# AN EVALUATION OF THE GRAVITY MODEL TRIP DISTRIBUTION 

by

Gary D. Long
Engineering Research Associate

Research Report Number 60-13

Research Study 2-8-63-60

Sponsored by

The Texas Highway Department

In Cooperation with the
U. S. Department of Transportation

Federal Highway Administration
Bureau of Public Roads

August, 1968

TEXAS TRANSPORTATION INSTITUTE
Texas A\&M University
College Station, Texas

This study was concerned with calibrating and testing the gravity model trip distribution using a small sized urban area - Waco, Texas. An analysis of trip purpose stratification was performed and it was concluded that no practical differences resulted between a seven purpose, a three purpose, and a single purpose model. Upon converting productions and attractions to origins and destinations for purposes of traffic assignment, the single purpose model was seen to differ slightly from the others. The source of this disparity was not definitely ascertainable due to entanglements involving the inappropriate conversion of nonhome-based trips. It was suggested that simply handling the home-based and nonhome-based trips separately as a two purpose model might be satisfactory.

Substantial river crossing bias was observed and corrected with a river time barrier. Inconsistencies associated with the use of the river time barrier suggested that "K-factors" may be the preferable means to compensate for river bias.

The study area itself presented no visible evidence to support any adjustments for bias attributed to social-economic characteristics. Contiguous zones were aggregated, nonetheless, on the basis of similar land use activity and residential age. Movements within the resulting regional structure displayed no distinct bias. A detailed examination of the movements linked with the region surrounding the Waco central busdness district revealed many gravity model volumes that differed from enrcesponding survey data volumes by significant percentages.

Intrazonal trips were recognized to be among the highest volume movements, and were observed to exhibit significant discrepancies between gravity model and survey data volumes. Since these trips are inactive in traffic assignment, it was recommended they be eliminated from the trip distribution.

The gravity model was observed to allocate some trips to nearly every conceivable zone pair combination. Conversely, about $80 \%$ of the survey data trip matrix was found to be vacant of trips. Into these vacant survey data cells, the gravity model located about onethird of the total trips. As a consequence, the gravity model shorted the movements observed in the survey data by this many trips. Particularly in the instance of high volume movements, this outcome was considered a major shortcoming since these are the most significant movements, and are also the ones measured with the greatest confidence. On an individual zone-to-zone basis, a virtually haphazard tendency was observed with regard to over or underestimation. Only about $80 \%$ of all zone pairs were found to have a gravity model volume within five trips of the survey data volume. Deviations as large as 17 trips had to be tolerated before $95 \%$ of the zone pairs fell within the tolerance range.

TABLE OF CONTENTS
Page
INTRODUCTION ..... 1
Objectives ..... 1
Study Area and Assignment Network ..... 1
Intrazonal Travel Times ..... 3
Stratification of Trip Purposes ..... 3
MODEL CALIBRATION ..... 7
Weighting Factors (F-factors) ..... 7
Topographical Barriers ..... 7
Social-Economic Bias ..... 12
River Time Barrier vs. K-factors ..... 18
ANALYSIS OF TRIP PURPOSE STRATIFICATION ..... 26
Trip Length Distributions ..... 26
Corridor Flows ..... 29
Assigned Volumes ..... 30
Screenlines ..... 33
Route Volume Profiles ..... 33
REPRODUCTION OF INTRAZONAL TRIPS ..... 38
TRIP MATRIX COMPARISONS ..... 43
Matrix Contrasts as a Function of Interchange Volume ..... 45
Major Cause of Gravity Model Shortcomings ..... 56
Matrix Contrasts as a Function of Spatial Separation ..... 57
Cell-by-Cell Matrix Comparisons ..... 63$\stackrel{i}{i}$
CONCLUSIONS AND RECOMMENDATIONS ..... 72
BIBLIOGRAPHY ..... 76
APPENDIX A ..... 77
APPENDIX B ..... 89

The opinions, findings, and conclusions expressed in this publication are those of the author and not necessarily those of the Bureau of Public Roads.
Figure

Title

Page12
Waco Normal-Detail Network Showing Screenlines . . . . . . 2
Regions Formed by Zone Aggregation15
CBD Oriented Corridor FlowsThree Purpose Composite with River Time Barrier17
Contrast of Regional Interchange Volumes ..... 19
CBD Oriented Corridor FlowsThree Purpose Composite Without River Time Barrier. . . 21
CBD Oriented Corridor FlowsThree Purpose Composite Showing River Time Barrier
Differences. ..... 22
RMS Differences in Link Volumes ..... 32
Waco Drive Route Volume Profile ..... 36
Valley Mills Road Route Volume Profile ..... 37
Single Purpose Model Intrazonal Trip Volumes ..... 40
Three Purpose Composite Model Intrazonal Trip Volumes. ..... 41
Seven Purpose Composite Model Intrazonal Trip Volumes ..... 42
Cumulative Distribution of Zone Pairs Over Entire Volume Range ..... 46
Cumulative Distribution of Zone Pairs for Small Volume Interchanges ..... 47
Survey Data Distribution for Small Volume Interchanges • ..... 51
Three Purpose Composite Model Distribution for Small Volume Interchanges ..... 54
Cumulative Distribution of Trips for Small Volume Inter- changes ..... 55
Distribution of Zone Pairs With Trip Interchanges ..... 58
Trip Length and Zonal Separation Distributions ..... 60

## LIST OF FIGURES (Continued)

| Figure: | Title |  |
| :--- | :--- | :--- |
| 20 | Average Trip Length Distribution . . . . . . . . . . . . . | 62 |
| 21 | Gravity Model Deviations from Survey Data. . . . . . . . . | 71 |

Table
Title
Page
Screenline Comparisons of Assigned Volumes with
Traffic Counts . . . . . . . . . . . . . . . 4
Summary of 24-Hour, Internal, Noncommercial, Vehicle Trips by Purpose . . . . . . . . . . . . . . . . . . . . 6
Screenline Comparisons of Survey Data with Three Purpose Composite Model. . . . . . . . . . . . . . . . . . . . . 9
Screenline Comparisons of Survey Data with Three Purpose Composite Model having River Time Barrier. . . . . . . . 10
Screenline Comparisons of Gravity Models having Different Trip Purpose Stratifications . . . . . . . . . . . . . . 34
Trips Distributed by Gravity Model to Vacant Survey Data Cells . . . . . . . . . . . . . . . . . . . 48
Comparisons of High Volume Movements (Over 300 Trips) . . . 64
Overall Comparison of Trip Interchanges . . . . . . . . . . 65
Distribution of Volume Differences. . . . . . . . . . . . 68

## Objectives

This report presents the results of research concerned with calibrating and testing gravity model trip distributions for a small urban area - Waco, Texas. The study objectives* were: 1) To determine the appropriate trip purpose stratification to be used with a gravity model trip distribution for Waco, Texas and other urban areas of similar character; and 2) To evaluate the ability of the gravity model to represent the actual trip behavior as estimated from a dwelling unit survey.

## Study Area and Assignment Network

The analysis was based on data compiled by the Waco Urban Transportation Study during 1964-65. At that time, 132,352 persons resided in 46,740 dwelling units located within the 248 square mile survey area. The study area included the city of Waco plus seven smaller incorporated urban entities. The Brazos River divided the area into two unequal parts. A majority of the urban development, including the Waco central business district, was located to the south of the river.

The traffic zone configuration and the assignment network used in the research were defined in the operational study. The assignment network (the central portion of which is shown in Figure 1 without centroid connectors) contained 631 nodes (including centroids), and

[^0]

WACO NORMAL-DETAIL NETWORK SHOWING SCREENLINES
FIGURE 1
required 916 link data cards for its description. There were 206 internal zones and 15 external stations for a total of 221 centroids. All internal and external vehicle trips compiled in the 1964 origindestination survey were assigned to the network. The resulting link volumes were compared to the 1964 traffic counts; speeds were adjusted and the assignment repeated until assigned volumes were in relative agreement with the counts. Assigned volumes for the screenlines shown in Figure 1 were compared with corresponding traffic counts. As indicated in Table 1, the screenline assigned volumes and ground counts were in close agreement, with the exception of screenline I-J.

## Intrazonal Travel Times

Intrazonal travel times for all zones were initially estimated by the procedure described in the Gravity Model Manual. (1)*. These estimates were then reviewed by inspecting the network map and examining the centroid connector linkage of all zones employed in each estimate. In several instances it was observed that the intrazonal travel times were not reasonable when considered with the network configuration, and were therefore adjusted.

## Stratification of Trip Purpose

Models were calibrated using 24-hour, internal, noncomercial vehicle trips. Three sets of models, each involving different trip purpose stratifications, were studied:

1. Single purpose model - all trips combined, regardless of purpose.
[^1]TABLE 1

SCREENLINE COMPARISONS OF ASSIGNED VOLUMES WITH TRAFFIC COUNTS

|  | Traffic <br> Count | Survey Data <br> Assignment <br> (All vehicle trips) | Percentage <br> (Assigned/Counted) |
| :--- | :---: | :---: | :---: |
| Y-Z | 24,600 | 23,000 | 93 |
| $\mathrm{~W}-\mathrm{X}$ | 53,500 | 53,100 | 99 |
| U-V | 50,400 | 49,100 | 97 |
| S-T | 79,800 | 82,200 | 103 |
| Q-R | 7,400 | 7,100 | 96 |
| O-P | 8,600 | 8,400 | 98 |
| M-N | 85,300 | 82,600 | 97 |
| K-L | 18,600 | 23,000 | 124 |
| I-J | 84,800 | 48,200 | 87 |
| G-H | 28,000 | 23,000 | 82 |

> 2. Three-purpose model - trips were stratified into home-based work, home-based nonwork, and nonhome-based.
> 3. Seven-purpose model - trips were stratified into home-based work, home-based shop, home-based socialrecreational, home-based business, home-based school, home-based other, and nonhome-based.

The trip sumary by purpose is shown in Table 2 . The values shown differ slightly from those in the origin-destination survey report (2) due to the deletion of trip records with data discrepancies. In addition, sample expansion factors wexe calculated and applied by traffic zones rather than by census tracts as used in the Waco Urban Transportation Study.

In the seven-purpose model, the home-based shop trips, home-based social-recreetiomal trips, and homenbased business trips were considered separately on the basis of each being a relatively large percentage of the total trip volume. Additionally, these three purposes generally exhibit different trip patterns and travel characteristics. The number of trips represented in the change travel mode, eat meal, and medicaldental categories was considered too insignificant to warrant calibration separately. These trips were combined with serve passenger trips since the latter category was regarded to consist of a multitude of purposes. Home-based school trips were retained separately, despite their small proportion, due to the presence of Baylor University which was located near the center of the study area. It was anticipated that, due to the influence of the University, these trips might have a unique pattern.

TABLE 2
SUMMARY OF 24-HOUR, INTERNAL, NONCOMMERCIAL VEHICLE TRIPS BY PURPOSE

| Survey İrip Purposes | Number of Trips | Percentage |
| :---: | :---: | :---: |
| Home-Based Work | 44,333 | 20.3 |
| Home-Based Shop | 26,725 | 12.3 |
| Home-Based Social-Recreational | 22,063 | 10.1 |
| Home-Based Business | 16,335 | 7.5 |
| Home-Based School | 4,878 | 2.3 |
| Home-Based Serve Passenger | 33,154 | 15.3 |
| Home-Based Change Travel Mode | 103 | . 0 |
| Home-Based Medical-Dental | 1,729 | 0.8 |
| Home-Based Eat Meal | 3,269 | 1.5 |
| Nonhome-Based | 65,026 | 29.9 |
| TOTAL | 217,615 | 100.0 |
| Three Purpose Model |  |  |
| Home-Based Work | 44,333 | 20.3 |
| Home-Based Nonwork | 108,256 | 49.8 |
| Nonhome-Based | 65,026 | 29.9 |
| TOTAL | 217,615 | 100.0 |
| Seven Purpose Model |  |  |
| Home-Based Work | 44,333 | 20.3 |
| Home-Based Shop | 26,725 | 12.3 |
| Home-Based Social-Recreational | 22,063 | 10.1 |
| Home-Based Business | 16,335 | 7.5 |
| Home-Based School | 4,878 | 2.3 |
| Home-Based Other | 38,255 | 17.6 |
| Nonhome-Based | 65,026 | 29.9 |
| TOTAL | 217,615 | 100.0 |

Rather than calibrate all models simultaneously, the trip stratification into three purposes was selected for initial development. This degree of trip stratification corresponds to that commonly used in similar cities, and proceeding with only one complete model set was thought to reduce confusion in handling the data.

Weighting Factors (F-factors)
The initial sets of weighting factors for each of the three component models were acquired from gravity model trip distributions for other cities that exhibited similar trip length characteristics. Comparison of the trip length distributions resulting from the first calibration revealed, however, that the initial weighting factors did not produce adequate correspondence between survey data and model results. The sets of factors were therefore revised by the procedures given In the Gravity Model Manual (1), and the distribution process was repeated in the normal manner for calibrating a gravity model.

Recalibration of each of the three models was continued until "reasonable" agreement was attained between the trip length distribution from the model with that from the survey data. A deviation of three percent was considered to be the tolerance limit on the average trip length for comparing the model results with the survey data.

Topographical Barriers
After the three models were calibrated within the designated tolerances, the effect of the Brazos River was considered. This was the
only distinct topographical feature from which bias would be expected. To facilittate the examination for bias, the set of screenlines, shown in Figure 1 , was used. The survey data trip matrix consisting of only the 24 -hour, internal, noncommercial vehicle trips was assigned to the network. This was to be compared with the assignment of the trip matrix resulting from the model. However, for the assignments to be compatible, the gravity model trip matrix had to be converted from nondirectional productions and attractions to directional origins and destinations. This was achieved in the conventional manner by applying a 50 percent conversion factor (assumes half of the trips interchange in one direction; the remaining half interchange in the other direction). While the error that is introduced by this conversion of a model trip matrix is not very critical with regard to screenline analyses, it plays a more important role in later comparisons of link volumes.

The screenline comparisons are shown in Table 3. The largest ratio of synthetically distributed trips to. survey data trips occurs on screenIine $U-V$ with an overestimate of about 25 percent. This is the river crossing screenline. Past experience has demonstrated the frequent existence of bias in gravity model river crossing estimates, and this condition has often been relieved by administering a river crossing time barrier.

In a manner similar to that described in the Gravity Model Manual (1), a time barrier of two minutes was initially selected and tested. The results are shown in Table 4. After careful examination with some simple

## TABLE 3

## SCREENLINE COMPARISONS OF SURVEY DATA

WITH THREE PURPOSE COMPOSITE MODEL

| Screenline | Survey Data* | Gravity Model* | Percentage <br> (Model/Survey) |
| :--- | :---: | :---: | :---: |
| T-Z | 10,300 | 11,300 | 110 |
| W-X | 27,500 | 31,900 | 116 |
| U-V | 25,900 | 32,500 | 125 |
| S-T | 57,500 | 55,100 | 96 |
| Q-R | 2,900 | 3,100 | 107 |
| O-P | 4,300 | 5,100 | 119 |
| M-N | 58,400 | 54,100 | 93 |
| K-L | 8,600 | 9,500 | 110 |
| I-J | 40,000 | 38,800 | 97 |
| G-H | 16,100 | 17,700 | 110 |

*Internal, noncommercial vehicle trips only.

WITH THREF PURPOSE COMPOSITE MODEL HAVING RIVER TIME BARRIER

|  | Survey Data* | Gzavity Model* | Percentage <br> With Time <br> Barrier | Percentage${ }^{*}$Without <br> Time Barrier |
| :--- | :---: | :---: | :---: | :---: |
| S-Z | 10,300 | 10,800 | 105 | 110 |
| W-X | 27,500 | 28,300 | 103 | 116 |
| U-V | 25,900 | 25,500 | 98 | 125 |
| S-T | 57,500 | 52,400 | 91 | 96 |
| Q-R | 2,900 | 3,100 | 107 | 107 |
| O-P | 4,300 | 5,000 | 116 | 119 |
| M-N | 58,400 | 53,400 | 91 | 93 |
| K-L | 8,600 | 9,400 | 109 | 110 |
| I-J | 40,000 | 37,900 | 95 | 97 |
| G-H | 16,100 | 18,100 | 112 | 110 |

[^2]interpolation, it was deduced that if the time barrier was constrained to integer values (whole minutes), then the optimum barrier had been selected. A barrier of one minute would not produce as much improvement as a two-minute barrier, but three minutes would be too large and cause overcompensation. Limiting the barrier value to whole minutes is deemed reasonable due to the current inability to forecast its magnitude. It is difficult to fustify the existence of the river bias, the fnitial estimate of the magnitude of the time barrier is little more than a guess, and very little is known with regard to projecting its future value. Therefore, refinement beyond integer values for use with data describing existing trips would surely be lost in conjuction with the distribution of future trips.

Table 4 shows that the river screenline volume was reduced from a volume 25 percent greater than the survey data to just slightly less than the survey data volume after administering the time barrier. Accompanying this improvement, most of the other screenlines with volumes greater than the survey data volumes were also' reduced. However, the underassigned screenlines, too, displayed slight reductions with the use of the river time barrier. From the screenline analysis it can be concluded that the implementation of the river time barrier, overall, resulted in some improvement, and was necessary to correct the river bias.

Since the river time barrier brought about oniy a minor volume increase in one screenline, it was apparent that the screenlines were not portraying the entire picture of the river time barrier effects. To compensate for a decrease in volume at one location, there must be
a corresponding increase in volume somewhere else. This was not displayed in the screenlines. However, the reason for the apparent inconsistency is associated with the screenline arrangement. When the largest portion of the volume tallied at any particular screenline crosses the river, this portion will be reduced, the remainder increased, but the reduced proportion may exceed the portion which was increased. This would bring about an overall reduction in the screenline volume. In Waco, the major artery of travel lies perpendicular to the river, and the screenlines generally intercept this principal flow. Therefore much of the traffic that crosses the screenlines also crosses the river, and these trips evidently outweigh the number of trips at each screenline that do not cross the river.

## Social-Economic Bias

Pronounced differences in the social and economic conditions of different segments of a community have been known to produce unusual trip patterns (1). The Waco study area was therefore thoroughly examined for such characteristics. Distinct discontinuities were not observed. The Waco urban area consisted of a relatively homogeneous population structure, with regard to social and economic status, and no unusual trip patterns were anticipated between any sections within the study boundary.

Indeed, differences exist in the population structure, as would be expected in any urban area. The pattern of growth was clearly evident; older and more dense residential development existed close to the CBD, and both the spatial density and age of the residential units decreased
at greater radial distances from the urban center. Some sections were predominantly residential, others industrial, commercial, agricultural, undeveloped, etc. These different regions undoubtedly exhibit vastly different trip generation properties. With regard to trip distribution, however, no pronounced disparities in model values would be expected to occur. The gravity model, by virtue of its input parameters (trip productions, relative attractiveness, and weighting factors based on the spatial separation of each zone), should account for different land use activity and intensity. If subsidiary adjustments are necessary, the modifications are enacted upon individual zone-to-zone movements through the use of 'K-factors'. When utilizing a calibrated model to distribute egtimates of future trips, future projections of the K-factors are necessary--and forecasting these factors is neither an elementary nor exact science (particularly when no visible evidence supporting their application was detected for the base year).

With the 206 internal zones within the study area, there is the possibility of applying 42,436 (206 x $206=42,436$ ) separate K-factors. Of course, if this many factors were required, there would be no benefit in using a gravity model distribution; developing the K-factors would be equivalent to distributing the trips without the model. The simplifying aspect lies in the observation that uniformity normally extends beyond the confines of single zones and, therefore, clusters of zones that exhibit similar social-economic characteristics can be grouped together. If K-factors are necessary between any two clusters, a constant value is simply applied to all zone pairs involved.

The 206 zones of the Waco urban area were therefore aggregated to form the 14 regions as shown in Figure 2. In aggregating the zones, every attempt was made to obtain relatively homogeneous regions, and this effort was weighted against the desire to keep the number of regions to a minimum. These zone aggregations were intended exclusively for use in establishing if K -factors were needed between any two clusters, and a supplementary analysis of the river barrier. An evaluation of the trip distribution at this level of areal subdivision was considered to be inappropriate. Certainly if the traffic assignment network of interest was of such a gross structure that a zone configuration of only 14 zones was both adequate and compatible with the network, the 14 zones would not be further subdivided for trip distribution but rather the gravity model engaged directly. In Waco, a zone structure of only 14 zones was of no interest with regard to traffic assignment. The full 206 zones were estab1ished in conjunction with the degree of detail desired in the traffic assignment network. Consequently, all 206 zones should participate in the trip distribution. Admittedly, after the trip distribution is performed with all 206 zones, the results can be presented on the basis of the 14 region aggregates, but by so doing glaring discrepancies may be camouflaged.

With continued aggregation to form larger and fewer agglomerates, agreement between observed and model values must improve. Consider the limiting case where the entire urban area is aggregated to form a single cluster. All trips become "intrazonal" and hence the observed and model


REGIONS FORMED BY ZONE AGGREGATION
FIGURE 2
values must be identical. A case not quite so trivial involves aggregation to form only two agglomerates using, say, the river as the dividing border. Satisfactory agreement at this level may be attained, but it does not imply that at each of the individual crossing points there will exist adequate correspondence between model and survey values. If an objective is to obtain individual link volumes by means of a traffic assignment using the trip matrix, then the evaluation criteria for the trip matrix should be on the level of zone-to-zone mevements (i.e. the same level as the assignment). The aggregation of zones to form only 14 regions is convenient for developing blanket $K$-factors to apply to zone pairs between regions, but agreement at this level implies little (disagreement at this level does yield noteworthy information). Furthermore, any comparisons employing these regions would be biased since they were delineated with the intention of isolating common social-economic characteristics.

For K-factor analysis, the regional aggregates were used and model interchanges were contrasted with the survey data for trips connected to the Waco central business region (Region 1). The principal direction of travel followed a northeast-southwest corridor with a secondary movement perpendicular to it. Therefore, the zone aggregates were linked together on this basis to enable suming the tributary volumes. Figure 3 illustrates the outcome of the three purpose composite model having the river time barrier imposed. On each branch, the survey data volume is shown with the corresponding model deviation.



CBD ORIENTED CORRIDOR FLOWS
THREE PURPOSE COMPOSITE WITH RIVER TIME BARRIER
FIGURE 3

The magnitudes of the volumes are small, as would be expected, since many trips become intraregional within these large agglomerates and, further, only one of the 14 sets of movements is being considered. Accompanying these small trip volumes, the magnitudes of the model deviations are also small. Small discrepancies can be misleading if viewed absolutely. They must be examined relative to the size of the interchange volume. While the magnitudes of the deviations are generally small, some of the deviation percentages, even at this level of zone aggregation, provide cause for concern. In view of the previously discussed aspects of $K$-factors, however, it was concluded that the regional volume deviations were not serious enough to warrant introducing $K$-factors in the Waco study area.

The foregoing discussion concerns only those trips connected with the central business region. To investigate the need for $K-f a c t o r s ~ r e-~$ lated to other movements, Figure 4 was prepared. With 14 regions there are a total of 105 nondirectional movements possible (including intraregional movements), and for each movement a point has been plotted on the graph corresponding the appropriate model and survey volumes. Ideally all points should lie on the line that represents identical volumes. Most points lie very close to it. Therefore, it was concluded that with respect to relatively large regions of common socialeconomic setting, the pattern of the gravity model trip distribution appeared acceptable without the use of K-factors.

River Time Barrier vs. K-factors
In the Gravity Model Manual (1) it is explained how river crossing


CONTRAST OF REGIONAL INTERCHANGE VOLUMES

FIGURE 4
bias can be compensated by the application of a time barrier at each of the river crossings. An alternative method to correct this bias would be to corisider the study area to be divided into two regions by the river, and apply blanket $K$-factors between the regions as suggested earlier. On the surface, these two methods appear equivalent. To examine this equivalence more thoroughly, Figure 5 was prepared analogously to Figure 3, only it represents the gravity model without the river time barrier. Both of these exhibits contrast model values with the survey data.

Figure 6 was prepared to illustrate the change in model values resulting from the incorporation of the river time barrier. This figure verifies, in part, the anticipated response from the river barrier. Trips to and from Regions 10, 11, 12 and 13 are the only ones that cross the river to reach Region 1 ; these show a volume reduction with the river barrier. None of the other regions cross the river to reach Region 1 , and all except Region 14 show a consequent increase in volume. The volume decrease for Region 14 was contradictory to the anticipated response. Since the total number of trips within the study area is constant, it follows that the reduction of one set of trip volumes (movements crossing the river) should bring about a conpensatory increase to the other set of trip volumes (movements not crossing the river). With the river time barrier imposed, no movement crossing the river would be expected to increase in volume. The barrier was deliberately imposed to lower them. Likewise, all movements not crossing the river would logically be expected to increase, but the movement associated with Region 14 did not.


FIGURE 5


CBD ORIENTED CORRTDOR FLOWS
THREE PURPOSE COMPOSITE SHOWING RIVER TIME BARRIER DIFFERENCES

FIGURE 6

As an aid to understanding the Region 14 behavior, it should be observed that the two-minute time barrier simply augments the separation of all movements crossing the river by two minutes. This shifts the river crossing movements through two one-minute cells in the trip length distribution but does not affect the other movements. As an example, suppose there exist 40 zone pairs with 400 trips, jointly, which are separated by 20 minutes. After imposing a two-minute time barrier, suppose 15 of the zone pairs separated by 20 minutes cross the river and become separated by 22 minutes. Suppose further that 20 of the zone pairs formerly separated by 18 minutes cross the river and become separated by 20 minutes. This changes the number of zone pairs separated by 20 minutes to 45 . The 400 trips must now be distributed among 45 zone pairs rather than 40 as before. The 25 original zone pairs not crossing the river that still remain separated by 20 minutes might now receive fewer trips than before because the 400 trips must now be shared among more zone pairs than before. Therefore, movements not crossing the river, which would normally be expected to experience a volume increase after imposing the river barrier, may instead be reduced due to having to share the capacity of their cell with additional zone pairs. Such may be the case with the aggregate movement between Regions 1 and 14.

It follows that individual movements crossing the river could be increased, in an analogous manner, with the imposition of a river barrier. The barrier may shift a set of movements into a trip length distribution cell that contains fewer total trips, but if the number of movements
removed exceeds the number added to this cell, the trips will be distributed among a smaller set of zone pairs than before. As a result the incoming zone pairs may be allocated more trips than before the time shift.

Thus it can be seen that a river barrier can cause individual movements crossing the river to either increase or decrease; noncrossing movements can behave likewise. Such an inconsistent treatment of individual zone pairs seems undesirable. This phenomenon probably escapes detection due to the common practice of aggregating zones to form larger regions for analysis; aggregation can totally conceal this response.

If instead of the river barrier a blanket K-factor had been applied to all zone pairs crossing the river, the inconsistency in individual trip volumes could not occur. Zone pairs not crossing the river would not be given a K-factor (1.0); zone pairs crossing the river would all be given the same value as a K-factor. Social-economic bias correcting K-factors could always be superimposed. Projecting a future value for the river crossing K-factor should be no more difficult than projecting a future value for a river time barrier.

The attractiveness of the K-factor approach comes from only the movements crossing the river will experience a volume decrease (K-factor smaller than unity). The remaining movements in each separation cell which do not cross the river will be the ones to absorb the conpensating trips. This is where the trips belong. With this foundation, the use of K -factors appears to be the preferable method for correcting river
crossing bias from a conceptual viewpoint, but no claims are made concerning the relative desirability in terms of the effort involved. No comparison at the zone level was performed to determine the severity of the inconsistencies assoclated with the use of the river time barrier.

It should be noted that for the particular instance cited, the reduction in interchange volume between Region 1 and Region 14 corresponded to an improvement in matching the survey data. There is no guarantee that this will always occur. It should also be emphasized that this entire analysis was performed using a river time barrier rather than K-factors to compensate for river crossing bias. This followed from complying with the procedural steps outlined in the Gravity Model Manual (1), and, thus, the conceptual inconsdstency allied with the river time barrier was not uncovered until late in the analysis when the trip matrices were compared. At this stage, the IBM 7094 computer at Texas A\&M University had been replaced with an IRM 360 and the programs could not be re-executed.

## Trip Length Distributions

The trip length distributions for each separate trip purpose, and for the composite models are shown in Appendix A. Each exhibit shows the survey data distribution, and the model distribution with the river time barrier. All gravity models were calibrated using survey data trip length distributions developed from the actual separation matrix that was obtained from the traffic assignment (i.e. no fictitious time barriers were imposed). The use of a trip length distribution representing observed trips with a deliberately distorted separation matrix would be pointless.

The survey data with the river barrier were also plotted in order to gain some insight into the effect of the time barrier on trip lengths. Contrasting the trip length distributions of the survey data including the time barrier with those excluding it, the differences observed are generally small. The proportion of trips of short lengths generally decreased with compensatory increases in the proportion of longer trips. In several instances, many trips were either added to or removed from a single separation interval resulting in a localized change in the trip length distribution curve. Some of the discontinuities in trip patterns are reduced with the river barrier (e.g. 9-minute separation in Figure A-9), and some are merely shifted (e.g. 9- to 16minute separations in Figure A-10). The generally small differences are somewhat reassuring in that it is believed the results presented in this report are not significantly distorted by the use of a time
barrier rather than $K$-factors to correct the river bias.
For the survey data, with no river barrier, the single purpose model shows a fairly regular trip length pattern with few pronounced changes between successive time intervals. The home-based nonwork and the nonhome-based trip length distributions (Figures $A-5$ and $A-6$, respectively) are similar and do not differ greatly from the trip length distribution of all trips, combined, shown in Figure A-1. The home-based work trips (Figure A-4) form a distinctly different distribution and show a number of sharp breaks in the curve. Further stratification of the home-based nonwork trips reveals that the Individual component distributions are not very similar. The home-based shop trips display a high frequency of short lengths with very few longer trips. The social-recreational purpose exhibits a much less peaked distribution than the shop purpose, and a relatively high proportion of moderate length trips. Business trips, along with the home-based other category, have distributions of the same form as the composite distribution (Figure A-1); but have more severe discontinuities.

The distribution of school trips displays three distinct peaks and bears little resemblance to any of the other distributions. It is difficult to distinguish the cause of the three peaks in the home-based school purpose between chance effects of a rather small sample or different characteristics between grade school, high school, and college trips (which are combined together in this classification).

From the standpoint of the survey trip length distributions, it would seem that stratification into seven separate purposes might be
excessive. Tre zore pronounced irregularities in the component distributions of the home-based nonwork category suggests that the sample size of these individual purposes might be insufficient to consider them separately. Yet it also appears that handing all trips collectively in a single purpose might not be refined enough. The homebased work trip length distribution is certainly distinct from the others and this'purpose comprises over $20 \%$ of the total trips. Therefore, on the basis of survey data trip length distributions alone, there is evidence supporting the stratification of trips into three categories: home-based work, home-based nonwork, and nonhome-based. It is advantageous to retain the nonhome-based trips as a distinct category due to their unique character and conceptual violation of the methodology developed for home-based trips.

The trip length distributions resulting from the gravity models display, in general, adequate conformity with those of the survey data. The greatest deviations are found in the vicinities of the most substantial irregularities in the survey data. As irregularities in the survey data trip length distribution become more severe, gravity model reproduction of the distribution becomes poorer. Even though the average trip length met the prescribed tolerance limit, the gravity model and survey data trip length distributions for home-based school trips. (Figure A-10) displayed only vague resemblances. Better agreement is not attained as long as weighting factors (F-factors) decrease with increased spatial separation in a continuous manner.

Even with the poorer agreement of the component purposes, the
aggregate forming the seven purpose composite model, shown in Figure A-3, displays the same excellent agreement as the three purpose composite model and single purpose model illustrated in Figures A-2 and A-1 respectively. This outcome emphasizes the relatively small influence, in terms of the number of trips, dictated by the purposes with the poorly modeled trip length distributions. There appears existence of two compensating effects which tend to offset each other. The tradeoff is between stratifying trips into several trip purpose categories in order to isolate patterns that are characteristically different, as opposed to using few categories to avoid the irregularities (attributed to small samples) that cause poor model reproduction.

## Corridor Flows

The same regional structure previously delineated was again used to examine the gravity model results for purposes of detecting bias in describing movements between regions. Figures showing the model results versus the survey data for central business region oriented movements are included in Appendix B.

Examining the effect of the river time barrier, it can be observed that the river crossing movements in both the single purpose and seven purpose composite model display much better agreement with the survey data after the time barrier was imposed. As in the case of the three purpose composite model, the single purpose and seven purpose composite models reflect improvements in agreement with the survey data in some of the individual movements and in other movements the agreement is poorer. The behavior of the seven purpose composite model was very
similar to that of the three purpose composite model. The response of the single purpose model differed slightly from the other two, but most values were in the same numerical neighborhood. Overall it can be concluded that, for each of the trip purpose stratifications, a river time barrier brought about improvement in the agreement of gravity model results with the survey data. This conclusion involves river crossing movements in addition to movements not crossing the river. Furthermore, little difference exists in the movements linked with the central business region between the three different trip purpose stratifications.

The home-based work trips (central business region oriented) display a slight overestimation at the river crossing even with the river time barrier, but the home-based nonwork and nonhome-based trips show near perfect agreement. No clear pattern is evident in contrasting the agreement between gravity model results and the survey data in the three component models (Figures $B-5, B-6$, and $B-7$ ). Again the discrepancies in terms of absolute magnitudes seem small, but the relative error percentages are alarmingly high.

## Assigned Volumes

This research is primarily concerned with trip distribution (calibrating the gravity model to produce a matrix of trip interchanges) rather than traffic assignment. It was not desired to influence the analysis of the distribution phases through shortcomings that might evolve in the assignment. Therefore, no extensive analysis of link volumes was undertaken. As an illustration of the potential introduction
of distortions, consider the conversion of production-attraction trip matrices to origins and destinations for assignment. Detection of this source of error arose when the single purpose, three purpose composite, and seven purpose composite gravity model trip matrices were converted to origins and destinations. This was achieved by applying a $50 \%$ conversion factor to all home-based trips. A cell-by-cell root-mean-square (RMS) difference was calculated for each matrix in conjunction with the origin-destination survey data matrix. These are plotted in Figure 7 as a function of interchange volume. The three purpose and seven purpose composite matrices are nearly identical in terms of RMS differences. The single purpose model displays somewhat larger values, but it is difficult to isolate the cause due to the trip conversion.

About one-third of the total trips are nonhome-based which should not be affected by a conversion from production-attractions to origins and destinations since the origin zone is always considered the production zone and the destination zone the attraction.

With the three and seven purpose matrices, the nonhome-based trips were retained separately and not converted, yet with the single purpose matrix there was no way to isolate them. After the conversion, the differences with the single purpose model could be due to true deviations, or else introduced by the inappropriate conversion of the nonhome-based trips. Furthermore, a certain amount of error is introduced into all model results as is evidenced by the residual in the conversion of survey data production-attractions to origin-destinations. In this


RMS ITFPFRFNCBS IN I.INK VGIUMPS
FIGURE 7

Instance, only the home-based trips were converted as appropriate. Indeed, some effort should be devoted to develop improved methods for conversion of trips, but that was not a part of this analysis.

## Screenlines

In view of the preceding discussion, screenlines are an exception and are basically uninfluenced by trip matrix conversion. Employing the same screenline configuration as described earlier, the contrasts between the three different trip stratifications are indicated in Table 5. All models are seen to produce similar results.

## Route Volume Profiles

If all network links were coded to permit travel in both directions, nondirectional link volumes could be analyzed with little concern for the conversion of production-attractions to origins and destinations. The path connecting each pair of zones would be the same in both directions (except perhaps in cases of ties). However in Waco, many network links were coded to represent one-directional travel and as a result differences as large as $6 \%$ were found to exist between the summation of travel times from a given zone to all others and the summation of travel times from all zones to the same given zone. With the all-or-nothing assignment and one-way links it becomes apparent that the trip direction cannot be disregarded. With indifference to this, two routes were selected in order to at least obtain an estimate of the agreement between the different stratifications. One route, Waco Drive, was selected because it represented the principal corridor

TABLE 5
SCREENLINE COMPARISONS OF GRAVITY MODELS
$\therefore$ HAVING DIFFERENT TRIP PURPOSE STRATIFICATIONS

| Screenline | Single Purpose* <br> Gravity Mode I | Three Purpose* <br> Composite <br> Gravity Model | Seven Purpose* <br> Composite <br> Gravity Model |
| :---: | :---: | :---: | :---: |
| Y-7. | 9,800 | 10,800 | 10,800 |
| $\mathrm{W}-\mathrm{X}$ | 27,300 | 28,300 | 28,200 |
| $\mathrm{U}-\mathrm{V}$ | 24,400 | 25,500 | 25,200 |
| S-T | 51,200 | 52,400 | 52,500 |
| $Q-R$ | 3,100 | 3,100 | 3,100 |
| $0-\mathrm{P}$ | 5,200 | 5,000 | 5,000 |
| $\mathrm{M}-\mathrm{N}$ | 53,400 | 53,400 | 53,300 |
| K-L | 9,800 | 9,400 | 9,400 |
| I-J | 37,000 | 37,900 | 38,100 |
| $\mathrm{G}-\mathrm{H}$ | 18,400 | 18,100 | 18,600 |

*Internal, noncommercial vehicle trips only.
through the study area. The other route, Valley Mills Road, was selected since it represented a major corridor which had no parallel facilities. The volume profiles for these two routes are shown in Figures 8 and 9 respectively.

Waco Drive crosses the river and, in the neighborhood of the crossing, all three full model volumes are seen to be lower than the survey data volume. This apparent underestimation appears somewhat ironical in view of the deliberate use of the river time barrier to reduce the overestimation that occurred at the river. However, two other crossings exist, and these alternatives were obviously overassigned the compensating trips. Exemplified in this illustration is the fact that balancing a river screenline with a time barrier in no way insures that individual crossing volumes will be in balance. Except for the vicinity obviously influenced by the river, each of the models seems to agree well with the survey data. On one section, the single purpose model appears slightly lower than the others, but with the many one-way links in this area the cause could again be associated with the inappropriate conversion of the nonhome-based trips.

Valley Mills Road shows close agreement between each of the models, but the pattern does not closely resemble that of the survey data. From this abbreviated overview of route volumes, it is concluded that the different trip purpose stratifications react virtually the same.



Intrazonal trips are recorded in the survey data but do not play an active role in traffic assignment. These trips are usually carried along with disregard until they finally vanish with the assignment of a trip matrix. Since they impose no impediment to the mechanics of the distribution process, but can be deleted as effortlessly as retained, the adequacy at their reproduction was assessed.

Figures 10 , 11 , and 12 present a visual display of the agreement between the survey data and the corresponding gravity model intrazonal volumes for each of the 206 zones. A line indicating the locus of equal volumes has been shown on all three exhibits.

Generally speaking, all three figures display the same pattern. However, on closer examination it can be seen that the points from the three purpose composite model usually lie slightly closer to the equal volume locus than those from the single purpose model. A similar improvement is not observed when.switching from the three purpose to the seven purpose composite model.

Taking a more detailed look at the three purpose composite model, many of the intrazonal movements appear to be of small volume. Indeed, many of the points are crowded into the corner at the origin, but it should be noted that only about $1.8 \%$ of all 42,436 zone pairs have survey data volumes in excess of 50 trips. Therefore, contrasted to the interzonal movements, the intrazonal movements are high in
volume. In light of this discussion, it should be observed that in terms of absolute error many of the discrepancies are quite noteworthy. . The gravity model seems to underestimate the high volume movements, and also overestimate some of the low volumes movements. The point that corresponds to a gravity model volume of 840 trips and a survey data volume of 190 trips represents an absolute error at 650 trips. This abundance of 650 intrazonal trips (which do not enter the traffic assignment) means that the interzonal movements (which do enter the traffic asaignment), corresponding to the related production zone, have been shorted by 650 trips.

An additional aspect to consider with regard to intrazonal trips is that their realm of influence is concentrated on zone pairs with short spatial separations (to which are attached the largest weighting factors). This condition, of course, is reflected by the relatively large intrazonal volumes, but the point in mention is that nothing guarantees that, overall, the gravity model intrazonal volumes will be in close proximity to the survey data intrazonal volumes. Instead of being rather evenly distributed about the equal volume locus as was observed, collectively the intrazonal volumes could lie to either side of it. This condition would produce efther an excess or deficit in the number of interzonal trips at the most significant portion of the distribution (i.e. in the zone pairs with short spatial separations). Therefore, on the basis of the observed disagreement with the survey data and the potential for additional discrepancies which in this instance were unobserved, it is recomended that intrazonal trips be eliminated from gravity model trip distributions.


SINGLE PURPOSE MODEL INTRAZONAL TRIP VOLUMES
FIGURE 10


THREE PURPOSE COKPOSITE MODEL INTRAZONAL TRIP VOLUMES
PIGURE 11


SEVEN PURPOSE COMPOSITR MODEL INTRAZONAL TRIP VOLUMES
FIGURE 12

## TRIP MATRIX COMPARISONS

Two advantages and motives favoring the direct comparison of trip matrices rather than the classical approaches to contrasting survey data with gravity model results have been discussed previously. The first aspect was related with not aggregating the zones before making comparisons. While zone aggregation does bring about some simplification by reducing the quantity of data to be digested, the resulting regional structure is not considered to be the appropriate level for analysis. Instead, the zone-to-zone level is deemed the desirable level for comparison since this is the level at which the trip distribution was performed, the level at which trip interchange data are needed for purposes of traffic assignment, and the level at which discrepancies are not hidden. The second aspect was related to comparing trip matrices rather than the link volumes from traffic assigments. Direct comparison of the trip matrices eliminates the potential for introduction of discrepancies attributable to the traffic assignment phase. A prime source of error of this nature is found in the conversion of productions and attractions to origins and destinations for traffic assignment. Instead, survey data and gravity model production-attraction trip matrices can be contrasted directly. Such contrasts, however, are still subject to one shortcoming which is related to the characteristics of the survey data.

The goal of the entire process is to calibrate a device (constructed as simply as possible) that will reliably forecast travel at some future time. To aid in achieving this goal, current travel characteristics
need to be inventoried. Due to the prohibitive cost of such a venture (assuming it is even possible) a sampling procedure is employed which allows examining only a small portion and, with this portion, estimating the parameters that actually describe the entire situation. Therefore, the sample survey provides only estimates of the various trip related parameters. In some instances, the estimates are very good (e.g. total trips within a study area, trip ends in each zone, etc). Some estimates bear very low confidence (e.g. low volume zone-to-zone movements). Some estimates are biased (e.g. the proportion of zone pairs having trip interchanges).

Likewise a predictive model provides estimates of parameters for some future time. This is accomplished by abstracting observed performances and trends, and making as many assumptions as are necessary to fill in the voids.

The gravity model is thus calibrated on the basis of the estimates of current behavior derived from the survey data. This is achieved while bearing in mind the shortcomings of sample survey estimates. Any contradictions or suspect values in the survey data should certainly be revised before model calibration. Therefore, if the model is to be expected to give reliable future estimates, it should surely be able to produce current estimates which compare favorably with the values which constitute our assessment of present conditions (i.e. the survey data). It is entirely possible that the survey data themselves should be modified, perhaps with a model, in order to improve their estimates. Research is needed in this area, and until something better is developed
the survey data still reign as the "best estimate".

Matrix Contrasts as a Function of Interchange Volume
Comparing the cumulative frequency distribution of the zone pairs appearing in successive increments of interchange volume, it is observed in Figure 13 that the three purpose composite model seems to agree fairly well with the survey data over the entire volume range. The model shows a slight tendency to exceed the survey data with the larger volume interchanges. However, the real discrepancies occur at low volume interchanges, and to more clearly illustrate this, the low volume portion of Figure 13 has been seplotted in Figure 14 with the scale of the abscissa greatly expanded. As can be seen from Figure 14, nearly $80 \%$ of the zone pairs contain no trips in the survey data as compared to only about $8 \%$ with the gravity model. This greatly larger number of zone pairs with trips resulting from the gravity model leads to concern for how many trips are involved. Therefore, opposite every vacant zone pair in the survey data, the corresponding zone pair in the gravity model trip matrix was examined. Table 6 presents this information for each trip purpose stratification. All three trip matrices that represent the entire collection of trips are again seen to be almost identical. In each, around $71 \%$ of the zone pairs, found vacant in the survey data, were allocated trips by the gravity model, and contained in these zone pairs was about $33 \%$ of the total trips in each model matrix. A more alarming aspect concerns the even greater percentages of mislocated trips in each of the component trip matrices. Since zone pairs are not mutually exclusive with regard


CUMULATIVE DISTRIBUTION OF ZONE PAIRS OVER ENTIRE VOLUME RANGE

FIGURE 13


CUMULATIVE DISTRIBU:ION OF ZONF PAIRS FOR SMALL VOLUME INTERCHANGES
FIGURE 14

TABLE 6

TRIPS DISTRIBUTED BY GRAVITY MODEL TO VACANT SURVEY DATA CELLS

| Model type | Misplaced <br> Trips | Total <br> Trips | Percentage <br> Misplaced | Zone Pairs <br> Number |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Single Purpose | 72,690 | 217,615 | 34 | 30,637 | $7:$ |


| Three Purpose Composite | 71,447 | 217,615 | 33 | 30,102 | 71 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Home-Based Work | 28,683 | 44,333 | 65 | 25,014 | 59 |
| Home-Based Nonwork | 44,752 | 108,256 | 41 | 26,878 | 64 |
| Nonhome-Based | 30,969 | 65,026 | 48 | 29,179 | 69 |


| Seven Purnose Comnosite | 71,132 | 217,615 | 33 | 30,012 | 71 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Home-Based Work | 28,683 | 44,333 | 65 | 25,014 | 59 |
| Home-Based Shop | 17,988 | 26,725 | 67 | 14,234 | 34 |
| Home-Based Social- | 16,292 | 22,063 | 74 | 22,168 | 52 |
| Recreational | 11,389 | 16,335 | 69 | 17,339 | 41 |
| Home-Based Business | 2,736 | 4,878 | 56 | 2,820 | 7 |
| Home-Based School | 22,511 | 38,255 | 59 | 20,907 | 49 |
| Home-Based Otner | 30,969 | 65,026 | 48 | 29,179 | 69 |

to trip purpose stratification, compensating errors fortunately reduce the overall error when the matrices are combined. From this standpoint, there seems to be little difference in the results with different trip purpose stratifications, and therefore discussion has been limited to the three purpose composite model throughout the remainder of the analysis.

Before leaving Table 6, there is another interesting aspect that should be discussed. The school purpose had the fewest total trips and correspondingly the gravity model produced numerous vacant cells. This resulted mainly because many of the zones produced and/or attracted no school trips. Therefore, only 7\% of the zone pairs that were vacant in the survey data school trip matrix were allocated trips by the gravity model. However, $56 \%$ of the trips were mislocated in these $7 \%$ of the vacant survey data zone pairs. A more alarming aspect involves the large portion (74\%) of social-recreational trips which were mislocated. This causes concern with regard to the gravity model's ability to describe social-recreational trips, and is especially upsetting since this trip purpose is rather significant. A disturbing extension of this aspect involves the mislocation of such a large portion of the highdy important purposes such as work trips ( $65 \% \mathrm{misplaced}$ ). The prime concern regarding thesemislocated trips is related to the fact that if they are allocated to places where no trips were observed, the places where trips were observed must suffer the loss. High volume interchanges, which are estimated with the greatest confidence in the sample survey, must compensatingly be shorted by the gravity model.

Referring back to Figure 14 , the survey data zone pairs are seen to undergo a concentrated increase in frequency at a volume in the neighborhood of 8 trips. In order to understand this more fully, Figure 15 has been prepared to illustrate the survey data distribution of zone pairs with low interchange volumes (truncated at 12 trips). A nominal sampling rate of $12.5 \%$ was used in the survey which results In a nominal expansion factor of 8 trips. Variations about the value of 8.0 occur due to the number of dwelling units contained in a zone not being evenly divisible by 8 , and other survey considerations. Low volumes result primarily from zones with few dwelling units (found mostly around the study area fringe) where the sample size was increased to improve the reliability of the estimates.

Keeping in mind that the punpose of the sample survey was to estimate what actually occurs with a reduced effort, it is obvious that certain aspects of sample survey data are unrealistic. Certainly interchange volumes of 5 trips, for instance, actually occur with a higher frequency than shown. Likewise, it is probable that the frequency of occurrence of interchange volumes in the neighborhood of 8 trips is excessive as shown. Indeed, any fraction of a trip is a fictitious quantity and could be rounded off. But the point being illustrated is that the survey data display trip frequency patterns which cluster around the nominal expansion factor, and multiples of it, with few observations in between. This is not a realistic portrayal of what actually occurs.

The cause of these peculiar low volume frequencies is related to


SURVEY DATA DISTRIBUTION FOR SMALL VOLUME INTERCHANGES
FIGURE 15
51
sampling probability. Only a portion of the dwelling units are sampled and it is hoped that the others display the same trip-making tendencies. Therefore, all of the trips recorded in the sample from a given zone are multiplied by the ratio of total dwelling units in the zone to the number of dwelling units sampled. In this manner it is hoped to represent the entire trip pattern of the zone. This means that if only a single trip, say, was detected to terminate at some other zone in the sample data, it would enter the matrix after expansion as 8 trips (with an expansion factor of 8). A final volume of four trips, for example, is impossible to attain in a sample expanded by a factor of 8.

Perhaps of even more importance than a sample reporting only a single trip interchange (for exampled, is the instance where in reality only one interchange exists. If the sample detects the trip, it enters the trip matrix after expansion as a volume of 8 trips rather than one, or else it goes undetected and results with a zero in the trip matrix. The probability it is detected is $1 / 8$; the probability it goes unnoticed is $7 / 8$ (this same reasoning holds for multiple trips from the same dwelling unit). Therefore, on the average $7 / 8$ of the zone pairs with exactly one actual trip interchange will indicate no interchanging trips, and the remaining $1 / 8$ of these zone pairs will display 8 trips. An analogous argument can be extended for zone pairs that in reality have $2,3,4 . . . e t c$. Interchanging trips. The conclusion that follows is nothing new: merely that many zone pairs which actually have one, two, or more interchanging trips appear with zeros in the survey data and, as such, account for the zero trip frequency to be distorted.

Frequencies neighboring around multiples of the nominal sample expansion factor will also be overestimated, to a perhaps lesser degree, and those spanning the peaks will be underestimated.

The gravity model results, on the other hand, display a much more continuous distribution as shown in Figure 16 . On the surface, this distribution appears to behave more like the physical situation would be expected to react. Yet, here the proportion of small volume interchanges appears excessive, especially for volumes smaller than one trip. Rounding of these trip volumes to the nearest trip can greatly affect the results. The tremendous difference in ordinate scales between Figures 15 and 16 should be emphasized in order to keep comparisons in their proper perspective.

The discussion thus far has centered around the distribution of zone pairs as a function of interchange volume. More appropriate, however, is the effect of these distributions on the allocated trips. Figure 17 shows the distribution of trips for the same truncated volume range as presented in the other figures. The ordinate in this exhibit reflects the percentage of the total trip volume ( 217,615 trips). It shows that about 7,000 trips are allocated in quanta of one or fewer trips by the gravity model, as opposed to no survey data trips. About 57,000 trips are allocated in quanta of 8 or fewer trips by the gravity model compared to only about 6,000 trips in the survey data; around 68,000 and 29,000 gravity model and survey data trips, respectively, are allocated in quanta of 10 or fewer trips; and 77,000 and 31,000 trips, respectively, in quanta of 12 or fewer trips. This


THREE PURPOSE COMPOSITE MODEL DISTRIBUTION FOR SMALL VOLUME INTERCHANGES
FIGURE 16


CUMULATIVE DISTRIBUTION OF TRIPS FOR SMALL VOLUME INTERCHANGES
FIGURE 17
55
covers over one-third of all trips for the gravity model, and up to this point the survey data never reached even one-half of the corresponding model allocation. It follows that since both total to 217,615 trips, the survey data must "catch up" with the higher volume interchanges.

It is quite likely that neither of these two curves is a very good representation of reality. Perhaps the real physical situation would be described by the uniform, almost linear, relationship expressed by the gravity model, but it would probably lie in closer proximity to the survey data.

## Major Cause of Gravity Model Shortcomings

The shortcomings in the gravity model trip distribution arise from the unlikelihood of every zone having trip interchanges with every other zone. Certainly the survey data present an underestimate of how many there actually should be, but on the other hand, it is unreasonable to suspect all possible combinations exist. With the gravity model formulation,

$$
T_{i j}=\frac{P_{i} \cdot A_{j}=F\left(S_{i j}\right)}{\sum_{k \neq 1}^{n} A_{k} \cdot F\left(S_{i k}\right)}
$$

it is apparent that no interchange volume will be estimated to be exactly zero unless either $P_{i}, A_{j}$, or $F\left(S_{i j}\right)$ is identically zero. The weighting factor, $F\left(S_{i f}\right)$, will never be zero unless a particular movement is prohibited. Therefore, either the $P_{i}$ or $A_{j}$ has to be zero or else a nonzero volume will be inserted in the trip matrix.

This condition rarely occurs when all trips are considered collectively, however when considering individual purposes (that represent rather few total trips), its occurrence is somewhat higher as previously observed with regard to school trips. The problem, then, involves identifying which zone pairs should be vacant.

## Matrix Contrasts as a Function of Spatial Separation

A frequent assumption in trip distribution is that a trip will be no longer than necessary. A driver making a trip for any given purpose will try to fulfill his need or desire without traveling any further than he has to. It is on this basis that the weighting factors (F-factors) for the gravity model are constrained to decrease with increasing spatial separation. This same assumption seems extendable to zone pairs without trip interchanges. It would appear most unreasonable to travel completely across a city to buy grocerles when an adequate grocery store was available nearby a residence. Therefore, the zone pairs that are separated by great distances would be expected to be void of trip interchanges more often than two zones which are nearby. With this in mind, Figure 18 was plotted to see if this expected relationship was reflected in the survey data. The corresponding gravity model results are also indicated. Both show the anticipated trend, but the curves are definitely characteristically different. The model results again verify that virtually all zone palrs are allocated some trips. The survey data conversely contain many void entries. Again, attention is focused on previously described considerations relative to biases. Neither


DISTRIBUTION OF ZONE PAIRS WITH TRIP INTERCHANGES
FIGURE 18
of these two estimates of the situation are likely to be correct. The survey data should understate the real occurrences. The survey data cannot overstate what happens since actual observations form the base. The real world relationship presumably lies to some extent above that described by the survey data, but certainly not nearly as far removed as the gravity model. There is no justification for suspecting the form of the survey data relationship to be biased. Therefore the real world situation would be expected to conform to this shape of curve rather than that of the model.

Closely allied to the relationship displayed in Figure 18 is the distribution of zonal separations shown in Figure 19. This curve describes the number of zone pairs that exist in each separation increment. As was discovered in another investigation and documented in Research Report 60-11, "The Effect of Network Detail on Traffic Assignment Results" (3), this curve was found to be dependent, to a certain degree, upon the configuration of the transportation network employed. It was deduced in that discussion that the distribution of trip lengths might subsequently be network dependent. This, of course, suggests that anticipated network alterations for a projected future year might negate any assumptions that weighting factors (F-factors) will remain constant, without even asserting anything about travel patterns. The point to be observed here, however, is that for a given network configuration in Waco, the ratio of trips to zone pairs, contained in any given separation interval, decreases substantially as separation increases. For example, the two-minute separation interval

trip lengeth and zonal separation distributions

FIGURE 19
shows nearly $10 \%$ of the trips are found in only about $2 \%$ of all possible zone pairs. Correspondingly, in the fifteen-minute interval only $1.5 \%$ of the trips are found in about $4.5 \%$ of the zone pairs. Yet referring back to Figure 18 , it should be observed that only $45 \%$ of the zone pairs at two minutes and $13 \%$ of those at 15 minutes have trip interchanges. This leads to the development of Figure 20 which presents the average number of trip interchanges between those zone pairs in each time interval which expereinced trip interchanges. Again the gravity model and survey data relationships are observed to depart. This further illustrates that survey data volumes, on the average, are larger than gravity model volumes because the gravity model ispreads the same quantity of trips (a constant) over more zone pairs.

The survey data curve in Figure 20 might be expected to approach an asymptote at a level of about eight trips since this value corresponds to the nominal sample expansion factor. The tail of the curve is seen to dip below this value due to the greater sample collected in the sparsely developed fringe zones. These fringe zones being located near the periphery of the study area have, overall, the longest separations from all other zones and therefore the effect of the higher sampling rate is principally concentrated in the tail of the curve. Here again, the real world behavior would be expected to deviate from both plotted curves since it ia not dependent upon sampling rate, yet it should exceed the diluted average of the gravity model.


AVERAGE TRIP LENGTH DISTRIBUTION
FIGURE 20

## Cell-by-Cell Matrix Comparisons

Before engaging in this section, attention is again focused on the fact that the survey data may not provide the most ideal estimate of what actually occurs in reality, but nonetheless is the best and only estimate available. Conclusions must therefore be tempered accordingly.

There is little opposition to the assertion that high volume trip interchanges are the movements most confidently estimated by the survey data. Additionally, these are the most significant movements and, hence, the most essential values to reproduce accurately with a model. Using a rather arbitrary dividing point set at a volume of 300 trips, Table 7 presents corresponding gravity model and survey data values for those zone pairs exceeding this limit. Observing the totals for this selected set of interchanges, the model is seen to be low. The difference column indicates an almost haphazard tendency for the model to over or underestimate the survey data on an individual basis; the magnitudes of the differences are cause for concern.

In an effort to contrast the remainder of the movements, those with both model and survey data values under 300 trips, Table 8 was prepared, This table reflects the number of zone pairs falling into each volume group. The groupings are all uniform with intervals equal to the nominal sample expansion of 8 trips. The groupings are also centered around integer multiples of the nominal expansion factor. Vacant cells are categorized separately, and it should be observed that the second grouping is irregular and represents the residue from establishing the above scheme.

TABLE: 7

COMPARISON OF HIGH VOLUNE: MOVEMENTS (OVER 300 'IRIPS)

| Survey Data | Gravity Model | Difference |
| :---: | :---: | :---: |
| 47 | 343 | +296 |
| 123 | 310 | +187 |
| 161 | 382 | +221 |
| 181 | 843 | +662 |
| 266 | 360 | +94 |
| 307 | 137 | -170 |
| 307 | 137 | -170 |
| 310 | 258 | - 52 |
| 315 | 327 | + 12 |
| 317 | 201 | -116 |
| 320 | 228 | - 92 |
| 327 | 193 | -134 |
| 333 | 304 | - 29 |
| 334 | 372 | + 38 |
| 336 | 305 | - 31 |
| 338 | 185 | -153 |
| 341 | 323 | - 18 |
| 343 | 249 | - 94 |
| 344 | 90 | -254 |
| 347 | 235 | -112 |
| 352 | 392 | $+40$ |
| 353 | 144 | -209 |
| 355 | 242 | -113 |
| 358 | 164 | -194 |
| 371 | 380 | + 9 |
| 376 | 176 | -202 |
| 386 | 335 | - 51 |
| 395 | 134 | -261 |
| 401 | 454 | + 53 |
| 401 | 108 | -293 |
| 420 | 694 | +274 |
| 428 | 191 | -237 |
| 441 | 278 | -163 |
| 490 | 422 | - 68 |
| 513 | 557 | + 44 |
| 537 | 564 | +27 |
| 541 | 223 | -318 |
| 547 | 325 | -222 |
| 573 | 366 | -207 |
| 782 | 646 | -136 |
| 911 | 542 | -369 |
| 1073 | 737 | -336 |
| 1136 | 1057 | - 79 |
| $\overline{17,837}$ | $\overline{14,911}$ | +1957 |
|  | 64 | -4883 |

```
    *)
```





```
        Mrsmeg, 00000000-00000...0000000000000000000q0000 - 
        P!2,30, 0000000000-0000000-000-00000000000000000..
```





```
        M(tymp, 00000000000000000000----0000000000000000-
```




```
        "/4,
        mmsm, 0000000~-0N000-00000000000|-00-000000000.
```




```
    4/4,4,
```



```
    M%M,
    N'140,0,000000-0--0-0000000-poom00000000000000-
```



```
        Mtr,0000--0--N--Noon-0of-00000-00000000000000
    *)
    M,
```



```
    OO
```




```
        B
```





```
        *)
```





```
        *)
```








If the gravity model was exactly reproducing the survey data, the underlined element in each column would numerically represent all of the zone pairs found in each column total. All other elements in each column would be zero. In this instance, the row totals would equal the column totals. Nonzero entries in elements above the underlined diagonal represent zone pairs underestimated by the gravity model; those appearing below the underlined diagonal represent zone pairs overestimated by the gravity model.

As an illustration in reading Table 8 , consider a survey data volume of between 268 and 276 trips. Scanning the appropriate column, the underlined element is seen to be zero, and the column total indicates three zone pairs fall within this specified volume range in the survey data. The gravity model volume for each of the three zone pairs is seen to be, respectively (1) between 52 and 60 , (2) between 100 and 108 , and (3) between 108 and 116 trips.

Table 8 serves to verify previous observations that the gravity model produces many more low volume movements than are found in the survey data, and fewer high volume movements. It underestimates the interchange volume for high volume movements.

Table 9 displays the distribution of volume differences between corresponding zone pairs for the survey data and gravity model. Illustrated are the number of zone pairs involved in each volume difference category. Although volumes were carried to tenths of a trip up to this point, for purposes of this comparison each volume was rounded to the nearest whole trip. Values under one-half of a trip
in the gravity model matrix were, of course, rounded to zero. Consequently, the rounding effectively destroyed 3,000 trips in that trip matrix. Involved with these trips were about 10,200 nonzero zone pairs which were set to zero. This naturally improved the number of zone pairs which had an exact match with the survey data. A total of 14,045 zone pairs (about one-third of the total) were found to have model values that agreed exactly with those of the survey data. For volume differences of up to about 13 trips, the model overestimated more often than it underestimated the corresponding survey data values. Beyond a difference of 13 trips the reverse was generally true. The first part of Table 9 continuously covers differences ranging between 1 and 100 trips. The second portion of Table 9 1ists only differences over 100 trips that were found to occur. In 65 of the 79 instances where the model differed from the survey data by over 100 trips, it underestimated. Differences of this magnitude are most alarming in view of the relatively small interchange volumes found in the survey data (only 38 of the 42,436 zone pairs had volumes over 300 trips in the survey data).

Figure 21 is a graphical representation of a portion of the data presented in Table 9. This figure displays the cumulative percentage of zone pairs that result as greater differences in trips, between survey and model values, are tolerated. For example, it indicates that differences as large as 17 trips must be tolerated before $95 \%$ of the zone pairs fall within the tolerance limits. If tolerance limits are set at $\pm 5$ trips; as another example, only $79.5 \%$ of the zone pairs

TABT,F 9
IISTRIBUTION OF VOLUME DIFFFRENCES


| $\begin{aligned} & \text { Trip } \\ & \text { Volume } \\ & \text { Difference } \end{aligned}$ | Model <br> Low | Model <br> High | Trip Volume Difference | $\begin{aligned} & \text { Model } \\ & \text { Low } \\ & \hline \end{aligned}$ | Mode 1 <br> High |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 101 |  | 1 | 219 | 1 |  |
| 102 | 1 |  | 221 | 1 | 1 |
| 103 | 3 |  | 236 | 1 |  |
| 105 | 2 | 1 | 253 | 1 |  |
| 106 | 3 |  | 260 | 1 |  |
| 107 | 1 |  | 275 | 1 |  |
| 109 | 1 |  | 291 | 1 |  |
| 110 |  | 1 | 296 | 1 |  |
| 111 | 1 |  | 318 | 1 |  |
| 112 | 1 |  | 336 | 1 |  |
| 115 | 1 |  | 368 | 1 |  |
| 116 | 2 |  | 662 |  | 1 |
| 117 | 1 |  |  | 6293 | 22098 |
| 118 | 3 | 1 |  |  |  |
| 119 |  | 1 |  | Model low | 6293 |
| 125 | 3 | 1 |  | Model high | 22098 |
| 126 | 1 | 1 |  | o difference | 14045 |
| 127 | 1 |  |  |  | 42436 |
| 127 | 2 |  |  | Lotal | 42436 |
| 132 | 1 |  |  |  |  |
| 133 | 2 |  |  |  |  |
| 134 | 1 |  |  | . |  |
| 135 | 1 |  |  |  |  |
| 140 |  | 1 |  |  |  |
| 142 | 3 |  |  |  |  |
| 144 |  | 1 |  |  |  |
| 147 | 1 |  |  |  |  |
| 151 | 1 |  |  |  |  |
| 152 | 1 |  |  |  |  |
| 154 | 1 |  |  |  |  |
| 155 | 1 |  |  |  |  |
| 159 | 1 |  |  |  |  |
| 161 | 1 |  |  |  |  |
| 164 | 1 |  |  |  |  |
| 165 |  | 1 |  |  |  |
| 166 | 1 |  |  |  |  |
| 169 | 3 |  |  |  |  |
| 171 | 1 |  |  |  |  |
| 177 |  | 1 |  |  |  |
| 187 |  | 1 |  |  |  |
| 192 | 1 |  |  |  |  |
| 193 | 1 |  |  |  | - |
| 197 | 1 |  |  |  |  |
| 201 | 1 |  |  |  |  |
| 206 | 1 | . |  |  |  |
| 207 | 1 |  |  |  |  |
| 215 | 1 |  |  |  |  |

occur within this range. Also, shown in Figure 21 is the percentage of zone paịs which the model overestimates and underestimates at each difference level. For any specified trip difference, corresponding values from the three curves should sum to $100 \%$ of the zone pairs.


GRAVITY MODEL DEVIATIONS FROM SURVEY DATA
FIGURE 21

With regard to trip purpose stratification, the principal findings of this study were:

1. Little practical difference existed between the production-attraction trip matrices that resulted from the seven purpose, three purpose, or single purpose gravity model.
2. After the conversion of productions and attractions to origins and destinations for purposes of traffic assignment, the single purpose model differed slightly from the others. This effect could be allied with the inappropriate conversion of nonhomebased trips.
3. The residual observed after "converting" survey data productions and attractions to origins and destinations, and comparing with survey data origins and destinations suggests the need for research in this area to develop improved conversion methods. Further work is definitely needed if single purpose models are to be satisfactorilly converted.
4. Stratification of trips into several categories was observed to isolate different travel patterns in terms of trip length distributions. However, each category consequently contains a reduced number of trips, and with
this maller sample size each trip length distribution is subject to greater sample variation.
5. Due to weighting factors (F-factors) being constrained to decrease in a continuous manner with increasing spatial separations, the gravity model does poorer in reproducing the observed trip length distribution as irregularities in the observed trip length distribution increase.
6. A two purpose model composed of home-based and nonhomebased trips, respectively, appears to be optimal for the Waco area.

The major findings related to the ability of the gravity model in representing actual trip behavior were:

1. The survey data suffer from several shortcomings and provide only an indicator of what actually occurs in reality. Since the purpose of the gravity model is to estimate actual traffic movements (rather than reproduce the survey data), rigorous comparison of the survey data with gravity model results is not necessarily desirable.
2. High volume movements, as reflected by the survey data, carry the greatest confidence in the estimates and are also the most important movements to be estimated by the gravity model. In general, the gravity model underestimated the volume of these movements.
3. The gravity model allocated some trips to nearly every possible zone pair combination. A distribution such as this is somewhat unrealistic when an urban area is divided into a large number of small zones (for purposes of traffic assignment) ; it is unlikely that each zone would have trip interchanges with every other zone. The survey data do not yield a realistic estimate of how many zone pairs having trip interchanges actually exist.
4. The gravity model does not exhibit a consistent tendency with regard to over or underestimating interchange volumes between individual zone pairs. With both high and low volume survey data movements, the gravity model sometimes overestimates, sometimes underestimates, sometimes the difference is large, and sometimes the difference is small.
5. Only about $80 \%$ of the total possible zone pairs were allocated a volume by the gravity model that fell within 5 trips of the corresponding survey data volume. Deviations as large as 17 trips had to be permitted before $95 \%$ of the zone pairs fell within the prescribed tolerance limits.
6. Research is needed in adjusting survey data information in order to provide better estimates of real world occurences. The matrix of trip interchanges developed by a gravity model distribution of trip ends does not display considerable similarity with
what is expected to actually exist in reality. Additional research effort should be expended for the refinement of existing, or development of new trip distribution methods.

In the use of the gravity model, it is recommended that K -factors be employed rather than river time barriers if river crossing bias corrections are necessary. Since intrazonal trips are inactive in traffic assignment, it is recommended that they be eliminated from the trip distribution.

## BIELIOGRAPHY

1. U.S. Department of Commerce, Calibrating and Testing a Gravity Model For lny Size Urban Area, (Hashington, D.C.: Bureall of Public Roads, Office of Planning; October; 1965.)
2. Texas Highway Department, Waco Urban Transportation Plan: Origin Destination Survey, Volume 1, (Austin,Texas: Texas Highway Department; 1964).
3. Long, Gary D. and Vergil G. Stover, 'The Effect of Network Detail on Traffic issignment Results," Research Report $60-11$; (College Station, Texas: Texas Transportation Institute; August, 1967).

## APPENDIX A

TRIP LENGTH DISTRIBUTIONS

| Figure | Description |
| :--- | :--- |
| A-1 | Single Purpose |
| A-2 | Three Purpose Composite |
| A-3 | Seven Purpose Composite |
| A-4 | Home-Based Work |
| A-5 | Home-Based Nonwork |
| A-6 | Nonhome-Based |
| A-7 | Home-Based Shop |
| A-8 | Home-Based Social-Recreational |
| A-9 | Home-Based Business |
| A-10 | Home-Based School |
| A-11 | Home-Based Other |



SINGLE PURPOSE TRIP LENGTH DISTRIbUTION
FIGURE A-1


THREE PURPOSE COMPOSITE TRIP LENGTH DJSTRIBUTION
FICORE A-2


SEVEN PURPOSE COMPOSITE TRIP LENGTH DISTRIBUTION

FIGURE A-3


HOME-BASED WORK TRIP LENGTH DLSTRIBUTION
EIGURE A-4


HOME-BASED NONWORK TRIP LENGTH DISTRIBUTION
FIGURE A-5


NONHOME--BASED TRIP LENGTH DISTRIBUTION
FIGURE A-6


HOME RASED SHOP TRIP LFNGIIL UISTRIBUTION
FLCURE A-7


HOME-BASED SOCIAL-RECREATIONAL TRIP LENGTH DISTRIBUTION
FIGURE A-8



HOME-BASED SOCIAL-RECREATIONAL TRIP LENGTH DISTRIBUTION FIGURE A-8



HOME-BASED SCHOOL TRIF LENG'LH DISTRIBUTION

FICURE A-10


HOME-BASED OTHER TRIP LENGTH DISTRIBUTION

FIGURE A-11

# APPENDTX B <br> CBD ORIENTED CORRIDOR FLOWS 

| Figure | Description |
| :--- | :--- |
| B-1 | Single Purpose Without River Time Barrier |
| B-2 | Seven Purpose Composite Without River Time Barrier |
| B-3 | Single Purpose With River Time Barrier |
| B-4 | Seven Purpose Composite With River Time Barrier |
| B-5 | Hone-Based Work With River Time Barrier |
| B-6 | Home-Based Nonwork With River Time Barrier |
| B-7 | Nonhome-Based With River Time Barrier |



CBD ORIENTED CORRIDOR FLOWS
SINGLE PURPOSE WITHOUT RIVER TIME BARRIER

FIGURE B-1


CBD ORIENTED CORRIDOR FLOWS
SEVEN PURPOSE COMPOSITE WITHOUT RIVER TIME BARRIER
FIGURE B-2


CBE ORIENTEIO CORREDOR FLOWS
single purpose with river time barkier

FIGORE: B-3


CBD ORIENTED CORRIDOR FLOWS
SEVEN PURPOSE COMPOSITE WITH RIVER TIME BARRIER
FIGURE B-4


CBD ORIENTED CORRIDOR FLOWS
HOME-BASED WORK WITH RIVER TIME BARRIER

FIGURE B-5


CBD ORIENTED CORRIDOR FLOWS
HOME-BASED NONWORK WITH RIVER TIME BARRIER

FIGURE B-6


CBD ORIENTED CORRIDOR FLOWS
NONHOME-BASED WITH RIVER TIME BARRIER

FIGURE B-7


[^0]:    *It should be noted that the scope of this analysis was partially truncated by a changeover in computers from an IBM 7094 to an IBM 360. This occurred while the analysis was in progress and prohibited some of the intended investigations.

[^1]:    *Numbers in parentheses refer to entries in the bibliography.

[^2]:    *Internal, noncommercial vehicle trips only.
    Fisted here for convenience from Table 3.

