

**Technical Report 118** 

# A Latent Class Multiple Constraint Multiple Discrete-Continuous Extreme Value Model of Time Use and Goods Consumption

Chandra Bhat Center for Transportation Research

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A Tier 1 USDOT University Transportation Center at The University of Texas at Austin





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D-STOP is a collaborative initiative by researchers at the Center for Transportation Research and the Wireless Networking and Communications Group at The University of Texas at Austin.

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## Chapter 1. Introduction

Several microeconomic theory-based utility maximization models have been proposed in the past few decades to explain individuals' activity participation and travel behavior. The most notable advance in this context is the extension of the traditional, goods consumptionbased consumer theory by including the time use dimension. Goods consumption-based consumer theory assumes that individuals derive (and maximize) utility from consuming goods they purchase using money available with them. Models based on this theory are formulated as utility maximization problems subject to a money budget constraint. Bringing the time dimension into these models requires the incorporation of time (along with goods) into the utility functions, defining how goods consumption and time allocation relate to activities, and the recognition of constraints on available time along with the money budget constraint. By simultaneously considering time allocation and monetary expenditures, these models are able to disentangle different values of time estimates: value of time as a resource, value of working time, and value of assigning time to an activity/travel. This capability is important for the evaluation of transportation policies, because the benefits of travel time reductions can be economically measured using the different estimated values of time. Earlier research along these lines includes Becker (1965), DeSerpa (1971), Evans (1972), Train and McFadden (1978), Gronau (1986), Jara-Díaz (2003), and Jara-Díaz et al. (2008).

Although the microeconomic time use models have been gaining traction in the recent past, they are still saddled with at least a couple of limitations. First, traditional microeconomic models were aimed at analyzing consumer behavior among broad consumption categories (housing, education, etc.) that are almost always consumed. In such analyses, allowing zero consumptions (or corner solutions) was not necessary. Therefore, extensions of traditional consumer behavior models to include time use analysis also did not consider corner solutions. However, modern activity-based, time use and goods consumption analysis requires a detailed categorization of activities and goods, due to which the consideration of corner solutions (i.e., zero consumptions of certain goods or no time allocation to certain activities) becomes important. Second, several model formulations do not allow the possibility that participation in activities might need a minimum necessary amount of time. For example, eating activity might need a minimum time to do so. The few microeconomic models that allow minimum time allocations in the form of technical constraints do not simultaneously allow for non-participation in those activities (see for example, DeSerpa, 1971; Jara-Díaz et al., 2008; Jara-Díaz and Astroza, 2013; or Jara-Díaz et al., 2016). These studies use simple utility functions, such as the Cobb-Douglas functions that do not allow zero consumptions or zero time allocations.

In the past decade, a separate stream of research has made significant advances in the context of using sophisticated utility functions for modeling individuals' activity participation and time-use choices while allowing corner solutions (i.e., non-participation in some activities). Notable among these advances is the multiple discrete-continuous extreme value (MDCEV) model proposed by Bhat (2005, 2008), which is based on a microeconomic utility maximization formulation with elegant random utility functions that are easy to interpret, accommodate corner solutions, feature the law of diminishing

marginal utility with increasing time allocation to an activity, and yield closed form probability expressions. Due to the theoretical appeal and the practical ease with which the model parameters can be estimated, the MDCEV model has now been used by a number of studies for time use analysis.

The multiple discrete-continuous (MDC) formulation, while very appealing for time use analysis, has been applied largely for contexts with: (1) time allocations to activities as the only decision variables entering the utility function and (2) a single budget constraint associated with time. This leaves goods consumption completely out of the picture in MDC models of time-use. As discussed earlier, however, there is an increasing recognition that both time allocations and goods consumption generate utility and that both time and money budget constraints govern time use and consumption decisions (Konduri et al., 2011; Castro et al., 2012; Jara-Díaz and Astroza, 2013). More generally, there has been limited research on the use of multiple types of decision variables and multiple constraints within the context of MDC models (Parizat and Shachar, 2010, Satomura et al., 2011, and Castro et al., 2012). Castro et al. (2012) presented the multiple constraint-MDCEV (or MC-MDCEV) model structure, considering two constraints: a monetary budget constraint and a time constraint. However, the formulation does not consider both time allocations and goods consumption separately as decision variables in the utility function, and does not accommodate technical constraints, such as minimum values for the decisions variables.

In view of the above discussion, the aim of this paper is to develop a microeconomic theory-based MDC choice model that considers: (a) both time allocations and goods consumption separately as decision variables in the utility function, (b) both time and money constraints as determinants of activity participation and goods consumption decisions, (c) a finite probability of zero-consumptions and zero time allocations (i.e., corner solutions), and (d) technical constraints in the form of minimum consumption levels for any good that is consumed and minimum time allocation for any activity pursued. In addition, following Jara-Diaz et al. (2016), our utility function includes time assigned to work also as a decision variable (i.e., work duration is endogenously determined) along with the time allocations to various non-work activities. The work activity provides the link between the two constraints (monetary budget and total available time) and represents the trade-offs portrayed in our model; individuals may assign more time to work to generate more money (for buying more goods), but less free-time to perform non-work activities. Alternatively, individuals may assign less time to work, thereby generate less income, but have more time for non-work activities.

The application of our proposed model to different segments of the population allows the analyst to capture demographic heterogeneity in preferences and to estimate values of time that vary based on observed demographic variables such as gender, age, and income. In this paper, however, we capture heterogeneity in preferences using the latent class model formulation that allows a discrete-mixture distribution for model parameters based on observed demographic variables. Unlike most previous microeconomic theory based time use models that consider heterogeneity through a priori-determined market segments (see Jara-Díaz et al., 2013, Konduri et al., 2011, Jara-Díaz and Astroza, 2013, and Jara-Diaz et al., 2016), the latent class framework allows the analyst to endogenously segment the population (see Bhat, 1997, Sobhani et al., 2013).

We apply the proposed model to a 2012 Dutch data set on weekly time use and goods consumption. The empirical model is used to understand the sociodemographic determinants of time allocation and goods consumptions as well as to derive different value of time – value of work time and value of leisure (non-work) time. We compare the values of time derived from our model with those from other time use models in the literature that: (1) ignore corner solutions and minimum consumptions and/or time allocations, and (2) ignore that goods consumptions also enter the utility functions along with time allocations. We also demonstrate that the latent class model helps identify different segments of the population, each one of them with distinct preferences and values of time.

### Chapter 2. Methodology

Consider an individual q (q = 1, 2, ..., Q) belonging to a segment g (g = 1, 2, ..., G) who maximizes his/her utility of consuming different goods k (k = 1, 2, ..., K) and time allocations to different non-work activities n (n = 1, 2, ..., N) and work activity w, subject to two binding constraints, as below:

$$\max\left(U_{q}(\boldsymbol{x}_{q}, \boldsymbol{t}_{q}, \boldsymbol{t}_{q}, t_{qw}) \mid (q \in g)\right) = \sum_{k=1}^{K} u_{gk}(x_{qk}) + \sum_{n=1}^{N} \widetilde{u}_{gn}(t_{qn}) + \widetilde{u}_{gw}(t_{qw})$$

$$\sum_{k=1}^{K} p_{qk} x_{qk} = E_{q} + \omega_{q} t_{qw},$$

$$\sum_{n=1}^{N} t_{qn} + t_{qw} = T_{q},$$
(1)

In the above equation,  $U_q(\mathbf{x}_q, \mathbf{t}_q, t_{qw}) | (q \in g)$  is a quasi-concave, increasing and continuously differentiable utility function with respect to consumption of goods and time allocation to activities, given that individual q belongs to market segment g. Specifically,  $\mathbf{x}_q$  (=  $x_{q1}, x_{q2}, ..., x_{qk}, ..., x_{qK}; \mathbf{x}_{qk} \ge 0, \forall k = 1, 2, ..., K$ ) is the vector of consumption of different goods,  $\mathbf{t}_q$  (=  $t_{q1}, t_{q2}, ..., t_{qn}, ..., t_{qN}; t_{qn} \ge 0, \forall n = 1, 2, ..., N$ ) is the vector of time allocation to different non-work activities, and  $t_{qw}$  is the time allocation to work.

The first of the two constraints in Equation (1) is the money budget constraint, where  $p_{qk}$  is the unit price of consuming good k for individual q,  $E_q$  is the total expenditure (or income) of individual q after removing all other fixed expenses such as housing and utilities, and  $\omega_q$  is the individual's wage rate. The second is the time budget constraint, where  $T_q$  is the total available time for individual q.

Note from Equation (1) that the utility function is defined as an additively separable function of sub-utilities derived from consuming goods,  $u_{gk}(x_{qk})$ , sub-utilities derived from allocating time to non-work activities,  $\tilde{u}_{gn}(t_{qn})$ , and sub-utility from the time allocated to work,  $\tilde{u}_{gw}(t_{qw})$ . The functional from of the sub-utilities follows the linear expenditure system (LES) utility form originally proposed by Bhat (2008), which was extended by Van Nostrand *et al.* (2013) to accommodate minimum required consumptions and time allocations, as below:

$$u_{gk}(x_{qk}) = \psi_{qgk} x_{qk} \qquad \text{if} \qquad x_{qk} \leq x_{qk}^{0}$$

$$= \psi_{qgk} x_{qk}^{0} + \gamma_{qgk} \psi_{qgk} \ln \left( \frac{x_{qk} - x_{qk}^{0}}{\gamma_{qgk}} + 1 \right) \qquad \text{if} \qquad x_{qk} > x_{qk}^{0}$$

$$\widetilde{u}_{gn}(t_{qn}) = \widetilde{\psi}_{qgn} t_{qn} \qquad \text{if} \qquad t_{qn} \leq t_{qn}^{0}$$

$$= \widetilde{\psi}_{qgn} t_{qn}^{0} + \widetilde{\gamma}_{qgn} \widetilde{\psi}_{qgn} \ln \left( \frac{t_{qn} - t_{qn}^{0}}{\widetilde{\gamma}_{qgn}} + 1 \right) \qquad \text{if} \qquad t_{qn} > t_{qn}^{0}$$

$$\widetilde{u}_{gw}(t_{qw}) = \widetilde{\psi}_{qgw} t_{qw}^{0} \qquad \text{if} \qquad t_{qw} \leq t_{qw}^{0}$$

$$= \widetilde{\psi}_{qgw} t_{qw}^{0} + \widetilde{\psi}_{qgw} \ln(t_{qw} - t_{qw}^{0}) \qquad \text{if} \qquad t_{qw} > t_{qw}^{0}$$

where  $x_{qk}^0$  is the minimum required consumption of good k (if it is consumed),  $t_{qn}^0$  is the minimum amount of time required to conduct activity n (if that activity is conducted), and  $t_{qw}^0$  is the minimum required duration for work.<sup>1</sup> As discussed in Van Nostrand *et al.* (2013), the utility derived from consuming a good (time allocation to a non-work activity) increases linearly until the minimum required amount of consumption (time) is allocated to that good (activity), after which the functional form takes a non-linear shape to allow diminishing marginal utility. Due to this functional form, if a good is consumed (time is allocated to an activity), the consumption (time allocation) has to be greater than the minimum values defined above. Note also that the functional form for  $\tilde{u}_{gw}$  implies that work plays the role of an '*essential alternative*' that is always allocated a positive amount of time by all workers. For all goods and non-work activities, the functional form allows corner solutions (*i.e.*, zero consumptions or time allocations) because of the presence of +1 in the utility form (see Bhat, 2008).<sup>2</sup>

For an individual q who belongs to segment g,  $\psi_{qgk}$ ,  $\tilde{\psi}_{qgn}$ , and  $\tilde{\psi}_{qgw}$  are the baseline utility parameters associated with good k, non-work activity type n, and work activity, respectively. A greater value of the baseline utility parameter for an alternative good or non-work activity suggests a greater likelihood of choice and a greater amount of consumption of that alternative.  $\gamma_{qgk}$  and  $\tilde{\gamma}_{qgn}$  are satiation parameters for good k and nonwork activity n, respectively; a greater value of the satiation parameter suggests a greater amount of consumption of that alternative.

<sup>&</sup>lt;sup>1</sup> We considered, as with many previous studies (Jara-Díaz and Guevara, 2003 and Jara-Díaz *et al.*, 2008), exogenously given minimum levels of good consumption and time allocation. Endogenously determining the minimum levels is beyond the scope of this paper. Specifically,  $x_{qk}^0$  is set as the observed minimum level of consumption of good k in the dataset,  $t_{qn}^0$  is set as the observed minimum level of time allocation to activity n, and  $t_{qw}^0$  is set as the observed minimum work duration minus 1. Note that the minus 1 in the utility function of work activity ensures that the function is defined and continuously differentiable at all values of  $t_{qw}$ .

 $<sup>^2</sup>$  Changing the utility formulation to treat all (or a subset of) other activities and/or goods as essential is straightforward.

The optimal values of goods consumption, non-work time allocation, and work time allocation may be solved by forming the following Lagrangian function for the optimization problem in Equation (1) and deriving the Karush-Kuhn-Tucker (KKT) conditions of optimality. The Lagrangian function is below:

$$l_{q} \mid q \in \text{segment } g = U_{q}(\boldsymbol{x}_{q}, \boldsymbol{t}_{q}, \boldsymbol{t}_{q}, \boldsymbol{t}_{qw}) + \lambda_{qg} \left( E_{q} + \omega_{q} t_{qw} - \sum_{k=1}^{K} p_{qk} x_{qk} \right) + \mu_{qg} \left( T_{q} - t_{qw} - \sum_{n=1}^{N} t_{qn} \right)$$
(3)

where  $\lambda_{qg}$  and  $\mu_{qg}$  are segment *g*-specific Lagrangian multipliers for the budget and time constraints, representing the marginal utilities of expenditure and time, respectively. The KKT conditions for optimal consumption and time allocations  $(x_{qk}^*, t_{qn}^* \text{ and } t_{qw}^*)$  are as below:

$$u'_{gk}(x^*_{qk}) - \lambda_{qg} p_{qk} = 0 \quad \text{if } x^*_{qk} > 0, \ k = 1, 2, ..., K$$

$$u'_{gk}(x^*_{qk}) - \lambda_{qg} p_{qk} < 0 \quad \text{if } x^*_{qk} = 0, \ k = 1, 2, ..., K$$

$$\widetilde{u}'_{gn}(t^*_{qn}) - \mu_{qg} = 0 \quad \text{if } t^*_{qn} > 0, \ n = 1, 2, ..., N$$

$$\widetilde{u}'_{gn}(t^*_{qn}) - \mu_{qg} < 0 \quad \text{if } t^*_{qn} = 0, \ n = 1, 2, ..., N$$

$$\widetilde{u}'_{gw}(t^*_{qw}) + \omega_q \lambda_{qg} - \mu_{qg} = 0$$
(4)

where,  $u'_{gk}(x^*_{qk}), \widetilde{u}'_{gn}(t^*_{qn})$ , and  $\widetilde{u}'_{gw}(t^*_{qw})$  are the marginal utility functions, defined as:

$$\begin{aligned} u'_{gk}(x^{*}_{qk}) &= \psi_{qgk} \quad \text{if } x^{*}_{qk} \leq x^{0}_{qk} \\ &= \psi_{qgk} \left( \frac{x^{*}_{qk} - x^{0}_{qk}}{\gamma_{qgk}} + 1 \right)^{-1} \quad \text{if } x^{*}_{qk} \geq x^{0}_{qk} \quad , \\ \widetilde{u}'_{gn}(t^{*}_{qn}) &= \widetilde{\psi}_{qgn} \quad \text{if } t^{*}_{qn} \leq t^{0}_{qn} \\ &= \widetilde{\psi}_{qgn} \left( \frac{t^{*}_{qn} - t^{0}_{qn}}{\gamma_{qgn}} + 1 \right)^{-1} \quad \text{if } t^{*}_{qn} \geq t^{0}_{qn} , \text{ and} \\ \widetilde{u}'_{gw}(t^{*}_{qw}) &= \widetilde{\psi}_{qgw} \quad \text{if } t^{*}_{qw} \leq t^{0}_{qw} \\ &= \widetilde{\psi}_{qgw}(t^{*}_{qw} - t^{0}_{qw})^{-1} \quad \text{if } t^{*}_{qw} > t^{0}_{qw} . \end{aligned}$$

The optimal consumptions (of goods) and time allocations (to activities) satisfy the KKT conditions in Equation (4) and the money budget and time constraints in Equation (1). Denote good 1 as the good to which the individual allocates non-zero consumption (the individual has to participate in at least 1 of the K purposes). The corresponding KKT

condition is:  $\psi_{qg1} \left( \frac{x_{q1}^* - x_{q1}^0}{\gamma_{qg1}} + 1 \right)^{-1} - \lambda_{qg} p_{q1} = 0$ , using which  $\lambda_{qg}$  may be expressed as:

$$\lambda_{qg} = \frac{\psi_{qg1}}{p_{q1}} \left( \frac{x_{q1}^* - x_{q1}^0}{\gamma_{qg1}} + 1 \right)^{-1} = \frac{u'_{g1}(x_{q1}^*)}{p_{q1}}$$
(6)

Now, since all individuals assign non-zero amount of time to work (and at least 1 unit above the minimum work duration), the KKT condition for working time is:

$$\widetilde{\psi}_{qgw} (t_{qw}^* - t_{qw}^0)^{-1} + \omega_q \lambda_{qg} - \mu_{qg} = 0$$
<sup>(7)</sup>

Replacing (6) in (7) the expression for  $\mu_{qg}$  may be written as:

$$\mu_{qg} = \widetilde{\psi}_{qgw} (t_{qw}^* - t_{qw}^0)^{-1} + \omega_q \frac{\psi_{qg1}}{p_{q1}} \left( \frac{x_{q1}^* - x_{q1}^0}{\gamma_{qg1}} + 1 \right)^{-1} = \widetilde{u}_{gw}' (t_{qw}^*) + \omega_q \frac{u_{g1}'(x_{q1}^*)}{p_{q1}}$$
(8)

Substituting  $\lambda_{qg}$  and  $\mu_{qg}$  into Equation (4), the KKT conditions may be rewritten as:

$$\frac{u_{gk}'(x_{qk}^{*})}{p_{qk}} = \frac{u_{g1}'(x_{q1}^{*})}{p_{q1}} \quad \text{if } x_{qk}^{*} > 0, \ k = 2,..., K$$

$$\frac{u_{gk}'(x_{qk}^{*})}{p_{qk}} < \frac{u_{g1}'(x_{q1}^{*})}{p_{q1}} \quad \text{if } x_{qk}^{*} = 0, \ k = 2,..., K$$

$$\widetilde{u}_{gn}'(t_{qn}^{*}) = \widetilde{u}_{gw}'(t_{qw}^{*}) + \omega_{q} \frac{u_{g1}'(x_{q1}^{*})}{p_{q1}} \quad \text{if } t_{qn}^{*} > 0, \ n = 1, 2, ..., N$$

$$\widetilde{u}_{gn}'(t_{qn}^{*}) < \widetilde{u}_{gw}'(t_{qw}^{*}) + \omega_{q} \frac{u_{g1}'(x_{q1}^{*})}{p_{q1}} \quad \text{if } t_{qn}^{*} = 0, \ n = 1, 2, ..., N$$
(9)

The above KKT conditions have an intuitive interpretation. For any good k, its optimal consumption will either be (a) positive such that its price-normalized marginal utility at optimal consumption is equal to the price-normalized marginal utility of good 1 (or any other consumed good) at its optimal consumption point, or (b) zero if the price-normalized marginal utility of good 1 or any other consumed good. Similar is the case of time allocation, where all the activities that are performed have the same marginal utility, following a common result in time use models since DeSerpa (1971), who proposed that all the freely chosen activities (activities that are assigned more time than the necessary minimum) have the same marginal utility. In the context of work, as in Equation (7), marginal utility of time allocated to work plus the wage rate multiplied by the marginal utility of money should be equal to the marginal utility of activities that are assigned more time than the minimum necessary.

The most interesting property of this model is the ability to calculate the value of time as a resource, or value of leisure (VL), and the value of allocating time assigned to work (VW):

$$VL = \frac{\mu_{qg}}{\lambda_{qg}} = \frac{\widetilde{u}'_{gw}(t^*_{qw}) + \omega_q \frac{u'_{g1}(x^*_{q1})}{p_{q1}}}{\frac{u'_{g1}(x^*_{q1})}{p_{q1}}} = \frac{\widetilde{u}'_{gw}(t^*_{qw})}{\frac{u'_{g1}(x^*_{q1})}{p_{q1}}} + \omega_q$$
(10)  
$$VW = \frac{\mu_{qg}}{\lambda_{qg}} - \omega_q = \frac{\widetilde{u}'_{gw}(t^*_{qw})}{\frac{u'_{g1}(x^*_{q1})}{p_{q1}}}$$
(11)

 $p_{q1}$ 

Note that VL is equal to the total value of work, *i.e.*, VW plus the wage rate, a common result for time use models in which work duration enters the utility function (DeSerpa, 1971, Jara-Díaz *et al.*, 2008).

#### 2.1 Model Estimation

We introduce observed heterogeneity across individuals within segment g and stochasticity through the baseline marginal utility functions, as below:

$$\begin{split} \psi_{qgk} &= \exp(\beta'_{qg} z_{qk} + \varepsilon_{qgk}), \\ \widetilde{\psi}_{qgn} &= \exp(\widetilde{\beta}'_{qg} \widetilde{z}_{qn} + \widetilde{\varepsilon}_{qgn}), \\ \widetilde{\psi}_{qgw} &= \exp(\widetilde{\varepsilon}_{qgw}). \end{split}$$
(12)

where  $\mathbf{z}_{qk}$  is a *D*-dimensional vector of observed attributes characterizing good *k* and individual *q*; and  $\boldsymbol{\beta}_{qg}$  is the corresponding vector of coefficients (of dimension *D*×1), including alternative-specific constants to capture intrinsic preferences for each good. Similarly,  $\tilde{\mathbf{z}}_{qn}$  is a  $\tilde{D}$ -dimensional vector of observed attributes characterizing individual *q*; and  $\tilde{\boldsymbol{\beta}}_{qg}$  is the corresponding vector of coefficients, including alternative-specific constants to capture intrinsic preferences for each activity. For identification purposes, for each individual attribute entering  $\mathbf{z}_{qk}$  in the goods consumption utility function, the coefficient for one good is normalized to zero. Similarly, the alternative-specific constant for one good is normalized to zero (*i.e.*, one good is treated as the base alternative). The time allocation utility function is normalized by treating the work activity as the base alternative (with no observed variables or a constant entering the utility function).

Using stochastic baseline utility expressions from Equation (12) in the KKT conditions of Equation (9) leads to the following stochastic KKT conditions:

$$\ln\left(\frac{V_{qgk}}{p_{qk}}\right) + \boldsymbol{\beta}'_{qg}\boldsymbol{z}_{qk} + \boldsymbol{\varepsilon}_{qgk} = -\ln\left(\frac{1}{p_{q1}}\left(\frac{x_{q1}^{*} - x_{q1}^{0}}{\gamma_{qg1}} + 1\right)\right) + \boldsymbol{\beta}'_{qg}\boldsymbol{z}_{q1} + \boldsymbol{\varepsilon}_{qg1} \text{ if } x_{qk}^{*} > 0 \text{ , } k = 2,...,K$$

$$\ln\left(\frac{V_{qgk}}{p_{qk}}\right) + \boldsymbol{\beta}'_{qg}\boldsymbol{z}_{qk} + \boldsymbol{\varepsilon}_{qgk} < -\ln\left(\frac{1}{p_{q1}}\left(\frac{x_{q1}^{*} - x_{q1}^{0}}{\gamma_{qg1}} + 1\right)\right) + \boldsymbol{\beta}'_{qg}\boldsymbol{z}_{q1} + \boldsymbol{\varepsilon}_{qg1} \text{ if } x_{qk}^{*} = 0 \text{ , } k = 2,...,K$$

$$\ln\left(\widetilde{V}_{qgn}\right) + \boldsymbol{\widetilde{\beta}}'_{qg}\boldsymbol{\widetilde{z}}_{qn} + \boldsymbol{\widetilde{\varepsilon}}_{qgn} = \ln\left(\widetilde{u}'_{gw}(t_{qw}^{*}) + \boldsymbol{\omega}_{q}\frac{u'_{g1}(x_{q1}^{*})}{p_{q1}}\right) \text{ if } t_{qn}^{*} > 0 \text{ , } n = 1,2,...,N$$

$$\ln\left(\widetilde{V}_{qgn}\right) + \boldsymbol{\widetilde{\beta}}'_{qg}\boldsymbol{\widetilde{z}}_{qn} + \boldsymbol{\widetilde{\varepsilon}}_{qgn} < \ln\left(\widetilde{u}'_{gw}(t_{qw}^{*}) + \boldsymbol{\omega}_{q}\frac{u'_{g1}(x_{q1}^{*})}{p_{q1}}\right) \text{ if } t_{qn}^{*} = 0 \text{ , } n = 1,2,...,N$$

where,

$$V_{qgk} = 1 \quad \text{if } x_{qk}^* \le x_{qk}^0$$
  
=  $\left(\frac{x_{qk}^* - x_{qk}^0}{\gamma_{qgk}} + 1\right)^{-1} \quad \text{if } x_{qk}^* \ge x_{qk}^0$   
 $\widetilde{V}_{qgn} = 1 \quad \text{if } t_{qn}^* \le t_{qn}^0$   
=  $\left(\frac{t_{qn}^* - t_{qn}^0}{\gamma_{qgn}} + 1\right)^{-1} \quad \text{if } t_{qn}^* \ge t_{qn}^0$ 

Assuming that the stochastic terms are IID type-1 extreme value distributed, the probability that an individual q (who belongs to segment g) consumes M of the K goods and assigns time to  $\tilde{M}$  of the N non-work activities is:

$$P_{qg}\left(x_{q1}^{*}, x_{q2}^{*}, \dots, x_{qM}^{*}, 0, \dots, 0, t_{q1}^{*}, t_{q2}^{*}, \dots, t_{q\tilde{M}}^{*}, 0, \dots, 0, t_{qw}^{*}\right) = \frac{1}{\sigma_{g}^{M-1}} \left[\prod_{k=2}^{M} c_{qgk}\right] \left[\prod_{n=2}^{\tilde{M}} \widetilde{c}_{qgn}\right]$$

$$\int_{\varepsilon_{qg1}=-\infty}^{\infty} \int_{\widetilde{\varepsilon}_{qgw}=-\infty}^{\infty} \prod_{k=2}^{M} h\left(\frac{W_{k}\left|(\varepsilon_{qg1}, \widetilde{\varepsilon}_{qgw})\right|}{\sigma_{g}}\right) \times \prod_{n=1}^{\tilde{M}} h\left(\frac{W_{n}\left|(\varepsilon_{qg1}, \widetilde{\varepsilon}_{qgw})\right|}{\sigma_{g}}\right)$$

$$\times \left\{\prod_{l=M+1}^{K} H\left(\frac{W_{l}\left|(\varepsilon_{qg1}, \widetilde{\varepsilon}_{qgw})\right|}{\sigma_{g}}\right) \times \prod_{r=\tilde{M}+1}^{N} H\left(\frac{W_{r}\left|(\varepsilon_{qg1}, \widetilde{\varepsilon}_{qgw})\right|}{\sigma_{g}}\right)\right\}$$

$$\times f_{g}(\varepsilon_{qg1})f_{g}(\widetilde{\varepsilon}_{qgw})d\varepsilon_{qg1}d\widetilde{\varepsilon}_{qgw}.$$
(14)

where,  $c_{qgk} = \frac{1}{x_{qgk}^* - x_{qg0}^* + \gamma_{qgk}}, \quad \widetilde{c}_{qgn} = \frac{1}{t_{qgn}^* - t_{qg0}^* + \widetilde{\gamma}_{qgk}},$   $W_k \left| (\varepsilon_{qg1}, \widetilde{\varepsilon}_{qgw}) \right| = -\ln \left( \frac{1}{p_{q1}} \left( \frac{x_{q1}^* - x_{q1}^0}{\gamma_{qg1}} + 1 \right) \right) \right| + \beta'_{qg} z_{q1} + \varepsilon_{qg1} - \ln \left( \frac{V_{qgk}}{p_{qk}} \right) - \beta'_{qg} z_{qk},$  $W_n \left| (\varepsilon_{qg1}, \widetilde{\varepsilon}_{qgw}) \right| = \ln \left( \widetilde{u}'_{gw}(t_{qw}^*) + \omega_q \frac{u'_{g1}(x_{q1}^*)}{p_{q1}} \right) - \ln (\widetilde{V}_{qgn}) - \widetilde{\beta}'_{qg} \widetilde{z}_{qn},$ 

*h* is the standard extreme value density function, *H* is the standard extreme value cumulative distribution function, and  $f_g(\varepsilon)$  is the probability density function of the extreme value distributed  $\varepsilon$  term with scale parameter  $\sigma_g \cdot \sigma_g$  is estimable if there is price variation across different goods; its value needs to normalized (typically to 1) if there is no price variation.

The derivation thus far was based on the assumption that individual q belongs to a single segment g. Now, consider the case when individual q belongs to a finite mixture of segments. That is, the actual assignment of individual q to a specific segment is not observed, but we are able to attribute different probabilities  $\pi_{qg}$  (g = 1, 2, ..., G) that the individual belongs to different latent segments. We require that  $0 \le \pi_{qg} \le 1$ , and  $\sum_{g=1}^{G} \pi_{qg} = 1$  using the logit link function below:

$$\pi_{qg} = \frac{\exp(\boldsymbol{\delta}'_{g}\boldsymbol{w}_{q})}{\sum_{g'=1}^{G}\exp(\boldsymbol{\delta}'_{g'}\boldsymbol{w}_{q})},$$
(15)

where  $w_q$  is a vector of individual exogenous variables and  $\delta_g$  is the vector of coefficients determining the influence of  $w_q$  on the membership of individual q in segment g, with all the elements in  $\delta_1$  set to zero for identification purposes. Using these latent segmentation probabilities, the overall likelihood for observation q may be written as:

$$P_{q} = \sum_{g=1}^{G} \pi_{qg} P_{qg} , \qquad (16)$$

and the likelihood function for the entire data may be written as:

$$P = \prod_{q} P_{q} . \tag{17}$$

The use of latent classes requires labeling restrictions for identifiability. In particular, the parameter space includes G! subspaces, each associated with a different way of labeling the mixture components. To prevent the interchange of the mixture components, we impose the restriction that the constants specific to the second alternative (good) are increasing across the segments. Such a labeling restriction is needed because the same model

specification results simply by interchanging the sequence in which the segments are numbered, so multiple sets of parameters result in the same likelihood function.

## Chapter 3. Empirical Application

We apply the modeling methodology presented above using a Dutch data set drawn from the Longitudinal Internet Studies for the Social Sciences (LISS) panel. It is worth noting here that survey datasets of the type needed for analysis in this paper, with both time use and goods consumption (and expenditures) information, are rare. Therefore, previous studies had to resort to alternative approaches to impute data or to merge data from separate time use surveys and consumer expenditure surveys (see, for example, Konduri *et al.*, 2011).

### 3.1 Data Description and Sample Selection

The LISS panel is based on a probability sample of Dutch households drawn from the country's population register. Administered in the form of monthly internet surveys in 2009, 2010, and 2012, the LISS panel included a survey of time use and expenditures (see Cherchye *et al.*, 2012 for a detailed description). In the current paper, we will focus on the data from the latest wave (October 2012). In this survey, respondents reported: (a) their time allocation to various activities (including work) during seven days before the survey, and (b) their average monthly monetary expenditure (in euros) in 30 expense categories for 12 months before the survey. In this analysis, the monetary expenditures were considered as a proxy for goods consumption. This is because the surveyed information did not include the amount of goods consumed. The database includes the monthly average gross and net income of the worker. For the analysis, only net income is considered. Henceforth, net income is referred to as income. To achieve consistency between activity durations and expenditures, monthly expenditures and monthly income were divided by 4 to obtain weekly expenditures and weekly income, respectively. Income was categorized into work income and non-work income; (1) work income was obtained from working (for an employer or independently) and used to compute the wage rate ( $\omega_q$ , which is work income divided by the time allocated to work), and (2) non-work income  $(E_q)$  corresponds to the money received from pensions, investments, annuities, governmental support, scholarships, tax reimbursement and others sources.

The survey contains information on 3,056 workers, who were considered in this analysis. The sample was further narrowed down in the following ways. <u>First</u>, we selected respondents who worked for at least 1 hour per week. <u>Second</u>, we selected only those workers who live in single-worker households, to avoid assumptions on how the household expenditures are shared among different income producers in the household. <u>Third</u>, we selected workers who reported expenditures in at least one expenditure category. <u>Fourth</u>, we removed workers who slept, on average, less than 4 hrs./day (28 hours per week), because of the very real possibility that they are underestimating their sleep time and may also be misestimating other time allocations. <u>Fifth</u>, we removed individuals who reported abnormally high activity durations such as working for 24 hrs./day. <u>Sixth</u>, in some cases the total expenditures are larger than the respondent's net income. If the difference between total expenditures and income was smaller than 20% of the worker's income, the difference was added to the worker's non-work income. Otherwise, if the difference was larger than 20%, the observation was removed from the sample. <u>Seventh</u>, respondents who spent less

than 2 euros per day were removed from the sample, along with those workers whose wage was less than 3 euros/hour (the minimum hourly wage in Netherlands was 8.4 euros in 2012). After this sample cleaning process, the final estimation sample has 1,193 workers.

### 3.2 Variable Specification and Model Formulation

From the various activities reported in The LISS panel, the following 11 categories of activities were constructed for the analysis: (1) work (any type of paid work as an employee or self-employed worker, including overtime hours), (2) commute (travel to/from work), (3) household chores (includes cleaning, shopping, cooking, and gardening), (4) personal care (includes washing, dressing, eating, visiting the hairdresser, and seeing the doctor), (5) education (includes day or evening courses, professional courses, language courses or other courses types, and doing homework), (6) activities with children (includes any activity with own children aged less than 16 years, such as washing, dressing, playing, taking child to see doctor, and taking child to school/hobby activities), (7) entertainment (includes in-home and out-of-home recreational activities, such as watching TV, reading, practicing sports, hobbies, computer as hobby, visiting family or friends, going out, walking the dog, sex, and recreational cycling), (8) assisting friends and family (includes any assistance to friends and family members that are not children, for example, helping with administrative chores, washing, dressing, seeing the doctor, voluntary work, or babysitting), (9) administrative chores and family finances, (10) sleeping and relaxing (includes sleeping, resting, thinking, and meditating), and (11) other activities (includes going to church, funeral, weddings, and any other activity).

The time allocations to all the above activities, except commute time enter the utility functions (*i.e.*, N = 10). We assume that individuals do not derive utility from commuting; individuals would simply allocate the minimum possible time they need to get to/from work based on the mode of travel to work.<sup>3</sup> Since our time frame of analysis is a week, the total weekly time available for any individual is 168 hours ( $24 \times 7$  hours). The total weekly time budget for all individuals  $T_q$  was obtained by subtracting the weekly commute time and travel time for activities from 168 hours. Three of the ten activities entering the utility function – work, sleeping and relaxing, and personal care – are treated as "essential alternatives" in that all working individuals participate and spend time in these activities (*i.e.*, the corresponding utility functions were specified not to allow corner solutions).

As indicated earlier, since we only have information about expenditures in composite categories, we assume that the expenditures enter the utility functions as a proxy for consumption of goods. Therefore, the same expenditures enter the money budget constraint with unit prices. To do so, the 30 categories of expenses recorded in the database were combined into the following six composite expense categories (see Jara-Diaz *et al.*, 2016 for details about the definition of these categories): (1) commute, (2) household chores, (3) personal care, (4) education, (5) activities with children, and (6) entertainment. Note that these six categories are in the activity type categorization (*i.e.*, in the context of time allocation) as well. Among the other activities, it is reasonable to assume that work activity

<sup>&</sup>lt;sup>3</sup> Arguably, other activities such as household chores and personal care might be considered as mere maintenance tasks that do not generate utility. However, individuals can derive utility from household chores such as cooking, gardening, and shopping. Similarly, personal care activities such as visiting the beauty salon may also provide utility.

has no expenditures (as it generates income). It is also reasonable that the remaining five activities – assisting friends and family, administrative chores, sleeping and relaxing, and other activities – do not have expenditures. Further, similar to the time-allocation case, we assume that individuals do not extract utility from commuting expenses. As a result, the expenditures (aka, goods consumption) in only the following five categories enter the goods consumption utility function: household chores, personal care, activities with children, education, and entertainment (*i.e.*, K = 5). Further, the monetary budget available for expenditures is computed by subtracting commute expenses from the individuals' available income. Finally, note that although all individuals in the participated in personal care activities, not all of them spent money on associated consumptions. Therefore, while time allocation to personal care was viewed as an essential alternative, expenditure in personal care was not treated as essential (*i.e.*, corner solutions were allowed).

The descriptive statistics of activity time allocations and goods expenditures are presented in Table 1. As discussed earlier, all individuals in the sample allocate time to work, sleeping and relaxing: on average, individuals work (6.6 hrs./day), sleep/relax (8.4 hrs./day), and personal care (1.3 hrs./day). Most workers allocate some time to commute, entertainment and personal care, while education and activities with children present the lowest participation rates suggesting the importance of accommodating corner solutions for (i.e., zero time allocations) to these activities. In the context of expenditures, personal care presents the highest average value and it is also the most expenditure-intensive activity (average of 10 euros/hour). Although people spend a relatively large amount of money in entertainment activities, these represent an expenditure rate of only 2.2 euros/hour, which is considerably lower than the average wage of 18 euros/hour. The values of Eq and  $\omega q$ 

were obtained as explained in Section 3.1. We set the minimum time allocations  $\binom{t_{qn}}{q_{qn}}$  and minimum consumption of goods  $\binom{x_{qk}^0}{q_{qk}}$  as equal to the minimum values observed in the sample for the corresponding categories (see the fifth and ninth columns of Table 1). The minimum work duration  $t_{qw}^0$  was set to be the observed minimum duration in the sample minus 1.

### 3.3 Estimation Results

A number of different empirical specifications were explored, with different sets of explanatory variables, different functional forms of variables, and different groupings. All the demographic variables available in the data were considered for characterizing the latent segments as well as the baseline preference specification. These variables include respondents' gender, age, presence of children in the household, income level, marital status, level of education, race, household size, household location (urban or rural area), and dwelling type (renter or owner). The final specification was based on the presence of adequate observations in each category of categorical explanatory variables, a systematic process of rejecting statistically insignificant effects, combining effects when they made sense and did not degrade fit substantially, and judgment and insights from earlier studies. To identify the appropriate number of latent segments (G), we estimated the model for increasing values of G until we reached a point where an additional segment did not

significantly improve model fit. The evaluation of model fit was based on the Bayesian Information Criterion (BIC):

$$BIC = -\frac{L(\boldsymbol{\theta})}{1} + 0.5 \cdot R \cdot \ln(N),$$

- / -

(18)

where  $L(\theta)$  is the log-likelihood value at convergence, *R* is the number of parameters estimated and *N* is the number of observations. The model with the number of segments corresponding to the lowest value of BIC is considered to provide the market segmentation that best fits the data. In our analysis, the three-segment model provided the least BIC value. The log-likelihood value at convergence for this model was -8,486.12 and, with 85 model parameters, the BIC was 8723. The BIC values for the model with one (*i.e.*, no latent segmentation), two, and four segments were 8810, 8793, and 8804, respectively. The log-likelihood value for the naïve model with no latent segmentation, and only alternative-specific constants in the baseline marginal utility and satiation parameters, was -15,757.84.

#### 3.3.1 Latent Segmentation Variables

The first row panel in Table 2 corresponds to the probabilistic assignment of individuals to each of the three latent segments (first segment is the base). The constants in this latent segmentation part of the model contribute to the size of each segment and do not have a substantive interpretation. The other parameter estimates in the top panel of Table 2 indicate that the second segment, relative to the other two segments, is likely to have proportions of individuals who are single and individuals aged 50 years or older between that of the first and third segments, and more likely to include individuals with children and be low-income. The third segment comprises individuals who tend to belong to the old age category (older than 50), who are unlikely to be single, and unlikely to have children. The first segment, on the other hand, is more likely than the other two segments to consist of younger individuals and those who live alone. Similar to the third segment, this segment also has a low proportion of individuals with children. A better way to characterize the different segments is to estimate the means of the demographic variables in each segment (see Bhat, 1997). The results are presented in Table 3, which shows the means of the demographic variables in each segment as well as the overall sample (and supports our observations from the model estimation results on the characteristics of the three market segments). Based on these results, we will refer to the first segment as the "young and singles" (YS) segment, the second as the "low income parents or single mothers" (LIPSM) segment, and the third as the "old couples without kids" (OCWOK) segment. The segment sizes are estimated and results show that LIPSM is the most prominent segment in the population (44.8%), followed by OCWOK (29.6%), and lastly YS (25.6%).

#### 3.3.2 Variables in the Utility Functions

The second panel of rows in Table 2 presents the parameter estimates corresponding to the baseline utility function specifications of the MDCEV model corresponding to each segment.<sup>4</sup> Within each segment, the baseline utility parameters corresponding to time and/or goods consumptions utility components are presented for each demographic

<sup>&</sup>lt;sup>4</sup> To conserve space, the alternative specific constants in the baseline utility functions and the satiation parameters are not presented in the table but they are available from the authors.

variable (depending on the utility functions the variable enters). The first demographic variable in the table, household size enters the utility functions of the time-allocation utility functions for two activities – assisting friends and family and administrative chores and family finances – and the expenditure (goods consumption) utility function corresponding to activities with children. As expected, people in the LIPSM segment are more likely to spend time assisting family/friends and doing administrative chores or family finances as their household size increases. A larger family implies a greater need to spend time on these activities, especially for families with children or single-mothers. Similarly, people from larger households are more likely to expend more money on activities with children.

Another variable that impacts the baseline utilities is the household's residential neighborhood type. Workers living in urban neighborhoods are likely to spend more time and money in entertainment, perhaps because of a greater proximity (than those living in rural neighborhoods) to activity centers such as restaurants, theaters, cinema, museums, or parks. Individuals who have completed graduate school are more likely to spend time and money in education (than those with lower levels of education), probably because they are more likely to continue their education or they spend for the education of other, non-workers in the household. Interestingly, well-educated individuals spend less time and less money on personal care, as can be observed from the negative coefficients on the graduate school variable in all three segments. Reasons behind this particular effect should be explored in detail in future research.

### 3.3.3 Values of Time

Average values of leisure time and work for each market segment identified from the latent class model are reported at the end of Table 3. Notably, the values of time for different market segments are quite different, highlighting the importance of the latent segmentation model. The OCWOK segment has the greatest value of work, followed by the YS segment, while the lowest value of work corresponds to the LIPSM segment. This is perhaps due to the following three reasons. First, workers who have children generally present a negative value of working time (Jara-Díaz et al., 2016), indicating that they do not derive pleasure from work at the margin; on the other hand, workers without children enjoy their work more. Also individuals who do not have to economically support children can choose a more satisfying job than workers who need to provide for their family. An alternative explanation is that parents prefer to spend time out of work to share it with their children (Sayer et al., 2004). Second, young workers (aged 50 years or less) have a smaller value of work, while old workers (aged more than 50 years) have a greater value. It is possible that young workers, compared to old workers, have more debt or commitments (college debt, mortgage) that, to some extent, force them to choose less satisfying jobs. Also, earlier studies have shown that older workers generally have more positive job attitudes (such as overall job satisfaction, satisfaction with work itself, satisfaction with pay, job involvement, or satisfaction with coworkers) than younger workers (see Mather and Johnson, 2000, and Ng and Feldman, 2010). Third, income is a relevant determinant of value of time. Our results show that low income workers (monthly income less or equal to 3,000 euros) have a lower valuation of time than high income workers.

#### 3.4 Comparison with Alternative Model Formulations

We compared our models results with those from three alternative model formulations. One of them is a simpler version of our model that does not allow corner solutions, called an "all essential alternatives model":

$$\max U(\mathbf{x}, t, t_{w}) = \sum_{k=1}^{K} \psi_{k} \ln(x_{k} - \vec{x}_{k}^{0}) + \sum_{n=1}^{N} \widetilde{\psi}_{n} \ln(t_{n} - \vec{t}_{n}^{0}) + \widetilde{\psi}_{w} \ln(t_{w} - t_{w}^{0})$$

$$\sum_{k=1}^{K} p_{k} x_{k} = E + \omega t_{w},$$

$$\sum_{n=1}^{N} t_{n} + t_{w} = T$$
(19)

In the above equation,  $\vec{x}_k^0$ ,  $\vec{t}_n^0$ , and  $\vec{t}_w^0$  correspond to exogenous minimum consumption for good *k*, exogenous minimum time allocation for activity *n*, and exogenous minimum duration for work respectively. These values are computed as the observed minimum in the sample minus one. Note that the minus one ensures that the utility function is defined at zero consumption values as well.

The second formulation is the MC-MDCEV model proposed by Castro *et al.* (2012) whose utility specification is only a function of time allocation (but not goods consumption) and does not allow for minimum time allocation, as below:

$$\max U(\mathbf{x}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left( \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right)$$
  
S.t. 
$$\sum_{k=1}^{K} p_k x_k = E$$
  

$$\sum_{k=1}^{K} g_k x_k = T$$
(20)

The third formulation is the Jara-Díaz *et al.* (2008) model which specifies utility as a Cobb-Douglas form which is function of both time allocation and goods consumption as well as allows for minimum time allocation but without allowing for corner solutions (zeros) in time allocation or consumptions, as below:

$$\max U(\mathbf{x}, \mathbf{t}, t_{w}) = \Omega t_{w}^{\theta_{w}} \prod_{n} t_{n}^{\theta_{n}} \prod_{k} x_{k}^{\varphi_{k}}$$

$$\sum_{k=1}^{K} p_{k} x_{k} = E + \omega t_{w},$$

$$\sum_{n=1}^{N} t_{n} + t_{w} = T,$$

$$x_{k} \ge x_{k}^{0},$$

$$t_{n} \ge t_{n}^{0}$$
(21)

For each individual in the sample, we computed the probability that he/she belongs to each of the three segments (see Equation 15) and we deterministically assigned the individual to one of the segments following those probabilities. Then we computed the values of time within each of the segments. The values of time implied from these alternative models are presented in the last three rows of Table 3 and those implied from our proposed model are presented in the last but fourth row of the table. It can be observed that all the three alternative models overestimate the values of time allocated to both work and leisure. The first alternative model and the Jara-Díaz et al. (2008) formulation do not allow corner solutions and do not allow minimum consumptions and minimum time allocations. In the Castro et al. formulation, a linear relation is assumed between time assigned to activities and the expense associated to those activities using money prices of time allocation to different activities. This not only creates a transformation between money and time that is not necessarily always true but also precludes the inclusion of goods consumed (or expenditures for consuming goods) in the utility functions. Also, the Castro et al. formulation does not consider minimum consumptions. Therefore, one can conclude that either ignoring corner solutions and minimum consumptions or ignoring goods consumption in time use models can lead to overestimation of the values of leisure and work times.

## Chapter 4. Conclusions

This paper develops a microeconomic theory-based MDC choice model that considers: (a) that both time allocations and goods consumption enter separately as decision variables in the utility function, (b) that both time and money constraints govern the activity participation and goods consumption decisions, (c) a finite probability of zero-consumptions and zero time allocations (*i.e.*, corner solutions), and (d) technical constraints in the form of minimum consumption levels for any good that is consumed and minimum time allocation for any activity conducted. The time allocation utility function includes time assigned to work also as a decision variable, along with time allocations to various non-work activities, to capture trade-offs between the generation of income for consuming goods and time allocation to non-work activities.

The proposed model is applied in the form of a latent class market segmentation model (to consider heterogeneity) on a Dutch dataset. The empirical model is used to understand the sociodemographic determinants of time allocation and goods consumptions behavior as well as to derive different values of time – value of work time and value of leisure (non-work) time. The latent class model helped identify three market segments – "young and singles", "low income parents or single mothers", and "old couples without kids" – based on differences in the time allocation and goods consumption preferences. The values of time implied by the model are notably different among these market segments. Comparison of the values of time implied by the proposed model with those from simpler models proposed earlier in the literature suggest that ignoring either corner solutions and minimum consumptions or ignoring goods consumption in time use models can potentially lead to overestimation of the values of leisure and work times.

A notable limitation of the model presented in this paper is that the technical constraints – minimum time allocation values and minimum consumption amounts – were treated as exogenous and not related to each other. Recognition of the relationships between goods consumption and time allocation in the form of technical constraints, for example a minimum required time allocation dependent on the amount of goods consumed, while considering corner solutions is an important avenue for future research (see Jara-Diaz *et al.*, 2016 who recognize such relationships albeit without considering corner solutions, and assuming *a priori* segmentation rather than allowing for endogenous segmentation).

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	Participation	Duration (hours/week)*				Expenditure (euros/week)*			
Activity	(%)	Mean	St. Dev.	Min.	Max.	Mean	St. Dev.	Min.	Max.
Work	100.0	33.4	13.7	1.0	100.0	-	-	-	-
Household chores	97.8	12.4	9.8	0.3	90.0	5.9	9.8	5.3	107.5
Personal care	100.0	9.1	5.8	0.5	49.0	96.9	66.5	7.2	1005.0
Education	24.7	7.4	9.3	0.2	87.7	1.4	7.4	8.0	125.0
Activities with children	31.2	14.3	11.7	0.5	65.0	17.6	29.1	9.5	166.3
Entertainment	99.8	31.9	16.1	1.0	102.0	38.7	63.1	7.8	725.0
Assisting friends and family	57.6	7.5	7.8	0.2	81.3	-	-	-	-
Administrative chores and family finances	86.6	3.1	3.5	0.2	50.0	-	-	-	-
Sleeping and relaxing	100.0	58.8	11.4	28.0	119.2	-	-	-	-
Other activities	42.5	11.7	12.5	0.3	71.0	-	-	-	-
Number of observations	1193								

 TABLE 1 Descriptive Statistics

(\*): Durations and expenditures are computed only for workers participating in the corresponding activity.

\_\_\_\_

Variable	First Segment (YS)		Second Segment (LIPSM)		Third Segment (OCWOC)	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Segment Probabilities						
Alternative specific constant	-	-	0.956	2.50	0.591	3.16
Gender: male	-	-	-0.175	-2.10	-	-
Age: 50 years or older	-	-	0.166	2.00	0.537	2.72
Single person household	-	-	-0.702	-3.00	-0.813	-3.20
Presence of children in the household	-	-	0.680	3.12	-	-
Income less than \$3,000 euros/month	-	-	1.204	2.25	0.322	4.51
Baseline utilities		-				-
Household size specific to						
Assisting friends and family time	-	-	0.328	2.56	-	-
Administrative chores & family finances time	-	-	0.290	2.49	-	-
Activities with children expenditure	-	-	0.210	4.78	-	-
Urban household specific to						
Entertainment expenditure	0.478	3.24	0.497	3.45	0.422	3.20
Entertainment time	0.326	2.07	0.590	3.00	0.371	3.49
Graduate school studies specific to						
Education expenditure	0.046	2.74	0.105	2.30	0.096	2.22
Education time	0.190	4.67	0.341	6.22	0.271	5.10
Personal care expenditure	-3.090	-3.40	-4.223	-5.12	-4.107	-2.60
Personal care time	-0.110	-4.75	-0.486	-9.11	0.214	3.61
Log-Likelihood at Convergence -8,486.12						

 TABLE 2
 Three Segments Model Estimation Results

\_\_\_\_\_

Segmentation Variable		First Segment (YS)	Second Segment (LIPSM)	Third Segment (OCWOC)	Overall Market
Gender	Male	51.1%	43.1%	50.4%	51.6%
Gender	Female	48.9%	56.9%	49.6%	48.4%
	Younger than 50	66.8%	58.6%	50.8%	59.9%
Age	50 years or older	32.2%	41.4%	49.2%	40.1%
	Single person	38.2%	26.1%	28.0%	27.6%
	Couple	29.9%	28.6%	37.2%	32.6%
Household structure	Single parent	3.8%	6.3%	3.3%	4.8%
	Nuclear family, multi family or non-family	28.1%	39.0%	31.5%	35.0%
Income	Less than 3,000 euros/month	55.8%	67.5%	57.4%	56.4%
	More than 3,000 euros/month	44.2%	32.5%	42.6%	43.6%
Segment s	Segment size		44.8%	29.6%	NA
Value of time from the proposed model (euros/hr)	Value of Leisure	4.6	2.1	5.0	4.4
	Value of Work	1.8	0.2	2.3	2.3
Value of time (euros/hr) using 'all essential alternatives' formulation	Value of Leisure	5.1	2.1	5.3	4.8
	Value of Work	2.9	0.5	2.8	2.6
Value of time (euros/hr) using Castro <i>et al.</i> (2012) formulation	Value of Leisure	6.3	3.0	7.9	6.1
	Value of Work	3.8	1.0	5.3	4.4
Value of time (euros/hr)	Value of Leisure	5.4	2.2	5.7	5.0
using Jara-Díaz <i>et al.</i> (2008) formulation	Value of Work	3.1	0.6	3.1	2.9

TABLE 3 Quantitative Characterization of the Three Segments

NA: Not applicable