THEORETICAL APPROACHES TO THE STUDY AND CONTROL OF FREEWAY CONGESTION

by

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INTRODUCTION

Among the important problems arising from the population explosion is that of congestion. Although this overcrowding manifests itself in virtually every aspect of modern life, nowhere is it as dramatically exhibited in our society as on our streets and highways. The most vigorous attempt to eliminate traffic congestion was the development of the "freeway," a concept based on (a) the reduction of vehicle-to-vehicle conflicts, (b) elimination of vehicle-to-pedestrian conflicts, and (c) elimination of delay producing traffic control devices. Still, practically all major cities are troubled with severe peak hour congestion on newly completed freeways.

Previous studies have shown that a relatively small increase in traffic demand on an already heavily loaded expressway can have a very detrimental effect on the operating conditions for all traffic on the facility. Speeds and volumes are reduced, densities and travel times are increased, and the highway immediately loses much of its efficiency. Theoretically, it seems desirable to either ration or completely deny access to the freeway at certain locations.

The automatic evaluation of freeway traffic flow will be a vital element of any future control system. Research must be directed toward the evaluation of the use of surveillance and sensing equipment, and the simultaneous investigation of those characteristics of traffic flow related to freeway congestion which can be determined and treated by such equipment. Inherent in the problem are the complexities and manifestations of freeway traffic congestion. Traffic inefficiency is reflected in such factors as changes in speed, the frequency of speed changes, a low over-all speed, time loss, and driver discomfort. These factors are influenced by such additional variables as traffic demand, traffic composition, lane occupancy, highway geometrics and the drivers' desired speeds. Before it can be decided just what level of efficiency is economically feasible, or stated another way, how much congestion should be tolerated during peak periods, congestion must be defined quantitatively in terms of known and measurable parameters of traffic flow theory.

In recent years a number of descriptive theories of vehicular traffic have been put forward. These theories are based on mathematical models of two basic types: deterministic and stochastic. Included in the first category are the continuous flow models and individual vehicle models which describe the macroscopic and microscopic properties of the traffic flow phenomena respectively. Included in the second group are the probability distribution hypotheses and queueing theory.

GENERALIZATION OF DETERMINISTIC MODELS OF TRAFFIC FLOW

If vehicular traffic is assumed to behave as a one dimensional compressible fluid of concentration (density), k, and fluid velocity, u, then the conservation of vehicles is explained by

$$\frac{\partial k}{\partial t} + \frac{\partial (ku)}{\partial x} = 0. \tag{1}$$

Taking the derivative of the product in the second term yields

$$\frac{\partial k}{\partial t} + \frac{u\,\partial k}{\partial x} + \frac{k\,\partial u}{\partial x} = 0. \tag{2}$$

It is well established in the theory of traffic flow that vehicular velocity varies inversely with the concentration of vehicles,

$$u = f\,(k). \tag{3}$$

As a consequence of (3),

$$\frac{\partial u}{\partial k} = \frac{\partial u}{\partial x}\,\frac{\partial x}{\partial k} = \frac{du}{dk} = u'. \tag{4}$$

Solving for $\partial u/\partial x$ from (4) and substituting in (2), one obtains the following equation of continuity for single lane vehicular traffic flow,

$$\frac{\partial k}{\partial t} + [u + ku']\,\frac{\partial k}{\partial x} = 0. \tag{5}$$

Now, if it is assumed that a driver adjusts his velocity at any instant in accordance with the traffic conditions about him as expressed by $k^n\,\partial k/\partial x$, the acceleration of the traffic stream at a given place and time becomes,

$$\frac{du}{dt} = -c^2\,k^n\,\frac{\partial k}{\partial x}\;. \tag{6}$$

Taking the total derivative of $u = f\,(x,\,t)$ gives

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\,\frac{dx}{dt} + \frac{\partial u}{\partial t}\,\frac{dt}{dt}\;, \tag{7}$$

where $dx/dt = u$ and $dt/dt = 1$. Substituting (7) in (6) yields

$$\frac{\partial u}{\partial x} u + \frac{\partial u}{\partial t} + c^2 k^n \frac{\partial k}{\partial x} = 0. \quad (8)$$

From (4), it is equally apparent that

$$\frac{\partial u}{\partial t} = u' \frac{\partial k}{\partial t} . \quad (9)$$

Solving for $\partial u/\partial k$ from (4) and substituting in (8), substituting for (9) in (8), then dividing through by u', equation (8) becomes

$$\frac{\partial k}{\partial t} + \left[u + \frac{c^2 k^n}{u'} \right] \frac{\partial k}{\partial x} = 0, \quad (10)$$

which is the generalized equation of motion. The nontrivial solution of equations (5) and (10) is obtained by equating the quantities within the brackets,

$$(u')^2 = c^2 k^{(n-1)}. \quad (11)$$

Finally, because of the inverse relation between velocity and concentration,

$$u' = -ck^{(n-1)/2}. \quad (12)$$

Greenberg[1] has solved (12) for $n = -1$ obtaining

$$u = c \ln (k_j/k). \quad (13)$$

The solution of (12) for $n > -1$ is as follows:

$$u = \frac{-2c}{(n+1)} k^{(n+1)/2} + C_1, \; n > -1, \quad (14)$$

where the constant of integration is to be evaluated by the boundary conditions inherent in the vehicular velocity-concentration relationship. Thus, since no movement is possible at jam concentration, k_j,

$$C_1 = \frac{2c}{(n+1)} k_j^{(n+1)/2}, \; n > -1, \quad (15)$$

and

$$u = \frac{2c}{(n+1)} \left[k_j^{(n+1)/2} - k^{(n+1)/2} \right], \; n > -1. \quad (16)$$

Similarly, the implication exists that a driver is permitted his free speed, u_f, only when there are no other vehicles on the highway ($k = 0$). Therefore,

$$u_f = \frac{2c}{(n+1)} k_j^{(n+1)/2}, \; n > -1. \tag{17}$$

and the constant of proportionality takes on the following physical significance -

$$c = \frac{(n+1)\, u_f}{2k_j^{(n+1)/2}}, \; n > -1. \tag{18}$$

Substitution of (18) in (16) yields the generalized equations of state,

$$u = u_f \left[1 - \left(\frac{k}{k_j}\right)^{(n+1)/2} \right], \; n > -1 \tag{19}$$

$$q = ku = ku_f \left[1 - \left(\frac{k}{k_j}\right)^{(n+1)/2} \right], \; n > -1. \tag{20}$$

Differentiation of (20) with respect k equated to zero gives the optimum concentration, k_m, which is that concentration yielding the maximum flow of vehicles:

$$\frac{dq}{dk} = \left[1 - \frac{(n+3) \; k^{(n+1)/2}}{2k_j^{(n+1)/2}} \right] u_f = 0,$$

$$k_m = [(n+3)/2]^{-2/(n+1)} \, k_j, \; n > -1. \tag{21}$$

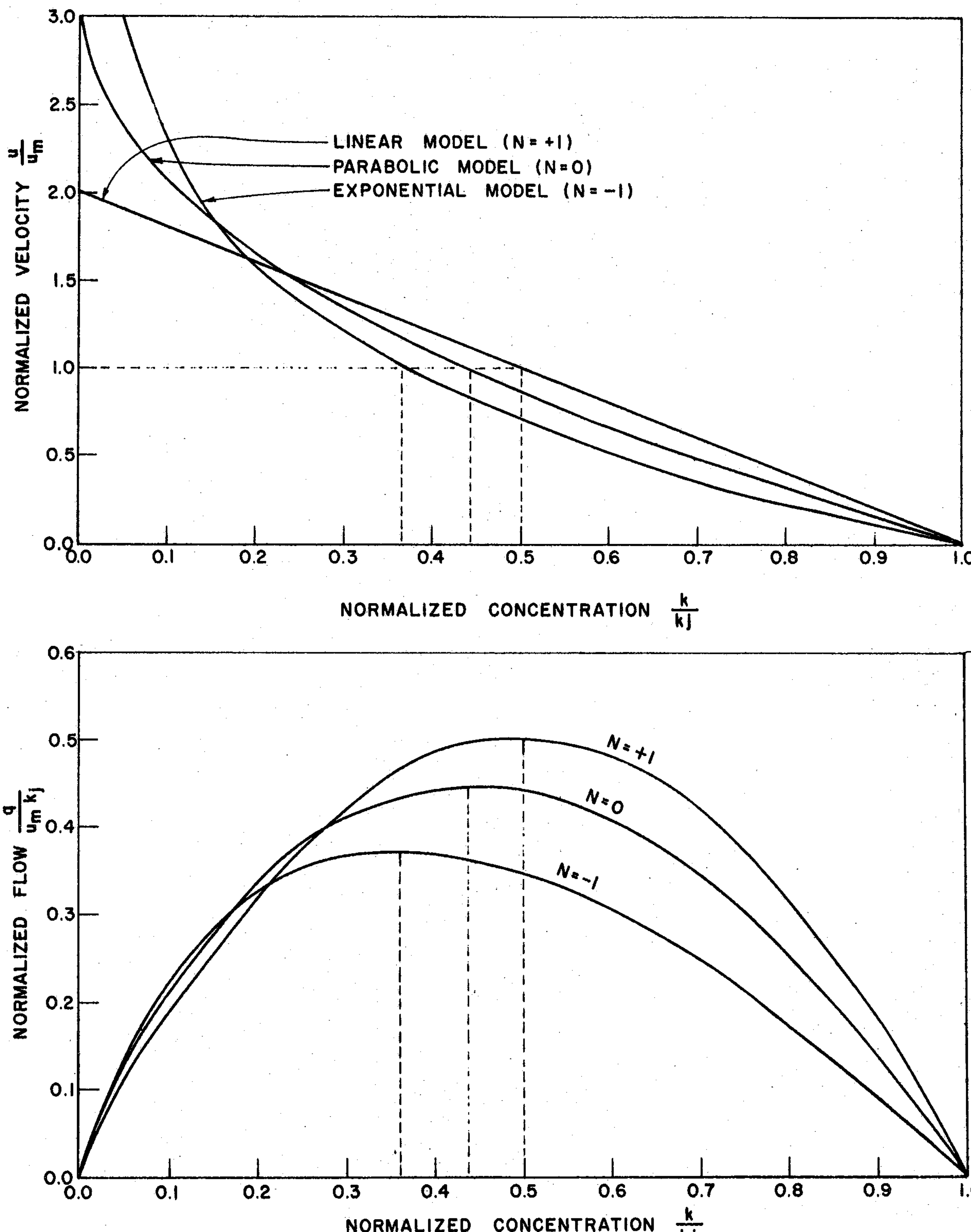
Substituting (21) in (19), one obtains the optimum velocity,

$$u_m = \left[\frac{n+1}{n+3}\right] u_f, \; n > -1. \tag{22}$$

The maximum flow of vehicles of which the highway lane is capable (capacity) is obtained from the product of (21) and (22),

$$q_m = \left[\frac{(n+1)}{(1/2)^{2/(n+1)} \; (n+3)^{[2/(n+1)]+1}} \right] u_f \, k_j, \; n > -1. \tag{23}$$

Some special cases of (19) through (23) have proven to be of significance. Greenshields'[2] linear model is obtainable by setting n = 1, while Drew[3] has discussed the case for n = 0. These cases, as well as Greenberg's model, are summarized in Figure 1 and Table 1.



SOLUTION OF GENERALIZED EQUATION OF TRAFFIC MOTION

$\frac{du}{dt} + c^2 k^n \frac{\partial k}{\partial x} = 0$, for N = −1, 0, +1

FIGURE 1

TABLE 1

COMPARISON OF MACROSCOPIC MODELS OF TRAFFIC FLOW

Element	General ($n>-1$)	Exponential ($n=-1$)	Parabolic ($n=0$)	Linear ($n=1$)
Eq. of Motion	$\frac{du}{dt} + c^2 k^n \frac{\partial k}{\partial x} = 0$	$\frac{du}{dt} + \frac{c^2}{k}\frac{\partial k}{\partial x} = 0$	$du + c^2 \frac{\partial k}{\partial x} = 0$	$\frac{du}{dt} + c^2 k \frac{\partial k}{\partial x} = 0$
Constant of Proportionality	$c = [(n+1)u_f]/2k_j^{(n+1)/2}$	u_m	$u_f/2k_j^{1/2}$	u_f/k_j
Eq. of State	$q = ku_f\left[1-\left(\frac{k}{k_j}\right)^{(n+1)/2}\right]$	$ku_m \ln\left(\frac{k_j}{k}\right)$	$ku_f\left[1-\left(\frac{k}{k_j}\right)^{1/2}\right]$	$ku_f\left[1-\frac{k}{k_j}\right]$
Optimum Concentration	$k_m = [(n+3)/2]^{-2/(n+1)} k_j$	k_j/e	$4k_j/9$	$k_j/2$
Optimum Speed	$u_m = [(n+1)/(n+3)]u_f$	c	$u_f/3$	$u_f/2$
Capacity	$q_m = \frac{(n+1)u_f k_j}{(1/2)^{2/(n+1)}(n+3)^{[2/(n+1)]+1}}$	$\frac{1}{e}u_m k_j$	$\frac{4}{27}u_f k_j$	$\frac{1}{4}u_f k_j$
Wave Vel.	$\frac{dq}{dk} = u_f\left[1-\frac{(n+3)}{2}\left(\frac{k}{k_j}\right)^{(n+1)/2}\right]$	$u_m\left[\ln\left(\frac{k_j}{k}\right)-1\right]$	$u_f\left[1-\frac{3}{2}\left(\frac{k}{k_j}\right)^{1/2}\right]$	$u_f\left[1-\frac{2k}{k_j}\right]$

Typical of some of the car-following laws that have been proposed are those that express the performance of a vehicle in terms of its velocity and position with respect to the vehicle immediately preceding it,

$$\ddot{x}_i (t+T) = a [\dot{x}_{i-1} (t) - \dot{x}_i (t)] [x_{i-1} (t) - x_i (t)]^{-m}. \tag{24}$$

Equation (24) states that the acceleration of a car, $\ddot{x}_i$, at a delayed time, T, is directly proportional to the relative speed of the car, $\dot{x}_i$, with respect to the one ahead, $\dot{x}_{i-1}$, and inversely proportional to the headway of the car, $x_{i-1} - x_i$. Since the right side of (1) is of the form dy/y^m, integration of (24) yields

$$\dot{x}_i (t+T) = a \ln [x_{i-1} (t) - x_i (t)] + C_1, \quad m = 1, \tag{25}$$

and

$$\dot{x}_i (t+T) = (-m+1)^{-1} a [x_{i-1} (t) - x_i (t)]^{-m+1} + C_2, \quad m > 1. \tag{26}$$

The constants of integration are evaluated by observing that the velocity of a car approaches zero as its headway approaches the effective length of each car, L;

$$C_1 = a \ln L, \tag{27}$$

$$C_2 = -(-m+1)^{-1} a L^{-m+1}, \quad m > 1. \tag{28}$$

Substituting for C_1 and C_2, equations (25) and (26) become

$$\dot{x}_i (t+T) = a \ln \; L^{-1} [x_{i-1} (t) - x_i (t)], \quad m = 1, \tag{29}$$

$$\dot{x}_i (t+T) = (m-1)^{-1} a \left\{ L^{-(m-1)} - [x_{i-1} (t) - x_i (t)]^{-(m-1)} \right\}, \quad m > 1. \tag{30}$$

Equation (29) is due to Gazis, Herman and Potts[4] who showed that the traffic equation of state could be derived from the microscopic car following law just as the gas equation of state can be derived from the microscopic law of molecular interaction. Since the space headway is the reciprocal of concentration, k, equations (29) and (30) become

$$u = a \ln (k_j/k) \tag{31}$$

and

$$u = (m-1)^{-1} a (k_j^{m-1} - k^{m-1}), \quad m > 1. \tag{32}$$

The constant of proportionality is evaluated at $u = u_f$ and $k = o$ giving

$$a = \left[\frac{(m - 1)}{k_j^{m-1}} \right] u_f, \quad m > 1. \tag{33}$$

Special cases of (32), as well as the relationship of the macroscopic parameteters "c" and "n" to the microscopic parameters "a" and "m" are shown in Table 2.

TABLE 2

COMPARISON OF MICROSCOPIC MODELS OF TRAFFIC FLOW

Element	General (m > 1)	m = 1	m = 3/2	m = 2
Eq. of Motion	$\ddot{x}_i = \dfrac{a(\dot{x}_{i-1} - \dot{x}_i)}{(x_{i-1} - x_i)^m}$	$\ddot{x}_i = \dfrac{a(\dot{x}_{i-1} - \dot{x}_i)}{(x_{i-1} - x_i)}$	$\ddot{x}_i = \dfrac{a(\dot{x}_{i-1} - \dot{x}_i)}{(x_{i-1} - x_i)^{3/2}}$	$\ddot{x}_i = \dfrac{a(\dot{x}_{i-1} - \dot{x}_i)}{(x_{i-1} - x_i)^2}$
Constant of Proportionality	$a = (m-1)\, u_f\, k_j^{-(m-1)}$	u_m	$u_f/2k_j^{1/2}$	u_f/k_j
Eq. of State	$q = ku_f\, [1 - \left(\frac{k}{k_j}\right)^{m-1}]$	$ku_m \ln\left(\frac{k_i}{k}\right)$	$ku_f\, [1 - \left(\frac{k}{k_j}\right)^{1/2}]$	$ku_f\, [1 - \left(\frac{k}{k_j}\right)]$
Macroscopic Counterpart (See Table 1)	$n = 2m - 3$	$n = -1$	$n = 0$	$n = +1$

APPLICATION OF DETERMINISTIC MODELS

The applicability of these deterministic models to freeway traffic was tested on the Gulf Freeway in Houston, Texas (Figure 2). Time-lapse aerial photography with a 60% overlap was utilized to ensure a given point on the freeway appearing on 3 consecutive photos (Figure 3). Six flight runs were made in the direction of the traffic being studied, inbound during the morning peak. Since a given vehicle appeared on at least 3 consecutive photos, individual vehicular speeds, accelerations, and space headways were measured. The observations were compared (on a lane basis) to the 3 macroscopic models in Table 1 and the 3 microscopic models in Table 2.

Regression analyses based on the macroscopic hypotheses of equations 13 and 19 ($n = 0$ and $n = +1$) are summarized in Table 3. Statistical tests were, in general, highly significant on each of the 3 freeway lanes, as well as on the total traffic on all 3 lanes. The microscopic analyses, however, were inconclusive. A constant of proportionality "a" was calculated for every freeway vehicle based on its performance and position with respect to the vehicle in front of it. The physical significance of "a" is indicated in Table 2 for the 3 microscopic models tested. The values obtained were extremely variable; approximately one-eighth of the values were negative indicating that, even under conditions of heavy traffic, the opportunity for changing lanes reduces a driver's necessity to respond to the performance of the car in front of him.

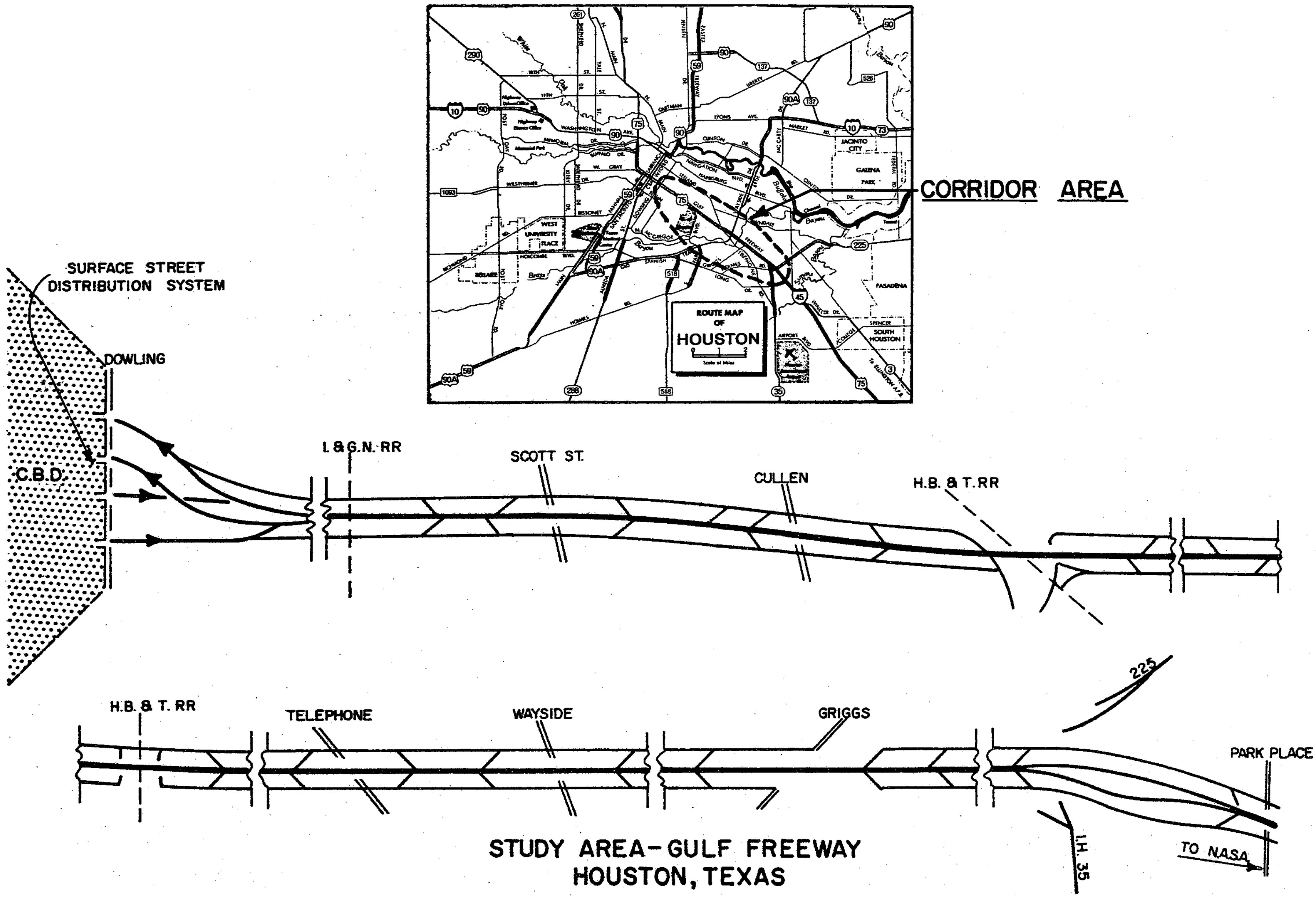
Essential to the development of freeway control techniques is the determination of suitable control parameters. Among the many techniques for controlling freeway traffic, ramp metering at entrance ramps and changeable-advisory speed limit signs located on the freeway itself offer the most promise. "Capacity", q_m, and "optimum speed," u_m, represent two ideal control parameters. Figures 4 and 5 illustrate continuous speed and capacity profiles for the outside lane of the 6-mile stretch of the Gulf Freeway. "Free speeds", u_f. are also shown on Figure 4 for the linear and parabolic models ($u_f = \infty$ for the exponential model).

Because the control of vehicles entering the freeway, as against the control of vehicles already on the freeway, offers a more positive means of preventing congestion, considerable emphasis is being placed on the technique of ramp metering. Entrance ramp metering may be oriented to either the freeway capacity or freeway demand. A capacity-oriented ramp control system restricts the volume rate on the entrance ramps in order to prevent the flow rates at downstream bottlenecks from exceeding the capacities of the bottlenecks. Figure 5 illustrates a capacity profile for traffic on all 3 inbound lanes of the Gulf Freeway. Bottleneck sections along with their respective control capacities are evident.

TABLE 3

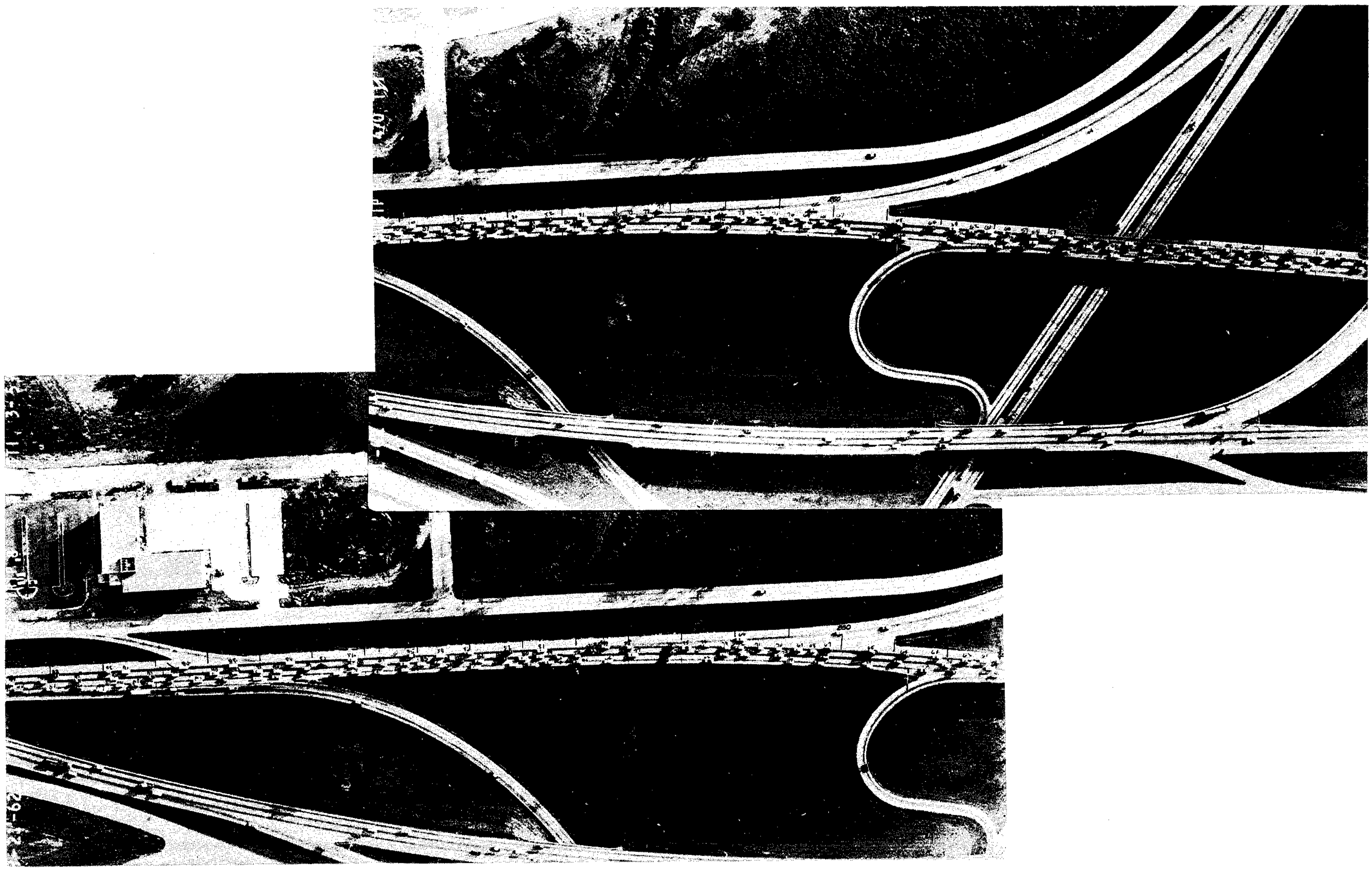
Regression Analyses of Equations of State (3 Lane Total)

Station	$u = a - bk$ b	a	t	$u = a - bk^{1/2}$ b	a	$\ln k = a - bu$ b	a
306 - 288	.129	52.8	7.52**	3.32	71.0	.044	6.20
299 - 281	.115	50.9	32.16**	3.14	69.2	.050	6.35
292 - 274	.112	52.0	18.37**	3.20	72.0	.047	6.41
286 - 268	.132	54.3	40.60**	3.47	74.6	.046	6.35
280 - 262	.131	53.3	30.60**	3.34	72.2	.048	6.38
273 - 255	.142	55.3	13.49**	3.74	78.1	.041	6.26
267 - 249	.141	56.0	11.87**	3.89	81.3	.038	6.25
261 - 243	.102	46.7	4.98**	2.09	54.4	.069	6.77
254 - 236	.143	58.0	7.79**	4.03	84.8	.035	6.20
248 - 230	.173	66.1	20.77**	4.66	95.5	.032	6.18
241 - 223	.175	64.6	11.53**	4.56	92.6	.032	6.15
235 - 217	.181	63.0	10.93**	4.67	91.9	.032	6.09
229 - 211	.167	59.7	4.85**	4.32	86.6	.032	6.06
223 - 205	.182	64.5	15.84**	4.91	96.5	.030	6.08
216 - 198	.205	67.8	10.00**	5.33	101.5	.028	6.00
210 - 192	.176	62.3	8.82**	4.45	89.2	.035	6.17
204 - 186	.190	65.5	19.03**	4.86	95.4	.032	6.12
197 - 179	.176	64.3	14.32**	4.55	92.6	.033	6.20
190 - 172	.197	66.4	7.99**	5.11	99.0	.028	6.04
183 - 166	.200	65.9	4.97**	5.01	97.0	.027	5.97
176 - 158	.181	63.2	7.12**	4.57	91.8	.032	6.16
169 - 151	.179	63.0	11.14**	4.31	88.5	.036	6.28
162 - 144	.154	60.0	8.78**	3.59	79.7	.043	6.51
155 - 137	.157	60.1	4.18*	3.78	82.3	.034	6.23
148 - 130	.167	61.3	6.06**	4.03	85.1	.035	6.25
141 - 123	.158	61.2	5.03**	3.68	82.2	.038	6.41
134 - 118	.140	58.3	4.00*	3.18	76.0	.042	6.50
128 - 110	.145	58.1	3.60*	3.19	75.4	.041	6.44
121 - 103	.051	45.6	.90	1.08	51.2	.029	5.91
115 - 97	.153	57.8	3.93*	3.14	73.7	.049	6.71
108 - 90	.222	66.4	4.85**	4.75	91.7	.034	6.12
101 - 83	.194	64.8	2.67	4.14	86.8	.028	5.94
95 - 77	.165	61.9	1.65	3.35	78.7	.023	5.67
89 - 71	.176	64.0	1.47	3.79	84.2	.021	5.62
82 - 64	.126	58.0	2.87*	2.75	72.7	.048	6.81
76 - 58	.121	58.0	3.16*	2.67	71.7	.053	6.99
69 - 51	.127	57.7	2.73*	2.98	74.8	.043	6.62
63 - 45	.114	55.7	2.80*	2.67	71.1	.049	6.86
56 - 38	.110	55.5	1.95	2.48	69.1	.039	6.51
50 - 32	.131	57.6	3.79*	2.88	73.1	.051	6.91
44 - 26	.122	54.2	3.37*	2.62	67.9	.053	6.88
37 - 19	.137	54.2	4.60**	2.96	69.8	.053	6.75



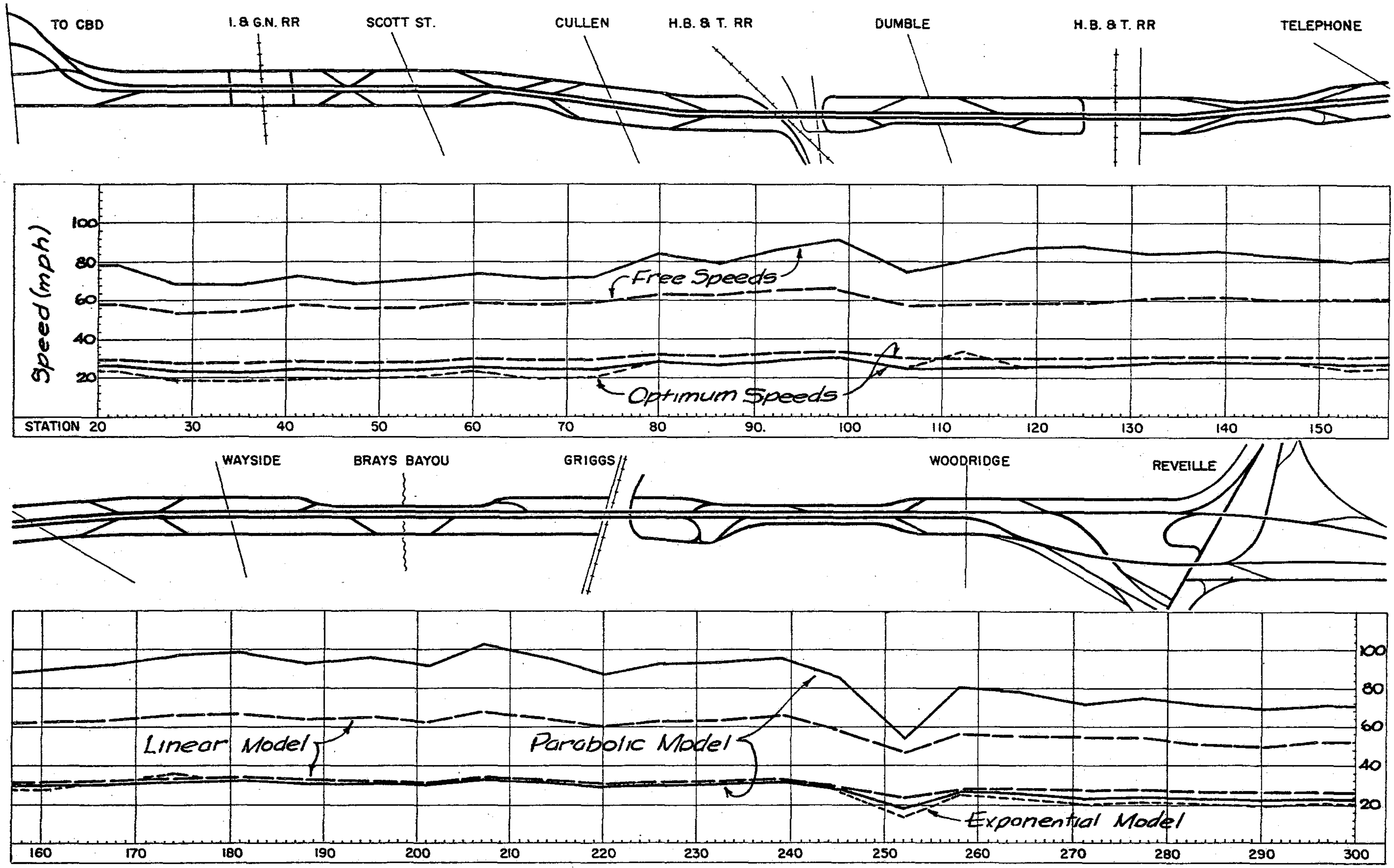
STUDY AREA-GULF FREEWAY
HOUSTON, TEXAS

FIGURE 2



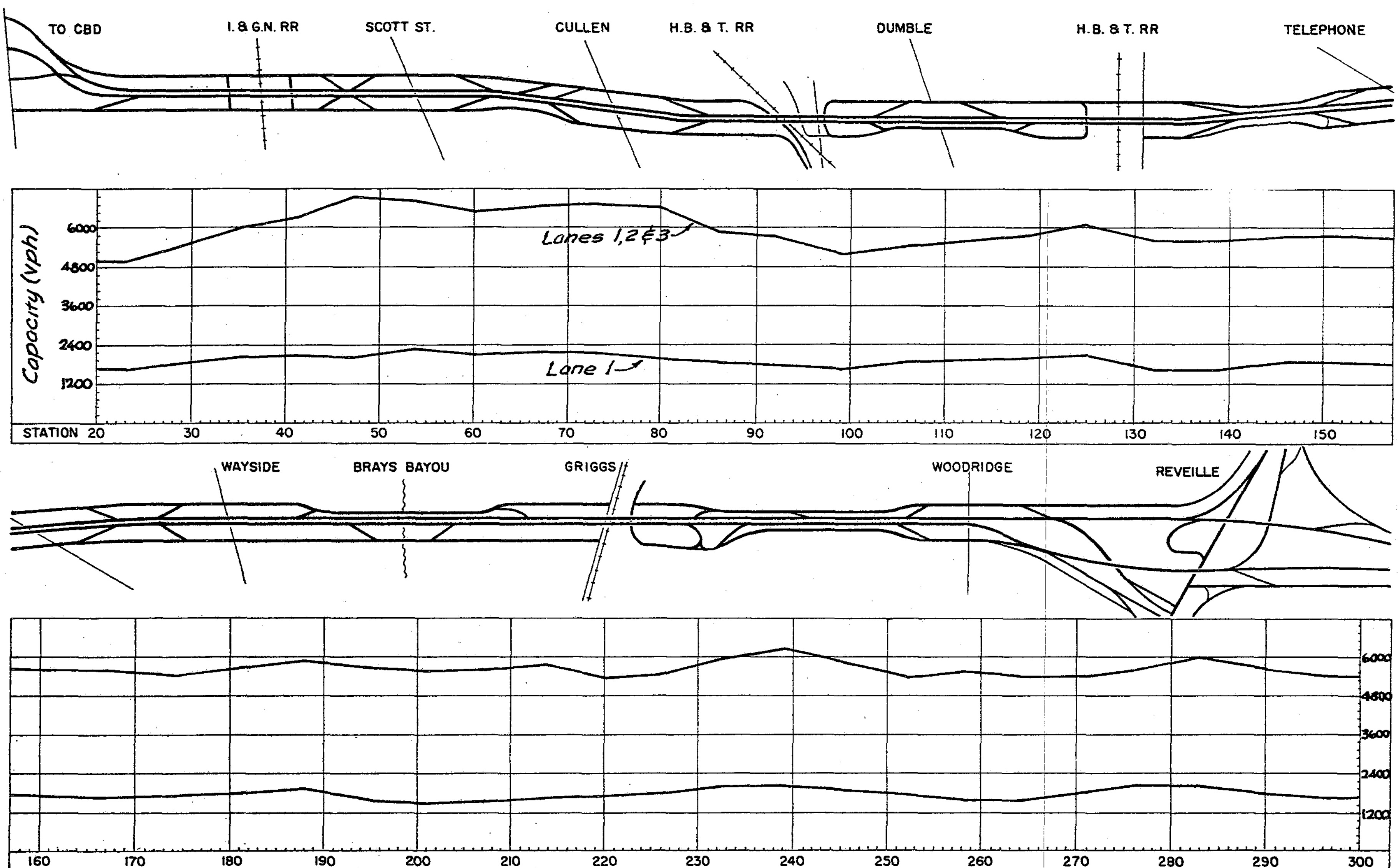
TIME LAPSE PHOTOGRAPHY

Figure 3



SPEED PROFILES (TOTAL INBOUND TRAFFIC)

FIGURE 4



CAPACITY PROFILES (LINEAR MODEL)

FIGURE 5

Entrance ramp metering may also be oriented to the distribution of freeway demand on the outside (merging) lane of the freeway. In the following section a queueing model is described which can be utilized in this type of metering.

FORMULATION OF MOVING QUEUES MODEL

Congestion is an expression first used in queueing theory to describe inefficiency in the operation of a system. Congestion in a system is usually produced by the combination of three circumstances: (l) a demand or flow of arrivals requiring service, (b) some restriction on the availability of service, and (c) irregularity in either the demand or in the servicing operation or in both. A system operating under these circumstances is called a queueing system.

As traffic volumes increase, vehicles tend to form platoons, or moving queues. The criterion in determining when two moving vehicles are queued is arbitrary. And, for that matter, the criterion in determining when two stationary units are queued in classical queueing systems is equally arbitrary since the concept of distance does not appear, the usual assumption being limited to independence of arrivals. Borrowing from car-following theory, a line of moving vehicles could be considered to be in a single queue if each must react instantly to the speed reductions of its predecessor. For the purposes of this discussion, it will be assumed that a vehicle is queued to the vehicle ahead if its headway is less than S, if space is the parameter, and T, if time is the parameter.

There are several traffic characteristics that can indicate congestion on a highway facility: low speeds, high flow to capacity ratios, high space densities, and high time densities (lane occupancy). It seems, however, that these various parameters ignore the distribution of traffic and therefore give incomplete descriptions of congestion and the state of a system. It is suggested that E(n) (which, in the case of moving queues, shall be called the queue length or number queued) is a logical measure of congestion on a highway facility, just as it is in conventional queueing systems. Figure 6 illustrates how the moving queue length might be a more sensitive indicator of congestion than density.

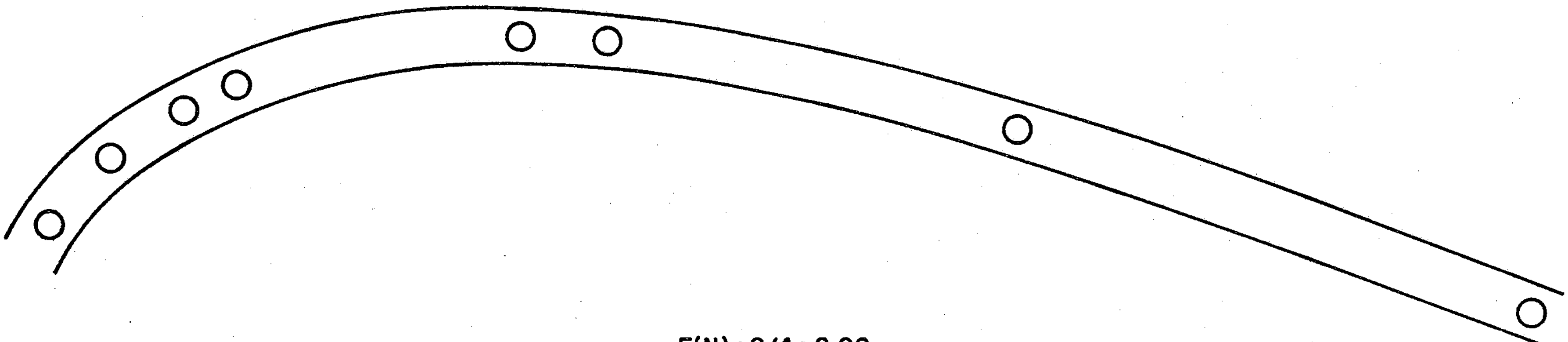
The formulation of the moving queues model is based on performing a Bernoulli test, with probabilities p and (1 - p), on each headway (either time headway or space headway). If headways between successive vehicles are assumed to be independent, then the probability of having a queue of exactly one vehicle is

$$P_1 = 1 - p \qquad (34)$$

where 1-p is the probability that the headway of vehicle number 2 (Figure 7a) is greater than the arbitrary queueing headway, S. Similarly the probability of a queue of exactly two vehicles is obtained by a combination of one "success" followed by a "failure," or

$$P_2 = p\,(1 - p) \qquad (35)$$

E(N) = 8/6 = 1.33

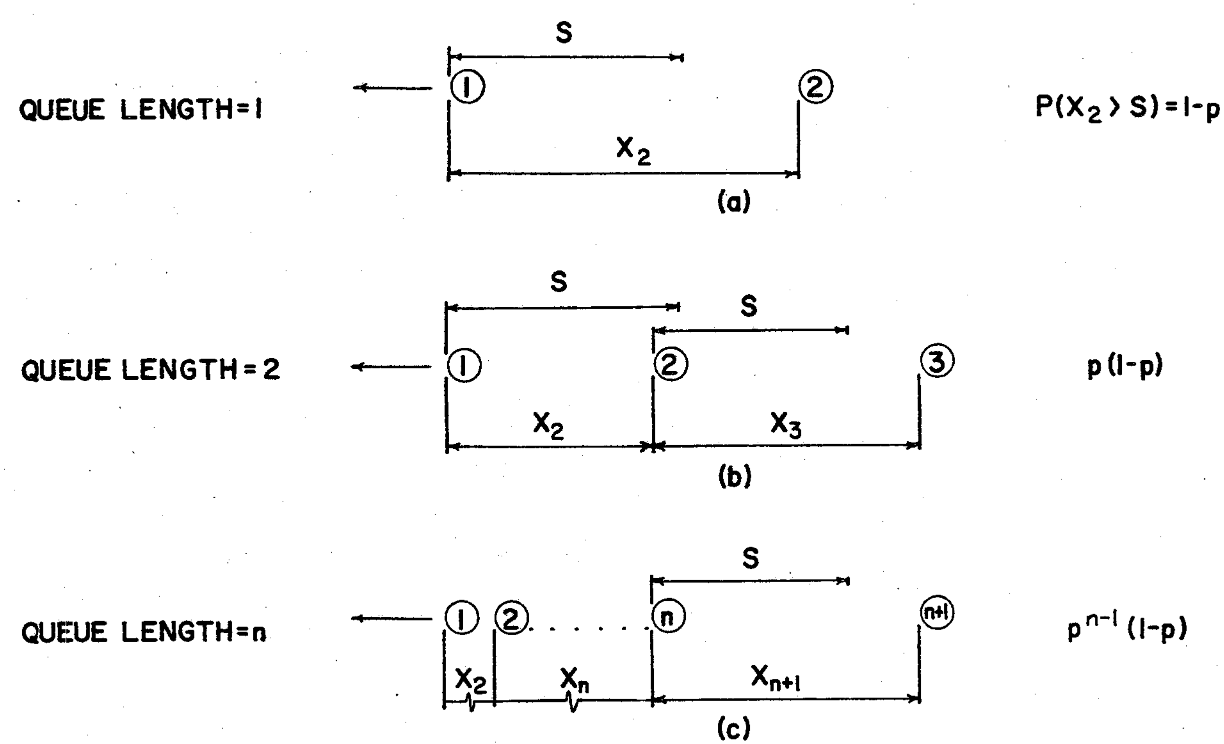


E(N) = 8/4 = 2.00

COMPARISON OF TWO HIGHWAY FACILITIES WITH EQUAL DENSITIES AND DIFFERENT CONGESTION INDICES

FIGURE 6

STATE OF SYSTEM | PROBABILITY OF OCCURRENCE



BERNOULLI TEST FOR DETERMINING THE PROBABILITY OF INDIVIDUAL QUEUE LENGTHS

FIGURE 7

where p is the probability that the headway of vehicle number 2 (Figure 7b) is greater than the arbitrary queueing headway, S. By induction, the individual queue lengths form a geometric distribution (Figure 7 c)

$$P_n = p^{n-1} (1 - p) , \quad n = 1, 2, \ldots \tag{36}$$

The expected queue length E(n) is given by

$$E(n) = \sum_{n=1}^{\infty} n P_n$$

which yields

$$E(n) = 1 P_1 + 2P_2 + 3P_3 + \ldots$$
$$= (1 - p)(1 + 2p + 3p^2 + \ldots). \tag{37}$$

The second factor of (37) is a telescoping geometric series whose sum is $1/(1-p)^2$; therefore

$$E(n) = (1 - p)^{-1}. \tag{38}$$

The probability (1 - p) of any vehicle headway, x, being greater than the arbitrary queueing headway, S, is of course dependent on the distribution of vehicles in space on the highway lane,

$$1 - p = P (x > S) = \int_S^{\infty} f \ (x; k, c) \, dx \tag{39}$$

The two parameter probability distribution implied in (39) is the Erlang distribution. Substituting in (39),

$$P (x > s) = \int_S^{\infty} \frac{(kc)^c}{(c-1)!} x^{c-1} e^{-ckx} \, dx \tag{40}$$

where k is the average concentration of vehicles, x is distance, and c = 1, 2, 3, ... Substituting (40) in (39) and then in (38) yields

$$E(n) = \left[\int_S^{\infty} \frac{(k\, c)^c}{(c-1)!} x^{c-1} e^{-ckx} \, dx \right]^{-1} \tag{41}$$

which is the fundamental relation between the moving queue length E(n), concentration (k), the arbitrary queueing headway S, and the distribution of concentration, c.

In the interest of brevity, the theory was developed for space headways only. However, it is apparent from the arbitrary nature of the queueing criteria that the results can be extended to time headways by replacing x, S and k in (41) by t, T and q.

Inherent in the development of the model is the hypothesis, stated in equation 40, that the distribution of traffic on a freeway conforms to an Erlang distribution. This is, in fact, a logical choice because the exponential distribution, well established in the description of traffic headways, may be considered as a special case of the Erlang distribution for which $c = 1$. However, the exponential distribution appears to be unduly restrictive for application to freeway traffic, partly because the exponential distribution implies that the smaller the headway the more likely it is to occur. On the other hand, use of the Erlang distribution for all values of c represents the distribution of vehicles for all cases between randomness ($c = 1$) to complete uniformity ($c = \infty$).

Using the same aerial surveys of traffic flow on the six-mile section of the Gulf Freeway in Houston, Figure 2, the distribution of space headways was examined. The individual areas studied covered approximately one-third of a mile. Figure 8 illustrates the observed distribution of space headways of inbound traffic during the A.M. peak period at location nine compared to the expected distribution of space headways according to the Erlang distribution. Employing the Chi Square test, it was found that significant relationships existed for varying values of c for the entire freeway as typified by the location reported in Figure 8.

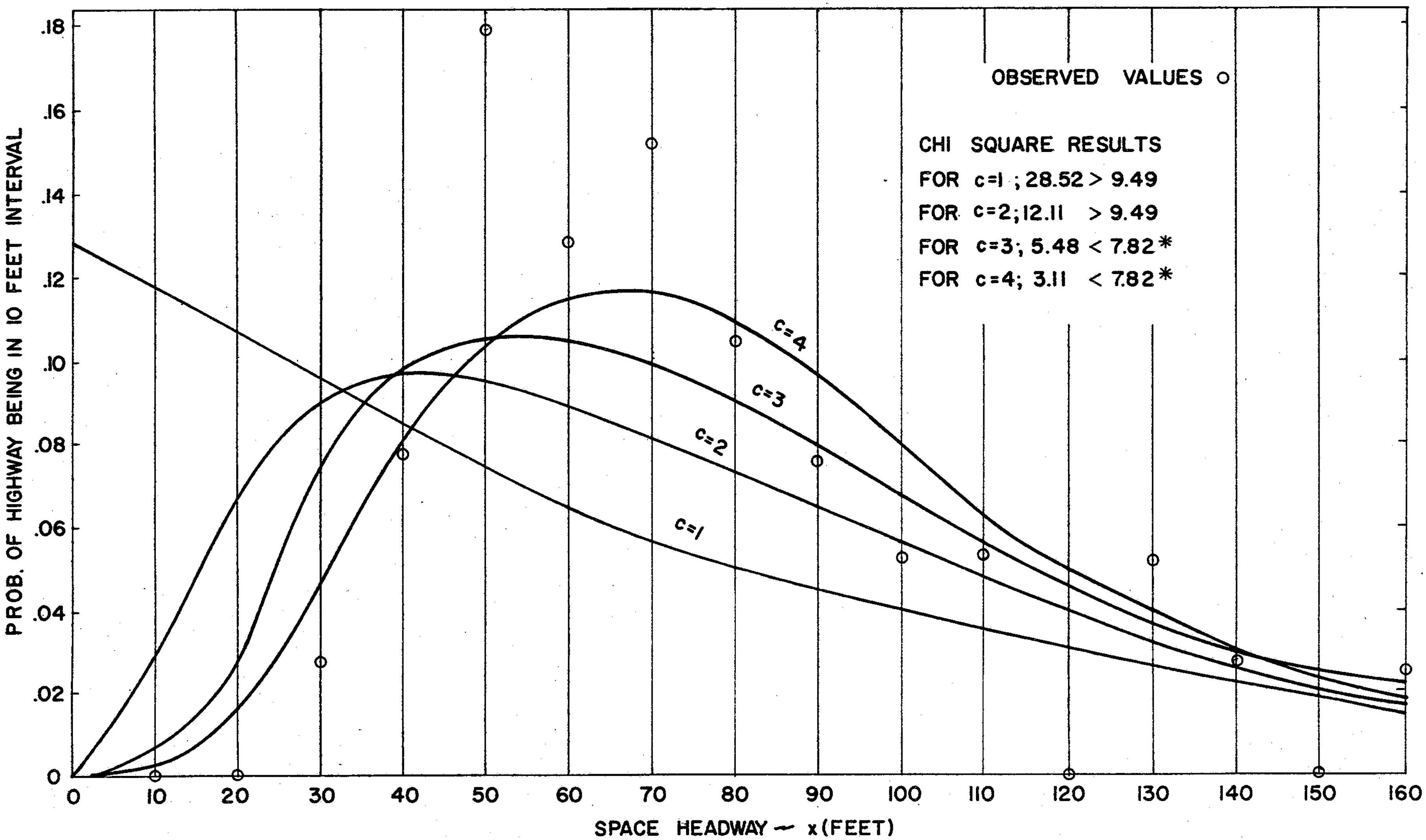
The distribution of headways having been established, the moving queues model states in equation 41 that the queue length (congestion) varies inversely with the percentage of large headways, or directly with the percentage of small headways for a given concentration k. Integration of (41) for $c = 1, 2, 3,$ and 4 yields

$$E(n)_{c=1} = e^{kS} , \tag{42}$$

$$E(n)_{c=2} = \frac{e^{2kS}}{2kS + 1} , \tag{43}$$

$$E(n)_{c=3} = \frac{e^{3kS}}{4.5\,(kS)^2 + 3kS + 1} , \tag{44}$$

$$E(n)_{c=4} = \frac{e^{4kS}}{10.67\,(kS)^3 + 8(kS)^2 + 4kS + 1} . \tag{45}$$



OBSERVED DISTRIBUTION OF SPACE HEADWAYS COMPARED TO THE ERLANG FREQUENCY DISTRIBUTION (c=1,2,3,& 4)

FIGURE 8

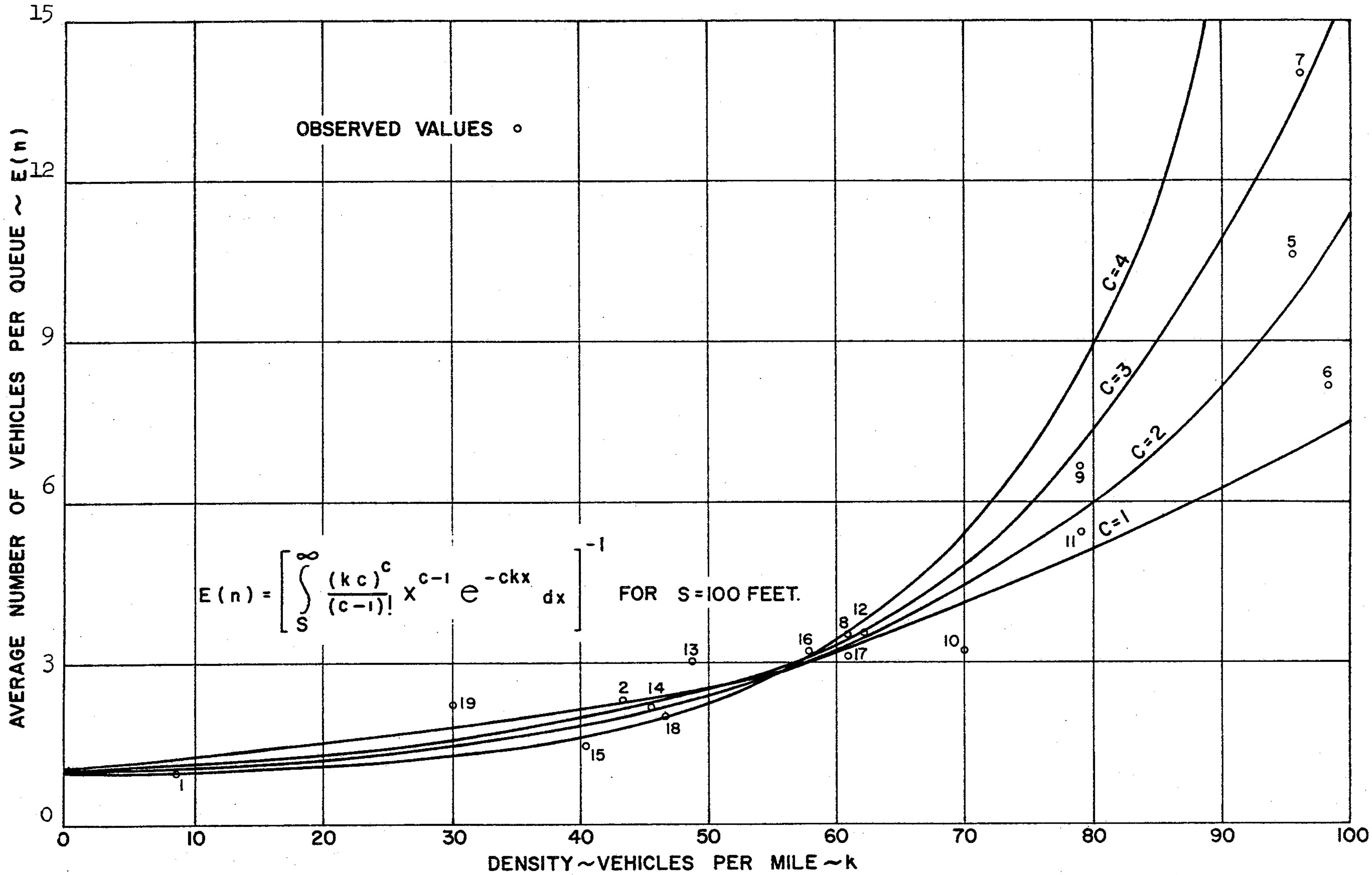
The curves of equations 42 thru 45 are shown in Figure 9. The numbered points plotted on the graph refer to the observed relationship of queue length to concentration for the one-third mile increments of the entire 6 miles of the outside lane of the Gulf Freeway obtained from the aerial photos. It is seen that the use of the first four curves of the Erlang family is sufficient to provide a very satisfactory fit to the varied conditions of congestion experienced on a long busy freeway.

APPLICATION OF MODEL TO FREEWAY CONTROL

The significance of some means of predicting congestion on a freeway subsystem lies in the utilization of this warning time to minimize its undesirable effects. The prediction of congestion ranges in sophistication from (1) the projection of peak period time patterns from one day to the next, (2) the extrapolation of one or more parameters of congestion from one small time period for use as a basis for controlling the next period, to (3) the evaluation and control of congestion all within the same period. Alternative (1) suggests some pre-timed control system in which, for example, ramp metering rates and the time of ramp closures would be fixed. Alternatives (2) and (3) are forms of automatic control, and while (3) is obviously more desirable, it is not always possible.

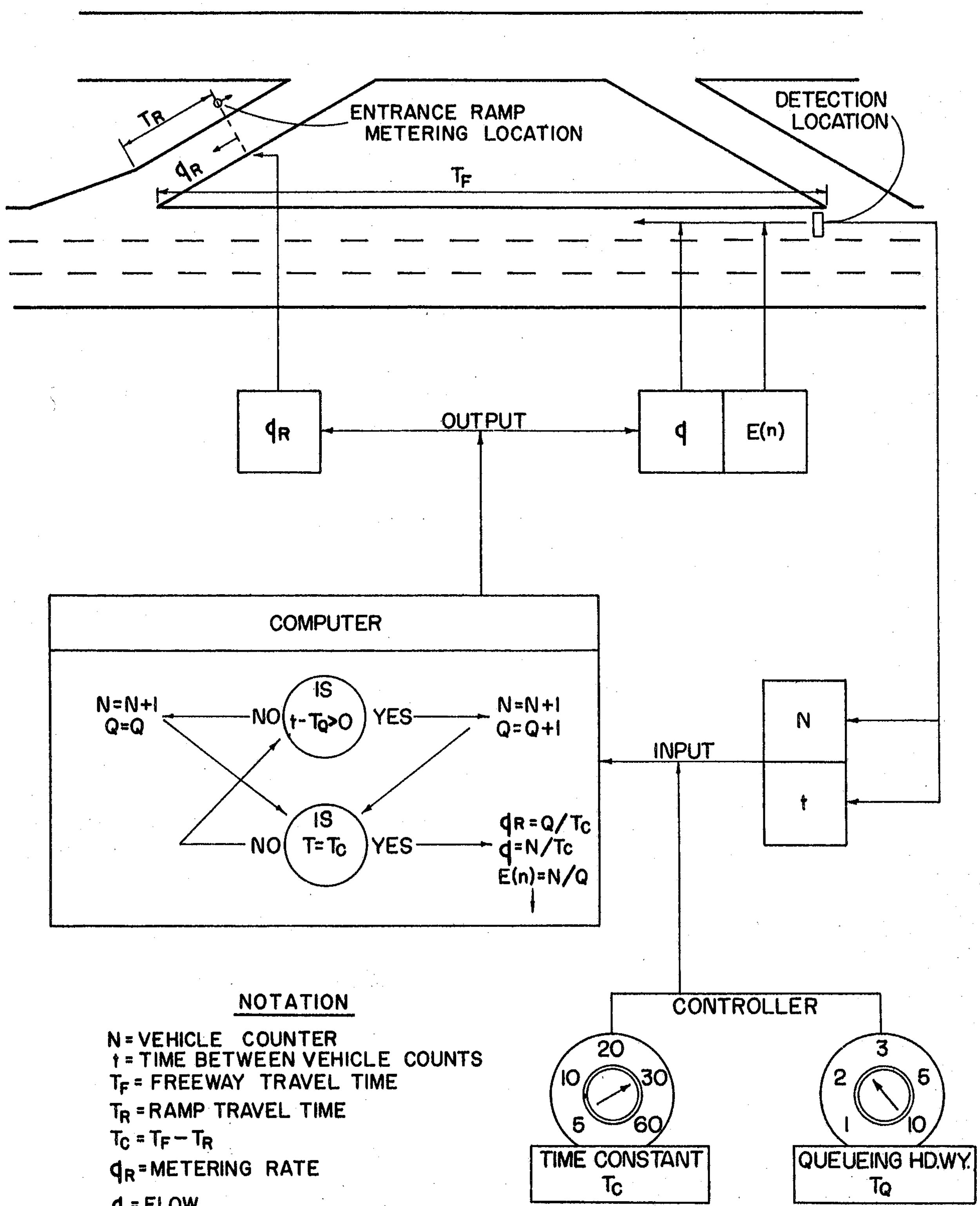
Figure 10 illustrates one application of the moving queues model to the control of a merging situation at an entrance ramp. It must be re-emphasized that although moving queues were defined in terms of space headways, it was noted that time headways are equally applicable. Thus, a vehicle with a time headway less than the arbitrary queueing headway T_Q is considered to be queued to the preceding vehicle. The control system illustrated consists of the flow of information from a detector located on the outside freeway lane to a computer and then to the metering signal on the ramp. For the "closed loop" system pictured, either a digital or analog computing device could be utilized. However, use of the former would necessitate a reduction in the "time constant" (time over which traffic conditions are averaged) by an interval equal to the time necessary for computation.

In considering an example, suppose the travel time during the peak period from detector to merging area is $T_F = 35$ sec. and from the metering station to the merging area is $T_R = 5$ sec. The critical gap (that headway in the outside freeway lane for which an equal percentage of ramp traffic will accept a smaller headway as will reject a larger one) is assumed to be 2.5 seconds. It is apparent that if control adjustment is to be made during the same period as detection, the time constant T_C cannot be greater than $T_F - T_R$. Moreover, if the arbitrary queueing headway T_Q is equated to the critical gap, the number of moving queues Q will equal the number of critical gaps. The latter determines q, the number of ramp vehicles that



OBSERVED RELATIONSHIP BETWEEN QUEUE LENGTH AND DENSITY COMPARED TO THE THEORETICAL RELATIONSHIP

FIGURE 9



APPLICATION OF QUEUEING MODEL TO FREEWAY CONTROL

FIGURE 10

can merge during T_c. Thus, in the example the dials on the controller would be set to T_c = 30 sec. and T_Q = 2.5 sec. If during T_c, N = 10 vehicles were detected and Q = 5 of the headways were greater than the queueing headway on the dial $(t > T_Q)$, the metering rate during T_c would be:

$$q_R = \frac{Q}{T_c} = \frac{5 \text{ veh.}}{30 \text{ sec.}} = 1 \text{ veh. every } 6 \text{ sec.}$$

The rate of flow and congestion index at the detection station during T_c would be:

$$q = \frac{N}{T_c} = \frac{10 \text{ veh.}}{30 \text{ sec.}} = 1 \text{ veh. every } 3 \text{ sec.},$$

$$E(n) = \frac{N}{Q} = \frac{10 \text{ vehicles}}{5 \text{ queues}} = 2 \text{ vehs./queue.}$$

It is apparent that controller settings would vary from entrance ramp to entrance ramp. For example the size of the critical gap for merging would depend on the angle of entry, availability of an acceleration lane, sight distance for evaluating oncoming gaps and the effect of the grade on acceleration capabilities. The geometrics of the freeway would also affect the detector location and hence influence the time constant.

CONCLUSIONS

Freeway traffic control both in the form of controls on the freeway and the metering of inputs on the freeway offers great possibilities for reducing freeway congestion. Such macroscopic parameters as capacity and the optimum speed provide good indices of conjestion and bases for control. Since capacity is difficult to obtain through direct field measurements, a theoretical capacity may be substituted. A capacity profile such as shown in Figure 5 can be outlined to establish metering rates on entrance ramps.

The expected number of vehicles per moving queue, E(n), provides a quantitative index of congestion. This parameter may be oriented either to space headways or time headways. It affords an index of congestion superior to volume if time headways are considered, or to density if space headways are considered, because E(n) takes into account the distribution of volume and density.

The congestion index E(n) developed offers more than a subjective means of evaluating freeway performance. An automatic control system, based on moving queues, would have many advantages. The control system pictured in Figure 10 is simpler than a typical semi-actuated signal system at an intersection. The parameter E(n) is sensitive and therefore ramp traffic need not be unduly penalized when gaps in the outside freeway lane are available. The system pictured could easily be expanded if more sophistication is desired. That is, a speed detector could be incorporated with the vehicle detector thereby adjusting T_F when speeds were reduced during the peak period. Moreover, several local controllers could be operated from a large computer, reducing the possibility of undetected congestion due to shock waves.

The field of freeway traffic control offers both an opportunity and a challenge to a multitude of manufacturers already engaged in traffic control, or with capabilities in this area. It is hoped that this model system will encourage more activity in the development of simple, rational, theoretically oriented hardware to control our freeways.

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