FATIGUE ANALYSIS OF THE CAST ALUMINUM BASE

By

Hayes E. Ross, Jr.
Assistant Research Engineer

Thomas C. Edwards
Assistant Research Engineer

Gerald R. Babb
Student Assistant

Research Report Number 75-11

Supplementary Studies in Highway Illumination Research

Research Study Number 2-8-64-75

Sponsored by
The Texas Highway Department

In cooperation with the
U. S. Department of Transportation, Federal Highway Administration
Bureau of Public Roads

August, 1968

TEXAS TRANSPORTATION INSTITUTE
Texas A&M University
College Station, Texas

FOREWORD

The object of this research was not to conduct an in-depth study on the general fatigue analysis of the cast aluminum transformer base. Rather, it was very limited in nature in that only an insight into the fatigue of the base due to wind-induced vibrations was desired so as to determine if trouble areas exist. The particular type of wind-induced vibration considered was that due to the vortex shedding along the vertical support pole. Two typical pole, luminaire, and base configurations were analyzed.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

FATIGUE ANALYSIS OF THE CAST ALUMINUM BASE

I. Introduction

Cast aluminum transformer bases and cast aluminum "shear" bases have helped reduce the hazard that roadway lighting presents to the motorist. This safety feature results from inherent breakaway characteristics of these two bases. The shear base is used primarily to modify existing lighting installations which have no breakaway features, e.g., it has been used in conjunction with the steel transformer base. The cast aluminum transformer base is used in many new installations and requires no additional breakaway features.

Wind induced vibration is a common, though not always readily apparent, phenomenon occurring in street lighting structures. In many cases it is insignificant while in other cases it might be responsible for lamps becoming loose in their sockets or cause a fatigue failure in the luminaire or its support structure. The vibration is usually a result of aerodynamic instabilities attributable to the geometry of the pole, luminaire, or a combination of the two. Vibration can also be caused by motion of a bridge on which the pole is mounted.

In this study, vibration due to the aerodynamic characteristics of the pole were investigated together with their effects on the fatigue life of a cast aluminum transformer base and a cast aluminum shear base. The other types of vibration mentioned may cause higher stress within the structure but their frequency of occurrence will usually be considerably less than the type considered here.

In order to analyze a structure for fatigue, its stress history is needed and in many cases this is difficult to obtain, experimentally or analytically. This is especially true of a light pole subjected to wind since the wind occurs randomly, in both magnitude and direction. The stress history would have to be determined based upon predicted wind velocity, direction and frequency of occurrence spectrums. This would entail extensive parameter studies for each type of pole and fixture design and was beyond the scope of this study.

II. Objectives

The objectives of this study were; (1) to develop a computer program, from a mathematical model, to be used in determining the dynamic response of light poles subjected to steady winds, (2) to determine the dynamic response of two typical light poles with typical luminaires attached for wind velocities between 0 and 50 miles per hour, and (3) from the response determine if a fatigue failure will occur in the cast aluminum transformer base or the cast aluminum shear base.

III. Mathematical Model

Many of the luminaire support structures are of the type shown in Figure 1. The vertical pole is usually a tapered tubular member and the mast consists of a tubular trussed structure or a single tubular member.

Shown in Figure 2 is the idealized structure, where the continuum is lumped into discrete masses interconnected by weightless elastic springs. The number of discrete masses selected represents a compromise between an exact representation of the real structure, which theoretically requires an infinite number, and the number of calculations necessary to reach a solution, which increases with increasing numbers of masses.

For the purpose of this investigation, only motions in the plane of the pole and mast (x-y plane) were considered. Observations of luminaire structures subjected to winds in their natural environment indicated this to be the plane in which the structure reached its most severe vibration.

When air flows around a cylindrical shaped member two types of loading are produced. A static load is produced in the direction of the wind and alternating forces are produced perpendicular to the wind. It is this second type, described initially by von Karman, that induces vibration and the type considered in this analysis.

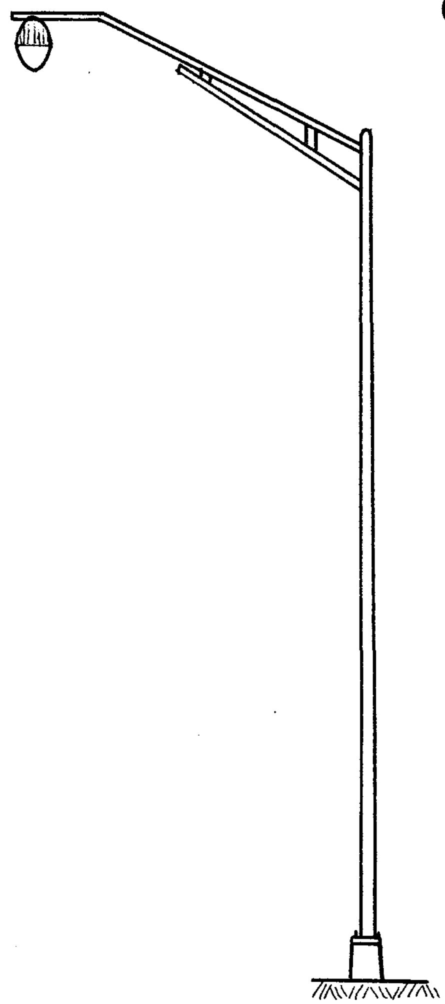

FIGURE 1. TYPICAL LUMINAIRE STRUCTURE

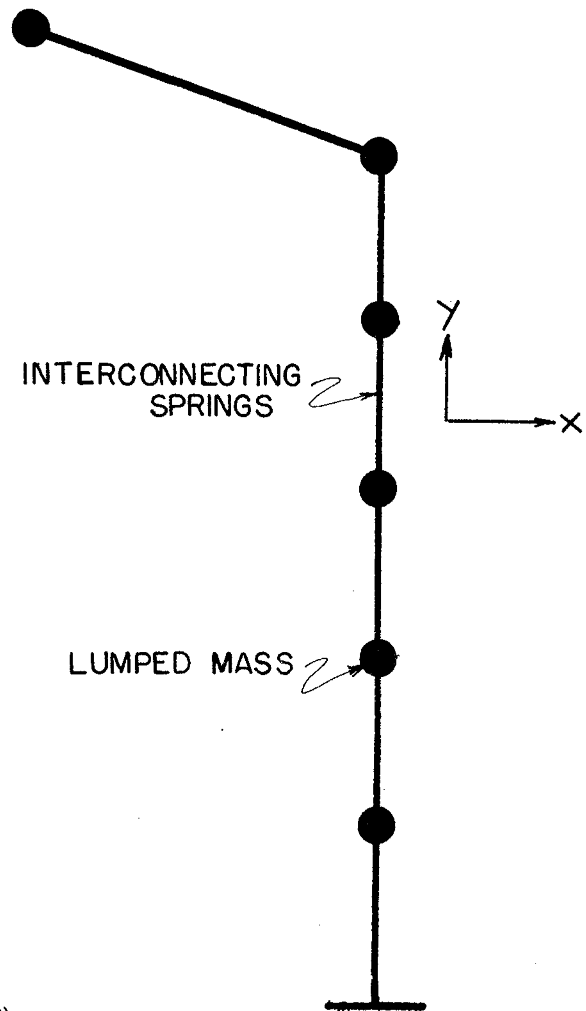

FIGURE 2. IDEALIZED POLE

The generally accepted expression [1*] for this attenuating force, F, is

$$F = (1/2)\rho V^2 A C_1 \operatorname{Sin}\Omega t$$

where A denotes the projected area of the cylinder, C_1 is a force coefficient which is dependent on Reynolds' number and the surface roughness, ρ is the mass density of air, V is the wind velocity, Ω is the frequency of alternating forces or vortex shedding, and t is time. The value of Ω is determined from the Strouhal [2] number which is dependent on the cylinder's diameter and the magnitude of wind. In applying this distributed force to the luminaire structure, the value of F was concentrated at the lumped masses, its value being computed over the spacing between the masses, using an average diameter for calculating purposes.

To solve the differential equations of motion matrix algebra was used in a modal analysis approach [3]. Solution of the resulting uncoupled equations of motion results in displacement versus time for each mass. Knowing the displacements with time and the stiffness of the structure one can determine the forces and moments at the base, and hence the stresses.

Solution by the above method was accomplished with the aid of a high speed computer. Input to the computer consisted

---

* Superscript numbers refer to corresponding items in the References.

of the pole and luminaire support properties, material damping coefficient, mass of the structure, and wind velocity.

IV. Pole Properties

Shown in Table 1 are the properties of the two structural configurations that were considered. These two are felt to be representative of the many different types in use.

To determine a value for the damping coefficient, pole B was instrumented with strain gages in order that its response could be recorded while undergoing free vibration. By measuring the logarithmic decrement between successive amplitudes of vibration a value of percent critical damping could be determined. Pole A was assumed to have the same percent critical damping as Pole B.

Table 2 lists the natural frequencies of both pole configurations for the seven different modes of vibration. Figures 3 and 4 show the first three mode shapes for the two poles. Natural frequencies and mode shapes are a by-product of the computer analysis since they are necessary in the modal analysis method.

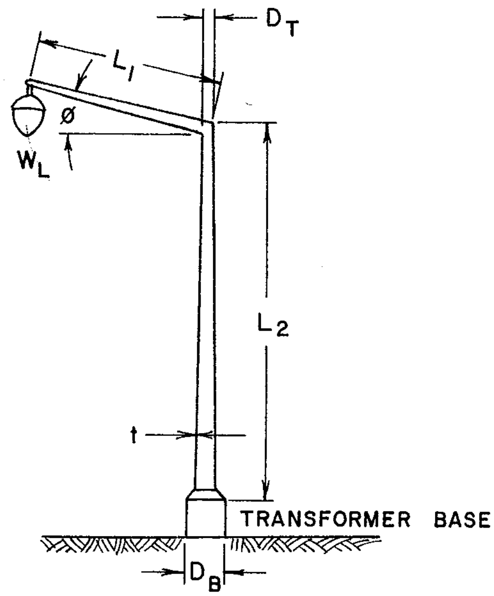

TABLE I. POLE PROPERTIES

Property	Pole A	Pole B
Material	Aluminum	Steel
Length, L_1	6 ft.	15.5 ft.
Length, L_2	30 ft.	35 ft.
Angle, ϕ	30 degrees	18.5 degrees
Luminaire Weight, W_L	67 lbs.	50 lbs.
Diameter at Top of Pole, D_T	4.5 in.	4.25 in.
Diameter at Base of Pole, D_B	7.5 in.	9.00 in.
Pole Thickness, t	0.188 in.	0.125 in.
Damping coefficient*, c	5%	5%

*Expressed as a percent of critical damping.

LEGEND: - - - - - Mode 1
—— - —— Mode 2
—— - - —— Mode 3

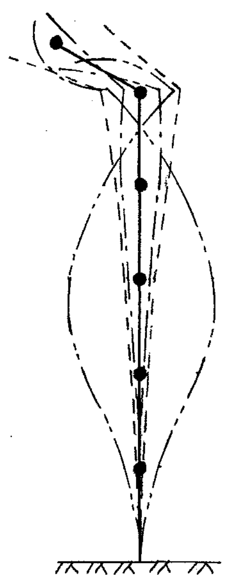

Figure 3. Mode Shapes-
Pole A

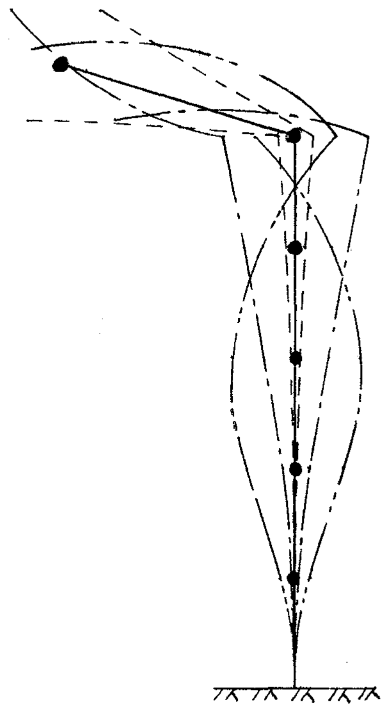

Figure 4. Mode Shapes-
Pole B

TABLE 2. NATURAL FREQUENCIES

Mode	Pole A	Pole B
1	1.00	0.92
2	3.24	1.90
3	9.54	7.14
4	25.37	18.29
5	26.35	18.84
6	48.44	36.18
7	78.23	60.04

<u>NOTE</u>: Frequencies are in cycles per second.

V. Stress Analysis of Base

Figures 5 and 6 show the calculated dynamic bending moment at the base of each pole versus time for poles A and B, respectively. These moments are those caused by vibratory motion only and do not include the static moment due to the luminaire weight. As can be seen, the bending moment in both cases is neither periodic or harmonic and the 0.6 seconds of response shown represents only a random sample of the total response. Table 3 lists the maximum and minimum moments at the base of the pole calculated over a 60-second response time and the corresponding values of shear load. These are the values used in the stress analysis.

The wind velocities for which the moments were computed are shown on the figures. These respective values result in the greatest dynamic response of all wind velocities considered (0 to 50 miles per hour).

For analysis the standard aluminum transformer base of Figure 7 was used in conjunction with the cast aluminum shear base of Figure 8. It is recognized that the aluminum shear base will probably not be used in conjunction with the aluminum transformer base.

Figure 9 shows the forces and moments acting on the base of a typical luminaire support structure for a wind blowing in the negative Z-direction. It is felt that this wind direction (or the +Z-direction), of all possible directions, will result in the greatest stress on the base.

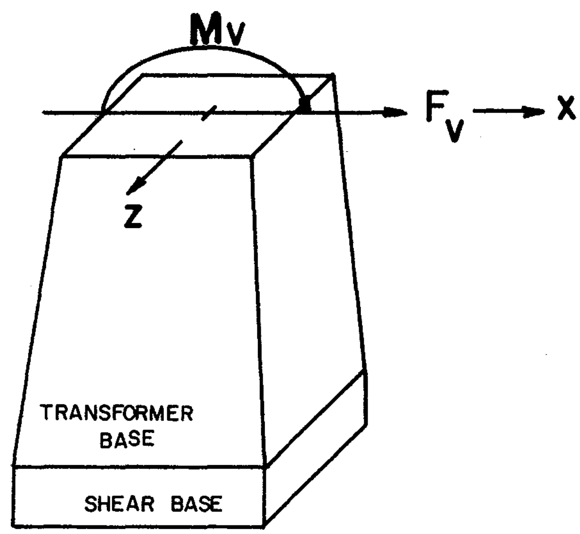

TABLE 3. MAXIMUM & MINIMUM DYNAMIC LOADS

Load	Pole A		Pole B	
	Max.	Min.	Max.	Min.
Moment, M_V in-lbs	12,000	-12,000	13,000	-13,000
Shear, F_V lbs.	220	-220	200	-200

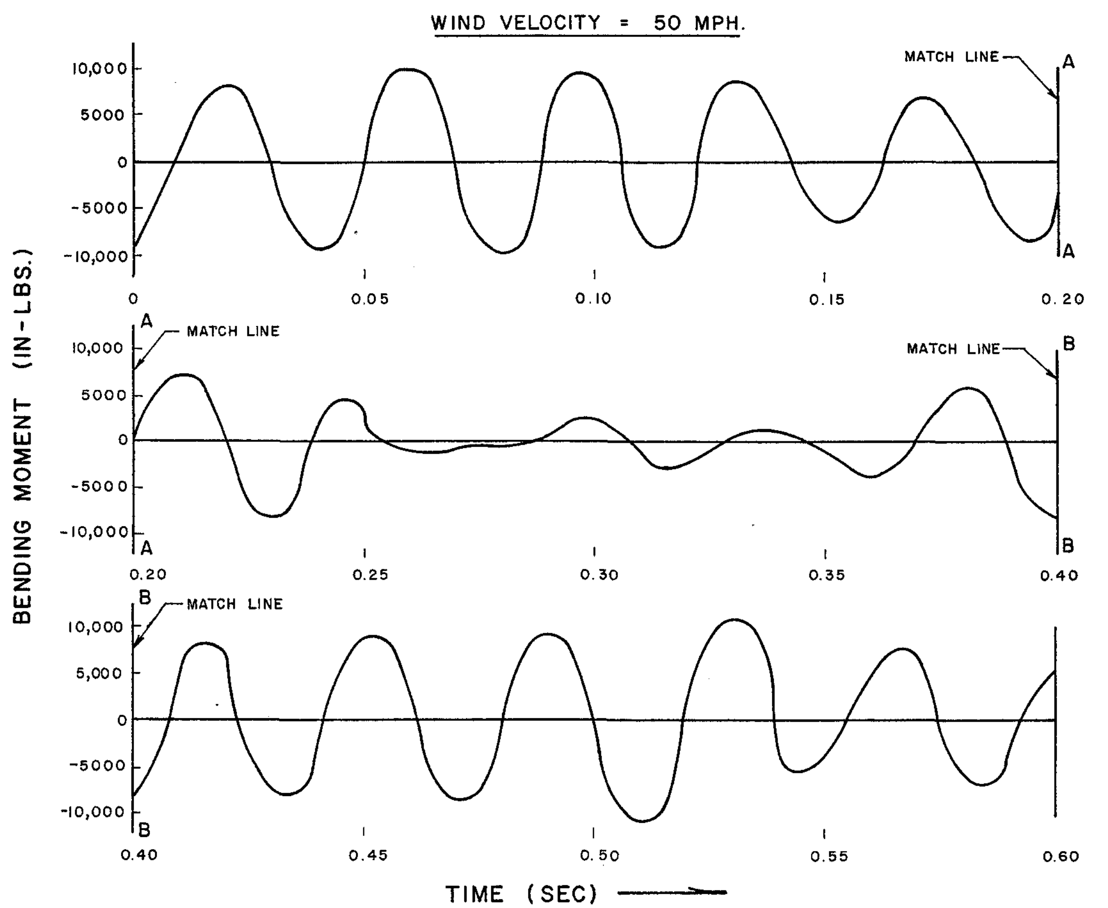

FIGURE 5. BASE BENDING MOMENT VERSUS TIME - POLE A

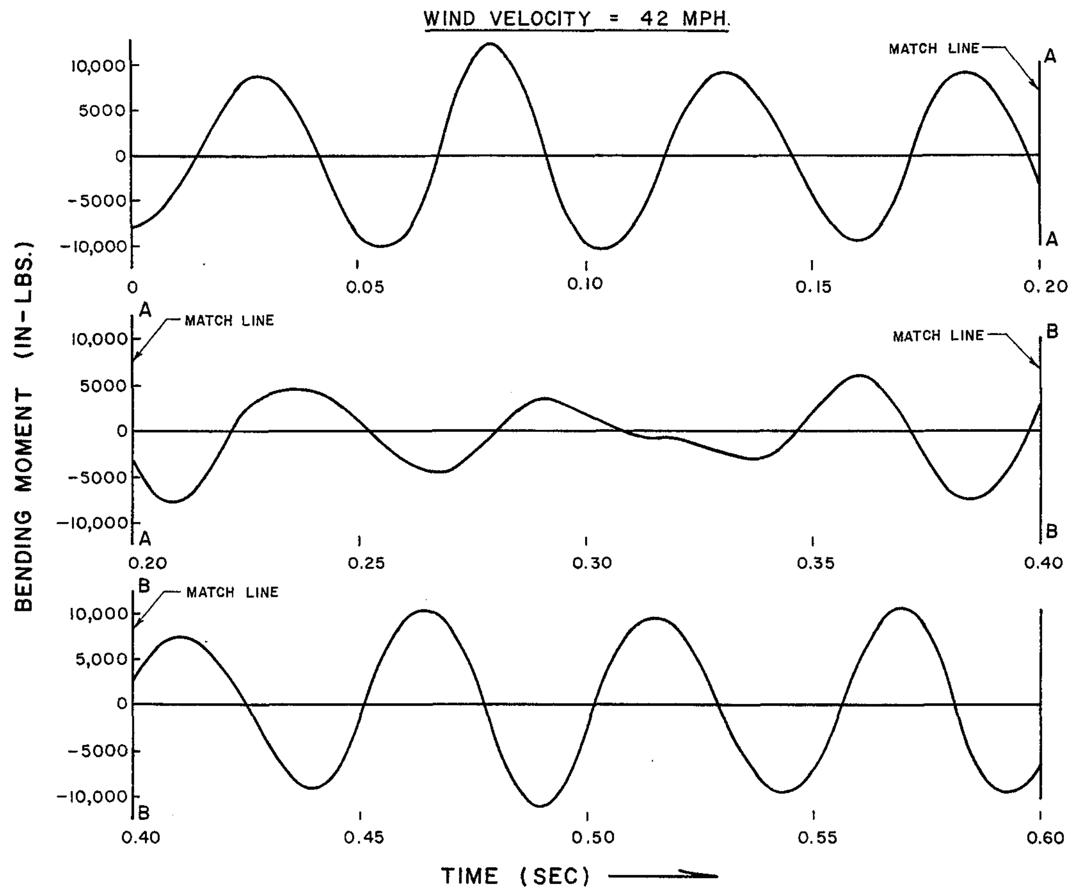

FIGURE 6. BASE BENDING MOMENT VERSUS TIME – POLE B

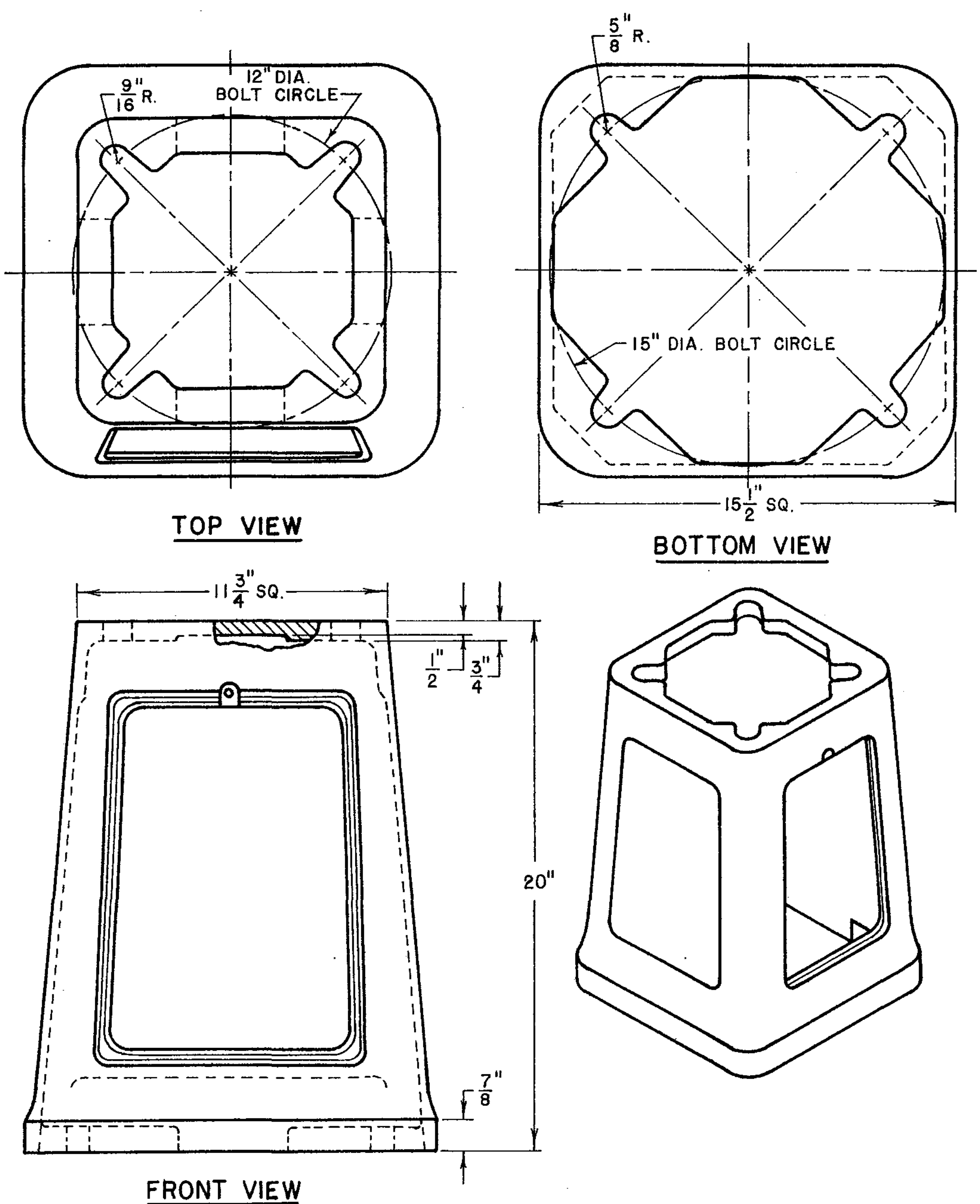

FIGURE 7. STANDARD ALUMINUM TRANSFORMER BASE

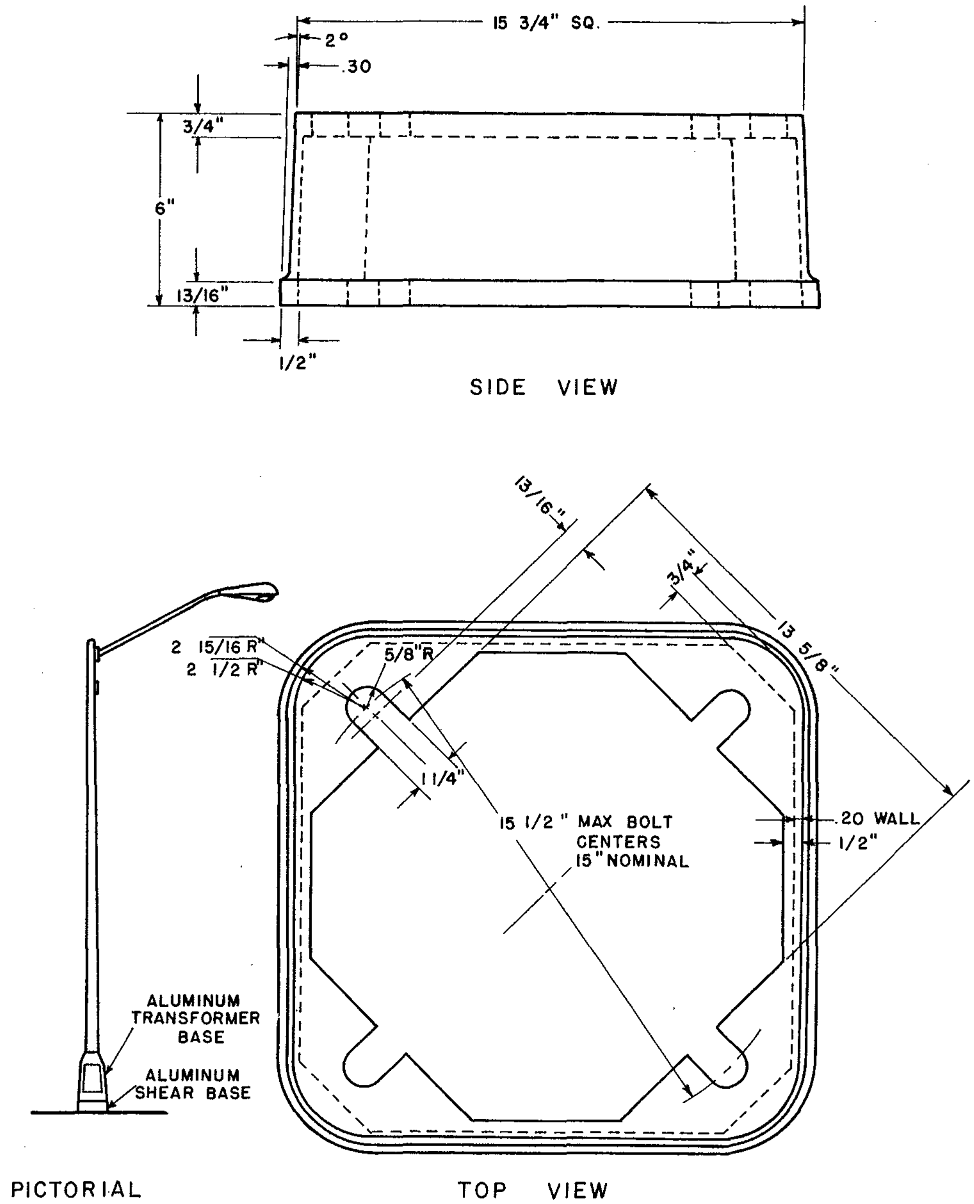

FIGURE 8 ALUMINUM SHEAR BASE

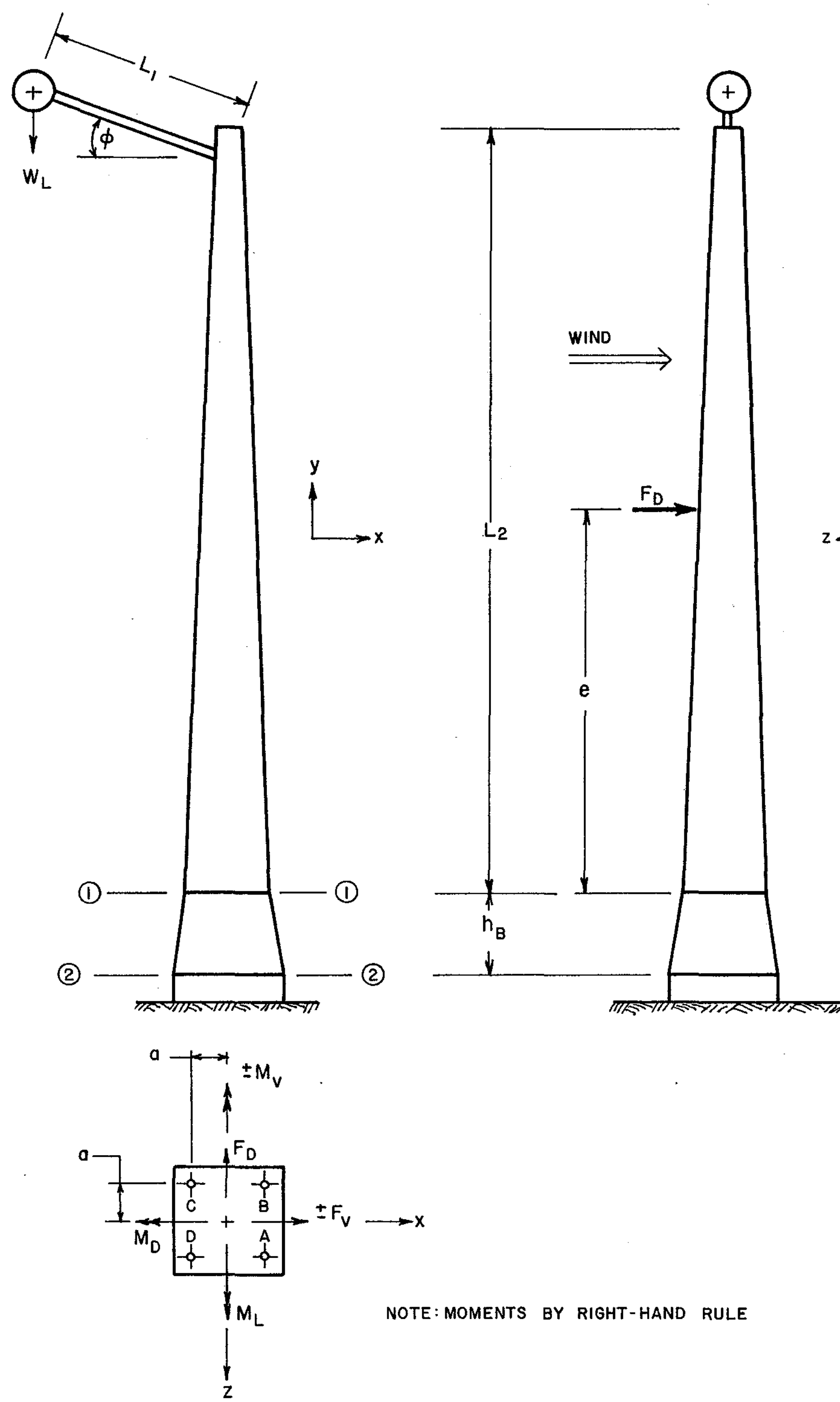

FIGURE 9 FORCES AND MOMENTS ON BASE

For this wind direction bolt "A" will experience the greatest tensile load. The bolt load P_A^1 at location 1 - 1 is computed by

$$P_A^1 = \frac{1}{4a} M_D^1 + \frac{1}{4a} M_L \pm \frac{1}{4a} M_V^1$$

$$= \frac{1}{4a} (M_D^1 + M_L \pm M_V^1)$$

where

M_V^1 = alternating dynamic bending moment at section 1 - 1

M_L = static bending moment caused by luminaire weight

$M_L = (L_1 \cos\phi) W_L$

M_D^1 = static bending moment at section 1 - 1 due to the drag force F_D in direction of wind (assumed to be caused by wind acting on pole, with the drag force of the luminaire and its mast arm assumed negligible.

The static drag force F_D is computed by

$$F_D = \frac{1}{2} C_D \rho V^2 L_2 \bar{D}$$

where

C_D = drag coefficient

ρ = mass density of air

V = velocity of the air

L_2 = length of pole

$\bar{D}$ = average diameter of pole

Upon obtaining F_D the drag moment M_D^1 is computed by

$$M_D^1 = F_D e$$

where

$e = L_2/2$

Moments at section 2 - 2 (interface of transformer and shear base) are computed as follows:

$$M_D{}^2 = M_D{}^1 + F_D(h_B)$$

$$M_V{}^2 = [\pm M_V{}^1 + F_V(h_B)]$$

where

F_V = alternating dynamic shear force at section 1 - 1

The bolt load $P_A{}^2$ at location 2 - 2 is then computed by

$$P_A{}^2 = \frac{1}{4a}\,(M_D{}^2 + M_L \pm M_V{}^2)$$

Maximum tensile bolt loads were computed for both configurations (A & B) and the results are listed in Table 4. Those constants, not defined previously or shown on the figures, used in determining the values in Table 4 are as follows:

C_D = 1.25 (for cylindrical shapes)

ρ = 0.00238 LB-Sec^2/ft^4

The bolt loads from pole configuration B are used in the analysis since they are the greater of the two types considered.

Table 4. BOLT LOADS

	POLE A					POLE B				
LOCATION	M_L	M_D	M_V	4(a)	P_A	M_L	M_D	M_V	4(a)	P_A
1 - 1	4176	21,600	±12,000	17.0"	1520 ±705	8820	22,890	±13,000	17.0	1865 ±765
2 - 2	4176	24,000	±16,400	21.2"	1330 ±775	8820	25,070	±19,000	21.2"	1600 ±895

NOTE: Moments are in inch-pounds.

P_A in pounds.

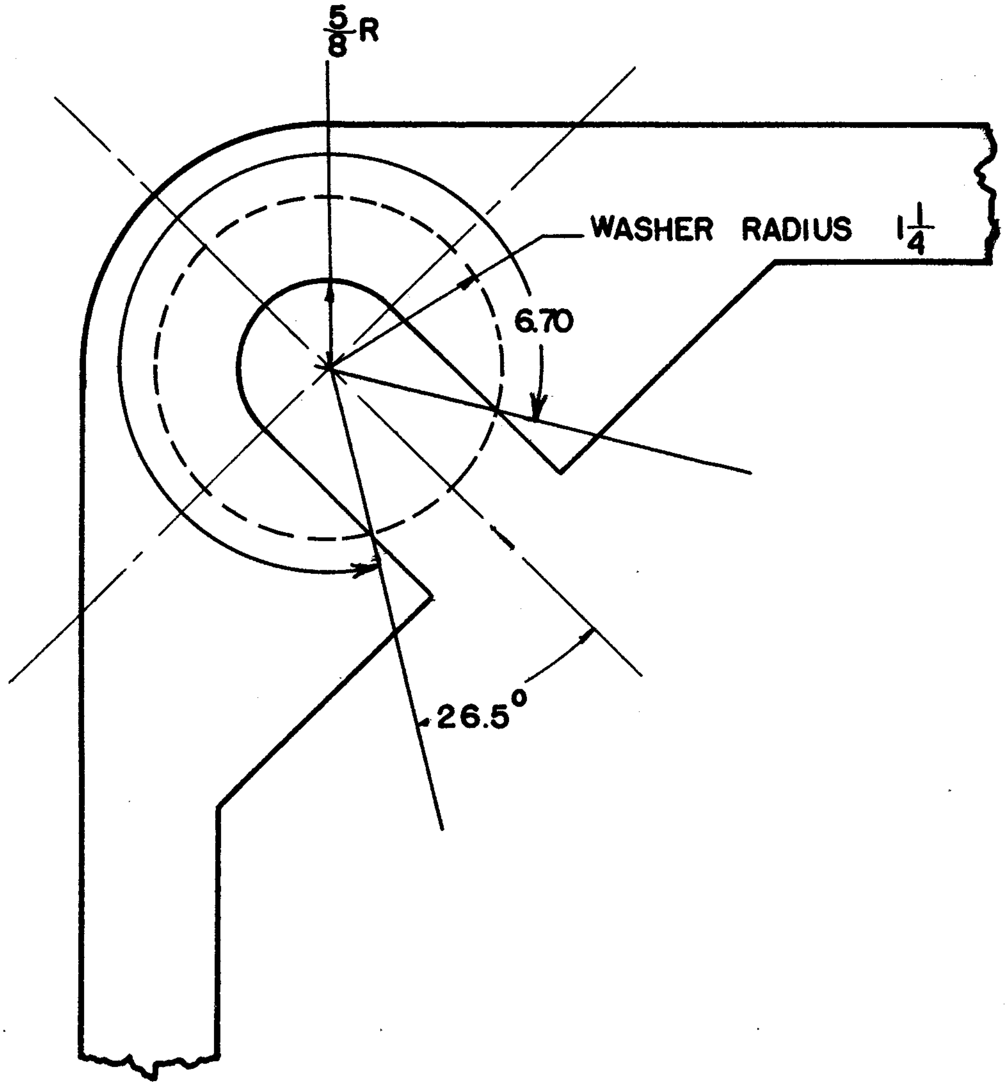

FIGURE 10 SLOTTED CONNECTION

It was assumed that a fatigue failure would occur in the region of the bolt slots of the transformer base (Figure 7) or the shear base (Figure 8) since this is an area of stress concentrations and an area in which the larger stress reversals take place. The connection was checked for shear around the slot, see Figure 10, with the shear area computed as the thickness of the plate times the circumferential distance along the washer's edge in contact with the plate. Shown in Table 5 are the shear stresses at the referenced locations.

TABLE 5. SHEAR STRESSES

Location	Material Thickness (in.)	Circumferential Distance (in.)	Shear Area ($in.^2$)	Bolt Load (lbs)	Shear Stress ($lb/in.^2$)
Top of transformer base	0.75	6.7	5.0	1865 ±765	373 ±153
Bottom of transformer base	0.875	6.7	5.8	1600 ±895	276 ±154
Top of shear base	0.75	6.7	5.0	1600 ±895	320 ±179

As shown in Table 5, the maximum stress reversal occurs at the top of the shear base. A stress concentration factor of 3[4] was used to modify the maximum stress to account for stress raisers which may be present, giving a maximum alternating shear stress of 537 psi and a mean shear stress of 960 psi.

Shearing strength of 356-T6 aluminum is given as 26,000 psi in the Alcoa Aluminum Handbook and the endurance limit as 8500 psi at 5×10^8 cycles. Endurance limit is defined as the maximum reversed stress that may be repeated an indefinite number of times without failure. It must be noted that this is a theoretical endurance limit and was taken on a standard, polished, nominal specimen in rotating bending. Several factors have to be taken into consideration which reduce the endurance limit. The percent reduction and its cause are as follows:[5]

5% - Size effects

50% - Shear failure

10% - Surface and casting defects

TOTAL = 65%

The resulting endurance strength, S_{ns}, is computed as

$$S_{ns} = 8500\ (1 - 0.65) = 3640 \text{ psi}$$

In checking the applied stress levels versus the endurance strength the "Goodman-Gerber Line" criteria[6] were used. This is expressed by

$$\frac{1}{N} = \frac{S_{ms}}{S_{us}} + \frac{S_{as}}{S_{ns}}$$

where,

N = factor of safety,

S_{ms} = mean shear stress,

S_{us} = ultimate shear strength,

S_{as} = alternating shear stress, and

S_{ns} = endurance strength (shear).

In this case the mean stress, S_{ms}, is assumed to equal the static shear stress, which is a conservative assumption since the compressive stresses due to the weight of the structure will increase the factor of safety. The resulting equation for N is

$$\frac{1}{N} = \frac{960}{26,000} + \frac{537}{3640} = 0.037 + 0.148$$

$$\frac{1}{N} = 0.185$$

$$N = \underline{\underline{5.40}}$$

VI. Discussion of Results

The minimum factor of safety for the two pole configurations considered was equal to 5.40. This value is greater than the normal scatter factor of 4.0 which is used extensively in the aircraft industry[7]. The safety factor also indicates that the maximum alternating stresses are well below the lowest endurance limit for sand cast aluminum at the 99% probability scatter bands, given in reference 8.

As noted previously, the assumed mean stress is not exact since the weight of the pole has been neglected in all calculations. This is a conservative assumption, however, because a compressive mean stress will increase the overall fatigue life. It was also assumed that the critical wind velocity (42 miles per hour) remained constant for an indefinite period of time. This is a very conservative assumption.

It was not within the scope of this study to verify this mathematical model with extensive full-scale tests. A full-scale outdoor test was made, however, on pole "B" (see Table 1) in which response frequencies were measured during winds of 25 to 35 miles per hour. The magnitude of the stresses resulting from the vibration was not measured, only frequencies. Frequency response was measured by electric resistance strain gages attached at the base of the pole. Two predominant frequency responses were observed superimposed on each other; the first, attributable to the alternating lift force on the weight attached at the end of the pole to stimulate the luminaire, occurred at a frequency of approximately 0.75 cycles per second; and, the second, due to the von Karman vortex street along the pole, occurred at a frequency of approximately 13 cycles per second. It is the second type that this study pertains to. The mathematical model's response frequency for pole "B" at the given wind velocities was approximately 15 cycles per second. This is felt to be an acceptable degree of verification, considering the actual conditions under which the full-scale test was made and the simulated ideal conditions under which the model was used.

VII. Conclusions

Based on the results of this investigation it appears that vibrations in light pole standards, due to the von Karman vortex street, will not result in a fatigue failure of the cast aluminum base. The minimum factor of safety of 5.40 found for the two pole configurations considered (one steel and one aluminum) forms a basis for this conclusion.

It has been demonstrated that the mathematical model of the luminaire support structure can be used to determine the dynamic response of light poles under wind loading.

VII. Recommendations for Future Research

Considered herein were vibrations in light pole standards due to the von Karman vortex street and their influence on the fatigue life of the cast aluminum base. Other causes of light pole vibration should be investigated together with their effects on the structural integrity of the entire standard. Aerodynamic instabilities due to the shape of the luminaire and the movement of bridges on which the light pole may be mounted are two major causes of vibration which should be investigated. More data are also needed on force coefficients and Strouhal numbers for light poles with cross-sections other than the circular type, such as the octagonal cross-section.

REFERENCES

1. Weaver, William Jr., "Wind Induced Vibrations in Antenna Members", Paper No. 3336, Vol. 127, Part I, American Society of Civil Engineers, Transactions, 1962, page 681.

2. Fung, Y. C., An Introduction to the Theory of Aeroelasticity, John Wiley & Sons, New York, 1955, page 67.

3. Biggs, John M., Introduction to Structural Dynamics, McGraw-Hill, 1964, page 329.

4. Peterson, R. E., Stress Concentration Design Factors, John Wiley & Sons, New York, 1947.

5. Faires, V. M., Design of Machine Elements, The McMillan Co., New York, 1962, page 109.

6. Ibid, page 107.

7. Buntin, W. D., "Aircraft Structural Mechanics, Part IX, Fatigue Analysis and Design", General Dynamics Manual, June 1960.

8. Butz and Nordmark, "Fatigue Resistance of Aluminum and Its Products", Reprinted by permission of SAE, distributed by Aluminum Company of America.