CTR D-STOP

Technical Report 108

# Improved Traffic Operations through Real-Time Data Collection and Control

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Center for Transportation Research

May 2016

# Data-Supported Transportation Operations & Planning Center (D-STOP)

A Tier 1 USDOT University Transportation Center at The University of Texas at Austin

D-STOP is a collaborative initiative by researchers at the Center for Transportation Research and the Wireless Networking and Communications Group at The University of Texas at Austin.

**Technical Report Documentation Page**

| 1. Report No. | 2. Government Accession No. | 3. Recipient's Catalog No. |
|---|---|---|
| D-STOP/2016/108 | | |

| 4. Title and Subtitle | 5. Report Date |
|---|---|
| Improved Traffic Operations through Real-Time Data Collection and Control | May 2016 |
| | 6. Performing Organization Code |
| 7. Author(s)<br>Stephen Boyles and Michael Levin | 8. Performing Organization Report No.<br>Report 108 |
| 9. Performing Organization Name and Address<br>Data-Supported Transportation Operations & Planning Center (D-STOP)<br>The University of Texas at Austin<br>1616 Guadalupe Street, Suite 4.202<br>Austin, Texas 78701 | 10. Work Unit No. (TRAIS)<br><br>11. Contract or Grant No.<br>DTRT13-G-UTC58 |
| 12. Sponsoring Agency Name and Address<br>Data-Supported Transportation Operations & Planning Center (D-STOP)<br>The University of Texas at Austin<br>1616 Guadalupe Street, Suite 4.202<br>Austin, Texas 78701 | 13. Type of Report and Period Covered<br><br>14. Sponsoring Agency Code |

15. Supplementary Notes

Supported by a grant from the U.S. Department of Transportation, University Transportation Centers Program.

16. Abstract

Intersections are a major source of delay in urban networks, and reservation-based intersection control for autonomous vehicles has great potential to improve intersection throughput. However, despite the high flexibility in reservations, existing control policies are fairly limited. To increase reservation throughput, we adapt two pressure-based policies for reservations in dynamic traffic assignment. The backpressure policy is throughput optimal in communications networks, but communications networks are significantly different from traffic networks. We propose that congestion propagation can be introduced by modeling each cell in the cell transmission model as a link in a communications network. The finite-buffer limitation on the maximum pressure per cell can be overcome by including queue spillback to previous cells and links. However, a counterexample shows that local pressure-based policies such as backpressure cannot be throughput optimal under user equilibrium route choice. Therefore, we also adapt the $P0$ policy to reservations. Its adaptation is more straightforward, although dynamic traffic assignment also prevents proving that $P0$ is throughput optimal. Nevertheless, results on the downtown Austin network show that both backpressure and $P0$ performed significantly better than first-come-first-served, which has been used in most previous work on reservations. Therefore, although backpressure and $P0$ cannot be proven to be throughput optimal, they provide a better alternative to existing policies.

| 17. Key Words | 18. Distribution Statement |
|---|---|
| autonomous vehicles, reservation-based intersection control, dynamic traffic assignment, pressure-based policy | No restrictions. This document is available to the public through NTIS (http://www.ntis.gov):<br>National Technical Information Service<br>5285 Port Royal Road<br>Springfield, Virginia 22161 |

| 19. Security Classif.(of this report) | 20. Security Classif.(of this page) | 21. No. of Pages | 22. Price |
|---|---|---|---|
| Unclassified | Unclassified | 21 | |

**Form DOT F 1700.7 (8-72)** **Reproduction of completed page authorized**

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## Acknowledgements

The authors gratefully acknowledge the support of the Data-Supported Transportation Operations & Planning Center and the National Science Foundation, Grant No. 1254921. The authors are also grateful for the suggestions from Tarun Rambha.

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# Chapter 1. Introduction

Autonomous vehicles (AVs) admit new traffic behaviors with the potential to greatly improve traffic efficiency such as reservation-based intersection control *(1, 2)*. Vehicles request a *reservation* from the computerized intersection manager for a specific turning movement starting at a specific time. The intersection manager simulates vehicle requests on a grid of space-time tiles representing the intersection, and accepts some subset of requests that do not conflict in any of the space-time tiles. Reservations offer more possibilities in intersection control than traffic signals because individual vehicle movements are directly controlled.

We define a *policy* for reservations to be a function that determines which vehicle requests to accept. Despite the large number of possibilities for control policies, optimizing reservations is a little-studied question. Reservations have been mostly studied with the first-come-first-serve (FCFS) policy. Fajardo et al. *(3)* and Li et al. *(4)* found that FCFS reservations reduced delays beyond optimized signals. However, in some situations FCFS will perform worse than signals *(5)*. Reservations can mimic traffic signal phases *(6)*, so reservations can always perform as least as well as signals. Other studied control policies are prioritizing emergency vehicles *(8)* and intersection auctions *(8, 9, 32)* but those do not optimize for efficiency either.

Traffic signal timings are often coordinated with signals at nearby intersections to reduce delays for through traffic. Although a coordinated policy for reservations to maximize throughput would likely result in substantial improvements in intersection capacity and delay compared with traffic signals, finding such an optimal policy is quite difficult. In fact, an optimal *decentralized* throughput policy has not yet been developed. A decentralized policy can be implemented at the level of individual intersections, and is therefore less complex and more computationally efficient than system policies that act on multiple intersections. This paper studies decentralized *pressure-based* policies to improve throughput for individual intersections. A pressure-based policy is a policy that is responsive to congestion in the form of queue lengths or high intersection delay.

We study two pressure-based policies for reservations. The *backpressure* policy for packet routing in communications networks of Tassiulas & Ephremides *(11)* and extensions *(12, 13)* have been proven to be throughput optimal for the entire network. Although communications networks are similar to traffic networks, there are significant differences that make applying backpressure to reservations difficult. One major difference is user equilibrium (UE) route choice, in which vehicles choose routes to minimize their own travel time. Backpressure policies assume system-determined route choice in their proofs of optimality. To maximize capacity under UE route choice, Smith *(16, 23)* proposed the $P_0$ policy which responds to intersection delay. However, $P_0$ was not designed for reservations or mesoscopic simulation-based models. Therefore, the purpose of this paper is to investigate the pressure-based policies of backpressure and $P_0$ for reservations in dynamic traffic assignment (DTA).

The contributions of this paper are as follows: we show how the backpressure algorithm designed for telecommunications networks can be applied to traffic networks. However, we also show that UE route choice can prevent stabilizing the network. Therefore, we also adapt the $P_0$ policy because of UE route choice behavior. Then, we compare both backpressure and $P_0$ on city networks using DTA. Results indicate that both improve significantly over FCFS, although the backpressure also had lower delays than $P_0$.

The remainder of this paper is organized as follows: We review previous work on the backpressure and $P_0$ policies. Then, we define the traffic network model and queue dynamics, and present the DTA model of reservations. Afterwards, we adapt backpressure and $P_0$ to CTM, and study the stability. Finally, we present results on city networks and our conclusions.

# Chapter 2. Literature Review

This section first discusses the backpressure policy for communications networks. Then, we review the $P_0$ policy for maximizing intersection throughput with UE route choice.

## 2.1 Backpressure policy

The backpressure policy originates from studies of multihop communication networks. Such networks typically involve packets traveling from some origin node to some destination node with unspecified routing. The seminal paper of Tassiulas & Ephremides *(11)* is concerned with developing a policy that is stable for the largest possible region of demands. A *stable policy* is a policy in which customer queues at each node remain bounded. Using a queueing model, Tassiulas & Ephremides *(11)* proposed a maximum throughput policy based on queue pressure – the difference between upstream and downstream queues. They proved that choosing the combination of packets that maximized the relieved pressure at each intersection resulted in maximum stability. Route choice was determined by the system at each node based on downstream queue lengths.

As the work of Tassiulas & Ephremides *(11)* is focused on communication routing, the assumptions and modeling are not standard to traffic literature. First, they modeled links as point queues without a free flow travel time. This is because in electronic communications, the transmission speed is typically fast (possibly the speed of light) relative to node processing speeds. Therefore, their packets are modeled as traversing a link in one time step. This may be applied to traffic by reversing the nodes and links: vehicles take relatively little time to traverse an intersection compared with the typical link travel time, and intersection controls determine intersection access. However, in traffic networks, queues require physical space. Later extensions to finite-buffer queues *(12, 13)* required a minimum buffer size, which cannot be guaranteed for arbitrary roads. As demonstrated by Daganzo *(17)*, queue spillback with UE route choice can create significant congestion issues. Furthermore, traffic queues place first-in-first-out (FIFO) restrictions on vehicle movement, whereas in communication networks the order of service may be arbitrary. Finally, Tassiulas & Ephremides *(11)* adaptively determine route choice in response to queue lengths, whereas vehicles typically choose routes individually, resulting in UE behavior. Although tolling can encourage a system-optimal route choice, the route choices specified by backpressure could change every time step, and current tolling models have not considered changing route choice at such high frequencies.

Nevertheless, several papers have applied the backpressure policy to traffic intersections. Zhang et al. *(18)* proposed a pressure-based algorithm for intersection control that determined the probability of a driver choosing a specific turning movement based on the difference in the upstream and downstream link queue lengths. This is challenging to resolve with UE routing, but Zhang et al. *(18)* modeled adaptive route choice on a hyperpath, similar to some stochastic UE models. Gregoire et al. *(19)* applied the pressure idea more conventionally with respect to route choice by using the difference between upstream and downstream queue lengths to choose which signal phase to activate. Wongpiromsarn et al. *(20)* also included lack of route control in their adaptation of the pressure-based algorithm to signal control, and provided an analytical treatment similar to that of Tassiulas & Ephremides *(11)*. Under the assumption of infinite queue capacities, they were able to show that their pressure-based policy maximized throughput. However, practical limitations such as link length require careful choice of the pressure function to avoid queue spillback. Therefore, Xiao et al. *(21)* proposed a pressure-releasing policy that accounts for finite queue capacities. Nonetheless, to more canonically apply the pressure-based routing they assumed that each turning movement has a separate queue, which is often not realistic.

A major limitation on signal control is the clearance intervals necessary to separate phases for human drivers. Some demand scenarios could result in frequent phase switching as the pressure relieved by one phase makes another phase have relatively higher pressure, and it does not appear that previous work on using backpressure policies to activate signal phases included lost time penalties in their models. Frequent phase switching for signalized intersections would result in considerable time lost to clearance intervals. Therefore, we apply the backpressure policy to reservation-based control, which does not require clearance intervals and has much greater flexibility in vehicle movements.

## 2.2 $P_0$ traffic signal policy

In contrast to the communication network pressure-based approach, the $P_0$ signal control policy by Smith *(16)* is designed for traffic intersection control with UE route choice. Smith *(22)* demonstrated that Webster's signal policy could significantly reduce network capacity due to UE route choice, and Smith *(23)* further derived properties about signal policies that resulted in a consistent equilibrium. For instance, Webster's policy and a delay-minimizing policy induce route choice counter to the objectives of the signal policy. This motivated the $P_0$ policy of Smith *(16)*, which was also derived from traffic assignment principles later discussed by Smith & Ghali *(24)*. The problem $P_0$ addresses is how to allocate green time to each signal phase. $P_0$ uses the principle that low pressure phases receive no green time to avoid encouraging vehicles to switch to low capacity routes. As specified by Smith & Ghali *(24)*, the pressure on a phase is the product of saturation flow and link travel delay. This favors links with two properties:

1. Links with high saturation flow have a greater ability to service demand. Providing more green time to high saturation flow links will encourage drivers to choose links that can better handle the demand.
2. Links with a high delay (due to unsatisfied flow) have a longer queue of demand waiting to be serviced by the intersection.

Whereas $P_0$ is capacity maximizing, follow-up work by Smith & Van Vuren *(25)* studied policies that are gradient, monotone, and/or capacity maximizing with respect to the BPR cost function. Smith & Ghali *(24)* also provided a method of modeling $P_0$ signal timing as a static traffic assignment problem. Meneguzzer *(26)* provided a review of papers considering signal timing and UE together. Liu & Smith *(27)* extended this work to a day-to-day bottleneck model and demonstrate that if the delay formula is non-decreasing and the $P_0$ policy is used for the signal control, then flow swapping among pairs will achieve equilibrium. Overall, in contrast to the work on backpressure, the work on the $P0$ signal policy is much more inclusive of UE route 2 choice effects, and we therefore consider $P0$ for reservations.

# Chapter 3. Traffic Network

This section describes the mesoscopic simulation model used to study the pressure-based policies. Although the model has been developed in previous work, a review is beneficial for framing the traffic network as a communications network for backpressure policy, and to provide context for some parts of the backpressure and $P_0$ policy algorithms. We model link flows through CTM *(14, 15)*, which is a Godunov approximation *(28)* of the kinematic wave theory of traffic flow *(29, 30)*.

Consider a traffic network $\mathcal{G} = (\mathcal{N}, \mathcal{A}, \mathcal{V})$ with nodes $\mathcal{N}$, links $\mathcal{A}$, and time-specific demand $\mathcal{V}$. All demand enters and exits from a centroid; let $\mathcal{Z} \subseteq \mathcal{N}$ denote the set of centroids. We consider discrete flow, referred to as vehicles. Each vehicle $v \in \mathcal{V}$ has a specific origin $r \in \mathcal{Z}$ and destination $s \in \mathcal{Z}$ and chooses a path from $r$ to $s$ before departing. Links are divided into two types: centroid connectors and ordinary links. *Ordinary links* connect two intersections (nodes in $\mathcal{N}/\mathcal{Z}$ with flows defined by CTM. *Centroid connectors* connect an intersection to a zone.

Each link is divided into cells via CTM. Cells for link $a \in \mathcal{A}$ have length $u_a^f \Delta t$, where $u_a^f$ is the free flow speed of link $a$ and $\Delta t$ is the simulation time step. Therefore, vehicles can traverse at most one cell per time step. Let $\Gamma_i^-$ and $\Gamma_i^+$ be the incoming and outgoing cells for $i$, respectively. Each cell is a first-in-first-out (FIFO) queue of vehicles. Although the hydrodynamic theory defines flow for continuous space and time, CTM approximates the hydrodynamic theory by constraining flow between cells. As $\Delta t \to 0$, the solution to CTM approaches the solution to the hydrodynamic theory. CTM is commonly used for large-scale or practical applications when solving the hydrodynamic theory exactly is not tractable.

## 3.1 Cell flow dynamics

Our CTM formulation differs somewhat from that of Daganzo *(14, 15)* due to the need to track individual vehicles. Let $x_i(t)$ be the set of specific vehicles, which will be necessary for defining which vehicles move at each time step. Let $S_i(t) \subseteq x_i(t)$ be the sending flow – the set of vehicles in cell $i$ at time $t$ that would leave $i$ if there were no downstream constraints. Let $R_i(t) \in \mathbb{R}_+$ be the receiving flow of cell $i$ at time $t$ – the number of vehicles that would enter if connected to a source of infinite demand. Let $y_{ij}^v(t) \in \{0,1\}$ indicate whether vehicle $v \in x_i(t)$ moves from cell $i$ to cell $j$ at time $t$. If $y_{ij}^v(t) = 1$, $v$ moves from $i$ to $j$ at $t$. $v$ will not move from $i$ to $j$ unless $j \in p_v$, which is important for intersection dynamics. Flow between $i$ and $j$ is further constrained: $v$ cannot leave $i$ at $t$ unless $v \in S_i(t)$. Also, the total flow into $j$ cannot exceed $R_j(t)$. Formally,

$$\sum_{i \in \Gamma_j^-} \sum_{v \in S_i(t)} y_{ij}^v(t) \leq R_j(t)$$

for all cells $j$. Also,

$$|S_i(t)| \leq Q_i$$

where $Q_i$ is the capacity of cell $i$, and

$$R_j(t) = \min\left\{Q_j, \frac{w_j}{u_j^{\mathrm{f}}}\left(K_j - |x_j(t)|\right)\right\}$$

where $u_j^{\mathrm{f}}$ is the free flow speed, $w_j$ is the congested wave speed, and $K_j$ is the maximum occupancy of cell $j$.

Vehicle movement is also constrained by the FIFO behavior of cell queues. Vehicles cannot exit if blocked by a vehicle in front. Finally, flow between links may be constrained by intersection conflicts. Let $\mathbf{y}_{ij}(t)$ denote a vector of vehicle movements for vehicles in $S_i(t)$. Let $Y_n(\mathbf{x}(t))$ denote the set of feasible vehicle movements across node $n \in \mathcal{N}$ at $t$ when cell occupancies are given by the vector $\mathbf{x}(t)$. $Y_n(\mathbf{x}(t))$ is constrained by sending flow, receiving flow, path constraints, intersection conflicts, and FIFO behavior.

Each $\mathbf{y}_{ij}(t) \in Y_n(t)$ is an action that may be taken for moving flow. Let $\mathbf{S}(t)$ be a vector of sending flows and $\mathbf{Y}(\mathbf{x}(t))$ be a vector of feasible movements across all nodes at time $t$. A policy $\pi$ determines which vehicles are moved when the sending flow is $\mathbf{S}(t)$.

The state of this system evolves according to conservation of flow:

$$x_j(t+1) = x_j(t) \cup \mathcal{V}_j(t) \cup \left(\bigcup_{i \in \Gamma_j^-} \{v \in S_i(t) : y_{ij}^v(t) = 1\}\right) / \left(\bigcup_{k \in \Gamma_j^+} \{v \in S_j(t) : y_{jk}^v(t) = 1\}\right)$$

where $\mathcal{V}_j(t) \subseteq \mathcal{V}$ is the set of vehicles departing from cell $j$ at time $t$.

Flow between two cells on a link (as opposed to flow across an intersection) is clearly defined by the CTM *(14, 15)* in accordance with the kinematic wave theory. Recall that vehicles on each cell are stored in a FIFO queue. CTM defines the quantity of flow, and a corresponding number of vehicles from the FIFO queue are moved. Therefore, for cells $i$, $j$ on the same link, $|Y_{ij}(t)| = 1$. Flow between two cells across an intersection may have more possibilities due to the intersection conflicts.

## 3.2 Reservation-based intersection control

A major challenge in modeling and optimizing reservations is the high computational requirements of simulating the tile grid at each intersection. Previous microsimulation studies of multiple intersections were limited in size *(31)* or made major simplifications that reduced reservation efficiency *(10, 32)*. Zhu & Ukkusuri *(33)* proposed simplifying the tiles into conflict points between turning movements for DTA modeling. However, the number of conflict points scales with the square of the number of turning movements. Therefore, Levin & Boyles *(34)* proposed aggregating tiles into capacity-constrained *conflict regions* to reduce the computational burden, and demonstrated that it was tractable for DTA. Levin et al. *(35)* developed an integer program and polynomial-time heuristic for the conflict region model, which we use for our pressure-based policies.

Levin et al. *(35)* developed the following integer program for determining which reservations requests to accept for a single intersection $n \in \mathcal{N}$ and time step $t$. We use it in both the backpressure and $P_0$ algorithms. Let $\Gamma_n^-$ and $\Gamma_n^+$ be the sets of incoming and outgoing cells to $n$, and let $\gamma_{v^-,n}$ and $\gamma_{v,n}^+$ be the incoming and outgoing cells for vehicle $v$ at $n$. To simplify the notation, let $y_n^v(t) = y^v_{\gamma_{v,n}^-,\gamma_{v,n}^+}(t)$ denote whether $v$ moves through $n$ at $t$. Also, let $\mathcal{C}_n$ be the set of conflict regions for $n$, and let $\delta_v^c$ indicate whether $v$ uses $c \in \mathcal{C}_n$. $\mathbf{z}(t)$ is the objective function, which is a vector of weights for moving individual vehicles. $\mathbf{z}(t)$ will be determined by the pressure-based policies. The integer program is

$$
\begin{aligned}
\max\ & \mathbf{z}(t) \cdot \mathbf{y}(t) & \\
\text{s.t.}\ & \sum_{v \in S_n(t)} \frac{y_n^v(t)\delta_v^c}{Q_{\gamma_{v,n}^-}} \leq 1 & \forall c \in \mathcal{C}_n \\
& y_n^v(t) \leq 1 + \frac{\tilde{Q}_{\gamma_{v,n}^-}(v) - 1}{M} & \forall v \in S_n(t) \\
& \sum_{v \in S_n(t): \gamma_{v,n}^+ = j} y_n^v(t) \leq R_j(t) & \forall j \in \Gamma_n^+ \\
& y_n^v(t) \in \{0,1\} & \forall v \in S_n(t)
\end{aligned}
\quad (2)
$$

where

$$
\tilde{Q}_{\gamma_{v,n}^-}(v) = \left( Q_{\gamma_{v,n}^-} - \sum_{v' \in \tilde{S}_{v,n}(t)} y_n^{v'}(t) \right) \left( \frac{\ell_{\gamma_{v,n}^-} - \left( \left| \tilde{S}_{v,n}(t) \right| - \sum_{v' \in \tilde{S}_{v,n}(t)} y_n^{v'}(t) \right)}{\ell_{\gamma_{v,n}^-}} \right)
$$

for any $v \in S_n(t)$. $\tilde{S}_{v,n}(t)$ is set of vehicles ahead of $v$ (based on FIFO order) on $\gamma_{v^-,n}$at time $t$ $\ell_i$ is the number of lanes on cell $i$, and $M$ is a large positive constant.

Because this integer program (1) is NP-hard, we use the greedy heuristic proposed by Levin et al. *(35)*. Each vehicle is given an *efficiency* $e_n^v(t)$ at $n$ defined as follows:

$$
e_n^v(t) = \frac{z_n^v(t)}{\frac{1}{R_{\gamma_{v,n}^+}(t)} + \sum_{c \in \mathcal{C}_n} \frac{1}{Q_{\gamma_{v,n}^-}}} \quad (2)
$$

At each time step, the algorithm creates a list of vehicles able to enter the intersection $\mathcal{W}$, consisting only of vehicles at the front of their lane. The algorithm iterates through $\mathcal{W}$ in order of greatest efficiency until it finds a vehicle $v$ that can feasibly move. $v$'s reservation is granted, resulting in $y_n^v(t) = 1$, and the vehicle behind $v$ is added to $\mathcal{W}$. The algorithm terminates when $\mathcal{W}$ is empty or no vehicles in $\mathcal{W}$ can move.

The purpose of the greedy heuristic is to efficiently find a solution to the integer program (1). This integer program is used in the solution of both the backpressure and $P_0$ policies, and therefore must be solved every time step. Levin et al. *(35)* showed that the greedy heuristic can find effective solutions and is tractable for solving DTA on city networks.

# Chapter 4. Backpressure Policy for Reservations

In this section, we adapt the backpressure policy *(11)* for the traffic network. Due to bounded queues and FIFO behavior, we cannot prove that this is a maximum throughput behavior. In fact, we demonstrate that UE route choice behavior can result in unbounded queues for stable demand. Nevertheless, results on a city network show significant improvement over the FCFS policy.

## 4.1 Traffic network as constrained queueing system

A major difference between communications networks and traffic networks is that in traffic networks, congestion creates regions of high-density, slower-moving traffic. Communications networks are essentially point queues, and the size of the queue does not affect link travel times. After a review of the communications network of Tassiulas & Ephremides *(11)*, we show that our CTM traffic network is similar to the constrained queueing systems that they studied. Each cell is a point queue, and shockwaves in traffic flow are modeled through cell transition flows. This model results in many queues – including multiple queues per link. Still, flows between cells within a link are simple to handle because the feasible region is determined exactly by cell transition flows. Of course, this relies on the CTM approximation to the kinematic wave theory; the kinematic wave theory itself is continuous and can be solved in continuous space *(36)*. Nevertheless, CTM is commonly used in large-scale DTA models, so using CTM to adapt the backpressure policy is reasonable.

Although this cell model is equivalent to a communications network, there are several issues that prevent proving that backpressure maximizes throughput. First, queue sizes are bounded due to network geometry, and previous work on communications networks has required large queue sizes to ensure stability *(12, 13)*. While arbitrary queue sizes are possible in computer storage, road lengths are not so arbitrary. Second, communications networks do not have FIFO behavior. Due to different destinations, FIFO behavior at intersections limits the feasible region of the control policy. For instance, a left-turning vehicle could block a right-turning vehicle behind it, even though the right-turning vehicle could otherwise move through the intersection. Finally, communications network policies assume route choice is controlled by the system. However, in traffic networks, vehicles typically choose routes individually, and UE route choice can reduce efficiency. Levin et al. *(5)* created an example showing that route choice can result in arbitrarily long queues and prevent stability with any local pressure-based policy, which we review here.

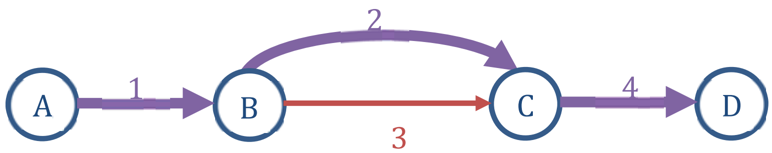


**FIGURE 1** Network for unbounded queueing due to UE route choice.

Figure 1 shows the network for the counterexample. Links 1, 2, and 4 have capacity of 2400vph, whereas link 3 has capacity of 1200vph. Demand from A to D is 1800vph. Clearly, if all vehicles take path [1,2,4], then queue lengths will be 0 for all links. However, suppose that link 2 is slightly longer than link 1, so the free flow travel time of link 2 is 10s longer. If C is controlled by a traffic signal with fixed phase lengths, the signal can be timed so that the expected travel time on 3 is higher than the expected travel time on 2 due to signal delay. Then, all demand on path [1,2,4] is the UE.

Suppose instead that C is controlled by a pressure-based policy. The controller allocates vehicle movement or green times in response to pressure (queue lengths) on links 2 and 3. Then, all demand taking path [1,3,4] is the UE. For vehicles reaching B, neither links 2 or 3 are uncongested due to the pressure-based policy at C, so all vehicles at B prefer link 3 because of its lower free flow travel time. However, link 3 has lower capacity than demand, so the queue on link 1 grows arbitrarily long.

Based on the above counterexample, it is not possible to prove that any *decentralized* pressure-based policy, including backpressure, is throughput optimal for a network under UE route choice. Any local pressure-based policy applied at C will allow movement by all vehicles on links 2 and 3, since they cannot become congested in this example. It is true that previous work on applying backpressure *(18-21)* were able to prove that backpressure was stable, if demand allowed it. However, they assumed that turning proportions remained fixed, which is not true under UE behavior *(22)*. This counterexample uses UE route choice to create a situation in which the network can be stabilized, but will not be stabilized under a pressure-based policy.

## 4.2 Maximum throughput heuristic

We adapt the backpressure policy of Tassiulas & Ephremides *(11)* to the CTM network. We cannot prove that backpressure maximizes throughput, but the insights of backpressure control are used for this heuristic. Backpressure is an algorithm executed each time step that determines intersection vehicle movements. As with the algorithm of Tassiulas & Ephremides *(11)*, backpressure consists of three stages. Stage 1 selects the weights on each vehicle based on cell queues. Stage 2 decides the combination of vehicles to move given the vehicle weights. Note that the decision of which vehicles to move can be separated by intersection: a system-wide controller is not necessary. However, computing the vehicle weights in Stage 1 requires communication of queue lengths between neighboring intersections.

The key insight is in the calculation of the pressure terms $D_n^v(t)$ for each vehicle $v$ at node $n$ at time $t$. For communications networks, this is simply the queue size because queues are unbounded. A key requirement of Tassiulas & Ephremides's proof *(11)* is that $D_n^v(t)$ can become arbitrarily large as the queue grows. However, cell queues have are bounded, so setting $D_n^v(t) = \left|x_{\gamma_{v,n}^-}(t)\right|$ does not provide sufficient pressure. Instead, we define a *congestion region* of connected congested cells, and sum the occupancies of all cells in the congestion region.

### 4.2.1 Stage 1

This stage determines the vehicle weights $D_n{}^v(t)$ for each vehicle $v$. Since the queue at cell $j$ could be bounded, to achieve unbounded pressures we must consider cells behind $j$. Even link queue lengths might be too small to provide sufficient pressure *(12, 13)*. Define $C_j$ to be the set of congested cells leading up to $j$. $C_j$ is defined recursively as

$$C_j = \{j\} \cup \left\{i \in \Gamma_{j'}^{-} : j' \in C_j \text{ and } \left|x_j(t)\right| > Q_j\right\}$$

This can be explained intuitively as follows: $C_j$ is the set of congested cells containing queued vehicles that might use cell $j$. We define cell $j$ to be *congested* if $n_j(t) > Q_j$, which means that not all vehicles in $j$ can exit in a single time step. The queue at $j$ is always considered, so $j \in C_j$. If $j$ is not congested, $C_j = \{j\}$. If $j$ is congested, then $C_j$ is the set of contiguous congested cells leading up to and including $j$. If the network is sufficiently congested, then $C_j$ will include one or more centroid cells, which have unbounded queues. The pressure from the queues from the centroid cell(s) will result in arbitrarily large pressure, which is one of the key features of the backpressure policy.

Let $p_i^j(t)$ be the proportion of vehicles in cell $i$ that have cell $j$ in their path. Clearly, $p_j^j(t) = 1$, and for any cell $i$ preceding $j$ on the same link, $p_i^j(t) = 1$ also. When $i$ is on a different link than $j$, $p_i^j(t) < 1$ is possible.

Define the queue length for cell $j$ at time $t$ to be

$$L_j(t) = \sum_{i \in C_j} |x_i(t)| p_i^j(t)$$

$L_j$ is the number of vehicles in the congested region $C_j$ waiting to use cell $j$. Now define $D_n^v(t)$ as follows:

$$D_n^v(t) = \left(L_{\gamma_{v,n}^{+}}(t) - L_{\gamma_{v,n}^{-}}(t)\right) \min\left\{Q_{\gamma_{v,n}^{-}}, Q_{\gamma_{v,n}^{+}}\right\}$$

$D_n^v(t)$ is the product of the difference in queue lengths for cells $\gamma_{v,n}^{-}$ and $\gamma_{v,n}^{+}$ and the maximum flow rate between $\gamma_{v,n}^{-}$ and $\gamma_{v,n}^{+}$. This product is taken directly from Tassiulas & Ephremides *(11)*. Note that when $\gamma_{v,n}^{+}$ is a sink cell, $Q_{\gamma_{v,n}^{+}} = \infty$ and $L_{\gamma_{v,n}^{+}}(t) = 0$ by definition. The difference is used because moving vehicles onto a congested cell (if possible) is intuitively less efficient than moving vehicles onto uncongested cells. $D_n^v(t)$ does not depend on properties of $v$ besides the path of $v$. The vehicle index is retained for vector notation; let $\mathbf{D}(t)$ be the vector of vehicle specific weights.

### 4.2.2 Stage 2

Find a vehicle movement vector $\mathbf{y}^*(t)$ satisfying the following:

$$\mathbf{y}^*(t) \in \underset{\mathbf{y}(t) \in \mathbf{Y}(t)}{\arg\max} \{\mathbf{D}(t) \cdot \mathbf{y}(t)\}$$

Note that this can be solved for individual intersections because the choice of flows at a single intersection does not affect the feasible flows for other intersections at the same time step.

### 4.2.3 Stage 3

If $y_n^{*v}(t) = 1$, then vehicle $v$ is moved from $\gamma_{v,n}^-$ to $\gamma_{v,n}^+$ at $t$. Otherwise, $v$ remains in $\gamma_{v^-,n}$. This flow is feasible because $\mathbf{y}^*(t) \in \mathbf{Y}(t)$.

### 4.2.4 Remarks

Note that Stages 1 and 2 only need to be computed for incoming and outgoing cells at nodes. For flow between two cells on the same link, there is only one feasible solution as defined by the CTM transition flows *(14, 15)*.

Stage 2 requires the solution of an integer program, which is NP-hard. For reservation-based intersection control, vehicles may be allowed to move individually, which could result in a large feasible region. $|Y_n(t)|$ is $O(2^{|S_n(t)|})$. For tractability, we use the polynomial-time greedy heuristic of Levin et al. *(35)* to find a decent solution. In calculating the efficiency, we set $\mathbf{z}(t) = \mathbf{D}(t)$ in equation (2).

## 4.3 A note on practical implementation

One potential concern is how to implement the backpressure policy in practice. CTM is itself an approximation to the hydrodynamic theory, and defining the policy in terms of cell queues may not seem completely realistic. However, as $\Delta t \to 0$, the predictions of CTM approach those of the hydrodynamic theory. Therefore, the calculation of the intersection queue length from the queues in contiguous congested cells becomes the length of the queues on intersection approaches. The size of these queues may be determined through loop detectors.

A second issue with implementation is calculating the total length of queues across queue spillback. In the backpressure, we assumed that we know vehicle routes, and whether they will use any given cell. In practice, vehicle routes may not be known, even for autonomous vehicles. Queues specific to a link could be estimated by turning fractions when queue spillback is present. However, these turning fractions may change over time due to UE route choice.

Our traffic network model also assumes that centroid queues will grow arbitrarily large if demand is sufficiently high. Realistically, travelers will probably choose to depart later if queues are backed up to their origin. However, when demand is modeled as elastic, boundedness of queue length is not an effective measure of stability.

# Chapter 5. $P_0$ Policy for Reservations

The backpressure policy is from a model where routing is determined by the system *(11)* and the counterexample to stability shows that UE route choice could prevent stability. In the worst case, policies relying on local information could result in unbounded queues despite a stabilizable demand. Therefore, we also adapt the $P_0$ policy *(16, 23)* to reservations for comparison. $P_0$ was designed to maximize network capacity under UE route choice. However, proving that $P_0$ maximizes capacity in the simulation-based CTM is difficult because link travel times are not continuous with respect to inflow or demand. $P_0$ also uses a congestion-increased pressure term, but the pressure is based on link travel times rather than queue lengths.

$P_0$ was designed for a model using link performance functions for delay. Specifically, $P_0$ assumes that the travel time $\tau_a$ for link $a \in \mathcal{A}$ is of the form

$$\tau_a = \tau_a^{\mathrm{f}} + f_a\big(\omega_a + \mu_a \hat{Q}_a\big)$$

where $\tau_a^{\mathrm{f}}$ is the free flow travel time, $f_a(\cdot)$ is the delay function, $\omega_a$ is the demand for the link, $\hat{Q}_a$ is saturation flow, and $\mu_a$ is the proportion of red time. For phase $k$ at node $n \in \mathcal{N}$, let $\mathcal{A}_n^k \subseteq \mathcal{A}$ be the set of links given green time. For a link travel time of this form, the resulting 5 pressure $\rho_n^k$ for phase $k$ is then

$$\rho_n^k = \sum_{i \in \mathcal{A}nk} \hat{Q}_a f_a(\omega_a + \mu_a \hat{Q}_a)$$

Applying this to DTA requires evaluating the function $f_a(\cdot)$, which is determined through simulation in DTA. However, previous travel times are observable. Let $\bar{\tau}_a(t)$ be the expected travel time for link $a$ at time $t$, based on estimates from vehicles that traversed $a$. Then we create an estimate of $f_a(\cdot)$ at $t$, $\bar{f}_a(t)$, by taking

$$\bar{f}_a(t) = \bar{\tau}_a(t) - \tau_a^{\mathrm{f}}$$

We also replace saturation flow $\hat{Q}_a$ with capacity $Q_a$. In practice, these may not be equivalent since many static models assume that link flows can exceed the saturation flow at the cost of high delay. However, capacity is the flow constraint parameter for DTA.

We also adapt this to reservation-based intersection control, meaning that pressure is specified for specific vehicles rather than phases. Since the pressure is based on the link travel time, let $a_{v^-,n} \in \mathcal{A}$ be the incoming link for vehicle $v$ at node $n$. (This differs from the incoming *cell* because the pressure for $P_0$ is based on the link travel time, not the cell travel time). This results in the following pressure $P_n^v(t)$ for vehicle $v$ at node $n$ at time $t$ using the $P_0$ policy:

$$P_n^v(t) = Q_{a_{\bar{v},n}}\big(\bar{\tau}_{a_{\bar{v},n}}(t) - \tau_{a_{\bar{v},n}}^{\mathrm{f}}\big)$$

$P_n^v(t)$ favors links with high capacity and/or with a high delay (travel time beyond the free flow time). Delay should greatly increase as the queue length increases.

Define the vector of pressures to be $\mathbf{P}(t)$ for all waiting vehicles. The objective is then to find

$$\mathbf{y}^*(t) \in \arg\max_{\mathbf{y}(t) \in \mathbf{Y}(t)} \{\mathbf{P}(t) \cdot \mathbf{y}(t)\}$$

As with the backpressure policy, this can be determined locally for individual intersections. We also approximately solve this integer program (1) using the greedy heuristic of Levin et al. *(35)*. To calculate the efficiencies, we set $\mathbf{z}(t) = \mathbf{P}(t)$ in equation (2).

# Chapter 6. Experimental Results

We compared four types of intersection controls – traffic signals and reservations with FCFS, backpressure, and $P_0$ – on the downtown Austin network, shown in Figure 2. The network has 171 zones, 546 intersections, and 1247 links. Data was from the Capital Area Metropolitan Planning Organization. The dynamic network loading used the cell transmission model with a 6s time step, and the conflict region model for reservation-based intersection control *(34, 35)*. Traffic signals were modeled by simulating phases and changing the capacity of turning movements proportional to green time at each time step. Flow was discretized and individual vehicles were tracked. We used the method of successive averages *(37)* to solve DTA to a 1% gap for all scenarios. To demonstrate robustness, we considered demand levels from 70% to 100% at 10% increments.

Table 1 compares the travel times for all four intersection control policies at different demand levels. Reservations using all policies (including FCFS) consistently had much lower total system travel time (TSTT) than traffic signals. Although Levin et al. *(5)* found several situations in which FCFS reservations would increase delay compared with signals, there are also scenarios (such as symmetric intersections) in which FCFS is likely to reduce delay *(3, 4)*. Both backpressure and $P_0$ made significant improvements over FCFS as well. This is not surprising because FCFS does not prioritize links with higher demand, which could cause queues to build up and spillback on such links. Backpressure also consistently performed slightly better than $P_0$. This is probably because backpressure is more responsive to current traffic conditions than $P_0$. $P_0$ was developed for a model with link performance functions, in which travel times could be easily calculated. However, in simulation-based DTA, travel times are determined by simulation. Therefore, high travel times were only observed after vehicles had exited the link, which delayed the effect of queuing on the $P_0$ prioritization. In contrast, backpressure prioritized based on queue lengths at the current time. Therefore, backpressure responded faster and more dynamically to congestion and queueing.

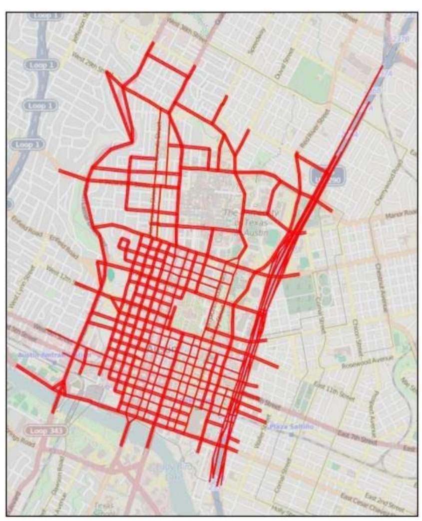

**FIGURE 2** Downtown Austin network

**TABLE 1** Intersection control results on downtown Austin network

| Demand | Intersection policy | TSTT (hr) | Avg. TT per vehicle (min) |
|---|---|---|---|
| 43965 (70%) | Traffic signals | 8552.2 | 11.67 |
| | FCFS | 4276.6 | 5.84 |
| | Backpressure | 3974.0 | 5.42 |
| | $P_0$ | 4003.1 | 5.46 |
| 50290 (80%) | Traffic signals | 10771.5 | 12.9 |
| | FCFS | 5550.4 | 6.62 |
| | Backpressure | 4819.7 | 5.74 |
| | $P_0$ | 4897.6 | 5.84 |
| 56592 (90%) | Traffic signals | 13776.0 | 14.61 |
| | FCFS | 7116.0 | 7.55 |
| | Backpressure | 6016.1 | 6.38 |
| | $P_0$ | 6285.6 | 6.66 |
| 62847 (100%) | Traffic signals | 16971.6 | 16.20 |
| | FCFS | 9334.2 | 8.91 |
| | Backpressure | 7815.5 | 7.46 |
| | $P_0$ | 8397.1 | 8.01 |

Results for signals and FCFS differ slightly from previously reported numbers for the same network because the discrete vehicle trips were recreated from a dynamic trip table, resulting in some stochasticity in the demand.

# Chapter 7. Conclusions

This paper adapted two pressure-based policies for reservation-based intersection control in dynamic traffic assignment. The backpressure policy is based on the work of Tassiulas & Ephremides *(11)* in communications networks. There are several significant differences between communications networks and traffic networks, including congestion propagation, finite buffers (where the buffer size is determined by the physical length of the road), and user equilibrium route choice. We found that a cell transmission model of the traffic network is similar to a communications network, modeling each cell as a link. To allow pressure to grow arbitrarily, we summed the cell occupancies with a congested region of cells leading up to the intersection. However, we also found that user equilibrium route choice could prevent *any* local pressure-based policy from stabilizing the network. (Previous work on pressure-based signal timings assumed fixed turning proportions in their proofs of stability.) Therefore, the backpressure policy cannot be proven to stabilize a traffic network, and was used as a heuristic.

The $P_0$ policy was developed by Smith *(16, 23)* for the user equilibrium route choice issue, and we therefore studied $P_0$ as well. However, $P_0$ was designed for signal timing with static traffic models with link performance functions, so the same counterexample to backpressure applies to $P_0$ for reservations. Nevertheless, results on the downtown Austin network showed that backpressure and $P_0$ performed significantly better than the first-come-first-served policy, which has been used in most previous work on reservations *(e.g. 1-4)*. Therefore, although backpressure and $P_0$ are not throughput-optimal, they provide a better alternative to existing policies.

As vehicle automation becomes increasingly available to consumers, optimizing autonomous vehicle technologies becomes more important. Future work on reservation-based intersection control might investigate whether non-local policies can be proven to be throughput optimal under user equilibrium route choice. In addition, it is possible that backpressure might be throughput optimal under system optimal route choice, although the finite buffer is still an issue to overcome. It is clear from this paper and others *(5)* that reservation control policies require further study before they are ready to replace traffic signals.

## References


*(1)* Dresner, Kurt, and Peter Stone. "Multiagent traffic management: A reservation-based intersection control mechanism." *Proceedings of the Third International Joint Conference on Autonomous Agents and Multiagent Systems-Volume 2*. IEEE Computer Society, 2004.

*(2)* Dresner, Kurt, and Peter Stone. "Traffic intersections of the future." *Proceedings of the National Conference on Artificial Intelligence*. Vol. 21. No. 2. Menlo Park, CA; Cambridge, MA; London; AAAI Press; MIT Press; 1999, 2006.

*(3)* Fajardo, David, Tsz-Chiu Au, S. Waller, Peter Stone, and David Yang. "Automated intersection control: Performance of future innovation versus current traffic signal control." *Transportation Research Record: Journal of the Transportation Research Board* 2259 (2011): 223-232.

*(4)* Li, Zhixia, Madhav Chitturi, Dongxi Zheng, Andrea Bill, and David Noyce. "Modeling reservation-based autonomous intersection control in VISSIM." *Transportation Research Record: Journal of the Transportation Research Board* 2381 (2013): 81-90.

*(5)* Levin, Michael W., Stephen D. Boyles, and Rahul Patel. "Paradoxes of reservation-based intersection controls in traffic networks." *Transportation Research Part A: Policy and Practice* 90 (2016): 14-25.

*(6)* Dresner, Kurt M., and Peter Stone. "Sharing the Road: Autonomous Vehicles Meet Human Drivers." *IJCAI*. Vol. 7. 2007.

*(7)* Dresner, Kurt, and Peter Stone. "Human-usable and emergency vehicle-aware control policies for autonomous intersection management." *Fourth International Workshop on Agents in Traffic and Transportation (ATT), Hakodate, Japan*. 2006.

*(8)* Schepperle, Heiko, and Klemens Böhm. "Auction-based traffic management: Towards effective concurrent utilization of road intersections." *2008 10th IEEE Conference on ECommerce Technology and the Fifth IEEE Conference on Enterprise Computing, E-Commerce and E-Services*. IEEE, 2008.

*(9)* Vasirani, Matteo, and Sascha Ossowski. "A market-inspired approach for intersection management in urban road traffic networks." *Journal of Artificial Intelligence Research* 43 (2012): 621-659.

*(10)* Carlino, Dustin, Mike Depinet, Piyush Khandelwal, and Peter Stone. "Approximately orchestrated routing and transportation analyzer: Large-scale traffic simulation for autonomous vehicles." *2012 15th International IEEE Conference on Intelligent Transportation Systems*. IEEE, 2012.

*(11)* Tassiulas, Leandros, and Anthony Ephremides. "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks." *IEEE transactions on automatic control* 37.12 (1992): 1936-1948.

*(12)* Giaccone, Paolo, Emilio Leonardi, and Devavrat Shah. “Throughput region of finitebuffered networks.” *IEEE Transactions on Parallel and Distributed Systems* 18.2 (2007): 251263.

*(13)* Le, Long Bao, Eytan Modiano, and Ness B. Shroff. “Optimal control of wireless networks with finite buffers.” *IEEE/ACM Transactions on Networking* 20.4 (2012): 1316-1329.

*(14)* Daganzo, Carlos F. “The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory.” *Transportation Research Part B: Methodological* 28.4 (1994): 269-287.

*(15)* Daganzo, Carlos F. “The cell transmission model, part II: network traffic.” *Transportation Research Part B: Methodological* 29.2 (1995): 79-93.

*(16)* Smith, M. J. “A local traffic control policy which automatically maximises the overall travel capacity of an urban road network.” *Traffic Engineering & Control* 21.HS-030 129 (1980).

*(17)* Daganzo, Carlos F. “Queue spillovers in transportation networks with a route choice.” *Transportation Science* 32.1 (1998): 3-11.

*(18)* Zhang, Rui, Zhijun Li, Cheng Feng, and Shouxu Jiang. “Traffic routing guidance algorithm based on backpressure with a trade-off between user satisfaction and traffic load.” *Vehicular Technology Conference (VTC Fall), 2012 IEEE*. IEEE, 2012.

*(19)* Gregoire, Jean, Emilio Frazzoli, Arnaud de La Fortelle, and Tichakorn Wongpiromsarn. “Back-pressure traffic signal control with unknown routing rates.” *IFAC Proceedings Volumes* 47.3 (2014): 11332-11337.

*(20)* Wongpiromsarn, Tichakorn, Tawit Uthaicharoenpong, Emilio Frazzoli, Yu Wang, and Danwei Wang. “Throughput optimal distributed traffic signal control.” *arXiv preprint arXiv:1407.1164* (2014).

*(21)* Xiao, Nan, Emilio Frazzoli, Yitong Li, Yu Wang, and Danwei Wang. “Pressure releasing policy in traffic signal control with finite queue capacities.” *53rd IEEE Conference on Decision and Control*. IEEE, 2014.

*(22)* Smith, M. J. “Traffic control and route-choice; a simple example.” *Transportation Research Part B: Methodological* 13.4 (1979): 289-294.

*(23)* Smith, M. J. “Properties of a traffic control policy which ensure the existence of a traffic equilibrium consistent with the policy.” *Transportation Research Part B: Methodological* 15.6 (1981): 453-462.

*(24)* Smith, M. J., and M. Ghali. “The dynamics of traffic assignment and traffic control: A theoretical study.” *Transportation Research Part B: Methodological* 24.6 (1990): 409-422.

*(25)* Smith, M. J., and T. Van Vuren. "Traffic equilibrium with responsive traffic control." *Transportation Science* 27.2 (1993): 118-132.

*(26)* Meneguzzer, Claudio. "Review of models combining traffic assignment and signal control." *Journal of Transportation Engineering* 123.2 (1997): 148-155.

*(27)* Liu, Ronghui, and Mike Smith. "Route choice and traffic signal control: a study of the stability and instability of a new dynamical model of route choice and traffic signal control." *Transportation Research Part B: Methodological* 77 (2015): 123-145.

*(28)* Godunov, Sergei Konstantinovich. "A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics." *Matematicheskii Sbornik* 89.3 (1959): 271-306.

*(29)* Lighthill, Michael J., and Gerald Beresford Whitham. "On kinematic waves. II. A theory of traffic flow on long crowded roads." *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*. Vol. 229. No. 1178. The Royal Society, 1955.

*(30)* Richards, Paul I. "Shock waves on the highway." *Operations research* 4.1 (1956): 42-51.

*(31)* Hausknecht, Matthew, Tsz-Chiu Au, and Peter Stone. "Autonomous intersection management: Multi-intersection optimization." *2011 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2011.

*(32)* Carlino, Dustin, Stephen D. Boyles, and Peter Stone. "Auction-based autonomous intersection management." *16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013)*. IEEE, 2013.

*(33)* Zhu, Feng, and Satish V. Ukkusuri. "A linear programming formulation for autonomous intersection control within a dynamic traffic assignment and connected vehicle environment." *Transportation Research Part C: Emerging Technologies* 55 (2015): 363-378.

*(34)* Levin, Michael W., and Stephen D. Boyles. "Intersection auctions and reservation-based control in dynamic traffic assignment." *Transportation Research Record: Journal of the Transportation Research Board* 2497 (2015): 35-44.

*(35)* Levin, Michael W., Hagen Fritz, and Stephen D. Boyles. "On optimizing reservation-based intersection controls." Accepted for publication in *IEEE: Transactions on Intelligent Transportation Systems*.

*(36)* Yperman, Isaak, Steven Logghe, Chris MJ Tampere, and Ben Immers. "Multicommodity link transmission model for dynamic network loading." *Transportation Research Board 85th Annual Meeting*. No. 06-1062. 2006.

*(37)* Levin, Michael W., Matt Pool, Travis Owens, Natalia Ruiz Juri, and S. Travis Waller. "Improving the convergence of simulation-based dynamic traffic assignment methodologies." *Networks and Spatial Economics* 15.3 (2015): 655-676.