PREDICTION OF MOISTURE MOVEMENT IN EXPANSIVE CLAYS

by

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PREFACE

This report is the third in a series of reports from Research Project 3-8-68-118 entitled "Study of Expansive Clays in Roadway Structural Systems." The report uses the theoretical results of the two previous research reports (Nos. 118-1 and 118-2) in developing one and two-dimensional computer programs for solving the concentration-dependent, partial differential equation for moisture movement in expansive clay.

A numerical method is used in which errors made at one time step do not grow with additional steps forward in time. This property, called stability, is very important in solution of the highly nonlinear flow problems encountered in unsaturated soil.

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LIST OF REPORTS

Report No. 118-1, "Theory of Moisture Movement in Expansive Clays" by Robert L. Lytton, presents a theoretical discussion of moisture movement in clay soil.

Report No. 118-2, "Continuum Theory of Moisture Movement and Swell in Expansive Clays" by R. Ray Nachlinger and Robert L. Lytton, presents a theoretical study of the phenomenon of expansive clay.

Report No. 118-3, "Prediction of Moisture Movement in Expansive Clays" by Robert L. Lytton and Ramesh K. Kher, uses the theoretical results of Research Report Nos. 118-1 and 118-2 in developing one and two-dimensional computer programs for solving the concentration-dependent, partial differential equation for moisture movement in expansive clay.

ABSTRACT

This report describes two computer programs for determining changing moisture distribution with time. Program FLOPIP2 is arranged to work one-dimensional problems and computer Program GCHPIP1 solves moisture distribution problems in two dimensions. The equation governing the flow of moisture is a concentration-dependent, parabolic, partial differential equation which is solved numerically using the implicit Crank-Nicolson method of marching forward in time.

Although it is stable in one-dimensional problems, the Crank-Nicolson method can become unstable in two-dimensional problems, depending upon the relative size of the components of the permeability tensor. This rare form of instability is predicted theoretically and observed in one of the example problems.

Example problems are worked to demonstrate the capabilities and breadth of application of the computer programs and to prove the validity of the approach. The one-dimensional example problems are concerned with matching measured field data and with presenting the results of a parameter study of various suction and permeability factors. The field data can be duplicated to within very close tolerances.

The two-dimensional example problems are arranged to demonstrate the versatility of computer Program GCHPIP1. Problems solved include a two-dimensional consolidation problem, ponding problems, and a problem of predicting moisture distribution within a concrete highway bridge girder.

KEY WORDS: moisture movement, expansive clays, discrete-element analysis, computers, permeability, suction, ponding, Crank-Nicolson method, unsaturated permeability, compressibility.

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CHAPTER 1. INTRODUCTION

The purpose of this report is to present numerical methods of solution to the differential equations which describe mathematically the movement of moisture in one and two-dimensional regions of clay soil.

The basic numerical method used is the discrete-element approach, which is similar in many respects to the numerical method used to solve for the deflections of beams, slabs, and grid beams described in several reports of Project 3-5-63-56 (Refs 26 and 27). The greatest difference between the two rests in the fact that a beam or slab differential equation is of fourth order, i.e., involves fourth derivatives, while the flow differential equation is of second order. The one-dimensional flow equation is solved herein by computer Program FLOPIP2 and the two-dimensional solution is accomplished by computer Program GCHPIP1.

This report consists of eight chapters. The second chapter presents briefly the moisture flow equation to be solved. A more detailed treatment of this subject is contained in Research Report 118-1, "Theory of Moisture Movement in Expansive Clays." The third chapter outlines the numerical technique used to form discrete-element analogs to the differential equations of flow. Chapters 4 and 5 discuss the two and one-dimensional moisture distribution computer programs, respectively, detailing the forms of input and output information. Chapter 6 presents the results of a study made of field experimental data collected by Donald R. Lamb and others at the University of Wyoming. These data were assembled from readings of moisture and density nuclear depth probes. The chapter is valuable because it shows a technique for using the computer to develop realistic field soils data. Chapter 7 presents results of two-dimensional problem solutions and demonstrates a rare form of instability of the numerical method used to march forward in time. This instability is predicted theoretically in Chapter 3. These two-dimensional problems involve solutions of flow problems in both rectangular and cylindrical coordinates. Chapter 8, the concluding chapter, summarizes the findings and capabilities presented in this report and suggests areas for use of the computer tools developed in Project 3-8-68-118, "Study of Expansive Clays in Roadway Structural Systems," for predicting moisture movement in clay.

CHAPTER 2. THE FLOW EQUATIONS

In Chapter 5 of Research Report 118-1, a detailed derivation of the flow equations was given. In this chapter, these equations will be summarized and their discrete-element forms will be given. In the latter part of the chapter, boundary conditions will be considered. These conditions will involve definitions of soil suction which are given in detail in Chapter 3 of Research Report 118-1.

<u>The Flow Equation in Rectangular Coordinates</u>

The flow equation is derived from a combination of the continuity equation and the tensor form of Darcy's law. The element used to derive the equations is given in Fig 1(a). The continuity equation developed from this element is

$$\frac{\partial}{\partial t}(\rho\theta) = -\frac{\partial}{\partial x_i}(\rho v_i) \tag{2.1}$$

where

ρ = the mass density of liquid,

θ = the volumetric water content of water,

v_i = the velocity in the i^{th} direction.

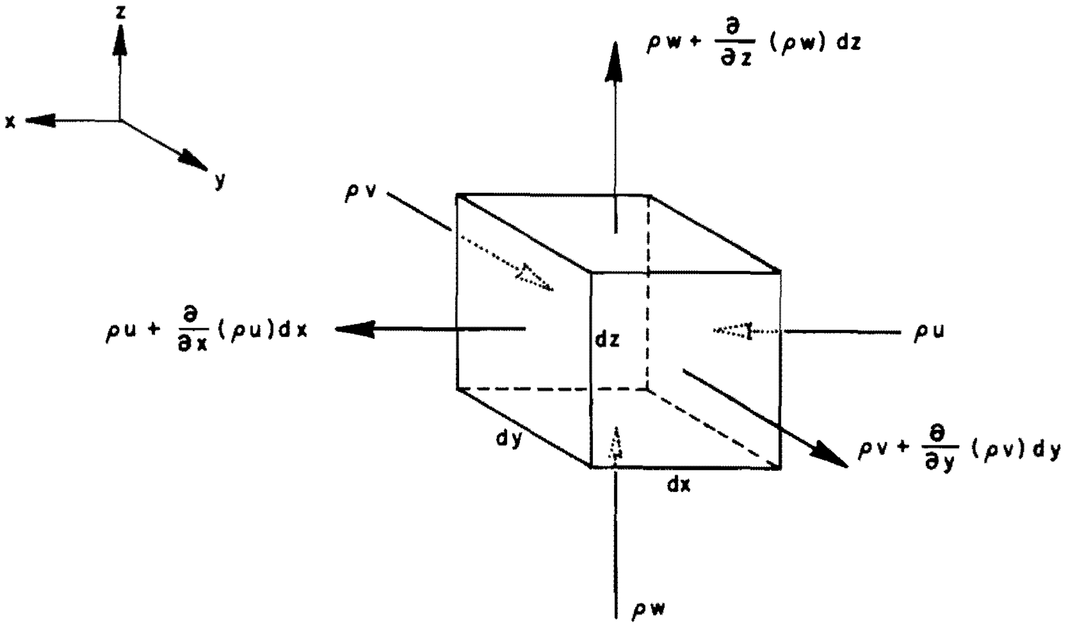
Darcy's law in rectangular coordinates is as follows:

$$v_i = -k_{ij}\frac{\partial H}{\partial x_j} \tag{2.2}$$

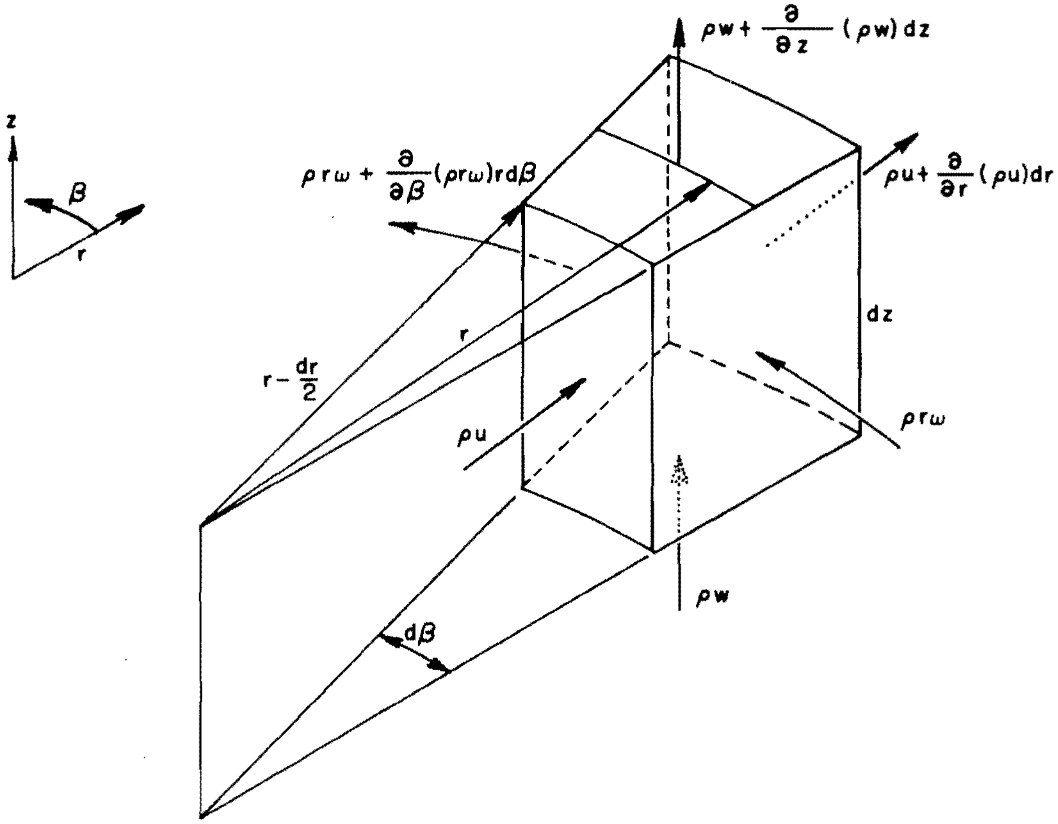
where

k_{ij} = the permeability tensor,

$\frac{\partial H}{\partial x_j}$ = the force potential head gradient in the j^{th} direction.



(a) Rectangular element.



(b) Cylindrical element.

Fig 1. Elements used to derive equations.

Although the total head is a function of all the variables included under the term "suction" and of temperature in addition, the moisture distribution programs given in this report use suction alone. Thus the force potential head used in this report is more restricted than the broadest possible definition which includes temperature effects. Using τ for the suction, designating the three-direction as the direction opposite to the pull of gravity, and assuming that the average water density does not change greatly within a soil region either in time or space, the flow equation in rectangular coordinates becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x_i} \left(k_{ij} \frac{\partial \tau}{\partial x_j} + k_{i3} \right) \tag{2.3}$$

A further assumption is that suction is a unique function of water content. Although this is not true, because of known hysteresis effects, it is certain that changes of suction with water content in a certain direction, say drying, do follow a unique curve so that long-term one-way changes of suction may be treated as if suction and water content were related by a single curve. Given this assumption, the time derivative is found to be

$$\frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial \tau} \frac{\partial \tau}{\partial t} \tag{2.4}$$

from which comes the form of the equation used in this report

$$\frac{\partial \tau}{\partial t} = \frac{\partial \tau}{\partial \theta} \frac{\partial}{\partial x_i} \quad k_{ij} \frac{\partial \tau}{\partial x_j} + k_{i3} \tag{2.5}$$

This equation is presented in its discrete-element form in a subsequent section of this report.

The Flow Equation in Cylindrical Coordinates

The element from which these relations are derived is shown in Fig 1(b). The continuity equation in cylindrical coordinates is

$$\frac{1}{r}\frac{\partial}{\partial r}(pur) + \frac{1}{r}\frac{\partial}{\partial \beta}(pv) + \frac{\partial}{\partial z}(pw) = -\frac{\partial(p\theta)}{\partial t} \qquad (2.6)$$

Darcy's law in cylindrical coordinates is of the form

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = - \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial r} \\ \frac{1}{r}\frac{\partial H}{\partial \beta} \\ \frac{\partial H}{\partial z} \end{bmatrix} \qquad (2.7)$$

The combination of these two equations with the assumptions made in developing the flow equation in rectangular coordinates, and the designation of the direction opposite to the pull of gravity as the three-direction, gives the three-dimensional flow equation in cylindrical coordinates:

$$\begin{aligned} \frac{\partial \tau}{\partial t} = {} & \frac{\partial \tau}{\partial \theta}\left(\frac{1}{r} + \frac{\partial}{\partial r}\right)\left(k_{11}\frac{\partial \tau}{\partial r} + \frac{k_{12}}{r}\frac{\partial \tau}{\partial \beta} + k_{13}\frac{\partial \tau}{\partial z} + k_{13}\right) \\ & + \frac{\partial \tau}{\partial \theta}\frac{1}{r}\frac{\partial}{\partial \beta}\left(k_{21}\frac{\partial \tau}{\partial r} + \frac{k_{22}}{r}\frac{\partial \tau}{\partial \beta} + k_{23}\frac{\partial \tau}{\partial z} + k_{23}\right) \\ & + \frac{\partial \tau}{\partial \theta}\frac{\partial}{\partial z}\left(k_{13}\frac{\partial \tau}{\partial r} + \frac{k_{32}}{r}\frac{\partial \tau}{\partial \beta} + k_{33}\frac{\partial \tau}{\partial z} + k_{33}\right) \end{aligned} \qquad (2.8)$$

The axially symmetric condition occurs when all derivatives with respect to β are equal to zero. This is the equation which is used in this report for flow in cylindrical coordinate systems:

$$\begin{aligned} \frac{\partial \tau}{\partial t} = {} & \frac{\partial \tau}{\partial \theta}\left(\frac{1}{r} + \frac{\partial}{\partial r}\right)\left[k_{11}\frac{\partial \tau}{\partial r} + k_{13}\left(\frac{\partial \tau}{\partial z} + 1\right)\right] \\ & + \frac{\partial \tau}{\partial \theta}\frac{\partial}{\partial z}\left[k_{31}\frac{\partial \tau}{\partial r} + k_{33}\left(\frac{\partial \tau}{\partial z} + 1\right)\right] \end{aligned} \qquad (2.9)$$

The discrete-element form of this equation is given below.

Discrete-Element Representation of Flow in a Rectangular Region

Each pipe segment shown in Fig 2 has one or two permeability coefficients. If a principal permeability is aligned with the pipe direction, then

$$k_{11} = k_1$$

and

$$k_{12} = 0$$

If the principal permeability is at some angle, the pipe increment will have two permeability coefficients.

The pipe increment i,j running in the y-direction, has permeability components

$$k_{21i,j} = k_{22i,j}$$

and

$$k_{22i,j}$$

The differential equation for transient flow in these pipes is as follows:

$$\frac{\partial \tau}{\partial t} = \frac{\partial \tau}{\partial \theta} \frac{\partial}{\partial x} \left[k_{11} \frac{\partial \tau}{\partial x} + k_{12} \left(\frac{\partial \tau}{\partial y} + 1 \right) \right]$$

$$+ \frac{\partial \tau}{\partial \theta} \frac{\partial}{\partial y} \left[k_{21} \frac{\partial \tau}{\partial x} + k_{22} \left(\frac{\partial \tau}{\partial y} + 1 \right) \right] \qquad (2.10)$$

The suction at a point i,j will be denoted $\tau_{i,j}$. With the suction and permeability conventions set, the finite-difference form of Eq 2.10 may be written virtually by inspection of the discrete-element representation in Fig 2. The superscripts k and k+1 indicate the time step.

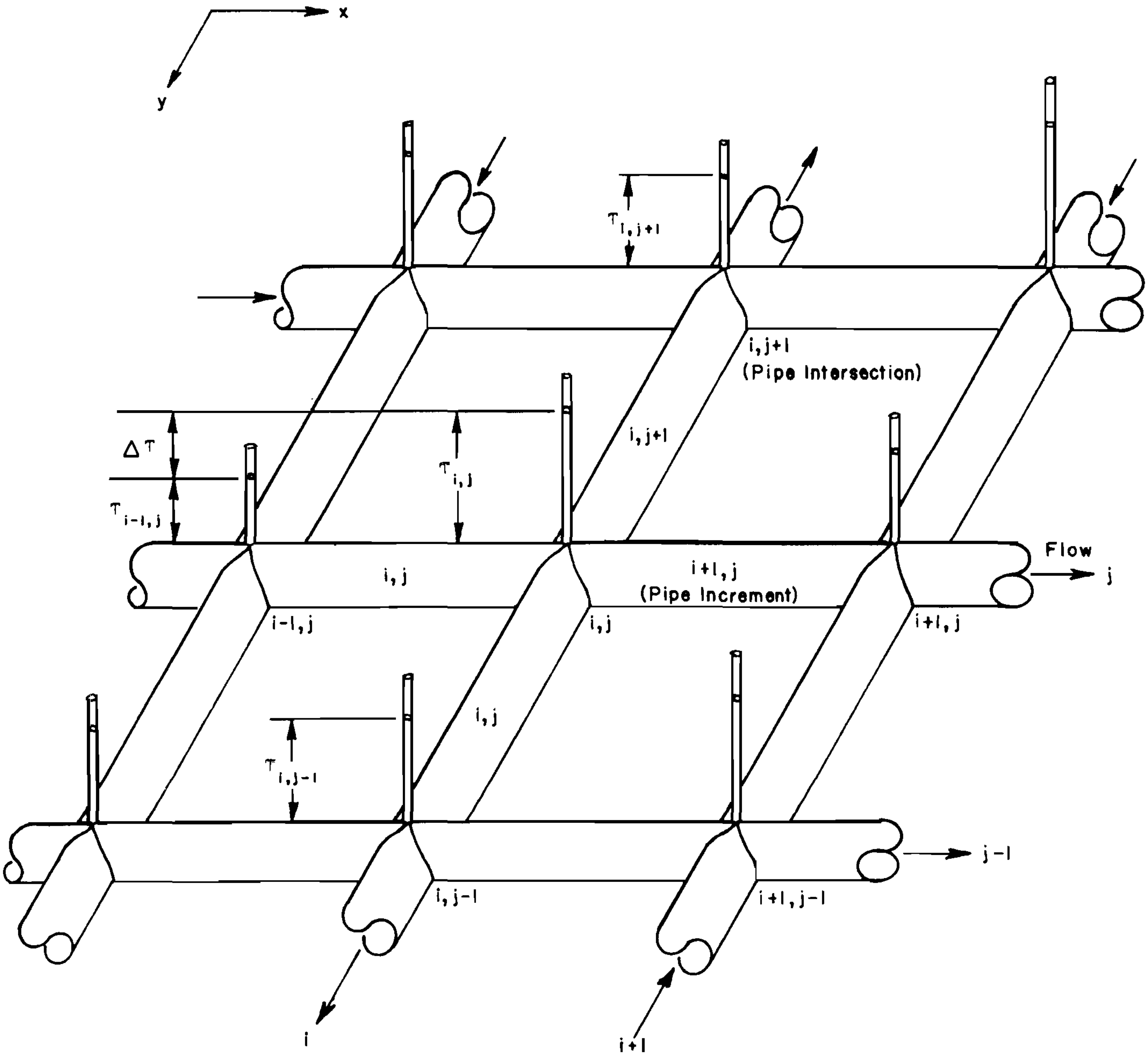


Fig 2. Discrete-element representation of flow in a rectangular region.

$$\frac{\tau^{k+1}_{i,j} - \tau^{k}_{i,j}}{h_t} = \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \frac{1}{h_x} \Bigg[- k_{11i,j} \quad \frac{- \tau^{k}_{i-1,j} + \tau^{k}_{i,j}}{h_x}$$

$$+ k_{11i+1,j} \left(\frac{-\tau^{k}_{i,j} + \tau^{k}_{i+1,j}}{h_x} \right) - k_{12i,j} \left(\frac{-\tau^{k}_{i,j-1} + \tau^{k}_{i,j}}{h_y} \right)$$

$$+ k_{12i+1,j} \left(\frac{-\tau^{k}_{i,j} + \tau^{k}_{i,j+1}}{h_y} \right) + \left(-k_{12i,j} + k_{12i+1,j} \right) \Bigg]$$

$$+ \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \frac{1}{h_y} \Bigg[-k_{21i,j} \left(\frac{-\tau^{k}_{i-1,j} + \tau^{k}_{i,j}}{h_x} \right)$$

$$+ k_{21i,j+1} \left(\frac{-\tau^{k}_{i,j} + \tau^{k}_{i+1,j}}{h_x} \right) - k_{22i,j} \left(\frac{-\tau^{k}_{i,j-1} + \tau^{k}_{i,j}}{h_y} \right)$$

$$+ k_{22i,j+1} \left(\frac{-\tau^{k}_{i,j} + \tau^{k}_{i,j+1}}{h_y} \right) + \left(-k_{22i,j} + k_{22i,j+1} \right) \Bigg] \tag{2.11}$$

For convenience, coefficients of like terms may be collected and the collection itself designated as follows:

$$A_{i,j} = \frac{h_t}{2} \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \left(\frac{k_{12i,j}}{h_x h_y} + \frac{k_{22i,j}}{h_y h_y} \right) \tag{2.12}$$

$$B_{i,j} = \frac{h_t}{2} \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \left(\frac{k_{11i,j}}{h_x h_x} + \frac{k_{21i,j}}{h_y h_x} \right) \tag{2.13}$$

$$CX_{i,j} = \frac{h_t}{2} \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \left(\frac{k_{11i,j}}{h_x h_x} + \frac{k_{11i+1,j}}{h_x h_x} \right.$$

$$+ \frac{k_{12i,j}}{h_x h_y} + \frac{k_{12i+1,j}}{h_x h_y} \Big) \tag{2.14}$$

$$CY_{i,j} = \frac{h_t}{2} \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \Big(\frac{k_{21i,j}}{h_y h_x} + \frac{k_{21i,j+1}}{h_y h_x}$$

$$+ \frac{k_{22i,j}}{h_y h_y} + \frac{k_{22i,j+1}}{h_y h_y} \Big) \tag{2.15}$$

$$D_{i,j} = \frac{h_t}{2} \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \left(\frac{k_{11i+1,j}}{h_x h_x} + \frac{k_{21i,j+1}}{h_y h_x} \right) \tag{2.16}$$

$$E_{i,j} = \frac{h_t}{2} \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \left(\frac{k_{12i+1,j}}{h_x h_y} + \frac{k_{22i,j+1}}{h_y h_y} \right) \tag{2.17}$$

$$F_{i,j} = \frac{h_t}{2} \left(\frac{\partial\tau}{\partial\theta}\right)_{i,j} \Big(- \frac{k_{12i,j}}{h_x} + \frac{k_{12i+1,j}}{h_x}$$

$$- \frac{k_{22i,j}}{h_y} + \frac{k_{22i,j+1}}{h_y} \Big) \tag{2.18}$$

If these substitutions are made into Eq 2.11, the result is

$$\tau^{k+1}_{i,j} - \tau^{k}_{i,j} = 2A_{i,j}\tau^{k}_{i,j-1} + 2B_{i,j}\tau^{k}_{i-1,j}$$

$$- 2(CX_{i,j} + CY_{i,j})\tau^{k}_{i,j} + 2D_{i,j}\tau^{k}_{i+1,j}$$

$$+ 2E_{i,j}\tau^{k}_{i,j+1} + 2F_{i,j} \tag{2.19}$$

The method used to solve systems of equations such as this is discussed in the next chapter.

Discrete-Element Representation of Flow in a Cylindrical Slice

The equation for flow in a cylindrical continuous medium is Eq 2.9. This development derives a finite-difference equation which corresponds to a discrete-element representation of the continuous medium, a sketch of which is shown in Fig 3. Again, very nearly by inspection, the finite-difference equation may be written as follows:

$$
\begin{aligned}
\frac{\tau^{k+1}_{i,j} - \tau^{k}_{i,j}}{h_t} = {} & \frac{\partial\tau}{\partial\theta_{i,j}} \; \frac{1}{r}\Bigg[k_{11i,j} \left(\frac{-\tau^{k}_{i-1,j} + \tau^{k}_{i,j}}{h_r} \right) \\
& + k_{13i,j} \left(\frac{-\tau^{k}_{i,j-1} + \tau^{k}_{k,j}}{h_z} \right) - k_{13i,j} \Bigg] \\
& + \left(\frac{\partial\tau}{\partial\theta} \right)_{i,j} \frac{1}{h_r} \Bigg[- k_{11i,j} \left(\frac{-\tau^{k}_{i-1,j} + \tau^{k}_{i,j}}{h_r} \right) \\
& + k_{11i+1,j} \left(\frac{-\tau^{k}_{i,j} + \tau^{k}_{i+1,j}}{h_r} \right) - k_{13i,j} \left(\frac{-\tau^{k}_{i,j-1} + \tau^{k}_{i,j}}{h_z} \right) \\
& + k_{13i+1,j} \left(\frac{-\tau^{k}_{i,j} + \tau^{k}_{i+1,j}}{h_z} \right) + \left(-k_{13i,j} + k_{13i+1,j} \right) \Bigg] \\
& + \left(\frac{\partial\tau}{\partial\theta} \right)_{i,j} \frac{1}{h_z} \Bigg[- k_{31i,j} \left(\frac{-\tau^{k}_{i,j-1} + \tau^{k}_{i,j}}{h_z} \right) \\
& + k_{31i,j+1} \left(\frac{-\tau^{k}_{i,j} + \tau^{k}_{i+1,j}}{h_r} \right) - k_{33i,j} \left(\frac{-\tau^{k}_{i,j-1} + \tau^{k}_{k,j}}{h_z} \right) \\
& + k_{33i,j+1} \left(\frac{-\tau^{k}_{i,j} + \tau^{k}_{i,j+1}}{h_z} \right) + \left(-k_{33i,j} + k_{33i,j+1} \right) \Bigg] \qquad (2.20)
\end{aligned}
$$

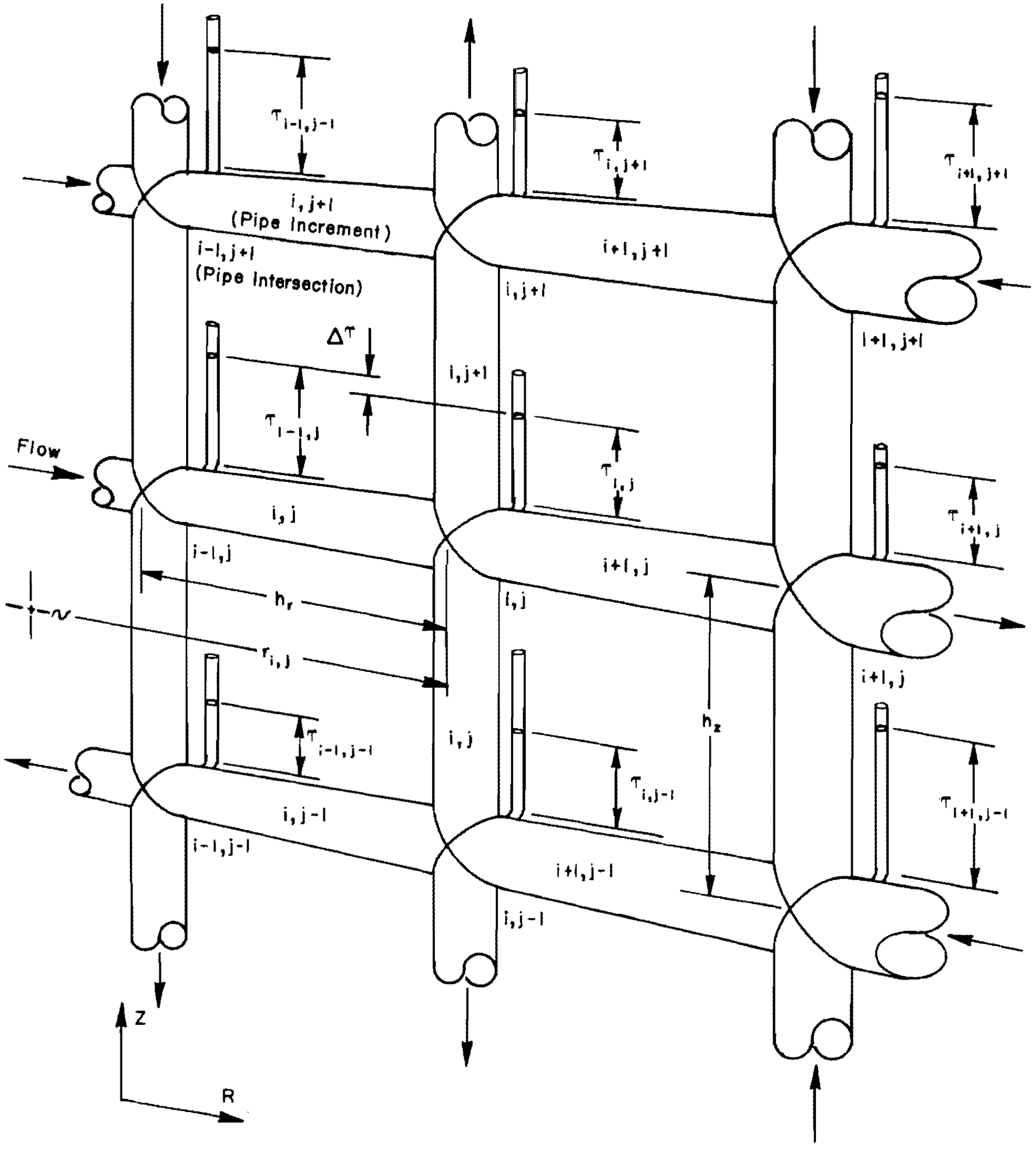


Fig 3. Discrete-element representation of flow in a cylindrical slice.

As in the rectangular case, coefficients of like terms may be collected and defined as given in Eqs 2.21 through 2.27.

$$A_{i,j} = \frac{h_t}{2}\left(\frac{\partial\tau}{\partial\theta}\right)_{i,j}\left[-\frac{k_{13i,j}}{rh_z}+\frac{k_{13i,j}}{h_rh_z}+\frac{k_{33i,j}}{h_zh_z}\right] \tag{2.21}$$

$$B_{i,j} = \frac{h_t}{2}\left(\frac{\partial\tau}{\partial\theta}\right)_{i,j}\left[-\frac{k_{11i,j}}{rh_r}+\frac{k_{11i,j}}{h_rh_r}+\frac{k_{31i,j}}{h_zh_r}\right] \tag{2.22}$$

$$\begin{aligned} CR_{i,j} = {} & \frac{h_t}{2}\left(\frac{\partial\tau}{\partial\theta}\right)_{i,j}\left[-\frac{k_{11i,j}}{rh_r}+\frac{k_{11i,j}}{h_rh_r}+\frac{k_{11i+1,j}}{h_rh_r}\right.\\ & \left.-\frac{k_{13i,j}}{rh_z}+\frac{k_{13i,j}}{h_rh_z}+\frac{k_{13i+1,j}}{h_zh_r}\right] \end{aligned} \tag{2.23}$$

$$\begin{aligned} CZ_{i,j} = {} & \frac{h_t}{2}\left(\frac{\partial\tau}{\partial\theta}\right)_{i,j}\left[+\frac{k_{31i,j}}{h_zh_r}+\frac{k_{31i,j+1}}{h_zh_r}\right.\\ & \left.+\frac{k_{33i,j}}{h_zh_z}+\frac{k_{33i,j+1}}{h_zh_z}\right] \end{aligned} \tag{2.24}$$

$$D_{i,j} = \frac{h_t}{2}\left(\frac{\partial\tau}{\partial\theta}\right)_{i,j}\left[\frac{k_{11i+1,j}}{h_rh_r}+\frac{k_{31i,j+1}}{h_zh_r}\right] \tag{2.25}$$

$$E_{i,j} = \frac{h_t}{2}\left(\frac{\partial\tau}{\partial\theta}\right)_{i,j}\left[\frac{k_{13i+1,j}}{h_rh_z}+\frac{k_{33i,j+1}}{h_zh_z}\right] \tag{2.26}$$

$$\begin{aligned} F_{i,j} = {} & \frac{h_t}{2}\left(\frac{\partial\tau}{\partial\theta}\right)_{i,j}\left[+\frac{k_{13i,j}}{r}-\frac{k_{13i,j}}{h_r}+\frac{k_{13i+1,j}}{h_r}\right.\\ & \left.-\frac{k_{33i,j}}{h_z}+\frac{k_{33i,j+1}}{h_z}\right] \end{aligned} \tag{2.27}$$

If Eq 2.20 is rewritten using these newly defined coefficients, it is found that the result, Eq 2.28, is of the same form as the rectangular coordinate Eq 2.19.

$$\tau^{k+1}_{i,j} - \tau^{k}_{i,j} = 2A_{i,j}\tau^{k}_{i,j-1} + 2B_{i,j}\tau^{k}_{i-1,j} - 2\left[CR_{i,j} + CZ_{i,j} \right]\tau^{k}_{k,j} + 2D_{i,j}\tau^{k}_{i+1,j} + 2E_{i,j}\tau^{k}_{k,j+1} + 2\tau_{i,j} \qquad (2.28)$$

This leads to the conclusion that both rectangular and cylindrical region problems may be solved with the same computer program, provided the coefficients $A_{i,j}$ through $F_{i,j}$ are appropriately computed. As mentioned before, discussion of the solution to a system of such equations is given in Chapter 3.

Boundary Conditions

In a mathematical sense, only two types of boundary conditions may be considered: a specified value of the variable on the boundary and a specified gradient of the variable perpendicular to the boundary. The first of these is termed a Dirichlet problem and the second a Neumann problem by mathematicians. The use of the term "boundary conditions" in its engineering sense requires determination of physical quantities which exist on the fringes of an area of interest. All of the engineering boundary conditions to be considered may be expressed as a boundary value or a boundary gradient. A set of typical problems, shown in Fig 4, will permit easier discussion of these boundary conditions.

(1) No Flow. This is a condition in which the gradient normal to the boundary is zero.

(2) Symmetry or Mirror Image. No flow will cross a line of symmetry. The normal gradient must be zero on such a boundary.

(3) Seal. A watertight seal will permit no flow. The normal gradient must be zero.

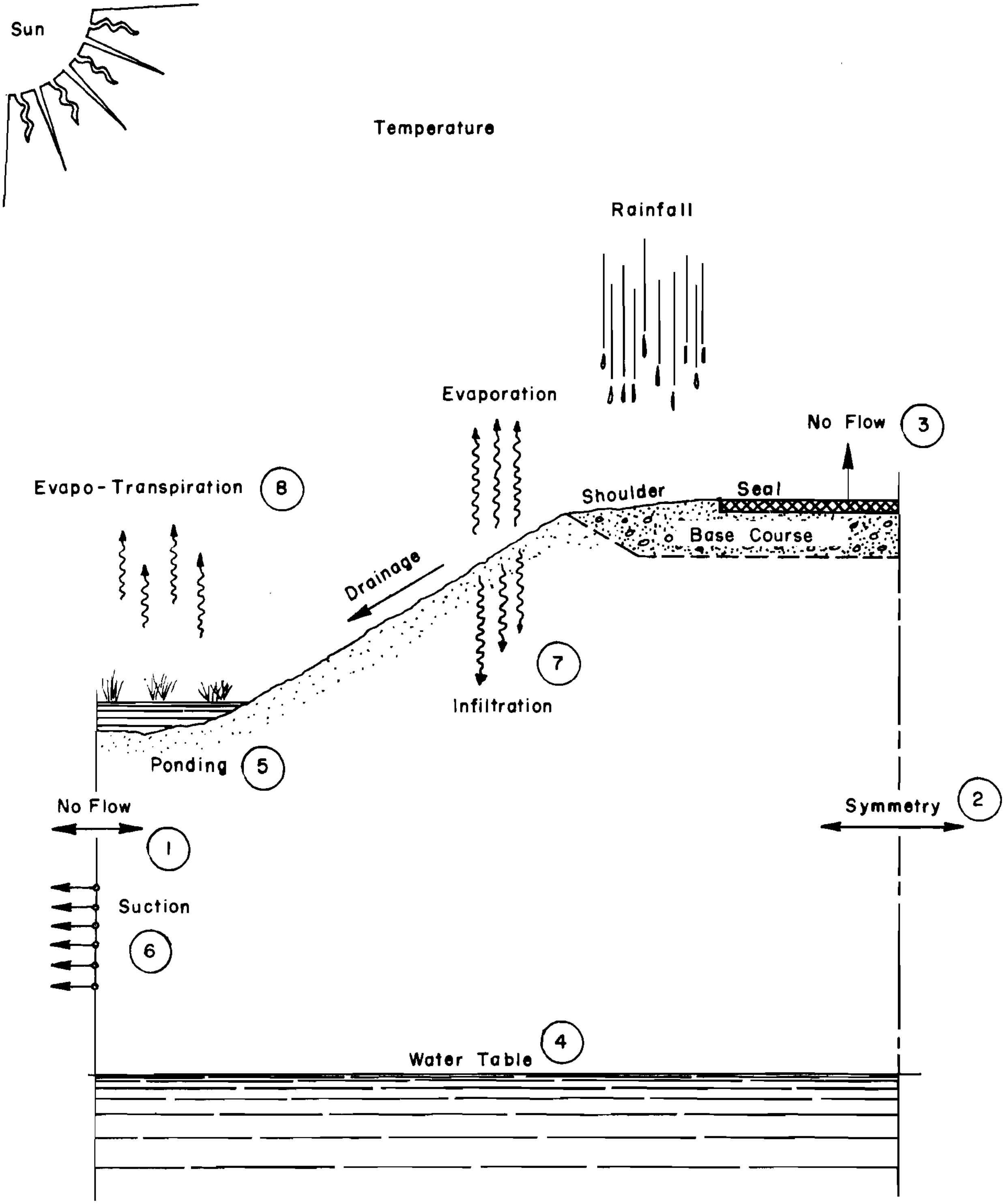


Fig 4. Boundary conditions.

(4) Water Table. At a water table no suction other than solute suction exists. If a water table is known to exist in a clay formation, a convenient, though not necessarily correct, assumption would be that the suction is zero on that boundary.

(5) Ponding. At the surface while water covers it, the suction in the soil can be assumed to be zero or some value dictated by a difference in ion concentration from ordinary soil water. The most convenient value is zero.

(6) Suction. If a constant water content is maintained at some depth below ground, then the suction will remain relatively stable. The value of suction corresponding to this constant moisture content may be set. If the moisture content on the boundary changes with time in a known way, then the corresponding suction may be set at the appropriate time.

(7) Evaporation and Infiltration. This condition can be handled in either of the two ways: by specifying a known suction which corresponds to the condition of soil moisture humidity or by specifying the gradient which corresponds to the net inflow or outflow. Richards (Ref 18) discusses this problem and chooses the gradient method. Some of the considerations he presented are given here.

The total moisture entering or leaving the soil is the algebraic sum of infiltration (+) and evaporation (-). This sign convention requires negative gradient into the soil. Infiltration will be denoted as I and evaporation as E, and each is expressed in units of length per time increment, e.g., in/hr. The time and length units should be the same as the units being used to express suction and permeability.

Infiltration is a topic studied by hydrologists who recognize that it is affected by soil type, surface roughness and vegetation cover, antecedent moisture conditions, and ground slope. Rainfall is disposed of on the surface as runoff, surface storage, and infiltration. Ideally, if there were no surface storage, then a runoff coefficient and an infiltration coefficient which add to one could be defined. The coefficients represent the fraction of rainfall which becomes that component of surface water disposition. No table of typical values is given here because of the many different methods used by hydrologists to estimate runoff characteristics of small areas. It is evident, however, that with a tight, dry, smooth clay soil on a moderate slope the infiltration factor is close to zero. On a rough-surfaced, open-structured soil with a flat slope and surface cracks and slickensides, the infiltration coefficient will be closer to 1.0. If the total rainfall is R and the infiltration coefficient is C_i, then $I = C_iR$.

Evaporation is more difficult. It is based on the difference between soil-moisture vapor pressure and atmospheric vapor pressure according to a statement attributed to Philip by Richards (Ref 18) in referring to smooth bare ground. For this condition

$$E = K(p - p_a) \tag{2.29}$$

where

K = mass transfer coefficient dependent on climatological considerations,

p = vapor pressure of soil moisture,

p_a = atmospheric vapor pressure.

Similarly, for saturated soil

$$E_{sat} = K(p_{sat} - p_a) \tag{2.30}$$

The ratio of the two equations gives an expression for evaporation:

$$E = E_{sat} \frac{(p - p_a)}{(p_{sat} - p_a)} \tag{2.31}$$

Dividing each term of the fraction by the saturated soil vapor pressure corresponding to 100 percent soil-moisture humidity gives

$$E = E_{sat} \left(\frac{H - H_a}{100 - H_a} \right) \tag{2.32}$$

where

H = relative humidity of soil moisture,

H_a = atmospheric relative humidity.

Attempts have been made among climatologists interested in the agricultural sciences to estimate $E_{sat} = 0.4\ E_{pan}^{0.75}$, which applies to a certain area of Australia. This equation is of the same form as proposed by Thornthwaite (Ref 21) to describe total evaporation including the effect of transpiration:

$$E_t = kT_e^n \tag{2.33}$$

where

T_e = temperature in degrees centigrade,

k, n = constants calculated from a temperature-efficiency index,

E_t = total evaporation.

Other work indicates that potential evaporation should be considered a function of wind speed in a form like Dalton's law of partial pressure:

$$E_o = f(u)(p - p_a) \tag{2.34}$$

One of the most recent approaches, which gives excellent prediction, is an energy balance method reported by van Bavel (Ref 22). This includes the factors of wind speed, latent heat of vaporization, sensible heat, and a term which lumps together all energy inputs such as radiative flux, soil heat flux, heat storage changes in vegetation or ponded water, and energy used in plant photosynthesis. Latent heat of vaporization is the quantity of heat required to change a unit weight of water into water vapor. This heat is absorbed by the water without change in temperature. On the other hand, a sensible heat change can be detected with a thermometer or other temperature measuring device.

The velocity with which moisture enters or leaves the ground is

$$v_2 = I - E = k_{21} \frac{\partial \tau}{\partial x} + k_{22} \left(\frac{\partial \tau}{\partial y} - 1 \right) \tag{2.35}$$

Set the x-gradient to zero and get

$$\frac{I - E + k_{22}}{k_{22}} = \frac{\partial \tau}{\partial y} \tag{2.36}$$

which gives the proper sign and magnitude for the required gradient.

The other method also uses Eq 2.35 but assumes that v_2 and I are known or can be estimated. Then Eq 2.32 is used to give an estimate of the soil-moisture humidity:

$$H = \frac{(I - v_2)}{E_{sat}} (100 - H_a) + H_a \tag{2.37}$$

The relative humidity is then used in the equation

$$\tau = \frac{RT_e}{mg} \ln \frac{H}{100} \tag{2.38}$$

where

R = the universal gas constant,

T_e = the absolute temperature,

m = the molecular weight of water,

g = the acceleration due to gravity,

τ = the suction.

This suction can be set on the boundary where infiltration and evaporation are taking place and can be changed as these conditions change. Equation 2.38 is taken from the condition of change of free energy in an isothermal process:

$$dF = vdp \tag{2.39}$$

and

$$mgpv = RT_e \tag{2.40}$$

$$F - F_o = \int_{p_o}^{p} \frac{RT_e}{mg} \frac{dp}{p} \tag{2.41}$$

$$= \frac{RT_e}{mg} \ln \frac{p}{p_o} \tag{2.42}$$

$$= \frac{RT_e}{mg} \ln \frac{H}{100} \tag{2.43}$$

In the equations written above

p = vapor pressure of soil water vapor,

v = volume occupied by the water vapor,

p_o = vapor pressure of free water,

$\frac{p}{p_o}$ = the relative humidity of soil water, $\frac{H}{100}$.

(8) Evapo-transpiration. Evapo-transpiration is the process of water transport from soil, through plants, to the atmosphere. This is a serial flow process in which the flow rate is controlled at the point of greatest resistance to water movement.

The same reasoning applies to this boundary as in Condition 7. Rainfall infiltration will generally be higher because the soil is more loose, but the transpiration from plants may counterbalance these, depending on the nature of the vegetation. Qualitatively, it is known that a large tree keeps the soil within and around its root zone in a rather dry condition. When the tree is cut down, the subsequent moisture gain causes a heave in the soil. This has been the sad experience with roads built across the location of old hedgerows. In attempting to derive vegetation moisture requirements, agricultural scientists have developed tables of transpiration ratios which give the weight of water transpired compared to the weight of dry plant material above ground. In a more recent development, Gardner (Ref 8) has proposed that the water intake rate of plants in volume of water per unit time per unit volume of soil $\frac{\Delta\theta}{\Delta t}$ is

$$\frac{\Delta\theta}{\Delta t} = \frac{\tau_p - \tau_m}{R_p + R_s} \tag{2.44}$$

where

τ_p = the matrix suction of the plant,

τ_m = the soil matrix suction,

R_p = the resistance to water movement in the plant,

R_s = the resistance to water movement in the soil.

His experimental results show fair agreement with his predicted results. Ehlig and Gardner (Ref 6) then showed experimental relations and some theoretical explanation of plant suction and transpiration rate. The plant suction is, of course, dependent on the soil suction and this system is tied together with continuity relations of water intake, storage, and transpiration. An analog model of the entire process has been proposed by Woo, Boersma, and Stone (Ref 23) in a paper which includes a thorough discussion of the transpiration problem.

The eight boundary conditions just discussed compose a fairly exhaustive list of conditions which may occur on the boundary of a soil region of interest.

Internal Conditions

Internal conditions are those which occur within a soil region of interest and in principle are no different from boundary conditions. For example, a known gradient or suction (such as from a root system) may occur within a soil region being studied and any computational process should be able to handle such interior complications.

One of the benefits of using the numerical solution process discussed in the next chapter is that it permits the inclusion of internal conditions with relatively little complication.

CHAPTER 3. THE NUMERICAL METHOD OF SOLUTION

In Chapter 2, it is shown that whether rectangular or cylindrical coordinates are chosen for a problem, the finite-difference equation representing change of suction with time is of the form

$$\begin{aligned} \tau^{k+1}_{i,j} - \tau^{k}_{i,j} &= 2A_{i,j}\tau^{k}_{i,j-1} + 2B_{i,j}\tau^{k}_{i-1,j} \\ &- 2\left[CX_{i,j} + CY_{i,j} \right]\tau^{k}_{i,j} + 2D_{i,j}\tau^{k}_{i+1,j} \\ &+ 2E_{i,j}\tau^{k}_{i,j+1} + 2F_{i,j} \end{aligned} \qquad (3.1)$$

The type of partial differential equation for this process of suction changing with time is called a parabolic equation. Two sets of information must be known for this type of equation to be solved: (1) the initial conditions and (2) the boundary conditions. Initial conditions specify the original value of τ at each point in a region at the time chosen for the start of the problem. Boundary conditions specify the value or gradient of τ on the boundaries of a region at each step in time. This parabolic partial differential equation is, of course, different from a Laplace equation in which the time derivative is zero. In the Laplace equation, values computed for the interior of a region do not change with time. Only one set of information is required for solution of a Laplace equation problem: the value or gradient of the variable of interest on the boundaries of an area. An example of a problem described by a Laplace equation is a steady-state seepage problem.

Forward-Difference Method

If Eq 3.1 is used to solve for $\tau^{k+1}_{i,j}$, then the expression becomes

$$\begin{aligned}\tau^{k+1}_{i,j} &= 2A_{i,j}\tau^{k}_{i,j-1} + 2B_{i,j}\tau^{k}_{i-1,j} \\ &\quad - 2\left[CX_{i,j} + CY_{i,j} - \frac{1}{2} \right]\tau^{k}_{i,j} \\ &\quad + 2D_{i,j}\tau^{k}_{i+1,j} + 2E_{i,j}\tau^{k}_{i,j+1} + 2F_{i,j} \qquad (3.2)\end{aligned}$$

If values for τ at time step k are known at each point, then τ at time step $k+1$ can be computed by Eq 3.2. This procedure is termed forward-difference method and is the method used by Richards (Ref 18) in his computer program. From a computational standpoint, this is a very convenient method, but it has the disadvantage that unless the coefficients like $A_{i,j}$ are less than 0.25, errors between actual and computed values can become very large--a condition termed "unstable." The terms $CX_{i,j}$ and $CY_{i,j}$ should be less than 0.125 for the numerical solution to remain stable as time increases. The value of τ at one time step depends solely on the five surrounding values of τ at the previous time step. A graphical representation of this method is shown in Fig 5. The coefficients of each applicable τ term are shown enclosed in the diagram.

Convergence of a numerical scheme is assured if the numerical values obtained approach the exact solution of the differential equation as the increment size is decreased. Though other definitions of convergence are used, this appears to be widely accepted. A clear discussion of both stability and convergence of a numerical approximation of a parabolic equation is given by Kunz (Ref 12). Although the difference equation considered by Kunz is a function of x and t alone, the method of proving convergence and finding the condition for stability is the same as is used when a function of x , y , and t is considered. The forward-difference method is convergent and stable for coefficient values less than the amounts previously mentioned.

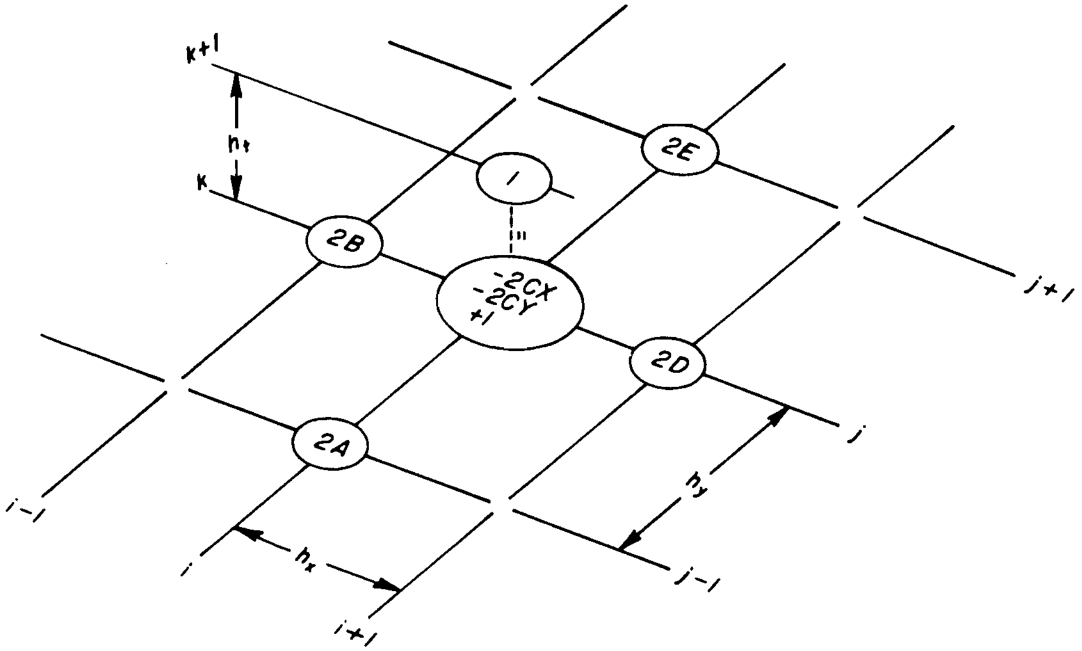


Fig 5. Forward-difference operator.

Crank-Nicolson Method

The Crank-Nicolson method was proposed for use in the solution of heat flow problems (Ref 4) and normally has the advantage that any size of time step may be chosen and the process will still remain stable. When compared with the forward-difference method, it has the disadvantage of being a more complicated computational procedure.

The Crank-Nicolson method requires a change from Eq 3.1 as shown in Eq 3.3. A graphical representation of the operator is given in Fig 6(a).

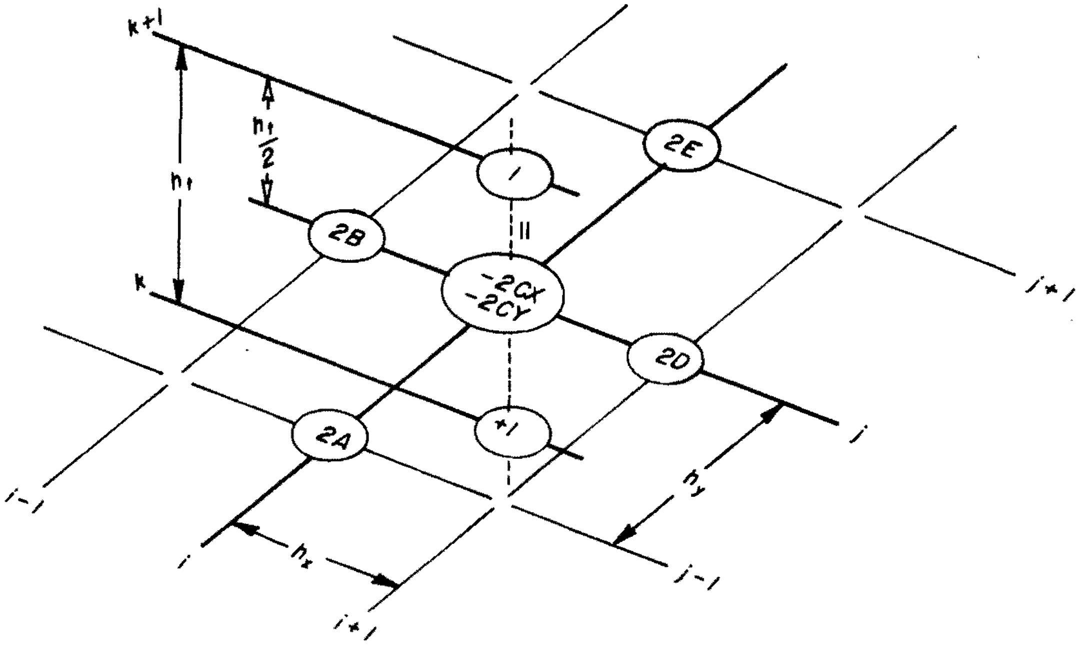
$$
\begin{aligned}
\tau^{k+1}_{i,j} - \tau^{k}_{i,j} &= 2A_{i,j}\tau^{k+\frac{1}{2}}_{i,j-1} + 2B_{i,j}\tau^{k+\frac{1}{2}}_{i-1,j} \\
&\quad - 2\left[CX_{i,j} + CY_{i,j} \right]\tau^{k+\frac{1}{2}}_{i,j} \\
&\quad + 2D_{i,j}\tau^{k+\frac{1}{2}}_{i+1,j} + 2E_{i,j}\tau^{k+\frac{1}{2}}_{i,j+1} + 2F_{i,j} \qquad (3.3)
\end{aligned}
$$

Because the values of τ are not computed at the half-time step, it is further assumed that

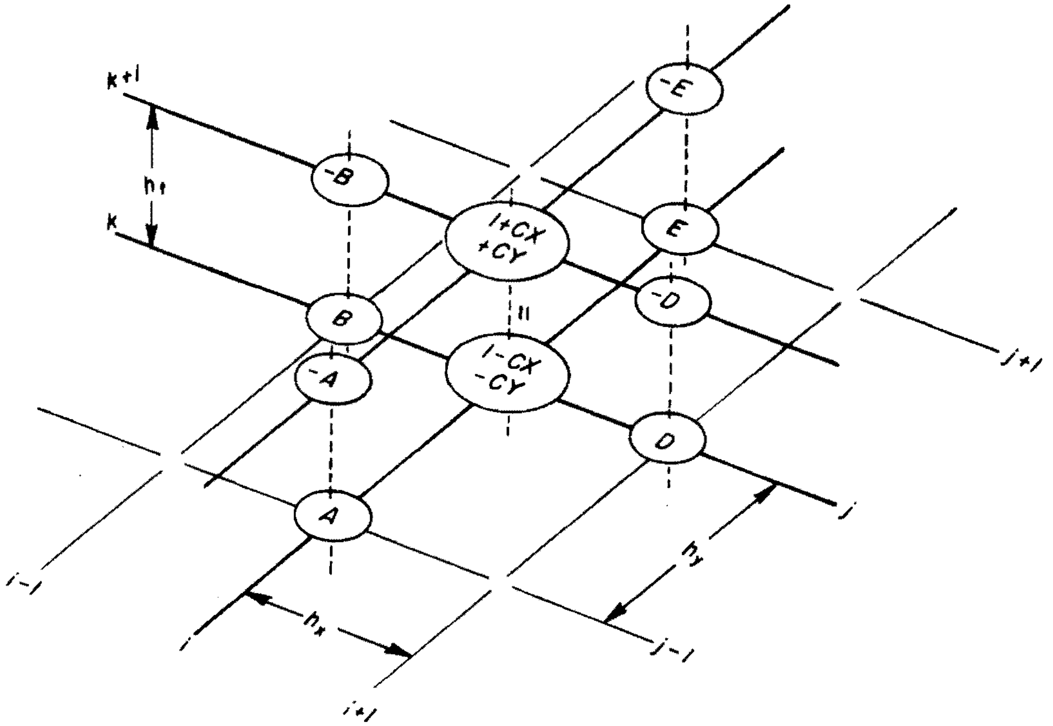
$$
\tau^{k+\frac{1}{2}} = \frac{1}{2}\left(\tau^{k+1} + \tau^{k} \right) \qquad (3.4)
$$

This approximation is inserted in Eq 3.3, and the form of the Crank-Nicolson method that is actually used in computations is found in Eq 3.5. The actual operator used is shown in Fig 6(b).

$$
\begin{aligned}
\tau^{k+1}_{i,j} - \tau^{k}_{i,j} &= A_{i,j}\tau^{k}_{i,j-1} + B_{i,j}\tau^{k}_{i-1,j} - \left[CX_{i,j} + CY_{i,j} \right]\tau^{k}_{i,j} \\
&\quad + D_{i,j}\tau^{k}_{i+1,j} + E_{i,j}\tau^{k}_{i,j+1} + A_{i,j}\tau^{k+1}_{i,j-1} \\
&\quad + B_{i,j}\tau^{k+1}_{i-1,j} - \left[CX_{i,j} + CY_{i,j} \right]\tau^{k+1}_{i,j} \\
&\quad + D_{i,j}\tau^{k+1}_{i+1,j} + E_{i,j}\tau^{k+1}_{i,j+1} + 2F_{i,j} \qquad (3.5)
\end{aligned}
$$



(a) Operator illustrating the Crank-Nicolson concept.



(b) Crank-Nicolson operator.

Fig 6. Crank-Nicolson operator as applied to discrete-element representation.

A demonstration of the stability of the forward-difference method is not given here. A demonstration of the stability of the Crank-Nicolson method will be sketched briefly. With the following simplifications, the demonstration will be more straightforward:

$$h_x = h_y = h$$

$$k_{12} = k_{21} = k_1$$

$$k_1 \frac{\partial \tau}{\partial \theta} = D$$

k = one time increment

In its simplified form, Eq 3.5 may be written

$$\begin{aligned} \tau(x,y,t+k) - \tau(x,y,t) = {} & + \frac{h_t}{h^2} \frac{D}{2} \Big[\tau(x,y-h,t) \\ & + \tau(x-h,y,t) + \tau(x+h,y,t) + \tau(x,y+h,t) \\ & - 4\tau(x,y,t) + \tau(x,y-h,t+k) + \tau(x-h,y,t+k) \\ & + \tau(x+h,y,t+k) + \tau(x,y,t+k) \Big] \end{aligned} \tag{3.6}$$

The following substitutions are made and the equation is manipulated into the form shown in Eq 3.8.

$$r = \frac{h_t}{h^2}$$

$$\tau(x,y,t) = e^{\gamma t} X(x,y) \tag{3.7}$$

$$\frac{e^{\gamma(t+k)} - e^{\gamma t}}{e^{\gamma(t+k)} \qquad e^{\gamma t}} = \frac{Dr}{2}\left[\frac{X(x,y-h) + X(x-h,y) + X(x+h,y) + X(x,y+h)}{X(x,y)}\right.$$

$$\left.\frac{-4X(x,y)}{X(x,y)}\right] = \phi \tag{3.8}$$

In Eq 3.8, ϕ is a constant. The function $X(x,y)$ must be found from the initial boundary conditions. Two somewhat austere cases are shown in Fig 7. For the condition shown in Fig 7(a), the function $X(x,y)$ is of the form:

$$X(x,y) = \frac{L_y C}{n\pi} \cos \frac{n\pi x}{L_x} \cos \frac{n\pi y}{L_y} \tag{3.9}$$

For the condition shown in Fig 7(b) the function is

$$X(x,y) = \cos \frac{n\pi x}{L_x} \left[\frac{CL_y}{n\pi} \sin \frac{n\pi y}{L_y} + \tau y \cos \frac{n\pi y}{L_y} \right] \tag{3.10}$$

which can be reduced to the following form:

$$X(x,y) = A \cos \frac{n\pi x}{L_x} \cos \left(\frac{n\pi y}{L_y} - \psi \right) \tag{3.11}$$

where

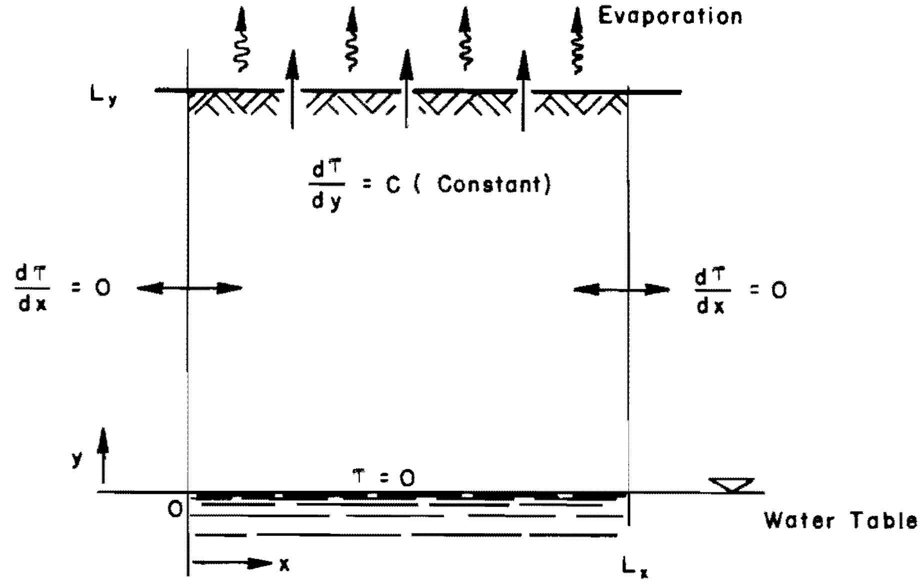
$$\psi = \text{arc tan} \left(\frac{CL_y}{n\pi\tau y} \right)$$

A = some constant.

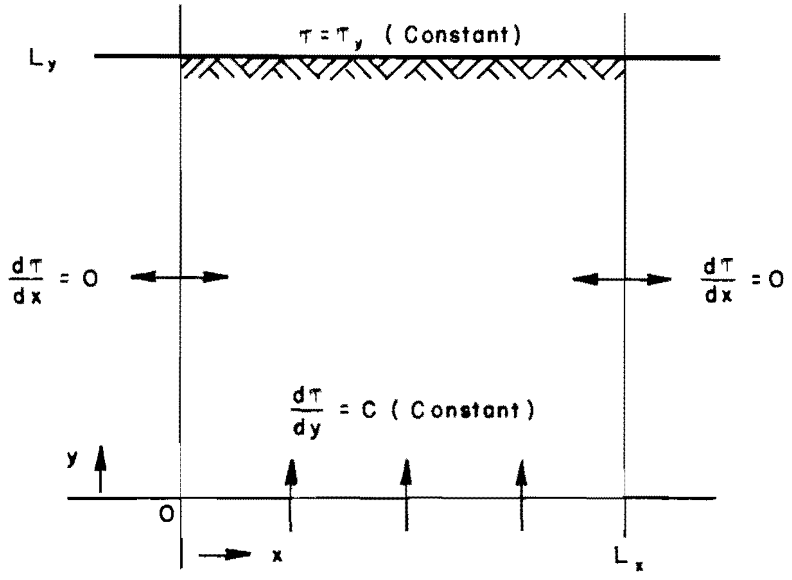
Generally speaking, the function $X(x,y)$ will be of the form

$$X(x,y) = A \cos \alpha x \cos \beta y \tag{3.12}$$

This general relation may be substituted into Eq 3.8 to find an expression for the constant ϕ



(a) High-water table.



(b) Low-water table.

Fig 7. Two cases of boundary conditions.

$$\phi = -2Dr\left[\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right] \tag{3.13}$$

The constant ϕ is set equal to the time-dependent fraction in Eq 3.8 to obtain

$$e^{\gamma t} = \left[\frac{1 - 2Dr\left(\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right)}{1 + 2Dr\left(\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right)}\right]^{\frac{t}{k}} \tag{3.14}$$

This form is substituted into Eq 3.7 to obtain Eq 3.15, a finite Fourier series which expresses τ as it varies with x, y, and t:

$$\tau(x,y,t) = \sum_{i=0}^{M}\sum_{j=0}^{N} A_{ij}\cos\alpha\left(\frac{i}{M}\right)\cos\beta\left(\frac{i}{N}\right) \times \ldots .$$

$$\ldots . \times \left[\frac{1 - 2Dr\left(\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right)}{1 + 2Dr\left(\sin^2\frac{\alpha h}{2} + \sin^2\frac{\beta h}{2}\right)}\right]^{\frac{t}{k}} \tag{3.15}$$

where

M = number of x-increments,

N = number of y-increments,

A_{ij} = constants.

In order for a method to be stable, it must produce bounded results as t approaches infinity. In most real cases, the constants A_{ij} are bounded and the only term which affects stability is that in brackets. Because the terms D, r, and $(\sin^2)$ are all positive, the term in brackets is always less than one. Thus, as t approaches infinity this term remains bounded.

Thus, the Crank-Nicolson method will allow stable solutions of this type of numerical, parabolic, partial difference equation regardless of the time step chosen. It must be recalled that the cross permeability terms were set at zero for this development. In the next section, the effect of including all of the terms of the permeability tensor will be shown.

Stability of Crank-Nicolson Method with Tensor Form of Permeability

As in the previous section, the difference equation form of the Crank-Nicolson method is written as a function of x, y, and t as follows:

$$\tau(x,y,t) = e^{\gamma t}X(x,y) \tag{3.16}$$

With the aid of the following definitions

$$r = \frac{h_t}{h^2}$$

$$D = k_{11}\left(\frac{\partial\tau}{\partial\theta}\right)$$

$$m = \frac{k_{12}}{k_{11}}$$

$$n = \frac{k_{22}}{k_{11}}$$

an equation similar to Eq 3.8 may be written in two parts:

$$\frac{e^{\gamma(t+k)} - e^{\gamma t}}{e^{\gamma(t+k)} + e^{\gamma t}} = \phi \tag{3.17a}$$

and

$$\phi = \frac{Dr}{2}\left[\frac{(1+m)X(x-h,y) + (m+n)X(x,y-h) + (1+m)X(x+h,y)}{X(x,y)}\right.$$

$$\left. + \frac{(m+n)X(x,y+h) - 2(1+2m+n)X(x,y)}{X(x,y)}\right] \tag{3.17b}$$

Again recognizing that the function $X(x,y)$ will be of the form

$$X(x,y) = A\cos\alpha x\cos\beta y \tag{3.18}$$

it is found that the constant ϕ is

$$\phi = -2Dr \left[\sin^2 \frac{\alpha h}{2} + n \sin^2 \frac{\beta h}{2} + m \left(\sin^2 \frac{\alpha h}{2} + \sin^2 \frac{\beta h}{2} \right) \right] \quad (3.19)$$

The term in brackets must always be positive in order for the method to be stable. It is apparent from a Mohr's permeability circle that m may be negative and thus the stability requirement becomes

$$m \left(\sin^2 \frac{\alpha h}{2} + \sin^2 \frac{\beta h}{2} \right) + \sin^2 \frac{\alpha h}{2} + n \sin^2 \frac{\beta h}{2} > 0 \quad (3.20)$$

The sine term ratio ψ is defined as follows

$$\psi = \frac{\sin^2 \frac{\beta h}{2}}{\sin^2 \frac{\alpha h}{2}} \quad (3.21)$$

In addition to ψ , the positive angle ξ is defined as the angle measured counterclockwise from the major principal permeability to the horizontal. With this definition, the cross-permeability term is

$$k_{12} = - \left(\frac{k_{11} - k_{22}}{2} \right) \tan 2\xi \quad (3.22)$$

and the quantity m may be written as a function of n and ξ .

$$m = \left(-\frac{1}{2} + \frac{n}{2} \right) \tan 2\xi \quad (3.23)$$

The stability condition becomes

$$\tan 2\xi < \frac{2(1 + n\psi)}{(1 + \psi)(1 - n)} \quad (3.24)$$

which indicates that instability is a function of n , ξ , and ψ . The following table gives ranges of angles ξ for which instability may be anticipated for various values of n and ψ .

TABLE 1. RANGES OF ANGLES FOR INSTABILITY OF THE METHOD

			VALUES OF ψ				
n	Angles Range from		1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	0
1	$+45^\circ$	to	$+45^\circ$	$+45^\circ$	$+45^\circ$	$+45^\circ$	$+45^\circ$
	-45°	to	-45°	-45°	-45°	-45°	-45°
$\frac{1}{2}$	$+45^\circ$	to	$35^\circ 47'$	$36^\circ 39'$	$37^\circ 14'$	$37^\circ 35'$	$37^\circ 59'$
	-45°	to	$-54^\circ 13'$	$-53^\circ 21'$	$-52^\circ 46'$	$-52^\circ 25'$	$-52^\circ 01'$
$\frac{1}{4}$	$+45^\circ$	to	$29^\circ 13'$	$31^\circ 43'$	$+33^\circ 06'$	$+33^\circ 52'$	$34^\circ 43'$
	-45°	to	$-60^\circ 29'$	$-58^\circ 17'$	$-56^\circ 54'$	$-56^\circ 08'$	$-55^\circ 17'$
$\frac{1}{8}$	$+45^\circ$	to	$26^\circ 04'$	$29^\circ 09'$	$31^\circ 02'$	$32^\circ 04'$	$33^\circ 11'$
	-45°	to	$-63^\circ 56'$	$-60^\circ 51'$	$-58^\circ 58'$	$-57^\circ 56'$	$-56^\circ 49'$
0	$+45^\circ$	to	$22^\circ 30'$	$26^\circ 34'$	$29^\circ 00'$	$30^\circ 19'$	$31^\circ 43'$
	-45°	to	$-67^\circ 30'$	$-63^\circ 26'$	$-61^\circ 00'$	$-59^\circ 41'$	$-58^\circ 17'$

In addition, Fig 8 shows graphically the safe ranges of direction for the maximum principal permeability with respect to the horizontal for $\psi = 1$.

The stability condition may be written as a function of the major and minor principal permeabilities for two different angle ranges. The first range is

$$-90^\circ < 2\xi < +90^\circ$$

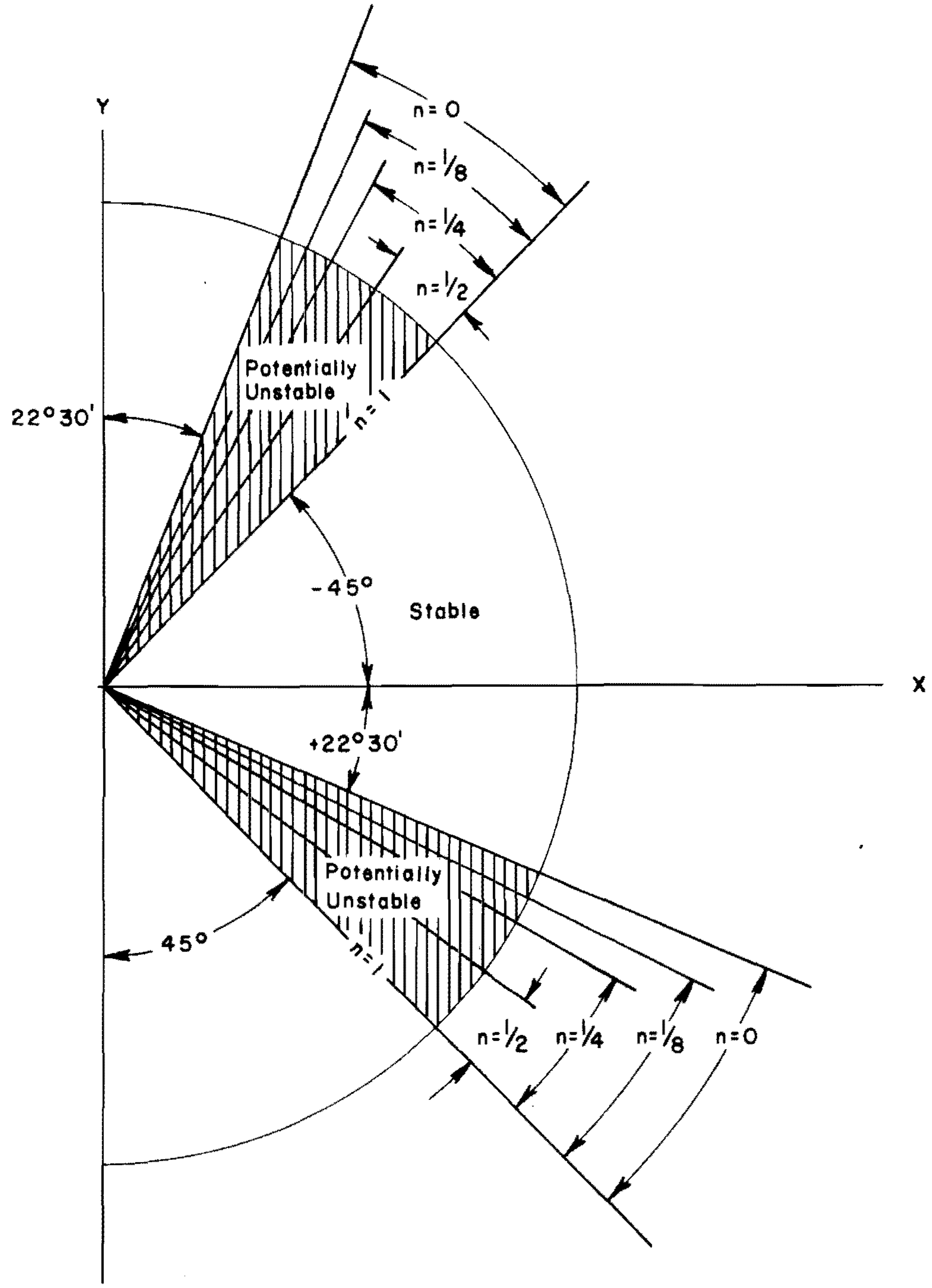


Fig 8. Stable directions for major principal permeability for $\psi = 1$.

in which the stability condition is

$$\frac{k_1 + k_2}{k_1 - k_2} \sec 2\xi > \tan 2\xi + \frac{2\psi}{1 + \psi} - 1 \qquad (3.25)$$

The second range is

$$90^\circ < 2\xi < 180^\circ \quad \text{and}$$

$$- 90^\circ > 2\xi > - 180^\circ$$

in which the stability condition is

$$\frac{k_1 + k_2}{k_1 - k_2} \sec 2\xi < \tan 2\xi + \frac{2\psi}{1 + \psi} - 1 \qquad (3.26)$$

These guidelines will point out conditions in which instability can develop before a problem is submitted for solution to the computer.

Method of Solution

At each mesh point of a region of interest, an equation like Eq 3.5 may be written. The complete collection of all such equations, with boundary conditions included, will form a system of linear algebraic equations which must be solved simultaneously. The methods proposed by mathematicians and engineers to solve systems containing large numbers of equations exhibit considerable variety and ingenuity. In discussing the classes of methods, Forsythe and Wasow (Ref 7) stated:

> Methods for solving a given computational problem are ordinarily divided into direct and iterative. Direct methods . . . are those which would yield the exact answer in a finite number of steps if there were no round-off error. Ordinarily the algorithm computation procedure of a direct method is rather complicated and non-repetitious. Iterative methods, on the other hand, consist of the repeated application of a simple algorithm, but ordinarily yield the exact answer only as the limit of sequence, even in the absence of round-off error. . . . Iterative methods are preferred for solving large "sparse" systems because they can usually take full advantage of the numerous zeros in the coefficient matrix , both in storage and in operation.

The term "sparse" refers to the fact that each equation written contains unknowns in the immediate vicinity of the point about which the equation is written. Thus, in each equation the coefficient of all other unknowns in a region is zero and the preponderant number in any such coefficient matrix is zero. Furthermore, if points are numbered row-wise and column-wise, then the nonzero coefficients will be arranged in diagonal fashion, symmetrically or nearly symmetrically positioned around the main diagonal. Out of the many available methods, an alternating-direction-implicit iterative method was chosen for this study.

The discussion of this method will be much clearer if operators are used. In the discussion to follow, the symbols defined below will be used.

$\underset{\sim}{\tau}$ = the collection of $\tau_{i,j}$ operated upon

$$\frac{\partial}{\partial x_i}\left(k_{i1} \frac{\partial \tau}{\partial x_1} \right) = \frac{1}{h^2} \delta_x^2 \underset{\sim}{\tau}$$

$$\frac{\partial}{\partial x_i}\left[k_{i2} \left(\frac{\partial \tau}{\partial x_2} - 1 \right) \right] \cong \frac{1}{h^2} \delta_y^2 \underset{\sim}{\tau}$$

$$r = \frac{h_t}{h^2} \left(\frac{\partial \tau}{\partial \theta} \right)$$

With this notation, Eq 3.5 may be written as

$$\underset{\sim}{\tau}_{k+1} - \underset{\sim}{\tau}_k = r \left(\delta_x^2 + \delta_y^2 \right) \left(\frac{\underset{\sim}{\tau}_{k+1} + \underset{\sim}{\tau}_k}{2} \right)$$

In this case, the subscripts k and k+1 indicate the time step. If all X-operators on τ_{k+1} are collected on one side of the equation, Eq 3.27 results

$$\left(1 - \frac{r}{2} \delta_x^2 \right) \underset{\sim}{\tau}_{k+1} = \left(1 + \frac{r}{2} \delta_x^2 + \frac{r}{2} \delta_y^2 \right) \underset{\sim}{\tau}_k + \left(\frac{r}{2} \delta_y^2 - \nu \right) \underset{\sim}{\tau}_{k+1}^{(n)} \tag{3.27}$$

Similarly, if all Y-operators on $\underset{\sim}{T}_{k+1}$ are collected on the left, Eq 3.28 is found

$$\left(1 - \frac{r}{2} \delta_y^2 \right) \underset{\sim}{T}_{k+1} = \left(1 + \frac{r}{2} \delta_x^2 + \frac{r}{2} \delta_y^2 \right) \underset{\sim}{T}_k + \left(\frac{r}{2} \delta_x^2 - \mu \right) \underset{\sim}{T}_{k+1}^{(n+\frac{1}{2})} \tag{3.28}$$

The problem to be solved involves marching a step forward in time: given $\underset{\sim}{T}_k$, find $\underset{\sim}{T}_{k+1}$. The coefficients represented by the operators are set. Any acceleration parameter must be added. The acceleration parameter is a number added to an operator to increase the speed of convergence of an iterative process. Addition of an acceleration operator ν to Eq 3.27 and μ to Eq 3.28 gives the iterative process used in this study and given in Eqs 3.29 and 3.30.

$$\left(1 - \frac{r}{2} \delta_x^2 - \nu \right) \underset{\sim}{T}_{k+1}^{(n+\frac{1}{2})} = \left(1 + \frac{r}{2} \delta_x^2 + \frac{r}{2} \delta_y^2 \right) \underset{\sim}{T}_k + \left(\frac{r}{2} \delta_y^2 - \nu \right) \underset{\sim}{T}_{k+1}^{(n)} \tag{3.29}$$

$$\left(1 - \frac{r}{2} \delta_y^2 - \mu \right) \underset{\sim}{T}_{k+1}^{(n+1)} = \left(1 + \frac{r}{2} \delta_x^2 + \frac{r}{2} \delta_y^2 \right) \underset{\sim}{T}_k + \left(\frac{r}{2} \delta_x^2 - \mu \right) \underset{\sim}{T}_{k+1}^{(n+\frac{1}{2})} \tag{3.30}$$

The superscripts n, $n+\frac{1}{2}$, and $n+1$ refer to iteration number. Iteration should continue until the difference between the values of $\underset{\sim}{T}$ computed on the first half iteration are within some specified tolerance of those calculated on the second half iteration in the y-direction. One cycle of this double-sweep process is termed an iteration. It is apparent that if $\bar{\underset{\sim}{T}}_{k+1}$ = the true value of $\underset{\sim}{T}$ at time step $k+1$, and the error in the computed value of $\underset{\sim}{T}$ is

$$\underset{\sim}{e}_{k+1}^{(n)} = \underset{\sim}{T}_{k+1}^{(n)} - \bar{\underset{\sim}{T}}_{k+1}$$

then the difference between the true solution and that computed by Eqs 3.29 and 3.30 is given in operator form by

$$\underset{\sim}{e}_{k+1}^{(n+\frac{1}{2})} = \frac{\left(\frac{r}{2}\,\delta_y^2 - \nu\right)}{\left(1 - \frac{r}{2}\ {}_{x}^{2} + \nu\right)}\,\underset{\sim}{e}_{k+1}^{(n)} \tag{3.31}$$

and

$$\underset{\sim}{e}_{k+1}^{(n+1)} = \frac{\left(\frac{r}{2}\,\delta_x^2 - \mu\right)}{\left(1 - \frac{r}{2}\,\delta_y^2 + \mu\right)}\,\underset{\sim}{e}_{k+1}^{(n+\frac{1}{2})} \tag{3.32}$$

In each case, if the numerator is set equal to zero, the error at the next half step would be zero. This is no mathematical proof, but in operator form it suggests a relation that has been found to be useful. If

$$\nu \cong +\frac{r}{2}\,\delta_y^2 \tag{3.33}$$

and

$$\mu \cong +\frac{r}{2}\,\delta_x^2 \tag{3.34}$$

then the error should decrease provided the approximation is good enough. To get the best approximation, one assumes that the latest computed values of $\tau_{i,j}$ are the best, applies the operator, and divides by $\tau_{i,j}$ at the point in question. Thus, if τ_{k+1} is the value of $\tau_{i,j}$ at a particular point i,j ,

$$\nu_{i,j}^{(n+\frac{1}{2})} = \frac{r}{2}\,\frac{\delta_y^2\,\underset{\sim}{\tau}_{k+1}^{(n)}}{\tau_{k+1}^{(n)}} \tag{3.35}$$

and

$$\mu_{i,j}^{(n+1)} = \frac{r}{2} \frac{\delta_{x}^{2}\tau_{k+1}^{(n+\frac{1}{2})}}{\tau_{k+1}^{(n+\frac{1}{2})}} \tag{3.36}$$

The operator form becomes complicated at this point and it is more convenient to return to explicitly stated formulas. The alternating-direction approach separately considers flow in the x-direction and then in the y-direction. The limit of this sequence of double sweeps of computation is the condition in which τ computed in the x-direction (τX) equals the τ computed in the y-direction (τY) . A physical representation of the alternating-direction process is shown in Fig 9. At each intersection of an X and Y-pipe at a mesh point, the two are connected by tubes with valves on them. Storage of water at each point is represented by a sump. It is interesting and perhaps significant to note that the dimensions of the parameters μ and ν are square inches or square centimeters per unit area of soil region. These parameters may be regarded as valve openings which allow flow from one pipe to another. For this reason, the parameters μ and ν have been termed "valve setting - x" and "valve setting - y" with the appropriate abbreviation. The expressions for these terms are shown in Eqs 3.37 and 3.38.

$$VSX_{i,j} = - \left[\frac{B_{i,j}\tau X_{i-1,j,k+1} - CX_{i,j}\tau X_{i,j,k+1} + D_{i,j}\tau X_{i+1,j,k+1}}{\tau X_{i,j,k+1}} \right] \tag{3.37}$$

$$VSY_{i,j} = - \left[\frac{A_{i,j}\tau Y_{i,j-1,k+1} - CY_{i,j}\tau Y_{i,j,k+1} + E_{i,j}\tau Y_{i,j+1,k+1}}{\tau Y_{i,j,k+1}} \right] \tag{3.38}$$

The equations for each half iteration take the form:

$$a_i\tau_{i-1,k+1} + b_i\tau_{i,k+1} + c_i\tau_{i+1,k+1} = d_i$$

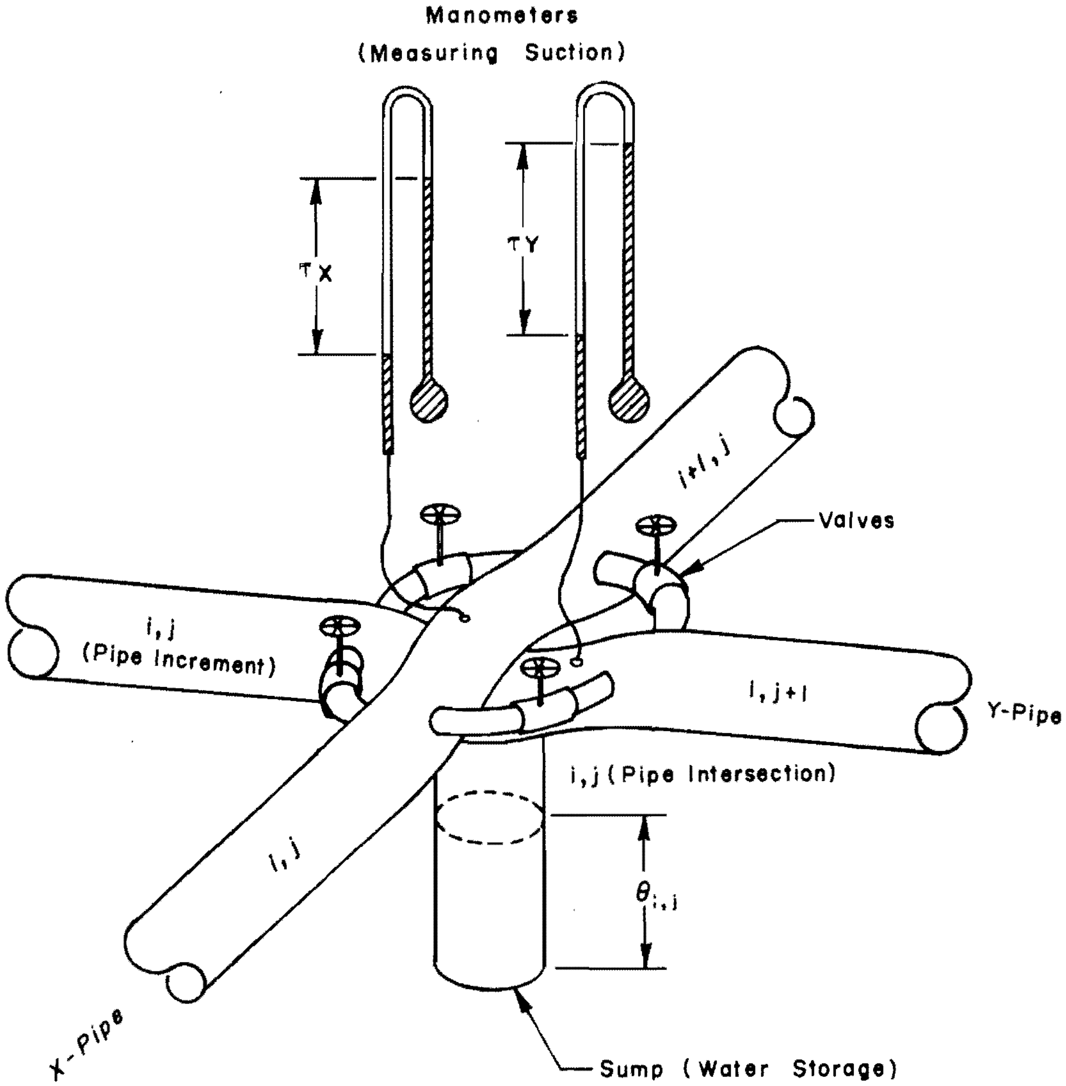


Fig 9. The grid pipe system.

in which, for the x-iterations, the coefficients are given in Eqs 3.39 through 3.42:

$$a_i = -B_{i,j} \tag{3.39}$$

$$b_i = 1 + CX_{i,j} + VSY_{i,j} \tag{3.40}$$

$$c_i = -D_{i,j} \tag{3.41}$$

$$\tau_{\ldots,k+1} = \tau X_{\ldots,k+1}$$

$$\begin{aligned} d_i = {} & A_{i,j}\tau Y_{i,j-1,k+1} + \left[VSY_{i,j} - CY_{i,j} \right] \tau Y_{i,j,k+1} \\ & + E_{i,j}\tau Y_{i,j+1,k+1} + A_{i,j}\tau_{i,j-1,k} \\ & + B_{i,j}\tau_{i-1,j,k} + (1 + CX_{i,j} + CY_{i,j})\tau_{i,j,k} \\ & + D_{i,j}\tau_{i+1,j,k} + E_{i,j}\tau_{i,j+1,k} + 2F_{i,j} \qquad (3.42 \end{aligned}$$

The coefficients for the y-iterations are given below in Eqs 3.43 through 3.46.

$$a_j = -A_{i,j} \tag{3.43}$$

$$b_j = 1 + CY_{i,j} + VSX_{i,j} \tag{3.44}$$

$$c_j = -E_{i,j} \tag{3.45}$$

$$\tau_{\ldots,k+1} = \tau Y_{\ldots,k+1}$$

$$\begin{aligned} d_j = {} & B_{i,j}\tau X_{i-1,j,k+1} + (VSX_{i,j} - CX_{i,j})\tau X_{i,j,k+1} \\ & + D_{i,j}\tau X_{i+1,j,k+1} + A_{i,j}\tau_{i,j-1,k} \\ & + B_{i,j}\tau_{i-1,j,k} + (1 + CX_{i,j} + CY_{i,j})\tau_{i,j,k} \\ & + D_{i,j}\tau_{i+1,j,k} + E_{i,j}\tau_{i,j+1,k} + 2F_{i,j} \end{aligned} \qquad (3.46)$$

The quantities involving τ_k are known and remain constant throughout the iteration process. The definitions of the valve-setting terms show that the τY terms in Eq 3.42 and the τX terms in Eq 3.46, when added to the appropriate valve-setting term, will be zero. The valve setting is not a mathematically precise quantity, however. It depends for its accuracy upon the degree of accuracy in the previously computed values of τX or τY as the case may be. Thus, in a computation process a little judgment must be built into the procedure. It has been found useful, by trial and error, never to allow the value of VSX or VSY to be negative. If a computed value of valve setting is negative, then it is set to zero and the τX or τY terms on the right side of Eqs 3.42 and 3.46 will add to a value other than zero. Some intuitive or empirical reasons can be given for not allowing the valve settings to become negative:

(1) There is no physical significance for a negative area.
(2) The negative factor appears to force τX and τY apart rather than pulling them together.

One other limitation should be followed at present. There is no reliable guideline to which kinds of problems may be worked using this "naturally determined valve setting" and thus it appears success can only be guaranteed if the problem to be solved is relatively well-behaved. If there is a problem in which τ is expected to change by a large amount in one time step, one may expect to have difficulty, even though at times he may be pleasantly surprised. In the case of more ill-behaved problems, it is safer at the present time to use the more established methods of computing valve settings such as the Peaceman-Rachford or Wachspress parameters. The formula for the P-R valve settings is

$$\mu_i = \nu_i = b\left(\frac{a}{b}\right)^{\frac{2i-1}{2m}} \qquad (3.47)$$

where

b = the largest eigenvalue of both the x and the y-coefficient matrix,

a = the smallest eigenvalue of both the x and the y-coefficient matrix,

m = an integer chosen so that

$$\frac{a}{b} \geq \left(\sqrt{2} - 1\right)^{2m} ,$$

i = an integer that varies from 1 to m .

The Wachspress formula

$$\mu_i = \nu_i = b\left(\frac{a}{b}\right)^{\frac{i-1}{m-1}} \qquad (3.48)$$

where in this case M = an integer chosen so that

$$\frac{a}{b} \geq \left(\sqrt{2} - 1\right)^{2m-1}$$

The computed valve settings are used cyclically until acceptable closure has been achieved.

The preceding discussion is deliberately not mathematical. The problem described in Chapter 2 is not susceptible to the precise analytical treatment that mathematicians have given to the alternating-direction method for a somewhat restricted set of conditions. For example, Forsythe and Wasow show that the "Peaceman-Rachford method," of which Eqs 3.39 to 3.46 are an example, will converge for any positive valve setting provided all of the eigenvalues of both the x and y-coefficient matrices are positive. Young and Wheeler (Ref 24) prove convergence for the process for any set of positive valve settings provided the x and y-coefficient matrices are commutative and are similar to diagonal matrices with positive diagonal elements in addition to meeting the requirements of Forsythe and Wasow's proof.

Commutative matrices will give the same result when they are multiplied together regardless of the order in which they are multiplied. Thus if there are two matrices M and N , then they commute if

$$MN = NM \qquad (3.49)$$

Forsythe and Wasow comment that "this commutativity is a very exceptional property, occurring only for rectangular [regions]."

The analytical problem is a difficult one. The results that have been achieved lend assurance that the alternating-direction scheme is a powerful method which is characterized by rapid convergence when compared with other iterative schemes. Additional assurance may be gained from the fact that alternating-direction methods have been used to solve a variety of problems involving both second and fourth-order partial difference equations (Refs 10, 11, and 19) for which no proof of convergence exists. Young and Wheeler state, ". . . the Peaceman-Rachford method has been found to be extremely effective even in cases where commutativity does not hold."

Even in those cases in which convergence can be proven, the positive valve settings may be chosen wisely to achieve a faster rate of convergence. The Peaceman-Rachford and Wachspress parameters computed from Eqs 3.47 and 3.48, respectively, are attempts at choosing values which will accelerate the convergence.

Systems of equations like Eqs 3.39 to 3.46 may be solved simultaneously by a procedure which Young and Wheeler credit to L. H. Thomas (Ref 20). Given a system like

$$a_i \tau_{i-1} + b_i \tau_i + c_i \tau_{i+1} = d_i \qquad (3.50)$$

a systematic method of applying Gauss elimination would give equations like

$$\tau_{i-1} = A_{i-1} + B_{i-1} \tau_i \qquad (3.51)$$

$$\tau_i = A_i + B_i \tau_{i+1} \qquad (3.52)$$

Substitution of Eq 3.51 into Eq 3.50 results in the following equations:

$$A_i = \frac{d_i - a_i A_{i-1}}{b_i + a_i B_{i-1}} \qquad (3.53)$$

$$B_i = \frac{-c_i}{b_i + a_i B_{i-1}} \qquad (3.54)$$

Boundary conditions are special cases of this general form as will be shown in the next section.

Representation of Boundary Conditions

As previously discussed in Chapter 2, boundary conditions may fall into two types: a specified value of τ and a specified gradient $\frac{\partial \tau}{\partial x}$ or $\frac{\partial \tau}{\partial y}$.

Suction specified. In this case, Eq 3.52 would show that

$$\tau_o = A_o + B_o \tau_1 \qquad (3.55)$$

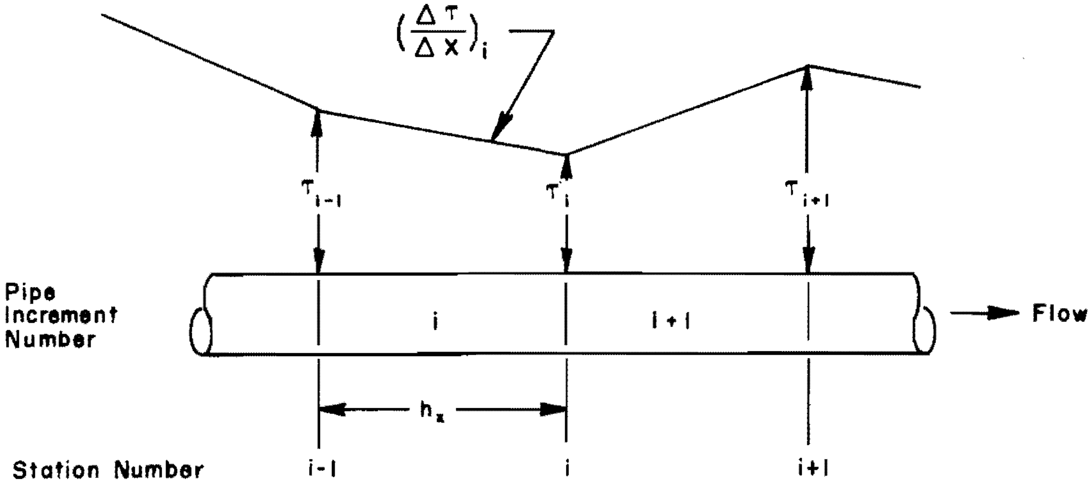
Because τ_o must remain the same regardless of what the numerical value of τ_1 is, this condition is enforced by setting

$$A_o = \tau_o \qquad (3.56)$$

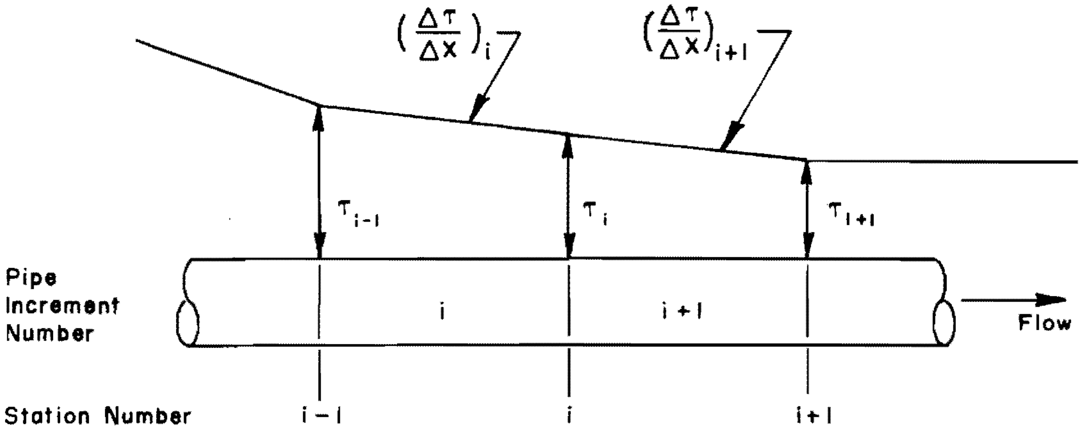
$$B_o = 0 \qquad (3.57)$$

The same reasoning applies to a value of suction set on the interior of a region.

Boundary Gradient Specified. Although the point seems trivial, it must be mentioned that in the discrete-element representation of the transient flow problem, gradient does not exist at a point. Rather, it occurs between mesh points. Thus, when a gradient is specified it must be taken to apply a certain



(a) Pipe increment gradient.



(b) Point gradient: average of gradients on each side of the point.

Fig 10. Discrete-element representation of point and increment gradients.

rise or drop of suction in a certain pipe increment. If gradient at a point is desired, it must be viewed as the average gradient on each side of the point. An illustration of the pipe increment and point gradient is given in Fig 10.

For representing boundary gradients, the point form was chosen for this study. Thus, it is found that

$$\left(\frac{\partial \tau}{\partial x} \right)_{AVG} \cong \frac{1}{2} \left[\frac{-\tau_{-1} + \tau_{o}}{h_{x}} + \frac{-\tau_{o} + \tau_{1}}{h_{x}} \right] \tag{3.58}$$

which produces the result

$$\tau_{-1} = \tau_{-1} = \tau_{1} - 2h_{x} \left(\frac{\partial \tau}{\partial x} \right) \tag{3.59}$$

If each of the pipe-increment gradients is also set equal to the average gradient, then the following two equations are derived:

$$\tau_{-1} = \tau_{o} - h_{x} \left(\frac{\partial \tau}{\partial x} \right)_{AVG} \tag{3.60}$$

$$\tau_{1} = \tau_{o} + h_{x} \left(\frac{\partial \tau}{\partial x} \right)_{AVG} \tag{3.61}$$

which in turn give the constant values

$$A_{-1} = -h_{x} \left(\frac{\partial \tau}{\partial x} \right)_{AVG} \tag{3.62}$$

$$B_{-1} = 1 \tag{3.63}$$

$$A_{o} = \tau_{o} \tag{3.64}$$

$$B_{o} = 0 \tag{3.65}$$

$$A_1 = B_{-1}A_o + A_{-1} + 2h_x \left(\frac{\partial \tau}{\partial x} \right)_{AVG} \qquad (3.66)$$

$$B_1 = 0 \qquad (3.67)$$

The same set of equations applies at the other boundary where x has its maximum value. Analogous equations may be developed for a specified gradient in the y-direction.

The value τ_o may be specified or it may be computed from flow conditions in the y-direction. This latter is the way the gradient boundary condition is used. The value of τ_o is allowed to change, but its relation to surrounding values of τ is not. An example of this is the use of the line of symmetry as a boundary. A mirror image is assumed to exist on each side of a line of symmetry. The point gradient is thus zero and no flow takes place across this type of boundary.

Internal Gradient Specified. Only the gradient along a particular pipe increment is considered here. If a point gradient is desired, then that gradient should be specified for the pipe increments on each side of the point of interest. The discussion to follow is concerned with specifying a gradient along pipe-increment i . This is shown in Fig 11.

According to the standard procedure given in Eqs 3.53 and 3.54, the coefficients A_{i-1} , B_{i-1} , A_i , and B_i will be computed. For convenience, a coefficient C_{i-1} is defined as

$$C_{i-1} = b_{i-1} + a_{i-1}B_{i-2} \qquad (3.68)$$

so that the other coefficients will be

$$A_{i-1} = \frac{d_{i-1}a_{i-1}A_{i-2}}{C_{i-1}} \qquad (3.69)$$

and thus

$$T_i = A_{i-1} + B_{i-1}\tau_i \qquad (3.70)$$

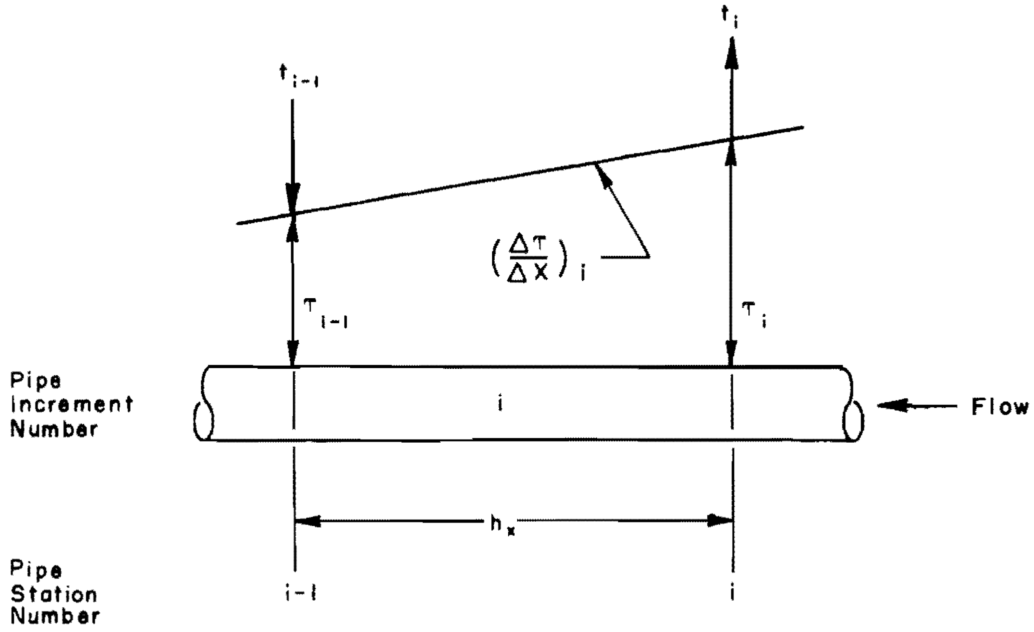


Fig 11. Internal suction gradient specified.

The final result that is desired is that

$$\tau_{i-1} = -\left(\frac{\partial\tau}{\partial x}\right)_i h_x + \tau_i \qquad (3.71)$$

The coefficients A_{i-1}, B_{i-1}, A_i, and B_i contain information carried from the boundary to the points i-1 and i by virtue of the elimination process. If the coefficients A_{i-1} and B_{i-1} were simply reset to reflect the relation given in Eq 3.71, the continuity of the elimination procedure from one boundary to the other would be interrupted. To avoid this difficulty a special procedure must be used. A fictitious suction t_{i-1} is added to τ_{i-1} and subtracted from τ_i. The fictitious suction is of sufficient size to cause the difference in τ_{i-1} and τ_i to be in accord with the desired gradient. The size of this fictitious suction is established from the relations which must be satisfied simultaneously: continuity of the elimination process and establishment of a desired gradient. These two relations are specified in the following two equations:

$$\tau_{i-1} = A_{i-1} + \frac{t_{i-1}}{C_{i-1}} + B_{i-1}\tau_i \qquad (3.72)$$

$$\tau_{i-1} = -\left(\frac{\partial\tau}{\partial x}\right)_i h_x + \tau_i \qquad (3.73)$$

Solving these two equations for t_{i-1} gives

$$t_{i-1} = C_{i-1}\left[-\left(\frac{\partial\tau}{\partial x}\right)_i h_x - A_{i-1} + (1 - B_{i-1})\tau_i\right] \qquad (3.74)$$

This same amount is subtracted from τ_i in the following fashion:

$$\tau_i = A_i - \frac{t_{i-1}}{C_i} + B_i\tau_{i+1} \qquad (3.75)$$

$$\tau_i = A_i + \frac{C_{i-1}}{C_i}\left[A_{i-1} + \left(\frac{\partial\tau}{\partial x} \right)_i h_x - (1 - B_{i-1})\tau_i \right]$$

$$+ B_i \tau_{i+1} \tag{3.76}$$

After some manipulation, new coefficients A_i' , B_i' , and C_i' are found to be

$$A_i' = \frac{1}{C_i'}\left[A_i + \frac{C_{i-1}}{C_i}\left(A_{i-1} + \right) \frac{\partial\tau}{\partial x}{}_i \, h_x \right] \tag{3.77}$$

$$B_i' = \frac{B_i}{C_i} \tag{3.78}$$

$$C_i' = 1 + \frac{C_{i-1}}{C_i}(1 - B_{i-1}) \tag{3.79}$$

The continuous "flow" of the elimination process is preserved with the computation of these coefficients. At this point, the new values of A_{i-1}' and B_{i-1}' may be set in accord with the requirements of Eq 3.71:

$$A_{i-1}' = -\left(\frac{\partial\tau}{\partial x} \right)_i h_x \tag{3.80}$$

$$B_{i-1}' = 1 \tag{3.81}$$

and both continuity and desired gradient are established.

Special Conditions for Large Suction Change

Practical experience with problems run on a computer have shown that it is possible to get answers that are obviously incorrect because of truncation. Truncation error is the amount by which the numerical answer fails to represent the exact answer. The large truncation errors have occurred in problems describing the sudden wetting of very dry soils where suction changes abruptly from a very low value to a very high value in a distance that is sometimes shorter than a "reasonable" increment length.

Three elements are involved in this truncation error:

(1) Permeability is highly (factor of 100 to 1000) dependent on suction.
(2) Suction gradients are large.
(3) The product of permeability and suction gradient is water velocity, which need not be very large.

Because diffusion of water is based on a gradient of water velocity it is necessary that water velocity be accurately determined. Where there are large changes of suction in a short distance it has become quite clear that unreasonable answers can result.

There are at least three ways to attempt to correct this situation:

(1) Use a smaller mesh size.
(2) Use a higher order difference equation to represent the gradient.
(3) Fit a polynomial through the points and get a gradient by differentiation.

The first method is always preferable because of its simplicity and should be used wherever possible. Variable increment lengths have aided in the solution of such problems.

All of the equations programmed for the CDC 6600 computer use the concepts stated in this chapter. In Chapters 6 and 7, example problems will be worked to demonstrate solutions to problems of concern to engineering in which the transient flow of water in unsaturated soils is an important factor.

CHAPTER 4. THE TWO-DIMENSIONAL COMPUTER PROGRAM

This chapter outlines the capabilities of the computer program developed for studying transient moisture movement through clay soils. It is the final program in a series which started with GRPIPE1 (GRid PIPE 1) and included CYLPIP1 (CYLindrical PIPe 1) before arriving at the present version which is GCHPIP1 (Grid-Cylindrical-Heavy Soil PIPe 1). The program is written in FORTRAN language for the Control Data Corporation 6600 computer at The University of Texas Computation Center. An austere version of FORTRAN has been maintained to permit easy conversion to other types of machines. A guide for data input is included as Appendix 2. As will be seen by referring to this appendix, there are nine tables of input data. Each of these tables will be explained in this chapter, in the order in which they appear.

Problem Identification Cards

These cards are included before the data for any table is read into the machine. The first card is in an alphanumeric format which allows 80 columns of run information. The second card includes five spaces for alphanumeric characters to be used as the problem number. The last 70 spaces on the card are for problem identification.

Table 1. Program Control Switches

The format for this card is seen in Appendix 2. In the first six spaces of five columns, the hold option for Tables 2 through 7 may be exercised by placing a 1 in the appropriate position. This keeps the data from the previous problem. The initial conditions put into the computer in Table 4 are not stored for recall. The data that is kept from the previous problem are the most recently calculated set of suction and water content values. As stated before, the keep options occupy the first thirty spaces on the control switch card.

The next six five-column-wide spaces specify the numbers of cards to be read in Tables 2 through 7. There is one exception: the number of cards in Table 4A is specified in that position reserved for Table 4.

In column 65, the switch KGRCL is set. This switch specifies whether the problem has rectangular or cylindrical coordinates. The number 1 specifies a rectangular grid, while a 2 tells the computer that the problem to be solved is in cylindrical coordinates.

In column 70, the switch KLH is specified. The number 1 in that column denotes a "light" soil. In this case, compressibility effects are neglected. If a 2 is inserted, Subroutine HEAVY is called, which permits consideration of the soil-suction change as a function of overburden pressure, soil compressibility, and porosity.

Table 2. Increment Lengths and Iteration Control

For the most part, this table is self-explanatory. (See the Input Format, Appendix 2.) The first card has space for the inside radius of a cylindrical problem to be specified. If KGRCL has been set at 1, however, this space may be left blank. Also, a closure tolerance is specified on this card. The closure tolerance is a relative one based on a fraction of the computed TY (FORTRAN for τY). That is, the error at each point must be within a specified fraction of the value of suction at that point. The closure signal printed at the successful conclusion of computations on a particular time step signifies one of two things:

(1) Actual closure has been achieved at each point of a region.

(2) The number of iterations allowed for each time step has been completed.

A glance at the monitor data will indicate which has occurred. If condition (2) occurs, then an explicit forward-difference estimation of the new T at each point not closed is made. This estimation uses both the values of T for the previous time step and the most recently computed values of TX and TY. If many such closures occur, it may be desirable to shorten the time increment to assure stability of the estimation process.

The second card in Table 2 requires a list of four monitor stations to be specified. The values of TX and TY at these points for each iteration will be printed out at each time step for which output is desired.

The third card in Table 2 permits some experimentation with the form of the equation which is being solved. If a 1 is set, the transient flow equation, Eq 3.5, is specified. If a 2 is inserted, the time derivative term is set to zero by making the 1's in Eqs 3.40, 3.42, 3.44, and 3.46 equal to zero. In most circumstances, the transient flow condition should be specified.

Table 3. Permeability

The tensor form of permeability has been programmed and provision has been made for using unsaturated permeability. A different set of principal permeabilities, directions, and coefficients for determining unsaturated permeability may be read in at each point of a soil region. There are three essential parts of the card which specifies permeability: (1) the specified rectangular region, (2) the two principal permeabilities and their directions, and (3) the coefficients for determining unsaturated permeability. Each of these will be discussed separately.

Specified Rectangular Region. The first four spaces give the corner coordinates of the region within which the permeability data applies. The first two numbers specify the smallest x and y-coordinates and the next two specify the largest x and y-coordinates. Permeability is a property of a pipe increment between mesh points. Because of this, permeability should be specified for all pipe increments that extend one increment beyond each boundary point. Thus, if a region extends from coordinates (0, 0) to coordinates (10, 10) , the permeabilities should be specified for pipe increments (0, 0) to (11, 11) . This is in accord with the stationing system illustrated in Figs 2 and 3 in Chapter 2.

Principal Permeabilities and Their Directions. The principal permeabilities are given in the next three spaces in order: P1 , P2 , and ALFA . The quantity P1 is the principal permeability nearest the x-direction and ALFA is the angle in degrees from P1 to the x-direction with counterclockwise angles positive. The quantity P2 is the principal permeability at right angles to P1 . The permeabilities specified should be the saturated permeabilities. They will be corrected downward by the three unsaturated coefficients found in the next part of the card if the water content of the soil drops below what has been termed in Research Report 118-1 as "final saturation."

Unsaturated Permeability Coefficients. The form of unsaturated permeability recommended by W. R. Gardner (Ref 9) has been programmed. This is of the form:

$$k_{unsat} = \frac{k_{sat}}{\frac{\tau^n}{b} + 1} \qquad (4.1)$$

Since much of the published data on unsaturated permeability are in the units of centimeters, a conversion factor may be included which transforms the inches of suction used in this program to the centimeters from which the constants b and n are derived. The expression programmed is

$$k_{unsat} = \frac{k_{sat}}{\frac{(a\tau)^n}{b} + 1} \qquad (4.2)$$

where a is normally equal to 2.54 cm/in.

One note of caution is required before leaving this section. The data read in at each point are added algebraically to the data already stored at that point. At the start of a problem all data at each point are set to zero. Either positive or negative values of permeability, angle, or unsaturated permeability may be read in at each point; but the computer will use the algebraic sum of all data furnished it for each point.

Table 4. Suction-Water Content Curves

Table 4 data consist of two parts: the first part is concerned with specifying numbered single-valued suction-volumetric-water-content (pF-θ) relations and other pertinent soils data; the second part establishes the rectangular regions within which each numbered pF-θ curve applies. No hysteresis effects are considered in these relations. This is not a serious limitation, however, because the pF-θ relation specified for a point may be an approximation of a scanning curve. The greatest difficulty introduced by this limitation occurs when the trend of moisture change is reversed, and a new pF-θ curve must be followed. This can be handled by stopping one problem, holding all previous data, and changing the appropriate pF-θ curves to represent the new scanning curve. B. G. Richards notes (Ref 18) that in many cases, changes of moisture content are in one direction over a long period of time and thus the hysteresis effect may be neglected. Young's (Ref 25) discussion of the infiltration problem gives an important exception to this rule. Scanning curves may be estimated from experimental data in the manner demonstrated in Research Report 118-1.

Input Soils Data. Certain soils data must be included on each card in Table 4. The computer assigns a number to each card in the order in which the cards are read. The data on each card include the following:

(1) number of separate rectangular regions to which the following data apply, LOC ,

(2) maximum pF , PFM ,

(3) pF at the inflection point, PFM - PFR ,

(4) exponent for pF-curve, BETA ,

(5) air entry gravimetric water content, WVA ,

(6) exponent for the water pressure - total pressure relation, Q . The shape of this curve could be assumed to be the same as that of the shrinkage curve,

(7) the slope of the water pressure - total pressure curve at zero water content, ALFO . It is probably safe to assume that this value will always be zero.

(8) porosity at air entry point, PN ,

(9) slope of the void ratio-log pressure (e-log p) curve AV ,

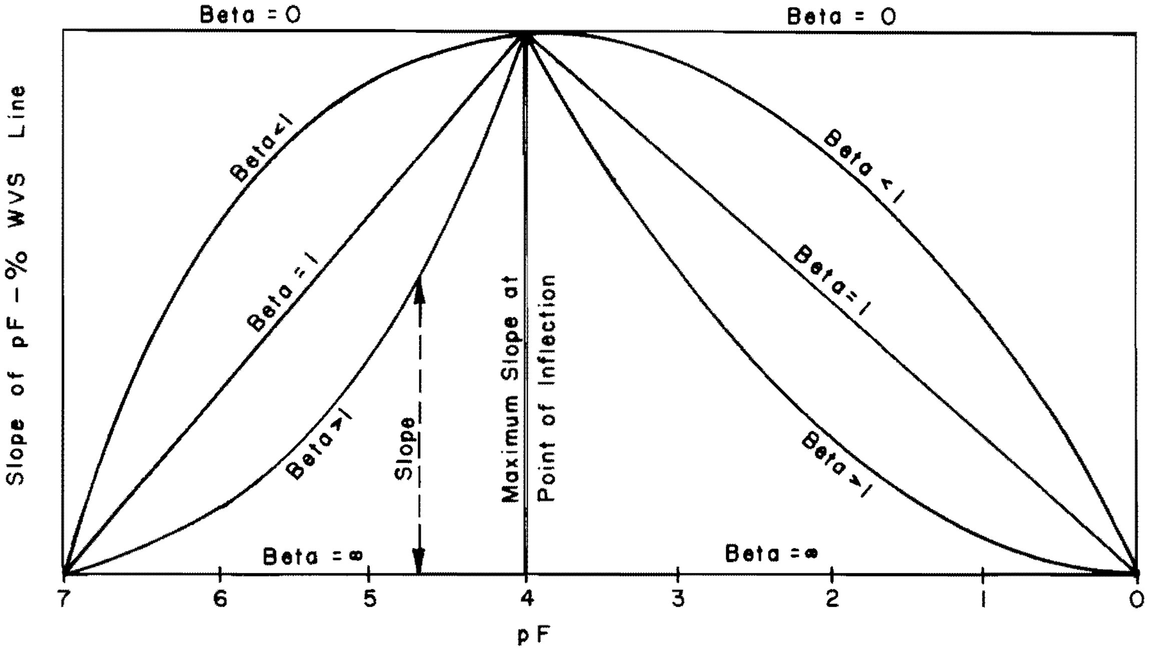
(10) saturation exponent relating the degree of saturation to the factor χ_E , which is assumed (perhaps erroneously in some cases) to range between zero and one, R ,

(11) the soil unit weight in pounds per cubic inch, GAM , and

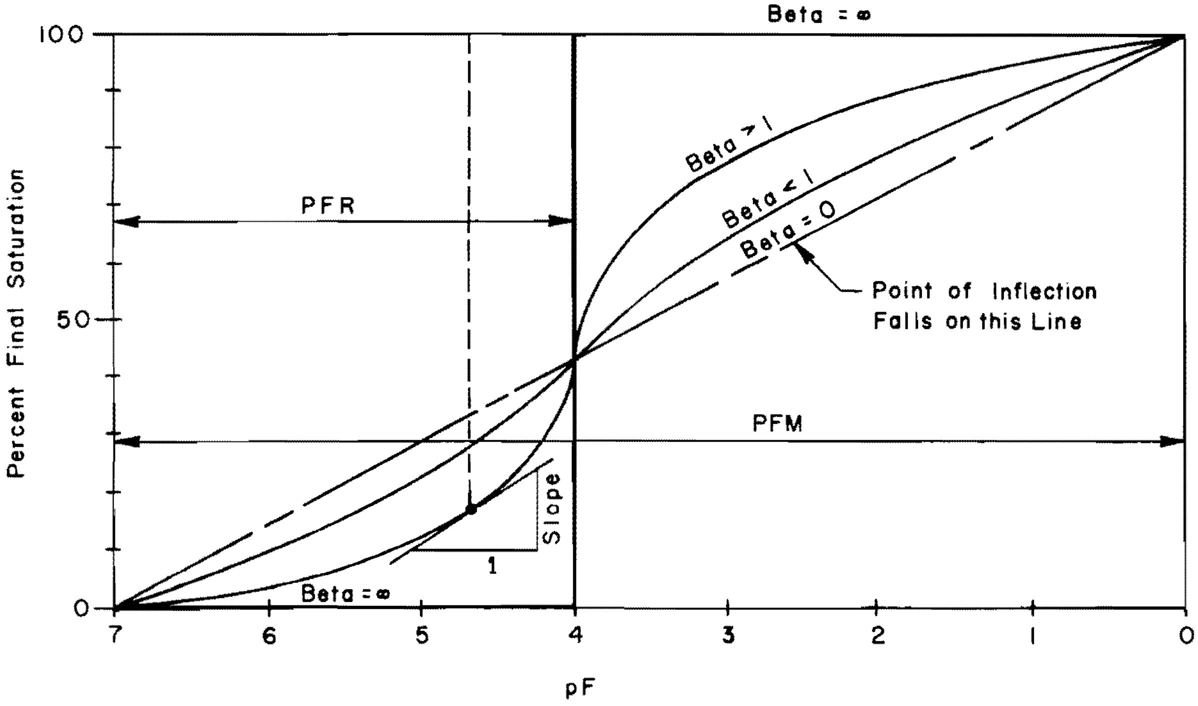
(12) the gravimetric water content at final (or suction-free) saturation, WVS .

If the overburden pressure and compressibility of the soil are not to be considered, i.e., if the switch KLH has been set to 1, then only items 1, 2, 3, 4, and 12 need to be read in. The form of the assumed relations among these soil variables is discussed below.

The PF - Water-Content Relation. The assumed form of the pF-θ relations is an exponential curve, the slope of which is the ordinate of a pF-slope curve. The cumulative area under the pF-slope curve is the percent of final saturation. Both curves are needed to explain the assumed pF-θ relations. The pF-slope curve is shown in Fig 12(a) and the pF-% final saturation curve is



(a) The pF - slope relation.



(b) The pF - % of final saturation curve.

Fig 12. Suction-moisture relations.

shown in Fig 12(b). The pF-slope curve may be intuitively related to the pore-size distribution of the soil. The point of inflection of the pF-% final saturation curve rests on the line between 100 percent final saturation and maximum pF . Any inflection-point pF , maximum pF , and exponent BETA , may be specified to give the shape of pF-θ curve desired. The final saturation water content must be specified as well.

Subroutines SUCTION and DSUCT have been written to deal with these relations. SUCTION operates when a water content is known and a value for suction, as well as $\frac{\partial \tau}{\partial \theta}$, is desired. DSUCT is called upon when a suction is known and a water content and $\frac{\partial \tau}{\partial \theta}$ is desired.

The Water Pressure - Total Pressure Relation. This relation is discussed in some detail in Chapter 4 of Research Report 118-1. The quantity α_{po} is defined in that report as follows:

$$\alpha_{po} = \left(\frac{\partial u}{\partial p}\right) t = 0 \qquad (4.3)$$

where

u = excess pore water pressure,

p = total pressure,

t = time after the initial change of water pressure.

It is assumed that the α_{po} relation has approximately the same shape as the slope of the shrinkage curve which is given in Chapter 4 of Research Report 118-1. The equation which has been programmed to express this relation is of the form

$$\alpha_{po} = \alpha_{pod} + \left(1 - \alpha_{pod}\right) \left(\frac{WV}{WVA}\right)^{Q-1} \qquad (4.4)$$

where

α_{pod} = the slope of the water pressure-total pressure relation, at zero water content,

WV = water content,

WVA = air entry water content,

Q = an exponent drawn from the shape of the shrinkage curve. Differentiation of this curve produces a slope and the Q-1 exponent given in Eq 4.4

The value of α_{po} is assumed to be 1.0 at water contents above air entry. All computations involving the water pressure-total pressure relation are programmed in Subroutine HEAVY which is called only when switch KLH is set at 2.

The χ-Saturation Curve. This computation is made in Subroutine HEAVY which is called only when switch KLH is set at 2. The limitations on the relation between the unsaturated stress parameter χ_E and the degree of saturation S is discussed in Chapter 4 in Research Report 118-1. The assumed form of the relation is undoubtedly too simple to include all cases, but it is programmed as the exponential function given below:

$$\chi_E = S^R = \left(\frac{V_W}{100 \times POR}\right)^R = \left(\frac{\theta}{n}\right)^R \tag{4.5}$$

where

χ_E = the equilibrium unsaturated stress parameter,

θ = the volumetric water content, decimal,

V_W = the volumetric water content, percent,

n, POR = the porosity of the soil, decimal,

S = the degree of saturation, decimal.

This calculation is made only if the water content is less than air entry water content. Although it is slightly in error, the porosity is assumed to remain constant once the water content falls below the air entry point. Above the air entry water content, the porosity is assumed to have the form

$$POR = \left(\frac{PN + \Delta\theta}{1 + \Delta\theta}\right) \tag{4.6}$$

where

$$\Delta\theta = \frac{V_W - V_{WA}}{100} \tag{4.7}$$

PN = the porosity at air entry,

V_{WA} = the volumetric water content at air entry.

An appropriate value of the exponent R should be determined after consulting experimental results, but a value between 0.5 and 2.0 would cover many cases reported in the literature. In all of these computations, the soil unit weight and a solid specific gravity of 2.70 are used to convert gravimetric into volumetric water content.

The Compressibility Relation. The computations involving this relation are contained in Subroutine HEAVY. The basic relation used is Eq 4.16. Some other equations must be considered first. The plot of void ratio and the logarithm of pressure gives a straight line over a fairly wide range of pressures as long as soils are either preconsolidated or normally consolidated and not in an intermediate pressure range. The relation normally used is

$$e - e_o = -C_c \log_{10} \frac{p}{p_o} \tag{4.8}$$

where

e = void ratio,

p = pressure,

C_c = slope of the e-log p curve.

The derivative of this expression gives

$$\frac{de}{dp} = -\frac{0.435C_c}{p} \tag{4.9}$$

In Chapter 4 of Research Report 118-1 reference was made to Blight's compressibility coefficient c (Ref 2), as defined in the following equation:

$$\frac{\Delta V_T}{V_T} = c\Delta p \qquad (4.10)$$

If it is assumed that the change of total volume is equal to the change of void volume, the equation can be rewritten as

$$(1 - n)\Delta e = c\Delta p \qquad (4.11)$$

and thus

$$\frac{\Delta e}{\Delta p} = \frac{C}{1 - n} \qquad (4.12)$$

Equations 4.9 and 4.12 may be combined to give an expression for Blight's compressibility c in terms of the slope of the e-log p curve:

$$c = \frac{0.435C_c(n - 1)}{p} \qquad (4.13)$$

This relation and one more to be developed below will be included in the compressibility correction term for the slope of the pressure-free suction-moisture curve which was discussed in Chapter 4 of Research Report 118-1.

The second relation deals with the ratio of air volume V_A to water volume V_W.

$$\frac{V_A}{V_W} = \frac{V_V - V_W}{V_W} = \frac{\frac{V_V}{V_T} - \frac{V_W}{V_T}}{\frac{V_W}{V_T}} \qquad (4.14)$$

$$\frac{V_A}{V_W} = \frac{n - \theta}{\theta} \qquad (4.15)$$

where

n = the porosity,

θ = the volumetric water content.

Equations 4.13 and 4.15 are to be used subsequently. It is explained in detail in Chapter 4 of Research Report 118-1 that the rate of change of suction with respect to water content varies with the compressibility of the soil. This was expressed by the following relation

$$\frac{\partial\tau}{\partial\theta} = \left(\frac{\partial\tau}{\partial\theta}\right)_o + \left(\frac{\partial\tau}{\partial\theta}\right)_p \tag{4.16}$$

where the o subscript stands for the pressure-free relation and the p subscript denotes the contribution of the compressibility of the soil. This latter term uses Eq 4.15 and is expressed in the following fashion for saturated soil:

$$\left(\frac{\partial\tau}{\partial\theta}\right)_p = -\frac{1}{c(1-\theta)\chi_E} \cdot \frac{1}{\gamma_W} \tag{4.17}$$

where

χ_E = the equilibrium effective stress factor,

γ_W = the unit weight of water: independent of pressure if soil is saturated,

In the effectively unsaturated case,

$$\frac{\partial\tau}{\partial\theta}_p = -\frac{1}{c(1-\theta)\chi_E} \quad 1 + F\,(\alpha_{FS} - 1) \quad \frac{1}{\gamma_W} \tag{4.18}$$

and

$$\frac{1}{\gamma_W} = \frac{1}{p_o}\,\frac{RT_e}{mg}\,e^{-\frac{\tau mg}{RT_e}} \tag{4.19}$$

where

p_o = saturated water vapor pressure,

R = universal gas constant,

T_e = absolute temperature,

m = gram-molecular weight of water vapor,

g = acceleration due to gravity,

α_{FS} = ratio of total volume to water volume change,

τ = suction,

F = a factor which includes air compressibility and solubility.

For the purpose of Subroutine HEAVY the F-factor is considered to be zero. It is not judged to cause serious error but this judgment is not based on quantitative results.

The form of the compressibility correction term as used in Subroutine HEAVY uses Eq 4.13 and may be expressed as

$$\left(\frac{\partial\tau}{\partial\theta}\right)_p = + \frac{p}{.435C_c(1 - n)(1 - \theta)\chi_E} \cdot \frac{1}{\gamma_W} \qquad (4.20)$$

This equation is used to adjust the value of $\frac{\partial\tau}{\partial\theta}$ computed from the pF-ϕ water-content curves. The value of p is taken as the total overburden pressure and is computed from the value of GAM read into the computer. It must be noted carefully that this equation neglects the effect of air compressibility, an exclusion which may be seriously in error in less saturated soils.

Location of Soils Data. The cards in Table 4 representing the different types of soils present in a soil region specify the number of rectangular regions occupied by the soil of each type. The soils data cards must then be followed by exactly the same number of cards as the total number of rectangular regions occupied by the different types of soils. These cards give the smallest x and y-coordinate and the largest x and y-coordinate of each region and specify the curve number which applies there.

As an example, assume that two soils are present in a soil region. One occupies two locations and the other occupies one. The total number of curve location cards should be three.

Table 5. Initial Conditions

Each card put into the computer has a rectangular distribution scheme for either of two cases: water content (Case 1) or suction (Case 2). The value at the upper right-hand corner of the specified rectangular region is given along with the x and y-slopes of these quantities. If the value in the upper right-hand corner is smaller than any other in the region, both slopes should be positive. If no slopes are read in, the machine will assume them to be zero and distribute the same value of either water content or suction over the entire region.

The values input in this manner are added algebraically to the values already stored at each point. To avoid any complications, when a new problem is read in, all initial values of water content and suction are set at zero. Any subsequent additions will start from that datum.

Initial conditions are replaced in the computer memory with new values at each time step. For this reason, the exercise of the hold option for Table 5 means simply that the most recently computed values of suction and moisture content will be retained. A new set of initial conditions must be input if a new start is required.

Table 6. Boundary and Internal Conditions

Five cases are permitted as boundary and internal conditions:

(1) gravimetric water content,
(2) suction,
(3) suction gradient in the x-direction,
(4) suction gradient in the y-direction, and
(5) temperature and humidity of soil water.

A rectangular distribution scheme is provided which distributes the specified quantity uniformly over the region outlined by its smallest and largest x and y-coordinates and adds algebraically to values already stored at each point in the region. Cases 1, 2, and 5 result in computation of a value of suction and a final setting of the switch KAS(I,J) to 2. Boundary and internal conditions are computed differently based on the value of the switch KAS(I,J) which is set for each point. The values of this switch recognized by the computer are given below:

KAS(I,J) = 1 , a regular point at which no value of suction or gradient is set,

= 2 , suction set,

= 3 , x-gradient set,

= 4 , y-gradient set.

A discussion of these conditions and the way they are computed is given in Chapters 2 and 3. The method of converting each of the five input conditions is discussed in the succeeding paragraphs.

Volumetric-Water-Content Set. When this quantity is specified, Subroutine SUCTION is called. It converts water content to suction according to the pF-ϕ water-content relations read in as Table 4. Values of pF and $\frac{\partial \tau}{\partial \theta}$ are also computed. Water content may be set at any point of a region.

Suction Set. The setting of this quantity requires that Subroutine DSUCT be called to compute volumetric water content, pF , and $\frac{\partial \tau}{\partial \theta}$ from the appropriate input soils data. Suction may be set at any point of a region.

x-Suction Gradient Set. The x-gradient must not be set at any point on the upper or lower boundary of the soil region. When a suction gradient is set on the right or left boundary (excluding the corner points), a line starting at the value of suction one station inside the boundary is projected outward to the boundary along the set gradient to establish a value of suction at the boundary point. Then Subroutine DSUCT is called to provide its information on water content, pF , and $\frac{\partial \tau}{\partial \theta}$. An x-gradient may be set at any interior pipe increment.

y-Suction Gradient Set. The y-gradient may be set at any point along the upper and lower boundaries of the region including the corner. The same projection scheme is used as was explained above and Subroutine DSUCT is called into operation. A y-gradient may be set along any interior pipe increment.

Temperature and Soil-Water Humidity Set. This option may be used at any point where these data are known. The option was intended for use primarily along the upper boundary where infiltration and evaporation rates may be used to establish a soil moisture humidity, but the condition is valid at any point of the region. Subroutine HUMIDY is used to compute suction according to the relative humidity formula presented in Chapter 3.

Units of suction in this program are inches, water content is in percent, angles in degrees, permeability in inches per second, time in seconds, and increment lengths in inches. Ordinary pF-ϕ water-content curves should be furnished, however, since there is a programmed internal conversion from centimeters to inches for computed suction values.

Table 7. Closure Acceleration Data

A different number of closure valve settings for the x and the y-directions may be read into the computer. The number of each is specified on the first card of Table 7.

The cards immediately following list the x-closure value settings and the cards after that list the y-closure value settings. A maximum of 10 of each may be used.

Table 8A. Time Steps for Boundary Condition Change

The options are permitted based on the value of KEY which is input on the first card of Table 8A. The values of KEY and their meanings are given below:

KEY = 1 , discontinuous boundary condition change (Read in a list of time steps for boundary condition changes.),

= 2 , continuous boundary condition change (A new boundary condition must be read in at each time step.),

= 3 , no boundary condition change.

If KEY is set at 1, then the same card should specify the number of time steps at which boundary conditions will change. This first card should then be followed by cards listing the time steps at which boundary conditions will change. The maximum number of time steps at which boundary conditions change should not be greater than the number of time steps for the problem nor greater than the dimensioned storage of KLOC , the array which tells the program whether to read a new set of boundary conditions.

Table 8B. Time Steps for Output

This table is included to save the amount of output that is produced by the computer. The first card of Table 8B specifies a value of KEYB . Values of KEYB and their explanations are given below:

KEYB = 1 , discontinuous output (Read in a list of time steps at which output is desired.),

= 2 , continuous output.

If KEYB is 1, then the same card should specify the number of time steps for output. Additional cards listing these time steps should follow.

If KEYB is 2, no other cards should be added. The maximum number of time steps for output should not exceed the maximum number of time steps for the problem or the dimensioned storage of array KPUT .

Table 9. Subsequent Boundary Conditions

This table is used only if KEY from Table 8A is set at 1 or 2. At the beginning of the specified time step, at least two cards are read in: (1) the time-step identifier and (2) the boundary-condition cards.

Time-Step Identifier. This card has two entries: (1) the time step and (2) the number of cards to be input at this time step.

Boundary-Condition Cards. These cards follow the same format as those used in Table 6. The same subroutines are called and all other explanations for Table 6 data apply to the data to be read in as Table 9.

This completes the outline of input procedures. All data that is put into the machine is echo printed by the computer to afford a check on the information actually being used in the computer.

Output

Output before each time step includes the station, suction, water content, $\frac{\partial\tau}{\partial\theta}$, and the elements of the unsaturated permeability tensor P11 , P12 , and P22 at each point of the region.

Output after each time step includes the station, suction, water content, pF , and closure value settings.

A guide for data input is included as Appendix 2. It should be consulted when preparing data because it gives the formats in which data is furnished to the computer.

CHAPTER 5. THE ONE-DIMENSIONAL COMPUTER PROGRAM

This chapter describes the differences between capabilities of the two-dimensional Computer Program GCHPIP1 and the one-dimensional Computer Program FLOPIP2. The latter was developed from the two-dimensional program by (1) extracting two important features (computation of suction change in the y-direction and alternating-direction-implicit iteration procedure at each time step), (2) by changing another important feature, namely that the doubly-dimensioned arrays are changed to single-dimensions, and (3) by adding one important feature, the switch to allow the choice of vertical or horizontal flow problems.

Familiarity with the contents of Chapter 4 is essential to an understanding of the discussion to be presented in this chapter. Input format will be discussed in the same order as in the previous chapter and only the differences will be noted. The entire input format may be reviewed in Appendix 7.

Problem Identification Cards

Three cards are used for problem and run identification: the first two of these have 80 columns of alphanumeric run information and the third has five spaces for the problem number and 70 spaces for problem identification. Only two cards are used in Computer Program GCHPIP1.

Table 1. Program Control Switches

Only six table switches are provided for input. Table 7 in GCHPIP1 is not included in FLOPIP2. One additional switch is provided, KVERT . This switch allows the choice between vertical flow (KVERT = 1) and horizontal flow (KVERT = 2). The initial conditions read into the computer in Table 4 are not kept. The most recently computed values of suction and moisture content are retained if the keep switch for Table 4 is set to 1.

Table 2. Increment Lengths

This table is substantially different from Table 2 in Computer Program GCHPIP1. Tables 2B and 2C have been eliminated entirely and Table 2A has been changed to include a smaller amount of input information. The only information input in the FLOPIP2 Table 2 includes the number of increments and time steps, the size of each, and the inside radius if a horizontal cylindrical flow problem is being worked.

Table 3. Permeability

The one-dimensional problem permits a change of saturated permeability in several different regions along the length being considered. No direction of principal permeability is considered in this program. The constants a , b , and n have the same meaning as in Computer Program GCHPIP1.

Table 4. Suction-Water Content Curves

The information on Table 4 given in Chapter 4 is identical for FLOPIP2 with one exception. Table 4B specifies the linear location of the places where each of the pF-water content curves apply.

Table 5. Initial Conditions

Several changes have been made in Table 5. Each card input in Table 5 has a linear distribution scheme for either of two cases: gravimetric water content (Case 1) or suction (Case 2). If the value at the right-hand (or up-station) side of the distribution is smaller than any other, then the slope specified should be positive. If no slope is read in, the machine will assume a zero slope and distribute the same value over the entire linear region.

All input values are added algebraically to those already stored at each point. New problems start with zero suction and water content values at each point along the line.

Table 6. Boundary and Internal Conditions

Boundary and internal conditions that may be specified are as follows:

(1) gravimetric water content,

(2) suction,

(3) suction gradient, and
(4) temperature and humidity of soil water.

The specified quantity is distributed uniformly over the linear region determined by the smallest and largest increment numbers.

In this program, a specified boundary or internal condition replaces any previously stored value. Otherwise the discussion of Table 6 in Chapter 4 is applicable.

Tables 7, 8, and 9 for FLOPIP2

The explanation of Tables 8A, 8B, and 9 given in Chapter 4 is identical for Computer Program FLOPIP2. There is no Table 7 for this computer program because its contents are applicable only to two-dimensional problems.

Output

Output before each time step includes the station, suction, water content, $\frac{\partial\tau}{\partial\theta}$, and the unsaturated permeability at each point along the line being considered.

CHAPTER 6. EXAMPLE PROBLEMS: ONE-DIMENSIONAL

The study in this chapter was undertaken to examine the validity of moisture distribution Computer Program FLOPIP1 by comparing the predicted pattern of moisture distribution with that measured in actual field experiments. The study is also aimed at fixing some reasonable properties and constants for the soils at the site of such field experiments so as to use them in the swell programs described in Research Report 118-4 to predict the swell potentials of _in situ_ soils with time. Since this study was made, minor modifications have been made in the program, and except in this chapter, where the results were obtained with FLOPIP1, the program reported is FLOPIP2.

Such studies cannot be made by laboratory observations on sampled and remolded soils because the distrubed structure of the soil in such samples has a profound effect on their behavior. Because an analysis needs to be made on an _in situ_ basis, a small project carried out at about 5 miles from Laramie north of Interstate 80 and reported in a Highway Engineering Research Publication H-18 of the University of Wyoming (Ref 13) was selected for this study.

The surface formation in the area overlies a small portion of the geological formation of Steele shale, which is a source of expansive clays. The 2-1/2 feet of overburden material at the test site area, a 40-foot square with the surface sloping at approximately 7 percent, was stripped to expose the clay shale. Within the cleared area moisture-density access tubes were installed in the pattern as shown in Fig 13. The site, including the side slopes meeting the normal ground level, was covered with two layers of 6 mil polyethylene sheeting to prevent intrusion and loss of water. Holes in the sheeting, made to facilitate the tubes, were sealed against moisture. Approximately 3 inches of soil was placed and compacted over the sheeting to secure it. The water supply and injection system was built as shown in the figure. From the 1-inch plastic pipes serving as feeder lines, water injection lines were inserted vertically into the ground for a depth of approximately 8 inches below the membrane. To facilitate the flow, penetration holes were drilled to depths 2 inches below the ends of the tubing and filled with sand. The membrane was sealed around the tubing.

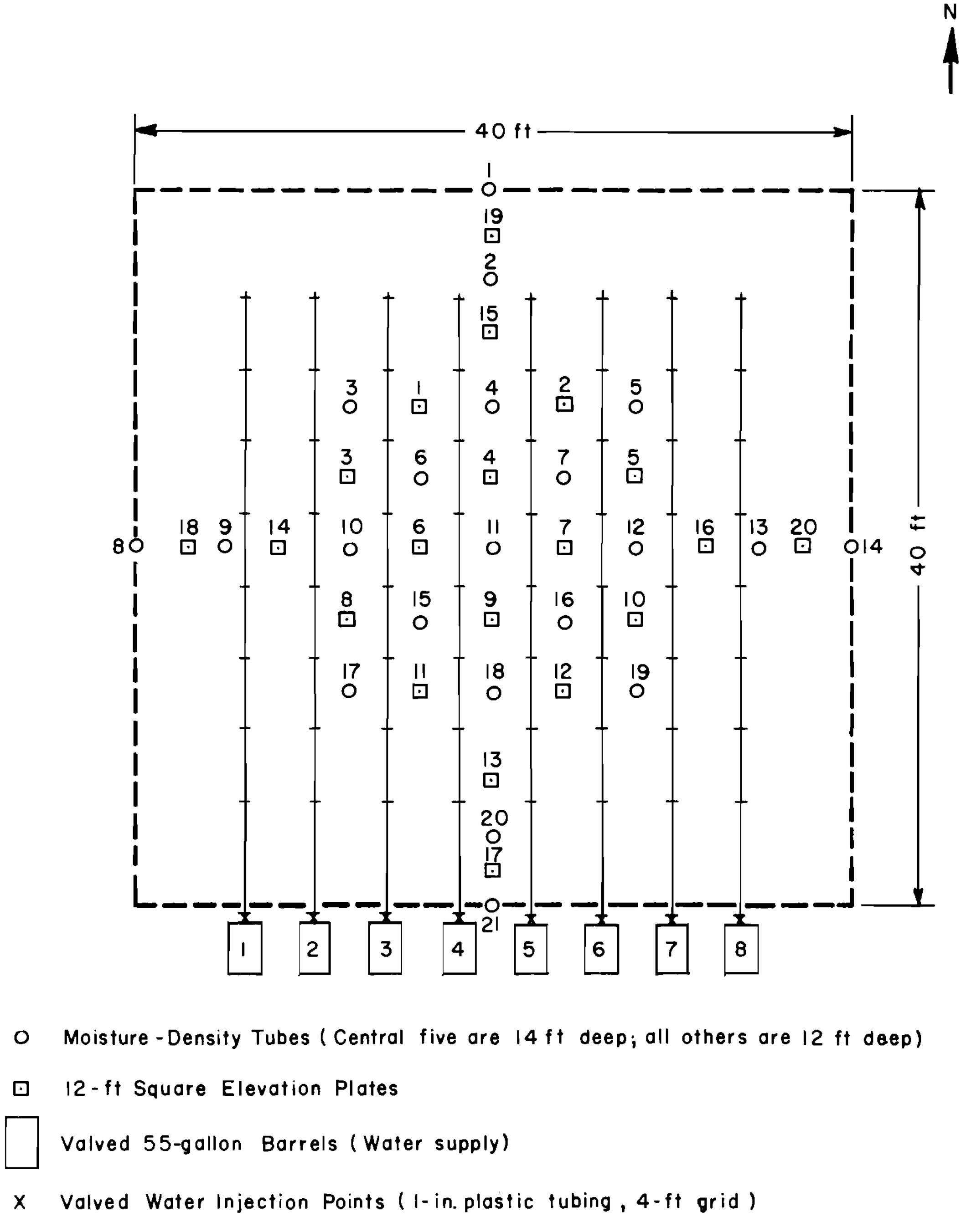


Fig 13. Experimental field site for _in situ_ study of swell of expansive clay.

For vertical movements of the surface elevations, control points were placed at 20 points as shown in the figure. These control points consisted of 2-inch vertical stems welded to the center of a 12 by 12 by 1/4-inch metal plate and held firmly by four corner spurs penetrating a few inches into the compacted soil but not through the membrane. Plates were covered with an inch of compacted soil. The stems served as the elevation control points. Elevations were measured with respect to the permanent bench marks set for the purpose, the relative elevations of which remained quite consistent throughout the study.

Operation of the experiment began in September 1966 when two partial sets and one complete set of data were obtained to establish the initial values of moisture and density. The site was closed for the winter months after the application of small volumes of water. Actual data collection was done for the period from April 27 through July 17, 1967, for a total of 80 days excluding the first four of the last date. At the start of the experiment in April, a partial set of moisture readings agreed closely with the sets taken the previous September and therefore those sets were established to be the initial values of moisture and density. Initial elevations were obtained on April 27, 1967, and the subsequent sets for moisture, density, and elevations were taken on May 19, June 19 and 20, and July 16 and 17.

Computer Simulation of the Problem

The problem was simulated in the computer with the help of the one-dimensional moisture distribution program. The program uses some of the unconventional properties of soil which are not commonly found in the laboratory. It also uses some of the common engineering properties as well as the initial conditions, boundary conditions, and subsequent changes in those conditions.

Moisture data for this experiment were taken to a depth of 13.2 feet in the field. Tube No. 11, which is in the center of the test site, was picked for the first computer study. The initial moisture values were plotted as shown in Fig 15. A depth of 13.5 feet was divided into 27 equal parts of 6 inches each with stations numbering zero at the bottom and 27 at the top surface. The moisture values at each of these stations points were measured from the plot in Fig 15 and taken as the initial values of moisture at these points for the computer solution.

General Computer Input. The test period was divided into time increments of 8 days each so that the tenth time increment fell at approximately the time when the final readings were observed. Soil was assumed to be a "light" soil and therefore the compressibility effects are neglected. In other words, it is assumed that changes in soil suction do not vary as a function of overburden pressure, soil compressibility, and porosity. It is shown in the ponding problems of Chapter 7 that such effects, if neglected, do not cause an appreciable change in the total moisture variation in the region. Therefore, the modification of soil suction due to overburden pressure can safely be neglected.

Boundary conditions were fixed by the condition which prevailed at the top and the bottom at the time of the field test. There was practically no change in moisture content at the depth of 13.5 feet. The surface was assumed to be kept completely saturated. There was no subsequent change in these boundary conditions with time.

Soil Properties. Some of the needed engineering properties of the soil at the site were measured by University of Wyoming project personnel; all other soils data were assumed.

Because the effects of overburden were neglected, only the following were the soil parameters selected for use in the one-dimensional example problems:

(1) saturation permeability PB ,
(2) unsaturated permeability constant BK1 ,
(3) unsaturated permeability exponent EN1 ,
(4) maximum pF , PFM ,
(5) pF at inflection pF1 ,
(6) pF-moisture content exponent BETA ,
(7) saturation water content WH , and
(8) constant AK1 with a value of 2.54 cms/in.

The values for these constants generally reported in literature or determined in this project are as follows:

(1) for soil permeability:

$$\text{BK1} = 1 \times 10^{6} - 1 \times 10^{14}$$

$$\text{EN1} = 2.0 - 4.0$$

(2) for soil suction:

pFM = 7.0

pF1 = 3.0 - 5.0

BETA = 1.0 - 4.0

For the problem being studied, these soil parameters, which at present are not firmly related to any of the common engineering properties of the soils, were established by solving a large number of problems using different values of these constants in different combinations with each other.

In the first phase of study, saturated soil permeability was used over the entire 13.5-foot depth and different soil parameters were tried in an attempt to match the values of moisture in the top 3 feet of soil. This gave the preliminary values of these constants for the more exact analysis which followed later on. A final moisture curve derived using saturated permeability is plotted in Fig 15 as curve No. 3.

The soil permeability was then allowed to change as a function of suction and numerous computer runs were made to establish the exact moisture distribution pattern as observed in the field. The predicted moisture distribution with the best fit is given in Fig 15 as curve No. 4. Predicted and observed values are given in Table 2. The accuracy of the solution is apparent from this table. The following values seem to be the best for the soil at the site of the experiment:

(1) saturation permeability PB = 1.050×10^{-6} ,
(2) BK1 = 1×10^{9} ,
(3) exponent EN1 = 3.0 ,
(4) maximum pF = 6.5 ,
(5) pF at inflection = 3.0 ,
(6) BETA exponent = 3.0 , and
(7) saturation moisture content = 40 percent .

Some of these values are plotted in Fig 14. Curve AOB is the pF-moisture content curve obtained by using data items 4, 5, 6, and 7. Curve AOB_f is a hypothetical curve typical of soils which are completely

TABLE 2. COMPUTED OBSERVED VALUES OF MOISTURE AT THE END OF THE TEST PERIOD FOR TOP 7 FEET OF SOIL (Tube No. 11, Central Tube).

Depth ft	Moisture Content, %	
	Observed	Computed
0	40.00	40.00
0.5	30.50	30.54
1.0	25.44	25.42
1.5	21.45	21.18
2.0	17.70	17.49
2.5	14.20	14.56
3.0	13.10	13.57
3.5	13.70	13.62
4.0	13.90	13.74
4.5	13.85	13.77
5.0	13.80	13.72
5.5	13.75	13.62
6.0	13.70	13.50
6.5	13.40	13.39
7.0	13.30	13.38

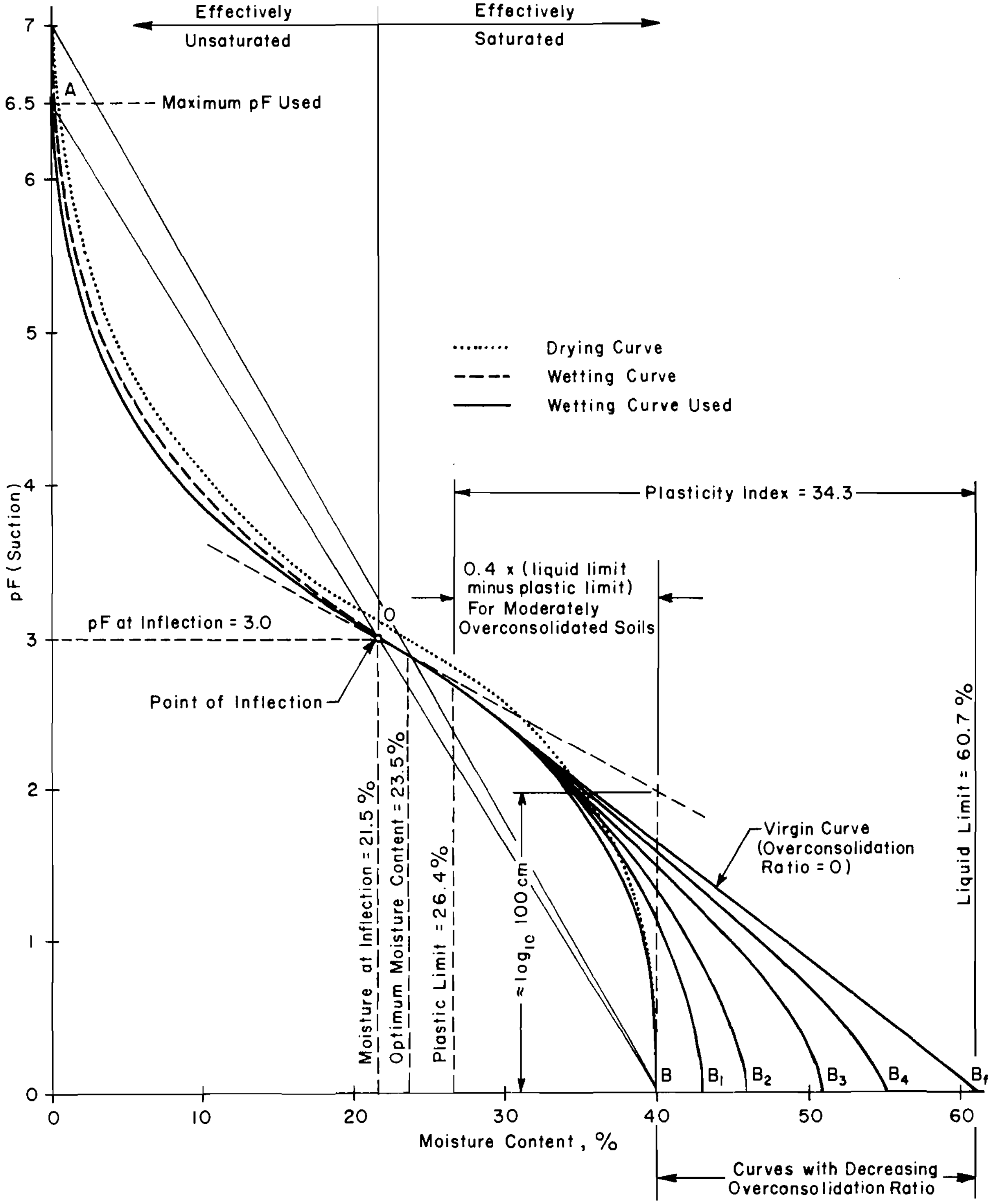


Fig 14. Suction moisture relationship.

disturbed. Curve branches OB_1, OB_2, etc. are curves for soils with higher overconsolidation or desiccation in the soil loading history. A final saturation moisture content of 40 percent proved to give best values in the computer studies of the moisture migration problem of this chapter. Figure 14 shows the difference between the wetting and drying curves and the hysteresis area that is included between them. The study of this chapter uses the wetting curve.

Several rough guidelines may be used in choosing appropriate values for the pF-water-content curve. Some of these guidelines are given below.

Final Saturation Water Content. This value will lie between the plastic limit and the liquid limit based on the drying and loading history of the soil. Soil with high antecedent drying conditions or high overconsolidation ratios will have a final saturation water content nearer the plastic limit. The 40 percent value found in this study can be calculated as the plastic limit plus 0.4 times the plasticity index.

Inflection Point Water Content. Although it is not always a reliable rule-of-thumb, the inflection point moisture content may fall close to the optimum moisture content reached with a relatively high compactive effort. The best inflection point moisture content for this study was 21.5 percent coinciding closely with the 23.5 percent optimum moisture content reported by the Wyoming project. Inflection point water content may be identical with the air entry water content from a shrinkage test.

In the pF-moisture curve used in this report, the point of inflection always falls on a straight line between the final saturation water content and the maximum pF. Thus, if either the pF or the water content at inflection are determined, then the other can be found by a simple proportion calculation.

Maximum pF. This value, as it is used in the computer, may be chosen by trial and error. It may or may not have any relation to the maximum measurable pF. It is chosen so that the pF-water-content curve fits very closely that of the actual soil in the moisture ranges being considered in a particular problem. Thus, if soil is very dry, a more accurate value of maximum pF will have to be assumed than if the soil is rather wet. The best value found in the computer studies of this chapter was 6.5.

Inflection Point pF. Experimental data reported by Croney, Coleman, and Black (Ref 5) and others place the inflection pF for the wetting curve between about 1.8 for a fine sand to about 3.4 for a heavy clay soil. A value of 3.0 was used throughout the studies of this chapter. This corresponds to a suction of -1000 cm or about an atmosphere of negative pressure.

Results of the Computer Study

Figure 15 shows the final results plotted with depth for Tube No. 11. Curve No. 3 gives the values of moisture which best fit the data with permeability kept as constant, and Curve No. 4 is the one with permeability as a function of depth. Curve No. 4 is in a very close approximation to the field observed data. It may be pointed out that the combination of different constants with a variable permeability follows the shape of the observed curve very closely whereas Curve No. 3 is a smooth curve which does not follow the observed curve well. This illustrates, in addition to the saturated permeability, the importance of the constants used in describing unsaturated permeability and suction-moisture relations.

The values of the constants established for the central tube were then used for analyzing the moisture patterns at the other tubes in the central area where boundary conditions on all the sides can fairly be assumed to be the same, e.g., Tube Nos. 5, 6, 15, and 16. Computer output at the Tube No. 16 is plotted in Fig 16. The solution again shows a striking agreement with the field observations.

It may be pointed out that the small deviations of the two curves can justifiably be attributed to the heterogeneous nature of any *in situ* soil, errors in field measurements, and the disturbance to the soil structure during the experimentation and observations.

Some Observations on Results of the Field Experiment

The following comments are excerpts from the Highway Engineering Research Publication No. H-18 from the University of Wyoming. They are listed here to emphasize the major findings of the field measurements.

(1) There were considerable changes in subsurface moisture and surface elevations.

(2) Having considered the possible losses due to leakage for some initial period of experiment, it can be stated that the site most likely did absorb water at a greater rate during the initial weeks of the study.

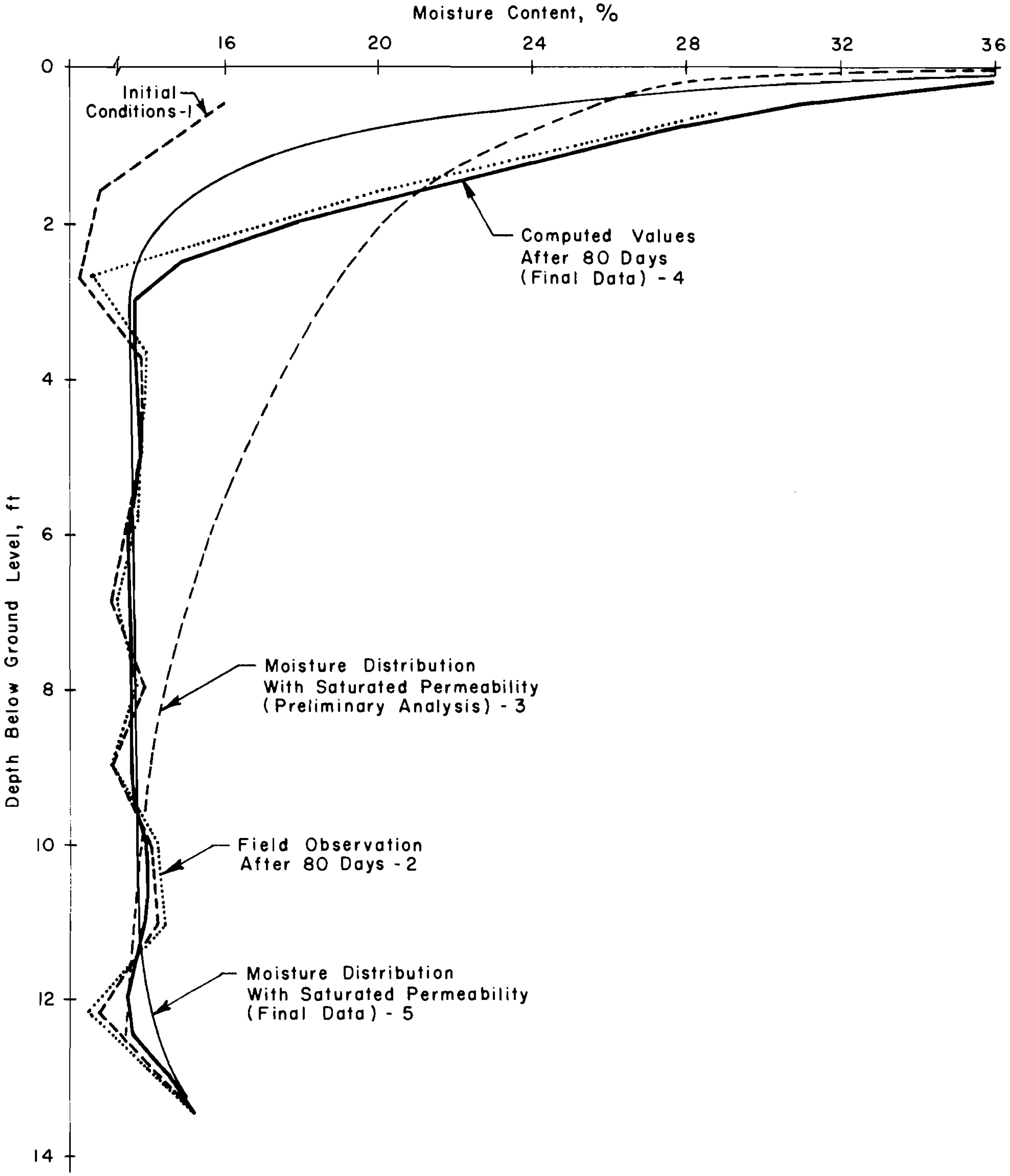


Fig 15. Moisture distribution study at the
Tube No. 11, central tube.

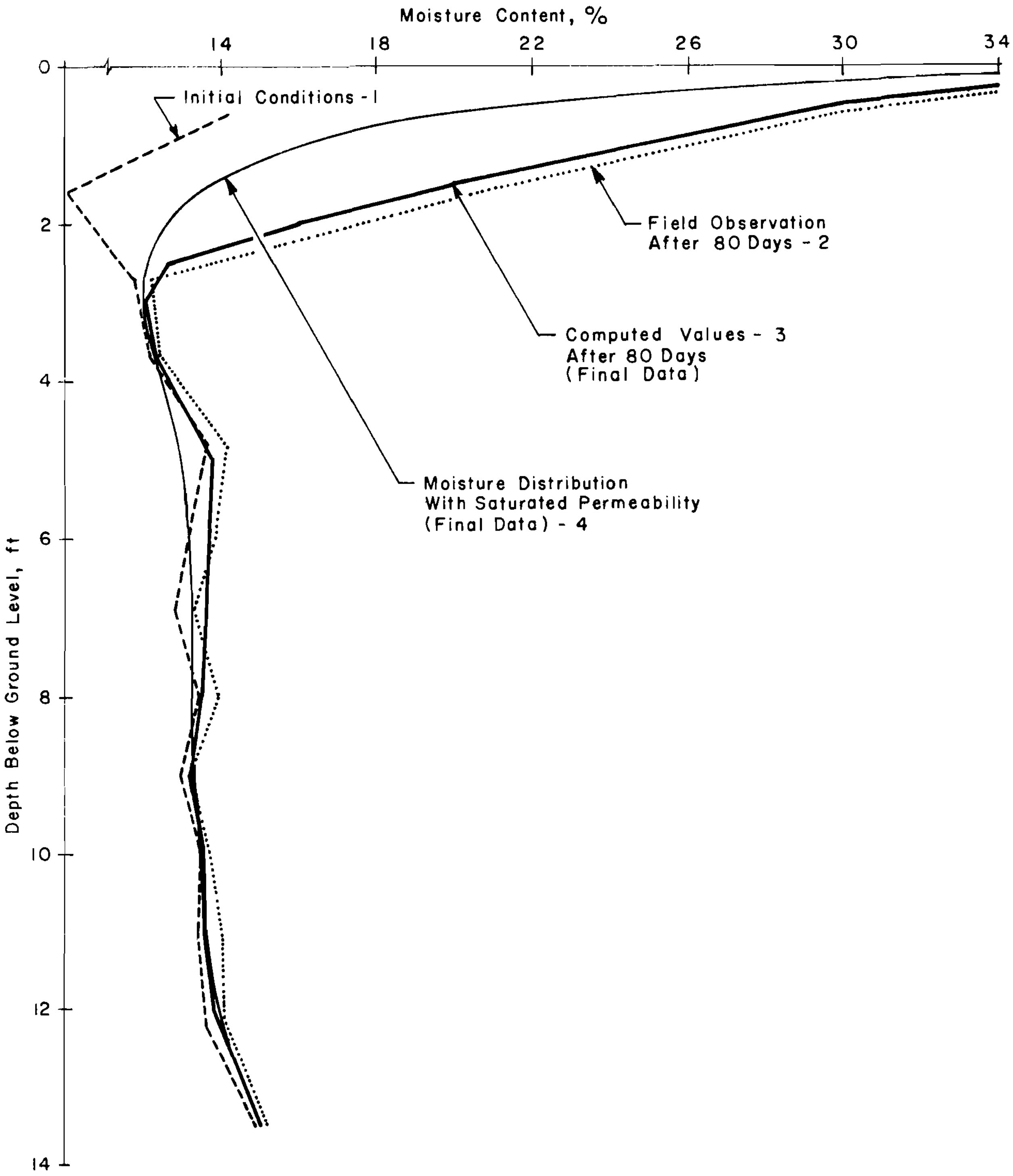


Fig 16. Moisture distribution study at the Tube No. 16.

(3) The site showed an appreciable gain in moisture 6 inches below the membrane within the first 22 days. Values of about 11 increased to about 23 lbs/cu ft. Points at the perimeter of the test site showed lesser amounts of increase as expected.

(4) After about 52 days the readings revealed no further gain in moisture at the 6-inch level but sizable increases to the depths of 18 inches.

(5) At the end of the test the readings showed only slight further increases of water content at 18-inch level but appreciable gains were noted at 24 inches below the membrane. A few of the central readings showed considerable gain to 36 inches.

(6) An average elevation increase of 1.3 inches occurred in this period due to the moisture changes.

(7) Notable deviations from the average were observed especially at the points lying on the perimeter of the test site.

The above results can roughly be summarized as follows. The most striking conclusion of this experimental study is that the top 6 inches or so became quite wet after a short period of time and the increase of water content was less and less rapid as the depth increased. The site absorbed water at a greater rate during the initial weeks of the study. With the exceptions accounting for the soil necessarily being a heterogeneous material, indications are that a uniform swelling occurred over the entire wetted area.

Computed moisture distributions, when observed continuously with time, followed very closely the pattern as indicated by (3), (4), and (5) above. The top 6 inches were observed to be fairly wet after three time increments, i.e., 24 days. The rate of moisture intrusion was very small at greater depths. On the whole, the rate of water absorption throughout the depth was computed to be quite high in the initial periods as compared to the rest of the time.

Parameter Studies

Some useful computer studies were conducted at The University of Texas at Austin to observe the effects of changing various coefficients on the distribution of moisture. Such studies prove to be useful in understanding these important soil parameters which are not easily determined in the conventional engineering analysis of soils. Virtually no correlations exist to connect them to engineering properties, and their behavior cannot be found in detail in the presently available literature. The results of these studies are given below.

Saturated Permeability PB . As seen in Table 3 an increase in permeability, on the average, increased the moisture values throughout the region under consideration. It may be noted that there is relatively a very high increase in the moisture contents for a small increase in permeability ranging between 1×10^{-6} and 3×10^{-6}. Such a phenomenon can cause truncation in the computational procedure, sometimes increasing the suction to the positive values. It may be further pointed out that such truncation errors depend not only on the specific permeability values but also on the other soil constants involved in the solution. Numerous problems solved with different soil parameters to study this phenomenon of suction gradient truncation reveal that its effects can be reduced to a reasonable level by a suitable selection of soil constants.

Unsaturated Permeability Constant BK1 . The effect of this constant is studied in Tables 4 and 5. It appears that the region wetted by an increase in this number shifts downwards as this number is increased. Of course, all other coefficients involved remain constant. This can be explained by the fact that for smaller values of this number the permeability is more dependent upon water content or suction. This dependence is shown by the large accumulation of water at 0.5 foot depth as this number is decreased and by the tendency for the moisture to spread throughout the depth as this number is increased. As the dependence of permeability on suction decreases, the moisture content near the surface decreases and moisture contents are increased at the lower levels.

Unsaturated Permeability Exponent EN1 . The effect of EN1 , as shown in Table 6 is of the same form as that of a decrease in unsaturated permeability constant BK1 . A higher exponent results in a greater dependence of permeability on moisture content. The lower values of this constant result in a condition of more even distribution of water in the entire region of soil. A more complete discussion of this phenomenon is presented in Chapter 2 of Research Report 118-1 and it is not considered further in this chapter.

Maximum pF , pFM . An increase in maximum pF value, as shown in Table 7, causes an increase in moisture roughly on the wetter side of the point of inflection and a decrease on the drier side. In addition, the slope of the suction-moisture curve increases as the value of maximum pF is increased. This slope is $\frac{\partial \tau}{\partial \theta}$ which is used to calculate changes of suction from one time

TABLE 3. COMPUTED SOIL MOISTURE VALUES TO SHOW THE EFFECTS OF THE VARIATIONS OF SATURATED PERMEABILITY PB .

Saturation moisture content = 38 percent

Maximum PF = 5.00

PF at inflection = 3.00

PF moisture content exponent = 3.00

Unsaturated permeability constant = 1×10^9

Unsaturated permeability exponent = 3.00

Depth, ft	Saturated Permeability				
	5×10^{-7}	9×10^{-7}	1×10^{-6}	3×10^{-6}	5×10^{-6}
0	38.00	38.00	38.00	38.00	38.00
0.5	25.76	27.72	28.06	31.36*	32.65
1.0	20.04	22.39	22.82	27.17*	28.98
1.5	16.28	18.72	19.19	24.09*	26.19
2.0	14.04	16.04	16.50	21.63*	23.94
2.5	13.31	14.38	14.70	19.58*	22.02
3.0	13.35	13.72	13.87	17.86*	20.36
3.5	13.55	13.59	13.64	16.44*	18.90
4.0	13.69	13.63	13.64	15.33*	17.63
4.5	13.74	13.67	13.66	14.56*	16.52
5.0	13.70	13.65	13.64	14.08	15.61
5.5	13.61	13.60	13.60	13.82	14.90
6.0	13.51	13.53	13.54	13.67	14.39

*Very high relative increase in the values.

TABLE 4. COMPUTED SOIL MOISTURE VALUES TO SHOW THE EFFECTS OF THE VARIATIONS OF UNSATURATED PERMEABILITY CONSTANT BK 1 .

Saturation moisture content = 39 percent

Maximum PF = 4.50

PF at inflection = 3.00

PF moisture content exponent = 3.00

Saturated permeability = 8×10^{-7}

Unsaturated permeability exponent = 3.00

Depth, ft	Unsaturated Permeability Constant			
	1×10^{6}	1×10^{8}	1×10^{10}	1×10^{12}
0	39.00	39.00	39.00	39.00
0.5	31.99	28.38**	26.95**	26.90**
1.0	18.23	22.35*	21.10**	21.04**
1.5	13.12	16.73*	17.63*	17.61**
2.0	12.60	13.29*	15.57*	15.60*
2.5	12.41	12.79*	14.43*	14.48*
3.0	13.74	13.43	13.88*	13.93*
3.5	13.55	13.52	13.67*	13.70*
4.0	13.90	13.80	13.63	13.63
4.5	13.85	13.82	13.62	13.62
5.0	13.75	13.74	13.61	13.61
5.5	13.65	13.64	13.58	13.58

* Increased value (left to right).
**Decreased value.

TABLE 5. COMPUTED SOIL MOISTURE VALUES TO SHOW THE EFFECTS OF THE VARIATIONS OF UNSATURATED PERMEABILITY CONSTANT BK 1 .

Saturation moisture content = 38 percent
Maximum PF = 5.0
PF at inflection = 3.0
PF moisture content exponent = 4.0
Saturated permeability = 9×10^{-7}
Unsaturated permeability exponent = 3.00

Depth, ft	Unsaturated Permeability Constant			
	1×10^{7}	1×10^{8}	1×10^{9}	1×10^{10}
0	38.00	38.00	38.00	38.00
0.5	31.83	30.37**	27.72**	28.75*
1.0	23.87	24.32*	22.39**	22.69*
1.5	13.54	17.38*	18.72*	18.72
2.0	12.61	13.12*	16.04*	16.19*
2.5	12.44	12.70*	14.38*	14.71*
3.0	13.71	13.47	13.72*	14.01*
3.5	13.57	13.62	13.59	13.73*
4.0	13.89	13.82	13.63	13.64*
4.5	13.85	13.83	13.67	13.62
5.0	13.75	13.75	13.65	13.61
5.5	13.65	13.64	13.60	13.58
6.0	13.50	13.49	13.53	13.55

* Increased value.
**Decreased value.

TABLE 6. COMPUTED SOIL MOISTURE VALUES TO SHOW THE EFFECTS OF THE VARIATIONS OF UNSATURATED PERMEABILITY EXPONENT EN1 .

Saturation moisture content = 40.0 percent

Maximum PF = 6.0

PF at inflection = 3.0

PF moisture content exponent = 3.0

Saturated permeability = 1×10^{-6}

Unsaturated permeability constant = 1×10^{9}

Depth, ft	Unsaturated Permeability Exponent				
	2.5	3.0	3.5	4.0	4.5
0	40.00	40.00	40.00	40.00	40.00
0.5	28.95	29.94*	32.06*	33.51*	34.81*
1.0	23.40	24.58*	26.77*	26.24*	21.19**
1.5	19.71	20.35*	17.61**	13.18**	13.10**
2.0	17.37	16.81**	12.68**	12.60**	12.60
2.5	15.94	14.41**	12.43**	12.40**	12.40
3.0	15.03	13.59	13.71	13.75	13.75
3.5	14.44	13.59	13.57	13.55	13.55
4.0	14.08	13.70	13.89	13.90	13.90
4.5	13.85	13.74	13.85	13.85	13.85
5.0	13.72	13.70	13.75	13.75	13.75
5.5	13.64	13.61	13.65	13.65	13.65
6.0	13.60	13.51	13.50	13.50	13.50

* Increased value.
**Decreased value.

TABLE 7. COMPUTED SOIL MOISTURE VALUES TO SHOW THE EFFECTS OF THE VARIATIONS OF MAXIMUM PF , PFM .

Saturation moisture content = 38 percent

PF at inflection = 3.00

PF moisture content exponent = 3.00

Saturated permeability = 9×10^{-7}

Unsaturated permeability constant = 1×10^{8}

Unsaturated permeability exponent = 3.00

Depth, ft	Maximum PF				
	4.0	4.5	5.0	6.0	6.5
0	38.00	38.00	38.00	38.00	38.00
0.5	27.47	28.22*	28.87*	29.91*	30.40*
1.0	21.60	22.60*	23.47*	24.85*	25.56*
1.5	16.85	17.48*	17.98*	18.56*	19.15*
2.0	13.73	13.68**	13.46**	13.15**	13.13**
2.5	12.99	12.88**	12.75**	12.56**	12.51**
3.0	13.36	13.40	13.45	13.59	13.64
3.5	13.61	13.62	13.63	13.61	13.60
4.0	13.76	13.79	13.81	13.85	13.86
4.5	13.79	13.81	13.82	13.84	13.84
5.0	13.73	13.74	13.74	13.75	13.75
5.5	13.63	13.63	13.64	13.64	13.65
6.0	13.49	13.49	13.49	13.49	13.50

* Increased value.
**Decreased value.

step to the next. An increase of this slope has the effect of increasing the permeability and making it more dependent upon suction.

pF at Inflection pFl . The effect of change in inflection pF must be considered along each of the two branches of suction moisture curve, one branch being above and the other being below the inflection pF . As shown in Table 8, the value of inflection pF of 3 shows a relatively higher increase in moisture over the entire soil region. The higher increases in the top 1 foot can be explained by the fact that the higher pF at inflection implies a greater water content for certain suction levels. In the drier range, pF of 3.0 appears to give wetter values throughout. This is explained by the fact that higher slopes of suction-moisture curve at inflection as the inflection pF is increased have the same effect as increasing permeability and making it more dependent upon suction.

pF Moisture Content Exponent BETA . The effects of BETA as shown by Table 9 imply that higher BETA values cause flatter slopes of suction-moisture curve in the vicinity of inflection and therefore moisture values are more widely divergent in this region as BETA increases.

Saturation Moisture Content WN . Greater values of saturation moisture contents in general imply a greater openness of the pores of the soils which in turn can take on greater amounts of moisture. This is reflected in the results shown in Table 10. A consistent decrease in water content noticed between the depths of 1.5 and 2.5 feet is due to the disparity of slopes of the suction-moisture curves in the vicinity of the inflection pF .

Concluding, this study suggests that a very high confidence can be placed in the one-dimensional moisture distribution Program FLOPIP2 and its capability of working problems such as reported in this chapter. The values of constants involved can be assumed in a better way with their different effects having been thoroughly studied and indicated. Thus, this study can furnish a valuable guide in future selection of constants for the solution of moisture distribution problems.

Output after each time step includes the station, suction, water content, and pF .

TABLE 8. COMPUTED SOIL MOISTURE VALUES TO SHOW THE EFFECTS OF THE VARIATIONS OF PF AT INFLECTION PF1 .

Saturation moisture content = 40 percent

Maximum PF = 6.5

PF moisture content exponent = 3.0

Saturated permeability = 1×10^{-6}

Unsaturated permeability constant = 1×10^{9}

Unsaturated permeability exponent = 3.0

Depth, ft	PF at Inflection		
	2.5	3.0	4.0
0	40.00	40.00	40.00
0.5	25.27	30.37*	31.52*
1.0	19.15	25.18*	25.54*
1.5	16.29	20.86*	20.23**
2.0	14.64	17.16*	16.03**
2.5	13.77	14.29*	13.65**
3.0	13.51	13.52*	13.41**
3.5	13.55	13.62*	13.60**
4.0	13.64	13.74*	13.73**
4.5	13.68	13.77*	13.77
5.0	13.66	13.72*	13.72
5.5	13.60	13.62*	13.62
6.0	13.53	13.50	13.50

* Increased value.
**Decreased value.

TABLE 9. COMPUTED SOIL MOISTURE VALUES TO SHOW THE EFFECTS OF THE VARIATIONS OF PF MOISTURE CONTENT EXPONENT BETA .

Saturation moisture content = 40 percent

Maximum PF = 6.5

PF at inflection = 3.0

Saturated permeability = 1×10^9

Unsaturated permeability constant = 1×10^9

Unsaturated permeability exponent = 3.0

Depth, ft	PF Moisture Content Exponent			
	2.5	3.0	3.5	4.0
0	40.00	40.00	40.00	40.00
0.5	29.90	30.37*	30.95*	31.52*
1.0	25.21	25.18**	25.34*	25.54*
1.5	21.48	20.86**	20.51**	20.23**
2.0	18.13	17.16**	16.53**	16.03**
2.5	15.20	14.29**	13.89**	13.65**
3.0	13.72	13.52	13.45	13.41
3.5	13.64	13.62	13.60	13.60
4.0	13.75	13.74	13.74	13.73
4.5	13.78	13.77	13.77	13.77
5.0	13.72	13.72	13.72	13.72
5.5	13.62	13.62	13.62	13.62
6.0	13.50	13.50	13.50	13.50

* Increased value.
**Decreased value.

TABLE 10. COMPUTED SOIL MOISTURE VALUES TO SHOW THE EFFECTS OF THE VARIATIONS OF SATURATION MOISTURE CONTENT WN .

Maximum PF = 5.0

PF at inflection = 3.0

PF moisture content exponent = 3.0

Saturated permeability = 9×10^{-7}

Unsaturated permeability constant = 1×10^{8}

Unsaturated permeability exponent = 3.0

Depth, ft	Saturation Moisture Content %			
	37 %	38 %	39 %	40 %
0	37.00	38.00	39.00	40.00
0.5	28.29	28.87*	29.45*	30.03*
1.0	23.14	23.47*	23.79*	24.11*
1.5	18.03	17.96**	17.90**	17.79**
2.0	13.60	13.46**	13.34**	13.24**
2.5	12.79	12.75**	12.72**	12.70**
3.0	13.43	13.45	13.46	13.48
3.5	13.63	13.63	13.63	13.62
4.0	13.80	13.81	13.81	13.82

* Increased value.
**Decreased value.

Computer Program FLOPIP2 is similar to the two-dimensional program in many respects but the differences in input are such that use of a separate input format as shown in Appendix 7 may be required.

The flow chart is identical except that only one direction is computed and no iteration is required for solution at a particular time step.

CHAPTER 7. EXAMPLE PROBLEMS: TWO-DIMENSIONAL

This chapter is broken into two parts: the first demonstrates the solution to a problem for which a theoretical answer can be obtained and the second gives the soil properties used and a summary of results for example problems involving transient moisture movement in different clay regions.

Theoretical Problems

This problem is the determination of the decay of positive pore pressure head in a square clay region 100 inches on each side. The steady-state hydrostatic pressure head on this clay region is 100 inches at the top and 200 inches at the bottom. A footing load is imposed which increases pore pressure head in the region by 100 inches. The region is surrounded by sand which immediately relieves the excess pore pressure head on the boundaries of the clay region to its original hydrostatic state. The decay of the excess pressure head in the region has been computed by Program GCHPIP1 and the results are compared with the exact solution to the problem determined by other computer programs especially written for the purpose. The problem is illustrated in Fig 17 and results of the computer solution are shown for points along the diagonal of the square in Fig 18. The compressibility coefficient is assumed to be 10^{-6} and the time steps used in GCHPIP1 were 10^{5} and 10^{6} seconds.

The exact solution of the problem is the product of two series, one representing the decay of pore pressure head in the x-direction and the other the decay of pore pressure head in the y-direction. For a unit initial excess pressure head, then, a one-dimensional solution for excess head U with respect to x and time t is

$$U(x,t) = \frac{4}{\pi} \sum^{\infty} \frac{1}{(2n-1)} \sin (2n-1) \frac{\pi x}{L} (e)^{-(2n-1)^2 \frac{\pi^2}{L^2} ct} \qquad (7.1)$$

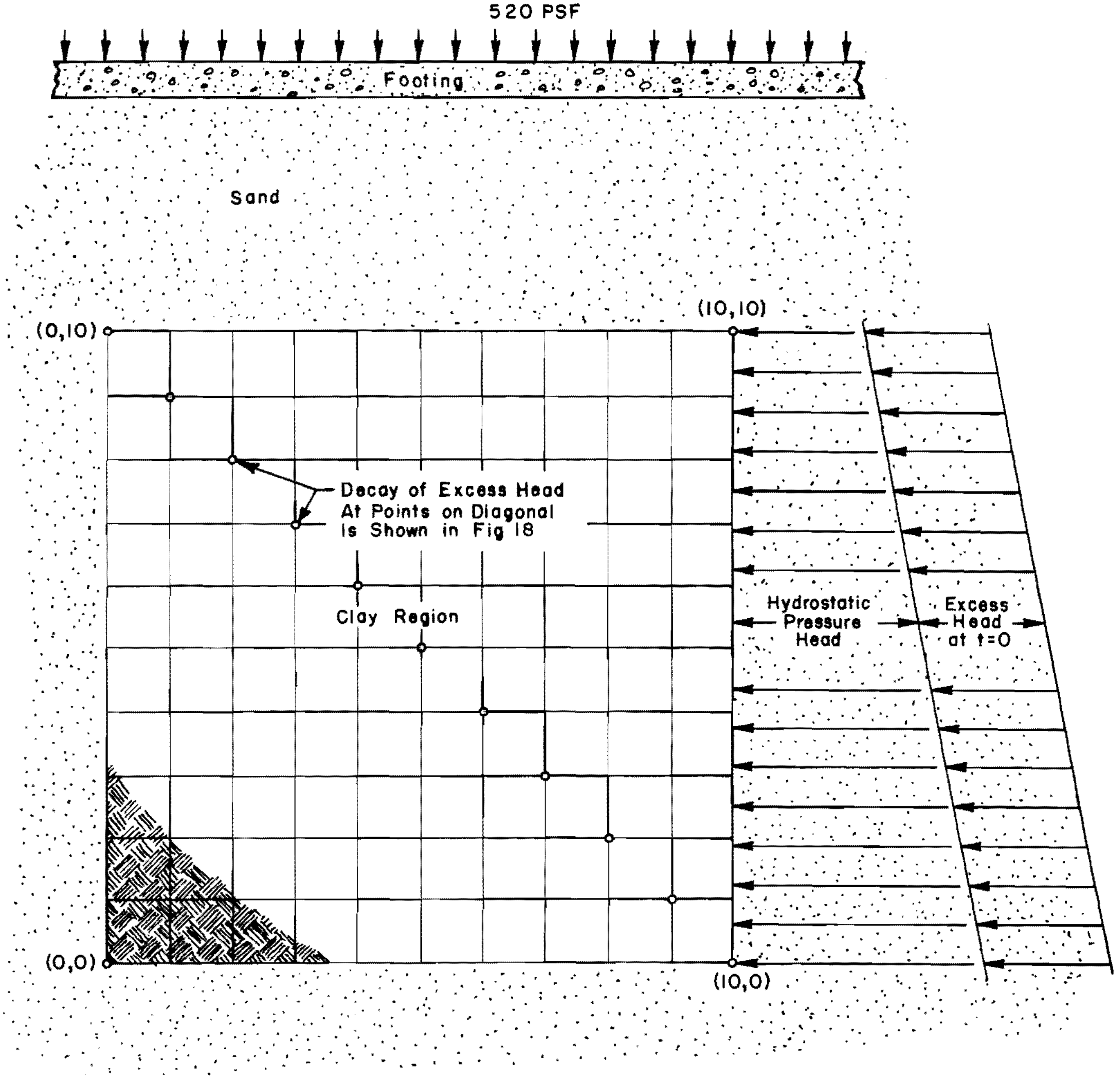


Fig 17. Example problem for decay of pore pressure head in two-dimensional clay region.

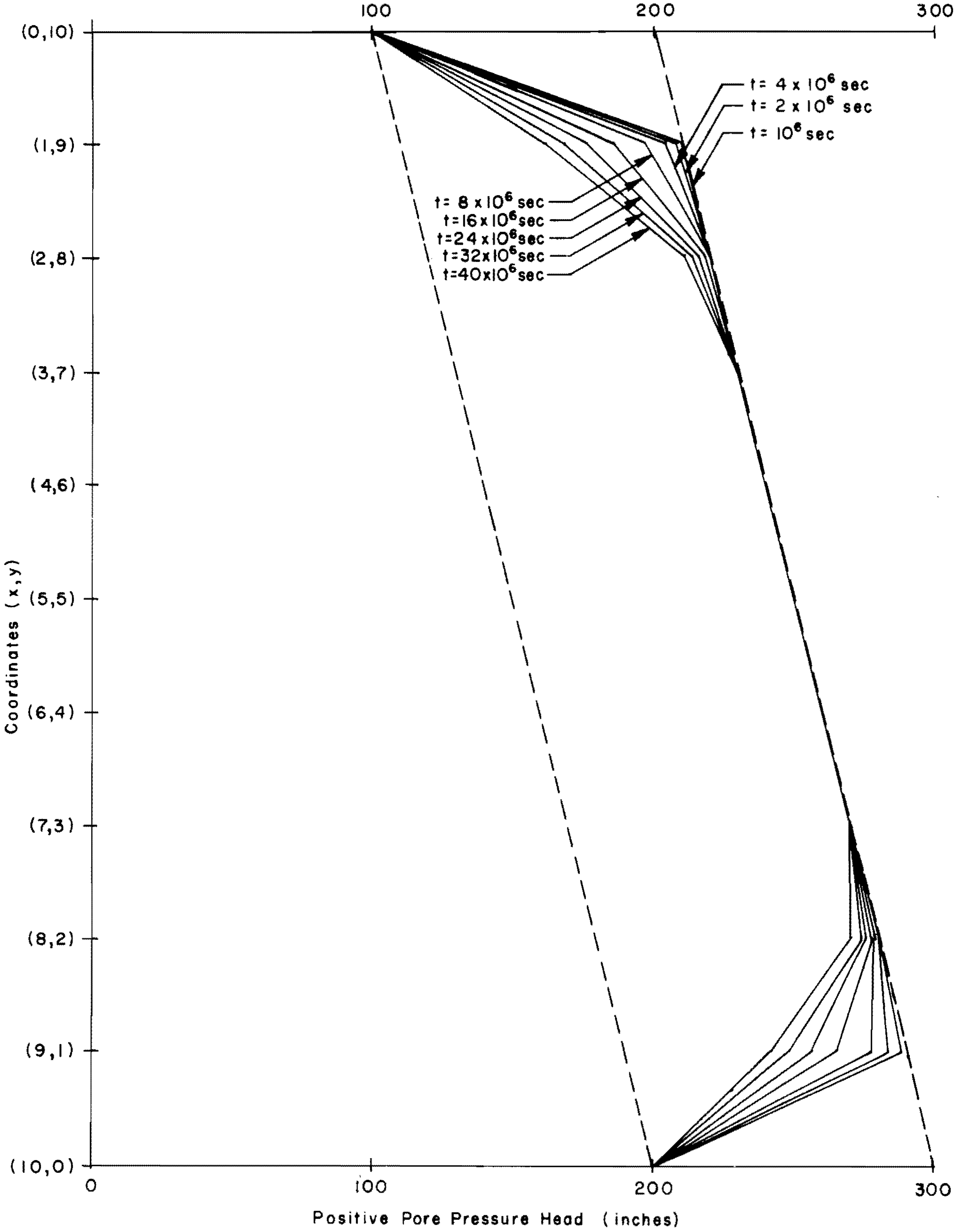


Fig 18. Decay of excess head along diagonal.

where

L = the length of a side,

c = the compressibility coefficient,

t = time.

If U(y,t) is understood as the symbol for an identical series in the y-direction and A is the initial excess head, then the expression for the two-dimensional decay of excess pressure head is

$$U(x,y,t) = AU(x,t)U(y,t) \tag{7.2}$$

The series in Eq 7.1 is very slowly convergent for small values of the time factor ct/L^2 . A special series which converges rapidly may be used to evaluate both U(x,t) and U(y,t) . The series is as follows:

$$U(x,t) = 1 - \sum_{n=1}^{\infty} (-1)^{n-1} \mathrm{erfc} \frac{(2n-1)\frac{L}{2} - x}{2\ ct}$$

$$- \sum_{n=1}^{\infty} (-1)^{n-1} \mathrm{erfc} \frac{(2n-1)\frac{L}{2} + x}{2\ ct} \tag{7.3}$$

where

$$x = x - \frac{L}{2}$$

and

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-u^2}\, du$$ is the complimentary error function.

The error function itself can be difficult to obtain for small values of ct/L^2 . Three methods were programmed. The first, suggested by Carslaw and Jaeger (Ref 3) involves a series

$$\operatorname{erfc}(g) = \frac{e^{-g^2}}{\sqrt{\pi}} \left(\frac{1}{g} - \frac{1}{2g^3} + \frac{1 \times 3}{2^2 g^5} - \frac{1 \times 3 \times 5}{2^3 g^7} + \cdots \right) \tag{7.4}$$

The computer program for this method is named SQFOURE. The second method used to evaluate the "exact" answer to this pore pressure head decay problem uses a polynomial approximation to the complimentary error function called Algorithm 209 (Ref 1). The program containing this algorithm is named SQFOURI. The third method, Program SQFOUR, uses Eqs 7.1 and 7.2 directly. Answers from these programs are compared with results of computations with GCHPIP1 with two sizes of time step. Table 11 shows values of excess pore pressure head computed for point (1,9) by the four methods for various amounts of time elapsed.

The truncation error in the Crank-Nicolson parabolic difference equation is of the order of $(h_x)^2$, $(h_y)^2$, and $(h_t)^2$. Thus, while the error in representing the time derivative is decreased by decreasing the time step, better overall results would be obtained only if there is a corresponding decrease in the size of the x and y-increments.

TABLE 11. VARIATION OF EXCESS PORE PRESSURE HEAD AT (1,9)

Time Elapsed, Seconds	GCHPIP1 $H_t = 10^6$ sec	GCHPIP1 $H_t = 10^5$ sec	SQFOUR Sine series	SQFOURE Carslaw & Jaeger Erfc series	SQFOURI Erfc series Algorithm 209
1×10^6	100.0	100.0	100.0*	100.0	100.0
2×10^6	98.0	98.0	100.0*	100.0	99.9
4×10^6	94.2	96.1	99.9	99.9	97.5
8×10^6	87.3	92.5	97.5	102.4	85.2
16×10^6	75.6	85.7	85.2	96.2	62.2
24×10^6	66.2	79.8	72.4	65.8×10^8	48.0
32×10^6	58.6	74.4	62.2	62.2	38.8
40×10^6	52.4	69.6	54.2	54.2	32.6

* 100 terms of series used to compute this number.

Point (1,9) is in a corner of the clay region and experiences the greatest amount of change in the period of time studied. The table above gives an idea of the difficulty encountered in arriving at "exact" solutions, as well as the kind of accuracy to expect from the numerical computations. In the exact solution programs, at least twenty terms of each series were used in computing the values reported. In SQFOUR, the first 100 terms of the Fourier series were used to compute the values of excess pressure head at times 1×10^6 and 2×10^6 seconds. The figures shown in the black-bordered section are probably incorrect because of two factors: (1) the erfc series converges very slowly for larger values of ct/L^2 and (2) the series used to evaluate erfc in SQFOURE diverges for values between 1 and 2.

The remaining part of the chapter presents example problems involving computations of unsaturated flow in clay soils.

Accumulation of Moisture Around a Bored Casing

A perplexing phenomenon related to the heave of pavement above a bored casing has been observed by men of the Texas Highway Department. Normally, cased utilities (gas, water, and electricity lines) are laid in an open trench before construction of a highway. Even though high-quality backfill is used (in some cases one-sack mix concrete) enough swelling can occur subsequently to require costly maintenance and repair work. It was thought that by boring a hole beneath the completed highway, casing the hole with a light steel pipe liner, and extending the utility lines under the pavement through the casing, the swelling would be eliminated. The swelling that occurred after these precautions were taken was both surprising and puzzling.

A partial explanation of this phenomenon is offered here as an example problem. Several factors can contribute to an accumulation of moisture around the casing which will cause or permit swelling. Some of these factors are

1. (1) difference in temperature between ground and casing,
2. (2) ion-concentration potential between ground and casing,
3. (3) presence and availability of boring water, and
4. (4) increase of soil suction in disturbed soil around the casing.

There are probably more factors, but these are the most significant. The example problem considers the effect of factor (4) alone. When preconsolidated, clay is disturbed, its suction increases even though the water content remains unchanged. This can be explained microscopically by the realignment of particles and breaking their internal bonds by shearing. Reference to Croney, Coleman, and Black's data (Ref 5) indicates that air entry pF and water content may remain very nearly the same, but disturbed soil pF can be expected to be higher at the same water content. The final saturation water content may be expected to be larger as well.

Problem Description. A disturbed area 2 feet on a side is centered 7 feet below the subgrade. This area is surrounded by soil in an undisturbed state and the entire area remains at the undisturbed water content. This arrangement is shown in Fig 19. Principal saturated permeabilities are assumed as follows: (1) horizontal, 1.0×10^{-7} in/sec; and (2) vertical, 0.5×10^{-7} in/sec, in both the disturbed and undisturbed soils. No attempt was made to model the casing hole. This problem simply assumes the entire 4-square-foot area to be composed of disturbed soil alone. The unsaturated coefficients used are

$$a = 2.54$$

$$b = 1.6 \times 10^{8}$$

$$n = 3.0$$

which are fairly representative of the Yolo light-clay data presented in Ref 17.

The following data for the disturbed and undisturbed soil pF-θ relations are presented in tabular form for comparison.

TABLE 12. pF-θ DATA FOR UNDISTURBED AND DISTURBED CLAY

	Undisturbed Soil	Disturbed Soil
Maximum pF	7.0	7.0
pF at inflection	3.0	4.0
Exponent for pF	1.0	1.5
Saturated volumetric water content	45.0	48.0

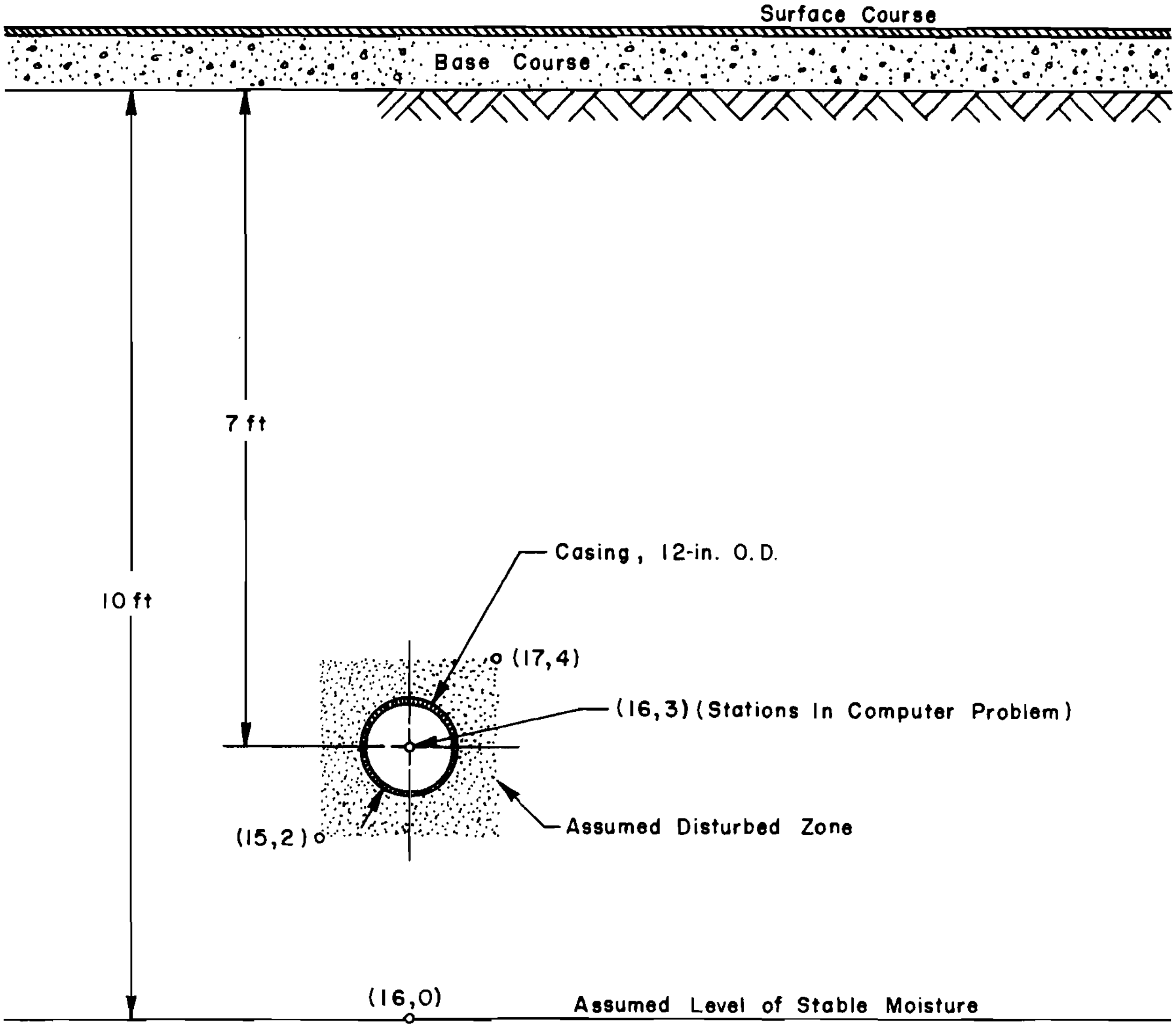


Fig 19. Example problem for moisture accumulation in disturbed soil zone.

No overburden pressures were considered in this problem. This effect will be illustrated in a later example problem. Initial volumetric water content is 42.8 percent at the top and 37.8 percent at the bottom with a linear variation between the two. If a specific gravity of solids of 2.70 is assumed, a volumetric water content of 37.8 percent corresponds to a gravimetric water content of 22.5 percent.

One-week time increments were used in this problem and 50 weeks of data were computed. The suction changed only in the immediate vicinity of the disturbed soil in that period of time. The initial and final values of suction are shown in Fig 20. Maximum suction change was recorded at station (15,2) from a value of -151 inches to -85 inches. Because $\frac{\partial \tau}{\partial \theta}$ at this point remained between 26 and 39 inches throughout the entire problem, the suction change represents a gain of volumetric water content of over 2 percent in 50 weeks. As can be seen in Fig 20, a suction potential still remains at the end of this period which, if brought to equilibrium, could account for another 1-1/2 percent. The increase of suction at points surrounding the disturbed area indicates that water has been sucked out of the surrounding soil. A 2-percent volume change in a 24-inch cube will produce between 0.17 and 0.5 inch of vertical heave. The first figure applies if there is equal swell in all directions. The latter figure is based on swell in the vertical dimension alone. A larger disturbed area and larger moisture change will have proportional results. This example problem demonstrates that the increased suction due to soil disturbance can be significant in causing the swell of a pavement surface above a bored casing.

Soils Data for Other Example Problems

There is a scarcity of published experimental data of the kind required in this study. Of the data that have been published, none include the complete list of permeability-suction, suction-moisture, and shrinkage curves; specific gravity; and e-log p data. Because of this it is necessary to collect data from several sources and use a kind of conglomerate soil in the example problems. Data for each item in the list will be presented along with the source of information.

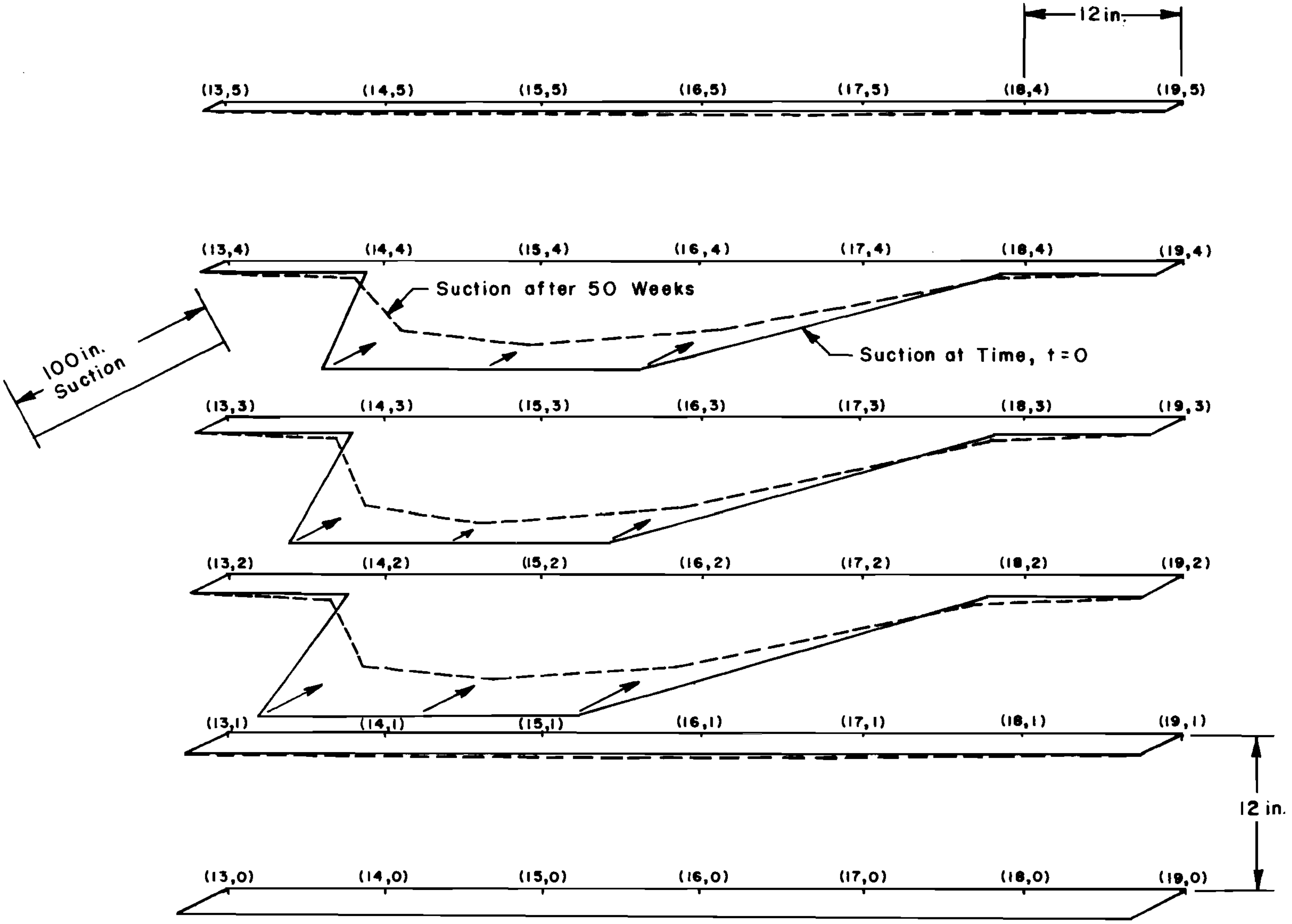


Fig 20. Change of suction in disturbed soil region.

Permeability Suction. The data for two clay soils reported in Ref 17 were used to get values of the unsaturated coefficients a , b , and n . The data used and the derived coefficients are given in the table below.

TABLE 13. CLAY PERMEABILITY DATA

Yolo Light Clay		Horsham Clay	
Suction, cm	Permeability, cm/sec	Suction, cm	Permeability, cm/sec
-200	2×10^{-7}	-200	6.3×10^{-10}
-500	2×10^{-8}	-500	5.6×10^{-10}
-900	6×10^{-9}	-900	2.6×10^{-10}
k_{sat}	$= 2.2 \times 10^{-7}$ cm/sec	k_{sat}	$= 6.3 \times 10^{-10}$ cm/sec
	$= 0.9 \times 10^{-7}$ in/sec		$= 2.5 \times 10^{-10}$ in/sec
a	= 2.54 cm/in	a	= 2.54 cm/in
b	$= 1.58 \times 10^{8}$	b	$= 1.08 \times 10^{12}$
n	= 3.0	n	= 4.1

In addition to these data, a rough value for silt permeability, 10^{-6} cm/sec, was drawn from Table 2.1 of Foundation Engineering, by Peck, Hanson, and Thornburn (Ref 14).

Suction-Moisture and Shrinkage. The curves used for guidance in this section were drawn from Croney, Coleman, and Black (Ref 5). A pF at inflection is assumed to be between 3.0 and 5.0 unless the soil is silty clay. For silty clay, an inflection pF of 2.5 is assumed. Maximum pF in all cases is 7.0. Other check points are shown in the table below. Gravimetric water contents are computed assuming a solids specific gravity of 2.70. The quantities shown for silty clay are purely assumed data.

TABLE 14. SOIL PROPERTIES USED IN EXAMPLE PROBLEMS

Property	Clay	Silty Clay
Final saturation water content (volumetric)	45.0%	35.0%
Final saturation water content (gravimetric)	30.3%	20.0%
α at 0 water content	0	0
Air entry water content (volumetric)	34.0 - 35.0%	30.0%
Air entry porosity	0.350 - 0.365	.310
α - exponent of shrinkage curve	2.0	1.5
χ - exponent of saturation curve	0.9	0.8

Compressibility. None of the problems attempted achieved low enough moisture content for the programmed equation containing e-log p data to be used. To be on the safe side, however, values of compressibility coefficient of 0.08 for clay and 0.04 for silty clay were used. The figure for clay is for a preconsolidated Boston blue clay. The silty clay figure was assumed to represent twice the clay stiffness, i.e., one-half of its compressibility.

Redistribution of Moisture and Suction Beneath a House Foundation

In this problem, an inclined silty clay lens intrudes downward into the horizontal clay layer on which a house is built. Outside of the house, at the surface of the soil, the soil-moisture humidity remains at 99.99 percent with an average temperature of 80° F throughout the period of the problem. The physical arrangement of the problem is shown in Fig 21 on which is indicated the direction and size of the assumed saturated permeabilities of each soil type. Two problems are worked: one with the major principal permeability at 45 degrees below the horizontal and the other with the major principal permeability at 45 degrees above the horizontal. The initial condition for this problem is a linear suction gradient from -167.3 inches at the top to -50.2 inches at the bottom.

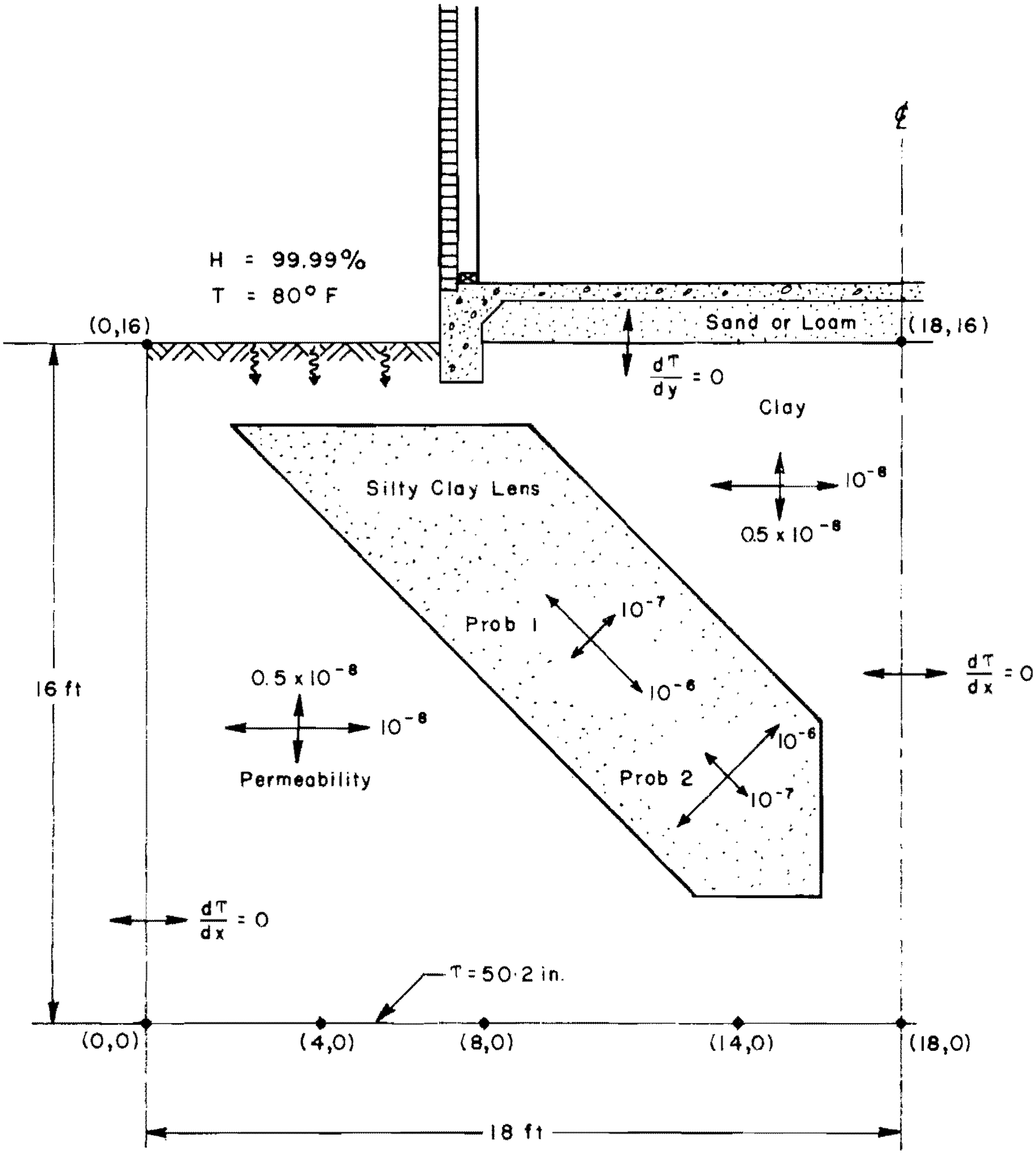


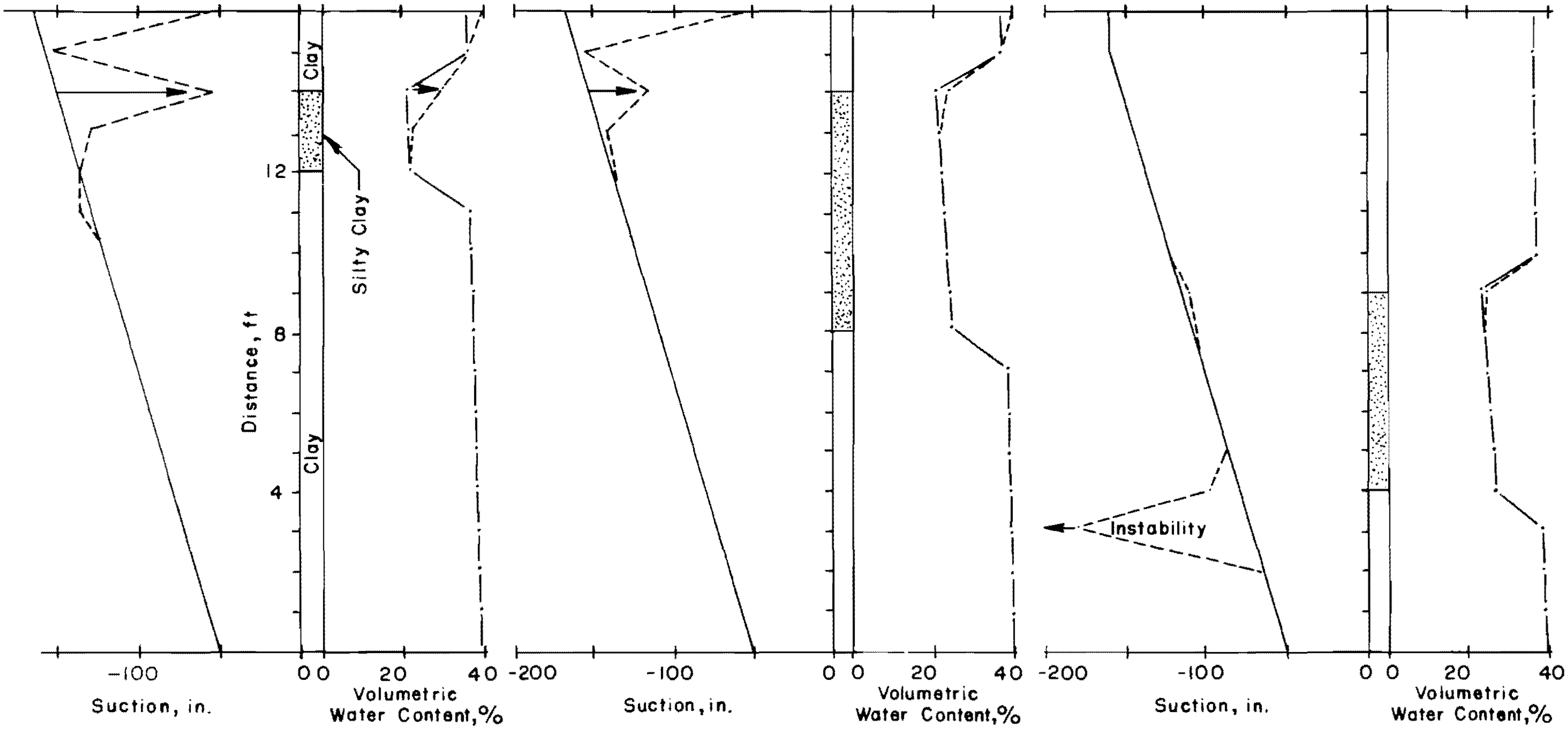
Fig 21. House foundation over an inclined silty clay lens.

Results. The results include output at time steps over a period of 400 weeks. The time steps for these problems are 4 weeks. Because it is impossible to catalog the results in their entirety, the initial and final suction and moisture are shown at three locations which may be represented as borings at x-stations 4, 8, and 14. These results are shown in Figs 22 and 23.

Two mechanisms of moisture movement are in evidence in these example problems: an upward transfer of water through the silty clay lens into the drier soil above and wetting of the soil from above by infiltration. In Problem 1, the second effect was predominant because of the low permeability in the lateral direction of the lens. In Problem 2, the greater permeability in an upward 45^{o} orientation allowed relatively rapid transfer of water from below, the effect dominating the increase of moisture due to infiltration. As is seen in Figs 22 and 23, much suction remains to be changed to a lower value at the end point of each problem. The consequent gain of moisture content can cause a considerable amount of swelling.

The instability predicted in Chapter 4 is a consequence of allowing a negative cross permeability term to become too large in magnitude. The instability noted in Fig 22, which occurred in the problem with the negative cross permeability terms, did not occur in the identical problem (Problem 2) with positive cross permeability terms. The negative cross permeability, the permeability discontinuity between clay and silty clay, and the use of the explicit estimation of suction at points not closed after a specified number of iterations account for the instability noticed in Problem 1 and not noticed in any other problem worked.

As a check on this idea, two variations of the theoretical problem of positive pore pressure head decay in a square clay region were run. One was run with a major principal permeability of 10^{-6} in/sec inclined at a positive 45^{o} and the other inclined at a negative 45^{o}. The minor principal permeability used was 10^{-7} in/sec. In neither of these problems did the solution become unstable. Thus it becomes apparent that in house foundation Problem 1 the negative cross-permeability term in vertical pipe increment (14,4) dominated the smaller positive permeability terms in the rest of the difference operator at point (14,3) and caused instability at that point. It is useful to note further that as the solution to Problem 1 marched farther in time, the suctions at the interfaces between clay and silty clay became larger and larger, some reaching exponents as high as 7 and 8 within the 400-week duration of the

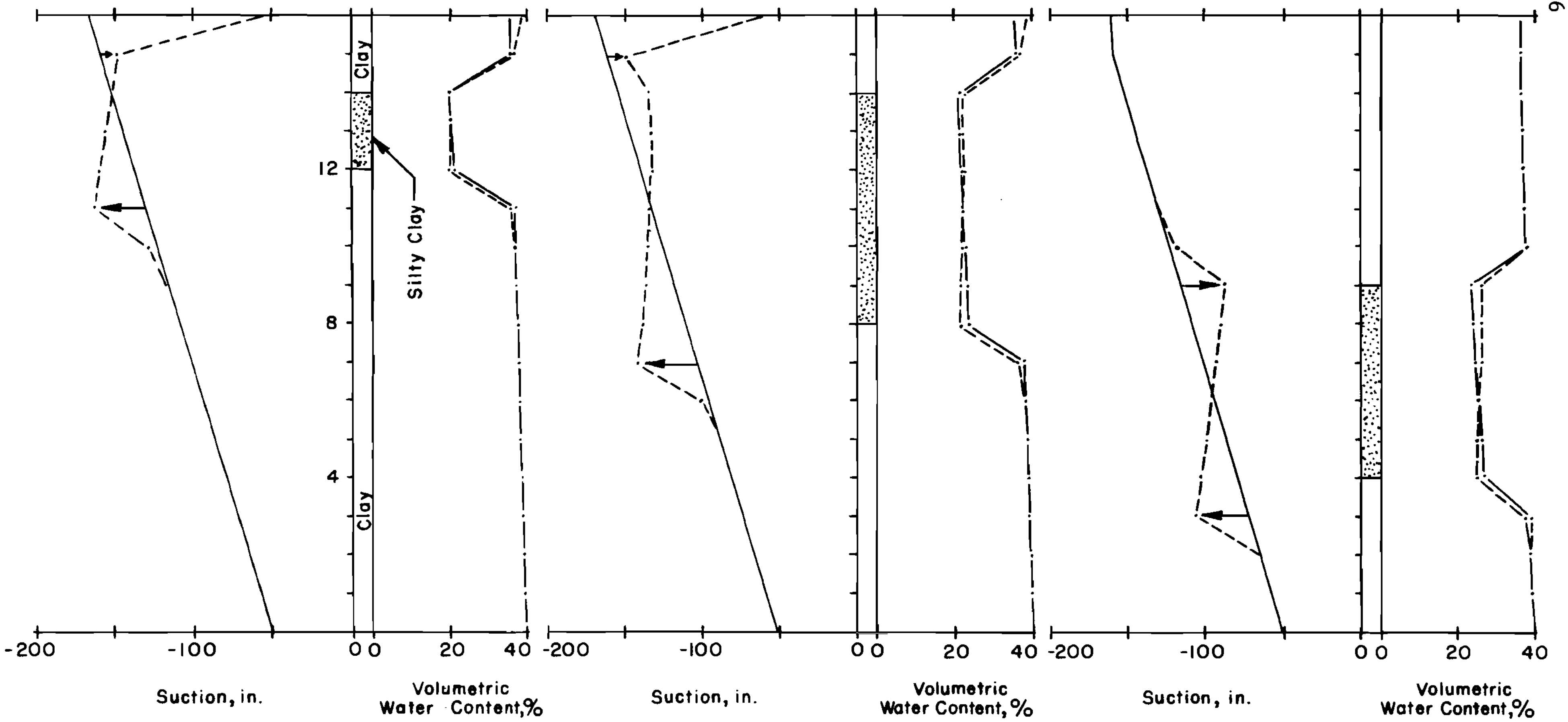


a. Boring at Station 4.

b. Boring at Station 8.

c. Boring at Station 14.

Fig 22. Suction and water content for Problem 1 after 96 weeks.



a. Boring at Station 4.

b. Boring at Station 8.

c. Boring at Station 14.

Fig 23. Suction and water content for Problem 2 after 400 weeks.

simulated problem. There is no physical interpretation of this instability; it is simply a weakness of the method used.

Ponding Problems

A technique used by engineers to reduce the amount of swell, ponding has been tried with varying degrees of success for many years. Methods employed vary: some use sand wells, others trenches, and still others simply pond the surface of a soil expected to cause trouble. The problems reported below serve both as examples of the method of solution and as a study of the efficiency of sand wells as opposed to surface ponding.

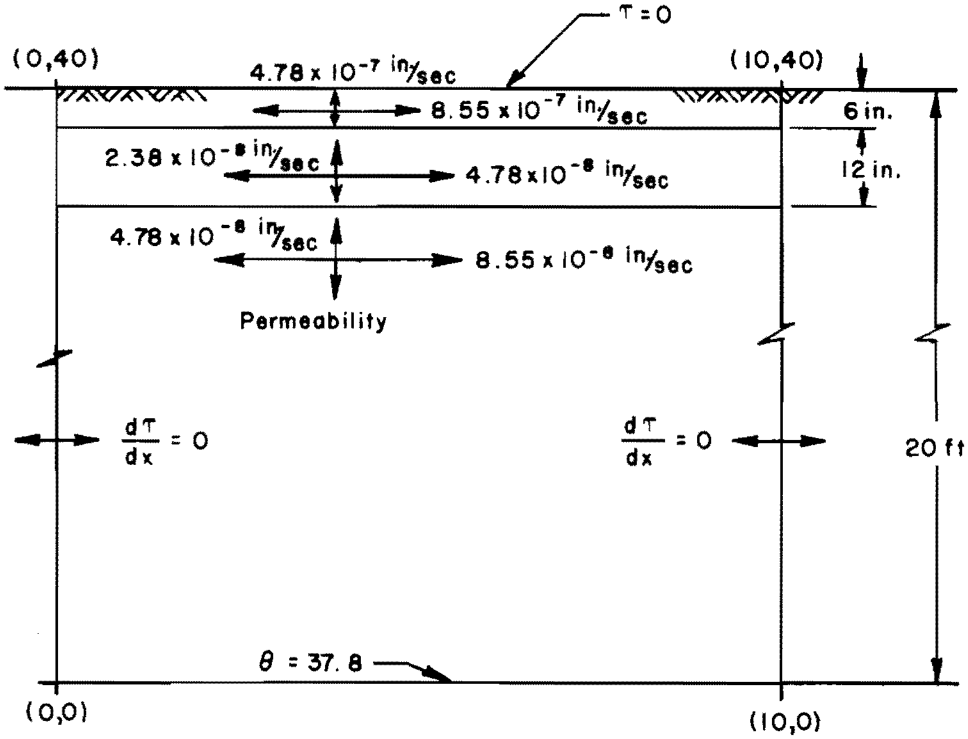
Six problems are presented: three with 12-inch diameter, 10-foot deep sand wells, and three with surface ponding. The soil in each case is the same with a final saturation volumetric water content of 45 percent and an inflection pF of 4.0. Other particulars are shown in Fig 24. Positive hydrostatic pressure head is set as a boundary condition in each sand well.

Of the three problems in each ponding category, one includes the effect of soil compressibility and two do not. Also, two problems have an initial condition in which all soil is at the plastic limit (assumed to be 37.8 percent, volumetric), and one is initially wetter than the plastic limit. Volumetric water contents specified for this problem range from 42 percent at the surface decreasing linearly to the plastic limit at a depth of 20 feet. The following table will show the variations presented and the corresponding problem number.

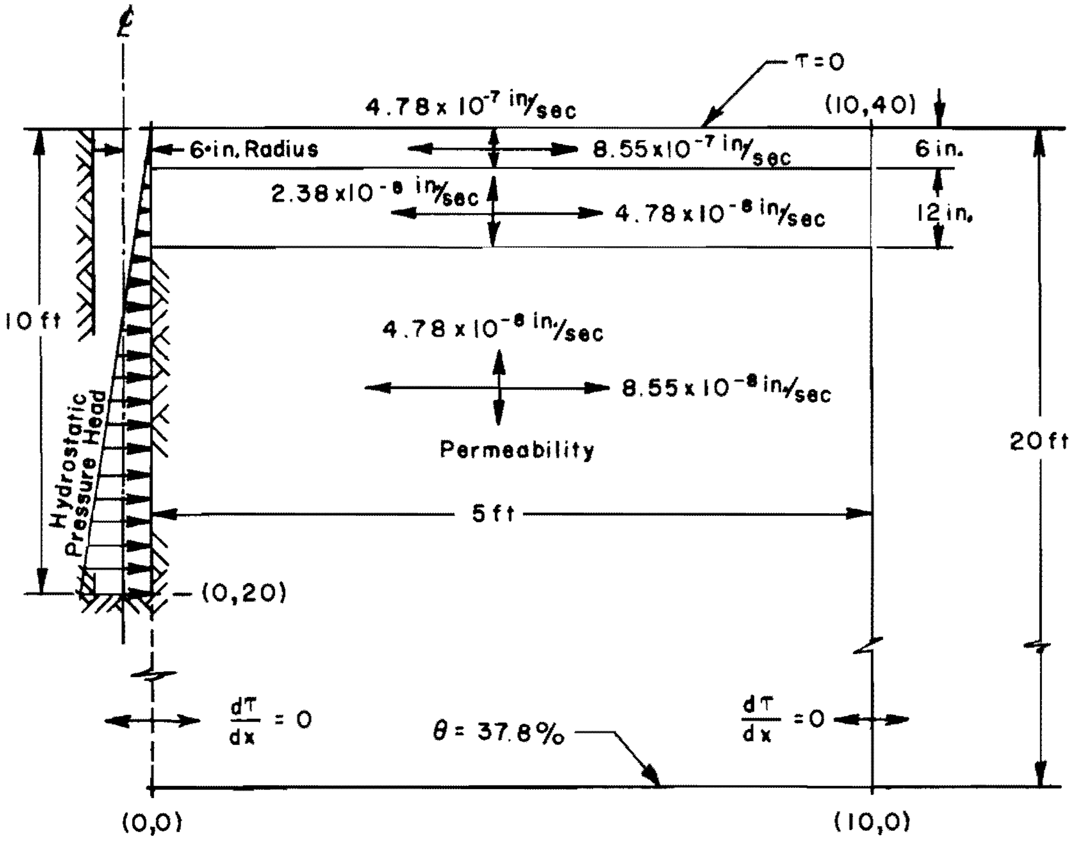
As is seen in Table 15, one other variation of each ponding category is possible: consideration of overburden pressure effects on the wetter soil. These problems are not presented because in this case the overburden pressure "overcomes" soil suction at a depth of 20 inches, indicating that all expansive moisture change must occur in the upper three 6-inch increments. This problem variation is not considered significant in the comparison of the efficiency of surface ponding as opposed to sand wells.

Figures. Figures 25 and 26 show the results of all of the surface ponding problems. Figure 25(b) presents the ponding problem which considers the effects of overburden pressure and soil expansibility. Figure 26 gives results of Problem POND2.

Sand well problems are presented in Figs 27 through 32. Results of each sand well problem are presented in two figures: one shows contours of total

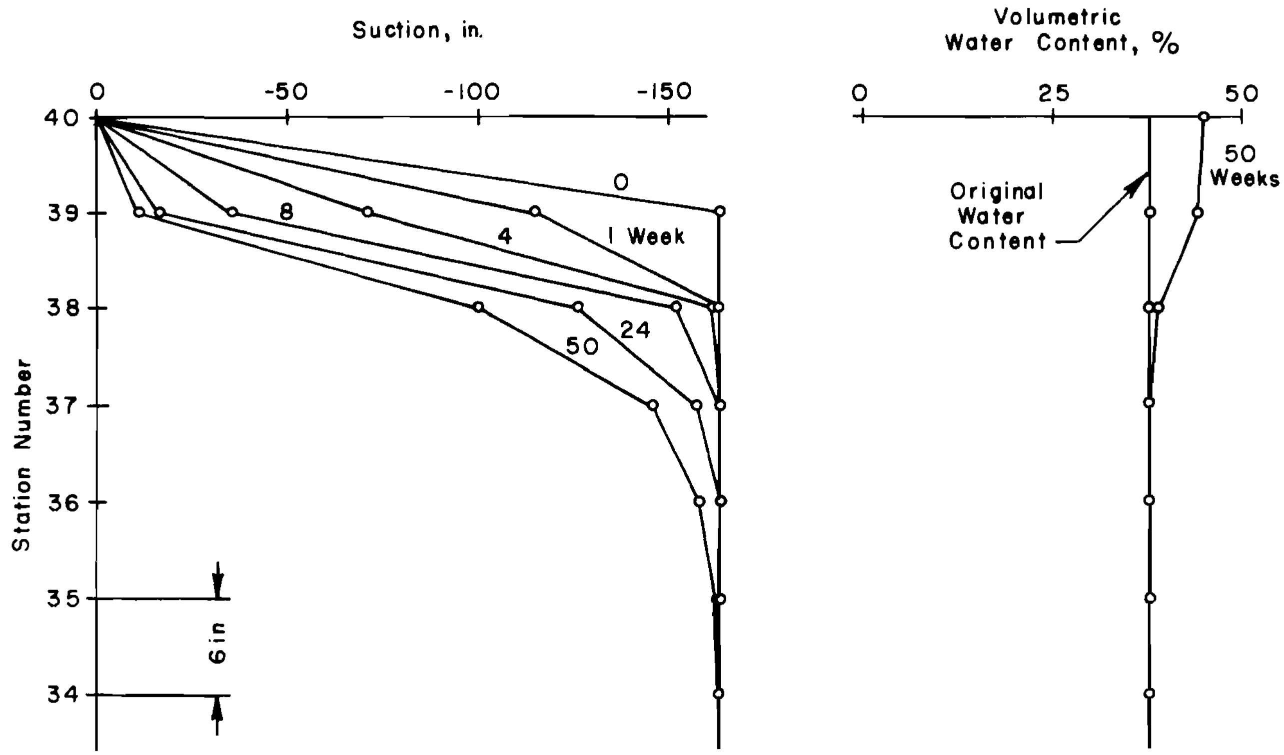


(a) Surface ponding problem.

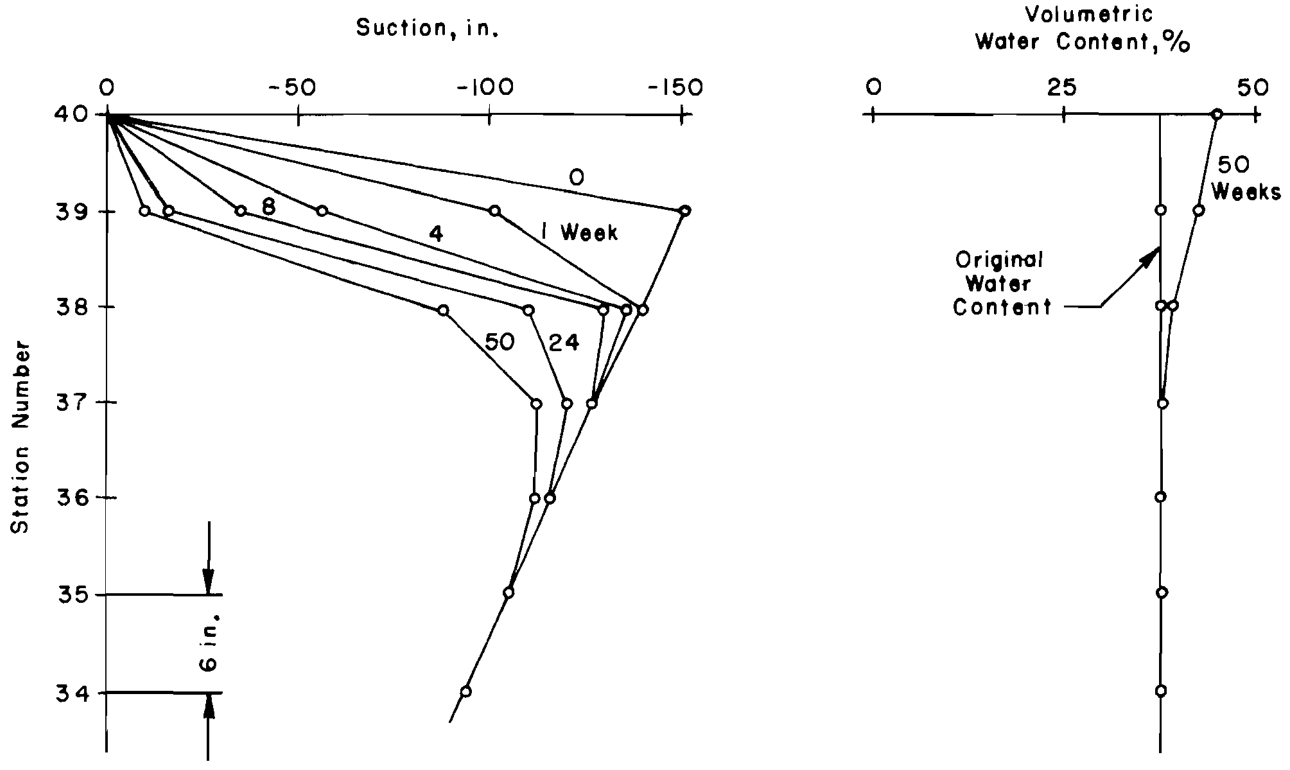


(b) Sand well ponding problem.

Fig 24. Ponding problems.



(a) POND1



(b) POND3 - expansive effects included.

Fig 25. Suction and moisture at Station 4, soil initially at plastic limit.

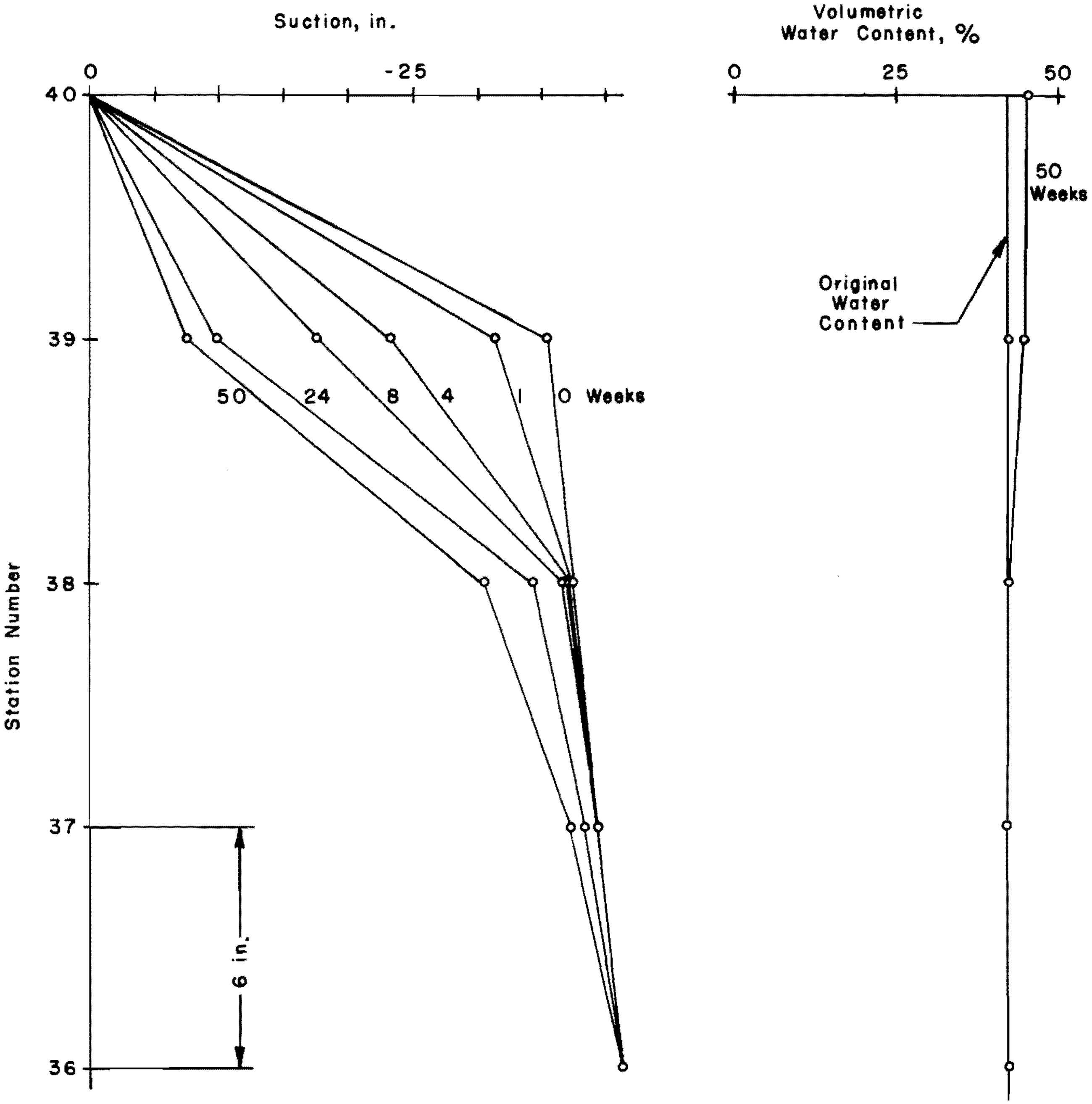


Fig 26. POND2 - soil initially wetter than plastic limit.

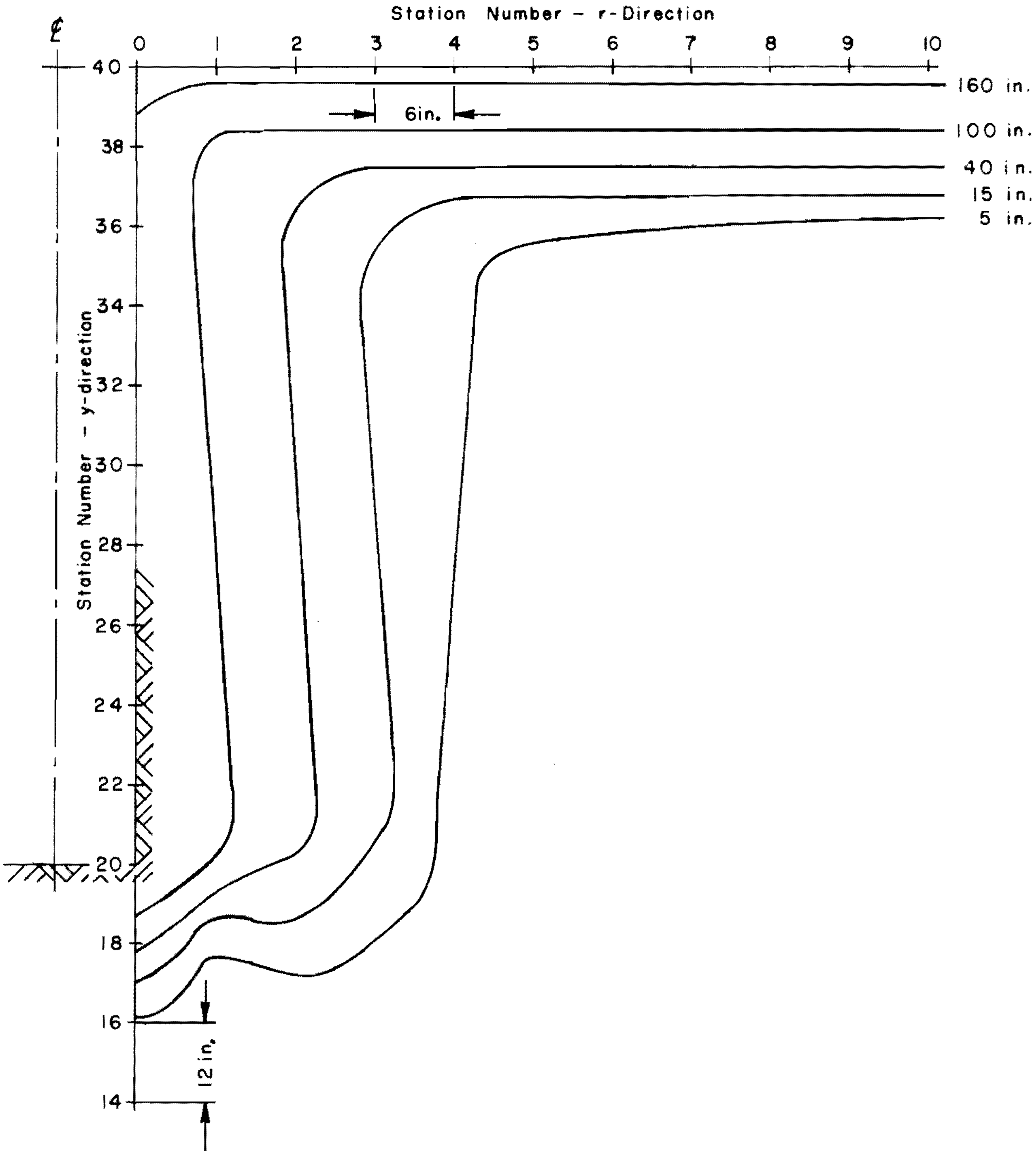


Fig 27. PW1 - change of suction in 50 weeks.

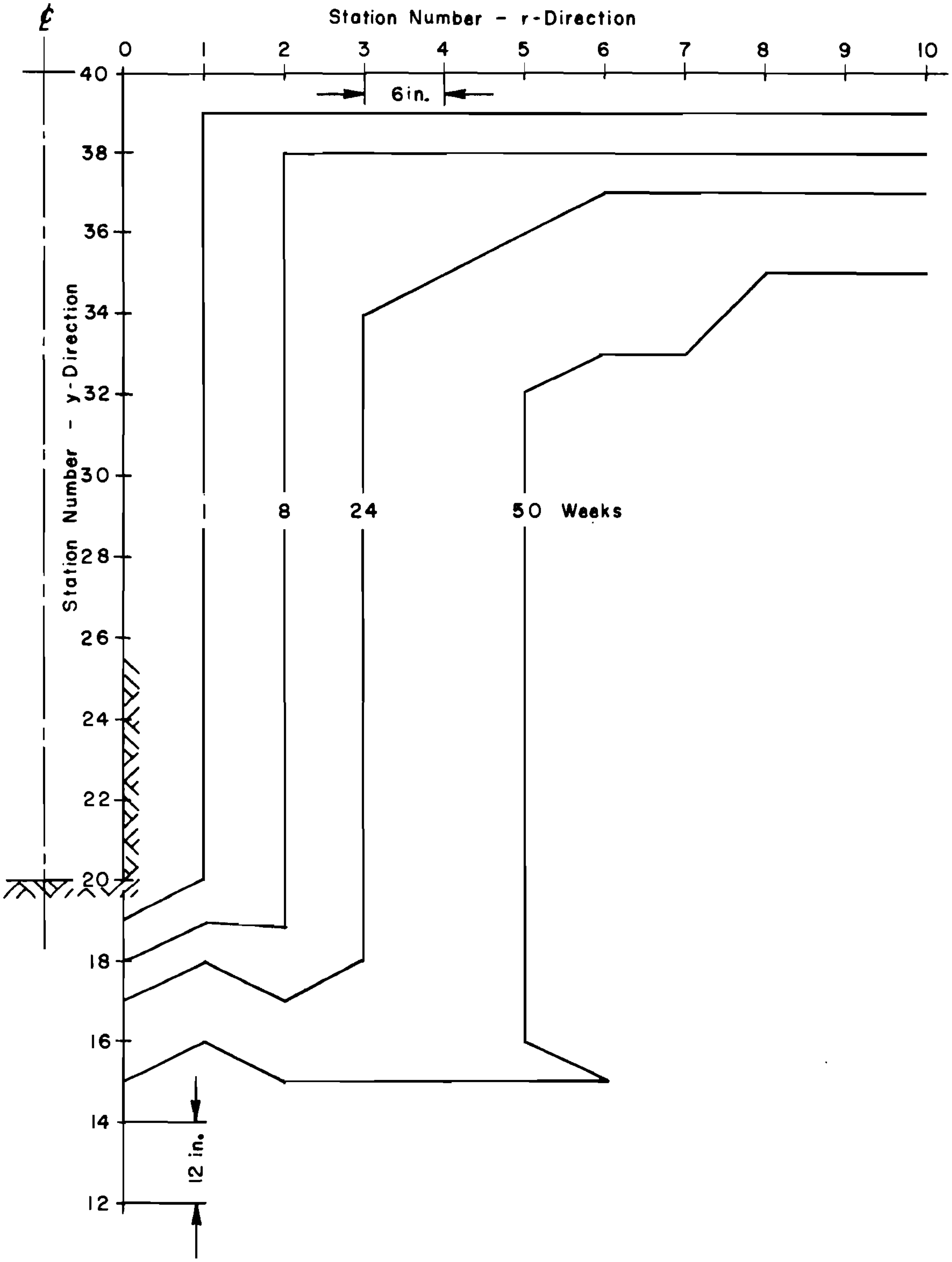


Fig 28. PW1 - isochrones for suction change of 1 inch or more.

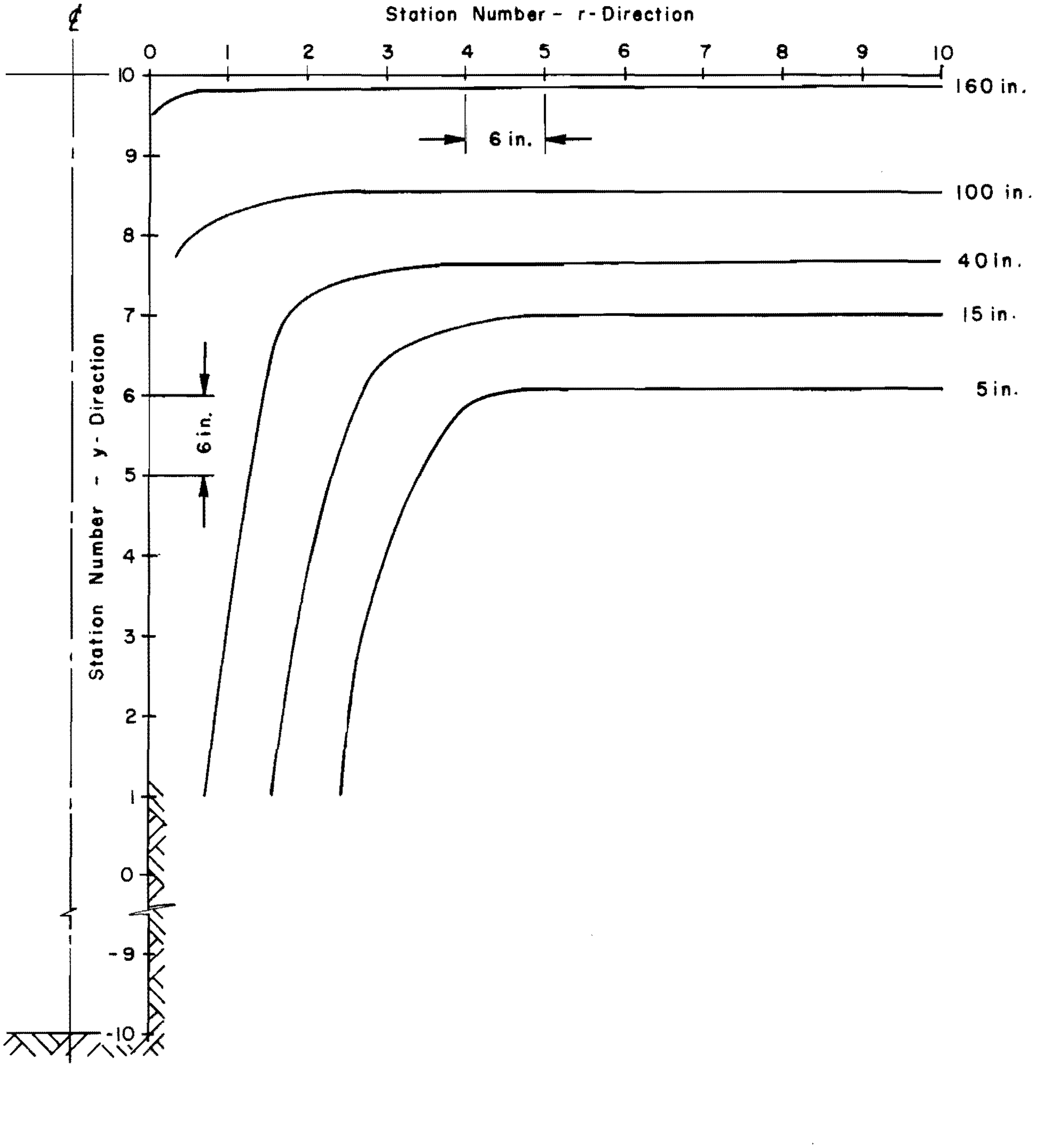


Fig 29. PW2 - sand well with expansive effects included, contours of suction change in 50 weeks.

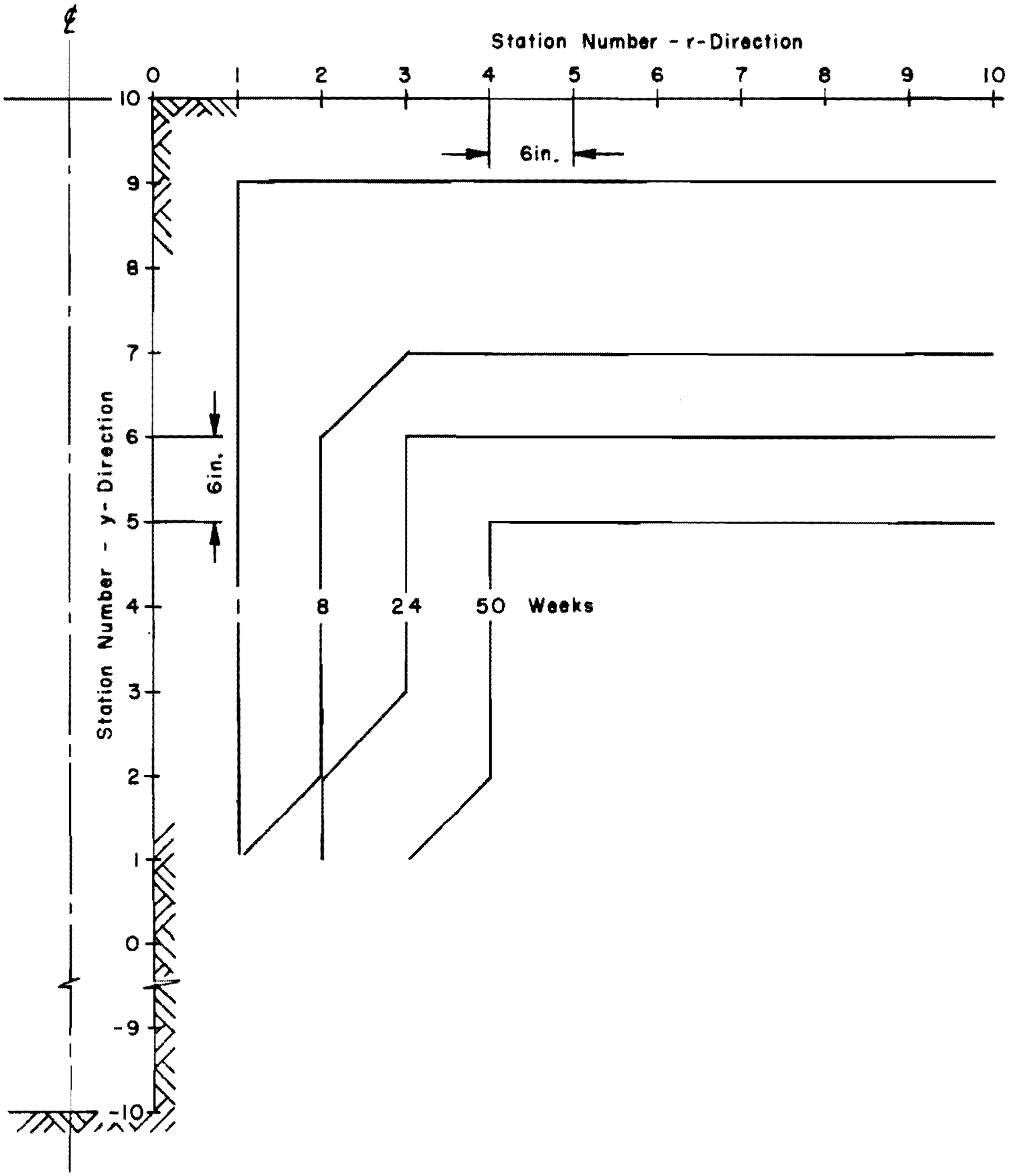


Fig 30. PW2 - isochrones for suction change of 1 inch or more.

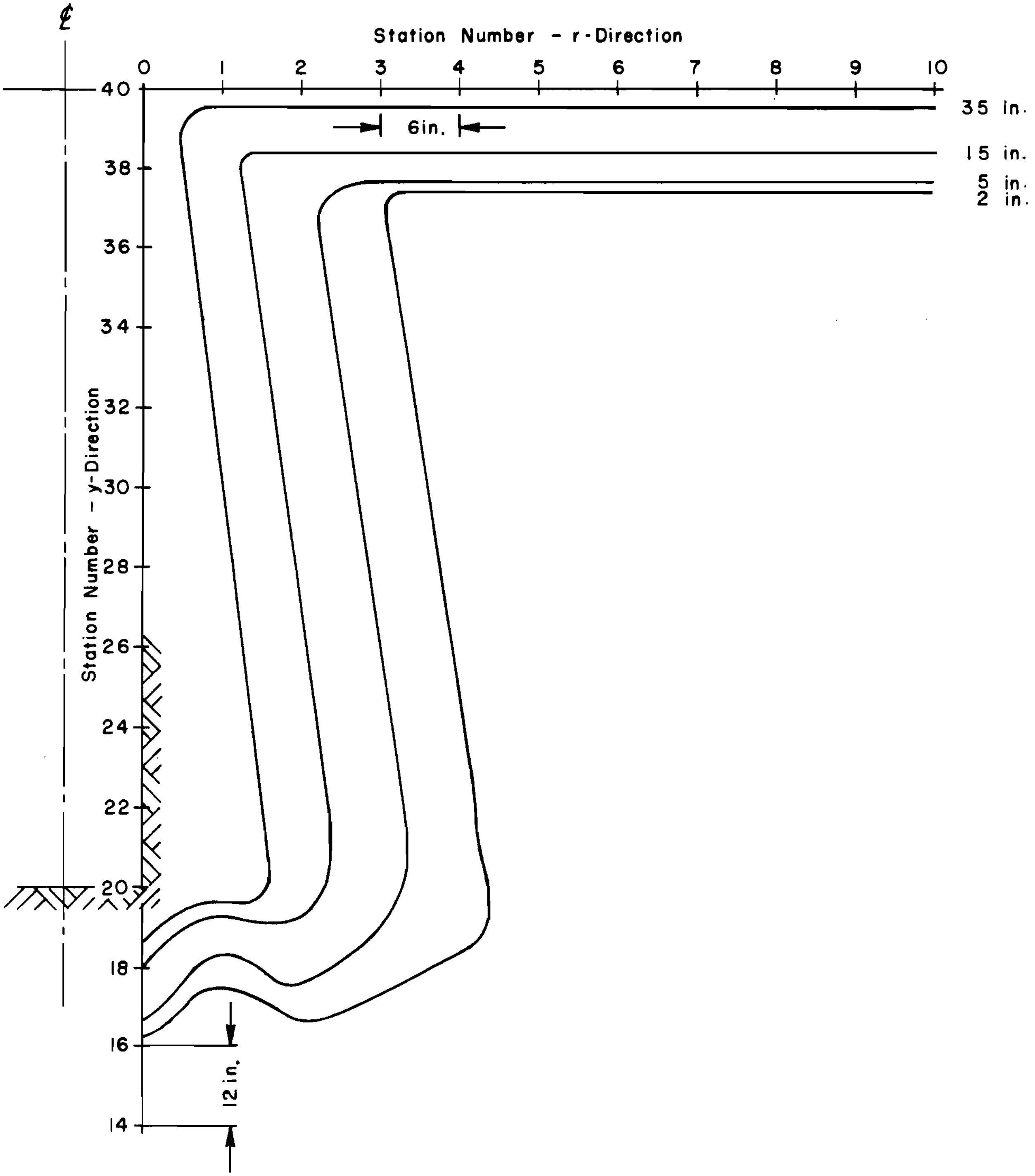


Fig 31. PW3 - change of suction in 50 weeks.

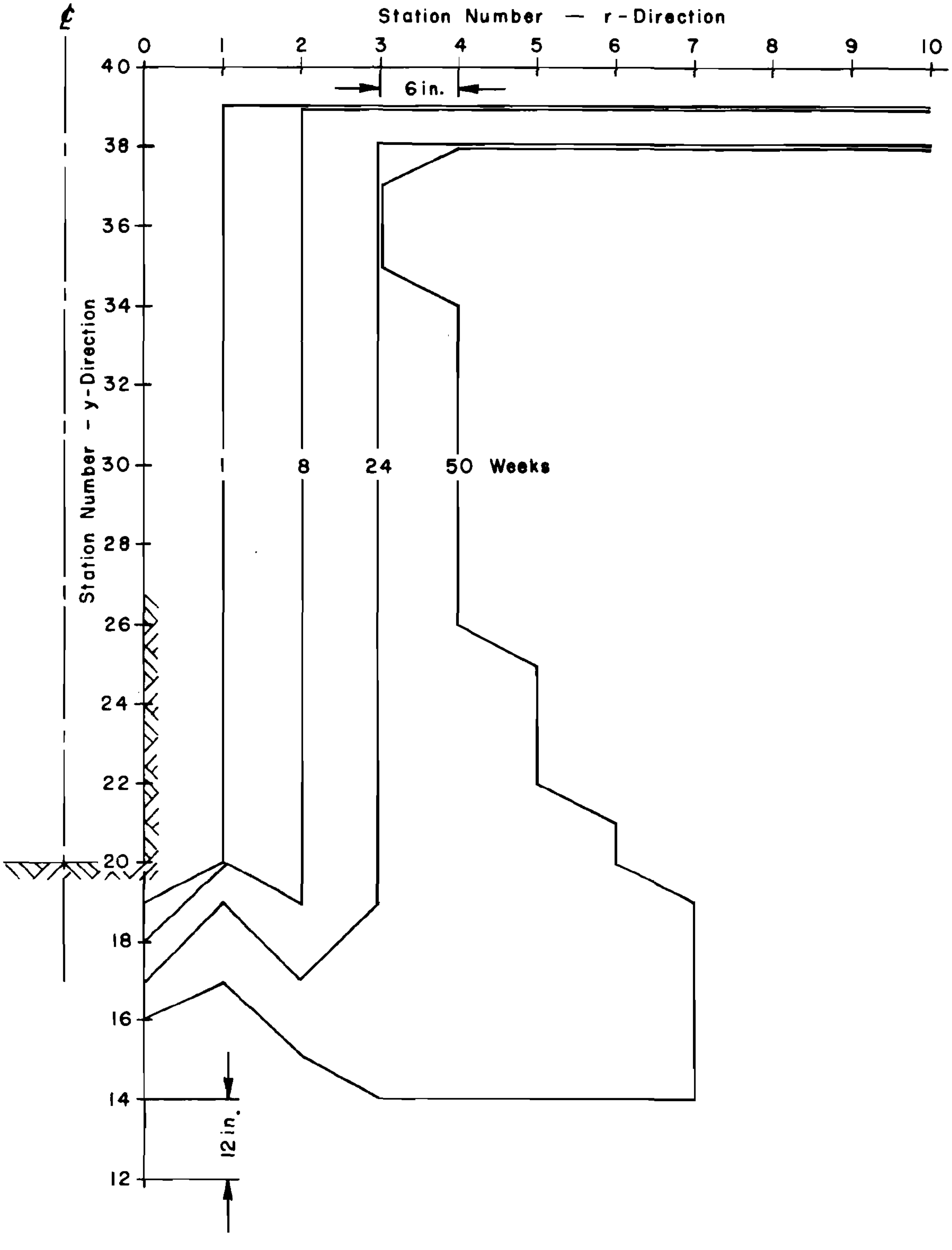


Fig 32. PW3 - isochrones for suction change of 1 inch or more.

TABLE 15. VARIATIONS OF PONDING PROBLEMS

Problem	Type of Ponding	Soil Initially at Plastic Limit	Soil Wetter than Plastic Limit	Overburden Pressure Considered
POND1	Surface	x		
POND2	Surface		x	
POND3	Surface	x		x
PW1	Surface and sand well	x		
PW2	Surface and sand well	x		x
PW3	Surface and sand well		x	

change of suction in 50 weeks and the other shows approximate isochrone lines for suction changes of at least 1 inch. Figures 29 and 30 show results of the expansive problem, PW2. Because overburden pressure "overcomes" suction at about 7 feet of depth, only results for the top 5 feet were computed.

The scale of Fig 25 is too fine to reveal the small details of moisture and suction change. Table 16 provides a list of moisture and suction changes over a period of 50 weeks for Problems POND1 and POND3.

Table 16 shows that there is a difference in the suction and moisture change depending on whether the soil weight and expansibility are considered. When these factors are considered, suction and moisture change are lower, but not by a relatively large amount. Because prediction of swell is related to moisture change, it appears that the analysis which excludes expansive effects would overestimate swell, but not by a substantial amount. Assuming swell to be equal to volumetric moisture content change and to occur in the vertical direction alone, POND3 predicts 0.84 inches and POND1 predicts 0.92 inches of swell in the upper 30 inches.

TABLE 16. COMPARISON OF RESULTS, POND1 VERSUS POND3, CHANGES IN 50 WEEKS

Depth Below Surface, inches	POND 1		POND 3	
	ΔT, inches	$\Delta\theta$ percent	ΔT, inches	$\Delta\theta$ percent
0	162.9	7.20	162.9	7.20
6	151.8	5.98	140.3	5.01
12	61.8	1.58	51.4	1.27
18	17.1	0.39	16.3	0.37
24	5.2	0.12	5.1	0.11
30	1.6	0.04	1.7	0.04
36	1.2	0.03	0.7	0.02
42	0.6	0.01	0.4	0.01

Effect of Expansibility of Soil. The imprecise term "overcoming suction" has been used and is clarified in this section. When water becomes available to an expansive soil, the suction in the soil draws the water in, and the soil tends to swell. If the soil is not restrained, it will swell freely. If it is restrained from changing its volume, it develops an internal excess pore pressure which can be very large in comparison with most building loads. If the soil is restrained by an isotropic stress p, then a portion of the suction in the soil must be converted into an excess pore pressure which is exactly equal to p before volume change or change of moisture can take place. This process could be viewed as an "overcoming" by the isotropic pressure p of an amount of suction equal to $\alpha_{po}\, p/\gamma_w$. This is precisely the point developed in Chapter 4 of Research Report No. 118-1 regarding the constant water content test. The experiments made by Croney, Coleman, and Black (Ref 5) showed that the water content of a soil sample does not change as long as change in suction from the free swell condition is equal to $\alpha_{po}\, p/\gamma_w$.

In both expansibility problems, it is assumed that the conversion of suction to an excess pore water pressure occurs instantaneously when water becomes available to the soil at any depth. The only suction that remains to change water content and volume is that which has not been "overcome" by isotropic pressure. In the expansive problems, this isotropic pressure is assumed to be the overburden pressure. This assumption is in error in the vicinity of the sand well because the horizontal pressure is decreased by the presence of the drilled hole. Similarly, the presence of shrinkage cracks can relieve horizontal pressure to an active state, reducing the confining effect of overburden pressure. In general, the confining conditions, both vertical and horizontal, should be considered in determining the isotropic pressure against which swelling occurs. Thus, the use of an inert surcharge over an expansive clay effectively reduces the suction available for changing volume. The amount of reduction depends on the level of suction in the soil.

The expansibility coefficient of a soil may be taken from the rebound curve in a drained triaxial test or consolidation test. If no experimental data are available this coefficient could be assumed equal to the preconsolidated compressibility coefficient.

The results of these six problems reinforce the qualitative opinions held by engineers on the technical superiority of sand wells. In addition, the fact that antecedent moisture conditions determine the amount of infiltration that will occur has been clearly demonstrated. The wetter the soil before ponding, the less water will be absorbed by the soil. Presence of weather cracks and slickensides in soil reduces the effective confining pressure so that suction changes which actually occur may be at some intermediate stage between those presented in this study.

Distribution of Moisture in a Concrete Girder

Although this problem does not apply strictly to soil, it does indicate the mathematical and physical similarity of moisture movement in clay and concrete. This problem also reveals another possibility for experimental verification and predictive use of the computer programs presented in this report.

Powers and Brownyard (Ref 15) of the Portland Cement Association presented data relating the water-cement ratio of hardened cement paste to the relative vapor pressure with which it comes into equilibrium. These vapor pressures have been converted to suction and included in Table 17 with the water content and relative vapor pressure data.

TABLE 17. SUCTION-MOISTURE RELATION FOR HARDENED CEMENT PASTE

Water Content, $\frac{\text{wt of water}}{\text{wt of cement}}$	Relative Vapor Pressure	Suction, cm of water	pF
15.47	0.00		-
17.98	0.08	-3.54×10^6	6.55
18.87	0.16	-2.66×10^6	6.43
20.10	0.32	-1.59×10^6	6.21
20.82	0.39	-1.32×10^6	6.12
21.44	0.46	-1.08×10^6	6.03
22.02	0.53	-0.89×10^6	5.95
22.82	0.60	-0.72×10^6	5.86
24.82	0.70	-0.50×10^6	5.70
27.52	0.81	-0.296×10^6	5.47
30.43	0.88	-0.179×10^6	5.25
35.31	0.96	-0.0575×10^6	4.76

These data are plotted in Fig 33 along with the approximate curve used in the computer program. The exponent for the pF-water-content curve is 2.13 and was determined from a plot of the experimental data on log-log graph paper.

An attempt has been made to input these data as exactly as possible into Computer Program GCHPIP1 to allow calculation of the evaporable water that is retained in a large concrete highway girder subjected to a drying atmosphere.

The size of the girder is chosen to be approximately the same overall dimensions as the largest standard Texas Highway Department prestressed highway bridge girder. An overall depth of 54 inches and flange width of 24 inches was assumed. The web thickness is unrealistically large at 9.6 inches.

The cross section of girder shown in Fig 34 is assumed to be cast at a water-cement ratio of 0.50 after bleeding. It is surrounded by an atmosphere that remains at 80° F and at 15 percent relative humidity. The problem does not completely model field conditions because the upper flange of the girder is ordinarily covered by a deck slab and is subjected to different atmospheric conditions than the rest of the girder. In addition, stress gradients will cause moisture migrations from the compression into the tensile zone. Both of these effects could be modeled with GCHPIP1, of course, but inclusion of these effects will detract from the use of this problem as an example.

Permeability of cement paste is assumed to be the permeability of the concrete. The value of 5.9×10^{-12} in/sec is drawn from a paper by Powers, Copeland, Hayes, and Mann (Ref 16).

Computation of moisture distribution with time in the concrete girder was attempted using two approaches:

(1) recognizing the fact that permeability of cement paste increases with drying, and

(2) assuming permeability is constant with decreasing water content.

Experimental observations reported in Ref (16) show that permeability of cement paste may increase by a factor of 70 as the paste is dried to 80 percent relative humidity. It was assumed that permeability would be 100 times larger at 8 percent relative humidity and the following set of a, b, and n were computed for the moisture dependent case.

$$a = 2.54 \text{ cm/in.}$$

$$b = -1.0858$$

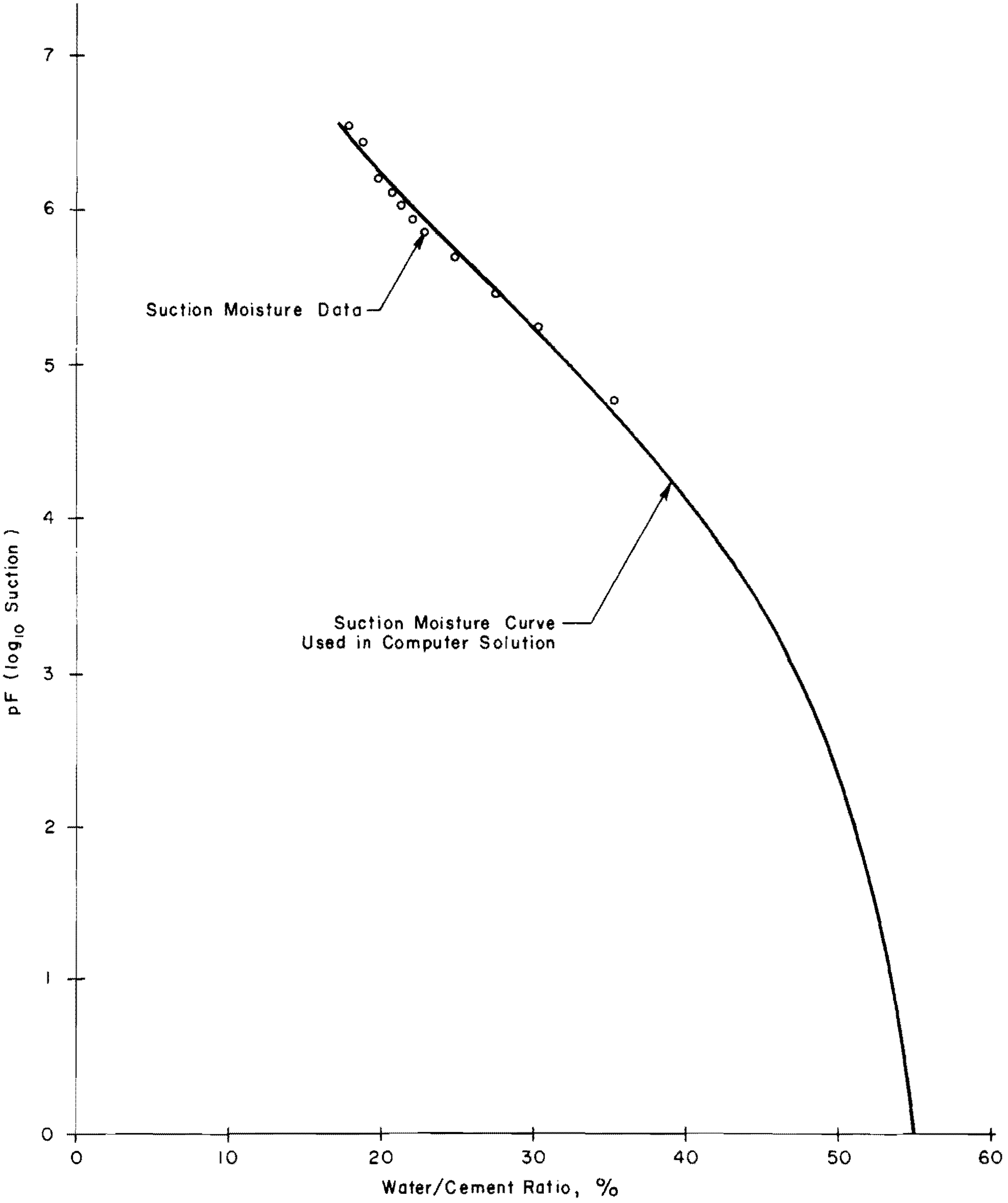


Fig 33. Graph of concrete suction-moisture curves.

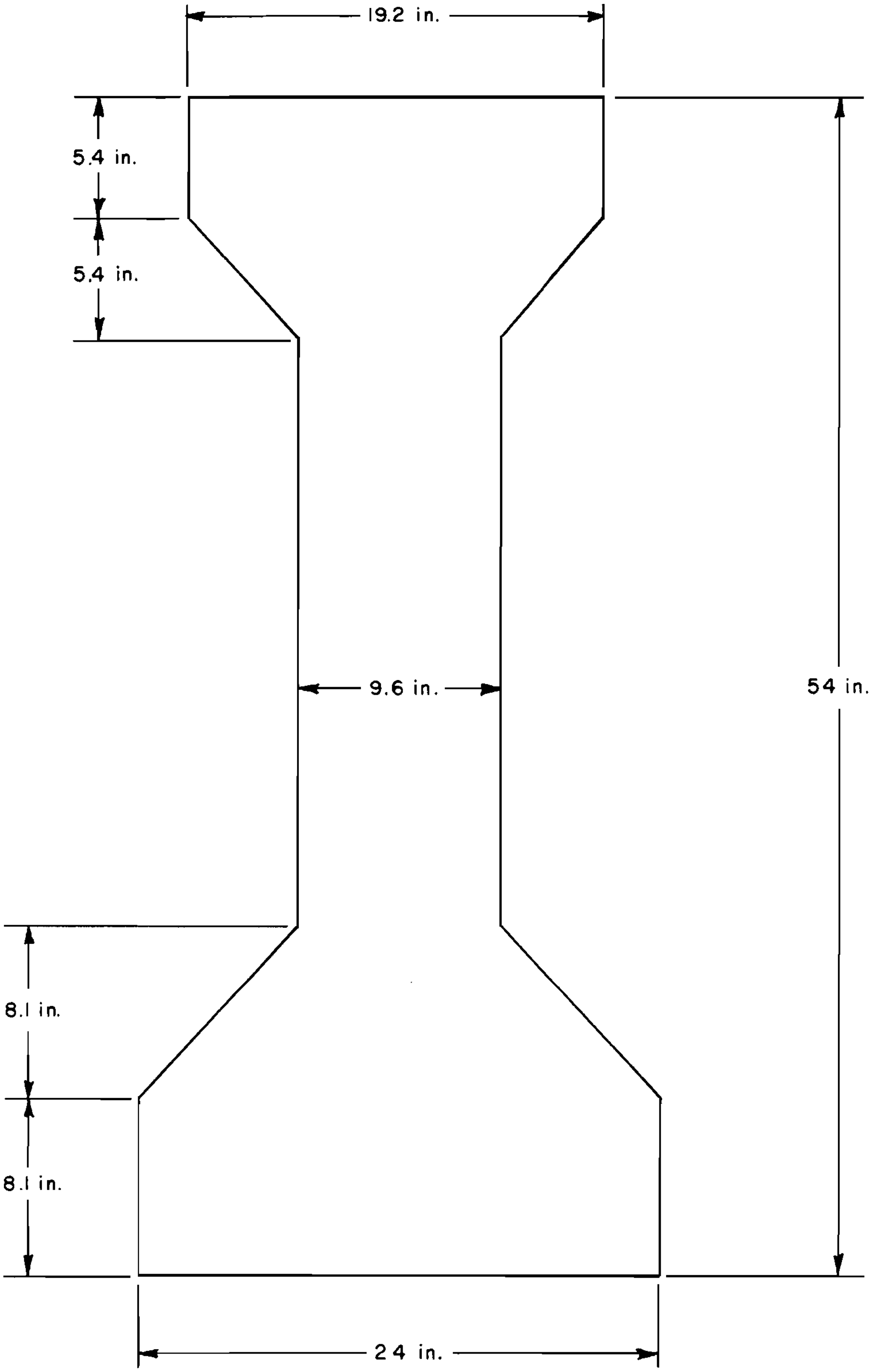


Fig 34. Concrete girder cross section.

$n = .0054$

The values of a, b, and n for the constant permeability case are as follows:

$a = 0.0$

$b = 1.0$

$n = 1.0$

The negative value of b and the small size of n in the moisture dependent case resulted in an unstable solution process in which negative permeabilities were computed which in turn induced larger errors in computed suction values. This instability is akin to the instability noticed in the house foundation problem and predicted analytically in Chapter 3. Because of the erratic results achieved, none of the data for this case are presented here.

Results of computations in the constant permeability case are shown in Fig 35 on the left. In Fig 35 are shown contours of equal water-cement ratio after a period of about one year. The right side of Fig 35 shows the same contours after three years. It is readily apparent that the concrete within the flanges remains substantially wetter than that at the exterior for long periods of time even in every dry climates. This higher water content in the interior of concrete structural members has been observed in thick concrete columns by personnel of the Portland Cement Association. This last example shows the versatility of Computer Program GCHPIP1 in solving unusual problems with odd geometry.

The most important finding of the studies reported in this chapter is that it is possible to make quantitative prediction of results such as these for a heterogeneous, anisotropic region of soil or concrete in which unsaturated water movement occurs.

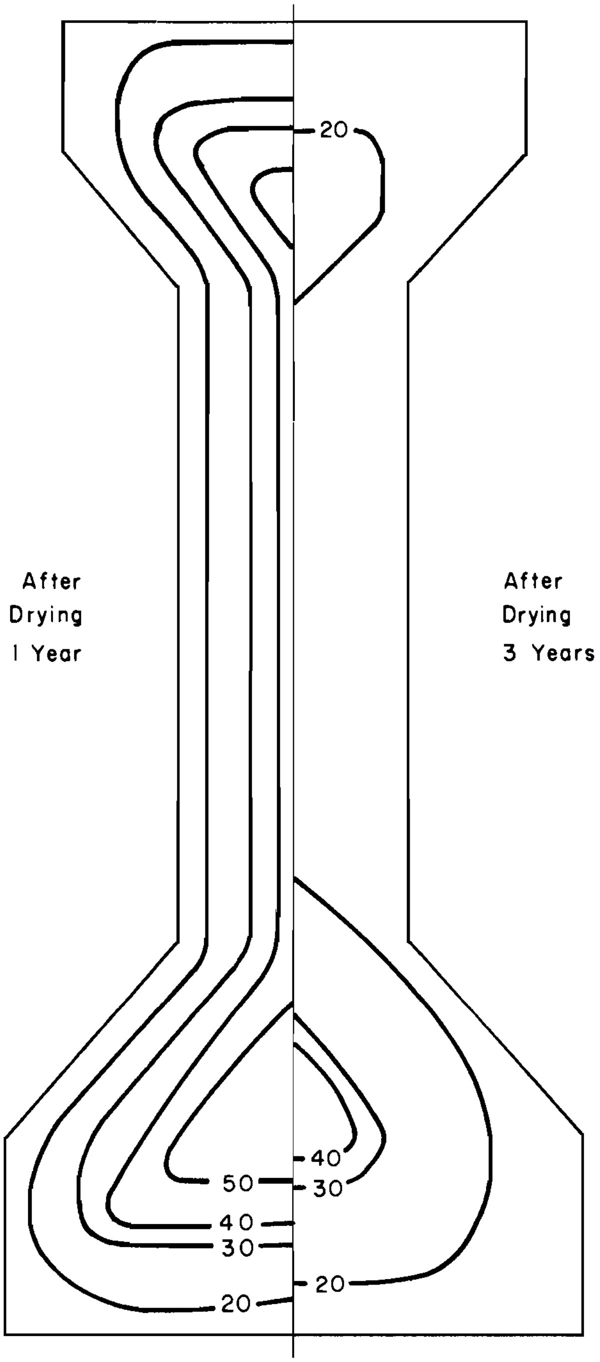


Fig 35. Water-cement ratios after drying in 15 percent relative humidity, 80° F atmosphere.

CHAPTER 8. CONCLUSIONS AND APPLICATIONS

The two computer programs described in this report, GCHPIP1 and FLOPIP2, are written to solve a nonlinear concentration-dependent partial differential equation for moisture movement in soil regions with irregular boundaries using a discrete-element model of the flow process.

The computer programs have been tested for accuracy and validity against theoretical series solutions and against field data. In both of these widely divergent cases, the methods used have been proven valid. The results of computer simulation of moisture change measured by University of Wyoming personnel at their West Laramie test site show that computer prediction of such results is not only practical but is quite accurate. In addition, the procedure has been demonstrated by which field data can be used by the computer to determine field permeability characteristics of the soil. If one has access to data such as these, it may no longer be necessary, except as a check, to make laboratory permeability tests, the results of which are questionable in many cases.

Chapter 2 contains an abbreviated discussion of the flow equations solved by the computer programs of this report. A more comprehensive treatment of the subject is given in Chapter 5 of Research Report 118-1. The flow equation is a nonlinear parabolic partial differential equation which is normally solved in one of two ways:

(1) a closed form series solution and

(2) a numerical method which starts with some initial condition and marches forward in time, computing incremental changes from one time step to the next.

Of these, the first method is applicable only in the most well-behaved problems. Several possibilities exist in the second approach.

Chapter 3 presents two of these possible approaches: the forward difference method and the implicit Crank-Nicolson method. The latter is used in both Computer Programs of this report because of its inherent stability. A method is defined as stable if errors do not normally grow with time regardless of the size of time step chosen. An interesting discovery in numerical analysis is described in this chapter in a demonstration of a rare form of instability in the Crank-Nicolson method associated with the angle between the horizontal and the direction of principal permeability.

Chapters 4 and 5 outline the details of the two computer programs, GCHPIP1 and FLOPIP2, respectively. A prospective user of these programs is advised to read these chapters carefully.

In Chapter 6, field data from tests by University of Wyoming personnel were studied in detail using the one-dimensional computer program. These data will be considered further in Research Report 118-4, in which computer programs to predict swell will be presented. It is impossible to present in a single chapter any but the most austere outline of the information and artificial experience gained from the many computer trials made in an effort to match the Wyoming field data. The outline presented is encouraging as it provides guidance for an efficient choice of the unsaturated permeability parameters b , n , and k_{sat} .

In Chapter 7, a number of example problems are presented which have been solved with the two-dimensional computer program. Solution of a two-dimensional consolidation problem is checked with series solutions, some of which were not within their region of convergence. This problem indicates two pertinent points:

(1) the accuracy of the computer method and
(2) the difficulty of achieving closed form solutions even in such fairly well-defined problems as this one.

Predictions of moisture accumulation around a pipe-casing, beneath a house, and in stratified clay due to ponding and sand wells were presented to emphasize the versatility of the computer program. The computation of moisture distribution in a concrete girder is presented as an example of the broad scope of applicability of the suction-moisture approach adopted as a basis in these computer programs. Based on these latter findings, it would be possible to use moisture distribution in concrete members to determine field permeability conditions in concrete.

The computer programs of this report were devised with a comprehensive theoretical development as a foundation and with an accurate and stable numerical method as a framework. Their demonstrably broad scope of applicability is a planned result and their use for practical analytical and predictive purposes is to be expected. The accuracy that can be achieved in using these computer programs is excellent provided the input data are of as high a quality.

High quality input data are normally difficult to achieve but, as shown in this report, a considerable amount of detailed information about the soil can be gained from field tests and this information can be used subsequently to make quite reliable predictions. In this way, the computer programs of this report can be used to improve the quality of their own input information.

Computer Programs GCHPIP1 and FLOPIP2 are expected to provide widely applicable and versatile tools for analysis, prediction, and data improvement of moisture movement in porous materials.

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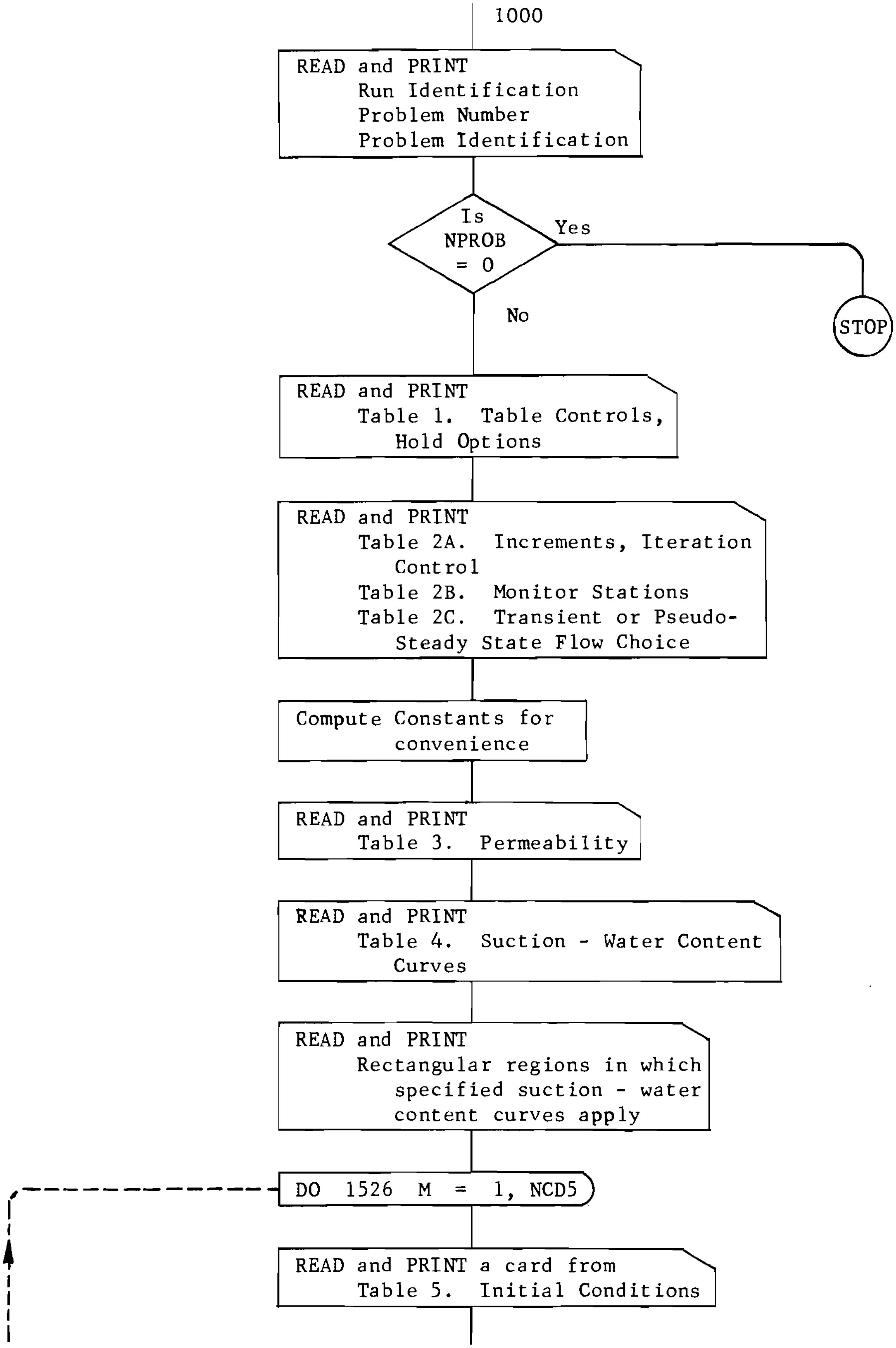
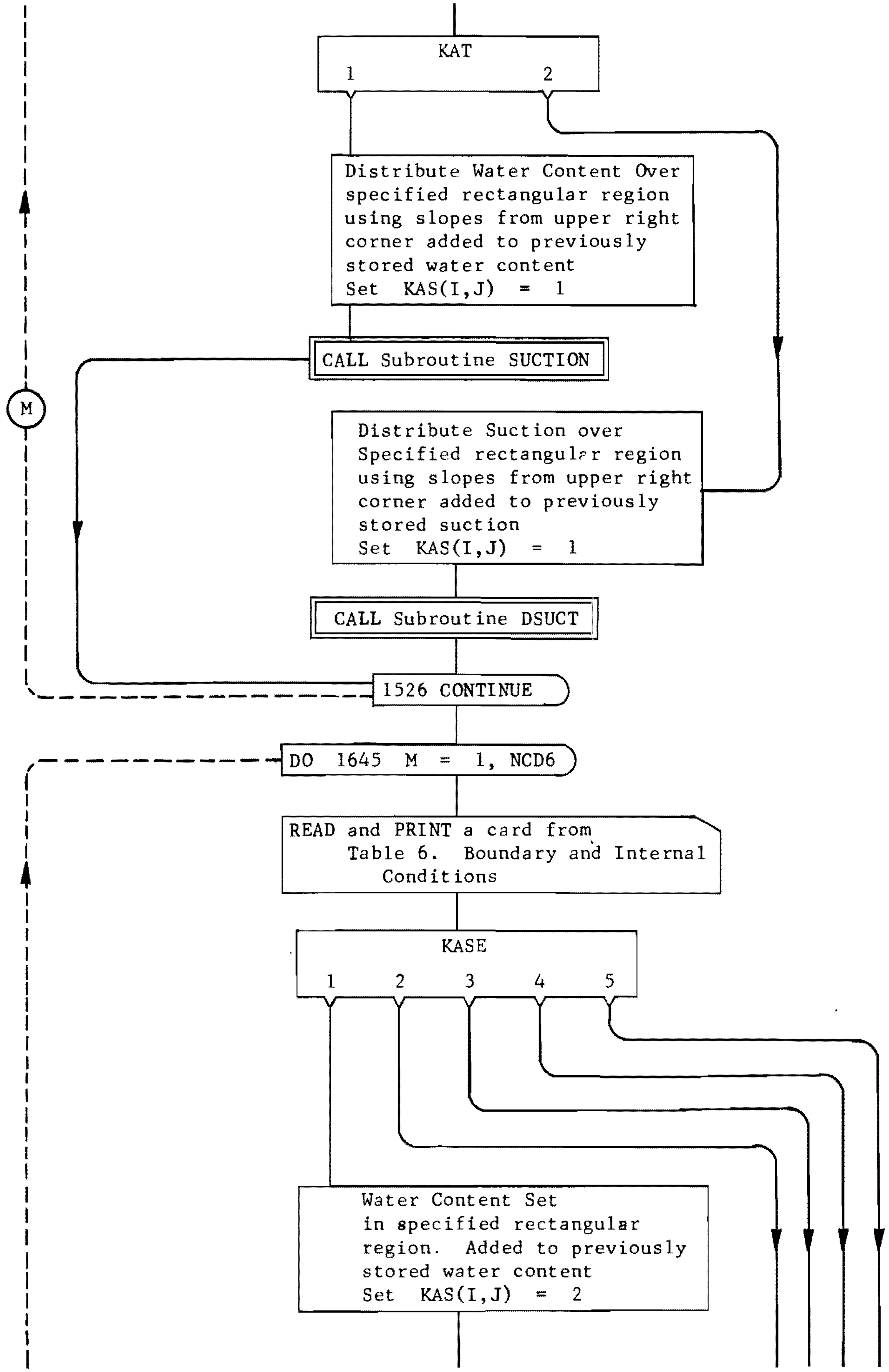
APPENDICES

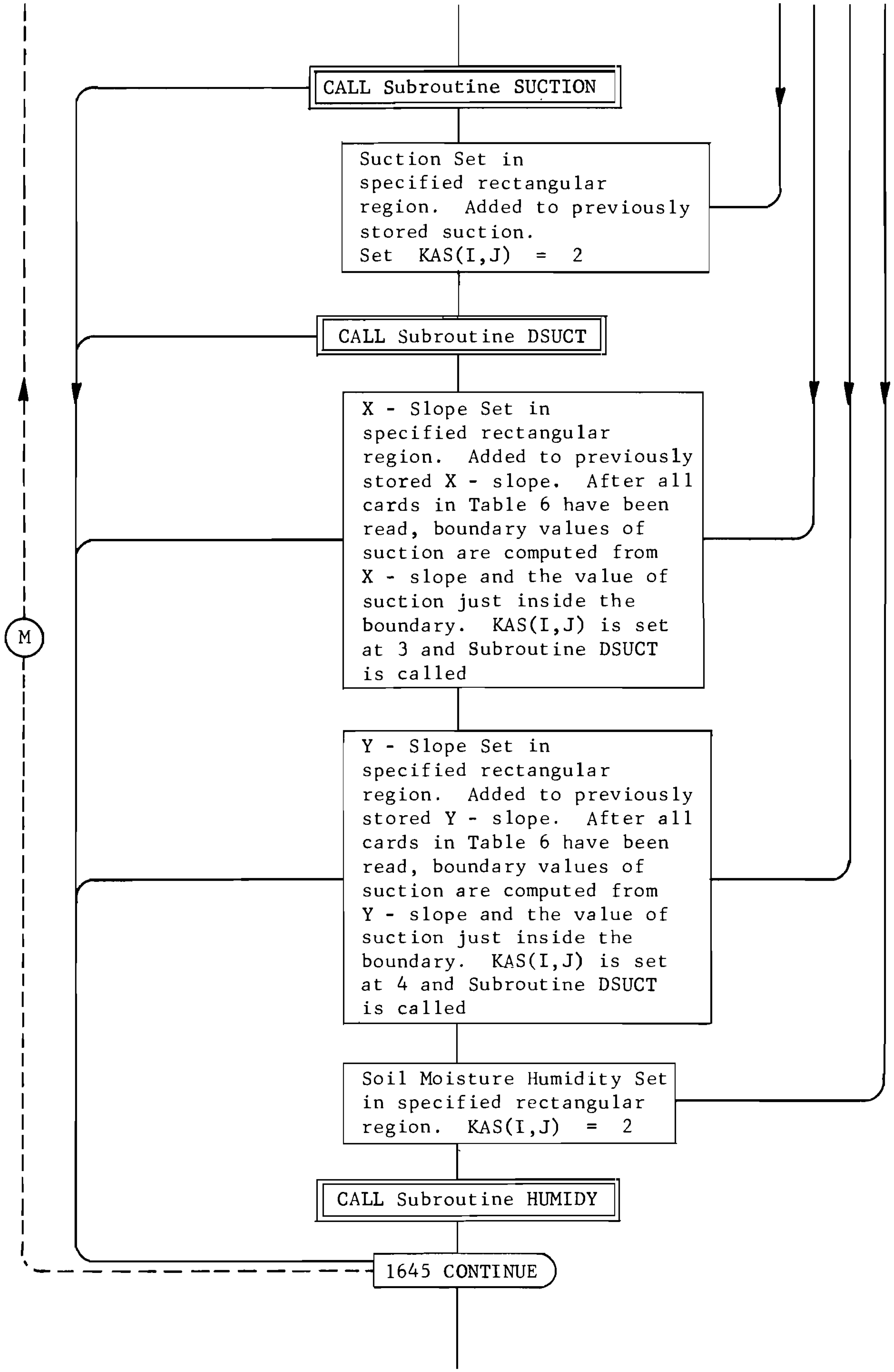
APPENDIX 1

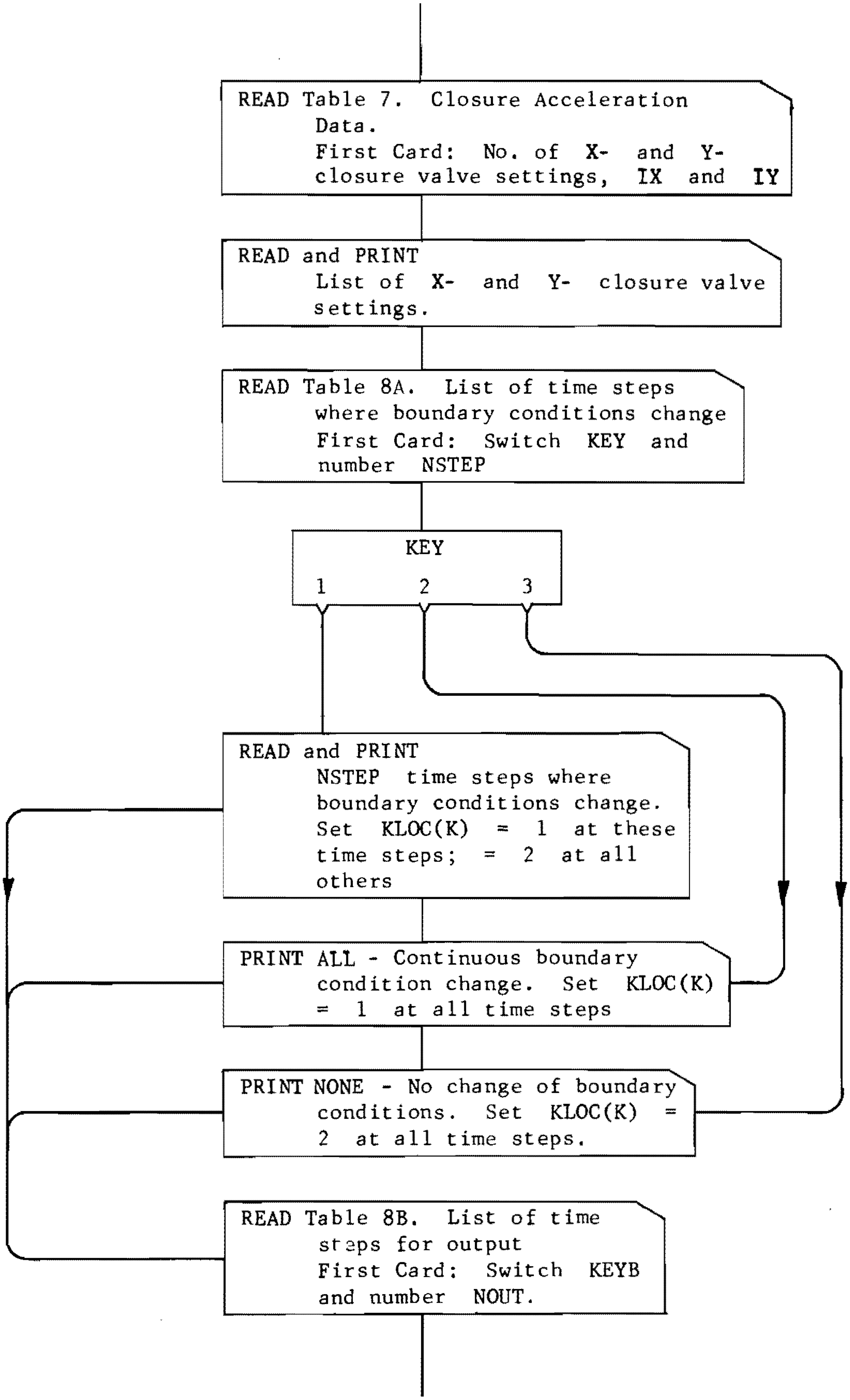
FLOW CHARTS FOR PROGRAM GCHPIP1

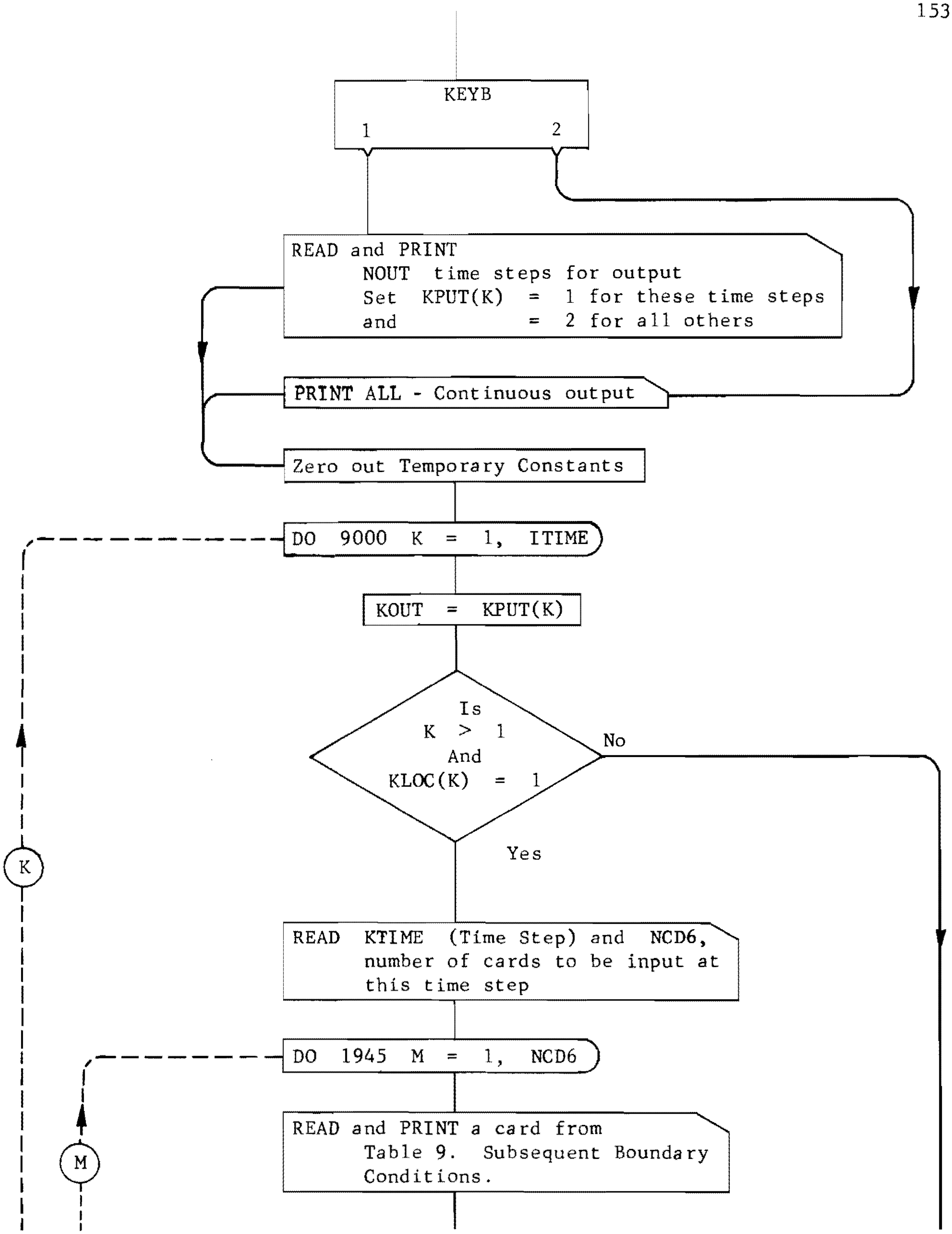
PROGRAM GCHPIP1

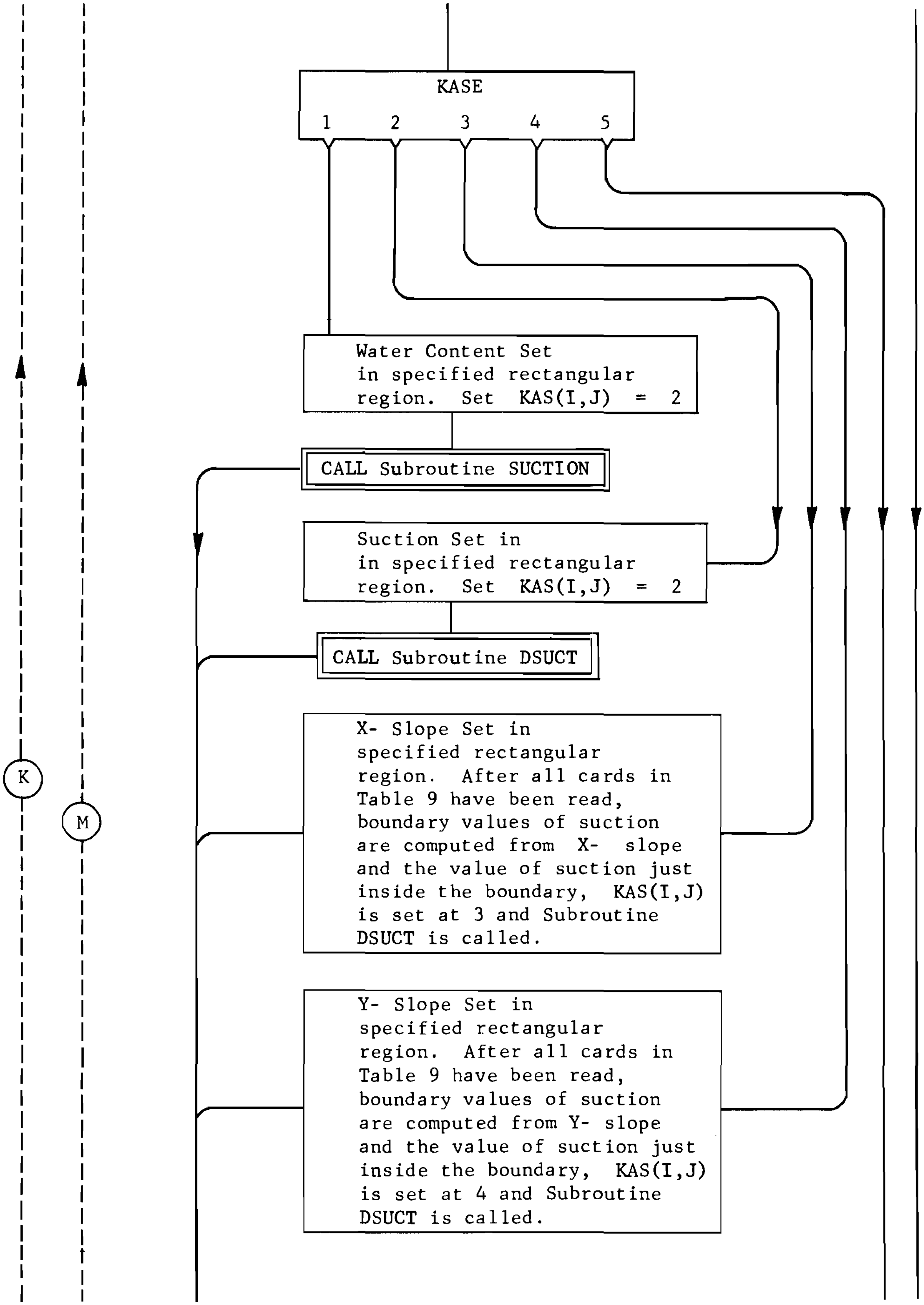



KAT
1
2
Distribute Water Content Over specified rectangular region using slopes from upper right corner added to previously stored water content
Set KAS(I,J) = 1
CALL Subroutine SUCTION
M
Distribute Suction over Specified rectangular region using slopes from upper right corner added to previously stored suction
Set KAS(I,J) = 1
CALL Subroutine DSUCT
1526 CONTINUE
DO 1645 M = 1, NCD6
READ and PRINT a card from Table 6. Boundary and Internal Conditions
KASE
1
2
3
4
5
Water Content Set in specified rectangular region. Added to previously stored water content
Set KAS(I,J) = 2


CALL Subroutine SUCTION
Suction Set in specified rectangular region. Added to previously stored suction. Set KAS(I,J) = 2
CALL Subroutine DSUCT
X - Slope Set in specified rectangular region. Added to previously stored X - slope. After all cards in Table 6 have been read, boundary values of suction are computed from X - slope and the value of suction just inside the boundary. KAS(I,J) is set at 3 and Subroutine DSUCT is called
Y - Slope Set in specified rectangular region. Added to previously stored Y - slope. After all cards in Table 6 have been read, boundary values of suction are computed from Y - slope and the value of suction just inside the boundary. KAS(I,J) is set at 4 and Subroutine DSUCT is called
Soil Moisture Humidity Set in specified rectangular region. KAS(I,J) = 2
CALL Subroutine HUMIDY
1645 CONTINUE
M


READ Table 7. Closure Acceleration Data.
First Card: No. of X- and Y- closure valve settings, IX and IY
READ and PRINT
List of X- and Y- closure valve settings.
READ Table 8A. List of time steps where boundary conditions change
First Card: Switch KEY and number NSTEP
KEY
1
2
3
READ and PRINT
NSTEP time steps where boundary conditions change. Set KLOC(K) = 1 at these time steps; = 2 at all others
PRINT ALL - Continuous boundary condition change. Set KLOC(K) = 1 at all time steps
PRINT NONE - No change of boundary conditions. Set KLOC(K) = 2 at all time steps.
READ Table 8B. List of time steps for output
First Card: Switch KEYB and number NOUT.


KEYB
1
2
READ and PRINT
NOUT time steps for output
Set KPUT(K) = 1 for these time steps
and = 2 for all others
PRINT ALL - Continuous output
Zero out Temporary Constants
DO 9000 K = 1, ITIME
KOUT = KPUT(K)
Is
K > 1
And
KLOC(K) = 1
No
Yes
K
READ KTIME (Time Step) and NCD6,
number of cards to be input at
this time step
DO 1945 M = 1, NCD6
M
READ and PRINT a card from
Table 9. Subsequent Boundary
Conditions.

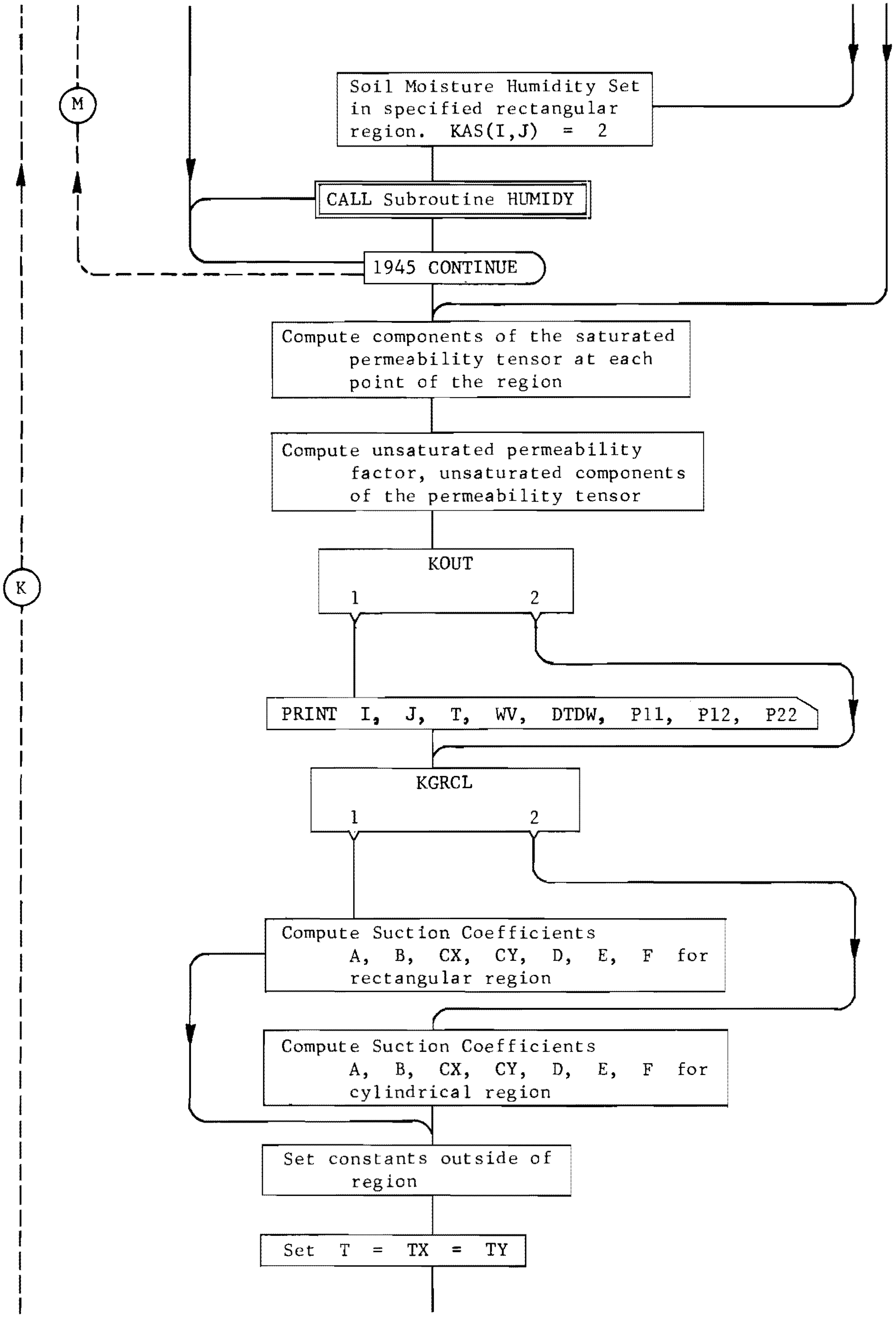


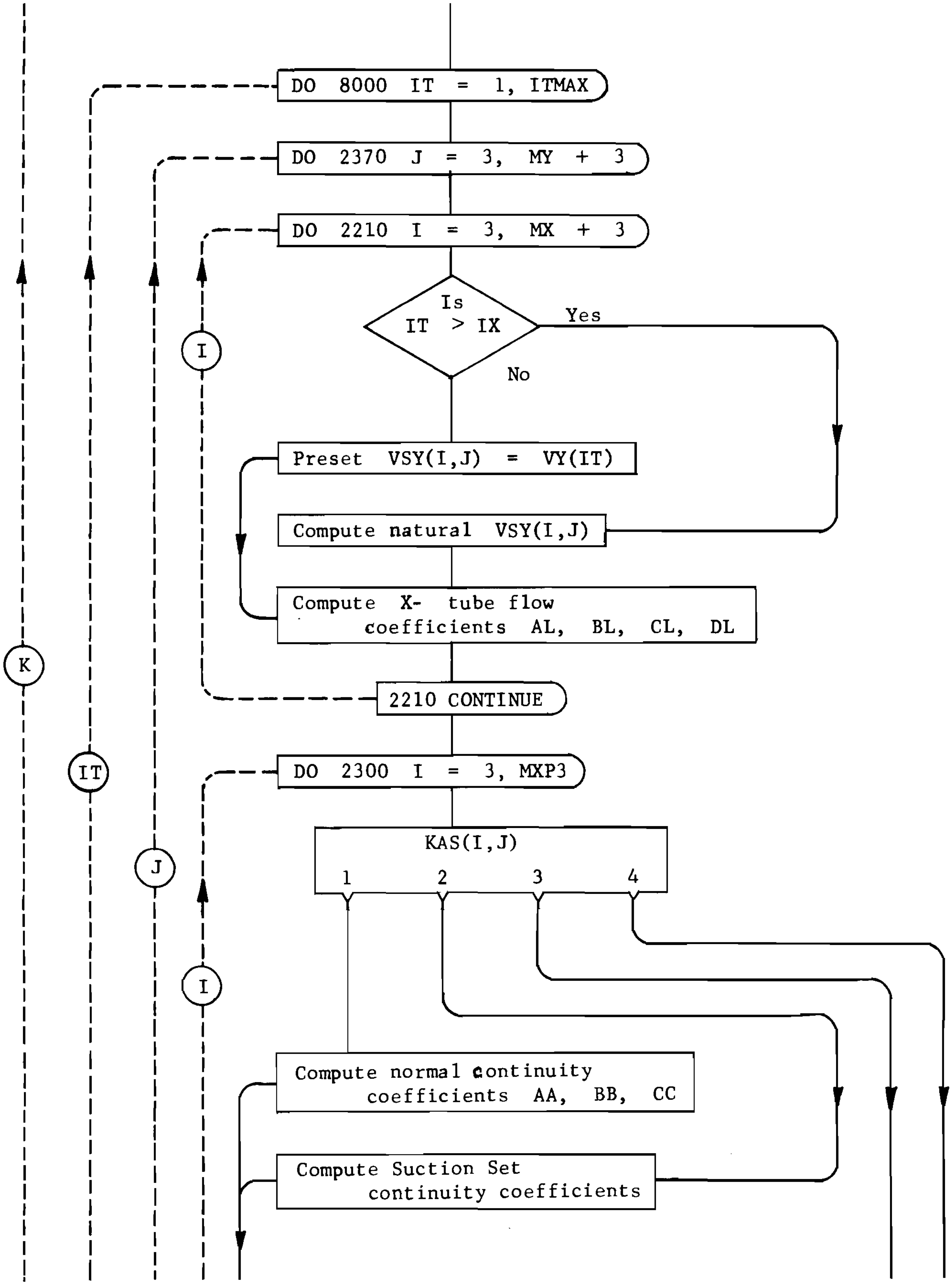
Water Content Set in specified rectangular region. Set KAS(I,J) = 2

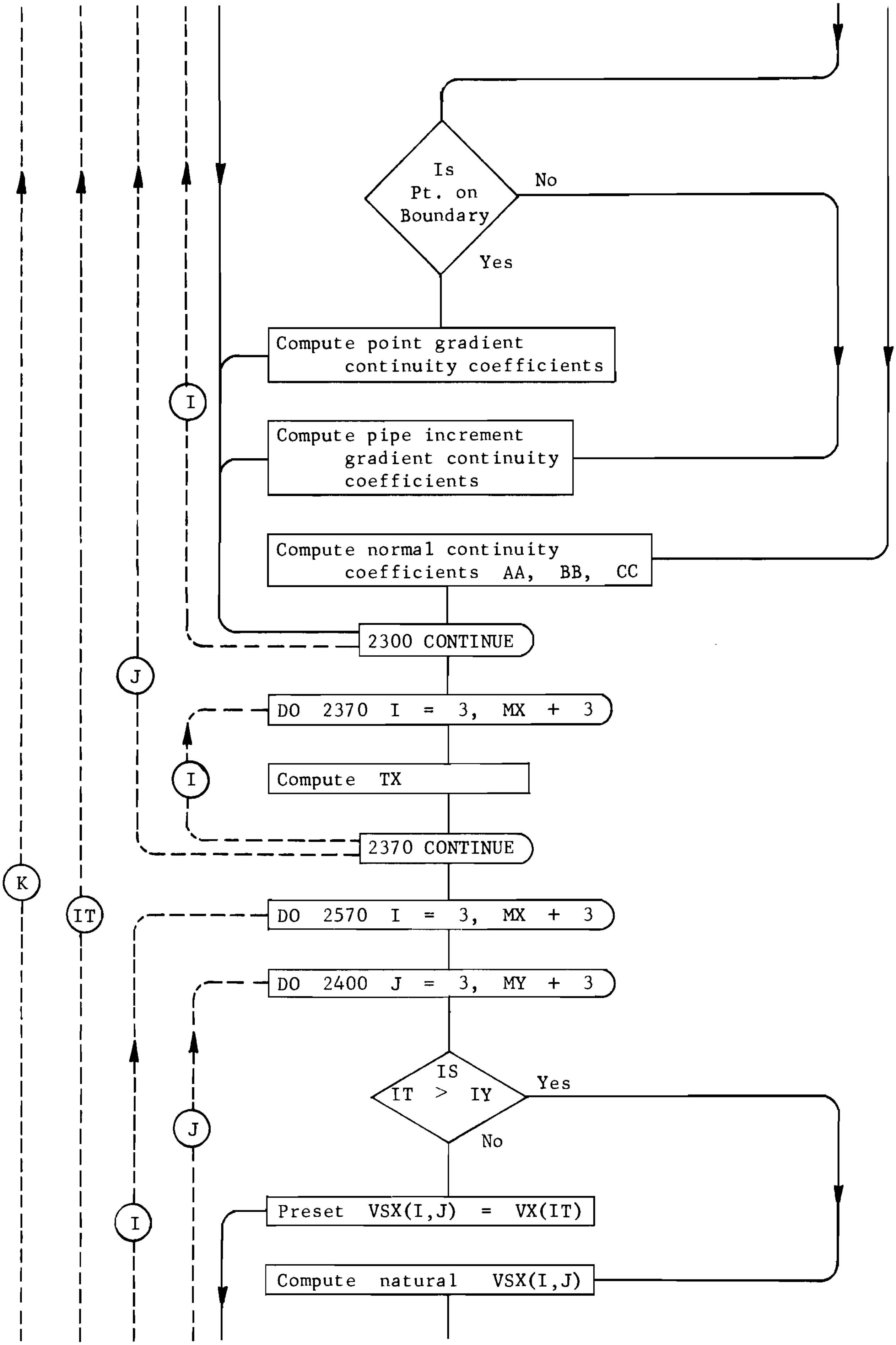
Suction Set in in specified rectangular region. Set KAS(I,J) = 2

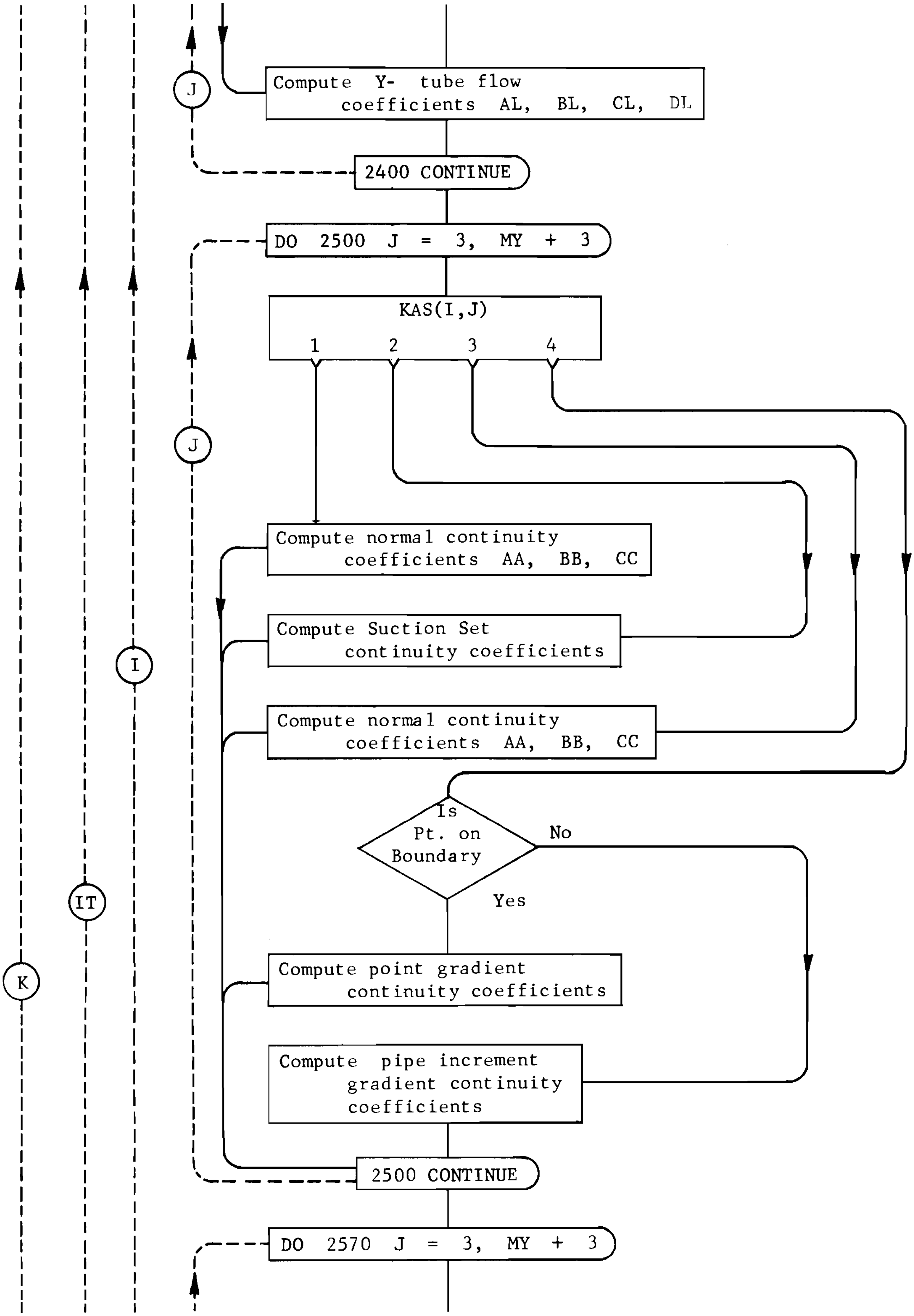
X- Slope Set in specified rectangular region. After all cards in Table 9 have been read, boundary values of suction are computed from X- slope and the value of suction just inside the boundary, KAS(I,J) is set at 3 and Subroutine DSUCT is called.

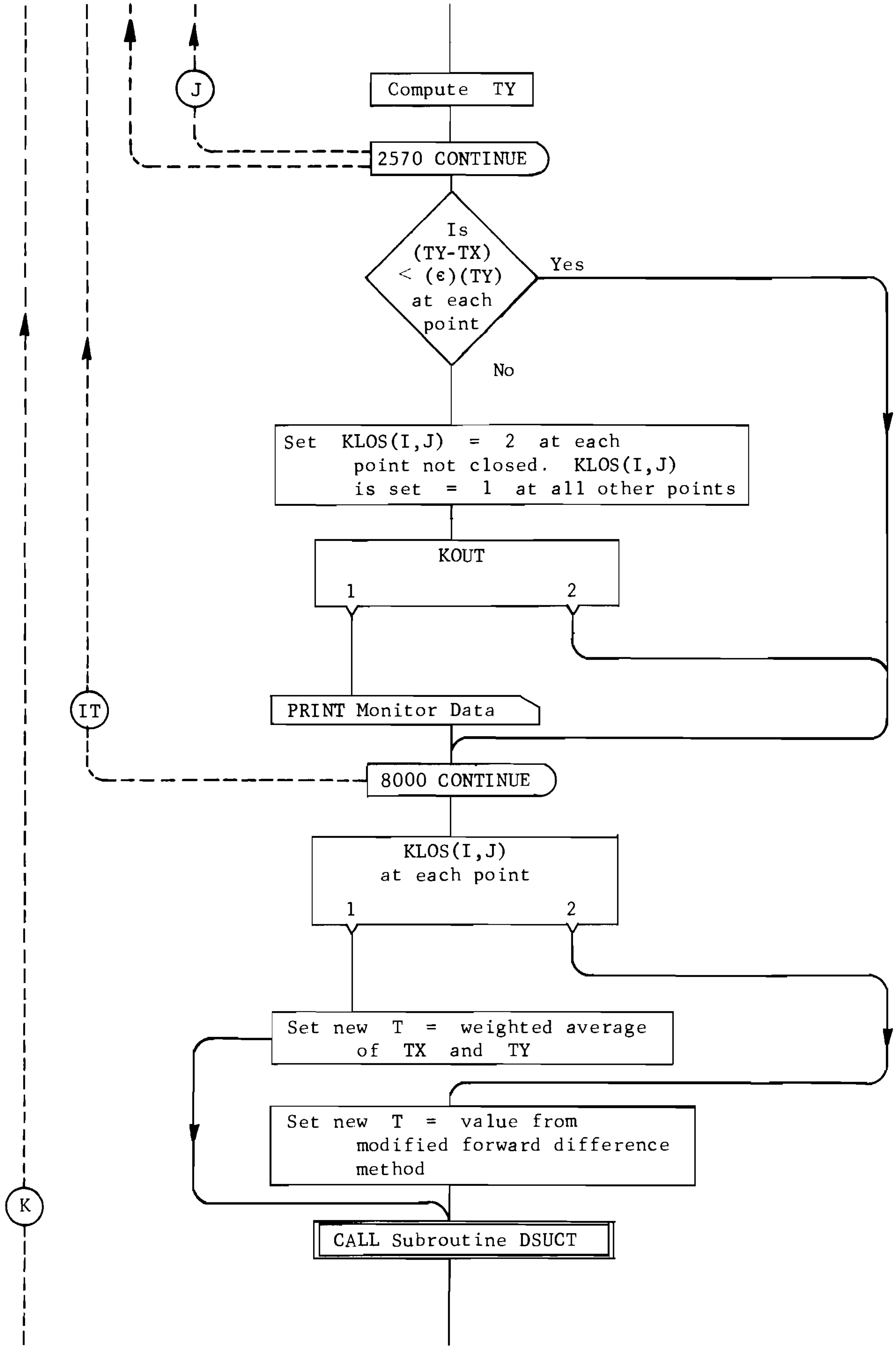
Y- Slope Set in specified rectangular region. After all cards in Table 9 have been read, boundary values of suction are computed from Y- slope and the value of suction just inside the boundary, KAS(I,J) is set at 4 and Subroutine DSUCT is called.

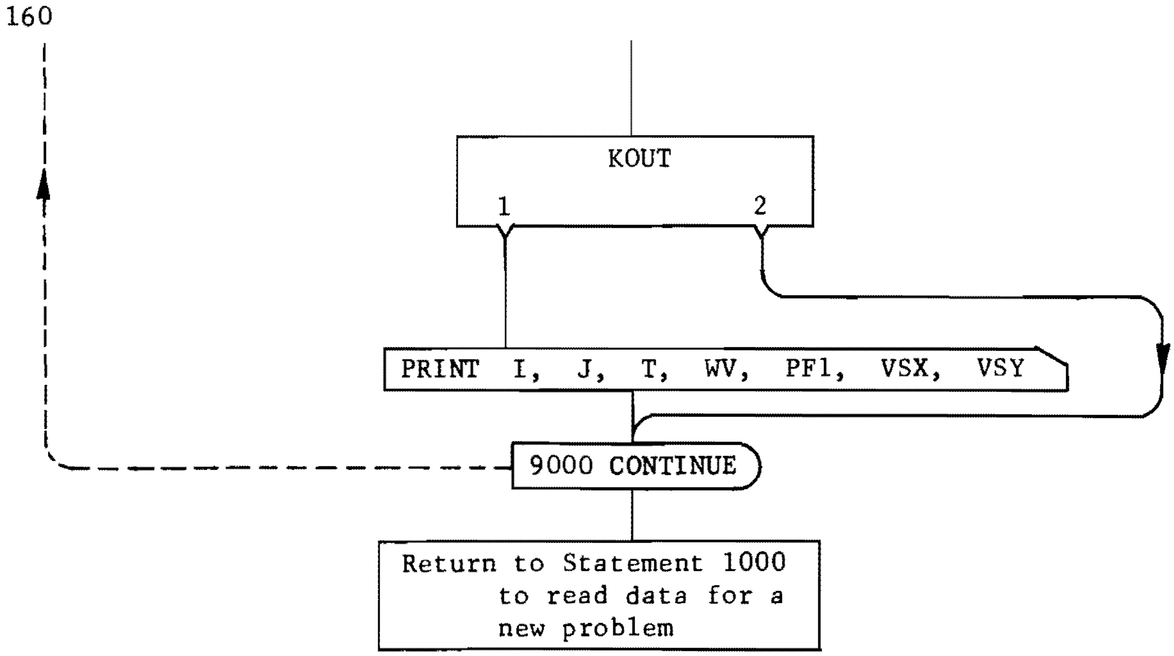

M
Soil Moisture Humidity Set in specified rectangular region. KAS(I,J) = 2
CALL Subroutine HUMIDY
1945 CONTINUE
Compute components of the saturated permeability tensor at each point of the region
Compute unsaturated permeability factor, unsaturated components of the permeability tensor
K
KOUT
1
2
PRINT I, J, T, WV, DTDW, P11, P12, P22
KGRCL
1
2
Compute Suction Coefficients A, B, CX, CY, D, E, F for rectangular region
Compute Suction Coefficients A, B, CX, CY, D, E, F for cylindrical region
Set constants outside of region
Set T = TX = TY


DO 8000 IT = 1, ITMAX
DO 2370 J = 3, MY + 3
DO 2210 I = 3, MX + 3
Is IT > IX
Yes
No
Preset VSY(I,J) = VY(IT)
Compute natural VSY(I,J)
Compute X- tube flow coefficients AL, BL, CL, DL
2210 CONTINUE
DO 2300 I = 3, MXP3
KAS(I,J)
1
2
3
4
Compute normal continuity coefficients AA, BB, CC
Compute Suction Set continuity coefficients
I
K
IT
J
I

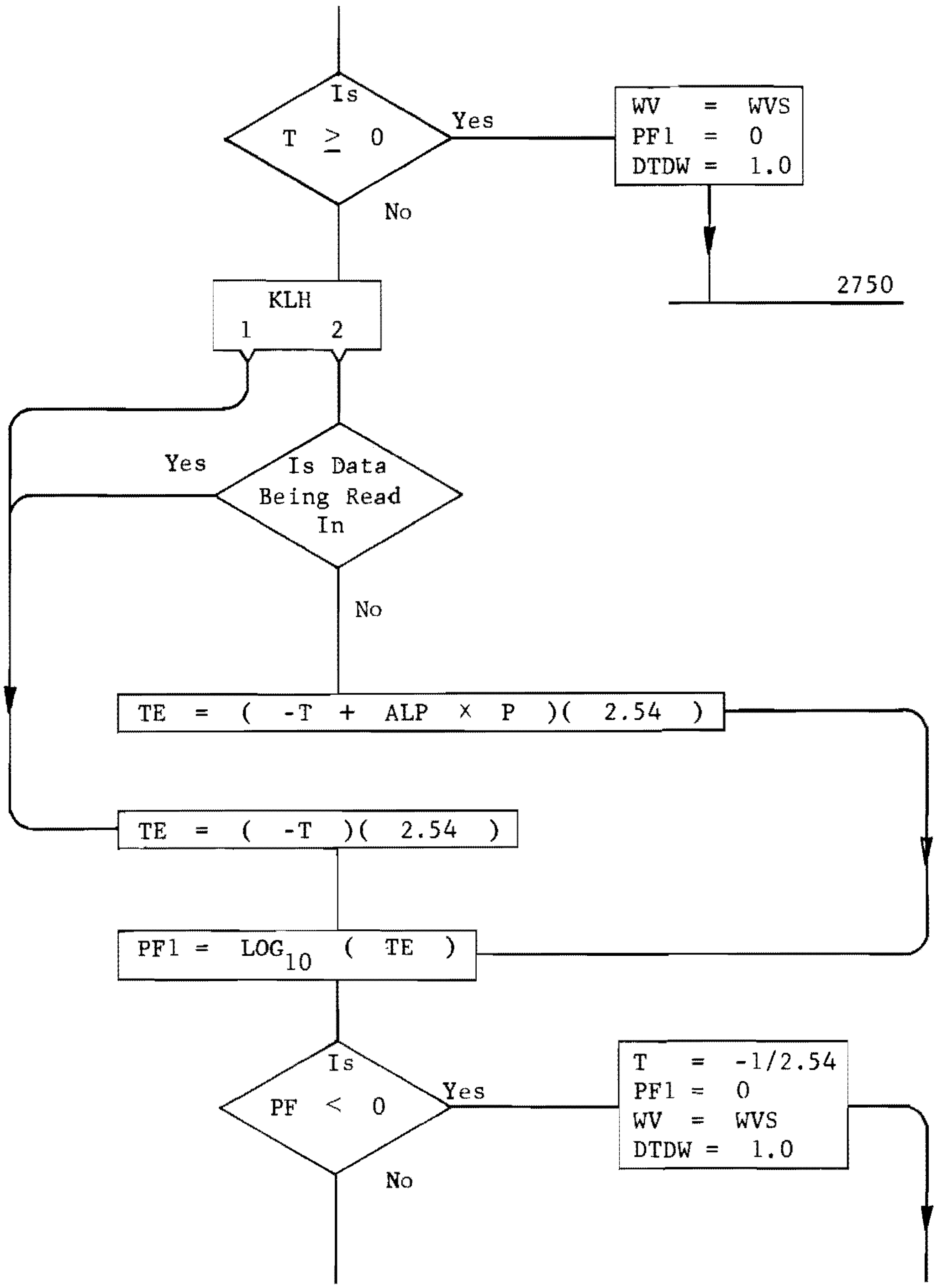

Is Pt. on Boundary
No
Yes
Compute point gradient continuity coefficients
I
Compute pipe increment gradient continuity coefficients
Compute normal continuity coefficients AA, BB, CC
2300 CONTINUE
J
DO 2370 I = 3, MX + 3
I
Compute TX
2370 CONTINUE
K
IT
DO 2570 I = 3, MX + 3
DO 2400 J = 3, MY + 3
IS IT > IY
Yes
No
J
I
Preset VSX(I,J) = VX(IT)
Compute natural VSX(I,J)

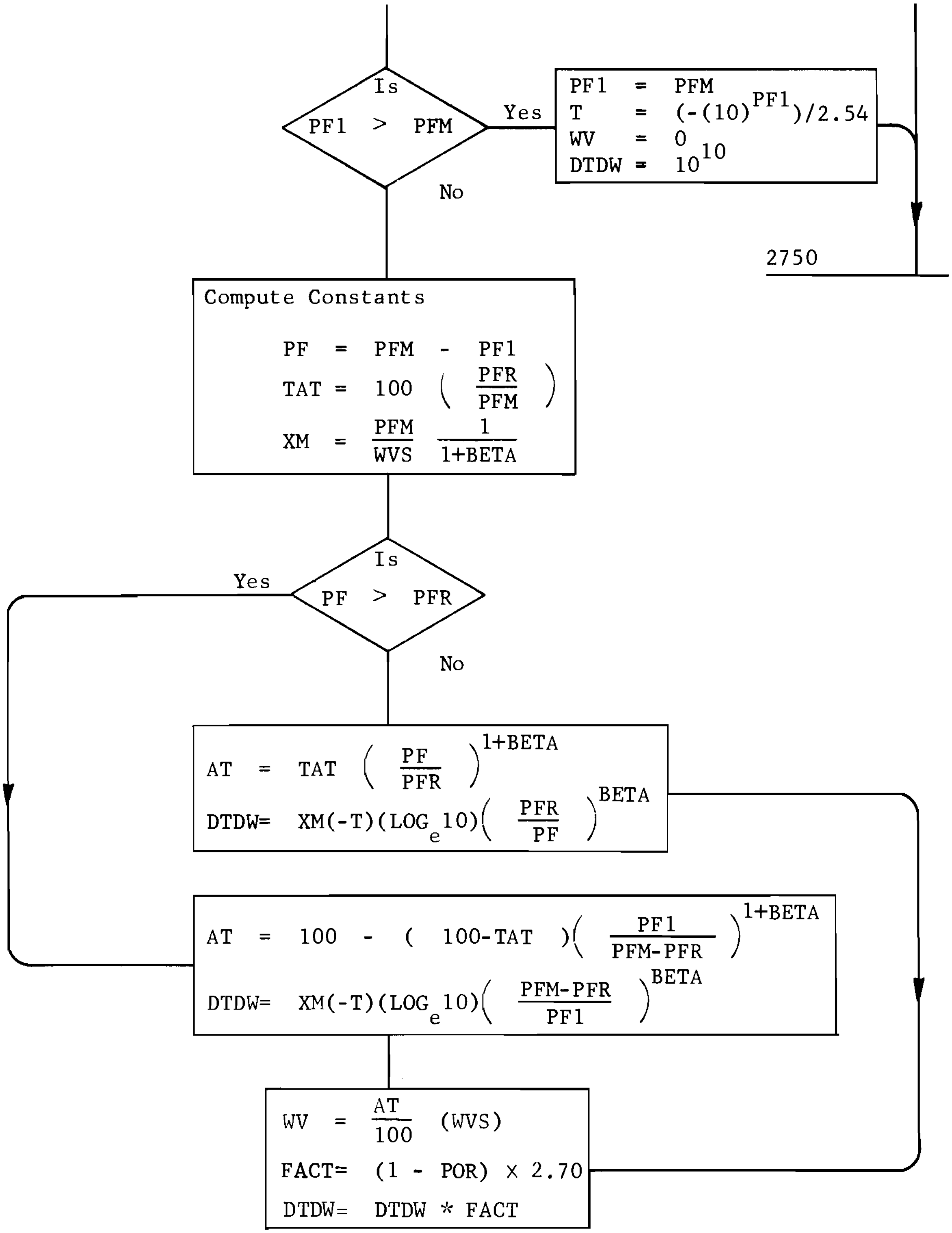

J
Compute Y- tube flow coefficients AL, BL, CL, DL
2400 CONTINUE
DO 2500 J = 3, MY + 3
KAS(I,J)
1
2
3
4
J
Compute normal continuity coefficients AA, BB, CC
Compute Suction Set continuity coefficients
I
Compute normal continuity coefficients AA, BB, CC
Is Pt. on Boundary
No
Yes
IT
K
Compute point gradient continuity coefficients
Compute pipe increment gradient continuity coefficients
2500 CONTINUE
DO 2570 J = 3, MY + 3

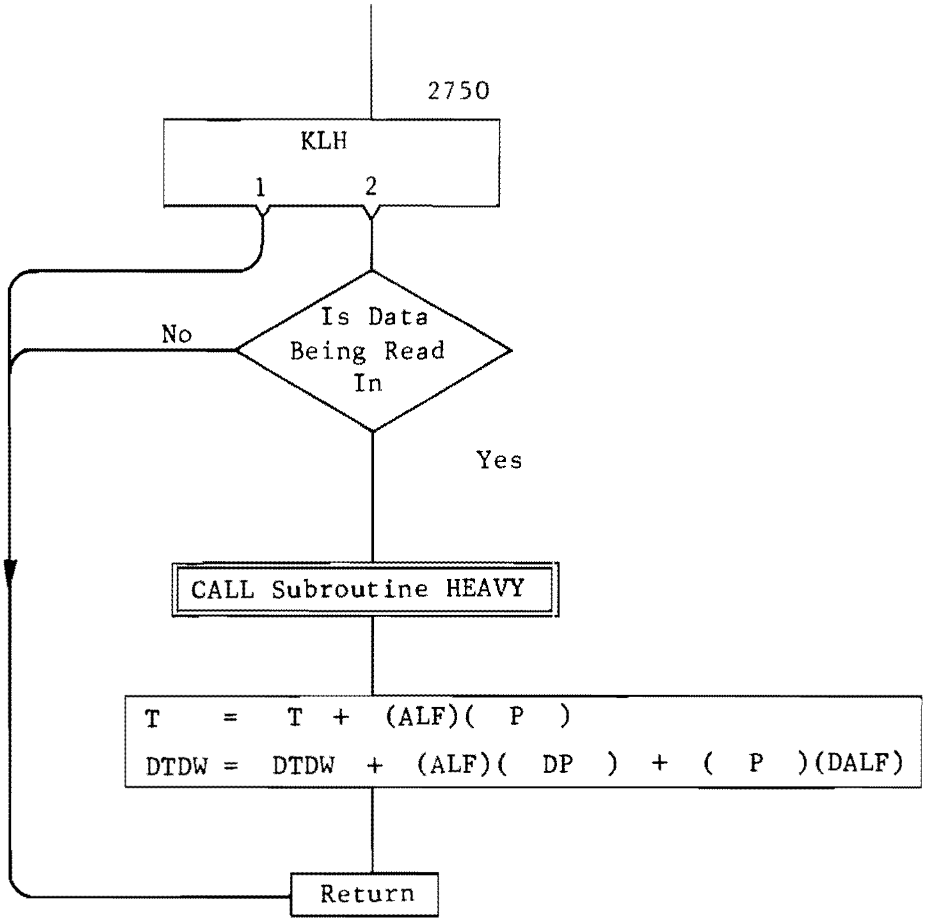

J
Compute TY
2570 CONTINUE
Is (TY-TX) < (ε)(TY) at each point
Yes
No
Set KLOS(I,J) = 2 at each point not closed. KLOS(I,J) is set = 1 at all other points
KOUT
1
2
IT
PRINT Monitor Data
8000 CONTINUE
KLOS(I,J) at each point
1
2
Set new T = weighted average of TX and TY
Set new T = value from modified forward difference method
K
CALL Subroutine DSUCT


160
KOUT
1
2
PRINT I, J, T, WV, PF1, VSX, VSY
9000 CONTINUE
Return to Statement 1000 to read data for a new problem

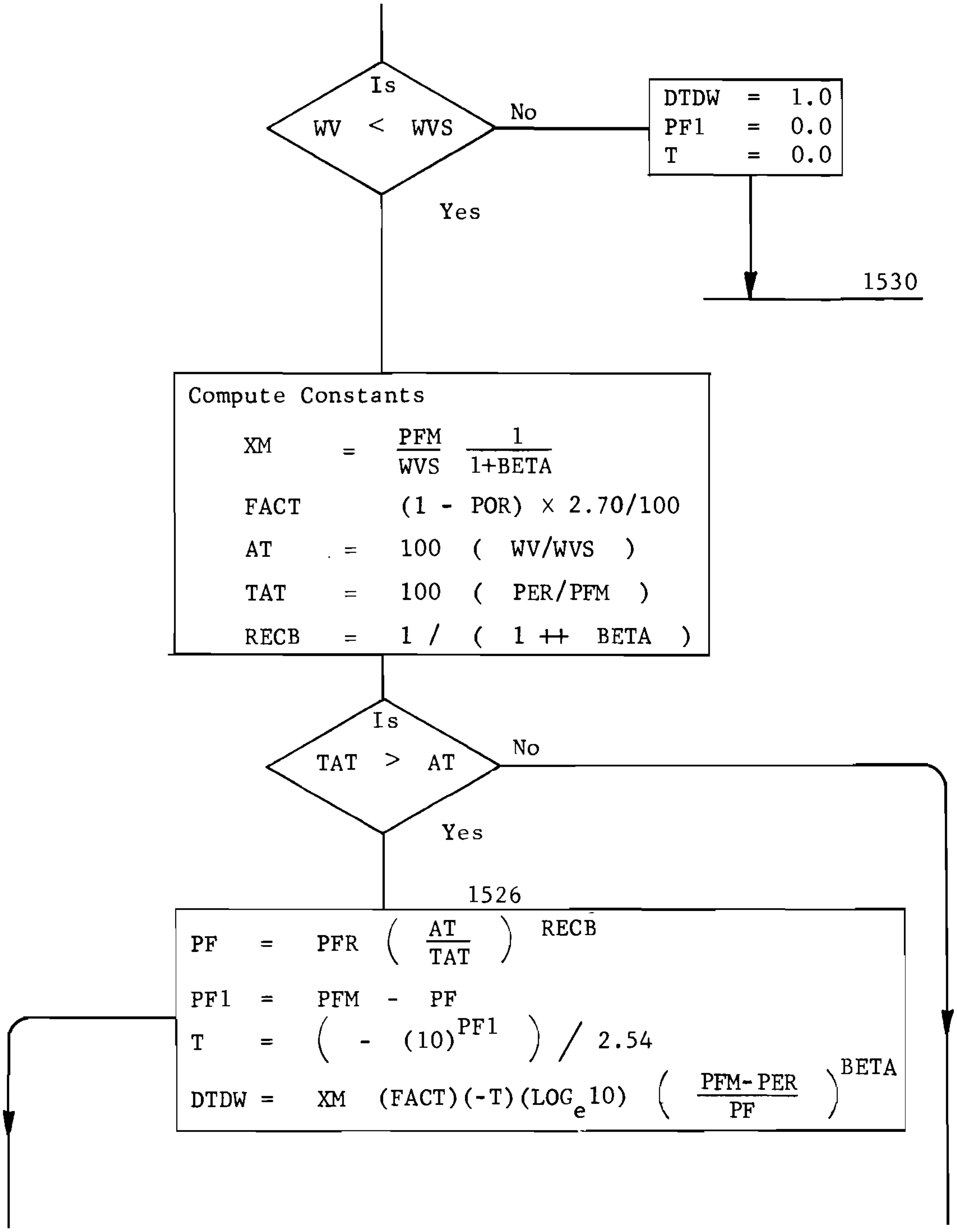
Subroutine DSUCT

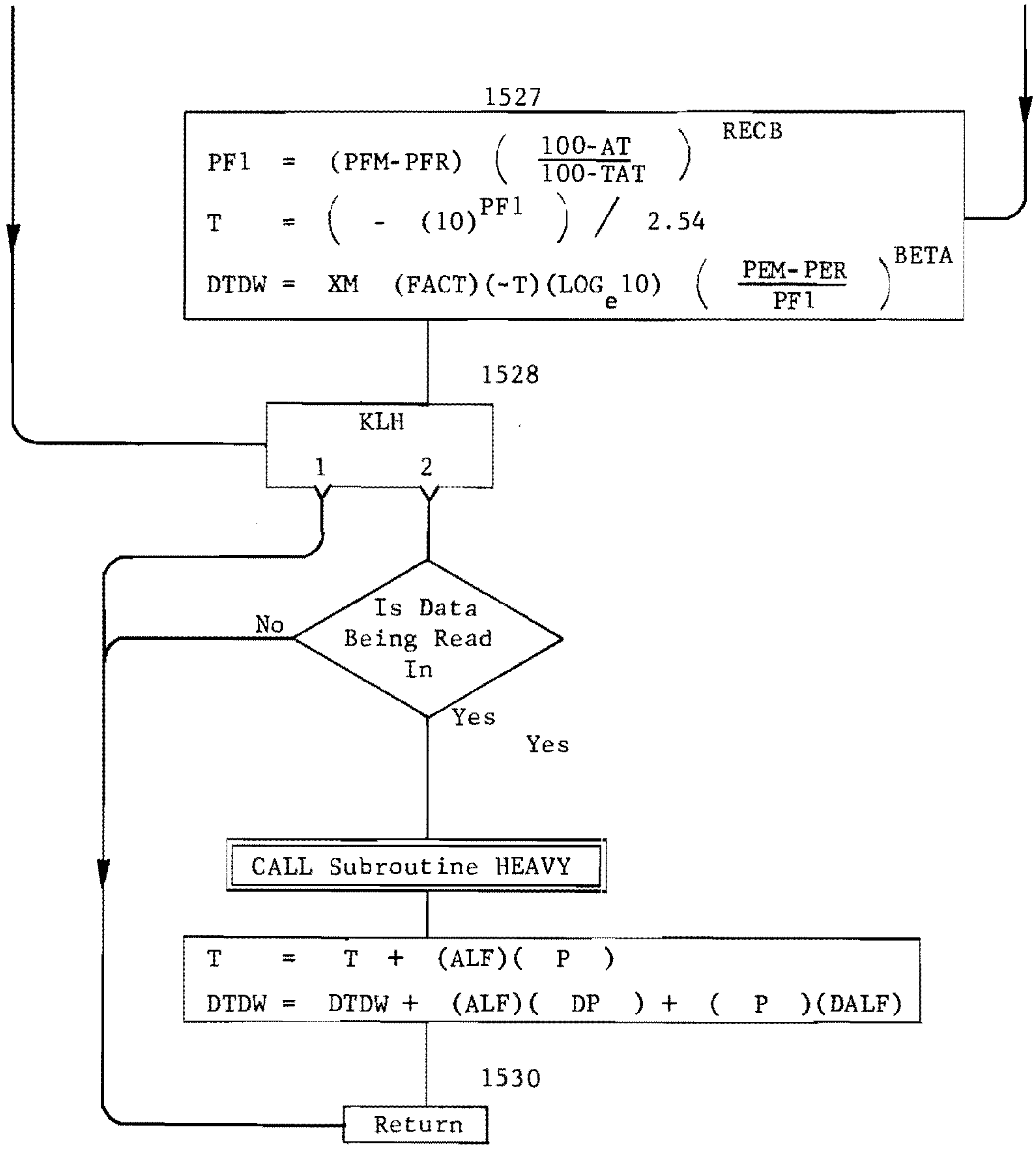



Is PF1 > PFM
Yes
No
PF1 = PFM
T = (-(10)^PF1)/2.54
WV = 0
DTDW = 10^10
2750
Compute Constants
PF = PFM - PF1
TAT = 100 (PFR/PFM)
XM = PFM/WVS 1/(1+BETA)
Is PF > PFR
Yes
No
AT = TAT (PF/PFR)^(1+BETA)
DTDW= XM(-T)(LOG_e 10)(PFR/PF)^BETA
AT = 100 - (100-TAT)(PF1/(PFM-PFR))^(1+BETA)
DTDW= XM(-T)(LOG_e 10)((PFM-PFR)/PF1)^BETA
WV = AT/100 (WVS)
FACT= (1 - POR) x 2.70
DTDW= DTDW * FACT

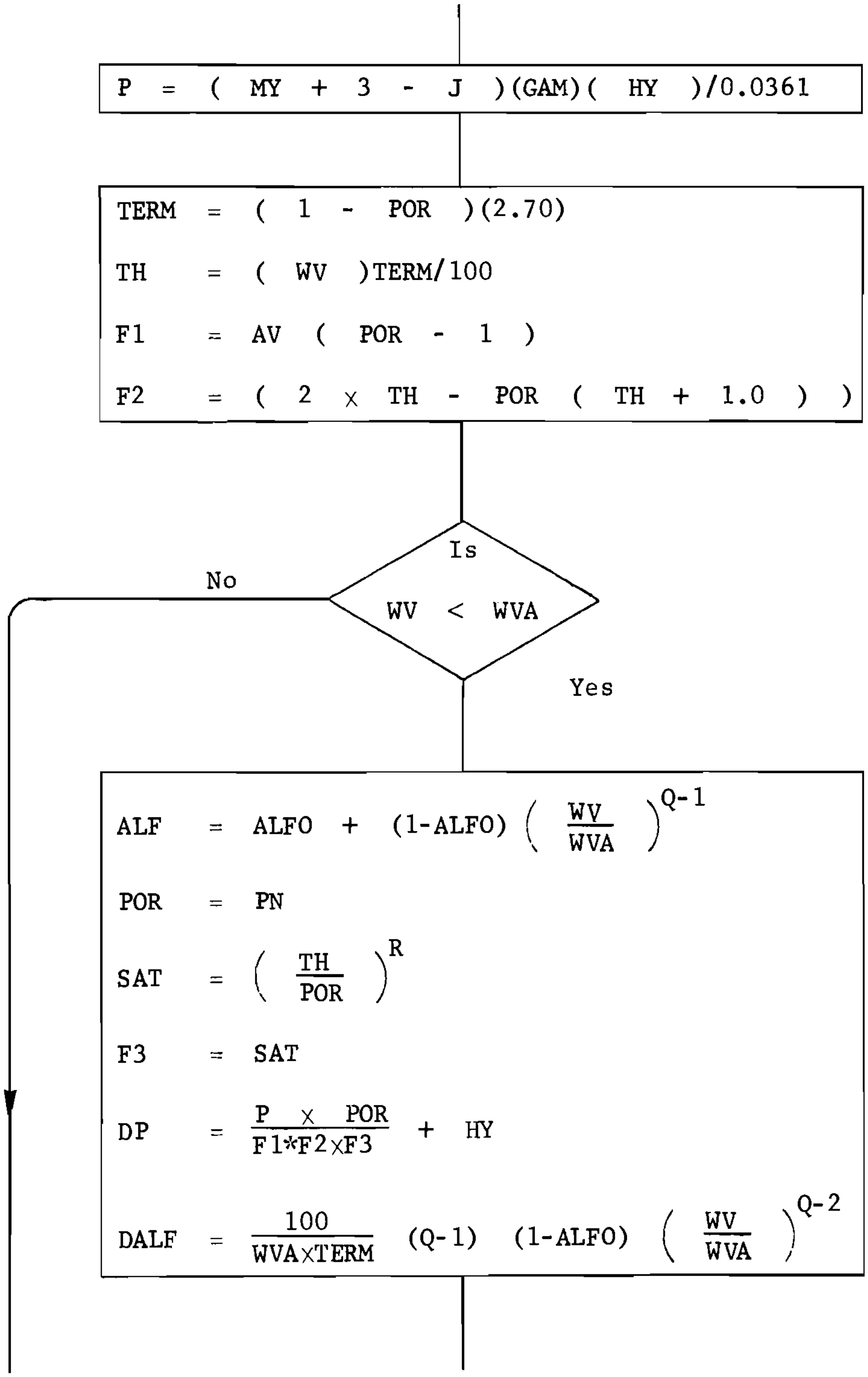

2750
KLH
1
2
Is Data Being Read In
No
Yes
CALL Subroutine HEAVY
T = T + (ALF)(P)
DTDW = DTDW + (ALF)(DP) + (P)(DALF)
Return

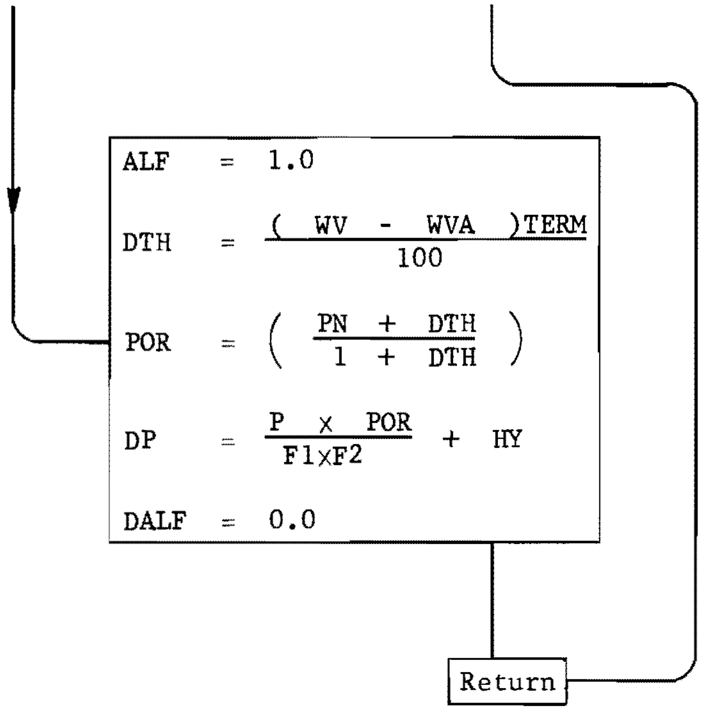
Subroutine SUCTION




1527
PF1 = (PFM-PFR) (100-AT / 100-TAT) RECB
T = (- (10) PF1) / 2.54
DTDW = XM (FACT)(-T)(LOG e 10) (PEM-PER / PF1) BETA
1528
KLH
1
2
Is Data Being Read In
No
Yes
Yes
CALL Subroutine HEAVY
T = T + (ALF)(P)
DTDW = DTDW + (ALF)(DP) + (P)(DALF)
1530
Return

Subroutine HEAVY




ALF = 1.0
DTH = (WV - WVA)TERM / 100
POR = (PN + DTH / 1 + DTH)
DP = P x POR / F1xF2 + HY
DALF = 0.0
Return

APPENDIX 2

GUIDE FOR DATA INPUT FOR PROGRAM GCHPIP1

GENERAL PROGRAM NOTES

A detailed discussion of all input data is given in Chapter Seven

All words not marked E or F are understood to be input as integers, the last number of which is in the farthest right space in the box 22

All words marked E or F are for decimal numbers which may be input at any position in the box with the decimal point in the proper position . . . . -19.36

0.0013

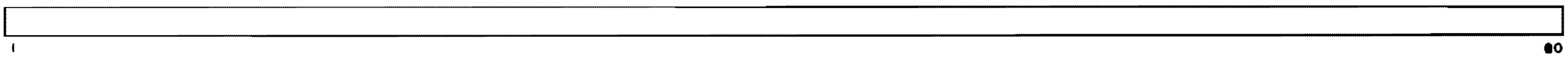
72.

The words marked E have been provided for those numbers which may require an exponential expression. The last number of the exponent should appear in the farthest right space in the box . . . . -3.142E-06

The program is arranged to compute quantities in terms of pounds, inches, and seconds. All dimensional input should be in these units.

GCHPIP1 GUIDE FOR DATA INPUT --- Card forms

IDENTIFICATION OF PROGRAM AND RUN (one alphanumeric card per problem)



IDENTIFICATION OF PROBLEM (one card each problem; program stops if NPROB is left blank)

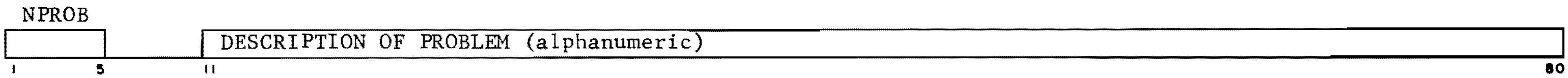
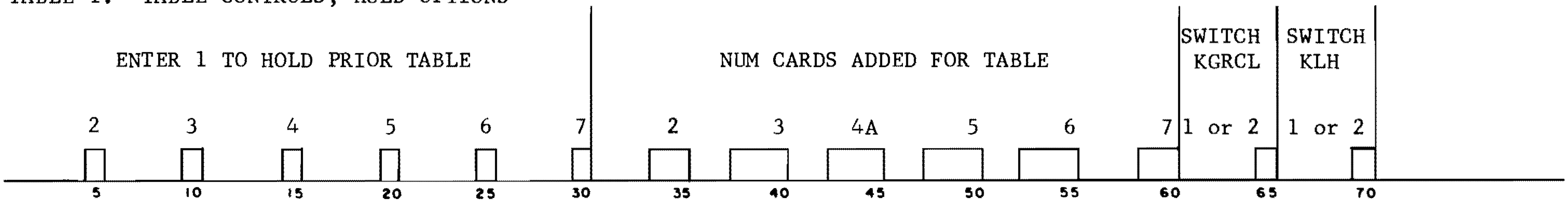


TABLE 1. TABLE CONTROLS, HOLD OPTIONS



IF KGRCL IS
1 Grid Coordinates
2 Cylindrical Coordinates

IF KLH IS
1 Light - overburden pressure and compressibility not considered
2 Heavy - overburden pressure and compressibility considered

Note: KLH SWITCH should be set to 2 only if data includes the soil compressibility effect on suction.

TABLE 2A. INCREMENTS, ITERATION CONTROL

NUM OF X-INCRS	NUM OF Y-INCRS	MAX ITERS PER TIME	NUM TIME STEPS	X-INCR LENGTH	Y-INCR LENGTH	INSIDE RADIUS	TIME STEP	CLOSURE TOLERANCE
				E	E	E	E	E
1 5	10	15	20	30	40	50	60	70

TABLE 2B. MONITOR STATIONS

COORDINATES OF MONITOR POINTS

I	J	I	J	I	J	I	J
1 5	10	15	20	25	30	35	40

TABLE 2C. CHOICE OF TRANSIENT OR PSEUDO-STEADY STATE FLOW

1 : TRANSIENT FLOW 2 : PSEUDO-STEADY STATE FLOW

☐
5

TABLE 3. PERMEABILITY

FROM		TO				ANGLE FROM	UNSATURATED PERMEABILITY COEFFICIENTS		
I	J	I	J	P1	P2	P1 TO HORIZ.	AK	BK	EN
				E	E	E	E	E	E
1 5	10	15	20	30	40	50	60	70	80

TABLE 4. SUCTION-MOISTURE-COMPRESSIBILITY

NUMBER LOCATIONS	MAX PF	INFL PF	PF-w EXPONENT	AIR ENTRY WATER CONTENT	ALFA EXPONENT	ALFA AT ZERO WATER CONTENT	POROSITY AT AIR ENTRY	E-LOG P COMPRESSIBILITY COEFFICIENT	X EXPONENT	UNIT WEIGHT OF SOIL	FINAL SATURATION WATER CONTENT
	F	F	F	F	F	E	E	E	F	F	E
1 5	10	15	20	25	30	40	50	60	65	70	80

FROM		TO		CURVE NUM
I	J	I	J	
1 5	10	15	20	25

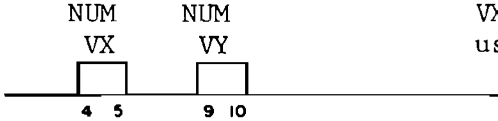
TABLE 5. INITIAL CONDITIONS

FROM		TO		KAS 1 OR 2	WATER CONTENT	SUCTION	Y-SLOPE A2	X-SLOPE C2
I	J	I	J					
					E	E	E	E
1 5	10	15	20	25	31 40	50	60	70
					KAS = 1	KAS = 2		

TABLE 6. BOUNDARY AND INTERNAL CONDITIONS

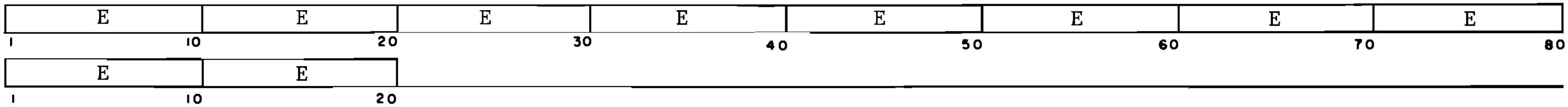
FROM		TO		KASE 1 TO 5	WATER CONTENT	SUCTION	X-GRADIENT OF SUCTION	Y-GRADIENT OF SUCTION	SOIL MOISTURE HUMIDITY	TEMP
I	J	I	J							
					E	E	E	E	F	F
1 5	10	15	20	25	31 40	50	60	70	75	77 80
					KASE = 1	KASE = 2	KASE = 3	KASE = 4	KASE = 5	

TABLE 7. CLOSURE ACCELERATION DATA



VX and VY are externally specified X and Y-closure valve settings which are all used before natural closure valve settings are computed.

X-CLOSURE VALVE SETTINGS (maximum number is 10)



Y-CLOSURE VALVE SETTINGS (maximum number is 10)

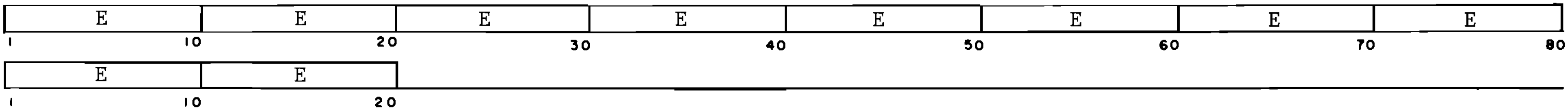
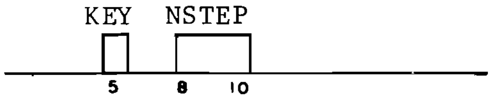


TABLE 8A. TIME STEPS FOR BOUNDARY CONDITION CHANGE



IF KEY IS
1 Read in a list of time steps for boundary condition change NSTEP is the number of these steps
2 Continuous boundary condition change. Read in a new boundary condition at each time step. NSTEP is left blank
3 No boundary condition change. NSTEP is left blank

LIST OF TIME STEPS (if KEY = 1 maximum is 50)

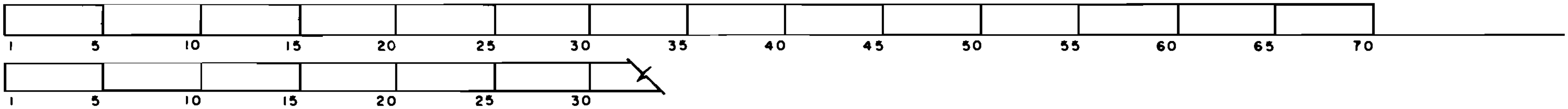
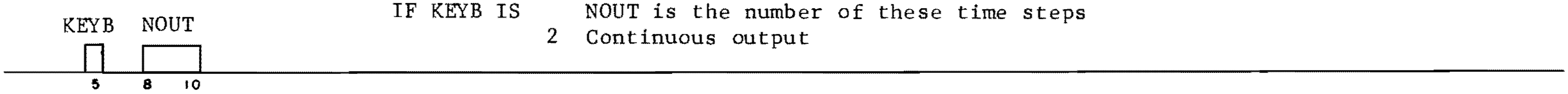


TABLE 8B. LIST OF TIME STEPS FOR OUTPUT

IF KEYB IS
1 Read in a list of output time steps
NOUT is the number of these time steps
2 Continuous output



LIST OF TIME STEPS (if KEYB = 1 maximum is 50)

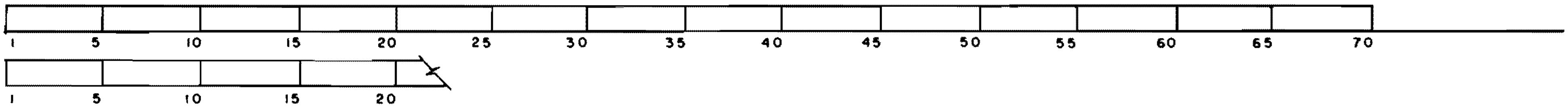
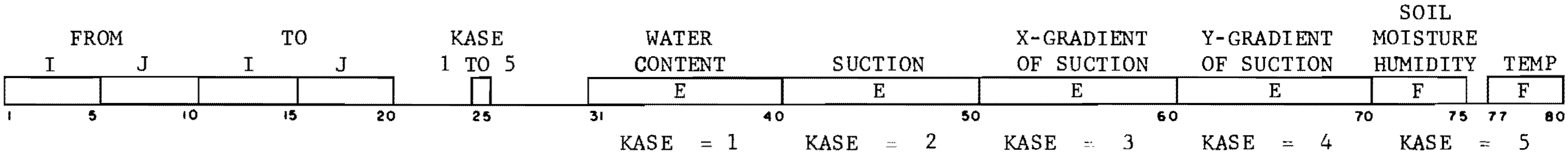


TABLE 9. SUBSEQUENT BOUNDARY CONDITIONS (if KEY = 1 or 2)



APPENDIX 3

LISTING FOR PROGRAM GCHPIP1

```
      PROGRAM GCHPIP1  (INPUT,OUTPUT)
C     NOTATION
C     T           SUCTION
C     TX          TRIAL SUCTION  IN  X - PIPES
C     TY          TRIAL SUCTION IN  Y - PIPES
C     P11         PERMEABILITY IN X-DIRECTION  AFFECTED BY  X- HEAD CHANGE
C     P12         PERMEABILITY IN X-DIRECTION  AFFECTED BY  Y- HEAD CHANGE
C     P21         PERMEABILITY IN Y-DIRECTION  AFFECTED BY  X- HEAD CHANGE
C     P22         PERMEABILITY IN Y-DIRECTION  AFFECTED BY  Y- HEAD CHANGE
C     P1          PRINCIPAL  PERMEABILITY NEAREST  X-DIRECTION
C     P2          PRINCIPAL  PERMEABILITY NEAREST  Y-DIRECTION
C     A           SUCTION               COEFFICIENT  OF  T( I,J-1)
C     B           SUCTION               COEFFICIENT  OF  T( I-1,J)
C     C           SUCTION               COEFFICIENT  OF  T( I , J)
C     D           SUCTION               COEFFICIENT  OF  T( I+1,J)
C     E           SUCTION               COEFFICIENT  OF  T(I,J+1)
C     F           GRAVITY  POTENTIAL COMPONENT  OF  PERMEABILITY
C     DTDW        RATE OF CHANGE OF SUCTION  WITH WATER  CONTENT
C     AL          TUBE  FLOW  MATRIX COEFFICIENT  OF   TX  OR  TY AT I -1
C     BL          TUBE  FLOW  MATRIX COEFFICIENT  OF   TX  OR  TY AT I
C     CL          TUBE  FLOW  MATRIX COEFFICIENT  OF   TX  OR  TY AT I +1
C     DL          TUBE  FLOW    CONSTANT
C     HX          INCREMENT  LENGTH  IN  THE  X-DIRECTION
C     HY          INCREMENT  LENGTH  IN  THE  Y-DIRECTION
C     HT          INCREMENT  LENGTH  IN  THE  TIME- DIRECTION
C     AA          CONTINUITY  COEFFICIENT  - A  CONSTANT
C     BB          CONTINUITY  COEFFICIENT  - B  CONSTANT
C     CC          CONTINUITY  COEFFICIENT  - C  CONSTANT
C     DD          CONTINUITY  COEFFICIENT  - A  DENOMINATOR
C     ALPHA       ANGLE BETWEEN  P1 AND  THE  X- DIRECTION
C     EPS         CLOSURE  TOLERANCE  ON  DIFFERENCE  IN TX AND  TY
C     WV          VOLUMETRIC WATER  CONTENT
C     WVS         SATURATED WATER CONTENT
C     VSX         CLOSURE  PARAMETER  FOR  THE   X-DIRECTION
C     VSY         CLOSURE  PARAMETER  FOR  THE   Y-DIRECTION
      DIMENSION P1(40,25),P2(40,25),ALFA(40,25),AK(40,25),BK(40,25),
     1EN(40,25),WV(40,25),T(40,25),P11(40,25),P12(40,25),P22(40,25),
     2DTDW(40,25),VSX(40,25),VSY(40,25),A(40,25),B(40,25),CX(40,25),
     3CY(40,25),D(40,25),E(40,25),F(40,25),AL(40),BL(40),CL(40),DL(40),
     4AA(40),BB(40),CC(40),TX(40,25),TY(40,25),KURV(40,25),
     5KLOC(1000),AN1(16),AN2(7),WVS(40,25),DTDX(40,25),DTDY(40,25),
     6KAS(40,25),VX(10),VY(10),PFM(10),PFR(10),BETA(10),WVA(10),Q(10),
     7ALFO(10),R(10),AV(10),PN(10),POR(40,25),KT(50),WN(10),KPUT(1000),
     8KLOS(40,25)
      COMMON/ONE/PFM,PFR,BETA,DTDW,PF1
     1/TWO/T,I2,J2
     2/THREE/WVS,KLH,K
     3/FOUR/WVA,Q,ALFO,R,AV,POR,KURV,WV,GAM,ALF,P,DP,DALF,MY,HY,PN
    1 FORMAT (// 50H  PROGRAM GCHPIP1   R.L.LYTTON  REVISION DATE
     1          15H  JUNE  , 1969  ,//)
   11 FORMAT(     5H1     ,80X ,10HI-----TRIM )
```

```
 12 FORMAT (  8A10)
 14 FORMAT ( A5,5X,7A10)
 15 FORMAT (///10H      PROB , /5X, A5, 5X, 7A10)
 20 FORMAT (16I5)
 21 FORMAT ( 4I5,5E10.3)
 22 FORMAT ( 4I5,6E10.3)
 23 FORMAT ( I5,5F5.2,3E10.3,2F5.1,E10.3)
 24 FORMAT (5I5,5X,4E10.3)
 25 FORMAT ( 5I5,5X,4E10.3,F5.3,1X,F4.1)
 26 FORMAT( 8E10.3)
 27 FORMAT ( 5X,I5,2(5X,E10.3))
 28 FORMAT( 2I4,2X,6(E10.3,2X))
 29 FORMAT (// 50H   I    J    T(I,J)        WV(I,J)     DTDW(I,J)    P11
   1            30H(I,J)    P12(I,J)     P22(I,J)      )
100 FORMAT (///40H  TABLE 1. PROGRAM CONTROL SWITCHES.
   1 / 50X,     25H           TABLES NUMBER
   2 / 50X,     35H     2     3    4A    5     6     7
   3        //  40H           PRIOR  DATA OPTIONS (1 = HOLD),11X ,6I5,
   4         /  41H           NUMBER CARDS INPUT THIS PROBLEM, 10X,6I5,
   5    //      41H           GRID = 1, CYLINDER = 2 SWITCH  , 10X,I5,
   6    //      41H           LIGHT = 1, HEAVY = 2  SWITCH   , 10X,I5 )
200 FORMAT (///50H  TABLE 2. INCREMENT LENGTHS, ITERATION CONTROL    )
201 FORMAT (// 35H            NUM OF X-INCREMENTS    = , 5X,I5,
   1      /   , 35H           X-INCREMENT LENGTH     = , E10.3,5H IN. ,
   2      /   , 35H           NUM OF  Y-INCREMENTS   = , 5X,I5 ,
   3      /   , 35H           Y-INCREMENT LENGTH     = , E10.3,5H IN. ,
   4      /   , 35H           NUM OF TIME INCREMENTS = , 5X, I5,
   5      /   , 35H           TIME INCREMENT LENGTH  = , E10.3,5H SECS,
   6      /   , 35H           ITERATIONS / TIME STEP = , 5X,I5,
   7      /   , 35H           INSIDE RADIUS          = ,E10.3,5H IN    ,
   8      /   , 35H           TOLERANCE              = ,E10.3)
202 FORMAT (// 30H            MONITOR STATIONS I,J ,5X, 4(I7,I3))
203 FORMAT (// 25H     TRANSIENT  FLOW        )
204 FORMAT (// 35H      PSEUDO-STEADY STATE FLOW        )
300 FORMAT (///30H  TABLE  3.  PERMEABILITY       )
301 FORMAT (// 50H  FROM        TO         P1         P2   ALFA(DEG.)
   1           30H  AK          BK     EXPONENT    )
400 FORMAT (///45H  TABLE  4. SUCTION - WATER CONTENT CURVES    )
401 FORMAT (// 35H            CURVE NUMBER            ,I5,
   1    /,     35H            NUM  LOCATIONS        = , I5,
   2    /,     35H            MAXIMUM  PF           = ,5X,F5.2,
   3    /,     35H            PF  AT  INFLECTION    = ,5X,F5.2,
   4    /,     35H            EXPONENT  FOR  PF     = ,5X,F5.2,
   5    /,     35H            AIR ENTRY WATER CONT  = ,5X,F5.2,
   6    /,     35H            DRYING CURVE EXPONENT = ,5X,F5.2,
   7    /,     35H            ALFA AT 0 WATER CONT  = , E10.3,
   8    /,     35H            INITIAL  POROSITY     = , E10.3,
   9    /,     35H            REFERENCE  AV         = , E10.3    )
402 FORMAT (   35H            SATURATION EXPONENT   = ,5X,F5.2,
   1    /,     35H            SOIL UNIT WT  PCI     = , E10.3 ,
   2    /,     35H            SATURATED WATER CONT. = , E10.3,// )
403 FORMAT (// 25H  NO.    FROM      TO      )
500 FORMAT (///30H  TABLE 5. INITIAL CONDITIONS  )
501 FORMAT (// 50H    FROM        TO      CASE     VOL. W.   PORE PR.
   1           20H  SLOPE Y  SLOPE X    )
```

```
  600 FORMAT (///45H  TABLE 6. BOUNDARY AND INTERNAL CONDITIONS  )
  601 FORMAT (// 50H  FROM STA   TO STA  CASE           WV          T
     1           40H  DT/DX        DT/DY      H  TEMP              )
  700 FORMAT (///40H  TABLE 7.  CLOSURE ACCELERATION DATA   )
  701 FORMAT (// 40H     FICTITIOUS CLOSURE VALVE SETTINGS  ,//,
     1           40H       NO.           VSX             VSY       )
  800 FORMAT (///40H  TABLE 8A. TIME STEPS FOR B.C. CHANGE    )
  801 FORMAT (// 50H  ITERATION  PTS.NOT CLOSED                  MONITOR
     1           10H STATIONS ,//,32X, 4(2I3,6X)   )
  802 FORMAT ( 2(5X,I5),10H   TX       ,4(E10.3,2X)   )
  803 FORMAT (    20X,  10H    TY      , 4(E10.3,2X),/ )
  804 FORMAT (// 50H  STATION       T(I,J)         WV(I,J)       PF(I,J)
     1           30HVSX(I,J)    VSY(I,J)              )
  805 FORMAT ( 2I4,5X,5(E10.3,2X)    )
  806 FORMAT (// 10H     ALL    )
  807 FORMAT (// 10H   NONE      )
  808 FORMAT (///40H  TABLE 8B. TIME STEPS FOR OUTPUT.        )
  809 FORMAT (// 15H   TIME STEP   =   , I5,//)
  810 FORMAT (// 20H   ***CLOSURE***      ,//)
  900 FORMAT (// 50H  TABLE 9. SUBSEQUENT BOUNDARY CONDITIONS          )
  905 FORMAT (// 40H    USING DATA FROM PREVIOUS PROBLEM     )
  906 FORMAT (// 45H    USING DATA FROM PREVIOUS PROBLEM PLUS     )
  907 FORMAT (// 25H     ERROR IN DATA          )
                 ITEST   =  5H
 1000 READ 12,(AN1(N), N =1, 8     )
 1010 READ 14, NPROB, ( AN2(N), N =1,7)
          IF (NPROB - ITEST) 1020, 9999, 1020
 1020 PRINT 11
      PRINT 1
      PRINT 12, (AN1(N), N = 1,8     )
      PRINT 15, NPROB, (AN2(N), N =1,7)
C     INPUT  OF  TABLE 1 . TABLE CONTROLS,  HOLD OPTIONS,
 1100 READ 20, KEEP2,KEEP3,KEEP4,KEEP5,KEEP6,KEEP7,NCD2,NCD3,NCD4,NCD5,
     1NCD6,NCD7,KGRCL,KLH
      PRINT 100,KEEP2,KEEP3,KEEP4,KEEP5,KEEP6,KEEP7,NCD2,NCD3,NCD4,
     1NCD5,NCD6,NCD7,KGRCL,KLH
C     INPUT OF  TABLE 2A  INCREMENTS, ITERATION CONTROL
 1200 PRINT 200
          IF(KEEP2)9980, 1210, 1230
 1210 READ 21, MX,MY,ITMAX,ITIME,HX,HY,RO,HT,EPS
      PRINT 201, MX,HX,MY,HY,ITIME,HT,ITMAX,RO,EPS
          GO TO 1240
 1230 PRINT 905
C     COMPUTE  CONSTANTS TO BE USED IN THE PROGRAM
 1240            MXP5   =  MX + 5
                 MYP5   =  MY + 5
                 MXP4   =  MX + 4
                 MYP4   =  MY + 4
                 MXP3   =  MX + 3
                 MYP3   =  MY + 3
                  MXP2       =  MX + 2
                 MYP2        =  MY + 2
C     READ IN THE TABLE  2B  MONITOR STATIONS
      READ 20, IM1,JM1,IM2,JM2,IM3,JM3,IM4,JM4
      PRINT 202,IM1,JM1,IM2,JM2, IM3, JM3, IM4, JM4
```

```
                IM1    =  IM1 + 3
                JM1    =  JM1 + 3
                IM2    =  IM2 + 3
                JM2    =  JM2 + 3
                IM3    =  IM3 + 3
                JM3    =  JM3 + 3
                IM4    =  IM4 + 3
                JM4    =  JM4 + 3
C     TABLE  2C.  CHOICE  OF  TRANSIENT OR  STEADY  STATE  FLOW
      READ 20,IN1
         GO TO (1250,1260) IN1
 1250 PRINT 203
                A4           =  1.0
         GO TO 1300
 1260 PRINT 204
                A4           =  0.0
C     INPUT  TABLE  3.  PERMEABILITY
 1300 PRINT 300
         IF(KEEP3) 9980,1310,1317
 1310    DO 1315  I = 1, MXP5
         DO 1315  J = 1, MYP5
                P1(I,J)    =   0.0
                P2(I,J)    =   0.0
                ALFA(I,J)  =   0.0
                AK (I,J)   =   0.0
                BK (I,J)   =   0.0
                EN (I,J)   =   0.0
                WVS(I,J)   =   0.0
 1315     CONTINUE
         GO TO 1319
 1317    IF (NCD3)9980,1330,1318
 1318 PRINT 906
 1319 PRINT 301
         DO 1320  K = 1, NCD3
      READ 22, IN1,JN1,IN2,JN2,PB,PL,ALF,AK1,BK1,EN1
      PRINT 22,IN1,JN1,IN2,JN2,PB,PL,ALF,AK1,BK1,EN1
                IN1    =  IN1 + 3
                JN1    =  JN1 + 3
                IN2    =  IN2 + 3
                JN2    =  JN2 + 3
         DO 1320  I  = IN1,IN2
         DO 1320  J  = JN1,JN2
                P1(I,J)    =   P1(I,J) +  PB
                P2(I,J)    =  P2(I,J) + PL
                ALFA(I,J)  =   ALFA(I,J)+ ALF
                AK (I,J)   =   AK  (I,J)+  AK1
                BK (I,J)   =   BK  (I,J)+  BK1
                EN(I,J)  =   EN(I,J)  +  EN1
 1320     CONTINUE
         GO TO 1400
 1330 PRINT 905
C     INPUT OF TABLE  4. SUCTION - WATER CONTENT CURVE
C     AT PRESENT,  THIS IS AN EXPONENTIAL SINGLE - VALUED CURVE.   IT
C    1SHOULD BE REPLACED BY NUMERICAL CURVES  FOR WETTING, DRYING, AND
C    2SCANNING BETWEEN THE TWO.
```

```
1400  PRINT 400
          IF (KEEP4) 9980,1410,1430
1410           NLOC   = 0
          DO 1415 M  =  1,NCD4
      READ 23,LOC,PFM(M), PF1  ,BETA(M),WVA(M),Q(M),ALFO(M),PN(M),AV(M),
     1R(M),GAM,WN(M)
      PRINT 401,M,LOC,PFM(M), PF1  ,BETA(M),WVA(M),Q(M),ALFO(M),
     1PN(M),AV(M)
      PRINT 402,R(M),GAM,WN(M)
               PFR(M)     =  PFM(M) - PF1
               NLOC   =  NLOC + LOC
1415      CONTINUE
      PRINT 403
          DO 1420 M  =  1,NLOC
      READ 20, IN1,JN1,IN2,JN2,KAT
      PRINT 20, KAT,IN1,JN1,IN2,JN2
               IN1   =  IN1 + 3
               JN1   =  JN1 + 3
               IN2   =  IN2 + 3
               JN2   =  JN2 + 3
          DO 1420 I  = IN1,IN2
          DO 1420 J  = JN1,JN2
               KURV(I,J)   =  KAT
               POR(I,J) = PN(KAT)
               WVS(I,J)   =  WN(KAT)
1420      CONTINUE
          GO TO 1500
1430  PRINT 905
C     INPUT OF TABLE 5. INITIAL CONDITIONS
1500  PRINT 500
          IF(KEEP5)9980,1510,1505
1505      IF(NCD5) 9980,1506,1507
1506  PRINT 905
          GO TO 1600
1507  PRINT 906
          GO TO  1520
1510      DO 1515 I = 1,MXP5
          DO 1515 J = 1,MYP5
               WV(I,J) =   0.0
               T(I,J)      =   0.0
1515      CONTINUE
1520  PRINT  501
          DO 1526 M =  1,NCD5
               K           =  0
      READ 24 , IN1,JN1,IN2,JN2,KAT,WV1,T1,A2 , C2
      PRINT 24, IN1,JN1,IN2,JN2,KAT,WV1,T1,A2 , C2
               IN1   = IN1 + 3
               JN1 = JN1 + 3
               IN2 = IN2 + 3
               JN2 = JN2 + 3
         GO TO (1522,1523), KAT
1522      DO 1525 I = IN1,IN2
          DO 1525 J  = JN1,JN2
               A1          =  JN2 - J
               C1          =  IN2 - I
```

```
                 WV(I,J)    =  WV(I,J) + WV1 + A1*A2*HY + C1*C2*HX
                 KAS(I,J) = 1
                 I2    =  I
                 J2    =  J
      CALL SUCTION
 1525      CONTINUE
          GO TO 1526
 1523     DO 1524 I  =  IN1,IN2
          DO 1524 J  =  JN1,JN2
                 A1          =  JN2 - J
                 C1          =  IN2 - I
                 KAS(I,J)    =  1
                 T(I,J)      =  A1*A2*HY   +  C1*C2*HX   +  T1 +  T(I,J)
          IF (A4) 1528,1527
 1527            WV(I,J)     =  WVS(I,J)
                 DTOW(I,J) =  1.0
                 PF1         =  0.0
           GO TO 1524
 1528            I2          =  I

                 J2          =  J
      CALL USUCT
 1524     CONTINUE
 1526      CONTINUE
C     INPUT OF TABLE 6. BOUNDARY AND INTERNAL CONDITIONS
 1600 PRINT 600
           IF(KEEP6) 9980,1610,1605
 1605      IF(NCD6) 9980,1606,1607
 1606 PRINT 905
           GO TO 1700
 1607 PRINT 906
           GO TO 1612
 1610 PRINT 601
           DO 1611 I   = 1, MXP5
           DO 1611 J   = 1, MYP5
                 KAS(I,J) =   1
                 DTLX(I,J)   =  0.0
                 DTDY(I,J)   =  0.0
 1611      CONTINUE
 1612      DO 1645 M   =   1,NCD6
                 K             =  0
      READ 25, IN1,JN1,IN2,JN2,KASE, WV1, T1, DTX1, DTY1,H1,IE
      PRINT 25,IN1,JN1,IN2,JN2,KASE, WV1, T1, DTX1, DTY1,H1,IE
                 IN1    =  IN1 + 3
                 JN1    =  JN1 + 3
                 IN2    =  IN2 + 3
                 JN2    =  JN2 + 3
           DO 1645 I    =  IN1,IN2
           DO 1645 J    =  JN1,JN2
                 I2     =  I
                 J2     =  J
                 KAS(I,J) =  KASE
           GO TO (1615,1620,1625,1630,1635) KASE
 1615            WV(I,J)   =  WV(I,J) + WV1
      CALL SUCTION
                 KAS(I,J)    =  2
```

```
          GO TO 1645
 1620            T(I,J)    =  T1   +  T(I,J)
      CALL DSUCT
          GO TO 1645
 1625            DTDX(I,J)=  DTDX(I,J) +  DTX1
          GO TO 1645
 1630            DTDY(I,J)=  DTDY(I,J) +  DTY1
          GO TO 1645
 1635 CALL HUMIDY (TE,H1)
      CALL DSUCT
                 KAS(I,J)   =  2
 1645     CONTINUE
                 K              =  1
         DO 1670 J            = 4,MYP2
          IF ( 3 - KAS(3,J)) 1655, 1650, 1655
 1650            T(3,J)      =   T(4,J)   - HX* DTDX(3,J)
                 I2          =  3
                 J2          =  J
      CALL DSUCT
 1655     IF ( 3 - KAS(MXP3,J)) 1670,1660,1670
 1660            L           =   MXP3   -  1
                 T(MXP3,J)  =   T(L,J)  +   HX *DTDX( MXP3,J)
                 I2          =  MXP3
                 J2          =  J
      CALL DSUCT
 1670     CONTINUE
          DO 1690 I    =   3, MXP3
          IF( 4   - KAS(I,3)) 1680, 1675, 1680
 1675            T(I,3)      =   T(I,4)   -  HY* DTDY(I,3)
                 I2          =  I
                 J2          =  3
      CALL DSUCT
 1680     IF (4  -  KAS(I,MYP3)) 1690, 1685, 1690
 1685            L           =   MYP3   -  1
                 T(I,MYP3)  =   T(I,L)  +   HY * DTDY(I,MYP3)
                 I2          =  I
                 J2          =  MYP3
      CALL DSUCT
 1690     CONTINUE
C     INPUT OF TABLE 7, CLOSURE ACCELERATION DATA
 1700 PRINT 700
          IF(KEEP7)9980,1710,1705
 1705     IF(NCD7) 9980,1706,1707
 1706 PRINT 905
          GO TO 2000
 1707 PRINT 906
 1710 PRINT 701
      READ 20, IX,IY
          IF(IX - IY) 1711,1712,1712
 1711            IV     =  IY
          GO TO 1715
 1712            IV      =  IX
 1715     DO 1720 I    = 1,IV
                 VX(I)  =  0.0
                 VY(I)  =  0.0
```

```
 1720     CONTINUE
      READ 26,( VX(N),N = 1,IX)
      READ 26,( VY(N),N = 1,IY)
          DO 1725 I  = 1,IV
      PRINT 27, I,  VX(I),  VY(I)
 1725     CONTINUE
 1800 PRINT 800
      READ 20, KEY   , NSTEP
          GO TO (1805,1840,1840) KEY
C     LIST OF TIME-STEPS WHERE B.C. CHANGE
 1805 READ 20, (KT(N), N = 1,NSTEP)
      PRINT 20, (KT(N), N  =  1,NSTEP)
                N    =  1
          DO 1830 K =  1, ITIME
          IF ( K  - KT(N))  1820,1815
 1815           KLOC(K)   =     1
                N         =   N + 1
          GO TO 1830
 1820           KLOC(K)   =     2
 1830     CONTINUE
          GO TO 1871
C     CONTINUOUS B.C. CHANGE (READ IN NEW B.C. FOR EACH TIME STEP)
 1840     DO 1850 K  =  1,ITIME
             KLOC(K) =  1
 1850     CONTINUE
      PRINT 806
          GO TO 1871
 1860 PRINT 807
         DO 1870 K    =  1, ITIME
                KLOC(K)    =  2
 1870    CONTINUE
 1871 PRINT 808
      READ 20,KEYB,NOUT
          GO TO ( 1872,1882) KEYB
C     LIST OF TIME STEPS FOR OUTPUT READ IN
 1872 READ 20,(KT(N),N = 1,NOUT )
      PRINT 20,(KT(N), N = 1,NOUT)
                N            =  1
          DO 1875 K  =  1, ITIME
          IF( K - KT(N))1874,1873
 1873           KPUT(K)    =  1
                N          =  N + 1
          GO TO 1875
 1874           KPUT(K)    =  2
 1875     CONTINUE
          GO TO 2000
C     CONTINUOUS OUTPUT
 1882     DO 1883 K  =  1,ITIME
                KPUT(K)    =  1
 1883     CONTINUE
      PRINT 806
C     ZERO-OUT OF ALL TEMPORARY CONSTANTS
 2000     DO 2005 I  = 1,MXP5
          DO 2005 J  = 1,MYP5
                A(I,J)     =   0.0
```

```
                  B(I,J)    =    0.0
                  CX(I,J)   =    0.0
                  CY(I,J)   =    0.0
                  D(I,J)    =    0.0
                  E(I,J)    =    0.0
                  F(I,J)    =    0.0
                  TX(I,J)   =    0.0
                  TY(I,J)   =    0.0
                  VSX(I,J)  =    0.0
                  VSY(I,J)  =    0.0
 2005        CONTINUE
             IF(MYP5 - MXP5) 2006,2006,2007
 2006             MMAX      =    MXP5
             GO TO 2008
 2007             MMAX      =    MYP5
 2008        DO 2009 I  =  1, MMAX
                  AL(I)     =    0.0
                  BL(I)     =    0.0
                  CL(I)     =    0.0
                  DL(I)     =    0.0
 2009        CONTINUE
C       START OF TIME STEP
             DO 9000 K  =  1, ITIME
                  KOUT        =  KOUT(K)
             IF (K - 1) 9980, 1980, 1900
 1900             KAT    =    KLOC(K)
             GO TO (1910,1980)  KAT
 1910 READ 20, KTIME, NCD6
      PRINT 900
      PRINT 906
      PRINT 601
             DO 1945 M  =  1, NCD6
      READ 25, IN1,JN1,IN2,JN2,KASE, WV1, T1, DTX1, DTY1, H1, TE
      PRINT 25,IN1,JN1,IN2,JN2,KASE, WV1, T1, DTX1, DTY1, H1, TE
                  IN1    =  IN1 + 3
                  JN1    =  JN1 + 3
                  IN2    =  IN2 + 3
                  JN2    =  JN2 + 3
             DO 1945 I  =  IN1,IN2
             DO 1945 J  =  JN1,JN2
                  I2     =  I
                  J2     =  J
                  KAS(I,J) =  KASE
             GO TO (1915,1920,1925,1930,1935) KASE
 1915             WV(I,J)    =  WV1
      CALL SUCTION
                  KAS(I,J)     =  2
             GO TO 1945
 1920             T(I,J)     =  T1
      CALL DSUCT
             GO TO 1945
 1925             DTDX(I,J) =  DTX1
             GO TO 1945
 1930             DTDY(I,J) =  DTY1
            GO TO 1945
```

```
1935 CALL HUMIDY (TE,H1)
     CALL USUCT
               KAS(I,J)   =  2
1945      CONTINUE
        DO 1970 J          =  4,MYP2
          IF   ( 3  -  KAS(3,J)) 1955, 1950, 1955
1950           T(3,J)      =    T(4,J)   - HX* DTDX(3,J)
               I2          =  3
               J2          =  J
     CALL USUCT
1955      IF ( 3  - KAS(MXP3,J)) 1970, 1960, 1970
1960           L           =   MXP3  -  1
               T(MXP3,J) =    T(L,J)  +   HX*DTDX(MXP3,J)
               I2          =  MXP3
               J2          =  J
     CALL DSUCT
1970      CONTINUE
          DO 1990 I  =  3, MXP3
          IF( 4  - KAS(I,3)) 1975, 1965, 1975
1965           T(I,3)      = T(I,4)    - HY* DTDY(I,3)
               I2          =  I
               J2          =  3
     CALL USUCT
1975      IF (4 -  KAS(I,MYP3)) 1990, 1985, 1990
1985           L           =  MYP3 - 1
               T(I,MYP3) =  T(I,L)  +  HY* DTDY(I,MYP3)
               I2          =  I
               J2          =  MYP3
     CALL DSUCT
1990      CONTINUE
C    ROTATION,COMPUTATION OF UNSATURATED PERMEABILITY
1980      DO  2010 I  = 3, MXP4
          GO TO ( 1982,1983) KOUT
1982 PRINT 809,K
     PRINT 29
1983      DO  2010 J  =  3,MYP4
          IF (ALFA(I,J))2014,2013,2014
2013           C1          =  1.0
               C2          =  0.0
               C3          =  0.0
          GO TO 2017
2014           A1          =  ALFA(I,J)/57.2957795
               C1      =  COS(A1)
               A2      =  (90.0 - ALFA(I,J))/57.2957795
               C2      =  COS(A2)
               A3      =  (90.0 + ALFA(I,J))/57.2957795
               C3      =  COS(A3)
2017           P11(I,J)   =  P1(I,J)*C1*C1 + P2(I,J)*C2*C2
               P22(I,J)   =  P1(I,J)*C3*C3 + P2(I,J)*C1*C1
               P12(I,J)   =  P1(I,J)*C1*C3 + P2(I,J)*C1*C2
          IF (WV(I,J) - WVS(I,J))2015,2020,2020
2015           TE          =  ABS(T(I,J))
               RE           =  FN(I,J)
               A1          =  AK(I,J)
               C1          =  BK(I,J)
```

```
                C2          =  1.0  +  ((TE*A1)**BE)/C1
                UNSAT       =  1.0 / C2
                P11(I,J)    =  P11(I,J)* UNSAT
                P22(I,J)    =  P22(I,J)* UNSAT
                P12(I,J)    =  P12(I,J)* UNSAT
 2020      GO TO (2025,2010)KOUT
 2025           I1          =  I - 3
                J1          =  J - 3
     PRINT 28,I1,J1,T(I,J),WV(I,J),DTDW(I,J),P11(I,J),P12(I,J),P22(I,J)
 2010      CONTINUE
           GO TO (2120,2140) KORCL
 2120      DO 2130 I  =  3, MXP3
           DO 2130 J  =  3, MYP3
                CONST       =  HT * DTDW(I,J) / (2.0)
                A(I,J)      = (( P12(I,J)/ HX  + P22(I,J)/HY )/HY) * CONST
                B(I,J)      = (( P11(I,J)/ HX  + P12(I,J)/HY )/HX) * CONST
                CX(I,J)     = (( P11(I,J)/ HX  + P11(I+1,J)/HX + P12(I,J)/HY
     1                          + P12(I+1,J)/HY )/HX)* CONST
                CY(I,J)     = ((P12(I,J)/HX + P12(I,J+1)/HX + P22(I,J)/HY
     1                          + P22(I,J+1)/HY)/HY )* CONST
                D(I,J)      =  (( P11(I+1,J)/HX + P12(I,J+1)/HY)/HX)*CONST
                E(I,J)      = (( P12(I+1,J)/HX + P22(I,J+1)/HY)/HY)*CONST
                F(I,J)      = -(( P12(I,J) - P12(I+1,J))/(HX)
     1                          + (P22(I,J) - P22(I,J+1))/(HY))* CONST
 2130      CONTINUE
           GO TO 2155
 2140      DO 2150 I  =  3,MXP3
                A1          =  I - 3
                R           =  RO + A1*HX
           DO 2150 J  =  3,MYP3
                CONST       =  HT*DTDW(I,J)*(0.5)
                A(I,J)      =  (( -P12(I,J)/R  +  P12(I,J)/HX
     1                             + P22(I,J)/HY)/HY)*CONST
                B(I,J)      =  (( -P11(I,J)/R  +  P11(I,J)/HX
     1                             + P12(I,J)/HY)/HX)*CONST
                CX(I,J)     =  (( -P11(I,J)/R  +  P11(I,J)/HX
     1                             + P11(I+I,J)/HX  +  P12(I,J)/HY
     2                             - P12(I+1,J)/HY)/HX
     3                             - P12(I,J)/(HY*R))*CONST
                CY(I,J)     =  (( P12(I,J)/HX  +  P12(I,J+1)/HX
     1                             + P22(I,J)/HY  +  P22(I,J+1)/HY)/HY)
     2                             *CONST
                D(I,J)      =  (( P11(I+1,J)/HX  +  P12(I,J+1)/HY)/HX)
     1                             *CONST
                F(I,J)      =  (( P12(I+1,J)/HX  +  P22(I,J+1)/HY)/HY)
     1                             *CONST
                F(I,J)      = -(  - P12(I,J)/R  +  P12(I,J)/HX
     1                             - P12(I+1,J)/HX  +  P22(I,J)/HY
     2                             - P22(I,J+1)/HY  ) * CONST
 2150      CONTINUE
 2155      DO 2195 I  =  1,MXP5
           DO 2195 J  =  1,MYP5
                TX(I,J)     =  T(I,J)
                TY(I,J)     =  T(I,J)
           IF (A4)2195,2181
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```
 2181           T(I,J)      =  0.0
 2195     CONTINUE
                I1          =     IM1 - 3
                J1          =     JM1 - 3
                I2          =     IM2 - 3
                J2          =     JM2 - 3
                I3          =     IM3 - 3
                J3          =     JM3 - 3
                I4          =     IM4 - 3
                J4          =     JM4 - 3
      PRINT 801, I1,J1,I2,J2,I3,J3,I4,J4
          DO 8000 IT   =  1, ITMAX
           DO 2370  J  =  3, MYP3
C     CLOSURE  PARAMETER  CHOICE
          IF ( IT - IY) 2197,2197,2212
C     PRESET PARAMETERS
 2197     DO 2210  I  =  3, MXP3
                VSY(I,J)    =  VY(IT)
 2210     CONTINUE
          GO TO 2215
C     SELF-DETERMINING PARAMETERS
 2212     DO 2214  I  =  3, MXP3
                DP              = - A(I,J)* TY(I,J-1) + CY(I,J)* TY(I,J)
     1                            - E(I,J)* TY(I,J+1)
          IF  (TY(I,J))2216,2217,2216
 2217           VSY(I,J)    =  VY(I)
          GO TO 2214
 2216           VSY(I,J)    =  DP/TY(I,J)
         IF(VSY(I,J))2213,2214,2214
 2213           VSY(I,J)    =  0.0
 2214     CONTINUE
 2215     DO 2200  I  =  3, MXP3
                AL(I)           = -B(I,J)
                BL(I)           =  CX(I,J) +  A4 + VSY(I,J)
                CL(I)           = -D(I,J)
                DL(I)       =  A(I,J)*T(I,J-1) + B(I,J)*T(I-1,J)
     1              - (CX(I,J)+ CY(I,J)-  A4)* T(I,J) + D(I,J)* T(I+1,J)
     2                       + F(I,J)* T(I,J+1) + 2.0 * F(I,J)
     3                      + A(I,J)* TY(I,J-1) + ( VSY(I,J) - CY(I,J))*
     4                        TY(I,J)   +  E(I,J)* TY(I,J+1)
 2200     CONTINUE
C     COMPUTE CONTINUITY COEFFICIENTS
          DO 2300 I  =  3, MXP3
          IF ( 3 - KAS(3,J)) 2305,2304
 2304     IF ( I - 4) 2305,2300
 2305           KAT         =  KAS(I,J)
          GO TO ( 2350,2320,2330,2350) KAT
C     SUCTION SET
 2320           CC(I)       =  1.0
                BB(I)       =  0.0
                AA(I)       =  T(I,J)
          IF ( I - 3) 2324,2322
 2322           BB(2)       =  1.0
                AA(2)       =  0.0
          GO TO 2300
```

```
2324      IF (I - MXP3) 2300,2326
2326           BB(I+1)    =  0.0
               AA(I+1)    =  T(I,J)
          GO TO 2300
C      SLOPE SET
2330      IF ( 2 - KAS(I-1,J)) 2334,2332
2332           CC(I)      =  1.0
               BB(I)      =  0.0
               AA(I)      =  T(I-1,J) + DTDX(I,J)*HX
          GO TO 2300
2334      IF ( I - 3) 2336,2338
2336      IF ( I - MXP3) 2340,2338
2338           AA(I-1)    =  -DTDX(I,J)*HX
               BB(I-1)    =  1.0
               BB(I)      =  0.0
               AA(I)      =  TY(I,J)
               CC(I+1)    =  1.0
               BB(I+1)    =  0.0
               AA(I+1)    =  AA(I)*BB(I-1) + HX*DTDX(I,J)
          GO TO 2300
C      PIPE INCREMENT SLOPE SET
2340           CC(I)      =  BL(I) + AL(I)*BB(I-1)
               BB(I)      =  -CL(I) / (CC(I))
               AA(I)      =  (DL(I) - AL(I)*AA(I-1))/(CC(I))
               CTEMP      =  1.0  +  CC(I-1)*(1.0 - BB(I-1))/(CC(I))
               BTEMP      =  BB(I)/CTEMP
               ATEMP      =  (AA(I) + CC(I-1)*(AA(I-1) + HX*DTDX(I,J))
     1                          /(CC(I)))/CTEMP
               AA(I-1)    =  -DTDX(I,J)*HX
               BB(I-1)    =  1.0
               AA(I)      =  ATEMP
               BB(I)      =  BTEMP
               CC(I)      =  CTEMP
          GO TO 2300
2350           CC(I)    =  BL(I) + AL(I)* BB(I-1)
               BB(I)    =  -CL(I)/ CC(I)
               AA(I)    = (DL(I) - AL(I)* AA(I-1))/ CC(I)
2300      CONTINUE
          DO  2360  IR =  2, MXP4
               I    =   MXP4  + 2  - IR
               TX(I,J)   =   AA(I)  +   BB(I)  *  TX  (I+1,J)
2360      CONTINUE
2370      CONTINUE
C      SOLUTION OF FLOW IN Y-PIPES
          DO 2570  I  =  3,MXP3
C      CLOSURE PARAMETER CHOICE
          IF (IT - IX) 2365,2365,2375
C      PRESET PARAMETERS
2365      DO  2367 J   =  3, MYP3
               VSX(I,J)   =   VX(IT)
2367      CONTINUE
          GO TO 2450
C      SELF-DETERMINING  PARAMETERS
2375      DO 2385   J  =   3, MYP3
               DP         =  -B(I,J)* TX(I-1,J) + CX(I,J)*TX(I,J)
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     1                          -D(I,J)* TX(I+1,J)
          IF (TX(I,J))2379,2376,2379
 2376          VSX(I,J)  =  VX(I)
          GO TO 2385
 2379          VSX(I,J)  =  UP/TX(I,J)
         IF (VSX(I,J))2380,2385,2385
 2380          VSX(I,J)  =  0.0
 2385     CONTINUE
 2450     DO 2400  J  =   3, MYP3
               AL(J)  =  - A(I,J)
               BL(J)  =  CY(I,J) +   A4  + VSX(I,J)
               CL(J)  =  - E(I,J)
               DL(J)  =  A(I,J)* T(I,J-1) + B(I,J)* T(I-1,J)
     1                   - (CX(I,J) + CY(I,J) -  A4)* T(I,J) + D(I,J)*
     2                     T(I+1,J)  +  E(I,J)* T(I,J+1) +  2.0* F(I,J)
     3                   + B (I,J)* TX(I-1,J) +   ( VSX(I,J) - CX(I,J))*
     4                   TX(I,J)  +  D(I,J)*TX(I+1,J)
 2400     CONTINUE
C     COMPUTE CONTINUITY COEFFICIENTS
          DO 2500 J  =  3, MYP3
          IF ( 4 - KAS(I,3)) 2505,2504
 2504     IF ( J - 4) 2505,2500
 2505          KAT          =  KAS(I,J)
          GO TO ( 2550,2520,2550,2530) KAT
C     SUCTION SET
 2520          CC(J)      =  1.0
               BB(J)      =  0.0
               AA(J)      =  T(I,J)
          IF ( J - 3) 2524,2522
 2522          BB(2)      =  1.0
               AA(2)      =  0.0
          GO TO 2500
 2524     IF ( J - MYP3) 2500,2526
 2526          BB(J+1)    =  0.0
               AA(J+1)    =  T(I,J)
          GO TO 2500
C     SLOPE SET
 2530     IF ( 2 - KAS(I,J-1)) 2534,2532
 2532          CC(J)      =  1.0
               BB(J)      =  0.0
               AA(J)      =  T(I,J-1) + DTDY(I,J)*HY
          GO TO 2500
 2534     IF ( J - 3) 2536,2538
 2536     IF ( J - MYP3) 2540,2538
 2538          AA(J-1)    =  -DTDY(I,J)*HY
               BB(J-1)    =  1.0
               BB(J)      =  0.0
               AA(J)      =  TX(I,J)
               CC(J+1)    =  1.0
               BB(J+1)    =  0.0
               AA(J+1)    =  AA(J)*BB(J-1) + HY*DTDY(I,J)
          GO TO 2500
C     PIPE INCREMENT SLOPE SET
 2540          CC(J)      =  BL(J) + AL(J) *AA(J-1)
               BB(J)      =  -CL(J) / (CC(J))
```

```
                AA(J)       =  (DL(J) - AL(J)*AA(J-1))/(CC(J))
                CTEMP       =  1.0 + CC(J-1)*(1.0 - BB(J-1))/(CC(J))
                BTEMP       =  BB(J)/CTEMP
                ATEMP       =  (AA(J) + CC(J-1)*(AA(J-1) + HY*DTDY(I,J))
     1                           /(CC(J)))/CTEMP
                AA(J-1)     =  -DTDY(I,J)*HY
                BB(J-1)     =  1.0
                AA(J)       =  ATEMP
                BB(J)       =  BTEMP
                CC(J)       =  CTEMP
           GO TO 2500
 2550           CC(J)     =  BL(J) +  AL(J) *BB(J-1)
                BB(J)     = - CL(J)/ CC(J)
                AA(J)     = (DL(J) -  AL(J)* AA(J-1))/ CC(J)
 2500      CONTINUE
           DO 2560 JR  =  2,MYP4
                J  =  MYP4  + 2  - JR
                TY(I,J)  =  AA(J) + BB(J) *TY(I,J+1)
 2560      CONTINUE
 2570      CONTINUE
C      CHECK CLOSURE TOLERANCE
                KOUNT     =  0
           DO 2600 I   =  3,MXP3
           DO 2600 J   =  3,MYP3
                KLOS(I,J) =  1
                ECL         =  ABS(EPS*TY(I,J))
                ERR         =  ABS(TY(I,J) - TX(I,J))
           IF( ECL - ERR) 2605,2600,2600
 2605           KOUNT =KOUNT + 1
                KLOS(I,J) =  2
 2600      CONTINUE
           IF(KOUNT)9980,2650,2608
 2608      GO TO (2610,8000)KOUT
 2610 PRINT 802, II,KOUNT,TX(IM1,JM1),TX(IM2,JM2),TX(IM3,JM3),
     1 TX(IM4,JM4)
      PRINT 803, TY(IM1,JM1),TY(IM2,JM2),TY(IM3,JM3),TY(IM4,JM4)
 8000      CONTINUE
C      OUTPUT OF TIME STEP RESULTS
 2650 PRINT 810
         DO 2700 I   =  3,MXP3
           GO TO (2625,2630)KOUT
 2625 PRINT 809, K
      PRINT 804
 2630      DO 2700 J   =  3,MYP3
                I2     =  I
                J2     =  J
                KAT           =  KLOS(I,J)
         GO TO (2653,2680) KAT
 2680           KAT           =  KAS(I,J)
           GO TO (2685,2653,2653,2653) KAT
 2685           T(I,J)        =  T(I,J) + A(I,J)*( T(I,J-1) + TY(I,J-1))
     1                             +  B(I,J)*( T(I-1,J) + TX(I-1,J))
     2                             -  CX(I,J)*( T(I,J) + TX(I,J))
     3                             -  CY(I,J)*( T(I,J) + TY(I,J))
     4                             +  D(I,J)*( T(I+1,J) + TX(I+1,J))
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```
     5                          +  E(I,J)*( T(I,J+1) + TY(I,J+1))
     6                          +  2.0*F(I,J)
        GO TO 2665
 2653          A1         =  CX(I,J) + CY(I,J)
        IF(A1)2652,2651,2652
 2651          A2         =  0.5
               A3         =  0.5
        GO TO 2660
 2652          A2         =  CX(I,J)/A1
               A3         =  CY(I,J)/A1
 2660          T(I,J)     =  A2*TX(I,J)  +  A3*TY(I,J)
 2665 CALL USUCT
        GO TO (2670,2700) KOUT
 2670          I1         =  I - 3
               J1    =  J - 3
      PRINT 805,I1,J1,T(I,J),WV(I,J),PF1,VSX(I,J),VSY(I,J)
 2700   CONTINUE
 4000   CONTINUE
        GO TO 1010
 9980 PRINT 907
 9999   CONTINUE
      END
```

```
      SUBROUTINF SUCTION
      COMMON/ONE/PFM(10),PFR(10),BETA(10),DTDW(40,25),PF1
     1/TWO/T(40,25),I2,J2
     2/THREE/WVS(40,25),KLH,K
     3/FOUR/WVA(10),Q(10),ALFO(10),R(10),AV(10),POR(40,25),
     4KURV( 0,25),WV(40,25),GAM,ALF,P,DP,DALF,MY,HY,PN(10)
                I           =  I2
                J           =  J2
                L           =  KURV(I,J)
          IF( WV(I,J) - WVS(I,J) ) 1525,1524,1524
 1524           DTDW(I,J) = 1.0
                PF1         =  0.0
                T(I,J)      =  0.0
          GO TO 1530
 1525           AT          =  (100.0*WV(I,J))/(WVS(I,J))
                TAT         =  (100.0*PFR(L))/(PFM(L))
                B           =  BETA(L)
                RECB        =  1.0/(1.0 + B )
                C           =  2.302585
                D           =  PFM(L) - PFR(L)
                XM          =  PFM(L) / ( WVS(I,J)*(1.0 + BETA(L)))
                FACT        =   1.0 / (( 1.0 - POR(I,J))*2.70 )
          IF(TAT - AT) 1527,1526,1526
 1526           PF          =  PFR(L)*(AT/TAT)**RECB
                PF1         =  PFM(L) - PF
                T(I,J)      =  (-(10.0)**PF1)/(2.54)
                TE          =  ABS(T(I,J))
                DTDW(I,J)   = (TE*XM*C*(PFR(L)/PF)**B) * FACT
          GO TO 1528
 1527           PF1         =  D*((100.0 - AT)/(100.0 - TAT))**RECB
                T(I,J)      =  (-(10.0)**PF1)/(2.54)
                TE          =  ABS(T(I,J))
                DTDW(I,J)   = (XM*TE*C*(D/PF1)**B) * FACT
 1528     GO TO (1530,1529) KLH
 1529     IF(K) 1530,1523
 1523 CALL HEAVY
                T(I,J)      =  T(I,J)   +  ALF*P
                DTDW(I,J)   =  DTDW(I,J) + ALF*DP + P*DALF
 1530 RETURN
      END
```

```
      SUBROUTINE DSUCT
      COMMON/ONE/PFM(10),PFR(10),BETA(10),DTOW(40,25),PF1
     1/TWO/T(40,25),I2,J2
     2/THREE/WVS(40,25),KLH,K
     3/FOUR/WVA(10),Q(10),ALFO(10),R(10),AV(10),POR(40,25),
     4KURV(40,25),WV(40,25),GAM,ALF,P,DP,DALF,MY,HY,PN(10)
            I          =  I2
            J          =  J2
            L          =  KURV(I,J)
         IF(T(I,J))2710,2705,2705
 2705       WV(I,J)    =  WVS(I,J)
            DTOW(I,J)  =  1.0
            PF1        =  0.0
         GO TO 2750
 2710    GO TO (2713,2711)KLH
 2711    IF(K) 2712,2713
 2712 CALL HEAVY
            TE         =  - T(I,J)*(2.54) + ALF*P*(2.54)
         GO TO 2714
 2713       TE         =  - T(I,J)*(2.54)
 2714       PF1        =  ALOG10(TE)
         IF(PF1)2715,2720,2720
 2715       PF1        =  0.0
            T(I,J)     =  -1.0/(2.54)
            WV(I,J)    =  WVS(I,J)
            DTOW(I,J) = 1.0
         GO TO 2750
 2720    IF ( PF1 - PFM(L)) 2724,2724,2722
 2722       PF1        =  PFM(L)
            T(I,J)     =  (-(10.0)**PF1)/2.54
            WV(I,J)    =  0.0
            DTOW(I,J)  =  1.0E+10
         GO TO 2750
 2724       PF         =  PFM(L) - PF1
            B          =  BETA(L)
            BP         =  1.0 + BETA(L)
            C          =  2.302585
            D          =  PFM(L) - PFR(L)
            TAT        =  (PFR(L)*100.0)/(PFM(L))
            XM         =  PFM(L) / ( WVS(I,J)*(1.0 + BETA(L)))
         IF (PF - PFR(L))2725,2725,2730
 2725       AT         =  TAT*(PF/PFR(L))**BP
            TE         =  -T(I,J)
            DTOW(I,J)  =  TE*C*XM*(PFR(L)/PF)**B
         GO TO 2735
 2730       AT         =  100.0 - (100.0 - TAT)*(PF1/D)**BP
            TE         =  -T(I,J)
            DTOW(I,J)  =  TE*C*XM*(D/PF1)**B
 2735       WV(I,J)    =  AT*WVS(I,J)/(100.0)
            FACT       =   1.0 / (( 1.0 - POR(I,J))*2.70 )
            DTOW(I,J) = DTOW(I,J) * FACT
 2750    GO TO (2760,2755) KLH
 2755    IF(K) 2760,2756
 2756 CALL HEAVY
            T(I,J)     =  T(I,J)  +  ALF*P
```

```
                  DTDW(I,J) = DTDW(I,J) + ALF*DP + P*DALF
 2760 RETURN
      END
```

```
      SUBROUTINE HEAVY                                                  941
      COMMON/TWO/T(40,25),I2,J2                                         942
     1/FOUR/WVA(10),Q(10),ALFO(10),R(10),AV(10),POR(40,25),             943
     2KURV(40,25),WV(40,25),GAM,ALF,P,DP,DALF,MY,HY,PN(10)              944
                I           =  I2                                       945
                J           =  J2                                       946
                L           =  KURV(I,J)                                947
C     DETERMINE OVERBURDEN PRESSURE HEAD                                948
                P           =  (MY + 3 - J)*GAM*HY / (0.0361)           949
                TERM = (1.0 - POR(I,J)) * 2.70
                TH          = (WV(I,J)/(100.0)) * TERM
                ENN         =  POR(I,J)                                 959
                F1          =   AV(L)*( 1.0 - FNN )
                F2          =  ( 1.0 - TH )                             961
          IF ( WV(I,J) - WVA(L)) 1540,1550,1550                         950
 1540           RP0         =  R(L)                                     951
                QM1         =  Q(L) - 1.0                               952
                QM2         =  Q(L) - 2.0                               953
                ALF         =  ALFO(L) + (1.0 - ALFO(L))*(WV(I,J)/WVA(L))  954
     1                         **QM1                                    955
                SAT         = ( TH / (POR(I,J))   )   ** RP0            956
                F3          = SAT                                       957
                DP          =  P / ( F1 * F2 * F3 )    +    HY          963
                DALF        =  QM1*(1.0 - ALFO(L))*(WV(I,J)/WVA(L))**QM2   964
                DALF        = (DALF* 100.0)/(WVA(L) * TERM)
          GO TO 1560                                                    966
 1550           ALF         =  1.0                                      967
                DALF        =  0.0                                      968
                DP          =  HY  +  P / ( F1 * F2 )                   969
                DTH         = ((WV(I,J) - WVA(L))/(100.0)) * TERM       970
                POR(I,J)    =  ( PN(L) + DTH)/(1.0 + DTH)               971
 1560 RETURN                                                            972
      END                                                               973
```

```
SUBROUTINE HUMIDY (TE,H1)
COMMON/TWO/T(40,25),I2,J2
          I      =  I2
          J      =  J2
          R           =  8.314E+07
          G           =  981.0
          EM          =  18.02
          AN          =  ALOG(H1)
          TM          =  (TE - 32.0)*5.0/(9.0)   +  273.0
          T(I,J)      =   R*TM*AN/(G*EM*2.54)
RETURN
END
```

APPENDIX 4

SAMPLE DATA FOR PROGRAM GCHPIP1

```
DECAY  OF  PORE  PRESSURE   COMPUTATIONS ON  A  SQUARE  REGION
PP 1     POSITIVE  PORE  PRESSURE  DECAY  IN  A  TWO-DIMENSIONAL  REGION
                                 3    1    1    1    4    3    1    1
   10   10   10  500   10.0      10.0             100000.0    0.001
    2    8    4    7    5    5    6    2
    1
    0    0   11   11 1.000E-06 1.000E-06   0.0       0.0       1.0       1.0
    1
    0    0   10   10    1
    0    1   10    9    2                  210.0     1.0       0.0
    0    0   10    0    2                  200.0
    0   10   10   10    2                  100.0
    0    1    0    9    2                 -100.0
   10    1   10    9    2                 -100.0
    2    2
  .0001    .001
  .0001    .001
    3    0
    1   10
    1   20   40   80  160  240  320  400  480  500
```

```
ACCUMULATION OF MOISTURE AROUND A BURIED CASING DUE TO DISTURBANCE OF SOIL
SC 1     TWO SQUARE FT DISTURBED AREA - CENTER CASING AT 7 FT BELOW SUBGRADE
                                  3    1    2    1    4    3    1    1
   32   10   10   50   12.0      12.0              604800.0    0.001
   16    3    8    4   28    6   16    8
    1
    0    0   33   11 1.000E-07 .5000E-07   0.0       2.54    1.6  E+08   3.0
    4 7.0  3.0  1.0  34.6 2.0    0.0      0.365     0.08    0.9 .0695   45.0
    1 7.0  4.0  1.5  34.6 2.0    0.0      0.365     0.08    0.9 .0695   48.0
    0    0   32    1    1
    0    2   14    4    1
    0    5   32   10    1
   18    2   32    4    1
   15    2   17    4    2
    0    0   32   10    1      42.8               -0.0417     0.0
    0    0   32    0    2                0.0
    0    1    0    9    2                0.0
   32    1   32    9    2                0.0
    0   10   32   10    4                                     0.0
    2    2
 0.0001   0.001
 0.0001   0.001
    3    0
    1    4
    4   16   40   50
```

```
 MOISTURE REDISTRIBUTION UNDER A HOUSE FOUNDATION
PROB1     CLAY REGION WITH  INCLINED SILT LENS - CLAY INITIALLY DRY 4 WK JUMP
                                         3   12    2    1    6    3    1    1
   18   16   10  100    12.0      12.0                2419200.0  0.01
    3    4    8    8   14   10   17   14
    1
    0    0   19   17 1.000E-08 0.500E-08   0.0        2.54     1.6  E+10    3.5
    2   14    9   14 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
    3   13   10   13 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
    4   12   11   12 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
    5   11   12   11 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
    6   10   13   10 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
    7    9   14    9 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
    8    8   15    8 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
    9    7   16    7 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
   10    6   16    6 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
   11    5   16    5 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
   12    4   16    4 9.9   E-07 9.5   E-08    45.0      0.0    -15999E+09    -1.5
    1 7.0   4.0   1.5   34.0 2.0     0.0       0.350      0.08   0.9  .0695    45.0
   11 7.0   2.5   4.0   30.0 1.5     0.0       0.310      0.04    0.8 .0695    35.0
    0    0   18   16    1
    2   14    9   14    2
    3   13   10   13    2
    4   12   11   12    2
    5   11   12   11    2
    6   10   13   10    2
    7    9   14    9    2
    8    8   15    8    2
```

```
    9    7   16    7    2
   10    6   16    6    2
   11    5   16    5    2
   12    4   16    4    2
    0    0   18   16    2                 -167.3    0.61
    0    0   18    0    2                   0.0
    0   16    8   16    5                                              .9999 80.0
    0    1    0   15    3                             0.0
   18    1   18   15    3                             0.0
    9   16   17   16    4                                       0.0
   18   16   18   16    2                   0.0
    2    2
  0.0001    0.001
  0.0001    0.001
    1    3
    6   14   25
    1    8
    1    4    8   24   48   72   96  100
    6    1
   18   16   18   16    1        36.25
   14    1
   18   16   18   16    1        36.50
   25    1
   18   16   18   16    1        36.75
```

```
 MOISTURE REDISTRIBUTION UNDER A HOUSE FOUNDATION
PROB2     CLAY REGION WITH  INCLINED SILT LENS - CLAY INITIALLY DRY 4 WK JUMP
                                         3   12    2    1    6    3    1    1
   18   16   10  100    12.0       12.0            2419200.0  0.01
    3    4    8    8   14   10   17   14
    1
    0    0   19   17 1.000E-08 0.500E-08   0.0        2.54     1.6  E+10    3.5
    2   14    9   14 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
    3   13   10   13 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
    4   12   11   12 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
    5   11   12   11 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
    6   10   13   10 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
    7    9   14    9 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
    8    8   15    8 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
    9    7   16    7 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
   10    6   16    6 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
   11    5   16    5 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
   12    4   16    4 9.9  E-07 9.5  E-08   -45.0      0.0     -15999E+09    -1.5
    1 7.0  4.0  1.5  34.0 2.0     0.0       0.350      0.08    0.9  .0695    45.0
   11 7.0  2.5  4.0  30.0 1.5     0.0       0.310      0.04     0.8 .0695    35.0
    0    0   18   16    1
    2   14    9   14    2
    3   13   10   13    2
    4   12   11   12    2
    5   11   12   11    2
    6   10   13   10    2
    7    9   14    9    2
    8    8   15    8    2
```

```
    9    7   16    7    2
   10    6   16    6    2
   11    5   16    5    2
   12    4   16    4    2
    0    0   18   16    2               -167.3    0.61
    0    0   18    0    2                 0.0
    0   16    8   16    5                                            .9999 80.0
    0    1    0   15    3                           0.0
   18    1   18   15    3                           0.0
    9   16   17   16    4                                     0.0
   18   16   18   16    2                 0.0
    2    2
  0.0001    0.001
  0.0001    0.001
    1    3
    6   14   25
    1    8
    1    4    8   24   48   72   96  100
    6    1
   18   16   18   16    1      36.25
   14    1
   18   16   18   16    1      36.50
   25    1
   18   16   18   16    1      36.75
```

```
 DEPTH OF WATER PENETRATION FOR PONDING - DIFFERING ANTECEDENT MOIST. COND.
POND1     ALL SOIL IS INITIALLY AT THE PLASTIC LIMIT YOLO LIGHT CLAY PERM.
                                  3    3    1    1    4    3    1    1
   10   40   10   50    6.0      6.0               604800.0   0.001
    5   35    5   30    5   25    5   15
    1
    0    0   11   37 .8550E-07 .4775E-07   0.0       2.54    1.575E+08   3.00
    0   38   11   39 .4775E-07 .2380E-07   0.0       2.54    1.575E+08   3.00
    0   40   11   41 .8550E-06 .4775E-06   0.0       2.54    1.575E+08   3.00
    1 7.0  4.0  2.0  34.6 2.0    0.0       .365     0.08     .9  .0695   45.0
    0    0   10   40    1
    0    0   10   39    1       37.8
    0    0   10    0    1         0.0
    0    1    0   39    3                            0.0
   10    1   10   39    3                            0.0
    0   40   10   40    2                  0.0
    2    2
   0.0001   0.001
   0.0001   0.001
    3    0
    1    6
    1    4    8   24   48   50
```

```
 DEPTH OF WATER PENETRATION FOR PONDING - DIFFERING ANTECEDENT MOIST. COND.
POND2     SOIL IS WETTER THAN THE PLASTIC LIMIT BY 4 PCT. AT THE TOP
                                      3    3    1    1    4    3    1    1
   10   40   10   50    6.0      6.0                 604800.0   0.001
    5   35    5   30    5   25    5   15
    1
    0    0   11   37 .8550E-07 .4775E-07   0.0      2.54    1.575E+08   3.00
    0   38   11   39 .4775E-07 .2380E-07   0.0      2.54    1.575E+08   3.00
    0   40   11   41 .8550E-06 .4775E-06   0.0      2.54    1.575E+08   3.00
    1  7.0  4.0  2.0  34.6 2.0    0.0      .365    0.08     .9  .0695    45.0
    0    0   10   40    1
    0    0   10   39    1        42.0              -0.017     0.0
    0    0   10    0    1         0.0
    0    1    0   39    3                           0.0
   10    1   10   39    3                           0.0
    0   40   10   40    2                  0.0
    2    2
   0.0001   0.001
   0.0001   0.001
    3    0
    1    6
    1    4    8   24   48   50
```

```
 DEPTH OF WATER PENETRATION FOR PONDING - DIFFERING ANTECEDENT MOIST. COND.
POND3     SOIL IS INITIALLY AT P.L. - COMPRESSIBILITY EFFECTS INCLUDED
                                   3    3    1    1    4    3    1    2
   10   10   10   50   6.0       6.0               604800.0   0.001
    5    8    5    6    5    4    5    2
    1
    0    0   11    7 .8550E-07 .4775E-07   0.0       2.54    1.575E+08   3.00
    0    8   11    9 .4775E-07 .2380E-07   0.0       2.54    1.575E+08   3.00
    0   10   11   11 .8550E-06 .4775E-06   0.0       2.54    1.575E+08   3.00
    1 7.0  4.0  2.0  34.6 2.0    0.0       .365     0.08     .9  .0695   45.0
    0    0   10   10    1
    0    0   10    9    1      37.8
    0    0   10    0    1         0.0
    0    1    0    9    3                            0.0
   10    1   10    9    3                            0.0
    0   10   10   10    2                  0.0
    2    2
   0.0001   0.001
   0.0001   0.001
    3    0
    1    6
    1    4    8   24   48   50
```

```
EFFECT OF WATER INJECTION WELLS IN CLAY  - 12 IN. DIAMETER
PW 1     12 IN DIAM WELL  10 FT DEEP IN SOIL AT THE PLASTIC LIMIT
                                  3    3    1    2    5    3    2    1
  10   40   10   50    6.0      6.0       6.0     604800.0   0.001
   5   38    5   35    5   30    5   24
   1
   0    0   11   37 .8550E-07 .4775E-07   0.0       2.54    1.575E+08    3.00
   0   38   11   39 .4775E-07 .2380E-07   0.0       2.54    1.575E+08    3.00
   0   40   11   41 .8550E-06 .4775E-06   0.0       2.54    1.575E+08    3.00
   1 7.0  4.0  2.0  34.6 2.0    0.0       .365     0.08     .9  .0695    45.0
   0    0   10   40    1
   1    1    9   39    1      37.8
   0   20    0   40    2                  0.0       1.0
  10    1   10   39    3                            0.0
   0    1    0   19    3                            0.0
   0    0   10    0    1       37.8
   0   20    0   40    2                  0.0
   0   40   10   40    2                  0.0
   2    2
  .0001    .001
  .0001    .001
   3    0
   1    6
   1    4    8   24   48   50
```

```
EFFECT OF WATER INJECTION WELLS IN CLAY  - 12 IN. DIAMETER
PW 2     12 IN  DIAM WELL  -  10 FT DEEP - SOIL AT P.L. COMPRESSIBILITY INCL.
                                 3    3    1    2    4    3    2    2
   10   10   10   50    6.0       6.0       6.0     604800.0   0.001
    5    8    5    6    5    4    5    2
    1
    0    0   11    7 .8550E-07 .4775E-07  0.0       2.54     1.575E+08   3.00
    0    8   11    9 .4775E-07 .2380E-07  0.0       2.54     1.575E+08   3.00
    0   10   11   11 .8550E-06 .4775E-06  0.0       2.54     1.575E+08   3.00
    1 7.0  4.0  2.0  34.6 2.0    0.0      .365     0.08      .9  .0695    45.0
    0    0   10   10    1
    1    1    9    9    1      37.8
    0    0    0   10    2                 0.0     -2.84
    0   10   10   10    2                 0.0
    0    0    0   10    2                 0.0
    0    0   10    0    1       37.8
   10    1   10    9    3                            0.0
    2    2
  .0001    .001
  .0001    .001
    3    0
    1    6
    1    4    8   24   48   50
```

```
EFFECT OF WATER INJECTION WELLS IN CLAY  - 12 IN. DIAMETER
PW 3     12 IN DIAM WELL - 10 FT DEEP - SOIL WETTER THAN P.L. BY 4 PCT. AT TOP
                                       3     3     1     2     5     3     2     1
 10    40    10    50    6.0        6.0         6.0      604800.0    0.001
  5    38     5    35     5    30     5    24
  1
  0     0    11    37 .8550E-07 .4775E-07    0.0         2.54     1.575E+08    3.00
  0    38    11    39 .4775E-07 .2380E-07    0.0         2.54     1.575E+08    3.00
  0    40    11    41 .8550E-06 .4775E-06    0.0         2.54     1.575E+08    3.00
  1  7.0   4.0   2.0   34.6  2.0      0.0         .365      0.08       .9  .0695    45.0
  0     0    10    40     1
  1     1     9    39     1        42.0                  -0.017       0.0
  0    20     0    40     2                   0.0         1.0
  0     0    10     0     1        37.8
  0    40    10    40     2                   0.0
  0     1     0    19     3                              0.0
 10     1    10    39     3                              0.0
  0    20     0    40     2                   0.0
  2     2
 .0001     .001
 .0001     .001
  3     0
  1     6
  1     4     8    24    48    50
```

APPENDIX 5

SAMPLE OUTPUT FOR PROGRAM GCHPIP1

```
 PROGRAM GCHPIP1   R.L.LYTTON  REVISION DATE        JUNE 1969

DEPTH OF WATER PENETRATION FOR PONDING - DIFFERING ANTECEDENT MOIST. COND.


    PROB
   POND3     SOIL IS INITIALLY AT P.L. - COMPRESSIBILITY EFFECTS INCLUDED


 TABLE 1. PROGRAM CONTROL SWITCHES.
                                                          TABLES NUMBER
                                                       2     3    4A    5     6     7

         PRIOR  DATA OPTIONS (1 = HOLD)               -0    -0    -0    -0    -0    -0
         NUMBER CARDS INPUT THIS PROBLEM               3     3     1     1     4     3

         GRID = 1, CYLINDER = 2 SWITCH                 1


         LIGHT = 1, HEAVY = 2  SWITCH                  2



 TABLE 2. INCREMENT LENGTHS, ITERATION CONTROL


         NUM OF X-INCREMENTS     =          10
         X-INCREMENT LENGTH      =  6.000E+00 IN.
         NUM OF  Y-INCREMENTS    =          10
         Y-INCREMENT LENGTH      =  6.000E+00 IN.
         NUM OF TIME INCREMENTS  =          50
         TIME INCREMENT LENGTH   =  6.048E+05 SECS
         ITERATIONS / TIME STEP  =          10
         INSIDE RADIUS           = -0.        IN
         TOLERANCE               =  1.000E-03


         MONITOR STATIONS I,J            5  8      5  6      5  4      5  2

  TRANSIENT  FLOW



 TABLE  3.  PERMEABILITY

  FROM        TO          P1          P2    ALFA(DEG.)      AK          BK     EXPONENT
   0    0     11      7 8.550E-08 4.775E-08 0.           2.540E+00 1.575E+08 3.000E+00
   0    8     11      9 4.775E-08 2.380E-08 0.           2.540E+00 1.575E+08 3.000E+00
   0   10     11     11 8.550E-07 4.775E-07 0.           2.540E+00 1.575E+08 3.000E+00
```

```
TABLE  4. SUCTION - WATER CONTENT CURVES
          CURVE NUMBER                    1
          NUM  LOCATIONS         =        1
          MAXIMUM  PF            =          7.00
          PF  AT  INFLECTION     =          4.00
          EXPONENT  FOR  PF      =          2.00
          AIR ENTRY WATER CONT   =         34.60
          DRYING CURVE EXPONENT  =          2.00
          ALFA AT 0 WATER CONT   =    0.
          INITIAL  POROSITY      =    3.650E-01
          REFERENCE  AV          =    8.000E-02
          SATURATION EXPONENT    =           .90
          SOIL UNIT WT  PCI      =    6.950E-02
          SATURATED WATER CONT.  =    4.500E+01



  NO.      FROM     TO
   1    0     0    10    10


 TABLE 5. INITIAL CONDITIONS

    FROM        TO      CASE     VOL. W.   PORE PR.   SLOPE Y  SLOPE X
  0     0    10     9     1      3.780E+01-0.          -0.        -0.


 TABLE 6. BOUNDARY AND INTERNAL CONDITIONS

 FROM STA    TO STA  CASE          WV         T         DT/DX       DT/DY        H  TEMP
  0     0    10    0     1      0.        -0.        -0.         -0.        *.000 -0.0
  0     1     0    9     3     -0.        -0.         0.         -0.        *.000 -0.0
 10     1    10    9     3     -0.        -0.         0.         -0.        *.000 -0.0
  0    10    10   10     2     -0.         0.        -0.         -0.        *.000 -0.0


 TABLE 7.  CLOSURE ACCELERATION DATA

    FICTITIOUS CLOSURE VALVE SETTINGS

     NO.          VSX            VSY
        1      1.000E-04      1.000E-04
        2      1.000E-03      1.000E-03
```

```
TABLE 8A. TIME STEPS FOR B.C. CHANGE

 NONE


TABLE 8B. TIME STEPS FOR OUTPUT.
  1     4     8    24    48    50

 TIME STEP  =     1



I   J    T(I,J)         WV(I,J)      DTDW(I,J)     P11(I,J)      P12(I,J)       P22(I,J)
0   0  -4.742E+01     3.780E+01     6.254E+03     8.456E-08     0.             4.723E-08
0   1  -5.898E+01     3.780E+01     1.009E+01     8.371E-08     0.             4.675E-08
0   2  -7.053E+01     3.780E+01     1.207E+01     8.249E-08     0.             4.607E-08
0   3  -8.208E+01     3.780E+01     1.405E+01     8.085E-08     0.             4.515E-08
0   4  -9.363E+01     3.780E+01     1.603E+01     7.877E-08     0.             4.399E-08
0   5  -1.052E+02     3.780E+01     1.800E+01     7.627E-08     0.             4.259E-08
0   6  -1.167E+02     3.780E+01     1.998E+01     7.336E-08     0.             4.097E-08
0   7  -1.283E+02     3.780E+01     2.196E+01     7.010E-08     0.             3.915E-08
0   8  -1.398E+02     3.780E+01     2.394E+01     3.717E-08     0.             1.853E-08
0   9  -1.514E+02     3.780E+01     2.591E+01     3.509E-08     0.             1.749E-08
0  10   0.            4.500E+01     7.000E+00     8.550E-07     0.             4.775E-07
0  11   0.            0.           -7.174E+57     8.550E-07     0.             4.775E-07

 TIME STEP  =     1



I   J    T(I,J)         WV(I,J)      DTDW(I,J)     P11(I,J)      P12(I,J)       P22(I,J)
1   0  -4.742E+01     3.780E+01     6.254E+03     8.456E-08     0.             4.723E-08
1   1  -5.898E+01     3.780E+01     5.848E+03     8.371E-08     0.             4.675E-08
1   2  -7.053E+01     3.780E+01     5.202E+03     8.249E-08     0.             4.607E-08
1   3  -8.208E+01     3.780E+01     4.555E+03     8.085E-08     0.             4.515E-08
1   4  -9.363E+01     3.780E+01     3.909E+03     7.877E-08     0.             4.399E-08
1   5  -1.052E+02     3.780E+01     3.263E+03     7.627E-08     0.             4.259E-08
1   6  -1.167E+02     3.780E+01     2.617E+03     7.336E-08     0.             4.097E-08
1   7  -1.283E+02     3.780E+01     1.971E+03     7.010E-08     0.             3.915E-08
1   8  -1.398E+02     3.780E+01     1.325E+03     3.717E-08     0.             1.853E-08
1   9  -1.514E+02     3.780E+01     6.786E+02     3.509E-08     0.             1.749E-08
1  10   0.            4.500E+01     7.000E+00     8.550E-07     0.             4.775E-07
1  11   0.            0.           -7.174E+57     8.550E-07     0.             4.775E-07

 TIME STEP  =     1



I   J    T(I,J)         WV(I,J)      DTDW(I,J)     P11(I,J)      P12(I,J)       P22(I,J)
2   0  -4.742E+01     3.780E+01     6.254E+03     8.456E-08     0.             4.723E-08
```

```
 2   1   -5.898E+01     3.780E+01     5.848E+03     8.371E-08      0.             4.675E-08
 2   2   -7.053E+01     3.780E+01     5.202E+03     8.249E-08      0.             4.607E-08
 2   3   -8.208E+01     3.780E+01     4.555E+03     8.085E-08      0.             4.515E-08
 2   4   -9.363E+01     3.780E+01     3.909E+03     7.877E-08      0.             4.399E-08
 2   5   -1.052E+02     3.780E+01     3.263E+03     7.627E-08      0.             4.259E-08
 2   6   -1.167E+02     3.780E+01     2.617E+03     7.336E-08      0.             4.097E-08
 2   7   -1.283E+02     3.780E+01     1.971E+03     7.010E-08      0.             3.915E-08
 2   8   -1.398E+02     3.780E+01     1.325E+03     3.717E-08      0.             1.853E-08
 2   9   -1.514E+02     3.780E+01     6.786E+02     3.509E-08      0.             1.749E-08
 2  10    0.            4.500E+01     7.000E+00     8.550E-07      0.             4.775E-07
 2  11    0.            0.           -7.174E+57     8.550E-07      0.             4.775E-07

 TIME STEP   =      1


 I   J     T(I,J)        WV(I,J)       DTOW(I,J)     P11(I,J)      P12(I,J)       P22(I,J)
 3   0   -4.742E+01     3.780E+01     6.254E+03     8.456E-08      0.             4.723E-08
 3   1   -5.898E+01     3.780E+01     5.848E+03     8.371E-08      0.             4.675E-08
 3   2   -7.053E+01     3.780E+01     5.202E+03     8.249E-08      0.             4.607E-08
 3   3   -8.208E+01     3.780E+01     4.555E+03     8.085E-08      0.             4.515E-08
 3   4   -9.363E+01     3.780E+01     3.909E+03     7.877E-08      0.
 3   5   -1.052E+02     3.780E+01     3.263E+03     7.62
 3   6   -1.167E+02     3.780E+01                                                 4.097E-08
 3   7   -1.2                                      .010E-08      0.             3.915E-08
                                      1.325E+03     3.717E-08      0.             1.853E-08
         -1.514E+02     3.780E+01     6.786E+02     3.509E-08      0.             1.749E-08
 9  10    0.            4.500E+01     7.000E+00     8.550E-07      0.             4.775E-07
 9  11    0.            0.           -7.174E+57     8.550E-07      0.             4.775E-07

 TIME STEP   =      1


 I   J     T(I,J)        WV(I,J)       DTOW(I,J)     P11(I,J)      P12(I,J)       P22(I,J)
10   0   -4.742E+01     3.780E+01     6.254E+03     8.456E-08      0.             4.723E-08
10   1   -5.898E+01     3.780E+01     1.009E+01     8.371E-08      0.             4.675E-08
10   2   -7.053E+01     3.780E+01     1.207E+01     8.249E-08      0.             4.607E-08
10   3   -8.208E+01     3.780E+01     1.405E+01     8.085E-08      0.             4.515E-08
10   4   -9.363E+01     3.780E+01     1.603E+01     7.877E-08      0.             4.399E-08
10   5   -1.052E+02     3.780E+01     1.800E+01     7.627E-08      0.             4.259E-08
10   6   -1.167E+02     3.780E+01     1.998E+01     7.336E-08      0.             4.097E-08
10   7   -1.283E+02     3.780E+01     2.196E+01     7.010E-08      0.             3.915E-08
10   8   -1.398E+02     3.780E+01     2.394E+01     3.717E-08      0.             1.853E-08
10   9   -1.514E+02     3.780E+01     2.591E+01     3.509E-08      0.             1.749E-08
10  10    0.            4.500E+01     7.000E+00     8.550E-07      0.             4.775E-07
10  11    0.            0.           -7.174E+57     8.550E-07      0.             4.775E-07

 TIME STEP   =      1


 I   J     T(I,J)        WV(I,J)       DTOW(I,J)     P11(I,J)      P12(I,J)       P22(I,J)
11   0    0.            0.            2.595-220     8.550E-08      0.             4.775E-08
```

```
11   1   0.          0.          2.218E-62   8.550E-08   0.          4.775E-08
11   2   0.          0.         -7.174E+57   8.550E-08   0.          4.775E-08
11   3   0.          0.         -7.174E+57   8.550E-08   0.          4.775E-08
11   4   0.          0.         -7.174E+57   8.550E-08   0.          4.775E-08
11   5   0.          0.         -4.503+115   8.550E-08   0.          4.775E-08
11   6   0.          0.          3.903E-32   8.550E-08   0.          4.775E-08
11   7   0.          0.         -7.174E+57   8.550E-08   0.          4.775E-08
11   8   0.          0.         -5.550E+24   4.775E-08   0.          2.380E-08
11   9   0.          0.         -7.230E+57   4.775E-08   0.          2.380E-08
11  10   0.          0.         -7.230E+57   8.550E-07   0.          4.775E-07
11  11   0.          0.         -7.230E+57   8.550E-07   0.          4.775E-07

ITERATION  PTS.NOT CLOSED                  MONITOR   STATIONS

                                    5  8        5  6        5  4        5  2
      1          88      TX    -1.397E+02  -1.163E+02  -9.321E+01  -7.019E+01
                         TY    -1.073E+02  -1.136E+02  -9.276E+01  -7.015E+01

      2          52      TX    -1.076E+02  -1.138E+02  -9.267E+01  -7.001E+01
                         TY    -1.075E+02  -1.137E+02  -9.271E+01  -7.007E+01

      3          23      TX    -1.073E+02  -1.137E+02  -9.272E+01  -7.006E+01
                         TY    -1.074E+02  -1.137E+02  -9.272E+01  -7.007E+01

      4          17      TX    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01
                         TY    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01

      5          17      TX    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01
                         TY    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01

      6          17      TX    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01
                         TY    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01

      7          17      TX    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01
                         TY    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01

      8          17      TX    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01
                         TY    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01


      9          17      TX    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01
                         TY    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01

     10          17      TX    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01
                         TY    -1.074E+02  -1.137E+02  -9.273E+01  -7.007E+01


 ***CLOSURE***


 TIME STEP  =     1


STATION      T(I,J)        WV(I,J)       PF(I,J)   VSX(I,J)     VSY(I,J)
```

```
 0   0      -4.742E+01     3.780E+01     2.617E+00     0.            0.
 0   1      -5.856E+01     3.781E+01     2.616E+00     0.            0.
 0   2      -6.994E+01     3.781E+01     2.615E+00     0.            1.648E-05
 0   3      -8.129E+01     3.782E+01     2.615E+00     0.            2.018E-05
 0   4      -9.263E+01     3.782E+01     2.614E+00     0.            2.372E-05
 0   5      -1.040E+02     3.783E+01     2.614E+00     0.            2.701E-05
 0   6      -1.153E+02     3.783E+01     2.613E+00     0.            3.179E-05
 0   7      -1.266E+02     3.784E+01     2.612E+00     0.            3.178E-04
 0   8      -1.387E+02     3.783E+01     2.614E+00     0.            7.791E-04
 0   9      -1.208E+02     3.852E+01     2.526E+00     0.            1.034E-01
 0  10       0.            4.500E+01     0.            1.000E-04     1.000E-04


 TIME STEP  =      1



STATION       T(I,J)        WV(I,J)        PF(I,J)     VSX(I,J)      VSY(I,J)
 1   0      -4.742E+01     3.780E+01     2.617E+00     0.            0.
 1   1      -5.713E+01     3.784E+01     2.612E+00     0.            9.137E-03
 1   2      -6.436E+01     3.794E+01     2.600E+00     0.            8.671E-03
 1   3      -6.840E+01     3.811E+01     2.579E+00     0.            1.098E-02
 1   4      -7.019E+01     3.834E+01     2.549E+00     0.            1.475E-02
 1   5      -7.305E+01     3.856E+01     2.521E+00     6.639E-04     2.112E-02
 1   6      -8.057E+01     3.866E+01     2.508E+00     9.847E-03     3.382E-02
 1   7      -9.105E+01     3.869E+01     2.504E+00     3.408E-02     6.562E-02
 1   8      -9.142E+01     3.899E+01     2.464E+00     7.231E-02     2.958E-01
 1   9       1.376E+02     4.500E+01     0.            3.069E-01     2.970E+00
 1  10       0.            4.500E+01     0.            1.000E-04     1.000E-04


 TIME STEP  =      1



STATION       T(I,J)        WV(I,J)        PF(I,J)     VSX(I,J)      VSY(I,J)
 2   0      -4.742E+01     3.780E+01     2.617E+00     0.
 2   1      -5.866E+01     3.781E+01     2.616E+00
 2   2      -7.001E+01
                                                                     6.452E-02
                                                       0.            2.894E-01
                           4.500E+01     0.            5.454E-01     2.998E+00
             0.            4.500E+01     0.            1.000E-04     1.000E-04


 TIME STEP  =      1



STATION       T(I,J)        WV(I,J)        PF(I,J)     VSX(I,J)      VSY(I,J)
 9   0      -4.742E+01     3.780E+01     2.617E+00     0.            0.
 9   1      -5.752E+01     3.783E+01     2.613E+00     0.            9.027E-03
 9   2      -6.541E+01     3.791E+01     2.603E+00     0.            8.485E-03
 9   3      -7.047E+01     3.806E+01     2.585E+00     0.            1.083E-02
 9   4      -9.258E+01     3.782E+01     2.614E+00     0.            1.468E-02
 9   5      -9.911E+01     3.794E+01     2.600E+00     0.            2.117E-02
 9   6      -1.029E+02     3.811E+01     2.578E+00     8.027E-03     3.398E-02
 9   7      -1.013E+02     3.843E+01     2.538E+00     3.206E-02     6.589E-02
 9   8      -8.359E+01     3.921E+01     2.433E+00     7.180E-02     2.954E-01
```

```
 9   9       1.754E+02    4.500E+01    0.              3.073E-01    2.970E+00
 9  10       0.           4.500E+01    0.              1.000E-04    1.000E-04


 TIME STEP  =     1



STATION       T(I,J)       WV(I,J)       PF(I,J)      VSX(I,J)     VSY(I,J)
10    0      -4.742E+01   3.780E+01    2.617E+00    0.           0.
10    1      -5.855E+01   3.781E+01    2.616E+00    0.           0.
10    2      -6.991E+01   3.781E+01    2.615E+00    0.           1.719E-05
10    3      -8.125E+01   3.782E+01    2.615E+00    8.745E-17    2.119E-05
10    4      -9.255E+01   3.782E+01    2.614E+00    0.           2.504E-05
10    5      -1.038E+02   3.783E+01    2.613E+00    0.           2.862E-05
10    6      -1.151E+02   3.784E+01    2.612E+00    0.           3.356E-05
10    7      -1.262E+02   3.785E+01    2.611E+00    0.           3.098E-04
10    8      -1.384E+02   3.763E+01    2.613E+00    0.           7.846E-04
10    9      -1.204E+02   3.853E+01    2.525E+00    0.           1.034E-01
10   10       0.          4.500E+01    0.           1.000E-04    1.000E-04

ITERATION  PTS.NOT CLOSED                MONITOR   STATIONS

                                      5   8         5   6         5   4         5   2


 ***CLOSURE***



 ITERATION  PTS.NOT CLOSED                MONITOR   STATIONS

                                      5   8         5   6         5   4         5   2


 ***CLOSURE***




 TIME STEP  =     4



 I    J    T(I,J)        WV(I,J)       DTDW(I,J)     P11(I,J)      P12(I,J)      P22(I,J)
 0    0   -4.742E+01    3.780E+01    8.119E+00    8.456E-08    0.            4.723E-08
 0    1   -5.772E+01    3.783E+01    9.910E+00    8.382E-08    0.            4.681E-08
 0    2   -6.869E+01    3.784E+01    1.181E+01    8.271E-08    0.            4.619E-08
 0    3   -7.952E+01    3.786E+01    1.369E+01    8.125E-08    0.            4.538E-08
 0    4   -9.022E+01    3.788E+01    1.557E+01    7.943E-08    0.            4.436E-08
 0    5   -1.009E+02    3.790E+01    1.744E+01    7.725E-08    0.            4.314E-08
 0    6   -1.116E+02    3.791E+01    1.933E+01    7.470E-08    0.            4.172E-08
 0    7   -1.223E+02    3.793E+01    2.123E+01    7.182E-08    0.            4.011E-08
 0    8   -1.350E+02    3.791E+01    2.336E+01    3.802E-08    0.            1.895E-08
 0    9   -8.048E+01    3.966E+01    1.715E+01    4.529E-08    0.            2.258E-08
 0   10    0.           4.500E+01    1.000E+00    8.550E-07    0.            4.775E-07
```

```
0  11    0.              0.              -7.174E+57    8.550E-07    0.             4.775E-07


TIME STEP  =     4


I     J     T(I,J)         WV(I,J)        DTOW(I,J)     P11(I,J)     P12(I,J)       P22(I,J)
1     0   -4.742E+01     3.780E+01      8.119E+00     8.456E-08    0.             4.723E-08
1     1   -5.713E+01     3.784E+01      9.824E+00     8.387E-08    0.             4.684E-08
1     2   -6.448E+01     3.793E+01      1.120E+01     8.318E-08    0.             4.645E-08
1     3   -6.878E+01     3.810E+01      1.217E+01     8.270E-08    0.             4.619E-08
1     4   -7.097E+01     3.832E+01      1.288E+01     8.243E-08    0.             4.604E-08
1     5   -7.424E+01     3.853E+01      1.379E+01     8.201E-08    0.             4.580E-08
1     6   -8.194E+01     3.863E+01      1.540E+01     8.087E-08    0.             4.516E-08
1     7   -9.220E+01     3.866E+01      1.740E+01     7.905E-08    0.             4.415E-08
1     8   -8.882E+01     3.906E+01      1.760E+01     4.451E-08    0.             2.218E-08
1     9    1.352E+02     4.500E+01      1.000E+00     4.775E-08    0.             2.380E-08
1    10    0.            4.500E+01      1.000E+00     8.550E-07    0.             4.775E-07
1    11    0.            0.            -7.174E+57     8.550E-07    0.             4.775E-07


TIME STEP  =     4


I     J     T(I,J)         WV(I,J)        DTOW(I,J)     P11(I,J)     P12(I,J)       P22(I,J)
2     0   -4.742E+01     3.780E+01      8.119E+00     8.456E-08    0.             4.723E-08
2     1   -5.863E+01     3.781E+01      1.004E+01     8.374E-08    0.             4.677E-08
2     2   -6.990E+01     3.781E+01      1.198E+01     8.257E-08    0.             4.611E-08
2     3   -8.112E+01     3.782E+01      1.392E+01     8.100E-08    0.
2     4   -9.223E+01     3.783E+01      1.584E+01
2     5   -1.032E+02     3.78                                                      4.775E-08
2     6                                              8.550E-08    0.             4.775E-08
                                        1.939+292     8.550E-08    0.             4.775E-08
      6    0.            0.             3.739+135     8.550E-08    0.             4.775E-08
11    7    0.            0.            -7.174E+57     8.550E-08    0.             4.775E-08
11    8    0.            0.             5.041+239     4.775E-08    0.             2.380E-08
11    9    0.            0.            -7.174E+57     4.775E-08    0.             2.380E-08
11   10    0.            0.            -7.174E+57     8.550E-07    0.             4.775E-07
11   11    0.            0.             4.940-223     8.550E-07    0.             4.775E-07

ITERATION  PTS.NOT CLOSED                 MONITOR    STATIONS

                                  5  8          5  6          5  4          5  2
      1          36      TX   -1.035E+02   -1.136E+02   -9.271E+01   -7.007E+01
                         TY   -1.035E+02   -1.136E+02   -9.271E+01   -7.007E+01

      2          18      TX   -1.035E+02   -1.136E+02   -9.271E+01   -7.007E+01
                         TY   -1.035E+02   -1.136E+02   -9.271E+01   -7.007E+01

      3          18      TX   -1.035E+02   -1.136E+02   -9.271E+01   -7.007E+01
                         TY   -1.035E+02   -1.136E+02   -9.271E+01   -7.007E+01

      4          18      TX   -1.035E+02   -1.136E+02   -9.271E+01   -7.007E+01
                         TY   -1.035E+02   -1.136E+02   -9.271E+01   -7.007E+01
```

```
    5          18    TX      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01
                     TY      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01

    6          18    TX      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01
                     TY      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01

    7          18    TX      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01
                     TY      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01

    8          18    TX      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01
                     TY      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01


    9          18    TX      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01
                     TY      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01

   10          18    TX      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01
                     TY      -1.035E+02  -1.136E+02  -9.271E+01  -7.007E+01


***CLOSURE***


TIME STEP  =     4


STATION      T(I,J)       WV(I,J)       PF(I,J)      VSX(I,J)      VSY(I,J)
 0    0    -4.742E+01    3.780E+01    2.617E+00    0.            0.
 0    1    -5.732E+01    3.784E+01    2.612E+00    6.109E-17     0.
 0    2    -6.809E+01    3.785E+01    2.610E+00    0.            2.490E-05
 0    3    -7.868E+01    3.788E+01    2.608E+00    1.355E-16     2.761E-05
 0    4    -8.908E+01    3.790E+01    2.605E+00    0.            2.292E-05
 0    5    -9.941E+01    3.793E+01    2.601E+00    0.            1.520E-05
 0    6    -1.098E+02    3.795E+01    2.598E+00    0.            2.372E-05
 0    7    -1.203E+02    3.798E+01    2.595E+00    0.            2.672E-04
 0    8    -1.329E+02    3.795E+01    2.598E+00    0.            2.521E-03
 0    9    -6.787E+01    4.008E+01    2.305E+00    0.            6.568E-02
 0   10     0.           4.500E+01    0.           1.000E-04     1.000E-04

TIME STEP  =     4


STATION      T(I,J)       WV(I,J)       PF(I,J)      VSX(I,J)      VSY(I,J)
 1    0    -4.742E+01    3.780E+01    2.617E+00    0.            0.
 1    1    -5.714E+01    3.784E+01    2.612E+00    0.            1.606E-04
 1    2    -6.453E+01    3.793E+01    2.601E+00    0.            2.027E-04
 1    3    -6.897E+01    3.810E+01    2.580E+00    0.            1.404E-04
 1    4    -7.135E+01    3.831E+01    2.553E+00    0.            0.
 1    5    -7.481E+01    3.851E+01    2.527E+00    0.            0.
```

```
 1   6     -8.260E+01     3.861E+01     2.515E+00     0.            0.
 1   7     -9.273E+01     3.865E+01     2.510E+00     0.            8.838E-04
 1   8     -8.757E+01     3.910E+01     2.449E+00     1.062E-03     8.704E-03
 1   9      1.341E+02     4.500E+01     0.            7.726E-04     4.342E-03
 1  10      0.            4.500E+01     0.            1.000E-04     1.000E-04


 TIME STEP   =     4


STATION      T(I,J)        WV(I,J)       PF(I,J)      VSX(I,J)      VSY(I,J)
 2   0     -4.742E+01     3.780E+01     2.617E+00     0.            0.
 2   1     -5.861E+01     3.781E+01     2.616E+00     1.443E-04     7.033E-06
 2   2     -6.984E+01     3.782E+01     2.615E+00     1.849E-04     2.035E-05
 2   3     -8.098E+01     3.782E+01     2.614E+00     2.210E-04     2.649E-05
 2   4     -9.199E+01     3.784E+01     2.612E+00     2.506E-04     3.042E-05
 2   5     -1.028E+02     3.785E+01     2.611E+00     2.776E-04     4.993E-05
 2   6     -1.132E+02     3.788E+01     2.607E+00     2.877E-04     1.592E-04
 2   7     -1.212E+02     3.796E+01     2.598E+00     1.408E-04     8.108E-04
 2   8     -1.100E+02     3.850E+01     2.529E+00     0.            6.089E-03
 2   9      6.286E+01     4.500E+01     0.            7.782E-04     4.561E-03
 2  10      0.            4.500E+01     0.            1.000E-04     1.000E-04


 TIME STEP   =     4


STATION      T(I,J)        WV(I,J)       PF(I,J)      VSX(I,J)      VSY(I,J)
 3   0     -4.742E+01     3.780E+01     2.617E+00     0.            0.
 3   1     -5.869E+01     3.781E+01     2.616E+00     8.358E-06     5.571E-06
 3   2     -7.004E+01     3.781E+01     2.616E+00     2.105E-05     1.361E-05
 3   3     -8.140E+01     3.781E+01     2.615E+00     4.617E-05     2.229E-05
 3   4     -9.270E+01     3.782E+01     2.614E+00     8.029E-05     3.693E-05
 3   5     -1.038E+02     3.783E+01     2.613E+00     1.074E-04     7.539E-05
 3   6     -1.139E+02     3.786E+01     2.609E+00     9.970E-05     2.096E-04
 3   7     -1.207E+02     3.797E+01     2.596E+00     1.797E-05     8.942E-04
 3   8     -1.049E+02     3.863E+01     2.512E+00     0.            6.155E-03
 3   9      7.162E+01     4.500E+01     0.            4.649E-05     4.504E-03
 3  10      0.            4.500E+01     0.            1.000E-04     1.000E-04


 TIME STEP   =     4


STATION      T(I,J)        WV(I,J)       PF(I,J)      VSX(I,J)
 4   0     -4.742E+01     3.780E+01     2.617E+00
 4   1     -5.871E+01
 4   2
                                                      0.            2.318E-04
                                        2.595E+00     0.            2.506E-03
           -8.736E+01     4.010E+01     2.302E+00     2.637E-17     6.546E-02
10  10      0.            4.500E+01     0.            1.000E-04     1.000E-04


 ITERATION   PTS.NOT CLOSED                    MONITOR    STATIONS
```

```
                                           5  8         5  6         5  4         5  2

  ***CLOSURE***


ITERATION  PTS.NOT CLOSED                        MONITOR   STATIONS
                                           5  8         5  6         5  4         5  2

  ***CLOSURE***


ITERATION  PTS.NOT CLOSED                        MONITOR   STATIONS
                                           5  8         5  6         5  4         5  2

  ***CLOSURE***


  TIME STEP  =     4


  I    J     T(I,J)       WV(I,J)      DTDW(I,J)     P11(I,J)      P12(I,J)      P22(I,J)
  0    0   -4.742E+01   3.780E+01    8.119E+00    8.456E-08    0.            4.723E-08
  0    1   -5.617E+01   3.786E+01    9.680E+00    8.395E-08    0.            4.689E-08
  0    2   -6.637E+01   3.789E+01    1.147E+01    8.298E-08    0.            4.634E-08
  0    3   -7.627E+01   3.793E+01    1.324E+01    8.173E-08    0.            4.564E-08
  0    4   -8.585E+01   3.797E+01    1.497E+01    8.022E-08    0.            4.480E-08
  0    5   -9.533E+01   3.802E+01    1.671E+01    7.843E-08    0.            4.380E-08
  0    6   -1.050E+02   3.806E+01    1.849E+01    7.631E-08    0.            4.262E-08
  0    7   -1.147E+02   3.811E+01    2.030E+01    7.390E-08    0.            4.127E-08
  0    8   -1.265E+02   3.810E+01    2.237E+01    3.944E-08    0.            1.966E-08
  0    9   -4.427E+01   4.100E+01    1.168E+01    4.732E-08    0.            2.359E-08
  0   10    0.          4.500E+01    1.000E+00    8.550E-07    0.            4.775E-07
  0   11    0.          0.          -7.174E+57    8.550E-07    0.            4.775E-07


  TIME STEP  =     4


  I    J     T(I,J)       WV(I,J)      DTDW(I,J)     P11(I,J)      P12(I,J)      P22(I,J)
  1    0   -4.742E+01   3.780E+01    8.119E+00    8.456E-08    0.            4.723E-08
  1    1   -5.714E+01   3.784E+01    9.825E+00    8.387E-08    0.            4.684E-08
  1    2   -6.470E+01   3.793E+01    1.123E+01    8.316E-08    0.            4.644E-08
  1    3   -6.950E+01   3.808E+01    1.227E+01    8.261E-08    0.            4.614E-08
  1    4   -7.243E+01   3.829E+01    1.309E+01    8.225E-08    0.            4.593E-08
```

APPENDIX 6

FLOW CHART FOR PROGRAM FLOPIP2

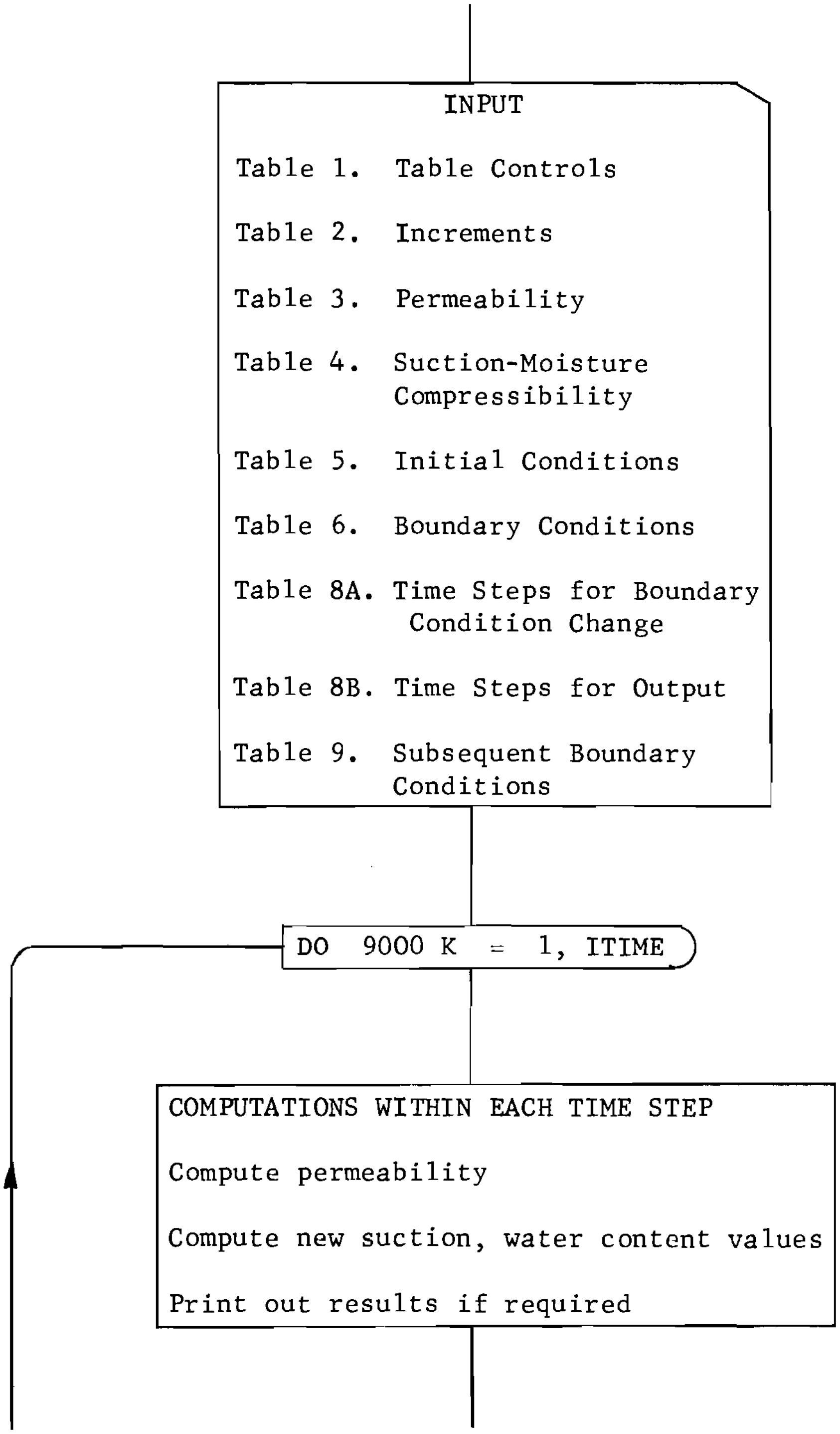
Commentary

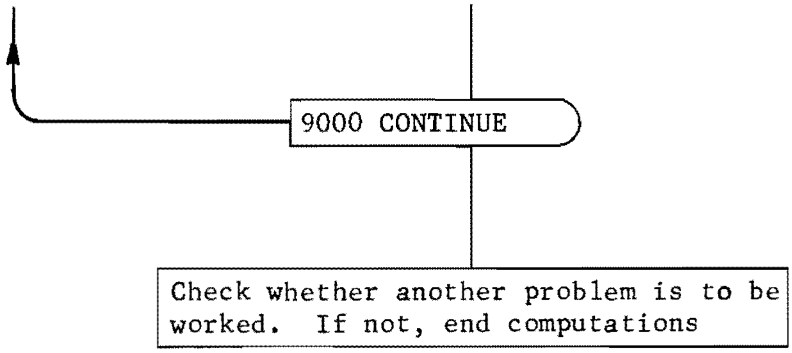
Although FLOPIP2 is a one-dimensional program, the input arrangement is identical in most respects with that of GCHPIP1. In addition, computations of new values of suction at each time step do not require iteration and closure. The computation procedure is identical with that for one direction in the two-dimensional computer program.

Because of the similarities, a detailed flow chart of FLOPIP2 is not presented here. Instead, a general flow diagram is included.

Flowcharts of the subroutines are not shown because of their similarity with those of GCHPIP1.

PROGRAM FLOPIP2





Check whether another problem is to be worked. If not, end computations

APPENDIX 7

GUIDE FOR DATA INPUT FOR PROGRAM FLOPIP2

GENERAL PROGRAM NOTES

A detailed discussion of all input data is given in Chapter Seven.

All words not marked E or F are understood to be input as integers, the last number of which is in the farthest right space in the box . . . . `22`

All words marked E or F are for decimal numbers which may be input at any position in the box with the decimal point in the proper position . . . . `-19.36`

`0.0013`

`72.`

The words marked E have been provided for those numbers which may require an exponential expression. The last number of the exponent should appear in the farthest right space in the box . . . . `-3.142E-06`

The program is arranged to compute quantities in terms of pounds, inches, and seconds. All dimensional input should be in these units.

FLOPIP2 GUIDE FOR DATA INPUT --- Card forms

IDENTIFICATION OF PROGRAM AND RUN (two alphanumeric cards per problem)

1	80

1	80

IDENTIFICATION OF PROBLEM (one card for each problem; program stops if NPROB is left blank)

NPROB		DESCRIPTION OF PROBLEM (alphanumeric)	
1	5	11	80

TABLE 1. TABLE CONTROLS, HOLD OPTIONS

ENTER 1 TO HOLD PRIOR TABLE					NUM CARDS ADDED FOR TABLE					SWITCH KGRCL	SWITCH KLH	SWITCH KVERT
2	3	4	5	6	2	3	4	5	6	1 or 2	1 or 2	1 or 2
5	10	15	20	25	30	35	40	45	50	55	60	65

IF KGRCL IS
1 Grid Coordinates
2 Cylindrical Coordinates

IF KLH IS
1 Light - overburden pressure and soil compressibility not considered
2 Heavy - overburden pressure and soil compressibility considered

IF KVERT IS
1 Vertical Flow
2 Horizontal Flow

Note: KLH SWITCH should be set to 2 only if data includes the soil compressibility effect on suction.

TABLE 2. INCREMENTS

NUM INCRS.	NUM TIME STEPS	INCREMENT LENGTH	INSIDE RADIUS	TIME STEP
		E	E	E
5	15	21 30	40	50

TABLE 3. PERMEABILITY

		SATURATED PERMEABILITY	UNSATURATED PERMEABILITY COEFFICIENTS		
FROM I	TO I	P	AK	BK	EN
		E	E	E	E
1 5	10	21 30	40	50	60

TABLE 4. SUCTION-MOISTURE-COMPRESSIBILITY

NUMBER LOCA- TIONS	MAX PF	INFL PF	PF- w EXPO- NENT	AIR ENTRY WATER CONTENT	ALFA EXPO- NENT	ALFA AT ZERO WATER CONTENT	POROSITY AT AIR ENTRY	E-LOG P COMPRESSIBILITY COEFFICIENT	X EXPONENT	UNIT WEIGHT OF SOIL	FINAL SATURATION WATER CONTENT
	F	F	F	F	F	E	E	E	F	F	E
1 5	10	15	20	25	30	40	50	60	65	70	80

FROM I	TO I	CURVE NUM
1 5	10	15

TABLE 5. INITIAL CONDITIONS

FROM I	TO I	KAS 1 or 2	WATER CONTENT	SUCTION	SLOPE C2
			E	E	E
1 5	10	15	21 30	40	50
			KAS = 1	KAS = 2	

TABLE 6. BOUNDARY CONDITIONS

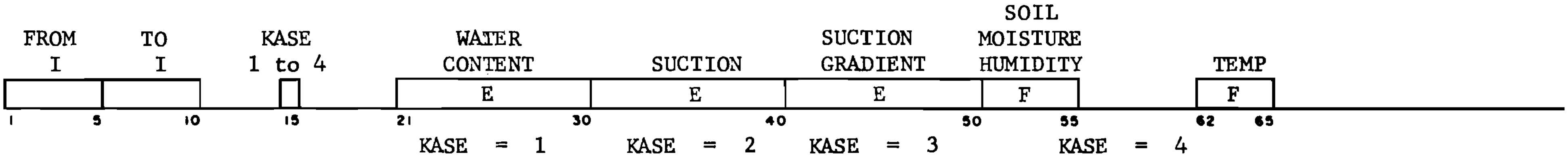
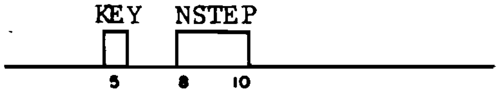


TABLE 8A. TIME STEPS FOR BOUNDARY CONDITION CHANGE



IF KEY IS
1 Read in a list of time steps for boundary condition change. NSTEP is the number of these steps
2 Continuous boundary condition change. Read in a new boundary condition at each time step. NSTEP is left blank
3 No boundary condition change. NSTEP is left blank

LIST OF TIME STEPS (if KEY = 1 maximum is 50)

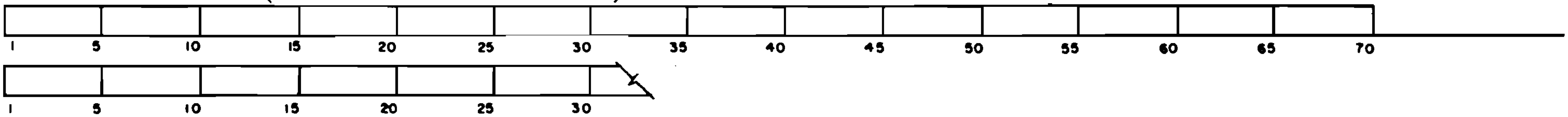
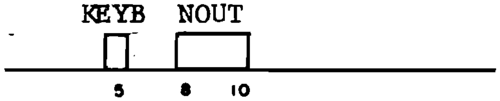


TABLE 8B. LIST OF TIME STEPS FOR OUTPUT

IF KEYB IS
1 Read in a list of output time steps
NOUT is the number of these time steps
2 Continuous output



LIST OF TIME STEPS (if KEYB = 1 maximum is 50)

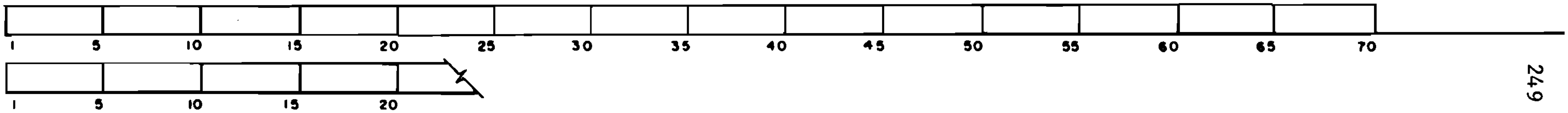


TABLE 9. SUBSEQUENT BOUNDARY CONDITIONS (if KEY = 1 or 2)

TIME STEP	NUMBER CARDS
1 5	10

FROM I	TO I	KASE 1 to 4	WATER CONTENT	SUCTION	SUCTION GRADIENT	SOIL MOISTURE HUMIDITY	TEMP
1 5	10	15	21 30	40	50	55	62 65
			KASE = 1	KASE = 2	KASE = 3	KASE = 4	

STOP CARD (one blank card to end run)
1 80

APPENDIX 8

LISTING FOR PROGRAM FLOPIP2

```
RAMESH,,037,070000,060.CE116001,LYTTON.
RUN(S,,,,,,,100000)
LGO.
-
      PROGRAM FLOPIP2  ( INPUT, OUTPUT )
C     NOTATION
C     T           SUCTION
C     TX          TRIAL SUCTION  IN  X - PIPES
C     P1          PRINCIPAL PERMEABILITY IN X-DIRECTION
C     B           SUCTION COEFFICIENT OF T(I-1)
C     C           SUCTION COEFFICIENT OF T(I)
C     D           SUCTION COFFFICIENT OF T(I+1)
C     F           GRAVITY  POTENTIAL COMPONENT  OF  PERMEABILITY
C     DTDW        RATE OF CHANGE OF SUCTION  WITH WATER  CONTENT
C     AL          TUBE FLOW MATRIX COEFFICIENT OF TX AT I-1
C     BL          TUBE FLOW MATRIX COFFFICIENT OF TX AT I
C     CL          TUBE FLOW MATRIX COEFFICIENT OF TX AT I+1
C     DL          TUBE  FLOW    CONSTANT
C     HX          INCREMENT  LENGTH  IN  THE  X-DIRECTION
C     HT          INCREMENT  LENGTH  IN  THE  TIME- DIRECTION
C     AA          CONTINUITY  COEFFICIENT  - A  CONSTANT
C     BB          CONTINUITY  COEFFICIENT  - B  CONSTANT
C     CC          CONTINUITY  COEFFICIENT  - C  CONSTANT
C     DD          CONTINUITY  COFFFICIENT  - A  DENOMINATOR
C     ALPHA       ANGLF BETWEEN  P1 AND  THE  X- DIRECTION
C     WV          VOLUMETRIC WATER  CONTENT
C     WVS         SATURATED WATER CONTENT
      DIMFNSION P1(40), P2(40),    AK(40), BK(40), EN(40), WV(40), T(40),
     1    DTDW(40), B(40), CX(40), D(40), F(40), AL(40), BL(40), CL(40),
     2    DL(40), AA(40), BB(40), CC(40), TX(40), KURV(40), KLOC(1000),
     3    AN1(16), AN2(7), WVS(40), DTDX(40), KAS(40), PFM(10), PFR(10),
     4    BETA(10), WVA(10), Q(10), ALFO(10), R(10), AV(10), PN(10),
     5    POR(40), KT(50), WN(10), KPUT(1000), KLOS(40), D1X(5)
      COMMON/ONE/PFM,PFR,BETA,DTDW,PF1
     1/TWO/T,I2
     2/THREE/WVS,KLH,K
     3/FOUR/WVA,Q,ALFO,R,AV,POR,KURV,WV,GAM,ALF,P,DP,DALF,MX,HX,PN
    1 FORMAT (// 50H PROGRAM FLOPIP2   R.L.LYTTON    REVISION DATE
     1           15H  DFC 02, 1968RK,//)
   11 FORMAT(     5H1     ,80X ,10HI-----TRIM )
   12 FORMAT (   8A10)
   14 FORMAT ( A5,5X,7A10)
   15 FORMAT (///10H      PROB , /5X, A5, 5X, 7A10)
   20 FORMAT (16I5)
   21 FORMAT ( I5, 5X, I5, 3E10.3 )
   22 FORMAT (2I5 , 10X, 4E10.3)
   23 FORMAT (I5, 5F5.2, 3E10.3, 2F5.1, E10.3 )
   24 FORMAT (3I5, 5X, 3E10.3)
   25 FORMAT (3I5, 5X, 3E10.3, F5.0, 6X, F4.1)
   28 FORMAT (I4, 2X, 4(E10.3, 2X))
   29 FORMAT (// 40H    I      T(I)         WV(I)        DTDW(I)
     1           15H    P1(I)         / )
  100 FORMAT (///40H  TABLE 1. PROGRAM CONTROL SWITCHES.
     1 / 50X,     25H           TABLES NUMBER
```

```
   2 / 50X,     35H      2     3    4A    5     6
   3        // 40H                PRIOR DATA OPTIONS (1 = HOLD) , 11X, 5I5,
   4         / 41H                NUMBER CARDS INPUT THIS PROBLEM, 10X, 5I5,
   5   //      41H                GRID = 1, CYLINDER = 2 SWITCH   , 10X,I5,
   6     //    41H                LIGHT = 1, HEAVY = 2  SWITCH    , 10X,I5,
   7     //    41H                VERT = 1, HORIZ = 2 SWITCH      , 10X, I5 )
 200 FORMAT (///50H  TABLE 2. INCREMENT LENGTHS, ITERATION CONTROL     )
 201 FORMAT (// 35H               NUM OF INCREMENTS       = , 5X,I5,
   1      /    , 35H              INCREMENT LENGTH        = , E10.3,5H IN   ,
   4      /    , 35H              NUM OF TIME INCREMENTS = , 5X, I5,
   5      /    , 35H              TIME INCREMENT LENGTH   = , E10.3,5H SECS,
   6      /    , 35H              INSIDE RADIUS           = , E10.3, 3H IN )
 202 FORMAT (// 30H               MONITOR STATIONS     , 5X, 4I7 )
 203 FORMAT (// 25H          TRANSIENT FLOW         )
 204 FORMAT (// 35H          PSEUDO-STEADY STATE FLOW         )
 300 FORMAT (///30H  TABLE  3. PERMEABILITY          )
 301 FORMAT (// 50H  FROM  TO               P1          AK          BK     ,
   1            10H EXPONENT )
 400 FORMAT (///45H  TABLE  4. SUCTION - WATER CONTENT CURVES    )
 401 FORMAT (// 35H               CURVE NUMBER                ,I7,
   1      /,    35H               NUM  LOCATIONS          =  ,I7,
   2      /,    35H               MAXIMUM  PF             =  ,5X,F5.2,
   3      /,    35H               PF  AT  INFLECTION      =  ,5X,F5.2,
   4      /,    35H               EXPONENT  FOR  PF       =  ,5X,F5.2,
   5      /,    35H               AIR ENTRY WATER CONT    =  ,5X,F5.2,
   6      /,    35H               DRYING CURVE EXPONENT  =  ,5X,F5.2,
   7      /,    35H               ALFA AT 0 WATER CONT    =  ,5X,E10.3,
   8      /,    35H               INITIAL  POROSITY       =  ,5X,E10.3,
   9      /,    35H               REFERENCE  AV           =  ,5X,E10.3 )
 402 FORMAT (    35H               SATURATION EXPONENT     =  ,5X,F5.2,
   1      /,    35H               SOIL UNIT WT  PCF       =  ,5X,E10.3,
   2      /,    35H               SATURATED WATER CONT. =  ,5X,E10.3,// )
 403 FORMAT (// 15H   NO. FROM  TO )
 500 FORMAT (///30H  TABLE 5. INITIAL CONDITIONS   )
 501 FORMAT (// 50H  FROM  TO   CASE     VOL. W.   PURE PR.  SLOPE X   )
 600 FORMAT (///45H  TABLE 6. BOUNDARY AND INTERNAL CONDITIONS  )
 601 FORMAT (// 50H  FROM  TO   CASE          WV          T         DT/DX    ,
   1            15H   H      TEMP )
 800 FORMAT (///40H  TABLE 8A. TIME STEPS FOR B.C. CHANGE    )
 804 FORMAT (// 45H   I          T(I)         WV(I)        PF(I)       / )
 805 FORMAT (I4, 5X, 3(E10.3,2X) )
 806 FORMAT (// 10H    ALL    )
 807 FORMAT (// 10H    NONE    )
 808 FORMAT (///40H  TABLE 8B. TIME STEPS FOR OUTPUT        )
 809 FORMAT (// 15H   TIME STEP  =   , I5,//)
 900 FORMAT (// 50H  TABLE 9. SUBSEQUENT BOUNDARY CONDITIONS            )
 901 FORMAT (///30H  TABLE  10. OUTPUT OF RESULTS )
 905 FORMAT (// 40H    USING DATA FROM PREVIOUS PROBLEM     )
 906 FORMAT (// 45H    USING DATA FROM PREVIOUS PROBLEM PLUS     )
 907 FORMAT (// 25H     ERROR IN DATA          )
             ITEST   =   5H
1000 READ 12,(AN1(N), N =1,16)
1010 READ 14, NPROB, ( AN2(N), N =1,7)
       IF (NPROB - ITEST) 1020, 9999, 1020
```

```
 1020 PRINT 11
      PRINT 1
      PRINT 12, (AN1(N), N = 1,16)
      PRINT 15, NPROB, (AN2(N), N =1,7)
C      INPUT  OF  TABLE 1 . TABLE CONTROLS,  HOLD OPTIONS,
 1100 READ 20, KEEP2, KEEP3, KEEP4, KEEP5, KEEP6, NCD2, NCD3, NCD4,
     1         NCD5, NCD6, KGRCL, KLH, KVERT
      PRINT 100, KEEP2, KEEP3, KEEP4, KEEP5, KEEP6, NCD2, NCD3, NCD4,
     1           NCD5, NCD6, KGRCL, KLH, KVERT
C      INPUT OF  TABLE 2A  INCREMENTS, ITERATION CONTROL
 1200 PRINT 200
          IF(KEEP2)9980, 1210, 1300
 1210 READ 21, MX, ITIME, HX, RO, HT
      PRINT 201, MX, HX, ITIME, HT, RO
          GO TO 1240
 1230 PRINT 905
C     COMPUTE  CONSTANTS TO BE USED IN THE PROGRAM
 1240          MXP5   =  MX + 5
               MXP4   =  MX + 4
               MXP3   =  MX + 3
                MXP2      =  MX + 2
               HXE2  = HX * HX
               A4        =  1.0
         GO TO 1300
 1260 PRINT 204
               A4        =  0.0
C      INPUT  TABLE  3.  PERMEABILITY
 1300 PRINT 300
          IF(KEEP3) 9980,1310,1317
 1310     DO 1315  I = 1, MXP5
               P2(I)   = 0.0
               AK(I)   = 0.0
               BK(I)   = 0.0
               EN(I)   = 0.0
               WVS(I)  = 0.0
 1315     CONTINUE
         GO TO 1319
 1317     IF (NCD3)9980,1330,1318
 1318 PRINT 906
 1319 PRINT 301
          DO 1320  K = 1, NCD3
      READ  22, IN1, IN2, PB,      AK1, BK1, EN1
      PRINT 22, IN1, IN2, PB,      AK1, BK1, EN1
               IN1   =  IN1 + 3
               IN2   =  IN2 + 3
          DO 1320  I  = IN1,IN2
               P2(I)   = P2(I) + PB   $   P1(I) = P2(I)
               AK(I)   = AK(I) + AK1
               BK(I)   = BK(I) + BK1
               EN(I)   = EN(I) + EN1
 1320     CONTINUE
         GO TO 1400
 1330 PRINT 905
C      INPUT OF TABLE  4. SUCTION - WATER CONTENT CURVE
```

```
C     AT PRESENT,  THIS IS AN EXPONENTIAL SINGLE - VALUED CURVE.   IT
C    1SHOULD BE REPLACED BY NUMERICAL CURVES  FOR WETTING, DRYING, AND
C    2SCANNING BETWEEN THE TWO.
 1400 PRINT 400
          IF (KEEP4) 9980,1410,1430
 1410          NLOC   = 0
          DO 1415 M   =  1,NCD4
      READ 23,LOC,PFM(M), PF1   ,BETA(M),WVA(M),Q(M),ALFO(M),PN(M),AV(M),
     1R(M),GAM,WN(M)
      PRINT 401,M,LOC,PFM(M), PF1   ,BETA(M),WVA(M),Q(M),ALFO(M),
     1PN(M),AV(M)
      PRINT 402,R(M),GAM,WN(M)
               PFR(M)     =  PFM(M) - PF1
               NLOC   =  NLOC + LOC
 1415     CONTINUE
      PRINT 403
          DO 1420 M   =  1,NLOC
      READ 20, IN1, IN2, KAT
      PRINT 20, KAT, IN1, IN2
               IN1   =  IN1 + 3
               IN2   =  IN2 + 3
          DO 1420 I   = IN1,IN2
               KURV(I) = KAT
               POR(I) = PN(KAT)
               WVS(I)   = WN(KAT)
 1420     CONTINUE
          GO TO 1500
 1430 PRINT 905
C     INPUT OF TABLE 5. INITIAL CONDITIONS
 1500 PRINT 500
          IF(KEEP5)9980,1510,1505
 1505     IF(NCD5) 9980,1506,1507
 1506 PRINT 905
          GO TO 1600
 1507 PRINT 906
          GO TO  1520
 1510     DO 1515 I = 1,MXP5
               WV(I) = 0.0
               T(I)   = 0.0
 1515     CONTINUE
 1520 PRINT  501
               DO 1526 M =   1,NCD5
               K          =  0
      READ 24, IN1, IN2, KAT, WV1, T1, C2
      PRINT 24,IN1, IN2, KAT, WV1, T1, C2
               IN1   = IN1 + 3
               IN2 = IN2 + 3
         GO TO (1522,1523), KAT
 1522     DO 1525 I = IN1,IN2
               C1          =  IN2 - I
               WV(I) = WV(I) + WV1 + C1 * C2 * HX
               KAS(I) = 1
               I2    =  I
      CALL SUCTION
```

```
1525     CONTINUE
         GO TO 1526
1523    DO 1524 I  =  IN1,IN2
              C1         =  IN2 - I
              KAS(I) = 1
              T(I)    = C1 * C2 * HX + T1 + T(I)
        IF (A4) 1528,1527
1527          WV(I)    = WVS(I)
              DTDW(I) = 1.0
              PF1        =  0.0
         GO TO 1524
1528          I2         =  I
     CALL DSUCT
1524    CONTINUE
1526     CONTINUE
C    INPUT OF TABLE 6. BOUNDARY AND INTERNAL CONDITIONS
1600 PRINT 600
         IF(KEEP6) 9980,1610,1605
1605     IF(NCD6) 9980,1606,1607
1606 PRINT 905
         GO TO 1700
1607 PRINT 906
         GO TO 1612
1610 PRINT 601
         DO 1611 I  = 1, MXP5
              KAS(I)  = 1
              DTDX(I) = 0.0
1611     CONTINUE
1612     DO 1645 M  =  1,NCD6
              K          =  0
     READ  25, IN1, IN2, KASE, WV1, T1, DTX1, H1, TE
     PRINT 25, IN1, IN2, KASE, WV1, T1, DTX1, H1, TE
              IN1   =  IN1 + 3
              IN2   =  IN2 + 3
         DO 1645 I  =  IN1,IN2
              I2    =  I
              KAS(I)  = KASE
         GO TO (1615,1620,1625,1630,1635) KASE
1615          WV(I) = WV1
     CALL SUCTION
              KAS(I)  = 2
         GO TO 1645
1620          T(I) = T1
     CALL DSUCT
         GO TO 1645
1625          DTDX(I) = DTX1
              L          =   MXP3  -  1
         GO TO 1645
1630     CONTINUE
1635 CALL HUMIDY (TE,H1)
     CALL DSUCT
              KAS(I)  = 2
1645     CONTINUE
              K          =  1
```

```
          IF ( 3 - KAS(3) ) 1655, 1650, 1655
 1650          T(3)     = T(4) - HX * DTDX (3)
               I2         =  3
      CALL DSUCT
 1655     IF ( 3 - KAS(MXP3) ) 1670, 1660, 1670
 1660          T(MXP3) = T(L) + HX * DTDX(MXP3)
               I2         =  MXP3
      CALL DSUCT
 1670     CONTINUE
 1700     CONTINUE
 1800 PRINT 800
      READ 20, KEY    , NSTEP
          GO TO (1805,1840,1860) KEY
C     LIST OF TIME-STEPS WHERE B.C. CHANGE
 1805 READ 20, (KT(N), N = 1,NSTEP)
      PRINT 20, (KT(N), N  =  1,NSTEP)
               N    =  1
          DO 1830 K =  1, ITIME
          IF ( K  - KT(N))  1820,1815
 1815          KLOC(K)   =    1
               N         =    N + 1
          GO TO 1830
 1820          KLOC(K)   =    2
 1830     CONTINUE
          GO TO 1871
C     CONTINUOUS B.C. CHANGE (READ IN NEW B.C. FOR EACH TIME STEP)
 1840     DO 1850 K   =  1,ITIME
             KLOC(K) =  1
 1850     CONTINUE
      PRINT 806
          GO TO 1871
 1860 PRINT 807
         DO 1870 K    =  1, ITIME
               KLOC(K)    =  2
 1870    CONTINUE
 1871 PRINT 808
      READ 20,KEYB,NOUT
          GO TO ( 1872,1882) KEYB
C     LIST OF TIME STEPS FOR OUTPUT READ IN
 1872 READ 20,(KT(N),N = 1,NOUT )
      PRINT 20,(KT(N), N = 1,NOUT)
               N          =  1
          DO 1875 K   =  1, ITIME
          IF( K - KT(N))1874,1873
 1873          KPUT(K)    =  1
               N          =  N + 1
          GO TO 1875
 1874          KPUT(K)    =  2
 1875     CONTINUE
          GO TO 2000
C     CONTINUOUS OUTPUT
 1882     DO 1883 K   =  1,ITIME
               KPUT(K)    =  1
 1883     CONTINUE
```

```
      PRINT 806
 2000 PRINT 11
      PRINT 901
C     ZERO-OUT OF ALL TEMPORARY CONSTANTS
          DO 2005 I  = 1,MXP5
               B(I)  = 0.0
               CX(I) = 0.0
               D(I)  = 0.0
               F(I)  = 0.0
               TX(I) = 0.0
 2005     CONTINUE
 2008     DO 2009 I = 1, MXP5
               AL(I)    =   0.0
               BL(I)    =   0.0
               CL(I)    =   0.0
               DL(I)    =   0.0
 2009     CONTINUE
C     START OF TIME STEP
          DO 9000 K  =  1, ITIME
               KOUT      =  KPUT(K)
          IF (K - 1) 9980, 1980, 1900
 1900          KAT   =   KLOC(K)
          GO TO (1910,1980)  KAT
 1910 READ 20, KTIME, NCD6
      PRINT 900
      PRINT 906
      PRINT 601
          DO 1945 M  =  1, NCD6
      READ  25, IN1, IN2, KASE, WV1, T1, DTDX1, H1, TE
      PRINT 25, IN1, IN2, KASE, WV1, T1, DTDX1, H1, TE
               IN1   =  IN1 + 3
               IN2   =  IN2 + 3
          DO 1945 I  =  IN1,IN2
               I2    =  I
               KAS(I) = KASE
          GO TO (1915,1920,1925,1930,1935) KASE
 1915          WV(I)    = WV1
      CALL SUCTION
               KAS(I)  = 2
          GO TO 1945
 1920          T(I)    = T1
      CALL DSUCT
          GO TO 1945
 1925          DTDX(I) = DTX1
          GO TO 1945
 1930     CONTINUE
 1935 CALL HUMIDY (TE,H1)
      CALL DSUCT
               KAS(I)  = 2
 1945     CONTINUE
          IF ( 3 - KAS(3) ) 1955, 1950, 1955
 1950          T(3)    = T(4) - HX * DTDX(3)
               I2        =  3
      CALL DSUCT
```

```
1955      IF ( 3 - KAS(MXP3) ) 1970, 1960, 1970
1960            L            =   MXP3   -  1
                T(MXP3) = T(L) + HX * DTDX(MXP3)
                I2          =  MXP3
      CALL DSUCT
1970      CONTINUE
1980      GO TO ( 1982,1983) KOUT
1982 PRINT 809,K
      PRINT 29
1983      CONTINUE
C     ROTATION,COMPUTATION OF UNSATURATED PERMEABILITY
          DO  2010 I   = 3, MXP4
          IF ( WV(I) - WVS(I) ) 2015, 2020, 2020
2015            TE     = ABS(T(I))
                BE     = EN(I)
                A1     = AK(I)
                C1     = BK(I)
                C2         =  1.0  +  ((TE*A1)**BE)/C1
                UNSAT      =  1.0 / C2
                P1(I) = P2(I) * UNSAT
2020      GO TO (2025,2010)KOUT
2025            I1         =  I - 3
      PRINT 28, I1, T(I), WV(I), DTDW(I), P1(I)
2010      CONTINUE
          GO TO (2120,2140) KGRCL
2120      DO 2130 I  =  3, MXP3
                CONST = HT * DTDW(I) * 0.5
                B(I)   =(P1(I) / HXE2)* CONST
                CX(I) = ( ( P1(I) + P1(I+1) ) / HXE2) * CONST
                D(I)   =(P1(I+1) / HXE2)* CONST
          GO TO ( 2121, 2122 ) KVERT
2121            F(I)=-( ( P1(I) - P1(I+1) )/ HX)* CONST
          GO TO 2130
2122            F(I)   = 0.0
2130      CONTINUE
          GO TO 2155
2140      DO 2150 I  =  3,MXP3
                A1         =  I - 3
                R          =  RO + A1*HX
                HXR    = HX * R
                CONST = HT * DTDW(I) * 0.5
                B(I)   = ( - P1(I) / HXR + P1(I) / HXE2 ) * CONST
                CX(I) = ( - P1(I) / HXR + ( P1(I) + P1(I+1))/HXE2)*CONST
                D(I)   =(P1(I+1) / HXE2)* CONST
                F(I)   = 0.0
2150      CONTINUE
2155      DO 2195 I  =  1,MXP5
                TX(I) = T(I)
          IF (A4)2195,2181
2181            T(I)   = 0.0
2195      CONTINUE
2215      DO 2200  I  =  3, MXP3
                AL(I) = - B(I)
                BL(I) = CX(I) + A4
```

```
                CL(I) = - D(I)
                DL(I) = B(I) * T(I-1) - ( CX(I) - A4 ) * T(I)
     1                   + D(I) * T(I+1) + 2.0 * F(I)
 2200       CONTINUE
C      COMPUTE CONTINUITY COEFFICIENTS
            DO 2300 I  =  3, MXP3
            IF ( 3 - KAS(3) ) 2305, 2304
 2304       IF ( I - 4) 2305,2300
 2305             KAT    = KAS(I)
            GO TO ( 2350,2320,2330,2350) KAT
C      SUCTION SET
 2320             CC(I)      =  1.0
                  BB(I)      =  0.0
                  AA(I) = T(I)
            IF ( I - 3) 2324,2322
 2322             BB(2)      =  1.0
                  AA(2)      =  0.0
            GO TO 2300
 2324       IF (I - MXP3) 2300,2326
 2326             BB(I+1)    =  0.0
                  AA(I+1) = T(I)
            GO TO 2300
C      SLOPE SET
 2330       IF ( 2 - KAS(I-1) ) 2334, 2332
 2332             CC(I)      =  1.0
                  BB(I)      =  0.0
                  AA(I)     = T(I-1) + DTDX(I) * HX
            GO TO 2300
 2334       IF ( I - 3) 2336,2338
 2336       IF ( I - MXP3) 2340,2338
 2338             AA(I-1) = - DTDX(I) * HX
                  BB(I-1)    =  1.0
                  BB(I)      =  0.0
                  AA(I)     = TX(I)
                  CC(I+1)    =  1.0
                  BB(I+1)    =  0.0
                  AA(I+1) = AA(I) * BB(I-1) + HX * DTDX(I)
            GO TO 2300
C      PIPE INCREMENT SLOPE SET
 2340             CC(I)      =  BL(I) + AL(I)*BB(I-1)
                  BB(I)      =  -CL(I) / (CC(I))
                  AA(I)      =  (DL(I) - AL(I)*AA(I-1))/(CC(I))
                  CTEMP      =  1.0  +  CC(I-1)*(1.0 - BB(I-1))/(CC(I))
                  BTEMP      =  BB(I)/CTEMP
                  ATEMP      =  (AA(I) + CC(I-1)*(AA(I-1) + HX*DTDX(I)   )
     1                            /(CC(I)))/CTEMP
                  AA(I-1)    =  -DTDX(I ) *HX
                  BB(I-1)    =  1.0
                  AA(I)      =  ATEMP
                  BB(I)      =  BTEMP
                  CC(I)      =  CTEMP
            GO TO 2300
 2350             CC(I)    =  BL(I) + AL(I)* BB(I-1)
                  BB(I)    =  -CL(I)/ CC(I)
```

```
                AA(I)    = (DL(I) - AL(I)* AA(I-1))/ CC(I)
 2300      CONTINUE
           DO  2360  IR =  2, MXP4
                I    =  MXP4   + 2   - IR
                TX(I)    = AA(I) + BB(I) * TX(I+1)
 2360      CONTINUE
C     OUTPUT OF TIME STEP RESULTS
           GO TO (2625,2630)KOUT
 2625 PRINT 809, K
      PRINT 804
 2630      CONTINUE
         DO 2700 I   =   3,MXP3
                T(I)     =   TX(I)
                I2     =   I
 2665 CALL DSUCT
           GO TO (2670,2700) KOUT
 2670           I1          =   I - 3
      PRINT 805, I1, T(I), WV(I), PF1
 2700      CONTINUE
 9000      CONTINUE
           GO TO 1010
 9980 PRINT 907
 9999      CONTINUE
      END
```

```
      SUBROUTINE SUCTION
      COMMON/ONE/PFM(10),PFR(10),BETA(10),DTDW(40),PF1
     1/TWO/T(40),I2
     2/THREE/WVS(40),KLH,K
     3/FOUR/WVA(10),Q(10),ALFO(10),R(10),AV(10),POR(40),
     4KURV(40),WV(40),GAM,ALF,P,DP,DALF,MX,HX,PN(10)
               I          =  I2
               L     = KURV(I)
         IF ( WV(I) - WVS(I) ) 1525, 1524, 1524
 1524          DTDW(I) = 1.0
               PF1         =  0.0
               T(I)     = 0.0
         GO TO 1530
 1525          AT          = 100.0 * WV(I) / WVS(I)
               TAT         =   (100.0*PFR(L))/(PFM(L))
               B           =   BETA(L)
               RECB        =   1.0/(1.0 + B )
               C           =   2.302585
               D           =   PFM(L) - PFR(L)
               XM       = PFM(L) / ( WVS(I) * ( 1.0 + BETA(L) ) )
               FACT = WV(I) * ( 1.0 - POR(I) ) * 2.70 / 100.0
         IF(TAT - AT) 1527,1526,1526
 1526          PF          =   PFR(L)*(AT/TAT)**RECB
               PF1         =   PFM(L) - PF
               T(I)     = - 10.0 ** PF1 / 2.54
               TE       = ABS ( T(I) )
               DTDW(I) =(TE * XM * C * ( PFR(L) / PF ) ** B ) * FACT
         GO TO 1528
 1527          PF1         =   D*((100.0 - AT)/(100.0 - TAT))**RECB
               T(I)     = - 10.0 ** PF1 / 2.54
               TE       = ABS ( T(I) )
               DTDW(I) =(XM * TE * C * ( D / PF1 ) ** B ) * FACT
 1528    GO TO (1530,1529) KLH
 1529    IF(K) 1530,1523
 1523 CALL HEAVY
               T(I)     = T(I) + ALF * P
               DTDW(I) = DTDW(I) + ALF * DP + P * DALF
 1530 RETURN
      END
```

```
       SUBROUTINE DSUCT
       COMMON/ONE/PFM(10),PFR(10),BETA(10),DTDW(40),PF1
     1/TWO/T(40),I2
     2/THREE/WVS(40),KLH,K
     3/FOUR/WVA(10),Q(10),ALFO(10),R(10),AV(10),POR(40),
     4KURV(40),WV(40),GAM,ALF,P,DP,DALF,MX,HX,PN(10)
                 I         =  I2
                 L         = KURV(I)
          IF(T(I  ))2710,2705,2705
2705             WV(I)     = WVS(I)
                 DTDW(I) = 1.0
                 PF1         =  0.0
          GO TO 2750
2710      GO TO (2713,2711)KLH
2711      IF(K) 2712,2713
2712 CALL HEAVY
                 TE        = - T(I) * 2.54 + ALF * P * 2.54
          GO TO 2714
2713             TE        = - T(I) * 2.54
2714             PF1         =  ALOG10(TE)
          IF(PF1)2715,2720,2720
2715             PF1         =  0.0
                 T(I)      = - 1.0 / 2.54
                 WV(I)     = WVS(I)
                 DTDW(I) = 1.0
          GO TO 2750
2720      IF ( PF1 - PFM(L)) 2724,2724,2722
2722             PF1         =  PFM(L)
                 T(I)      = - 10.0 ** PF1 / 2.54
                 WV(I)     = 0.0
                 DTDW(I) = 1.0E+10
          GO TO 2750
2724             PF          =  PFM(L) - PF1
                 B           =  BETA(L)
                 BP          =  1.0 + BETA(L)
                 C           =  2.302585
                 D           =  PFM(L) - PFR(L)
                 TAT         =  (PFR(L)*100.0)/(PFM(L))
                 XM        = PFM(L) / ( WVS(I) * ( 1.0 + BETA(L) ) )
          IF (PF - PFR(L))2725,2725,2730
2725             AT          =  TAT*(PF/PFR(L))**BP
                 TE        = - T(I)
                 DTDW(I) = TE * C * XM * ( D / PF1 ) ** B
          GO TO 2735
2730             AT          =  100.0 - (100.0 - TAT)*(PF1/D)**BP
                 TE        = - T(I)
                 DTDW(I  ) =  TE*C*XM*(D/PF1)**B
2735             WV(I)     = AT * WVS(I) / 100.0
                 FACT = WV(I) * ( 1.0 - POR(I) ) * 2.70 / 100.0
                 DTDW(I) = DTDW(I) * FACT
2750      GO TO (2760,2755) KLH
2755      IF(K) 2760,2756
2756 CALL HEAVY
                 T(I)      = T(I) + ALF * P
```

```
             DTDW(I) = DTDW(I) + ALF * DP + P * DALF
2760  RETURN
      END
```

```
      SUBROUTINE HEAVY                                                   611
      COMMON/TWO/T(40),I2                                                612
     3/FOUR/WVA(10),Q(10),ALFO(10),R(10),AV(10),POR(40),                 613
     4KURV(40),WV(40),GAM,ALF,P,DP,DALF,MX,HX,PN(10)                     614
                I          =  I2                                         615
                L          = KURV(I)                                     616
C     DETERMINE OVERBURDEN PRESSURE HEAD                                 617
                P          = ( MX + 3 - I ) * GAM * HX / 0.0361          618
                TERM = ( 1.0 - POR(I) ) * 2.70                           TO
                TH         =( WV(I) / (100.0) ) * TERM                   625
                ENN        = POR(I)                                      628
                F1         =  AV(L)*(ENN - 1.0)                          629
                F2         =  ( 2.0*TH - ENN*(TH + 1.0))                 630
           IF ( WV(I) - WVA(L) ) 1540, 1550, 1550                        619
 1540           RPO        =  R(L)                                       620
                QM1        =  Q(L) - 1.0                                 621
                QM2        =  Q(L) - 2.0                                 622
                ALF        =  ALFO(L) + (1.0 - ALFO(L))*(WV(I   )/WVA(L))  623
     1                        **QM1                                      624
                SAT = ( TH / POR(I) ) ** RPO                             TO
                F3         =  SAT                                        630
                DP         = P * ENN / ( F1 * F2 * F3 ) + HX             632
                DALF       = QM1 * ( 1.0 - ALFO(L) ) * ( WV(I) / WVA(L) )  633
     1                      ** QM2                                       634
                DALF       =  (DALF*100.0)/(WVA(L) * TERM )              TO
           GO TO 1560                                                    636
 1550           ALF        =  1.0                                        637
                DALF       =  0.0                                        638
                DP         = HX + P * ENN / ( F1 * F2 )                  638
                DTH        =(( WV(I) - WVA(L) ) / 100.0 ) * TERM         TO
                POR(I)     = ( PN(L) + DTH ) / ( 1.0 + DTH )             641
 1560 RETURN                                                             642
      END                                                                643
```

```
SUBROUTINE HUMIDY (TE,H1)
COMMON/TWO/T(40),I2
          I      =  I2
          R          =  8.314E+07
          G          =  981.0
          EM         =  18.02
          AN         =  ALOG(H1)
          TM         =  (TE - 32.0)*5.0/(9.0)   +  273.0
          T(I)     = R * TM * AN / ( G * EM * 2.54 )
RETURN
END
```

APPENDIX 9

SAMPLE DATA FOR PROGRAM FLOPIP2

```
   MOISTURE DISTRIBUTION  VERIFICATION OF  WEST LARAMIE TEST SITE
   WYOMING  HIGHWAY  DEPARTMENT                      RAMESH KHER
111     A SAMPLE PROBLEM FOR MOISTURE DISTRIBUTION AFTER 80 DAYS
  0    0    0    0    0    1    1    1   26    2    1    1    1
 27          10+6.000E+00          +6.912E+05
  0   28             +1.050E-06+2.540E+00+1.000E+09+3.000E+00
  1 6.50 3.00 3.0023.50 2.00 0.000E+00 4.000E-01 8.000E-02 0.90.0580 4.000E+01
  0   28    1
  1    1    1        1.450E+01
  2    2    1        1.330E+01
  3    3    1        1.285E+01
  4    4    1        1.360E+01
  5    5    1        1.420E+01
  6    6    1        1.415E+01
  7    7    1        1.400E+01
  8    8    1        1.355E+01
  9    9    1        1.310E+01
 10   10    1        1.350E+01
 11   11    1        1.390E+01
 12   12    1        1.355E+01
 13   13    1        1.315E+01
 14   14    1        1.330E+01
 15   15    1        1.350E+01
 16   16    1        1.365E+01
 17   17    1        1.375E+01
 18   18    1        1.385E+01
 19   19    1        1.390E+01
 20   20    1        1.355E+01
 21   21    1        1.375E+01
 22   22    1        1.240E+01
 23   23    1        1.260E+01
 24   24    1        1.310E+01
 25   25    1        1.470E+01
 26   26    1        1.590E+01
  0    0    1        1.520E+01
 27   27    1        4.000E+01
  3    0
  1    1
 10
```

```
116     SAMPLE STUDY OF MOISTURE DISTRIBUTION ALONG A VERTICAL, CONT. OUTPUT
  0     0     0     0     0     1     1     1    26     2     1     1     1
 27         10+6.000E+00            +6.912E+05
  0   28           +1.000E-06+2.540E+00+1.000E+09+3.000E+00
  1 6.50 3.00 3.0023.50 2.00 0.000E+00 4.000E-01 8.000E-02 0.90.0580 4.000E+01
  0   28     1
  1    1     1        1.442E+01
  2    2     1        1.390E+01
  3    3     1        1.355E+01
  4    4     1        1.345E+01
  5    5     1        1.340E+01
  6    6     1        1.345E+01
  7    7     1        1.345E+01
  8    8     1        1.330E+01
  9    9     1        1.300E+01
 10   10     1        1.320E+01
 11   11     1        1.340E+01
 12   12     1        1.315E+01
 13   13     1        1.290E+01
 14   14     1        1.300E+01
 15   15     1        1.320E+01
 16   16     1        1.340E+01
 17   17     1        1.355E+01
 18   18     1        1.325E+01
 19   19     1        1.260E+01
 20   20     1        1.210E+01
 21   21     1        1.190E+01
 22   22     1        1.140E+01
 23   23     1        1.060E+01
 24   24     1        1.040E+01
 25   25     1        1.380E+01
 26   26     1        1.420E+01
  0    0     1        1.500E+01
 27   27     1        4.000E+01
  3    0
  2
```

APPENDIX 10

SAMPLE OUTPUT FOR PROGRAM FLOPIP2

```
PROGRAM FLOPIP2  R.L.LYTTON    REVISION DATE       DEC 02, 1968RK

   MOISTURE DISTRIBUTION  VERIFICATION OF  WEST LARAMIE TEST SITE
   WYOMING  HIGHWAY  DEPARTMENT                    RAMESH KHER



    PROB
     111     A SAMPLE PROBLEM FOR MOISTURE DISTRIBUTION AFTER 80 DAYS


TABLE 1. PROGRAM CONTROL SWITCHES.
                                                            TABLES NUMBER
                                                     2     3     4A    5     6

          PRIOR DATA OPTIONS (1 = HOLD)              0     0     0     0     0
          NUMBER CARDS INPUT THIS PROBLEM            1     1     1    26     2

          GRID = 1, CYLINDER = 2 SWITCH              1

          LIGHT = 1, HEAVY = 2  SWITCH               1


          VERT = 1, HORIZ = 2 SWITCH                 1


TABLE 2. INCREMENT LENGTHS, ITERATION CONTROL


          NUM OF INCREMENTS       =          27
          INCREMENT LENGTH        =  6.000E+00 IN
          NUM OF TIME INCREMENTS  =          10
          TIME INCREMENT LENGTH   =  6.912E+05 SECS
          INSIDE RADIUS           = -0.        IN


TABLE  3. PERMEABILITY

FROM  TO              P1        AK         BK      EXPONENT
  0   28          1.050E-06 2.540E+00 1.000E+09 3.000E+00


TABLE  4. SUCTION - WATER CONTENT CURVES


          CURVE NUMBER                    1
          NUM  LOCATIONS        =         1
          MAXIMUM  PF           =         6.50
          PF  AT  INFLECTION    =         3.00
          EXPONENT  FOR  PF     =         3.00
          AIR ENTRY WATER CONT  =        23.50
          DRYING CURVE EXPONENT =         2.00
          ALFA AT 0 WATER CONT  =         0.
```

```
        INITIAL  POROSITY         =        4.000E-01
        REFERENCE  AV             =        8.000E-02
        SATURATION EXPONENT       =         .90
        SOIL UNIT WT  PCI         =        5.800E-02
        SATURATED WATER CONT. =            4.000E+01


 NO. FROM  TO
  1    0   28


TABLE 5. INITIAL CONDITIONS


FROM   TO    CASE      VOL. W.    PORE PR.  SLOPE X
  1     1     1        1.450E+01-0.        -0.
  2     2     1        1.330E+01-0.        -0.
  3     3     1        1.285E+01-0.        -0.
  4     4     1        1.360E+01-0.        -0.
  5     5     1        1.420E+01-0.        -0.
  6     6     1        1.415E+01-0.        -0.
  7     7     1        1.400E+01-0.        -0.
  8     8     1        1.355E+01-0.        -0.
  9     9     1        1.310E+01-0.        -0.
 10    10     1        1.350E+01-0.        -0.
 11    11     1        1.390E+01-0.        -0.
 12    12     1        1.355E+01-0.        -0.
 13    13     1        1.315E+01-0.        -0.
 14    14     1        1.330E+01-0.        -0.
 15    15     1        1.350E+01-0.        -0.
 16    16     1        1.365E+01-0.        -0.
 17    17     1        1.375E+01-0.        -0.
 18    18     1        1.385E+01-0.        -0.
 19    19     1        1.390E+01-0.        -0.
 20    20     1        1.355E+01-0.        -0.
 21    21     1        1.375E+01-0.        -0.
 22    22     1        1.240E+01-0.        -0.
 23    23     1        1.260E+01-0.        -0.
 24    24     1        1.310E+01-0.        -0.
 25    25     1        1.470E+01-0.        -0.
 26    26     1        1.590E+01-0.        -0.


TABLE 6. BOUNDARY AND INTERNAL CONDITIONS


FROM   TO    CASE         WV          T        DT/DX        H          TEMP
  0     0     1        1.520E+01-0.           -0.          -0         -0.0
 27    27     1        4.000E+01-0.           -0.          -0         -0.0


TABLE 8A. TIME STEPS FOR B.C. CHANGE


   NONE


TABLE 8B. TIME STEPS FOR OUTPUT
 10
```

TABLE 10. OUTPUT OF RESULTS

```
 TIME STEP  =    10


 I      T(I)          WV(I)         DTDW(I)       PI(I)

 0   -7.713E+02    1.520E+01     1.344E+01     1.232E-07
 1   -8.455E+02    1.446E+01     1.352E+01     9.630E-08
 2   -9.688E+02    1.341E+01     1.363E+01     6.604E-08
 3   -1.020E+03    1.303E+01     1.367E+01     5.708E-08
 4   -9.458E+02    1.359E+01     1.361E+01     7.063E-08
 5   -8.874E+02    1.407E+01     1.356E+01     8.421E-08
 6   -8.832E+02    1.412E+01     1.356E+01     8.543E-08
 7   -9.020E+02    1.395E+01     1.357E+01     8.062E-08
 8   -9.496E+02    1.356E+01     1.361E+01     6.985E-08
 9   -9.936E+02    1.322E+01     1.365E+01     6.150E-08
10   -9.566E+02    1.350E+01     1.362E+01     6.842E-08
11   -9.228E+02    1.378E+01     1.359E+01     7.566E-08
12   -9.519E+02    1.354E+01     1.362E+01     6.939E-08
13   -9.929E+02    1.323E+01     1.365E+01     6.161E-08
14   -9.811E+02    1.331E+01     1.364E+01     6.373E-08
15   -9.579E+02    1.349E+01     1.362E+01     6.817E-08
16   -9.393E+02    1.364E+01     1.361E+01     7.201E-08
17   -9.262E+02    1.375E+01     1.359E+01     7.489E-08
18   -9.157E+02    1.384E+01     1.359E+01     7.730E-08
19   -9.151E+02    1.384E+01     1.359E+01     7.745E-08
20   -9.428E+02    1.361E+01     1.361E+01     7.128E-08
21   -9.475E+02    1.358E+01     1.361E+01     7.028E-08
22   -1.084E+03    1.255E+01     1.372E+01     4.734E-08
23   -1.066E+03    1.271E+01     1.370E+01     5.042E-08
24   -8.783E+02    1.416E+01     1.355E+01     8.675E-08
25   -4.982E+02    1.913E+01     1.306E+01     3.469E-07
26   -2.437E+02    2.616E+01     1.199E+01     8.487E-07
27     0.          4.000E+01     1.000E+00     1.050E-06
28     0.          0.           -1.833-270     1.050E-06

 TIME STEP  =    10


 I          T(I)          WV(I)         PF(I)


 0        -7.713E+02    1.520E+01    3.292E+00
 1        -8.457E+02    1.446E+01    3.332E+00
 2        -9.674E+02    1.342E+01    3.390E+00
 3        -1.018E+03    1.304E+01    3.413E+00
 4        -9.459E+02    1.359E+01    3.381E+00
 5        -8.890E+02    1.406E+01    3.354E+00
 6        -8.837E+02    1.411E+01    3.351E+00
 7        -9.024E+02    1.395E+01    3.360E+00

 8        -9.495E+02    1.356E+01    3.382E+00
```

```
 9      -9.922E+02    1.323E+01    3.401F+00
10      -9.566E+02    1.350E+01    3.386F+00
11      -9.240E+02    1.377E+01    3.371F+00
12      -9.520E+02    1.354E+01    3.383F+00
13      -9.921E+02    1.323E+01    3.401F+00
14      -9.809E+02    1.332E+01    3.396F+00
15      -9.580E+02    1.349E+01    3.386F+00
16      -9.394E+02    1.364E+01    3.378F+00
17      -9.263E+02    1.375E+01    3.372F+00
18      -9.159E+02    1.383E+01    3.367F+00
19      -9.156E+02    1.384E+01    3.367F+00
20      -9.424E+02    1.362E+01    3.379F+00
21      -9.492E+02    1.356E+01    3.382F+00
22      -1.087E+03    1.257E+01    3.441F+00
23      -1.061E+03    1.274E+01    3.431F+00
24      -8.485E+02    1.443E+01    3.333F+00
25      -4.772E+02    1.955E+01    3.084F+00
26      -2.343E+02    2.649E+01    2.775F+00
27       0.           4.000E+01    0.
```

```
PROGRAM FLOPIP2  R.L.LYTTON     REVISION DATE        DEC 02, 1968RK

    MOISTURE DISTRIBUTION  VERIFICATION OF  WEST LARAMIE TEST SITE
    WYOMING  HIGHWAY  DEPARTMENT                   RAMESH KHER


     PROB
      116      SAMPLE STUDY OF MOISTURE DISTRIBUTION ALONG A VERTICAL, CONT. OUTPUT


TABLE 1. PROGRAM CONTROL SWITCHES.
                                                              TABLES NUMBER
                                                         2     3     4A    5     6

         PRIOR DATA OPTIONS (1 = HOLD)                   0     0     0     0     0
         NUMBER CARDS INPUT THIS PROBLEM                 1     1     1     26    2

         GRID = 1, CYLINDER = 2 SWITCH                   1

         LIGHT = 1, HEAVY = 2  SWITCH                    1

         VERT = 1, HORIZ = 2 SWITCH                      1


TABLE 2. INCREMENT LENGTHS, ITERATION CONTROL

         NUM OF INCREMENTS        =         27
         INCREMENT LENGTH         =  6.000E+00 IN
         NUM OF TIME INCREMENTS   =         10
         TIME INCREMENT LENGTH    =  6.912E+05 SECS
         INSIDE RADIUS            = -0.        IN


TABLE  3. PERMEABILITY

FROM  TO                 P1        AK        BK        EXPONENT
  0   28           1.000E-06 2.540E+00 1.000E+09 3.000E+00


TABLE  4. SUCTION - WATER CONTENT CURVES

         CURVE NUMBER                      1
         NUM  LOCATIONS          =         1
         MAXIMUM  PF             =         6.50
         PF  AT  INFLECTION      =         3.00
         EXPONENT  FOR  PF       =         3.00
```

```
AIR ENTRY WATER CONT   =        23.50
DRYING CURVE EXPONENT  =         2.00
ALFA AT 0 WATER CONT   =         0.
INITIAL  POROSITY      =         4.000E-01
REFERENCE  AV          =         8.000E-02
SATURATION EXPONENT    =          .90
SOIL UNIT WT  PCI      =         5.800E-02
SATURATED WATER CONT.  =         4.000E+01
```

```
NO. FROM  TO
 1    0   28
```

```
TABLE 5. INITIAL CONDITIONS
```

```
FROM  TO   CASE    VOL. W.    PORE PR.  SLOPE X
  1    1    1     1.442E+01-0.        -0.
  2    2    1     1.390E+01-0.        -0.
  3    3    1     1.355E+01-0.        -0.
  4    4    1     1.345E+01-0.        -0.
  5    5    1     1.340E+01-0.        -0.
  6    6    1     1.345E+01-0.        -0.
  7    7    1     1.345E+01-0.        -0.
  8    8    1     1.330E+01-0.        -0.
  9    9    1     1.300E+01-0.        -0.
 10   10    1     1.320E+01-0.        -0.
 11   11    1     1.340E+01-0.        -0.
 12   12    1     1.315E+01-0.        -0.
 13   13    1     1.290E+01-0.        -0.
 14   14    1     1.300E+01-0.        -0.
 15   15    1     1.320E+01-0.        -0.
 16   16    1     1.340E+01-0.        -0.
 17   17    1     1.355E+01-0.        -0.
 18   18    1     1.325E+01-0.        -0.
 19   19    1     1.260E+01-0.        -0.
 20   20    1     1.210E+01-0.        -0.
 21   21    1     1.190E+01-0.        -0.
 22   22    1     1.140E+01-0.        -0.
 23   23    1     1.060E+01-0.        -0.
 24   24    1     1.040E+01-0.        -0.
 25   25    1     1.380E+01-0.        -0.
 26   26    1     1.420E+01-0.        -0.
```

```
TABLE 6. BOUNDARY AND INTERNAL CONDITIONS
```

```
FROM  TO   CASE       WV          T        DT/DX       H        TEMP
  0    0    1     1.500E+01-0.          -0.         -0       -0.0
 27   27    1     4.000E+01-0.          -0.         -0       -0.0
```

```
TABLE 8A. TIME STEPS FOR B.C. CHANGE
```

```
    NONE


 TABLE 88. TIME STEPS FOR OUTPUT


    ALL
```

TABLE 10. OUTPUT OF RESULTS

```
 TIME STEP  =     1



 I      T(I)          WV(I)        UTOW(I)       P1(I)

 0  -7.904E+02    1.500E+01    2.357E+01    1.100E-07
 1  -8.496E+02    1.442E+01    2.508E+01    9.051E-08
 2  -9.081E+02    1.390E+01    2.657E+01    7.535E-08
 3  -9.508E+02    1.355E+01    2.764E+01    6.629E-08
 4  -9.635E+02    1.345E+01    2.796E+01    6.387E-08
 5  -9.699E+02    1.340E+01    2.812E+01    6.269E-08
 6  -9.635E+02    1.345E+01    2.796E+01    6.387E-08
 7  -9.635E+02    1.345E+01    2.796E+01    6.387E-08
 8  -9.830E+02    1.330E+01    2.844E+01    6.037E-08
 9  -1.024E+03    1.300E+01    2.945E+01    5.381E-08
10  -9.963E+02    1.320E+01    2.877E+01    5.811E-08
11  -9.699E+02    1.340E+01    2.812E+01    6.269E-08
12  -1.003E+03    1.315E+01    2.894E+01    5.701E-08
13  -1.038E+03    1.290E+01    2.980E+01    5.176E-08
14  -1.024E+03    1.300E+01    2.945E+01    5.381E-08
15  -9.963E+02    1.320E+01    2.877E+01    5.811E-08
16  -9.699E+02    1.340E+01    2.812E+01    6.269E-08
17  -9.508E+02    1.355E+01    2.764E+01    6.629E-08
18  -9.896E+02    1.325E+01    2.861E+01    5.923E-08
19  -1.082E+03    1.260E+01    3.088E+01    4.597E-08
20  -1.162E+03    1.210E+01    3.282E+01    3.748E-08
21  -1.196E+03    1.190E+01    3.365E+01    3.446E-08
22  -1.288E+03    1.140E+01    3.585E+01    2.779E-08
23  -1.458E+03    1.060E+01    3.985E+01    1.935E-08
24  -1.505E+03    1.040E+01    4.095E+01    1.758E-08
25  -9.200E+02    1.380E+01    2.687E+01    7.267E-08
26  -8.737E+02    1.420E+01    2.570E+01    8.383E-08
27    0.          4.000E+01    1.000E+00    1.000E-06
28    0.          0.          -1.833-270    1.000E-06

 TIME STEP  =     1



 I         T(I)          WV(I)        PF(I)

 0      -7.904E+02    1.500E+01    3.303E+00
 1      -8.492E+02    1.442E+01    3.334E+00
 2      -9.073E+02    1.391E+01    3.363E+00
 3      -9.497E+02    1.356E+01    3.382E+00
 4      -9.633E+02    1.345E+01    3.389E+00
 5      -9.695E+02    1.340E+01    3.391E+00
 6      -9.637E+02    1.345E+01    3.389E+00
 7      -9.641E+02    1.345E+01    3.389E+00
 8      -9.835E+02    1.330E+01    3.398E+00
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 9      -1.022E+03    1.301E+01    3.414F+00
10      -9.963E+02    1.320E+01    3.403F+00
11      -9.718E+02    1.339E+01    3.392F+00
12      -1.003E+03    1.315E+01    3.406F+00
13      -1.030E+03    1.291E+01    3.420F+00
14      -1.023E+03    1.300E+01    3.415F+00
15      -9.963E+02    1.320E+01    3.403F+00
16      -9.702E+02    1.340E+01    3.392F+00
17      -9.527E+02    1.354E+01    3.384F+00
18      -9.907E+02    1.324E+01    3.401F+00
19      -1.081E+03    1.260E+01    3.439F+00
20      -1.160E+03    1.211E+01    3.469F+00
21      -1.197E+03    1.189E+01    3.483F+00
22      -1.288E+03    1.140E+01    3.515F+00
23      -1.456E+03    1.061E+01    3.568F+00
24      -1.472E+03    1.054E+01    3.573F+00
25      -9.315E+02    1.371E+01    3.374F+00
26      -5.331E+02    1.848E+01    3.132F+00
27       0.           4.000E+01    0.


TIME STEP  =      2


I      T(I)          WV(I)          DTDW(I)        P1(I)

0   -7.904E+02    1.500E+01     1.347E+01     1.100E-07
1   -8.492E+02    1.442E+01     1.352E+01     9.063E-08
2   -9.073E+02    1.391E+01     1.358E+01     7.552E-08
3   -9.497E+02    1.356E+01     1.361E+01     6.650E-08
4   -9.633E+02    1.345E+01     1.36
5   -9.695E+0
                                      E+01     1.006E-07
                  2.265E+01     1.273E+01     5.788E-07
27   0.           4.000E+01     1.000E+00     1.000E-06
28   0.           0.           -1.833-270     1.000E-06


TIME STEP  =      3


I          T(I)         WV(I)         PF(I)

0       -7.904E+02    1.500E+01    3.303F+00
1       -8.483E+02    1.443E+01    3.333F+00
2       -9.058E+02    1.392E+01    3.362F+00
3       -9.478E+02    1.357E+01    3.382F+00
4       -9.627E+02    1.346E+01    3.388F+00
5       -9.687E+02    1.341E+01    3.391F+00
6       -9.641E+02    1.345E+01    3.389F+00
7       -9.653E+02    1.344E+01    3.389F+00
8       -9.844E+02    1.329E+01    3.398F+00
9       -1.018E+03    1.304E+01    3.413F+00
10      -9.962E+02    1.320E+01    3.403F+00
11      -9.751E+02    1.336E+01    3.394F+00
12      -1.003E+03    1.315E+01    3.406F+00
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13     -1.034E+03    1.293E+01    3.419F+00
14     -1.022E+03    1.301E+01    3.414F+00
15     -9.962E+02    1.320E+01    3.403F+00
16     -9.707E+02    1.339E+01    3.392F+00
17     -9.561E+02    1.351E+01    3.385F+00
18     -9.928E+02    1.323E+01    3.402F+00
19     -1.080E+03    1.261E+01    3.438F
20     -1.159E+03    1.212E+01
21     -1.198E+03
22     -1.28
23
                                                      6.248E-08
                                          62E+01      6.496E-08
                           E+01     1.365E+01         5.853E-08
             E+03     1.262E+01     1.371E+01         4.625E-08
      -1.158E+03    1.212F+01     1.376E+01         3.782E-08
21    -1.199E+03    1.188E+01     1.379E+01         3.422E-08
22    -1.284E+03    1.139F+01     1.384E+01         2.767F-08
23    -1.450E+03    1.063E+01     1.391E+01         1.963E-08
24    -1.373E+03    1.098E+01     1.388E+01         2.305E-08
25    -6.772E+02    1.630E+01     1.333E+01         1.642E-07
26    -3.030E+02    2.418E+01     1.247E+01         6.868E-07
27      0.          4.000E+01     1.000E+00         1.000E-06
28      0.          0.           -1.833-270         1.000E-06

 TIME STEP  =     8



 I          T(I)          WV(I)         PF(I)

 0     -7.904E+02    1.500E+01    3.303F+00
 1     -8.477E+02    1.444E+01    3.333F+00
 2     -9.046E+02    1.393E+01    3.361F+00
 3     -9.463E+02    1.359E+01    3.381F+00
 4     -9.622E+02    1.346E+01    3.388F+00
 5     -9.682E+02    1.341E+01    3.391F+00
 6     -9.644E+02    1.344E+01    3.389F+00
 7     -9.661E+02    1.343E+01    3.390F+00
 8     -9.849E+02    1.329E+01    3.398F+00
 9     -1.016E+03    1.306E+01    3.412F+00
10     -9.962E+02    1.320E+01    3.403F+00
11     -9.773E+02    1.334E+01    3.395F+00
12     -1.003E+03    1.315E+01    3.406F+00
13     -1.032E+03    1.294E+01    3.419F+00
14     -1.022E+03    1.302E+01    3.414F+00
15     -9.961E+02    1.320E+01    3.403F+00
16     -9.712E+02    1.339E+01    3.392F+00
17     -9.585E+02    1.349E+01    3.386F+00
18     -9.943E+02    1.321E+01    3.402F+00
19     -1.079E+03    1.262E+01    3.438F+00
20     -1.157E+03    1.212E+01    3.468F+00
21     -1.199E+03    1.188E+01    3.484F+00
22     -1.290E+03    1.139E+01    3.515F+00
23     -1.448E+03    1.064E+01    3.566F+00
24     -1.342E+03    1.113E+01    3.533F+00
25     -6.420E+02    1.677E+01    3.212F+00
26     -2.911E+02    2.456E+01    2.869F+00
27      0.           4.000E+01    0.
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TIME STEP  =     9



I      T(I)          WV(I)         DTUW(I)       P1(I)

0   -7.904E+02    1.500E+01     1.347E+01     1.100E-07
1   -8.477E+02    1.444E+01     1.352E+01     9.106E-08
2   -9.046E+02    1.393E+01     1.358E+01     7.616E-08
3   -9.463E+02    1.359E+01     1.361E+01     6.717E-08
4   -9.622E+02    1.346E+01     1.362E+01     6.410E-08
5   -9.682E+02    1.341E+01     1.363E+01     6.299E-08
6   -9.644E+02    1.344E+01     1.363E+01     6.370E-08
7   -9.661E+02    1.343E+01     1.363E+01     6.338E-08
8   -9.849E+02    1.329E+01     1.364E+01     6.003E-08
9   -1.016E+03    1.306E+01     1.367E+01     5.502E-08
10  -9.962E+02    1.320E+01     1.365E+01     5.814E-08
11  -9.773E+02    1.334E+01     1.364E+01     6.137E-08
12  -1.003E+03    1.315E+01     1.366E+01     5.701E-08
13  -1.032E+03    1.294E+01     1.368E+01     5.256E-08
14  -1.022E+03    1.302E+01     1.367E+01     5.414E-08
15  -9.961E+02    1.320E+01     1.365E+01     5.815E-08
16  -9.712E+02    1.339E+01     1.363E+01     6.245E-08
17  -9.585E+02    1.349E+01     1.362E+01     6.481E-08
18  -9.943E+02    1.321E+01     1.365E+01     5.845E-08
19  -1.079E+03    1.262E+01     1.371E+01     4.628E-08
20  -1.157E+03    1.212E+01     1.376E+01     3.786E-08
21  -1.199E+03    1.188E+01     1.379E+01     3.420E-08
22  -1.290E+03    1.139E+01     1.384E+01     2.766E-08
23  -1.448E+03    1.064E+01     1.391E+01     1.969E-08
24  -1.342E+03    1.113E+01     1.386E+01     2.460E-08
25  -6.420E+02    1.677E+01     1.329E+01     1.874E-07
26  -2.911E+02    2.456E+01     1.239E+01     7.122E-07
27    0.          4.000E+01     1.000E+00     1.000E-06
28    0.          0.           -1.833-270     1.000E-06

TIME STEP  =     9



I          T(I)          WV(I)          PF(I)

0       -7.904E+02    1.500E+01     3.303E+00
1       -8.475E+02    1.444E+01     3.333E+00
2       -9.042E+02    1.393E+01     3.361E+00
3       -9.459E+02    1.359E+01     3.381E+00
4       -9.621E+02    1.346E+01     3.388E+00
5       -9.681E+02    1.341E+01     3.391E+00
6       -9.645E+02    1.344E+01     3.389E+00
7       -9.664E+02    1.343E+01     3.390E+00
8       -9.851E+02    1.328E+01     3.398E+00
9       -1.015E+03    1.306E+01     3.411E+00
10      -9.962E+02    1.320E+01     3.403E+00
11      -9.780E+02    1.334E+01     3.395E+00
12      -1.003E+03    1.315E+01     3.406E+00
13      -1.032E+03    1.294E+01     3.418E+00
14      -1.021E+03    1.302E+01     3.414E+00
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