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EFFECTS OF CHEMICAL ADMIXTURES IN CONCRETE AND MORTAR

in cooperation with the
Department of Commerce
Bureau of Public Roads

RESEARCH REPORT 70-3 (FINAL)
STUDY 2-5-63-70
CHEMICAL ADMIXTURES FOR CONCRETE

EFFECTS OF CHEMICAL ADMIXTURES IN CONCRETE AND MORTAR

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Research Report 70-3 (Final)

Chemical Admixtures for Concrete
Research Project 2-5-63-70

Sponsored by
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TEXAS TRANSPORTATION INSTITUTE
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The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Bureau of Public Roads.

SYNOPSIS

This paper reports physical effects of chemical admixtures on concrete and cement mortars, compares the variability of mortar tests with the variations encountered in concrete tests, and shows the degree of correlation of these tests with tests on concrete. The data presented provide a basis for utilizing a standard mortar for quality control tests of chemical admixtures. Most of the work is concentrated on compressive strength, shrinkage, and time of set. Also included are the results of durability tests on admixture concrete and a section on the control of chemical admixture uniformity.

A theoretical solution for restrained shrinkage crack spacing is developed and a comparison of this theory with limited test data is shown.

Chapter I.

Introduction

The controversy over the necessary performance specifications for chemical admixtures has been intense since the entrance of these admixtures in the field of concrete construction. Within the past ten years the major areas of controversy have been the requirements on limitations of concrete shrinkage, the effect of chemical admixtures on concrete durability, and the control of the uniformity of these admixtures.

In the area of shrinkage control, ASTM C494-63T, Chemical Admixtures for Concrete, required that admixture concrete be limited to a shrinkage of 100 micro-inches per inch over that of a control concrete without admixture. It was first shown by Tremper and Spellman (1)* and later by Torrans et al. (2), that this was a highly arbitrary specification depending to a large extent on the selection of the coarse and fine aggregate to be used in the test. The revisions to ASTM C494-63T now specify a maximum increase in shrinkage of 100 micro-inches per inch and not more than 135% of control for concretes with control shrinkage of over 300 micro-inches per inch.

The bridge deck deterioration problems highway engineers in some sections of the country are presently experiencing have caused increasing concern about the effect of chemical admixtures on concrete durability. Whether or not these admixtures, or their improper use, may, in some cases, contribute to the problem is a controversial question.

Since it is highly impractical to run physical acceptance tests on concretes made with every different aggregate that may be used on various jobs, the mortar tests that are reported in this paper represent an effort to standardize admixture performance tests. Control of uniformity of chemical admixtures has been subject to considerable discussion within the profession, and a portion of the latest revision to ASTM C494-63T is devoted to this problem. The sample tests reported in this paper give indications of ways in which the uniformity may be verified. In the use of cement mortars to determine product variations, a precedent was set by Walker and Bloem (3) in an extensive study of cement variations.

The objectives of this study as stated in the Plan of Research for the final fiscal year are:

1. The development of standard mortar tests to show the major performance characteristics of admixtures such as their effect on shrinkage, time of set, and strength.
2. Determination of the effects of different cements on the physical properties of mortars used to test chemical admixtures.
3. Determination of the effects of delayed addition of set retarding agents on the time of set of cement mortars.

It was not expected that the effect of chemical admixtures on a standard cement mortar could be extrapolated to accurately predict effects on various job concretes, but the relative performance of admixtures should be indicated, and performance variations in excess of predetermined testing variations should indicate variations in the admixture. The data presented in this paper provide a basis for utilizing a standard mortar for quality control tests of chemical admixtures.

A segment of this paper reports the results of infrared analysis, percent solids, and specific gravity tests on samples of admixtures received from ready-mix concrete producers as well as admixture manufactures.

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*Refers to numbers in Selected References.

Chapter II.

Testing Program

Mortar Tests. The mortars used in these tests were composed of Atlas Type I cement (Table 1-A* for mill analysis), Ottawa 20-30 silica sand, and water. The cement to sand ratio on all mortars was 0.366. The water-cement ratio was nominally 0.55 for the control (no admixture) mortar and varied from 0.41 to 0.56 in the batches utilizing admixtures, depending on the water reducing qualities of the admixture. In determining the water reduction characteristic of each admixture, the amount of water necessary to produce a flow of 75% was used. This allows the admixture the benefit of its characteristic water reduction when compared to the control (no admixture) mortar. There are several exceptions to this procedure when constant water batches are used to separate the water reduction effect from the effects due to chemical and dispersion action of the admixture.

Mortar tests were designed to determine the effects of the various admixtures on time of set, water reduction, shrinkage, and strength. In some tests an air detraining agent was used to cut down entrained air variations. Other test series were used to show some of the coefficients of variation necessary in developing standard test procedures.

Concrete Tests. Forty, 1.5 ft.^{3} batches of natural siliceous sand gravel concrete were mixed to compare results of the mortar tests with concrete test results. These included ten control (no admixture) batches and three batches for each of the ten admixtures under test.

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*Table numbers followed by an upper case A may be found in the Appendix.

Shrinkage, compressive strength, and durability specimens were cast from each concrete batch. In addition, a quantity of mortar was separated from the coarse aggregate using a No. 4 screen and specimens cast to determine mortar shrinkage, compressive strength, and time of set. The flow table and mortar mold used conform to ASTM C290-61T.

Admixtures. The admixtures tested were five lignosulfonates (AL*, DL$_1$, DL$_2$, DL$_3$, and DL$_4$), three organic acids (DO$_1$, DO$_2$, and DO$_3$), two polymers (AP and DP), and calcium chloride (CC). A description of these admixtures and the dosage used is given in Table 2-A.

Product Uniformity. In this phase of the program, eighty-seven ready-mix concrete producers were asked to supply samples of admixtures to be tested for chemical uniformity. More than 50% of these returned samples for inclusion in the program. Of the samples returned the largest number was the DO$_1$ admixture, followed by DL$_2$, and DO$_2$.

Fifteen samples of DO$_1$, nine of DL$_2$, and six samples of DO$_2$ were subjected to infrared spectrographic (IRS) analysis and a determination of percent solids. A study of specific gravity and percent solids on admixtures AL, DL$_1$, DL$_2$, DO$_1$, and DO$_2$ is also included for use as a simple field quality control test.

Chapter III.

Results and Discussions

Compressive Strength

The development of comparative compressive strength data was divided into three major parts as follows:

1. Comparative compressive strength determinations on mortars using admixtures AL, AP, DP, DL$_1$ through DL$_4$, DO$_1$ through DO$_3$, and CC holding flow fundamentally constant by variation of water content and maintaining a rough control on air content by the use of an air detraining agent (tributyl phosphate) with the lignosulfonates.

2. Comparative compressive strength on both the concrete and the mortar screened from the concrete using the same admixtures listed above. Slump was held between 2½ and 3½ inches by variation of water content while holding a fundamentally constant cement factor (5.45 to 5.55 sacks/yard). Air was held within the limits of 4.5 to 5.5%.

3. Comparative compressive strength determinations between twelve different cements holding water content constant for admixtures DO$_1$, DL$_2$, and CC. The mill analyses of these cements are given in Table 3-A.

In all cases compressive strength was determined using 2 in. diameter by 4 in. length cylinders for mortar, and 6 in. by 12 in. cylinders for concrete. The strength specimens were continually moist cured before testing for the time period indicated.

Strength determinations on mortar cylinders 2 in. in diameter by 4 in. in length have shown an extremely low variability. A within-batch coefficient of variation of 2.4% was found for the seven -day compressive strengths that are tabulated in Table 4-A.

All admixtures tested in both the mortar and concrete programs gave either a fundamentally equal or improved compressive strength at both the 7- and 28-day level when they were allowed to utilize their characteristic water reductions, as equated by flow in the mortar batches and by slump in the concrete batches.

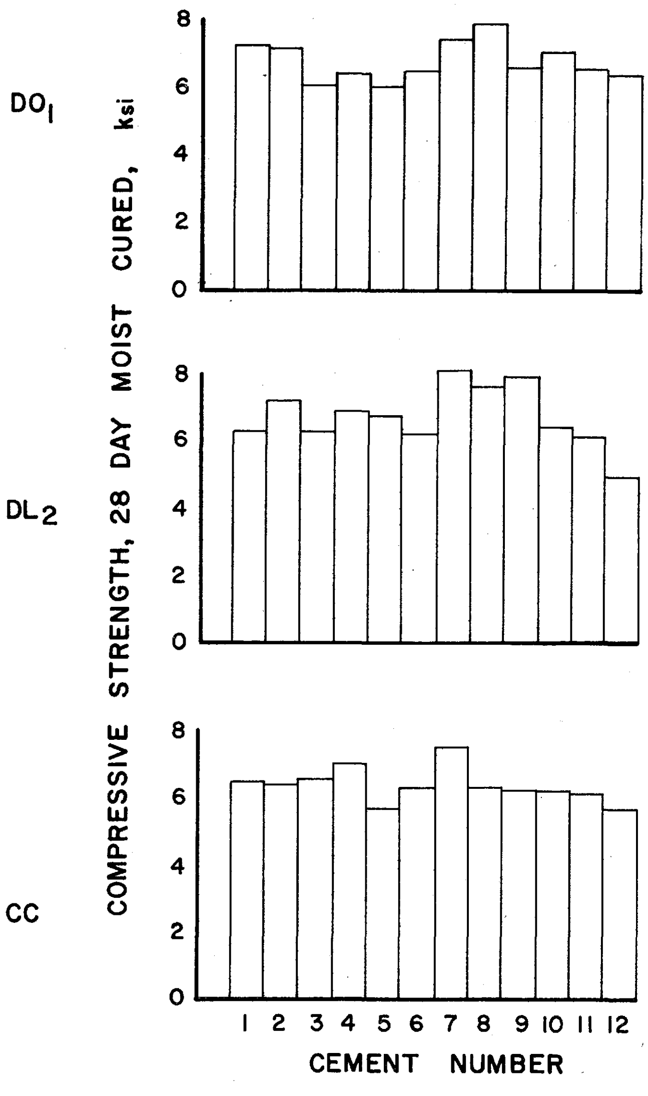

Figure 1. Variation of mortar strength with various cements. (Batch data: Tables 14-A, 15-A and 16-A.)

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*The first letter of the admixture designations is the designation given in ASTM C494-63T (A—Water reducing, C—Accelerating, D—Water reducing and retarding). The second letter refers to chemical type (L-Lignosulfonate, O—Organic Acid, P—Polymer, C—Calcium Chloride). The subscript which may occur differentiates between commercial products of the same ASTM designation and chemical type.

Table 1
COMPRESSIVE STRENGTH COMPARISON OF CONCRETE AND MORTARS

Admixture	(1) 7 day Concrete	(2) 28 day Concrete	(3) 7 day Mortar (from Conc.)	(4) 28 day Mortar (from Conc.)	(5) 7 day Mortar	(6) 28 day Mortar	(7) Ratio (3)/(1)	(8) Ratio (5)/(1)	(9) Ratio (4)/(2)	(10) Ratio (6)/(2)	(11) Ratio (5)/(3)	(12) Ratio (6)/(4)
Control	3.75*	4.53	5.32	6.28	4.41	6.10	1.42	1.18	1.36	1.33	.84	.98
DL$_1$	4.46	5.30	5.75	7.08	5.63	6.72	1.29	1.26	1.34	1.27	.98	.95
DL$_2$	4.16	4.96	5.79	6.98	5.68	7.01	1.39	1.37	1.41	1.41	.98	1.00
DL$_3$	4.31	5.04	6.33	7.67	5.58	7.15	1.47	1.29	1.52	1.42	.88	.93
DL$_4$	4.27	5.27	5.72	6.44	5.44	6.96	1.34	1.27	1.22	1.32	.95	1.08
DO$_1$	4.23	4.96	6.19	7.25	5.54	6.89	1.46	1.31	1.46	1.39	.89	.95
DO$_2$	4.06	4.63	5.37	6.81	5.69	7.14	1.32	1.40	1.47	1.54	1.06	1.05
DO$_3$	4.12	5.21	5.38	7.16	5.86	7.00	1.31	1.42	1.37	1.34	1.09	.98
DP	4.27	5.49	6.06	7.11	5.47	6.42	1.42	1.28	1.30	1.17	.90	.90
AP	4.14	5.04	5.72	6.90	5.45	6.33	1.38	1.32	1.37	1.26	.95	.92
AL	4.01	4.73	5.49	7.00	5.80	7.01	1.37	1.45	1.48	1.48	1.06	1.00
CC	3.93	4.71	5.75	6.99	5.74	6.65	1.46	1.46	1.48	1.41	1.00	.95
						Average —	1.39	1.34	1.40	1.36	.97	.97
						Range —	1.29-1.47	1.19-1.46	1.22-1.52	1.17-1.54	.84-1.09	.90-1.08
						Coefficient of Variation —	4.2%	6.3%	6.4%	9.1%	8.2%	5.3%

*Units of compressive strengths are ksi.

The comparison in compressive strengths found by mortar batches and concrete batches is best seen by an analysis of the data given in Table 1. Columns 1 and 2 are the average concrete strengths from Table 7-A for each admixture. Columns 3 and 4 are the average mortar strengths determined on the mortar screened from the same concretes. The individual determinations are given in Tables 8-A and 9-A. Columns 5 and 6 are the compressive strengths from the mortar batches given in Table 12-A. The ratios of the designated columns are given in columns 7 through 12. The average ratio, range of the ratios, and C. V.'s* of these columns are given.

The smallest variation is in the ratio of 7-day concrete mortar to 7-day concrete (4.2%), while the largest variation is in the ratio of 28-day mortar to 28-day concrete (9.1%). One correlation which seems fairly good (5.3% C. V.) is that of the ratio of the mortar batches to the mortar screened from concrete batches.

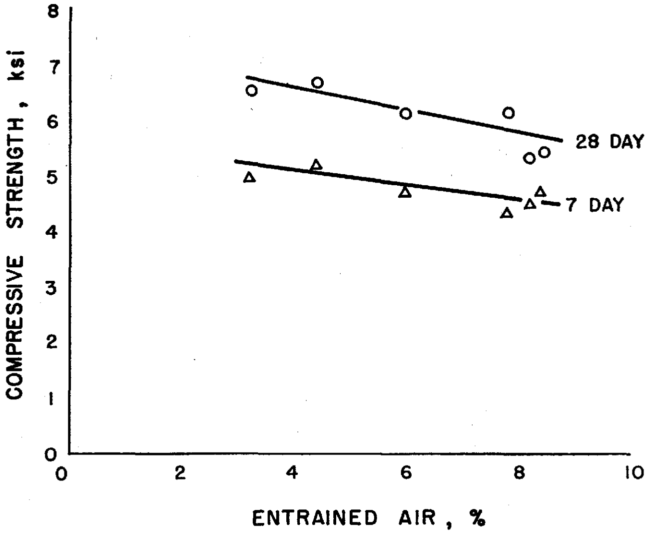

Figure 2. Effect of entrained air on mortar strength. (Batch data: Table 17-A.)

The third portion of the compressive strength study is illustrated by the bar graphs of Figure 1. These graphs show for three different admixtures the variation in compressive strengths one can expect in the type 1 cements produced in Texas. The range of strengths is seen to be considerable, with an average C. V. of 9.9%. This graphically points out that in strength testing for admixture uniformity, a single supply of cement from one mill run must be stored for use throughout the time the admixture is to be tested.

Figure 2 illustrates the effect of entrained air on the compressive strength. By using an air detraining agent (tributyl phosphate) with lignosulfonates the air content was held within the limits of 3.4 and 6.2% (Table 7-A) for the control batch and ten admixture batches, excluding calcium chloride, which had 2.7% air. According to the data developed in Figure 2 this would represent a variation in compressive strength due to entrained air of 8.8%. This is a considerable variation, and would seem to indicate an air detraining agent should be used with the organic acids as well as the lignosulfonates if comparisons between admixtures are to be made.

Shrinkage

Shrinkage specimens from both concrete and mortar batches were cast to provide information on the degree of variability of both test methods. The concrete shrinkage specimens were 3 in. x 3 in. x 11 in. prisms while mortar specimens were 1 in. x 1 in. x 11 in. prisms. Both types of specimens had a nominal gage length of 10 inches and were cured for seven days in a fog room prior to their removal to the 50% relative humidity room.

The comparison of shrinkage values obtained on both concrete and mortar batches for the various admix-

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*Coefficient of variation.

Table 2
SHRINKAGE COMPARISON

Admixture	(1)* Concrete 60 day µin./in.	(2)** Mortar 28 day µin./in.	(3)** Mortar 7 day µin./in.	(4) Ratio (2)/(1)	(5) Ratio (3)/(1)	(6) Ratio (2)/(3)
Control	430*	700	440	1.628	1.022	1.591
DL$_1$	450	880	540	1.956	1.200	1.630
DL$_2$	500	840	500	1.680	1.000	1.680
DL$_3$	510	920	590	1.805	1.157	1.559
DL$_4$	650	950	580	1.462	.892	1.638
DO$_1$	380	650	440	1.711	1.158	1.477
DO$_2$	430	740	490	1.721	1.140	1.510
DO$_3$	500	860	540	1.720	1.080	1.590
DP	420	850	620	2.024	1.476	1.371
AP	480	870	630	1.813	1.313	1.381
AL	470	930	580	1.979	1.234	1.603
CC	580	870	660	1.500	1.138	1.318
			Average	1.750	1.151	1.529
			Range	1.46-2.02	0.89-1.48	1.32-1.68
			C. V.	10.1%	13.1%	7.7%

*For batch quantities see Table 7-A.
**For batch quantities see Table 12-A.

tures is shown by Figure 3 and Table 2. The mortar shrinkages, as would be expected since the restraining effect of the coarse aggregate is removed, are considerably in excess of concrete shrinkages. The average ratio of 28-day mortar shrinkage to 60-day concrete shrinkage is 1.753 while the average shrinkage ratio of 7-day mortar to 60-day concrete is 1.153. The ratios of mortar to concrete show fairly high variations, however. Table 2 shows a C. V. of 10% in the shrinkage ratios of 28-day mortar to 60-day concrete and 13% in the ratio of 7-day mortar to 60-day concrete.

A comparison of the effect on shrinkage of the various chemical types of admixtures does seem indicated. As a class, even though they have in general higher water reductions, the lignosulfonates show higher drying shrinkage than the organic acids and polymers

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*Micro-inches/inch.

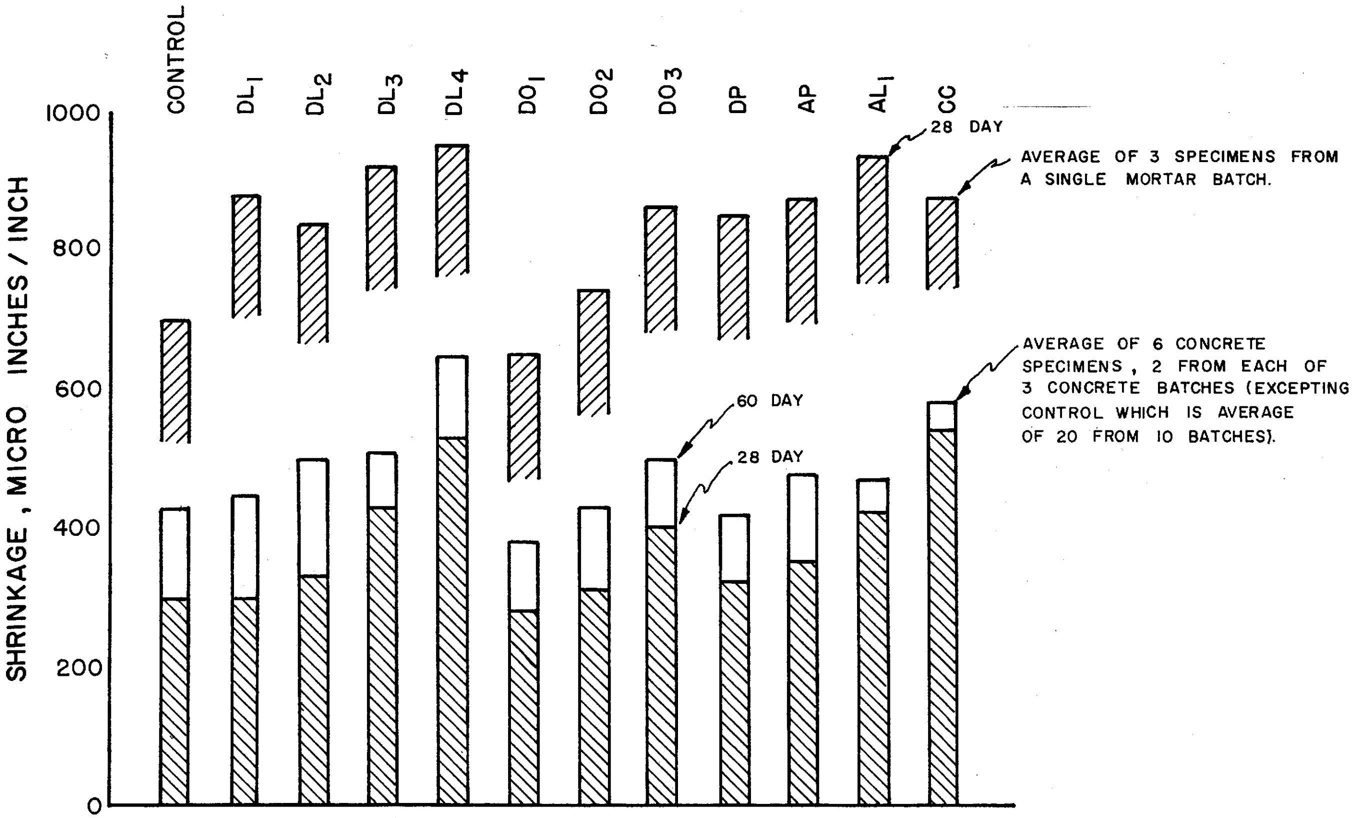

Figure 3. Comparison of concrete and mortar shrinkage.

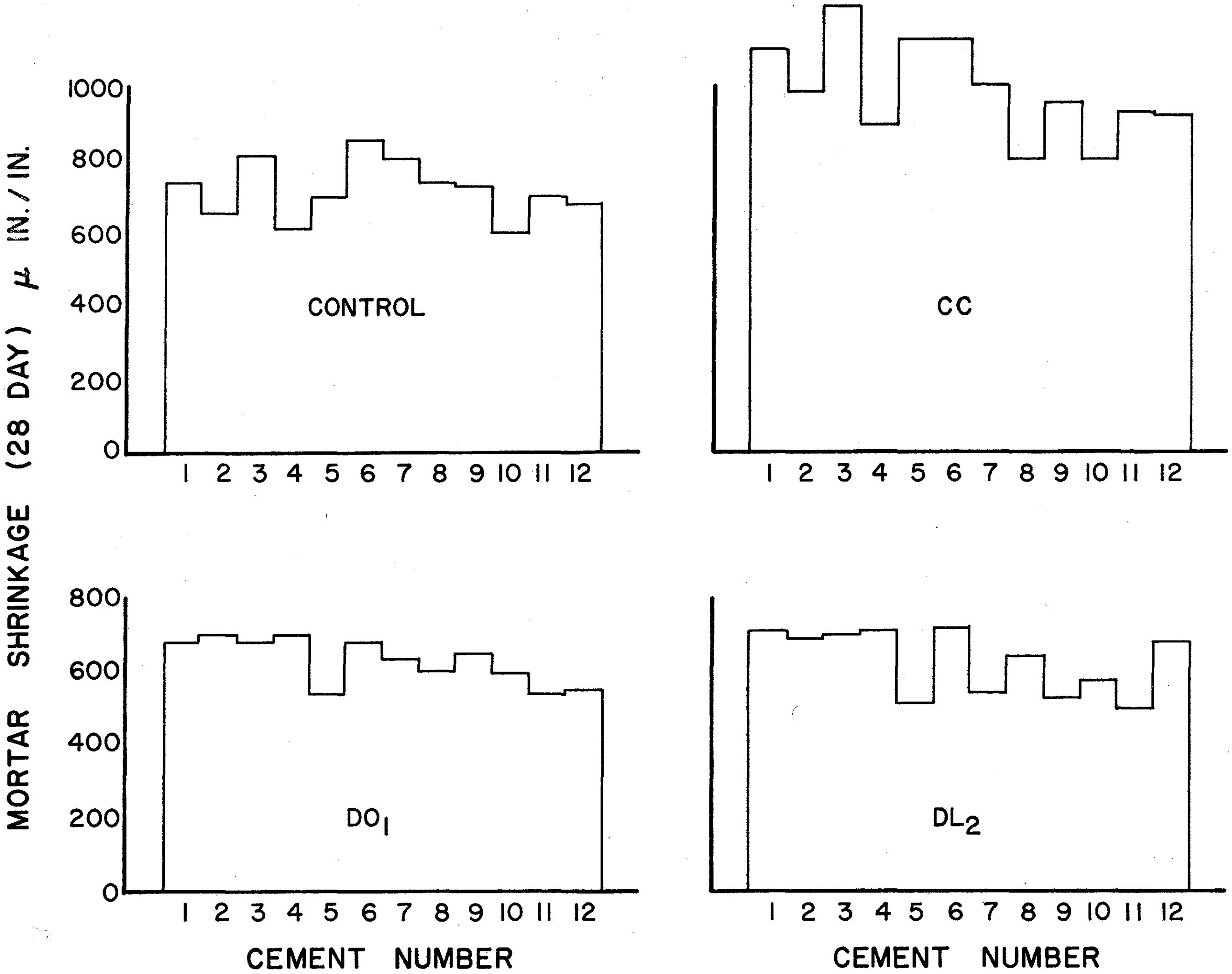

Figure 4. Comparison in mortar shrinkage for twelve cements . (Batch data: Table 13-A.)

tested. Concrete 60-day shrinkage varies from 450* to 650 for four lignosulfonates, 380 to 500 for three organic acids and 420 to 480 for two polymers. In mortars the 28-day shrinkage of the lignosulfonates varied from 840 to 950, organic acids 650 to 860, and polymers 850 to 870.

Only one retarder (DL_4) failed to meet the ASTM requirement (C494-63T) that the admixture concrete produce drying shrinkages less than 135% of the control (no admixture) batches. Admixture DL_4 produced 60-day concrete drying shrinkage that was 154% of control. It should be recognized, however, that three of the lignosulfonates increased concrete drying shrinkage by only 7 to 19% of the control concrete. Calcium chloride also failed to meet the criteria, having a drying shrinkage of 138% of control.

The only admixture consistently reducing drying shrinkage as compared to the control in both concrete and mortar was the organic acid, DO_1.

The greater within-batch specimen to specimen variability of concrete shrinkage specimens with a cross-sectional area to gage length ratio of 1.6 in. was observed as compared to the relatively small variation between mortar shrinkage specimens which had a cross-sectional area to gage length ratio of 0.1 in. The within-batch C. V. of mortar specimens was 3.5%, as compared to 6.5% for the concrete. The batch to batch shrinkage C. V. for the concrete was 10.4%. Batch to batch shrinkage C. V. was not determined for the mortar but it is the author's opinion that it would be considerably reduced over that of concrete due to the greater degree of control that can be maintained on smaller batches using the standard 20-30 Ottawa sand.

The shrinkage variation that can be expected using the cements produced in Texas is indicated by the bar charts of Figure 4. An equal amount of mix water was used for all twelve cements in conjunction with a particular admixture. No effort was made to equalize flow by water variation, but an indication of the various water requirements is given by the flow variation shown with the batch data.

In order to determine just what effect entrained air might have on shrinkage, six batches of mortar were mixed using the tributyl phosphate detraining agent and progressively increasing dosages of vinsol resin. Batches with air entrainments from 3 to 8% were achieved.

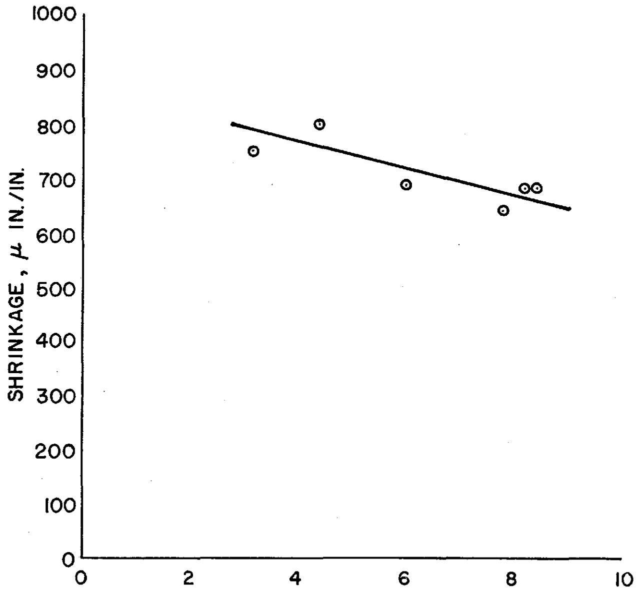

Figure 5. Effect of entrained air on shrinkage. (Batch data: Table 17-A.)

Mortar shrinkage versus air entrainment is plotted in Figure 5 showing an apparent decrease in shrinkage with increased air entrainment. If this trend is indeed valid for air variations between lignosulfonate batches, it certainly has an effect on shrinkage results when proper air control is not maintained. In increasing the entrained air from 4 to 8% a decrease in 28-day drying shrinkage of approximately 100 micro-inches/in., or a 13% decrease is observed.

Set Retardation and Acceleration

The ability of an admixture to influence setting time is one of their main uses, and as such was a subject of prime consideration in this study. In this discussion the time of set from the Ottawa sand mortar batches will be referred to as mortar time of set, while the time of set on the mortars screened from the concrete batches will be referred to as concrete time of set. Time of set determinations on all mortars were by the Proctor penetration test described in ASTM C403-62T. The testing program can be divided into five parts:

1. Comparison of mortars of equal flow with mortars sieved from concretes of equal slump.
2. Comparison of the influence of twelve different cements on the time of set of mortar batches.
3. Effect of entrained air on setting time of mortars.
4. Effect of variation in water on setting time of mortars.
5. Effect of delayed addition of admixtures on setting time of mortars.

The comparison of the mortar with concrete tests is illustrated in Figures 6 and 7. The influence of the admixture on setting time is much more pronounced in the mortar tests. With the exception of one lignosulfonate (DL_1), the retarders are all fairly close to a line with a slope of 4/1 in Figure 7. That is, the effect on initial set retardation is approximately four times as great in the mortars as it is in the concrete. This ratio could not be expected to hold for all concretes.

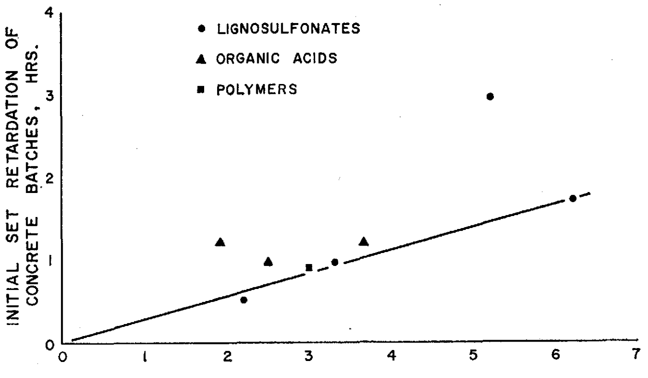

Figure 6. Comparison of mortar and concrete initial set retardation. (Batch data: Tables 8-A, 9-A and 18-A.)

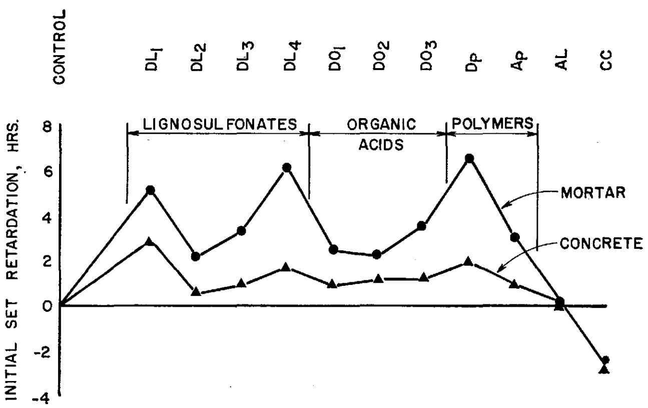

Figure 7. Initial set retardation of mortar batches, hrs. (Data: Tables 8-A, 9-A, and 18-A.)

The influence of twelve different cements on the setting time of the control (no admixture) batch is illustrated by Figure 8. Two separate determinations were made for each cement with approximately a month between determinations. The analysis of these batch repetitions shows an average batch to batch C. V. of 6.71%. If cement number 4 is left out of the calculation, this C. V. is reduced to 4.08%. The two initial set determinations for cement number 4 differed by 1.5 hrs. and are not thought to be representative. Earlier tests shown in Table 19-A yielded a batch to batch C. V. of 3.5%. The range in values was 1.8 hours for time of initial set and 7.4 hours for final set which indicates again the importance of using a single cement when using mortars as quality control test for admixtures.

Figure 9 shows the effect of entrained air on time of set is very slight in the 2% to 4% range. The effect becomes considerable, from 6% to 8%, and would be expected to show a marked effect at higher air contents.

The effect of mortar water content on time of set is illustrated by Figure 10. In the test mortars an increase in water cement ratio of 32% increases setting time by 36%. The importance of comparing retarders at their characteristic water reduction levels is evident.

Small delays in the addition of the retarder beyond the time of cement and water contact has a marked effect on the effectiveness of a retarder. This was first pointed out by Dodson and Farkas (4). They concluded that,

> "Set retarding admixtures are usually added to concrete with the mixing water. Their addition

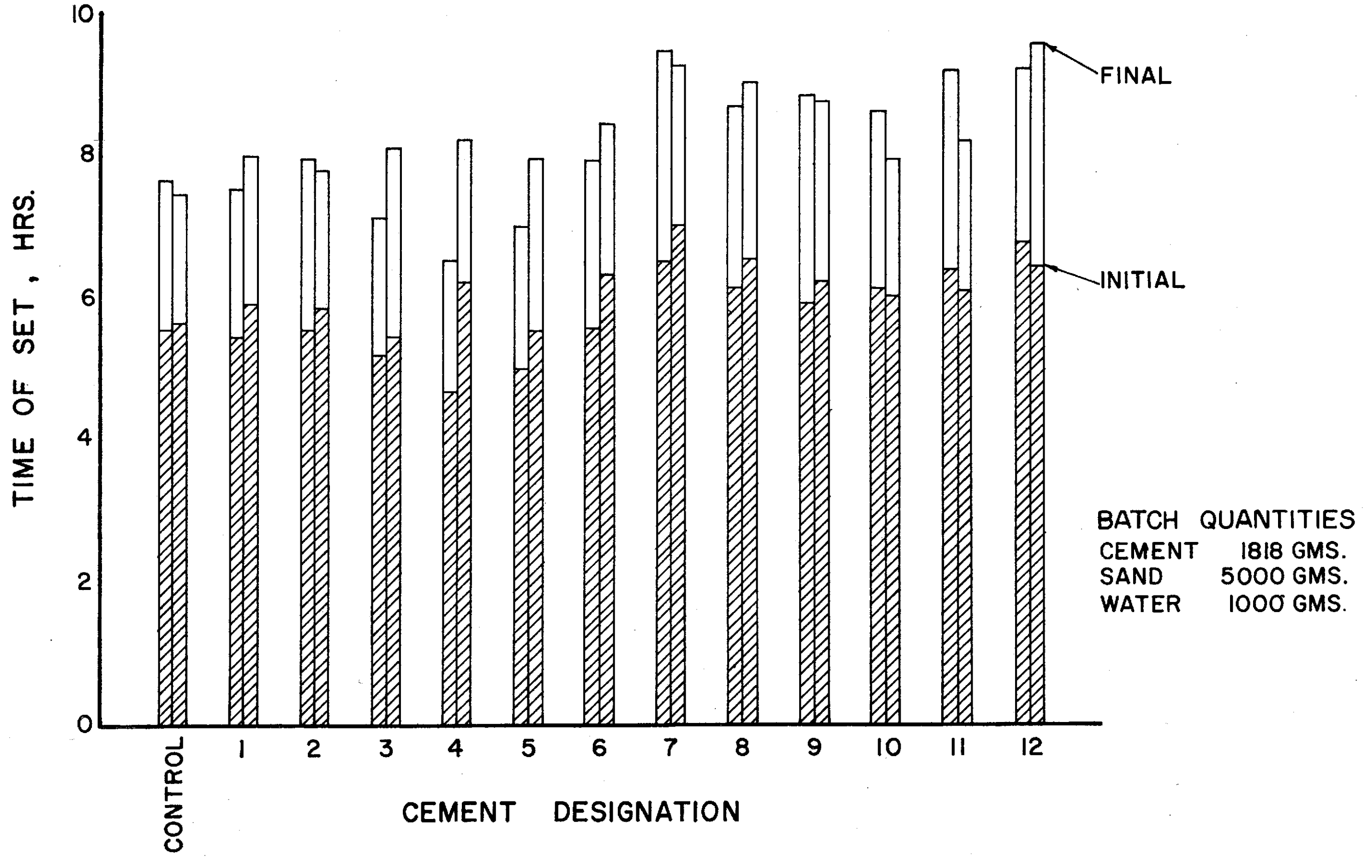

Figure 8. Comparison in setting time of cements.

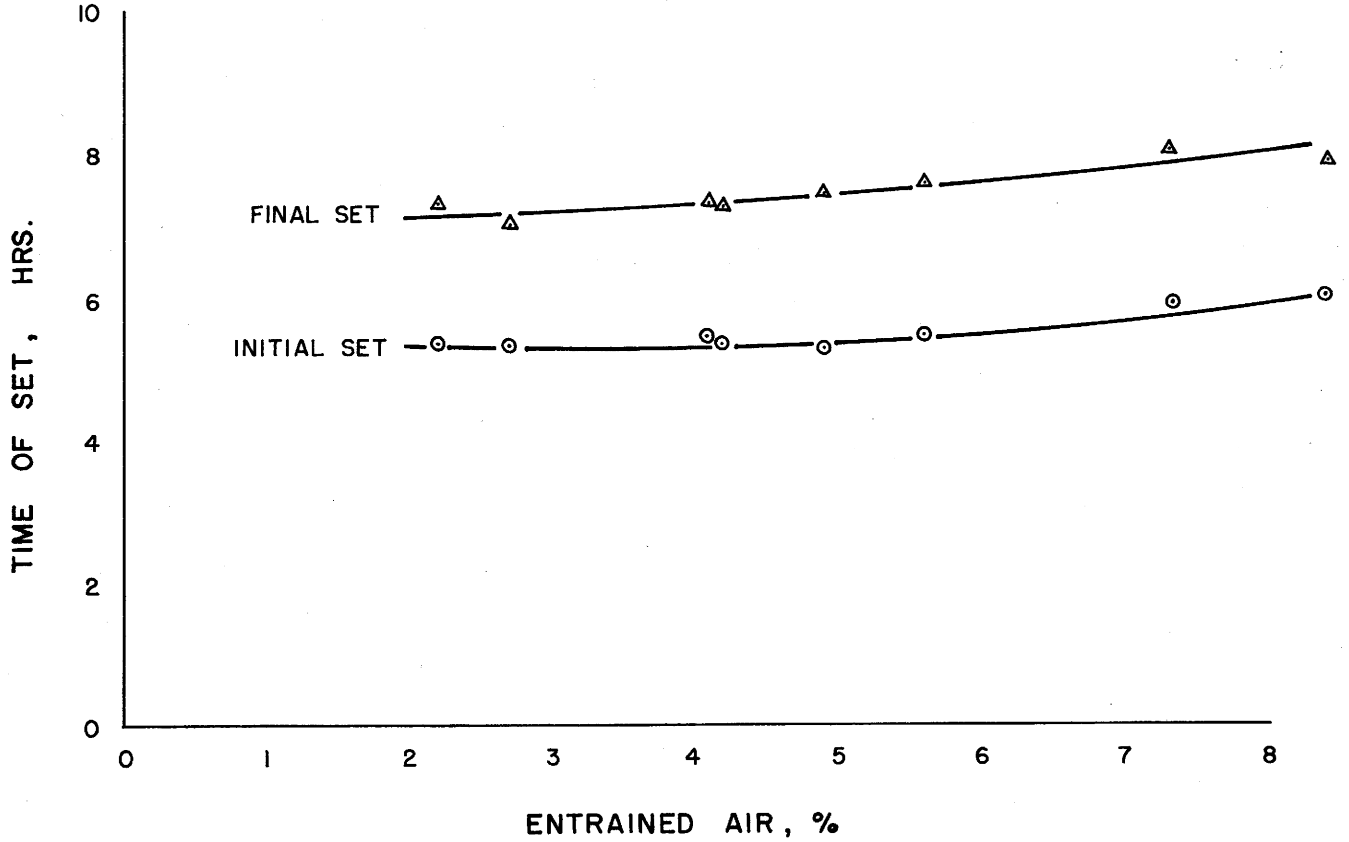

Figure 9. Mortar time of set, air variable. (Batch data: Table 20-A.)

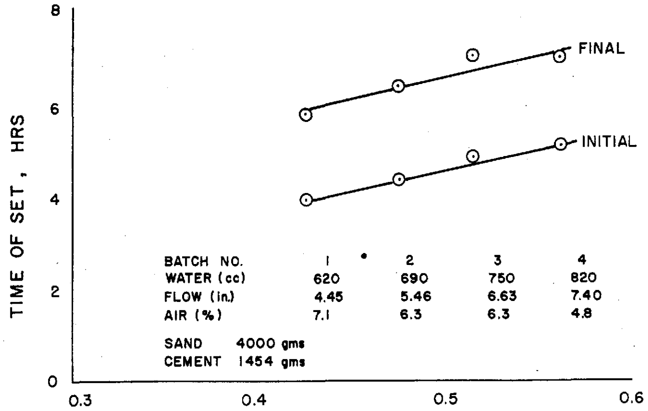

Figure 10. Effect of W/C on setting time.

> after mixing has been started (delayed addition) increases their efficiency as set retarders and increases their capacity to entrain air and improves their water-reducing properties."

This action has been explained by Bruere (5) in the following way,

> "When a retarder is added to cement with the mixing water, it is adsorbed on the C3A before any appreciable amount of gypsum can dissolve in the aqueous phase and make itself available for reaction with the C3A. This leaves a relatively small amount of the retarder available for adsorption on the other components of the cement and thereby retard their hydration reaction.
>
> "The delayed addition of the retarder allows the gypsum time to dissolve and to coat the C3A. When the retarder is finally added, the C3A is unable to adsorb it and a large amount of the retarder is available to retard the silicate hydration reactions."

Some of the data developed by Dodson and Farkas on concrete has been plotted in Figure 11 along with data developed on mortars in this program.

The data from concrete batches in the Dodson and Farkas paper indicates a much smaller delay effect on retardation for the organic acid as compared to the lignosulfonate. In testing the effect of delayed addition using admixtures DO_1 and DL_2 in mortars, the difference shown by the Dodson and Farkas data is not indicated. At delays of 5 seconds and 1 minute the effect on set using admixture DO_1 is slightly greater than that indicated by lignosulfonate DL_2. After two minutes delay however, both organic acid DO_1 and lignosulfonate DL_2 have increased their set retardation by slightly more than 100%. The importance of controlling this factor in testing admixtures in mortars is thereby illustrated.

Air Entrainment

The air entrainment of all mortar batches was determined by a test very similar to that described in ASTM C185-59. The cylindrical measure was 2.5 in. in diameter by 3.625 in. in depth. A 0.25 in. diameter steel rod instead of the prescribed spatula was used to rod each layer. Typical variations in the amounts of air entrained by the various admixtures, when no effort is made to control air by detraining agents, is shown in the mortar batches in Table 22-A. The entrained air varies from 4.3% for the control batch to 12.3% for admixture DL_2.

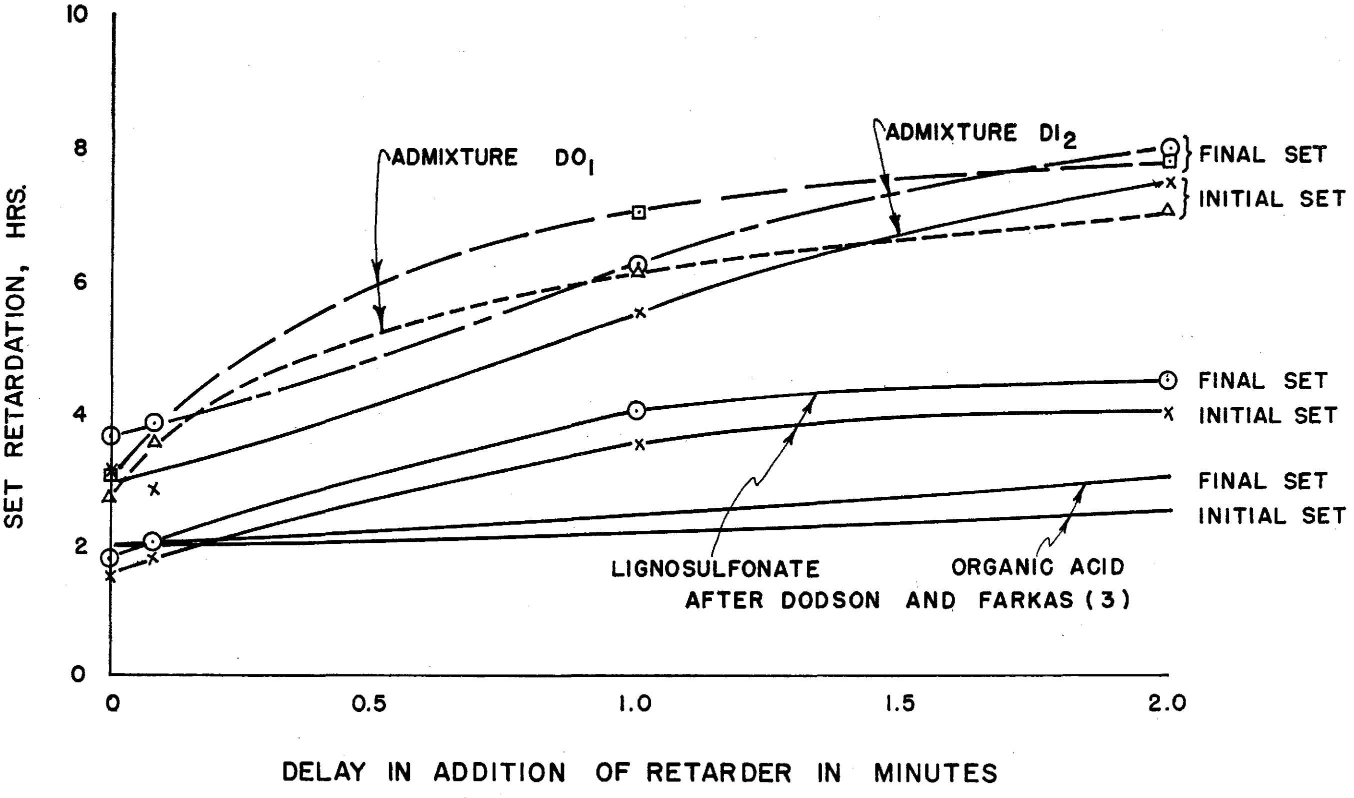

Figure 11. Effect of delay in addition of retarder. (Batch data: Table 21-A.)

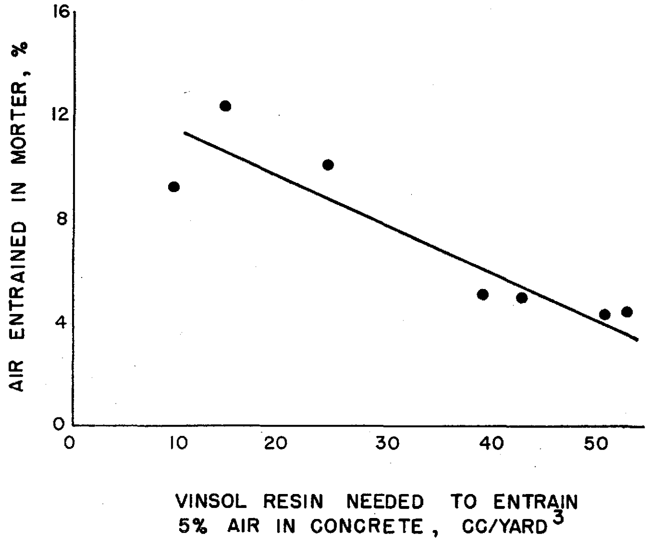

Figure 12. Mortar air entrainment as a function of vinsol resin demand in concrete. (Data: Tables 22-A and 23-A.)

This wide variation is also seen in the average amount of vinsol resin needed to entrain 5% air in the concrete batches. This tabulation is given in Table 23-A.

Figure 12 compares mortar air entrainment with the vinsol resin demand in concrete batches. Although it was not specifically determined, the batch to batch variation of air contents in the lignosulfonate batches seems to be rather high, possibly accounting for the scatter of data in Figure 12. The flow of the mortars in Table 23-A was not controlled as accurately as desired, which again would contribute to data scatter.

In comparing the various admixtures in mortar batches with control batches or with each other it is difficult to stabilize precisely the amount of entrained air. In this program the lignosulfonates tended to entrain considerable amounts of air. This problem was overcome to some extent by the use of an air detraining agent, but still a very precise control of air in the mortars was not achieved. It appears that even the organic acids entrain, or contribute to the entrainment of, small amounts of air (up to 2% over control shown in Table 12-A). The solution may be in using an air detraining agent with all batches, but the organic acid producers were not able to recommend an air detraining agent for use with their product. In the absence of a detraining agent for use with organic acids, a comparison with control batches could be achieved by increasing slightly the air content of the control batch with vinsol resin. A closer control of the lignosulfonate air could be achieved by using a detraining agent in the control batch (lignosulfonates used with detraining agents yield mortars with a smaller air content than the control batch).

Variations in air content have a definite effect on the various mortar properties. Increasing entrained air increases flow, thereby decreasing the necessary water for a set value of flow. These effects are summarized as follows:

1. Water reduction—The change in flow due to variations in air content directly influences mortar water reduction (Table 17-A).

2. Compressive strength—At constant water, increasing air decreases compressive strength (Figure 2). Decreasing water, decreases W/C, increasing compressive strength. These effects are compensating but the direct effect of air is probably larger than the indirect effect on W/C.

3. Shrinkage—At constant water, increasing air decreases shrinkage (Figure 5). Decreasing water content decreases shrinkage. These effects are accumulative.

4. Time of set—At constant water, increasing air increases setting time (Figure 9). Decreasing water content decreases setting time (Figure 10). These effects are compensating with the water reduction effect the larger of the two.

Water Reduction

The ability of chemical admixtures to reduce the necessary batch water has been based on the comparison of concretes of equal slump, with and without the admixture. The equating factor that would be preferred is workability. Although slump is a rather arbitrary measure of workability, the water reduction criteria used here, equating flow of mortars, would be expected to be an even more arbitrary measure. The development of curves of water requirements vs. flow for various admixtures is shown in Figure 13. A considerable difference in apparent water reduction is indicated by the lignosulfonate mortars depending on whether or not an air detraining agent is used. The data developed in this program do not indicate a well defined relationship between water reductions obtained from mortar and concrete batches. This was also indicated by Walker and Bloem

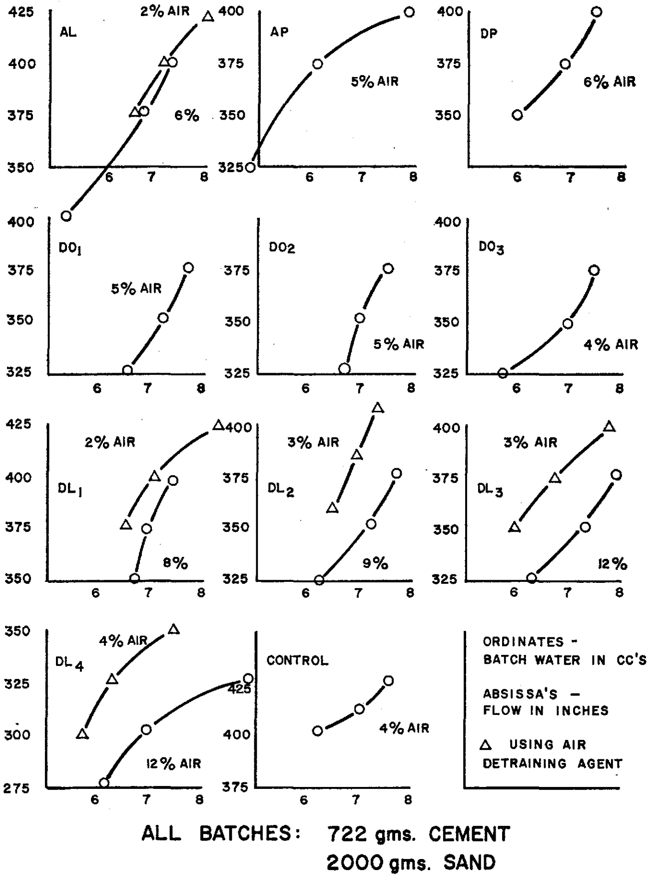

Figure 13. Determination of water quantity for 7" flow.

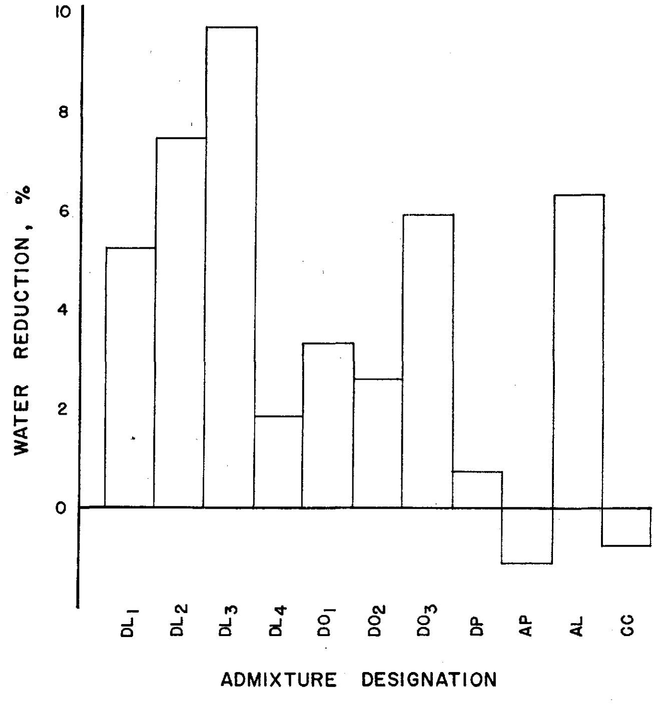

Figure 14. Water reductions of admixtures in concrete. (Batch data: Table 7-A.)

(3) in their comparison of cement water requirements in mortar and concrete. They state, in talking about a plot of mixing water requirements on ASTM C109 mortar and mixing water requirements of a constant slump concrete: "Were it not for the erratic sample No. 26 from source 5, no relationship would be discernible, and the one shown is of doubtful validity."

The water reductions calculated from the concrete batch water requirements are shown in bar graph form in Figure 14. It should be noted that none of the admixtures tested meet the Texas Highway Department specification of 10% for water reduction. Due to the inaccuracies in determining the exact moisture content of the concrete sand, and the influence of this factor on the indicated water requirements, it is not believed these determinations are more accurate than about ± 1%. For instance, it is not the author's opinion (based on other tests on mortars, Figure 13) that admixture AP increases water demand, as is indicated by the concrete batch data, but the water reduction capabilities of this admixture are apparently rather insignificant.

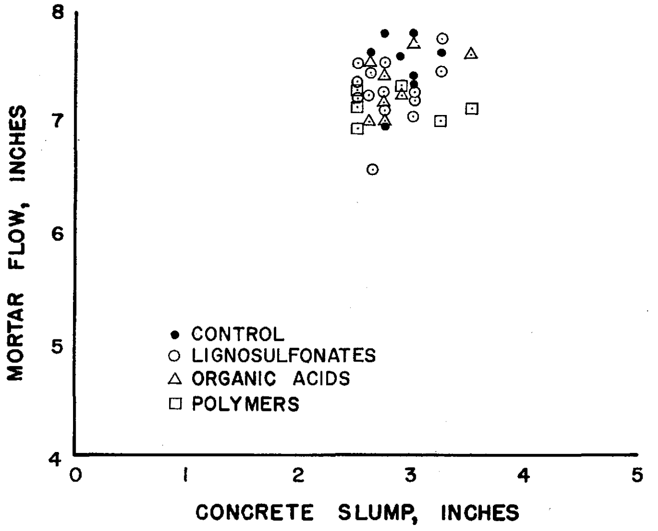

Figure 15. Comparison of mortar flow and concrete slump.

The comparison of concrete slumps and the flow of mortar screened from the concrete is shown by Figure 15. In concretes with slumps varying between 2½ and 3½ inches the variation in mortar flow was from 6.57 to 7.81 inches. The average flow was 7.30 as compared to an average slump of 2.82 inches. The coefficient of variation of the ratio of flow to slump was 8.8%. This comparison justified to some extent the use of the 7-inch flow as a basis of comparison of admixtures in mortars.

Durability

Freeze-thaw durability testing (ASTM C290) was conducted on one 3 in. x 3 in. x 16 in. prism from each of the forty concrete batches cast in the final year's work. All concrete batches had air contents of 5 ± 0.5% with the average of the three batches for each admixture very close to 5%. All specimens were subjected to 300 cycles of rapid freezing and thawing in water with some very definitive results obtained. Although only one durability specimen was obtained from each batch of concrete, the C. V. between the three specimens for each admixture was only 9.1%. The average durability factors were calculated for concrete containing each admixture at both 200 and 300 cycles of freezing and thawing. Figure 16 compares the average durability factors for the various admixtures at the 200 cycle level. The durability factor is defined by the following equations, according to ASTM C290.

$$DF = \frac{PN}{M}$$

$$P_c = \frac{n_1^2}{n^2}\ (100)$$

Where:

DF = durability factor of the test specimen,

P = relative dynamic modulus of elasticity at N cycles, percent,

N = number of cycles at which P reaches the specified minimum value for discontinuing the test or the specified number of cycles at which the exposure is to be terminated, whichever is less, and

M = specified number of cycles at which the exposure is to be terminated.

P_c = relative dynamic modulus of elasticity, percent, after c cycles of freezing and thawing,

n = fundamental transverse frequency at 0 cycles of freezing and thawing, and

n_1 = fundamental transverse frequency after c cycles of freezing and thawing.

Both organic acid and polymer admixtures produced concrete with durability greater than control (no admixture) concrete. Admixtures DO_1, DO_2, DP, and AP indicated considerable increases in durability over the control batches. Admixtures DO_3, DL_1, and DL_2 produced concrete with fundamentally the same durability as the control batches while DL_4, DL_3, AL, and CC were

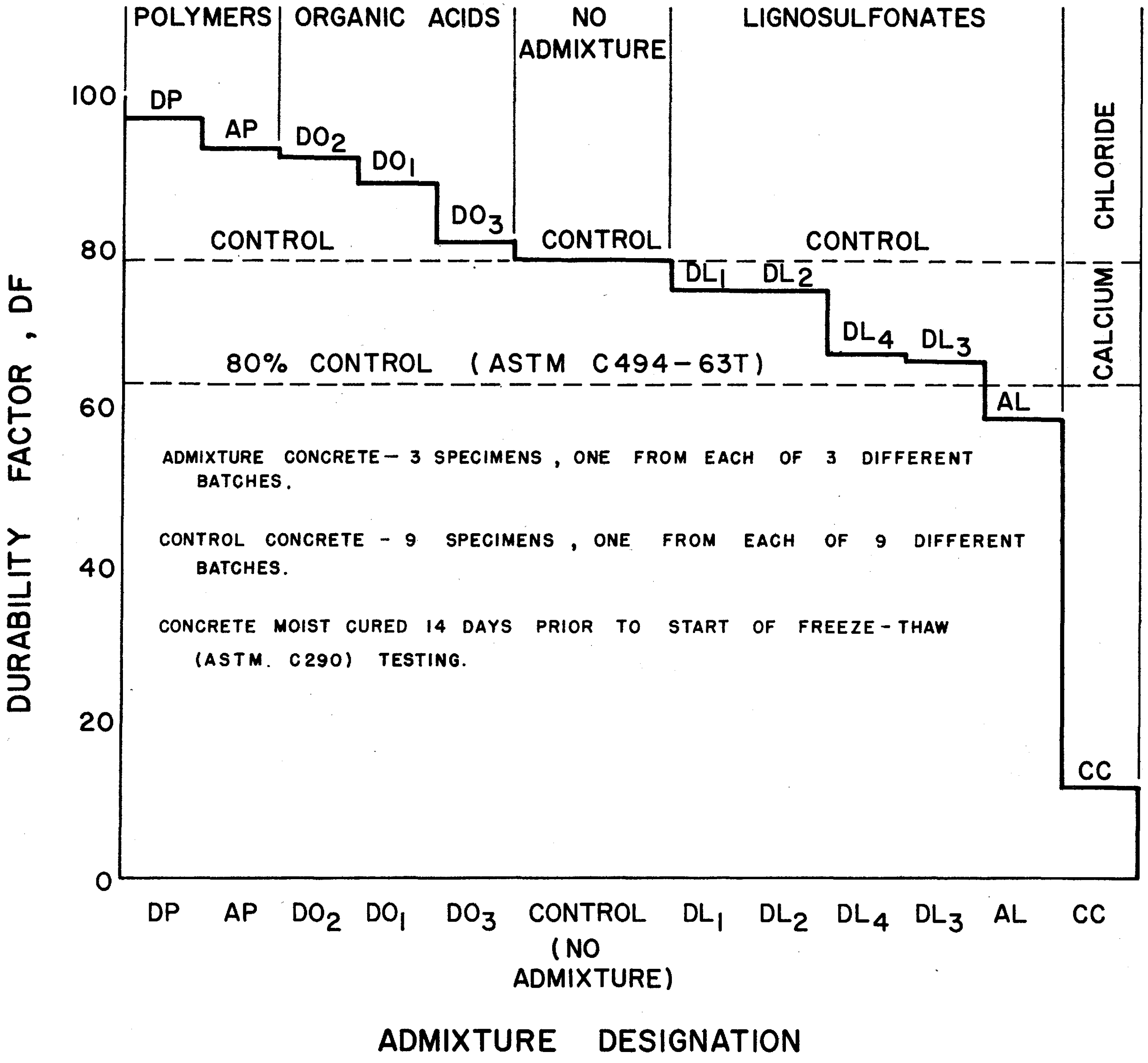

Figure 16. Relative durability of concrete using different admixtures; (N = 200) batch data: Table 7-A, durability data: Tables 10-A and 11-A.

increasingly detrimental to durability in that order. Admixtures AL and CC show durability factors less than 80% of control, the lower boundary set for durability in ASTM C494-63T.

The durability factors calculated at the 300 cycle level are shown in Figure 17. This figure again shows that organic acid and polymer admixtures produced concrete with durability greater than control concrete. Admixture DL_2 produced concrete with fundamentally the same durability as control concrete and admixtures DL_1, DL_3, DL_4, AL, and CC show durability factors less than 80% of control.

Extensibility

By means of a restrained shrinkage test, an indication of the extensibility was achieved. The various admixture mortars were cast in 2 in. x 2 in. x 36 in. prisms centric with a No. 5 bar. For the first 7 days the specimens were subjected to continuous moist curing at 73°F. The specimens were then dried for 21 days at 73°F and 50% R. H. At the age of 28 days the visible cross-sectional cracks were counted, and the average crack spacing for each specimen determined. The crack spacing in inches is plotted as the ordinate in Figure 18. Shown on the abscissa is 28 day shrinkage in percent. It should be noted that as shrinkage increases by 41% (.08% to .113%) average crack spacing decreases by 76% (7 in. to 1.67 in.). This tends to indicate that allowing arbitrary percentage increases in shrinkage of admixture batches over control batches may yield concrete with severe cracking problems.

The following equation was derived (appendix) to predict the crack spacing as a function of shrinkage and

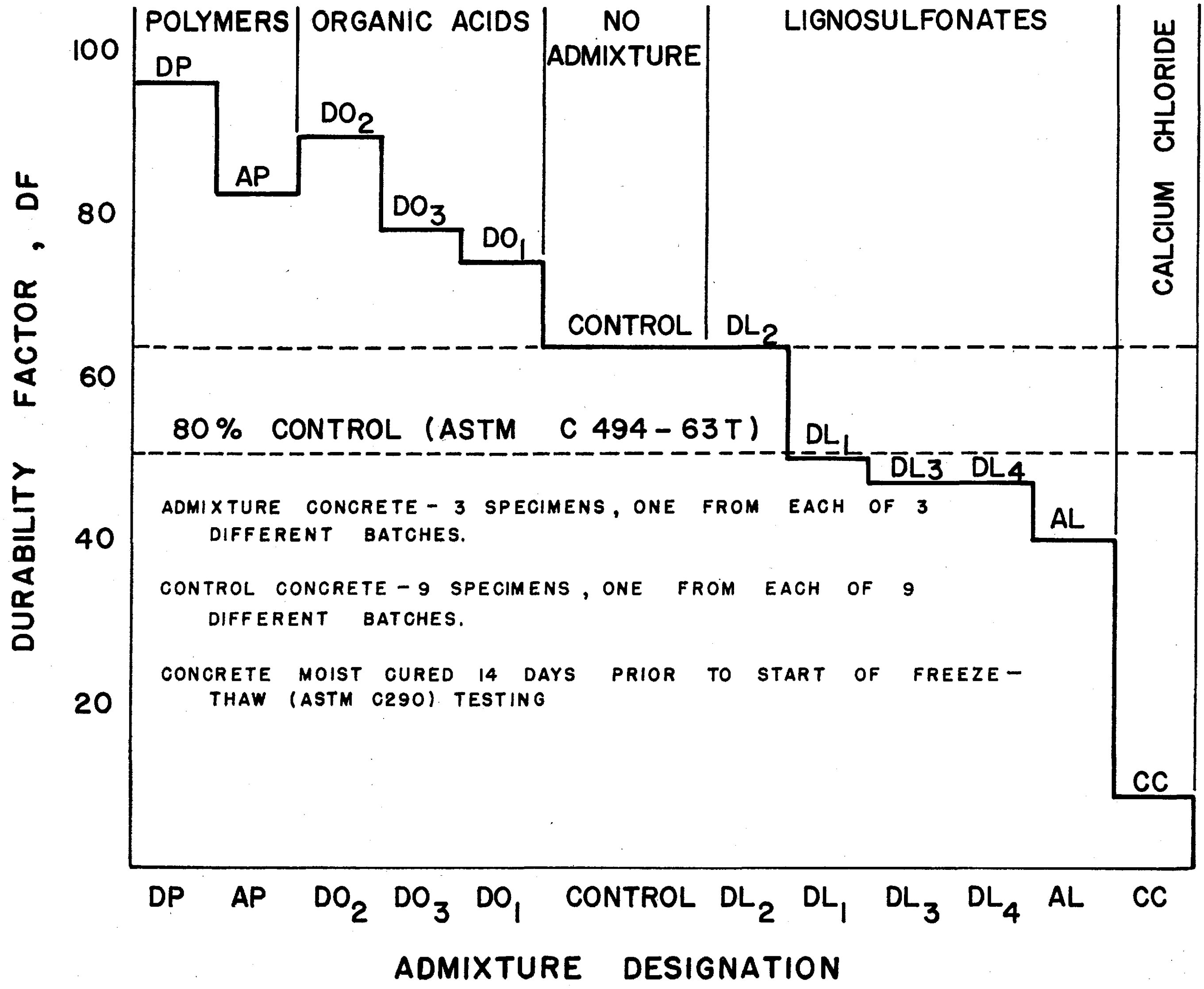

Figure 17. Relative durability of concrete using different admixtures (n = 300).

the geometric and elastic properties of restrained shrinkage specimens.

$$N = \frac{3}{4}\,\frac{L}{l_o}\left\{1 - \frac{f'^{\,2}_t\left(\frac{1}{E_c} + \frac{Ac}{E_sA_s}\right)^2}{\epsilon_{sh}^2}\right\}$$

Where

N = total number of cracks forming in a restrained shrinkage specimen of length L,

L = length of specimen,

l_o = distance from point of cracking to point where longitudinal stress due to shrinkage is not reduced by the crack formation,

f'_t = tensile strength of the concrete,

E_c = concrete modulus of elasticity,

E_s = steel modulus of elasticity,

A_c = concrete cross-sectional area,

A_s = steel cross-sectional area, and

ϵ_{sh} = unrestrained shrinkage strain.

Figures 18 and 19 show the correlation of theoretical and observed data points. There is a high specimen to specimen variability, as shown by the data points on Figure 19.

Admixture Uniformity

Infrared spectrographs (IRS) of samples submitted to Texas Transportation Institute by ready-mix concrete producers and admixture producers have not indicated variations in chemical constituents within given commercial products. Figure 20 shows multiple IRS scans of admixtures DL_2, DO_1, and DO_2 and single IRS scans of admixtures AP and DP. Although peak intensity variations are present, the variations shown in these figures can be accounted for in the inaccuracies involved in producing IRS pellets from highly absorptive materials.

Occurrence of new peaks, disappearance of peaks, or large variations in relative peak intensities can be interpreted as probable variations in the chemical constituents of admixtures. It should be understood, however, that variations in IRS scans can only be evaluated

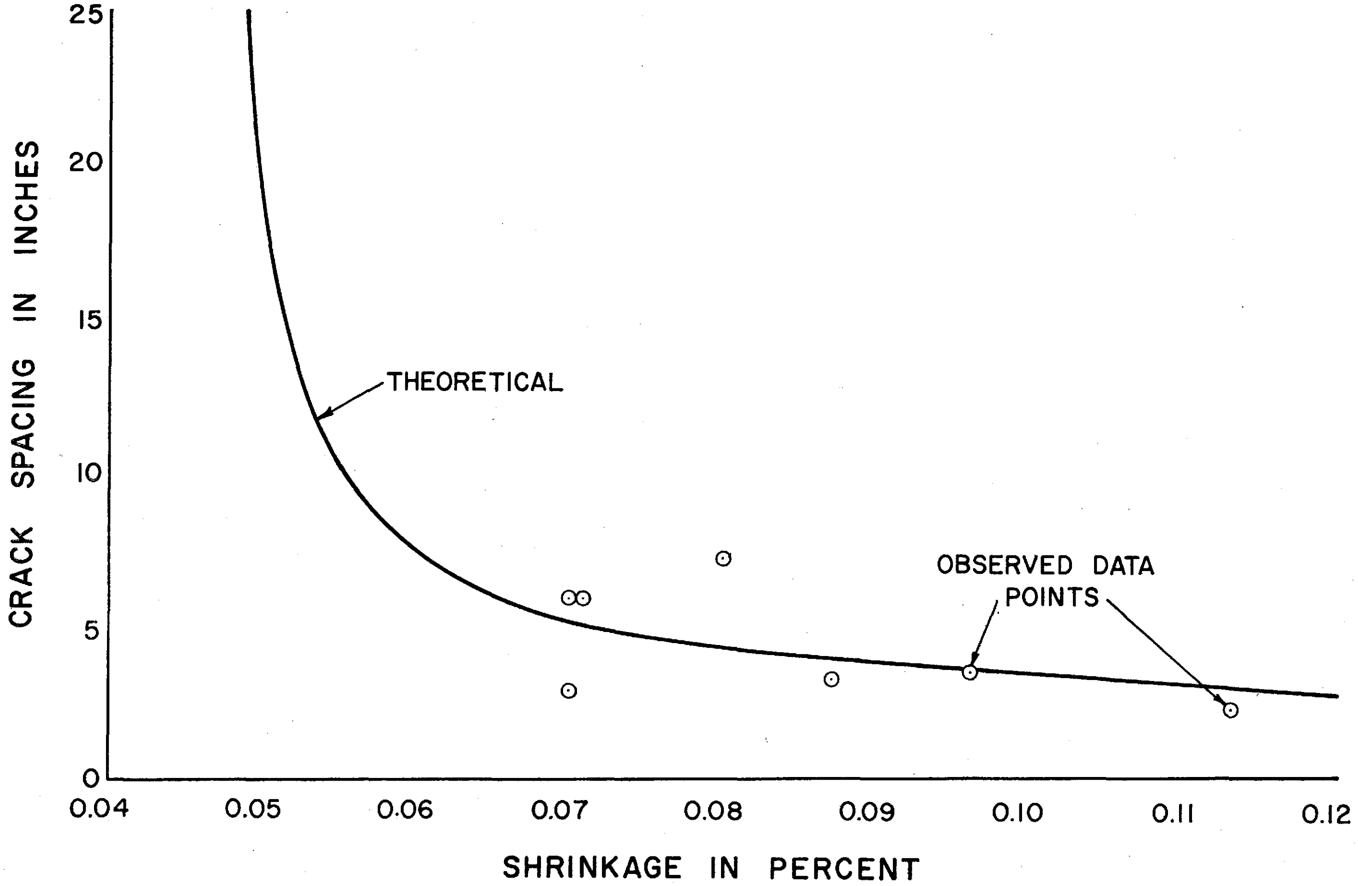

Figure 18. Comparison of theoretical crack spacing with observed data points, *using $l_o = 2''$, $E_c = 4 \times 10^6$ psi, $E_s = 30 \times 10^6$ psi, $f'_t = 500$ psi, $L = 36''$, $A_s = 0.2$ in^2, $A_c = 3.8'$ in^2. The theoretical curve is obtained from equation 11.

by persons skilled in IRS technology. No simple rules of allowable variation in magnitude or position of peaks can be set forth to allow interpretation of IRS scans by persons unfamiliar with IRS technology. This would seem to present no real problem however, since the samples must be sent to an IRS laboratory where skilled personnel would be available to interpret variations in the scans.

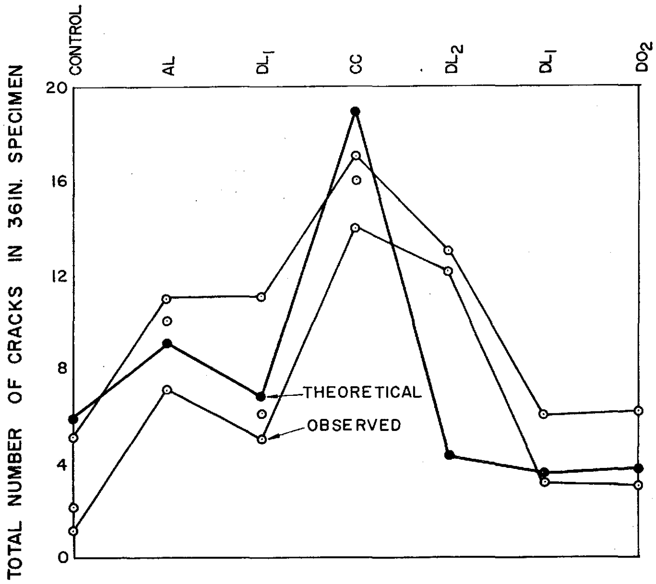

Figure 19. Comparison of theoretical solution and observed data points. (Batch data: Table 22-A.)

In view of the apparent need for a field quality control test, a specific gravity determination on fluid admixtures is a quick and easily accomplished test for admixture concentration that could easily be run by inspectors using a hydrometer. It is recognized that all solids in chemical admixtures are not active in modifying concrete properties. This is especially true in the case of admixtures supplied in powder form, where various inactive constituents are added to make convenient powder to water mixing ratios. Nevertheless, a hydrometer check during batching operations would furnish a convenient means of assuring accurate mixing of admixture powder and water and prevention of variations due to settlement of the liquid admixture while in storage.

The fluid specific gravity and a determination of the percent solids by weight has been used to calculate a specific gravity of solids value for five of the admixtures tested in this project. This value was calculated from the results of 5 separate hydrometer readings and 3 separate percent solids determinations on a single sample of each admixture. These data are given in Table 26-A. Using this calculated value, the graphs of Figure 21 show what variations in specific gravity of the liquid will mean in terms of the percent variation of solid admixture material. Each black point represents the average of the 5 hydrometer readings and the average of the 3 percent solids determinations for a particular admixture. The open points represent values of specific gravity and percent solids obtained from samples of admixtures sent in by ready-mix concrete producers in the state-wide sampling that was conducted. The values used in plotting the lines shown were calculated from the equation,

$$G_{ad} = \frac{1}{1 - \frac{Ps}{100}\left(1 - \frac{1}{G_s}\right)}$$

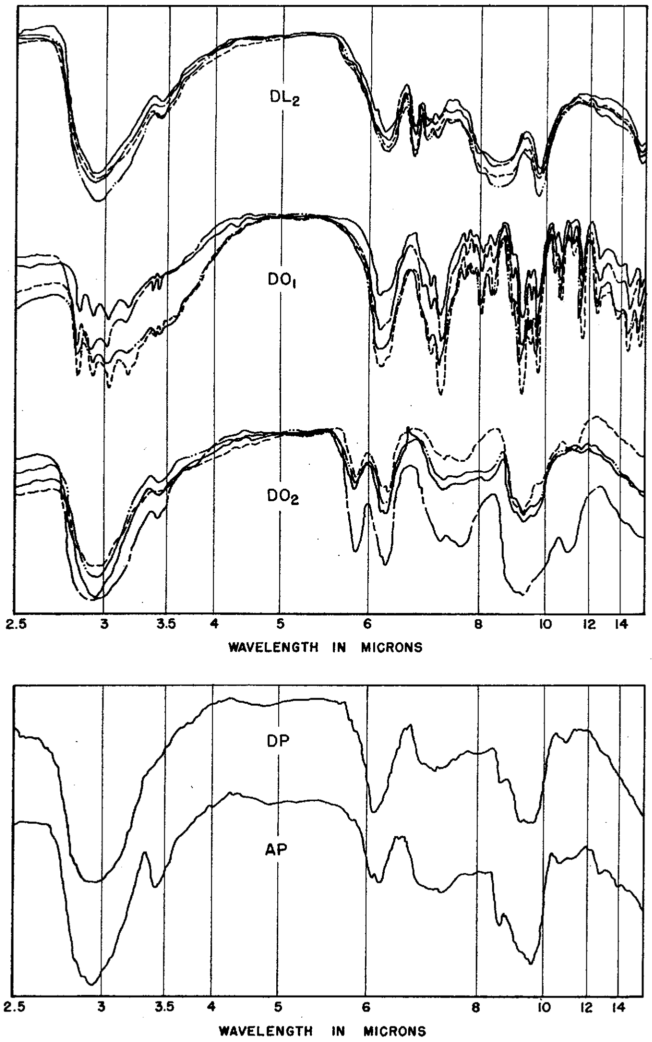

Figure 20. Infra-red spectographs.

Where

G_{ad} — Specific Gravity of Liquid Admixture

G_s — Specific Gravity of Admixture Solids

P_s — Percent Solids in Liquid Admixture by Weight.

As can be seen in the figure for admixture AL, a variation of only .01 in the specific gravity of the liquid will result in a change of 22 percent in the amount of solid admixture added to a concrete.

Thus, a specific gravity determination in the field could be used either as an acceptance test or as a basis for increasing or decreasing the admixture dosage in order to make the amount of admixture solids added to the concrete in accordance with the approved amount.

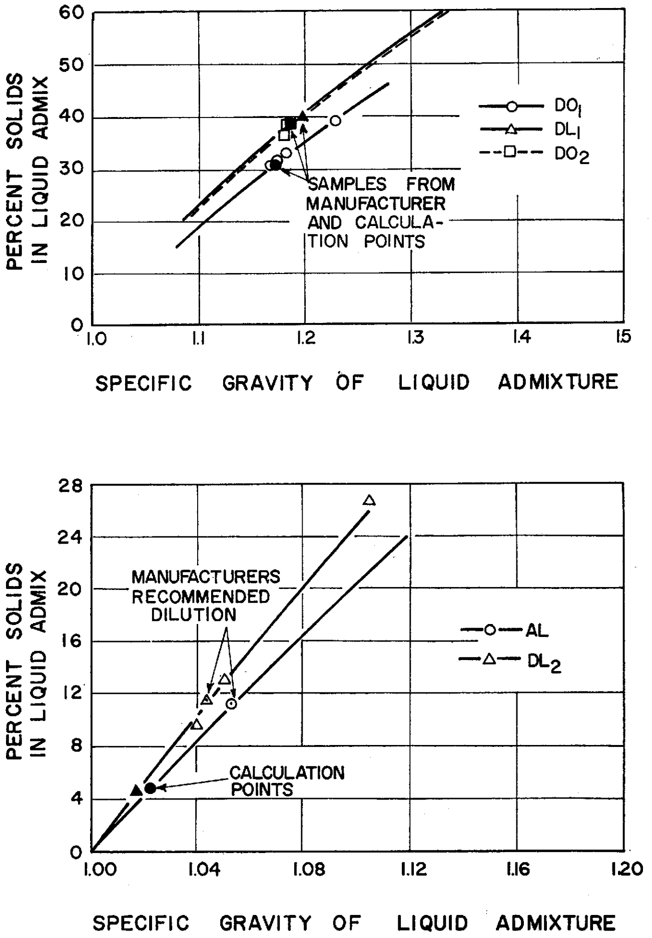

Figure 21. Variation of percent solids with specific gravity of liquid admixture.

Chapter IV.

Summary and Conclusions

The data and test procedures presented in this paper provide a basis for the use of cement mortars as quality control tests for chemical admixtures. Comparisons between mortar tests and concrete tests in the areas of compressive strength, shrinkage, and setting time show some promise for the use of mortar tests as indicators of the relative performance between admixtures.

Compressive Strength. All admixture concretes and mortars tested showed higher 7- and 28-day compressive strength than control concretes or mortars when they were allowed to utilize their characteristic water reductions.

Comparison of the compressive strengths of Ottawa sand mortar batches and mortar from concrete batches yielded a coefficient of variation in the ratios of the two values for each admixture of only 5.3%. The C.V. of the ratio of 28-day mortar compressive strengths to 28-day concrete was 9.1% (Table 1).

Shrinkage. The results of shrinkage tests indicate some differences in the effect of different chemical types in both concrete and mortar (Figure 3). As a class, even though they have in general higher water reductions, the lignosulfonates show higher drying shrinkage than the organic acids and polymers tested. Concrete, 60-day shrinkage varies from 450* to 650 for four lignosulfonates, 380 to 500 for three organic acids and 420 to 480 for two polymers. In mortars the 28-day shrinkage of lignosulfonates varied from 840 to 950, organic acids 650 to 860, and polymers 850 to 870.

Only one retarder (DL_4) failed to meet the ASTM requirement (C494-63T) that the admixture concrete produce drying shrinkages less than 135% of control concrete. Calcium chloride also failed this criteria. The only admixture consistently reducing drying shrinkage as compared to the control in both concrete and mortar was DO_1.

The ratios of 28-day mortar shrinkage to 60-day concrete shrinkages showed a C. V. of 10.1% (Table 2).

Setting Time. The effect of retarders on setting time is significantly increased by using Ottawa sand mortars as opposed to mortar screened from concrete batches. Comparison of these mortars with mortar from the siliceous aggregate concrete batched in this program indicated an increase in setting time of approximately four to one (Figures 6 and 7).

Batch to batch coefficients of variation from 3.5 to 4.1% were indicated for mortar batches.

Delayed addition of retarders to mortar batches increased set retardation by more than 100% over mortar batches with the admixture added with the gage water (Figure 11).

Air Entrainment. High variations in the air entrainment characteristics of different admixtures were observed (Figure 12). Lignosulfonates entrain significant amounts of air while organic acids and polymers may contribute to the entrainment of small amounts of air (Tables 12-A and 18-A). The relationship between mortar air entrainment and the vinsol resin needed to entrain 5% air in concrete is illustrated by Figure 12.

In mortar testing, variations in entrained air have an effect on water reduction, strength, shrinkage, and setting time. These effects are summarized in Chapter III.

Water Reduction. A lack of correlation between water reductions indicated by mortars and concretes was observed. Two factors are believed to contribute to this observation: (1) Difficulty in maintaining precise control of entrained air in mortars and the resulting effect on water demand. (2) Difficulty in precisely determining sand moisture contents when batching concrete, and the resulting effect on calculated batch water.

Water reductions calculated from concrete batch data indicate that none of the admixtures reduced the water requirements by as much as 10%. Lignosulfonates showed higher water reductions, as a class, than did organic acids. Polymers did not show significant water reductions.

Concrete Durability. The freeze-thaw testing of concrete according to ASTM C290 indicated significant differences in the durability of concrete using the various chemical types of admixtures. Polymers and organic acids produced concretes of higher durability than control concrete, while lignosulfonates, as a class, produced concretes with lower durability. Admixture DL_2 produced concrete with durability not significantly different from the control concrete. Admixtures DL_1, DL_3, DL_4, AL, and CC did not meet the ASTM requirement that admixture concrete show durability not less than 80% of control concrete. Admixtures DP, AP, DO_1, DO_2 and DO_3 improved the durability of the test concrete (Figure 17).

Cements. Twelve type 1 cements manufactured in Texas showed significant differences in mortar tests of compressive strength, shrinkage, and setting time (Chapter III).

Extensibility. The extensibility test used in this study has shown some correlation with a theoretical solution for restrained shrinkage crack spacing (Chapter III). It was noted that shrinkage increases of 41% decreased average crack spacing by 76%. This tends to indicate that allowing arbitrary percentage increases in shrinkage may have a marked effect on concrete cracking.

Admixture Uniformity. Infrared spectrographs of admixture samples obtained on a state-wide basis did not show significant variations in chemical constituents (Figure 20).

Hydrometer specific gravity tests can be used to determine admixture concentration in the field (Figure 21).

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*Micro-inches/inch.

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Appendix

Table 1-A
UNIVERSAL ATLAS CEMENT
Mill Test Report
Atlas T-1 Portland Cement

CHEMICAL ANALYSIS

SiO_2	20.8
Al_2O_3	5.76
Fe_2O_3	2.44
CaO	66.0
MgO	1.0
SO_3	2.3
Loss on Ignition	1.0

Gilmore Setting Time
Initial 3 hrs 20 min
Final 6 hrs 10 min

Autoclave Expansion 0.09
Specific Surface 1950

Compressive Strength (2″ cubes)
1:2.75 G.O.S. by wt.
3 Day ....3200 psi
7 Day ....4550 psi

Tensile Strength
1:3 S.O.S. by wt.
3 Day ....340 psi
7 Day ....460 psi

Original report certified by:
H. W. Husst, Inspector
February 8, 1965

Table 2-A
DESCRIPTION OF ADMIXTURES

Admixture	Type	Standard Dosage Used in Program	Description of Admixture
AL	Water Reducing	¼ lb./sack	Powder lignosulfonate, contains approx. 20% chlorides.
DL_1	Water Reducing Set Retarder	8.0 oz./sack	Liquid lignosulfonate
DL_2	Set Retarder	¼ lb./sack	Powder lignosulfonate
DL_3	Set Retarder	0.35 lb./100 lb.	Liquid lignosulfonate
DL_4	Set Retarder	0.35 lb./sack	Liquid lignosulfonate
DO_1	Set Retarder	3 oz./sack	Liquid organic acid
DO_2	Set Retarder	3 oz./sack	Liquid inorganic complex of hydroxy carboxylic acid.
DO_3	Set Retarder	3 oz./sack	Liquid organic acid
DP	Set Retarder	5 oz./sack	Liquid hydroxylated polymer
AP	Water Reducing	5 oz./sack	Liquid hydroxylated polymer Contains approx. 11% chlorides
CC	Set Accelerator	2 lb./100 lb.	Flakes, calcuim chloride, anhydrous granular, 8 mesh.

Table 3-A
CEMENT MILL ANALYSIS
(From Manufacturer)

Cement	Chemical Constituents									
	SiO_2	Al_2O_3	Fe_2O_3	MgO	SO_3	Loss on Ign.	CaO	Na_2O	K_2O	Res.	Free Lime
Control	20.80	5.76	2.44	1.00	2.30	1.00	66.00				
1	20.50	5.10	3.70	1.10	2.00	1.40	65.60	0.40	0.10	0.10	
2	21.80	4.90	2.90	1.20	2.20	0.80	66.20			0.20	
3	21.50	5.10	2.60	1.80	2.60	1.90	63.50				0.60
4	21.06	5.70	2.12	1.15	2.42	0.68	66.36	0.37	0.10	0.20	
5	21.96	4.16	2.92	1.14	2.02	0.55	66.80	0.18	0.35	0.41	
6	21.70	4.90	2.20	1.40	2.30	0.80	66.70	0.56	0.05		
7	20.94	6.12	2.11	1.08	2.02		66.70	0.28		0.02	0.86
8	20.20	6.52	2.38	0.80	1.78	0.90	66.88				
9	21.90	5.66	2.08	0.85	2.52	1.24	65.76				
10	20.88	5.90	3.24	0.90	2.31	0.39	66.10				
11				0.90	2.33	1.24				0.10	
12	21.26	5.87	2.85	1.09	2.32	1.00	65.80	0.11	0.42	0.14	

Table 4-A
VARIATION IN SHRINKAGE AND STRENGTH, MORTAR

Batch Data, Table 5-A Shrinkage			Batch Data, Table 6-A Compressive Strength		
Batch	Shrinkage[1]		Batch	Compressive Strength[2]	
J1	820	810	R1	4450	4010[4]
	790	810		4380	
J2	1000	970	R2	4990	4990
	970	930		4930	
J3	900	890	R3	5250	5310
	860	830		5200	
J4	1170	1150	R4	5560	4040[4]
	1140	1110		5250	
J5	750	740	R5	5570	5540
	750	710		5540	
J6	760	720	R6	6260	5370
	650			6070	
J7	730	720	R7	5200	4880
	690	660		5250	

Average within-batch[3] coefficients of variation
Shrinkage — 3.5%
Compressive Strength — 2.4%

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[1] 28 day shrinkage of individual mortar specimens in micro-in./in., 1 in. x 1 in. x 11 in. prisms.

[2] 7 day compressive strength of mortar cylinders in psi, 2 in. diameter x 4 in. length.

[3] Batch to batch variations are not indicated since each of these batches contains a different chemical admixture.

[4] Left out of averages in Table 6-A because of faulty caps. Each number under "Compressive Strength" is a single cylinder test.

Table 5-A
MORTAR SHRINKAGE

Date—March 30, 1965
Temperature—72°F
Humidity—53%

Cement—727
Sand—2000

Batch No.	Admix-ture	Dosage	Water cc	Flow Inches	Air %	Batch Temp. °F
J-1	None		410	7.05	4.57	70.88
J-2	AL	38 cc 1/20	345	7.30	13.41	71.96
J-3	DL_1	80 cc 1/20	355	7.04	8.96	69.44
J-4	CC	20 gms	405	7.12	6.96	72.68
J-5	DL_2	38 cc 1/20	300	7.20	14.62	73.04
J-6	DO_1	30 cc 1/20	370	6.53	5.87	70.16
J-7	DO_2	30 cc 1/20	380	6.86	7.48	66.56

Table 6-A
MORTAR STRENGTH BATCHES
Cement 1454
Sand 4000

Batch No.	Admixture	Dosage	Water cc	Flow Inches	Air %	Batch Temp. °F	Compressive strength 7 day ksi	Compressive strength 28 day ksi
R-1	None	None	820	7.09	4.15	74.84	4.42	5.27
R-2	AL	77 cc 1/20	690	6.53	12.07	74.48	4.98	6.02
R-3	DL$_1$	160 cc 1/20	710	7.09	7.48	74.12	5.23	6.20
R-4	CC	40 gms	810	7.06	7.04	75.92	5.41	6.69
R-5	DL$_2$	77 cc 1/20	600	6.51	14.22	75.20	5.56	7.03
R-6	DO$_1$	60 cc 1/20	740	7.16	5.51	75.20	5.91	7.51
R-7	DO$_2$	60 cc 1/20	760	7.31	5.51	75.20	5.12	7.05

Table 7-A
CONCRETE DATA

Batch	Cement sks./c.y.	Aggregate Coarse lbs./c.y.	Aggregate Fine lbs./c.y.	Total Water lbs./c.y.	Air %	Slump in.	Shrinkage 7 day μin./in.	Shrinkage 28 day μin./in.	Compressive strength 7 day ksi	Compressive strength 28 day ksi
C1	5.51	1790	1360	267	4.8	2⅝	265	410	3.87	4.60
C2	5.51	1790	1360	270	4.7	2¾	245	380	3.80	4.54
C3	5.54	1800	1360	270	4.8	3	360	480	3.71	4.53
C4	5.46	1770	1360	266	4.8	3¼	270	420	3.75	4.69
C5	5.47	1770	1360	267	4.5	2⅞	240	425	4.00	4.95
C6	5.45	1770	1360	276	4.6	3	250	480	3.89	4.93
C7	5.47	1770	1380	268	4.7	2¾	330	420	3.61	3.96
C8	5.52	1790	1360	276	4.8	3¼	320	470	3.66	4.24
C9	5.48	1780	1360	274	4.8	3	420	450	3.86	4.58
C10	5.48	1780	1360	280	5.3	3	300	360	3.32	4.30
AP1	5.47	1770	1360	275	5.0	3¼	350	495	4.24	5.06
AP2	5.48	1780	1360	268	5.3	2⅞	355	480	4.14	5.13
AP3	5.47	1770	1360	278	4.8	2½	330	470	4.03	4.92
AL1	5.46	1770	1360	256	5.3	3¼	420	450	4.17	4.72
AL2	5.51	1790	1370	252	4.8	2¾	420	490	4.05	4.65
AL3	5.52	1790	1370	254	4.7	2½	430	470	3.80	4.83
CC1	5.49	1780	1360	269	5.5	2½	600	640	3.82	4.76
CC2	5.48	1780	1360	275	5.0	3	530	550	3.89	4.77
CC3	5.48	1780	1360	276	4.8	2½	500	550	4.07	4.60
DO11	5.51	1790	1360	261	5.0	2¾	260	380	3.94	4.81
DO12	5.53	1790	1360	266	4.8	2½	280	390	4.62	5.13
DO13	5.55	1800	1370	259	4.7	3	290	360	4.14	4.93
DO21	5.50	1780	1390	261	4.5	2⅝	300	420	4.16	4.83
DO22	5.49	1780	1380	271	5.0	2⅞	320	450	4.10	4.54
DO23	5.51	1790	1390	261	5.2	3	310	430	3.92	4.51
DO31	5.49	1780	1360	259	5.3	3	370	500	4.14	5.08
DO32	5.46	1770	1360	257	5.2	2¾	385	470	4.05	5.25
DO33	5.48	1780	1370	248	5.6	2¾	435	525	4.16	5.29
DP1	5.49	1780	1370	270	4.8	3½	370	460	4.54	5.48
DP2	5.52	1790	1370	262	5.0	2½	285	400	4.30	5.66
DP3	5.51	1790	1360	275	4.5	2½	305	410	3.96	5.34
DL11	5.53	1790	1370	256	5.0	2⅝	310	490	4.49	5.20
DL12	5.56	1800	1380	257	4.6	2¾	270	440	4.44	5.30
DL13	5.54	1800	1380	259	5.0	2⅝	320	410	4.46	5.39
DL21	5.49	1780	1370	244	4.9	3	360	560	4.33	5.20
DL22	5.48	1780	1370	251	5.0	3¼	320	500	4.05	4.77
DL23	5.50	1780	1370	257	5.5	2½	320	450	4.10	4.92
DL31	5.52	1790	1360	247	5.0	3	510	630	4.19	5.04
DL32	5.53	1790	1370	245	5.0	2¾	400	440	4.21	5.27
DL33	5.51	1780	1360	244	5.2	2½	380	450	4.53	4.81
DL41	5.47	1770	1350	269	5.3	2¾	530	690	4.26	5.79
DL42	5.51	1790	1350	265	5.5	3	590	670	4.17	4.83
DL43	5.49	1780	1360	263	5.2	3¼	480	600	4.37	5.20

Table 8-A
DATA ON MORTAR FROM CONCRETE BATCHES

Batch	Flow in.	Time of Set Initial hrs.	Time of Set Final hrs.	Compressive Strength 7 day	Compressive Strength 28 day
C1	7.61	4.40	6.15	5.60	6.18
C2	7.80	4.95	6.50	5.38	6.25
C3	7.81	4.45	6.45	5.03	5.59
C4	7.64	4.55	6.20	5.86	6.89
C5	7.60	4.75	6.55	5.76	6.73
C6	7.31	4.85	6.55	5.35	7.03
C7	6.95	4.75	6.30	5.12	6.88
C8	7.75	4.40	5.95	4.42	6.38
C9	7.40	4.15	5.45	5.57	5.09
C10	7.35	4.70	6.45	5.09	5.79
AP1	7.00	5.50	7.20	6.14	6.56
AP2	7.30	5.65	7.25	5.44	6.92
AP3	7.32	5.55	7.15	5.57	7.21
AL1	7.47	4.50	6.13	5.79	7.10
AL2	7.54	4.65	6.23	5.16	6.97
AL3	7.35	4.55	6.10	5.52	6.94
CC1	7.03		2.25	5.41	7.61
CC2	6.95	1.83	2.48	5.89	6.21
CC3	6.83	1.70	2.25	5.95	7.16
DO11	7.30	5.60	7.17	6.11	7.22
DO12	7.55	5.70	7.13	6.44	7.38
DO13	7.35	5.55	7.05	6.02	7.16

Table 9-A
DATA ON MORTAR FROM CONCRETE BATCHES
(Continued)

Batch	Flow in.	Time of Set Initial hrs.	Time of Set Final hrs.	Compressive Strength 7 day	Compressive Strength 28 day
DO21	6.98	5.80	7.55	5.59	6.91
DO22	7.27	6.15	7.80	5.46	6.88
DO23	7.30	5.40	6.80	5.06	6.65
DO31	7.68	5.80	8.10	5.01	7.10
DO32	7.04	5.65	7.10	5.63	7.45
DO33	7.20	5.90	7.45	5.51	6.92
DP1	7.11	6.15	7.65	6.08	7.19
DP2	7.15	6.80	8.20	6.24	6.91
DP3	6.95	6.55	8.20	5.86	7.24
DL11	6.57	7.45	9.05	5.95	6.94
DL12	7.14	7.90	9.80	5.57	7.00
DL13	7.26	7.30	9.30	5.73	7.29
DL21	7.28	5.05	6.70	5.73	7.43
DL22	7.80	5.10	6.90	5.86	6.94
DL23	7.24	5.15	6.70	5.79	6.56
DL31	7.20	5.50	7.20	6.52	7.54
DL32	7.25	5.45	6.95	6.06	7.89
DL33	7.55	5.70	7.20	6.40	7.58
DL41	7.10	6.20	7.90	6.18	6.14
DL42	7.03	6.50	8.20	6.21	6.56
DL43	7.47	6.40	8.35	4.77	6.62

Table 10-A
CONCRETE DURABILITY DATA

Specimen	$n^2 \times 10^{-6}$ (1)	$n_1^2 \times 10^{-6}$ (2)	N (3)	P_c (4)	D.F. (5)
Control	3.3969	2.6768	200	78.8	78.8
AP	2.7735	2.5683	200	92.6	92.6
AL	3.5494	2.1296	198	60.0	59.4
CC	3.4447	2.0668	41	60.0	12.3
DO$_1$	3.4969	3.1052	200	88.8	88.8
DO$_2$	3.5494	3.3187	200	93.5	93.5
DO$_3$	3.5494	2.8679	200	80.8	80.8
DP	3.4969	3.3920	200	97.0	97.0
DL$_1$	3.4969	2.6297	200	75.2	75.2
DL$_2$	3.4188	2.5675	200	75.1	75.1
DL$_3$	3.5948	2.4085	200	66.0	66.0
DL$_4$	3.5948	2.4085	200	67.0	67.0

[1] Fundamental transverse frequency at zero cycles of freezing and thawing.

[2] Fundamental transverse frequency at termination of test.

[3] Total number of freeze-thaw cycles at termination of test.

[4] $P_c = \frac{n_1^2}{n^2}(100)$

[5] $DF = \frac{P_c N}{M}$ $M = 200$ cycles. This factor was calculated using $N = 200$ cycles or $P_c = 60\%$ (whichever occurred first) as the test termination points.

Table 11-A
CONCRETE DURABILITY DATA

Specimen	$n^2 \times 10^{-6}$	$n_1^2 \times 10^{-6}$	N	P_c	D.F.[1]
Control	3.3969	2.1614	300	63.6	63.6
AP	2.7735	2.2680	300	82.1	82.1
AL	3.5494	2.1296	198	60.0	39.6
CC	3.4447	2.0668	41	60.0	8.2
DO$_1$	3.4969	2.5772	300	73.7	73.7
DO$_2$	3.5494	3.1767	300	89.5	89.5
DO$_3$	3.5494	2.7508	300	77.5	77.5
DP	3.4969	3.3570	300	96.0	96.0
DL$_1$	3.4969	2.0981	250	60.0	50.0
DL$_2$	3.4188	2.1744	300	63.6	63.6
DL$_3$	3.5948	2.1569	234	60.0	46.8
DL$_4$	3.5948	2.1569	234	60.0	46.8

[1] $DF = \frac{P_c N}{M}$ $M = 300$ cycles. This factor was calculated using $N = 300$ cycles or $P_c = 60\%$ (whichever occurred first) as the test termination points.

Table 12-A
MORTAR SHRINKAGE AND STRENGTH BATCHES

Cement 1454 gms.
Sand 4000 gms.

Batch Number	Water gms.	Admix	Dosage	Air Detraining Agent cc.	Flow Inches	Air	Shrinkage 14 Day	Shrinkage 28 Day	Compressive Strength 7 Day ksi	Compressive Strength 28 Day ksi
1	820	None		0.0	7.17	4.3	440	700	4.41	6.10
2	742	AL	¼ lb/sack	2.0	7.05	3.6	580	930	5.80	7.01
3	750	DL_1	8 oz/sack	1.4	7.14	3.8	540	880	5.63	6.72
4	726	DL_2	¼ lb/sack	2.6	7.00	3.6	500	840	5.68	7.01
5	732	DL_3	0.35 lb/100 lb.	2.3	6.85	3.8	590	920	5.58	7.15
6	724	DL_4	0.35 lb/sack	3.9	6.85	3.4	580	950	5.44	6.96
7	702	DO_1	3 oz/sack	0.0	6.88	6.2	440	650	5.54	6.89
8	704	DO_2	3 oz/sack	0.0	6.93	5.3	490	740	5.69	7.14
9	706	DO_3	3 oz/sack	0.0	7.10	3.8	540	860	5.86	7.00
10	764	DP	5 oz/sack	0.0	7.17	5.3	620	850	5.47	6.42
11	784	AP	5 oz/sack	0.0	7.13	4.1	630	870	5.45	6.33
12	820	CC	2 lb/100 lb.	0.0	7.18	2.7	660	870	5.74	6.65

Table 13-A
MORTAR SHRINKAGE AND TIME OF SET, 12 CEMENTS NO ADMIXTURE

Cement	Flow in.	Air %	Shrinkage 28 day μin./in.	Time of Set Initial hrs.	Time of Set Final hrs.
1	7.15	2.4	740*	5.85	7.95
2	7.15	2.8	650	5.78	7.78
3	6.95	2.6	810	5.43	8.05
4	7.12	2.5	610	6.13	8.16
5	7.79	2.0	700	5.50	7.90
6	7.10	3.7	850	6.30	8.40
7	6.55	2.8	800	6.85	9.20
8	7.15	3.0	740	6.50	8.95
9	6.91	2.6	720	6.20	8.70
10	7.58	1.6	600	6.00	7.90
11	7.60	2.0	700	6.10	8.15
12	7.14	3.0	670	6.40	9.50

Batch Quantities:
Cement 1818 gms.
Sand 5000 gms.
Water 1000 gms.

*Average of 3 specimens.

Table 14-A
MORTAR SHRINKAGE AND COMPRESSIVE STRENGTH, 12 CEMENTS WITH ADMIXTURE DO_1

Cement	Flow in.	Air %	Shrinkage 28 day μin./in.	Compressive Strength 28 day psi
1	6.98	3.3	680*	7350*
2	6.73	4.8	700	7230
3	6.81	5.3	680	6130
4	6.81	4.2	700	6480
5	6.99	4.8	540	6090
6	7.22	6.0	680	6570
7	6.63	5.3	630	7530
8	7.08	5.3	600	7980
9	6.59	5.2	650	6620
10	6.95	4.7	590	7110
11	7.32	4.0	540	6510
12	6.48	5.0	550	6520

Batch Quantities:
Cement 1818 gms.
Sand 5000 gms.
Water 868 ml.

*Average of 3 specimens.

Table 15-A
MORTAR SHRINKAGE AND COMPRESSIVE STRENGTH, 12 CEMENTS WITH ADMIXTURE DL_2

Cement	Flow in.	Air %	Shrinkage 28 day µin./in.	Compressive Strength 28 day psi
1	6.51	12.5	710*	6330*
2	8.32	9.0	690	7190
3	6.18	13.4	700	6300
4	6.55	13.4	710	6860
5	5.66	14.4	510	6770
6	6.61	13.6	720	6230
7	6.23	10.5	540	8040
8	6.48	15.6	630	7680
9	6.88	13.9	530	7960
10	5.90	14.2	580	6440
11	6.91	15.6	500	6180
12	5.68	14.2	680	4760

Batch Quantities:
Cement 1818 gms.
Sand 5000 gms.
Water 703 cc.

*Average of 3 specimens.

Table 16-A
MORTAR SHRINKAGE AND COMPRESSIVE STRENGTH, 12 CEMENTS WITH ADMIXTURE CC

Cement	Flow in.	Air %	Shrinkage 28 day µin./in.	Compressive Strength 28 day psi
1	7.68		1100*	6400*
2	7.62		980	6330
3	7.06		1220	6450
4	7.15		890	6920
5	8.10		1130	5660
6	8.12		1130	6250
7	7.37		1000	7409
8	7.93		800	6260
9	8.15		950	6190
10	8.30		800	6170
11	8.29		930	6120
12	8.22		920	5610

Batch Quantities:
Cement 1818 gms.
Sand 5000 gms.
Water 703 cc.

*Average of 3 specimens.

Table 17-A
EFFECTS OF AIR ENTRAINMENT ON MORTAR BATCHES

Batch	Vinsol Resin Dosage	Air Detrainer Dosage	Flow in.	Air %	Compression 7 day	Compression 28 day	Shrinkage 28 day
SA1		1 cc	7.10	3.2	4950*	6540	750*
SA2		—	6.85	4.4	5240	6640	800
SA3	¼ cc	—	7.10	6.0	4670	6180	690
SA4	½ cc	—	7.25	7.8	4360	6170	640
SA5	¾ cc	—	7.50	8.2	4500	5370	680
SA6	1 cc	—	8.05	8.4	4790	5400	680

Batch Quantities:
Cement 1454 gms.
Sand 4000 gms.
Water 760 cc

*Average of 3 specimens.

Table18-A
MORTAR TIME OF SET BATCHES
Cement 1454 gms.
Sand 4000 gms.
Standard Admixture Dosage

Batch	Water gms.	Admixture	Air Detraining Agent cc.	Flow in.	Air %	Setting Time Initial hrs.	Setting Time ISR hrs.	Setting Time Final hrs.	Setting Time FSR hrs.
1	750	None	0	7.03	4.5	5.15		7.40	
2	700	AL	2.0	6.99	3.6	5.20	0.05	7.23	0.17
3	720	DL_1	1.4	6.98	3.1	10.35	5.20	13.90	6.50
4	722	DL_3	2.3	7.17	3.0	8.45	3.30	10.55	3.15
5	671	DO_3	0	6.89	6.6	8.83	3.68	11.17	3.77
6	739	DP	0	7.05	5.7	11.70	6.55	14.25	6.85
7	729	AP	0	7.15	5.5	8.14	2.99	10.20	2.80
8	760	CC	0	6.96	5.2	2.60	—2.45	3.50	—3.90
9	800	None	0	7.03	3.3	4.95		6.95	
10	800	DL_2	2.6	7.04	2.2	7.15	2.20	9.70	2.75
11	724	DL_4	3.9	6.82	3.4	11.10	6.15	14.45	7.50
12	702	DO_1	0	6.95	5.9	7.50	2.55	9.85	2.90
13	704	DO_2	0	6.80	5.0	7.10	1.95	9.35	2.40

Batches 2 through 8 compared to control batch 1.
Batches 10 through 13 compared to control batch 9.
These two groups were run on different days.

Table 19-A
MORTAR TIME OF SET
Test Variation

Temperature 73°F — Water 800 cc
R.H. 55% — Cement 1454 gm
No Admixture — Sand 4000 gm

Batch No.	Flow	Initial Set (TIS) hrs.	Final Set (TFS) hrs.
2	7.15	4.8	6.8
2	7.16	5.1	7.0
3	7.04	4.8	6.9
4	7.04	5.0	6.8
5	7.24	4.9	6.7
6	7.18	5.0	6.6
7	7.15	5.3	6.8
8	7.20	5.1	6.6
9	7.26	5.1	7.1
10	7.05	5.3	7.2
Average[10]		5.0	6.9

Variance, $S^2_{TIS} = .0316$
$CV_{TIS} = 3.53\%$

Variance, $S^2_{TFS} = .0405$
$CV_{TFS} = 2.94\%$

Table 20-A
MORTAR TIME OF SET, AIR VARIABLE

Sand 4000 gms.
Cement 1454 gms.
Water 760 cc

Batch	Vinsol Resin	Air Detraining Agent	Flow in.	Air %	Initial Set hrs.	Final Set hrs.
1	—	1 cc	6.67	2.7	5.30	7.05
2	—	—	7.33	4.2	5.30	7.25
3	¼ cc	—	7.20	5.6	5.45	7.55
4	1 cc	—	7.83	8.4	6.00	7.85
1′	—	1 cc	7.13	2.2	5.35	7.30
2′	—	—	7.09	4.1	5.40	7.30
3′	¼ cc	—	7.35	4.9	5.25	7.42
4′	1 cc	—	7.95	7.3	5.90	8.00

Table 21-A
DELAYED ADDITION OF ADMIXTURE IN MORTAR

Batch Quantities:
Cement 1454 gms
Sand 4000 gms
Water 700 gms

Method of Retarder Addition	Retarder	Flow in.	Air %	ISR hrs.	FSR hrs.
None		6.36	6.5	—	—
With batch water	DL_2	7.08	5.1	3.1	3.6
5 second delay	DL_2	7.39	10.6	2.8	3.8
1 minute delay	DL_2	8.37	10.9	5.5	6.2
2 minute delay	DL_2	8.33	10.6	7.5	8.0
(Dosage ¼ lb./bag)					
None		6.33	4.0	—	—
With batch water	DO_1	7.08	5.1	2.7	3.0
5 second delay	DO_1	7.22	4.1	3.5	3.8
1 minute delay	DO_1	7.27	5.4	6.1	7.0
2 minute delay	DO_1	7.00	5.5	7.1	7.8
(Dosage 3 oz./sack)					

Table 22-A
EXTENSIBILITY

Date: E1, E2, E3 ———April 19, 1965
E4, E5, E6, E7 —April 21, 1965
Cement 11.2 lbs.
Sand 30.8 lbs.

Batch	Admix	Dosage	Water + Admix lbs.	Flow in.	Air %	Observed Cracks		
						Spec. 1	Spec. 2	Spec. 3
E-1	None	None	6.32	7.78	4.3	2	1	5
E-2	AL	13.5 grams	5.32	7.01	10.5	7	10	11
E-3	DL$_1$	38.0 cc	5.47	7.27	9.2	5	11	6
E-4	CC	108.0 grams	6.24	7.65	4.5	17	16	14
E-5	DL$_2$	13.5 grams	4.62	7.60	12.3	13	13	12
E-6	DO$_1$	10.5 cc	5.70	7.40	5.1	3	6	3
E-7	DO$_2$	10.5 cc	5.86	7.45	5.0	0	6	3

Table 23-A
VINSOL RESIN DEMAND IN CONCRETE

Batch	Concrete Air Content %	Vinsol Resin Used cc/yd^3
Control	4.78*	51.08
AP	5.03	41.33
AL	4.97	24.40
CC	5.10	53.32
DO$_1$	4.83	39.44
DO$_2$	4.90	43.33
DO$_3$	5.37	53.25
DP	4.77	42.83
DL$_1$	4.87	8.99
DL$_2$	5.13	14.24
DL$_3$	5.07	12.52
DL$_4$	5.33	17.80

*Each number in this table is the average of three concrete batches with the exception of the "Control" which is the average of ten batches.

Table 24-A
FLOW-WATER CONTENT TESTS

Standard Cement/Sand Ratio

Date Tested 12-2-65 Cement 727 gm
Temperature 73°F Sand 2000 gm
R.H. 55% No Admixture

	Batch No.	Water cc	Avg. Flow in.
Water added in successive increments to test mortar	1	300	4.55
	2	325	5.50
	3	350	6.36
	4	375	6.31
	5	400	6.93
	6	425	7.45
	7	450	7.55
	8	475	8.04
	9	500	8.45
Tests on same mortar over 15 minute interval	10	400	7.18
	11	400	6.90
	12	400	7.15
	13	400	6.55
Repeat initial test points	3a	350	5.72
	5a	400	7.34
	7a	450	7.50

Table 25-A
AIR ENTRAINMENT IN MORTAR

Date Batched 3-24-65 Water 360 cc
Temperature 71°F Cement 727 gm
R.H. 55% Sand 2000 gm

Batch No.	Vinsol Resin* Stock 1/10 cc	Flow in.	Air Content %		
			1	2	3**
1	0.0	5.45	5.5	4.8	5.4
2	1.0	6.60	8.6	8.7	8.2
3	1.5	6.70	9.5	9.8	9.3
4	2.0	7.01	10.7	10.4	10.1
5	2.5	6.85	10.7	11.2	9.8
6	3.0	6.98	12.4	11.9	11.0

*Note a dosage of 1.7 corresponds to 54 cc Vinsol Resin in 1 yd^3 of concrete and produces approximately 5% entrained and entrapped air by the pressure measurement method.

**Three successive air content determinations were made over a period of approximately 15 minutes with the mortar re-mixed between determinations.

Table 26-A
SPECIFIC GRAVITY DATA

Admixture	Specific Gravity by Hydrometer	Percent Solids by Weight	Calculated S.G. of Solids
DO$_2$	1.188	38.03	1.71
DO$_1$	1.171	30.97	1.89
DL$_2$	1.018	4.87	1.57
AL	1.026	5.76	1.79
DL$_1$	1.198	40.06	1.70

Derivation of Crack Spacing Equation

Consider a concrete specimen having a rectangular cross-section with a steel reinforcing bar imbedded centrically as shown in Figure 1′. The length (L) is long compared to its cross-section dimensions so that end effects can be neglected. It is required that the volume of concrete be small compared to its surface area so that uniform shrinkage throughout the concrete specimen can be expected under slow drying conditions. As shown in Figure 1′ (c), the deformation of the element cut by planes A and B will be considered. The problem will be treated as a case of plane stress, and radial stresses induced by shrinkage will be neglected.

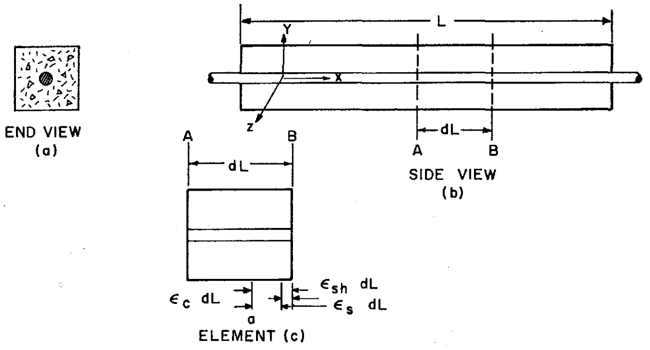

Figure 1′. Extensibility specimen.

In the absence of steel reinforcement, the element would change in length by the amount ϵ_{sh} dL (ϵ_{sh}, concrete shrinkage strain), to point a. The presence of the steel resists this movement however, and deforms only the amount ϵ_s dL (ϵ_s, steel compressive strain) leaving the distance ϵ_c dL (ϵ_c, concrete tensile strain) to be accommodated by the concrete.

This consideration of continuity yields equation (1),

$$\epsilon_{sh} = \epsilon_c + \epsilon_s \; . \qquad (1)$$

Not considering the forces acting on the plane A, static equilibrium requires that

$$\sigma_c \, A_c = \sigma_s \, A_s \qquad (2)$$

σ_c = Concrete tensile stress,

σ_s = Steel compressive stress,

A_c = Cross-sectional area of concrete,

A_s = Cross-sectional area of steel.

Using the relationships

$$E_c = \frac{\sigma_c}{\epsilon_c} \qquad E_s = \frac{\sigma_s}{\epsilon_s}$$

and equation (2), ϵ_s can be eliminated from equation (1) yielding

$$\epsilon_c = \frac{\epsilon_{sh}}{\left(1 + \dfrac{A_c}{n \, A_s}\right)} \; , \; n = \frac{E_s}{E_c} \; . \qquad (3)$$

When creep is of significance, it can be accommodated by modification of E_c. Creep will not be treated in this derivation.

Now, considering the condition of strain prior to formation of any cracks, the amount of strain energy in the concrete can be found as follows:

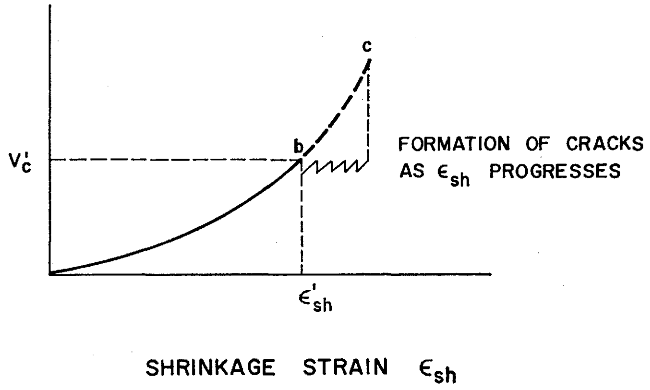

Figure 2′. Strain energy vs. shrinkage.

$$V_c = \int\!\!\int\!\!\int_{\text{Volume}} \tfrac{1}{2} \, \sigma_x \, \epsilon_x \, dx \, dy \, dz = \tfrac{1}{2} \, A_c \, L \, E_c \, \epsilon_c^2 \qquad (4)$$

It will now be postulated that the total amount of strain energy in the concrete due to restrained shrinkage will not exceed some critical value (V_c'). As the shrinkage progresses past ϵ_{sh} (the shrinkage corresponding to the ultimate concrete tensile strength, f'_t) progressive cracking will occur preventing the increase of V_c above the value corresponding to point b on the V_c vs. ϵ_{sh} curve. The actual number of cracks necessary to prevent V_c from exceeding its V'_c plateau will be found as follows. Figure 3′ shows a section of an extensibility specimen containing a crack.

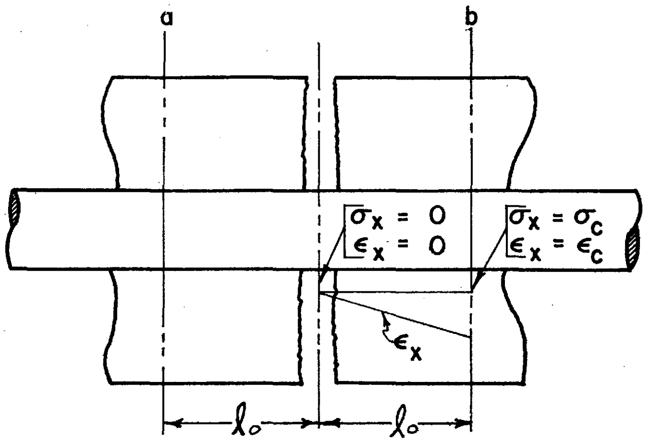

Figure 3′. Cracked section.

It will be assumed that after cracking the stress and therefore the strain in the concrete varies linearily from zero at the point of the crack to values of σ_c and ϵ_c a distance l_o away from the crack. Neglecting surface energy on the surface of tensile failure, the amount of strain energy lost in the concrete on formation of the crack will be numerically equal to the amount of strain energy in the concrete between points a and b before cracking, minus the amount of strain energy in the concrete between a and b after the crack has formed. This can be expressed as

$$\triangle V_{cr} = \frac{A_c\ 2\ l_o\ E_c\ \epsilon_c^2}{2} - \tfrac{1}{2} \int\int\int^{*} \sigma_x\ \epsilon_x\ dx\ dy\ dz$$

From Equation 4 substituting 2 l_o for L and substituting for σ_x, $E_x\ \epsilon_x$ $\left(\text{where } \epsilon_x = \frac{x}{l_o}\ \epsilon_c \right)$ yields

$$\triangle V_{cr} = A_c\ l_o\ E_c\ \epsilon_c^2 - \frac{E_c\ A_c\ \epsilon_c^2}{2} \int_0^{l_o} x^2\ d_x$$

$$= \frac{2}{3}\ E_c\ A_c\ l_o\ \epsilon_c^2\ . \qquad (5)$$

Now using equation (3) to eliminate ϵ_c yields

$$\triangle V_{cr} = \frac{2}{3}\ A_c\ E_c\ l_o \left\{ \frac{\epsilon^2_{sh}}{\left(1 + \frac{A_c}{nA_s} \right)^2} \right\}. \qquad (6)$$

It will then be seen that the total number of cracks forming will be the number necessary to prevent the strain energy of the system from exceeding V'_c (Figure 2').

Then

$$N = \frac{V_c - V'_c}{\triangle V_{cr}} \qquad (7)$$

or

$$N = \frac{\left\{ \frac{A_c\ L\ E_c\ \epsilon_{sh}^2}{2\left(1 + \frac{A_c}{nA_s} \right)^2} \right\} - \left\{ \frac{A_c\ L\ E_c\ \epsilon'^2_{sh}}{2\left(1 + \frac{A_c}{nA_s} \right)^2} \right\}}{\left\{ \frac{2\ A_c\ E_c\ l_o\ \epsilon_{sh}^2}{3\left(1 + \frac{A_c}{nA_s} \right)^2} \right\}}$$

which reduces to

$$N = \frac{3}{4}\ \frac{L}{l_o} \left\{ 1 - \left(\frac{\epsilon'_{sh}}{\epsilon_{sh}} \right)^2 \right\}. \qquad (8)$$

From equation (1), substituting σ_c/E_c for ϵ_c and $\sigma_c A_c/E_s A_s$ for ϵ_s,

$$\sigma_c = \frac{\epsilon_{sh}}{\left(\frac{1}{E_c} + \frac{A_c}{E_s A_s} \right)}. \qquad (9)$$

Now since $\sigma_c = f'_t$ when $\epsilon_{sh} = \epsilon'_{sh}$,

$$f'_t = \frac{\epsilon'_{sh}}{\left(\frac{1}{E_c} + \frac{A_c}{E_s A_s} \right)}. \qquad (10)$$

Replacing ϵ'_{sh} in equation (8) yields

$$N = \frac{3}{4}\ \frac{L}{l_o} \left\{ 1 - \frac{f'^{\,2}_t \left(\frac{1}{E_c} + \frac{A_c}{E_s A_s} \right)^2}{\epsilon_{sh}^2} \right\}. \qquad (11)$$

Thus the total number of cracks due to shrinkage is expressed in terms of tensile strength, shrinkage and the geometric and elastic properties of the specimen.

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*Integral over the volume from $- l_o$ to $+ l_o$.