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# Traffic State Estimation for Connected Vehicles using the Second-Order Aw-Rascle-Zhang Traffic Model


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## Introduction

- Legacy traffic sensors such as inductive loop detectors are expensive to install and maintain which makes them infeasible for covering the entire network.
- Present work focuses on the problem of using both fixed legacy sensors along with moving sensors in the form of connected vehicles (CVs) to perform state estimation for unmeasured segments of the highway.

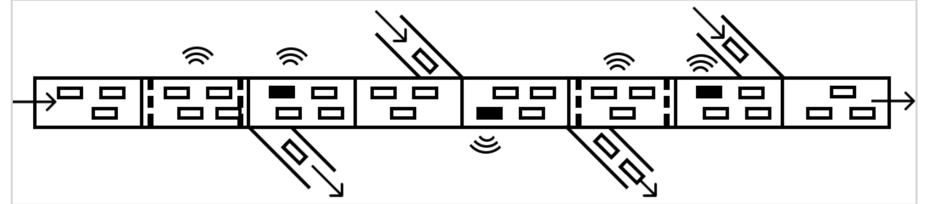

**Fig 1.** Fixed and moving sensors on the highway.

- Moving Horizon Estimation (MHE) is implemented for Traffic State Estimation (TSE) and compared with popular techniques in the literature namely EKF, UKF, and EnKF.
- The second-order Aw-Rascle-Zhang (ARZ) traffic model is utilized for TSE as opposed to first order models due to its ability to model speeds and densities separately.

## Research Goals

- Present a discrete-time nonlinear state-space model for highway networks having multiple on- and off-ramps based on the ARZ model.
- Linearize the nonlinear traffic model to implement linear state estimation methods for scalability.
- Implement MHE for TSE using the linearized ARZ model and compare with popular techniques like EKF, UKF and EnKF with respect to accuracy, parameter tuning and computational tractability.

## Traffic Dynamical System

- The second-order Aw-Rascle-Zhang (ARZ) model is utilized to describe the traffic dynamics. The highway is divided into segments of length $l$. Given time-step duration $T$, the discrete-time conservation equations for any highway segment can then be written as

$$\rho_i[k+1] = \rho_i[k] + \frac{T}{l}(q_{i-1}[k] - q_i[k]),$$

$$\psi_i[k+1] = \left(1 - \frac{1}{\tau}\right)\psi_i[k] + \frac{T}{l}(\phi_{i-1}[k] - \phi_i[k]) + \frac{v_f}{\tau}\rho_i[k].$$

where $\rho_i$ and $\psi_i$ denote the density and relative flow respectively and are the states of the system; $q_i$ and $\phi_i$ are nonlinear fluxes between segments which depend on the segment connections. In general, $\psi_i = \rho_i(v_i + p_i(\rho_i))$ where $v_i$ is the segment speed and $p(\cdot)$ is the pressure function which accounts for the equilibrium traffic speed. $v_f$ and $\tau$ are parameters of the model. $(v_i{+}p_i(\rho_i))$ is also called driver characteristic.
- The measurements are the segment traffic densities and speeds. The measurement model which converts measured densities and speeds to the states is also nonlinear.
- The nonlinear ARZ model and the measurement model are linearized using the first-order Taylor Series approximation by differentiating with respect to the state vector to obtain the following system:

$$\boxed{\begin{aligned} \boldsymbol{x}[k+1] &\approx \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{B}\boldsymbol{u}[k] + \boldsymbol{c}_1, \\ \boldsymbol{y}[k] &\approx \boldsymbol{C}[k]\boldsymbol{x}[k] + \boldsymbol{c}_2[k] \end{aligned}}$$

where $\boldsymbol{x}[k]$ is an array of the segment densities and relative flows while the input vector $\boldsymbol{u}[k]$ consists of the demands and driver characteristic for entry segments and densities ahead of the exit segments; $\boldsymbol{y}[k]$ is the vector of measured densities and speeds. Note that some of the state space parameters $\boldsymbol{A}$, $\boldsymbol{C}$, and $\boldsymbol{c}_2$ are time dependant since the linearization is always performed using a state vector near the current time step.

## Moving Horizon Estimation

### Optimization Problem for MHE

- The objective function of the optimization problem is given as

$$\begin{aligned} J[k] = \mu||\boldsymbol{x}_k[k-N] - \tilde{\boldsymbol{x}}[k-N]||^2 + w_1\sum_{i=k-N}^{k}||\boldsymbol{y}[i] - (\boldsymbol{C}_i\boldsymbol{x}_k[i] + \boldsymbol{c}_{2_i})||^2 \\ + w_2\sum_{i=k-N}^{k-1}||\boldsymbol{x}_k[i+1] - (\boldsymbol{A}_i\boldsymbol{x}_k[i] + \boldsymbol{B}_i u[i] + \boldsymbol{c}_{1_i})||^2. \end{aligned}$$

Here, $\tilde{x}[k-N]$ is a prediction of $x[k-N]$ based on a previously obtained state estimate. The constraints include upper and lower bounds on the states.

### Quadratic Programming Formulation

- The MHE optimization problem can be transformed into a standard Quadratic Program that can be solved using available solvers.

$$\begin{aligned} &\underset{\boldsymbol{z}_k}{\text{minimize}} && \boldsymbol{z}_k^T\boldsymbol{H}\boldsymbol{z}_k + \boldsymbol{q}^T\boldsymbol{z}_k \\ &\text{subject to} && \boldsymbol{z}_{\min} \leq \boldsymbol{z}_k \leq \boldsymbol{z}_{\max}. \end{aligned}$$

## Case Study

- We perform state estimation under various settings of sensors, initial states and measurement noise on a highway of length 900 m with 2 off-ramps and 1 on-ramp of 100 m each divided into a total of 12 segments.

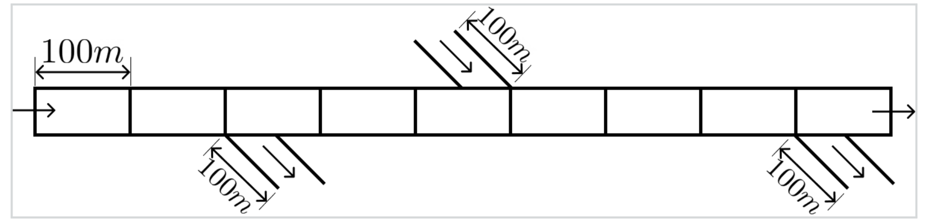


**Fig 2.** Schematic diagram of considered highway.

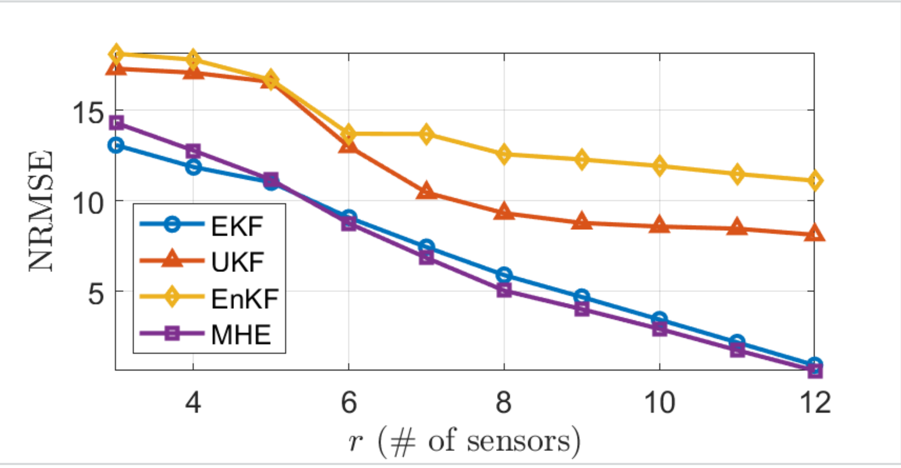


**Fig 3.** Estimation error with increasing number of fixed sensors.

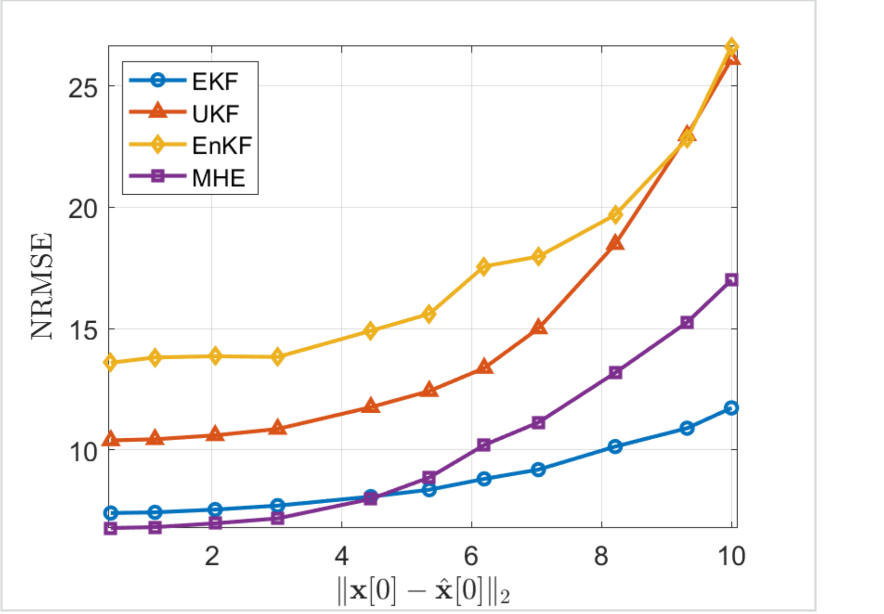


**Fig 4.** Estimation error with increasing gap in initial state guesses.

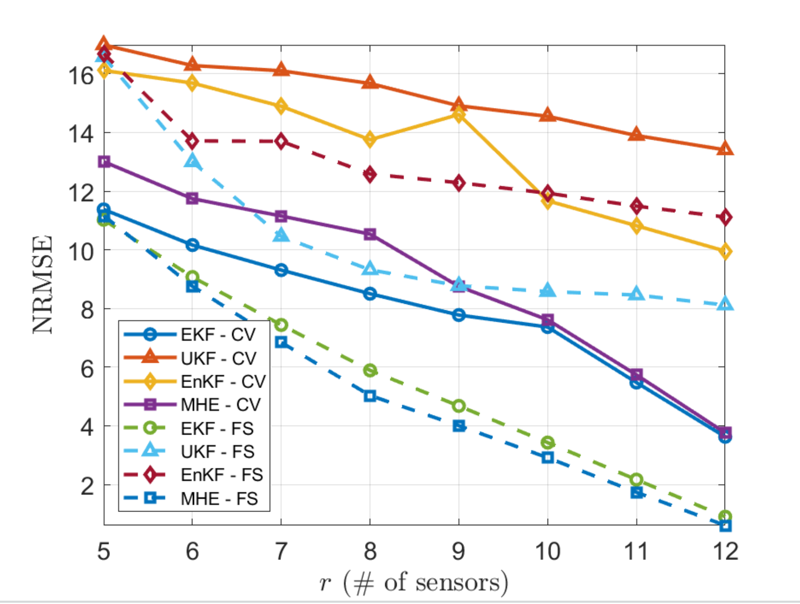


**Fig 5.** Estimation error with increasing number of CVs and fixed sensors.

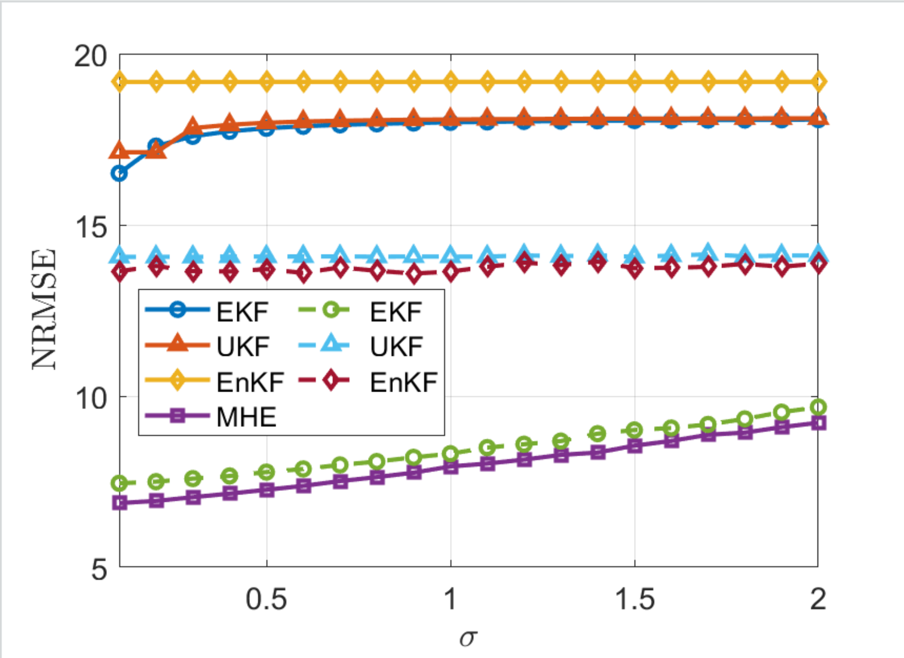


**Fig 6.** Estimation error with increasing noise in measurement data with un-tuned (solid-lines) versus tuned (dashed-lines) error covariance matrices for the KFs.

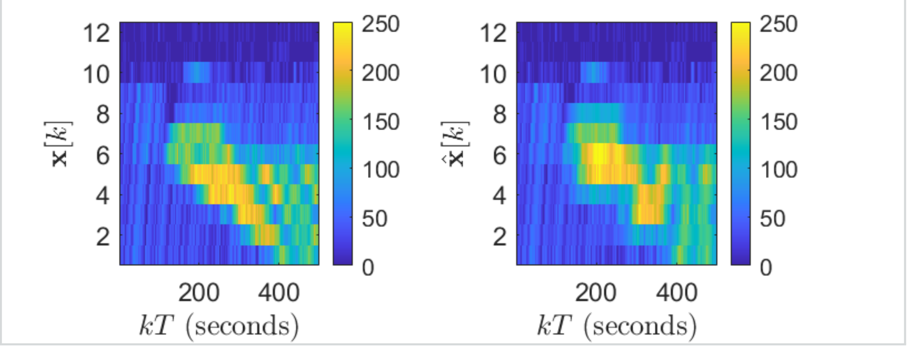


**Fig 7.** Comparison of real densities [left] and estimated densities using MHE [right] for all segments.

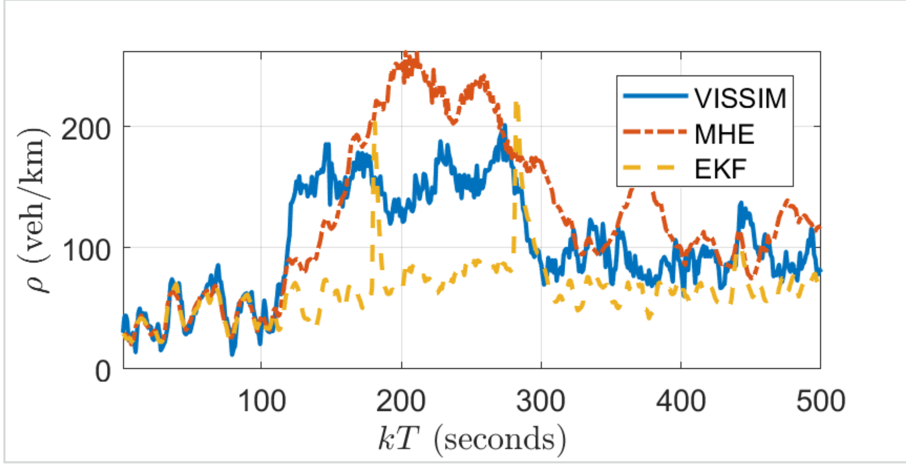


**Fig 8.** Estimated density trajectories for an unmeasured segment of the highway.

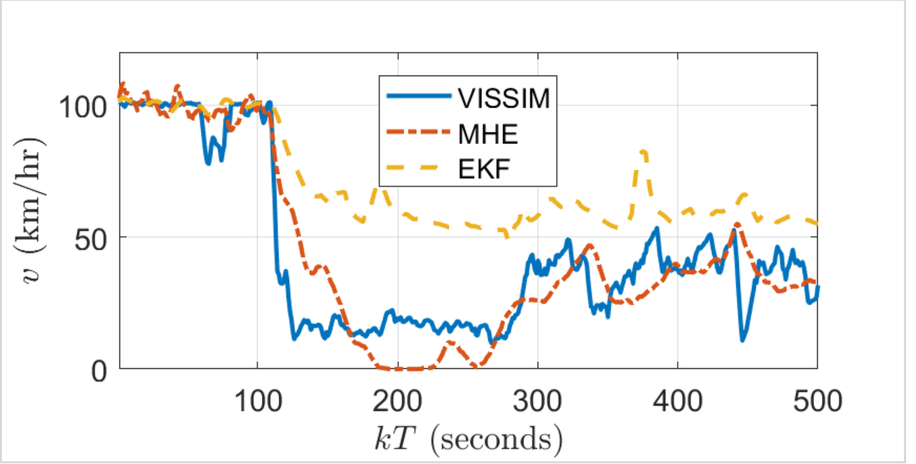


**Fig 9.** Estimated speed trajectories for an unmeasured segment of the highway.

## Key Takeaways

- As expected, TSE performance improves with the number of sensors. State estimation accuracy with CVs is slightly worse performance than an equal number of fixed sensors but they offer a cost effective alternative to fixed sensors.
- UKF performance is deteriorated most with CVs since UKF requires fine tuning of error covariance matrices with changing configuration of sensors which is not possible in practice with moving sensors. Other methods can work with a single fine tuned set of values for most configurations. MHE works with quite general values of parameters.
- EKF is fastest to converge to the actual state with increasing error in initial state guess followed by MHE.
- EKF and MHE perform similarly in different scenarios with the exception that MHE is more computationally intensive but can handle arbitrary constraints and is easier to tune. UKF and EnKF are less reliable than the former.


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