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16. Abstract

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FORECASTING SERVICEABILITY LOSS OF FLEXIBLE PAVEMENTS

by

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William M. Moore

Research Report Number 57-1F
Development of Improved Method for
Pavement Rehabilitation Forecasting
Research Project 2-8-74-57

conducted for

The Texas Highway Department
in cooperation with the
U.S. Department of Transportation
Federal Highway Administration

by the

Texas Transportation Institute
Texas A&M University
College Station, Texas 77843

November, 1974

DISCLAIMER

PREFACE

This is the final report issued under Research Study 2-8-74-57, "Development of Improved Method for Pavement Rehabilitation Forecasting," conducted at the Texas Transportation Institute as part of the cooperative research program with the Texas Highway Department and the Department of Transportation, Federal Highway Administration.

The authors are grateful to Mr. James L. Brown of the Texas Highway Department for his active interests in and supports of the project.

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ABSTRACT

Pavement performance data collected in Research Project 2-8-62-32, "Extension of AASHO Road Test Results," are analyzed in this study. Serviceability loss of three flexible pavement types due to fatigue, swelling, shrinkage and thermal cracking are correlated to many environmental, traffic, time, design and construction material variables. A "two-step constrained select regression procedure" is developed to find the functional relationships. A sensitivity analysis method is developed to examine the effect that each variable has on pavement serviceability loss. Stochastic reliability concepts are applied to evaluate the expected value and variance of the serviceability loss.

KEYWORDS: Select regression, differential sensitivity, stochastic reliability, serviceability loss, pavement performance, flexible pavement system, fatigue, swelling, shrinkage, thermal cracking.

SUMMARY

This report presents a methodology for (1) building more "rational" pavement performance models, (2) analyzing the sensitivity of these models, and (3) implementing these models at a reliability level that is specified by the user.

A "Two-Step Constrained Select Regression Procedure" was developed for the curve-fitting of pavement serviceability loss as a function of environmental impacts, traffic conditions, aging effects, design variables and construction material properties. It was observed that each pavement type required a separate performance equation. Three flexible pavement types were investigated in this study: (1) surface treatment pavement, (2) hot mix asphaltic concrete (HMAC) pavement without overlay construction, and (3) HMAC overlaid pavement. Pavement serviceability was analyzed based on integrated effects of many distress mechanisms, such as: fatigue, swelling, shrinkage and thermal cracking.

The performance equations derived in this study fit the data collected in Texas Study 2-8-62-32 better than the Scrivner's equation (5) based on AASHO Road Test data which is currently implemented in the Texas Flexible Pavement Design System, FPS-11. However, the Texas data were not collected in an experiment that was well-designed for regression analysis purposes, a sharp contrast with the AASHO Road Test.

A differential analysis method derived from the Taylor's series expansion was developed to examine the sensitivity of pavement serviceability in terms of each of the environment, traffic, time, design, and paving material variables. Also, the significance of each variable with

respect to pavement performance can be examined using the differential analysis method.

Probabilistic design concepts were incorporated to indicate how to design reliable pavements which would provide satisfactory service to the user throughout the design life at designer-specified confidence levels. Equations to calculate the expected value and variance of pavement serviceabilities were derived from the Taylor's series expansion.

IMPLEMENTATION STATEMENT

Pavement performance equations presented in this report are not recommended for immediate implementation. This report outlines a research procedure for future pavement performance studies to be conducted in Study 2-8-75-207," Flexible Pavement Evaluation and Rehabilitation," which has the ultimate goal of implementing more "rational" performance equations in the pavement design systems developed in Texas Study 2-8-62-32, "Extension of AASHO Road Test Results," and Study 1-8-69-123, "A System Analysis of Pavement Design and Research Implementation."

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CHAPTER I

INTRODUCTION

This report is intended to document the empirical pavement performance equations developed in Research Study 2-8-74-57 "Development of Improved Method for Pavement Rehabilitation Forecasting."

These equations explain loss in pavement serviceability due to fatigue, swelling, shrinkage, and thermal cracking. The complexity of serviceability loss is fully recognized because of the interactive characteristics of many construction, traffic and environmental variables. Included in these equations are: axle applications, surface deflections, temperature, Thornthwaite index (an indicator of climatic moisture balance), top layer thickness, overlay thickness, pavement stiffness coefficient, time after construction or rehabilitation, number of freeze-thaw cycles, percent lime and percent of fines in base course and percent fines in the subgrade.

The intermediate products of this project are seven serviceability loss equations that include: (1) three equations for serviceability loss due to fatigue - one equation for surface treatment pavement, another for hot mixed asphaltic concrete (HMAC) pavement without overlay construction, and the third for pavements with HMAC overlay construction, (2) one equation for swelling clay serviceability loss for all flexible pavement types, (3) one equation for serviceability loss due to base course shrinkage for all flexible pavement types, and (4) two equations for thermal cracking serviceability loss - one for HMAC pavement without overlay construction, and another for pavements with HMAC overlay construction. It has been found that thermal cracking is insignificant in surface treatment sections. Thus, no serviceability loss due to thermal cracking is assumed for surface treatment pavements. Also, serviceability loss due to swelling and shrinkage are assumed independent of pavement types.

The final products are three performance equations for the three pavement types investigated in this project: (1) surface treatment pavement, (2) HMAC pavement without overlay construction, and (3) pavements with HMAC overlay construction. In essence, the final equations are the integration of the intermediate serviceability loss equations due to fatigue, swelling, shrinkage and thermal cracking.

This report presents a discussion of how the equations were developed and how the equations could be used to solve practical design problems. A sensitivity analysis method has been developed to establish how reasonable the pavement performance equations are and how significant each construction, traffic, and environmental variable is in the equations. In addition, probabilistic design concepts are incorporated into the performance equations in order to allow consideration of the effects of the inherent uncertainty and variation of the variables. This report outlines a research procedure for future pavement performance studies to be conducted in Study 2-8-75-207 "Flexible Pavement Evaluation and Rehabilitation" which has the ultimate goal of implementing more "rational" performance equations in the pavement design systems developed in Texas Study 2-8-62-32, "Extension of AASHO Road Test Results," and Study 1-8-69-123, "A System Analysis of Pavement Design and Research Implementation."

Following this introductory chapter, Chapter II reports a brief review of literature. Descriptions are centered around the effects of construction, traffic, and environmental variables to the pavement performance. Chapter III describes the data collected for analysis. Performance equations are derived in Chapter IV by a "Two-Step Constrained Select Regression Methodology." Sensitivity analysis and stochastic considerations are discussed in Chapter V and VI, respectively. Finally, Chapter VII concludes the findings in this study. A glossary of symbols used in this report is presented in Appendix.

CHAPTER II

REVIEW OF LITERATURE

Road construction in the early days was primarily based on past experiences. The resulting pavements varied widely in their performance: many of them failed short of the anticipated life, requiring costly rehabilitation earlier than planned or were found to be grossly overdesigned, resulting in unnecessary initial costs. In order to improve the prediction of pavement service life, several road tests and laboratory experiments were conducted to advance the knowledge of pavement design and performance related to construction, traffic and environmental effects. This chapter reviews the development of pavement performance equations from several of the significant flexible pavement performance studies.

The AASHO Road Test, which has been the most comprehensive highway pavement performance study, was conducted from 1958 to 1961. This test studied the performance of highway structures of known layer thickness under known loads (1). The following equation was developed by regression to relate the level of serviceability of the surviving flexible pavement sections to various measures of pavement distress:

$$p = 5.03 - 1.91 \log (1 + SV) - 0.01 \sqrt{C + P} - 1.38 \, RD^2 \quad (2.1)$$

where

p = present serviceability index;

SV = mean of the slope variance in the two wheelpaths;

$C+P$ = a measure of cracking and patching in the pavement surface; and

RD = a measure of rutting in the wheelpaths.

Eq. 2.2 gave the principal relationships showing flexible pavement performance as a function of design and load variables. The initial serviceability trend value was 4.2 for flexible pavements, and the serviceability level at which a test section was taken out of the test and no longer observed was 1.5.

$$p = 4.2 - (4.2 - 1.5) \left(\frac{W}{\rho}\right)^{\beta} \tag{2.2}$$

where

p = serviceability trend value;

W = accumulated axle load applications at the time when p is to be observed;

β and ρ are functions of design and load.

$$\beta = 0.4 + \frac{0.081 \; (L_1 + L_2)^{3.23}}{(DD + 1)^{5.19} \; L_2^{\,3.23}} \tag{2.3}$$

$$\rho = \frac{10^{5.93} \; (DD + 1)^{9.36} \; L_2^{\,4.33}}{(L_1 + L_2)^{4.79}} \tag{2.4}$$

in which

$DD = 0.44D_1 + 0.14D_2 + 0.11D_3$;

D_1 = top layer thickness;

D_2 = second layer thickness;

D_3 = third layer thickness;

L_1 = nominal load axle weight in kips;

L_2 = 1 for single axle vehicles,
2 for tandem axle vehicles.

Following completion of the AASHO Road Test, the AASHO Interim Design Guide (2) was developed. Although the Road Test represented the most comprehensive development of the relationships between performance, structural thickness, and traffic loadings, the results were limited by the scope of the test and the conditions under which it was conducted. In applying the road test equations to mixed traffic conditions and to those situations where soil materials and climate differed from those that prevailed at the test site, certain assumptions had to be made. An NCHRP project (3) was thus initiated to evaluate the AASHO Interim Guide for Design of Pavement Structures. The revised Interim Guide (4) which incorporated the updated practice, experience, and research results provided the following performance equation for flexible pavements.

$$\log N = 9.36 \log (SN+1) - 0.20 + \frac{G}{0.40 + \dfrac{1094}{(SN+1)^{5.19}}} + \log \frac{1}{RF} + 0.372(SS-3.0) \qquad (2.5)$$

where

N = total load applications

SN = structural number

$G = \log \left(\dfrac{4.2 - p}{4.2 - 1.5}\right)$

p = present serviceability

RF = regional factor

SS = soil support value of subgrade material

Eq. 2.5 can be rewritten as follows:

$$p = 4.2 - (4.2-1.5) \left[\frac{10^{0.20}\ N\ RF}{(SN+1)^{9.36} 10^{0.372(SS-3.0)}}\right]^{0.40 + \frac{1094}{(SN+1)^{5.19}}} \qquad (2.6)$$

Following completion of the AASHO Road Test in 1962, several states, including Texas, initiated research directed toward extending the AASHO Road Test, and adapting the Road Test results to local environmental conditions. The Texas study resulted in the following performance equation for flexible pavements (5):

$$Q = \frac{53.6\ N\ SCI^2}{\alpha} + Q_2(1 - e^{-bt}) \tag{2.7}$$

where

Q = pavement serviceability loss

$= \sqrt{5-p} - \sqrt{5-p_1}$

p_1 = expected maximum serviceability index, occurring only immediately after initial or overlay construction.

p = present serviceability index

N = total number of equivalent applications of an 18-kip axle that will have been applied in one direction

SCI = surface curvature index

α = district temperature constant

Q_2 = pavement serviceability loss due to swelling

$= \sqrt{5-p'} - \sqrt{5-p_1}$

p' = swelling clay parameter, the assumed serviceability index in the absence of traffic

b = swelling clay parameter (5)

t = time after initial construction or rehabilitation

Eq. 2.7 can be rewritten as follows:

$$p = 5 - \{\sqrt{5-p_1} + \frac{53.6\ N\ SCI^2}{\alpha} + (\sqrt{5-p'} - \sqrt{5-p_1})\ [1 - e^{-bt}]\}^2 \tag{2.8}$$

Due to difficulties in estimating swelling clay parameters, the Texas flexible pavement performance equation (Eq. 2.8) was revised later to include a more rational swelling clay sub model (6). This is shown in Eq. 2.9.

$$p = 5 - [\sqrt{5-p_1} + \frac{53.6\ N\ SCI^2}{\alpha}]^2 - 0.335\ C_1 C_2 e^{-\Theta t} \qquad (2.9)$$

where

p = present serviceability index;

p_1 = serviceability index after initial or overlay construction;

N = one direction cumulated number of equivalent applications of 18-kip single axle;

SCI = surface curvature index;

α = district temperature constant;

C_1 = probability of encountering expansive clay on a troublesome site along a given project length;

C_2 = potential vertical rise of swelling soil;

Θ = swelling rate constant;

t = time after initial construction or rehabilitation.

Also, the swelling clay submodel was integrated into the AASHO flexible pavement performance equation (Eq. 2.6) in NCHRP Project 1-10A (7). This is shown in Eq. 2.10.

$$p = p_1 - (p_1 - 1.5) \{RF\ N\ [\frac{1.051}{(SN+1)\ 10^{0.03973(SS-3)}}]^{9.3633}\}^{0.4 + \frac{0.081(19)^{3.23}}{(SN+1)^{5.19}}} + 0.335 C_1 C_2 (e^{-\Theta t}) \qquad (2.10)$$

where

p = present serviceability index at time t;

p_1 = initial serviceability index either after construction or after an overlay;

RF = regional factor;

N = total 18-kip equivalent axles;

SN = structural number;

SS = soil support value of subgrade material;

C_1 = the probability of surface activity;

C_2 = potential vertical rise of swelling soil;

Θ = swelling rate constant;

t = time after initial construction or rehabilitation.

In attempting to verify these performance equations by observations of real pavements in Texas, several unusual discrepancies were found:

1. Despite their greater stiffness, pavements on stabilized base courses did not seem to perform as well as expected and in some cases, even appeared to perform worse than flexible pavements on water-bound base courses. Shrinkage cracking was the suspected cause.
2. Overlaid pavements in their second or third performance period did not appear to perform as well as predicted by the equations.
3. Reduction in service life due to the effects of the climate was far more prevalent than would be predicted from the performance equations.
4. There appeared to be a need to write a separate performance equation for each kind of pavement: surface treatment, hot mix asphaltic concrete, pavements with stabilized base courses, pavements overlaid with hot mix, and others.

All of these considerations led to a re-study of the data collected in Study 2-8-62-32 "Extension of AASHO Road Test Results" to determine if statistical methods could extract reliable models of pavement performance for the kinds of pavement represented and could determine equations which can predict the effect on pavement performance of some of the climatic variables such as moisture balance, shrinkage, and freeze-thaw cycling.

CHAPTER III

DESCRIPTION OF DATA

Pavement performance data which were collected on 133 Texas sections in study 32 (5) are analyzed in this report. The 133 sections can be divided into three pavement types: (1) 45 surface treatment sections, (2) 61 HMAC pavement sections without overlay construction and (3) 27 HMAC overlaid pavement sections. Using data from these sections, this study set out to develop a pavement performance equation for each pavement type.

Pavement damage is a complicated phenomenon which usually results from the combined effects of fatigue due to load, roughness due to swelling, cracking due to shrinkage and thermal cycling, and other effects. Rutting was not considered in this study. Generally, field measurements of rut depth were very small; thus in the opinion of the researchers the application of the Texas Triaxial Test procedures developed by the Texas Highway Department has reduced this problem to a minor factor in pavement performance.

Without doubt, a more exhaustive list of independent variables could have been written. The following were chosen for this investigation of Texas Study 2-8-62-32 data, and, as far as possible, each of them were determined for each test site represented.

Environmental Effects

- air temperature
- Thornthwaite moisture index
- number of freeze-thaw cycles

amount of solar radiation

Traffic Condition

number of load applications

Time Variable

time after construction or rehabilitation

Design Variable

top layer thickness

base course thickness

overlay thickness (if any)

composite pavement stiffness

surface curvature index

Base Course Property

percent fines

percent lime

Triaxial class

Subgrade Property

plasticity index

percent fines

permeability index

liquid limits

The collection of these data is detailed in subsequent sections. The dependent variable in the statistical studies made was the serviceability loss function, Q.

Serviceability Loss

Pavement Serviceability can be measured by several instruments such as GM Profilometer, PCA Road meter, Mays Ride Meter, etc. The serviceability index of a new (or rehabilitated) pavement usually begins at a level somewhere between 4.0 and 5.0 and then decreases with time as a result of traffic and environmental influences. When the serviceability index has dropped to a minimum acceptable level, then some major maintenance effort must be applied to restore the riding quality.

As described in Report 32-13(5), the high degree of variability of the measurements made it clear that it was not possible to estimate the initial serviceability index for an individual section with any degree of confidence in the result. The initial serviceability index of HMAC surfaced pavements in Texas averaged 4.3, close to the average value of 4.2 measured at the AASHO Road Test. The general level of the initial serviceability of surface treatment pavements averaged 2.9 on 11 sections in Texas Study 2-8-62-32. However, subsequent measurements on around 100 sections of District 7 (26) have indicated that a value of 3.9 is a more appropriate estimate of the initial serviceability index.

The serviceability loss of a pavement was defined as a function of initial serviceability index, p_1, and present serviceability index, p (5).

$$Q = \sqrt{5-p} - \sqrt{5-p_1} \tag{3.1}$$

In this study, p_1 is assumed a value of 4.3 for HMAC surfaced pavements and 3.9 for surface treatment pavements. Q is thus defined in this report as a function of p. For the HMAC surfaced pavements,

$$Q = \begin{cases} \sqrt{5-p} - \sqrt{5-4.3} & \text{if } p \leq 4.2 \\ = 0.05 & \text{if } p > 4.2 \end{cases} \tag{3.2}$$

For the surface treatment pavements,

$$Q = \sqrt{5-p} - \sqrt{5-3.9} \quad \text{if } p \leq 3.8$$
$$= 0.04 \quad \text{if } p > 3.8 \qquad (3.3)$$

Typical values of p and Q by Eq. 3.2 and 3.3 are shown in Table 3.1.

TABLE 3.1

TYPICAL PAVEMENT SERVICEABILITY LOSS VALUES

p	Q	
	HMAC Surfaced	Surface Treatment
0.0	1.399	1.187
1.0	1.163	0.951
2.0	0.895	0.683
3.0	0.578	0.365
3.6	0.347	0.134
3.8	0.259	0.047
4.0	0.163	0.040
4.2	0.058	0.040
4.4	0.050	0.040
5.0	0.050	0.040

Three successive measurements of the serviceability index, p, were made on 45 surface treatment sections, 61 HMAC pavement sections without overlay construction, and 27 HMAC overlaid pavement sections. The time between the first and second measurements averaged 2.1 years;

the time between the second and third averaged 2.5 years. It was observed that serviceability of 49% of HMAC sections and 64% of surface treatment sections increased as time passes. As described in Report 32-13, the gains in serviceability were due to the following reasons: (1) the time between successive measurements was too short to allow the development of significant trends, or (2) routine maintenance of the test sections prevented the development of significant trends, or (3) measurement errors masked the actual trends. In order to overcome the difficulty in applying these data for pavement performance analysis, the serviceability of the three measurements of each pavement section were averaged in Study 2-8-62-32. In this report, the averaged serviceability index was thus used in Eq. 3.2 or 3.3 to calculate the serviceability loss, Q. Since the initial serviceability index, p_1, was assumed a value of 4.3 for HMAC surfaced pavements, and 3.9 for surface treatment pavements, the averaged present serviceability index may be higher than the initial serviceability index. This is the reason that Q is given a value of 0.05 and 0.04 in Eq. 3.2 and 3.3 if the present serviceability is higher than $(p_1-0.1)$.

Environmental Factors

Included in this section are discussions of four environmental factors: (1) air temperature, (2) Thornthwaite moisture index, (3) number of freeze-thaw cycles and (4) amount of solar radiation.

The Texas district temperature constant, α, is defined (5) as follows:

$$\alpha = \frac{12}{\sum_{i=1}^{12} \frac{1}{\alpha_i}}$$

where α_i is the mean value of the mean daily temperature (^{0}F) less 32^0F for the i^{th} month averaged over a ten-year period. A table of district temperature constants for each of the 25 Texas districts (5) has been adopted by the Texas Highway Department as a parameter to evaluate pavement performance.

The Thornthwaite moisture index, TI, is defined (21)as follows:

$$TI = \frac{100S - 60D}{E} \tag{3.5}$$

where

S = surplus of water in inches,

D = deficit of water in inches, and

E = potential Evapo-Transpiration in inches.

A moisture surplus will store water in the subsoil water region, thus making more water available to deep rooted plants, lessening the effect of a drought. In this manner a surplus of six inches in one season will counteract ten inches deficiency in another season. The potential evapo-transpiration is defined as the amount of water which would be returned to the atmosphere by evaporation from the ground surface and transpiration by plants if there was an unlimited supply of water to the plants and ground. A map of Thornthwaite moisture index as it is distributed across Texas is shown in Report 18-1 (21).

A Texas map of annual average number of freeze-thaw cycles based on air-temperature is also shown in Report 18-1. The number of freeze-thaw cycles will be indicative of the level of thermal-fatigue the pavement will undergo; while not actually being measured pavement temperatures.

The annual average daily solar radiation is extremely influential in changing pavement temperatures, heating the surrounding air, and causing aging and brittleness in the asphalt. A Texas map of annual average daily solar radiation is also included in Report 18-1. The west Texas area receives the largest amount of solar radiation in Texas.

Traffic, Time and Design Variables

Time factor, t, is defined as the number of years after initial construction or rehabilitation.

Let

t_0 = time of initial construction or rehabilitation

t_1 = time of first measurement of serviceability

t_2 = time of second measurement of serviceability

t_3 = time of third measurement of serviceability

As mentioned above, the value used for the present serviceability index is the average of three measurements. The time at which the averaged serviceability will occur is defined by:

$$t = \frac{1}{3}(t_1+t_2+t_3) - t_0 \qquad (3.6)$$

Traffic is represented in this study by the accumulated number of equivalent applications of an 18-kip single axle load in one direction after construction or rehabilitation. A simple relationship is assumed.

Let

N_1 = accumulated number of 18 KSA at time t_1

N_2 = accumulated number of 18 KSA at time t_2

N_3 = accumulated number of 18 KSA at time t_3

then

$$\frac{N_1}{t_1-t_0} = \frac{N_2}{t_2-t_0} = \frac{N_3}{t_3-t_0} \qquad (3.7)$$

Given N_1, the cumulative number of equivalent applications of 18 KSA at time t (denoted by N) can thus be calculated by:

$$N = \left(\frac{t}{t_1-t_0}\right) N_1 \qquad (3.8)$$

Design variables included in this study are: (1) top layer thickness, (2) base course thickness, (3) overlay thickness (if any), (4) composite pavement stiffness, and (5) surface curvature index.

The surface curvature index (denoted by SCI), which defines the surface deflection basin near the load, was calculated based on the measurements of surface deflection by Dynaflect (22, 23).

$$SCI = W_1 - W_2 \qquad (3.9)$$

where W_1 is the surface deflection caused by cyclic loading of Dynaflect at a central point between the dual-wheel loads of Dynaflect, and W_2 is the deflection at a point, 12 inches apart from the eentral point, in the direction perpendicular to the plane of the dual-wheel loads.

The composite pavement stiffness was calculated based on the surface curvature index. Consider a n+1 layer pavement, in which n layers are above the subgrade. Now compose the top n layers as one layer and regard the subgrade as the second layer. Given the stiffness coefficient of the subgrade, A_2, the composite pavement stiffness coeffcient, A_1, was calculated by Eq. 3.10 (24).

$$SCI = 0.891087 \{A_1^{-4.50292}[\frac{1}{100} - \frac{1}{244}] + (A_2^{-4.50292} - A_1^{-4.50292}) [\frac{1}{100+6.25(A_1D_1)^2} - \frac{1}{244 + 6.25(A_1D_1)^2}]\} \quad (3.10)$$

where D_1 is the composite thickness of the pavement above the subgrade.

Base Course and Subgrade Properties

Base course properties studied in this project are: (1) percent fines, (2) percent lime and (3) Triaxial class. This information was found from construction records in the Districts where the test sections were located. The percent fines and percent lime were included in an attempt to find variables that would correlate well with serviceability loss due to shrinkage cracking, as well as to determine the effect of stabilization upon performance. There were not many test sections which were lime-stabilized and consequently, a better model could be expected to be found if more data were available. Stabilizing additives other than lime were not used in the test sections and were therefore not included. Nevertheless, the shrinkage cracking model which resulted showed that more serviceability loss occurred with higher percent fines and the loss was reduced by the addition of lime. The Texas Triaxial class of the base course was added to the list of variables in an attempt to represent the stiffness of the base course. This factor did not prove to be a strong variable in any of the models composed.

Subgrade properties included in this study are: (1) plasticity index (2) percent fines, (3) permeability index and (4) liquid limits. In most cases, none of this information was available from THD District records, so they were taken from the soil maps of the Soil Conservation Service. All four variables were expected to be strongly correlated with swelling activity and it was expected in addition that the plasticity index and percent fines would correlate well with damage due to thermal cracking.

Data described in this chapter were used in the regression analyses described in chapter IV.

CHAPTER IV

PERFORMANCE EQUATIONS

Data collected for this pavement performance study have been described in Chapter III. Huge accumulations of data usually become a cumbersome burden to engineers. Statistical regression analysis is a useful technique of extracting, from masses of data, the main features of the relationships hidden or implied in the tabulated figures. In the pavement performance system, in which quantities of the construction, traffic and environmental variables change, it is of interest to examine the effects that some variables exert on the pavement performance; in order that highway engineers and researchers can predict the pavement life based on specific construction alternatives, estimates of traffic and environmental impacts.

In any system, there may in fact be a simple functional relationship between variables; often there exists a relationship which is too complicated to grasp or to describe in simple terms. Usually, a "good" regression equation requires: (1) simple expression, (2) high multiple correlation, (3) small prediction error, and (4) satisfaction of all physical constraints. Several regression models and techniques have been utilized to analyze the pavement performance data. None of the existing methods could be qualified as a good regression procedure based on the previously mentioned criteria. Especially, conventional regression procedures provided no restrictions on the regression coefficients to satisfy physical constraints. A "Two-step Constrained Select Regression Methodology" was thus developed to analyze pavement performance data. This method requires two successive regression analysis steps. Regression models, sub-models and dependent variables are selected based on user-oriented decisions subject to physical constraints.

TWO-STEP CONSTRAINED SELECT REGRESSION METHODOLOGY

The first step of this method is essentially a selection regression procedure (8, 9) using a multiplicative model in order to obtain the approximate exponents of each individual independent variable. The results of the first step of the regression procedure, which are called intermediate products in this report, are several sub-models, which are selected based on the criteria (2), (3), and (4) mentioned on the previous page. The second step determines the coefficients of linear combinations of the intermediate products. The final model is selected from these linear combinations based on the four criteria. The following example will illustrate how this two-step constrained select regression method works.

Example: Let y = dependent variable and x_1, x_2, x_3 = independent variables, such that

$$y = f(x_1, x_2, x_3) \qquad (4.1)$$

The first step of the regression assumes a multiplicative model

$$y = e^{a_0} x_1^{a_1} x_2^{a_2} x_3^{a_3} \qquad (4.2)$$

This model is equivalent to

$$\ln y = a_0 + a_1 \ln x_1 + a_2 \ln x_2 + a_3 \ln x_3 \qquad (4.3)$$

The SELECT regression program (10) uses the Hocking-La Motte-Leslie selection strategy (8, 9) and a linear regression technique to determine the constants in Eq. 4.3, and select the best models using n, n-1, and so on down to 3, 2 and 1 variables. In this example, in which there are only three variables to start with, there is only one 3-variable model, three 2-variable models, and three 1-variable models.

$$\text{Model 1:} y = e^{a_{01}} x_1^{a_{11}} x_2^{a_{21}} x_3^{a_{31}}$$

$$\text{Model 2:} y = e^{a_{02}} x_1^{a_{12}} x_2^{a_{22}}$$

$$\text{Model 3:} y = e^{a_{03}} x_1^{a_{13}} x_3^{a_{33}}$$

$$\text{Model 4:} y = e^{a_{04}} x_2^{a_{24}} x_3^{a_{34}} \qquad (4.4)$$

$$\text{Model 5:} y = e^{a_{05}} x_1^{a_{15}}$$

$$\text{Model 6:} y = e^{a_{06}} x_2^{a_{26}}$$

$$\text{Model 7:} y = e^{a_{07}} x_3^{a_{37}}$$

The first subscript of regression coefficient, a, is the same subscript shown in Eq. 4.2 and 4.3; the second subscript is the model number. Now physical constraints can be examined by inspecting the signs of resulting regression coefficients. The constant e^{a_0} is always positive no matter whether a_0 is positive or negative. Therefore, if $a_i > o$ ($i = 1, 2, 3$), then y will increase as x_i increases; if $a_i < o$, then y will decrease as x_i increases. Physical considerations usually will show which sign a_i should have in order to be physically realistic, and consequently the signs (+ or −) of these constants are usually known beforehand. In case one or more coefficients of a model has the wrong sign, the model is discarded from further consideration. Meanwhile, the multiple correlation coefficient and standard error of each feasible model should be examined. To illustrate the screening procedure using signs of the constants, R^2, and standard error, Table 4.1 shows assumed regression coefficients, multiple correlation coefficients (R^2), and standard errors (SE) of the intermediate models (Eq. 4.4). Assume the following constraints are given:

TABLE 4.1 Example Intermediate Models

Model	a_0	a_1	a_2	a_3	R^2	SE
1	3	5	-1	2	0.80	0.01
2	4	5	-1	0	0.70	0.02
3	2	5	0	-1	0.70	0.02
4	8	0	-4	8	0.40	0.08
5	5	5	0	0	0.70	0.02
6	6	0	2	0	0.20	0.10
7	9	0	0	2	0.10	0.15

(1) y increases as x_1 increases, i.e. $dy/dx_1 > o$.

(2) y decreases as x_2 increases, i.e. $dy/dx_2 < o$.

(3) y increases as x_3 increases, i.e. $dy/dx_3 > o$.

Slopes of the basic multiplicative model can be derived from Eq. 4.2

$$\frac{dy}{dx_1} = a_1 e^{a_0} x_1^{a_1 - 1} x_2^{a_2} x_3^{a_3}$$

$$\frac{dy}{dx_2} = a_2 e^{a_0} x_1^{a_1} x_2^{a_2 - 1} x_3^{a_3} \qquad (4.5)$$

$$\frac{dy}{dx_3} = a_3 e^{a_0} x_1^{a_1} x_2^{a_2} x_3^{a_3 - 1}$$

In each case the sign of the derivatives depend upon the signs at the coefficients, a_1, a_2, and a_3 respectively. That is, $a_1 > o$, $a_2 < o$, and $a_3 > o$. Examining a_1, a_2, and a_3 values in Table 4.1, models 3 ($a_3 < o$) and 6 ($a_2 > o$) violate these rules and are thus discarded. Also, from Table 4.1, models 4, 6 and 7 are discarded because of poor R^2 and SE. Models 1, 2, and 5 are selected for second step regression analysis. The following linear combination model is assumed.

$$\begin{aligned} y &= b_0 + b_1 z_1 + b_2 z_2 + b_5 z_5 \\ &= b_0 + b_1 (x_1^5 x_2^{-1} x_3^2) + b_2 (x_1^5 x_2^{-1}) + b_5 (x_1^5) \end{aligned} \qquad (4.6)$$

It must be noted that the constant term, a_0, is not included in Eq. 4.6, since the linear combination regression will provide new regression coefficients. The same select regression program is utilized to produce the following models:

Model 1 $\quad y = b_{01} + b_{11} z_1 + b_{21} z_2 + b_{31} z_3$

Model 2 $\quad y = b_{02} + b_{12} z_1 + b_{22} z_2$

$$
\begin{aligned}
&\text{Model 3} \quad y = b_{03} + b_{13}z_1 + b_{33}z_3 \\
&\text{Model 4} \quad y = b_{04} + b_{24}z_2 + b_{34}z_3 \\
&\text{Model 5} \quad y = b_{05} + b_{15}z_1 \\
&\text{Model 6} \quad y = b_{06} + b_{26}z_2 \\
&\text{Model 7} \quad y = b_{07} + b_{37}z_3
\end{aligned}
\tag{4.7}
$$

The first subscript of regression coefficient, b, is the same subscript in Eq. 4.6; the second subscript is the model number. As described previously, a "good" regression function requires: (1) simple expression, (2) high multiple correlation, (3) small prediction error, and (4) satisfaction of all physical constraints. The last criterion is a necessary condition and should be checked first. The slopes of Eq. 4.6 are as follows:

$$
\begin{aligned}
\frac{dy}{dx_1} &= 5b_1x_1^4x_2^{-1}x_3^2 + 5b_2x_1^4x_2^{-1} + 5b_5x_1^4 \\
\frac{dy}{dx_2} &= -b_1x_1^5x_2^{-2}x_3^2 - b_2x_1^5x_2^{-2} \\
\frac{dy}{dx_3} &= 2b_1x_1^5x_2^{-1}x_3
\end{aligned}
\tag{4.8}
$$

Physical constraints require $\frac{dy}{dx_1} > 0$, $\frac{dy}{dx_2} < 0$, and $\frac{dy}{dx_3} > 0$. Usually Eq. 4.8 is not easy to solve. A quick and obvious method is to examine the sign of b's. Since z_1, z_2 and z_3 satisfy physical constraints, thereby if b_1, b_2, $b_3 > 0$ then $\frac{dy}{dx_1} > 0$, $\frac{dy}{dx_2} < 0$, and $\frac{dy}{dx_3} > 0$. Any of the seven models shown in Eq. 4.7 with any $b_i < 0$, i = 1, 2, 5, is thus discarded from final selection. It is possible, however, that $\frac{dy}{dx_1} > 0$, $\frac{dy}{dx_2} < 0$ and $\frac{dy}{dx_3} > 0$ hold feasible even if one of the b's is negative. This case is not considered due to the complexity in evaluation. Models which satisfy the physical constraints are then compared using the other three selection criteria. Often, compromises of the first three selection criteria are needed in the selection of the final product, since more complicated expression of a regression function is usually accompanied by higher R^2 and smaller SE; while simpler expressions result in lower R^2 and higher SE.

There is no guarantee that the Two-Step Constrained Select Regression Methodology provides the best fit regression equation with best R^2 and SE. However, this method provides a "good" fit regression equation which satisfies all user-specified physical constraints with reasonable R^2 and SE, as well as simple expressions adaptable for practical application. The following sections of this chapter will illustrate the utilization of this regression procedure to fit the flexible pavement performance data described in Chapter III.

SERVICEABILITY LOSS DUE TO FATIGUE

Analysis (5) of AASHO Road Test data has shown that serviceability loss due to fatigue is a function of traffic, surface curvature, and temperature. The functional relationship is as follows:

$$Q = \frac{53.6 \text{ N SCI}^2}{\alpha} \qquad (4.9)$$

where

Q = serviceability loss resulting from the repeated application of an 18-kip single axle load,

N = number of 18-kip single axle loads applied during a period for which SCI is relatively constant,

SCI = surface curvature index in mils determined by the Dynaflect (11), and

α = harmonic mean of the daily temperature (°F) less 32°F (5).

This equation implies that serviceability loss increases as (1) traffic increases, (2) surface curvature increases, or (3) temperature decreases. Meanwhile pavement surface fatigue can be written as a function of pavement stiffness, top layer thickness and second layer thickness.

$$Q = \frac{b_o}{A^{b_1} D_2^{b_2} [(D_1 - 2)^2]^{b_3}} \qquad (4.10)$$

where

Q = serviceability loss due to surface fatigue,

A = composite pavement stiffness coefficient,

D_1 = top layer thickness in inches,

D_2 = second layer thickness in inches, and

b_o, b_1, b_2, b_3 = constants.

Eq. 4.10 is based on a recent study of asphaltic concrete surfacing materials (12), which discovered that: (1) more stiff pavement results in less serviceability loss, (2) thicker base course results in less tensile stress at the bottom of the surface course due to tire loading and this results in less fatigue damage, and (3) the maximum tensile stress at the bottom of the surface course begins to occur when the top layer thickness is around 2 inches. In addition, surface course fatigue is increased if the subgrade is more compressible. Subgrade compressibility varies directly with its plasticity and moisture content and it is expected the serviceability loss is similarly correlated.

$$Q = c_o \, PI^{c_1} \, (TI + 35)^{c_2} \qquad (4.11)$$

where

Q = serviceability loss due to subgrade fatigue,

PI = plasticity index of subgrade,

TI = Thornthwaite moisture index, and

c_o, c_1, c_2 = constants.

The Thornthwaite moisture index ranges from -30 in west Texas to +15 in east Texas. The positive sign indicates that rainfall outruns potential evapo-transpiration and it is assumed that the more positive the Thornthwaite index (the farther east) the wetter the subgrade soils may be expected to be. The constant, 35, is added to the Thornthwaite index so that the variable (TI + 35) will always be positive within the state but will be smaller where there is less likelihood of fatigue damage to the pavement due to a wet, compressible subgrade.

The time factor, t (in years), has been included in the traffic term, N, since the cumulative number of axle loads is proportional to time. However, pavement deterioration could be directly correlated to the time factor due to the effects of aging and exposure to the elements. The overlay thickness, OV (in inches), could affect the serviceability of rehabilitated pavements. Combining all these variables, the serviceability loss due to fatigue can be approximated by multiplicative characteristics such as the following

$$Q_1 = a_0\left(\frac{N^{a_1} SCI^{a_2}}{\alpha^{a_3}}\right)\left(\frac{1}{A^{a_4} D_2^{a_5}\,[(D_1-2)^2]^{a_6}}\right)\left(PI^{a_7}[TI + 35]^{a_8}\right)$$
$$(t^{a_9})\quad(OV^{a_{10}}) \qquad (4.12)$$

in which a_i, i=o, 1 ..., 10, are expected to be positive constants. Not all of the variables listed above are expected to make their way into the final models.

Step 1 of the Two-Step Constrained Select Regression Methodology was applied to fit the performance data of three flexible pavement types for which data were available. Resulting equations after user-oriented selections are as follows:

For surface treatment pavements,

$$Q_1 = e^{-3.918}\ SCI^{0.602}\ \alpha^{0.543}\ N^{0.127}\ (TI + 35)^{0.519} \tag{4.13}$$

$$(\text{Obs.} = 45,\ R^2 = 0.296,\ SE = 0.624)$$

For HMAC pavements without overlay construction,

$$Q_1 = e^{-2.911}\ SCI^{0.499}\ N^{0.197}\ \alpha^{0.610}\ [(D_1 - 2)^2]^{-0.018} \tag{4.14}$$

$$(\text{Obs.} = 61,\ R^2 = 0.271,\ SE = 0.839)$$

where $D_1 \neq 2$.

For pavements with HMAC overlay construction,

$$Q_1 = e^{-6.290}\ N^{0.166}\ OV^{1.408}\ A^{-0.717}\ (TI + 35)^{1.404} \tag{4.15}$$

$$(\text{Obs.} = 22,\ R^2 = 0.480,\ SE = 0.522)$$

Obs., R^2 and SE are, respectively, number of observations in each data group under investigation, multiple correlation coefficient and standard error of regression.

By approximation, Eq. 4.13, 4.14 and 4.15 become Eq. 4.16, 4.17 and 4.18, respectively.

Surface Treatment Pavements

$$Q_1 = \frac{1}{50}\ N^{\frac{1}{8}}\ [SCI\ \alpha\ (TI + 35)]^{\frac{1}{2}} \tag{4.16}$$

HMAC pavements without overlay construction

$$Q_1 = \frac{1}{18.4} \frac{SCI^{\frac{1}{2}} N^{\frac{1}{5}} \alpha^{\frac{3}{5}}}{(|D_1-2|)^{.036}}, \quad D_1 \neq 2 \tag{4.17}$$

Pavements with HMAC overlay construction

$$Q_1 = \frac{1}{539} N^{\frac{1}{6}} \left[\frac{OV^2 \ (TI + 35)^2}{A}\right]^{\frac{7}{10}} \tag{4.18}$$

Examining the three equations, specific findings are as follows:

1. Every equation includes the term, N. At time, t=0, cumulative number of axle loads, N=0; this implies that serviceability loss, Q_1=0, at t=0.
2. Contribution of N to Q_1 depends on pavement types. The exponent of N is $\frac{1}{8}$ for surface treatment pavements, $\frac{1}{5}$ for HMAC pavements without overlay construction, and $\frac{1}{6}$ for pavements with HMAC overlay construction. Previous research (5) assumed the exponent of N to be one.
3. The surface curvature index, SCI, appears in Eq. 4.16 and 4.17 with an exponent of $\frac{1}{2}$, other than 2 in a previous study (5). SCI is a function of layer thicknesses and material stiffnesses. Eq. 4.18 shows that Q_1 is directly correlated to composite pavement stiffness coefficient, A, rather than SCI.
4. District temperature constant, α, appears in Eq. 4.16 and 4.17 with an exponent of $\frac{1}{2}$ and $\frac{3}{5}$, respectively, other than -1 as in the previous study (5), which discovered that high temperature reduced pavement deterioration in AASHO Road Test site. However, this finding does not appear to fit Texas pavement condition. Both Eq. 4.16 and 4.17 reveal that high temperature results in

more pavement damage.

5. The district temperature constant is not correlated to serviceability loss in HMAC overlaid pavement. However, the Thornthwaite moisture index, TI, plays a relatively significant role in Eq. 4.18 (overlaid pavements) with an exponent of 1.4. It also appears in Eq. 4.16 (surface treatment) with an exponent of 0.5. The positive exponents imply more fatigue loss in east Texas than west Texas.
6. Equation 4.17 shows that critical pavement damage begins to occur at around D_1 = 2 inches. This finding agrees with the suggestion by the Texas Highway Department (13), that top layer thickness should not be in the range from 2 inches to 7 inches.
7. Equation 4.18 reveals that damage of HMAC overlaid pavements is also a function of overlay thickness; thicker overlays result in more serviceability loss, with an exponent of 1.4. It should be noted carefully that the overlays from which these data were taken were generally less than 3 inches thick. Thus the equation is valid only for OV < 3 inches.
8. Subgrade plasticity index, thickness of base course, and time after construction (or rehabilitation) are not directly correlated to flexible pavement fatigue damage.

SERVICEABILITY LOSS DUE TO SWELLING

The loss of serviceability due to swelling clay was assumed to be a function of subgrade plasticity, moisture, percent fines, permeability, liquid limits, and time after construction or rehabilitation. This relationship is shown as follows:

$$Q_2 = a_0 (PI)^{a_1} (TI + 35)^{a_2} (F_s)^{a_3} (P_s)^{a_4} (L_s)^{a_5} (t)^{a_6} \qquad (4.19)$$

where

Q_2 = serviceability loss due to swelling,

PI = subgrade plasticity index,

TI = Thornthwaite moisture index,

F_s = percent fines of subgrade. "fines" are defined here as the percent passing the #200 sieve,

P_s = permeability index of subgrade,

L_s = liquid limits of subgrade,

t = time after construction or rehabilitation in years, and

a_i, i=0, 1, ..., 6, are expected to be positive constants. This model (Eq. 4.19) is based on two assumptions: (1) serviceability loss due to swelling is independent of flexible pavement types, and (2) no swelling occurs if the PI is less than 25. The step 1 select regression results in the following swelling loss model:

$$Q_2 = e^{-5.246} (TI + 35)^{0.918} (F_s)^{0.140} (t)^{0.510}$$

$$(\text{Obs.} = 19,\ R^2 = 0.337,\ SE = 0.661) \tag{4.20}$$

which is good only for subgrade soils with PI $\geq$ 25.

By approximation, Eq. 4.20 becomes

$$Q_2 = \frac{1}{190} (TI + 35) (F_s)^{\frac{1}{7}} (t)^{\frac{1}{2}} \tag{4.21}$$

This equation can be interpreted as follows:

1. At t=0 swelling loss Q_2=0. Q_2 increases as t increases.
2. The higher Thornthwaite indexes (TI) in east Texas are correlated with a greater supply of water to the subgrade soil and thus a greater amount of serviceability loss due to swelling.
3. A higher percent of fines in the subgrade (F_s) causes more swelling damage to the pavement.

4. Q_2 is not directly correlated to PI, when the PI is greater than 25. This model assumes that there will be no swelling loss when the PI is less than 25.

5. The permeability index P_s and the liquid limit L_s are not directly correlated with Q_2.

SERVICEABILITY LOSS DUE TO SHRINKAGE CRACKING

The pavement serviceability loss due to shrinkage cracking can be represented as the following equation:

$$Q_3 = a_0(TI + 35)^{-a_1} (F_B)^{a_2} (L_B + 1)^{-a_3} (PI)^{a_4} (D_1)^{a_5} (D_2)^{a_6} (TC)^{a_7} (t)^{a_8} \qquad (4.22)$$

where

Q_3 = serviceability loss due to shrinkage cracking,

TI = Thornthwaite moisture index,

F_B = percent fines in base course,

L_B = percent lime in base course,

PI = subgrade plasticity index,

D_1 = thickness of the top layer in inches,

D_2 = thickness of the base course in inches,

TC = triaxial class of the base course,

t = time after construction or rehabilitation,

and a_i, i=0, 1, ..., 8, are positive constants. It has been assumed that shrinkage cracking is independent of flexible pavement types. The step 1 select regression results in the following equation:

$$Q_3 = e^{-3.857} (TI + 35)^{-0.081} (F_B)^{0.576} (L_B + 1)^{-0.205} (t)^{0.316}$$

$$(\text{Obs.} = 90,\ R^2 = 0.129,\ SE = 0.772) \qquad (4.23)$$

By approximation, Eq. 4.23 becomes

$$Q_3 = \frac{1}{47} \left[\frac{F_B{}^6\ t^3}{(TI + 35)\ (L_B + 1)^2}\right]^{\frac{1}{10}} \tag{4.24}$$

This equation can be interpreted as follows:

1. At t=0, there is no shrinkage cracking, i.e., Q_3=0. The serviceability loss due to shrinkage cracking increases as t increases.

2. A greater percent of fines in base course (F_B) results in more shrinkage cracking.

3. A smaller value of TI + 35 which corresponds with west Texas is associated with more shrinkage cracking. This equation would predict a lower percentage of shrinkage cracking in east Texas.

4. A greater percent of lime in the base course results in less shrinkage cracking.

5. PI, D_1, D_2, and TC are not directly correlated to Q_3.

SERVICEABILITY LOSS DUE TO THERMAL CRACKING

The following functional relationship is assumed for the serviceability loss due to thermal cracking

$$Q_4 = a_0 \left(\frac{N_{FT}}{10}\right)^{a_1} \left(\frac{S_R}{100}\right)^{a_2} \left(\frac{F_B}{10}\right)^{a_3} (D_1)^{a_4} (D_2)^{a_5} (PI)^{a_6} (TI + 35)^{-a_7} (LB + 1)^{a_8} (t)^{a_9} \tag{4.25}$$

where

Q_4 = serviceability loss due to thermal cracking,

N_{FT} = number of freeze-thaw cycles,

S_R = amount of solar radiation,

F_B = percent fines in base course,

D_1 = top layer thickness in inches,

D_2 = thickness of base course in inches,

PI = subgrade plasticity index,

TI = Thornthwaite moisture index,

L_B = percent lime in base course,

t = time after construction or rehabilitation,

and a_i, i=0, 1, ..., 9, are positive constants. Two restrictions were applied in arriving at this equation: (1) It was assumed that there will be no thermal cracking if the number of freeze-thaw cycles in air temperature (N_{FT}) is less than 50; and (2) for HMAC overlaid pavements, a_8=0, since none of these pavements had lime stabilized base courses. The first step of the select regression results in two thermal cracking models. For surface treatment pavements no physically realistic correlations could be found; it is thus assumed that virtually no thermal cracking occurs in these pavements. For HMAC pavements without overlay construction,

$$Q_4 = e^{-2.784} \left(\frac{N_{FT}}{10}\right)^{0.027} \left(\frac{F_B}{10}\right)^{0.397} (TI + 35)^{-0.439} (t)^{1.084} \tag{4.26}$$

(Obs. = 26, R^2 = 0.694, SE = 0.567)

For HMAC overlaid pavements,

$$Q_4 = e^{-2.698} \left(\frac{N_{FT}}{10}\right)^{0.091} \left(\frac{F_B}{10}\right)^{0.440} (TI + 35)^{-0.173} (t)^{0.689} \tag{4.27}$$

(Obs. = 16, R^2 = 0.323, SE = 0.640)

By approximation, Eq. 4.26 and 4.27 can be written as follows:

HMAC pavements without overlay construction

$$Q_4 = \frac{1}{16} \frac{\left(\frac{N_{FT}}{10}\right)^{\frac{1}{37}} \left(\frac{F_B}{10}\right)^{\frac{2}{5}} t}{(TI + 35)^{\frac{1}{2}}} \tag{4.28}$$

where $N_{FT} > 50$

HMAC overlaid pavements

$$Q_4 = \frac{1}{15} \frac{\left(\frac{N_{FT}}{10}\right)^{\frac{1}{11}} \left(\frac{F_B}{10}\right)^{\frac{2}{5}} t^{\frac{7}{10}}}{(TI + 35)^{\frac{1}{6}}} \tag{4.29}$$

where $N_{FT} > 50$

Equation 4.28 and Eq. 4.29 can be interpreted as follows:

1. At t=0, there is no serviceability loss due to thermal cracking i.e., Q_4=0. Q_4 increases as t increases. The exponent of t is 1 and 0.7. respectively, in Eq. 4.28 and 4.29.
2. More freeze-thaw cycles result in more thermal cracks. The exponent of the number of freeze-thaw cycles is $\frac{1}{37}$ and $\frac{1}{11}$ respectively, in Eq. 4.28 and 4.29, indicating that freeze-thaw cycles cause more cracking in overlays than in pavements in their first performance period.
3. A greater percent of fines in the base course causes more thermal cracks. The exponent of the percent fines in base course is $\frac{2}{5}$ for both Eq. 4.28 and 4.29.
4. A smaller value of (TI + 35) (west Texas) is associated with more thermal cracking.

5. S_R, D_1, D_2, PI and L_B are not directly correlated to Q_4.

INTEGRATED SERVICEABILITY LOSS DUE TO FATIGUE, SWELLING, SHRINKAGE AND THERMAL CRACKING

The ultimate goal of this pavement performance study is to develop rational equations which can be easily adopted as a practical tool to forecast the life of pavements with reasonable accuracy. Design and construction engineers can compare feasible construction alternatives based on the estimates of traffic, environmental effects, future maintenance and economics. At the same time significant causes of specific types of pavement failures in certain districts can be prevented in the design stage.

The serviceability loss of pavements is a complex phenomenon that results from the integrated effects of fatigue, swelling, shrinkage, thermal cracking, and other natural distress mechanisms.

The serviceability loss equations, derived in previous sections by statistical regression analysis, do not provide high multiple correlations. However, these equations, which represent approximate functional relationship, containing what are judged to be appropriate variables. These equations were integrated into a final performance equation for a specific pavement type by applying the second step of the two-step constrained select regression procedure, thus lumping together the four major causes of pavement damage assumed in this report. The following sections of this chapter show how the final performance equations were regressed and what these equations imply.

Performance of Surface Treatment Pavements. Included in the regression analysis of pavement serviceability of surface treatment pavements are fatigue, swelling, and shrinkage cracking. Thermal cracking has been found insignificant for this pavement type. The second step constrained select regression assumes the following model:

$$Q = a_1 x_1 + a_2 x_2 + a_3 x_3 \tag{4.30}$$

where

$$x_1 = N^{\frac{1}{8}} [SCI\ \alpha\ (TI + 35)]^{\frac{1}{2}} \qquad \text{(From Eq. 4.16)}$$

$$x_2 = (TI + 35)\ (F_s)^{\frac{1}{7}}\ (t)^{\frac{1}{2}} \qquad \text{(from Eq. 4.21)}$$

$$x_3 = [\frac{F_B^{\ 6}\ t^3}{(TI + 35)\ (L_B + 1)^2}]^{\frac{1}{10}} \qquad \text{(from Eq. 4.24)}$$

and a_1, a_2, and a_3 are positive constants. The final selection, based on the criteria mentioned above, is

$$Q = 0.01703x_1 + 0.00716x_3$$

$$= 0.01703N^{\frac{1}{8}} [SCI\ \alpha\ (TI + 35)]^{\frac{1}{2}}$$

$$+ 0.00716 [\frac{F_B^{\ 6} t^3}{(TI + 35)\ (L_B + 1)^2}]^{\frac{1}{10}} \tag{4.31}$$

(Obs. = 33, R^2 = 0.880, SE = 0.099, Q range (0.040, 0.451))

Specific findings of this equation is as follows:

1. There is no serviceability loss at t = 0, since N = 0 at t = 0.
2. This equation satisfies physical constraints on N, SCI, TI, F_B, t, TI, and L_B. α violates the rule concluded from the AASHO Road Test data (5), as has been discussed previously.

3. The swelling term, x_2, is not included in this equation. Examining the three variables included in x_2, TI and t have been included in x_1 and x_3, except that TI + 35 has a positive exponent in x_1 and a negative exponent in x_3. The variable left is F_S, the fines in the subgrade. Comparing the exponents of the three variables in x_2, F_s is relatively insignificant.

4. Eq. 4.16 and Eq. 4.24 can be rewritten as follows:

$$Q_1 = \frac{1}{50} x_1 \tag{4.32}$$

$$Q_3 = \frac{1}{47} x_3 \tag{4.33}$$

By substitution, Eq. 31 becomes

$$Q = 0.01703\ (50\ Q_1) + 0.00716\ (47\ Q_3)$$

$$= 0.852\ Q_1 + 0.337\ Q_3 \tag{4.34}$$

This equation indicates that serviceability loss due to fatigue is 2.5 times the serviceability loss due to shrinkage for surface treatment pavements. The multiple correlation coefficient is quite acceptable and the standard error of residuals (S.E.) is small.

<u>Performance of HMAC Pavements Without Overlay Construction</u>. The second step constrained select regression assumes the following model:

$$Q = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \tag{4.35}$$

where

$$x_1 = \frac{SCI^{\frac{1}{2}}\ N^{\frac{1}{5}}\ \alpha^{\frac{3}{5}}}{(|D_1-2|)^{0.036}} \quad \text{(from Eq. 4.17)}$$

$$x_2 = (TI + 35)\ (F_s)^{\frac{1}{7}}\ (t)^{\frac{1}{2}} \qquad \text{(from Eq. 4.21)}$$

$$x_3 = \left[\frac{F_B^{\ 6}\ t^3}{(TI + 35)\ (L_B + 1)^2}\right]^{\frac{1}{10}} \qquad \text{(from Eq. 4.24)}$$

$$x_4 = \frac{\left(\frac{N_{FT}}{10}\right)^{\frac{1}{37}} \left(\frac{F_B}{10}\right)^{\frac{2}{5}} t}{(TI + 35)^{\frac{1}{2}}} \qquad \text{(from Eq. 4.28)}$$

and a_1, a_2, a_3 and a_4 are positive constants. The final selection of the performance equation is as follows:

$$Q = 0.04200\ x_1 + 0.00002\ x_2 + 0.03862\ x_4$$

$$= 0.04200\ \frac{SCI^{\frac{1}{2}}\ N^{\frac{1}{5}}\ \alpha^{\frac{3}{5}}}{(|D_1-2|)^{0.036}}$$

$$+ 0.00002\ (TI + 35)\ (F_s)^{\frac{1}{7}}\ (t)^{\frac{1}{2}}$$

$$+ 0.03862\ \frac{\left(\frac{N_{FT}}{10}\right)^{\frac{1}{37}} \left(\frac{F_B}{10}\right)^{\frac{2}{5}} t}{(TI + 35)^{\frac{1}{2}}} \qquad (4.36)$$

(Obs. = 42, R^2 = 0.802, SE = 0.157, Q range (0.050, 0.730))

Eq. 4.36 is interpreted as follows:

1. There is no serviceability loss at t=0 since N=0 at t=0.
2. This equation satisfies all physical constraints, except α.
3. The shrinkage term, x_3, is not included in this equation. However, F_B, t and TI of the shrinkage term have been included in the thermal crack term, x_4. Meanwhile, t and TI appear in

the swelling term, x_2. The variable, TI + 35, has a positive exponent in x_2 and a negative exponent in x_3. L_B is the only variable which does not appear in Eq. 4.36.

4. Eq. 4.17, 4.21 and 4.28 can be rewritten as follows:

$$Q_1 = \frac{1}{18.4} x_1 \tag{4.37}$$

$$Q_2 = \frac{1}{190} x_2 \tag{4.38}$$

$$Q_4 = \frac{1}{16} x_4 \tag{4.39}$$

By substitution, Eq. 4.36 becomes

$$\begin{aligned} Q &= 0.04200\ (18.4Q_1) + 0.00002\ (190\ Q_2) + 0.03862\ (16\ Q_4) \\ &= 0.773\ Q_1 + 0.004\ Q_2 + 0.618\ Q_4 \end{aligned} \tag{4.40}$$

This equation indicates that swelling is insignificant. The serviceability loss of HMAC pavements without overlays, due to fatigue, swelling and thermal cracks, has the following respective percentages: 55.4, 0.2 and 44.3. The multiple correlation coefficient is acceptable but the standard error is larger than the equation for the surface treatment pavement, indicating a wider scatter in the data.

Performance of HMAC Overlaid Pavements. The second step of the constrained select regression assumes the following model for HMAC overlaid pavements:

$$Q = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \qquad (4.41)$$

where

$$x_1 = N^{\frac{1}{6}} \left[\frac{OV^2 (TI + 35)^2}{A}\right]^{\frac{7}{10}} \qquad \text{(from Eq. 4.18)}$$

$$x_2 = (TI + 35)\ (F_s)^{\frac{1}{7}}\ (t)^{\frac{1}{2}} \qquad \text{(from Eq. 4.21)}$$

$$x_3 = \left[\frac{F_B^{\,6}\ t^3}{(TI + 35)\ (L_B + 1)^2}\right]^{\frac{1}{10}} \qquad \text{(from Eq. 4.24)}$$

$$x_4 = \frac{\left(\frac{N_{FT}}{10}\right)^{\frac{1}{11}} \left(\frac{F_B}{10}\right)^{\frac{2}{5}} t^{\frac{7}{10}}}{(TI + 35)^{\frac{1}{6}}} \qquad \text{(from Eq. 4.29)}$$

and a_1, a_2, a_3 and a_4 are positive constants. Final selection of the performance equation is as follows:

$$Q = 0.00058\ x_1 + 0.00259\ x_2 + 0.00114\ x_3$$

$$= 0.00058\ N^{\frac{1}{6}} \left[\frac{OV^2\ (TI + 35)^2}{A}\right]^{\frac{7}{10}}$$

$$+ 0.00259\ (TI + 35)\ (F_s)^{\frac{1}{7}}\ (t)^{\frac{1}{2}} \qquad (4.42)$$

$$+ 0.00114 \left[\frac{F_B^{\,6}\ t^3}{(TI + 35)\ (L_B + 1)^2}\right]^{\frac{1}{10}}$$

(Obs. = 21, R^2 = 0.811, SE = 0.178, Q range (0.050, 0.611))

This equation indicates that:

1. There is no serviceability loss at t=0, since N=0 at t=0.
2. This equation satisfies all physical constraints.
3. The thermal crack term, x_4, has been discarded from the final model. However, F_B, t, and TI of the thermal crack term have

been included in the shrinkage term, x_3. Also t and TI appear in the swelling term, x_2, except that TI + 35 has a positive exponent in x_2 rather than a smaller negative exponent in x_4. The number of freeze thaw cycles, N_{FT} is the only variable which does not appear in Eq. 4.42. However, N_{FT} is the least significant variable in X_4.

4. Eq. 4.18, 4.21 and 4.24 can be rewritten as follows:

$$Q_1 = \frac{1}{539} x_1 \qquad (4.43)$$

$$Q_2 = \frac{1}{190} x_2 \qquad (4.44)$$

$$Q_3 = \frac{1}{47} x_3 \qquad (4.45)$$

By substitution, Eq. 4.42 becomes

$$Q = 0.00058\ (539\ Q_1) + 0.00259\ (190\ Q_2) + 0.00114\ (47\ Q_3)$$

$$= 0.313\ Q_1 + 0.492\ Q_2 + 0.054\ Q_3 \qquad (4.46)$$

Eq. 4.46 implies that serviceability loss due to fatigue, swelling, and shrinkage in HMAC overlaid pavement has the following respective percentages: 36.4, 57.3, and 6.3. The multiple correlation coefficient is acceptable but the standard error for this model is largest of all pavement types, indicating a wider variation in the measured performance of overlaid pavements.

In the foregoing discussion of the regression models for the three pavement types, relative weights of the four major pavement distress mechanisms were found. It would be misleading to claim that these weights should apply to <u>all</u> pavements for obviously the weights should change with variations in climate, soil type, and so on. The weights simply reflect the bias that is in the data and consequently, no general conclusions should be drawn from the regression models obtained. This study, which was conducted on the data available from Texas Study 2-8-62-32, has pointed up the need for a careful experimental design to be followed in choosing a set of pavement sections from which to develop practical performance equations.

CHAPTER V

DIFFERENTIAL SENSITIVITY

Pavement behavior is such a complex phenomenon that it cannot be completely described by a single mathematical equation or model. Instead, a total coordinated and systematic approach based on the coupling of different subsystems defined for specific physical, and economical considerations can combine all the fundamental relationships into a systems optimization procedure.

In addition to the optimum solution of a systems problem, it is of interest to secure, whenever possible, additional information concerning the behavior of the solution due to changes in the system's parameters. This is a "sensitivity analysis."

The sensitivity analysis is valuable in this pavement performance study for the following reasons:

1. Performance equations are derived from regression analysis. Poor correlations due to lack of well-designed experiments and information do not provide true functional relationships. The sensitivity analysis can be utilized to check these equations by physical conditions.

2. Performance equations are to be integrated into pavement design systems. Sensitivity of pavement performance will ultimately affect the sensitivity of system performance.

3. There exists some skepticism toward the use of mechanistic theory in a field heretofore dominated by the exercise of experience, empirical rules and engineering judgement. The sensitivity analysis provides the designer with information about the change in pavement performance when other variables are changed.

The sensitivity of pavement performance will be formulated in this chapter by a mathematical method, named "differential analysis."

Differential Analysis

Consider the following problem. Suppose y is a function of x_i, i=1, 2, . . . , n; then

$$y = f(x_1, x_2, . . ., x_n) \tag{5.1}$$

There are two questions which can be answered by differential analysis: (1) what is the change in y when one or more of the variables, x_i, i=1, 2, . . ., n, are changed by a small amount? (2) which of the variables are relatively significant in change in y?

To find the answers to these questions, the function, f, must first be expanded as a Taylor's series (25):

$$f(x_1+\Delta x_1, x_2+\Delta x_2, . . ., x_n+\Delta x_n)$$

$$= f(x_1, x_2, . . ., x_n) + \sum_{i=1}^{n} \Delta x_i \frac{\partial}{\partial x_i} f(x_1, x_2, . . . x_n)$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \Delta x_i \Delta x_j \frac{\partial^2}{\partial x_i \partial x_j} f(x_1, x_2, . . ., x_n) + . . . \tag{5.2}$$

Taking the only linear term of the Taylor's series expansion gives:

$$f(x_1+\Delta x_1, x_2+\Delta x_2, . . . x_n+\Delta x_n) - f(x_1, x_2, . . ., x_n)$$

$$= \sum_{i=1}^{n} \Delta x_i \frac{\partial}{\partial x_i} f(x_1, x_2, . . ., x_n) \tag{5.3}$$

The left hand side of Eq. 5.3 is Δy, and it is equal to

$$\Delta y = \Delta x_1 \frac{\partial f}{\partial x_1} + \Delta x_2 \frac{\partial f}{\partial x_2} + . . . + \Delta x_n \frac{\partial f}{\partial x_n} \tag{5.4}$$

Eq. 5.4 is called a sensitivity equation and is used for differential analysis.

Sensitivity of Intermediate Serviceability Loss Models

Applying Eq. 5.4 to Eq. 4.16, 4.17, 4.18, 4.21, 4.24, 4.28 and 4.29, sensitivity analysis equations are summarized herein.

A. Serviceability Loss Due to Fatigue

(1) Surface Treatment Pavement

$$\Delta Q_1 = \frac{1}{50} N^{\frac{1}{8}} [SCI \cdot \alpha \cdot (TI+35)]^{\frac{1}{2}} [\frac{\Delta N}{8N} + \frac{\Delta SCI}{2SCI} + \frac{\Delta\alpha}{2\alpha} + \frac{\Delta TI}{2(TI+35)}] \quad (5.5)$$

(2) HMAC Pavement Without Overlay Construction

$$\Delta Q_1 = \frac{1}{18.4} \frac{SCI^{\frac{1}{2}} N^{\frac{1}{5}} \alpha^{\frac{3}{5}}}{(|D_1-2|)^{0.036}} [\frac{\Delta N}{5N} + \frac{\Delta SCI}{2SCI} + \frac{3\Delta\alpha}{5\alpha} - \frac{0.036\Delta D_1}{|D_1-2|}] \quad (5.6)$$

(3) HMAC Overlaid Pavement

$$\Delta Q_1 = 1.86N^{\frac{1}{6}} [\frac{OV^2 (TI+35)^2}{A}]^{\frac{7}{10}} [\frac{\Delta N}{6N} + \frac{1.4\Delta OV}{OV} + \frac{1.4\Delta TI}{TI+35} - \frac{0.7\Delta A}{A}] \quad (5.7)$$

B. Serviceability Loss Due to Swelling

$$\Delta Q_2 = \frac{1}{190} (TI+35)(F_S)^{\frac{1}{7}} (t)^{\frac{1}{2}} [\frac{\Delta TI}{TI+35} + \frac{\Delta F_S}{7F_S} + \frac{\Delta t}{2t}] \quad (5.8)$$

C. Serviceability Loss Due to Shrinkage

$$\Delta Q_3 = \frac{1}{47} [\frac{F_B{}^6 \, t^3}{(TI+35)(L_B+1)^2}]^{\frac{1}{10}} [\frac{0.6\Delta F_B}{F_B} + \frac{0.3\Delta t}{t} - \frac{0.1\Delta TI}{TI+35} - \frac{0.2\Delta L_B}{L_B+1}] \quad (5.9)$$

D. Serviceability Loss Due to Thermal Cracking

(1) HMAC Pavement Without Overlay Construction

$$\Delta Q_4 = \frac{1}{16} \frac{\left(\frac{N_{FT}}{10}\right)^{\frac{1}{37}} \left(\frac{F_B}{10}\right)^{\frac{2}{5}} t}{(TI+35)^{\frac{1}{2}}} \left[\frac{\Delta N_{FT}}{37N_{FT}} + \frac{2\Delta F_B}{5F_B} + \frac{\Delta t}{t} - \frac{\Delta TI}{2(TI+35)}\right] \quad (5.10)$$

(2) HMAC Overlaid Pavement

$$\Delta Q_4 = \frac{1}{15} \frac{\left(\frac{N_{FT}}{10}\right)^{\frac{1}{11}} \left(\frac{F_B}{10}\right)^{\frac{2}{5}} t^{\frac{7}{10}}}{(TI+35)^{\frac{1}{6}}} \left[\frac{\Delta N_{FT}}{11N_{FT}} + \frac{2\Delta F_B}{5F_B} + \frac{7\Delta t}{10t} - \frac{\Delta TI}{6(TI+35)}\right] \quad (5.11)$$

It is of interest to know how these sensitivity equations answer the two questions mentioned above. The serviceability loss of surface treatment pavement due to fatigue (Eq. 5.5) is used for illustration. In order to answer the first question, suppose there is no change in SCI, α and TI, that is, $\Delta SCI = \Delta\alpha = \Delta TI = 0$; then

$$\Delta Q_1 = \frac{1}{400} N^{-\frac{7}{8}} [SCI \cdot \alpha \cdot (TI+35)]^{\frac{1}{2}} \cdot \Delta N \quad (5.12)$$

This implies that when N is increased (or decreased) by a small amount, say one unit, that is, $\Delta N=1$; Q_1 is increased (or decreased) by ΔQ_1 units where

$$\Delta Q_1 = \frac{1}{400} N^{-\frac{7}{8}}[SCI \cdot \alpha \cdot (TI+35)]^{\frac{1}{2}} \qquad (5.13)$$

The same procedure can be applied to examine the sensitivity of Q_1 when more than one variables at the right hand side of Eq. 5.5 are changed. For instance, if there is no change in N and TI, that is, $\Delta N = \Delta TI = 0$; then

$$\Delta Q_1 = \frac{1}{50} N^{\frac{1}{8}}[SCI \cdot \alpha \cdot (TI+35)]^{\frac{1}{2}}[\frac{\Delta SCI}{2SCI} + \frac{\Delta \alpha}{2\alpha}] \qquad (5.14)$$

Suppose SCI is increased by 0.05 units and α is decreased by 0.5 units; that is, $\Delta SCI=0.05$ and $\Delta\alpha=-0.5$; then Q_1 is increased by ΔQ_1 units (or decreased if ΔQ_1 is negative), where

$$\Delta Q_1 = \frac{1}{50} N^{\frac{1}{8}}[SCI \cdot \alpha \cdot (TI+35)]^{\frac{1}{2}}[\frac{0.05}{2SCI} - \frac{0.5}{2\alpha}] \qquad (5.15)$$

To answer the second question, significance of N, SCI, α and TI in change in Q_1 can be compared by absolute values of $\frac{\Delta N}{8N}$, $\frac{\Delta SCI}{2SCI}$, $\frac{\Delta\alpha}{2\alpha}$ and $\frac{\Delta TI}{2(TI+35)}$. Suppose the change in N, SCI, α and TI is, respectively, 1, 0.05 , -0.5 and -0.2 units. The significance of N, SCI, α and TI in change in Q_1 follows the ratio: $\left|\frac{1}{8N}\right| : \left|\frac{0.05}{2SCI}\right| : \left|\frac{-0.5}{2\alpha}\right| : \left|\frac{-0.2}{2(TI+35)}\right|$.

<u>Sensitivity of Final Serviceability Loss Models</u>

Applying Eq. 5.4 to Eq. 4.31, 4.36 and 4.42, sensitivity analysis equations are summaried herein.

A. Surface Treatment Pavement

$$\Delta Q = 0.01703 N^{\frac{1}{8}} [SCI \cdot \alpha \cdot (TI+35)]^{\frac{1}{2}} [\frac{\Delta N}{8N} + \frac{\Delta SCI}{2SCI} + \frac{\Delta \alpha}{2\alpha} + \frac{\Delta TI}{2(TI+35)}] + 0.00716$$

$$[\frac{F_B^{\ 6} \ t^3}{(TI+35)(L_B+1)^2}]^{\frac{1}{10}} [\frac{0.6\Delta F_B}{F_B} + \frac{0.3\Delta t}{t} - \frac{0.1\Delta TI}{TI+35} - \frac{0.2\Delta L_B}{L_B+1}] \qquad (5.16)$$

B. HMAC Pavement Without Overlay Construction

$$\Delta Q = 0.04200 \ \frac{SCI^{\frac{1}{2}} \ N^{\frac{1}{5}} \ \alpha^{\frac{3}{5}}}{(|D_1-2|)^{0.036}} [\frac{\Delta N}{5N} + \frac{\Delta SCI}{2SCI} + \frac{3\Delta\alpha}{5\alpha} - \frac{0.036\Delta D_1}{|D_1-2|}] + 0.00002(TI+35)$$

$$(F_S)^{\frac{1}{7}} (t)^{\frac{1}{2}} [\frac{\Delta TI}{TI+35} + \frac{\Delta F_S}{7F_S} + \frac{\Delta t}{2t}] + 0.03862 \ \frac{\left(\frac{N_{FT}}{10}\right)^{\frac{1}{37}} \left(\frac{F_B}{10}\right)^{\frac{2}{5}} t}{(TI+35)^{\frac{1}{2}}} [\frac{\Delta N_{FT}}{37N_{FT}} + \frac{2\Delta F_B}{5F_B}$$

$$+ \frac{\Delta t}{t} - \frac{\Delta TI}{2(TI+35)}] \qquad (5.17)$$

C. HMAC Overlaid Pavement

$$\Delta Q = 0.00058 N^{\frac{1}{6}} [\frac{OV^2 (TI+35)^2}{A}]^{\frac{7}{10}} [\frac{\Delta N}{6N} + \frac{1.4\Delta OV}{OV} + \frac{1.4\Delta TI}{TI+35} - \frac{0.7\Delta A}{A}]$$

$$+ 0.00259(TI+35)(F_S)^{\frac{1}{7}} (t)^{\frac{1}{2}} [\frac{\Delta TI}{TI+35} + \frac{\Delta F_S}{7F_S} + \frac{\Delta t}{2t}] + 0.00114 [\frac{F_B^{\ 6} \ \ t^3}{(TI+35)(L_B+1)^2}]^{\frac{1}{10}}$$

$$[\frac{0.6\Delta F_B}{F_B} + \frac{0.3\Delta t}{t} - \frac{0.1\Delta TI}{TI+35} - \frac{0.2\Delta L_B}{L_B+1}] \qquad (5.18)$$

The change in Q when one or more of the independent variables are changed by a small amount and relative significance of these variables in change in Q can be determined as shown previously.

This chapter presents a method of evaluating the sensitivity of pavement performance. However, this study is not complete unless typical values of each construction, traffic and environment variable are used in each equation to estimate ΔQ. Numerical computations are not included in the scope of this report.

CHAPTER VI

STOCHASTIC RELIABILITY

The inherent uncertainty and variation of the estimates of traffic, interactive characteristics of materials, as well as environmental and human factors, result in overdesigned and underdesigned pavements. In turn, either overdesigned or underdesigned pavements result in higher construction or rehabilitation costs, especially in terms of today's inflated cost of construction materials and labor.

The concept of probabilistic design has been applied to pavement studies since the late 1960's. Many computerized pavement design systems (7, 14, 15, 16, 17, 18) have adopted this concept as an evaluation of pavement reliability. Reliability is a statistical measure of the probability that a pavement will provide satisfactory service to the user through its design service life. In general, reliability is one of many factors influencing the effectiveness of complex systems. Specification of system reliability requires a trade-off between reliability and all other parameters that affect system effectiveness. The trade-off is recognized in the pavement design systems where reliability is balanced against future maintenance, economics and pavement performance. In fact, the choice of a reliability figure is ultimately a management decision.

The reliability (denoted by R) of the Texas Pavement Design Systems has been formulated (15, 17) as the probability that the predicted number of load applications that the pavement can withstand, N, is greater than the expected number of actual load applications, n.

$$R = P[(\log N - \log n) > 0] \tag{6.1}$$

This equation is based on the assumption that log N and log n are normally distributed.

The fatigue - associated performance equations developed in this study have the number N as a factor but the environmentally associated performance equations such as swelling, shrinking, and thermal cracking, do not include N. Consequently, the reliability of a pavement with both traffic and non-traffic distress is difficult to compute on the basis shown in Eq. 6.1. Because of this, this chapter will evaluate the reliability of each individual pavement performance equation and will show the development of the expected value and variance of pavement serviceability loss.

Estimation of Expected Value and Variance

Before the derivation of stochastic models to predict pavement life, some basic statistical concepts and methods are discussed herein. Included in this section are definitions (19) of expected value and variance as well as how they can be estimated by a Taylor's series expansion.

Definition 6.1 If X is a continuous random variable with probability density function $f_X(x)$, the expected value of H(X) is defined to be

$$E[H(X)] = \int_{-\infty}^{\infty} H(x) f_X(x) dx \tag{6.2}$$

so long as the integral is absolutely convergent. If the integral is not absolutely convergent, we simply say the expected value does not exist.

Definition 6.2 The expected value of X itself is called the mean or average value of X and is denoted by μ_x; that is, $\mu_x = E[X]$.

Definition 6.3 The variance of a random variable X (denoted by σ_x^2) is defined to be

$$\sigma_x^2 = E[(X - \mu_x)^2]; \tag{6.3}$$

its positive square root is denoted by σ_x and is called the standard deviation of X. Thus, $\sigma_x = \sqrt{\sigma_x^2}$.

The operation of taking expected values of random variables has several convenient properties. If X is a random variable, then

1. $E[c] = c$, where c is a constant (6.4)
2. $E[cH(X)] = c\,E[H(X)]$, (6.5)
3. $E[H(X) + G(X)] = E[H(X)] + E[G(X)]$, (6.6)

so long as the expected values involved exist. Proof of these properties are simple and can be found in many fundamental statistics textbooks (19). Occasionally taking the expected value of a complicated function is a painstaking process. In order to overcome this difficulty, the expected value can be approximated by taking Taylor's series expansion

$$f(x) = f(x-\Delta x) + f'(x-\Delta x)\Delta x + \frac{1}{2}f''(x-\Delta x)\cdot\Delta x^2 + \ldots \tag{6.7}$$

Take the first three terms on the right hand side and let $\Delta x = x - \mu_x$, then

$$f(x) = f(\mu_x)+f'(\mu_x)\cdot(x-\mu_x) + \frac{1}{2}f''(\mu_x)\cdot(x-\mu_x)^2 \tag{6.8}$$

Expected value of f(x) is

$$\begin{aligned} E[f(x)] &= E[f(\mu_x)] + E[f'(\mu_x)\cdot(x-\mu_x)] + E[\frac{1}{2}f''(\mu_x)\cdot(x-\mu_x)^2] \\ &= f(\mu_x) + f'(\mu_x)E[(x-\mu_x)] + \frac{1}{2}f''(\mu_x)E[(x-\mu_x)^2] \\ &= f(\mu_x) + \frac{1}{2}f''(\mu_x)\sigma_x^2 \end{aligned} \tag{6.9}$$

since $E[(x-\mu_x)] = 0$, and $E[(x-\mu_x)^2] = \sigma_x^2$. Three examples are used to illustrate the operation.

Example 6.1 If $f(x) = \log x$, then

$$f'(x) = \frac{0.4343}{x}$$

$$f''(x) = -\frac{0.4343}{x^2}$$

By substitution, Eq. 6.9 becomes

$$E[\log x] = \log \mu_x - \frac{0.4343}{2}\left(\frac{\sigma_x}{\mu_x}\right)^2 \tag{6.10}$$

Example 6.2 If $f(x) = \log(x+c)$, where c is a constant, then

$$f'(x) = \frac{0.4343}{x+c}$$

$$f''(x) = -\frac{0.4343}{(x+c)^2}$$

By substitution, Eq. 6.9 becomes

$$E[\log(x+c)] = \log\ (\mu_x+c) - \frac{0.4343}{2}\left(\frac{\sigma_x}{\mu_x+c}\right)^2 \tag{6.11}$$

<u>Example 6.3</u> If $f(x) = g^2(x)$, then

$$f'(x) = 2g(x)g'(x)$$
$$f''(x) = 2[g'(x)]^2 + 2g(x)g''(x)$$

By substitution, Eq. 6.9 becomes

$$E[g^2(x)] = g^2(\mu_x) + \frac{1}{2}\{2[g'(\mu_x)]^2 + 2g(\mu_x)g''(\mu_x)\}\sigma_x^{\ 2} \tag{6.12}$$

Taking the variance is also a painstaking process if the function is complicated. The Taylor's series expansion is thus applied to approximate the estimate of variance. By Eq. 6.3, the variance of a function, $f(x)$, (denoted by $V[f(x)]$) is as follows:

$$\begin{aligned} V[f(x)] &= E[\{f(x) - E[f(x)]\}^2] \\ &= E[f^2(x)] - \{E[f(x)]\}^2 \end{aligned} \tag{6.13}$$

Substitution of Eq. 6.9 and 6.12 into Eq. 6.13 gives

$$V[f(x)] = f^2(\mu_x) + \{[f'(\mu_x)]^2 + f(\mu_x)\quad f''(\mu_x)\}\sigma_x^2 - \{f^2(\mu_x) + f(\mu_x)f''(\mu_x)\sigma_x^2 + \frac{1}{4}[f''(\mu_x)]^2\sigma_x^{\ 4}\}$$

$$= [f'(\mu_x)]^2 \sigma_x^2 - \frac{1}{4}[f''(\mu_x)]^2 \sigma_x^4 \quad (6.14)$$

The second term on the right hand side can usually be neglected without significant loss of accuracy. Thus

$$V[f(x)] = [f'(\mu_x)]^2 \sigma_x^2 + \sigma_{1of}^2 \quad (6.15)$$

in which, σ_{1of}^2 is the variance of lack-of-fit. The following example illustrates the operation.

<u>Example 6.4</u> Let $f(x) = 5 + 4x_1{}^2 + 3x_2{}^2 + 2x_1x_2$.

$$V[f(x)] = [f'(\mu_x)]^2 \sigma_x{}^2 + \sigma_{1of}^2$$

Since

$$\frac{\partial f}{\partial x_1} = 8x_1 + 2x_2$$

and

$$\frac{\partial f}{\partial x_2} = 2x_1 + 6x_2$$

then

$$V[f(x)] = (\frac{\partial f(\mu_x)}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f(\mu_x)}{\partial x_2})^2 \sigma_{x_2}^2 + \sigma_{1of}^2$$

$$= (8\mu_{x_1} + 2\mu_{x_2})^2 \sigma_{x_1}^2 + (3\mu_{x_1} + 6\mu_{x_2})^2 \sigma_{x_2}^2 + \sigma_{1of}^2$$

The following sections will derive the expected value and variance of the pavement performance equations developed in chapter IV.

Expected Value and Variance of Serviceability Loss Models

The serviceability loss models can be represented by the following basic multiplicative equation:

$$y = a_o \left[\prod_{i=1}^{n} x_i^{a_i} \right] \left[\prod_{i=n+1}^{N} (x_i + c_i)^{a_i} \right] \qquad (6.16)$$

where y = dependent variable,

x_i = i^{th} independent variable,

n = number of x_i terms,

N - n = number of $(x_i + c_i)$ terms,

N = total number of independent variables, and,

a_o, $a_i (i = 1, 2, \ldots N)$, and $c_i (i = n+1, n+2, \ldots, N)$ are constants, in which, a_o is positive; a_i and c_i are not restricted. Eq. 6.16 is equivalent to Eq. 6.17.

$$\log y = \log a_o + \sum_{i=1}^{n} a_i \log x_i + \sum_{i=n+1}^{N} a_i \log(x_i + c_i) \qquad (6.17)$$

Take the expected value of log y,

$$E[\log y] = \log a_o + \sum_{i=1}^{n} a_i E[\log x_i] + \sum_{i=n+1}^{N} a_i E[\log(x_i + c_i)] \qquad (6.18)$$

Applying Eq. 6.10 and 6.11 to Eq. 6.18,

$$\log \mu_y - \frac{0.4343}{2} \left(\frac{\sigma_y}{\mu_y}\right)^2 = \log a_o + \sum_{i=1}^{n} a_i \left[\log \mu_{x_i} - \frac{0.4343}{2} \left(\frac{\sigma_{x_i}}{\mu_{x_i}}\right)^2\right]$$

$$+ \sum_{i=n+1}^{N} a_i \left[\log (\mu_{x_i} + c_i) - \frac{0.4343}{2} \left(\frac{\sigma_{x_i}}{\mu_{x_i} + c_i}\right)^2\right] \qquad (6.19)$$

Take variance of y (Eq. 6.16) by Eq. 6.15,

$$V[y] = \sum_{j=1}^{N} \left(\frac{\partial y}{\partial x_j}\right)^2 \sigma_{x_j}^2 + \sigma_{lof}^2$$

$$= \sum_{j=1}^{n} \left\{\frac{a_o a_j}{\mu_{x_j}} \left[\prod_{i=1}^{n} \mu_{x_i}^{a_i}\right]\left[\prod_{i=n+1}^{N} (\mu_{x_i} + c_i)^{a_i}\right]\right\}^2 \sigma_{x_j}^2 + \sum_{j=n+1}^{N} \left\{\frac{a_o a_j}{\mu_{x_j} + c_j}\right.$$

$$\left.\left[\prod_{i=1}^{n} \mu_{x_i}^{a_i}\right]\left[\prod_{i=n+1}^{N} (\mu_{x_i} + c_i)^{a_i}\right]\right\}^2 \sigma_{x_j}^2 + \sigma_{lof}^2$$

$$= \left\{a_o \left[\prod_{i=1}^{n} \mu_{x_i}^{a_i}\right]\left[\prod_{i=n+1}^{N} (\mu_{x_i} + c_i)^{a_i}\right]\right\}^2 \left\{\sum_{j=1}^{n} \left[a_j \frac{\sigma_{x_j}}{\mu_{x_j}}\right]^2\right.$$

$$\left. + \sum_{j=n+1}^{N} \left[a_j \frac{\sigma_{x_j}}{\mu_{x_j} + c_j}\right]^2\right\} + \sigma_{lof}^2 \qquad (6.20)$$

where σ_{lof}^2 is the mean square residual due to lack-of-fit of Eq. 6.16 for predicting the serviceability loss.

Applying Eq. 6.18 and 6.20 to Eq. 4.16, 4.17, 4.18, 4.21, 4.24, 4.28, and 4.29, the expected value and variance of intermediate serviceability loss are determined and the results are summarized below.

A. Serviceability Loss Due to Fatigue

(1) Surface Treatment Pavement

$$\log \mu_Q - \frac{0.4343}{2}\left(\frac{\sigma_Q}{\mu_Q}\right)^2 = \log \frac{1}{50} + \left[\frac{1}{8}\log \mu_N + \frac{1}{2}\log \mu_{SCI} + \frac{1}{2}\log \mu_\alpha + \frac{1}{2}\log(\mu_{TI} + 35)\right] - \frac{0.4343}{2}\left[\frac{1}{8}\left(\frac{\sigma_N}{\mu_N}\right)^2 + \frac{1}{2}\left(\frac{\sigma_{SCI}}{\mu_{SCI}}\right)^2 + \frac{1}{2}\left(\frac{\sigma_\alpha}{\mu_\alpha}\right)^2 + \frac{1}{2}\left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right] \quad (6.21)$$

$$\sigma_Q^2 = \left\{\frac{1}{50}\mu_N^{\frac{1}{8}}[\mu_{SCI}\mu_\alpha(\mu_{TI}+35)]^{\frac{1}{2}}\right\}^2\left\{\left(\frac{1}{8}\frac{\sigma_N}{\mu_N}\right)^2 + \left(\frac{1}{2}\frac{\sigma_{SCI}}{\mu_{SCI}}\right)^2 + \left(\frac{1}{2}\frac{\sigma_\alpha}{\mu_\alpha}\right)^2 + \left(\frac{1}{2}\frac{\sigma_{TI}}{\mu_{TI}+35}\right)^2\right\} + \sigma_{lof}^2 \quad (6.22)$$

(2) HMAC Pavement without Overlay Construction

$$\log \mu_Q - \frac{0.4343}{2}\left(\frac{\sigma_Q}{\mu_Q}\right)^2 = \log\left(\frac{1}{18.4}\right) + \left[\frac{1}{2}\log \mu_{SCI} + \frac{1}{5}\log \mu_N + \frac{3}{5}\log \mu_\alpha - 0.036\log(|\mu_{D_1} - 2|)\right] - \frac{0.4343}{2}\left[\frac{1}{2}\left(\frac{\sigma_{SCI}}{\mu_{SCI}}\right)^2 + \frac{1}{5}\left(\frac{\sigma_N}{\mu_N}\right)^2 + \frac{3}{5}\left(\frac{\sigma_\alpha}{\mu_\alpha}\right)^2 - 0.036\left(\frac{\sigma_{D_1}}{|\mu_{D_1} - 2|}\right)^2\right] \quad (6.23)$$

$$\sigma_Q^2 = \left\{\frac{1}{18.4}\frac{\mu_{SCI}^{\frac{1}{2}}\mu_N^{\frac{1}{5}}\mu_\alpha^{\frac{3}{5}}}{(|\mu_{D_1} - 2|)^{0.036}}\right\}^2\left\{\left(\frac{1}{2}\frac{\sigma_{SCI}}{\mu_{SCI}}\right)^2 + \left(\frac{1}{5}\frac{\sigma_N}{\mu_N}\right)^2 + \left(\frac{3}{5}\frac{\sigma_\alpha}{\mu_\alpha}\right)^2 + \left(0.036\frac{\sigma_{D_1}}{|\mu_{D_1} - 2|}\right)^2\right\} + \sigma_{lof}^2 \quad (6.24)$$

(3) HMAC Overlaid Pavement

$$\log \mu_Q - \frac{0.4343}{2}\left(\frac{\sigma_Q}{\mu_Q}\right)^2 = \log(1.86) + \left[\frac{1}{6}\log \mu_N + \frac{14}{10}\log \mu_{OV} + \frac{14}{10}\log(\mu_{TI} + 35) - \frac{7}{10}\log \mu_A\right] - \frac{0.4343}{2}\left[\frac{1}{6}\left(\frac{\sigma_N}{\mu_N}\right)^2 + \frac{14}{10}\left(\frac{\sigma_{OV}}{\mu_{OV}}\right)^2 + \frac{14}{10}\left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2 - \frac{7}{10}\left(\frac{\sigma_A}{\mu_A}\right)^2\right] \quad (6.25)$$

$$\sigma_Q^2 = \left\{1.86\ \mu_N^{\frac{1}{6}}\left[\frac{\mu_{OV}^2\ (\mu_{TI} + 35)^2}{\mu_A}\right]^{\frac{7}{10}}\right\}^2\left\{\left(\frac{1}{6}\frac{\sigma_N}{\mu_N}\right)^2 + \left(\frac{14}{10}\frac{\sigma_{OV}}{\mu_{OV}}\right)^2 + \left(\frac{14}{10}\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2 + \left(-\frac{7}{10}\frac{\sigma_A}{\mu_A}\right)^2\right\} + \sigma_{lof}^2 \quad (6.26)$$

B. Serviceability Loss Due to Swelling

$$\log \mu_Q - \frac{0.4343}{2}\left(\frac{\sigma_Q}{\mu_Q}\right)^2 = \log\left(\frac{1}{190}\right) + \left[\frac{1}{2}\log \mu_t + \frac{1}{7}\log \mu_{F_S} + \log(\mu_{TI} + 35)\right] - \frac{0.4343}{2}\left[\frac{1}{2}\left(\frac{\sigma_t}{\mu_t}\right)^2 + \frac{1}{7}\left(\frac{\sigma_{F_S}}{\mu_{F_S}}\right)^2 + \left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right] \quad (6.27)$$

$$\sigma_Q^2 = \left\{\frac{1}{190}\ \mu_t^{\frac{1}{2}}\ \mu_{F_S}^{\frac{1}{7}}\ (\mu_{TI} + 35)\right\}^2\left\{\left(\frac{1}{2}\frac{\sigma_t}{\mu_t}\right)^2 + \left(\frac{1}{7}\frac{\sigma_{F_S}}{\mu_{F_S}}\right)^2 + \left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right\} + \sigma_{lof}^2 \quad (6.28)$$

C. Serviceability Loss Due to Shrinkage

$$\log \mu_Q - \frac{0.4343}{2}\left(\frac{\sigma_Q}{\mu_Q}\right)^2 = \log\frac{1}{47} + \left[\frac{6}{10}\log \mu_{F_B} + \frac{3}{10}\log \mu_t - \frac{2}{10}\log\right.$$

$$(\mu_{L_B} + 1) - \frac{1}{10}\log(\mu_{TI} + 35)\Big] - \frac{0.4343}{2}$$

$$\left[\frac{6}{10}\left(\frac{\sigma_{F_B}}{\mu_{F_B}}\right)^2 + \frac{3}{10}\left(\frac{\sigma_t}{\mu_t}\right)^2 - \frac{2}{10}\left(\frac{\sigma_{L_B}}{\mu_{L_B} + 1}\right)^2\right.$$

$$\left. - \frac{1}{10}\left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right] \qquad (6.29)$$

$$\sigma_Q^2 = \left\{\frac{1}{47}\left[\frac{\mu_{F_B}^{6}\ \mu_t^{3}}{(\mu_{L_B} + 1)^2(\mu_{TI} + 35)}\right]^{\frac{1}{10}}\right\}^2\left\{\left(\frac{6}{10}\frac{\sigma_{F_B}}{\mu_{F_B}}\right)^2 + \left(\frac{3}{10}\frac{\sigma_t}{\mu_t}\right)^2 + \left(-\frac{2}{10}\frac{\sigma_{L_B}}{\mu_{L_B} + 1}\right)^2\right.$$

$$\left. + \left(-\frac{1}{10}\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right\} + \sigma_{lof}^2 \qquad (6.30)$$

D. Serviceability Loss Due to Thermal Cracking

(1) HMAC Pavement Without Overlay Construction

$$\log \mu_Q - \frac{0.4343}{2}\left(\frac{\sigma_Q}{\mu_Q}\right)^2 = \log\frac{1}{16} + \left[\frac{1}{37}\log\frac{\mu_{N_{FT}}}{10} + \frac{2}{5}\log\frac{\mu_{F_B}}{10} + \log \mu_t\right.$$

$$- \frac{1}{2}\log(\mu_{TI} + 35)\Big] - \frac{0.4343}{2}\left[\frac{1}{37}\left(\frac{\sigma_{N_{FT}}}{\frac{\mu_{N_{FT}}}{10}}\right)^2\right.$$

$$+ \frac{2}{5} \left(\frac{\sigma_{F_B}}{\frac{\mu_{F_B}}{10}}\right)^2 + \left(\frac{\sigma_t}{\mu_t}\right)^2 - \frac{1}{2} \left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2] \qquad (6.31)$$

$$\sigma_Q^2 = \{\frac{1}{16} \frac{\left(\frac{\mu_{N_{FT}}}{10}\right)^{\frac{1}{37}} \left(\frac{\mu_{F_B}}{10}\right)^{\frac{2}{5}} \mu_t}{(\mu_{TI} + 35)^{\frac{1}{2}}}\}^2 \{ \left(\frac{1}{37} \frac{\sigma_{N_{FT}}}{\mu_{N_{FT}}}\right)^2 + \left(\frac{2}{5} \frac{\sigma_{F_B}}{\mu_{F_B}}\right)^2 \left(\frac{\sigma_t}{\mu_t}\right)^2$$

$$+ \left(-\frac{1}{2} \frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\} + \sigma_{lof}^2 \qquad (6.32)$$

(2) HMAC Overlaid Pavement

$$\log \mu_Q - \frac{0.4343}{2} \left(\frac{\sigma_Q}{\mu_Q}\right)^2 = \log \frac{1}{15} + [\frac{1}{11} \log \left(\frac{\mu_{N_{FT}}}{10}\right) + \frac{2}{5} \log \left(\frac{\mu_{F_B}}{10}\right) + \frac{7}{10} \log \mu_t$$

$$- \frac{1}{6} \log (\mu_{TI} + 35)] - \frac{0.4343}{2} [\frac{1}{11} \left(\frac{\sigma_{N_{FT}}}{\frac{\mu_{N_{FT}}}{10}}\right)^2 + \frac{2}{5} \left(\frac{\sigma_{F_B}}{\frac{\mu_{F_B}}{10}}\right)^2$$

$$+ \frac{7}{10} \left(\frac{\sigma_t}{\mu_t}\right)^2 - \frac{1}{6} \left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2] \qquad (6.33)$$

$$\sigma_Q^2 = \{\frac{1}{15} \frac{\left(\frac{\mu_{N_{FT}}}{10}\right)^{\frac{1}{11}} \left(\frac{\mu_{F_B}}{10}\right)^{\frac{2}{5}} \mu_t^{\frac{7}{10}}}{(\mu_{TI} + 35)^{\frac{1}{6}}}\}^2 \{ \left(\frac{1}{11} \frac{\sigma_{N_{FT}}}{\mu_{N_{FT}}}\right)^2 + \left(\frac{2}{5} \frac{\sigma_{F_B}}{\mu_{F_B}}\right)^2 + \left(\frac{7}{10} \frac{\sigma_t}{\mu_t}\right)^2$$

$$+ \left(-\frac{1}{6} \frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\} + \sigma_{lof}^2 \qquad (6.34)$$

Procedures to solve these equations for expected value, μ_Q, and variance, σ_Q^2, of pavement serviceability loss are summarized below. Thermal cracks of HMAC overlaid pavement are used for illustration.

1. Given expected value and variance of N_{FT}, F_B, t and TI, σ_Q^2 can be calculated by Eq. 6.34.
2. Since expected value and variance of N_{FT}, F_B, t and TI, as well as σ_Q^2 are known, Eq. 6.33 can be simplified as follows:

$$\log \mu_Q - \frac{c_1}{\mu_Q^2} = c_2 \qquad (6.35)$$

in which c_1 and c_2 are constants.

3. The Newton-Raphson search (20) can thus be applied to calculate μ_Q by iterating with Eq. 6.35.

<u>Expected Value and Variance of Serviceability Loss Models for Pavement Types</u>

The final serviceability loss model is essentially the linear combination of intermediate models. Since there are at most four intermediate models for a specific pavement type: fatigue, swelling, shrinkage, and thermal cracking, the pavement type model has the following form:

$$Y = \sum_{i=1}^{4} a_i y_i \qquad (6.36)$$

where

Y = pavement type model,

a_i = constant, and

y_i = intermediate model.

Expected value of Y, E[Y], can be derived by

$$E[Y] = \sum_{i=1}^{4} a_i \mu_{y_i} \qquad (6.37)$$

Since y_i is a multiplicative model as shown in Eq. 6.16, μ_{y_i} can be calculated by iterating Eq. 6.19. Variance of Y can be calculated directly from Eq. 6.15. The expected value and variance of three final models, Eq. 4.31, 4.36, and 4.42, are summarized below.

A. Surface Treatment

$$\mu_Q = \mu_{K_1} + \mu_{K_2} \qquad (6.38)$$

$$\sigma_Q^2 = \sigma_{K_1}^2 + \sigma_{K_2}^2 + 2(\tfrac{1}{2})(\tfrac{-1}{10})\, \tilde{K}_1 \tilde{K}_2 (\frac{\sigma_{TI}}{\mu_{TI} + 35})^2 + 0.00988 \qquad (6.39)$$

$$\log \mu_{K_1} - \frac{0.4343}{2} (\frac{\sigma_{K_1}}{\mu_{K_1}})^2 = \log (0.01703) + [\tfrac{1}{8} \log \mu_N + \tfrac{1}{2} \log \mu_{SCI} + \tfrac{1}{2} \log \mu_\alpha + \tfrac{1}{2} \log (\mu_{TI} + 35)] - \frac{0.4343}{2} [\tfrac{1}{8}(\frac{\sigma_N}{\mu_N})^2 + \tfrac{1}{2}(\frac{\sigma_{SCI}}{\mu_{SCI}})^2 + \tfrac{1}{2}(\frac{\sigma_\alpha}{\mu_\alpha})^2 + \tfrac{1}{2}(\frac{\sigma_{TI}}{\mu_{TI} + 35})^2] \qquad (6.40)$$

$$\sigma_{K_1}^2 = \tilde{K}_1^2 \, [(\frac{1}{8} \frac{\sigma_N}{\mu_N})^2 + (\frac{1}{2} \frac{\sigma_{SCI}}{\mu_{SCI}})^2 + (\frac{1}{2} \frac{\sigma_\alpha}{\mu_\alpha})^2 + (\frac{1}{2} \frac{\sigma_{TI}}{\mu_{TI} + 35})^2] \qquad (6.41)$$

$$\log \mu_{K_2} - \frac{0.4343}{2} \left(\frac{\sigma_{K_2}}{\mu_{K_2}}\right)^2 = \log (0.00716) + \left[\frac{6}{10} \log \mu_{F_B} + \frac{3}{10} \log \mu_t - \frac{2}{10} \log (\mu_{L_B} + 1) - \frac{1}{10} \log (\mu_{TI} + 35)\right]$$

$$- \frac{0.4343}{2} \left[\frac{6}{10} \left(\frac{\sigma_{F_B}}{\mu_{F_B}}\right)^2 + \frac{3}{10} \left(\frac{\sigma_t}{\mu_t}\right)^2 - \frac{2}{10} \left(\frac{\sigma_{L_B}}{\mu_{L_B} + 1}\right)^2 - \frac{1}{10} \left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right] \quad (6.42)$$

$$\sigma_{K_2}^2 = \tilde{K}_2^2 \left[\left(\frac{6}{10} \frac{\sigma_{F_B}}{\mu_{F_B}}\right)^2 + \left(\frac{3}{10} \frac{\sigma_t}{\mu_t}\right)^2 + \left(\frac{-2}{10} \frac{\sigma_{L_B}}{\mu_{L_B} + 1}\right)^2 + \left(\frac{-1}{10} \frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right] \quad (6.43)$$

$$\tilde{K}_1 = 0.01703 \, \mu_N^{\frac{1}{8}} \left[\mu_{SCI} \, \mu_\alpha (\mu_{TI} + 35)\right]^{\frac{1}{2}} \quad (6.44)$$

$$\tilde{K}_2 = 0.00716 \left[\frac{\mu_{F_B}^{6} \quad \mu_t^{3}}{(\mu_{L_B} + 1)^2 \, (\mu_{TI} + 35)}\right]^{\frac{1}{10}} \quad (6.45)$$

B. HMAC Pavement Without Overlay Construction

$$\mu_Q = \mu_{K_1} + \mu_{K_2} + \mu_{K_3} \quad (6.46)$$

$$\sigma_Q^2 = \sigma_{K_1}^2 + \sigma_{K_2}^2 + \sigma_{K_3}^3 + 2\left(\frac{1}{2}\right) \tilde{K}_2 \tilde{K}_3 \left(\frac{\sigma_t}{\mu_t}\right)^2 + 2\left(-\frac{1}{2}\right) \tilde{K}_2 \tilde{K}_3 \left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2 + 0.02450 \quad (6.47)$$

$$\log \mu_{K_1} - \frac{0.4343}{2} \left(\frac{\sigma_{K_1}}{\mu_{K_1}}\right)^2 = \log (0.04200) + [\frac{1}{5} \log \mu_N + \frac{1}{2} \log \mu_{SCI}$$

$$+ \frac{3}{5} \log \mu_\alpha - 0.036 \log |\mu_{D_1} - 2|] - \frac{0.4343}{2}$$

$$[\frac{1}{5} \left(\frac{\sigma_N}{\mu_N}\right)^2 + \frac{1}{2} \left(\frac{\sigma_{SCI}}{\mu_{SCI}}\right)^2 + \frac{3}{5} \left(\frac{\sigma_\alpha}{\mu_\alpha}\right)^2 - 0.036$$

$$\left(\frac{\sigma_{D_1}}{|\mu_{D_1} - 2|}\right)^2] \tag{6.48}$$

$$\sigma_{K_1}^{\ 2} = \tilde{K}_1^{\ 2}[(\frac{1}{5} \frac{\sigma_N}{\mu_N})^2 + (\frac{1}{2} \frac{\sigma_{SCI}}{\mu_{SCI}})^2 + (\frac{3}{5} \frac{\sigma_\alpha}{\mu_\alpha})^2 + (-0.036 \frac{\sigma_{D_1}}{|\mu_{D_1} - 2|})^2] \tag{6.49}$$

$$\log \mu_{K_2} - \frac{0.4343}{2} \left(\frac{\sigma_{K_2}}{\mu_{K_2}}\right)^2 = \log (0.00002) + [\log (\mu_{TI} + 35) + \frac{1}{7} \log \mu_{F_S}$$

$$+ \frac{1}{2} \log \mu_t] - \frac{0.4343}{2} [\left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2 + \frac{1}{7} \left(\frac{\sigma_{F_S}}{\mu_{F_S}}\right)^2$$

$$+ \frac{1}{2} \left(\frac{\sigma_t}{\mu_t}\right)^2] \tag{6.50}$$

$$\sigma_{K_2}^{\ 2} = \tilde{K}_2^{\ 2}[(\frac{\sigma_{TI}}{\mu_{TI} + 35})^2 + (\frac{1}{7} \frac{\sigma_{F_S}}{\mu_{F_S}})^2 + (\frac{1}{2} \frac{\sigma_t}{\mu_t})^2] \tag{6.51}$$

$$\log \mu_{K_3} - \frac{0.4343}{2}\left(\frac{\sigma_{K_3}}{\mu_{K_3}}\right)^2 = \log(0.03862) + \left[\frac{1}{37}\log\left(\frac{\mu_{N_{FT}}}{10}\right) + \frac{2}{5}\log\left(\frac{\mu_{F_B}}{10}\right)\right.$$

$$\left. + \log \mu_t - \frac{1}{2}\log(\mu_{TI} + 35)\right] - \frac{0.4343}{2}$$

$$\left[\frac{1}{37}\left(\frac{\sigma_{N_{FT}}}{\frac{\mu_{N_{FT}}}{10}}\right)^2 + \frac{2}{5}\left(\frac{\sigma_{F_B}}{\frac{\mu_{F_B}}{10}}\right)^2 + \left(\frac{\sigma_t}{\mu_t}\right)^2 - \frac{1}{2}\left(\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right] \tag{6.52}$$

$$\sigma_{K_3}^{\ 2} = \tilde{K}_3^{\ 2}\left[\left(\frac{1}{37}\frac{\sigma_{N_{FT}}}{\mu_{N_{FT}}}\right)^2 + \left(\frac{2}{5}\frac{\sigma_{F_B}}{\mu_{F_B}}\right)^2 + \left(\frac{\sigma_t}{\mu_t}\right)^2 + \left(-\frac{1}{2}\frac{\sigma_{TI}}{\mu_{TI} + 35}\right)^2\right] \tag{6.53}$$

$$\tilde{K}_1 = 0.04200\,\frac{\mu_N^{\frac{1}{5}}\ \mu_{SCI}^{\frac{1}{2}}\ \mu_\alpha^{\frac{3}{5}}}{(|\mu_{D_1} - 2|)^{0.036}} \tag{6.54}$$

$$\tilde{K}_2 = 0.00002(\mu_{TI} + 35)(\mu_{F_S})^{\frac{1}{7}}(\mu_t)^{\frac{1}{2}} \tag{6.55}$$

$$\tilde{K}_3 = 0.03862\,\frac{\left(\frac{\mu_{N_{FT}}}{10}\right)^{\frac{1}{37}}\left(\frac{\mu_{F_B}}{10}\right)^{\frac{2}{5}}\ \mu_t}{(\mu_{TI} + 35)^{\frac{1}{2}}} \tag{6.56}$$

C. HMAC Overlaid Pavement

$$\mu_Q = \mu_{K_1} + \mu_{K_2} + \mu_{K_3} \tag{6.57}$$

$$\sigma_Q^2 = \sigma_{K_1}^2 + \sigma_{K_2}^2 + \sigma_{K_3}^2 + 2(\frac{1}{2})(\frac{3}{10})\tilde{K}_2\tilde{K}_3(\frac{\sigma_t}{\mu_t})^2 + [2(\frac{14}{10})\tilde{K}_1\tilde{K}_2 + 2(\frac{14}{10})(-\frac{1}{10})\tilde{K}_1\tilde{K}_3 + 2(-\frac{1}{10})\tilde{K}_2\tilde{K}_3](\frac{\sigma_{TI}}{\mu_{TI}+35})^2 + 0.03156 \tag{6.58}$$

$$\log \mu_{K_1} - \frac{0.4343}{2}(\frac{\sigma_{K_1}}{\mu_{K_1}})^2 = \log(0.00058) + [\frac{1}{6}\log \mu_N + \frac{14}{10}\log \mu_{OV} + \frac{14}{10}\log(\mu_{TI}+35) - \frac{7}{10}\log \mu_A] - \frac{0.4343}{2}[\frac{1}{6}(\frac{\sigma_N}{\mu_N})^2 + \frac{14}{10}(\frac{\sigma_{OV}}{\mu_{OV}})^2 + \frac{14}{10}(\frac{\sigma_{TI}}{\mu_{TI}+35})^2 - \frac{7}{10}(\frac{\sigma_A}{\mu_A})^2] \tag{6.59}$$

$$\sigma_{K_1}^2 = \tilde{K}_1^2[(\frac{1}{6}\frac{\sigma_N}{\mu_N})^2 + (\frac{14}{10}\frac{\sigma_{OV}}{\mu_{OV}})^2 + (\frac{14}{10}\frac{\sigma_{TI}}{\mu_{TI}+35})^2 + (-\frac{7}{10}\frac{\sigma_A}{\mu_A})^2] \tag{6.60}$$

$$\log \mu_{K_2} - \frac{0.4343}{2}(\frac{\sigma_{K_2}}{\mu_{K_2}})^2 = \log(0.00259) + [\frac{1}{2}\log \mu_t + \frac{1}{7}\log \mu_{F_S} + \log(\mu_{TI}+35)] - \frac{0.4343}{2}[\frac{1}{2}(\frac{\sigma_t}{\mu_t})^2 + \frac{1}{7}(\frac{\sigma_{F_S}}{\mu_{F_S}})^2 + (\frac{\sigma_{TI}}{\mu_{TI}+35})^2] \tag{6.61}$$

$$\sigma_{K_2}{}^2 = \tilde{K}_2{}^2[(\frac{1}{2}\frac{\sigma_t}{\mu_t})^2 + (\frac{1}{7}\frac{\sigma_{F_S}}{\mu_{F_S}})^2 + (\frac{\sigma_{TI}}{\mu_{TI}+35})^2] \qquad (6.62)$$

$$\log \mu_{K_3} - \frac{0.4343}{2}(\frac{\sigma_{K_3}}{\mu_{K_3}})^2 = \log(0.00114) + [\frac{6}{10}\log\mu_{F_B} + \frac{3}{10}\log\mu_t - \frac{2}{10}\log(\mu_{L_B}+1) - \frac{1}{10}\log(\mu_{TI}+35)] - \frac{0.4343}{2}[\frac{6}{10}(\frac{\sigma_{F_B}}{\mu_{F_B}})^2 + \frac{3}{10}(\frac{\sigma_t}{\mu_t})^2 - \frac{2}{10}(\frac{\sigma_{L_B}}{\mu_{L_B}+1})^2 - \frac{1}{10}(\frac{\sigma_{TI}}{\mu_{TI}+35})^2] \qquad (6.63)$$

$$\sigma_{K_3}{}^2 = \tilde{K}_3{}^2[(\frac{6}{10}\frac{\sigma_{F_B}}{\mu_{F_B}})^2 + (\frac{3}{10}\frac{\sigma_t}{\mu_t})^2 + (-\frac{2}{10}\frac{\sigma_{L_B}}{\mu_{L_B}+1})^2 + (-\frac{1}{10}\frac{\sigma_{TI}}{\mu_{TI}+35})^2] \qquad (6.64)$$

$$\tilde{K}_1 = 0.00058\ \mu_N^{\frac{1}{6}}[\frac{\mu_{OV}{}^2(\mu_{TI}+35)^2}{\mu_A}]^{\frac{7}{10}} \qquad (6.65)$$

$$\tilde{K}_2 = 0.00259\ \mu_t^{\frac{1}{2}}\ \mu_{F_S}^{\frac{1}{7}}\ (\mu_{TI}+35) \qquad (6.66)$$

$$\tilde{K}_3 = 0.00114[\frac{\mu_{F_B}^{6}\ \mu_t^{3}}{(\mu_{L_B}+1)^2\ (\mu_{TI}+35)}]^{\frac{1}{10}} \qquad (6.67)$$

Procedures to solve these equations for expected value, μ_Q, and variance, σ_Q^2, of pavement serviceability loss are summaried below. HMAC overlaid pavement is used for illustration.

1. Given expected value and variance of N, OV, TI, A, t, F_S, F_B and L_B, $\tilde{K}_1$, $\tilde{K}_2$ and $\tilde{K}_3$ can be calcualted, respectively, from Eq. 6.65, 6.66 and 6.67.

2. Since $\tilde{K}_1$, $\tilde{K}_2$, and $\tilde{K}_3$ are known, $\sigma_{K_1}^2$, $\sigma_{K_2}^2$, and $\sigma_{K_3}^2$ can be calcualted, respectively, from Eq. 6.60, 6.62, and 6.64.

3. Eq. 6.59, 6.61, and 6.63 can be simplified as follows:

$$\log \mu_{K_1} - \frac{c_1}{\mu_{K_1}^2} = c_2 \tag{6.68}$$

$$\log \mu_{K_2} - \frac{c_3}{\mu_{K_2}^2} = c_4 \tag{6.69}$$

$$\log \mu_{K_3} - \frac{c_5}{\mu_{K_3}^2} = c_6 \tag{6.70}$$

in which c_i, i=1, 2, ..., 6, are constants.

4. The Newton-Raphson search can thus be applied to calculate μ_{K_1}, μ_{K_2}, and μ_{K_3} by iterating Eq. 6.68, 6.69, and 6.70, respectively.

5. μ_Q and σ_Q^2 can be determined by Eq. 6.57, and 6.58, respectively.

The complexity of numerical computations required to evaluate the expected value and variance of pavement serviceability loss has been recognized. Equations developed in this chapter, especially in terms of cumbersome iteration scheme involved, must be coded for high-speed computer operation, if these equations are adopted to predict pavement performance.

CHAPTER VII

CONCLUSIONS

Specific conclusions of this study are summarized herein.

1. A well-designed experiment is needed to provide adequate information for pavement performance analysis.

2. The two-step constrained select regression methodology, developed in Chapter IV, can be applied to approximate the true functional relationship of pavement performance information collected from experiments. This allows pavement life to be predicted based on the construction of alternatives, estimates of traffic, and environmental effects.

3. Pavement serviceability loss due to fatigue, swelling, shrinkage and thermal cracking can be integrated into a simple performance equation.

4. Performance equations, derived in Chapter IV, fit the Texas data collected in Texas Study 2-8-62-32 better than the equation currently implemented in Texas Flexible Pavement Design System, FPS-11(14). The better fit is due to two factors:

   a. a better physical explanation of the real data including more effects of the climate

   b. more terms are used in the model

   Regression analyses of the data using the current FPS performance equation to predict serviceability loss resulted in R^2 values of around 0.02 to 0.1.

5. Differential analysis can be applied to examine the sensitivity of pavement serviceability loss. Sensitivity study evaluates the significance of design, traffic and environment variables.

6. Probabilistic design concepts can be utilized to design a reliable pavement which will provide satisfactor service to the user through its design service life at a designer-specified confidence limit.

7. Products of this study are not recommended for immediate implementation. More information is needed for confirmation of the models. However, the methodology developed and utilized in this report can be applied to future pavement performance study.

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APPENDIX

GLOSSARY OF SYMBOLS

The following symbols are used in this report:

A = composite pavement stiffness coefficient;

A_1 = top layer stiffness coefficient;

A_2 = second layer stiffness coefficient;

b = a swelling clay parameter;

$C+P$ = a measure of cracking and patching in the pavement surface;

C_1 = probability of swelling clay;

C_2 = potential vertical rise of swelling soil;

D = deficit of water in inches;

DD = a parameter of design;

D_1 = top layer thickness in inches;

D_2 = second layer thickness in inches;

D_3 = third layer thickness in inches;

E = potential evapo-transpiration in inches;

$E[\,]$ = expected value;

F_B = percent fines in base course;

F_s = percent fines of subgrade;

G = a parameter of pavement serviceability;

L_B = percent lime in base course;

L_s = liquid limits of subgrade;

L_1 = nominal load axle weight in kips;

L_2 = 1 for single axle vehicles,
2 for tandem axle vehicles;

N = accumulated number of equivalent applications of an 18-kip single axle load in one direction;

N_{FT} = number of freeze-thaw cycles;

N_1 = accumulated number of 18-kip single axle at time t_1;

N_2 = accumulated number of 18-kip single axle at time t_2;

N_3 = accumulated number of 18-kip single axle at time t_3;

Obs = number of observations;

OV = overlay thickness in inches;

p = present serviceability index;

p^1 = a swelling clay parameter, the assumed serviceability index in absence of traffic;

PI = plasticity index of subgrade;

P_s = permeability index of subgrade;

P_1 = initial serviceability index;

Q = pavement serviceability loss;

Q_1 = pavement serviceability loss due to fatigue;

Q_2 = pavement serviceability loss due to swelling;

Q_3 = pavement serviceability loss due to shrinkage cracking;

Q_4 = pavement serviceability loss due to thermal cracking;

R^2 = multiple correlation coefficient;

RD = a measure of rutting in wheel paths;

RF = regional factor;

S = surplus of water in inches;

SCI = surface curvature index in mils;

SE = standard error;

SN = structural number;

S_R = amount of solar radiation;

SS = soil support value of subgrade material;

SV = mean of slope variance in two wheel paths;

t = time in years after construction or rehabilitation;

TC = triaxial class of the base course;

TI = Thornthwaite moisture index;

t_0 = time of initial construction or rehabilitation;

t_1 = time of first measurement of serviceability;

t_2 = time of second measurement of serviceability;

t_3 = time of third measurement of serviceability;

V[] = variance;

W = accumulated axle load applications;

W_1 = surface deflection measured by Dynaflect at geophone 1;

W_2 = surface deflection measured by Dynaflect at geophone 2;

α = district temperature constant;

α_i = mean value of the mean daily temperature (°F) less 32°F for the ith month averaged over a ten-year period;

β = a parameter of design and load;

ρ = a parameter of design and load;

θ = a swelling rate constant;

μ = expected value; and

σ^2 = variance.