

INVESTIGATION OF AN INTERNAL ENERGY MODEL
FOR EVALUATING
FREEWAY LEVEL OF SERVICE

by

Donald R. Drew
Assistant Research Engineer

and

Conrad L. Dudek
Research Assistant

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PREFACE

For the past several years researchers have attempted to describe "level of service" in both qualitative and quantitative terms. They have struggled with data taken at a given point, or series of points, or from short sections of roadway. Since "point survey" data are merely a volume-speed history at that point, and because of the dampening effects of averaging over a time period, these data become extremely difficult to interpret.

Travel time, the "old faithful" of traffic parameters, is not a good indicator of operating conditions throughout a trip unless delays are quite large or quite frequent.

There is a need to establish parameters that: (1) reflect the quality of the trip or "level of service" afforded the motorist, and (2) are predictable for a given set of conditions.

The energy-acceleration noise model is an attempt to apply a quantitative value to the many of the qualitative characteristics of traffic flow considered in the "level of service" concept. Also an attempt is made to find a much needed quantitative value for measuring the effect of geometrics on the quality of traffic flow.

As far back as 1948, the late Thomas H. MacDonald, former Commissioner, Bureau of Public Roads and later one of the founders of the Texas Transportation Institute, emphasized the need for correlating design and operation of traffic facilities. In the Beecroft Memorial Lecture to the Society of Automotive Engineers, Mr. MacDonald said:

"We have reached the point in our knowledge of the manner in which highways are used by the mass of traffic, to coordinate driver behavior under prevailing traffic conditions, and the geometric details of highway design. The degree to which the criteria so determined are accepted and intelligently applied in practice will determine the degree of safety efficiency of our future highways."

This correlation of traffic behavior and design is the key to the Level of Service concept.

This report follows the pattern characteristic of research reports of the Texas Transportation Institute, i.e., development of theoretical concepts, verification through experimentation and finally a discussion of the application of the findings.

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INTRODUCTION

The Problem

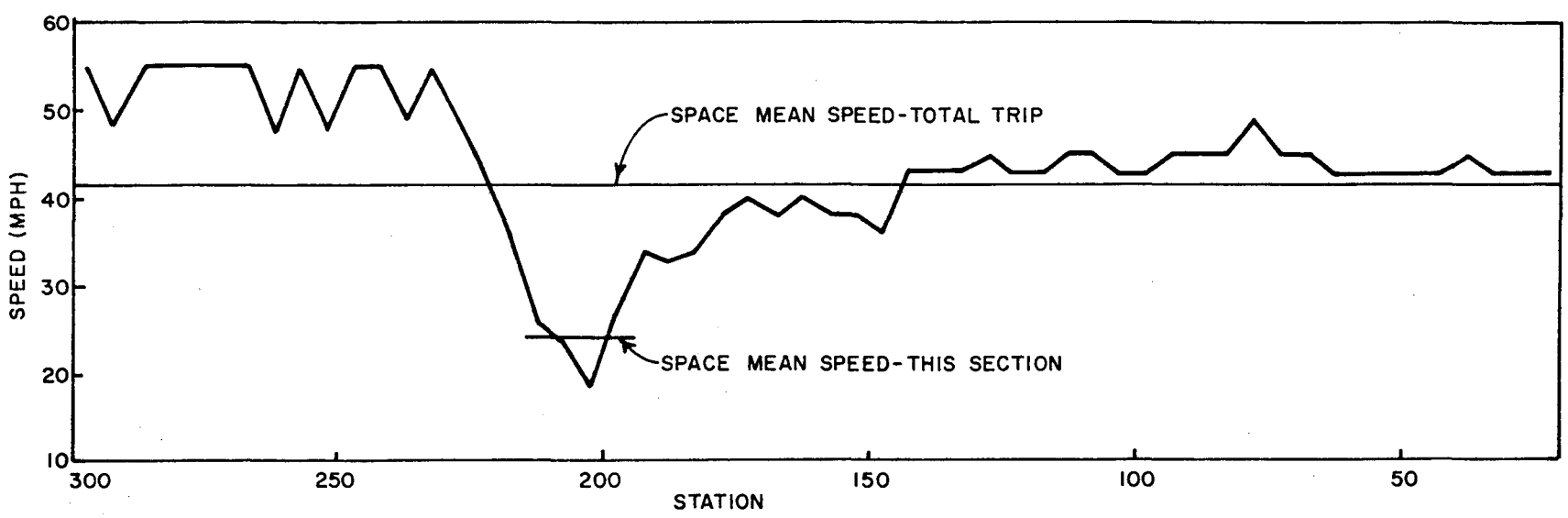
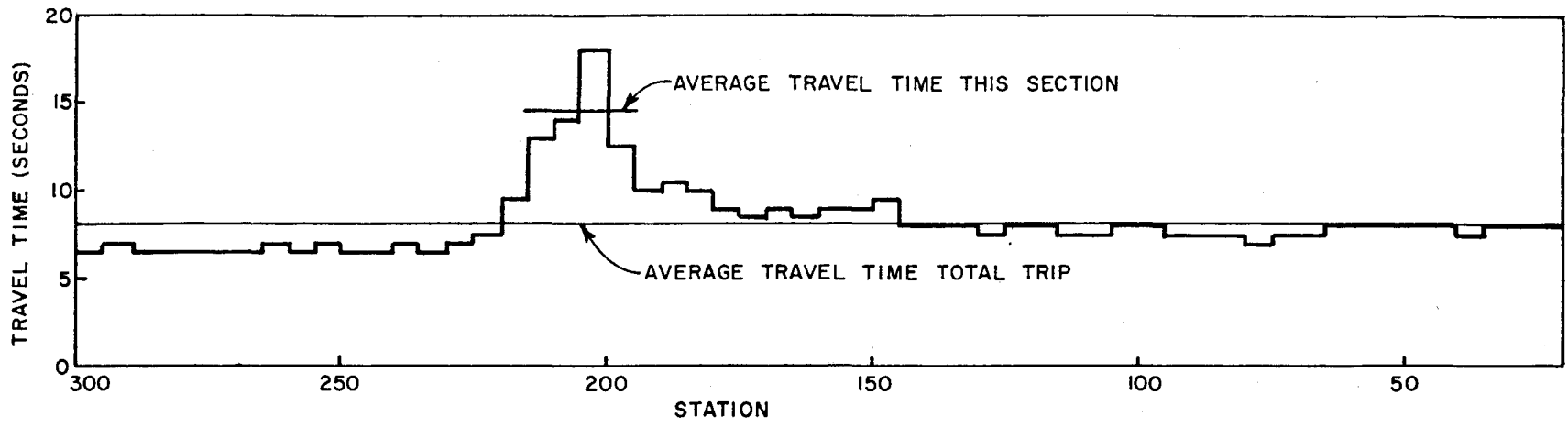
An evaluation of any change in operation, design, or system analysis requires a thorough study of those traffic parameters which will reflect changes in traffic flow not only quantitatively but also qualitatively. Drew¹ has categorized the traffic parameters of volume, density, speed, and queueing of vehicles on the Gulf Freeway to present a complete description of the quantity of flow on the facility. Wattleworth² has used a systems approach in his description of the quantity of flow in terms of capacity. The measurement of other similar parameters such as travel time and delay, to complete the quantitative analysis, present no immediate problem. The need exists, however, for means of qualitatively evaluating controls, geometric design, and system operation which can be expressed in quantitative terms and which is correlated to known measurable traffic parameters. Geometric design, for instance is largely dependent upon pure judgment since there are no positive methods of comparing the effects of various designs on the quality of flow, or level of service.

"Level of Service," as applied to the traffic operation on a particular roadway, refers to the quality of driving conditions afforded a motorist by a particular facility. Factors which are involved in the level of service are: (1) speed and travel time, (2) traffic interruption, (3) freedom to maneuver, (4) safety, (5) driving comfort and convenience, and (6) vehicular operating costs.

Many approaches to the problem of measuring level of service provided to the motorists have been introduced in an attempt to obtain a qualitative measurement of traffic flow. A universal standard has not yet been adopted, however, perhaps due to the uncertainty of what traffic parameters should be measured that truly indicate the quality of flow.

The need for an objective evaluation of the level of service on a facility has long been recognized. Travel time on a roadway has often been used as a criterion for this evaluation.^{3,4,5} However, travel time may not always reflect the true conditions on parts of a facility and therefore may not always indicate the congestion and discomfort experienced by the motorist.

For example, Figure 1 represents a travel time and speed pattern between successive 500-foot sections of the inbound Gulf Freeway for one trip made near the end of the peak period. The figure helps to illustrate the delay in one section of the Freeway which is somewhat concealed by the value of the average travel time. The average speed for the entire trip was 41.5 mph which indicates good flow. However, it is noticed that a speed differential of more than 30 miles per hour (mph) existed between some of the sections.



COMPARISON OF TRAVEL TIME AND SPEEDS
WITHIN SECTIONS TO TOTAL TRIP

FIGURE 1

Also, the speed in one section of the freeway averaged only 24 mph and dropped below 20 mph. Therefore, the congestion in this section is dampened by the smooth flow in the remaining portions of the freeway when the flow is evaluated on the basis of travel time.

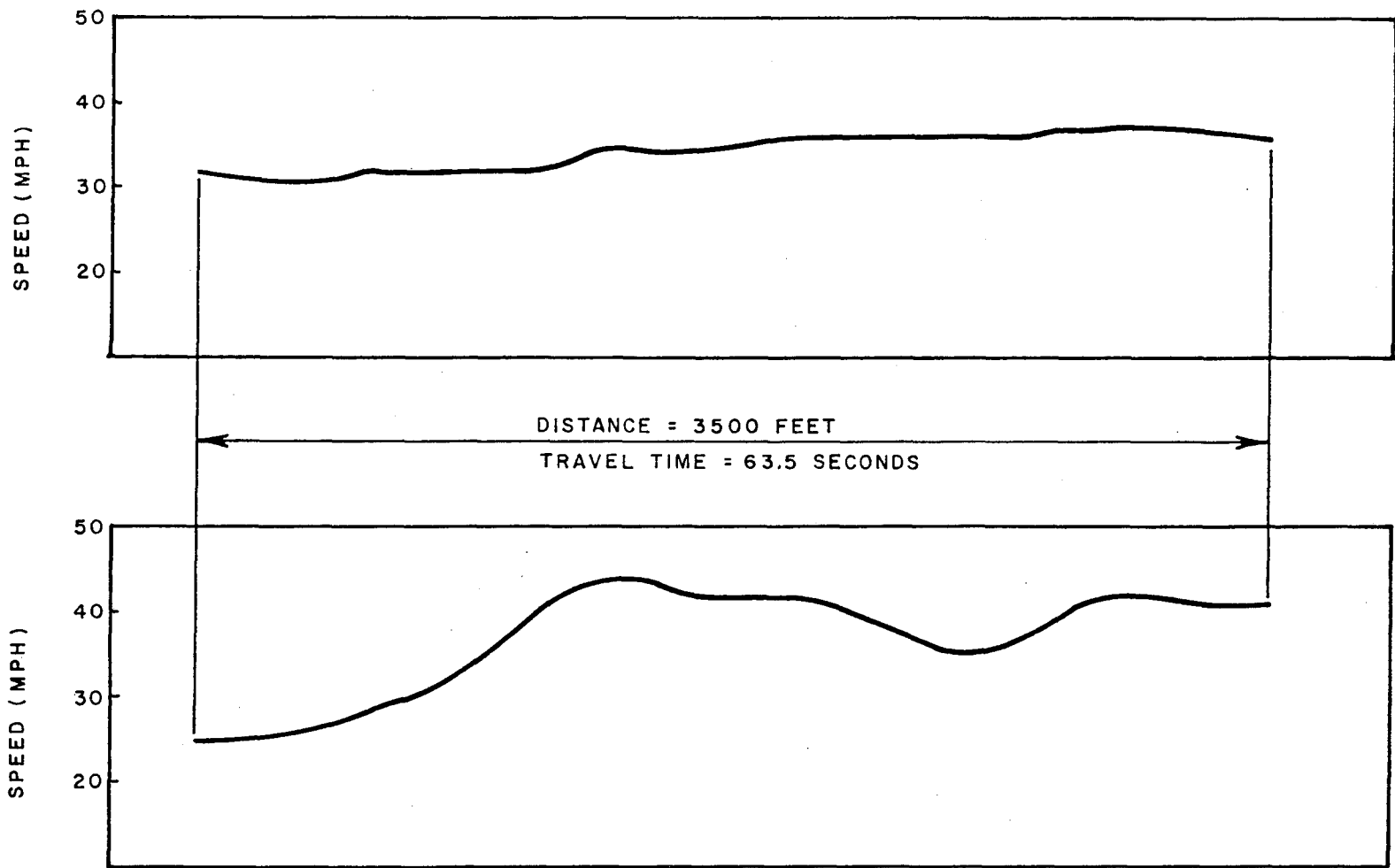
The fact that the discomfort experienced by the motorist is not necessarily reflected in travel time is further exemplified by Figure 2. This figure represents a comparison of a section of the Gulf Freeway 3500 feet in length, with identical travel times but with two distinct patterns of speed. The upper chart shows relatively smooth flow; however, the lower chart illustrates rapid decelerations which are indicative of motorist discomfort. Violent braking operations indicate dangerous conditions on the facility and add to the annoyance of the motorist, yet these maneuvers are not necessarily reflected by travel time as is illustrated in Figure 2.

The criterion of a "systems" approach⁶ to increase the efficiency of traffic flow on a facility (or facilities) is based primarily on maximizing the system output. Since maximizing the volume output rate is equivalent to minimizing the travel time within the system, travel time is also used as the measurement of service provided to the motorist.

With a relatively large system and high volumes a small saving in travel time per vehicle, after geometric improvements have been made or during some control procedure, may result in a substantial reduction in travel time within a system. But will the reduction in travel time be appraised equally by both the motorist and the traffic engineer? For example, a saving of two minutes per vehicle within a large closed system may reflect considerable improvement to the traffic engineer. However, this saving may not be noticed by the average motorist because this may only represent a small percentage of his over-all travel time. Therefore, unless a noticeable reduction of his travel time is provided, the motorist may very well conclude that the expenditures for the improvements or control system were not justified. However, if he were also provided with a smoother operation in addition to a saving in travel time, it most likely would be apparent to him.

There is evidence that the motorist personally evaluates a facility by the speed at which he can operate his vehicle and also by the uniformity of speed. Greenshields^{7,8} proposed a quality index with these factors in mind. His contentions in the development of the quality of Flow Index are that the over-all speed determines travel time and is therefore proportional to the quality of flow. Also, the amount of speed changes and the frequency of speed changes are undesirable factors which irritate the motorist, increase the cost of operation, and are therefore inversely proportional to the quality of flow. The Quality of Flow Index as formulated by Greenshields is

$$Q = KS / \Delta s \sqrt{f}$$



COMPARISON OF SPEED PROFILES OF A SECTION OF FREEWAY HAVING IDENTICAL TRAVEL TIMES

FIGURE 2

where S = average speed (mph)

Δs = absolute sums of speed changes per mile

f = number of speed changes per mile

K = constant of 1000

Platt⁹ observed that traffic delays and traffic control devices are more annoying to the driver than slow moving traffic because they cause the motorist to stop. Also it was established that driver satisfaction and driver effort do not vary linearly with speed but vary as complex functions. For these reasons two additional terms were added by Platt to Greenshields' Quality of Flow Index. Some of the factors measured in the additional terms were speed change rate, steering reversal rate, accelerator reversal rate and brake application rate.

The Highway Capacity Committee of the Highway Research Board has proposed six levels of service as a basis of design. The basic speed-volume curve is divided arbitrarily into six levels of service as related to volumes and freedom to maneuver.

The aforementioned indices developed by Greenshields and Platt seem to possess excellent attributes for evaluating the over-all efficiency of a long stretch of highway. However, it has been established that the varying nature of the geometrics is accompanied by a varied degree of optimum speed, volume, and density for short successive sections of freeway.¹ If the parameter measuring the quality of flow is to be related to these known quantitative parameters, it must reflect the qualitative efficiency of small roadway sections. This parameter must not only reflect the engineer's evaluation of level of service but, more important, that measure of service which is considered by the individual motorist.

The Parameter

Because of recent research results, a traffic parameter referred to as acceleration noise has received attention as a possible measurement of traffic flow quality for two basic reasons. First of all, it is dependent upon the three basic elements of the traffic stream, namely: (1) the driver, (2) the road, and (3) the traffic condition. Secondly, it is, in effect, a measurement of the smoothness of flow in a traffic stream. A presentation of the history, definition and manifestations of this parameter is made in the following paragraphs.

Definition and History. On a given highway with traffic volumes so light as to not restrict his maneuverability, a motorist normally attempts to drive at a uniform and comfortable speed. Unconsciously, however, he accelerates and decelerates occasionally and deviates from a uniform speed throughout his journey. When traffic volumes have increased to a level which restricts his desirable speed, the motorist is forced to perhaps change lanes and

increase speed to overcome slower moving vehicles resulting in higher and more frequent deviations from a uniform speed. The accelerations during his trip can be considered random components of time and the acceleration distributions essentially follow a normal distribution.¹⁰ The smoothness of his journey can be described by the amount that the individual random accelerations disperse about the mean acceleration. This deviation is measured by determining the standard deviation (also referred to as the root-mean-square deviation) of the accelerations.

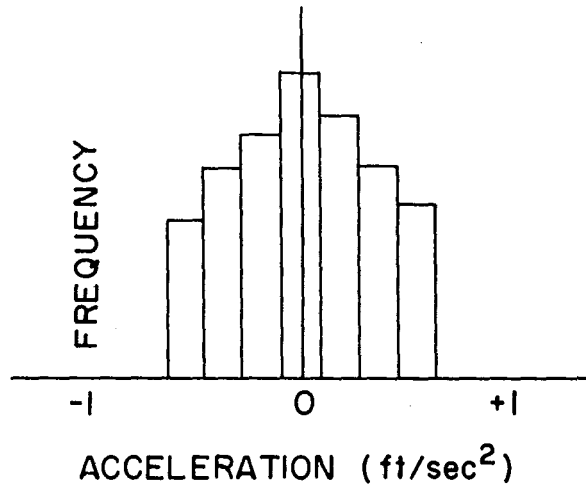
The standard deviation of the accelerations is called acceleration noise. This parameter can be considered as the disturbance of the vehicle's speed from a uniform speed or can be identified as a measurement of the smoothness of traffic flow. The term "noise" is used to indicate that disturbance of the flow comparable to the coined phrase "video noise" which is used to describe the fluttering of the video signal on a television set.

The accelerations of a vehicle can either be measured directly by an accelerometer or approximated from a speed-time graph of the vehicle's trip. This latter method is fully explained later in the report.

The distribution of the accelerations of a vehicle which has been driven at almost a uniform speed throughout its trip can be identified as being similar to that shown in Figure 3. A vehicle experiencing higher deviations from a uniform speed might result in an acceleration distribution similar to that presented in Figure 4. Acceleration noise then varies with the amount and frequency of acceleration and deceleration. The more violent and more frequent that these maneuvers are, the higher is the noise. Since violent deceleration seriously affects the acceleration noise, this parameter also reflects potentially dangerous traffic conditions.

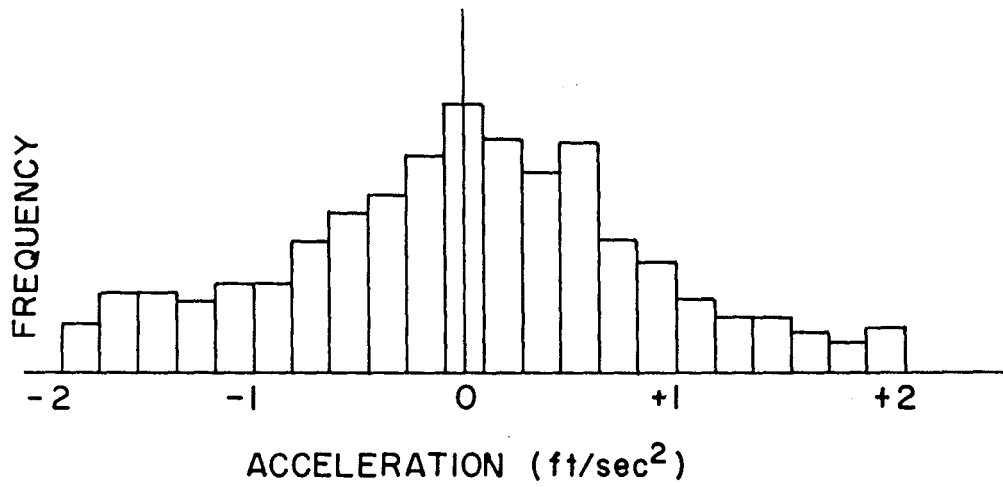
The concept of acceleration noise was first introduced in conjunction with the car-following equations.^{11,12} Additional investigations by Jones and Potts¹³ brought rise to its possible usefulness in helping to evaluate the quality of traffic flow in quantitative terms. The mathematical equation for approximating acceleration noise is derived in Appendix A. The significance of this measurement as mentioned before is that it gives a qualitative comparison of the factors which form the basic elements of the traffic stream, namely: (1) the driver, (2) the road, and (3) the traffic condition. When these factors increase the acceleration and deceleration of the vehicle, the acceleration noise increases.

Driver - One of the most critical factors which affects the characteristics of traffic flow is the driver. However, because of the many external



DISTRIBUTION OF ACCELERATIONS WITH
MINOR DEVIATIONS

FIGURE 3



DISTRIBUTION OF ACCELERATIONS WITH
LARGER DEVIATIONS

FIGURE 4

and internal elements which affect his decisions,¹⁴ evaluation of his behavior in a traffic stream presents a most complex problem.

Acceleration noise seems to be a useful parameter in helping to evaluate the behavior of various drivers in a traffic stream in terms of danger potential. Since this parameter is affected by acceleration and deceleration, a reckless driver who attempts to drive faster than the stream of traffic will accelerate and decelerate violently and perhaps frequently and will experience a much larger acceleration noise than the motorist that is content (or patient) with present flow conditions.

Road - It might be conjectured that if a motorist was able to operate a vehicle on a perfect roadbed without the influence of traffic, the acceleration noise would be zero. However tests¹¹ conducted using four drivers attempting to maintain constant speeds between 20 and 60 mph on the General Motors proving grounds resulted in acceleration noise of 0.32 ft/sec². It is quite obvious then that although a motorist desires to maintain a constant speed with ideal conditions, he unconsciously is unsuccessful.

Other experiments¹¹ using the same drivers and without the influence of traffic but on roads with varied geometric design produced an increased acceleration noise. The noise determined from runs in the Holland Tunnel of the New York Port Authority was 0.73 ft/sec² and preliminary runs on poorly surfaced winding country roads was even twice this amount. The increase of noise in the tunnel was ascribed to the narrow lanes, artificial lighting and confined conditions. Investigations on winding country roads by Jones and Potts¹³ showed values between 0.79 and 1.41. The acceleration noise decreased with improved road design features in the latter studies.

The acceleration noise which is present on a road in the absence of traffic is called the natural noise of the driver on the road.¹¹ Based on the above findings it might be rationalized that the acceleration noise experienced on a road in the absence of traffic, is some factor times the natural noise which would be experienced on a perfect roadbed. The factor can be ascribed basically to geometrics of the facility.

Comparison of acceleration noise on similar roadways might establish whether or not the intrinsic design features of the facilities have equal effects on the flow of traffic. In other words, a comparison of acceleration noise could perhaps reflect the effects to the smoothness of flow due to the difference in geometric design of the similarly classed roads to the quality of flow. One might expect a higher level of service from the facility with the lowest acceleration noise.

Of equal significance is the attribute which this parameter seems to possess in comparing the smoothness of flow before and after changes in the geometric configuration of a facility. It has been established that each roadway

exhibits natural noise in an amount greater than that of a near perfect roadbed in relation to its intrinsic design features. Any improvement in design could perhaps be evaluated qualitatively by determining the amount that the acceleration noise approaches the noise of the ideal roadway. In other words, an attempt might be made to minimize acceleration noise in design procedures if this parameter shows signs of merit.

Traffic Condition - The third factor which influences the amount of acceleration noise is congestion. Results of preliminary studies¹³ on suburban highways showed that the noise increased when congestion increased due to higher volumes and influence of parked cars. The amount of acceleration noise in excess of the natural noise of the facility was basically due to the existing traffic condition.

Reiterating, although stopped time or delay is a popular parameter used in evaluation congestion, it is not necessarily a satisfactory means of evaluating congestion. Results of studies¹³ on a suburban road passing through a shopping center which incidentally experienced a high accident rate (80 accidents per million vehicle miles) showed a stopped time of only 7 seconds in an over-all travel time of 128 seconds. But the acceleration noise during the peak period was 1.43 compared to 0.77 ft/sec² during the off-peak period. The latter comparison no doubt gives a better indication of the degree of congestion.

The Model

One of the most difficult tasks facing the traffic researcher is the translation of a real traffic problem situation, comprising drivers, vehicles, control devices and the highway, into a set of mathematical symbols and relationships that reproduces their behavior. The fundamental conceptual device which enables one to regard this interaction as a whole is a model.

A model is an idealized representation of reality. It must be constructed in such a way as to reproduce the behavior of the real world with acceptable accuracy, recognizing that no abstraction can be identical to reality. This attempt to establish a correspondence between the problem and rational thought may be realized by forming a physical model or a theoretical model. Physical models may be either scalar or analogs of the proto-type; such as a wind tunnel for testing aircraft, and an hour glass for measuring time. Kirchoff's dynamical analogy illustrates that the critical load on an axially loaded structural column may be determined by studying the oscillations of a pendulum of equal length. Numerically oriented analog models may be even more subtle. The slide rule and nomograph are numerical analogs; the odometer of a vehicle gives us the area under a vehicular speed vs. time curve and as such is an integrator.

A theoretical model is essentially a hypothesis. For example, Newton's three laws of motion provide a theoretical model of our physical world. Most theoretical models are mathematically oriented for obvious reasons. Unless the hypothesis and the situation it describes are very simple, the only practical method of studying the many manifestations of a complex system is with the aid of mathematics. The trick, in representing a traffic situation, is to define the relevant parts and arrive at a set of relationships between them which, while simple, will result in meaningful predictions.

The traffic engineer has only a limited influence on the traffic variables. True, he can add traffic lanes so as to reduce the rate of flow per lane; he can set speed limits to discourage high speeds; and he can erect traffic signals to alternate the right-of-way between conflicting streams. However, traffic demand, vehicular capabilities and driver performance are a few of the variables over which he has little influence.

As in the case of other disciplines, many problems can be reduced to finding the maximum or minimum of some function. Thus, the determination of the number of traffic lanes is contingent upon finding the maximum rate of flow (capacity) whereas traffic signal cycle apportionment may be based on minimizing delay. If the mathematical model chosen enables one to compute precisely what will happen to one variable if a specified value is chosen for another variable, the model is said to be deterministic. This may be contrasted with stochastic models in which allowance is made for the probability of a variable assuming various values.

In very general terms, any sequence of experiments that can be subjected to a probability analysis is called a stochastic, or random process. Such a process can be either independent or Markov. An independent process varies when experiments are performed in such a way that the outcome of any one experiment does not influence the outcome of any other experiment. If the outcome of any particular trial or experiment depends on the outcome of the immediately preceding trial, it is called a Markov Process.

The deterministic aspects of vehicular traffic may be explored by studying a single unit of traffic or the traffic stream as a whole. The effect of the motion or the headway of one vehicle on another vehicle is referred to as the local or microscopic properties of traffic. On the other hand, the relation between traffic flow, over an extended period of time, to traffic concentration, over a portion of the roadway, define the over-all or macroscopic properties of traffic. In the energy model discussed in this report, the term microscopic which applies to aggregate conditions or behavior in the system.

Most situations in nature are so complicated that they cannot be dealt with exactly by mathematics. The regular procedure is to apply mathematics to an ideal situation having only important features of the actual one. The results are approximations having a practical importance which depends on the

closeness of the approximation as verified by reasoning and experiment. The deterministic approach to traffic theory involves analyzing the pertinent traffic characteristics, devising a theory, and then applying methods in the development of which differential equations usually play a prominent role. In this treatment the relationships obtained by observation, experimentation and reasoning are given; the researcher is required to express them in mathematical symbols, solve the resulting differential equations, and interpret the solutions.

Heat Flow Analogy - Consider the problem of one-dimensional heat flow in a long slender insulated rod satisfying the differential equation *

$$\frac{\partial \theta}{\partial t} = a \frac{\partial^2 \theta}{\partial x^2} \quad (1)$$

where θ is the temperature, x is the distance, t is time and $a = C/sk$ where C is the conductivity of the material, s the specific heat and k the density. The boundary conditions are statements of the initial conditions at the ends of the rod at any time, and the initial conditions throughout the rod at time zero. What is usually desired is a description of the temperature at any time and place along the rod, or, in other words, a solution of the differential equation expressed in the form

$$\theta(x, t) = f(L, a, x, t) \quad (2)$$

On the other hand if a continuous record in time were kept of the temperature at various points along the rod, one could solve for "a" and thus determine some property of the material such as its conductivity, C .

In many traffic situations, a single traffic lane acts as a long, slender, insulated rod (controlled access and no opportunity for lane change). If θ in (1) is allowed to assume the role of some parameter related to such conventional traffic variables as speed, concentration and flow, the solution, Equation 2, provides the means for evaluating some property of the highway--call it the "trafficability." All that need be done would be to measure the traffic characteristics along the stretch of highway defined by L .

Harr and Leonards¹⁴ postulate, for example, that the movement of traffic results from a motivating "pressure potential" analogous to θ in Equation 1. In their solution the parameter θ was eliminated leaving vehicular speed as a function of what we have defined above as "trafficability", thus affording a means of rating such various geometric features as curve, grade, entrance, ramp, etc.

It should be pointed out that the difficulty in utilizing the deterministic

* Symbols are defined in Appendix B.

approach is not in solving a differential equation, but in finding one that expresses the physical condition realistically. For example, Harr and Leonards¹⁴ define θ so that

$$\frac{\partial \theta}{\partial x} = -c_1 u \quad (3)$$

and

$$\frac{\partial k}{\partial \theta} = c_2 k \quad (4)$$

where u and k are speed and concentration. Solving (3) and (4) simultaneously would suggest the following speed-concentration relationship:

$$u = - (c_1 c_2 k)^{-1} \partial k / \partial x \quad (5)$$

This implies that speed might be negative if the change in vehicular density with respect to distance is increasing. This is not logical.

Fluid Flow Analogy - It seems more realistic to express an equation of motion such as given in (5) in terms of acceleration rather than velocity or speed, since the sign (positive or negative) would not specify forward or backward movement, but merely speeding up or slowing-down. Consider the following equation of motion¹⁵ which expresses the acceleration of the traffic stream at a given place and time as

$$\frac{du}{dt} = \frac{-c^2}{k} \frac{\partial k}{\partial x} \quad (6)$$

This states that a driver adjusts his velocity at any instance in accordance with the traffic conditions about his as expressed by $k^{-1} \partial k / \partial x$. In traffic is thinning out (negative $\partial k / \partial x$) the driver accelerates (positive du/dt) and vice-versa.

Equation 6 is of the form of the equation of motion of a one-dimensional compressible fluid with a concentration k and a fluid velocity u .¹⁰ This is illustrative of several new theories of traffic flow described in terms of fluid or hydrodynamic flows. Their analyses are based on a partial differential equation expressing the conservation of matter and an assumed relation between flow and concentration such as equations (5) and (6).

In fluid mechanics, fluids are commonly divided into liquids and gases. The chief differences between liquids and gases are that liquids are practically incompressible whereas gases are compressible and must be so treated. A single stream of traffic offers a striking analogue to the flow of a compressible gas in a constant area duct. Both consist of discrete particles: individual molecules in the case of a fluid, and individual vehicles in the case of traffic stream.

Lighthill and Whitham¹⁶ applied fluid dynamic principles to various highway occurrences and concluded that discontinuities in traffic flow are propagated in a manner similar to shock waves in the theory of compressible fluids. Greenberg¹⁵ developed the fluid dynamic analogy still further, resulting in functional relations for the basic interactions between vehicles. Herman and Potts¹⁷ emphasized the macroscopic nature of the traffic quantities, flow and concentration, by referring to the somewhat analogous properties of a gas, such as pressure, volume and temperature. Thus, pressure can be computed from the average number of molecular collisions on the containing wall over a long interval of time. Since the relation between the pressure, volume and temperature is called the gas equation of state, the relation between the traffic flow and traffic concentration has come to be known as the traffic equation of state.

Because of the widespread interest in recent years in high-speed gas flow, the dimensionless parameter called the "Mach number" has become very significant in fluid dynamics. The Mach number is defined as the ratio of the actual fluid velocity to the acoustic velocity or velocity of sound propagation, under the conditions of the flow where the velocity in question is measured. Following the logic of compressible flows wherein local sonic speed represents the condition of maximum flow per unit area,¹⁸ the critical velocity for traffic flow corresponds to the optimum velocity, u_m , or the velocity of traffic at capacity, q_m .

The analogy between fluid dynamics and traffic dynamics is seemingly endless. For example, just as the gas equation of state can be derived from the microscopic law of molecular interaction for two molecules, it has been shown that the traffic equation of state can be derived from the microscopic car-following law governing the motion of two cars.¹⁹

In a previous publication,²⁰ utilizing the hydrodynamic analogy, an energy-momentum concept of traffic flow was formulated in which momentum ku was likened to traffic flow q , and the kinetic energy of the traffic stream was defined as ku^2 . Some interesting comparisons based on optimizing both momentum and kinetic energy were made. It was further suggested that acceleration noise might represent the internal energy or lost energy of the traffic stream. This research deals with the theoretical and practical implications of the acceleration noise parameter and the energy model, as related to the level of service concept.

Objectives

The objectives of this research fall into three phases: (1) theory formulation, (2) measurement of appropriate traffic characteristics for theory verification, and (3) recommendations for application. They include:

1. The formulation of a complete "energy" model of traffic flow which includes both "kinetic energy" and "internal energy."
2. Measurement of the acceleration noise on the Gulf Freeway to test the hypothesis that acceleration noise represents the "internal energy" of a traffic stream.
3. Determination of the effects of such geometrics as grades of the facility on acceleration noise.
4. Determination of the effects of operational control procedures such as ramp metering on acceleration noise.
5. Recommendations for application of energy parameters in freeway design and operations.
6. The application of the energy concept to the quantitative description of freeway level of service.

DEVELOPMENT OF ENERGY MODEL

Flow Oriented Parameters

Fluid mechanics is based on, among other things, the principle of conservation of mass. Imagine a volume in space: if the outflow of mass is greater than the inflow, the principle of conservation of mass requires that there is an equal decrease of mass stored in the volume. When this principle is stated mathematically as an equation, it is called the equation of continuity.

Considering traffic flow as a conserved system, the change in the number of vehicles on a length of road dx , in a interval of time dt (see Figure 5), must equal the difference between the number of vehicles entering the section at x and the number of vehicles leaving the section at $x + dx$. If the number of vehicles on the length dx at time t is kdx and the number of vehicles entering in time dt at x is expressed as qdt , the conservation of vehicles can be expressed symbolically as

$$kdx - (k + \frac{\partial k}{\partial t} dt) dx = qdt - (q + \frac{\partial q}{\partial x} dx) dt \quad (7)$$

Making use of $q = ku$ yields the equation of continuity for traffic flow

$$\frac{\partial k}{\partial t} + \frac{\partial(ku)}{\partial x} = 0 \quad (8)$$

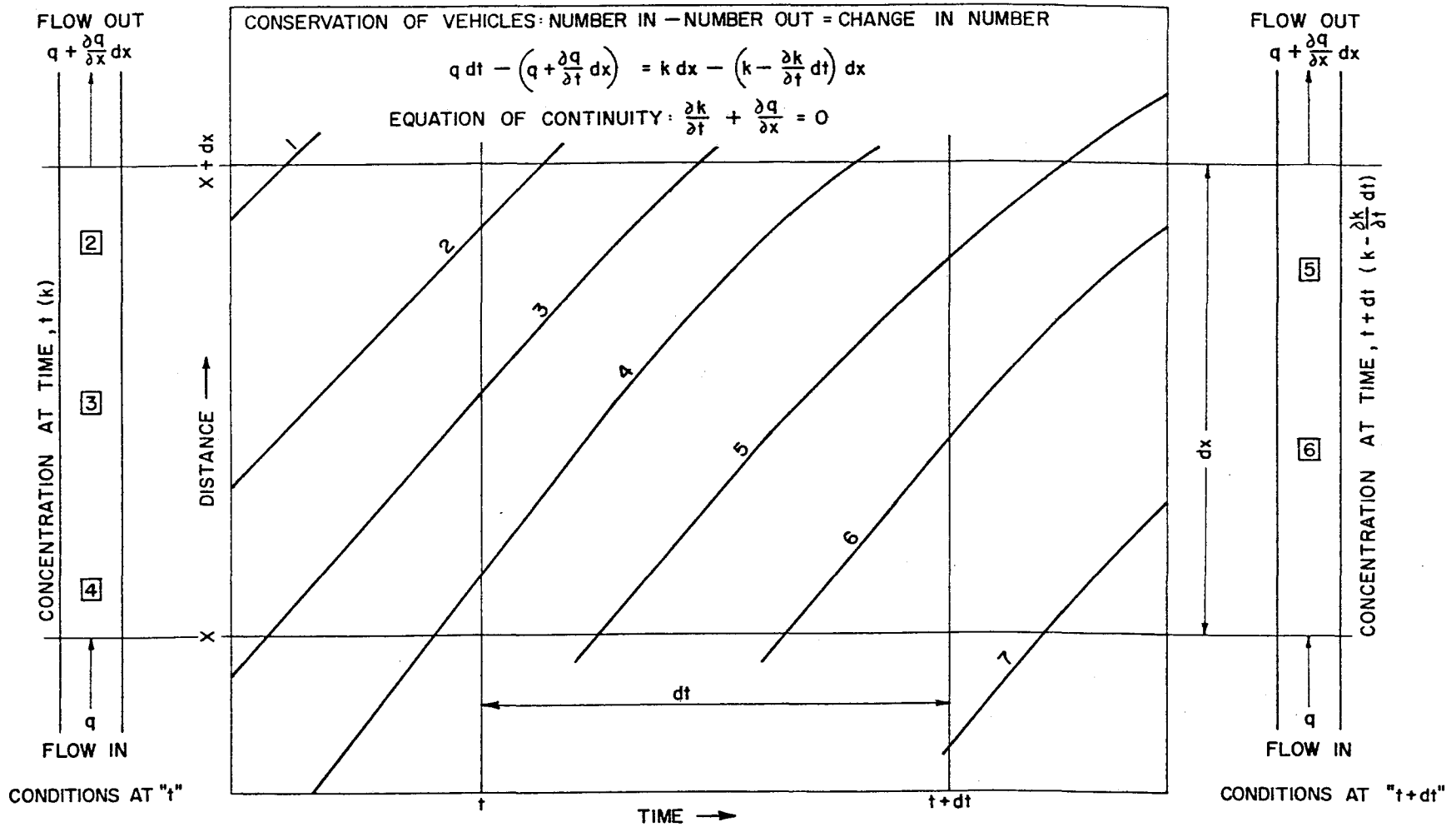
If it is assumed that drivers adjust their speed in accordance with the traffic conditions about them as expressed by the general expression $k^n \partial k / \partial x$, the equation of motion of the traffic stream in terms of acceleration becomes

$$\frac{du}{dt} = -c^2 k^n \frac{\partial k}{\partial x} \quad (9)$$

Solving equations (8) and (9) for $u = f(k)$ and utilizing $q = ku$ yields the generalized equation of state for a traffic stream,

$$q = ku_f \left[1 - \frac{k}{k_j} \right]^{(n+1)/2}, \quad n > -1 \quad (10)$$

where u_f is the free speed and k_j is the jam concentration. The exponent of proportionality n has been shown to provide flexibility in fitting a theoretical flow-concentration curve to a particular highway.¹



DERIVATION OF THE EQUATION OF CONTINUITY FOR A TRAFFIC STREAM

FIGURE 5

The values of concentration and speed for which flow is a maximum are obtained by setting the derivation of $q = f(k)$ and $q = f(u)$ equal to zero:

$$k_m = (n + 3)/2^{-2/(n + 1)} k_j, \quad n > -1, \quad (11)$$

$$u_m = n + 1 / (n + 3) u_f, \quad n > -1. \quad (12)$$

Using the compressible fluid analogy wherein the conditions at maximum flow are termed "critical", we shall refer to k_m and u_m as the critical concentration and critical speed of the traffic stream. Their product q_m is the maximum flow of which the highway lane is capable, the critical flow, or capacity. These critical values are the experimentally determined maximums, and should not be confused with a statistical "extreme value."

Momentum-Kinetic Energy

The speed of waves carrying continuous changes of flow through the stream of vehicles is given by the derivative q' of the q - k equation defined in (10). Discontinuities in traffic flow are propagated in a manner similar to "shock waves" in the theory of compressible fluids. The speed of a shock wave U is given by the slope of the chord joining the two points on the flow-concentration curve which represent the conditions ahead of and behind the shock waves. Application of the mean value theorem suggests that the speed of the shock wave is approximately the mean of the speeds of the waves running into it from either side.

$$U = 1/2 (q'_1 + q'_2) . \quad (13)$$

The very strong analogy between traffic flow and fluid flow suggests that the conditions of continuity of momentum and energy should be fulfilled at the surface of a traffic shock wave, just as the equations of dynamic compatibility must be fulfilled in fluid dynamics. Multiplying equation (8) by u and equation (9) by k , then adding the two equations gives

$$\frac{\partial(ku)}{\partial t} = \frac{-\partial[ku^2 + k^{n+2} c^2/(n+2)]}{\partial x} . \quad (14)$$

Equation (14) is the law of conservation of momentum in the differential form as applied to traffic flow. Comparing equation 8 and 14 with the classical forms in hydrodynamics, we can add to the analogy between the fluid and traffic quantities. This correspondence, which has been discussed in the Introduction and in this section, is summarized in Table 1.

It is apparent that the equations of continuity, motion and momentum are identical in the two systems when the exponent of proportionality n is set equal to -1 . Because, in classical systems, the conservation of momentum equation serves to establish the form for momentum, the quantity ku in

TABLE 1

Correspondence Between Physical Systems

Characteristic	Hydrodynamic System	Traffic System
Continuum	1-dimensional compressible fluid	single-lane traffic stream
Discrete Unit	Molecule	Vehicle
Variables	Mass density, ρ Velocity, v	Concentration, k Speed, u
Equations	<p>Continuity $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$</p> <p>Motion $\frac{dv}{dt} + \frac{c^2}{\rho} \frac{\partial \rho}{\partial x} = 0$</p> <p>Momentum $\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho v^2 + \rho c^2)}{\partial x} = 0$</p> <p>State $P = c \rho T$</p>	<p>$\frac{\partial k}{\partial t} + \frac{\partial(ku)}{\partial x} = 0$</p> <p>$\frac{du}{dt} + c^2 k^n \frac{\partial k}{\partial x} = 0$</p> <p>$\frac{\partial(ku)}{\partial t} + \frac{\partial(ku^2 + k^{n+2} c^2 / (n+2))}{\partial x} = 0$</p> <p>$q = ku$</p>
Parameters	<p>Critical velocity, V_c</p> <p>Critical Flow, Q_c</p> <p>Momentum, ρv</p> <p>Kinetic energy, $\rho v^2 / 2$</p> <p>Internal energy, ϵ</p> <p>Friction factor, f</p>	<p>Critical speed, u_m</p> <p>Critical concentration, k_m</p> <p>Capacity Flow, q_m</p> <p>Flow, ku</p> <p>Kinetic energy, αku^2</p> <p>Optimum speed, u'_m</p> <p>Optimum concentration, k'_m</p> <p>Optimum flow, q'_m</p> <p>Internal energy, σ</p> <p>Natural noise, σ_n</p>

Equation 14 is defined here as the "momentum" of the traffic stream. If in fact, traffic "momentum" is equal to ku , it is apparent that it is also equivalent to traffic flow and that the flow oriented parameters (u_m , k_m , and q_m) discussed in the previous section are based on optimizing this "momentum."

It is well known that the kinetic energy of a fluid is $(1/2 \rho v^2)$. However, because of the generalized equation of motion utilized in the formulation of the traffic system model, the kinetic energy of the traffic stream will be defined as $\alpha k u^2$ where α is a constant. Squaring (10) and then dividing by k yields

$$E = \alpha k u_f^2 \left[1 - 2 \left(\frac{k}{k_j} \right)^{(n+1)/2} + \left(\frac{k}{k_j} \right)^{(n+1)} \right], \quad n > -1 \quad (15)$$

The critical depth of flow y in an open channel may be obtained from optimizing either specific momentum M or specific energy E by setting either $dM/dy = 0$ or $dE/dy = 0$. Therefore, differentiating (15) with respect to concentration and speed, $dE/dk = dE/du = 0$, to get the appropriate "energy" parameters gives:

$$k'_m = (n+2)^{-2/(n+1)} k_j, \quad n > -1 \quad (16)$$

$$u'_m = \left[(n+1)/(n+2) \right] u_f, \quad n > -1$$

$$q'_m = k'_m u'_m \quad (18)$$

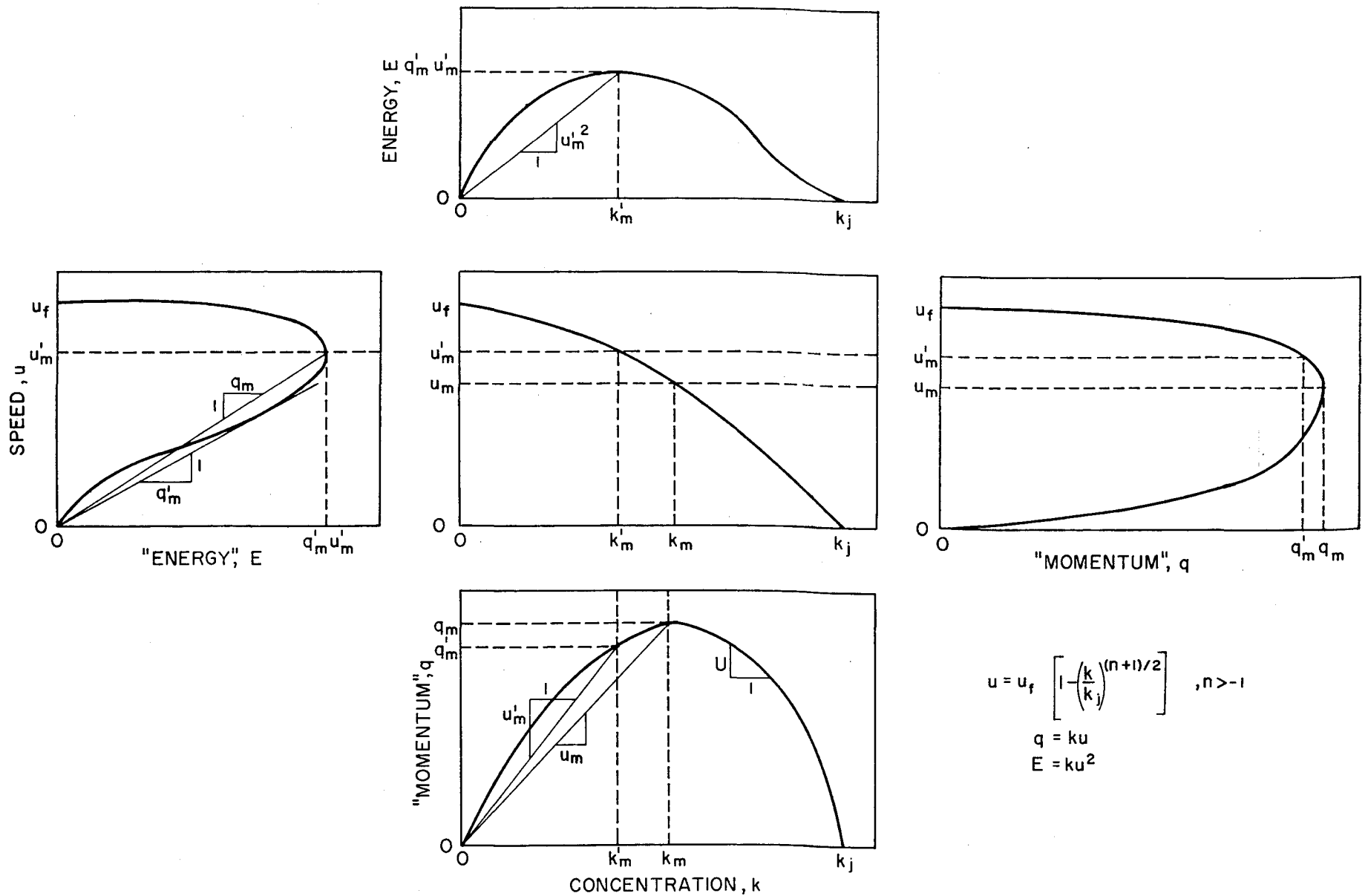
and

$$E'_m = \alpha k'_m u'_m{}^2 \quad (19)$$

In comparing (11) and (12) with (16) and (17), it is apparent that for vehicular traffic, quite unlike the case of the hydraulics of open channel flow, the parameters obtained from optimizing momentum are not equivalent to the parameters obtained from optimizing energy. The significance of this will be discussed in the applications of the model to level of service. The relationship between the fundamental traffic variables and parameters discussed so far and summarized in Table 1 is illustrated in Figure 6.

Internal Energy

In the same manner as the principle of conservation of mass, the energy conservation law merely states that within the confines of a given system or control section, energy is neither created nor destroyed, although it may appear in several forms (kinetic energy, potential energy, internal energy, etc.), and it may be transformed from one type to another. Energy appears in forms associated with a given mass (kinetic energy) or as transitory energy in the familiar forms of heat and work (internal energy).



$$u = u_f \left[1 - \left(\frac{k}{k_j} \right)^{(n+1)/2} \right], n > -1$$

$$q = ku$$

$$E = ku^2$$

RELATIONSHIP BETWEEN FUNDAMENTAL TRAFFIC VARIABLES AND PARAMETERS

FIGURE 6

In classical dynamics, the general energy equation is concerned only with changes in energy content and form, and hence only those forms of energy that undergo change in a given system need be considered. For this reason, in this discussion of energy conservation in a traffic stream we shall obviously not be concerned with chemical, electrical, or atomic forms of energy. The forms of energy that will be considered are kinetic energy and internal energy.

Kinetic energy, ku^2 , is the energy of motion of the traffic stream. If in fact, there is an internal energy or lost energy associated with a traffic stream, it should manifest itself as either lost or erratic motion due to adverse geometrics and traffic interaction. In the introduction it was seen that the measure of the jerkiness of the driving in this stream is given by acceleration noise. The units of both kinetic energy and acceleration noise are those of acceleration, which adds credibility to the hypothesis that acceleration noise represents internal energy.

The conservation of energy for the traffic stream over a section of road x is simply a case, then, of the total energy T being equal to the kinetic energy E plus the internal energy I of the traffic stream. With this notation, an energy balance or accounting becomes

$$T = E + I \quad (20)$$

$$T = \alpha ku^2 + \sigma$$

where σ represents the acceleration noise for a vehicle on a section of highway of length x .

It is common to say that there is a loss of energy due to the effects of friction in a system, whether it is a classical system or traffic system. The general energy equation tells us that there can be no loss in energy from the system if the principle of conservation of energy is to hold true. Re-examination of the energy equation will yield a clue as to the true meaning of friction. Energy is not really lost, it is simply converted from one of the mechanical forms (kinetic energy) to internal energy, a thermal form of energy. One recalls from the second law of thermodynamics that the mechanical forms of energy, such as kinetic energy, are more valuable than an equivalent amount of thermal energy or internal energy. This is certainly true in the case of traffic flow. Thus, we can say that the forces of friction (adverse geometrics and traffic interaction) tend to convert the desirable forms of energy (traffic motion) into the less valuable forms (traffic interaction).

Two boundary conditions become important in evaluating the parameter, T and α , in (21). As the internal energy approaches zero, kinetic energy approaches the total energy of the stream and

$$T = \alpha k'_m u'_m{}^2 = (4/27) \alpha k_j u_f{}^2 \quad (22)$$

The converse leads to an expression for total energy in terms of acceleration noise,

$$T = \sigma_{\max} \quad (23)$$

where σ_{\max} is the maximum acceleration noise. Equating (22) and (23), we can express α as

$$\alpha = (k_j x)^{-1} \quad (24)$$

where

$$x = \frac{27}{4} \frac{\sigma_{\max}}{u_f{}^2} \quad (25)$$

The units of x are those of feet. Intuitively this would be the length of section, x , over which acceleration noise should be averaged. Tests runs yielding such representative values as $\sigma_{\max} \sim 2 \text{ fps}^2$ $u_f \sim 60 \text{ mph}$ for $x = 500$ feet tended to substantiate this interpretation. The significance of α would be the reciprocal of the maximum number of vehicles possible on the section of highway x . Again, this seems logical; looking at (20) and (21) it is apparent that the internal energy of the stream is being estimated by a single average vehicle whereas the kinetic energy is being estimated by all traffic. The parameter α serves to adjust E and I so that their sum is equal to the total energy T which is a constant in keeping with the concept that energy cannot be created or destroyed.

Acceleration noise is the standard deviation of changes in speed. The form of $\sigma = f(u)$ may be deduced from (21) and (23)

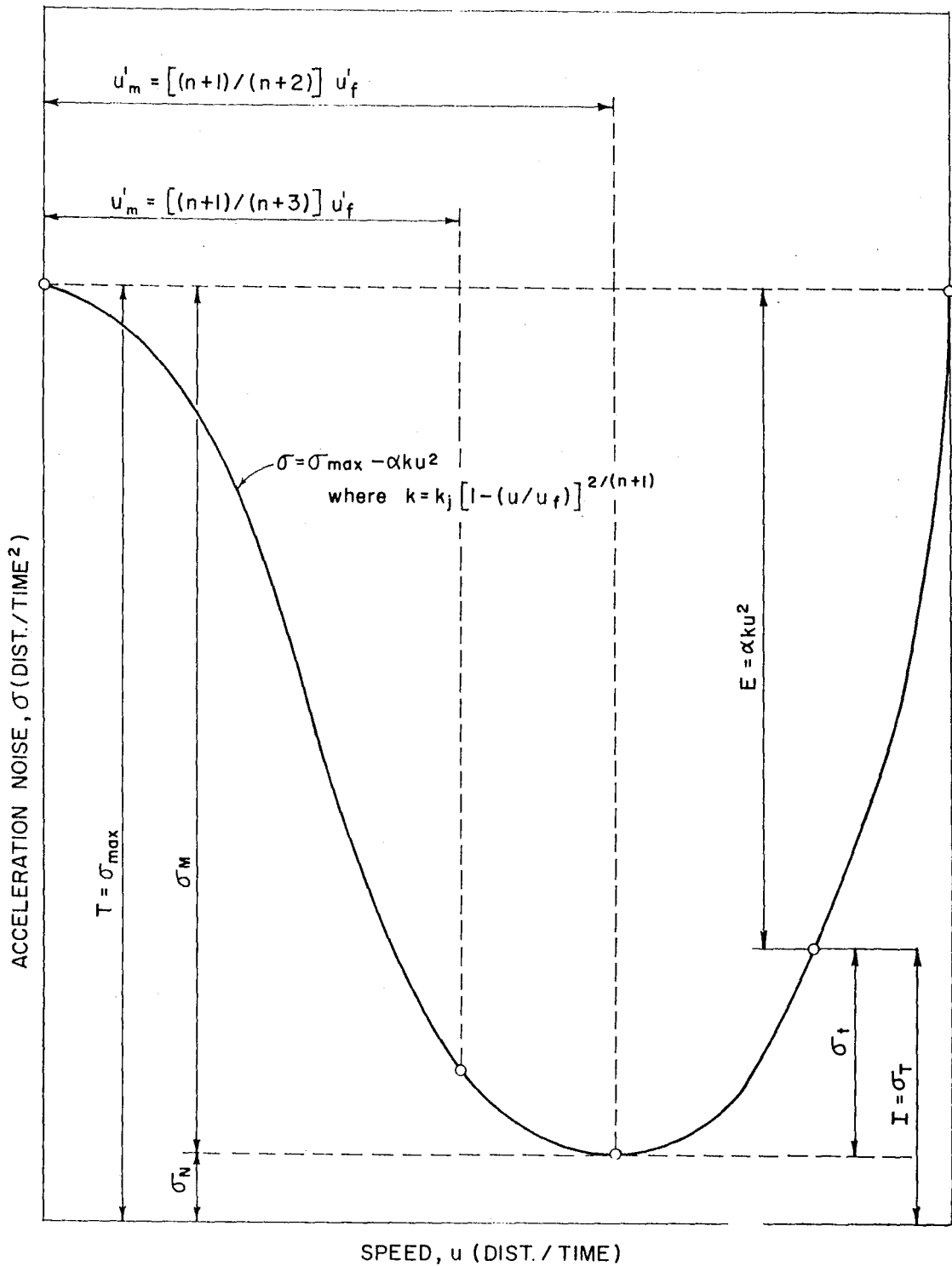
$$\sigma = \sigma_{\max} - \alpha k u^2 \quad (26)$$

From (10) it is apparent that

$$k = k_j [1 - (u/u_f)]^{2/(n+1)} \quad (27)$$

Substituting (27) in (26) and differentiating (26) with respect to u gives the vehicular speed u'_m at which the acceleration noise or lost energy is a minimum. It is

$$u'_m = [(n+1)/(n+2)] u_f, \quad n > -1 \quad (28)$$



RELATIONS BETWEEN ACCELERATION NOISE
 PARAMETERS FOR INTERNAL ENERGY MODEL

FIGURE 7

This is identical to the speed given in (17) that maximizes the motion or kinetic energy of the traffic stream. For example, letting $n=1$ in (27) and substituting in (26) gives

$$\sigma = \sigma_{\max} - \alpha k_j u^2 + \alpha k_j u^3 / u_f \quad (29)$$

for which the optimum speed would be from (28)

$$u'_m = (2/3) u_f$$

Finally, σ_{\min} would be obtained by using (28) and (26)

$$\sigma_{\min} = \sigma_{\max} - \alpha k'_m u'^2_m \quad (30)$$

We know that the total measured acceleration noise σ_T is made up of a certain contribution due to the interaction of traffic σ_t plus the natural noise σ_N due to the geometrics of the facility. The natural noise σ_N could be obtained either by measurements made by a test vehicle under conditions when there was no traffic interaction (say at 3 AM) or by fitting a regression line to acceleration noise vs. speed data to obtain σ_{\min} . The relations between the parameters derived here are summarized in Figure 7.

The significance of the internal energy-acceleration noise model developed is that it provides estimates of all the parameters summarized in Table 1 and Figure 7 based on data collected with a single test vehicle equipped with a recording speedometer. The study procedure, data reduction and broad applications of the model is discussed in the following section.

STUDY PROCEDURES

Study Location: The Gulf Freeway

The study site is the Gulf Freeway in Houston, Texas. It is a six-lane divided facility having 12-foot lanes and a 4-foot concrete median with 6-inch barrier type curbs. The freeway is an at grade type and is carried over the major intersections by grade crossings producing a "roller coaster" effect of the thru lanes.

The study section of the Gulf Freeway (Figure 8) begins at the Reveille Interchange at the intersection of Highways U. S. 75, State 35 and State 225, and extends to the downtown distribution system. The interchanges within the study area are either full or partial diamond with the exception of the Reveille Interchange and the distribution system to the C. B. D. which are directional. The distribution system consists of two four-lane one-way streets and thus affords for high capacity movement. Frontage roads in the study section are continuous except at two locations for the inbound movement and at three locations outbound.

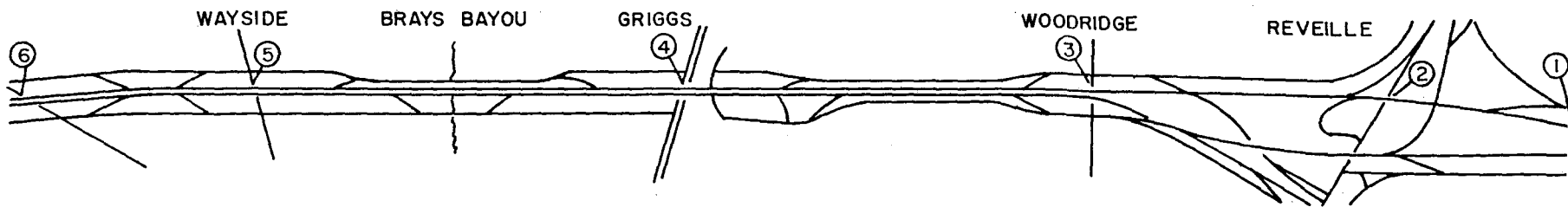
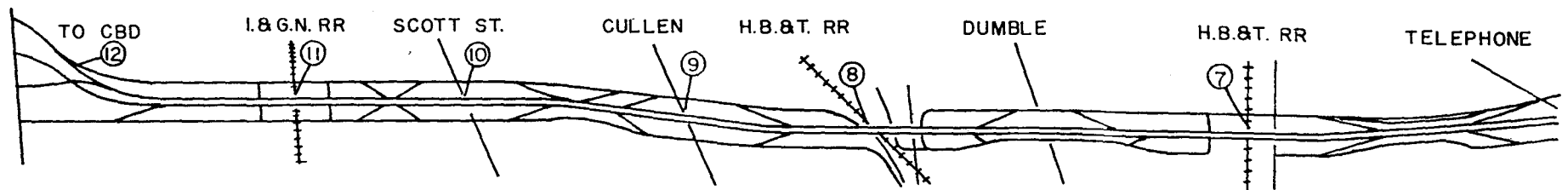
Method of Study

In the study of traffic flow on a busy freeway, it is necessary to describe the motion of a great number of vehicles. Continuing with the hydrodynamic analogy, this task is similar to that of describing the motion of an infinite number of fluid particles. A quantity such as velocity must be measured relative to some convenient coordinate system. The two methods commonly used in fluid mechanics are the Euler and Lagrangian methods of analysis.

One may choose to remain fixed in space and observe the fluid pass by a given point. This method, whereby a fixed coordinate system is established, is referred to as the Euler method analysis. The lagrangian method of analysis involves establishing a coordinate system relative to a moving fluid particle as it flows through the continuum and measuring all quantities relative to the moving particle.

Similar methods of analysis may be utilized to describe a traffic system. They are summarized in Figure 8b. By careful design of a freeway study; point, instant or floating studies can be used to give essentially continuous coverage in both time and space.²⁰ The point and instant methods are more applicable to the measurement of the macroscopic properties of the traffic stream. Moving vehicle methods are microscopic in nature, and quite analogous to the Lagrangian method of analysis used in fluid mechanics.

The use of the floating car method has long been established as providing accurate determination of the average speed of traffic on a roadway. This method was also applied in this study with the assumption made that the accelerations of



STATION POINTS WITHIN THE STUDY AREA

FIGURE 8a

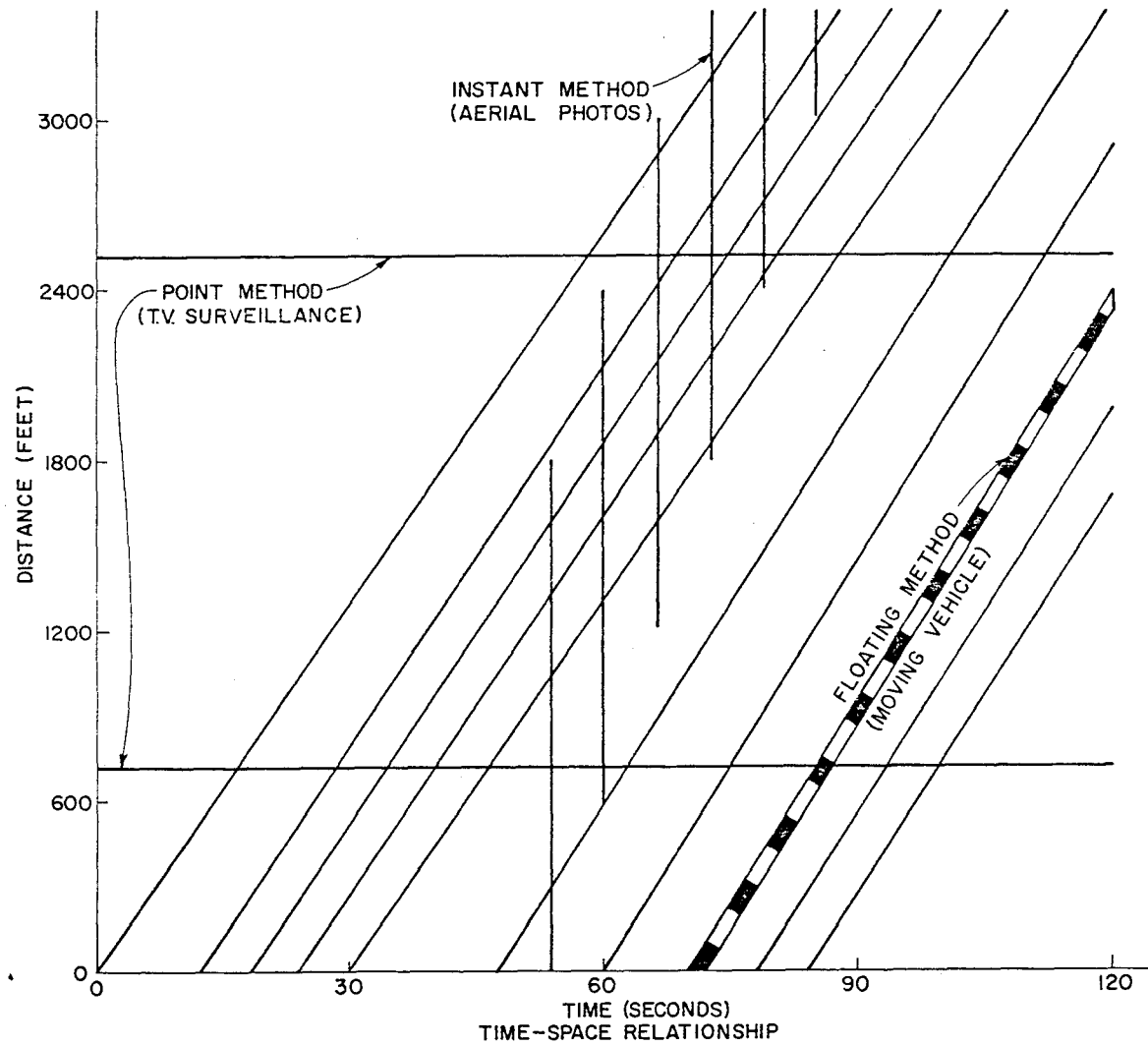
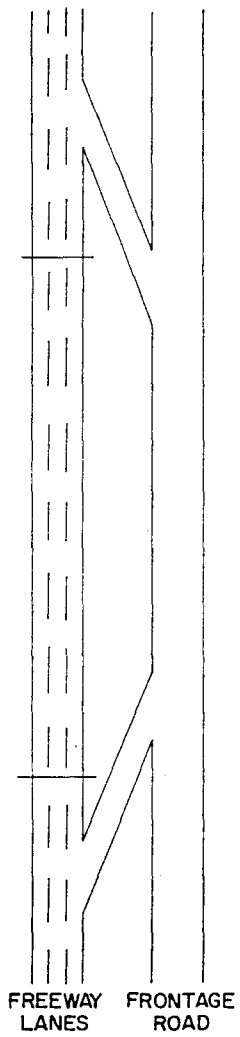


ILLUSTRATION OF STUDY PROCEDURES

FIGURE 8b

a floating car would also represent a good average of the accelerations of the traffic stream.

The total acceleration noise of vehicles at different locations in a platoon has been previously measured in single lane experiments²¹ and it was noted that traffic broadens the acceleration distribution function of the lead car. It was found that the dispersion down the platoon increases up to about the 5th vehicle at which location the noise reaches about three times the lead car. However, the broadening factor then remains relatively constant down the rest of the platoon. During the study periods, care was exercised to position the test vehicle in a platoon so as to represent the majority of vehicles, that is, not closer than the fifth vehicle in a platoon during peak period runs so that the full effects of the broadening factor could be measured.

Although there may be some variance in the acceleration noise experienced by different drivers on a given road and traffic conditions, this variance was not measured as part of this study. The variance was eliminated however, by employing the same motorist throughout this study.

In order to accomplish the objectives of this study, speed recordings were made:

1. During off-peak traffic flow periods (about 12 midnight) to determine the effects of such geometrics as grades of the facility on acceleration noise.
2. During peak periods of traffic flow to evaluate the effects of traffic interaction and to document the congestion and levels of service on the facility.
3. During on-ramp control studies to determine the difference in the smoothness of flow between normal and controlled operation.

Natural Acceleration Noise - In order to obtain a means of rating the various geometric features of a freeway using the acceleration noise parameter, it is necessary to eliminate the effects of traffic interaction. Thus, one method of obtaining the natural acceleration noise on a facility would be to drive the test vehicle on the facility in the absence of traffic. However, since this feat is almost an impossibility on the Gulf Freeway, it was decided to record the data at about midnight on several days. The extremely light volumes at this hour reduced the probability of any traffic interference with the test vehicle which would affect the natural noise measurement. A total of 20 runs were made for this measurement. Data recorded in freeway sections where the speed of the test vehicle was influenced by traffic interference were discarded.

Traffic Interaction - A total of 81 runs were made during periods between 6:20 a.m. and 8:20 a.m. over the course of 16 days on the inbound facility to evaluate the effects of acceleration noise due to freeway traffic interaction. During these two hour periods, it was possible to make one complete round trip approximately every 20 minutes. The starting times of each run were altered daily so that the typical cross section of the total period could be measured.

An on-ramp control study²² initiated on the Gulf Freeway in August, 1964 afforded an excellent opportunity to measure the acceleration noise due to traffic interaction during controlled conditions and thereby compare the results with those obtained during normal operation. The control area with a summary of the control plans is shown in Figure 9.

Runs were made on the inbound lanes in similar fashion to the uncontrolled periods. The study periods began at 6:20 a.m. and extended to 8:20 a.m. each day. Again, starting times were staggered each day in an attempt to sample the entire study period.

Equipment

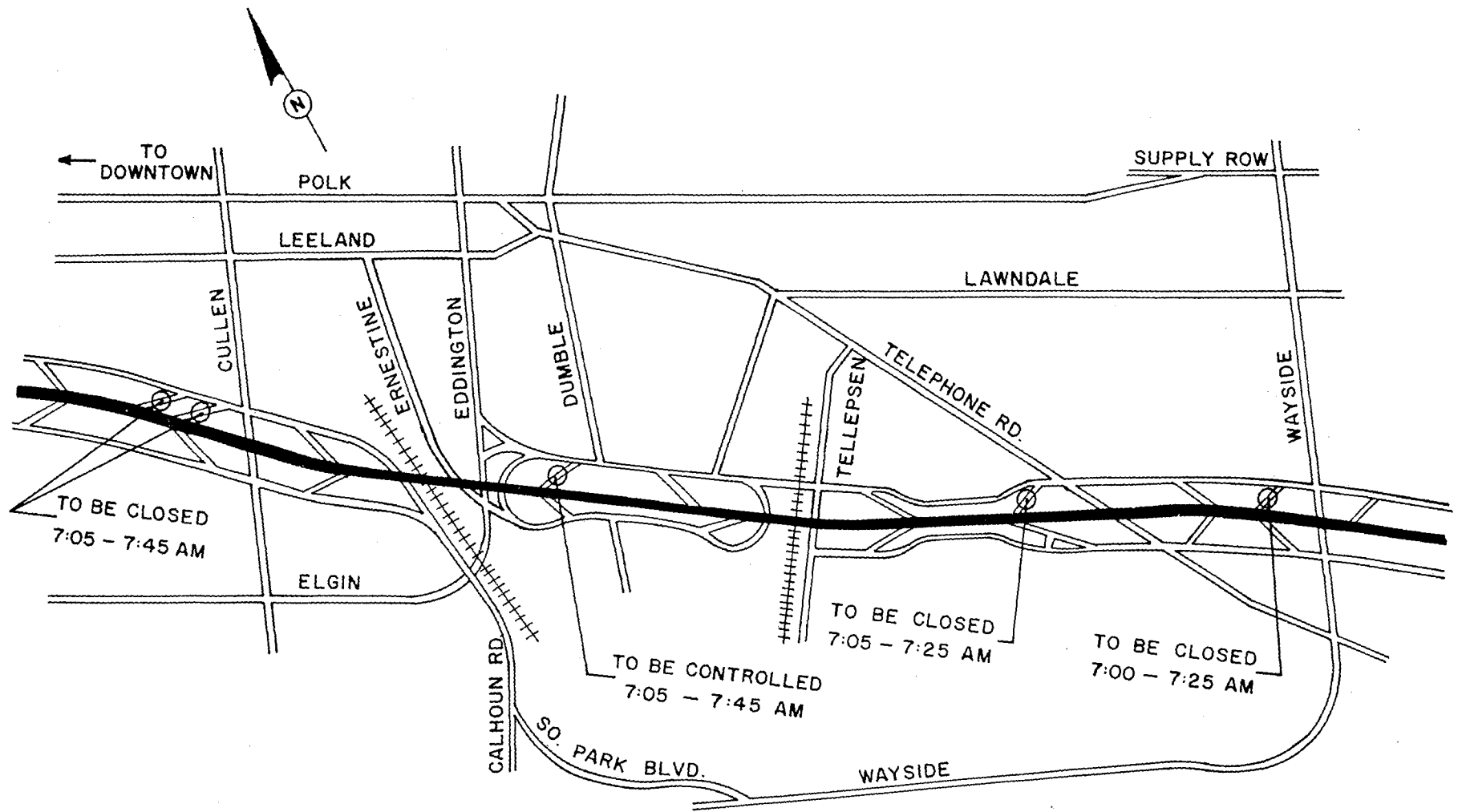
In previous studies of acceleration noise, two different methods of recording the necessary data to calculate the parameter were employed. In one study¹¹ an accelerometer mounted in the test vehicle was photographed and σ was determined by analysis of an acceleration-time curve. Other researchers,¹³ realizing that this method was too time consuming, utilized a tachograph and recorded the speed of the test vehicle as it progressed in the traffic stream. Basic equations of motion were employed to approximate the formula for acceleration noise.

The tachograph had several advantages; however, a recording speedometer manufactured by Esterline-Angus, shown in Figure 10, was more suitable for the study on the freeway because of its flexibility. This recorder is cable connected to the transmission of the vehicle in similar manner as the odometer and is capable of recording an analogue graph of the vehicle's speed in the traffic stream.

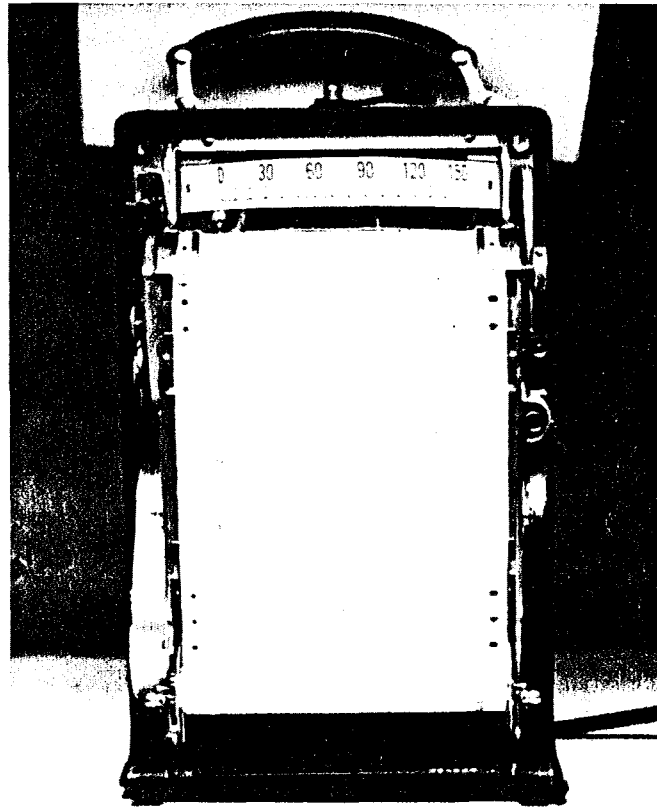
In addition to the speed recording, a special event mark, which is indicated by either an upward or downward sweep of the speed recording pen, was used to identify certain reference station points along the route. These stations, which in most cases were the center of each grade separation on the freeway, were marked manually through a switching box connected to the speed recorder. The reference stations are shown in Figure 8A.

In addition to the speed pen, a second pen recorded 100-foot distance marks automatically.

The chart speed was also very flexible. A set of gears accompanying the speed recorder allowed a varied selection of chart advance rates. For this study, a chart advance rate of 1 inch per 10 seconds was used to allow a more accurate reading of time, which was one of the two measured variables.



ON-RAMP CONTROL AREA
FIGURE 9



SPEED RECORDER

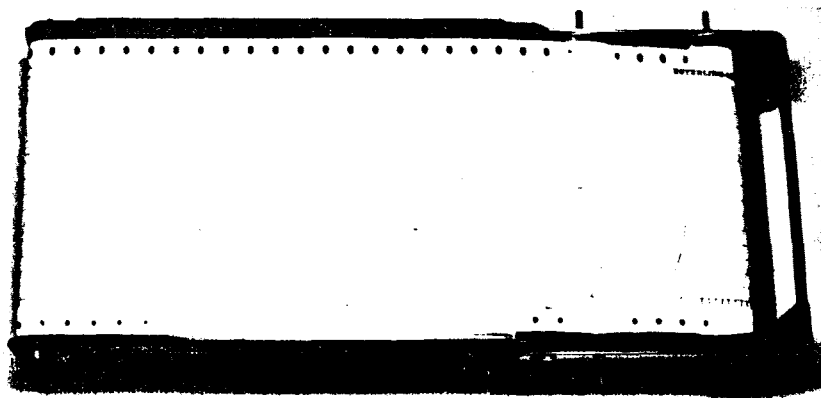


CHART AND CHART INSPECTOR

FIGURE 10

Again the flexibility of the equipment permitted modifications which also increased the reading accuracy of the second variable speed. A rheostat on the switching box was electronically connected to the recorder in such a fashion as to vary the range of the vertical scale. The normal chart scale is graduated in increments of 2 mph. However, the rheostat permitted the use of 1 mph increments with a full scale deflection of 75 rather than the normal 150 mph.

Data Reduction

Choice of Speed Increment - From these speed profiles obtained with the test vehicle, the acceleration noise was easily estimated using Equation (6) in Appendix A: A transparent template graduated in 1.0 second increments was placed over the speed graph to determine Δt to the nearest 0.5 seconds for every Δv . The term " $1/\Delta t$ " was accumulated on a calculator for the entire section under study. Thus this method provided a fast means of reducing the data.

The approximating equation for acceleration noise (Appendix A) is formulated on the principle that the speed increment Δv is constant and could be used throughout the evaluation of a speed-time curve while Δt is measured for every Δv . It is obvious that a selection of two separate values of Δv could result in different values for the approximation of σ . For instance if Δv were chosen as 2 mph, the measurement would be much more sensitive than if it was taken to be 6 mph. It is conceivable that a traffic stream could experience several speed fluctuations of up to 5 mph without exceeding the 6 mph limit. The resulting calculations would produce values of σ when $\Delta v=2$ is used, but no values with $\Delta v=6$. Some aspects and comparisons regarding the selection of Δv for the data reduction are discussed in Appendix C.

The selection of the constant value Δv was given careful consideration for this study. One would not expect a motorist to maintain an absolutely uniform speed but would expect minor fluctuations. It also seems apparent that a motorist does not even sense a speed change of 2 mph or less and therefore this degree of fluctuation would not effect the quality of his trip.

From still another practical standpoint based on the limitations of the equipment and data reduction methods employed in this study, an accuracy of no more than 2 mph could be expected. A constant of $\Delta v=2$ was therefore used throughout this study. For all practical purposes then, acceleration noise was calculated to be zero unless a speed change of 2 mph occurred.

Acceleration Noise on a Short Section of Freeway - The fundamental equation for estimating acceleration noise from Appendix A (Equation 4) is

$$\sigma^2 = \left[\frac{(\Delta v)^2}{T} \sum_{i=0}^T \frac{n^2}{t_i} \right] - \left[\frac{(V_T - V_0)}{T} \right]^2 \quad (31)$$

It has been emphasized that on a long stretch of roadway the last term in this equation is zero since in most cases the starting speed and the ending speed are the same. In other cases T is very large which makes the second term insignificant. When an investigation is made of a short section of free-way (say 500 feet), however, it is not uncommon to have conditions where the speed at the beginning of the section varies considerably with the speed at the end of that same section. In these cases, the second term of the equation is of significant value and therefore, according to the definition of acceleration noise, it must be considered in the calculation.

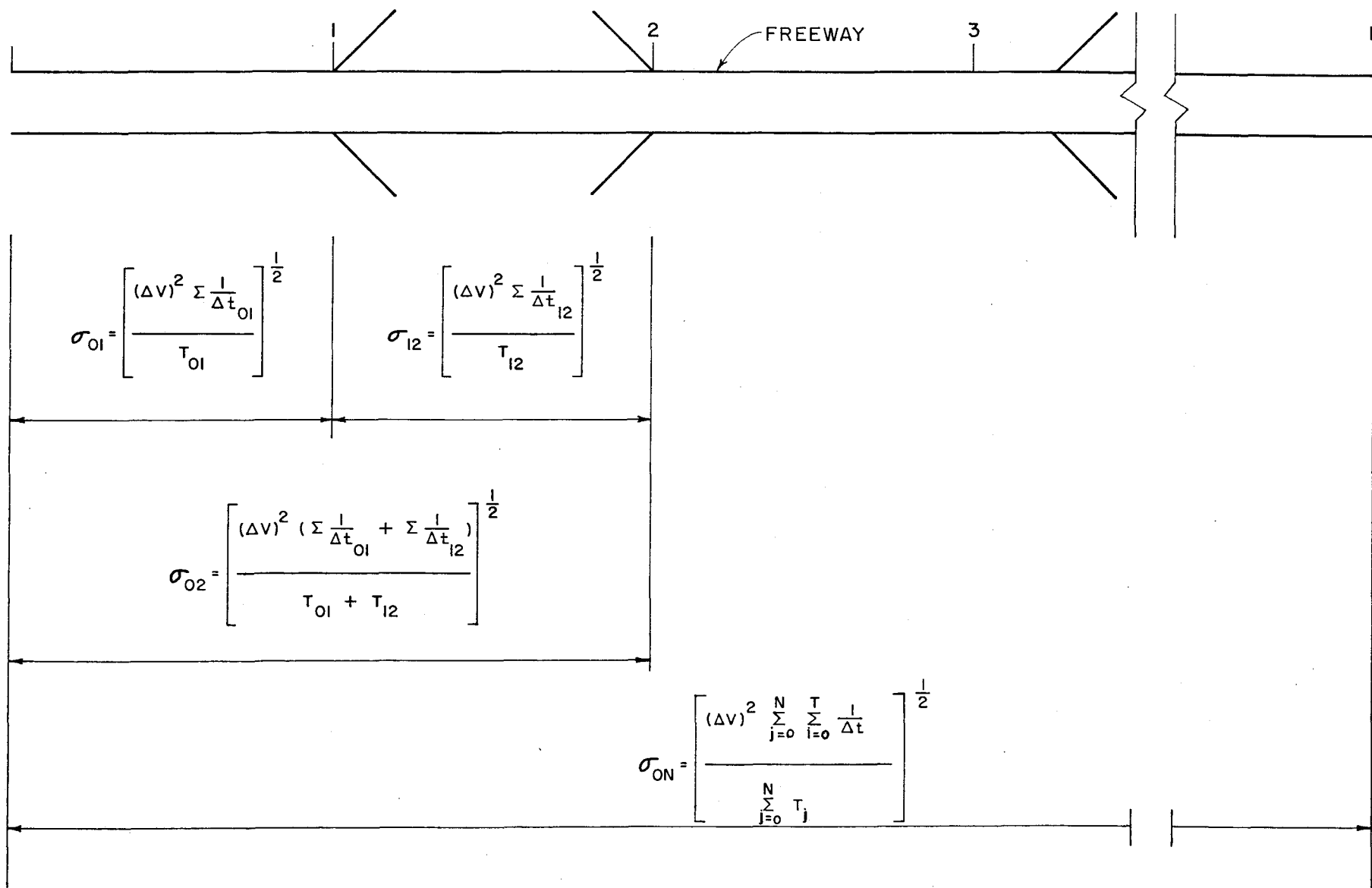
Dudek²³ has derived a general expression for estimating acceleration noise which does not depend on the average acceleration being zero (Appendix D). It is based on finding the acceleration noise about the origin rather than the mean. The relative acceleration noise for short sections of roadway using this approach becomes

$$\sigma_0 = \left[\frac{(\Delta v)^2}{T} \sum_{i=0}^T \frac{1}{\Delta t_i} \right]^{1/2} \quad (32)$$

It is seen that Equation 31 is identical to Equation 32 ($V_T = V_0$), which defines acceleration noise for a relatively long trip. This identity makes it possible to use a simple systematic means of evaluating σ on any desirable roadway section of any length based on calculations from successive short sections. This property is discussed in the next section.

Additive Property of Acceleration Noise - Suppose acceleration noise data were obtained over a long continuous roadway section. The acceleration noise on short successive sections (say 500 to 1000 feet) could be calculated using Equation 32. If it should be desirable to increase the length of the study sections for reasons of evaluating a larger problem area, σ could easily be determined from the data readily available for the short sections. The additive property of the standard "variance" (standard deviation squared) permits the "pooling" of numerators and denominators in the equation.²⁴ Therefore, the data for the variance on the accelerations, or acceleration noise squared, can be pooled. For example, in Figure 11, if the values of σ were known from 0 to 1 and from 1 to 2, σ from 0 to 2 becomes

$$\sigma_{02} = \left\{ \frac{(\Delta v)^2 \sum_{i=0}^1 \frac{1}{\Delta t_i} + (\Delta v)^2 \sum_{i=1}^2 \frac{1}{\Delta t_i}}{T_{01} + T_{12}} \right\}^{1/2}$$



SCHMATIC SHOWING THE ADDITIVE PROPERTY OF ACCELERATION NOISE

FIGURE 11

which can be written as

$$\sigma_{02} = \left\{ \frac{(\Delta v)^2 \sum_{i=0}^1 \frac{1}{\Delta t_i} + \sum_{i=0}^2 \frac{1}{\Delta t_i}}{\sum_{j=0}^2 T_j} \right\}^{1/2}$$

Generalizing the above equation, if σ has been determined for n successive sections from $j = 0, \dots, N$, then σ for the entire section is

$$\sigma_{0N} = \left\{ \frac{(\Delta v)^2 \sum_{j=0}^N \sum_{i=0}^T \frac{1}{\Delta t_i}}{\sum_{j=0}^N T_j} \right\}^{1/2} \quad (33)$$

There are at least two important advantages of measuring σ in several successive short sections rather than measuring the noise over the total length of roadway under study. First of all, one can easily determine locations where problems are inherent due to either geometric deficiencies or congestion. By measuring σ on several short sections, these trouble locations can be isolated for future extensive study. Secondly, if σ can be related to known parameters such as speed, volume, or concentration, each of which vary between short sections of freeway, then it is also important to measure acceleration noise in short sections.

RESULTS OF ACCELERATION NOISE STUDIES

Natural Acceleration Noise

The objective of determining the natural acceleration noise (σ_N) on the Gulf Freeway was to determine the amount of disturbance to a vehicle's trip on the facility that can be ascribed to a driver's natural speed changes in the absence of traffic. Twenty trips were made on the freeway with the driver instructed to drive at a comfortable speed throughout the 6-mile study section. Although all the test runs were made about midnight, occasionally the speed of the test vehicle was influenced by another vehicle in certain sections. These sections were thus excluded from the total sample runs.

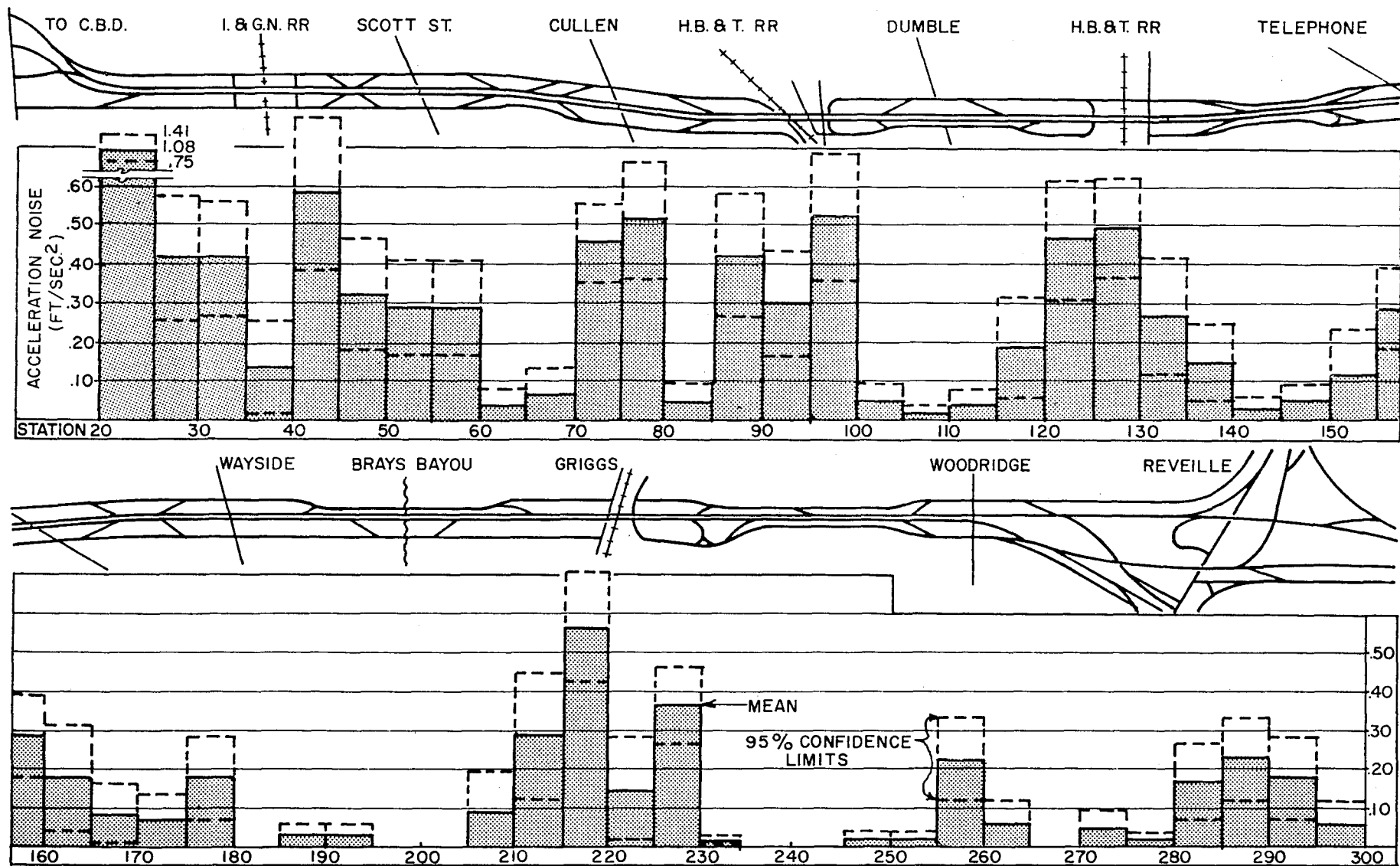
The natural acceleration noise on the inbound Gulf Freeway for the entire length of the study section was found to be $0.38 \pm .02$ ft/sec². Even though the freeway has a rolling grade, the value of the natural noise is not drastically above the value (0.32 ft/sec²) measured on an almost perfect roadbed.¹¹

This value, however, could be deceiving if the tangent sections of the freeway were sufficiently long to dampen the effects of geometrics. The intrinsic or natural effects in each of 500-foot continuous freeway sections were therefore determined and are presented in Figure 12.

Each of the roads which are marked on the Figure as crossing under the freeway represent locations where grades exist with two exceptions. Dumble is not continuous and therefore does not pass under the freeway. Also, Brays Bayou passes under the freeway but well below the existing grade. The remaining roads constitute locations of grades ranging from 1.0 to 5.0 percent.

It is noted that σ_N is approximately zero on the tangent freeway sections. However, the pattern of increased noise is evident on the grades. Further study of the relationship of acceleration noise to grades was made and the results are presented later in this same chapter. The high values of σ_N at the distribution system, of course, are attributable to the rapid deceleration in anticipation of traffic signals at upcoming intersections.

An important factor in the measurement of σ_N is the variability of the parameter. It was noticed that after the 20 test runs on the freeway, the 95 percent confidence limits of the estimate of σ_N was within $\pm .02$ ft/sec². However, by dividing the freeway into short segments, the variability of σ_N was much higher within the segments. For the majority of the sections the 95 percent confidence limits of the estimate of σ_N was within $\pm .15$ ft/sec² of the observed mean (Figure 12).



NATURAL ACCELERATION NOISE (σ_N)

FIGURE 12

Evaluation of Geometrics: Grades - The level of service concept is predicated on the theory of providing the motorist a quality of driving conditions which includes safety and comfort. Jones and Potts¹³ have clearly presented the increase of acceleration noise due to winding country roads. It is obvious that on the basis of safety and comfort, the large values of acceleration noise on these roads indicated very poor service to the motorists.

The magnitude of grade effects has been generally documented in terms of capacity reduction with respect to percentage of trucks in a traffic stream. However little attention has been given to the effects on passenger vehicles. It is generally conceded²⁵ that passenger vehicles can negotiate long grades up to 7 percent at speeds exceeding 30 miles per hour and that grades up to 7 percent have a negligible effect on the performance of passenger cars.

The level of service concept, however, places emphasis upon driver comfort and safety as well as roadway capacity. Accident studies by Mullins and Keese²⁶ found no significant relationship between the algebraic difference in grade and the frequency of accidents and likewise no apparent relationship between the sight distance on the vertical curves and frequency of accidents. Further evaluation of high accident frequency locations, however, showed that rear-end type accidents accounted for 70 percent of all accidents on high frequency crest-sag locations and that driver tendency to follow too closely "probably" constituted a "primary" or a contributing causative factor."

A major reason why motorists follow too close on vertical alignment might perhaps be explained by their natural tendency to decelerate upgrade and to accelerate downgrade. Upon reaching the crest, the drivers realize the reduction in speed due to the incline and have a tendency to accelerate downgrade sometimes to speeds beyond their desirable comfortable speeds.

For this phase of the investigation the hypothesis that the irregular pattern in driving speeds on grades can be described in terms of acceleration noise and that acceleration noise on vertical curves is dependent upon the linear combination of the two independent variables of algebraic difference in adjoining grades and length of grade is to be tested. In other words, it is hypothesized that these two variables contribute to the naturalness of speed irregularities on vertical curves which can be measured in terms of acceleration noise. The dependency of acceleration noise on both or any one of these variables would enable a basic evaluation of vertical curves as related to danger potential.

The data collected to determine the natural acceleration noise were also used to test the hypothesis concerning grades. Acceleration noise over each of the nine grades within the study section was determined and the measurements were then averaged to define the mean acceleration noise over each grade. Although only nine grades were located within the study area, six of which were identical, the analyses are presented to illustrate some general trends. The data used for this analysis, confidence limits of the average acceleration noise and statistical

tests are set forth in Appendix E.

Although the lack of a sufficient sample size prohibits any definite conclusions, the results in Appendix E, illustrated in Figure 13 are indicative. It is evident that if a relationship between grades and acceleration noise can be further substantiated, the element of grades as related to level of service could be documented in terms of acceleration noise. Although there did not seem to be any direct relationship between grade and accident experience in the study by Mullins and Keese²⁶, there is indication that there is some relationship between grade and accident potential as defined in terms of acceleration noise.

Comparison of σ_{\min} and σ_N - The total measured acceleration noise (σ_T) is the sum of the noise due to traffic interaction (σ_t) and the natural noise (σ_N). Since σ_N is a constant, σ_T becomes a minimum when σ_t is zero. Therefore, the minimum measured acceleration noise (σ_{\min}) is equal to σ_N .

As a means of substantiating the above equality, data from each of five freeway sections was used to determine the parameters of Equation (29). Minimum acceleration noise was then computed and compared to σ_N which was measured over the same sections. Results of an analysis of variance confirmed acceptance at the .01 level of the null hypothesis that the difference between the two parameters is zero, that is $\sigma_{\min} = \sigma_N$.

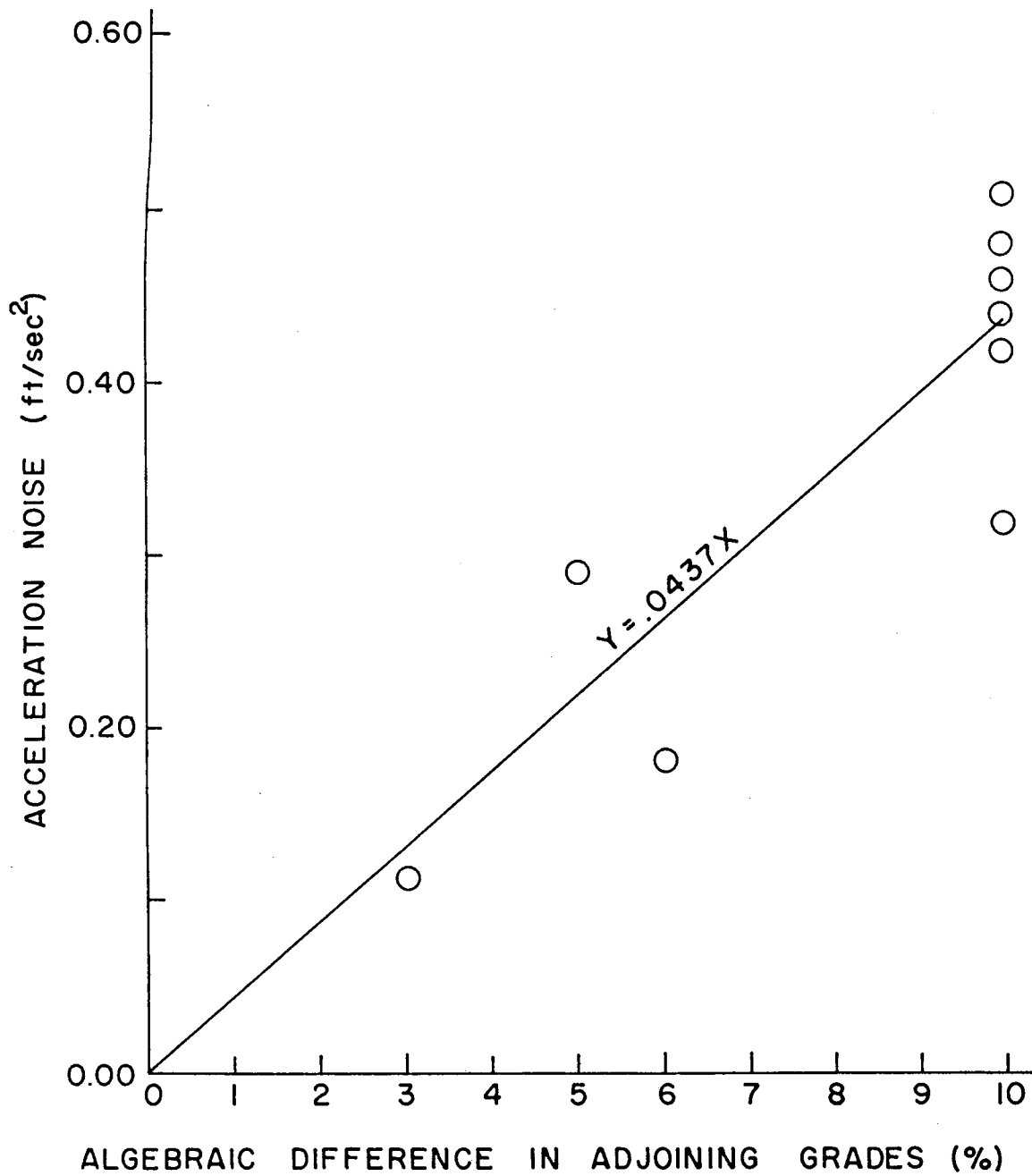
The significance of this equality is the fact that both σ_t and σ_N can be determined by only measuring the total acceleration noise in the traffic stream (σ_T). Thus geometric effects can be assessed by measuring the acceleration noise in the traffic stream in lieu of measurements in the absence of traffic. Refer to Figure 7.

Acceleration Noise Due to Traffic Conditions

In this phase of the investigation, the acceleration noise data collect during the morning peak period were reduced for each 500-foot section as previously established for the natural noise phase of this study. Data were collected during periods in which the entrance ramps were being metered (controlled operation) as well as during periods of normal freeway operation.

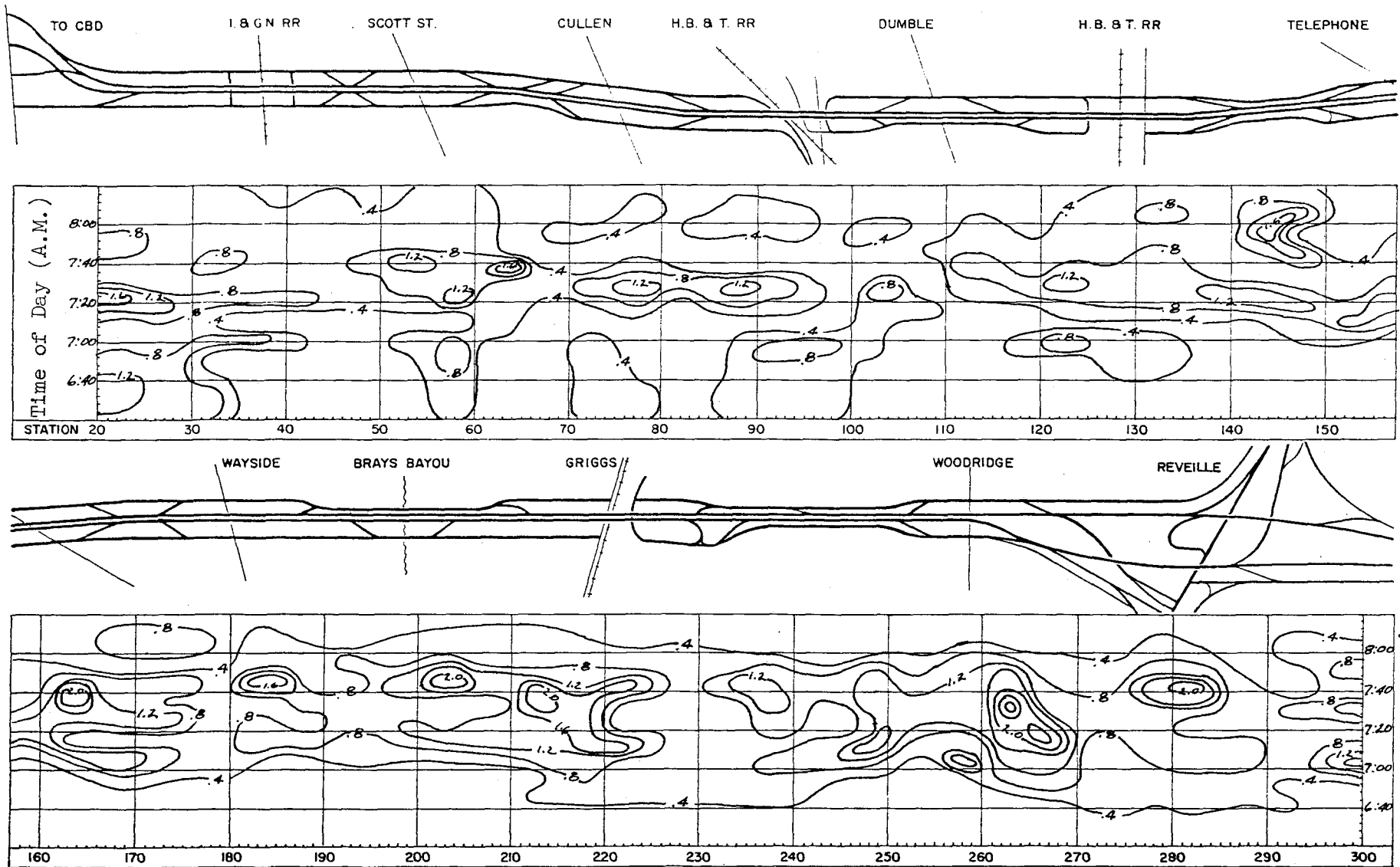
Normal Operation - The total acceleration noise on the inbound Gulf Freeway as measured by the test vehicle can best be illustrated in terms of contours as in Figure 14. The acceleration noise due to traffic conditions (natural noise removed) is presented in Figure 15.

Figure 13 reveals that the most violent speed changes as a result of vehicle interaction occur generally from the Reveille Interchange to the south H.B. & T. Railroad overpass. The flow downstream of this area is relatively smooth. Within the former area, three critical areas are predominant: (1) downstream of Telephone Rd., (2) at Brays Bayou, and (3) at Woodridge.



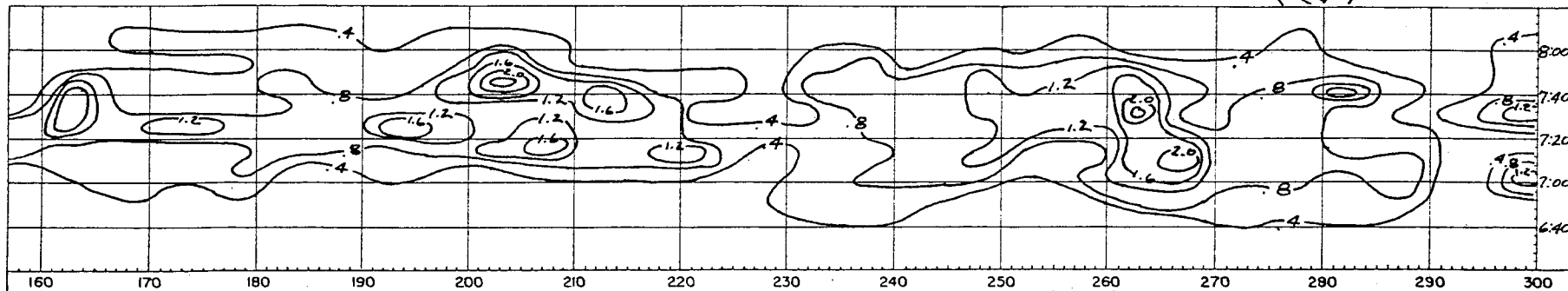
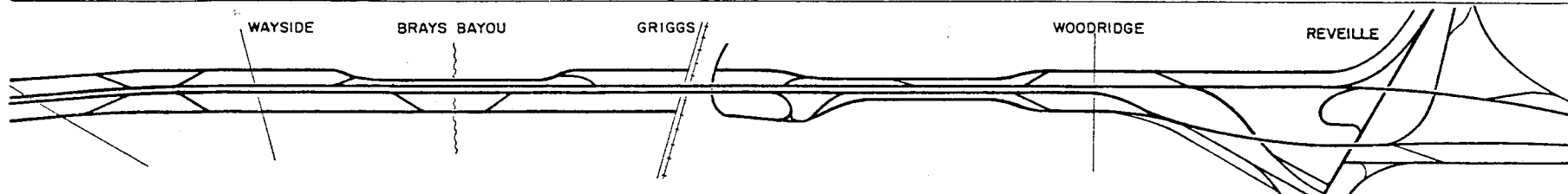
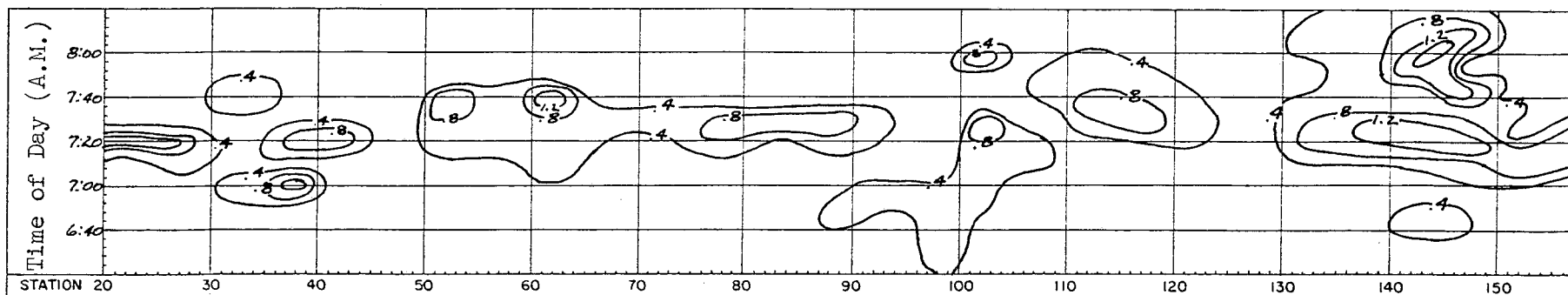
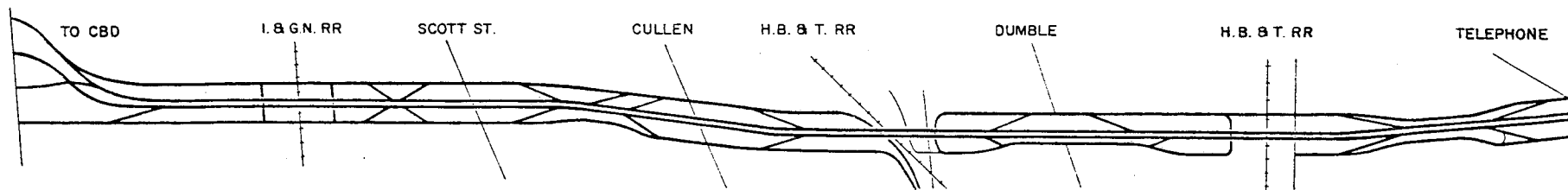
RELATIONSHIP BETWEEN ALGEBRAIC DIFFERENCE
IN ADJOINING GRADES AND ACCELERATION
NOISE

FIGURE 13



TOTAL ACCELERATION NOISE (σ_T) CONTOURS

FIGURE 14



CONTOURS OF ACCELERATION NOISE DUE TO
TRAFFIC INTERACTION (σ_t)

Downstream of Telephone, three conditions exist which could influence the instability of flow. One factor is the existence of a 5 percent grade downstream of the area and the other two factors are the presence of on and off ramps within the area. It cannot be concluded from the available data however, whether these conditions alone or in part were the sole causes of the large amount of acceleration noise.

The second critical area is in the vicinity of Brays Bayou. The significance of this area is the fact that is not considered a problem spot. A bottleneck actually exists at the Griggs on-ramp which is located downstream of the Bayou. However, vehicle queues, resulting primarily from inefficiency at the Griggs ramp, usually extend beyond the Bayou during the critical peak traffic period. This condition often extends beyond the time after which the majority of the remaining freeway is moving relatively smoothly. The resulting effect of the queue buildup is an area of rapid deceleration.

The third area of high acceleration noise, at Woodridge, can largely be explained by the continuous congestion at the Reveille Interchange. Beyond the bottleneck at State Highway 35, is an area in which the motorists have the opportunity to discharge from the queue but are suddenly met with another stopped platoon. The acceleration noise at this area is ascribed to a combination of both rapid acceleration and deceleration.

The acceleration noise contour map does not actually identify the exact location of bottlenecks; it merely illustrates the danger potential areas which result from critical bottlenecks. High acceleration noise, of course, would normally occur at the end of queues which in many cases are far removed from the actual bottlenecks. However, it would not be difficult to locate the cause of bottlenecks after the critical areas have been located from the acceleration noise contours.

Controlled Operation - In the past the basic concept of improving freeway operations by ramp control has been to maximize volume throughout with little consideration given to the quality of freeway operation. It is an accepted fact that most freeway bottlenecks can be reduced or eliminated by on-ramp control techniques but little has been done to measure the qualitative changes due to the controls other than a measurement of travel time. It is very possible to reduce the over-all travel time and yet create hazardous locations where rapid decelerations occur, far removed from the original bottlenecks. As was previously stated, travel time would not reflect these conditions. However, a measurement of acceleration noise could locate any hazardous locations and would indicate whether additional controls would be necessary to maintain smooth operation throughout the study area.

In an attempt to compare acceleration noise before and during the on-ramp control study conducted in the summer of 1964, the data collected on days during which any vehicular incident occurred, such as an accident or stall, were not used. As a result, four days of usable data were averaged and are presented in the forthcoming figures.

Figures 16 and 17 represent contour maps of the total acceleration noise (σ_T) and the acceleration noise due to traffic interaction (σ_t), respectively, measured during the on-ramp control studies. Comparison of Figure 17 with the acceleration noise contours of the data measured before the controls, as presented in Figure 15, indicated that the flow during controls was much smoother. To further illustrate the difference, a cross section of the contours taken at time 7:30 a.m., identified as Section 1-1 on Figure 17, is presented as profiles in Figure 18. It is noticed that the acceleration noise during controls was generally much less than without controls.

Additional investigations were made of specific locations. Sections were passed through four critical locations which were measured before controls. The resulting profiles of these sections are compared to the acceleration noise during the control study measured at the identical locations. This presentation is made in Figure 19. It is very obvious that acceleration noise was considerably less during the control studies.

One objective on the on-ramp control study was to improve the flow upstream of the Wayside interchange. Also, minor improvement was expected downstream of Wayside. Comparison of Figures 18 and 19 indeed confirm the fulfillment of the ramp control study objectives. It is very obvious that the flow from Wayside to the distribution system was much smoother during the ramp controls. The high acceleration noise normally present between the two H. B. & T. Railroad tracks did not occur during the controls. Also, the smoothness of flow was noticeably better upstream of Wayside during the study. Finally, the sensitivity of the acceleration noise parameter as a means of evaluating before and after conditions is apparent.

Verification of Internal Energy Model

In the development of the internal energy model, a functional relationship between acceleration noise and speed was deduced (Equations 26 and 27). This relationship is expressed in Equation 29 for the special case in which $n = 1$. Substituting the identities

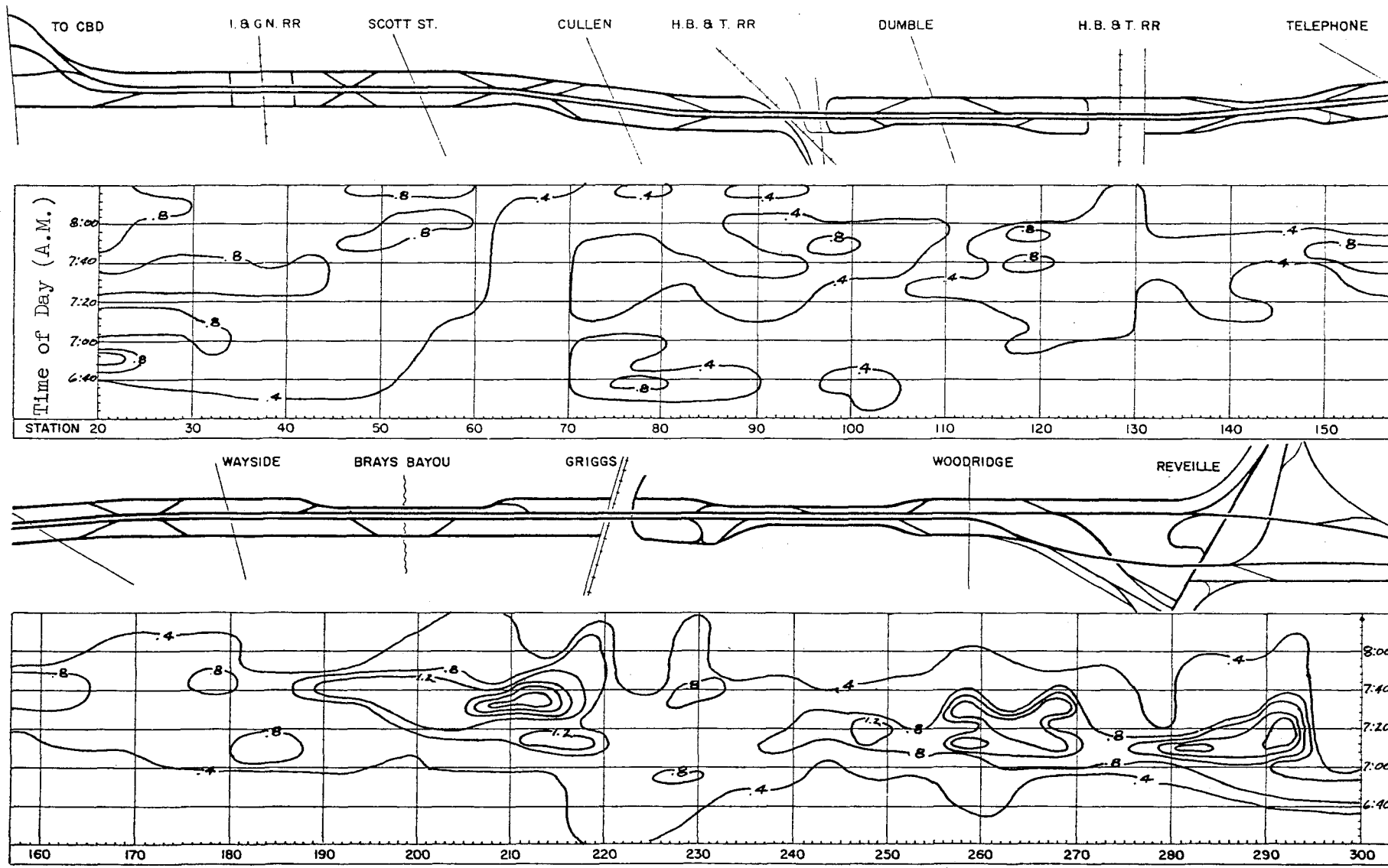
$$\sigma_{\max} = \sigma_M + \sigma_N$$

and

$$\sigma_t = \sigma - \sigma_N$$

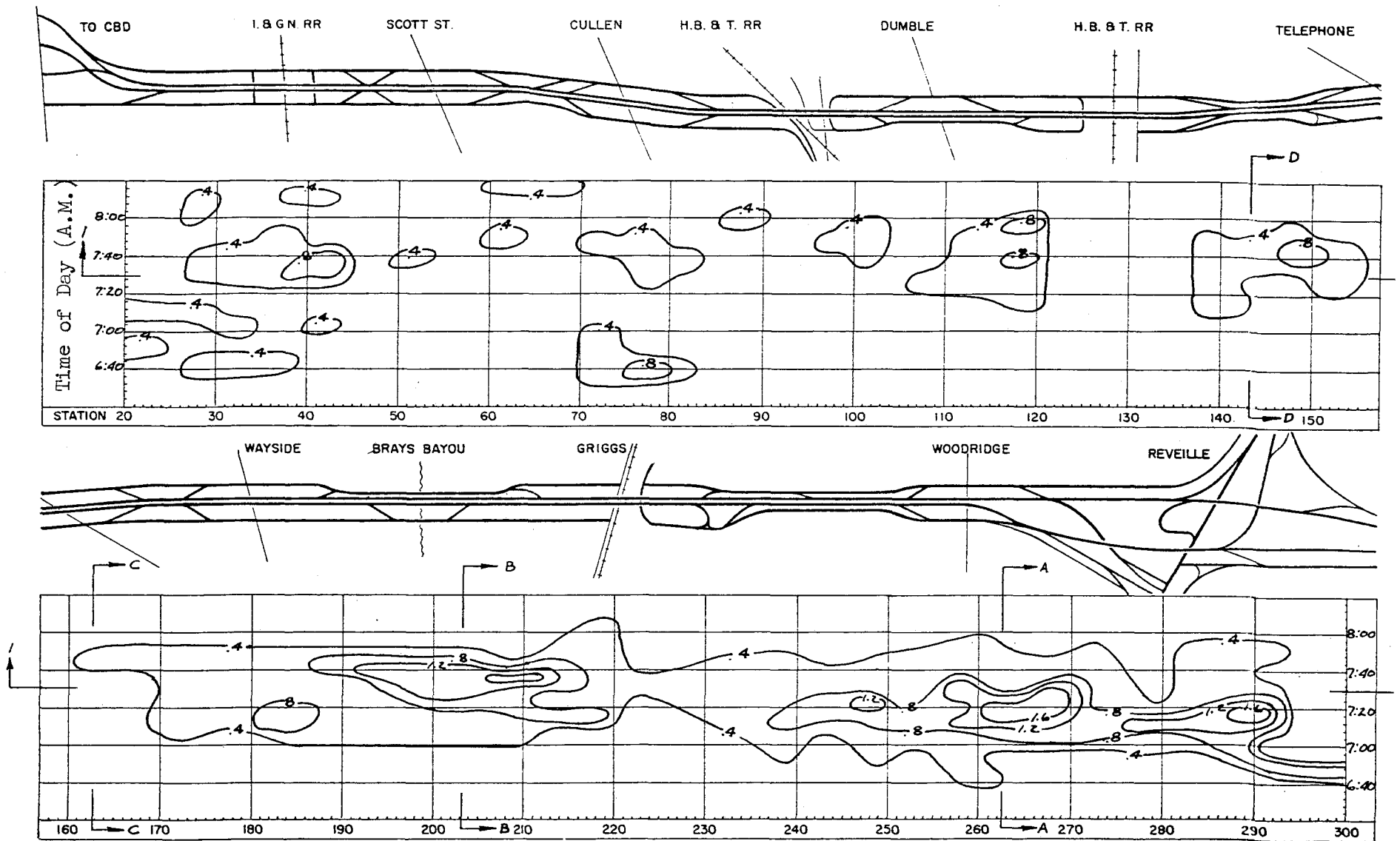
in (29) gives

$$\sigma_t = \sigma_M - \alpha k_j u^3 / u_f \quad (34)$$



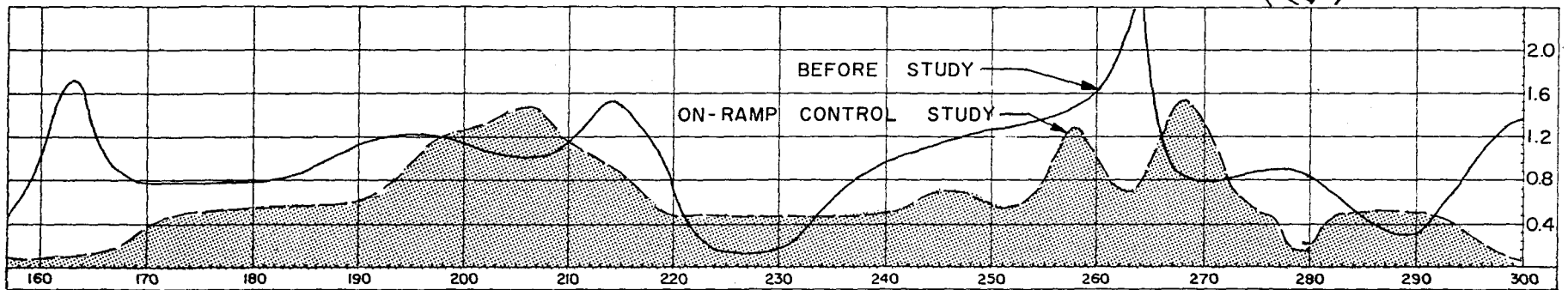
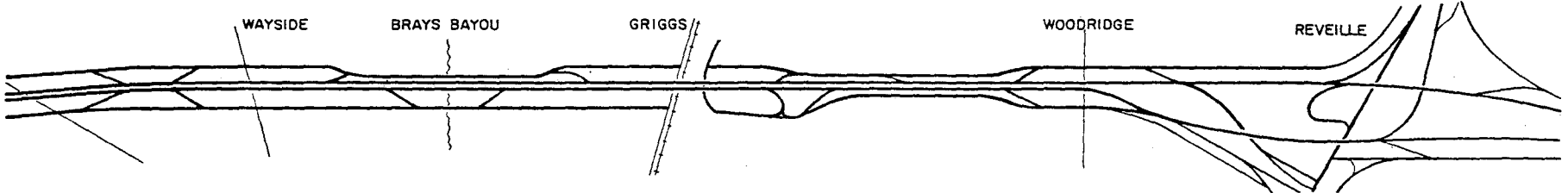
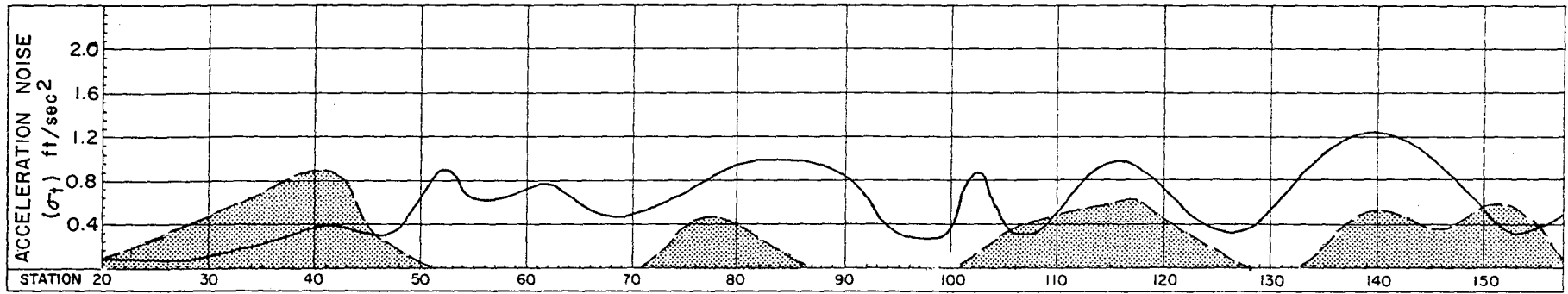
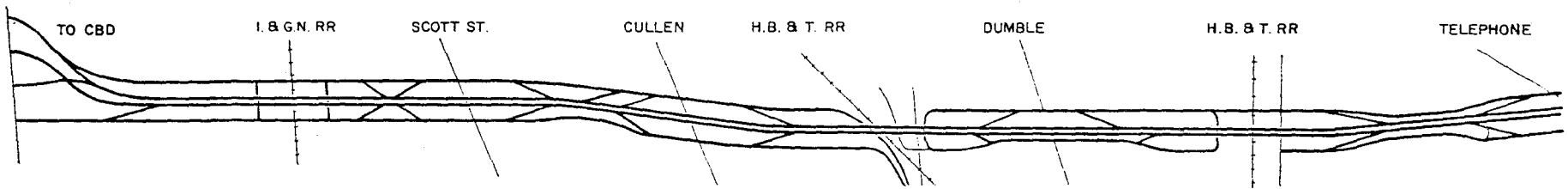
TOTAL ACCELERATION NOISE (σ_T) CONTOURS
CONTROL STUDY

FIGURE 16



CONTOURS OF ACCELERATION NOISE DUE TO TRAFFIC INTERACTION (σ_t) - CONTROL STUDY

FIGURE 17



ACCELERATION NOISE PROFILES - 7:30 A.M.
SECTION 1 - 1

FIGURE 18

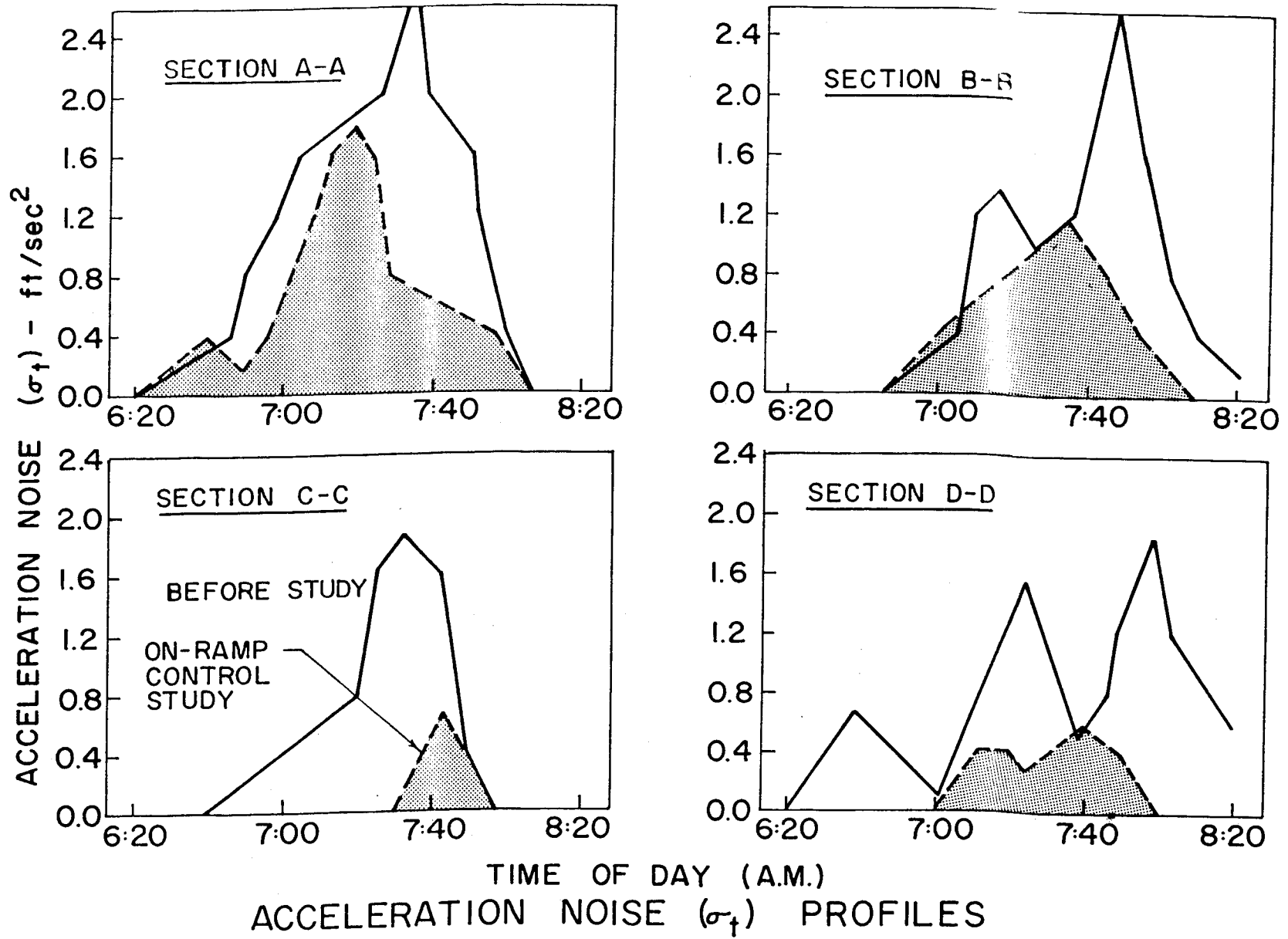


FIGURE 19

Standard regression techniques were employed to test the hypothesized relationship between acceleration noise and speed expressed in Equation 34, and thereby evaluate the internal energy model. A discussion of the statistical tests associated with the regressions are presented in Appendix 6.

A review of the results tabulated in Table 2 reveal that the regressions and the partial regression coefficients are generally highly significant (.01 level indicated by double asterisks). These and the R^2 values tend to verify the internal energy model. Two distinct continuous areas of the freeway, however, showed no significant relationship to the model. The first of these areas was between stations 300 and 285 which included the entire Reveille Interchange. A second area was located between the north and the south H. B. & T. Railroad tracks. The only explainable reason that could be formulated at this time is that the lack of data within the entire range of the regression curve resulted in an insignificant relationship. That is, the data points were generally confined to two concentrated sections of the curves which resulted in poor correlation. Further investigation of these locations would be necessary to determine the exact reason for this lack of correlation.

Figure 20 is a presentation of the final regression curve for one of the 56 freeway sections. The results show that acceleration noise increases very rapidly at the onset of congestion. Also, it is very evident that the maximum quality of flow is not realized at maximum volume output, a fact that will be discussed in more detail in the next section.

Table 2

REGRESSION ANALYSES OF ACCELERATION NOISE

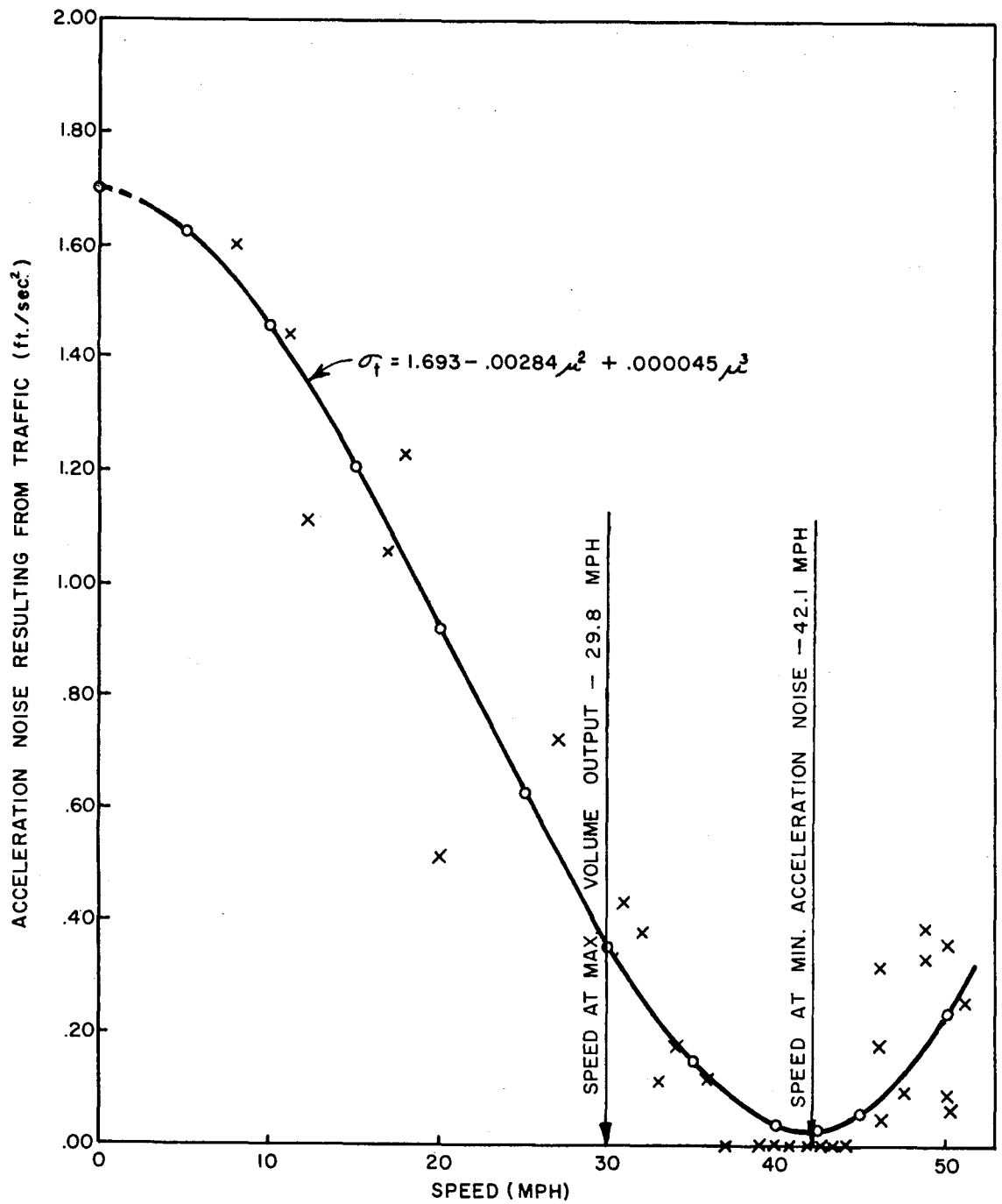
$$\text{Model: } \sigma_t = \sigma_M - C_1 u^2 + C_2 u^3$$

Section	σ_M	$C_1 \times 10^2$	Sign	$C_2 \times 10^4$	Sign	R ²	"F" Test
300-295	1.566	.0974	**	.0798		48.1	**
295-290	.953	.0119		.0724		11.4	**
290-285	.921	.0317		.0007		22.0	**
285-280	1.287	.0047		.0808		37.9	**
280-275	1.404	.0927	**	.0843	*	54.0	**
275-270	1.289	.0657	*	.0419		37.5	**
270-265	1.806	.1617	**	.1907	**	56.1	**
265-260	2.016	.1947	**	.2399	**	56.0	**
260-255	1.183	.0646		.0504		24.2	**
255-250	1.163	.2020	**	.2818	**	54.7	**
250-245	1.836	.2333	**	.3262	**	63.5	**
245-240	1.361	.1289	**	.1515	**	45.3	**
240-235	1.454	.1197	**	.1257	*	58.6	**
235-230	1.201	.1277	**	.1665	**	43.4	**
230-225	1.387	.2208	**	.3454	**	30.2	**
225-220	1.737	.2783	**	.4371	**	44.1	**
220-215	1.693	.2838	**	.4509	**	52.6	**
215-210	1.902	.2219	**	.3040	**	64.3	**
210-205	2.217	.2351	**	.3093	**	51.8	**
205-200	1.814	.1675	**	.2011	**	51.7	**
200-195	1.413	.0639		.0188		40.1	**
195-190	1.342	.1264	**	.1529	*	41.1	**
190-185	.963	.0625		.0611		28.6	**
185-180	1.659	.2388	**	.3268	**	43.1	**
180-175	1.113	.0969	*	.1191		18.7	**
175-170	2.054	.2788	**	.4026	**	53.6	**
170-165	1.841	.2256	**	.3133	**	51.4	**
165-160	2.086	.3181	**	.4876	**	52.4	**

REGRESSION ANALYSES OF ACCELERATION NOISE

$$\text{Model: } \sigma_t \quad \sigma_M - C_1 u^2 + C_2 u^3$$

Section	σ_M	$C_1 \times 10^2$	Sign	$C_2 \times 10^4$	Sign	R^2	"F"
160-155	1.954	.3080	**	.4830	**	50.2	*
155-150	.998	.0984	**	.1248	*	24.2	*
150-145	2.428	.3599	**	.5395	**	42.4	*
145-140	2.087	.2453	**	.3497	**	47.9	*
140-135	1.930	.2783	**	.4203	**	52.2	*
135-130	2.049	.3133	**	.4738	**	54.4	*
130-125	.678	.8079	*	.1402	*	4.4	
125-120	.092	-.0230		-.0473		3.2	
120-115	.154	-.0304		-.0613		2.1	
115-110	.213	-.0290		-.0709		8.4	
110-105	1.234	.1512	*	.2144		9.0	*
105-100	1.098	.1034		.1240		17.5	*
100- 95	.145	-.0535		-.0965		3.3	
95- 90	1.221	.1180		.1428		16.8	
90- 85	2.314	.3159	**	.4607	**	14.1	*
85- 80	1.798	.2179	*	.3041	*	6.8	*
80- 75	.832	.0768		.0914		4.8	
75- 70	1.851	.2778	**	.4194	**	23.0	
70- 65	1.094	.0922		.1066		11.2	*
65- 60	3.037	.3476	**	.4576	**	37.2	*
60- 55	3.047	.3764	**	.5281	**	42.3	*
55- 50	1.695	.2105	**	.2988	**	35.5	*
50- 45	.546	.0576		.0940		1.4	
45- 40	1.333	.1600	*	.2183		8.0	*
40- 35	2.959	.4254	**	.6296	**	51.7	*
35- 30	1.090	.0919		.1073		10.0	*
30- 25	2.467	.3514	**	.5357	**	24.5	*
25- 20	1.949	.4358	**	.8022	**	36.0	*



RELATIONSHIP BETWEEN SPEED AND ACCELERATION NOISE DUE TO TRAFFIC INTERACTION (σ_t)

FIGURE 20

APPLICATIONS

Freeway Operations

It seems apparent that the maximum satisfaction that could be experienced by a motorist would result from a uniform uninterrupted freeway journey. This condition can be achieved by minimizing acceleration noise in the stream. If a relationship exists between acceleration noise and any of the above quantitative traffic parameters, the optimum operating conditions based on smoothness of flow can be determined.

When the volumes on a freeway are extremely light, an individual motorist can select a desirable speed because of the freedom of movement which he experiences. The acceleration noise of his trip is due primarily to his driving abilities and the geometrics of the freeway. As volumes increase, a level is reached at which an individual motorist finds it difficult to adjust laterally. Thus his speed in effect is influenced by the vehicle in front of him, which in turn is influenced by the vehicle in front of it, and so on downstream to the lead car in the platoon. The flow at this point approaches a uniform speed and uniform headways. Consequently, the operation is smooth and the acceleration noise is a minimum (the value of which would be dictated by the freeway geometrics). A further increase in demand is accompanied by an increase in internal friction which promotes some instability in the stream and increases acceleration noise. It is during this unstable flow condition that driver discomfort is beginning to be realized. Additional demand increases result in inevitable freeway breakdown and a rapid decrease in driver convenience and comfort.

It has been shown (Equation 28) that the optimum speed (u'_m) at minimum acceleration noise can be determined by calculating the points on the curve of zero slope. This is accomplished by setting the first derivative of the regression equation to zero and solving for u . The equation for the curve presented in Figure 20 was determined to be

$$\delta t = 1.693 - .00428u^2 + .000045u^3$$

The optimum speed is thus

$$u'_m = 42.1 \text{ mph}$$

The results of similar calculations of optimum quality speed are presented in Table 3. A study of Table 3 reveals that the quality of flow based on minimizing acceleration noise is a maximum at speeds ranging generally between 40 and 50 mph. A previous study¹ on the Gulf Freeway has shown that the speeds at maximum volume output range between 25 and 35 mph.

These maneuvers would naturally have a tendency to increase acceleration noise. As volumes continue to increase, a level is reached at which an individual motorist finds it difficult to adjust laterally. Thus his speed in effect is influenced by the vehicle in front of him, which in turn is influenced by the vehicle in front of it, and so on downstream to the lead car in the platoon. The flow at this point approaches a uniform speed and uniform headways. Consequently, the operation is smooth and the acceleration noise is a minimum (the value of which would be dictated by the freeway geometrics). A further increase in demand is accompanied by an increase in internal friction which promotes some instability in the stream and increases acceleration noise. It is during this unstable flow condition that driver discomfort is beginning to be realized. Additional demand increases result in inevitable freeway breakdown and a rapid decrease in driver convenience and comfort.

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$$\delta t = 1.693 - .00428u^2 + .000045u^3$$

The optimum speed is thus

$$u'_m = 42.1 \text{ mph}$$

The results of similar calculations of optimum quality speed are presented in Table 3. It is noticed that only those sections of the freeway exhibiting high correlation to the model were analyzed. A study of Table 3 reveals that the quality of flow based on minimizing acceleration is a maximum at speeds ranging generally between 40 and 50 mph. A previous study¹ on the Gulf Freeway has shown that the speeds at maximum volume output range between 25 and 35 mph.

Table 3

OPTIMUM SPEED (u') AND VOLUME (q'_m)

AT MINIMUM ACCELERATION NOISE

Section	u'_m (mph)	q'_m (vph)	Section	u'_m (mph)	q'_m (vph)
300-295			160-155	43	4710
295-290			155-150		
290-285			150-145	44	4440
285-280			145-140	47	3980
280-275			140-135	44	4530
275-270			135-130	44	4780
270-265			130-125		
265-260			125-120		
260-255			120-115		
255-250	48	4260	115-110		
250-245	48	3350	110-105		
245-240	56	3240	105-100		
240-235			100- 95		
235-230	51	4360	95- 90		
230-225	43	5400	90- 85	46	4500
225-220	42	4880	85- 80	48	
220-215	42	4300	80- 75		
215-210	48	3510	75- 70	44	4840
210-205	50	4390	70- 65		
205-200	55	3200	65- 60	51	2690
200-195			60- 55	47	3790
195-190			55- 50	47	3610
190-185			50- 45		
185-180	48	4380	45- 40		
180-175			40- 35	45	4330
175-170	46	4550	35- 30		
170-165	48	4030	30- 25	44	3350
165-160	43	4740	25- 20	36	4760

The relationship between location and optimum speed at minimum acceleration noise can best be illustrated in a profile map as in Figure 21. The distinction between optimum speed u'_m and critical speed u_m is also made in this presentation. Again, only locations where the regression equations were highly significant are presented. Examination of the speed profiles shows that the speeds at minimum acceleration noise are much higher than the speeds for maximizing volume output.

Increasing attention has been given to the possibility of and need for using variable speed control on urban freeway sections as a means of easing the accordion effects in a traffic stream as congestion develops. The speed limit signs may display normal speed limit signs or multivalued reduced speeds and would be controlled by command signals from a traffic regulating center. The optimum speed u'_m offers an ideal parameter as a basis for this type of control. Two neon matrices would be utilized with the left-hand or tens matrix capable of displaying digits 2, 3, 4, or 5. The right-hand or units matrix would display digits 0 and 5. The signs would be mounted over each lane. The advisory speeds would be determined at a given location from the speed profiles (see Figure 21). The idea would be to induce drivers to travel voluntarily at a speed that would minimize acceleration noise and perhaps postpone or eliminate breakdowns due to congestion.

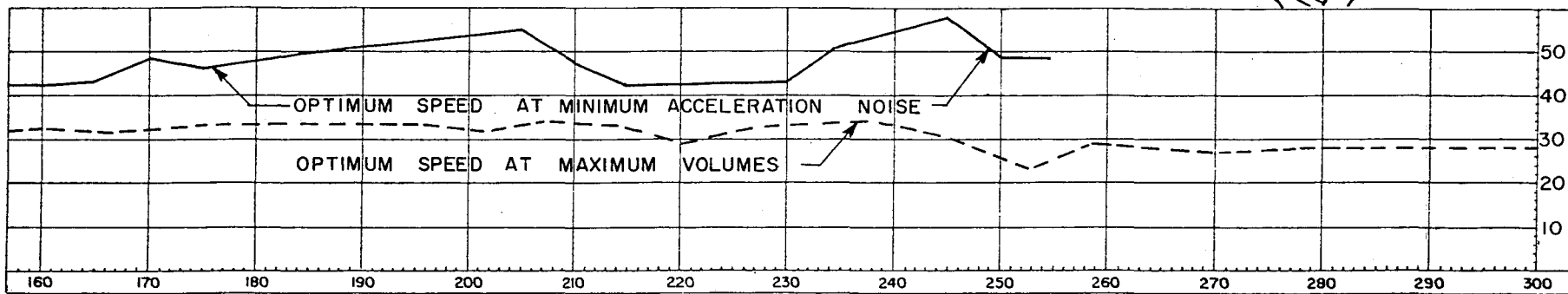
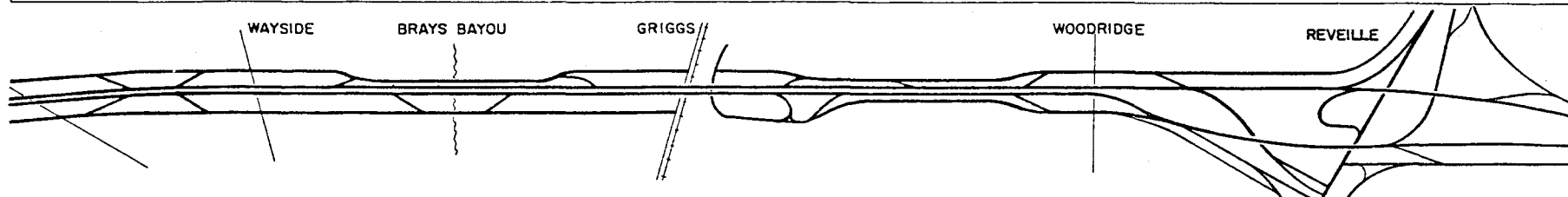
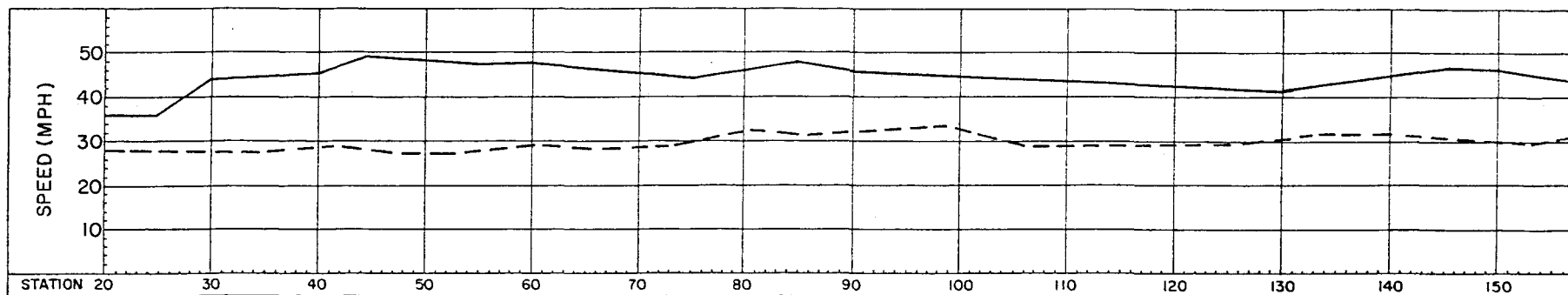
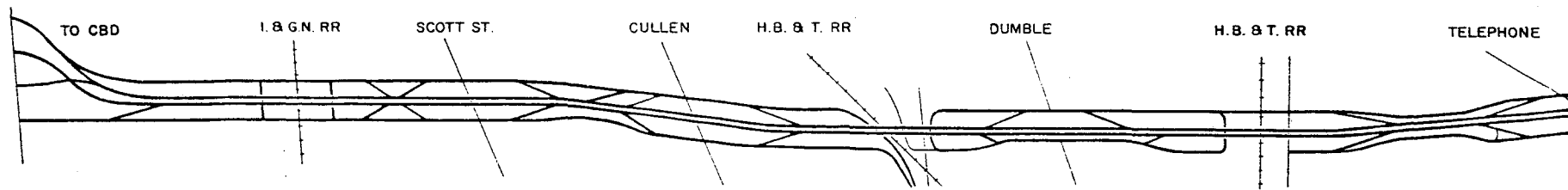
Because the control of vehicles entering the freeway, as against the control of vehicles already on the freeway, offers a more positive means of preventing congestion, considerable emphasis has been placed on the technique of ramp metering.

The utilization of the acceleration noise parameter as a means of evaluating freeway control and before-and-after studies has been demonstrated in this study as reported in the previous section. It is anticipated that this parameter will play an important role in the future in determining which of the many procedures for ramp and freeway control are the most effective.

Capacity and Level of Service

Greater dependency on motor vehicle transportation has brought about a need for greater efficiency in traffic facilities. The ability to accommodate vehicular traffic is a primary consideration in the planning, design, and operation of streets and highways. It is, however, not the only consideration. The individual motorist, for example, seldom interprets the efficiency of a facility in terms of the volume accommodated. He evaluates efficiency in terms of his trip--the service to him.

The original edition of the Highway Capacity Manual defined three levels of roadway capacity--basic capacity, possible capacity, and practical capacity. It was considered of prime importance that traffic volumes be accurately related to local operating conditions so that particular agencies could decide on the "practical" capacities for facilities within their jurisdiction. The manual recognized that



OPTIMUM SPEED

FIGURE 21

"practical" capacity would depend on the basis of a subjective evaluation of the quality of service provided.

The present Capacity Committee of the Highway Research Board has elected, in the new edition, to define a single parameter--possible parameter--for each facility. Possible capacity is simply the maximum number of vehicles that can be handled by a particular roadway component under prevailing conditions. The practical capacity concept has been replaced by several specific "service volumes" which are related to a group of desirable operating conditions referred to as level of service.

Ideally, all the pertinent factors--speed, travel time, traffic interruptions, freedom to maneuver, safety, comfort, convenience and economy should be incorporated in a level of service evaluation. The Committee has, however, selected speed and the service volume-to-capacity "v/c" ratio as the factors to be used in identifying level of service because "there are insufficient data to determine either the values or relative weight of the other factors listed."

Six levels of service, designated A through F from best to worst, are recommended for application in describing the conditions existing under the various speed and volume conditions that may occur on any facility. Level of Service A describes a condition of free flow; Level of Service E describes an unstable condition at or near capacity; Level of Service F gives a condition of forced flow. Levels of Service B, C, and D describe the zone of the stable flow with the upper limit set by the zone of free flow and the lower limit defined by Level of Service F. Although definitive values are assigned to these zone limits for each type of highway in the new manual, no explanation is given as to how these values were obtained. This is in no way intended as a criticism since it is recognized that the function of any manual is essentially that of a handbook and therefore, should not include a methodical discussion of the facts and principles involved and conclusions reached for every value between its covers.

The authors feel that much of the traffic engineer's dilemma can be attributed to his inability to relate capacity and level of service. If these volumes cannot be related quantitatively for an existing facility, there can be little hope for the designer to relate them for a facility that is still on the drafting board! The acceleration noise - internal energy model developed in this report provides a simple, rational means for measuring freeway capacity and level of service.

If equations (10) and (15) are expressed in terms of speed only, and then normalized, they become (for the special case of $n = 1$)

$$\frac{q}{q_m} = 4 \left[\left(\frac{u}{u_f} \right) - \left(\frac{u}{u_f} \right)^2 \right] \quad (35)$$

and

$$\frac{E}{T} = \frac{27}{4} \left[\left(\frac{u}{u_f} \right)^2 - \left(\frac{u}{u_f} \right)^3 \right] \quad (36)$$

The curves of equations (35) and (36) are plotted in Figure 22. The right side of the graph is the well known volume-speed relationship normalized so that the abscissa is the ratio of flow to capacity ("v/c" ratio) and the ordinate the ratio of speed to free speed. Substituting (20) in (36) gives

$$\frac{I}{T} = 1 - \frac{27}{4} \left[\left(\frac{u}{u_f} \right)^2 - \left(\frac{u}{u_f} \right)^3 \right] \quad (37)$$

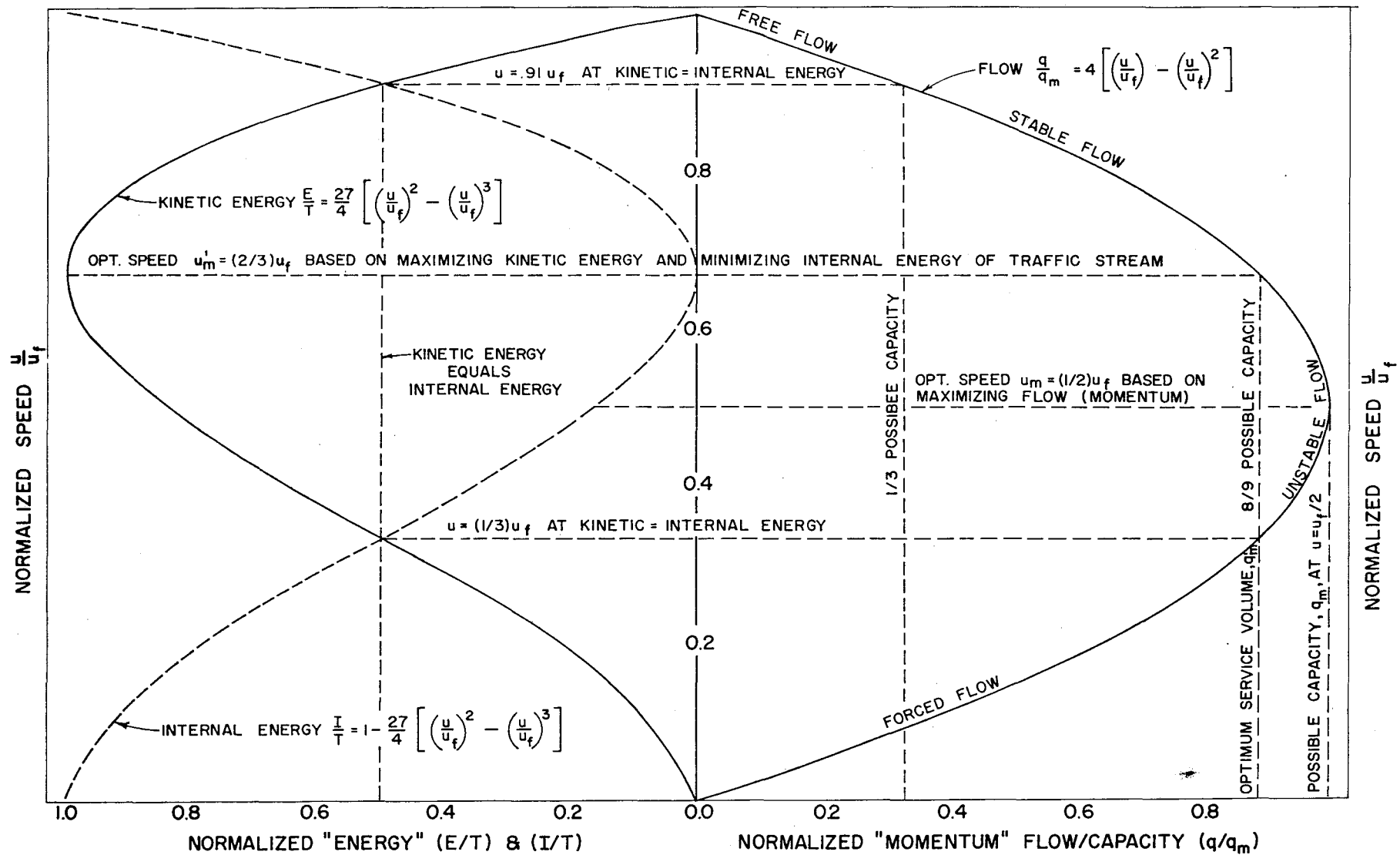
which is also plotted in Figure 22. Equating (36) and (37) gives the two speed parameters for which the kinetic energy equals the internal energy of the traffic stream. These values are $u = 1/3 u_f$ and $u = .91 u_f$. Proceeding to the right side of the graph, it is seen that the corresponding values for flow are $q = 8/9 q_m$ and $q = .328 q_m$ rounded off to $q = .35 q_m$. These two points plus the point defined as u'_m , q'_m serve to establish the four levels of service zones defined by the 1965 Highway Capacity Manual--free, stable, unstable and forced flow. In Table 4, the foregoing fundamental level of service criteria are summarized.

The significance of Figure 22 is that it provides a rational basis for defining level of service and relating it to the other traffic variables--speed, flow, and density. The relationships between level of service and traffic volume (flow) are analogous to the relationships in classical hydrodynamics between energy and momentum. Efficiency in a classical system is measured by the ratio of useful energy to total energy of E/T . Optimum operation occurs when lost energy I is at a minimum. In a traffic system, this concept of efficiency is manifest by maximizing the kinetic energy of the stream as a whole and minimizing the acceleration noise of the individual vehicles (internal energy).

The objective of freeways and other expressways is to provide high levels of service for high volumes of traffic. The traffic conditions existing at maximum E/T and minimum I/T , therefore, might logically be termed "critical level of service." Referring to the right side of Figure 22, it is seen that a small increase in demand above the volume existing at this critical level of service tends to greatly increase the density of the traffic stream accompanied inevitably by a sharp decrease in operating speed. That the traffic conditions k'_m and u'_m at the critical level of service are superior to those at possible capacity can be shown explicitly by dividing Equations 16 and 17 by 11 and 12 for $n = 1$.

$$k'_m = (2/3) k_m \quad (38)$$

$$u'_m = (4/3) u_m \quad (39)$$



QUANTITATIVE APPROACH TO LEVEL OF SERVICE USING THE "TOTAL ENERGY" – MOMENTUM ANALOGY

FIGURE 22

Table 4

LEVELS OF SERVICE AS ESTABLISHED BY ENERGY-MOMENTUM CONCEPT

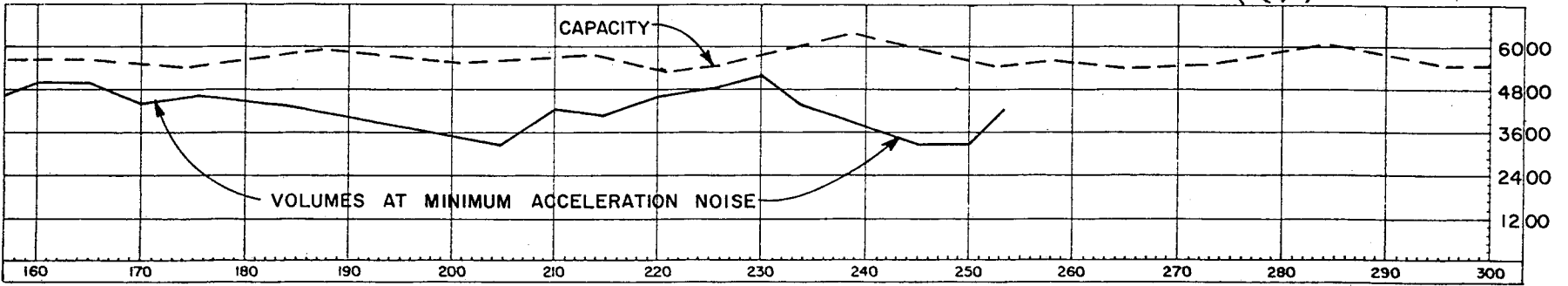
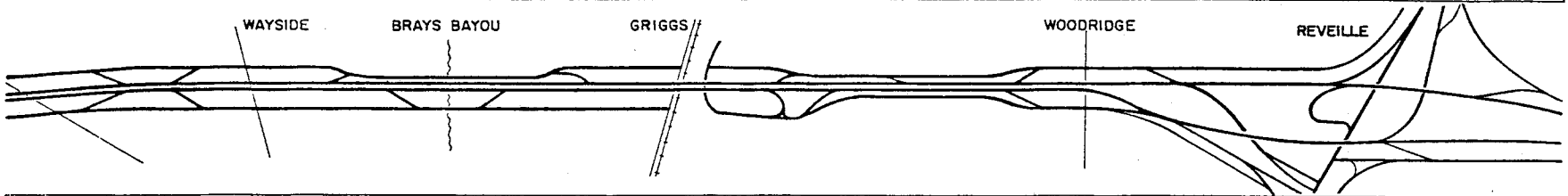
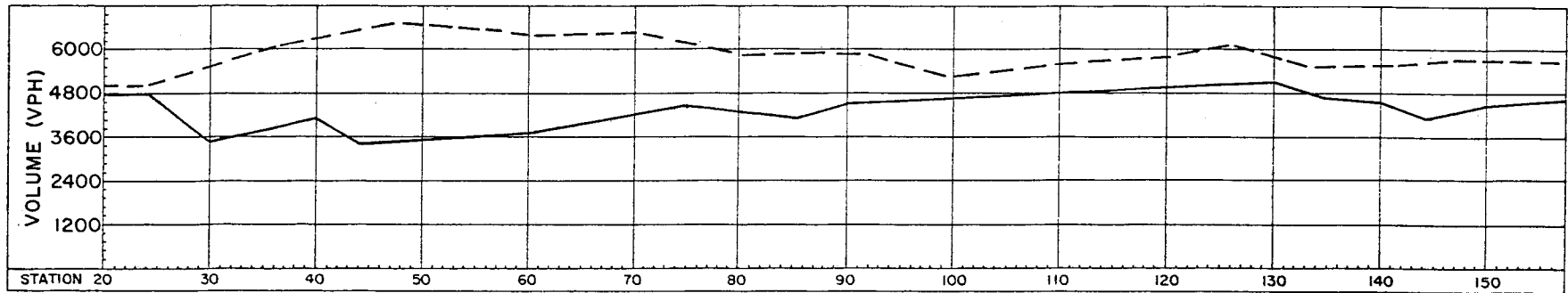
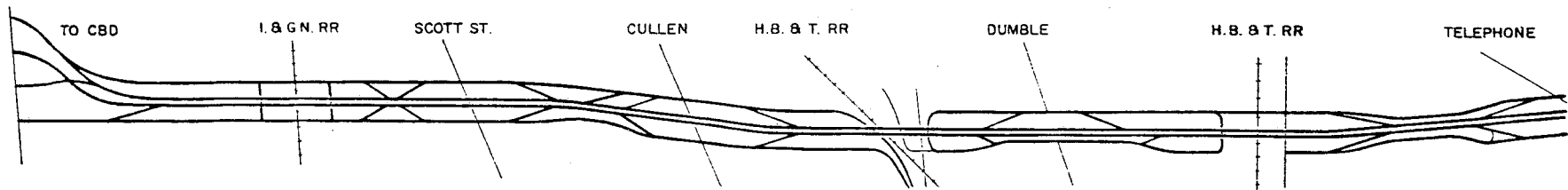
Level of Service Zone	Zone Limits (See Figure 22)		Description
	Upper	Lower	
Free Flow - A	u_f	$.91u_f, .35q_m$	Speeds controlled by driver desires and physical roadway conditions. This is the type of service expected in rural locations.
		$.83u_f, .55q_m$	Speed primarily a function of traffic density.
Stable Flow		$.75u_f, .75q_m$	The conditions in this zone are acceptable for freeways in suburban locations.
		u'_m, q'_m	The conditions in this zone are acceptable for urban design practice. The lower limit u'_m, q'_m represents the critical level of service.
Unstable Flow		u_m, q_m	A small increase in demand (flow is accompanied by a large decrease in speed leading to high densities and internal friction.
		$.33u_f, q'_m$	This type of high density operation can not persist and leads inevitably to congestion.
Forced Flow		0	Flows are below capacity and storage areas consisting of queues of vehicles form. Normal operation is not achieved until the storage queue is dissipated.

The density at the critical level of service is only 2/3 the density at possible capacity and the operating speed is 1/3 higher. Of course this is accomplished at a sacrifice in the traffic volume accommodated since

$$q'_m = (8/9) q_m \quad (40)$$

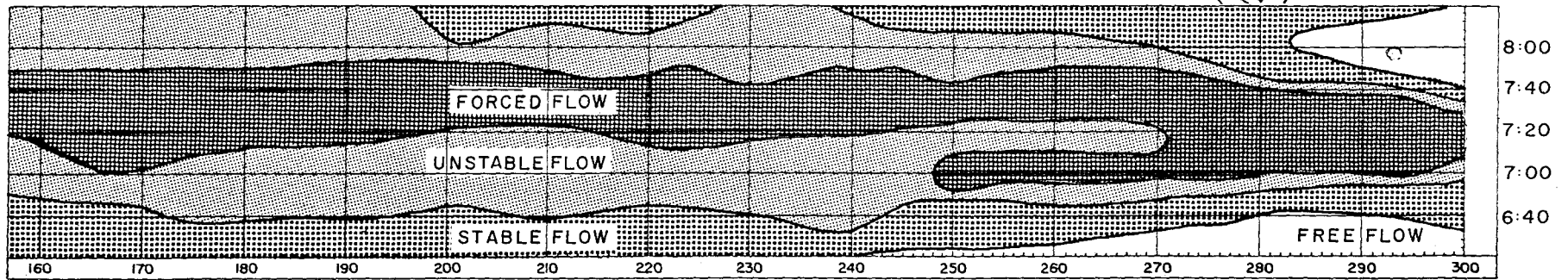
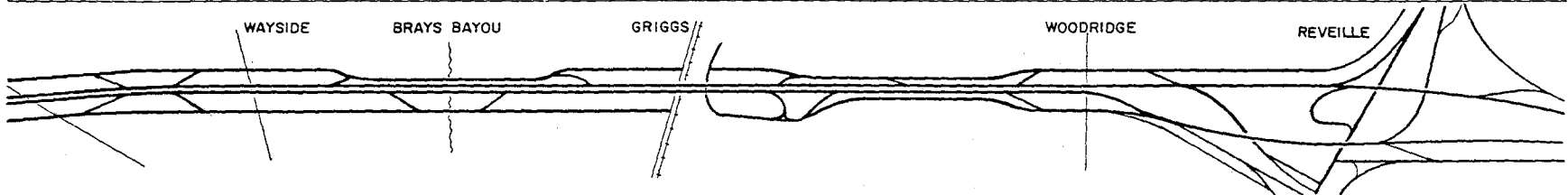
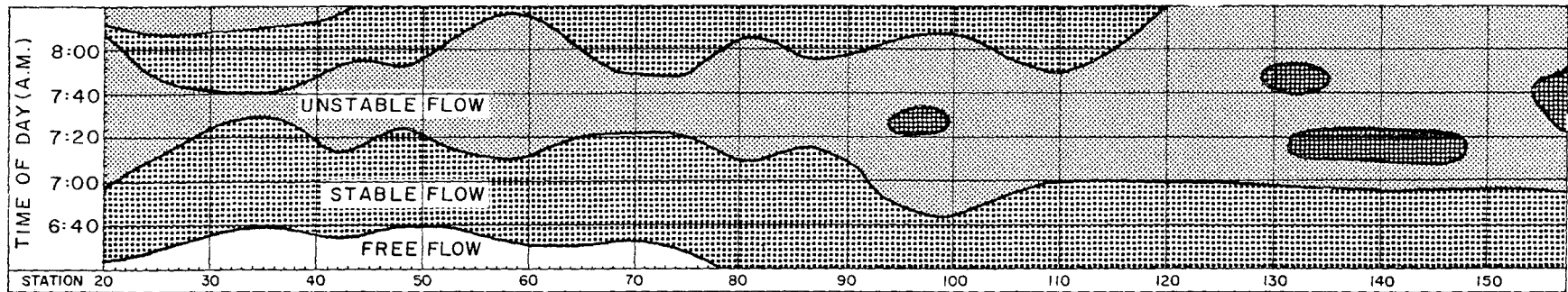
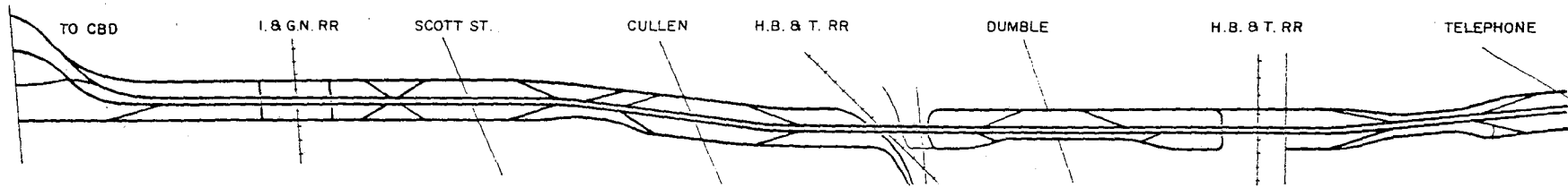
The acceleration noise measurements taken on the Gulf Freeway plotted in Figure 21 tend to substantiate the relationship stated in Equation 39 regarding the speed u'_m at critical level of service as compared to the speed u_m at critical flow (capacity). Figure 23 illustrates a similar comparison between the critical service volume q'_m and the capacity q_m as a basis for ramp metering, for example, places operation of the facility in the unstable zone of operation, and provides absolutely no safety factor against breakdowns due to statistical variability in demand.

Figure 24 illustrates the operation of the three inbound lanes of some six miles of the Gulf Freeway during the morning peak two hours as obtained using the moving vehicle study procedure described in this report. The figure represents a complete documentation of level of service in both time and space. This simple procedure affords a rational means for describing the level of operation on the facility--free flow, stable flow, unstable flow and forced flow--as established by the energy-momentum concepts illustrated in Figure 22.



OPTIMUM VOLUMES

FIGURE 23



LEVEL OF SERVICE CONTOURS
TUESDAY - INBOUND
FIGURE 24

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions may be drawn:

1. The standard deviation of the acceleration of a vehicle is called acceleration noise, σ . This parameter can be calculated from a speed-time graph of a vehicle's trip. If the acceleration noise for the vehicle is obtained in the absence of traffic, this factor can be ascribed to the geometrics of the facility, and is therefore called the natural noise, σ_N . If the acceleration noise for the vehicle is obtained during periods of normal freeway operation, the amount of acceleration noise in excess of the natural noise of the facility is due to the existing traffic interaction, σ_t .

2. Kinetic energy $\alpha k u^2$, is the energy of motion of the traffic stream.* Internal energy is the lost energy of the traffic stream and it is given by the acceleration noise, σ . The sum of kinetic energy E internal energy I is the total energy T; the units of traffic "energy" are those of acceleration. Energy, as expressed in terms of E and I is consistent with the level of service concept described in the 1965 Highway Capacity Manual. Thus, the kinetic energy of the stream fulfills the first level of service factor (speed and travel time) whereas internal energy of acceleration noise takes into account such level of service factors as traffic interruption, freedom to maneuver, safety, comfort, and operation costs.

3. If any two of the traffic variables -- k , u , q , or σ -- can be measured, then by using the energy model the graphs of Figure 22 can be drawn for short homogeneous freeway sections and the following traffic parameters calculated: k_m , u_m , q_m , k_m^E , u_m^I , q_m^I , and σ_N . Thus, it was possible for a driver in a test vehicle equipped with a recording speedometer measuring u and σ to obtain these 7 parameters for a six-mile section of the Gulf Freeway, plus a complete description of the level of service for the facility over the two-hour morning peak, by simply driving over the facility.

4. Acceleration noise can be a useful tool for measuring the changes in smoothness of flow resulting from on-range control and metering procedures. Since there is a good indication that acceleration noise is linearly related to the absolute difference in adjoining grades, it is apparent that this parameter might be useful in measuring the effects of geometric changes. Thus, acceleration noise has practical application in both operations and design.

Recommendations for future research include:

1. Further research is needed to determine the reason for lack of correlation to the tested models of acceleration noise within certain locations of the inbound Gulf Freeway.

*Symbols are defined in Appendix B.

2. More extensive study should be made to substantiate the relationship between freeway grades and acceleration noise.

3. The effects of other geometric considerations such as entrance and exit ramps as related to acceleration noise should be studied to determine the efficiency of various configurations.

4. Special studies should be made at location with high acceleration noise measurements especially in relation to accident experience.

5. To establish the level of service on parallel arterials to the Gulf Freeway during the peak period using the energy model employed for establishing the level of service on the freeway.

6. To measure the level of service on parallel arterials in conjunction with control or ramp metering experiments in order to develop data which can be compared to normal periods of operation on the Gulf Freeway. This would provide a cost-benefit approach to measuring the improvements provided by automatic surveillance and control for use by highway engineers and administrators.

APPENDICES

Appendix A

Mathematical Approach

The following is a development of an equation used by Jones and Potts¹² for approximating acceleration noise. If $v(t)$ and $a(t)$ are the speed and acceleration of a car at time t , then the average acceleration of the car for a trip of time T is

$$\alpha_{ave} = \frac{1}{T} \int_0^T a(t_i) dt = \frac{1}{T} [v(T) - v(0)]$$

The standard deviation of a set of n numbers x_1, x_2, \dots, x_n is denoted by s and is defined by

$$s = \left[\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right]^{1/2}$$

where \bar{x} is the mean of the x 's.

From Figure 25 it is seen that if $a(t_i)$ denotes the acceleration of a vehicle at time t_i , the square of the difference between any acceleration and the average acceleration is denoted by

$$\left[a(t_i) - \alpha_{ave} \right]^2$$

The summation of these differences over a time T becomes

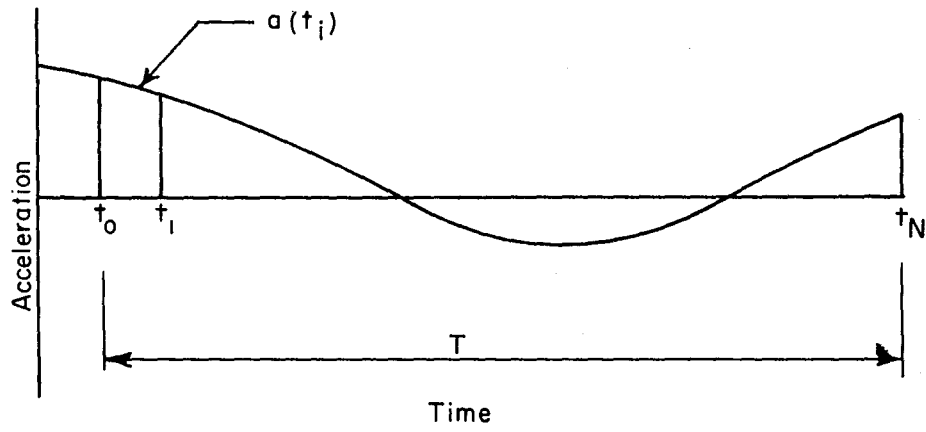
$$\int_0^T \left[a(t_i) - \alpha_{ave} \right]^2 dt$$

The number of t 's in the sample is equal to $\sum_{i=0}^T t_i = T$. Therefore the acceleration noise (σ) can be written

$$\sigma = \left[\frac{1}{T} \int_0^T \left[a(t_i) - \alpha_{ave} \right]^2 dt \right]^{1/2} \quad (1)$$

and

$$\sigma^2 = \frac{1}{T} \int_0^T \left[a(t_i) - \alpha_{ave} \right]^2 dt \quad (2)$$



ACCELERATION VS TIME

FIGURE 25

Expanding (2)

$$\begin{aligned} \sigma^2 &= \frac{1}{T} \int_0^T [a(t_i)]^2 dt - \frac{1}{T} \int_0^T 2a(t_i) \alpha_{ave} dt + \frac{1}{T} \int_0^T [\alpha_{ave}]^2 dt \\ \sigma^2 &= \frac{1}{T} \int_0^T [a(t_i)]^2 dt - 2 \alpha_{ave} \frac{1}{T} \int_0^T a(t_i) dt - [\alpha_{ave}]^2 \\ \sigma^2 &= \frac{1}{T} \int_0^T [a(t_i)]^2 dt - 2 [\alpha_{ave}]^2 + [\alpha_{ave}]^2 \\ \sigma^2 &= \frac{1}{T} \int_0^T [a(t_i)]^2 dt - [\alpha_{ave}]^2 \end{aligned} \quad (3)$$

The value of σ^2 can be estimated by approximating Equation (3) with

$$\sigma^2 \approx \frac{1}{T} \sum_{i=0}^T \left[\frac{\Delta v}{\Delta t} \right]^2 \Delta t - \left[\frac{(V_T - V_0)}{T} \right]^2$$

where V_0 and V_T are the initial and final velocities, respectively. If Δv is taken as a constant (say 2 mph) throughout the measurement, then

$$\sigma^2 = \frac{(\Delta v)^2}{T} \sum_{i=0}^T \frac{n^2}{\Delta t_i} - \left[\frac{(V_T - V_0)}{T} \right]^2 \quad (4)$$

where n is the number of speed changes of 2 mph in time Δt .

If the final velocity is the same as the initial velocity, the average acceleration is zero and the second term of Equation (4) drops out. When the acceleration noise is measured on a long stretch of highway where T is very large, the second term is very small and can be ignored.¹³ Therefore for relatively long trips

$$\sigma^2 = \frac{(\Delta v)^2}{T} \sum_{i=0}^T \frac{n^2}{\Delta t_i}$$

If Δt is measured for each successive speed change of 2 mph, that is n is equal to 1, the approximation equation of acceleration noise as developed by Jones and Potts¹³ becomes

$$\sigma = \left[\frac{(\Delta v)^2}{T} \sum_{i=0}^T \frac{1}{\Delta t_i} \right]^{1/2} \quad (5)$$

To calculate σ in units of feet/second/second (ft/sec²), Equation (5) becomes

$$\sigma = \left[\frac{(1.465)^2 (\Delta v)^2}{T} \sum_{i=0}^T \frac{1}{\Delta t_i} \right]^{1/2} \quad (6)$$

where Δv is in units of miles/hour
 T is in units of seconds
 Δt is in units of seconds

The deviations of the accelerations when a vehicle is stopped in traffic is zero. Since this type of situation would reduce the value of σ even though it adds to the annoyance and frustration of the motorist, σ is measured only while the vehicle is in motion and therefore T is taken as the running time of the vehicle.

If a continuous analogue record is made of a vehicle's speed vs. time while traversing on a highway, the acceleration noise of a vehicle can be determined using Equation (6). For every change in speed (Δv) of say 2, Δt is measured from the chart and the value $1/\Delta t$ can be accumulated up to time T .

Appendix . B

Traffic Flow Notation

Term	Symbol	Units
acceleration	a	dist./ time ²
acceleration noise (internal energy)	σ	"
maximum noise due to interaction	σ^M	"
natural noise	σ^N	"
total noise	σ^T	"
traffic interaction	σ^t	"
concentration (density)	k	veh./ dist.
critical concentration	k_m	"
jam concentration	k_j	"
optimum concentration	k'_m	"
constant of proportionality	c	--
energy (kinetic)	E	veh-dist./time ²
internal energy(acceleration noise)	E	"
total energy (max. accel. noise)	T	"
flow (volume, demand, momentum)	q	veh./time
critical flow (capacity	q_m	"
optimum flow	q'_m	"
velocity (speed), individual vehicle	x	dist./time
velocity (speed), traffic stream	u	"
critical speed	u_m	"
free speed	u^f	"
optimum speed	u'_m	"

Appendix C

Comparison of Speed Increments

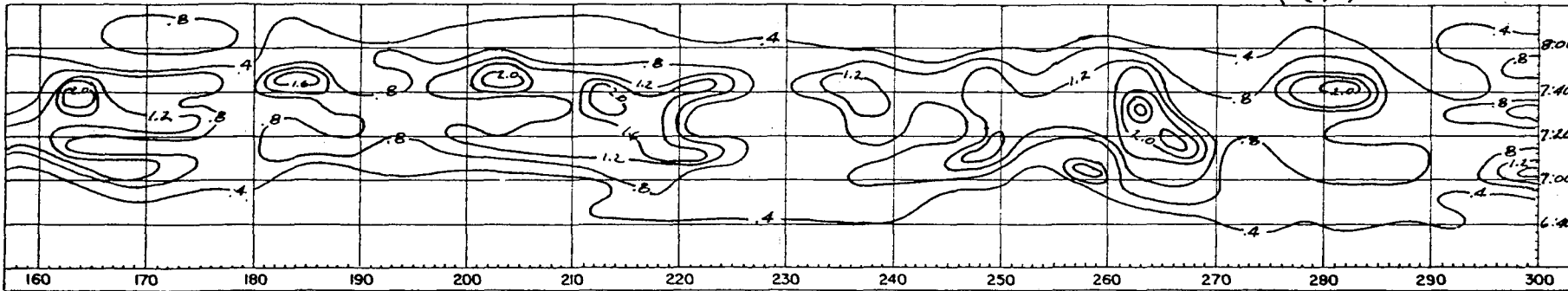
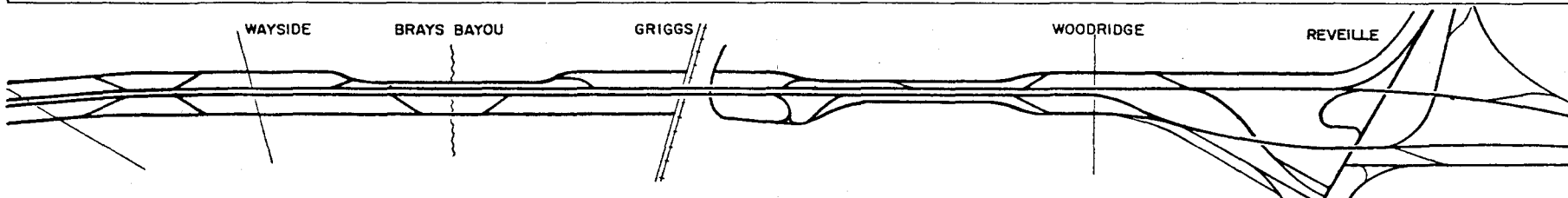
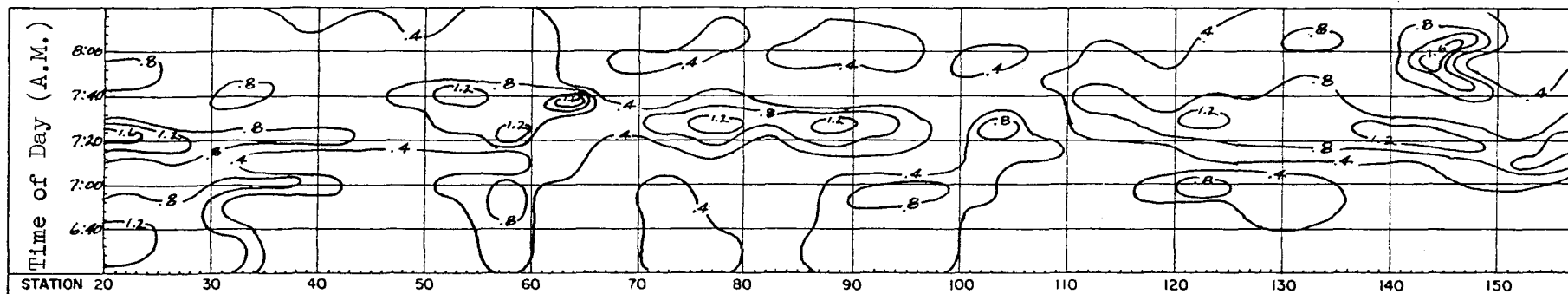
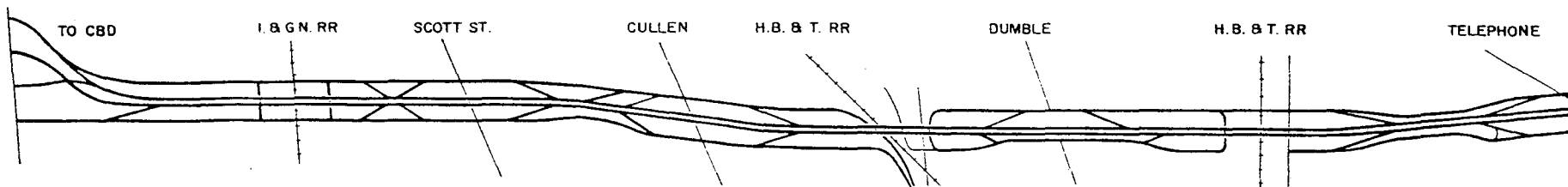
A problem arises regarding the selection of Δv for Equation (4) in Appendix A. It must be realized that the equation is merely an approximating equation and, therefore, the smaller the value of Δv selected, the closer the values of σ will approach the true condition.

The determinants which will affect the choice of this constant are obviously the limitations of the recording equipment and the accuracy needed for the study. Several types of speed recording equipment are available with varied degrees of speed ranges and with varied degrees of accuracy associated with reading the graphs. For example, one type of recorder is limited to 45 mph hour with increments of 5 mph on the chart recording. Other equipment can be adjusted to record speeds to any desired reasonable maximum with speed increments as low as 1 mph.

The purpose of the study might also be influenced in the decision of the accuracy of the σ estimate. One may be concerned about reaching preliminary estimates for a quick comparison during certain freeway control operations. Using a small value for Δv may be too time consuming at the time for the purpose in mind. A larger value would result in larger error but would present a quick impression of where additional control measures could be taken.

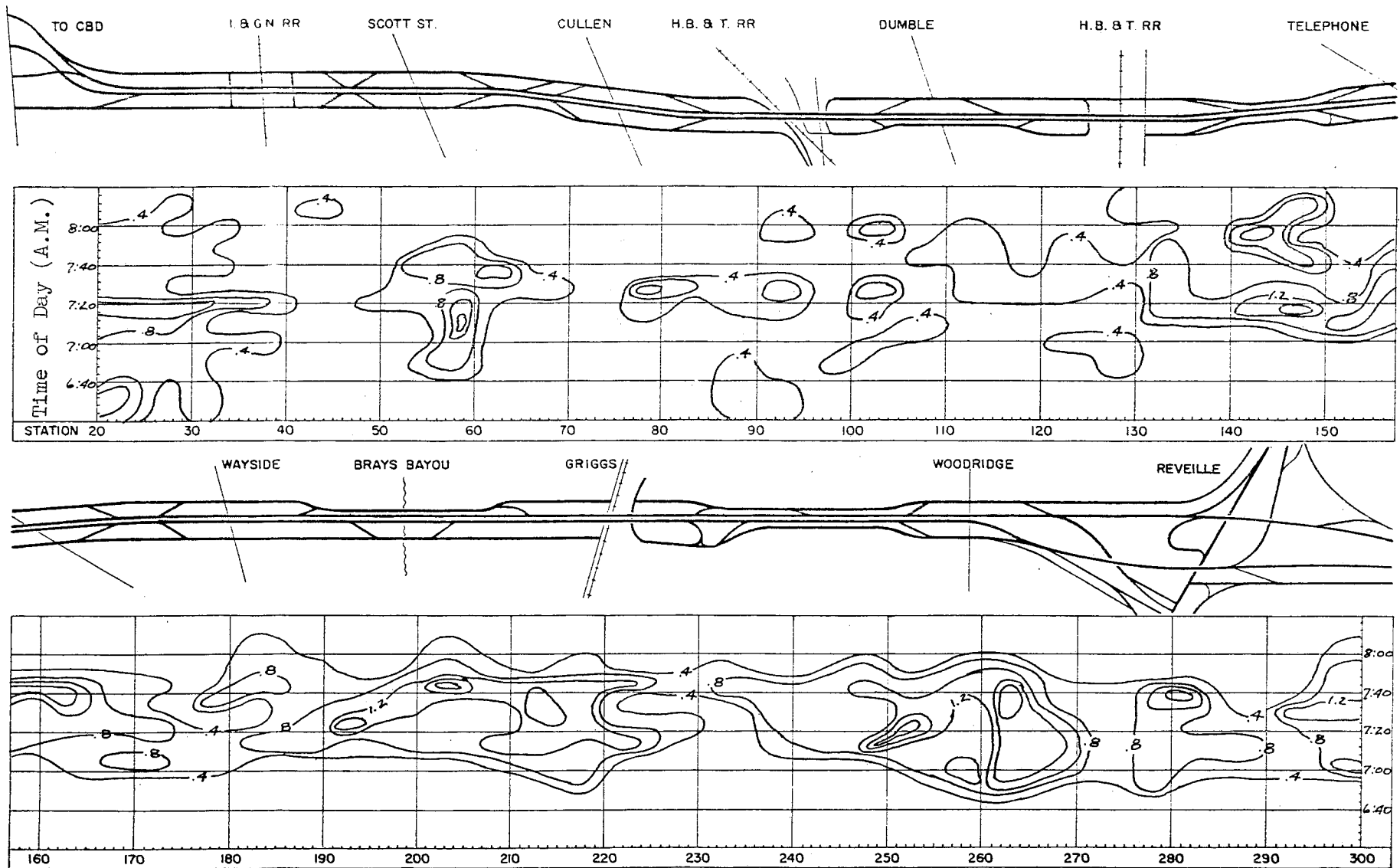
It must be emphasized that the selection of Δv to be used in Equation (4) will influence the accuracy of the final results. Therefore, care must be exercised when comparing results of other studies. Also, the values of σ determined from studies should be related to the constant chosen.

Contour maps of acceleration noise are illustrated in Figures 26, 27, and 28 based on varied constants of $\Delta v = 2, 4, \text{ and } 6$ respectively. It is noticed that as Δv increases, the contours are less dense because of the decreased sensitivity of σ . Figure 28 with $\Delta v = 6$ for example, illustrates the sections of freeway where the speed changed by as much as 6 mph. Speed deviations less than this amount resulted in zero acceleration noise. Figure 26 with $\Delta v = 2$, however, indicated areas with speed changes of 2 mph or greater.



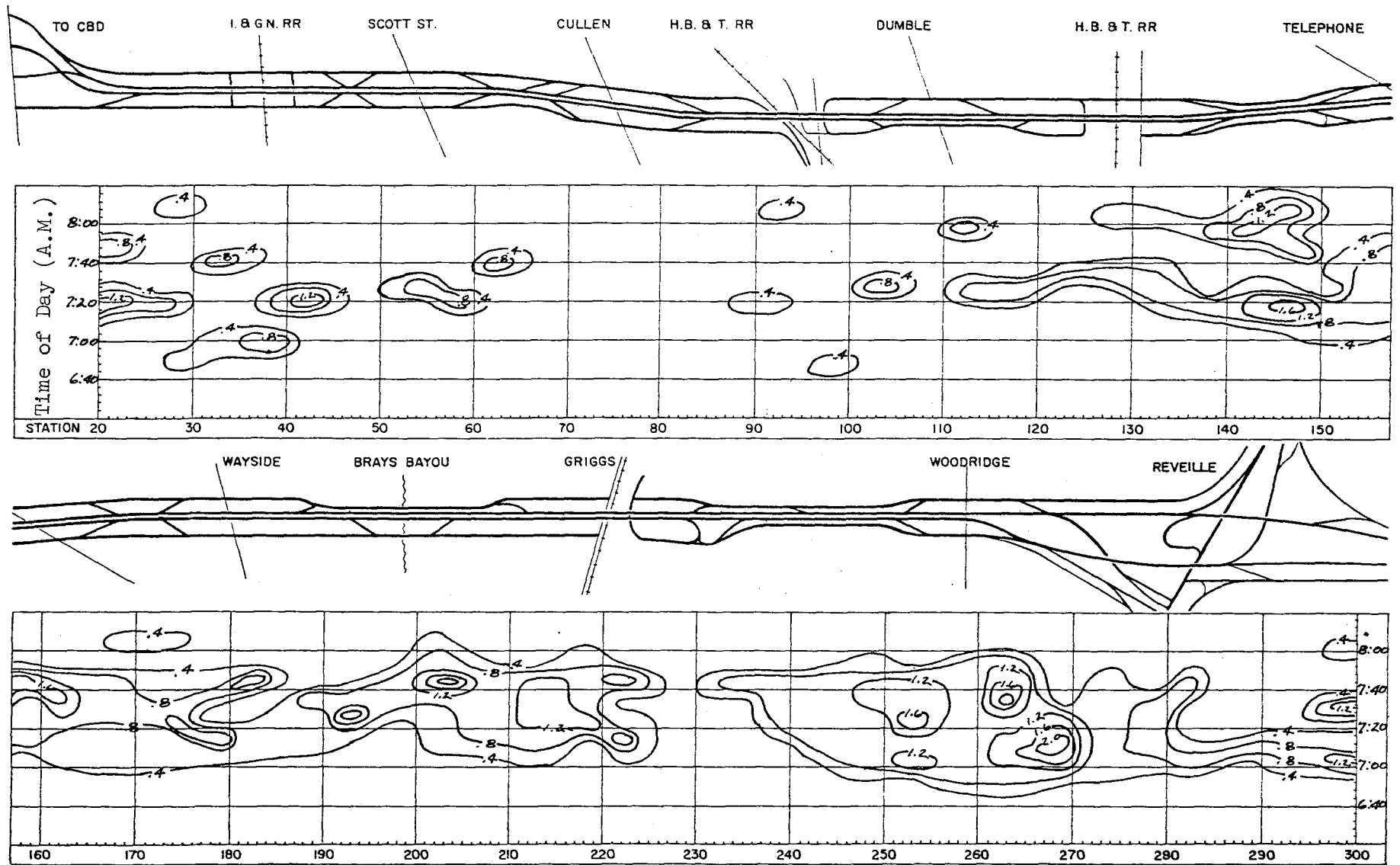
TOTAL ACCELERATION NOISE (σ_T) CONTOURS
 $\Delta V = 2$ MPH

Figure 26



TOTAL ACCELERATION NOISE (σ_T) CONTOURS
 $\Delta V = 4$ MPH

FIGURE 27



TOTAL ACCELERATION NOISE (σ_T) CONTOURS
 $\Delta V = 6$ MPH

FIGURE 28

Appendix D

Comparison of Acceleration Noise in Short Freeway Sections

Assume the speed data recorded are as shown in Figure 29. Two distinct conditions are presented. In the first section the speed drops 8 mph and then returns to the original speed. The second section illustrates a 16 mph reduction in speed. It is noted that the average acceleration in the first section is zero; however, the average acceleration in the second section is $.84 \text{ ft/sec}^2$ and is therefore very significant and cannot be ignored.

It is obvious that if only an acceleration or deceleration takes place within the short section, the mean acceleration takes on some value other than zero. The distribution of the accelerations is about this mean and the acceleration noise (by definition) may be very small. However, if there is a reversal of speeds within the section so that the speed at the end of the section approaches the speed at the beginning of the section, the average acceleration approaches zero and the second term of Equation (4) in Appendix A vanishes.

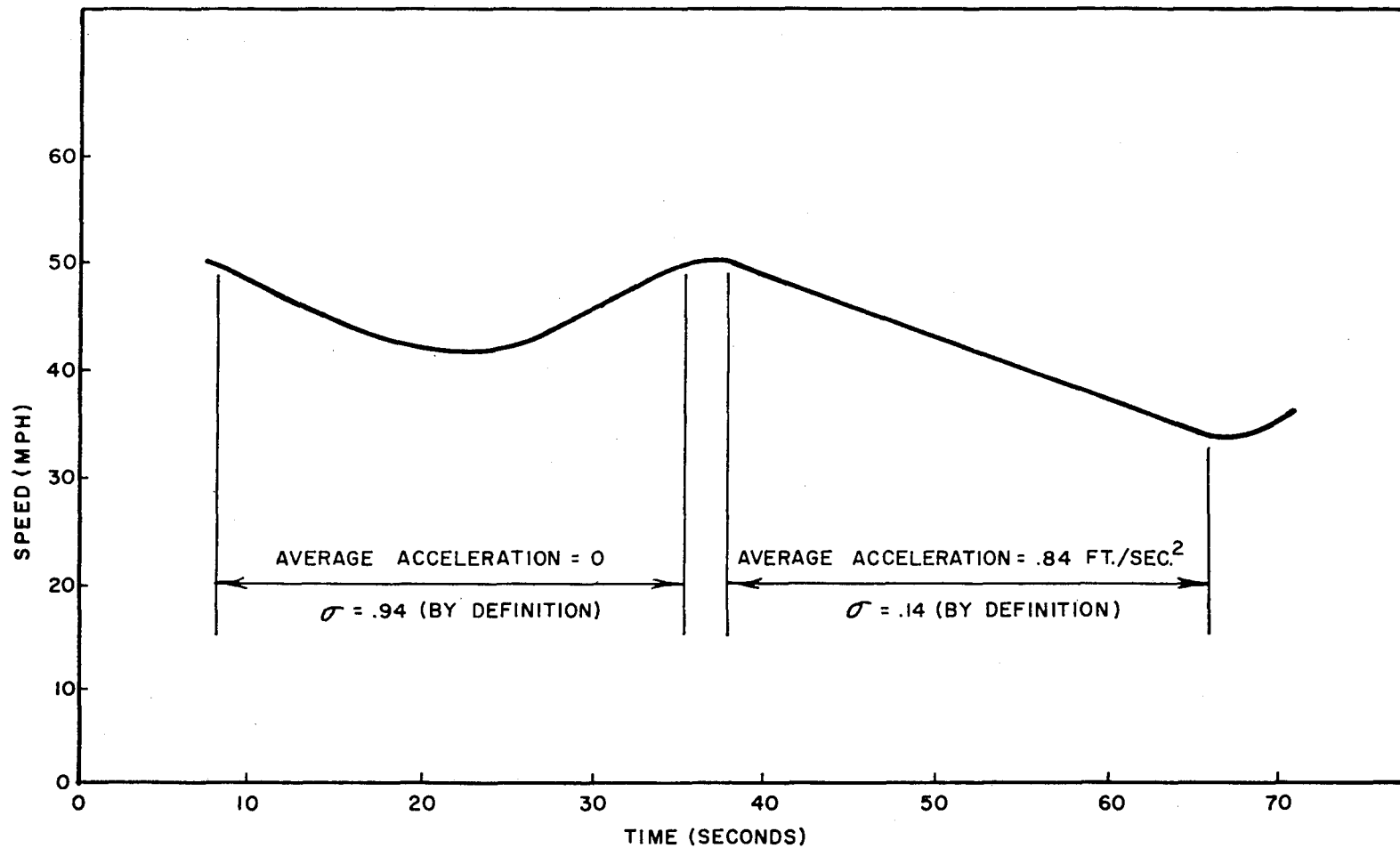
The acceleration noise calculated in both sections as shown in Figure 29 reveals that σ is much larger in the first section even though the speed reduction in the second section was twice as much. The reason for this contradiction can be explained by the fact that although the variance of the accelerations in the second section was lower, the distribution of the accelerations is about a mean value of $.84 \text{ ft/sec}^2$, whereas the distribution of the accelerations of the first section is about a mean of zero. One cannot conclude, however, that the flow in the second section is better because of a lower σ value.

In order to evaluate the disturbance of flow in both sections the acceleration distribution must have a relative base for comparison. An appropriate base would be a common mean acceleration. Since it is intended to use acceleration noise to measure the smoothness of flow it is desirable to evaluate the sections about an ordinate of zero acceleration.

The transformation of the distribution from any mean c , to the ordinate is accomplished by the following technique. The standard deviation in statistics is similar to the radius of gyration of an area under a frequency distribution curve about the ordinate through the centroid of that area is equal to the standard deviation of the distribution.

The meaning of the radius of gyration of an area can be interpreted as a length which when squared and multiplied by the area assumed to be concentrated at a point would have the same rotational effect as the actual area with respect to the given axis.

$$I_a = Ak_a^2$$



ACCELERATION NOISE IN SHORT FREEWAY SECTIONS
DETERMINED BY DEFINITION

FIGURE 29

where I_a = moment of inertia of the area
 A = area
 k_a = radius of gyration

The standard deviation of a frequency distribution considered as a set of n equal particles of area is the square root of the arithmetic mean of the squares of radial distances of these particles from the centroidal axis. In other words, the standard deviation is the radius of gyration k with respect to the centroidal axis.

$$\sigma = k$$

By use of the parallel axis theorem it is evident that the moment of inertia of an area about any axis b other than the centroidal axis is equal to the moment of inertia about the centroidal axis c plus the area of rotation times the square of the distance of b from c .

$$I_b = I_c + Ad^2$$

$$Ak_b^2 = Ak_c^2 + Ad^2$$

$$k_b^2 = k_c^2 + d^2$$

Substituting σ for k in (7)

$$\sigma_b^2 = \sigma_c^2 + d^2$$

when b is equal to zero the square of the acceleration noise about zero becomes

$$\sigma_0^2 = \sigma_c^2 + d^2 \quad (1)$$

where d is the distance from the origin to the mean of the simple distribution, that is, the distance to the average acceleration. Substituting Equation (4) from Appendix A into (1) and noting that d is equal to the average acceleration, Equation (1) can be written as

$$\sigma_0^2 = \left[\frac{(\Delta v)^2}{T} \sum_{i=0}^T \frac{n^2}{\Delta t_i} \right] - \left[\frac{(V_T - V_0)^2}{T} \right] + \left[\frac{(V_T - V_0)^2}{T} \right]^2$$

which reduces to

$$\sigma_0^2 = \frac{(\Delta v)^2}{T} \sum_{i=0}^T \frac{n^2}{\Delta t_i}$$

Assuming n is equal to 1, the relative acceleration noise for short sections of roadway becomes

$$\sigma_0 = \left[\frac{(\Delta v)^2}{T} \sum_{i=0}^T \frac{1}{\Delta t_i} \right]^{1/2} \quad (2)$$

Appendix E

Relationship Between Grades and Acceleration Noise

The data used for this analysis are listed in Table 5. Confidence limits of the average acceleration noise are set forth in Table 6. The principle of least squares was again applied to resolve the best fit to the available data. The hypothesized equation to be tested was as follows

$$Y = B_0 + B_1X_1 + B_2X_2$$

where Y acceleration noise (σ_N)

X_1 = absolute difference in adjoining grades

X_2 = length of grade

The estimating equation for regression becomes

$$y = b_0 + b_1X_1 + b_2X_2$$

The standard regression program from the Data Processing Center, Texas A&M University was utilized to determine the partial regression coefficients and to make the proper tests of significance.

The results of the regression analysis indicated that the independent variable length of grade (X_2) was not significant at the .05 level. The absolute value of the grades however was highly significant. These results prompted a second analysis of only the first independent variable X_1 . The results of this second analysis showed a very significant linear relationship between the absolute difference in adjoining grades and acceleration noise. The final regression equation was as follows:

$$Y = -.01415 + .04523X_1$$

A summary of these results is presented in Table 7.

The constant term b_0 in the above equation represents y intercept of the regression curve or the value of acceleration noise at zero grade. Based on the data available for the natural noise phase of this study, it is logical to hypothesize that the noise at tangent grade is zero, that is $b_0 = 0$. A procedure for testing the hypothesis that the intercept is zero was used to determine whether the acceleration noise at tangent grade is zero. The test was not significant at the .05 level and the conclusion was drawn that the regression line passes through

the origin. Results of this test are presented in Appendix F.

A final regression was made to determine the value of B_1 with $B_0 = 0$. The final relationship was determined to be

$$Y = .4367X_1 \quad (19)$$

where $Y =$ natural acceleration noise (σ_N)

$X_1 =$ absolute difference in adjoining grades.

Regression results are tabulated in Table 8.

Table 5

TABLE OF GRADES WITHIN THE STUDY AREA*

Location	Percent Plus Grade	Percent Minus Grade	Absolute Difference	Length (ft.)	Acceleration Noise (σ_N) (ft./sec. ²)
Woodridge Rd	3.0	3.0	6.0	1200	0.18
Griggs Rd.	5.0	5.0	10.0	1900	0.44
Wayside Dr.	1.5	1.5	3.0	1200	0.11
Telephone Rd.	2.5	2.5	5.0	1200	0.29
South H.B. & T. RR	5.0	5.0	10.0	1550	0.42
North H.B. & T. RR	5.0	5.0	10.0	1500	0.49
Cullen Rd.	5.0	5.0	10.0	1200	0.51
Scott St.	5.0	5.0	10.0	1000	0.36
I. & G. N. RR	5.0	5.0	10.0	1500	0.40

* Excluding grades at the major interchanges.

Table 6

CONFIDENCE LIMITS OF NATURAL ACCELERATION NOISE (σ_N) ON GRADES

Location	Acceleration Noise (σ_N)	Variance	Confidence Limits
Woodridge Rd.	.18	.030	.18 \pm .08
Griggs Rd.	.44	.009	.44 \pm .04
Wayside Dr.	.11	.024	.11 \pm .07
Telephone Rd.	.29	.060	.29 \pm .11
81 South H. B. & T. RR	.42	.019	.42 \pm .07
North H. B. & T. RR	.49	.039	.49 \pm .10
Cullen Road	.51	.046	.51 \pm .11
Scott St.	.36	.021	.36 \pm .07
I. & G. N. RR	.40	.018	.40 \pm .07

Table 7

RESULTS OF REGRESSION ANALYSIS OF ACCELERATION NOISE (Y) ON GRADES

$$Y = b_0 + b_1X_1 + b_2X_2$$

Analysis	"F" Test	b_0	b_1	b_2	R^2
Absolute difference in adjoining grades and length of grade	12.3**	-.09827	.04177**	.00008 ⁻	80.4
Absolute difference in adjoining grades only	25.3**	-.01415	.04523**	N/A	78.4

Table 8

RESULTS OF REGRESSION ANALYSIS OF ACCELERATION NOISE (Y)
ON GRADES, $b_0 = 0$

$$Y = b_1X_1$$

Analysis	"F" Test	b_1	R^2
Absolute difference in adjoining grades	255.8**	.04367**	97.3

- X_1 Absolute difference in adjoining grades.
 X_2 Length of grade.
⁻ Not significant at .05 level.
 ** Highly significant relationship.
 N/A Not applicable.
 R^2 Percent of variance explained by multiple regression.

Appendix F

Test to Determine Whether the "Y" Intercept of the Linear Expression of Grades (X_1) and Acceleration Noise (Y) is Zero

For a given linear equation of the form $Y = A + BX$ where A represents the Y intercept of the line and B is the slope of the line, the hypothesis that $A = 0$ is rejected whenever

$$|t| = \left| \frac{a - A}{S_{Y/X} \sqrt{\frac{1}{n} + \frac{x^2}{\sum_{i=1}^n (x_i - \bar{x})^2}}} \right| \geq t_{\infty/2, n-2}$$

where a is an estimate of A, and $S_{Y/X}$ is an estimate of the variability about the line. $S_{Y/X}^2$ is given by

$$S_{Y/X}^2 = \left(\sum_{i=1}^n (y_i - \bar{y})^2 - b \left[\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) \right] \right) 1/n-2$$

where b is an estimate of B.

For the relationship $Y = -.01415 + .04523X_1$

$$t_{.025, 7} = 2.365$$

$$|t| = .1715 < t_{.025, 7}$$

therefore, conclude that $A = 0$.

Appendix G

Regression of Speed Versus Acceleration Noise

The resulting equation of the model was written as

$$Y = B_0 - B_1X_1 + B_2X_2$$

The problem of determining whether the variability in Y can be explained by the variability in X_1 and X_2 is basically a problem of finding an equation that best explains the data. This is accomplished by estimating the parameters (B_0 , B_1 , and B_2) of the curve. The best method of estimating these parameters is known as the theory of least squares. This principle in application to the above equation would result in a curve with the statistics b_0 , b_1 , and b_2 (partial regression coefficients) such that the sum of the squares of the deviations (residuals) between the ordinates of the data points and the ordinates of the curve are minimized.

$$R^2 = \sum_{i=0}^N (Y_i - B_0 + B_1X_1 - B_2X_2)^2 \quad (20)$$

where R = residual.

The normal equations found by differentiating (20) and replacing B_i by their estimates b_i are

$$Nb_0 + b_1 \Sigma X_1 + b_2 \Sigma X_2 = \Sigma Y_i \quad (21)$$

$$b_0 \Sigma X_1 + b_1 \Sigma X_1^2 + b_2 \Sigma X_1 X_2 = \Sigma X_1 Y_i \quad (22)$$

$$b_0 \Sigma X_2 + b_1 \Sigma X_1 X_2 + b_2 \Sigma X_2^2 = \Sigma X_2 Y_i \quad (23)$$

The constant b_1 is termed the net regression of Y holding X_2 constant and b_2 is the net regression of Y holding X_1 constant. These coefficients can therefore be conveniently written

$$b_1 = b_{y1.2}$$

$$b_2 = b_{y2.1}$$

Equations (21) through (23) can thus be written

$$Nb_y + b_{y1.2} \Sigma X_1 + b_{y2.1} \Sigma X_2 = \Sigma Y_i \quad (24)$$

$$b_y \Sigma X_1 + b_{y1.2} \Sigma X_1^2 + b_{y2.1} \Sigma X_2 = \Sigma X_1 Y_i \quad (25)$$

$$b_y \Sigma X_2 + b_{y1.2} \Sigma X_1 X_2 + b_{y2.1} \Sigma X_2^2 = \Sigma X_2 Y_i \quad (26)$$

Several techniques have been devised for the solution of a system of equations. For this study, a standard regression program from the Data Processing Center, Texas A&M University was utilized. This program employs an inverse matrix technique.

After the statistics b_i have been determined, two other significant tests must be made. First of all, an analysis of variance must be made to test the over-all significance of the regression. Secondly, a more detailed test will be required of each regression coefficient in order to assess the contribution of each.

An F-ratio calculated from an analysis of variance table, is used to test the significance of the apparent dependence of the dependent variables. The null hypothesis tested by the F-ratio is $B_{y1.2} = B_{y2.1} = 0$.

The quantity $(b_{y1.2} - B_{y1.2})/S_{by1.2}$ is distributed as t. To test the significance of each partial regression coefficient, the hypothesis $B_{y1.2} = B_{y2.1} = 0$ is tested by the Students t-test:

$$t_1 = b_{y1.2} / S_{by1.2}$$

$$t_2 = b_{y2.1} / S_{bt2.1}$$

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