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16. Abstract

The object of this study is to propose and evaluate a design procedure for shear and torsion in reinforced and prestressed concrete beams, with the aim of clarifying and simplifying current design requirements and AASHTO requirements.

In previous reports in this series a three-dimensional space truss model with variable angles of inclination of the diagonals was introduced as a design model and shown by comparison with test data to be a conservative yet more accurate model than current ACI/AASHTO design approaches.

The general nature of this variable angle truss model makes it extremely useful to the designer in treating complex shear and torsion problems. Several examples of such applications are included in this report. Specific recommendations for incorporating such models are presented in language and expressions consistent with the type of language used in AASHTO Bridge Specifications. Several design examples are included both to clarify the application of the design model and to provide a comparison of the reinforcement using both the proposed changes and the current AASHTO requirements.

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PROPOSED DESIGN PROCEDURES FOR SHEAR AND TORSION IN REINFORCED AND PRESTRESSED CONCRETE

by

J. A. Ramirez and J. E. Breen

Research Report No. 248-4F

Research Project 3-5-80-248

"Reevaluation of AASHTO Shear and Torsion Provisions for Reinforced and Prestressed Concrete"

Conducted for

Texas

State Department of Highways and Public Transportation

In Cooperation with the
U. S. Department of Transportation
Federal Highway Administration

by

CENTER FOR TRANSPORTATION RESEARCH
BUREAU OF ENGINEERING RESEARCH
THE UNIVERSITY OF TEXAS AT AUSTIN

November 1983

P R E F A C E

This report is the fourth and final report in a series which summarizes a detailed evaluation of AASHTO design procedures for shear and torsion in reinforced and prestressed cocrete beams. The first report summarized an exploratory investigation of the shear transfer between joints using details commonly found in segmental box girder construction. The second report reviewed the historical development of design procedures for shear and torsion in concrete members as found in American practice and presented the background and equilibrium relationships for use of a space truss with variable inclination diagonals as a design model. The third report in this series summarized special considerations required for the practical usage of the variable inclination truss model. It also compared the theoretical capacity as computed by the truss model to experimental results for a great variety of previously reported tests as well as the results of tests run in this program to investigate several variables. This report draws on the analytical and experimental results presented in the earlier reports. It uses these results to develop design procedures and suggested AASHTO Specification procedures for girder shear and torsion. This report also contains several examples to illustrate the application of the design criteria and procedures.

This work is part of Research Project 3-5-80-248, entitled "Reevaluation of AASHTO Shear and Torsion Provisions for Reinforced and Prestressed Concrete." The studies described were conducted at the Phil

M. Ferguson Structural Engineering Laboratory as part of the overall research program of the Center for Transportation Research of The University of Texas at Austin. The work was sponsored jointly by the Texas State Department of Highways and Public Transportation and the Federal Highway Administration under an agreement with The University of Texas at Austin and the State Department of Highways and Public Transportation.

Liaison with the State Department of Highways and Public Transportation was maintained through the contact representatives, Mr. Warren A. Grasso and Mr. Dean W. Van Landuyt; the Area IV Committee Chairman, Mr. Robert L. Reed; and the State Bridge Engineer, Mr. Wayne Henneberger. Mr. T. E. Strock was the contact representative for the Federal Highway Administration.

The overall study was directed by Dr. John E. Breen, who holds the Carol Cockrell Curran Chair in Engineering. The project was under the immediate supervision of Dr. Julio A. Ramirez, Research Engineer. He was assisted by Mr. Thomas C. Schaeffer and Mr. Reid W. Castrodale, Assistant Research Engineers.

SUMMARY

The object of this study is to propose and evaluate a design procedure for shear and torsion in reinforced and prestressed concrete beams, with the aim of clarifying and simplifying current design requirements and AASHTO requirements.

In previous reports in this series a three-dimensional space truss model with variable angles of inclination of the diagonals was introduced as a design model and shown by comparison with test data to be a conservative yet more accurate model than current ACI/AASHTO design approaches.

The general nature of this variable angle truss model makes it extremely useful to the designer in treating complex shear and torsion problems. Several examples of such applications are included in this report. Specific recommendations for incorporating such models is presented in language and expressions consistent with the type of language used in AASHTO Bridge Specifications. Several design examples are included to both clarify the application of the design model and to provide a comparison of the reinforcement using both the proposed changes and the current AASHTO requirements.

IMPLEMENTATION

This report is the final in a series which summarizes a major experimental and analytical project aimed directly at suggesting new design recommendations for treating shear and torsion in reinforced and prestressed concrete girders. The detailed recommendations for possible changes in AASHTO Bridge Specifications are included in this report.

This report contains background information of interest both to those responsible for deciding on specifications and codes and to designers. In addition, it contains detailed examples of the application of the space truss with variable angle of inclination of the diagonals to shear and torsion design. Such information will be of particular value to designers interested in specific application of the variable angle truss model in new and unfamiliar situations.

The report shows the new proposal to be conservative, accurate and more versatile than existing procedures. In some cases it can result in reduction of web reinforcement and congestion but in other cases will produce around the same designs as currently found. However, the designer will have substantially improved knowledge regarding the design process and will be able to treat many design cases now not covered by the AASHTO Specifications.

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CHAPTER 1

INTRODUCTION

1.1 General

Design provisions for shear and torsion for reinforced and prestressed concrete members and structures in both the AASHTO Specifications (1) and the ACI Building Code (2) have evolved into complex procedures in recent revisions. The complexity of such procedures results from their highly empirical basis and the lack of a unified treatment of shear and torsion. Ironically, such design procedures seem better suited for analysis, since they become cumbersome and obscure when used for design.

In the case of continuous bridges, the designer must consider several different loading combinations to obtain maximum shear and flexural effects. The use of different loading combinations in the current design procedures is unclear and contradictory. This highly complicates the design of such members.

Both current ACI recommendations and AASHTO specifications superimpose reinforcement required for torsion to that required for bending and shear without specific consideration of the interactions. The practice of superimposing these effects is due to the lack of a unified approach to design for shear and torsion which would permit the correct evaluation of the combined actions. There is a total absence of design regulations for the case of prestressed concrete members subjected to torsion or combined torsion, shear, and bending. Current

American design practices do not emphasize the importance of adequate detailing for members subjected to shear and torsion. Furthermore, due to the empirical nature of such design procedures, it is not clear to the designer how to adequately detail such members.

Such deficiencies could be overcome if the design procedures in the shear and torsion areas were based on behavioral models rather than on detailed empirical equations. The designer would be able to envision the effects of the forces acting on the member, and then provide structural systems capable of resisting those forces. Furthermore, design provisions based on a conceptual model would become more simple and would not require as much test verification.

1.2 Problem Statement

The June 1973 report of ACI-ASCE Committee 426, "The Shear Strength of Reinforced Concrete Members" (3), indicated that for the next decade the Committee

> . . . hoped that the design regulations for shear strength can be integrated, simplified, and given a physical significance so that designers can approach unusual design problems in a rational manner.

Procedures for dimensioning cross sections for reinforced and prestressed concrete members subjected to axial load, or moment, or combined axial load and moment, are generally well established. These procedures can be explained in a few pages of text, and are based on rational, simple general design models which can be embodied in a few paragraphs of code or specification documents.

Such failure models provide the designer with means to evaluate the ultimate moment capacity of quite irregular sections in both reinforced and prestressed concrete. In addition, the same basic models can be used to study the interaction between axial load and moment, making the related design process relatively simple and straightforward. Unfortunately, design provisions in the areas of shear and torsion are not of the same level of rationality and general applicability. The absence of rational models has resulted in highly empirical design procedures characterized by large scatter when compared to test results.

The lack of fundamental behavioral models for concrete members subjected to shear and torsional loading seems to be the prime reason for the unsatisfactory nature of the current highly empirical design procedures used in North American codes and standards.

In the late 1960's, researchers in Europe were working with the idea of a conceptual model to properly represent the behavior of concrete members subjected to torsion and shear. The main objectives were to rationalize and at the same time simplify the design procedures in these areas. In Switzerland, Lampert and Thürlimann (4) developed a conceptual model based on theory of plasticity. The model was a Space Truss with variable angle of inclination of the diagonal compression members. This model was a refined version of the Truss Model with a constant 45 degree angle of inclination of the diagonal compression members originally introduced in Switzerland at the beginning of this century by Ritter (5) for the case of shear in reinforced concrete members. Thürlimann (6-9) refined the model and it has been used in the

Swiss Code (10). During the late 1970's, in Canada, Mitchell and Collins (11-17) also proposed a generalized design approach based on a theoretical compression field model. This was a major departure from the highly empirical approach followed in American practice. Mitchell and Collins were able to treat general problems of shear and torsion in both prestressed and reinforced concrete members in a unified rational fashion. However, the authors fell short of providing the designer with a simple and easy to apply design method. The advantages of the procedure proposed by Mitchell and Collins were obscured because of the complex approach followed in the proposed design recommendations to indirectly ensure suitable behavior at service load levels.

Designers are generally not eager to adopt complex new design methods, even if accurate, when they previously have ignored effects such as torsion without disastrous consequences. For this reason, a rational and easy to apply approximate design approach based on a simplified model which considers only the main variables is needed.

1.3 Objectives and Scope of the Study

The present study attempts to develop such a simplified approach based on an acceptable model. An overall review of the current AASHTO Specifications and the ACI Building Code in the areas of shear and torsion was summarized in Report 248-2. This study showed that design procedures have become more and more complex with every revision. The highly empirical provisions are difficult to use in many design situations.

The main objective of this study is to propose and evaluate a design procedure for shear and torsion in reinforced and prestressed concrete beams. The goal is to clarify and simplify current design recommendations and AASHTO requirements in such areas. The basic reevaluation of the current procedures and development of new procedures are to be carried out using a conceptual structural model rather than detailed empirical equations wherever practical.

The theoretical background of the space truss model with variable angle of inclination of the diagonal elements was summarized in Report 248-2. This model was selected as one which best represents the behavior of reinforced and prestressed concrete beams subjected to shear and torsion. This conceptual model was developed over the past 20 years. Principal contributions were made by Thürlimann (4,6-9), Lampert (6,19), Nielsen (19-21), Müller (39-40), Marti (41), Collins and Mitchell (11-17), and code provisions have been adopted by Switzerland (10) and CEB-FIP (22). Much of the work has been based on highly complex proofs of the application of plasticity theorems in the fields of shear and torsion. The complete formulations are generally not in English and are quite complex. The more limited reports which are in English have not had wide American readership. The apparent complexity of the proofs of the plasticity theorems as applied to shear and torsion can cause the more design-oriented reader to lose sight of the fact that the authors use these proofs only as a theoretical basis for proving the application of a refined truss model. The model is shown to be a lower bound equilibrium solution giving the same result as the much more rigorous kinematic upper bound solution. Hence, it is a valid solution which correctly represents the failure load.

However, it was felt that before the generalized refined truss model approach could be used as the basic design procedure in American practice, a complete evaluation of the accuracy of the model using a significant body of the available test data reported in the American literature was necessary. In companion Report 248-3, thorough comparisons of the space truss model with a wide range of test data and with predicted failure loads from other design procedures are presented. It is shown to be accurate and conservative.

In this report the general procedures derived from the space truss model are translated into design recommendations and draft AASHTO requirements are recommended. Design applications for typical highway structures using the proposed design recommendations as well as the current AASHTO approach are presented for comparison.

CHAPTER 2

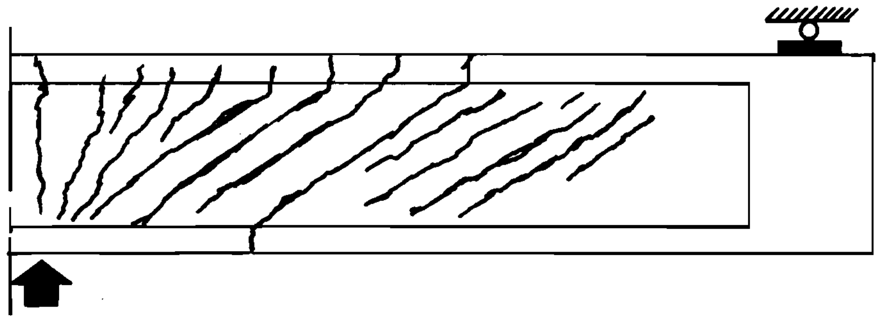
BACKGROUND FOR DESIGN RECOMMENDATIONS

2.1 Introduction

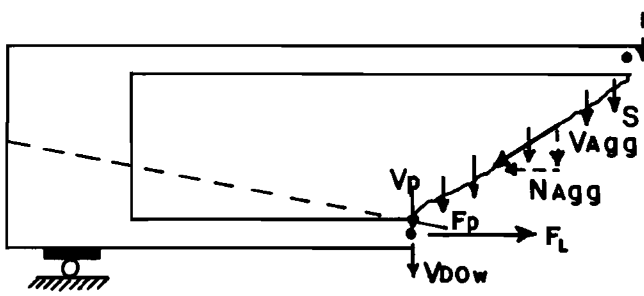
Historically, design models and rules for beams with web reinforcement have been oriented to the crack pattern and strains of the beam at failure. What failure means is subject to definition, but normally is defined as the maximum load of a test beam.

The basic design model selected in this study after review of currently proposed design approaches is based on the generally familiar truss model. However, it includes a less familiar extension that provides for compression diagonals with variable angle of inclination (see Fig. 2.1). The general background and the derivation of the equilibrium equations for the space truss under combinations of bending, shear, and/or tension were given in Report 248-2. The accuracy of the model was documented in Report 248-3.

The general design approach ensures a reasonably ductile behavior by incorporating relationships to preclude shear and torsion failure without yielding of web reinforcement and to force any shear or torsion failure to occur in combination with yielding of the longitudinal reinforcement. In this way brittleness is prevented. One of the benefits of this approach is that both prestressed and reinforced beams, with steel percentages limited to those values which qualify as underreinforced sections and with premature failures due to poor



(a) Crack pattern at failure



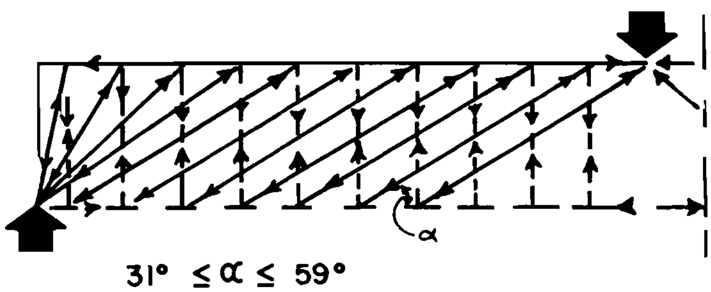
(b) Free body diagram

V_N: Vertical component of the compression chord

S: Stirrup forces

V_p: Vertical component of prestressing force

V_{Dow}: Dowel action forces



(c) Truss model

Fig. 2.1 Basic actions of a beam in shear and the truss model for beams with web reinforcement at failure detailing prevented, show the same characteristics of behavior at failure. Therefore, both types can be treated with the same model.

All of the other current design procedures or proposals discussed in the next sections have, in one form or another, the truss model as the fundamental design model for the cases of torsion and/or shear. The basic differences between them lie in the limitations of the truss model.

An examination of Fig. 2.1 reveals that some of the components of the failure mechanism in a beam are not considered in the truss model in favour of simplicity of the design model. These other components of the failure mechanism must be considered indirectly either in the geometry of the truss (compression strut angle) or by additional rules (e.g., V_c-term).

In this chapter a review of some of the recent design procedures for shear and torsion available in codes other than ACI and AASHTO is carried out. After that review, further background for the proposed design recommendations for reinforced and prestressed concrete members subjected to shear and/or torsion are given. The detailed recommendations based on the truss model with variable angle of inclination of the diagonals as the fundamental design model are given in suggested design specification language in the next chapter.

The design recommendations proposed in Chapter 3 are illustrated through a series of design examples worked in Chapter 4. The results are compared with similar examples designed under the current AASHTO and ACI design recommendations.

2.2 Review of Some Design Procedures Available or Recommended for Other Codes

All the design procedures discussed in this report are based on the variable inclination truss model. The main difference between these design methods is in the way the actions that are not directly considered in the truss model are introduced in the design procedure.

These actions are introduced either indirectly in the geometry of the truss model (by modifying the compression strut angle) or by allowing an additional concrete contribution (V_c-term) to supplement the truss contribution, but only at certain stages.

2.2.1 CEB-Refined. The CEB-Refined method (22) is based on the truss model with variable angle of inclination of the diagonal compression struts at failure. The design procedure can be used for the design of reinforced and prestressed concrete members subjected to bending and shear. It can also be applied to the case of torsion and to the combined cases of bending, shear, and torsion.

In the CEB-Refined procedure the actions neglected in the truss model are considered indirectly in the geometry of the truss model (variable angle of inclination of the compression strut) and also by allowing an additional diminishing concrete contribution, which approaches zero as the nominal shear stress increases.

The inclination of the compression strut is limited to values of α between

$$31^\circ \le \alpha \le 59^\circ \tag{2.1}$$

The lower limit on the angle α is introduced to control excessive cracking in the web under service load conditions. Another reason to limit the range within which the angle α is allowed to vary is that for yield to be developed in both the longitudinal and transverse reinforcement, very high strains are required in the reinforcement which yields first. There are also possibilities of excessive inclined crack widths when the angle α deviates too greatly from 45 degrees (see Report 248-2). The initial shear cracks in reinforced concrete beams are often inclined at approximately 45 degrees. The development of failure cracks at other angles requires the transmission of forces across the initial cracks. Since the capacity for this transmission may be limited, excessive redistribution of internal forces required by designing for angles which deviate too greatly from 45 degrees should be avoided.

In the CEB refined design method the design shear force V_u must be equal to or less than the sum of the nominal shear resistance V_s carried by the truss action (inclined concrete struts and steel reinforcement) and the resistance V_c attributed to the shear resistance of the concrete flexural compression zone and secondary effects.

$$V_u \leq V_n = V_s + V_c \tag{2.2}$$

The shear carried by the truss is computed using Eq. 2.3.

$$V_s = [A_s/s]f_{wd}(0.9d)(\cot\alpha + \cot\theta)\ \sin\theta \tag{2.3}$$

where A_s is the cross-sectional area of web reinforcement, f_{wd} is the design stress of the web reinforcement, i.e., the yield stress divided by a resistance factor, "s" represents the spacing of web reinforcement (stirrups), θ is the angle of inclination of the web reinforcement, and α is the angle of inclination at ultimate of the concrete compression diagonals. Equation 2.3 for the case of vertical stirrups (θ = 90 degrees) follows directly from Eq. 3.63 of Report 248-2.

$$V_s = \frac{A_s}{s} f_{wd} (0.9d) \cot\alpha \tag{2.4}$$

The value of the angle α has to be chosen within the limits presented in Eq. 2.1. The truss model shows that the chosen angle α will have direct influence on the design of the longitudinal reinforcement. An area of longitudinal steel required to balance the horizontal component of the diagonal compression field due to the presence of shear must be provided in addition to that required for flexure.

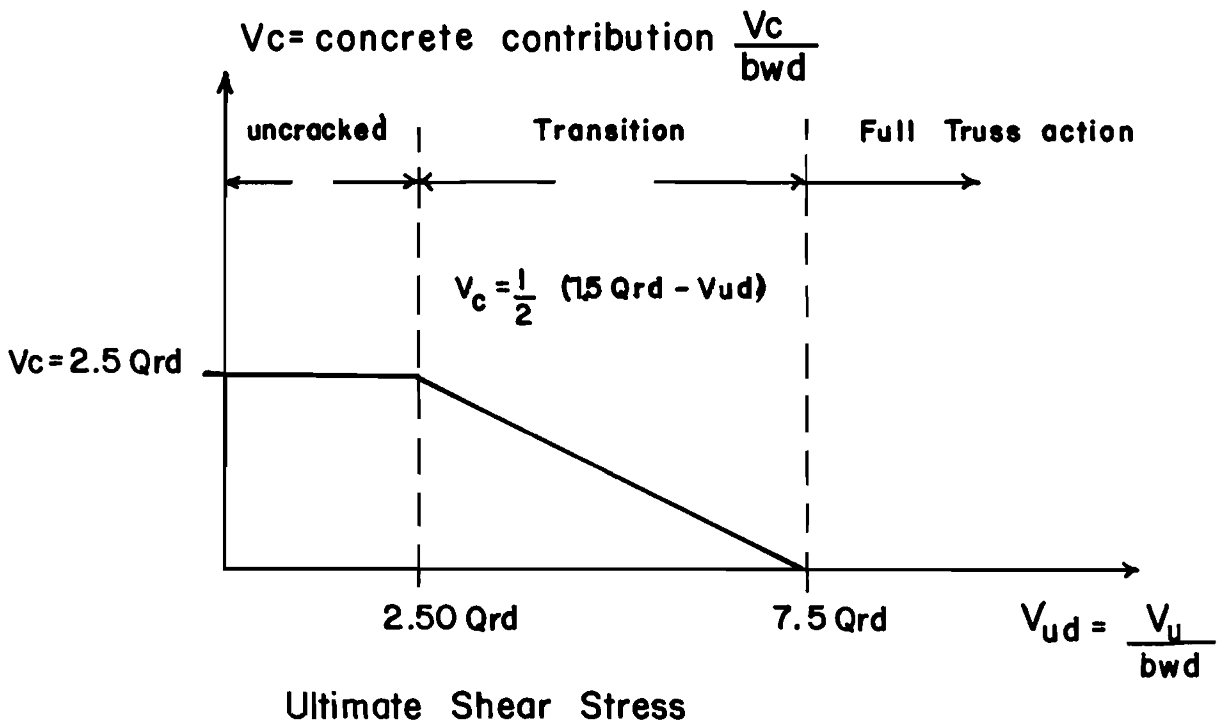
$$A_1(V) = \frac{V_u^2 s}{2A_s f_{wd}(d) f_{1d}} = \frac{1}{2} \frac{V_u}{f_{1d}} \cot\alpha \tag{2.5}$$

where f_{1d} is the design stress of the longitudinal steel, i.e. the value of the yield stress divided by the appropriate safety factor. Eq. 2.5 follows directly from Eq. 3.65 in Sec. 3.5.2 of Report 248-2 from equilibrium considerations for the truss model with variable angle of inclination.

The concrete contribution term V_c varies linearly with the intensity of the nominal shear stress [V/b_w*d] in the transition range between the uncracked state and the fully developed truss action in a manner similar to that discussed in Report 248-3. The CEB proposed values for the concrete contribution in this transition zone are presented in Fig. 2.2.

In Fig. 2.2 the values of Q_{rd} include a material safety factor of 1.5 as recommended by the CEB Code for the case of concrete. Therefore, the nominal concrete contribution in shear in the uncracked low shear stress range is 1.5 * 2.5 * Q_{rd} or 3.75 * Q_{rd}. In terms of $k\sqrt{f'_c}$ for the values of Q_{rd} given in Fig. 2.2, this lower range 3.75 Q_{rd} yields values of k ranging from 2.4 to 3.2. These values are between the values of $2\sqrt{f'_c}$ and $3.5\sqrt{f'_c}$ which are currently recommended in the ACI Building Code (2) and AASHTO Standard Specifications (1) as the simplified and maximum values respectively of the nominal concrete contribution in shear depending on the moment to shear ratio on a section. These values then decrease for members with higher values of shear. Such a provision gives substantial relief in shear design of lightly loaded members.

For the case of prestressed concrete members, the same type of linear concrete contribution in the transition zone is suggested. However, the values of v_c of 2.5 Q_{rd} are increased by a factor $K = 1 + [M_o/M_{sdu}] \leq 2$, where M_{sdu} is the maximum design moment in the shear region under consideration, and M_o denotes the decompression moment at transfer related to the extreme tensile fiber, for the section where M_{sdu} is acting. This moment is equal to that which produces a tensile stress that cancels the compression stresses due to the applied prestress force and other design axial forces.



where:

f'_c (psi)	Q_{rd} (psi)
1740	26.1
2320	31.9
2900	37.7
3625	43.5
4350	49.3
5075	55.1
5800	60.9
6525	66.7
7250	72.5

Fig. 2.2 Concrete contribution in the transition range CEB-Refined Method

In order to avoid failures due to crushing of the web, an upper limit on the shear resistance of

$$V_{max} = 0.3 f_{cd} b_w d \sin\alpha \tag{2.6}$$

is required. f_{cd} represents the design concrete compression stress, i.e. characteristic value of the concrete compression strength divided by a resistance safety factor. In terms of f_c' the maximum shear stress $V_{max}/[b_w d]$, with a resistance safety factor of 1.5 as suggested in the CEB Code, would become equal to $0.2\, f_c' \sin 2\alpha$. A comparison between the CEB upper limit and AASHTO and ACI upper limit of $10\sqrt{f_c'}$ is shown in Fig. 2.3 for α of 45 and 30 degrees.

The design procedure for torsion in the CEB-Refined method is also based on the Truss model with variable angle of inclination of the

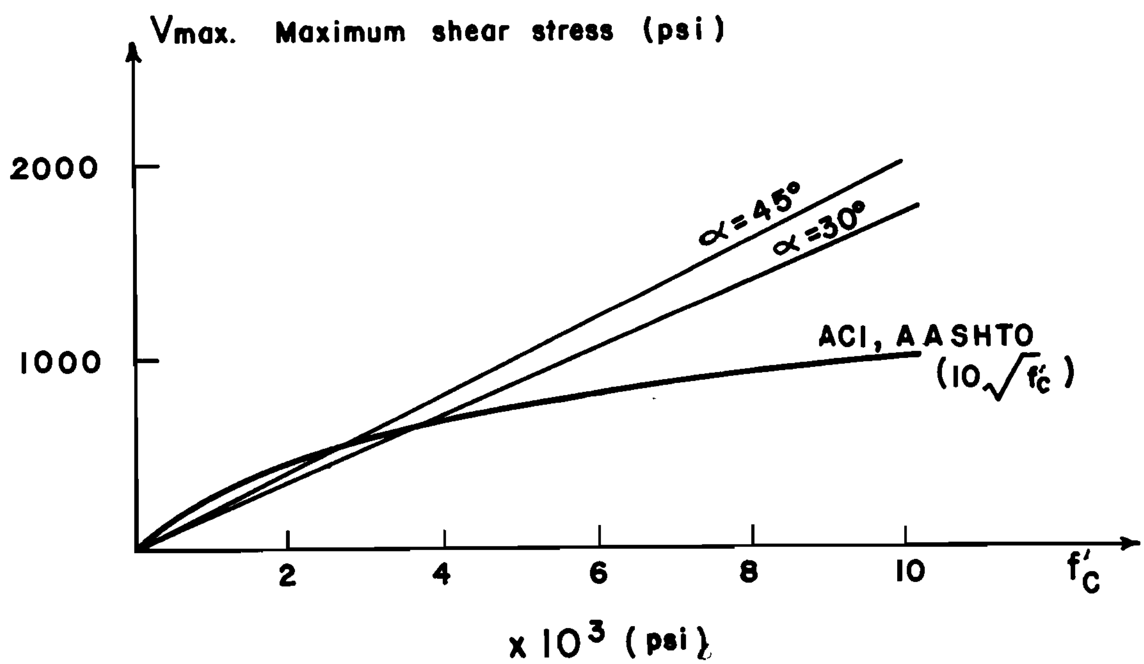


Fig. 2.3 Upper limit of the shear stress in the section

diagonal compression strut. A very important differentiation is made between the cases of equilibrium and compatibility torsion. The CEB-refined procedure in the case of torsion neglects compatibility torsion. In this design procedure it is assumed that since compatibility torsion is caused by deformations of adjacent members in statically indeterminate structures it will produce secondary effects which should be considered in evaluating servicability, but can be neglected in the ultimate strength design of the section. Therefore, in the ultimate strength design of the section only the cases of equilibrium torsion are considered.

For the same reasons given in the case of shear the limits for the angle of inclination of the diagonal compression strut remain controlled by the values proposed in Eq. 2.1.

In the case of torsion, the ultimate torsional moment T_u must be equal or less than the resistance value. The resistance value T_n is made up of the resistance T_s carried by the truss and the additional resistance of the concrete T_c in the transition range between the uncracked state and full truss action.

The torsion carried by the truss action is evaluated using Eq. 2.7.

$$T_s = \frac{A_s}{s} 2 A_0 f_{wd} \cot\alpha \qquad (2.7)$$

Equation 2.7 follows directly from Eq. 3.31 derived in Sec. 3.4 of Report 248-2 from the equilibrium conditions in the truss model. A_o is the area enclosed by the perimeter conecting the centers of the longitudinal chords in the truss model. A design carried out on the basis of the truss model requires an area of longitudinal steel, in addition to the one required for flexure, due to the presence of torsion.

$$A_1(T) = \frac{T_u \cdot u}{f_{\ell d} \; 2\,A_0} \cot\alpha \qquad (2.8)$$

Equation 2.8 results from Eq. 3.30 derived in Sec. 3.4 of Report 248-2. $A_1(T)$ is the total area of longitudinal steel required to resist the torsional moment T_u, "u" represents the perimeter connecting the centers of the longitudinal chords of the truss model.

As in the case of shear, the concrete contribution to the torsional resistance of the section varies linearly depending upon the magnitude of the nominal shear stress produced by the torsional moment T_u. Figure 2.4 illustrates the concrete contribution as suggested in the CEB-Refined method.

As can be seen from Fig. 2.4 the concrete contributions to the torsional and shear capacity of the section are the same in terms of shear stresses. In Fig. 2.4 the values of the shear stresses Q_{rd} remain the same as those values given in Fig. 2.2.

The nominal shear stress due to torsion (v) is given by Eq. 2.9

$$v = T/[2A_0b_e] \qquad (2.9)$$

which is the shear stress produced in a thin walled tube by the presence of a torsional moment assuming a constant shear flow around the perimeter of the cross section. In Eq. 2.9 "b_e" represents the

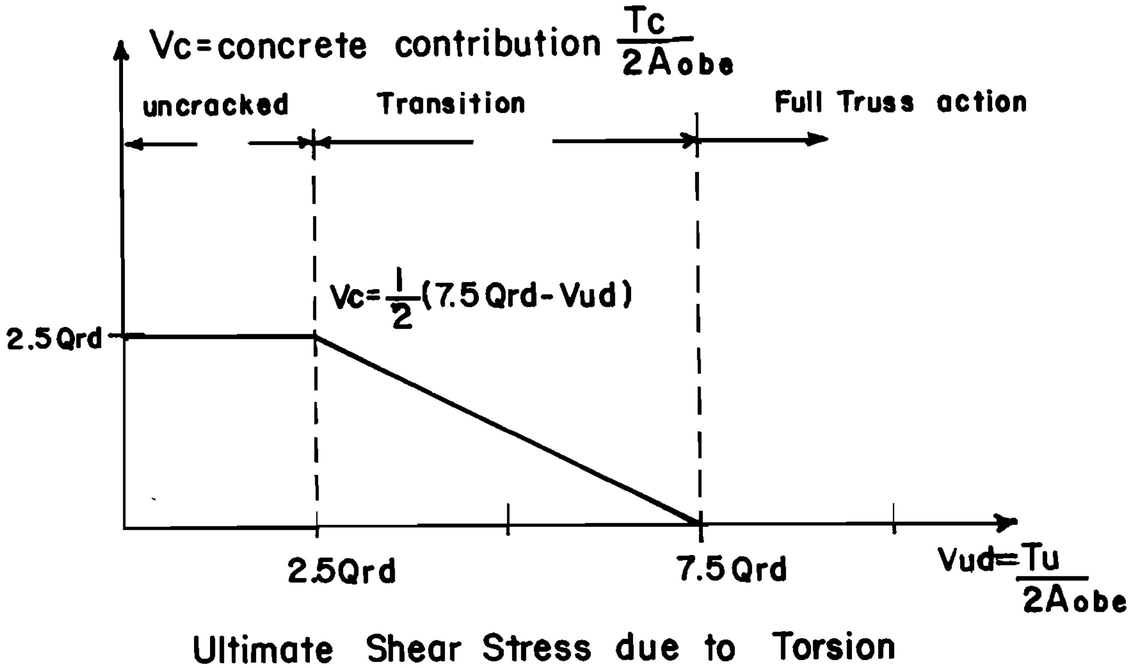


Fig. 2.4 Additional concrete contribution in the transition range

effective wall thickness of this assumed thin-walled tube. This term has been discussed in Sec. 2.6.1 of Report 248-3. In the CEB-Refined method, "b_e" is taken equal to $d_e/6$, where d_e is the diameter of the circle inscribed into the perimeter "u" formed by the centroids of the longitudinal bars forming the truss model of the cross section in consideration.

From Fig. 2.4, it can be seen that the maximum value of the concrete contribution occurs at low torsional shear stress levels and is given by

$$T_c = 5Q_{rd}A_ob_e \tag{2.10}$$

Hence, the concrete contribution [v_c] in terms of shear stress and f'_c can be obtained from Eqs. 2.9, 2.10 and the resistance safety factor for the concrete, 1.5 introduced in the CEB-Refined method.

$$v_c = T_c/[2A_ob_e] = 1.5*2.5Q_{rd} \qquad (2.11)$$

Expressing Eq. 2.11 in terms of $k\sqrt{f'_c}$ and substituting the values of Q_{rd} yields k values ranging from 2.3 to 3.4 which are the same magnitude as those in the case of shear. ACI 318-77 (2) and the AASHTO Standard Specifications (1) allow a nominal concrete contribution in the case of pure torsion of $v_t = 2.4\ \sqrt{f'_c}$, which is a lower bound value for the CEB-Refined method.

In order to prevent failures due to crushing of the web, an upper limit to the torsional strength of

$$T_{max} = [f_{cd}A_ob_e\sin 2\alpha]/2 \qquad (2.12)$$

is given. In terms of f'_c and with a resistance safety factor of 1.5 as suggested by the CEB Code, this yields a maximum shear stress of

$$V_{max} = \frac{T_{max}}{2A_0b_e} = 0.167\ f'_c\ \sin 2\alpha \qquad (2.13)$$

A comparison of this upper limit with the ACI (2) and AASHTO (1) limit of $12\sqrt{f'_c}$ for values of α of 45 and 30 degrees is shown in Fig. 2.5.

In the case of combined torsion and shear the reinforcement for torsion and for shear are determined separately and then added. However, when torsion and shear interact on a section the additional

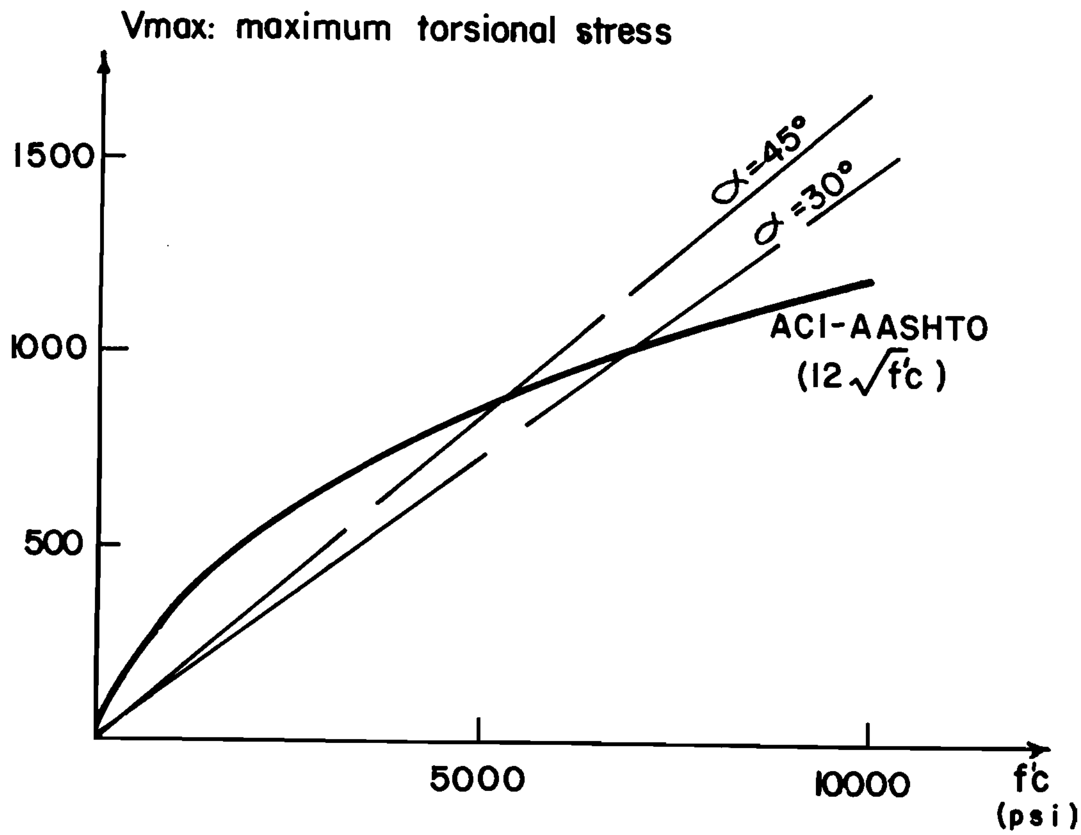


Fig. 2.5 Upper limits for the ultimate torsional stress acting on a cross section

resistance of the concrete V_c for shear and T_c for torsion, are considered equal to zero. The acting design torque and shear force T_u and V_u must meet the condition

$$[T_u/T_{max}] + [V_u/V_{max}] \leq 1.0 \tag{2.14}$$

where T_{max} and V_{max} are given by Eqs. 2.13 and 2.6, respectively.

Finally, the reinforcement must meet the following detailing requirements. The minimum percentage of web reinforcement must be equal to 0.0013 for web reinforcement made out of high strength steel or

0.0030 for mild steels in members where the characteristic compressive strength of the concrete (5% fractile) is between 5800 and 7250 psi. In members where the concrete has a characteristic compressive strength between 3600 and 5000 psi the minimum percentage of web reinforcement must be equal to 0.0011 for web reinforcement made out of high strength steel or 0.0024 for mild steels. The maximum stirrup spacing is 0.5*d if $V_u \leq (2/3)\ V_n$ or 0.3*d if $V_u \geq (2/3)\ V_n$. The transverse spacing of legs in each stirrup group under no circumstance should be greater than "d" or 800 mm (32 in.) whichever is the smaller.

In the case of members subjected to torsion, the minimum percentage of web reinforcement is the same as in the case of shear. The minimum area of longitudinal reinforcement must be 0.0015 $b_t d$ for high strength steels and 0.0025 $b_t d$ for mild steel where b_t is the width of the member in the tension zone. However, the total tension area should not exceed 0.04*A_g, where A_g is the cross-sectional area of the member. The stirrup spacing shall not exceed the value of u/8. The longitudinal bars can be uniformly distributed around the interior perimeter formed by the stirrups, but spacing shall not exceed 350 mm (14 in.), and at least one bar must be placed at each corner.

2.2.2 Swiss Code. The design procedure in the case of reinforced and prestressed concrete members in the Swiss Code, Structural Design Code SIA 162 (10), is based on the truss model with variable angle of inclination of the diagonal compression struts.

In the Swiss Code (10) the actions not considered directly in the truss model are introduced in a manner similar to the one followed in the CEB-Refined method:

a. In the geometry of the truss model: The angle of inclination of the diagonal strut is allowed to vary between the limits suggested in Eq. 2.1.

b. In an allowance for an additional diminishing concrete contribution to the shear and torsion carrying capacity of the member. The concrete contribution approaches zero as the nominal shear stress due to shear and/or torsion increases.

In the case of shear, the design procedure followed in the Swiss Code specifies that the shear force at calculated ultimate load minus the vertical component of the prestressing force under service load conditions when inclined tendons are utilized, must be equal or less that the sum of the nominal resistance V_s carried by the truss action, and the resistance V_c attributed to the concrete in the transition state. The shear carried by the truss is computed using Eq. 2.15.

$$V_s = [A_v f_y z \cot \alpha]/s \tag{2.15}$$

where f_y is the yield stress of the stirrup reinforcement (2% permanent strain), "z" is the distance between the centroids of the top and bottom longitudinal reinforcement enclosed by the stirrups and α is the angle of inclination of the compression diagonal. Under the Swiss Code $3/5 \leq \tan\alpha \leq 5/3$.

Due to the inclination of the concrete compression field in the truss model an area of longitudinal steel in addition to that required for flexure must be provided. The horizontal component of this diagonal compression field is assumed to act at the web center (z/2). If the resultant from the combined action of bending and shear is compressive in the chord of the truss model where the applied moment induces compression, then the following additional reinforcement should only be placed on the truss chord where the applied moment induces tension.

$$A_1(V) = [V_u \cot \alpha]/[2f_y] \qquad (2.16)$$

where f_y is the yield stress of the longitudinal reinforcement (2% permanent strain).

The concrete contribution V_c in the transition range between the uncracked shear resistance of the member and its shear resistance with the fully developed truss action is assumed to vary linearly as the nominal shear stress increases. The Swiss Code proposed values for this concrete contribution are shown in Fig. 2.6. The nominal shear stress "v_u" is taken to be

$$v_u = V_u/[b_w z] \qquad (2.17)$$

or

$$v_u = V_u/[b_w * 0.8 * H] \qquad (2.18)$$

where b_w is the minimum web thickness, z is the distance between centers of the top and bottom longitudinal reinforcement enclosed by the stirrup reinforcement, and H is the member depth.

The proposed values for the concrete contribution shown in Fig. 2.6 are based on the limits originally suggested by Thürlimann for the different stages in the behavior of a beam subjected to shear, i.e. 1/6 v_{max} for the uncracked range and 1/2 v_{max} for the limit between the transition and the full truss action stages. In determining the values

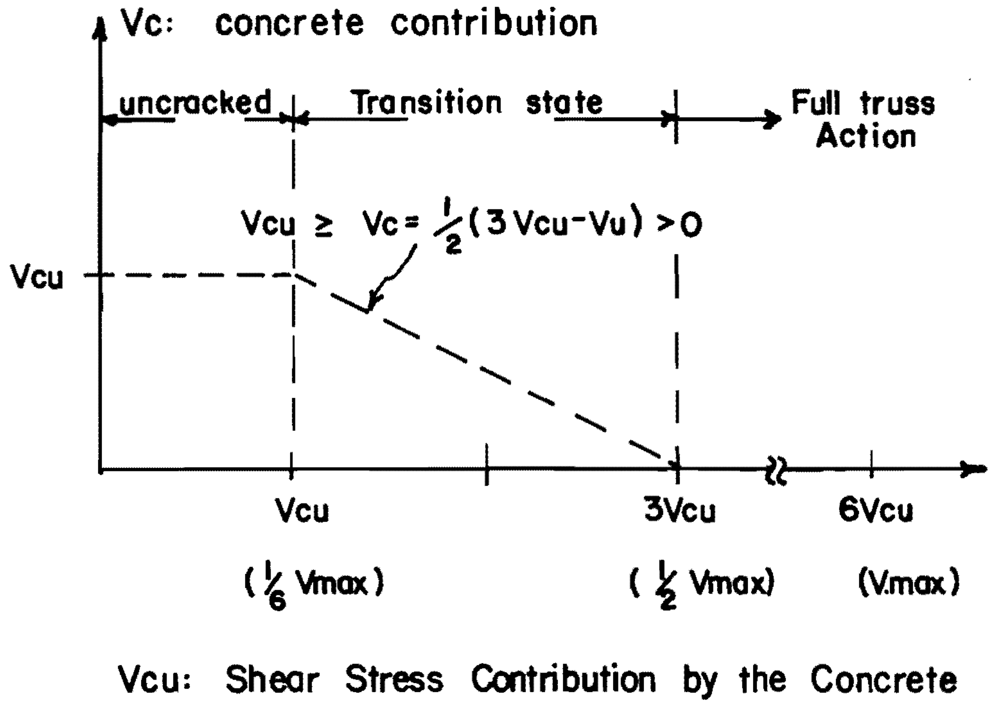


Fig. 2.6 Concrete contribution in the case of reinforced concrete members

of the shear stress that the concrete can carry in the transition state of the member, the following values of the uncracked shear stress carrying capacity of the section are recommended.

- If f'_c = 1400 psi then v_{cu} = 112 psi
- f'_c = 2100 psi then v_{cu} = 140 psi
- f'_c = 3000 psi then v_{cu} = 168 psi
- f'_c > 3500 psi then v_{cu} = 196 psi

If the preceding values for v_{cu} are put in a $k\sqrt{f'_c}$ form, k would be found to vary between 3.0 and 3.3, which is in the higher range of the values suggested in both the ACI Building Code (2) and AASHTO Standard Specifications (1) of $2\sqrt{f'_c}$ but no more than $3.5\sqrt{f'_c}$.

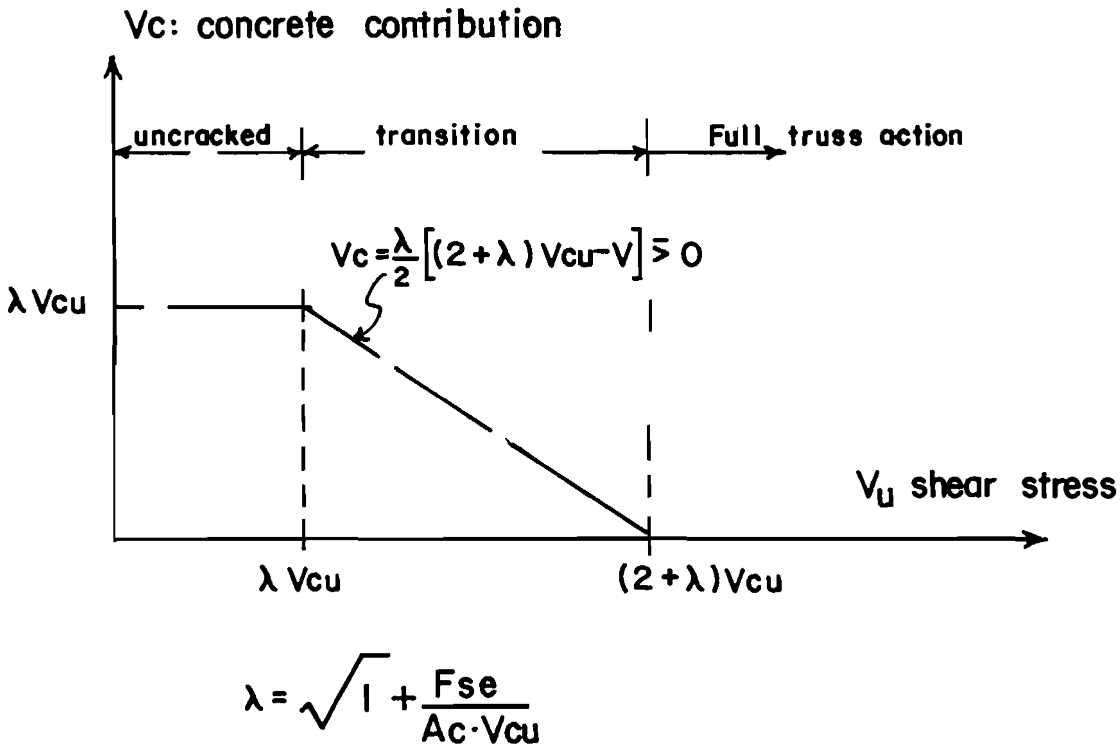
In the case of prestressed concrete members an increased value of the concrete contribution in the transition state is allowed, if for the calculated ultimate load and for a given applied prestressing force the resulting extreme fiber stress does not exceed the value of $2v_{cu}$ (e.g. at the support regions of a pretensioned beam). Note that this is similar to a tensile stress of 6 to $6.6\sqrt{f'_c}$. This limit in effect introduces a V_{ci} check into the Swiss procedure. The allowed concrete contribution in the case of prestressed concrete members is shown in Fig. 2.7.

In order to avoid failures due to crushing of the web, the nominal shear stress v_n evaluated using the nominal shear force $V_n = V_s + V_c$ must not exceed the values v_{max}, which are dependent upon the concrete strength and the maximum stirrup spacing.

- $v_{max} = 5v_{cu}$ for $s_{max} = z/2$ but $s \leq 12$ in.
- $v_{max} = 6v_{cu}$ for $s_{max} = z/3$ but $s \leq 8$ in.

A comparison between these two limits and the upper limit suggested in the ACI Code (24) and AASHTO Standard Specs. (1) of $10\sqrt{f'_c}$ is shown in Fig 2.8. The Swiss Code allows much higher shear stresses.

The design procedure for the case of torsion in the Swiss Code follows the same lines as the truss model. The Swiss Code design procedures are applicable to both reinforced and partially prestressed or fully prestressed concrete members, provided their warping resistance is neglected. As in the CEB-Refined method, torsional moments, as a



F_{se} : Prestressing force under service load condition

A_c : Cross-sectional area of the concrete

Fig. 2.7 Concrete contribution in the case of prestressed concrete members

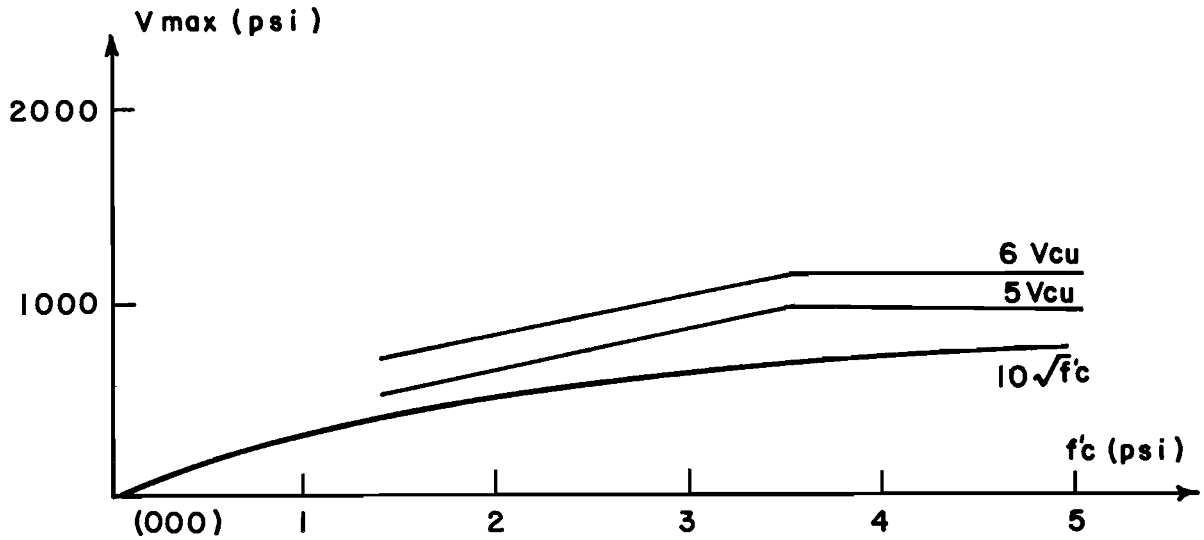


Fig. 2.8 Comparison between the upper limit for the shear stress in a section

rule, are only to be taken into account in the design if they are necessary for equilibrium. For compatibility torsion, the only requirement is that some reinforcement be placed to control crack development. No specific information is given as to how to evaluate this required amount of reinforcement.

The limits for the angle of inclination of the diagonal strut remain those presented in Eq. 2.1.

The torsional moment for the calculated ultimate load must be equal or less than the resistance value. The resistance value is made up of the resistance T_s carried by the truss, and the additional resistance of the concrete T_c in the transition range between the uncracked state and the full truss action.

The amount of torsion carried by the truss with vertical stirrups is given by Eq. 2.19.

$$T_s = [A_t f_{ys} 2A_o \cot\alpha]/s \tag{2.19}$$

where A_o is the area described by the perimeter enclosing the longitudinal reinforcement.

Due to the inclination of the compression field in the truss model an additional area of longitudinal reinforcement is required to resist the horizontal component of the inclined compression field which is assumed to be acting at the centroid of the perimeter u around the area A_o. The additional area is evaluated using Eq. 2.20.

$$A_l(T) = [T_u u \cot\alpha]/[2A_o f_{yl}] \tag{2.20}$$

where $A_l(T)$ is the total area of longitudinal steel required to resist the tension force produced by the torsional moment T_u. Eq. 2.20 follows directly from Eq. 3.30 derived in Sec. 3.4 of Report 248-2 from equilibrium considerations in the truss model.

The concrete contribution in the transition state is the same as the one assumed for the case of shear shown in Figs. 2.6 and 2.7 for reinforced and prestressed concrete respectively.

The shear stress due to torsion is evaluated using Eq. 2.21

$$v = T/[2A_o b_e] \tag{2.21}$$

which as in the CEB-Refined method, is derived from the theory of thin-walled cross sections. The value "b_e" represents the effective wall thickness of the assumed thin-walled cross section. This term has been previously discussed in Sec. 2.6.1 of Report 248-3. In the Swiss Code "b_e" is taken as $d_e/6$ for a solid cross section. For a hollow cross section $b_e = t$, where t represents the wall thickness of the cross section, but $b_e \leq d_e/8$. The term d_e is defined in Fig. 2.9.

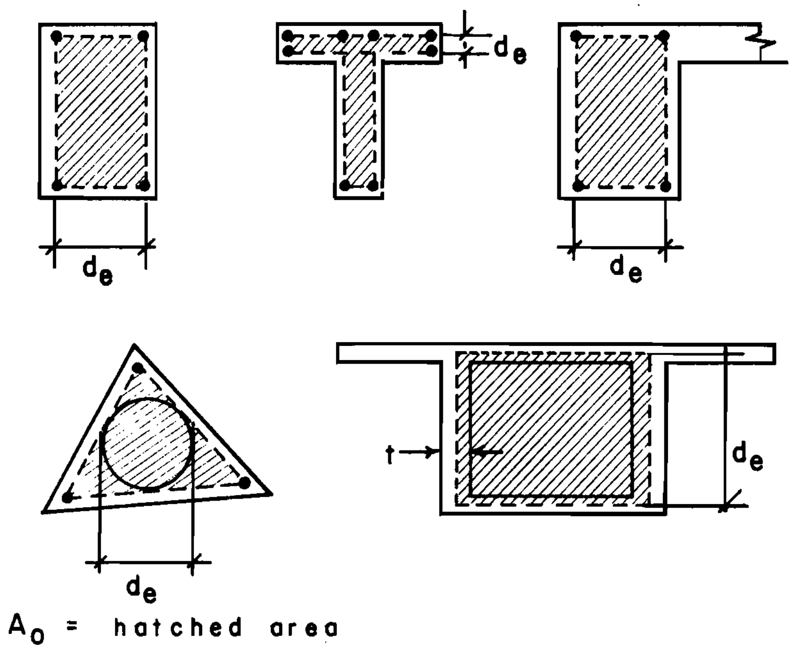


Fig. 2.9 Definition of the term d_e in the effective wall thickness b_e (from Ref. 10)

To avoid failures due to web crushing an upper limit for the nominal shear stress due to torsion v_n must not exceed the value of v_{max}, which is a function of the concrete strength and the maximum stirrup spacing s.

- $v_{max} = 5v_{cu}$ for $s_{max} = d_e/2$ but $s \leq 12$ in.
- $v_{max} = 6v_{cu}$ for $s_{max} = d_e/3$ but $s \leq 8$ in.

In the above expressions for s, for small solid cross sections (rectangle, T-section) with side ratios greater than 3:1 "d_e" can be replaced by $2*d_e$.

A comparison of these upper limits and the limit of $12\sqrt{f'_c}$ suggested in the ACI Code (2) and AASHTO Standard Specs. (1) for the case of pure torsion is shown in Fig. 2.10. Again, the Swiss limits allow higher torsional stresses.

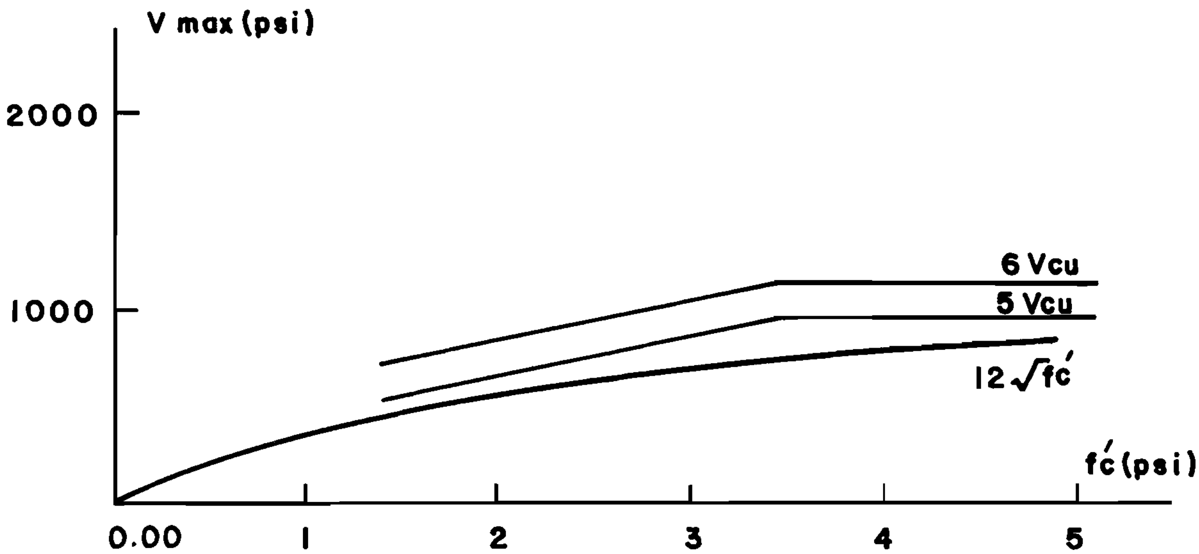


Fig. 2.10 Comparison between the upper limits of the shear stress in the case of pure torsion

For the case of combined actions the nominal shear stress due to shear and torsion must not exceed the prescribed values of v_{max}.

$$v(V + T) = v(V) + v(T) \leq v_{max} \tag{2.22}$$

The concrete contribution in the transition zone v_c is to be evaluated for the simultaneous action of shear and torsion and then is to be distributed in accordance with the respective levels of shear and torsion so that $v_c(V) = (v(V)/v(V + T))v_c$ and $v_c(T) = (v(T)/v(V + T))v_c$. The stirrup reinforcement required for shear and torsion are to be determined separately and then superimposed. The longitudinal reinforcement for shear and torsion must be determined separately and then added to the reinforcement for bending. If at a cross section the tensile force due to shear or torsion is counteracted by a compression force due to bending, the longitudinal reinforcement required will only be that required for the remaining tensile force.

The reinforcement for shear and torsion must meet the following requirements. The minimum area of shear reinforcement must be equal or greater to

$$A_{vmin} = [v_{cu}b_w s]/[2f_y] \tag{2.23}$$

for the case of shear, and

$$A_{vmin} = [v_{cu}b_e s]/[2f_y] \tag{2.24}$$

in the case of torsion. The stirrup reinforcement is to be continued on past the design region for at least the distance of the stirrup spacing. Stirrups must enclose the longitudinal reinforcement, and be properly anchored so that their required strength is effective over the depth z.

The additional longitudinal reinforcement required for shear and/or torsion is to be placed uniformly around the perimeter "u" formed by the stirrups. Furthermore, the longitudinal steel at the corners should be arranged so as to prevent pushing out of the concrete compression field. Proper detailing also calls for sufficient anchorage of the longitudinal reinforcement particularly at the support regions.

2.2.3 Proposed Canadian Code--General Method. The General Method design procedure proposed in the Canadian Code Draft of August 1982 (23) is based on the compression field theory developed by Collins and Mitchell (17) and uses equilibrium relations from the truss model. The General Method is applicable to both reinforced and prestressed concrete members subjected to shear and/or torsion. Collins and Mitchell further developed the truss model in the compression field theory by introducing a compatibility condition for the strains of the transverse and longitudinal steel members and the diagonal concrete compression strut. This condition was derived only for a constant strain profile over the section such as in the case of pure torsion, leading to the equation

$$\tan^2\alpha = [\epsilon_{ds} + \epsilon_l]/[\epsilon_{ds} + \epsilon_s] \qquad (2.25)$$

where α is the angle of inclination of the diagonal strut, ϵ_{ds} is the compressive strain in the diagonal strut, ϵ_l is the longitudinal tensile strain, and ϵ_s is the transverse tensile strain. Eq. 2.25 allows the evaluation of the inclination of the diagonal compression struts for a given state of strain in the shear field element. Using Eq. 2.25, the stress-strain relationships of the concrete and the steel, and the equilibrium equations of the truss model, the compression field theory attempts to predict the full behavioral response of reinforced and prestressed concrete members subjected to torsion or shear.

In the General Method in contrast with the CEB-Refined and Swiss Code approaches, the actions not considered in the truss model are introduced indirectly only in the geometry of the truss model (variable angle of inclination of the diagonal compression strut).

The compression field theory has not yet been extended to the design for combined shear and torsion. Thus, a somewhat alternative simplified approach is taken in the General Method proposed in the 1982 Draft of the Canadian Code. In the General Method some concepts of the diagonal compression field theory are mixed with the truss model principles.

For design purposes the use of an equation is suggested for the strut inclination which simplifies the different relations for shear and torsion. The design limits for the angle α(in degrees) of the diagonal compression strut are given in Eq. 2.26.

$$10 + 110K \leq \alpha \leq 80 - 110K \qquad (2.26)$$

K in the case of shear is given by Eq. 2.27

$$K = V_u/[\phi f'_c b_v d_v \qquad (2.27)$$

where b_v is the stirrup center to center dimension in the direction of the web resisting shear, but need not be less than 1/2 b_w. It must be noted that, the value of b_v, not the minimum web width b_w, is used to compute the level of shear stress acting on the member. This proposition seems more logical in the case of torsion where the high tension stresses induced in the outer shell of the member would induce the unrestrained cover to spall off. However, in the case of shear, even at high shear stresses, this assumption seems too conservative. It would unduly penalize thin web members not subject to torsion.

In determining the minimum effective web width b_v, the diameters of ungrouted ducts or one half the diameters of the grouted ducts, shall be subtracted from the web width at the level of these ducts. The term d_v represents the effective shear depth and can be taken as the flexural lever arm but need not be taken less than the vertical distance between centers of bars or prestressing tendons in the corners of the stirrups. The term is simply a capacity reduction factor. Suggested value would be 0.85 for both shear and torsion.

For the case in which torsion interacts with shear the term K in Eq. 2.26 is defined as

$$K = \left(\frac{V_u}{\phi f'_c\, b_v\, d_v} + \frac{T_u\, P_h}{\Phi f'_c\, A^2_{oh}}\right) \qquad (2.28)$$

where P_h is the outer perimeter of the centerline of the closed transverse torsion reinforcement, A_{oh} is the area enclosed by the centerline of the exterior closed torsion reinforcement, and ϕ is a capacity reduction factor. The cross-sectional dimensions are considered adequate to avoid crushing of the concrete in the web if it is possible to choose a value of K between the limits suggested in Eq. 2.26.

In the General Method the shear force V_u minus the vertical component of the effective prestressing force, in the case of prestressed concrete members with inclined tendons, must be equal to or less than the nominal shear resistance V_n of the section.

$$\phi V_n \geq V_u - V_p \tag{2.29}$$

where V_p is the vertical component of the effective prestressing force, and ϕ is a capacity reduction factor.

The nominal shear resistance V_n is entirely provided by the truss action (inclined concrete struts and steel reinforcement). The shear carried by the truss is given by

$$V_n = [A_v f_y d_v \cot \alpha]/s \tag{2.30}$$

Equation 2.30 follows directly from Eq. 3.63 of Report 248-2 and is derived from equilibrium conditions in the truss model, with the exception that d_v is the effective shear depth measured center-to-center of the horizontal legs of the stirrup reinforcement instead of being measured between the centroids of the longitudinal bars.

Due to the inclination of the diagonal compression field it is necessary to provide an additional area of longitudinal reinforcement to take care of the horizontal component N of the diagonal compression field. This horizontal component produces a longitudinal tension force which is assumed to be acting mid-depth of the truss model. If a top and bottom chord capable of resisting the applied tension force are provided the tension force per chord becomes

$$N = [V_u \operatorname{cotan} \alpha]/2 \tag{2.31}$$

Thus, the additional area of longitudinal steel required in the tension chord is

$$A_l(V) = V_u/[2f_{yl} \tan \alpha] \qquad (2.32)$$

which follows directly from Eq. 3.65 derived in Sec. 3.5.2 of Report 248-2 and is from the equilibrium equations of the truss model.

The design procedure for the case of torsion in the General Method considers the case of compatibility and equilibrium torsion. In the case of compatibility torsion, this is to say in the case of a statically indeterminate structure where reduction of torsional moment in a member can occur due to redistribution of internal forces, the design moment T_u need not be greater than 0.67 ϕT_{ocr}, where T_{ocr} represents the torsional strength of the uncracked cross section. It is suggested that T_{ocr} be taken as

$$T_{ocr} = [A_c^2\, 4\lambda \sqrt{f'_c}]/P_c \qquad (2.33)$$

for nonprestressed members, and

$$T_{ocr} = \left(\frac{A_c^2}{P_c}\right) 4\, \lambda\sqrt{f'_c}\, \left[1 + \frac{f_{pc}}{4\lambda\sqrt{f'_c}}\right]^{0.5} \qquad (2.34)$$

for prestressed members. A_c is the cross-sectional area of the member, p_c is the perimeter of the member, λ is a modification factor to account for different types of concrete (λ= 1 for normal density concrete), and f_{pc} is the compressive stress in the concrete (after allowance for all prestress losses) at centroid of the cross section resisting externally applied loads or at the junction of the web and the flange when the centroid lies within the flange.

For all other cases, i.e. equilibrium torsion, the section must be designed to resist the full applied torsional moment.

The limits for the angle of inclination of the diagonal compression strut remain the same limits proposed in Eq. 2.26.

The ultimate torsional moment T_u must be equal or less than the nominal torsional resistance T_n

$$T_u \leq \phi T_n \tag{2.35}$$

where ϕ is a capacity reduction factor. The resistance T_n is entirely provided by the truss action. The torsion carried by the truss action is evaluated by means of Eq. 2.36

$$T_n = [A_t f_{ys} 2A_q \cot\alpha]/s \tag{2.36}$$

Equation 2.36 follows directly from Eq. 3.31 derived in Sec. 3.4 of Report 248-2 from equilibrium considerations in the truss model. A_q is the area enclosed by the torsional flow and is evaluated as

$$A_q = A_{oh} - [a_o p_h]/2 \tag{2.37}$$

where a_o is the equivalent torsional depth of the compression block, derived from the compression field theory approach (12), and can be computed as

$$a_o = \frac{A_{oh}}{p_h} \left[1 - \left[1 - \frac{T_u\ p_h}{0.7\phi f'_c A^2{o_h}} (\tan\alpha + \frac{1}{\tan\alpha})\right]^{0.5}\right] \tag{2.38}$$

A_{oh} is the area enclosed by the centerline of the exterior closed transverse torsion reinforcement, and p_h is the perimeter formed by the centerline of the closed transverse torsion reinforcement.

Due to the inclination of the compression field in the truss model, an area of longitudinal steel due to torsion must be provided.

$$A_l(T) = [T_u p_q \cot\alpha]/[2A_q f_{yl}] \qquad (2.39)$$

Equation 2.39 follows directly from equilibrium consideration in the truss model. However, p_q is the perimeter enclosed by the shear flow path, and may be computed as

$$p_q = p_h - 4a_o \qquad (2.40)$$

In the case of combined torsion and shear the required amount of transverse reinforcement is assumed to be the sum of the amount required for shear and the amount required for torsion.

The amount of longitudinal steel required due to the presence of torsion and shear, is evaluated in an approximate form. It is suggested that a simple conservative procedure for determing the required tension area under combined loading is to take the square root of the sum of the squares of the individually calculated tensions. Thus, the equivalent total area of longitudinal steel due to shear and torsion can be computed as

$$A_l\ (V+T) = \frac{1}{f_{yl}}\ [[V_u^2 + (\frac{T_u\ p_q}{2\ A_q})^2]^{0.5}]\ \cot\alpha \qquad (2.41)$$

Since in the General Method the angle of inclination of the diagonal compression field, as computed from Eq. 2.26, is allowed to take very low values (much less than 31 degrees), it is then necessary to introduce service load checks to ensure adequate crack control at this limit state. The service load check in the General Method is carried out by means of an additional empirically found condition for the lower limit of the diagonal strut angle, which ensures that at service loads, the strains in the transverse reinforcement do not exceed the value of 0.001 for interior exposure, and 0.0008 for exterior exposure. In the General Method the strain in the transverse reinforcement at service load ε_{se} is evaluated in the following manner.

$$\varepsilon_{se} = \left[\frac{V_{se}\ s}{A_v\ E_s\ dv} + \frac{T_{se}\ s}{1.6 A_t E_s A_{oh}}\right] \left[\left(1 - \frac{f_y f_{pc}}{30\ f'_c}\right)\tan\alpha\right]^{0.5} \left[1 - \left(\frac{V_{cr}}{V_{se}}\right)^3\right] \quad (2.42)$$

where V_{se} is the service load shear force, V_{cr} is the shear force causing diagonal cracking (23), T_{se} is the torsional moment at service load conditions, A_v and A_t are the amount of transverse reinforcement provided for shear and torsion respectively, and E_s is the modulus of elasticity of the transverse reinforcement.

Finally, the detailing of the steel reinforcement has to satisfy the following requirements. The spacing of the transverse reinforcement placed perpendicular to the axis of the member shall not exceed, in the case of shear, the smaller of $d_v/(3 \tan \alpha)$or d_v. In the case of torsion, the spacing cannot exceed $P_h/(8 \tan \alpha)$. A minimum area of transverse reinforcement has to be provided in all regions of flexural members where the shear force exceeds $0.5\,\phi V_c$ or the torsion exceeds $0.25\ \phi T_{ocr}$.

The shear capacity of the uncracked concrete section V_c is taken as $2\lambda\sqrt{f_c'}\ b_w d$. The minimum area of transverse shear reinforcement is

$$A_v = 50\ b_w s/f_{ys} \tag{2.43}$$

For prestressed concrete members with an effective prestress force not less than 40% of the tensile strength of the flexural reinforcement, the minimum area of shear reinforcement can be computed by Eq. 2.43 or by Eq. 2.44.

$$A_{vmin} = \frac{A_{ps}}{80}\,\frac{f_{pu}}{f_{ys}}\,\frac{s}{d}\left[\frac{d}{b_w}\right]^{0.5} \tag{2.44}$$

where A_{ps} is the area of prestressed reinforcement in the tension zone, f_{pu} represents the specified tensile strength of the prestressing tendons, and "d" is the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement.

In calculating the term A_s in the Eqs. 2.43 and 2.44, the transverse reinforcement used to resist torsion may be included. The stirrup reinforcement provided for torsion has to be provided in the form of closed ties. Adequate anchorage of the transverse reinforcement is required. In the case of stirrups and other wires or bars used as shear reinforcement, they must be anchored at both ends to develop the design yield strength of the reinforcement. In the case of torsion the transverse reinforcement has to be anchored by means of 135 degrees hooks where the concrete surrounding the anchorage is unrestrained against spalling.

The yield strength used in the design calculations of the shear and torsion transverse reinforcement shall not exceed 60000 psi.

The longitudinal reinforcement has to be adequately anchored, and at least one longitudinal bar or prestressing tendon shall be placed in each corner of the closed transverse reinforcement required for torsion. The nominal diameter of the bar or tendon has to be larger than s/16, in order to prevent pushing out of the concrete compression diagonals.

The Canadian code draft seems too complex for general use. The truss model is obscured by the complex equations required for deformations and service load strain checks.

2.3 Concrete Contribution in the Transition State

After comparing a very wide range of test results with the predictions of the variable angle truss model as a failure model for reinforced and prestressed concrete members subjected to shear and torsion, it becomes clear that although the truss model conservatively represents the behavior of members subjected to shear and/or torsion, it is not a completely satisfactory failure model for design purposes. While it is safe and extremely useful for visualizing behavioral and detailing trends, the model is very conservative for members with low levels of shear and torsion. This results in higher requirements for web reinforcement than some current codes and imposes an economic penalty.

For the sake of simplicity in the design model, some of the actions that exist in the actual failure mechanism are not considered in the truss model. Components of the shear carrying mechanism of a reinforced concrete member such as the shear carrying capacity of the concrete compression zone, the dowel action of the longitudinal reinforcement, the aggregate interlock mechanisms, and the tensile strength of the concrete, are implicitly included for redistribution of forces at ultimate in the truss model with variable angle of inclination of the diagonals. These components are of increased significance at the lower levels of shear and torsion loading. Recognition of this contribution by introduction of the transition state should improve the economics of the procedure by removing unnecessary conservatism.

Since only flexurally underreinforced sections are encouraged under American design practices, yielding of the longitudinal steel in the tension chord should always occur at failure in the case of members subjected to bending and shear. Thus, the dowel action effect of that reinforcement is neglected in the truss theory. At shear or torsion failure the truss theory assumes that the shearing stresses on the section due to shear and torsion are of such magnitude that they would produce considerable diagonal cracking in the web of the member. Under these circumstances wide cracks in the web would prevent any further redistribution of forces due to aggregate interlock mechanisms.

Furthermore, at this level of shear stress, all the tensile capacity of the web concrete would be depleted.

In actual practice however, often because of the design procedures, loading conditions, clear span length, or even architectural constraints, flexure will control the design of a given member. In such case the shear stresses on the cross section defined as

$$v_u = V_u/[b_w z] \tag{2.45}$$

for shear, and

$$v_u = T_u/[2A_o b_e] \tag{2.46}$$

for torsion, might be of such low magnitude that the shear stresses in the member at failure would be in a transition state between the uncracked condition, and the behavioral state where the truss action would provide the entire resistance of the member. Moreover, the limits proposed

$$26^o < \alpha < 63^o \tag{2.47}$$

for the inclination of the diagonal strut, and in particular the lower limit of 26 degrees, which is established in order to prevent extensive web cracking under service load conditions, might sometimes force a member into this transition state.

For members in the transition state, components of the shear failure mechanism such as aggregate interlock and the concrete tensile strength, become of importance. The contribution of these mechanisms to the ultimate strength of the member can be reflected by an inclusion of an additional concrete contribution to the shear and/or torsional capacity in this transition state.

The review of other available design procedures conducted in Sec. 2.2 has shown the different ways in which this additional concrete contribution has been introduced in the overall design process.

In general, the shear capacity of a reinforced concrete member in its uncracked state is taken to be somewhere around 2 to 3 $\sqrt{f'_c}$.

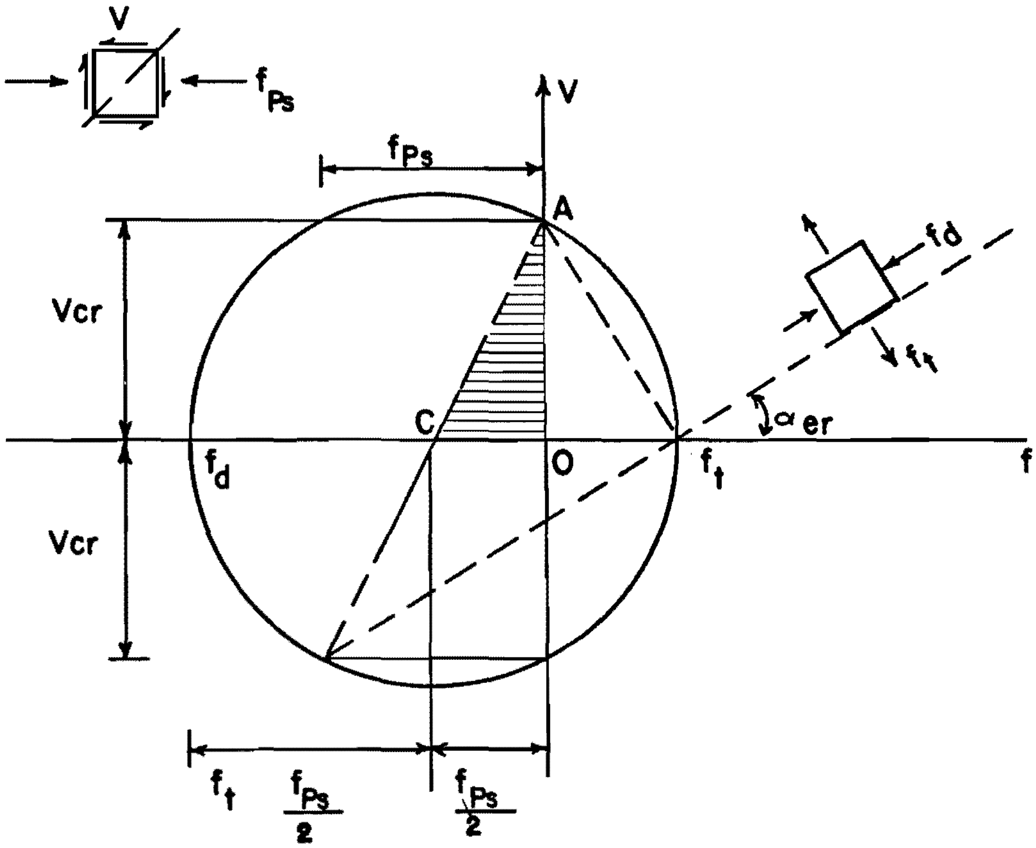
The beneficial effect of the presence of prestress on the shear strength of a concrete member in its uncracked state is introduced by increasing the uncracked strength of a reinforced concrete member. The shear capacity of a reinforced concrete member, before diagonal cracking occurs, is multiplied by a factor K, which is dependent upon the level of prestress force in the member. As was shown in Report 248-36, the presence of prestress in the elastic range has the effect of shifting the radius of the Mohr circle, causing a reduction in the principal diagonal tension stress.

This factor K can be derived from the Mohr circle representation of an element at the neutral axis of a prestressed concrete member, prior to initial diagonal cracking (see Fig. 2.11).

From Fig. 2.11 the factor K is found to be

$$K = [1 + (f_{ps}/f_t)]^{0.5} \qquad (2.48)$$

where f_{ps} is the compression stress at the neutral axis (i.e. the effective prestress force devided by the area of the cross section), and f_t is the principal diagonal tension stress. The value shown in Eq.



$$\overline{AO} = [\overline{cA}^2 - (\overline{c0})^2]^{0.5} = v_{cr}$$

$$v_{cr} = [(f_t + \frac{f_{ps}}{2})^2 - (\frac{f_{ps}}{2})^2]^{0.5}$$

$$= f_t\sqrt{1 + (f_{ps/ft})}$$

$$v_{cr} = kf_t$$

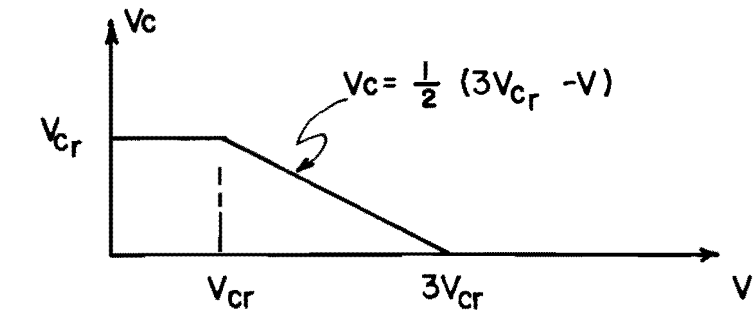
Fig. 2.11 Derivation of K-factor for prestressed concrete members

2.48 is the same as used in both the ACI Building Code 318-77 (2), and AASHTO Standard Specifications (1) as the basis for the web shear cracking criteria (V_{cw}). It is also used in the Swiss Code. In these codes, the value of f_t, is approximated by an expression for the diagonal tension cracking strength of the concrete. The ACI-AASHTO value is $4 \sqrt{f'_c}$.

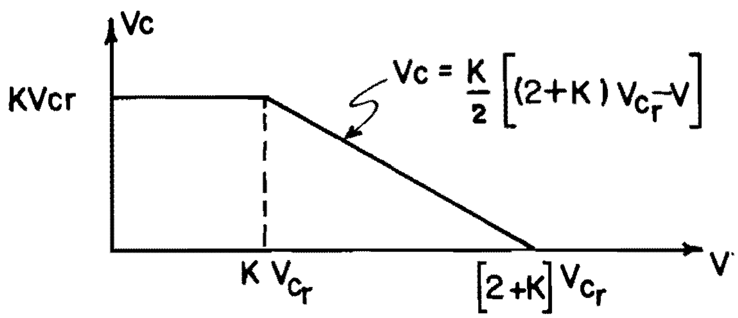
The CEB-Refined Method and Swiss Code suggested values for the additional concrete contribution v_c in the transition state are shown in Fig. 2.12 in terms of the shear strength of a reinforced concrete beam prior to diagonal cracking v_{cr} and the K term, for both reinforced and prestressed concrete members. The term β in the CEB-Refined method is based on the same principles used to derived the K term. Thürlimann (24) suggested a concrete contribution v_c in the uncracked and transition states as shown in Figs. 2.13a and 2.13b for reinforced and prestressed concrete members respectively.

The additional concrete contribution in the transition state has been discussed in Report 248-3. A complete evaluation of the concrete contribution in the uncracked and transition states has been conducted in Sec. 3.8 of Report 248-3 with test data from reinforced and prestressed concrete members with no or very small amounts of web reinforcement subjected to shear or torsion.

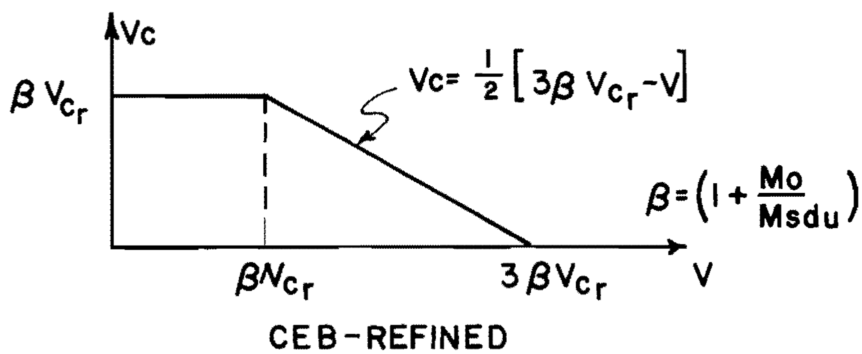
Based on this evaluation, a value of $2 \sqrt{f'_c}$ is suggested as an approximation of the shear strength of a reinforced concrete beam prior to diagonal cracking v_{cr}. It was shown in the evaluation of the Swiss (10) and CEB-Refined Method (22) proposed additional concrete



(a) Reinforced Concrete Swiss Code,
CEB Refined

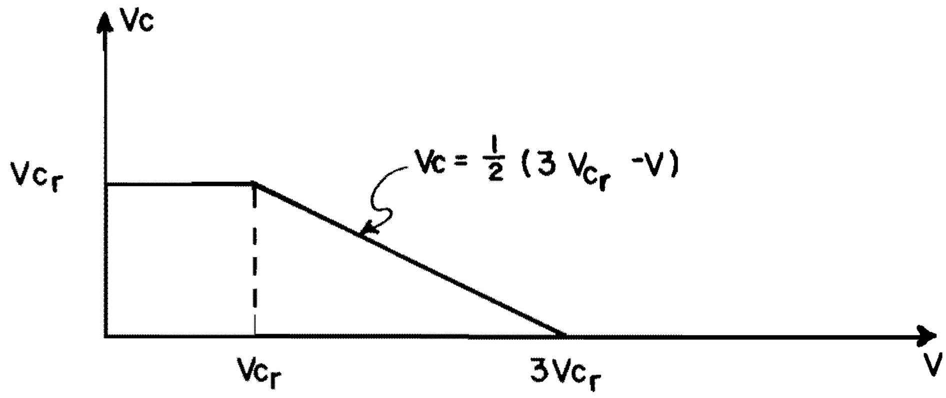


Swiss Code

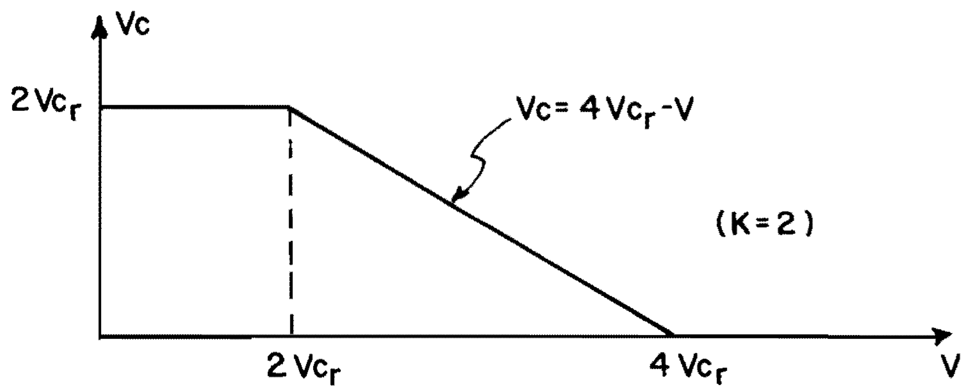


(b) Prestressed Concrete

Fig. 2.12 CEB-Refined and Swiss Code proposed concrete contribution in the case of prestressed concrete beams



(a) Reinforced concrete

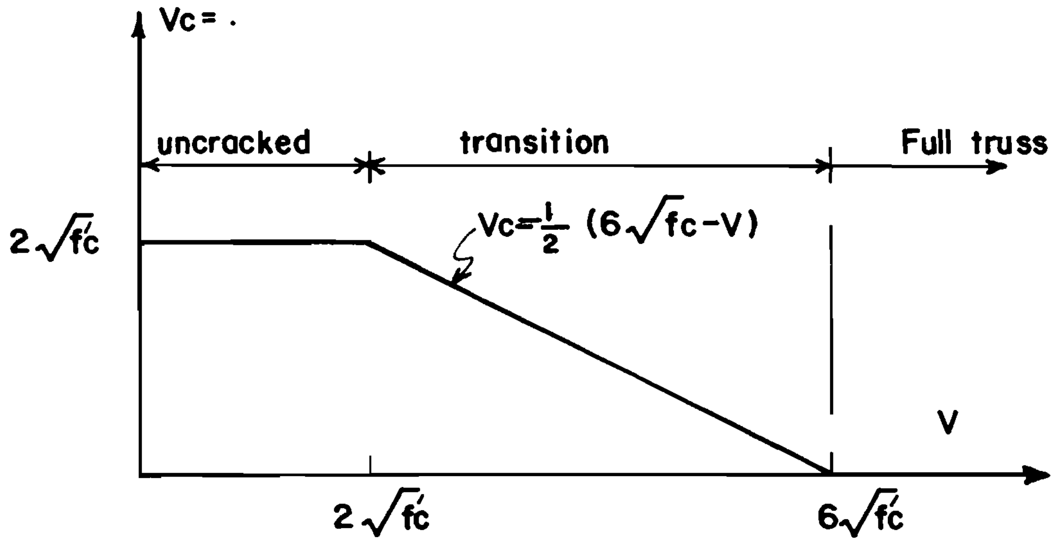


(b) Prestressed concrete

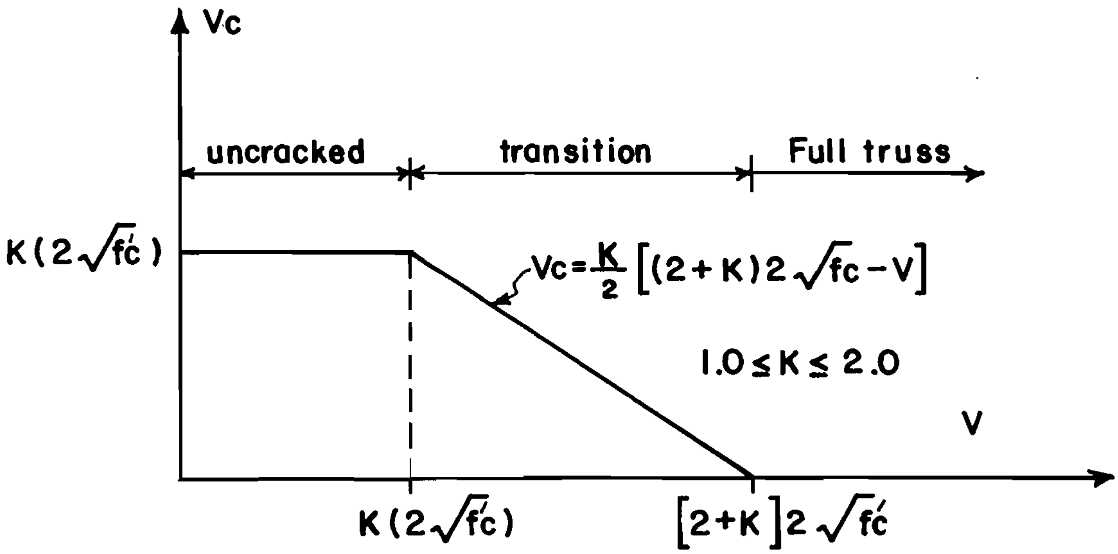
Fig. 2.13 Thürlimann's suggested concrete contribution in the transition state contribution in the transition state that the value of $2\sqrt{f'_c}$ may be used as an approximation of the shear strength of reinforced concrete beams without web reinforcement prior to diagonal cracking. Thus, this constitutes a safe lower bound approximation for the concrete contribution in the uncracked state.

In Fig. 2.13b, the concrete diagonal tensile strength v_{cr} is influenced by the factor K. Thürlimann (24) suggests a value of $(1/3)v_{max}$ as the limiting value between the uncracked and the transition state. This implies that a value of K equal to 2 should be used.

However, based on the evaluation of the concrete contribution in the uncracked and transition states conducted in Sec. 3.8 of Report 248-3 for the case of prestressed levels in various members, it seems more appropriate to maintain the level of K as a variable function of the prestress level in the cross section, such that K is then evaluated using Eq. 2.48 but should not be taken larger than 2.0. Shown in Figs. 2.14a and 2.14b are the proposed additional concrete contributions in the uncracked and transition states for reinforced and prestressed concrete members. These values are based on the evaluation of test results conducted in Sec. 3.8 of Report 248-3 and are slightly more conservative than the values proposed in the CEB Refined Method and the Swiss Code, but have the same general form. These values may be used for combined actions of shear and torsion but the contribution to each action must be prorated and the sum of these contributions must not exceed the additional concrete contribution.



(a) Reinforced concrete one-way members



(b) Prestressed concrete one-way members

Fig. 2.14 Proposed concrete contribution in the uncracked and transition states

As previously mentioned in Report 248-3, the introduction of other regulatory provisions such as requirement of a minimum amount of web reinforcement tend to obscure the actual additional concrete contribution to the shear strength of the member in the uncracked and transition states. However, this confusion can be avoided by recognizing that the minimum amount of web reinforcement requirement is introduced for a completely different purpose. Such reinforcement greatly increases ductility and provides toughness and warning. It serves as a backup to the concrete tensile contribution in lightly loaded members.

2.3.1 Reevaluation of the Truss Model Predictions with the Additional Proposed Concrete Contribution in the Transition State. The proposed concrete contribution in the uncracked state, thoroughly evaluated in Sec. 3.8 of Report 248-3, was shown to be an adequate and safe value for members with no web reinforcement.

Since the concrete contribution is set equal to zero for members in the full truss state, the evaluation of the accuracy of the truss model predicted ultimate strength has been already conducted in Report 248-3 for those members in the full truss state at failure.

The evaluation of members in the transition state is conducted in the following manner:

1. The shearing stresses due to shear and/or torsion are computed for each member using Eqs. 2.45 and 2.46 with the respective test values of the shear force and/or the torsional moment.

2. The computed value of the shearing stress at failure is then compared with the proposed concrete contribution shown in Fig. 2.14 and the additional concrete contribution to the shear strength of the member is computed. For the case of combined

actions, the shear stresses due to shear and torsion are added and the concrete contribution v_c, is evaluated. The concrete contribution for the case of combined shear and torsion is then prorated part to shear and part to torsion as a function of the relative shear and torsional stresses acting on the member.

3. The computed values of shear force and/or torsional moment resisted by the concrete, computed in step 2, are then subtracted from the test values of the shear force and/or torsional moment. With these reduced values of shear and/or torsion an evaluation procedure similar to the one used in Secs. 3.2, 3.3, 3.4 and 3.5 of Report 248-3 for the cases of torsion, torsion-bending, torsion-bending-shear, and bending-shear is then utilized so as to show that in fact by using the proposed values of the concrete contribution for reinforced and prestressed concrete members failing in the transition state the truss model design approach yields adequate safe results.

The analysis conducted in Chapter 3 of Report 248-3 on test data of 104 members subjected to pure torsion revealed that all of them were in the full truss state. Thus, the results presented in Sec. 3.2 and 3.7 of Report 248-3 remain the same.

In the case of combined torsion and bending, the analysis of the test data from 54 specimens shown in Secs. 3.3 and 3.7 of Report 248-3 revealed that 18 specimens were in the transition state. The results of the evaluation of the truss model including the concrete contribution in the transition state are shown in Tables 2.1 and 2.2.

As can be seen by the values of the mean and standard deviation of the dispersion index I, the truss model together with the proposed values of the concrete contribution in the transition state are in excellent agreement in the case of members subjected to torsion and bending failing in the transition state.

In the case of combined torsion-bending-shear, the test data of the 80 specimens analyzed in Secs. 3.4 and 3.7 of Report 248-3 was

Tests reported by Rangan and Hall (25) on prestressed concrete box beams								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Member ID	r	T_c (in-k)	$T_{TEST} - T_c / T_{uo}$	M_{TEST} / M_{uo}	K	K_{actual}	I	Level of prestress (σ/f'_c)
A1	0.33	144	0.64	1.16	1.55	1.55	1.27	0.03
A2	0.33	99	0.74	0.88	1.63	1.63	1.05	0.04
A3	0.33	39	1.01	0.82	1.63	1.63	1.12	0.04
A4	0.33	18	1.15	0.74	1.65	1.65	1.12	0.04
B1	0.33	146	0.61	1.14	1.66	1.66	1.24	0.04
B2	0.33	147	0.62	0.86	1.66	1.66	0.99	0.04
B3	0.33	120	0.75	0.74	1.67	1.67	0.95	0.04
B4	0.33	63	0.98	0.70	1.66	1.66	1.00	0.04
B5	0.33	15	1.15	0.55	1.75	1.75	0.99	0.04
C1	0.33	36	0.81	1.11	1.81	1.81	1.28	0.05
C2	0.33	42	0.81	0.84	1.82	1.82	1.04	0.05
						x	1.10	
						s	0.12	
Tests reported by Mitchell and Collins (17) on prestressed concrete box beams								
TB3	1.0	92	0.35	0.98	2.0	3.01	1.09	0.23
		Overall Table 2.1				x	1.10	N = 12
						s	0.11	

Table 2.1 Evaluation of beams subjected to torsion and bending failing in the transition state

Tests reported by Johnston and Zia (26) on prestressed concrete box beams								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Member ID	r	T_c (in-k)	$T_{TEST} - T_c / T_{uo}$	M_{TEST} / M_{uo}	K	K_{actual}	I	Level of prestress (σ/f'_c)
H-0-6-3	0.5	11	0.73	0.84	2.0	2.27	1.08	0.13
H-0-6-5	0.4	7	0.92	0.50	2.0	2.18	0.95	0.10
H-0-6-6	0.5	92	0.03	1.08	2.0	2.18	1.08	0.10
						x	1.04	
						s	0.08	
Tests reported by Warwaruk and Taylor (27) on prestressed concrete double celled beams								
R2	0.24	93	0.69	1.13	1.85	1.85	1.38	0.07
T1	0.36	4	0.85	0.68	2.0	2.19	0.95	0.12
T2	0.36	108	0.26	1.09	2.0	2.22	1.11	0.12
						x	1.15	
						s	0.22	
			Overall Table 2.2			x	1.09	N = 6
						s	0.16	

Table 2.2 Evaluation of the truss model procedure with test data of beams failing in the transition state subjected to combined torsion bending

reevaluated taking into account the concrete contribution in the transition state. The analysis of these specimens revealed that all of them were in the full truss state at failure and therefore the results presented in Secs. 3.4 and 3.7 remain unaltered.

In the case of members subjected to combined bending and shear the reevaluation of the 141 specimens with various amounts of web reinforcement analyzed in Secs. 3.5 and 3.8 of Report 248-3 showed that of all those specimens only 34 failed in the transition state. The data for these specimens are shown in Tables 2.3 and 2.4.

As can be seen from the value of the mean and the standard deviation from Table 2.3, the truss model approach with the addition of the concrete contribution to the shear strength of the member is in good agreement with test obtained values and yields conservative results in all cases. The failure of specimen C2A1 previously discussed in Sec. 3.5 of Report 248-3 was due to poor detailing of the longitudinal reinforcement which produced a premature failure and thus should not be considered in the overall evaluation.

Shown in Table 2.4 are data on beams with light amounts of web reinforcement ($\rho <$ 100 psi). These beams were previously studied in Sec. 3.8 of Report 248-3 to evaluate the proposed concrete contribution in members which failed right after first diagonal cracking, i.e., at the limit value between the uncracked and the transition state.

It might seem from the values of the dispersion index I shown in Table 2.4 that the proposed concrete contribution would be unsafe for members with very light amounts of web reinforcement failing in shear.

Tests reported by Hernandez (28) on prestressed concrete I-beams									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Member (ID)	V_c (kips)	$V_{TEST} - V_c / V_{uo}$	M_{TEST} / M_{uo}	$\rho_v f_y$ (psi)	$\tan\alpha$	I	K	K_{actual}	Level of prestress σ/f'_c
G28	0.16	0.71	0.99	120	0.24	1.36	2.0	2.31	0.14
Tests reported by Moayer, Regan (29) on prestressed concrete T-beams									
P4	3.66	0.49	1.21	105	0.52	1.38	1.91	1.91	0.07
P13	1.24	0.94	1.11	104	0.14	1.64	1.7	1.7	0.05
P18	0.16	0.97	1.0	104	0.13	1.59	2.0	2.49	0.13
P24	1.47	0.51	1.11	155	0.49	1.31	1.73	1.73	0.05
P25	10.13	0.39	1.19	104	0.21	1.31	1.73	1.73	0.05
P27	10.72	0.41	1.09	104	0.18	1.23	2.0	2.50	0.13
P29	7.00	0.47	1.06	104	0.23	1.24	2.0	2.52	0.13
Test reported by Rodriguez, Bianchini, Viest, Kesler (30) on two-span continuous reinforced concrete beams									
C2A1	0.65	0.45	0.58	190	0.58	0.83	1	1	0.0
Overall for Table 2.3					x	1.32		N = 9	
					s	0.23			
Overall for Table 2.3 without specimen C2A1					x	1.38		N = 8	
					s	0.15			

Table 2.3 Evaluation of beams under bending and shear failing in the transition state

Tests reported by Krefeld and Thurston (31) on reinforced concrete T-beams.									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Member ID	V_c (kips)	$\frac{V_{TEST}-V_c}{V_{uo}}$	$\frac{M_{TEST}}{M_{uo}}$	$\rho_v f_y$ (psi)	tan α	I	K	K_{actual}	Level of prestress (σ/f'_c)
26-1	12.5	0.44	0.91	79	0.27	1.09	1.0	1.0	0.0
29a-1	16.9	0.31	0.71	53	0.23	0.82			
29b-1	16.4	0.32	0.71	53	0.23	0.83			
213.5-1	18.4	0.29	0.65	35	0.17	0.76			
29a-2	9.6	0.58	0.97	62	0.20	1.24			
213.5a-2	16.0	0.36	0.71	42	0.18	0.86			
318-1	10.7	0.48	0.99	93	0.29	1.18			
321-1	16.4	0.27	0.73	79	0.33	0.82			
313.5-2	8.8	0.56	1.04	65	0.25	1.37			
318-2	14.9	0.37	0.79	64	0.25	0.93			
321-2	15.5	0.36	0.75	55	0.22	0.89			
218-2	15.8	0.44	0.73	31	0.13	0.94			
39-3	9.6	0.53	1.06	55	0.28	1.28			
313.5-2	12.6	0.52	0.95	65	0.21	1.17			
318-3	17.8	0.36	0.77	48	0.19	0.91			
321-3	21.2	0.19	0.63	42	0.20	0.69			
					x =	0.99	N = 16		
					s =	0.21			

Tests reported by Palaskas, Attiogbe and Darwin (32) on reinforced concrete T-beams.									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Member ID	V_c (kips)	$\frac{V_{TEST}-V_c}{V_{uo}}$	$\frac{M_{TEST}}{M_{uo}}$	$\rho_v f_y$ (psi)	tan α	I	K	K_{actual}	Level of prestress (σ/f'_c)
A25	11.4	0.22	0.46	32	0.17	0.58	1.0	1.0	0.0
A25a	10.6	0.29	0.50	32	0.15	0.67			
A50	6.0	0.39	0.62	74	0.29	0.84			
A50a	7.3	0.33	0.58	75	0.31	0.77			
A75	5.2	0.45	0.75	97	0.31	1.00			
B25	11.9	0.20	0.55	32	0.19	0.65			
B50	8.3	0.35	0.76	76	0.32	0.94			
C25	10.2	0.21	0.31	32	0.18	0.43			
C50	5.1	0.40	0.50	76	0.26	0.75			
					x =	0.74	N=9		
					s =	0.18			
			Overall for Table 2.4		x =	0.90	N=25		
					s =	0.23			

Table 2.4 Evaluation of reinforced concrete members with light amounts of web reinforcement under bending and shear failing in the transition state

However, on close examination of these specimens, it was found that poor detailing of the reinforcement was the cause for these premature failures.

In the case of the specimens from Ref. 31 all but 26-1 had stirrup spacings in the longitudinal direction in excess of d/2 and in some instances larger than d. As previously explained in Sec. 2.4.2 of Report 248-3, large stirrup spacings do not allow the formation of a uniform diagonal compression field. Instead, those large spacings cause the excessive concentration of diagonal compression forces in the joints of the truss formed by the longitudinal and transverse reinforcement which then produced premature failures by pushing out of the longitudinal corner bars. Furthermore, when the stirrup spacing is even larger than d, the first diagonal crack which opens at 45 degrees in reinforced concrete members will run untouched by a single stirrup producing a sudden failure of the member.

For those members from Ref. 32 the cause of failure was the inadequate detailing of the longitudinal reinforcement. The longitudinal reinforcement consisted of ASTM A416 Grade 270 seven-wire stress-relieved strand. The yield strength of this type of strand is usually defined as the value of stress corresponding to a strain of 0.01 and is usually about 240-250 ksi. The transverse reinforcement used in these specimens was made out of low carbon, smooth wires. These wires were annealed and the yield stress obtained was between 60 and 70 ksi. The longitudinal reinforcement was left unstressed, thus creating an enormous difference between the yield strengths of both reinforcements which then led to an excessive redistribution of forces causing very large strains in transverse reinforcement and in the diagonal compression strut leading to a premature failure.

This problem does not exist in prestressed concrete members because the initial tensioning of the strand eliminates the difference between the strain required to produce yield in the transverse reinforcement which is usually made out of deformed reinforcing bars (40-60 ksi) and that required to yield the longitudinal prestressed reinforcement (Grades 250-270).

The excessive redistribution of forces required in these members from Ref. 32 is illustrated by the very low values of the angle of inclination of the diagonal strut required at failure in those members. The values of $\tan\alpha$ for each member are shown in column (6) of Table 2.4. As can be seen they differ considerably from the $\tan\alpha = 1.0$ equivalent to the 45 degree angle corresponding to initial diagonal cracking of the concrete member. Of even more importance they fall well below the lower limit of $\tan\alpha > 0.5$ introduced into the design provisions. These specimens violate that limit severely.

Finally, it must be noted that for the case of prestressed concrete members subjected to bending and shear, the current AASHTO/ACI Specifications (1,2) require that the concrete contribution shall be given by the smaller of the two values v_{cw} and v_{ci} where v_{cw} represents the shear required to produce first inclined cracking in the web of the member, and v_{ci} is the shear stress required to produce first flexure cracking and then cause this flexural crack to become inclined. These two shear mechanisms have been previously explained in Report 248-2.

The web shear cracking mechanism, v_{cw}, is the shear stress in a nonflexurally cracked member at the time that diagonal cracking occurs in the web. The design for web shear cracking in prestressed concrete members is based on the computation of the principal diagonal tension stress in the web and the limitation of that stress to a certain specified value. The ACI/AASHTO Specifications indicate that a value $3.5\sqrt{f'_c}$ should be used as the limit value of this principal diagonal tension stress. As seen in Fig. 2.11 from a Mohr's circle it can be shown that the value of the shear stress at the centroid of the web of a prestressed concrete beam prior to cracking, v_{cr}, is given by

$$v_{cr} = f_t[1 + (f_{ps}/f_t)]^{0.5} \qquad (2.49)$$

where f_t is the principal diagonal tension stress and f_{ps} is the compressive stress due to prestress. In the current AASHTO/ACI recommendation, f_t is substituted by the limiting value $3.5\sqrt{f'_c}$ and for simplification the expression is reduced to the generally equivalent (see Fig. 2.10 of Report 248-2) straight line function

$$v_{cw} = v_{cr} = 3.5\sqrt{f'_c} + 0.3\ f_{ps} \qquad (2.50)$$

In the derivation of the proposed concrete contribution for prestressed concrete members the same approach was followed (see Fig. 2.11) to obtain the value of the shear stress required to produce initial diagonal cracking in the web of a member uncracked in flexure

$v_{cr} = K(2\sqrt{f'_c})$, where K should be between the limits $1.0 \leq K \leq 2.0$. For the case of fully prestressed concrete members, K is approximately equal to 2.0, thus v_{cr} becomes $4\sqrt{f'_c}$ which is essentially the same as the value of v_{cw} given in Eq. 2.50.

The other shear mechanism, v_{ci} (flexure shear cracking), is the shear necessary to cause a flexure crack at a distance d/2 from the section under consideration, plus an increment of shear assumed to develop it into an inclined crack.

The value of v_{ci} proposed in the ACI/AASHTO Specifications (2,1) was based on the results of a series of tests reported by Sozen, Zwoyer, and Siess (33) on prestressed concrete beams with no web reinforcement, and tests reported by MacGregor, Sozen, and Siess on prestressed concrete I beams (34,35).

The proposed concrete contribution in the uncracked and transition state was evaluated in Sec. 3.8 of Report 248-3 using the results from Ref. 33 and was shown to be an adequate value of the concrete contribution. Those specimens from Ref. 35 failing in shear were examined in Sec. 3.5 of Report 248-3 and the evaluation of those results in this section showed that with the proposed concrete contribution all of them were in the full truss state at failure. Hence, the ultimate strength of those members as evaluated in Sec. 3.5 remains unaltered.

However, all those specimens had the longitudinal prestressed reinforcement in the form of straight wires or strands.

MacGregor, Sozen and Siess (34), as a result of a study conducted on prestressed concrete beams with the longitudinal prestressed reinforcement in the form of draped wires, reported that in general draping of the longitudinal wires did not increase either the inclined cracking load or the ultimate shear strength of prestressed concrete which developed flexure-shear cracks. Instead, the trend of the test results indicated a reduction in both the inclined cracking load and the ultimate strength of those beams.

The test data from that study (34) on members failing in shear, is shown in Table 2.5. All the specimens failing in shear had no web reinforcement, and the longitudinal reinforcement consisted of straight and draped cold drawn, stress relieved high tensile strength single wires Grade 250. As can be seen from the values of the mean and standard deviation of the ratio v_u (Test)/[K($2\sqrt{f'_c}$)] shown in column (9), the proposed concrete contribution in general seems to be a safe lower bound value. However, the test results of specimens AD.14.37a, AD.14.37b, and BD.14.23 with draped wires and where failure was triggered by the flexure shear mechanism give very unconservative results.

The case of beam AD.14.37b is of special interest. In this member, subjected to two equal concentrated loads at 1/3 points, one shear span had draped wires and the other had straight wires. The shear span in which the wires were draped developed a flexural crack before the other span, as would be expected. This crack initially rose as high as the longitudinal steel. With further loading, the crack progressed

Tests reported by Macgregor, Sozen and Siess (34) on prestressed concrete I-beams.								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Member ID	f'_c (psi)	K_{actual}	K	$K (2\sqrt{f'_c})$ (ksi)	V_u Test (ksi)	$\rho_v f_y$ (psi)	a/d	V_u Test/ $K(2\sqrt{f'_c})$
AD.14.37a	3260	2.03	2.0	0.23	0.12	0.0	3.55	0.53
AD.14.367b	3260	2.03	2.0	0.23	0.16		3.55	0.68
B.14.34	2640	2.22	2.0	0.20	0.33		3.50	1.64
B.14.41	2890	2.41	2.0	0.21	0.36		3.60	1.73
BD.14.18	6280	2.11	2.0	0.32	0.48		3.56	1.49
BD.14.19	6780	2.05	2.0	0.32	0.43		3.53	1.34
BD.14.23	3873	1.92	1.92	0.24	0.21		3.56	0.88
BD.14.26	3460	2.08	2.0	0.24	0.24		3.56	1.00
BD.14.27	3400	2.05	2.0	0.24	0.36		3.66	1.50
BD.14.34	2700	2.14	2.0	0.21	0.33		3.52	1.55
BD.14.35	2610	2.14	2.0	0.20	0.25		3.52	1.24
BD.14.42	2870	2.35	2.0	0.21	0.37		3.56	1.76
BD.24.32	3800	1.99	1.99	0.25	0.34		3.56	1.76
C.13.23b	3730	2.17	2.00	0.24	0.60		2.60	2.49
CD.13.24b	3670	2.21	2.00	0.24	0.62		2.56	2.57
CD.13.25	3460	2.19	2.00	0.24	0.68		2.58	2.84
CD.13.34	2560	2.22	2.00	0.20	0.35		3.52	1.74
							x	1.55
							s	0.63

Table 2.5 Evaluation of the proposed concrete contribution with test data of prestressed concrete beams from Ref. 34

toward the load, splitting occurred, and a flexure-shear crack developed in the draped span. Failure occurred by crushing of the compression zone over this crack. At failure there were no inclined cracks in the shear span with straight wires, showing that the inclined cracking load was lower in the shear span with draped wires than it was in the shear span with straight wires.

This seems to indicate that the reduction in the flexural capacity of the member due to the draping of the longitudinal reinforcement, causing the appearance of flexural cracks in the shear span prior to inclined cracking of the web, tends to reduce the shear carrying capacity of the concrete in its uncracked state.

Based on these considerations, it is suggested that for the case of prestressed concrete members, the value of K in the proposed concrete contribution can be taken greater than 1.0 only in those regions of the member where flexural cracking does not occur prior to diagonal tension cracking. This is to say, for those regions of the member where the stress in the extreme tension fiber does not exceed the value of $6\sqrt{f'_c}$. This requirement is similar to the ones suggested in the Swiss Code where a value of $K > 1$ is only allowed in those regions where the extreme tensile stresses due to the calculated ultimate load and the applied prestressed force does not exceed the value of $2v_{cu}$, where v_{cu} varies between 3 and $3.3\sqrt{f'_c}$. Note that this is similar to a tensile stress of 6 to $6.6\sqrt{f'_c}$.

If this limit is then applied to members AD.14.37a, AD.14.37b, and BD.14.23, the ratio of v_u(Test)/[K2 $\sqrt{f'_c}$], with K = 1.0, becomes 1.05, 1.40, and 1.69, respectively.

In the subsequent sections the design recommendations based on the truss model are introduced. These recommendations are applicable to both reinforced and prestressed concrete members, subjected to shear and/or torsion in the transition state as well as in the full truss action state. However, in the uncracked and in the transition state the design shear force should be adjusted in accordance with proposed values (see Fig. 2.14) to recognize the concrete contribution. However, in the case of prestressed concrete members a value of K greater than 1.0 is only allowed in those sections of the member where the stress in the extreme tension fiber does not exceed $6\sqrt{f'_c}$.

2.4 General Assumptions and Design Procedures in the Truss Model Approach

The design approaches for the cases of bending-shear and torsion-bending-shear were treated separately in Report 248-2 in Secs. 3.6.1 and 3.6.2.

In this section, the variable angle truss model design approaches developed in Report 248-2 and the specific problems and limits in application, as well as the results from the evaluation of the truss model using a wide variety of published data in Report 248-3, are translated into detailed design recommendations. These design recommendations are applicable to either prestressed or normally reinforced concrete sections containing web reinforcement. They are suitable for the design of sections subjected to:

a. Shear and Bending

b. Shear and Torsion

c. Shear, Torsion and Bending

These provisions do not consider certain areas of shear such as two-way or punching shear and shear friction. Current provisions for such special cases would have to be added.

The general assumptions for the application of the truss model in the design procedure are:

1. Prior to failure, yielding of the longitudinal reinforcement is required. This limits consideration to underreinforced sections.

2. Diagonal crushing of the concrete does not occur prior to yielding of the transverse reinforcement. This requires an upper limit for the concrete stresses as well as limits on the angle of inclination of the diagonal compression struts.

3. Only uniaxial forces are present in the reinforcement (thus dowel action is neglected).

4. The steel reinforcement must be properly detailed so as to prevent premature local crushing and bond failures.

The general design procedure based on the truss model is easy to conceptualize and use. Basically the procedure consists of 6 steps:

1. Select an appropriate truss system for the load pattern and structural constraints.

2. Assume a compression diagonal inclination that is within the limits which are based on Sec. 3.3 of Report 248-2 ($25^\circ \leq \alpha \leq 65^\circ$).

3. Check the web concrete stress f_d in the diagonal compression elements of the truss to guard against web crushing.

4. Compute the area of transverse reinforcement required as truss tension verticals. Select spacing to satisfy both equilibrium and practical spacing limits. Check to see if the amount provided satifies the minimum web reinforcement requirement.

5. Determine the area of longitudinal reinforcement required for the combined actions. The additional longitudinal reinforcement required for shear and for torsion should be added to flexural requirements.

6. Provide adequate detailing of the steel reinforcement. Adequate detailing of the longitudinal and transverse reinforcement is of utmost importance in the Truss Model design approach since the reinforcement is required to develop its full yield strength prior to failure.

2.4.1 Selection of the Truss System. This step implies the selection of a truss model which is in equilibrium with the applied loads and structural constraints.

Examples of the truss model selection have been given in Report 248-3 for the case of deep beams and brackets, and in Report 248-2 for the case of members of constant depth cross section with rectangular, solid and hollow, L, T, and I shapes.

In this step of the design procedure lies the real advantage of the truss model approach. In the case of very complex situations, the truss model approach helps the designer to visualize internal structural patterns which can adequately carry the loads.

Once the designer has chosen a truss model which is suitable to carry the applied loads, he then can analyze the internal forces using the chosen truss model. He then can proceed to dimension the truss members so that those internal forces can be carried safely. If necessary, the initial truss model can be revised. Finally, using the chosen truss model, he can draw the necessary conclusions for the adequate detailing of the reinforcement.

Further examples on the selection of truss systems are given in Chapter 4.

2.4.2 Inclination of the Compression Diagonal Members of the Truss System. The Space Truss Model with variable angle of inclination of the compression diagonals departs from the traditional truss model with constant 45 degree angle diagonals proposed by Ritter (5) and generally adapted by Morsch (34) (who did recognize the variable angle of inclination). Hence, it is a more realistic truss model.

However, as explained in the earlier reports, limits on the angle of inclination of the diagonal concrete compression struts must be introduced. The proposed limits allow the angle of inclination to vary between 25 and 65 degrees. These lower and upper limits help to:

1. Provide adequate inclined crack width control at service load levels.

2. Maintain the compression diagonal stresses within prescribed limits helping to prevent diagonal crushing of the concrete prior to yielding of the transverse reinforcement.

3. Prevent excessive redistribution of forces. First inclined shear cracks in ordinary reinforced concrete members occur at about 45 degrees and the development of cracks at other angles requires the transmission of forces across the first cracks. Since the capacity for this transmission may be limited, excessive redistribution of internal forces caused by designing for angles which deviate too much from 45 degrees must be avoided.

4. Avoid excessive strains in the reinforcement and prevent extremely wide crack openings. As shown in Sec. 3.3 of Report 248-2, when the angle deviates too greatly from 45 degrees, in order for yield to be developed in both longitudinal and transverse reinforcement, very high strains are required in the

reinforcement which yields first in addition to large crack openings.

2.4.3 <u>Dimensioning of the Transverse Reinforcement</u>. In Secs. 3.6.1 and 3.6.2 of Report 248-2, the dimensioning of the transverse reinforcement for the cases of bending-shear and combined torsion-bending-shear was illustrated using the equilibrium conditions in the truss model ($\Sigma F_V = 0$).

However, dimensioning of the transverse reinforcement based entirely on the equilibrium conditions of the truss model may unduly penalize members subjected to low levels of shear stress.

As explained in Sec. 2.3, many times because of the design process followed, loading conditions, clear span length or even architectural constraints, flexure will control the design of a given member. In such case the shear stress on the cross section, defined as $v_u = V_u/b_w z$ for shear, and $v_u = T_u/2A_o b_e$ for torsion, might be of such low magnitude that as far as shear stresses are concerned the member at failure would be in an uncracked state or in a transition state between its uncracked condition and the behavioral state where the truss action would provide the entire resistance of the member. Moreover, sometimes the lower limit of 25 degrees on the inclination of the diagonal strut, which is established to prevent extensive web cracking under service load conditions, might force a member into this transition state.

For members in the uncracked and transition states, components of the shear failure mechanism such as aggregate interlock, and the concrete tensile strength become of importance. The contribution of these mechanisms to the ultimate strength of the member is reflected by recognition of an additional concrete contribution to the shear and/or torsional capacity of the member. For economy, such additional contribution by the concrete should be considered in the design process.

For members in the uncracked or transition state the design shear stress should be adjusted in accordance with the proposed values shown in Fig. 2.14a for reinforced concrete and 2.14b for prestressed concrete. There is an additional limitation that K can only be taken larger than 1.0 in those regions of the prestressed member where the stress in the extreme tension fiber due to the calculated ultimate load and applied prestressing does not exceed $6\sqrt{f'_c}$. Thus, the design shear stress used to compute the required amount of web reinforcement v_{TR} shall be taken as $[v_u/\phi - v_c]$, where ϕ is a capacity reduction factor, equal to 0.85, similar to the one required in the current ACI and AASHTO Specifications (2,1).

In the case of combined shear and torsion the computed concrete contribution must be distributed part to shear and part to torsion as a function of the relative shear and torsion acting on the member. This procedure is similar to the one suggested in the Swiss Recommendations (10).

Finally, recognizing the sudden nature of shear failure, it is suggested that a minimum amount of transverse reinforcement be provided for ductility whenever the value of the applied shearing stress exceeds 1/2 of the cracking shearing stress of the concrete section. This is in order to avoid sudden type failures, since in an unreinforced web the sudden formation of inclined cracking might lead directly to failure without warning. The minimum amount of web reinforcement then serves as a back up to the concrete contribution. Since the minimum amount is required as soon as the value of the shearing stress exceeds 1/2 of the cracking shearing stress (1.0 $\sqrt{f'_c}$), then it is reasonable to suggest that an amount equal 1.0 $\sqrt{f'_c}$, which would allow the member to at least reach its cracking shear stress (2 $\sqrt{f'_c}$), should be provided. In Fig. 2.15 the proposed value for the minimum amount 1.0 $\sqrt{f'_c}$ is compared with the ACI Code and AASHTO Specifications (2,1) recommended minimum of 50 psi.

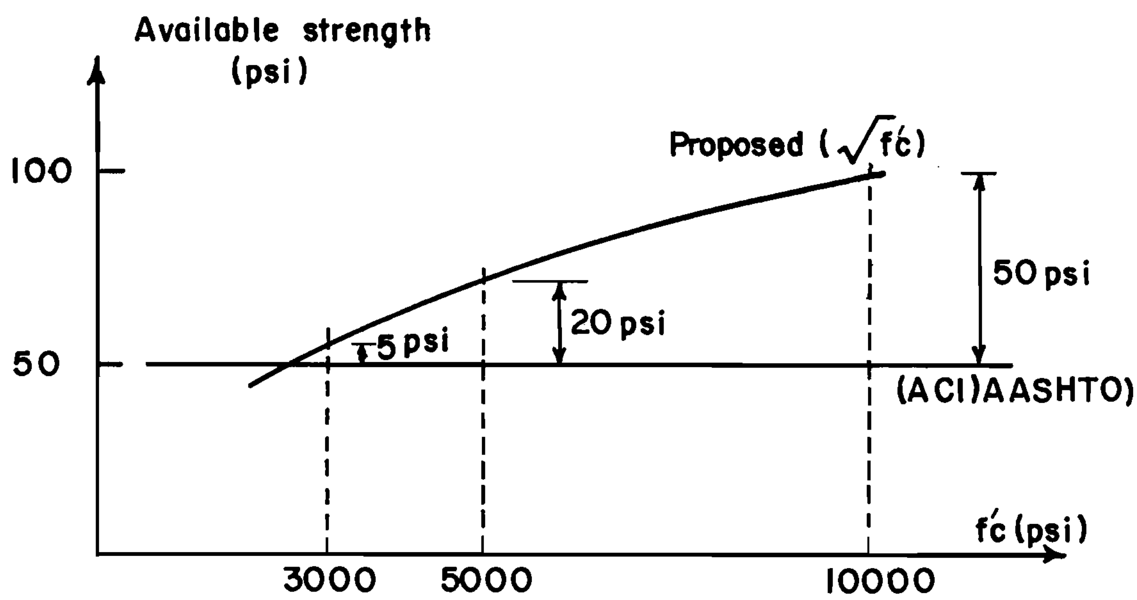


Fig. 2.15 Minimum amounts of web reinforcement

As can be seen from Fig. 2.15 both amounts are approximately the same plus or minus 20 psi in the 3000 to 5000 psi concrete compressive strength range. However, the suggested value of $1.0\sqrt{f'_c}$ reflects an increase of about 25 to 50 psi for the upper range of concrete compressive strength (f'_c greater than 5000 psi). This seems highly desirable to reflect the intent of the requirement for minimum web reinforcement. Although in high strength concretes the shear stress required to produce diagonal cracking increases, the mechanisms of aggregate interlock diminish. The crack surfaces become smoother, thus reducing the aggregate interlock which counts heavily on bearing between the jagged surfaces of the crack in order to transmit shear stresses between those cracks. Consequently, the concrete contribution does not increase directly with compressive strength. Since more shear is allowed to be carried by the concrete contribution in high strength concrete members, more minimum reinforcement should be provided.

2.4.4 Dimensioning of the Longitudinal Reinforcement. Due to the presence of an inclined compression field formed by the diagonal compression struts of the truss model, an area of longitudinal steel in addition to that required for flexure is necessary to resist the horizontal component of the diagonal compression struts.

The additional area of longitudinal reinforcement can be determined from the equilibrium conditions of the truss model ($\Sigma F_H = 0$). If a uniform compression field is assumed and the stirrup spacing is constant within the design zone equal to the horizontal projection of the inclined crack ($z^*\cot\alpha$), then the total horizontal component of the diagonal strut is located at the web center (z/2). Thus, it may be resisted by equal additional forces in the top and bottom chords of the chosen truss model.

2.4.5 Checking the Web Concrete Stresses. The use of the truss model with variable angle of inclination of the diagonal struts in the design of reinforced and prestressed concrete members requires that the steel reinforcement yield prior to failure of the concrete in compression. Concrete failure can be due to crushing of the bending compression zone or of the concrete diagonals.

As explained in Sec. 2.3 of Report 248-3, the restrictions on longitudinal reinforcement as a fraction of balanced reinforcement based on simultaneous yielding of the longitudinal steel and crushing of the concrete in the case of pure bending constitutes a safe lower bound for the case of combined torsion and bending.

The concrete compression diagonal struts carry the diagonal forces necessary for truss equilibrium. As shown in Sec. 2.3 of Report 248-3, the stress in the diagonal strut can be found from geometric considerations, and is given by the relation:

$$f_d = q/[b_w \sin\alpha \cos\alpha] \qquad (2.51)$$

where "q" is the shear flow due to shear or shear and torsion. The term q/b_w becomes the average shear stress "v".

In Sec. 2.3 of Report 248-3, it was demonstrated that the compression stress f_d in the diagonal strut does not vary significantly within the limits proposed for the inclination of the diagonal strut.

As a result, the average diagonal compression stress f_d could be controlled by limiting the nominal shear stress independently of the inclination of the compression diagonals.

In Chapter 3 of Report 248-3, a complete evaluation of the strength of the diagonal compression strut as a function of the maximum applied shear stress was conducted. The test data used in this evaluation belonged to reinforced and prestressed concrete members with web reinforcement subjected to shear and/or torsion failing in a web crushing mode. As a result of this evaluation it was suggested that failures due to crushing of the web concrete could be prevented by limiting the maximum nominal shear stress due to shear and/or torsion to a value equal or less than $15\sqrt{f'_c}$.

Therefore, in order to avoid premature failures due to web crushing, the stress f_d in the diagonal compression strut should always be kept equal to or less than $30\sqrt{f'_c}$.

2.4.6 Adequate Detailing of the Steel Reinforcement. The space truss model design approach is based on the assumption that all tensile forces have to be carried through yielding of the web and flexural tension reinforcement. Thus, reinforced and prestressed concrete members not only have to be designed as underreinforced sections, but in addition premature failures due to improper detailing of the reinforcement must be avoided.

In the design of reinforced and prestressed concrete members using the truss model it is clear to the designer that not only an adequate amount of reinforcement is necessary but its distribution is also of great significance.

2.4.6.1 Torsion. In Report 248-3, the differences in the design of members to resist torsional moments produced either by equilibrium or by compatibility torsion was established.

In the case of compatibility torsion the distribution of the reinforcement is more important than the amount. When designing members to resist compatibility torsion it is recommended that a minimum amount of reinforcement be provided for two reasons:

1. Minimum reinforcement (both transverse and longitudinal) helps at service load level to maintain adequate crack control.
2. Minimum amount of torsional reinforcement might raise the ultimate load of the entire structure since after the onset of yield in the flexural reinforcement of the adjacent members, further redistribution of forces can take place.

In the ACI Code and AASHTO Standard Specifications (2,1) in the case of reinforced concrete members subjected to compatibility torsion the nominal torsion shear stress need not exceed 1.67 times the torsion shear stress required to produce first diagonal cracking. The proposed Canadian General Method (23) specifies that in the case of compatibility torsion the maximum nominal shear stress produced by torsion may be reduced to 0.67 times the pure torsional cracking strength of the section, provided that the member and adjoining members are adequately detailed to account for the redistribution of forces after cracking. The Swiss Code (10) specifies that torsional moments produced by compatibility torsion can be neglected. In addition, as in the CEB-Refined Method, torsional moments as a rule are only to be taken into account in the design if they are necessary for equilibrium. However, in the case of compatibility torsion it is suggested that some reinforcement should be placed to control crack development. No level of nominal torsion shear stress is specified.

In the proposed design recommendations it is suggested that in the case of members subjected to compatibility torsion, the members should meet the minimum detailing requirements for transverse and longitudinal reinforcement as given for the case of equilibrium torsion. Such a member should:

a. Exhibit good service load behavior.

b. Have enough additional strength to allow further redistribution of forces after the onset of yield in the flexural reinforcement of adjacent members.

The case of equilibrium torsion is different. Here the amount of reinforcement becomes equally as important as its distribution. In designing a member subjected to torsion it is necessary to provide a uniform distribution of the longitudinal reinforcement around the perimeter of the cross section in order to provide adequate crack control. It is suggested that the longitudinal bars distributed around the perimeter should not be spaced farther apart than 8 in. center-to-center. At the same time, in order to satisfy ductility and strength requirements at ultimate, it is recommended that a considerable amount of the longitudinal reinforcement required for torsion be placed at the corners of the cross section and inside the closed stirrups. It is recommended based on studies by Collins and Mitchell (11) that under no circumstances should the corner bar diameter be less than either 1/16 of the stirrup spacing or that of a #3 bar.

Due to the overall lengthening effect in the member caused by the torsional moment, the longitudinal reinforcement for torsion acts as tension ties between the ends of the member. Therefore, it is necessary to provide it with adequate end anchorage and splices to allow it to develop its full yield strength everywhere along the length subjected to the torsional moment.

Since torsion produces cracking on all sides of the beam, the transverse reinforcement must be provided in the form of closed hoops. Because of the torsionally induced tensile stresses acting on the outer shell of the section, it is expected that at high torsional stresses the outer shell of concrete will spall off. Thus, in order for the stirrup to be properly detailed it is recommended that the free ends must be bent into the concrete contained within the stirrups with at least a 105 degree bend. (see Sec. 2.4.1 of Report 248-3). Furthermore, so that truss like behavior exists and to prevent the compression diagonals from breaking out between the stirrups, it is necessary to limit the maximum spacing of closed hoops "s" to a value $s_{max} \leq h_2/2$ but no more than 8 in., where h_2 is the shortest dimension of the cross section.

2.4.6.2 Shear. As explained in Report 248-3, detailing for shear strength also requires that both the longitudinal and the transverse reinforcement be properly anchored to allow the development of their full yield strength. Required anchorage can be provided by means of adequate straight embedment length, standard hooks or even mechanical anchorage.

In Sec. 2.4.2 of Report 248-3, it was shown that the longitudinal steel acts as a tension chord as required for flexure and at the same time balances the horizontal components of the diagonal compression struts. In addition, it must provide adequate end support for the stirrup reinforcement. In the truss model the longitudinal tension chords must tie the beam together along its longitudinal axis and be properly anchored at the ends.

In Sec. 2.4.2 of Report 248-3, the adequate anchorage of the longitudinal steel in the end region of simply supported beams where the reaction induces compression was examined. It was established that the tensionchord requires an anchorage length such that a force equal to $V^{*}\cot\alpha/2$ is adequately developed. The question of curtailment of the longitudinal tension reinforcement was also examined. As a result of this study, it is recommended that the longitudinal tension steel should be extended a distance l_s beyond the point at which it is no longer required for flexure. The distance l_s is given by

$$l_s = l_d - \frac{\Delta A_l f_{yl} z}{V} \tag{2.52}$$

for the case of concentrated loads "l_d" is the anchorage length required to develop yielding, of the bar, "V" is the shear force at the section, and ΔA_l is the area of longitudinal steel to be terminated. This equation is also applicable when detailing positive moment tension reinforcement at points of inflection and simple supports. For the case of distributed loading "l_s" is given as

$$l_s = l_d - \frac{\Delta A_l f_{yl}}{\frac{V}{z} + \frac{w}{2} \cot\alpha} \tag{2.53}$$

where "w" is the uniformly distributed load, "V" is the shear at the theoretical cut-off point, α is the chosen angle of inclination of the diagonal strut, and "l_s" represents the supplemental length required beyond the teoretical cut-off point.

As explained in Sec. 2.4.2 of Report 248-3 the transverse reinforcement provides the vertical tension ties to resist the vertical component of the diagonal compression struts. All stirrups must be properly anchored in the compression and tension zones of the member. The cracking of the concrete in the tension zone demands that the stirrup be continuous throughout this zone. No splicing of stirrups should be permitted.

The hooks of stirrups should be anchored around large longitudinal bars in order to distribute the concentrated force in the stirrups. A highly desirable recommended practice is to always bend stirrups around longitudinal bars, and terminate them only in the compression zone with always at least a 135 degree hook at the ends.

In the case of members having large web widths, and where more than two longitudinal bars are used to resist flexure it is recommended that multiple stirrup legs be used. In the case of members subjected to shear streses in excess of $6\sqrt{f'_c}$, it is suggested that the transverse spacing of stirrup legs should not exceed 7.5 inches. In the case of members with smaller nominal shear stresses it is suggested that the transverse spacing of stirrup legs can be as much as 18 inches but should not exceed the effective depth z of the truss model (see Sec. 2.4.2 of Report 248-3).

An upper limit on the maximum longitudinal stirrup spacing must be imposed to avoid the concentration of large compression forces at the joints between the stirrups and the longitudinal chords and to ensure that all the compression struts have effective reactions to bear against. The space truss model assumes a uniform distribution of the diagonal compression struts over the length of the beam. With overly large stirrup spacings these inclined struts react almost exclusively at the stirrup locations. These local concentrations may induce premature failures due to crushing of the diagonal strut or bulging out of the corner longitudinal bars. Furthermore, since in reinforced concrete members first diagonal cracking generally occurs at 45 degrees, there could be the possibility that if the member had been designed using the lower limit of 25 degrees, the initial diagonal crack would not be crossed by a single stirrup. Therefore, it is recommended that the maximum stirrup spacing be limited to a value of $s_{max} \leq z/2$ but no more than 12 inches, and for members with nominal shearing stresses in excess of $6\sqrt{f'_c}$ a value of $s_{max} \leq z/4$ but no more than 12 in. is suggested.

2.5 Summary

In Chapter 2, an overall review of some of the other current design procedures for reinforced and prestressed concrete one-way members was made. As a result it was shown that all of those procedures have the variable angle truss model as the fundamental design model. However, in the codification of these procedures, the simplicity and fundamental truss approach has been hidden. The Swiss Code and CEB Refined Recommendations are expressed in straightforward equations which are easy to use for familiar beam cases but give little guidance for more complex cases. The proposed Canadian recommendations contain very complex requirements for service load checks, are overly influenced by torsion considerations, and distort the limits on the angles of inclination to permit indirect inclusion of a V_c term. This approach does not seem suitable for codification if the goal is to make the designer more aware of the use of truss models so that he can apply the general truss concept in less familiar design situations. More emphasis should be given to the basic application of the truss model and to the proper detailing requirements for struts, ties and the nodes at which they join (41,42,43,44).

An examination of the truss model shows that for the sake of simplicity in the design approach, not all of the mechanisms that may transmit shear or torsion in a beam at failure are directly considered in the truss model. In this chapter, it was shown how these mechanisms which are not directly considered in the truss model may be indirectly introduced in the design approach either through limits on the geometry of the truss model (compression strut angle), or by allowing an additional concrete contribution (V_c-Term) with values which depend on the failure state of the member.

Finally, the general outline for shear and torsion design recommendations for reinforced and prestressed concrete one-way members was presented. These recommendations have the space truss model with variable angle of inclination of the diagonal compression struts as the fundamental design model. In the next chapter, a proposed text based on these design recommendations for revised AASHTO Design Specifications in this area is formulated. These design recommendations stress the general assumptions and limitations of the space truss model and present the basic model as the fundamental approach. A deliberate attempt is made to parallel the general approach for combined axial load and flexure, where the Code or Specification contains general principles and relegates specific application equations to commentaries, textbooks, or design aids. In the long run this should greatly simplify the design process, since designers will be able to readily envision how the different components of the members resist the applied shear force and/or torsional moment. Such a better understanding should lead to a simpler and more rational design process when the designer becomes familiar with the approach.

CHAPTER 3

PROPOSED DESIGN RECOMMENDATIONS

3.1 Recommended AASHTO Design Specifications for Shear and Torsion in Reinforced and Prestressed Concrete One-Way Members with Web Reinforcement

The design recommendations presented in this section are given in specification format and apply only to the ultimate strength design of reinforced and prestressed concrete one-way flexural members subjected to shear and/or torsion.

These proposed recommendations are to replace Secs. 1.5.10 (A),(B),(C), 1.5.13 (B)(3), 1.5.21 (B)(3), 1.5.21 (C) and (E), 1.5.35 (A),(B),(C), and Secs. 1.6.13 (A),(B),(C), in the current AASHTO Standard Specifications (12,13,14,15,16,17).

The sections in the current AASHTO Standard Specifications dealing with the shear-friction design as well as the design for two-way shear in slabs and footings would have to be added to these proposed recommendations (Secs. 1.5.35 (D),(E),(F)).

1.0 Notation

NOTATION

a = shear span, distance between concentrated load and face of support.

A_s = area of nonprestressed tension reinforcement, sq.in.

A_v = area of shear reinforcement within a distance s, sq.in.

A_o = area enclosed by the centroids of the longitudinal chords of the space truss model resisting the applied ultimate torsional moment and shear force, sq.in.

A_t = area of one leg of a closed stirrup resisting torsion within a distance s, sq.in.

A_l = total area of longitudinal reinforcement to be terminated at given section, sq.in.

b = width of compression face of the member, in.

b_e = effective web width of the member resisting the torsional shear stresses, in.

b_w = effective web width of the member resisting the applied shear force, in.

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

d_d = diameter of prestressing duct, in.

f'_c = specified compressive strength of concrete, psi.

$\sqrt{f'_c}$ = square root of specified compressive strength of concrete, psi.

f_y = specified yield strength of the nonprestressed reinforcement, psi.

f_{ps} = compression stress at the neutral axis of the section due to applied axial forces (including effective prestressing), or at junction of web and flange when the centroid lies within the flange, psi.

f_{py} = specified yield strength of prestressing tendons, psi.

K = $[1 + (f_{ps}/2\sqrt{f'_c})]^{0.5}$

l_d = anchorage length required to develop yielding of the bar, in.

l_s = additional embedment length beyond theoretical cut-off point, in.

q = shear flow due to shear and/or torsion, lb/in.

R = diameter of the largest inscribed circle in the cross section, in.

R_o = diameter of the largest inscribed circle in the area A_o, in.

T_c = nominal torsional strength provided by the concrete in the uncracked and transition states, in.-lb.

T_u = factored torsional moment, in.-lb.

T_{TR} = nominal torsional strength provided by the truss system, in.-lb.

V_u = factored shear force, lb.

V_c = nominal shear strength provided by the concrete in the uncracked and transition states, lb.

V_p = component in the direction of applied shear of effective prestress force at section, lb.

V_{TR} = nominal shear strength provided by the truss system, lb.

v_u = $v_u(V) + v_u(T)$

$v_u(V)$ = $V_u/[b_w*z]$, psi.

$v_u(T)$ = $T_u/[2A_o b_e]$, psi.

z = distance betwen the centroids of the longitudinal chords of the truss model, in.

α = angle of inclination of the diagonal compression members of the truss model at failure.

ρ = ratio of longitudinal flexural tension reinforcement, $A_s/[bd]$

ρ_b = reinforcement ratio producing balanced strain conditions.

ϕ = strength reduction factor, taken as 0.85. for shear and torsion.

1.1 Scope

These provisions shall apply for design of reinforced and prestressed concrete one-way members with web reinforcement subjected to shear, or torsion, or to combined shear and torsion. The design of slabs, footings and horizontal shear connectors is outside the scope of these provisions.

1.1.1 In a statically indeterminate structure where significant reduction of torsional moment in a member can occur due to redistribution of internal forces upon cracking, a design for torsional ultimate strength is not required. However, the detailing requirements for the transverse and the longitudinal reinforcement specified in Section 1.4 shall be met.

1.1.2 Torsion effects may be neglected in members where the factored torsional moment T_u is less than $0.5\phi T_c$.

1.1.3 Shear effects may be neglected in members where the factored shear force V_u is less than $0.5\phi V_c$.

1.1.4 For the case of combined shear and torsion, the design of reinforced and prestressed concrete members shall be conducted using the superposition of the shearing stresses due to shear and torsion.

1.2 Design Assumptions

1.2.1 The nominal strength of a member subjected to shear, torsion, or combined shear and torsion shall be determined from the analysis of the variable angle of inclination truss model based on the assumptions given in Sections 1.2.2 through 1.2.11. For members with relatively low levels of shear and torsion stresses, an additional concrete contribution to the nominal strength may be included as specified in Section 1.3.6.

1.2.1.1 Design of sections subjected to shear shall be based on

$$V_u < \phi V_n \quad (1\text{-}1)$$

where V_u is the factored shear force at the section, and V_n is the nominal shear strength computed by

$$V_n = V_c + V_{TR} + V_p \tag{1-2}$$

V_c is the nominal shear strength provided by the concrete in the uncracked and transition states, evaluated in accordance with Secs. 1.3.6(a), V_{TR} is the nominal shear strength provided by the truss model, and V_p is the component in the direction of the applied shear of the effective prestress force at section.

1.2.1.2 Design of sections subjected to torsion shall be based on

$$T_u < \phi T_n \tag{1-3}$$

where T_u is the factored torsional moment at the section, and T_n is the nominal torsional strength computed by

$$T_n = T_c + T_{TR} \tag{1-4}$$

T_c is the nominal torsional strength provided by the concrete in the uncracked and transition states, evaluated in accordance with Secs. 1.3.6 (c) and (d), and T_{TR} is the nominal torsion strength provided by the truss model.

1.2.1.3 Design of sections subjected to combined shear and torsion shall be based on the nominal strength indicated from the truss model considering any contribution of the concrete in the uncracked or transition state distributed as provided in Sec. 1.3.6(e).

1.2.2 The general design procedure shall be based on a truss model with variable angle of inclination of the compression struts. The basic components of the truss model consist of upper and lower longitudinal chords, stirrups or welded wire fabric perpendicular to axis as tension ties between chords, and a continuous compression field made up of the concrete compression diagonals inclined at an angle α.

1.2.3 Prior to failure, yielding of the longitudinal reinforcement is assumed. Chord capacities shall be based on underreinforced sections for flexure as specified in ACI 318 Sec. 10.3.3 or AASHTO Sec. 1.5.32 (A).

1.2.4 Crushing of the inclined compression struts shall be prevented prior to yielding of the transverse reinforcement.

1.2.5 The angle of inclination of the diagonals in the truss model shall be selected between the limits

$$25^\circ \leq \alpha \leq 65^\circ \qquad (1\text{-}5)$$

1.2.6 Tensile strength of the concrete shall be neglected in shear and torsion except as provided in Section 1.3.6.

1.2.7 Only uniaxial forces shall be considered in the reinforcement.

1.2.8 At ultimate load, uniaxial yielding of the steel reinforcement is assumed.

1.2.9 For strains in the reinforcement greater than that corresponding to the specified yield strength f_y, stress in the reinforcement shall be considered independent of strain and equal to f_y.

1.2.10 The model shall apply directly to both reinforced and prestressed concrete members. The area of prestressed reinforcement shall be transformed into an equivalent area of nonprestressed reinforcement based on computed yield force capacity.

1.2.11 Adequate detailing of the reinforcement shall be provided to prevent premature failures prior to yielding of this reinforcement.

1.3 General Principles and Requirements

1.3.1 Design of members subjected to shear, or torsion, or to combined shear and torsion, shall be based on a truss model with variable angle of inclination of the diagonals resulting from use of the assumptions in Section 1.2. For members with low levels of shear and torsion stresses an additional concrete contribution to the nominal strength may be included as specified in Sec.1.3.6.

1.3.2 The ratio of longitudinal reinforcement ρ provided shall not exceed 0.75 of the ratio ρ_b that would produce balanced strain conditions for the section under pure flexure without axial load.

1.3.3 The compression stress in the diagonal members of the truss model shall not exceed the value:

$$30\sqrt{f'_c} \qquad (1\text{-}6)$$

1.3.4 For members subjected to torsion the truss analogy shall be based on a space truss with variable angle of inclination of the diagonals. The torsional resistance of the space truss may be computed as the resistance of an equivalent thin walled tube. An applied torsional moment may be considered to produce a constant shear flow around the cross section.

$$q = T/2A_o \tag{1-7}$$

1.3.4 For the case of solid cross sections subjected to torsion an effective web thickness b_e shall be used. The effective web thickness b_e shall be taken as the smaller of the two values

$$b_e = R/6 \text{ or } b_e = R_o/5$$

1.3.5 In members with ducts in the webs having a diameter d_d greater than 1/10 of the web, the effective web width shall be taken as

$$b_w - \Sigma d_d \tag{1-9}$$

for ungrouted ducts, and

$$b_w - 0.67\ \Sigma d_d \tag{1-10}$$

for the case of grouted ducts. In determining Σd_d, only the ducts in a single critical plane should be considered.

1.3.6 An additional concrete contribution to the shear and torsional strength of the member may be recognized in the design of the transverse reinforcement as follows:

(a) For the case of shear in reinforced concrete members

$$V_c = (1/2)[6\sqrt{f'_c} - v_u]\ b_w z \tag{1-11}$$

but $0 \leq V_c \leq 2\sqrt{f'_c}\ b_w z$

(b) For the case of shear in prestressed concrete members

$$V_c = (K/2)[(4 + 2K)\sqrt{f'_c} - v_u] b_w z \tag{1-12}$$

but $0 \leq V_c \leq 2K\ \sqrt{f'_c}\ b_w z$.

(c) For the case of torsion in reinforced concrete members

$$T_c = (1/2)[6\sqrt{f'_c} - v_u]2A_o b_e \qquad (1\text{-}13)$$

but $0 \leq T_c \leq 2\sqrt{f'_c}\,[2A_o b_e]$.

(d) For the case of torsion in prestressed concrete members

$$T_c = (K/2)[(4 + 2K)\sqrt{f'_c} - v_u]2A_o b_e \qquad (1\text{-}14)$$

but $0 \leq T_c \leq 2K\sqrt{f'_c}\,[2A_o b_e]$

(e) K shall be computed as $[1 + f_{ps}/2\sqrt{f'_c}]^{0.5}$ but $1.0 \leq K \leq 2.0$. Furthermore, K shall be taken equal to 1.0 at all sections of the member where the stress in the extreme tension fiber due to the computed ultimate load and the applied effective prestress force exceeds the value of $6\sqrt{f'_c}$.

(f) For the case of combined shear and torsion, the concrete contribution shall be distributed in part to shear, and in part to torsion, as a function of the levels of shearing ($v_u(V)$) stress and torsional ($v_u(T)$) stress in accordance with the following:

The value of V_c given by Sec. 1.3.6(a) or (b) shall be multiplied by

$$v_u(V)/[v_u(V) + v_u(T)] \qquad (1\text{-}15)$$

in the presence of combined shear and torsion.

The value of T_c given by Sec. 1.3.6(c) or (d) shall be multiplied by

$$v_u(T)/[v_u(V) + v_u(T)] \qquad (1\text{-}16)$$

in the presence of combined shear and torsion.

1.3.7 In the design of the longitudinal steel required for shear and/or torsion, the concrete contribution shall be taken as zero when the factored shearing stress due to shear and/or torsion exceeds the values of

$$2\sqrt{f'_c} \tag{1-17}$$

in reinforced concrete, and

$$K(2\sqrt{f'_c}) \tag{1-18}$$

in prestressed concrete members. The value of K shall be limited as specified in Sec. 1.3.6(e).

1.4 Detailing of the Reinforcement

1.4.1 Torsion

1.4.1.1 Members in which the torsional shearing stress exceeds the value of $\phi 1.0\sqrt{f'_c}$ shall have a minimum amount of web reinforcement equal to

$$A_t = 1.0\sqrt{f'_c}[b_e s/f_y] \tag{1-19}$$

1.4.1.2 Where the factored torsional moment T_u exceeds the torsional moment strength ϕT_c, torsional reinforcement shall be provided to satisfy Eqs. (1-3) and (1-4).

1.4.1.3 Longitudinal reinforcement required for torsion shall be distributed around the perimeter formed by the closed stirrups. At least one longitudinal bar shall be placed in each corner of the stirrups. The minimum diameter of the corner bar shall be taken as 1/16 of the stirrup spacing but no less than that of a #3 bar.

1.4.1.4 Longitudinal reinforcement required for torsion shall be adequately anchored to develop its full yield strength everywhere within the section subjected to the torsional moment.

1.4.1.5 The transverse reinforcement required for torsion shall be provided as closed hoops formed by closed stirrups, closed ties or spirals.

1.4.1.6 The closed stirrup or tie shall be made out of a single piece. The free ends must be bent into the concrete contained within the stirrups with at least a 105 degree bend.

1.4.1.7 Spacing of closed hoops shall not exceed one half of the shortest dimension of the cross section, nor 8 in.

1.4.1.8 Design yield strength of the transverse reinforcement shall not exceed 60,000 psi.

1.4.1.9 The transverse reinforcement shall be continued on past the section where it is no longer required for torsion for at least a distance equal to the stirrup spacing.

1.4.2 Shear

1.4.2.1 Members in which the shear stress $v_u(V)$ exceeds the value of $\phi 1.0\sqrt{f'_c}$ shall have a minimum amount of web reinforcement equal to

$$A_v = 1.0\sqrt{f'_c}[b_w s/f_y] \qquad (1\text{-}20)$$

1.4.2.2 Where the factored shear force V_u exceeds the shear strength ϕV_c shear reinforcement shall be provided to satisfy Eqs. (1.1) and (1.2).

1.4.2.3 The longitudinal tension reinforcement at the end support regions of simply supported members where the reaction induces compression shall be provided with an anchorage length such that a force equal to V_u*cotα/2 is adequately developed.

1.4.2.4 The longitudinal tension reinforcement no longer required for flexure shall be continued a distance l_s beyond the theoretical cut-off point. The supplemental distance l_s shall satisfy the following

for members subjected to concentrated loads:

$$l_s = l_d - \frac{A_v f_y z}{V_u} \qquad (1\text{-}21)$$

for members subjected to a uniformly distributed load w:

$$l_s = l_d - \frac{A_v f_y}{\frac{V_u}{z} + \frac{w}{2}\cot\alpha} \qquad (1\text{-}22)$$

1.4.2.5 Any transverse reinforcement stirrup or hoop shall be formed as a single continuous piece of reinforcement. Transverse reinforcement shall be provided in the form of deformed bars, or welded wire fabric, or deformed wire perpendicular to the axis of the member. The transverse reinforcement shall be terminated only in the compression zone with a 135 degree hook at the ends. Hooks of the transverse reinforcement shall be anchored around longitudinal reinforcement. Transverse reinforcement can also be provided in the form a continuous spiral.

1.4.2.6 The maximum longitudinal spacing of stirrups shall not exceed the smaller of z/2 or 12 in. for members subjected to a factored shear stress less than $\phi 6\sqrt{f'_c}$. For members where the factored shear stress exceeds $\phi 6\sqrt{f'_c}$, the maximum longitudinal spacing of stirrups shall not exceed the smaller of z/4 or 12 in.

1.4.2.7 In members subjected to shear stress in excess of $6\sqrt{f'_c}$, the transverse spacing of stirrup legs shall not exceed 7.5 in.. In members subjected to smaller shear stresses, the transverse spacing of stirrup legs shall not exceed 15 in. nor the effective depth z.

1.4.2.8 The transverse reinforcement shall be continued past the section where it is no longer required for shear for at least a distance equal to the stirrup spacing.

1.4.2.9 In the case of members subjected to bending and shear where the support reaction induces compression, no longitudinal tensile reinforcement within a distance $[z\cot\alpha]/2$ from the centerline of the support is required in the top compression face of the member due to effects of shear.

1.4.2.10 The design yield strength of shear reinforcement shall not exceed 60,000 psi.

1.4.2.11 The longitudinal reinforcement required to resist shear and torsional actions are to be added to the reinforcement required to resist bending or bending with axial forces.

1.4.2.12 The most restrictive requirements for detailing of the reinforcement in regard to spacing, placement, yield strength, and minimum amount shall be met in the case of combined shear and torsion.

3.2 Summary

The design recommendations presented in this chapter have the space truss model as the fundamental structural design model. This should simplify the design process. Once designers are familiar with the overall design procedure, they will be able to readily envision how the different components of the members resist the applied shear force and/or torsional moment. Such a better understanding should lead to a simpler and more rational design process.

It must be pointed out that a commentary for the proposed design recommendations would be of tremendous help in clarifying certain aspects of the overall design process. In particular, the adequate selection of the truss model and the subsequent solution of the equilibrium conditions to evaluate the internal design forces should be shown initially in such a commentary. Such commentary material could be drawn from this report series.

In the initial years after adoption of this approach to code or specification language, the commentary type document could include example equations for simple cases to speed design. However, it was felt that such a powerful design approach as the one based on the truss model should not be translated to a series of design equations in the code or specification itself. Such an approach would hide the truss model. It is precisely the truss model and its applicability to several different design situations which are the real strength of the proposed design procedure. Presentation in the form of empirical equations would obscure this powerful model.

In Chapter 4, the proposed design recommendations are illustrated through a series of design examples. Parallel designs using current ACI and AASHTO recommendations are carried out, and the resulting designs are compared.

CHAPTER 4

APPLICATIONS OF THE PROPOSED DESIGN RECOMMENDATIONS AND COMPARISON WITH CURRENT AASHTO PROCEDURES

4.1 Introduction

A design procedure and proposed AASHTO design recommendations were presented in Chapter 3. The proposed design recommendations have the truss model with variable angle of inclination of the diagonals as the fundamental design model.

In this chapter the first step of the design procedure suggested in Sec. 2.4 (the selection of an appropriate truss system) is illustrated through several different examples. The strength of the truss model design procedure lies in its versatility. The truss model approach allows the designer to handle unusual design situations without great difficulty. This versatility of the truss model is illustrated in Sec. 4.2.

The design recommendations proposed in Sec. 3.1 are applied in a series of design examples.

a. A reinforced concrete rectangular box beam under combined torsion, bending and shear.

b. A prestressed concrete I-girder under bending and shear.

Finally, the amounts of reinforcement obtained using the proposed design recommendations are compared with those obtained using the current AASHTO procedures (2).

4.2 Selection of an Appropriate Truss System

As suggested in Sec. 2.4, the first step in the design procedure is a critical one. It calls for the selection of a truss system which is in equilibrium with the applied loads and the structural constraints. Further information on such truss models has been given by Marti (42,44) and Schlaich (43).

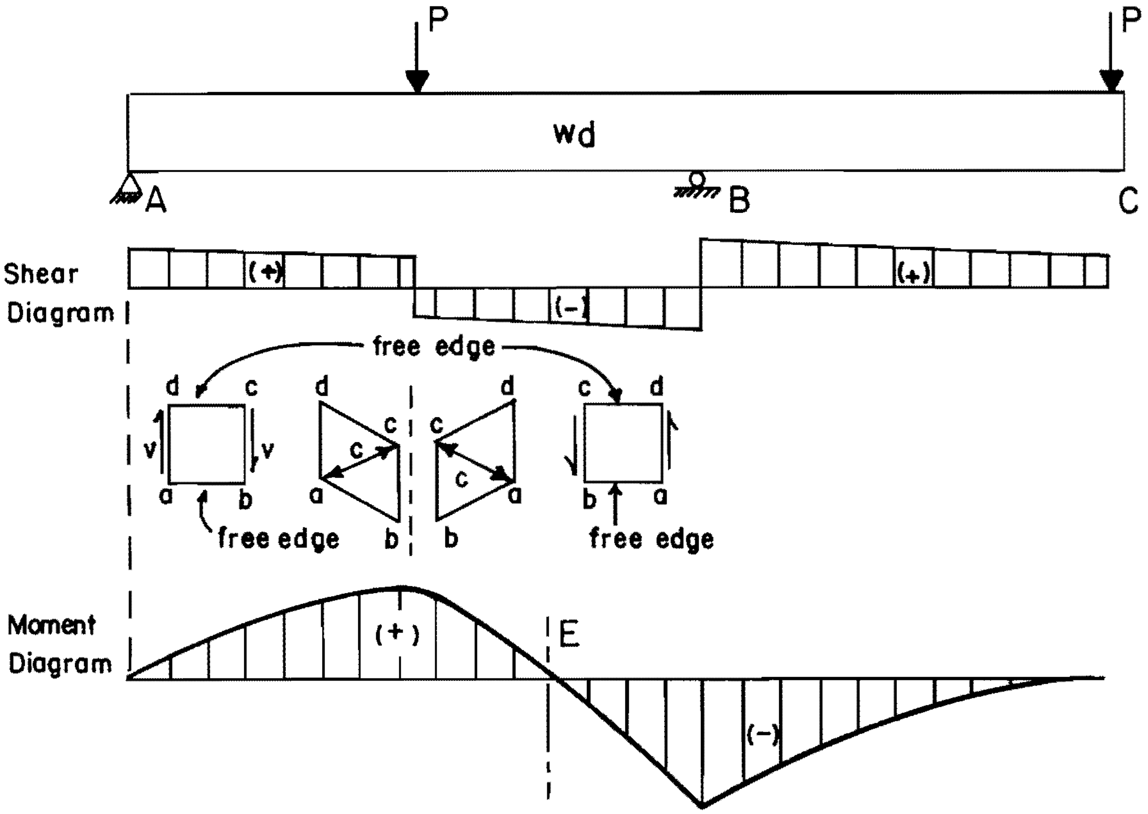
The selection of adequate truss systems is illustrated in the following examples.

4.2.1 Truss Model for a Semicontinuous Beam. Shown in Fig. 4.1b is the truss system selected to represent the semicontinuous beam shown in Fig. 4.1a.

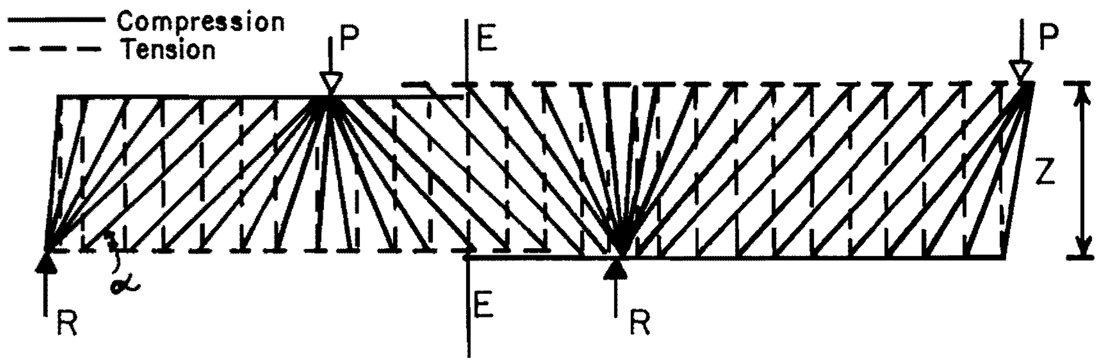
In the selection of an appropriate truss model several basic rules have to be followed. First, the selected truss system has to be in equilibrium and has to be compatible with the applied loads and support conditions.

It is also of importance to correctly determine the direction that the compression diagonals of the truss must follow. This direction can be found from the shear diagram due to the applied loads. The elements directly below the shear diagram in Fig. 4.1a show how the direction of the shear force and the resulting deformation of the elements determines the general direction of the inclination of the compression diagonals in the truss model.

Another important condition is set by the limits imposed on α, the inclination of the compression diagonals in the truss model. The



(a) Shear and moment diagrams for semicontinuous beam



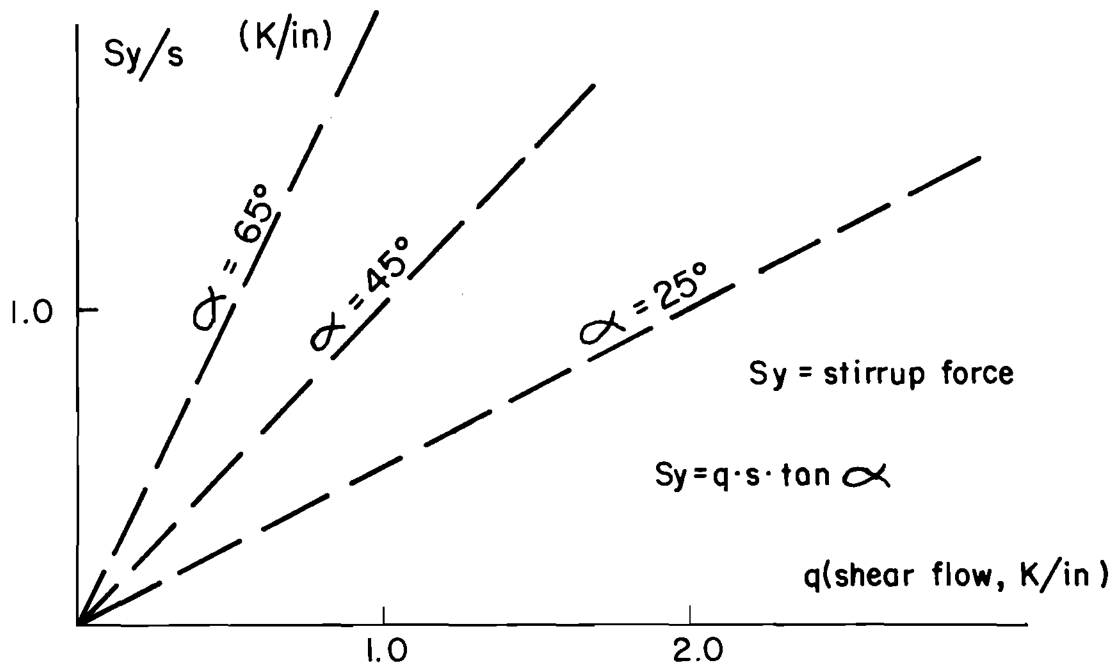
(b) Truss model for semicontinuous beam

Fig. 4.1 Selection of an adequate truss model in the case of a semicontinuous member inclination of these diagonal members must be within the limits $25^\circ \leq \alpha \leq 65^\circ$.

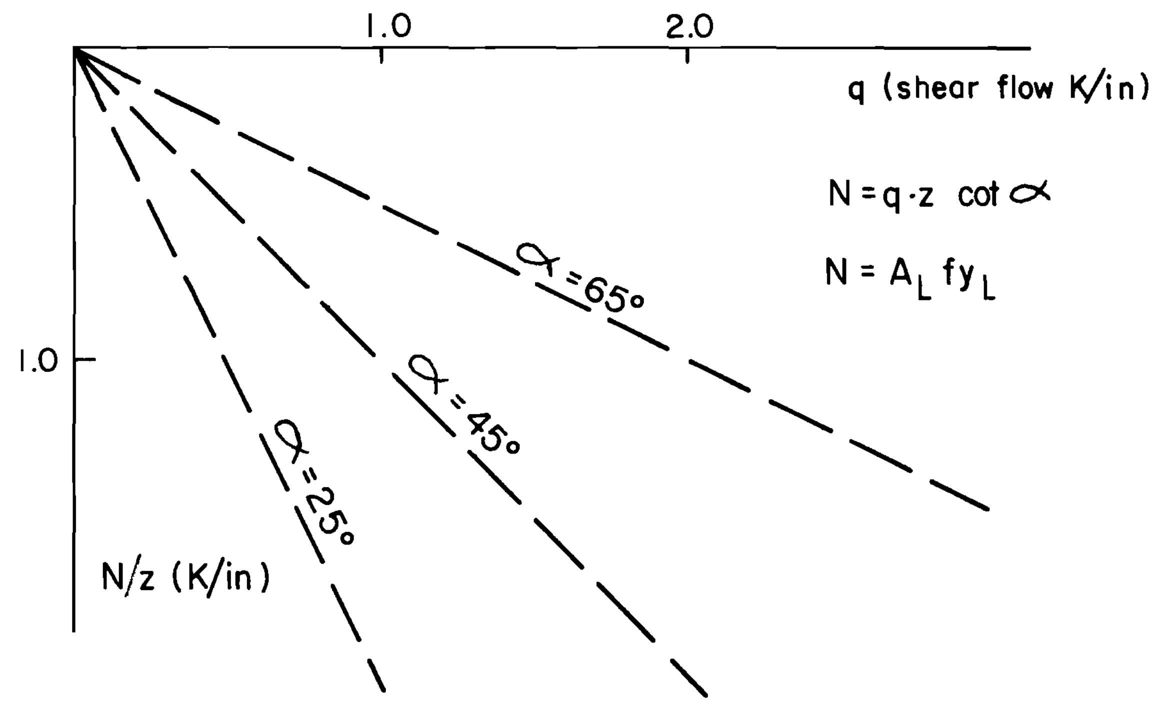
The effects of the value assumed for the angle of inclination of the diagonal compression strut on the required amounts of transverse and longitudinal reinforcement are illustrated in Fig. 4.2.

As can be seen from Fig. 4.2, small angles reduce the amount of transverse reinforcement required for a given level of shear and/or torsion. At the same time, the requirement for longitudinal reinforcement is increased. On the other hand, large values of the angle at inclination lead to smaller amounts of additional longitudinal reinforcement being required for shear and/or torsion but require that larger amounts of transverse reinforcement be provided.

The freedom in the selection of the angle of inclination of the diagonal struts in the truss model gives the designer several different design schemes to choose from. The designer can then select the one that best fits the requirements of the particular design situation. Sometimes architectural constraints, loading conditions, and general economy may lead the designer towards using low values of the angle α to minimize congestion of transverse reinforcement. However, in the cases of members where shear and/or torsion is not critical, the selection of larger values of the angle α, such as 45 degrees, may be more advisable. The angle selected does not have to be constant. When the shear on a member varies linearly, selection of values of α which vary along the span can result in uniform stirrup spacing which may simplify



(a) Transverse reinforcement



(b) Longitudinal reinforcement

Fig. 4.2 Longitudinal and transverse reinforcement requirements as a function of the chosen angle of inclination α

construction. However, in every case the truss model must be consistent with the requirements of equilibrium and have proper detailing (43).

A basic assumption that must be followed, and which is common to any simple truss system, is that the load can only be transmitted at the joints of the truss. This implies that the diagonal compression members must be anchored at the joints formed between the longitudinal chords and the vertical tension ties of the truss system.

The selected truss system has to be compatible with the applied loads and support conditions. As can be seen in Fig. 4.1b, compression fans will form under the applied concentrated loads and the support reactions. This phenomenon was analyzed in Sec. 2.2.2 of Report 248-3, and, as indicated, the effect of these fans vanishes as soon as the inclination of the diagonal members of the truss reaches the inclination of the chosen angle α.

In that section it was shown that the force in the vertical numbers of the truss remains the same in the fans as in the regular truss. However, the presence of fans influences the design of the longitudinal chords of the truss.

It was also shown how the presence of fans in the support regions of members subjected to bending and shear (where the support reaction induces compression) eliminates the need for longitudinal tension reinforcement due to effects of shear at the top compression face of the member within a distance [$z\cot\alpha$] from the centerline of the support. The presence of a compression fan requires that the longitudinal tension reinforcement in the noncontinuous end regions of simply supported members where the reaction induces compression be provided with an anchorage length such that a force equal to $[V_u \cot\alpha]/2$ is adequately developed.

Similar to the case of the compression fan at the support, in the zone of the compression fan under an applied concentrated load, the forces in the vertical members of the truss are the same in the fan region as in the regular truss.

Directly under the applied load the angle of inclination of the crack is equal to 90 degrees. Hence, shear will not cause any increase in the tensile force of the longitudinal chord. As a consequence, the area of longitudinal tension steel in this region need not exceed the area required for maximum flexure. However, because of the presence of the compression fan under the applied load, when dimensioning the tension chord reinforcement using the truss model approach, the calculations should be made at a distance $z\cot\alpha/2$ from the concentrated load.

As required by the corresponding moment diagram shown in Fig. 4.1a, the top chord of the truss model is in compression near support A and changes to tension as it approaches the support "B" when it crosses the point of inflection "E". The lower chord does just the opposite. This implies that the top of the diagonal compression struts will have to switch from bearing on a compression chord to a tension chord. This transition must be considered when detailing the longitudinal reinforcement in these regions. As indicated in Sec. 2.4.2 of Report 248-3, in order to allow this transition, the longitudinal tension reinforcement has to be continued an additional distance l_s beyond the theoretical cut-off point.

Once the truss model has been selected the design procedure becomes very simple and straight forward.

1. Determine the internal forces in the members of the truss.
2. Check compression stresses in the diagonal members of the truss to prevent web crushing failures.
3. Using these internal forces dimension the truss members.
4. From the chosen truss model draw the necessary conclusions for the adequate detailing of the reinforcement.

4.2.2 <u>Truss Model for a Simply Supported Member with Distributed Loading</u>. In the case of members where the shear force is not constant as in the case of members subjected to uniformly distributed loads, the angle of inclination of the compression diagonals of the truss may remain constant throughout the span of the member.

In Chapter 3 of Report 248-2, it was shown from the equilibrium condition of the truss model $\Sigma F_V = 0$, that the yield force in the stirrups ($A_v f_y = S_y$) and the shear flow "q" were related as $S_y = q{*}s{*}\tan\alpha$.

For the case of a member subjected to bending and shear "q" is equal to V/z where V is the applied shear force and z is the effective depth of the truss model. Thus, $V = A_v f_y z\cot\alpha/s$.

If the angle of inclination α remains constant, the change of the applied shear force within the design region $z\cot\alpha$ implies that at least one of the following conditions is satisfied:

1. The stirrup size or yield strength is changed.

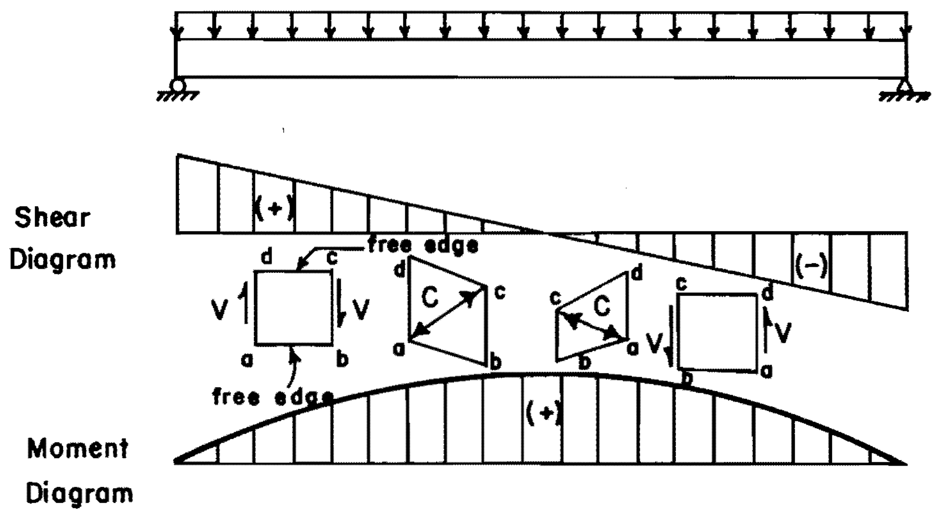
2. The stirrup spacing is changed as in current design procedure.

It is reasonable to suggest that the stirrup spacing be changed. Fig. 4.3b shows a typical truss model for the case of members subjected to distributed loads. The same principles are applied in this case. The directions of the compression diagonals are obtained directly from the shear diagrams shown in Fig. 4.3a. The inclination of the diagonals has to be within the limits $25^{o} \leq \alpha \leq 65^{o}$.

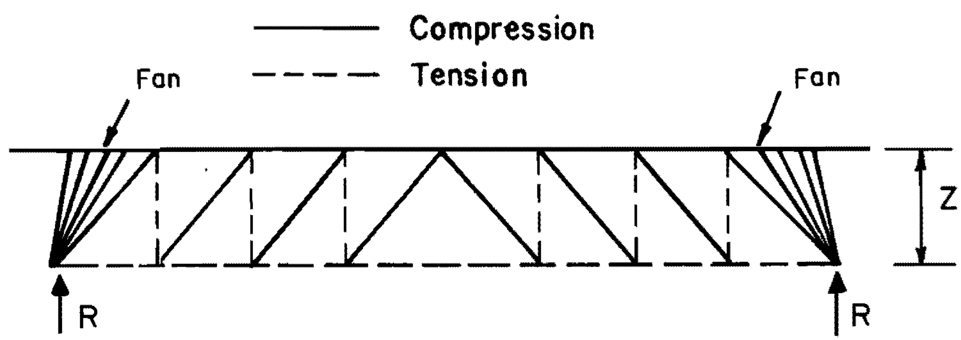
Once the truss model has been selected the design procedure is the same as the one presened in the previous section. The determination of the sectional forces that should be used in the design procedure for the case of bending and shear in reinforced and prestressed concrete members subjected to distributed loads has been illustrated in Sec. 2.2.1 of Report 248-3.

4.2.3 Truss Model for the Flange Region of Inverted T-Bent Caps. Figures 4.4a and 4.4b show the case of a simply supported inverted T-bent cap loaded on its bottom chord. Figures 4.5b and 4.5c show the transverse truss systems selected to design the transverse reinforcement for the flange region of the inverted T-bent cap loaded through the bottom flange.

The same basic concepts have to be applied in this case. Again the direction of the compression diagonals is obtained directly from the shear diagram shown in Fig. 4.5a. The inclination of the compression diagonals has to be within the limits $25^{o} \leq \alpha \leq 65^{o}$, where α is the angle of inclination of the truss compression diagonals (see Fig. 4.5b).

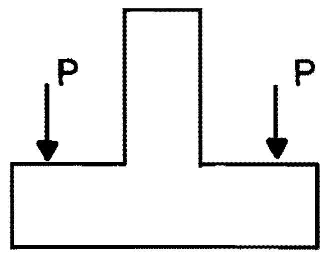


(a) Shear and moment diagrams

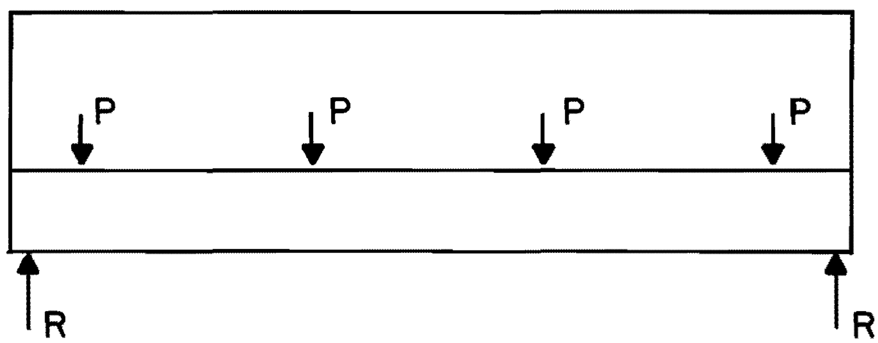


(b) Truss model

Fig. 4.3 Truss model in the area of a simply supported member with distributed loading

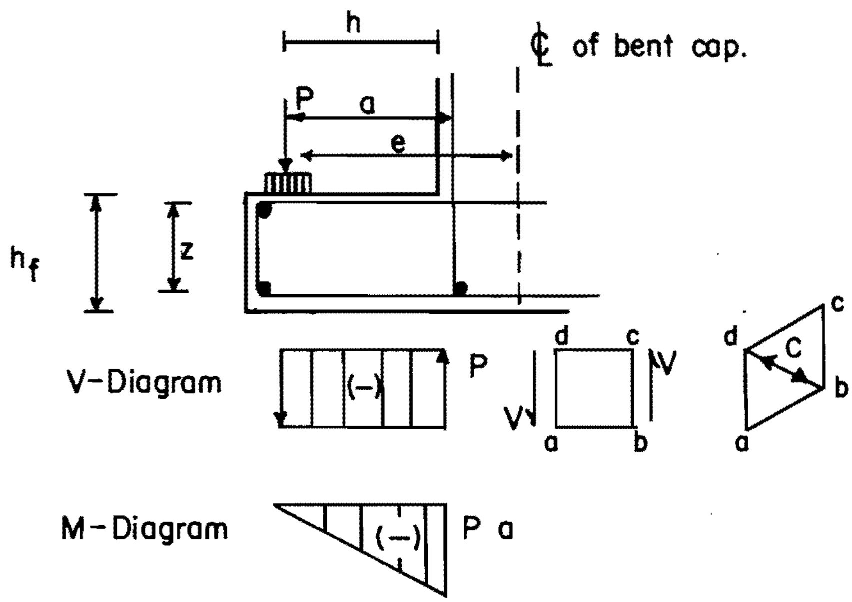


(a) End view

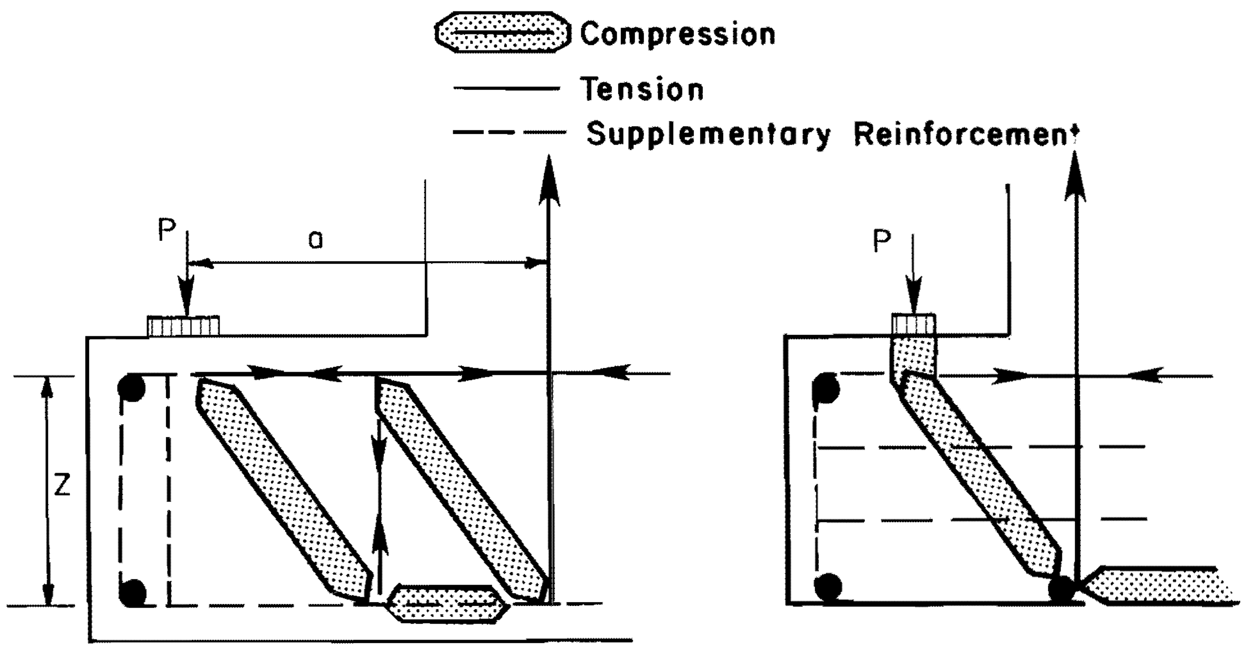


(b) Longitudinal view

Fig. 4.4 Simply supported inverted T-bent cap subjected to bending and shear



(a) Flange shear and moment diagrams



(b) Truss model
$0.5 \leq a/z \leq 2.0$

(c) Truss model
$a/z \leq 0.5$

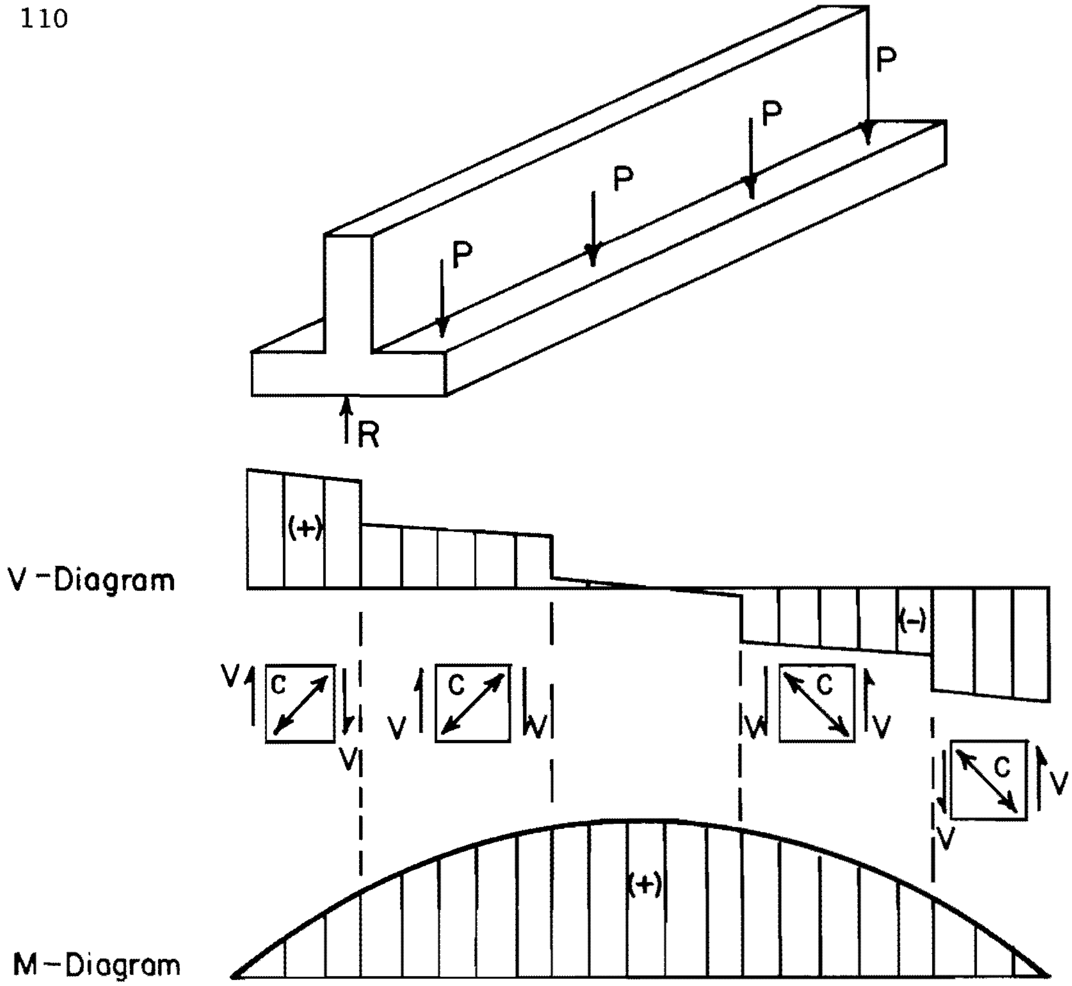
Fig. 4.5 Truss model for the flange region of an inverted T-bent cap loaded on the bottom flange

Sometimes, because of the particular geometry of these members and the type of loading, the angle of inclination of the diagonal strut will be larger than 65 degrees, corresponding to values greater than $\tan\alpha = 2.0$. In such cases, the flange becomes a bracket loaded at its tip. The truss model shown in Fig. 4.5c corresponds to such a case.

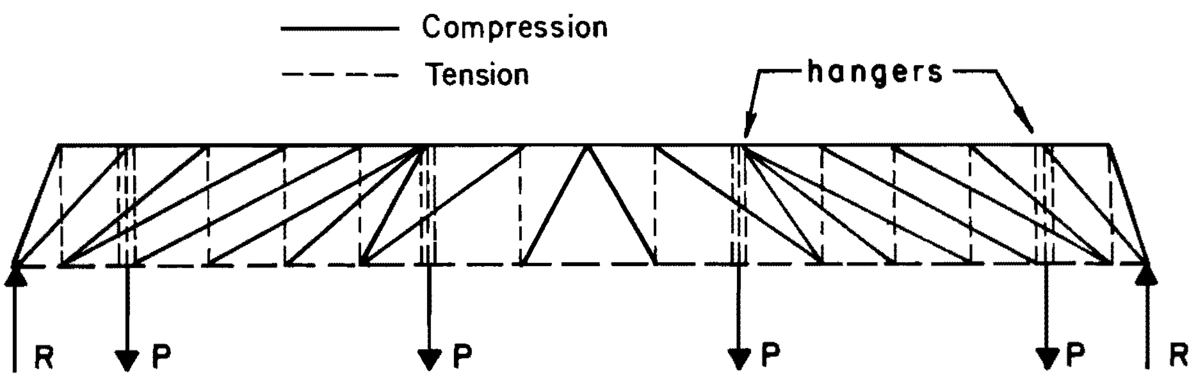
The application of the truss model to bracket design has been examined in Sec. 2.2.3 of Report 248-3. As previously indicated, in members with a/z less than 0.5 the pattern of cracks at failure shows an inclination which is very close to 90 degrees from the horizontal. Hence, as the truss model clearly shows, vertical stirrups will not be effective. As in the case of brackets, shear acts along a vertical plane and vertical slip of one crack face can occur with respect to the other. If the crack faces are rough and irregular, this slip is accompanied by a horizontal separation of the crack faces. Thus, supplementary horizontal web reinforcement (shear friction reinforcement) should be provided to control the crack opening.

As suggested in Sec. 2.2.3 of Report 248-3, the design of this type of members can be based on a simple truss analogy consisting of the main reinforcement acting as tension ties and the concrete struts acting as inclined compression members, such as shown in Fig. 4.5c.

Another interesting effect observed in both truss models of Figs. 4.5b and 4.5c is the so-called "hanger effect" produced in the vertical tension ties located in the longitudinal member web. This effect was shown in Sec. 2.2.4 of Report 248-3. Figure 4.6a shows an inverted T-bent cap subjected to a series of concentrated loads applied



(a) Shear and moment diagrams



(b) Truss model

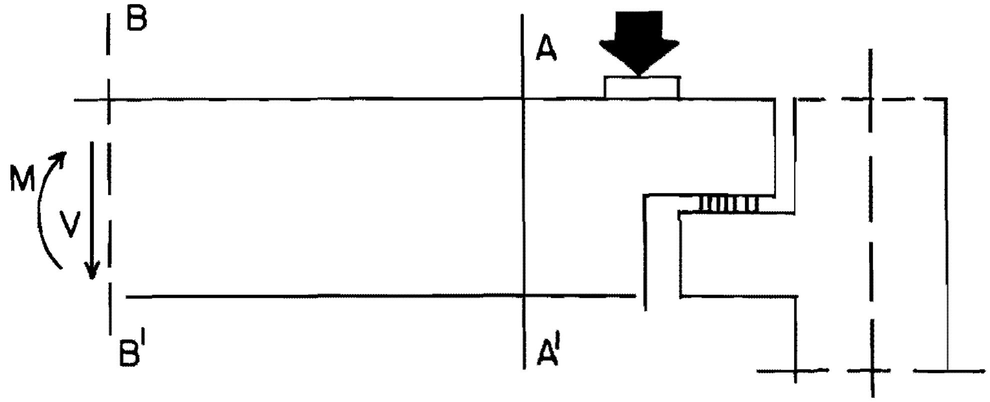
Fig. 4.6 Longitudinal truss analogy for an inverted T-bent cap loaded at the bottom chord along the bottom chord. Figure 4.6b shows the longitudinal truss model for this inverted T-bent cap. As explained in Sec. 2.2.4 of Report 248-3, members loaded on the bottom chord experience an increase in the tension force acting on the vertical ties of the member web. The additional area required in the verticals of the truss is that necessary for "hangers" for the load P. These hangers pick up the load from the bottom (tension face) and transfer it to the top compression chord of the truss. The truss model has been selected to provide different load paths for the heavy concentrated forces. Note the pattern of the diagonals tends to keep the center concentrated loads from passing through the truss joints where the outer loads are acting.

After the truss model has been selected, the design procedure is essentially the same one presented in the previous sections.

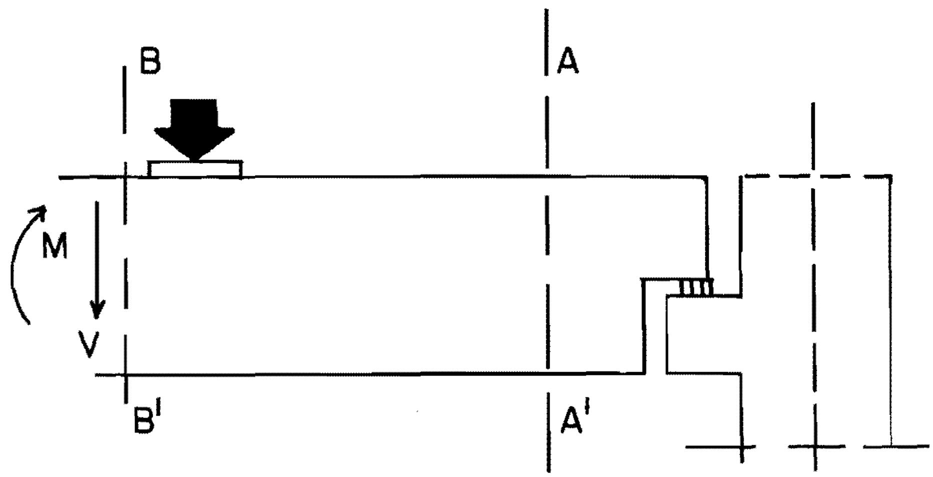
4.2.4 Dapped-End Beams. Shown in Figs. 4.7a and 4.7b are the problems of a member with an abrupt change in depth at the support region, with and without heavy concentrated loads near the support.

The freedom in the selection of a truss system to adequately carry the loads allows the designer to handle this difficult problem. Shown in Fig. 4.8a is the truss model selected to analyze the internal forces at the end region of this member.

As in the previous cases, the direction of the main diagonals can be found from the shear diagram (see Fig. 4.3a). As previously mentioned, the selected truss model has to satisfy the particular loading and structural constraints. In this case, a second truss system is constructed within the main truss system to handle the heavy

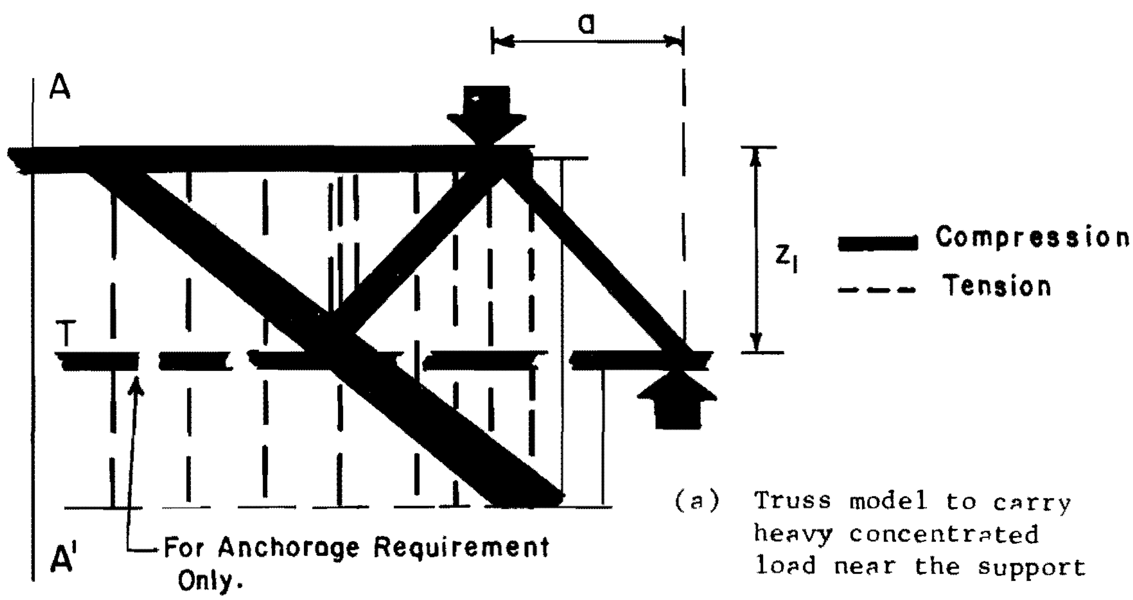


(a) Dapped end beam

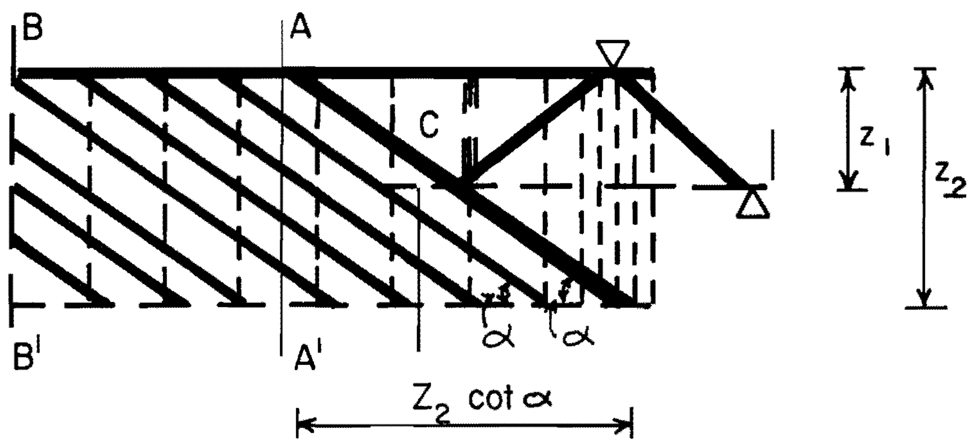


(b) Dapped end beam

Fig. 4.7 Dapped end beam with and without a heavy concentrated load near the support



(a) Truss model to carry heavy concentrated load near the support



(b) Truss model to carry loads out on span

Fig. 4.8 Truss analogies for dapped end beams concentrated load near the support region. This second truss system is needed even in the case where the heavy concentrated load is not present in order to provide for an orderly transfer of depth. Several other truss models could be used. In Fig. 4.8a the additional truss system is shown in heavy dotted and solid lines. In this particular situation due to the proximity of the heavy concentrated load to the support, similar to the case of brackets, vertical reinforcement is not effective in carying the heavy concentrated load to the support. However, closely spaced stirrups are necessary at the end region of the members to provide hanger support for the diagonal compression strut"C" shown in Fig. 4.8b. Because of the change in depth of the member outside the support region, the diagonal compression strut "C" of the main truss system would not have an effective support to bear against if closely spaced vertical stirrups were not provided in the end region of the full depth section. These stirrups then support the diagonal compression strut "C" and prevent the type of failure shown in Fig. 4.9.

The proposed truss systems shown in Fig. 4.8 consist of a strut and tie system. In this case the geometry of the member together with the loading condition at the end region dictate the geometry of the truss model. If the ratio a/z_1 shown in Fig. 4.8a is less than 0.5, then ductility and crack control are better served by distributing the horizontal ties over the entire depth z_1.

Similar to the case of brackets the controlling failure mechanisms would be either crushing of the concrete diagonal struts in compression or yielding of the longitudinal tension reinforcement

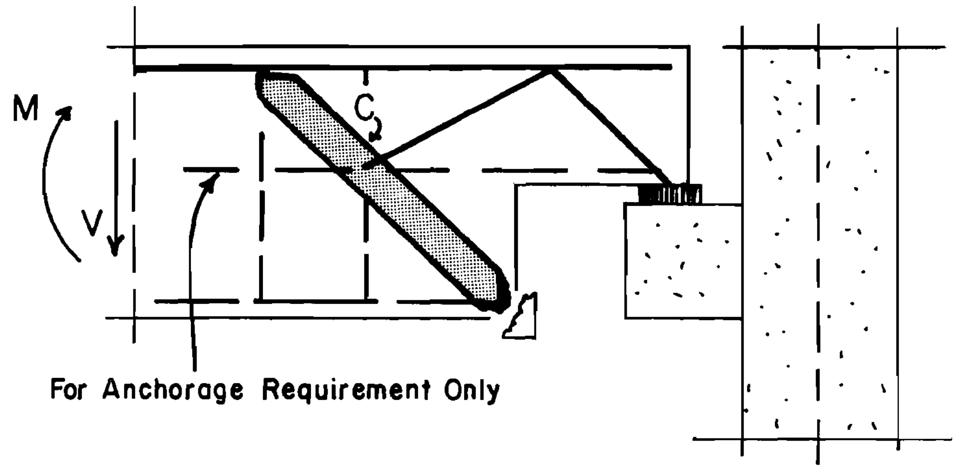


Fig. 4.9 Failure due to inadequate support of the diagonal compression strut "C"

assuming that adequate anchorage of the longitudinal steel used as tension ties is provided.

The first failure mechanism is prevented by limiting the stresses in the diagonal strut to less than $30\sqrt{f'_c}$. Then, a ductile type failure due to yielding of the longitudinal tension ties would be achieved by adequately detailing this reinforcement.

Once the truss model is chosen, the design procedure is similar to the one suggested in the previous cases.

4.2.5 Box Girder Bridge with Cantilever Overhang. Figure 4.10 shows the case of a box girder bridge with cantiliver overhangs subject to combined torsion-bending-shear. The same basic concepts applied in the previous section are valid in this case to determine an adequate truss analogy.

Due to the presence of a torsional moment, which as previously explained is assumed to produce a constant shear flow "q" around the cross section, the truss model becomes a space truss model. However, the design of each of the walls forming the truss model representation of the box section remains the same one presented in the previous sections. The direction of the compression diagonals in the space truss depends upon the relative magnitudes of the shear flows due to shear and torsion present on each of the walls.

Figure 4.11b shows the resultant shear flows due to shear and torsion on each of the side walls of the box section for the span 1 of the box girder bridge. Assume counterclockwise shearing flows as positive. As can be seen from Fig. 4.11b, the inclination of the

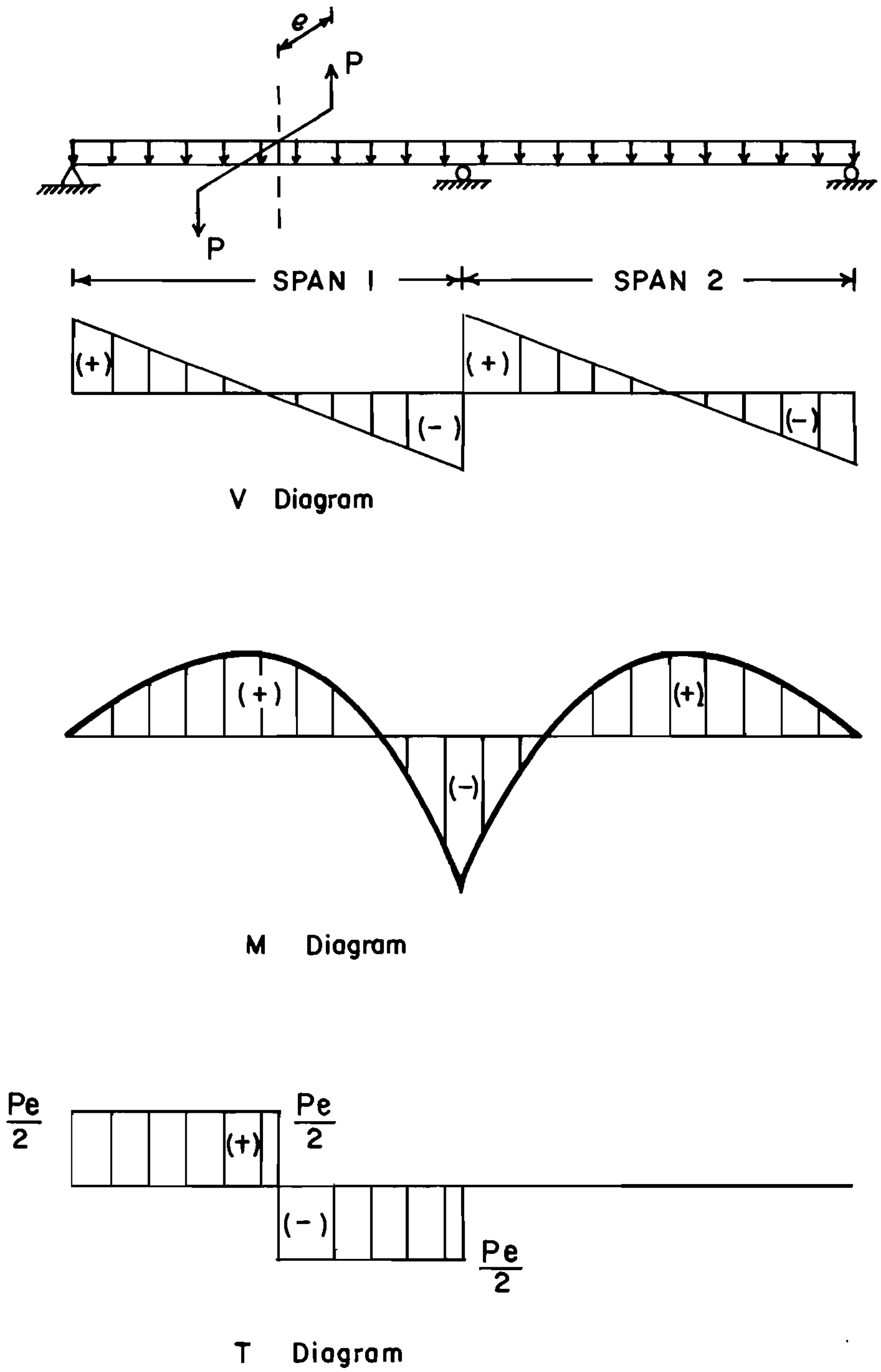
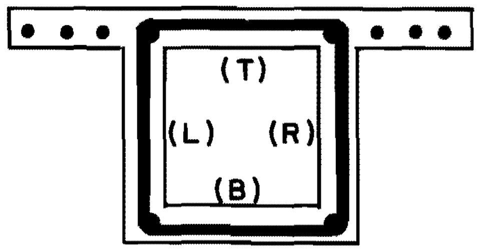
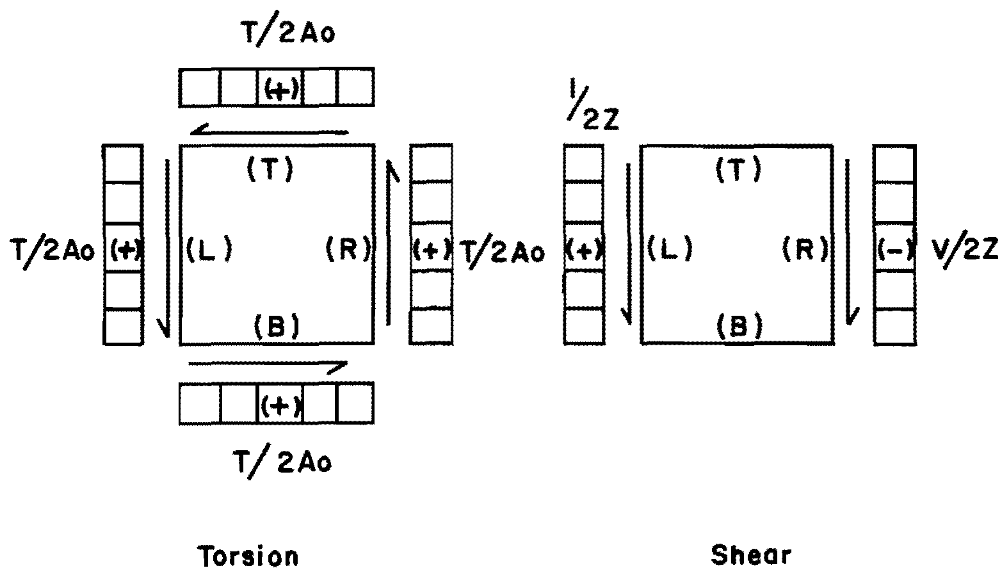


Fig. 4.10 Box girder bridge under combined torsion-bending-shear



(a) Box section



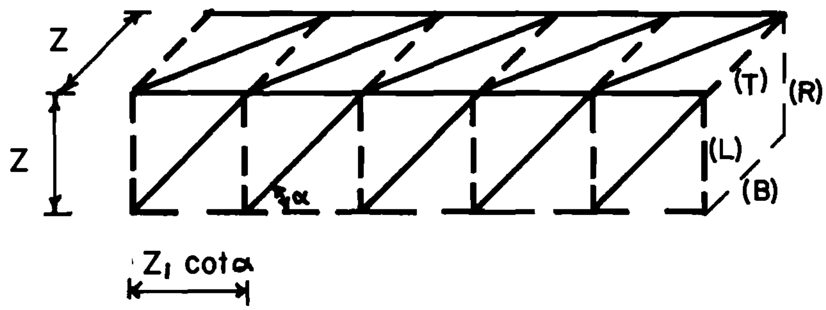
(b) Resultant shearing flows "q" due to shear and torsion

Fig. 4.11 Box section subjected to combined torsion and shear diagonal struts in the top and bottom walls depends only on the torsion shear flow since V = 0 in these plates.

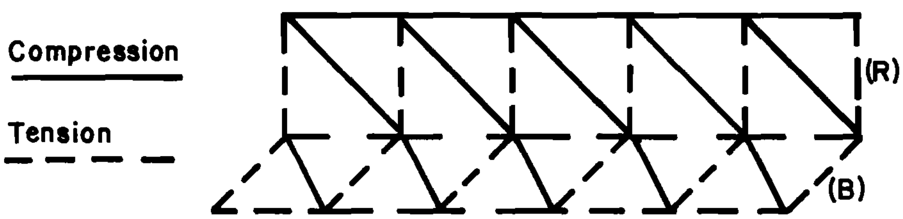
In the side wall (L) the shear flow due to shear and torsion will be additive. However, in the side wall (R) the shear flows due to shear and torsion will counteract each other. The compression diagonal will follow the direction indicated by the maximum of the two shear flows at any section.

Shown in Fig. 4.12 is the space truss analogy for span 1 of the box girder bridge. Figure 4.12a shows the truss analogy for the top (T) and left (L) side walls at any section in span 1 of the continuous bridge. Figure 4.12b shows the truss analogy for the bottom wall (B) and the side wall (R) when the shear flow due to torsion (T) is larger than the shear flow due to shear (V). Figure 4.12c illustrates the truss analogy for the bottom wall (B) and the side wall (R) when the shear flow due to shear (V) is larger than the shear flow due to torsion.

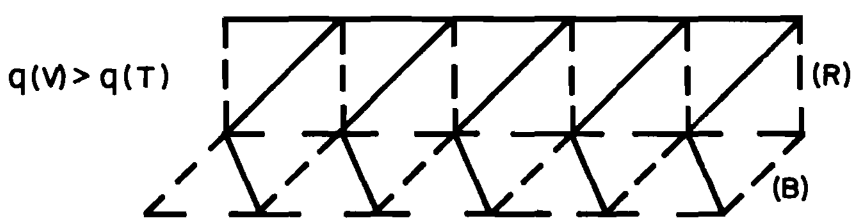
As shown in Figs. 4.12b and 4.12c, the orientation of the compression diagonals on the side wall (R) will change depending upon the relative magnitude of the shear flows due to shear and torsion. However, in the actual design of the member it is recommended that the design of the walls (L) and (R) be carried out assuming that both shear flows are always additive unless there is absolute certainty that the direction of the applied torsional moment will remain unchanged.



(a) Truss analogy for walls (L) and (T).



(b) Truss analogy for walls (R) and (B) when q(T) is greater than q(V).



(c) Truss analogy for walls (R) and (B) when q(V) is larger than q(T).

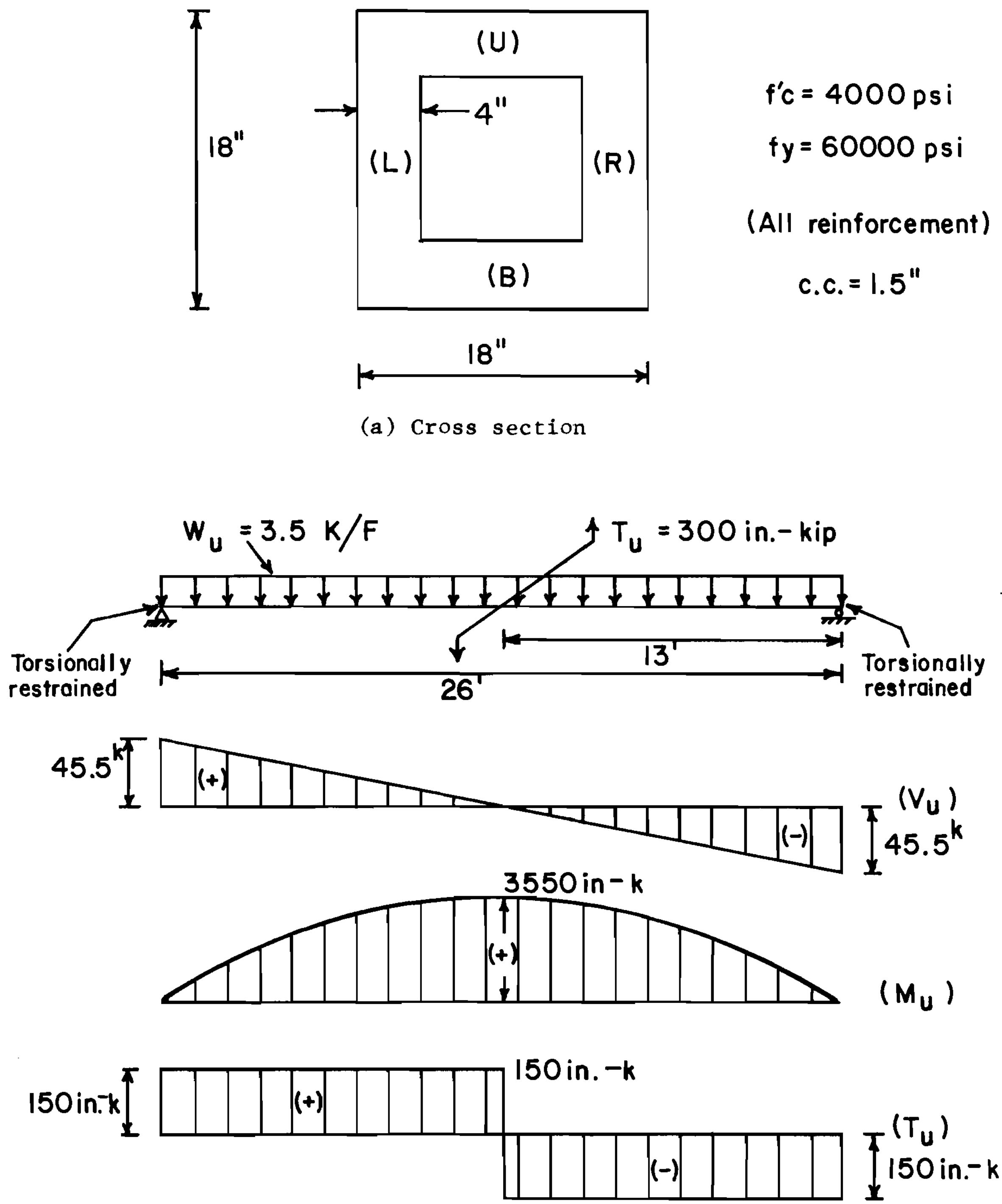
Fig. 4.12 Space truss analogy for span 1 of the box girder bridge

The inclination of the compression diagonals in all the walls of the space truss has to be within the limits $25^{\circ} \leq \alpha \leq 65^{\circ}$ where α is the angle of inclination of the truss compression diagonals.

After the truss model has been selected, the design procedure is essentially the same one presented in the previous sections. Although the computations can be carried out separately for each wall of the member, the designer must always keep in mind the overall system and must add all effects for the overall system. This is of special significance for example when dimensioning the longitudinal chords of the truss model as well as in the overall detailing of the member since it must behave as a unit.

4.3 Design Example of a Reinforced Concrete Rectangular Box Beam under Combined Torsion, Bending, and Shear

In this secton the design of the reinforced concrete box section shown in Fig. 4.13a subjected to combined shear, bending and torsion, as shown in Fig. 4.13b, is carried out using the truss approach. The amounts of reinforcement required using the truss model design procedure are compared with those obtained using the current AASHTO Standard Specifications (2). Exhibit 4-1 shows the detailed calculations required for design of this member using the truss analogy. The calculations presented in this exhibit are amplified and explained in this section to introduce the reader to the design method in full detail. However, in practical application by an experienced designer only the calculations shown in the exhibit and the referenced tables would be needed.



(a) Cross section

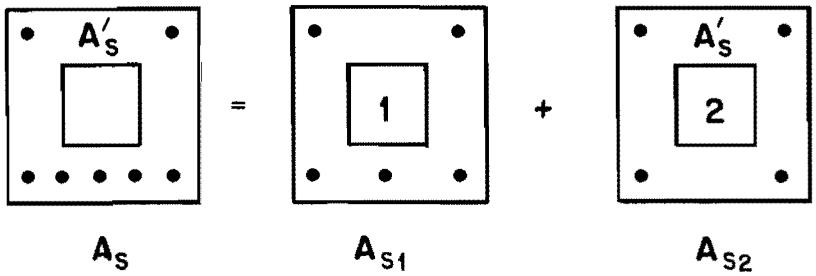
(b) Actions

Fig. 4.13 Actions on the reinforced concrete box beam

4.3.1 Preliminary Flexure Design. As in any usual design procedure proportioning and selection of reinforcement as controlled by flexure will be conducted first (1,2). The flexure design procedure will be the same in the Truss Model and ACI/AASHTO design approaches. A section containing both tension and compression reinforcement will be selected because the presence of compression reinforcement helps to adequately anchor web reinforcement as well as to control creep deflections. In addition, the presence of a torsional moment might require some tension reinforcement in the flexural compression face (Top (U)) of the member.

In this flexural design example it is assumed that the overall dimension are known (18 x 18 in.), as well as the material properties f'_c (4000 psi) and f_y (60,000 psi). The effective depth d, taken as the distance between the extreme compression fiber and the centroid of the longitudinal tension reinforcement, is evaluated assuming a 1.5 in. clear cover, a #4 stirrup, and a #9 longitudinal bar. Thus, d = 18 - 0.56 - 0.5 - 1.5 = 15.44 in. The distance between the centroid of the compression reinforcement and the extreme compression fiber "d" is evaluated assuming a clear cover of 1.5 in., a #4 stirrup and a #8 longitudinal bar. Hence, d' = 0.5" + 0.5" + 1.5" = 2.5 in. Detailed calculations for flexure are shown in Fig. 4.14.

The preliminary flexure design for the midspan region of the simply supported box beam is shown in Fig. 4.15. Detailing of this longitudinal steel at other sections along the span of the member will



$$A_s = A_{s1} + A_{s2}; \; M_N = M_{N1} + M_{N2}$$

Assume $A_{s1} = 0.014\ b_d = 0.014(18)(15.44) = 3.89$ in.^{2}

Check ductility $\quad 0.75\,\rho_{bal} = 0.016 > 0.014 \quad$ OK

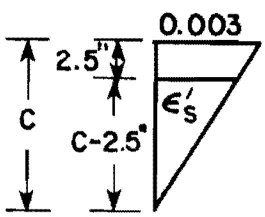
C = T in(1): $\; a = A_{s1}f_y/0.85f'_c b = (3.89)(60)/0.85(4)(18) - 3.81'' <$ 4" rect. sec.

$M_{N1} = A_{s1}f_y(d-a/2) = (3.89)(60)(15.4-3.81/2) = 3149$ in-k

$M_{N2} = M_N - M_{N1} = 3550/0.9 - 3149 - 795$ in-k

$d-d' = 15.44 - 2.5 = 12.94$ in; $c = a/B_1 = 3.81/0.85 = 4.48''$

Strain diagram in compression zone:



$$\epsilon'_s = 0.003/c\ (c-2.5) - 0.003/4.48\ (4.48-2.5) = 0.0014$$

$$f'_s = E\ \epsilon'_s = (29 \times 10^3)(0.0014) = 38.5 \text{ ksi}$$

Thus: $\quad M_{N2} = A_{s2}f_y(d-d')$ and $A_{s2} = M_{N2}/f_y(d-d')$

$$A_{s2} = 795/60\ (12.94) = 1.02 \text{ in}^2$$

C = T in (2): $\; A'_s f'_s = A_{s2}f_y; \; f'_s = 38.5$ ksi

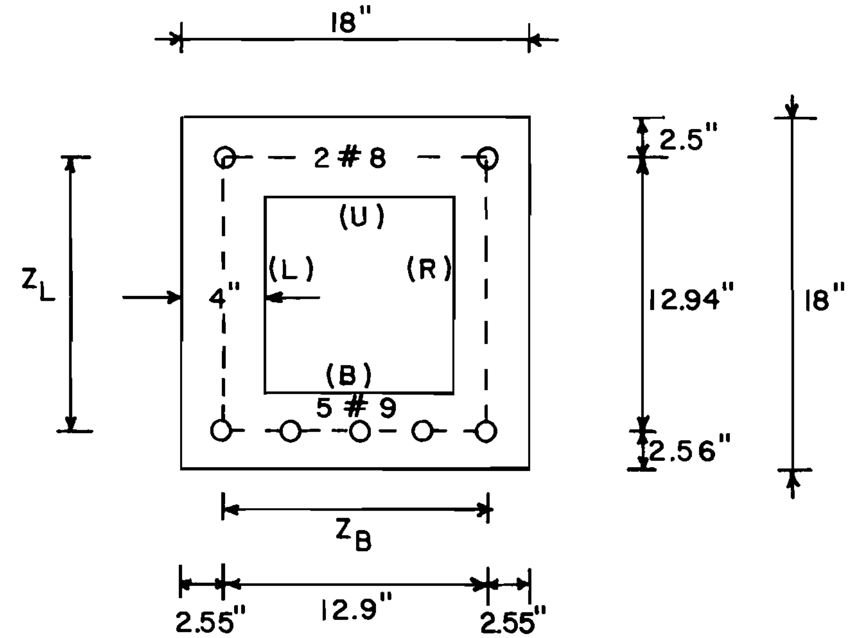
$$A'_s = A_{s2}f_y/f'_s = 1.02\ (60)/38.5 = 1.58 \text{ in}^2$$

Thus use 2 #8 $A'_s = 1.58$ as top compression steel

$A_s = A_{s1} + A_{s2} = 3.89 + 1.02 = 4.91$ in^2

Thus use 5 #9 $A_s = 5.00$ in^2 as bottom tension steel.

Fig. 4.14 Calculations for flexure at midspan section of the reinforced concrete box beam



$Z_L = Z_R$: $Z_B = Z_U$

Fig. 4.15 Preliminary flexure design for the midspan region of the reinforced concrete box beam be conducted after the shear and torsion design of the member has been carried out.

After the preliminary flexure design has been conducted, the section will be checked to satisfy the shear and torsion requirements.

This section presents a summary of the design steps involved in the truss model approach. Detailed explanation of each of the steps as well as numerical calculations are shown in subsequent subsections.

1. Carry out preliminary flexure design

2. Select an adequate truss system

3. Select angle of inclination between the limits

$$25^{\circ} \leq \alpha \leq 65^{\circ}$$

4. Develop truss system

   4.1 Compute length of design panel

$$z_L \cot\alpha$$

   4.2 Determine direction of the compression diagonals in each of the design panels (zones) of the truss model (see Fig. 4.17b)

$$q(T,V) = q(T) \pm q(V) = \frac{T_n}{2A_o} \pm \frac{V_n}{2z_L}$$

5. Evaluation of the diagonal compression stresses $f_d(T,V)$

$$f_d(T,V) = f_d(T) + f_d(V) = \frac{1}{\sin\alpha\cos\alpha}\left[\frac{q(T)}{be} + \frac{q(V)}{bw}\right]$$

6. Design of the web reinforcement

   6.1 Evaluation of the concrete contribution in accordance with the proposed Specification Sec. 1.3.6f which was presented in Section 3.1.

Additional concrete contribution to the shear capacity $= \frac{v_u(V)}{[v_u(V)+v_u(T)]} * V_c$

Additional concrete contribution to the torsional capacity $= \frac{v_u(T)}{[v_u(V)+v_u(T)]} * T_c$

where

$$v_u(V) = \frac{V_u}{2b_w z_L}$$

$$v_u(T) = \frac{T_u}{2A_o b_e}$$

$$V_c = \frac{1}{2}\,[6\sqrt{f'_c} - [v_u(V) + v_u(T)]]\; b_w\, z_L$$

$$T_c = \frac{1}{2}\,[6\sqrt{f'_c} - [v_u(V) + v_u(T)]]\, 2A_o\, b_e$$

6.2 Evaluation of the amount of web reinforcement required to resist the factored shear force

$$\frac{A_v}{s} = [\frac{(V_u - w_u\, z_L\, \cot\alpha)}{2\phi} - V_c\;(T,V)]\,\frac{\tan\alpha}{z_L\, f_y}$$

6.3 Evaluation of the amount of web reinforcement required to resist the factored torsional moment

$$\frac{A_t}{s} = [\frac{T_u}{\phi} - T_c\;(T,V)]\,\frac{\tan\alpha}{2f_y\, A_o}$$

6.4 Evaluation of the minimum amount of web reinforcement

$$(\frac{A_t}{b_e s} + \frac{A_v}{b_w s})_{min} = 1.0\,\frac{\sqrt{f'_c}}{f_y}$$

7. Evaluation of the compression stresses in the fan regions f_{di}

$$f_{di} = \frac{D_i}{b_w \, z_L \, \cos\alpha_i} \leq 30\sqrt{f'_c}$$

where

$$D_i = \frac{[S_{(i)} + w_n s]}{\sin\alpha_{(i)}}$$

8. Dimensioning of the longitudinal reinforcement required for shear and torsion

$$A_L(T_n, V_n) = [\frac{(T_n * u)}{(4\ A_o)} + \frac{V_n}{2}] \frac{\cot\alpha}{2f_y}$$

$$u = 2z_L + 2z_B$$

$$A_o = z_L \times z_B$$

9. Detailing of the longitudinal reinforcement

Once the required amounts of longitudinal reinforcement for shear, torsion and bending are known, the detailing of this reinforcement can be conducted. Detailed calculations which include both curtailment and anchorage of the longitudinal reinforcement are shown in Sec. 4.3.8.

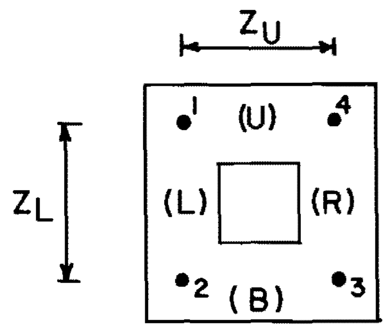
4.3.2 Selection of an Adequate Truss System. The first step would be to select an adequate truss section of the given load pattern and structural constraints.

Due to the presence of a torsional moment, which as previously explained is assumed to produce a constant shear flow "q" around the cross section, the truss model becomes a space truss model. The direction of the compression diagonals in the space truss model will depend upon the relative magnitudes of the shear flows due to shear and torsion acting on each of the walls of the box section. Figure 4.16 shows the resultant shear flows due to shear and torsion on each of the side walls of the box section, assuming counterclockwise shear flows as positive. Shown is the section between the left support and midspan of the box girder beam.

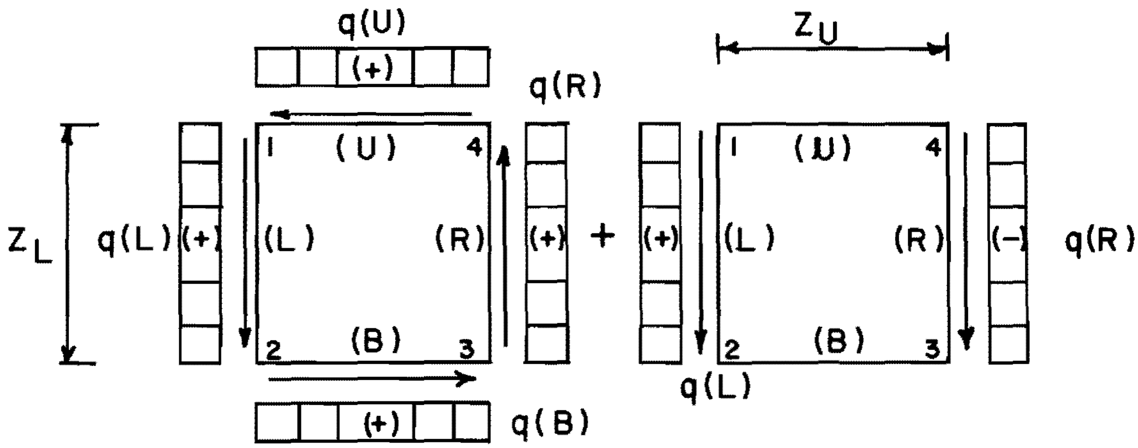
As can be seen from Fig. 4.16, the inclination of the diagonal struts in the bottom (B) and top (U) walls depends only on the torsion shear flow since V = 0 in these plates. In the side walls (L) and (R), however, the direction that the diagonals must follow will depend on the relative magnitude of the shear flow due to shear and torsion acting at any section.

In order to determine the relative magnitudes of the shear flow due to shear and torsion, it is first necessary to determine the number of design zones or panels that the chosen truss model is going to have. To determine the number of design zones it is necessary to choose the angle of inclination of the diagonal compression elements of the truss model, since design zone will have a length equal to the horizontal projection ($z_L \cot\alpha$) of the compression diagonal members of the truss.

The designer had a complete freedom in the selection of the angle of inclination between the limits $25^{\circ} \leq \alpha \leq 65^{\circ}$. As previously explained in Sec. 4.2, the selection of the angle of inclination of the diagonal compression struts in the truss model has a strong influence in



(a) Box section



$q(L) = q(B) = q(R) = q(U) = q(T)$

$q(T) = T_n/2A_o$

Torsion

$q(L) = q(R) = q(V)$

$q(V) = V_n/2z_L$

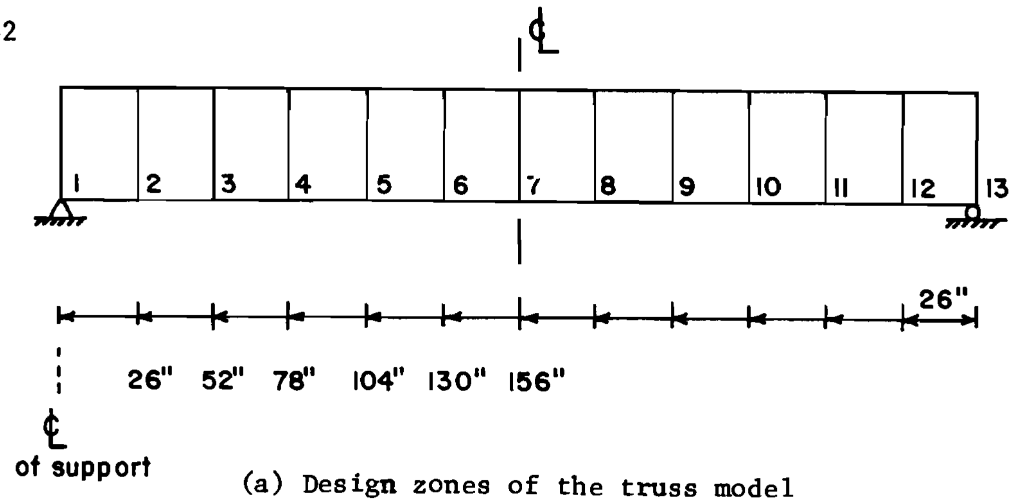
Shear

(b) Shear flows due to torsion and shear

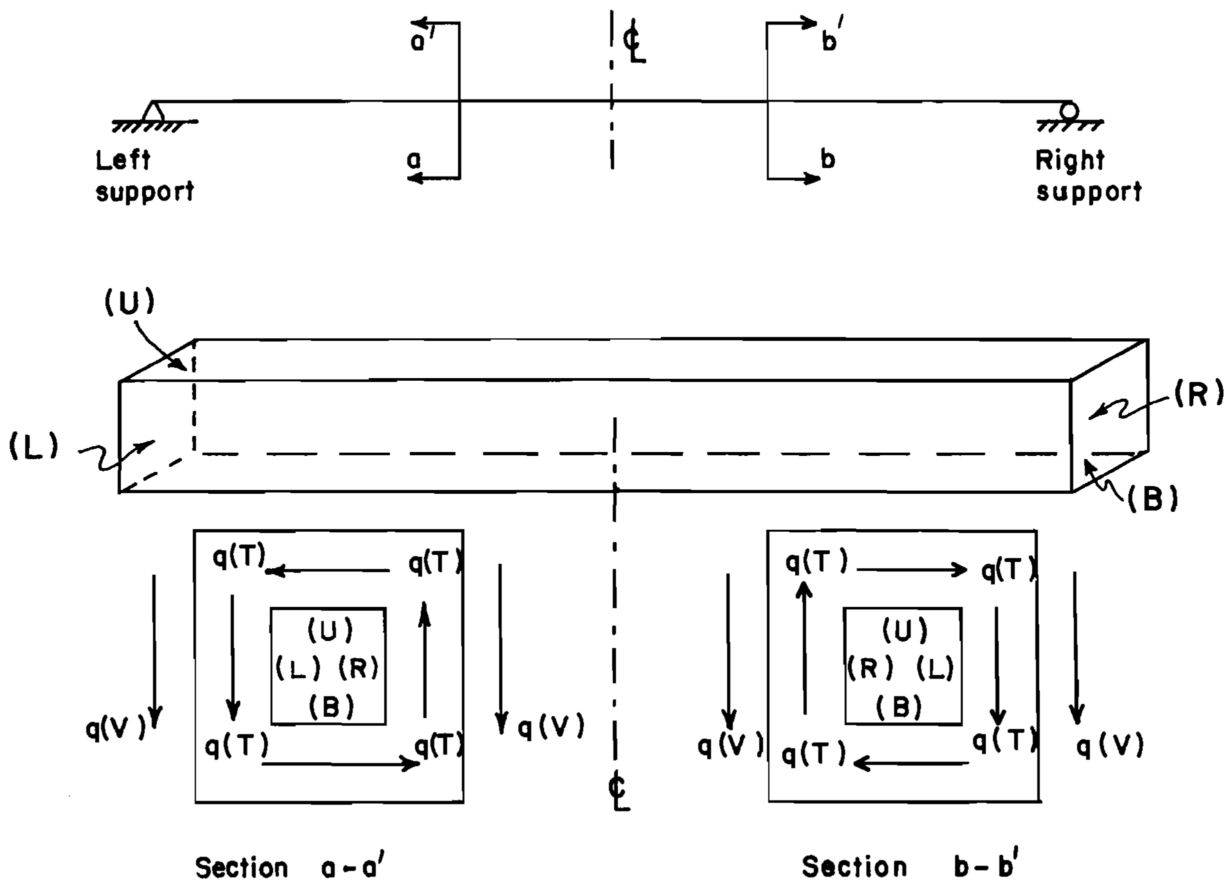
Fig. 4.16 Box section in the case of combined shear and torsion the final relative amounts of transverse and longitudinal reinforcement in the member. Low values of the angle of inclination reduce the amount of transverse reinforcement required. At the same time they increase the requirement for longitudinal reinforcement due to shear and/or torsion. On the other hand, large values of the angle of inclination lead to smaller amounts of longitudinal reinforcement for shear and/or torsion, but larger amounts of transverse reinforcement must be provided.

In this design example the maximum nominal shearing stress due to shear evaluated as $v_n = V_u/\phi 2 b_w z_L$ at the support results in a shearing stress equal to 520 psi. The number 2 in the formula for v_n represents the two vertical walls, (L) and (R), resisting the applied vertical shear. The magnitude of the maximum nominal shearing stress (520 psi), is in excess of a shearing stress of $6\sqrt{f'_c}$, 380 psi, thus indicating that high shearing stresses are acting on the member. Hence, it seems to be advisable to use a lower value for the angle of inclination of the diagonal compression strut in order to avoid congestion of the web reinforcement. It is also convenient to select a value that would yield a convenient whole number of design panels for the overall truss model. In this case an initial value of 26.5 degrees, which yields a length for the design zone $z_L \cot\alpha$ of 26 in., will be selected. This selection of the angle of inclination then results in 12-26 in. design panels for the total length of the member.

Figure 4.17a shows the box beams with the resulting design zones.



(a) Design zones of the truss model



(b) Directions of the shear flows

Fig. 4.17 Determination of the direction of the shearing flows for the box section in the case of combined shear and torsion

Figure 4.17b shows the relative direction of the shear flows due to shear, q(V), and torsion, q(T). Section a-a' shows the relative direction of both shear flows for any section between the midspan and the left support while looking towards the left support. Section b-b' shows the respective direction of the shear flows for any section between the midspan and the right support, while looking towards the right support. These directions are the ones corresponding to the shear and torsion diagrams shown in Fig. 4.13b.

Once the design zones have been determined, the respective shear flows due to shear and torsion are determined at each of the sections bounding the design zones. As shown in Fig. 4.16b, the shear flow due to torsion q(T) is evaluated as

$$q(T) = T_n/2A_o \qquad (4.1)$$

where A_o is the area enclosed by the perimeter correcting the centroids of the longitudinal chords of the space truss model resisting the applied ultimate torsional moment T_u. For this design example, A_o is equal to $z_L * z_B$ = (12.94)(12.9) = 166.7 in.^{2} T_n is the nominal torsional moment T_u/ϕ where ϕ = 0.85. The shear flow due to shear is evaluated as

$$q(V) = V_n/2z_L \qquad (4.2)$$

where z_L is the vertical dimension of the truss model (12.94 in.). V_n is the nominal shear force at the section V_u/ϕ. V_u is the ultimate shear force at the section and ϕ is taken as 0.85. The number 2 in the denominator indicates that there are two vertical walls resisting the applied ultimate vertical shear force.

Table 4.1 shows the resultant shearing force due to shear and torsion q(V) and q(T) for each of the walls of the box section, evaluated at the boundary of each design zone.

As can be seen from Fig. 4.17b and Table 4.1, the shear flows due to shear and torsion in the region between the left support and the midspan will be additive on the side wall (L) and will oppose each other on the side wall (R). On the side wall (R), as shown by column (7) of Table 4.1, in the design zones 1-2, 2-3, 3-4, 4-5, and 5-6 the direction of the diagonal compression members in the truss will be determined by the direction of the shear flow due to shear (V). However, in the design zone 6-7 of this side wall (R) the direction of the compression diagonal will be determined by the shear flow due to torsion (T).

In the region of the member between the midspan and the right support, the situation is similar. In the side wall (L), the shear flows due to shear and torsion are additive. As can be seen from column (7) in Table 4.1, in the side wall (R) the direction of the compression diagonals is controlled by the relative magnitude of the shear flows due to shear and torsion. In the design zones 6-7 and 7-8, the direction of the diagonals is determined by the resultant shear flow due to torsion (T). In the design sections 8-9, 9-10, 10-11, 11-12, and 12-13, the direction of the diagonals is determined by resultant shear flow due to shear (V).

(1) Jt (N)	(2) Sec. from CL of Left Support	(3) Design Zone	(4) q(T) (k/in)	(5) q(V) (k/in)	(6) q(T+V) Wall(L) (k/in)	(7) q(T+V) Wall(R) (k/in)	(8) q(T) Wall(U) (k/in)	(9) q(T) Wall(B) (k/in)
1	0.0							
		1-2	0.5	2.1	2.6	1.6(V)	0.5(T)	0.5(T)
2	2.17							
		2-3	0.5	1.7	2.2	1.2(V)	0.5	0.5
3	4.33							
		3-4	0.5	1.4	1.9	0.9(V)	0.5	0.5
4	6.50							
		4-5	0.5	1.0	1.5	0.5(V)	0.5	0.5
5	8.67							
		5-6	0.5	0.7	1.2	0.2(V)	0.5	0.5
6	10.83							
		6-7	0.5	0.3	0.8	0.2(T)	0.5	0.5
7	13.0 (CL)							
		7-8	0.5	0.3	0.8	0.2(T)	0.5	0.5
8	15.17							
		8-9	0.5	0.7	1.2	0.2(V)	0.5	0.5
9	17.33							
		9-10	0.5	1.0	1.5	0.5(V)	0.5	0.5
10	19.50							
		10-11	0.5	1.4	1.9	0.9(V)	0.5	0.5
11	21.67							
		11-12	0.5	1.7	2.2	1.2(V)	0.5	0.5
12	23.83							
		12-13	0.5	2.1	2.6	1.6(V)	0.5	0.5
13	26.00							

Table 4.1 Resultant shearing flows due to shear and torsion at each of the walls of the box section

In the top (U) and bottom (B) walls the direction of the compression diagonals is entirely dependent on the shear flow due to torsion since the applied shear force V is equal to zero in these plates.

The resultant truss model for this case is shown in Fig. 4.18 for the entire member length. As previously stated, each design zone $z_L \cot\alpha$ is equal to 26 in.

The chosen truss system of Fig. 4.18 can be compared with the resulting crack patterns of reinforced concrete beams subjected to the same combination of shear force and torsional moment. Figure 4.19 shows types of crack patterns to be expected in a reinforced concrete member subjected to combinations of shear force and torsional moment similar to those applied to the box section of this design example.

4.3.3 Evaluation of the Diagonal Compression Stresses. Once the angle of inclination has been selected and the design zones defined, diagonal compression stresses should be checked before detailed dimensioning of reinforcement is carried out. This step should be taken early so that if there is a problem the web width or the assumed inclination angle α can be changed. It was recommended in Sec. 2.5 that this type of failure be eliminated by limiting the compression stresses f_d in the diagonal members of the truss to a value less than or equal to $30\sqrt{f'_c}$. Since in this design example f'_c = 4000 psi, then $f_d \leq 1.9$ ksi.

As shown in Chapter 3 of Report 248-2, the compression stress in the diagonal strut can be obtained from equilibrium of the truss model, and is given by the relationship

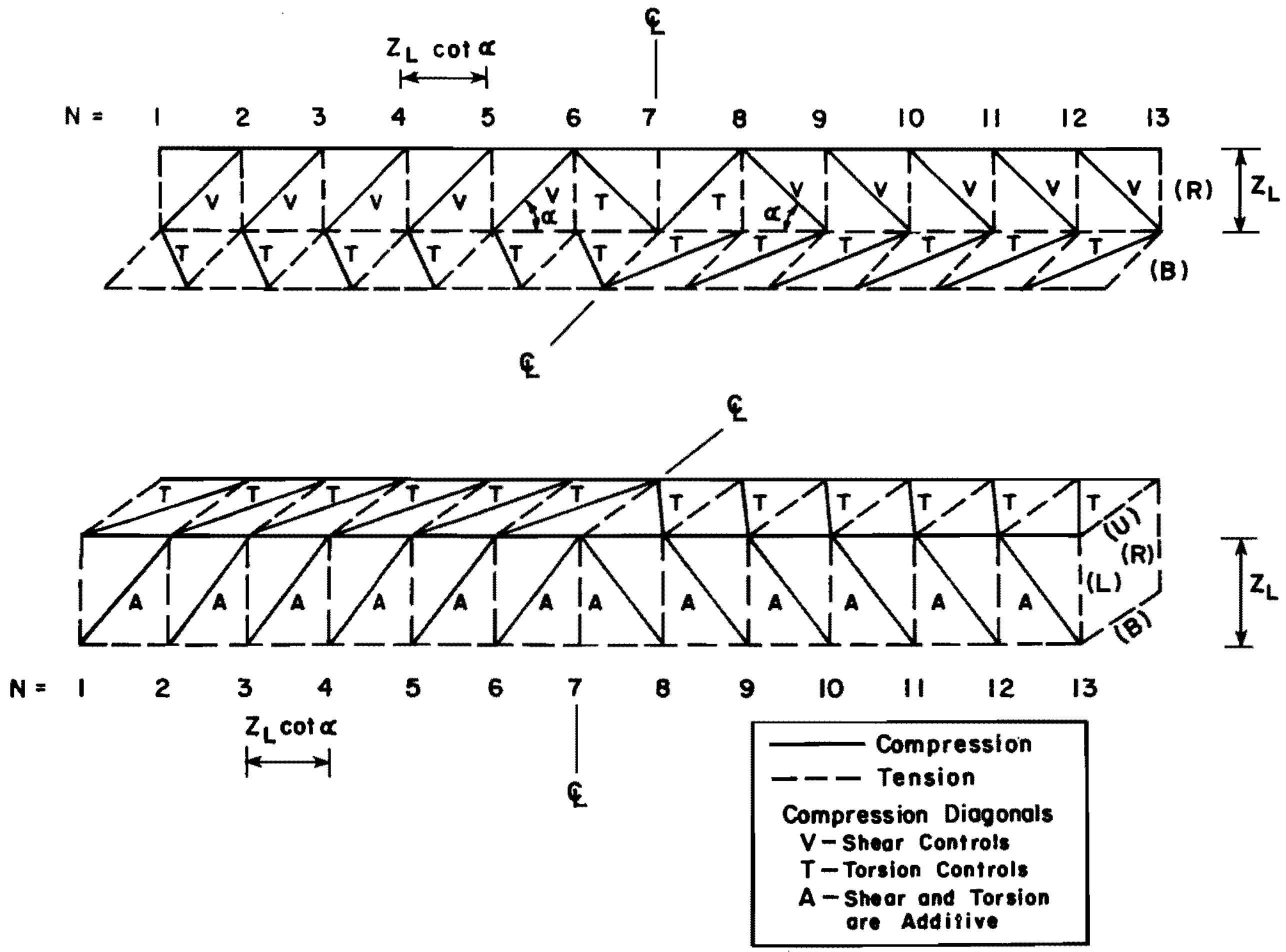


Fig. 4.18 Truss model

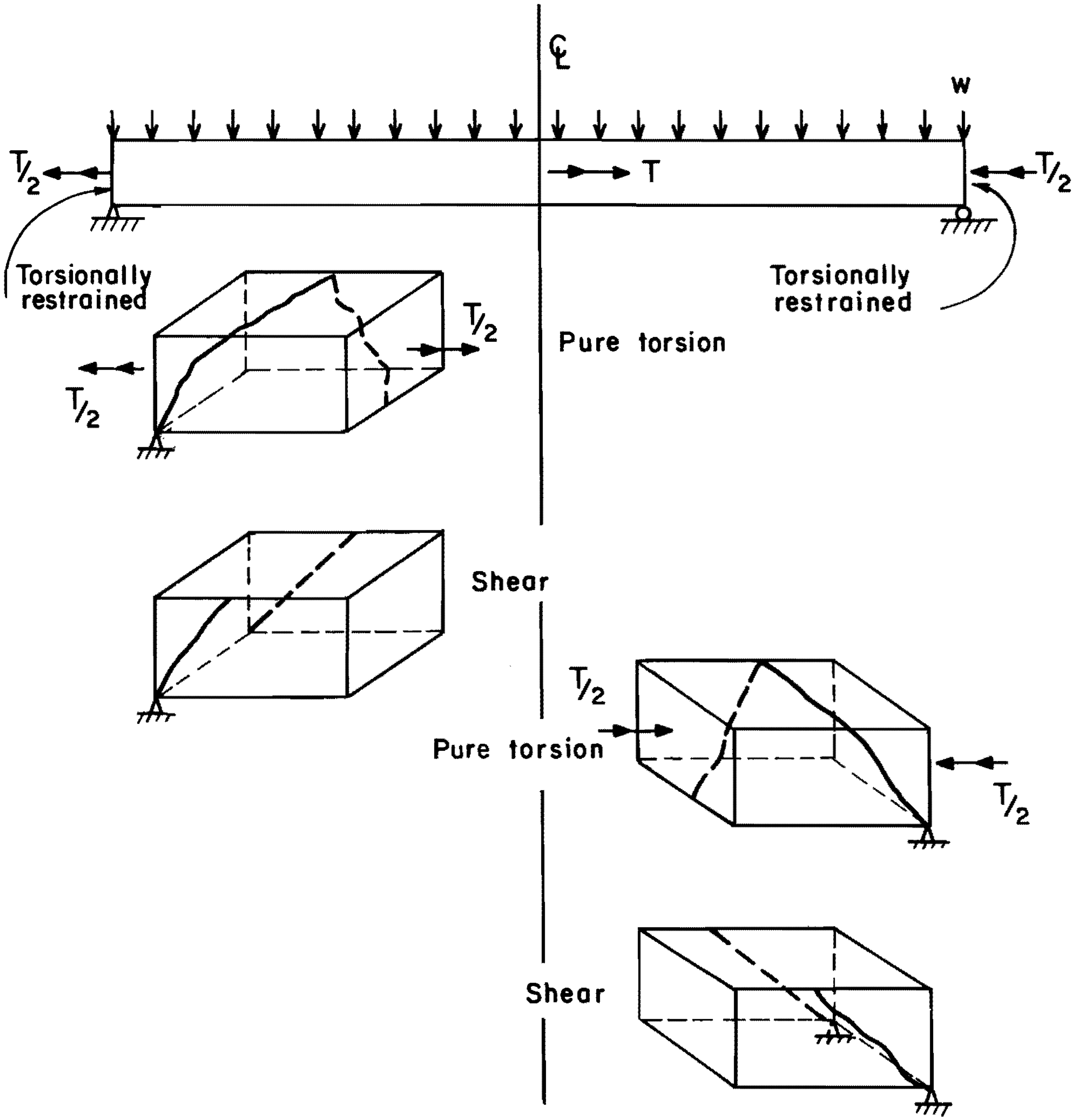


Fig. 4.19 Typical crack patterns for beams subjected to shear or torsion

$$f_d(V) = q(V)/b_w \sin\alpha \cos\alpha \tag{4.3}$$

For the case of a shear flow due to a shear force, $q(V)$, b_w is the effective web width resisting the applied shear force, and α is the angle of inclination of the diagonal truss member. For the case of torsion the diagonal stress in the compression strut is given by

$$f_d(T) = q(T)/b_e \sin\alpha \cos\alpha \tag{4.4}$$

$q(T)$ is the shear flow due to a torsional moment, and b_e is the effective web width resisting the applied torsional moment. It is given by the smaller of the two values $R_o/5$ or $R/6$. R_o is the diameter of the largest circle inscribed in the area A_o and R is the diameter of the largest circle inscribed in the cross section.

In this design example, for the case of vertical shear b_w for each plate is the actual width of the web (4 in.). In the case of torsion as shown in Fig. 4.20, b_e = 12.9/5 = 2.6 in.

In order to determine the total compression stress acting in the diagonal members due to the presence of shear and torsion it is suggested that both values be computed separately as given by Eqs. 4.3 and 4.4, and then superimposed

$$f_d(T,D) = f_d(T) + f_d(V) \tag{4.5}$$

where $f_d(T)$ is the diagonal compression stress due to torsion and $f_d(V)$ is the diagonal compression stress due to shear.

Hence, the total diagonal compression stress for this case is given as

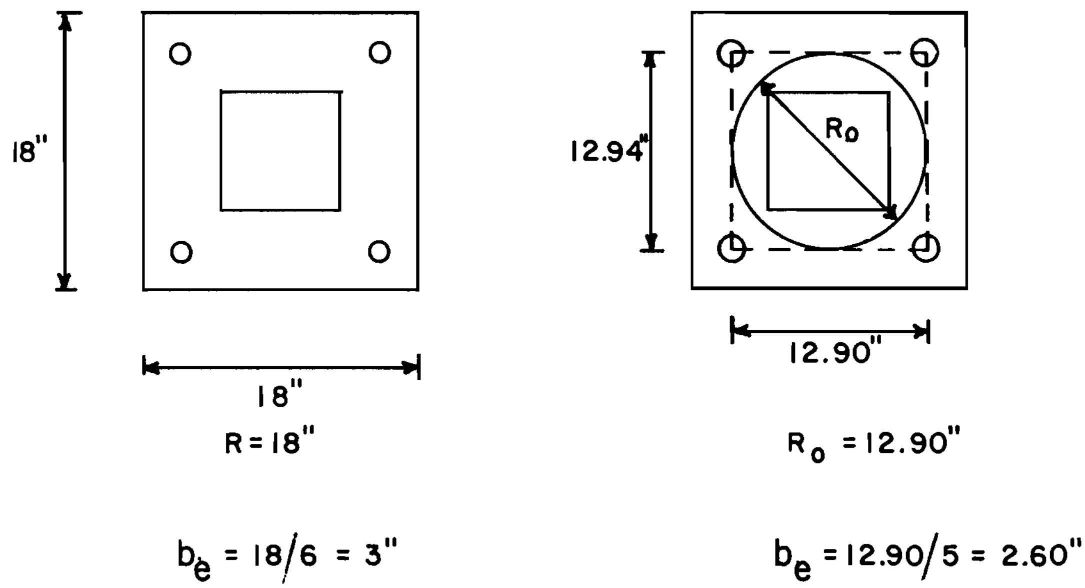


Fig. 4.20 Determination of the effective web width b_e resisting the applied ultimate torsional moment

$$f_d(T,V) = (1/\sin\alpha \cos\alpha)[q(T)/b_e + q(V)/b_w] \qquad (4.6)$$

where q(T) is the shear flow due to torsion evaluated using Eq. 4.1 and q(V) is the shear flow due to shear as given by Eq. 4.2. Values of the compression stress, $f_d(T,V)$, are tabulated in column (8) of Table 4.2.

The maximum value is 1.78 ksi, which is below the maximum value $30\sqrt{f'_c}$ = 1.9 ksi. The maximum value of 1.78 ksi is somewhat close to the maximum allowed value; if it exceeded the minimum a steeper α could be chosen which would then reduce the maximum value.

4.3.4 Design of Transverse Reinforcement. Once the truss model has been selected and the compression stresses in the diagonal members of the truss model have been evaluated to ensure that premature failure due to crushing of the concrete in the web is prevented, the internal forces for the chosen truss model can be evaluated and the design process becomes relatively simple and straightforward.

Shown in Figs. 4.21 and 4.22 is a typical design zone (panel 2-3) of the truss model shown in Fig. 4.18.

The vertical dimension of the truss model z_L is determined as the vertical distance between the centroids of the longitudinal chords of the truss model. In this case, as shown in Fig. 4.15, z_L is equal to 12.94 inches. Hence, the horizontal dimension of the typical truss panel $z_L \cot\alpha$ shown in Fig. 4.18 becomes 26 inches. Note that conveniently there are then six panels or design zones between the support and the centerline and twelve panels in the overall structure.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Jt (N)	Sec. from Left Support CL (ft)	Design Zone	V_n (k)	T_n (in-k)	q(V) (k/in)	q(T) (k/in)	f_d(T,V) (ksi)
1	0.0						
		1-2	53.5	176	2.1	0.5	1.78
2	2.17						
		2-3	44.6	176	1.7	0.5	1.53
3	4.33						
		3-4	35.7	176	1.4	0.5	1.35
4	6.50						
		4-5	26.8	176	1.0	0.5	1.10
5	8.67						
		5-6	17.8	176	0.7	0.5	0.91
6	10.83						
		6-7	8.9	176	0.3	0.5	0.66
7	13.0 (CL)						
		7-8	8.9	176	0.3	0.5	0.66
8	15.17						
		8-9	17.8	176	0.7	0.5	0.91
9	17.33						
		9-10	26.8	176	1.0	0.5	1.10
10	19.50						
		10-11	35.7	176	1.4	0.5	1.35
11	21.67						
		11-12	44.6	176	1.7	0.5	1.53
12	23.83						
		12-13	53.5	176	2.1	0.5	1.78
13	26.00						

Table 4.2 Evaluation of the compression stresses in the diagonal members of the truss

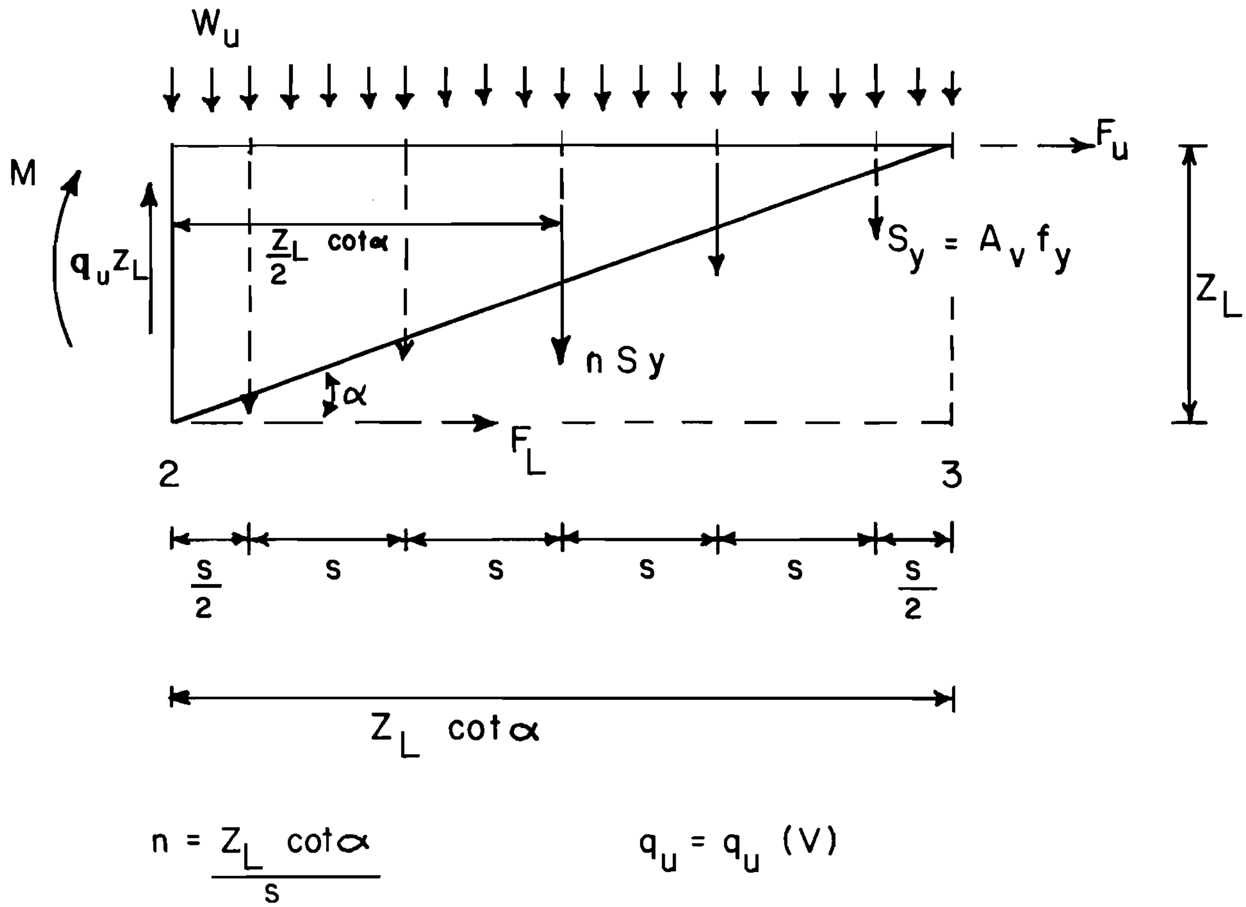


Fig. 4.21 Dimensioning of the stirrup reinforcement required to resist the applied shear force

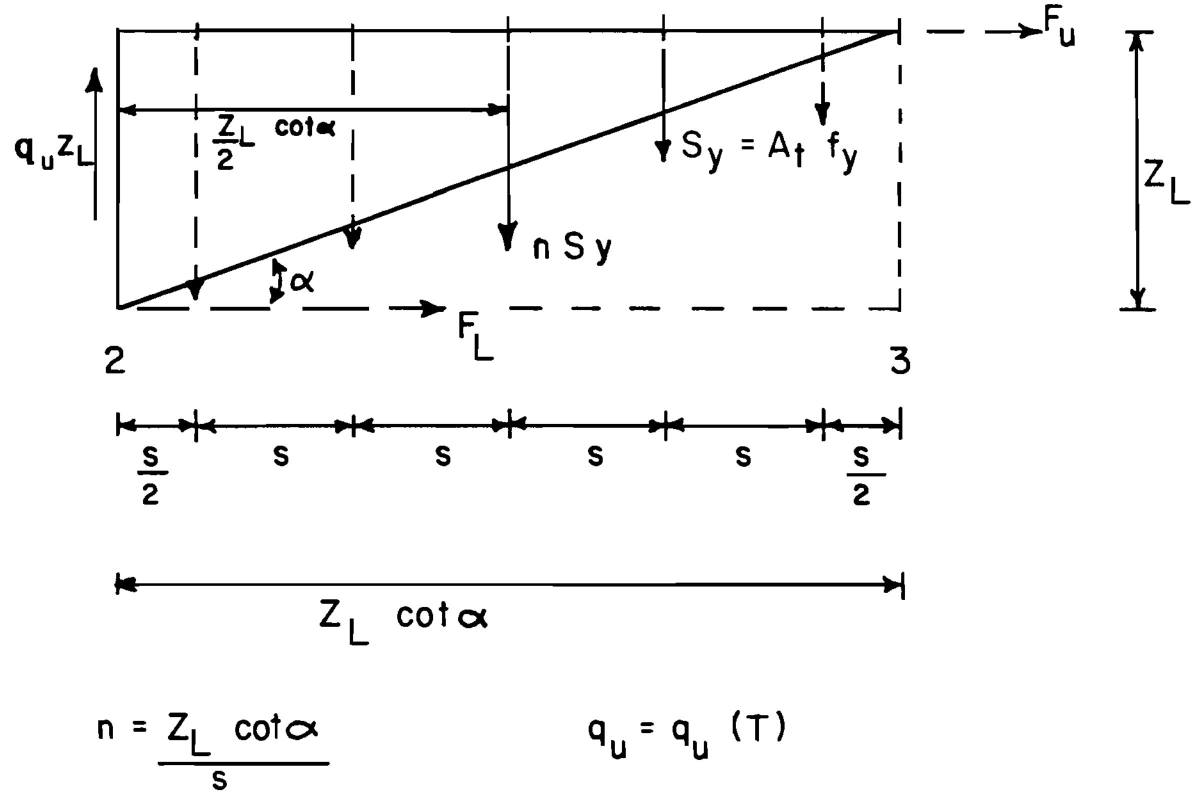


Fig. 4.22 Dimensioning of the stirrup reinforcement required to resist the applied torsional moment

Web reinforcement within a panel is assumed to be spaced uniformly and all at yield. This greatly simplifies detailing.

Since in this case torsion stresses exist, closed hoops formed of a single piece of reinforcement should be used. The area of web reinforcement computed for each design zone ($z_L \cot\alpha = 26$ in.) for the side wall, where $q_v = q_v(T) + q_v(V)$ is a maximum, will be provided in all four sides of the member. This is not only practical but is especially recommended where the direction of the applied torsional moment might change. Therefore, in the design of the transverse reinforcement for this design example the maximum of the four resultant values of the shear flow due to shear and torsion evaluated for each section will be used to determine the required amount of web reinforcement in the design zone starting at such section.

The suggested AASHTO revisions proposed that in members subjected to low shear stresses the concrete in the web may provide an additional contribution to the shear and torsional capacity of the member. This contribution may be easily reflected in the design procedure by using a reduced value of the shear force and the torsional moment when computing the required amounts of web reinforcement. However, this additional concrete contribution is only allowed where the member is in the uncracked or transition state. The proposed concrete contribution (see Sec. 2.3, Fig. 2.14) in the case of reinforced concrete members is assumed to disappear when the level of shearing stresses due to combined shear and torsion in the member exceeds $6\sqrt{f'_c}$. The total shearing stress due to shear and torsion $v_u(V,T)$ can be evaluated by computing separately the shearing stress due to shear $v_u(V) = V_u/2b_w z_L$ and the shearing stress due to torsion $v_u(T) = T_u/2A_o b_e$ and then superimposing the two effects. The values of the additional concrete contribution to the shear (V_c) and torsional (T_c) capacity evaluated in accordance with the provisions presented in Sec. 3.1 (1.3.6(c)) for the case of combined actions are given in columns (7) and (8) of Table 4.3.

In order to simplify the design procedure the amounts of web reinforcement required to resist the applied shear and torsion are computed separately and then superimposed.

First, the amount of web reinforcement required to resist the factored shear force is evaluated using the typical truss panel wall element shown in Fig. 4.21. The equilibrium condition $\Sigma F_V = 0$ yields the relation

$$q_u z_L - w_u z_L \cot\alpha = \phi n S_y \qquad (4.7)$$

For the case of shear $q_u = V_u/z_L$. Since there are two vertical webs (L), (R), resisting the applied vertical shear Eq. 4.7 becomes

$$1/2\ (V_u - w_u z_L \cot\alpha) = \phi n S_y \qquad (4.8)$$

The left-hand side of Eq. 4.8 represents the ultimate load actions, the right hand is the design strength (ϕV_{TR}) provided by the vertical members of the truss. Since $V_u \leq \phi V_n$ (Sec. 1.2.1.1 in Sec. 3.1), where $\phi = 0.85$, then

$$(V_u - w_u z_L \cot\alpha)\,/\,(2\phi) = V_{TR} = nS_y \qquad (4.9)$$

(1) Jt (N)	(2) Sec. from Left Support CL (ft)	(3) Design Zone	(4) $v_u(V)$ (ksi)	(5) $v_u(T)$ (ksi)	(6) $v_u(V,T)$ (ksi)	(7) V_c (kips)	(8) T_c (in--kip)
1	0.0						
		1-2	0.45	0.17	0.62	0	0
2	2.17						
		2-3	0.37	0.17	0.54	0	0
3	4.33						
		3-4	0.30	0.17	0.47	0	0
4	6.50						
		4-5	0.21	0.17	0.38	0	0
5	8.67						
		5-6	0.15	0.17	0.32	0.72	14
6	10.83						
		6-7	0.07	0.17	0.24	1.05	43
7	13.0 (CL)						
		7-8	0.07	0.17	0.24	1.05	43
8	15.17						
		8-9	0.15	0.17	0.32	0.72	14
9	17.33						
		9-10	0.21	0.17	0.38	0	0
10	19.50						
		10-11	0.30	0.17	0.47	0	0
11	21.67						
		11-12	0.37	0.17	0.54	0	0
12	23.83						
		12-13	0.45	0.17	0.62	0	0
13	26.00						

Table 4.3 Evaluation of the ultimate shearing stresses due to shear and torsion, and the concrete contributions V_c and T_c to the shear and torsional capacity of the member

For those regions of the member in the uncracked or transition state, where the concrete in the web provides additional shear strength, V_c, Eq. 4.9 becomes

$$(V_u - w_u z_L \cot\alpha)/(2\phi) = V_n = V_{TR} + V_c \qquad (4.10)$$

Rearranging Eq. 4.10 results in

$$(V_u - w_u z_L \cot\alpha)/(2\phi) - V_c = V_{TR} = nS_y \qquad (4.11)$$

Since $ns = z_L \cot\alpha$ and $S_y = A_v f_y$, then

$$A_v/s = [(V_u - w_u z_L \cot\alpha)/(2\phi) - V_c]\tan\alpha/z_L f_y \qquad (4.12)$$

where A_v/s is the area of stirrups resisting the factored shear force per inch of the stirrup spacing "s", f_y is the yield stress of the stirrup reinforcement, V_u represents the ultimate shear force in the section at the beginning of the design zone, and w_u is the ultimate (factored) distributed load. For this design example, f_y = 6000 psi, and α = 26.5 degrees; hence, $\tan\alpha$ = 0.5.

Using Eq. 4.12, the design of the web reinforcement required to resist the factored shear force is carried out.

Shown in column (5) of Table 4.4a are the amounts of web reinforcement per wall element for each of the design zones required to resist the applied factored shear force.

The amount of web reinforcement required to resist the applied factored torsional moment is evaluated using the typical truss panel

(1) Design Zone	(2) $\frac{(V_u - w_u z_L \cot\alpha)}{2\phi}$ (kips)	(3) V_c (kips)	(4) tan α	(5) A_v/s (required) (in^2/in)
1-2	22.3	0	0.5	0.014
2-3	17.8	0	0.5	0.011
3-4	13.4	0	0.5	0.009
4-5	8.9	0	0.5	0.005
5-6	4.4	0.72	0.5	0.002
6-7	0.0	1.05	0.5	0.000

a) Dimensioning of web reinforcement for shear

(1) Design Zone	(2) T_u/ϕ (k-in)	(3) T_c (k-in)	(4) tanα	(5) A_t/s (required) (in^2/in)
1-2	176	0	0.5	0.004
2-3	176	0	0.5	0.004
3-4	176	0	0.5	0.004
4-5	176	0	0.5	0.004
5-6	176	14	0.5	0.004
6-7	176	43	0.5	0.003

b) Dimensioning of web reinforcement for torsion

(1) Design Zone	(2) = (5) + (5) $A_v/s + A_t/s$ (in^2/in)	(3) Min. amt. of web reinforcement (in^2/in)	(4) s for #3 St. log. (in)	(5) s for #4 St. Log (in)	(6) s_{max} (in)
1-2	0.018	0.004	6.1	11.1	3.25
2-3	0.015	0.004	7.3	13.3	3.25
3-4	0.013	0.004	8.5	15.4	3.25
4-5	0.010	0.004	11.0	20.0	6.5
5-6	0.006	0.004	18.3	33.3	6.5
6-7	0.003	0.004	36.7	66.7	6.5

c) Resultant amounts of web reinforcement

Table 4.4 Dimensioning of web reinforcement wall element shown in Fig. 4.22. The equilibrium condition $\Sigma F_y = 0$ yields

$$q_u z_L = \phi n S_y \qquad (4.13)$$

For the case of torsion $q_u = T_u/2A_o$. The lefthand side of Eq. 4.13 represents the ultimate action produced by the factored torsional moment. The righthand side is the nominal strength provided by the truss system. Since $T_u \leq \phi T_n$ (Sec. 1.2.1.2 in Sec. 3.1) where ϕ= 0.85, then

$$T_u/\phi = T_n = (n/z_L)S_y 2A_o \qquad (4.14)$$

For those regions of the member in the uncracked or transition state, where the concrete in the web provides additional torsional strength (T_c), Eq. 4.14 becomes

$$T_u/\phi = T_n = (n/z_L)S_y 2A_o + T_c \qquad (4.15)$$

Since $ns = z_L \cot\alpha$ and $S_y = A_t f_y$, then Eq. 4.15 yields the following relationship

$$A_t/s = [T_u/\phi - T_c] \tan\phi / f_y 2A_o \qquad (4.16)$$

where A_t/s is the area of vertical stirrups resisting the applied torsional moment per inch of the stirrup spacing "s", f_y is the yield strength of the web reinforcement (60,000 psi), T_u represents the factored torsional moment in the section at the beginning of the design zone $z_L \cot\alpha$, $\tan\alpha$ is equal to 0.5.

With Eq. 4.16 the design of the web reinforcement required to resist the factored torsional moment is carried out for each of the truss panels (design zones). Shown in column (5) of Table 4.4b are the amounts of web reinforcement, A_t/s, required to resist the factored torsional moment.

Column (2) of Table 4.4c shows the total amount of web reinforcement per inch of stirrup spacing required for each wall of the member at each of the design zones $z_L \cot\alpha$.

Shown in column (3) of Table 4.4c is the minimum amount of web reinforcement which must be provided whenever the combined shearing stress due to shear and torsion in the wall exceeds the value of $1.0\sqrt{f'_c}$, where ϕ = 0.85. The minimum amount is evaluated in accordance with the requirements suggested in Sec. 1.4 of the proposed design recommendations presented in Sec. 3.1.

Hence

$$(A_t/b_e s + A_v/b_w s)_{min} = (1.0/f_y)\sqrt{f'_c} \qquad (4.17)$$

Since b_w/b_e = 4/2.60 = 1.5, then $b_e = b_w/1.5$, therefore

$$(1.5\ A_t/s + A_v/s)_{min} = 1.0\sqrt{f'_c}\ b_w/f_y \qquad (4.18)$$

Columns (4) and (5) of Table 4.4c show the respective spacings for a #3 and a #4 closed hoop. Column (6) contains the maximum allowable stirrup spacing evaluated in accordance with Sec. 1.4.3 of the proposed design recommendations of Sec. 3.1. For the design zones 1-2, 2-3 and 3-4, $z_L/4$ controls. The requirement of $z_L/2$ = 6.5 in. controls the maximum spacing of web reinforcement in zones 4-5, 5-6, 6-7.

At this point it would be possible to revise the assumed angle of inclination α and possibly choose a steeper value (say α approximately equal 45 degrees) since in this case the advantage of having larger spacings for the web reinforcement by using values of α close to the lower limit is eliminated by the maximum stirrup spacing requirement. At the same time, the selection of a steeper value of the angle would reduce the requirements for longitudinal reinforcement due to shear and torsion.

A change to a steeper value of α (45 degrees) reduces the compression stresses in the diagonal members of the truss model since

$$f_d = q/b \sin\alpha \cos\alpha \qquad (4.19)$$

thus for a value of α equal to 45 degrees Eq. 4.19 yields

$$f_d = 2q/b \qquad (4.20)$$

whereas with α of 26.5 degrees, Eq. 4.19 resulted in

$$f_d = 2.5q/b \qquad (4.21)$$

Therefore, there would be no need to recheck web crushing stresses.

Assuming a value of α approximately equal to 45 degrees, the design zone $z_L \cot\alpha$ (z_L equal to 12.94 in.) becomes equal to 13 in. Now there would be twelve design zones between the centerline of the support and the midspan of the member with a total of twenty-four zones for the overall member.

The dimensioning of the web reinforcement is conducted with the same procedure previously followed. Table 4.5 shows the concrete contribution to the shear strength of the member V_c and to the torsional strength T_c together with the values of the ultimate shearing stresses due to shear $v_u(V)$ and torsion $v_u(T)$ for each of the design zones between the centerline of the support and the midspan of the member. The amount of web reinforcement for the other half of the member is the same since there is symmetry about the midspan of the section.

Table 4.6 shows the revised dimensioning of the web reinforcement for shear, Table 4.7 shows the amount of web reinforcement now required for torsion, and Table 4.8 shows the final superposition of both amounts of reinforcement. Column (2) in Table 4.8 shows the superposition of the required web reinforcement amounts from column (5) in Table 4.6 and column (5) in Table 4.7. Column (3) shows the minimum amount of web reinforcement required. Since there is a constant torsional moment of 150 in.-kip the shearing stress at any section of the member would be at least $150/0.85*2*A_o*2.6 = 0.198$ ksi which is in excess of $1.0\sqrt{f'_c} = 0.053$ ksi. Thus, minimum web reinforcement would be required at any section of the member.

Column (4) in Table 4.8 shows the value of the web reinforcement spacing for a #3 closed hoop. It is obvious that the selection of a steeper angle α produced a closer stirrup spacing. From comparison of columns (4) and (5) the actual web reinforcement in the member is

(1) Jt (N)	(2) Sec. from Left Support CL (ft)	(3) Design Zone	(4) $v_u(V)$ (ksi)	(5) $v_u(T)$ (ksi)	(6) $v_u(V,T)$ (ksi)	(7) V_c (kips)	(8) T_c (in-kip)
1	0.0						
		1-2	0.45	0.17	0.62	0.0	0.0
2	1.08						
		2-3	0.40	0.17	0.57	0.0	0.0
3	2.17						
		3-4	0.37	0.17	0.54	0.0	0.0
4	3.25						
		4-5	0.33	0.17	0.50	0.0	0.0
5	4.33						
		5-6	0.29	0.17	0.46	0.0	0.0
6	5.42						
		6-7	0.26	0.17	0.43	0.0	0.0
7	6.5						
		7-8	0.22	0.17	0.39	0.0	0.0
8	7.58						
		8-9	0.18	0.17	0.35	0.40	6.0
9	8.67						
		9-10	0.15	0.17	0.32	0.73	14.0
10	9.75						
		10-11	0.11	0.17	0.28	1.01	26.0
11	10.83						
		11-12	0.07	0.17	0.24	1.05	43.0
12	11.92						
		12-13	0.04	0.17	0.21	0.84	60.0
13	13.0 (midspan)						

Table 4.5 Evaluation of the ultimate shearing stress due to shear and torsion, and the additional concrete contribution, V_c and T_c

(1) Design Zone	(2) $\frac{(V_u - w_u z_L \cot\ )}{2\phi}$ (kips)	(3) V_c (kips)	(4) $\tan\alpha$	(5) A_v/s (required) (in^2/in)
1–2	24.0	0	1.0	0.030
2–3	22.3	0		0.029
3–4	20.1	0		0.026
4–5	17.9	0		0.023
5–6	15.6	0		0.020
6–7	13.4	0		0.017
7–8	11.2	0		0.014
8–9	8.9	0.40		0.011
9–10	6.7	0.73		0.008
10–11	4.5	1.01		0.004
11–12	2.2	1.05		0.001
12–13	0.0	0.84		0.000

Table 4.6 Dimensioning of web reinforcement for shear

(1) Design Zone	(2) T_u/ϕ (k-in)	(3) T_c (k-in)	(4) $\tan\alpha$	(5) A_t/s (required) (in^2/in)
1-2	176	0.0	1.0	0.009
2-3		0.0		0.009
3-4		0.0		0.009
4-5		0.0		0.009
5-6		0.0		0.009
6-7		0.0		0.009
7-8		0.0		0.009
8-9		6.0		0.008
9-10		14.0		0.008
10-11		26.0		0.007
11-12		43.0		0.006
12-13		60.0		0.006

Table 4.7 Dimensioning of web reinforcement for torsion

(1) Design Zone	(2) $A_v/s + A_t/s$ (in^2/in)	(3) Minimum amount (in^2/in)	(4) s for #3 (in)	(5) s_{max} (in)	(6) $s_{provided}$ (in)
1-2	0.039	0.004	2.8	3.25	2.75
2-3	0.038		2.9	3.25	2.75
3-4	0.035		3.1	3.25	3.00
4-5	0.032		3.4	3.25	3.25
5-6	0.029		3.8	3.25	3.25
6-7	0.026		4.2	6.5	4.0
7-8	0.023		4.8	6.5	4.75
8-9	0.020		5.5	6.5	5.5
9-10	0.017		6.5	6.5	6.5
10-11	0.013		8.5	6.5	6.5
11-12	0.010		10.0	6.5	6.5
12-13	0.009		12.25	6.5	6.5

Table 4.8 Dimensioning of web reinforcement for combined shear and torsion

selected. Column (6) shows the longitudinal spacing chosen for the #3 closed hoops in the member. The closed #3 hoop is to be made out of a single piece. The free ends must be bent into the concrete contained within the stirrups with at least a 105 degree bend.

4.3.5 Evaluation of the Compression Stresses in the Fan Regions. As was explained in Sec. 2.2.2 of Report 248-3, in the truss model approach it is assumed that the compression diagonals of the truss form a continuous uniform compression field with a constant angle of inclination throughout the span of the member. However, the development of such a regular truss action in beams is disturbed by the introduction of concentrated loads. The presence of a concentrated load introduces a series of diagonal compressive forces which "fan out" from the concentrated load. Hence, for the design example of this section compression fans will form at both supports where the reaction introduces compression. For simplicity here, the reaction will be assumed as a point support. In actuality a bearing pad would have to be designed and then the lower part of the fan would be checked once the strut action was detailed (42). As previously explained in Sec. 2.2.2 of Report 248-3, the geometry of the compression fan depends on the spacing of the transverse reinforcement and the chosen angle α. Figure 4.23 shows the compression "fan" generated at the supports of the box section. Column (5) of Table 4.9 shows the compression forces generated at each of the joints of the truss in the compression fan zone. Column (6) shows the diagonal compression stresses induced by the diagonal compression forces shown in column (5). As previously illustrated in

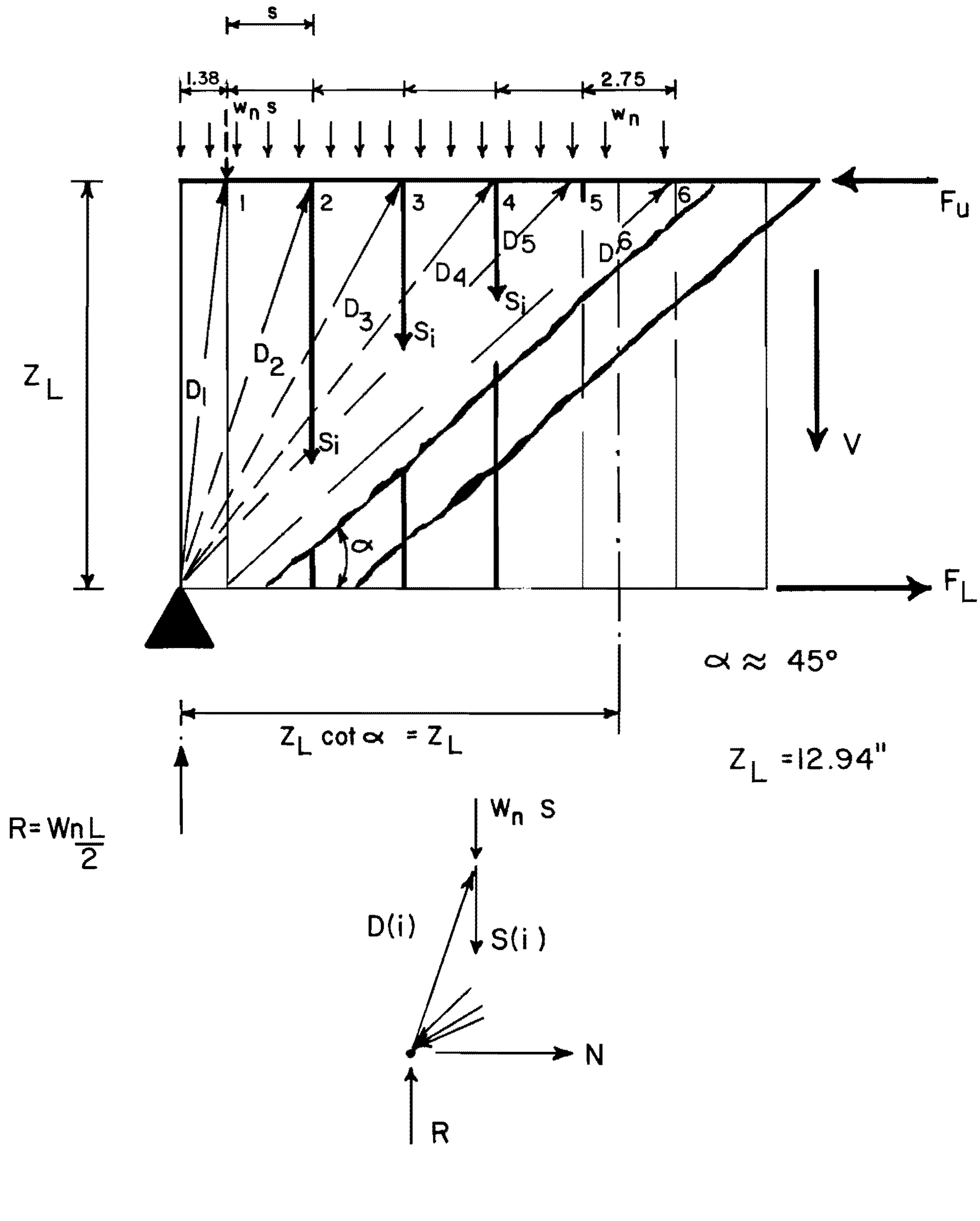


Fig. 4.23 Compression fan at support

Sec. 2.3 of Report 248-3, the diagonal compression stress at each of the joints (i) of the truss is given as:

$$f_{di} = D_i / b_w z_L \cos\alpha_i \qquad (4.22)$$

In this case b_w = 4" and z_L = 12.94". For this design example, #3 closed hoops Grade 60 are used as web reinforcement, thus $S_y = A_v f_y$ = (0.11)(60) = 6.6 kips.

The compression stress evaluated using Eq. 4.22 must be then compared with the maximum allowable compression stress in the diagonal member of truss of $30\sqrt{f'_c}$ given on column (7). As can be seen from the comparison of columns (6) and (7) of Table 4.9, the compression stresses in the fan region are within the allowed limit of $30\sqrt{f'_c}$.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Point (i)	α (i) (degrees)	tan α (i)	$S_{(i)}$	$D_{ci} = \frac{[S_{(i)} + w_n s]}{\sin\alpha(i)}$	$f_d(i)$ (psi)	$30\sqrt{f'_c}$ (psi)
1	83.9	9.4	S_y	1.16 S_y	1400	1897
2	72.32	3.14	S_y	1.21 S_y	508	1897
3	62.01	1.88	S_y	1.31 S_y	356	1897
4	53.36	1.34	S_y	1.44 S_y	308	1897
5	46.28	1.05	S_y	1.60 S_y	295	1897

Table 4.9 Diagonal compression stresses in the fan region

4.3.6 Dimensioning of the Longitudinal Reinforcement Required for Shear and Torsion. As previously explained in Sec. 2.4.4, the presence of the diagonal compression field induced by the applied shear force and torsional moment requires that an area of longitudinal steel in addition to the area required for flexure be provided. Figure 4.24 illustrates how to evaluate this additional area of longitudinal steel. Since a uniform compression field and a constant stirrup spacing are assumed throughout the design region $z_L \cot\alpha$, the horizontal component of the compression diagonals can be taken as concentrated at the midheight of each compression field element ($z_L/2$). Thus, it is reasonable to assume that the horizontal forces due to the diagonal compression fields are equally resisted by the two corner chords of each wall element.

The additional longitudinal reinforcement required for each side web is then determined from the horizontal components of the shear flows due to shear and torsion, $N_T = \Sigma N_i = \Sigma q_i z_i \cot\alpha_i$ (see Fig. 4.24). The value will be computed for the case where the plates have the highest combined shear flow at joint 2, which is identical to joint 1.

Using the equilibrium condition $\Sigma F_H = 0$ in the truss model yields for the side wall (L) the relation

$$A_{L2}(T_n, V_n) = [q_L z_L \cot\alpha + q_B z_B \cot\alpha]/2f_y \qquad (4.23)$$

Substituting the values of the shear flows q_L and q_B (see Fig. 4.16) gives the amount of additional longitudinal steel required due to shear and torsion in the truss chord (2) A_{L2}

$$A_{L2} = [(T_n * u)/(4A_o) + V_n/2]\cot\alpha/2f_y \qquad (4.24)$$

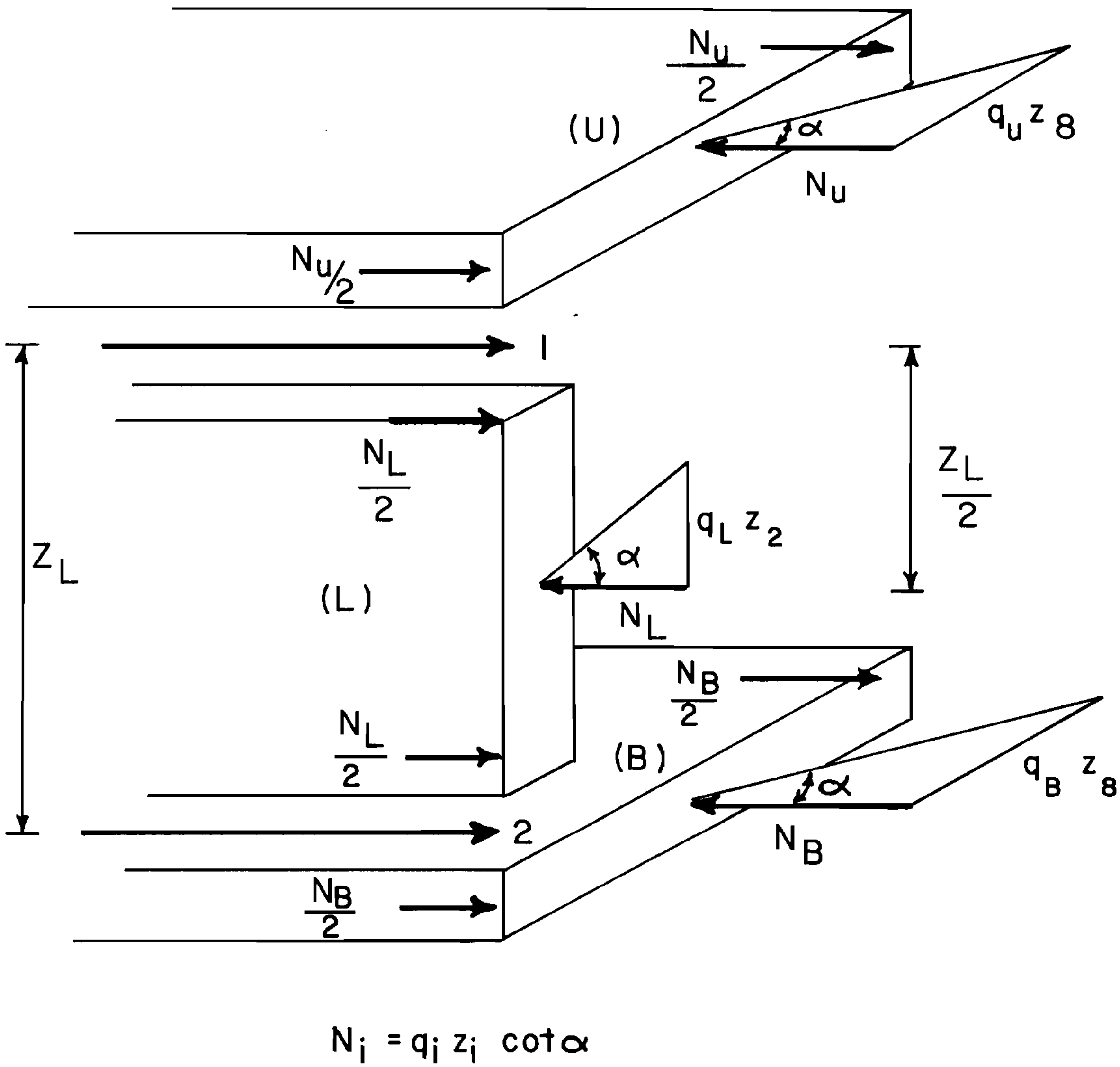


Fig. 4.24 Determination of the additional longitudinal reinforcement due to shear and torsion

where "T_n" is the nominal torsional moment T_u/ϕ with $\phi = 0.85$, "u" is the perimeter connecting the centroids of the longitudinal chords of the truss model (in this case $u = 2z_L + 2z_B$), A_o is the area enclosed by the centroids of the longitudinal chords of the space truss model resisting the applied ultimate torsional moment and shear force, V_n is the nominal shear force V_u/ϕ, $\phi = 0.85$, and f_y is the yield stress of the longitudinal reinforcement resisting the horizontal component of the diagonal compression field produced by the shear and torsional shearing flows.

In this design example

- $u = 2z_L + 2z_B = 51.64$ in.
- $A_o = z_L * z_B = 166.7$ in.^{2}
- $\cot\alpha = 1.0$
- $f_y = 60{,}000$ psi

Similar procedures could be followed to compute the areas of longitudinal steel required due to shear and torsion in the longitudinal chords 1, 3, and 4. However, it is recommended to simply take the area required in the other truss chords as equal to the area computed using Eq. 4.24. Since Eq. 4.24 represents the highest possible combination of shear and torsion, this practice would be a simple and conservative assumption. Furthermore, the applied bending moment will produce tension at the lower chords 2 and 3. This tension force due to flexure combines with the tension force due to shear and torsion to make the situation in chord 2 the most critical one for design.

Therefore, the design of the longitudinal reinforcement required for shear and torsion will be conducted for each design zone $z_L\cot\alpha$ for the tension chord of the truss where the effects of the shear torsion and flexure are additive. The additional area required in the other truss chords will simply be taken equal to the additional area required in the truss chord where the effects of shear, torsion and bending are additive. Shown in column (4) of Table 4.10 are the amounts of additional longitudinal reinforcement evaluated at each design zone, $z_L\cot\alpha$ = 13 in. = 1.08 ft., using Eq. 4.24. The areas of steel required for flexure in the corner where the applied bending moment produces tension are shown in column (5). The values shown in columns (4) and (5) are used to evaluate whether the amount of longitudinal reinforcement provided at the corners of the cross section satisfies the requirements of combined torsion, shear and bending.

The area required for flexure for each of the design zones of the truss, shown in column (5) of Table 4.10, is evaluated using the relationship

$$A_{L\ total}(M) = M_n/z_L f_y \qquad (4.25)$$

where M_n is the nominal moment M_n/ϕ at the section where the design zone starts, z_L is the vertical dimension of the truss model (12.94 in.), and f_y is the yield strength of the longitudinal reinforcement. Eq. 4.25 was previously derived in Sec. 3.5.1 of Report 248-2 and represents the flexural capacity of the truss model.

(1) Design Zones	(2) V_n (kips)	(3) T_n (in-kip)	(4) A_L (T,V) for each corner (Eq. 4.24) (in^2)	(5) A_L (M) for entire tension chord (in^2)
1-2	53.5	176	0.34	0.00
2-3	49.0		0.32	0.81
3-4	44.6		0.30	1.55
4-5	40.1		0.28	2.22
5-6	35.7		0.26	2.82
6-7	31.2		0.24	3.35
7-8	26.7		0.22	3.81
8-9	22.3		0.21	4.20
9-10	17.8		0.19	4.51
10-11	13.4		0.17	4.76
11-12	8.9		0.15	4.94
12-13	4.4		0.13	5.00

Table 4.10 Dimensioning of the longitudinal reinforcement

Once the required amounts of longitudinal reinforcement for shear, torsion and bending are known, the detailing of this reinforcement can be conducted.

4.3.7 Detailing of the Longitudinal Reinforcement. The area of longitudinal reinforcement required for shear and torsion was evaluated assuming a space truss model with four longitudinal chords, one in each corner of the box section. The areas of steel shown in column (4) of Table 4.10 have to be provided at each corner of the box section in the respective design zone and are in addition to flexural reinforcement requirements.

However, the amounts of longitudinal reinforcement shown in column (5) of Table 4.10 do not have to be concentrated at the corners of the box section. They can be distributed throughout the entire face of the member where the applied bending moment induces tension. In the midspan region of the beam the total tension steel requirement thus becomes 5.00 + 0.13 + 0.13 = 5.26 si.

From the preliminary flexure design of the member in Sec. 4.3.1, it was found that five #9 longitudinal bars had to be provided at the midspan region on the tension side (bottom) of the member. Therefore, if the area of longitudinal reinforcement required for flexure shown in column (5) of Table 4.10 is assumed to be equally distributed between the five longitudinal bars, the area of longitudinal steel required for combined shear, torsion and bending in the corner chords of the truss located in the face of the member (Bottom [B]) where the applied bending moment causes tension, becomes greater than that provided by the #9 bar which would be located at the corner of the truss model in the design zones 8-9, 9-10, 10-11, 11-12, and 12-13.

Therefore, the area of longitudinal reinforcement provided will be increased from 5.0 in.^{2} (5-#9) to 5.58 in.^{2} (4-#9 and 2-#8). The resulting cross section at midspan is shown in Fig. 4.25.

The longitudinal reinforcement of the cross section shown in Fig. 4.25 can now be detailed in the longitudinal direction satisfying both flexural, and shear and torsion requirement.

Starting in the design zone 12-13 in Table 4.10, if 4#9 and 2#8 are provided, the equivalent number of bars (n) in the tension face is computed as the total area of reinforcement provided divided by the area of the largest bar used. Thus, n = 5.58/1 = 5.58 and the area required for flexure in the corner truss bar is 5.00/5.58 = 0.89 in.^{2} Thus, the total area required is 0.89 + 0.13 = 1.0 in.^{2} which is equal to the area of the #9 bar (1.0 in.^{2}) provided at each of the two corners on the tension face of the member (bottom wall [B]).

A similar procedure is followed for zone 11-12. Hence, 4.94/5.58 = 0.88 in.^{2} and the total area required for the corner truss bar is 0.88 + 0.15 = 1.0 in.^{2} which is equal to the area provided by the 1#9 bar at each corner of the section. If a similar procedure is followed for zones 10-11, 9-10, 8-9, 7-8, and 6-7, the required areas are found to be 1.0 in.^{2}, 0.99 in.^{2}, 1.0 in.^{2}, 0.90 in.^{2}, and 0.8 in.^{2}, respectively, these values are less than the area provided (1.0 in.^{2} by the #9 bar located at the corner of the section.

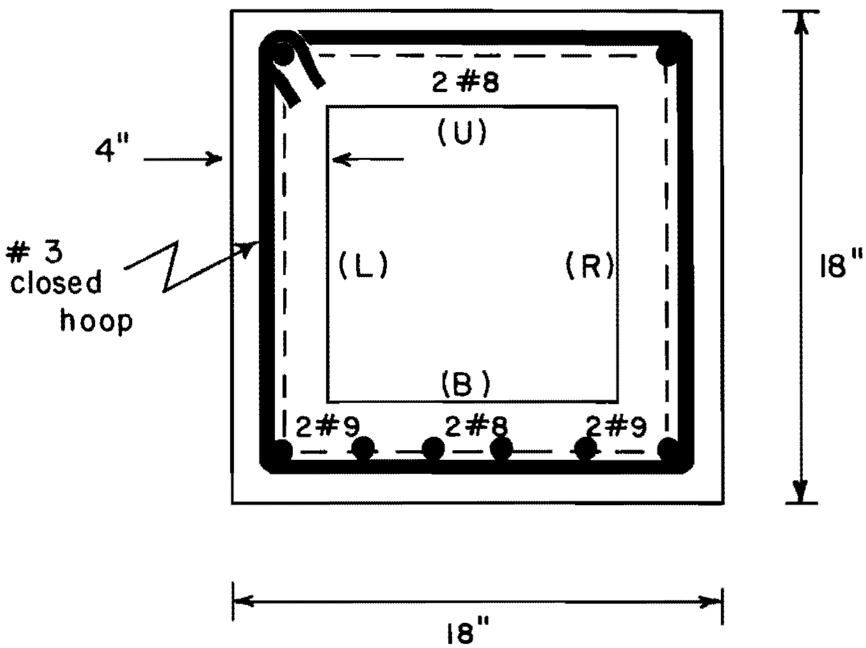


Fig. 4.25 Section for combined torsion, bending and shear at midspan

Assuming that both #8 bars are terminated so that only 4 #9 bars at the bottom wall (B) of the member are effective in the design zone 5-6, the area of longitudinal steel due to flexure required for the corner truss bar is 2.82/4 + 0.26 = 0.97 in.^{2}, which is once again less than the area of the #9 bar provided at the corners of the section.

The four #9 bars are then continued into the support and as a consequence the longitudinal reinforcement requirements of the design zones 4-5, 3-4, 2-3, and 1-2 would be satisfied. Figure 4.26 shows the final detailing of the longitudinal steel in the member. The flexure requirements for the two #8 tension bars are examined next. The area required for flexure at midspan in the tension face of the member is 5 in.^{2} Neglecting the excess area of longitudinal steel the distance at which the two #8 bars could be terminated is $X = [(5-4)(156/5)^2/5]^{1/2}$ = 70 in. Since the bar is going to be terminated without bending it into the compression zone then the total distance from the centerline of the span at which the two #8 bars could be terminated is 70 + 12 d_b = 70 + 12(1) = 82 in., where d_b is the bar diameter or 70 + d = 70 + 15.44 = 85.4", whichever is greater. Thus the two #8 bars could be terminated at 86 inches from the midspan. Since the two bars are going to be continued up to the section 6, then the distance from the midspan at which those two #8 bars are going to be terminated is 156 - 65 = 91 inches. Therefore, this satisfies the flexural requirements.

Due to the presence of a vertical shear force the longitudinal reinforcement which is terminated in the flexural tension face of the member (Bottom face [B]) must be provided with an additional embedment

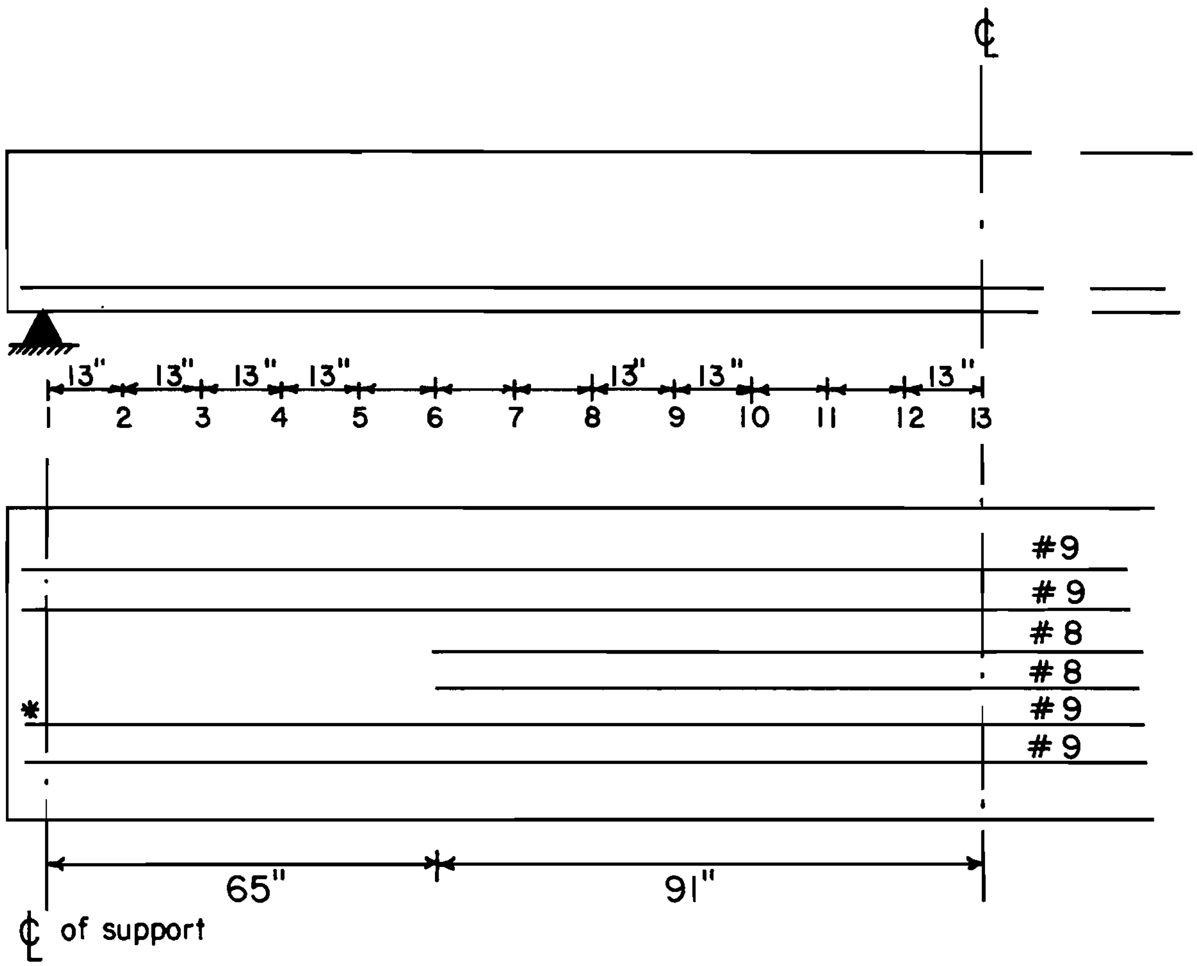


Fig. 4.26 Detailing of the longitudinal reinforcement

length l_s beyond the theoretical cut-off point. The additional embedment length l_s for the case of members subjected to distributed loading is

$$l_s = l_d - A_l f_y/[(V_u/2) + w_u \cot\alpha/2] \qquad (4.26)$$

where A_l is the total area of longitudinal reinforcement to be terminated, V_u is the factored ultimate shear force at the section, w_u is the factored distributed load, and l_d is the anchorage length required to develop yielding of the bar. The basic development length of a #8 bar evaluated in accordance with the ACI Building Code (2) is 24 in. The area of steel to be terminated is that corresponding to two number 8 bars or 1.58 $in.^2$ V_u at the section where the bars are no longer required for flexure (70 inches from the midspan section) is 33 kips, $\cot\alpha$ equals 1.0 and z_L is 12.94 in. Therefore, l_s = 2.0 - [(1.58)(60)/{(33/1.08) + (3.5/2)}] = - 0.9'. The negative value indicates that the magnitude of the shear force is such that for the amount of longitudinal steel to be terminated at that particular zone, no additional embedment length would be required past the theoretical cut-off point for flexure located at 70 inches from the midspan section. Since the 2 #8 bars would be continued up to 91 inches from the midspan section, all requirements would be satisfied.

Finally, the longitudinal tension reinforcement continued into the support (4 #9) because of the presence of compression fans at the support regions has to be provided with an anchorage length such that a force $V_u/2\cot\alpha$ is adequately developed. In this case $V_u/2\cot\alpha$ is equal to 45.5/2 = 22.8 kips.

The truss model resisting the applied shear and torsion has two vertical walls (L) and (R). Hence, each one takes 1/2 of the applied shear force. Thus, the force that needs to be anchored in the truss chord located at the corners of the wall where the applied bending moment induces tension is 1/2(22.75) = 11.4 kips. Although 4 #9 bars are coming into the support region, only one of them will actually be located at each of the bottom corners of the truss model. Hence, the force of 11.4 kips has to be totally taken by the 1 #9 bar at the corner of the section.

From column (4) of Table 4.10, due to the presence of shear and torsion, an area of longitudinal steel of 0.34 in.^{2} working at its full yield strength has to be developed at each bottom corner of the truss model. Thus, the force that has to be developed in the corner bottom #9 bar of the section at the support region is (0.34)(60) + (11.4) or approximately equal to 32 kips. The #9 corner bars have to be provided with an embedment length such that a force of 32 kips is adequately developed.

Since all the longitudinal bars anchored into the support region will be provided with a 6 in. straight embedment length past the support centerline, it is then necessary to check if this 6 in. straight embedment length is enough to adequately develop the 32 kip force, or if a standard hook is necessary for the two #9 bars located at each of the bottom corners of the member.

The embedment length requirements in the current ACI Building Code (2) for the reinforcement are based on the hypothesis that the tension force T that can be developed in a bar is a function of the perimeter bond stresses u_o, the perimeter of the bar to be developed Σ_o, and the embedment length of the bar such that

$$T = u_o \Sigma_o L \tag{4.27}$$

where the ultimate perimeter bond stress u_o is a function of the concrete strength f'_c and the bar diameter d_b

$$u_o = 9.5\sqrt{f'_c}/d_b \leq 800 \text{ psi} \tag{4.28}$$

For this design example u_o is $9.5\sqrt{4000}/1.125 = 534$ psi, since T = 32 kips and Σ_0 for a #9 bar is 3.53 in. The required straight embedment length in order to develop a tension force of 32 kips is

$$L = T/u_o\Sigma_0 = 32/(0.534)(3.53) = 17'' \tag{4.29}$$

Thus, the straight embedment length of 6" is not enough. Therefore, it is necessary to provide the #9 corner bars at the bottom of the reinforced concrete box beam with a standard 90 degree hook at the ends. If a standard 90 degree hook in accordance with the requirements given in the ACI Building Code (2) is provided, then the stress that can be developed by the hook f_h is

$$f_h = k\sqrt{f'_c} \tag{4.30}$$

where k = 540 for #9 bar. Thus, $f_h = 540*\sqrt{4000}/1000 = 34$ ksi. The required stress is 32 kips divided by the area of a #9 bar (1 in.$_2$) or 32 ksi. Hence, if a 90 degree standard hook is provided, the 32 kip force would be adequately developed.

For the case of the 2 #8 compression bars only the 20 kip force due to the presence of shear and torsion would have to be developed. However, as illustrated in Sec. 2.2.2 of Report 248-2, due to the presence of the compression fan in the support region no longitudinal tension reinforcement is required due to the effects of shear in the top compression face of the member within a distance $[(z_L\cot\alpha)/2]$ from the centerline of the support. Thus, at the support region only that area of longitudinal reinforcement required for torsion would have to be developed to its full yield strength. From Eq. 4.24 taking $V_n = 0$, the resultant area required only for torsion is

$$A_l(T) = [T_n u\cot\alpha/4A_o 2f_y] = 0.11 \text{ in.}^2 \qquad (4.31)$$

Thus, T = 0.11(60) = 6.8 kips, u_o is given as $9.5\sqrt{4000}/1 = 601$ psi and $L = T/u_o\Sigma_0 = 6.8/(.601)(3.1415) = 3.60"$. Since it is a top bar L_d = 1.4(L) = (1.4)(3.6) = 5.0". The 6 in. straight embedment length provided past the centerline of the support is then adequate to develop the required tension force.

Finally, the anchorage of the top compression reinforcement has to be evaluated at $(z_L\cot\alpha)/2 = 6.5$ in. from the centerline of the support.

The area of longitudinal reinforcement required for shear and torsion can be evaluated using Eq. 4.24, $A_L(T,V)$ = [(176)(51.64)/4(166.7) + 51.3/2]/(2)(60) = 0.32 in.^{2}, hence T = (0.32)(60) = 19 kips. The compression force produced by the applied moment can be evaluated using as C = T = M_u/z. M_u at a distance zcot /2 from the centerline of the support is 289 in.-k, thus C = 289/12 = 24 kips. Therefore, the net resultant tension at the section is zero. Therefore, the theoretical required embedment length would equal to zero.

The final detailing of the longitudinal reinforcement at the support region is shown in Fig. 4.27.

4.3.8 Design of the Reinforced Concrete Box Section Following the ACI/AASHTO Design Procedure. In order to show the difference in design procedures, the same example used in Section 4.3.7 is reworked using current design procedures.

The first step in the design procedure is to evaluate the magnitude of the torsional moment to find out if torsional effects can be neglected. The ACI/AASHTO design procedure (1,2) states that torsion effects shall be included with shear and flexure where the factored torsional moment T_u exceeds $(0.5\sqrt{f'_c}\,\Sigma x^2y)$. Otherwise, the torsion effects may be neglected. In this design example T_u = 150 in.-kip and f'_c = 4000 psi. The term Σx^2y represents the torsional section properties, where x is the shorter overall dimension of the rectangular part of the cross section and y is the longer overall dimension of the retangular part of the cross section. For this design example, x = y =

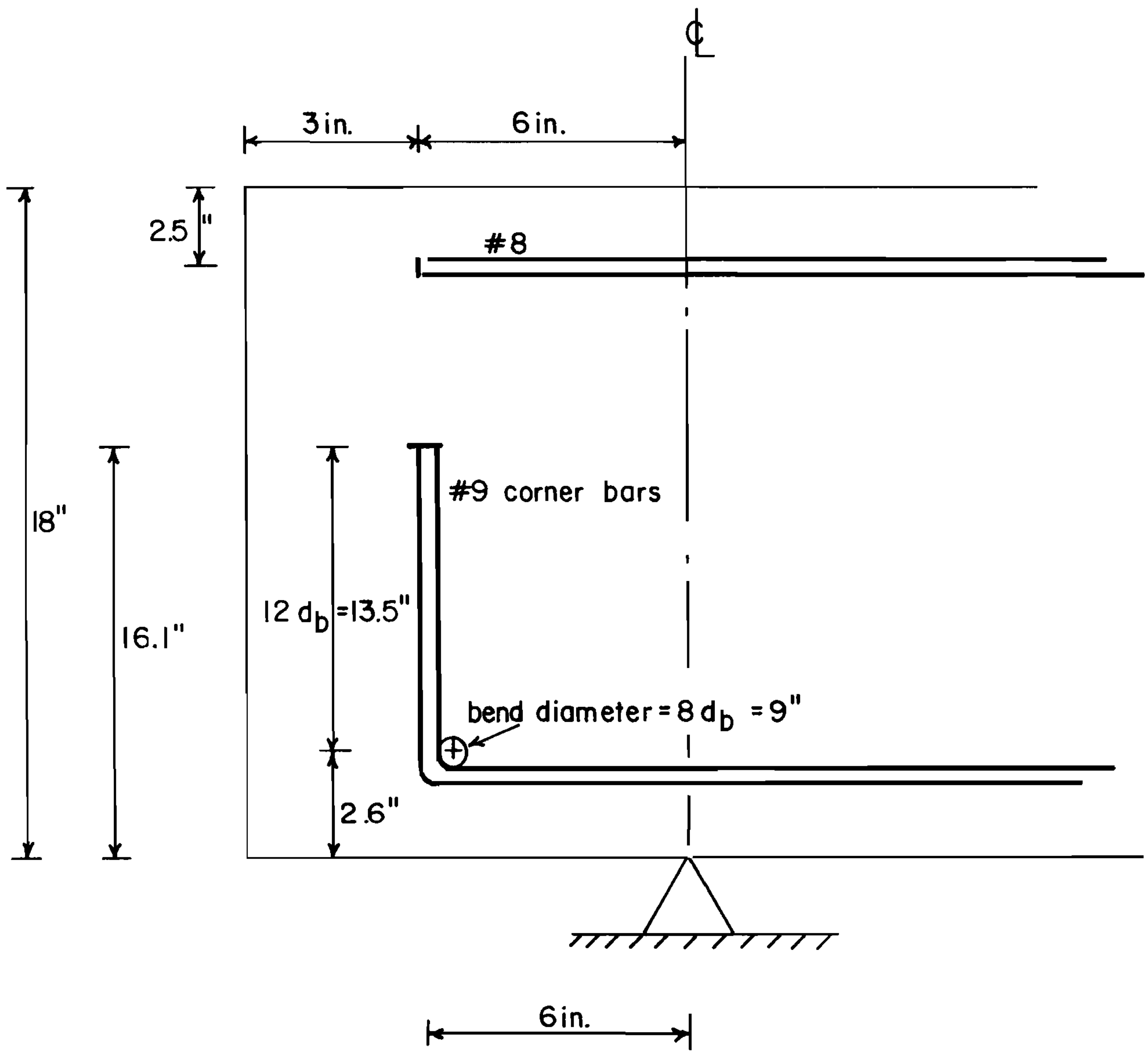


Fig. 4.27 Detailing of the longitudinal reinforcement at the support region

18 in. For the case of rectangular box sections having a wall thickness less than $x/4 = 18/4 = 4.5$ in. but greater than $x/10 = 1.8$ in., such as in this design example ($t = 4$ in.), the factor $\sum x^2y$ has to be multiplied by $4h/x$, where h is the wall thickness. For this example $(x^2y)(4h/y)$ yields the value 5184 $in.^3$ Thus, $\phi(0.5\ 4000\ [5184])$, where $\phi = 0.85$, results in a torsional moment of 139 k-in. Since 139 in.-k < 150 in.-k, the torsional effects cannot be neglected in the design of this member.

The preliminary flexural design is essentially the same as the one conducted for the Space Truss Model Approach. Then the consideration of shear and torsion effects should begin.

The member is divided into 12 design sections of 26/12 = 2.17 ft. each. However, following the ACI/AASHTO provisions the first critical section is located at a distance d from the centerline of the support (15.44/12 = 1.29 ft). The other sections are located at the midpoints of the design sections at 3.25 ft., 5.42 ft., 7.58 ft., 9.75 ft. and 11.92 ft. from the centerline of the support, respectively. Since the loading and structure are symmetrical, the design of the other half of the member would be essentially the same. Table 4.11 shows the design of the transverse and longitudinal reinforcement for shear and torsion in accordance with the ACI/AASHTO requirements. Table 4.11a shows the design of the transverse reinforcement required for shear and torsion. The ACI/AASHTO design procedures (1,2) recognize a concrete contribution to the shear strength V_c and to the torsional strength T_c at all load levels. The concrete contribution to the torsional

(1) Dist. from C.L. (ft)	(2) V_u (kips)	(3) T_u (in-k)	(4) T_c (in-k)	(5) A_t/s (in^2/in)	(6) V_c (kip)	(7) V_s (kip)	(8) A_v/s (in^2/in)	(9) $A_v/2s + A_t/s$ (in^2/in)	(10) Min. $A_v/2s + A_t/s$ (in^2/in)	(11) s #3 closed hoop (in)	(12) Max. Spac. (in)	(13) Spacing Provided (in)
d = 1.29	41	150	56	0.008	15.3	32.9	0.036	0.026	0.003	4.25	3.75	3.75
3.25	34	150	67	0.007	15.2	25	0.027	0.021	0.003	5.25	7.5	5.25
5.42	26	150	84	0.006	14.8	16.4	0.018	0.015	0.003	7.3	7.5	7.25
7.58	19	150	112	0.004	14.2	8.2	0.009	0.009	0.003	12.2	7.5	7.50
9.75	11	150	163	0.001	12.4	1.0	0.001	0.001	0.003	110	7.5	7.50
11.92	4	150	241	-----	6.1	-----	-----	-----	0.003	110	7.5	7.50

(a) Dimensioning of the web reinforcement

(1) Distance from supp. C.L. (ft)	(2) A_L Eq. 4.34 (in^2)	(3) A_L Ex. 4.35 (in^2)	(4) A_L Eq. 4.35 $=50 \frac{b_w}{f_y} \frac{2A_t}{s}$ (in^2)	(5) A_L reqd. due to shear and torsion (in^2)
d = 1.29	0.51	0.29	0.59	0.51
3.25	0.45	0.48	0.72	0.48
5.42	0.39	0.73	0.91	0.73
7.58	0.26	1.14	1.19	1.14
9.75	0.07	1.82	1.67	1.67
11.92	---	1.87	2.65	1.87

(b) Dimensioning of the longitudinal steel

Table 4.11 Dimensioning of the reinforcement required for shear and torsion in accordance with ACI/AASHTO requirements strength, T_c shown in column (4) of Table 4.11a, is evaluated using Eq. 4.32.

$$T_c = \frac{0.8\sqrt{f'_c}\Sigma x^2 y}{[1+(\frac{0.4V_u}{c_t T_u})^2]^{0.5}} \tag{4.32}$$

where C_t is a torsional constant defined as $b_w d/\Sigma x^2 y$, V_u is the factored shear force at the section and T_u is the factored torsional moment at the section. The concrete contribution to the shear strength V_c is related to the torsional strength T_c as $T_u/V_u = T_c/V_c$; thus $V_c = T_c(V_u/T_u)$. The values of the concrete contribution to the shear strength of the member V_c are shown in column (6) of Table 4.11a. The nominal torsional strength of the member $T_n = T_u/\phi$, where $\phi = 0.85$, is given as $T_n = T_c + T_s$. T_s is the torsional moment that is carried by the reinforcement given as $T_s = A_t \alpha_t x_1 y_1 f_y/s$, where A_t = area of one leg of a closed stirrup resisting torsion within a distance s, sq. in., x_1 is the shorter center-to-center dimension of the closed rectangular stirrup, and y_1 is the longer center-to-center dimension of the closed rectangular stirrup. α_t is a coefficient of x_1 and y_1 given as $[0.66 + 0.33(y_1/x_1)]$, f_y is the yield strength of the closed stirrup used as web reinforcement and s is the spacing of the torsion reinforcement in the direction parallel to the longitudinal reinforcement. From the relation for T_s, the area of web reinforcement required to resist torsion can be obtained as $A_t/s = T_s/\alpha_t x_1 y_1 f_y$. For the design example $x_1 = y_1 =$ 16.13", $\alpha_t = 0.99$ and $f_y = 60000$ psi. The required amounts of web reinforcement due to torsion are shown in column (5) of Table 4.11a.

The nominal shear strength required of the section $V_n = V_u/\phi$, where $\phi = 0.85$, is given as $V_n = V_c + V_s$. V_c is the concrete contribution to the shear strength of the member shown in column (6) of Table 4.11a, and V_s is the shear strength as provided by the web reinforcement. $V_s = A_v f_y d/s$, where A_v is the area of shear reinforcement within a distance s, f_y is the yield strength of the shear reinforcement, d is the effective depth of the section, and s is the spacing of the shear reinforcement in the direction parallel to the longitudinal axis. From the relation for V_s, the area of web reinforcement per inch of stirrup spacing, A_v/s, can be obtained as

$$A_v/s = V_s/f_y d = (V_n - V_c)/f_y d \qquad (4.33)$$

For this design example, f_y = 60000 psi and d = 15.44". The required amounts of web reinforcement for shear are shown in column (8) of Table 4.11a. Column (9) shows the total amount of web reinforcement required for shear and torsion. The amount of web reinforcement shown in column (9) of Table 4.11 is the area of one leg of closed stirrup resisting the combined shear and torsion per inch of stirrup spacing. Column (10) shows the specified minimum amount of web reinforcement required for shear and torsion. The minimum amount has to be provided at any section where the factored torsional moment T_u exceeds $\phi(0.5\sqrt{f'_c}\,\Sigma x^2 y)$. The box section in this design example is subjected to a constant factored torsional moment T_u = 150 in.-kip. $\phi(0.5\sqrt{f'_c}\,\Sigma x^2 y)$ is equal to 139 in.-kips. Thus, a minimum area of web reinforcement at least equal to $(A_v/2s) + (A_t/s) = 50\ b_w/f_y$ would have to be provided at any section of the member. For this design example, $(A_v/2s + A_t/s)_{min}$ = 50(4)/60000 = 0.003.

Column (11) of Table 4.11a shows the required spacing if a #3 closed hoop is used as web reinforcement. Column (12) shows the maximum spacing allowed in the ACI/AASHTO design procedures for shear and torsion. In this case the requirement that the maximum stirrup spacing be d/4 = 15.44/4 = 3.86 in., if $V_n \geq 6\sqrt{f'_c}b_wd = 6\sqrt{4000}\,2(4)(15.44)$ = 46.88 kips, controls the section located at a distance of 1.29 ft. from the face of the support (since the shear is taken by two walls b_w = 2(4) = 8"). From there on the maximum stirrup spacing is controlled by the requirement that $S < d/2$ = 15.44/2 = 7.7 in. for shear. Column (13) shows the selected spacing of web reinforcement for each of the design sections.

Table 4.11b shows the required amount of longitudinal reinforcement for torsion. In the ACI/AASHTO design procedures the total area of longitudinal reinforcement required for torsion A_l is evaluated as

$$A_l = 2A_t[x_1 + y_1]/s \qquad (4.34)$$

or

$$A_l = \left[400\,\frac{xs}{fy}\left(\frac{T_u}{T_u + \frac{V_u}{3C_t}}\right) - 2A_t\right]\left(\frac{x_1+y_1}{s}\right) \qquad (4.35)$$

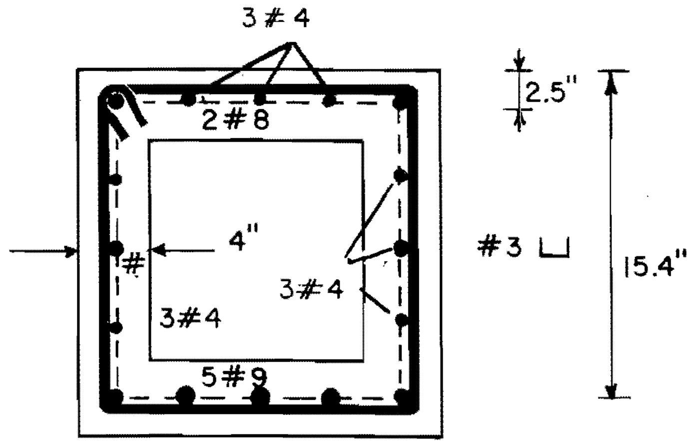
whichever is the greatest but A_l from Eq. 4.35 need not exceed that amount obtained by substituting in Eq. 4.35

$$2A_t = 50b_ws/f_y \qquad (4.36)$$

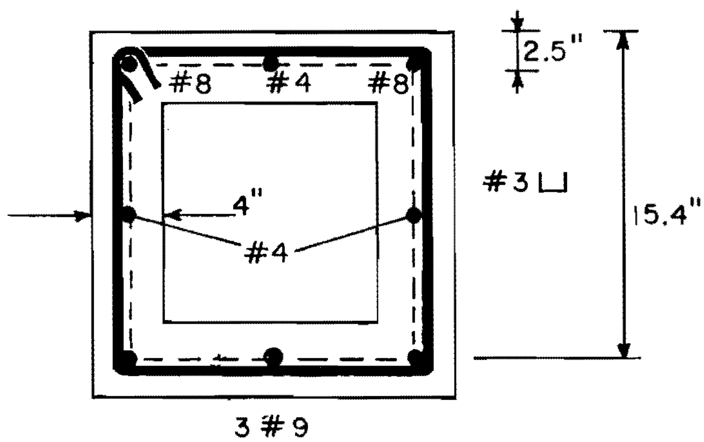
Column (2) shows the total area of longitudinal reinforcement required to resist torsion evaluated using Eq. 4.34. Column (3) shows the values of A_l evaluated with Eq. 4.35, and column (4) contains the amount of A_l required using Eq. 4.35, but with $2A_t = 50b_w s/f_y$. Finally, column (5) shows the total amount of longitudinal reinforcement required to resist shear and torsion. Note that the results of the application of current specification values in this example result in substantially more longitudinal reinforcement being added at midspan where the shear is lowest. This is a direct contradiction to the truss analogy results.

Failure due to crushing of the concrete in the web of the member is presented in the case of shear by limiting the nominal shear strength of the member V_n to a values less or equal to $10\sqrt{f'_c}\, b_w d$. For this case $10\sqrt{4000}$ (4) (15.44) = 39 kips on each wall. Since $V_{n\ max}$ at a distance "d" from the support centerline on each wall is given as $V_u/2\phi$ = 24.1 kips, then failure due to crushing of the concrete in the web would not take place prior to yielding of the reinforcement. In the case of torsion this failure is prevented by limiting the torsional shear strength of the member $T_n = T_u/\phi$ to a value less than $5{*}T_c$. For this case T_c = 56 in.-kip at the design section a distance d away from the support; thus, since $T_u/\phi = 150/0.85 = 176 < 5(56) = 280$ in.-kips, failures due to crushing of the concrete in the web prior to yielding of the stirrups are prevented.

4.3.9 <u>Detailing of the Longitudinal Flexural Reinforcement in the ACI/AASHTO Design Procedure</u>. Figure 4.28a shows the final design for midspan section of the box beam. The two #8 longitudinal



(a) Cross section at midspan



(b) Cross section at support

Fig. 4.28 Detailing of the box beam

compression bars will remain continuous through the entire member length to provide adequate anchorage for the stirrup reinforcement in addition to helping in controlling creep deflections. Three of the five #9 longitudinal tension bars will be kept continuous throughout the entire length of the member. The theoretical distance from the midspan where the other two #9 bars can be terminated in order to satisfy flexure requirements in $X = [(5-3)(156)^2/5]^{0/5} = 99$ inches. Since the bar is going to be terminated without bending it into the compression zone then the total length will be given by $A = x + 12d_b$, where d_b = diameter of the bar to be terminated, or $A = x + d$ whichever is greater. Then $A = 99 + 15.4 = 114.4$, say $A = 115$ inches.

However, in order for the 2 #9 bars to be terminated in the tension zone at least one of the following three requirements must be satisfied (1,2):

1. The shear at cutoff point does not exceed 2/3 of that permitted, including shear strength of reinforcement provided.

2. Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the termination point equal to 3/4d. Excess stirrup area shall not be less than $60b_w s.f_y$, Spacing s shall not exceed $d/8B_b$, where B_b is the ratio of area of reinforcement cut off to total area of tension reinforcement at the section.

3. For #11 bar and smaller, continuing reinforcement provides double the area required for flexure at the cut-off point and shear does not exceed 3/4 of that permitted.

For the first condition V_u at the cut-off point has to be less than $2/3V_n$. The factored shear force at the cut-off point is equal to 16.8 kips on each vertical web, the nominal shear strength of the cross section at the cut-off point is $(7.8) + V_s$, where V_s is the shear strength provided by the web reinforcement #3 U stirrups at 7.5 inches center-to-center. So using Eq. 4.34, V_s = (0.11)(60)(15.44)/7.5 = 13.6 kips per vertical web of the box section. Thus, V_n = 7.8 + 13.6 = 21.4 kips. Since 16.8 > 2/3(0.85)(21.4) = 12.1 kips, then condition 1 is not met.

Since at the theoretical cut-off point the continuing reinforcement would be working at its full yield strength, condition 3 is not met either.

Therefore, in order to be able to terminate the two #9 bars, condition 2 would have to be met. This is to say, an area of web reinforcement in excess of that required for shear would have to be provided along each terminated bar over a distance from the termination point equal to 3/4(d) = 3/4(15.4) = 11.5 in. This excess area of web reinforcement is taken equal to $A_v/s = 60b_w/f_y$ = (60)(4)/60000 = 0.004 per wall element resisting the applied vertical shear force. The area of web reinforcement required at the cut-off point for shear can be determined from column (5) in Table 4.11. A_v/s = 0.013. Thus, the total area that has to be provided over a distance of 11.5 inches for the termination point is 0.013 + 0.004 = 0.017. The amount provided is #3 stirrups at 7.5 inches is A_v/s = 0.11/7.5 = 0.015. Therefore, a closer stirrup spacing has to be utilized s = 0.11/0.017 = 6.5 inches. However, the maximum spacing of this extra reinforcement should not exceed the value $d/8B_b$, since 2 of the 5 #9 bars are being terminated. B_b = 2/5, thus s_{max} = 15.4/8(2/5) = 4.8 in. Since the termination point is at a distance equal to (13-9.6) = 3.4 ft. from the centerline of the support, in order to satisfy this requirement and terminate the 2 #9 bars in the tension zone, extend the stirrup spacing of 3.75 inches up to a distance of 4 ft. for the support centerline.

The three #9 bars which remain continuous would have to be able to develop their full yield strength within the 3.4' distance from the support centerline plus the 6 inches of straight embedment length past the support centerline. The required development length for a #9 bar is determined as (2):

$$l_d = 0.04A_b f_y \sqrt{f'_c} \tag{4.37}$$

where A_b is the area of the bar, in.^{2} Thus, l_d for a #9 bar is 38 inches, which is less than the 3.4 * 12 + 6 = 47 inches provided. So the 3 #9 bars will be adequately anchored. Finally, the anchorage of the tension reinforcement at the support region has to be checked. The box beam cross section at the support region is shown in Fig. 4.28b. The ACI/AASHTO design procedure requires that at the support region of noncontinuous members

$$l_d < 1.3M_n/V_u + l_a \tag{4.38}$$

where l_d is the development length of the bar, M_n is the nominal moment strength at the section, V_u is the factored shear force at the section and l_a is the additional embedment length past the centerline of the support. For this design example V_u = 45.5 kips, M_n = 2513 in.-k, l_a = 6 inches, and l_d for a #9 bar is 38 inches. Since 38 in. <

(1.3)2513/45.5 + 6 = 77.8 in., the longitudinal reinforcement will be adequately detailed at the support region.

4.3.10 Comparison between the Amounts of Reinforcement Required by the Truss Model and the ACI/AASHTO Design Procedure. Figure 4.29 shows a comparison between the amounts of web reinforcement A_v/s per wall element required by the truss method and the current AASHTO/ACI procedures.

In this case the truss model procedure requires more web reinforcement than the AASHTO procedures. This is due to the fact that the chosen angle for design was 45 degrees which is the one currently assumed in the ACI/AASHTO procedures. The difference comes about because the ACI/AASHTO procedures recognize a concrete contribution to the shear and torsional strength of the member regardless of the level of shearing stress due to shear and torsion. In contrast, the truss model transition zone expression diminishes the concrete contribution with increasing levels of shearing stress.

Figure 4.30 shows a comparison between the additional amounts of longitudinal reinforcement due to shear and torsion required by the Truss Model approach and the current AASHTO procedures. The areas of longitudinal steel shown in Fig. 4.30 are areas per longitudinal tension chord, assuming that there are 4 chords in the members. Since the area of longitudinal steel obtained using the ACI/AASHTO procedures in given in terms of total area of longitudinal steel required for torsion, it is divided by 4 and then plotted for a direct comparison of the increase in each chord. The values shown are those from Table 4.10 column (4) for

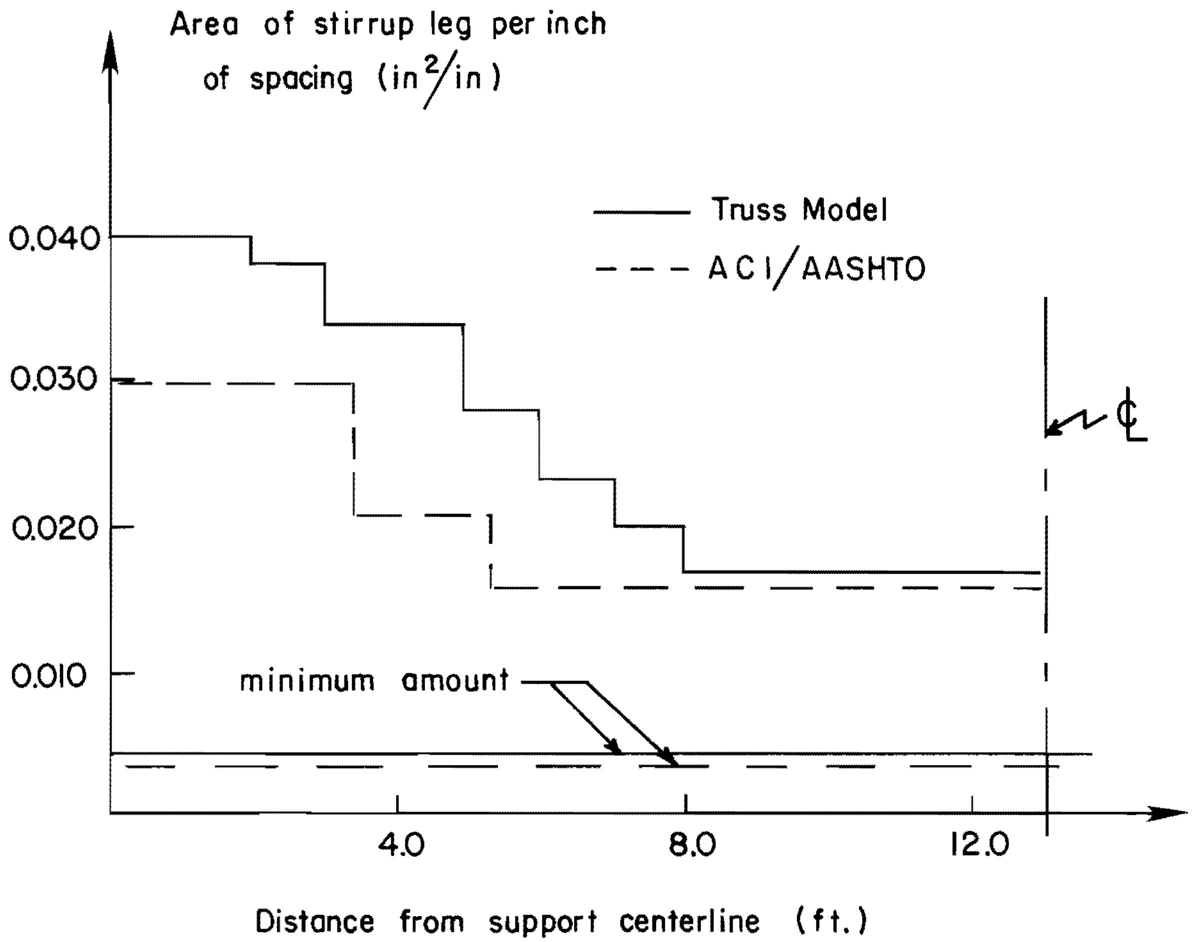


Fig. 4.29 Comparison of required amounts of web reinforcement

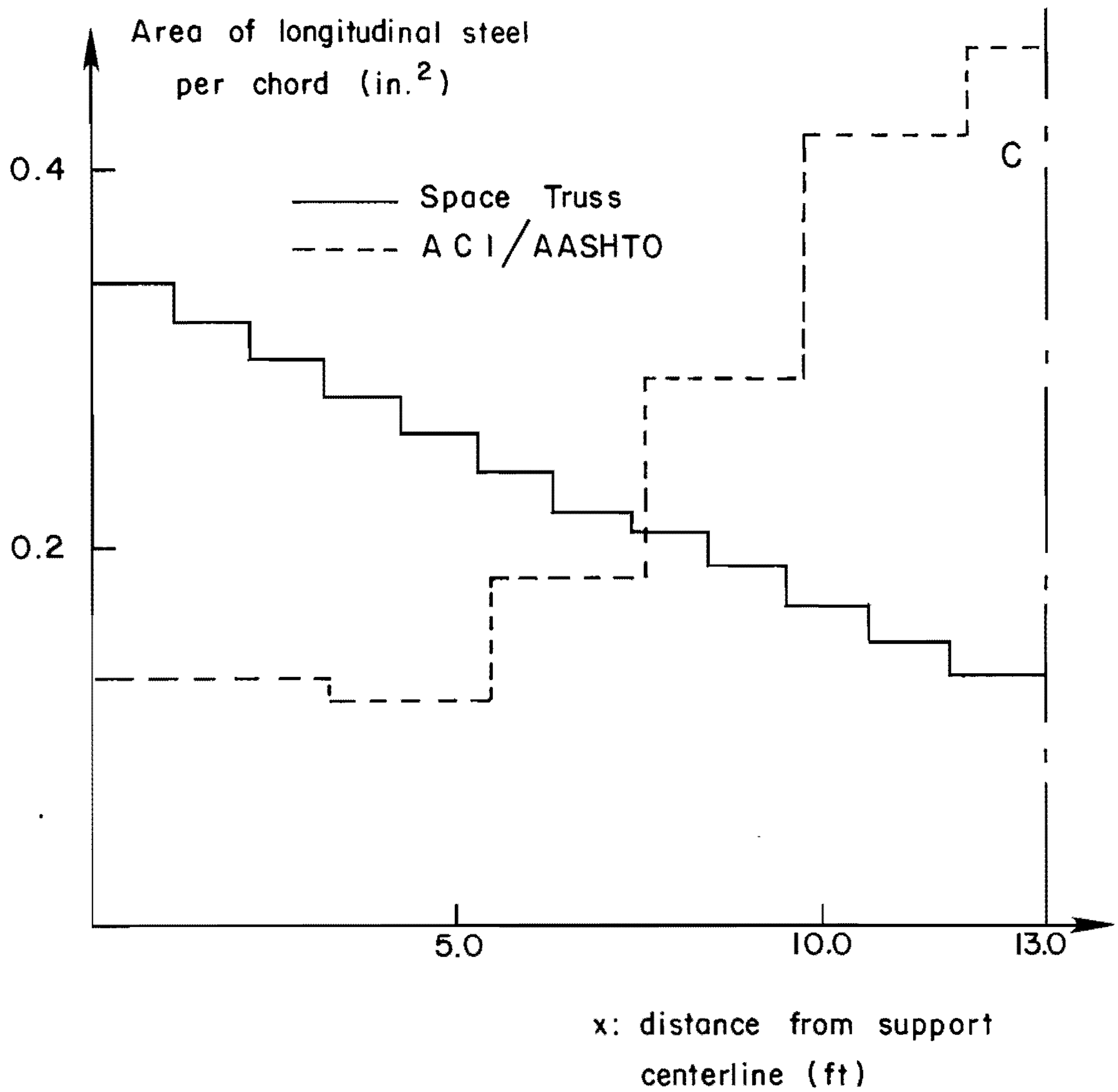


Fig. 4.30 Comparison of the additional amounts of longitudinal reinforcement required for shear and torsion

the case of the truss model and Table 4.11b column (5) for the case of the ACI/AASHTO procedures.

The differences shown here indicate a very substantial difference in philosophy between the two approaches. The rationale in the truss model seems very preferable. In addition, since the flexural steel requirements are maximum at midspan but diminish towards the ends and since the ACI/AASHTO requirements for additional steel are greatest at midspan, a greater amount of additional steel would have to be added to the member to satisfy the empirical ACI/AASHTO requirements.

4.4 Design of a Prestressed Concrete I-Girder under Bending and Shear

In this section, the design of a SDHPT Type C prestressed concrete girder for a 40 ft. span subjected to bending and shear is carried out using the proposed truss model design procedure. A comparison of the amounts of web reinforcement required by the proposed truss model approach and the current AASHTO design procedures (1) is given.

The prestressed I girder forms part of a three lane composite beam and slab highway bridge to be designed to resist HS 20 live loading. In this section the design of an interior girder will be shown.

The beams are spaced 6.5 ft. apart in the transverse direction of the bridge. The composite slab has a 7.25 in. thickness with a slab concrete strength f'_c equal to 3600 psi. The girder has a concrete strength f'_c of 5000 psi. The longitudinal prestressed reinforcement consists of 1/2 in., 270 ksi 7-wire strands. A single interior diaphragm is located at midspan of the girder. The composite slab width per girder is 78 in.

As in any normal design procedure the flexure design is conducted first. Figures 4.31 and 4.32 show details of the cross section at midspan and end region of the member. Ten 1/2 in. diameter 270 ksi strands are required for flexure, two of those 10 strands are draped. Figure 4.33 shows a detail of the beam strand pattern. Since the calculations for flexure are unchanged no further details will be given.

The first step in the Truss Model design procedure for the case of combined bending and shear is the selection of an appropriate truss system for the given load patterns and structural constraints.

Table 4.12 shows the determination of dead loads on the girder. The bridge is designed to resist an HS 20-44 live loading (1). The fraction of the wheel load applied to each girder is determined from Table 1.3.1(B) of AASHTO-1977 (1).

$$[S/5.5 = \text{Girder spacing in ft}/5.5] \qquad (4.39)$$

AASHTO Sec. 1.3.1 (B)(1) specifies that the live load bending moment for each interior girder shall be determined by applying to the girder the fraction of a wheel load (both front and rear) determined from this equation. Since a truck loading has two sets of wheels, the truck live load distribution factor (fraction of truck load applied to each girder) is 1.18/2 = 0.59.

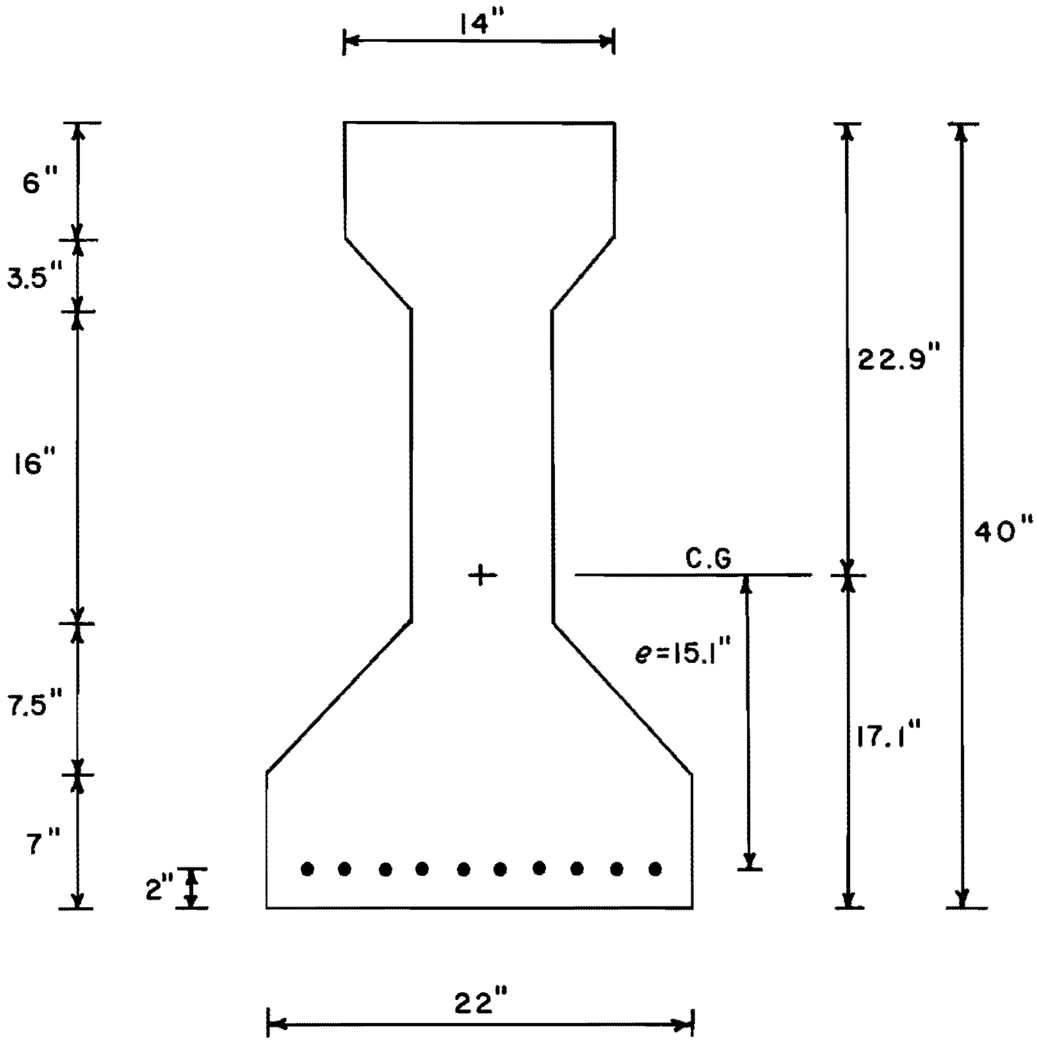


Fig. 4.31 Cross section at centerline

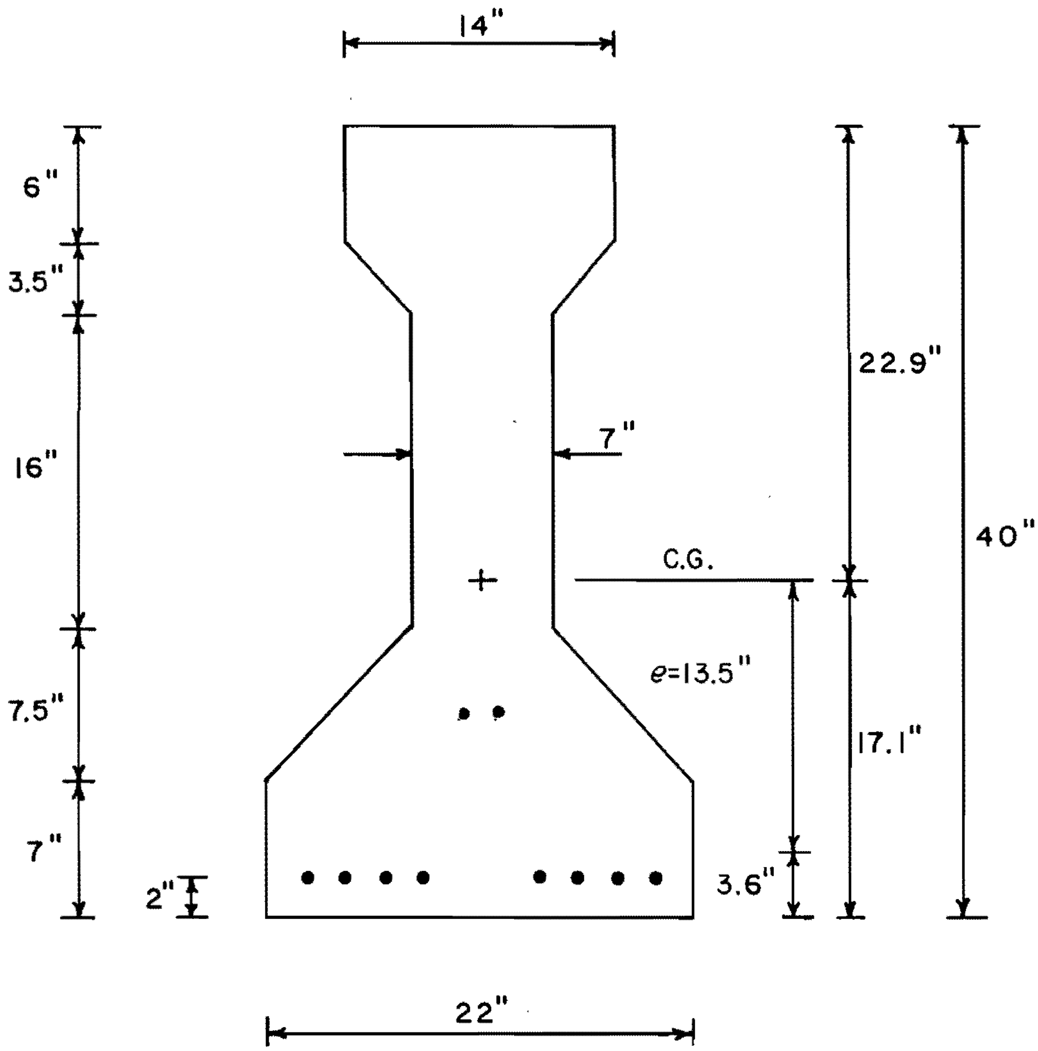
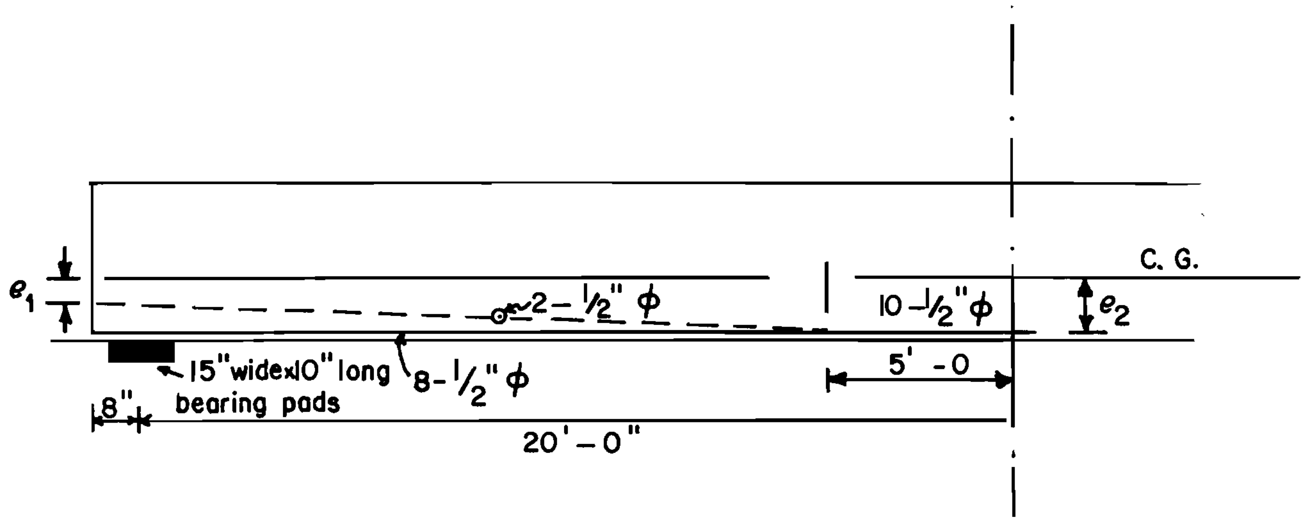


Fig. 4.32 Cross section at the support



10-1/2" diameter strands

Grade 270k

A_{str} = 0.153 in.^{2}

e_1 = 13.5"

e_2 = 15.1"

Fig. 4.33 Beam strand pattern

The impact allowance for truck load moments is determined from AASHTO Section 1.2.12

$$I = \frac{50}{(L + 125)} \tag{4.40}$$

where "L" is the span length. However, for computing shear due to truck loads, "L" is taken as the length of the loaded portion of the span from the point under consideration to the far reaction. Hence, for shear "I" is a function of position along the span. However, "I" should not exceed 30%.

For this design example, the portion of the wheel load applied to the girder including impact is for the 32 kip axle so P_{LL+I} = 32*0.59*1.3 = 24.54 kips. For the 8 kip axle, P_{LL+I} = 8*0.59*1.3 = 6.14

Dead loads supported by naked girder

- Girder $= \dfrac{(494.94)(150)}{1000 \times 144}$ 0.52 k/F

- Slab $= 6.5 \times 12 \times 7.25 = 565.5$ in.^{2}

  $= \dfrac{565.5 \times 150}{1000 \times 144} =$ 0.59 k/F

- Diaphragms at centerline 1

  $(8/12) \times 1.83 \times (6.5 - 7/12) \times 0.15 =$ <u>1.08 kips</u>

Dead loads supported by composite section

- No barrier walls

- No asphalt overlay

- Rails: $T5 = \dfrac{(0.324)(2)}{7} =$ 0.09 k/F

Table 4.12 Determination of dead loads

kips. In this example, the impact factor "I" limit of 30% controls for any section of the member in the case of shear.

This section presents a summary of the design steps involved in the truss model approach. Detailed explanation of each of the steps, as well as numerical computations are shown in subsequent sections.

1. Select angle of inclination between the limits

   $25^{\circ} \leq \alpha \leq 65^{\circ}$

2. Develop truss system

   2.1 Determine horizontal dimension of the truss panels (design zones)

   $z \cot\alpha$

   2.2 Evaluate envelope of the maximum live load shears, the dead load shear diagram, and determine the direction of the compression diagonals in each of the design panels (zones) of the truss model (see Figs. 4.34, 4.35, 4.36, and 4.37).

   This member will be designed as a symmetric section, since the truck loading can approach the bridge from either side. The design conducted for one-half of the span for the worst possible combination of live and dead load shears and moments will simply be repeated on the other half (see Table 4.13).

3. Evaluation of the diagonal compression stresses $f_d(V)$

$$f_d(V) = \frac{V_n}{[b_w z \sin\alpha \cos\alpha]}$$

   where V_n is the nominal shear force $V_u/\phi, \phi = 0.85$, at the section

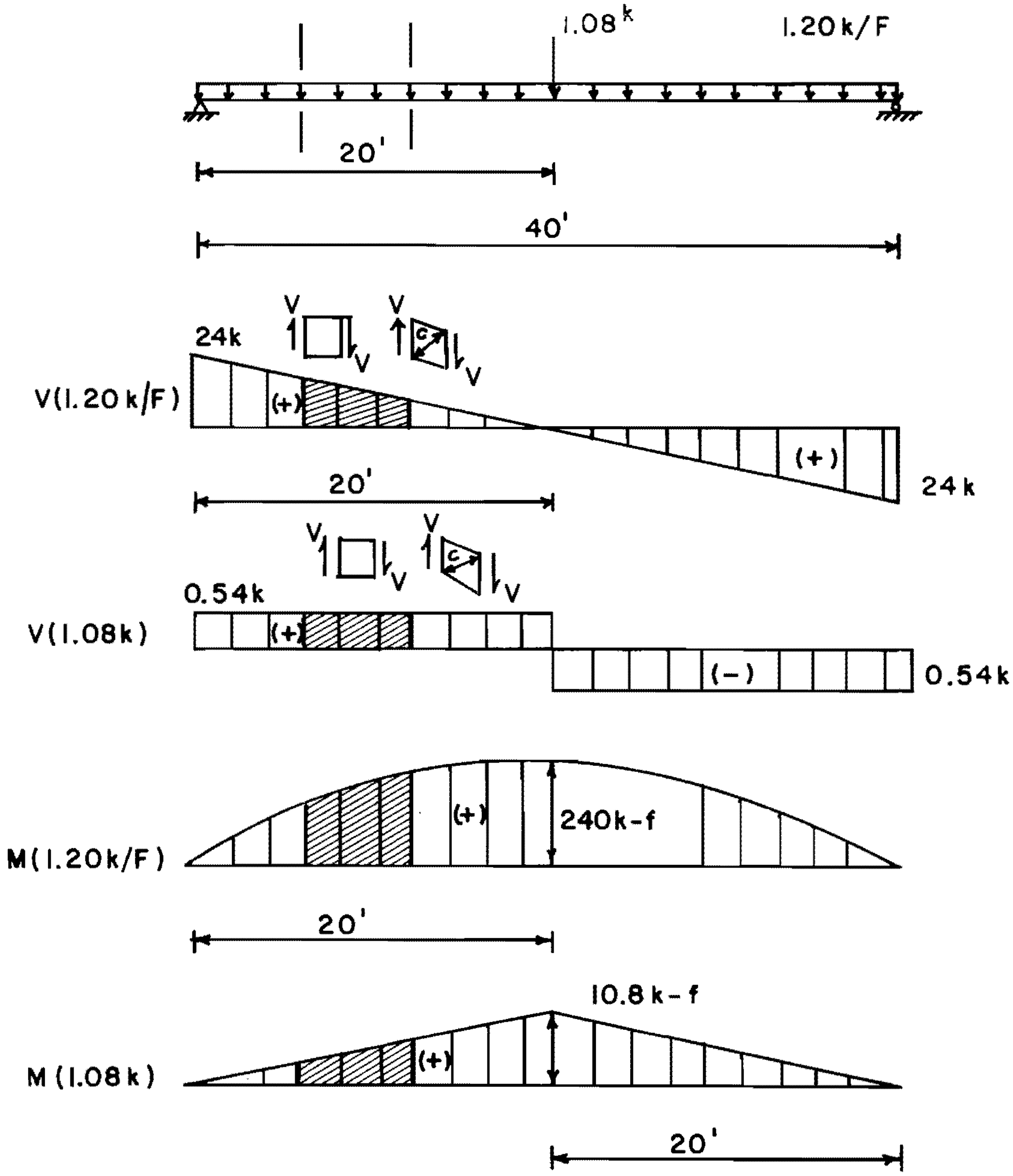


Fig. 4.34 Dead load shear and moment diagrams for the prestressed concrete bridge girder design example

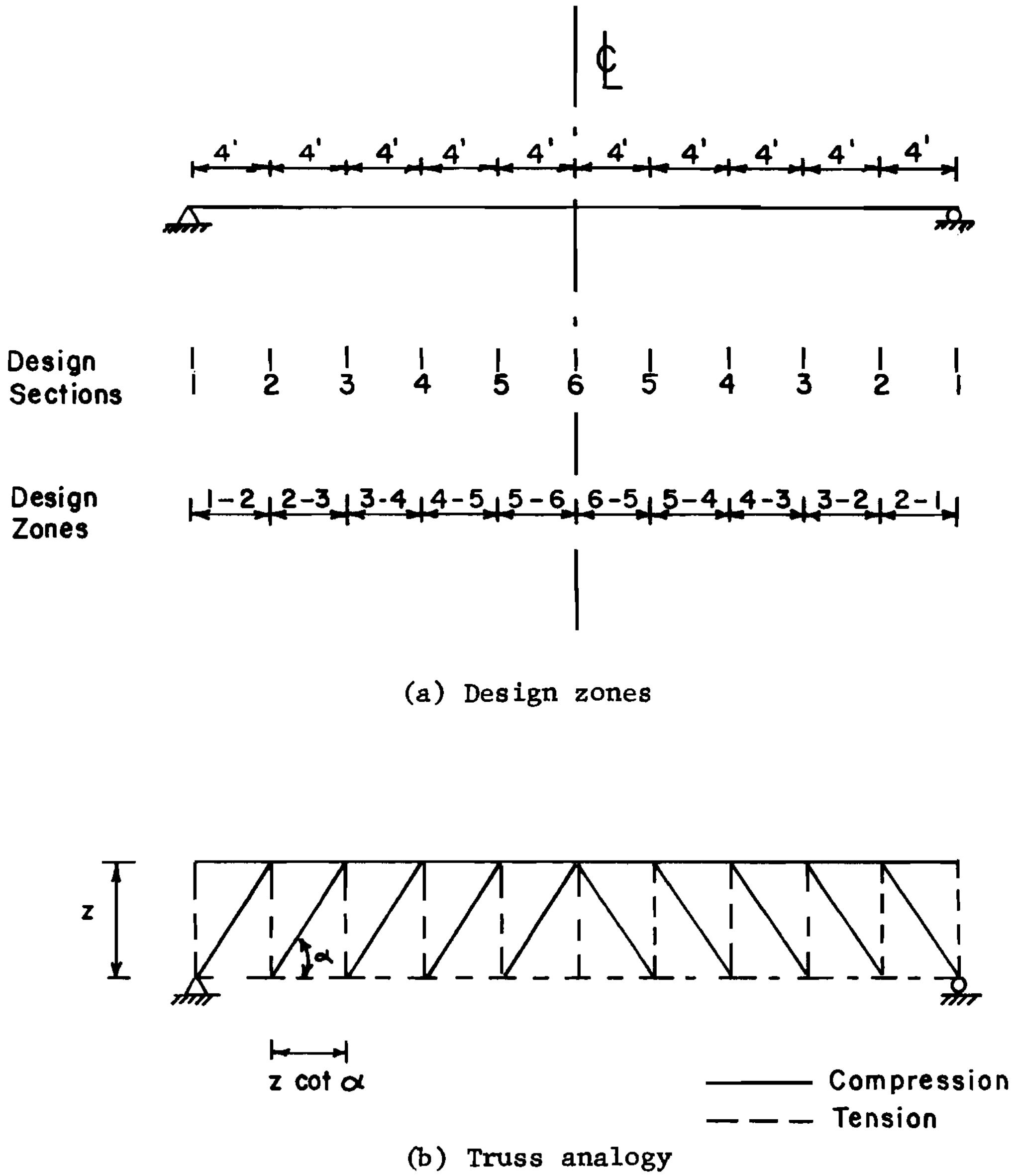
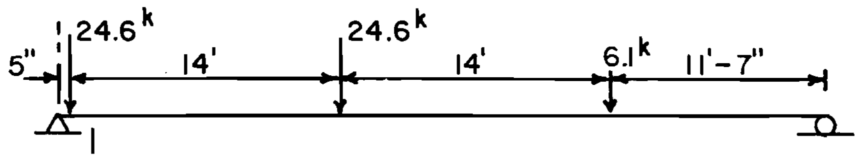
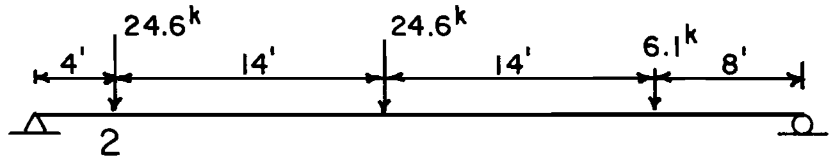


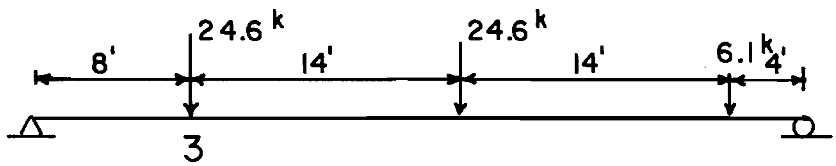
Fig. 4.35 Design zones of the composite prestressed concrete I-girder



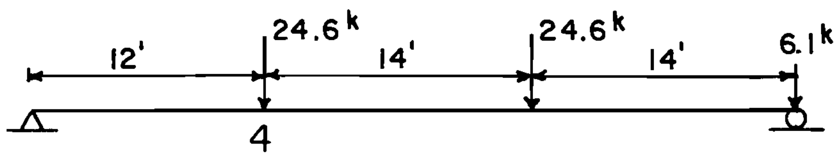
(a) Section 1



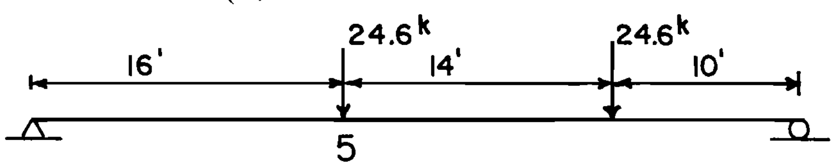
(b) Section 2



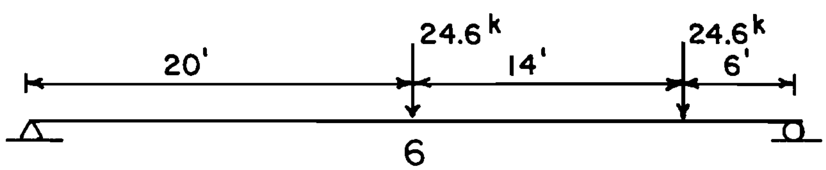
(c) Section 3



(d) Section 4



(e) Section 5



(f) Section 6

Fig. 4.36 Live loading cases

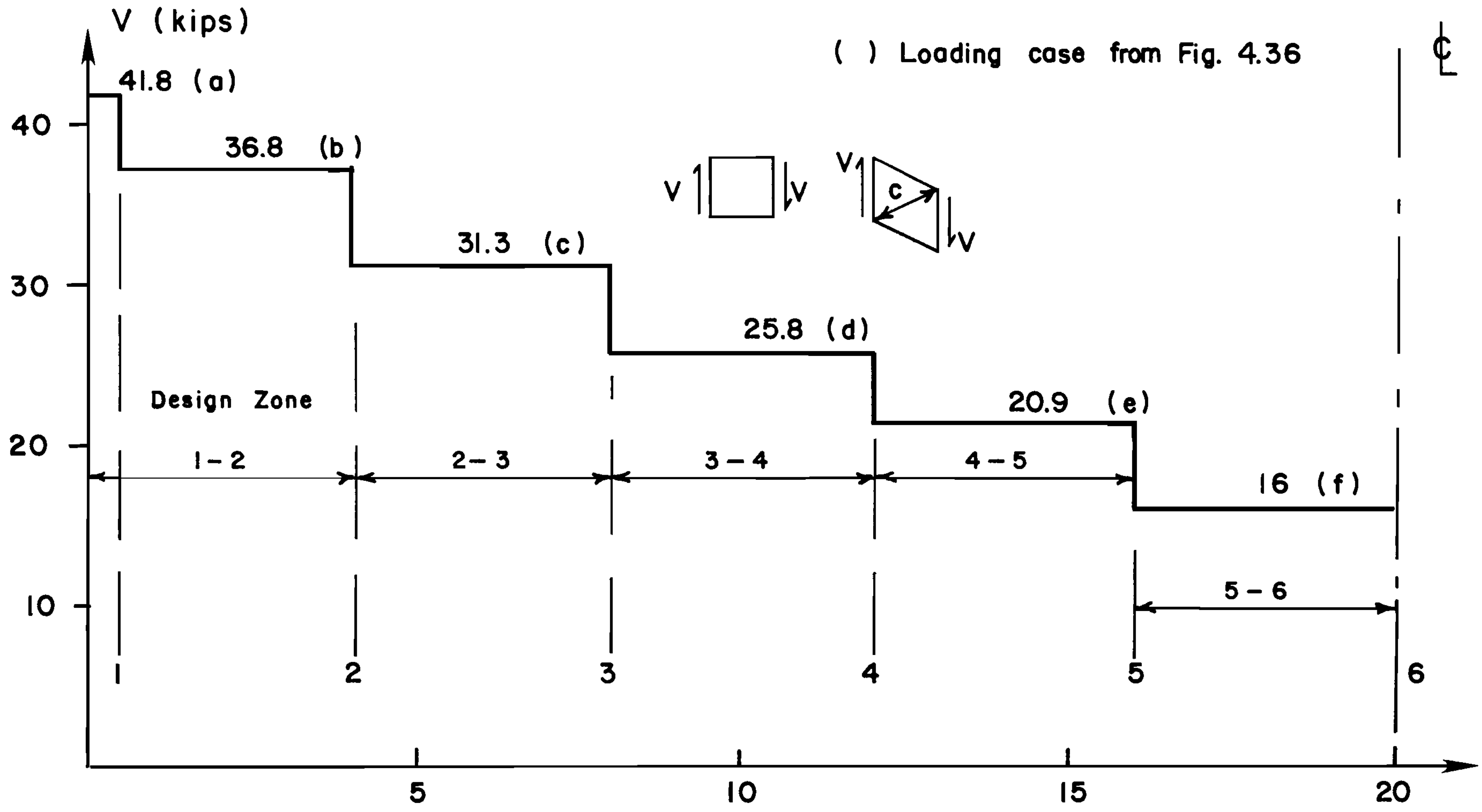


Fig. 4.37 Shear envelope for live load cases

Design Zones		1-2	2-3	3-4	4-5	5-6
Distance from the centerline of the support		0'-0"	4'-0"	8'-0"	12'-0"	16'-0"
Service Loads	Total D.L.	0.0	89	158	208	239
M_{se} (ft-k)	LL + I	0.0	125	206	251	256
V_{se}	Total D.L.	24.5	19.7	15	10	5.3
	I_1 (%)	30	30	30	30	30
(kips)	LL + I	36.8	31.3	25.8	20.9	16
Factored Loads M_u (ft-k)	1.3 [D + 5/3 (L + I)]	0.0	387	652	814	865
V_u (kips)	1.3 [D + 5/3 (L + I)]	112	93	75	58	42

Table 4.13 Calculated moments and shears for bridge girder

where the truss panel (design zone) starts, $b_w = 7$ in., $z = 43$ in., and $\alpha = 41.8$ degrees (see Table 4.14).

4. Design of transverse reinforcement

4.1 Evaluation of the concrete contribution in accordance with Sections 1.3.6b and 1.3.6e of Section 3.1.

Additional concrete contribution to the shear capacity $= V_c(V)$

$$V_c(V) = K/2[(4 + 2K)\sqrt{f'_c} - v_u]\, b_w z$$

but

$$0 \leq V_c \leq K * 2\sqrt{f'_c}\, b_w z$$

where

$$v_u = \frac{V_u}{b_w z}$$

$$K = [1 + (f_{ps}/2\sqrt{f'_c})]^{0.5}$$

with $1.0 \leq K \leq 2.0$

and $f_{ps} = F_{se}/A_b$ = Effective prestress force after losses/Area of the beam, but K = 1.0 if the stresses in the extreme tension fiber due to the computed ultimate load and the applied effective prestress force exceeds the value of $6\sqrt{f'_c}$ (see Fig. 4.14 and Table 4.15).

4.2 Evaluation of the amount of web reinforcement required to resist the factored shear force.

$$\frac{A_v}{s} = \{[(V_u - w_u z \cot\alpha)/\phi] - V_c - V_p\} \frac{\tan\alpha}{z f_y}$$

where A_v/s is the area of stirrups resisting the factored shear force per inch of the stirrup spacing "s" in each of the

Design Zones	1-2	2-3	3-4	4-5	5-6
Distance from support centerline	0'-0"	4'-0"	8'-0"	12'-0"	16'-0"
$V_n = V_u/\phi$ (kips)	132	109	88	68	49
f_d (Eq. 4.41) (ksi)	0.90	0.70	0.60	0.50	0.30

Table 4.14 Evaluation of the compression stresses in the diagonal members of the truss

Design Section	Support Centerline	2	3	4	5
Distance from the support centerline (ft)	0'-0"	4'-0"	8'-0"	12'-0"	16'-0"
Design Zone	1-2	2-3	3-4	4-5	5-6
f_{ps} (psi)	470	470	470	470	470
k (actual)	2.08	2.08	2.08	2.08	2.08
Stress in the extreme tension fiber (ksi)	-1.12 com-pression	-0.18 com-pression	0.45 tension	0.84 tension	0.85 tension
k design	2.0	2.0	1.0	1.0	1.0
v_u (ksi)	0.37	0.31	0.25	0.19	0.14
$[2 + k_{design}] * 2\sqrt{f'_c}$ (ksi)	0.57	0.57	0.42	0.42	0.42
V_p (kips)	2	2	2	2	0
V_c (kips)	60	78	26	35	42

Table 4.15 Evaluation of the additional concrete contribution to the shear strength of the member

design panels of the truss model (see Table 4.16).

4.3 Evaluation of the minimum amount of web reinforcement (see Table 4.16)

$A_v = [1.0\sqrt{f'_c}]b_w s/f_y$

5. Evaluation of the compression stresses in the fan regions f_{di} (see Table 4.17 and Figs. 4.45, 4.46, and 4.47).

$$f_{di} = \frac{D_i}{b_w z \cos\alpha_i} \leq 30\sqrt{f'_c}$$

where

$$D_i = \frac{[S_{(i)} + w_n s]}{\sin \alpha(i)}$$

6. Dimensioning of the longitudinal reinforcement required for shear

$A_l(V) = [V_n \cot\alpha]/[2f_y$

where V_n is the nominal shear force V_n/ϕ, $\phi = 0.85$, at the start of each design zone (truss panel) (see Fig. 4.48) and Table 4.18.

7. Detailing of the longitudinal reinforcement

Once the required amounts of longitudinal reinforcement for shear and bending are known, the detailing of this reinforcement can be conducted (see Sec. 4.4.6 and Table 4.18). Finally, the adequate anchorage of the longitudinal prestressed reinforcement at the support regions must be checked.

The longitudinal reinforcement going into the support has to be provided with an anchorage length such that a force $\frac{V_u \cot\alpha}{2}$ is adequately developed. The development length l_d of strand required to achieve the effective prestressing force is

$$l_d = \frac{f_{se}}{3d_b}$$

(1) Design Section	Support Centerline	2	3	4	5
(2) Distance from the support centerline (ft)	0'-0"	4'-0"	8'-0"	12'-0"	16'-0"
(3) Design Zone	1-2	2-3	3-4	4-5	5-6
(4) V_c (kips)	60	78	26	35	42
(5) V_u-w_u (4')(kips)	106	87	69	52	36
(6) nS_y (Eq. 4.47)(kips)	62	22	53	24	0.0
(7) A_v/s (Eq. 4.48) (in^2/in)	0.023	0.008	0.018	0.009	0.0
(8) A_v/s min (in^2/in)	0.008	0.008	0.008	0.008	0.008
(9) S: stirrup spacing for #3 bar (U) (in)	9.5	27.5	12.0	24.25	27.5
(10) Maximum stirrup spacing s_{max} (in)	10.75	12	12	12	12

Table 4.16 Dimensioning of the web reinforcement

4.4.1 Selection of the Truss Model. Figure 4.34 shows the shear and moment diagrams due to the applied dead loads on the girder.

The design of this type of member in general is controlled by flexure. Therefore, as explained in Sec. 3.2, in this situation the selection of a low angle of inclination of the diagonal members of the truss would not be very advantageous because the maximum stirrup spacing would probably control. Futhermore, the selection of a low angle also increases the amount of logitudinal reinforcement required for shear. Thus, for this design example a value of the angle α in the vicinity of 45 degrees will be chosen.

In the truss model approach the design zones are determined by the horizontal projection $z\cot\alpha$ of the inclined members of the truss. For this design example the depth of the truss model "z" is taken as the effective "d" of the precast section at the midspan section plus 5 inches. The 5 inches are added by assuming that the stirrup reinforcement would be anchored 5 inches plus a standard hook above the top face of the precast girder to enable, say, a #4 stirrup to be developed at the interface of the composite slab and the top face of the precast I girder. As can be seen in Fig. 4.31 "d" of the precast girder equals 38 inches, thus "z" is 38 + 5 = 43 inches. In this example, an angle of inclination of the diagonal members equal to 41.8 degrees is chosen, such that the length of each truss panel (design zone) $z\cot\alpha$ is equal to 4 ft. This divides the member into a convenient number of design zones (five) between the suppport face and the centerline (see Fig. 4.35). Once the design zones are determined, the envelope of the maximum live load shears has to be completed. Figure 4.36 shows the critical live loading cases for each of the design sections. For example, the loading case shown in Fig. 4.36a produces the maximum live load shear force at the design section 1. The loading cases shown in Figs. 4.36b through 4.36f produce the maximum live load shear for the design sections 2 through 6, respectively. The resultant envelope of maximum live load shears for the loading cases shown in Fig. 4.36 is shown in Fig. 4.37. Figure 4.38 shows the corresponding moment diagrams for each load case.

The resultant envelope of maximum live load shears shown in Fig. 4.37 together with the corresponding bending moments and the dead load shears and moments shown in Fig. 4.34 are used to determine the truss model for this design example. The section will be designed as a symmetric section, since the truck loading can approach the bridge from either side. The design conducted for one-half of the span for the worst possible combination of live and dead load shears and moments will simply be repeated on the other half.

From the dead load shear diagrams of Fig. 4.34 and the envelope of maximum live load shears shown in Fig. 4.36 the directions of the diagonal compression elements of the selected truss model are determined. The resulting general truss model for the member is shown in Fig. 4.35b. However, since in this case an envelope of maximum shears is used to design the section, special care should be exercised in using the appropriate free bodies when proportioning the

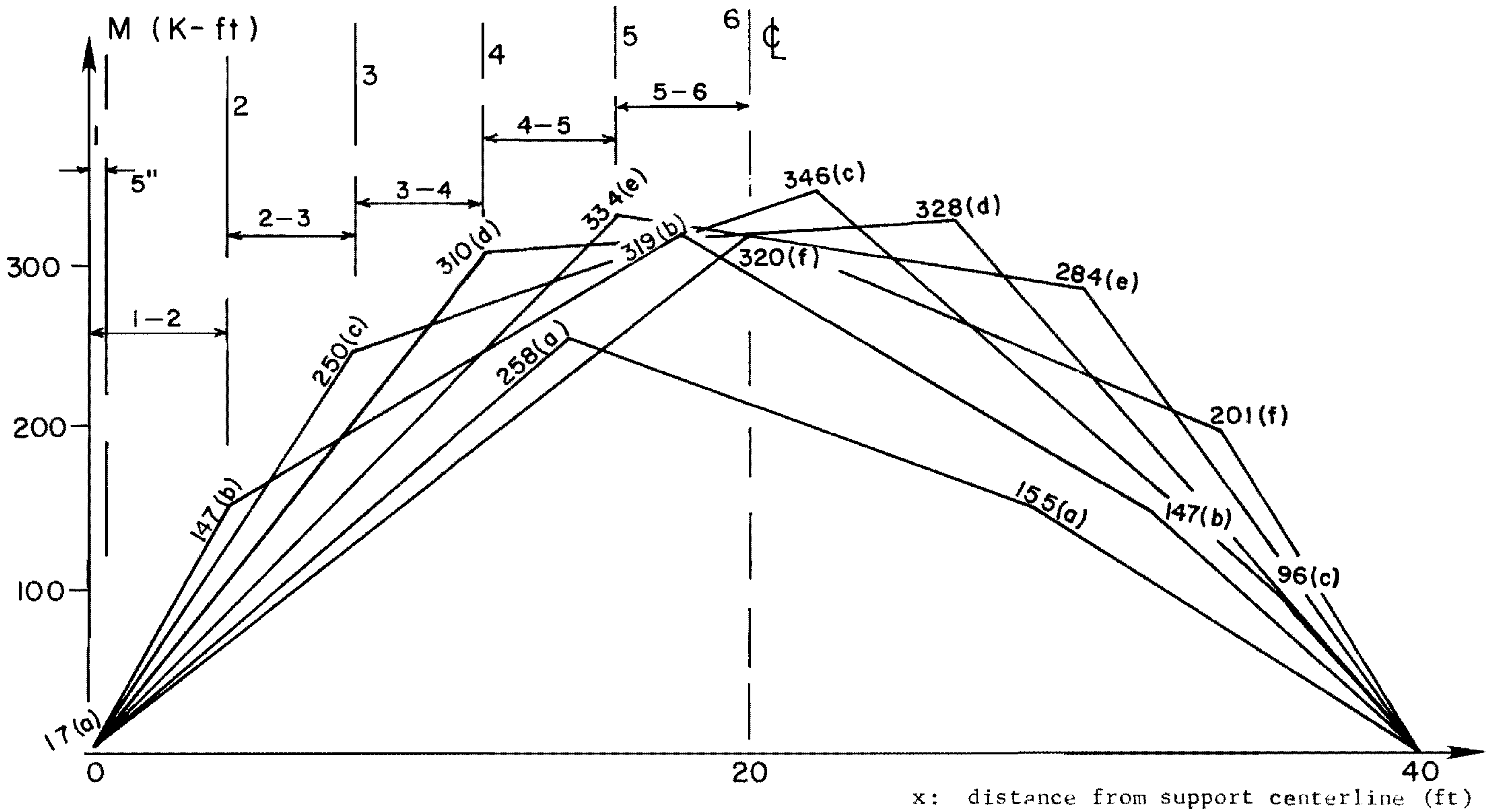


Fig. 4.38 Live load moment diagrams for each load case

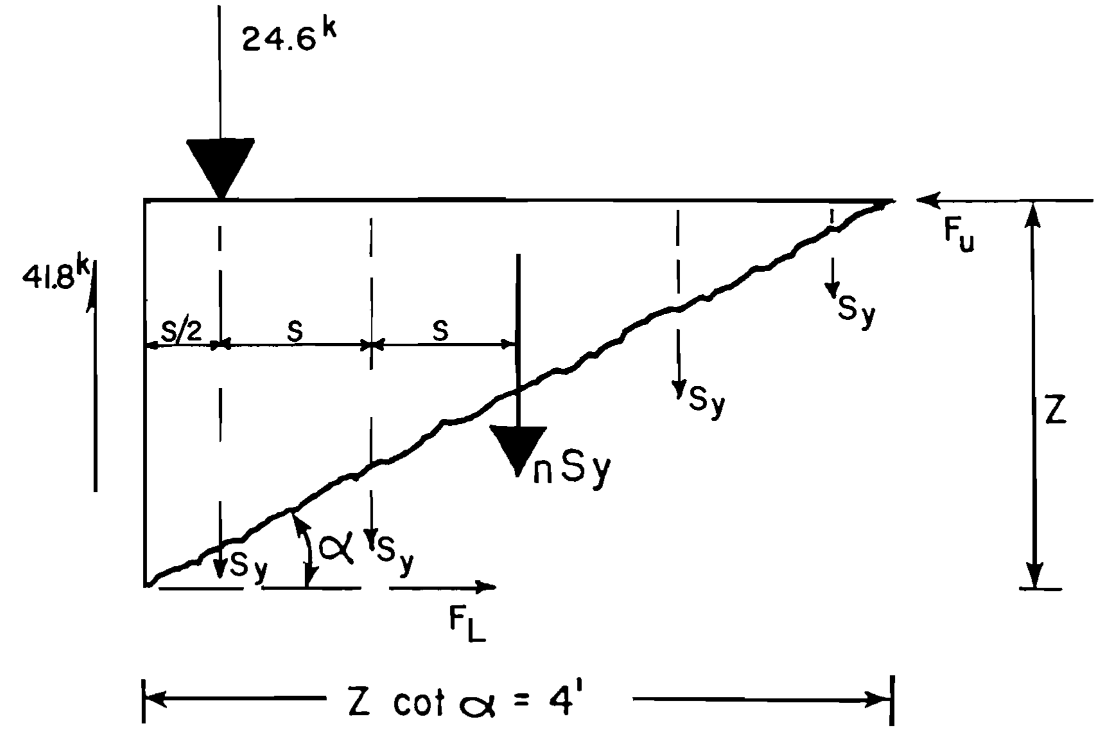
reinforcement and checking the web compression stress at each of the design zones.

Consider the case of design zone 1-2. Figure 4.37 indicates that the design live load shear could be taken either as 41.8 kips from the live load case shown in Fig. 4.36a or as 36.8 kips from live load case shown in Fig. 4.36b. In order to adequately select the design live load shear, it is necessary to look at the respective free bodies for each of the two loading cases. Figure 4.39a shows a free body for the loading case shown in Fig. 7.36a.

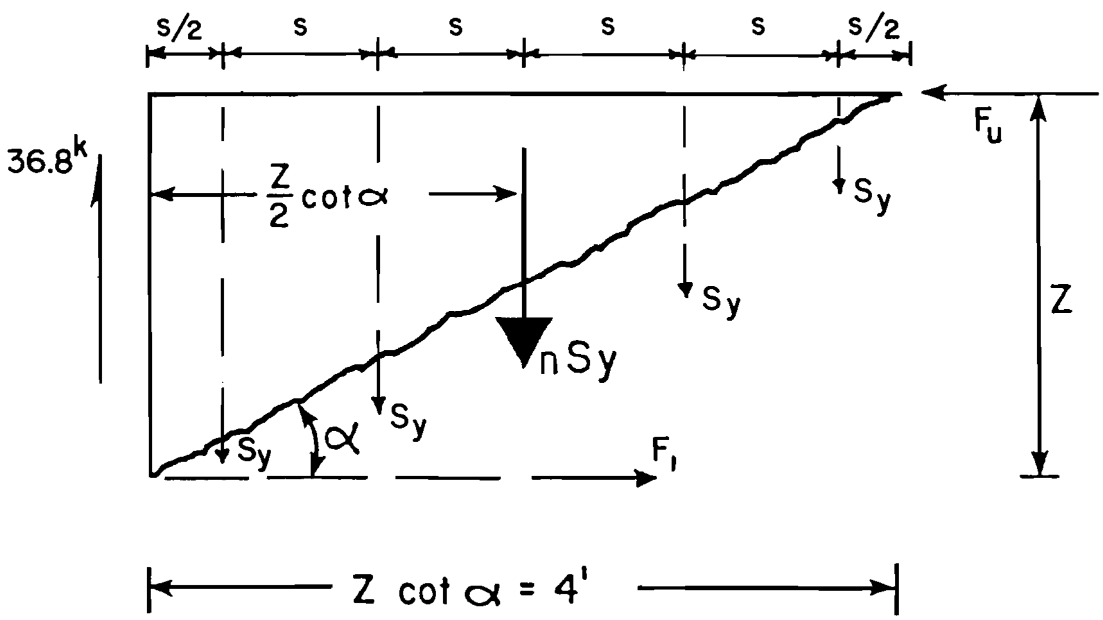
As can be seen from Fig. 4.39a, the equilibrium condition ΣF_y = 0 yields the actual force that will be carried by vertical reinforcement of the truss nS_y as $41.8 - 24.6 - nS_y = 0$; therefore, $nS_y = 17.2$ kips. Figure 4.38b shows a free body zone 1-2 for the loading case shown in Fig. 4.36b. Again, the equilibrium condition $\Sigma F_y = 0$ yields the vertical force that is carried by the vertical reinforcement of the the truss nS_y. In this case, $nS_y = 36.8$ kips. Thus, the maximum design live load shear force for the design zones 1-2 will be given by the loading case shown in Fig. 4.36b. The overall design free body for the design zone 1-2, including dead load effects, is shown in Fig. 4.40a. Figures 4.40b, 4.41a, 4.41b, and 4.42 show the design free bodies for zones 2-3, 3-4, 4-5, and 5-6, respectively.

Table 4.13 shows the calculated unfactored and factored maximum shears and respective moments for the five different design sections.

4.4.2 Evaluation of the Diagonal Compression Stresses. Once the angle of inclination has been selected and the design zones defined,

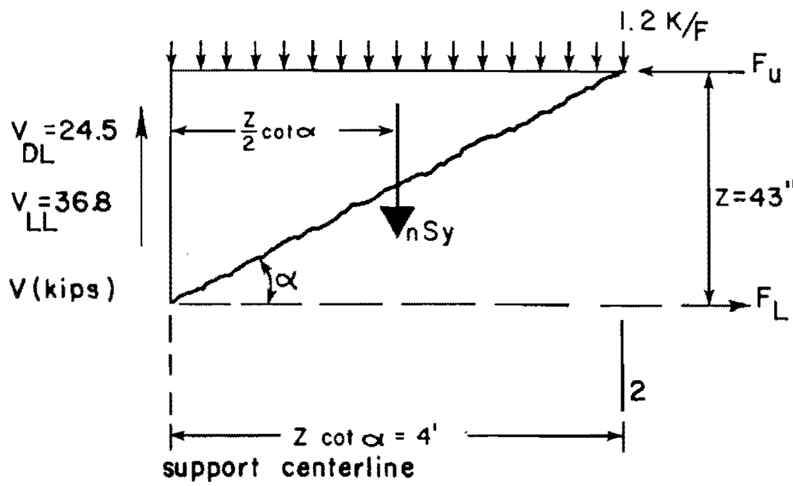


(a) Live load case from Fig. 4.36a

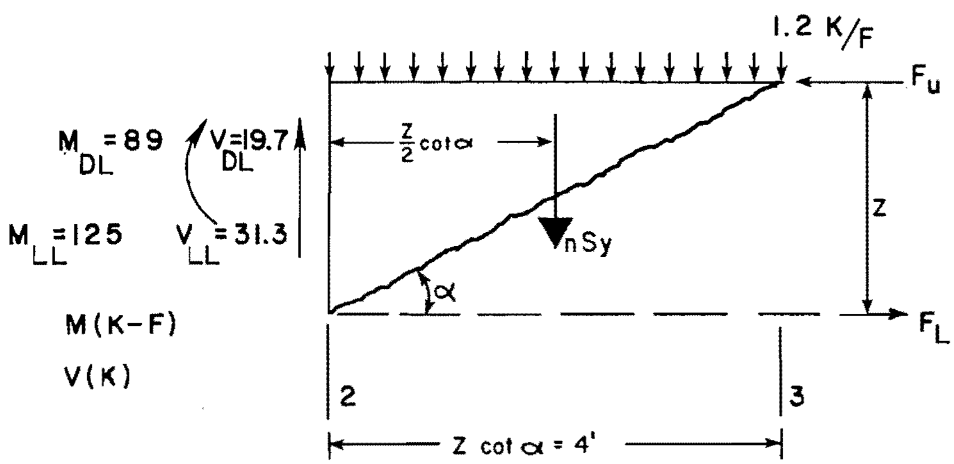


(b) Live load case from Fig. 4.36b

Fig. 4.39 Determination of design live load shear force at design section 1-2

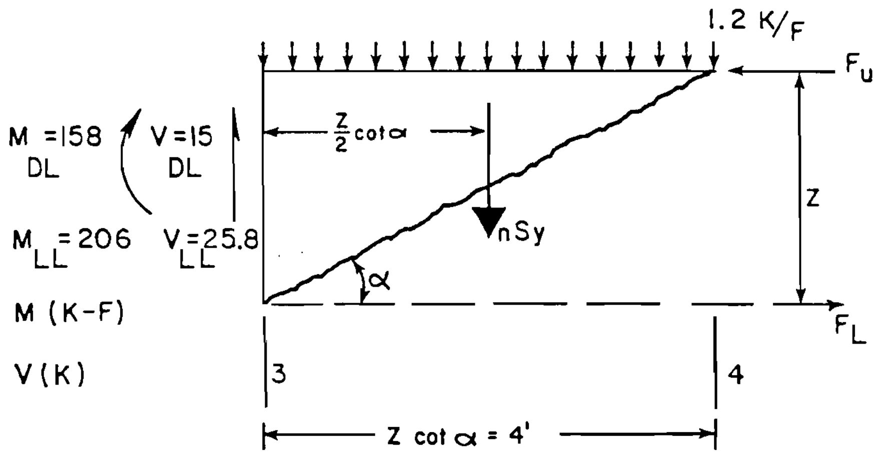


(a) Design free body for zone 1-2

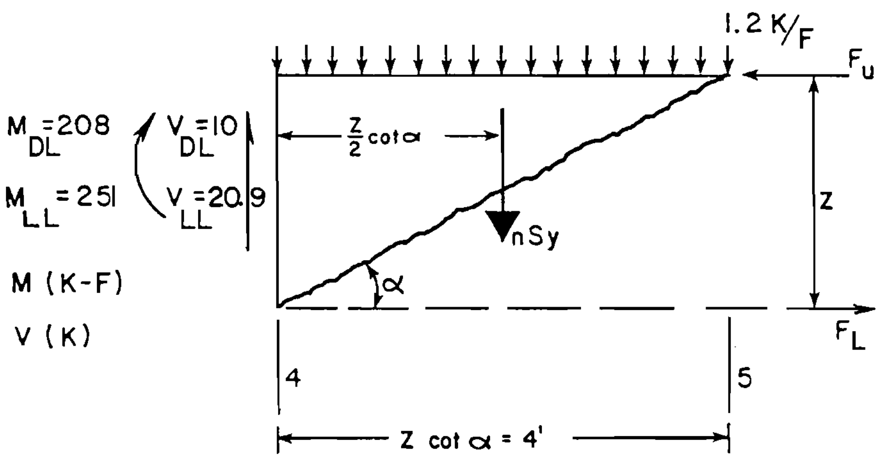


(b) Design free body for zone 2-3

Fig. 4.40 Design free bodies



(a) Design free body for zone 3-4



(b) Design free body for zone 4-5

Fig. 4.41 Design free bodies

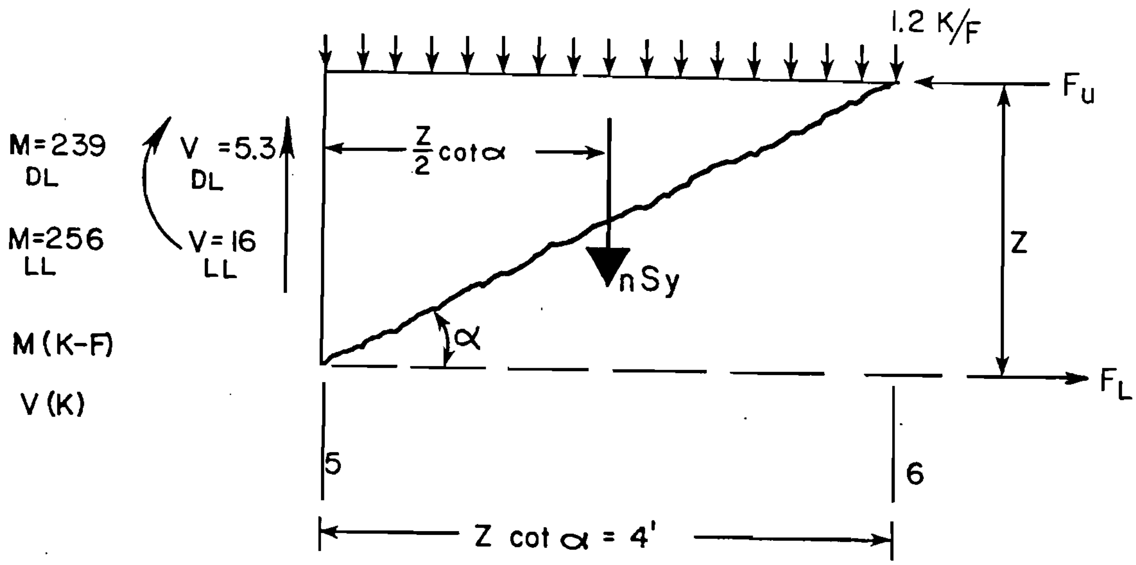


Fig. 4.42 Design free body for zone 5-6

the next step is the design procedure is to evaluate the diagonal compression stresses in order to prevent premature failures due to crushing of the diagonal members of the truss. This should be taken before detailed dimensioning of the reinforcement. This type of failure can be eliminated by limiting the compression stress f_d in the diagonal members of the truss to a value less than or equal to $30\sqrt{f'_c}$. Since in this design example f'_c = 5000 psi, then $f_d \leq$ 2.12 ksi.

As shown in Report 248-2, the compression stress in the diagonal strut obtained from geometric and equilbrium considerations using the truss model for the case of bending and shear is

$$f_d(V) = V_n/[b_w z \sin\alpha \cos\alpha] \quad (4.41)$$

where V_n is the nominal shear force V_u/ϕ where ϕ = 0.85, at the section where the design zone in consideration starts, b_w is the effective web width resisting the applied shear force, "z" is the depth of the truss model, and is the angle of inclination of the diagonal truss member. For this design example b_w = 7 in., z = 43 in., and α = 41.8 degrees.

Shown in Table 4.14 are the values of the compression stress, $f_d(V)$ in the diagonal strut, at the support centerline, and at sections 2, 3, 4, and 5.

As can be seen from Table 4.14, the maximum value of the compression stress f_d in the diagonal strut occurs at the support centerline and is equal to 0.90 ksi, which is way below the maximum allowed value of 2.12 ksi. This ensures that failures due to web crushing will not take place prior to yielding of the reinforcement.

4.4.3 Design of Transverse Reinforcement. Once the truss model has been selected and the compression stresses in the diagonal struts of the truss model have been evaluated to prevent premature failures due to web crushing, the internal forces in the truss can be evaluated to proportion the reinforcement.

The required amount of web reinforcement will be determined first. Shown in Fig. 4.43 is a typical design free body for one of the design zones shown in Figs. 4.40 through 4.42. V_u and M_u are the maximum factored shear force and the corresponding factored bending moment at section A where each of the design zones start. Those values are shown in Table 4.13. w_u is the factored distributed load since in this case there is only distributed dead load, w_u is equal to 1.3*1.2 = 1.6 k/ft. The vertical dimension of the truss model was found equal to 43 inches. Hence, the horizontal dimension of each of the design zones

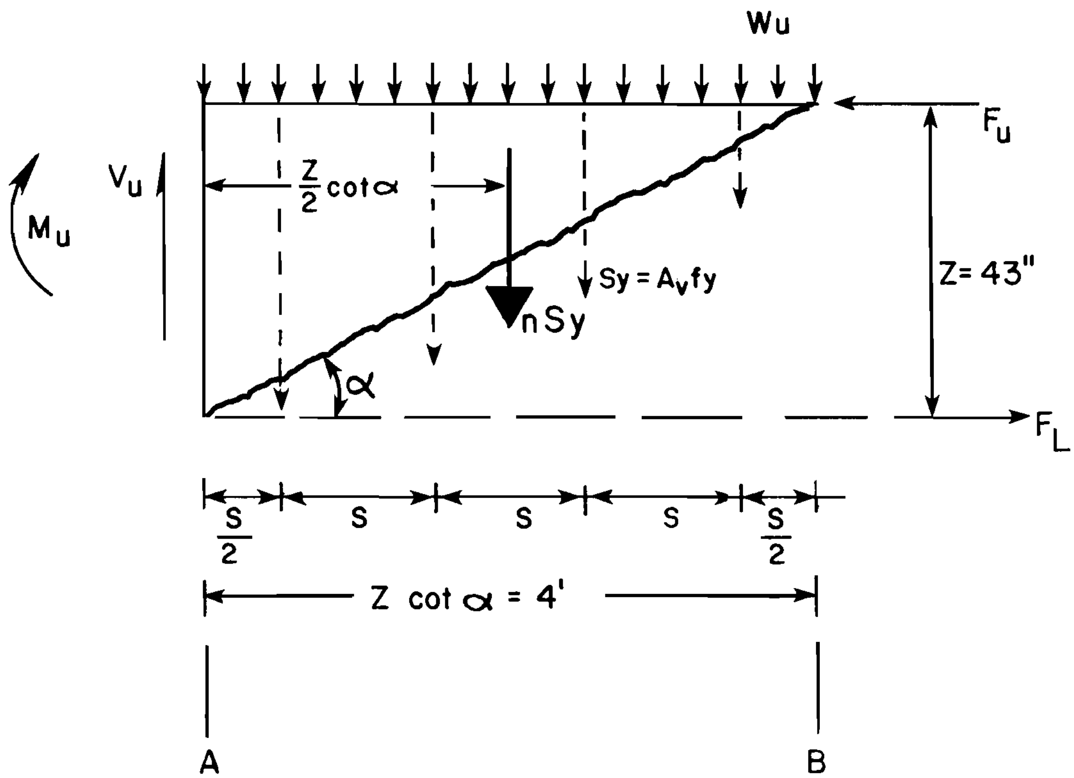


Fig. 4.43 Dimensioning of the stirrup reinforcement required to resist the applied shear force

$z\cot\alpha$, with chosen angle α of 41.8 degrees becomes 4 ft. Note that conveniently there are then 6 design zones between the support centerline and the midspan of the member. S_y is the stirrup yield force. Stirrup reinforcement within a design zone is assumed to be spaced uniformly and all at yield. Therefore, if several vertical tension ties (stirrups) of the truss cross the same diagonal strut, the shear carried by the truss is given by $V = nS_y$, where $n = z\cot\alpha/s$.

The amount of vertical web reinforcement is determined from the equilibrium condition $\Sigma F_V = 0$ in the free body of the design zone shown in Fig. 4.43, which yields the relation

$$V_u - w_u z\cot\alpha = \phi n S_y \qquad (4.42)$$

The lefthand side of Eq. 4.42 represents the ultimate load actions. The righthand side is the design strength provided by the vertical members of the truss system. Since $V_u \leq \phi V_n$ (Sec. 1.2.1.1 in Sec. 3.1), where $\phi = 0.85$, then

$$[V_u - w_u z\cot\alpha]/\phi = V_{TR} = nS_y \qquad (4.43)$$

The suggested AASHTO revisions proposed in Sec. 3.1 proposed that the concrete in the web may provide an additional contribution to the shear capacity of the member. This contribution then may be reflected in the design procedure by using a reduced value of the shear force when computing the required amounts of web reinforcement. However, this additional concrete contribution is only allowed where the member is in the uncracked or transition state. The proposed concrete contribution to the shear strength of prestressed concrete members is shown in Fig. 2.14b. This additional concrete contribution disappears when the level of shearing stress due to shear $v_u(V)$ in the member exceeds $[2 + K]2\sqrt{f'_c}$. The shearing stress due to shear $v_u(V)$ is given as $V_u/[b_w z]$. K is a factor which represents an increase in the shear strength provided by the concrete due to the presence of prestress. As indicated in Sec. 1.3.6b of the proposed AASHTO design recommendations of Sec. 3.1, the value of K has to be within the limits $1.0 \leq K \leq 2.0$, but it shall be taken equal to 1.0 at those sections of the member where the stress in the extreme tension fiber exceeds the value $6\sqrt{f'_c}$, in this case $6\sqrt{5000} = 420$ psi. To evaluate the additional concrete contribution to the shear strength of the member the prestress factor K has to be evaluated at each of the design sections using Eq. 4.44.

$$K = [1 + (f_{ps}/2\sqrt{f'_c})]^{0.5} \qquad (4.44)$$

where f_{ps} is the compression stress at the neutral axis of the section due to applied axial forces (including effective prestressing) or at the junction of web and flange when the centroid lies within the flange. As can be seen from Fig. 4.44a, the centroid of the composite section lies within the web of the member. Thus, the compression stress should be evaluated at the neutral axis of the beam as

$$f_{ps} = F_{se}/A_b \qquad (4.45)$$

where F_{se} is the effective prestress force after all loses, A_b is the area of the beam. For this design example $F_{se} = f_{se}A_{ps} =$

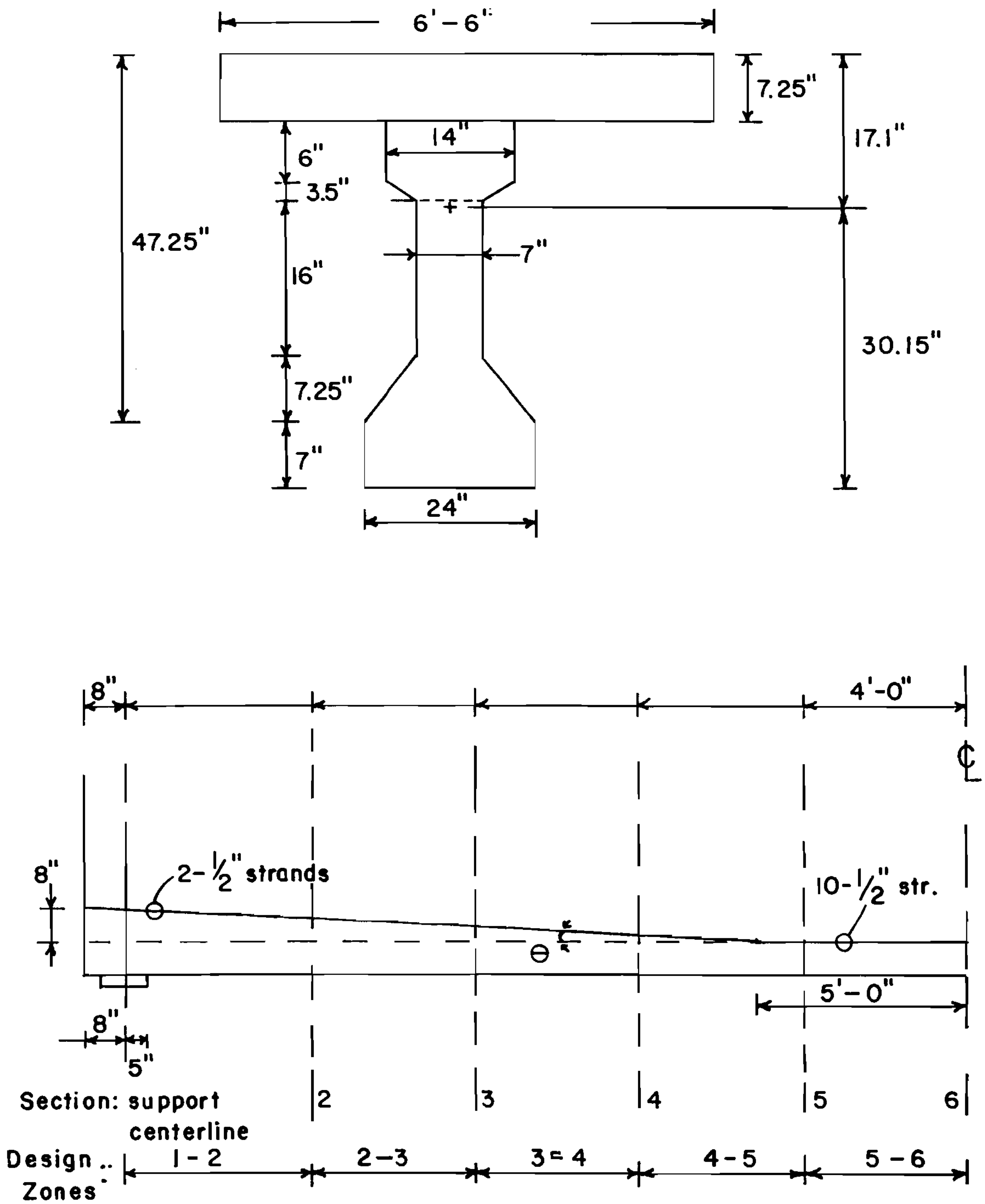


Fig. 4.44 Evaluation of the additional concrete contribution

(152.5)(10)(.153) = 233 kips. The area of the beam is 495 in.^{2} Thus, f_{ps} is 0.470 ksi. Shown in Table 4.15 are the actual K values for each design section evaluated using Eq. 4.44. However, as indicated in Sec. 1.3.6b of the proposed design recommendation the value of K has to be within the limits $1.0 \leq K \leq 2.0$, but must be taken equal to 1.0 at all those design sections of the member where the stress in the extreme tension fiber exceeds the value of $6\sqrt{f'_c}$. For this design example, $6\sqrt{f'_c}$ is equal to 0.420 ksi. Shown in Table 4.15 are the values of the stress in the extreme tension fiber at each of the design sections. Whenever that value exceeds 0.420 ksi (tension), K is taken equal to 1.0. Also shown in Table 4.15 are the design values of K. The values of the additional concrete contribution to the shear capacity (V_c) evaluated in accordance with the provisions presented in Sec. 3.1 (1.3.6b) for the case of shear in prestressed concrete members are given in Table 4.15.

As previously explained in Sec. 2.6.3 of Report 248-3, for the case of prestressed concrete members with draped strands there is an additional contribution to the shear strength of the member provided by the component of the effective prestress force V_p at the section in the direction of the applied shear force. As can be seen from Fig. 4.44, two of the 10-1/2" diameter strands are draped up into the web of the member. The vertical component of the prestressing force can be determined from the geometry of the figure or $V_p = F_{se}\sin\theta$, where θ is the angle of draping. In this case θ = arctan 8/(15*12 + 8) = 2.44 degrees. F_{se} is the effective prestressing force of the two strands F_{se}

= 2(.153)(152.5) = 46.6 kips. Thus, the vertical component V_p is 46.6*sin(2.44) = 1.98 kips. Shown in Table 4.15 are the values of the additional contribution to the shear strength of the member due to the presence of draped strands V_p.

For those regions of the member in the uncracked or transition state, where the concrete in the web provides additional shear strength V_c, Eq. 4.43 becomes

$$(V_u - w_u z\cot\alpha)/\phi = V_n = V_{TR} + V_c + V_p \tag{4.46}$$

Rearranging Eq. 4.44 results in

$$[(V_u - w_u z\cot\alpha)/\phi] - V_c - V_p = V_{TR} = nS_y \tag{4.47}$$

Since ns = $z\cot\alpha$ and $S_y = A_v f_y$

$$A_v/s = [(V_u - w_u z\cot\alpha)/\phi - V_c - V_p] \tan\alpha / zf_y \tag{4.48}$$

where A_v/s is the area of stirrups resisting the factored shear force per inch of the stirrup spacing "s", and f_y is the yield strength of the stirrup reinforcement. For this design example, f_y = 60000 psi, α = 41.8 degrees, hence $\tan\alpha$ = 0.90.

Eq. 4.48 is used to design the web reinforcement required to resist the factored shear force. Shown in row (7) of Table 4.16 are the required amounts of web reinforcement per inch of stirrup spacing "s" for each of the design zones. Row (8) contains the minimum amount of web reinforcement which must be provided whenever the factored shear stress (V_u) exceeds the value $1.0\phi\sqrt{f'_c}$, where ϕ = 0.85. The minimum amount of web reinforcement is evaluated in accordance with the requirements of Sec. 1.4.2.1 of tne proposed design recommendations. Hence,

$$(A_v/s)\min = 1.0\sqrt{f'_c}\ b_w/f_y \qquad (4.49)$$

As can be seen from Table 4.15, the value of the shear stress due to the factored shear force ($v_u[V]$) at all the design sections exceeds the value of $1.0\,\phi\sqrt{f'_c} = 1.0(0.85)\sqrt{5000} = 0.06$ ksi, hence at least the minimum amount of web reinforcement must be provided in all the design zones.

Row (9) of Table 4.16 shows the required stirrup spacing if a Grade 60 #3 U stirrup is used as web reinforcement. Row (10) indicates the maximum allowed stirrup spacing as required by Sec. 1.4.2.6 of the proposed design recommendations. Therefore, in the design zone 1-2 #3 U stirrups at 9.5 inches center-to-center should be provided. In the design zones 2-3, 3-4, 4-5, and 5-6, #3 U's at 12 inches must be provided. The U stirrups shall be terminated in the compression zone with a 135 degree hook at the ends.

4.4.4 Evaluation of the Compression Stresses in the Fan Regions. As explained in Secs. 4.3.5, the presence of concentrated load disturbs the continuous uniform compression field of the truss. The presence of a concentrated load introduces a series of diagonal compression forces which fan out from the concentrated load. Hence, in this design example compression fans will form at both supports where the reaction introduces compression, and under the concentrated truck wheel loads. As previously explained, the geometry of the compression fan depends on the spacing of the transverse reinforcement and the chosen angle α. Figure 4.45 shows the compression "fan" generated at the supports of the composite I girder. Column (5) of Table 4.17 shows the compression forces generated at each of the joints of the truss in the compression fan zone. Column (6) shows the diagonal compression

(1) Point (i)	(2) α (i) (degrees)	(3) tan α (i)	(4) S_i	(5) $D_i = \frac{[S_i + w_n s]}{\sin \alpha (i)}$ (kips)	(6) f_{di} (ksi)
1	83.7	9.1	S_y	14.79	0.450
2	71.4	2.97	S_y	15.51	0.160
3	61.1	1.81	S_y	16.79	0.120
4	52.3	1.29	S_y	18.58	0.10
5	45.2	1.01	S_y	20.72	0.10

Table 4.17 Diagonal compression stresses in the fan region

stresses induced by the diagonal compression forces shown in column (5). As previously explained, the diagonal compression stresses at each of the joints (i) of the truss is given as:

$$f_{di} = D_i/b_w \, z\cos\alpha_i \tag{4.50}$$

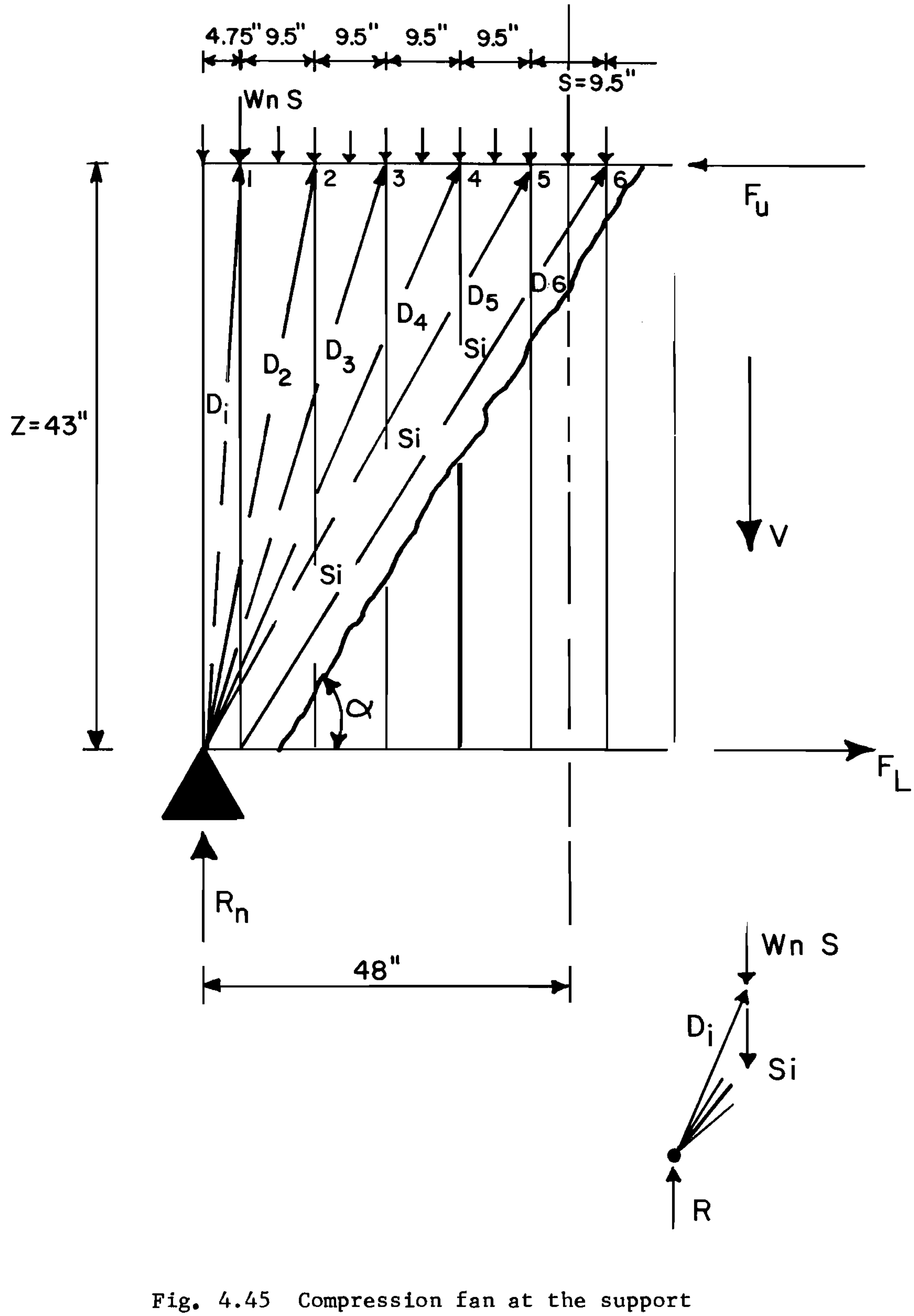


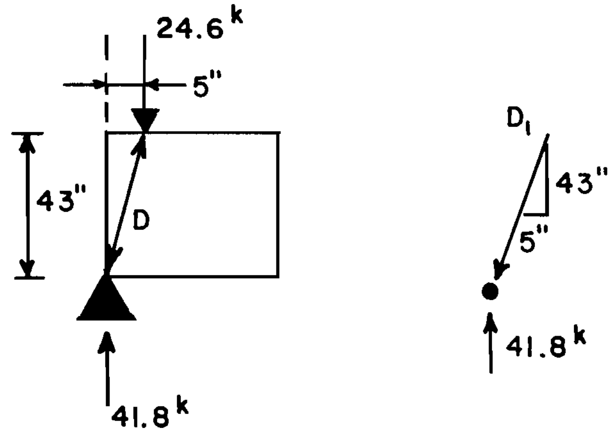
Fig. 4.45 Compression fan at the support

where b_w = 7 inches, z = 43 inches. For this design example #3 U stirrups Grade 60 are used as web reinforcements; thus, $S_y = 2(0.11)(60) = 13.2$ kips, $w_n = w*L.F./\phi = 1.2*1.3/0.85 = 1.8$; hence, $w_n*s = 1.8*9.5/12 = 1.5$ kips.

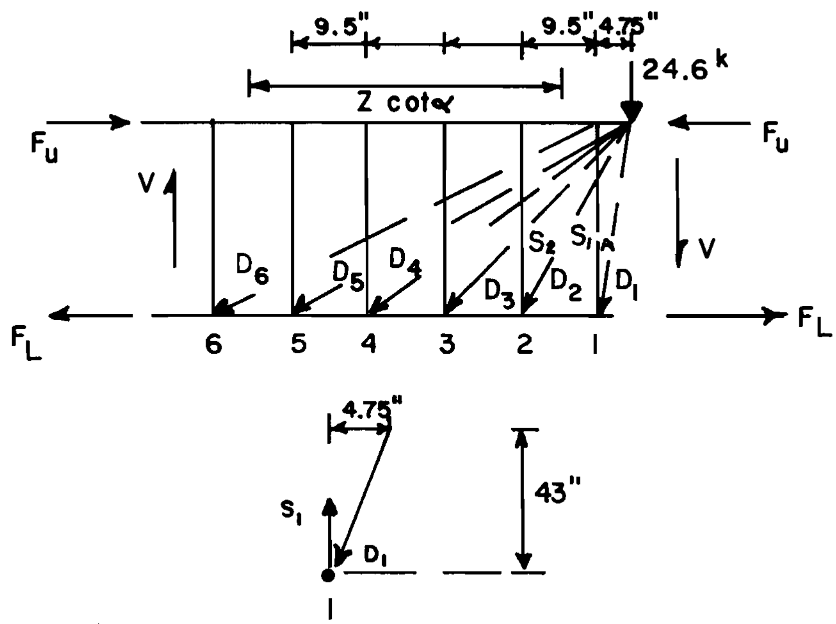
The compression stresses evaluated using Eq. 4.50 must be below the allowed maximum value of $30\sqrt{f'_c}$ in this case $f_{di} < 2.12$ ksi. As can be seen from the values shown in column (6) of Table 4.17, the compression stresses in the fan region are always below the maximum allowed compression stress.

Compression fans will also form under the concentrated truck loads, and the compression stresses need to be checked at these regions. In reality the truck can be at any position on the girder. It should be sufficient to check the cases where wheels are at panel points. As can be seen from Fig. 4.36, there will be 6 different compression fans to check. For the live loading case shown in Fig. 4.36a, the geometry of the diagonal compression strut is defined by the distance between the centerline of the support and the point of application of the load. Figure 4.46a shows the geometry of the diagonal compression strut for this case. From Fig. 4.46a the diagonal force in the compression strut is $D_i = R/\sin\alpha_i = 41.8/\sin 83.4 = 42.1$ kips. Thus, the compression stress in the diagonal strut results in $f_{di} = D_i/b_w z\cos\alpha_i = 42.1/(7)(43)(\cos 83.4) = 1.22$ ksi, which is less than the maximum allowed of 2.12 ksi.

For the live loading case of Fig. 4.36b the geometry of the compression fan is shown in Fig. 4.46b. As can be seen from Fig. 4.46b,



(a) Geometry of the diagonal compression strut for live load case from Fig. 4.36a



(b) Compression fan under the concentrated load for live load case from Fig. 4.36b

Fig. 4.46 Evaluation of the compression stresses in the fan region under the applied concentrated loads the geometry of this compression fan is the same as for the case of the compression fan at the support shown in Fig. 4.45. However, the diagonal compression for D_i is now given as $S_i/\sin\alpha$. The compression stresses are below those shown in Table 4.17 and, therefore, also below the specified limit of $30\sqrt{f'_c}$.

The compression fan under the applied load for the live loading case of Fig. 4.36c is shown in Fig. 4.47. As can be observed from Tables 4.9 and 4.17, the first (steeper) diagonal compression strut in the fan region is always the critical one. The compression stress drops in the subsequent less steeper struts. Thus, it will suffice to check if the stress in the first (steeper) diagonal compression strut in the fan region is below the maximum allowed value of $30\sqrt{f'_c}$. For the case shown in Fig. 4.47, $D_1 = S_1/\sin\alpha_1$, α_1 = arctan 43/6 = 82.06 degrees, $S_1 = A_v f_y$ = 2(0.11)(60) = 13.2 kips. Thus, D_1 = 13.2/sin82.06 = 13.3 kips. Therefore, $f_{d1} = D_1/b_w z\cos 82.06$ = 13.3/(7)(43)(0.138) = 0.32 ksi, which is below the maximum allowed 2.12 ksi. Following the same procedure for the loading case shown in Fig. 4.36d, $D_1 = S_1/\sin\alpha_1$, α_1 = arctan 43/12/2 = 82.06, $S_1 = A_v f_y$ = 2(0.11)(60) = 13.2. Hence, D_1 = 13.2/sin82.06 13.33 kips. Thus, f_{d1} = 13.33/(7)(43)cos 82.06 = 0.320 ksi, which again is below the maximum allowed of 2.12 ksi.

The geometry of the compression fans under the applied load for the loading cases in Figs. 4.36e and 4.36f is similar to the case shown in Fig. 4.47 since the angle alpha is the same as well as the stirrup spacing. Thus, the compression stresses in the diagonal struts for these two cases would also be under the maximum allowed value of 2.12

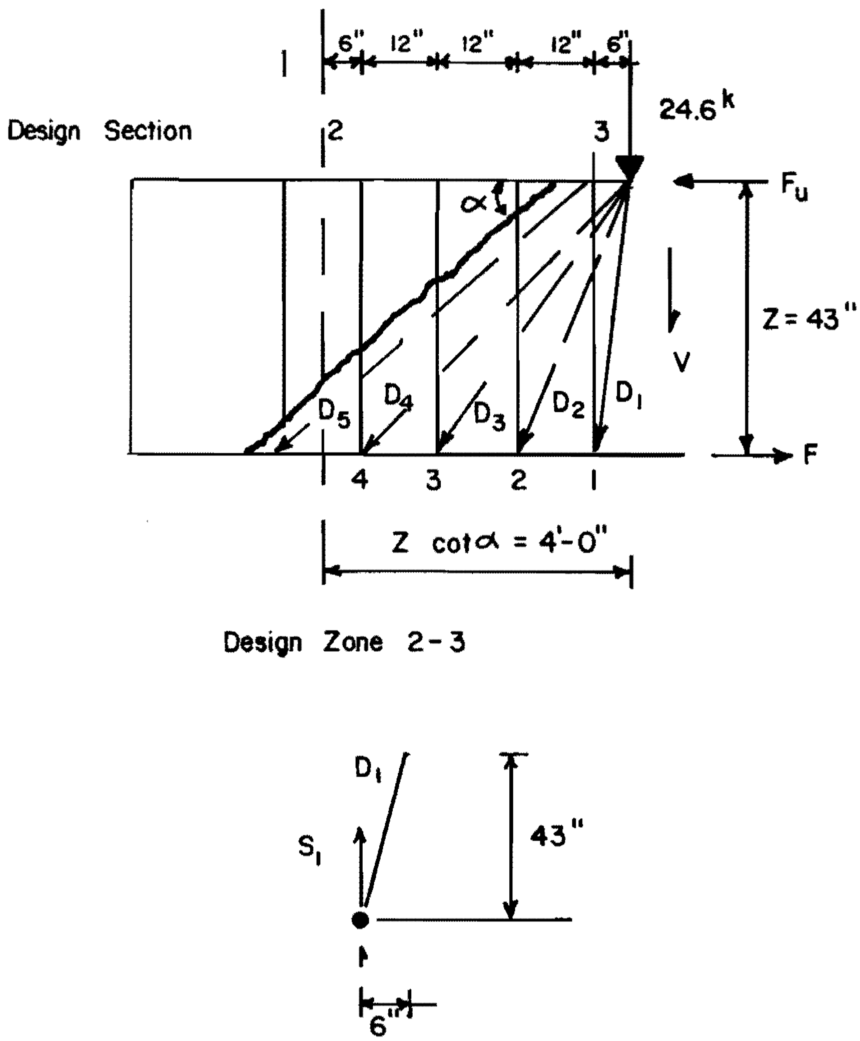
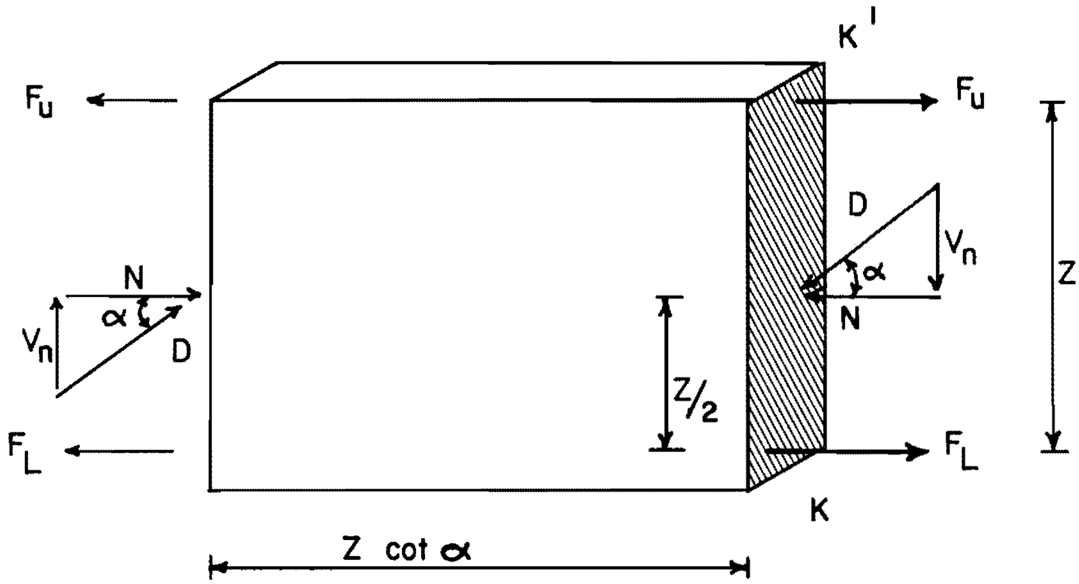


Fig. 4.47 Compression fan under the concentrated load for live load case shown in Fig. 4.36c

ksi. This example indicates that with experience the check of fan stresses can be minimized by checking the most critical case or cases.

4.4.5 Dimensioning of the Longitudinal Reinforcement Required for Shear. As previously explained in Sec. 3.6.1 of Report 248-2, the resultant diagonal compression force due to the presence of shear induces vertical and horizontal compression components which must be balanced by vertical and horizontal tension forces. Hence, the presence of a shear force induces not only vertical tension forces which must be resisted by the stirrup reinforcement, but longtitudinal tension forces as well. The area of longitudinal reinforcement required due to the presence of shear V_n is in addition to the area required for bending and is determined from the equilibrium condition $\Sigma F_H = 0$ in the truss model. If a constant stirrup spacing "s" is used throughout the design region $z\cot\alpha$, and a uniform compression field exists, then the horizontal component of the diagonal compression field is located at midheight of the member (z/2). In Fig. 4.48 the equilibrium condition for summation of horizontal forces is applied to determine the additional reinforcement required due to the presence of shear. If the horizontal component of the diagonal compression field is located at midheight of the member, then it is reasonable to assume that it will be equally resisted between the upper and lower chords of the truss model. Hence, the additional area of longitudinal steel due to shear $A_l(V)$ for each chord is

$$A_l(V) = [V_n \cot\alpha]/[2f_y] \qquad (4.51)$$



$$\Sigma F_H (k-k') = 0 = F_u + F_L - N$$

$$N = V_n \cot \alpha; \quad F_u = F_L$$

$$A_L = \frac{V_n \cot \alpha}{2 f_y}$$

Fig. 4.48 Determination of the additional amount of longitudinal reinforcement required due to shear where "V_n" is the nominal shear force V_u/ϕ, $\phi = 0.85$, and f_y is the yield stress of the longitudinal reinforcement forming the truss chord. For this design example, α = 41.8 degrees and f_y is the yield strength of the strands used as prestressed reinforcement (f_y = 259 ksi). Therefore, the design of the longitudinal reinforcement required for shear should be conducted at each design zone, $z\cot\alpha$ = 4'-0". Table 4.18 shows the required amounts of longitudinal reinforcement to resist shear and bending at both the top and the bottom truss chords. The tension force due to shear, $A_1(V)f_y$, at the top is balanced by the compression force produced by the applied moment, and only where the resultant is tension would an additional area of steel have to be provided. As shown in Sec. 2.2.2 of Report 248-3, the presence of compression fans where the reaction induces compression eliminates the need for the additional area of longitudinal steel due to shear within a distance $\frac{z}{2}\cot\alpha$(2 ft for this design example) from the centerline of the support in the face of the member where the applied moment induces compression. For this reason, the values computed in rows (4) and (5) of Table 4.18 for the design zones 1-2 are evaluated at a distane of 2' from the support centerline. As can be seen by the comparison of rows (4) and (5) shown in row (6) of Table 4.18, only at the design zone 1-2 does an unbalanced tension force exist at the top face of the member. The unbalanced tension force at the top face of the member in the design zone 1-2 is equal to 10 kips. In the standard detailing of this member's two #5 bars f_y = 60 ksi are always provided at the top compression face of the member. Thus, the available tension force is

(1) Design Section	Support Centerline	2	3	4	5
(2) Distance from the support centerline (ft)	0'-0"	4'-0"	8'-0"	12'-0"	16'-0"
(3) Design Zone	1-2	2-3	3-4	4-5	5-6
(4) Tension force of the top face of the member A_L f_y due to shear (kips) (f_y = 259 ksi)	71	61	49	38	27
(5) Compression resultant flexure (kips)	61	108	182	227	241
(6) Net amount of tension force (4-5) (kips)	10	—	—	—	—
(7) $A_L(v)$ required at the bottom face of the member due to shear (in^2) (f_y = 259 ksi)	0.29	0.24	0.19	0.15	0.11
(8) A_L required at the bottom due to flexure (f_y = 259 ksi)	0.0	0.49	0.82	1.04	1.10
(9) A_L required due to shear and bending (7+8) (in^2)	0.29	0.73	1.01	1.19	1.21
(10) A_L provided (in^2) (f_y = 259 ksi)	1.22	1.22	1.22	1.22	1.53

Table 4.18 Longitudinal reinforcement requirements

2(0.31)(60) = 37.2 kips, which is enough to take care of the additional requirement due to shear.

Shown in row (7) of Table 4.18 is the additional area of longitudinal reinforcement due to shear $A_l(V)$, evaluated using Eq. 4.51. Shown in row (8) is the area required for flexure for each of the design zones of the truss. The area is evaluated using the relation

$$A_{l\ total}(M) = M_n/zf_y \tag{4.52}$$

where M_n is the nominal moment M_n/ϕ at the section where the design zone starts, z is the vertical dimension of the truss model (43 in.), and f_y is the yield strength of the longitudinal reinforcement (f_y = 257 ksi). Equation 4.52 was previously derived in Sec. 3.5.1 of Report 248-2.

Row (9) shows the total area of longitudinal reinforcement due to shear and bending (row (7) + row (8)) required for each of the design zones. A comparison of the value shown in row (9) with the total area of longitudinal reinforcement provided at each of the design zones shown in row (10) indicates that the requirements for longitudinal reinforcement at all the design zones would be adequately satisfied.

Finally, the adequate anchorage of the longitudinal prestressed reinforcement at the support regions must be checked. As was previously shown, because of the presence of compression fans at the support regions, the longitudinal reinforcement which continues into the support has to be provided with an anchorage length such that a force $V_u \cot\alpha/2$ is adequately developed. In this case $V_u \cot\alpha/2$ is equal to

112*cot(41.8)/2 = 63 kips. The ACI Building Code Commentary (2) in Sec. 12.10 indicates that the transfer length l_d of strand required to achieve the effective prestressing stress is given as

$$l_d = (f_{se}/3)d_b \qquad (4.53)$$

where f_{se} is the effective prestressing stress in the strand after all loses, in this case f_{se} = 152.7 ksi, d_b is the nominal diameter of the strand. It is also indicated that this stress varies linearly with the distance from free end of strand to the distance where the stress f_{se} is developed in the strand. Thus, for this design example l_d = 152.7*0.5/3 = 25.5". As shown in Fig. 4.33, the distance between the centerline of the support and the end of the beam is 8 in. Thus, the stress that can be developed in the strand up to that point is f_{se} = 8*152.7/25.5 = 48 ksi and the force that could be developed per strand is 48*(.153) = 7.4 kips. Since eight 1/2 in. strands are continued straight into the support, the total required force of 63 kips should be equally developed between those eight strands. Hence, the force to be developed at each strand is equal to 63/8 = 7.8 kips. Since the available anchorage force (7.4 kips) is very close to the required (7.8 kips) and due to the empirical nature of Eq. 4.53, it is then suggested that no special provision be taken and assume that the required force can be adequately developed.

4.4.6 Design of the Prestressed Concrete Bridge Girder Following the ACI/AASHTO (1,2) Design Procedure. To show the difference in design procedures, the same example previously studied is reworked using current design procedures.

The member is divided in 5 design zones. The first one is located at a distance h/2 = 47.25/2 = 23.63" = 1.97 ft from the face of the support and then 4 ft, 8 ft, 12 ft, and 16 ft from the centerline of the support, respectively. Since in this design example maximum shear envelopes and corresponding moments are used, and since the truck live loading can approach the bridge from either side, the design of the other half of the bridge girder would be essentially the same. Table 4.19 shows the design of the transverse reinforcement according to the ACI/AASHTO requirements.

In the ACI/AASHTO recommendations, the first critical region for shear in the case of prestressed concrete members where the support reaction induces compression is located at a distance h/2 from the face of the support. Sections located less than a distance h/2 from the face of the support may be designed for the same shear V_u as that computed at a distance h/2, except when there are heavy concentrated loads within the distance h/2 such as the loading case shown in Fig. 4.36a. In such cases, the member should be designed for the actual shear at that critical section taking into account the heavy concentrated load. Thus, the first design region is located 5" from the support centerline which is the face of the support.

The current AASHTO/ACI procedures define the additional concrete contribution to the shear strength of the member in the case of prestressed concrete sections, as given by the smaller of the two values

(1) Design Section	Face of Support	2	3	4	5
(2) Distance from the support centerline (ft)	0'-5"	4'-0"	8'-0"	12'-0"	16'-0"
(3) V_{ci} (kips)	572	258	128	83	65
(4) V_{cw} (kips)	104	105	105	103	103
(5) $V_n = V_u/\phi$ (kips)	144	109	88	68	49
(6) $V_s = V_n - V_c$ (kips)	40	4	---	---	---
(7) $A_v/s = V_s/f_y d$ (in^2/in) f_y = 60 ksi	0.018	0.002	---	---	---
(8) Min. amt. (in^2/in)	0.006	0.006	0.006	0.006	0.006
(9) Spacing for a #3 U stirrup (in)	12.5	37.5	37.5	37.5	37.5
(10) Max. allowed stirrup spacing (in)	12	12	12	12	12

Table 4.19 Dimensioning of the web reinforcement for the bridge girder following ACI/AASHTO recommendations

V_{ci} or V_{cw}. V_{ci} is the shear force required to produce first flexural cracking and then cause this flexural crack to become inclined. V_{cw} represents the shear force required to produce first inclined cracking in the web of the member. These two shear mechanisms have been previously explained in Sec. 2.3.1.

Row (3) shows the values of V_{ci} for each of the design sections. These values are evaluated using Eq. 4.54.

$$V_{ci} = 0.6\sqrt{f'_c}\, b_w d + V_D + [V_i M_{cr}/M_{max}] \qquad (4.54)$$

where V_D is the shear force at the section due to the unfactored dead load, V_i is the factored shear force at the section due to externally applied loads occurring simultaneously with M_{max}, M_{max} is the maximum factored moment at the section due to externally applied loads. The evaluation of the ratio V_i/M_{max} causes a great deal of confusion in the shear design of prestressed concrete bridge members because it has to be evaluated at several sections along the span of the member. In addition, in the cases of members subjected to moving loads such as this design example, the loading combinations used to evaluate the maximum shear at a section are different than those used to evaluate the maximum moment. Hence, the question arises about which of the two loading combinations should be used in the evaluation of the V_{ci}. It would seem apparent that since the mechanism which is represented by the V_{ci} equation is that of the shear force required to produce first flexural cracking this would be associated with the maximum moment at the section. Therefore, the loading combination used should be that which produces maximum moment at the section under consideration. However, the amount of web reinforcement V_s would be determined from the relation $V_s = V_n - V_{ci}$, where $V_n = V_u/\phi$. Since V_u is the maximum factored shear force at the section then it would be determined from the loading case producing maximum shear at the section. Thus, in this case the reinforcement would be designed with the combination of two effects from two different loading cases. This discrepancy has led designers to simply use for the value of M_{max} the bending moment at the section which is associated with the loading case producing the maximum shear force at that section. Such a procedure is followed in this design example.

Row (4) of Table 4.19 shows the values of V_{cw} for each of the design sections. These values are evaluated using Eq. 4.55.

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3f_{pc})b_w d + V_p \qquad (4.55)$$

where f_{pc} is the compressive stress in the concrete (after allowance for all prestress losses) at the centroid of the cross section resisting the externally applied loads or at junction of web and flange when the centroid lies within the flange. In this design example the centroid of the composite cross section lies within the web. Thus, f_{pc} is simply given as F_{se}/A_b, where $F_{se} = f_{se}A_{ps}$, and A_b is the area of the precast bridge girder. V_p is the vertical component of the effective prestress force at the section.

Row (6) shows the amount of shear strength that has to be provided by the web reinforcement V_s. The amount of shear carried by the web reinforcement is evaluated using Eq. 4.56.

$$V_s = A_v f_y d/s \tag{4.56}$$

Rearranging Eq. 4.56 yields

$$A_v/s = V_s/f_y d \tag{4.57}$$

where A_v/s is the area of web reinforcement required per inch of longitudinal spacing s, f_y is the yield strength of the reinforcement (in this case f_y = 60000 psi). Shown in row (7) of Table 4.19 are the required amounts of web reinforcement at each of the design sections in accordance with the ACI/AASHTO requirements.

Row (8) shows the minimum amount of web reinforcement that has to be provided whenever the factored shear force at the section exceeds the value of 1/2 V_c. As can be seen from comparing rows (3), (4), and (5), a minimum amount would have to be provided at all the design sections. This minimum amount for this design example is evaluated using Eq. 4.58.

$$(A_v/s)_{min} = 50 b_w/f_y \tag{4.58}$$

which for this design example is equal to 0.006.

Row (9) shows the required stirrup spacing if a Grade 60 #3 U stirrup is used as web reinforcement. Row (10) shows the maximum allowed stirrup spacing. In this case the requirement that the spacing of vertical stirrups cannot exceed the 12" spacing required for adequate horizontal shear transfer controls at all design sections.

4.4.7 Comparison between the Amounts of Web Reinforcement Required by the Truss Model and the ACI/AASHTO Design Procedure. Figure 4.49 shows a comparison between the amounts of web reinforcement required by the truss method and the current ACI/AASHTO design procedures. As can be seen from Fig. 4.49 is obvious that minimum spacing requirements almost totally controls the shear design of this member. The fact that minimum requirements controlled the design of this specimen, in spite of the short span (40 feet) intended to maximize the shear, supports the idea that flexure would always control the design of this type of member. Futhermore, while the design using the truss model appears far more rational, it can be seen that the end product is virtually identical to that given by the current ACI/AASHTO procedures.

4.5 Summary

In this chapter several numerical design examples have been given to show the application of the truss model to different design situations .

In Sec. 4.2.4 it was shown how with the aid of the truss model the designer is able to handle complex design situations. The adaption of the truss model would give the most benefit in treating such complex cases of irregular sections, unusual loading or complex combined loading conditions. Once the truss model has been selected for the particular case, the design procedure becomes relatively simple and straight forward. Experience with the solution of the truss model would

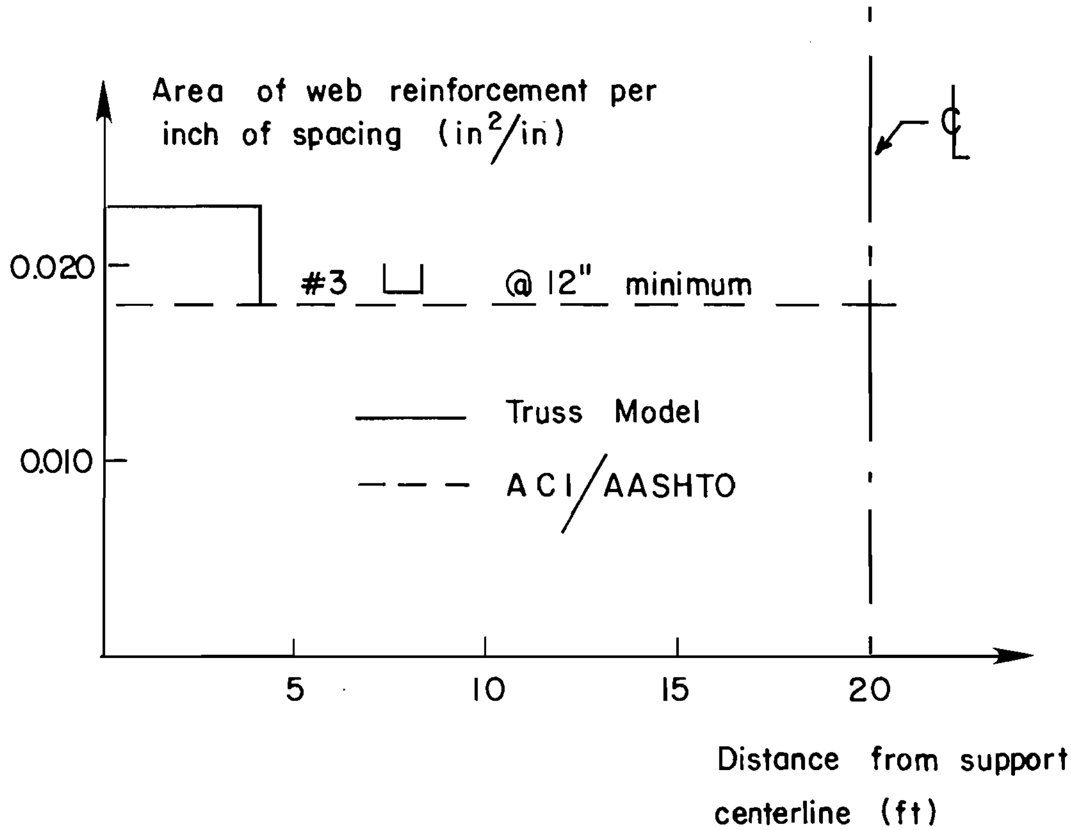


Fig. 4.49 Comparison of transverse reinforcement for bridge girder by the space truss model and the AASHTO specifications

greatly simplify the computations illustrated which went into great detail to show all facets of the solution.

Finally, a comparison between the amounts of reinforcement indicated by the truss model approach and the current AASHTO Specifications was given for two examples. Both procedures resulted in approximately the same amounts of reinforcement. A significant change in the distribution of the additional longitudinal reinforcement for combined shear and torsion was indicated. The truss model distribution seemed far preferable and much more rational. In spite of the generally equal amounts of longitudinal steel used, the empirical equations for A_l in the current AASHTO requirements are clearly in error because of the relation with V_u. The present AASHTO requirement results in low amounts of longitudinal steel in high shear zones. The opposite seems to be the true requirement. The versatility and rationality of the truss model approach make this method a preferable one.

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 Summary

The principal objective of the investigation reported herein was to propose and to evaluate a design procedure for shear and torsion in reinforced and prestressed concrete beams, with the aim of clarifying and simplifying current design recommendations and AASHTO requirements in such areas. The scope was limited to the design of reinforced and prestressed concrete one-way flexural members subjected to shear and/or torsion.

A comprehensive review of the current AASHTO and ACI design procedures for shear and torsion in reinforced and prestressed concrete beams was reported in Report 248-2. An effort was made to try to illustrate the factors that previous researchers considered to be of great influence in the overall behavior of members subjected to shear and/or torsion.

Because of the more abrupt nature of shear and torsion failures, and the difficulty of formulating reliable mathematical models for the behavior of beams in shear and torsion, research has usually tended to concentrate on predicting the collapse load of those members on an empirical basis.

Unfortunately, from a scientific standpoint an empirical approach is only correct if the separation and control of the main variables in the test program is assured, and if sufficient tests are conducted to allow a valid statistical treatment of the results. In testing structural components or entire structures of reinforced or prestressed concrete these conditions are almost impossible to fullfill because of the time and financial constraints. Furthermore, diverted by the large amount of test studies required to substantiate the empirical approaches, more basic studies of the behavior and modeling of the overall system carrying shear and torsional forces have been neglected.

In this study, a basic reevaluation of the current procedures and development of alternate design procedures is carried out using a conceptual structual model rather than detailed empirical equations wherever practical. The structural model used in this evaluation consists of a space truss with variable angle of inclination of the diagonal elements. This model was selected as the one which best represents the behavior of reinforced and prestressed concrete beams subjected to shear and torsion. This conceptual model was suggested in the early part of this century by Ritter, generalized by Morsch, and refined by a number of European engineers in the past 20 years. Deformation procedures were added by Canadian researchers. Much of the previous work has been based on highly complex proofs of the application of plasticity theorems in the fields of shear and torsion. The apparent complexity of the proofs of the plasticity theorems as applied to shear and torsion can cause the more design oriented reader to lose sight of the fact that the authors use these proofs only as a theoretical basis for proving the application of a refined truss model. The model has been shown to be a lower bound solution which gives the same result as the upper bound solution. Hence, it is a mathematically valid solution which correctly represents the failure load.

The variable angle truss model provides the designer with a conceptual model to analyze the behavior of members subjected to combined actions. The designer can visualize the effects that such actions will have on the different components of the member. A more complete understanding of this behavior should lead to a simpler and more effective design process.

A thorough evaluation of the space truss model using test data available in the literature and results from beams tested during this research project at the Ferguson Laboratory was reported in Report 248-3. The truss model predicted ultimate values, computed using the relations and interaction equations derived from equilibrium conditions in the truss model, were compared with test obtained results. Very good and uniformly conservative correlations were found.

Once the general interaction behavior and expected ultimate strength were confirmed by test results, the general procedures derived from the truss model were translated into design recommendation and draft AASHTO requirements. A review of some of the current design procedures available in other codes was also conducted.

Finally, the proposed design procedure based on the truss model was applied in a series of design examples. A comparison with the current AASHTO requirements, wherever available, was conducted and a comparison of the results using the two design methods was presented.

5.2 Conclusions

The conclusions described in this section are based on the overall study of reinforced and prestressed concrete one-way flexural members subjected to shear and/or torsion.

In this study only underreinforced beams are being considered. In such members the stirrups and longitudinal reinforcement yield prior to failure of the concrete, and premature failures due to poor detailing are prevented. The conclusions of this study should then be restricted to such members. The findings of the investigation can be summarized as follows:

1. Due to the complexity involved in explaining the behavior of concrete members subjected to shear and torsion, and the lack of adequate knowledge in this area, most research has tended to concentrate on predicting the collapse load of such members on an almost totally empirical basis. Unfortunately, the empiricism of the analytical methods has led to design procedures which are cumbersome and obscure.

2. It seems obvious that designers are not too eager to adopt new complex design methods, even if these are accurate, when for example they previously have ignored torsion without disastrous consequences. For this reason, a rational and easy to apply approximate design approach based on a simplified model, considering only the main variables is necessary.

3. A design procedure for shear and torsion in reinforced and prestressed concrete one-way flexural members based on equilibrium conditions of a refined truss model with variable inclination of the diagonal members is rational, simple, and conservative.

4. The variable angle truss model provides the designer with a conceptual model to analyze the behavior of members subjected to shear and/or torsion. The designer can visualize the effects that such actions will have on the different components of the member. A more complete understanding of this behavior leads to a simpler and more effective design process. It also shows some possible economical advantages over the present AASHTO procedures.

Other conclusions based on the study of reinforced and prestressed concrete members subjected to shear and/or torsion with the aid of the space truss model include:

a. Limits for the variation of the angle of inclination of the diagonal compression members in the truss model must be introduced to compensate for the fact that procedures based on plastic analysis, such as this one, cannot distinguish between underreinforcement and overreinforcement, i.e. yielding of the reinforcement prior to crushing of the concrete, because they do not predict total deformations. Furthermore, the lower limit of α = 25 degrees, which is intended to ensure adequate inclined crack width control at service load levels, made it necessary to introduce a transition region between uncracked and fully cracked behavior in order to avoid requiring excessive amounts of transverse reinforcement in members subjected to low shear stresses.

b. In the truss model approach, the inclination of the diagonal compression strut is the inclination at ultimate and not first inclined cracking. The inclination at ultimate may coincide with the inclination at first diagonal cracking, but this does not necessarely have to be the case. The change in the angle of inclination or redistribution of forces in the members is possible if contact forces act between the crack surfaces. These contact forces will induce tensile stresses in the compression struts, which must be taken by the concrete. Thus, the change in the inclination of the diagonal compression strut is possible due to the aggregate interlock forces and the concrete tensile strength. Thus, crack limits must be introduced indirectly by restrictions on α, or else a much more complex check of strain compatibility must be included as suggested by Collins and Mitchell (17).

c. In the behavior of reinforced and prestressed concrete beams subjected to shear and/or torsion, three failure states are distinguished. The first is the uncracked state. This state is limited in the case of shear, by the shear force at which first inclined cracking of the web occurs. A second failure state is the transition state in the section between the uncracked state and the full truss action state. When a member fails in the transition state, more cracking takes place and there is a redistribution of internal forces in the member. With failure at higher shear stresses in the transition state more cracking takes place and/or the previously existing cracks grow and become wider. As the crack width increases the mechanisms of aggregate interlock diminish, the contact forces become smaller and no further redistribution of forces in the member is possible. Therefore, in the transition state the concrete in the web provides an additional continuously diminishing resistance as failure occurs at higher shear stresses. In practical terms the concrete contribution can be significant and design for members with low shear stresses would be very conservative if a concrete contribution was not allowed when failure is the uncracked or transition states.

d. The use of the truss model with variable angle of inclination of the diagonal struts in the design of reinforced and prestressed concrete members requires that the steel reinforcement yield prior to failure of the concrete in compression. Concrete failure can be due to crushing of the bending compression zone or the concrete compression diagonals. A check on web crushing must be included in any design procedure.

e. The stresses in the bending compression zone can be determined using the well-known bending theory. In the case where torsion exists together with bending the situation is even less critical. Since a torsional moment introduces longitudinal tension in the member, it will raise the neutral axis in the case of positive bending moment (tension at the bottom of the member), therefore, reducing the compression stresses in the bending compression zone. The same holds true for the case of a negative bending moment (tension at the top), since now the torque will lower the neutral axis, hence reducing the compression stresses in the bending compression zone. Therefore, the flexural balanced reinforcement limits ensuring yielding of the longitudinal steel prior to crushing of the concrete in the case of pure bending constitute a safe lower bound for the case of combined torsion and bending.

f. The space truss model approach is based on the assumption that yielding of the reinforcement must take place at ultimate. Thus, reinforced and prestressed concrete members not only have to be designed as underreinforced sections, but in addition, premature failures due to improper detailing of the reinforcement must be avoided.

5.3 Recommendations for Further Research

Since the recent reinterest in the variable angle truss model, considerable research has been conducted and only partly assimilated in American practice. Substantial new research has been reported, particularly in German language reports and papers. In addition to the complete evaluation of this work for pertinent material, the following areas of further research on the truss model with variable angle of inclination of the diagonals may be useful:

- The effect of high strength concrete (f'_c > 7000 psi) on the behavior of reinforced and prestressed concrete beams subjected to shear and/or torsion.

- The effect of lightweight concrete members subjected to shear and/or torsion on the truss model design approach.

- The effects of restrained torsion in the case of members where warping restraint becomes significant.

- All the conclusions presented in this study apply to members subjected to an static type loading. Further research is needed to evaluate the effects of load reversals, and dynamic loading on the truss model design approach.

- The effects of fatigue were not considered and research may be necessary in this area.

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