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16. Abstract

The economic advantages of precasting can be combined with the structural efficiency of prestressed concrete box girders for long span bridge structures when erected by segmental construction. The complete superstructure is precast in box segments of convenient size for transportation and erection. These precast segments are erected in cantilever and post-tensioned together to form the complete superstructure.

This report details the application of design, analysis, and optimization techniques to segmentally precast prestressed concrete box girder bridges. These were classified into two main types according to their method of construction, namely those constructed on falsework and those erected in cantilever.

Design procedures are developed for both types of construction. Both ultimate strength and service load design criteria are satisfied under all loading conditions.

Sample designs are carried out for the case of a hypothetical two-span bridge constructed on falsework and that of an actual three-span bridge erected in cantilever.

Optimization techniques are used to find the optimal cross sections, i.e., those having minimum cost for such bridges.

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THE DESIGN AND OPTIMIZATION OF SEGMENTALLY PRECAST PRESTRESSED BOX GIRDER BRIDGES

by

G. C. Lacey

and

J. E. Breen

Research Report No. 121-3

Research Project No. 3-5-69-121

"Design Procedures for Long Span Prestressed Concrete
Bridges of Segmental Construction"

Conducted for

The Texas Highway Department
In Cooperation with the
U. S. Department of Transportation
Federal Highway Administration

by

CENTER FOR HIGHWAY RESEARCH
THE UNIVERSITY OF TEXAS AT AUSTIN

August 1975

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

P R E F A C E

This report is the third of a series which summarizes the detailed investigation of the various problems associated with the design and construction of long span prestressed concrete bridges of precast segmental construction. The initial report in this series summarized the general state-of-the-art for design and construction of this type bridge as of May 1969. The second report stated requirements for and reported test results of epoxy resin materials for jointing large precast segments. This report summarizes design criteria and procedures for bridges of this type and includes two design examples. One of these examples is the three-span segmental bridge constructed in Corpus Christi, Texas, during 1972-73. Later reports in this series will detail the development of an incremental analysis procedure and computer program which can be used to analyze segmentally erected box girder bridges and will summarize the results of an extensive physical test program of a one-sixth scale model of the Corpus Christi structure. Comparisons with analytical results using the computer model and verification of the design procedures will be presented in those reports.

This work is a part of Research Project 3-5-69-121, entitled "Design Procedures for Long Span Prestressed Concrete Bridges of Segmental Construction." The studies described were conducted as a part of the overall research program at The University of Texas at Austin, Center for Highway Research. Work was sponsored jointly by the Texas Highway Department and the Federal Highway Administration under an agreement with The University of Texas at Austin and the Texas Highway Department.

Liaison with the Texas Highway Department was maintained through the contact representative, Mr. Robert L. Reed, and the State Bridge Engineer Mr. Wayne Henneberger. Extensive detailed liaison in the design phase was maintained with Mr. Harold J. Dunlevy and Mr. Alan B. Matejowsky of the Bridge Division; Mr. Donald E. Harley and Mr. Robert E. Stanford were the contact representatives for the Federal Highway Administration.

The overall study was directed by Dr. John E. Breen, Professor of Civil Engineering. He was assisted by Dr. Ned H. Burns, Professor of Civil Engineering. The design phase and the optimization studies were developed by Dr. Geoffrey C. Lacey, who at that time was a Research Engineer for the Center for Highway Research. Valuable assistance was contributed by Dr. Robert C. Brown, Jr., Dr. Satoshi Kashima, and Mr. Tsutomu Komura, Assistant Research Engineers, Center for Highway Research. The authors are appreciative of the contributions of Dr. D. M. Himmelblau and Dr. W. G. Lesso of the College of Engineering, The University of Texas at Austin, for their advice regarding optimization techniques.

S U M M A R Y

The economic advantages of precasting can be combined with the structural efficiency of prestressed concrete box girders for long span bridge structures when erected by segmental construction. The complete superstructure is precast in box segments of convenient size for transportation and erection. These precast segments are erected in cantilever and post-tensioned together to form the complete superstructure.

This report details the application of design, analysis, and optimization techniques to segmentally precast prestressed concrete box girder bridges. These were classified into two main types according to their method of construction, namely those constructed on falsework and those erected in cantilever. The prestressing cable patterns and the design procedures required are very different in the two types of construction.

Design procedures are developed for both types of construction. Both ultimate strength and service load design criteria are satisfied under all loading conditions. The effect of the cable force on the concrete section is calculated using an equivalent load concept. A computer program is used to check all service level stresses.

Sample designs are carried out for the case of a hypothetical two-span bridge constructed on falsework and that of an actual three-span bridge erected in cantilever. In the former case, full length draped cables are used, the profile consisting of three parabolas. In the case of the bridge erected in cantilever, each stage of construction is a separate design condition and a pattern of cables of varying lengths is required.

Optimization techniques are used to find the optimal cross sections, i.e., those having minimum cost for such bridges. In each case, the problem is treated as an unconstrained nonlinear programming problem and a subroutine is developed to compute the objective function. Numerical

methods of solution that do not require derivatives of the objective function are used. From contour plots of the objective function it is found that the optimal dimensions can be varied substantially with small increase in cost.

IMPLEMENTATION

This report presents the background and detail of design, analysis, and optimizing procedures recommended for use with precast segmental prestressed concrete box girders erected on falsework or by balanced cantilevering. The design procedures illustrate the use of both ultimate strength and service load design techniques and consider a wide range of loading conditions. The interaction of manual calculations and various computerized analysis procedures is illustrated. The report includes a brief summary of important factors to be considered in initial design, treats analytical procedures which are especially useful in dealing with the types of tendon layouts and erection schemes utilized with this construction, and provides two major example problems illustrating the numerical calculations and procedures to be utilized. One of the example problems is based on the box girder bridge erected over the Intracoastal Waterway at Corpus Christi, Texas, and is essentially a documentation of the preliminary design procedure used in development of the structure.

While the design examples consider box girders of constant depth, the minor variations required in dealing with members of variable depths are indicated. The design and analysis procedures should be extremely useful in analysis of proposed structures in the 100 to 300 ft. span range, can be easily extended to structures up to 450 ft. and can deal with a wide variety of cross sections.

C O N T E N T S

Chapter | Page

Chapter		Page

LIST OF TABLES

LIST OF FIGURES

Figure		Page

Figure		Page

N O T A T I O N

a	=	depth of concrete stress block (Chapter 4)
	=	average concrete section area (Chapters 6 and 7)
a_1	=	minimum concrete section area
a_2	=	maximum concrete section area
A_s	=	area of prestressing cables or unprestressed reinforcement
A_v	=	area of web reinforcement
b	=	width of compression face of member (Chapters 3 and 4)
	=	width of superstructure (Chapters 6 and 7)
b'	=	total width of girder webs
$b_1,\ldots b_6$	=	widths of superstructure elements
b_L	=	width of lower lab of box
b_U	=	width of deck slab per box
C	=	objective function
d	=	distance from compression edge to centroid of cables or reinforcement (Chapters 3 and 4)
	=	depth of girder (Chapters 2, 6, and 7)
d_a	=	depth of stress block
d_{ec}	=	cable eccentricity
d_m	=	distance between center of compression and cable centroid
e	=	cable eccentricity
$f(\underline{x})$	=	objective function
f_c	=	compressive stress in concrete
f'_c	=	compressive strength of concrete
f_d	=	stress at top of girder due to dead load
f_{pc}	=	compressive stress at centroid of girder
f_{pe}	=	compressive stress at top of girder due to prestress
f_s	=	stress in steel
f'_s	=	ultimate strength of prestressing steel
f_{se}	=	effective steel prestress after losses

f_{su} = calculated stress in prestressing steel at ultimate load

f_t = stress at top of girder

f_y = yield stress of unprestressed reinforcement

f_y^* = yield point stress of prestressing steel

F = cable force

h = cable drape

I = moment of inertia

j = ratio of distance between center of compression and center of tension to the effective depth, d

k = ratio of distance between compression edge of slab and neutral axis to the effective depth, d

k_2, k_4 = slab stiffnesses

m = concentrated moment from cable

M = bending moment

M_{cr} = flexural cracking moment

M_P = primary cable moment

M_R = resultant cable moment

M_S = secondary cable moment

M_u = ultimate moment

n_w = number of webs of girder

p = A_s/bd

P = transverse force or load

P_u = cable force at ultimate load

q = fraction of span over which bottom slab is thickened

R = concrete moment resistance coefficient

s = longitudinal spacing of web reinforcement (Chapters 3 and 4)

= sloping height of web between slabs (Chapters 6 and 7)

S = effective span of deck slab

t = thickness of slab

V = shear force

V_c = shear carried by concrete

V_p = vertical component of cable force

V_u = shear at ultimate load

w = load per unit length

x	=	horizontal coordinate
$x_i (i=1,n)$	=	variables
$\underline{x}$	=	column vector of variables
y	=	vertical coordinate
z	=	ratio of distance between pier center and section considered to the span
θ	=	cable slope
φ	=	capacity reduction factor

CHAPTER 1

INTRODUCTION

1.1 General

Bridge engineers continually face requirements for safer, more economical bridge structures. In response to many requirements imposed by traffic considerations, natural obstacles, more efficient use of land in urban areas, safety, and aesthetics, the trend is to longer span structures. At present, the most commonly used structural system for highway bridge structures in Texas consists of prestressed concrete I-girders combined with a cast-in-place deck slab. This system has practical limitations for spans beyond the 120 ft. range. With the fluctuating costs and maintenance requirements of steel bridges, there exists a need to develop an economical approach to achieve precast, prestressed concrete spans in the 120-400 ft. range.

In the United States, spans in the 160 ft. range have been achieved by the use of post-tensioned, cast-in-place girders.[23*] The box, or cellular, cross section shown in Fig. 1.1 is ideal for bridge superstructures since its high torsional stiffness provides excellent transverse load distributing properties. Construction experience along the West Coast has indicated that this type bridge is a very economical solution to many long span challenges.

In Europe, Japan, and Australia, during the late 1960's and early 1970's, the advantages of the cellular cross section were combined with the substantial advantages obtained by maximum use of prefabricated components.[20] By precasting the complete box girder cross section in short segments of a convenient size for transportation and erection, the entire

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*Numbers refer to references listed at the end of this report.

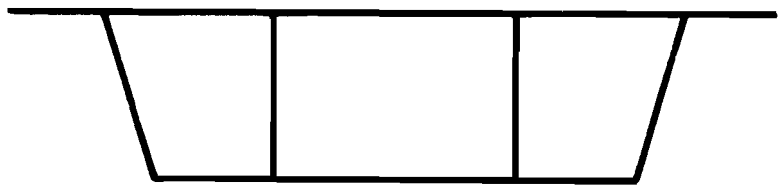


Fig. 1.1. Typical cellular cross section

bridge superstructure may be precast. These precast units are subsequently assembled on the site by post-tensioning them longitudinally. A number of extremely long span precast and cast-in-place box girder bridges have been segmentally constructed in Europe[16,20] and interest in this construction concept is rapidly growing in the United States. A three-span precast segmentally constructed box girder bridge with a 200 ft. maximum span was completed in Corpus Christi, Texas, in 1973.

When construction of large numbers of prestressed concrete bridges is envisaged, precasting has a number of advantages over cast-in-place construction, e.g.

(1) Mass production of standardized girder units is possible. This is done at present with precast I-girders for shorter spans.

(2) High quality control can be attained through plant production and inspection.

(3) Greater economy of production is possible by precasting the girder units at a plant site rather than casting in place.

(4) The speed of erection can be much greater. This is very important when construction interferes with existing traffic and is most critical in an urban environment.

In segmental box girder construction utilizing the cantilever erection procedure, precasting has several other advantages over cast-in-place construction, e.g.

(1) Strength gain of the concrete is essentially taken out of the erection critical path. This allows faster erection times and higher concrete strength at time of stressing.

(2) Shrinkage strains can substantially develop prior to erection and stressing if adequate lead times and stockpiling are used.

(3) Creep rates can be substantially reduced since the segments are considerably more mature at time of stressing.

The major advantages frequently cited for utilization of cast-in-place segmental construction are:

(1) Provision of positive nonprestressed reinforcement across the joints is easier.

(2) Continuous correction of girder grade and line is possible to compensate for deformations.

Extensive utilization of epoxy joints, grouted tendons, and shear keys has reduced the emphasis on the positive bonded joint reinforcement while the versatility of the match casting procedure on numerous major projects involving complex horizontal and vertical alignment has illustrated the ability of the precast procedures to deal with geometrical problems.

At present, precast I-section girders are widely used in highway bridge construction for spans up to about 120 ft. They are cast in a manufacturing plant and transported to the bridge site for erection. While their span can be stretched by using "drop in" girders, they cannot be used for significantly greater spans, because this length is approximately the upper limit that can be transported by road. In addition, I-sections are not the most structurally suitable form for long span bridge structures. A better structural unit is the box girder.

The box girder is a very compact structural member, which combines high flexural strength with high torsional strength and stiffness. It is superior to the I-section girder for long spans in that (a) there is no lateral buckling problem so that the compressive capacity of the bottom flange is fully utilized, and (b) the torsional rigidity brings about a more even distribution of flexural stresses across the section, under a variety of live loads. A further advantage of the box girder in precast structures is that it is possible to precast the full cross section (apart from a longitudinal joining strip in some cases), whereas with I-sections the deck slab must largely be cast in place.

A considerable number of long span bridges have been constructed throughout the world using prestressed concrete box girders. Both cast-in-place construction and segmental precasting have been widely utilized. The suitability of box girders even for extremely long spans can be seen from the Bendorf Bridge in West Germany, which was cast in place and has a span of 682 ft. In the United States, cast-in-place box girder bridges are being widely used by the California Division of Highways, as well as in several other states.

When box girder bridges are precast, the casting is generally segmental, i.e., the girders are cast in short, full width units or "segments". The reason for manufacture in short segments is essentially that box girders, unlike I-girders which have narrow width, cannot be readily transported in long sections. In addition, the short units are suited to fairly simple methods of assembly and post-tensioning. During erection the segments are joined together, end to end, and post-tensioned to form the completed superstructure. The segmental pattern for a typical bridge is shown in Fig. 1.2. The length and weight of the segments are chosen so as to be most suitable for transportation and erection.

1.2 Segmental Construction

Figure 1.3 illustrates some of the wide range of cross-sectional shapes which can be used. Various techniques have been used for jointing between the precast segments, with thin epoxy resin joints being the widest used. The most significant variation in construction technique is the method employed to assemble the precast segments. The most widely used methods may be categorized as construction on falsework and cantilever construction. Construction on falsework is the simplest method of erecting precast, segmental bridges. It also leads to the simplest design approach. The joints used are usually cast-in-place concrete or mortar. This method is particularly applicable to locations where access by construction

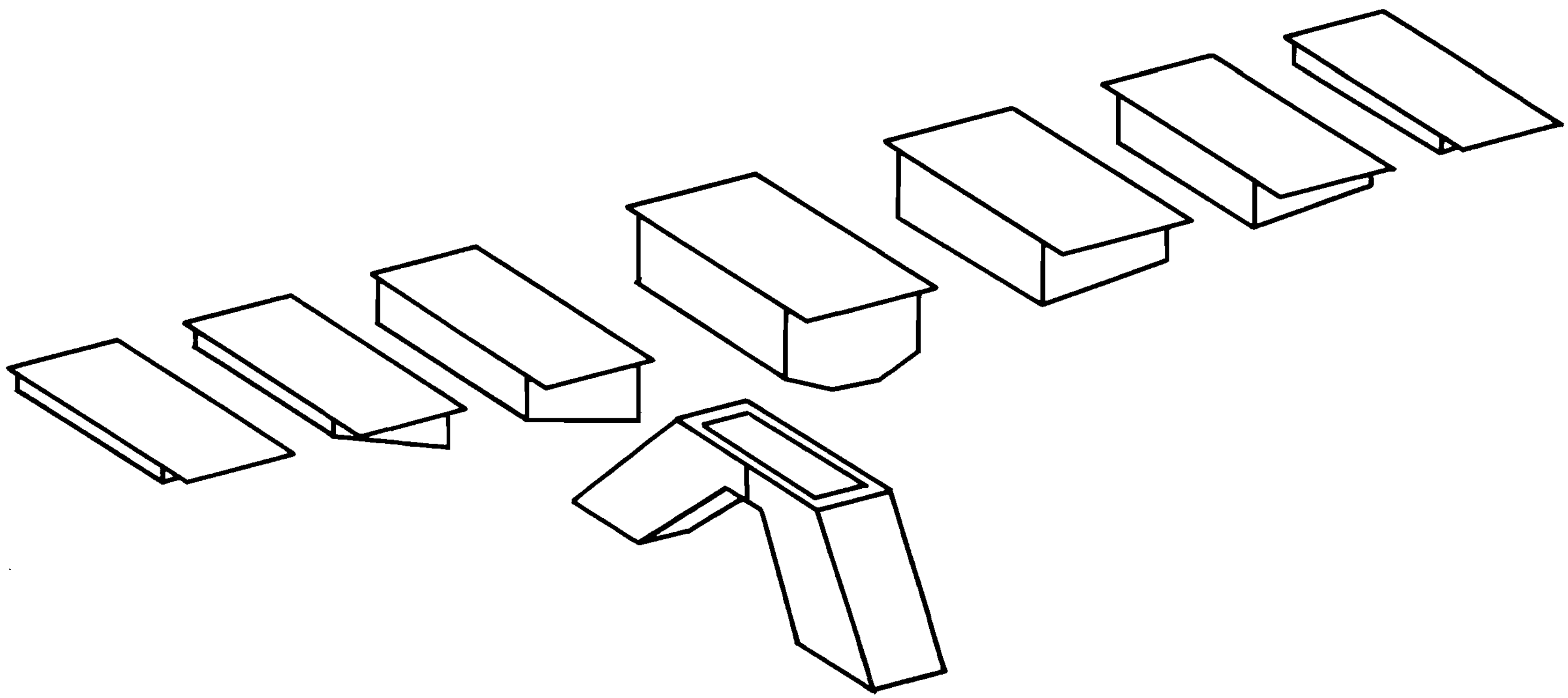
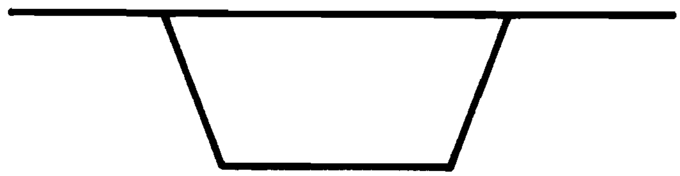
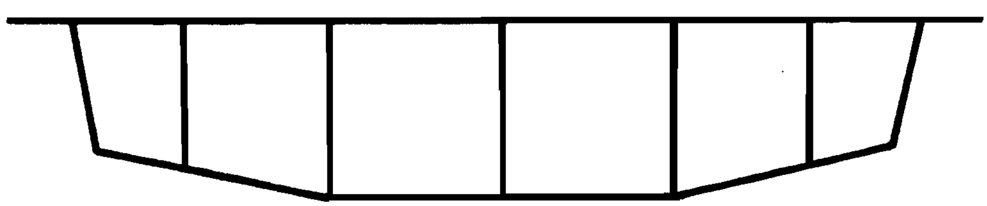


Fig. 1.2. Superstructure of the Oosterschelde Bridge, The Netherlands



(a) Single cell girder

(b) Single cells joined by deck



(c) Multicell girder

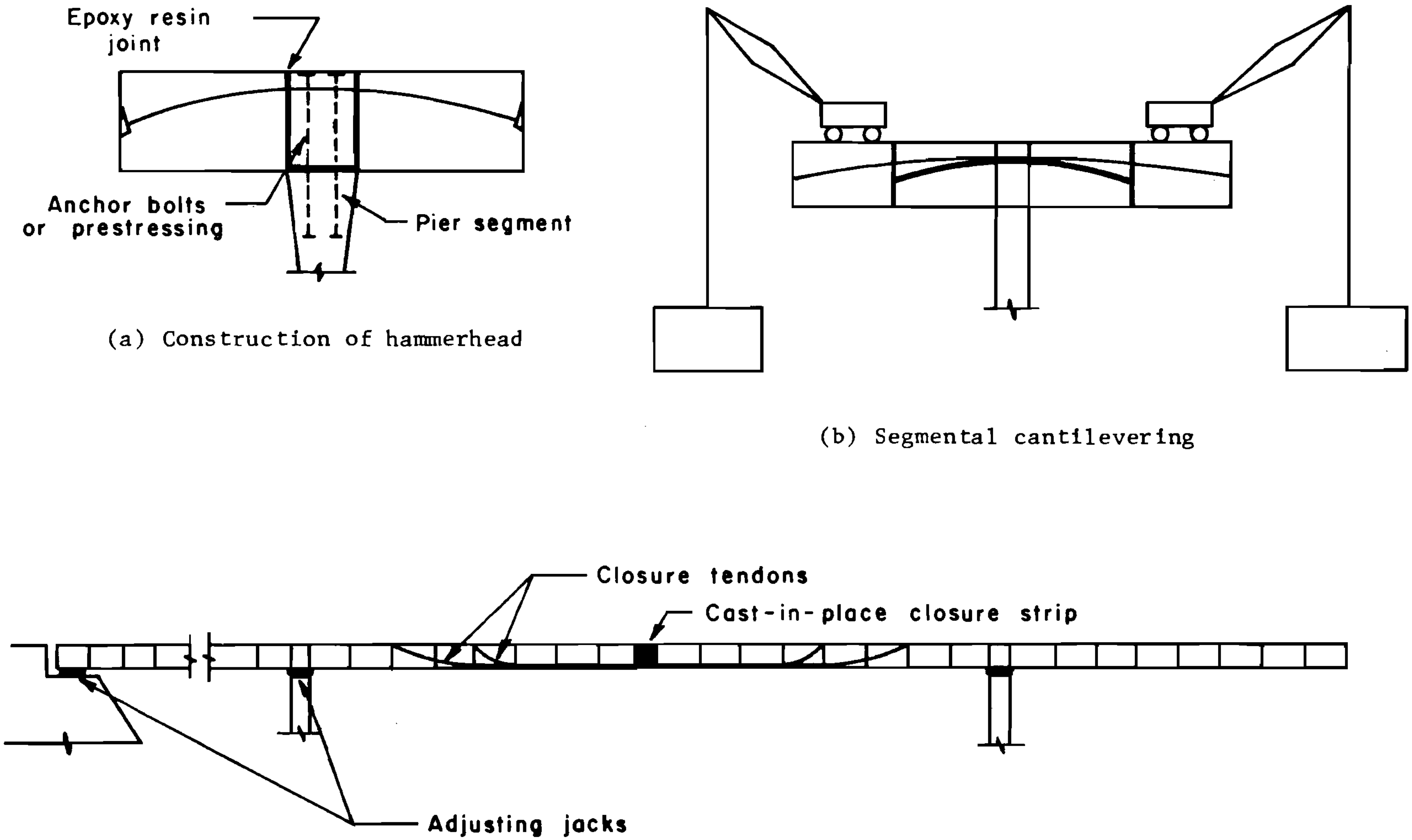
Fig. 1.3. Girder cross-sectional configurations

equipment is difficult, the project is of limited scope and traffic interruptions due to falsework are acceptable, and where single or twin spans are to be used so that balanced cantilevering is not feasible. The prestressing system for the superstructure will normally consist of long draped cables in the box girder webs. If the overall length of the bridge is moderate, it is possible to set all the segments in place and join them before inserting and tensioning full length cables. This method tends to economize on prestressing steel and hardware. Stressing operations are minimized but at the expense of falsework costs.

The outstanding advantage of the cantilever approach to segmental construction lies in the fact that the complete construction may be accomplished without the use of falsework and hence minimizes traffic interruption.

Assembly of the segments is accomplished by sequential balanced cantilevering outward from the piers toward the span centerlines. Initially the "hammerhead" is formed by erecting the pier segment and attaching it to the pier to provide unbalanced moment capacity. The two adjoining segments are then erected and post-tensioned through the pier segment, as shown in Fig. 1.4(a). Auxiliary supports may be employed for added stability during cantilevering or to reduce the required moment capacity of the pier. Each stage of cantilevering is accomplished by applying the epoxy resin jointing material to the ends of the segments, lifting a pair of segments into place, and post-tensioning them to the standing portion of the structure [see Fig. 1.4(b)]. Techniques for positioning the segments vary. They may be lifted into position by means of a truck or floating crane, by a traveling lifting device attached to (or riding on) the completed portion of the superstructure, or by using a traveling gantry. In the latter case the segments are transported over the completed portion of the superstructure to the gantry and then lowered into position.

The stage-by-stage erection and prestressing of precast segments is continued until the cantilever arms extend nearly to the span centerline. In this configuration the span is ready for closure. The term



(a) Construction of hammerhead

(b) Segmental cantilevering

(c) Closure of span

Fig. 1.4. Construction of complete span

closure refers to the steps taken to make the two independent cantilever arms between a pair of piers one continuous span. In earlier segmentally constructed bridges there was no attempt to ensure such longitudinal continuity. At the center of the span, where the two cantilever arms meet, a hinge or an expansion joint was provided. This practice has been largely abandoned in precast structures, since the lack of continuity allows unsightly creep deflections to occur.[19,24] Ensuring continuity is advised and usually involves:

(1) Ensuring that the vertical displacements of the two cantilever ends are essentially equal and no sharp break in end slope exists.

(2) Casting in place a full width closure strip, which is generally from 1 to 3 ft. in length.

(3) Post-tensioning through the closure strip to ensure structural continuity.

The exact procedures required for closure of a given structure must be carefully specified in the construction sequence. The final step in closure is to adjust the distribution of stress throughout the girder to ensure maximum efficiency of prestress. Adjustment is usually necessary to offset undesirable secondary moments induced by continuity prestressing. The final adjustments may involve[19]

(1) Adjusting the elevation of the girder soffit, at the piers, to induce supplementary moments. The adjustment may be accomplished by means of jacks inserted between the pier and the soffit of the girder with subsequent shimming to hold the girder in position.

(2) Insertion of a hinge in the gap between the two cantilever arms to reduce the stiffness of the deck while the continuity tendons are partially stressed. The hinge is subsequently concreted before the continuity tendons are fully stressed.

(3) A combination of hinges and jacks inserted in the gap to control the moment at the center of the span while the continuity tendons are stressed. The final adjustment is made by further incrementing the jack force and finally concreting the joint in.

The first of these possibilities is the widest used. After final adjustments are complete, the operation is moved forward to the next pier and the erection sequence begins again.

1.3 Research Program in Segmentally Constructed Bridges

In 1969, a comprehensive research effort dealing with segmental construction of precast concrete box girder bridges was initiated at The University of Texas at Austin. This report summarizes part of this multiphase project which had the following objectives:

(1) To investigate the state-of-the-art of segmental bridge construction.

(2) To establish design procedures and design criteria in general conformance with provisions of existing design codes and standards.

(3) To develop optimization procedures whereby the box girder cross section dimensions could be optimized with respect to cost to assist preliminary design.

(4) To develop a mathematical model of a prestressed box girder, and an associated computer program for the analysis of segmentally constructed girders during all stages of erection.

(5) To verify design and analysis procedures using a highly developed structural model of a segmental box girder bridge.

(6) To verify model techniques by observance of construction and service load testing of a prototype structure.

Various phases of this work were reported in previous reports in this series and in several dissertations.[7-11, 14-16] Objective (1) was initially accomplished with publication of Report 121-1. It has subsequently been updated and advanced by Muller's excellent paper.[20] Objectives (2) and (3) are the direct areas of interest in the present report. Objective (4) was accomplished with the development of the program SIMPLA2 as documented in Report 121-4. Objectives (5) and (6) were accomplished as documented in Report 121-5.

1.4 Objectives and Scope of this Report

The object of this report is to document proper design procedures and to develop practical optimization techniques for the application of the segmentally precast box girder in long span highway bridge superstructures.

Segmentally precast box girders should be designed and analyzed considering the construction process. In contrast to many concrete structures, it is essential that erection conditions and stresses be carefully checked at all stages for balanced cantilever construction. Analysis at each stage can represent a monumental task unless the problem is simplified considerably. Effects of simplification for purposes of design are often difficult to evaluate, since the degree to which a box girder behaves as an element depends on many variables and it is difficult to determine.[26]

In developing the design procedures, the authors used adaptations of methods recommended in AASHO specifications for the design of normal bridge cross sections under the action of wheel loads wherever possible. This AASHO method was somewhat simplified and utilized in the development of a computerized optimization scheme to determine preliminary box girder cross section dimensions for minimum cost.

In Chapter 2 a general design procedure is outlined. Interaction of the various steps of preliminary proportioning, optimization studies, detailed transverse and longitudinal design, box girder analysis for warping effects, checks of erection stresses, and development of post-tensioning system details are interrelated. Construction trends influencing preliminary proportioning are examined and references are made to several helpful summaries of physical properties of completed structures.

Detailed procedures for the design of bridges constructed on falsework are developed in Chapter 3 and a design example is utilized. Chapter 4 gives similar material and an example for bridges erected in cantilever. Both ultimate strength and service load design criteria are satisfied. Utilization of existing computer programs to thoroughly check the stresses under various loadings is illustrated.

Mathematical methods of optimization are briefly reviewed in Chapter 5. Appropriate methods are applied to illustrate factors affecting the optimal cross section for bridges constructed on falsework (Chapter 6) and erected in cantilever (Chapter 7). In the optimization studies, the function which is minimized is an approximate cost index for the bridge.

CHAPTER 2

DESIGN PROCEDURES

2.1 General

While over one hundred long span bridges have been constructed throughout the world using segmentally precast box girders, their utilization in the United States has been very slow in developing. With completion of the first U.S. project in 1973, there has been a heightened interest and a number of projects are now actively underway. While there are undoubtedly many reasons for the slow development of this type of construction in the United States, probably one of the most significant is the general division of the engineering and construction responsibilities that has existed in the concrete industry. The segmental precast box girder bridge requires extensive consideration of construction methods and procedures during the design phase. In the same way, the erector must be responsible for substantial calculations for control of stresses and deflections throughout the erection phase. While such interaction has been very common in construction of long span steel bridges, it has not been as usual in long span concrete structures.

A successful design of a precast segmental box girder bridge must consider carefully the constructability of the project, must leave room for competitive systems and constructor improvements, and must consider the stability of the structure in all of its embryo stages as well as performance of the completed structure.

Division of responsibility must be very carefully developed, so that the constructor is not forced into undertaking an unrealistic or unsafe construction procedure by orders of the designer and, conversely, the designer is not responsible for errors or lapse in judgment by the contractor.

The authors feel that the main reason for the lag in development of this type of structure in the United States has been the technological transfer gap, not of highly involved analysis procedures, but rather of efficient construction procedures suited for the engineering, constructor, laborer, and material practices of the United States.

In this chapter a very brief outline of the design procedures which should lead to a successful project is given. The material in this chapter is intended to outline the general framework of the design process. Two very specific technical designs are included in subsequent chapters to provide specific guidance on "detailing". Substantial information on factors affecting optimization of the cross sections are included in subsequent chapters to assist in preliminary designs.

2.2 State-of-the-Art

Report 121-1 summarized the state-of-the-art in precast segmental box girder technology as of 1970. The ensuing five years have seen rapid developments in this technology. Foreign experience by one of the world's foremost builders of this type structure was summarized in 1974 at the FIP/PCI Congress in New York City by Jean Muller. His report has been printed for distribution in the United States in the January 1975 PCI Journal.[20] One of the most interesting aspects of the developmental period of the last decade has been the evolution of the jointing and erection process. Epoxy joints are still the foremost type of jointing, but less reliance is being placed on the strength of the epoxy and more jointing surface is being provided for mechanical interlock keys between units. Muller shows pictures of recent French bridges with castellated or serrated web keys for a long portion of the web length. The multiple key designs certainly decrease reliance on long-term epoxy integrity and should be carefully studied. In the same light, numerous projects are going to procedures which move the negative moment (cantilevering) tendon anchorages out of the web end surface and provide internal stiffeners attached to the webs for locating anchorages. By moving the anchorage from the end surface of the unit, several units can be placed using temporary fasteners before stressing has to be accomplished. In this way threading of cables and stressing of tendons has been removed from the critical path operations in erection.

Another obvious tendency in foreign practice is to the use of wider sections resting on single piers, rather than double box sections supported by parallel twin piers. While the cost of the superstructure is somewhat higher for the wider single section, substantial economies have been achieved in pier costs.

The large number of projects currently underway in the United States as summarized by Koretzky and Kuo[13] indicates that this type of construction is emerging rapidly. Their survey indicates that as of December 1973 sixteen states were involved on a total of 56 bridges with almost half of these appearing to be on a fairly firm basis.

2.3 Design Sequence

Probably because of the relative unfamiliarity with the segmental construction procedures, but largely because of the close relationship which must exist between design and construction concept, the design sequence for a precast segmental box girder bridge is a highly interactive one. Figure 2.1 shows the various stages of the design sequence and the usual paths between sequences. The main elements are:

(1) Conceptual design--basic decisions regarding type of construction, span lengths and ratios, and cross section types.

(2) Preliminary design--choice of basic dimensions for cross section elements, tendon and reinforcement patterns, slab and web thicknesses, and optimization studies of the span and cross section layout. Analysis procedures are usually approximate.

(3) Detailed design--specific proportioning of a tentative cross section considering both construction sequence loads and normal design loads on the completed structure, sizing of tendons, reinforcement, structural member dimensions, and planning of the erection and closure sequences. Relatively detailed analysis to consider all major loads and conditions which will affect behavior of a structure.

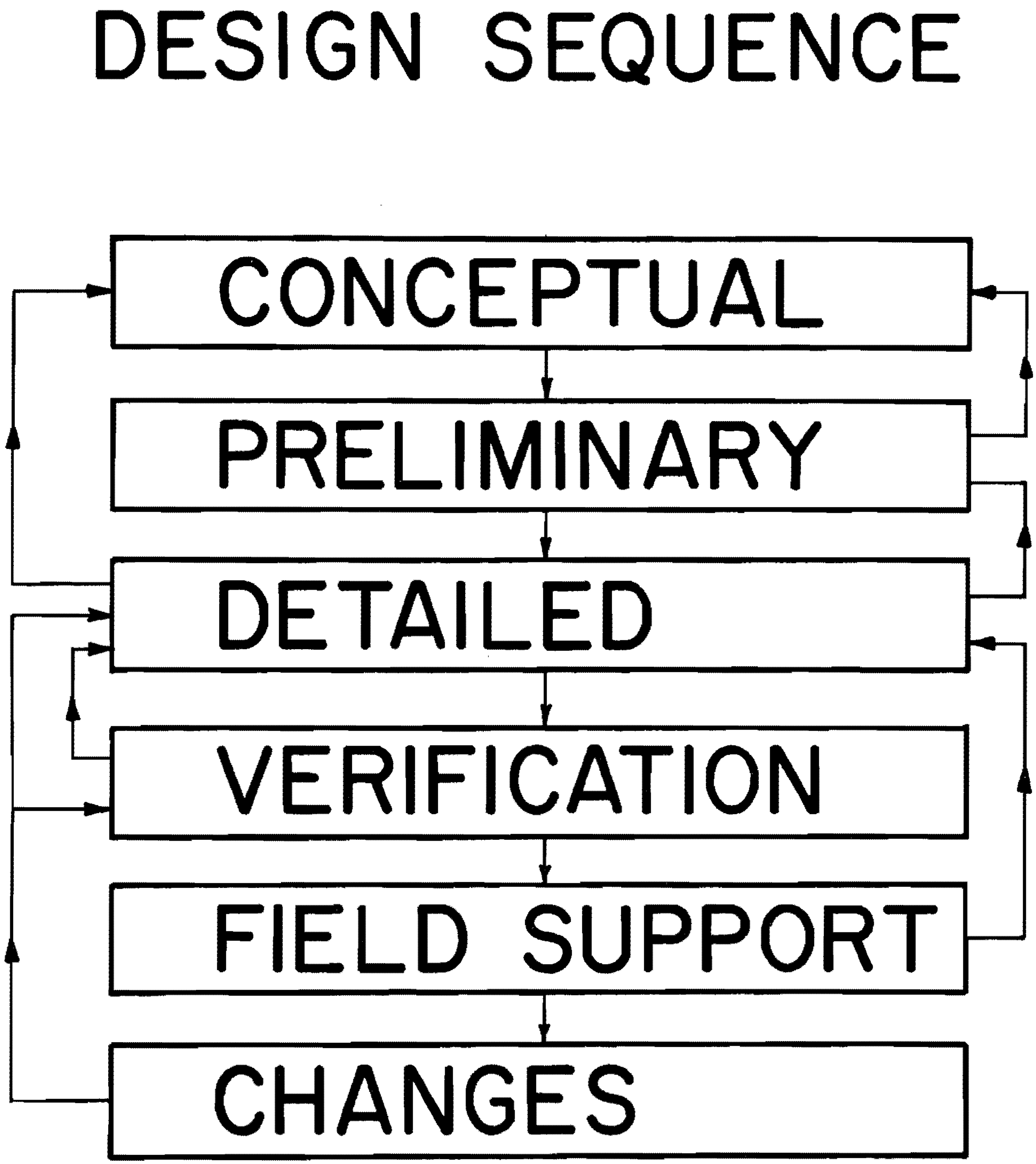


Fig. 2.1. Interactive design sequence

(4) Verification Analysis--studies undertaken after most elements of the design are substantially fixed to check construction stresses and deformations and behavior under all critical design load conditions.

(5) Field Support Analyses--checks of working drawings, contractor's erection stresses, detailed stressing sequences, and development of deflection and closure information for guidance of field forces.

(6) Change Order Evaluation--providing rapid information to field forces and contractor on technical advisability of proposed changes in design requires quick response in technical decisions.

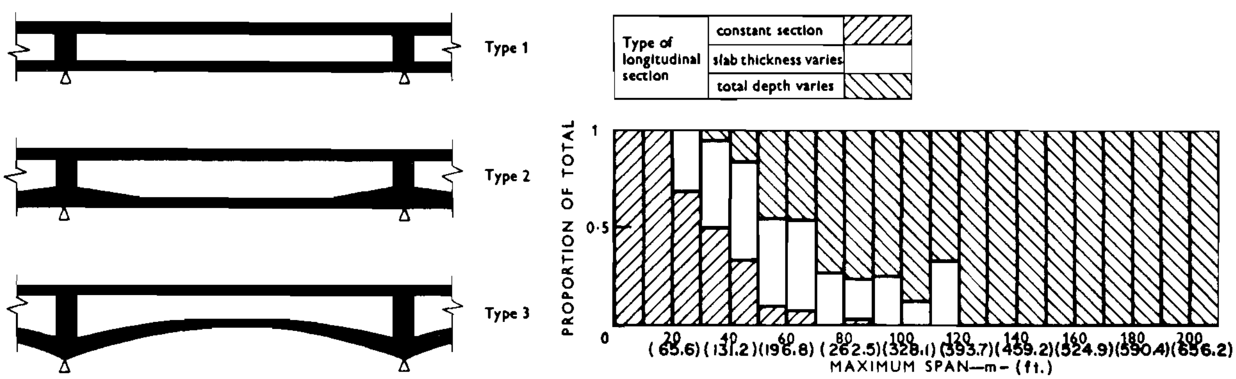
Some specific details for each of these stages will be developed in the following sections. The large number of interactions indicated in Fig. 2.1 shows that such a breakdown is extremely artificial, since often the same person will be handling several of the items in the sequence. The schematic is useful in organizing a discussion of the important elements in the design sequence.

2.4 Conceptual Design

The most important decisions in the project are generally made at the start when major questions have to be answered with relatively little hard information. Major decisions usually involve:

A. Span lengths
B. Span ratios
C. Box girder versus alternate structural system
D. Cast-in-place versus precast
E. Erection on falsework versus cantilever erection
F. Single box versus multiple box versus multicell cross section
G. Constant depth versus variable depth

These important questions are best decided after a careful review of the state-of-the-art, consultation with experts who have been involved in the design and construction of successful projects, and intensive study. However, a substantial body of information is available to assist in these decision-makings. Excellent summaries by Muller[19] and Swann[25] as well as a summary by Lacey, Breen, and Burns[16] describe many successful projects and can help one develop a feel for the "possible". In particular, the compilation by Swann[25] of detailed dimensions of 173 concrete box girder bridges (segmental, nonsegmental, precast, and cast-in-place) is very useful. Figure 2.2 illustrates the distribution of constant section, constant depth with variable slab thickness, and variable depth bridges reported by Swann. As in all studies, this distribution must be examined carefully, since it contains a variety of experiences. The majority of the short span structures were not built segmentally. In addition, most of the structures are located in Europe and are undoubtedly colored by the design criteria and economic experience of that region.



(a) Types of longitudinal section (b) Distribution of longitudinal section types

Fig. 2.2. Types of longitudinal profiles (from Swann[25])

Discussion with several other designers of segmental bridges indicates that the following rough "rule of thumb" represents the current state-of-the-art:

Span	Bridge Type
0-150 ft.	I-type pretensioned girder
125-300 ft.	Precast segmental constant depth
275-450 ft.	Precast segmental variable depth
400-600 ft.	Cast-in-place segmental
600-1200 ft.	Cable-stayed with precast segmental girders
1200 ft. up	Suspension

Obviously, such a rule-of-thumb is only a crude indicator of the appropriate type structure. Decision between precast and cast-in-place segmental units must consider not only span length but ease of access to the site of heavy handling equipment, construction seasons, and size of project.

Segmentally precast box girder bridges may be classified into two main types according to the method of erection, namely those constructed on falsework and those erected in cantilever. The third method, assembly on shore, will generally be too cumbersome to have widespread use.

The different methods of construction will require different prestressing cable patterns and different design procedures. In bridges constructed on falsework, long draped cables, traversing one or more spans, can be used. If the cables run the full length of the bridge, only one structural system, namely the completed continuous superstructure, need be considered in design.

For bridges erected in cantilever, a set of cables in the top of the girder is required for each length of the cantilever arm. Each stage of erection constitutes a separate design condition, with different bending moments in the cantilever. The completed superstructure contains additional cables in the bottom of the girder and constitutes an indeterminate continuous system. It is designed to withstand the dead and live loads under service conditions.

Erection on falsework with close-spaced supports is the simplest method of construction when conditions permit, as in the case of viaducts over land and not passing over existing roads. Lifting and placing techniques will depend on the exact site conditions. For bridges having three or more spans over water or over existing roads, where intermediate support is not possible, the cantilever method will probably be the most suitable. There will be a critical span length, however, below which it will be more economical to use a falsework truss. For two-span bridges over an existing highway, erection on a falsework truss or girder or with temporary braces or ties is probably the simplest procedure.

The superstructures of the box girder bridges generally conform to three main types: (a) single-cell box girder, (b) pair of single-cell box girders connected by the deck slab, and (c) multi-cell box girder. These types are sketched in Fig. 2.3. The simplicity, economy, and good appearance of these sections is evident.

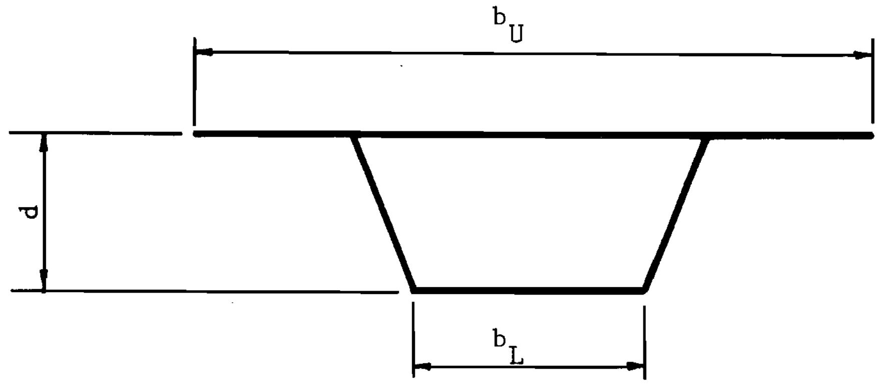
Single-cell box girders are generally used in relatively narrow bridges. As the width increases, the bending moments in the deck slab increase and hence the thickness must increase. Beyond some critical width it becomes more economical to use a multicell box or multiple single-cell boxes.

In the case of multiple single-cell box girders, the basic single-cell units are cast separately and are connected after erection with a concrete joint. Usually the deck is post-tensioned transversely, but it is possible to use nonprestressed reinforcement only and to make the joint width sufficient for splicing. In general, it is possible to have smaller basic units with multiple single-cell boxes than with a multi-cell box girder. The smaller units are easier to transport and erect. The bridge can be easily widened by the addition of another box. On the other hand, with a multi-cell box the cast-in-place longitudinal joint is not required. Also, a multi-cell box, of relatively small base width may be advantageous when narrow piers are desired.

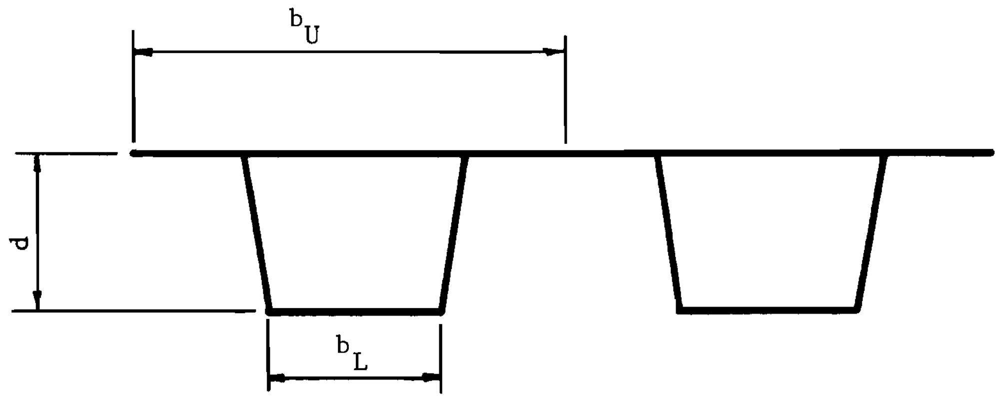
2.5 Preliminary Design

In the preliminary design stage, the important structural parameters are determined. Such factors as span-to-depth ratios, minimum web thickness, upper and cantilever flange thicknesses, and preliminary tendon requirements can be fairly readily determined by conventional elastic analyses, determination of cantilever moments prior to closure, and utilization of normal or "beam" theory for stress analysis of sections.

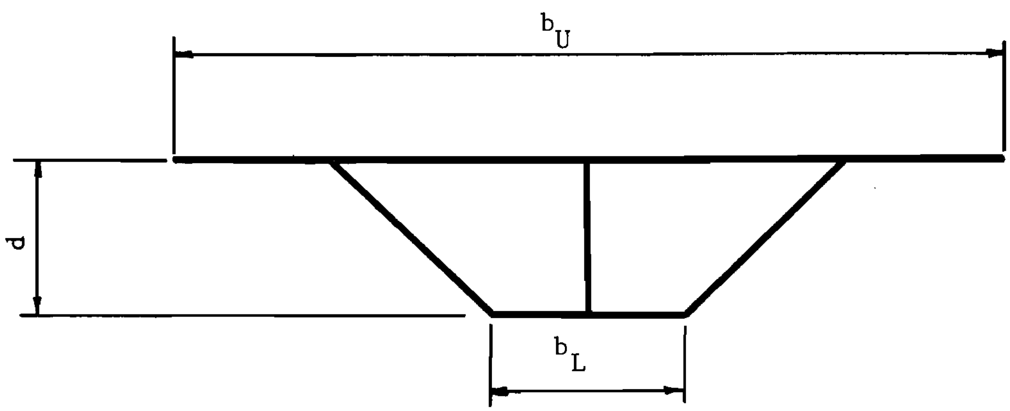
The recent report of the PCI Committee on Segmental Construction [21] suggests span-to-depth ratios from 18:1 to 25:1 are currently considered practical and economical for constant depth segmental bridges. They suggest that variable depth bridges may have span-to-depth ratios of 40 to 50, based



(a) Single-cell box girder.



(b) Single-cell boxes connected by deck slab.



(c) Two-cell box girder.

Fig. 2.3. Box girder cross section types.

on the depth at the center of the span. Figure 2.4 from Swann[25] indicates a wider variation in span-to-depth ratio. The optimizing studies in Chapter 6 and Chapter 7 indicate that very efficient structures can be obtained in the 25 to 30 span-to-depth ratio range. Past experience, as reflected in Fig. 2.4 may be colored by the much heavier live loads used in European design. Figure 2.5 is from a study by Rajagopalan[22] which indicates that for 140 ft. spans, live load design moments in some European countries will vary from 150 to 300 percent of those used in the United States. Use of lower span-to-depth ratio values are indicated when shears are heavy, little load balancing is utilized, or when preliminary design indicates extreme congestion of tendons. The experience with the Corpus Christi segmental bridge, which had a span-to-depth ratio of 25, indicates that even higher values could be used without substantial deflection difficulty.

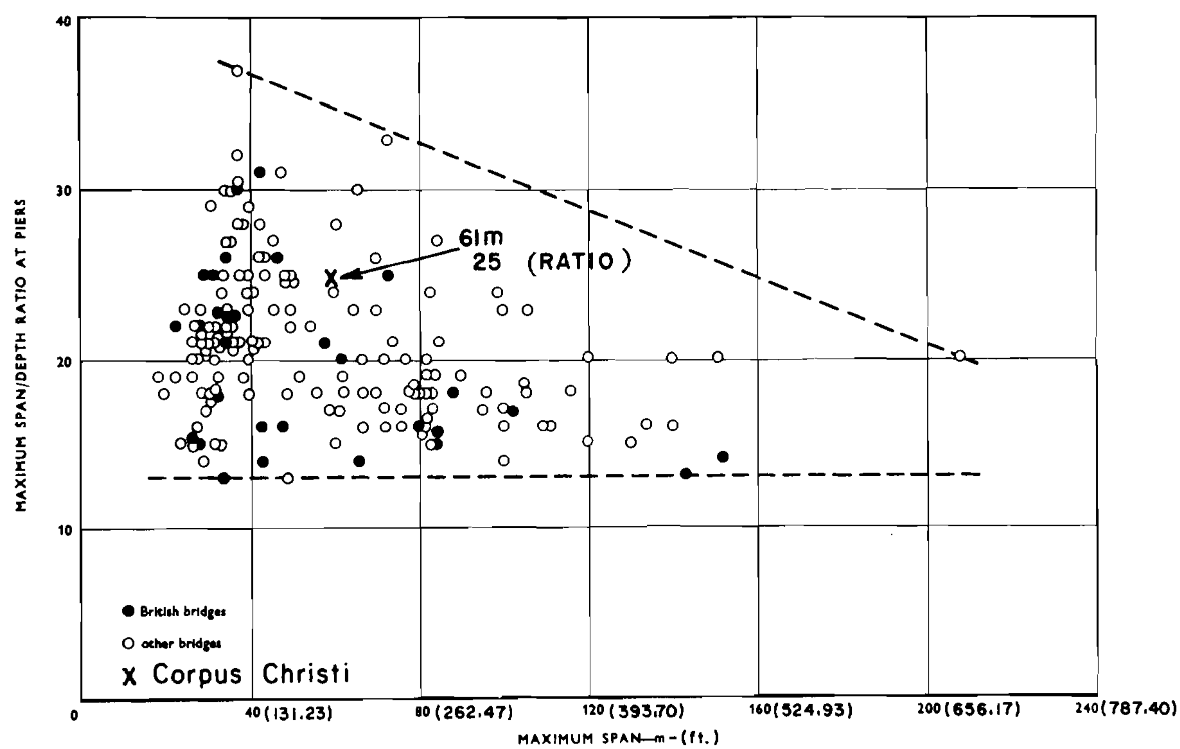


Fig. 2.4. Span/depth ratios (from Swann[25])

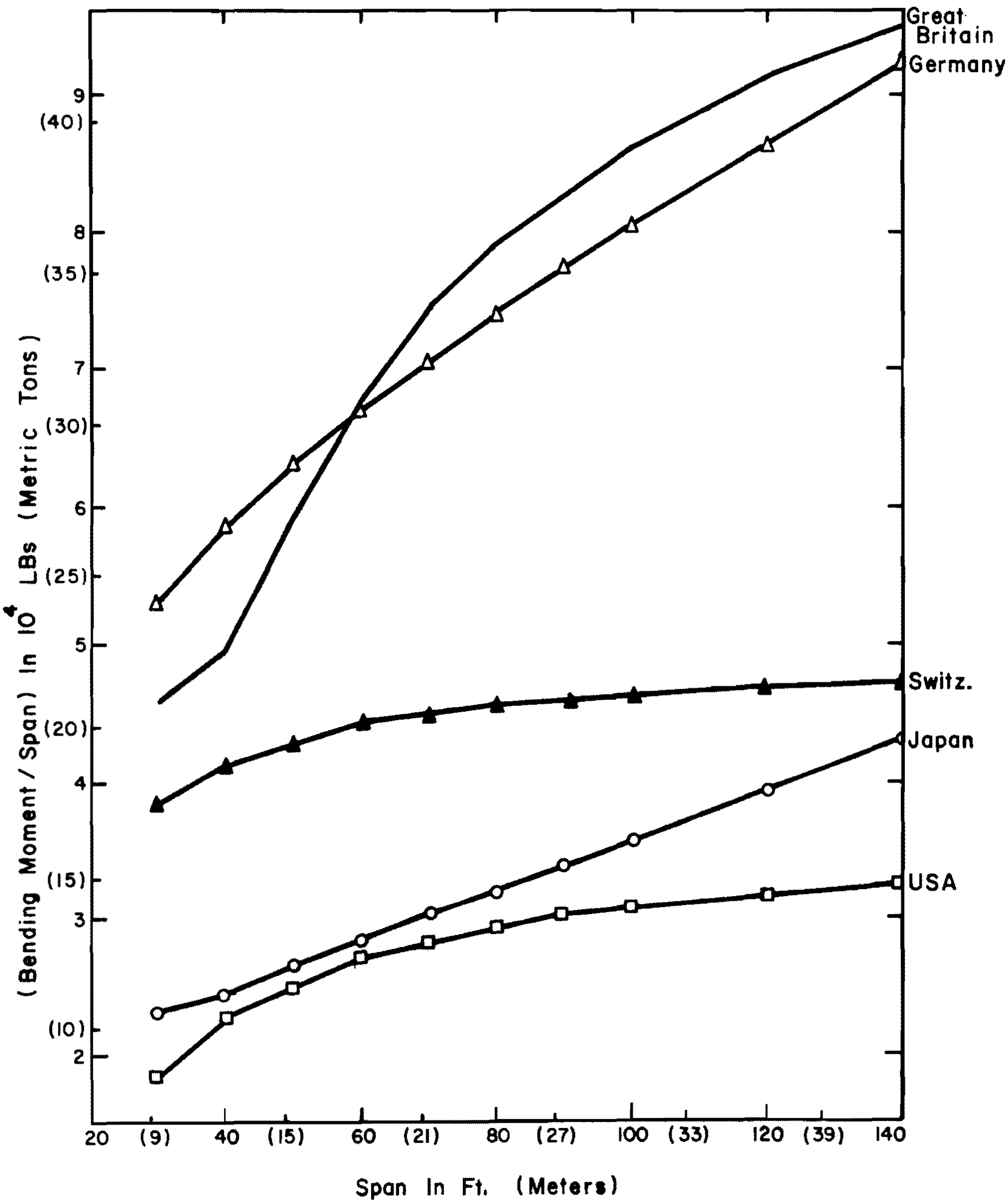


Fig. 2.5. (Bending moment/span) versus span in various countries (Ref. 22)

In many structures the web thickness will be based more on "placeability" considerations and providing adequate room for anchorages than on shear considerations. Based on successful French experience, the Corpus Christi structure was designed with a minimum web thickness of 12 in. In retrospect, the congestion of the webs hindered placement and made detailing of anchorages difficult. Figure 2.6(a) from Swann's study shows a web thickness parameter for a wide range of bridges. It can be seen that the value of the parameter for the Corpus Christi bridge is one of the lowest, with a value of 2.88×10^{-3}. Retrospect would indicate that the webs should have been increased to about 14 in. minimum, which would give a parameter value of 3.36×10^{-3} and essentially plot on Swann's curve. This indicates that such a graph could be quite useful in preliminary design.

While many of the cross-sectional elements can be designed utilizing normal slab design under AASHO specifications, the lower flange near the piers in narrow bridges is very critically affected by the cantilever moments and particularly span-to-depth ratios. This slab often has to be thickened and may indicate the desirability of a greater cross section depth to increase the lever arm and cut down the thickness of the lower flange. This will be more prevalent on double box cross sections than on single box cross sections.

Dimensions of successful projects are often one of the best indicators of the practical market place. However, the use of more formal optimization techniques can indicate important trends to be investigated in design. In Chapters 5 through 7 of this study, an attempt is made to illustrate how relatively simple optimization techniques can be used in preliminary design to give the designer information as to the cost "trade offs" of his basic parameter decisions. Unfortunately, these optimization examples only include a relatively narrow number of span lengths and roadway widths. These studies should be extended to give more information as to the effect of variations in these important parameters. There seems to be a systematic relationship between length and width in the choice of cross section, as indicated in Fig. 2.7. The relatively narrower

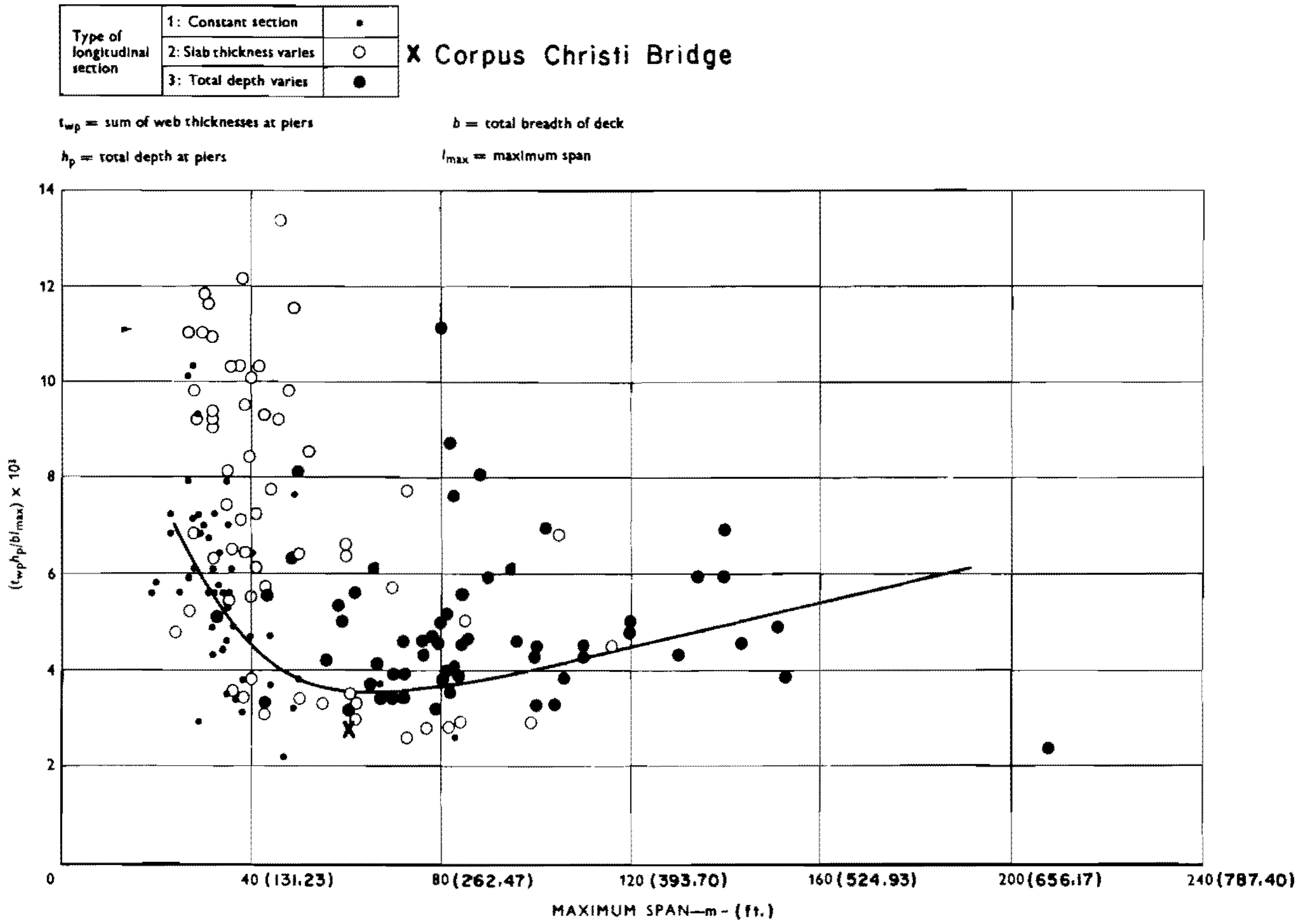


Fig. 2.6 (a) Web thickness parameter at piers (from Swann[25])

$$\frac{t_{wp} h_p (10^{-3})}{b\, L_{max}}$$

t_{wp} = sum of web thickness at piers

h_p = overall depth at piers

b = overall width at deck

L_{max} = maximum span

t_{wp} = 48 in. = 122cm = 1.22m

h_p = 8 ft. = 96 in. = 2.44m

b = 56 ft. = 17.2m $\quad \therefore \quad \dfrac{(1.22)(2.44)}{(17.2)(60)} = 0.00288 \therefore 2.88 \times 10^{-3}$

L_{max} = 200 ft. = 66 yd = 60m

Fig. 2.6 (b) Corpus Christi Bridge parameter calculations

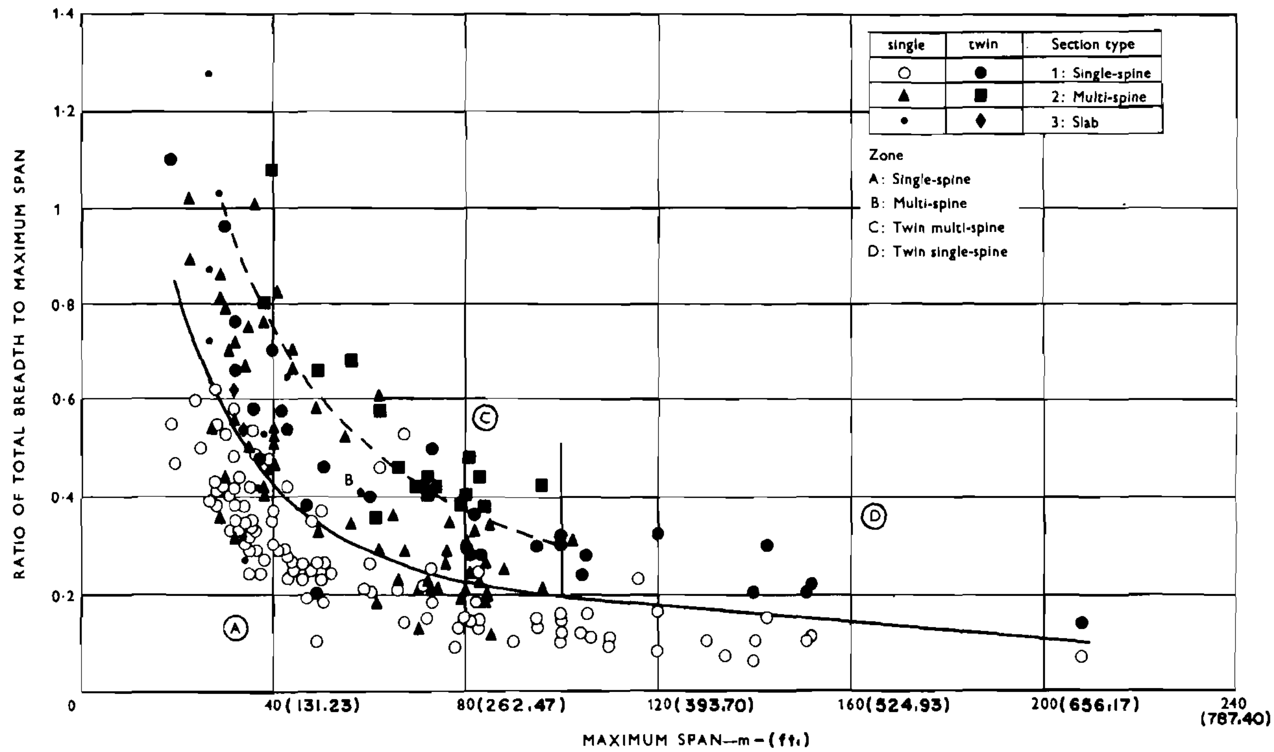


Fig. 2.7. Usage of basic section types (from Swann[25])

bridges go to single box units, while the wider structures go to twin box units. Further studies of variables would clarify the practical boundaries for these decisions.

2.6 Detailed Analysis

After a basic construction scheme, span arrangement, cross-sectional type and important section properties have been at least preliminarily decided upon, a detailed analysis can be made to determine tendon sizes and patterns, flange and web thicknesses, transverse and shear reinforcement, and stressing details. For the initial detailed analysis, ordinary equilibrium equations, elastic analysis, and normal "beam" design procedures are utilized. In many box girders, there will be substantial deviations from such stresses due to shear lag, section warping, and torsion due to unsymmetrical loading. After completion of a detailed design which considers both construction and normal live load effects, it is advisable to check the structure with a "folded plate" type analysis. The authors recommend the use of computer analysis programs such as MUPDI for constant depth sections or FINPLA2 for variable depth bridges. These programs were developed by A. Scordelis at The University of California under sponsorship of the California Department of Transportation and are widely available.

In Chapter 3 and Chapter 4, comprehensive examples of a segmental box girder erected on falsework and a structure erected by balanced cantilever are used to illustrate typical design procedures. These examples illustrate the interaction between the preliminary and the detailed design phase and typical changes made in the detailed design phase to satisfy normal design requirements.

2.7 Verification Analysis

Particularly when cantilever erection is to be used, it is advisable to run a check analysis which will verify the suitability of the proposed construction sequence and check for stresses and deflections to be expected during all stages of erection. In order to facilitate such an analysis, a program SIMPLA2 was developed in this study. Detailed information is given in Report 121-4. Such a program can be used to determine longitudinal and transverse stresses, deformations, tendon friction losses, tendon incremental stressing losses, and track the structure through all unbalanced states and closure operations. The program uses a "folded plate" analysis and so also gives indications of excessive shear lag or other effects. Because of the complexity of inputting the problem into this program and the high cost of the analysis, it is ordinarily only undertaken at the completion of the design as a final check.

In a similar way it is advisable to make a final check of any structure where substantial shear lag or warping effects are suspected to verify all design load conditions. The MUPDI program is an excellent one, and indicated very high correlation with the measurements in the companion test program involving a model study of the Corpus Christi Bridge.

2.8 Field Support

Depending on the contractual arrangements, the designer, the owner, or the contractor will need to carefully control the erection and have substantial technical information to check on the adequacy of construction. Upon completion of design and award of contract, erection stresses, deflection profiles at various stages, tendon stressing patterns and limits, tendon elongation values, and closure computations will have to be developed and transmitted to the appropriate parties. Many of the procedures are repetitive and a computer analysis is often advantageous. Because of the complexity of input into the SIMPLA2 program, it will be advisable to utilize simpler "beam theory" programs to develop the less critical values.

It is especially important that working drawings be cross-referenced and compared so that careful coordination exists in placing reinforcing, post-tensioning tendons, and post-tensioning anchorages. It is advisable to develop high modularity in details to make maximum use of precast technology.

2.9 Change Order Evaluation

After the contractor begins his work, numerous items will come up requiring technical decisions. Some of these will be major, such as submission by a contractor of a major revision in the tendon layout, stressing sequence, or erection plan. One of the great advantages of program SIMPLA2 is that it can be reprogrammed relatively quickly to handle such changes and give a complete reanalysis of all stages of construction. In this way the designer will be able to see the overall effect of the plan change in a clearer fashion.

2.10 Pier Design

In most of the existing literature on precast segmental box girders, insufficient attention is given to pier design. Since the cantilever erection procedure imposes substantial moment requirements on the pier, it can greatly increase the cost of the piers. Several cases have been reported where the increase in pier cost to permit balanced cantilever construction amounted to 25 percent of the superstructure cost. Careful attention should be given to the possibility of providing for the unbalanced moment with temporary struts, ties, or shoring, as shown in Fig. 2.8, so that the permanent pier does not have to have the built-in capability of resisting the full moment. In addition, considerable saving can be obtained by using hollow piers which can develop the required strength and stiffness, but which will not need as much material as the solid piers nor require as many additional vertical supports. In difficult water crossings, the pier costs may be of the same magnitude as the superstructure costs and it is extremely important that careful attention be paid to the pier design. Several recent examples have indicated that erection on falsework is practical even in long spans if pier costs are high.

2.11 Applicable Specifications and Regulations

In design of relatively modest (up to 400 ft. span) segmental box girder bridges, existing design regulations are reasonably adequate. The examples in Chapter 3 and Chapter 4 utilize the 1973 AASHO regulations[2], the ACI Building Code 318-71 provisions for shear and prestressed concrete as allowed by AASHO for prestressed concrete shear, and the 1969 Ultimate Design Criteria of the Bureau of Public Roads. This latter was used rather than the 1973 AASHO because the authors are leery of the combined load and φ factors permitted for this type of construction in the AASHO regulations.

In the 1969 Bureau of Public Roads ultimate design criteria, the basic load factors are 1.35 DL + 2.5 LL. In addition, the values of φ for flexure are 0.9 and for shear are 0.85. For this bridge type in the critical stage when cantilevering is almost complete, the structure is almost 100 percent dead load. The "safety factor" in flexure under the BPR criteria would then be $1.35 \div 0.9 = 1.5$. Using the 1973 AASHO, the load factor would be 1.3 dead load and a φ factor of 1 could be used, since this could be interpreted as "factory produced precast prestressed concrete members". This would give a total safety factor of 1.3 at this critical stage. The authors considered this as insufficient.

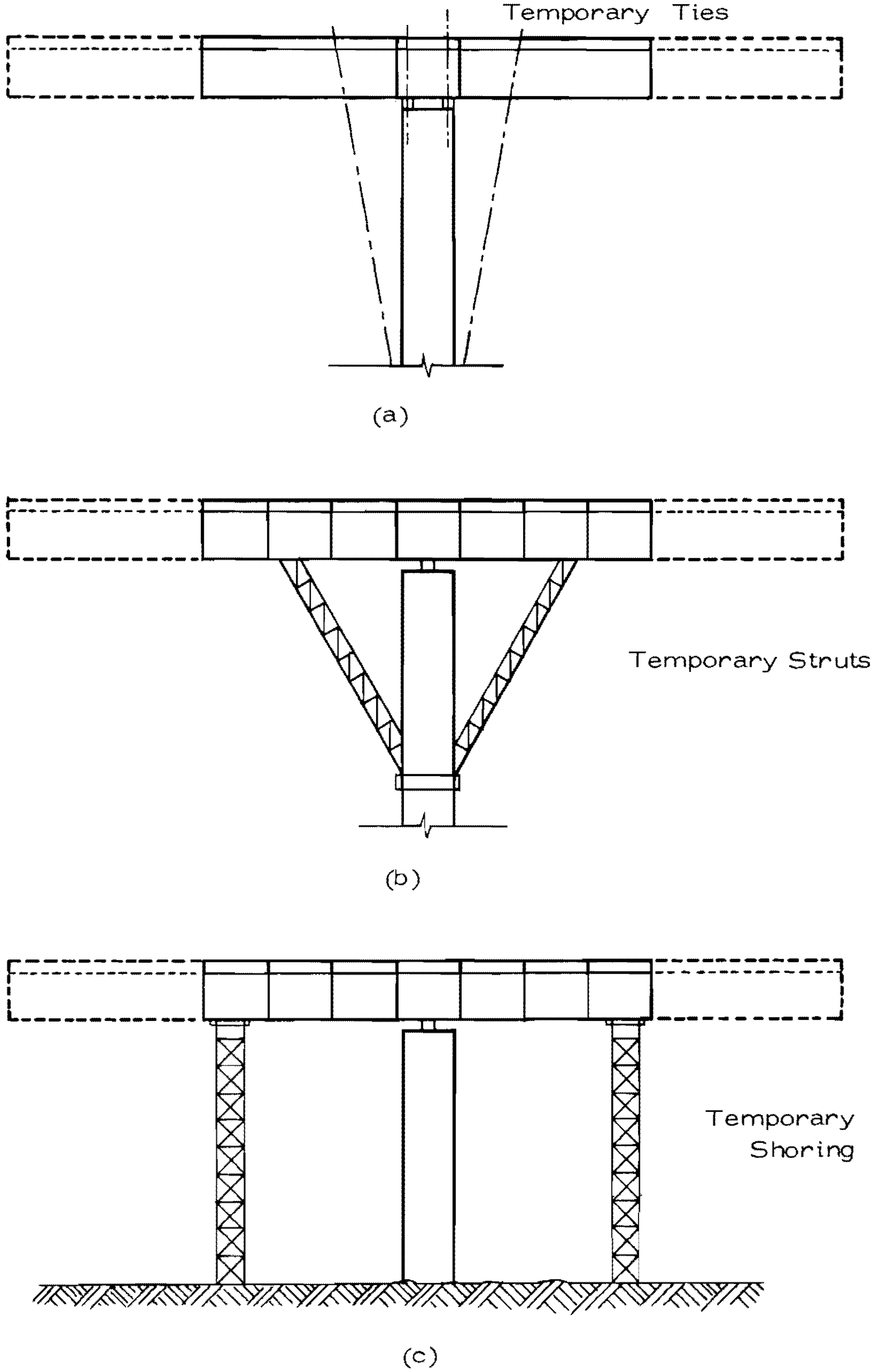


Fig. 2.8. Provisions for unbalanced moment

CHAPTER 3

DESIGN PROCEDURE FOR BRIDGES CONSTRUCTED ON FALSEWORK

Construction on falsework is the simplest method of erecting precast, segmental bridges. It also leads to the simplest design approach. The prestressing system for the superstructure will normally consist of long draped cables in the webs of the box girder. If the overall length of the bridge is moderate, say two to four spans, it is possible to set all the segments in place and join them before inserting and tensioning full-length cables. However, for very long bridges, especially viaducts having many spans, it will be necessary to erect and tension one or two spans at a time.

The design procedure in this chapter is developed using as a particular design example a two-span continuous bridge with spans of 180 ft.-180 ft. The basic steps in the design of the superstructure are as follows:

(a) An approximate cross section is chosen, on the basis of a preliminary design or an optimization study.

(b) The cross section is designed in detail.

(c) The prestressing cables are designed to balance the dead load.

(d) The ultimate strength is calculated.

(e) The concrete service load stresses are calculated from beam theory.

(f) The ultimate shear strength is checked.

(g) The final structure is analyzed using the computer program MUPDI to check for shearing, warping, and unsymmetrical loading effects and to verify the design.

The same procedure can be applied directly to other span lengths. Extension to bridges having more than two spans and to viaducts will be discussed.

It is to be noted that both ultimate strength criteria and service load stress criteria are applied in this design procedure.

In the design of continuous prestressed concrete structures there are different ways of considering the effect of the prestressing cables on the concrete stresses. The approach adopted here is to utilize an equivalent load concept, as described below.

3.1 Equivalent Load Concept

In a prestressed concrete girder the prestressing cables exert forces and moments on the concrete and so produce stresses in the concrete which are added to those produced by the dead loads and applied loads.

In a statically determinate girder, the cable moment at any point is simply equal to the product of the cable force and the eccentricity about the girder centroid. However, in a continuous girder the cables generally modify the external reactions and so the determination of the stresses produced in the concrete by the cables is more complex. The concrete stresses in a continuous prestressed girder may be determined most efficiently by means of the equivalent load concept,[17] which will be described below.

3.1.1 Cable Moments. In a continuous beam it is convenient to distinguish between the different components of the cable moments as follows.

The primary moment (M_P) at any point in the girder is equal to the product of the cable force (F) and the eccentricity about the girder centroid (e).

$$M_P = F \times e$$

The secondary moment (M_S) is the moment produced by the cable induced reaction. This moment will vary linearly between the supports.

The resultant cable moment on the concrete section (M_R) is the sum of these two.

$$M_R = M_P + M_S$$

The concrete stresses produced by the cables at any point in the girder can be determined from M_R and F at that point.

Normally M_R is determined directly, without first determining M_S. M_R will here be determined using the equivalent load concept.

3.1.2 Equivalent Load. Wherever there is a change in direction of the cable, a transverse force is exerted on the concrete section. Also, wherever a cable is anchored, it exerts a concentrated longitudinal force on the section. If the anchorage is not at the centroid, this force has a moment about the centroid. The equivalent load is here defined as the transverse load (and also the concentrated moment) exerted by the cable on the concrete.

The equivalent loads for some important cable configurations will now be determined. First consider a general configuration, $y = y(x)$, shown in Fig. 3.1. The slope is $\theta(x) = dy/dx$. The equivalent load per unit length is $w = w(x)$. The transverse cable force on the element dx is given by

$$\begin{aligned} w.dx &= F(\theta(x + dx) - \theta(x)) \\ &= F(\theta(x) + \frac{d}{dx}(\theta)dx - \theta(x)) \\ \therefore\ w &= F(d\theta/dx) \\ &= F(d^2y/dx^2) \end{aligned}$$

Consider now the parabolic cable shown in Fig. 3.2. Its equation is $y = ax^2 + bx + c$. The equivalent load is

$$\begin{aligned} w &= F(d^2y/dx^2) \\ &= 2aF \end{aligned}$$

i.e., a parabolic cable gives a uniform equivalent load. The total load along the length is given by

$$\begin{aligned} \int_A^B w\,.\,dx &= \int_A^B F(d\theta/dx)dx \\ &= F(\theta_B - \theta_A) \end{aligned}$$

i.e., the product of the cable force and the total change in slope.

It is also useful to obtain the equivalent load in terms of the cable drape, h, and the length, L.

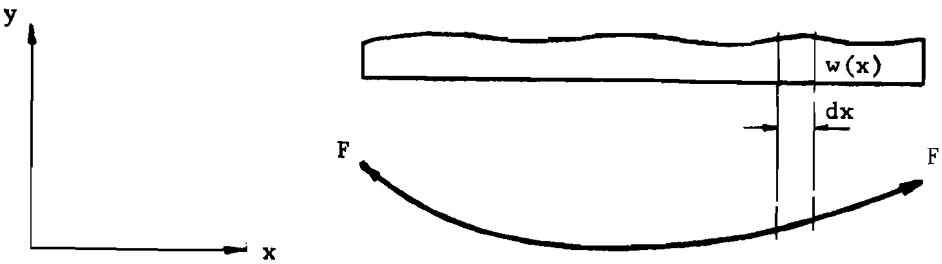


Fig. 3.1. Equivalent cable load

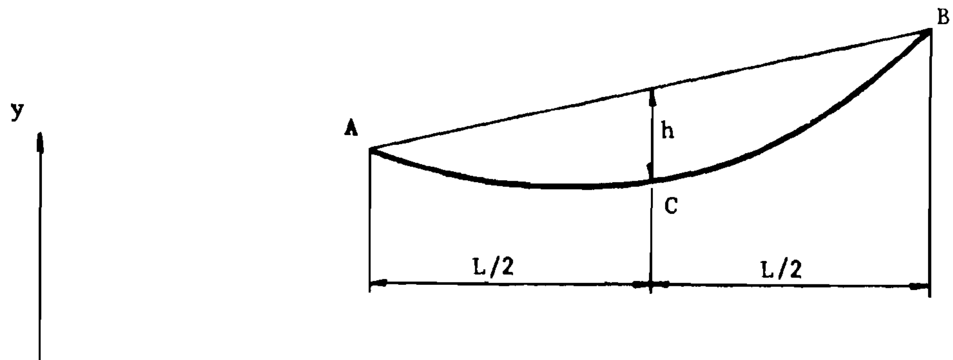


Fig. 3.2. Parabolic cable

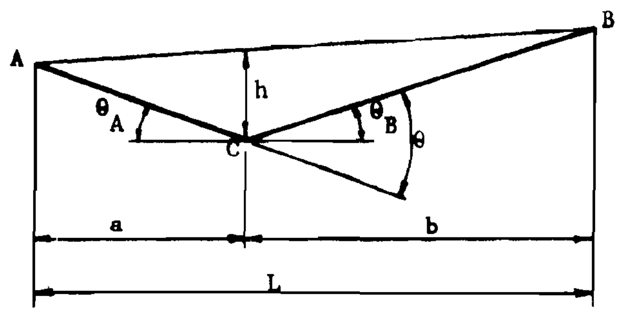


Fig. 3.3. Straight cable

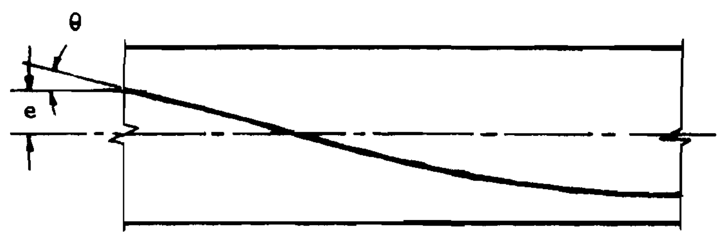


Fig. 3.4. Cable anchorage point

$$
\begin{aligned}
h &= (y_B + y_A)/2 - y_c \\
&= a(x_A + L)^2/2 + b(x_A + L)/2 + c/2 + ax_A^2/2 + bx_A/2 \\
&\quad + c/2 - a(x_A + L/2)^2 - b(x_A + L/2) - c \\
&= aL^2/4 \\
\therefore\ a &= 4h/L^2 \\
w &= 2aF \\
&= 8Fh/L^2
\end{aligned}
$$

Consider next a straight cable with a sharp bend, as shown in Fig. 3.3. The equivalent load, P, at C will be a concentrated load given by

$$
\begin{aligned}
P &= F(\sin\theta_A + \sin\theta_B) \\
&\approx F(\theta_A + \theta_B) \\
&= F\theta
\end{aligned}
$$

i.e., the product of the cable force and the change in slope, as before. In terms of the cable drape, h,

$$
\begin{aligned}
P &= F(h/a + h/b) \\
&= Fh(a + b)/(ab) \\
&= FhL/(ab)
\end{aligned}
$$

Finally consider the anchorage point of a cable, as shown in Fig. 3.4. The transverse equivalent load is given by

$$P = F.\theta$$

as above. There is also an equivalent moment

$$m = F.e$$

and an axial load

$$F\cos\theta \approx F$$

which will produce uniform compression but no bending if the girder cross section is uniform.

3.1.3 Determination of Cable Moments. When all the equivalent cable loads on a girder have been calculated, the resultant cable moments can be determined. This is done by analyzing the girder under the equivalent cable loads, treating these in the same way as externally applied loads. The secondary moments are obtained by subtracting the primary moments from the resultant moments:

$$M_S = M_R - M_P$$

3.1.4 Load Balancing. In the design of prestressed concrete girders it is often convenient to treat the equivalent cable load as an external load that can counteract or "balance" other applied loads; for example, the dead load and live load. A parabolic cable, for instance, exerts an upward uniform load, which will counteract a portion of a uniform dead load. The cable force may be determined so as to balance some definite proportion of the dead load. Similarly, a concentrated applied load could be balanced by a straight cable with a sharp bend. This design technique[17] is called load balancing.

3.2 Design Example - Two-Span Bridge

The design criteria will be developed, using as a design example the two span bridge shown in Fig. 3.5. The precast segments are 10 ft. long. The cross section, shown in Fig. 3.6, consists of a pair of single-cell boxes connected by the deck slab. Each box is cast separately and the 2-ft. wide longitudinal cast-in-place joint connecting them is not made until the two separate girders have been erected and fully tensioned. The completed superstructure is supported on simple neoprene pads on the piers and abutments. Diaphragms are provided inside the box sections at all supports. The prestressing system consists of a group of full length draped cables in the webs of the box girders. This particular example was chosen to correspond to an approximate maximum length of a two-span highway crossover.

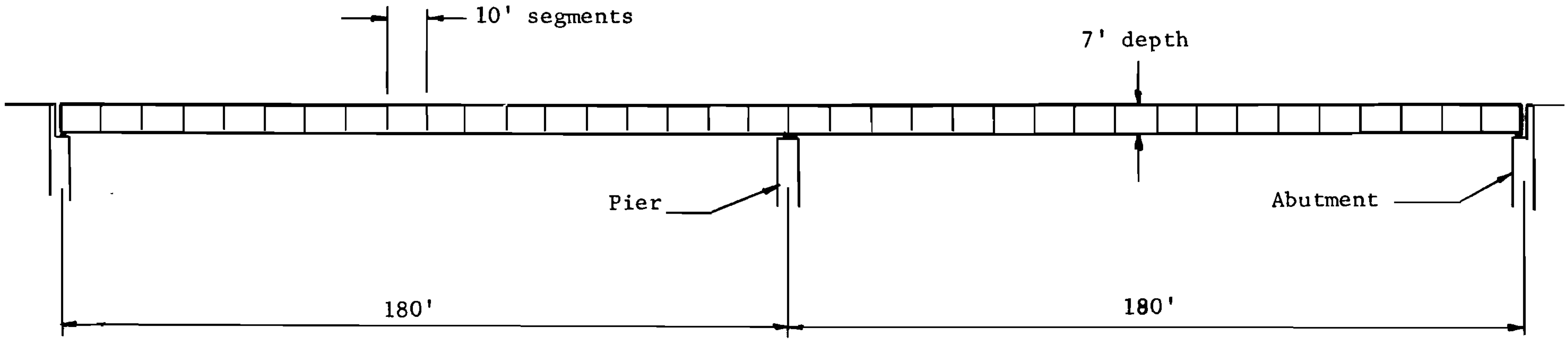
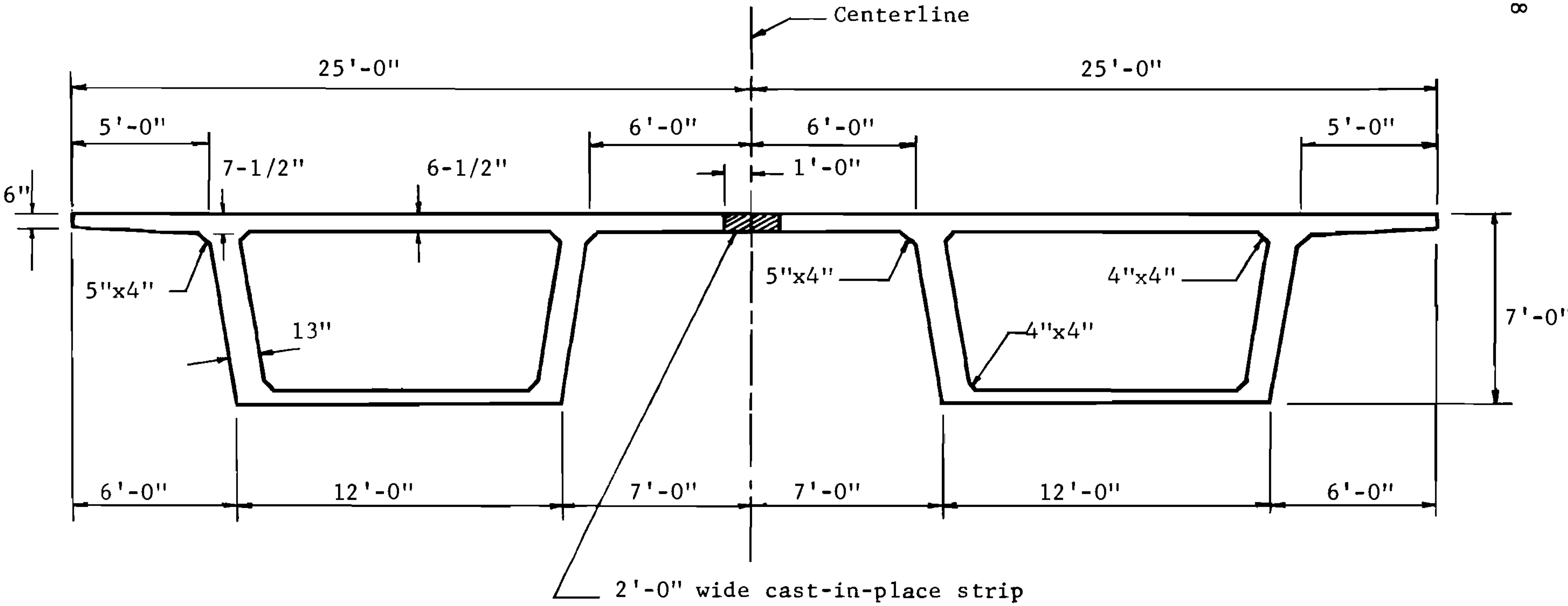


Fig. 3.5. Elevation of two-span bridge



NOTE: Bottom slab thickness varies from 10" at pier to 6" at 45'-0" from pier center.

Fig. 3.6. Cross section of superstructure

The overall width of the superstructure, 50 ft., allows a four-lane roadway width of 48 ft. The 7-ft. depth was chosen to give a conservative span/depth ratio of about 25, a value typical of existing segmental bridges. The other dimensions shown in Fig. 3.6 are trial dimensions for the design and were selected on the basis of a preliminary design similar to that described in Chapter 6, which was carried out to determine approximate dimensions and an approximate cable quantity. The 10-ft. segment length was chosen as a convenient dimension for handling and highway transportation. The cross section consisting of a pair of single-cell boxes was chosen, rather than a full-width multi-cell box girder, to provide basic units that conform with highway transportation weight requirements. The 2-ft. wide cast-in-place strip allows for splicing of the reinforcement between the two halves of the cross section. If the deck were post-tensioned transversely, this width could be reduced.

The joints between the segments may be either concrete or epoxy resin. Ordinarily segmental bridges erected on falsework have used concrete or mortar joints.

3.3 Construction Procedure

The details of the construction procedure depend on whether concrete joints or epoxy resin joints are used. In either case the falsework must be very rigid.

In the case of concrete jointing, the segments are cast short of the 10 ft. nominal length to allow for the cast-in-place joints. A 3-in. joint thickness is suitable. All of the segments are lifted onto the falsework and set in their exact positions, after which the joints between the segments are cast at one time. The cables are then inserted in their ducts and tensioned.

With epoxy resin jointing, the segments are lifted into place on the falsework and are glued together one by one, starting from the central pier. A number of single strand, 20-ft. long prestressing cables inserted in the top and bottom slabs, or alternatively temporary external strands, must be used to tie each pair of segments together. Great care must be taken to ensure that the segments are firmly supported on the falsework in such a way that no differential settlement will occur and cause cracking. When all the segments have been connected, the main cables are inserted and tensioned.

Finally, in either case, the falsework is removed so that the superstructure is supported on the neoprene bearing pads, and the longitudinal deck joint is cast between the two box girders.

Epoxy joints require more care in setting the segments in position than do concrete joints and also require the use of short cables to tie each pair of segments together. On the other hand, they require no forms and should make possible faster construction, better quality control and a better finished appearance.

3.4 Material Properties

The material properties assumed are as follows:

Concrete:	Compressive strength: $f'_c = 6$ ksi.
Reinforcement:	Yield strength: $f_y = 40$ ksi. (This choice is arbitrary; 60 ksi could be used.)
Prestressing:	Each cable consists of a bundle of 1/2 in. diameter strands. Ultimate strength: $f'_s = 270$ ksi.

3.5 Cross Section and Reinforcement

Details of the transverse design of the cross section will not be given here. The procedure used was identical with that to be described in Chapter 4 for a three-span bridge. The reinforcement details are shown in Fig. 3.16.

The deck slab thickness and reinforcement were designed to comply with the 1969 and 1973 AASHO specifications.[1,2] The live load used was AASHO HS20-44. The transverse reinforcement adopted in the top and bottom of the slab consists of #8 bars at 6-1/2 in. spacing.

Since this design envisaged cables anchored in the webs, the thickness of the web must be adequate to accommodate the cables and their anchors and, together with the reinforcement, to withstand the transverse bending moments and the shear force. In this case, a thickness of 13 in. was chosen as a minimum thickness which could accommodate cables consisting of 20 strands, arranged in rows of two. The vertical reinforcement was designed to resist the maximum transverse bending moment in the web. The shear capacity will be checked later.

A minimum thickness of 6 in. was chosen for the bottom slab, this being considered a practical minimum for placing of concrete with two layers of reinforcement. The thickness is increased linearly to 10 in. at the pier over a distance of 45 ft., i.e., a quarter of the span, to withstand the higher longitudinal compression in that region. The 10-in. maximum thickness was chosen on the basis of the preliminary design, using procedures outlined in Sec. 6.2.4 and 6.2.5. The compressive capacity will be checked later.

The transverse reinforcement in the bottom slab and the longitudinal reinforcement throughout the section were provided to comply with the minimum requirements specified by AASHO.

3.6 General Design Criteria

The following design criteria must be satisfied by the bridge superstructure.

3.6.1 Ultimate Strength. The concrete section and the prestressing cables must provide adequate ultimate flexural strength at all sections. The concrete in the webs, together with the vertical reinforcement, must provide adequate ultimate shear strength.

The load factors and φ factors used are the more conservative values specified in the BPR Criteria for Reinforced Concrete Bridge Members,[6] Clause 2.A.1, and are as follows:

LF Dead Load:	1.35	φ Flexure	0.9
LF Live Load and Impact:	2.25	φ Shear	0.85

As permitted by the AASHO specifications, ACI Standard 318-71[4] will be used for all of the ultimate shear strength calculations.

3.6.2 Service Load Stresses. Under all possible loads the stresses at service load after losses have occurred must not exceed the allowable limits specified by AASHO, Clause 1.6.6(B)(2).

The allowable concrete compressive stress at design load is

$$0.4f'_c = 2.4 \text{ ksi}$$

Since the nonprestressed longitudinal reinforcement does not cross the joints between the segments, no tension will be allowed on the concrete.

The effective prestress in the cables after losses is assumed to be

$$f_{se} = 0.6f'_s = 162 \text{ ksi [Equivalent to } 0.8f^*_y]$$

3.6.3 Deflections. Deflections in the completed bridge will be examined to ensure that they are compatible with proper functioning and good appearance of the bridge.

3.7 Design of Superstructure

After completion, the bridge superstructure will consist of a pair of connected box girders, continuous over two spans and seated on simple supports.

The section properties of the superstructure have been calculated by a program, for which a listing and a diagram showing the notation are given in Appendix B. The properties of the section at the pier and of the minimum section are presented in Table 3.1. Properties of the half section (i.e., one box girder) and of the full section (i.e., two box girders joined by the 2 ft. width of a cast-in-place deck) are given for each position. The former are required for computing stresses under dead load and cable forces and the latter for computing stresses under live load.

3.7.1 Loading. Dead Load. The dead load compreses the weight of the girder section and that of an asphalt road surface. The unit weights are as follows:

(a) Concrete: A gross density, including reinforcement and cables, of 0.150 kip/ft.^{3} is assumed.

(b) Asphalt: The asphalt surface, weighing 0.017 kip/ft.^{2}, covers the 48-ft. width of roadway.

TABLE 3.1. SECTION PROPERTIES OF SUPERSTRUCTURE--TWO-SPAN BRIDGE

	Maximum Section	Minimum Section
Properties of Half Section (1 box)		
Area (ft.^{2})	35.84	32.55
Distance from top to centroid (ft.)	3.118	2.797
Second moment of area (ft.^{4})	261.9	225.4
Section modulus (top) (ft.^{3})	83.98	80.58
Section modulus (bottom) (ft.^{3})	67.47	53.61
Properties of Full Section (2 boxes)		
Area (ft.^{2})	72.77	66.19
Distance from top to centroid (ft.)	3.076	2.755
Second moment of area (ft.^{4})	532.4	457.5
Section modulus (top) (ft.^{3})	173.1	166.0
Section modulus (bottom) (ft.^{3})	135.7	107.8

The load per ft. length is as follows, using the area of the completed section obtained from Table 3.1.

Dead Load at Pier:

Concrete section:	0.150 x 72.77	=	10.91
Asphalt:	0.017 x 48	=	0.82
			11.73 kip/ft.

Dead Load at Minimum Section:

Concrete section:	0.150 x 66.19	=	9.92
Asphalt:		=	0.82
			10.74 kip/ft.

Live Load. The live load is AASHO HS20-44. The lane load will be critical. When four lanes are loaded simultaneously, a 25 percent reduction in load intensity is allowed (AASHO Clause 1.2.9).

Impact. The impact factor, specified in Clause 1.2.12, is as follows:

$$I = 50/(180 + 125) = 0.164$$

3.7.2 Bending Moments. The bending moments are calculated at the pier, i.e., the point of maximum negative moment, and at a distance of 72 ft. from the end of the girder (i.e., 0.4 x span) which is approximately the point of maximum positive moment. The influence coefficients used in the following calculations were obtained from Ref. 5.

Dead Load Moments

Moment at pier:

$$-0.125 \times 10.74 \times 180^2 = -43{,}500$$

$$-(263/960) \times (11.7 - 10.7) \times 45^2/2 = -280$$

-43,780 k.ft.

Moment at 72 ft. from end:

$$0.0700 \times 10.74 \times 180^2 = 24{,}360$$

$$(19/1280) \times (11.7 - 10.7) \times 45 \times 72/2 = 20$$

24,380 f.ft.

Live Load Moments

Uniform lane load on 4 lanes = 0.75 x 4 x 0.640 = 1.92 kip/ft.

Concentrated lane load on 4 lanes = 0.75 x 4 x 18 = 54 kip

Moment at pier:

$$-0.125 \times 1.92 \times 180^2 = -7{,}776$$
$$-2 \times 0.0962 \times 54 \times 180 = \underline{-1{,}870}$$
$$-9{,}650 \text{ kip-ft.}$$

Moment at 72 ft. from end:

$$0.0950 \times 1.92 \times 180^2 = 5{,}910$$
$$0.2064 \times 54 \times 180 = \underline{2{,}006}$$
$$7{,}920 \text{ kip-ft.}$$

Moments Due to the Cables

These will be determined after the cable area has been calculated.

3.7.3 Cable Area. In an initial preliminary design the cable area was calculated by the criterion of adequate ultimate strength at the pier similar to that in Sec. 6.2.4. However, when service load stresses were determined, it was found that this cable area was insufficient to prevent some tension in the concrete. Accordingly, it was decided instead to select the cable area by the relatively simple procedure of balancing the dead load. The service load stresses and ultimate strength are then checked in detail.

The cable profile will take the form of three parabolas having points of tangency at 22.5 ft. (i.e., span ÷ 8) from the center of the pier, as shown in Fig. 3.7. An "ideal" profile, consisting of two parabolas and a sharp bend at the pier (Fig. 3.8) would balance a uniform load along the full length of the girder. However, it is impossible to have a sharp bend in practice, and so the third parabola is fitted. The position of the tangent point, at a distance of span ÷ 8 from the pier, is sufficient to avoid excessive curvature.

If five cables per web are assumed as suggested by the preliminary design, then

Minimum distance from edge of girder to cable center = 0.610 ft.

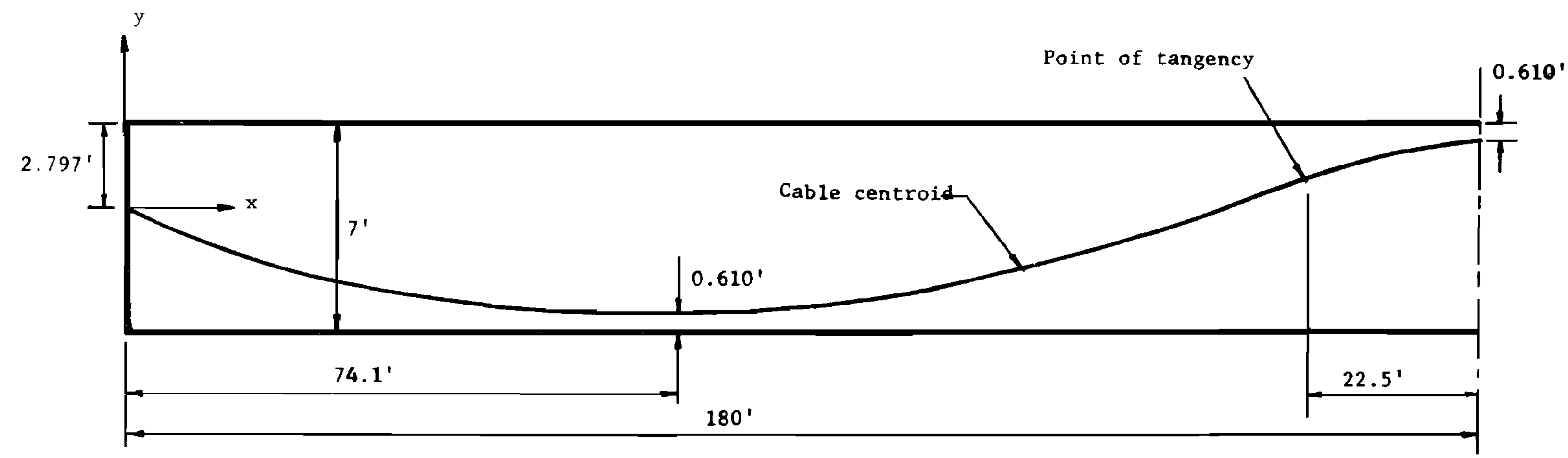


Fig. 3.7. Cable profile

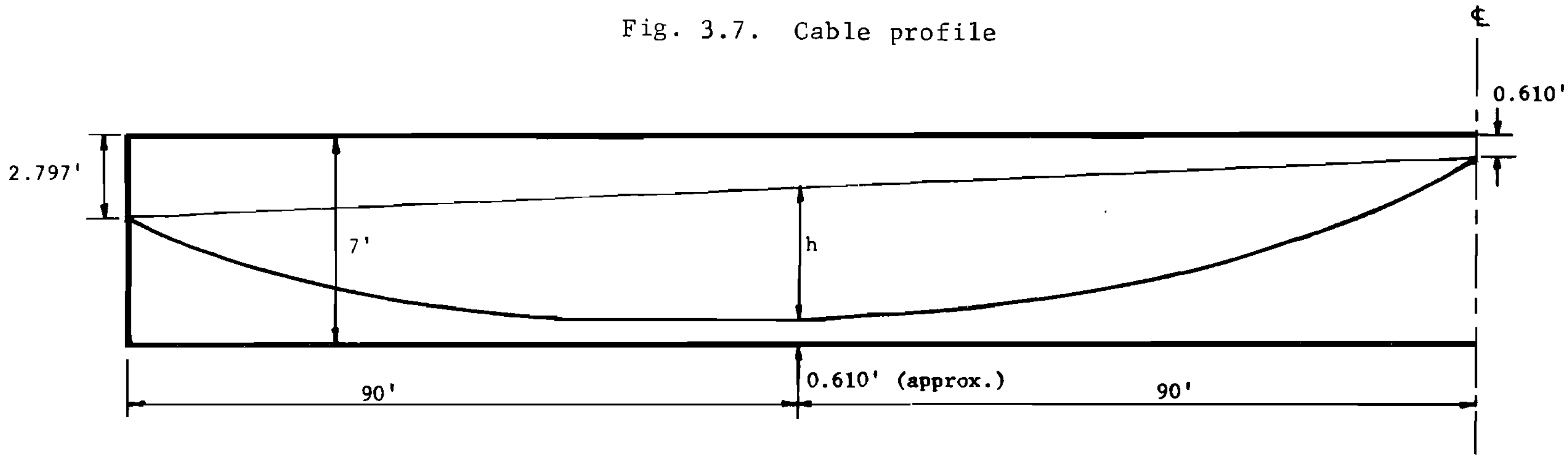


Fig. 3.8. Idealized cable profile

At the end of the girder the centroid of the cable anchors will be made to coincide with the centroid of the concrete section, to produce no bending at that point. From Table 3.1:

Distance from top of girder to centroid of end section = 2.797 ft.

The cable area will now be chosen as that required to balance the dead load of the superstructure, assuming the idealized cable profile of Fig. 3.8. The cable drape is:

$$h \approx (7 - 0.610) - (2.797 + 0.610)/2 = 4.686 \text{ ft.}$$

Cable force required to balance dead load:

$$F = wL^2/8h \qquad \text{(From Sec. 3.1.2)}$$
$$= 10.8 \times 180^2/(8 \times 4.686) = 9{,}330 \text{ kip}$$

Effective prestress: $f_{se} = 162$ ksi

Cable area required: $A_s = 9{,}330/162 = 57.6 \text{ in}^2.$

Adopt 20 cables each 20 strands (i.e., 5 cables per web) [A possible alternate would be 32 cables, each 12 strands.]

$$A_s = 61.2 \text{ in}^2. \text{ (Total - 4 webs)}$$

The cables could of course have been chosen to balance some other fraction of dead load, either greater or less than unity. All that is necessary is that the concrete stresses in the girder be satisfactory under all service loads and that the ultimate strength be adequate. The choice of the factor of unity, used here, was again suggested by a preliminary design in which the cables balanced about 0.9 of the dead load and were insufficient since some tensile stress occurred in the concrete.

The idealized cable profile was used to simplify the "load balancing" calculation. However, in all checks on the concrete stresses and the ultimate strength in the following sections, the actual profile is used.

3.7.4 Equivalent Cable Loads. In order to determine the stresses in the concrete the equivalent cable loads must be calculated for the actual tendon profile. The total cable force is given by:

<u>Cable Force</u>: $F = A_s \times f_{se}$

$= 61.2 \times 162 = 9{,}914$ kip

The equations of the parabolas forming the cable profile (Fig. 3.7) are as follows:

$x = 0$ to $x = 157.5$ ft.:

$$y = 0.00065440x^2 - 0.096980x$$

$x = 157.5$ ft. to 180 ft.:

$$y = -0.0024257x^2 + 0.87325x - 76.406$$

where y is measured upwards from the centroid of the minimum cross section (i.e., the centroid of the cables at the end). Reference 12 (Table 1A) was utilized in deriving these equations.

The equivalent transverse cable loads are obtained as follows, using the approach of Sec. 3.1.2.

$x = 0$ to $x = 157.5$ ft.:

Uniform equivalent load, $w = F(d^2y/dx^2)$

$= 9{,}914 \times (2x.00065440)$

$= 12.975$ kip/ft.

Total load $= 12.975 \times 157.5 = 2{,}043.6$ kip

$x = 157.7$ ft. to $x = 180$ ft.:

Uniform equivalent load $= 9{,}914 \times (2x-0.0024257)$

$= -48.097$ kip/ft.

Total load $= -48.097 \times 22.5 = -1{,}082.2$ kip

$x = 0$:

Concentrated load, $P = F(dy/dx)$

$= 9{,}914(-0.096980)$

$= -961.46$ kip

These transverse loads acting on the concrete section are shown in Fig. 3.9. An axial load, equal to the cable force, is also shown acting on either end.

3.7.5 <u>Cable Moments</u>. The bending moments produced by these cable loads on the concrete section are now determined. The moment at the pier is obtained by elastic analysis utilizing Fig. 21 of Ref. 12.

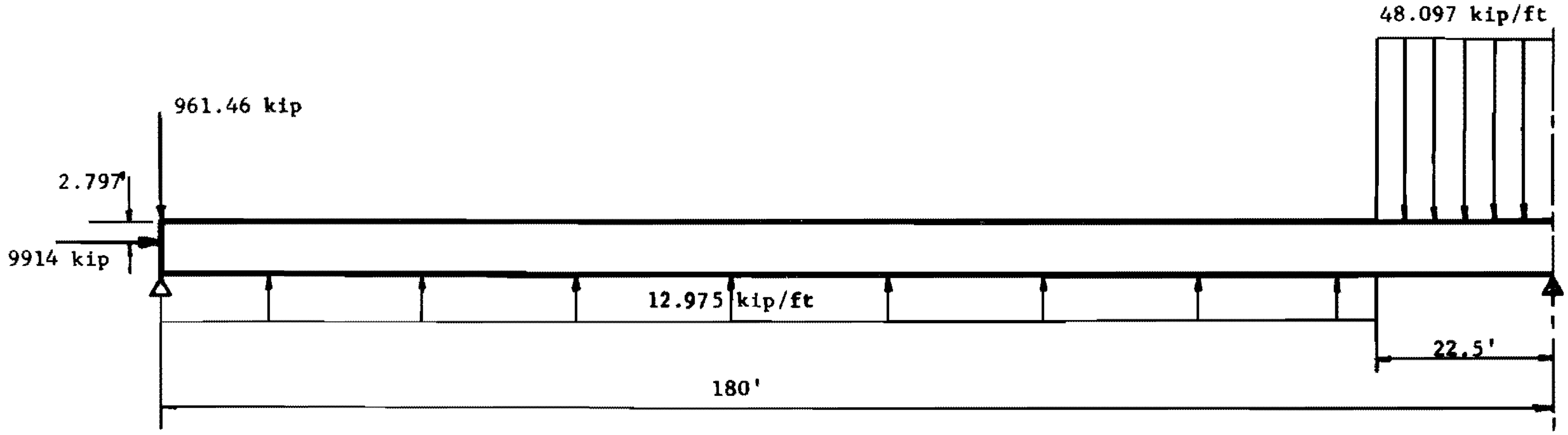


Fig. 3.9. Equivalent cable loads on superstructure

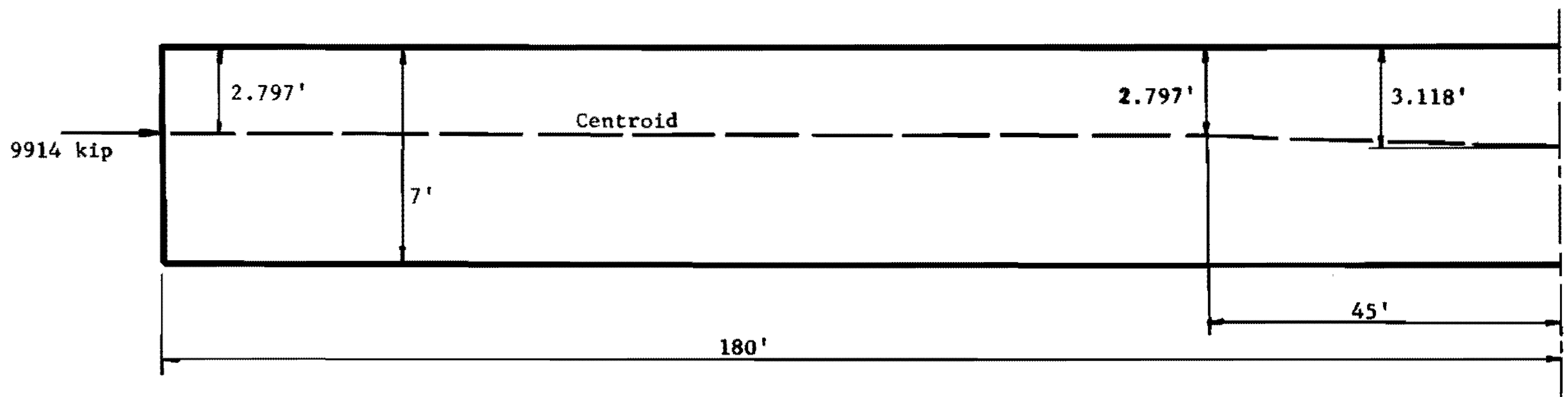


Fig. 3.10. Variation in position of section centroid

Resultant cable moment at pier (about line of action of F) = 39.761 k-ft.

This value is not quite "exact" because the effect of the axial force was ignored. The position of the section centroid varies over a part of the span, as shown in Fig. 3.10, and in that portion the axial force will have a moment about the section centroid, which will cause bending of the girder and modify the moment determined above. This small error is ignored here, but will be taken into account when a computer analysis of stresses is made using the program MUPDI.

The following moments and reactions are required in the calculation of ultimate strength and concrete stresses.

Resultant Cable Moment at Pier (about centroid of section) =

$$39{,}761 + 9{,}914(3.118 - 2.797) = 42{,}943 \text{ k-ft.}$$

Secondary Cable Moment at Pier =

Resultant moment - Primary moment =

$$42{,}943 - 9{,}914(3.118 - 0.610) = 18{,}080 \text{ k-ft.}$$

Since the secondary moment is produced by the cable-induced reactions, it will vary linearly across the span from zero at the end of the girder to the above value at the pier.

Cable-induced End Reaction =

Secondary moment at pier $\div$ span =

$$18{,}080/180 = 100.44 \text{ kip}$$

This is an upward reaction.

Resultant Cable Moment at 72 ft. from End =

$$(100.44 - 961.46) \times 72 + 12.975 \times 72^2/2 = -28{,}362 \text{ k-ft.}$$

3.7.6 Ultimate Flexural Strength at Pier. In calculating ultimate flexural strength, no moment redistribution will be assumed. The secondary cable moments are included in the calculations, because these are produced by real external reactions caused by the cables. The ultimate moment at the pier is given by

$$M_u = 1.35(\text{DL moment}) + 2.25(\text{LL + Impact moment})$$
$$+ (\text{secondary cable moment})$$
$$= 1.35(-43{,}780) + 2.25(1.164)(-9{,}650) + 18{,}080$$
$$= -59{,}100 - 25{,}270 + 18{,}080$$
$$= -66{,}300 \text{ k-ft.}$$

The capacity of the cables and the bottom slab are now determined as follows:

Effective depth: $d = 7 - 0.61 = 6.39$ ft.

Total bottom slab width: $b = 2 \times 12 = 24$ ft.

Cable area: $A_s = 61.2 \text{ in}^2$.

$$p = A_s/bd$$
$$= 61.2/(24 \times 6.39 \times 144) = 0.00277$$
$$0.5pf'_s/f'_c = 0.5 \times 0.00277 \times 270/6 = 0.0623$$

Cable stress at ultimate load, given by AASHO Sec. 1.6.9(C) for bonded members:

$$f_{su} = f'_s(1 - 0.5pf'_s/f'_c)$$
$$= 270(1 - 0.0623) = 253 \text{ ksi}$$

Bottom slab thickness: $t = 10$ in. $= 0.833$ ft.

Since $C = T$ indicates $A = 10.5$ in. $\approx t$

Moment arm $= d - 0.5a = d - 0.5t$

$$= 6.39 - 0.5(0.833) = 5.97 \text{ ft.}$$

Cable force at ultimate load: $P_u = M_u/(d - 0.5t)$

$$= 66{,}300/5.97 = 11{,}100 \text{ kip}$$

Capacity of cables $= \varphi f_{su} A_s$

$$= 0.9 \times 253 \times 61.2 = 13{,}900 \text{ kip}$$

Capacity of bottom slab $= \varphi(0.85 f'_c bt)$

$$= 0.9 \times 0.85 \times 6 \times 144 \times 24 \times 0.833 = 13{,}200 \text{ kip}$$

The capacities of both the cables and the slab are greater than P_u. Hence the ultimate flexural strength is adquate.

3.7.7 Ultimate Flexural Strength at 72 ft. from End. The ultimate moment at 72 ft. from the end is given by

$$M_u = 1.35(\text{DL moment}) + 2.25(\text{LL + Impact moment}) + (\text{Secondary moment})$$

$$= 1.35(24{,}380) + 2.25(1.164)(7{,}920) + (0.4 \times 18{,}080)$$

$$= 32{,}910 + 20{,}740 + 7{,}230$$

$$= 60{,}880 \text{ k-ft.}$$

The capacity of the cables was checked in a manner similar to that used in the preceding section and found to be adequate.

3.7.8 Service Load Stresses. The service load stresses in the concrete at the critical sections are first determined using beam theory. The cable moments used are the resultant moments on the concrete section. In the calculation of dead load stresses the properties of the two unjoined box girders are used and for live load stresses the properties of the full section are used. These properties are obtained from Table 3.1.

Concrete stresses at pier

Stress at centroid = P/A =

-9,914/(2 x 35.84 x 144) = -0.960 ksi

Stresses under dead load

Dead load moment = -43,780

Cable moment = 42,940

-840 k-ft.

Top stress = -0.960 + 840/(2 x 83.98 x 144) = -0.925 ksi

Bottom stress = -0.960 - 840/(2 x 67.47 x 144) = -1.003 kdi

Stresses under full load

LL + Impact moment = -1.164 x 9,650 = -11,230 k-ft.

Top stress = -0.925 + 11,230/(173.1 x 144) = -0.474 ksi

Bottom stress = -1.003 - 11,230/(137.5 x 144) = -1.570 ksi

Concrete stresses at 72 ft. from end

Stress at centroid = P/A =

-9,914/(2 x 32.55 x 144) = -1.057 ksi

Stresses under dead load

Dead load moment = 24,380

Cable moment = -28,360

-3,980 k-ft.

Top stress = -1.057 + 3,980/(2 x 80.58 x 144) = -0.885 ksi

Bottom stress = -1.057 - 3,980/(2 x 53.61 x 144) = -1.315 ksi

Stresses under full load

LL + Impact moment = 1.164 x 7,920 = 9,220 k-ft.

Top stress = -0.885 - 9,220/(166 x 144) = -1.270 ksi

Bottom stress = -1.315 + 9,220/(107.8 x 144) = -0.721 ksi

All of the stresses calculated are within the acceptable 2.4 ksi compression and 0 ksi tension limits. The stresses at all sections and under various critical loadings will also be checked by computer analyses. However, before that the shear strength will be investigated to determine web adequacy.

3.7.9 Shear. The maximum shear forces on the full width of the superstructure are calculated using the influence coefficients obtained from Ref. 5.

Shear force at pier:

Dead load:	0.625 x 10.74 x 180 =	1,208
	(1,261/1,280) x (11.7 - 10.7) x 45/2 =	22
		1,230 kip
Live Load:	4[(0.625 x 0.640 x 180) + 26] =	392 kip

The 25 percent reduction in live load intensity for loading on four lanes is not used here, because it was found in the computer analysis of a double box girder bridge (to be described in the next chapter) that, if this reduction is made, the critical shear loading case will then be live load on two lanes only.

Cable-induced shear: -100 kip

This is equal to the external end reaction induced by the cables, as calculated in Sec. 3.7.5.

<u>Shear force at end of bridge</u>

Dead load:	$0.375 \times 10.74 \times 180 =$		725
	$(19/1{,}280) \times (11.7 - 10.7) \times 45/2$	=	0
			725 kip
Live load:	$4[(0.4375 \times 0.640 \times 180) + 26]$	=	306 kip
Cable-induced shear		=	100 kip

<u>Ultimate shear at pier</u>

$$V_u = 1.35(\text{DL shear}) + 2.25(\text{LL + Impact shear}) + (\text{cable shear})$$
$$= 1.35(1{,}230) + 2.25(1.164)(392) - 100$$
$$= 1{,}660 + 1{,}027 - 100$$
$$= 2{,}587 \text{ kip}$$

The shear capacity of the webs at this section will now be determined. The concrete stresses f_{pc} and $(f_{pe} - f_d)$ are obtained from Sec. 3.7.8.

Compressive stress at centroid: $f_{pc} = 0.960$ ksi

Compressive stress at top of girder under dead load and prestress: $(f_{pe} - f_d) = 0.925$ ksi

$$6\sqrt{f'_c} = 6\sqrt{6{,}000}/1{,}000 = 0.465 \text{ ksi}$$

Top section modulus (from Table 3.1):

$$(I/y) = 173.1 \times 12^3 \text{ in}^3$$

Cracking Moment:
$$M_{cr} = (I/y)[6\sqrt{f'_c} + (f_{pe} - f_d)]$$
$$= 173.1 \times 12^3(0.465 + 0.925)/12$$
$$= 34{,}600 \text{ k-ft.}$$

Live load shear/moment ratio:

$$(V_\ell/M_{max}) = 0.75 \times 392/9{,}650 = 0.0304$$

$$\frac{V_\ell M_{cr}}{M_{max}} = (0.0304)(34{,}600) = 1052$$

Effective depth: $d = (7 - 0.61) \times 12 = 76.7$ in.

Total web width: $b' = 4 \times 13 = 52$ in.

Vertical component of cable force: $V_p = 0$

since the cables are horizontal at the pier.

The shear carried by the concrete, $V_c = v_c b' d$ is the lesser of $V_{ci} = v_{ci} b' d$ and $V_{cw} = v_{cw} b' d$, where

$$V_{ci} = (0.6\sqrt{f'_c})b'd + V_d + V_\ell M_{cr}/M_{max}$$
$$= (0.0465 \times 52 \times 76.7) + (1230 - 100) + 1052$$
$$= 185 + 1130 + 1052$$
$$= 2{,}367 \text{ kip}$$

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3f_{pc})b'd + V_p$$
$$= (0.271 + 0.288) \times 52 \times 76.7 + 0$$
$$= 2{,}230 \text{ kip}$$

Hence, $V_c = 2{,}230$ kip.

The AASHO Specifications allow the use of the above expressions for the shear capacity of the concrete, taken from Eqs. 11-11 and 11-12 of the ACI Standard 318-71. These expressions were actually developed for prestressed I-sections, but are considered also applicable for box girders, when the webs are loaded approximately uniformly as in this calculation.

In the expression for V_{ci} the cable-induced shear must be included with the dead load shear.

The vertical shear reinforcement required in the webs of the box girders will now be determined.

Shear reinforcement required: Using Eq. 11-13 of ACI 318-71 as permitted by AASHO Specifications

$$A_v = \frac{(v_u - v_c)b's}{f_y} = \frac{(V_u - \varphi V_c)b's}{\varphi b'd\, f_y}$$
$$= (V_u - \varphi V_c)s/(\varphi d f_y)$$
$$= (2{,}587 - 0.85 \times 2{,}230) \times 12/(0.85 \times 76.7 \times 40)$$
$$= 3.18 \text{ in.}^2 \text{ per ft. length of bridge}$$

This exceeds the minimum $A_v = 100b's/f_y = 1.56$ in.^{2} per ft. required by AASHO.

The vertical web reinforcement provided for transverse bending moment is #7 at 13 in. in both faces of each web. The total area (4 webs) is

$$A_v = 4.43 \text{ in.}^2/\text{ft. length}$$

In this particular case it is not considered necessary to add the reinforcement requirements for maximum shear and maximum transverse moment, because the amount required for shear falls off rapidly away from the pier, whereas the diaphragm over the support ensures that the transverse moment near the pier will be small. So the web reinforcement will remain as shown in Fig. 3.16.

The shear strength at the end of the bridge was also investigated. It was found that the concrete webs have adequate strength at that section without utilizing additional reinforcement.

3.7.10 Computer Analysis. With the basic proportions, reinforcement, and tendons designed, the completed superstructure was analyzed by computer to determine the stresses in the concrete section under dead load and under various live load patterns. The MUPDI program of A. Scordelis, described in Ref. 23, was used. This program analyzes the structure using elastic folded plate theory.

For dead load (including the cable forces) one box girder (i.e., half of the superstructure cross section) was analyzed, but for live load the full cross section was used. In this way the actual behavior of the structure is best represented.

The effect of the prestressing cables was simulated by treating their equivalent transverse loads and axial forces as applied loads in the webs of the girder. Thus, the cable forces for input consist of the distributed lateral loads and the axial forces shown in Fig. 3.9. In addition, the effect of the change in the position of the girder centroid over the 45-ft. distance either side of the pier (Fig. 3.10), was taken into account. In that region the axial force has a moment about the centroid, which causes additional bending. This effect was treated as an applied moment varying linearly from a value of (cable force) x (shift in centroid position) at the pier to zero at a distance of 45 ft. from the pier.

The following two simplifications were made, so that analysis with the MUPDI program would be feasible.

(a) The MUPDI program cannot handle the variation in the thickness of the bottom slab. Separate analyses were made with two different idealized sections, one having the properties of the maximum section (Fig. 3.11), the other the minimum section (Fig. 3.12). The first is used to obtain the stresses near the pier and the second to obtain the stresses elsewhere throughout the superstructure.

(b) With this program, all concentrated loads must be applied at the node points of the idealized section. The moment of the end axial force about the centroid of the real section was calculated and this force and moment replaced by a pair of forces at the node points at the top and bottom of the web of the idealized section. This pair of forces was determined so as to give the same resultant force and moment about the centroid of the idealized section as occur in the real section. The applied moment near the pier was also replaced by a pair of forces in the same way.

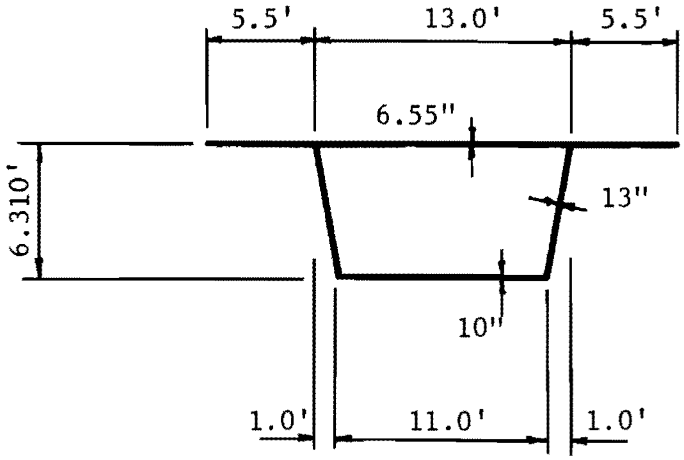
The following live load cases were investigated:

(a) Full lane loads on all lanes of one span

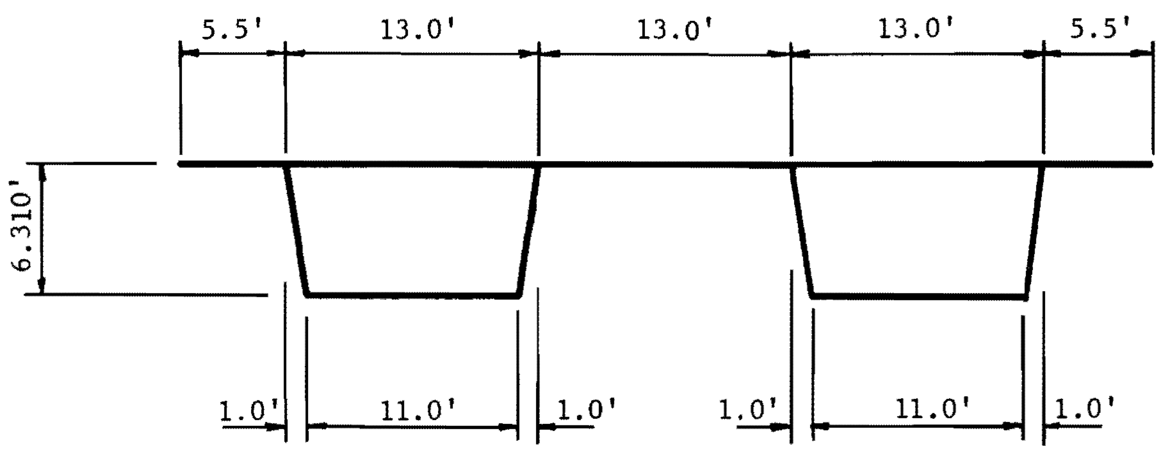
(b) Lane load on one side (2 lanes) of one span

(c) Full lane loads on all lanes of two spans

Examination of the computer output revealed that under dead load and also under full dead and live load (with impact) all stresses in the concrete were within the permissible limits. Stress distributions across the section at the pier and at a distance of 70 ft. from the end of the bridge are shown in Figs. 3.13 and 3.14. It can be seen that the stresses vary across the section at the pier because of shear lag, but are almost uniform over each slab for the section 70 ft. from the end, where the shear is small. Comparison of these stresses with those calculated using beam theory in Sec. 3.7.8 shows generally good agreement. The increased local stress at the piers due to shear lag results in an approximately 25 percent increase over beam theory calculations. This indicates a need for conservatism in design based on beam theory analyses.

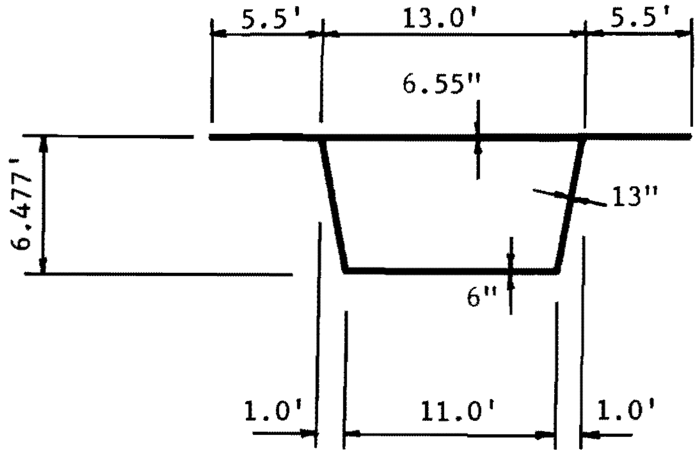


(a) Section for dead load

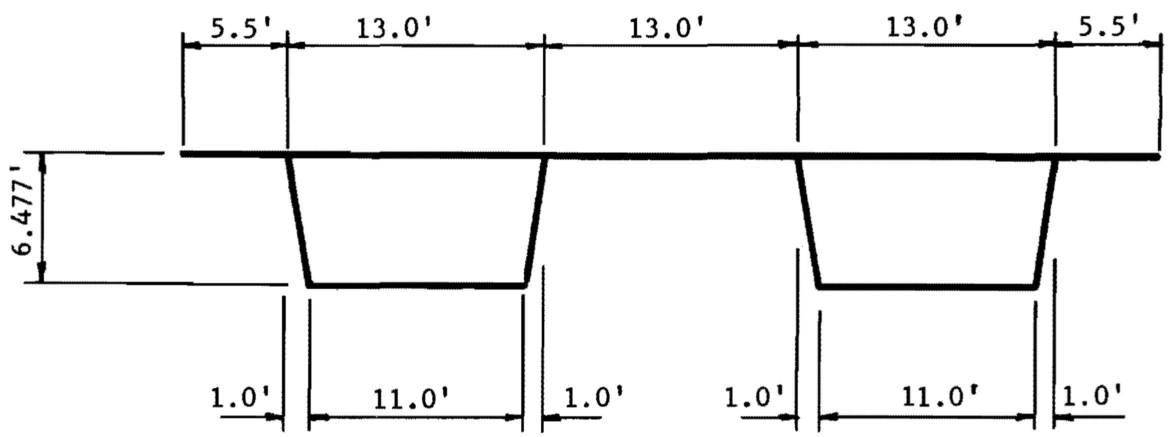


(b) Section for live load

Fig. 3.11. Idealized maximum sections

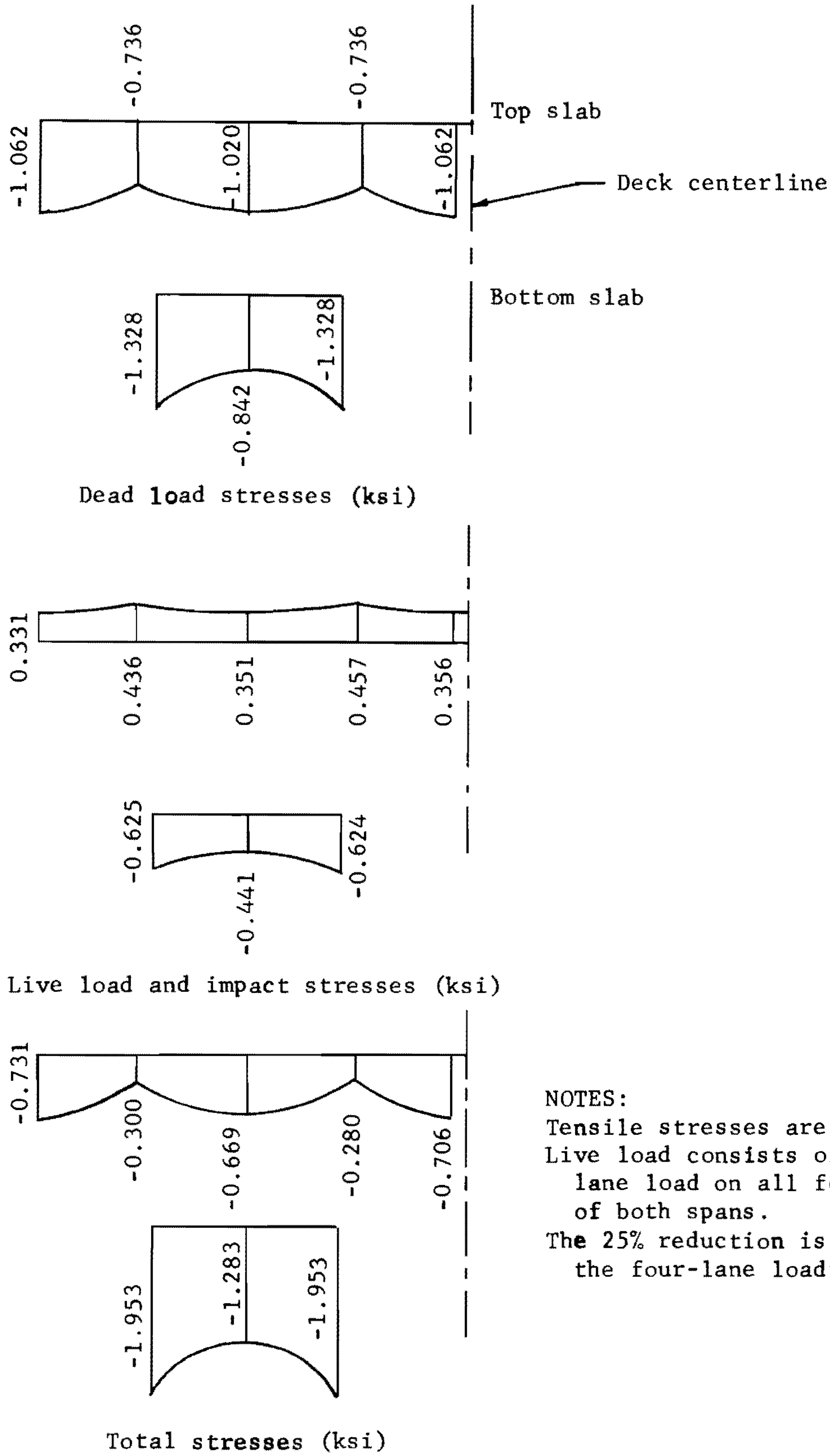


(a) Section for dead load



(b) Section for live load

Fig. 3.12. Idealized minimum sections



NOTES:
Tensile stresses are positive.
Live load consists of AASHO lane load on all four lanes of both spans.
The 25% reduction is made for the four-lane loading.

Fig. 3.13. MUPDI analysis of stress distributions in two-span bridge at pier

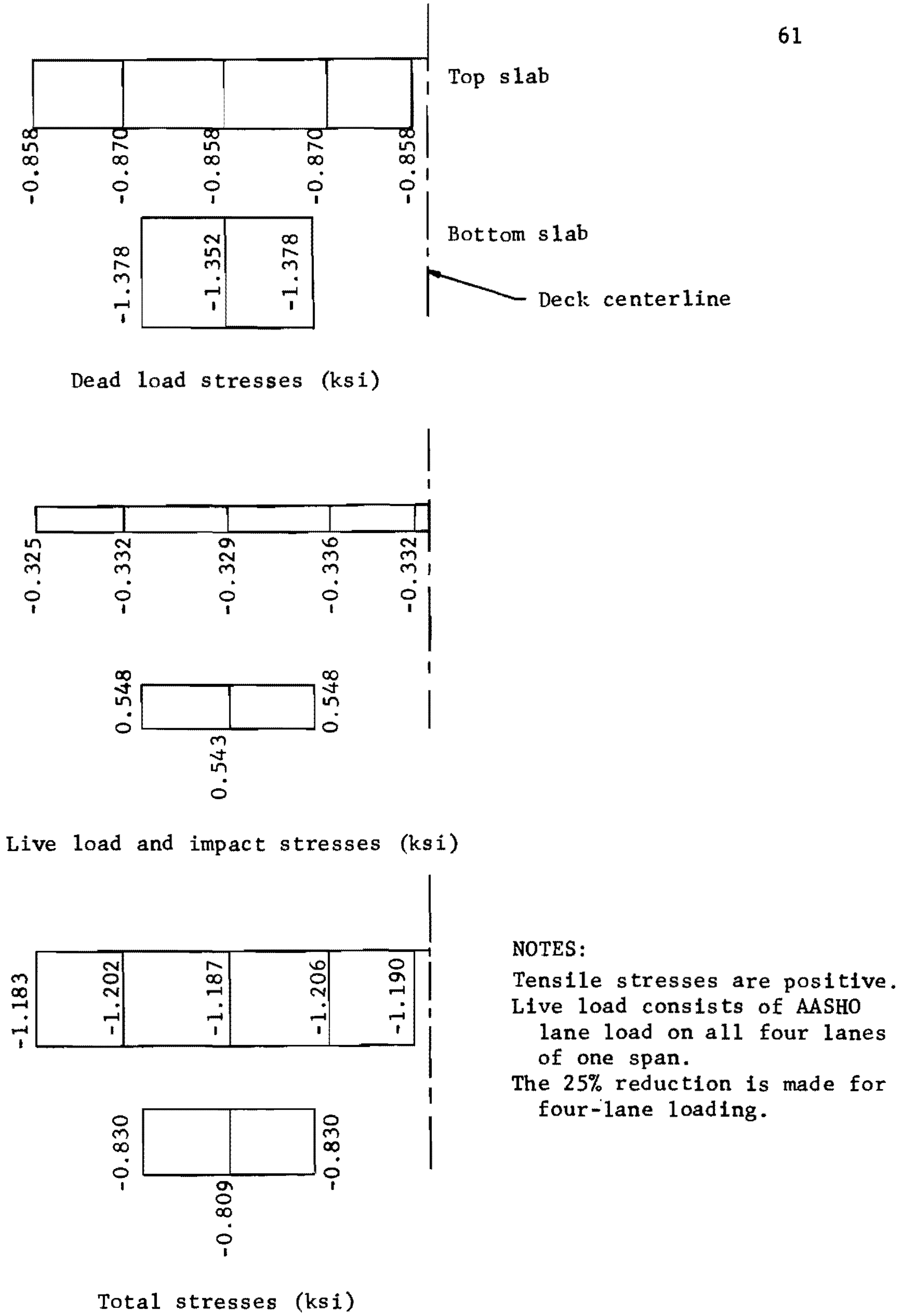


Fig. 3.14. MUPDI analysis of stress distributions in two-span bridge at 70 ft. from end support

The stresses in the cables and hence in the concrete alter with time under the effects of creep and shrinkage. Creep tends to reduce the effective stress in the cables. However, the value of the effective stress used in the calculations, namely $f_{se} = 0.6f'_s$, allows for this loss. The effect of shrinkage is similar to that of creep, but will probably be small, because much of the shrinkage occurs in the precast units before they are erected. This minimization of creep and shrinkage effects is an important advantage of precast segmental construction.

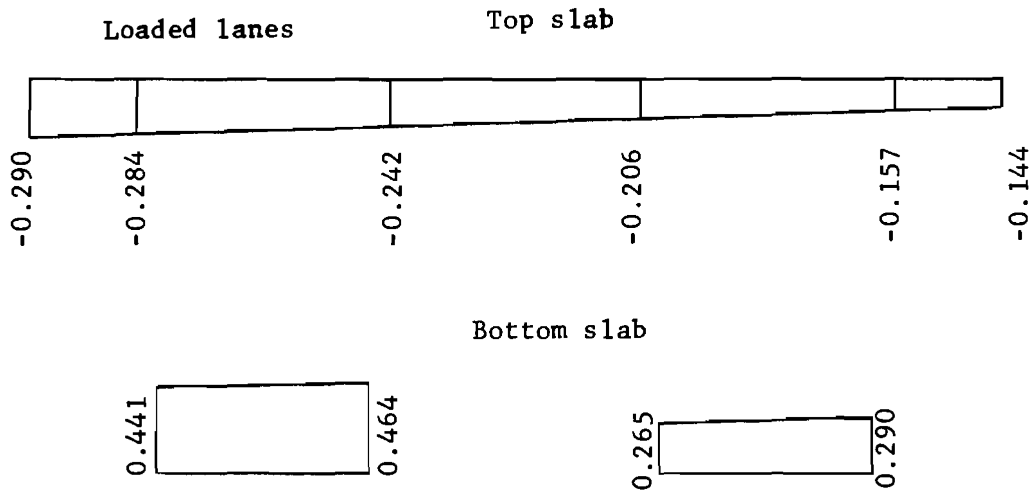
Comparison of the results obtained for (a) full live load on one span and (b) live load on one side of one span showed that the full (4 lane) loading is critical (for stresses in the positive moment region), even though a 25 percent reduction in load intensity is made in this case. When the load is uneven, the superstructure tends to even out the stresses to some extent. The stress distribution for case (b) is shown in Fig. 3.15.

Deflections are also given in the computer output. The maximum deflections occur at 80 ft. from the end of the bridge and are as follows:

Deflection under dead load	-0.481 in.
Deflection under live load (with impact) on one span	1.128 in.
Total deflection	0.647 in.

The effect of creep on the concrete modulus is not included. If creep, shrinkage, or temperature seem significant, the MUPDI analysis could be changed to examine these effects. The deflection/span ratio under live load is approximately 1/2000. This is well within the limit of 1/300 which is normally considered acceptable.

The results of the computer analysis indicate that no alterations are required to the girder section or the cables. Since the units meet all service load stress conditions comfortably and have approximately 25 percent more ultimate moment capacity than required, another iteration in design could be attempted. However, as the previous trial did not meet tensile stress limits with approximately 10 percent less tendon area, little is to be gained from further refinement. Full details of the section, including cables and reinforcement, are shown in Fig. 3.16.



NOTES: Stresses are in ksi. Tensile stresses are positive. Live load consists of AASHO lane load on two lanes (one side) of one span. Impact is included.

Fig. 3.15. MUPDI analysis live load stress distribution in two-span bridge at 70 ft. from end support under live load on one side only

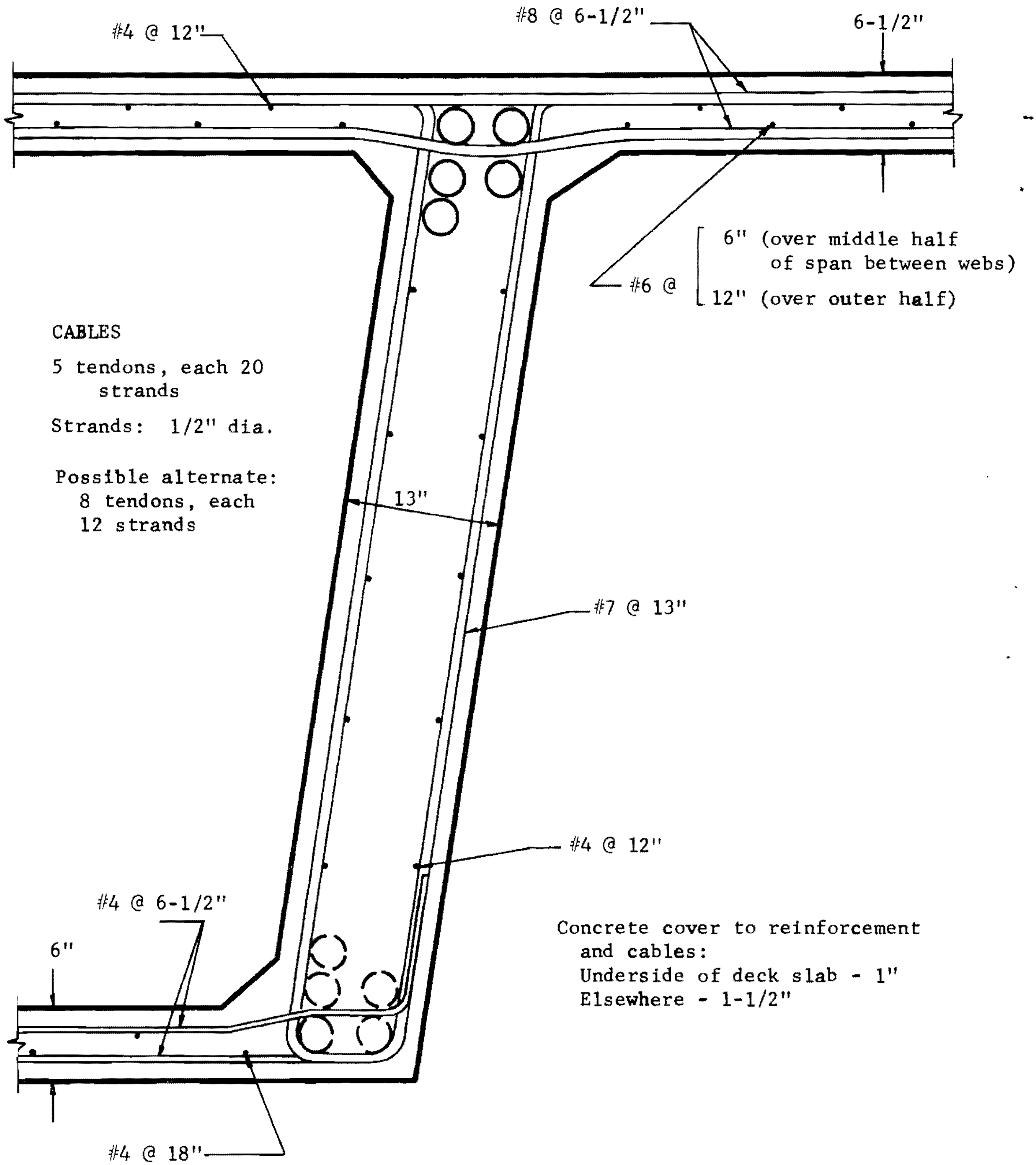


Fig. 3.16. Cable and reinforcement details

3.7.11 Friction Losses. The friction losses in the cables were calculated using the SIMPLA2 program developed by R. Brown. It was found that the assumed effective prestress of $0.6f'_s$ was realistic with normal stressing if the conduits consist of rigid thin wall metal tubing.

3.7.12 Diaphragms. Diaphragms inside the box sections are required at each of the bearings to maintain the shape of the cross section and to provide concrete bearing capacity. A thickness of 6 in. is chosen as a practical minimum. This provides adequate bearing capacity. No intermediate diaphragms are indicated as necessary from the results of the MUPDI analysis.

3.7.13 Prestressing System Details. When the actual post-tensioning system is selected for the project (usually following selection of a contractor), the prestressing system details will have to be closely examined. Anchorage locations, dimensions, and auxiliary reinforcement to control bursting, spalling, and splitting stresses should be checked by the designer.

3.8 Summary of Design Procedure

The principal stages of design are as follows:

(a) An approximate cross section shape is chosen.--This can be based on the result of an optimization study, as described in Chapter 6. Alternatively, a preliminary design may be carried out.

(b) The cross section is designed in detail.--The deck slab thickness and reinforcement are determined by wheel load moments. The web thickness must be sufficient to accommodate the cables and their anchors. A preliminary shear check is advisable to ensure adequate web thickness is provided.

(c) The cables are designed to balance the dead load.--To calculate the cable area, an ideal cable profile consisting of two parabolas is assumed. The actual profile consists of three parabolas, fitted to avoid excessive curvature at the pier.

(d) The ultimate strength of the superstructure is checked.--The capacities of the cables and the bottom slab are checked at the pier. The capacity of the cables is also checked at the section of maximum positive moment. If necessary the cross section or the cables are revised.

(e) The concrete service load stresses are calculated from beam theory.--The stresses under dead and live load are checked at both critical sections. The cross section or cables are revised if necessary.

(f) The ultimate shear strength is checked.--The ultimate shear force, the capacity of the webs, and the reinforcement required are calculatd at all the critical sections. Web thickness is adjusted upward if necessary.

(g) The superstructure is analyzed with the MUPDI program.--The stresses are determined under dead load and under various live loads. If necessary the cross section or the cables are revised and the analysis repeated.

3.9 Other Examples of Bridges Constructed on Falsework

Other cases of bridges constructed on falsework, requiring variations in the design procedure adopted in the example chosen, are considered briefly.

3.9.1 Multi-Cell Box Girder. An alternative cross section for the bridge considered is a three-cell box girder cast in full width sections.

The design procedure for this case is almost identical with that already outlined. However, a different program must be prepared or manual computations used to compute the cross section properties. A preliminary design and optimization program for a multi-cell box girder bridge is included in Chapter 6.

The advantage of a multi-cell box is that the cast-in-place longitudinal joint is not required. The disadvantage is that the basic units are twice as heavy and, therefore, more difficult to transport and erect. Also, the lower flange tends to have excess capacity and results in longer cantilever overhangs on the top flange.

3.9.2 Bridges with More Than Two Spans. With bridges having three or four spans, it may still be possible to use full length draped cables. Friction losses constitute the main factor limiting the feasible length of cables.

If full length cables are used, the design procedure can be essentially the same as that for two-span bridges. The number of critical sections for investigation of service load stresses and ultimate strength is, of course, greater. If all spans are equal, the load that is balanced by the cables in the outer spans will be less than that balanced in the inner spans, unless the drape is adjusted.

If the cables do not run the full length of the bridge, the cable pattern and, hence, the determination of the cable loads will become more complex. Apart from this, it should be possible to follow a generally similar design procedure. Concrete stresses must also be checked during construction, i.e., as each separate group of cables is tensioned.

3.9.3 Continuous Viaducts. In viaducts, comprising a large number of equal spans, the cables will generally extend across one or two spans. The strength of the superstructure must be checked during the different stages of construction of a span, especially as each set of cables is tensioned.

The determination of ultimate strength and service load stresses in the completed structure can be generally similar to that for the two-span example. The MUPDI program can handle a maximum of five spans, but this should provide an adequate approximation to the stresses in the multi-span superstructure.

Provision must be made for expansion of the superstructure and careful consideration given to location of joints and to design of the piers.

CHAPTER 4

DESIGN PROCEDURE FOR BRIDGES CONSTRUCTED IN CANTILEVER

The design of a bridge to be constructed in cantilever is considerably more complex than that of a bridge constructed on falsework. The bridge must be designed for each stage of the segmental construction, as well as in its completed state. Besides, unlike construction on falsework, where full-length draped cables can be used, cantilever construction requires a large number of cables of various lengths anchored at various points along the girder.

The design procedure in this chapter is developed using the particular example of a continuous bridge with spans 100 ft.-200 ft.-100 ft. The basic steps in the design of the superstructure are as follows:

(a) An approximate cross section is chosen on the basis of a preliminary design or an optimization study.

(b) The cross section is designed in detail.

(c) The top cables are designed for cantilever erection.

(d) The bottom cables are designed for ultimate load on the completed structure.

(e) The concrete service load stresses are calculated from beam theory.

(f) The ultimate shear strength is checked.

(g) The completed superstructure is analyzed using the computer program MUPDI to check for shear lag, warping, and unsymmetrical loading effects, and to verify the design.

(h) The final design is analyzed using the computer program SIMPLA2 to check stresses, deflections, and forces at each construction stage to verify the design and construction plan.

The same procedure can be directly applied to other three-span bridges with span ratios 1:2:1 but with varying lengths. Extension of the method to other span ratios and to bridges of more than three spans will be discussed.

The equivalent cable load concept, developed in Chapter 3, will again be extensively used. However, first this concept will be extended to apply to a system with cables of varying length as used in a cantilever constructed bridge.

4.1 Equivalent Load of a Cable System

4.1.1 Moment Balancing. In a continuous girder prestressed by a system of cables of varying length, it is often desirable or convenient to design the cable forces such that the cable moment at each point will counteract or "balance" the bending moment due to some applied load. By analogy with load balancing, this design technique will be called moment balancing.

As an example, consider the problem of balancing a uniform load on the three-span bridge shown in Fig. 4.1. This can be done by setting the primary bending moment of the cables about the centroid equal and opposite to the elastic bending moment due to the uniform load w, at each point along the girder [Fig. 4.1(a)]. With cantilever construction the cables are generally straight over the greater part of their length and the eccentricity from the centroid is approximately constant. Variation in cable moment is achieved by varying the number of cables of each size from section to section. The resultant of the bending moments due to the applied load and the cables will be zero at all points. The deflection, consequently, will also be zero.

The cable (primary) moment diagram shown in Fig. 4.1(a), however, is not the only one that will achieve balance. There is no need for the primary cable moment diagram to correspond to an elastic distribution; any moment diagram that is statically compatible with the applied load will do. Two other suitable primary cable moment diagrams are shown in Figs. 4.1(b) and 4.1(c). In 4.1(b) the moment diagram is the same as in a simply supported system, and in 4.1(c) it is the same as in a cantilever system, i.e., the end reactions are zero. However, in all cases under the applied load w, there will be zero resultant moment at all points in the concrete section. There will be zero deflection also and no interaction between the different spans.

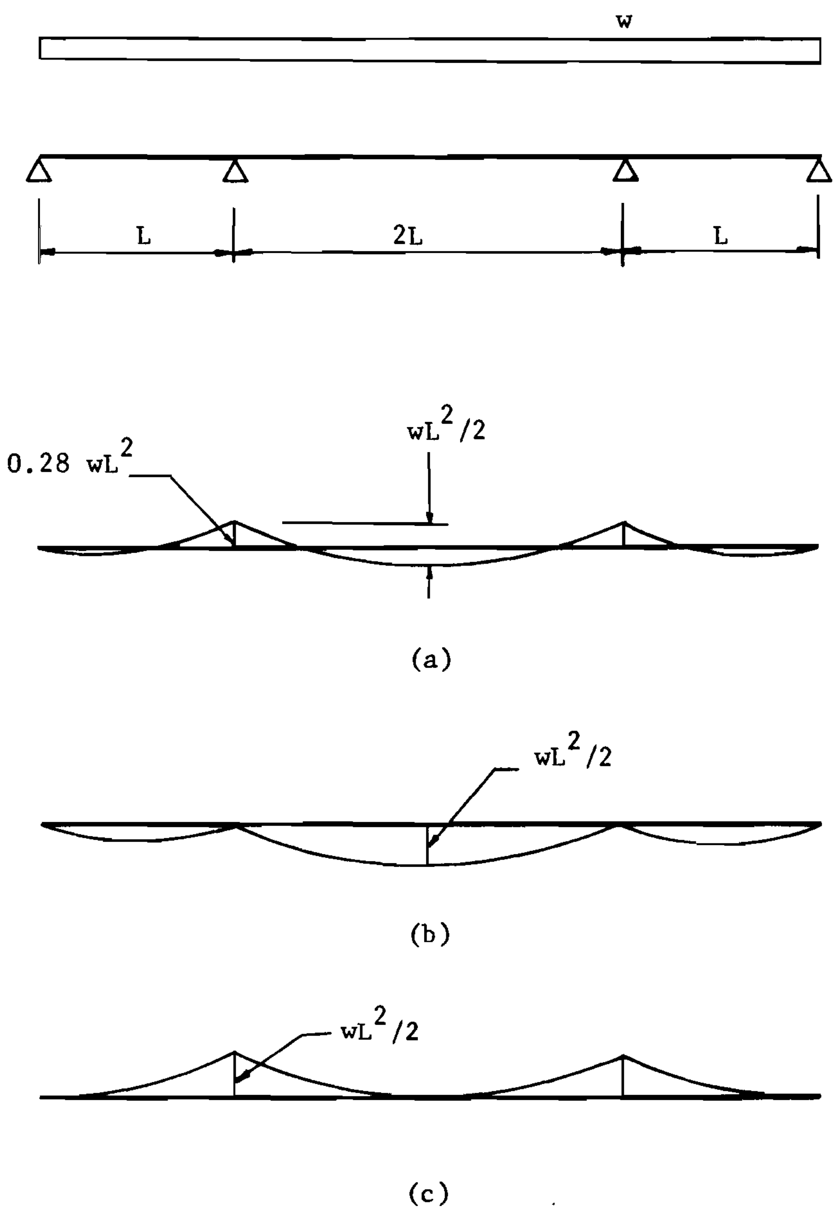


Fig. 4.1. Moment balancing for a uniform load

If the cable systems of Figs. 4.1(a), (b), and (c) are further examined, it will be found that the external reactions at the supports are different in each case. Also, it may be noted that the primary and secondary cable moments differ in each case but the resultant cable moments are the same.

One further important distinction between load balancing with a single draped cable and moment balancing with a system of cables should be pointed out. With a draped cable the shears, as well as the bending moments, can be balanced. However, with a system of cables of varying length it is generally possible to balance the bending moments only but not the shears, because the cables normally run horizontally except near their anchorages.

4.1.2 Equivalent Load. The equivalent load of a cable system will be defined as the applied load which produces the same resultant bending moments on the concrete girder section as are produced by the cable system.

The equivalent load is equal and opposite to the load that would be balanced by the cable system. Consequently, in view of the preceding section, the following principle provides one way of determining the equivalent load of a cable system.

> If a system of cables produces a primary bending moment diagram that can statically balance some applied load, then the negative of that load is the equivalent load of the system.

Thus, when the cable system is designed to directly balance some specific applied loading, the equivalent load will be known. However, sometimes a cable system will be designed by some other criterion, and the above principle cannot be readily used to obtain the equivalent load. In such cases, the following alternative approach will be useful.

> Another way to determine the equivalent load of the system is to take each cable separately and calculate its individual equivalent load. Then combine all the separate loads for the different cables along the length of the girder to give the equivalent load of the system.

As an example, consider a 200-ft. span of a continuous girder, as shown in Fig. 4.2(a), which has a system of ten cables in the bottom flange, all of equal size and stopping off at 10-ft. intervals starting 5 ft. from the span center. The cable force is the same in all cables and the eccentricity is constant. The equivalent load for each individual cable consists of a pair of concentrated moments, m(k-ft.), one at each anchor, where

$$m = (\text{cable force}) \times (\text{eccentricity})$$

Summing for all cables, the equivalent load will be a set of concentrated moments, m, at 10-ft. intervals, as shown in Fig. 4.2(b).

The girder could now be analyzed to obtain the resultant cable moments. However, a simplification can be made that will greatly facilitate the design procedure. Each concentrated moment can be replaced by the statically equivalent couple of forces, P(kip), as shown in Fig. 4.2(c), where

$$P = m/10$$

Now it can be seen that these forces all cancel out, except at midspan, where there is a resultant upward force of 2P, and at the supports where there is a downward force P[Fig. 4.2(d)]. So, ignoring the forces at the supports, which are equivalent reactions, we have an equivalent load of 2P at midspan.

In a simple case like this, the equivalent load can also be obtained using the first method and the two results can be compared. The primary moment diagram for the cable system is triangular (smoothing out the steps) with a maximum moment

$$M = 10m = 100P$$

Such a moment diagram will statically balance a downward concentrated load Q (kip) at midspan, where

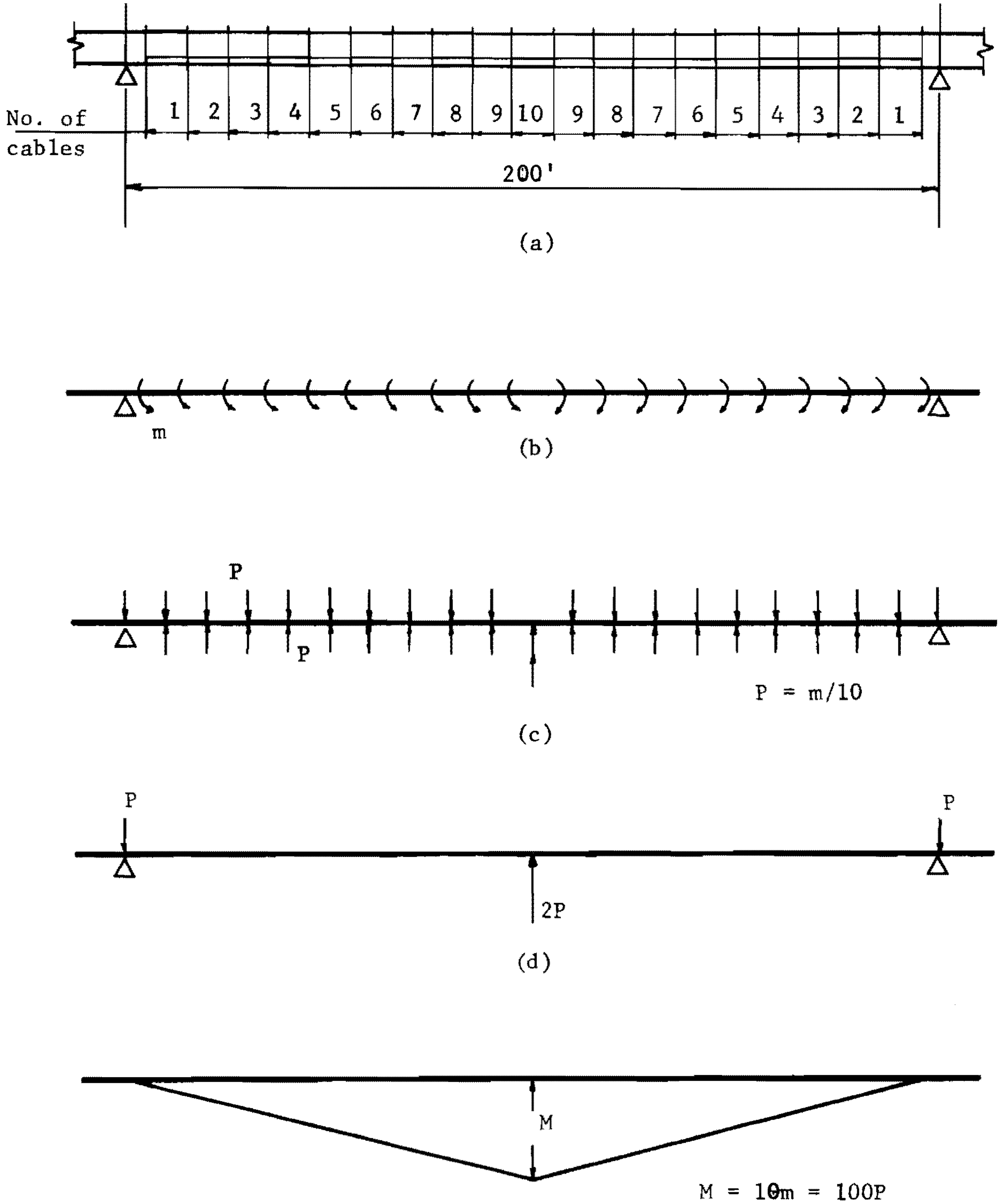


Fig. 4.2. Equivalent load of a cable system

$$M = QL/4$$
$$\approx Q \times 200/4$$

i.e.,
$$Q = M/50$$
$$\approx 2P$$

Hence, again there is obtained an equivalent load of 2P acting upwards at midspan.

4.2 Design Example - Three-Span Bridge

The design criteria will be developed using as a design example the three-span bridge shown in Fig. 4.3. The precast segments are 10-ft. long, except for the two end segments which are 5 ft. and the central closing segment which is made 1 ft. short to allow for a concrete joint. The cross section, shown in Fig. 4.4, consists of a pair of single-cell boxes connected by the deck slab. Each box is cast separately and the 2-ft. wide longitudinal cast-in-place joint connecting them is not made until the two separate girders have been erected and fully tensioned. The completed superstructure is supported on simple neoprene pads on the piers. Diaphragms are provided inside the box section at all supports.

This particular example was chosen to meet the requirements for an actual prototype, envisaged by the Texas Highway Department, to cross the Gulf Intracoastal Waterway at Corpus Christi, Texas. This example was used as the preliminary design of the structure. Details of the final design are shown in Appendix A.

The overall width of the superstructure, 56 ft., allows a four lane roadway width of 54 ft. The 8-ft. depth was chosen to give a conservative span/depth ratio of 25. The other dimensions shown in Fig. 4.4 are trial dimensions for the design and are selected on the basis of a preliminary design similar to that described in Chapter 7, which was carried out to determine approximate dimensions of the cross section and approximate cable layouts and quantities.

Epoxy resin jointing is used between the segments.

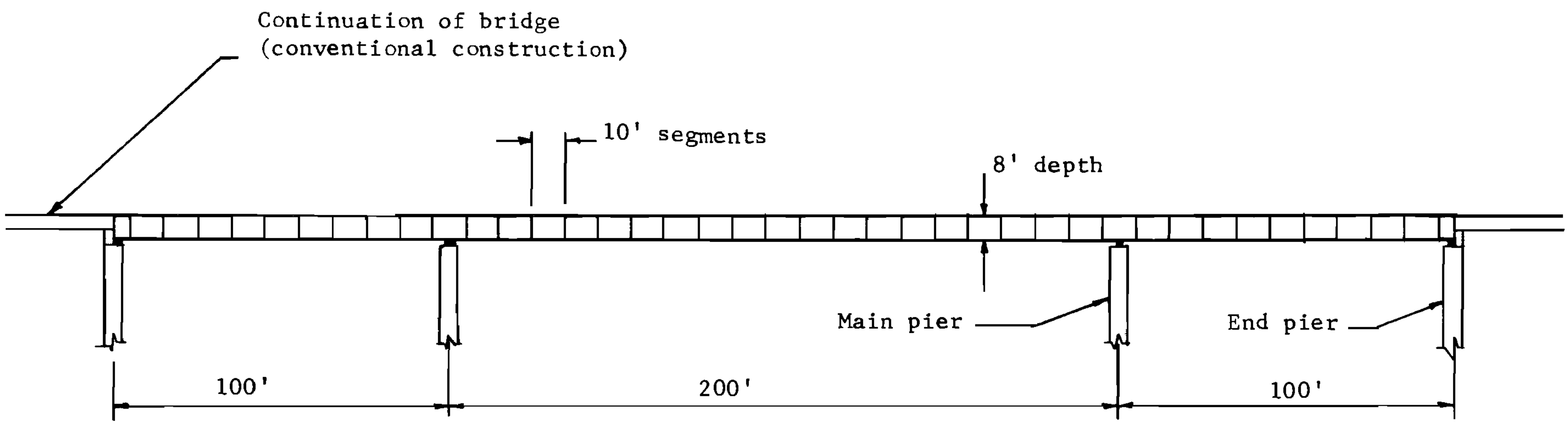
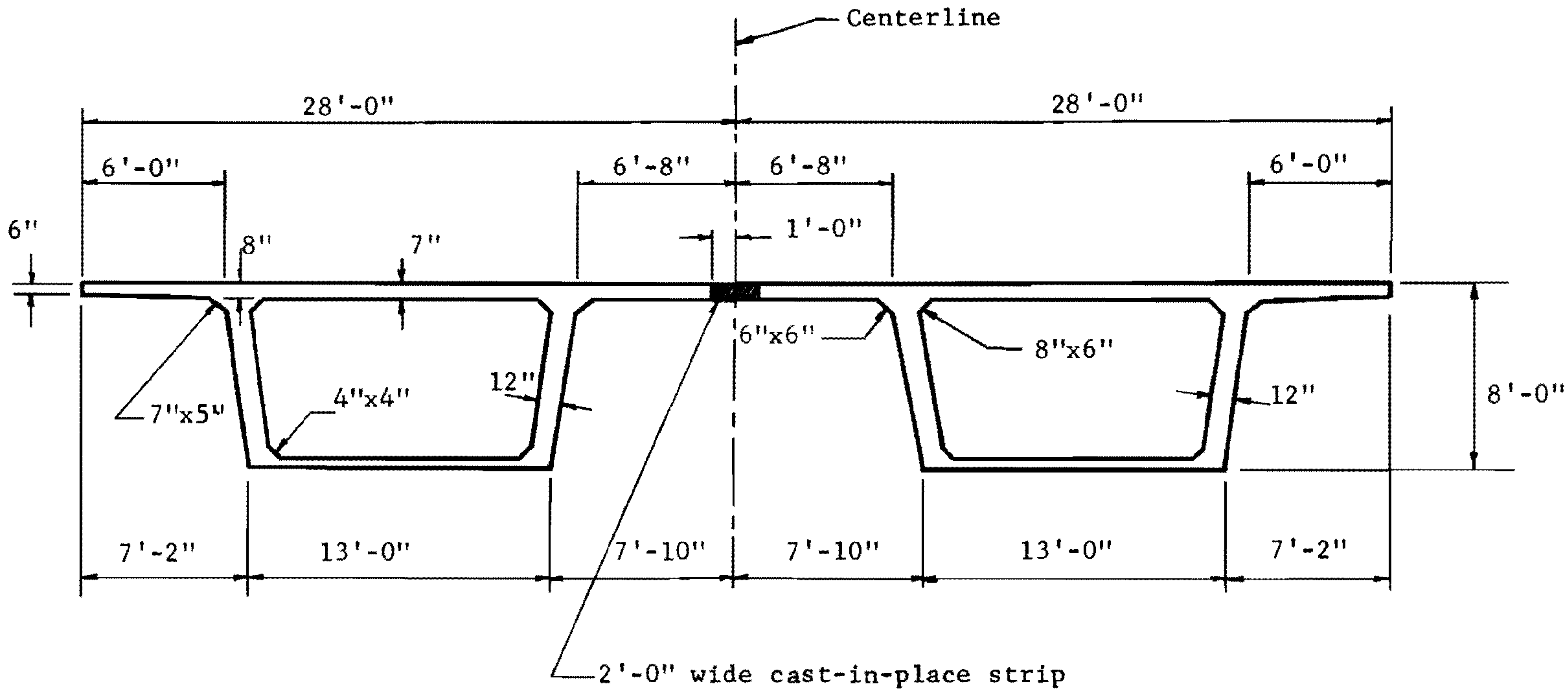


Fig. 4.3. Elevation of three-span bridge



NOTE: Bottom slab thickness varies from 10" at pier to 6" at 25'-0" from pier center.

Fig. 4.4. Cross section of superstructure

4.3 Construction Procedure

The stages of construction are as follows:

The first 10-ft. segment is set in place directly above the main pier, as in Fig. 4.5. Temporary support blocks are used instead of the neoprene pads, to provide rigidity during construction. Temporary holding down bolts are also used to keep the segment fixed in place. To provide fixity against any unbalanced moments on the superstructure the pier must have a high moment capacity or temporary struts supported on the pier foundation can be set at either end of this first segment.

The superstructure is erected by the symmetric cantilever procedure, to a distance of 95 ft. either side of the pier [Fig. 4.6(a)]. Top cables are inserted and tensioned as each pair of segments is set in place and jointed. This procedure is carried out for both halves of the cross section and for both of the main piers.

The final 5 ft. segment is placed in each side span [Fig. 4.6(b)]. The bottom cables in this span are inserted and tensioned, thereby completing this span. Jacks are set on the end piers under the ends of the girder.

The closing segment is placed at the center of the bridge [Fig. 4.6(c)]. This segment is made 9 ft. long (or two 4 ft., 6 in. segments are used) to allow a 6 in. cast-in-place concrete joint at either end. The girder is required to have zero slope at this point for continuity. This could be assured either by initially setting the pier segment at an appropriate slope or else by camber.

The bottom cables in the main span are then inserted and tensioned, starting with the longest cables. The temporary struts or bolt connections at the piers can be removed after the first (longest) set of cables have been tensioned [Fig. 4.6(d)]. The jacks at the ends of the bridge will provide restraint. At some stage during the insertion of the bottom cables, these jacks will have to be raised to give an increment in the reaction sufficient to prevent tension in the concrete at the top of the girder at midspan. The required stage and increment will be determined in Sec. 4.8.11.

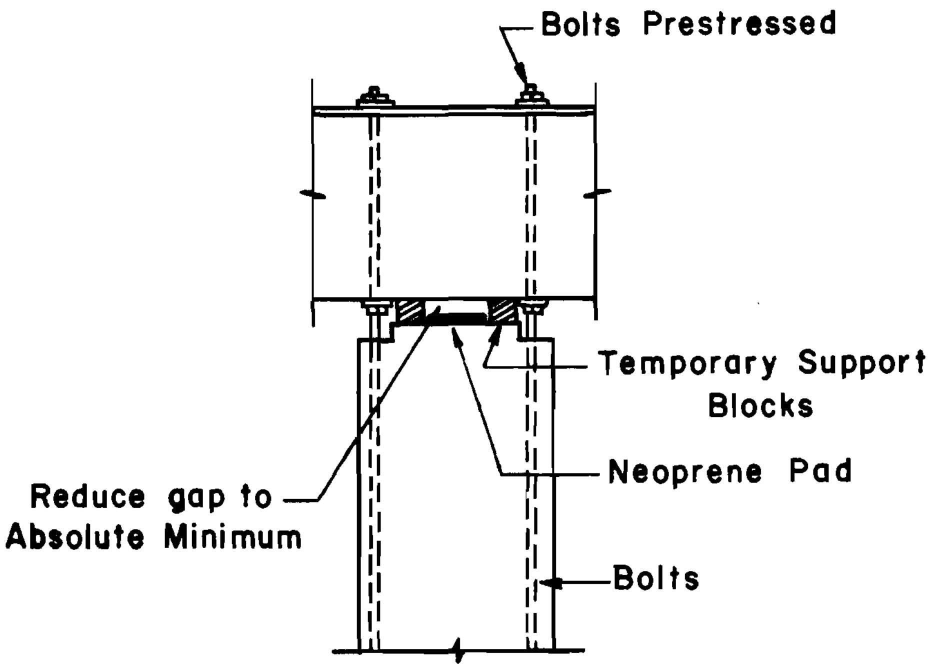


Fig. 4.5. Suggested improvement for temporary support

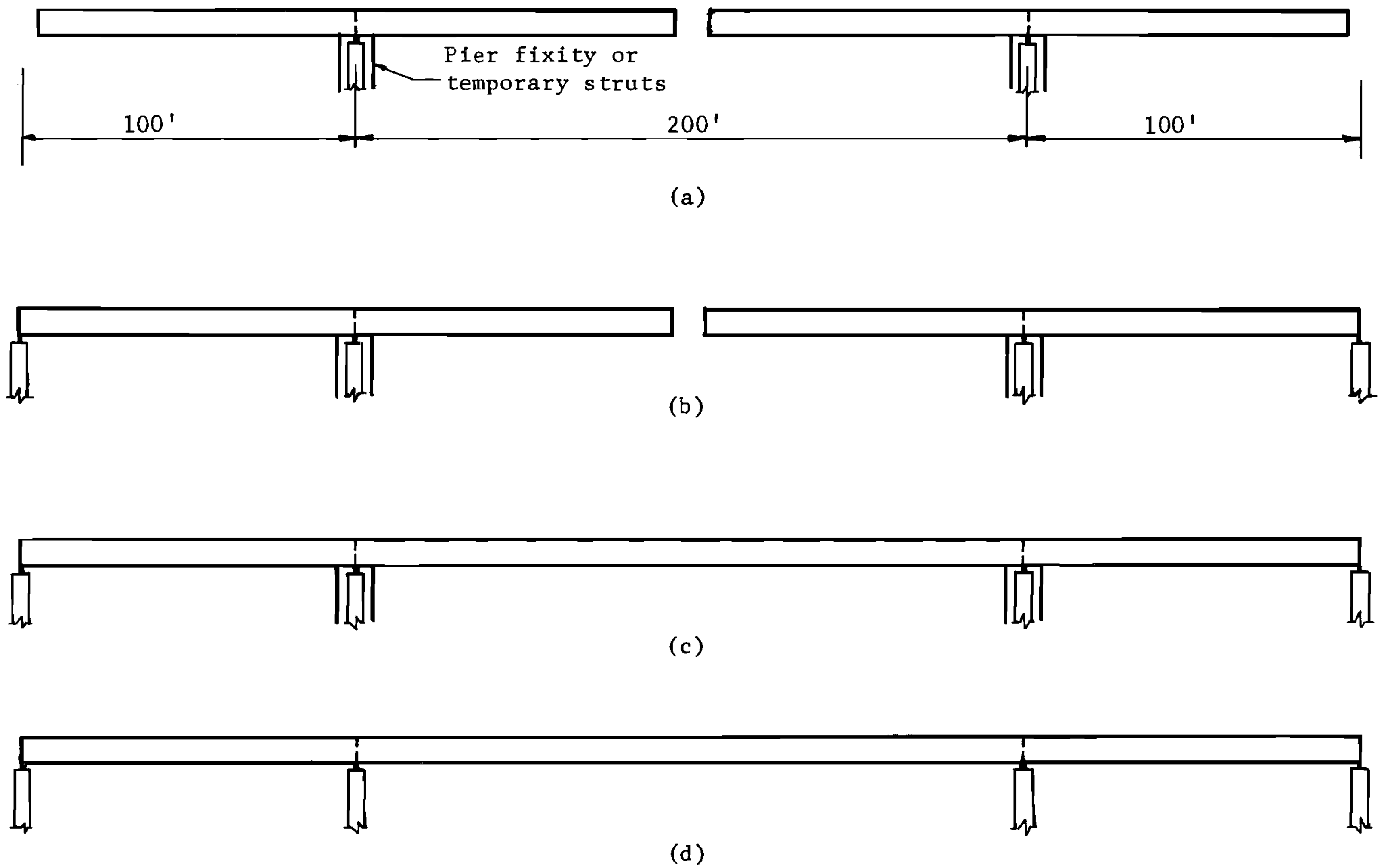


Fig. 4.6. Stages of construction

After all the cables have been inserted and tensioned, the bridge is jacked up slightly at the main piers and the temporary supports are replaced by the neoprene bearing pads. The end jacks are then adjusted to exert the correct reaction for the completed continuous bridge as determined in Sec. 4.8.9. The neoprene bearing pads at the ends are set firmly in position to provide this reaction ensuring continuous behavior.

Finally, the longitudinal deck joint between the two box girders is cast in place.

4.4 Materal Properties

The material properties assumed are the same as in Chapter 3, i.e.,

Concrete:	Compressive strength:	$f'_c = 6$ ksi
Reinforcement:	Yield Strength:	$f_y = 40$ ksi
Prestressing:	Each cable consists of a bundle of 1/2 in. dia. strands	
	Ultimate strength:	$f'_s = 270$ ksi

4.5 Design of Cross Section and Reinforcement

4.5.1 Deck Slab. The deck is designed according to the 1969 and 1973 AASHO specifications.[1,2] The loading on the deck is as follows:

Dead load

(a) Concrete: A density of 0.150 kips/ft.^{3} is assumed.

(b) Asphalt: An asphalt surface, weighing 0.017 kip/ft.^{2}, covers the 54-ft. width of roadway.

Live load and impact. The live load is AASHO HS20-44. The impact factor is 30 percent (Clause 1.2.12).

Allowable stresses (Clause 1.5.1)

Concrete: $f_c = 0.4f'_c = 2.4$ ksi

Reinforcement: $f_s = 20$ ksi

Modular ratio (Clause 1.5.2): $n = 6$

The distance from the surface of the slab to the neutral axis is kd, where

$$k = f_c/(f_c + f_s/n)$$
$$= 2.4/(2.4 + 3.33) = 0.419$$

The moment arm is jd, where

$$j = 1 - k/3 = 0.860$$

The concrete moment resistance coefficient

$$R = 0.5 f_c jk$$
$$= 0.5 \times 2.4 \times 0.860 \times 0.419 = 0.432 \text{ ksi}$$

Concrete cover. The concrete cover for reinforcement and cables will be as specified by AASHO (Clause 1.5.6), i.e.,

(a) Underside of deck slab: 1 in.

(b) Elsewhere: 1-1/2 in.

Live load moments. The live load moments in the deck slab are calculated by the method given in Clause 1.3.2 of the AASHO specifications. To determine if this method is sufficiently accurate for a double box girder bridge, a typical superstructure was analyzed, using the MUPDI program, with truck loads in various critical positions. The maximum negative moment computed by the program for the interior portion of the deck slab exceeded that given by the AASHO method by 9 percent under the worst loading condition. The maximum positive moment computed was 20 percent below the AASHO value. Allowing for a small amount of inelastic moment redistribution, and the conservatism of the design procedures selected, the AASHO method may be considered sufficiently accurate.

4.5.2 Cantilever Portion of Deck Slab. The slab dimensions and the critical wheel load position are shown in Fig. 4.7. For design purposes, assume a 6 in. x 6 in. fillet to allow for possible variations. The critical section is at the root of the fillet. The weight of a curb and railing is neglected in this design example. In general it should be included, although the effect on the total bending moment will be small.

Dead load moment

Asphalt:	$0.017 \times 4.5^2/2$	$= 0.172$
Concrete:	$(6/12) \times 0.150 \times 5.5^2/2$	$= 1.134$
	$(1.833/12) \times 0.150 \times 5.5^2/6$	$= \underline{0.116}$
		1.422 k-ft./ft.

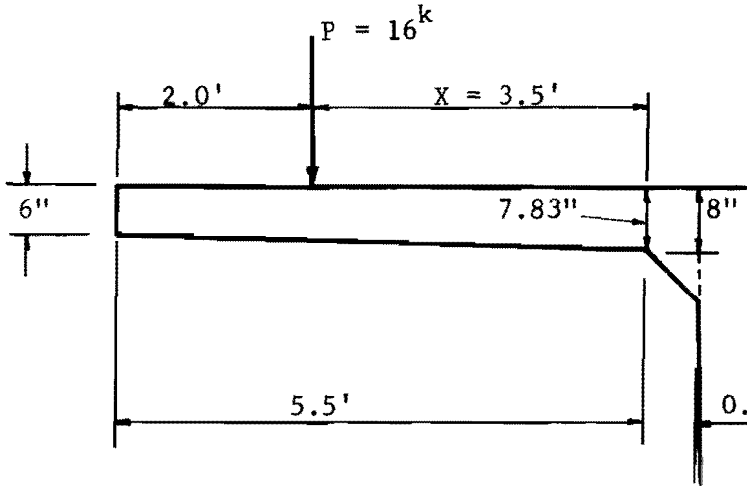


Fig. 4.7. Cantilever portion of deck slab

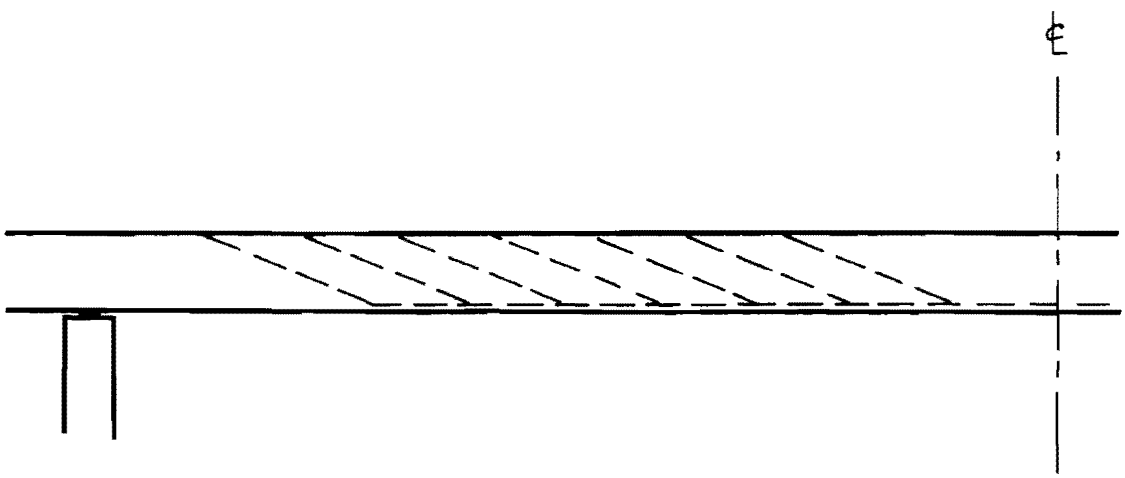


Fig. 4.8. Main span bottom cable pattern in preliminary design

(Live load and impact) moment

$X = 3.5$ ft.

Width of distribution [Clause 1.3.2(H)]:

$E = 0.8X + 3.75 = 6.55$ ft.

Moment:

$M = 1.3\ PX/E$ (30% Impact Factor)

$= 1.3 \times 16 \times 3.5/6.55 = 11.114$ k-ft./ft.

Total moment: 12.54 k-ft./ft.

Required effective depth

$d = \sqrt{[M/(R \times 1)]}$

$= \sqrt{[12.54/(0.432 \times 1)]} = 5.39$

Actual d = 5.83 in., assuming #8 bars and 1-1/2 in. concrete cover.

Reinforcement required

$A_s = M/f_s jd$

$= 12.54 \times 12/(20 \times 0.86 \times 5.83) = 1.50\ in.^2/ft.$

Adopt #8 bars at 6 in. in top of slab

$A_s = 1.57\ in.^2/ft.$

4.5.3 Interior Portion of Deck Slab. To be on the safe side, the fillets are ignored and the full clear span between the webs is used:

$$S = 13.33 \text{ ft.}$$

Dead load moment

$(0.017 + 0.150 \times 7/12) \times 13.33^2/12 = 1.55$ k-ft./ft.

Live load moment

Wheel load: $P = 16$ kip

The maximum positive and negative moment is given according to Clause 1.3.2(C)

$M = 1.3 \times 0.8(S + 2)P/32$

$= 1.3 \times 0.8(13.33 + 2) \times 16/32 = 7.97$ k-ft./ft.

(0.3 represents the impact factor and 0.8 is a continuity factor)

Total moment = 9.52 f-ft./ft.

This moment must be corrected for carry-over from the cantilever portion. As an approximation, the top slab is considered fixed at the interior web and the outer web pinned at the bottom slab.

$$\begin{array}{lll} \text{Top slab stiffness:} & (7/12)^3/14.33 & = 0.014 \\ \text{Web stiffness:} & (3/4)(1)^3/7.5 & = \underline{0.100} \\ & & \ 0.114 \end{array}$$

The corrected moment is then obtained from moment distribution, thus

$$M = [(9.52 \times 0.100) + (12.54 \times 0.014)]/0.114$$
$$= 9.89 \text{ k-ft./ft.}$$

Required effective depth

$d = \sqrt{[9.89/(0.432 \times 1)]} = 4.78$ in.

Actual d = 5 in.

Reinforcement required

$A_s = 9.89 \times 12/(20 \times 0.86 \times 5) = 1.38$ in.^{2}/ft.

To match the spacing in the cantilever portion, adopt #8 bars at 6 in. in top and bottom of slab.

$A_s = 1.57$ in.^{2}/ft.

4.5.4 Web Thickness. The maximum cable size to be used in the girder was envisioned as 20 strands. The contractor later submitted a construction plan based on 12 strand tendons. A minimum web thickness of 12 in. is required to accommodate the anchorages for these cables. Construction experience with the congested webs later indicated that a 14 in. thickness would have been more appropriate.

4.5.5 Bottom Slab. A minimum thickness of 6 in. is chosen for the bottom slab, as in Chapter 3. The thickness is increased linearly or in steps to 10 in. at the main pier, over a distance of 25 ft., to withstand the higher longitudinal compression in that region. The 10 in. maximum thickness and the 25 ft. taper length are chosen on the basis of a previous preliminary design similar to that outlined in Sec. 7.2.3. The compressive capacity will be checked later.

4.5.6 Reinforcement of the Girder Section. The transverse reinforcement in the deck slab has been determined. The reinforcement for the full cross section is shown in Fig. 4.14. The following criteria were used to determine this reinforcement.

Vertical reinforcement in the webs. The results of the MUPDI analysis of a bridge under truck loads, referred to earlier, indicated that the maximum bending moments in the webs are approximately equal to those in the interior portion of the deck slab. Vertical reinforcement is provided to withstand these moments. The shear capacity will be checked later.

Transverse reinforcement in the bottom slab. Minimum reinforcement of 0.5 percent of the flange section is specified by AASHO, Clause 1.5.12(F). This is adequate to withstand the maximum bending moments computed in the MUPDI analysis.

Longitudinal reinforcement in bottom of deck slab. Reinforcement to distribute the wheel loads is provided as specified by AASHO Clause 1.3.2(E).

Longitudinal reinforcement in top of deck slab. The BPR Criteria for Reinforced Concrete Bridge Members[6] specify temperature and shrinkage reinforcement in the top of the deck slab equal to 0.25 percent of the concrete area.

Longitudinal reinforcement in webs and bottom slab. Shrinkage reinforcement equal to 0.125 $in.^2$ per foot of each surface is specified by AASHO, Clause 1.5.6(H).

4.6 General Design Criteria

The design criteria set out in Chapter 3, Sec. 3.6, are again applied. They must be satisfied by the bridge superstructure at all stages of construction and also in the completed state.

4.7 Design of Superstructure during Cantilever Construction

As each pair of segments is set in place, a set of cables is inserted and anchored (see Fig. 4.13). The cables at each section must be adequate to resist all moments applied at that section. The highest bending moment at each section during the construction phase occurs when the full cantilever arm, a length of 100 ft. from the pier centerline, is completed. The two box girders comprising the superstructure are constructed separately and are not joined till after completion.

It was initially assumed that the segments would be lifted into place with a pair of floating cranes and that the only live load on the bridge would be the weight of equipment for fixing the segments in position, placing the cables, etc., and the weight of the persons who would carry out these operations. To include all these loads, a concentrated live load (including impact) of 25 kips could be assumed at the end of the cantilever for each (27 ft. wide) box girder. The maximum distance of this load from the pier centerline would be 95 ft. Further reflection indicated that even if two cranes were used, unrealistic coordination would be required to maintain symmetrical loading at all times. Both to reflect construction realities and to allow use of only one crane, design should be based on temporary unbalance of one segment during an erection stage. Each completed stage should be able to support a segment plus holding equipment with reasonable impact.

If, instead of using a floating crane, the segments were to be lifted by traveling hoists moving outward on the superstructure, the live load would be even greater. It would have to include the weight of the hoist and the segment being lifted, and a high impact factor of at least 100 percent should be used. Since segments will almost certainly be temporarily unbalanced, the temporary unbalanced moments must also be considered in pier design.

The section properties of the superstructure were calculated by the program listed in Appendix B. The properties of the section at the pier and of the minimum section are presented in Table 4.1. Properties of the half section (i.e., one box girder) and of the full section (i.e., two box girders joined by the 2-ft. width of cast-in-place deck) are given. The former are required for design during construction and the latter for design of the completed bridge under live load.

The approximate positions of the prestressing cables, which are required to carry out the design, were obtained from a preliminary layout

TABLE 4.1. SECTION PROPERTIES OF SUPERSTRUCTURE--THREE-SPAN BRIDGE

	Maximum Section	Minimum Section
<u>Properties of Half Section (One box)</u>		
Area (ft.^{2})	40.60	36.93
Distance from top to centroid (ft.)	3.442	3.058
Second moment of area (ft.^{4})	398.2	338.0
Section modulus (top)(ft.^{3})	115.7	110.5
Section modulus (bottom)(ft.^{3})	87.36	68.39
<u>Properties of Full Section (Two boxes)</u>		
Area (ft.^{2})	82.37	75.02
Distance from top to centroid (ft.)	3.397	3.015
Second moment of area (ft.^{4})	807.8	684.8
Section modulus (top)(ft.^{3})	237.8	227.2
Section modulus (bottom)(ft.^{3})	175.5	137.4

TABLE 4.2. TOP CABLE ECCENTRICITIES

Distance from center of main pier (ft.)	Distance from top of girder to Centroid of section (ft.)	Distance from top of girder to Center of cables (ft.)	Cable eccentricity about section centroid (ft.)
0	3.442	0.460	2.982
5	3.373	0.439	2.934
15	3.224	0.455	2.769
25	3.058	0.402	2.656
35 to 85	3.058	0.313	2.745

based on the preliminary design. Table 4.2 gives the position of the section centroid, the position of the center of the top cables, and the cable eccentricity for various locations along the girder, corresponding to the ends of the segments. The section centroid was obtained from the program mentioned above.

4.7.1 Loading. The loading on each box girder is as follows:

Dead load. This is the weight of the girder section.

Maximum section weight (i.e., at pier):

$40.60 \times 0.150 = 6.09$ kip/ft.

Minimum section weight:

$36.93 \times 0.150 = 5.54$ kip/ft.

Live load and impact. 25 kip concentrated load as an allowance for erection equipment and personnel plus a construction load equal to the weight of a unit being temporarily supported from the end of the cantilever arm plus a 50 percent impact factor.

4.7.2 Top Cables Required at Pier Center. The bending moment on each box girder at the pier for the full 100 ft. cantilever is as follows:

Dead load moment

$$5.54 \times 100^2/2 = 27{,}700$$

$$(6.09 - 5.54) \times 25^2/6 = \underline{\quad 60}$$

$$27{,}760$$

Live load moment

$$25 \times 95 = \underline{2{,}380}$$

$$30{,}140 \text{ k-ft.}$$

(NOTE: This case does not require temporary support of one unit since it represents the terminal and worst condition.)

Cable eccentricity (from Table 4.2) = 2.982 ft.

Effective prestress: $f_{se} = 162$ ksi (A very conservative assumption during construction, since losses will not have fully occurred.)

For a cable force F (per box girder) the concrete stress in the top of the girder can be found from

$$f_t = -F/A \pm M/S = -F/A - F_e/S_t + M/S_t$$

$$f_t = (30{,}140 - F \times 2.982)/115.7 - F/40.60$$

in ksf units (tension positive).

The prestress force furnished F must be at least sufficient to ensure that this stress is compressive, since no tension is desired across the fresh epoxy joints.

$$F \geq 30{,}140/(2.982 + 115.7/40.60)$$
$$= 5{,}170 \text{ kip}$$

However, a more severe case may be the construction condition when individual units are being temporarily supported from the existing cantilevering structure prior to completion of stressing. As an example, the condition with 95 ft. of cantilever completed on each side of the pier and with the last 5 ft. length segment near the side pier being applied will be checked.

Dead load moment - completed section		
	$5.54 \times 95^2/2$	= 25,000
	$(6.09 - 5.54) \times 25^2/6$	= 60
		25,060
Live load moment		
Lifting equipment	25×90	= 2,250
Segment	(5.54)(5)(97.5)(1.5)	= 4,050
Note: 1.5 includes 50% impact factor		31,360 k-ft.

Substituting this moment in the equation for prestress force, F = 5380 kip or approximately a 4 percent increase for construction loads.

The design of the completed superstructure will be greatly simplified if the top cable pattern is such as to balance some simple applied load, for instance a uniform load. Within some range the value of the uniform load to balance may be chosen arbitrarily. In choosing such a load, the following considerations are relevant: (a) The more cables there are in the top of the girder, the fewer cables will be required in the bottom; (b) it is desirable to make the balanced load less than the dead load to ensure that there will be adequate reaction at the outer supports of the completed girder without excessive jacking. This is particularly important with short side spans, since live loading in the center span causes the end reactions to decrease and it is undesirable to have the girder rise off its outer supports; (c) the balanced load should be sufficiently high that it will be the controlling criterion for the top cable quantity at each section rather than the no tension criterion or the ultimate load criterion.

Based on trial designs, it was found that a balanced load of about 60 percent of the dead load is suitable. The force F was chosen to balance a uniform load of 3.5 kip/ft. on each box girder.

$$w = 8Fh/L^2$$
$$F = wL^2/8h$$
$$F = 0.125 \times 3.5 \times 200^2/2.982 = 5{,}870 \text{ kip}$$

The cable area required

$$A_s = F/f_{se}$$
$$= 5{,}870/162 = 36.2 \text{ in.}^2$$

The ultimate strength must now be checked. The ultimate moment at the pier center with a 50 percent impact factor on the suspended segment during lifting is

$$M_u = 1.35(25{,}060) + 2.25(2{,}250) + 2.25(4{,}050)$$
$$\approx 48{,}000 \text{ k-ft.}$$

Effective depth:

$$d = 8 - 0.460 = 7.540 \text{ ft.}$$

Bottom slab width:

$$b = 13 \text{ ft.}$$
$$p = A_s/bd$$
$$= 36.2/(13 \times 7.54 \times 144) = 0.00256$$
$$0.5pf'_s/f'_c = 0.5 \times 0.00256 \times 270/6 \approx 0.0577$$

The cable stress at ultimate load, given by AASHO Sec. 1.6.9(C) for bonded members is

$$f_{su} = f'_s(1 - 0.5pf'_s/f'_c)$$
$$= 270(1 - 0.0577) = 254 \text{ ksi}$$

Bottom slab thickness: $t = 10 \text{ in.} = 0.833 \text{ ft.}$

$$T = C$$
$$A_s f_{su} = 0.85f'_c A_c$$
$$(36.2)(254) = (0.85)(6)(A_c)$$
$$A_c = 1802 \text{ in.}^2 \text{ required}$$

Area of lower flange $= (10)(156) = 1560$

Therefore, part of web is in compression zone

$$\frac{1802 - 1560}{24} \approx 10 \text{ in.}$$

Centroid of compression is 0.53 ft. from bottom

Moment arm: $\approx d - 0.53$

$= 7.540 - 0.53 \approx 7.01$ ft.

Ultimate moment capacity $= (\phi)(A_s)(f_{su})$(Moment arm)

$= (0.9)(36.2)(254)(7.01)$

$= 58010$ k-ft. [Approximately 1.21 M_u]

Hence, the ultimate strength of the section at the pier is adequate.

This reserve in ultimate strength (approximately 20 percent considering the impact allowance) was fortunate, since the contractor decided to erect the structure with one crane. Similar calculations at each section based on a one-segment imbalance indicated this was permissible. Provision for imbalance should be considered in initial design.

4.7.3 Top Cable Pattern throughout Girder. The above procedure is followed to determine the cables required for sections at distances of 5, 15, 25, . . . and 85 ft. from the pier center, corresponding to the ends of each of the segments in the cantilever. The cable force and cable area required at each section are shown in Table 4.3.

TABLE 4.3. TOP CABLES REQUIRED

(For each box girder)

Distance from pier ft.	Cable force required kip	Cable area required sq. in.
0	5868	36.2
5	5383	33.2
15	4566	28.2
25	3706	22.9
35	2694	16.6
45	1929	11.9
55	1291	8.0
65	781	4.8
75	398	2.5
85	152	0.9

In the case of all sections except the last (85 ft. from the pier center), the governing criterion for the cable force and area was the requirement to balance the 3.5 kip/ft. load. In the latter case, the no-tension criterion was critical by a small amount. The ultimate load capacity was quite adequate at all sections.

The system of cables adopted in the design, which provided the required area at each section, is given in Table 4.4. Each set of cables extends beyond the point indicated in the table to the end of the next segment, where the cables are anchored in the webs. An elevation of the bridge showing the cables is given in Fig. 4.13

At any stage of erection, before the cantilever arms are completed, the cable area at each section is less than the value indicated in Table 4.4. Beam theory calculations were carried out to ensure that at each section and for each length of the cantilever arm during erection, the cables inserted up to that stage are sufficient to provide a compressive stress in the top of the girder and adequate ultimate strength. In these calculations, the dead load moment was conservatively taken as that corresponding to the length of cantilever arm completed at the stage considered together with an extra segment added to allow for the possibility of accidental loss of crane support during placing of a segment or unsymmetrical placement of segments. It was found that the cables are adequate at all sections during all stages of erection.

A check was made on the concrete stresses in the bottom of the girder for the different stages of completion of the cantilever arm. The compressive stresses are highest when the cantilever arm is completed and these do not exceed the allowable value of 2.4 ksi. However, it was found that some tensile stresses can occur near the pier. The highest tensile stresses occur at a distance of 15 ft. from the pier center when the cantilever arm length is 25 ft. or 35 ft. They disappear when the length becomes 45 ft. These stresses do not exceed 50 psi and must be controlled by the use of some temporary external prestressing strands. Us of such external prestressing force is advantageous on all units to keep positive contact on the epoxy surfaces prior to completion of stressing.

Prior to actual construction, the post-tension supplier asked that an alternative cable arrangement be allowed so that the maximum tendon

TABLE 4.4. TOP CABLES ADOPTED
(For each box girder)

Distance from pier (ft.)	No. of Cables		No. of strands per cable		Design cable area (sq.in.)	Design total cable area (sq.in.)	Cable force (kip)		Total cable force (kip)	
	Design	Supplied	Design	Supplied			Design	Supplied	Design	Supplied
85	2	2	6	6	1.837	1.84	298	298	298	298
75	2	2	6	6	1.837	3.67	298	298	596	596
65	2	2	6	7	1.837	5.51	298	348	894	944
55	2	2	13	12	3.981	9.49	645	596	1538	1540
45	2	2	13	12	3.981	13.47	645	596	2182	2136
35	2	2	13	12	3.981	17.45	645	596	2827	2732
25	2	4	20	11	6.124	23.58	992	1092	3819	3824
15	2	4	20	11	6.124	29.70	992	1092	4812	4916
5	2	2	13	12	3.981	33.68	645	595	5456	5511
0	2	2	13	12	3.981	37.66	645	595	6101	6106

size would be based on 12 - 1/2 in. diameter 270 ksi strands. In this way anchorages could be used more efficiently and the maximum anchorage size reduced. The proposed pattern was approved, as shown in Table 4.4, and used in the construction. At most stages force equivalents were virtually the same except for the initial segments. This illustrates again the desirability of flexibility in design procedures which can be had if the initial design philosophy does not take the design to absolute limits.

4.8 Design of Completed Superstructure

In the completed bridge it is assumed as a trial that the support reactions are set to provide the "beam on unyielding supports" condition. If there were no camber the supports could all be set at the same level to obtain the correct reactions. However, camber will be provided and at the completion of construction the end supports will be set in position using jacks to ensure that the correct reactions are obtained. These reactions will be determined in Sec. 4.8.9. Correct application of these reactions allows analysis of the completed structure to be based on elastic analysis of a continuous bridge.

If it is found in the process of design that this condition does not lead to suitable behavior of the structure, the end reactions may be modified as necessary and the resulting effect on the calculated bending moments and shears taken into account.

Strictly speaking, the sequence of construction should be considered in the design and analysis of the completed superstructure.[9,19] While dead load moments are determined for a three-span continuous beam using normal elastic influence lines, the structure has carried its full dead load as a cantilever span and its moment pattern is set by its stage completion (including jacking of the reactions). This will be considered in the ultimate moment performance, but is questionable refinement at the service load stage. Action of creep will put the true values of moment somewhere intermediate between those of full cantilever dead load and fully continuous dead load. In each case, a reasonably conservative assumption will be made.

Throughout this section, loads, shears, and moments calculated refer to the full 56 ft. width of the superstructure unless otherwise noted.

4.8.1 Loading. Dead load. The dead load consists of the weight of the girder section and the asphalt. The area of the completed section is obtained from Table 4.1.

Dead load at main pier:

Concrete section:	0.150×82.37	$= 12.36$
Asphalt:	0.017×54	$= 0.92$
		13.28 kip/ft.

Dead load at minimum section:

Concrete section:	0.150×75.02	$= 11.25$
Asphalt:		0.92
		12.17 kip/ft.

Live load. The live load is AASHO HS20-44. Generally the lane load will be critical, rather than the truck load. When four lanes are loaded simultaneously, a 25 percent reduction in load intensity is allowed (Clause 1.2.9).

Impact. The impact factors, specified in Clause 1.2.12, are as follows:

Positive moment--main span:	$50/(200 + 125)$	$= 0.154$
Positive moment--side span:	$50/(100 + 125)$	$= 0.222$
Negative moment--both spans:	$50/(150 + 125)$	$= 0.182$

4.8.2 Bending Moments. The influence coefficients used in the following calculations were obtained from a program prepared by T. Komura. This program was checked against tables in Ref. 5 for the case of a continuous beam with three equal spans.

Dead load moment (if fully continuous)

Moment at main pier:

$-0.070 \times 12.17 \times 200^2$	$= -34{,}080$
$-(13.28 - 12.17) \times 25^2 \times 375/(12 \times 200)$	$= -100$
	$-34{,}180$ k-ft.

Moment at center of bridge:

$0.055 \times 12.17 \times 200^2$	$= 26{,}780$
$(13.28 - 12.17) \times 25^2/6 - 100$	$= 10$
	26,790 k-ft.

Moments in side span (at x' from end):

$(x = 10)$ $0.0043 \times 12.17 \times 200^2 = 2{,}090$ k-ft.

$(x = 20)$ $0.0061 \times 12.17 \times 200^2 = 2{,}970$ k-ft.

$(x = 30)$ $0.0054 \times 12.17 \times 200^2 = 2{,}630$ k-ft.

$(x = 40)$ $0.0022 \times 12.17 \times 200^2 = 1{,}070$ k-ft.

$(x = 50)$ $-0.0036 \times 12.17 \times 200^2 = -1{,}750$ k-ft.

<u>Live load moments</u>

Uniform lane load on four lanes $= 0.75 \times 4 \times 0.640$

$= 1.92$ kip/ft.

Concentrated lane load on four lanes $= 0.75 \times 4 \times 18$

$= 54$ kip

Moment at main pier:

$-0.074 \times 1.92 \times 200^2 = -5{,}680$

$-(0.1024 + 0.0360) \times 54 \times 200 = \underline{-1{,}495}$

$-7{,}175$ k-ft.

Moment at center of bridge:

$0.0625 \times 1.92 \times 200^2 = 4{,}800$

$0.1563 \times 54 \times 200 = \underline{1{,}690}$

$6{,}490$ k-ft.

Moments in side span:

(Truck load is critical)

$(x = 10)$

$[(0.0441 + 0.0353)96 + (0.0269)24] \times 200 = 1650$ k-ft.

$(x = 20)$

$[(0.0764 + 0.0593)96 + (0.0430)24] \times 200 = 2810$ k-ft.

$(x = 30)$

$[(0.0973 + 0.0725)96 + (0.0492)24] \times 200 = 3500$ k-ft.

$(x = 40)$

$[(0.1074 + 0.0758)96 + (0.0663)24] \times 200 = 3840$ k-ft.

<u>Moments due to top cables</u>

The equivalent load from the top cables, ignoring the turned down section at the anchorages, for the full width of the superstructure is $-2 \times 3.5 = -7.0$ kip/ft. (Two boxes with 0.6DL.) The moments on the concrete section produced by this load are as follows:

Moment at main pier:

$$0.070 \times 7.0 \times 200^2 = 19{,}600 \text{ k-ft.}$$

Moment at center of bridge:

$$-0.055 \times 7.0 \times 200^2 = -15{,}400 \text{ k-ft.}$$

The above requirements are the resultant cable moments. The secondary moments will also be required, where

Secondary moment = Resultant moment - Primary moment

At the center of the bridge the primary moment for the top cables is zero, because there are no top cables at that section. Hence,

Secondary moment at center of bridge = -15,400 k-ft.

Since the secondary moment is that produced by the cable-induced reactions, it will be constant in the main span and will vary linearly in the side spans from the above value at the main piers down to zero at the ends of the bridge. (This moment is induced during the closure by jacking of the end reactions. Stressing top cables during cantilevering does not introduce secondary moments.)

Moments due to bottom cables

For ultimate load calculations the secondary moments due to the bottom cables will also be required. Let the values of these moments in the main span be denoted as follows.

M_{S1} = Secondary moment from bottom cables in main span

M_{S2} = Secondary moment from bottom cables in side spans

Again, these moments will be constant in the main span and in the side spans will vary linearly down to zero at the ends.

The values of M_{S1} and M_{S2} are not known until the bottom cable areas have been determined. Initial values must therefore be assumed, so that the ultimate moments can be calculated. After the cable areas have been determined, the assumed values can be corrected and the ultimate moments rechecked if necessary. The initial values could be zero. However, on the basis of a preliminary design the following trial values will be assumed.

M_{S1} = +5000 k-ft.

M_{S2} = +1000 k-ft.

4.8.3 Ultimate Strength at Main Pier. In calculating ultimate strength, no moment redistribution will be assumed. The secondary cable moments are included in the calculations, because these are produced by real external reactions caused by the cables.

Conventional calculation of the ultimate moment at the pier, assuming the structure was completely constructed as a three-span beam, is given by

$$
\begin{aligned}
M_u &= 1.35(\text{DL moment}) + 2.25(\text{LL + Impact moment}) \\
&\quad + (\text{Secondary moments}) \\
&= 1.35(-34{,}180) + 2.25(1.182)(-7{,}175) \\
&\quad + (-15{,}400 + M_{S1} + M_{S2})
\end{aligned}
$$

$$
\begin{aligned}
|M_u| &< 46{,}140 + 19{,}080 + (15{,}400 - 6000) \\
&= 74{,}620 \text{ k-ft.}
\end{aligned}
$$

which is less than the ultimate moment capacity found in Sec. 4.7.2

$$|M_u| = 2 \times 58{,}010 = 116{,}020 \text{ ft.}$$

However, based on the recommendations following the model test program contained in Report 121-5, a more severe computation of ultimate moment is recommended. The structural ultimate moment capacity should exceed $M_{u1} + M_{u2}$. M_{u1} is computed for 1.35 DL_1 as a balanced cantilever. DL_1 is the dead load at time of cantilevering construction. M_{u2} is computed for 1.35 DL_2 + 2.25(LL + I) + S. DL_2 is any subsequent dead load placed on the completed structure, LL is the design live load, I is impact, and S is secondary moments induced by stressing of cables and reactions provided on closure. (See Sec. 4.8.9.)

On this basis

$$M_{u1} = -\frac{(1.35)(11.25)(100)^2}{2} - \frac{(1.35)(1.11)(25)^2}{6} = -76{,}090 \text{ ft.-k}$$

$$
\begin{aligned}
M_{u2} &= (1.35)(0.92)(-0.070)(200)^2 - (2.25)(7175)(1.182) \\
&\quad +(185)(100) = -4060 \text{ ft.-k}
\end{aligned}
$$

$$M_u = -76{,}090 - 4060 = -80{,}150 \text{ ft.k}$$

which is less than the ultimate moment capacity of 116,020 k-ft.

4.8.4 Bottom Cables in the Main Span. The bottom cable pattern used in the preliminary design is shown in Fig. 4.8. All of the cables are the same size and there are seven cables per web. However, under detailed analysis with the MUPDI program, it was found that this pattern did not give a satisfactory stress distribution along the length of girder. Excessive cable area was required to prevent tensile stresses from occurring in the bottom of the girder in a region about 20 ft. to 40 ft. from the bridge center under live loads.

Accordingly, the cable pattern was revised to that shown in Fig. 4.13, in which the inner set of cables is removed, leaving six cables per web all the same size.

The design procedure used to determine the bottom cable quantity is as follows. First, the cable area required to give adequate ultimate strength at the bridge center is determined. Later, after design of the bottom cables in the side span, stress analyses are carried out and the cable area is revised if necessary.

In this case it will be more conservative to compute the moment as if a three-span continuous beam for all loads. Such a condition will be closely approximated if correct reactions are jacked in during closure. A check will be made in Sec. 4.8.9 to ensure that the ends of the bridge will not lift off the neoprene pads under design ultimate load. The ultimate moment at the center of the bridge is given by

$$
\begin{aligned}
M_u &= 1.35(\text{DL moment}) + 2.25(\text{LL + Impact moment}) \\
&\quad + (\text{Secondary moments}) \\
&= 1.35(26{,}800) + 2.25(1.154)(6{,}490) \\
&\quad + (-15{,}400 + M_{S1} + M_{S2})
\end{aligned}
$$

Using the assumed values of M_{S1} and M_{S2} from Sec. 4.8.2

$$
\begin{aligned}
&= 36{,}180 + 16{,}850 - 15{,}400 + 5{,}000 + 1{,}000 \\
&= 43{,}630 \text{ k-ft.}
\end{aligned}
$$

The cable area A_s required to satisfy this moment will be determined.

On the basis of the preliminary design, the following two quantities are first assumed.

Distance from bottom of girder to center of cables = 0.437 ft.

$A_s \approx 25$ in.^{2}

Then the exact determination of A_s proceeds as follows:

Effective depth: $d = 8 - 0.437 = 7.563$ ft.

Top slab width: $b = 56$ ft.

$$p = A_s/bd$$
$$= 25/(56 \times 7.563 \times 144) = 0.00041$$
$$0.5pf'_s/f'_c = 0.5 \times 0.00041 \times 270/6 = 0.00922$$

Cable stress at ultimate load:

$$f_{su} = f'_s(1 - 0.5pf'_s/f'_c)$$
$$= 270(1 - 0.00922) = 267.5 \text{ ksi}$$

Concrete stresss block depth:

$$a = A_s f_{su}/0.85f'_c b$$
$$= 25 \times 267.5/(0.85 \times 6 \times 56 \times 144) = 0.163 \text{ ft.}$$

Cable force at ultimate load:

$$P_u = M_u/(d - 0.5a)$$
$$= 43{,}630/(7.563 - 0.163/2) = 5{,}830 \text{ kip}$$

Cable area required:

$$A_s = P_u/(\varphi f_{su})$$
$$= 5{,}830/(0.9 \times 267.5) = 24.2 \text{ in.}^2$$

<u>Adopt 24 cables each 7 strands</u>

$$A_s = 25.72 \text{ in.}^2$$

Actual ultimate moment capacity:

$$M_u \approx \frac{25.72}{24.2}(43{,}630) = 46{,}370 \text{ ft.-k}$$

<u>Equivalent load</u>

In order to determine the cable moments, the equivalent cable load will be calculated.

Total cable force: $F = 25.72 \times 162 = 4166.8$ kip

Cable eccentricity about section centroid at center of bridge:

$$e = 7.563 - 3.058 = 4.505 \text{ ft.}$$

The equivalent load is calculated using the second approach described in Sec. 4.1.2. As a (conservative) simplification,* each group of similar cables is treated as stopping off at the bend-up point, i.e., at distances of 20 ft., 30 ft., 40 ft., 50 ft., 60 ft., and 70 ft., from the bridge center. Thus, the equivalent load for each group is a moment m at either end, as shown in Fig. 4.9(a), where

$$\begin{aligned} m &= F \cdot e/6 \\ &= 4{,}167 \times 4.505/6 = 3{,}130 \text{ ft.-k} \end{aligned}$$

and this moment can be replaced by a pair of loads P [Fig. 4.9(b)], where

$$P = m/10 = 313 \text{ kip}$$

These loads cancel out except for the four shown in Fig. 4.9(c), which will be considered as the equivalent load diagram.

Moments due to bottom cables

The resultant moments on the concrete section from the bottom cables are determined using influence line values as follows:

Moment at main pier:

$$\begin{aligned} &(0.1005 + 0.0823 - 0.0609 - 0.0206) \times 313 \times 200 \\ &= 0.1013 \times 313 \times 200 \qquad = 6{,}340 \text{ k-ft.} \end{aligned}$$

Moment at center of bridge:

$$\begin{aligned} &(-0.1211 + 0.0217) \times 2 \times 313 \times 200 \\ &= -0.1988 \times 313 \times 200 \qquad = -12{,}440 \text{ k-ft.} \end{aligned}$$

Since the primary moment due to the bottom cables at the main pier is zero, the secondary moment is given by

$$M_{S1} = 6{,}340 \text{ k-ft.}$$

Since this is greater than the initial assumed value, the ultimate moment at the bridge center must be rechecked. Using M_{S1} = 6,340, the design moment increases to 44,970 ft.-k which is still less than the 46,370 ft.-k capacity provides. Thus, these cables were found to be adequate.

4.8.5 Bottom Cables in Side Span. The bottom cable pattern chosen for the side span is shown in Fig. 4.13. There are two cables per web. The cable size required to give adequate ultimate strength in this

*Consideration of the curved portion of the tendons would increase equivalent loads less than 10 percent.

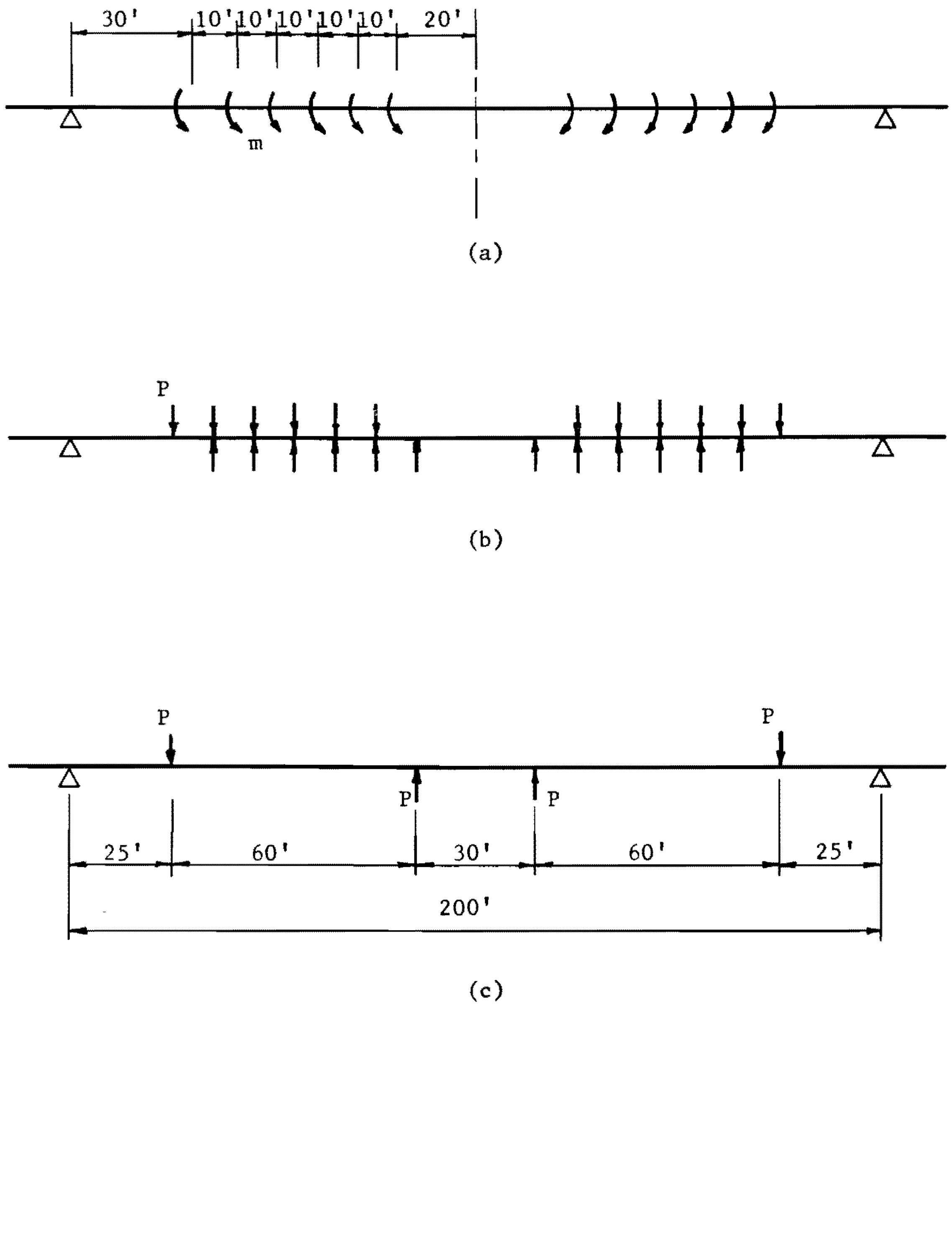


Fig. 4.9. Equivalent load for main span bottom cables

span will be determined first. Later, stress analyses will be carried out and the size revised if necessary. Again, it will be more conservative to compute design moments as if a three-span continuous beam for all loads. It was found in the preliminary design that the critical section is about 30 ft. from the end support. The ultimate moment at this section is given by

$$\begin{aligned} M_u &= 1.35(\text{DL moment}) + 2.25(\text{LL + Impact moment}) \\ &\quad + (\text{Secondary moments}) \\ &= 1.35(2{,}630) + 2.25(1.222)(3{,}500) \\ &\quad + 0.3(-15{,}400 + 6{,}340 + 1{,}000) \end{aligned}$$

(Note: The last term is the linear proportion of the secondary moment at the pier which is effective at the critical section. The value of M_{S1} determined in the previous section is used along with an assumed $M_{S2} = 1{,}000$.)

$$\begin{aligned} &= 3{,}550 + 9{,}620 - 4{,}620 + 1{,}900 + 300 \\ &= 10{,}750 \text{ k-ft.} \end{aligned}$$

From preliminary design, $A_s \approx 6$ in.^{2}, and
Distance from bottom of girder to center of cables = 0.292 ft.
Effective depth: $d = 8 - 0.292 = 7.708$ ft.

$$\begin{aligned} p &= A_s/bd \\ &= 6/(56 \times 7.708 \times 144) = 0.0001 \\ 0.5pf'_s/f'_c &= 0.5 \times 0.0001 \times 270/6 = 0.0022 \\ f_{su} &= 270(1 - 0.0022) = 269 \text{ ksi} \\ a &= A_s f_{su}/0.85f'_c b \\ &= 6 \times 269/(0.85 \times 6 \times 56 \times 144) = 0.04 \text{ ft.} \end{aligned}$$

Cable force at ultimate:

$$\begin{aligned} P_u &= M_u/(d - 0.5a) \\ &= 10{,}750/(7.708 - 0.02) = 1{,}400 \text{ kip} \end{aligned}$$

Cable area required:

$$\begin{aligned} A_s &= P_u/(\varphi f_{su}) \\ &= 1{,}400/(0.9 \times 269) = 5.8 \text{ in.}^2 \end{aligned}$$

<u>Adopt 8 cables each 5 strands</u>

$$A_s = 6.124 \text{ in}.^2$$

Actual ultimate moment capacity:

$$M_u = \frac{6.124}{5.8}(10,750) = 11,350 \text{ k-ft.}$$

<u>Equivalent load</u>

Cable force: $F = 6.124 \times 162 = 992.1$ kip

Cable eccentricity: $e = 7.708 - 3.058 = 4.650$ ft.

The equivalent load is calculated in the same way as for the main span bottom cables, and the different steps are shown in Fig. 4.10. The moments m and the load P are as follows.

$$m = F \cdot e/2$$

$$= 992 \times 4.65/2 = 2,300 \text{ k-ft.}$$

$$P = m/10 = 230 \text{ kip}$$

<u>Moments due to side span bottom cables</u>

The resultant moments on the concrete section from the bottom cables in both side spans are calculated using influence lines as follows.

Moment at center of bridge:

$$[2(0.0031) + 0.0120 - 0.0090] \times 2 \times 230 \times 200 = 850 \text{ k-ft.}$$

This moment is constant over the main span.

Since the primary moment due to the side span bottom cables is zero in the main span, the secondary moment is

$$M_{S2} = 850 \text{ k-ft.}$$

Since this is very close to the 1,000 k-ft. assumed, no further check is required.

4.8.6 <u>Additional Cables in Side Span</u>. When the 100 ft. cantilever arm in the side span is completed, the bottom cables in this span are to be inserted and tensioned as the last segment is added and thus before the end supports are set in place. To avoid tensile stresses in the top of the girder near the end at this stage, it is necessary to include some additional cables at the centroid. Trial designs were made in which top

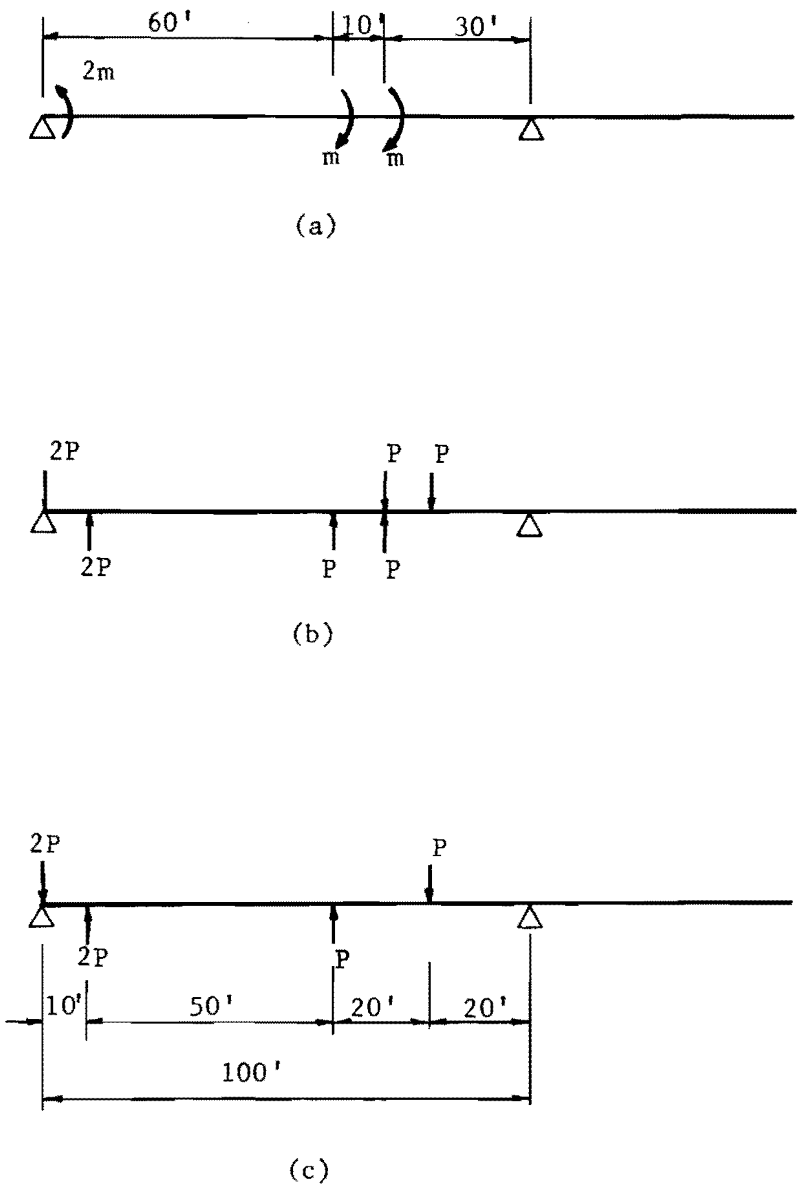


Fig. 4.10. Equivalent load for side span bottom cables

tensile stress at several critical sections was calculated and necessary compressive stress to restore the top fiber to compression was determined. These showed that the following set of centroidal cables will give satisfactory stresses.

Centroidal cables adopted: 8 cables each 5 strands (i.e., two cables per web), half to extend from the end a distance of 25 ft., half to extend from the end a distance of 45 ft. All to be placed along the section centroid.

These cables produce no moments on the concrete section but could provide resistance to ultimate moment loading in the side span. This contribution will be ignored although the design could recycle to reduce the bottom cables.

4.8.7 Service Load Stresses for Completed Structure. The service load stresses at the critical sections will now be determined using beam theory. Elastic analysis for a three-span continuous beam will be generally applicable if the correct reactions are jacked in during closure. Similar service load stresses were previously checked for the cantilever stage and determined acceptable in Sec. 4.7.3. The cable moments used are the resultant moments on the concrete section. In the calculation of dead load stresses the properties of the two unjoined box girders are used and for live load stresses the properties of the full section are used. In inch units these properties are as follows:

Maximum section

Properties of section for dead load

Area	11,690 $in.^2$
Section Modulus (Top)	(33,320 x 12) $in.^3$
Section Modulus (Bottom)	(25,160 x 12) $in.^3$

Properties of section for live load

Section Modulus (Top)	(34,240 x 12) $in.^3$
Section Modulus (Bottom)	(25,270 x 12) $in.^3$

Minimum section

Properties of section for dead load

Area	10,640 $in.^2$

Section Modulus (Top) (31,820 x 12) in.^{3}

Section Modulus (Bottom) (19,700 x 12) in.^{3}

Properties of section for live load

Section Modulus (Top) (32,720 x 12) in.^{3}

Section Modulus (Bottom) (19,790 x 12) in.^{3}

Concrete stresses at main pier assuming full continuous three-span beam for all stresses

Stress at centroid

Top cable force = 2 x 6,101 = 12,200 kip

Stress = 12,200/11,690 = -1.044 ksi

Stresses under dead load

Dead load moment:	-34,180
Top cable moment:	19,600
Main span bottom cable moment:	6,340
Side span bottom cable moment:	850
	- 7,390 k-ft.

Top stress = -1.044 + 7,390/33,320 = -0.822 ksi

Bottom stress = -1.044 - 7,390/25,160 = -1.338 ksi

Stresses under full load

Live load + impact moment = -1.182 x 7,175 = 8,480 k-ft.

Top stress = -0.822 + 8,480/34,240 = -0.574 ksi

Bottom stress = -1.338 - 8,480/25,270 = -1.673 ksi

Concrete stresses at main pier assuming dead load of concrete is carried as a cantilever

This calculation represents the most conservative stress calculation possible in case end reactions are incorrectly applied during closure. The true state of stress is somewhere intermediate between the cantilever state and the fully continuous state when creep is considered.

Stress at centroid

Top cable force = 2 x 6,100 = 12,200 kip

Stress = -12,200/11,690 = -1.044 ksi

Stresses under dead load

Concrete cantilever dead load moment:

-27,760 x 2	= -55,520
Asphalt topping dead load moment:	
0.92/12.17 x (-34,180)	= - 2,500
Top cable moment:	19,600
Main span bottom cable moment:	6,340
Side span bottom cable moment:	850
	-31,310 k-ft.

$$\text{Top stress} = -1.044 + \frac{31,310}{33,320} = -0.104 \text{ ksi}$$

$$\text{Bottom stress} = -1.044 - \frac{31,310}{25,160} = -2.288 \text{ ksi}$$

Stresses under full load

Top stress = -0.104 + 8,480/34,240 = 0.143 ksi

Bottom stress = -2.288 - 8,480/25,270 = -2.623 ksi

The stresses are within acceptable limits under dead load even by this calculation. Stresses would be reduced when applied end reactions are considered. However, under this extreme calculation procedure, full service load stresses are high. The tensile stress at the top is within the $6\sqrt{f'_c}$ tension allowed by Clause 1.6.6(B)(2). The compressive stress is 9 percent above the $0.4f'_c$ limit. In view of the extreme conservatism of the calculation procedure these were judged acceptable.

Concrete stresses at center of bridge

These stresses can be checked conservatively for fully continuous action only.

Stress at centroid

Bottom cable force = 4,167 kip

Stress = -4,167/10,640 = -0.392 ksi

Stresses under dead load

Dead load moment	26,790
Top cable moment	-15,400
Main span bottom cable moment	-12,440
Side span bottom cable moment	850
	-200 k-ft.

Top stress = -0.392 + 200/31,820 = -0.386 ksi

Bottom stress = -0.392 - 200/19,700 = -0.402 ksi

Stress under full load

Live load + impact moment = 1.154 x 6,490 = 7,490 k-ft.

Top stress = -0.386 - 7,490/32,720 = -0.615 ksi

Bottom stress = -0.402 + 7,490/19,790 = -0.023 ksi

Concrete stresses 30 ft. from end of bridge

Stress at centroid

Top cable force:	2 x 893 =	1,786
Bottom cable force:		992
Center cable force:	992/2 =	496
		3,274 kip

Stress = -3,274/10,640 = -0.308 ksi

Stresses under dead load

Dead load moment:	= 2,630
Top cable moment: $-0.0054 \times 7.0 \times 200^2$	= -1,510
Main span bottom cable moment: 0.3 x 6,340	= 1,900
Side span bottom cable moment:	
-(992 x 4.65) + (0.3 x 850)	= -4,360
	-1,340 k-ft.

Top stress = -0.308 + 1,340/31,820 = -0/266 ksi

Bottom stress = -0.308 - 1,340/19,700 = -0.376 ksi

Stresses under full load

Live load + impact moment = 1.222 x 3,500 = 4,277 k-ft.

Top stress = -0.266 - 4,277/32,720 = -0.397 ksi

Bottom stress = -0.376 + 4,277/19,790 = -0.160 ksi

All of the other stresses calculated are within the acceptable limits. The stresses at all sections and under all loadings will also be checked with the MUPDI program for the completed structure and the SIMPLA2 program for erection stresses. However, first the shear strength will be investigated.

4.8.8 Shear. The shear forces on the full width of the superstructure are as follows:

Shear force at main pier during construction

Dead load:	$2 \times 5.54 \times 95$	=	1053
	$2 \times 0.5(6.09 - 5.54) \times 25$	=	14
			1067 kip
Live load:	2×25	=	50 kip
	$2 \times 5.54 \times 5 \times 1.5$	=	83
			133 kip
	Total		1200 kip

Shear force in main span at pier (after completion

Dead load:	$(0.5 \times 12.17 \times 200) + 14$	=	1231 kip
Live load:	$4[(0.5155 \times 0.640 \times 200) + 26]$	=	368 kip
	Total		1599 kip

(Note: The 26 kip load is the AASHO concentrated load with shear lane loads.) The 25 percent reduction in live load intensity for loading on four lanes is not used here, because it was found in the MUPDI analysis of the completed bridge (to be described later) that, if this reduction is made, the critical shear loading will then be live load on two lanes only.

Shear force in side span at main pier

Dead load:	$(0.3902 \times 12.17 \times 200) + 14$	=	964 kip
Live load:	$4[(0.398 \times 0.640 \times 200) + 26]$	=	308 kip
Top cables:	15,400/100	=	154 kip
Main span bottom cables:	-6340/100	=	-63 kip
Side span bottom cables:	-850/100	=	-9 kip
	Total		1354 kip

For live load shear, the lane load is critical. The cable shears computed above are the shears due to the external reactions induced by the cables, where

End reaction = (Secondary moment at main pier) $\div$ (Side span)

Shear force at end of bridge

Dead load:	$0.1098 \times 12.17 \times 200$	=	267 kip
Live load:	(truck load critical)		
	$4[32(1 + 0.822) + 8(0.649)]$	=	254 kip
Top cables:		=	-154 kip
Main span bottom cables:		=	63 kip
Side span bottom cables:		=	9 kip
	Total		439 kip

From examination of the various service load shear conditions, the shear face in the main span after completion is clearly the critical condition.

Ultimate shear in main span at pier

(Disregarding allowable reduction to the shear at critical section at d from support)

$$V_u = 1.35(\text{DL shear}) + 2.25\ (\text{LL + Impact shear})$$
$$= 1.35(1{,}231) + 2.25(1.154)(368)$$
$$= 2{,}617 \text{ kip}$$

The shear capacity of the webs at the pier will now be determined. Using ACI 318-71, Eqs. 11-11 and 11-12, as allowed by AASHO, the concrete stresses, f_{pc} and $(f_{pe} - f_d)$ are obtained from Sec. 4.8.7.

Compressive stress at centroid: $f_{pc} = 1.044$ ksi

Compressive stress at top of girder under dead load and prestress:

$$(f_{pe} - f_d) = \frac{0.822 \text{ ksi} + 0.104 \text{ ksi}}{2} = 0.461$$

(Note: This assumes an actual stress midway between cantilever and fully continuous conditions.)

$$6\sqrt{f'_c} = 6\sqrt{6{,}000}/1{,}000 = 0.465 \text{ ksi}$$

Top section modulus (from Table 4.1): $(I/y) = 237.8 \times 12^3 \text{ in.}^3$

Cracking moment: $M_{cr} = (I/y)\ [6\sqrt{f'_c} + (f_{pe} - f_d)]$

$$= [(237.8 \times 12^3)(0.465 + 0.461)]/12$$
$$= 31{,}700 \text{ k-ft.}$$

Live load shear/moment ratio:

$$(V_\ell/M_{max}) = 0.75 \times 368/7{,}175 = 0.0385$$

$$\frac{V_\ell M_{cr}}{M_{max}} = (0.0385)(31{,}700)$$
$$= 1220$$

Effective depth: $d = (8 - 0.46) \times 12 = 90.5$ in.

Total web width: $b' = 4 \times 12 = 48$ in.

The shear carried by the concrete $V_c = v_c b' d$ is the lesser of $V_{ci} = v_{ci} b' d$ and $V_{cw} = v_{cw} b' d$, where

$$V_{ci} = (0.6\sqrt{f'_c})b' d + V_d + (V_\ell M_{cr}/M_{max})$$
$$= (0.0465 \times 48 \times 90.5) + 1231 + 1220$$
$$= 2653 \text{ kip}$$

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3f_{pc})b'd$$
$$= (0.271 + 0.313) \times 48 \times 90.5$$
$$= 2{,}537 \text{ kip}$$

Hence, $V_c = 2{,}537$ kip.

Shear reinforcement required using Eq. 11-13 of ACI 318-71 as permitted by AASHO:

$$A_v = \frac{(v_u - v_c)b's}{f_y} = \frac{(V_u - \varphi V_c)s}{\varphi\, d\, f_y}$$
$$= (2{,}617 - 0.85 \times 2{,}537) \times 12/(0.85 \times 90.5 \times 40)$$
$$= 1.80 \text{ in.}^2 \text{ per ft. length of bridge.}$$

This exceeds the minimum $A_v = 100b's/f_y = 1.44$ in.^{2} per ft. required by AASHO. This is much less than the reinforcement required for bending moment in the webs. Thus, the web reinforcement will remain as shown in Fig. 4.14.

The shear strengths in the side span at the main pier and at the end support were investigated, and also the shear strength at the main pier during construction. None of these cases was critical.

4.8.9 Reaction at End of Bridge. The reaction over the full width of the superstructure at each end of the fully continuous completed bridge is obtained as follows:

Dead load: $0.1098 \times 12.17 \times 200$	=	267 kip
Maximum live load: (truck load critical)		
$0.75 \times 4[32(1 + 0.0822) + 8(0.649)]$	=	190 kip
Minimum live load: (lane load critical)		
$-0.75 \times 4[(0.1247 \times 0.640 \times 200) + (0.2047 \times 26)]$	=	-64 kip
Top cables		-154 kip
Main span bottom cables		63 kip
Side span bottom cables		9 kip

The total reactions under dead load and under maximum and minimum live loads are as follows:

Reaction due to dead load and cables =

$267 - 154 + 63 + 9 \quad = 185$ kip/bridge = 92.5 kip/box

Maximum reaction = $185 + 1.222(190) = 417$ kip/bridge = 208.5 kip/box

Minimum reaction = $185 - 1.222(64) = 107$ kip/bridge = 53.5 kip/box

Checking ultimate live load conditions the minimum reaction

$= 185 - (2.25)(1.222)(64) = 9^k$

The minimum reaction is adequate (barely) to maintain proper seating on the bearings. Therefore, the "beam on unyielding supports" condition will be adopted as initially assumed. At the completion of erection, the reaction at each end of the bridge will be set to the correct value for dead load and cable forces, i.e., 185 kip, or 92.5 kip/box.

In a subsequent check, Kashima[9,11] showed that consideration should be given to the effect of the end reaction on cracking moment as well. His calculations indicated the optimum value of the end reaction as 176 kips per bridge, or 88 kips/box. This is very close to the value above.

4.8.10 Computer Analysis of Completed Structure. With the basic proportions, reinforcement, and tendons designed, the completed bridge was analyzed by the MUPDI program to obtain the stresses in the concrete section under dead load and under various live load patterns. For dead load (including the cable forces) one box girder (i.e., half of the superstructure cross section) was considered and for live load the full cross section was analyzed to correspond to the real conditions. The structure was assumed fully continuous at all times. This corresponds to the completed structure with the correct end reactions applied.

In this analysis the effect of the cables was simulated by considering each cable individually, determining the load it exerts on the concrete section and including this load with the input. However, the number of cable loads far exceeded the number that can be handled by the program in one run. The following simplifying assumptions were made regarding these loads in order that the dead load and cable forces could be handled in three runs. The output for the runs was then added to give the total dead load stresses.

(a) Each cable is treated as a straight cable stopping off approximately at the bend point. Thus, the top cables are considered to stop off at distances of 5 ft., 10 ft., 20 ft., 30 ft., . . . and 90 ft. from the main pier center; the main span bottom cables at 20 ft., 30 ft., . . . and 70 ft. from the bridge center: and the side span bottom cables at 60 ft. and 70 ft. from the end support. The load exerted by each cable is a pair of equal and opposite longitudinal forces, one at each end.

(b) In cases where a number of successive cables along the span are the same size and have approximately the same eccentricity about the centroid, the set of concentrated longitudinal forces corresponding to these cables is replaced by a single linearly varying longitudinal force in each web.

(c) The beneficial compressive effect of the turned-up portions of the main span bottom cables is taken into account by treating these overlapping portions as a single horizontal cable at the centroid in each web, extending between 15 ft. and 80 ft. from the bridge center.

The following further simplifications were made in order to make the analysis feasible:

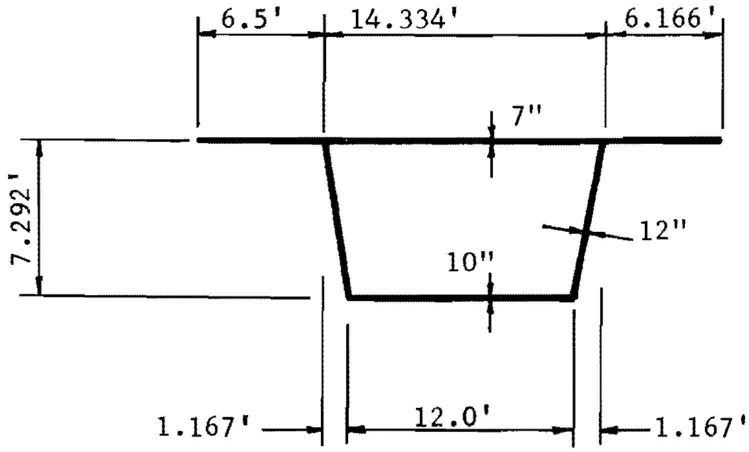
(d) The MUPDI program cannot handle the variation in the thickness of the bottom slab. Separate analyses were made with two different idealized sections, one having the properties of the maximum section (Fig. 4.11), the other of the minimum section (Fig. 4.12). The first is used to obtain the stresses near the main pier and the second to obtain the stresses elsewhere throughout the superstructure.

(e) With this program, all concentrated loads must be applied at the node points of the idealized section. The moment of each cable force about the centroid of the real section was calculated and this force and moment replaced by a pair of forces at the node points at the top and bottom of the web of the idealized section. This pair of forces was determined so as to give the same resultant force and moment about the centroid of the idealized section as occur in the real section.

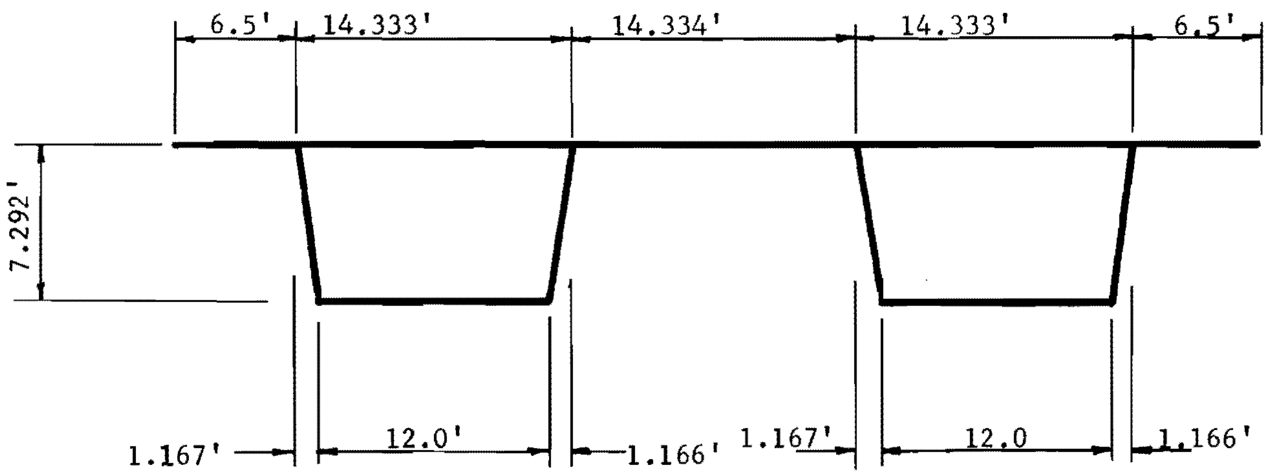
The following live load cases were investigated:

(a) Full lane loads on main span

(b) Lane loads on one side (two lanes) of main span

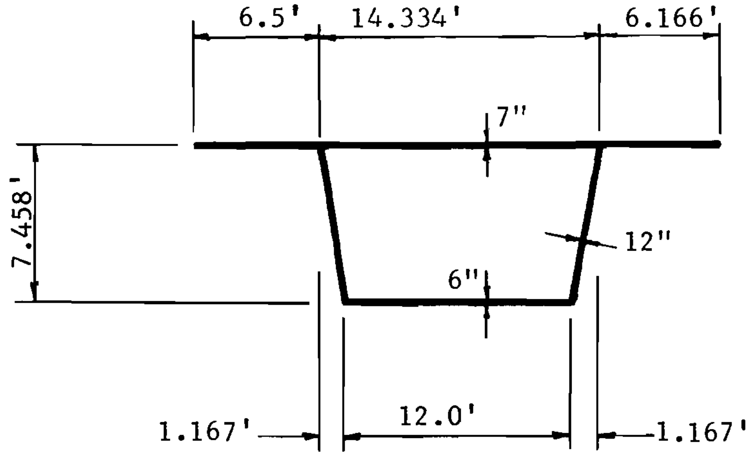


(a) Section for dead load

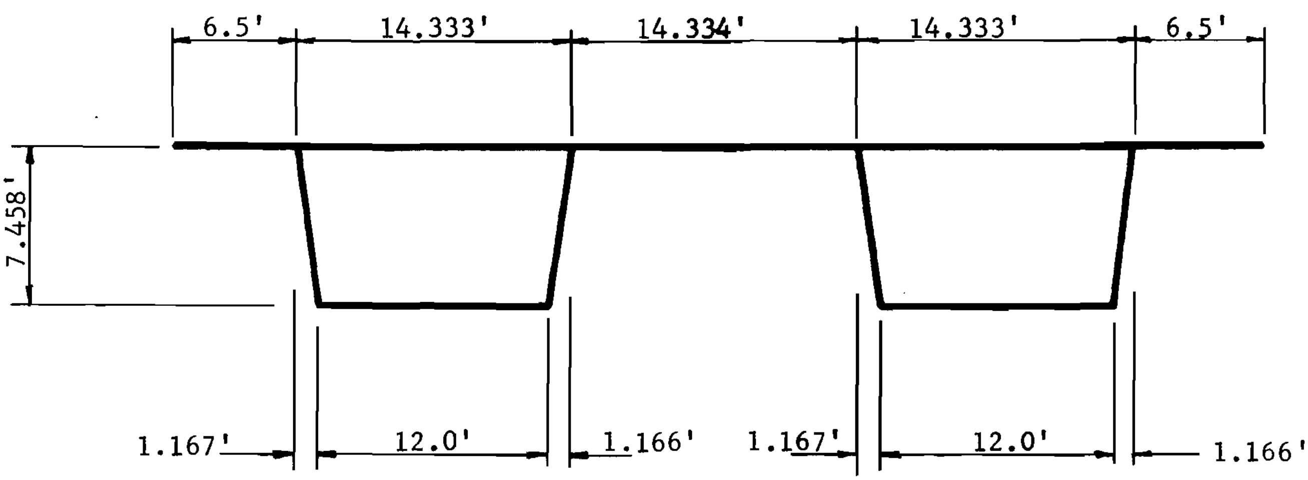


(b) Section for live load

Fig. 4.11. Idealized maximum sections



(a) Section for dead load



(b) Section for live load

Fig. 4.12. Idealized minimum sections

(c) Full lane loads on two adjacent spans
(d) Lane loads for maximum moment 40 ft. from center of bridge
(e) Full truck loads for maximum moment in side span
(f) Truck loads on one side (two lanes) of side span for maximum moment
(g) Lane loads on main span for maximum shear
(h) Lane loads on one side (two lanes) of main span for maximum shear
(i) Lane loads on two inner lanes of main span for maximum shear

Examination of the computer output revealed that under each of the loadings considered, all stresses in the concrete were within the permissible limits. However, in order to provide a greater factor of safety against tensile cracks, it was decided to increase the size of the bottom cables in the main span to 8 strands and that of the bottom and centroidal cables in the side span to 6 strands. In this way the minimum compressive stresses in the bottom slab under the most severe live load conditions are increased from about 0.023 to 0.118 ksi in the main span and from 0.035 to 0.077 ksi in the side span.

The final layout of the cables is shown in elevation in Fig. 4.13 and in section in Fig. 4.14.

The dead load analysis was then repeated for the altered cable sizes. All stresses were found to be satisfactory. Flexural stress distributions across the section at the main pier and the section at the bridge center are shown in Figs. 4.15 and 4.16 for dead and live load. The characteristics of these stress distributions are similar to those in Chapter 3; shear lag is evident at the pier section, whereas the stresses are almost uniform over each slab at the center of the bridge. Pier section stresses increased by as much as 25 percent over the beam theory computations due to shear lag. The increased stresses on the lower flange at midspan reflect the increased cable sizes.

The critical live load conditions for bending moments are as follows:

Bending moment at center of bridge: Full lane load on main span
Bending moment in side span: Truck load on two lanes

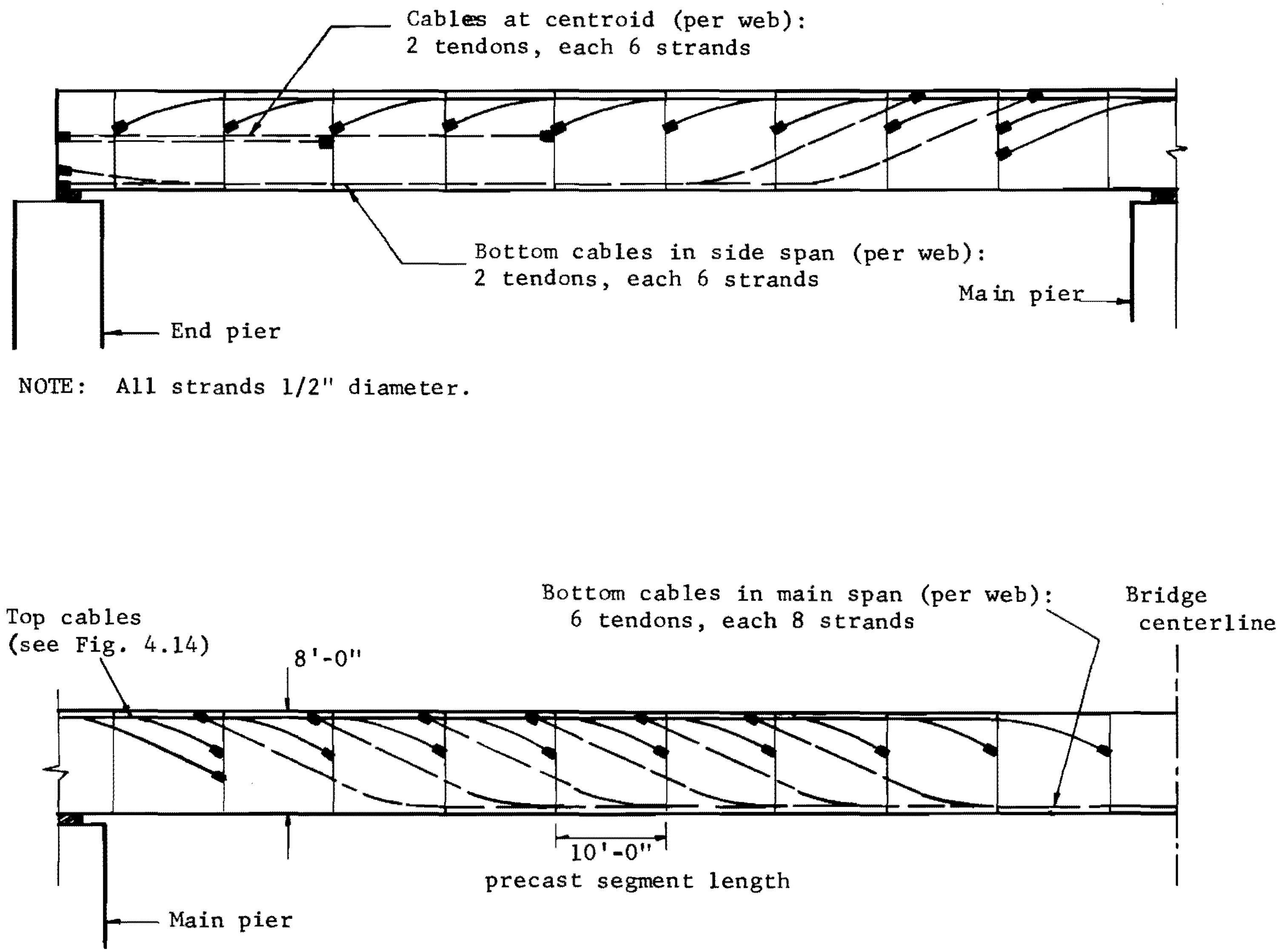


Fig. 4.13. Elevation showing design cable profile

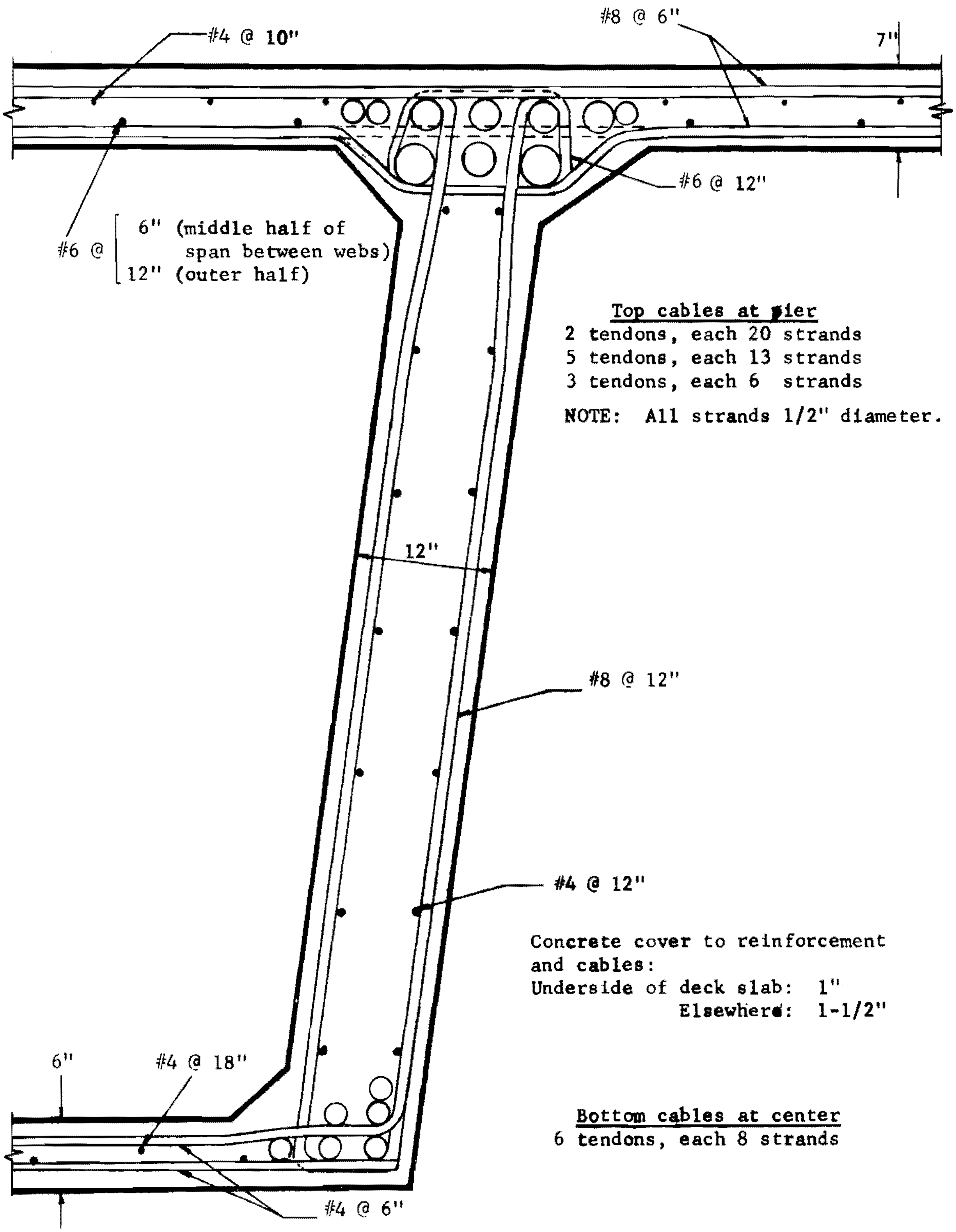
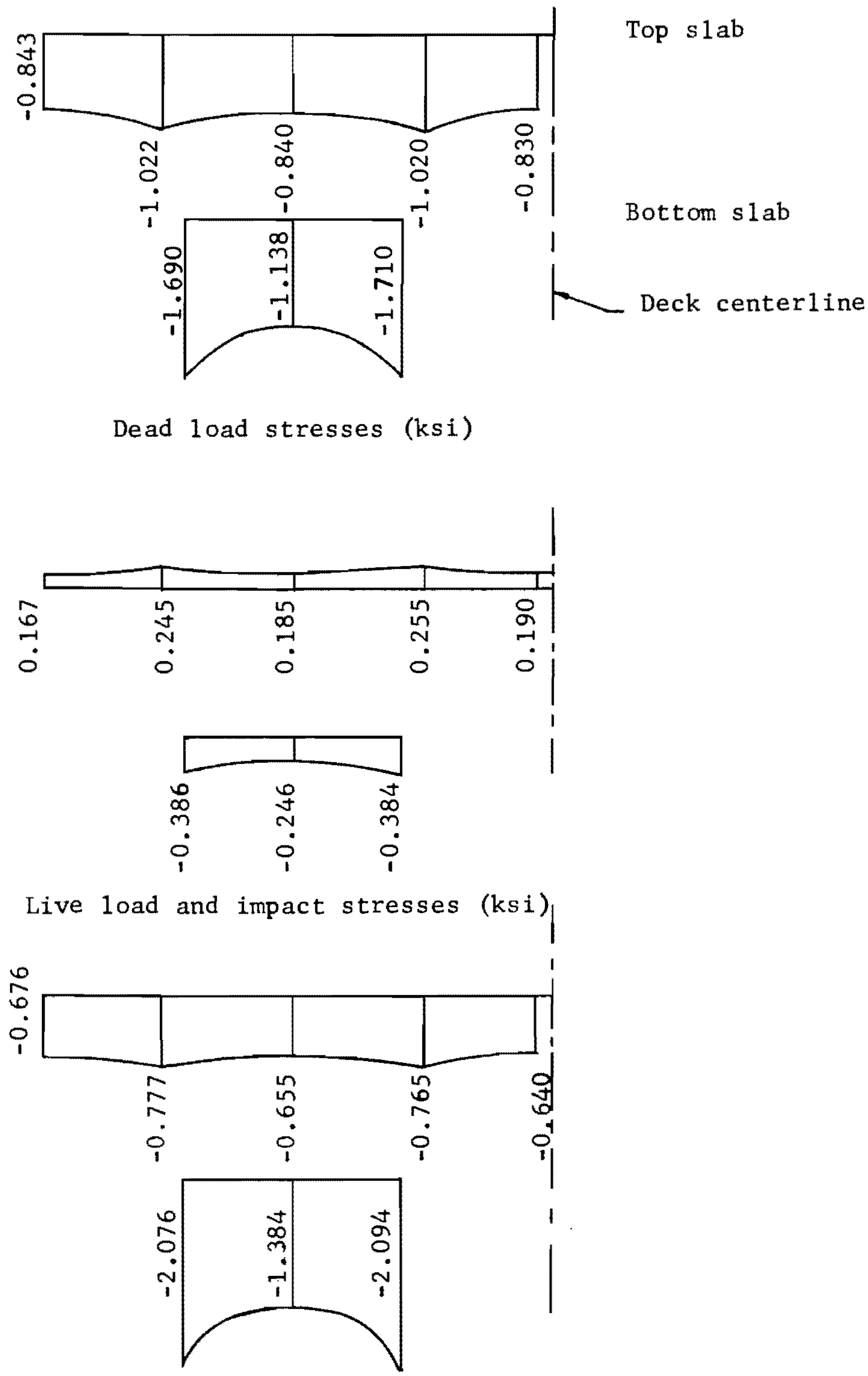
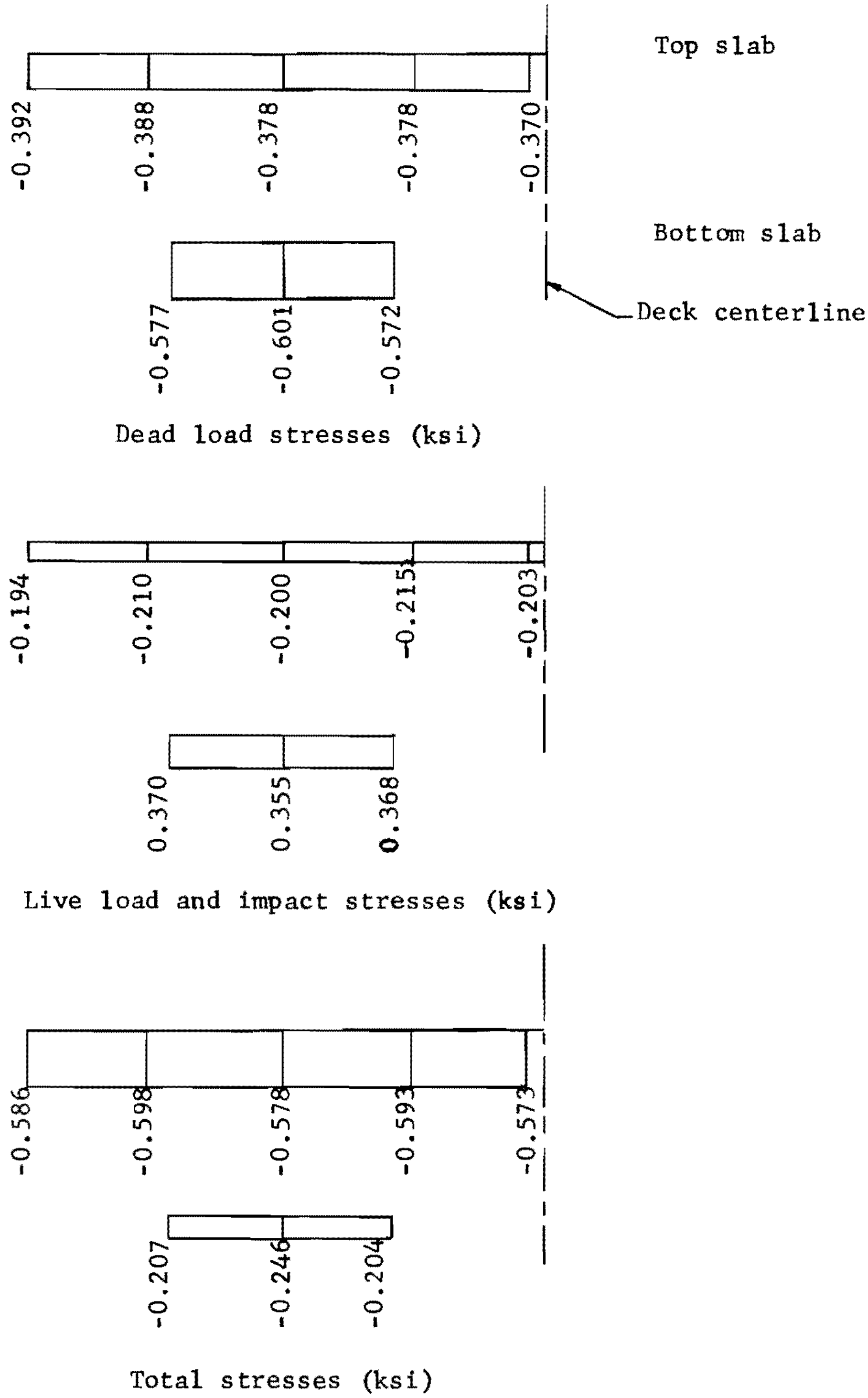


Fig. 4.14. Design cable and reinforcement details



NOTES: Tensile stresses are positive.
Live load consists of AASHO lane load on all 4 lanes of 2 adjacent spans.
The 25% reduction is made for 4-lane loading.

Fig. 4.15. MUPDI analysis stress distributions in three-span bridge at main pier



NOTES: Tensile stresses are positive.
Live load consists of AASHO lane load on all 4 lanes of main span.
The 25% reduction is made for 4-lane loading.

Fig. 4.16. MUPDI analysis stress distributions in three-span bridge at center of main span

If it were not for the 25 percent reduction, four lane loading would be critical in all cases.

The shear stresses in the four webs are approximately equal under dead load and under live load on all four lanes. For live load on two lanes only they are not equal. The maximum shear stress in the main span occurs under two-lane loading on one side of the deck.

When the maximum shear stress occurs under a load condition that produces unequal shears, as in this case, then the reinforcement for each web may be designed individually for the maximum shear possible in that web. An alternative, simpler procedure is to design all four webs together under a uniform load that produces shears in all of the webs equal to the maximum value that can occur in any one of them. This latter approach was adopted in the design of the webs in Sec. 4.8.8. Four-lane loading without the 25 percent reduction was used as this loading causes shears slightly greater than the maximum occurring under the critical two-lane loading condition.

It is to be noted that although dead load and uniform live load produced an equal distribution of shears among the webs in this particular design example, this may not always occur. In general, the distribution will depend on the geometry of the cross section.

Deflections in the bridge are also given in the computer output. The deflections at the center of the bridge are as follows:

Deflection under dead load:	0.216 in.
Deflection under live load on main span (with impact):	0.665 in.
Total deflection:	0.881 in.

The effect of creep on the concrete modulus is not included, although if felt significant it could be examined in a MUPDI analysis. The deflection/span ratio under full load is approximately 1/2,700. This is well within the limit of 1/300 normally considered acceptable. The structure is quite stiff.

Examination of behavior under minimum positive moment conditions in the main span indicated satisfactory behavior. Muller[19] indicates midspan top cables are sometimes needed across the closure section if moment reversal is possible.

4.8.11 Stresses during Tensioning of Main Span Bottom Cables. As described in Sec. 4.3, the first of the main span bottom cables is inserted and tensioned after jacks have been set under the ends of the superstructure and the closing segment has been placed at the center. At this stage the bridge behaves as a continuous girder, although the end supports have not been raised to their final position.

As the bottom cables are tensioned, there is a tendency to produce a tensile stress in the top of the girder at midspan. To reduce this tendency the longest cables are placed first. The stress produced by the cables at this point was calculated as in Sec. 4.8.7, after determining the equivalent load of the cables as in Sec. 4.8.4.

It was found that in order to prevent tensile stresses in the concrete during the placing of the last two sets of cables, the reaction at each end of the superstructure had to be increased by 20 kip (10 kip/box) by means of the jacks after the fourth set of cables had been tensioned.

4.8.12 Friction Losses. The friction losses in the longest cables were calculated using the program developed by R. Brown.

It was found that the assumed effective prestress of $0.6f'_s$ was realistic if the conduits consist of rigid thin wall metal tubing.

4.8.13 Diaphragms. Diaphragms inside the box sections are required at each of the bearings to maintain the shape of the cross section and to provide concrete bearing capacity. A 6-in. thickness is adequate. no intermediate diaphragms were indicated as necessary from the MUPDI analysis.

4.8.14 Prestressing System Details. When the actual post-tensioning system is selected for the project (usually following selection of a contractor), the prestressing system details will have to be closely examined. Anchorage locations, dimensions, and auxiliary reinforcement to control bursting, spalling, and splitting stresses should be checked by the designer.

4.8.15 Incremental Computer Analysis of Construction Sequence Stresses. With the completed structural plans, a final check was made of the structure to examine stresses and deflections under each stage of construction through closure and setting of permanent supports. The sequential analysis program SIMPLA2 was utilized as described in Report 121-4. All proposed details and operations were input to the program and an incremental folded plate analysis used to include effects of shear lag, warping, and construction sequence. This program transitions smoothly from cantilever to continuous structure so that its results are more consistent than the MUPDI analysis. The complexity of input procedures and extensive running time required restrict practical use of the program to the final check stages.

Typical stress calculation results are shown in Figs. 4.17 through 4.20. These calculations indicate that dead load stresses are much closer to those calculated for beam theory assuming continuity than for complete cantilever action. Figures 4.19 and 4.20 are particularly informative, showing the critical stages for flange stress to occur at widely different stages for top and bottom flanges.

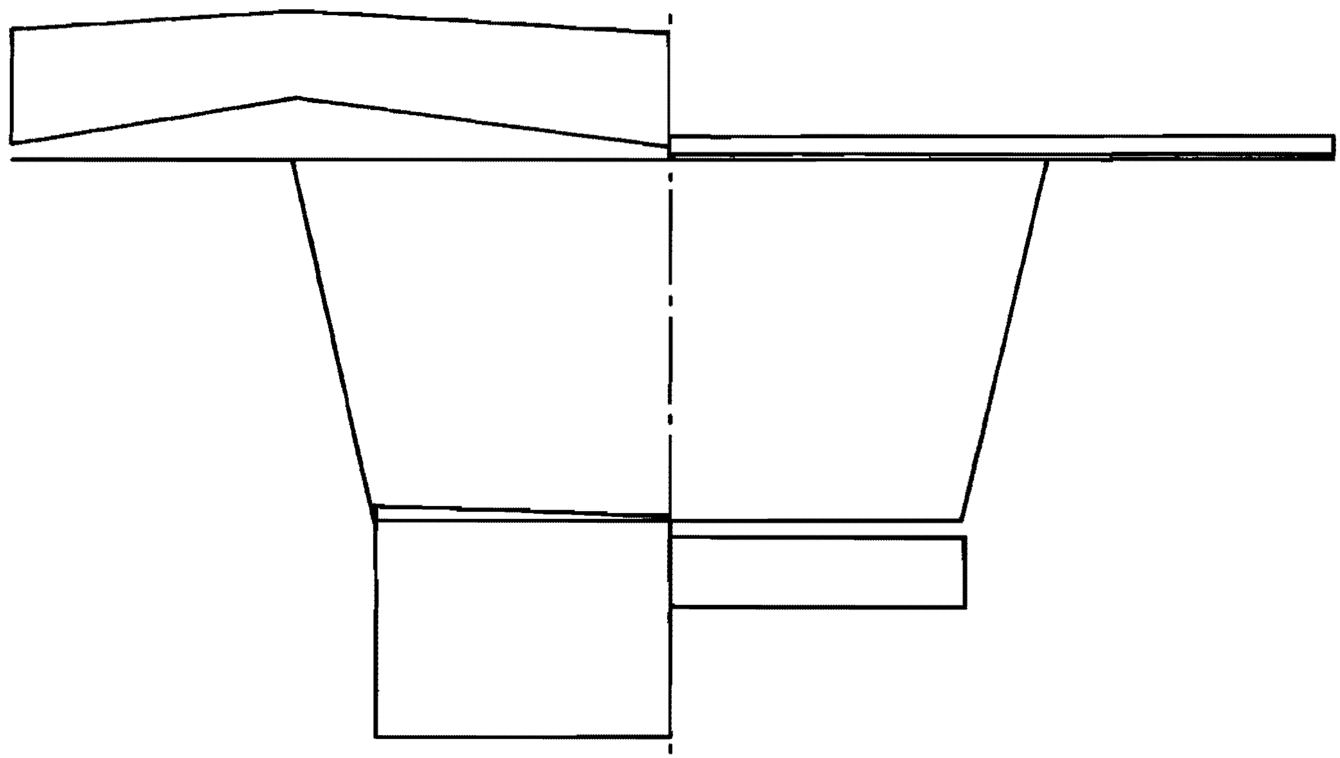
Typical displacement results are shown in Figs. 4.21 and 4.22. These figures illustrate the advantage of the incremental analysis which has the ability to track all major construction operations. This analysis indicated a slightly lower closure reaction desirable for geometric compatability.

4.9 Summary of Design Procedure

The principal stages of design are as follows:

(a) An approximate cross section shape is chosen.--This can be based on the result of an optimization study as described in Chapter 7. Alternatively, a preliminary design may be carried out.

		Pier			Closure Strip	
Maximum	-.924	-1.042	-.903	-.174	-.174	-.174
Minimum	-.125	-.417	-.118	-.021	-.021	-.021



Maximum	-1.250	-1.264	-.660	-.660
Minimum	+.069	+.014	-.104	-.104

Fig. 4.17. Stress envelopes during erection (ksi)

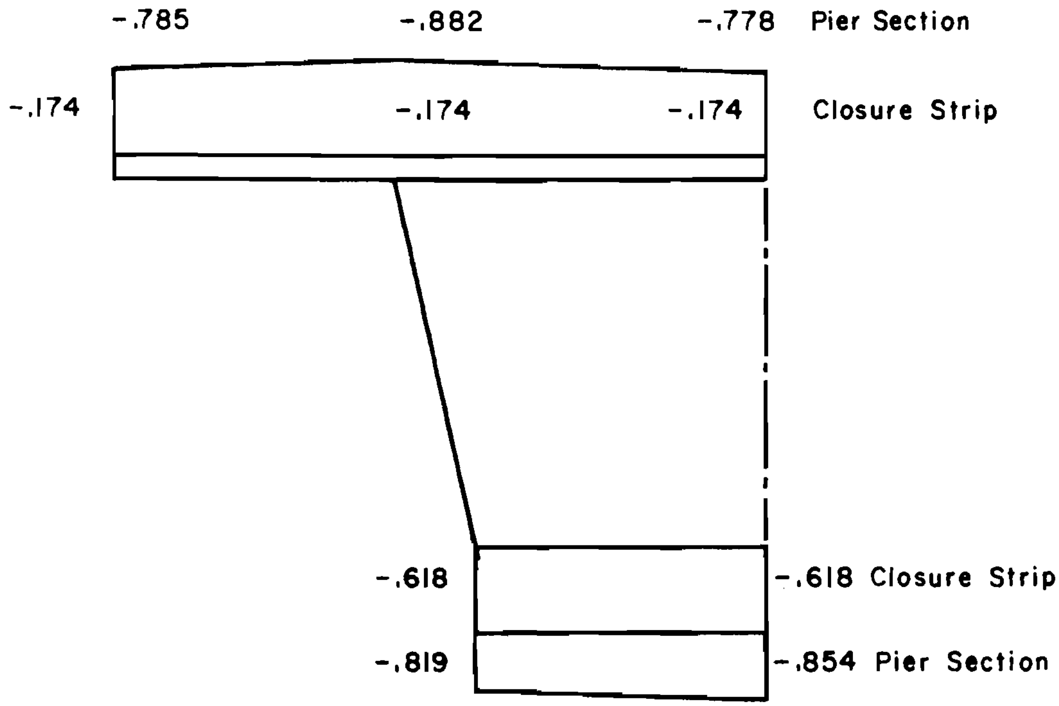


Fig. 4.18(a) Stress distribution at completion (ksi)

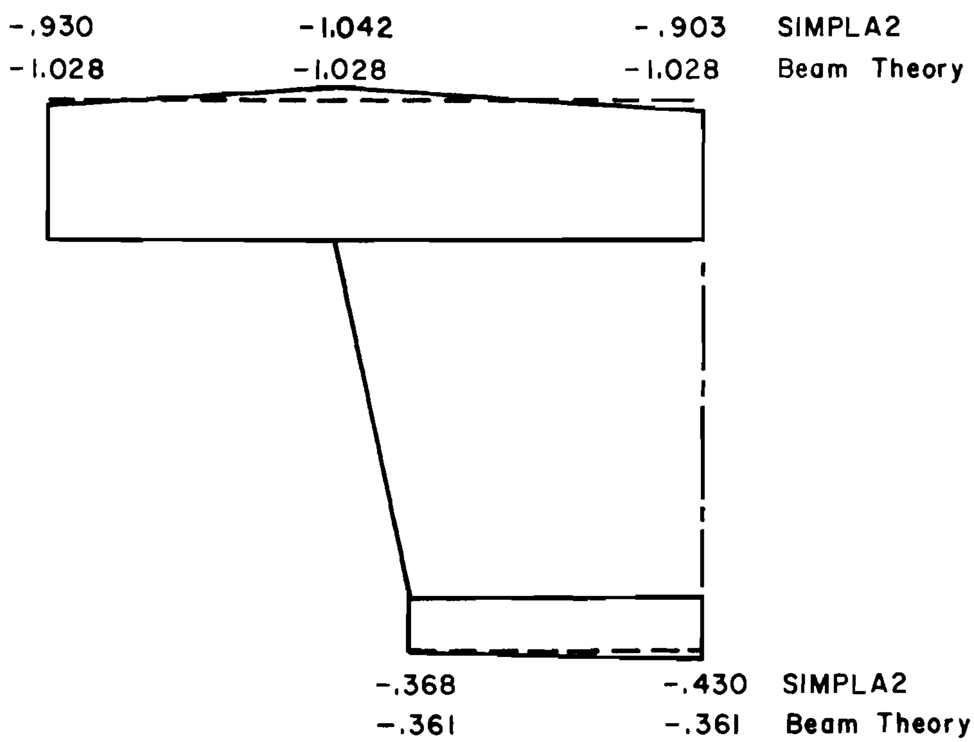


Fig. 4.18(b) Stress at pier section at completion of Stage 6 - comparison to beam theory (ksi)

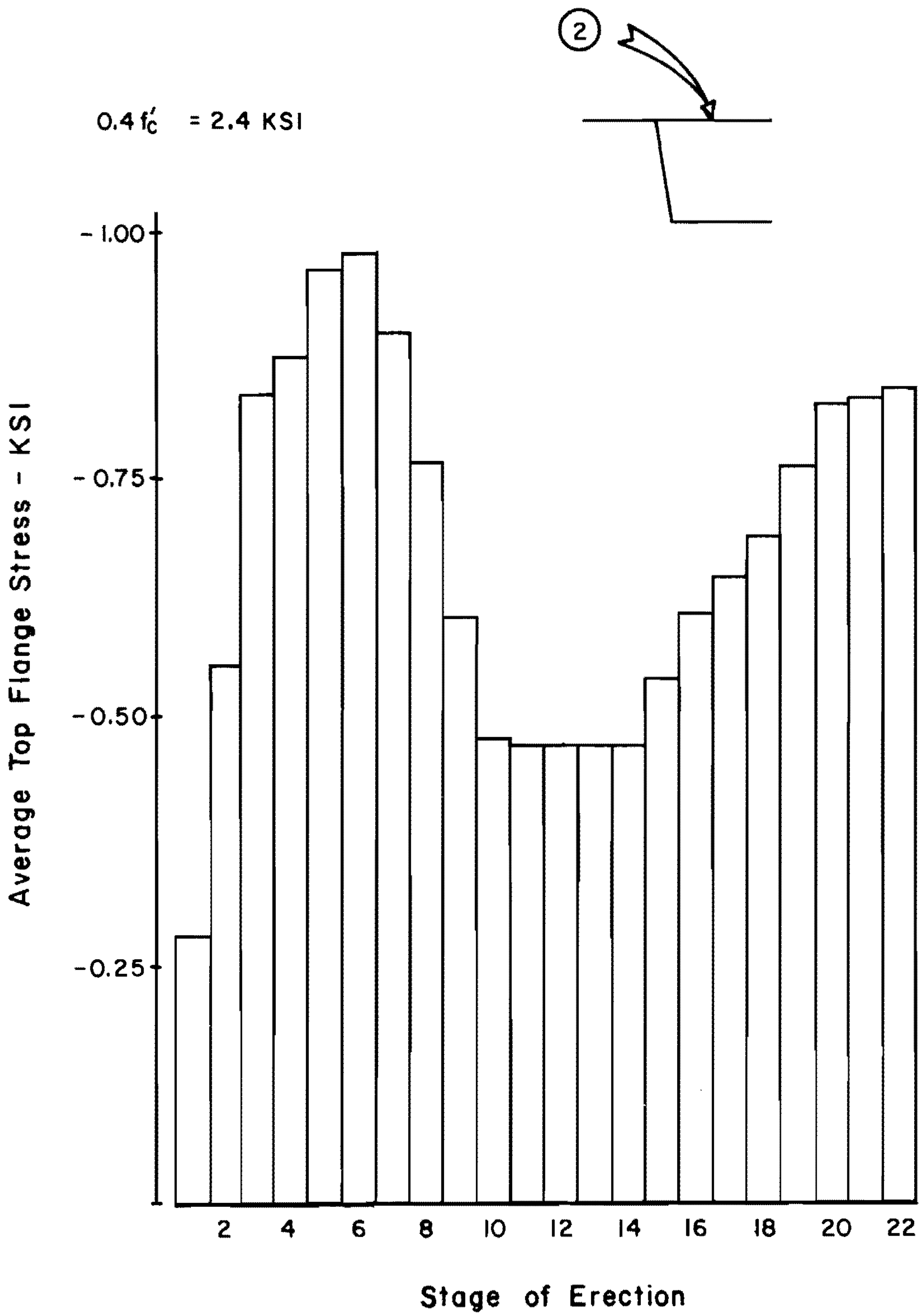


Fig. 4.19. Variation of average top flange stress at pier section

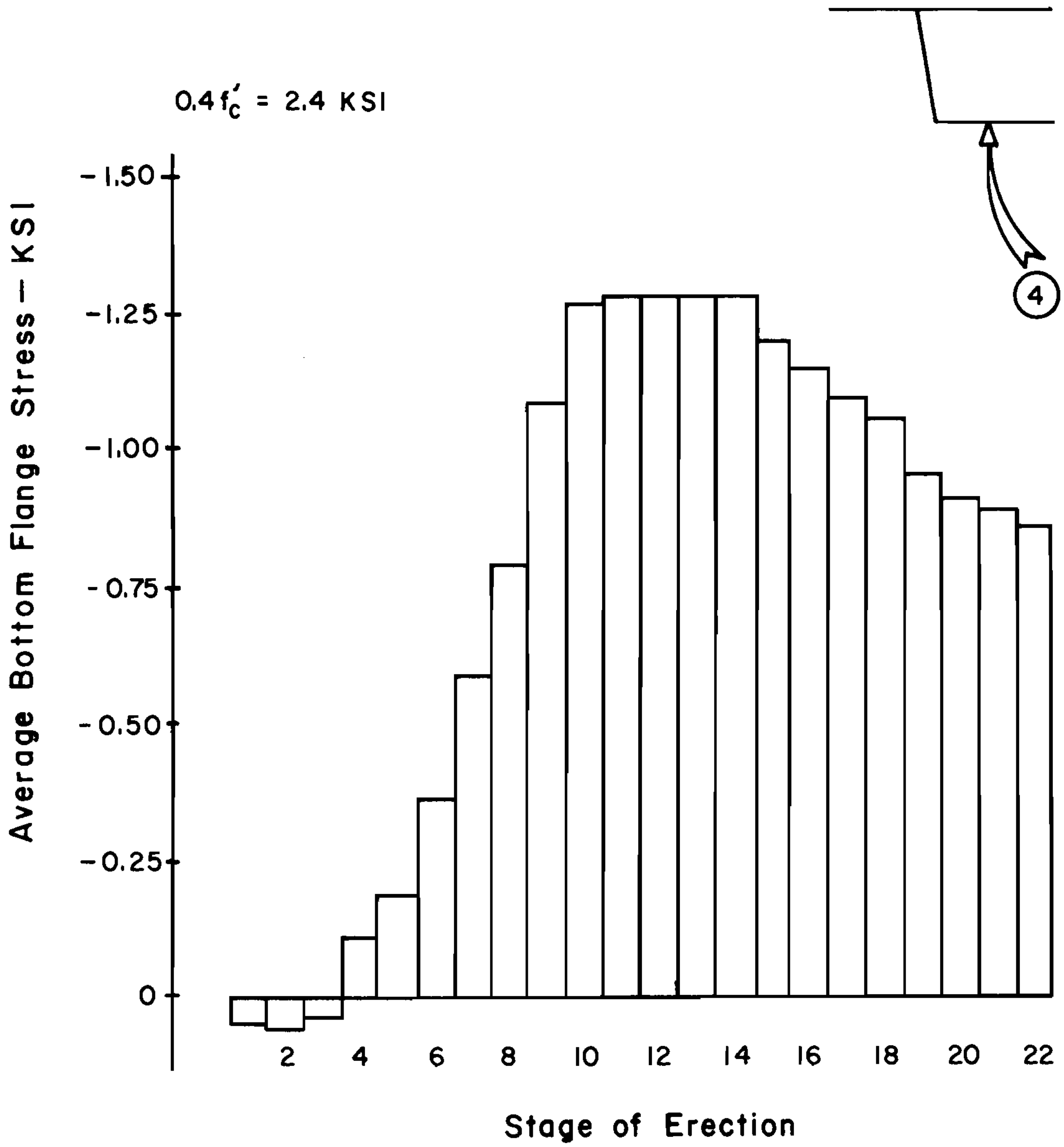


Fig. 4.20. Variation of average bottom flange stress at pier section

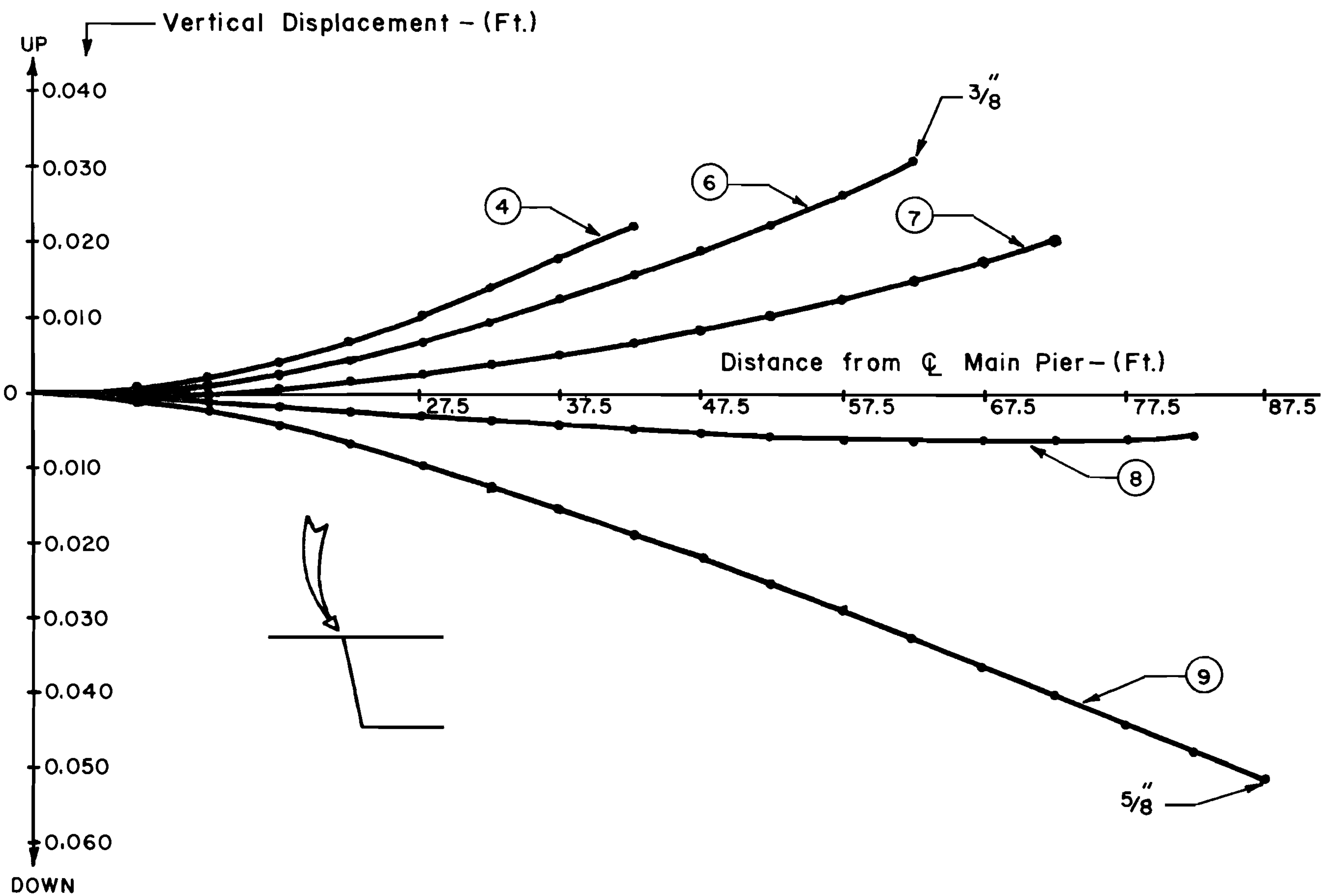


Fig. 4.21. Intracoastal Canal Bridge--displacements during erection

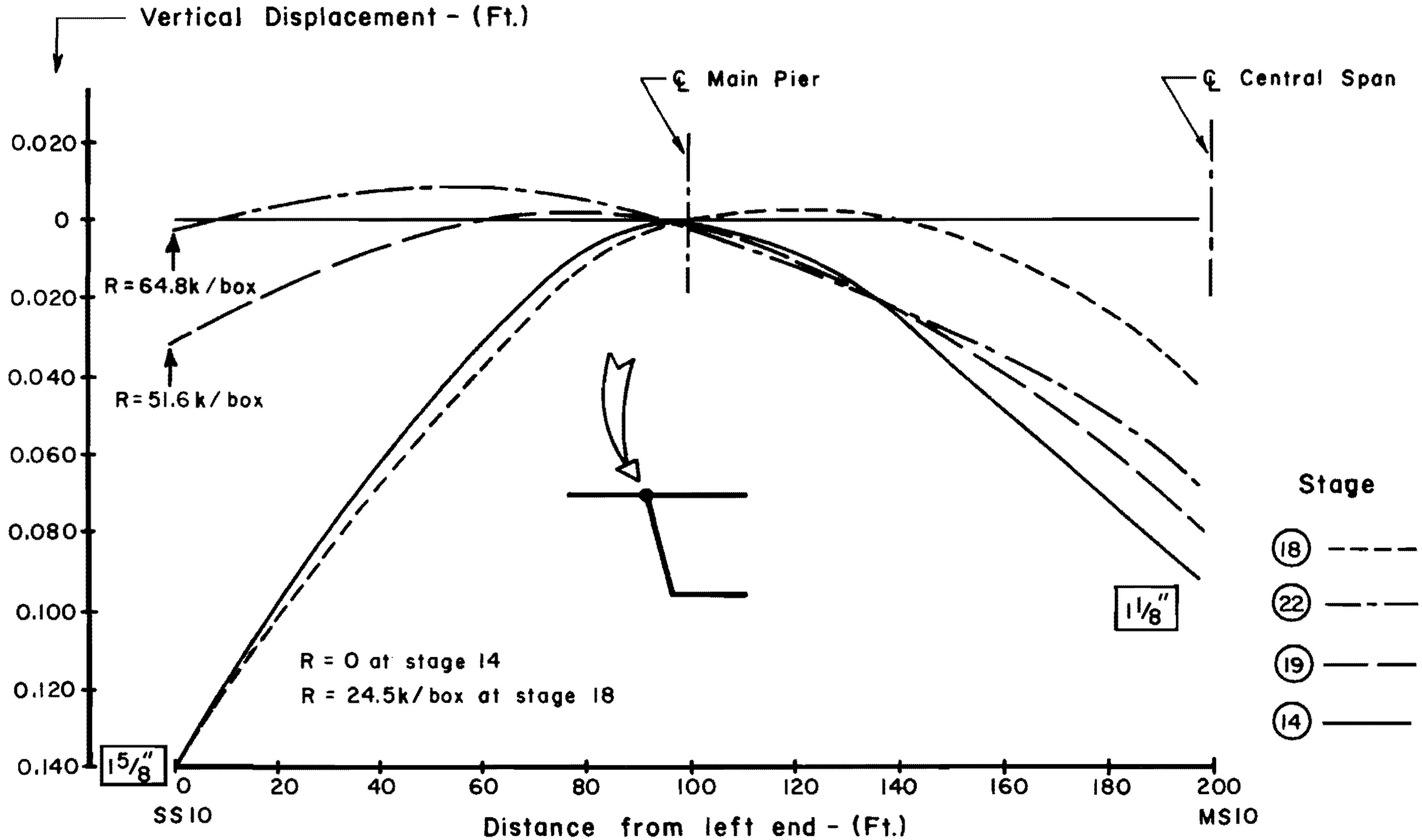


Fig. 4.22. Elastic curve at various stages of closure

(b) The cross section is designed in detail.--The deck slab thickness and reinforcement are determined by wheel load moments. The web thickness must be sufficient to accommodate the cable anchors. A preliminary shear check is advisable to ensure adequate web thickness is provided.

(c) The top cables are designed for cantilever construction.--The cables are chosen at each section to balance a uniform load, about 60 percent of the dead load. They must provide adequate ultimate load capacity and ensure acceptable service load stresses in the concrete. The bottom slab of the girder must be made thick enough to give adequate ultimate load capacity.

(d) The bottom cables are designed for ultimate load on the completed superstructure.--A simple pattern of bottom cables is chosen for each span and the quantity adjusted to give adequate ultimate strength under the critical live load.

(e) The concrete service load stresses are computed from beam theory.--The stresses under dead and live load are obtained for the critical sections. If necessary the bottom cables are revised.

(f) The ultimate shear strength is checked.--The ultimate shear force, the capacity of the webs and the reinforcement required are calculated at all of the critical sections. Web thickness is adjusted upward if necessary.

(g) The completed superstructure is analyzed with the MUPDI program.--The stresses are determined under dead load and under various live loads. If necessary the bottom cables are revised and the analysis repeated.

(h) The completed design is analyzed with the SIMPLA2 program.--Stresses and deflections are determined at all stages of construction to verify the design.

4.10 Other Examples of Bridges Constructed in Cantilever

Other cases of bridges constructed in cantilever, requiring variations in the design procedure adopted in the example chosen are considered briefly.

4.10.1 Multi-Cell Box Girder. An alternative cross section for the bridge considered is a three-cell box girder, cast in full width sections.

The design procedure for this case is almost identical with that already outlined. However, a different program or manual computations must be prepared to compute the cross section properties. Other advantages and disadvantages are as given in Sec. 3.9.1.

4.10.2 Segments Lifted from Bridge Superstructure. In the construction procedure considered in Sec. 4.3, the segments were lifted into position by a floating crane. An alternative method is to lift them by a hoist on the partially completed superstructure.

In this case the impact load on the cantilever should be much higher (possibly 100 percent) and the live load and impact moments may constitute a substantial fraction of the total moment during construction. If this is so, it is probably best to design the top cables to balance a uniform load together with a concentrated load at the ends of the cantilevers (i.e., at the center and ends of the completed bridge).

4.10.3 Superstructure Rigidly Connected to Pier. Instead of the final simple support system considered so far, it is possible to have the segments above the main piers permanently rigidly fixed by vertical prestressing cables.

This will considerably modify the construction procedure and hence the design. The cantilever erection process will not be altered, but because of the fixity at the main piers it will no longer be possible to adjust the moments in the completed structure simply by jacking at the ends. Before closure at midspan, flat jacks will have to be inserted between the final segments at deck level and pressure applied to induce a positive moment.

Closure and placing of the main span bottom cables may be done before placing the side span bottom cables if desired.

4.10.4 Side Span Greater Than Half Main Span. If the side span is greater than half the main span, the final segments in the side span cannot be readily erected by the cantilever method.

The simplest procedure is to erect the superstructure by cantilever on either side of the main piers to a distance of half the main span (minus the gap for the closing segment). The remaining segments in the side spans can be erected on falsework. Closure and insertion of the main span bottom cables can be done either before or after completion of the side spans, depending on the details of the structural system.

4.10.5 Continuous Viaducts. The construction and design of viaducts, comprising a large number of equal continuous spans presents no special difficulties. However, provision must be made for expansion and careful attention paid to joint location and pier-girder connections. Muller[20] treats this problem in some detail.

CHAPTER 5

METHODS OF OPTIMIZATION

In the previous two chapters criteria were developed for the design of bridges, for which the spans, overall width, and construction method were specified. The basic dimensions of the cross section apart from the overall width were chosen somewhat arbitrarily on the basis of experience and initial trial designs. The problem to be considered in this and the following two chapters will be that of determining the dimensions that will lead to a design having "minimum" cost.

To obtain this minimum cost design, i.e., to "optimize" the design, the structural problem must be expressed in mathematical terms. A standard mathematical problem is that of minimizing a function $f(x_1 \ . \ . \ . \ x_n)$ of a set of variables $x_1 \ . \ . \ . \ x_n$. Such a problem is called "mathematical programming" or "nonlinear programming". The function f is called the "objective function".[30,31,34]

Nonlinear programming problems may be subdivided into unconstrained and constrained problems. In an unconstrained problem the variables can take on any value. In a constrained problem the constraints may be either equality constraints, of the form

$$h(x_1 \ . \ . \ . \ x_n) = 0$$

or inequality constrains, of the form

$$g(x_1 \ . \ . \ . \ x_n) \geq 0$$

A simple example of an inequality constraint would be the following:

$$x_1 \geq \text{a constant}$$

i.e., there is a lower limit to x_1(for instance, a minimum web thickness).

In this chapter only unconstrained problems will be considered, as it will be shown later that the optimization of the bridge superstructure can be treated as such a problem. This is fortunate, because the solution of constrained problems is much more complex.

5.1 Notation and Definitions

$\underline{x}$ is the column vector $[x_1 \; . \; . \; . \; x_n]^T$

The optimal point: $\underline{x}^*$, is the vector $\underline{x}$ which minimizes the objective function, $f(\underline{x})$.

The optimal value: $f(\underline{x}^*)$, is the corresponding value of the objective function.

The optimal solution: comprises the optimal point and the optimal value.

A global optimal solution represents the smallest value of $f(\underline{x})$ for all $\underline{x}$.

A local optimal solution represents the smallest value of $f(\underline{x})$ in a limited region only (see Fig. 5.1).

A unimodal function has only one optimum.

A contour of the objective function is the set of points for which this function has a constant value.

The gradient, $\nabla f(\underline{x})$ of the objective function at any point is a vector pointing in the direction of maximum increase of the function and is given by

$$\nabla f(\underline{x}) \;=\; [\partial f/\partial x_1 \; . \; . \; . \; \partial f/\partial x_n]^T$$

The gradient exists if the objective function is continuous and differentiable.

5.2 Unconstrained Minimization Using Derivatives

There are various numerical methods of solving the nonlinear programming problem without constraints. These methods may be divided into two classes, those that use derivatives of the objective function and those that use values of the function only. The methods using derivatives are generally the more efficient, provided of course the derivatives exist and can be easily calculated. These methods will be examined first.

5.2.1 Gradient Methods. In gradient (or "steepest descent") methods,[28,30,31] an initial starting point is chosen and the gradient

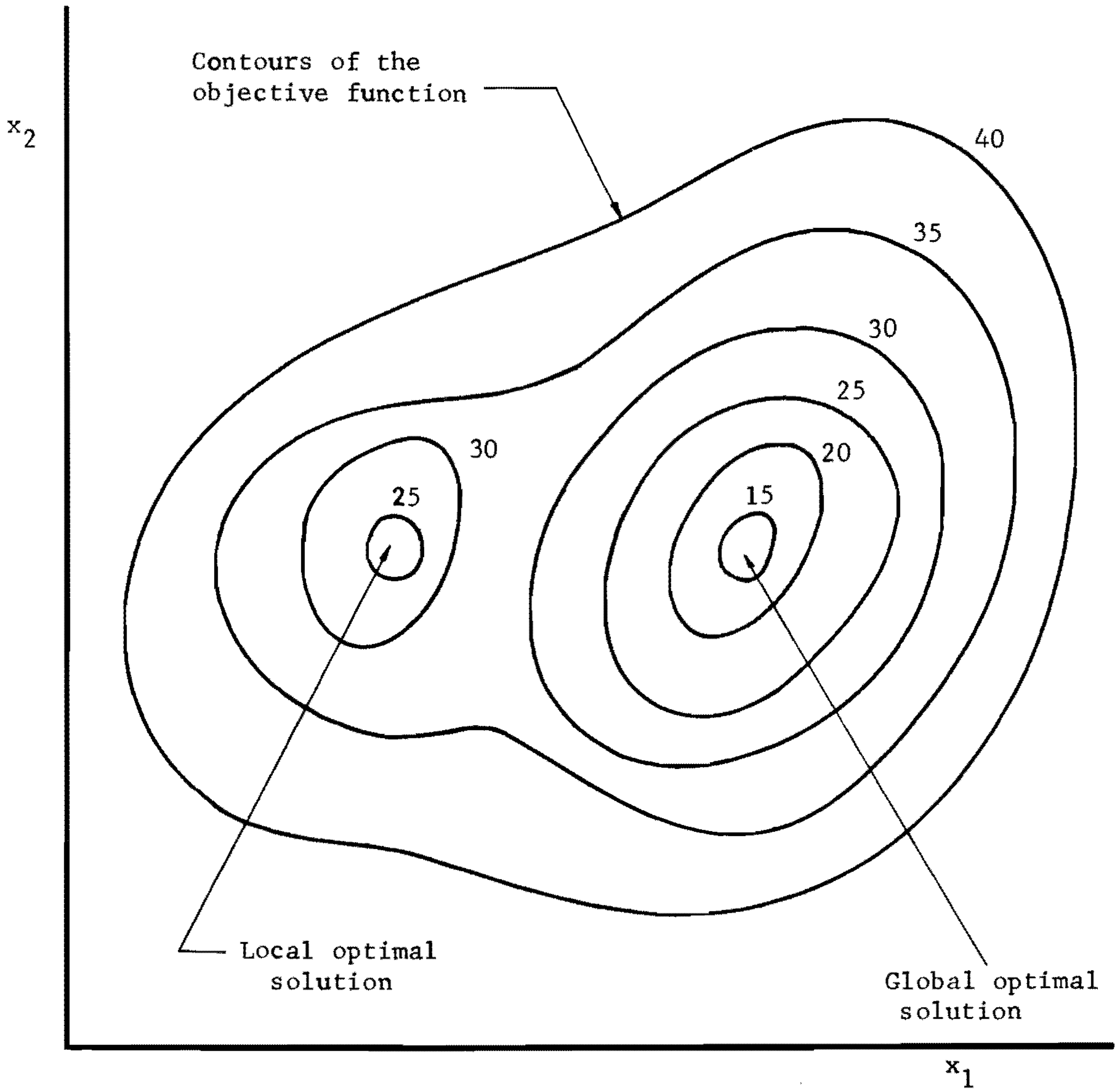


Fig. 5.1. Local and global optimal solutions

$\nabla f(\underline{x})$ at that point calculated. A step is taken in the direction of steepest descent, i.e., the reverse of the gradient, and a new point obtained. This point may be chosen, for example, by searching along the direction of steepest descent until a minimum value of $f(\underline{x})$ along this line is reached. The gradient is then calculated at the new point and another step taken. The process continues until an optimal is reached to within some tolerance, i.e., until some appropriate stop criterion is satisfied.

The gradient methods are relatively simple. However, they converge very slowly and are inefficient. With some functions which have very irregular contours, they can never reach a solution.

5.2.2 Second Order Methods. Second order methods are those which will minimize a quadratic function in n steps or less, where n is the number of variables.[30,31] Such methods are much more efficient than the gradient methods and will usually converge for a general objective function.

Some of the most powerful of these methods make use of "conjugate directions. It can be shown that a quadratic function

$$f(\underline{x}) = \underline{x}^T \underline{A}\, \underline{x} + \underline{b}^T \underline{x} + c$$

can be minimized in n steps by searching along each of a set of $\underline{A}$-conjugate directions (in a manner similar to that described for gradient methods in the preceding section). The Fletcher-Powell method,[29] is perhaps the most powerful of the second order methods.

5.3 Unconstrained Minimization without Using Derivatives

Although the methods using derivatives are generally the most efficient, sometimes continuous derivatives may not exist or may not be readily calculated as in the box girder optimization problems. For such cases, derivative-free methods, also known as "search" methods, must be used to find the optimal solution of the nonlinear programming problem.[30] Two of the most efficient of the search methods are Powell's method and the Nelder-Mead method.

5.3.1 Powell's Method. In Powell's method[33] the minimum of an objective function of n variables is located through a series of iterations, each of which involves a search for a minimum along a set of n linearly independent directions. These directions are the coordinate directions initially, but at each iteration a new direction is defined to replace one of the initial directions. The new directions formed after a series of iterations will be mutually conjugate if the objective function is quadratic.

One iteration of the procedure is as follows. Let $\underline{x}_o$ be the starting point and let $\underline{s}_1$, $\underline{s}_2$, . . . $\underline{s}_n$ be the search directions.

(a) Carry out the search for a minimum along each of the n directions; i.e., for r = 1, 2 . . . n calculate λ_r so that $f(\underline{x}_{r-1} + \lambda_r \underline{x}_r)$ is a minimum. Define $\underline{x}_r = \underline{x}_{r-1} + \lambda_r \underline{x}_r$ where $\lambda_r \underline{x}_r$ is the step size.

(b) Define a new direction to replace one of the initial directions, thus: for r = 1,2, . . . (n-1) replace $\underline{s}_r$ by $\underline{s}_{r+1}$ and replace $\underline{s}_n$ by $(\underline{x}_n - \underline{x}_o)$.

(c) Define a new starting point for the next iteration, thus: choose λ so that $f[\underline{x}_n + \lambda(\underline{x}_n - \underline{x}_o)]$ is a minimum and replace $\underline{x}_o$ by $\underline{x}_o + \lambda(\underline{x}_n - \underline{x}_o)$.

It can be shown that, if the objective function is quadratic, after k iterations the last k of the n directions chosen for the (k+1)th iteration is mutually conjugate. After n iterations all the directions are mutually conjugate and the exact minimum of the quadratic is found.

Powell has added some modifications to the procedure to ensure rapid convergence for more difficult objective functions and poor starting points. The method appears to be very efficient in general. The method has been programmed in FORTRAN as program OPTMSE and listed in Appendix C.1.

5.3.2 Nelder-Mead Method. In the Nelder-Mead method[32] the optimal solution for a problem involving n variables is obtained by a search procedure using a "simplex". This is defined by a set of (n+1) points in the n-dimensional space of the variables. In a two-dimensional space the simplex is defined by three points forming a triangle; in a three-dimensional space by four points forming a tetrahedron, etc. Over successive iterations the simplex is modified by processes of "reflection", "expansion", and "contraction", to be defined; eventually it becomes smaller and smaller and converges on the optimal point.

The following definitions are made:

Let P_o, P_1, . . . P_n be the (n+1) points defining the current simplex,

y_i = the value of the objective function at each P_i,

H = the suffix such that $y_H = \max (y_i)$,

L = the suffix such that $y_L = \min (y_i)$,

$\bar{P}$ = the centroid of the points excluding P_H,

$[P_iP_j]$ = the distance from P_i to P_j.

The steps in one iteration of the method are as follows.

(a) Reflection. The reflection of P_H, denoted by P', is obtained. Its coordinates are defined by the relation

$$P' = (1 + a) \bar{P} - a P_H$$

where the "reflection coefficient", a, is usually taken as 1.0. If y' lies between y_H and y_L, the P_H is replaced by P' to form a new simplex.

(b) Expansion. If $y' < y_L$, then P' is expanded to P'' by the relation

$$P'' = cP' + (1 - c) \bar{P}$$

where the "expansion coefficient", c, is generally taken as 2.0. If $y'' < y_L$, replace P_H by P'' and start the next iteration. Otherwise, replace P_H by P' and restart.

(c) Contraction. If in stage (a) it occurs that $y' > y_i$ for all $i \neq H$, define a new P_H to be either the old P_H or P', whichever has the lower y value, and contract P_H, thus

$$P'' = bP_H + (1 - b) \bar{P}$$

where the "contraction coefficient", b, is usually taken as 0.5. Then replace P_H by P'' and start the next iteration, unless $y'' > \min (y_H, y')$, in which case replace all the P_i's by $(P_i + P_L)/2$ and restart.

The iteration process continues until a stop criterion is satisfied, indicating convergence on a minimum. A flow chart taken from Ref. 32 is shown in Fig. 5.2.

The method appears to be very efficient for a wide range of objective functions. The method has been programmed in FORTRAN as program SIMPLEX and is listed in Appendix C.2.

5.4 Limitations of the Methods

The second order methods using derivatives and the two search methods presented generally give rapid convergence and good solutions for problems in which the contours of the objective function are fairly regular. When the contours are very irregular, solution will be more difficult. However, these methods have been found to give a reasonable rate of convergence, even in several test cases having irregular contours such as the function

$$f(x_1, x_2) = 100(x_2 - x_1)^2 + (1 - x_1)^2$$

known as Rosenbrock's parabolic valley.

When the objective function is not unimodal, there is no guarantee that the solution obtained will be the global optimum rather than a local optimum.[30] The Nelder-Mead method is considered most likely to terminate at a global optimum, provided the initial points defining the simplex are widely dispersed. In general, when it is known or suspected that there is more than one optimum, a safe procedure is to compare solutions obtained using widely different starting points.

In some cases the gradient of the objective function is very flat in a wide region around the optimal point. This makes exact determination of the optimal point very difficult, although the actual optimal value of the objective function can be found quite accurately. In such cases it is useful to obtain two-dimensional plots of contours of the objective function, and these may be of more significance than the value of the optimal point itself. When gradients are very flat, a wide choice of dimensions is usually possible.

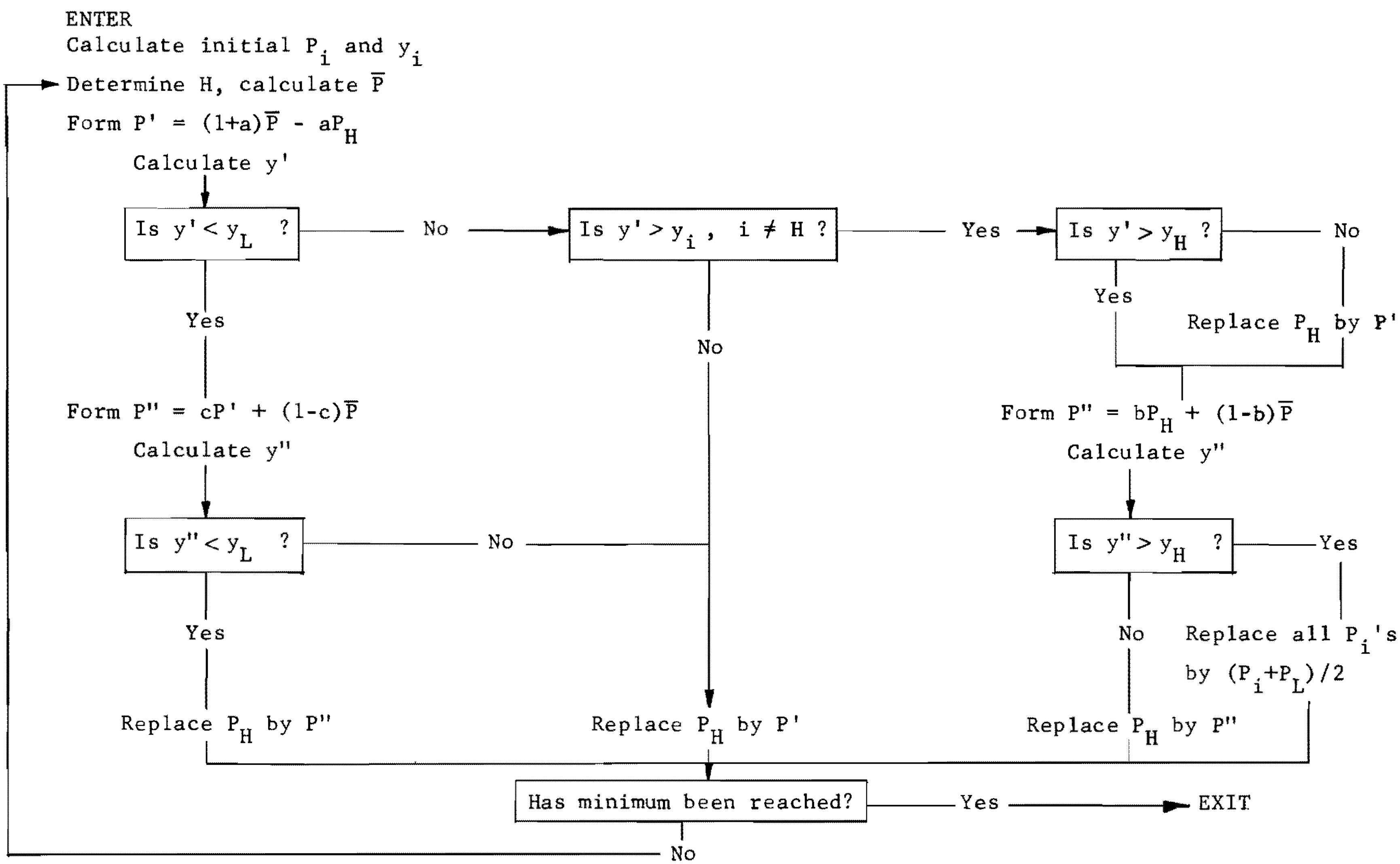


Fig. 5.2 Flow Chart for the Nelder-Mead Method

The choice of a starting point can be important in some cases. If the starting point is far removed from the optimal point, solution may be difficult in the case of very irregular contours. If there are local optima or a very flat gradient, the starting point may influence the solution.

CHAPTER 6

OPTIMIZATION OF BRIDGES CONSTRUCTED ON FALSEWORK

In this chapter the problem of optimizing the cross section of bridges constructed on falsework will be considered. The function to be minimized will be the cost of the bridge. The dimensions defining the basic geometry of the cross section to produce this minimum cost will be determined.

The optimization will be carried out for the sample case of a two-span, four-lane crossover, having the same length as the bridge designed in Chapter 3. Two cases will be solved: (a) a superstructure consisting of a pair of single-cell boxes and (b) one consisting of a multi-cell box girder. The procedure used in these examples can be readily extended to other spans. Extension of the method to bridges having more than two spans will be discussed.

The steps in the optimization procedure are as follows:

(a) A mathematical model of the structure is set up.--The constants, independent variables, and dependent variables for a nonlinear programming problem are defined.

(b) A computer subroutine is developed to calculate the objective function.--The objective function is calculated by carrying out a simplified design of the superstructure and summing the costs of the various components. This process is programmed to form the computer subroutine.

(c) The nonlinear programming problem is solved by the Nelder-Mead method and the Powell method.--Computer programs for both methods are used, together with the objective function subroutine, to obtain the optimal solution of the problem.

6.1 First Example--Two-span, Double Box Girder Bridge

The first example for optimization is shown in cross section in Fig. 6.1. The cross-sectional type, the cable pattern, and the support system are the same as in the example designed in Chapter 3. Each span is 180 ft.

To obtain the optimum cross section the problem must be expressed in mathematical terms, i.e., as a nonlinear programming problem. The various dimensions and quantities will first be somewhat artibrarily classified into constants, independent variables, and dependent variables as follows. All dimensions are in foot units.

(a) Constants:

The spans of the bridge: 180 ft. - 180 ft.

b: overall width

b_6: width of cast-in-place strip

t_4: web thickness (based on anchorage and placement requirements)

Minimum thickness of bottom slab: 6 in.

Thickness of outer edge of deck slab: 6 in.

Segment length: 10 ft.

(b) Independent Variables:

b_1: width of cantilever portion of deck slab

b_2: width of outer internal spans of deck slab

b_3: width of lower slab of each box

d : depth of concrete cross section

(c) Dependent Variables:

b_5: width of central span of deck slab

s : sloping height of web between slabs

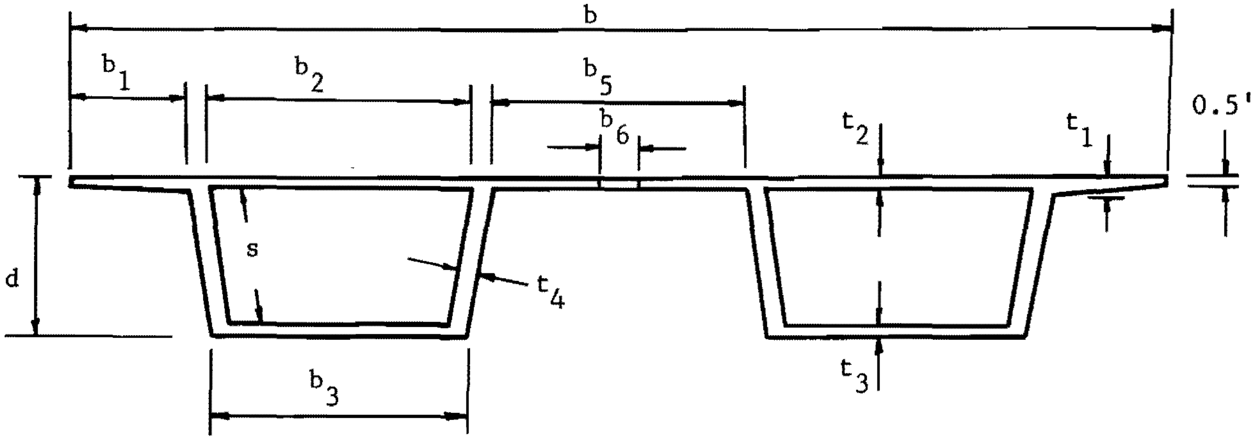
t_1: thickness at root of cantilever portion of deck slab

t_2: thickness of interior portion of deck slab

t_3: thickness of bottom slab at pier

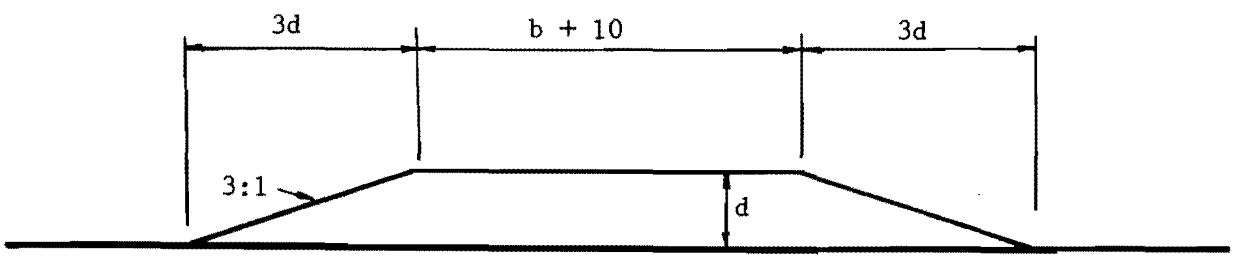
q : fraction of span over which bottom slab is thickened

A_s: cable area



NOTES: Bottom slab thickness tapers from t_3 at pier to 0.5 ft. at 180q ft. from pier.
All dimensions are in feet.
Spans: 180 ft.-180 ft.

Fig. 6.1. Cross section of first example



NOTE: Dimensions in feet.

Fig. 6.2. Profile of roadway embankment

The objective function, denoted by C, will be the cost of the bridge per foot length of superstructure. It is a function of the independent variables, i.e.,

$$C = f(b_1, b_2, b_3, d)$$

The problem will be treated as an unconstrained nonlinear programming problem, i.e., no limits will be set on the values taken by the independent variables. This is the simplest approach possible, and will be justified if a meaningful solution is obtained. From physical considerations it seems likely that there will be such a solution. However, if the resulting dimensions turned out to be physically unfeasible (i.e., d having a negative value), the problem would have to be reformulated as a constrained problem; limits would have to be set on the dimensions, and very different and more complex solution techniques used.

The procedure outlined above was not the only approach considered. Initially the quantities t_1, t_2, and t_3 were included with the independent variables and a constrained nonlinear programming problem formulated. Constraints would take the form of inequalities expressing allowable limits for the concrete stresses. However, it became apparent that with this approach the problem would be so complex as to be probably insoluable. Besides, it became clear that the thicknesses could be treated correctly as dependent variables.

6.2 The Objective Function

A computer subroutine listed in Appendix C.2 was developed to calculate the objective function. This calculation involves the two stages of performing an approximate design of the superstructure and summing the costs of the various components.

The procedure used in the approximate design follows closely that outlined in Chapter 3, except that the later stages are omitted. The basic steps adopted here are as follows:

(a) The deck is designed for wheel loads.

(b) The cables are designed to balance the dead load.

(c) The ultimate moment is calculated, the bottom slab thickness is determined, and the cable area adjusted.

A number of simplifications are made to facilitate the design and to reduce the time taken by the computer to evaluate the objective function. Reducing this time is important since the computer solution of the problem may involve over a hundred evaluations of the objective function. Some of the main simplifications are as follows; others will appear in the course of the computations.

(a) Fillets in the cross section are ignored.

(b) The nonprestressed reinforcement is not calculated separately, but is considered as a fixed percentage of the concrete quantity.

(c) The positions of the cable centers are treated as being independent of the cable quantity.

These simplifications may significantly affect the design and hence the cost estimate. However, the purpose is not to develop a computerized final design but to determine the basic dimensions of the cross section for minimum cost. In other words, what is important is to obtain correct values of the independent variables at the optimal point. It is considered that the simplifications will have little effect on the solution value of these variables.

The material properties used in the calculations are the same as those given in Chapter 3.

6.2.1 <u>Design of Deck</u>. The deck slab thicknesses at the base of the centilever portion, t_1, and in the interior portion, t_2, are calculated using the procedure of Sec. 4.5.

<u>Cantilever portion of deck slab</u>

$$\text{Dead load moment} = 0.5 \times 0.150 \times b_1^2/2 + (t_1 - 0.5) \times 0.150 \times b_1^2/6$$
$$= 0.025(1 + t_1)b_1^2$$

$$\text{Live load and impact moment} = 1.3\ PX/E$$
$$= 1.3 \times 16 \times (b_1 - 2)/[0.8(b_1 - 2) + 3.75]$$
$$= 26(b_1 - 2)/(b_1 + 2.6875)$$

$$\text{Total moment: } M_1 = 0.025(1 + t_1)b_1^2 + 26(b_1 - 2)/(b_1 + 2.6875)$$

If $b_1 > 8.0$ ft., the live load moment will include an additional term corresponding to a second wheel load. This is done in the subroutine.

The concrete moment resistance coefficient is

$$R = 0.430 \text{ ksi}$$

The distance from the top of the slab to the center of the reinforcement is assumed to be 2 in. (corresponding to #8 bars and 1-1/2 in. concrete cover). The slab thickness required, t_1 (feet), is then given by

$$\begin{aligned} t_1 &= [2 + \sqrt{(M_1/R)}]/12 \\ &= [2 + \sqrt{(M_1/0.432)}]/12 \\ &= 0.167 + 0.1268 \sqrt{M_1} \end{aligned}$$

Since M_1 depends on t_1, an initial value of t_1 is assumed in the subroutine and the correct value obtained by iteration.

Interior portion of deck slab

The interior portion of the deck has two different spans, b_2 and b_5. The latter is a dependent dimension given by

$$b_5 = b - 2b_1 - 2b_2 - 4t_4$$

The thickness required for each span will be calculated and t_2 set to the larger value.

(a) Span, $S = b_5$

$$\begin{aligned} \text{Dead load moment} &= t_2 \times 0.150 \times b_5^2/12 \\ &= 0.0125\ t_2 b_5^2 \end{aligned}$$

$$\begin{aligned} \text{Live load and impact moment} &= 1.3 \times 0.8(S + 2)P/32 \\ &= 1.3 \times 0.8(b_5 + 2) \times 16/32 \\ &= 0.52(b_5 + 2) \end{aligned}$$

Total moment: $M_5 = 0.0125\ t_2 b_5^2 + 0.52(b_5 + 2)$

The slab thickness required, t_2(feet), is given by

$$t_2 = 0.167 + 0.1268 \sqrt{M_5}$$

(b) Span, $S = b_2$

Total moment: $M_2 = 0.0125\ t_2 b_2^2 + 0.52(b_2 + 2)$

However, this moment must be corrected for carry-over from the cantilever portion, as in Sec. 4.5.3.

Clear sloping height of web:

$$s = \sqrt{([d - t_2 - 0.5]^2 + [(b_2 - b_3)/2 + t_4]^2)}$$

Top slab stiffness: $k_2 = t_2^3/b_2$

Web stiffness: $k_4 = 0.75t_4^3/s$

Corrected moment: $M = (k_4M_2 + k_2M_1)/(k_4 + k_2)$

The slab thickness required is

$$t_2 = 0.167 + 0.1268\sqrt{M}$$

Minimum thickness

The minimum value allowed for all slab thicknesses will be 0.5 ft.

6.2.2 Properties of Concrete Girder Section. The following properties of the minimum cross section (i.e., away from the pier) are next computed by the subroutine.

a_{11} = area of the half-section (i.e., one box girder)
a_1 = area of the full section
d_{c1} = distance from top of girder to centroid of the half-section

6.2.3 Cable Area. The cables are designed to balance the dead load of the concrete section, assuming the idealized double-parabolic profile.

The minimum distance from the edge of the girder to the center of the cables is assumed to be 0.67 ft. (corresponding to six cables per web, 20 strands).

Cable drape: $h = (d - 0.67) - (d_{c1} + 0.67)/2$
$= d - 0.5d_{c1} - 1.005$

Dead load per unit length of half-section = $0.150a_{11}$

Cable force (per half-section) required to balance dead load:

$$F = 0.150a_{11} \times 180^2/8h$$
$$= 607.5a_{11}/h$$

Effective prestress: $f_{se} = 0.6f'_s = 162$ ksi

The cable area per full section in ft.^{2} is given by

$$\begin{aligned} A_s &= 2F/f_{se} \\ &= 2 \times 607.5 \times a_{11}/(162 \times h \times 144) \\ &= a_{11}/(19.2h) \end{aligned}$$

6.2.4 Ultimate Strength at Pier and Bottom Slab Thickness. The load factors are the same as in Sec. 3.6.1. The bending moments are calculated in the same way as in Sec. 3.7.2. The moments at the pier are as follows:

$$\begin{aligned} \text{Dead load moment} &= -0.125 \text{ (Concrete DL + Asphalt DL)} \times 180^2 \\ &= -0.125\ [0.150a_1 + 0.017(b - 2)] \times 180^2 \\ &= -607.5[a_1 + 0.113(b - 2)] \end{aligned}$$

$$\text{Live load moment} = -9{,}650 \text{ (k-ft.)}$$

The secondary cable moment is obtained by calculating that for the idealized cable profile and multiplying by a correction factor of 0.7. This factor takes account of the difference between the actual and idealized profiles and was obtained during the preliminary design for the bridge of Chapter 3.

$$\begin{aligned} \text{Secondary moment: } M_S &= 0.7\ [\text{Resultant moment (ideal profile) - Primary moment (ideal profile)}] \\ &= 0.7\ [(2 \times 0.150a_{11} \times 180^2/8) - 2F(d_{c1} - 0.67)] \\ &= 0.7\ [(12{,}150a_{11}) - (2 \times 607.5a_{11}/h)(d_{c1} - 0.67)] \\ &= 850.5a_{11}[1 - (d_{c1} - 0.67)/h] \end{aligned}$$

The ultimate negative moment at the pier is then given by

$$\begin{aligned} M_u &= -1.35(\text{DL moment}) - 2.25(\text{LL + Impact moment}) - (\text{Secondary moment}) \\ &= 1.35(607.5)[a_1 + 0.113(b - 2)] \\ &\quad + 2.25(1.164)(9650) - M_S \\ &= 820[a_1 + 0.113(b - 2)] + 25{,}260 - M_S \end{aligned}$$

The thickness of the bottom slab at the pier, t_3, is chosen to give adequate compressive capacity at ultimate load.

Moment arm: $d_m = d - 0.67 - 0.5t_3$

The required value of the thickness is given by

$$t_3 = M_u/(d_m \times \varphi \times 0.85f'_c \times 2b_3)$$

$$= M_u/(d_m \times 0.9 \times 0.85 \times 6 \times 144 \times 2b_3)$$

$$= M_u/(1322\ d_m b_3)$$

Since d_m depends on t_3, an initial value of t_3 must be assumed in the subroutine and the correct value obtained by iteration.

The cable area required for ultimate strength is now determined. The following (conservative) value is assumed for the cable stress at ultimate load.

$$f_{su} = 240 \text{ ksi}$$

Cable area required:

$$A_s(\text{ult}) = M_u/(dm \times \varphi \times f_{su})$$

$$= M_u/(d_m \times 0.9 \times 240 \times 144)$$

$$= M_u/(31{,}100 d_m)$$

If the value of A_s computed previously is less than this value, then it must be replaced by this value.

6.2.5 Extent of Bottom Slab Taper. The value of q, the fraction of the span over which the bottom slab must be thickened, is also calculated.

The effective depth d_e (i.e., the distance from the center of the cables to the bottom of the girder) varies along the span from a maximum value of (d - 0.67) at the pier. A linear approximation to d_e was obtained by drawing a straight line through points on the cable profile at the pier and at the point of contraflexure, in the case of the example in Chapter 3, giving

$$d_e = (d - 0.67) - 1.7z(d - 1.34)$$

where z = (the distance from the pier center)/(span).

At the end of the taper in the bottom slab, the thickness is 0.5 ft. and the ultimate moment capacity is obtained as follows.

Moment arm: $d_m = d_e - 0.5/2$

$$= d - 0.92 - 1.7z(d - 1.34)$$

Moment capacity:

$$M_u(\min) = d_m \times \phi \times 0.85f'_c \times 2b_3 \times 0.5$$

$$= d_m \times 0.9 \times 0.85 \times 6 \times 144 \times 2b_3 \times 0.5$$

$$= 661\ d_m b_3$$

The actual moments at z are obtained as follows. The dead load moment varies parabolically, thus

Dead load moment at z = (DL moment at pier) $\times$ $(1 - 5z + 4z^2)$

The live load moment for this case will be obtained by loading only the span that does not contain the section considered, and so will vary linearly as follows.

Live load moment at z = 0.5 $\times$ (LL moment at pier) $\times$ (1 - z)

The secondary moment varies linearly along the span. Hence,

Secondary moment at z = $M_S \times (1 - z)$

The ultimate negative moment at z is then given by

$$M_u(z) = -1.35\ (\text{DL moment}) - 2.25(\text{LL moment}) - (\text{Secondary moment})$$

$$= 820[a_1 + 0.113(- 2)](1 - 5z + 4z^2)$$

$$+ 12{,}630(1 - z) - M_S(1 - z)$$

The subroutine computes the value of $M_u(z)$ for successive values of z, starting from z = 1/9, until a value less than $M_u(\min)$ is obtained. The fraction q is set equal to the value of z at that point.

6.2.6 <u>Average Section Area</u>. The subroutine next computes the area of the full cross section at the pier, a_2.

The average cross-sectional area is given by

$$a = a_1 + (a_2 - a_1)q/2$$

6.2.7 Unit Costs. The following typical unit costs (1970) were obtained from manufacturing firms and from the Texas Highway Department. The costs are relatively of a correct magnitude, although a wide range was indicated. Objective functions can easily be updated for new or more accurate cost information.

(a) Concrete

Cost of concrete without reinforcement = \$75 per yd^3.

= \$2.78 per ft^3.

Cost of reinforcement = \$0.20 per lb.

Both figures include an allowance for the placing of the materials.

A cost analysis of the bridge designed in Chapter 4 gave the following average figure for the cost of the concrete + reinforcement, which will be used in this optimization study.

Cost of reinforced concrete = \$4.92 per ft^3.

(b) Prestressing cables

Cost of cables (including tensioning) = \$0.70 per lb.

= \$343 per ft^3.

(c) Epoxy resin joints

Cost of epoxy (including application) = \$80 per ft^3.

(d) Earth fill

Cost of fill (including labor) = \$1.0 per yd^3.

= \$0.037 per ft^3.

(e) Transportation of segments

Seventy-five miles is chosen as a likely average distance for transportation. For this distance,

Transportation cost = \$2.55 per kip

(f) Forms - The cost of the forms is considered likely to be a constant, for the range of dimensions possible in this analysis, and is not taken into account.

(g) Asphalt surface - The asphalt cost is also ignored, as it is a constant.

(h) Erection cost - There are insufficient data available about erection costs. The cost will depend on the weight of the segments, but will probably increase with discrete increments of weight rather than in a linear manner. Hence, it will not be taken into account.

Final cost objective functions will thus correctly reflect variables such as costs of concrete, steel, transportation of segments, etc., but will not give correct cost estimates, since items such as forms and stressing labor are not included.

6.2.8 Total Cost per Foot Length. The costs of the various items, per foot length of superstructure, are obtained as follows:

(a) Concrete

Cost per ft. length = (Unit cost) x (Average section area)

= 4.92a

(b) Cables

Cost per ft. length = (Unit cost) x (Cable area)

= $343A_s$

(c) Joints

An epoxy joint thickness of 1/16 in. is assumed

Cost per ft. length = (Unit cost) x (Joint thickness) x (Average section area)/(Segment length)

= (80) x [1/(16 x 12)] x (a)/(10)

= 0.042a

(d) Earth fill - The quantity of earch fill, forming the roadway embankment leading up to the abutment, is a function of the girder depth, d. The profile of the embankment at the abutment (above the level of the bottom of the girder) is shown in Fig. 6.2. The width and slopes shown are those required by the Texas Highway Department. A 150 ft. length of embankment is assumed, giving a volume of fill (above that required for a value of d = 0) at each abutment, as follows:

Vol. = (b + 10 + 3d) x d x 150

Cost per ft. length of superstructure = (Unit cost) x (vol.)/(1/2 length of bridge)

= 0.037 x (b + 10 + 3d) x d x 150/180

= 0.031(b + 10 + 3d)d

(e) Transportation

Cost per ft. length = (Unit cost) x (Unit weight) x (Average section area)

$$= 2.55 \times 0.150 \times a$$

$$= 0.383a$$

(f) Total cost

The total cost per ft. length is given by

$$C = 4.92a + 343A_s + 0.042a + 0.031(b + 10 + 3d)d + 0.383a$$

$$= 5.345a + 343A_s + 0.031(b + 10 + 3d)d$$

The quantity C constitutes the objective function for the problem.

6.2.9 Subroutine. A listing of the subroutine which carries out the design outlined above and computes the objective function C is given in Appendix C.3.

The constants b, b_6, and t_4 are set to the following values, corresponding to the design in Chapter 3.

$$b = 50$$

$$b_6 = 2$$

$$t_4 = 1.0833$$

6.3 The Optimal Solution

6.3.1 Optimization Methods. To solve the unconstrained nonlinear programming problem the methods of Nelder-Mead and Powell will be used. As explained in Chapter 5, these methods do not require derivatives of the objective function. It is probable that continuous derivatives of the object function C do not exist at some points. In any case, to calculate the derivatives would be extremely difficult.

Computer programs for both the Nelder-Mead and the Powell methods were used in order to obtain solutions by both methods in order to see which is the more efficient for this problem.

6.3.2 Solution. Optimal solutions to the problem were obtained using both optimization programs together with the subroutine for determining the objective function.

As a precaution against obtaining a local optimum rather than a global optimum, several solutions were obtained using different starting points.

In order to determine the sensitivity of the optimal point to relative changes in the unit costs of the materials, a second problem was solved in which the tendon cost was increased by 50 percent. Thus, the objective functions for the two problems are as follows:

First problem: $C = 5.345a + 343A_s + 0.031(b + 3d + 10)d$

Second problem: $C = 5.345a + 514.5A_s + 0.031(b + 3d + 10)d$

The solutions obtained for both problems, using the two optimization methods and various starting points, are shown in Table 6.1. The best solution for each problem, i.e., the one having the lowest value of the objective function, is indicated.

6.3.3 Contour Plot. To obtain an estimate of the sensitivity of the objective function to small changes in the variables near the optimum, a contour plot of this function is useful.

In the case of the first problem, a two-dimensional contour plot of the objective function was obtained by computer. The two variables chosen for the axes of the plot were b_1 and d. The other independent variables, b_2 and b_3, must be set at constant values or else made dependent on b_1 and d. They were made functions of b_1 in the following way.

b_2 was set at a fraction of the sum of the interior spans of the top slab, i.e., $(b - 2b_1 - 4t_4)$, and b_3 was set at a fraction of the bottom slab width for vertical webs, i.e., $(b_2 + 2t_4)$. The fractions were chosen to give the correct values of b_2 and b_3 at the optimal point; thus,

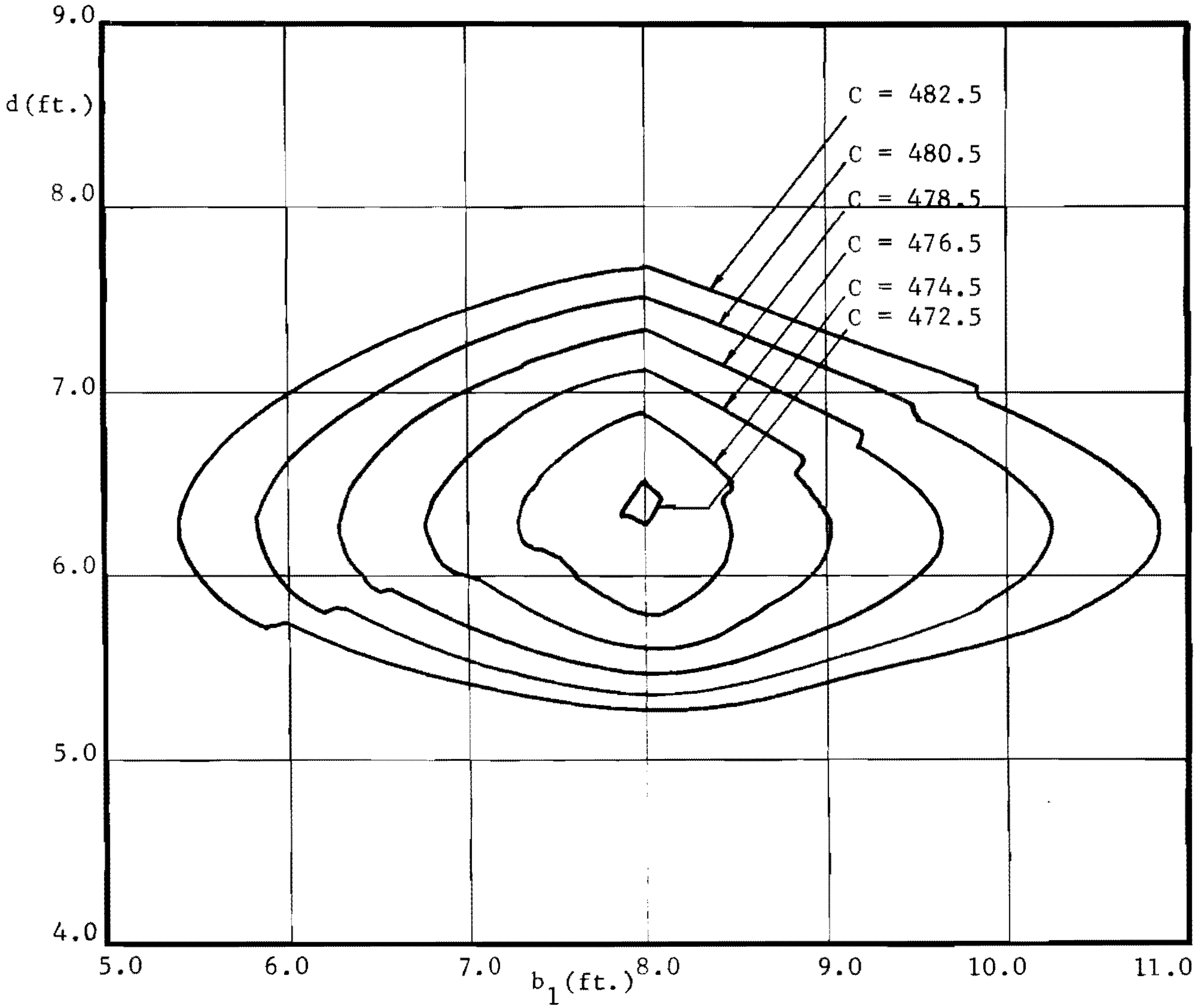
$$b_2 = 0.319(b - 2b_1 - 4t_4)$$
$$b_3 = 0.700(b_2 + 2t_4)$$

The contour plot is shown in Fig. 6.3.

6.3.4 Comments. With each of the two problems solved, there is some variation in the values obtained for the variables at the optimal point, using the different optimization methods and different starting points. However, the variation in the optimal value of the objective

TABLE 6.1. OPTIMAL SOLUTION FOR TWO-SPAN DOUBLE BOX GIRDER BRIDGE

Method	Starting Point				Solution					
	Variables (feet)				Variables (feet)					Objective Function ($ per ft.)
	b_1	b_2	b_3	d	b_1	b_2	b_3	d	L/d	
	Objective function: $C = 5.345a + 343A_s + 0.031(b + 3d + 10)d$									
Nelder-Mead	6.0	10.0	10.0	8.0	8.00	9.47	8.14	6.30	28.6	472.04
Powell	10.0	8.0	7.0	5.0	9.35	8.06	6.85	6.38	28.2	474.00
Powell	6.0	10.0	7.0	5.0	8.00	9.46	7.73	6.32	28.5	472.50
	Best Solution				8.00	9.47	8.14	6.30	28.6	472.04
	Objective function: $C = 5.345a + 514.5A_s + 0.031(b + 3d + 10)d$									
Nelder-Mead	6.0	10.0	10.0	8.0	8.00	9.38	7.15	7.27	24.8	535.95
Nelder-Mead	6.0	10.0	5.0	8.0	8.00	9.38	7.12	7.30	24.6	535.96
Powell	6.0	10.0	10.0	8.0	7.23	10.00	7.39	7.20	25.0	538.59
Powell	9.0	9.0	9.0	8.0	8.22	9.14	6.83	7.29	24.7	536.72
	Best Solution				8.00	9.38	7.15	7.27	24.8	535.95



Objective Function

$$C = 5.345a + 343A_s + 0.031(b + 3d + 10)d$$

Fig. 6.3. Objective function contours for two-span double box girder bridge.

function is less than 0.5 percent. Examination of the contour plot reveals that the gradient of the objective function is quite flat near the optimum. Consequently, the optimal point is not sharply defined.

The best solutions to both problems were obtained using the Nelder-Mead method. In the case of the Powell method, some starting points were tried that did not lead to a solution at all. The reason for this was that in one of the early iterations the method would arrive at a point having a value of b_3 or d so small that the objective function subroutine could not obtain a value for t_3. On the other hand, with the Nelder-Mead method, the computer output always revealed a slow but steady convergence.

The value of b_1 (the width of the cantilever portion of the deck slab) has a value of 8.00 ft. at the optimum in both problems. The apparent reason for this "round figure" value is that for greater widths a second wheel load would be acting on the cantilever, giving a discontinuity in the gradient of the objective function at this value. The value of b_2 (the width of the outer interior span of the deck slab) is such that this span is less than the central span, b_5. The value of b_3 (the width of the lower slab) is such as to give sloping webs for the box girders.

The effect of increasing the unit cost of the cables as done in the second problem is to increase the value of the depth d at the optimum and to decrease the value of b_3. The increase in d was expected because the required cable area decreases with the depth of the superstructure. Optimal L/d ratios decreased from 29 to 25 with increased tendon costs. The bottom flange area required for negative moment also decreases with the depth; hence the decrease in b_3.

The contour plot for the first problem (Fig. 6.3) shows that the objective function is not very sensitive to small changes in the variables near the optimal point. In other words, as already noted, the gradient is flat. A range of values of b_1 and d to give values of the objective function within 1 percent and 2 percent of the optimal value are as follows:

Objective Function	Range of b_1(ft)	Range of d(ft)	Range of L/d
1 percent above optimal value	6.8 to 9.0	5.6 to 7.1	32 to 25
2 percent above optimal value	5.3 to 10.8	5.2 to 7.7	35 to 23

6.4 Second Example--Two-Span, Multi-cell Box Girder Bridge

The cross section of the second example bridge to be optimized is shown in Fig. 6.4. The spans, the support system, and the cable pattern are the same as for the first example. The constants and the variables for the nonlinear programming problem are the same as in the previous example, apart from the following exceptions to the constants.

The dimension b_6 does not appear in this example. The following item is added:

$$n_w = \text{number of webs}$$

Four webs are shown in Fig. 6.4. However, the subroutine is set up to handle any arbitrary number. If there are more than four webs, the interior spans of the deck slab, apart from those adjacent to the cantilever portion, are all set equal to b_5.

The procedure for computing the objective function is similar to that used in the previous case. A listing of the subroutine is given in Appendix C.4. The constants b, t_4, and n_w are set to the following values:

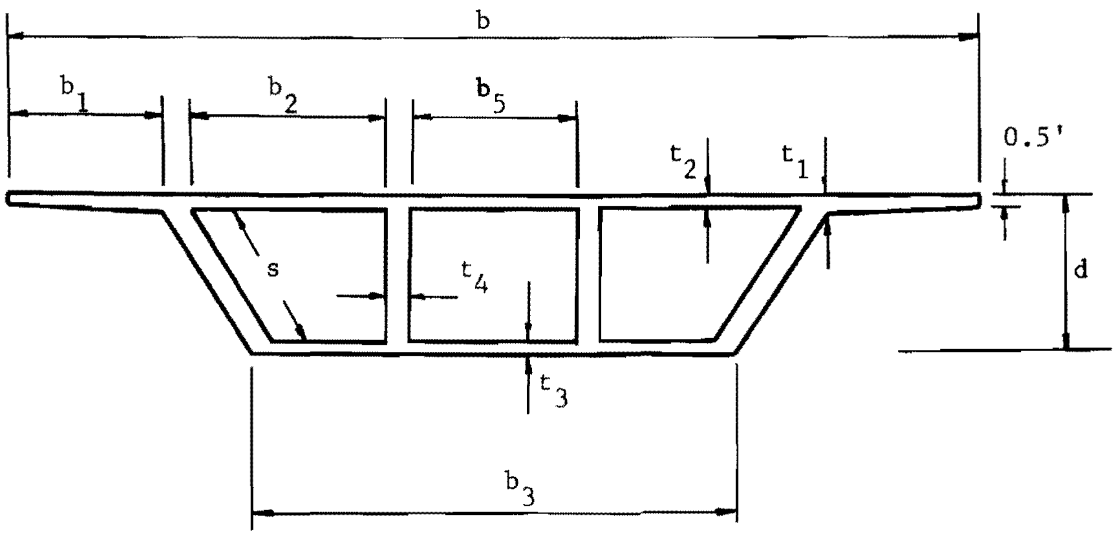
$$b = 50$$
$$t_4 = 1.0833$$
$$n_w = 4$$

Two problems are again considered, corresponding to two values of cable cost. As before, the objective functions are given by:

First problem: $C = 5.345a + 343A_s + 0.031(b + 3d + 10)d$

Second problem: $C = 5.345a + 514.5A_s + 0.031(b + 3d + 10)d$

The optimal solutions obtained from the Nelder-Mead method and the Powell method are shown in Table 6.2.



NOTES: Bottom slab thickness tapers from t_3 at the pier to 0.5 ft. at 180q ft. from the pier.
All dimensions are in feet.
Spans: 180 ft.-180 ft.

Fig. 6.4. Cross section of second example

TABLE 6.2. OPTIMAL SOLUTION FOR TWO-SPAN THREE-CELL BOX GIRDER BRIDGE

Method	Starting Point				Solution					
	Variables (feet)				Variables (feet)					Objective Function ($ per ft.)
	b_1	b_2	b_3	d	b_1	b_2	b_3	d	L/d	
	Objective function: $C = 5.345a + 343A_s + 0.031(b + 3d + 10)d$									
Nelder-Mead	6.0	10.0	20.0	8.0	10.83	6.46	24.39	5.95	30.2	509.63
Powell	6.0	10.0	20.0	8.0	11.00	6.29	24.39	5.91	30.4	509.68
	Best Solution				10.83	6.46	24.39	5.95	30.2	509.63
	Objective function: $C = 5.345a + 514.5A_s + 0.031(b + 3d + 10)d$									
Nelder-Mead	6.0	10.0	20.0	8.0	10.89	6.10	23.25	6.32	28.5	582.45
Powell	6.0	10.0	20.0	8.0	14.30	8.12	17.28	6.83	26.3	581.20
Powell	10.0	8.0	20.0	8.0	13.78	7.68	18.30	6.69	26.9	581.28
	Best Solution				14.30	8.12	17.28	6.83	26.4	581.20

A contour plot for the first problem is given in Fig. 6.5. The axes correspond to the variables b_1 and d. As in Sec. 6.3.3, the variables b_2 and b_3 are expressed in terms of b_1 in such a way that the correct values are obtained at the optimal point; thus,

$$b_2 = 0.269(b - 2b_1 - 4t_4)$$
$$b_3 = 0.861(b - 2b_1)$$

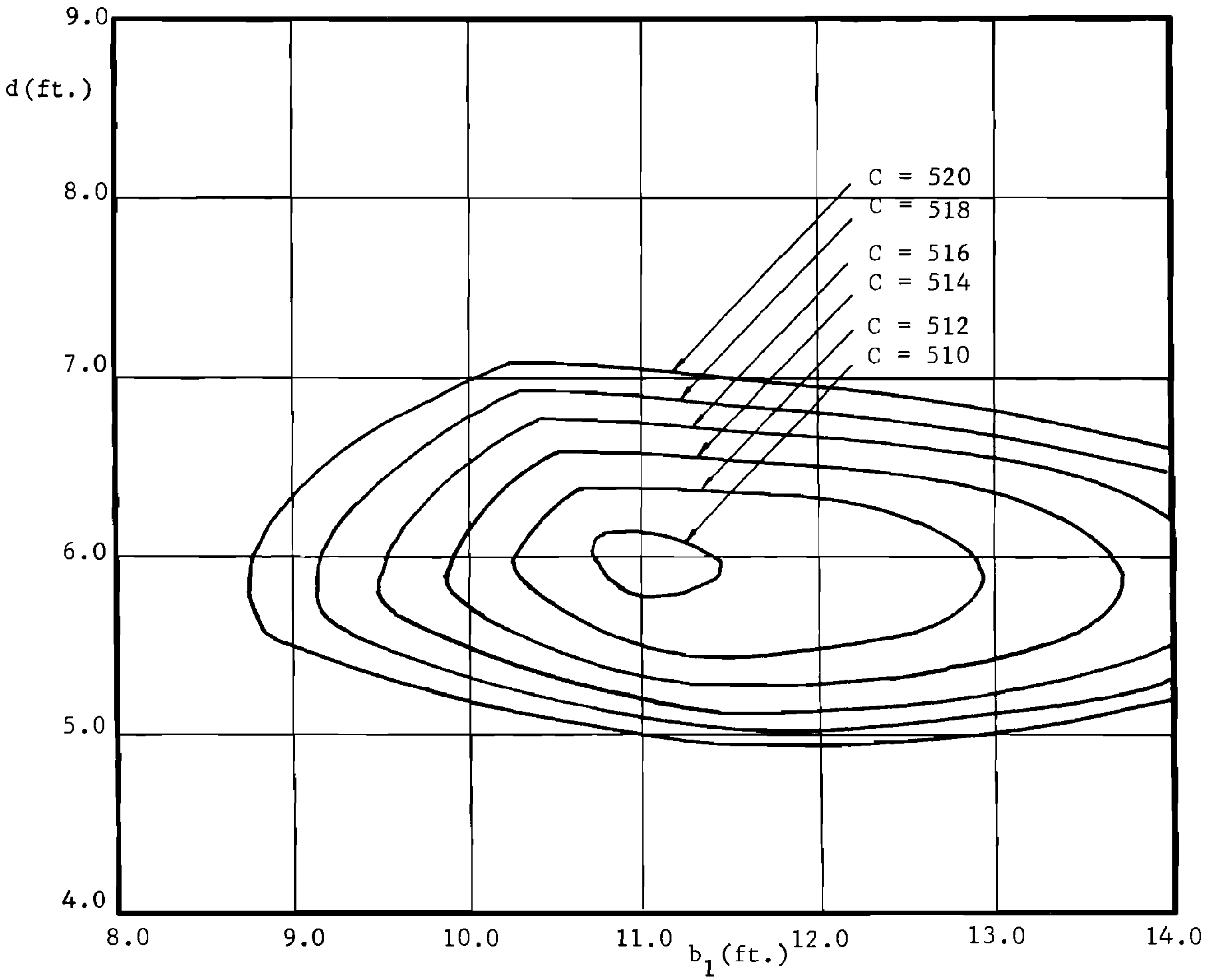
6.4.1 Comments on the Optimal Solution. As in the previous example, there is some variation in the values of the variables at the optimal point, obtained with the different methods and different starting points. The variation in the optimal value of the objective function does not exceed 0.2 percent. Again, the flat gradient of this function is the reason why the optimal point is not sharply defined.

The best solution for the first problem was obtained by the Nelder-Mead method and for the second problem by the Powell method. The Powell method yielded solutions with all of the starting points tried. In view of the small variation in the solution values of the objective function, the two methods can be considered about equally effective in this example.

The values of b_1 and b_3 in the optimal solutions indicate a large cantilever overhang for the top slab and a narrow bottom slab. The apparent reason for this is that a large area of bottom slab is required only in the negative moment region near the pier, thus leading to a narrow width for the optimum, and in consequence the cantilever portion of the top slab becomes large, avoiding excessive slope of the outside webs.

The effect of increasing the unit cost of the cables is again to increase the value of the depth d at the optimum. L/d decreases from 30 to 26 with increasing tendon costs. Also the value of b_3 decreases and that of b_1 increases.

The contour plot for the first problem (Fig. 6.5) again indicates that the objective function is not very sensitive to changes in the variables near the optimal point. The range of values of b_1 and d giving values of the objective function within 1 percent and 2 percent of the optimal value are as follows.



<u>Objective Function</u>

$$C = 5.345a + 343A_s + 0.031(b + 3d + 10)d$$

Fig. 6.5. Objective function contours for two-span three-cell box girder bridge

Objective Function	Range of b_1(ft)	Range of d(ft)	Range of L/d
1 percent above optimal value	9.8 to 13.7	5.3 to 6.6	34 to 27
2 percent above optimal value	8.8 to >14.0	4.9 to 7.2	36 to 25

6.5 The Optimal Solution as a Basis for Design

The procedure developed in this chapter makes possible the determination of the optimal basic dimensions (b_1, b_2, b_3, and d) for the cross section of a two-span bridge, of given length, width, and sectional type. These optimal dimensions can form the basis for the design of the bridge.

As noted in Chapter 2 and Sec. 3.8, the first step in the design of the superstructure is to select an approximate cross section. The full dimensions, including the various thicknesses, are required. These latter dimensions, classified as dependent dimensions in Sec. 6.1, may be otained as follows.

The subroutine developed to compute the objective function can easily be converted into a program to compute the full dimensions of the section. The variables b_1, b_2, b_3, and d can be made input items for this program and the various dimensions computed by the subroutine, e.g., the slab thicknesses, the section area, and the cable area, can be printed as output. Thus, this program performs the function of a preliminary design.

6.6 Possible Limitations of the Optimal Solution

The question must finally be raised as to whether the cross section determined by the optimal solution, i.e., having the minimum cost, is in fact the most appropriate one to use in design. The following considerations are relevant.

The optimal depth, d, obtained using the best estimate of cable cost, came to 6.30 ft. in the case of the double box girder and 5.95 ft. for the three-cell box girder. These correspond to a span/depth ratio of about 30, which is higher than that for most box girder bridges actually constructed abroad. It is to be noted that the smaller the depth the greater the number and the crowding of the cables. However, there is no reason to consider this excessive for these values of d. The higher span/depth ratios may be reflecting both lighter U.S. live loads and different economic conditions.

Deflections are also greater for smaller depth. A deflection limitation was not built into the subroutine for the objective function. This could be done but would add significantly to the complexity. In view of the very small deflections obtained for the bridge designed in Chapter 3 and considering that the dead load was balanced, it is unlikely that the deflections obtained for the optimal values of depth, given above, will be excessive.

In the case of the three-cell box girder, the optimal values of b_1 indicate a large cantilever overhang. The larger the overhang the more crowded the transverse reinforcement becomes in the top of the slab. It is also possible that with a large value of b_1 the shear stresses in the outer webs of the girder may be considerably higher than those in the inner webs. A large inequality of shears will probably result in a greater quantity of web reinforcement being required.

The flat gradient of the objective function near the optimum allows a reasonable latitude in the choice of the dimensions without large variation in this function. So an increase in depth or a decrease in the cantilever overhang (in the case of the three-cell box) can be made with small increase in cost.

6.7 Other Examples for Optimization

The methods developed in this chapter for optimizing two-span bridges can be extended to bridges having a greater number of spans. The changes in procedure will follow the changes outlined in the design method.

In the case of long viaducts, the span could also be made a variable to be optimized. This will be discussed further in the next chapter.

CHAPTER 7

OPTIMIZATION OF BRIDGES CONSTRUCTED IN CANTILEVER

The problem of optimizing the cross section of bridges constructed in cantilever will be considered in this chapter. The difference in the design procedure for a bridge of this type as compared to one constructed on falsework will be reflected in the subroutine to compute the objective function for the problem. Otherwise, the optimization procedure is essentially similar to that described in the preceding chapter and the basic steps are as follows:

(a) A mathematical model of the structure is set up.

(b) A subroutine is developed to calculate the objective function.

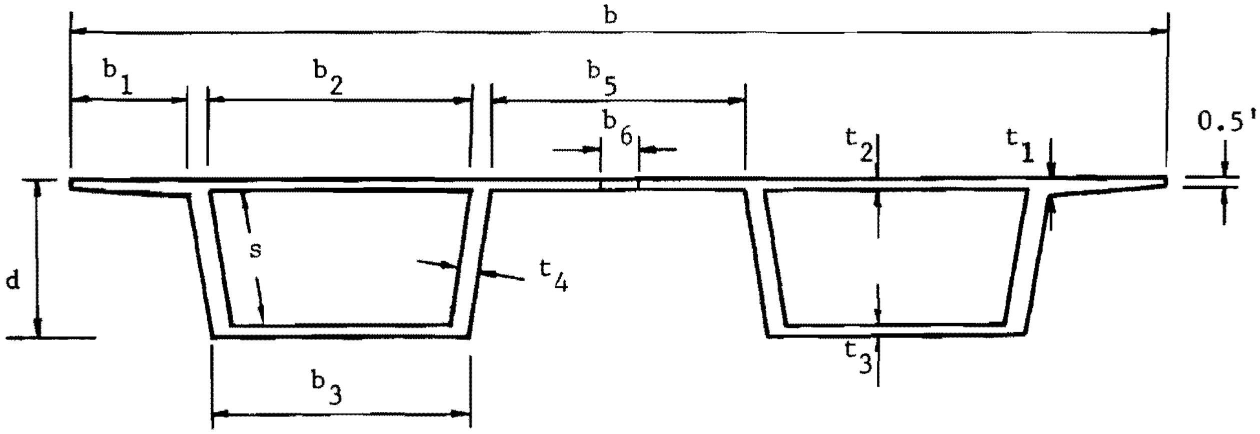
(c) The nonlinear programming problem is solved using the Nelder-Mead method and/or the Powell method.

The sample case for which the optimization is carried out in full is that of a three-span bridge, having the same length and span ratio as that designed in Chapter 4. The function to be minimized will again be the cost of the bridge. The method used in this example can be readily extended to other lengths of bridge. Extension of the method to bridges with other span ratios and to viaducts having many spans will be discussed.

7.1 Example Case--Three-Span Double Box Girder Bridge

The cross section of the example bridge is shown in Fig. 7.1. It consists of a pair of connected, single-cell box girders. The segmental pattern, the cable pattern and the support system are the same as for the bridge designed in Chapter 4. The spans are 100 ft. - 200 ft. - 100 ft.

The constants, independent variables, and dependent variables for the nonlinear programming problem are the same as in Sec. 6.1. The



NOTES: Bottom slab thickness varies from t_3 at main pier to 0.5 ft. at 100q ft. from pier.
All dimensions are in feet.
Spans: 100 ft.-200 ft.-100 ft.

Fig. 7.1. Cross section of example for cantilever erection

objective function C, the cost of the bridge per foot length, is a function of the independent variables; thus

$$C = f(b_1, b_2, b_3, d)$$

The problem will again be treated as unconstrained.

7.2 The Objective Function

The subroutine given in Appendix C.5 is developed to calculate the objective function, by performing an approximate design of the superstructure and summing the costs of the various components.

The basic steps adopted in the approximate design are as follows:

(a) The deck is designed for wheel loads.

(b) The bottom slab thickness and the top cables are designed for cantilever erection.

(c) The bottom cables are designed for ultimate load on the completed superstructure.

This procedure closely follows that outlined in Chapter 4, except that some steps are omitted. Various simplifications are made to facilitate the design. These include the ones outlined in Sec. 6.2.

The material properties assumed are as given in Chapter 3.

7.2.1 Design of Deck. This is exactly the same as in Sec. 6.2.1.

7.2.2 Properties of Concrete Section at Center of Bridge. The following properties of the minimum section are next computed by the subroutine.

a_{11} = area of the half section (i.e., one box girder)
a_1 = area of the full section
d_{c1} = distance from top of girder to centroid of the half section

7.2.3 Bottom Slab Thickness. The bottom slab thickness at the main piers, t_3, is designed to give adequate compressive capacity for ultimate strength during cantilever erection. The ultimate load factors are the same as in Sec. 3.6. The approximate bending moments are calculated as in Sec. 4.7.2 and the ultimate moment is as follows for each box girder.

$$\begin{aligned} M_u &= 1.35(\text{Dead load moment}) + 2.25(\text{Live load moment}) \\ &= 1.35(0.150a_{11} \times 100^2/2) + 2.25(25 \times 95) \\ &= 1012.5a_{11} + 5{,}344 \end{aligned}$$

The distance from the top of the girder to the center of the top cables is assumed to be 0.46 ft., as in the bridge designed in Chapter 4.

Moment arm: $d_m = d - 0.46 - 0.5t_3$

The required value of the thickness is given by

$$\begin{aligned} t_3 &= M_u/(d_m \times \varphi \times 0.85f'_c \times b_3) \\ &= M_u/(d_m \times 0.9 \times 0.85 \times 6 \times 144 \times b_3) \\ &= M_u/(661\ d_m b_3) \end{aligned}$$

The value of q, the fraction of the cantilever span over which the bottom slab must be thickened, is now calculated. At the end of the taper the bottom slab thickness is 0.5 ft. and the ultimate moment capacity is obtained as follows.

Moment arm: $d_m = d - 0.46 - 0.5(0.5)$

$= d - 0.71$

Moment capacity: $M_u(\min) = d_m \times \varphi \times 0.85f'_c \times b_3 \times 0.5$

$= d_m \times 0.9 \times 0.85 \times 6 \times 144 \times b_3 \times 0.5$

$= 330.5\ d_m b_3$

The actual ultimate moment at a distance (100z) ft. from the pier center is given by

$$\begin{aligned} M_u(z) &= 1.35(\text{DL moment}) + (\text{LL moment}) \\ &= 1.35 \times 0.5 \times 0.150a_{11}[100(1 - z)]^2 + \\ &\qquad 2.25 \times 25(95 - 100z) \\ &= 1012.5a_{11}(1 - z)^2 + 5{,}625(0.95 - z) \end{aligned}$$

The subroutine computes the value of $M_u(z)$ for successive values of z, starting from z = 0.10 until a value less than $M_u(\min)$ is obtained. The fraction q is set equal to the value of z at that point.

7.2.4 Properties of Concrete Section at Main Piers. The following properties of the maximum section can now be computed by the subroutine.

a_{21} = area of the half section

a_2 = area of the full section

d_{c2} = distance from top of girder to centroid of the half section

7.2.5 Top Cable Area. The top cables are designed to balance 60 percent of the dead load during cantilever erection.

The moment to be balanced at the pier for each box girder (i.e., the half section) is given by

$$\begin{aligned} M &= 0.6(\text{Dead load moment}) \\ &= 0.6 \times 0.150a_{11} \times 100^2/2 \\ &= 450a_{11} \end{aligned}$$

The eccentricity of the cables about the section centroid is

$$d_{ec} = d_{c2} - 0.46$$

The effective prestress: $f_{se} = 0.6f'_s = 162$ ksi

The top cable area required for the full cross section at the pier is given by

$$\begin{aligned} A_{s1} &= 2 \times M/(d_{ec} \times f_{se}) \\ &= 2 \times 450a_{11}/(d_{ec} \times 162 \times 144) \\ &= 0.0386a_{11}/(d_{c2} - 0.46) \end{aligned}$$

The capacity of the cables at ultimate load is now checked. From Sec. 7.2.3 the ultimate moment at the pier is given by

$$M_u = 1012.5a_{11} + 5{,}344$$

and the moment arm is

$$d_m = d - 0.46 - 0.5t_3$$

The following (conservative) value is assumed for the cable stress at ultimate load

$$f_{su} = 240 \text{ ksi}$$

Hence, the top cable area required for ultimate strength is given by

$$A_{s1}(\text{ult}) = M_u/(d_m \times \phi \times f_{su})$$
$$= M_u/(d_m \times 0.9 \times 240 \times 144)$$
$$= M_u/(31{,}100\ d_m)$$

If the value of A_{s1} computed previously is less than this value, then it must be replaced by this value.

The average top cable area along the length of the girder will be required for the cost estimate. This was calculated in the case of the bridge designed in Chapter 4 and found to be 43 percent of the cable area at the pier. This proportion will be assumed here. Hence,

Average top cable area: $A_{st} = 0.43A_{s1}$

7.2.6 Bottom Cable Area. The bottom cable area is designed to provide adequate ultimate strength at the center of the completed bridge.

The bending moments (on the full cross section) at the center of the bridge are calculated as in Sec. 4.8.2 and are as follows:

$$\text{Dead load moment} = 0.055(\text{Concrete DL} + \text{Asphalt DL}) \times 200^2$$
$$= 0.055[0.150a_1 + 0.017(b - 2)] \times 200^2$$
$$= 330[a_1 + 0.113(b - 2)]$$

Live load moment = 6,490 (k-ft.)

Secondary moment due to top cables

$$= -0.055(\text{balanced load}) \times 200^2$$
$$= -0.055(2 \times 0.6 \times 0.150a_{11}) \times 200^2$$
$$= -396a_{11}$$

In order to determine the secondary moment due to the bottom cables, an initial value of the bottom cable area at the bridge center, A_{s2}, is assumed. The cable force and the eccentricity of the cables about the centroid of the section are obtained as follows:

Cable force: $F = f_{se} \times A_{s2}$

$$= 162 \times 144 \times A_{s2}$$

$$= 23{,}330A_{s2}$$

The distance from the bottom of the girder to the center of the bottom cables is assumed (conservatively) to be 0.5 ft.

Cable eccentricity: $d_{ec} = (d - d_{c1} - 0.5)$

Since the cable pattern and segmental pattern are the same as in Chapter 4 the equivalent cable load and the secondary moment may be obtained in the same way as in Sec. 4.8.4. The equivalent load for the main span bottom cables is given by

$$\begin{aligned} P &= F \times d_{ec}/(6 \times 10) \\ &= 23{,}330A_{s2}d_{ec}/60 \\ &= 388.8A_{s2}d_{ec} \end{aligned}$$

The secondary moment due to the main span bottom cables is (0.1013 $\times$ P $\times$ 200). In Chapter 4 the secondary moment due to the side span bottom cables amount to 14 percent of this value. So it is assumed here that the secondary moment due to all of the bottom cables is given by

$$\begin{aligned} M_S &= 1.14 \times 0.1013 \times P \times 200 \\ &= 1.14 \times 0.1013 \times 388.8A_{s2}d_{ec} \times 200 \\ &= 8{,}980A_{s2}d_{ec} \end{aligned}$$

The ultimate moment at the center of the bridge can now be determined; thus

$$\begin{aligned} M_u &= 1.35(\text{DL moment}) + 2.25(\text{LL + Impact moment}) \\ &\qquad + (\text{Secondary moments}) \\ &= 1.35[330(a_1 + 0.113(b - 2)) + 2.25(1.154)(6{,}490) \\ &\qquad + (-396a_{11} + M_S)] \\ &= 445.5[a_1 + 0.113(b - 2)] + 16{,}850 - 396a_{11} + M_S \end{aligned}$$

The required cable area is now determined. In Sec. 4.8.4 a value of 267.5 ksi was calculated for the cable stress at ultimate load at the bridge center. The following conservative value will be assumed here.

$$f_{su} = 265 \text{ ksi}$$

Depth of stress block: $d_a = A_{s2}f_{su}/(0.85f'_c b)$

$$\begin{aligned} &= A_{s2} \times 265/(0.85 \times 6 \times b) \\ &= 52A_{s2}/b \end{aligned}$$

Moment arm: $d_m = d - 0.5 - 0.5d_a$

$$= d - 0.5 - 26A_{s2}/b$$

Bottom cable area required at center of bridge:

$$A_{s2} = M_u/(d_m \times \phi \times f_{su})$$
$$= M_u/(d_m \times 0.9 \times 265 \times 144)$$
$$= M_u/(34,340d_m)$$

The subroutine thus computes the correct value of A_{s2} by an iterative procedure, starting with the assumed value.

The average bottom cable area over the full length of the bridge was calculated in the case of the bridge designed in Chapter 4 and found to be 47.5 percent of the area at the center. The same proportion will be assumed in this calculation.

Average bottom cable area: $A_{sb} = 0.475A_{s2}$

7.2.7 Average Section Area and Cable Area. The average cross-sectional area is

$$a = a_1 + (a_2 - a_1)q/2$$

The average area of top and bottom cables is

$$A_s = A_{st} + A_{sb}$$

7.2.8 Cost per Foot Length of Bridge. The unit costs for this bridge are the same as in Sec. 6.2.7. However, earth fill is not considered in this case, as it is assumed that a conventional short span structure leads up to each end pier of the bridge (as in Chapter 4) rather than an earth embankment.

The objective function, i.e., the total cost per foot length, is given by the same expression as in Sec. 6.2.8, except that the last term (corresponding to earth fill) is dropped. Thus,

$$C = 5.345a + 343A_s$$

A listing of the subroutine that computes the objective function, C, is given in Appendix C.5. The constants b, b_6, and t_4 are set to the following values corresponding to the design in Chapter 4.

$$b = 56.0$$
$$b_6 = 2.0$$
$$t_4 = 1.0$$

7.3 The Optimal Solution

The nonlinear programming problem having the above objective function was again solved using both the Nelder-Mead method and the Powell method.

A second problem was also solved in which the cable cost was increased by 50 percent. The objective functions for the two problems are as follows:

First problem: $C = 5.345a + 343A_s$

Second problem: $C = 5.345a + 514.5A_s$

The solutions obtained are shown in Table 7.1.

A contour plot obtained by computer for the first problem is presented in Fig. 7.2. The axes correspond to the variables b_1 and d. As in Sec. 6.3.3, the variables b_2 and b_3 are expressed in terms of b_1 in such a way that the correct values are obtained at the optimal point; thus

$$b_2 = 0.322(b - 2b_1 - 4t_4)$$
$$b_3 = 0.783(b_2 + 2t_4)$$

7.3.1 Comments. As in the examples of the previous chapter, there is some variation in the values of the variables at the optimal point, obtained with different methods and different starting points. The flat gradient of the objective function is again the reason for this. The variation in the optimal value does not exceed 0.2 percent.

The best solution to both problems was obtained by the Nelder-Mead method. As in the first example of the previous chapter, the Powell method failed to give a solution for some starting points that were tried.

The geometry of the cross section defined by the optimal solution for each problem is similar to that obtained in the first example of

TABLE 7.1. OPTIMAL SOLUTION FOR THREE-SPAN DOUBLE BOX GIRDER BRIDGE

Method	Starting Point				Solution					
	Variables (feet)				Variables (feet)					Objective Function ($ per ft.)
	b_1	b_2	b_3	d	b_1	b_2	b_3	d	L/d	
	Objective function: $C = 5.345a + 343A_s$									
Nelder-Mead	6.0	10.0	10.0	8.0	8.00	11.62	10.14	5.72	34.9	463.64
Nelder-Mead	6.0	10.0	5.0	8.0	8.00	11.60	10.65	6.13	32.6	463.62
Powell	10.0	8.0	7.0	5.0	8.00	11.61	10.07	5.76	34.7	463.66
	Best Solution				8.00	11.60	10.65	6.13	32.6	463.62
	Objective function: $C = 5.345a + 514.5A_s$									
Nelder-Mead	6.0	10.0	10.0	8.0	8.00	11.54	9.76	6.78	29.5	521.79
Nelder-Mead	6.0	10.0	5.0	8.0	8.00	11.54	9.32	6.75	29.6	522.71
Powell	10.0	8.0	7.0	5.0	8.00	11.53	9.08	6.86	29.2	522.75
	Best Solution				8.00	11.54	9.76	6.78	29.5	521.79

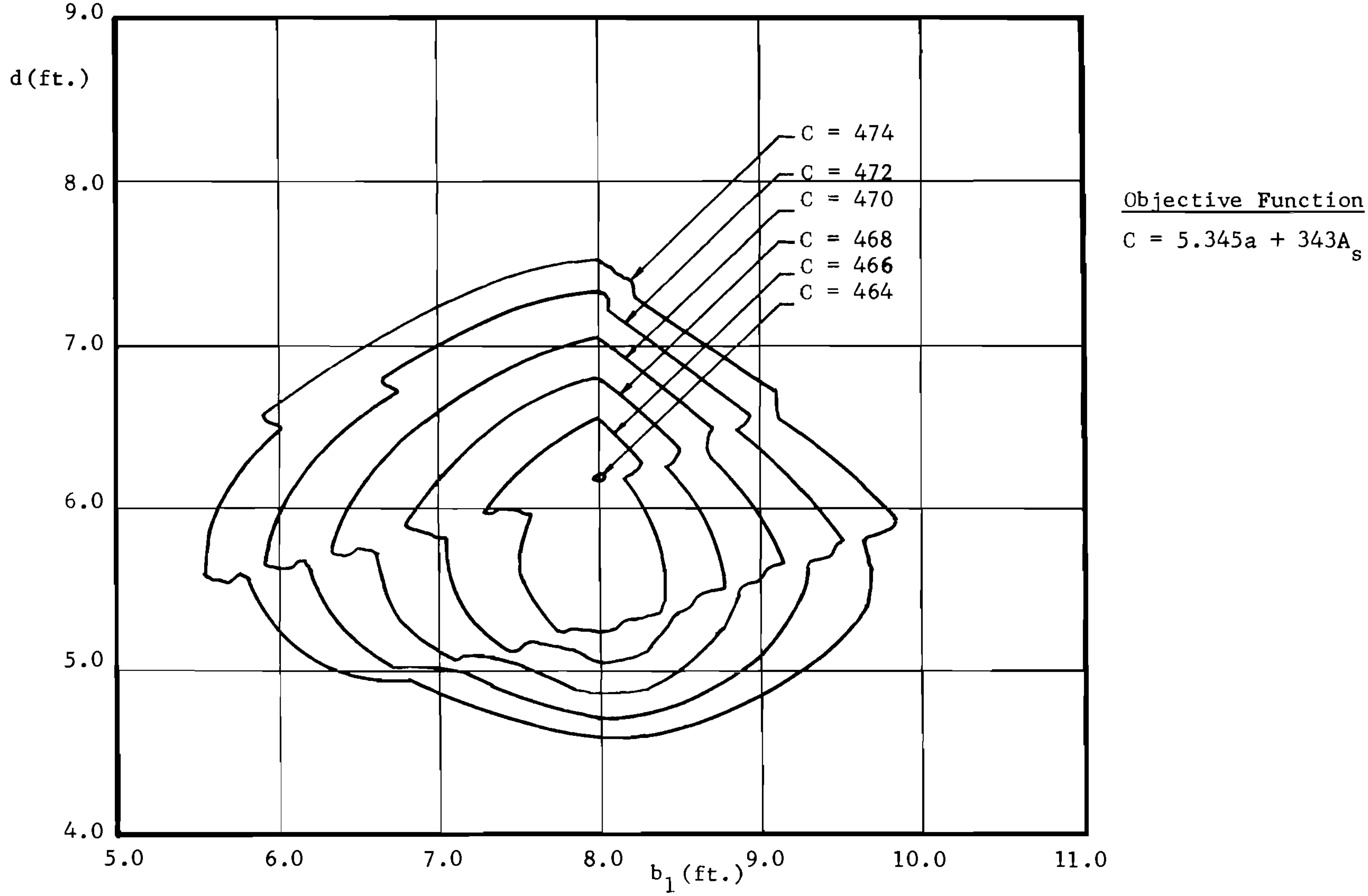


Fig. 7.2. Objective function contours for three-span double box girder bridge

Chapter 6. The value of b_1 (the width of the cantilever portion of the deck) has a "round figure" value of 8.00 ft. The value of b_2 (the width of the outer interior span of the deck slab) is such that this span is slightly less than the central span, b_5. The value of b_3 (the width of the lower slab) is such as to ensure sloping webs for the box girders.

The effect of increasing the unit cost of the cables is again to increase the value of the depth d at the optimal point. The value of b_3 decreases also as before.

The contour plot for the first problem (Fig. 7.2) again indicates that the objective function is not very sensitive to changes in the variables near the optimal point. It may also be noted that the contours are more irregular than in the previous cases. The range of values of b_1 and d to give values of the objective function within 1 percent and 2 percent of the optimal value are as follows.

Objective Function	Range of b_1(ft)	Range of d(ft)	Range of L/d
1 percent above optimal value	6.8 to 8.7	5.1 to 6.8	39 to 29
2 percent above optimal value	5.6 to 9.8	4.6 to 7.5	43 to 26

7.3.2 <u>Possible Limitations of the Solution</u>. The optimal depth d, obtained in the first problem (i.e., using the lower value of cable cost) is 6.1 ft. This corresponds to a span/depth ratio of 33, which is higher than that occurring in any of the existing bridges recorded in Ref. 16 except for those having variable depth. So the question arises as to whether or not the optimal depth should be used for design in this case.

First, deflections must be considered. As in the previous chapter, no deflection limitation was built into the subroutine for the objective function. However, a rough estimate of the deflection at the center of the bridge can be obtained by a comparison with the bridge designed in Chapter 4. The maximum dead and live load deflection (excluding creep effects) in that case was 0.88 in. Assuming that the deflection is inversely proportional to the square of the box girder depth, the approximate deflection in the present case is $0.88 \times (8.0/6.1)^2 = 1.52$ in. = span/1600. This is quite acceptable.

The size of the top and bottom cables required will be roughly inversely proportional to the depth of the girder. With larger cables bigger fillets may be required to provide adequate concrete cover. Fillets were ignored in the optimization process, as noted in Sec. 6.2, and it is considered that they would not significantly affect the optimum depth.

The girder section corresponding to the optimal solution is the one having minimum cost. However, in view of the flat gradient of the objective function, a design having some improved features, namely smaller deflections and smaller cable and fillet sizes, can be obtained at small increase in cost by increasing the depth above the optimal. The actual value used in construction of the example bridge (L/d = 25) is near the 2 percent above optimal limit. This bridge was so stiff that higher L/d ratios seem indicated.

7.4 Effectiveness of the Optimization Techniques

The following is a summary of the previous comments regarding the effectiveness of the optimization techniques:

(a) Treatment as an Unconstrained Problem.--In each of the three examples considered, the problem of optimizing the dimensions of the bridge cross section was set up as an unconstrained nonlinear programming problem. The cost per foot length was chosen as the objective function and expressed in terms of the basic variables, b_1, b_2, b_3, and d. Physically acceptable solutions were obtained for these variables in all cases, thereby justifying the treatment as an unconstrained problem.

(b) The Nelder-Mead and Powell Methods of Optimization.--Calculation of derivatives of the objective function was not feasible. The Nelder-Mead and the Powell methods were accordingly chosen, being the most efficient of the methods that do not require derivatives. In the examples considered, the Nelder-Mead method was generally superior to the Powell method. In two of the three examples, with some starting points the Powell method would not reach a solution. With the Nelder-Mead method steady convergence to the solution was always attained.

(c) Design Simplifications.--In calculating the objective function an approximate design of the superstructure was carried out by the subroutine. Various simplifications were made in this design, as outlined in Sec. 6.2. It was considered that these would have little effect on the solution value of the basic variables at the optimal point.

(d) Contours of the Objective Function.--Two-dimensional plots of contours of the objective function were obtained by computer for each of the three examples. In each case the plot showed that the gradient of the objective function near the optimal point was quite flat. Variations in the values of the variables b_1 and d over a considerable range produce only small changes in the value of the objective function.

(e) Effect of Changes in Unit Costs.--By varying the cost of the cables by 50 percent, it was found that the optimal point is sensitive to relative changes in the unit costs. A 50 percent increase in cable cost causes an increase of about 1 ft. in the optimal value of d. The value of b_3 decreases.

(f) Deflections.--No deflection limitation was built into the objective function subroutines. Approximate estimates indicate that for the examples considered deflection is not critical. However, if bridges with much greater spans were being optimized, it would be best to have a deflection limitation in the subroutines.

(g) Design Dimensions.--The optimal solution defines the basic dimensions of the cross section for the bridge having minimum cost. However, by utilizing the flat gradient of the objective function, the design dimensions can be varied to some extent with only small increase in cost. An increase in the depth d will result in smaller deflections and also fewer or smaller cables. In the case of a multi-cell girder, a decrease in the cantilever overhang b_1 will result in less crowding of the transverse reinforcement and a better distribution of shears.

7.5 Other Examples for Optimization

The optimization of other cases of bridges constructed in cantilever will now be considered briefly.

7.5.1 Multi-Cell Box Girder. The procedure for a multi-cell box girder bridge is similar to that for the example considered in this chapter. Some minor changes are required in the objective function subroutine, as was the case for the second example in Chapter 6.

7.5.2 Side Span Greater than Half Main Span. In the case of a three-span bridge in which the span lengths are specified, the optimization process can be generally similar to that used in the example of this chapter. However, when the side span is greater than half the main span, the design procedure and the calculation of the objective function will be a little more complex.

More generally the different spans will not be specified but only the total length of the bridge. The side span can be made one of the independent variables to be optimized. An alternative case is that of a bridge crossing a navigation canal, where the main span is specified and the side span can be a variable. The two cases can be treated in essentially the same way. They differ from the problems already considered in that there is an extra variable, the side span length.

In Sec. 4.10.4 it was pointed out that when the side span exceeds half the main span, the final outer portion of the superstructure cannot readily be erected by the cantilever process and that some falsework will normally be required. The amount of this falsework will depend on the length of the side span in excess of half the main span. Its cost must be included in the objective function, when the side span is a variable.

7.5.3 Continuous Viaducts. With continuous viaducts, comprising many spans, the span length should be included among the independent variables in the optimization problem. The number of piers will depend on the span and so the pier and bearing costs must be included in the objective function.

CHAPTER 8

CONCLUSIONS

8.1 General Conclusions

The prestressed concrete box girder constitutes an effective and economical member for the superstructure of a long span bridge. Segmental precasting provides an economical means of manufacture, with high quality control, and leads to rapid methods of erection. A number of long span bridges have been constructed throughout the world using segmental precast box girders and techniques of precasting and jointing are available for high strength and precision.

There are two principal methods of construction, namely erection on falsework and cantilever erection. The former is generally simpler provided support for the falsework is feasible at fairly close intervals. Cantilever erection is usually more suitable for river crossings and for viaducts over water or heavily traveled roadways.

Procedures for the design of such bridges erected on falsework or erected in cantilever have been developed. These procedures involve using "beam" theory analysis procedures to satisfy both service load criteria and ultimate strength criteria under all conditions of loading. The effect of the cable forces on the concrete stresses is calculated using an equivalent load concept. Service load level stresses considering possible warping effects and effects of unsymmetrical loading throughout the structure are then checked using the MUPDI program developed by A. Scordelis. Erection stresses in bridges erected in cantilever are also checked using the SIMPLA2 program developed by R. Brown.

For bridges erected on falsework, the prestressing system can consist of long, draped cables. If the number of spans is small, these can run the full length of the bridge and are inserted and tensioned at the end of construction. A cable profile consisting of a series of parabolas will produce a series of uniform equivalent loads along the girder. The cables and the concrete section must provide adequate ultimate strength and acceptable service load stresses under dead load and the various live load patterns on the continuous superstructure.

In the case of bridges erected in cantilever, each stage of construction is a separate design condition, for which the ultimate strength and service load criteria must be satisfied. The cables are of varying length. Those in the top of the girder are inserted and tensioned as each pair of segments is erected and must withstand the dead load and possible unbalanced moments in the cantilever arm. The cables in the bottom of the girder are inserted after completion of the cantilever arms and casting of the closure and are designed to ensure adequate ultimate strength and satisfactory service load stresses under dead and live load on the completed bridge. To obtain the effect of the cable forces on the concrete stresses, the equivalent load concept has been further developed for application to a system of cables of varying length.

Mathematical methods of optimization have been adapted to the problem of finding the optimal cross sections, i.e., those having minimum cost, for bridges constructed on falsework and bridges erected in cantilever. Two types of cross section were considered--a pair of connected single-cell box girders and a multi-cell box girder. In each case the problem was treated as an unconstrained nonlinear programming problem in four variables that define the geometry of the cross section. A subroutine was developed to compute the objective function, taken as the relative cost of the bridge per foot length, and a solution obtained by the Nelder-Mead method and the Powell method.

The optimal solution obtained for each problem defines the basic dimensions of a cross section having minimum cost. However, to some extent the dimensions can be varied with small increase in cost, because it was found from two-dimensional contour plots that the gradient of the objective function is quite flat near the optimum. The range of variation of the variables for a given increase in cost can be readily obtained from the contour plots.

The dimensions of the section for minimum cost are sensitive to relative changes in the unit costs of the materials. An increase in the cost of the cables, for example, causes an increase in the optimal depth. Studies using the optimization programs were limited to the general spans and roadway widths of the example problems. Results should not be widely generalized until further parameter studies are made.

8.2 Particular Conclusions

The following conclusions apply to the particular examples treated in Chapters 3, 4, 6, and 7, and may not necessarily apply to other cases, such as bridges with different spans, widths, and loadings.

For the bridges designed in Chapters 3 and 4, the more accurate stress analyses obtained with the MUPDI and SIMPLA2 programs confirmed the adequacy of the basically simple design procedures adopted. These procedures utilized hand calculations and beam theory to determine the basic concrete section and the layout and size of the prestressing cables required.

In the case of the bridge constructed on falsework (Chapter 3), the cable profile consisted of three parabolas and the equivalent cable loads were determined accurately. However, in the case of the bridge erected by the cantilever method (Chapter 4), the design procedure utilized approximate estimates of the equivalent loads of the system of cables. This procedure resulted in a satisfactory design, as confirmed by the MUPDI and SIMPLA2 analysis.

In the design of the bridge constructed on falsework, full length draped cables were adopted and their size determined by a simple load-balancing procedure. The cable area was set equal to that required to balance the dead load, assuming an idealized, double-parabolic profile for this purpose. The area so determined gave satisfactory service load stresses and ultimate strength.

For the bridge constructed by the cantilever method, it was found that the top cables could be designed to balance a uniform load of about 60 percent of the dead load on the completed cantilever. The quantity of cables so determined gave acceptable service load stresses and adequate ultimate strength during construction. The MUPDI program was used to determine a suitable pattern for the bottom cables. These were designed to give adequate ultimate strength under live load on the completed bridge and their quantity was adjusted after analysis to give better service load stresses.

Stress distributions obtained by the MUPDI analyses for both bridges designed indicated a variation of stress across the sections at the main piers, because of shear lag, and an almost uniform stress distribution across the top and bottom slabs at sections of zero shear.

Studies in Chapters 6 and 7 showed that the problem of optimizing the cross section of the bridge could be treated successfully as an unconstrained nonlinear programming problem. Physically acceptable solutions were obtained in all cases.

Of the two optimization methods used, the Nelder-Mead method was generally the more effective. It always gave steady convergence to a solution. On the other hand, in two of the three examples considered, the Powell method did not reach a solution for some starting points. The Nelder-Mead method is also the simpler of the two methods.

For sections of the double box girder type with an approximate overall width of 56 ft., whether constructed on falsework or by the cantilever method, the optimal width obtained for the cantilever overhang of the deck was 8.0 ft. This is the maximum width for which there is a single wheel load on this portion of the deck. The values of bottom slab width were such as to give sloping webs.

In the case of the three-cell box girder, the optimal solution indicated a large value of the cantilever overhang, a narrow bottom slab and sloping webs. The apparent reason for this configuration being optimal is that a large area of bottom slab is not required structurally except in the short region of negative moment.

The span/depth ratios obtained in the optimal solutions were approximately 30 for the two-span bridges constructed on falsework, and

33 for the three-span bridge constructed in cantilever. These values are higher than those generally occurring in similar bridges, excluding those having variable depth. The flatness of the cost gradient indicated relatively little cost change (2%) for span/depth ratios as low as 26. This indicates that the "optimum" ratio will be somewhat above 25 but factors such as fillet size and tendon congestion should be considered in selecting proportions.

8.3 Recommendations

Important design cases not considered in detail in this report include skew bridges and bridges with variable depth. These should be investigated more fully.

Highway crossovers will often be skewed. The design and analysis of such bridges will require modifications to the procedures developed for normal bridges. A few computer programs are now available for analyzing box girder skew bridges and these could be used in the way MUPDI was used in this study. Skew will also create some difficulties with segmental precasting. One solution is to cast a few segments near the ends of the span on the skew and the remaining segments normal to the roadway axis. Much will depend on the pier location details in this case.

Bridges having spans greater than 250 to 300 ft. generally vary in depth from a maximum at the piers to a minimum at midspan. In this way greater economy can be achieved. To a large extent the design procedure developed is applicable to bridges with variable depth. The finite element analysis program FINPLA2, developed by Scordelis, can be used for these bridges in place of the MUPDI analysis. Unfortunately, no program corresponding to SIMPLA2 exists for this case.

The optimization techniques should also be extended to cases where the bridge span is a variable, as well as the cross-sectional dimensions. Examples of such cases include three-span river crossings, in which the total length is specified but not the span ratio, and also viaducts. A wider study of effects of variables such as span and roadway width should be undertaken using the optimization techniques to provide further guidance for preliminary designs.

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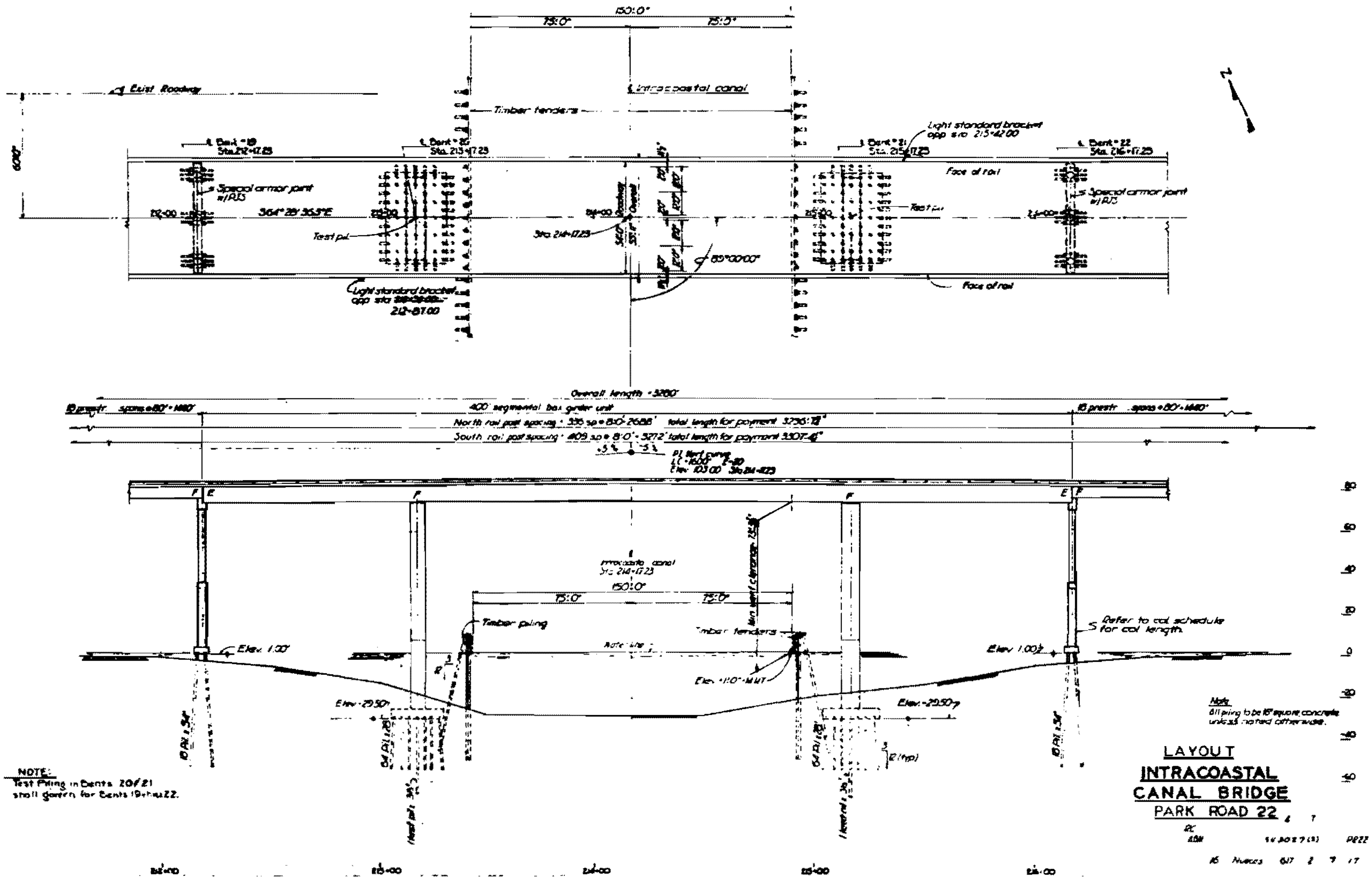
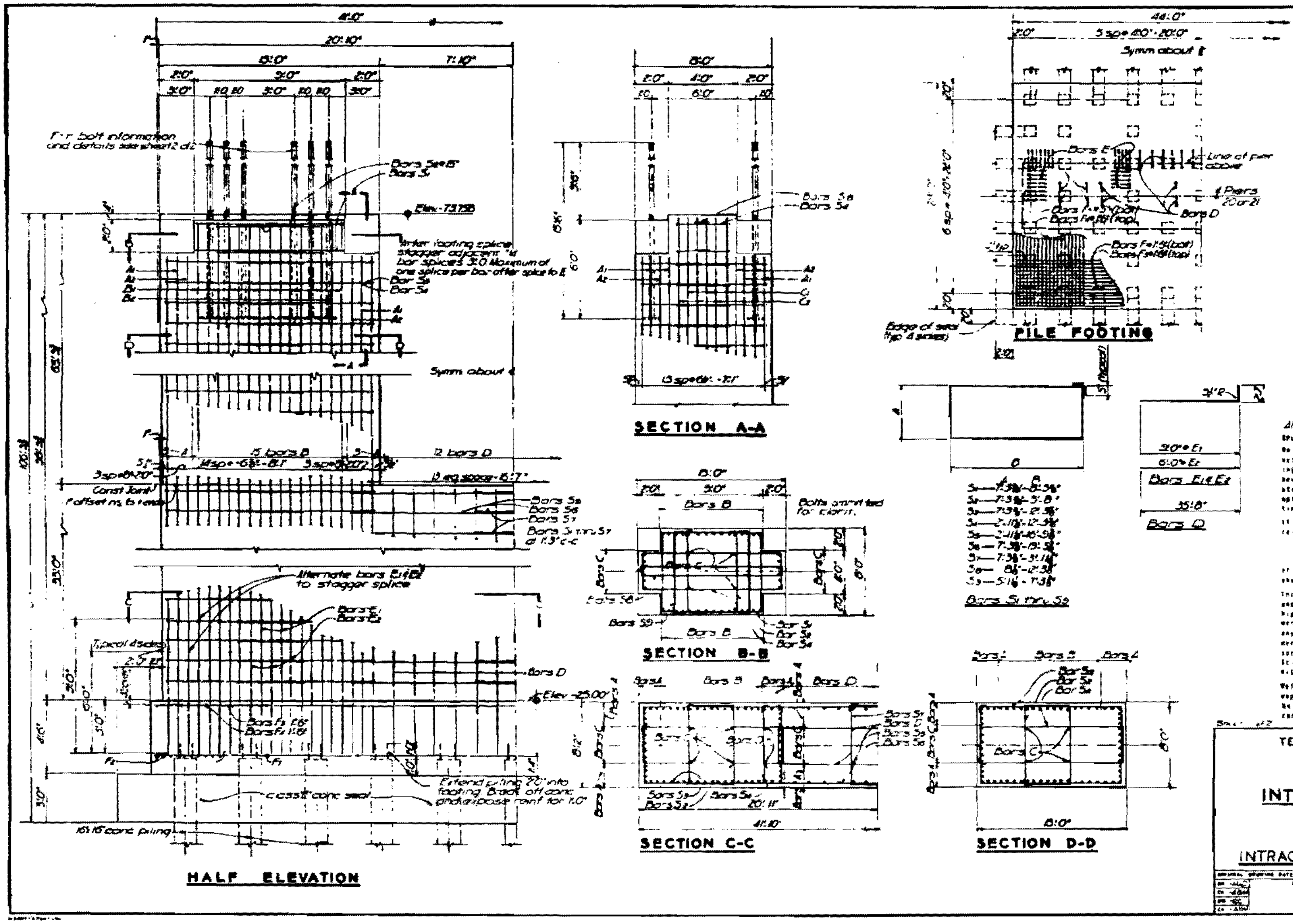
APPENDIX A

PROTOTYPE BRIDGE PLANS


150'-0"
75'-0"
75'-0"
Exist Roadway
Intracoastal canal
Timber fenders
Light standard bracket
Special armor joint
Test pile
Face of rail
Overall length
400' segmental box girder unit
Timber piling
Timber fenders
Elev. 1.00'
Elev. -29.50'
Refer to col schedule for col length.
NOTE:
Test Piling in Bents 20 & 21 shall govern for Bents 19 thru 22.
Note
All piling to be 18" square concrete unless noted otherwise.
LAYOUT
INTRACOASTAL
CANAL BRIDGE
PARK ROAD 22



BILL OF REINF & EST QUANT

Bar	No	Size	Length	Weight
A₁	24	#14	50'-2"	16555
A₂	24	#14	55'-2"	17105
B₁	30	#14	52'-2"	21152
B₂	30	#14	55'-2"	21641
C₁	16	#14	52'-6"	1522
C₂	16	#14	55'-6"	11689
D	32	#14	37'-1"	[illegible]
E₁	70	[illegible]	11'-[illegible]	[illegible]
E₂	70	[illegible]	8'-[illegible]	[illegible]
F₁	19	#10	43'-6"	[illegible]
F₂	104	#10	27'-[illegible]	[illegible]
F₃	19	#5	[illegible]	[illegible]
F₄	30	#5	27'-8"	6[illegible]5
S₁	4	#5	32'-0"	[illegible]
S₂	160	#5	22'-9"	37[illegible]
S₃	156	#5	40'-0"	65[illegible]
S₄	160	#5	31'-3"	521[illegible]
S₅	27	#5	40'-3"	1133
S₆	27	#5	54'-4"	1530
S₇	27	#5	23'-3"	655
S₈	4	#5	26'-11"	112
S₉	4	#5	27'-3"	114
Reinf steel			Lb	156,018
Conc cl "C"			Cu Yd	1,265
Conc cl "E"			Cu Yd	1707
Uncl struct excv			Cu Yd	567

All quantities shown are for one pier

SPLICE NOTE

No. 14 Bars B, A, C, & E may be spliced by full butt welding in accordance with item [illegible] (Structural Welding), or by the Cadweld process. All splices shall develop [illegible] psi which is [illegible] of the required yield strength of reinforcing steel. At the Contractor's option Bars A, B, C & E may be furnished in one continuous length.

If full butt welds are used, the chemistry of the reinforcing steel shall not exceed the following:

Carbon [illegible]
Manganese 1.50

If Cadweld process is used reinforcing bars shall be shear cut or saw cut at splice ends.

Three test specimens shall be [illegible] at the job site and shipped to Materials & Tests Laboratory, Texas Highway Department, Austin, for testing. Test specimens must comply with specifications prior to making any splices in the structure. If any one test specimen fails to meet specification, two additional test specimens shall be furnished for testing. [illegible] in the structure shall be made in conformity with test specimens.

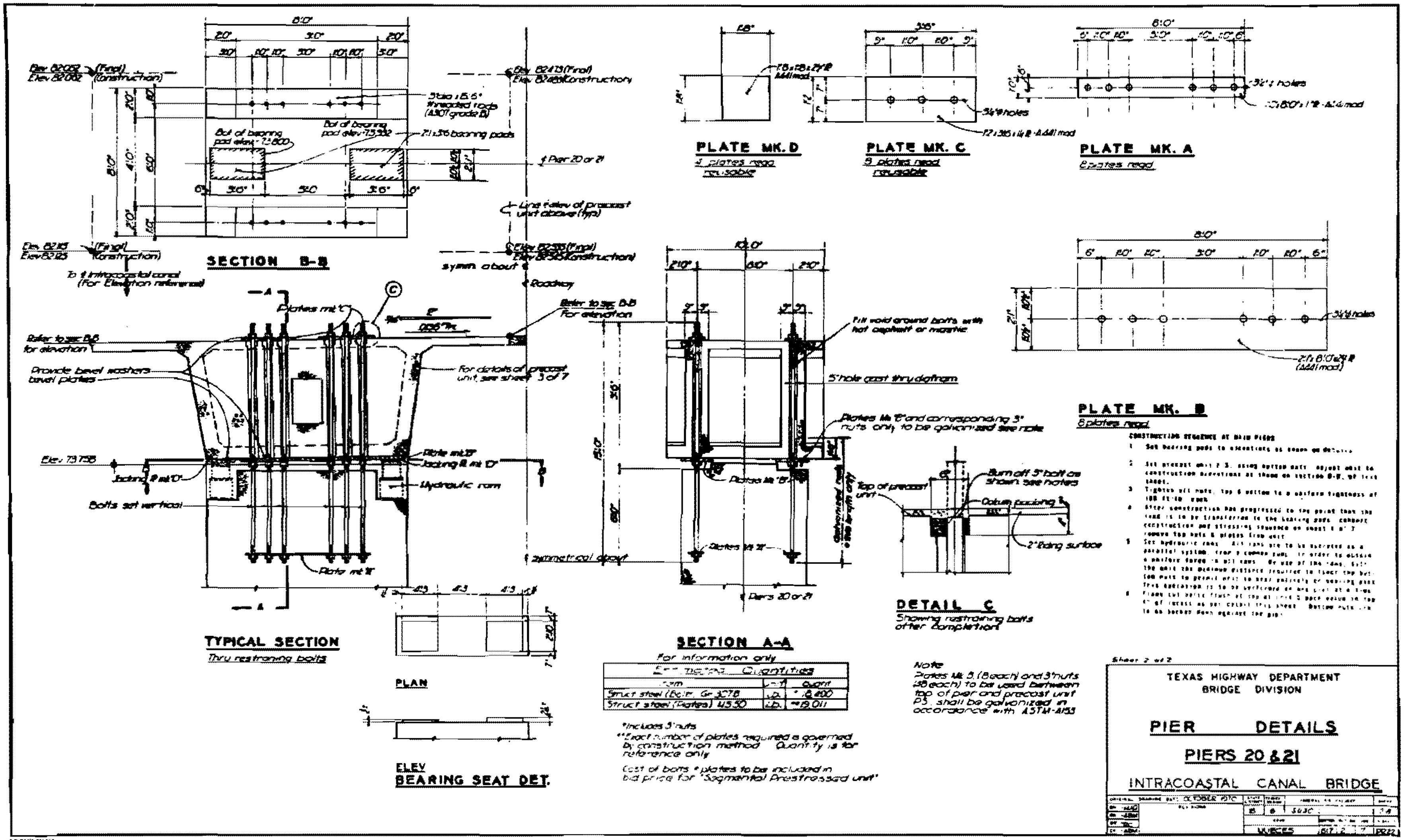
Welded splices will be radiographed by the Texas Highway Department at no expense to the Contractor. No direct payment will be made for splices but such cost shall be incidental to bid item "Reinforcing Steel".

TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION

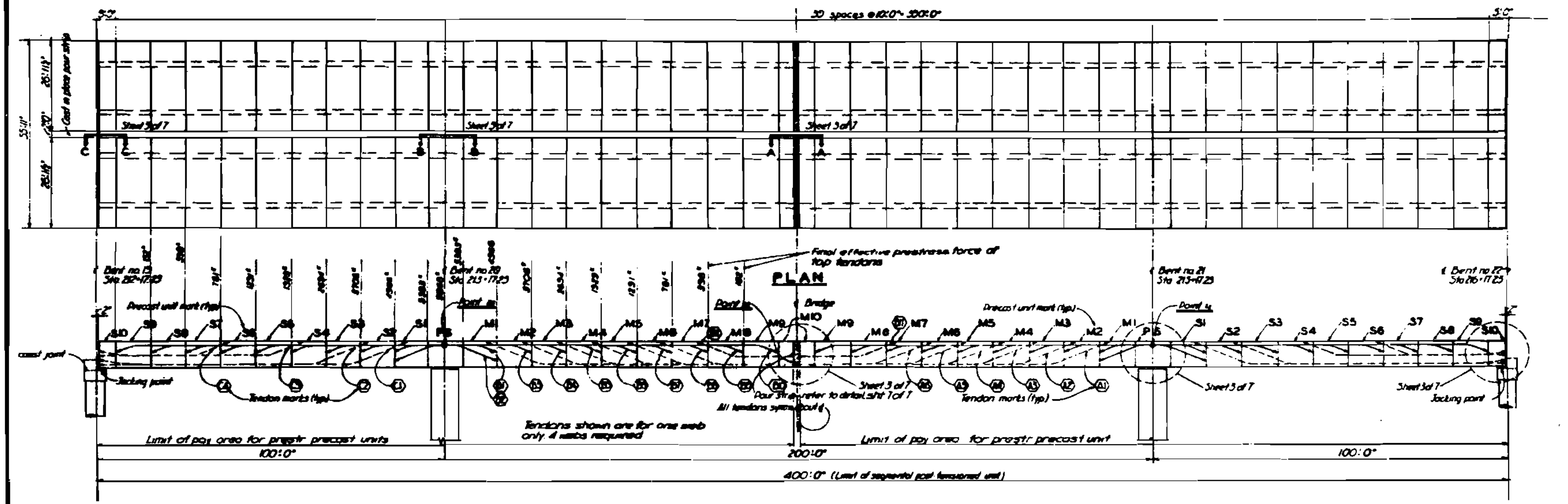
INTERIOR PIERS
20 & 21

INTRACOASTAL CANAL BRIDGE

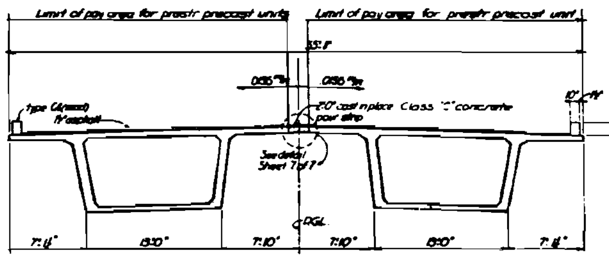
197



SECTION B-B
PLATE MK. D
PLATE MK. C
PLATE MK. A
PLATE MK. B
TYPICAL SECTION
Thru restraining bolts
PLAN
ELEV
BEARING SEAT DET.
SECTION A-A
For information only
DETAIL C
Showing restraining bolts after Completion
Provide bevel washers bevel plates
Bolts set vertical
Hydraulic ram
5" hole cast thru diafram
Fill void around bolts with hot asphalt or mastic
Sheet 2 of 2
TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION
PIER DETAILS
PIERS 20 & 21
INTRACOASTAL CANAL BRIDGE
OCTOBER 1970
NUECES



ELEVATION



TYPICAL SECTION

Apply epoxy joint sealer to all mating surfaces of precast units in accordance with specifications.

Grout all anchorage blockouts & match exposed surfaces to precast units.

The design will accomodate a 25^k load at the end of the cantilever, in addition to the weight of a segment. This value will be used as a guide in evaluating the effects of any erection or working apparatus attached to the cantilever.

Other tendon forces will be considered provided the minimum design forces shown in the elevation are provided.

Friction factors for calculating effective prestress will be as follows:
Rigid Conduit - k = .0002 μ = .25
Flexible Conduit - k = .0015 μ = .25

CONSTRUCTION & STRESSING SEQUENCE FOR LEFT & RIGHT [illegible] SEGMENTS

1. The superstructure is erected by the symmetric cantilever method to a distance of [illegible] either side of the pier, by stressing tendons with [illegible] on each of the precast units [illegible] erected.
2. After precast [illegible] in place [illegible] and stress tendons with [illegible] in the sequence C1, C2, C3 and C4. [illegible]
3. Place [illegible] closure strip.
4. Stress tendons A1, A2, A3 and A4 in that order.
5. [illegible] restraining [illegible] and transfer load to bearing pads. [illegible]
6. Adjust [illegible] at bents no. 19 & 22 [illegible] of 15 kips for each segment at each bent.
7. Stress tendons [illegible] then [illegible].
8. [illegible] jacking reactions at bents 19 & 22 [illegible] segment at each bent. [illegible] bearing pads & shims [illegible]
9. Place 2'-0 longitudinal closure strip.

Suggested Tendon Schedule

all forces shown are final effective prestress force after losses

Tendon Mark	Min. Force (Kips)	Tendon Mark	Min. Force (Kips)
B1	322.4*	A1	138.4*
B2	322.4*	A2	138.4*
B3	496.0***	A3	138.4*
B4	496.0***	A4	138.4*
B5	322.4*	A5	138.4*
B6	322.4*	A6	138.4*
B7	322.4*	**Tendon Mark**	**Min. Force**
B8	148.8*	C1	148.8*
B9	148.8*	C2	148.8*
B10	148.8*	C3	148.8*
B11	148.8**	C4	148.8*

* Minimum force reqd in tendon
** Optional - 2 tendons @ 248^k

Sheet 1 of 7

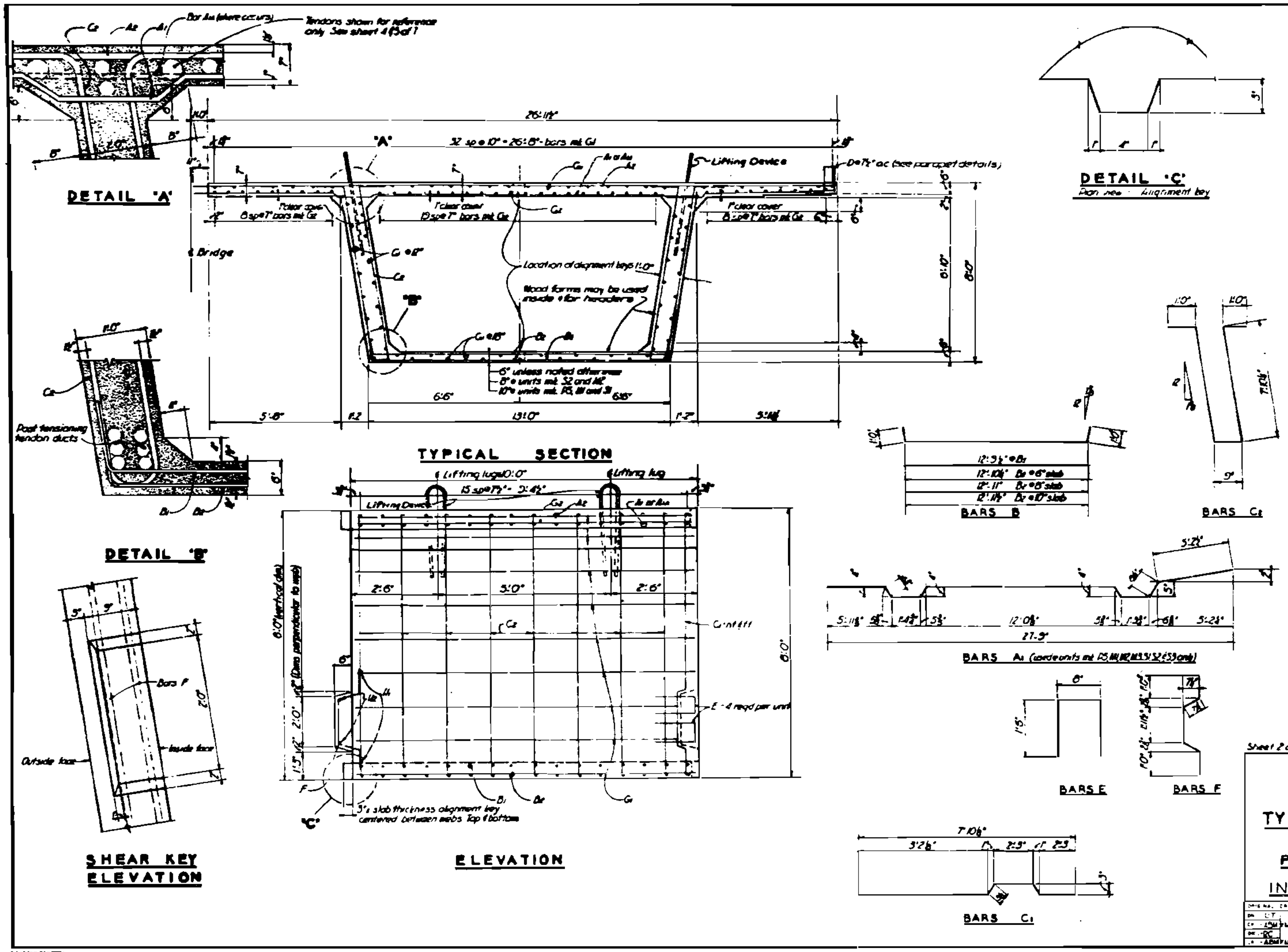
TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION

PLAN AND ELEVATION

400 FT. SEGMENTAL POST TENSIONED UNIT

INTRACOASTAL CANAL BRIDGE

Original drawing date OCTOBER 1970



BILL OF REINF. & EST. QUANT.
Typical pre-cast units

Bar	no.	size	length	weight
A_1	16	#8	[illegible]	1207
A_2	16	#8	[illegible]	1182
B_1	16	#4	14'-5½"	158
B_2 (avg)	16	#4	14'-10½	159
C_1	4	#8	8'-2½"	88
C_2	16	#8	18'-5½"	789
E	4	#4	3'-8"	10
F	4	#8	5'-2½"	56
G_1	77	#6	9'-9"	502
G_2	38	#6	9'-9"	556
H_1	4	#3	0'-9"	1
H_2	4	#3	0'-6"	1
Reinforcing (using A_{1A} bars)			Lb	4729
Reinforcing (using A_1 bars)			Lb	4798
Concrete-class ... #6" bottom slab			CY	13.7
Concrete-class ... #8" bottom slab			CY	14.3
Concrete-class ... #10" bottom slab			CY	15.1

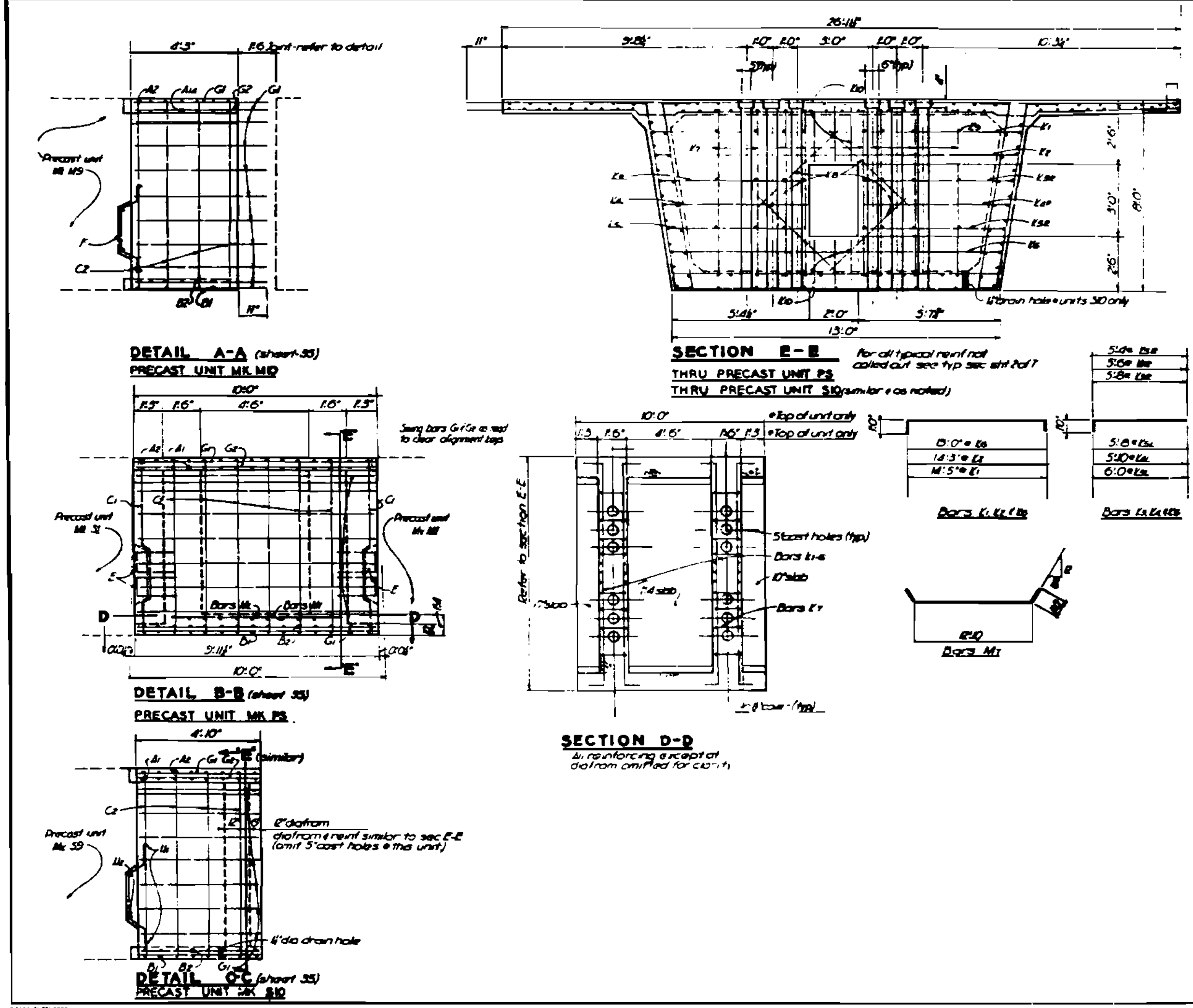
Tensioning Strength = 6000 p.s.i.
Handling Strength = 3000 p.s.i.

Sheet 2 of 7

TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION

TYPICAL PRECAST UNIT
400 FT. SEGMENTAL
POST TENSIONED UNIT
INTRACOASTAL CANAL BRIDGE

Date OCTOBER 1970
NUECES



BILL OF REINF & EST. QUANT.
PRECAST UNIT MK. MIO

For information only

Bar	no	size	length	weight
A1A	7	#8	27'-8	517
A2	7	#8	27'-8	517
B1	7	#4	14'-9½	69
B2	7	#4	14'-10½	69
C2	8	#8	18'-5½	394
F	4	#8	5'-2½	56
G1	77	#4	5'-0	257
G2	38	#6	5'-0	285
Reinforcing			Lb	2164
Concrete - class H			Cu Yd	58

BILL OF REINF & EST. QUANT.
PRECAST UNIT MK. PS

For information only

Bar	no	size	Length	weight
A1	16	#8	28'-3	207
A2	16	#8	27'-8	[illegible]
B1	[illegible]	#4	12'-9½	58
B2	[illegible]	#4	12'-11½	160
C1	8	#8	8'-3½	175
C2	12	#8	18'-5½	591
E	9	#4	3'-8	20
G1	77	#4	9'-9	502
G2	38	#6	9'-9	556
K1,2&6-MG	12	#6	15'-9	284
K3-5,5ᵃ MG	24	#6	5'-8	204
K7	16	#5	17'-2	286
K8	16	#6	4'-6	108
K9	48	#5	2'-3	113
K10	24	#5	2'-3	56
ML	23	#6	7'-0	242
Mr	11	#6	14'-10	245
Reinforcing			Lb	[illegible]
Concrete - class H			Cu Yd	[illegible]

BILL OF REINF & EST. QUANT.
PRECAST UNIT MK. S10

For information only

Bar	no	size	Length	weight
A1A	8	#8	27'-8	591
A2	8	#8	27'-8	591
B1	8	#4	14'-9½	79
B2	8	#4	14'-10½	79
C2	4	#8	18'-5½	197
F	4	#8	5'-2½	33
G1	77	#4	4'-7	236
G2	38	#6	4'-7	262
H1	4	#3	0'-9	1
H2	4	#3	0'-6	1
K1,2&6 MG	6	#6	15'-9	142
K3-5,5ᵃ MG	12	#6	5'-8	102
K7	8	#5	16'-10	140
K8	8	#6	4'-6	54
K9	24	#5	1'-9	44
K10	12	#5	2'-3	28
Reinforcing			Lb	2602
Concrete - class H			Cu Yd	94

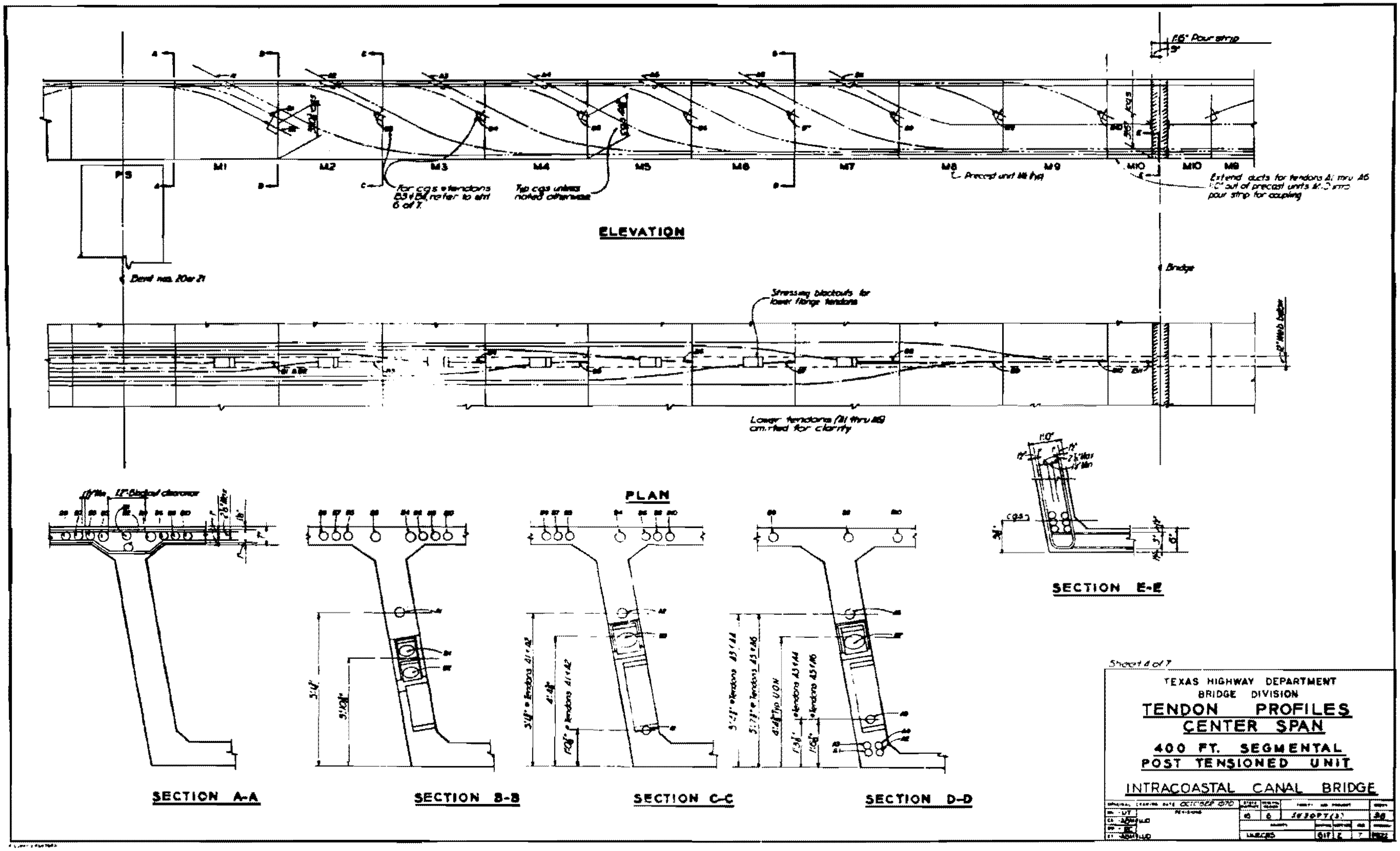
Sheet 5 of 7

TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION

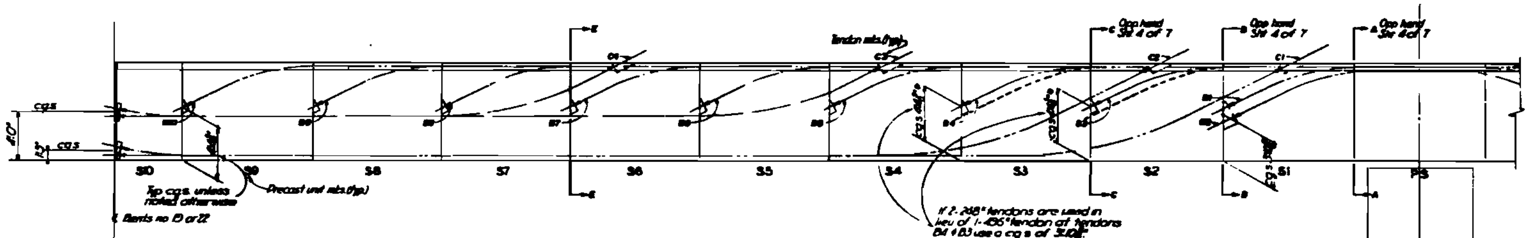
PRECAST UNITS CS-PS-S10

400 FT. SEGMENTAL POST TENSIONED UNIT

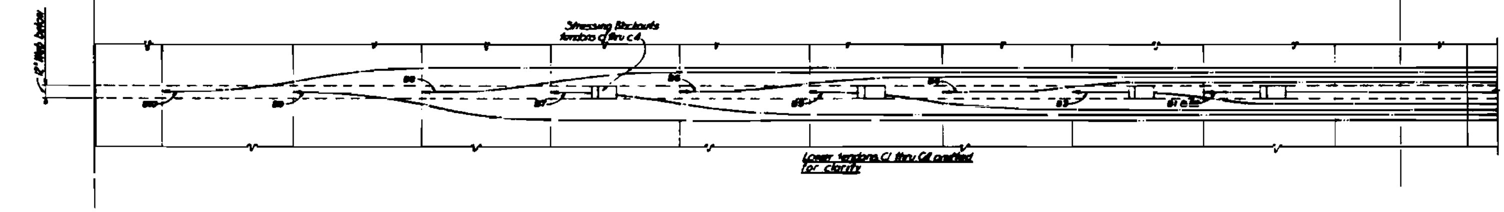
INTRACOASTAL CANAL BRIDGE



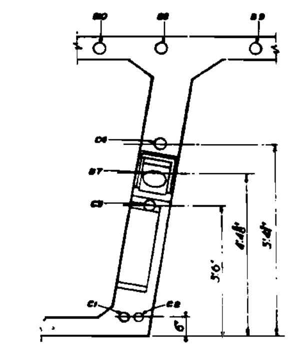
ELEVATION
PLAN
SECTION A-A
SECTION B-B
SECTION C-C
SECTION D-D
SECTION E-E
Lower tendons (B1 thru B9) omitted for clarity
Stressing blockouts for lower flange tendons
Extend ducts for tendons A1 thru A6 1'-0" out of precast units M-D into pour strip for coupling
Sheet 4 of 7
TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION
TENDON PROFILES
CENTER SPAN
400 FT. SEGMENTAL
POST TENSIONED UNIT
INTRACOASTAL CANAL BRIDGE



ELEVATION



PLAN



SECTION E-E

Sheet 5 of 7

TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION

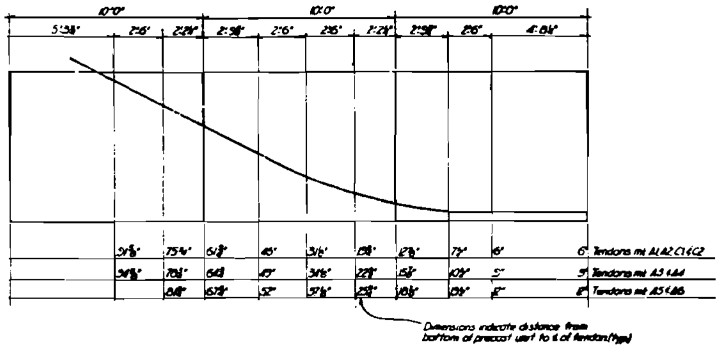
TENDON PROFILES
SIDE SPANS

400 FT. SEGMENTAL
POST TENSIONED UNIT

INTRACOASTAL CANAL BRIDGE

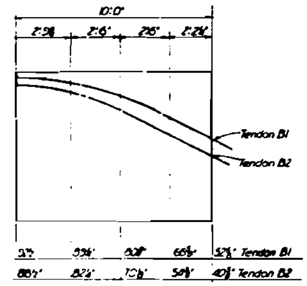
ORIGINAL DRAWING DATE OCTOBER 1970

203



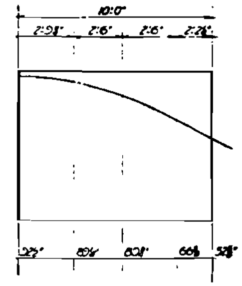
TENDON PROFILES

TENDONS MK A1 THRU A6 – C1 & C2



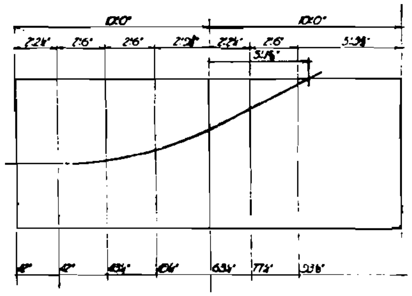
TENDON PROFILES

TENDON MK B1 & B2



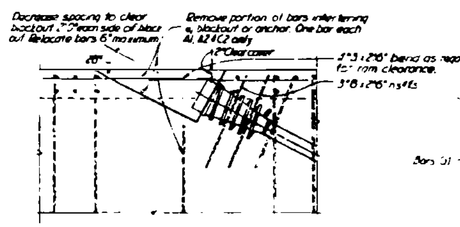
TENDON PROFILES

TENDONS MK B3 THRU B10



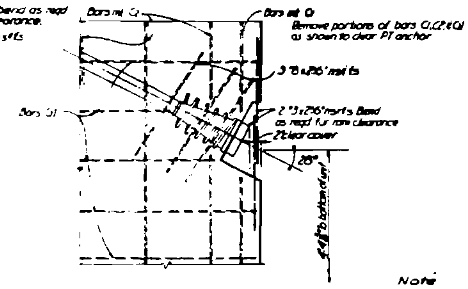
TENDON PROFILES

TENDONS–MK C3 – C4 & B11



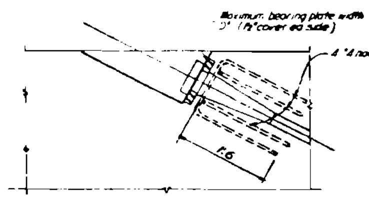
TENDON ANCHORAGE

WEDGE TYPE ANCHOR
TENDONS MK A1 THRU A6 & C1 THRU C4



TENDON ANCHORAGE

WEDGE TYPE ANCHOR
TENDONS MK B5 THRU B10



TENDON ANCHORAGE

BEARING PLATE TYPE ANCHOR
TYPICAL DETAIL

Note
Tendon anchorage details are not intended to limit types of strand systems used.

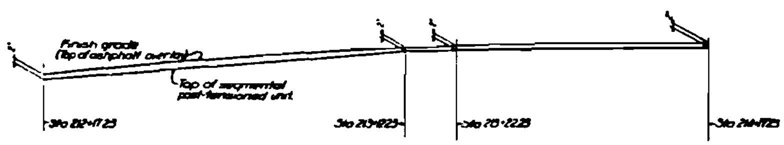
Sheet 6 of 7

TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION

TENDON PROFILES AND DETAILS

400 FT. SEGMENTAL POST TENSIONED UNIT

INTRACOASTAL CANAL BRIDGE

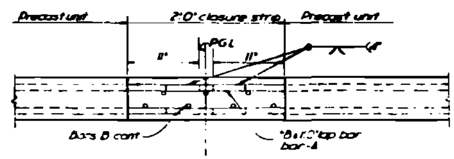


BLOCKING DIAGRAM

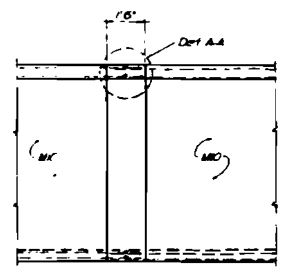
Dead load + post-tensioned prestressed unit in place

ELEVATION TABLE

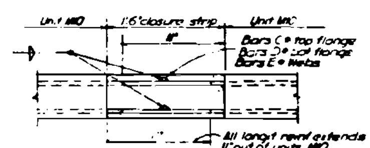
Sta no	Top of conc	Fin grade	Sta no	Top of conc	Fin grade
212+17.23	81.6250	81.7500	213+22.23	82.5990	82.7100
22.23	81.6727	[illegible]	32.23	82.6227	82.7142
32.23	81.7080	81.9505	42.23	82.6912	82.8242
42.23	81.8655	82.0430	52.23	82.6821	82.8680
52.23	81.9996	82.1492	62.23	82.7117	82.9055
62.23	82.0919	82.2492	72.23	82.7414	82.9367
72.23	82.1492	82.3430	82.23	82.7711	82.9617
82.23	82.2445	82.4305	92.23	82.8008	82.9805
92.23	82.3559	82.5117	214+02.23	82.8305	82.9930
213+02.23	82.4332	82.5867	12.23	82.8602	82.9992
12.23	82.5305	82.6555	17.23	82.8750	83.0000
17.23	82.5616	82.6875			



SECTION THRU LONGITUDINAL JOINT



ELEV. TRANSVERSE JOINT



DETAIL A-A
Bottom flange & webs similar

4.3 CY of C.I.P. conc and 227# of reinforcing for transverse joint are to be included in bid price for segmental Prestressed unit

BILL OF REINF & EST. QUANT. CLOSURE STRIP (Longitudinal)

Bar	no	size	length	weight
A	1182	#8	1'-0"	3156
* B	7	#5	406'-0"	2964
Reinforcing (Gr 60)			Lb	6120
Conc (CIP)			CY	17.3

* Includes 6-1'-0" laps.

Sheet 1 of 7

TEXAS HIGHWAY DEPARTMENT
BRIDGE DIVISION

BLOCKING & DETAILS

400 FT. SEGMENTAL POST TENSIONED UNIT

INTRACOASTAL CANAL BRIDGE

A P P E N D I X B

LISTING OF PROGRAM BOX2 TO CALCULATE SECTION PROPERTIES OF DOUBLE BOX GIRDER BRIDGE

Notation

N: Problem Number

B, B1, B3, B5, B7, D, T1, T2, T3, T4, T6, H1, V1, H2, V2, H3, V3, H4, V4:

Dimensions defined in Fig. B.1

Form of Input Data

1. First Card -- Format (I1)

   Column 1 - N

2. Second Card -- Format (8E10.3)

   Col. 1 to 10 - B
   Col. 11 to 20 - B1
   Col. 21 to 30 - B3
   Col. 31 to 40 - B5
   Col. 41 to 50 - B7
   Col. 51 to 60 - D
   Col. 61 to 70 - T1
   Col. 71 to 80 - T2

3. Third Card -- Format (8E10.3)

   Col. 1 to 10 - T3
   Col. 11 to 20 - T4
   Col. 21 to 30 - T6
   Col. 31 to 40 - H1
   Col. 41 to 50 - V1
   Col. 51 to 60 - H2
   Col. 61 to 70 - V2
   Col. 71 to 80 - H3

4. Fourth Card -- Format (8E10.3)

   Col. 1 to 10 - V3
   Col. 11 to 20 - H4
   Col. 21 to 30 - V4

5. All above data cards are repeated for next problem.
6. One blank card will terminate program.

Output Description

The output includes (a) a printout of the input quantities and (b) a list of the following section properties for both the half section (i.e., each unconnected box) and the full section:

Section area

Distance from top of girder to centroid of section

Second moment of area

Section modulus (top of girder)

Section modulus (bottom of girder)

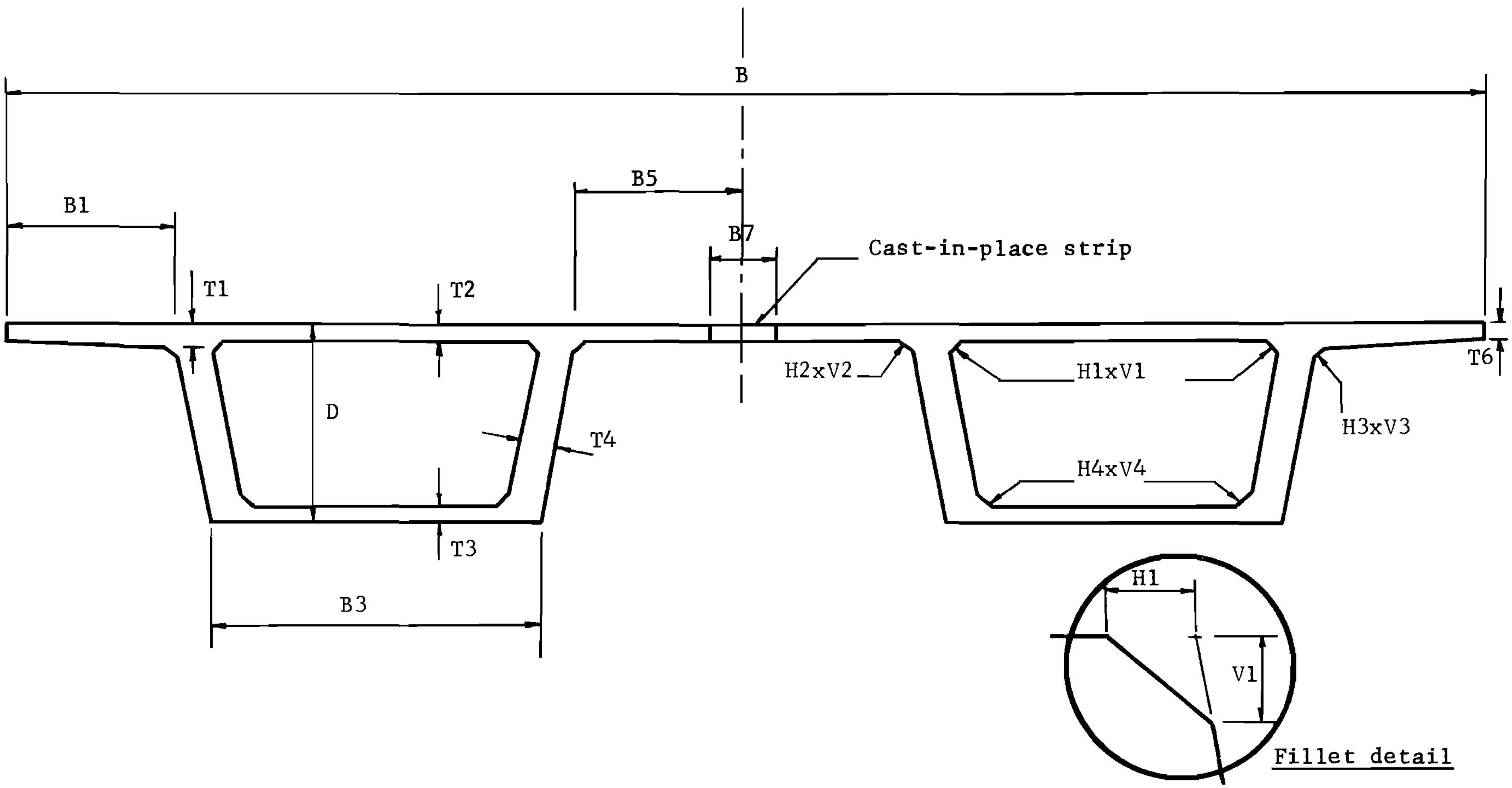


Fig. B.1. Notation for Program BOX2

```
      PROGRAM BOX2 (INPUT, OUTPUT)
      PRINT 1000
1000 FORMAT (1H1,  9X, *BOX GIRDER SECTION PROPERTIES*)
  10 READ 1010,N
1010 FORMAT (I1)
      IF (N.EQ.0) CALL EXIT
      READ 1020,  B, B1, B3, B5, B7, D, T1, T2, T3, T4, T6,
     1     H1, V1, H2, V2, H3, V3, H4, V4
1020 FORMAT (8E10.3)
      PRINT 1030, B, B1, B3, B5, B7, D, T1, T2, T3, T4, T6,
     1     H1, V1, H2, V2, H3, V3, H4, V4
1030 FORMAT (/// 10X, *B =*, E10.3, 5X, *B1 =*, E10.3, 5X, *B3 =*,
     1     E10.3, 5X, *B5 =*, E10.3/ 10X, *B7 =*, E10.3, 5X,
     1     *D =*, E10.3, 5X, *T1 =*, E10.3, 5X, *T2 =*, E10.3/
     1     10X, *T3 =*, E10.3, 5X, *T4 =*, E10.3, 5X, *T6 =*,
     1     E10.3, 5X, *H1 =*, E10.3/ 10X, *V1 =*, E10.3, 5X,
     1     *H2 =*, E10.3, 5X, *V2 =*, E10.3, 5X, *H3 =*, E10.3/
     1     10X, *V3 =*, E10.3, 5X, *H4 =*, E10.3, 5X, *V4 =*, E10.3)
      AA = 0.
      AY = 0.
      AYY = 0.
      A = B1*(T1+T6)/2.
      Y = (T1**2+T6**2+T1*T6)/(T1+T6)/3.
      AA = AA + A
      AY = AY + A*Y
      AYY = AYY + A*Y*Y
      A = ((B-B7)/2.-B1)*T2
      Y = T2/2.
      AA = AA + A
      AY = AY + A*Y
      AYY = AYY + A*Y*Y
      A = B3*T3
      Y = D-T3/2.
      AA = AA + A
      AY = AY + A*Y
      AYY = AYY + A*Y*Y
      SY = D-T2-T3
      SX = (B/2.-B1-B5-B3)/2.
      S = SQRT(SY**2+SX**2)
      A = 2.*S*T4
      Y = T2+SY/2.
      AA = AA + A
      AY = AY + A*Y
      AYY = AYY + A*Y*Y + A*SY**2/12.
      A = H1*V1
      Y = T2+V1/3.
      AA = AA + A
      AY = AY + A*Y
      AYY = AYY + A*Y*Y
      A = H2*V2/2.
      Y = T2+V2/3.
      AA = AA + A
```

```
      AY = AY + A*Y
      AYY = AYY + A*Y*Y
      A = H3*V3/2.
      Y = T1+V3/3.
      AA = AA + A
      AY = AY + A*Y
      AYY = AYY + A*Y*Y
      A = H4*V4
      Y = D-T3-V4/3.
      AA = AA + A
      AY = AY + A*Y
      AYY = AYY + A*Y*Y
      YM = AY/AA
      R = AYY - AA*YM*YM
      ST = R/YM
      SB = R/(D-YM)
      PRINT 1040, AA, YM, R, ST, SB
 1040 FORMAT (/ 10X, *PROPERTIES OF HALF SECTION*/
     1      15X, *AREA =*, E10.3/
     1      15X, *DISTANCE FROM TOP TO CENTROID =*, E10.3 /
     1      15X, *SECOND MOMENT OF AREA =*, E10.3/
     1      15X, *SECTION MODULUS (TOP) =*, E10.3/
     1      15X, *SECTION MODULUS (BTM) =*, E10.3)
      AA = 2.*AA
      AY = 2.*AY
      AYY = 2.*AYY
      A = B7*T2
      Y = T2/2.
      AA = AA + A
      AY = AY + A*Y
      AYY = AYY + A*Y*Y
      YM = AY/AA
      R = AYY - AA*YM*YM
      ST = R/YM
      SB = R/(D-YM)
      PRINT 1050, AA, YM, R, ST, SB
 1050 FORMAT (/10X, *PROPERTIES OF FULL SECTION*/
     1      15X, *AREA =*, E10.3/
     1      15X, *DISTANCE FROM TOP TO CENTROID =*, E10.3 /
     1      15X, *SECOND MOMENT OF AREA =*, E10.3/
     1      15X, *SECTION MODULUS (TOP) =*, E10.3/
     1      15X, *SECTION MODULUS (BTM) =*, E10.3)
      GO TO 10
      END
```

A P P E N D I X   C

OPTIMIZING PROGRAMS

SUBROUTINES TO CALCULATE OBJECTIVE FUNCTIONS

C.1 OPTMSE
C.2 SIMPLEX
C.3 OBJECTIVE FUNCTION 2 SPAN BRIDGE
C.4 OBJECTIVE FUNCTION MULTIWEB 2 SPAN BRIDGE
C.5 OBJECTIVE FUNCTION 3 SPAN BRIDGE

C.1

Unconstrained NLP Algorithm OPTMSE
(Powell's Method)

These instructions describe how to use the computer code to execute Powell's algorithm as programmed by Alan Brown.

1. Structure of the Program

1.1 Main Program (OPTMSE). The main program initializes the counting indices, introduces the initial step size by

STEP = 1.0

and causes the following function values and time printouts to be activated after completion:

(a) Time beginning

(b) Time ending

(c) Number of stages

(d) Number of functional evaluations in linear searches

(e) Value of $f(\underline{x})$, the objective function

(f) Value of the components of $\underline{x}$.

Place ICONVG = 1 if one pass through the Powell algorithm is sufficient. Place ICONVG = 2 if the final solution is to be perturbed, a new solution sought, and an extrapolation between the two solutions carried out.

1.2 Subroutine POWELL. This subroutine carries out the Powell algorithm. Place the desired accuracy in the fractional change in $f(\underline{x})$ and $\underline{x}$ as the first statement, such as ACC = 0.00001 (see TEST below).

1.3 Subroutine TEST. Executes the test for convergence on both $[f(\underline{x}^{k+1}) - f(\underline{x}^k)]/f(\underline{x}^k)$ and $[\underline{x}^{k+1} - \underline{x}^k]/\underline{x}^k$ (as well as $\Delta f(\underline{x})$ and $\Delta\underline{x}$ if $f(\underline{x})$ or $\underline{x}$ 0.

1.4 Subroutine COGGIN. Executes the unidimensional search to minimize $f(\underline{x})$ in a selected search direction. The initial step size is fixed by

STEP = 1.0

Termination is based on the fraction change in $f(\underline{x})$ in statement 27.
Note: Subroutine GOLDEN in the Davidon-Fletcher-Powell code is compatible with this code and may be used in lieu of COGGIN.

1.5 Subroutine FUN. Contains the function to be minimized. Place $f(\underline{x})$ below the comment card FUNCTION 1 as

FX = ...

2. User Supplied Information

2.1 Place the objective function, $f(\underline{x})$, to be minimized in subroutine FUN (see 1.5).

2.2 Place the initial values of the elements of $\underline{x}$ and the number of independent variables in BLOCK DATA as follows

```
DATA X(1), X(2), X(3)/3.0, -1.0, 0.0/
DATA N/3/
```

(or add, N/3/ to the end of first data entry).

NOTATION

Comment cards in the code explain most of the pertinent notation.

```
      PROGRAM OPTMSE(INPUT,OUTPUT)
      COMMON /ONE/  X,Y,S,FX,FY,N,KOUNTS,LIN,NDRV,H,SIG,DELG
      DIMENSION     X(10),Y(10),S(10),SIG(10),DELG(10),H(10,10)
C.....OPTIMIZATION BY THE POWELL METHOD.....
C *** NDRV IS A REDUNDANT PARAMETER WITHIN THE POWELL METHOD
      KOUNTS=0
      ICONVG=2
      STEP=1.0
      LIN=0
      CALL SECOND(A1)
      CALL POWELL(STEP,ICONVG)
      CALL SECOND(A2)
      PRINT 600,A1
      PRINT 601,KOUNTS,LIN,FX,(X(I),I=1,N)
      PRINT 602,A2
      CALL EXIT
  600 FORMAT(14H TIME IS NOW =,F20.3)
  601 FORMAT (/// I10,48H FUNCTION EVALUATIONS WITHIN POWELL ROUTINE AND
     1             I10,47H FUNCTION EVALUATIONS DURING THE LINE SEARCHES.
     2        //     18H FUNCTION VALUE = ,E20.8
     3        //     18H VARIABLE VALUES:- / (X,5E20.8 ))
  602 FORMAT(/// 14H TIME IS NOW = , F20.8)
      END
      SUBROUTINE POWELL (STEP,ICONVG)
      COMMON /ONE/  X,Y,S,FX,FY,N,KOUNT,LIN,NDRV,DIRECT,BEFORE,FIRST
      DIMENSION     X(10),Y(10),S(10),DIRECT(10,10),BEFORE(10),FIRST(10)
     1              ,W(10),SECND(10)
      EQUIVALENCE   (W,SECND)
C *** N = THE NUMBER OF VARIABLES.
C     KOUNT = THE NUMBER OF FUNCTIONS EVALUATIONS NOT IN LINEAR SEARCH.
C     ICONVG = THE FINAL CONVERGENCE TEST DESIRED.
C            = 1, TERMINATE AS SOON AS TESTING IS SATISFIED.
C            = 2, AS SOON AS THE TESTING CRITERIA ARE SATISFIED INCREASE
C                 ALL THE VARIABLES BY 10*ACC AND SOLVE PROBLEM AGAIN.
C     THEN PERFORM A LINE SEARCH BETWEEN THE SOLUTIONS IF DIFFERENT
C     SOLUTIONS ARE DEEMED TO BE FOUND.
C     STEP = THE INITIAL STEP SIZE.
C     ACC = THE REQUIRED ACCURACY IN THE FUNCTION AND VECTOR VALUES.
C     INSERT IPRINT= 1 FOR COMPLETE PRINT OUT OR IPRINT = 2 FINAL
C    1ANSWER ONLY
      ACC = .0001
      IPRINT=1
      NTRY=1
      N1=N-1
      STEPA=STEP
C *** SET UP THE INITIAL DIRECTION MATRIX (USING UNIT VECTORS).
      DO 2  I=1,N
      DO 1  J=1,N
    1 DIRECT(J,I)=0.
    2 DIRECT(I,I)=1.
C *** EVALUATED THE FUNCTION AT THE INITIAL VARIABLE VALUES.
  100 CALL FUN(X,FX)
      KOUNT=KOUNT+1
C *** SAVE THE FINAL FUNCTION VALUE (F1) AND THE FINAL VARIABLE VALUES
C     (BEFORE) FROM THE PREVIOUS CYCLE.
      PRINT 36
   36 FORMAT(8X,*FX*,12X,*X(1)*,14X,*X(2)*,12X,*X(3)*,13X,*X(4)*,12X,*X(
     15)*,14X,*X(6)*,)
    3 F1=FX
      DO 4  I=1,N
    4 BEFORE(I)=X(I)
```

```
      GO TO (801,802), IPRINT
  801 PRINT 901,FX,(X(I),I=1,N)
  901 FORMAT (/(5E16.8))
C *** START SEARCHING HERE.
  802 SUM=0.
C     AT THE END OF THE CYCLE, SUM WILL CONTAIN THE MAXIMUM CHANGE IN
C     THE FUNCTION VALUE FOR ANY SEARCH DIRECTION, AND ISAVE INDICATES
C     THE DIRECTION VECTOR TO WHICH IT CORRESPONDS.
      DO 9  I=1,N
C     S CONTAINS THE INITIAL STEP SIZES IN THE I-TH DIRECTION.
      DO 5  J=1,N
    5 S(J)=DIRECT(J,I)*STEP
C     FIND THE MINIMUM IN THE I-TH DIRECTION, AND THE CHANGE IN FUNCTION
C     VALUE.
      CALL COGGIN
      A=FX-FY
      IF(A-SUM) 7,7,6
    6 ISAVE=I
      SUM=A
C     TRANSFER THE NEW FUNCTION AND VARIABLE VALUES TO FX AND X.
    7 DO 8  J=1,N
    8 X(J)=Y(J)
    9 FX=FY
C *** NOW INVESTIGATE WHETHER A NEW SEARCH DIRECTION SHOULD BE INCORPOR-
C     ATED INSTEAD OF THE ISAVE DIRECTION.
      F2=FX
      DO 10  I=1,N
   10 W(I)=2.0*X(I)-BEFORE(I)
      CALL FUN(W,F3)
      KOUNT=KOUNT+1
      A=F3-F1
      IF(A) 11,19,19
   11 A=2.0*(F1-2.0*F2+F3)*((F1-F2-SUM)/A)**2
      IF(A-SUM) 12,19,19
C *** A NEW SEARCH DIRECTION IS REQUIRED. FIRST REMOVE ROW ISAVE.
   12 IF(ISAVE-N) 13,15,15
   13 DO 14  I=ISAVE,N1
      II=I+1
      DO 14  J=1,N
   14 DIRECT(J,I)=DIRECT(J,II)
C     SET THE N-TH DIRECTION VECTOR EQUAL TO THE NORMALISED DIFFERENCE
C     BETWEEN THE INITIAL AND FINAL VARIABLE VALUES FOR LAST CYCLE.
   15 A=0.
      DO 16  J=1,N
      DIRECT(J,N)=X(J)-BEFORE(J)
   16 A=DIRECT(J,N)**2+A
      A=1.0/SQRT(A)
      DO 17  J=1,N
      DIRECT(J,N)=DIRECT(J,N)*A
   17 S(J)=DIRECT(J,N)*STEP
      CALL COGGIN
      FX=FY
      DO 18  I=1,N
   18 X(I)=Y(I)
C *** TEST FOR CONVERGENCE.
   19 CALL TEST(F1,FX,BEFORE,X,FLAG,N,ACC)
      IF(FLAG) 22,22,20
C *** CONVERGENCE NOT YET ACHEIVED, COMPUTE A NEW STEP SIZE AND
C     GO BACK TO 3.
   20 IF(F1-FX)121,120,120
  121 STEP=-0.4*SQRT(ABS(F1-FX))
      GO TO 123
```

```
  120 STEP=0.4*SQRT(F1-FX)
  123 IF(STEPA-STEP) 21,3,3
   21 STEP=STEPA
      GO TO 3
C *** CONVERGENCE ACHEIVED. IF ICONVG=2, INCREASE ALL VARIABLES BY
C     10*ACC AND GO BACK TO 3.
   22 GO TO (23,24),ICONVG
   23 RETURN
   24 GO TO (25,27),NTRY
   25 NTRY=2
      DO 26  I=1,N
      FIRST(I)=X(I)
   26 X(I)=X(I)+ACC*10.
      FFIRST=FX
      GO TO 100
C *** CONVERGENCE ATTAINED USING TWO DIFFERENT STARTING POINTS. CONSTRUC
C     UNIT VECTOR BETWEEN SOLUTIONS AND SEARCH DIRECTION FOR A MINIMUM.
   27 FSECND=FX
      A=0.
      DO 28  I=1,N
      SECND(I)=X(I)
      S(I)=FIRST(I)-SECND(I)
   28 A=A+S(I)**2
      IF(A) 23,23,29
   29 A=STEP/SQRT(A)
      DO 30  I=1,N
   30 S(I)=S(I)*A
      CALL COGGIN
C *** TEST IF NEW POINT IS SUFFICIENTLY CLOSE TO EITHER OF THE TWO
C     SOLUTIONS. IF SO RETURN.
      CALL TEST(FFIRST,FY,FIRST,Y,FLAG,N,ACC)
      IF(FLAG) 32,32,31
   31 CALL TEST(FSECND,FY,SECND,Y,FLAG,N,ACC)
      IF(FLAG) 32,32,34
   32 DO 33  I=1,N
   33 X(I)=Y(I)
      FX=FY
      RETURN
C *** FINAL SOLUTION NOT ACCURATE ENOUGH.  REPLACE THE FIRST DIRECTION
C     VECTOR BY INTER-SOLUTION VECTOR (NORMALISED) AND RECYCLE
   34 A=A/STEP
      DO 35  I=1,N
      DIRECT(I,1)=(FIRST(I)-SECND(I))*A
   35 FIRST(I)=SECND(I)
      GO TO 3
      END
      SUBROUTINE TEST(FI,FF,RI,RF,FLAG,N,ACC)
      DIMENSION  RI(10),RF(10)
      FLAG=+2.
      IF(ABS(FI)-ACC) 2,2,1
    1 IF(ABS((FI-FF)/FI)-ACC) 3,3,7
    2 IF(ABS(FI-FF)-ACC) 3,3,7
    3 DO 6  I=1,N
      IF(ABS(RI(I))-ACC) 5,5,4
    4 IF(ABS((RI(I)-RF(I))/RI(I))-ACC) 6,6,7
    5 IF(ABS(RI(I)-RF(I))-ACC) 6,6,7
    6 CONTINUE
      FLAG=-2.
    7 RETURN
      END
      SUBROUTINE COGGIN
      COMMON /ONE/  X,Y,S,FX,FY,N,KOUNTS,LIN,NDRV,H,SIG,DELG
```

```
      DIMENSION     X(10),Y(10),S(10),SIG(10),DELG(10),H(10,10)
C *** THE INITIAL VARIABLE VALUES ARE IN X, AND THE CORRESPONDING
C *** FUNCTION VALUE IS FX.
C *** THE SEARCH DIRECTION VECTOR IS S, AND THE INITIAL STEP SIZE STEP.
C *** LIN IS USED TO COUNT THE NUMBER OF FUNCTION EVALUATIONS AND N IS
C *** THE NUMBER OF VARIABLES.
      FA=FB=FC=FX
      DA=DB=DC=0.
      STEP=1.0
      D=STEP
      K=-2
      M=0
C *** START THE SEARCH THE BOUND THE MINIMUM
    1 DO 2  I=1,N
    2 Y(I)=X(I)+D*S(I)
      CALL FUN(Y,F)
      K=K+1
      LIN=LIN+1
      IF(F-FA) 5,3,6
C *** NO CHANGE IN FUNCTION VALUE.  RETURN WITH VECTOR CORRESPONDING TO
C     FUNCTION VALUE OF FA, BECAUSE IF THE FUNCTION VALUE IS INDEPENDENT
C     OF THIS SEARCH DIRECTION, THEN CHANGES IN THE VARIABLE VALUES MAY
C     UPSET THE MAIN PROGRAM CONVERGENCE TESTING.
    3 DO 4  I=1,N
    4 Y(I)=X(I)+DA*S(I)
      FY=FA
      RETURN
C *** THE FUNCTION IS STILL DECREASING.  INCREASE THE STEP SIZE BY
C     DOUBLE THE PREVIOUS INCREASE IN STEP SIZE.
    5 FC=FB  $  FB=FA  $  FA=F
      DC=DB  $  DB=DA  $  DA=D
      D=2.0*D+STEP
      GO TO 1
C *** MINIMUM IS BOUNDED IN AT LEAST ONE DIRECTION.
    6 IF(K) 7,8,9
C     MINIMUM IS BOUNDED IN ONE DIRECTION ONLY.  REVERSE THE SEARCH
C     DIRECTION AND RECYCLE.
    7 FB=F
      DB=D  $  D=-D  $  STEP=-STEP
      GO TO 1
C     MINIMUM IS BOUNDED IN BOTH DIRECTIONS AFTER ONLY TWO FUNCTION
C     EVALUATIONS (ONE EITHER SIDE OF THE ORIGIN).  PROCEED TO THE
C     PARABOLIC INTERPOLATION.
    8 FC=FB  $  FB=FA  $  FA=F
      DC=DB  $  DB=DA  $  DA=D
      GO TO 21
C     THE MINIMUM IS BOUNDED AFTER AT LEAST TWO FUNCTION EVALUATIONS IN
C     THE SAME DIRECTION.  EVALUATE THE FUNCTION AT STEP SIZE=(DA+DB)/2.
C     THIS WILL YEILD 4 EQUALLY SPACED POINTS BOUNDING THE MINIMUM.
    9 DC=DB  $  DB=DA  $  DA=D
      FC=FB  $  FB=FA  $  FA=F
   10 D=0.5*(DA+DB)
      DO 11  I=1,N
   11 Y(I)=X(I)+D*S(I)
      CALL FUN(Y,F)
      LIN=LIN+1
C *** NOW HAVE THAT  FA>FB<FC  AND THAT  FA>F<FC  ASSUMING THAT THE
C     FUNCTION IS UNIMODAL.  REMOVE EITHER POINT A OR POINT B IN SUCH A
C     WAY THAT THE FUNCTION IS BOUNDED AND  FA>FB<FC  (THE CORRESPONDING
C     STEP SIZES ARE  DA>DB>DC  OR  DA<DB<DC  ).
   12 IF((DC-D)*(D-DB)) 15,13,18
C *** LOCATION OF MINIMUM IS LIMITED BY ROUNDING ERRORS. RETURN WITH B.
```

```
   13 DO 14  I=1,N
   14 Y(I)=X(I)+DB*S(I)
      FY=FB
      RETURN
C *** THE POINT D IS IN THE RANGE DA TO DB.
   15 IF(F-FB) 16,13,17
   16 FC=FB  $  FB=F
      DC=DB  $  DB=D
      GO TO 21
   17 FA=F
      DA=D
      GO TO 21
C *** THE POINT D IS IN THE RANGE DB TO DC
   18 IF(F-FB) 19,13,20
   19 FA=FB  $  FB=F
      DA=DB  $  DB=D
      GO TO 21
   20 FC=F
      DC=D
C *** NOW PERFORM THE PARABOLIC INTERPOLATION.
   21 A=FA*(DB-DC)+FB*(DC-DA)+FC*(DA-DB)
      IF(A) 22,30,22
   22 D=0.5*((DB*DB-DC*DC)*FA+(DC*DC-DA*DA)*FB+(DA*DA-DB*DB)*FC)/A
C     CHECK THAT THE POINT IS GOOD. IF SO, EVALUATE THE FUNCTION.
      IF((DA-D)*(D-DC)) 13,13,23
   23 DO 24  I=1,N
   24 Y(I)=X(I)+D*S(I)
      CALL FUN(Y,F)
      LIN=LIN+1
C *** CHECK FOR CONVERGENCE.  IF NOT ACHEIVED, RECYCLE.
      IF(ABS(FB)-0.00001) 25,25,26
   25 A=1.0  $  GO TO 27
   26 A=1.0/FB
   27 IF(ABS((FB-F)*A)-.0001) 28,28,12
C *** CONVERGENCE ACHEIVED.  RETURN WITH THE SMALLER OF F AND FB.
   28 IF(F-FB) 29,13,13
   29 FY=F
      RETURN
C *** THE PARABOLIC INTERPOLATION WAS PREVENTED BY THE DIVISOR BEING
C     ZERO.  IF THIS IS THE FIRST TIME THAT IT HAS HAPPENED, TRY AN
C     INTERMEDIATE STEP SIZE AND RECYCLE; OTHERWISE GIVE UP AS IT LOOKS
C     LIKE A LOST CAUSE.
   30 IF(M) 31,31,13
   31 M=M+1
      GO TO 10
      END
      BLOCK DATA
      COMMON /ONE/  X,Y,S,FX,FY,N,KOUNTS,LIN,NDRV,H,SIG,DELG
      DIMENSION     X(10),Y(10),S(10),SIG(10),DELG(10),H(10,10)
      DATA X(1)/40.0/,N/1/
      END
      SUBROUTINE FUN(X,FX)
      DIMENSION X(10)
C     FUNCTION 1
      P=0.02516
      T=X(1)
C        INSERT SUBROUTINE TO COMPUTE C
      B=3600./(44.1*P*T**2-310.5*(P*T)**2-1.562*T)
      C=((0.2421+11.91*P)*B*T+3.33*T+1.667*B)/25.0  $ FX=C
      PRINT 1,P,T ,B,C
    1 FORMAT(5X,*P=*,F10.5,5X,*INITIAL T=*,F10.2,*B=*,E10.3,*C=*,E10.3/)
      RETURN
```

Optimization Method of Nelder and Mead (SIMPLEX)
Instructions to Introduce Data into the Work

The Data Cards are as follows:

1. 7
   8
   9

2. Punch NX, the number of variables in the objective function in format (I5) in col. 1-5 right justified, and STEP, the step size in format (F10.5) in columns 6-15. In the absence of other information select

   $$\text{STEP} = \min \left(\frac{0.2}{n} \sum_{i=1}^{n} d_i \ , \ d_1, d_2, \ldots, d_n\right)$$

   where n = no. independent variables and d_i is the possible region for search for the variable x_i.

3. Punch the initial guess for each variable in format (F10.5).
   Cards 2,3 can be repeated changing the step size and intial variables as desired, but after last card of type 3 must come:

4. Blank card

5. 6
   7
   8
   9

The card (SUM(IN) = f(x)), the third last card in the deck, must be changed for each different function. Allowance has been made for a 50 dimension problem, i.e., X(1) to X(50).

```
      PROGRAM SIMPLEX(INPUT,OUTPUT)
C.....OPTIMIZATION BY THE NELDER-MEAD METHOD.....
C     NX IS THE NUMBER OF INDEPENDENT VARIABLES.
C     STEP IS THE INITIAL STEP SIZE.
C     X(I) IS THE ARRAY OF INITIAL GUESSES.
C     DATA CARDS ARE AS FOLLOWS.
C     CARD NO.          PARAMETERS          FORMAT          COLUMNS
C        1                NX                  I5            1 THRU 5
C        1                STEP                F10.5         6 THRU 15
C        2                X(I)                F10.5         1 THRU 10
C     CARD 3 IS BLANK.
C     TO OPTIMIZE THE OBJECTIVE FUNCTION FOR ANOTHER SET OF PARAMETERS
C     REPEAT CARDS 1 AND 2 ONLY.
C     FOR PROPER PRINTOUT OF DESIRED X(I) ARRAY, FORMAT STATEMENTS 103
C     AND 101 MUST BE REVISED ACCORDINGLY.
      DIMENSION X1(50,50), X(50), SUM(50)
      COMMON/1/ X,X1,NX,STEP,K1,SUM,IN
    1 FORMAT(I5,F10.5)
  100 READ1, NX,STEP
      IF(NX) 998,999,998
  998 READ 2, (X(I),I=1,NX)
    2 FORMAT(10F10.5)
      ALFA=1.0
      BETA=0.5
      GAMA=2.0
      DIFER = 0.
      XNX = NX
      IN = 1
      CALL SUMR
       PRINT 102,SUM(1),(X(I),I=1,NX)
      PRINT 1002,STEP
      CALL SECOND(TIME)
      PRINT 105,TIME
  105 FORMAT(/50X,11HTIME IS NOW,F10.3,8H SECONDS/)
      PRINT 103
  103 FORMAT(4X,14HFUNCTION VALUE,15X,3HX1=,20X,3HX2=,20X,3HX3=,20X,3HX4
     1=,16X,12HFUNC. CHANGE)
  102 FORMAT(1H1,12X,23HFUNCTION STARTING VALUE,F10.5,/,*THE X ARRAY IS*
     1,/,5X,10(E11.4,2X))
 1002 FORMAT(12X,*STEP=*,F6.2)
      K1 = NX + 1
      K2 = NX + 2
      K3 = NX + 3
      K4 = NX + 4
      CALL START
   25 DO 3 I = 1, K1
      DO 4 J = 1, NX
    4 X(J) = X1(I ,J)
      IN = I
      CALL SUMR
    3 CONTINUE
C  SELECT LARGEST VALUE OF SUM(I) IN SIMPLEX
   28 SUMH = SUM(1)
      INDEX = 1
      DO 7 I = 2, K1
      IF(SUM(I).LE.SUMH) GO TO 7
      SUMH = SUM(I)
      INDEX = I
    7 CONTINUE
C  SELECT MINIMUM VALUE OF SUM(I) IN SIMPLEX
      SUML = SUM(1)
```

```
      KOUNT = 1
      DO 8 I = 2, K1
      IF(SUML.LE.SUM(I)) GO TO 8
      SUML = SUM(I)
      KOUNT = I
    8 CONTINUE
C  FIND CENTROID OF POINTS WITH I DIFFERENT THAN INDEX
      DO 9 J = 1, NX
      SUM2 = 0.
      DO 10 I = 1, K1
   10 SUM2 = SUM2 + X1(I,J)
      X1(K2,J) =1./XNX*(SUM2 - X1(INDEX,J))
C  FIND REFLECTION OF HIGH POINT THROUGH CENTROID
      X1(K3,J) = (1. + ALFA)*X1(K2,J) - ALFA*X1(INDEX,J)
    9 X(J) = X1(K3,J)
      IN = K3
      CALL SUMR
      IF(SUM(K3).LT.SUML) GO TO 11
C   SELECT SECOND LARGEST VALUE IN SIMPLEX
      IF(INDEX.EQ.1) GO TO 38
      SUMS = SUM(1)
      GO TO 39
   38 SUMS = SUM(2)
   39 DO 12 I = 1, K1
      IF((INDEX - I).EQ.0) GO TO 12
      IF(SUM(I).LE.SUMS) GO TO 12
      SUMS = SUM(I)
   12 CONTINUE
      IF(SUM(K3).GT.SUMS) GO TO 13
      GO TO 14
C  FORM EXPANSION OF NEW MINIMUM IF REFLECTION HAS PRODUCED ONE MINIMUM
   11 DO 15 J = 1, NX
      X1(K4,J) = (1  - GAMA)*X1(K2,J) + GAMA*X1(K3,J)
   15 X(J) = X1(K4,J)
      IN = K4
      CALL SUMR
      IF(SUM(K4).LT.SUML) GO TO 16
      GO TO 14
   13 IF(SUM(K3).GT.SUMH) GO TO 17
      DO 18 J = 1, NX
   18 X1(INDEX,J) = X1(K3,J)
   17 DO 19 J = 1, NX
      X1(K4,J) = BETA*X1(INDEX,J) + (1. - BETA)*X1(K2,J)
   19 X(J) = X1(K4,J)
      IN = K4
      CALL SUMR
      IF(SUMH.GT.SUM(K4)) GO TO 16
C  REDUCE SIMPLEX BY HALF IF REFLECTION HAPPENS TO PRODUCE A LARGER VAL
C  LUE THAN THE MAXIMUM
      DO 20 J = 1, NX
      DO 20 I = 1, K1
   20 X1(I,J) = 0.5*(X1(I,J) + X1(KOUNT,J))
      DO 29 I = 1, K1
      DO 30 J = 1, NX
   30 X(J) = X1(I,J)
      IN = I
      CALL SUMR
   29 CONTINUE
      GO TO 26
   16 DO 21 J = 1, NX
      X1(INDEX,J) = X1(K4,J)
   21 X(J) = X1(INDEX,J)
```

```
      IN = INDEX
      CALL SUMR
      GO TO 26
   14 DO 22 J = 1, NX
      X1(INDEX,J) = X1(K3,J)
   22 X(J) = X1(INDEX,J)
      IN = INDEX
      CALL SUMR
   26 DO 23 J = 1, NX
   23 X(J) = X1(K2,J)
      IN = K2
      CALL SUMR
      DIFER = 0.
      DO 24 I = 1, K1
   24 DIFER = DIFER + (SUM(I) - SUM(K2))**2
      DIFER =1./XNX*SQRT(DIFER)
      PRINT 101, SUML, (X1(KOUNT,J), J= 1,NX), DIFER
  101 FORMAT(2(2X,E16.6),3(7X,E16.6),12X,E16.6)
      IF (DIFER.GE..0001) GO TO 28
      CALL SECOND(TIME)
      PRINT 105,TIME
      GO TO 100
  999 CONTINUE
      END
      SUBROUTINE START
      DIMENSION A(50,50), X1(50,50), X(50), SUM(50)
      COMMON/1/ X,X1,NX,STEP,K1,SUM,IN
      VN = NX
      STEP1 = STEP/(VN*SQRT(2.))*(SQRT(VN + 1.) + VN - 1.)
      STEP2= STEP/(VN*SQRT(2.))*(SQRT(VN + 1.) - 1.)
      DO 1 J = 1, NX
    1 A(1,J) = 0.
      DO 2 I = 2, K1
      DO 2 J = 1, NX
      A(I,J) = STEP2
      L = I - 1
      A(I,L) = STEP1
    2 CONTINUE
      DO 3 I = 1, K1
      DO 3 J = 1, NX
    3 X1(I,J) = X(J) + A(I,J)
      RETURN
      END
      SUBROUTINE SUMR
      COMMON/1/ X,X1,NX,STEP,K1,SUM,IN
      DIMENSION X1(50,50), X(50), SUM(50)
      B1 = X(1)
      B2 = X(2)
      B3 = X(3)
      D = X(4)
C        INSERT SUBROUTINE TO COMPUTE C
      SUM(IN) = C
      RETURN
      END
```

SUBROUTINE TO CALCULATE THE OBJECTIVE FUNCTION
FOR 2-SPAN DOUBLE BOX GIRDER BRIDGE

```
NOTATION

   CONSTANTS
      B     TOTAL WIDTH
      B6    WIDTH OF CAST-IN-PLACE STRIP
      T4    WEB THICKNESS

   VARIABLES FROM MAIN PROGRAM
      B1    WIDTH OF CANTILEVER OVERHANG
      B2    FIRST INTERIOR SPAN OF DECK
      B3    WIDTH OF BOTTOM SLAB
      D     DEPTH OF SECTION

   QUANTITIES CALCULATED
      A     AVERAGE SECTION AREA
      AS    CABLE AREA
      C     OBJECTIVE FUNCTION


      B = 50.
      B6 = 2.
      T4 = 1.0833
      GO TO 5
    2 PRINT 2000, B1, B2, B3, D
 2000 FORMAT (*B1 B2 B3 D =*, 4E12.2)
      CALL EXIT
    5 CONTINUE
      T11 = .7
   10 F1 = .025*(1.+T11)*B1**2 + 26.*(B1-2.)/(B1+2.6875)
      IF (B1.LE.8.) GO TO 20
      F1 = F1 + 26.*(B1-8.)/(B1-3.3125)
   20 T1 = .167 + .1268*SQRT(F1)
      E = ABS(T1-T11)
      IF (E-.001) 40,40,30
   30 T11 = T1
      GO TO 10
   40 IF (T1.LT..5) T1=.5
      W = B-2.*B1-4.*T4
      B5 = W-2.*B2
      T5 = 0.
      IF (B5.LE.B2) GO TO 70
      T51 = .5
   50 F5 = .0125*T51*B5**2 + .52*(B5+2.)
      T5 = .167 + .1268*SQRT(F5)
      E = ABS(T5-T51)
      IF (E-.001) 70,70,60
   60 T51 = T5
      GO TO 50
   70 CONTINUE
```

```
      T21 = .5
   80 F2 = .0125*T21*B2**2 + .52*(B2+2.)
      ST2 = T21**3/B2
      S = SQRT((D-T21-.5)**2+((B2-B3)/2.+T4)**2)
      ST4 = .75*T4**3/S
      F2 = (ST4*F2+ST2*F1)/(ST2+ST4)
      T2 = .167 + .1268*SQRT(F2)
      E = ABS(T2-T21)
      IF (E-.001) 100,100,90
   90 T21 = T2
      GO TO 80
  100 IF (T2.LT..5) T2=.5
      IF (T2.LT.T5) T2=T5
      AA1 = B1*(T1+.5)/2.
      Z1 = (T1**2+.5*T1+.25)/(T1+.5)/3.
      AA2 = ((B-B6)/2.-B1)*T2
      Z2 = T2/2.
      AAT = AA1 + AA2
      AZT = AA1*Z1 + AA2*Z2
      AA3 = B3*.5
      Z3 = D-.25
      AA4 = 2.*S*T4
      Z4 = (D-.5+T2)/2.
      A11 = AAT + AA3 + AA4
      AZ = AZT + AA3*Z3 + AA4*Z4
      DC1 = AZ/A11
      H = D-.5*DC1-1.005
      AS = A11/(19.2*H)
      A1 = 2.*A11 + B6*T2
      BMD = 820.*(A1+.113*(B-2.))
      BMS = 850.*A11*(1.-(DC1-.67)/H)
      BMU = BMD + 25260. - BMS
      T31 = 1.
      J = 0
  110 DM = D-.5*T31-.67
      T3 = BMU/(1322.*DM*B3)
      E = ABS(T3-T31)
      J = J+1
      IF (J.GT.20) GO TO 2
      IF (E-.001) 130,130,120
  120 T31 = T3
      GO TO 110
  130 IF (T3.LT..5) T3=.5
      ASU = BMU/(31100.*DM)
      IF (ASU.GT.AS) AS=ASU
      Q = 1./12.
  140 Q = Q+1./36.
      DM = D-.92-1.7*Q*(D-1.34)
      BMU = 661.*DM*B3
      BMX = BMD*(1.-5.*Q+4.*Q**2) + 12630.*(1.-Q) - BMS*(1.-Q)
      IF (BMX.GT.BMU) GO TO 140
      AA3 = B3*T3
      S = SQRT((D-T2-T3)**2+((B2-B3)/2.+T4)**2)
      AA4 = 2.*S*T4
      AT = 2.*AAT + B6*T2
      A2 = AT + 2.*(AA3+AA4)
```

```
A = A1 + (A2-A1)*Q/2.
C = 5.345*A + 343.*AS + .031*(B+3.*D+10.)*D
```

C.4

SUBROUTINE TO CALCULATE THE OBJECTIVE FUNCTION
FOR 2-SPAN MULTI-CELL BOX GIRDER BRIDGE

```
      NOTATION

         CONSTANTS
            B     TOTAL WIDTH
            NW    NUMBER OF WEBS
            T4    WEB THICKNESS

         VARIABLES FROM MAIN PROGRAM
            B1    WIDTH OF CANTILEVER OVERHANG
            B2    FIRST INTERIOR SPAN OF DECK
            B3    WIDTH OF BOTTOM SLAB
            D     DEPTH OF SECTION

         QUANTITIES CALCULATED
            A     AVERAGE SECTION AREA
            AS    CABLE AREA
            C     OBJECTIVE FUNCTION


      B = 50.
      NW = 4
      T4 = 1.0833
      GO TO 5
    2 PRINT 2000, B1, B2, B3, D
 2000 FORMAT (*B1 B2 B3 D =*, 4E12.2)
      CALL EXIT
    5 CONTINUE
      T11 = .7
   10 F1 = .025*(1.+T11)*B1**2 + 26.*(B1-2.)/(B1+2.6875)
      IF (B1.LE.8.) GO TO 20
      F1 = F1 + 26.*(B1-8.)/(B1-3.3125)
   20 T1 = .167 + .1268*SQRT(F1)
      E = ABS(T1-T11)
      IF (E-.001) 40,40,30
   30 T11 = T1
      GO TO 10
   40 IF (T1.LT..5) T1=.5
      W = B-2.*B1-NW*T4
      B5 = (W-2.*B2)/(NW-3)
      T5 = 0.
      IF (B5.LE.B2) GO TO 70
      T51 = .5
   50 F5 = .0125*T51*B5**2 + .52*(B5+2.)
      T5 = .167 + .1268*SQRT(F5)
      E = ABS(T5-T51)
      IF (E-.001) 70,70,60
   60 T51 = T5
      GO TO 50
   70 CONTINUE
```

```
      T21 = .5
   80 F2 = .0125*T21*B2**2 + .52*(B2+2.)
      ST2 = T21**3/B2
      S = SQRT((D-T21-.5)**2+((B-B3)/2.-B1)**2)
      ST4 = .75*T4**3/S
      F2 = (ST4*F2+ST2*F1)/(ST2+ST4)
      T2 = .167 + .1268*SQRT(F2)
      E = ABS(T2-T21)
      IF (E-.001) 100,100,90
   90 T21 = T2
      GO TO 80
  100 IF (T2.LT..5) T2=.5
      IF (T2.LT.T5) T2=T5
      AA1 = B1*(T1+.5)
      Z1 = (T1**2+.5*T1+.25)/(T1+.5)/3.
      AA2 = (B-2.*B1)*T2
      Z2 = T2/2.
      AT = AA1 + AA2
      AZT = AA1*Z1 + AA2*Z2
      AA3 = B3*.5
      Z3 = D-.25
      AA4 = ((NW-2)*(D-T2-.5)+2.*S)*T4
      Z4 = (D-.5+T2)/2.
      A1 = AT + AA3 + AA4
      AZ = AZT + AA3*Z3 + AA4*Z4
      DC1 = AZ/A1
      H = D-.5*DC1-1.005
      AS = A1/(38.4*H)
      BMD = 820.*(A1+.113*(B-2.))
      BMS = 425.*A1*(1.-(DC1-.67)/H)
      BMU = BMD + 25260. - BMS
      T31 = 1.
      J = 0
  110 DM = D-.5*T31-.67
      T3 = BMU/(661.*DM*B3)
      E = ABS(T3-T31)
      J = J+1
      IF (J.GT.20) GO TO 2
      IF (E-.001) 130,130,120
  120 T31 = T3
      GO TO 110
  130 IF (T3.LT..5) T3=.5
      ASU = BMU/(31100.*DM)
      IF (ASU.GT.AS) AS=ASU
      Q = 1./12.
  140 Q = Q+1./36.
      DM = D-.92-1.7*Q*(D-1.34)
      BMU = 330.5*DM*B3
      BMX = BMD*(1.-5.*Q+4.*Q**2) + 12630.*(1.-Q) - BMS*(1.-Q)
      IF (BMX.GT.BMU) GO TO 140
      AA3 = B3*T3
      S = SQRT((D-T2-T3)**2+((B-B3)/2.-B1)**2)
      AA4 = ((NW-2)*(D-T2-T3)+2.*S)*T4
      A2 = AT + AA3 +AA4
      A = A1 + (A2-A1)*Q/2.
      C = 5.345*A + 343.*AS + .031*(B+3.*D+10.)*D
```

C.5

SUBROUTINE TO CALCULATE THE OBJECTIVE FUNCTION
FOR 3-SPAN DOUBLE BOX GIRDER BRIDGE

```
      NOTATION

         CONSTANTS
            B     TOTAL WIDTH
            B6    WIDTH OF CAST-IN-PLACE STRIP
            T4    WEB THICKNESS

         VARIABLES FROM MAIN PROGRAM
            B1    WIDTH OF CANTILEVER OVERHANG
            B2    FIRST INTERIOR SPAN OF DECK
            B3    WIDTH OF BOTTOM SLAB
            D     DEPTH OF SECTION

         QUANTITIES CALCULATED
            A     AVERAGE SECTION AREA
            AS    AVERAGE CABLE AREA
            C     OBJECTIVE FUNCTION


      B = 56.
      B6 = 2.
      T4 = 1.
      GO TO 5
    2 PRINT 2000, B1, B2, B3, D
 2000 FORMAT (*B1 B2 B3 D =*, 4E12.2)
      CALL EXIT
    5 CONTINUE
      T11 = .7
   10 F1 = .025*(1.+T11)*B1**2 + 26.*(B1-2.)/(B1+2.6875)
      IF (B1.LE.8.) GO TO 20
      F1 = F1 + 26.*(B1-8.)/(B1-3.3125)
   20 T1 = .167 + .1268*SQRT(F1)
      E = ABS(T1-T11)
      IF (E-.001) 40,40,30
   30 T11 = T1
      GO TO 10
   40 IF (T1.LT..5) T1=.5
      W = B-2.*B1-4.*T4
      B5 = W-2.*B2
      T5 = 0.
      IF (B5.LE.B2) GO TO 70
      T51 = .5
   50 F5 = .0125*T51*B5**2 + .52*(B5+2.)
      T5 = .167 + .1268*SQRT(F5)
      E = ABS(T5-T51)
      IF (E-.001) 70,70,60
   60 T51 = T5
      GO TO 50
   70 CONTINUE
```

```
      T21 = .5
   80 F2 = .0125*T21*B2**2 + .52*(B2+2.)
      ST2 = T21**3/B2
      S = SQRT((D-T21-.5)**2+((B2-B3)/2.+T4)**2)
      ST4 = .75*T4**3/S
      F2 = (ST4*F2+ST2*F1)/(ST2+ST4)
      T2 = .167 + .1268*SQRT(F2)
      E = ABS(T2-T21)
      IF (E-.001) 100,100,90
   90 T21 = T2
      GO TO 80
  100 IF (T2.LT..5) T2=.5
      IF (T2.LT.T5) T2=T5
      AA1 = B1*(T1+.5)/2.
      Z1 = (T1**2+.5*T1+.25)/(T1+.5)/3.
      AA2 = ((B-B6)/2.-B1)*T2
      Z2 = T2/2.
      AAT = AA1 + AA2
      AZT = AA1*Z1 + AA2*Z2
      AA3 = B3*.5
      Z3 = D-.25
      AA4 = 2.*S*T4
      Z4 = (D-.5+T2)/2.
      A11 = AAT + AA3 + AA4
      AZ = AZT + AA3*Z3 + AA4*Z4
      DC1 = AZ/A11
      BMU = 1012.5*A11 + 5344.
      T31 = 1.
      J = 0
  110 DM = D-.5*T31-.46
      T3 = BMU/(661.*DM*B3)
      E = ABS(T3-T31)
      J = J+1
      IF (J.GT.20) GO TO 2
      IF (E-.001) 130,130,120
  120 T31 = T3
      GO TO 110
  130 IF (T3.LT..5) T3=.5
      ASU = BMU/(31100.*DM)
      DM = D-.71
      BMU = 330.5*DM*B3
      Q = .05
  140 Q = Q+.05
      BMX = 1012.5*A11*(1.-Q)**2 + 5625.*(.95-Q)
      IF (BMX.GT.BMU) GO TO 140
      AA3 = B3*T3
      Z3 = D-T3/2
      S = SQRT((D-T2-T3)**2+((B2-B3)/2.+T4)**2)
      AA4 = 2.*S*T4
      Z4 = (D-T3+T2)/2.
      A21 = AAT + AA3 + AA4
      AZ = AZT + AA3*Z3 + AA4*Z4
      DC2 = AZ/A21
      A1 = 2.*A11 + B6*T2
      A2 = 2.*A21 + B6*T2
      A = A1 + (A2-A1)*Q/2.
```

```
      AS1 = .0386*A11/(DC2-.46)
      IF (ASU.GT.AS1) AS1=ASU
      AST = .43*AS1
      DEC = D-.5-DC1
      AS21 = .15
      J = 0
  150 BMS = 8980.*AS21*DEC
      BMU = 445.5*(A1+.113*(B-2.)) + 16850. - 396.*A11 + BMS
      DM = D-.5-26.*AS21/B
      AS2 = BMU/(34340.*DM)
      E = ABS((AS2-AS21)/AS21)
      J = J+1
      IF (J.GT.20) GO TO 2
      IF (E-.002) 170,170,160
  160 AS21 = AS2
      GO TO 150
  170 ASB = .475*AS2
      AS = AST + ASB
      C = 5.345*A + 343.*AS
```