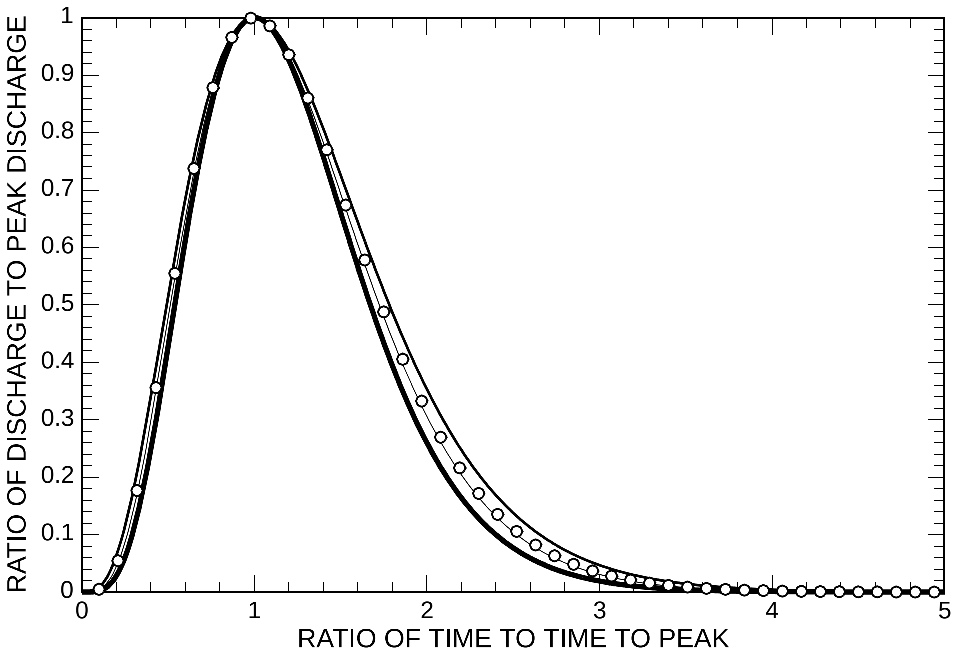


Figure 19. Comparison of gamma dimensionless hydrographs from LP approach.

The inverted covariance matrix is shown in figure 18 as `invcov`, where column 1 (row 1) is for regression intercept, column 2 (row 2) is for main channel length, and column 3 (row 3) is for the watersheds development classification. For brevity, the computer output resolution of the matrix has been rounded; although to mitigate for round-off errors, the values in figure 18 should be used. Specifically, h_o is equal to

$$h_o = x_o(X^TWX)^{-1}x_o^T \text{ and} \tag{42}$$

$$h_o = \begin{bmatrix}1 & \log(L) & D\end{bmatrix}\begin{bmatrix}0.11435 & -0.10597 & -0.04130 \\ -0.10597 & 0.12870 & 0.01723 \\ -0.04130 & 0.01723 & 0.04978\end{bmatrix}\begin{bmatrix}1 \\ \log(L) \\ D\end{bmatrix}, \tag{43}$$

where x_o is a row vector of the intercept and D for the watershed, and x_o^T is a column vector.

It is useful to demonstrate application of the K equation. Suppose a hypothetical undeveloped watershed ($D = 0$) has an L of 10 miles. The estimate of K thus is $10^{0.481}L^{0.140} = 4.18$. The df of the equation is 83 and σ is 0.1460. Suppose the 95th-percentile prediction limits are needed. These limits have an $\alpha/2$ = (1-0.95)/2 = 0.025. The upper tail quantile of the t-distribution for

$1 - \alpha/2 = 0.975$ nonexceedance probability with 83 degrees of freedom is 1.989 (see `qt()` function in the R system). Finally, the leverage of the hypothetical watershed is

$$h_o = \begin{bmatrix} 1 & \log(10) & 0 \end{bmatrix} \begin{bmatrix} 0.11435 & -0.10597 & -0.04130 \\ -0.10597 & 0.12870 & 0.01723 \\ -0.04130 & 0.01723 & 0.04978 \end{bmatrix} \begin{bmatrix} 1 \\ \log(10) \\ 0 \end{bmatrix} = 0.0311 . \quad (44)$$

Therefore the 95th-percentile prediction limits of K for the watershed are

$$10^{\log(4.18) - 1.989 \times 0.1460\sqrt{1 + 0.0311}} \leq K \text{ and} \quad (45)$$

$$K \leq 10^{\log(4.18) + 1.989 \times 0.1460\sqrt{1 + 0.0311}} . \quad (46)$$

Therefore the 95th-percentile prediction limits are

$$2.12 \leq K \leq 8.24 . \quad (47)$$

Estimation of Time to Peak

Regression analysis was conducted, as previously described, between T_p of the 5-minute GUH and the watershed characteristics for the 88 watersheds. Interpretation of the residual and diagnostic analysis consistently indicated that stations 08158820 Bear Creek at FM 1626, Manchaca, Texas; 08177600 Olmos Creek tributary at FM 1535, Shavano Park, Texas; and 08178690 Salado Creek tributary at Bitters Road, San Antonio, Texas, should be considered outliers. These three watersheds have high leverage and contribute excessive influence on regression coefficients relative to the other watersheds and should be removed from the regression analysis of T_p; PRESS and residual standard errors were substantially reduced by removal of the outliers. These stations represent a small component of the overall database, and the authors believe that their removal is appropriate. For the final T_p regression equation, main channel length, dimensionless channel slope, and watershed development classification were used. The final steps of the regression computations using the R system are depicted in figure 20 (user inputs shown in bold type).

The most appropriate equation from the analysis for T_p is

$$T_p = 10^{(-1.41 - 0.313D)} L^{0.612} S^{-0.633} , \quad (48)$$

where T_p is time to peak in hours; D is 0 for an undeveloped watershed and 1 for a developed watershed; L is main channel length of watershed in miles; and S is the dimensionless main channel slope.

```
R : Copyright 2004, The R Foundation for Statistical Computing
Version 2.0.1  (2004-11-15), ISBN 3-900051-07-0
> WLS2.OUT <- lm(log10(MLR2_MeanTp) ~ log10(MLR2_MCL) + log10(MLR2_MCS2) +
MLR2_DU, weights=MLR2_WEIGHTS)
> summary(WLS2.OUT)
Call:
lm(formula = log10(MLR2_MeanTp) ~ log10(MLR2_MCL) + log10(MLR2_MCS2) +
    MLR2_DU, weights = MLR2_WEIGHTS)

Residuals:
     Min       1Q   Median       3Q      Max
-0.25274 -0.09356 -0.01283  0.07110  0.37111

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      -1.40990    0.15225  -9.260 2.43e-14 ***
log10(MLR2_MCL)   0.61201    0.05901  10.372  < 2e-16 ***
log10(MLR2_MCS2) -0.63313    0.08054  -7.861 1.41e-11 ***
MLR2_DUU         -0.31254    0.02825 -11.064  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1266 on 81 degrees of freedom
Multiple R-Squared: 0.875,Adjusted R-squared: 0.8703
F-statistic:   189 on 3 and 81 DF,  p-value: < 2.2e-16
> W <- diag(MLR2_WEIGHTS)
> X  <- model.matrix(WLS2.OUT)
> Xt <- t(X)
> invcov <- chol2inv( chol( Xt %*% W %*% X ) )
> invcov
             [,1]       [,2]       [,3]         [,4]
[1,]  1.446357872 0.18595626 0.73143100 -0.003700808
[2,]  0.185956264 0.21725380 0.16926954  0.022034429
[3,]  0.731431000 0.16926954 0.40470699  0.018845421
[4,] -0.003700808 0.02203443 0.01884542  0.049787240
> max(diag(X %*% tmp %*% Xt))
[1] 0.1418
```

Figure 20. Summary of regression execution and output for final weighted least-squares regression on time to peak of 5-minute gamma unit hydrograph from LP approach.

The residual standard error of the equation is 0.1266 $\log(T_p)$ units and the adjusted R-squared is 0.870. The equation has three regressor variables; two represent watershed characteristics that could be a considerable source of multicollinearity. The VIF values, in which values greater than 10 indicate that multicollinearity is a serious problem in the equation, are 1.53, 1.48, and 1.05, for L, S, and D, respectively. These VIF values are small and indicate that multicollinearity between the predictor variables is not of concern. The PRESS statistic is 1.45. The maximum leverage value is about 0.142.

The relation between observed T_p and main channel length is depicted in figure 21. Unlike the equation for K, dimensionless main channel slope is present in the equation for T_p; therefore the lines for the equation can not be depicted directly in figure 21. The results of the regression analysis on T_p are seen in figure 22, in which the vertical variation in the values is the influence of dimensionless main channel slope on T_p. In both figures 21 and 22, a distinction between undeveloped and developed watersheds is made; there is considerable difference in average of typical T_p depending on the watershed development classification. Comparison of the two figures shows less variation in T_p in the estimated values, as expected from a regression equation. Comparison of the figures also indicates that the T_p equation is reliable.

The inverted covariance matrix is shown in figure 20 as `invcov`, where column 1 (row 1) is for regression intercept, column 2 (row 2) is for main channel length, column 3 (row 3) is for the dimensionless main channel slope, and column 4 (row 4) is for the watershed development classification.

It is useful to demonstrate application of the T_p equation. Suppose a hypothetical developed watershed ($D = 1$) has an L of 10 miles and a S of 0.004. The estimate of T_p thus is $T_p = 10^{(-1.41 - 0.313)}10^{0.612}0.004^{-0.633}$ = 2.55 hours. The df of the equation is 81 and σ is 0.1266. Suppose the 95th-percentile prediction limits are needed. These limits have an $\alpha / 2$ = (1-0.95)/2 = 0.025. The upper tail quantile of the t-distribution for $1 - \alpha / 2$ = 0.975 nonexceedance probability with 81 degrees of freedom is 1.990. Finally, the leverage of the hypothetical watershed is

$$h_o = \begin{bmatrix} 1 & \log(10) & \log(0.004) & 1 \end{bmatrix} \begin{bmatrix} 1.4464 & 0.18596 & 0.73143 & -0.00370 \\ 0.18596 & 0.21725 & 0.16927 & 0.02203 \\ 0.73143 & 0.16927 & 0.40471 & 0.01885 \\ -0.00370 & 0.02203 & 0.01885 & 0.049787 \end{bmatrix} \begin{bmatrix} 1 \\ \log(10) \\ \log(0.004) \\ 1 \end{bmatrix} \quad (49)$$

$$h_o = 0.0391 . \quad (50)$$

Truncation in $(X^T WX)^{-1}$ is shown in eq. 49, but to mitigate round-off errors, the values in figure 20 should be used. Finally, the 95th-percentile prediction limits for T_p are

$$10^{\log(2.55) - 1.990 \times 0.1266\sqrt{1 + 0.0391}} \le T_p \text{ and} \quad (51)$$

$$T_p \le 10^{\log(2.55) + 1.990 \times 0.1266\sqrt{1 + 0.0391}} . \quad (52)$$

Therefore, the 95th-percentile prediction limits of T_p for the watershed are

$$1.41 \le T_p \le 4.61 . \quad (53)$$

The T_p equation has three parameters and simple two-dimensional representation is problematic. However, it is useful to construct separate diagrams for T_p as a function of main channel length and dimensionless channel slope for both undeveloped and developed watersheds. These diagrams are depicted in figures 23 and 24. The grid lines are specifically included to ease numerical lookup.

Figure 21. Relation between observed time to peak of 5-minute gamma unit hydrograph and main channel length for undeveloped and developed watersheds from LP approach.

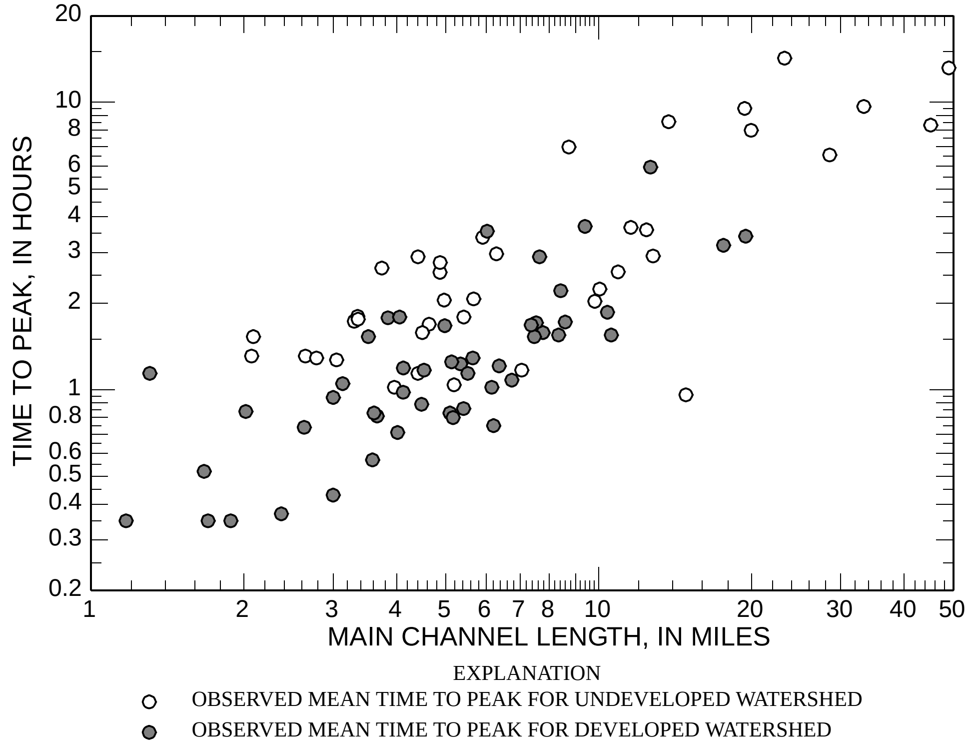
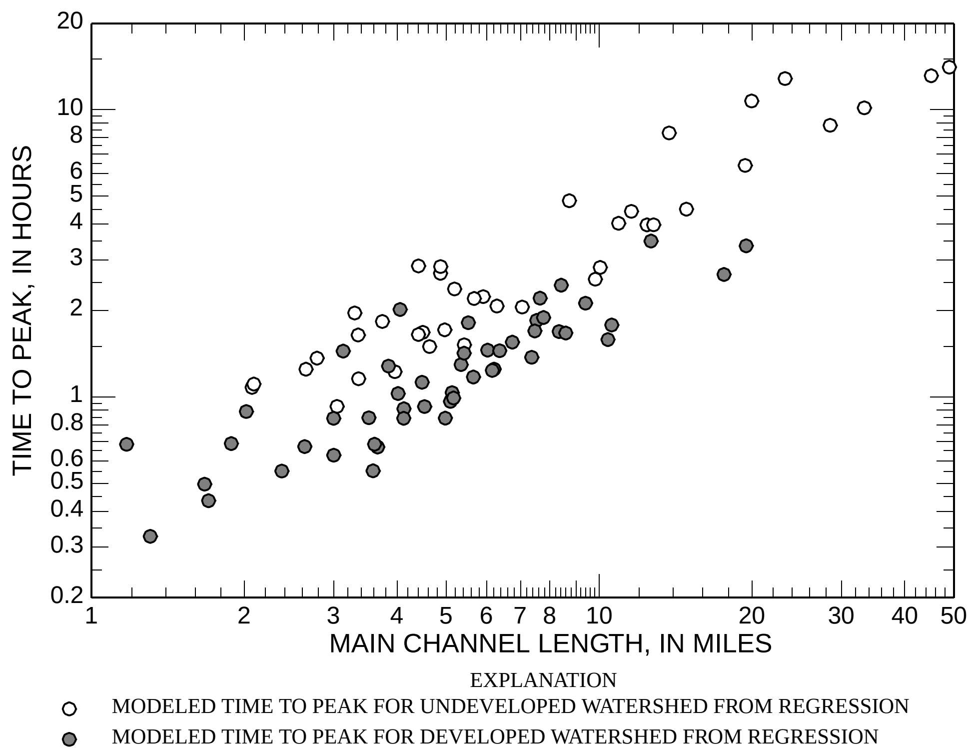


Figure 22. Relation between modeled time to peak of 5-minute gamma unit hydrograph and main channel length for undeveloped and developed watersheds from LP approach.



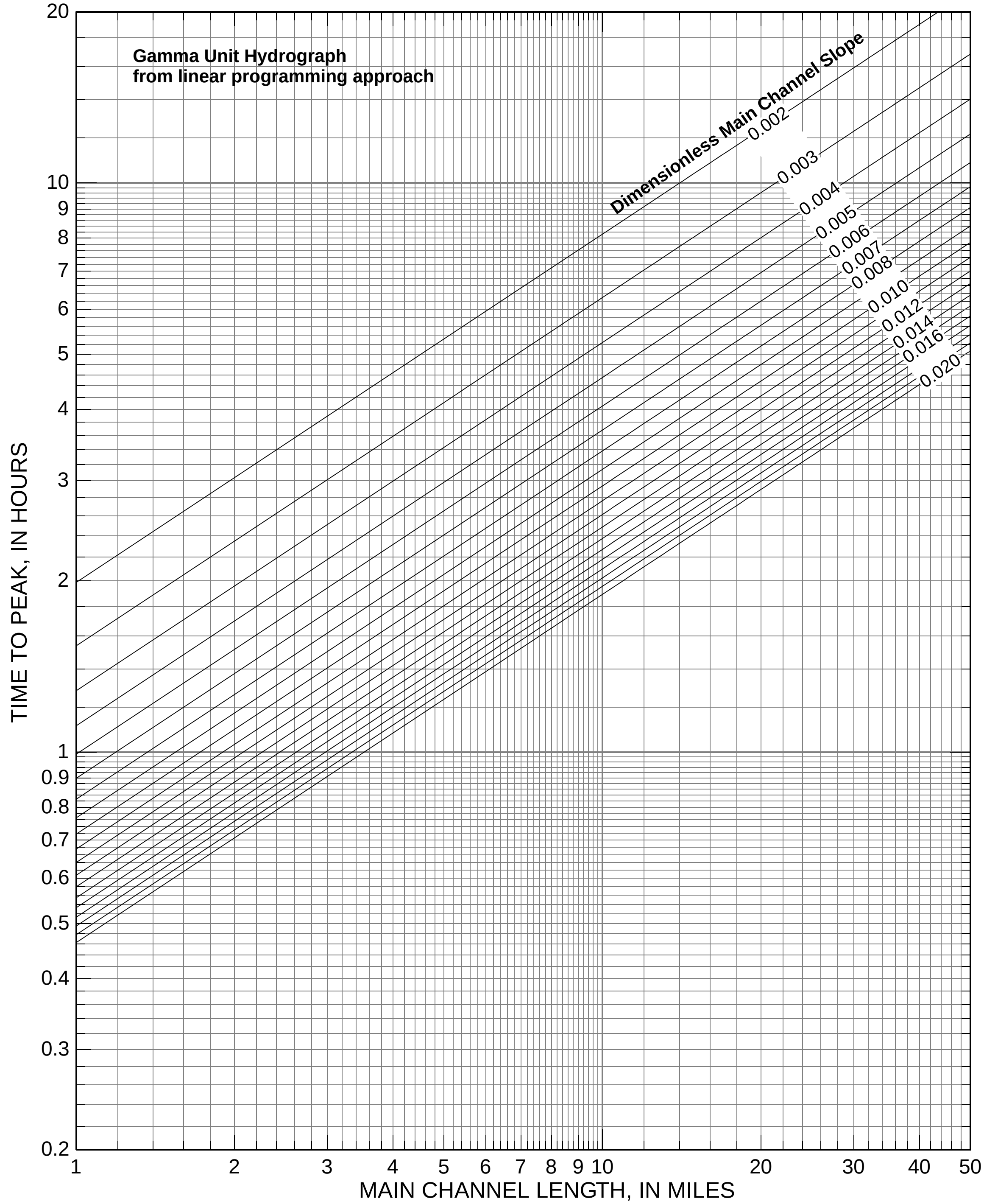


Figure 23. Time to peak of 5-minute gamma unit hydrograph as function of main channel length and dimensionless main channel slope for undeveloped watersheds in Texas from LP approach.

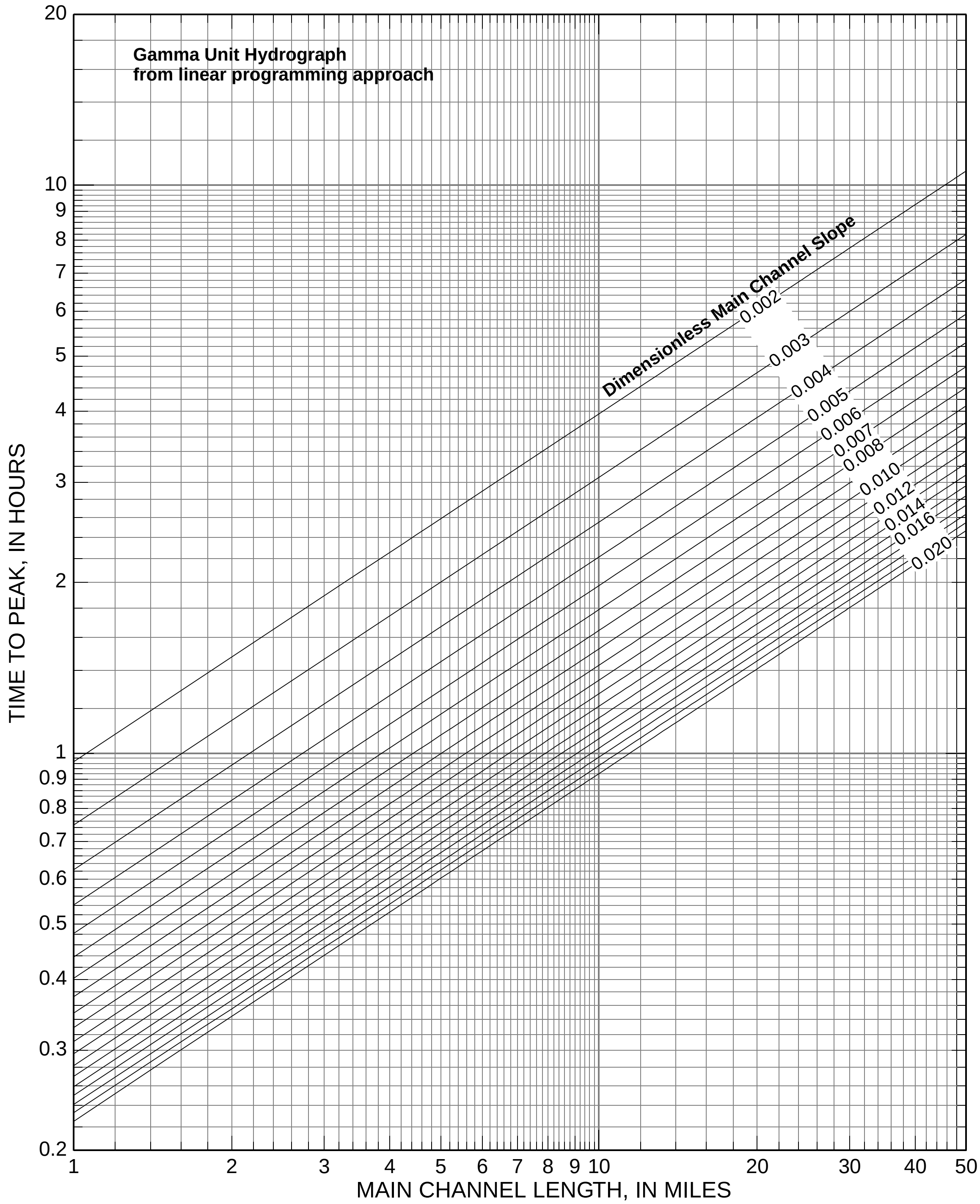


Figure 24. Time to peak of 5-minute gamma unit hydrograph as function of main channel length and dimensionless main channel slope for developed watersheds in Texas from LP approach.

Rayleigh Unit Hydrograph Parameters from IUH Approach

The IUH approach uses eq. 7 for a hydrograph, which is referred to as the Rayleigh hydrograph. The Rayleigh hydrograph is made dimensionless by division of the left-hand side of eq. 7 by q_p, and dividing t by T_p. The analysis produced mean values of Rayleigh dimensionless hydrograph shape parameter (N) and time to peak (T_p) for each watershed. These were used as regressor variables. The IUH approach provides a 1-minute Rayleigh unit hydrograph.

For 92 of the 93 watersheds, 30-meter DEM watershed characteristics were available for regression analysis. The 30-meter DEM watershed characteristics were not available for the Seminary South Shopping Center watershed in Fort Worth, Texas. Therefore, 92 watersheds were considered for the analysis reported here. In total, 1,545 storms were analyzed for the 92 watersheds. The average number of events per watershed is about 17.

Estimation of Rayleigh Dimensionless Hydrograph Shape

The relation between N and main channel length is depicted in figure 25. An upper limit of 4 was artificially imposed during the computation runs on the database. A distinction between undeveloped and developed watersheds is made; there is a statistically significant difference in average or typical N depending on the binary watershed development classification. The Welch Two Sample t-test (R Development Core Team, 2004) shows that N for undeveloped watersheds is greater than N for developed watersheds with a p-value of .0101. However, in a weighted least-squares context, the watershed development classification was not statistically significant as a predictor of hydrograph shape.

Regression analysis was conducted as previously described. Interpretation of the residual and diagnostic analysis consistently indicated that station 08178300 Alazan Creek at St. Cloud Street, San Antonio, Texas, should be considered an outlier. This watershed has high leverage and contributes excessive influence on regression coefficients relative to the other 91 watersheds and should be removed from the regression analysis of N; PRESS and residual standard errors were substantially reduced by removal of the outlier. This single station represents a small component of the overall database, and the authors believe that its removal is appropriate. The final steps of the regression computations using the R system are depicted in figure 26 (user inputs shown in bold type). The most appropriate equation from the analysis for N is

$$N = 2.39L^{0.0722}, \qquad (54)$$

where N is the shape parameter, and L is main channel length of watershed in miles. The residual standard error for the equation is 0.0632 $\log(N)$ units and the adjusted R-squared is 0.107. The maximum leverage value is about 0.110.

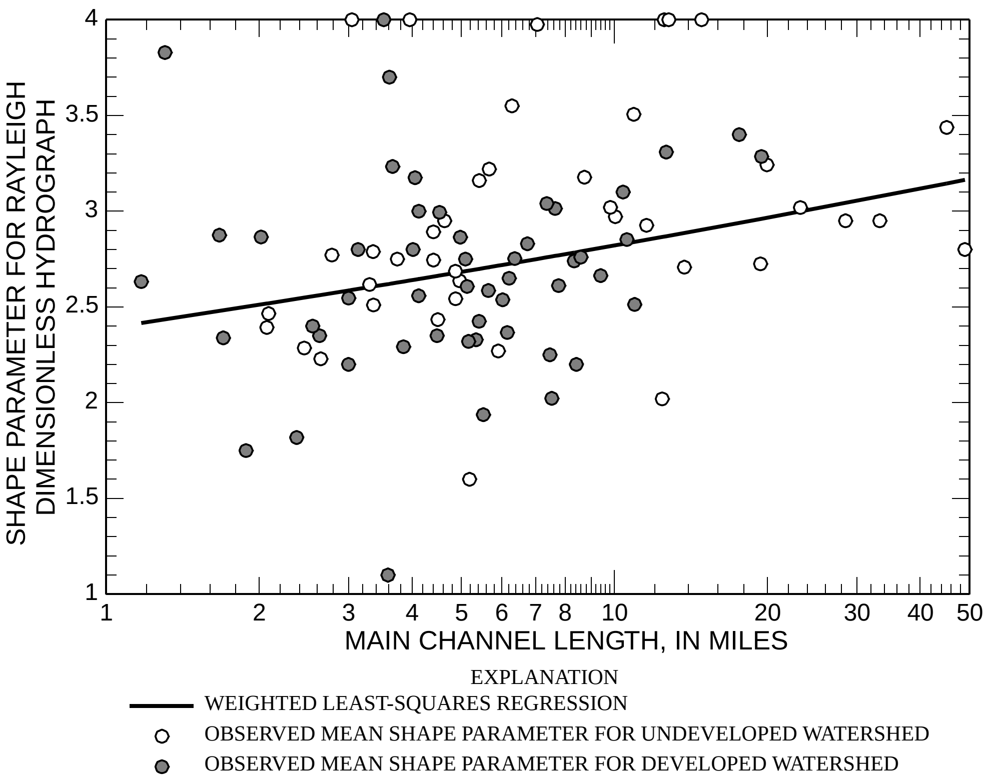


Figure 25. Relation between observed shape parameter of Rayleigh dimensionless hydrograph and main channel length for undeveloped and developed watersheds from IUH approach.

Superimposed on the data points in figure 25 is the line derived from the N regression equation. The regression was performed in log-log space, but a linear N axis is depicted. The line has been terminated at the minimum and maximum values for main channel length.

The weighted mean N (no log transformation) without regard to the development classification is about 2.70. The weighted mean N with regard to the development classification are 2.80 and 2.62 for undeveloped and developed, respectively—a difference that was statistically significant. A comparison of the Rayleigh dimensionless hydrograph for the weighted mean shape factor is shown in figure 27. Also shown on the figure are the gamma dimensionless hydrographs from the GUHAS analysis (K=3.9) and the NRCS equivalent (K=3.77).

```
R : Copyright 2004, The R Foundation for Statistical Computing
Version 2.0.1  (2004-11-15), ISBN 3-900051-07-0
> NWLS3.OUT <-lm(log10(NWLS_Mean_N)~log10(NWLS_MCL),
weights=NWLS_WEIGHTS)
> summary(NWLS3.OUT)

Call:
lm(formula = log10(NWLS_Mean_N) ~ log10(NWLS_MCL), weights =
NWLS_WEIGHTS)

Residuals:
      Min        1Q    Median        3Q       Max
-0.167231 -0.027648  0.003586  0.045873  0.180108

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       0.37816    0.01698  22.267  < 2e-16 ***
log10(NWLS_MCL)   0.07219    0.02106   3.428 0.000922 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.06324 on 89 degrees of freedom
Multiple R-Squared: 0.1167,Adjusted R-squared: 0.1067
F-statistic: 11.75 on 1 and 89 DF,  p-value: 0.0009218

> W <- diag(NWLS_WEIGHTS)
> X <- model.matrix(NWLS3.OUT)
> Xt <- t(X)
> invcov <- chol2inv( chol( Xt %*% W %*% X ) )
> invcov
            [,1]       [,2]
[1,]  0.07212783 -0.0823375
[2,] -0.08233750  0.1108864
> max(diag(X %*% invcov Xt))
[1] 0.110
```

Figure 26. Summary of regression execution and output for final weighted least-squares regression on shape parameter of Rayleigh dimensionless hydrograph from IUH approach.

The prediction limits of N from the equation can be useful for expressing the uncertainty when the equation is used. The prediction limits are computed by

$$10^{\log(N) - t_{\alpha/2,df}\,\sigma\sqrt{1+h_o}} \le N \le 10^{\log(N) + t_{\alpha/2,df}\,\sigma\sqrt{1+h_o}}, \tag{55}$$

where N is the estimate from eq. 54; $t_{\alpha/2,df}$ is the t-distribution with probability $\alpha/2$ and df degrees of freedom; σ is the residual standard error; and h_o is the leverage for the watershed. Leverage for the watershed is computed from the inverted covariance matrix $(X^T WX)^{-1}$ of the regression.

The inverted covariance matrix is shown in figure 26 as `invcov`, where column 1 (row 1) is for regression intercept and column 2 (row 2) is for main channel length. For brevity, the computer output resolution of the matrix has been rounded; to mitigate for round-off errors, the values in figure 26 should be used. Specifically, h_o is equal to

$$h_o = x_o(X^T WX)^{-1} x_o^T \text{ and} \tag{56}$$

$$h_o = \begin{bmatrix} 1 & \log(L) \end{bmatrix} \begin{bmatrix} 0.07213 & -0.08234 \\ -0.08234 & 0.11089 \end{bmatrix} \begin{bmatrix} 1 \\ \log(L) \end{bmatrix}, \tag{57}$$

where x_o is a row vector of the intercept, L, and D for the watershed; and x_o^T is a column vector.

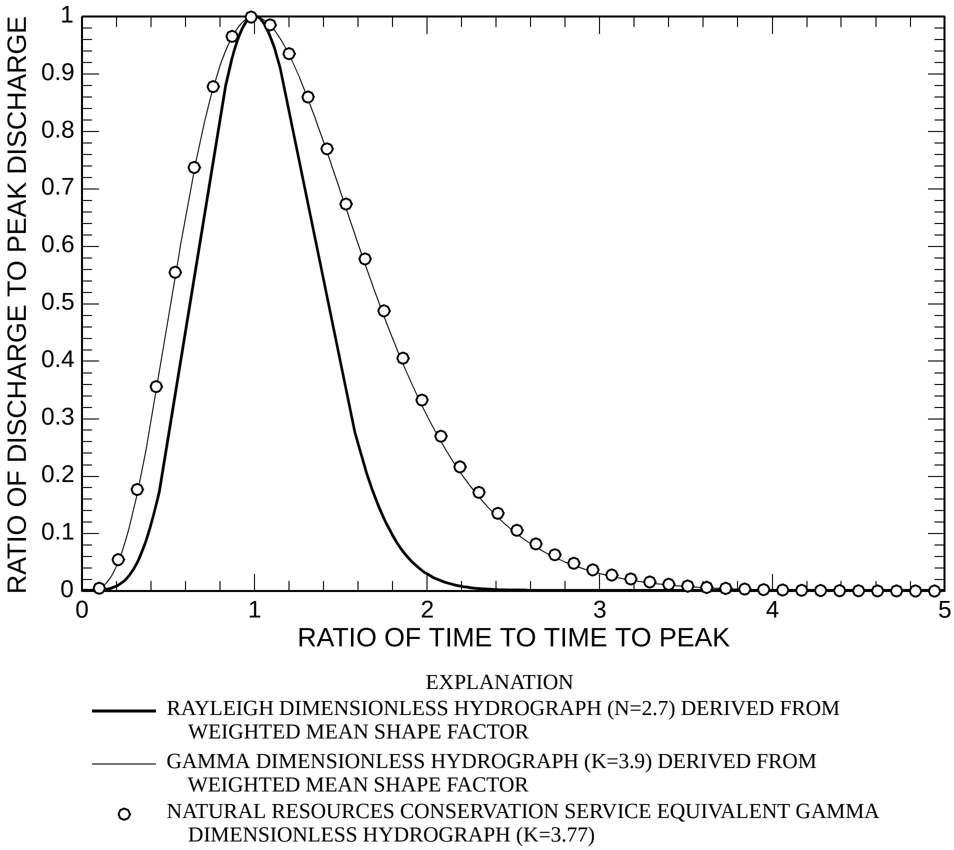


Figure 27. Comparison of Rayleigh dimensionless hydrograph to gamma dimensionless hydrograph from IUH approach.

It is useful to demonstrate application of the N equation. Suppose a hypothetical watershed has an L of 10 miles. The estimate of N thus is $2.39L^{0.0722} = 2.82$. The df of the equation is 89 and σ is 0.0632. Suppose the 95th-percentile prediction limits are needed. These limits have an $\alpha/2 = (1\text{-}0.95)/2 = 0.025$. The upper tail quantile of the t-distribution for $1 - \alpha/2 = 0.975$ non-exceedance probability with 89 degrees of freedom is 1.987 (see `qt()` function in the R system). Finally, the leverage of the hypothetical watershed is

$$h_o = \begin{bmatrix}1 & \log(10)\end{bmatrix} \begin{bmatrix} 0.07213 & -0.08234 \\ -0.08234 & 0.11089 \end{bmatrix} \begin{bmatrix} 1 \\ \log(10) \end{bmatrix} = 0.0184\,. \tag{58}$$

The 95th-percentile prediction limits are

$$10^{\log(2.82) - 1.987 \times 0.0632\sqrt{1 + 0.0184}} \le N \text{ and} \tag{59}$$

$$N \le 10^{\log(2.82) + 1.987 \times 0.0632\sqrt{1 + 0.0184}}\,. \tag{60}$$

Therefore, the 95th-percentile prediction limits of N for the watershed are

$$2.11 \leq N \leq 3.78\,. \tag{61}$$

Estimation of Time to Peak

Regression analysis was conducted, as previously described, between T_p and the watershed characteristics for the 92 watersheds. Interpretation of the residual and diagnostic analysis consistently indicated that stations 08177600 Olmos Creek tributary at FM 1535, Shavano Park, Texas, and 08178620 Lorence Creek at Thousand Oaks Boulevard, San Antonio, Texas, should be considered outliers. These two watersheds have high leverage and contribute excessive influence on regression coefficients relative to the other 90 watersheds and should be removed from the regression analysis of T_p; PRESS and residual standard errors were substantially reduced by removal of the outliers. These stations represent a small component of the overall database, and the authors believe that their removal is appropriate. For the final T_p regression equation, main channel length, dimensionless channel slope, and watershed development classification were used. The final steps of the regression computations using the R system are depicted in figure 28 (user inputs shown in bold type).

```
R : Copyright 2004, The R Foundation for Statistical Computing
Version 2.0.1  (2004-11-15), ISBN 3-900051-07-0
> WLS2.OUT <- lm(log10(MLR2_MeanTp)~log10(MLR2_MCL) + log10(MLR2_MCS) +
MLR2_DU, weights=MLR2_WEIGHTS)
> summary(WLS2.OUT)

Call:
lm(formula = log10(MLR2_MeanTp) ~ log10(MLR2_MCL) + log10(MLR2_MCS) +
    MLR2_DU, weights = MLR2_WEIGHTS)

Residuals:
      Min         1Q     Median         3Q        Max
-0.2829419 -0.1130612  0.0001643  0.1079055  0.3311406

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     -1.27027    0.15610  -8.137 2.79e-12 ***
log10(MLR2_MCL)  0.66322    0.05481  12.101  < 2e-16 ***
log10(MLR2_MCS) -0.50296    0.07960  -6.319 1.12e-08 ***
MLR2_DUU        -0.29763    0.03017  -9.864 8.60e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.14 on 86 degrees of freedom
Multiple R-Squared: 0.8441,Adjusted R-squared: 0.8387
F-statistic: 155.2 on 3 and 86 DF,  p-value: < 2.2e-16

> W <- diag(MLR2_WEIGHTS)
> X <- model.matrix(WLS2.OUT)
> Xt <- t(X)
> invcov <- chol2inv( chol( Xt %*% W %*% X ) )
> invcov
          [,1]      [,2]       [,3]       [,4]
[1,] 1.2439214 0.1054626 0.60929523 0.00621630
[2,] 0.1054626 0.1533311 0.10568024 0.01650440
[3,] 0.6092952 0.1056802 0.32342248 0.01938166
[4,] 0.0062163 0.0165044 0.01938166 0.04647707
> max(diag(X %*% invcov %*% Xt))
[1] 0.1289
```

Figure 28. Summary of regression execution and output for final weighted least-squares regression on time to peak of 1-minute Rayleigh unit hydrograph from IUH approach.

The most appropriate equation from the analysis for T_p is

$$T_p = 10^{(-1.27 - 0.298D)} L^{0.663} S^{-0.503}, \tag{62}$$

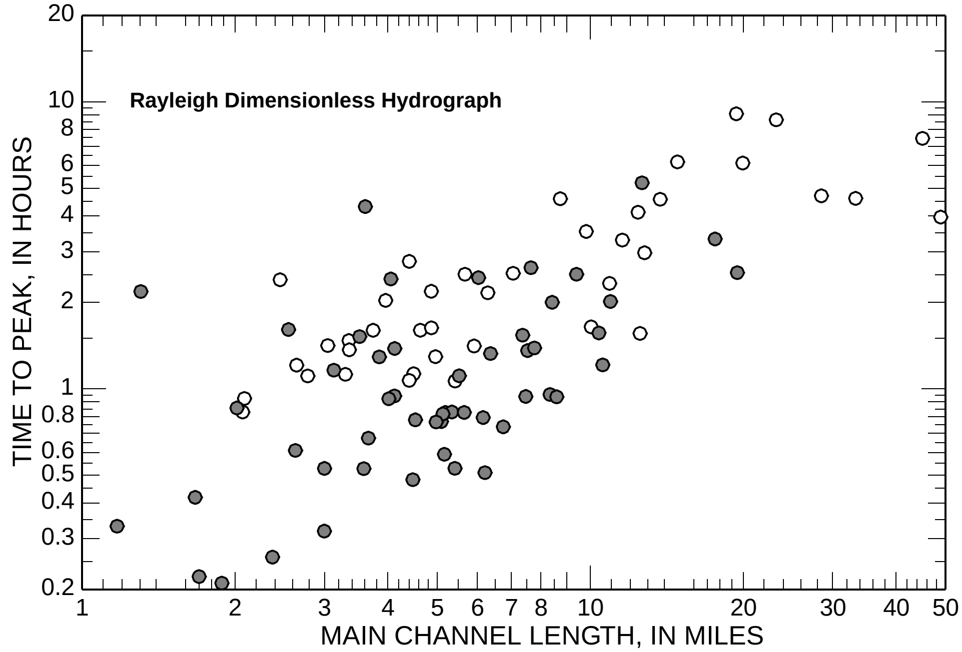
where T_p is time to peak in hours; D is 0 for an undeveloped watershed and 1 for a developed watershed; L is main channel length of watershed in miles; and S is the dimensionless main channel slope.

The residual standard error of the equation is 0.1400 $\log(T_p)$ units and the adjusted R-squared is 0.839. The equation has three regressor variables; two represent watershed characteristics that could be a considerable source of multicollinearity. The VIF values, in which values greater than 10 indicate that multicollinearity is a serious problem in the equation, are 1.32, 1.30, and 1.05, for L, S, and D, respectively. These VIF values are small and indicate that multicollinearity between the predictor variables is not of concern. The PRESS statistic is 1.88. The maximum leverage value is about 0.129.

The relation between T_p and main channel length is depicted in figure 29. Unlike the equation for N, dimensionless main channel slope is present in the equation for T_p; therefore the lines for the equation can not be depicted directly in figure 29. The results of the regression analysis on T_p are seen in figure 30, in which the vertical variation in the values is the influence of dimensionless main channel slope on the estimates of T_p. In both figures 29 and 30, a distinction between undeveloped and developed watersheds is made; there is considerable difference in average of typical T_p depending on the watershed development classification. Comparison of the two figures shows less variation in T_p in the estimated values, as expected from a regression equation. Comparison of the figures also indicates that the T_p equation is reliable.

The inverted covariance matrix is shown in figure 28 as `invcov`, where column 1 (row 1) is for regression intercept, column 2 (row 2) is for main channel length, column 3 (row 3) is for the dimensionless main channel slope, and column 4 (row 4) is for the watershed development classification.

Figure 29. Relation between observed time to peak of 1-minute Rayleigh unit hydrograph and main channel length for undeveloped and developed watersheds from IUH approach.

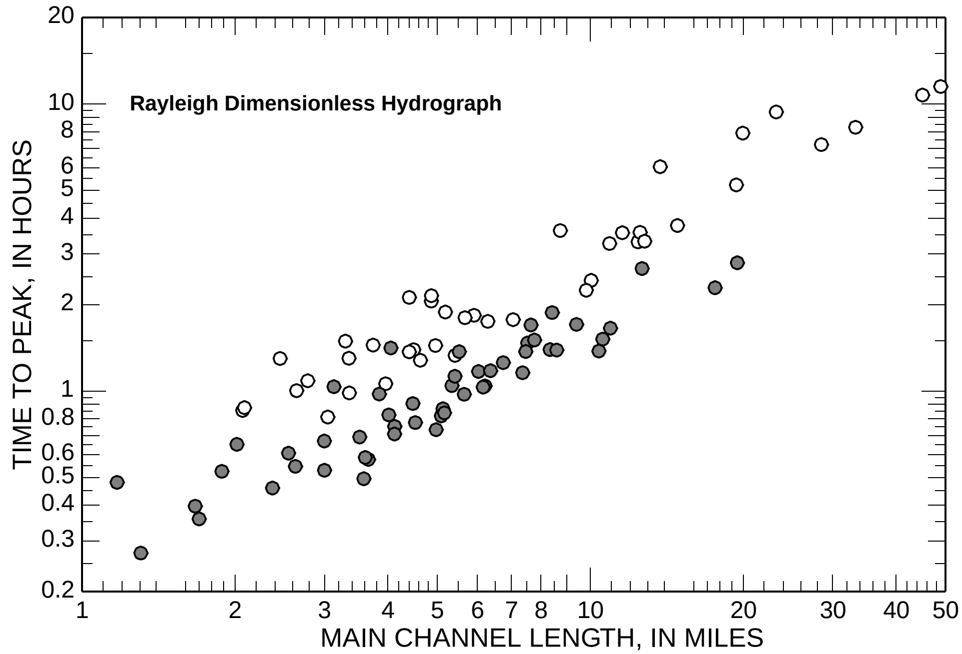


EXPLANATION

○ OBSERVED MEAN TIME TO PEAK FOR UNDEVELOPED WATERSHED

● OBSERVED MEAN TIME TO PEAK FOR DEVELOPED WATERSHED

Figure 30. Relation between modeled time to peak of 1-minute Rayleigh unit hydrograph and main channel length for undeveloped and developed watersheds from IUH approach.



EXPLANATION

○ MODELED TIME TO PEAK FOR UNDEVELOPED WATERSHED FROM REGRESSION

● MODELED TIME TO PEAK FOR DEVELOPED WATERSHED FROM REGRESSION

It is useful to demonstrate application of the T_p equation. Suppose a hypothetical developed watershed (D = 1) has an L of 10 miles and a S of 0.004. The estimate of T_p thus is $T_p = 10^{(-1.27 - 0.298)}10^{0.663}0.004^{-0.503}$ = 2.00 hours. The df of the equation is 86 and σ is 0.140. Suppose the 95th-percentile prediction limits are needed. These limits have an $\alpha/2$ = (1-0.95)/2 = 0.025. The upper tail quantile of the t-distribution for $1 - \alpha/2$ = 0.975 nonexceedance probability with 86 degrees of freedom is 1.988. Finally, the leverage of the hypothetical watershed is

$$h_o = \begin{bmatrix} 1 & \log(10) & \log(0.004) & 1 \end{bmatrix} \begin{bmatrix} 1.2439 & 0.10546 & 0.60930 & 0.00622 \\ 0.10546 & 0.15333 & 0.10568 & 0.01650 \\ 0.60930 & 0.10568 & 0.32342 & 0.01938 \\ 0.00622 & 0.01650 & 0.01938 & 0.04648 \end{bmatrix} \begin{bmatrix} 1 \\ \log(10) \\ \log(0.004) \\ 1 \end{bmatrix} \tag{63}$$

$$h_o = 0.0379\,. \tag{64}$$

Truncation in $(X^T WX)^{-1}$ is shown in eq. 63, but to mitigate round-off errors, the values in figure 28 should be used. Finally, the 95th-percentile prediction limits for T_p are

$$10^{\log(2) - 1.988 \times 0.1400\sqrt{1 + 0.0379}} \le T_p \text{ and} \tag{65}$$

$$T_p \le 10^{\log(2) + 1.988 \times 0.1400\sqrt{1 + 0.0379}}\,. \tag{66}$$

Therefore, the 95th-percentile prediction limits of T_p for the watershed are

$$1.04 \le T_p \le 3.84\,. \tag{67}$$

The T_p equation has three parameters and simple two-dimensional representation is problematic. However, it is useful to construct separate diagrams for T_p as a function of main channel length and dimensionless channel slope for both undeveloped and developed watersheds. These diagrams are depicted in figures 31 and 32. The grid lines are specifically included to ease numerical lookup.

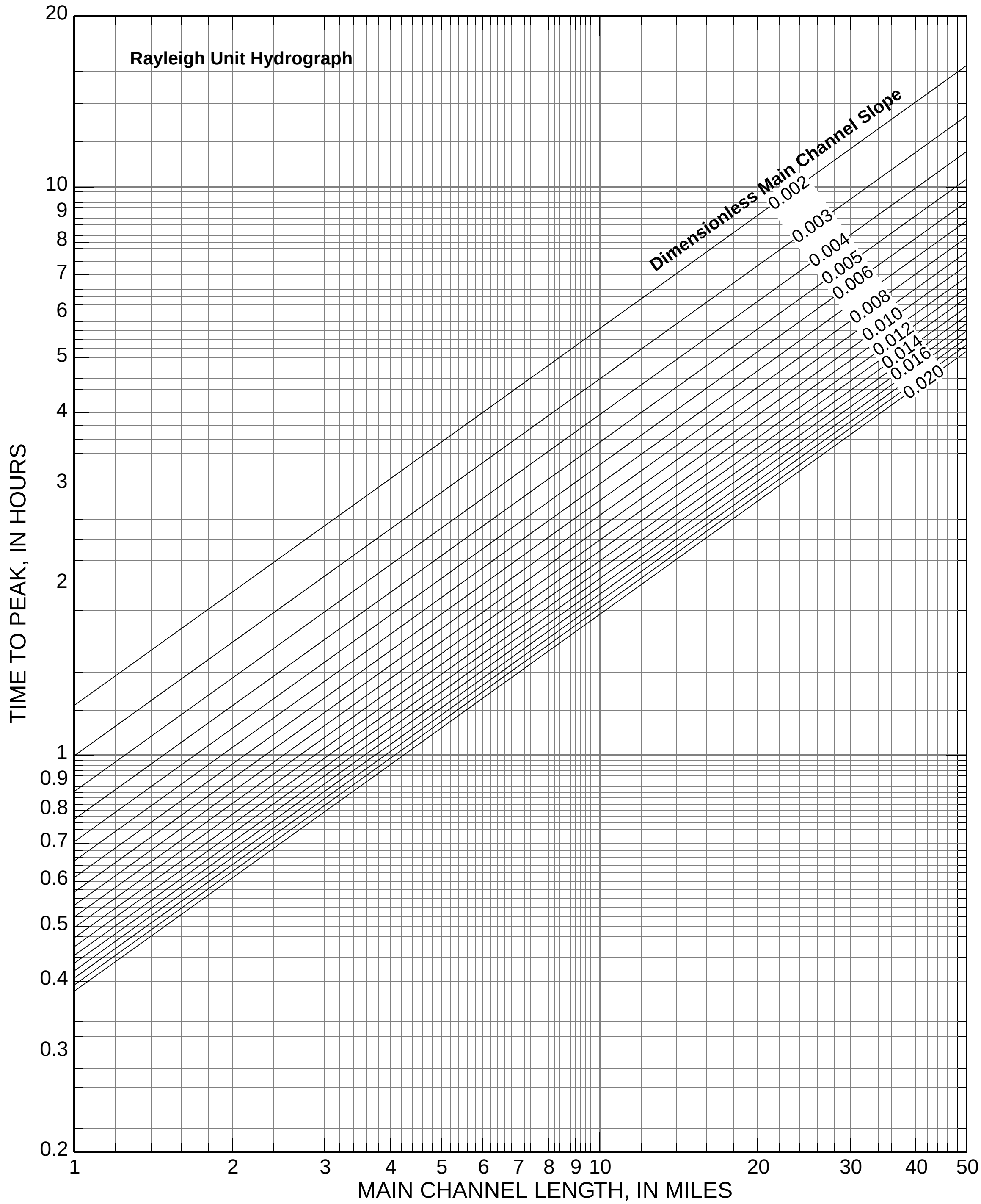


Figure 31. Time to peak of 1-minute Rayleigh unit hydrograph as function of main channel length and dimensionless main channel slope for undeveloped watersheds in Texas from IUH approach.

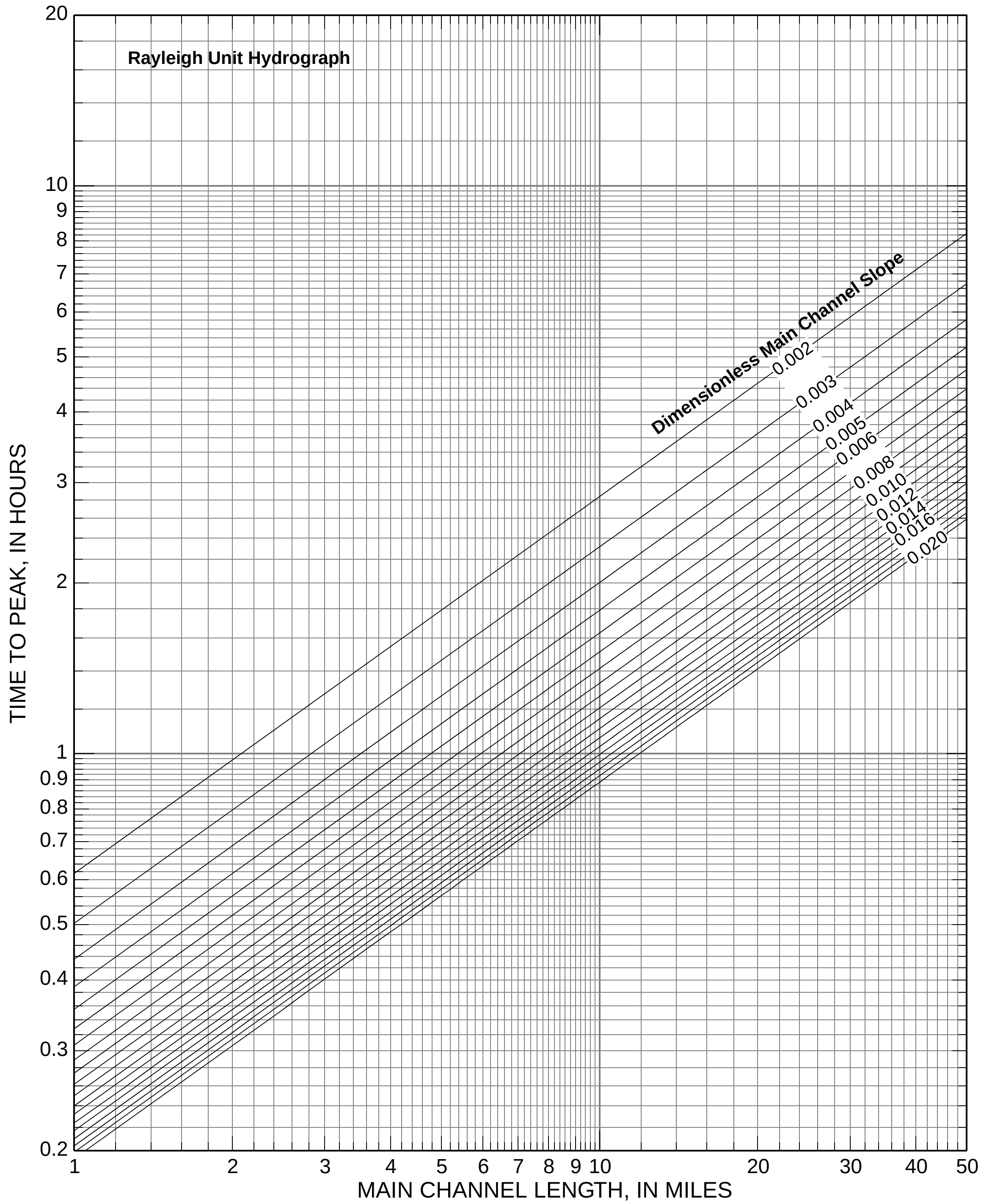


Figure 32. Time to peak of 1-minute Rayleigh unit hydrograph as function of main channel length and dimensionless main channel slope for developed watersheds in Texas from IUH approach.

UNIT HYDROGRAPH ESTIMATION FOR APPLICABLE TEXAS WATERSHEDS

A comparison between the four independent approaches for unit hydrograph estimation is provided in this section. A comparison is made in this section of the regionalization of unit hydrograph parameters (K or N and T_p) results as expressed by the regression equations in section "Regional Analysis of Unit Hydrograph Parameters" in this report. This section also assesses equation applicability.

It is important to clarify that the IUH approach produces Rayleigh unit hydrographs having 1-minute durations and the remaining three approaches produce gamma unit hydrographs having 5-minute durations. The duration distinction is important for unit hydrograph implementation in hydrologic engineering practice, but comparatively unimportant for the basic comparison made here.

Because each unit hydrograph approach was independent in terms of data analysis as well as procedure algorithms, it is informative to compare mean q_p and T_p for each of the watersheds. Comparison of shape parameters are not made because the shape parameter is not consistent in formulation and hence numerical values between the approaches.

A comparison of q_p among the methods is provided in figure 33. The mean q_p values from the GUHAS approach are plotted on the horizontal axis. This is an arbitrary decision. An equal value line is superimposed on the figure. The association of T_p to main channel length also is depicted in the figure. From the figure it, is evident that the approaches all produce q_p of similar order. However, there are differences.

Relative to GUHAS, the IUH approach produces q_p values that are systematically larger by about one-third of a log cycle. The traditional approach also produces q_p values that are systematically larger than GUHAS values and generally larger than IUH values. A reason attributable for the larger q_p of the traditional method is that the T_p values of the traditional method are smaller—as T_p is reduced, q_p must increase to maintain unit depth for the unit hydrograph. There is considerably more variation in q_p values for the traditional approach relative to the other three approaches. The q_p values from the LP approach generally are consistent with those from the GUHAS approach.

A comparison of T_p among the methods is provided in figure 34. The mean T_p values from the GUHAS approach are plotted on the horizontal axis. This is an arbitrary decision. An equal value line is superimposed on the figure. The association of T_p to main channel length also is depicted in the figure. From the figure, it is evident that the approaches all produce T_p of similar order. However, there are differences.

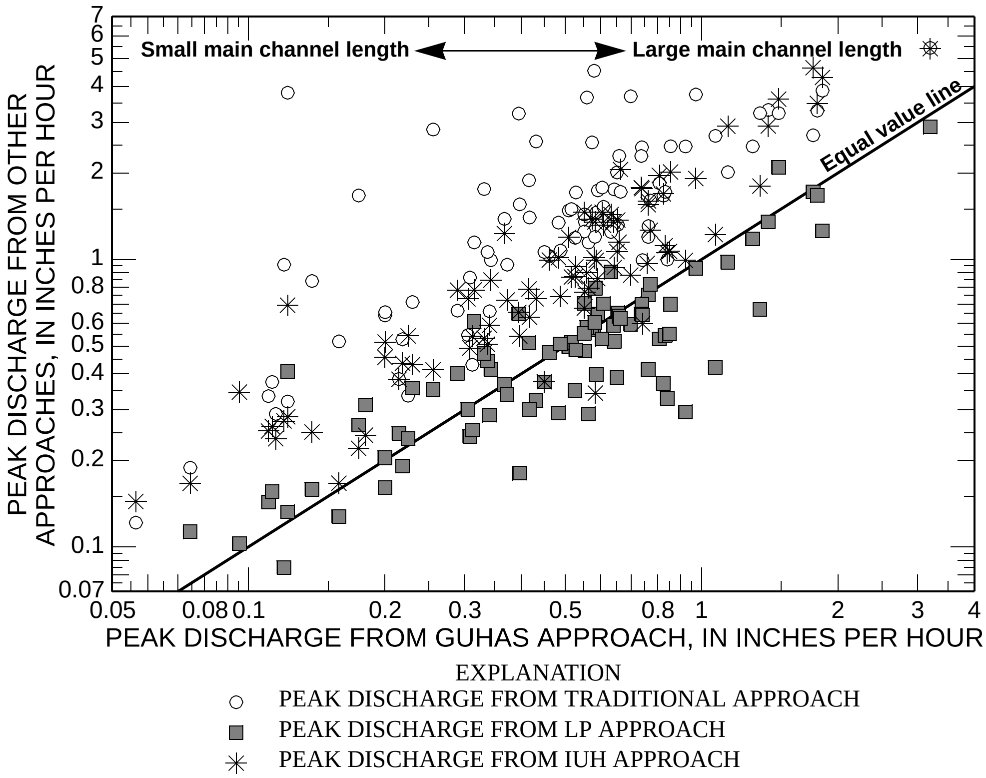


Figure 33. Comparison of peak discharge by watershed from the four unit hydrograph approaches.

Relative to GUHAS, the IUH approach produces T_p values that are similar with a trend towards IUH values less than GUHAS for the largest main channel lengths. The LU approach also produces similar T_p values but with slightly longer times relative to GUHAS for the smallest main channel lengths and shorter times relative to GUHAS for the longest main channel lengths. The LU approach appears to produce T_p values that are larger relative to GUHAS for the smallest main channel lengths and produces smaller T_p values relative to GUHAS for the largest main channel lengths. The T_p values from the traditional approach are systematically smaller than those from the other three approaches. One reason attributable to the smaller T_p for the traditional approach is the use of larger storms in the database.

Some of the differences in mean q_p and T_p for each of the watersheds among the methods are attributable to differences in approach including minimization differences, model selection or constraint differences, and differences in loss rate models. An additional consideration for differences are sample-size differences per watershed (table 3). Minimization techniques included peak discharge and observed peak time (GUHAS approach), range of deviations (LP approach), and maximum absolute deviation at peak discharge (IUH approach). Two of the approaches assume a unit hydrograph model prior to minimization; these models are the gamma unit hydrograph (GUHAS approach) and Rayleigh unit hydrograph (IUH approach). The irregular unit hydrographs derived directly from the traditional and LP approaches were smoothed by fitting a GUH only to the q_p and T_p.

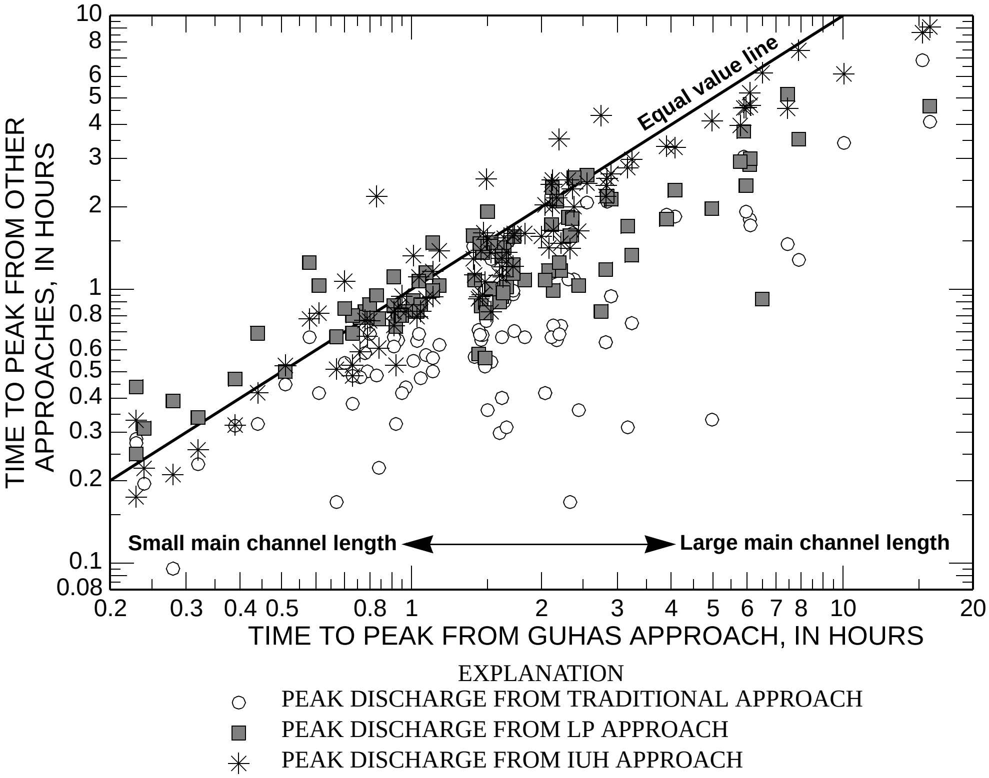


Figure 34. Comparison of time to peak by watershed for the four unit hydrograph approaches.

Finally, supported by figures 33 and 34, it is logical to conclude the basic computational methods implemented by the four approaches are somewhat consistent—gross errors in algorithms are unlikely. In general, the GUHAS, LP, and IUH approaches support each other. The traditional approach produces systematically different values. The traditional approach is very different from the other approaches including the focus on the largest events; therefore, considerable differences are expected. The traditional approach results are reasonable, so there is confidence that the procedure was implemented properly considering the heavy dependence on analyst input.

A comparison of the regression equations for hydrograph shape parameters from the four approaches is informative. A summary of the equations and weighted mean values is listed in table 4. No statistically significant equation for K estimation from the traditional approach was possible, and no significant differences in K according to watershed development classification are evident. These observations suggest that K might be a poor surrogate for the general shape of the unit hydrograph from the traditional approach.

The weighted mean shape factors for the undeveloped watersheds are all larger for the developed watersheds for the GUHAS, LP, and IUH approaches. The consistency among the three methods is important and basically consistent with the corresponding regression equations (note that for the N equation for the IUH approach, watershed development is not statistically significant).

The GUHAS and LP approaches both use K, and the equations are directly comparable. The intercept and exponent on main channel length for the GUHAS and LP equations are remarkably similar. However, watershed development classification has a larger influence in the GUHAS K equation than in the LP equation. Reasons for the difference are difficult to identify. Watershed development was not a significant variable for the 1-minute Rayleigh unit hydrograph shape in a weighted least-squares context from the IUH approach. The exponent on main channel length for the IUH N equation is positive, which is consistent in direction with both the GUHAS and LP approaches. The regression coefficients for the GUHAS, LP, and IUH approaches suggest that dimensionless hydrograph shape is more symmetrical with a shortening recession as main channel length increases. The range of adjusted R-squared for the equations is approximately 0.11 to 0.29; the range of residual standard error is approximately 0.06 to 0.21. The adjusted R-squared values are not large; there is much uncertainty in hydrograph shape estimation from watershed characteristics.

A comparison of the regression equations for hydrograph T_p from the four approaches is informative. A summary of the equations is listed in table 5. Statistically significant equations for T_p estimation from all four approaches are available. The signs on the exponent for main channel length and dimensionless main channel slope are consistent between the approaches and consistent with expectation; T_p is proportional to main channel length and inversely proportional to dimensionless main channel slope. The intercepts among the equations are similar as is the coefficient on the watershed development classification. The exception might be the intercept for the IUH approach. Of special recognition is that the GUHAS and LP T_p equations are essentially identical, with the LP equation having slightly better regression diagnostics such as residual standard error. The T_p equation from the traditional approach is substantially different from those of the other three approaches. The adjusted R-squared values indicate that there is comparatively less uncertainty in time to peak estimation from watershed characteristics than for unit hydrograph shape.

Table 4. Comparison of dimensionless hydrograph shape regression equation coefficients and summary statistics

[*a*, *b*, and *c*, regression coefficients; *K*, gamma dimensionless hydrograph shape; *N*, Rayleigh dimensionless hydrograph shape; nd, no statistically significant difference between weighted mean shape parameter on basis of watershed development classification; *D*, watershed development classification (*D* = 0, undeveloped; *D* = 1, developed); *L*, main channel length in miles; --, not applicable]

Equation form	Intercept coef-ficient (*a*)	Watershed develop-ment classifi-cation coef-ficient (*b*)	Main channel length coef-ficient (*c*)	adjusted R-squared	Residual standard error log(*K* or *N*)	Maxi-mum leverage	Weighted mean parameter for all water-sheds	Weighted mean parameter for undevel-oped water-sheds	Weighted mean parameter for devel-oped water-sheds
Traditional Unit Hydrograph Approach—84 watersheds and 602 events									
no statistically significant equation developed							6.3	nd	nd
Gamma Unit Hydrograph Analysis System Approach—92 watersheds and 1,984 peaks									
$K = 10^{a+bD}L^c$	0.560	-0.249	0.142	0.292	0.205	0.132	3.9	5.2	2.9
Linear Programming Approach—88 watersheds and 1,248 events									
$K = 10^{a+bD}L^c$	.481	-.0782	.140	.145	.146	.124	3.8	4.4	3.4
Instantaneous Unit Hydrograph Approach—92 watersheds and 1,545 events									
$N = 10^{a}L^c$	.378	--	.722	.107	.0632	.110	2.7	2.8	2.6

Table 5. Comparison of time to peak regression equation coefficients and summary statistics

[*a*, *b*, *c*, and *d*, regression coefficients; T_p, time to peak in hours; *D*, watershed development classification (*D* = 0, undeveloped; *D* = 1, developed); *L*, main channel length in miles; *S*, dimensionless main channel slope]

Equation form	Intercept coef-ficient (*a*)	Watershed development classification coefficient (*b*)	Main channel length coef-ficient (*c*)	Main channel slope coef. (*d*)	Adjusted R-squared	Residual standard error log(T_p)	Maxi-mum leverage
Traditional Unit Hydrograph Approach—80 degrees of freedom							
$T_p = 10^{a+bD}L^cS^d$	-1.63	-0.142	0.659	-0.497	0.698	0.188	0.138
Gamma Unit Hydrograph Analysis System Approach—87 degrees of freedom							
$T_p = 10^{a+bD}L^cS^d$	-1.49	-.354	.602	-.672	.858	.138	.136
Linear Programming Approach—81 degrees of freedom							
$T_p = 10^{a+bD}L^cS^d$	-1.41	-.313	.612	-.633	.870	.127	.142
Instantaneous Unit Hydrograph Approach—86 degrees of freedom							
$T_p = 10^{a+bD}L^cS^d$	-1.27	-.298	.663	-.503	.839	.140	.129

A comparison of the regression exponents on main channel length and dimensionless main channel slope for the time to peak equations to established watershed time of concentration equations is informative. Dingman (2002, table 9-9) through citation of Loucks and Quick (1996) summarizes time of concentration equations derived from earlier sources. Three of the equations only use main channel length and dimensionless channel slope. Therefore, these three equations are comparable to the T_p equations listed in table 5 when an assumption of a linear relation between T_p and time of concentration is made. This assumption commonly is made in hydrologic-engineering practice. The two exponents on main channel length and dimensionless channel slope can be compared. The mean exponent on main channel length from table 5 is about 0.63 and the typical exponent from the literature considered is about 0.73—the two values are similar in both sign and general magnitude. The mean exponent on dimensionless main channel slope from table 5 is about -0.58 and the typical exponent from the literature considered is about -0.37—the two values are similar in both sign and general magnitude.

It is illustrative to compute a unit hydrograph from each of the four approaches. Suppose a 10-square-mile watershed has a main channel length of 8 miles and dimensionless main channel slope of 0.006. The equations for hydrograph shape and T_p coupled with the GUH (eqs. 2 and 3) and the Rayleigh unit hydrograph (eqs. 7, 8, and 9) produce unit hydrographs. For the example watershed and an undeveloped watershed classification, the four unit hydrographs are shown in figure 35; whereas, for a developed watershed, the four unit hydrographs are shown in figure 36. As expected, the GUHAS and LP approaches produce similar unit hydrographs. The IUH approach produces a unit hydrograph that is more peaked and has a shorter T_p than the GUHAS or LP approaches as anticipated. Some of the IUH difference is attributable to a different unit hydrograph duration.

The regression equations are applicable in or near the multivariable parameter space as defined by the watershed characteristics (table 2). The range of main channel length is approximately 1.2 to 49 miles. The range of dimensionless main channel slope is approximately 0.002 to 0.020. For watersheds having characteristics increasingly outside these ranges, equation applicability is reduced. Another method to assess equation applicability is leverage h_o. Computation of h_o for arbitrary watersheds for each equation is described in section "Regional Analysis of Unit Hydrograph Parameters" in this report. An estimate for a regression equation can be considered an interpolation when the following condition is met:

$$h_o \leq h_{max}, \tag{68}$$

where h_{max} is the maximum leverage for the equation or the parameter space. For the four T_p equations, h_{max} is approximately 0.136. For the three shape parameter equations, h_{max} is approximately 0.122.

Figure 35. Estimated 1-minute Rayleigh and three 5-minute gamma unit hydrographs for an undeveloped 10-square-mile watershed (main channel length of 8 miles and dimensionless channel slope of 0.006) from each of the four unit hydrograph approaches.

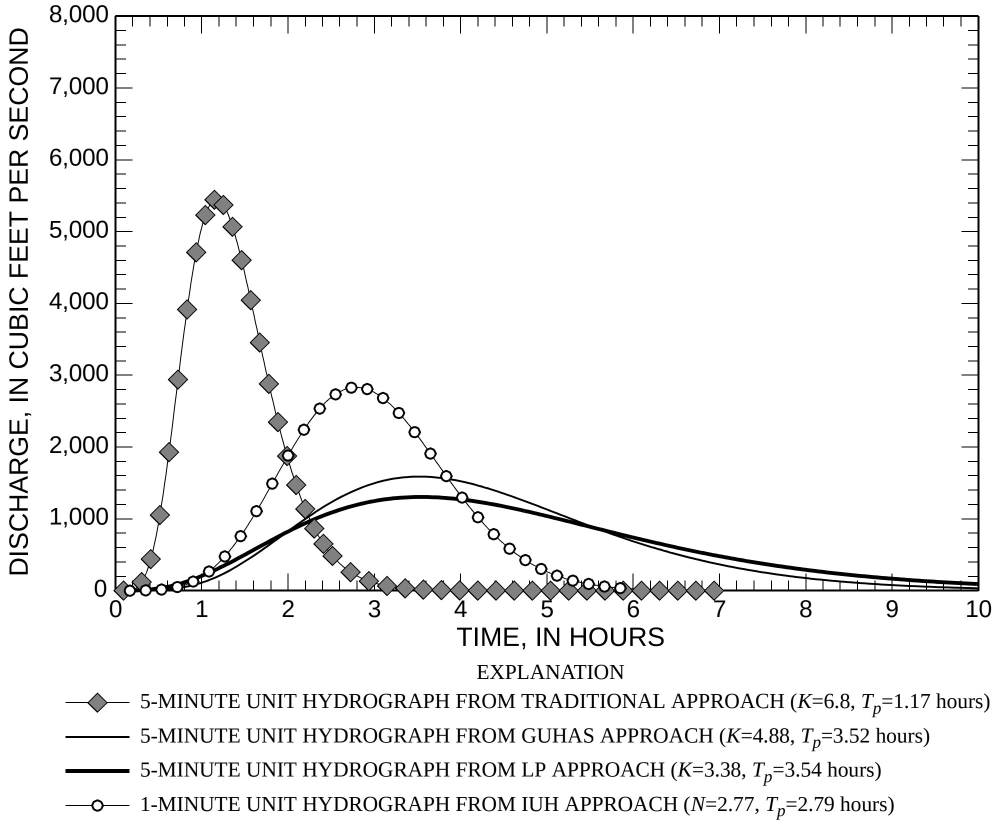
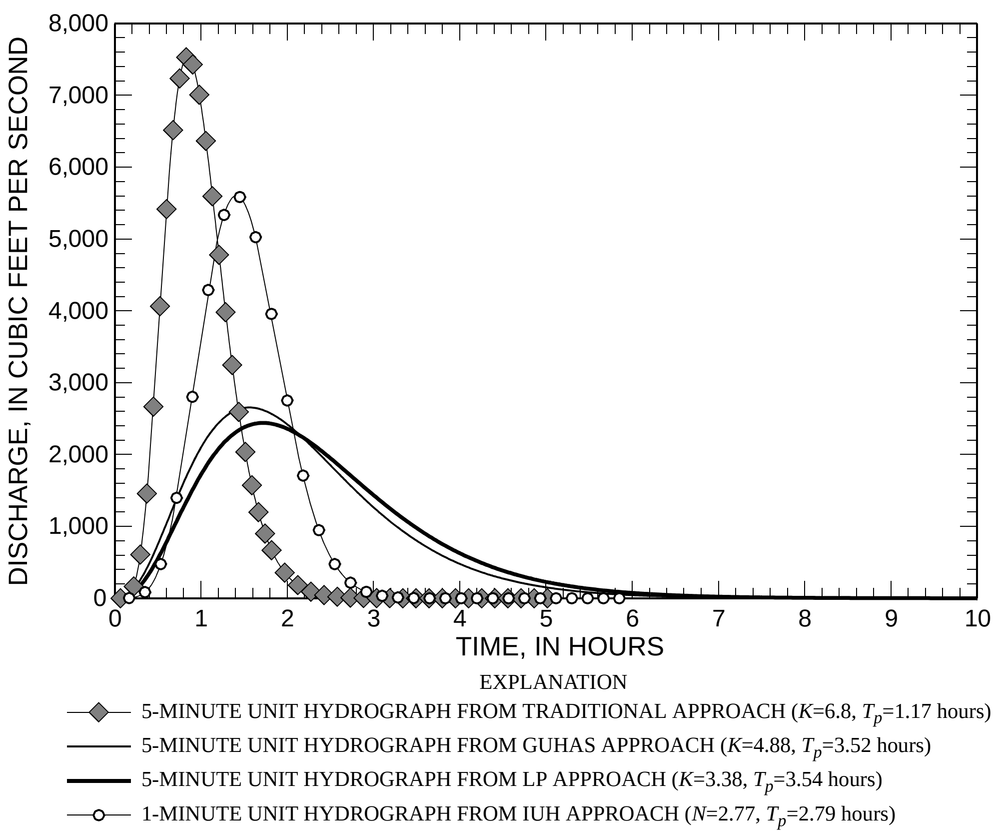


Figure 36. Estimated 1-minute Rayleigh and three 5-minute gamma unit hydrographs for a developed 10-square-mile watershed (main channel length of 8 miles and dimensionless channel slope of 0.006) from each of the four unit hydrograph approaches.



SUMMARY

The unit hydrograph is defined as a direct runoff hydrograph resulting from a unit pulse of excess rainfall generated uniformly over the watershed at a constant rate for an effective duration. The unit hydrograph method is a well-known hydrologic-engineering technique for estimation of the runoff hydrograph given an excess rainfall hyetograph. Methods to estimate the unit hydrograph for ungaged watersheds using common watershed characteristics are useful to hydrologic engineers. A large database of more than 1,600 storms with both rainfall and runoff data for 93 watersheds in Texas is used for four unit hydrograph investigation approaches.

Four separate approaches are used to extract unit hydrographs from the database on a per watershed basis: (1) the traditional approach that depends heavily on analyst input with a gamma unit hydrograph (GUH) model fit to the results, (2) the analyst-directed GUH analysis system (GUHAS) approach that matches observed peak discharge and time of peak, (3) the automated linear programming (LP) approach that minimizes on range of deviations with a GUH model fit to the results, and (4) the instantaneous unit hydrograph (IUH) approach based on a Rayleigh unit hydrograph that minimizes on the maximum absolute deviation at peak discharge. The unit hydrographs by watershed from the approaches are represented by shape and time to peak parameters.

Weighted least-squares regression is independently performed for unit hydrograph shape and time to peak for each approach. Main channel length and dimensionless main channel slope derived from 30-meter digital elevation models are the principal watershed characteristics considered. A binary watershed development classification (developed and undeveloped) also is considered for the regression analysis. The range of watershed area is approximately 0.32 to 167 square miles. The range of main channel length is approximately 1.2 to 49 miles. The range of dimensionless main channel slope is approximately 0.002 to 0.020.

A comparison of unit hydrograph peak discharge by watershed for the four approaches is made. A similar comparison for time to peak also is made. The comparison demonstrates that there is considerable intra-approach variability for these two unit hydrograph characteristics. The comparison also demonstrates that there are systematic differences in peak discharge and time to peak by approach by watershed.

Three equations are developed for estimation of hydrograph shape (GUH or Rayleigh as appropriate) for the GUHAS, LP, and IUH approaches. A statistically significant equation could not be developed for the traditional approach—the weighted mean shape factor is reported instead. Main channel length and watershed development classification (except for IUH approach) are represented in the equations. The range of adjusted R-squared values for the equations is approximately 0.11 to 0.29; the range of residual standard error is approximately 0.06 to 0.21. The adjusted R-squared values are not large; there is much uncertainty in hydrograph shape estimation from watershed characteristics. The weighted mean values for the GUH shape parame-

ter from the traditional, GUHAS, and LP approaches are consistent with values previously considered in the literature.

Four equations are developed for estimation of time to peak for each approach. Main channel length, dimensionless main channel slope, and watershed development classification are represented in the equations. The range of adjusted R-squared values for the equations is approximately 0.70 to 0.87; the range of residual standard error is approximately 0.13 to 0.19 log(hours). The adjusted R-squared values indicate that there is comparatively less uncertainty in time to peak estimation from watershed characteristics than for unit hydrograph shape.

A comparison of the regression exponents on main channel length and dimensionless main channel slope for the time to peak equations to established time of concentration equations is made. Assuming that there is a linear relation between time to peak and time of concentration, the two exponents can be compared. The mean exponent on main channel length is about 0.63 and the typical exponent from the literature considered is about 0.73—the two values are similar in both sign and general magnitude. The mean exponent on dimensionless main channel slope is about -0.58 and the typical exponent from the literature considered is about -0.37—the two values are similar in both sign and general magnitude.

The watershed characteristic ranges and the maximum leverage statistic of the parameter space on which the equations are developed provided a framework by which equation applicability for an arbitrary watershed can be assessed. For example, for watersheds having characteristics increasingly outside the watershed characteristic ranges, equation applicability is reduced. The use of the leverage statistics for estimation of prediction limits for the equations is demonstrated.

Finally, the equations provide a framework by which hydrologic engineers can estimate hydrograph shape, time to peak, and hence peak discharge of the unit hydrograph with assessment of equation applicability and uncertainty for a given watershed. The authors explicitly do not identify a preferable approach and hence equations for unit hydrograph estimation. Each equation is appropriate for a specific approach, and each approach represents the optimal unit hydrograph solution on the basis of the details of approach implementation including unit hydrograph model, objective functions, loss model assumptions, and other factors.

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