FLEXIBLE PAVEMENT PERFORMANCE RELATED TO DEFLECTIONS, AXLE APPLICATIONS, TEMPERATURE AND FOUNDATION MOVEMENTS

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1. INTRODUCTION

This report is intended to document the empirical equation for predicting pavement performance that was given in Section 3.3 of Research Report 32-11, "A Systems Approach To the Flexible Pavement Design Problem" (1).

The equation explains loss in serviceability in terms of deflections, axle applications, temperature, and foundation movements due to swelling clays. This report is limited to a discussion of how the equation was developed. How it is used to solve practical problems of design is described briefly in a summary report, "The Design of Flexible Pavements (A Systems Approach)", and in detail in Research Report 32-11 (1).

2. A FLEXIBLE PAVEMENT PERFORMANCE HYPOTHESIS

The phrase "pavement performance" is used herein in the sense first proposed by Carey and Irick (2) in connection with the AASHO Road Test. That is, we define the performance of a pavement as its serviceability history, as illustrated in Figure 1. Thus, in discussing pavement performance we shall be concerned with the gradual loss in serviceability that occurs over a period of time and is attributed to the combined effect of traffic and environment on the pavement structure and its foundation.

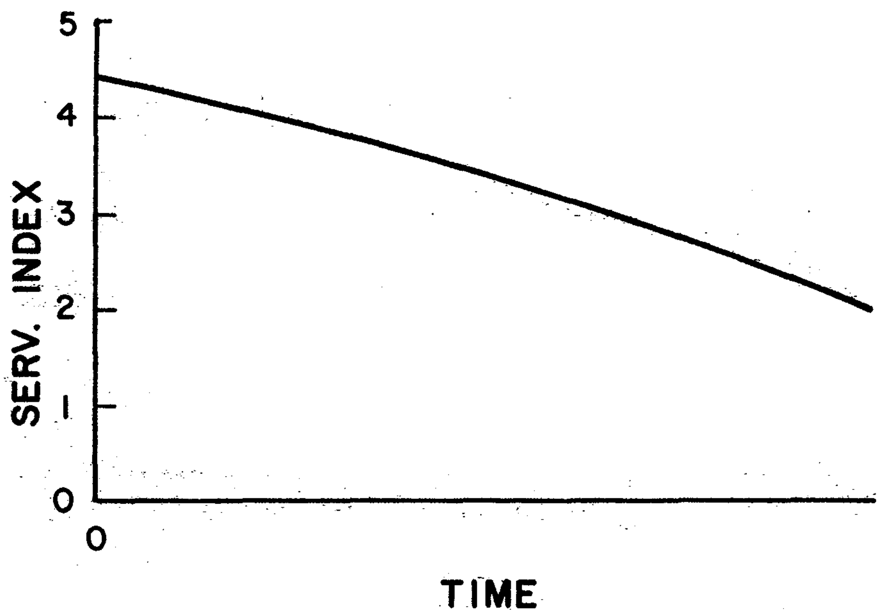

FIGURE 1 - The Serviceability History of a Pavement.

2.1 A Simple Case of Flexible Pavement Performance

To consider a simplified case of the factors affecting pavement performance, suppose that a wheel load of fixed magnitude is repeatedly applied at a point on a flexible pavement over a period of time during which the mean daily air temperature remains relatively constant but above the freezing point of water, and during which the physical state of the materials remains relatively constant at every point in the structure and its foundation. Suppose also that the load is well below the ultimate, so that after each application of load the pavement surface rebounds to a position very nearly the same as its original position.

Under these conditions we assume that the curvature--and therefore the stress--produced in the pavement structure by any one load application will be practically the same as that produced by any other application. We also assume that although pavement rebound is very nearly 100% after each load application, in reality it is usually less than 100% by an immeasurably small amount. If this is granted, then it follows that each application of load produces a very small but real permanent deformation, that such deformations are of varying magnitude along the wheel paths because of inherant variations in the materials, and that they accumulate with load applications until the resulting surface distortions are not only measurable, but are large enough to be felt by passengers in a moving vehicle.

The increase in roughness just described may be equated to a loss in the serviceability index of the pavement, and it seems reasonable to assume that for a fixed number of load applications and a fixed mean daily temperature, the serviceability loss will be relatively great if the curvature produced by the load is great, and relatively small if the curvature is small. Furthermore, for a fixed curvature and a fixed mean daily temperature, the serviceability loss will increase as the number of load applications increases. And finally, because the rupture strain of asphaltic concrete is sensitive to temperature, we assume that if both the curvature and the number of load applications are fixed, the loss in serviceability will be relatively high if the mean daily temperature is low, and relatively low if the mean daily temperature is high. As a first approximation, we can express these ideas in mathematical form as follows:

$$Q = \frac{C_1 N S'^{C_2}}{\alpha^{C_3}}, \text{ if } \alpha > 0, \text{ and}$$

$$Q = 0, \text{ if } \alpha \leq 0. \qquad (1)$$

In Equation 1 Q is some (as yet undefined) measure of the loss in serviceability occurring while N wheel loads are applied; S' is a (as yet undefined) measure of the curvature of the pavement structure caused by an application of the load; α is the mean daily air temperature minus 32°F; and C_1, C_2 and C_3 are constants all greater than zero.

It is important to note that neither the magnitude of the load, nor a description of the design of the pavement appears in Equation 1: these have been replaced by the curvature variable, S'. It is equally important to observe that the equation is limited to periods during which mean daily temperatures are constant and above 32°F.

Limiting the application of the performance hypothesis to periods having mean daily temperatures above freezing may, at first glance, seem unduly restrictive. However, it is believed that increases in pavement roughness usually occur not during sub-freesing weather (when sub-surface layers are solidly frozen), but during the subsequent thaw. Thus, our hypothesis applies to that portion of the annual temperature cycle when serviceability loss can be expected to occur, and excludes those periods when changes in serviceability index seem likely to be negligible.

2.2 A Measure of Surface Curvature

The critical wheel load stress acting in the pavement structure, particularly the tensile stress acting at the bottom of the asphaltic concrete layer, is believed to be approximately proportional to the curvature of the surface produced by the load.

The quantity adopted for use in this report to represent surface curvature is the deflection difference, S', defined in Figure 2. It is similar to the "partial deflection" measured at the AASHO Road Test. The partial deflection was defined as ". . . the depth of the deflection basin measured under a 2-ft. chord at the bottom of the basin. . ." The basin was determined by means of a Benkelman Beam especially equipped to record a complete analogue of the deflection as the load truck moved forward at creep speed over the beam's probe (Reference 3, Page 96).

In Figure 2, W_1' is the maximum deflection measured between the dual tires of a truck at the center of gravity of the dual wheel load, W_2' is the deflection measured at a longitudinal distance of one foot from the center of gravity of the dual wheel load, and S' is equal to the difference, $W_1' - W_2'$.

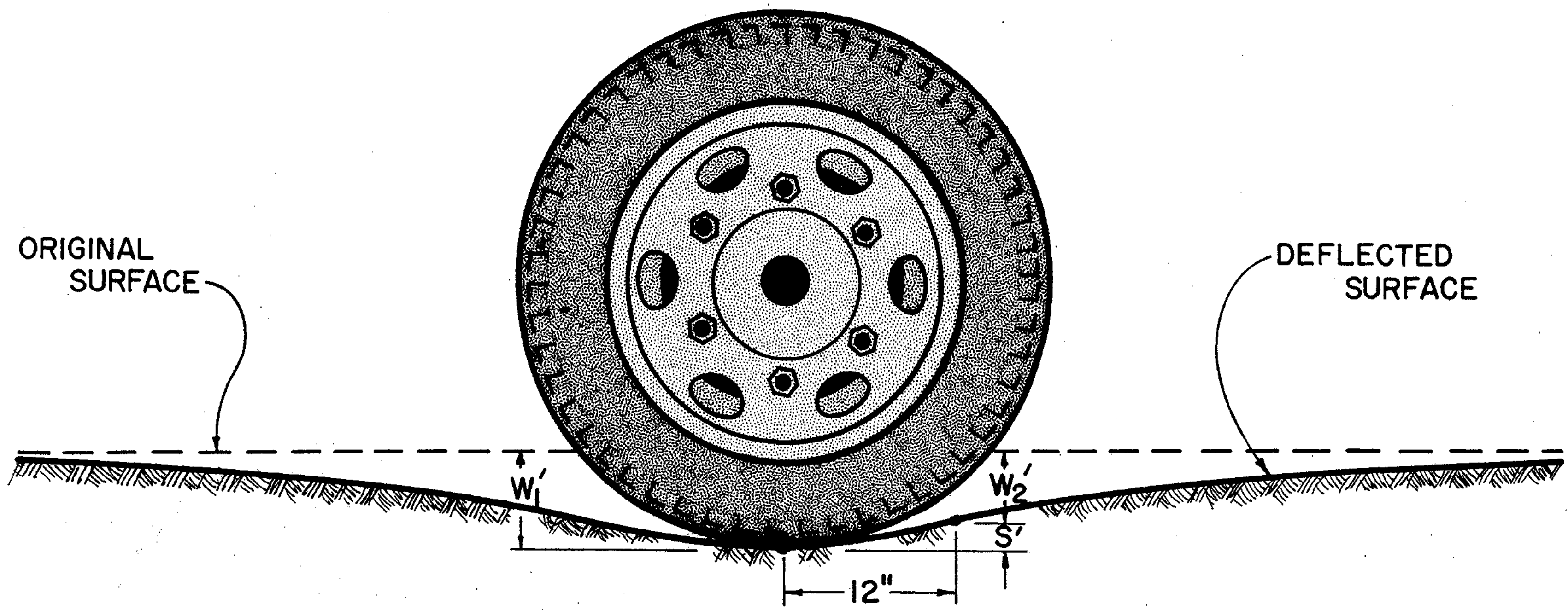

FIGURE 2 - The Deflection Difference, S', Used as a Measure of Surface Curvature.

2.3 The Serviceability Loss Function, Q

The serviceability loss function, Q, is defined in general terms as a function of P_1 and P, where P_1 is the value of the serviceability index at the beginning of a time period during which α and S' are constant, and P is its value at any time during the period. In mathematical notation,

$$Q = F(P_1, P) \tag{2}$$

where F is read, "a function of."

In order to arrive at the exact form of the serviceability loss function (the right side of Equation 2) it was first necessary to examine the serviceability histories of test sections at the AASHO Road Test (the only available source of performance data) and determine, if possible, the general shape of these curves for the special conditions of constant α and constant S' assumed in our performance hypothesis. Since very large seasonal variations in both deflections and temperatures were observed at the Road Test over the two-year traffic period, it became necessary to divide the two-year period into segments of varying durations, ranging from two to fourteen weeks, during each of which both α and the measured deflections were reasonably constant. These periods, ten in number, and the number of test sections available for study within each period, are displayed in Table 1. The analysis periods will be discussed later; for the present it is sufficient to say that because of the wide scatter of the serviceability index data, it was very difficult to determine what might be a typical shape of the curve of serviceability index versus load applications within any period. It was finally decided,

TABLE 1 - The Ten Analysis Periods

Analysis Period	Date From	Date To	Length of Period (Weeks)	Accumulated Axle Applications (Millions) Beginning	Accumulated Axle Applications (Millions) End	No. Test Sections Observed
1	3-11-59	3-24-59	2	.0762	.0797	88
2	3-25-59	4-7-59	2	.0797	.0860	79
3	4-8-59	4-21-59	2	.0860	.0978	70
4	4-22-59	5-19-59	4	.0978	.1205	51
5	5-20-59	8-25-59	14	.1205	.2331	46
6	8-26-59	11-3-59	10	.2331	.3239	45
7	4-6-60	4-19-60	2	.6014	.6340	26
8	4-20-60	5-3-60	2	.6340	.6692	24
9	5-4-60	5-17-60	2	.6692	.7047	23
10	5-18-60	8-9-60	12	.7047	.9014	21
Total			52			473

after examining dozens of plotted curves, to approximate the data with a curve that would always be concave downward, as illustrated in Figure 3, and defined by the differential equation

$$\frac{dP}{dN} = -k\ (5 - P)^{1/2} \qquad (3)$$

where P is the serviceability index, and k is a constant assumed to be dependent upon the load and the properties of the pavement--or alternatively, upon the surface curvature variable, S'--as well as upon the air temperature.

The number, 5, in Equation 3 is the defined upper limit of the serviceability index, P. Thus, the slope, dP/dN, of the P versus N curve is always negative (downward in Figure 3), and the steepness of the slope increases as the loss term, $(5 - P)^{1/2}$, increases.

Again referring to Figure 3, let N_1 be the value of N at the beginning of a period during which α and S' are constant, and let N_2 be its value at the end of the period. Then, from our previous definition of P_1 and P_2 it follows that $P = P_1$ when $N = N_1$, $P = P_2$ when $N = N_2$, and Equation 3 applies during the period from $N = N_1$ to $N = N_2$.

We now separate the variables in Equation 3, and integrate between the limits of P and N, as indicated below:

$$-\int_{P_1}^{P_2} \frac{dP}{(5 - P)^{1/2}} = k \int_{N_1}^{N_2} dN \qquad (4)$$

Performing the integration indicated in Equation 4, we obtain -

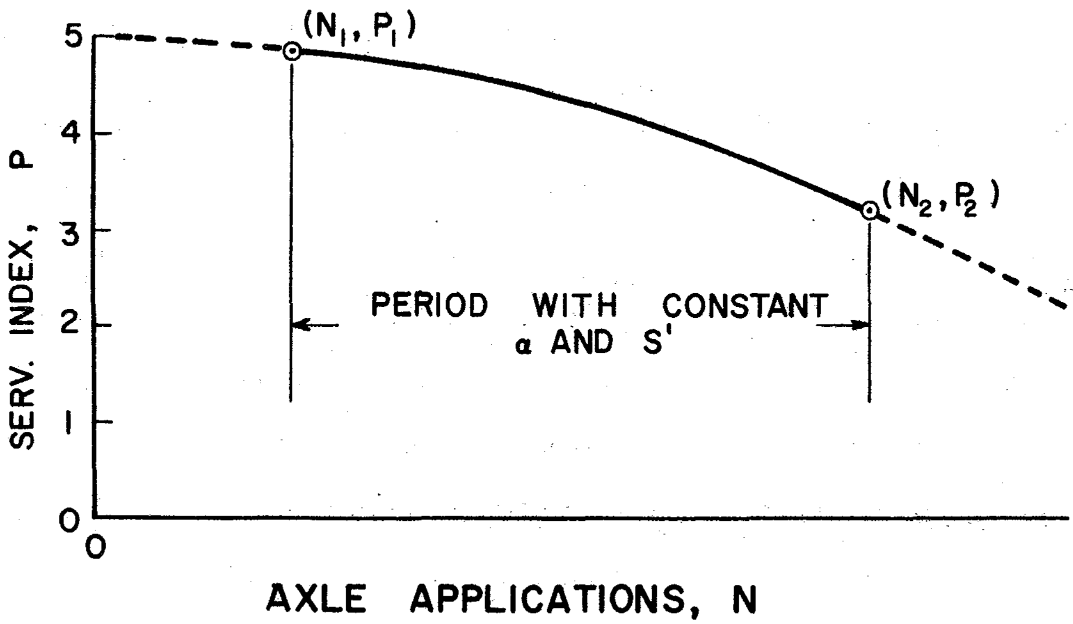

FIGURE 3 - Segment of P versus N curve with α and S' Constant.

$$(5 - P_2)^{1/2} - (5 - P_1)^{1/2} = \frac{k}{2}(N_2 - N_1) \qquad (5)$$

Equation 5 can be made consistent with our performance hypothesis (Equation 1) by introducing the following definitions:

$$Q_2 \equiv (5 - P_2)^{1/2} - (5 - P_1)^{1/2} = Q \text{ when } P = P_2$$

$$\frac{k}{2} \equiv \frac{C_1 S'^{C_2}}{\alpha^{C_3}} \qquad (6)$$

By utilizing the definitions of Equations 6 in Equation 5, we arrive at --

$$Q_2 = \frac{C_1 (N_2 - N_1) S'^{C_2}}{\alpha^{C_3}}, \qquad (7)$$

where $Q_2 \equiv \sqrt{5 - P_2} - \sqrt{5 - P_1}$,

P_1 = the value of P when $N = N_1$,

P_2 = the value of P when $N = N_2$,

N = number of load applications,

S' = a quantity assumed to be proportional to the surface curvature, and assumed constant in the interval $N_1 \leq N \leq N_2$,

α = the mean daily temperature (°F) less 32°F, assumed constant in the interval $N_1 \leq N \leq N_2$, and

C_1, C_2, C_3 = constants.

Note that the difference, $N_2 - N_1$, appearing in Equation 7 is simply the number of load applications occurring during the period of constant α and constant S', and we have dispensed with the need for defining the point, N = 0.

2.4 The Effective Value of α

If the asphaltic surfacing layer is relatively thin when compared to the total thickness of the pavement above subgrade level, as is the usual case in Texas, we assume that changes in α--which produce changes in the stiffness of the surfacing--will not, as a rule, appreciablly affect surface deflections nor the surface curvature variable, S'. This assumption makes it possible to link together, into one period, several successive days during which observed deflections are relatively constant but α is not, and to compute for the whole period a single, or effective, value of α, designated as $\bar{\alpha}$. The derivation of the formula for $\bar{\alpha}$ is given below.

We begin by considering a period of four successive days.

Let P_{1i} = the value of P at the beginning of the i^{th} day (i = 1, 2, 3 or 4).

P_{2i} = the value of P at the end of the i^{th} day = $P_{1,i+1}$

N_{1i} = the value of N at the beginning of the i^{th} day.

N_{2i} = the value of N at the end of the i^{th} day = $N_{1,i+1}$

α_i = the value of α for the i^{th} day.

From Equation 7 we have, for the first day --

$$\sqrt{5 - P_{21}} - \sqrt{5 - P_{11}} = C_1(N_{21} - N_{11})\ S'^{C_2}/\alpha_1{}^{C_3}$$

For the second day --

$$\sqrt{5 - P_{22}} - \sqrt{5 - P_{12}} = C_1(N_{22} - N_{12})\ S'^{C_2}/\alpha_2{}^{C_3}$$

For the third day -- (8)

$$\sqrt{5 - P_{23}} - \sqrt{5 - P_{13}} = C_1(N_{23} - N_{13})\ S'^{C_2}/\alpha_3{}^{C_3}$$

Finally, for the fourth day --

$$\sqrt{5 - P_{24}} - \sqrt{5 - P_{14}} = C_1(N_{24} - N_{14})\ S'^{C_2}/\alpha_4{}^{C_3}$$

Note that the following equalities exist among certain quantities in the four equations

$$P_{21} = P_{12};\ P_{22} = P_{13};\ P_{23} = P_{14};$$
$$N_{21} = N_{12};\ N_{22} = N_{13};\ N_{23} = N_{14}. \tag{9}$$

(These equations arise from the fact that P (or N) at the end of one day is equal to P (or N) at the beginning of the next day).

By adding Equations 8, and by taking into account the equalities in P (Equation 9), we obtain an equation for the four-day period, as follows:

$$\sqrt{5 - P_{24}} - \sqrt{5 - P_{11}} = C_1 S'^{C_2}\left[\frac{N_{21} - N_{11}}{\alpha_1{}^{C_3}} + \frac{N_{22} - N_{12}}{\alpha_2{}^{C_3}} + \frac{N_{23} - N_{13}}{\alpha_3{}^{C_3}} + \frac{N_{24} - N_{14}}{\alpha_4{}^{C_3}}\right] \tag{10}$$

The total number of load applications during the four-day period is equal to $N_{24} - N_{11}$. Let it be assumed that the number of load applications per day is constant throughout the four-day period. Then

$$N_{21} - N_{11} = N_{22} - N_{12} = N_{23} - N_{13} = N_{24} - N_{14} = \frac{N_{24} - N_{11}}{4} \tag{11}$$

By appropriate substitutions of Equations 11 in Equation 10, we obtain--

$$\sqrt{5 - P_{24}} - \sqrt{5 - P_{11}} = C_1 S'^{C_2}\left(\frac{N_{24} - N_{11}}{4}\right)\left(\frac{1}{\alpha_1{}^{C_3}} + \frac{1}{\alpha_2{}^{C_3}} + \frac{1}{\alpha_3{}^{C_3}} + \frac{1}{\alpha_4{}^{C_3}}\right) \tag{12}$$

Equation 12 can be simplified and generalized to represent an n-day period simply by dropping the second subscript on P and N. Then we have, according to Equation 12,

$$Q_2 = C_1 S'^{C_2}(N_2 - N_1)\,\frac{1}{n}\left(\frac{1}{\alpha_1{}^{C_3}} + \frac{1}{\alpha_2{}^{C_3}} + \ldots + \frac{1}{\alpha_n{}^{C_3}}\right) \tag{13}$$

where $Q_2 = \sqrt{5 - P_2} - \sqrt{5 - P_1}$,

P_1, P_2 = the value of P at the beginning and end of the n-day period,

N_1, N_2 = the value of N at the beginning and end of the n-day period, and,

$\alpha_1, \alpha_2, \ldots, \alpha_n$ = the value of α for the first, second, ..., n^{th} day.

We now define $\bar{\alpha}$, the effective value of α, by the equation,

$$Q_2 = \frac{C_1 S'^{C_2}(N_2 - N_1)}{\bar{\alpha}^{C_3}} \tag{14}$$

where the symbols are as defined in connection with Equation 13.

Equation 14 can be written in the following form:

$$\frac{Q_2}{C_1 S'^{C_2}(N_2 - N_1)} = \frac{1}{\bar{\alpha}^{C_3}} \quad . \tag{14a}$$

From Equation 13:

$$\frac{Q_2}{C_1 S'^{C_2}(N_2 - N_1)} = \frac{1}{n}\left(\frac{1}{\alpha_1{}^{C_3}} + \frac{1}{\alpha_2{}^{C_3}} + \ldots + \frac{1}{\alpha_n{}^{C_3}}\right) \tag{13a}$$

From Equations 13a and 14a we see that

$$\frac{1}{\bar{\alpha}^{C_3}} = \frac{1}{n}\left(\frac{1}{\alpha_1{}^{C_3}} + \frac{1}{\alpha_2{}^{C_3}} + \ldots + \frac{1}{\alpha_n{}^{C_3}}\right) .$$

By inverting the last equation we have

$$\bar{\alpha}^{C_3} = \left(\frac{n}{\frac{1}{\alpha_1{}^{C_3}} + \frac{1}{\alpha_2{}^{C_3}} + \ldots + \frac{1}{\alpha_n{}^{C_3}}}\right), \quad \text{or}$$

$$\bar{\alpha} = \left(\frac{n}{\frac{1}{\alpha_1^{C_3}} + \frac{1}{\alpha_2^{C_3}} + \ldots + \frac{1}{\alpha_n^{C_3}}} \right)^{\frac{1}{C_3}} \tag{15}$$

Thus, Equations 14 and 15 can be used to compute the serviceability loss over a period during which the deflections of a pavement remain reasonably constant, but α varies significantly, provided that (1) the time rate of load application is constant throughout the period, (2) the asphaltic concrete layer is relatively thin when compared to the total thickness of the pavement structure, and (3) the values of C_1, C_2 and C_3 are known.

The values of the constants C_1, C_2 and C_3 were estimated from AASHO Road Test data. The data required for the evaluations were, according to Equations 14 and 15 --

(a) Serviceability index data, P_1 and P_2, for computing Q.

(b) Deflection data, W_1' and W_2', for computing S'.

(c) Axle load application data, N_1 and N_2.

(d) Air temperature data for computing $\bar{\alpha}$.

How the constants were evaluated will be explained in the chapters to follow.

3. CALCULATION OF S' FOR AASHO ROAD TEST SECTIONS

The "partial deflection," or S', was measured at the AASHO Road Test in only a few cases, but the deflection W_1' was measured approximately every two weeks. Thus, before evaluation of the constants in Equation 14 could be accomplished, it became necessary to devise a method for estimating S' from W_1'. The empirical deflection equation described in Research Report 32-12 (4), in conjunction with certain assumptions with regard to the stiffness coefficients of the AASHO Road Test materials, was used for estimating the value of S', as described in the following sections. The deflection equation, modified to predict the deflection caused by a given axle load (instead of the Dynaflect load as in Research Report 32-12 (4)), is given below. The equation applies to a system of n layers above the foundation layer, which is assumed to be of infinite thickness.

$$S' = W_1' - W_2' \tag{16}$$

where

$$W_j' = \sum_{k=1}^{n+1} \Delta'_{jk}$$

$$\Delta'_{jk} = \frac{B_o}{a_k^{c_1}} \left[\frac{1}{r_j^2 + C_2 \left(\sum_{i=0}^{k-1} a_i D_i\right)^2} - \frac{1}{r_j^2 + C_2 \left(\sum_{i=0}^{k} a_i D_i\right)^2} \right]$$

$$B_o = C_o \frac{20L}{18}$$

$$C_o = .891$$

L = single axle load (kips)

$C_1 = 4.503$

$C_2 = 6.25$

$a_o = D_o = 0$

a_i = stiffness coefficient of i^{th} layer

D_i = thickness (inches) of i^{th} layer ($D_{n+1} = \infty$)

$r_1{}^2 = 100$, $r_2{}^2 = 244$

If C_o is substituted for B_o in Equation 16, the equation takes the form for predicting Dynaflect deflections. The number, 20, appearing in the factor 20L/18 in the expression for B_o converts Dynaflect deflections to deflections caused by an 18-kip single axle load, as determined from field correlation studies (6). The additional factor, L/18, converts the 18-kip axle deflection to the deflection caused by an axle load, L. The latter conversion is based on the assumption that deflection is proportional to axle load.

3.1 Assumptions in Computing Initial Stiffness Coefficients of AASHO Road Test Materials

The use of the deflection equation (Equation 16) in connection with sections at the AASHO Road Test required that estimates be made of the stiffness coefficients, a_i, of the Road Test materials. (The stiffness coefficients used in Equation 16 should not be confused with the surface, base and subbase coefficients used in the official AASHO Road Test flexible pavement performance equations). The following simplifying assumptions were made with regard to the initial (fall of 1958) stiffness coefficients.

(1) The stiffness coefficient of the asphaltic concrete surfacing material used at the AASHO Road Test was the same as the coefficient of the asphaltic concrete used at the Texas A&M University Pavement Test Facility described in Research Report 32-12 (4), and is given by the equation

$$a_1 = 0.52 + .00284\ (62 - T) \qquad (17)$$

where a_1 is the coefficient of the asphaltic concrete and T is the mean daily temperature in degrees Farenheit at the time initial deflections were taken in the fall of 1958. (Equation 17 is based on experimental work done by Manke (5), and is discussed in the appendix of this report.)

(2) The stiffness coefficients of the crushed limestone base and the gravel subbase at the AASHO Road Test were initially the same as the initial stiffness coefficients of the crushed limestone base and the gravel embankment materials used at the A&M Pavement Test Facility as determined in March, 1966. These were 0.4716 and 0.3988.

(3) The initial ratio of the clay embankment stiffness coefficient to the

foundation stiffness coefficient at the AASHO Road Test was the same as the initial ratio of the clay embankment coefficient (0.2709) to the foundation coefficient (0.1980) at the A&M Pavement Test Facility.

Figure 4 illustrates the application of the foregoing assumptions to a particular section at the AASHO Road Test, Section 257, which was located in the single axle lane of Loop 6 and was subjected to a 30-kip axle load. During the time initial deflections were measured in the fall of 1958, the average daily temperature was 55°F; thus, according to Equation 17, the estimated value of a_1 was .54, as shown in the figure. The values of a_2 and a_3 given in the figure are the values found at the A&M Pavement Test Facility for the untreated crushed limestone base and the gravel embankment materials (see Table 4 of Reference 4). The value of a_4 and a_5 shown in the figure is the coefficient for the compacted clay embankment at the A&M Facility, multiplied by a constant B, to be discussed later. Layers 4 and 5 were assumed initially to have the same coefficient because in the fall of 1958 the upper portion of the embankment had been recompacted and the pavement placed since the last freeze-thaw cycle. The value of a_6 is that of the foundation clay at the A&M Facility multiplied by the constant B.

For Section 257, as for all sections included in this study, the constant B was varied by a convergent process until a value was found such that the resulting stiffness coefficients, when substituted in Equation 16, resulted in a deflection, W_1', that matched the measured deflection of the section. When the matching of the computed with the observed deflection had been accomplished, the resulting array of coefficients, a_1, a_2,..., a_6 was assumed to be the correct initial (fall of 1958) array.

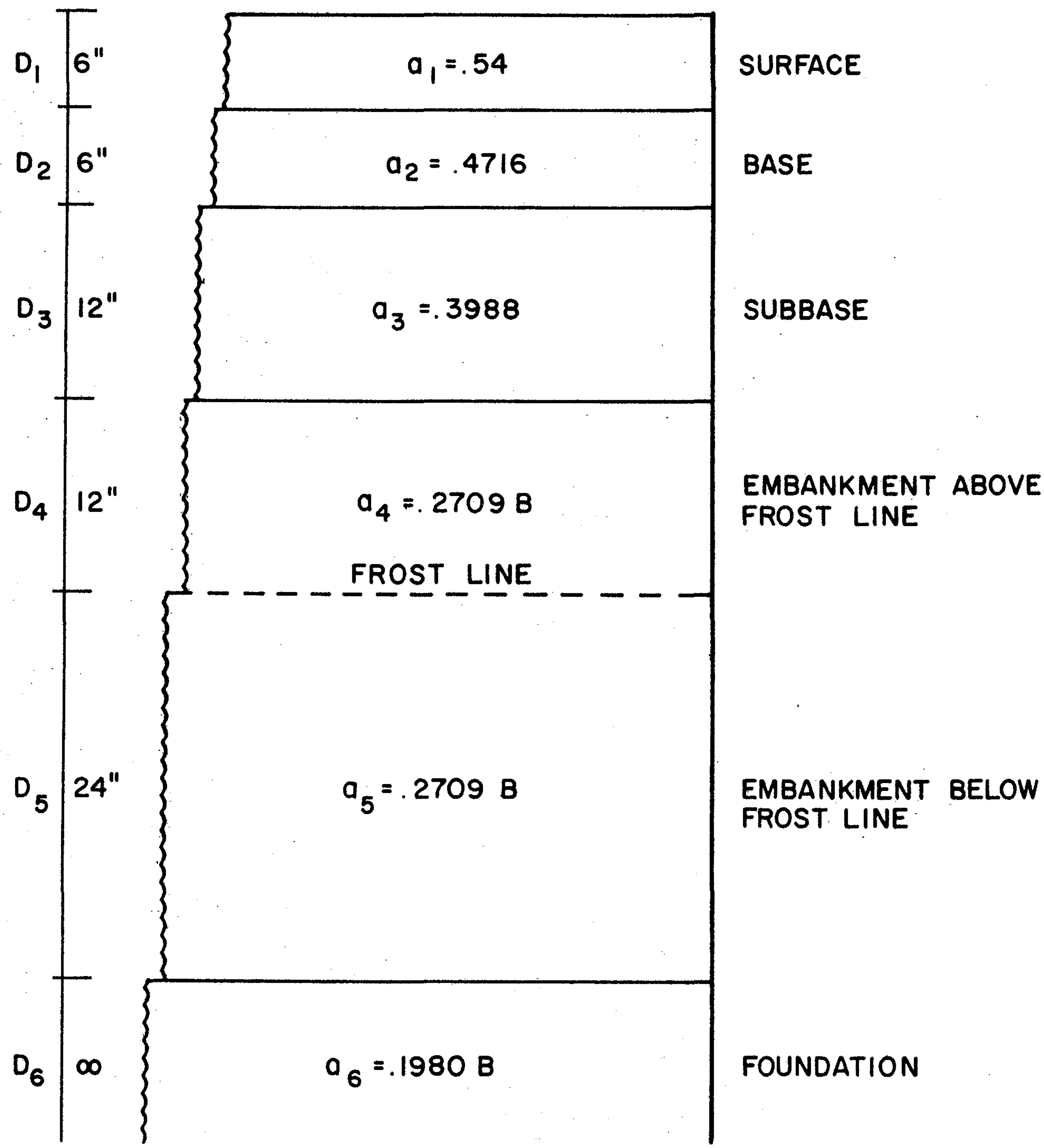

FIGURE 4 - "As Constructed" Stiffness Coefficients of AASHO Road Test Section 257.

The factor B was computed for 88 test sections in the single axle lanes of the main loops (Loops 3 through 6), as indicated in Table 1, Section 2.3. Its value ranged from a minimum of 0.88 to a maximum of 1.13, the mean value being 0.97. The variation in B from section to section appeared to be random, apparently resulting from random variations in the deflection measurements.

3.2 Assumptions in Computing Coefficients After the First Freeze-Thaw Cycle

The following simplifying assumptions were made in the computation of stiffness coefficients for AASHO Road Test sections after the first (1958-59) freeze-thaw cycle.

(1) Any changes with time in the deflection, W_1', observed on any section after the first freeze-thaw cycle could be attributed solely to a change in the coefficients of the materials above the frost penetration line, which was taken as 36 inches below the surface of the pavement.

(2) The ratios of the coefficients of the base, subbase and embankment materials above the 36-inch frost line remained constant, regardless of changes in the coefficients themselves.

Figure 5 illustrates the application of these two assumptions to Section 257, the section previously treated in Figure 4. The coefficient, a_1, in Figure 5, was computed from Equation 17, using the mean daily air temperature existing at the time deflections were measured. The initial coefficients a_2, a_3 and a_4 (above the frost line) were multiplied by a constant, C, while the coefficients a_5 and a_6 (below the frost line) were assumed to remain at their initial (fall 1958) values. The constant, C, was adjusted by a convergent process until the deflection, W_1', computed from Equation 16 was equal to the measured deflection. When the matching of computed and observed deflections had been accomplished, the resulting array of coefficients, a_1, a_2, ..., a_6, was assumed to be the correct array.

With the correct array of stiffness coefficients in hand, the deflection, W_2', was computed from Equation 16, after which S' was calculated from the formula, $S' = W_1' - W_2'$.

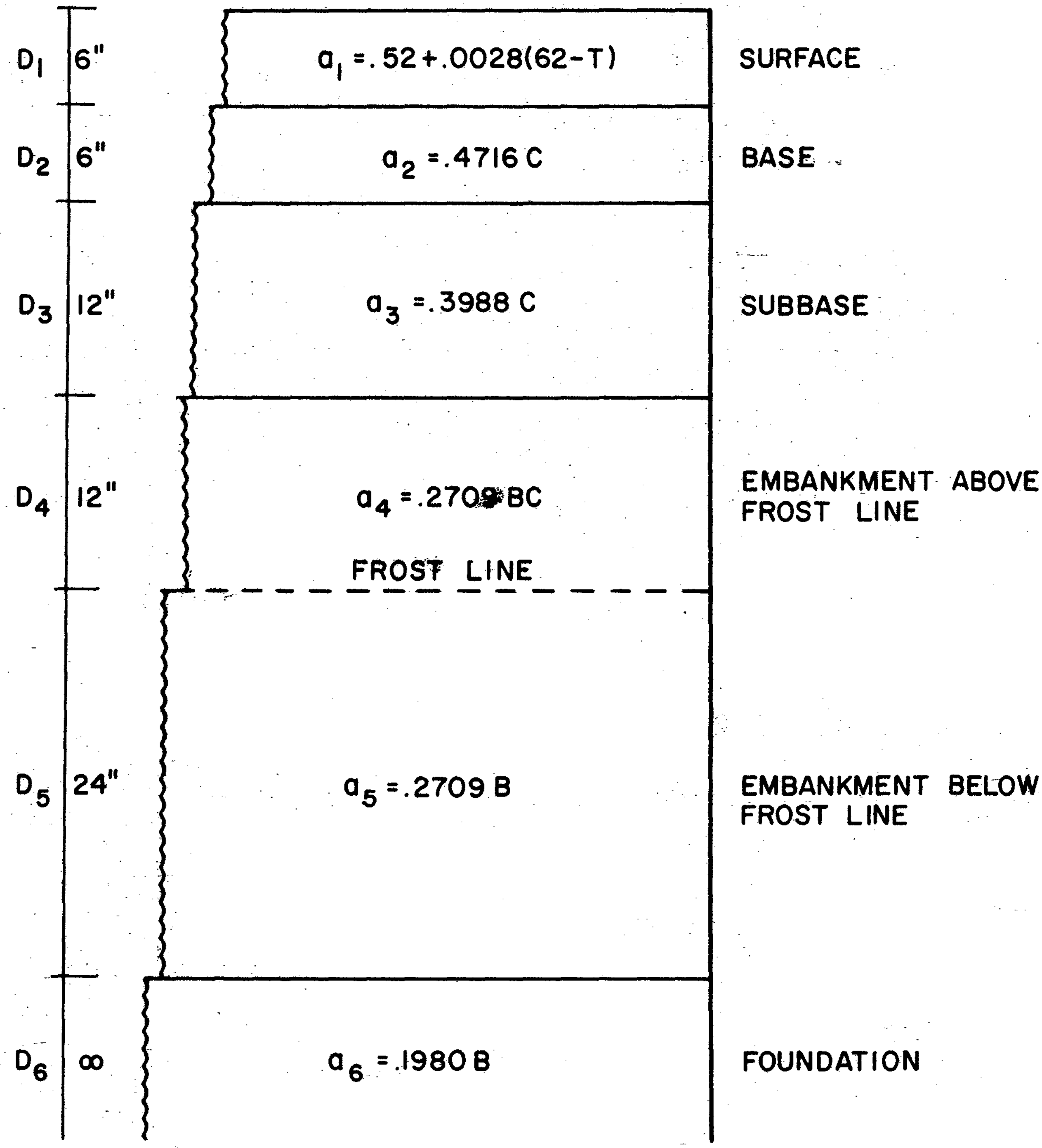

FIGURE 5 - Stiffness Coefficients for AASHO Road Test Section 257 at Anytime After the 1958-59 Freeze-Thaw Cycle.

The stiffness coefficients for Section 257 are given in Table 2 for the fall of 1958 and the subsequent ten analysis periods. Also shown in the table are the measured deflections in thousandths of an inch (mils), the average daily temperature, and the value of the surface curvature variable, S', computed in accordance with the procedure described above.

In Figure 6 (a) the coefficients for Section 257 of the base, the subbase, and that portion of the embankment above the frost penetration line, have been plotted against the mid-date of the ten analysis periods. The trend lines of these coefficients are typical of most of the sections. Some similarity will be observed between these trend lines and those of the CBR values shown in Figure 6 (b). The CBR data--reproduced from Figure 96, Page 119, of Reference 3--were gathered on the non-traffic loop (Loop 1) at the AASHO Road Test. The qualitative agreement between the two sets of graphs in Figure 6 appears to lend some credence to the procedure used in computing stiffness coefficients.

TABLE 2 - STIFFNESS COEFFICIENTS FOR AASHO ROAD TEST SECTION 257

Analysis Period	Date From	Date To	Season	T (°F)	W_1' (mils)	a_1	a_2	a_3	a_4	a_5	a_6	S' (mils)
Fall 1958	10-8-58	11-19-58	Fall	55	38	.	.472	.399	.268	.268	.196	8
1	3-11-59	3-24-59	Spring	39	53	.59	.403	.341	.229	.268	.196	13
2	3-25-59	4-7-59	Spring	48	63	.56	.385	.325	.219	.268	.196	17
3	4-8-59	4-21-59	Spring	48	61	.56	.389	.329	.221	.268	.196	16
4	4-22-59	5-19-59	Spring	62	59	.52	.401	.339	.228	.268	.196	16
5	5-20-59	8-25-59	Summer	74	56	.49	.414	.351	.236	.268	.196	15
6	8-26-59	11-3-59	Fall	61	49	.52	.429	.363	.244	.268	.196	12
7	4-6-60	4-19-60	Spring	52	70	.55	.373	.315	.212	.268	.196	19
8	4-20-60	5-3-60	Spring	62	60	.52	.398	.337	.226	.268	.196	16
9	5-4-60	5-17-60	Spring	55	63	.54	.388	.328	.221	.268	.196	17
10	5-18-60	8-9-60	Summer	71	56	.49	.414	.351	.236	.268	.196	15

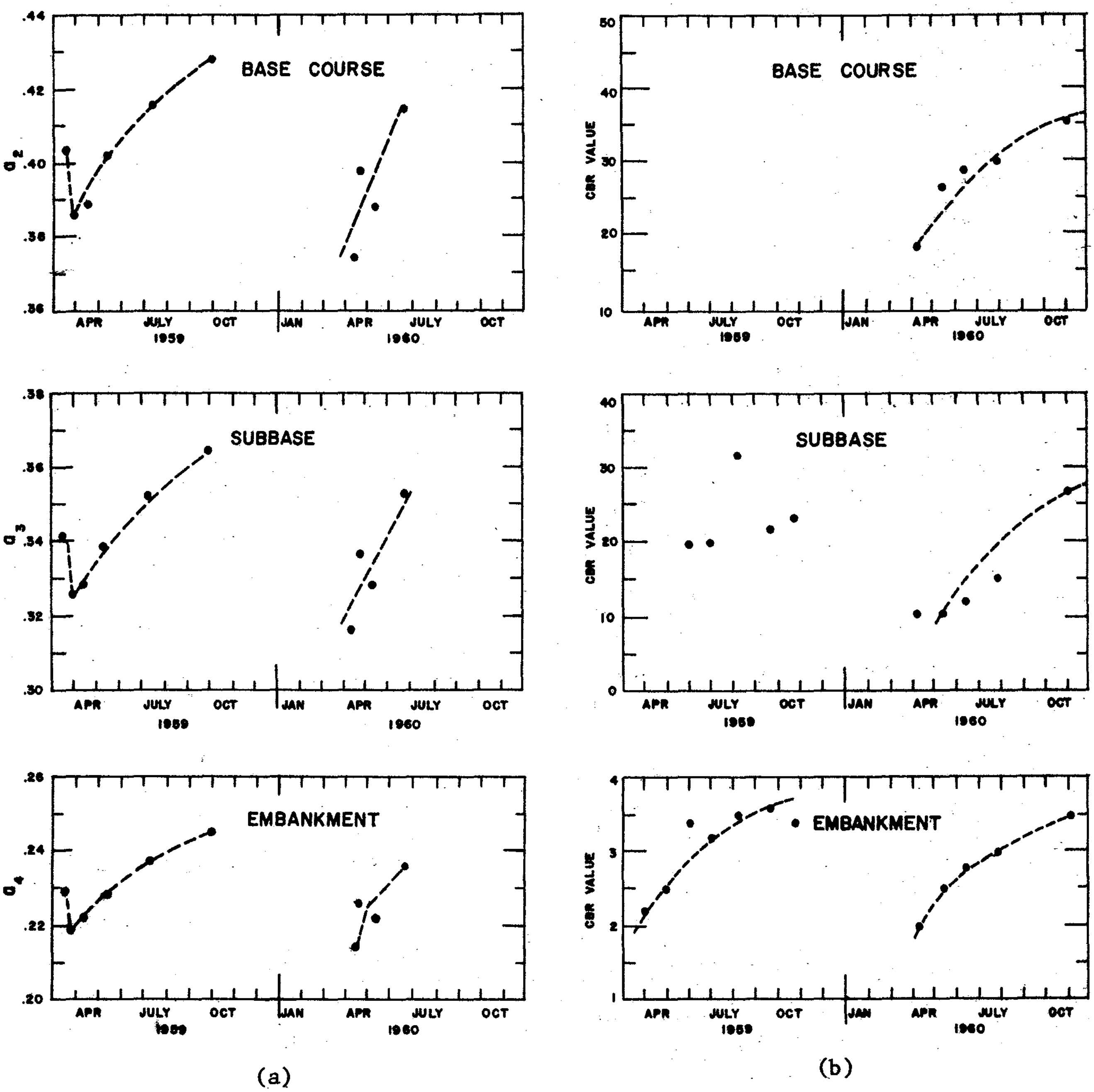

FIGURE 6 - Variation with Time of Stiffness Coefficients Compared with Variation of CBR Values at AASHO Road Test.

4. ESTIMATING THE CONSTANTS C_1, C_2, C_3 FROM AASHO ROAD TEST DATA

As pointed out in Section 2.4, the data necessary for evaluating the constants C_1, C_2 and C_3 appearing in the performance equation (Equation 14) were P_1, P_2, W_1', N_1, N_2 and α; furthermore, these data were required for each of several periods during the spring, summer and fall seasons. It was further required that α and W_1' be relatively constant during each period.

The first task, then, in selecting data was the delineation of the analysis periods. This job was accomplished through study of Loop 1 deflections (Appendix B, Reference 3) in conjunction with temperature data from the Ottawa, Illinois, weather station and frost data from Figure 11 of Reference 3. All periods involving intermittent or deep frost penetration, and one period showing no loss in serviceability, were eliminated from further consideration. The elimination of these periods cut the total length of time available for study from two years to one year (see Table 1).

Once the analysis periods had been defined, the required data were obtained from the sources indicated in Table 3.

TABLE 3 - Sources of AASHO Road Test Data

Variable	Period	Source of Data
P_1, P_2	1 thru 10	Data System 7132*
W_1'	Fall 1958	Appendix C, Reference 3
W_1'	1 thru 10	Data System 5121*
N_1, N_2	1 thru 10	Appendix B, Reference 3
α	All	Ottawa, Ill., Weather Station

* AASHO Road Test Data Systems were loaned to the researchers by the Highway Research Board.

4.1 The Analysis Procedure

An example of the performance data used in estimating C_1, C_2 and C_3 is the data for Section 257, shown in Table 4. The reader will note that negative values of Q_2, resulting from an apparent increase in the serviceability index, occurred in two of the analysis periods. Such increases occurred frequently throughout the mass of serviceability index data used in the analysis, and may be partly due to random variations in the measurement of the serviceability index. In any event, the model used (Equation 14) cannot account for negative values of Q_2, i.e., increases in serviceability index; this capability was deliberately excluded because the reasons for such increases--if they are real--are still a matter of speculation that cannot be quantified. Therefore, the negative values of Q_2 were attributed to random error and were "averaged out" in the analysis procedure.

The analyses procedure is described below, step by step.

(1) Values were assigned to C_2 and C_3.

(2) With C_3 known, $\bar{\alpha}$ was computed from Equation 15 for each of the ten analysis periods.

(3) With C_2, C_3 and ten values of $\bar{\alpha}$ known, C_1 was computed from the following formula based on Equation 14:

$$C_1 = \left(\sum_{j=1}^{10} \sum_{i=1}^{n_j} Q_{ij}\right) / \left(\sum_{j=1}^{10} \sum_{i=1}^{n_j} X_{ij}\right) \qquad (18)$$

where $X_{ij} = \dfrac{(S'_{ij})^{C_2} \, (N_{2j} - N_{1j})}{\bar{\alpha}_j^{\,C_3}}$,

TABLE 4 - PERFORMANCE DATA FOR AASHO ROAD TEST SECTION 257

N Period	Fall 1958	1	2	3	4	5	6	7	8	9	10
$\bar{\alpha}$*	--	3.5	11.7	11.9	27.3	41.0	22.3	9.5	27.8	16.1	38.0
N_2-N_1	--	.0035	.0063	.0118	.0227	.1126	.0908	.0326	.0352	.0355	.1967
W_1'	38	53	63	61	59	56	49	70	60	63	56
W_2'	30	40	46	45	43	41	37	51	44	46	41
S'	8	13	17	16	16	15	12	19	16	17	15
P_1	--	4.2	4.3	4.2	4.0	2.9	3.3	3.2	2.8	2.6	2.5
P_2	--	4.3	4.2	4.0	2.9	3.3	3.3	2.8	2.6	2.5	2.0
Q_2	--	-0.058	0.058	0.106	0.449	-0.145	0.0	0.142	0.066	0.032	0.151

* Computed from Equation 15 with C_3 = 1.

Note: All data from AASHO Road Test sources except W_2' and S', which were computed as explained in text.

Q_{ij} = the value of the i^{th} observation of Q in the j^{th} period,

S'_{ij} = the value of the i^{th} observation of S' in the j^{th} period,

$\bar{\alpha}_j$ = the value of $\bar{\alpha}$ in the j^{th} period,

$N_{2j} - N_{1j}$ = the number of axle applications in the j^{th} period (see Table 4),

n_j = the number of sections in the j^{th} period (see Table 1).

(4) With C_1, C_2, and C_3 known, individual errors, E_{ij}, were computed from the formula

$$E_{ij} = \hat{Q}_{ij} - Q_{ij}$$

where $\hat{Q}_{ij}$ is the value computed from Equation 14, and Q_{ij} is the observed value of Q.

(5) The root-mean-square-residual was calculated from the errors, E_{ij}.

Table 5 gives the combinations of C_2 and C_3 that were investigated, and shows the root-mean-square-residual for each combination. The residual error associated with the combination, $C_2 = 2$ and $C_3 = 1$, was very nearly the minimum error, and that combination was chosen as a sufficiently accurate determination of C_2 and C_3. The corresponding value of C_1 was 0.1340. Thus, Equation 14, with C_1, C_2 and C_3 evaluated, becomes

$$Q_2 = \frac{0.134(N_2 - N_1)\, S'^2}{\bar{\alpha}} \tag{19}$$

and Equation 15 becomes

$$\bar{\alpha} = \frac{n}{\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \dots + \frac{1}{\alpha_n}} \tag{20}$$

TABLE 5 - ERRORS IN COMPUTED VALUES OF Q FOR VARIOUS COMBINATIONS OF THE EXPONENTS C_2 AND C_3

C_2	C_3						
	0.50	0.75	1.00*	1.25	1.50	1.75	2.00
1.00	.108	.102	.097	.094	.097	.106	.136
1.25	.105	.099	.094	.092	.096	.105	.134
1.50	.101	.096	.091	.090	.095	.104	.132
1.75	.099	.094	.090	.090	.096	.105	.130
2.00*	.098	.094	.092	.092	.098	.106	.129
2.25	.099	.095	.094	.096	.101	.107	.128
2.50	.100	.098	.097	.100	.105	.110	.128

*Selected value. Corresponding value of C_1 was 0.1340.

where all symbols are as previously defined in connection with Equation 14 and 15. Equation 20 defines $\bar{\alpha}$ as the <u>harmonic</u> <u>mean</u> of α.

Plots of averaged Q_2 data within periods showed that S' was a significant variable. To test the significance of $\bar{\alpha}$, a second analysis, similar to the analysis just described, was performed, except that α was eliminated from the model. The resulting least residual was approximately double the value associated with Equation 19. Therefore, it was concluded that $\bar{\alpha}$ was a significant variable.

4.2 Prediction Error of Equation 19

The observed serviceability history of Section 257 over the ten analysis periods is compared, in Figure 7, with the serviceability history predicted by Equation 19, using the performance data in Table 4. The observed value of the serviceability index at the beginning of Period 1 was substituted for P_1 in Equation 19 for computing P_2 for that period; P_2 for each subsequent period in 1959 (Periods 2 through 6) was computed using the last calculated value of the serviceability index as P_1. A similar procedure was followed in computing the second branch of the curve (Periods 7 through 10 in the year 1960). The data plotted in Figure 7 are given in Table 6.

As indicated in Figure 7, the interval between the end of Period 6 and the beginning of Period 7 was eliminated from consideration in the analysis because of the intermittent occurrence of frost in the pavement and subgrade. A portion of the interval between the end of Period 10 and the termination of test traffic was frost free, but the 21 surviving test sections eshibited no decrease in serviceability index during that time--hence that interval also had to be excluded from the analysis.

The procedure just described in connection with Section 257 was used in computing the serviceability history of all test sections. Predicted values of serviceability index are plotted against observed values in Figure 8. The correlation coefficient between the predicted and observed values was 0.79. The mean absolute residual was 0.28. This residual may be compared with the average deviation of the measured serviceability

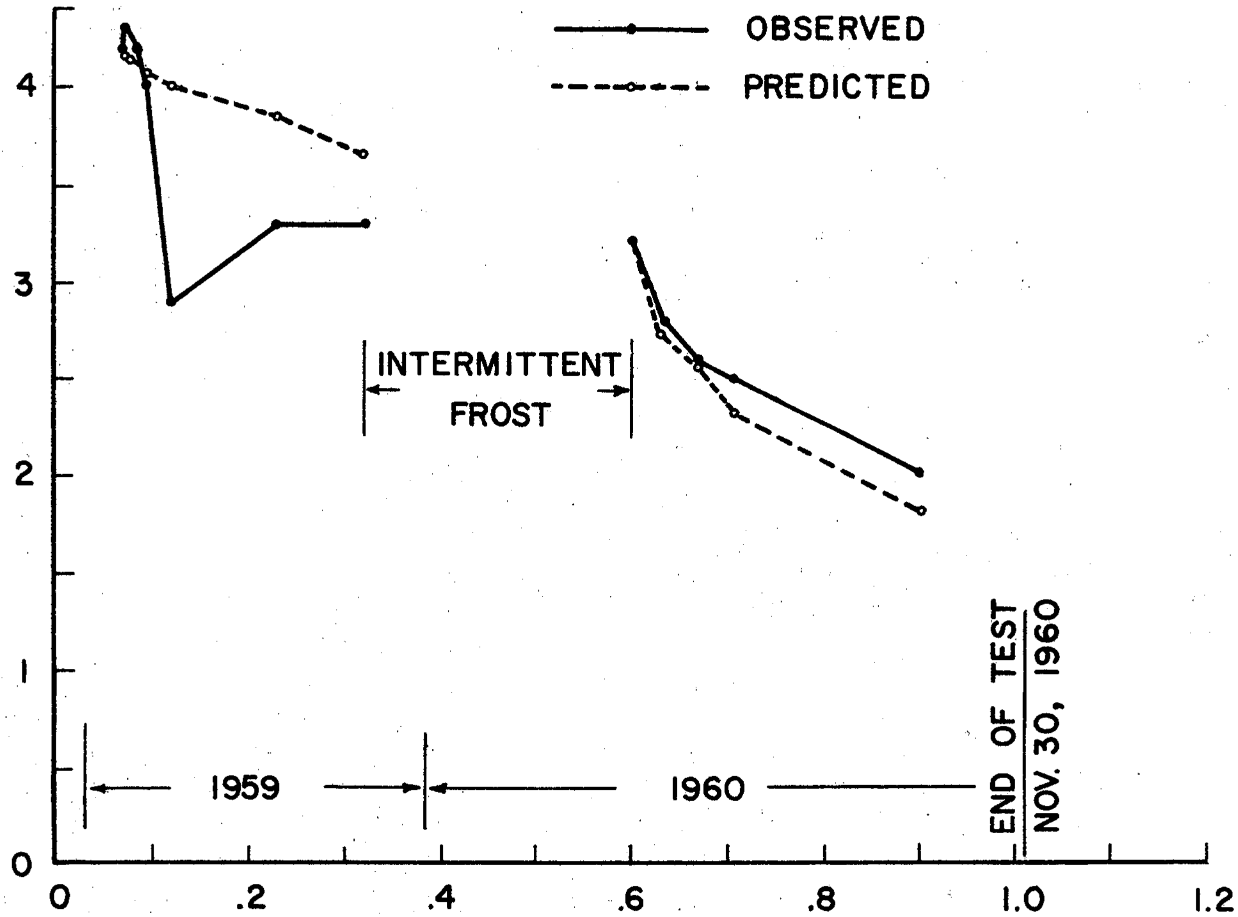

FIGURE 7 - Observed Serviceability History of AASHO Road Test Section 257
Compared with Predictions of Equation 19.

TABLE 6 - OBSERVED AND COMPUTED VALUES OF SERVICEABILITY INDEX PLOTTED IN FIGURE 7

Year	Period	N_1	N_2	Observed Values P_1	Observed Values P_2	Computed Value P_2
1959	1	.0762	.0797	4.2	4.3	4.16
1959	2	.0797	.0860		4.2	4.12
1959	3	.0860	.0978		4.0	4.06
1959	4	.0978	.1205		2.9	4.00
1959	5	.1205	.2331		3.3	3.83
1959	6	.2331	.3239		3.3	3.65
1960	7	.6014	.6340	3.2	2.8	2.73
1960	8	.6340	.6692		2.6	2.59
1960	9	.6692	.7047		2.5	2.32
1960	10	.7047	.9014		2.0	1.79

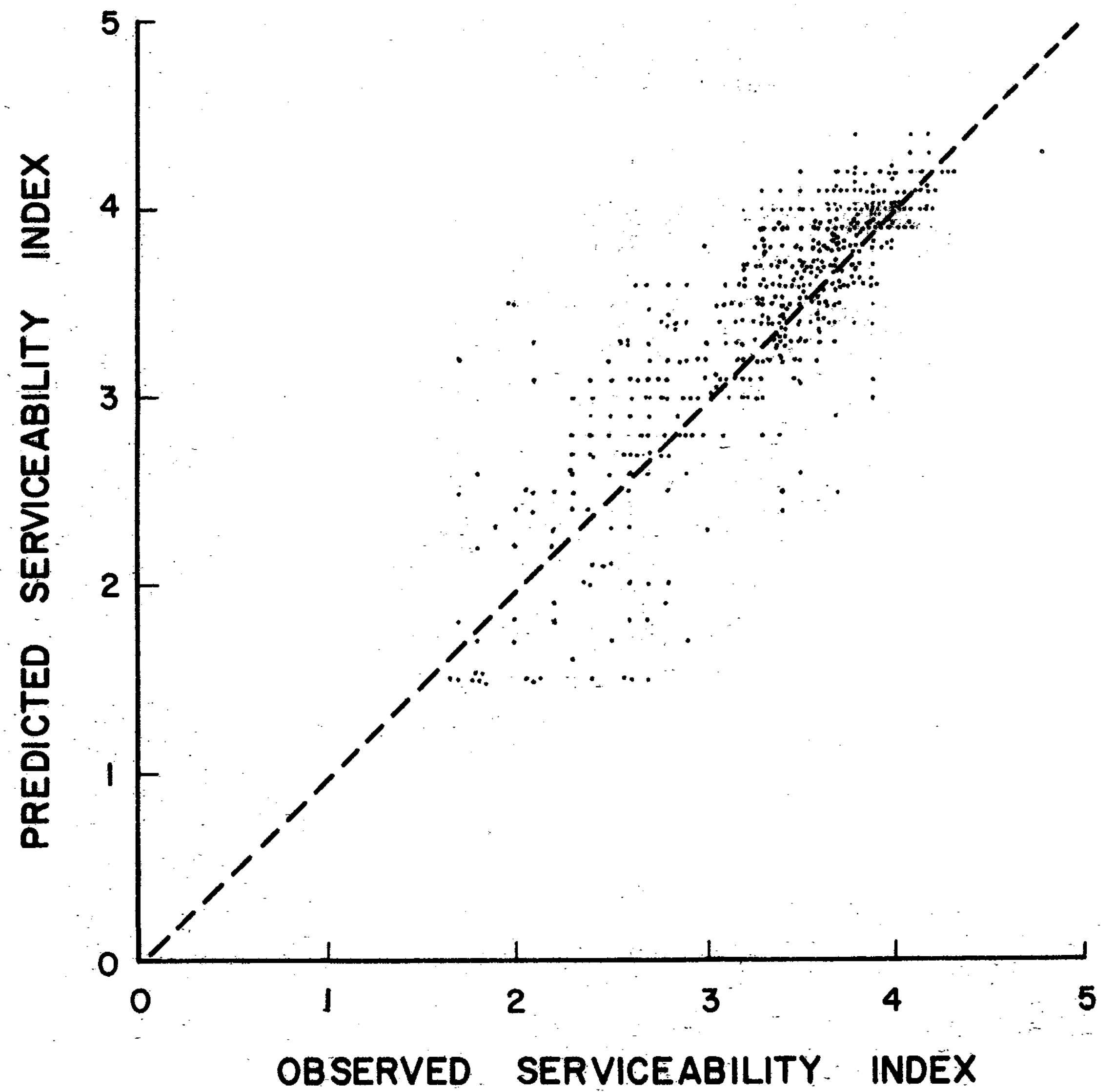

FIGURE 8 - Predictions of Serviceability Index from Equation 19, Compared with Observed Values. Data from AASHO Road Test.

index of a pair of replicate sections from the mean of the pair, which was 0.23 (see Table 11 of Reference 3).

4.3 Performance of a Pavement Subjected to an 18-Kip Single Axle Load

We now introduce the surface curvature index, S, measured by the Dynaflect and defined in Report 32-11 (1) by the equation

$$S = W_1 - W_2$$

where W_1 is the reading of Sensor No. 1 and W_2 is the reading of Sensor No. 2.

For the special case where S' is produced by an 18-kip single axle load, S and S' are related, according to extensive correlation studies (6), by the equation

$$S'_{18} = 20S \qquad (21)$$

where S'_{18} is the partial deflection caused by an 18-kip axle load.

By substituting the right side of Equation 21 for S' in Equation 19, we arrive at a performance equation applicable to the standard 18-kip axle load:

$$Q_2 = \frac{53.6\ (N_2 - N_1)S^2}{\bar{\bar{\alpha}}} \qquad (22)$$

where $N_2 - N_1$ = the number of 18-kip single axle loads applied during a period for which S is relatively constant,

S = the surface curvature index (mils) determined by the Dynaflect,

$\bar{\bar{\alpha}}$ = the harmonic mean of the daily values of α existing during the period (°F),

Q_2 = the serviceability loss resulting from the repeated application of an 18-kip single axle load = $\sqrt{5 - P_2} - \sqrt{5 - P_1}$,

P_1, P_2 = the serviceability index at the beginning and end of the period.

The right side of Equation 22 is the first term of the performance equation given in Research Report 32-11 (1).

The use of Equation 22 for predicting serviceability history over a typical year is restricted to regions where significant seasonal variations in deflections are not expected to occur. If they are expected to occur, then the year must be divided into r equal time intervals during each of which the surface curvature index, S, is assumed to be constant; then the following equation can be used:

$$Q_2 = 53.6 \ (N_2 - N_1)\left(\frac{S^2}{\alpha^2}\right) \tag{23}$$

where $N_2 - N_1$ = the number of 18-kip single axle loads applied during the year,

Q_2 = the serviceability loss resulting from the repeated application of an 18-kip axle load = $\sqrt{5 - P_2} - \sqrt{5 - P_1}$,

P_1, P_2 = the serviceability index at the beginning and end of the year,

$$\frac{S^2}{\alpha^2} = \frac{1}{r}\left[\frac{S_1{}^2}{\bar{\alpha}_1} + \frac{S_2{}^2}{\bar{\alpha}_2} + \ldots + \frac{S_r{}^2}{\bar{\alpha}_r}\right] \quad , \tag{24}$$

$S_1, S_2, \ldots, S_r$ = the expected value of S in the first, second, ..., r^{th} period of the year,

$\bar{\alpha}_1, \bar{\alpha}_2, \ldots, \bar{\alpha}_r$ = the expected harmonic mean of α for the first, second, ..., r^{th} period of the year,

r = the number of equal time intervals into which the year has been divided.

Equation 24 for S^2/α^2 was derived by a procedure paralleling that used in the derivation of Equation 15 for $\bar{\alpha}$.

An investigation of seasonal variations in deflections in Texas is underway in Phase 1 of Research Project 2-8-69-136. Meanwhile Equation 22, in which S is assumed to be constant throughout, is being used in the system for the design of flexible pavements described in Research Report 32-11 (1).

5. THE EFFECT OF SWELLING CLAYS

The performance equation introduced in Research Report 32-11 (1) included a term assumed to represent the effect on serviceability loss of differential foundation movements. The full equation is repeated below.

$$Q_2 = \frac{53.6\ (N_k - N_{k-1})S^2}{\bar{\alpha}} + Q_2'(1 - e^{-b_k(t_k-t_{k-1})}) \qquad (25)$$

The definitions of the variables appearing in Equation 25 are quoted below from Report 32-11. A performance period, a phrase occurring frequently in the definitions, is defined as the interval between the time of initial or overlay construction to the time the next overlay is required. The performance periods are numbered consecutively, the first period being the time interval between initial construction and the first overlay. Equation 25 applies to the k^{th} performance period.

Performance Variables -

t = time (years) since original construction.

P = the serviceability index at time, t.

P_1 = the expected maximum value of P, occurring only immediately after initial or overlay construction.

P_2 = the specified value of P at which an overlay will be applied.

P_2', swelling clay parameter = the assumed value of P at $t = \infty$, in the absence of traffic. In general, $0 \leq P_2' \leq P_1$.

b_k is a swelling clay parameter applying to the k^{th} performance period.

t_k = the value of t at the end of the k^{th} performance period, or the beginning of the next period. $t_o = 0$.

N_k = the value of N at the end of the k^{th} performance period. $N_o = 0$.

Q, the serviceability loss function,= $\sqrt{5 - P} - \sqrt{5 - P_1}$.

Q_2 = Q when P = P_2.

$Q_2' = \sqrt{5 - P_2'} - \sqrt{5 - P_1}$

α, a daily temperature constant = 1/2 (maximum daily temperature + minimum daily temperature) - 32°F.

$\bar{\alpha}$ = the effective value of α for a typical year in a given locality, defined by the formula for the harmonic mean--

$$\bar{\alpha} = \frac{n}{\sum_{i=1}^{n} \left(\frac{1}{\alpha_i}\right)}$$

where n is the number of days in a year, and α_i is the value of α for the i^{th} day of the year. To obtain an approximate value of $\bar{\alpha}$ for this report, the formula was used with n = 12, and α_i = the mean value of α for the i^{th} month averaged over a ten-year period.

5.1 The Swelling Clay Parameters b_1 and P_2'

As may be inferred from the definitions of b_k and Q_2' given above, the second term in Equation 25 represents the assumed effect of the swelling of foundation clays on the serviceability loss term, Q_2. To isolate this term and study its behavior, consider the case of a highway subjected to traffic so light that its effect on the pavement can be neglected. For this case we may set $N_k = N_{k-1} = 0$ in Equation 25, with the following result:

$$Q_2 = Q_2'(1 - e^{-b_k(t_k - t_{k-1})}) \qquad (26)$$

For the first performance period (k = 1) Equation 26 takes the following form:

$$Q_2 = Q_2'(1 - e^{-b_1 t_1}) \qquad (27)$$

By substituting Q for Q_2, and t for t_1 in Equation 27, we obtain the following equation for the serviceability loss at any time, t:

$$Q = Q_2'(1 - e^{-b_1 t}) \qquad (28)$$

By referring to the definitions of Q and Q_2', we see that Equation 28 can be written--

$$\sqrt{5 - P} - \sqrt{5 - P_1} = (\sqrt{5 - P_2'} - \sqrt{5 - P_1})(1 - e^{-b_1 t}) \qquad (28a)$$

By solving Equation 28a for P we obtain

$$P = 5 - [\sqrt{5 - P_1} - (\sqrt{5 - P_2'} - \sqrt{5 - P_1})(1 - e^{-b_1 t})]^2 \qquad (28b)$$

Equation 28b predicts the serviceability index, P, at any time, t, for a pavement subjected to traffic so light that its effect on P can be neglected.

If we put $t = 0$, Equation 28b reduces to $P = P_1$; that is, the equation correctly predicts the initial value of P. If we put $t = \infty$, the equation predicts that $P = P_2'$. If we differentiate the equation with respect to t, the result shows that the slope, dP/dt, is always negative, but approaches zero as t approaches infinity. It follows that P approaches P_2' asymptotically as t approaches infinity. These characteristics of Equation 28b are illustrated in Figure 9 where computed values of P have been plotted against time for selected values of P_1, P_2' and b_1.

The reader will note from Figure 9 that the value assigned to P_2' controls the ultimate value of the serviceability index, P, while the value assigned to b_1 controls the rate at which P approaches its ultimate value.

It is well known that some foundation clays in Texas adversely affect pavement serviceability, but quantitative data are lacking. It is recommended that research be initiated as soon as practicable with the aim of collecting such data so that accurate estimates can be made of the swelling clay parameters b_1 and P_2' for use in the design of pavements. Meanwhile, the assignment of values to these parameters in any particular case must remain a matter of judgement.

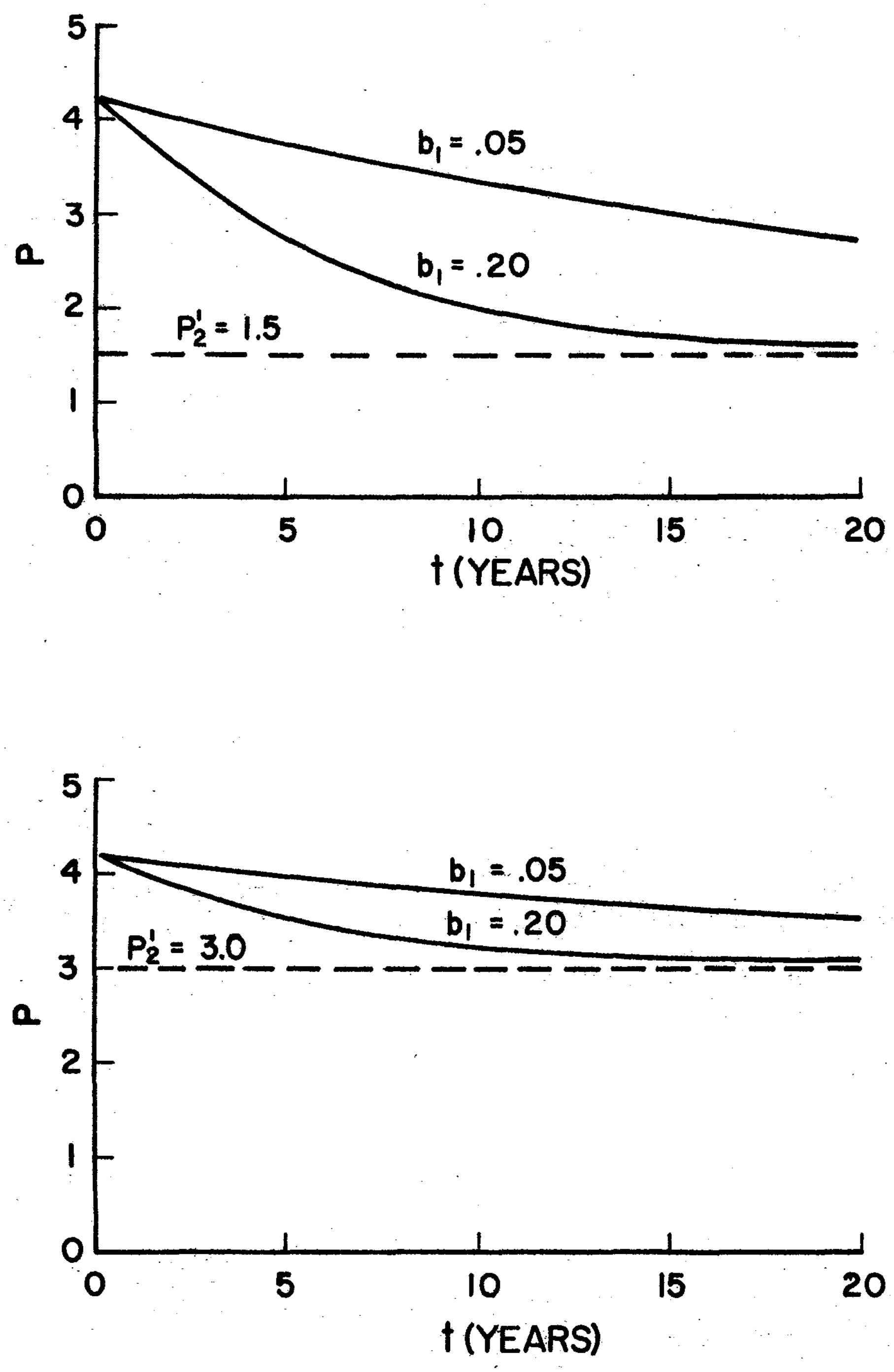

FIGURE 9 - Serviceability History Predicted by Equation 28B for a Pavement Carrying Very Light Traffic.

5.2 The Parameter b_k for k>1

In the previous section we discussed the parameters, b_1 and P_2'. We now turn to the parameter b_k for the general case, $k > 1$.

If Q is substituted for Q_2' and t for t_k in Equation 26, that equation takes the following form:

$$Q = Q_2'(1 - e^{-b_k(t - t_{k-1})}). \tag{29}$$

Equation 29 predicts the value of Q for any time, t, within the k^{th} performance period, for a highway subjected to traffic so light that its effect on the pavement may be neglected.

If we substitute $t = t_{k-1}$, Equation 29 predicts $Q = 0$, the correct value at the beginning of the k^{th} performance period. If we substitute $t = \infty$, the equation predicts $Q = Q_2'$. If we differentiate the equation, we find that dQ/dt is always positive, and that as t approaches infinity, dQ/dt approaches zero. Therefore, Q approaches Q_2' asymptotically as t approaches infinity. Figure 10 illustrates these characteristics of Equation 29, for selected values of t_{k-1}, b_k, P_1 and P_2'. These values were $t_{k-1} = 5$, $b_k = .20$, $P_1 = 4.2$ and $P_2' = 1.5$. The corresponding value of Q_2' was 0.976.

Suppose now that the pavement represented in Figure 10 is overlayed at t = 15 years. This situation is illustrated in Figure 11. It was necessary to form an hypothesis with regard to the equation for the second branch of the curve. The hypothesis is stated below:

> The slope, dQ_k/dt, at the end of period k (Point A in Figure 11) is equal to the slope, dQ_{k+1}/dt, at the beginning

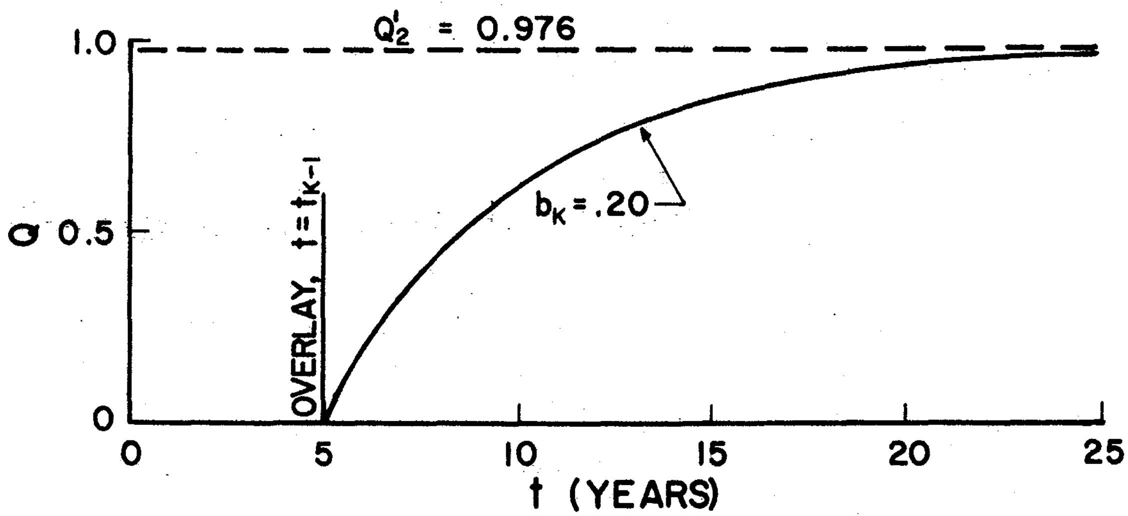

FIGURE 10 - Variation of Q with Time for a Pavement Carrying Very Light Traffic.

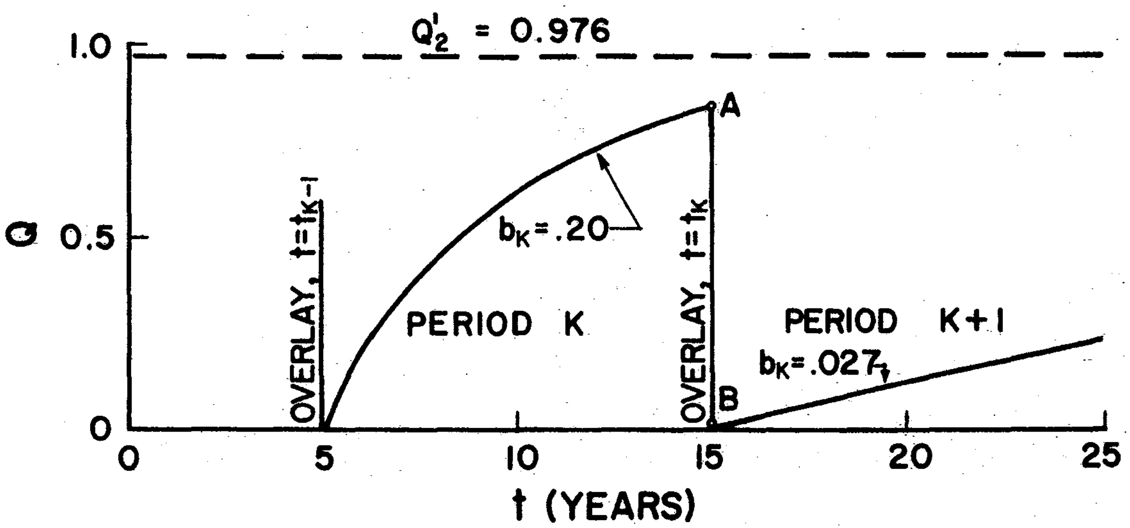

FIGURE 11 - Effect of an Overlay on the Q Versus Time Curve for a Pavement Carrying Very Light Traffic

of period k+1 (Point B in Figure 11), where Q_k is Q in the k^{th} performance period.

The above hypothesis leads to an equation for b_{k+1}, as shown below. The equation for Q in period k is, according to Equation 29,

$$Q_k = Q_2'(1 - e^{-b_k(t - t_{k-1})}) \tag{30}$$

The equation for Q in period k + 1 is

$$Q_{k+1} = Q_2' \ (1 - e^{-b_{k+1}(t - t_k)}) \tag{31}$$

By differentiating Equation 30, and substituting $t = t_k$, we find that

$$\left.\frac{dQ_k}{dt}\right]_{t=t_k} = b_k Q_2' e^{-b_k(t_k - t_{k-1})} . \tag{32}$$

Similarly, from Equation 29 we find that

$$\left.\frac{dQ_{k+1}}{dt}\right]_{t=t_k} = b_{k+1} Q_2' . \tag{33}$$

By equating the right sides of Equations 32 and 33 in accordance with our hypothesis, and then dividing by Q_2', we arrive at the following recurrence formula for b_{k+1}:

$$b_{k+1} = b_k e^{-b_k(t_k - t_{k-1})} \tag{34}$$

Thus, if b_1 is known, b_2 can be calculated from Equation 34. With b_2 known, b_3 can be calculated, etc.

It should be noted that although $b_{k+1} \neq b_k$, both Q_k and Q_{k+1} approach the same asymptote, Q_2', as t approaches infinity. This can be verified by substituting ∞ for t in Equations 30 and 31.

6. APPLICATION TO TEXAS TEST SECTIONS

A thorough testing of the performance equation (Equation 25) against Texas data would require the following data from each Texas test section:

(1) The initial serviceability index.

(2) One or more later measurements of the serviceability index.

(3) The elapsed time between the measurements.

(4) The equivalent number of 18-kip axle loads applied between the measurements.

(5) The surface curvature index, measured by Dynaflect.

(6) Mean daily temperature data for computing $\bar{\alpha}$.

(7) An estimate of the swelling clay parameters b_1 and P_2'.

6.1 Quality of the Texas Data

The lack of quantitative data required by the last item listed at the beginning of this chapter, the swelling clay parameters, has already been discussed.

The first item, the initial serviceability index, was measured in only a few cases, and the high degree of variability of the measurements made it clear that it was not possible to estimate the initial value for an individual section with any degree of confidence in the result. To document this point, comparisons of the serviceability index measured on relatively new and relatively old sections are given in Table 7. As may be seen from this table, the serviceability index of 16 relatively new sections surfaced with asphaltic concrete averaged 4.3, close to the average value of 4.2 measured at the AASHO Road Test. However, since the standard deviation was 0.4, one may conclude that about two-thirds of any group of new Texas pavements surfaced with asphaltic concrete will have a serviceability index ranging from 3.9 to 4.7, while the index for the remaining one-third will lie outside that range. Coupled with this unexpectedly high degree of variability was the fact that approximately 25% of the older asphaltic concrete sections summarized in Table 7 had a serviceability index exceeding 4.3, the average value for <u>new</u> sections.

Also shown in Table 7 are comparisons of new and old sections with surface treatments. It can be seen that the general level of the initial serviceability of these sections was significantly lower than for the

TABLE 7 - COMPARISONS OF THE SERVICEABILITY INDEX OF NEW AND OLD TEXAS SECTIONS

			Range in Traffic and Age		Serviceability Index		
Type of Surfacing	Age Class	No. Sections	Equivalent 18-Kip Axle Applications	Age (Years)	Range	Average	Standard Deviation
Asph. Conc.	New	16	0-25,000	0-1.4	3.6-4.8	4.3	0.4
	Old	178	26,000-1, 937,000	1.4-26.8	2.3-4.9	3.7	.6
Surf. Trt.	New	11	0-15,000	0-1.5	2.3-3.6	2.9	0.4
	Old	128	16,000-1,508,000	1.5-26.9	2.2-4.5	3.1	.4

asphaltic concrete sections. Since the average value and standard deviation were 2.9 and 0.4 respectively, it appears that about two-thirds of any group of new surface-treatment pavements will have a serviceability index ranging from 2.5 to 3.3, while the serviceability of the remaining one-third will lie outside that range. In addition, it was found that among the older sections well over two-thirds had a serviceability index exceeding 2.9, the average value of the new sections.

It is of interest to note that the difference of 0.5 in serviceability between new and old asphaltic concrete sections (Table 7) was found from an analysis of variance to be statistically highly significant, reflecting the already well known effect of time and traffic on the serviceability index. No such conclusion can be inferred from the surface treatment section data, since the apparent difference in the average serviceability index of new and old sections was found to be meaningless (statistically not significant).

Except for the initial serviceability index, which is lacking entirely, and the swelling clay parameters, about which little is known currently, the data described at the beginning of this chapter are assumed to be sufficiently accurate for use in an analysis.

6.2 Serviceability Index Trends, Texas Data

Three successive measurements of the serviceability index were made on 87 Texas sections surfaced with asphaltic concrete and on 48 surface treatment sections. The time between the first and second measurements averaged 2.1 years; the time between the second and third averaged 2.5 years. Thus, there were 174 opportunities to observe a change in the serviceability of the asphaltic concrete sections and 96 opportunities to observe a change in the surface treatment sections.

Among the asphaltic concrete sections, losses and gains were nearly evenly divided: there were 88 losses and 86 gains. On the other hand among the surface treatment sections, gains in serviceability predominated: there were 35 losses and 61 gains.

Among the asphaltic concrete sections the change in serviceability ranged from a loss of 1.68 to a gain of 0.98, the average change being a loss of 0.05. Among the surface treatment sections the change ranged from a loss of 0.53 to a gain of 1.26, the average being a gain of 0.13.

With the number of gains in serviceability index equalling or exceeding the number of losses, it seems that (1) the time between successive measurements was too short to allow the development of significant trends, or (2) routine maintenance of the test sections prevented the development of significant trends, or (3) measurement errors masked the actual trends.

Whatever the reason, a consistent downward trend in serviceability index was not observed on the Texas sections in the 4.6-year period over which they were observed.

6.3 Predictions of the Performance Equation Compared with Texas Data

Faced with the lack of initial serviceability index data, but with three later measurements of the serviceability index on hand for the 135 sections treated in the preceeding section, the researchers followed the step-by-step procedure outlined below for each Texas section, for the purpose of studying the prediction characteristics of Equation 25 when applied in Texas:

(1) Using air temperature data from a centrally located weather station in the Texas Highway Department's District within which the section was located, a value of $\bar{\alpha}$ was estimated from Equation 20 for a typical year by setting n = 12 and defining α_i as the mean value of α for the i^{th} month averaged over a ten year period. Table 8 gives the estimate of $\bar{\alpha}$ for each District.

(2) If the test section had been overlayed prior to the time it was originally selected for study, a value of .0195 was assigned to b_1. If the section had not been overlayed, a value of .025 was assigned to b_1.

(3) A value of 1.5 was assigned to P_2' for all sections.

(4) The value of N at the time of the first measurement of the serviceability index was obtained from the Planning Survey Division of the Texas Highway Department.

(5) The value of N at the time of the second and third measurements of serviceability were estimated by extrapolation, assuming that N was directly proportional to the time, t.

(6) Since no consistent trends in the serviceability index had been observed, the P, N and t data for the three measurements of serviceability

TABLE 8 - DISTRICT VALUES OF $\bar{\alpha}$

Dist.	Temp. Const. ($\bar{\alpha}$)	Dist.	Temp. Const. ($\bar{\alpha}$)	Dist.	Temp. Const. ($\bar{\alpha}$)	Dist.	Temp. Const. ($\bar{\alpha}$)	Dist.	Temp. Const. ($\bar{\alpha}$)
1	21	6	23	11	28	16	36	21	38
2	22	7	26	12	33	17	30	22	31
3	22	8	26	13	33	18	26	23	25
4	9	9	28	14	31	19	25	24	24
5	16	10	24	15	31	20	32	25	19

index were averaged to obtain a single data point with the coordinates $\bar{P}$, $\bar{N}$ and $\bar{t}$.

(7) In Equation 25 $\bar{P}$ was substituted for P_2, $\bar{N}$ for N_2, $\bar{t}$ for t_2, zero for N_1, zero for t_1, 1.5 for P_2', either .0195 or .025 for b_1, and the District value from Table 8 for $\bar{\alpha}$. The value of the surface curvature index measured by Dynaflect in 1964 was substituted for S. The equation was then solved for P_1, which was the <u>predicted value of the initial serviceability index</u>.

(8) In Equation 25, the initial serviceability index computed in Step 7 was substituted for P_1, N and t data for the third measurement of serviceability index were substituted for N_2 and t_2, and the equation was solved for P_2, which was the <u>predicted value of the final serviceability index</u>, designated $\hat{P}_3$. The prediction error, $E_3 = P_3 - \hat{P}_3$, was then computed, where P_3 was the third observed value of the serviceability index.

The effect of the computations described in the preceeding eight paragraphs was to pass a theoretical serviceability--time curve through the mean data point as illustrated in Figure 12, which shows data for Section 282-2-2 in District 3.

In the case of some sections it was not possible to pass the theoretical serviceability history curve through the mean data point because of the theoretical upper limit of 5 imposed on the serviceability index in the performance equation. In such cases the theoretical curve, if started at $P = 5$ when $t = 0$, would have passed <u>below</u> the mean data point. This usually happened for sections having an unusually high value of S combined with large values of $\bar{N}$ and $\bar{t}$ and a low value of $\bar{\alpha}$. Of the 135 sections

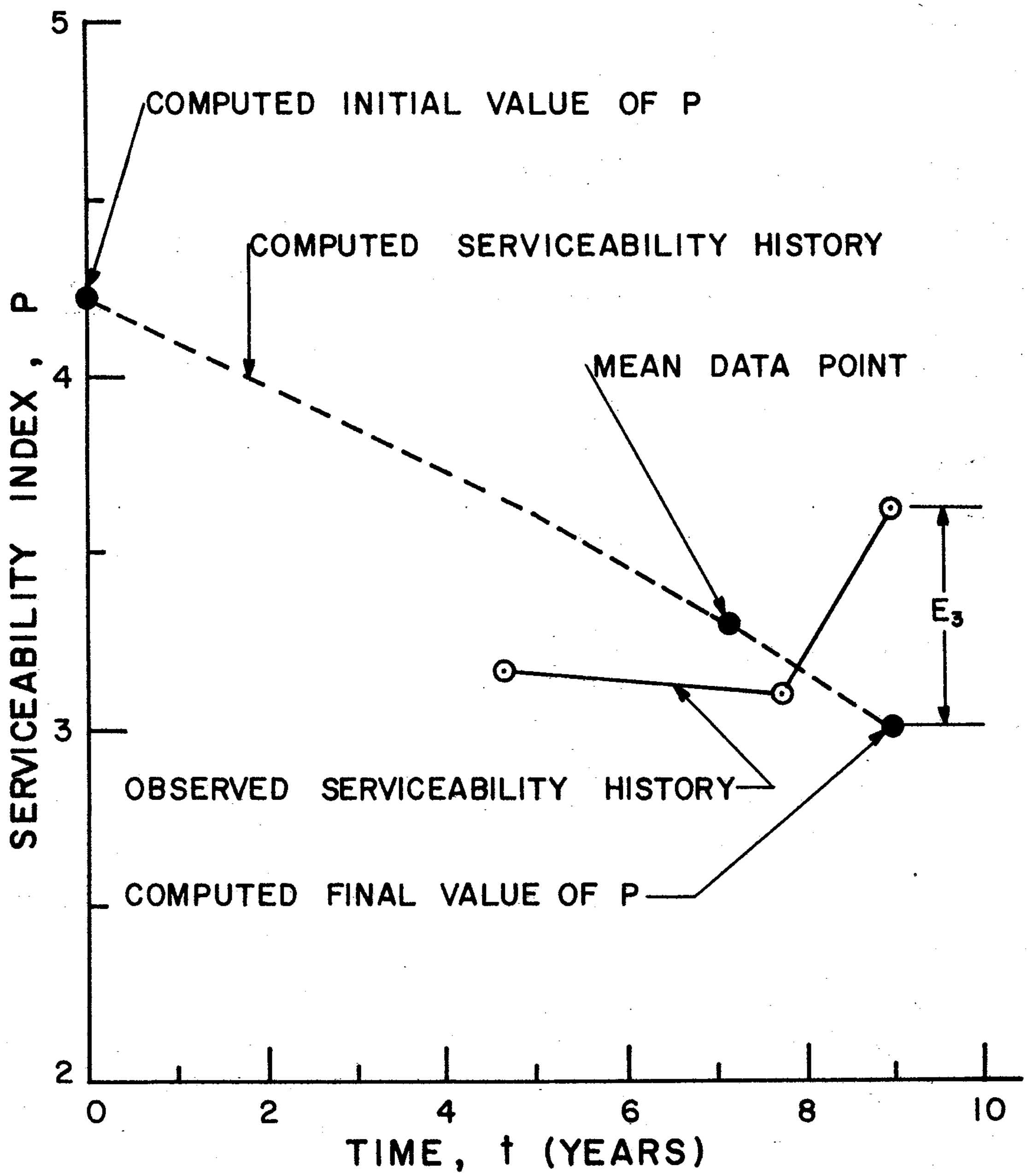

FIGURE 12 - Predicted and Observed Serviceability Histories of Texas Section 282-2-2 in District 3.

investigated, the curve could not be passed through the mean data point of 20 sections, or 15% of the total. For these 20 sections estimates of the initial serviceability index and of the final prediction error, E_3, could not be made.

Among the 87 asphaltic concrete sections, values of the initial serviceability index and the prediction error, E_3, were computed for 74 sections. Similar computations were made for 41 surface treatment sections.

The range, average and standard deviation of predicted values of initial serviceability are given in Table 9. Also shown in this table for comparison are the measured results given earlier in Table 7. It can be concluded from Table 9 that the distribution of predicted and measured values was practically the same for the asphaltic concrete sections. In the case of the surface treatment sections, however, the average predicted value was much higher than the average measured value.

A study of the prediction error, E_3, in the case of the 74 asphaltic concrete sections showed that the predicted value of the final serviceability index was greater than the observed value in 17 cases (23%) and less than the observed value in 57 cases (77%). The value of E_3 ranged from -0.85 to +0.76 and had a mean value of 0.16. From these results it is concluded that predictions of the performance equation are conservative--that is, "on the safe side"--more often than not.

In the case of the 41 surface treatment sections, a study of the prediction error, E_3, showed that the predicted value of the final service-

TABLE 9 - PREDICTED VERSUS MEASURED VALUES OF INITIAL SERVICEABILITY INDEX

Type of Surfacing	Pred. or Measured	No. Sections	Range	Average	Standard Deviation
Asph. Concr.	Measured*	16	3.6-4.8	4.3	0.4
	Predicted	74	3.4-5.0	4.3	0.4
Surf. Trt.	Measured*	11	2.3-3.6	2.9	0.4
	Predicted	41	3.1-4.6	3.9	0.5

*From Table 7.

ability index was greater than the observed value in only 2 cases (5%) and less than the observed value in 39 cases (95%). The value of E_3 ranged from -0.05 to +1.12 and had a mean value of 0.31. These results, together with the data presented in Table 9, indicate that predictions of the performance equation are definitely "on the safe side" when applied to surface treatment sections.

CHAPTER 7 - CONCLUSIONS

The following conclusions were drawn from the data presented in this report:

1. The first term of the performance equation (Equation 25) reproduces AASHO Road Test performance data as well as could be expected, considering the replication error measured at the Road Test. The prediction error (average absolute residual) was 0.28; the replication error was 0.23.

2. The second term of the performance equation, assumed to represent the effect of foundation movements caused by changes in moisture content, can be validated only through further research; meanwhile the assignment of values to the parameters in that term must depend on experience and judgement.

3. The predictions of the performance equation, when checked against Texas data, appear to be conservative, i.e. "on the safe side."

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APPENDIX - EFFECT OF TEMPERATURE ON THE STIFFNESS COEFFICIENT OF ASPHALTIC CONCRETE USED ON THE A&M PAVEMENT TEST FACILITY

It was indicated in Research Report 32-12 (Page 19 of Reference 4) that a rough estimate of the stiffness coefficient, a_1, of a material could be made by dividing 10,000 into the wave velocity (ft./sec.) of the material, measured in situ. In-place measurements of the asphaltic concrete used on the A&M Pavement Test Facility were not made; however, measurements on laboratory compacted specimens were made at temperatures ranging from 40°F to 140°F. The measured velocities are plotted against temperature in Figure 13 (5).

The stiffness coefficient of the asphaltic concrete surfacing, determined from Dynaflect measurements made at an average air temperature of 62°F, was 0.52, corresponding to an in situ wave velcity of 5200 ft/sec. (3).

As can be seen by reference to Figure 13, the laboratory values of wave velocity were considerably higher than the estimated in situ value of 5200 ft/sec, throughout the range of temperatures investigated in the laboratory. Nevertheless, for the purpose of estimating the effect of temperature on the stiffness coefficient of this material, it was assumed that the derivative dV/dT (where V is the laboratory determined wave velocity in feet per second) was equal to 10,000 times the derivative da_1/dT, where a_1 is the stiffness coefficient estimated from Dynaflect measurements. This assumption is expressed by the following equation:

$$\frac{da_1}{dT} = \frac{dV}{dT} \times 10^{-4} \qquad (35)$$

The average slope of the three curves in Figure 13, within the interval

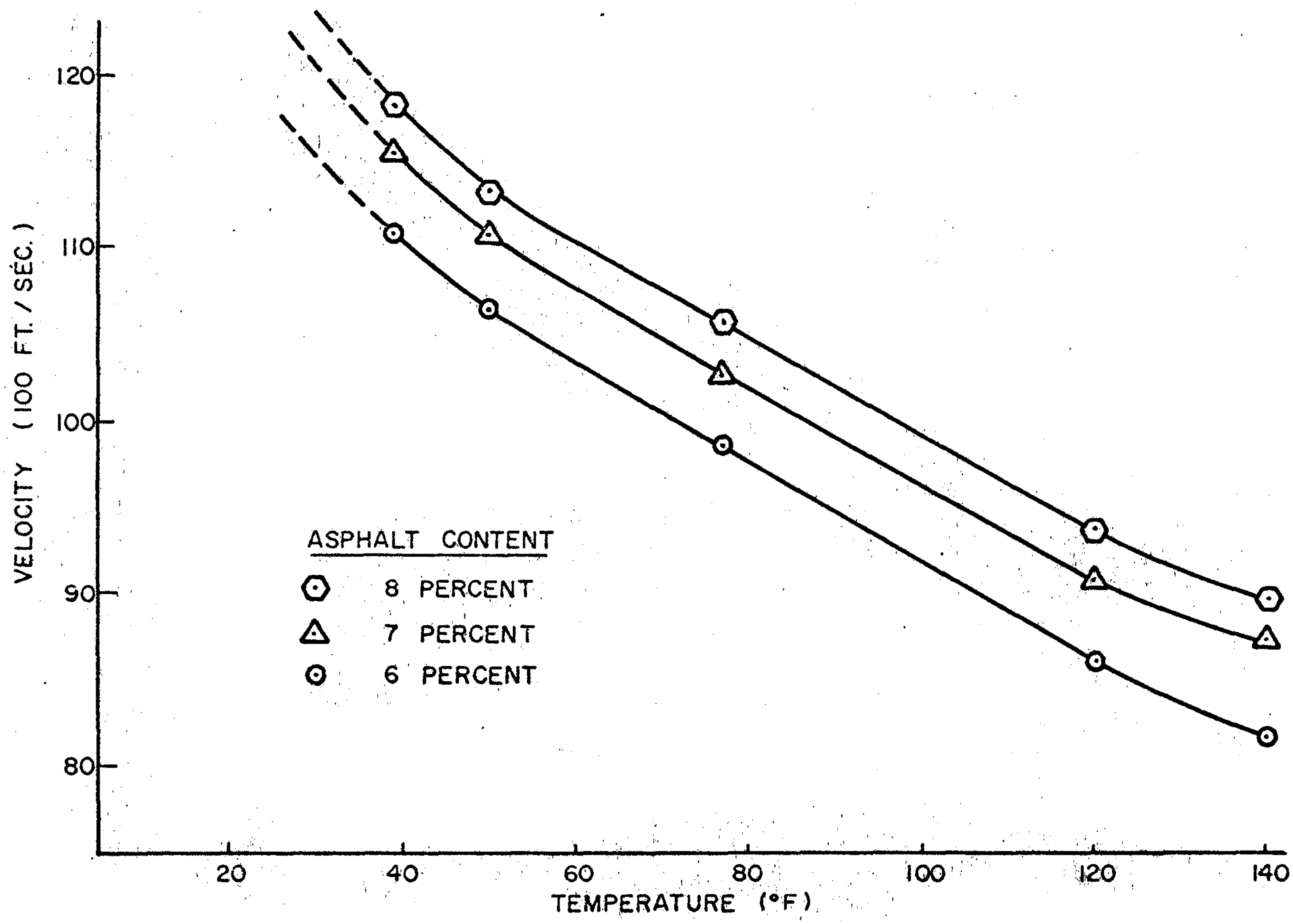

FIGURE 13 - Effect of Temperature on Wave Velocity of Laboratory Compacted Specimens of Asphaltic Concrete (after Figure 22 of Reference 5).

$40 \leq T \leq 140$, was estimated to be -28.4 feet per second per degree F. change in temperature. Thus, according to Equation 35

$$\frac{da_1}{dT} = -.00284 \qquad (36)$$

Assuming that the average air temperature of 62°F existing during the Dynaflect measurement program at the A&M Pavement Test Facility was equal to the average temperature of the asphaltic concrete, we have the following condition to be satisfied by the integral of Equation 36:

$$a_1 = 0.52, \text{ when } T = 62.$$

It can be seen by inspection that Equation 17, below, satisfies this condition, and also satisfies Equation 36.

$$a_1 = 0.52 + 0.00284(62 - T) \qquad (17)$$

Equation 17 appears in Section 3.1 of this report.