ANALYTICAL METHODS APPLIED TO THE MEASUREMENTS OF DEFLECTIONS AND WAVE VELOCITIES ON HIGHWAY PAVEMENTS: PART 1, MEASUREMENTS OF DEFLECTIONS

By

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The opinions, findings, and conclusions expressed in this publication are those of the author and not necessarily those of the Bureau of Public Roads.

Summary

The apparatus known as the Dynaflect has been used to measure a number of deflection basins on highway pavements throughout the country. The initial goal of this work was to determine the elastic parameters of the materials composing the highway structure using the results of these measurements. A method has been developed which is believed to be generally applicable. It has been tested on a group of relatively complex highway structures.

1. INTRODUCTION

The need is recognized to obtain meaningful parameters for the materials composing a highway structure. The purposes of obtaining such parameters are (1) to enable calculations to be made of the stresses in the materials caused by vehicle loadings and (2) to permit the control of materials and methods during the construction of a pavement. The first part of this report will describe a method which has been developed in order to interpret the results of the measurements of deflection basins. Such basins have been measured in the neighborhood of two point loads at standard distances from the loads. The apparatus used to obtain these results is known as the Dynaflect. The method to be described is a numerical one. It determines values of the Young's moduli E for the materials composing the layers of a highway structure, assuming a value of Poisson's ratio, and supposing that the thicknesses of the layers in the structure are know.

Two primary requirements of this work are the means of loading a pavement and measuring the resulting deflections and a computational procedure relating the elastic parameters of the materials to the measured deflections. Both of these were available and will be described.

2. DYNAFLECT

The Dynaflect has been described elsewhere (1). It is an apparatus which enables the deflection basin of a pavement to be determined under a known load. The load is applied to the pavement by means of a set of out-of-balance weights acting through relatively rigid metal wheels. The weights rotate at a speed of eight revolutions per second. The load applied is then nominally 1,000 pounds weight, peak to peak. The deflection basin is measured by means of five geophones which are placed at standard distances from the load. The apparatus has been automated, and it is possible to obtain a set of measurements along a radius of the deflection basin within two minutes of positioning the Dynaflect. Scrivner and Moore (3) have shown that a relationship exists between the deflections of the five geophones and certain empirical parameters of the materials composing the highway structure.

It is possible also to determine the elastic parameters of the materials composing the structure. This can be done, for instance, by trial and error employing successive iterations; the elastic parameters of the materials can then be related to the deflections at the free surface under a load equivalent to that exerted by the Dynaflect.* This has been done and it appears that the deflection basins can be predicted to an order of accuracy which is similar to that obtained by Scrivner and Moore. The parameters obtained are different.

A subjective trial and error method is, however, not satisfactory for determining the elastic parameters of the materials. High accuracy

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*Suitable procedures will be discussed.

cannot be achieved. An objective method is needed in order to determine values of the elastic parameters which yield a deflection basin closely approximating the measured one.

3. PROCEDURES FOR COMPUTING PAVEMENT DEFLECTIONS UNDER A STATIC LOAD

There is no direct method for calculating the elastic parameters of a layered structure such as a highway pavement. However, there are two distinct methods available for calculating the surface displacements when the parameters of the materials composing the structure are given. These methods will be discussed in the sections which follow. The first, a finite difference method, enables the required quantities to be determined at discrete mesh points within the system, but readily incorporates non-linear stress-strain behavior. The second, an analytical method, provides a solution which is defined at all points in the coordinate system, although the associated definition of the applied load is arduous. The method of solution and the handling of data vary. The choice of method is governed by the input data available and the results which are required.

3.1 The Finite Difference Method

The differential equations of elasticity, which are assumed to govern the displacements within a highway structure, are solved by considering the particle displacements at the modal points of a mesh (4). For this purpose the differential equations are expressed not in terms of continuous space coordinates (x, y, z), but in terms of the spacing between the mesh points. The particle displacements (u, v, w) in the direction of the axes are, therefore, regarded as varying not continuously but in discrete steps when passing from one mesh point to the next. The difference between the coordinates of neighboring mesh points is finite, not infinitesimal as in the solution by analytical methods.

The problem becomes the solution of a large number of simultaneous equations (three for each mesh point), in order that the differential equations of elasticity may be satisfied.

As the mesh points are made successively closer, the finite difference solution approaches the solution obtained by analytical means. The finite difference method can be used in cases where the elastic parameters vary in an arbitrary way; variation from one nodal point to the next can be specified as part of the problem data. This can be done with no essential complication to the computing program. Non-linearity in a stress-strain response can be similarly handled, or the stress-strain relationship can be defined in a subroutine to save the labor of data punching.

The spacing between the mesh points can be varied to provide a greater density of information in significant regions. In the transition from a region of coarse to one of fine spacing, however, the displacements at points at the zone interface need careful consideration; these represent the boundary conditions of a zone of relatively fine mesh, and the accuracy requirements may differ from those within a coarse mesh.

3.2 The Application of an Analytical Method

The deflections on the surface of a semi-infinite isotropic elastic solid may be determined by analytical means. If the solid is uniform, closed analytical solutions are available for certain simple loading systems. The case of a point load applied at the free surface has been discussed by Timoshenko (5). The result is obtained by using a trial solution of the equations of elasticity, which involves two arbitrary constants. One of these is eliminated by means of the physical condition that the shearing stress in a vertical plane at the free surface must be zero. In order to determine the second in terms of the applied load, elementary annuli of wedge-shaped cross section are considered; these annuli form a hemisphere centered on the point at the surface at which the load is applied. The net vertical forces acting on these annuli are summed and equated to the applied load, thus fixing the second of the constants. The deflection at the free surface is inversely proportional to the distance from the point at the surface at which the load acts.

By means of a formal solution of the equations of elastic equilibrium, Love (6) obtained expressions for the displacements at any point on or within such a medium; the force is considered to be applied to a point on the free surface. The deflection at the free surface is inversely proportional to the distance from the line of action of the load.

Perhaps the clearest solution to the problem is given by Filenko-Borodich (7). As in the solution given by Timoshenko, a hemisphere is considered which is centered on the load point at the free surface. The sum of the vertical forces acting on the elements of this surface are equated to the applied load. Only one arbitrary constant is introduced throughout. Its value is found, as indicated, in terms of the applied load. The vertical deflection at the free surface is, as before, inversely proportional to the distance from the point at which the load is applied.

This solution for the elementary case of a continuous homogeneous semi-infinite isotropic elastic solid suggests a method of plotting the results (8). If the logarithm of the deflection is plotted against the logarithm of the spacing (the distance between the applied load and the point at which deflection is measured on the free surface), the graph will have a slope of 45°; If however (deflection x spacing) is plotted against spacing on double logarithmic paper, the graph will have a constant ordinate for the (deflection x spacing) product. This provides a useful basis for judging the departure from homogeneity of any real structure.

3.3 The General Analytical Solution to the Problem of a Loaded Semi-Infinite Layered Structure Composed of Isotropic Elastic Materials

Using the equations of elasticity in cylindrical coordinates, Burmister (9) obtained expressions for the stresses and displacements in a structure composed of layers of materials having dissimilar elastic parameters. The vertical stress at the free surface is assumed to be a Bessel function of the radial spacing. This choice of stress as a boundary condition makes it possible to satisfy the differential equations of elasticity using relatively simple stress functions. Particular applications of the resulting solutions to two- and three-layer structures are considered. Any form of surface loading can be represented by adding a series of Bessel functions, such as Burmister assumed as a basic surface stress. The loading can be synthesised analytically by means of a Fourier-Bessel transform (10). Alternatively, it can be represented by a sum of a number of Bessel functions, each with a different weighting coefficient.

3.4 The Chevron Program for Computing the Displacements and Stresses in a System Composed of Layers of Isotropic Elastic Media

The Chevron program (11, 12) for the computation of stresses and displacements in a layered medium such as a highway structure has been written using Burmister's theory as the starting point. The authors of the program have extended Burmister's work to permit the analysis of a structure consisting of any number of layers supported by a foundation which is of infinite depth and infinite lateral extent. The radially symmetric differential equation of elastic equilibrium in cylindrical coordinates (r, z), namely

$$\nabla^4 \phi = 0$$

is satisfied by a potential function $\phi_i(r, z, m)$ of the form

$$\phi_i(r,z,m) = J_o(mr)\left[(A_i + B_i z)e^{mz} + (C_i + D_i a)e^{-mz}\right]$$

where m is a parameter, as it is in Burmister's papers (9), and i denotes the i^{th} medium in the layered structure.

The stresses and displacements in the system can then be expressed in terms of the potential function $\phi(r,z,m)$ by means of standard expressions of elasticity (5, 6, 7). Beyond this point the approach adopted by the authors of the Chevron program differs from that of Burmister. The stresses and displacements, in terms of the arbitrary constants A_i, B_i, C_i and D_i are written in matrix form in order to make the algebra more easy to grasp; there is one set of constants A_i, B_i, C_i and D_i for each layer denoted by the subscript "i". The matrix expressions for the stresses and displacements at the bottom of layer "i" are equated to those for the top of layer (i + 1). This process can be continued until the underlying semi-infinite medium is reached when i = n. The matrix expression for the stresses and displacements in the n^{th} medium has only two arbitrary constants; the remaining two must be zero because both stresses and displacements vanish at infinite depth. All of the A_i's, B_i's, C_i's and D_i's may then be written in terms of the two arbitrary constants associated with the semi-infinite medium C_n and D_n; A_n and B_n are zero for the reason just given. In particular, the A_1, B_1, C_1 and D_1 can be written in terms of C_n and D_n. (Because of the complexity of the expression, we should not like--indeed we should not attempt--to write down the expressions for A_1, B_1, C_1 and D_1 in full: fortunately the computer does it for us, in carrying out the successive matrix operations.) We are able to say something about the arbitrary constants associated with the first layer, remembering that they are now expressed in terms of only two unknown quantities, C_n and D_n. They are related to the shear stress at the free surface in the (r,z) plane. This shear stress must everywhere be zero, because of the way in which the system is loaded. Second, the vertical stress at the free surface is defined at all points on the surface. It is equal to a Bessel function of the radius, multiplied by a weighting function, p(m). Using these two additional boundary conditions the problem is solved.

It is possible to repeat this calculation using a series of values of m; the vertical surface stresses are then superposed in proportion to their weighting functions p(m). Using a method similar to that employed to determine a Fourier series, it is possible to determine a spectrum of weighting functions p(m) which will represent any desired surface loading (10). This is done in a subsequent stage of the Chevron program. The surface load which is represented is a circular area on which the vertical pressure is everywhere the same, and the surface shear stresses are zero. The load thus corresponds with that exerted by a flexible plate which is able to follow the deformed shape of the surface of the structure being loaded.

When the weighting coefficients have been determined, the stresses and displacements obtained for each value of "m" are superposed. This yields the final result. See the Appendix for a description of the operating method for the Chevron program.

4. THE ANALYSIS OF DEFLECTION BASINS MEASURED ON A GROUP OF HIGHWAY PAVEMENTS, WHICH FORM PART OF A CONTROLLED EXPERIMENT IN THE CONSTRUCTION OF PAVEMENTS

There is a need to determine the elastic constants of the materials used in constructing highway and airfield pavements. These depend on the state of compaction, as well as on the moisture content of the materials; they are affected by physical and chemical changes which occur during the service life, such as comminution of the granular substances composing the structure and the weathering action of the atmosphere. Laboratory tests to determine the elastic parameters are, therefore, difficult to apply in the field, as the exact field conditions and the history after a period of use are so complex that they cannot be simulated.

Methods have been developed for the purpose of determining the elastic parameters of layered systems. Most of these depend, in principle, on the measurement of a mechanical admittance (the measurement of a displacement caused by a load). In the case of the Benkelman Beam, for example, the load is effectively static (it is maintained at a fixed value for long enough to allow any vibrations which it causes in the pavement to decay), and the deflection is measured at a point in the neighborhood of the load. The deflection basin caused by a static load can be measured by the same or similar apparatus. This yields more information about the pavement than a single point deflection.

4.1 Experimental justification for applying the calculations made by means of the Chevron program to the measurements made by means of the Dynaflect

The calculations made by means of the Chevron program are based on the assumption of isotropic, elastic substances under static load. The effects of intertia and "viscous" damping do not need to be considered as they do not affect the results. However, the Dynaflect applies a load to the surface of the pavement which is not static but varies with time. The variation is approximately sinusoidal, at a repetitive rate of eight applications per second. The effects of inertia and material damping are not necessarily negligible, and some indication of their effect is needed before applying calculations based on static conditions to the Dynaflect measurements.

In order to estimate the effect of the applied load being other than static, some experiments were carried out on a few highway structures which have been constructed for experimental purposes. The surfaces of selected sections were loaded by means of the Dynaflect and the deflection basins measured according to the normal procedure used with the apparatus. After modifications to the loading and measuring equipment, the same load was applied at repetition rates varying from 4 c/s to 12 c/s. The deflection basins were measured at a number of frequencies of loading. The deflections for a given spacing were plotted as a function of frequency of application of the load. Typical results are shown in Figure 1. It appears that the vertical deflections at the free surface are independent of the frequency within the range of frequencies employed. The local increase of deflection near 12 c/s is believed to be due to a mechanical resonance in the loading

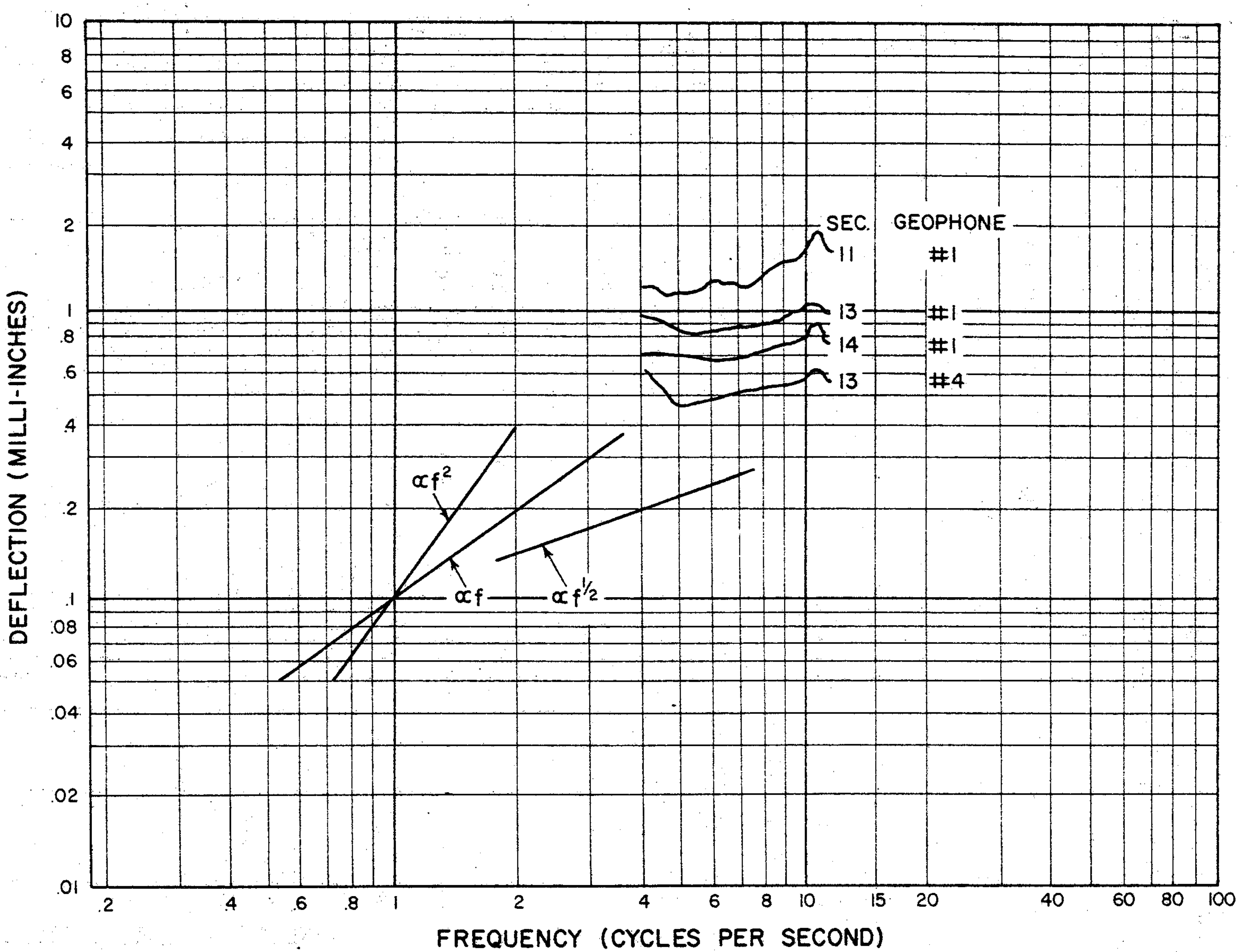

FIGURE 1 - Results of Pavement Deflection Measurements obtained by Means of the Dynaflect Operation over a Range of Frequencies.

system of the Dynaflect. If effects of inertia were dominant, the deflections should decrease with increasing frequency according to the square of the frequency. The slope of the deflection-frequency curve should then be downward and have a value of two decades vertical to one decade horizontal. If a viscous type of damping were dominant, the curves would have a downward slope of one decade vertical to one decade of horizontal change. No such changes in deflections are observed, however. It is deduced that both effects are unimportant within the range of frequencies investigated. In particular, this is evidence that such effects are unimportant below the upper limit of frequency investigated here; we shall, therefore, apply the results of calculations, based on the assumption of static conditions, to the measurements obtained by means of the Dynaflect.

In what follows, we shall investigate what is effectively the iteration of the Chevron program, altering the Young's moduli of the materials composing the layers until the calculated and measured deflection basins agree; the final values of the Young's moduli obtained are used as the moduli for the materials. In order to save execution time, the actual Chevron program was not iterated. Instead an approximation to the Chevron results was established, which is simpler in form than the complete expressions for deflections used in the Chevron program. This approximation, valid over a limited range of values of the Young's moduli of the materials, was used as the basis of an iterative solution for the moduli.

4.2 Application of the Chevron Program to Measurements Obtained by Means of the Dynaflect

In theory, it is of course possible to determine the elastic parameters of the materials composing the structure; this can be done, for instance, by trial and error employing successive applications of the Chevron program. As previously mentioned, this has been done, and it appears that the deflections basins can be predicted to an order of accuracy which is similar to that obtained by Scrivner and Moore. The results are shown in Figure 2. In Figure 3 a comparison is shown between the empirical coefficients determined by Scrivner and Moore and the Young's moduli estimated by this means. This figure indicates that a functional relationship may exist between the two sets of quantities.

The trial and error method is, however, not satisfactory for determining the elastic parameters of the materials. Good accuracy cannot be achieved when trials are made which are merely subjective. Also, the problem of how the parameters should be varied is difficult for a two layer system, complicated for a three-layer and highly complex for a five-layer. An objective method is needed in order to determine values of the elastic parameters which yield a deflection basin most closely approximating the measured one.

NOTE ON FIGURE 2

This figure should be read in conjunction with the data contained in reference (2). The theoretical curves of deflection shown are based on the following values of Young's moduli (Poisson's ratio is 0.45).

Material Index	Material	Young's modulus (lb./sq. inch)
0	Plastic clay (undisturbed)	18000
1	Plastic clay (compacted)	20000
2	Sandy clay	25000
3	Sandy Gravel	60000
4	Crushed limestone	70000
5	Crushed limestone + 2% lime	110000
6	Crushed limestone + 4% cement	800000
7	Asphaltic concrete	60000

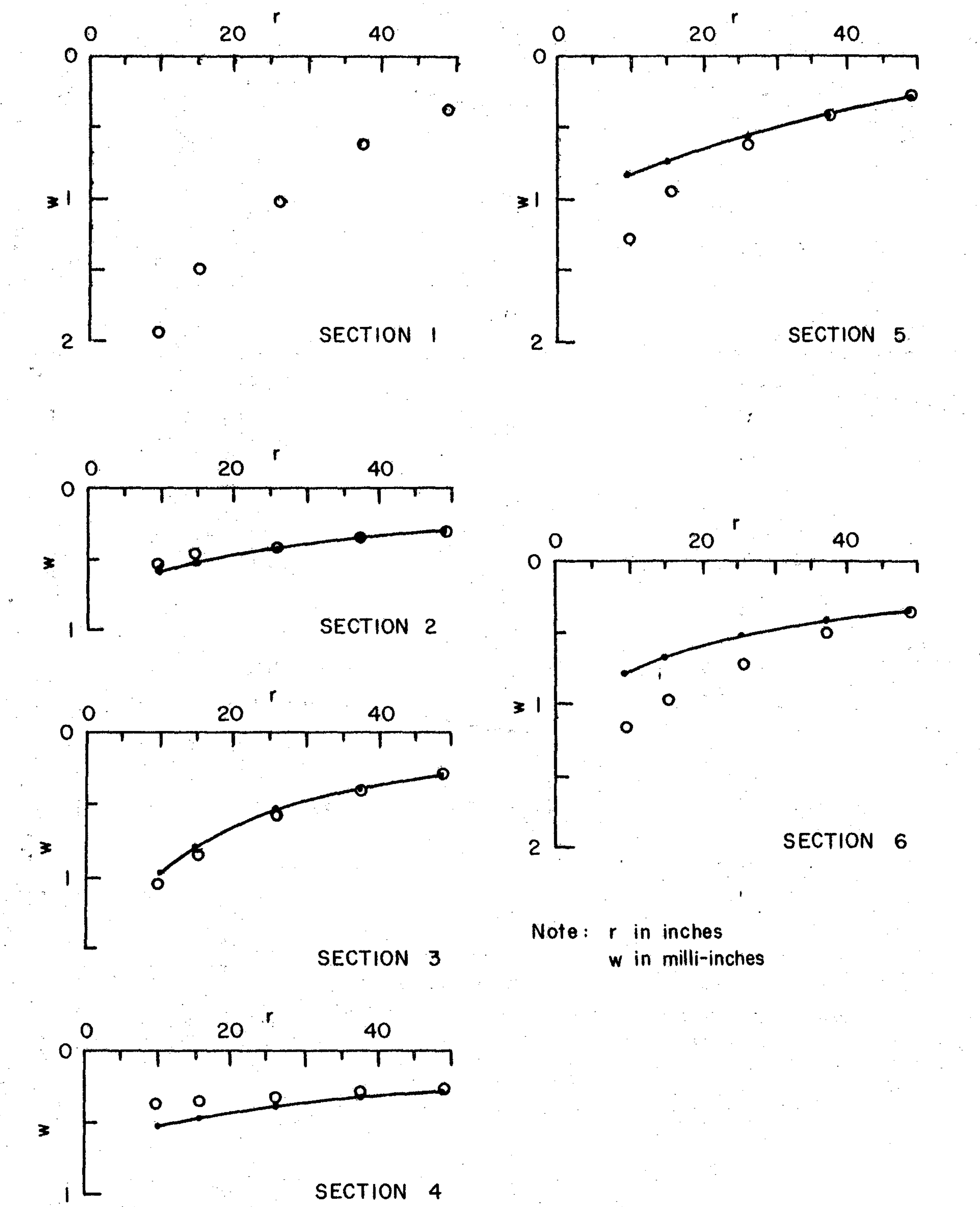

FIGURE 2 - Observed Pavement Deflections (circled points) Compared With Calculations Made by Means of the Chevron Program, Using the Values shown for the Young's Moduli of the Materials Used in the Construction.

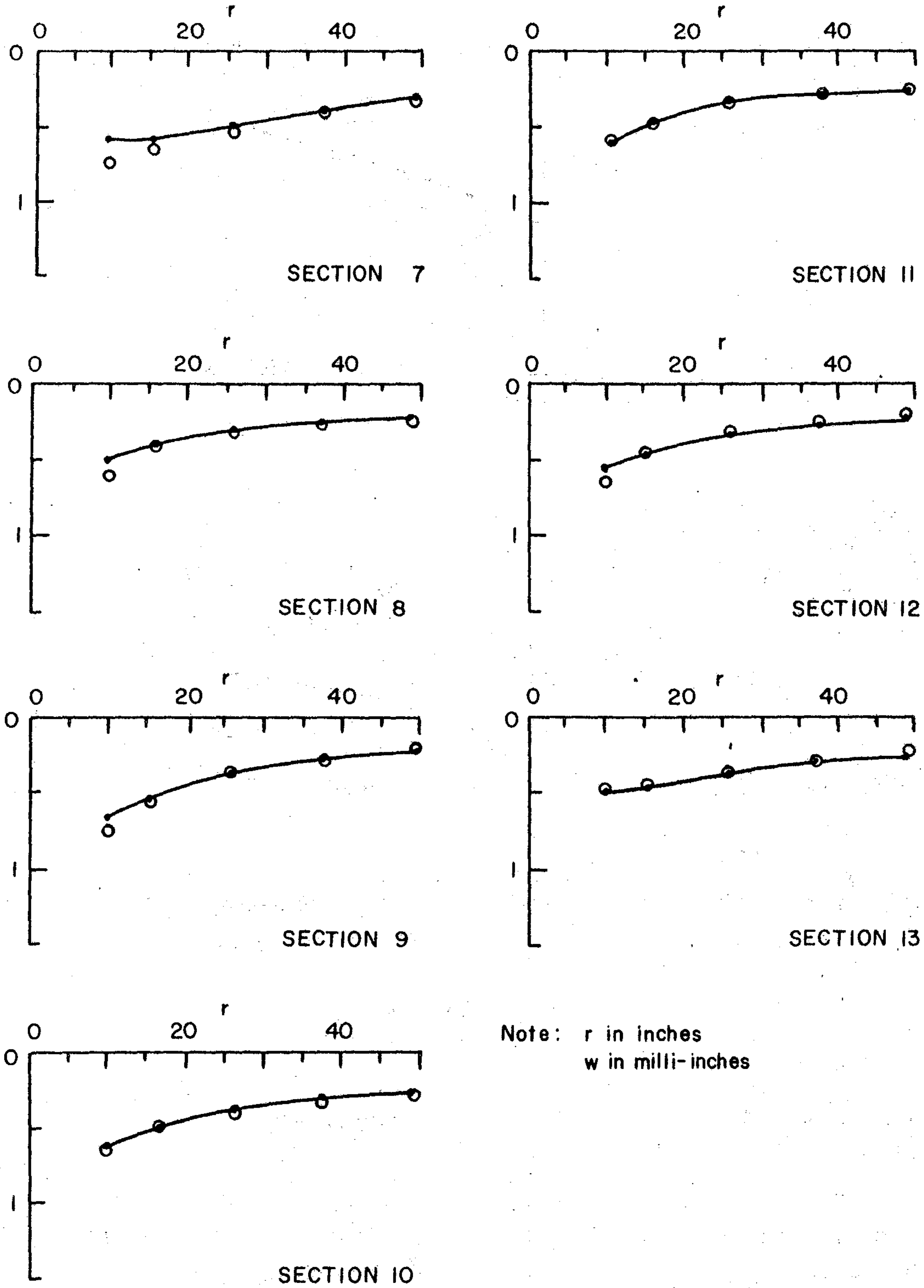

FIGURE 2 continued - Observed Pavement Deflections (circled points) Compared With Calculations Made by Means of the Chevron Program, Using the Values shown for the Young's Moduli of the Materials Used in the Construction.

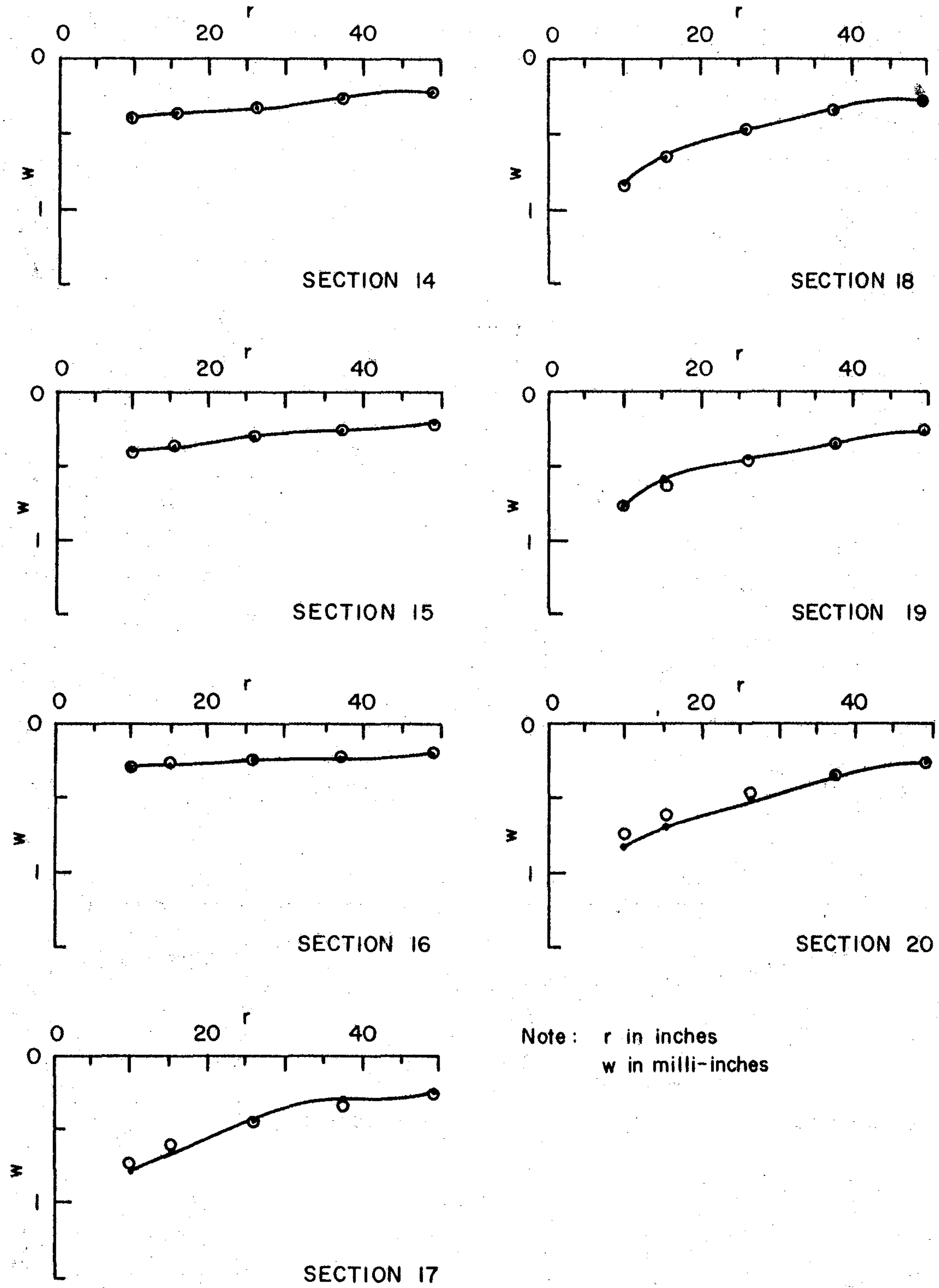

FIGURE 2 continued - Observed Pavement Deflections (circled points) Compared With Calculations Made by Means of the Chevron Program, Using the Values shown for the Young's Moduli of the Materials Used in the Construction.

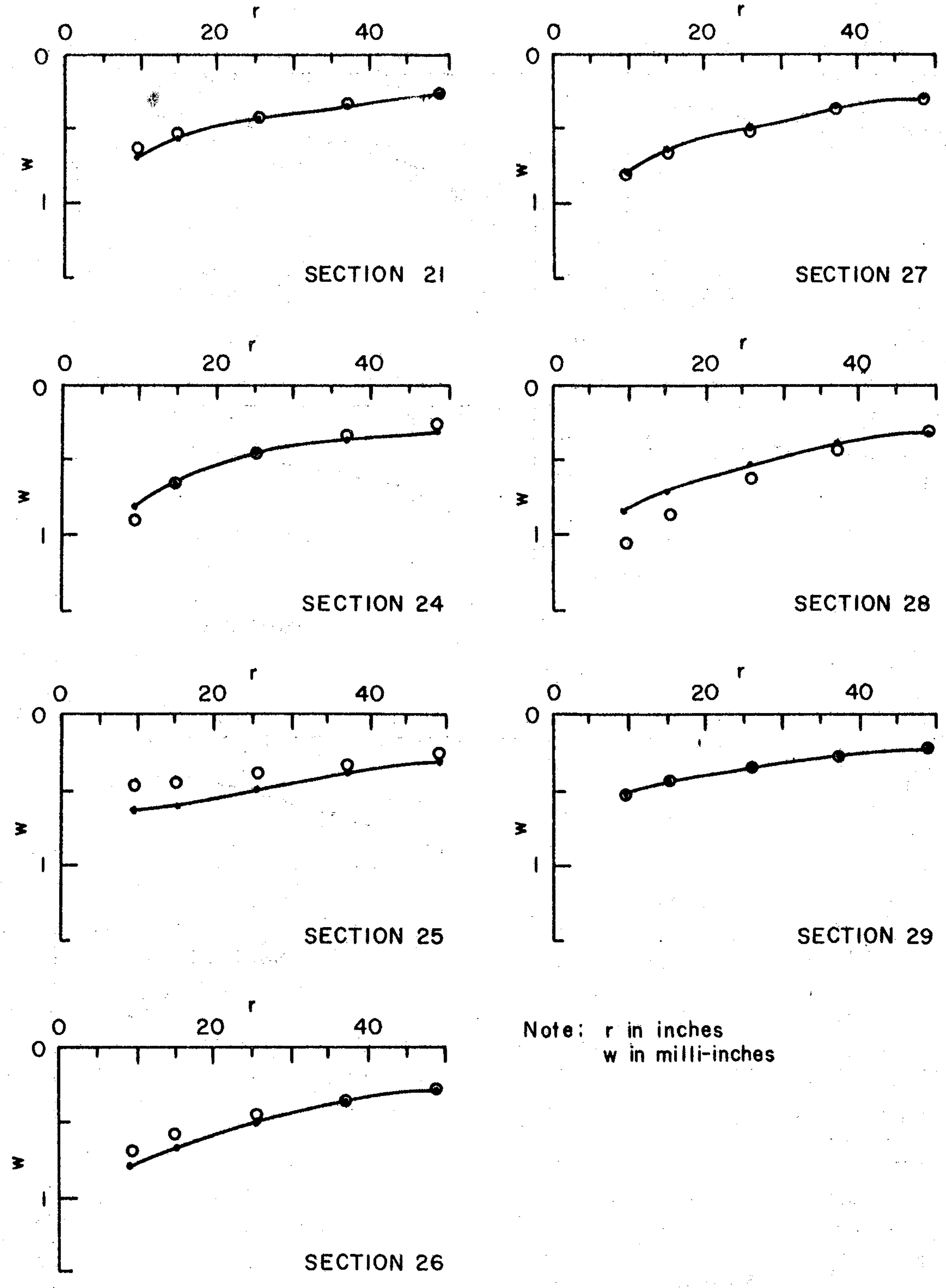

FIGURE 2 continued - Observed Pavement Deflections (circled points) Compared With Calculations Made by Means of the Chevron Program, Using the Values shown for the Young's Moduli of the Materials Used in the Construction.

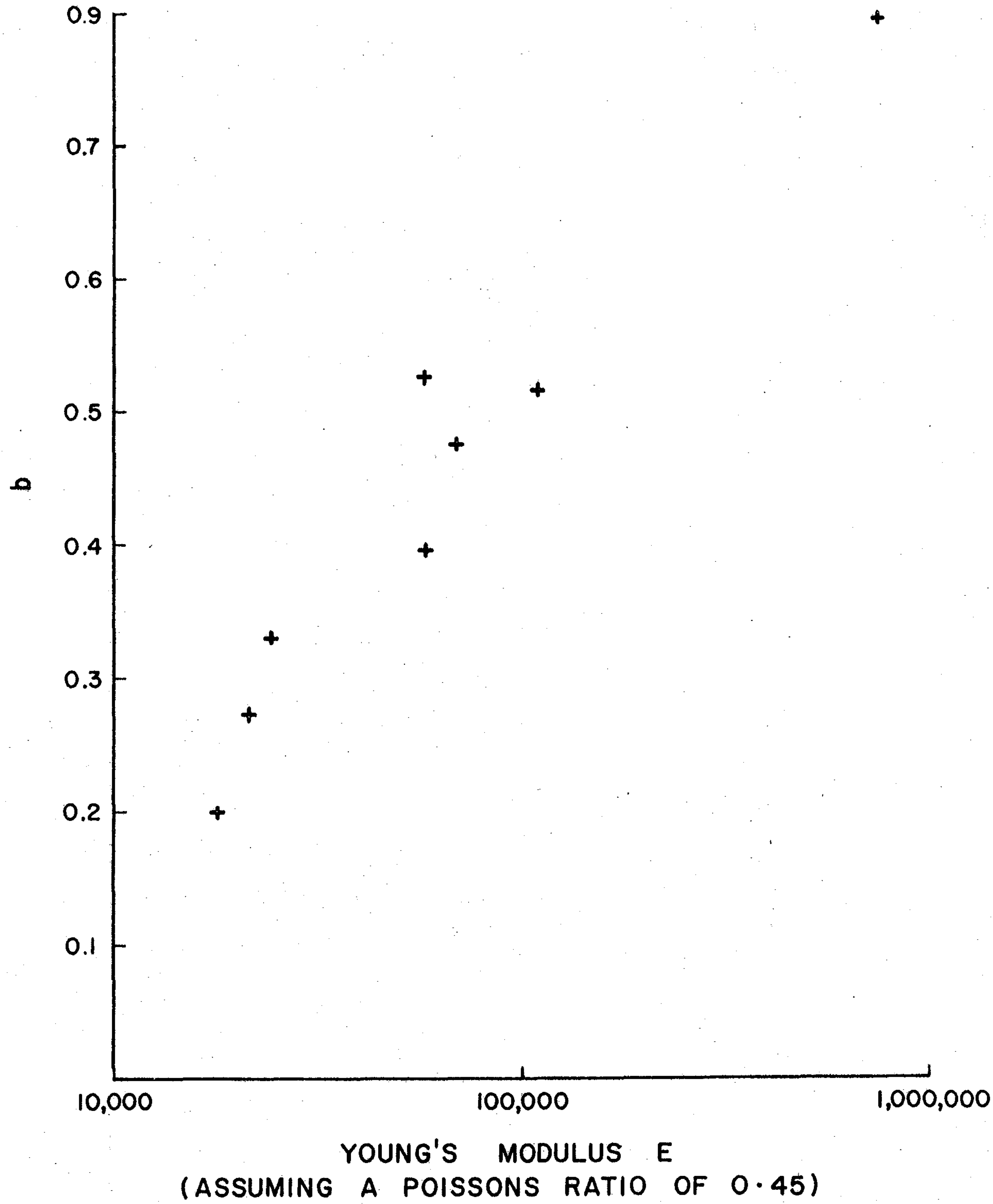

FIGURE 3 - Values of the Estimated E's Plotted Against the "b's" Determined by Scrivner and Moore.

4.3 A Program which Computes the Young's Moduli of the Materials Composing a Layered Structure, Using the Deflection Basin as Data

It was found that, provided the variation in Young's modulus is not greater than about a hundred to one (or not greater than ten to one about the geometric mean), the logarithms of the deflections at the various geophone positions can be represented by the following:

$$\log_e y_i = a_i + \sum_j (c_{ij} + d_{ij} \log_e E_j) \log_e E_j, \qquad (1)$$

where y_i = vertical deflection at the i^{th} geophone station ($i = 1, 2, \ldots 5$)

a_i = constant

c_{ij}, d_{ij} = matrices to be determined

E_j = Young's modulus in the j^{th} medium from the free surface.

Some extrapolation is permissible although accuracy is lost very rapidly; an additional factor of two to one beyond the range of the computed values was generally found to be the limit. An example of the computed deflections are shown, with their fitted parabolae, in Figure 4. The values of a_i, c_{ij}, and d_{ij} were determined for each layer and for each geophone position by means of eleven Chevron runs.* An example of the elastic parameters selected for these runs is shown in Table 1. The inverse Chevron program then determines by iteration which values of the Young's moduli provide the best fit between the measured deflection basin and that calculated by means of the Chevron program. The values obtained by this iterative procedure are used as data for a further run of the Chevron program to provide confirmation. The agreement between the final calculated values of the deflections

*The case shown, which includes the results obtained from extra values of E_2, required thirteen Chevron runs.

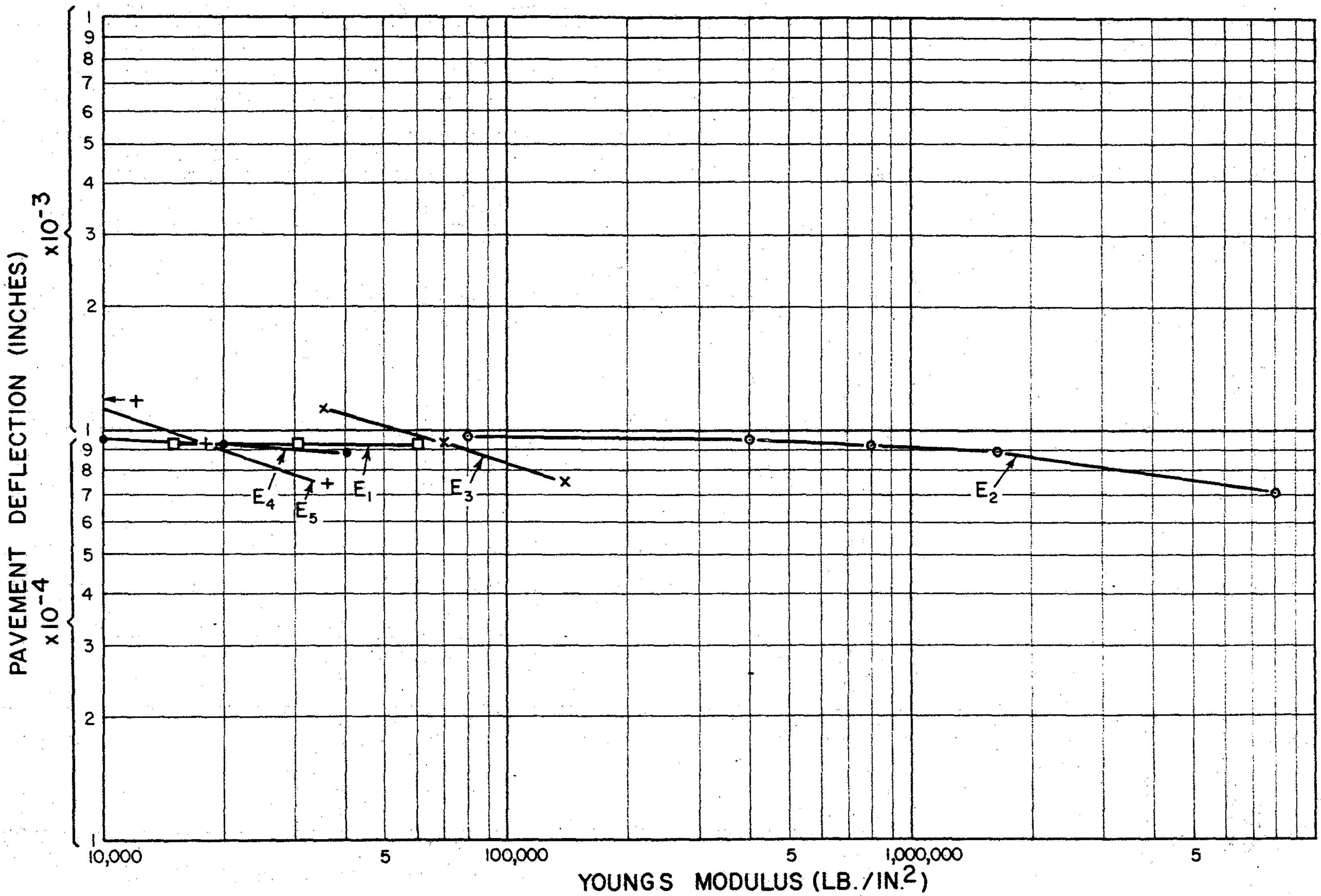

FIGURE 4 - The Effect of Varying the Moduli of the Materials Composing Individual Layers on the Deflection at a Distance of 10" From a Circular Load of Radius 1.60".

TABLE 1 - TABLE OF VALUES OF E SELECTED FOR THE INITIAL COMPUTATIONS REQUIRED FOR SECTION 3 OF THE A&M TEST FACILITY

Table of values of Young's Modulus selected for section 3 central values:

17,000 psi	1"
530,000 psi	4"
55,000 psi	12"
17,000 psi	36"
19,000 psi	Semi-Infinite

These values were varied, one at a time, while the remaining four were held constant. The whole scheme is shown below, with a typical combination indicated.

5,667	17,000	51,000
265,000	530,000	1,060,000
27,500	55,000	110,000
8,500	17,000	34,000
9,500	19,000	38,000

There are eleven such combinations, including the central values. A typical grouping is:

Layer 1	17,000
Layer 2	1,060,000
Layer 3	55,000
Layer 4	17,000
Layer 5	19,000

and those measured in the field indicates the sufficiency of the model represented by equation 1. Greater accuracy could be achieved by employing terms of the third order in log E_j, or by introducing terms involving cross products of the log E_j's. Comparison between the final calculated and the observed values of the deflections shows the agreement is better than a unit in the least significant digit in the measured values. The accuracy achieved by means of the present expression is adequate.

4.3.1 Description of the Program

A block diagram of the main routine of the convergence program WHC36 is shown in Figure 5. The program consists of a main routine and two subroutines DIMEQN and COEFFT. DIMEQN is a subroutine which inverts a matrix, and is used to solve five simultaneous equations; COEFFT is a subroutine which calculates the elements of the matrices C(I,J) and D(I,J) using as input data the grid of values of deflections Y(I,J) which are calculated by means of the Chevron program.

The program operates as follows. The vector A(I) and the matrices C(I,J) and D(I,J) are calculated as described. The trial values of E(J) are used to calculate the Y(I) vector. The calculated Y vector is subtracted from the measured Y vector. The result, the R vector, is used in the following set of simultaneous equations:

```
(C(1,1)+2*D(1,1)*EM(1))*DELTA(EM(1))+...
...+C(1,J)+2*D(1,J)*EM(J))*DELTA(EM(J))=-R(1)

(C(J,1)+2*D(J,1)*EM(J)*DELTA(EM(J))+...
...+(C(J,J)+2*D(J,J)*EM(J)*DELTA(EM(J))=-R(J),
```

which are solved for DELTA (EM(K)), (K = 1,..,J) the correction to be applied to EM(K), the logarithm of E_k. On the first cycle of iteration, the trial E vector is placed in the EM(K)'s, and the change DELTA(EM(K)) in the EM(K)'s is calculated which is necessary to make the R vector zero, i.e. to make the calculated Y's agree with the measured Y's. A fraction of the calculated changes in the EM(K)'s is added to the trial values of EM(K),

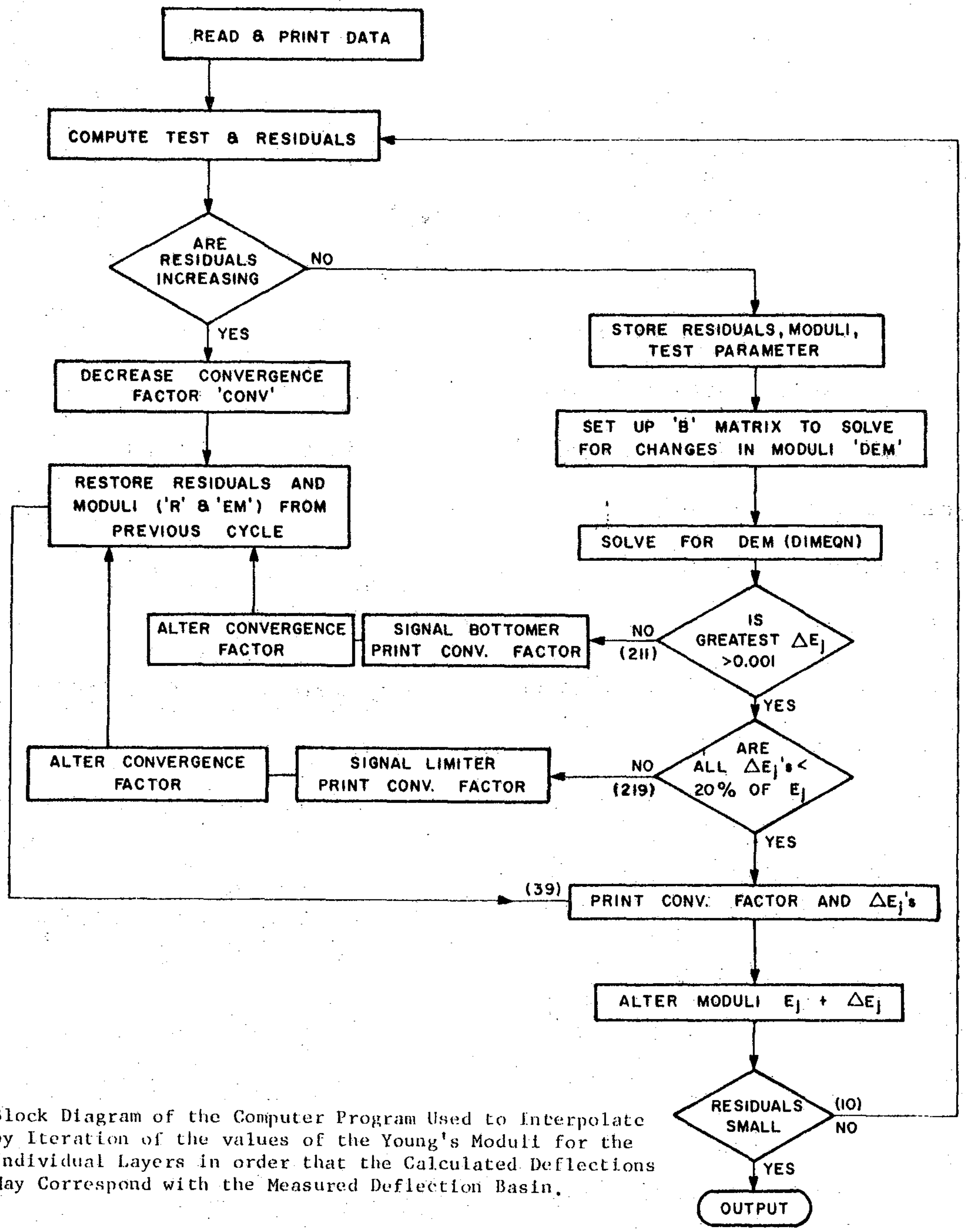

FIGURE 5 - Block Diagram of the Computer Program Used to Interpolate by Iteration of the values of the Young's Moduli for the Individual Layers in order that the Calculated Deflections May Correspond with the Measured Deflection Basin.

and the operation is repeated, until the solution criterion is satisfied.

It is necessary to use only a small fraction of the calculated changes in the EM(K)'s as a correction to the trial EM(K)'s in order to prevent oscillation. Other safeguards were included, such as discontinuing the program when the changes in the calculated Y become too small, and also when some varieties of saddle point are reached.

The input data are complicated, and a program has been written (WHC35) in order to assist with checking it for numerical and sorting errors. The input data are as shown in Table 2. A typical output from the checking program is shown in Table 3.

Normal functioning of the program leads to values of EM(K)'s for the materials composing the layers; the values yield the correct deflections (those measured in the field)wwhen used as input data to the Chevron program. If the discrepancies are larger than the errors of the observed (field) deflections, then the program is not functioning properly. See the section concerning fault tracing.

TABLE 2 - DATA CARDS FOR INPUT TO THE CONVERGENCE PROGRAM WHC36 AND THE DATA CHECKING PROGRAM WHC35

Card No.	Description
1	Title card, using all 80 columns; the final 40 columns are printed as running identification during each cycle of iteration of WHC36
2	Columns 1-3 contain the number of media present in the system. Columns 7-12, 19-24, 31-36, 43-48, and 55-60 contain the values of central E's of the successive layers starting from the top layer. The remaining columns of this card are not read.
3-7	Five fields of ten characters each; each field is the value of Y(I,N) for the selected low value of E(N); the first card has on it the computed Y's (from the Chevron grid) obtained from the low value of E(1), the second card contains the values obtained by using the low E(2) in computing the Chevron grid, and so on.
8-12	Five fields of ten characters each; each field is the value of Y(I) for the central values of the E's. All five cards are therefore the same. Five cards are read in at this point instead of only one, in order to preserve the symmetry of the data stream, which is already complicated.
13-17	Five fields of ten characters each; each field is the value of Y(I,N) for the selected high value of E(N); the first card has on it the computed Y's (from the Chevron grid) obtained from the high value of E(1), the second card contains the values obtained by using the high E(2) in computing the Chevron grid, and so on.
18	Five fields of ten characters each; each field contains the factor which represents the departure of the "high" and "low" values of E(J) from the central value of E(J): the first field contains the factor pertaining to the top layer, and so on.
19	Five fields of ten characters each; the measured values of the deflections Y(I) are punched, starting with the 10" geophone.

TABLE 2 - DATA CARDS FOR INPUT TO THE CONVERGENCE PROGRAM WHC36 AND THE DATA CHECKING PROGRAM WHC35 (Continued)

Card No.	Description
20	Columns 1-3 contain the number of media present in the system. Columns 7-12, 19-24, 31-36, 43-48 and 55-60 contain the trial E's with which the iteration is to be started; the first E is the trial E for the top layer. The remaining columns of this card are not read.

Note: If modified values of the C and the D matrices are to be read in, the data are punched in the same format as above, with all the E's distributed about their central E's; modified values of the C and D matrices are read in between cards 19 and 20 - two additional cards are read in for each E(J) which is modified relative to the original central values of the E's.

A & M TEST FACILITY - CHEVRON INVERSE - SECTION 2 - 68338

Y-VALUES USED FOR THE INTERPOLATION

Geophone Position	10-INCH	15.6-INCH	26-INCH	37.4-INCH	49-INCH
CENTRAL E'S	0.0004765	0.0004495	0.0003954	0.0003338	0.0002856
E 1= 6638.	0.0003265	0.0004722	0.0004382	0.0003360	0.0002740
E 1= 663840.	0.0004687	0.0004338	0.0003809	0.0003277	0.0002829
E 2= 235614.	0.0005233	0.0004827	0.0004155	0.0003437	0.0002912
E 2= 942454.	0.0004224	0.0004145	0.0003714	0.0003225	0.0002783
E 3= 81138.	0.0005695	0.0005255	0.0004318	0.0003478	0.0002889
E 3=8113800.	0.0003876	0.0003796	0.0003521	0.0003161	0.0002752
E 4= 18867.	0.0005548	0.0005334	0.0004596	0.0003829	0.0003273
E 4= 75466.	0.0003898	0.0003721	0.0003301	0.0002869	0.0002499
E 5= 8669.	0.0007023	0.0006638	0.0006094	0.0005465	0.0004855
E 5= 34676.	0.0003427	0.0003207	0.0002593	0.0002030	0.0001690
MEASURED Y'S	0.0005200	0.0004700	0.0004100	0.0003400	0.0002900

THE CENTRAL E-VALUES ARE

66384.	471227.	811380.	37733.	17338.

THE INITIAL TRIAL E-VALUES ARE

66384.	235613.	811380.	37733.	17338.

TABLE 3 - Typical Output from the Data Checking Program WHC35

5. AN INVESTIGATION OF THE A&M TEST FACILITY - SECTIONS 1 TO 8

Sections 1 through 8 of the A&M Test Facility (2) were investigated in detail using the program described. Assuming a value for Poisson's ratio of 0.45, values of E were obtained by the methods given in Chapter 4 for materials composing each of the layers in the sections; these values of E were then used as input data to the Chevron program and the calculated values of the deflections compared with the measured values. The measured values were used as obtained in the field, even when it appeared that gross errors may have been present. The agreement between the calculated and measured deflections is usually closer than one unit in the least significant digit. This indicates that the iteration has been carried at least as far as justified by the data. The results are shown in Table 4. The root mean square residuals of the deflections, which are used for checking the performance of the program, are shown in Table 5.

The values of the E's for the materials composing the layers of the structures are shown in Table 6. The results are shown diagrammatically in Figure 6. The most consistent materials appear to be the clay subgrade and the hot-mix asphaltic concrete in a 5" layer; the hot-mix asphaltic concrete appears to have a lower modulus when the layer is 1" thick.* The sensitivity of the deflections to a change in the E of the material composing a layer depends on the depth of the layer and its thickness. For example, the effect of decreasing the E of each material composing the structure of section 2 on the deflection of the geophone placed at a distance of 10"

---

*An isolated low value of modulus in the material of the embankment (Section 1) may be due to local water seepage.

```
A & M TEST FACILITY  -  CHEVRON INVERSE -  SECTION  2  -  68338

CENTRAL E VALUES

   66384.       471227.       811380.       37733.       17338.

              E VECTOR
                  78089.       338900.       873102.       31127.       18127.
              C MATRIX                          J
                 I
                  0.904755      0.574910     -0.011177      0.813440     -1.701573
                 -0.047139      0.156668     -0.037765      0.131484     -1.585543
                 -0.167426      0.273434      0.027339      0.420241     -0.833043
                  0.019436      0.095776      0.013631      0.102737     -0.626126
                  0.113667      0.143351      0.055164     -0.255028     -0.880954
              D MATRIX                          J
                 I
                 -0.037208     -0.027919     -0.002660     -0.050675      0.060652
                  0.001293     -0.010202     -0.001207     -0.018563      0.054340
                  0.006169     -0.013564     -0.002633     -0.031266      0.011099
                 -0.001120     -0.005424     -0.001263     -0.014753     -0.004520
                 -0.004806     -0.006738     -0.002415      0.002866      0.006133
INITIAL Y VECTOR   0.0005200   0.0004700   0.0004100   0.0003400   0.0002900
              A VECTOR      -7.3234       4.0633      -1.7617      -1.2621       0.5269
              Y VECTOR   0.0005150   0.0004740   0.0004093   0.0003405   0.0002896

TRIAL E VECTOR        78089.       338900.       873102.       31127.       18127.VERSE -  SECTION  2  -  68338
DELTA E'S      0.662D 01    -0.610D 02     0.150D 03    -0.378D 02     0.990D 01
LIMITER OPERATED.  CONV =      0.5000D-01
CONV =      0.6683D-03

              E VECTOR
                  78089.       338900.       873102.       31127.       18127.
```

TABLE 4 - Typical Output from the Convergence Program WHC36

A & M TEST FACILITY - CHEVRON INVERSE - SECTION 2 - 68338

FINAL LOG E VALUES

11.2656	12.7335	13.6798	10.3458	9.8052

C MATRIX (I = rows, J = columns)

0.904755	0.574910	-0.011177	0.813440	-1.701573
-0.047139	0.156668	-0.037765	0.131484	-1.585543
-0.167426	0.273434	0.027339	0.420241	-0.833043
0.019436	0.095776	0.013631	0.102737	-0.626126
0.113667	0.143351	0.055164	-0.255028	-0.880954

D MATRIX (I = rows, J = columns)

-0.037208	-0.027919	-0.002660	-0.050675	0.060652
0.001293	-0.010202	-0.001207	-0.018563	0.054340
0.006169	-0.013564	-0.002633	-0.031266	0.011099
-0.001120	-0.005424	-0.001263	-0.014753	-0.004520
-0.004806	-0.006738	-0.002415	0.002866	0.006133

A VECTOR	-7.3234	4.0633	-1.7617	-1.2621	0.5269
E VECTOR	78089.	338900.	873102.	31127.	18127.
Y VECTOR	0.0005150	0.0004740	0.0004093	0.0003405	0.0002896

TABLE 4 (cont.) - Typical Output from the Convergence Program WHC36

TABLE 5 - ROOT MEAN SQUARE RESIDUALS USED FOR CHECKING THE PERFORMANCE OF THE CONVERGENCE PROGRAM. DATA FROM THE A&M TEST FACILITY, SECTIONS 1 THROUGH 8

Section	Chevron To Measured Values	Convergence Program (WHC36) To Chevron	WHC36 To Measured Values
1	±0.0000320	±0.0000296	±0.0000034
2	±0.0000070	±0.0000042	±0.0000029
3	±0.0000029	±0.0000027	±0.0000006
4	±0.0000051	±0.0000027	±0.0000073
5	±0.0000072	±0.0000072	±0.0000001
6	±0.0000135	±0.0000135	±0.0000005
7	±0.0000261	±0.0000021	±0.0000241
8	±0.0000069	±0.0000035	±0.0000042

TABLE 6 - VALUES OF YOUNG'S MODULUS FOR THE MATERIALS COMPOSING THE LAYERS OF THE STRUCTURES, SECTIONS 1-8
(Values are in thousands of pounds per square inch)

Material	Thick. (In.)	Section 1	2	3	4	5	6	7	8	Mean	(Overall Mean)	Std. Dev.
Asph. Conc.	1		78	29			15	130		63	135	99
Asph. Conc.	5	240			260	85			240	206		
L.S. + 4% Cem.	4	85		960		45	940			508	439	370
L.S. + 4% Cem.	12		340		670			160	310	370		
L.S.	4	30	870			100		38		260	179	292
L.S.	12			50	280		16		46	98		
Comp. Clay	24				37				27	32		
Comp. Clay	36		31	16			11	22		20	21	10
Comp. Clay	40	5.9				16				11		
Found. Clay	∞	20	18	20	19	20	16	16	21	19	19	2

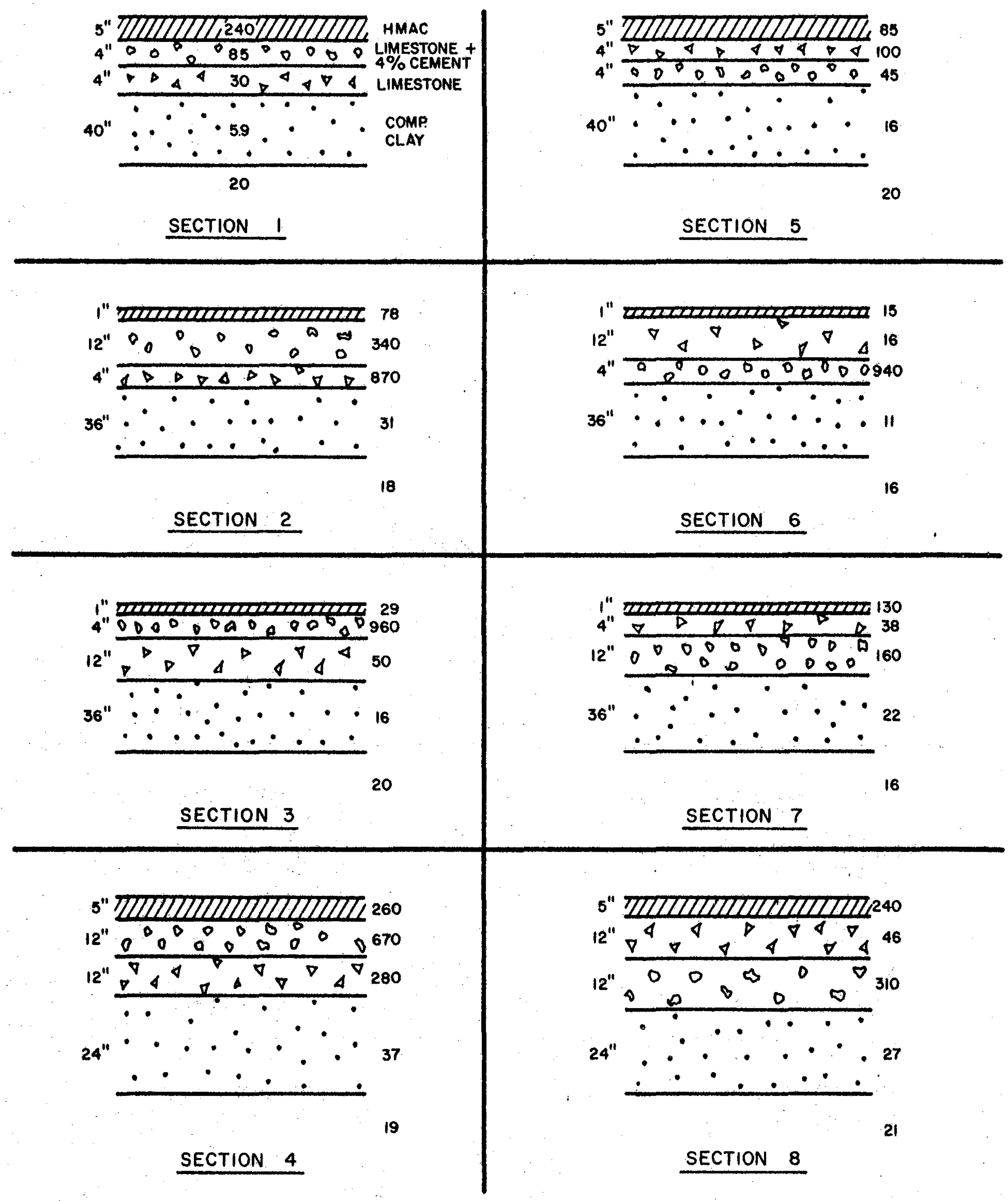

Figure 6 - Values of Young's Modulus for the Materials Composing the Layers of the Structures, Sections 1 Through 8 (values are in thousands of pounds per square inch).

from the load can be calculated; this is done by means of the elements C(1,J) and D(1,J) for values of J from one to five. The results are shown in Table 7(a). Decreasing the Young's modulus of the material composing the top layer (the hot-mix asphaltic concrete) actually decreases the deflection at a distance of 10"--in all other cases the deflection is increased. Section 6 has the same thickness distribution and placement of the layers but the moduli of all the materials except that composing the subgrade are different from those composing the structure of Section 2. Table 7(b) shows that a decrease in the E of the top layer of Section 6 leads to a decrease in the deflection measured at ten inches from the load; decreases in the moduli of any of the materials composing the other layers leads to an increase in the value of this deflection. This is qualitatively the same as in the case of Section 2.

The corresponding ratios obtained using the deflections at 49" from the load are as follows:*

	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
Section 2:	0.988,	1.019,	0.998,	1.156,	1.696
Section 6:	0.997,	0.919,	0.998,	1.191,	1.839

These figures indicate that the 49" deflection is relatively sensitive to changes in the moduli of the material composing the subgrade; the sensitivity decreases for the upper layers.

By comparing the figures above with those in Tables 7(a) and 7(b), it can be seen that the deflection of the 10" geophone is less sensitive than

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*The meaning of these figures is, for example, "The ratio of the 49" deflection for half the calculated value of E(5) in Section 2 to the deflection for the calculated value of E(5) is 1.696".

TABLE 7(a) - THE EFFECT OF HALVING THE YOUNG'S MODULUS OF THE MATERIALS COMPOSING EACH OF THE LAYERS ON PAVEMENT DEFLECTION MEASURED AT A DISTANCE OF 10" FROM THE LOAD - SECTION 2

Layer No. (J)	Thickness Inches	Calculated E psi	E*=1/2E psi	[C(1,J)+D(1,J) lnE] lnE	[C(1,J)+D(1,J) lnE*] lnE*	Difference	$y_{10"}^*/y_{10"}$
1	1	66,384	33,192	+7.116	+7.013	0.103	0.902
2	12	471,227	235,613	-4.054	-3.944	-0.110	1.116
3	4	811,380	405,690	-0.808	-0.768	-0.040	1.041
4	36	36,733	18,366	-6.490	-6.236	-0.253	1.288
5		17,338	8,669	-13.586	-13.202	-0.384	1.468

TABLE 7(b) - THE EFFECT OF HALVING THE YOUNG'S MODULUS OF THE MATERIALS COMPOSING EACH OF THE LAYERS ON PAVEMENT DEFLECTION MEASURED AT A DISTANCE OF 10" FROM THE LOAD - SECTION 6

Layer No. (J)	Thickness Inches	Calculated E psi	E*=1/2E psi	[C(1,J)+D(1,J)lnE]lnE	[C(1,J)+D(1,J)lnE*]lnE*	Difference	$y_{10''}^{*}/y_{10''}$
1	1	14,566	7,283	+0.242	+0.237	+0.005	0.995
2	12	15,662	7,833	-3.220	-3.084	-0.136	1.146
3	4	937,423	468,712	-6.197	-6.127	-0.070	1.073
4	36	11,065	5,533	-7.756	-7.515	-0.241	1.273
5		16,076	8,038	-13.364	-13.044	-0.320	1.377

the deflection of the 49" geophone to changes in the subgrade modulus. Both the 10" and the 49" geophone are relatively insensitive to changes in the moduli of the upper three layers.

5.1 Conclusion

The need exists to determine the elastic moduli of the materials composing the structure of a highway pavement. The knowledge of the moduli provides a means of calculating the stresses in the subsurface layers caused by loads on the surface. In this part of the report a method of estimating the required moduli from the measured surface deflections has been discussed. The surface deflections were obtained from the results of measurements made by means of the Dynaflect apparatus, discussed in Chapter 2. A deflection basin, as determined by elastic theory, was fitted to the measured deflections. When a good fit was achieved, the values of the elastic parameters were assumed to be those of the materials composing the pavement structure.

The loading system of the Dynaflect may be a cause of significant error. Owing to resonances in the Dynaflect chassis, it is likely that the load actually applied to the pavement is less than calculated from the simple dynamics of the loading system. The elastic moduli for the materials would then be lower than those obtained here.

5.2 Suggestions for Further Work

The work reported herein suggests the desirability of certain further work.

5.2.1 Improving the Accuracy of the Moduli of the Materials Composing The Layers Nearest to the Surface

It is a weakness of the present work that the moduli of the materials composing the layers close to the surface cannot be found with accuracy; this is evidenced by the elements of the C and D matrices relating to these layers - large changes in the moduli of the materials composing the layers lead to only small changes in the measured points of the deflection basin. The changes in the moduli of the deeper layers have, however, an appreciable effect on the measured deflections. Generally it appears that the effectiveness of a change in modulus at depth d is a maximum at a spacing of approximately d.

In order, therefore, to determine accurately the moduli of the materials within the layers close to the surface, basin points should be determined at spacings much smaller than those so far employed. The program discussed here is directly applicable if shorter spacings are used. The matrix elements will, however, be such that the measured deflections will have a far greater influence on the values of the surface moduli than is at present the case.

5.2.2 Extension of the work in order that the loading conditions may correspond with those induced by traffic

The deflection basins could be measured over a range of frequencies up to the highest of those which occur due to traffic loading. This will enable the moduli to be determined under conditions which are more truly representative of the actual traffic loading conditions. If this were done, inertia and damping would probably have to be taken into account. The Chevron program used in the present work would not necessarily be adequate as a confirmation for trial values of the moduli. A program which could be adapted for use under these conditions will be discussed in Part 2 of this report.

APPENDIX A - THE OPERATION OF THE CHEVRON PROGRAM

A.1 The Accuracy of the Chevron Program

The following are two main causes of error in the results obtained from the Chevron program. The accuracy of the Bessel function in the subroutine BESSEL may be insufficient; this can be improved if required by taking additional terms in the numerical integration. Second, the number of terms ITN of Bessel functions used to approximate the applied load can be increased. This is set at 46, and any increase would lead to an increase in execution time in proportion to ITN, as the integration is the major operation of the program. The accuracy is probably better than the 5% claimed by the authors, as shown by the following checks.

A.1.1 The Accuracy of Representing the Applied Load

The Chevron program was used to calculate the vertical direct stresses at the free surface of a semi-infinite medium, loaded by means of a circular load with a radius of 7.07 inches and exerting a constant pressure over the loaded area of 70 pounds per square inch. The results of the calculations are shown in Figure 7. The figure shows the actual load represented by a rectangle, and the approximation to it represented by a smooth curve. It appears that the accuracy of the calculations is sufficient for almost all engineering purposes.

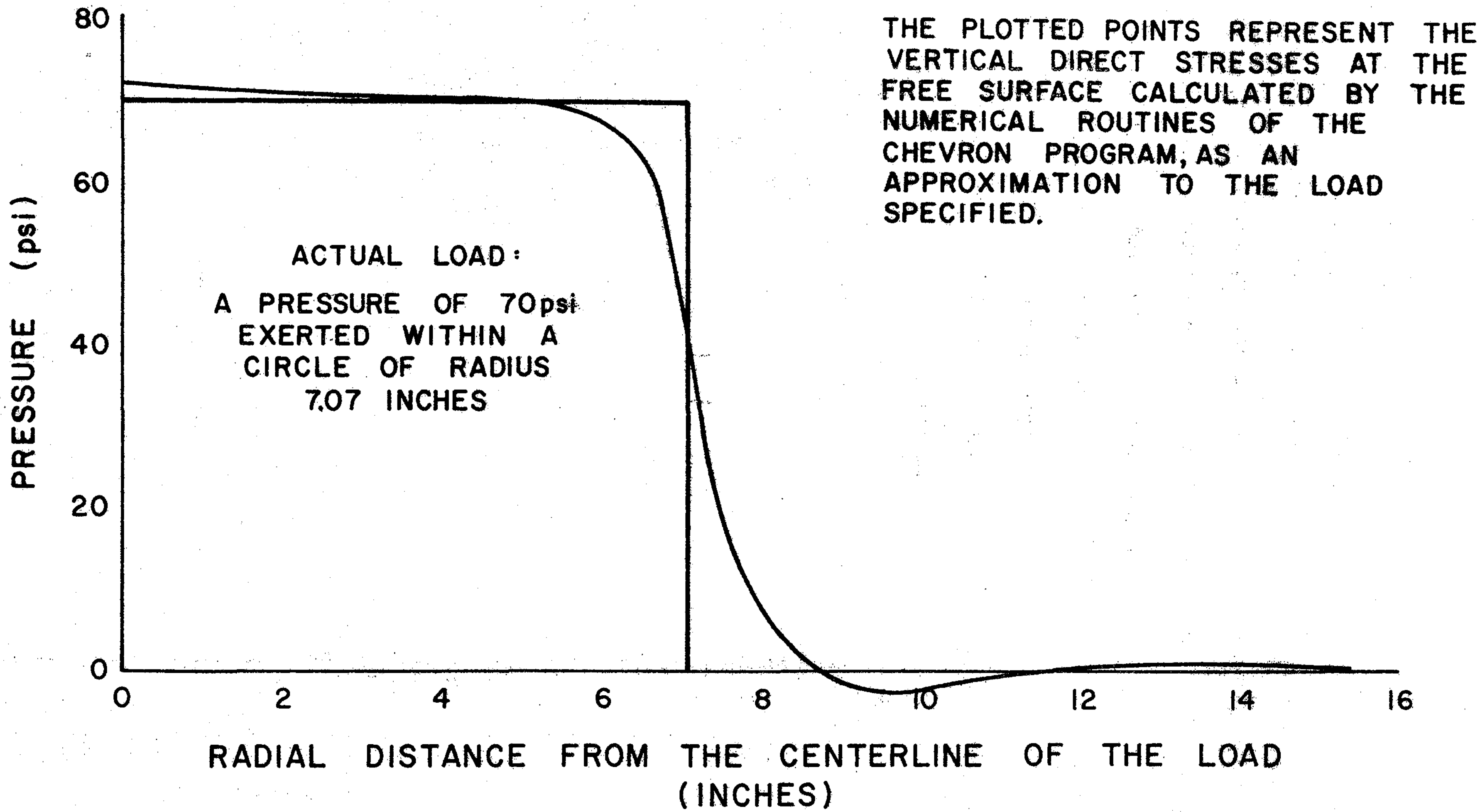


FIGURE 7 - The Correspondence Between A Specified Vertical Load and the vertical Direct Stresses Calculated by the Chevron Program in the Vicinity of the Load.

A.1.2 <u>The Accuracy of Representing the Vertical Settlements at the Free Surface</u>

The accuracy of representing the vertical settlements at the free surface is less easy to establish, because it is less easy to determine what these should be. However, certain closed solutions are available which may be used to check in a limited way the results of the Chevron computations. Biot (14) has determined the stresses within an elastic layer bounded by a rigid surface at its base. These results were presented in a more general form by Pickett (15). The stresses in such systems were expressed by means of integrals which can be evaluated by approximate methods or by numerical integration. As a check on the accuracy of the vertical settlements calculated by means of the Chevron program, it is suggested that the Boussinesq solution should be used; the spacing should be large compared with the radius of the loaded circle for which the Chevron calculations are made. Table 8 is an example of the use indicated.

The results given in Table 8 indicate that the calculations made by means of the Chevron program are probably accurate to 5% or less within a spacing equal to ten radii of the applied load; the accuracy is better at small spacings than at large spacings.

TABLE 8 - A COMPARISON BETWEEN THE VERTICAL SETTLEMENTS AT THE SURFACE OF A LAYERED STRUCTURE CALCULATED BY MEANS OF THE CHEVRON PROGRAM, AND THE TRUE SETTLEMENTS FOR A SIMILAR SYSTEM

The Chevron calculations are based upon:

Load: 1000 lbs., on a circle 1.60" radius.

A layer of material with a Young's Modulus of 9990 psi, a Poisson's ratio of 0.330 and a thickness of 18" is supported by a semi-infinite medium with a Young's Modulus of 10,000 psi, a Poisson's ratio of 0.330 and of infinite depth.

Radius Inches	Vertical Settlement* Inches	Vertical settlement due to a concentrated load of 1,000 lb., at the surface of a semi-infinite homogeneous isotropic elastic medium, with a Poisson's ratio of 0.330 and a Young's Modulus of 10,000 psi (Inches)**
15.6	0.001823	0.0018182
26.0	0.001081	0.0010909
37.4	0.0007730	0.0007666
49.0	0.0006035	0.0005789

*Calculated by means of the Chevron Program.
**The results in this column were obtained by means of Boussinesq's formula.

APPENDIX B - FAULT TRACING IN THE CONVERGENCE PROGRAM

If the central values of the E_j's are chosen such that they coincide with the E's which the Chevron program requires to produce the measured deflections, then the following will occur:

(1) The Convergence Program (WHC36) will converge on the required values of the E's with infinitesimally small error; the predicted values of the deflections will be the same as the measured values within very close limits.

(2) If a confirmation of the predicted E's is run using the Chevron, it too will yield deflections which agree (within very small limits) with the measured deflections.

However, it is not the object of the work to use trial E's which coincide exactly with those occurring in the field. Instead, we wish to select the trial E's which are rough approximations to the real ones and to use the convergence program to seek the final values. Errors are to be expected in the predicted deflections and in the predicted E's which are obtained at output from this program. The performance of the program is assessed by three root mean square residuals. These are:

(1) Between the measured deflections and those predicted by the convergence program.

(2) Between the deflections calculated by the Chevron program, using the converged E's, and those predicted by the convergence program.

(3) Between the measured deflections and those calculated by the Chevron program, using the values of the E's obtained as output from the convergence program.

If the program does not perform well, its faults may be diagnosed by reference to those three root mean square residuals. See Table 5.

First, there must be good agreement between the measured deflections and those predicted by the convergence program. The extent of this agreement is shown by comparing the Y - vector obtained at the final cycle of iteration with the "INITIAL Y VECTOR" which is printed before iteration commences. See Table 4. If the root mean square residual between those two sets of Y's is greater than the experimental error of the measured deflections, i.e. if the predictions are less accurate than the experimental results, there are two possible causes:

(a) The central E's may have been chosen too far from those E's which the Chevron program requires to yield the measured deflections. In a very bad case of this error, in which the error is more than 20% of the measured deflections, the solution is to use the predicted values of the E's (the E's obtained as the result of the iteration) as central values for a new grid of calculations--in other words, to start again.

(b) The iteration may have reached a saddle point, rather than a true minimum. If this is the case, one or more of the values of E is incorrect; the process can be continued by providing a trial E which is more nearly the true one, a decision requiring engineering judgement.

The mean square error at this stage must be made small before any further fault tracing is undertaken.

Second, there should be good agreement between the deflections predicted by the convergence program and those calculated by the Chevron program. If this is not the case, the fault is assumed to be in the convergence program and must be due to the inherent inaccuracies of the logarithmic model which is employed. To correct the fault, one of the following improvements may be made.

B.1 First Operational Suggestion

One or more of the factors governing the choice of the "high E" and the "low E" are incorrect. These factors may be either too high, causing the convergence program to calculate a model which does not represent the actual effect produced by the variation of the particular E; this case occurs particularly when attempting to find the value of E in the top layer--a change in the factor often alters the concavity of the particular element of the model from upwards to downwards, or the other way round. The factors may be too low, forcing the convergence program to select E's which are outside the calculated grid, i.e. forcing extrapolation. The allowable extrapolation depends on the accuracy required. It seems advisable to reject extrapolations outside a factor of two beyond the calculated grid. This type of error can be corrected by altering the factor, and recalculating the portion of the grid affected by means of the Chevron program. To be sure that this fault is eliminated, the predicted E should be near a grid point but see the following operational suggestion.

B.2 Second Operational Suggestion

If a particular E is outside the calculated Chevron grid, it is possible to converge upon the correct value without recalculating the complete grid. This may be done as follows. Use the Chevron program to calculate three sets of

Y's, centered on the expected value of E for the particular layer. Then calculate the C(I,J)'s and the D(I,J)'s by means of the Wang program 684. The subscript J here denotes the layer number, increasing downwards with the top layer as number one, and I denotes the deflection station increasing outwards with the 10" geophone as station number one. Therefore, if the N'th layer is the one with the faulty value of E, the elements of the C and D matrices which are calculated will be the C(I,N)'s and the D(I,N)'s where I varies from one to five. These matrix elements are then read in by the READ statements following line 217 in the subroutine COEFFT. The results of the calculations of the C(I,N)'s and the D(I,N)'s are punched on two cards (for each N), with the C's on one card and the D's on the second card; the two cards are inserted in the data stream after the card containing the measured Y's and before the card containing the trial values of the E's. The program will then calculate an A vector different from that which would have been obtained using the originally chosen central E's and the corresponding factors; apart from this addition, the data is read in as before, using those values of C(I,J) and D(I,J) which led previously to malfunctioning.

Third in the process of fault-tracing, the mean square error between the measured deflections and those calculated by means of the Chevron program should be re-calculated. This error will be small if the two errors discussed above have been made small.

APPENDIX C - A NOTE ON THE SOLUTIONS OF THE EQUATIONS REPRESENTED BY THE 'C' and 'D' MATRICES

In the system of equations

$$y_i = a_i + \sum_j (c_{ij} + d_{ij}E_{ij})E_j \qquad (i = 1,..,5;\ j = 1,..,5)$$

each $$(c_{ij} + d_{ij}E_j)E_j = b_{ij}$$

can be chosen equal to any one of five numbers which are in general different.

There are five and only five independent b_{ij}; this is because

$$b_{ij} = f(b_{1j}) \qquad i = 2, 3, 4, 5 \text{ all } j,$$

where $f(b_{1j})$ denotes some function of b_{1j} only. There are 5 mutually independent b_{1j}'s, because the y_1 is linear in the b_{1j}'s.

For each b_{1j} there are two possible E_j's, because b_{1j} is quadratic in E_j. Therefore, there are 2^5 possible solutions for the group of E_j's.

APPENDIX D - COMPARISON BETWEEN THE DEFLECTIONS CAUSED IN A STRUCTURE REPRESENTING A HIGHWAY PAVEMENT BY A TRUCK WHEEL WITH THOSE CAUSED IN THE SAME PAVEMENT BY THE DYNAFLECT AND BY A 12" PLATE

The effect of varying the moduli of the materials composing the layers of the pavement was considered.

Computations were performed for two structures:

(1) a 5" layer overlying a semi-infinite medium.

(2) a 25" layer overlying a semi-infinite medium.

The moduli of the materials composing the layer and the semi-infinite medium were 200,000 and 10,000 psi respectively.

The loadings were represented as follows:

Truck Wheel - Two loads of 4,500 pounds exerting uniform pressures of 100 psi, spaced 24" apart; the deflection was calculated at the mid-point of the line joining the load centers.

Dynaflect - A total load of 1,000 pounds exerted on a circle of radius 1.60"; the deflection was calculated at a distance of 10" from the load.

12" Plate - A uniform pressure of 100 psi exerted on a plate 12" in diameter; the deflection was calculated at the center of the plate.

For each layer, the effect was investigated of decreasing the modulus of elasticity of the materials composing the layers by 5%. The decrease was applied to each layer separately and then to both the layers simultaneously. The results are plotted in Figure 8. From this figure the

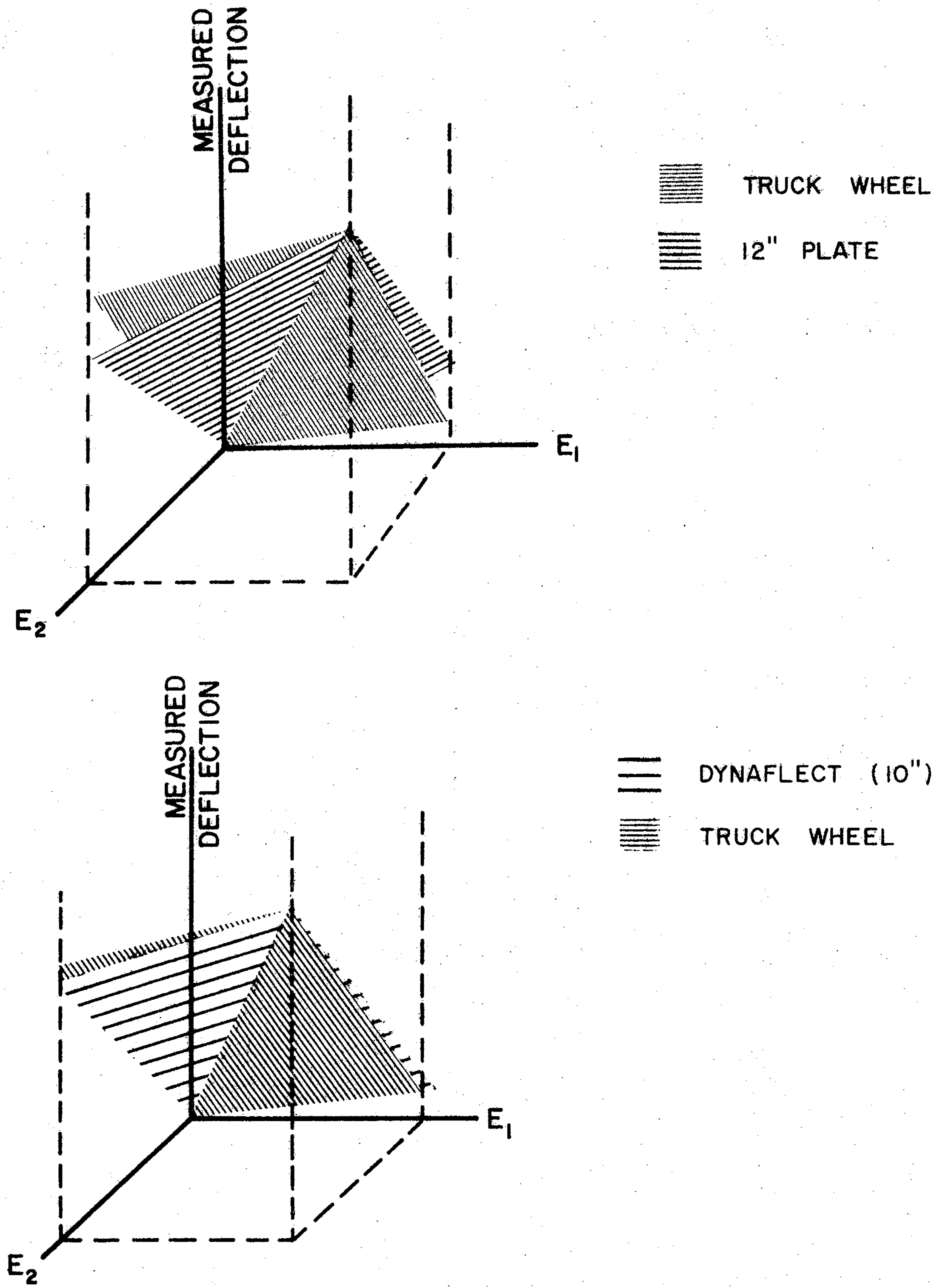
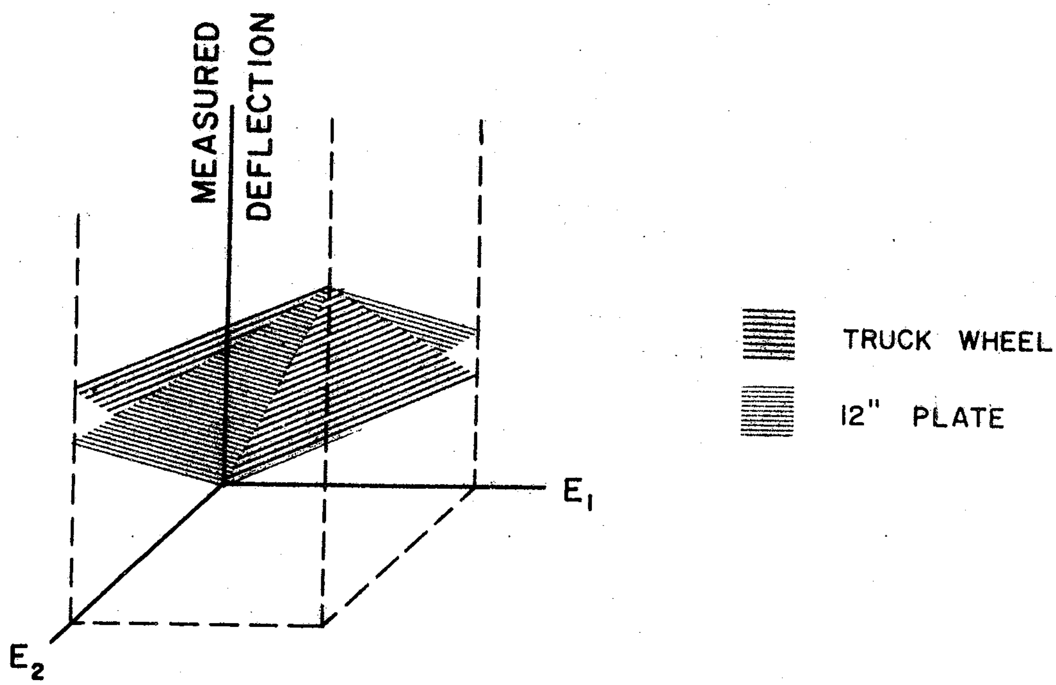


FIGURE 8 - The Effect of Changes in the Young's Moduli of the Materials Composing a Two-Medium Structure on the Deflections Measured by Means of (1) a Truck Wheel (2) a Dynaflect (3) a 12-Inch Plate. Top Layer 5".



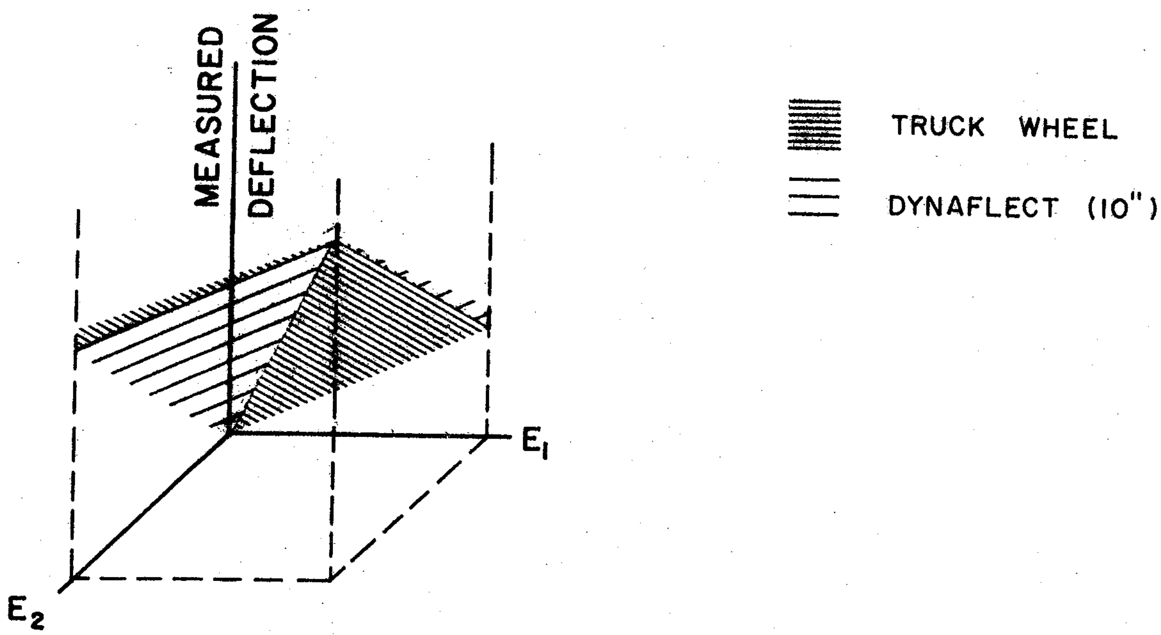


FIGURE 8 continued - The Effect of Changes in the Young's Moduli of the Materials Composing a Two-Medium Structure on the Deflections Measured by Means of (1) a Truck Wheel (2) a Dynaflect (3) a 12-Inch Plate. Top Layer 25".

following appears:

(1) A change in the modulus of the top layer (E_1) produces a greater effect on all measurements if the top layer is thicker.

(2) The Dynaflect represents the behavior of the truck wheel more closely than the measurements obtained from the 12" plate.

(3) The measurements of the deflections of the truck wheel are less affected than those obtained from either the Dynaflect or the 12" plate by a change in E_1.

(4) The measurements of the deflections of the truck wheel are more affected than those obtained from either the Dynaflect or the 12" plate by a change in E_2.

Conclusions (3) and (4) are not necessarily related.

APPENDIX E - LIST AND DESCRIPTIONS OF THE COMPUTER PROGRAMS ASSOCIATED WITH PART I OF THIS REPORT

Fortran Programs

The Fortran programs used in this work are intended for use with a Watfor compiler but can be operated with only minor changes on an IBM/360/65 Fortran IV compiler.

Designation	Description
WHC35	Data checking program for the input to WHC 36. Supplies a print-out of the input data in tabular form suitable for rapid checking; requires less than a tenth of a second for operation, and is an insurance against wasted time when the longer running WHC 36 is operated.
WHC36	Interpolates values of E_j (j = 1 through 5), the Young's moduli of the materials composing the layers of a system corresponding with a highway pavement. Requires about twenty seconds to converge and contains some safety features to prevent non-productive running.

Wang Programs

(See Part 2 of this report for a description of the Wang computer)

Designation	Description
684.xx	where xx is a two digit parameter. This program has been written in order to compute the elements of the C and D matrices. As input to the program, a central

Description

value of E is required. Values of Y are also needed; they are computed by means of the Chevron program for the central E and also for two neighboring E's which are evenly spaced about the central E on a logarithmic scale of E. The spacing is, therefore, denoted by a factor, xx in the program designation. This factor is to be punched in the Wang program card in columns 2 and 3, and again in columns 18 and 19. Two columns are thus available for the factor. If only a single digit factor is required, punch the factor followed by a decimal point, or a zero followed by the factor.

Details of operation: All input is in natural logarithms. The central E value is placed in SR_0; the Y's are placed in successively higher storage registers, starting with the Y which corresponds with the lowest E. At the end, the C element is read, and the D element from A_R, if the Y's are inserted in the reverse order from that specified above, the D elements will be correct for use with WHC36, although the C's will not.

APPENDIX F - THE PLOTTING OF THE CROSS SECTION OF THE DEFLECTION BASIN IN A PAVEMENT WHICH IS SUBJECTED TO A POINT LOAD

It has been mentioned in the text that, for the case of a semi-infinite homogeneous medium subjected to a point load perpendicular to the free surface, the plot of (deflection x spacing) against spacing is a horizontal straight line of ordinate equal to the product (deflection x spacing). This is due to the relationship between deflection and spacing, one of inverse proportionality (13). The plot obtained is indicated in Figure 9.

The systems which are of practical interest are not homogeneous however. They consist of layers of materials which may be approximated by isotropic elastic substances having a wide range of elastic moduli. The layers may be composed of stiff and compliant materials, and may be superposed in any order of stiffness.

Consider first the case of a layer of a compliant material overlying a semi-infinite medium which is composed of a stiffer material. At spacings which are large compared with the thickness of the layer, a horizontal straight line is obtained; the ordinate of this line represents a compliance parameter of the material composing the underlying semi-infinite medium.* Similarly, at spacings which are small compared with the thickness of the layer, a horizontal straight line is obtained the ordinate of which represents the same parameter relating to the layer. At spacings which are of the order

*This parameter is $(1 - \nu^2)/\pi E$ per unit load (13), where E is the Young's modulus of the material and ν is its Poisson's ratio.

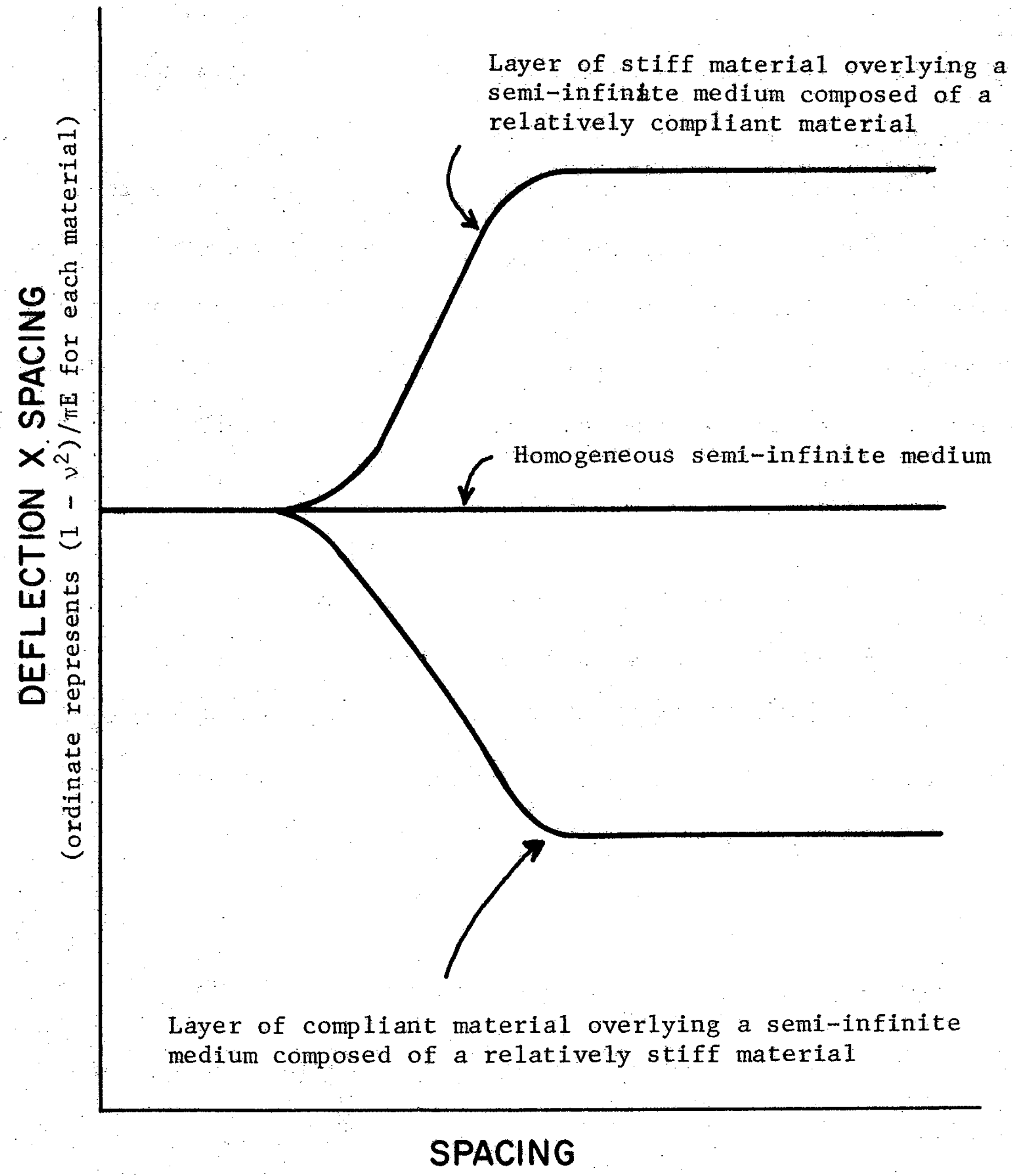


FIGURE 9 - Hypothetical Plot of the Results of the Measurements of the Deflections on the Surface of a Pavement Subjected to a Point Load.

of the thickness of the layer a transition occurs; points on the curve, including the transition, may be calculated by means of the Chevron program (11, 12). The relationship is sketched in Figure 9.

If the overlying layer is composed of a stiffer material than the semi-infinite medium, the form of the plot is reversed. At short spacings the ordinate of the line represents the same parameter as before related to the compliance of the layer. For large spacings, the ordinate represents the parameter relating to the underlying medium. The shape of the curve is sketched in Figure 9, and points on the curve may be calculated if required.

Although highway structures are not usually as simple as the ideal system discussed here, some may be made to approximate such a structure. Alternatively, this method may be applied to highway structures provided the errors induced (usually by treating all the constructed layers as a single one) are not important.

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