CALCULATION OF THE ELASTIC MODULI

of a

TWO LAYER PAVEMENT SYSTEM

from

MEASURED SURFACE DEFLECTIONS
PART II

by

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Research Report Number 123-6A
(A supplement to Research Report 123-6)

conducted

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Federal Highway Administration

by the

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Texas Highway Department

Texas Transportation Institute
Texas A&M University

Center for Highway Research
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Preface

This is a supplement to Research Report 123-6, which was the sixth report issued under Research Study 1-8-69-123, A Systems Analysis of Pavement Design and Research Implementation. The study is being conducted jointly by principal investigators and their staffs in three agencies -- The Texas Highway Department, The Center for Highway Research, and the Texas Transportation Institute -- as a part of the cooperative research program with the Department of Transportation, Federal Highway Administration.

Previous reports emanating from Study 123 are the following:

Report No. 123-1, "A Systems Approach Applied to Pavement Design and Research," by W. Ronald Hudson, B. Frank McCullough, Frank H. Scrivner, and James L. Brown, describes a long-range comprehensive research program to develop a pavement systems analysis and presents a working systems model for the design of flexible pavements.

Report No. 123-2, "A Recommended Texas Highway Department Pavement Design System Users Manual," by James L. Brown, Larry J. Buttler, and Hugo E. Orellana, is a manual of instructions to Texas Highway Department personnel for obtaining and processing data for flexible pavement design system.

Report No. 123-3, "Characterization of the Swelling Clay Parameter Used in the Pavement Design System," by Arthur W. Witt, III, and B. Frank McCullough, describes the results of a study of the swelling clay parameter used in pavement design system.

Report No. 123-4, "Developing a Pavement Feedback Data System," by R. C. G. Haas, describes the initial planning and development of a pavement feedback data system.

Report No. 123-5, "A Systems Analysis of Rigid Pavement Design," by Ramesh K. Kher, W. R. Hudson, and B. F. McCullough, describes the development of a working systems model for the design of rigid pavements.

Report No. 123-6, "Calculation of the Elastic Moduli of a Two Layer Pavement System from Measured Surface Deflections," by Frank H. Scrivner, Chester H. Michalak and William M. Moore, describes a method for converting Dynaflect deflections to the Young's moduli of a simple pavement-subgrade (two-layer elastic) system.

Report No. 123-7, "Annual Report on Important 1970-71 Research Needs," by B. Frank McCullough, James L. Brown, W. Ronald Hudson and F. H. Scrivner, was produced mainly for the information of the Research Area III Advisory Committee of the Texas Highway Department.

Report No. 123-8, "A Sensitivity Analysis of Flexible Pavement System FPS2," by Ramesh K. Kher, B. Frank McCullough and W. Ronald Hudson, presents a sensitivity analysis performed to establish the plausibility of solutions and relative importance of some of the variables in FPS2.

The opinions, findings and conclusions expressed in this publication are those of the authors and not necessarily those of the Department of Transportation, Federal Highway Administration.

Abstract

This report is a sequel to a previous one that gave the details of a computer program capable of calculating in situ values of the Young's moduli of a pavement-subgrade (two-layer elastic) system from surface deflections measured at two points located at specified distances from the load wheels of a Dynaflect. The present report describes the effect of a change in the specified location of one of the deflected points on (a) the calculated moduli, (b) the shape of the deflection basin calculated from these moduli and (c) certain wheel load stresses calculated from the moduli at the pavement-subgrade interface. Dynaflect data from flexible highway pavements and rigid airport pavement are used in the calculations.

Key Words: PAVEMENT, DESIGN, MODULUS, DEFLECTION.

Summary

One of the tasks undertaken in Study 123 is to investigate the feasibility of the use of linear elasticity theory in a subsystem of the flexible pavement design system now on trial in the Texas Highway Department (2). Such a subsystem would provide estimates of stresses, strains and displacements at critical points within the pavement structrue and subgrade, resulting from passing wheel loads.

But to calculate stresses, strains and displacements, the designer must have at hand estimates of -- among other things -- the in situ value of the elastic modulus of each material to be used in the pavement.

A previous report (1) describes how such moduli may be estimated, by the computer program ELASTIC MODULUS, from surface deflections measured on a simple (two-layer, or pavement-subgrade) type of flexible pavements at two points located at distances of zero and one foot from the centroid of the loaded areas provided by a Dynaflect. In the present report, analyses are presented of the differences in computed moduli encountered when the distance of one of the deflection points is increased from its original value of one foot to two feet. Also presented are the moduli resulting from the use of the Dynaflect on certain rigid pavements. On these the increase in spread between deflection points is found to be necessary to achieve sufficient contrast in the data.

Included with the report is a listing of the computer program, ELASTIC MODULUS II, used to estimate the moduli of pavement and subgrade from the deflections measured at zero and two feet from the centroid of the Dynaflect loaded area. Also included is a listing of the program, POINT LOAD, which accepts as inputs the moduli of a two-layer pavement system, and the thickness of the upper layer, and computes deflections at points at any desired distances from the centroid of a Dynaflect load.

It was possible to compute from ELASTIC MODULUS II (or from ELASTIC MODULUS) the moduli of pavement and subgrade, using Dynaflect deflections observed at only two points, and then, by using these moduli, to predict (by use of POINT LOAD) the entire five-point Dynaflect basin. The predicted basin could then be plotted and compared with the plotted basin actually observed. From plots of this type, a subjective judgement could be made of the validity of elasticity theory when applied to simple, two-layer pavement structures.

Besides comparing directly the moduli computed from deflections measured at different pairs of points, and the resulting predicted Dynaflect deflection basins, it was also possible, by use of the computer program BISTRO (supplied by Koniklijke/Shell-Laboratorium, Amsterdam) to compute certain stresses at the pavement-subgrade interface resulting from use of the two sets of moduli. It is these stresses that are of interest in design.

Sources of flexible pavement deflection data were seven 500-foot sections near College Station, Texas. Rigid pavement data were obtained at the Houston Intercontinental Airport by courtesy of airport officials and their consultants.

The report contains many detailed conclusions, which may be summed up as follows.

The Dynaflect in its present form, combined with the computer programs ELASTIC MODULUS II and POINT LOAD, has the potential of becoming a useful method for material characterization in a pavement design system using linear elastic theory as a subsystem, provided that proper heed is paid to instrument error, the inevitable variability of highway materials in place, and the many other uncertainties that enter into a complete pavement design system. ELASTIC MODULUS II, rather than

ELASTIC MODULUS, is recommended in order to achieve compatibility between moduli found for rigid pavements and those determined for flexible pavements.

Implementation Statement

The programs ELASTIC MODULUS II and POINT LOAD were written in the expectation that eventually the Texas Highway Department's Flexible Pavement Design System will, in the prediction of pavement life, use the stresses, strains and displacements computed at critical points within the structure from the theory of linear elastic layered systems, instead of solely the surface deflections calculated by the present empirical equation. When such a change occurs in the design system, in situ values of elastic moduli will be needed. This need probably can be met, at least to some degree, by the computer programs described herein.

Acknowledgments

For their assistance in obtaining the deflection data indicated below, the writers are especially grateful to the following personnel and their organizations.

Data from flexible pavements: Dr. Robert E. Long of the Texas Highway Department, Bryan, Texas.

Data from rigid pavements: Col. Harry Fischer of the Houston Intercontinental Airport, Mr. D. E. Aviles of Marillo Engineering and Testing Service, Houston, Texas, and Mr. H. P. Carothers of Lockwood, Andrews and Newnam of Houston.

Thanks are also due Mr. Rudell Poehl and Mr. Neil K. Holley, both of Texas Transportation Institute, for their expert operation of the Dynaflect on the pavements tested.

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FOREWARD

The primary objective of this research project is to develop techniques for the optimal operation of a linked system of multi-purpose reservoirs. Linkage of the system may be through normal river reaches, canals, or through pumping in pipelines. In this report a model is developed which utilizes stochastic inflows with the total system subject to certain constraints. This model will be utilized later in an operational study of an existing system.

1. Introduction

Research Report 123-6, "Calculation of the Elastic Moduli of a Two Layer Pavement System from Measured Surface Deflections" (1), describes a computer program, ELASTIC MODULUS, that accepts as inputs the deflections w_1 and w_2, indicated by Geophones 1 and 2, respectively, of a Dynaflect (see Figure 1) acting on the surface of an idealized, linear elastic, two layer pavement system such as that illustrated in Figure 2. An additional input is the thickness, h, of the top layer. From the three inputs -- w_1 (mils), w_2 (mils) and h (inches) -- and the constant distances r_1 (inches) and r_2 (inches) from Geophone 1 and Geophone 2 to either load wheel, ELASTIC MODULUS calculates the moduli E_1 and E_2 (both in pounds per square inch) of the two layers.

Because the geometry of a real pavement departs from that assumed in the theory at a lateral distance of only a few feet from the location of the Dynaflect load wheels, it was felt that the two geophones nearest the load would be likely to yield data more consistent with theory than more distant geophones -- hence, the selection of Geophones 1 and 2 in Research Report 123-6 for use in estimating the moduli. However, in the case of portland cement concrete pavements experience has shown that in many cases the deflection basin created by the Dynaflect is so flat that w_1 and w_2 frequently differ by an amount only 1 to 3 times the 0.00001 inch sensitivity of the instrument. For this reason, it was decided to investigate the use of Geophones 1 and 3 in ELASTIC MODULUS, since the values of w_1 and w_3 had been found to be significantly different, even for rigid airport pavements as thick as 14 inches.

This report, a supplement to Research Report 123-6, describes the investigation and lists the computer program ELASTIC MODULUS II, which

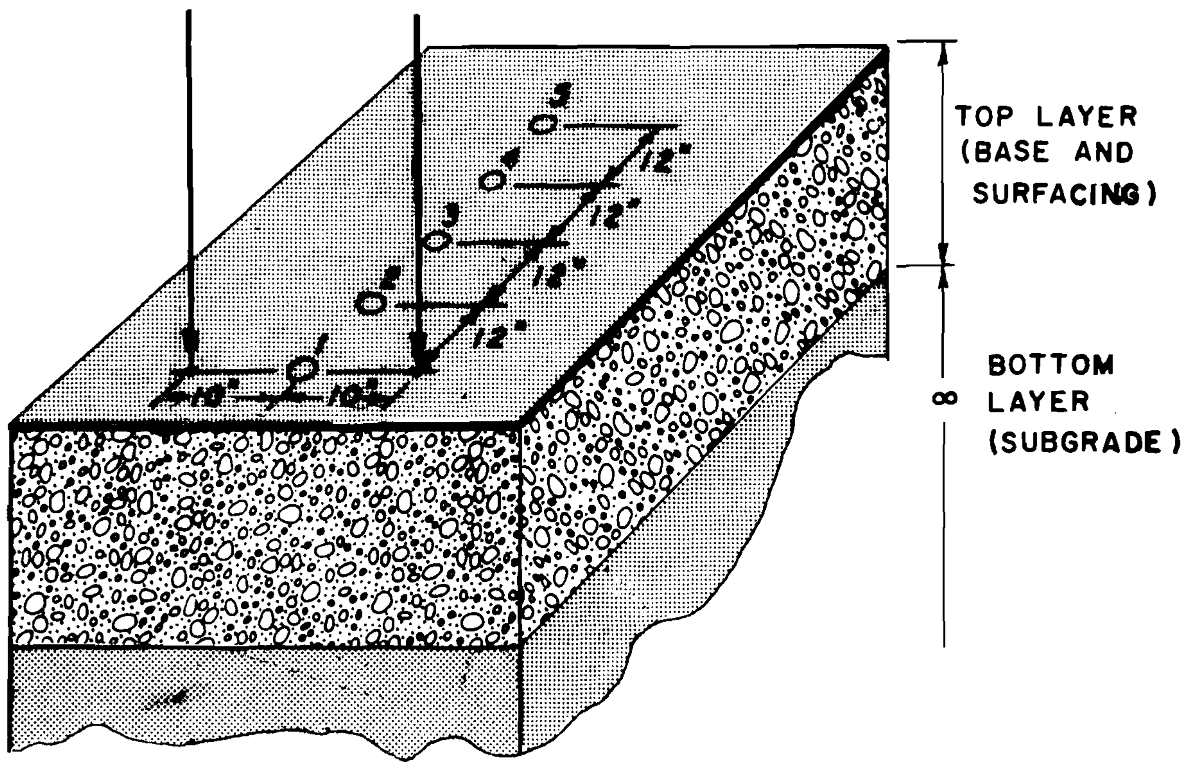

Figure 1: Relative position of Dynaflect loads and sensors. The sensors are usually placed in the outer wheel path, on a line paralleling the center line of the highway.

closely resembles the program described in Research Report 123-6 (see Appendix A). It also lists the computer program POINT LOAD (see Appendix B) which accepts as inputs the moduli E_1 and E_2 of a two layer system, the thickness h of the top layer, and the distance r to a point on the surface. From these inputs POINT LOAD computes the surface deflection, w, at the distance r from the load, using the same basic equations and numerical integration methods as those employed in ELASTIC MODULUS.

The program POINT LOAD was found useful in determining the degree of agreement between

(1) ELASTIC MODULUS II and the older and more comprehensive program BISTRO, and between

(2) a Dynaflect deflection basin computed from deflections observed at only two points on the pavement, and the entire (five point) basin actually observed.

2. Accuracy Check

As indicated above, the equations and computational procedures used in ELASTIC MODULUS II are the same as those described in Research Report 123-6, and therefore will not be discussed in this report. However, because there were some numerical changes, it was considered prudent to make an accuracy check of ELASTIC MODULUS II against the computer program BISTRO, similar to the check described in Reference (1) for ELASTIC MODULUS. The results are given in Table 1. As was expected from previous experience with ELASTIC MODULUS, the agreement between ELASTIC MODULUS II and BISTRO was excellent except in the improbable case where the modulus of the pavement layer was assumed to be only one-tenth of the subgrade modulus.

The changes made in ELASTIC MODULUS to produce ELASTIC MODULUS II are summarized below.

MAIN PROGRAM: The restraints listed in Table 2 (next chapter) are used instead of those listed in Table 3 of Reference 1.

SUBROUTINE EMOD: r_2 (value = 15.62 inches) was changed to r_3 (value = 26 inches); also w_1 and w_3 are used in the computations instead of w_1 and w_2.

(The numbers listed in Table 1 in the column headed "ELASTIC MODULUS II" were actually computed from the program, POINT LOAD. However, since both programs use the same equations and methods for computing surface deflections, the deflections printed out by POINT LOAD are precisely the same as those computed internally, but not printed out, by ELASTIC MODULUS II.)

Table 1: Comparison of ELASTIC MODULUS II with BISTRO

E_1 (psi)	E_2 (psi)	E_1/E_2	h (in.)	Computed Deflections (mils)			
				w_1		w_3	
				ELASTIC MODULUS II	BISTRO	ELASTIC MODULUS II	BISTRO
10,000,000	10,000	1,000	5	0.99	0.99	0.81	0.81
			10	0.52	0.52	0.48	0.48
			20	0.26	0.26	0.26	0.26
			40	0.13	0.13	0.13	0.13
1,000,000	10,000	100	5	1.86	1.85	1.09	1.09
			10	1.07	1.07	0.84	0.84
			20	0.57	0.57	0.51	0.51
			40	0.30	0.30	0.28	0.28
100,000	10,000	10	5	2.65	2.65	0.98	0.98
			10	1.94	1.93	1.06	1.06
			20	1.20	1.20	0.86	0.86
			40	0.74	0.74	0.56	0.56
10,000	10,000	1	5	2.39	2.39	0.92	0.92
			10	2.39	2.39	0.92	0.92
			20	2.39	2.39	0.92	0.92
			40	2.39	2.39	0.92	0.92
1,000	10,000	0.1	5	-0.01	-0.04	0.80	0.80
			10	-0.15	-0.06	0.35	0.35
			20	7.45	7.52	0.42	0.42
			40	14.90	14.90	1.60	1.60

Note: ELASTIC MODULUS II: Point load of 1000 lbs.
BISTRO: Circular loaded area with radius of 1.41 in., pressure of 160 psi, load of 1000 lbs.
Both programs: Vertical deflection computed at the points r = 10", z = 0 and r = 26", z = 0.

3. Non-Unique Solutions

As in the case of ELASTIC MODULUS, the posibility exists that non-unique solutions will sometimes arise from the Dynaflect data processed through ELASTIC MODULUS II, and occasionally no solution at all will be possible. These possibilities were investigated by (a) preparing the graph shown in Figure 3 (comparable to Figure 3 of Reference (1)) and -- based on conclusions drawn from the graph -- (b) arriving at the constraints shown in Table 2 (comparable to Table 3 of Reference (1)) to be included in ELASTIC MODULUS II. The logic followed in choosing these restraints from a study of Figure 3 is the same as previously described in Reference (1), and need not be repeated here. The coordinates of the points used in plotting the curves in Figure 3 were computed by the program POINT LOAD.

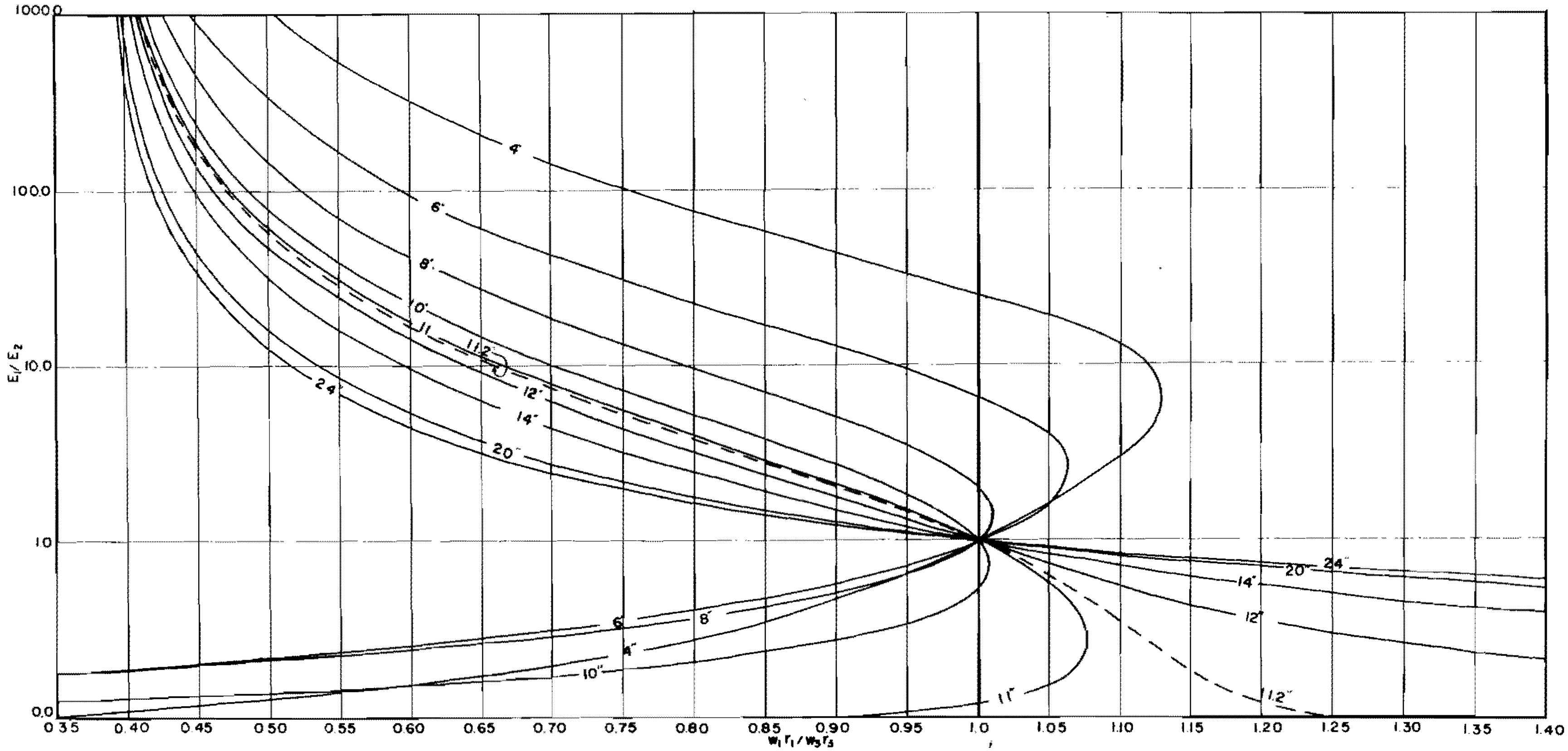

Figure 3: Contours of pavement thickness, h, plotted as a function of the ratios E_1/E_2 and w_1r_1/w_3r_3.

Table 2: Summary of Information from Figure 3 Used in the Control of the Program, ELASTIC MODULUS II

Measured Input Data		Unique Solution	Layer Having The Greater Modulus	Program Printout
$w_1 r_1 / w_3 r_3$	Thickness, h (in.)			
Greater than 1	Greater than 11.2	Yes	Subgrade	Subgrade and pavement moduli
Greater than 1	Less than 11.2	No	May be either	"NO UNIQUE SOLUTION"*
Less than 1	Greater than 11.2	Yes	Pavement	Subgrade and pavement moduli
Less than 1	Less than 11.2	No	May be either, but the more probable of two possible solutions is selected	Subgrade and pavement moduli for solution having $E_1/E_2 > 1$

* When the experimental data $w_1 r_1 / w_3 r_3$ exceeds unity, and h is less than 11.2", some cases can arise for which no solution at all is possible.

4. Examples of Solutions Provided by ELASTIC MODULUS II for Flexible Pavements

In May, 1968, Dynaflect deflections were measured at ten points in the outer wheel path on each of several 500-ft. sections of highways in the vicinity of College Station, Texas. Originally this was done for the purpose of obtaining the "stiffness coefficient" used to characterize materials in the systems approach to the design of flexible pavements now on trial in the Texas Highway Department (2, 3, 4). Later the 1968 data from Geophones 1 and 2 were processed through ELASTIC MODULUS and the resulting moduli were given in Reference (1). Finally, in Tables 3 and 4 of this report, average moduli for each test section resulting from the use of Geophone 1 and 3 data in ELASTIC MODULUS II are given, together with a verbal description of the materials involved. The computer printouts -- one for each test section -- are reproduced in Tables 5a through 5g.

The moduli computed by ELASTIC MODULUS II, and presented in Tables 3 and 4, will be discussed in the next two chapters, with the main emphasis being placed on comparisons of those moduli with corresponding values previously computed by ELASTIC MODULUS and reported previously in Reference 1.

Table 3: Average Pavement Modulus, E_1, for Each of Seven 500-ft. Flexible Pavement Sections of Highways near College Station Texas, using w_1 and w_3 Data

(Deflection measurements made May 21, 1968)

Test Section	Pavement Materials and Thicknesses: Surfacing	Pavement Materials and Thicknesses: Base	Pavement Thickness, h: Average Value (In.)	Pavement Thickness, h: Standard Deviation	Pavement Modulus, E_1: No.* Solutions	Pavement Modulus, E_1: Average Value (PSI)	Pavement Modulus, E_1: Standard Deviation	Pavement Modulus, E_1: Coefficient of Variation (percent)
15	1.2" Asph. Conc.	14.0" Cement stabilized limestone	15.2	1.2	10	314,100	75,200	24
4	0.5" Seal Coat	7.5" Asphalt stabilized gravel	8.0	0.4	4	110,500	90,400	82
16	1.0" Asph. Conc.	6.5" Asph. emulsion stab. gravel	7.5	0.4	10	109,300	19,700	18
17	0.5" Seal Coat	7.8" Iron ore gravel	8.3	0.7	10	81,900	47,700	58
5	0.5" Seal Coat	11.5" Lime stabilized sandstone	12.0	2.8	10	23,800	15,400	64
3	0.5" Seal Coat	12.0" Red sandy gravel	12.5	1.0	10	23,700	11,600	49
12	3.7" Asph. Conc.	16.2" Sandstone	19.9	0.5	10	14,900	3,300	22

* Measurements were made at 10 locations in each section. Less than 10 solutions occur in cases where $w_1 r_1 / w_3 r_3 > 1$ and $h < 11.2"$, as indicated in Table 2.

Table 4: Average Subgrade Modulus, E_2, for Each of Seven 500-ft. Flexible Pavement Sections of Highways near College Station, Texas, using w_1 and w_3 Data

(Deflection measurements made May 21, 1968)

		Subgrade Material		Subgrade Modulus, E_2			
Test Section	Thickness Investigated	Description	Formation	No.* Solutions	Average Value (PSI)	Standard Deviation	Coefficient of Variation (percent)
15	32"	Red sandy clay, some gravel	Stone City	10	19,120	793	4
3	23"	Sand over clay	Spiller Sandstone Member of Cook Mountain Formation	10	18,980	1297	6
5	24"	Tan sandy clay	Caddell	10	14,840	1597	11
12	22"	Black stiff clay	Lagarto	10	14,010	978	7
4	25"	Grey sandy clay	Spiller Sandstone Member of Cook Mountain Formation	4	11,800	1268	11
17	21"	Grey sandy clay	Spiller Sandstone Member of Cook Mountain Formation	10	11,400	1201	11
16	18"	Brown clay	Alluvium deposit of Brazos River	10	11,110	528	5

* Measurements were made at 10 locations in each section. Less than 10 solutions occur in cases where $w_1 r_1 / w_3 r_3 > 1$ and $h < 11.2"$, as indicated in Table 2.

TEXAS HIGHWAY DEPARTMENT

DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULI

THIS PROGRAM WAS RUN - 07/15/71

DIST.	COUNTY
17	BRAZOS

CONT.	SECT.	JOB	HIGHWAY	DATE	DYNAFLECT
1560	1	1	FM 1687	5-21-68	1

PAV. THICK. = 12.50 INCHES

SEAL COAT 0.50 RED SANDY GRAVEL 12.00

GREY & BRWN SAND SUB 0.0

STATION	W1	W2	W3	W4	W5	SCI	** ES **	** EP **	REMARKS
1 - A	1.170	0.770	0.520	0.310	0.219	0.400	19100.	39400.	
1 - B	1.140	0.770	0.510	0.310	0.213	0.370	19600.	41600.	
2 - A	1.290	0.340	0.490	0.300	0.204	0.450	18600.	17500.	
2 - B	1.200	0.840	0.490	0.300	0.201	0.360	19500.	26500.	
3 - A	1.140	0.770	0.470	0.300	0.195	0.370	20400.	29000.	
3 - B	1.110	0.770	0.460	0.300	0.201	0.340	20900.	30400.	
4 - A	1.470	0.960	0.490	0.320	0.222	0.510	16500.	8400.	
4 - B	1.380	0.900	0.470	0.310	0.213	0.480	17500.	9800.	
5 - A	1.290	0.870	0.500	0.340	0.231	0.420	18500.	19200.	
5 - B	1.260	0.890	0.460	0.310	0.219	0.460	19200.	14800.	
AVERAGES	1.245	0.829	0.486	0.310	0.212	0.416	18980.	23660.	
STANDARD DEVIATION						0.057	1297.	11609.	
NUMBER OF POINTS IN AVERAGE =						10	10	10	

W1	DEFLECTION AT GEOPHONE 1
W2	DEFLECTION AT GEOPHONE 2
W3	DEFLECTION AT GEOPHONE 3
W4	DEFLECTION AT GEOPHONE 4
W5	DEFLECTION AT GEOPHONE 5
SCI	SURFACE CURVATURE INDEX (W1 MINUS W2)
ES	ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W3
EP	ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W3

Table 5a: Computer print-out for Section 3.

TEXAS HIGHWAY DEPARTMENT

DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULI

THIS PROGRAM WAS RUN - 07/15/71

DIST.	COUNTY
17	BRAZOS

CONT.	SECT.	JOB	HIGHWAY	DATE	DYNAFLECT
2824	2	1	FM 2776	5-21-68	1

PAV. THICK. = 8.00 INCHES

SEAL COAT 0.50 ASPHALT STAB. GRAVEL 7.50

GREY SANDY CLAY SUBG 0.0

STATION	W1	W2	W3	W4	W5	SCI	** ES **	** EP **	REMARKS
1 - A	1.650	1.200	0.870	0.660	0.500	0.450	12400.	188900.	
1 - B	1.560	1.110	0.810	0.610	0.490	0.450	13300.	188500.	
2 - A	2.310	1.470	0.930	0.710	0.530	0.840	10700.	36900.	
2 - B	2.310	1.410	0.900	0.670	0.510	0.900	10800.	27600.	
3 - A	2.430	1.500	0.930	0.670	0.490	0.930	NO UNIQUE	SOLUTION	
3 - B	2.490	1.530	0.930	0.670	0.500	0.960	NO UNIQUE	SOLUTION	
4 - A	2.490	1.470	0.900	0.640	0.480	1.020	NO UNIQUE	SOLUTION	
4 - B	2.430	1.410	0.840	0.610	0.470	1.020	NO UNIQUE	SOLUTION	
5 - A	2.340	1.440	0.870	0.620	0.450	0.900	NO UNIQUE	SOLUTION	
5 - B	2.430	1.470	0.930	0.650	0.470	0.960	NO UNIQUE	SOLUTION	
AVERAGES	2.244	1.401	0.891	0.651	0.489	0.843	11800.	110475.	
STANDARD DEVIATION						0.214	1268.	90406.	
NUMBER OF POINTS IN AVERAGE =						10	4	4	

W1 DEFLECTION AT GEOPHONE 1
W2 DEFLECTION AT GEOPHONE 2
W3 DEFLECTION AT GEOPHONE 3
W4 DEFLECTION AT GEOPHONE 4
W5 DEFLECTION AT GEOPHONE 5
SCI SURFACE CURVATURE INDEX (W1 MINUS W2)
ES ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W3
EP ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W3

Table 5b: Computer print-out for Section 4.

TEXAS HIGHWAY DEPARTMENT

DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULI

THIS PROGRAM WAS RUN - 07/15/71

DIST.	COUNTY
17	BURLESON

CONT.	SECT.	JOB	HIGHWAY	DATE	DYNAFLECT
1399	1	1	FM 1361	5-21-68	1

PAV. THICK. = 12.00 INCHES

SEAL COAT 0.50 LIME STAB. SANDSTONE 11.50

TAN SANDY CLAY SUBGR 0.0

STATION	W1	W2	W3	W4	W5	SCI	** ES **	** EP **	REMARKS
1 - A	1.500	1.110	0.710	0.470	0.330	0.390	14400.	43700.	
1 - B	1.560	1.230	0.780	0.480	0.330	0.330	13300.	52900.	
2 - A	1.650	1.200	0.670	0.400	0.243	0.450	14200.	19400.	
2 - B	1.440	1.050	0.640	0.380	0.246	0.390	15600.	34700.	
3 - A	1.500	1.050	0.600	0.370	0.267	0.450	15800.	19700.	
3 - B	1.440	0.990	0.580	0.370	0.261	0.450	16400.	21400.	
4 - A	1.500	1.050	0.560	0.340	0.216	0.450	16000.	13400.	
4 - B	1.380	0.990	0.540	0.330	0.213	0.390	17200.	19000.	
5 - A	1.920	1.260	0.650	0.400	0.280	0.660	12300.	5800.	
5 - B	1.800	1.140	0.630	0.420	0.310	0.660	13200.	7600.	
AVERAGES	1.569	1.107	0.636	0.396	0.270	0.462	14840.	23760.	
STANDARD DEVIATION						0.112	1597.	15352.	
NUMBER OF POINTS IN AVERAGE =						10	10	10	

W1	DEFLECTION AT GEOPHONE 1
W2	DEFLECTION AT GEOPHONE 2
W3	DEFLECTION AT GEOPHONE 3
W4	DEFLECTION AT GEOPHONE 4
W5	DEFLECTION AT GEOPHONE 5
SCI	SURFACE CURVATURE INDEX (W1 MINUS W2)
ES	ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W3
EP	ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W3

Table 5c: Computer print-out for Section 5.

TEXAS HIGHWAY DEPARTMENT

DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULI

THIS PROGRAM WAS RUN - 07/15/71

DIST.	COUNTY
17	WASHINGTON

CONT.	SECT.	JOB	HIGHWAY	DATE	DYNAFLECT
186	5	1	SH 36	5-21-68	1

PAV. THICK. = 19.90 INCHES

HOT MIX ASPH. CONC. 3.75 SANDSTONE 16.15

BLACK CLAY SUBGRADE 0.0

STATION	W1	W2	W3	W4	W5	SCI	** ES **	** EP **	REMARKS
1 - A	1.680	1.020	0.610	0.420	0.300	0.660	14800.	13100.	
1 - B	1.830	1.080	0.610	0.420	0.310	0.750	14300.	10800.	
2 - A	1.740	1.080	0.670	0.470	0.360	0.660	13700.	13700.	
2 - B	1.950	1.170	0.690	0.490	0.370	0.780	13000.	10900.	
3 - A	1.680	1.080	0.680	0.500	0.380	0.600	13700.	15400.	
3 - B	1.710	1.080	0.670	0.480	0.370	0.630	13800.	14300.	
4 - A	1.680	1.110	0.750	0.570	0.460	0.570	12700.	18300.	
4 - B	1.560	1.080	0.730	0.550	0.440	0.480	13200.	21600.	
5 - A	1.500	0.960	0.590	0.440	0.330	0.540	15700.	16500.	
5 - B	1.590	0.990	0.600	0.430	0.330	0.600	15200.	14600.	
AVERAGES	1.692	1.065	0.660	0.477	0.365	0.627	14010.	14920.	
STANDARD DEVIATION						0.091	978.	3286.	
NUMBER OF POINTS IN AVERAGE =						10	10	10	

W1	DEFLECTION AT GEOPHONE 1
W2	DEFLECTION AT GEOPHONE 2
W3	DEFLECTION AT GEOPHONE 3
W4	DEFLECTION AT GEOPHONE 4
W5	DEFLECTION AT GEOPHONE 5
SCI	SURFACE CURVATURE INDEX (W1 MINUS W2)
ES	ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W3
EP	ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W3

Table 5d: Computer print-out for Section 12.

TEXAS HIGHWAY DEPARTMENT

DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULI

THIS PROGRAM WAS RUN - 07/16/71

DIST.	COUNTY
17	ROBERTSON

CONT.	SECT.	JOB	HIGHWAY	DATE	DYNAFLECT
49	8	1	US 190	5-21-68	1

PAV. THICK. = 15.20 INCHES

HOT MIX ASPH. CONC. 1.25 CEM. STAB. LIMESTONE 13.95

RED SANDY CLAY SUBGR 0.0

STATION	W1	W2	W3	W4	W5	SCI	** ES **	** EP **	REMARKS
1 - A	0.680	0.590	0.490	0.390	0.310	0.090	18600.	312700.	
1 - B	0.680	0.600	0.490	0.390	0.310	0.080	18600.	312700.	
2 - A	0.720	0.630	0.510	0.390	0.310	0.090	18200.	271900.	
2 - B	0.700	0.620	0.490	0.390	0.310	0.080	19100.	264500.	
3 - A	0.750	0.650	0.520	0.390	0.300	0.100	18200.	235500.	
3 - B	0.760	0.650	0.510	0.390	0.300	0.110	18900.	201700.	
4 - A	0.600	0.540	0.450	0.350	0.280	0.060	19500.	433000.	
4 - B	0.580	0.520	0.430	0.330	0.880	0.060	20600.	422300.	
5 - A	0.620	0.550	0.450	0.350	0.910	0.070	20100.	355600.	
5 - B	0.650	0.570	0.470	0.360	0.280	0.080	19400.	331100.	
AVERAGES	0.674	0.592	0.481	0.373	0.419	0.082	19120.	314100.	
STANDARD DEVIATION						0.016	793.	75213.	
NUMBER OF POINTS IN AVERAGE =						10	10	10	

W1 DEFLECTION AT GEOPHONE 1
W2 DEFLECTION AT GEOPHONE 2
W3 DEFLECTION AT GEOPHONE 3
W4 DEFLECTION AT GEOPHONE 4
W5 DEFLECTION AT GEOPHONE 5
SCI SURFACE CURVATURE INDEX (W1 MINUS W2)
ES ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W3
EP ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W3

Table 5e: Computer print-out for Section 15.

TEXAS HIGHWAY DEPARTMENT

DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULI

THIS PROGRAM WAS RUN - 07/15/71

DIST.	COUNTY
17	BRAZOS

CONT.	SECT.	JOB	HIGHWAY	DATE	DYNAFLECT
1560	1	1	FM 1687	5-21-68	1

PAV. THICK. = 7.50 INCHES

ASPHALT SURFACING 1.00 ASPH EMUL STAB GRAVL 6.50

BROWN CLAY SUBGRADE 0.0

STATION	W1	W2	W3	W4	W5	SCI	** ES **	** EP **	REMARKS
1 - A	2.160	1.500	0.960	0.660	0.520	0.660	10900.	86100.	
1 - B	2.130	1.530	0.960	0.650	0.510	0.600	10900.	93000.	
2 - A	1.920	1.410	0.930	0.640	0.490	0.510	11500.	140500.	
2 - B	1.860	1.350	0.900	0.630	0.500	0.510	11800.	144300.	
3 - A	2.040	1.470	0.930	0.630	0.490	0.570	11300.	102300.	
3 - B	2.070	1.500	0.960	0.650	0.500	0.570	11000.	109200.	
4 - A	2.220	1.620	1.020	0.670	0.490	0.600	10300.	97000.	
4 - B	2.220	1.590	1.020	0.650	0.490	0.630	10300.	97000.	
5 - A	1.980	1.380	0.900	0.610	0.470	0.600	11700.	103800.	
5 - B	1.980	1.440	0.930	0.610	0.460	0.540	11400.	120100.	
AVERAGES	2.058	1.479	0.951	0.640	0.492	0.579	11110.	109330.	
STANDARD DEVIATION						0.049	528.	19723.	
NUMBER OF POINTS IN AVERAGE =						10	10	10	

W1 DEFLECTION AT GEOPHONE 1
W2 DEFLECTION AT GEOPHONE 2
W3 DEFLECTION AT GEOPHONE 3
W4 DEFLECTION AT GEOPHONE 4
W5 DEFLECTION AT GEOPHONE 5
SCI SURFACE CURVATURE INDEX (W1 MINUS W2)
ES ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W3
EP ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W3

Table 5f: Computer print-out for Section 16.

TEXAS HIGHWAY DEPARTMENT

DISTRICT 17 - DESIGN SECTION

DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULI

THIS PROGRAM WAS RUN - 07/15/71

DIST.	COUNTY
17	BRAZOS

CONT.	SECT.	JOB	HIGHWAY	DATE	DYNAFLECT
540	3	1	FM 974	5-21-68	1

PAV. THICK. = 8.30 INCHES

SEAL COAT 0.50 IRON ORE GRAVEL 7.80

GREY SANDY CLAY SUBG 0.0

STATION	W1	W2	W3	W4	W5	SCI	** ES **	** EP **	REMARKS
1 - A	2.400	1.530	0.960	0.680	0.500	0.870	10300.	29800.	
1 - B	2.250	1.440	0.900	0.630	0.480	0.810	11000.	31800.	
2 - A	1.770	1.170	0.820	0.600	0.480	0.600	12800.	92900.	
2 - B	1.800	1.200	0.820	0.620	0.490	0.600	12700.	84600.	
3 - A	1.650	1.170	0.840	0.640	0.510	0.480	12700.	148100.	
3 - B	1.590	1.170	0.840	0.610	0.510	0.420	12800.	177900.	
4 - A	2.250	1.470	0.990	0.750	0.600	0.780	10400.	57600.	
4 - B	2.340	1.590	1.050	0.790	0.630	0.750	9900.	60600.	
5 - A	2.220	1.470	0.990	0.710	0.550	0.750	10500.	62000.	
5 - B	2.100	1.410	0.960	0.680	0.530	0.690	10900.	73800.	
AVERAGES	2.037	1.362	0.917	0.671	0.528	0.675	11400.	81910.	
STANDARD DEVIATION						0.146	1201.	47676.	
NUMBER OF POINTS IN AVERAGE =						10	10	10	

W1	DEFLECTION AT GEOPHONE 1
W2	DEFLECTION AT GEOPHONE 2
W3	DEFLECTION AT GEOPHONE 3
W4	DEFLECTION AT GEOPHONE 4
W5	DEFLECTION AT GEOPHONE 5
SCI	SURFACE CURVATURE INDEX (W1 MINUS W2)
ES	ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W3
EP	ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W3

Table 5g: Computer print-out for Section 17.

5. Comparison of Moduli Estimated from Geophone 1 and 2 Data with Those Estimated from Geophone 1 and 3 Data

In Table 6 the average moduli, E_1 (w_1, w_2)and E_2 (w_1, w_2) for each test section, and the within-section standard deviation, as computed previously by ELASTIC MODULUS using w_1 and w_2 data (1), are compared with similar quantities computed by ELASTIC MODULUS II using w_1 and w_3 data. Of course, if all assumptions used in the theory were completely valid, and if the instrument error were zero, the average moduli E_1 (w_1, w_2) and E_1 (w_1, w_3) (or E_2 (w_1, w_2) and E_2 (w_1, w_3)) for each section in Table 6 would be practically identical, and the corresponding within-section standard deviations would be negligible. That such ideal results were not obtained will be revealed at once by a glance at Table 6.

The results actually obtained will probably surprise no one with previous experience in researching the deflection behavior of real pavements. However, granting that the in situ properties of real base and subgrade materials inevitably change in relatively short distances along and across a highway, one may legitimately ask the question: for a given test section, should E_1 (w_1, w_2) be considered really different from E_1 (w_1, w_3) (or should E_2 (w_1, w_2) be considered really different from E_2 (w_1, w_3)), when compared to the variations of these moduli encountered along the 500-foot test section?

In an effort to answer these questions subjectively, the data in Table 6 were plotted in Figures 4a and 4b to display graphically the difference between the two average values of E_1 (Figure 4a) and the two values of E_2 (Figure 4b) obtained for each test section. The number adjacent to each point identifies the test section. The rectangle surrounding each point is an indication of the scatter of the data

Table 6: Comparison of Moduli Computed from w_1 and w_2 with Those Computed from w_1 and w_3

	E_1 (psi)				E_2 (psi				Pavement Thickness (In.)	
	Average Value		Standard Deviation		Average Value		Standard Deviation			
Section	(w_1, w_2)	(w_1, w_3)	(w_1, w_2)	(w_1, w_3)	(w_1, w_2)	(w_1, w_3)	(w_1, w_2)	(w_1, w_3)	h	Standard Deviation
3	24,700	23,700	6,000	11,700	19,000	19,000	1,600	1,300	12.5	1.0
4	78,900	110,500	8,200	90,400	14,900	11,800	850	1,300	8.0	0.4
5	32,300	23,800	15,100	15,400	14,500	14,800	1,400	1,600	12.0	2.8
12	13,900	14,900	2,700	3,300	14,400	14,000	900	1,000	19.9	0.5
15	283,200	314,100	76,000	75,200	20,000	19,100	900	800	15.2	1.2
16	73,900	109,300	13,800	20,000	11,700	11,100	680	500	7.5	0.4
17	36,600	81,900	25,000	47,700	12,700	11,400	1,700	1,200	8.3	0.7
Overall Average	77,600	96,900	21,000	37,700	15,300	14,500	1,100	1,100		

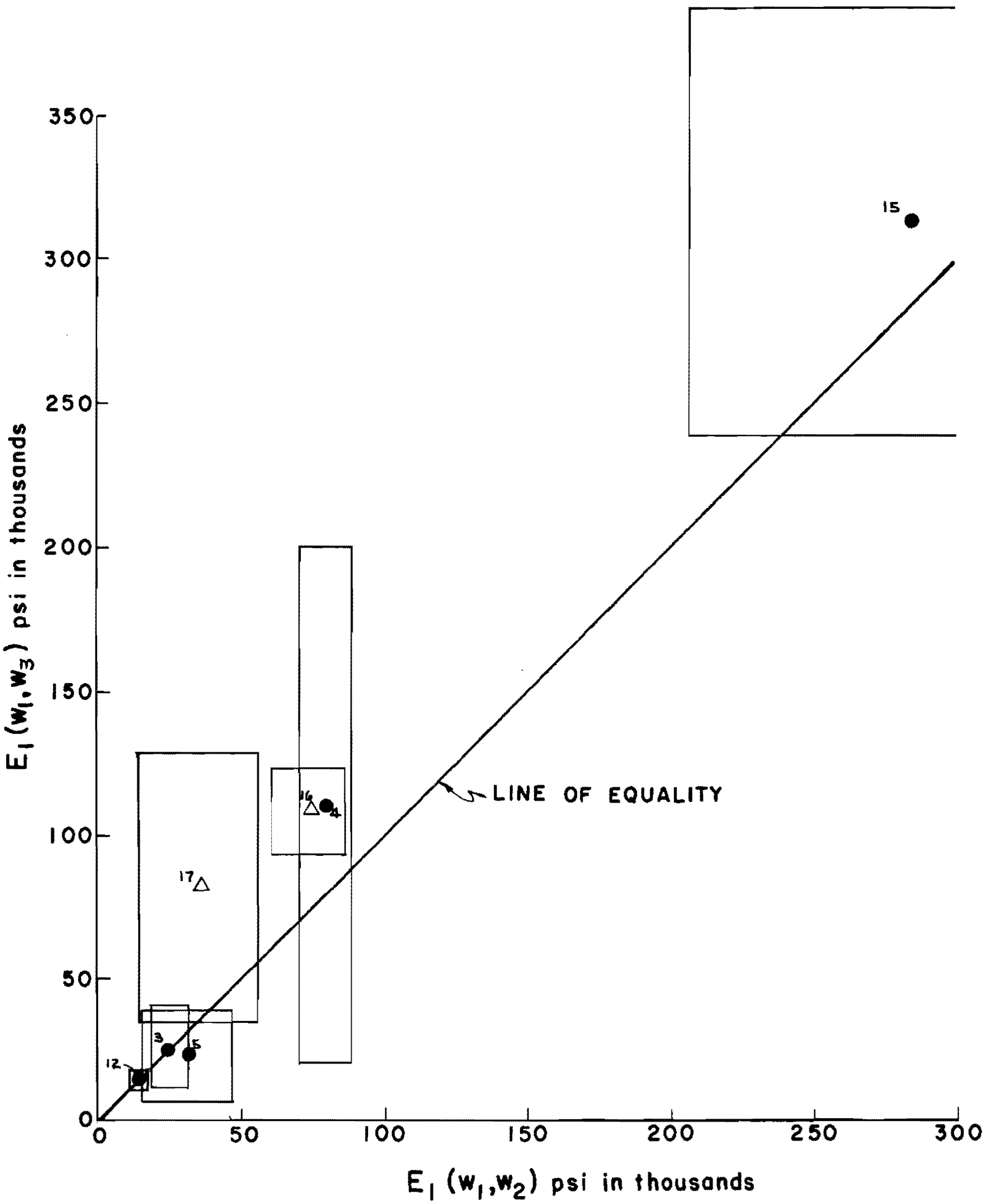

Figure 4a: Comparison of E_1 computed from w_1 and w_2, with E_1 computed from w_1 and w_3. The rectangles represent within-section standard deviations of E_1, with the mean value plotted at the center. Points shown as triangles indicate that the two moduli are significantly different according to a statistical test.

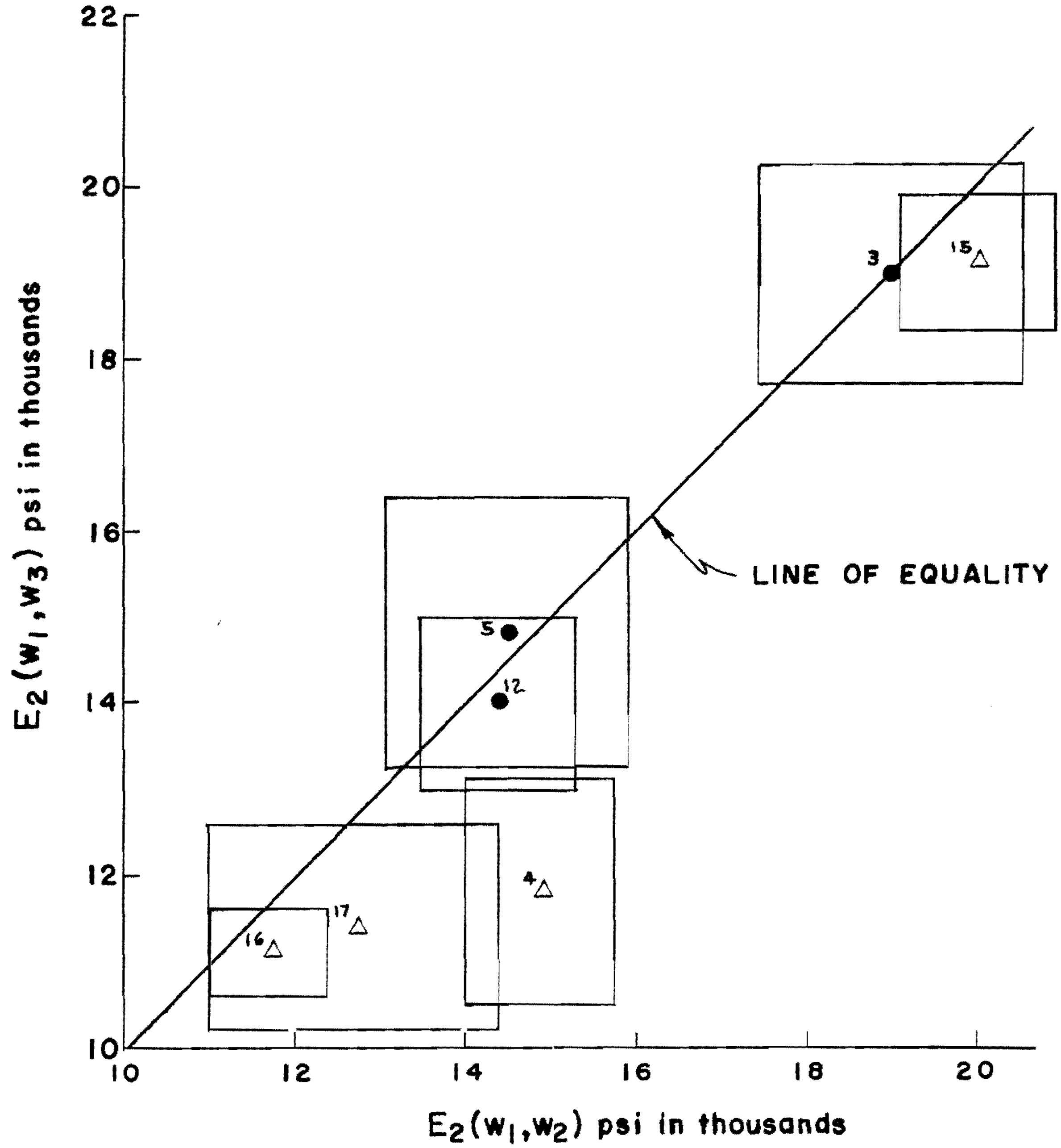

Figure 4b: Comparison of E_2 computed from w_1 and w_2, with E_2 computed from w_1 and w_3. The rectangles represent within-section standard deviations of E_2, with the mean value plotted at the center. Points shown as triangles indicate that the two moduli are significantly different according to a statistical test.

obtained along the test section: each dimension of a rectangle is equivalent to two standard deviations of the data used to obtain the average value of the corresponding coordinate of the point shown at the center of the box.

For any test section where none -- or only a small proportion -- of a rectangle overlaps the line of equality drawn on Figures 4a and 4b, it is reasonable to assume that the difference between a modulus computed from w_1, w_2 data, and one computed from w_1, w_3 data is significantly greater than can be accounted for by random variations in the materials (see, for example, the rectangles for Sections 16 and 17 in Figure 4a, and those for Sections 4, 15, 16 and 17 in Figure 4b, the central points of which have been plotted as triangles to indicate a significant difference between E (w_1, w_2) and E (w_1, w_3).

To confirm subjective conclusions drawn from Figures 4a and 4b, analyses of variance were performed. The results are given in Table 7, where E_1 (w_1, w_2) is compared with E_1 (w_1, w_3), and in Table 8, where E_2 (w_1, w_2) is compared with E_2 (w_1, w_3). In these comparisons, significance was judged at a confidence level of 90%. It is of interest to note from the last columns of these tables that, on the average, E_1 (w_1, w_3) exceeded E_1 (w_1, w_2) in the two cases that were significant in Table 7, while the order was reversed -- E_2 (w_1, w_2) exceeded E_2 (w_1, w_3) -- in the four cases that were significant in Table 8.

Another point to be noted from Table 8 is that although the two methods for computing E_2 gave statistically different results in four sections out of seven, the differences as judged from a practical or engineering point of view appear small -- at least to these writers.

On the other hand, the difference between the two average values of E_1

Table 7: Results of analyses of variance to determine whether the difference between the average values of E_1 (w_1, w_2) and E_1 (w_1, w_3) for a test section are significantly different when compared to within-section variation.

Section	Number Test Stations Used	Average Value of Modulus (psi)		Computed F Ratio	DF for Numerator	DF for Denominator	Required F Ratio**	Are the Moduli Significantly Different?	Standard Dev. (psi)	Coeff. of Var. (%)	Significantly Larger Modulus
		E_1 (w_1, w_2)	E_1 (w_1, w_3)								
3	10	24,720	23,660	0.07	1	18	3.01	No	9,239	38	Neither
4	2, 4*	78,900	110,475	0.02	1	4	4.54	No	78,402	78	Neither
5	10	32,340	23,760	1.59	1	18	3.01	No	15,230	54	Neither
12	10	13,900	14,920	0.58	1	18	3.01	No	2,990	21	Neither
15	10	283,180	314,100	0.83	1	18	3.01	No	75,664	25	Neither
16	10	73,910	109,330	21.61	1	18	3.01	Yes	17,038	19	E_1 (w_1, w_3)
17	8, 10*	36,600	81,910	5.91	1	16	3.05	Yes	39,306	64	E_1 (w_1, w_3)

* First of the two numbers is associated with first modulus, E_1 (w_1, w_2), while the second number applies to the second modulus, E_1 (w_1, w_3).

** For 10% level of significance (90% confidence level).

Table 8: Results of analyses of variance to determine whether the difference between the average values of $E_2(w_1, w_2)$ and $E_2(w_1, w_3)$ for a test section are significantly different when compared to within-section variation.

Section	Number Test Stations Used	Average Value of Modulus (psi)		Computed F Ratio	DF for Numberator	DF for Denominator	Required F Ratio**	Are the Moduli Significantly Different?	Standard Dev. (psi)	Coeff. of Var. (%)	Significantly Larger Modulus
		$E_2(w_1, w_2)$	$E_2(w_1, w_3)$								
3	10	18,970	18,980	0.00	1	18	3.01	No	1,429	8	Neither
4	2, 4*	14,900	11,800	9.25	1	4	4.54	Yes	1,177	9	$E_2(w_1, w_2)$
5	10	14,480	14,840	0.28	1	18	3.01	No	1,508	10	Neither
12	10	14,420	14,010	0.99	1	18	3.01	No	922	6	Neither
15	10	19,990	19,120	5.10	1	18	3.01	Yes	862	4	$E_2(w_1, w_2)$
16	10	11,740	11,110	5.37	1	18	3.01	Yes	608	5	$E_2(w_1, w_2)$
17	8, 10*	12,700	11,400	3.59	1	16	3.05	Yes	1,446	12	$E_2(w_1, w_2)$

* First of the two numbers is associated with first modulus, $E_2(w_1, w_2)$.

** For 10% level of significance (90% confidence level).

found for a test section was generally larger (see Table 7), but within-section variations were also larger, as can be seen by comparing the coefficients of variation given in Table 7 with those shown in Table 8.

Based on the data presented in Figures 4a and 4b, and in Tables 6, 7 and 8, it is concluded that for the seven flexible pavements studied herein and consisting essentially of two layers (granular base -- some stabilized -- and subgrade),

(1) the elastic modulus of the base of a short, apparently uniform test section, as estimated from Dynaflect data (either Geophones 1 and 2 or Geophones 1 and 3) was much more variable than the subgrade modulus, probably because of the assumption that the subgrade was homogeneous to an infinite depth;

(2) in several cases (2 out of 7 in the case of the base, 4 out of 7 in the case of the subgrade) the average section modulus estimated from Geophones 1 and 2 differed significantly, according to statistical tests, from the modulus estimated from Geophones 1 and 3; but

(3) the differences mentioned in conclusion (2) were not very significant in most cases when considered from a practical or engineering point of view.

6. Computed Versus Observed Dynaflect Deflection Basins for Flexible Pavements

Another method of studying the engineering significance of variations in moduli estimated from Dynaflect data, as well as providing a means for helping the reader to judge the degree of validity of linear elasticity as applied to the flexible pavements discussed herein, is to use the estimated moduli E_1 and E_2 to compute a Dynaflect "deflection basin" that can be compared directly with the real basin observed. At the risk of some repetition the method is described below in step-by-step fashion for clarity.

(1) ELASTIC MODULUS was used to compute E_1 and E_2 from Geophone 1 and 2 data at each of the ten test stations (subject to the constraints shown in Table 3 of Reference (1)) in each 500-ft. section. From these results section averages, designated E_1 (w_1, w_2) and E_2 (w_1, w_2), were calculated. These values appear in Tables 7 and 8, respectively.

(2) In a similar manner (but subject to the restraints shown in Table 2 of this report), the section averages E_1 (w_1, w_3) and E_2 (w_1, w_3) were computed using ELASTIC MODULUS II. These values also appear in Tables 7 and 8, respectively.

(3) E_1 (w_1, w_2) and E_2 (w_1, w_2) were used in POINT LOAD to calculate the deflections w_1, w_2, w_3, w_4 and w_5 for each section at the geophone positions 1, 2, 3, 4 and 5, respectively shown in Figure 1. The results are plotted in Figures 5a through 5g as curves labeled "computed from w_1 and w_2". In drawing each curve, or basin, the portion to the left of the vertical axis was ommitted, since it was assumed that the ommitted portion

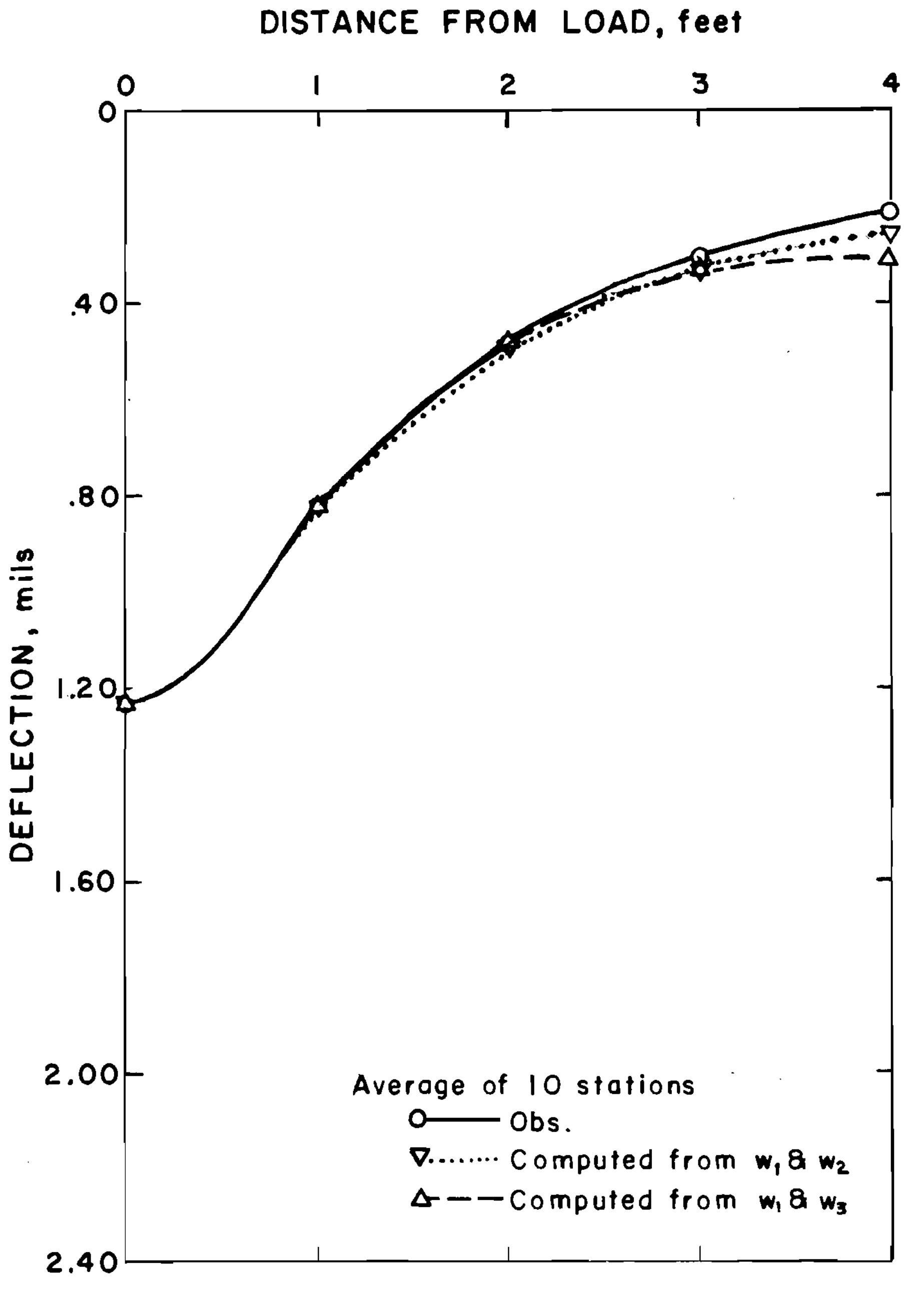

Figure 5a: Computed and average observed Dynaflect deflection **basins**, Section 3. E_1 (w_1, w_2) ≈ E_1 (w_1, w_3) and E_2 (w_1, w_2) ≈ E_2 (w_1, w_3) by statistical test.

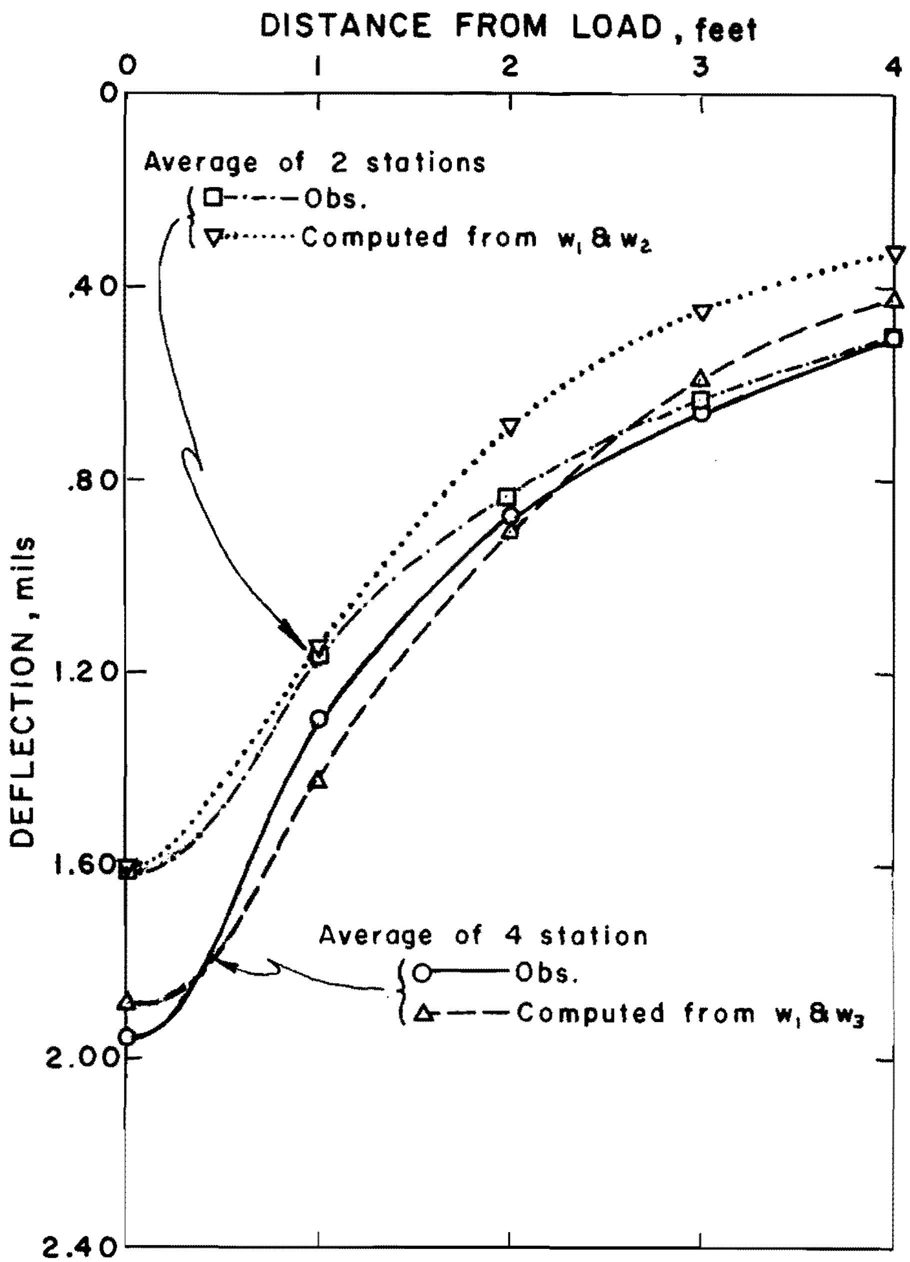

Figure 5b: Computed and average observed Dynaflect deflection basins, Section 4. E_1 $(w_1, w_2) \approx E_1$ (w_1, w_3) but E_2 $(w_1, w_2) > E_2$ (w_1, w_3) by statistical test.

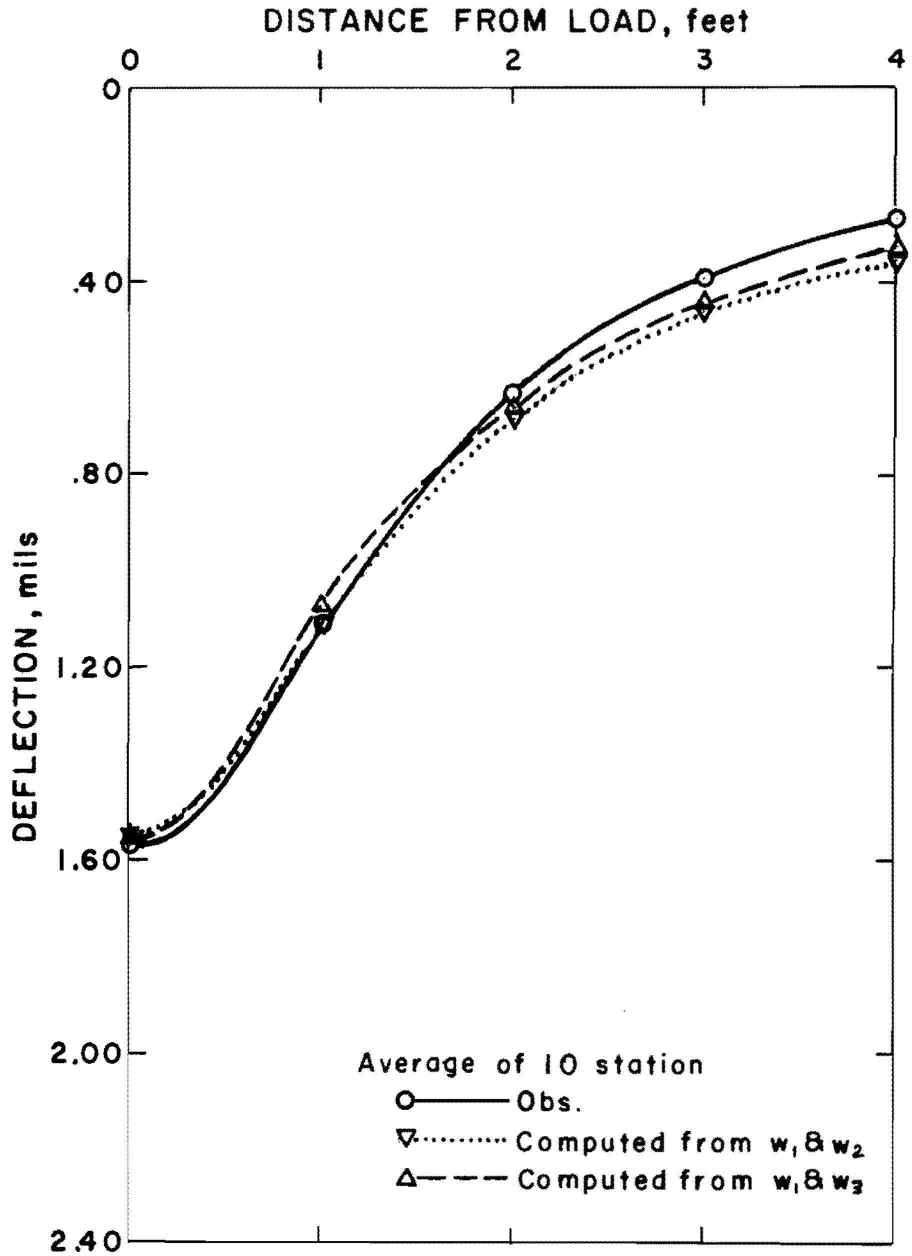

Figure 5c: Computed and average observed Dynaflect deflection basins, Section 5. E_1 (w_1, w_2) = E_1 (w_1, w_3) and E_2 (w_1, w_2) = E_2 (w_1, w_3) by statistical test.

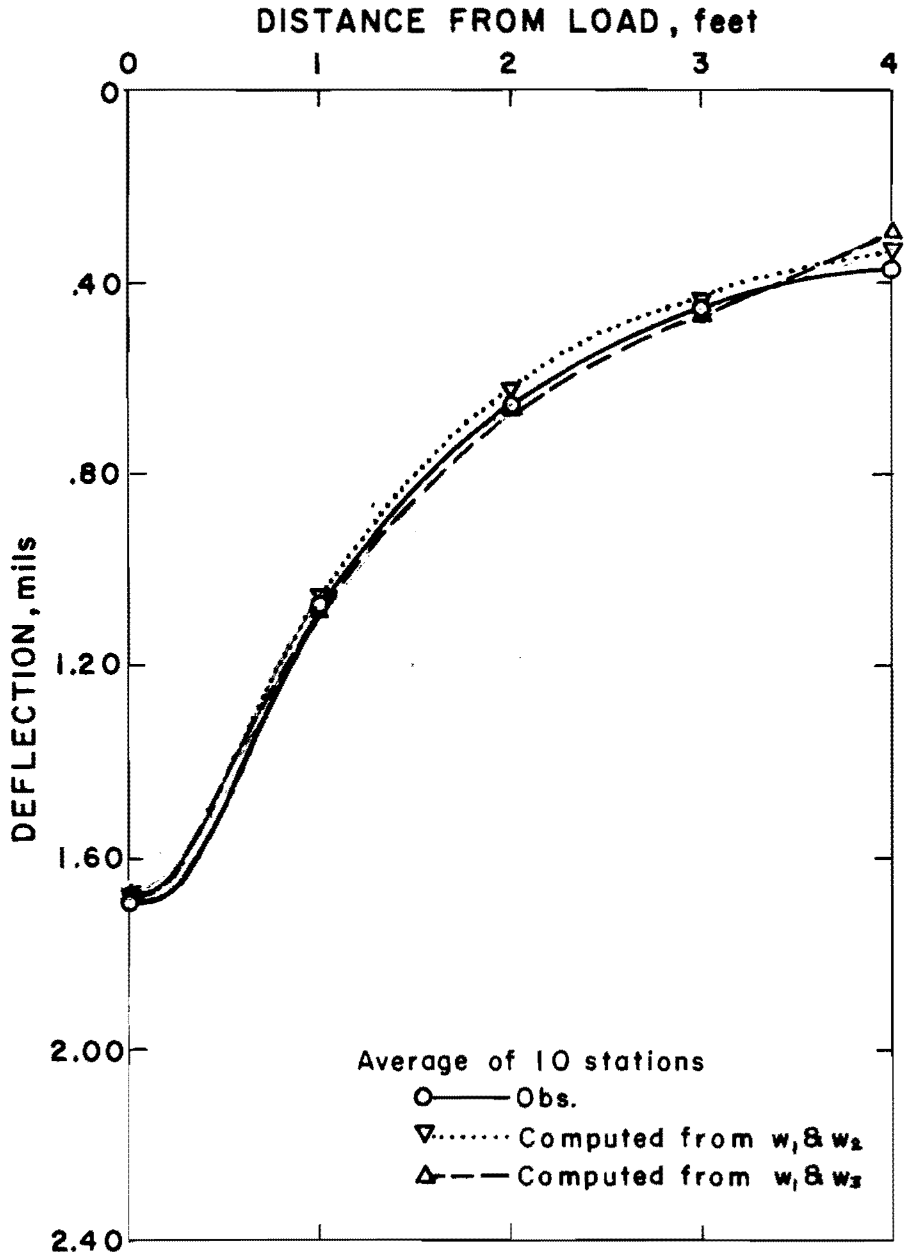

Figure 5d: Computed and average observed Dynaflect deflection basins, Section 12. E_1 (w_1, w_2) ≈ E_1 (w_1, w_3) and E_2 (w_1, w_2) ≈ E_2 (w_1, w_3) by statistical test.

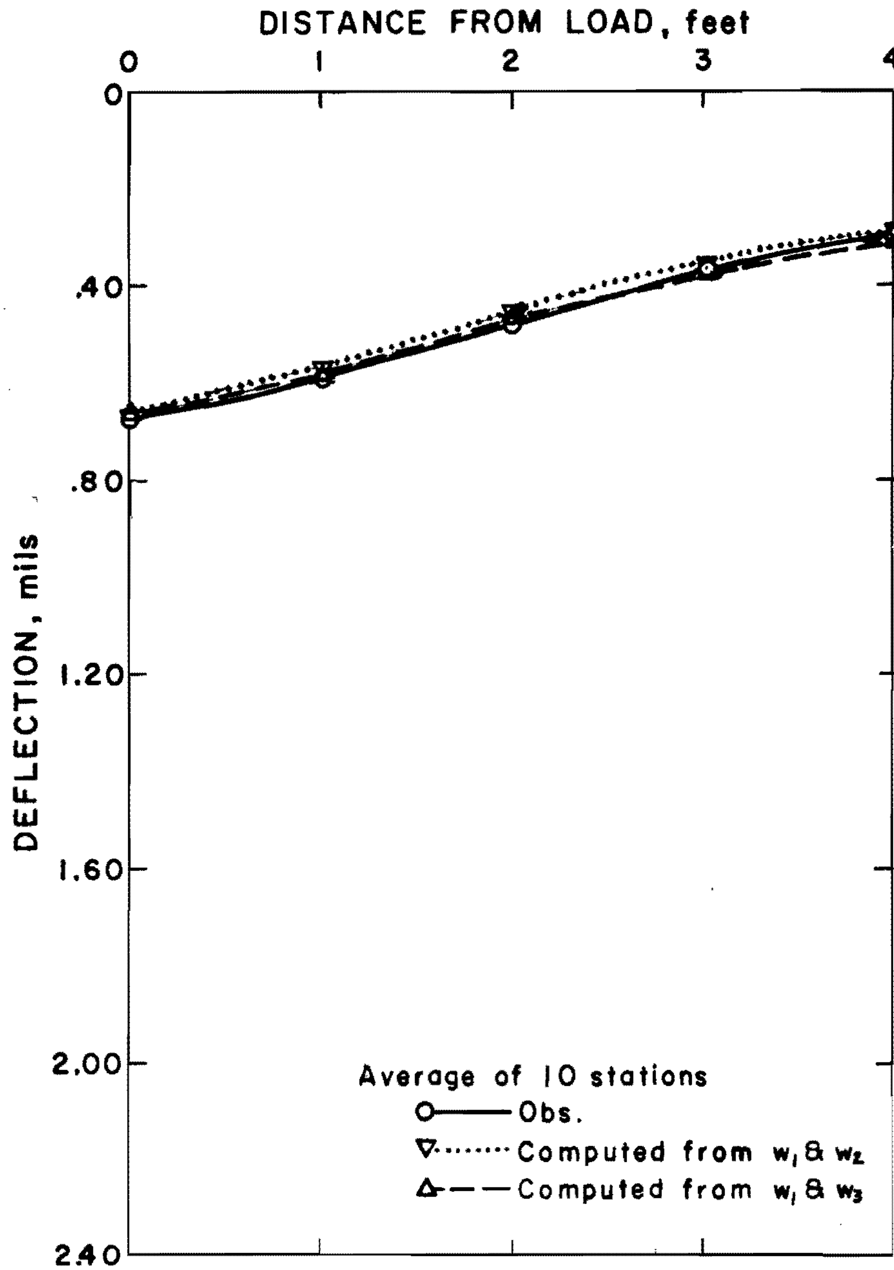

Figure 5e: Computed and average observed Dynaflect deflection basins, Section 17. E_1 (w_1, w_2) ≈ E_1 (w_1, w_3) but E_2 (w_1, w_2) > E_2 (w_1, w_3) by statistical test.

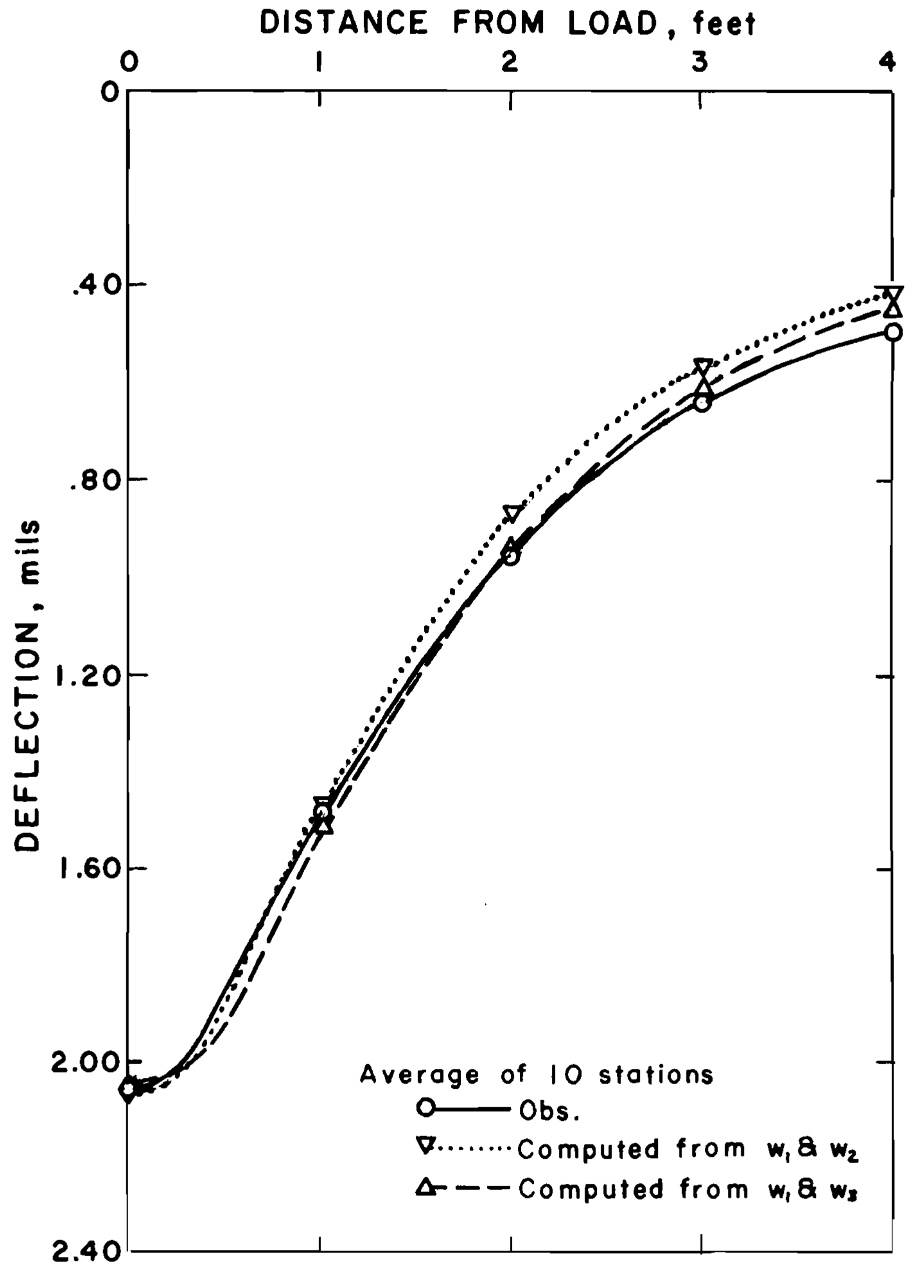

Figure 5f: Computed and average observed Dynaflect deflection basins, Section 16. E_1 (w_1, w_2) < E_1 (w_1, w_3) but E_2 (w_1, w_2) > E_2 (w_1, w_3) by statistical test.

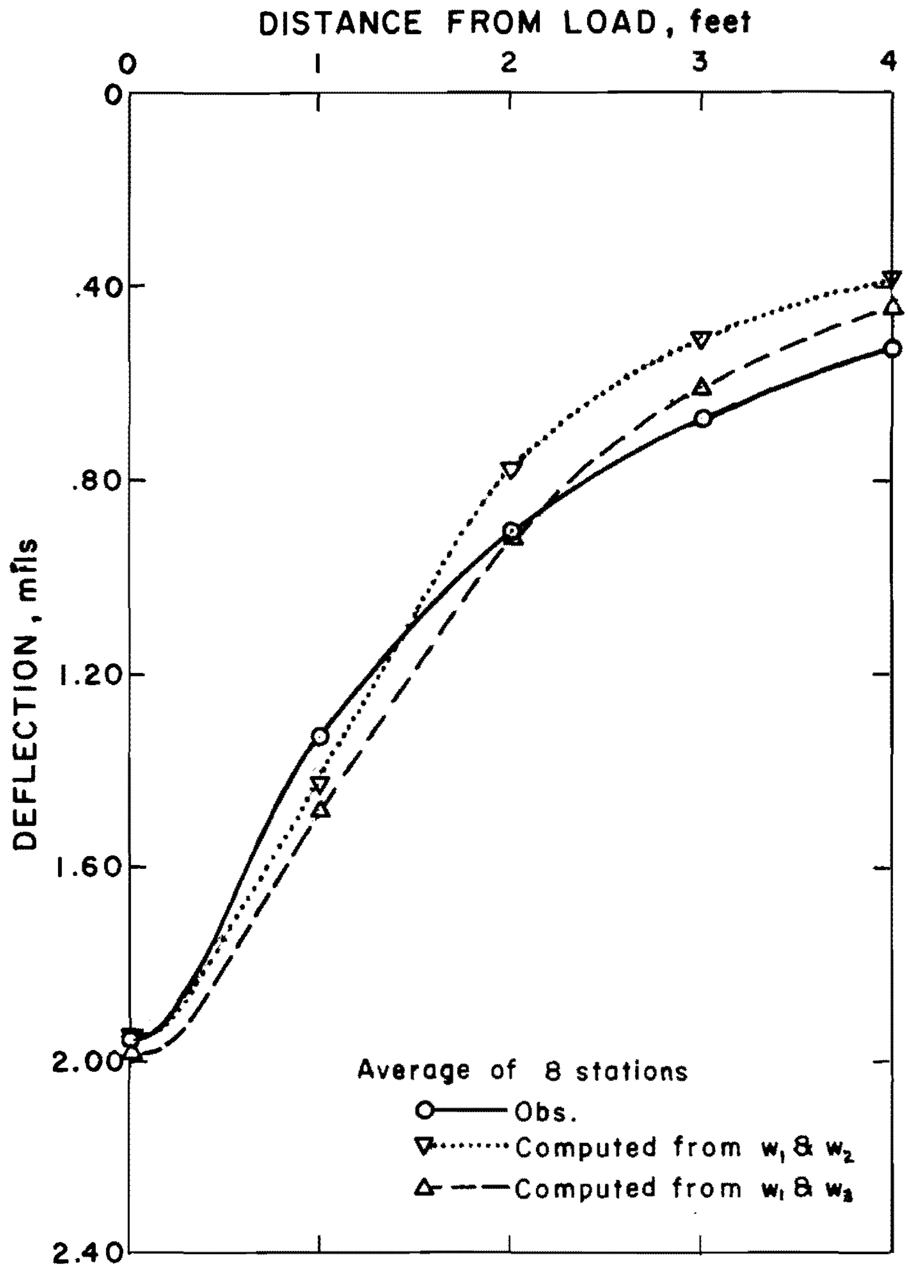

Figure 5g: Computed and average observed Dynaflect deflection basins, Section 17. E_1 (w_1, w_2) < E_1 (w_1, w_3) but E_2 (w_1, w_2) > E_2 (w_1, w_3) by statistical test.

would be a mirror image of the portion shown in the figure. Thus, in theory, the tangent of the curve where it crosses the vertical axis should be horizontal, as indicated in the figures.

(4) E_1 (w_1, w_3) and E_2 (w_1, w_3) were then used in POINT LOAD to calculate the five deflections at the five geophone positions. These results were also plotted in Figures 5a through 5g as curves labeled "computed from w_1 and w_3".

(5) Finally the average values of w_1, w_2, w_3, w_4 and w_5 actually observed at the test stations where solutions for the moduli had been obtained, were plotted on Figures 5a through 5g as curves labeled "Obs.". In the case of Section 4 (Figure 5b) only two solutions were found using Geophone 1 and 2 data, while four solutions were obtained using Geophone 1 and 3 data: thus for comparing observed with computed data in Figure 5b, it was necessary to plot two "observed" curves, one being the average of two stations (for comparison with the curve computed from Geophone 1 and 2 data) and the other being the average of four stations (for comparison with the curve computed from Geophone 1 and 3 data). In each of the other figures it was necessary to plot only one "observed" curve.

The distance scale on each of the above mentioned figures has its origin at the position of Geophone 1 between the Dynaflect load wheels, and extends along the line of geophones -- that is, longitudinally along the outer wheel path of the highway. In the caption of each figure the sign, $\approx$, between two symbols means that, according to an analysis of variance (see Table 7 or 8), the moduli represented by the symbols are statistically the same (though actually somewhat different) while the symbols > and < mean "significantly greater than" and "significantly less than", respectively, as determined from an analysis of variance.

Examination of Figures 5a through 5g shows that, with the exception of Sections 4 and 17 (Figures 5b and 5g),

(1) the deflection basins predicted by use of Geophone 1 and 2 data are nearly the same as those predicted by Geophone 1 and 3 data,

(2) the predicted basins lie reasonably close to the observed curves, and

(3) notwithstanding pavement edge effects and other differences between the assumptions underlying the theory and actual site conditions, linear elasticity may be sufficiently accurate for use as a subsystem in a pavement design system, especially if one considers the many other uncertainties that inevitably enter into such a system.

7. Comparison of Certain Stresses Computed from Geophone 1 and 2 Data with Those Computed from Geophone 1 and 3 Data

Given E_1, E_2, and h for a linear elastic two-layer system, it is possible, from the computer program BISTRO mentioned earlier, to estimate -- for any point in the structure -- the state of stress resulting from the combined effect of two or more circular loaded areas on the surface of the pavement. This capability of the program was used to make a limited comparison of stresses within the structure caused by the simulated dual wheel load, totalling 9000 lbs., illustrated in Figure 6. Each circular area is acted on by a constant pressure.

As indicated in the figure, BISTRO requires that a set of rectangular coordinates, x and y, be used to identify the position of the center of each load on the plane surface, z = 0. In addition the radius of each loaded area must be supplied, as well as the coordinates x, y, z of each point within the structure where stresses, strains and displacements are to be computed.

For making a limited comparison of the stress resulting from the use of Geophone 1 and 2 data with that found from Geophones 1 and 3, neighboring points were selected in Layer 1 and Layer 2, located at the base-subbase interface directly beneath the center of one of the loaded areas. Thus, the coordinates used for both points were the same: x = 0, y = 0, z = h. (Obviously, use of the coordinates x = 12, y = 0, z = h, would have given the same results).

For the point located in the base material, the major principal stress (the algebraically greatest stress) was chosen for investigation because of the tendency of this stress to be a tension in any case where $E_1/E_2 > 1$, believed to be the usual case in two-layer pavement

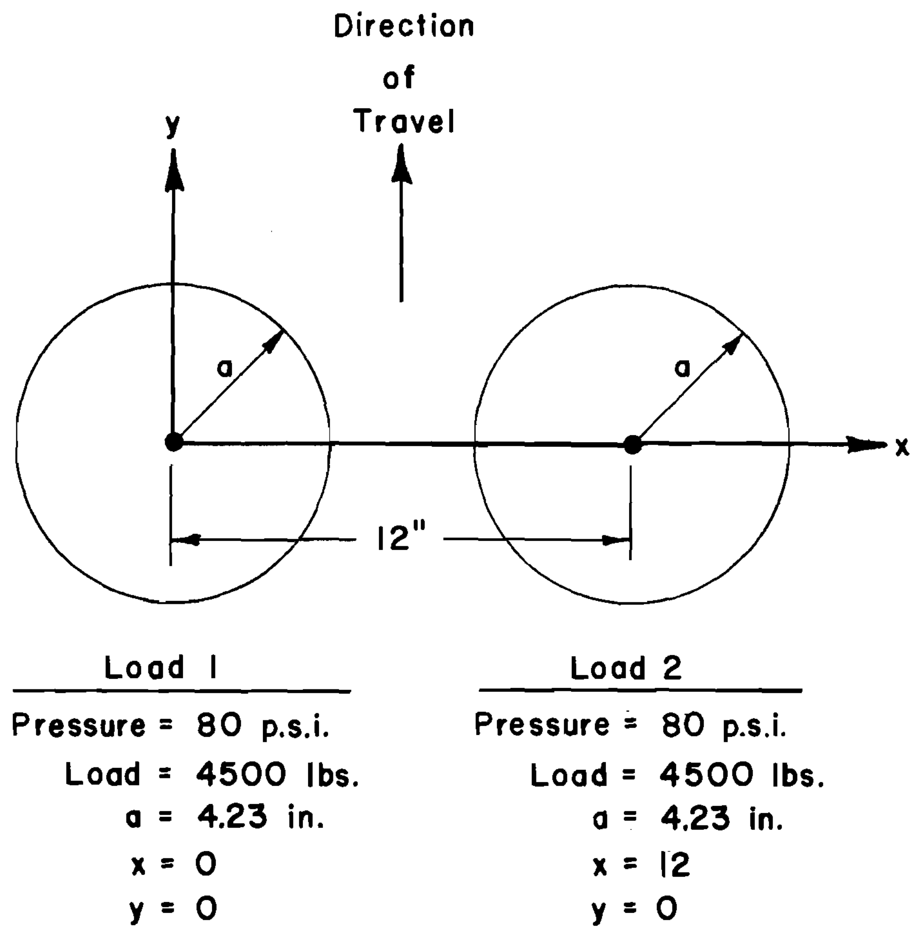

Figure 6: Plan view of simulated dual wheel tire-pavement contact areas, with parameters used in BISTRO for computing principal stresses at the base-subbase interface.

structures. For the neighboring point in the subgrade, however, the minor principal stress was chosen, as the state of stress there, for the usual case, is compressive, and the stress having the greatest absolute value there is the minor principal stress. The results of the computations, which are based on the average moduli and thickness for each test section given in Table 6, are shown in Table 9, and are also plotted in Figures 7, 8 and 9. The identification number of the test section represented by each point in these figures is shown adjacent to the point.

Figure 7 was included here because of the strong influence of the ratio, E_1/E_2, on the distribution of stress in a two-layer system. There is a striking similarity between Figure 7 and Figure 4a which demonstrates that lack of agreement between the two sets of ratios, E_1/E_2, in Figure 7 results almost entirely from lack of agreement between the base moduli E_1 (w_1, w_2) and E_1 (w_1, w_3) apparent in Figure 4a.

The point symbols used in Figures 7, 8 and 9 reflect the results of the analyses of variance given in Tables 7 and 8. As might be expected, it is apparent from Figure 7 that the greatest lack of agreement between the two sets of modular ratios occurred in the four test sections (Sections 4, 15, 16 and 17) where a statistically significant difference was found between one or both of the two moduli computed from Geophone 1 and 2 data, and those computed from Geophone 1 and 3 data. The obvious trend was for the Geophone 1 and 3 data to result in higher estimates of the ratio E_1/E_2 in the case of these four sections. It also should be pointed out that these four sections had the highest values of E_1 (w_1, w_3), ranging from 81,900 psi to 314,100 psi, while the remaining values varied from 14,900 to 23,800, as may be verified by reference to Table 3.

Table 9: Comparison of Certain Stresses at the Base-Subbase Interface Computed from Geophone 1 and 2 Data, with Those Computed from Geophones 1 and 3.

Test Section	Base Material	Base Thick. (In.)	Modulus Ratio E_1/E_2 Computed From		Major Principal Stress (psi)* In Base Material, Computed From		Minor Principal Stress (psi)* In Subgrade Material, Computed From	
			w_1, w_2	w_1, w_3	w_1, w_2	w_1, w_3	w_1, w_2	w_1, w_3
3	Red sandy gravel	12.0	1.30	1.25	3.3	2.7	-14.3	-14.4
4	Asphalt stabilized gravel	7.5	5.30	9.36	47.4	66.9	-15.6	-12.1
5	Lime stabilized sandstone	11.5	2.23	1.60	12.4	6.7	-12.6	-14.2
12	Sandstone	16.2	0.96	1.06	-3.4	0.4	-8.0	-7.8
15	Cement stabilized limestone	14.0	14.17	16.43	32.6	34.4	-3.6	-3.4
16	Asph. emulsion stab. gravel	6.5	6.30	9.84	57.5	74.3	-15.8	-13.0
17	Iron ore gravel	7.8	2.88	7.19	26.6	55.2	-18.9	-13.0

* Tensile stresses are positive, compressive stresses negative.

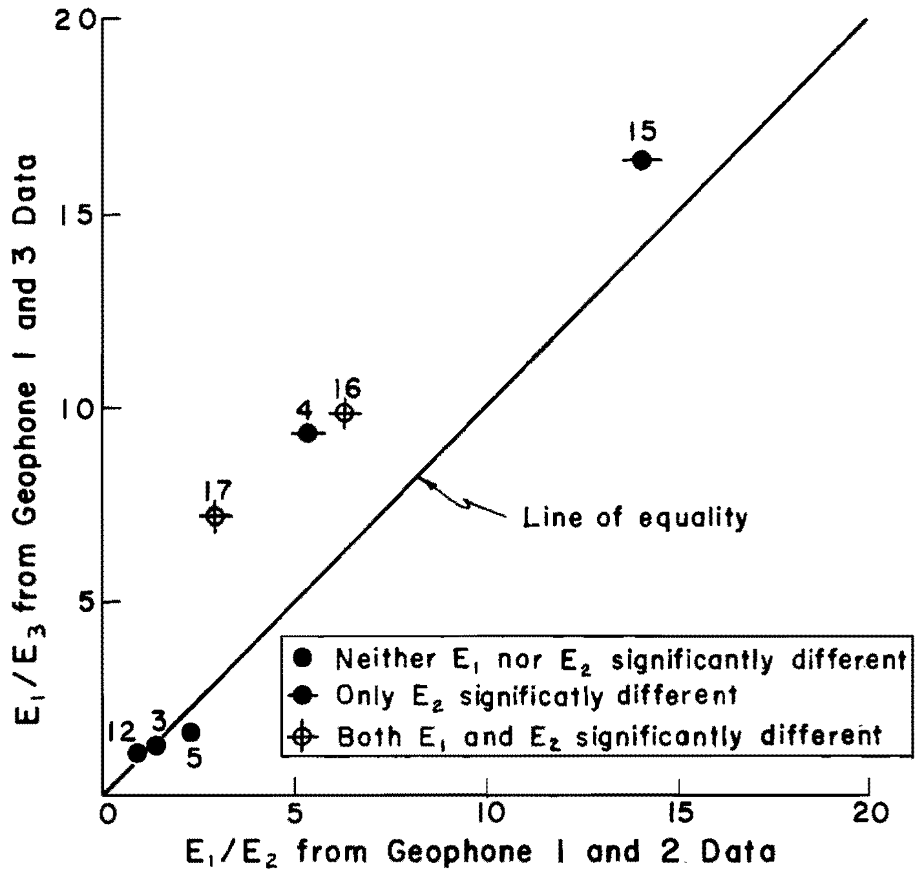

Figure 7: Comparison of the ratio, E_1/E_2, computed from data from different pairs of geophones. Significance data from Tables 7 and 8.

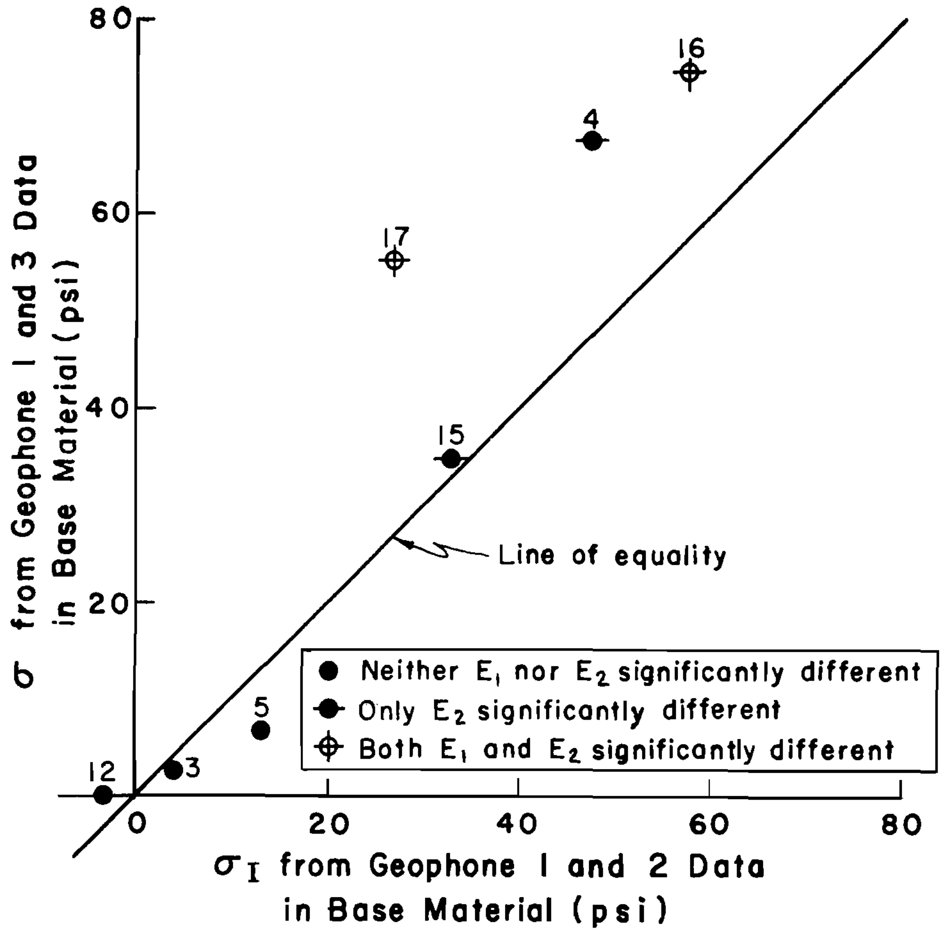

Figure 8: Comparison of the major principal stresses in the base material at the point x = 0, y = 0, z = h, as computed from data from different pairs of geophones. Significance data from Tables 7 and 8.

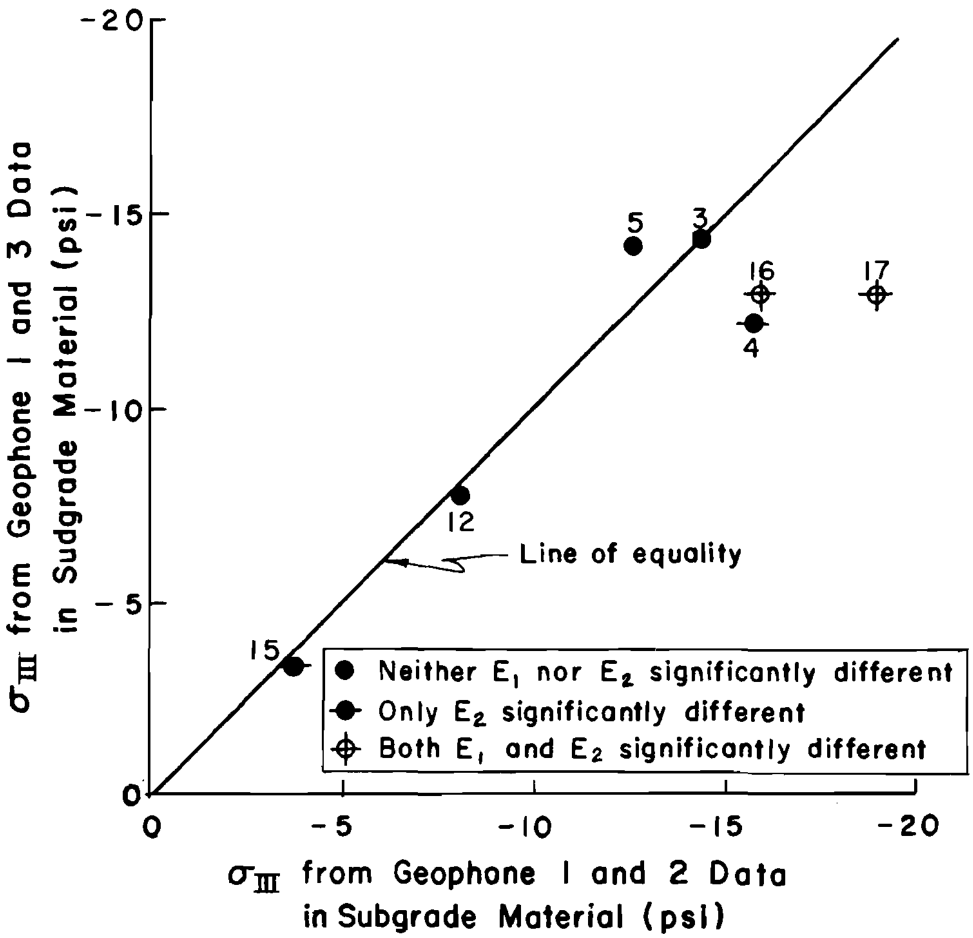

Figure 9: Comparison of the minor principal stresses in the subgrade at the point $x = 0$, $y = 0$, $z = h$, as computed from data from different pairs of geophones. Significance data from Tables 7 and 8.

The effect of the choice of geophone data on the computed major principal stress, σ_I, in the base material at the base-subgrade interface is illustrated in Figure 8. Here only three of the four test sections mentioned in the preceding paragraph show an apparently significant lack of agreement -- Sections 4, 16 and 17. The same three sections also exhibit lack of agreement in Figure 9, where the minor principal stress, σ_{III}, computed from Geophone 1 and 2 data, is compared with that computed from Geophones 1 and 3.

In summary, it is concluded from the information presented in this chapter that the use of Geophone 1 and 3 data in lieu of Geophones 1 and 2, resulted, in most cases, in

(1) a greater ratio, E_1/E_2, which, in turn, led to

(2) a greater tensile stress in the base material at the base-subbase interface, and

(3) a somewhat smaller compressive stress in the subgrade material at the base-subbase interface.

(4) However, in the writers' opinion, the differences in <u>stresses</u> mentioned above were not especially significant when viewed from the vantage point of design and materials enginers familiar with the frequently observed large differences in the measured <u>strength</u> of apparently similar laboratory specimens of base and subbase materials.

8. Examples of Solutions, Rigid Pavements

Site Description: Through the courtesy of airport officials and their consultants, a Texas Transportation Institute team measured Dynaflect deflections on concrete pavements at the Houston International Airport on June 22, 1971.

The airport is situated on the Pleistocene Montgomery formation. The subgrade soils are described as silty sand, sandy silt, clayey sand, clayey silt and expansive silty clay.

Test Details and Results: Dynaflect tests were made at eleven locations, or "test points", on the airport pavements. At each location the load was applied at the center of a 25 x 25-foot concrete slab. Test points were selected on one runway, three taxi-ways and one apron. Slab thicknesses, substructures, Geophone 1 and 3 data, and the moduli computed by ELASTIC MODULUS II at each test point are given in Table 10. The average values of E_1 and E_2, their standard deviations from their averages, and their coefficients of variation, are given at the bottom of the table.

It can be seen from Table 10 that -- as was previously pointed out in the case of flexible pavements -- the computed values of E_1 were somewhat more variable than those of E_2.

Sensitivity of Computed Moduli to Variations in Measured Deflections: A careful examination of the data given in the columns headed "w_1", "w_3", "E_1", and "E_2" in Table 10 will show the extreme sensitivity of the calculated moduli to small changes in either of the measured deflections, w_1 and w_3. For example, if Test Points 34 and 49 are compared, it will be seen that an increase of 0.01 mil -- or 1/100,000 in. -- in w_1 (with h and w_3 held constant) resulted in a decrease in the computed value of

Table 10: Data from Houston Intercontinental Airport

(Taken June 22, 1971)

Test Point	General Lodation	Substructure	h (in.)	Deflections (mils)		Computed Moduli (psi)		Figure Showing Deflection Basin
				w_1	w_3	E_1	E_2	
6	Runway 14-32	6 in. sand-shell subbase	12.0	0.40	0.37	7,494,800	13,000	10a
10	Runway 14-32	on soil excavated to approximately 4 ft. and	12.0	0.50	0.44	3,066,500	14,300	10b
13	Runway 14-32	re-compacted.	12.0	0.52	0.47	4,137,400	11,700	10c
25	Taxiway A	6 in. sand-shell subbase	12.0	0.40	0.36	5,085,500	15,600	10d
28	Taxiway A	on soil excavated to approximately 4 ft. and	12.0	0.43	0.39	5,154,900	14,000	10e
32	Taxiway A	re-compacted.	12.0	0.41	0.36	3,674,300	17,600	10f
34	Taxiway A		12.0	0.40	0.37	7,494,800	13,000	10g
49	Taxiway B	6 in. sand-shell subbase on soil excavated to approximately 4 ft. and re-compacted	12.0	0.41	0.37	5,099,500	15,100	10h
56	Taxiway K	9 in. soil-cement subbase	14.0	0.33	0.31	7,831,000	13,600	10i
63	Taxiway K	on soil excavated to approximately 4 ft. and re-compacted	12.0	0.39	0.35	4,952,600	16,400	10j
69	North Apron	12 in. soil-cement subbase on soil excavated to approximately 6 ft. and re-compacted	12.0	0.24	0.22	10,975,400	23,000	10k
					Average	5,906,000	15,200	
					Standard Deviation	2,311,000	3,100	
					Coefficient of Variation (%)	39	20	

E_1 from approximately 7,500,000 psi to about 5,100,000 psi, accompanied by an increase in E_2 from 13,000 psi to 15,100 psi. And by comparing Test Points 25 and 34, it appears that an increase of 0.01 mils in w_3 (with h and w_1 held constant) caused an increase in E_1 from approximately 5,100,000 psi to approximately 7,500,000 psi, with a corresponding decrease in E_2 from 15,600 psi to 13,000 psi. Now 0.01 mil is generally considered by Dynaflect users to be the limit of instrument and operator error under ideal conditions: we must conclude, therefore, that the values of moduli -- particularly the value of E_1 -- found under routine conditions by the method described in this report must be regarded as rather crude estimates in individual tests, and that instrument error should be included as a component of the total variability involved in a pavement design system using the Dynaflect to characterize the materials in terms of elastic constants.

Computed Versus Observed Deflection Basins, Concrete Pavements: A method similar to that described in Chapter 6 for computing w_1, w_2, w_3, w_4 and w_5 from given values of E_1, E_2 and h, was employed for comparing computed deflection basins with those measured on the concrete pavements at the Houston Intercontinental Airport. The results are shown in Figures 10a through 10k. Since Geophone 1 and 3 data were used in determining the values of E_1 and E_2 employed in computing the theoretical (dashed) curve in each figure, this curve passes through the circled points representing the output of those geophones, i.e., the points plotted at zero and two feet on the distance scale. Thus, in any of these figures, the departure of the dashed line from the remaining three circled points is a partial measure of the difference between the assumptions made in the theory and the actual conditions at the test site, including instrument error.

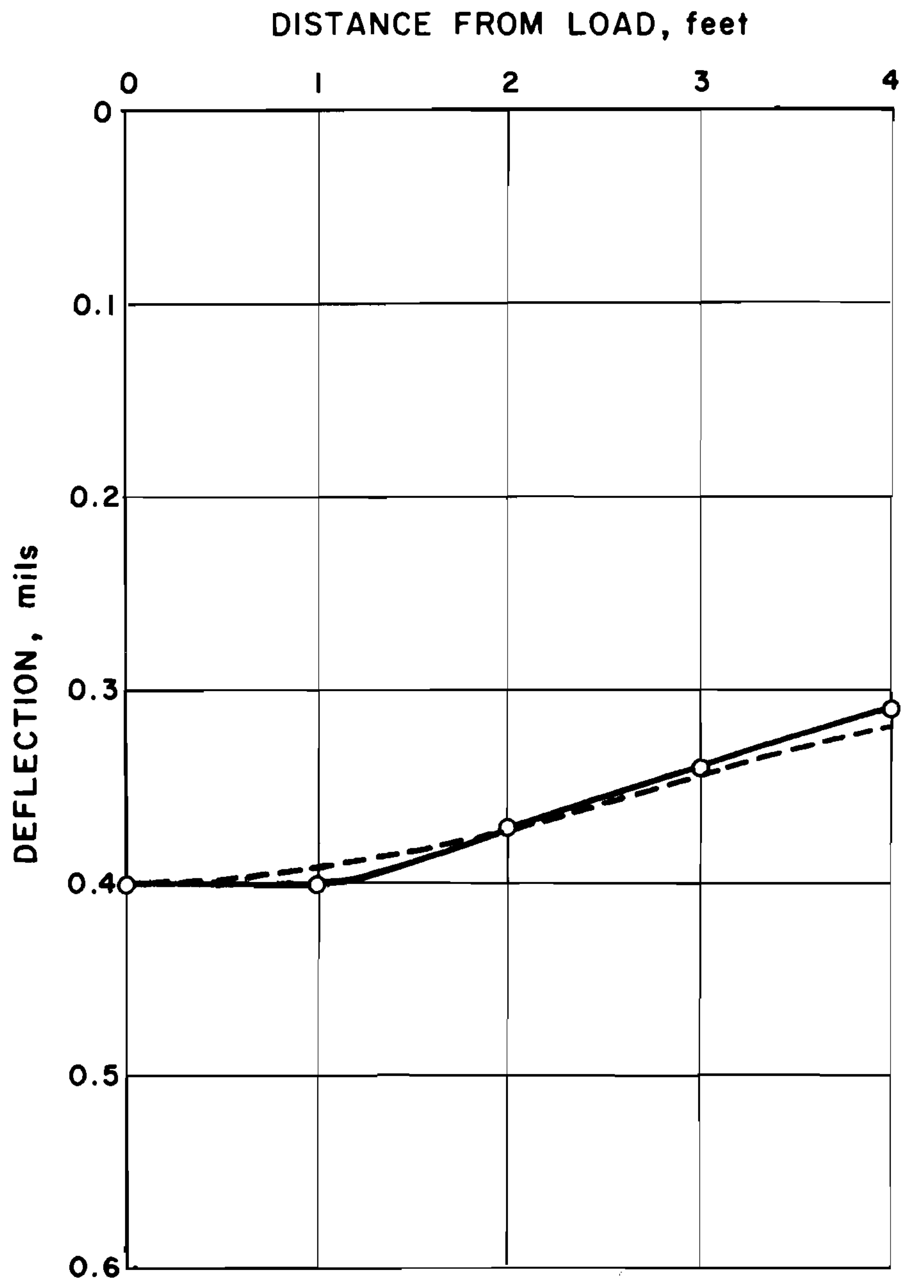

Figure 10a: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 6. Moduli are given in Table 10.

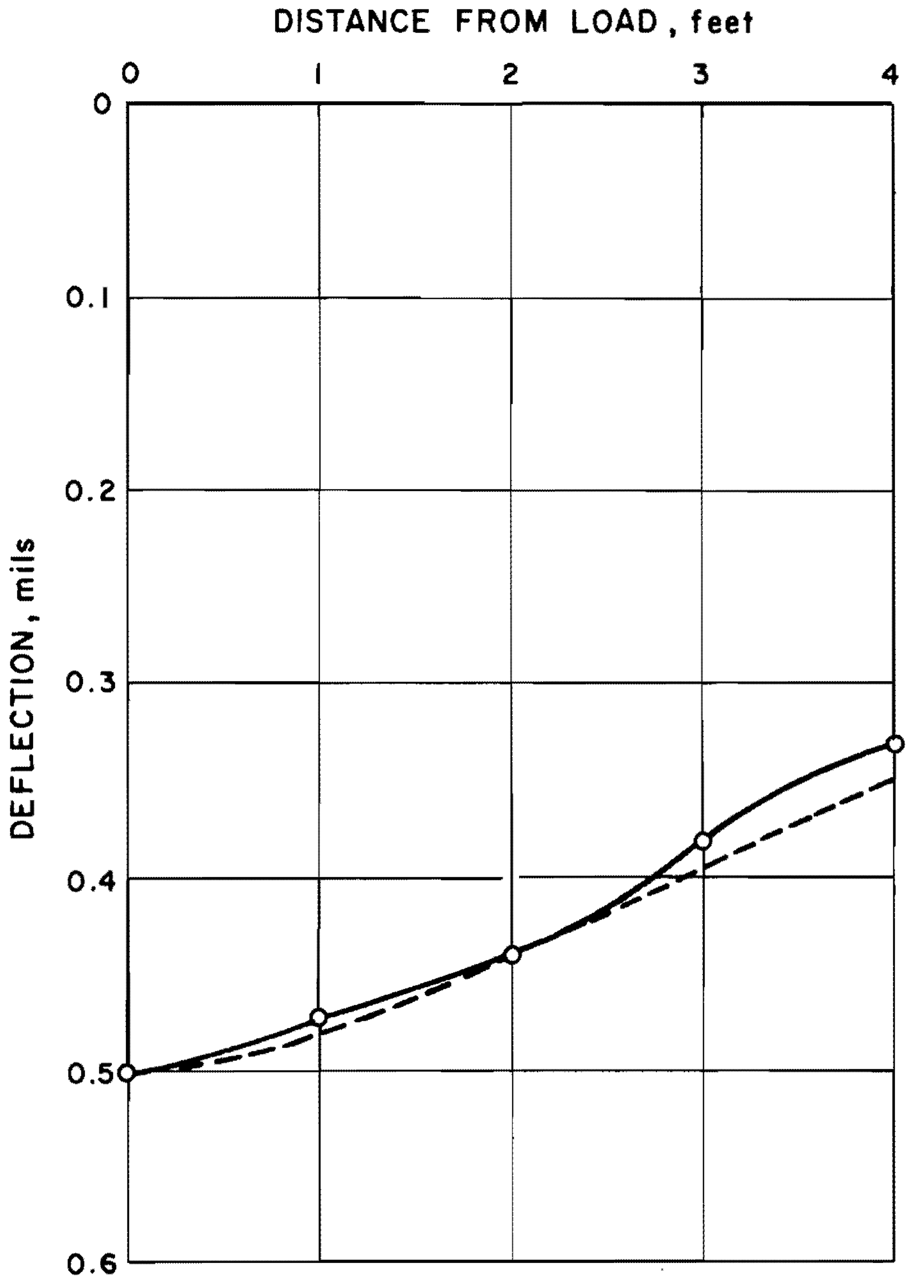

Figure 10b: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 10. Moduli are given in Table 10.

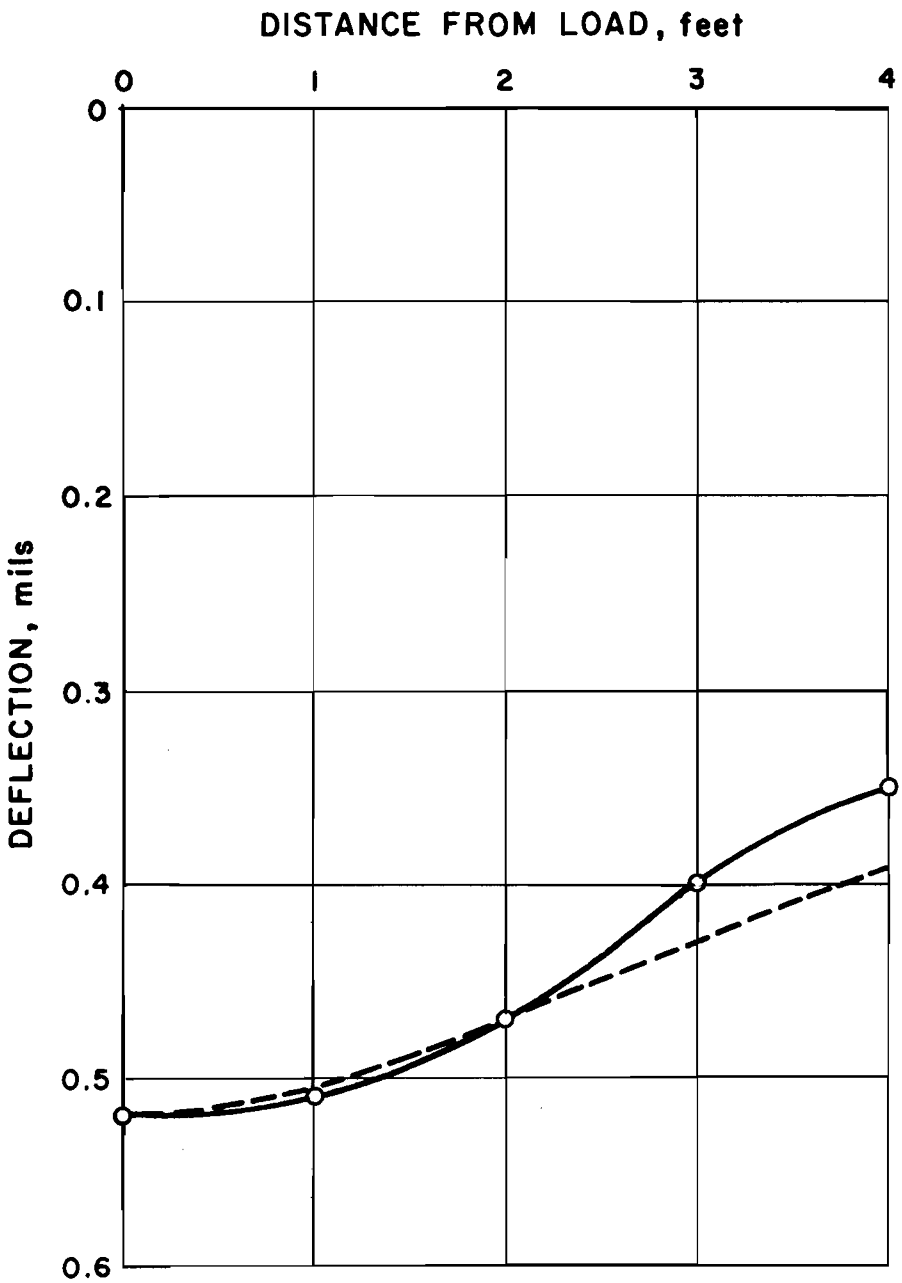

Figure 10c: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 13. Moduli are given in Table 10.

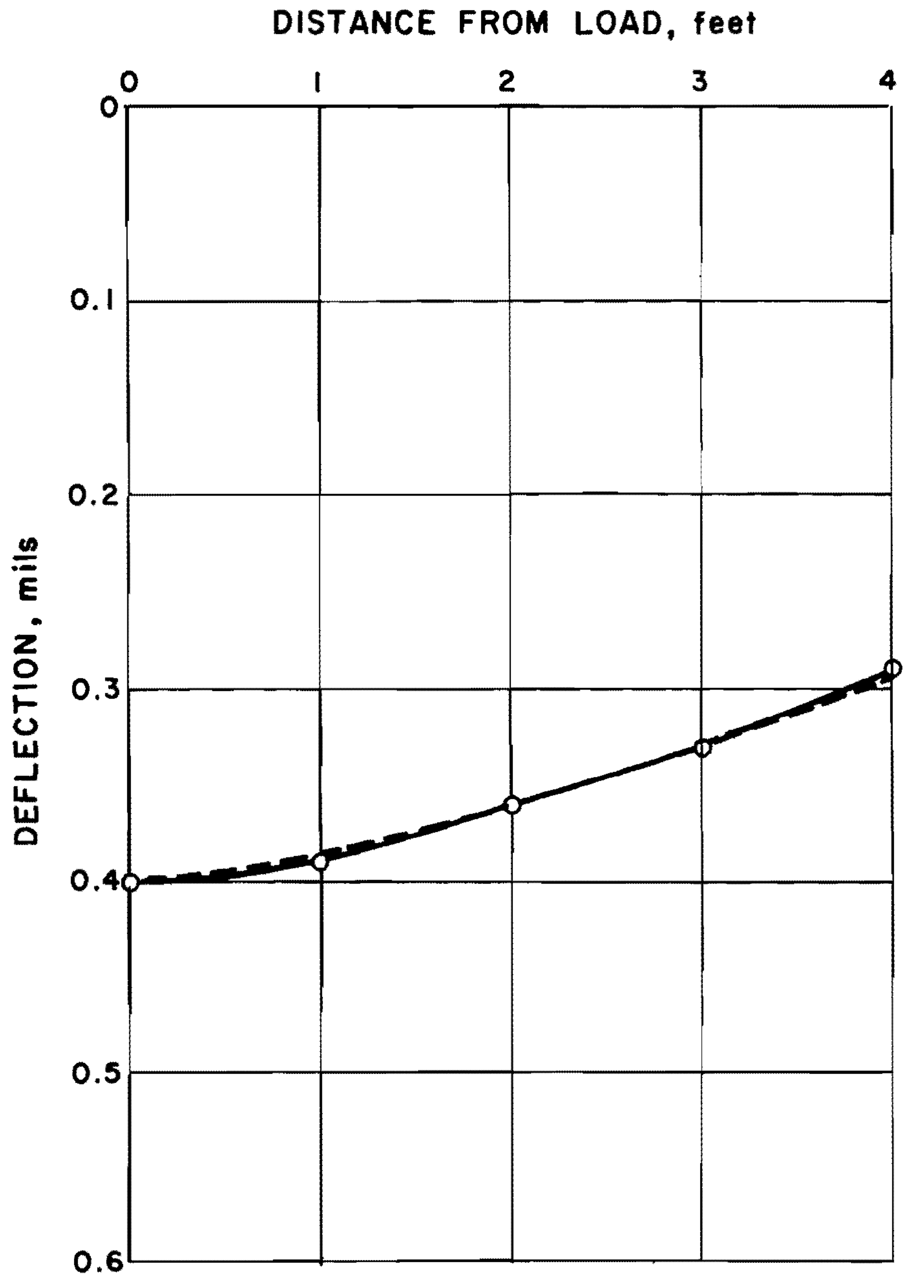

Figure 10d: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 25. Moduli are given in Table 10.

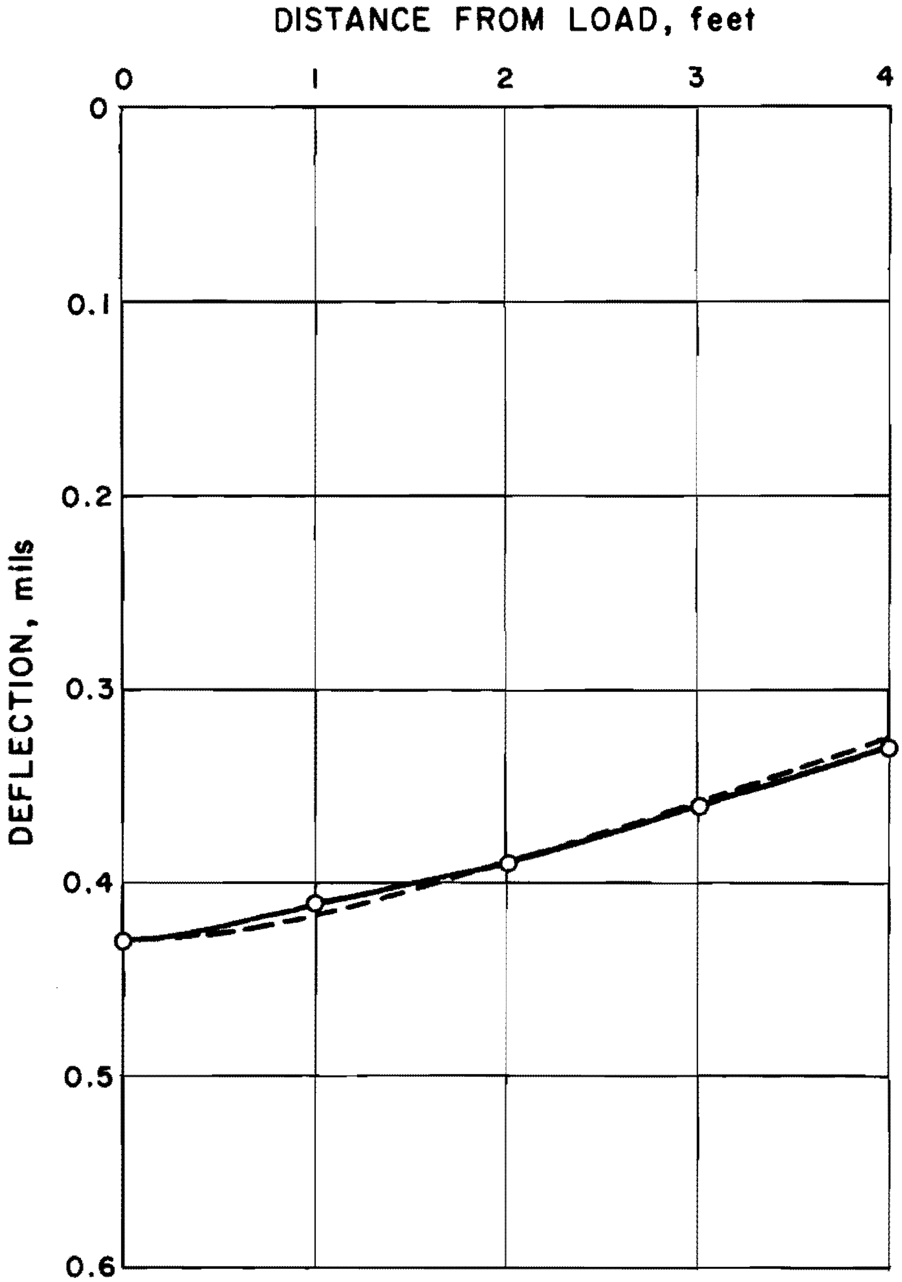

Figure 10e: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 28. Moduli are given in Table 10.

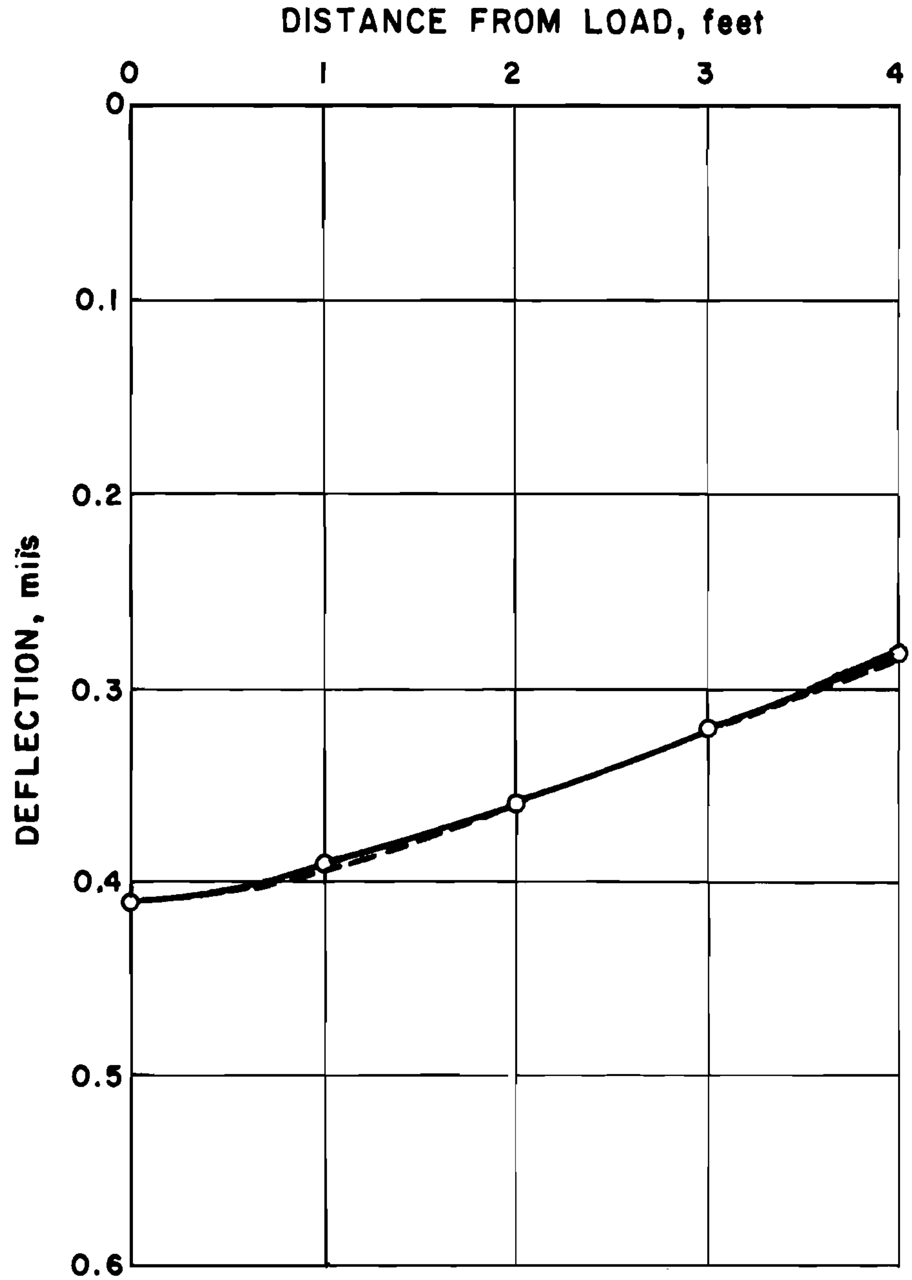

Figure 10f: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2(w_1, w_3) at Test Point 32. Moduli are given in Table 10.

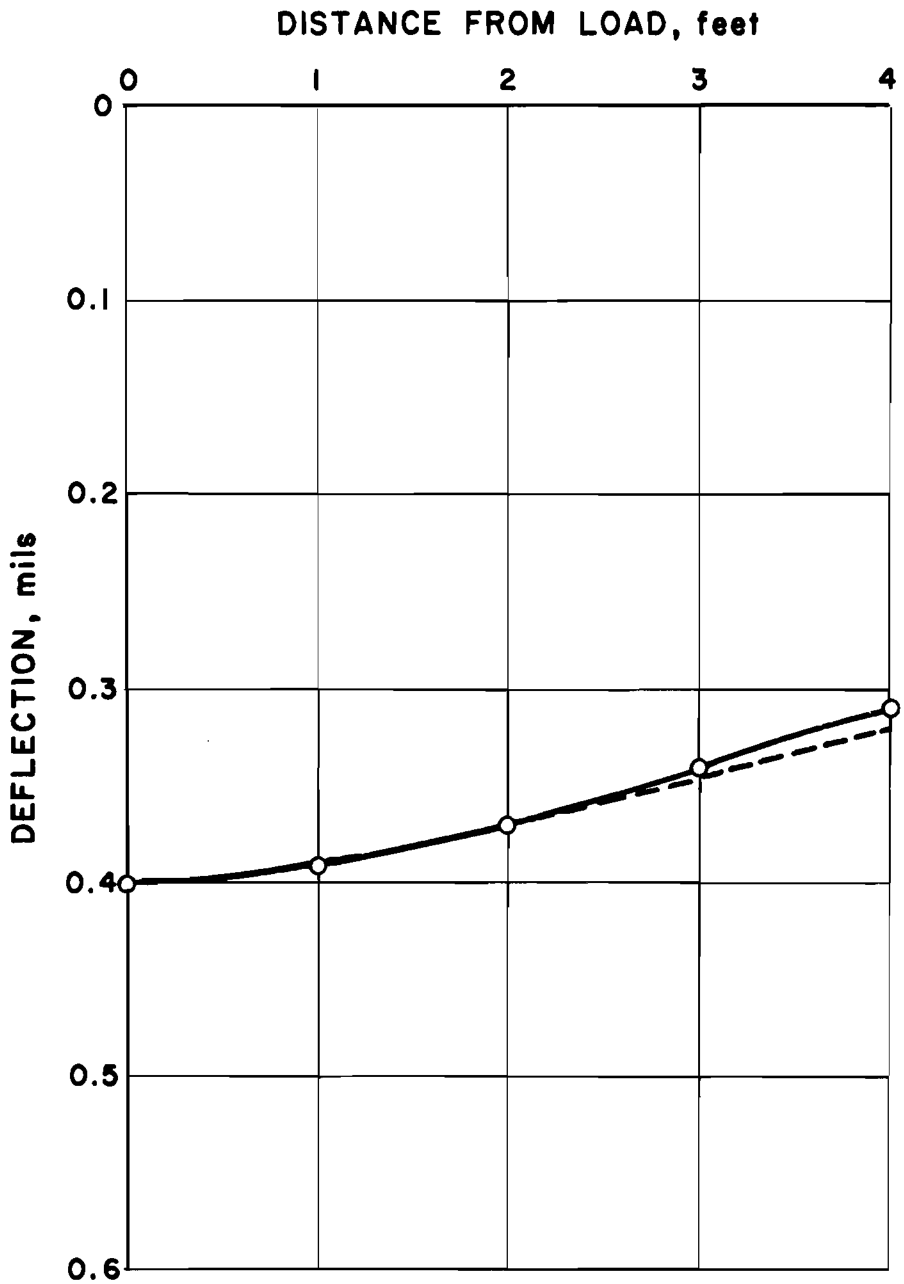

Figure 10g: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 34. Moduli are given in Table 10.

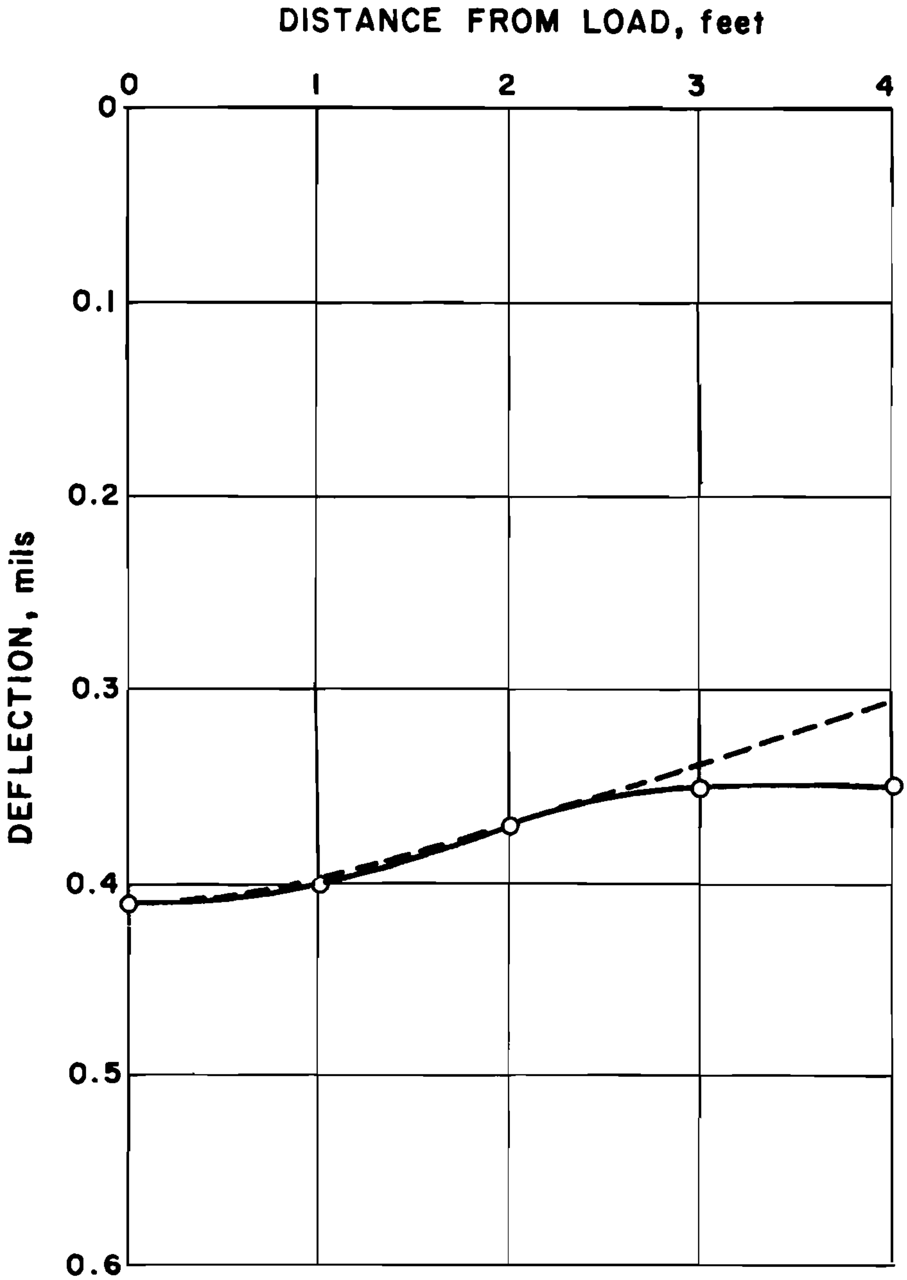

Figure 10h: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 49. Moduli are given in Table 10.

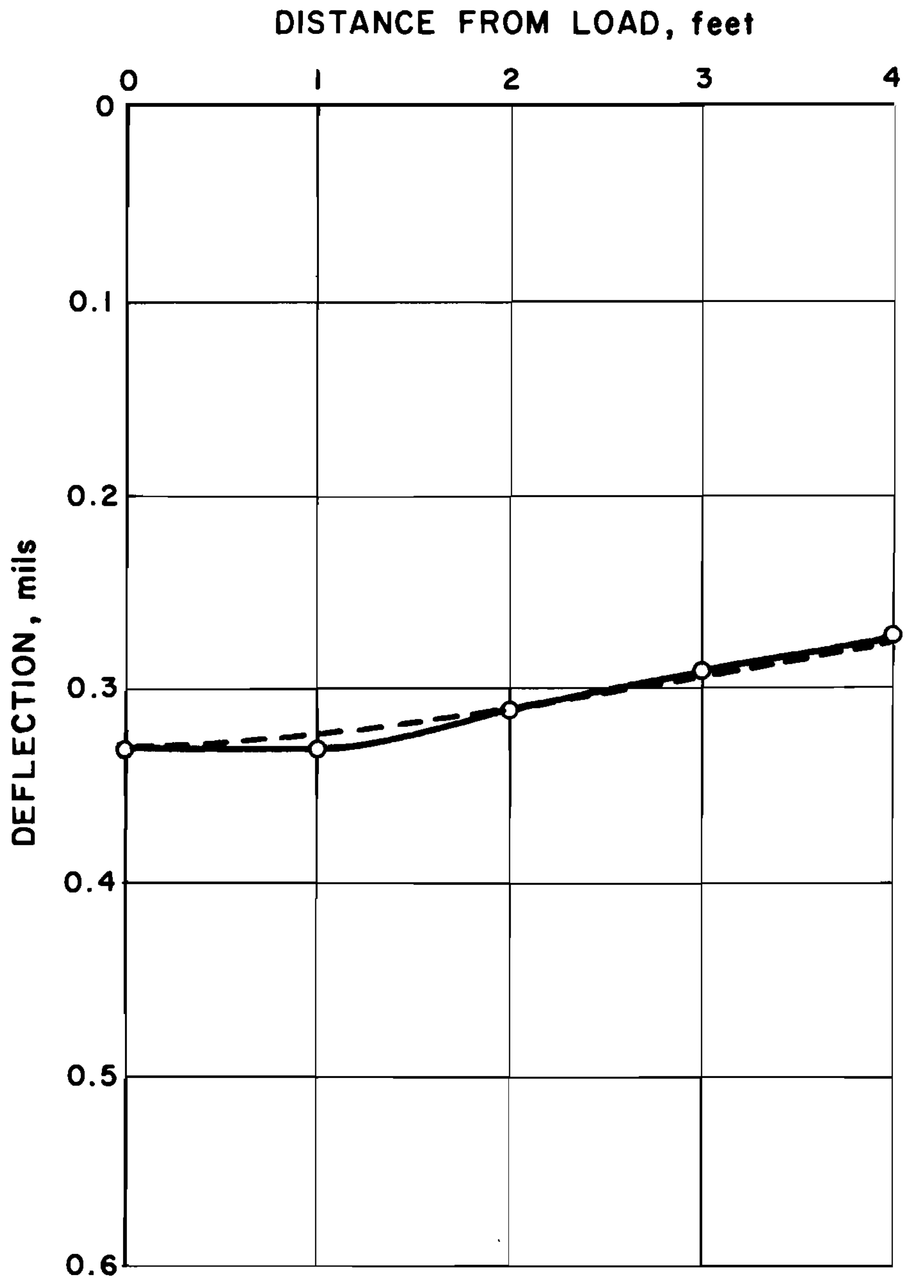

Figure 10i: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 56. Moduli are given in Table 10.

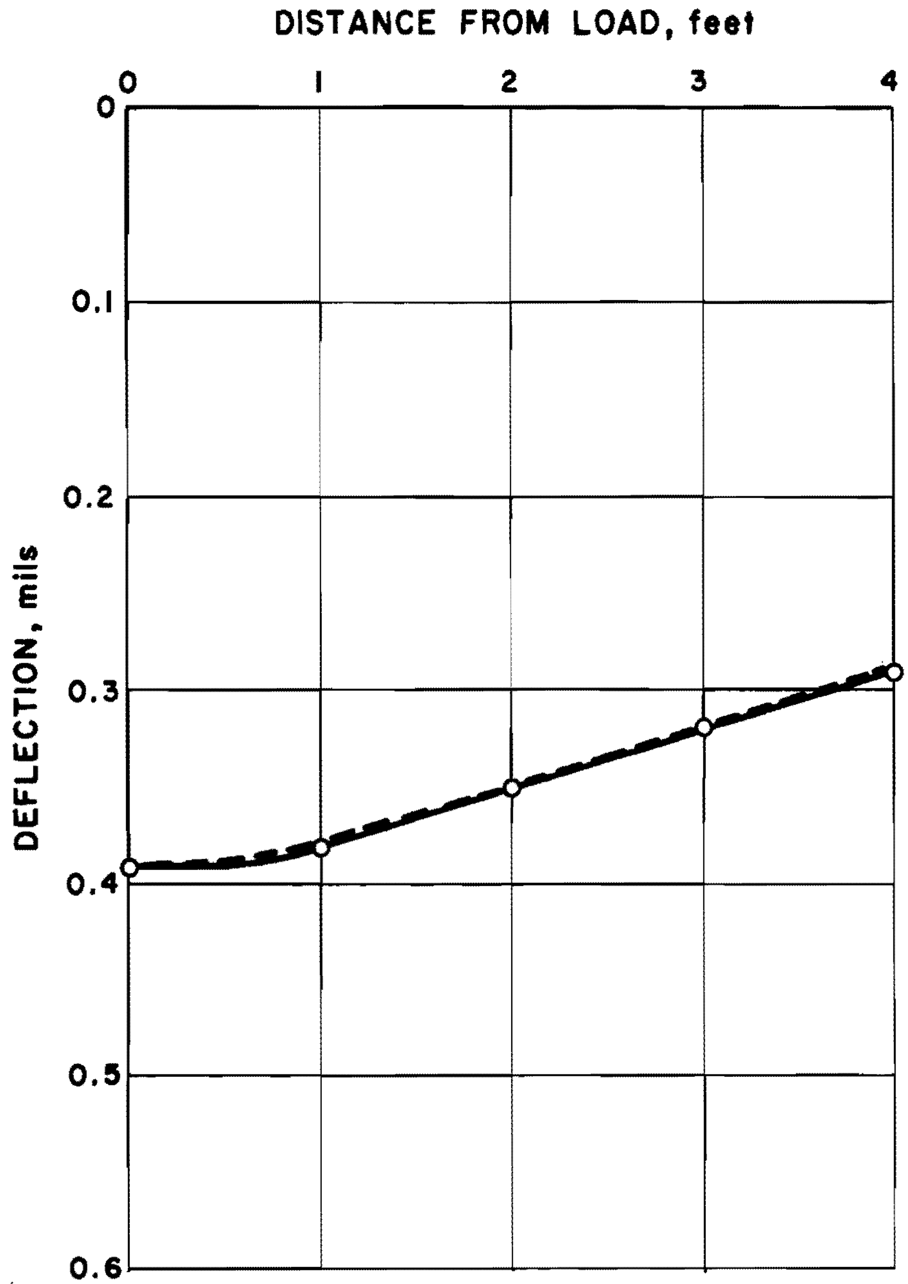

Figure 10j: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 63. Moduli are given in Table 10.

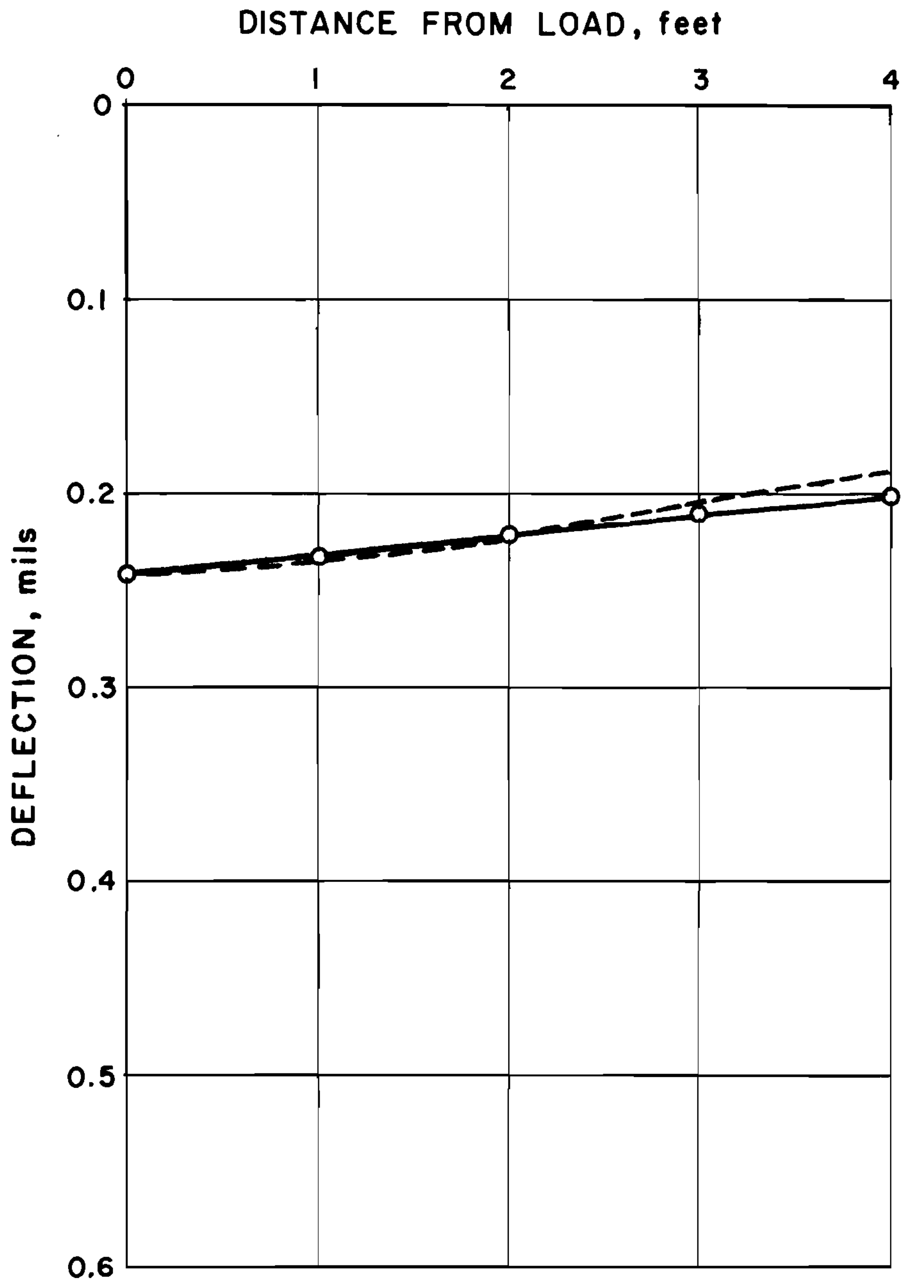

Figure 10k: Observed deflection basin (solid line through circled points) compared with theoretical basin (dashed line) computed from E_1 (w_1, w_3) and E_2 (w_1, w_3) at Test Point 69. Moduli are given in Table 10.

After examining Figures 10a through 10k, the writers assigned a subjective rating -- "good" or "bad" -- to the degree of agreement between each theoretical curve and the corresponding measured basin. The results are given in Table 11. As shown at the bottom of the table, 8 of the 11 theoretical curves were rated "good". The greatest disagreement between theoretical and observed basins was found in Figure 10c and 10h while near perfect agreement appeared in Figures 10d, 10f and 10j.

Table 11: Subjective Rating of "Goodness of Fit" of Theoretical Deflection Basins to Experimental Data, Figures 10A through 10K

Figure Number	Test Point Number	Subjective Rating
10A	6	Good
10B	10	Bad
10C	13	Bad
10D	25	Good
10E	28	Good
10F	32	Good
10G	34	Good
10H	49	Bad
10I	56	Good
10J	63	Good
10K	69	Good

Summary

Rating	Number	Percent
Good	8	73
Bad	3	27
Total	11	100

9. Conclusions

With regard to certain technical aspects of the computer program ELASTIC MODULUS II, the following conclusions were drawn.

1. As judged by its agreement with the older and more comprehensive computer program BISTRO, the program ELASTIC MODULUS II is as accurate as required for the job it was designed to perform.
2. Occasionally the possibility of a non-unique solution -- or no solution at all -- will arise from Dynaflect data processed through ELASTIC MODULUS II; however, restraints built into the program inform the user of such cases by printing out appropriate messages.

From Dynaflect data taken on flexible pavement sections, the following conclusions were drawn.

3. The elastic modulus of the base of a 500-ft., apparently uniform test section, as estimated from Dynaflect data, was much more variable than the subgrade modulus, probably because of the assumption that the subgrade was homogeneous to an infinite depth.
4. In several test sections (2 out of 7 in the case of the base, 4 out of 7 in the case of the subgrade) the average section modulus estimated from Geophone 1 and 2 data differed significantly, by statistical tests, from the modulus estimated from Geophone 1 and 3 data.
5. The differences mentioned in Conclusion 4, above, were not, in the opinion of the writers, very significant when considered from a practical or engineering point of view.

6. In five out of the seven sections tested, indications were that

   a. Dynaflect deflection basins predicted by use of Geophone 1 and 2 data were nearly the same as those predicted by use of Geophone 1 and 3 data,

   b. the predicted basins lie reasonably close to the observed basins, and

   c. linear elasticity may be sufficiently accurate for use as a subsystem in a flexible pavement design system.

7. The use of a Geophone 1 and 3 data, in lieu of Geophones 1 and 2, for computing base and subgrade moduli resulted, in most cases, in

   a. a greater ratio, E_1/E_2, which, in turn led to

   b. a greater computed tensile stress in the base material at the base-subbase interface resulting from the application of a 9000-lb. dual wheel load to the pavement surface, and

   c. a somewhat smaller compressive stress in the subgrade material at the base-subbase interface.

8. The differences in stress mentioned in Conclusion 7 did not appear to be especially significant when viewed from the vantage point of design and materials engineers familiar with the frequently observed large differences in the measured strength of apparently similar laboratory specimens of base and subbase materials.

From Dynaflect data taken on concrete pavements at the Houston Intercontinental Airport, the following conclusions were drawn.

9. After noting the dramatic effect of very small changes in the output of either Geophone 1 or Geophone 3 on the computed modulus of a concrete slab, it was concluded that the values of moduli -- particularly the value of E_1 -- found by the method described in this report must be regarded as rather crude estimates in individual tests, and that instrument error should be included as a component of the total variability involved in a pavement design system using the Dynaflect to characterize the materials in terms of their elastic constants.

10. Dynaflect deflection basins computed from Geophone 1 and 3 data at 11 test points agreed with the observed data nearly perfectly in three cases, very well in six cases, and rather poorly in two cases.

From all the data studied the following conclusion was drawn.

11. The Dynaflect in its present form, combined with the computer programs ELASTIC MODULUS II and POINT LOAD, has the potential of becoming a useful method for material characterization in a pavement design system using linear elastic theory as a subsystem, provided that proper heed is paid to instrument error, the inevitable variability of highway materials in place, and the many other uncertainties that enter into a complete pavement design system. ELASTIC MODULUS II, rather than ELASTIC MODULUS, is recommended in order to achieve compatibility between moduli found for rigid pavements and those determined for flexible pavements.

List of References

1. Scrivner, Frank H.; Chester H. Michalak and William M. Moore, "Calculation of the Elastic Moduli of a Two Layer Pavement System from Measured Surface Deflections," Research Report 123-6, Texas Transportation Institute, Texas A&M University, College Station, Texas, 1971.

2. Scrivner, Frank H.; W. M. Moore; W. F. McFarland and G. R. Carey, "A Systems Approach to the Flexible Pavement Design Problem," Research Report 32-11, Texas Transportation Institute, Texas A&M University, College Station, Texas, 1968.

3. Hudson, W. Ronald; B. Frank McCullough; F. H. Scrivner and James L. Brown, "A Systems Approach Applied to Pavement Design and Research," Research Report 123-1, Highway Design Division Research Section, Texas Highway Department, Austin, Texas; Texas Transportation Institute, Texas A&M University, College Station, Texas; and Center for Highway Research, The University of Texas at Austin, Austin, Texas, 1970.

4. "Texas Highway Department Pavement Design System, Part I, Flexible Pavement Designer's Manual," Highway Design Division, Texas Highway Department, Austin, Texas, 1970.

Appendix A

Listing of ELASTIC MODULUS II

```
          C
          C
          C      ELASTIC MODULUS II -- MAIN PROGRAM (FOR W1 AND W3)
          C
          C
0001             DIMENSION STA(200),W1(200),W2(200),W3(200),W4(200),
                *  AP2(200),LA1(5),LA2(5),LA3(5),LA4(5),LA5(5),LA6(5),
                * W5(200), AS2(200), A(20), SCI(200),
                *  IXDATE(3),COMM(7),REM(4)
0002             REAL * 8 STA, DAS , DAP, DBLE
          C
          C
          C
          C      NOTE -- THE PRINT & FORMAT STATEMENTS ARE FOR
          C      OUTPUT ON 8 1/2 X 11 PAPER.  FOR OUTPUT ON 11 X 14
          C      PAPER USE THE PRINT & FORMAT STATEMENTS WITH 'C' IN
          C      COLUMN 1.
          C
          C
          CCC    STATEMENT FUNCTION TO ROUND 'X' TO NEAREST 'EVEN'
          C
0003             ROUND( X, EVEN )  =  AINT( ( X + EVEN * .5 ) / EVEN  )
                * *  EVEN
          C
0004          10 CONTINUE
          C
          C      READ CARD CODE & REMAINDER OF CARD INTO A - ARRAY
          C
0005             READ(5,1,END=1000)  NCARD, ( A(I), I = 1 , 20 )
          C
0006           1 FORMAT( I3, 19A4, A1 )
0007             CALL  CORE  ( A,  80 )
          C
          C      TEST FOR DATA CARD 1
          C
0008             IF(NCARD.EQ.100) GO TO 11
          C
          C      TEST FOR DATA CARD 2
          C
0009             IF(NCARD.EQ.200) GO TO 12
          C
          C      TEST FOR DATA CARD 3
          C
0010             IF(NCARD.EQ.300) GO TO 13
          C
          C      I IS A POINTER TO DATA IN STORAGE
          C
0011          14 I=N+1
```

```
             C
             C      READ DATA CARD 4
             C
 0012               READ(5,6) ICONT,ISECT,M,IDAY,IYEAR,STA(I),D1,D2,D3,
                   * D4, D5, D6, D7, D8,
                   *D9,D10,(REM(J),J=1,4),ICK
             C
 0013             6 FORMAT(     I4,4I2,A7,3X, 5(F2.1,F3.2),8X,4A4,I2)
             C
 0014               IF( N.GT. 0 ) GO TO 555
 0015               IF(NG.GT.0) GO TO 555
             C
             C      PRINT OUTPUT COLUMN HEADINGS
             C
 0016               PRINT 61
             C   61 FORMAT(/,1X,'STATION     W1      W2      W3      W4      ',
             C     = 'W5      SCI    ** ES   **   ** EP   **      REMARKS' / )
             C
 0017            61 FORMAT(/ 7X,'STATION    W1     W2     W3     W4     W5',
                   = '     SCI   ** ES   ** ** EP   **  REMARKS'/)
             C
             C      CALCULATE DEFLECTIONS & SCI ( DEFLECTIONS IN MILS )
             C
 0018           555 W1(I)=D1*D2
 0019               W2(I)=D3*D4
 0020               W3(I)=D5*D6
 0021               W4(I)=D7*D8
 0022               W5(I)=D9*D10
 0023               SCI(I)=W1(I)-W2(I)
             C
             C      TEST FOR W1 OR W2 = 0, AND W1 LESS THAN W2
             C
 0024               IF (W1(I).EQ.0.OR.W2(I).EQ.0) GO TO 64
 0025               IF(W1(I).LT.W2(I)) GO TO 66
             C
 0026               AW1 =AW1 +W1 (I)
 0027               AW2 =AW2 +W2 (I)
 0028               AW3 =AW3 +W3(I)
 0029               AW4 =AW4 +W4(I)
 0030               AW5 =AW5 +W5(I)
 0031               ASCI=ASCI+SCI(I)
 0032               AS2(I)   =   0.0
 0033               AP2(I)   =   0.0
             C
             C
             C      TEST FOR NO UNIQUE SOLUTION
             C
             C
```

```
0034             IF( ( W1(I) * 10.0 ) / ( W3(I) * SQRT( 676.0 ) )
               =  .GT.  1.0  .AND.  DP  .LE.  11.1 )  GO  TO  60
          C
          C
          C      CONVERT W1 & W3 TO INCHES
          C
0035             W1(I)  =  W1(I) / 1000.
0036             W3(I)  =  W3(I) / 1000.
          C
          C      PASS W1, W3, & TOTAL PAVEMENT THICKNESS TO EMOD,
          C      EMOD RETURNS UNROUNDED VALUES OF PAVEMENT & SUBGRADE
          C      MODULI  AS DAP & DAS
          C
0037             CALL EMOD ( DBLE(W1(I)), DBLE(W3(I)), DBLE(DP),DAP, DAS)
          C
          C      CONVERT W1 & W3 TO MILS
          C
0038             W1(I) = W1(I) * 1000.
0039             W3(I) = W3(I) * 1000.
          C
          C
          C
          C
          C      ROUND PAVEMENT & SUBGRADE MODULI  TO NEAREST 100
          C
0040             DAS  =  ROUND( DAS, 100. )
0041             DAP  =  ROUND( DAP, 100. )
          C
          C      PUT PAVEMENT & SUBGRADE MODULI  IN STORAGE
          C
0042             AS2(I)  =  DAS
0043             AP2(I)  =  DAP
          C
          C      ADD TO THE SUMS OF THE DEFLECTIONS, SCI, PAVEMENT,
          C      AND SUBGRADE MODULI
          C
0044             AAS2=AAS2+AS2(I)
0045             AAP2=AAP2+AP2(I)
          C
          C      ADD TO N, THE NUMBER OF VALID TEST POINTS
          C
0046             N=N+1
          C
          C      PRINT A LINE OF OUTPUT
          C
          C      PRINT 63,STA(I),W1(I),W2(I),W3(I),W4(I),W5(I),SCI(I),
          C     = AS2(I), AP2(I), ( REM(J),J=1,4 )
          C   63 FORMAT(1X, A7, 3X, 5(F5.3,2X ), F5.3,2F11.0,5X,4A4 )
```

```
                C
0047                  PRINT 63,STA(I),W1(I),W2(I),W3(I),W4(I),W5(I),SCI(I),
                    * AS2(I), AP2(I), (REM(J),J=1,2)
0048               63 FORMAT( 7X,A7,1X,6(F6.3), 2F10.0,2X, 2A4   )
0049                  N1 = N1 + 1
0050                  IF(N1.LT.30) GO TO 88
                C
                C     SKIP TO NEXT PAGE & PRINT OUTPUT COLUMN HEADINGS IF
                C     THIRTY LINES HAVE BEEN PRINTED
                C
0051               84 CONTINUE
0052                  PRINT 51
                C     PRINT 56
                C     PRINT 57,IDIST,C01,C02,C03,C04,ICONT,ISECT,IJOB,HWY1,
                C    * HWY2, XLANE, M, IDAY, IYEAR, IDYNA
                C
0053                  PRINT 56, IDIST, C01, C02, C03, C04
0054               56 FORMAT( T35,'DIST.        COUNTY'/ T36, I2,9X, 3A4,A2 /)
0055                  PRINT 57, ICONT, ISECT,IJOB,HWY1,HWY2,M,IDAY,IYEAR,IDYNA
0056               57 FORMAT( T19, 'CONT.    SECT.    JOB    HIGHWAY     DATE',
                     '        DYNAFLECT' / T19,I4,2I7,4X,A4,A3,I4,2('-',I2),I9 /)
0057                  PRINT 61
0058                  N1 = 0
0059               88 CONTINUE
                C
                C     CHECK FOR LAST DATA CARD 4
                C
0060                  IF (ICK.EQ.0) GO TO 10
0061                  GO TO 80
                C
                C     READ DATA CARD 1
                C
0062               11 READ(5,2) IDIST,C01,C02,C03,C04,ICONT,ISECT,IJOB,HWY1,
                    * HWY2, XLANE, DP,M,IDAY,IYEAR,IDYNA,(COMM(I),I=1,7)
0063                2 FORMAT(    I2,3A4,A2,I4,2I2,A4,A3,A3,F5.2,4I2,7A4)
                C
                C     PRINT HEADING
                C
0064                  PRINT 51
0065               51 FORMAT( '1' )
                C
0066                  PRINT 52
                C  52 FORMAT(33X,'TEXAS HIGHWAY DEPARTMENT',/)
                C
0067               52 FORMAT(35X,'TEXAS HIGHWAY DEPARTMENT' /)
                C
0068                  PRINT 53,IDIST
                C  53 FORMAT(31X,'DISTRICT ',I2,' - DESIGN SECTION',/)
```

```
               C
 0069             53 FORMAT(33X,'DISTRICT ',I2,' - DESIGN SECTION' /)
               C
 0070                PRINT 54
               C  54 FORMAT(19X,'DYNAFLECT DEFLECTIONS AND CALCULATED ',
               C    = 'ELASTIC MODULI '  /  )
               C
 0071             54 FORMAT(21X,'DYNAFLECT DEFLECTIONS AND CALCULATED ',
                    * 'ELASTIC MODULI ' / )
               C
               C     GET CURRENT DATE
               C
 0072                CALL DATE ( IXDATE(1), IXDATE(2), IXDATE(3)  )
               C
 0073                PRINT 55,IXDATE
               C  55 FORMAT(30X,'THIS PROGRAM WAS RUN - ', 2A3,A2  /     )
               C
 0074             55 FORMAT(32X,'THIS PROGRAM WAS RUN - ', 2A3,A2 /  )
               C
               C     PRINT 56
               C  56 FORMAT( 1X,'DIST.       COUNTY       CONT.   SECT.',
               C    *'   JOB   HIGHWAY    DATE       DYNAFLECT')
               C
               C
 0075                PRINT 56, IDIST, C01, C02, C03, C04
               C
               C     PRINT CONTROL INFORMATION FROM DATA CARD 1
               C
               C     PRINT 57,IDIST,C01,C02,C03,C04,ICONT,ISECT,IJOB,HWY1,
               C    * HWY2, XLANE, M, IDAY, IYEAR, IDYNA
               C  57 FORMAT( 2X,I2,5X,3A4,A2,3X,I4,4X,I2,5X,I2,2X,A4,A3,
               C    * A3, 2X, I2, '-', I2, '-', I2, 6X, I2 / )
               C
 0076                PRINT 57, ICONT, ISECT,IJOB,HWY1,HWY2,M,IDAY,IYEAR,IDYNA
               C
 0077                PRINT 58,(COMM(I),I=1,7),DP
 0078             58 FORMAT(10X,7A4,2X,'PAV. THICK. = ',F5.2,' INCHES' /)
               C
               C     INITIALIZE ALL SUMS & COUNTERS
               C
 0079                N=0
 0080                N1 = 0
 0081                N0 = 0
 0082                AW1= 0.
 0083                AW2=0.
 0084                AW3=0.
 0085                AW4=0.
 0086                AW5=0.
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```
0087              ASCI=0.
0088              AAS2=0.
0089              AAP2=0.
0090              SR1= 0.
0091              SR2= 0.
0092              SR3= 0.
            C
0093              GO TO 10
            C     READ & PRINT INFORMATION ON DATA CARD 2
            C
0094           12 READ(5,3) (LA1(I),I=1,5),T1,(LA2(I),I=1,5),T2,
                 * (LA3(I),I=1,5), T3
0095            3 FORMAT(     5A4,F4.2,5A4,F4.2,5A4,F4.2)
            C     PRINT 59,(LA1(I),I=1,5),T1,(LA2(I),I=1,5),T2,
            C    * (LA3(I),I=1,5), T3
            C  59 FORMAT(  1X,5A4,1X,F5.2,2X,5A4,1X,F5.2,2X,5A4,1X,F5.2)
            C
0096              PRINT 59,(LA1(I),I=1,5),T1,(LA2(I),I=1,5),T2
0097              PRINT 59, ( LA3(I), I=1,5), T3
0098           59 FORMAT(16X, 5A4, 1X, F5.2, 5X, 5A4, 1X, F5.2/)
0099              GO TO 10
            C
            C     READ & PRINT INFORMATION ON DATA CARD 3, IF PRESENT
            C
0100           13 READ(5,3)  (LA4(I),I=1,5),T4,(LA5(I),I=1,5),T5,
                 * (LA6(I),I=1,5), T6
            C     PRINT 59,(LA4(I),I=1,5),T4,(LA5(I),I=1,5),T5,
            C    * (LA6(I),I=1,5), T6
            C
0101              PRINT 59,(LA4(I),I=1,5),T4,(LA5(I),I=1,5),T5
0102              PRINT 59, ( LA6(I), I=1,5), T6
0103              GO TO 10
0104           66 NO = NO+1
            C
            C     PRINT NEGATIVE SCI MESSAGE
            C
0105              PRINT 82,STA(I),W1(I),W2(I),(REM(J),J=1,4)
0106           82 FORMAT(1X,A7,3X,F5.3,2X,F5.3,2X,'NEGATIVE SCI OTHER ',
                 * 'CALCULATIONS OMITTED', 4X, 4A4)
            C
0107              N1=N1+1
0108              IF( N1  .LT.  30 )  GO  TO  88
0109              GO  TO  84
0110           64 NO = NO + 1
            C
            C     PRINT ERROR MESSAGE
            C
0111              PRINT 81,STA(I),(REM(J),J=1,4)
```

```
0112           81 FORMAT( 1X,A7,3X,'DATA ERROR ASSUMED A ZERO VALUE RE',
                 * 'AD FOR W1 OR W2', 5X, 4A4 )
             C
0113              N1 = N1+ 1
0114              IF( N1  .LT.  30 )  GO  TO  88
0115              GO  TO  84
             C
             C
0116           60 CONTINUE
0117              N  =  N  +  1
0118              NO  =  NO  +  1
0119              PRINT 85,  STA(I), W1(I), W2(I), W3(I), W4(I),
                   W5(I),  SCI(I)
             C 85 FORMAT(1X, A7, 3X, 5(F5.3,2X ), F5.3, 2X,
             C
0120           85 FORMAT( 7X, A7, 1X, 6F6.3, 2X,
                   'NO UNIQUE SOLUTION'   )
0121              N1  =  N1  +  1
0122              IF( N1  .LT.  30 )  GO  TO  88
0123              GO  TO  84
             C
             C
             C
             C    ALL CARDS READ FOR AN ANALYSIS, CALCULATE AVERAGE
             C    DEFLECTIONS, AVERAGE SCI, AVERAGE PAVEMENT MODULUS,
             C    AND AVERAGE SUBGRADE MODULUS
             C
0124           80 PN=N
0125              N1  =  N  -  NO
             C
             C    N1  IS THE NUMBER OF TEST POINTS THAT HAD
             C    VALID SOLUTIONS
             C
0126              IF( N1  .LE.  0 )  N1  =  1
0127              AW1V= AW1/PN
0128              AW2V= AW2/PN
0129              AW3V= AW3/PN
0130              AW4V= AW4/PN
0131              AW5V= AW5/PN
0132              ASCIV=ASCI/PN
0133              AAS2V=AAS2/N1
0134              AAP2V=AAP2/N1
             C
             C    CALCULATE VARIANCE OF SCI, SUBGRADE MODULUS & PAVE-
             C    MENT MODULUS
             C
0135              DO 62 I=1,N
0136              IF(W1(I).EQ.0.OR.W2(I).EQ.0) GO TO 62
```

```
0137              SR1= SR1+((ASCIV- SCI(I))**2)
0138              IF( AS2(I)  .EQ.  0.0 )  GO  TO  62
0139              SR2= SR2+((AAS2V-AS2(I))**2)
0140              SR3= SR3+((AAP2V- AP2(I))**2)
0141           62 CONTINUE
            C
            C     PRINT AVERAGES
            C
0142              PRINT 65,AW1V,AW2V,AW3V,AW4V,AW5V,ASCIV,AAS2V,AAP2V
            C  65 FORMAT(/1X,'AVERAGES', 6(2X, F5.3 ), 2F11.0    )
            C
0143           65 FORMAT(/ 7X, 'AVERAGES', 6(F6.3), 2F10.0   )
            C
            C     CALCULATE STANDARD DEVIATION OF SCI, SUBGRADE
            C     MODULUS, AND PAVEMENT MODULUS
            C
            C
            C     SKIP THE CALCULATION IF ONLY ONE OBSERVATION
            C
0144              IF( PN  .EQ.  1 )  GO  TO  90
0145              SE1  =   SQRT(SR1/(PN-1))
            C
0146              IF( N1  .LE.  1 )  GO  TO  90
            C
0147              SE2  =   SQRT(SR2/(N1-1))
0148              SE3  =   SQRT(SR3/(N1-1))
            C
            C     PRINT STANDARD DEVIATIONS
            C
0149              PRINT 71,SE1,SE2,SE3
            C  71 FORMAT( 1X,'STANDARD DEVIATION',27X,F5.3, 2F11.0     )
            C
0150           71 FORMAT(  7X,'STANDARD DEVIATION', 20X,F6.3,2F10.0)
            C
0151           90 CONTINUE
0152              IF( N  .EQ.  1 )  N1  =  1
0153              PRINT 99,N, N1,  N1
            C  99 FORMAT(1X,'NUMBER OF POINTS IN AVERAGE = ',
            C    =  10X, 2I10, I11  )
            C
0154           99 FORMAT(  7X,'NUMBER OF POINTS IN AVERAGE = ',
                 =  I14, I9, I10 )
            C
0155              PRINT 91
            C  91 FORMAT( /,5X,'W1    DEFLECTION AT GEOPHONE 1')
            C
0156           91 FORMAT(/10X,'W1    DEFLECTION AT GEOPHONE 1')
            C
```

```
0157                PRINT 92
             C   92 FORMAT(   5X,'W2   DEFLECTION AT GEOPHONE 2')
             C
0158             92 FORMAT( 10X,'W2    DEFLECTION AT GEOPHONE 2')
             C
0159                PRINT 93
             C   93 FORMAT(    5X,'W3   DEFLECTION AT GEOPHONE 3')
             C
0160             93 FORMAT( 10X,'W3    DEFLECTION AT GEOPHONE 3')
             C
0161                PRINT 94
             C   94 FORMAT(    5X,'W4   DEFLECTION AT GEOPHONE 4')
             C
0162             94 FORMAT( 10X,'W4    DEFLECTION AT GEOPHONE 4')
             C
0163                PRINT 95
             C   95 FORMAT(    5X,'W5   DEFLECTION AT GEOPHONE 5')
             C
0164             95 FORMAT( 10X,'W5    DEFLECTION AT GEOPHONE 5')
             C
0165                PRINT 96
             C   96 FORMAT(  5X, 'SCI  SURFACE CURVATURE INDEX ( W1 MIN',
             C     = 'US W2)' )
0166             96 FORMAT( 10X,'SCI   SURFACE CURVATURE INDEX ( W1 MIN',
                  * 'US W2)'   )
             C
0167                PRINT 97
             C   97 FORMAT( 5X,'ES   ELASTIC MODULUS OF THE SUBGRADE FRO',
             C     = 'M W1 AND 32'  )
             C
0168             97 FORMAT( 10X,'ES    ELASTIC MODULUS OF THE SUBGRADE FRO',
                  * 'M W1 AND W3' )
             C
0169                PRINT 98
             C   98 FORMAT( 5X,'EP   ELASTIC MODULUS OF THE PAVEMENT FROM',
             C     = ' W1 AND W3'    )
             C
0170             98 FORMAT( 10X,'EP    ELASTIC MODULUS OF THE PAVEMENT FRO',
                  * 'M W1 AND W3' )
             C
0171                GO TO 10
             C
0172          1000 CONTINUE
0173                END
```

```
               C
               C
               C      SUBROUTINE EMOD
               C
               C
0001                  SUBROUTINE  EMOD (  W1, W3, H, E1, E2 )
               C
               C
0002                  IMPLICIT REAL * 8  ( A-H, O-Z )
0003                  DIMENSION  RH( 2), FF( 2), Y(4000), DELM1(2),
                        DELM2(2), DELX1(2), DELX2(2), N(4)
               C
0004                  DATA P  /   1000.0D0 / , EP / .001D0/
0005                  DATA XNO / 61.0D0 /, XK1 / 0.01D0 /, XK2 / 0.10D0 /
               C
               C      P, XNO, XK1, XK2, R1 & R2 CAN BE CHANGED IF DESIRED
               C
0006                  INTEGER  PLUS
               C
               C      INITIALIZE SWITCHES & SAVE
               C
0007                  R1   =  10.0D0
0008                  R3   =  DSQRT( 676.0D0 )
0009                  MINUS  =  1
0010                  PLUS  =  0
0011                  ISW  =  0
0012                  SAVE  =  0.0D0
               C
               C      CALCULATE R/H, RATIO, & ACC ( ACC IS THE CONVERGENCE
               C      CRITERION )
               C
0013                  RH(1)  =  R1 / H
0014                  RH(2)  =  R3 / H
0015                  RATIO  =  ( W1 * R1 ) / ( W3 * R3 )
0016                  ACC  =  EP * RATIO
               C
               C
0017                  DO   2  KL  =  1 ,  2
               C
               C      CALCULATE AND TEST DELM1
               C
0018                  DELM1(KL) = ( 1.0D0 / RH(KL)) * ( 3.0D0 /
                    *  ( XNO - 1.0D0 ) )
               C
0019                  IF( XK1  .LE.  DELM1(KL) )  DELM1(KL)  =  XK1
               C
               C      CALCULATE DELX1
               C
```

```
0020              DELX1(KL)  =  DELM1(KL)  *  RH(KL)
            C
            C     CALCULATE AND TEST DELM2
            C
0021              DELM2(KL) = ( 1.0D0 / RH(KL)) * ( 3.0D0 /
                 *  ( XN0 - 1.0D0 ) )
            C
0022              IF( XK2  .LE.  DELM2(KL) )  DELM2(KL)  =  XK2
            C
            C     CALCULATE DELX2
            C
0023              DELX2(KL)  =  DELM2(KL)  *  RH(KL)
            C      CALCULATE NO. OF INTERVALS FOR SIMPSON'S RULE FOR
            C     EACH INTEGRATION.  N1 & N2 MUST BE ODD INTEGERS.
            C
0024              N(KL) = ( 3.0D0 * RH(KL) ) / DELX1(KL) + 1.0D0
0025              IF( (N(KL) / 2 ) * 2  .EQ.  N(KL) )
                    N(KL)  =  N(KL)  +  1
            C
0026              N(KL+1) = ( 7.0D0 * RH(KL) ) / DELX2(KL) + 1.0D0
0027              IF( (N(KL+1) / 2 ) * 2  .EQ.  N(KL+1) )
                   N(KL+1)  =  N(KL+1)  +  1
            C
0028            2 CONTINUE
            C
            C
            C     GET INITIAL VALUE OF E2/E1 AND DELTA
            C
0029              DELTA  =  0.5D0
0030              E2E1  =  0.001D0
            C
            C
            C     START ITERATION LOOP FOR EACH E2/E1 VALUE USED
            C
0031            4 CONTINUE
0032              XN  =  ( 1.0D0  -  E2E1 )  /  ( 1.0D0  +  E2E1 )
            C
            C     THE FUNCTIONS FF(1) AND FF(2) (SEE EQN. 5) ARE
            C     CALCULATED IN THE FOLLOWING DO LOOP.
            C
0033              DO  29  KK  =  1  ,  2
            C
            C      CALCULATE ORDINATES FOR SIMPSON'S RULE FOR FIRST
            C     INTEGRATION
            C
0034              N1  =  N(KK)
            C
0035              XM1  =  0.0D0
```

```
0036              X1  =  0.0D0
0037                DO  28   JJ  =  1, N1
0038              Y(JJ)  =  ( V( XN, XM1 )  -  1.0D0 )  *  BESJO( X1 )
0039              XM1  =  XM1  +  DELM1(KK)
0040                 X1  =  X1  +  DELX1(KK)
0041           28   CONTINUE
           C
           C       CALCULATE ORDINATES FOR SIMPSON'S RULE FOR SECOND
           C      INTEGRATION
           C
0042              N2  =  N(KK + 1 )
           C
0043                 XM2 =  XM1
0044                 X2  =  X1
0045                 DO  27   KL =  1,N2
0046              Y(N1 + KL)  =  ( V( XN, XM2 )  -  1.0D0 )  *  BESJO( X2 )
0047                 XM2 = XM2 + DELM2(KK)
0048                 X2  =  X2  +  DELX2(KK)
0049           27    CONTINUE
           C
           C      SUM ORDINATES TO CALCULATE AREA UNDER THE CURVE OF FIRST
           C      INTEGRATION
           C
0050              PART1  =  0.0D0
0051              PART3  =  0.0D0
           C
           C      N4 IS NO. OF INTERIOR ORDINATES OF FIRST INTEGRATION
           C
0052                N4 =  N1 - 3
           C
           C      SUM INTERIOR ORDINATES
           C
0053                DO  26  LL = 2 , N4, 2
0054           26 PART1  =  PART1  +  ( 2.0D0 * Y(LL)  +  Y(LL+1) )
           C
           C      SUM END ORDINATES
           C
0055              PART2  =  Y(1)  +  4.0D0  *  Y(N1-1)  +  Y(N1)
           C
           C       CALCULATE AREA OF FIRST INTEGRATION.
           C
0056              AREA1 = ((2.0D0 * DELX1(KK)) / 3.0D0) *
                    PART1 + ( DELX1(KK) / 3.0D0) * PART2
           C
           C       SUM ORDINATES TO CALCULATE AREA UNDER THE CURVE OF
           C       SECOND INTEGRATION
           C
           C
```

```
            C     THE LAST ORDINATE OF THE FIRST INTERVAL OF INTEGRAT-
            C     ION IS ALSO THE FIRST ORDINATE OF THE SECOND INTERVAL
            C
            C
            C     N5 IS THE POSITION IN THE Y VECTOR OF THE FIRST
            C     INTERIOR ORDINATE OF THE SECOND INTEGRATION INTERVAL
            C
0057                N5 = N1 + 2
            C
            C     N6 IS THE POSITION IN THE Y VECTOR OF THE LAST
            C     INTERIOR ORDINATE OF THE SECOND INTEGRATION INTERVAL
            C
0058                N6 =  N2 - 3  + N1
            C
            C     SUM INTERIOR ORDINATES
            C
0059                DO  25   LM =  N5 , N6 , 2
0060             25 PART3  =  PART3  +  ( 2.0D0 * Y(LM)  +  Y(LM+1) )
            C
            C     SUM END ORDINATES
            C
0061              PART4  =  Y(N1+1) + 4.0D0 * Y(N1 + N2 - 1) + Y(N1 + N2)
            C
            C      CALCULATE AREA OF SECOND INTEGRATION.
            C
0062              AREA2 = ((2.0D0 * DELX2(KK)) / 3.0D0) *
                    PART3 + ( DELX2(KK) / 3.0D0) * PART4
            C
            C      CALCULATE THE FUNCTION.
            C
0063              FF(KK)  =  AREA1  +  AREA2  +  1.0D0
0064           29  CONTINUE
            C
            C     CALCULATE F1/F3 AND CHECK FOR CONVERGENCE
            C
0065              F1F3  =  FF(1)  /  FF(2)
0066              ERROR  =  F1F3  -  RATIO
0067              IF(DABS( ERROR ) .LT. ACC )  GO  TO  31
            C
            C     SET ISW AND SAVE ON FIRST TIME THROUGH ITERATION LOOP
            C
0068              IF( ISW .NE. 0 )  GO  TO   6
0069              ISW  =  1
0070              SAVE  =  ERROR
0071              IF( ERROR  .LT.  0.0D0 )  GO  TO  6
            C
            C     SIGN OF FIRST ERROR IS '+'
            C
```

```
0072              PLUS  =  1
0073              MINUS  =  0
0074            6 CONTINUE
            C
            C     TEST FOR SIGN OF ERROR
            C
0075              IF( ERROR )  30, 31, 32
            C
            C     SIGN OF ERROR IS '+'
            C
0076           32 PLUS  =  1
0077              IF( MINUS .NE. 0 )  GO  TO  40
            C
            C     ERROR IS POSITIVE, DECREASE E2E1 FOR NEXT TRIAL
            C
0078              E2E1  =  E2E1  -  DELTA
0079              IF( E2E1  .LE.  0.0D0 )  E2E1  =  0.0001D0
0080              GO  TO   4
            C
            C
            C     ERROR WAS NEGATIVE, NOW POSITIVE, CHANGE DELTA
            C
0081           40 DELTA  =  0.5D0  *  DELTA
0082              IF( SAVE  .LT.  0.0D0 )  GO  TO  42
            C
            C     SET SAVE = ERROR, DECREASE E2E1 FOR NEXT TRIAL
            C
0083           41 SAVE  =  ERROR
0084              E2E1  =  E2E1  - DELTA
0085              IF( E2E1  .LE.  0.0D0 )  E2E1  =  0.0001D0
0086              GO  TO  4
            C
            C     ERROR IS INCREASING IN POSITIVE DIRECTION, DECREASE
            C     E2E1 FOR NEXT TRIAL
            C
0087           42 IF(DABS( SAVE ) .GT. ERROR )  GO  TO  41
0088              E2E1  =  E2E1  - DELTA
0089              IF( E2E1  .LE.  0.0D0 )  E2E1  =  0.0001D0
0090              GO  TO  4
            C
            C     SIGN OF ERROR IS '-'
            C
0091           30 MINUS  =  1
0092              IF( PLUS .NE. 0 )   GO  TO  45
            C
            C     ERROR IS NEGATIVE, INCREASE E2E1 FOR NEXT TRIAL
            C
0093              E2E1  =  E2E1  +  DELTA
```

```
0094              IF( E2E1  .GT.  1.000 )  GO  TO  44
0095              GO  TO  4
            C
            C
0096           44 CONTINUE
            C
            C
            C     CHECK FOR A DIVERGENT CONDITION FOR THE SITUATION
            C     WHEN RATIO IS LESS THAN 1.0 AND H IS LESS THAN  11.2 IN.
            C
            C
0097              IF( H  .GE.  11.200 )  GO  TO  4
0098              DELTA  =  0.500  *  DELTA
0099              E2E1  =  E2E1  -  DELTA
0100              GO  TO  4
            C
            C     ERROR IS NEGATIVE NOW, WAS POSITIVE BEFORE, CHANGE
            C     DELTA
            C
0101           45 DELTA  =  0.500  *  DELTA
0102              IF( SAVE  .GT.  0.000 )  GO  TO  47
            C
            C     TEST FOR ERROR LESS THAN SAVE
            C
0103           46 IF(DABS ( SAVE )  .GT.DABS ( ERROR ) )SAVE = ERROR
            C
            C     INCREASE E2E1 FOR NEXT TRIAL
            C
0104              E2E1  =  E2E1  +  DELTA
0105              IF( E2E1  .GT.  1.000 )  GO  TO  44
0106              GO  TO  4
            C
            C     TEST FOR ERROR GREATER THAN SAVE
            C
0107           47 IF(DABS ( ERROR )  .GT.  SAVE )  GO  TO  46
            C
            C     ERROR IS APPROACHING CONVERGENCE FROM NEGATIVE SIDE,
            C     SET SAVE = ERROR, INCREASE E2E1 FOR NEXT TRIAL
            C
0108              SAVE  =  ERROR
0109              E2E1  =  E2E1  +  DELTA
0110              IF( E2E1  .GT.  1.000 )  GO  TO  44
0111              GO  TO  4
0112           31 CONTINUE
            C
            C     CONVERGENCE CRITERION IS MET, CALCULATE E1 & E2
            C
0113              E1 = (3.000 * P * FF(1))/ (4.000 * 3.1415900 *W1*R1)
```

```
0114          E2  =  E2E1  *  E1
        C
        C
        C
0115          RETURN
0116          END
```

```
0001              REAL FUNCTION BESJO * 8 ( X )
            C
            C
            C     A FUNCTION TO CALCULATE BESSEL FUNCTION JO(X) USING
            C     POLYNOMIAL APPROXIMATION - REFERENCE HANDBOOK OF MATH.
            C     FUNCTIONS, BUREAU OF STANDARDS, PAGES 369-370
            C
0002              DOUBLE PRECISION X3, X32, X33, X34, X35, X36,DCOS,
                 *  DSQRT, DABS, X
            C
            C     CALCULATE X/3 OR 3/X
            C
0003              X3 = X/3.0
0004              IF( X.GT. 3.0) X3 = 3.0/ X
            C
            C     CALCULATE POWERS OF X
            C
0005              X32= X3*X3
0006              X33=X32*X3
0007              X34=X32*X32
0008              X35=X32*X33
0009              X36=X33*X33
            C
0010            2 IF ( DABS (X)  .LE. 3.000 )  GO  TO  3
            C
            C     CALCULATE BESJO(X) FOR VALUES OF X GREATER THAN 3
            C
0011              BESJO=((.79788456-.77E-6   * X3   -   0.552740D-02 *
                 * X32 - .9512E-04 * X33 + .137237D-02 * X34 -
                 * .72805E-03 * X35 + .14476E-03 * X36 ) / DSQRT(X) )
                 * * DCOS( X - .78539816 - .04166397 * X3 - .3954E-04
                 * * X32 + .262573D-02 * X33 - .54125D-03 * X34 -
                 * .29333E-03 * X35 + .13558E-03 * X36 )
            C
0012              RETURN
            C
            C     CALCULATE BESJO(X) FOR VALUES OF X LESS THAN 3
            C
0013          3   BES JO= 1.0  -  2.2499997 * X32  +  1.2656208 * X34
                 * - .3163866 * X36 + .0444479 * ( X34 * X34 ) -
                 * .0039444 * ( X35 * X35 ) + .000210 * ( X36 * X36)
            C
0014              RETURN
0015              END
```

```
0001                 REAL FUNCTION V * 8 ( XN , XM )
              C
              C
0002                 DOUBLE PRECISION  XN, XM, EXPM2M, EXPM4M, DEXP
              C
              C      V - A FUNCTION OF 'E2E1', AND 'M'
              C      'E2E1' IS THE E2/E1 RATIO, TESTED FROM .001 TO 1000.
              C      'M' TESTED USING VALUES FROM 0.0 TO 150. WHICH IS
              C      10 * (R/H)
              C
              C
              C      V APPROACHES 1 FOR LARGE VALUES OF M
              C
0003                       V  =  1.0
0004                       IF( XM .GT.  30 )    RETURN
              C
              C
              C      CALCULATE EXPONENTIALS
              C
0005                 EXPM2M  =  DEXP ( -2.0D0 * XM )
0006                 EXPM4M= EXPM2M*EXPM2M
              C
              C      CALCULATE FUNCTION V FOR THE XN & XM1 OR XM2 VALUES
              C
0007                 V  =  ( 1.0D0 + ( 4.0D0 * XN * XM * EXPM2M ) -
                    * ( XN * XN * EXPM4M ) )  /  ( 1.0D0 - ( 2.0D0 * XN
                    * * ( 1.0D0 + 2.0D0 * XM * XM )  * EXPM2M )  +
                    *  ( XN * XN * EXPM4M )  )
              C
0008                 RETURN
0009                 END
```

Appendix B

Listing of POINT LOAD

```
******************************************
*                                        *
*                                        *
*     TEXAS TRANSPORTATION INSTITUTE     *
*                                        *
*       PAVEMENT DESIGN DEPARTMENT       *
*                                        *
*          SURFACE  DEFLECTIONS          *
*                                        *
*                  OF A                  *
*                                        *
*            TWO-LAYER SYSTEM            *
*                                        *
*          LOADED  AT  A  POINT          *
*                                        *
*        (POISSON'S RATIO = 1/2)         *
*                                        *
******************************************
```

TWO-LAYER SYSTEM--POINT LOAD--SURFACE DEFLECTIONS

P IS THE POINT LOAD.
H IS THE THICKNESS OF LAYER 1.
E1 AND E2 ARE YOUNG'S MODULI FOR LAYERS 1 AND 2 RESPECTIVELY.
POISSON'S RATIO IS 0.5 THROUGHOUT.
W IS THE SURFACE DEFLECTION AT THE DISTANCE R FROM THE LOADED POINT.

THE FUNCTION F IS DEFINED AS F(E2/E1,R/H) = (4 * PI * E1 / 3 * P) * (W * R).
IT IS CALCULATED BY MEANS OF THE FOLLOWING EQUATIONS --

1. N = (1 - E2/E1) / (1 + E2/E1).
2. V(M,N) = (1 + 4*N*M * EXP(-2*M) - N*N * EXP(-4*M)) /
   (1 - 2*N * (1 + 2*M*M) * EXP(-2*M) + N*N * EXP(-4*M)).
3. X = MR / H.
4. U(M,N,R/H) = (V(M,N) - 1)* J0 (X), WHERE J0 IS A BESSEL FUNCTION, FIRST KIND, ZERO ORDER.
5. AREA 1 = THE INTEGRAL OF U * DX FROM X = 0 TO 3R/H.
6. AREA 2 = THE INTEGRAL OF U * DX FROM X = 3R/H TO 10R/H.
7. F = 1 + AREA 1 + AREA 2.

THE PROGRAM OUTPUT INCLUDES THE E2/E1 AND R/H RATIOS USED IN THE CALCULATION, THE FUNCTION F, AND THE DEFLECTION W.

THE PROGRAM INPUTS ARE AS FOLLOWS --

1. NO, THE MINIMUM NUMBER OF ORDINATES CALCULATED BETWEEN ZEROS OF J0(X), IN THE CALCULATION BY SIMPSON'S RULE OF AREA 1 AND AREA 2. NO IS AN ODD NUMBER, USUALLY 61.
2. K1, THE MAXIMUM VALUE OF DELTA M IN THE INTERVAL M = 0 TO M = 3. K1 USUALLY = 0.01.
3. K2, THE MAXIMUM VALUE OF DELTA M IN THE INTERVAL M = 3 TO M = 10. K2 USUALLY = 0.10.
4. E1, DEFINED ABOVE.
5. E2, DEFINED ABOVE.
6. R, DEFINED ABOVE.
7. H, DEFINED ABOVE.
8. P, DEFINED ABOVE.

FOR PREDICTING DYNAFLECT DEFLECTIONS INPUT THE FOLLOWING--

R (INCHES) = 10.0, 15.62, 26.0, 37.36, 49.0.
P (POUNDS) = 1000.

```
          C
          C
          C       PROGRAM TO COMPUTE SURFACE DEFLECTIONS, GIVEN THE
          C       ELASTIC MODULI AND THE THICKNESS OF THE PAVEMENT LAYER
          C
          C
01                IMPLICIT REAL * 8 ( A-H, O-Z )
02                  DIMENSION E2E1(30), RH(20), Y(4000), R(15), H(50), FF(20),      MAIN
                     FRAT(10) ,  DATA(20) , E1(30), E2(30), W(15)
03                DATA P / 1000.0D0 /
04                DATA CHK /'END '/
05                  WRITE(6,215)
06            215  FORMAT('1',21(/),45X,42('*'),2(/45X,'*',40X,'*')/45X,
               + '*      TEXAS TRANSPORTATION INSTITUTE      *'/45X,
               + '*',40X,'*'/45X,'*',7X,'PAVEMENT DESIGN DEPARTMENT',7X,'*'/45X,
                 '*',40X,'*'/45X,'*',10X,'SURFACE  DEFLECTIONS',10X,'*'/45X,
                   '*',40X,
               / '*'/45X,'*',18X,'OF A',18X,'*'/45X,'*',40X,'*'/45X,
                 '*',12X, 'TWO-LAYER SYSTEM',12X,'*'/45X,'*',40X,'*'/45X,'*',10X,
               = 'LOADED  AT  A  POINT',10X,'*'/45X,'*',40X,'*'/45X,
                  '*',8X,'(POISSON''S RATIO = 1/2)',9X,'*'/45X,'*',40X,'*'/45X,
                  42('*') )
07                WRITE(6,230)
08            230 FORMAT('1',  41X, 'TWO-LAYER SYSTEM--POINT LOAD--SURFACE DEFLECTIO
               )NS'        /        )
          C         SET UP NO. OF INTERVALS AND ACCURACY FOR EACH INTEGRATION.      MAIN
09            225    FORMAT('1')                                                    MAIN
          C
          C       THE VALUES OF RADIUS CAN BE CHANGED TO ANY DISTANCE
          C       AT WHICH A DEFLECTION IS TO BE CALCULATED
          C
10                R( 1)  =  10.0D0
11                R(2) = DSQRT( 244.0D0 )
12                R(3) = DSQRT( 676.0D0 )
13                R(4) = DSQRT( 1396.0D0  )
14                R(5) = DSQRT( 2404.0D0 )
          C
15              6 READ(5,103)  ( DATA(I), I  =  1 , 20 )
16            103 FORMAT( 20A4 )
17                IF( DATA(1) .EQ. CHK )  GO TO 1
18                WRITE(6,231)  ( DATA(I), I  =  1 , 20 )
19            231 FORMAT(15X, 20A4 )
20                GO TO 6
21              1 READ(5,100,END=50)  XNO, XK1, XK2
22            100   FORMAT( 3F10.5)                                                 MAIN
          C       READ IN E1, E2 & H VALUES.
23              4 CONTINUE
24                READ(5,101,END=50)  NE1,  (E1(J), J = 1 ,  9 )
```

```
              IF(NE1 .LE.  9)  GO TO 11
              READ(5,102)  (E1(L), L = 10, NE1 )
           11 READ(5,101,END=50)  NE2,     (E2(J), J = 1 ,  9 )
          101 FORMAT(  I3, 3X, 9F8.0    )
              IF( NE2 .LE.  9)  GO TO 2
              READ(5,102)  (E2(L), L = 10 , NE2 )
            2   CONTINUE                                                      MAIN
          102 FORMAT(  6X, 9F8.0     )
              READ(5,101)  NH,  (H(K), K  =  1 ,  9  )
              IF( NH .LE.  9  )  GO TO  3
              READ(5,102)  (H(M), M  =  10, NH )
            3     CONTINUE                                                    MAIN
      C       CALCULATE E2/E1 RATIOS
              DO 12  M  =  1 ,  NE1
              E2E1(M)   =  E2(M) / E1(M)
           12 CONTINUE
              WRITE(6,225)
              WRITE(6,206)
      C         SELECT EACH E2/E1 RATIO AND CALCULATE BIG N FOR EACH RATIO.    MAIN
                        KOUNT = 0                                             MAIN
                DO   30   JI   = 1, NE2                                       MAIN
              DO 30 JJI = 1, NH
                  LSW = 0                                                     MAIN
                XN  =  (1.0 - E2E1(JI))/ (1.0 + E2E1(JI))                     MAIN
      C        CALCULATE DELTA M1, DELTA M2, DELTA X1, & DELTA X2 FOR EACH R/H RMAIN
      C
      C       THIS DO LOOP SHOULD GO FROM ONE TO THE NUMBER
      C       OF DEFLECTIONS TO BE CALCULATED
      C
              DO 32 LK = 1, 5
           32   RH(LK) = R(LK) / H(JJI)                                       MAIN
      C
      C       THIS DO LOOP SHOULD GO FROM ONE TO THE NUMBER
      C       OF DEFLECTIONS TO BE CALCULATED
      C
              DO 29 KK = 1, 5
                 DELM1 =  ( 1.0 / RH(KK)) *  ( 3.0 / (XNO - 1.0) )             MAIN
                 IF(XK1 .LE. DELM1)  DELM1 = XK1                              MAIN
                 DELX1 =  DELM1 * RH(KK)                                      MAIN
                 DELM2  =  ( 1.0 / RH(KK)) *  ( 3.0 / (XNO - 1.0) )            MAIN
                 IF( XK2 .LE. DELM2)  DELM2 = XK2                             MAIN
                DELX2 =  DELM2 *  RH(KK)                                      MAIN
      C        CALCULATE NO. OF INTERVALS FOR SIMPSON'S RULE FOR EACH INTEGRATIOMAIN
                N1  =   (3.0 *  RH(KK)) / DELX1  + 1.0                        MAIN
               IF((N1 / 2) *  2  .EQ. N1)  N1 = N1 + 1                        MAIN
                N2  =     (7.0 * RH(KK)) / DELX2  + 1.0                       MAIN
                IF((N2 /2) * 2 .EQ. N2)  N2 = N2 + 1                          MAIN
      C        CALCULATE ORDINATES FOR SIMPSON'S RULE FOR FIRST INTEGRATION.   MAIN
```

```
                 XM1  =  0.0                                               MAIN
                 X1   = 0.0                                                MAIN
                DO  28   JJ  =  1, N1                                      MAIN
              Y(JJ)  =  (V( XN, XM1 ) - 1.0D0 ) * BESJO ( X1 )
                 XM1 =  XM1 + DELM1                                        MAIN
                 X1  =  X1  +  DELX1                                       MAIN
         28   CONTINUE                                                     MAIN
    C        CALCULATE ORDINATES FOR SIMPSON'S RULE FOR SECOND INTEGRATION.   MAIN
              XM2  =  XM1 - DELM1
              X2  =  X1  -  DELX1
                 DO  27   KL =  1,N2                                       MAIN
              Y( N1 + KL )  =  (V(XN , XM2 ) - 1.0D0 ) * BESJO ( X2 )
                 XM2 = XM2 + DELM2                                         MAIN
                 X2  =  X2  +  DELX2                                       MAIN
         27      CONTINUE                                                  MAIN
    C        SUM ORDINATES TO CALCULATE AREA UNDER THE CURVE OF FIRST INTEGRATMAIN
               PART1 = 0.0                                                 MAIN
               PART3 = 0.0                                                 MAIN
               N4 =  N1 - 3                                                MAIN
               DO  26  LL = 2 , N4, 2                                      MAIN
         26    PART1  =  PART1 + ( 2.0 * Y(LL) + Y(LL+1) )                 MAIN
               PART2  =  Y(1)  + 4.0 *  Y(N1-1)  +  Y(N1)                  MAIN
    C        CALCULATE AREA OF FIRST INTEGRATION.                          MAIN
               AREA1  =  ((2.0 * DELX1) / 3.0) * PART1 +  (DELX1 /3.0) * PART2 MAIN
    C        SUM ORDINATES TO CALCULATE AREA UNDER THE CURVE OF SECOND INTEGRAMAIN
               N5 = N1 + 2                                                 MAIN
               N6 =  N2 - 3  + N1                                          MAIN
               DO  25   LM =  N5 , N6 , 2                                  MAIN
         25    PART3 =  PART3 + ( 2.0 * Y(LM) +  Y(LM+1) )                 MAIN
               PART4 =  Y(N1+1) + 4.0 * Y(N1 + N2 - 1) + Y(N1 + N2)        MAIN
    C        CALCULATE AREA OF SECOND INTEGRATION.                         MAIN
             AREA2  = ((2.0 * DELX2) / 3.0) * PART3 +  (DELX2 / 3.0) * PART4  MAIN
    C        CALCULATE THE FUNCTION.                                       MAIN
                FF(KK) = AREA1 + AREA2 + 1.0                               MAIN
              W(KK) = ( 3. * P * FF(KK) ) / ( 4. * 3.14159 * E1(JI) * R(KK) )
                  IF(LSW .EQ. 1)  GO TO 42                                 MAIN
              WRITE(6,200) E2E1(JI), RH(KK), FF(KK), E1(JI), E2(JI), H(JJI),
             )  R(KK), W(KK)
        200 FORMAT(F18.3,  F11.3, 3X, G14.6, F10.0, F11.0, F12.2, F17.2, 6X,
               G14.6    /      )
                  LSW = 1                                                  MAIN
              GO  TO  29
         42 WRITE(6,201)  RH(KK), FF(KK), R(KK), W(KK)
        201 FORMAT(  F29.3, 3X, G14.6,  33X, F17.2, 6X, G14.6    /      )
         29  CONTINUE                                                      MAIN
             LSW  =  0
         31          CONTINUE                                              MAIN
         41      WRITE(6,225)                                              MAIN
```

```
          43    WRITE(6,206)                                                          MAIN
         206 FORMAT( 10X,'* E2/E1 *  ** R/H **  ***** F   *****  **  E1 **  **
             E2 **  ***** H  *****  ***** R  *****  ***** W  *****'   /    )
          30  CONTINUE                                                                MAIN
             GO TO  4
          50   WRITE(6,225)                                                           MAIN
                STOP                                                                  MAIN
                END                                                                   MAIN
```

```
0001            REAL FUNCTION BESJO * 8 ( X )
          C
          C
          C     A FUNCTION TO CALCULATE BESSEL FUNCTION JO(X) USING
          C     POLYNOMIAL APPROXIMATION - REFERENCE HANDBOOK OF MATH.
          C     FUNCTIONS, BUREAU OF STANDARDS, PAGES 369-370
          C
0002            DOUBLE PRECISION X3, X32, X33, X34, X35, X36,DCOS,
               *  DSQRT, DABS, X
          C
          C     CALCULATE X/3 OR 3/X
          C
0003            X3 = X/3.0
0004            IF( X.GT. 3.0) X3 = 3.0/ X
          C
          C     CALCULATE POWERS OF X
          C
0005            X32= X3*X3
0006            X33=X32*X3
0007            X34=X32*X32
0008            X35=X32*X33
0009            X36=X33*X33
          C
0010          2 IF ( DABS (X)  .LE. 3.0D0 )  GO  TO  3
          C
          C     CALCULATE BESJO(X) FOR VALUES OF X GREATER THAN 3
          C
0011            BESJO=((.79788456-.77E-6   * X3  -  0.552740D-02 *
               * X32 - .9512E-04 * X33 + .137237D-02 * X34 -
               * .72805E-03 * X35 + .14476E-03 * X36 ) / DSQRT(X) )
               * * DCOS( X - .78539816 - .04166397 * X3 - .3954E-04
               * * X32 + .262573D-02 * X33 - .54125D-03 * X34 -
               * .29333E-03 * X35 + .13558E-03 * X36 )
          C
0012            RETURN
          C
          C     CALCULATE BESJO(X) FOR VALUES OF X LESS THAN 3
          C
0013       3    BES JO= 1.0  -  2.2499997 * X32  +  1.2656208 * X34
               * - .3163866 * X36 + .0444479 * ( X34 * X34 ) -
               * .0039444 * ( X35 * X35 ) + .000210 * ( X36 * X36)
          C
0014            RETURN
0015            END
```

```
 0001                  REAL FUNCTION V * 8 ( XN , XM )
              C
              C
 0002                  DOUBLE PRECISION  XN, XM, EXPM2M, EXPM4M, DEXP
              C
              C        V - A FUNCTION OF 'E2E1', AND 'M'
              C        'E2E1' IS THE E2/E1 RATIO, TESTED FROM .001 TO 1000.
              C        'M' TESTED USING VALUES FROM 0.0 TO 150. WHICH IS
              C        10 * (R/H)
              C
              C
              C        V APPROACHES 1 FOR LARGE VALUES OF M
              C
 0003                        V  =  1.0
 0004                        IF( XM .GT.  30 )    RETURN
              C
              C
              C        CALCULATE EXPONENTIALS
              C
 0005                  EXPM2M  =  DEXP ( -2.0D0 * XM )
 0006                  EXPM4M= EXPM2M*EXPM2M
              C
              C        CALCULATE FUNCTION V FOR THE XN & XM1 OR XM2 VALUES
              C
 0007                  V  =  ( 1.0D0 + ( 4.0D0 * XN * XM * EXPM2M ) -
                     * ( XN * XN * EXPM4M ) )  /  ( 1.000 - ( 2.0D0 * XN
                     * * ( 1.0D0 + 2.0D0 * XM * XM )  * EXPM2M )  +
                     *  ( XN * XN * EXPM4M )  )
              C
 0008                  RETURN
 0009                  END
```

* E2/E1 *	** R/H **	***** F *****	** E1 **	** E2 **	***** H *****	***** R *****	***** W *****
0.139	1.205	6.82302	81910.	11400.	8.30	10.00	0.198862D-02
	1.882	7.98281				15.62	0.148948D-02
	3.133	8.20060				26.00	0.919279D-03
	4.502	7.74196				37.36	0.603926D-03
	5.907	7.40308				49.03	0.440068D-03